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Unit 10 Number Sense: Division

Introduction

In this unit, students will learn about division as a method of sharing. They will model two ways of sharing: when the number of sets is known, and when the number of items in each set is known. They will learn how to divide by skip counting and how to write an addition sentence for each division sentence. They will discover the relationship between division and multiplication and learn when it is appropriate to multiply or divide. They will also solve word problems involving multiplication and division.

Meeting Your Curriculum

Alberta—All lessons in this unit are required. Extension 3 of Lesson NS3-54 is required for Alberta students.

British Columbia—All lessons in this unit are required.

Manitoba—All lessons in this unit are required. Extension 3 of Lesson NS3-54 is required for Manitoba students.

Ontario—All lessons in this unit are required.

Vocabulary and concepts. The names for the terms in a division sentence are as follows:

\[
20 \div 4 = 5
\]

- **dividend**
- **divisor**
- **quotient**

These names are used occasionally in the lesson plans, for your benefit only. They do not need to be shared with students and probably shouldn’t be. With students, we will refer to what the numbers represent using the words **item**, **group**, and **set**. For example:

\[
20 \div 4 = 5
\]

- **total number of items**
- **number in each set/group**
- **number of sets/groups**

Students will model division using pictures, circles and dots, arrays, and number lines. The various models will help students understand the following:

- Division sentences can be interpreted different ways. For example:
  
  \[
  20 \div 4 = 5 \]
  
  20 items divided into 4 groups, with 5 in each group, or
  20 items divided into groups of 4, so 5 groups in total.

  In other words, the labels for “4” and “5” in the second equation above can be transposed.

- Multiplication and division are related and lead to **fact families**, such as:
  
  \[
  20 \div 5 = 4
  \]
  \[
  20 \div 4 = 5
  \]

  \[
  20 \times 4 = 20
  \]
  \[
  5 \times 4 = 20
  \]
**Signalling.** In these lessons, we often suggest that all students signal their answers simultaneously (e.g., by flashing thumbs up or thumbs down for “yes” or “no,” or by holding up the number of fingers that corresponds to their answer). For a complete description of signalling, see p. A-15.

**Quizzes and Tests**

The following table indicates the lessons covered by a quiz or test for each curriculum.

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**Goals**

Students will learn how to share equally when the number of sets is known.

**PRIOR KNOWLEDGE REQUIRED**

Can add
Can multiply

**MATERIALS**

ball
raisins

**Mental math minute.** Review the doubles: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. Remind students that multiplying by 2 finds one of the doubles. For example, $5 \times 2$ is the fifth double, 10. Remind students that the commutative property tells us that $5 \times 2$ and $2 \times 5$ are equal. Have the class stand up. Call out a multiplication question in which one of the factors is 2; for example, $6 \times 2$. Toss a ball to a student. The student will give the answer (12), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

**Sharing by distributing items one at a time into groups.** Ask for four volunteers who like raisins to come to the front of the room. Ask for another volunteer to be the distributor of food. Give the distributor 12 raisins. SAY: We are going to share the raisins equally between the four people. Ask the distributor to give one raisin to the first person, two raisins to the second person, and three raisins to the next person. Pause for a few seconds to see the class react. Then, ask the distributor to give one raisin to the fourth person, two raisins to the first person, and three raisins to the second person. Turn to the distributor and ASK: Do you have any raisins left? (no) ASK: Did everyone get some raisins? (yes) Does it seem fair? (no) Why not? (not everyone got the same number of raisins) SAY: We want to share equally. We want each volunteer to get the same number of raisins. ASK: What is a better way of sharing raisins equally? (hand them out one at a time) SAY: Okay volunteers, you can eat your raisins.

Ask for four new volunteers and another distributor to come to the front of the room. This time, give the distributor 20 raisins. Ask the distributor to give one raisin to the first person, one raisin to the next person, and so on, until there are no more raisins. Ask the class to count out the total number of raisins as the distributor does his or her job. ASK: When did we stop distributing raisins? (when we reached 20) How many raisins did each volunteer get? (5) Does this seem fair? (yes) Why? (each volunteer got the same number of raisins) SAY: Okay volunteers, you can eat your raisins.
Draw five circles on the board and SAY: We want to share 15 raisins among five people. Each circle represents a person. I am going to place a dot in a circle for each raisin I give out.

○ ○ ○ ○ ○

Draw one dot in each circle, and then continue adding one dot per circle. Count the dots out loud as you add them to the circles, and have students say “stop” when you reach 15. The final picture should look like this:

● ● ● ● ●

ASK: What does each circle represent? (a person) What does each dot represent? (a raisin) If five people share 15 raisins, how many raisins does each person get? (3) SAY: We call the collection of dots in each circle a group or set.

ASK: If five people share 15 apples, how many apples does each person get? (3) If I use circles and dots to represent this sharing, what does each circle represent? (a person) What does each dot represent? (an apple)

Have the class do the following exercises. Take time to walk around the class to check on students.

**Exercises:** Draw circles and dots to find how many raisins each person gets.

a) 15 raisins, 5 people  
   b) 12 raisins, 3 people  

c) 24 raisins, 4 people  
   d) 3 people, 18 raisins  

e) 4 people, 20 raisins  
   f) 6 people, 24 raisins

**Answers:** a) 3, b) 4, c) 6, d) 6, e) 5, f) 4

**Solving word problems that involve sharing.** Write on the board:

18 apples were shared by 3 people.

ASK: What was divided or shared into groups or sets? (apples) How many groups or sets? (3) If we use circles and dots to model this, what does each circle represent? (a person) What does each dot represent? (an apple)

Draw on the board:

○ ○ ○

Ask a volunteer to come to the board and draw dots one at a time in the three circles. Ask the volunteer to count out loud while drawing each dot, and ask the class to say “stop” at 18 dots. ASK: How many apples did each person get? (6)
Do the word problems in the following exercises with the class. Ask volunteers to read the questions. For each question, ASK: What is being shared? How many groups are there? Have students draw a circle to represent each group and then draw dots one at a time in each circle, until all the items have been shared. ASK: How many items are in each group?

**Exercises**

a) 24 tennis balls are shared by 4 tennis players. How many balls does each person get?

b) 15 people are divided equally into 5 cars. How many people are in each car?

c) 24 apples are divided equally into 3 baskets. How many apples are in each basket?

d) A student gets 20 hours of homework in 5 days. If the amount of homework is shared equally over the 5 days, how many hours of homework are done each day?

e) A teenager has an allowance of $35 each week. How much can he spend each day?

**Answers:** a) 6, b) 3, c) 8, d) 4, e) 5

**Extensions**

1. In each situation, the items cannot be shared equally. Find the smallest number of items that can be removed from the total so that the items that are left can be shared equally.

   a) 23 pears on 4 trees  
   b) 14 books on 3 shelves  
   c) $31 between 5 friends  
   d) 52 cards between 8 players

   **Answers:** a) 3, b) 2, c) 1, d) 4

2. For each part in Extension 1, find the smallest number of items that can be added to the total so that the items can be shared equally.

   **Answers:** a) 1, b) 1, c) 4, d) 4
Goals
Students will learn how to find the number of sets when the number of items in each set is known.

PRIOR KNOWLEDGE REQUIRED
Can add
Can multiply
Can share equally when the number of sets is known

MATERIALS
ball
overhead projector
counters

Mental math minute. Review counting by 5s: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. Remind students that multiplying by 5 finds one of these numbers. For example, 7 \times 5 is the seventh number in this list, 35. Remind students that the commutative property tells us that 7 \times 5 and 5 \times 7 are equal. Point out that when we multiply 5 by an even number, the answer ends in zero. When we multiply 5 by an odd number, the answer ends in 5. Have the class stand up. Call out a multiplication question in which one of the factors is 5; for example, 6 \times 5. Toss a ball to a student. The student will give the answer (30), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

Sharing by finding the number of groups. Using an overhead projector, display 20 counters that have been placed randomly. SAY: I want to share these counters so that each person gets five counters. Ask for a volunteer to come to the overhead and arrange the counters into groups of five. ASK: How many people can each get five counters? (4) How many counters does each person get? (5) How many counters are there? (20)

Draw a row of 12 dots on the board. Have students copy the row of 12 dots in their notebooks. Tell students that these dots represent counters. Ask them to circle the dots to find how many people can share the counters so that each person gets the following numbers of counters:

a) 2 counters each  
   b) 3 counters each  
   c) 4 counters each  
   d) 6 counters each

(see answers below)

a) \[\text{\tiny \includegraphics{soldiers1}}\], 6 people
b) \[\text{\tiny \includegraphics{soldiers2}}\], 4 people
c) \[\text{\tiny \includegraphics{soldiers4}}\], 3 people
d) \[\text{\tiny \includegraphics{soldiers6}}\], 2 people
Repeat the exercise on the board with the following arrangements of dots and numbers of counters. Ask volunteers to come up and write their answers on the board.

a)  
   ● ● ● ● ● ● ● ● ● ● ●
   2 counters each

b)  
   ● ● ● ● ● ●
   2 counters each

c)  
   ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● 

(see answers below)

a)  
   ● ● ●  ● ● ●, 5 people

b)  
   ● ● ●  ● ● ●  ● ● ●  ● ● ●, 4 people

c)  
   ● ● ●  ● ● ●, 3 people

ASK: What do the circled dots represent? (people) What do the dots represent? (counters)

Exercises: Draw dots for the things being divided equally. Draw circles for the sets. How many sets are there?

a) 15 dots, 3 dots in each set  
   b) 24 dots, 8 dots in each set  
   c) 20 dots, 5 dots in each set  

Bonus: 50 dots, 5 dots in each set

Answers: a) 5, b) 3, c) 4, Bonus: 10

Solving word problems that involve finding the number of groups. Write on the board:

18 apples are placed in baskets.  
Each basket has 6 apples.  
How many baskets are used?

ASK: What are the items being shared? (apples) What are the groups? (baskets) What are we being asked to find? (the number of groups)

Draw a row of 18 dots on the board. ASK: What do the dots represent? (apples) When we circle the dots to show groups, what will the groups represent? (baskets) How can we solve the problem? (count the groups) Ask for a volunteer to come to the board and circle the dots so that six dots are in each group. The final picture should look like this:

   ● ● ● ● ● ● | ● ● ● ● ● ● | ● ● ● ● ● ● | ● ● ● ● ● ●

ASK: How can we find the number of baskets? (count the groups) How many baskets are there? (3)
Exercises: Draw a picture to solve the problem.

a) A photo album has 30 pictures with 6 pictures on each page. How many pages are there?

b) A bike courier has to deliver 28 packages. He can deliver 7 packages per day. How many days will it take him to deliver the packages?

c) A bookcase has 24 books. 8 books fit on one shelf. How many shelves are needed to store the books?

Answers: a) 5, b) 4, c) 3

Extensions

1. Find the number of sets when 24 items are shared.
   a) 2 in each set  b) 3 in each set  c) 4 in each set
   d) 6 in each set  e) 8 in each set  f) 12 in each set

   Answers: a) 12, b) 8, c) 6, d) 4, e) 3, f) 2

2. What happens to the number of sets in Extension 1 as the number in each set increases?

   Answer: the number of sets decreases
Goals

Students will identify in a word problem what is being divided into sets or groups, the number of sets, and the number of items in each set.

PRIOR KNOWLEDGE REQUIRED

Can share equally when the number of sets is known
Understands multiplication as finding the total number of items when the number of sets and the number of items in each set are known

MATERIALS

ball

Mental math minute. Review counting by 10s: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100. Remind students that multiplying by 10 finds one of these numbers. For example, $7 \times 10$ is the seventh number in this list, 70. Remind students that the commutative property tells us that $7 \times 10$ and $10 \times 7$ are equal. Point out that when we multiply a number by 10, the answer ends in zero. For example, $6 \times 10$ is 6 followed by a 0, or 60. Have the class stand up. Call out a multiplication question where one of the factors is 10; for example, $4 \times 10$. Toss a ball to a student. The student will give the answer (40), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

Identifying the set and what is being divided into sets. SAY: In division problems, drawing a picture helps us to find out what the sets are and what the items being divided into sets are. Draw on the board:

10 apples in each basket

ASK: What is the set? (basket) What are the items in each set? (apples)

Write on the board:

a) 4 campers in each tent
b) 8 rowers in each boat
c) 25 students in each class
d) 10 pages in a notebook
e) 3 prizes for every person
For each of the situations above, ask students to draw a picture to represent the problem, and then identify the set and the items being divided into sets. (see sample pictures for parts a) and b) and answers in the table below)

<table>
<thead>
<tr>
<th>Set</th>
<th>Items Being Divided into Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>tent</td>
</tr>
<tr>
<td>b)</td>
<td>boat</td>
</tr>
<tr>
<td>c)</td>
<td>class</td>
</tr>
<tr>
<td>d)</td>
<td>notebook</td>
</tr>
<tr>
<td>e)</td>
<td>person</td>
</tr>
</tbody>
</table>

a) For part a), tent campers
b) For part b), boat rowers

c) class students
d) notebook pages
e) person prizes

Refer back to the situations you wrote on the board. SAY: Instead of drawing pictures, we can use a circle to represent the set and dots to represent the items. If we have four campers in each tent, we can draw this. Draw the picture in the margin on the board. Have students draw circles and dots to represent the situations in parts b) to e).

Identifying what is being divided into sets, the number of sets, and the number of items in each set. SAY: I have 12 quarters to share among three people. Draw on the board:

Ask for a volunteer to come to the board and, using dots to represent the quarters, share the quarters equally. ASK: What is being shared? (quarters) How many sets or groups are there? (3) How many items are in each set or group? (4) As an added challenge, ASK: How much money does each person get? (75¢)

Exercises: Identify the sets, the items being shared, and the number of items in each set.

a) 15 people, 3 cars, 5 people in each car
b) 20 stickers, 4 stickers on each page, 5 pages
c) 24 pencils, 4 people, 6 pencils for each person

Bonus

d) 15 people, 3 tables
e) 16 players, 4 teams
f) 12 jars, 4 cases
Answers

<table>
<thead>
<tr>
<th>Sets</th>
<th>Items Being Shared</th>
<th>Number of Items in Each Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>cars</td>
<td>people</td>
</tr>
<tr>
<td>b)</td>
<td>pages</td>
<td>stickers</td>
</tr>
<tr>
<td>c)</td>
<td>people</td>
<td>pencils</td>
</tr>
</tbody>
</table>

Bonus

<table>
<thead>
<tr>
<th>Sets</th>
<th>Items Being Shared</th>
<th>Number of Items in Each Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>d)</td>
<td>tables</td>
<td>people</td>
</tr>
<tr>
<td>e)</td>
<td>teams</td>
<td>players</td>
</tr>
<tr>
<td>f)</td>
<td>cases</td>
<td>jars</td>
</tr>
</tbody>
</table>

Extensions

1. A teacher shared 24 raisins among 6 students. Later, the teacher found 18 more raisins to share. What is the total number of raisins each student got?

   Answer: 7

2. A farmer has 40 apples to share among 8 people. How many more apples would each person get if there are only 5 people sharing them?

   Answer: 3
Goals

Students will share items equally, or divide, when given either the number of sets or the number of objects in each set.

Students will interpret the divisor in a division sentence as either the number of sets or the number of objects in each set.

PRIOR KNOWLEDGE REQUIRED

Can find the number of items in each set when given the number of items and the number of sets
Can find the number of sets when given the number of items and the number of items in each set

MATERIALS

ball
raisins
cups

Mental math minute. Review counting by 4s: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40. Remind students that multiplying by 4 finds one of these numbers. For example, $5 \times 4$ is the fifth number in this list, 20. Remind students that the commutative property tells us that $5 \times 4$ and $4 \times 5$ are equal. Point out that one way of multiplying by 4 is to double twice. For example, to multiply $6 \times 4$, you double 6 to get 12 and then double 12 to get 24, so $6 \times 4$ is 24.

Have the class stand up. Call out a multiplication question in which one of the factors is 4; for example, $7 \times 4$. Toss a ball to a student. The student will give the answer (28), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

Sharing by finding the number of sets. Ask for a volunteer, Volunteer A, to come to the front of the class. Give Volunteer A 24 raisins. Ask for six more volunteers to come to the front of the class to receive raisins. Have Volunteer A distribute the raisins one at a time to each person until they are gone. ASK: How many raisins did each person get? (4) Tell the volunteers they can eat the raisins and sit down. Tell the class you want to show what just happened by drawing dots and circles. ASK: What do the circles represent? (people) How many circles should I draw? (6) Draw on the board:

Ask for a volunteer to come to the board and draw dots one at a time in each circle. Have the volunteer count out loud, and ask the class to say “stop” when he should stop. ASK: How many dots are in each circle? (4)
How many raisins did each person eat? (4) The final picture should look like this:

![Raisins distribution](image)

**Sharing by finding the number in each set.** Ask another volunteer, Volunteer B, to come to the front of the class. Give Volunteer B 24 raisins and ask for six more volunteers to come up to the front of the class. Have Volunteer B give each person four raisins until they have run out. ASK: How many students got raisins? (6) How many raisins did each student get? (4) Tell the volunteers they can eat the raisins and sit down. SAY: We want to show what just happened by drawing dots and circles. Draw on the board:

![Dots and circles](image)

Ask for a volunteer to come to the board and draw four dots in the first circle. Ask her to continue to draw four dots at a time in the circles, until she has drawn 24 dots in total. ASK: How many circles did we use? (6) How many students got to eat raisins? (6) The final picture should look like this:

![Dots and circles](image)

Ask the class to skip count by 4s to 24, and write the total below each circle as you go:

```
4 8 12 16 20 24
```

SAY: Volunteer A and Volunteer B distributed the raisins in different ways. ASK: Did Volunteer A know the number of sets or the number of objects in each set? (the number of sets) Did Volunteer B know the number of sets or the number of objects in each set? (the number of objects in each set) Write on the board:

You can share in two ways.

a) Decide how many sets.  
   Example:
   Share 12 apples among 4 people.
   ![Dots](image)

b) Decide how many items in each set.  
   Example:
   Share 12 apples by giving each person 2 apples until you run out of apples.
   ![Dots](image)
Ask for a volunteer to come to the board and draw dots for part a). They should draw one dot at a time in each circle until they have drawn 12 dots. Ask for a different volunteer to draw dots for part b). They should draw two dots at a time in each circle until they have drawn 12 dots. The final pictures should look like this:

![Diagram of dots]

**Exercises**

1. Draw a picture to find the number of items in each group.
   
   a) 24 dots divided into 6 groups   b) 18 dots divided into 3 groups
   c) 12 dots divided into 3 groups   d) 35 dots divided into 7 groups

   **Answers:** a) 4, b) 6, c) 4, d) 5

2. Draw a picture to find the number of groups.
   
   a) 28 dots divided into groups of 7 each
   b) 36 dots divided into groups of 9 each
   c) 42 dots divided into groups of 6 each
   d) 48 dots divided into groups of 8 each

   **Answers:** a) 4, b) 4, c) 7, d) 6

3. Draw a picture to solve the problem. Use dots for objects and circles for groups.
   
   a) 24 bottles are in 3 cases. How many bottles are in each case?
   b) There are 5 rowers in each boat. There are 15 rowers. How many boats are there?
   c) 6 pencils are in each pencil case. There are 42 pencils. How many pencil cases are there?
   d) 42 players are on 7 teams. How many players are on each team?

   **Bonus:** Find the number of groups when 100 items are divided into groups of 25 each.

   **Answers:** a) 8, b) 3, c) 7, d) 6, Bonus: 4
Extensions

1. Find all the ways of sharing 24 cookies equally among different numbers of friends.

   **Answers**

<table>
<thead>
<tr>
<th>Number of Friends</th>
<th>24</th>
<th>12</th>
<th>8</th>
<th>6</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cookies</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>12</td>
<td>24</td>
</tr>
</tbody>
</table>

2. Find the number of items in each group when 1000 items are divided into 10 groups. Hint: Let each dot represent 10 items.

   **Answer:** 100
Goals

Students will solve division word problems that provide two pieces of information: the total number of items being divided and either the number of groups or the number in each group.

PRIOR KNOWLEDGE REQUIRED

Can share equally when the number of groups is known
Can share equally when the number of items in each group is known

MATERIALS

ball

Mental math minute. Review counting by 9s: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90. Remind students that multiplying by 9 finds one of these numbers. For example, $6 \times 9$ is the sixth number in this list, 54. Remind students that the commutative property tells us that $6 \times 9$ and $9 \times 6$ are equal. Point out some patterns in the nine times table. One pattern is to get the next multiple of 9, you can add 10 then subtract 1. For example, to get the next multiple after 36, add 10 to 36 to get 46, and then subtract 1 to get 45. Another pattern is that the sum of the digits in every multiple of 9 is 9. For example, if you know $9 \times 7$ starts with a 6, ask yourself, “What can I add to 6 to get 9?” (3) So, $9 \times 7$ is 63. Have the class stand up. Call out a multiplication question in which one of the factors is 9; for example, $4 \times 9$. Toss a ball to a student. The student will give the answer (36), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

Identifying the items that have been shared, the number of groups or sets, and the number of items in each group or set. Write on the board:

a) 15 people in 5 cars  
  3 people in each car  

b) 15 people  
  5 people in each car  
  3 cars

Have students draw a picture in their notebooks to show each situation. (see sample answers below)

ASK: In part a), what is being divided into groups? (people) What are the groups? (cars) How many groups are there? (5) How many people are in each group? (3) In part b), what is being divided into groups? (people)
What are the groups? (cars) How many groups are there? (3) How many people are in each group? (5)

Exercises: Fill in what you know. Write a question mark for what you don’t know.

<table>
<thead>
<tr>
<th>What has been shared or divided into sets?</th>
<th>How many sets?</th>
<th>How many in each set?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 24 riders on a roller coaster. 6 cars. 4 riders in each car.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 12 pencils placed in 4 boxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 20 cookies. 5 cookies for each person.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 24 hockey players on 4 teams</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) 40 rowers in 5 rowboats</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers

a) riders 6 4
b) pencils 4 ?
c) cookies ? 5
d) hockey players 4 ?
e) rowers 5 ?

Solving word problems that involve sharing. Write on the board:

A basketball league has 35 players and 5 players on each team. How many teams are there?

Ask students to copy the problem into their notebooks. ASK: What is the group or set? (team) What is being divided here? (players) How many are in each set? (5)

Draw a circle on the board. SAY: We are going to use circles to show the teams and dots to show the players. Ask a volunteer to come to the board and draw five dots in the circle. Ask the volunteer to continue drawing circles with five dots inside each circle. Have them count out loud the total number of players being placed in teams as they go along. (5, 10, 15, and so on) Ask the class to say “stop” when the volunteer should stop (i.e., at 35).

The final picture should look like this:

Number Sense 3-52
ASK: How many teams are there in the basketball league? (7)

Write on the board:

At a family barbecue, 24 hot dogs are being shared among 8 people. How many hot dogs does each person get?

Ask students to copy the problem into their notebooks. ASK: What is the group or set? (people) What is being divided here? (hot dogs) How many groups are there? (8) SAY: We are going to draw dots and circles for this problem. What will the circles represent? (people) How many circles will we need? (8) What will the dots represent? (hot dogs) Ask for a volunteer to come to the board and draw the eight circles. Ask the volunteer to draw dots one at a time in each of the circles until 24 dots total are placed. The final picture should look like this:

![Diagram of dots and circles]

ASK: How many dots are in each circle? (3) How many hot dogs does each person get? (3)

**Exercises:** Find the missing information.

a) 32 marbles, 8 children, ___ marbles for each child

b) $48, $6 for each winner, ___ winners

c) 24 desks, 6 desks in each row, ___ rows

d) 72 apples, 8 trees, ___ apples on each tree

e) 40 pictures in a photo album, 8 pictures on each page, ___ pages

**Answers:** a) 4, b) 8, c) 4, d) 9, e) 5

**Extensions**

1. A batch of bran muffins is shared among 6 people. If it had been shared among 5 people, each person could have had 1 more bran muffin. How many muffins were in the batch?

   **Answer:** 30

2. A pencil case has fewer than 15 pencils inside. The pencils can be shared equally among 2, 3, 4, or 6 people. How many pencils are there?

   **Answer:** 12

3. A basket of tennis balls has fewer than 20 tennis balls. The tennis balls can be shared equally among 2, 3, 6, or 9 people. How many tennis balls are there?

   **Answer:** 18
Goals
Students will represent a division sentence as repeated addition in which the divisor is added repeatedly until the total is equal to the dividend.

PRIOR KNOWLEDGE REQUIRED
Can share equally when the number of groups is known
Can share equally when the number of items in each group is known

MATERIALS
ball

Mental math minute. Review counting by 8s: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80. Remind students that multiplying by 8 finds one of these numbers. For example, 7 × 8 is the seventh number in this list, 56. Remind students that the commutative property tells us that 7 × 8 and 8 × 7 are equal. Point out some patterns in the eight times table. For example, to multiply a number by 8, you can double the number 3 times. So, 7 × 8 is 7 doubled (14), doubled again (28), and doubled one more time (56). Have the class stand up. Call out a multiplication question in which one of the factors is 8; for example, 5 × 8. Toss a ball to a student. The student will give the answer (40), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

Introduce division sentences. Draw on the board:

SAY: We want to share 12 items into four groups so that there are three items in each group. Ask for a volunteer to come to the board to draw dots in the circles to represent the sharing. The final picture should look like this:

SAY: A division sentence is a way of summarizing the sharing we have been doing. In the example, we are dividing 12 items into four groups of three items each. Write on the board:

**division sentence**

\[ 12 \div 3 = 4 \]

This many times

Add this number
Point at the *division sign* in the division sentence and SAY: Just as we use a plus sign to show addition and a minus sign to show subtraction, we use this sign to show *division*. Write on the board:

\[ 15 \div 3 = 5 \]

SAY: I want to draw a picture to show this division sentence. ASK: Which number tells me how many circles I need to draw? (5; the number after the equal sign; the answer) Have a volunteer draw five circles on the board. ASK: What number tells me how many dots should be in each circle? (3; the number after the division sign) Have the volunteer draw three dots in each circle. ASK: What number tells me how many dots there should be in total? (15; the number before the division sign)

**Exercises:**

1. Write a division sentence for the picture.
   
   a) 
   ![Image of six circles](image)
   b) 
   ![Image of three circles](image)
   c) 
   ![Image of eight circles](image)

   **Answers:** a) \(24 \div 6 = 4\), b) \(12 \div 4 = 3\), c) \(35 \div 7 = 5\)

2. The answer to the division sentence shows the number of sets. Draw a picture for the division sentence.
   
   a) \(10 \div 5 = 2\)  
   b) \(8 \div 2 = 4\)  
   c) \(18 \div 3 = 6\)

   **Answers**
   
   a) 
   ![Image of two circles](image)
   b) 
   ![Image of four circles](image)
   c) 
   ![Image of six circles](image)

**Writing division as repeated addition.** Draw on the board:

\[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]

SAY: I want to share 15 pencils among five students. Ask a volunteer to draw one dot in each circle to represent the pencils. Repeat with other volunteers until 15 pencils have been shared. SAY: Let’s check that we have the correct number of dots. ASK: How can we check quickly, without counting every single dot? (by skip counting) Skip count as a class to check, and write the addition sentence below the picture at the same time. The final picture should look like this:

\[ 3 + 3 + 3 + 3 + 3 = 15 \]
ASK: What division sentence does this picture show? \((15 ÷ 3 = 5)\) Write the division sentence on the board.

**Exercises:** Draw a picture to show the division sentence. Write the repeated addition sentence.

a) \(10 ÷ 5 = 2\)  

b) \(14 ÷ 2 = 7\)  

c) \(18 ÷ 3 = 6\)

**Answers**

a) 2 circles with 5 dots in each, \(5 + 5 = 10\)

b) 7 circles with 2 dots in each, \(2 + 2 + 2 + 2 + 2 + 2 = 14\)

c) 6 circles with 3 dots in each, \(3 + 3 + 3 + 3 + 3 = 18\)

Draw on the board:

<table>
<thead>
<tr>
<th>Division Sentence</th>
<th>Picture</th>
<th>Addition Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (10 ÷ 2 = 5)</td>
<td>![Picture]</td>
<td></td>
</tr>
<tr>
<td>b) (24 ÷ 8 = 3)</td>
<td>![Picture]</td>
<td></td>
</tr>
<tr>
<td>c) (12 ÷ 6 = 2)</td>
<td>![Picture]</td>
<td></td>
</tr>
<tr>
<td>d) (20 ÷ 5 = 4)</td>
<td>![Picture]</td>
<td></td>
</tr>
</tbody>
</table>

Complete the first row with the class as a whole, and then have students work on the remaining rows independently. Tell them that there might be more circles than they need in the second column. After the class has had time to work on each part, ask for volunteers to draw the dots in the circles and then write the addition sentences. (a) 5 circles, 2 dots in each circle, \(2 + 2 + 2 + 2 + 2 = 10\); b) 3 circles, 8 dots in each circle, \(8 + 8 + 8 = 24\); c) 2 circles, 6 dots in each circle, \(6 + 6 = 12\); d) 4 circles, 5 dots in each circle, \(5 + 5 + 5 = 20\)

**Writing a division sentence for an addition sentence.** Draw on the board:

\[5 + 5 + 5 = 15\]

\[
\begin{array}{c}
\circ \\
\circ \\
\circ \\
\circ \\
\circ \\
\end{array}
\]

ASK: How many dots go in each circle? (5) How many circles do we need? (3) How many dots are there in total? (15) Ask for a volunteer to come to the board to fill in the dots. (3 circles with 5 dots in each circle)
Draw on the board:

<table>
<thead>
<tr>
<th>Division Sentence</th>
<th>Picture</th>
<th>Addition Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 4 + 4 + 4 = 12</td>
<td><img src="4" alt="Picture" /></td>
<td></td>
</tr>
<tr>
<td>b) 7 + 7 + 7 + 7 = 28</td>
<td><img src="7" alt="Picture" /></td>
<td></td>
</tr>
<tr>
<td>c) 3 + 3 + 3 + 3 = 15</td>
<td><img src="3" alt="Picture" /></td>
<td></td>
</tr>
<tr>
<td>d) 9 + 9 = 18</td>
<td><img src="9" alt="Picture" /></td>
<td></td>
</tr>
<tr>
<td>Bonus: 60 ÷ 10 = 6</td>
<td><img src="60" alt="Picture" /></td>
<td></td>
</tr>
</tbody>
</table>

Complete the first row with the class, and then have students work on the remaining rows independently. As before, there might be more circles than they need in the second column. After the class has had time to work on each part, ask for eight volunteers to come to the board to draw the dots in the circles and write the division sentences. (a) 3 circles, 4 dots in each circle, 12 ÷ 4 = 3; b) 4 circles, 7 dots in each circle, 28 ÷ 7 = 4; c) 5 circles, 3 dots in each circle, 15 ÷ 3 = 5; d) 2 circles, 9 dots in each circle, 18 ÷ 9 = 2; Bonus: 6 circles, 10 dots in each circle, 10 + 10 + 10 + 10 + 10 + 10 = 60)

**Extension**

The different ways 10 items can be shared equally are shown by the following addition sentences:

\[
10 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \\
10 = 2 + 2 + 2 + 2 + 2 \\
10 = 5 + 5
\]

Find all the additions sentences that show how 12 items can be shared equally.

**Answers**

\[
12 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \\
12 = 2 + 2 + 2 + 2 + 2 \\
12 = 3 + 3 + 3 + 3 \\
12 = 4 + 4 + 4 \\
12 = 6 + 6
\]
**Goals**

Students will use skip counting to divide.

**PRIOR KNOWLEDGE REQUIRED**

Can skip count on a number line
Can skip count with fingers
Can divide by sharing given the total number of items and the number of items in each set

**MATERIALS**

ball
transparency of BLM Number Lines to 20 (p. M-63)
overhead projector
BLM Number Lines to 20 (p. M-63)
BLM 9 × 9 Multiplication Chart (p. M-64, see Extension 4)

**Mental math minute.** Review counting by 3s: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. Remind students that multiplying by 3 finds one of these numbers. For example, 4 × 3 is the fourth number in this list, 12. Remind students that the commutative property tells us that 4 × 3 and 3 × 4 are equal. Point out the following pattern in the list: the sum of the digits in every multiple of 3 is either 3, 6, or 9. For example, if you know 3 × 7 starts with a 2, ask yourself, “Why can’t the answer be 22?” (because the sum of the digits here is 2 + 2 = 4) Have the class stand up. Call out a multiplication question where one of the factors is 3; for example, 3 × 8. Toss a ball to a student. The student will give the answer (24), toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

**Relating division, addition, and skip counting.** Draw four circles, and then draw three dots in each circle, as shown below. After each circle, ask the class to count the total dots aloud and then write the total beneath each circle. The final picture is shown in the margin.

ASK: What division sentence does this show? (12 ÷ 3 = 4) What addition sentence does this show? (3 + 3 + 3 + 3 = 12) Explain that the division 12 ÷ 3 can be solved by skip counting on a number line. Demonstrate by projecting a number line from BLM Number Lines to 20, or draw a number line from 0 to 20 on the board. The final picture should look like this:
ASK: How many jumps of 3 does it take to reach 12? (4) SAY: So, the answer to $12 \div 3$ is 4. ASK: How can I tell that from the number line? (count the number of arrows) Ask volunteers to demonstrate how to use skip counting to find $12 \div 4$ and $12 \div 6$.

Give students BLM Number Lines to 20, and assign the following exercises.

**Exercises:** Use skip counting forwards to divide.

a) $15 \div 3$  

b) $15 \div 5$  

c) $20 \div 4$  

d) $20 \div 5$

e) $20 \div 10$  

f) $18 \div 6$  

g) $16 \div 2$  

h) $10 \div 2$

**Answers:** a) 5, b) 3, c) 5, d) 4, e) 2, f) 3, g) 8, h) 5

**Finding the division sentence.** Draw on the board (or display using BLM Number Lines to 20):

$0 \hspace{1cm} 1 \hspace{1cm} 2 \hspace{1cm} 3 \hspace{1cm} 4 \hspace{1cm} 5 \hspace{1cm} 6 \hspace{1cm} 7 \hspace{1cm} 8 \hspace{1cm} 9 \hspace{1cm} 10 \hspace{1cm} 11 \hspace{1cm} 12 \hspace{1cm} 13 \hspace{1cm} 14 \hspace{1cm} 15 \hspace{1cm} 16 \hspace{1cm} 17 \hspace{1cm} 18 \hspace{1cm} 19 \hspace{1cm} 20$

ASK: What number is being divided? (20) How can you tell from the picture? (20 is where the arrows end) What number are we dividing by? (4) How can you tell? (4 is the size of the jump) So, what is the answer to $20 \div 4$? (5) How can you tell from the picture? (the number of arrows) Write on the board:

\[
\begin{align*}
\text{20} & \div \text{4} = \text{5} \\
\text{number} & \hspace{1cm} \text{size of the jumps} \\
\text{being divided} & \hspace{1cm} \text{number of arrows}
\end{align*}
\]

**Exercises:** Use the number line to find the division sentence.

a)

b)

c)

d)

**Answers:** a) $18 \div 3 = 6$, b) $14 \div 7 = 2$, c) $12 \div 2 = 6$, d) $15 \div 5 = 3$
Skip counting using fingers. Explain that skip counting can also be performed with fingers. Draw on the board:

15 ÷ 3

ASK: What number should we skip count by? (3) Write “3” below each circle, add plus signs between the numbers, and write “= 15” at the end of the addition sentence, as shown below:

15 ÷ 3

3 + 3 + 3 + 3 + 3 = 15

SAY: We are going to skip count by 3s, and I want you to keep track of how many 3s we count, using your fingers. Begin skip counting aloud and hold up your own fingers, as shown below:

ASK: How many fingers do you have up? (5) How many numbers did we say? (5) So, what is 15 divided by 3? (5) Write “= 5” to complete the division sentence above the picture.

Do one more example together. Write on the board:

20 ÷ 5

Ask students to keep track of the number of times you skip count, using their fingers. Skip count by 5s to 20 aloud. The sequence should look like this:

ASK: How many fingers do you have up? (4) How many numbers did we say? (4) So, what is 20 divided by 5? (4)
Exercises: Divide by skip counting on your fingers.

a) \(14 \div 2\)  

b) \(20 \div 4\)  

c) \(27 \div 9\)

d) \(32 \div 8\)  

e) \(16 \div 4\)  

f) \(35 \div 7\)

g) \(42 \div 6\)  

h) \(30 \div 5\)  

Bonus: \(90 \div 10\)

Answers: a) 7, b) 5, c) 3, d) 4, e) 4, f) 5, g) 7, h) 6, Bonus: 9

Tell the class they can also skip count by picturing the numbers at the ends of their fingers. Draw the picture in the margin on the board.

SAY: To calculate 12 divided by 3, we skip count by 3s, starting with the thumb. The number 12 appears on the fourth finger, so 12 divided by 3 is 4.

ASK: What is 15 divided by 3? (5) Point out that when skip counting by 3s, 15 appears on the fifth finger.

Draw the picture in the margin on the board. Invite a volunteer to come to the board to skip count by 7s aloud and write the numbers in the boxes, starting with the thumb. \((7, 14, 21, 28, 35)\) ASK: What is 21 divided by 7? (3)

SAY: When skip counting by 7s, 21 appears on the third finger. ASK: What division sentence can we write from the fourth finger? \((28 \div 7 = 4)\)

Ask students to use more such “hand drawings” to complete the following exercises. (They don’t have to draw the hands each time—they can just draw and fill in the boxes.) Walk around and check to make sure they have entered the correct numbers.

Exercises

1. Use a drawing of a hand to skip count.

   a) count by 6s  

   b) count by 7s  

   c) count by 8s  

   d) count by 9s

   Answers: a) 6, 12, 18, 30; b) 7, 14, 21, 28, 35; c) 8, 16, 24, 32, 40; d) 9, 18, 27, 36, 45

2. Use a drawing of a hand to divide.

   a) \(36 \div 9\)  

   b) \(24 \div 8\)  

   c) \(28 \div 7\)  

   d) \(30 \div 6\)

   e) \(18 \div 6\)  

   f) \(7 \div 7\)  

   g) \(8 \div 8\)  

   h) \(45 \div 9\)

   Answers: a) 4, b) 3, c) 4, d) 5, e) 3, f) 1, g) 1, h) 5

NOTE: Extension 3 is required to meet the Alberta and Manitoba curricula.

Extensions

1. Skip count by the given number to fill in the boxes.

   a) count by 11s  

   b) count by 12s

   \[11\]  

   \[12\]
2. Use a hand drawing to divide.
   a) 44 ÷ 11  b) 36 ÷ 12  c) 33 ÷ 11  d) 24 ÷ 12
   e) 55 ÷ 11  f) 48 ÷ 12  g) 11 ÷ 11  h) 60 ÷ 12
   Answers: a) 4, b) 3, c) 3, d) 2, e) 5, f) 4, g) 1, h) 5

3. You can divide by repeated subtraction. For example, to calculate 12 ÷ 3, subtract 3 repeatedly from 12 until you get zero. Then, count the number of times you subtracted.

   \[
   \begin{align*}
   12 & \rightarrow 9 \\
   9 & \rightarrow 6 \\
   6 & \rightarrow 3 \\
   3 & \rightarrow 0
   \end{align*}
   \]

   Subtract 4 times so 12 ÷ 3 = 4

   Divide by repeated subtraction.

   a) 24 ÷ 4  24
      \[
      \begin{array}{cccccccc}
      \text{4} & \text{4} & \text{4} & \text{4} & \text{0} \\
      \text{20} & \text{16} & \text{12} & \text{8} & \text{4} & \text{0}
      \end{array}
      \]
   \[
   24 ÷ 4 = __
   \]
   b) 30 ÷ 6  30
      \[
      \begin{array}{cccc}
      \text{6} & \\
      \text{20} & \text{16} & \text{12} & \text{8} & \text{4} & \text{0}
      \end{array}
      \]
   \[
   30 ÷ 6 = __
   \]
   c) 28 ÷ 7  28
      \[
      \begin{array}{cccc}
      \text{7} & \\
      \text{16} & \text{12} & \text{8} & \text{4} & \text{0}
      \end{array}
      \]
   \[
   28 ÷ 7 = __
   \]
   d) 56 ÷ 8  56
      \[
      \begin{array}{cccc}
      \text{8} & \\
      \text{40} & \text{32} & \text{24} & \text{16} & \text{8} & \text{4} & \text{0}
      \end{array}
      \]
   \[
   56 ÷ 8 = __
   \]
   e) 54 ÷ 9  54
      \[
      \begin{array}{cccc}
      \text{9} & \\
      \text{27} & \text{24} & \text{16} & \text{8} & \text{4} & \text{0}
      \end{array}
      \]
   \[
   54 ÷ 9 = __
   \]
   Answers
   a) 6
   b) 30, 24, 18, 12, 6, 0; 5
   c) 28, 21, 14, 7, 0; 4
   d) 56, 48, 40, 32, 24, 16, 8, 0; 7
   e) 54, 45, 36, 27, 18, 9, 0; 6

4. Use BLM 9 \times 9 Multiplication Chart to find the missing number.

   a) 4 × __ = 32  b) 7 × __ = 28
   c) 9 × __ = 27  d) 3 × __ = 27
   e) 2 × __ = 12  f) 6 × __ = 42
   g) 3 × __ = 15  h) 1 × __ = 2
   Answers: a) 8, b) 4, c) 3, d) 9, e) 6, f) 7, g) 5, h) 2
Goals

Students will learn that division problems can be interpreted as having to find either the number of groups or the number of items in each group.

PRIOR KNOWLEDGE REQUIRED

Can find the number of items in each group, given the number of groups
Can find the number of groups, given the number of items in each group

MATERIALS

ball
counters (optional)

Mental math minute. Review counting by 6s: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60. Remind students that multiplying by 6 finds one of these numbers. For example, 5 × 6 is the fifth number in this list, 30. Remind students that the commutative property tells us that 5 × 6 and 6 × 5 are equal. Point out the following pattern in the list: the last digit in each multiple follows the pattern 6, 2, 8, 4, 0, 6, 2, 8, 4, 0. Also, point out that you can find a multiple of 6 by doubling the multiple of 3. For example, 5 × 6 is double 5 × 3. Since 5 × 3 is 15, 5 × 6 is double 15, or 30. Have the class stand up. Call out a multiplication question in which one of the factors is 6; for example, 4 × 6. Toss a ball to a student. The student will give the answer, toss the ball back to you, and then sit down. Continue until all students have had a chance to answer.

Dividing where the divisor represents the number of groups.

NOTE: Avoid using the word “divisor” for now. Draw on the board:

ASK: How many lines are there? (6) SAY: We want to divide the lines into two equal groups. Ask a volunteer to erase each line one at a time and redraw the lines one at a time in each circle. The pictures below show how the circles should look after every two lines are redrawn:

As the volunteer does this, have the class skip count by 2s to keep track of how many are placed. (2, 4, 6) If students skip count on their fingers, the sequence is shown in the margin.
ASK: How many times did we skip count? (3) How many lines are in each group? (3) SAY: So, 6 divided by 2 is 3.

Repeat the steps above, starting with 12 lines and three circles.

**Exercises:** Count the lines. Then divide the lines into equal groups. Skip count by the number of groups to find how many lines to put in each group.

- a) | | | | | | 4 groups
- b) | | | | | | 2 groups
- c) | | | | | | 5 groups

**Answers:** a) 12 lines, 3 lines in each group; b) 10 lines, 5 lines in each group; c) 20 lines, 4 lines in each group

**Dividing where the divisor represents the number of items in each group.** Draw six dots on the board. ASK: How many dots are in the picture? (6) SAY: We want to divide the dots so there are two dots in each group. Ask a volunteer to draw circles so that two dots are in each circle. As the volunteer does this, have the class skip count by 2s to keep track of how dots are placed and say “stop” when all the dots have been circled. (2, 4, 6, stop) The final picture should look like this:

If students use their fingers to skip count, the sequence should look like this:

ASK: How many groups are there? (3) SAY: So, 6 divided by 2 is 3.

**Exercises:** Count the lines. Then circle the lines so that the given number of lines are in each group. Count the number of groups.

- a) | | | | | | 4 lines in each group
- b) | | | | | | 2 lines in each group
- c) | | | | | | 5 lines in each group

**Answers:** a) 12 lines, 3 groups; b) 10 lines, 5 groups; c) 20 lines, 4 groups

**Using arrays to show the two meanings of division.** Show students how the same division sentence can have two different meanings and lead to two different pictures. Write on the board:

\[
12 \div 3 \text{ can mean dividing 12 into groups of 3, or dividing 12 into 3 groups.}
\]

\[
\begin{array}{c}
\bullet \bullet \bullet \\
\bullet \bullet \bullet
\end{array}
\]
Have a volunteer come to the board and circle groups of three in the first array. (see margin) ASK: How many groups are there? (4) What division sentence can we write for this picture? (12 ÷ 3 = 4)

Have a volunteer come to the board and use the second array to divide the dots equally into three groups. (see margin) ASK: How many dots are in each group? (4) What division sentence can we write for this picture? (12 ÷ 3 = 4) SAY: So, 12 ÷ 3 can mean finding the number of groups when there are three dots in each group or finding the number of dots in each group when there are three groups. In each case, the answer is 4.

Exercises: For the division sentence, show how the answer can represent the number of dots in each group or the number of groups.

a) 6 ÷ 2 = 3

b) 15 ÷ 3 = 5

Answers: a) b)

Writing two sentences from a picture. Now show students how the same picture can lead to two different division sentences. Draw on the board:

SAY: We can say 12 items are divided into groups of three each, to get four groups. So, 12 ÷ 3 = 4. We can also say 12 items are divided into four groups, to get three items in each group. So, 12 ÷ 4 = 3.

Exercises: Write two division sentences for the picture.

a) b) c)

Answers: a) 20 ÷ 4 = 5 and 20 ÷ 5 = 4, b) 15 ÷ 3 = 5 and 15 ÷ 5 = 3, c) 21 ÷ 3 = 7 and 21 ÷ 7 = 3
Solving word problems by first drawing a picture. Write on the board:

- 20 players
- 5 players on a team
- How many teams?

SAY: Let’s use stick figures for the players and rectangles for the teams. Ask for a volunteer to come to the board and draw a team with five players. Ask the volunteer to continue drawing teams until all the players are used. The final drawing should look like this:

![Carroll Diagram](image)

SAY: We can show this using circles and dots. Draw on the board:

![Carroll Diagram](image)

ASK: What division sentence can we write for this picture? (20 ÷ 5 = 4)
- How many teams are there? (4)

**Exercises:** Draw a picture and write a division sentence to solve the problem.

a) 24 apples  
   6 apples in a basket  
   How many baskets?

b) 12 granola bars  
   2 bars to a person  
   How many people?

c) 24 people  
   4 vans  
   How many people in a van?

**Answers:** a) 4, b) 6, c) 6

Solving story problems involving division. Write on the board and read aloud to the class:

The pet store has 15 birds and 5 cages. The owner wants to place the same number of birds in each cage. How many birds will be in each cage?

Draw on the board:

![Carroll Diagram](image)

ASK: What can these circles represent for the story? (cages) Have a volunteer draw one dot in each circle to represent the birds. ASK: How many birds have we placed in cages? (5) Repeat with another volunteer. ASK: Now how many birds have we placed in cages? (10) Repeat with one more volunteer. The final picture should look like this:

![Carroll Diagram](image)

ASK: How many birds have we placed in cages? (15) How do we know we can stop? (all the birds have been placed) How many birds are in each cage? (3) What division sentence can I write for the story? (15 ÷ 5 = 3)
SAY: If we look at the picture, we can say that we have divided 15 objects into five sets, with three objects in each set. The division sentence is $15 \div 5 = 3$. We can also say that when we divide 15 objects into sets of size 3, there are five sets. The division sentence is $15 \div 3 = 5$.

Exercises

1. Draw a picture or use counters to solve the story problem. Write a division sentence.
   
   a) Jun has 24 apples. He wants to give 4 apples to each of his friends. How many of his friends will get apples?
   
   b) Maria has 15 movie passes. She decides to share the passes with two of her friends. How many movie passes will each person get?

   **Answers**
   
   a) $24 \div 4 = 6$, he can give 6 friends 4 apples each
   
   b) $15 \div 3 = 5$, they will each get 5 movie passes

2. Write two division sentences for each picture.

   **Answers:** a) $8 \div 2 = 4$, $8 \div 4 = 2$; b) $18 \div 6 = 3$, $18 \div 3 = 6$

**Extension**

Three people are sharing 24 apples. How many apples would each person have to give up so that one more person can share the apples equally?

**Answer:** 2
Goals
Students will write two division sentences for each multiplication sentence.

PRIOR KNOWLEDGE REQUIRED
Can use skip counting to multiply
Can use skip counting to divide

MATERIALS
- playing cards from 1 (ace) to 10 in two suits or number cards, per pair of students
- BLM Multiplication Review (p. M-65)
- BLM Division Review (pp. M-66–67)
- BLM Fluency Practice—Division (p. M-68)
- BLM Fluency Practice—Multiplication and Division (p. M-69)

Mental math minute. Divide the class in half so that there is a left side and right side. Write the numbers from 1 to 9 on separate sheets of paper, and give one sheet to each student on the left side of the class. If there are more than 9 students on the left side, start again at 1. Repeat with the right side. Have the left side stand along one side of the classroom and the right side stand along the opposite side. Call out an answer from the times table. For example, call out 54. Students are to think of a pair of numbers that multiply to 54 (for example: 6 and 9). If any student from the left side holds a 6 or 9, they should step forwards. For each person from the left side, a student from the right side should step forwards with a number that will multiply to 54. Alternate between the left side and the right side stepping forwards first. Repeat until every student has had several opportunities to step forwards.

Reviewing multiplication facts. Review the times tables using any of the methods used earlier in the year or the activities below. A powerful technique for improving automatic recall of number facts involves incrementally increasing the number of facts being used. The activities can be repeated as desired. A suggested technique is to repeat reviewing the times table on Day 1, Day 2, Day 4, Day 8, Day 16, and so on.

ACTIVITIES 1–2
1. Player 1 pulls two cards from a deck and multiplies the numbers on the cards. Player 2 checks the answer. Players switch roles.
2. Students who need to practise recalling number facts in sequence can use BLM Multiplication Review. Students work in pairs. Starting with the 2 times table, Student A recites all the products aloud and in sequence, using the following technique.
Say the answer to $2 \times 1$: 2
Say the answers to $2 \times 1$ and $2 \times 2$, three times: 2, 4; 2, 4; 2, 4
Say the answers to $2 \times 1$, $2 \times 2$, and $2 \times 3$, three times: 2, 4, 6; 2, 4, 6; 2, 4, 6

Student B follows along with the BLM and prompts Student A if he or she needs help. If a mistake is made, Student A again repeats the answers three times. If another mistake is made, Student A goes back one step in the process. For example, if Student A makes two mistakes at $2 \times 6$, she or he says all answers three times, up to and including $2 \times 5$ (2, 4, 6, 8, 10; 2, 4, 6, 8, 10; 2, 4, 6, 8, 10) before attempting $2 \times 6$ again. When Student A completes the 2 times table, Student B takes a turn and Student A checks and prompts as necessary. Students then move to the 3 times table, but this time Student B goes first. Students continue the process until all the times tables are complete.

Relating division to multiplication. Ask a volunteer to come to the board and draw dots inside circles to calculate $12 \div 3$. Ask the volunteer to draw three dots in each circle, and have the class skip count by 3 aloud. Ask the class to say "stop" when the volunteer should stop drawing dots in circles. (class calls out: 3, 6, 9, 12, stop) Write "3" under each circle. The final picture should look like this:

![3 3 3 3]

ASK: What is 12 divided by 3? (4) How can we find the answer from the picture? (4 circles had dots drawn in them) What addition sentence can we use to check this calculation? ($3 + 3 + 3 + 3 = 12$) SAY: We have four groups of three dots for a total of 12 dots. ASK: How can we write this using multiplication? ($4 \times 3 = 12$) Write on the board:

$$12 \div 3 = 4$$

leads to $$4 \times 3 = 12$$

SAY: Notice that if you read the division sentence backwards, you can get the multiplication sentence.

Draw on the board:

![\bigcirc \bigcirc \bigcirc \bigcirc]

Ask for another volunteer to show $12 \div 4 = 3$ by drawing one dot in each circle. Ask the class to skip count by 4s each time the volunteer adds one more dot to every circle and to say "stop" when the volunteer should stop drawing dots, as shown on the next page. (class calls out: 4, 8, 12, stop)
ASK: What is 12 divided by 4? (3) How can we find the answer from the picture? (each circle has 3 dots in it) What addition sentence can we use to check this calculation? (4 + 4 + 4 = 12) What multiplication sentence can be used as a short form for this addition? (3 × 4 = 12) Write on the board:

\[ 12 ÷ 4 = 3 \text{ leads to } 3 \times 4 = 12 \]

SAY: If you read the division sentence backwards, you can get the multiplication sentence. The same division picture leads to two multiplication sentences and two division sentences. Write on the board:

\begin{align*}
4 \times 3 & = 12 \\
3 \times 4 & = 12 \\
12 ÷ 3 & = 4 \\
12 ÷ 4 & = 3
\end{align*}

Exercises: Write two multiplication sentences and two division sentences for the picture.

a) \[
\begin{array}{c}
\bullet \bullet \\
\bullet \bullet \bullet \\
\bullet \bullet \bullet \bullet \\
\end{array}
\]

b) \[
\begin{array}{c}
\bullet \bullet \\
\bullet \bullet \\
\bullet \bullet \\
\bullet \bullet \\
\bullet \bullet \\
\end{array}
\]

c) \[
\begin{array}{c}
\bigtriangleup \\
\bigtriangleup \\
\bigtriangleup
\end{array}
\]

Answers

a) \[5 \times 3 = 15, \; 3 \times 5 = 15, \; 15 ÷ 3 = 5, \; 15 ÷ 5 = 3\]
b) \[6 \times 4 = 24, \; 4 \times 6 = 24, \; 24 ÷ 4 = 6, \; 24 ÷ 6 = 4\]
c) \[3 \times 6 = 18, \; 6 \times 3 = 18, \; 18 ÷ 6 = 3, \; 18 ÷ 3 = 6\]

Writing division sentences from pictures. Draw on the board:

\[
\begin{array}{c}
\bullet \bullet \\
\bullet \bullet \\
\bullet \bullet \\
\bullet \bullet
\end{array}
\]

ASK: How many lines are there? (20) How many sets are there? (5) How many lines are in each set? (4) SAY: We can say that if 20 lines are grouped with four lines each set, there will be five sets. We can also say that if 20 lines are grouped into five sets, there will be four lines in each set. ASK: What are two division sentences for this picture? (20 ÷ 4 = 5 and 20 ÷ 5 = 4).
Exercises

1. Draw a picture and then write two division sentences for the picture.
   a) 12 dots, 4 dots in each circle, 3 circles
   b) 10 triangles, 2 triangles in each square, 5 squares

   **Answers**
   a) $12 \div 4 = 3$ and $12 \div 3 = 4$
   b) $10 \div 2 = 5$ and $10 \div 5 = 2$

2. Draw a picture to find the missing information.
   a) 28 circles, 4 dots in each circle, ___ circles
   b) 15 lines, 3 groups, ___ lines in each group

   **Answers**
   a) 7 circles
   b) 5 lines in each group.

**Dividing using multiplication facts.** SAY: Remember, there is a multiplication sentence for every division sentence. If you learn your multiplication facts, you can find the answer to any division sentence. Write on the board:

To find $15 \div 3 = ?$, think of the multiplication sentence $? \times 3 = 15$.

**ASK:** What number, when multiplied by 3, gives 15? (5) **SAY:** So, $15 \div 3 = 5$.

**ACTIVITIES 3–5**

3. Give each student a copy of **BLM Division Review**. This two-page BLM should be photocopied as a two-sided sheet if possible.
   
   Students work in pairs and begin with division by 2. Student A reads any division sentence from the division-by-2 table on BLM Division Review (1) without the answer. For example, “What is $18 \div 2$?” Student B looks only at the corresponding facts on BLM Division Review (2) and scans the 2 times table to find “$18 = 2 \times ?$”.
   
   Using knowledge of the multiplication facts, Student B answers both problems. ($18 = 2 \times 9$, so $18 \div 2 = 9$) Student A confirms the answers and prompts Student B when he or she needs help. If several mistakes are made, Student B may need to practise the 2 times table using the technique described in Activities 1 and 2.
When Student A has read all the division sentences from the division-by-2 table, students switch roles and repeat: Student B reads the division sentences and supplies prompts if necessary while Student A uses multiplication facts to complete the division sentences. Then students move to the 3 times table, with Student B taking the role of reader and prompter to start.

Students continue in this manner until all the division tables are complete. This activity can be repeated as desired, such as on Day 1, Day 2, Day 4, Day 8, Day 16, and so on.

4. Distribute **BLM Fluency Practice—Division**. Students who have shown fluency can work on improving their speed by completing this BLM. After they have checked their answers to the BLM against the division-by charts (BLM Division Review (1)), have pairs of students take turns asking and answering questions aloud from BLM Fluency Practice—Division. They can check their answers by referring to BLM Division Review (1).

5. Distribute **BLM Fluency Practice—Multiplication and Division**. This BLM has a mixture of multiplication and division problems, and can be used in the same way as BLM Fluency Practice—Division (in Activity 4).

**Extensions**

1. A collection of dots can be divided into 10 groups of 5 dots each. How many groups would there be if the collection were divided into groups of 25 dots?

   **Answer:** 2

2. Chairs in a room are organized into 10 rows with 10 chairs in each row. If we wanted only 5 rows, how many chairs would be in each row?

   **Answer:** 20

3. a) Explain using groups why $48 \div 6$ should be greater than $48 \div 8$.

   b) Without calculating, fill in the box with “>” or “<”. Check your answer by calculating.

   - i) $42 \div 6 \quad 42 \div 7$
   - ii) $45 \div 9 \quad 45 \div 5$
   - iii) $63 \div 7 \quad 63 \div 9$
   - iv) $72 \div 8 \quad 72 \div 9$

   **Answers:** a) $48 \div 6$ involves sharing 48 items with fewer people (groups), so each person should get more; b) i) $>$, ii) $<$, iii) $>$, iv) $>$
Goals

Students will understand when to use multiplication or division to find missing information.

PRIOR KNOWLEDGE REQUIRED

Understands the relationship between multiplication and division

MATERIALS

transparency of BLM 10 × 10 Multiplication Chart (p. M-70)
overhead projector
ball

Mental math minute. Have students stand up. Display a transparency of BLM 10 × 10 Multiplication Chart. For a given number in the chart, show how to create a multiplication sentence and a related division sentence for the number. For example, point to 35. SAY: 7 × 5 = 35 and 35 ÷ 5 = 7. Pick an answer from the multiplication chart and say it out loud. Toss a ball to a student. The student will say a multiplication sentence and a related division sentence that involves the number. The student will toss the ball back to you and sit down. If there is more than one answer, call out the same number again. For example, if you say 12, students could choose 4 × 3 = 12 and 12 ÷ 3 = 4, or 6 × 2 = 12 and 12 ÷ 2 = 6. Continue until all the students have had a chance to answer a question.

The relationship between the total number of items, the number of items in each group, and the number of groups. Draw on the board:

ASK: How many things or items are in each group? (3) How many groups or sets are there? (4) What is the total number of items? (12) What multiplication sentence connects these numbers? (3 × 4 = 12) What division sentences can be written because 3 × 4 = 12? (12 ÷ 4 = 3 and 12 ÷ 3 = 4) Write on the board:

\[
\text{Number of Items} \times \text{Number of Groups} = \text{Total Number of Items}
\]

SAY: There are three types of problems involving division. In each type, one of these pieces of information is missing. Write on the board:

Type 1:

\[
\text{Number of Items} \times \text{Number of Groups} = ?
\]
ASK: If there are four items in each group and six groups, what is the total number of items? SAY: Fill in the numbers you know in the sentence. Write on the board:

\[ 4 \times 6 = ? \]

ASK: What is \( 4 \times 6 \)? (24) SAY: So, there are 24 items.

Exercises: Find the total number of items.

<table>
<thead>
<tr>
<th>Number of Items in Each Group</th>
<th>×</th>
<th>Number of Groups</th>
<th>=</th>
<th>Total Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 4</td>
<td>×</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 6</td>
<td>×</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 8</td>
<td>×</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: a) 28, b) 30, c) 24

Write on the board:

**Type 2:**

\[ ? \times \text{Number of Groups} = \text{Total Number of Items} \]

ASK: If there are 30 items and five groups, how many items are in each group? SAY: Fill in the numbers you know in the sentence. Write on the board:

\[ ? \times 5 = 30 \]

ASK: What division sentence can be written for this multiplication sentence? If a prompt is needed, tell students to read the sentence backwards. (30 \( ÷ 5 \) = ?) ASK: What is 30 \( ÷ 5 \)? (6) SAY: So, there are six items in each group.

Exercises: Find the number of items in each group.

<table>
<thead>
<tr>
<th>Number of Items in Each Group</th>
<th>×</th>
<th>Number of Groups</th>
<th>=</th>
<th>Total Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ?</td>
<td>×</td>
<td>6</td>
<td></td>
<td>42</td>
</tr>
<tr>
<td>b) ?</td>
<td>×</td>
<td>8</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>c) ?</td>
<td>×</td>
<td>9</td>
<td></td>
<td>36</td>
</tr>
</tbody>
</table>

Answers: a) 7, b) 5, c) 4

Write on the board:

**Type 3:**

\[ \text{Number of Items in Each Group} \times ? = \text{Total Number of Items} \]
ASK: If there are seven items in each group and 35 items, how many groups are there? SAY: Fill in the numbers you know in the sentence. Write on the board:

\[ 7 \times \ ? = 35 \]

ASK: What division sentence can be written for this multiplication sentence? \((35 \div 7 = ?)\) What is \(35 \div 7\)? (5) SAY: So, there are five groups.

**Exercises:** Find the number of groups.

<table>
<thead>
<tr>
<th>Number of Items in Each Group</th>
<th>(\times)</th>
<th>Number of Groups</th>
<th>=</th>
<th>Total Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 6</td>
<td>(\times)</td>
<td>?</td>
<td>(_)</td>
<td>48</td>
</tr>
<tr>
<td>b) 3</td>
<td>(\times)</td>
<td>?</td>
<td>(_)</td>
<td>27</td>
</tr>
<tr>
<td>c) 4</td>
<td>(\times)</td>
<td>?</td>
<td>(_)</td>
<td>24</td>
</tr>
</tbody>
</table>

**Answers:** a) 8, b) 9, c) 6

**Writing multiplication or division sentences to find the missing information.** Write on the board:

24 items
4 groups
How many items in each group?

ASK: What is the number of groups? (4) What is the total number of items? (24) What information is missing? (the number of items in each group) Write on the board:

\[ \text{Number of Items in Each Group} \times \text{Number of Groups} = \text{Total Number of Items} \]

Ask for a volunteer to write the information given in the problem under the correct label, using a question mark for information that is missing. \((? \times 4 = 24)\) ASK: What division sentence can be written for this multiplication sentence? \((24 \div 4 = ?)\) What is \(24 \div 4\)? (4) SAY: So, there are four items in each group.

**Exercises:** Write a multiplication or division sentence to find the missing information.

a) 3 groups, 5 items in each group. How many items?

b) 32 items, 4 groups. How many items in each group?

c) 30 items, 6 items in each group. How many groups?

**Answers:** a) \(3 \times 5 = 15\), b) \(32 \div 4 = 8\), c) \(30 \div 6 = 5\)
Extension

For each word problem, underline the information you need to find and circle the information you would use to find the missing information. Ignore the extra information.

a) There are 5 apples in each basket and there are 4 baskets. How many apples are there altogether?

b) There are 24 people on a trip. They used 4 vans. How many people are in each van?

c) 4 tennis courts are used in a tournament. There are 6 tennis balls on each court. How many tennis balls are used in the tournament?

Answers

a) There are \textbf{5 apples in each basket} and there are \textbf{4 baskets}. How many apples are there altogether?

b) There are \textbf{24 people} on a trip. They used \textbf{4 vans}. How many people are in each van?

c) \textbf{4 tennis courts} are used in a tournament. There are \textbf{6 tennis balls on each court}. How many tennis balls are used in the tournament?
Goals

Students will use multiplication or division sentences to solve word problems.

PRIOR KNOWLEDGE REQUIRED

Can write division sentences for a multiplication sentence

MATERIALS

transparency of BLM 10 \times 10 Multiplication Chart (p. M-70) 
overhead projector

Mental math minute. Have students stand up. Display a transparency of BLM 10 \times 10 Multiplication Chart. Call out a single digit number. Ask a student to create a multiplication sentence and a related division sentence that involves the number. For example, if you choose 7, a student could say \(7 \times 2 = 14\) and \(14 \div 2 = 7\). The student then does three jumping jacks and sits down. Continue until all students have had a chance to answer.

Identifying the total number of items, the number of groups, and the number of items in each group from a word problem. Write on the board:

- 24 chairs
- 6 rows

ASK: What is the total number of items or things? (24) What are the groups or sets? (rows) How many groups? (6) What information is missing? (the number of items in each row or group) Write on the board:

\[
\text{Number of Items in Each Group} \times \text{Number of Groups} = \text{Total Number of Items}
\]

Ask for a volunteer to write the information in the correct spot under the sentence, using a question mark to represent missing information. \(? \times 6 = 24\) ASK: What division sentence can we write for this multiplication sentence? \(24 \div 6 = ?\) What is \(24 \div 6?\) (4) SAY: So, there are four chairs in each row.
Exercises

1. Fill in the table. Use a question mark for information you don’t know.

<table>
<thead>
<tr>
<th>Total Number of Items</th>
<th>Number of Groups</th>
<th>Number of Items in Each Group</th>
<th>Multiplication or Division Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 30 people</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6 vans</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>How many people</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>in each van?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8 chairs in each row</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 rows</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>How many chairs?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>24 rowers</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8 in each rowboat</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>How many rowboats?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: a) 30, ?, 30 ÷ 6 = ?; b) ?, 5, 8, 5 × 8 = ?; c) 24, ?, 8, 24 ÷ 8 = ?

2. Find the missing number in each part of Exercise 1.

Answers: a) 5, b) 40, c) 3

Finding members of a fact family. Draw on the board:

ASK: How many dots are there in the array? (20) What are two multiplication sentences to help count the number of dots in the array? (4 × 5 = 20 and 5 × 4 = 20) On the first array, circle five dots in each row, as shown below:

\[
\begin{array}{c}
\cdot \cdot \cdot \\
\cdot \cdot \cdot \\
\cdot \cdot \cdot \\
\cdot \cdot \cdot \\
\cdot \cdot \cdot \\
\end{array}
\]

SAY: I have grouped the dots so that there are five in each group. ASK: What division sentence can I use to find the number of groups? (20 ÷ 5 = 4) On the second array, circle four dots in each column, as shown below:

\[
\begin{array}{c}
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\cdot \cdot \\
\end{array}
\]
SAY: I have grouped the dots so that there are four in each group.
ASK: What division sentence can I use to find the number of groups?
\((20 \div 4 = 5)\) SAY: The same array leads to two multiplication sentences and two division sentences.

Write on the board:

\[
\begin{align*}
4 \times 5 &= 20 \\
5 \times 4 &= 20 \\
20 \div 5 &= 4 \\
20 \div 4 &= 5
\end{align*}
\]

SAY: Together, these sentences are called a fact family.

**Exercises:** Find the other three members of the fact family.

\begin{align*}
a) \quad 6 \times 4 &= 24 \\
b) \quad 28 \div 7 &= 4 \\
c) \quad 36 \div 4 &= 9
\end{align*}

**Answers:**

\begin{align*}
a) \quad 4 \times 6 &= 24, \quad 24 \div 4 &= 6, \quad 24 \div 6 &= 4; \\
b) \quad 4 \times 7 &= 28, \quad 7 \times 4 &= 28, \\
28 \div 4 &= 7; \\
c) \quad 9 \times 4 &= 36, \quad 4 \times 9 &= 36, \quad 36 \div 9 &= 4
\end{align*}

**Solving word problems by using the fact family.** Write on the board:

56 people are divided into teams with 8 people on each team. How many teams are there?

SAY: The noun after the word “each” is usually the group. The noun before the word “each” is usually the item. ASK: What are the groups? (teams) What are the items? (people) How many items are there? (56) How many items in each group? (8) 

Write on the board:

\[
\begin{array}{ccc}
\text{Number of Items} & \times & \text{Number of Groups} \\
\text{in Each Group} & & \text{= Total Number of Items}
\end{array}
\]

Have a volunteer write the information in the correct spot under the sentence, using a question mark for any missing information. \((8 \times ? = 56)\) ASK: What division sentence can be written for the multiplication sentence \(8 \times ? = 56?\) \((56 \div 8 = ?)\) What is \(56 \div 8?\) \((7)\) SAY: So, there are seven items in each group, which means there are seven people on each team.

**Exercises:** Solve the word problem.

\begin{align*}
a) \quad 45 \text{ stamps are arranged in 5 rows. How many stamps are in each row?} \\
b) \quad \text{A photo album with 7 pages has 6 photos on each page. How many photos are in the album?} \\
c) \quad \text{A student earned $72 for 8 hours of dog sitting. How much does the student earn each hour?}
\end{align*}

**Answers:**

\begin{align*}
a) \quad 9, \\
b) \quad 42, \\
c) \quad 9
\end{align*}
Extensions

1. An array of 24 dots has 8 dots in each row. How many more rows of dots are needed to make a total of 40 dots?

   **Answer:** 2 more rows

2. A bookshelf has 36 books on 3 shelves. How many shelves are needed if only 6 books can be put on a shelf?

   **Answer:** 6 shelves

3. A stamp book has 45 stamps with 5 stamps on each page. A second stamp book has 32 stamps with 4 stamps on each page. Which book has more pages?

   **Answer:** The first stamp book (the first stamp book has 9 pages and the second has 8 pages).

4. A basketball league has 42 players with 6 players on each team. A second basketball league has 5 teams with 8 players on each team.

   a) Which league has more players?

   b) Which league has more teams?

   **Answers:** a) the first league (the first has 42 players and the second has 40 players); b) the first league (the first has 7 teams and the second has 5 teams)
Goals

Students will use multiplication or division sentences to solve word problems.

Prior Knowledge Required

Knows the multiplication facts up to the 10 times table
Knows the division facts up to the 10 times table
Can find the members of a fact family
Can identify even and odd numbers

Mental math minute. Divide the class in half so that there is a left side and right side. Write the numbers from 1 to 9 on separate sheets of paper and give one sheet to each student on the left side. If there are more than nine students on the left side, start again at 1. Repeat with the right side. Have the left side stand along one side of the classroom and have the right side stand along the opposite side. Call out a number for the left side and a number for the right side. Students whose numbers have been called step forwards. If more than one person on either side steps forwards, choose one of them. The student on the left side must say a multiplication sentence involving the two chosen numbers. The student on the right side must say a division sentence involving the two numbers. Repeat several times so that every student has had a chance to step forwards.

Solving problems involving multiples. SAY: We say that 35 is a multiple of 5 because we can find a number that, when multiplied by 5, gives 35. ASK: What is that number? (7) Why is 36 a multiple of 9? (because $9 \times 4 = 36$) Write on the board:

I am a multiple of 5.
I am between 29 and 37
I am even.
What number am I?

Ask a volunteer to write the first 10 multiples of 5 in order on the board. (5, 10, 15, 20, 25, 30, 35, 40, 45, 50) Ask another volunteer to come to the board and cross out all the multiples that are not between 29 and 37. (the only multiples left are 30, 35) ASK: What are even numbers? (numbers you say when you count by 2s) What are the possible ones digits of an even number? (0, 2, 4, 6, or 8) Ask for a volunteer to come to the board and cross out the multiples left that are not even. ASK: What is the only number left? (30) SAY: So, the mystery number is 30.

Solving word problems involving multiplication or division. SAY: In a word problem involving multiplication or division, there are three things to look for: the total number of items, the number of items in each group, and the number groups. The noun after the word “each” or a similar word is
usually the group. The noun before the word “each” is usually the item that belongs to each group. Write on the board:

\[
\text{Number of Items} \quad \times \quad \text{Number of Groups} \quad = \quad \text{Total Number of Items}
\]

There are 6 trees in each row.
There are 24 trees.
How many rows of trees are there?

Prompt students for information from the word problem. ASK: What are the groups? (rows) What are the items? (trees) How many items are there? (24) How many items are in each group? (6) What information is missing? (the number of groups) Ask a volunteer to write the information for the template on the board. Tell the volunteer to use a question mark for the missing information. \((6 \times ? = 24)\) ASK: What division sentence can be written for this multiplication sentence? \((24 \div 6 = ?)\) What is \(24 \div 6\)? (4) SAY: So, there are 4 rows of trees.

**Extensions**

1. I am a multiple of 2, 3, and 5. I am greater than 50, but less than 70. What number am I?
   **Answer:** 60

2. Fill in the boxes with either +, −, ×, or ÷ to make the number sentence true.
   a) \(6 \big[\big] 4 \big[\big] 8 \big[\big] 7 = 10\)
   b) \(30 \big[\big] 6 \big[\big] 2 \big[\big] 7 = 3\)
   c) \(48 \big[\big] 8 \big[\big] 3 \big[\big] 2 = 4\)
   **Answers:** a) ×, ÷, +; b) ÷, ×, −; c) ÷, ÷, +

3. Use four 3s and any of the signs +, −, ×, and ÷ to make a number sentence with answer 24.
   **Answer:** \(3 \times 3 \times 3 - 3 = 24\)

4. Use five 4s and any of the signs +, −, ×, and ÷ to make a number sentence with answer 4.
   **Answer:** \(4 \div 4 \times 4 + 4 - 4 = 4\)
Goals

Students will write multiplication and division sentences to find the number of rows, the number of columns, or the total number of items in an array given the other two pieces of information.

PRIOR KNOWLEDGE REQUIRED

Can distinguish between rows and columns in an array
Can count the number of rows and columns in an array
Can multiply the number of rows and columns to find the total number of items in an array

MATERIALS

BLM 10 x 10 Multiplication Chart (p. M-70)

Mental math minute. Cut out the white squares on BLM 10 x 10 Multiplication Chart. Distribute the squares one at a time to all the students. When you run out of students, start again and continue until all the squares have been handed out. Ask students who have a single-digit square (such as 8) to put up their hands. Choose two of these students, and have them say their numbers out loud. Tell the class to multiply the two numbers mentally. Ask students who have the slips with the correct answer to put up their hands. Choose one student to read the number aloud. Each of the three students then do three jumping jacks. Repeat several times until all the students have had a chance to participate.

Review arrays of dots. Draw on the board:

```
  ●   ●   ●   ●   ●
  ●   ●   ●   ●   ●
  ●   ●   ●   ●   ●
```

ASK: Do you remember what this is called? (an array of dots) Trace your finger along the top row of dots and ASK: Is this called a row or a column? (row) Remind students that rows go across. ASK: How many rows are there in this array? (3) Number the rows on the left-hand side. Trace your finger down the fourth column and ASK: Is this called a row or a column? (column) Remind students that columns go up and down. ASK: How many columns does this array have? (5)

Number the columns above the array. The array is shown in the margin.

Review arrays of squares. Draw on the board, to the right of the array of dots:
ASK: Do you remember what this is called? (an array of squares) Students might remember that this can also be called a table. Remind students that they can make an array of dots, an array of squares, or an array of any kind of object they wish. Ask a volunteer to number the rows in the array, and ask another volunteer to number the columns. The final array is shown in the margin.

Exercises

1. Number the rows in the array. Write how many rows there are.

   a)    rows
   b)    rows
   c)    rows

   Answers: a) 4, b) 5, c) 3

2. Number the columns in the array. Write how many columns there are.

   a)    columns
   b)    columns
   c)    columns

   Answers: a) 6, b) 8, c) 5

Finding the total number of items in an array. Draw a 3 by 6 array of dots on the board. Have volunteers number the rows and then the columns, as shown below. ASK: How would you find the total number of dots in the array? (sample answers: counting the dots one by one, skip counting the rows, skip counting the columns) Write the skip counting by rows (6 + 6 + 6 = 18) and by columns (3 + 3 + 3 + 3 + 3 + 3 = 18) on the board, as shown below:

   1 2 3 4 5 6
   6 + 6 + 6 = 18
   2 3 4 5
   3 + 3 + 3 + 3 + 3 + 3 = 18
Write on the board below the array:

___ rows
___ columns
Total = _______ or _______

ASK: How many rows are there? (3) How many columns? (6) Write “3” in the first blank and “6” in the second blank. ASK: What multiplication sentence can we write to find the total number of dots? (3 \times 6 = 18 or 6 \times 3 = 18) Have a volunteer write the two multiplication sentences in the blanks beside the total. Remind students that the order doesn’t matter when you multiply two numbers. ASK: Does 3 \times 6 give the same answer as 6 \times 3? (yes, 18) Emphasize that because there are six columns, there are six dots in each row. SAY: So, there are three rows of six dots each, and we multiply three and six to find the total number of dots. Similarly, the answer can be found by first figuring out how many dots are in each column. Since there are three rows, that means that each of the six columns has three dots, and so the numbers three and six are multiplied to find the total number of dots. Repeat the process with an array of squares.

Have students complete Questions 1–2 on AP Book 3.2 p. 25.

Writing a division sentence from a multiplication sentence. Remind students how to write division sentences from a given multiplication sentence by reading the sentence backwards. Write on the board:

\[ 6 \times 3 = 18 \text{ leads to } 18 \div 3 = 6 \]

SAY: But if \( 6 \times 3 = 18 \), then \( 3 \times 6 = 18 \). Write on the board:

\[ 3 \times 6 = 18 \text{ leads to } 18 \div 6 = 3 \]

SAY: So, \( 6 \times 3 = 18 \) leads to two division sentences: \( 18 \div 3 = 6 \) and \( 18 \div 6 = 3 \). Write a new example on the board:

\[ 6 \times 7 = 42 \text{ so } 42 \div ___ = ___ \text{ and } 42 \div ___ = ___ \]

Have students signal the numbers that belong in the blanks. (42 \div 7 = 6 and 42 \div 6 = 7)

Multiplication and division sentences for arrays. Draw on the board:

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
- & - & - & - & - & - & - \\
2 & 3 & 4 & 5 & 6 & 7 & 8 \\
- & - & - & - & - & - & - \\
3 & 4 & 5 & 6 & 7 & 8 & 9 \\
- & - & - & - & - & - & - \\
4 & 5 & 6 & 7 & 8 & 9 & 10 \\
- & - & - & - & - & - & - \\
\end{array}
\]

___ rows
___ columns
Total = _______

Have students signal the answers before you fill in the number of rows and columns. (4, 7) Have a volunteer write the multiplication sentence for the total. (4 \times 7 = 28) ASK: What other multiplication sentence can we write for this array? (7 \times 4 = 28) What two division sentences can we write for this array? PROMPT: What division sentences do we get by reading each
multiplication sentence backwards? (28 ÷ 7 = 4 and 28 ÷ 4 = 7) Have a volunteer write the two division sentences on the board. Repeat this process with a 5 by 8 array of squares.

**Exercises:** Count the rows and columns in the array. Write a multiplication sentence to find the total number of squares or dots. Write one more multiplication sentence and two division sentences for the array.

a) ![Array Image]

b) ![Array Image]

c) ![Array Image]

**Answers:** a) 4 rows, 5 columns; 4 × 5 = 20, 5 × 4 = 20, 20 ÷ 5 = 4, 20 ÷ 4 = 5; b) 5 rows, 9 columns; 5 × 9 = 45, 9 × 5 = 45, 45 ÷ 9 = 5, 45 ÷ 5 = 9; c) 3 rows, 7 columns; 3 × 7 = 21, 7 × 3 = 21, 21 ÷ 7 = 3, 21 ÷ 3 = 7

**Writing multiplication and division sentences for arrays without pictures.** SAY: Now suppose we are given the number of rows and number of columns in an array, as well as the total number of objects in the array. The objects might be dots, squares, or something else. We do not have a picture of the array, but we can still write the number sentences for the array. Draw on the board:

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Number Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>40</td>
<td>4 × 10 = 40, 10 × 4 = 40</td>
</tr>
</tbody>
</table>

ASK: What multiplication sentences can we write for the array? (4 × 10 = 40 and 10 × 4 = 40) Ensure both possible answers are provided. Have a volunteer write the multiplication sentences in the table, on the left side of the box. ASK: What division sentences can we write? (40 ÷ 10 = 4 and 40 ÷ 4 = 10) Have a volunteer write the division sentences, on the right side of the box. Keep the table on the board for use later on. The final table should look like this:

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Number Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>40</td>
<td>4 × 10 = 40, 40 ÷ 10 = 4, 10 × 4 = 40, 40 ÷ 4 = 10</td>
</tr>
</tbody>
</table>

Explain to students that in the exercises below, each line of the table gives information about one single array.
Exercises: The table gives the number of rows and columns for arrays. Write two multiplication sentences and two division sentences for each array.

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Number Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>2</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>b)</td>
<td>9</td>
<td>7</td>
<td>63</td>
</tr>
<tr>
<td>c)</td>
<td>10</td>
<td>9</td>
<td>90</td>
</tr>
<tr>
<td>Bonus:</td>
<td>3</td>
<td>40</td>
<td>120</td>
</tr>
</tbody>
</table>

Answers: a) $2 \times 6 = 12$, $6 \times 2 = 12$, $12 \div 6 = 2$, $12 \div 2 = 6$; b) $9 \times 7 = 63$, $7 \times 9 = 63$, $63 \div 7 = 9$, $63 \div 9 = 7$; c) $10 \times 9 = 90$, $9 \times 10 = 90$, $90 \div 9 = 10$, $90 \div 10 = 9$; Bonus: $3 \times 40 = 120$, $40 \times 3 = 120$, $120 \div 40 = 3$, $120 \div 3 = 40$

Writing sentences for arrays with a missing number. Return to the earlier example on the board of $4 \times 10$. Point to each number, from left to right, in the sentence “$4 \times 10 = 40$” and ask what it stands for. (4 is the number of rows, 10 is the number of columns, 40 is the total number of objects) Repeat this process with each of the division sentences.

ASK: Which number sentence shows the number of rows as the answer? In other words, which sentence has the number of rows all by itself on one side of the equal sign? ($40 \div 10 = 4$) Which sentence gives the number of columns as the answer? ($40 \div 4 = 10$) Which sentences give the total as the answer? (the multiplication sentences, $4 \times 10 = 40$ and $10 \times 4 = 40$)

Add another row to the table on the board, as shown below:

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Number Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>40</td>
<td>$4 \times 10 = 40$ $40 \div 10 = 4$ $10 \times 4 = 40$ $40 \div 4 = 10$</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

SAY: In this new example, we know the number of rows and the total number of objects in the array, but we do not know the number of columns. We have drawn a question mark for the missing number. Ask students what other symbols they have used to show a missing number. (blank box, letter) Have volunteers write two multiplication sentences and two division sentences, using question marks for the missing number of columns. ($5 \times ? = 35$, $? \times 5 = 35$, $35 \div ? = 5$, $35 \div 5 = ?$) ASK: Which number sentence has the missing number of columns as the answer? ($35 \div 5 = ?$) Circle that sentence. Tell students that for the following exercises they only need to write one number sentence—a sentence that gives the missing number as the answer. In other words, the question mark should be all by itself on one side of the equal sign. Tell students that mathematicians usually write the missing number all by itself, on the left side of the sentence; for example, $? = 35 \div 5$. 
**Exercises:** A question mark (?) stands for the number we do not know. Write a number sentence that gives the missing on the left side of the sentence.

<table>
<thead>
<tr>
<th></th>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Number Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>?</td>
<td>9</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>6</td>
<td>?</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>8</td>
<td>9</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>Bonus:</td>
<td>25</td>
<td>?</td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) \(? = 54 ÷ 9\), b) \(? = 48 ÷ 6\), c) \(? = 8 \times 9\) or \(? = 9 \times 8\), Bonus: \(? = 75 ÷ 25\)

**Word problems with rows and columns.** Write on the board:

Mary arranges coins in 5 rows.
She puts 6 coins in each row.
How many coins does Mary use?

Have a volunteer read the question aloud. SAY: Let’s start by drawing an array of dots. Each dot will stand for a coin. Have a volunteer draw the array on the board. Near the array, make blanks for rows, columns, and total, as shown below:

```
  ●  ●  ●  ●  ●
  ●  ●  ●  ●  ●
  ●  ●  ●  ●  ●
  ●  ●  ●  ●  ●
  ●  ●  ●  ●  ●
  ___ rows    ___ columns    total ___
```

ASK: Do we know how many rows there are? (yes, 5) Write “5” in the blank for rows. ASK: Do we know how many columns there are? (yes, 6) Write “6” in the blank for columns. ASK: Do we know the total? (no) Write “?” in the blank for the total. ASK: How can we find the total number of coins Mary uses? Which number sentence should we use? (\(? = 5 \times 6\)) Have a volunteer write the sentence on the board. ASK: What is the total number of coins Mary uses? (30) PROMPT: What is 5 × 6? Write the answer on the board, as shown below:

\(? = 5 \times 6\)
\(? = 30\)
Mary uses 30 coins.

Repeat this process for part a) in the exercises below, but without drawing the array. Then have students complete the remaining exercises independently. Explain to students that they do not have to draw a picture for each question.
Exercises: Write a number sentence that gives the missing number as the answer. Solve the problem.

a) Sam plants 24 flowers in 3 columns. How many flowers are in each column?

b) Rani arranges 7 rows of chairs with 8 chairs in each row. How many chairs does she use?

c) Marko bakes 42 cookies on a tray. If there are 7 cookies in each row, how many rows fit on the tray?

Bonus

d) Ed has 8 rows of nickels with 9 in each row. Tessa has 7 rows of nickels with 10 in each row. Who has more nickels?

e) Zara arranges 35 oranges with 7 in each row. Jayden arranges 42 oranges with 6 in each row. Who makes more rows?

Answers: a) \( ? = 24 \div 3, \ ? = 8 \); b) \( ? = 7 \times 8, \ ? = 56 \); c) \( ? = 42 \div 7, \ ? = 6 \); Bonus: d) Ed: \( ? = 8 \times 9, \ ? = 72 \), Tessa: \( ? = 7 \times 10, \ ? = 70 \), Ed has more nickels; e) Zara: \( ? = 35 \div 7, \ ? = 5 \), Jayden: \( ? = 42 \div 6, \ ? = 7 \), Jayden makes more rows.

Extensions

1. A question mark (?) stands for the number we do not know. We have used the letters R, C, or T for numbers we do know. Write a sentence that gives the missing number by itself on the left of the equal sign.

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Sentence</th>
</tr>
</thead>
</table>
a)    ?     C     T                        
b)    R     ?     T                        
c)    R     C     ?                        

Bonus: R ? 999

Answers: a) \( ? = T \div C \), b) \( ? = T \div R \), c) \( ? = R \times C \), Bonus: \( ? = 999 \div R \)
2. Write a sentence that has the underlined number or letter on its own to the left of the equal sign.

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>7</td>
<td>10</td>
<td>70</td>
</tr>
<tr>
<td>b)</td>
<td>8</td>
<td>9</td>
<td>72</td>
</tr>
<tr>
<td>c)</td>
<td>3</td>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

Bonus:

<table>
<thead>
<tr>
<th>Rows</th>
<th>Columns</th>
<th>Total</th>
<th>Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>d)</td>
<td>R</td>
<td>C</td>
<td>T</td>
</tr>
<tr>
<td>e)</td>
<td>R</td>
<td>C</td>
<td>T</td>
</tr>
<tr>
<td>f)</td>
<td>R</td>
<td>C</td>
<td>T</td>
</tr>
</tbody>
</table>

Answers: a) $7 = \frac{70}{10}$, b) $9 = \frac{72}{8}$, c) $18 = 3 \times 6$,
Bonus: d) $R = \frac{T}{C}$, e) $C = \frac{T}{R}$, f) $T = R \times C$

3. Bill arranges 5 rows of coins with 8 in each row. Alexa arranges 8 rows of coins with 6 in each row. How many coins do they have altogether?

Solution: Bill: $5 \times 8 = 40$; Alexa: $8 \times 6 = 48$; $40 + 48 = 88$

4. Kate plants 40 flowers with 8 in each row. Jin plants 56 flowers with 7 in each row. Sara plants 20 flowers with 2 in each row. Who plants the most rows?

Solutions: Kate: $40 \div 8 = 5$ rows; Jin: $56 \div 7 = 8$ rows; Sara: $20 \div 2 = 10$ rows; Sara plants the most rows.

5. Amir has 12 coins. He wants to put them in an array. Find all the possible numbers of rows and columns he can use for his array.

Answer: 1 row and 12 columns, 2 rows and 6 columns, 3 rows and 4 columns, 4 rows and 3 columns, 6 rows and 2 columns, 12 rows and 1 column

6. A square array has the same number of rows as columns. Which numbers of coins can be placed into a square array—8, 9, 10, 15, 16, 17, 24, or 25?

Answers: 9, 16, and 25
Goals

Students will write multiplication and division sentences to find the number of groups, the number of items in each group, or the total number of items given the other two pieces of information.

PRIOR KNOWLEDGE REQUIRED

Knows how to obtain two division sentences from a multiplication sentence
Understands division as making equal groups

MATERIALS

BLM 10 × 10 Multiplication Chart (p. M-70)

Mental math minute. Cut out the white squares in BLM 10 × 10 Multiplication Chart. Distribute the squares one at a time to all the students. When you run out of students, start again and continue until all the squares have been handed out. Ask students who have a two-digit square (such as 24) to put up a hand. Choose one of these students, and have the student say her or his number aloud. Tell the class to think of a pair of numbers that multiply to make that number. Students in the class who have slips with one of the factors should put their hands up. Ask a pair of students to read their answers aloud. Note that sometimes there will be more than one pair of factors. For example, $6 \times 4 = 24$ and $8 \times 3 = 24$.

Repeat several times until all students have had a chance to participate.

Writing sentences for equal-group pictures. Draw on the board:

ASK: How many dots did I draw in each circle? (4) How many circles did I draw? (2) Write on the board:

___ in each group
___ in total

Tell students that the circles show the groups. ASk: How many groups did I draw? (2) Write “2” in the first blank. ASK: How many dots are in each group? (4) Write “4” in the second blank. ASK: How would you find the total number of dots? (sample answers: counting the dots, skip counting by 4s, or multiplying $2 \times 4$) Ensure that the last method, multiplication, comes out of the discussion. ASK: What is the total number of dots? (8) Write “8” in the third blank.
Tell students that we can write two multiplication sentences and two division sentences for the picture. **ASK:** What multiplication sentences can we write? \((2 \times 4 = 8\) and \(4 \times 2 = 8\)) What division sentences can we write? \((8 \div 4 = 2\) and \(8 \div 2 = 4\)) Write on the board:

\[
2 \times 4 = 8 \text{ leads to } 8 \div 4 = 2
\]

\[
4 \times 2 = 8 \text{ leads to } 8 \div 2 = 4
\]

If necessary, review the method for reading a multiplication sentence backwards and changing it to obtain a related division sentence, just as you did in Lesson NS3-60.

**Exercises:** Fill in the blanks. Then write two multiplication sentences and two division sentences for the picture.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:**

a) 3, 4, 12; \(3 \times 4 = 12\), \(4 \times 3 = 12\), \(12 \div 4 = 3\), \(12 \div 3 = 4\);

b) 3, 5, 15; \(3 \times 5 = 15\), \(5 \times 3 = 15\), \(15 \div 5 = 3\), \(15 \div 3 = 5\);

c) 4, 3, 12; \(4 \times 3 = 12\), \(3 \times 4 = 12\), \(12 \div 3 = 4\), \(12 \div 4 = 3\)

**Writing sentences for equal groups without pictures.** Write on the board:

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Number in Each Group</th>
<th>Total</th>
<th>Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

**SAY:** Now, we are given the number of groups, the number in each group, and the total number of objects. It doesn’t matter what the objects are. We do not have a picture, but we can still write the sentences. **ASK:** What multiplication sentences can we write? \((6 \times 7 = 42\) and \(7 \times 6 = 42\)) What division sentences can we write? \((42 \div 7 = 6\) and \(42 \div 6 = 7\)) Have a volunteer write the division sentences in the table as you go. The final table should look like this:

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Number in Each Group</th>
<th>Total</th>
<th>Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7</td>
<td>42</td>
<td>(6 \times 7 = 42) (42 \div 7 = 6) (7 \times 6 = 42) (42 \div 6 = 7)</td>
</tr>
</tbody>
</table>
Exercises: Write two multiplication sentences and two division sentences for each row in the table.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Number in Each Group</th>
<th>Total</th>
<th>Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3</td>
<td>6</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>b) 8</td>
<td>2</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>c) 7</td>
<td>9</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>Bonus:</td>
<td>30</td>
<td>10</td>
<td>300</td>
</tr>
</tbody>
</table>

Answers: a) \(3 \times 6 = 18\), \(6 \div 3 = 18\), \(18 \div 3 = 6\); b) \(8 \times 2 = 16\), \(2 \times 8 = 16\), \(16 \div 2 = 8\), \(16 \div 8 = 2\); c) \(7 \times 9 = 63\), \(9 \times 7 = 63\), \(63 \div 9 = 7\), \(63 \div 7 = 9\); Bonus: \(30 \times 10 = 300\), \(10 \times 30 = 300\), \(300 \div 10 = 30\), \(300 \div 30 = 10\)

Writing sentences for equal groups with an unknown. Write on the board:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number of Groups</th>
<th>Number in Each Group</th>
<th>Total</th>
<th>Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 bananas in each bag</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 bananas How many bags?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: I have a problem involving groups and numbers in each group, and I will use the information in the problem to fill in the chart. Read the problem together aloud. ASK: What are the objects or items in this problem? (bananas) What are the groups? (bags) Read the first line in the problem and ASK: Does this tell us the number of groups? (no) Does this tell us the number in each group? (yes) How many are in each group? (4) SAY: In this problem, the groups are bags and there are four bananas in each bag. Write “4” under “Number in Each Group.” Read the second line of the problem, and ASK: What does this tell us? (the total is 20) Write “20” under “Total.” Finally, point to the last line of the problem (“How many bags?”) and SAY: The problem is asking for the number of bags or groups. SAY: Since we don’t know the number of groups, we put a question mark under “Number of Groups.” Write the question mark.

ASK: What sentence gives the number of groups? \(? = 20 \div 4\) or \(20 \div 4 = ?\) PROMPT: Which sentence will have the “?” all by itself on one side of the equal sign?

Complete part a) from the following exercises as a class before having students complete the rest individually.
**Exercises:** Fill in the table. Write a question mark (?) for the amount you do not know. Write a number sentence that solves the problem.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number of Groups</th>
<th>Number in Each Group</th>
<th>Total</th>
<th>Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3 bowls, 27 berries</td>
<td>3</td>
<td>?</td>
<td>27</td>
<td>? = 27 ÷ 3</td>
</tr>
<tr>
<td>b) 8 pencils in each case, 5 cases</td>
<td>5</td>
<td>8</td>
<td>?</td>
<td>? = 5 × 8</td>
</tr>
<tr>
<td>c) 32 students, 4 students on each team</td>
<td>?</td>
<td>4</td>
<td>32</td>
<td>? = 32 ÷ 4</td>
</tr>
</tbody>
</table>

**Answers**

a) 3 ? 27 ? = 27 ÷ 3
b) 5 8 ? ? = 5 × 8
c) ? 4 32 ? = 32 ÷ 4

**Equal groups and one-step word problems.** Write on the board:

Each dog house fits 2 dogs.
How many dogs fit in 7 dog houses?

ASK: What are the objects in this problem? (dogs) What are the groups? (dog houses) Write on the board:

<table>
<thead>
<tr>
<th>groups</th>
<th>in each group</th>
<th>in total</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>?</td>
</tr>
</tbody>
</table>

ASK: How many groups do we have? (7) Write “7” in the first blank. ASK: How many objects in each group? (2) Write “2” in the second blank. ASK: Do we know how many dogs fit in a total of seven dog houses? (no) Point to the third blank and ASK: What should we write in this blank? (a question mark) Write “?” and ASK: What number sentence will give us the unknown total? (? = 2 × 7) Write the sentence on the board. ASK: What is 2 × 7? (14) So, how many dogs fit in seven dog houses? (14) Write the answer on the board, as shown below:

? = 2 × 7
? = 14
14 dogs fit in 7 dog houses.

Tell students to use the same process to solve the problems in the following exercises; i.e., write the number of groups, the number in each group, and the total, using a question mark (?) for the unknown; and then write a sentence to find the unknown number. Students can also draw a picture with circles and dots if they wish.
**Exercises:** Use a number sentence to solve the problem.

a) Nora bought 18 erasers. There are 3 erasers in each pack. How many packs did she buy?

b) There are 24 cookies. Jin puts the same number of cookies on each of 6 plates. How many cookies are on each plate?

c) Luc’s bookshelf has 5 shelves. There are 9 books on each shelf. How many books are there in total?

**Answers:** a) \( ? = 18 \div 3, ? = 6; \) b) \( ? = 24 \div 6, ? = 4; \) c) \( ? = 5 \times 9, ? = 45 \)

**Equal groups and two-step word problems.** Write on the board:

Rob planted 42 flowers with 6 in each row.
Sara planted 56 flowers with 7 in each row.

How many more rows did Sara plant than Rob?

After reading the problem aloud, ASK: What does the question ask for? (how many more rows Sara plants than Rob) What do you need to find first? (the number of rows Rob plants and the number of rows Sara plants)

SAY: Let’s start with Rob. Write on the board:

Rob: ___ groups ___ in each group ___ in total

Ask students to identify the objects and the groups. (flowers, rows) ASK: Do we know how many rows Rob planted? (no) Write “?” in the blank for groups. ASK: What can we fill in for Rob? (42 flowers in total and 6 in each group) Fill in these blanks. ASK: Which number sentence can we write that will give the unknown as the answer? (\( ? = 42 \div 6 \)) Write the sentence on the board.

SAY: Because there will be another sentence for the number of rows that Sara plants, there will be another unknown. Let’s use the letter \( R \) for the unknown number of rows Rob plants. Replace the question mark on the board with the letter \( R \). Then, have students find the value of \( R \). (\( R = 42 \div 6, R = 7 \))

Repeat the process for Sara, but use the letter \( S \) for the number of rows that she plants. The results for Rob and Sara will look like this:

Rob: \( R \) groups 6 in each group 42 in total \( R = 42 \div 6, R = 7 \)
Sara: \( S \) groups 7 in each group 56 in total \( S = 56 \div 7, S = 8 \)

SAY: Let’s look back at the last line of the problem. ASK: What do we need to find? (how many more rows Sara plants than Rob) How do we find that? (\( 8 - 7 = 1 \)) Write the final answer on the board:

\( 8 - 7 = 1 \)
Sara plants 1 more row than Rob.
Exercises: Use number sentences to solve each problem.

1. A volleyball team has 6 players. School A sent 24 players to a volleyball tournament. School B sent 42 players.
   a) How many teams did School A send?
   b) How many teams did School B send?
   c) How many more teams did School B send than School A?

   **Answers:** a) $24 ÷ 6 = 4$ teams; b) $42 ÷ 6 = 7$ teams,
   c) $7 - 4 = 3$ teams

2. Dory bought 10 packs of markers with 8 markers in each pack. Ivan bought 9 packs of markers with 7 markers in each pack. How many more markers did Dory buy than Ivan?

   **Solution:**
   Dory bought $10 \times 8 = 80$ markers and Ivan bought $9 \times 7 = 63$ markers, so Dory bought $80 - 63 = 17$ more markers than Ivan.

   **Bonus:** It costs $10 to rent a canoe on a pond. A canoe can hold 3 children. There are 27 children who want to go canoeing.
   a) How many canoes should be rented for the children?
   b) How much will it cost to rent all the canoes?

   **Answers:** a) $27 ÷ 3 = 9$ canoes, b) $9 \times 10 = 90$ dollars

Extensions

1. Glen has 8 stacks of paper with 9 sheets in each stack. Abella has 7 stacks of paper with 10 sheets in each stack. Tina has 10 stacks of paper with 10 sheets in each stack. How many sheets of paper do they have altogether?

   **Solution:**
   Glen has $8 \times 9 = 72$ sheets of paper. Abella has $7 \times 10 = 70$ sheets of paper. Tina has $10 \times 10 = 100$ sheets of paper. Altogether they have $72 + 70 + 100 = 242$ sheets of paper.

2. Write number sentences to solve the problems. Use the letters $A$, $B$, and $C$ for the unknowns.

   a) A basketball team has 5 players. School A sends 35 players to a basketball tournament. School B sends 40 players. School C sends 25 players. How many teams do the three schools send altogether?

   b) A baseball team has 9 players. School A sends 7 teams to a baseball tournament. School B sends 9 teams. School C sends 10 teams. How many players do the three schools send altogether?
Solutions
a) $A = 35 \div 5 = 7$, $B = 40 \div 5 = 8$, $C = 25 \div 5 = 5$,
$A + B + C = 7 + 8 + 5 = 20$ teams sent altogether
b) $A = 9 \times 7 = 63$, $B = 9 \times 9 = 81$, $C = 9 \times 10 = 90$,
$A + B + C = 63 + 81 + 90 = 234$ players sent altogether

3. Each student has 2 textbooks in their desk. There are 4 rows of student
desks with 5 desks in each row. How many textbooks are in the desks?

Answer: $2 \times 4 \times 5 = 8 \times 5 = 40$
Number Lines to 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
### 9 × 9 Multiplication Chart

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>56</td>
<td>63</td>
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<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
<td>56</td>
<td>64</td>
<td>72</td>
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<tr>
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<td>1. Divide.</td>
<td>2. Divide.</td>
<td></td>
</tr>
<tr>
<td>a) 63 ÷ 9 = ____</td>
<td>a) 9 ÷ 1 = ____</td>
<td>b) 20 ÷ 5 = ____</td>
</tr>
</tbody>
</table>
Fluency Practice—Multiplication and Division

I. Multiply or divide.

a) \(21 \div 7 = \) ____
b) \(9 \times 7 = \) ____
c) \(3 \div 3 = \) ____
d) \(4 \times 7 = \) ____
e) \(18 \div 9 = \) ____
f) \(7 \times 9 = \) ____
g) \(45 \div 9 = \) ____
h) \(2 \times 2 = \) ____
i) \(36 \div 9 = \) ____
j) \(9 \times 1 = \) ____
k) \(48 \div 6 = \) ____
l) \(1 \times 7 = \) ____
m) \(24 \div 6 = \) ____
n) \(6 \times 8 = \) ____
o) \(28 \div 4 = \) ____
p) \(1 \times 6 = \) ____
q) \(5 \div 1 = \) ____
r) \(8 \times 4 = \) ____
s) \(10 \div 2 = \) ____
t) \(9 \times 4 = \) ____

2. Multiply or divide.

a) \(40 \div 8 = \) ____
b) \(8 \times 1 = \) ____
c) \(12 \div 4 = \) ____
d) \(7 \times 6 = \) ____
e) \(9 \div 1 = \) ____
f) \(5 \times 2 = \) ____
g) \(8 \div 2 = \) ____
h) \(3 \times 8 = \) ____
i) \(8 \div 8 = \) ____
j) \(4 \times 6 = \) ____
k) \(16 \div 8 = \) ____
l) \(5 \times 5 = \) ____
m) \(12 \div 2 = \) ____
n) \(6 \times 4 = \) ____
o) \(12 \div 3 = \) ____
p) \(7 \times 4 = \) ____
q) \(5 \div 5 = \) ____
r) \(2 \times 6 = \) ____
s) \(2 \div 1 = \) ____
t) \(3 \times 9 = \) ____
### $10 \times 10$ Multiplication Chart

<table>
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<tr>
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<td>50</td>
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<td>70</td>
<td>80</td>
<td>90</td>
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</tbody>
</table>
Unit 11 Patterns and Algebra: Patterns and Equations

Introduction

In this unit, students will make connections between different representations of patterns—numeric and geometric patterns, patterns on number lines, and patterns in hundreds charts and calendars. They will use patterns to skip count by 5s forwards and backwards, starting at any number within 1000 (not just multiples of 5).

Students will learn that a number sentence with an equal sign is called an equation, and equations can include some unknown numbers. Students will learn to represent these unknown numbers with blanks, boxes, symbols, or letters. Students will also explore various methods to solve an equation for the unknown number. Methods will include:

- performing calculations
- drawing a picture
- guessing and checking

Meeting Your Curriculum

Alberta—All lessons in this unit are required.
British Columbia—All lessons in this unit are required.
Manitoba—All lessons in this unit are required.
Ontario—All lessons in this unit are required.

NOTE: Some exercises and mental math minutes in this unit use multiplication within 9 × 9. If you have limited multiplication facts to 5 × 5 or 7 × 7, you will need to adjust the numbers in the examples given.

Materials. In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section V, are used in Unit 11:

BLM Empty Spinners (p. V-1)
BLM Multiplication Chain (pp. V-2–7)

Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tbody>
<tr>
<td>Quiz</td>
<td>PA3-13 to 14</td>
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<tr>
<td>Quiz</td>
<td>PA3-15 to 19</td>
<td>PA3-15 to 19</td>
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<td>PA3-13 to 19</td>
<td>PA3-13 to 19</td>
<td>PA3-13 to 19</td>
<td>PA3-13 to 19</td>
</tr>
</tbody>
</table>
PA3-13 Geometric Patterns
Pages 31–33

Goals
Students will describe geometric patterns and represent them using number patterns.
Students will represent number patterns using geometric patterns.
Students will determine the pattern rule for both number and geometric patterns.

PRIOR KNOWLEDGE REQUIRED
Can extend growing and shrinking patterns made by adding or subtracting a constant gap
Can determine the rule for a growing or shrinking pattern
Can determine the perimeter of a shape
Can draw a T-table and extend a pattern in a T-table

MATERIALS
blocks of different shapes, including many cubes or cylinders (optional)
3 shapes of pattern blocks per student (at least 4 blocks of each shape)
BLM Patterns with Increasing Gaps (p. N-50, see Extension 3)

Mental math minute. Have students stand in a line. Give the first student an addition problem that does not need regrouping, such as 21 + 13. Students in line repeatedly add a number, in this case 13, with each student saying one addition aloud. When a student says an addition that involves regrouping, emphasize that this addition is a bonus. Example: Student 1 says, “21 + 13 = 34”; Student 2 says, “34 + 13 = 47”; Student 3 says, “47 + 13 = 60” (note that this is a bonus). Continue for a few questions before starting a new chain.

Introduce geometric patterns. Draw the following sequence of figures on the board and tell students that the pictures show several stages in the construction of a castle made of blocks. Alternatively, if blocks are available, build a similar pattern from actual blocks. Use cubes or cylinders for towers.

Ask students to imagine that they want to keep track of the number of blocks used in each stage of the construction of the castle. SAY: We can use a T-table to keep track of how many blocks are needed for each stage of construction. Draw the following table on the board and ask students to help you fill in the number of blocks used in each figure.
ASK: What patterns do you see in the columns of the table? (the figure number grows by 1, and the number of blocks grows by 2 each time) To prompt students to see the second pattern, draw circles on the right side of the table and remind students that they can write the gap between the numbers in the second column in the circles, as shown below:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Add a few more rows to the table and have volunteers fill them in. If you are working with blocks for demonstration, invite other volunteers to build the next figures in the pattern to check the numbers predicted in the table. Keep the table on the board for later use.

**NOTE:** Have students complete Exercise 1 below, and alert them that in Exercise 2 the patterns are shrinking, or decreasing. This means that students will need to subtract the number each time. Remind students that in such cases they can write the difference in the circles with a minus sign in front.

**Exercises**

1. Make a T-table for the number of blocks in the pattern. Extend the table to show how many blocks will be in Figure 6.

   a) ![Figure 1](image1.png) ![Figure 2](image2.png) ![Figure 3](image3.png)

   b) ![Figure 1](image1.png) ![Figure 2](image2.png) ![Figure 3](image3.png)

   **Bonus:** Draw the figures in the pattern to check your answers in the table.
Selected answers

2. Make a T-table for the number of blocks in the pattern. Extend the table to show how many blocks will be in Figure 6.

a) Figure 1 Figure 2 Figure 3

b) Figure 1 Figure 2 Figure 3

Bonus: Draw the figures in the pattern to check your answers in the table.

Answers

a) Figure Number Number of Blocks
1 14
2 12
3 10
4 8
5 6
6 4

b) Figure Number Number of Blocks
1 20
2 18
3 16
4 14
5 12
6 10
Review writing rules for number patterns. Remind students that to write a rule for a number pattern, you need to say what number to start with and what number to add or subtract. Draw students' attention to the table on the board for the number of blocks in the castle. ASK: What number do you start with? (4) Do you add or subtract to get the next number of blocks? (add) How many blocks do you add each time? (2) How do you see that from the pictures? (there are two towers in the castle, each time we add one block to each tower) What is the rule for the number pattern? (start at 4 and add 2 each time) Write the rule on the board.

Exercises: Write the rule for the number patterns in Exercises 1 and 2 above.

Answers
1. a) start at 8 and add 3 each time, b) start at 4 and add 4 each time;
2. a) start at 14 and subtract 2 each time, b) start at 20 and subtract 2 each time

Describing the geometric pattern. SAY: I want to tell my friend in Quebec City about the castle we built. I would like to describe the castle for her so that she can build a castle just like it. Remind students that the blocks that make the towers are called cylinders. Ask students to try to describe the first castle in the pattern. (each castle is made from a gate, a triangular roof, and two towers—one on each side; the first castle in the pattern has towers that are 1 block tall) ASK: How do you make each next castle? (add 1 block on top of each castle tower)

Draw on the board:

Ask students to try to describe the pattern. PROMPT: Are the squares arranged in a row, a column, or an array of several columns or rows? (a column) How many cubes are in the column? (5) How many cubes do you add each time? (3) Do you add the cubes to the left side or to the right side of the column? (to the left side) Do you add the cubes at the top, at the bottom, or at the middle? How many cubes do you add at each place? (1 at the top, 1 at the bottom, 1 in the middle) Summarize: Start with a
column of 5 cubes. Each time, add 3 cubes to the left side, 1 at the top, 1 at the bottom, and 1 at the middle cube. Point out to students that sometimes patterns resemble familiar shapes and it makes sense to mention the resemblance when describing a pattern. For example, starting from Figure 2, the figures resemble a backwards capital E.

**Exercise:** Describe the pattern.

![Figures 1, 2, 3]

**Sample answer:** Start with 14 blocks that form the letter U. The figure is 4 blocks wide and 6 blocks tall. Remove 2 blocks each time, one block from the top of each column of the letter U.

Draw on the board:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
</tr>
</tbody>
</table>

SAY: This T-table shows how many blocks I used at each stage of building another castle. The castle has several towers, and I added one block to the top of each tower at each stage. There is a gate with a roof between each pair of towers. **ASK:** How many towers does the castle have? (5) **How do you know?** (the gap is 5, so if 1 block is added to each tower each time, there must be 5 towers) If you are using blocks, have a volunteer build the first figure, and check that the correct number of blocks is used. The first castle will have towers 1 block tall, with 4 gates and 4 triangular roofs. You can also ask students to sketch a picture of the castle. Then ask students to help you extend the T-table to five terms by adding the gap to successive terms. (28, 33)

**Review writing rules for number patterns that are not in a table.** Write on the board:

25, 23, 21, 19, __

**ASK:** Is this a growing or a shrinking number pattern? (shrinking) **How do you know?** (the numbers get smaller each time) **Should you add or subtract to continue the pattern?** (subtract) **What number do you subtract each time?** (2) **Have volunteers check different pairs of numbers and write “−2” in the corresponding circle.** **ASK:** What is the rule for the pattern? (start at 25 and subtract 2 each time)
Exercises: Which number is added or subtracted each time? Write the rule for the pattern.

a) 78, 74, 70, 66  
b) 32, 37, 42, 47  
c) 107, 102, 97, 92, 87

Answers: a) 4 is subtracted, start at 78 and subtract 4 each time; b) 5 is added, start at 32 and add 5 each time; c) 5 is subtracted, start at 107 and subtract 5 each time

Creating geometric patterns for a number pattern.

ACTIVITIES 1–2

Provide each student with a large number of different pattern blocks.

1. Create a growing pattern from pattern blocks. Build the first three or four figures in the pattern. Write the number pattern showing the number of blocks in the pattern and predict the number of blocks in the next figure. Create the next figure in the pattern to check your answer. Describe the pattern you made.

2. Write a rule for the number pattern. Make a pattern of blocks that matches the number pattern.

a) 7, 11, 15  
b) 17, 14, 11, 8  
c) 1, 5, 9

Selected sample answer

a) Start at 7 and add 4 each time.

Answers: b) start at 17 and subtract 3 each time, c) start at 1 and add 4 each time

Review perimeter. Draw on the board:

Remind students that the distance around a shape is called perimeter.

ASK: If each side of a triangle is 1 unit long, how long is the perimeter of this figure? (5 units) Invite a volunteer to show how to find the perimeter. (count the outer edges or add the side lengths)

Producing different number patterns from geometric patterns. Draw on the board:

SAY: I want to find the perimeter of each of the three figures. Imagine that each triangle is made from 3 toothpicks, and all the toothpicks are the same length. Have students sketch the figures in their notebooks and then
find the perimeter of each figure in toothpicks. Have a volunteer write the perimeter underneath the figure on the board. (5, 6, 7 toothpicks)

ASK: Do the perimeters make a number pattern? (yes) Explain that you can make number patterns from geometric patterns not only by counting the number of shapes, but also by counting or measuring other things, such as perimeter. For example, if you make this pattern by placing toothpicks to form triangles, you can also count the total number of toothpicks, or the number of triangles. Have students write the pattern for the number of toothpicks (7, 9, 11) and the number of triangles for each figure (3, 4, 5).

SAY: Imagine I have 20 toothpicks. I want to find the largest figure that I can make in this pattern. ASK: Should I draw larger and larger figures and count all the toothpicks, or is there a shorter, easier way to find it out? PROMPT: Can one of the number patterns we wrote help me find what the figure number will be? (yes) Which pattern? (the number of toothpicks)

Start a T-table on the board, as shown below:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Toothpicks</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
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<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
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</tbody>
</table>

Have students copy the table and extend it. Do the same on the board and have students help you fill it in, until you reach the 8th row—Figure 8 and 21 toothpicks. ASK: If I have 20 toothpicks, can I make Figure 8? (no) Why not? (you need 21 toothpicks, and you only have 20 toothpicks) Can I make Figure 7? (yes) Will I have any toothpicks leftover? (yes) How many triangles will I have in this figure? (9) How do you know? (answers may vary; students might notice that the number of triangles is always 2 more than the figure number, or they might extend the pattern for the number of triangles to 7 terms)

Exercises: Anika designs a pattern of long rectangles with toothpicks.

a) Make a T-table for the number of toothpicks in each figure.

b) Anika has 27 toothpicks. How many toothpicks long is the longest rectangle she can make in her pattern?

Bonus: Make a T-table for the perimeter of Anika’s rectangles. What is the perimeter of the longest rectangle she can make with 27 toothpicks?
Answers

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Toothpicks</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>6</td>
<td>25</td>
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<td>7</td>
<td>29</td>
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</table>

The longest rectangle Anika can make is Figure 6, which uses 25 toothpicks. Figure 1 is 1 toothpick long, Figure 2 is 2 toothpicks long, and so on, so Figure 6 is 6 toothpicks long.

**Bonus:**

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Perimeter</th>
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<tbody>
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<td>1</td>
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<td>5</td>
<td>12</td>
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<tr>
<td>6</td>
<td>14</td>
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</table>

The perimeter of the longest rectangle she can make is 14 toothpicks long.
Extensions

1. Matt makes a castle by adding 1 block at a time to each of 4 towers. He has a gate with a triangular roof between each pair of towers. Matt uses 22 blocks altogether.
   a) How many blocks are not used in the towers?
   b) How many blocks are used in the towers?
   c) How tall is each tower?

   Answers: a) 6 blocks, b) 16 blocks, c) 4 blocks

2. Cathy uses one kind of block to build a pattern. She adds the same number of blocks to make each new figure. She writes the number of blocks in the figure in a T-table. Cathy makes one mistake in the table. Find and correct her mistake.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
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<td>2</td>
<td>7</td>
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<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

   Answer: Figure 2 should have 8 blocks.

3. Have students complete BLM Patterns with Increasing Gaps.

   Answers
   1. a) gaps: +2, +3, +4, +5, +6, next terms: 17, 23; b) gaps: +1, +2, +3, +4, +5, +6, next terms: 19, 25; c) gaps: +2, +4, +6, +8, +10, +12, next terms: 36, 48; d) gaps: +3, +5, +7, +9, +11, next terms: 34, 45

   2. a–c)
3. | Figure Number | Number of Squares |
<table>
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<th></th>
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</thead>
<tbody>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4 (+3)</td>
</tr>
<tr>
<td>3</td>
<td>9 (+6)</td>
</tr>
<tr>
<td>4</td>
<td>16 (+7)</td>
</tr>
<tr>
<td>5</td>
<td>25 (+9)</td>
</tr>
</tbody>
</table>

4. Armand makes a pattern starting at 2. He multiplies each term by the same number to get the next number in the pattern. His pattern is 2, 4, 8, 16.

a) What number does Armand multiply each term by?

b) Write 3 more numbers in Armand’s pattern.

c) Find the gaps between the numbers in Armand’s pattern. What do you notice about the pattern in the gaps?

Answers: a) 2; b) 32, 64, 128; c) The pattern in the gaps is 2, 4, 8, 16, 32, 64. It is the same as the pattern itself.
**Goals**

Students will represent number patterns, including numeric representations of geometric patterns, on number lines.

Students will represent patterns given on number lines as number patterns and describe the pattern rule for number patterns.

**PRIOR KNOWLEDGE REQUIRED**

- Can add and subtract two numbers within 1000
- Can extend a number pattern by adding or subtracting the same number
- Can extend a geometric pattern
- Can write a number pattern based on a geometric pattern
- Can write a rule for a number pattern
- Can represent an addition or subtraction sentence on a number line
- Can create a geometric pattern based on a number pattern

**MATERIALS**

- ball
- transparency of BLM Number Lines to 100 (p. N-51)
- overhead projector
- BLM Number Lines (p. N-52), several copies per student
- paper clip, pencil, and BLM Empty Spinners (p. V-1) per pair of students
- BLM Number Lines with Large Numbers (p. N-53)
- pencil crayons
- large number of beads, cubes, or pattern blocks

**Mental math minute.** Give students problems that require subtracting one-digit numbers from one- and two-digit numbers, such as 35 − 8. Students can use any method, such as counting up using 1s and multiples of 10, or using number facts within 20. Toss a ball to the student you want to answer the question and have the student toss the ball back to you after answering.

**Review adding and subtracting on a number line.** Draw a number line starting at 20 and ending at 30 on the board. Draw an arrow from 20 to 23 and ASK: What addition sentence does this picture show? (20 + 3 = 23)

How does the arrow show this? (the start of the arrow is the number we start with, or the first addend; the length of the arrow itself shows the number we are adding, or the second addend; the end of the arrow shows the answer, or the sum)

Reverse the direction of the arrow so that it points from 23 to 20, and ask students to describe the subtraction sentence the model shows. (23 − 3 = 20; the start of the arrow is the number we subtract from; the length of
the arrow is the number subtracted; the end of the arrow is the result, or the difference

**Writing a pattern shown on a number line.** Draw on the board:

![Number Line Diagram]

SAY: This picture shows adding 3 repeatedly. ASK: What number do we start with? (20) Point to the first arrow and SAY: First we add 3 and get 23. Then we add another 3 (point to the second arrow) and get 26. Then we add another 3 (point to the third arrow) and get 29. Write on the board underneath the number line:

\[20, 23, 26, 29\]

SAY: The picture on the number line shows a pattern. ASK: Is this a growing pattern or a shrinking pattern? (growing) How do you know? (we add 3, the numbers get larger) How do you see that from the picture on the number line? (the arrows point to the right, towards larger numbers) SAY: We usually make growing patterns by adding the same number over and over. ASK: How do we see from the picture that we add the same number over and over? (the arrows are the same length and point in the same direction)

Repeat the discussion with the picture below, showing the shrinking pattern 29, 27, 25, 23, 21.

![Number Line Diagram]

Point out that when you add or subtract the same number over and over, you are actually skip counting forwards or backwards.

**Exercises:** Write the number pattern the picture shows.

a)

![Number Line Diagram]

b)

![Number Line Diagram]

c)

![Number Line Diagram]
Representing number patterns on a number line. Project BLM Number Lines to 100 on the board. Explain that all the numbers on the number lines are marked, but only the multiples of 10 are labelled. Point to several different marks that are not numbered and have students say what number the mark shows. Have volunteers explain how they know. One possible strategy is to look at the multiple of 10 before the number and count up by 1s marks. Students can also look at the next multiple of 10 and count back. Discuss when to use each strategy. The second strategy is best when the mark is closer to the larger multiple of 10. However, if the mark is about the same distance from either side, it is better to count up, because people tend to make more mistakes when counting back than when counting on.

Write the pattern “0, 25, 50, 75, 100” on the board and invite volunteers to place dots on those numbers on the number line. Ask another volunteer to draw the arrows to show the pattern. Repeat with 91, 86, 81, 76, 71. Then draw arrows showing the pattern 22, 32, 42, 52 and have students write the pattern the number line shows. Keep the patterns displayed for the next explanation.

Review rules for patterns. Remind students that to describe a pattern, we need to say what number we start with and what number we add or subtract each time. For example, the pattern 0, 25, 50, 75, 100 has the rule “Start at 0 and add 25 each time.” Write the description underneath the pattern on the board and have volunteers describe the other two patterns on the board. (start at 91 and subtract 5 each time, start at 22 and add 10 each time)

Writing rules for patterns represented on a number line. Draw the pattern below on the board again:

ASK: Can you find the rule for a pattern if the pattern is shown on a number line? (yes) What is the starting number? (29) How does the pattern on a number line show the starting number? (this is the start of the first arrow) Write “start at 29” underneath the picture. ASK: How do you know if you need to add or to subtract? (the arrows point to the left, so you need to subtract) Write “subtract ___ each time” underneath the picture.
ASK: How do you know which number to subtract? (the arrow is 2 units long, so we need to subtract 2) Write “2” in the blank.

Exercises: Write a rule for the patterns in the previous exercises.

Answers: a) start at 55 and subtract 2 each time; b) start at 20 and add 10 each time; c) start at 10 and add 15 each time; d) start at 92 and subtract 4 each time; e) start at 93 and subtract 3 each time

ACTIVITY 1

1. Give each pair of students several copies of BLM Number Lines, one copy of BLM Empty Spinners, a paper clip to act as a pointer for the spinner, and a pencil to anchor the paper clip. Have students use the spinner with 8 regions and label the regions “+ 2, + 3, + 4, + 5, − 2, − 3, − 4, − 5.” Player 1 spins the spinner so that Player 2 does not see the result. Player 1 uses the result of the spin as the pattern, chooses the first number of the pattern, and draws the pattern on a number line. Player 2 writes the rule for the pattern that Player 1 drew. Players switch roles and repeat. Students will use the same materials in Activity 2.

Representing patterns on a number line. Draw on the board:

Start at 24 and subtract 4 each time.

Explain that you want to show this pattern on the number line. Have students tell you how to do so. PROMPT: What number should the arrows start at? (24) How long should the arrows be? (4 units) How do you know? (you need to subtract 4 each time, the gap in the pattern is 4) Should the arrows point right or left? (left) How do you know? (you need to subtract each time) How many arrows do you need to draw to show the pattern? (3) Draw the arrows, as shown below:

ACTIVITY 2

2. Use the same materials as in Activity 1. Player 1 spins the spinner to determine what number to add or subtract each time and picks the number the pattern starts at. Player 1 then writes a rule for the pattern. Player 2 draws the pattern on the number line. Both players write the numerical pattern and compare answers. Players switch roles after each turn.
Distribute **BLM Number Lines with Large Numbers** and have students do the exercises below. Students might use different-coloured pencil crayons for patterns that appear on the same number lines.

**Exercises:** Show the first 5 numbers of each pattern on the number line.

a) Use Line 1. Start at 219 and subtract 2 each time.

b) Use Line 1. Start at 202 and add 2 each time.

c) Use Line 2. Start at 481 and add 5 each time.

d) Use Line 2. Start at 498 and subtract 5 each time.

e) Use Line 3. Start at 227 and add 5 each time.

f) Use Line 4. Start at 718 and add 5 each time.

g) Use Line 5. Start at 889 and subtract 5 each time.

**Bonus**

h) Start at 818 and add 10 each time.

i) Start at 996 and subtract 10 each time.

**Answers:** a) 219, 217, 215, 213, 211; b) 202, 204, 206, 208, 210; c) 481, 486, 491, 496, 501; d) 498, 493, 488, 483, 478; e) 227, 232, 237, 242, 247; f) 718, 723, 728, 733, 738; g) 889, 884, 879, 874, 869, 864; Bonus: h) 818, 828, 838, 848, 858; i) 996, 986, 976, 966, 956

**Representing a geometric pattern on a number line.** Draw on the board:

![Figure 1](image1.png) ![Figure 2](image2.png) ![Figure 3](image3.png)

Remind students that there are many ways to make a number pattern from a geometric pattern. For example, you can write how many shapes are in each figure of the pattern or how many of a specific type of shape, such as squares, are in each figure of the pattern. Other ways include finding the lengths, heights, or perimeters of each figure. Explain that as soon as you produce a number pattern, you can show it on a number line.

Point to the drawing on the board and SAY: Let’s make a pattern for the number of blocks in this pattern. Have students count the blocks and have a volunteer write the number pattern underneath the geometric pattern, clearly labelling it as the pattern in the number of blocks. (number of blocks in each figure: 16, 14, 12) Draw a number line from 0 to 20, have students copy it in their notebooks, and have them draw the pattern on the number line using arrows.
ASK: Can you use the number line to check how many blocks will be in the 6th figure? (yes, extend the pattern on the number line) SAY: The number of blocks in the first figure is the beginning of the first arrow, and the number of blocks in the second figure is the end of the first arrow. The number of blocks in the third figure is the end of the second arrow. ASK: How many arrows do you need to draw to find the number of blocks in the 6th figure? (5 arrows) Have students draw the arrows and find how many blocks will be in the 6th figure. (6 blocks) Invite volunteers to draw the next figures in the pattern and check the answer.

**ACTIVITIES 3–4**

Provide students with a copy of BLM Number Lines and a large number of pattern-making materials, such as beads, cubes, or pattern blocks.

1. Player 1 makes a growing or a shrinking pattern with the pattern-making materials. Player 2 shows the pattern on a number line. Players switch roles.

2. Player 1 draws a growing or a shrinking pattern on a number line. Player 2 makes the pattern that has the same number of shapes in each figure as the pattern on the number line.

**Extensions**

1. Teach students to use number lines to solve word problems. Write on the board:

   A caterpillar on a branch is 24 cm away from the tree trunk. The caterpillar crawls towards the trunk, 4 cm every hour. How far from the trunk is the caterpillar after 1 hour? 2 hours? 4 hours?

   SAY: We could make a T-table to solve this problem or we could make a number pattern for how far the caterpillar is from the tree trunk. We could also show this pattern on a number line and use this number line to solve the problem. Draw the number line below on the board, but do not draw the arrows.

   SAY: This number line shows the distance from the tree trunk. At the start, the caterpillar is 24 cm away from the trunk. Let’s start the arrows at 24. ASK: How long should each arrow be? (4 cm) How do you know? (the caterpillar travels 4 cm every hour) Should we draw arrows pointing to the left or right? (left) How do you know? (the caterpillar is crawling towards the trunk, so the distance to the trunk gets smaller) Have a volunteer draw the arrows, and have other volunteers decide how far from the trunk the caterpillar is after 1 hour (20 cm), 2 hours (16 cm), 4 hours (8 cm).
Have students use the same method to solve the problems below.

a) Jin can walk 5 km in 1 hour. He is 20 km away from home. Jin starts walking home. How far from home is Jin after 3 hours?

b) Anna is 15 km away from her campsite. She starts hiking towards her campsite. She can hike 4 km each hour. How far from her campsite is Anna after 3 hours?

Answers: a) 5 km, b) 3 km

2. Carl plants 4 apple trees in a row. The first tree is 5 m away from his house. Each tree after that is 2 m farther from the house than the tree before it. Draw a number line for the distance from the house. Put the house at 0. How far from the house is the 4th tree?

Answer
The 4th tree is 11 m away from the house.

3. A snail is at the bottom of a well on Monday morning. Every day the snail climbs up 3 m and every night the snail slides back 1 m. The well is 7 m deep. On what day does the snail reach the top of the well? Draw a number line for the depth of the well and show the snail’s journey.

Answer
The snail climbs out of the well on Wednesday evening.
PA3-15 Patterns in Charts
Pages 37–39

Goals
Students will identify and describe number patterns in hundreds charts and in calendars.
Students will use the patterns seen on hundreds charts to skip count forwards and backwards by 5s starting at any number within 1000.

PRIOR KNOWLEDGE REQUIRED
Can multiply one-digit numbers
Can extend a number pattern by adding or subtracting the same number
Can extend a geometric pattern
Can write a number pattern based on a geometric pattern
Can write the rule for a number pattern
Can represent an addition or subtraction sentence on a number line
Can create a geometric pattern based on a number pattern

MATERIALS
transparency of a hundreds chart or BLM Hundreds Charts (p. N-54)
transparency of a calendar or BLM Calendars (p. N-55)
overhead projector
BLM Multiplication Chain (pp. V-2–7)
hundreds chart or BLM Hundreds Charts (p. N-54) per student
pencil crayons
small token per pair of students
calendar or BLM Calendars (p. N-55) per student
BLM Empty Calendar (p. N-56)

NOTE: Throughout this lesson you will need to shade different patterns on a hundreds chart and a calendar page. If you do not have convenient manipulatives (such as a commercial erasable hundreds chart), you can photocopy BLM Hundreds Charts and BLM Calendars onto transparencies and either display new charts each time, or project enlarged copies on the board and shade the numbers on the board. This would allow you to erase the shading or circles without erasing the charts themselves.

Mental math minute. Give each student a card from BLM Multiplication Chain. Call a volunteer to the front of the class. The volunteer reads the card (e.g., I have $3 \times 4$ and 25). Students who have 12 or $5 \times 5$ on their cards should come to the front of the class and stand beside the volunteer, showing their cards. If there is more than one student with a card that matches (for example, 12 appears on multiple cards), pick who joins the chain at this moment and who will join the chain later. The students who just joined the chain read the unmatched halves of their cards, and new students with matches join the chain. If the number called from one side...
of the chain matches the multiplication sentence on the other side of the chain, and there is no third student who can join either side of the chain, the chain is complete. The remaining students should try to make a new chain of their own. The game ends when everyone has come to the front.

**Review patterns in charts.** Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
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<td>22</td>
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<td>40</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>44</td>
<td>46</td>
<td>48</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

Shade the top row. **ASK:** What pattern do you see in this row? (2, 4, 6, 8, 10) What kind of pattern is that? (increasing pattern, growing pattern) What is the rule for this pattern? (start at 2 and add 2 each time, skip count by 2s) How do you know? (the difference, or the gap between the numbers, is always 2) Have a volunteer write the rule on the board. Repeat with the pattern in the third row. (start at 22 and add 2 each time) **ASK:** Do you think in this chart all rows are made by adding 2? (yes) Have students check different rows.

Repeat the discussion with columns, concluding that the columns are all made by adding 10 each time.

**Identifying patterns in rows and columns of a hundreds chart.** Display a large hundreds chart and shade the third row. **ASK:** If I look at these numbers from left to right, do they show a pattern? (yes) What rule does this pattern have? (start at 21 and add 1 each time) Repeat with the 6th column going down. (start at 6 and add 10 each time)

Distribute **BLM Hundreds Charts.** Have students work with a partner. Each person uses pencil crayons to shade a row and a column on the first hundreds chart on the BLM. (Ask students not to shade the same column and the same row as you did, and to choose a row and a column different from that of their partners.) Have each student write the rules for their pattern going to the right and their pattern going down. Partners trade BLMs and write the rules for the patterns in the rows their partners shaded, this time going to the left, and the columns their partners shaded, this time going up from the bottom row.

**ASK:** What is the gap in the patterns in the rows of a hundreds chart? (1) Did anyone get a different gap? (no) Does everyone have the same starting number? (no) Point out that a hundreds chart is made by counting up by 1s in rows, so it makes sense that the patterns all have a gap of 1. **ASK:** What is the gap in the patterns in the columns of a hundreds chart? (10) Did anyone get a different gap? (no) Does everyone have the same starting number? (no) Why is the gap always 10 in any column? (there are 10 columns in the chart, so to get to the number directly below any number, you need to count up 10, or add 10) **PROMPT:** If you fill in the
hundreds chart counting by 1s, how many numbers are between a number and the number right below it? Do you need to count a whole row to get to the number right below? How many cells, or boxes, are in the whole row? How many columns are in the hundreds chart? Students should keep their copies of the BLM for later in the lesson.

**Identifying diagonal patterns on a hundreds chart.** Explain that when you go 1 row down and 1 column right or left on a chart, the cells are diagonal from each other. Diagonal cells have only one corner in common. Draw two diagonal arrangements on the board to illustrate, as shown in the margin.

Display a fresh hundreds chart. ASK: Which cells are diagonally beside 23? (12, 14, 32, 34) Point out that you can go both right and left, and up and down. However, when you want to make a pattern on a hundreds chart, you need to choose a pair of directions, say, right and down, and go only in that direction to make a pattern. Shade 23, then shade the cells diagonally, one down and one right, to illustrate. (34, 45, 56, 67, 78, 89, 100) Ask students to write the shaded numbers in order, from top to bottom, and write on the board:

23, 34, 45, 56, 67, 78, 89, 100

SAY: Let’s check if this is a pattern. Have students find the gaps between the numbers and help you to fill them in. SAY: All gaps are +11, so it is a pattern. On the second hundreds chart on BLM Hundreds Charts, have students shade another diagonal pattern that goes right and down, starting from a number of their choice. Have students check what the gaps are. Students will see that they all got another pattern that requires adding 11 each time.

Ask students why they all got gaps of 11 when going 1 cell down and 1 cell right. To prompt students to see the answer, have them recall how they get from any number to the number directly below. SAY: You need to add 10 to get to the number directly below, and then add 1 more to get the number to the right of it. This means that in total, you always add 11.

Point out that to make a pattern that decreases by 11, students need to start at the bottom of the chart and go in the opposite direction. ASK: Do you need to go up and right or up and left? (up and left)

Repeat the whole discussion with patterns that go 1 row down and 1 column left. Conclude that these patterns are made by adding 9 each time—when you go down a row, you add 10, but when you go 1 column to the left, you subtract 1.

**Multiples of 9 on a hundreds chart.** Remind students that multiples of, say, 4 are numbers we say when counting up by 4s starting at 0. So the multiples of 4 are 0, 4, 8, 12, and so on. ASK: What are multiples of 9? (numbers you say when counting up by 9s when starting at 0) Have students write down the multiples of 9 up to 90 and have a volunteer write them on the board. (0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90)
SAY: The word “multiple” reminds us of the word “multiply.” ASK: Do you think there is a connection between multiples and multiplying? (yes) Write on the board:

\[1 \times 9 =\]
\[2 \times 9 =\]
\[3 \times 9 =\]

Have students help you fill in the numbers. Have more volunteers continue the pattern of multiplication. ASK: Are the answers you are writing down multiples of 9? (yes) Explain that multiples of 9 are also numbers that are products of any number—like 1, 2, 3, and so on—multiplied by 9. SAY: Zero is also a multiple of 9. ASK: What number should we multiply by 9 to get zero? (0) Remind students that multiplication is adding the same number again and again, and that zero times any number is zero because you are simply not adding any numbers.

ASK: Where on a hundreds chart are all multiples of 9? Ask students to shade all the multiples of 9 on the third hundreds chart, if they have not done so already. Ask students to describe where the multiples of 9 are located on a hundreds chart. (diagonally, from 9 down and left)

**Skip counting forwards by 5s on a hundreds chart.** Write “3, 8, 13, 18, 23” on the board. Ask students to describe this pattern. (start at 3 and add 5 each time) Explain that another way to describe this pattern is “Start at 3 and skip count forwards by 5s.” Point out that these numbers are not the multiples of 5, because you started at a different number, not at 0, and not at 5. Ask students to shade the numbers in the pattern on the fourth hundreds chart, and have a volunteer do that on a fresh hundreds chart on the board. Have them continue for a few more numbers. Ask students to describe the location of the shaded numbers on the chart. (they are all in two columns, the 3rd and the 8th column) ASK: What do you notice about the ones digits of numbers in the same column of a hundreds chart? (they are all the same) What do you notice about the ones digits of the numbers in this pattern? (they make a pattern: 3, 8, repeat)

Have a volunteer write out the tens digits of the numbers in the pattern 3, 8, 13, 18, 23. You might need to remind students that a one-digit number has a tens digit of 0. The pattern in the tens digits is 0, 0, 1, 1, 2, 2, and so on. Remind students that such patterns repeat and grow at the same time: you repeat the number once, then add 1, then repeat the number again.

ASK: Did we skip any numbers in the tens digits? (no) If I continue to count up by 5s in this pattern, will I eventually say the number 93? (yes) How do you know? (you will say 90, because we do not skip numbers in the tens digit, and you will say 3 or 8 in the ones digit) Will I eventually say the number 87? (no) Why not? (the ones digits are 3 and 8, never 7) Do you think this pattern will continue beyond 100? (yes) What number will you say after 93? (98) After 98? (103) Have students continue writing out a few more numbers in the pattern. ASK: Will you eventually say 139? (no) Why not? (the ones digit does not fit the pattern) Will you eventually say 763? (yes) How do you know? (we say all numbers with ones digits of 3 and 8)
Exercises

a) Skip count forwards by 5s starting at 1. Write the first 10 numbers of the pattern.

b) Describe the pattern in the ones digits.

c) Describe the pattern in the tens digits.

d) Circle the numbers you say in the pattern if you count long enough.

\[
\begin{align*}
71 & \quad 86 & \quad 90 & \quad 91 & \quad 99 & \quad 101 & \quad 105 & \quad 106 & \quad 276 & \quad 394 \\
\text{Bonus:} & \quad 996 & \quad 1000
\end{align*}
\]

Answers: a) 1, 6, 11, 16, 21, 26, 31, 36, 41, 46; b) 1, 6, repeat; c) 0, 0, 1, 1, 2, 2, repeat a number, then add 1; d) these numbers should be circled: 71, 86, 91, 101, 106, 276, 996

Skip counting backwards by 5s on a hundreds chart. Repeat the discussion, this time for counting backwards, using the pattern 99, 94, 89, 84, and so on. You can have students circle or underline the numbers on the same hundreds chart as before, since the patterns also are in two columns. Students should see that the pattern in the ones digits is similar, 9, 4, repeat, and the pattern in the tens digits is a repeating and shrinking pattern, where we say all the numbers two times.

Exercises

a) Skip count backwards by 5s starting at 97. Write the first 10 numbers of the pattern.

b) Describe the pattern in the ones digits.

c) Describe the pattern in the tens digits.

d) Circle the numbers you say in the pattern if you count long enough.

\[
\begin{align*}
54 & \quad 37 & \quad 21 & \quad 12
\end{align*}
\]

Bonus: What is the smallest number in this pattern?

Answers: a) 97, 92, 87, 82, 77, 72, 67, 62, 57, 52; b) 7, 2, repeat; c) repeat a number, then subtract 1; d) these numbers should be circled: 37, 12; Bonus: 2

Missing numbers on a hundreds chart.

**ACTIVITY 1**

1. Have students work with a partner. Students use a hundreds chart from BLM Hundreds Chart and a small token that can cover a number on a hundreds chart. Player 1 closes their eyes. Player 2 covers one of the numbers on a hundreds chart. Player 1 opens their eyes and says what number is covered. Players switch roles after each round.
Identifying patterns on calendars. Display a calendar or BLM Calendars. Distribute a copy of the BLM to students as well. Discuss how calendars are similar to a hundreds chart, and how they are different. (months start at different days of the week; there are only 7 columns in a calendar, not 10 as on a hundreds chart; the columns in a calendar have labels for weekdays; a calendar can have a different number of days and sometimes a different number of rows)

Shade a row and a column on the calendar and have students describe the pattern in the row and the pattern in the column. ASK: Why are the gaps in the row 1? (a calendar is made by writing the numbers in order, each next number to the right of the previous one, until a row ends, similar to a hundreds chart) Why are the gaps in the column 7? (there are 7 columns on a calendar, so you need to add 7 to get to the number directly below) Have students shade a column and a row on the first calendar on the BLM and check that the patterns are as discussed.

Have students make a list of the multiples of 7 and find them on a calendar. The multiples of 7 are always in one column. Have students check that in all four calendars on the BLM. Then have students make a list of the multiples of 6 and multiples of 8. Have them describe the locations of these multiples on different calendars. Students should notice that these multiples are located diagonally on a calendar, though sometimes the diagonal breaks and starts over at the other side of the calendar. Have students explain why multiples of 8 go 1 row down and 1 column to the right (add 7 to go 1 row down and add 1 to go 1 column right, so add 8 in total) and why multiples of 6 go 1 row down and 1 column to the left. (add 7 to go 1 row down and subtract 1 to go 1 column left, so add $7 - 1 = 6$ in total)

ACTIVITY 2

2. Give students BLM Empty Calendar. Students create a calendar for the current month or the next month. They write the dates matching the month, and mark the dates of personal events, such as lessons, chores, and family activities. Encourage students to think of both special events, like birthdays or parties, and recurring events, such as cleaning their room, feeding a pet, going to the library, taking out the garbage, or visiting a relative.

Extensions

NOTE: Students should use BLM Calendars for the extensions.

1. Rob cleans his hamster’s cage on November 4th, and every 4th day after that. How many times does he clean the cage in November?

   Answer: 7 times

2. Liz gets her allowance of 2 dollars every Monday. How much money does she get in November?

   Answer: 8 dollars
3. Ivan brings in firewood every 4th day, starting on December 4th. He goes ice fishing every 6th day starting on December 6th. What days in December does he do both?

**Answer:** December 12th and 24th

4. On any calendar, draw a box around 4 days as shown below.

   ![Box around 4 days](image)

Add the numbers in the diagonal cells in the box. What do you notice about the sums? Try this 3 more times. Try a different month. Explain what happens and why.

**Solution:** The numbers always add to the same sum. The sum depends on the location of the box. Imagine placing counters in the cells, so that we have as many counters in each cell as the number written in it. For example, if we have a table, as shown below, we place 2 counters in the top-left cell, 3 counters in the top-right cell, and so on.

   ![Table with counters](image)

Let’s place 2 and 10 counters in the shaded cells. We now have 12 counters to place in the unshaded cells. We need to place $2 + 1$ counter in the top-right cell, because that number is always 1 more than the number in the top-left shaded cell. This means we have 1 counter less than in the bottom-right cell, but that is exactly the number we need in the bottom-left cell, because it is always 1 less than the number in the bottom-right. This means we need the same number of counters for both the shaded cells and the unshaded cells.

5. Explain to students that different calendars are used in many parts of the world. Have students pick a calendar from another part of the world and find more information about it. They can make a poster and report their findings. Questions to consider: How many months are in the year? Is the number of months the same every year? How long is the year? How long are the months? What defines the months and the year (movement of the sun, the moon, or anything else)? Is there a leap year? What is a leap year—an additional day or an additional month? How often does a leap year happen?
**Goals**

Students will learn that an equation is a number sentence with an equal sign.

Students will identify when two simple expressions are equal or not equal and when an equation is true or false.

**PRIOR KNOWLEDGE REQUIRED**

Can add and subtract two numbers within 100

Can multiply and divide two numbers involving factors no larger than 10

Understands what a number sentence is

**MATERIALS**

ball (optional)

2 small tables or desks (or 1 table or desk with a dividing line made from masking tape)

about 12 identical objects to use as counters

**Mental math minute.** Ask students to solve multiplication questions within the range of $1 \times 1$ to $5 \times 5$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 1$, $2 \times 3$, $6 \div 3$, and so on to $5 \times 3$ and $15 \div 3$. Then progress to a different number.

Next try questions out of order, but keep multiplication and corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

**Introduce equal sides.** Place two small tables or empty desks (or one table with a dividing line) at the front of the class. Place three counters on the left table and three counters on the right table. Point to the left table and ASK: How many counters are on this side? (3) Point to the table on the right and ASK: How many are on this side? (3) Do we have the same number on both sides? (yes) Draw on the board:

```
  o o o
  3
```

SAY: Because the left side and the right side have the same number, we can say the left side and right side are equal. We write an equal sign to show that the amount on the left equals the amount on the right. Have a volunteer draw an equal sign in the box on the board.

**Introduce unequal sides.** Now place three counters on the left table and two counters on the right table. As before, ask students to identify the number of counters on each side. ASK: Do we have the same number
on both sides? (no) Modify the picture on the board to create the one shown below:

```
\[ \begin{array}{c}
\text{3} \\
\text{2}
\end{array} \]
```

SAY: Because the left side and the right side do not have the same number, we say the left side and right side are not equal. Mathematicians have a special sign to show when two amounts are not equal. Write “≠” in the box. SAY: The not equal sign looks like a crossed out equal sign.

Draw several similar pictures on the board, one at a time, with equal and not equal sides. For example: 2 and 2 (equal), 5 and 4 (not equal), 1 and 3 (not equal), 5 and 5 (equal). For each picture, ask students to signal thumbs up for “equal” or thumbs down for “not equal” and then have a volunteer write the correct sign in the box.

**Identifying equal and unequal sides that include addition.** Turn back to the tables at the front of the class and place four counters on the left table in two groups, and four counters on the right table in one group, as shown below:

```
\[ \begin{array}{c}
\text{〇 〇} \\
\text{〇 〇 〇 〇}
\end{array} \]
```

Point to the left table and ASK: How many counters are in the first group? (1) How many are in the second group? (3) How many counters in total are on the left table? (4) PROMPT: What is 1 + 3? (4) Point to the table on the right and ASK: How many counters are on the right table? (4) Do we have the same number on both sides? (yes) Draw on the board:

```
\[ \begin{array}{c}
\text{〇 〇} \\
\text{〇 〇 〇 〇}
\end{array} \]
```

Have students signal which numbers belong in the blanks. (1, 3, 4) ASK: Are the two sides equal? (yes) What sign can we put in the box? (equal sign, =) Have a volunteer write an equal sign in the box. Repeat with the following examples, using only a picture on the board and not physical counters on tables; ask students to signal the numbers for the blanks and either thumbs up for “equal” or thumbs down for “not equal.” Examples: 4 + 1 and 4 (not equal), 3 + 2 and 5 (equal), 6 and 4 + 2 (equal), 3 and 4 + 1 (not equal).

**Identifying correct and incorrect addition sentences.** Write on the board:

```
10 = 9 + 3 \\
10 ≠ 9 + 3
```

Point to the first addition sentence and ASK: Is 10 the same as 9 + 3? (no) PROMPTS: What is 9 + 3? (12) Is 10 the same as 12? (no) ASK: Is this addition sentence correct? (no) Point to the second addition sentence and
ASK: Is this addition sentence correct? (yes) PROMPT: Is 10 different from 9 + 3? (yes) Point to each addition sentence and ASK: So, is it correct to say 10 is equal to 9 + 3? (no) Is it correct to say 10 is not equal to 9 + 3? (yes) Circle “10 ≠ 9 + 3.”

Write on the board:

\[ 8 + 5 = 13 \quad 8 + 5 ≠ 13 \]

Point to the first addition sentence and ASK: Is 8 + 5 the same as 13? (yes) PROMPT: What is 8 + 5? (13) ASK: Is this addition sentence correct? (yes) Then point to the second addition sentence and ASK: Is this addition sentence correct? (no) PROMPT: Is 8 + 5 different from 13? Point to each sentence in turn and ASK: So, is it correct to say 8 + 5 is equal to 13? (yes) Is it correct to say 8 + 5 is not equal to 13? (no) Circle “8 + 5 ≠ 13.”

**Exercises:** Circle the correct addition sentence in the pair.

a) \[ 15 = 13 + 2 \quad b) \quad 11 + 2 = 15 \quad c) \quad 21 + 3 = 25 \]

**Bonus**

\[ d) \quad 21 + 34 = 55 \quad e) \quad 513 + 201 = 724 \]

\[ 21 + 34 ≠ 55 \quad 513 + 201 ≠ 724 \]

**Answers:** a) 15 = 13 + 2, b) 11 + 2 ≠ 15, c) 21 + 3 = 25,

**Bonus:** d) 21 + 34 = 55, e) 513 + 201 ≠ 724

**Introduce the word “equation.”** SAY: A number sentence that has an equal sign is called an equation. Write on the board:

\[ 4 + 5 = 9 \]

Have a volunteer read the number sentence: “Four plus five equals nine.” ASK: Does this number sentence have an equal sign? (yes) So, is this number sentence called an equation? (yes) Point to the equal sign and SAY: The equal sign tells you that the part of the number sentence on the left side of the equal sign, 4 + 5 (point to 4 + 5), has the same value as the part of the number sentence on the right side of the equal sign, 9 (point to the 9).

Write the words “equal” and “equation” on the board. ASK: How many starting letters do these two words have in common? Have students signal the answer. (4) Underline the common starting letters on the board, as shown below:

\[ \underline{equal} \quad \underline{equation} \]

**Identifying equations.** Write on the board:

\[ 5 + 3 < 11 \quad 16 - 2 = 19 \quad 3 \times 4 = 12 \]

Point to the first number sentence and ASK: Is this number sentence an equation? (no) Why not? (because it does not have an equal sign) Point to the next number sentence and repeat the questions. (no, because it does...
not have an equal sign) Emphasize that this number sentence has a “not equal” sign, which is different from an equal sign. Point to the final number sentence and repeat the questions. (yes, because it has an equal sign)

**Exercises:** Circle the number sentences that are equations.

A. \(8 \times 6 \neq 50\)  
B. \(35 + 2 < 40\)  
C. \(23 - 4 = 19\)

D. \(9 = 72 \div 8\)  
E. \(100 > 42\)  
F. \(25 \neq 30 - 4\)

**Answers:** C, D

**Equations can be true or false.** SAY: When something is not true, we can say that it is false. For example, “pigs can fly” is a false sentence. Write on the board:

\[3 + 4 = 10\]

ASK: Is this number sentence correct? (no) PROMPT: Is \(3 + 4\) the same amount as \(10\)? (no, \(3 + 4\) is 7) ASK: Is this number sentence an equation? (yes) PROMPT: Does the number sentence have an equal sign? (yes) SAY: Even though the number sentence is incorrect, or false, it is still called an equation because it has an equal sign. An equation can be true or false.

Write on the board:

\[
\begin{align*}
2 + 3 &= 6 \, \_ \\
8 + 3 &= 11 \, \_ \\
5 + 6 &= 12 \, \_
\end{align*}
\]

SAY: Let’s check each equation to see if it is true or false. Point to the first equation and ASK: Is this equation true or false? (false) PROMPT: What is \(5 \times 7\)? (35) Is 35 the same as 36? (no) Have a volunteer write “F” for false in the blank. Repeat with the remaining equations. (T, F)

**Exercises:** Write “T” if the equation is true or “F” if the equation is false.

a) \(6 + 4 = 12 \, \_\)  
b) \(5 + 3 = 6 \, \_\)  
c) \(7 + 2 = 9 \, \_\)

**Bonus**

d) \(4 + 3 = 185 \, \_\)  
e) \(20 + 20 = 40 \, \_\)

**Answers:** a) F, b) F, c) T, Bonus: d) F, e) T

Repeat the process with equations that involve multiplication or division. Write on the board:

\[
\begin{align*}
5 \times 7 &= 36 \, \_ \\
18 \div 3 &= 6 \, \_ \\
15 \div 4 &= 12 \, \_
\end{align*}
\]

Point to the first equation and ASK: Is this equation true or false? (false) PROMPT: What is \(5 \times 7\)? (35) Is 35 the same as 36? (no) Have a volunteer write “F” for false in the blank. Repeat with the remaining equations. (T, F)

**Exercises:** Write “T” if the equation is true or “F” if the equation is false.

a) \(26 - 14 = 12 \, \_\)  
b) \(9 + 3 = 6 \, \_\)  
c) \(15 - 4 = 19 \, \_\)

d) \(15 \div 3 = 18 \, \_\)  
e) \(24 \div 3 = 8 \, \_\)  
f) \(6 \times 9 = 54 \, \_\)
Bonus

- g) $14 + 16 = 3 \times 10$
- h) $25 \div 5 = 5 + 1$
- i) $18 - 12 = 48 \div 8$

Answers: a) T, b) F, c) F, d) F, e) T, f) T, Bonus: g) T, h) F, i) T

Extensions

1. Which number sentence in the pair is an equation?
   - a) $15 + 9 = 8 \times 7$
   - b) $34 - 25 = 3 + 7$
   - c) $25 - 19 \neq 60 \div 10$
   - d) $600 - 1 < 999$

   Answers: a) $15 + 9 = 8 \times 7$, b) $35 = 7 + 21 + 8$, c) $7 \times 3 = 40 - 19$, d) $10 + 2 = 14 - 3$

2. For each part of Extension 1, which number sentence is correct?

   Answers: a) $24 - 13 > 81 \div 9$, b) $34 - 25 \neq 3 + 7$, c) $7 \times 3 = 40 - 19$, d) $600 - 1 < 999$

3. Write “T” if the equation is true or “F” if the equation is false.
   - a) $326 - 214 = 112$
   - b) $189 + 203 = 501 - 109$
   - c) $25 \times 2 = 10 \times 6$
   - d) $321 + 200 + 289 = 990 - 108$
   - e) $15 \div 3 = 583 - 578$
   - f) $9 \times 8 = 801 - 654$

   Answers: a) T, b) T, c) F, d) F, e) T, f) F

4. Use each of the four signs $=, \neq, <, >$ once in the boxes below to make all the number sentences true. There are two solutions.

   - $8 \square 3 \times 2$
   - $7 \square 5 + 2$
   - $6 \square 8 + 4$
   - $9 \square 56 \div 7$

   Answers: $\neq, =, <, >$; or $>, =, <, \neq$
Goals
Students will use pictures, guessing and checking, and subtraction to write and solve simple addition equations that include an unknown.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract within 20 mentally
Can add and subtract two-digit numbers
Understands the connection between addition and subtraction

MATERIALS
ball or relay race baton (optional)
about 12 identical objects to use as counters
one small table or desk with a dividing line (made from masking tape, for example)
cards with plus (+), minus (−), and equal (=) signs
cardboard box or opaque bag

NOTE: Demonstrations throughout this lesson and others in the unit feature apples (to match the pictures in AP Book 3.2). In place of real apples, you could use paper cut-outs of apples, counters, connecting cubes, or any other roughly identical objects.

Mental math minute. Arrange students in a line and give them addition problems within 20. Students can pass a ball or a relay race baton to each other, so that the person who receives the baton answers the next question.

Review equality of two sides. Set up a table at the front of the class with a dividing line. Place 5 apples on one side of the line and 3 apples on the other side. Show the card with the equal sign and ASK: Are the sides equal? (no) When students say that the sides are not equal, make a point of moving the card with the equal sign away from the demonstration; in other examples, when the sides are equal, place the card upright on the table on top of the dividing line so that students can see it clearly. Repeat with 4 apples on one side and 3 apples on the other, and then with 3 apples arranged differently on both sides. Then repeat with other similar situations, this time placing the apples on one side into two separate piles, with the plus card in between the piles. Draw pictures on the board to model the equality or inequality. For example, you would represent $3 + 4 = 7$ like this:
Remind students that in an addition equation, the numbers you add are called addends and the result of adding the two numbers is called the sum. ASK: What are the addends in this equation? (3 and 4) What is the sum? (7)

**Solving addition equations presented as models.** Have 5 apples ready in a cardboard box or an opaque bag. Show students the box or bag and explain that sometimes we do not know how many apples there are in an addition equation. Place the box or bag on the table on the left side of the dividing line. On the right side, place a group of 2 apples, the card with the plus sign, and then a group of 3 apples to represent the addition 2 + 3. Explain to students that the number of apples is the same on the left side of the dividing line as on the right side. ASK: So, can you tell how many apples are in the box? (yes) How many? (5) Have a volunteer verify the answer by removing the apples from the box and counting them. Repeat with 5 + 2 apples on one side and 7 apples in the box on the other side, and then repeat again with similar examples.

When students have mastered this, increase the challenge by making three groupings of apples on one side: for example, 2 + 4 + 3 apples on one side and 9 apples in the box on the other side. ASK: Since both sides have the same number of apples, what symbol can we put between the two sides to show this? (equal sign, =) Place the card with the equal sign on the dividing line and do a few more examples in which students have to figure out how many apples are in the box.

**Exercises:** How many apples are in the box? Write the number.

a) ![Apples](apples_a.png)

b) ![Apples](apples_b.png)

**Bonus:** ![Apples](apples_bonus.png)

**Answers:** a) 7, b) 9, Bonus: 10

Present the following picture using the table, apples, and cards:

![Apples](apples_asking.png)

ASK: Can you tell how many apples are in the box? (yes) How many? (4) How did you figure this out? (several solutions are possible: counting up from 3 to 7, subtracting 7 — 3, matching apples in the picture on one side to the other and circling the extra) Repeat with a few more examples, placing the box or bag on different sides of the dividing line. When students have
mastered such questions, increase the challenge by including an extra addend on one side of the equal sign, as shown in the example below:

\[
\begin{array}{c}
\text{\includegraphics[width=0.2\textwidth]{apples.png}} \\
+ \\
\text{\includegraphics[width=0.1\textwidth]{box.png}} = \\
\text{\includegraphics[width=0.2\textwidth]{apples.png}} + \\
\text{\includegraphics[width=0.1\textwidth]{apples.png}}
\end{array}
\]

Guide students to solve examples such as this one by first adding the apples on the right-hand side. \((6 + 4 = 10)\) Students can then count up to 10 from 7 to find the missing quantity on the left-hand side of the equal sign \((7 + 3 = 10)\) or use subtraction \((10 - 7 = 3)\) to find the answer.

**Exercises:** How many apples are in the box? Write the number.

\begin{align*}
a) \quad & \text{\includegraphics[width=0.2\textwidth]{apples.png}} + \\
& \text{\includegraphics[width=0.1\textwidth]{box.png}} = \text{\includegraphics[width=0.2\textwidth]{apples.png}} \\

b) \quad & \text{\includegraphics[width=0.2\textwidth]{apples.png}} = \text{\includegraphics[width=0.1\textwidth]{box.png}} + \\
& \text{\includegraphics[width=0.1\textwidth]{apples.png}}
\end{align*}

**Bonus**

\[
\begin{array}{c}
\text{\includegraphics[width=0.2\textwidth]{apples.png}} + \\
\text{\includegraphics[width=0.1\textwidth]{box.png}} = \\
\text{\includegraphics[width=0.2\textwidth]{apples.png}} + \\
\text{\includegraphics[width=0.2\textwidth]{apples.png}} + \\
\text{\includegraphics[width=0.2\textwidth]{apples.png}}
\end{array}
\]

**Answers:** a) 6, b) 2, Bonus: 6

**Using pictures to represent problems.** Draw attention to the pictures you have drawn on the board to represent the addition equations. Explain that people draw different pictures for different purposes. SAY: In art, we might try to draw apples as realistically as possible. We would pay attention to colour, shape, and other details. ASK: Does colour help us to answer the mathematical problem of how many apples are in the box? (no) Does including leaves on the apples help us to answer the mathematical problem? (no) SAY: These details do not help us to answer mathematical questions, so we do not need to include them. In mathematics, we want to use simple pictures that help us to answer problems but that do not take too much time to draw.

ASK: What should we pay attention to in the pictures we draw to help answer mathematical questions? (the number of objects; creating a picture that is not messy; drawing the objects so they are easy to count; in these specific examples, drawing circles that are about the same size from each other and not bigger than the box so that we are not distracted)

Ask students to copy the pictures from the previous exercises with the apples and complete the pictures by drawing the necessary number of apples in the box. Encourage students to draw circles or large dots for apples.
Writing addition equations from pictures. Point out that it is inconvenient to draw apples or circles all the time. ASK: What if you have a box and 79 apples on one side of the line and 125 apples on the other side? What would be more convenient to use than a picture? (numbers)

Remind students that a number sentence is called an equation because it has an equal sign. Draw on the board:

\[ 4 + \phantom{5} = 9 \]

ASK: How can we show this picture as an equation with numbers? (write the number of apples instead of drawing apples) Have students tell you the number for each group as you write the equation, as shown below:

\[ 4 + \phantom{5} = 9 \]

Explain that the box in the equation with numbers can be smaller since you are not drawing the apples inside, you are just writing the number of apples. SAY: Think of how you have found missing numbers in equations so far. You have counted on from one number to the next, you have used subtraction, or you have matched pictures to show the extra. Here, count up from 4 until you get to 9, or subtract 9 \(-\) 4. Pointing to the picture, ASK: How many apples should we draw in the box? (5) Draw the 5 apples in the box. Point to the box underneath in the numerical equation and ASK: What number should we write in this box? (5)

Exercises: Draw the missing apples in the box and then write the missing number in the box.

a) \[ 4 + \phantom{5} = 8 \]

b) \[ 7 + \phantom{5} = 10 \]

Answers: a) 4, b) 3

Drawing pictures to solve addition equations. Write on the board:

\[ 6 + \phantom{5} = 8 \]

Explain that there is a number missing in the equation and it is shown by the box. SAY: The missing number is called the unknown number, because we don’t know what it is right away. I want to draw a picture for the equation that will help find the unknown number. ASK: How many apples should
I draw under the number 6? (6) Draw the 6 apples. Write a plus sign underneath the plus sign of the numerical equation, and then draw a large box under the small box of the numerical equation. ASK: Why should we draw a larger box for the picture? (because we need more space to draw apples) Write an equal sign under the equal sign of the numerical equation, and then ASK: How many apples should I draw for the number 8? (8) The final picture should look like this:

\[ \begin{array}{cccc}
6 & + & \boxed{} & = & 8 \\
\end{array} \]

Point to the large box and ASK: How many apples should I draw here? (2) Point to small box in the equation and ASK: What number should I write here? (2) Write “2” in the box. SAY: We just found the missing number in the equation. Finding the missing number in an equation is called solving the equation. When you are asked to solve an equation, it means you need to find the missing number in the equation. Remind students that for these drawings they should keep the apples simple: just circles or circles with a small line to show the stem if they wish.

**Exercises:** Draw a picture for the equation. Use your picture to solve the equation.

a) \( 7 + \square = 9 \)  

b) \( 8 = 1 + \square \)

**Bonus**

c) \( 5 + \square = 5 \)  
d) \( 10 = 10 + \square \)

**Answers:** a) 2, b) 7, Bonus: c) 0, d) 0

**Using guessing and checking to solve an equation.** Write on the board:

\[ 8 + \square = 17 \]

SAY: We can use a picture to solve this equation, but we will need to draw 8 apples on one side, and 17 apples on the other side. That’s a lot of drawing! Let’s try to solve this equation without drawing. ASK: What number do you need to add to 8 to get 17? (9) How do you know? (answers will vary: use doubles, \( 8 + 8 = 16 \) so \( 8 + 9 = 17 \); count on; use memorized addition facts) Write “9” in the box and ASK: Does \( 8 + 9 \) equal 17? (yes) So, is the equation true? (yes)

Write on the board:

\[ 8 + \square = 15 \]

SAY: I think the missing number is 6. Write “6” in the box and ASK: Is this equation true? (no) Why not? (\( 8 + 6 = 14 \), not 15) ASK: Should I try a larger number or a smaller number next? (larger) How do you know? (14 is too small; we need 15, so we need a larger addend) Write “7” in the box and
ASK: Is this equation true? (yes) SAY: The method we are using to solve the equation is called **guessing and checking**. You first try to guess the correct number, then you check if your guess is correct. Use your knowledge of number facts because the closer your guess is to the answer, the better. Although it is best if you guess the correct number right away because you know your number facts.

**Exercises:** Solve by guessing and checking.

a) $\square + 4 = 10$  
b) $5 + \square = 11$  
c) $15 = 7 + \square$

d) $6 + 8 = \square$  
e) $\square = 7 + 9$  
f) $18 = \square + 9$

g) $19 = 10 + \square$  
**Bonus:** $100 = 20 + \square$

**Answers:** a) 6, b) 6, c) 8, d) 14, e) 16, f) 9, g) 9, Bonus: 80

**Introduce fact families.** Draw on the board:

[Circle diagram with 8 circles, 3 dark and 5 light]

SAY: This is another picture we can draw for addition and subtraction sentences. ASK: How many dark circles do we have? (3) How many light circles do we have? (5) How many circles do we have in total? (8) How do you get 8 from 5 and 3? (add) What addition or subtraction equations can you write for this model, using the total number of circles? (3 + 5 = 8, 5 + 3 = 8, 8 − 5 = 3, 8 − 3 = 5) If students need help thinking about subtraction equations, SAY: There are 8 circles. Three of them are dark. ASK: How many light circles do you have? (5) What equation can you write for that problem? (8 − 3 = 5)

Write all four equations underneath the picture. SAY: These four equations together are called a **fact family**. The fact family shows the addition and the subtraction equations you can write for a picture. You can have the first addend be the number of dark circles (point to 3 + 5 = 8) or the number of light circles (point to 5 + 3 = 8). Point to 8 − 3 = 5 and ASK: Is this subtraction giving us the number of light circles or the number of dark circles? (the number of light circles) Repeat with 8 − 5 = 3, showing the number of dark circles.

Draw on the board:

[Circle diagram with 8 circles, 3 dark and 5 light]

Have volunteers come to the board and write the equations forming the fact family for the picture. (4 + 6 = 10, 6 + 4 = 10, 10 − 6 = 4, 10 − 4 = 6) Point out the structure of the equations: the parts can come in any order in the addition equations, and the two subtraction equations can have either part as the number that is being subtracted.
**Exercise:** Write the fact family for the picture.

```
○○○○○○○○○
```

**Answers:** 3 + 6 = 9, 6 + 3 = 9, 9 - 3 = 6, 9 - 6 = 3

Reverse the task. Write "2 + 1 = 3" on the board and ask students to draw a model for the equation in their notebooks. Have a volunteer draw the model on the board. The model should show two circles of one colour and one circle of another colour. Have students write the rest of the equations in the fact family in their notebooks. (1 + 2 = 3, 3 - 2 = 1, 3 - 1 = 2)

Write "4 - 3 = 1" on the board. **ASK:** How many circles should be in the model for this equation? (4) **How do you know?** (the total, the largest number in the equation, is 4) Draw 4 circles and **ASK:** How many circles should I shade? (3 or 1) Does it matter for the fact family if I shade 3 circles or 1 circle? (no) **Why not?** (in a fact family you will have two subtraction equations, showing how to find the parts; both pictures will produce the same fact family) Shade 1 circle and have students write the fact family for the model in their notebooks. Have a volunteer write the fact family on the board. (4 - 3 = 1, 4 - 1 = 3, 3 + 1 = 4, 1 + 3 = 4)

**Exercises:** Draw the model for the equation. Write the rest of the equations in the fact family.

a) 2 + 3 = 5  
b) 7 - 2 = 5

**Answers**

a) ○○○○○

3 + 2 = 5, 5 - 2 = 3, 5 - 3 = 2  
b) ○○○○○○○○

2 + 5 = 7, 5 + 2 = 7, 7 - 5 = 2

**Using subtraction to find the missing addend.** **SAY:** We can use pictures with circles to solve equations, too. Imagine that some circles are covered by a box. Draw on the board:

```
○○○
```

**SAY:** There are 8 circles in total in this picture; you can see 3 of the circles and the rest are hidden in the box. I can write an equation for this picture. Write on the board:

```
3 + = 8
```

**ASK:** What is the missing number here? (5) **How can you get 5 from 8 and 3?** (subtract) Can you always subtract when you need to find the missing addend? (yes) Write "5" in the box and have volunteers write the fact family for the equation 3 + 5 = 8 on the board. (5 + 3 = 8, 8 - 5 = 3, 8 - 3 = 5) **SAY:** For each number in each of these equations you can make a problem...
where this is the missing number. For example, there are 8 circles, 3 you can see, and the rest are in the box. How many circles are in the box? To solve this problem, you can write “3 + box = 8” (point to the equation on the board) or you can write “8 – 3 = box.” Write “8 – 3 = box” on the board as well.

Point to the two equations with the boxes, 3 + box = 8 and 8 – 3 = box, and ASK: How are the equations the same? (they describe the same situation or picture, they have the same numbers, in both of them 5 is missing) How are the equations different? (in 3 + box = 8 you need to guess the number, in 8 – 3 = box you just need to calculate) SAY: For any problem where an addend, meaning the number you add, is missing, you can write a subtraction equation. You just need to subtract the other addend from the total.

Write the equation in the margin on the board. SAY: An addend is missing here. ASK: What subtraction equation can I write so that I can find the missing number? (5 – 2 = box) Write “5 – 2 = box” on the board. ASK: What is 5 – 2? (3) Does writing 3 in the box make the first equation true as well? (yes)

**Exercises:** Write the subtraction equation to find the missing number.

a) □ + 4 = 12   b) 2 + □ = 11   c) 15 = 9 + □

d) 18 = □ + 9   **Bonus:** 100 = 50 + □

**Answers:** a) 12 – 4 = 8, b) 11 – 2 = 9, c) 15 – 9 = 6, d) 18 – 9 = 9,

**Bonus:** 100 – 50 = 50

SAY: Let’s try this method for larger numbers. Write “□ + 36 = 52” on the board. SAY: It would take a lot of time to solve this equation by drawing circles. Let’s write a subtraction equation to find the missing number. Have a volunteer write the subtraction equation (52 – 36 = □) and have students solve it. (16) You may want to remind students of some mental math strategies they learned, such as counting up by 1s to get to 40 and then by 10s to get to 50 and by 1s to get to 52. Write “16” in the blank of the initial equation and have students check that the addition equation is true. Emphasize that checking the answer by doing the addition is important because it allows students to find out if they are correct without depending on anybody to check their answers.

**Exercises:** Write the subtraction equation to find the missing number.

a) □ + 43 = 72   b) 52 + □ = 99

c) 75 = 9 + □   d) 88 = □ + 79

**Bonus:** 999 = 520 + □

**Answers:** a) 72 – 43 = 29, b) 99 – 52 = 47, c) 75 – 9 = 66, d) 88 – 79 = 9,

**Bonus:** 999 – 520 = 479
Extensions

1. Write +, −, or = in each blank to make a true equation.
   a) 5 ___ 4 ___ 9  
   b) 12 ___ 2 ___ 10  
   c) 16 ___ 20 ___ 4  
   d) 35 ___ 22 ___ 57  

   **Bonus**
   e) 416 ___ 515 ___ 99  
   f) 82 ___ 12 ___ 90 ___ 20  

   **Answers:** a) 5 + 4 = 9,  b) 12 − 2 = 10 or 12 = 2 + 10,  
   c) 16 = 20 − 4,  
   d) 35 + 22 = 57, **Bonus:** e) 416 = 515 − 99, f) 82 − 12 = 90 − 20

2. Which part in Extension 1 has two possible answers? Write the two equations.

   **Answer:** part b), 12 − 2 = 10 and 12 = 2 + 10

3. Draw a picture for the equation. Use your picture to solve the equation.
   a) 7 + 2 + ___ = 19  
   b) 11 = 1 + 5 + ___  
   c) 3 + 2 + ___ = 14  
   d) 13 = 1 + 5 + 3 + ___  

   **Selected answers:** a) 10, b) 5, c) 9, d) 4

4. Beth shows an equation using apples and two boxes:

   Box A  +  Box B
   
   There are fewer than 10 apples on each side of the equal sign. List the number of apples that could go into Box A and Box B that would make the equation true.

   **Answers:** 2, 0; 3, 1; 4, 2
Goals
Students will use pictures, guessing and checking, addition, and subtraction to write and solve simple subtraction equations that include an unknown.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract within 20 mentally
Can add and subtract two-digit numbers
Can solve equations using guessing and checking
Knows that a box can represent an unknown number
Can write the equations in a fact family

MATERIALS
ball
about 12 identical objects to use as counters
cardboard box or opaque bag

NOTE: Demonstrations at the beginning of this lesson and others in the unit feature apples (to match the pictures in AP Book 3.2). In place of real apples, you could use paper cut-outs of apples, counters, connecting cubes, or any other roughly identical objects.

Mental math minute. Give students subtraction problems within 20. Toss a ball to a student who you want to answer, and have the student toss the ball back to you as he or she answers the question. Repeat until all students have had a chance to answer a subtraction problem.

Review vocabulary. Write on the board:

\[ -3 = 4 \]

ASK: Is this number sentence an equation? (yes) How do you know? (it has an equal sign) Point to the box and ASK: What does this box stand for? (a missing number or an unknown number) Tell students that you would like to find the missing number. ASK: What is it called when we find the missing number in an equation? (solving the equation)

Introducing subtraction equations with unknowns. Place 7 apples in a box so that students do not see how many apples are in the box. Show them the box and SAY: There are some apples in this box, but I won’t tell you how many. Remove 3 apples from the box. SAY: Now there are 4 apples left in the box. Draw on the board:

\[ \square - \bigcirc = \bigcirc \bigcirc \]
SAY: There were some apples in the box. I took away 3. Now there are 4 apples left in the box. ASK: How can you find how many apples were in the box before you took away 3 apples? Students might suggest guessing and checking, or adding the number of apples that were taken away to the number left over. Ensure both ideas arise from the discussion.

**Solving subtraction equations using guessing and checking.** Point to the equation you wrote on the board and ASK: What could be the number in the box? Suggest 6 as a first guess. Write “6” in the box and SAY: We guess that $6 - 3 = 4$. So let’s check our guess. ASK: Does $6 - 3 = 4$? (no, $6 - 3 = 3$) SAY: So, we erase the 6 in the box, mark our first guess of 6 to the side, and cross that 6 out because it does not solve the equation.

ASK: Now, what number can you guess next? Students will likely say 7. Write “7” in the box and ASK: Does $7 - 3 = 4$? (yes) SAY: So the missing number is 7. Write $[7] - 3 = 4$ on the board to demonstrate that 7 is the unknown number. Emphasize that it is okay to guess the correct number right away; moreover, if students learn the addition and subtraction facts up to $9 + 9$, they will be able to say what the answer is right away without guessing. However, they still need to check their answers.

**Exercises:** Solve the equation by guessing and checking.

a) $\square - 3 = 6$  
b) $8 - \square = 2$

c) $9 = \square - 3$  
d) $\square = 13 - 6$

**Bonus**

e) $0 = \square - 5$  
f) $7 = 7 - \square$

**Answers:** a) 9, b) 6, c) 12, d) 7, Bonus: e) 5, f) 0

**Drawing a picture to solve an equation with a missing total.** Write on the board:

$$\square - 5 = 6$$

SAY: I want to draw a picture showing this equation, similar to the picture we had at the beginning of the lesson. The first number is the total number of apples, before I took some out, and it is the number we do not know. So let’s draw a box for it. Draw a large box and have students copy the equation and draw a large box in their notebooks.

SAY: The second number shows the apples I took out of the box. We drew them with a minus sign. Draw 5 circles or symbolic apples with a minus sign in front of them. SAY: The last number, the number after the equal sign, shows how many apples are left. Draw the equal sign and 6 apples. Have students do the same. ASK: How many apples were in the box in the beginning? (11) How do you know? (this is the total, the apples you took out plus the apples that are left in the box) Point out that you add the numbers to find the missing total number of apples.
**Exercise:** Draw a picture for the equation \( 7 = \square - 5 \).

**Answer**

![Picture showing 7 apples on the left, 5 apples taken out, and 2 apples remaining]

Using addition to solve subtraction equations with a missing total.

**ASK:** How do you know how many apples to draw in a box? (you add the apples that are left and the apples that were taken out)

**SAY:** When the missing number in an equation is the total number of apples, you use addition to find the missing number. Write on the board:

\[ \square - 15 = 6 \]

**ASK:** What quantities do we know in this equation? (the number of apples left in the box and the number of apples taken out)

**SAY:** Is the missing number the total? (yes)

**SAY:** Let's add the numbers in the equation. Write “\( 15 + 6 = \_ \)” on the board.

**ASK:** What is \( 15 + 6 \)? (21)

**Write “21” in the blank and in the box.**

**SAY:** Let's check if this equation is true. **ASK:** Does 21 − 15 equal 6? (yes)

**SAY:** We checked, so we know we solved the equation correctly.

**Exercises:** Solve the equation by writing an addition sentence. Check your answer.

a) \( \square - 23 = 61 \)  
b) \( \square - 35 = 12 \)

c) \( 9 = \square - 73 \)  
d) \( 38 = \square - 46 \)

**Bonus**

e) \( 0 = \square - 125 \)  
f) \( \square - 200 = 777 \)

**Answers:** a) \( 61 + 23 = 84 \), check: \( 84 - 23 = 61 \); b) \( 35 + 12 = 47 \), check: \( 47 - 35 = 12 \); c) \( 73 + 9 = 82 \), check: \( 82 - 73 = 9 \); d) \( 46 + 38 = 84 \), check: \( 84 - 46 = 38 \); Bonus: e) \( 0 + 125 = 125 \), check: \( 125 - 125 = 0 \); f) \( 777 + 200 = 977 \), check: \( 977 - 200 = 777 \)

**Review fact families.** Draw on the board:

![Picture showing 6 apples and 2 apples left]

**ASK:** What addition and subtraction equations can we write for this picture? (\( 2 + 3 = 5, 3 + 2 = 5, 5 - 2 = 3, 5 - 3 = 2 \))

**Have volunteers write the equations on the board.**

**ASK:** What do we call these four equations together? (a fact family)

**Exercise:** Write the fact family for the picture.

![Picture showing 8 apples and 2 apples taken out]

**Answer:** \( 2 + 6 = 8, 6 + 2 = 8, 8 - 2 = 6, 8 - 6 = 2 \)
Using subtraction to solve subtraction equations. Draw on the board:

\[ \begin{array}{c}
\bullet \bullet \bullet \bullet \square \\
7 - 5 = \square \\
7 - \square = 5
\end{array} \]

SAY: Some circles are hidden in the box. The total number of circles is 7. Write “Total = 7” on the board. SAY: I want to write the equations this picture shows before we figure out how many circles are hidden. ASK: What addition equations did we write for pictures like this in the last lesson? (5 + box = 7, box + 5 = 7) Invite volunteers to write the equations on the board. SAY: I want to write subtraction equations from the addition equations. ASK: What number do we start with? (the total) Write “7 − ” on the board twice, and ASK: What number do we subtract? (5 or the number of circles hidden in the box) Write both options on the board. Point to the first option and ASK: What is this subtraction equal to? (the number of circles hidden in the box) Finish writing the equation. Then have students help you finish writing the second equation. The equations are shown in the margin.

ASK: Which of these equations is just a calculation? (the first equation) Which is easier to solve? (the first equation) What is the missing number? (2) Have a volunteer draw the circles in the box in the picture above and check that there are indeed 7 circles in total.

Ask students to look at all four equations with the unknown number. ASK: How are the equations the same? (they describe the same situation, they have the same numbers, in all of them the unknown number is 2) How are the equations different? (in three of them you need to guess the number, in the fourth you just need to calculate) SAY: A subtraction equation shows a situation with a total and one part is subtracted. If the subtracted part is missing, you can just subtract the other part from the total. If you do not know the total, add the parts.

Write “12 − \square = 4” on the board. ASK: What is the unknown, the total or a part? (part) What is the total in this situation? (12) What subtraction should you write to find the missing part? (12 − 4 = box) Write that equation underneath the first one. Write “8” in the box for the missing number and ASK: Is this equation true? (yes) SAY: This means we solved the equation correctly.

Exercises: Solve the equation by writing the subtraction sentence. Check your answer.

a) 12 − \square = 9  
b) 35 − \square = 12  
c) 9 = 46 − \square  
d) 38 = 75 − \square  
e) 35 = 48 − \square  
f) 100 − \square = 77

Bonus: 450 − \square = 120

Answers: a) 12 − 9 = 3, b) 35 − 12 = 23, c) 46 − 9 = 37, d) 75 − 38 = 37, e) 48 − 35 = 13, f) 100 − 77 = 23, Bonus: 450 − 120 = 330
Solving different equations. Emphasize again that if a total is unknown in a subtraction equation, you can add the parts to find the total. If a part is missing, you can subtract the other part from the total to find the unknown number. Remind students to check their answers.

Exercises: Solve the equation.

a) $\bigsquare - 37 = 62$  
b) $78 - \bigsquare = 2$

c) $49 = \bigsquare - 7$  
d) $\bigsquare = 93 - 36$

Bonus

e) $0 = \bigsquare - 45$  
f) $700 = 800 - \bigsquare$

Answers: a) $62 + 37 = 99$, b) $78 - 2 = 76$, c) $49 + 7 = 56$, d) $93 - 36 = 57$

Bonus: e) $0 + 45 = 45$, f) $800 - 700 = 100$

Extensions

1. You can write a question for an equation. Example:

For $\bigsquare - 37 = 62$, you can ask: What number is 37 more than 62?

Write a question for the equation.

a) $\bigsquare - 17 = 6$  
b) $7 - \bigsquare = 2$

c) $9 = \bigsquare - 7$  
d) $\bigsquare = 93 - 39$

Sample answers: a) What number is 6 more than 17? b) How much more than 2 is 7? c) What number is 7 more than 9? d) How much more than 39 is 93?

2. You can make a story for any equation. Example

For $\bigsquare - 37 = 62$, you can make a problem: Emma has some stickers. She gives 37 stickers to her brother. She has 62 stickers left. How many stickers did Emma start with?

Write a story for the equation.

a) $\bigsquare - 17 = 6$  
b) $7 - \bigsquare = 2$

c) $9 = \bigsquare - 7$  
d) $\bigsquare = 93 - 39$

Sample answers

a) Ren has some marbles. He loses 17 of them and has 6 marbles left. How many marbles did he start with?

b) Alice has 7 dollars. She pays some money for lunch and has 2 dollars left. How much did she pay for lunch?

c) Eric has some apples. He makes a pie with 7 apples and has 9 apples left. How many apples did he start with?

d) Jasmin’s book has 93 pages. She reads 39 pages. How many pages are left?

3. Make your own subtraction equation and create a story for it.


PA3-19 **Using Letters for Unknown Numbers**

Pages 48–49

**Goals**

Students will represent an unknown number in an equation with a letter or a symbol.

Students will solve simple addition and subtraction equations.

**PRIOR KNOWLEDGE REQUIRED**

Can add and subtract numbers within 20 mentally

Can add and subtract two-digit numbers

Can solve an addition or subtraction equation involving an unknown

**MATERIALS**

ball (optional)

about 12 identical objects to use as counters

cardboard box

**Mental math minute.** Ask students to solve multiplication questions within the range of $1 \times 1$ to $9 \times 9$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on to $9 \times 3$ and $27 \div 3$. Then progress to a different number. Next try questions out of order, but keep multiplication and corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

**Review equations.** Write on the board:

\[
\begin{align*}
\text{a)} & \quad -7 = 6 \\
\text{b)} & \quad 29 - \square = 12 \\
\text{c)} & \quad 30 = \square + 17
\end{align*}
\]

ASk: What are these number sentences called? (equations) How do you know? (they all have an equal sign) What do the boxes stand for? (unknown numbers) In equation a), is the unknown one of the parts or the total? (total) How do you know? (because you are subtracting from the unknown) How do you find the unknown total? (by adding the parts) What is the unknown number? (13) Have a volunteer write the addition equation ($13 = 7 + 6$) underneath the initial equation, then have students check that 13 is the correct solution to the subtraction equation.

Repeat with the two other equations. In both equations, a part is missing so students will need to subtract to find the number. (b) 17, c) 13)

**Using letters to stand for unknown numbers.** Ask: What did we use to show the unknown number in the equation? (a box) Say: Mathematicians also use letters to stand for the unknown numbers in an equation. Write on the board:

\[
\begin{align*}
\square + 33 &= 65 \\
x + 33 &= 65
\end{align*}
\]
SAY: I can write “box + 33 = 65” or I can replace the box with the letter \( x \) and write “\( x + 33 = 65 \).” I can use the letter \( x \) or any other letter. For example, I could use the letter \( m \). Write on the board:

\[
m + 33 = 65
\]

Use subtraction to solve this equation. The solution should look like this:

\[
m + 33 = 65 \\
65 - 33 = 32 \\
m = 32
\]

Have students check the solution. SAY: You do not have a box to write the solution in, but you can write the equation again, writing the number instead of the letter or the symbol. Show how to do this:

Check: \( m + 33 = 65 \)

\[
32 + 33 = 65
\]

ASK: Is this equation true? (yes) Place a checkmark beside the equation.

Repeat with the equation \( y - 37 = 9 \). (\( y = 46 \))

Have students solve the following exercises the same way. Remind them to write the final answer with the letter, for example \( x = 3 \).

**Exercises:** Solve the equation.

a) \( x + 8 = 11 \)  
   b) \( n - 8 = 2 \)  
   c) \( 7 = y + 6 \)  

d) \( 9 + n = 16 \)  
   e) \( b - 8 = 5 \)  
   f) \( 7 = x - 6 \)  

g) \( 11 = m - 28 \)  
   h) \( 32 - x = 18 \)  
   i) \( 37 = a + 26 \)  

**Bonus**

j) \( 100 + w = 350 \)  
   k) \( 799 - u = 799 \)  
   l) \( 123 = r - 654 \)

**Answers:** a) \( x = 3 \), b) \( n = 10 \), c) \( y = 1 \), d) \( n = 7 \), e) \( b = 13 \), f) \( x = 13 \), g) \( m = 39 \), h) \( x = 14 \), i) \( a = 11 \), Bonus: j) \( w = 250 \), k) \( u = 0 \), l) \( r = 777 \)

**Solving equations that require rewriting before solving.** Explain that sometimes you need to rewrite an equation before you can solve it. Write on the board:

\[
13 + 4 + x = 22
\]

SAY: Imagine that I have 13 apples, another 4 apples, a box with some apples, and in total there are 22 apples. ASK: How many apples are outside the box? (17) How do you know? (13 + 4 = 17) SAY: We need to rewrite this equation before we can properly solve it. Write on the board:

\[
\underline{13 + 4} + x = 22 \\
17 + x = 22
\]

ASK: How can you find the unknown number? (subtract 22 – 17) What is the unknown number? (5) PROMPT: What number makes the equation true? (5)
How do you know? (17 + 5 is 22) Write the solution on the board. (see margin)

Repeat with the equation 35 – a = 23 – 3. (35 – a = 20, 35 – 20 = 15, a = 15)

**Exercises:** Rewrite the equation, then solve it.

a) 6 + 2 + y = 18  
   b) n – 6 = 15 + 7  
   c) 21 + a = 56 – 20

**Bonus:** n – 9 = 4 × 5

**Answers:**

a) + 8 = 11  
   b) – 8 = 2  
   c) 7 = ? + 6

**Using symbols in equations.** SAY: I can also use other symbols to represent unknown numbers, such as smiley faces, question marks, or any other things that are easy to draw and will not be confusing in an equation. Write on the board:

● + 3 = 22  
  22 – 3 = 19

SAY: All these equations are the same. They all have the same solution, the same numbers, and the same operation; the only difference is the symbol they use for the unknown number. We solve equations with symbols exactly the same way we solve equations with boxes or letters. Write on the board:

● + 3 = 22  
  22 – 3 = 19

SAY: If you use a question mark instead of a smiley face, write a question mark in the last line instead.

**Exercises**

1. Rewrite the equation using ● instead of the letter.
   
   a) x + 8 = 11  
   b) n – 8 = 2  
   c) 7 = y + 6

   **Answers:** a) + 8 = 11, b) – 8 = 2, c) 7 = ? + 6

2. Solve the equation.
   
   a) 11 = ● – 5  
   b) 12 – = 8  
   c) 17 = * + 16

   d) 61 + △ = 96  
   e) 5 = ? – 38  
   f) 77 = – 9

   **Bonus**

   g) 100 + ◆ = 100  
   h) 100 – = 100  
   i) 0 = ? – 654

   **Answers:** a) = 16, b) = 4, c) * = 1, d) △ = 35, e) ? = 43,  
   f) = 86, Bonus: g) = 0, h) = 0, i) ? = 654
Discuss how many solutions an equation has. Write on the board:

\[ 4 + x = 7 \]

Invite a volunteer to solve the equation. **ASQ**: Is there any other number besides 3 that will solve this equation? (no) How do you know? (sample answers: \( x \) should be equal to \( 7 - 4 = 3 \), any other number added to 4 will give a different sum—not 7) If students suggest numbers other than 3, check the sum of all answers and point out that to get a sum of 7, there is only one number that can be added to 4.

Write on the board:

\[ \smiley + 0 = \smiley \]

**SAY**: I have to use the same number for both smiley faces. Can you tell me a number that will solve the equation? Have students make suggestions. For each number that students suggest, check by writing the equation, such as \( 2 + 0 = 2 \). **ASQ**: Is the equation true? (yes) Does any other number make this equation true? (yes) Point out that this equation is very different from the other equation, \( 4 + x = 7 \), because the equation with the two smiley faces can have many solutions, but the other equation can have only one solution.

**Exercises**: How many solutions, one or many, does the equation have? If there is only one, write the solution.

a) \( 5 - a = 3 \)  

b) \( 25 + \smiley = 25 \)

c) \( \smiley - \smiley = 0 \)

**Answers**: a) one, \( a = 2 \); b) one, \( \smiley = 0 \), c) many

**Extensions**

1. Nina writes two equations. She uses \( x \) for the unknown number in the first equation and \( y \) for the unknown number in the second equation.

\[ x + 6 = 9 \qquad 3 + y = 8 \]

a) What does \( x \) stand for? Solve the first equation by guessing and checking. Write your answer as \( x = \) ___.

b) What does \( y \) stand for? Solve the second equation by guessing and checking. Write your answer as \( y = \) ___.

c) Which number is larger, \( y \) or \( x \)?

d) What is \( x + y \)? Add the two unknown numbers to find out.

e) What is \( y - x \)?

**Answers**: a) \( x = 3 \), b) \( y = 5 \), c) \( y \), d) \( x + y = 8 \), e) \( y - x = 2 \)
2. Bill has some apples in a box and 7 apples outside the box. Altogether he has 11 apples. He writes an equation, using the letter $b$ for the unknown number of apples in his box.

$$b + 7 = 11$$

Rani has 3 apples outside a box and some apples in the box. Altogether she has 12 apples. She writes an equation using the letter $r$ for the unknown number of apples in her box.

$$3 + r = 12$$

a) How many apples are in Bill’s box? Solve Bill’s equation by guessing and checking. Write your answer as $b =$ 

b) How many apples are in Rani’s box? Solve Rani’s equation by guessing and checking. Write your answer as $r =$

c) Are there more apples in Bill’s box or Rani’s box? How many more are there?

d) How many apples are in both boxes in total?

**Answers:** a) $b = 4$; b) $r = 9$; c) there are more apples in Rani’s box, 5; d) $r + b = 9 + 4 = 13$

3. In the equation, the letters $x$ and $y$ stand for two unknown numbers. The numbers $x$ and $y$ can be different, or they can be the same.

$$x + y = 4$$

Each unknown number is a whole number between 0 and 4. List all pairs of whole numbers that make the equation true.

**Answers:** 0, 4; 1, 3; 2, 2; 3, 1; 4, 0
Patterns with Increasing Gaps

In some patterns, the gap between the numbers makes a growing pattern.

\[ 1, 2, 4, 7, 11 \]

The next gap is +5. The next number in the pattern is 16.

\[ 1, 2, 4, 7, 11, 16 \]

1. Find the pattern in the gaps. Extend the number pattern.

   a) 3, 5, 8, 12, ___ , ___

   b) 4, 5, 7, 10, 14, ___ , ___

   c) 6, 8, 12, 18, 26, ___ , ___

   d) 10, 13, 18, 25, ___ , ___

2. a) Complete the T-table for Figure 3 and Figure 4.

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   b) Write the number of squares added each time in the circles.

   c) Use the pattern in the circles to find the number of squares in Figure 5 and Figure 6.

3. Make a T-table to predict how many squares are needed for Figure 5 in the pattern. Draw Figure 4 and Figure 5 to check your answer.
Number Lines to 100

0 10 20 30 40 50 60 70 80 90 100

0 10 20 30 40 50 60 70 80 90 100

0 10 20 30 40 50 60 70 80 90 100
Number Lines

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

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Number Lines with Large Numbers

200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220

480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500


700 705 710 715 720 725 730 735 740 745 750 755 760 765 770 775 780 785 790 795 800

800 810 820 830 840 850 860 870 880 890 900 910 920 930 940 950 960 970 980 990 1000

Blackline Master — Patterns and Algebra — Teacher Resource for Grade 3

N-53
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## Calendars

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Month:
Goals

Students will use structure (place value, properties of operations) to reduce the extent of the search needed to solve a problem.

PRIOR KNOWLEDGE REQUIRED

Can add within 1000
Can subtract within 1000 (for Problem Banks 4–8)
Can write the related addition sentence for a given subtraction sentence (for Problem Banks 6–8)

Comparing additions with regrouping to additions without regrouping.

Write the following questions on the board and have volunteers write the answers:

\[
\begin{align*}
34 &+ 25 \\
59 &
\end{align*}
\quad
\begin{align*}
34 &+ 27 \\
61 &
\end{align*}
\]

ASK: What is different about how you find the answers to these questions? (the first problem doesn’t need regrouping, the second problem does)

SAY: When there is no regrouping, you can just add the ones and tens to get the digits in the answer. Write on the board:

\[
\begin{align*}
4 + 5 &= 9 \\
3 + 2 &= 5
\end{align*}
\]

SAY: But when there is regrouping, you can’t do that. Write on the board:

\[
\begin{align*}
4 + 7 &\text{ is not 1} \\
3 + 2 &\text{ is not 6}
\end{align*}
\]

Finding missing digits in additions. Write on the board:

\[
\begin{align*}
2 &+ 15 \\
3 &7
\end{align*}
\]

SAY: We are going to find missing digits in addition puzzles, starting with problems that don’t need regrouping. ASK: How can you tell right away that the ones don’t need regrouping? (5 is less than 7, or \(2 + 1 = 3\) so nothing was added to the tens) To ensure students understand both ways, SAY: You can’t add any one-digit number to 5 to get 17, so the sum must be 7, not 17. So, you can tell just by looking at the ones. But you can also tell that there is no regrouping by looking at the tens. If any ones were regrouped, you would have to add them to the \(2 + 1\) to get 3, but you can’t add anything to \(2 + 1\) to get 3. So, there are two ways to tell that there is no regrouping in the ones: you can look at the ones or you can look at the tens.
ASK: What digit goes in the box? (2) How do you know? \((2 + 5 = 7)\) Write “2” in the box. SAY: Add the ones digits first and then the tens digits. Let’s show the two additions separately. Write on the board:

\[
\begin{align*}
2 + 5 &= 7 \\
2 + 1 &= 3
\end{align*}
\]

SAY: Checking the ones and the tens separately helps you to make sure you found the correct answer.

**Exercises:** Write the additions for the ones digits and tens digits separately. Then fill in the box.

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<tbody>
<tr>
<td>[2] + 5 = 7</td>
<td>[2] + 1 = 3</td>
<td>[2] + 5 = 3</td>
<td>[2] + 1 = 4</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>8</td>
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**Answers:**
- a) 2 + \[\square\] = 9 and 3 + 4 = 7, so 7 goes in the box;
- b) 7 + 1 = 8 and 2 + \[\square\] = 5, so 3 goes in the box;
- c) 3 + 5 = 8 and \[\square\] + 4 = 9, so 5 goes in the box;
- d) \[\square\] + 1 = 5 and 4 + 3 = 7, so 4 goes in the box

**Using letters for missing numbers.** SAY: Sometimes puzzles use letters instead of boxes for missing digits. This is different from a letter that stands for a number. Here the letter only stands for one digit of a number. Write “3A” on the board. SAY: This means the number with tens digit 3 and ones digit A. You might read it as “three-A” but you really mean “thirty-A.” If A is 4, then 3A is 34.

**Exercises**

1. A is 4. What is the number?

   a) 5A  b) 4A  c) A0  d) A8

   **Bonus**
   e) 35A  f) 444A

   **Answers:** a) 54, b) 44, c) 40, d) 48, Bonus: e) 354, f) 4444

2. What is 5A?

   a) A is 1  b) A is 6  c) A is 5  d) A is 0

   **Answers:** a) 51, b) 56, c) 55, d) 50

Write on the board:

\[
\begin{align*}
3 & \quad A \\
+ & \quad 4 \quad 5 \\
7 & \quad \quad 8
\end{align*}
\]
ASK: What is the addition for the ones digits? (A + 5 = 8) Write “A + 5 = 8” on the board. ASK: So what is A? (3) Erase A in the two-digit sum and write “3” instead. Have a volunteer do the addition to check that you solved the puzzle correctly.

**Exercises:** Find A. Then check your answer by doing the addition.

a) \[ \begin{array}{c} A \ 5 \\ + \ 3 \ 2 \end{array} \]
\[ \begin{array}{c} 5 \ 7 \end{array} \]

b) \[ \begin{array}{c} 4 \ 3 \\ + \ A \ 6 \end{array} \]
\[ \begin{array}{c} 9 \ 9 \end{array} \]

c) \[ \begin{array}{c} 5 \ A \\ + \ 1 \ 4 \end{array} \]
\[ \begin{array}{c} 6 \ 8 \end{array} \]

d) \[ \begin{array}{c} 2 \ 2 \\ + \ 6 \ A \end{array} \]
\[ \begin{array}{c} 8 \ 7 \end{array} \]

**Bonus:**

\[ \begin{array}{c} 3 \ 4 \ A \ 1 \\ + \ 2 \ 5 \ 1 \ 2 \end{array} \]
\[ \begin{array}{c} 5 \ 9 \ 7 \ 3 \end{array} \]

**Answers:** a) 2, b) 5, c) 4, d) 5, Bonus: 6

**Solving puzzles with two digits missing.** SAY: Sometimes two digits are missing, but you can do the question in exactly the same way. Write on the board:

\[ \begin{array}{c} 3 \ A \\ + \ B \ 4 \end{array} \]
\[ \begin{array}{c} 7 \ 5 \end{array} \]

ASK: What is the addition for the ones digits? (A + 4 = 5) So what is A? (1) Erase A and write “1” in its place. ASK: What is the addition for the tens digits? (3 + B = 7) So what is B? (4) Erase B and write “4” in its place. Have a volunteer do the addition to check the answer.

**Exercises:** Solve the puzzle.

a) \[ \begin{array}{c} 2 \ A \\ + \ B \ 6 \end{array} \]
\[ \begin{array}{c} 7 \ 9 \end{array} \]

b) \[ \begin{array}{c} 1 \ 3 \\ + \ A \ B \end{array} \]
\[ \begin{array}{c} 8 \ 4 \end{array} \]

c) \[ \begin{array}{c} A \ 7 \\ + \ 3 \ B \end{array} \]
\[ \begin{array}{c} 5 \ 8 \end{array} \]

**Bonus**

\[ \begin{array}{c} A \ 1 \ 5 \\ + \ 2 \ B \ C \end{array} \]
\[ \begin{array}{c} 9 \ 5 \ 8 \end{array} \]

e) \[ \begin{array}{c} 3 \ 4 \ 2 \ 6 \ 7 \ A \ 5 \\ + \ 2 \ B \ 1 \ 1 \ 2 \ 6 \ 4 \end{array} \]
\[ \begin{array}{c} 5 \ 7 \ 3 \ 7 \ 9 \ 8 \ 9 \end{array} \]

**Answers:** a) 23 + 56 = 79, so A = 3 and B = 5; b) 13 + 71 = 84, so A = 7 and B = 1; c) 27 + 31 = 58, so A = 2 and B = 1; Bonus: d) 715 + 243 = 958, so A = 7, B = 4, and C = 3; e) 3 426 725 + 2 311 264 = 5 737 989, so A = 2, B = 3
Write on the board:

\[
\begin{array}{c}
2 \ A \\
+ 3 \ A \\
\hline
5 \ 8
\end{array}
\]

SAY: In this type of puzzle, different letters must stand for different digits and the same letters must stand for the same digit. So \(A\) has to be the same digit for both numbers. Let’s try the possible digits in order starting with 1.

Write on the board:

\[
\begin{array}{c}
2 \ 1 \\
+ 3 \ 1 \\
\hline
2 \ 2
\end{array}
\]
\[
\begin{array}{c}
2 \ 2 \\
+ 3 \ 2 \\
\hline
2 \ 3
\end{array}
\]
\[
\begin{array}{c}
2 \ 3 \\
+ 3 \ 3 \\
\hline
2 \ 4
\end{array}
\]
\[
\begin{array}{c}
2 \ 4 \\
+ 3 \ 4 \\
\hline
2 \ 8
\end{array}
\]

Have volunteers do the additions. (52, 54, 56, 58) ASK: So what is the missing digit? (4) Is there an easier way to see that \(A\) is 4 without trying all the possibilities? \((A + A = 8\) or 18, so try 4 or 9) SAY: Four works right away, so we know that \(A\) is 4 with a lot less checking.

**Exercises:** Solve the puzzle. Make sure the same letter stands for the same digit and different letters stand for different digits.

a) \[
\begin{array}{c}
3 \ A \\
+ A \ 5 \\
\hline
7 \ 9
\end{array}
\]

b) \[
\begin{array}{c}
\ A \ 4 \\
+ 2 \ B \\
\hline
3 \ 8
\end{array}
\]

c) \[
\begin{array}{c}
3 \ A \\
+ 3 \ A \\
\hline
6 \ 4
\end{array}
\]

d) \[
\begin{array}{c}
\ A \ 5 \\
+ A \ 5 \\
\hline
7 \ 7
\end{array}
\]

e) \[
\begin{array}{c}
3 \ A \\
+ B \ 3 \\
\hline
9 \ 7
\end{array}
\]

f) \[
\begin{array}{c}
\ A \ 4 \\
+ 7 \ B \\
\hline
8 \ 6
\end{array}
\]

**Bonus:** \[
\begin{array}{c}
1 \ A \\
+ \ A \ 4 \\
\hline
7 \ 9
\end{array}
\]

**Answers:** a) \(A = 4\); b) \(A = 1, B = 4\); c) \(A = 2\); d) \(A = 2\); e) \(A = 4, B = 6\); f) \(A = 1, B = 2\); Bonus: \(A = 3\)

**Regrouping to solve puzzles with one unknown.** Tell students that you are going to give them a puzzle that is a bit trickier, but you’re not going to tell them how it is trickier. Write on the board:

\[
\begin{array}{c}
1 \ A \\
+ 3 \ A \\
\hline
5 \ 6
\end{array}
\]

Give students a few minutes to solve the puzzle. Tell students to check their answers by adding the numbers and making sure they get 56. SAY: If you are having trouble, you can try all the possible digits in order from zero.

Write on the board:

\[
\begin{array}{c}
1 \ 0 \\
+ 3 \ 0 \\
\hline
4 \ 0
\end{array}
\]
\[
\begin{array}{c}
1 \ 1 \\
+ 3 \ 1 \\
\hline
4 \ 2
\end{array}
\]
\[
\begin{array}{c}
1 \ 2 \\
+ 3 \ 2 \\
\hline
4 \ 4
\end{array}
\]
\[
\begin{array}{c}
1 \ 3 \\
+ 3 \ 3 \\
\hline
4 \ 6
\end{array}
\]
\[
\begin{array}{c}
1 \ 4 \\
+ 3 \ 4 \\
\hline
4 \ 8
\end{array}
\]
\[
\begin{array}{c}
1 \ 5 \\
+ 3 \ 5 \\
\hline
5 \ 0
\end{array}
\]
\[
\begin{array}{c}
1 \ 6 \\
+ 3 \ 6 \\
\hline
5 \ 2
\end{array}
\]
\[
\begin{array}{c}
1 \ 7 \\
+ 3 \ 7 \\
\hline
5 \ 4
\end{array}
\]
\[
\begin{array}{c}
1 \ 8 \\
+ 3 \ 8 \\
\hline
5 \ 6
\end{array}
\]
\[
\begin{array}{c}
1 \ 9 \\
+ 3 \ 9 \\
\hline
5 \ 8
\end{array}
\]
Have volunteers do the additions simultaneously. ASK: In the addition $1A + 3A = 56$, what is $A$? (8) Why was this problem harder? ($A + A$ isn’t 6; instead $A + A = 16$ regrouped) Did anyone try 3 at first to see if it worked? (answers may vary) Why doesn’t 3 work? ($3 + 3$ is 6, but $1 + 3$ in the tens digits is 4 tens, not 5) Did trying 3, even though it didn’t work, help you solve the puzzle? (answers may vary) If some students answer yes, ASK: How did it help you solve it? (it helped me rule out 3; it made me realize that the only way to get 5 tens and 6 ones is to regroup)

Write on the board:

$$
\begin{array}{c}
1 \ A \\
+ \ 3 \ A \\
\hline
5 \ 6
\end{array}
$$

SAY: You can add the tens and the ones separately, just like you do when all the digits are known. ASK: If the tens add to 40, what do the ones have to add to? (16) So what is $A$? (8) SAY: When you add the tens and then the ones separately, you don’t have to try all the numbers in order, and that can save you a lot of work.

**Exercises:** Find $A$. Hint: Add the tens and ones separately.

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<td>A 7</td>
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**Answers:** a) 7, b) 8, c) 3, d) 2, e) 7, Bonus: 7

**Regrouping to solve puzzles with two unknowns.** Write on the board:

$$
\begin{array}{c}
3 \ A \\
+ \ B \\
\hline
6 \ 2
\end{array}
$$

SAY: Let’s look at the ones digits. ASK: Can $A + 5$ be 2? (no) Why not? (2 is less than 5) So how can $A + 5$ have the ones digit 2? (the sum must be 12) PROMPT: What is the next number with the ones digit 2? (12) Write on the board:

$$
A + 5 = 12
$$

ASK: What number works here for $A$? (7) SAY: If you are not sure how to guess, you can guess the digits in order.
Write on the board:

\[
\begin{align*}
0 + 5 &= \\
1 + 5 &= \\
2 + 5 &= \\
3 + 5 &= \\
4 + 5 &= \\
5 + 5 &= \\
6 + 5 &= \\
7 + 5 &= \\
8 + 5 &= \\
9 + 5 &= 
\end{align*}
\]

Have a volunteer write the answers to all these questions. Circle \(7 + 5 = 12\).

SAY: So A is 7. When you’re trying to find more than one letter, it is a good idea to rewrite the problem as soon as you find one letter because it might help you find the next letter. Write on the board:

\[
\begin{array}{c}
\phantom{+} 3 \ 7 \\
+ B \ 5 \\
\hline
6 \ 2 \\
\end{array}
\]

SAY: When I add 7 and 5 in the ones digits, I get 12. I regroup and include the 1 in the tens digits. Write on the board:

\[
1 + 3 + B = 6
\]

ASK: What works for B? (2) SAY: So now we have solved the puzzle completely. Write on the board:

\[
\begin{array}{c}
3 \ 7 \\
+ 2 \ 5 \\
\hline
6 \ 2 \\
\end{array}
\]

Have a volunteer do the addition to check that the answer is 62.

**Exercises:** Solve the puzzle.

\[
\begin{array}{cccccc}
a) & 2 & A & + & B & 7 \\
    & 4 & 6 & & & \\
b) & A & 4 & + & 5 & B \\
    & 9 & 1 & & & \\
c) & 3 & A & + & 8 & \\
    & B & 6 & & & \\
d) & 5 & A & + & 2 & 9 \\
    & B & 3 & & & \\
\end{array}
\]

**Bonus**

\[
\begin{array}{cccccc}
e) & 7 & 6 & 4 & A & 3 & 2 & 1 & 5 \\
    & 1 & 3 & 8 & 2 & 9 & 1 & 6 & 4 & 7 \\
f) & 3 & A & 5 & + & 4 & 7 \\
    & 9 & 2 & C & & & \\
g) & A & 8 & + & 5 & B \\
    & 1 & 6 & 2 & & & \\
\end{array}
\]

**Answers:** a) \(A = 9, B = 1\); b) \(A = 3, B = 7\); c) \(A = 8, B = 4\); d) \(A = 4, B = 8\);

**Bonus:** e) \(A = 3, B = 1\); f) \(A = 7, B = 5, C = 2\); g) \(A = 5, B = 7\)
Problem Bank

1. Solve the puzzle.

   a) \[ \begin{array}{c}
   3 & 6 \\
   + & 5 & A & B \\
   \hline
   9 & 7 & 8
   \end{array} \]

   b) \[ \begin{array}{c}
   7 & \_ & \_ \\
   + & A & B \\
   \hline
   6 & 9
   \end{array} \]

   c) \[ \begin{array}{c}
   1 & 3 & A \\
   + & B & 5 & A \\
   \hline
   6 & 9 & 4
   \end{array} \]

   d) \[ \begin{array}{c}
   6 & \_ \\
   + & A & B \\
   \hline
   7 & 4
   \end{array} \]

   e) \[ \begin{array}{c}
   2 & \_ \\
   + & 1 & A & B \\
   \hline
   5 & 7 & 6
   \end{array} \]

   Bonus: \[ \begin{array}{c}
   1 & \_ \\
   + & A & B \\
   \hline
   7 & 0
   \end{array} \]

   **Answers:** a) \( A = 4, B = 2 \); b) \( A = 3, B = 2 \); c) \( A = 7, B = 5 \); d) \( A = 3, B = 8 \); e) \( A = 4, B = 8 \); Bonus: \( A = 2, B = 4 \)

2. Write the puzzle vertically, then solve.

   a) \[ \begin{array}{c}
   3A \\
   + 4A \\
   \hline
   88
   \end{array} \]

   b) \[ \begin{array}{c}
   A4 \\
   + A7 \\
   \hline
   91
   \end{array} \]

   c) \[ \begin{array}{c}
   5A \\
   + B7 \\
   \hline
   89
   \end{array} \]

   d) \[ \begin{array}{c}
   7A \\
   + AB \\
   \hline
   9A
   \end{array} \]

   e) \[ \begin{array}{c}
   AA \\
   + AB \\
   \hline
   8A
   \end{array} \]

   **Answers:** a) \( A = 9 \); b) \( A = 4 \); c) \( A = 2, B = 3 \); d) \( A = 3, B = 5 \); e) \( A = 8 \); f) \( A = 4, B = 5 \); g) \( A = 3, B = 1 \)

3. Solve the puzzle. Hint: Write the puzzle vertically.

   a) \[ \begin{array}{c}
   A3 \\
   + 2B \\
   \hline
   83
   \end{array} \]

   b) \[ \begin{array}{c}
   A7 \\
   + 6B \\
   \hline
   97
   \end{array} \]

   c) \[ \begin{array}{c}
   36 \\
   + AB \\
   \hline
   86
   \end{array} \]

   d) \[ \begin{array}{c}
   7A \\
   + AB \\
   \hline
   9A
   \end{array} \]

   e) \[ \begin{array}{c}
   AA \\
   + AB \\
   \hline
   8A
   \end{array} \]

   Did solving parts a) to c) help you to solve parts d) and e)? Explain.

   **Answers:** a) \( A = 6, B = 0 \); b) \( A = 3, B = 0 \); c) \( A = 5, B = 0 \); d) \( A = 2, B = 0 \); e) \( A = 4, B = 0 \); Yes, it helped me recognize that if B does not change the ones digit, then B is 0.

4. Solve the subtraction puzzle.

   a) \[ \begin{array}{c}
   5A \\
   - 2 \\
   \hline
   B5
   \end{array} \]

   b) \[ \begin{array}{c}
   5A \\
   - 12 \\
   \hline
   B5
   \end{array} \]

   c) \[ \begin{array}{c}
   5A \\
   - 22 \\
   \hline
   B5
   \end{array} \]

   d) \[ \begin{array}{c}
   5A \\
   - 32 \\
   \hline
   B5
   \end{array} \]

   **Answers:** a) \( A = 7, B = 5 \); b) \( A = 7, B = 4 \); c) \( A = 7, B = 3 \); d) \( A = 7, B = 2 \); A is always the same because it is always true that \( A - 2 = 5 \), so A is always 7; B decreases by 1 because you are always subtracting 10 more, so the tens digit of the answer decreases by 1 each time.

Problem-Solving Lesson 3-4

N-63
5. Solve the puzzle.

\[
\begin{array}{c}
A & 7 & 8 \\
- & 1 & 4 & B \\
\hline
6 & C & 3 \\
\end{array}
\]

**Answer:** \(A = 7, B = 5, C = 3\)

6. Look at the two puzzles.

\[
\begin{array}{c}
3 & A \\
- & 7 \\
\hline
2 & 4 & A \\
\end{array}
\]

a) How are the two puzzles the same?
b) How are the two puzzles different?
c) If you can solve one puzzle, can you solve the other? How do you know?
d) Solve the easier puzzle. Explain your choice.

**Answers:** a) they use all the same numbers, even 3A is common to both; b) one is subtraction and the other is addition; c) they will have the same answer because the addition and subtraction are related; d) \(A = 1\), sample explanation: I picked the addition because I just had to add 24 + 7 and the answer is right there

7. In the puzzle, how can you get \(A\) from 7 + 5? Explain.

\[
\begin{array}{c}
6 & A \\
- & B & 7 \\
\hline
1 & 5 \\
\end{array}
\]

**Answer:** Write \(6A - B7 = 15\) as \(15 + B7 = 6A\). The ones digits add to \(5 + 7 = 12\), so \(A = 2\).

8. Turn the subtraction puzzle into an addition puzzle and solve.

\[
\begin{array}{c}
a) & 7 & A \\
- & B & 4 \\
\hline
1 & 8 \\
b) & 5 & A \\
- & 2 & 6 \\
\hline
B & 9 \\
c) & 9 & A \\
- & B & 2 \\
\hline
6 & 8 \\
\end{array}
\]

**Bonus:**

\[
\begin{array}{c}
A & 7 & 3 \\
- & 3 & B & 9 \\
\hline
6 & 1 & C \\
\end{array}
\]

**Answers:** a) \(A = 2, B = 5\); b) \(A = 5, B = 2\); c) \(A = 0, B = 2\);
**Bonus:** \(A = 9, B = 5, C = 4\)
Introduction

In this unit, students are introduced to fractions using paper folding. Beginning with unit fractions, students will learn that the denominator in a fraction is the number of equal parts in the whole and the numerator is the number of selected or shaded parts. They will write a fraction for the shaded region in a shape that has been divided into equal parts and they will shade the equal parts for a given fraction. Students will use fraction strips to compare two fractions and determine which is larger or smaller. Students will compare fractions when either the denominators or the numerators are the same and they will learn how to find the area of shaded regions using half squares.

Meeting Your Curriculum

Alberta—Lesson NS3-68 is recommended, as it expands on the meaning of fractions, demonstrating that fractions are not limited to area and length, but also describe parts of sets or numbers. Lesson NS3-70 is optional. All other lessons are required.

British Columbia—Lesson NS3-68 is recommended, as it expands on the meaning of fractions, demonstrating that fractions are not limited to area and length, but also describe parts of sets or numbers. Lesson NS3-70 is optional. All other lessons are required.

Manitoba—Lesson NS3-68 is recommended, as it expands on the meaning of fractions, demonstrating that fractions are not limited to area and length, but also describe parts of sets or numbers. Lesson NS3-70 is optional. All other lessons are required.

Ontario—Lesson NS3-68 is recommended for Ontario students. Understanding fractions of a set and fractions of a number is essential for understanding the concept of fair game in probability and predicting the probability of events with equal outcomes. Lesson NS3-69 is optional; students will study this material in Grade 4. All other lessons are required.

Fraction notation. We show fractions in two ways in our lesson plans:

- Stacked: \( \frac{1}{2} \)
- Not stacked: 1/2

Remember to only show students the stacked form when teaching fractions.

NOTE: Students in Ontario are not required to write and read fraction notation. However, fraction notation clearly shows the number of parts the whole is divided into and the number of parts used in the fraction. For many students, fraction notation is easier to understand than verbal notation, such as one half. Therefore, we recommend teaching fraction notation to all students.

Materials. Students will use BLM Random Number Spinner (p. O-41) in each lesson. With the BLM lying flat on the desk, have students hold a pencil vertically with the sharpened end going through one end of a paper clip and pressing against the centre of the spinner. Students flick the paper
clip so that it spins around the pencil and eventually stops on one of the sectors of the spinner.

In addition to the BLMs provided at the end of this unit, the following Generic BLM, found in section V, is used in Unit 12:

**BLM 1 cm Grid Paper** (p. V-8)

**Quizzes and Tests**

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>NS3-62 to 67</td>
<td>NS3-62 to 67</td>
<td>NS3-62 to 67</td>
<td>NS3-62 to 67</td>
</tr>
<tr>
<td>Quiz</td>
<td>NS3-68 to 69</td>
<td>NS3-68 to 69</td>
<td>NS3-68 to 69</td>
<td>NS3-68, 70</td>
</tr>
<tr>
<td>Test</td>
<td>NS3-62 to 67, 69</td>
<td>NS3-62 to 67, 69</td>
<td>NS3-62 to 67, 69</td>
<td>NS3-62 to 67, 70</td>
</tr>
</tbody>
</table>
NS3-62  Equal Paper Folding

Pages 50–51

Goals

Students will fold paper to create and identify unit fractions.

PRIOR KNOWLEDGE REQUIRED

none

MATERIALS

- paper clips
- BLM Random Number Spinner (p. O-41) per pair of students
- scissors
- BLM Folding Paper (pp. O-42–43)
- large piece of paper or construction paper
- BLM More Folding Paper (p. O-44, see Extension)

Mental math minute. Students work in pairs. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10 (see the unit introduction for instructions on using the spinners). Each pair creates two multiplication sentences using the numbers. For example, if Student A spins 7 and Student B spins 8, they create the multiplication sentences $8 \times 7 = 56$ and $7 \times 8 = 56$. When pairs are done working, ask all students to stand up. One at a time, ask each pair of students to say the multiplication sentences they created. Then the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Creating unit fractions by folding paper. Provide scissors and BLM Folding Paper to students. The dotted lines in each diagram indicate where to fold the shape. Some students may lack the dexterity to fold neatly along the dotted lines, so you may wish to circulate and provide assistance as needed.

Ask students to cut out the first figure on the BLM. Demonstrate how to fold along the dotted line using a large piece of paper or construction paper. Ask students to fold along the dotted line and then open the figure. ASK: Into how many parts did you fold the paper? (2) Are the parts equal or are they different? (equal) How many parts make up the whole? (2) SAY: When we break up the whole into parts, each part is called a fraction. When there are two equal parts in the whole, each part is called one half. The plural of half is halves. Draw on the board:

```
  one half
  
  one half

There are 2 halves in a whole.
```
SAY: We can fold more than once to get more equal parts. Hold up the paper and repeat the first fold. While holding the folded paper, fold it a second time by bringing the top down to the bottom. Ask students to copy what you did with their paper. Then unfold the paper, as shown in the margin.

ASK: How many parts are there? (4) Are the parts equal? (yes) SAY: When there are four equal parts in the whole, we call each part one fourth, or one quarter. Label the fourths on your picture on the board.

Draw on the board:

<table>
<thead>
<tr>
<th>Picture</th>
<th>Number of Equal Parts</th>
<th>Name of Each Fraction Part</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="unfolded.png" alt="Diagram" /></td>
<td>2</td>
<td>one half</td>
</tr>
</tbody>
</table>

Have a volunteer come to the board and fill in the number of equal parts for each picture. ASK: If the whole is divided into four equal parts, what do we call each part? (one fourth or one quarter) SAY: If the whole is divided into three equal parts, each part is called one third. Write “one third” in the third column for that picture. Repeat for six parts (one sixth) and eight parts (one eighth).

SAY: It is important that each equal part is the same size. Holding up a new sheet of paper, perform the folds shown below:
Unfold the paper and hold it up, as shown below:

ASK: Is each part one fourth? (no) Why not? (the parts are not the same size)

NOTE: Students can use the figures they cut out from BLM Folding Paper to help them answer the questions on AP Book 3.2 pp. 50–51.

Extension

Distribute BLM More Folding Paper. Have students cut out each figure and fold the paper to produce the desired number of equal parts.

Answers

a)

b)

c)
Goals

Students will identify unit fractions by first counting the total number of equal parts in the whole.

PRIOR KNOWLEDGE REQUIRED

none

MATERIALS

paper clips
BLM Random Number Spinner (p. O-41) per pair of students

Mental math minute. Students work in pairs. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10. Each pair creates two division sentences using the numbers. For example, if Student A spins 7 and Student B spins 8, they create the division sentences $56 \div 8 = 7$ and $56 \div 7 = 8$. When pairs are done working, ask all students to stand up. One at a time, ask each pair of students to say the division sentences they created. Then the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Review names of fractions with denominators 2, 3, 4, 6, and 8. Draw on the board:

\[
\begin{array}{c}
\text{number of parts shaded} \\
1 \\
2
\end{array}
\]

ASK: How many folds did we use to make this picture? (1) Describe the fold. (fold down) How many equal parts are in the whole? (2) Shade one part. ASK: What do we call one of these two equal parts? (one half) Continue drawing on the board:

\[
\begin{array}{c}
\text{number of parts shaded} \\
1 \\
2
\end{array}
\]

SAY: We write the fraction “one half” using two numbers separated by a fraction sign. The top number tells us how many parts are shaded. The bottom number tells us how many parts are in the whole. Draw on the board:

\[
\begin{array}{c}
\text{number of parts shaded} \\
1 \\
2
\end{array}
\]

ASK: How many folds did we use to make this picture? (2) Describe two ways we can make the folds. (fold down, then across; or fold across, then down) How many equal parts are in the whole? (4) Shade one part.
ASK: What do we call one of these four equal parts? (one fourth) How do we write the fraction with numbers? (1/4)

Draw on the board:

ASK: How many folds did we use to make this picture? (3) Describe two ways we can make the folds. (fold down, then fold across twice; or fold across twice, then down) Demonstrate both methods using two different sheets of paper. ASK: How many equal parts are in the whole? (8) Shade one part. ASK: What do we call one of these eight equal parts? (one eighth) How do we write the fraction with numbers? (1/8)

Draw on the board:

ASK: How many folds did we use to make the picture on the left? (2) How many equal parts are in the whole? (3) Shade one part. ASK: What do we call one of these three equal parts? (one third) How do we write the fraction with numbers? (1/3) ASK: How can we continue folding the diagram on the left to get the diagram on the right? (fold down) How many equal parts are in the whole? (6) What do we call one of these six equal parts? (one sixth) How do we write the fraction with numbers? (1/6)

Have students complete Question 1 on AP Book 3.2 p. 52.

Introduce unit fractions. ASK: In each picture in Question 1, how many parts are shaded? (1) What is the top number in each fraction? (1) SAY: If the top number in a fraction is 1, we call the fraction a unit fraction. Draw the picture in the margin on the board. ASK: How many parts are shaded? (1) How many equal parts are in the whole? (4) SAY: A unit fraction has only one equal part shaded. Ask a volunteer to write the fraction for the picture using words. (one fourth) Ask another volunteer to write the fraction using numbers. (1/4)

SAY: If the picture has five parts, we call each part “one fifth” and write 1/5. If the picture has seven parts, we call each part “one seventh” and write 1/7.

Write on the board:

5 parts ——— each part is called one fifth \( \frac{1}{5} \)
7 parts ——— each part is called one seventh \( \frac{1}{7} \)
9 parts ——— each part is called ______

ASK: What do you think each part is called if the whole is divided into nine parts? (one ninth) How do we write the fraction with numbers? (1/9) Write the answers in the blank on the board.
Sorting fractions into unit fractions and non-unit fractions.

SAY: Remember, a unit fraction has only one equal part shaded. Draw on the board:

```
  ___     ___
 /     /   |
```

ASK: How many equal parts are in the whole? (5) How many parts are shaded? (1) What fraction can we write for the shaded area? (1/5) Is this a unit fraction? Explain. (yes, only one part is shaded) How can we tell by looking at the fraction that it is a unit fraction? (the top part of the fraction is 1) Write on the board:

```
1/3, 2/3, 3/4, 1/5, 2/7
```

ASK: Which fractions are unit fractions? (1/3, 1/5) How can you tell just by looking at the numbers in the fraction? (the top number is 1)

Exercises

a) Circle the unit fractions:

```
3/11, 3/8, 2/16
```

b) Explain why 3/1 is not a unit fraction.

Answers: a) circle 1/8, 1/2, 1/6; b) the number 1 is in the bottom of the fraction instead of the top of the fraction

Extensions

1. Draw a line in the picture so that it shows the written fraction. Hint: The parts in the whole must be equal parts.

   a) \( \frac{1}{4} \)

   b) \( \frac{1}{3} \)

   c) \( \frac{1}{6} \)

   d) \( \frac{1}{8} \)

   
   Answers: a) b) c) d)

2. Draw one line so that \( \frac{1}{8} \) of the picture can be shaded.

   
   Answer:
Goals

Students will write fractions for a given picture, including fractions that are not unit fractions.

Students will shade in a picture to show a given fraction.

PRIOR KNOWLEDGE REQUIRED

Can write a unit fraction for a picture
Can shade a picture to represent a unit fraction

MATERIALS

paper clips
BLM Random Number Spinner (p. O-41) per pair of students

Mental math minute. Students work in pairs. Each student makes a two-digit number by spinning a paper clip twice on BLM Random Number Spinner: once to make the tens digit, and again to make the ones digit. If the paper clip lands on 10, students should spin again until they get a one-digit number. The pair creates two addition sentences using the numbers they made. For example, if Student A spins 7 and 3 and Student B spins 8 and 2, they will create the addition sentences $73 + 82 = 155$ and $82 + 73 = 155$. When pairs are done working, ask all students to stand up. One at a time, ask each pair of students to say the addition sentences they created. Then the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Introduce numerator and denominator. Draw on the board:

![Fraction Image]

ASK: How many equal parts are there? (4) How many parts are shaded? (3)

SAY: We write the fraction as $\frac{3}{4}$. Write on the board:

$$\frac{3}{4}$$

numerator
denominator

SAY: We call the top number of a fraction the numerator. The numerator tells how many equal parts are shaded. We call the bottom number of a fraction the denominator. The denominator tells how many equal parts are in the whole.
Exercises

1. Count the number of shaded parts and the number of equal parts in the whole. Then write the fraction shown by the shaded parts.

   a)  
   _____ number of shaded parts  
   _____ number of parts in the whole
   The fraction is ____. 

   b)  
   _____ number of shaded parts  
   _____ number of parts in the whole
   The fraction is ____. 

   c)  
   _____ number of shaded parts  
   _____ number of parts in the whole
   The fraction is ____. 

   d)  
   _____ number of shaded parts  
   _____ number of parts in the whole
   The fraction is ____. 

   Answers: a) 4, 5, 4/5; b) 2, 3, 2/3; c) 5, 8, 5/8; d) 4, 6, 4/6

2. Write a fraction for the picture.

   a)  
   b)  

   c)  

   Answers: a) 6/8, b) 3/5, c) 5/6

Shading a picture for a given fraction. Draw on the board:

   3
   8

   ASK: What is the top number of a fraction called? (numerator) What is the numerator here? (3) What does the numerator tell us? (how many parts should be shaded) What is the bottom number of a fraction called? (denominator) What is the denominator here? (8) What does the denominator tell us? (how many equal parts are in the whole) Count the parts aloud to make sure it’s correct. Ask a volunteer to shade 3/8 on the board.
Exercises: Shade parts to show the fraction.

a) \(\frac{3}{4}\)  

b) \(\frac{7}{10}\)  

c) \(\frac{2}{5}\)

Sample answers

a)  
b)  
c)  

Writing a fraction for the unshaded part. Draw on the board:

ASK: How many parts are shaded? (1) How many equal parts are there? (4) What fraction can we write for the shaded part? (1/4) How many parts are not shaded? (3) How many equal parts are there? (4) What fraction can we write for the unshaded parts? (3/4)

Exercises: Write a fraction for the parts that are not shaded.

a)  
b)  
c)  

Bonus: A circle is divided into 100 equal parts. \(\frac{93}{100}\) of the circle is shaded. What fraction of the circle is not shaded?

Answers: a) 2/6, b) 3/10, c) 3/4, Bonus: 7/100

Writing fractions in words for a picture. Draw on the board:

Each part is ________.

_____ parts are shaded.

The fraction is ________.

ASK: How many equal parts are in the whole? (4) What do we call each of the equal parts? (one fourth) Fill in the answer in words in the spaces on the board. ASK: How many parts are shaded? (3) Write “three” in the space on the second line on the board. ASK: How do we write the fraction using words? (three fourths) Write the answer in words in the spaces provided on the third line.
Exercises: Fill in the blanks to write a fraction in words for the shaded area of the picture.

a) Each part is one _____. ____ parts are shaded. The fraction is _____ _____.
b) Each part is one _____. ____ parts are shaded. The fraction is _____ _____.

Answers: a) sixth, five, five sixths; b) eighth, three, three eighths

Reinforcing equal parts. Draw on the board:

SAY: This is a pizza. I divided the pizza for four students. ASK: Does it seem fair? (no) Why? (some students will get a bigger slice than others) Erase the two lines dividing the pizza and ask a volunteer to draw a line to divide the pizza so that two students can share the pizza equally. Ask another volunteer to add a line to divide the pizza so that four students can share the pizza equally. The picture is shown in the margin.

ASK: Does the division seem fair now? (yes) Why? (each student gets the same size slice) SAY: For fractions, all the parts must be equal.

Identifying fractions and non-fractions. SAY: Remember that fractions are written using two numbers. The numerator tells you how many parts are shaded. The denominator tells you how many equal parts are in the whole. There is a horizontal line between the numerator and the denominator.

Write \( \frac{3}{8} \) on the board. ASK: Which number is the numerator? (3) Which number is the denominator? (8) How many equal parts are in the whole? (8) How many parts are shaded? (3) Continue writing on the board:

\[
\begin{align*}
\frac{3}{8} & \quad 2 & \frac{4}{5} & \quad 3 & \frac{7}{1} & \frac{1}{4}
\end{align*}
\]

ASK: Which numbers are fractions? (3/8, 4/5, 1/4) Why are the other numbers not fractions? (they don’t have a numerator and a denominator)

Finding fractions in everyday life. SAY: Wherever a whole is divided into parts, we can find fractions. An example is found in baking. Draw on the board:
SAY: This is a cup used in baking. A recipe calls for 3/4 of a cup of flour. ASK: How many parts does the cup have? (4) Is this the numerator or denominator? (denominator) If I want to shade 3/4 of the cup, how many parts of the picture should I shade? (3) Shade in three parts, as shown in the margin. SAY: To follow the recipe, I have to fill the shaded part of the cup with flour.

Exercises

1. Tom and Sally are shovelling snow from the driveway. Tom shovelled the shaded area.

   ![](image)

   a) How many equal parts are in the whole driveway?
   b) How many parts has Tom shovelled?
   c) What fraction of the driveway has Tom shovelled?
   d) How many parts are not shovelled?
   e) What fraction of the driveway does Sally have to shovel?

   **Answers:** a) 8, b) 5, c) 5/8, d) 3, e) 3/8

2. A Ferris wheel is in the shape of a circle. Cathy and her friends are sitting in the shaded area of the Ferris wheel.

   ![](image)

   a) How many equal parts are in the whole?
   b) How many parts are occupied by Cathy and her friends?
   c) What fraction of the seats do Cathy and her friends occupy?
   d) How many parts are not shaded?
   e) What fraction of the seats are not occupied by Cathy and her friends?

   **Answers:** a) 20, b) 8, c) 8/20, d) 12, e) 12/20
Extensions

1. A circle is divided into equal parts that are shaded, dotted, or white. The fraction $\frac{2}{8}$ represents the shaded parts. The fraction $\frac{5}{8}$ represents the dotted parts. What fraction represents the white parts? Hint: Draw a picture.

   \[ \text{Answer: } \frac{1}{8} \]

2. Jin divides a triangle into four equal parts and shades three parts. Then he divides each equal part into two equal parts. What fraction represents the shaded parts?

   \[ \text{Answer: } \frac{6}{8} \]

3. Tessa divides a rectangle into equal parts that are shaded or white. Then she shades more parts to double the number of shaded parts. The fraction $\frac{6}{10}$ represents the shaded parts. How many parts were white at the start?

   \[ \text{Answer: } 7 \]

4. The medicine wheel is an important symbol in many First Nation cultures. It can be used to show the four parts of a person, as shown below:

   ![Medicine Wheel Diagram]

   a) Shade the parts of the circle that are not physical.

   b) What fraction of the circle shows the parts of a person that are not physical?

   \[ \text{Answers: a) } \text{b) } \frac{3}{4} \]
Goals
Students will use pattern blocks to find the fraction of a bigger pattern block.

PRIOR KNOWLEDGE REQUIRED
Can write a fraction for a shape with equal parts
Can shade a picture to represent a given fraction

MATERIALS
paper clips
BLM Random Number Spinner (p. O-41) per pair of students
paper labels with names of shapes written on them (triangle, circle, rhombus, parallelogram, square, trapezoid, hexagon, rectangle)
pattern blocks or BLM Pattern Blocks (p. O-45)

Mental math minute. Students work in pairs. Each student makes a two-digit number by spinning a paper clip twice on BLM Random Number Spinner: once to make the tens digit, and again to make the ones digit. If the paper clip lands on 10, students should spin again until they get a one-digit number. Then the pair subtracts the smaller number from the larger number to create two subtraction sentences. For example, if Student A spins 4 and 2 and Student B spins 2 and 3, they will create the subtraction sentences 42 − 23 = 19 and 42 − 19 = 23. When pairs are done working, ask all students to stand up. One at a time, ask each pair of students to say the subtraction sentences they created. Then the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Review the vocabulary for geometric shapes. Write on the board:

quadrilateral

ASK: How many sides does a quadrilateral have? (4) SAY: “Quad” means four and “lateral” means sides. Draw on the board:

△ □ □ □ ○ ○ ○ □ □ □ □ □

Ask different volunteers to circle the shapes that are quadrilaterals. (all shapes with four sides) Ask other volunteers to affix a label with the correct name of the shape below each picture. (triangle, rectangle, parallelogram, circle, rhombus, trapezoid, square, hexagon)

ASK: Which shape has three sides? (triangle) Six sides? (hexagon)
SAY: Remember, parallel lines are lines that never meet. ASK: Which quadrilaterals have two pairs of sides that are parallel? (rectangle, square,
parallelogram, rhombus) Which shapes have right angles? (rectangle, square) What properties does a rhombus have that other parallelograms do not have? (a rhombus has all sides of equal length)

Provide students with pattern blocks or BLM Pattern Blocks for the following exercises. Demonstrate how to form a shape such as a rhombus using two triangles joined along one side.

**Exercises**

1. Form the shape using only triangles. Count the number of triangles you used.
   a) a rhombus
   b) a trapezoid
   c) a parallelogram that is not a rhombus
   d) a hexagon

   **Sample answers**
   a) 2, b) 3, c) 4, d) 6

2. a) Form a hexagon that matches the yellow hexagon block using only trapezoid blocks.
   b) Form a hexagon that matches the yellow hexagon block using only rhombus blocks.

   **Answers:** a) , b)

3. a) How many trapezoid blocks did you need to form a hexagon in Exercise 2?
   b) How many rhombus blocks did you need to form a hexagon in Exercise 2?

   **Answers:** a) 2, b) 3

**Writing fractions using pattern blocks.** Draw on the board:

ASK: How many triangles will cover the trapezoid? (3) Ask a volunteer to draw a line to show how the triangles would fit, as shown in the margin.

ASK: How many parts are there? (3) Are the parts equal? (yes) How many parts are shaded? (1) What fraction can we write for the shaded part of the picture? (1/3)
**Exercises:** Draw a line so the parts are equal. Then write a fraction for the shaded part using numbers and words.

a) ![Triangle]

b) ![Geometric Shape]

c) ![Hexagon]

**Answers:** a) 1/4, one fourth; b) 2/6, two sixths; c) 1/3, one third

**Equal fractions but different shapes.** Draw on the board:

- ![Rectangle 1]
- ![Rectangle 2]

Ask a volunteer to draw a line to connect the dots in the first rectangle. Ask a different volunteer to do the same for the second rectangle. When they are done, shade one part in each picture, as shown below:

- ![Shaded Rectangle 1]
- ![Shaded Rectangle 2]

ASK: In each rectangle, what is the total number of equal parts? (2) How many parts are shaded? (1) What is the fraction for the shaded part? (1/2) Is the shaded part in each rectangle the same shape? (no) SAY: For some shapes, you can shade the same fraction in different ways.

**Exercises**

1. Find a different way to divide a rectangle in two equal parts. Each part needs to have a different shape than the ones you have already seen.

   **Sample answer:**

   ![Different Rectangle Division]

2. Josh divided a triangle in two different ways to show the fraction $\frac{1}{2}$:

   ![Different Triangle Division]

   Is he correct? Explain.

   **Answer:** no, the parts are not equal in the second picture

3. a) Abella divided a hexagon to show the fraction $\frac{1}{2}$:

   ![Different Hexagon Division]

   Is she correct? Explain.

   b) Draw a different way to divide the hexagon to show the fraction $\frac{1}{2}$.

   **Sample answer:** b) ![Different Hexagon Division]

   **Answers:** a) yes, the parts are equal
4. Shade the parts to show \( \frac{1}{4} \) of a square in different ways.
   a)   b)

   **Sample answers:** a)   b)

5. Add a line to the picture to make 4 equal parts. Then shade \( \frac{1}{4} \).
   a)   b)

   **Answers:** a)   b)

**Extensions**

1. What fraction of the shapes are rectangles?
   
   Answer: \( \frac{2}{3} \) (note: a square is also a rectangle)

2. What fraction of the shapes are parallelograms?
   
   Answer: \( \frac{2}{3} \) (note: a square is also a parallelogram)

3. Bees store honey in cells that have the shape of a hexagon.
   
   a) How many hexagons does the picture of the honeycomb show?
   b) How many rhombuses would it take to build the honeycomb?
   c) How many triangles would it take to build the honeycomb?
   d) What fraction of the honeycomb is shaded?

   **Answers:** a) 7, b) 21, c) 42, d) \( \frac{1}{7} \)
**Goals**

Students will divide a shape into equal parts in more than one way.

**PRIOR KNOWLEDGE REQUIRED**

Can recognize when a shape is divided into equal parts
Can shade a picture to represent a given fraction

**MATERIALS**

- paper clips
- BLM Random Number Spinner (p. O-41) per pair of students

**Mental math minute.** Students work in pairs. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10. One student creates a multiplication sentence from the two numbers and the other student creates a corresponding addition sentence for the multiplication. For example, if students spin the numbers 5 and 3, Student A creates the multiplication sentence $5 \times 3 = 15$ and Student B creates the addition sentence $3 + 3 + 3 + 3 + 3 = 15$. (Alternatively, Student A creates the multiplication sentence $3 \times 5 = 15$ and Student B creates the addition sentence $5 + 5 + 5 = 15$.) When pairs are done working, ask all students to stand up. One at a time, ask each pair of students to say the sentences they created. Then the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

**Shading the same fraction in different ways.** Draw on the board:

```
  
  
  
```

SAY: I want to divide this shape into three equal parts. Add two vertical lines to divide the shape into three equal parts, as shown below:

```
  
  
  
```

ASK: How many equal parts are there? (3) Shade one part. ASK: How many of the equal parts are shaded? (1) What fraction can we write for the shaded part? (1/3) SAY: This is not the only way to divide the rectangle into three equal parts. Erase the vertical lines and the shaded region to get the original rectangle. Add two horizontal lines to divide the shape into three equal parts, as shown in the margin.

Shade one part. ASK: How many of the equal parts are shaded? (1) What fraction can we write for the shaded part? (1/3) SAY: We used the same rectangle and were able to show the fraction 1/3 in two different ways.
Exercises

1. Shade $\frac{1}{6}$ of the shape in different ways.

2. Shade $\frac{1}{8}$ of the shape in different ways.

Dividing a shape into equal parts. Draw on the board:

SAY: I divided this party-sized pizza into two parts to share between our class and the class next door. ASK: Does it seem fair to both classes? (no) Why? (one class will get a lot more pizza) How can I move the dividing line to make it fair for both classes? (move the line to the right until it reaches the middle) Ask a volunteer to erase the dividing line and then divide the rectangle into two equal parts, as shown below:

Erase the dividing line. SAY: Suppose both classes want a long, thin pizza. I want to draw a horizontal line to divide the rectangle into two equal parts. Ask a volunteer to draw the new line, as shown below:

Erase the dividing line. SAY: We used a horizontal line and a vertical line to divide the pizza into two equal parts in different ways. ASK: What other kind of line can we draw to divide the pizza into two equal parts? (diagonal) Ask a volunteer to draw the new line—there are two possibilities, as shown in the margin.

Exercises: Add one line to divide the shape into four equal parts.

a)  

b)  

c)  

Answers: a)  

b)  

c)  
Extensions

1. An octagon has eight sides.

a) Draw lines to divide the shape into eight equal parts.
   Then shade \( \frac{1}{8} \).

b) Draw a different way to divide the octagon into eight equal parts.

Answers: a) \( \) \( \) b) \( \)

2. Find four different ways to divide the shape in half by drawing only one line.
   a) square   b) hexagon
   c) octagon   d) a shape with 10 sides

Sample answers
   a)
   b)
   c)
   d)

3. Find the name for a polygon with 10 sides.

Answer: decagon

4. How many ways can you divide a circle in half by drawing only one line?

Answer: infinite (unlimited number)
Goals
Students will shade the same fractions using different shapes.

PRIOR KNOWLEDGE REQUIRED
Can write a fraction for a shape with equal parts
Can divide a shape into equal parts
Knows that equal fractions of the same whole can have different shapes

MATERIALS
paper clips
BLM Random Number Spinner (p. O-41) per pair of students
transparency of BLM 1 cm Grid Paper (p. V-8)
overhead projector
erasable marker

Mental math minute. Students work in pairs. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10. One student creates a division sentence from the two numbers and the other student creates a corresponding addition sentence for the division. For example, if students spin the numbers 5 and 3, Student A creates the division sentence 15 ÷ 3 = 5 and Student B creates the addition sentence 3 + 3 + 3 + 3 + 3 = 15. (Alternatively, Student A creates the division sentence 15 ÷ 5 = 3 and Student B creates the corresponding addition sentence 5 + 5 + 5 = 15.) When pairs are done working, ask all students to stand up. One at a time, ask each pair of students to say the sentences they created. Then the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Identifying common characteristics of a given set of fractions. Write on the board:

\[
\begin{array}{cccc}
\frac{1}{4} & \frac{3}{4} & \frac{4}{4} & \frac{2}{4} \\
\end{array}
\]

ASK: What is the denominator in each fraction? (4) What are the numerators? (1, 3, 4, 2) What do the fractions have in common? (same denominator) Ask for volunteers to draw pictures for each fraction on the board. Ask them to use a different shape for each picture. (see sample answers in margin)

Draw on the board:
ASK: What is a fraction for each shaded area? (2/4, 2/5, 2/6, 2/8) Which part of the fraction in the answers changes: the numerator or the denominator? (denominator) What is the numerator for each fraction? (2)

**Drawing equal parts for different shapes and then shading the same unit fraction.** Write on the board:

\[
\begin{array}{c}
\frac{1}{4} \\
\end{array}
\]

ASK: What number is the denominator? (4) What does the denominator tell you? (the number of equal parts in the whole) Ask a volunteer to draw a vertical line to divide the shape into two equal parts. Ask a different volunteer to draw a horizontal line to divide the shape into four equal parts. The picture should look like this:

Ask another volunteer to shade the picture to show 1/4. The final picture is shown in the margin.

**Exercises**

1. Draw one line to create four equal parts. Shade \( \frac{1}{4} \) of the whole.

   a)  
   b)  
   c)  

   **Sample answers:**

   a)  
   b)  
   c)  

2. Draw one line to create three equal parts. Shade \( \frac{1}{3} \) of the whole.

   a)  
   b)  
   c)  

   **Sample answers:**

   a)  
   b)  
   c)  

**Drawing equal parts for different shapes and then shading the same fraction (not a unit fraction).** Write on the board:

\[
\begin{array}{c}
\frac{2}{5} \\
\end{array}
\]

ASK: What is the denominator? (5) What does it tell us? (the number of equal parts in the whole) Is the pentagon divided into equal parts? (yes)
What is the numerator? (2) What does it tell us? (the number of equal parts to shade) Ask a volunteer to shade the fraction.

**Exercises:** Draw one line to create eight equal parts. Shade \( \frac{3}{8} \) of the whole.

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
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</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Shaded Fraction" /></td>
<td><img src="image2.png" alt="Shaded Fraction" /></td>
<td><img src="image3.png" alt="Shaded Fraction" /></td>
</tr>
</tbody>
</table>

**Sample answers:**

<table>
<thead>
<tr>
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<th>b)</th>
<th>c)</th>
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<tbody>
<tr>
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<td><img src="answer2.png" alt="Shaded Fraction" /></td>
<td><img src="answer3.png" alt="Shaded Fraction" /></td>
</tr>
</tbody>
</table>

**Bonus:**

Shading the whole when given a unit fraction. SAY: Rob drew a picture of a whole. He started with a unit fraction and then he drew the rest. Draw on the board:

\[
\begin{array}{c}
\frac{1}{2} \\
\frac{1}{3}
\end{array}
\]

ASK: How many parts are in the picture with 1/2? (2) How many parts are in the picture with 1/3? (3) How is the fraction related to the number of parts in the whole picture? (the denominator tells us the number of parts) Draw on the board:

\[
\frac{1}{4}
\]

ASK: What is the denominator in this picture? (4) How many parts do we need to make a whole picture? (4) Ask a volunteer to draw the other three parts. The final picture is shown in the margin.

**Exercises:** Draw the remaining parts to make a whole.

<table>
<thead>
<tr>
<th>a)</th>
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<th>c)</th>
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<td><img src="image4.png" alt="Remaining Fractions" /></td>
<td><img src="image5.png" alt="Remaining Fractions" /></td>
<td><img src="image6.png" alt="Remaining Fractions" /></td>
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</table>

**Sample answers**

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<th>b)</th>
<th>c)</th>
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<tr>
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<td><img src="answer5.png" alt="Remaining Fractions" /></td>
<td><img src="answer6.png" alt="Remaining Fractions" /></td>
</tr>
</tbody>
</table>

**Bonus:**

Shading the whole when given a fraction, using a grid. Project **BLM 1 cm Grid Paper** on the board. Shade squares and write the fraction, as shown below:

\[
\begin{array}{c}
\frac{1}{4}
\end{array}
\]
SAY: The shaded region is $\frac{1}{4}$ of the whole. To make it easier to see, I will draw a circle around the shaded region. The picture should look like this:

```
  __________
  _________
  _________
  _________
  _________
```

ASK: For the fraction $\frac{1}{4}$, how many equal parts are in the whole? (4) How many of the unit fractions $\frac{1}{4}$ will I need to make the whole? (4) Ask a volunteer to circle three more equal parts. If students have difficulty, ask them to count the number of squares in the unit fraction. The final picture should look like this:

```
  __________
  _________
  _________
  _________
  _________
  _________
  _________
  _________
```

**Exercises:** The shaded region is $\frac{1}{3}$ of the whole. Draw the outline of the whole.

a) 

```
  __________
  _________
  _________
  _________
```

b) 

```
  __________
  _________
  _________
  _________
```

c) 

```
  __________
  _________
  _________
  _________
```

**Bonus:** The shaded region is $\frac{1}{7}$ of the whole. Draw the outline of the whole.

```
  _________
  _________
  _________
  _________
  _________
  _________
```

**Answers**

a) 

```
  __________
  _________
  _________
  _________
```

b) 

```
  __________
  _________
  _________
  _________
```

c) 

```
  __________
  _________
  _________
  _________
```

**Bonus:**

```
  _________
  _________
  _________
```

**Extension**

What fraction of the shape is shaded?

Answer: $\frac{1}{30}$
Goals
Students will name fractions of a set.

Prior Knowledge Required
Can write a fraction for a shape with equal parts
Can divide a shape into equal parts
Knows that equal fractions of the same whole can have different shapes

Materials
paper clips
BLM Random Number Spinner (p. O-41)

Mental math minute. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10. Ask all students to stand up. One at a time, ask each student to recite the column from the multiplication table for that number. For example, if the spinner lands on 3, the student says $3 \times 1 = 3$, $3 \times 2 = 6$, $3 \times 3 = 9$, and so on until $3 \times 9 = 27$. Then the student does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Review writing a fraction for a shaded area. Draw on the board:

ASK: If we want to write a fraction for the shaded area, how do we find the denominator of the fraction? (find the number of equal parts) How do we find the numerator? (find the number of shaded parts) What is the fraction of the shaded area? (3/5) Draw on the board:

ASK: How many equal parts are in the whole? (5) How many parts have the letter T? (2) SAY: We say that 2/5 of the rectangles have the letter T. ASK: What fraction of the rectangles have the letter S? (3/5) Explain. (five equal parts, three parts have the letter S) SAY: We say that 3/5 of the rectangles have the letter S.

Exercises: Find the fraction of the rectangles that have the letter T, and the fraction of the rectangles that have the letter S.

a)    b)

Answers: a) T: 3/7, S: 4/7; b) T: 6/10, S: 4/10
Writing fractions of a set. Draw on the board:

\[ \begin{array}{cccc}
\triangle & \triangle & \square & \square & \square \\
\end{array} \]

ASK: What shapes do you see? (triangles, squares) How many shapes are there? (5) SAY: These shapes are like the equal parts in the rectangles earlier. ASK: How many of the shapes are triangles? (2) SAY: We say that \( \frac{2}{5} \) of the shapes are triangles. ASK: How many of the shapes are squares? (3) SAY: We say that \( \frac{3}{5} \) of the shapes are squares. Write on the board:

\[ \begin{array}{cccc}
T & T & S & S \\
\end{array} \]

SAY: Notice that if we write T for triangle and S for square, we get the same pattern as we had with the rectangles earlier. We have a rectangle with five equal parts. Two of the parts have the letter T. Three of the parts have the letter S. Draw on the board:

\[ \begin{array}{cccc}
\triangle & \triangle & \triangle & \square & \square \\
\end{array} \]

ASK: How many shapes are there? (5) What will be the denominator for our fractions? (5) How many shapes are triangles? (3) What fraction of the shapes are triangles? (3/5) How many shapes are squares? (2) What fraction of the shapes are squares? (2/5) How many shapes are shaded? (4) What fraction of the shapes are shaded? (4/5) SAY: Remember that 5 is the denominator for all our fractions because there are five shapes.

Exercises

1. Fill in the fraction.

\[ \begin{array}{cccc}
\triangle & \triangle & \square & \square & \square \\
\end{array} \]

a) \( \square \) of the shapes are triangles.

b) \( \square \) of the shapes are squares.

c) \( \square \) of the shapes are shaded.

d) \( \square \) of the shapes are not shaded.

Answers: a) 2/5, b) 3/5, c) 3/5, d) 2/5
2. Fill in the fraction.

\[ \text{\begin{array}{c c c c c c c c} \square & \circ & \square & \square & \circ & \circ & \square \end{array}} \]

a) \[ \_ \] of the shapes are triangles.

b) \[ \_ \] of the shapes are squares.

c) \[ \_ \] of the shapes are circles.

d) \[ \_ \] of the shapes are not shaded.

**Answers:** a) 1/8, b) 4/8, c) 3/8, d) 4/8

Finding fractions of a set in word problems. Write on the board:

Ren wears running shoes on Monday, Tuesday, and Friday. He wears sandals on the other days of the week.

What fraction of the week does he wear running shoes?

SAY: It sometimes helps to write letters as short forms for problems. If we use the letter R for running shoes and the letter S for sandals, we can write the problem in a short form. Write on the board:

\[ \text{Sunday Monday Tuesday Wednesday Thursday Friday Saturday} \]
\[ \text{S R R S S R S} \]

ASK: How many days are in a week? (7) What do the letters R on the board show? (the days Ren wears running shoes) How many days does he wear running shoes? (3) What fraction of the week does he wear running shoes? (3/7) What do the letters S on the board show? (the days Ren wears sandals) How many days does he wear sandals? (4) What fraction of the week does he wear sandals? (4/7)

**Exercises:** The Toronto Blue Jays baseball team won six games and lost four games.

a) Use W and L to write the problem in short form.

b) How many games did the Toronto Blue Jays play?

c) How many games did they win?

d) What fraction of the games played did they win?

e) How many games did they lose?

f) What fraction of the games played did they lose?

**Answers:** a) W W W W W W L L L L, b) 10, c) 6, d) 6/10, e) 4, f) 4/10
Writing fraction statements for a picture. Draw on the board:

ASK: What shapes do you see? (square, circle, triangle) How many shapes are there altogether? (7) What fraction statement can you write about the squares? (3/7 of the shapes are squares) Explain. (there are seven shapes altogether and three of them are squares) What fraction sentence can you write about the circles? (3/7 of the shapes are circles) Explain. (there are seven shapes altogether and three of them are circles) What fraction statement can you write about the triangle? (1/7 of the shapes are triangles) Explain. (there are seven shapes altogether and one of them is a triangle) What fraction statement can you write about the shaded shapes? (2/7 of the shapes are shaded) Explain. (there are seven shapes altogether and two of them are shaded)

Exercise: Write as many fraction statements as you can about the picture.

Sample answers: 2/5 of the shapes are squares, 3/5 of the shapes are circles, 4/5 of the shapes are shaded, 1/5 of the shapes are not shaded

Drawing pictures to match fraction statements. Write on the board:

There are six shapes. \( \frac{4}{6} \) of the shapes are triangles.
\( \frac{2}{6} \) of the shapes are squares. \( \frac{5}{6} \) of the shapes are shaded.

ASK: How many shapes are there altogether? (6) Point out that this is the denominator of the fractions. ASK: How many shapes are triangles? (4) Explain. (it is the numerator of the fraction 4/6) How many shapes are squares? (2) Explain. (it is the numerator of the fraction 2/6) Ask for a volunteer to draw the six shapes: four triangles and two squares in any order. SAY: 5/6 of the shapes are shaded. ASK: How many of the shapes should be shaded? (5) Ask for a volunteer to shade any five of the shapes on the board. (see sample answer below)

Exercises: There are nine shapes. \( \frac{4}{9} \) of the shapes are squares, \( \frac{2}{9} \) are circles, and \( \frac{3}{9} \) are triangles. \( \frac{5}{9} \) of the shapes are shaded. Draw a picture to match the description.

Sample answer:
Extensions

1. There are six shapes. \( \frac{4}{6} \) of the shapes are squares. \( \frac{3}{4} \) of the squares are shaded. \( \frac{1}{2} \) of the triangles are shaded. What fraction of all the shapes are shaded?

Sample solution: There are four squares and two triangles. Three of the four squares are shaded. One of the two triangles is shaded. There are four shapes shaded altogether. So \( \frac{4}{6} \) of the shapes are shaded, as shown in the picture below:

\[
\begin{array}{ccc}
\square & \square & \square \\
\triangle & \triangle
\end{array}
\]

2. Fill in the fractions.

\[
\begin{array}{ccccccccc}
\triangle & \bigcirc & \square & \square & \square & \bigcirc & \square
\end{array}
\]

a) \[
\square
\]

b) \[
\square
\]

c) \[
\square
\]

d) \[
\square
\]

e) \[
\square
\]

f) \[
\square
\]

Answers: a) \( \frac{4}{8} \), b) \( \frac{2}{8} \), c) \( \frac{2}{8} \), d) \( \frac{4}{8} \), e) \( \frac{2}{4} \), f) \( \frac{1}{2} \)
Goals
Students will use fraction strips to compare fractions with the same denominator.

PRIOR KNOWLEDGE REQUIRED
Can write a fraction for a shape with equal parts

MATERIALS
paper clips
BLM Random Number Spinner (p. O-41)

Mental math minute. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10. Ask all students to stand up. One at a time, ask each student to say three division statements involving the number. For example, if the spinner lands on 3, the student might say 15 ÷ 3 = 5, 27 ÷ 3 = 9, 18 ÷ 3 = 6. Then the student does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Review shading fractions of a strip. Draw on the board:

ASK: What fraction can we write for the shaded parts? (3/4) What does the fraction mean? (4 equal parts, 3 parts are shaded) Ask a volunteer to show the same fraction using a rectangle instead of a circle, as shown below:

SAY: When we use a rectangle to show fractions, it is called a fraction strip.

Exercises: Shade the fraction of the fraction strip.

a) \( \frac{2}{3} \)  

b) \( \frac{3}{5} \)  

c) \( \frac{5}{8} \)

Answers
a)  

b)  

c)  

Comparing fractions using fraction strips. Draw on the board:

Rick

Tina

SAY: Rick and Tina bought identical protein bars. Each ate the shaded part of their protein bar. ASK: How many parts did Rick eat? (3) How many parts did Tina eat? (5) Who ate more? (Tina) SAY: We know Tina ate more because five is greater than three. ASK: How can you tell just by looking at the picture that Tina ate more? (more is shaded)

ASK: How many parts are in each whole? (8) What fraction can you write for the parts Rick ate? (3/8) What fraction can you write for the parts Tina ate? (5/8) SAY: We know 5/8 is greater than 3/8 because more is shaded.

Exercises

1. Shade the fraction of the strip. Then circle the greater fraction.

   a) [3/5] b) [1/4]
   c) [5/8] [3/8]

   Answers: a) 3/5, b) 3/4, c) 5/8

2. Shade the fraction of the strip. Then circle the smaller fraction.

   a) [2/3] b) [2/6] c) [1/2]

   Answers: a) 1/3, b) 2/6, c) 1/2

Review the signs for greater than (>) and less than (<).

SAY: Mathematicians sometimes use signs instead of words. Write on the board:

> is greater than
< is less than
Pointing to each sign, SAY: We can use the sign for “is greater than” instead of writing the words. We can use the sign for “is less than” instead of writing the words. Write on the board:

7 is greater than 5
\[ 7 \ > \ 5 \]

SAY: We read the number sentence from left to right just as we do with words. Point to the 7 and say “seven.” Point to the sign and say “is greater than.” Point to the 5 and say “five.” Write on the board:

4 is less than 10
\[ 4 \ < \ 10 \]

Ask a volunteer to rewrite the sentence and replace the words “is less than” with the correct sign. (4 < 10)

SAY: The open part of the sign is always facing the bigger number. Write on the board:

7 > 5 \quad 4 < 10

Point out that the open part of the sign is facing the 7 in the first pair of numbers, and facing the 10 in the second pair of numbers.

**Exercises:** Use the correct sign (\(>\) or \(<\)) to compare the numbers.

a) \[ 5 \ > \ 7 \]

b) \[ 10 \ < \ 3 \]

c) \[ 9 \ > \ 4 \]

**Bonus:**

\[ 113 \ > \ 245 \]

**Answers:** a) \(<\), b) \(>\), c) \(>\), Bonus: \(<\)

SAY: We can also use the greater than and less than signs to compare fractions. Draw on the board:

\[ \frac{3}{4} \quad \frac{1}{4} \]

Ask volunteers to shade the fraction of each strip. ASK: Which fraction strip has more shaded? (\(\frac{3}{4}\)) Which fraction is greater? (\(\frac{3}{4}\)) Write on the board:

\[ \frac{3}{4} \ is \ greater \ than \ \frac{1}{4} \]

Ask a volunteer to replace “is greater than” with the correct sign. (\(>\))
Exercises: Shade the fraction of the strip. Circle the greater fraction. Then write the correct sign to compare the fractions.

a) \[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\end{array}
\]
\[
\begin{array}{c}
\hline
5
\hline
6
\end{array}
\]
\[
\begin{array}{c}
\hline
2
\hline
6
\end{array}
\]

b) \[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\end{array}
\]
\[
\begin{array}{c}
\hline
2
\hline
5
\end{array}
\]
\[
\begin{array}{c}
\hline
4
\hline
5
\end{array}
\]

c) \[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\end{array}
\]
\[
\begin{array}{c}
\hline
2
\hline
8
\end{array}
\]
\[
\begin{array}{c}
\hline
7
\hline
8
\end{array}
\]

Bonus: Write the correct sign:
\[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\end{array}
\]
\[
\begin{array}{c}
\hline
99
\hline
100
\end{array}
\]
\[
\begin{array}{c}
\hline
7
\hline
100
\end{array}
\]

Answers: a) \(>\), b) \(<\), c) \(<\), Bonus: \(>\)

Making sure fractions are of the same whole. Draw on the board:

SAY: I drew two pizzas that are divided. Ask volunteers to write a fraction for the shaded part of the pizza in each picture. (1/4, 3/4) SAY: Clara is really hungry. She says that 3/4 is greater than 1/4, so she wants the shaded pizza slices from the second picture. ASK: Will she get more pizza? (no) Why? (the pizzas are not the same size) SAY: We cannot compare fractions if the wholes are not the same size.

Comparing fractions using circles. Draw on the board:

ASK: Are the two wholes the same size? (yes) How many equal parts are in each whole? (4) In the first picture, how many parts are shaded? (1) What fraction is shown? (1/4) In the second picture, how many parts are shaded? (3) What fraction is shown? (3/4) Which shaded area is bigger? (second picture) Which fraction is bigger? (3/4) Write on the board:
\[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\hline
\end{array}
\]
\[
\begin{array}{c}
\hline
1
\hline
4
\end{array}
\]
\[
\begin{array}{c}
\hline
3
\hline
4
\end{array}
\]

ASK: Which sign should I put in between to make the statement true, a greater than sign or a less than sign? (less than) Write “<” in the box.
**Exercises:** Shade the fraction of each circle. Use the correct sign (< or >) to compare them.

a) \[
\begin{array}{cc}
\text{7/8} & \text{3/8} \\
\end{array}
\]

b) \[
\begin{array}{cc}
\text{2/5} & \text{4/5} \\
\end{array}
\]

**Answers:** a) 7/8 > 3/8, b) 2/5 < 4/5

**Extensions**

1. Draw fraction strips for each fraction. Then circle the larger fraction.

a) \[
\begin{array}{cc}
\text{2/5} & \text{4/5} \\
\end{array}
\]

**Answers**

a) [Fraction strips shaded 2/5 and 4/5]

b) [Fraction strips shaded 5/8 and 3/8]

2. In Extension 1 part a), which part of the fraction tells you how much of the strip to shade, the numerator or the denominator?

**Answer:** the numerator

3. Use fractions strips to sort the fractions from smallest to largest.

a) \[
\begin{array}{cc}
\text{1/4} & \text{3/4} & \text{2/4} \\
\end{array}
\]

b) \[
\begin{array}{cc}
\text{5/7} & \text{1/7} & \text{3/7} \\
\end{array}
\]

**Answers:** a) 1/4, 2/4, 3/4; b) 1/7, 3/7, 5/7
Goals

Students will count on by a unit fraction until one whole.
Students will find the total area of a figure by counting half squares and whole squares.

PRIOR KNOWLEDGE REQUIRED

Can write a fraction for a shape with equal parts
Understands what the numerator and denominator represent
Understands the concept of area
Can measure area by covering a shape with units of area

MATERIALS

paper clips
BLM Random Number Spinner (p. O-41)
10 squares of construction paper
overhead projector (optional)
erasable marker (optional)

Mental math minute. Each student spins a paper clip on BLM Random Number Spinner to get a random number from 1 to 10. Ask all students to stand up. One at a time, have each student say two multiplication statements and two division statements involving the number. For example, if the spinner lands on 3, the student might say $8 \times 3 = 24$, $7 \times 3 = 21$, $15 \div 3 = 5$, $27 \div 3 = 9$. The student then does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Counting on by a fraction. Draw on the board:

\[
\begin{array}{cccc}
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\end{array}
\]

ASK: How many equal parts are in each picture? (4) In the first picture, how many parts are shaded? (1) What fraction is shown by the first picture? (1/4)
Have volunteers write a fraction for each of the other pictures. (2/4, 3/4, 4/4)
ASK: Which fraction shows the entire whole shaded? (4/4) SAY: We say that 4/4 is the same as one whole. Write on the board:

\[
\frac{4}{4} = 1
\]

SAY: We counted 1/4, 2/4, 3/4, 4/4 until we reached one whole. We call this counting on by the fraction 1/4. ASK: How do we count on by the fraction 1/5 until one whole? (1/5, 2/5, 3/5, 4/5, 5/5)
Exercises: Count on by the fraction.

a) \(\frac{1}{3}\), 

\[
\begin{array}{ccc}
\square & \square & \square \\
\end{array}
\]

b) \(\frac{1}{8}\), 

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\end{array}
\]

c) \(\frac{1}{5}\), 

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\end{array}
\]

Answers: a) 2/3, 3/3; b) 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 8/8; c) 2/5, 3/5, 4/5, 5/5

Counting on by fractions to count the equal parts in a picture. Draw on the board:

\[
\begin{array}{c}
\bigcirc \\
\end{array}
\]

ASK: How many equal parts are there in the circle? (8) How many parts are shaded? (1) What fraction can you write for the shaded region? (1/8) How do you count on by the fraction 1/8 until you get to one whole? (2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 8/8)

Exercises: What fraction is the shaded part of the shape? Count on by the fraction to count all the equal parts of the shape.

a) 

\[
\begin{array}{ccc}
\square & \square & \square \\
\square & \square & \square \\
\end{array}
\]

b) 

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\end{array}
\]

c) 

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\end{array}
\]

d) 

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\end{array}
\]

Answers: a) 1/6; 1/6, 2/6, 3/6, 4/6, 5/6, 6/6; b) 1/4; 1/4, 2/4, 3/4, 4/4; c) 1/2; 1/2, 2/2; d) 1/8; 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 8/8
Review finding the area by counting squares. Draw on the board:

![Diagram of a grid of squares]

SAY: To find the area of the picture, you can count the unit squares inside the picture. Have a volunteer count the squares. (12) SAY: The area of the picture is 12 square units. ASK: What addition equation can you use to find the area? (4 + 4 + 4 = 12) What other addition equation could you use? (3 + 3 + 3 + 3 = 12) Write both addition equations on the board. ASK: What multiplication equations could you use to find the area? (3 × 4 = 12, 4 × 3 = 12) Write both multiplication equations on the board. Point out that each multiplication equation corresponds to an addition equation written on the board.

Counting half squares. Take 10 squares of construction paper and cut along the diagonal to get 20 triangular half squares. Display six triangles by either affixing them to the board or using an overhead projector. The picture should look like this:

![Diagram of six triangles]

SAY: Each shape is a triangle. Have a volunteer count the triangles. (6) Have another volunteer form a square using two of the triangles, as shown below:

![Diagram of two triangles forming a square]

SAY: Notice that each triangle is half of a square. We can call one triangle a half square. Ask another volunteer to pair up as many of the other half squares as possible. The squares should look like this (without the numbers):

![Diagram of three paired half squares]

SAY: We can group each pair and count it as a whole. Point to each square and count aloud with the class as you write the number below each square.

Finding the area by counting half squares. Draw on the board:

![Diagram of a grid with half squares shaded]
SAY: You can find the total area of the shaded parts by counting half squares. ASK: How many half squares do you need to make one whole? (2) Ask volunteers to circle as many pairs of half squares as they can. The picture should look like this:

![Half squares diagram]

SAY: Let’s find the area of the shaded parts by counting the half squares. Pointing to each circled pair, count aloud with the class: 1, 2, 3. SAY: The area of the shaded parts is three squares.

**Exercises:** Find the total area of the shaded parts by counting half squares.

a) ![Half squares diagram a)](image)

b) ![Half squares diagram b)](image)

**Answers:** a) 4 squares, b) 5 squares

**Finding the area by counting whole squares and half squares.** Draw on the board:

![Whole squares and half squares diagram]

SAY: We want to find the area of the shaded part of the picture. Some of the shaded parts are whole squares and some are half squares. ASK: How many whole squares are there? (3) How many half squares are there? (4) If we count by half squares, what is the total area of the half squares? (2) What is the total area of the whole squares? (3) Write on the board:

\[
\text{Area of half squares} = 2 \\
\text{Area of whole squares} = 3
\]

ASK: How can we find the total area of the shaded region? (add the area of the half squares and the area of the whole squares) What is \(2 + 3\)? (5) Continue writing on the board:

Total area = 5
**Exercises:** Count the shaded whole squares and half squares to find the total area of the shaded parts.

a) ![Diagram A]

b) ![Diagram B]

\[
\begin{align*}
\text{Area of half squares} &= 3 \\
\text{Area of whole squares} &= 2 \\
\text{Total area} &= 5
\end{align*}
\]

\[
\begin{align*}
\text{Area of half squares} &= 5 \\
\text{Area of whole squares} &= 3 \\
\text{Total area} &= 8
\end{align*}
\]

**Answers:** a) 3, 2, 5; b) 3, 5, 8

**Extensions**

1. Find the total area of the shaded parts by counting the quarter squares.

a) ![Diagram C]

b) ![Diagram D]

**Answers:** a) 3 squares, b) 13 squares

2. Two triangles in the picture below make a rectangle together.

a) Are the triangles the same?

b) What is the area, in squares, of the rectangle?

c) What fraction of the rectangle is shaded?

d) What is the area of the shaded part?

**Answers:** a) yes, b) 2 squares, c) 1/2, d) 1 square

3. Find the area, in squares, of the shaded part.

![Diagram E]

**Answer:** 4 squares
Random Number Spinner

10
1
9
6
8
7
4
3
5
2

Blackline Master — Number Sense — Teacher Resource for Grade 3

O-41
Folding Paper (I)
Folding Paper (2)
More Folding Paper

I. Fold the shape to make the equal parts.

a)

b)

c)
Pattern Blocks

- Triangles
- Squares
- Rectangles
- Parallelograms
- Hexagons
Unit 13 Measurement: Time

Introduction

In this unit, students will tell and write time to the nearest minute using digital clocks and to the nearest 5 minutes using analog clocks. Students will convert between different units of measuring time, including seconds, minutes, hours, days, weeks, months, and years.

Meeting Your Curriculum

Alberta—Lessons ME3-14, ME3-21, and ME3-22 are required to cover the curriculum. All the rest of the lessons are optional.

British Columbia—Lessons ME3-14, ME3-21, and ME3-22 are required to cover the curriculum. Extensions 5 and 6 of Lesson ME3-22 are also required. All the rest of the lessons are optional.

Manitoba—Lessons ME3-14, ME3-21, and ME3-22 are required to cover the curriculum. All the rest of the lessons are optional.

Ontario—Lessons ME3-14 and ME3-16 to ME3-21 are required to cover the curriculum. Extensions 1 and 2 in Lesson ME3-21 are also required. Lesson ME3-15 reviews the components of analog clocks that students studied in Grade 2, so we recommend teaching this lesson as well. Lesson ME3-22 is optional.

Materials. BLM Time Memory Cards contains many cards that show different ways to tell time, including analog clock faces, digital clock faces, and text descriptions of time. To help students combine these different ways to show time, have them play Picking Pairs or Memory (described below) using the cards from the BLM. Cut out and laminate the cards to allow students to use them multiple times, whenever they have a few spare minutes.

In addition to the BLMs provided at the end of this unit, the following Generic BLM, found in section V, is used in Unit 13:

BLM Multiplication Chain (pp. V-3–8)

Recurring Games

Picking Pairs. Place cards face up in an array (the deck of cards used and the dimensions of the array will depend on the lesson). Students play individually or in teams and take turns picking pairs of matching cards and placing them into a common discard pile. When there are no more pairs in the array, students can add more cards to it. The goal is to match all the cards.

Memory. Set up an array of cards face down on the table. Students turn over two cards at a time. If the cards match by time, students set these cards aside; otherwise, they turn them face down again and continue playing. Play this first as a class, with volunteers taking turns turning over cards. Students can then play individually or co-operatively in pairs. In either case, the goal is to find all the matching pairs. If playing with a partner, Player 1 leads by choosing and turning over a card and Player
2 follows by choosing and turning over another card. After all pairs are found, players switch roles and play again. Players can help each other by asking questions or making suggestions (for example, “I think I know which two cards show nine o’clock. Should I turn one of them over?”), but they are not allowed to tell each other where specific cards are.

**NOTE:** It is a good idea for students to play Picking Pairs first—to practise making and recognizing matches—before they play Memory.

**Quizzes and Tests**

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quiz</strong></td>
<td>ME3-14, 21 to 22</td>
<td>ME3-14, 21 to 22</td>
<td>ME3-14, 21 to 22</td>
<td>ME3-14, 16 to 21</td>
</tr>
<tr>
<td><strong>Test</strong></td>
<td>ME3-14, 21 to 22</td>
<td>ME3-14, 21 to 22</td>
<td>ME3-14, 21 to 22</td>
<td>ME3-14, 16 to 21</td>
</tr>
</tbody>
</table>
Goals

Students will tell time to the nearest 1 minute from a digital clock.

PRIOR KNOWLEDGE REQUIRED

Can add two-digit numbers with and without regrouping ones.
Can read and write two-digit numbers

MATERIALS

large digital clock
scissors
BLM Reading Digital Times (p. P-50)
BLM Time Memory Cards (1) to (2) (pp. P-51–52)

NOTE: This lesson assumes you will use a large digital clock that shows the hours with two digits and the minutes with two digits (for example, the time 5 minutes past 9 is shown as 09:05). If the digital clock you have does not show the first zero, draw students’ attention to the difference between the clock you use and the clocks in the AP Book.

Mental math minute. Arrange students in a line and have groups of three students add two-digit numbers by adding tens and adding ones. Give an addition problem, such as 35 + 46. The first student in line adds the tens (30 + 40 = 70), the second student adds the ones (5 + 6 = 11), and the third student finishes the addition (70 + 11 = 81, so 35 + 46 = 81). Then, give the next three students in the line a new problem. Start with problems that do not require regrouping, such as 25 + 34, then continue on to problems that require regrouping ones.

Discuss the need for clocks. Show students a large digital clock.

SAY: This is a digital clock. It shows the time in digits only—that is, only numbers, no hands. ASK: Why do we need clocks? (sample answer: to know the time of the day, to measure time) Record students’ ideas.

ASK: Why do we need to know the time of the day? (sample answer: you need a way to tell the time so you can get to school before lessons start) Again, record students’ ideas. Have students give examples of events that happen at a precise time. (sample answers: school starts at 8:30, lunch is at 12:25, karate class starts at 5:15)

Exercise: Write three events that happen at a certain time. At what time do they happen?

Sample answer: I get up at 7 o’clock. School starts at 9:05. School ends at 3:35.

Point out that sometimes you need to know the time precisely. For example, you need to know precisely the time a train leaves so that you
will not be late; if a train leaves at 7:33, you will miss it if you arrive at the station at 7:35.

**Introduce hours and minutes.** SAY: We use *hours* and *minutes* to show time and to measure time. A day is divided into hours, and hours are divided into minutes. There are 24 hours in a day and 60 minutes in an hour. A clock shows 12:00 at noon and at midnight. **NOTE:** The concepts of “a.m.” and “p.m.” will be introduced later in the unit. If students mention them, explain that you will use these labels later.

**Hours and minutes on digital clocks.** Point out the parts on the digital clock that show the hours and the minutes. Set the time on the clock to 07:05. Explain that digital clocks show hours using two-digit numbers and also minutes using two-digit numbers. Even when the number of hours or minutes is a one-digit number, such as 5, the clock still uses two digits. **ASK:** How many tens are in 5? (0) **SAY:** There are zero tens in 5, so the clock shows 5 minutes as 05. **ASK:** What hour does the clock show? (7 hours) What does the minute part of the clock show? (5 minutes)

**Exercises:** What hour does the clock show? What does the minute part of the clock show?

a) 12:15  

b) 04:23  

c) 10:01  

d) 09:02

**Answers:** a) 12 hours, 15 minutes; b) 4 hours, 23 minutes; c) 10 hours, 1 minute; d) 9 hours, 2 minutes

**Writing the time from a digital clock.** Draw students’ attention to the symbol dividing the hours and the minutes on the digital clock. **SAY:** This symbol is called a **colon**. When we write time, we use a colon to separate the numbers that show the hours from those that show the minutes, just like on the digital clock. We do not need to write the hours as a two-digit number, but we do write the minutes as a two-digit number. Using the example of 07:05 again, **SAY:** The hour (write “7” on the board) is separated from the minutes (write “05” to the right of the 7) by a colon (insert the colon between 7 and 05). Have students write the times from the previous exercises. (a) 12:15, b) 4:23, c) 10:01, d) 9:02)

**Reading the time from a digital clock.** Remind students that when you ask somebody what the time is, they usually do not answer “9 hours 5 minutes.” They say something like “5 minutes after 9.” Have students think of other ways to say the time. You might wish to record the answers. Explain that in this lesson you will read the time in the form “5 minutes past 9.” Emphasize that we say the minutes first. Have students read the times in the exercises above using this format. (a) 15 minutes past 12, b) 23 minutes past 4, c) 1 minute past 10, d) 2 minutes past 9)

**Exercises:** Write the time in numbers and words.

a) 11:27  

b) 08:46  

c) 10:08

d) 04:07  

e) 02:04  

f) 01:03

**Bonus:** Ren thinks that 02:10 is 2 minutes past 10. Explain his mistake.
**Measurement 3-14**

**Answers:** a) 27 minutes past 11; b) 46 minutes past 8; c) 8 minutes past 10; d) 7 minutes past 4; e) 4 minutes past 2; f) 3 minutes past 1; Bonus: Ren read the hours first instead of reading the minutes first, so the time is 10 minutes past 2

Students might need more practice with times in which the number of minutes is 12 or less, because there is a greater chance of mixing up minutes and hours. Remind students to start at the second number, the number of minutes, when saying the time. Students who repeatedly read 9:05 as 9 minutes past 5 can benefit from Activity 1.

**ACTIVITY 1**

1. Have students cut out the cards from the top half of BLM Reading Digital Times. Students pick two of the cards at random and place them face up in the top row of the remaining BLM, creating a time as it is shown on a digital clock. To get the time in words, they switch the order of the cards, place them in the second row of the BLM, and then read the time. After using all six cards in random order, students mix up the cards and carry out the activity in reverse; i.e., they try to say the time in words first, then check the answer by creating the time as it is shown on a digital clock.

**Showing the given time on a digital clock.** Remind students that when they say the time in words and numbers, they say the minutes first. SAY: When we write the time in numbers, or as it is shown on a digital clock, we write the hours first and then the minutes, so 12 minutes past 3 is 03:12, not 12:03. Write “12 minutes past 3 is 03:12.” on the board.

Remind students to write a zero in front of any one-digit numbers, both hours and minutes. Volunteers can show the times in the following exercises on the digital clock.

**Exercises:** How would the time look on a digital clock?

a) 15 minutes past 12  
  b) 9 minutes past 11  
  c) 7 minutes past 10  
  d) 12 minutes past 6  
  e) 10 minutes past 7  
  f) 8 minutes past 9

**Answers:** a) 12:15, b) 11:09, c) 10:07, d) 06:12, e) 07:10, f) 09:08

Students who forget to use two digits for both hours and minutes might benefit from a template with spots for two digits, as shown below:

```
  :  
```

**ACTIVITY 2**

2. Play Picking Pairs or Memory (see unit introduction) using cards from BLM Time Memory Cards (1) to (2). Cards that match show the same time.
Extensions

1. Explain that minutes are divided into seconds. Some digital clocks also show seconds, after minutes—for example, 09:08:07. The first two digits show the hour, the next two show the minutes, and the final two show the seconds. Explain that when you say the time on a clock with seconds, you say minutes first, seconds after, and only then say “past the hour.” For example, 09:08:07 is “8 minutes, 7 seconds past 9” (or “8 minutes and 7 seconds past 9”).

Have students write the time on each digital clock as they would say it, using words and numbers.

   a) 10:27:32  b) 07:46:15  c) 12:08:21
   d) 04:07:21  e) 02:04:09  f) 01:03:02

Answers
   a) 27 minutes, 32 seconds past 10
   b) 46 minutes, 15 seconds past 7
   c) 8 minutes, 21 seconds past 12
   d) 7 minutes, 21 seconds past 4
   e) 4 minutes, 9 seconds past 2
   f) 3 minutes, 2 seconds past 1

2. These clocks are broken. Explain what is wrong.
   a) 25:04   b) 11:4   c) 5:68

   Answers: a) the hour number is too high, the hours cannot be greater than 12; b) the minutes are shown with only one digit, but should be shown with two digits; c) the number of minutes is too high, the minutes cannot be greater than 59

3. Ask several students at what time they get up on a school morning. Write on the board several intervals of time that include all students’ answers, so that they can act as column headings. Example:
   Before 6:45  From 6:45 to 6:59  From 7:00 to 7:14
   From 7:15 to 7:30  After 7:30

   Have students line up according to the time they wake up, creating a concrete graph. Convert the headings into a table and have students write their names in the appropriate column. Discuss which interval of time is the most common and which is the least common wake-up time. You might remind students that the most common data value is called the mode. Have students determine the mode for their set of data.
Goals
Students will identify elements of an analog clock face.
Students will distinguish between the exact hour (o’clock) and not-exact hour, and will write time to the hour as both ___ o’clock and ___:00.

PRIOR KNOWLEDGE REQUIRED
Can subtract two-digit numbers
Can read and write two-digit numbers
Can read a number line
Has experience with direct (object to object) and indirect (using a third object) comparisons of length

MATERIALS
ball
digital clock
analog clock with hour, minute, and second hands
paper plates
paper fasteners
pencils
scissors
BLM Make Your Own Clock (p. P-59)
BLM Numbers on a Clock Face (p. P-60, optional)
BLM Empty Clock Faces (p. P-61)
BLM Time Memory Cards (1) to (4) (pp. P-51–54)
ruler

NOTE: In Activity 2, students will need to copy the hands on the classroom clock at different times throughout the day. You might want to start the lesson as early in the day as possible, and return to the material later in the day.

Mental math minute. Give students subtraction problems involving subtraction of close two-digit numbers, such as 43 – 38. Pass a ball to the student you want to answer the question, who then passes the ball back to you after answering.

Introduce analog clocks. Have students think of tools that are used to show the time of the day. (sample answers: clocks, watches, cell phones) SAY: There are two types of clocks people often use. One type shows the time with digits only; it is called a digital clock. Show a digital clock as an example. SAY: A clock with hands, such as the classroom clock, is called an analog clock. These clocks usually have three hands. It is easy to see one of the hands moving. We will call this hand “the fast-moving hand,” or “the fast hand” for short. NOTE: Since students will not learn about
seconds in this unit and the word “second” can be confused with the ordinal number, we use “fast hand” instead of “second hand.”

Explain that all the hands on an analog clock move, but two of them move so slowly that they don’t look like they are moving. Compare the slower hands to the sun in the sky; you don’t see the position of the sun move in the sky, but it is in a different place after school than it was when school started. You might have students note where the sun is in the sky at the start of the day and then again at the end. To help students develop a sense of the hands moving, you might do Activity 2 several times throughout the day.

**Comparing clocks to number lines.** Point out that the clock face has numbers all around, beginning with 1 and ending with 12. Draw a number line on the board and label it from 1 to 15. Discuss how the clock is like a number line that goes in a circle. ASK: What comes right before 3 on the number line? (2) What comes right before 3 on the clock? (2) Repeat with other numbers between 2 and 12. Tell students a number between 1 and 11, and ask them to tell you the number that comes right after. Repeat several times. ASK: What comes after 12 on the number line? (13) What comes after 12 on the clock? (1) Emphasize that clocks are like number lines, except the clock has numbers from 1 to 12 and then starts over again at 1.

**Adding the numbers to a clock face.** Point out the positions of the numbers 12, 6, 3, and 9 on the clock face. SAY: To write the numbers on a clock face, remember that the numbers go to 12 and start again at 1. The number 12 is at the top. Draw a large clock face without numbers on the board and have a volunteer mark the 12. ASK: What number is at the bottom of the clock? (6) Have another volunteer mark the 6 on the clock face. ASK: What number comes on the clock after 12? (1) Where does 1 go on the clock face? (to the right of 12) Have a volunteer mark the 1, then have more volunteers write the rest of the numbers. Remind them to check that 6, which is already marked on the bottom, goes where it should—between 5 and 7.

**ACTIVITY 1**

1. **Make your own clock.** Give each student a paper plate, a paper fastener, a pencil, scissors, and [BLM Make Your Own Clock](#). Have them follow these steps to make their own clocks:

   1. Cut out the circle and the hands from the BLM.
   2. Glue the circle to the inside of the paper plate.
   3. Use a sharp pencil to poke a hole in the centre of the clock face.
   4. Write the numbers in the correct positions on the clock face.
   5. Attach the hands to the plate with the paper fastener.

   Have students keep the clocks they make for use in later lessons.

   **NOTE:** Students who are struggling with filling in the numbers on a clock face will benefit from doing [BLM Numbers on a Clock Face](#).
Introduce hour and minute hands. Point out to students that the clock they made in Activity 1 has only two hands. Explain that many clocks do not have the fast-moving hand, including the clocks on the AP book. Explain that the short and thick hand on an analog clock is called the *hour hand* and the long, thin hand is called the *minute hand*. Show the hands in different positions on an analog clock, making sure at least one of the hands points directly at a number, and have students identify which hand points to the number. For example, set the clock to show 6:15 and ASK: Which hand points at 3? (minute hand) Repeat with other times.

**ACTIVITY 2**

2. Give each student **BLM Empty Clock Faces**. Direct students’ attention to the classroom clock and ASK: What number is the hour hand pointing at now? Show how to draw the hour hand on the clock, making it thick and short, and have students do the same. Repeat with the minute hand, but make it thin and extending all the way to the number. Explain that the fast hand moves too fast to copy it. Repeat the activity throughout the day several times, so that students copy the hour hand and the minute hand at various times. Discuss how the hands have moved. Example: From the start of the math lesson (e.g., at 1:15) to the end of the math lesson (e.g., at 1:50), both hands moved but the short hand didn’t move very much—it is still between the 1 and the 2, but closer to the 2 than the 1.

Distinguishing between hands on a clock. Show various times on an analog clock and ask students what number the hour hand is closest to. Students can signal the answer by holding up the corresponding number of fingers.

Introduce hours. Explain that it always takes the same amount of time for the hour hand to move from one number to the next, exactly one hour. For this reason, this hand is called the hour hand. Explain that it takes the minute hand one hour to do a full circle around the clock, so while the minute hand turns a full circle, the hour hand moves only from one number to the next. For example, it moves from 3 to 4.

Introduce “o’clock.” Explain that when the minute hand is at 12, the hour hand points exactly at one of the numbers on the clock face. SAY: We call a time like this o’clock. For example, when the minute hand is at 12 and the hour hand is at 9, we say the time is 9 o’clock. In other words, it is an exact hour or the time is on the hour. To check if the time is an exact hour, you need to look at the minute hand. Show different positions of hands on the clock and have students say whether the time is an exact hour or not—for example, 10:00, 3:15, 4:00, 7:30, and so on. Vary your questions: Is it an exact hour? Is the time on the hour? Is it o’clock? Students can signal the answers with thumbs up for yes and thumbs down for no.
Identifying time to the hour. Show 7:00 on the analog clock. ASK: Is this o’clock? (yes) Explain that when it is the exact hour, you need to look at the hour hand to tell what the time is. ASK: What number is the hour hand pointing to? (7) SAY: So, the clock shows 7 o’clock. Show different times (using exact hours only) on the clock and have students tell you what “o’clock” it is.

Reverse the task. Have students use the clocks they made in Activity 1 to show the time. Say specific “o’clock” times and have students show the hour hand in the correct position (say times sequentially at first and then in random order).

Writing ___ o’clock with numbers. Remind students that digital clocks don’t have hands but instead show the time using only numbers. Explain that they show 9 o’clock as 09:00. Show students a digital clock to illustrate. Remind students that when they write the time with numbers, they would write this time as 9:00. Explain that the o’clock times always have two zeros for minutes. Show 3 o’clock on an analog clock, and ASK: How do you write this time with numbers? (3:00) Repeat with other o’clock times.

Summarize the four different ways of showing or writing the same time: 9 o’clock, 9:00, 09:00 on a digital clock, and the hour hand at 9 with the minute hand at 12 on an analog clock. Again, show different times on the analog clock and have students write the time both ways, as ___:00 and as ___ o’clock.

ACTIVITY 3

3. Play Picking Pairs and then Memory (see unit introduction) with cards from BLM Time Memory Cards (3) to (4). Students who need a further challenge can also use some of the cards from BLM Time Memory Cards (1) to (2).

Identifying the hour when the hour hand is between two numbers. ASK: How do you know the time is not “o’clock?” (the minute hand is not at 12) Remind students that the hour hand moves the same way as the other hands. So, when the minute hand is not at the 12, the hour hand is somewhere between two numbers. SAY: When the hour hand is between the numbers 4 and 5, we say that the hour is 4, because the time hasn’t passed 5:00 yet. Point at the classroom clock and ASK: What hour is it now? Then show students different times on the analog clock and have them say what hour it is.

To help students who struggle with deciding what hour it is, draw a clock with an hour hand pointing a little before the 6. Use a ruler to extend the hour hand by drawing a dotted line to the clock boundary. ASK: What two numbers is the line between? (5 and 6) What hour is it? (5) Have students read various clocks by first drawing a dotted line to help them. Students can also use the same method when doing Questions 5–6 on AP Book 3.2 p. 73.
Extensions

1. **Measuring time.** Draw students' attention to the fast-moving hand on the analog clock and have them watch it move. Point out how the hand moves at a steady pace, or makes regular jumps to the next mark, all the way around the clock. **SAY:** When people measure length, they use units that are all the same length. When they measure time, they also need units that are the same. The time it takes the fast hand to move from one number to the next is always the same, so we can use the time it takes the fast-moving hand to get from one number to the next to measure time. We will call these "time units."

   Have a volunteer start doing 10 jumping jacks when the fast hand is at the 12 and have the other students tell you where the hand is pointing when the volunteer is done. Remind students that the length of an object is not usually an exact number of centimetres. For example, a pencil might be almost 5 cm, so we record the length as "about 5 cm." **SAY:** We can do the same with time. When telling time, we will always say the last number the clock hand passed. So, if the hand starts at 12 and, later, is close to but not yet at 5, we will say that four time units have passed. This makes sense because only four full time units have passed, and it might have taken some time to turn your head to look at the clock.

   Have a volunteer clap 10 times starting when the fast hand is at the 12, and have the other students tell you where the hand is pointing when the volunteer has finished the 10 claps. **ASK:** Which activity took more time or more time units, clapping 10 times or doing 10 jumping jacks? How do you know? How many time units passed while clapping 10 times or doing 10 jumping jacks? Repeat with other tasks that take less than a minute to complete and note the results on the board. **Examples:** count to 30, hop on one foot 10 times, count to 20, say the alphabet, walk around the room. **ASK:** Which task took the longest time? Which took the shortest time? Did any two tasks take about the same amount of time? How can you tell? (the measurements are close)

   Have students work in pairs to time each other doing first one task and then another. **Examples:** write your first and then your last name; write the first five letters of the alphabet (a to e) and then the last five (v to z); find page 3 of a book and then page 43. Students might predict which task they think will take longer (i.e., which takes more time units) before starting. Partners should choose different activities to avoid competing. **ASK:** Who took longer to write their first name than their last name? Who took longer to write their last name? Why do you think some people needed more time to write their first name and some needed more time to write their last name? (depends on the length of the names) Discuss why some activities might take more time than others.
2. Give students BLM Empty Clock Faces. Have them draw the hands to show the time.

   a) 4:00  b) 9:00  c) 12:00  d) 10:00
   e) 5 o’clock  f) 8 o’clock  g) 1 o’clock  h) 11 o’clock

   **Answers**

   a) ![Clock Image]
   b) ![Clock Image]
   c) ![Clock Image]
   d) ![Clock Image]
   e) ![Clock Image]
   f) ![Clock Image]
   g) ![Clock Image]
   h) ![Clock Image]

3. **Hour hand between two numbers.** At various times of the day, stop and look at the clock when the hour hand is not pointing directly at a number, but is closer to one number than another. **ASK:** Which number is the hour hand closest to? Is it pointing a little before or a little after the number?

   Explain that we can say times such as “a little before 2 o’clock” or “a little after 11 o’clock.” Show a clock with only an hour hand (for example, from BLM Make Your Own Clock) and have students tell you first which number the hour hand is closest to and then read the time as “a little before (or after) ___ o’clock.”

   Say more such times (for example, a little after 3 o’clock; a little before 7 o’clock) and ask students to show you on their clocks where the hour hand will be pointing at that time.
Goals
Students will use skip counting and multiplication to tell how many minutes passed from the last hour.

PRIOR KNOWLEDGE REQUIRED
Is familiar with analog clock faces
Can distinguish between the hour hand and the minute hand
Can write time using numbers
Can skip count by 5s
Can multiply one-digit numbers by 5
Knows that multiplication is repeated addition
Is familiar with multiplication strategies, such as adding on

MATERIALS
ball (optional)
2 analog clocks
clock with only the minute hand, for example, from BLM Make Your Own Clock (p. P-59)
clocks made in Activity 1 of Lesson ME3-15
2 dice for each pair of students

Mental math minute. Ask students to solve multiplication questions within the range of $0 \times 1$ to $5 \times 5$. For each number, first go through the questions in order, (such as $0 \times 3$, $1 \times 3$, and so on to $5 \times 3$), in reverse order, and out of order. Then, progress to a different number. You can pass a ball to the student you want to answer the question, who then passes the ball back to you after answering.

How the minute hand moves every minute. Draw students’ attention to the minute markings on an analog clock. Explain that the marks show divisions of an hour into minutes. SAY: When the fast hand moves all the way around the clock, the minute hand (point to the minute hand) moves from one tick mark to the next. Verify this by watching the clock for a minute.

Compare the motion of the minute hand to that of the hour hand. SAY: The hour hand moves past a number every hour; the minute hand moves past a tick mark every minute, and that’s why it (point to the minute hand) is called the minute hand.

How the minute hand moves every hour. Explain that the minute hand goes all the way around the clock in one hour. Demonstrate with two analog clocks, one showing 4:00 and the other showing 5:00. Explain that it looks like the minute hand didn’t move from 4:00 to 5:00, but the minute hand actually just moved around the circle and back to where it started. Show the movement of both hands from 4:00 to 5:00 on an analog clock.
Counting by 5s to determine the number of minutes after the hour.

Show 9:00 on an analog clock, and then move the hands to show 9:15.
SAY: I want to know how many minutes passed when the minute hand moved from the 12 to the 3. Draw on the board the clock below, using a different colour for the minute hand pointing at the 12:

![Clock Diagram]

SAY: This picture shows two minute hands. The grey hand shows where the minute hand was at 9 o’clock. The black minute hand shows where the minute hand moved to. (Substitute grey and black with the colours you used.) Count together as a class the number of minutes that passed, emphasizing every fifth number (when the minute hand passed numbers 1, 2, and 3 on the clock face).

Show 9:00 on the analog clock. SAY: Like the hour hand, the minute hand takes the same amount of time to pass from any number to the next.
ASK: How many minutes pass when the minute hand moves from the 12 to the 1? (5) Draw an arrow showing a jump and write “5” next to the 1. Add an arrow from 1 to 2 and ASK: How many minutes have passed now? (10) Write “10” next to the 2. Repeat to add arrows from 2 to 3, and so on until 11. The final picture should look like this:

![Arrows Diagram]

ASK: What are we counting by? (5s) Explain that by noticing the pattern, we have made it easier to count the number of minutes after the hour.

Use a clock with only a minute hand (for example, from BLM Make Your Own Clock) to show the minute hand pointing at various numbers. Have students tell how many minutes passed from when the minute hand was at the 12. Then add the hour hand and ask students to concentrate on the minute hand only. Show 7:15 on the clock and write on the board:

7: ___

Have students count by 5s to tell you the number of minutes. Write “15” in the blank. Repeat with other times, always providing the hour.

Multiplying to find the number of minutes after the hour. Remind students that when they skip count by 5s, they are actually adding
5 repeatedly. For example, when they skip count 5, 10, 15, it is the same as adding $5 + 5 + 5$. **ASK:** How can you write this calculation in a shorter way? (using multiplication) Have students write the multiplication sentence. ($3 \times 5 = 15$) Remind students that this is called a multiplication equation.

Show 10:15 on the analog clock. **ASK:** How many minutes past the hour is it? (15) How many numbers do you say when you skip count by 5s to get to the answer? (3) What number is the minute hand pointing at? (3) Repeat with 10:20 and 10:30. **ASK:** How can you use the number the minute hand is pointing at to write a multiplication sentence for the number of minutes that passed after the last o’clock or exact hour? (multiply the number the minute hand points at by 5)

**Exercises:** Write the multiplication equation for the minutes passed after 8 o’clock. Then write the time.

![Clock images](image)

**Answers:** a) $3 \times 5 = 15$, 8:15; b) $6 \times 5 = 30$, 8:30; c) $2 \times 5 = 10$, 8:10; d) $7 \times 5 = 35$, 8:35

Show 8:50 on a clock and SAY: A student I know thinks that the time is 8:10. **ASK:** Is he correct? (no) SAY: The hour hand is very close to 9, but it is not at 9 yet. So, the hour is 8. **ASK:** How many minutes passed after 8 o’clock? (50) What multiplication sentence gives 50 as the product? ($10 \times 5 = 50$) What mistake did the student make? (he did not use multiplication—he just wrote the number the minute hand is pointing at; the student might have also counted the minutes in the wrong direction, from 12 to 11 to 10)

Move the minute hand to 11. **ASK:** What multiplication should you use for the number of minutes in this case? ($11 \times 5$) Write on the board:

$$11 \times 5 =$$

Have students think about how they can find the answer. Students might suggest counting by 5s or adding 5 to 50. Have students find the answer both ways, and remind them that the answers should agree. Have a volunteer fill in the answer. (55)

**ASK:** What if five more minutes passed? What multiplication will that give us? ($12 \times 5$) Write on the board:

$$12 \times 5 =$$

Again, have students think of how to find the answer. Have a volunteer fill in the answer. (60) Then, point out that the minute hand has moved around the whole circle. This means that one hour has passed. **ASK:** How many minutes are in an hour? (60) So, is the new time 8:60? (no) What is the new time? (9:00)
ACTIVITY

Each pair of students will need two dice and a clock that was made in Activity 1 of Lesson ME3-15. Player 1 rolls the dice, adds the results, and points the minute hand at the sum. (For example, if Player 1 rolls 3 and 4, she adds $3 + 4 = 7$ and sets the minute hand to point at 7.) Player 1 writes down the minutes past the hour given by the minute hand. ($7 \times 5 = 35$ minutes) Player 2 checks Player 1’s answers, and then players switch roles.

Extensions

1. Have students use a clock and count by 5s to the number of minutes (given below) to determine where the minute hand points. Students can keep track on their fingers. For example, if 20 minutes passed from the last hour, students should count 5, 10, 15, 20. They have four fingers up, so the minute hand points at 4.

   a) 25 minutes  
   b) 40 minutes  
   c) 30 minutes  
   d) 45 minutes

   **Answers:** a) 5, b) 8, c) 6, d) 9

2. Have students use division by 5 to determine where the minute hand points for the minutes given below. For example, if 20 minutes passed from the last hour, students can divide $20 \div 5 = 4$ to conclude that the minute hand points at 4.

   a) 15 minutes  
   b) 10 minutes  
   c) 35 minutes  
   d) 50 minutes

   **Answers:** a) 3, b) 2, c) 7, d) 10
**Goals**

Students will tell time from an analog clock when the minute hand shows a multiple of 5 minutes.

**PRIOR KNOWLEDGE REQUIRED**

- Is familiar with analog clock faces
- Can distinguish between the hour hand and the minute hand
- Can write time using numbers
- Can skip count by 5s
- Can multiply one-digit numbers by 5
- Can divide by 5 up to $50 \div 5$
- Knows that multiplication is repeated addition
- Is familiar with multiplication strategies such as adding on

**MATERIALS**

- ball (optional)
- analog clock
- 2 dice for each pair of students
- clocks made in Activity 1 of Lesson ME3-15
- BLM Time Memory Cards (3) to (5) (pp. P-53–55)
- BLM Telling Time (The Second Hand) (p. P-62, see Extension 3)

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**Mental math minute.** Ask students to solve questions that require multiplying and dividing by 5. First, go through the questions in order, such as $1 \times 5$, $5 \div 5$, $2 \times 5$, $10 \div 5$, and so on, to $10 \times 5$ and $50 \div 5$. Then, ask the same questions out of order, but keep each multiplication and its corresponding division together. Finally, ask both types of questions separately. You can pass a ball to the student you want to answer the question, who then passes the ball back to you after answering.

**Review analog clocks.** Ask students to explain how to distinguish between the hands on a clock. (the hour hand is short and thick, the minute hand is longer) Review how to tell what hour it is using an analog clock. Show 9:00 on the analog clock and ASK: Is it o’clock? (yes) How do you know? (the minute hand points at the 12) What time is it? (9 o’clock) How do you know the hour? (the hour hand is pointing at the 9) Show 9:15 on the clock. Point out that the hour hand is now between 9 and 10. ASK: What hour is it? (9) Show 9:45 on the clock. Remind students that, even when the hour hand is closer to 10 than to 9, the hour is still 9. Review how to tell how many minutes passed after the last hour. Remind students that they can get the number of minutes by either counting by 5s around the clock starting at 12, or multiplying the number the minute hand points at by 5. Demonstrate both methods for 9:15, then have students tell how many minutes past the hour it is for 9:45.
**Writing times using the number of minutes after the hour.** Show 7:10 on the analog clock. Explain that you want students to write the time in numbers. Remind them that they should write the hour first. ASK: Which hand is close to 7? (hour hand) Which hand points at the 2? (minute hand) What is the hour? (7) SAY: So, when we write the time as numbers, we start by writing the hour. Write on the board:

7: ___

ASK: How many minutes after 7 o’clock is it? (10) How do you know? (the minute hand points at 2, and 2 × 5 = 10) Write “10” in the blank. Repeat with 7:50. Finally, show 7 o’clock and ask how many minutes after 7 o’clock it is. Explain that because it is 0 minutes after 7 o’clock, we just write 7:00.

**Exercises:** Write the time on the clock in numbers.

a)  b)  c)  d)  

**Answers:** a) 1:30, b) 4:35, c) 10:10, d) 6:15

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**ACTIVITY**

Each pair of students will need two dice and a clock made in Activity 1 of Lesson ME3-15. Player 1 rolls the dice, adds the results, and points the hour hand at the sum. Player 2 rolls the dice, adds the results, and points the minute hand at the sum. Both players write the time in numbers. Partners compare answers and switch roles.

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**Review writing the time in the format of a digital clock.** Remind students that digital clocks show both hours and minutes as two-digit numbers, so if the number of hours or minutes is a one-digit number, we write 0 first. For example, 5 minutes after 7:00 is shown as 07:05. Show several times on an analog clock and have students write the times in digital format. For more practice, you can either repeat the activity above, having partners write the time in digital format, or have them play Picking Pairs and then Memory (see unit introduction) using the cards from BLM Time Memory Cards (5). Include a few pairs from BLM Time Memory Cards (3) to (4) to make the game more challenging and increase the number of pairs.

**Review saying time in words.** Remind students that when the time is, say, 7:10, we say it as 10 minutes after 7, which is short for “10 minutes past (or after) 7 o’clock.” This means we say the minutes first and the hours after.

**Exercises:** Say the time in words.

a) 8:20  b) 7:45  c) 9:10  d) 12:40

**Answers:** a) 20 minutes past 8, b) 45 minutes past 7, c) 10 minutes past 9, d) 40 minutes past 12
Show 3:05 on an analog clock and have students first write the time in numbers, then say it in words. Repeat with other times, such as 6:15, 10:30, 1:25, and 12:35.

**Extensions**

1. Each group of three students will need two dice and the clock made in Activity 1 of Lesson ME3-15. Player 1 rolls the two dice. The player adds the results and writes them down as the hour (for example, 5 + 6 = 11, so 11:__). Player 2 then rolls a single die, multiplies the result of the roll by 10, and chooses whether or not to add a bonus 5 for the minutes (for example, roll 4, 4 × 10 = 40, add 5 if wanted, so the minutes could be :40 or :45). If the roll is 6, the player should write :00 or :05 instead of :60 or :65. Player 3 sets the clock to this time. Players rotate roles after each turn.

2. You can find the number of seconds after the minute the same way you find the number of minutes after the hour. Example: The clock shows 1:30:20.

   Write the time in numbers.

   a)  
   b)  
   c)  
   d)  

   **Answers:** a) 1:30:45, b) 4:35:05, c) 10:10:30, d) 6:15:40

3. Have students complete BLM Telling Time (The Second Hand).

   **Answers**
   1. a) 5:00:35, b) 8:25:55, c) 2:15:45, d) 1:30:50, e) 7:15:20, f) 10:40:10
   2. b)  
   c)  

**Measurement 3-17**

P-19
Goals
Students will tell time in the format of “half past” and “quarter past” the hour.

PRIOR KNOWLEDGE REQUIRED
Can mentally add one-digit numbers to two-digit numbers
Can tell time to the 5 minutes on an analog clock
Can tell time to the minute on a digital clock
Can write time using numbers
Can skip count by 5s
Can multiply one-digit numbers by 5
Can identify halves and quarters

MATERIALS
analog clock
digital clock
BLM Time Memory Cards (6) to (8) (pp. P-56–58)

Mental math minute. Give students problems that require adding one-digit numbers to one- and two-digit numbers, such as 35 + 8. Students can count on to perform the addition, and hop as they count. Students who do not need to count up can just hop the number of times equal to the number added.

Review “whole” and “half.” Draw some basic shapes on the board, such as a square, a rectangle, and a circle, and ask volunteers to show wholes and halves by shading or re-drawing the shapes. Then, draw an analog clock face on the board and ask students to shade half of the clock face. If students do not draw the division line vertically from 12 to 6, ask them specifically to start the line at the 12. Leave the picture on the board for later reference.

Introduce “half past” an hour. Show 2:00 on an analog clock and ask students to say or write the time in different ways. (2 o’clock, 2:00, and 02:00) Remind students that when the minute hand is at 12, the time is “o’clock,” as in 2 o’clock. SAY: In one hour, the hour hand will move from 2 to 3. ASK: How does the minute hand move? (It turns a full circle)

Return to the clock face on the board and trace your finger around the shaded half. SAY: In half an hour, the minute hand turns half a circle. ASK: What number will the minute hand point at when half an hour has passed? (6) Change the clock so that it shows 2:30 and explain that we can read a time like this as half past 2, because half an hour has passed after 2 o’clock. Draw hands on the clock face on the board to show 2:30 and write the time below it two ways, as shown on the following page.
The time is half past 2.

Show 4:30 on the analog clock. ASK: What hour is it? (4) SAY: The minute hand is at 6, so we know half an hour passed after 4. ASK: What is the time? (half past 4) Repeat with 7:30, 9:30, and 12:30.

Writing "half past" in two ways. Remind students that they have two methods to find how many minutes past the hour it is. One is to count minutes by 5s. Count by 5s as a class while tracing the five-minute skips from 12 through 6 on a clock showing 12:30. ASK: How many minutes are in half an hour? (30) Remind students that a faster way to say how many minutes passed from the last hour is to multiply by 5. ASK: What number do we multiply by 5 to get the number of minutes? (the number the minute hand points at) What number is the minute hand pointing at? (6) Have students write the multiplication equation for the number of minutes. (6 × 5 = 30) ASK: How would you write this time in numbers? (12:30) Have students say the time as "half past" and write it in numbers for the following times shown on an analog clock: 8:30, 3:30, 5:30, 10:30.

Exercises: Write the time in numbers.

a) half past 3  b) half past 6  c) half past 11  d) half past 9

Answers: a) 3:30, b) 6:30, c) 11:30, d) 9:30

Reading time in the "half past" form from a digital clock. Show a digital clock. ASK: How can you tell on a digital clock if the time is half past an hour? (the number of minutes is 30) Show several different times (for example, 04:30, 07:30, 02:30) and have students tell the time in the form "half past" the hour.

Exercises: Write the time in words with numbers.

a) 9:30  b) 3:30  c) 11:30  d) 5:30

Answers: a) half past 9, b) half past 3, c) half past 11, d) half past 5

Distinguishing between exact hours and half past times. ASK: How can you tell if the clock is showing "half past" or "o’clock"? (the minute hand points to the 12 for o’clock and to the 6 for half past) Where does the hour hand point for o’clock? (directly at the number for the hour) And for half past? (halfway between two numbers) Show times on the hour and on the half hour on an analog clock and have students identify each time (for example, 7:00, 4:30, 12:30, 6:00, 3:30). Then, have students write the
time in two ways—in numbers and in words with numbers. Emphasize that for o’clock—in other words, for the exact hour—it is zero minutes after the hour, since it is exactly on the hour, but for half past, it is 30 minutes after the hour, so o’clock is written as “:00” and half past is written as “:30.” Finally, show several times on a digital clock and have students identify the time (for example, 8:00, 05:30, 11:00, 01:00, 02:30).

**ACTIVITY 1**

1. Play **Picking Pairs** and then **Memory** (see unit introduction) with cards from BLM Memory Cards (6) to (7).

**Quarters on a clock.** Draw some basic shapes on the board, such as a square, a rectangle, and a circle, and ask volunteers to show quarters by shading or drawing. Draw a clock face on the board, divide it in half (draw a line from the 12 to the 6), and then have a volunteer divide it into quarters by drawing a line from 9 to 3. Shade the quarter between 12 and 3. Explain that when the minute hand passes the shaded part of the clock face, one quarter of an hour has passed. When it is a quarter of an hour after 7 o’clock, the hour hand is pointing a little after 7 and the minute hand is a quarter of the way around the clock and pointing at the 3.

**Quarter past.** SAY: When a quarter of an hour has passed after 7:00, we say it is *quarter past* 7. ASK: What time is a quarter of an hour after 9 o’clock? (quarter past 9) Repeat with various “quarter past” times.

**Writing “quarter past” times.** ASK: When it is quarter past 4, where does the minute hand point? (at the 3) How many minutes after 4 o’clock is that? (15) How do you know? (count the five-minute skips starting at the 12 and going to the 3: 5, 10, 15; multiply $3 \times 5 = 15$) Have students write the multiplication equation. ASK: How do we write 15 minutes after 4 o’clock as a time? (4:15) How do we read the time? (15 minutes past 4, quarter past 4) How would we write quarter past 6 on a digital clock? (6:15 or 06:15) Quarter past 9? (9:15 or 09:15) Repeat with various “quarter past” times, having students both write and say the time (e.g., 8:15, 15 minutes past 8).

Show times on a clock face and have students identify the time. Use various times that are all a quarter past the hour. Give times sequentially at first, from 12:15 to 11:15, and then in random order. Then, show the same times, first sequentially and then in random order, but have students say or write the digital times. Finally, include o’clock and half past times as well (for example, 7:15, 3:00, 5:30, 9:15, 4:30, and 11:00).

**ACTIVITY 2**

2. Students play **Picking Pairs** and then **Memory** (see unit introduction) with selected matching cards from BLM Time Memory Cards (6) to (8). Playing with all the Time Memory Cards might be too much, so use only those from the specified pages.
Saying times in different formats. Ask students for what times they know special ways to say the time. (exact hour, such as 1 o'clock; 30 minutes past the hour, such as half past 2; and 15 minutes past the hour, such as quarter past 3) Remind students that in other cases they simply say how many minutes past the hour it is. Remind them to multiply the number the minute hand points at on the analog clock by 5 to get the number of minutes past the hour. Students who struggle with multiplication can use skip counting by 5s to say the number of minutes past the hour. Show different times on the clock and have students say the time in all formats that they can think of. Use as many volunteers as possible. Repeat with a digital clock, including times that are not multiples of 5 minutes (for example, 5:15, 2:37, 9:00, 6:30, 12:01, 3:00, 4:14, 4:15).

Extensions
1. Write the times in order.
   a) 4:30, 2:30, 5:30
   b) 3:15, 11:15, 8:15
   c) 6:15, 3:00, 5:30
   d) 4:30, quarter past 5, 10 minutes after 5
   e) 6 o'clock, 5:50, 14 minutes past 6
   Bonus: 1:30, 5:00, 4:30, 2:15
   Answers: a) 2:30, 4:30, 5:30; b) 3:15, 8:15, 11:15; c) 3:00, 5:30, 6:15; d) 4:30, 10 minutes after 5, quarter past 5; e) 5:50, 6 o'clock, 14 minutes past 6; Bonus: 1:30, 2:15, 4:30, 5:00
2. Record the start and finish times of a favourite television show. Express the times using o'clock, half past, or quarter past if you can.
Goals

Students will tell time from an analog or digital clock using the format “25 minutes to 5.”

Prior Knowledge Required

- Can tell time to the minute on a digital clock
- Can tell time to the 5 minutes on an analog clock
- Can read and write time using numbers
- Can add and subtract mentally and using the standard algorithm

Materials

- Ball or relay race baton (optional)
- Analog clock
- Digital clock
- 2 dice for each pair of students

Mental Math Minute. Arrange students in a line, and have them subtract two-digit numbers by subtracting tens and ones. Use problems with minuends that are multiples of 10. Demonstrate the method first: 50 − 17 can be done by subtracting 50 − 10 = 40, then 40 − 7 = 33, and so 50 − 17 = 33. Have students solve each problem in groups of three: the first student in line subtracts the tens, the second student in line subtracts the ones, the third student in line finishes the problem. Then, give the next three students in line a new problem. Students can pass a ball or a relay race baton to each other, with the person who receives the ball or baton answering the next part of the question.

Review telling time using an analog clock. Show several times on an analog clock and have students tell the time. Examples: 3:00, 3:15, 4:20, 5:35. Have students explain how they tell the time from the clock. Make sure students mention that if the hour hand is between two numbers, we say the number it just passed for the hour. Review both multiplying by 5 and skip counting by 5s to find the number of minutes after the hour.

Time left to the next hour. Explain that when the minute hand has passed 6, there is less time left before the next hour than has passed after the last hour. In this case, people often tell the time by saying what is left until the next hour. For example, if the clock shows 4:55 (show this time on the analog clock), only 5 minutes are left until 5 o’clock, which is the next hour, so people often say that the time is 5 minutes to 5, rather than 55 minutes after 4.

Show 2:55 on an analog clock. ASK: How many minutes are left to the next hour? (5) What is the next hour? (3) What is the time? (5 minutes to 3)
Show 5:40 on the analog clock. SAY: It is a lot of work to count every minute. ASK: How can we find how many minutes are left to the next hour more quickly? (skip count by 5s) If students mention multiplying the number the minute hand points at by 5, remind them that the numbers count the minutes passed from the last hour, not the minutes to the next hour.

Trace your finger counterclockwise from 12 to the position of the minute hand, and have students count by 5s as a class: 5, 10, 15, 20. Write on the board:

20 minutes to ___

ASK: What hour should I put in the blank? (6) Why not 5? (the hour hand is between 5 and 6, and we counted the time to the next hour, which is 6) Write “6” in the blank.

Show 2:35 on the analog clock. Count the minutes left to the next hour as a class and record them on the board. (5, 10, 15, 20, 25) Again, have students explain how to say the hour and what the time is. (25 minutes to 3)

Show several different times on the analog clock (with the minute hand in the second half of the clock face) and have students write the time in the format “ ____ minutes to ____.” Then, continue with a few more times, and have students write the time both ways, as “ ____ minutes to ____” and “ ____ minutes past ____.” Example: 10:35 is 25 minutes to 11 and 35 minutes past 10. Have volunteers record the times and the answers on the board.

**Using subtraction to tell time to the hour.** Have students count by 5s all the way around a clock face. ASK: How many minutes are in one hour? (60) Ask students to look at the number of minutes in their answers written on the board. Have students add the number of minutes in both ways to say each time. For example, for 10:35 they should make the addition equation 25 + 35 = 60. ASK: What do the minutes add to? (60) Why does this happen? (because we count the minutes past the hour and the minutes that are left to the next hour, so in total we should get one full hour, 60 minutes)

Show 6:35 on a digital clock. Ask students to write the subtraction sentence to say how many minutes are left to the next hour. (60 – 35 = 25) PROMPT: There are 60 minutes in an hour. 35 of them passed. How many are left? Have students write the time in words with numbers both ways: past the hour and to the hour. (6:35, 35 minutes past 6, 25 minutes to 7) Repeat with 8:42 on a digital clock. (60 – 42 = 18, so the time is 42 minutes past 8 and 18 minutes to 9)
ACTIVITY

Give each pair of students two dice. Player 1 rolls the dice, adds the numbers, and writes the sum as the hour. The player rolls the dice again, multiplies the numbers, and writes the product as the minutes. Player 2 tells the time in two ways: past the hour and to the next hour. Player 1 checks the answers, then players switch roles. (For example, roll 2 and 6, 8 hours; roll 3 and 5, so \(3 \times 5 = 15\) minutes; the time 8:15 is 15 minutes past 8 and 45 minutes to 9.)

Variation: Have students use the product of the numbers rolled on the dice as the number of minutes to the hour. For example, if students roll the same numbers as in the example above, the time is 15 minutes before 8. Player 2 writes the time using numbers, so Player 2 should write 7:45 and say the time as 45 minutes past 7.

Quarter to the hour. Remind students that there is another way to say some special times such as 4:15. ASK: How else can we say this time? (quarter past 4) Remind students that 15 minutes is one quarter of an hour, and 30 minutes is one half of an hour.

Show an analog clock that shows 3:45. ASK: What time is it? (45 minutes past 3, 15 minutes to 4) Explain that if a quarter of an hour is left before 4, we say the time is “quarter to 4.” Show various “quarter to” times on an analog clock and have students tell the time. First have students only use the “quarter to” format, and then encourage them to say the time in as many different ways as possible. Then, include other times, and have students give as many different forms for each as they can. Examples: 5:45, 6:15, 7:20, 8:00, 9:35, 10:55, 11:30.

Discuss how students can tell which times on a digital clock have special ways to say the time. Make sure students recall that times that end with 00 are said as “o’clock,” times that end with 30 are “half past,” times that end with 15 are “quarter past,” and times that end with 45 are “quarter to.” Then, show different times on a digital clock and have students say the time in many different ways. Examples: 3:45, 2:25, 7:30, 4:00, 9:15, 10:47, 1:55.

Extensions

1. Show the hour hand at various positions between two hours and have students predict where the minute hand will be—closer to 12, 3, 6, or 9? Examples: The hour hand is about halfway between 7 and 8, so the minute hand will be near the 6; the hour hand is just a little after the 4, so the minute hand will be near the 3.

2. a) A music lesson starts at quarter past 5 and ends at 5 minutes to 6. How long is the lesson? Write both times using numbers. Subtract the start time from the end time vertically to get the answer.

b) A birthday party starts at half past 2 and ends at quarter to 6. How long does the party last? How do you know?
Answers

a) Start time: 5:15, end time 5:55. Subtraction:

\[
\begin{array}{c}
\text{5:55} \\
- \text{5:15} \\
\hline
\text{0:40}
\end{array}
\]

The lesson lasts 40 minutes.

b) The party starts at 2:30 and ends at 5:45. Subtraction:

\[
\begin{array}{c}
\text{5:45} \\
- \text{2:30} \\
\hline
\text{3:15}
\end{array}
\]

The party lasted for 3 hours 15 minutes or 3 and a quarter hours.

3. Cody played soccer from quarter past 6 until 20 minutes to 8. How long did he play for? How do you know?

Answer: Cody played soccer from 6:15 to 7:40. Subtraction:

\[
\begin{array}{c}
\text{7:40} \\
- \text{6:15} \\
\hline
\text{1:25}
\end{array}
\]

Cody played for 1 hour 25 minutes.
ME3-20 Timelines
Pages 85–86

CURRICULUM REQUIREMENT
AB: optional
BC: optional
MB: optional
ON: required

VOCABULARY
a.m.
analog clock
colon (:)
half past
hour
hour hand
midnight
minute
minute hand
multiplication
noon
o’clock
p.m.
quarter past
quarter to
timeline

Goals
Students will identify times of the day using a.m. and p.m.
Students will draw and use timelines to solve simple problems related to time.

PRIOR KNOWLEDGE REQUIRED
Can multiply numbers up to $7 \times 7$
Can tell time to the 5 minutes on an analog clock
Can read and write time using numbers
Can read and draw a number line

MATERIALS
BLM Multiplication Chain (pp. V-3–8)
analog clock

Mental math minute. Give each student a card from BLM Multiplication Chain (1) to (6)—use the cards up to $7 \times 7$. Call a volunteer to the front of the class. The volunteer reads the card (for example, “I have $3 \times 4$ and $25$”). Students who have, in this case, $12$ or $5 \times 5$ on their cards come to the front of the class and stand beside the volunteer, showing their cards. If there is more than one student with a card that matches (for example, $12$ appears on multiple cards), the teacher picks who joins the chain at this moment and who will join the chain later. The students who just joined the chain read the second half of their cards, and new students join the chain. If the number called from one side of the chain matches the multiplication sentence on the other side of the chain, and there is no third student who can join either side of the chain, the chain is complete. The remaining students try to make a new chain of their own. The game ends when everyone has come to the front.

Introduce a.m. and p.m. Write “12:00” on the board. ASK: What time is it? (12 o’clock) Is it nighttime or daytime? (it could be either) Ask students if anyone knows what we can write next to the time to tell whether it is 12:00 at night or 12:00 during the day. Write “a.m.” and “p.m.” on the board. SAY: We add a.m. to times between midnight and just before noon, and p.m. to times from noon to just before midnight. You might want to mention that these names or labels are short forms for expressions that originated in ancient Rome: a.m. stands for “ante meridiem” (before noon) and p.m. stands for “post meridiem” (after noon). Draw on the board:

\[\begin{array}{c}
\text{midnight} \quad \text{a.m.} \quad \text{noon} \quad \text{p.m.} \quad \text{midnight} \\
\end{array}\]
Writing times using a.m. and p.m. List several events with times of day on the board and ask students to say whether each time should have a.m. or p.m. after the numbers. (see examples below—the answers are in brackets) Students can signal the answer by pointing to the correct side of the picture on the board.

- Breakfast at 8:00 (a.m.)
- Plane takes off at 3:15 in the afternoon (p.m.)
- Train arrives at 11:45 in the morning (a.m.)
- Library visit at 5:30 (p.m.)
- Dentist appointment at 1:15 (p.m.)
- 3 hours after midnight is 3:00 (a.m.)
- 7 o’clock in the morning is 7:00 (a.m.)
- I ate ice cream at 4:30 (p.m.)
- Half past 9 in the morning (a.m.)

Show the times below on the analog clock and have students write the time in their notebooks, in numbers and using a.m. or p.m. Invite volunteers to write their answers on the board.

- Lunch time: 11:55 (a.m.)
- School ends: 3:15 (p.m.)
- Swimming pool closes: 9:30 (p.m.)
- School bus leaves: 8:05 (a.m.)

Remind students that when the minute hand is at 6, the time is half past the hour, and when the minute hand is at 3, they say the time is quarter past the hour. SAY: We also say “quarter to” the next hour when the minute hand points at 9. For example, in the exercises above, school ends at quarter past 3 p.m. and the swimming pool closes at half past 9 p.m. ASK: If I leave home to go to work at half past 7, what is the time in numbers? (7:30 a.m.) If I return home after school at quarter past 6, what is the time in numbers? (6:15 p.m.)

Exercises: Write the time in numbers using a.m. or p.m.

a) Math lesson ends at half past 10.

b) Karate class starts at quarter past 7.

Bonus: Mandy’s little sister has a nap during the day from half past 11 to quarter past 1.

Answers: a) 10:30 a.m., b) 7:15 p.m., Bonus: 11:30 a.m. to 1:15 p.m.

Ask students to list six things they do and the time they do each thing, making sure that three times are before noon and three times are after noon. Example: I wake up at 6:50 a.m., and I go to bed at 9:50 p.m.

Introduce timelines. Ask students to tell you some of the things they do in the a.m. or p.m. Write down several of the suggestions, but make sure that they are out of chronological order. Explain that you would like to organize the list of the activities students suggested so that they are in the order that they happen. For example, waking up should come before breakfast.
Point out that the times students indicated provide a natural order of things. Remind students that when they try to order numbers, they can mark them on a number line, and then read them from left to right. SAY: We can use a similar tool with times. It is called a timeline. Draw on the board:

| 7:00 a.m. |   |   | 8:00 a.m. |   |   | 9:00 a.m. |

SAY: This is a timeline from 7 o’clock in the morning to 9 o’clock in the morning. Ask a volunteer to show the part that shows one hour from 7:00 to 8:00 and the part that shows one hour from 8:00 to 9:00. ASK: How many parts is each hour divided into? (four parts) What fraction of an hour is each part? (quarter, fourth) How many minutes is one quarter of an hour? (15 minutes) PROMPT: If a quarter hour passed from 7 o’clock, what time is it? (7:15 a.m.) SAY: This means the timeline shows marks for every 15 minutes. Have students copy the timeline and tell you what times to put in each blank. Repeat with a timeline from 2:00 p.m. to 5:00 p.m. with half-hour increments.

Using a timeline to solve problems. Draw a timeline for a school morning with 15-minute increments, but label only the increments for hours—for example, from 9:00 a.m. to 11:00 a.m. ASK: How many parts is each hour divided into? (four) How many minutes is each part? (15 minutes) Point to different marks and ask students to say what time each mark shows.

NOTE: Revise the following as needed, based on the times at your school. SAY: This is the timeline of a school morning. School starts at 9 o’clock. Label that on the timeline. SAY: The first lesson is math. It starts at quarter past 9. Where is this time on the number line? Have a volunteer mark it. Present the events in the exercises below or ones that reflect your school morning, and point to different increments. Have students signal thumbs up if you are showing the correct position on the number line and thumbs down if not.

Exercises

a) Math lesson ends at quarter past 10 a.m.
b) Science lesson starts at half past 10 a.m.
c) Quiz starts at 9:45 a.m.

Answers

<table>
<thead>
<tr>
<th>School starts</th>
<th>Math lesson starts</th>
<th>Pop quiz starts</th>
<th>Math lesson ends</th>
<th>Science lesson starts</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 a.m.</td>
<td></td>
<td></td>
<td></td>
<td>10:00 a.m.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11:00 a.m.</td>
</tr>
</tbody>
</table>

ASK: How long is the break between the end of the math lesson and the start of the science lesson? (15 minutes) How do you know? (one quarter of an hour, one increment of the number line is between the arrows, so the
break is 15 minutes) How much time has passed from the start of the math lesson to the start of the quiz? (30 minutes) Again, have students explain how they determined the answer. SAY: The quiz was only 15 minutes long.

ASK: When did the quiz end? (at 10:00) How do you know? (15 minutes is one quarter of an hour, so the next mark on the timeline after the start of the quiz should be the time the quiz ends)

Extend the timeline for another hour. SAY: The science lesson was 45 minutes long. ASK: When did it end? (11:15 a.m.) To help students to see that 45 minutes are three increments of 15 minutes each, you can draw a clock face and have a volunteer shade 45 minutes on the clock.

ASK: What fraction of the clock face is shaded? (three quarters) SAY: This means 45 minutes equals three quarters of an hour. ASK: How many spaces on the timeline is that? (three) Have students count three spaces on the timeline, starting at 10:30. ASK: When did the science lesson end? (11:15)

**ACTIVITY**

Students record when they do different things over one day on the weekend. Then, they make a timeline of the day, marking the things they did. Students invent problems for their peers to solve. Example: I read from 8:15 a.m. to 9:00 a.m. How long did I read?

**Extensions**

1. **Time in ancient civilizations.** Tell students that our 12-hour clock comes from ancient Egypt, where people divided the day into 12 equal parts and the night into 12 equal parts. This meant that the length of each part or hour would change over the seasons, because the day would get longer in the summer and shorter in the winter. The Egyptians used sundials to measure time during the day and water clocks to measure time during the night. Students can research ways to construct sundials and water clocks, as well as their origins. Good online search phrases are “how to make a sundial for kids” and “how to make a water clock for kids.”

The Ancient Romans also divided the day into 12 equal parts, starting at around what we call 6 a.m. and ending at around 7 p.m. Here are more examples of times in Ancient Rome and their modern-day equivalents:

<table>
<thead>
<tr>
<th>Time in Ancient Rome</th>
<th>Time Today</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st hour</td>
<td>about 7 a.m.</td>
</tr>
<tr>
<td>6th hour</td>
<td>about 12 p.m.</td>
</tr>
<tr>
<td>7th hour</td>
<td>about 1 p.m.</td>
</tr>
<tr>
<td>12th hour</td>
<td>about 6 p.m.</td>
</tr>
</tbody>
</table>

Have students extend the table above to show all hours before noon or create a timeline that shows all day times (from 7 a.m. to 6 p.m.) as they would have been written in Ancient Rome and now. Ask students...
to look for patterns. What number do you need to subtract from modern a.m. times to get the Roman day time? (6 hours) What number do you need to add to modern p.m. times to get the Roman day time? (6 hours) What Roman hour is an exception to this rule? (6th hour: you still need to subtract 12 – 6 to get the Roman time)

2. Imagine a free day when you can do anything you want. Write six things you would do. Write the start time and the end time for each, using a.m. or p.m. for each time. Make a timeline and find the time each activity takes.
ME3-21 Intervals of Time
Pages 87–88

Goals
Students will convert between different units of time.
Students will solve simple problems that require converting time intervals.

PRIOR KNOWLEDGE REQUIRED
Can multiply and divide numbers up to $9 \times 9$
Can skip count by different numbers
Can multiply using skip counting
Can describe relationships between different base-ten blocks
Is familiar with place value
Can add and subtract three-digit numbers
Can extend a growing number pattern

MATERIALS
ball (optional)

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $9 \times 9$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on to $9 \times 3$ and $27 \div 3$. Then, use a different number. Next, try questions out of order, but keep each multiplication and its corresponding division together. You can pass a ball to the student you want to answer the question, who then passes the ball back to you after answering.

Converting weeks to days. Explain that there are a few weeks before an event you are eagerly awaiting. For example, in two weeks there is a big concert. You are so excited about it that you start counting days before this happens. ASK: How many days are in a week? (7) How many days are in two weeks? (14) How do you know? ($7 + 7 = 14$) Draw on the board:

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Ask students to help you fill in the last empty cell, then to extend the table. Have students describe the pattern in the rightmost column of the table. (start at 7 and add 7 each time) ASK: What other name can we use for the numbers in the right column? (multiples of 7) How can we use multiplication to get these numbers? (multiply the number of weeks by 7) Keep the table on the board for use in Lesson ME3-22.
Exercises

a) Ray’s birthday party is in 5 weeks. How many days are there until Ray’s birthday party?

b) Holidays start in 2 weeks. How many days are left before the holidays?

c) The full moon is in 3 weeks. How many days is that?

Answers: a) 35, b) 14, c) 21

Converting mixed measurements in weeks and days to days. SAY: A new book in a series I like is coming out in 3 weeks and 2 days. I want to know how many days are left before that. ASK: How can I find out? Have students suggest options. PROMPT: How many days are in 3 weeks? (21) How do you know? (from the table or by using multiplication: $3 \times 7 = 21$) How many more days will there be before the book comes out in 3 weeks? (2 more days) How many days are left in total? ($21 + 2 = 23$ days) Write the calculation on the board as you go through the prompts, as shown below:

$$3 \text{ weeks } 2 \text{ days}$$
$$3 \text{ weeks} = 21 \text{ days}$$
$$3 \text{ weeks } 2 \text{ days}$$
$$= 21 + 2 \text{ days}$$
$$= 23 \text{ days}$$

Write on the board:

$$5 \text{ weeks } 6 \text{ days}$$
$$5 \text{ weeks} = \_ \_ \_ \text{ days}$$
$$5 \text{ weeks } 6 \text{ days}$$
$$= \_ \_ \_ \_ \_ \_ \text{ days}$$
$$= \_ \_ \_ \_ \_ \_ \_ \text{ days}$$

Ask students to help you fill in the blanks. (35, 35 + 6, 41) Students might also notice that 5 weeks 6 days is just one day short of 6 weeks, so they can find the answer by multiplying $6 \times 7 = 42$ and subtracting 1 day. If students notice this, suggest that they use this method to check their answers.

Exercises: Multiply by 7 to convert weeks to days. Add leftover days.

a) 4 weeks 2 days

b) 5 weeks 4 days

c) 2 weeks 5 days

Bonus: 10 weeks 10 days

Answers: a) 30, b) 39, c) 19,

Converting days to hours. SAY: Sometimes I get so excited about some event that I start counting hours. ASK: How many hours are in 1 day? (24) Write on the board:

$$1 \text{ day} = 24 \text{ hours}$$

Draw on the board a table like the one used for weeks and days and have students help you fill it in, as shown on the following page.
Days | Hours
---|---
1 | 24
2 | 48
3 | 72
4 | 96
5 | 120

ASK: How many hours are in 3 days? (72) In 5 days? (120) Keep the table on the board.

**Solving problems that require converting days to hours.** SAY: A train ride from Toronto, ON, to Edmonton, AB, takes 68 hours. Write on the board:

Toronto, ON, to Edmonton, AB: 68 hours
between 1 and 2 days  between 2 and 3 days  between 3 and 4 days

SAY: An interval is part of a number line between two marks on a number line. The marks can be one beside each other or far apart. For example, we can talk about an interval between 1 and 2 (point to it), or we can talk about the interval between 3 and 5 (trace it with your finger). Label the number line above it as “days” and SAY: Now this line is a timeline. A time interval is part of a timeline. For example, when something takes between 1 and 2 days, we can say that the length of time it takes falls somewhere in this interval (trace the interval between 1 and 2 with your finger). SAY: So “between 1 and 2 days,” “between 2 and 3 days,” and “between 3 and 4 days” are all time intervals.

ASK: In which of these three intervals does 68 hours fit? (between 2 and 3 days) PROMPT: Is 68 hours more than 1 day? (yes) How do you know? (1 day = 24 hours, 68 is more than 24) Write “hours” underneath the timeline and label the increments for 0 as 0 and for 1 as 24. ASK: Is 68 hours more than 2 days? (yes) How do you know? (2 days = 48 hours, 68 is more than 48) Label the increment for 2 as 48. ASK: Is 68 hours more than 3 days? (no) How do you know? (3 days = 72 hours, which is more than 68) Label the increment for 3 as 72 and point to a point between 2 and 3. SAY: 68 is between 48 and 72 on the timeline that shows hours. We can clearly see on this timeline that 68 hours fall between 2 and 3 days.

Finish labelling the timeline with hours, as shown below. ASK: What number do we skip count by in the labels below the number line? (24) Why do we use 24 and not any other number? (there are 24 hours in 1 day)
Exercises

a) A train ride from Edmonton, AB, to Vancouver, BC, is 27 hours. Is it between 1 and 2 days, between 2 and 3 days, or between 3 and 4 days?

b) A bus ride from Ottawa, ON, to Vancouver, BC, is about 73 hours. Is it between 1 and 2 days, between 2 and 3 days, or between 3 and 4 days?

Bonus: Anna takes a bus trip from Ottawa, ON, to Winnipeg, MB. To start, she goes 8 hours by bus to Sudbury, ON, waits for 2 hours in Sudbury, and then takes a bus to Winnipeg. The second bus ride is 25 hours long. Is her total trip between 1 and 2 days, between 2 and 3 days, or between 3 and 4 days long?

Answers: a) between 1 and 2 days, b) between 3 and 4 days, Bonus: between 1 and 2 days

Converting hours to minutes. Ask: How many minutes are in 1 hour? (60)

Write on the board:

1 hour = 60 minutes

Start a table as in the previous cases and have students help you fill it in—keep it on the board for later use. Ask students to explain how they know what to put in each cell of the table. (add 60 to the previous number of minutes) The table should look like this:

<table>
<thead>
<tr>
<th>Hours</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
</tr>
</tbody>
</table>

Say: A movie is 72 minutes long. Ask: Is it shorter than 1 hour? (no) How do you know? (1 hour is 60 minutes, 72 is more than 60) Is it longer than 1 hour? (yes) Is it longer than 2 hours? (no) How do you know? (2 hours = 120 minutes, 120 > 72) Say: the movie is between 1 and 2 hours long.

Exercises

a) A birthday party lasts 100 minutes. Is it between 1 and 2 hours long, between 2 and 3 hours long, or between 3 and 4 hours long?

b) A flight from Whitehorse, YK, to Vancouver, BC, lasts 132 minutes. Is it between 1 and 2 hours long, between 2 and 3 hours long, or between 3 and 4 hours long?
c) A flight from Ottawa, ON, to Iqaluit, NU, lasts 190 minutes. Is it between 1 and 2 hours long, between 2 and 3 hours long, or between 3 and 4 hours long?

**Answers:** a) between 1 and 2 hours, b) between 2 and 3 hours, c) between 3 and 4 hours

**Converting mixed measurements in hours and minutes to minutes.**

SAY: If you look on a DVD to see the length of a movie, it is usually written in minutes, such as 72 minutes or 97 minutes. Write these two lengths of time on the board. SAY: The length of a flight is usually given in hours and minutes, for example, 2 hours 10 minutes. A flight attendant needs to decide between movies of these two lengths. She knows that she needs 20 minutes at the beginning of the flight and 20 minutes at the end of the flight for announcements and landing preparation, and she can put on a movie for the rest of the flight. ASK: Which movie should she choose? Write on the board:

Announcements: 20 minutes  
Landing preparation: 20 minutes  
Flight length: 2 hours 10 minutes

ASK: How can you find out how much time the attendant has for the movie? (subtract the time needed at the beginning and at the end of the flight from the total length of the flight) How much time does the attendant need for the announcements and landing preparation altogether? (40 minutes) How do you know? (20 + 20 = 40) Continue writing on the board:

Announcements: 20 minutes  
Landing preparation: 20 minutes  
Flight length: 2 hours 10 minutes

SAY: I cannot subtract 40 minutes from 10, and I cannot subtract 40 from 2. ASK: What should I do? (change hours to minutes, change the mixed measurement to minutes only) How many minutes are in 2 hours? (120) How many minutes are leftover? (10 minutes) How many minutes in total? (130 minutes) Write the solution on the board as you ask the questions, as shown below:

\[
\begin{align*}
2 \text{ hours} &= 120 \text{ minutes} \\
2 \text{ hours 10 minutes} &= 120 + 10 \text{ minutes} \\
&= 130 \text{ minutes}
\end{align*}
\]

ASK: If 40 minutes are needed for the announcements and landing preparation, how much time is left for the movie? (90 minutes) How do you know? (130 – 40 = 90 minutes) Write the subtraction equation on the board. ASK: Which movie should the flight attendant play—the movie that is 72 minutes long or the movie that is 97 minutes long? (72-minute-long movie)

SAY: Another flight is 2 hours 35 minutes long. Let’s check if the attendant can use the 97-minute-long movie on that flight. Have a volunteer show
how to calculate the length of the flight in minutes. (2 hours = 120 minutes, 2 hours 35 minutes = 120 + 35 = 155 minutes) ASK: How much time is left for the movie? (155 – 40 = 115 minutes) Is that enough time for the longer movie? (yes)

Leave the two movie times on the board for use in the bonus exercise below.

**Exercises:** Change the hours to minutes. Add the leftover minutes.

a) 3 hours 5 minutes
b) 5 hours 25 minutes
c) 4 hours 6 minutes

**Bonus:** A flight is 3 hours 43 minutes long. If the same rules apply as in the other flights and there should be a 5 minute break between movies, can the flight attendant use both movies?

**Answers:** a) 185 minutes, b) 325 minutes, c) 246 minutes, Bonus: yes

**Solving problems with minutes, hours, and days.** Have students share activities they do every day (on weekdays and weekends) and how much time each activity takes. For example, “reading, 20 minutes” or “walking the dog, 15 minutes, 3 times a day.” SAY: I want to know how much time you spend on these activities each week. ASK: How many days are in a week? (7) If you spend 20 minutes reading every day, how can you find out how much time you spend reading in one week? (multiply 7 × 20) Skip count by 20s as a class to find 7 × 20 = 140. ASK: Is 140 minutes less than 1 hour, between 1 and 2 hours, or between 2 and 3 hours? (between 2 and 3 hours) How do you know? (2 hours is 120 minutes, 3 hours is 180 minutes, 140 is between 120 and 180) Repeat with “exercising, 25 minutes each day.” (skip count by 25s to find 7 × 25 = 175 minutes, between 2 and 3 hours) Repeat with some of the activities students listed, using easy numbers to skip count or multiply by (for example, brushing teeth 2 × 3 minutes = 6 minutes every day, so 7 × 6 = 42 minutes, less than 1 hour each week).

**Exercises**

a) Edmond showers for 3 minutes every day. How much time does he spend showering in one week?

b) Ansel reads for 30 minutes every day. How much time does he spend reading each week? Between which two hours is that?

c) Tina walks her dog 25 minutes 2 times a day. How much time does she spend walking the dog each day? How much time does she spend walking the dog each week? Between which two hours is that?

**Bonus:** Ansel’s parents think he should read 4 hours every week, so he reads a little more on Sunday. How much more time should he spend reading to get to 4 hours every week?
Answers: a) 21 minutes; b) 210 minutes, between 3 and 4 hours; c) 50 minutes each day, 350 minutes each week, between 5 and 6 hours; Bonus: 30 minutes

Converting between years and weeks. SAY: There are 52 full weeks in 1 year. Actually, a regular year is 52 weeks and 1 day long. ASK: How many weeks are in 2 years? (104) How many weeks are in 3 years? (156) SAY: Alexa is exactly 3 years old. ASK: How many weeks old is Alexa? (156 weeks) Write on the board:

Alexa is 3 years old = 156 weeks old
Sam is 120 weeks old.
Karen is 35 weeks older than Sam.

SAY: Sam is 120 weeks old. Karen is 35 weeks older than Sam. ASK: Who is older, Alexa or Karen? (Alexa) PROMPT: How many weeks old is Karen? (120 + 35 = 155 weeks) How much older is Alexa than Karen? (1 week older) Keep this information on the board for the next exercise.

Exercises

a) Ken is 2 years and 3 weeks old. Who is older, Ken or Sam?
b) Avril is 16 weeks younger than Karen. Who is older, Avril or Ken?
Bonus: Order all five children from youngest to oldest.

Answers: a) Ken is 107 weeks old, so Sam is older; b) Avril is 139 weeks old, so Avril is older than Ken; Bonus: Ken, Sam, Avril, Karen, Alexa

Introduce decade, century, millennium. SAY: Long periods of time also have names. Ten years is called a decade, 100 years is called a century, and 1000 years is called a millennium. Write on the board:

1 decade = 10 years
1 century = 100 years
1 millennium = 1000 years

ASK: How many tens are in 100? (10) How many decades are in a century? (10) How many hundreds are in 1000? (10) How many centuries are in a millennium? (10) How many decades are in a millennium? (100) How do you know? (10 decades in a century, 10 centuries in a millennium, 10 × 10 = 100)

ASK: If I wanted to show these periods of time with base-ten blocks, using a ones block for 1 year, what would a tens block be? (decade) How do you know? (there are 10 ones blocks in a tens block, and 10 years in a decade) Repeat with hundreds block (century) and a thousands block (millennium).

NOTE: Extensions 1 and 2 are needed to cover the Ontario curriculum.
Extensions

1. Subtract the hours and minutes separately to find how much time passed from one reading of the clock to the next. Write the answer in hours and minutes. For example, to find 8:35 – 8:15:

\[
\begin{array}{c}
  8 : 3 \ 5 \\
- 8 : 1 \ 5 \\
\hline
  0 : 2 \ 0 \\
\end{array}
\]

0:20 is 0 hours 20 minutes, or just 20 minutes.

a) 3:25 – 3:05  
b) 4:55 – 4:24  
c) 2:48 – 2:12  
d) 12:55 – 9:15  
e) 10:54 – 6:25  
f) 6:58 – 1:32

**Answers:** a) 20 minutes, b) 31 minutes, c) 36 minutes, d) 3 hours 40 minutes, e) 4 hours 29 minutes, f) 5 hours 26 minutes

2. How much time passed between the time shown on Clock A to the time shown on Clock B? Write the answer in hours and minutes and in minutes only. Remember, 1 hour is 60 minutes.

a) Clock A  
   ![Clock A]

   Clock B  
   ![Clock B]

**Answers:** a) 8:35 – 8:15 = 0:20, so 0 hours 20 minutes, or 20 minutes; b) 4:50 – 1:30 = 3:20, so 3 hours 20 minutes or 200 minutes

3. Solve the problem by subtracting the length of time from the end time. Remember to write "a.m." or "p.m." in the answer.

a) It takes Braden 1 hour 15 minutes to cook dinner. Dinner needs to be ready by 6:30 p.m. When should he start cooking?

b) It takes Ms. B. 23 minutes to drive to work. She needs to be at the office at 8:55 a.m. When should she leave home?

c) It takes Liz 1 hour 33 minutes to get to the concert hall. The concert starts at 8:00 p.m., but she wants to be there 10 minutes early. When should she leave home?

**Answers:** a) 5:15 p.m., b) 8:32 a.m., c) 6:17 p.m.

4. Find the missing numbers.

a) \[
\begin{array}{c}
  8 : 3 \\
- 8 : 9 \\
\hline
  \quad : 2 \ 3 \\
\end{array}
\]

b) \[
\begin{array}{c}
  6 : 5 \\
- 4 : 7 \\
\hline
  : 2 \ 3 \\
\end{array}
\]

c) \[
\begin{array}{c}
  \quad : 5 \ 1 \\
- 5 : 4 \\
\hline
  7 : 4 \\
\end{array}
\]

**Answers:** a) 8:32 – 8:09 = 0:23, b) 6:50 – 4:27 = 2:23, c) 12:51 – 5:47 = 7:04
Goals

Students will convert between different units of time. Students will use calendars to determine the number of days in a month and use this information to solve problems. Students will solve problems that require converting between units of time.

PRIOR KNOWLEDGE REQUIRED

Can multiply and divide numbers up to $9 \times 9$
Can add and subtract two-digit numbers
Can extend a growing number pattern
Knows the units of time from seconds to millennia
Can change hours to minutes, and weeks to days

MATERIALS

ball or relay race baton (optional)
one-minute timer, such as clock with second hand, sand timer, or stopwatch
calendars from different years, one per student

Mental math minute. Arrange students in a line and have them add two-digit numbers by adding tens and adding ones. For each addition problem, such as $35 + 46$, the student needs to follow three steps: add the tens ($30 + 40 = 70$), add the ones ($5 + 6 = 11$), and finish the addition ($70 + 11 = 81$, so $35 + 46 = 81$). The next student in line gets a new problem. Students can pass a ball or a relay race baton to each other, so that the person who receives the ball or baton answers the next question. Start with problems that do not require regrouping, such as $25 + 34$, and continue to questions that require regrouping ones.

Introduce seconds. Remind students that in the last lesson they changed units of time to other units: hours to minutes, weeks to days, days to hours, and so on. ASK: What is the smallest unit of time you know? If students do not mention seconds, PROMPT: Is there a unit of time smaller than a minute? If your class clock has a second hand, point it out and SAY: The hour hand moves from number to number in 1 hour. The minute hand moves from dash to dash in 1 minute and makes a full circle in 1 hour. The fast hand makes a full circle in 1 minute. ASK: How long does it take the fast hand to move from dash to dash? (1 second) SAY: A second is the smallest unit of time we use. This is the time it takes the fastest hand on an analog clock to move from dash to dash. There are 60 seconds in 1 minute. Draw a table on the board, as shown in the margin and have students help you fill it in.
ASK: What other table does this look like? (changing hours to minutes) Why are the tables so similar? (there are 60 minutes in an hour and 60 seconds in a minute) Explain that changing minutes to seconds works exactly the same as changing hours to minutes. Keep the table on the board.

**Converting mixed measurements in minutes and seconds to minutes.**

Write “3 minutes 5 seconds” on the board. ASK: How many seconds are in 3 minutes? (180) How many seconds do we have left over? (5 seconds) How many seconds are in 3 minutes and 5 seconds? (185) How do you know? (180 + 5 = 185) Write the calculation on the board, as shown below:

\[
3 \text{ minutes} = 180 \text{ seconds} \\
3 \text{ minutes 5 seconds} = 180 + 5 \text{ seconds} = 185 \text{ seconds}
\]

ASK: How many seconds are there in 4 minutes 15 seconds? Write on the board:

\[
4 \text{ minutes} = \quad \text{seconds} \\
4 \text{ minutes 15 seconds} = \quad \text{seconds} = \quad \text{seconds}
\]

Have volunteers help you fill in the blanks. (240, 240 + 15, 255)

**Exercises:** Change the minutes to seconds. Add the leftover seconds.

a) 2 minutes 36 seconds  

b) 3 minutes 50 seconds 

c) 1 minute 8 seconds 

**Answers:** a) 156 seconds, b) 230 seconds, c) 68 seconds

Remind students that to compare lengths or distances in different units, they converted lengths to the same unit. For example, to compare 2 m 30 cm to 250 cm, they would write both lengths in centimetres only.

**Exercises:** Which time period is shorter?

a) 150 seconds or 2 minutes 29 seconds  

b) 150 seconds or 2 minutes 50 seconds 

**Bonus:** 3 minutes or 2 minutes 55 seconds 

**Answers:** a) 2 minutes 29 seconds, b) 150 seconds, Bonus: 2 minutes 55 seconds

**Choosing the best unit of time.** Review all units of time learned to date. Have students name various units of time and write them on the board out of order. Then, have students make a list of the units from longest to shortest. (millennium, century, decade, year, month, week, day, hour, minute, second) Point out that when there is more than one millennium, we say millenia.
Write on the board:

A TV show lasts 30 _________.

ASK: Which unit should we use in this sentence? Would years work? 
(no) SAY: 30 years is an age of a person, not the length of a TV show! 
ASK: Will seconds work? (no) SAY: 30 seconds is about the length of a 
long commercial. ASK: Which unit should we use for the length of a TV 
show? (minutes)

Ask students to give examples of something that is measured with each unit 
and write the examples on the board in the form of sentences, underlining 
the unit. Sample answers:

Dinosaurs lived many millennia ago. 
Vikings first came to North America about 10 centuries ago. 
Canada is many decades old. 
Jessica is 8 years old. 
There are 3 months left until the summer holidays. 
Cousin Rick is coming to visit in 2 weeks. 
The math test is in 3 days. 
I spend about 9 hours sleeping each night. 
A lesson lasts 45 minutes. 
I can run 100 m in 20 seconds.

Exercises: What unit of time should you use in the answer? Choose from 
seconds, minutes, hours, days, months, and years.

a) How much time do you spend reading in class? 
b) How old is your grandmother? 
c) How much time do you spend at school today? 
d) How long is one period of a soccer game? 
e) How much time will it take you to run around the classroom once? 
f) How much time is left until Christmas? 
g) How old is Canada? 
h) How much time passed since Saturday?

Answers: a) minutes, b) years, c) hours, d) minutes, e) seconds, f) months, 
g) years, h) days

NOTE: In the next part of the lesson, you need some kind of device 
that allows students to determine whether 1 minute has passed. If your 
classroom clock has a second hand, you can use the time it takes a second 
hand to make one full circle as a benchmark for 1 minute. A one-minute 
sand timer or a stopwatch can also work. You can either distribute sand 
timers to students or announce clearly when a minute starts or stops. 
Online countdown timers can be used as well.
Comparing time periods to 1 minute. Discuss options to determine that 1 minute passed. Explain that there are different tools to do that, such as a clock with a fast-moving hand called a second hand, a sand timer, or a stopwatch. Show the tools that you have. Explain that a second hand makes a full circle in 1 minute. It takes the sand in the sand timer exactly the same time to flow from the top chamber to the bottom of the sand timer. You can demonstrate that by turning over a sand timer when the second hand is exactly at the 12 and waiting for the sand timer to finish. Point out that the second hand returns to 12 exactly.

Demonstrate how the instrument you are going to use shows that 1 minute passed. (sand flows out of the sand timer; the second hand returns to the 12, if started at the 12; the stopwatch shows 1 minute, for example as 00:01:00; a countdown timer rings; and so on) Activity 1 will help students develop a sense of how long 1 minute is.

**ACTIVITY 1**

1. Students work in pairs. Partners check how many jumping jacks, burpees, squats, and toe touches they can do in a minute, how many times they can say the alphabet in 1 minute, and how many times they can write their names in 1 minute. For example, Partner 1 does jumping jacks for 1 minute and counts the jumping jacks. Partner 2 says when to start and when to stop. Partners switch roles for the next task, i.e., burpees, so that each partner does one active task and one quiet task and times the other partner twice.

Have students suggest activities that take less than 1 minute. (sample answers: making a funny face, running around the classroom once, writing your full name, drawing a stick person, doing 10 jumping jacks) Repeat with activities that take more than 1 minute, but still do not take more than a few minutes. (sample answers: brushing your teeth, eating breakfast, drawing a picture)

Students can signal the answers for the following exercises by showing thumbs up for something that takes longer than 1 minute and thumbs down for something that takes shorter than 1 minute.

**Exercises:** Does the activity take less than 1 minute or more than 1 minute?

a) diving into a pool  

b) singing the alphabet song  

c) eating dinner  

d) walking a dog  

e) baking cookies  

f) putting on a jacket

**Answers:** a) less than 1 minute, b) less than 1 minute, c) more than 1 minute, d) more than 1 minute, e) more than 1 minute, f) less than 1 minute

Comparing periods of time to 1 hour. Give students an example of a school activity that takes about 1 hour, such as the lesson between lunch and recess. Have students suggest activities that take less than 1 hour.
(sample answers: eating dinner, preparing your schoolbag for school, a TV show, a math lesson) Repeat with activities that take more than 1 hour.
(sample answers: sleeping at night, a full soccer game, watching a movie)

Students can signal the answers for the following exercises by showing thumbs up for something that takes longer than 1 hour and thumbs down for something that takes less than 1 hour.

**Exercises:** Does the activity take less than 1 hour or more than 1 hour?

a) running around the school one time
b) shampooing your hair
c) a full hockey game with extra time
d) reading 1 page of a book
e) getting dressed
f) washing a car inside and out

**Answers:** a) less than 1 hour, b) less than 1 hour, c) more than 1 hour, d) less than 1 hour, e) less than 1 hour, f) more than 1 hour

### ACTIVITY 2

2. **Determining the number of days in a month.** Distribute calendars from different years to students. Assign three months to each student, so that each group of four has a whole year assigned to them. Have students determine how many days are in each month. Have students make two tables—one showing the number of days in each month and the other showing which months have 31 days, 30 days, or fewer than 30 days. Have students who were assigned the month of February check the number of days in February in calendars for other years as well. Explain that once in four years there is a leap year, when February has 29 days instead of 28.

**Solving problems using calendars.** Return to the table showing the number of days in weeks (from Lesson ME3-21). **ASK:** Which month has the number of days that matches one of the numbers in the table? (February) How many weeks long is February? (4 weeks long) How many weeks long is February in a leap year? (4 weeks and 1 day long, or between 4 and 5 weeks) How many weeks long are the other months? (between 4 and 5 weeks)

Ask students to find June on their calendars. Ask them to put their fingers on June 15. **ASK:** What day is one week before June 15? (June 8) How do you know? (count back 7 days or move up 1 week) Repeat with two weeks after June 15. (June 29) Point out that calendars have 7 columns. **ASK:** Why are there seven columns for each month and not eight or six? (there are 7 days in 1 week, calendars show days of the week as well as dates) Point out that this makes it very easy to find any date that is a whole number of weeks before or after a given date.
ASK: What date is three weeks after June 15? (July 6) Have a volunteer explain how they found the answer. Point out that since we are talking about a whole number of weeks, we can find the answer by going down the column in a calendar. SAY: Two weeks later than June 15 is June 29, and the day that is one week after that is the day we are looking for. ASK: Is this day in June? (no) What month is this day in? (July) What day of the week is it in your calendar? (answers will vary) Will this day be the same day of the week as June 15? (yes) What day is one week after June 29? (July 6)

**Exercises:** Find May 10 on a calendar.

a) What day is 3 weeks after May 10?

b) What day is 2 weeks before May 10?

c) How many weeks are between May 10 and June 14?

**Answers:** a) May 31, b) April 26, c) 5 weeks

Finding the date before or after a given date without using calendars (and without regrouping weeks as days). ASK: What day is 11 days after June 15? (June 26) How do you know? (sample answers: count up days on a calendar; use the fact that $11 = 7 + 4$, so move down 1 week and count 4 more days; add $15 + 11 = 26$) Make sure all three of these methods come up and write the last solution on the board. ASK: What is the easiest method to find the date that is 11 days after June 15? (adding $15 + 11$) What day is 6 days before June 15? (June 9) How do you know? ($15 - 6 = 9$) Have a volunteer write the solution on the board.

**Exercises**

a) What day is 10 days after September 14?

b) What day is 9 days before October 30?

**Answers:** a) September 24, b) October 21

Finding the date before or after a given date without using calendars by regrouping weeks as days. ASK: What day is 14 days after April 16? (April 30) How do you know? ($16 + 14 = 30$) What day is two weeks after April 16? (April 30) How do you know? (2 weeks = 14 days, 14 days after April 16 is April 30) What day is two weeks before April 16? (April 2) How do you know? (2 weeks is 14 days, $16 - 14 = 2$) Have a volunteer write the solution on the board.

**Exercises**

a) What day is 2 weeks after January 10?

b) What day is 3 weeks before December 31?

**Bonus:** What day is 2 weeks and 3 days after May 7?

**Answers:** a) January 24, b) December 10, Bonus: May 24
Finding the date after the given date with regrouping weeks and changing months. SAY: I want to find out what day is three weeks after April 10. ASK: How many days are in three weeks? (21) How do you know? (3 × 7 = 21) Write on the board:

What day is 3 weeks after April 10?
3 weeks = 3 × 7 days
= 21 days

SAY: Let’s add the days as we did before. Write on the board:

10 + 21 = __

ASK: What is 10 + 21? (31) Fill in the blank and ASK: Is the day we are looking for April 31? (no) Have students check their calendars or tables to see how many days are in April. (30) This is the day that comes after April 30, so what day is that? (May 1) Point out that if you subtract the 30 days in April from the number you got by adding 10 + 21, you get 1, which is the date in May. Have students check the answer using calendars.

Repeat with “What day is 3 weeks after April 15?” Explain that by adding 15 + 21, you get April 36, which is clearly incorrect. SAY: April has 30 days. The 31st day is already May 1. So, what date is the 36th day? (May 6) Write on the board:

April has 30 days.
36 − 30 = 6
May 6 is 3 weeks after April 15.

Have students check the answer using calendars. Repeat with “What day is 4 weeks after May 25?” Have students tell you what to do at each next step. (see solution below)

4 weeks = 4 × 7 days
= 28 days
25 + 28 = 53
May has 31 days.
53 − 31 = 22
June 22 is 4 weeks after May 25.

Exercises

a) What day is 2 weeks after August 23?
b) What day is 5 weeks after April 1?

Bonus: What day is 8 weeks after May 15?
Answers: a) September 6, b) May 6, Bonus: July 10

Finding the date before the given date with regrouping weeks and changing months. SAY: I want to know what day is two weeks before April 12. ASK: How many days are in two weeks? (14) Can we subtract 14 from 12? (no) When you subtract, say, 18 from 56, what do you do?
Write the subtraction on the board and have students lead you step by step through the subtraction using the standard algorithm, including regrouping. Emphasize that when you do not have enough ones, you regroup tens.

Point out that you can use a similar strategy to the one you used before. SAY: We do not have enough days in April, so we need to use days in the month before it. ASK: What month comes before April? (March) How many days are in March? (31) Explain that you can write a fake date for April 12, as if it was a day in March. Write on the board:

What day is 2 weeks before April 12?
March has 31 days
31 + 12 = 43
April 12 = March 43

SAY: There is no such date as March 43, so it is a fake date. However, we now have enough days to subtract 14 days for 2 weeks! ASK: What is 43 − 14? (29) Continue writing on the board:

2 weeks = 2 × 7 = 14 days
43 − 14 = 29, so March 29 is 2 weeks before April 12.

Have students check that this is the correct answer by finding the day that is two weeks after March 29. (29 + 14 = 43, 43 − 31 = 12, so the answer is April 12) Have students also check the answer using their calendars.

**Exercises**

a) What day is 3 weeks before May 8th?

b) What day is 5 weeks before May 25th?

**Bonus:** What day is 9 weeks before May 25th?

**Answers:** a) April 17, b) April 20, c) March 23

**NOTE:** Extensions 5 and 6 are required to meet the British Columbia curriculum.
Extensions

1. Ella was born on April 7, 2008. Josh is 6 weeks older than Ella. When is Josh’s birthday? Hint: 2008 was a leap year, so February had 29 days in 2008.

Solution: 6 weeks = 6 \times 7 \text{ days} = 42 \text{ days} 
March has 31 days and February had 29 days in 2008. 
So, April 7 = March 38 = February 67 
67 − 42 = 25, so Josh was born on February 25, 2008.

2. Write the correct unit. Use years, decades, centuries, or millennia.
   a) Canada is about 15 ___ old.
   b) Christopher Columbus arrived in North America about 5 ___ ago.
   c) Maya is an ancient civilization. First Mayan cities appeared about 3 ___ ago.
   d) Archeologists excavated many Mayan temples that are over 1000 ___ old.

Answers: a) decades, b) centuries, c) millennia, d) years

3. Have students create a calendar that includes days of the week, dates, and personal events.

4. Find the date in a leap year.
   a) 15 days after December 23 
   b) 16 days before January 5 
   c) 7 days after February 24 
   d) 15 days before March 13 

Answer: a) January 7, b) December 20, c) March 2, d) February 27

5. Have students use a calendar (paper or traditional) to estimate how many months are between two events. For example, National Aboriginal Day is on June 21 and school starts in September, so there are about 3 months between them.

6. As in some other cultures, many First Nations celebrate the Lunar New Year. The Lunar New Year begins on the day of the first full moon of the calendar year.

In 2015, the Lunar New Year began on February 19. In 2016, the Lunar New Year began on February 8. How many weeks were there from the Lunar New Year in 2015 to the Lunar New Year in 2016?

Answer: about 51 weeks
Reading Digital Times

02 04 06
09 11 12

: :

minutes past
# Time Memory Cards (I)

<table>
<thead>
<tr>
<th>Time</th>
<th>Description</th>
<th>Time</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>01:05</td>
<td>5 minutes past 1</td>
<td>02:10</td>
<td></td>
</tr>
<tr>
<td>03:12</td>
<td>12 minutes past 3</td>
<td>10 minutes</td>
<td>past 2</td>
</tr>
<tr>
<td>04:30</td>
<td>30 minutes past 4</td>
<td>05:01</td>
<td></td>
</tr>
<tr>
<td>10:02</td>
<td>2 minutes past 10</td>
<td>1 minute</td>
<td>past 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Time Memory Cards (2)

- **06:50**: 50 minutes past 6
- **07:14**: 14 minutes past 7
- **11:07**: 7 minutes past 11
- **11:11**: 11 minutes past 7
- **08:42**: 42 minutes past 8
- **09:03**: 3 minutes past 9
- **12:22**: 22 minutes past 12
Time Memory Cards (3)

12:00

06:00

03:00

01:00

07:00

02:00

08:00
Time Memory Cards (4)
Time Memory Cards (5)

06:05

02:35

08:15

03:45

11:20

03:45

12:25
Time Memory Cards (6)

- 12:30 half past 12
- 05:30
- 01:30 half past 1
- half past 5
- half past 8
- half past 9
- half past 4
Time Memory Cards (7)

1 o’clock 09:00

2 o’clock 9 o’clock

4 o’clock

5 o’clock 8 o’clock
Time Memory Cards (8)

- Quarter past 12
- Quarter past 1
- Quarter past 5
- Quarter past 8
- Quarter past 4
- Quarter past 9
- Quarter past 1
- Quarter past 5

Clocks and digital times are shown with the corresponding phrases.
Make Your Own Clock
Numbers on a Clock Face

Analog clock faces show numbers from 1 to 12 in a circle. To label a clock face, write in the numbers 12, 6, 3, and 9 first. Then fill in the rest of the numbers.

I. Fill in the missing numbers on the clock face.

a) b) c) d) e) f)
Empty Clock Faces

[Diagram of empty clock faces with numbers 1 through 12 and positions for hands]
Telling Time (The Second Hand)

The second hand is longer and thinner than both the minute and hour hands.
We read seconds the same way we read minutes.
The exact time with seconds is:

1. Write the time in numbers under the clock.
   a)   b)   c)
   d)   e)   f)

2. Draw the hands on the analog clock to show the time.
   a) 3:50:35   b) 7:05:25   c) 9:15:05

   [Clock images for each time]
PS3-5 Using Number Lines

Teach this lesson after: Unit 13

VOCABULARY
- counting backwards
- counting forwards
- decreasing
- hour
- increasing
- kilometre (km)
- minute
- number line
- second
- time
- vertical
- week
- year

Goals
Students will use number lines to solve problems.

PRIOR KNOWLEDGE REQUIRED
- Can tell and write time to the nearest minute
- Can measure time intervals in minutes
- Can draw a number line with increasing numbers from left to right for horizontal number lines
- Can use a number line to solve word problems involving addition and subtraction of time intervals
- Can skip count forward
- Can add several two-digit numbers
- Can use organized search (for Problem Bank 7)
- Can use number lines that skip count (for Problem Banks 3, 4)

MATERIALS
- BLM Number Line Word Problems (p. P-70)
- BLM Phone Rings (p. P-72, see Extended Problem)
- BLM Clock Word Problems (pp. P-73–74, see Extended Problem)

Using number lines is easier than counting backwards. Write on the board:

36, 31, 26, 21

ASK: Is this pattern increasing or decreasing? (decreasing) PROMPT: Do the numbers get bigger or smaller? (smaller) SAY: The numbers get smaller, so this is a decreasing pattern. ASK: How much do the numbers get smaller by each time? (5) Ask a volunteer to write the next term in the pattern on the board. (16) ASK: How did you get that? (I counted backwards; I subtracted 21 – 5)

Write on the board:

Karen decides to hike from her home to Tea Lake. On Monday morning, she is 20 km away from the lake. She hikes 6 km each day. How far from the lake will she be by Wednesday evening?

ASK: What is the farthest she is from the lake? (20 km) SAY: She started 20 kilometres away and she is getting closer each day, so 20 kilometres is the farthest. So, let’s make a number line up to 20. Number lines are easier to use than counting backwards because you can write the numbers going up in order. Draw on the board:
Point to the dot at 20 and SAY: On Monday, she starts 20 kilometres away from the lake. Ask different volunteers to show her progress each day, as shown below:

SAY: Because Karen is getting closer to the lake each day, you just have to show moving back 6 spaces at a time. You could count back 6 to find out how far she is after each day, but you don’t have to. The number line does the counting back for you, because you’ve already written the numbers in order. ASK: How far will Karen be from the lake by the end of the day Wednesday? (2 km) If students say just “2,” ASK: Two what? (kilometres) Point out that in order to answer how far, you have to say whether Karen is 2 centimetres, 2 metres, or 2 kilometres from where she wants to be—otherwise, you really don’t know how far she is.

In the exercises below, students will solve real-life word problems by counting backwards on a number line.

Exercise: Complete BLM Number Line Word Problems.

Selected solution
3. c)

Answers: 1. 5 km; 2. 1 km; 3. a) 2 km, b) 6 minutes, c) 5 km

Bonus
1. A snail crawls 3 cm in one hour. It starts 20 cm away from the end of the branch and crawls towards the end of the branch.
   a) How far from the end of the branch will it be after 4 hours of crawling?
   b) A cherry hangs 5 cm before the end of the branch. How long after the snail starts crawling will it reach the cherry?

2. A messenger pigeon flies 1 km every 3 minutes.
   a) How long will it take the pigeon to carry a letter to a person who lives 6 km away?
   b) The message needs to arrive by 9:00 a.m. What time should the pigeon leave?

Answers: 1. a) 8 cm, b) after 5 hours; 2. a) 18 minutes, b) 18 minutes to 9 or 8:42
**Jumps in two directions on a number line.** Tell students that Amy is riding an elevator in her apartment building. Draw a vertical number line on the board, as shown below:

```
10
9
8
7
6
5
4
3
2

Ground level = G = 1
Basement = B = 0
```

Tell students that number lines can be vertical too, and this vertical number line shows the floor numbers in the building. SAY: Amy starts at the second floor. Have a volunteer show where she starts by drawing a dot on the number line. SAY: Amy goes up five floors. Have a volunteer draw an arrow to show where Amy travels to. (to the 7th floor) SAY: Amy now goes down two floors. Have a volunteer draw an arrow to show Amy’s movement. (to the 5th floor) The completed number line is shown below:

```
10
9
8
7
6
5
4
3
2

Ground level = G = 1
Basement = B = 0
```

SAY: So, Amy ends on the fifth floor.

**Exercises:** Draw a vertical number line in your notebook to answer the question.

a) Amy starts at the 5th floor. She goes up 4 floors, then down 7 floors. What floor does she end on?

b) Amy starts at the 10th floor. She goes down 6 floors, then up 4 floors. What floor does she end on?

c) Amy starts at the 8th floor. She goes up 2 floors, then down 5 floors. What floor does she end on?

d) Amy is on the 3rd floor. She goes up to the 9th floor. How many floors did she go up?

e) Amy pushes the button to go to the 10th floor. She travels up four floors before getting off. What floor did she start on?

**Bonus:** Amy starts at the 7th floor. She goes down 5 floors, up 6 floors, and then down 3 floors. What floor does she end on?

**Answers:** a) 2nd, b) 8th, c) 5th, d) 6, e) 6th, Bonus: 5th
Using number lines with unknown numbers. Write on the board:

Anton is 2 years older than Megan.
Kathy is 7 years older than Megan.
Evan is 3 years younger than Kathy.
Who is older, Anton or Evan? How much older?

Read the problem aloud. SAY: A good way to start a problem like this is to draw a number line so that you can see how far apart everyone’s ages are on the line. Draw on the board:

SAY: On the number line, we’ll use one place to the right to mean one year older. Look at the first sentence. ASK: Who is older, Anton or Megan? (Anton) Underline “Anton is” and “older than Megan,” as shown below:

Anton is 2 years older than Megan.

ASK: How much older? (2 years) SAY: That means we should place Anton two places to the right of Megan. Add “Megan” and “Anton” to the number line on the board, as shown below:

SAY: We can’t put numbers on the number line, because we don’t know anyone’s actual age. We just know how they compare to each other—that Anton is two years older than Megan. We don’t know yet if we need more places on the left or the right, so I’ve written their names in the middle of the number line. If there is not enough room, I might have to add to the line. Read the second sentence of the problem aloud, and then have a volunteer extend the number line to show Kathy’s age, as shown below:

Show the two comparisons done so far, using arrows, as shown below:

Read the third sentence of the problem aloud, and have a volunteer show Evan’s age. Show this comparison with an arrow, as shown below:
SAY: Now we can answer the problem. ASK: Who is older, Anton or Evan? (Evan) How do you know Evan is older? (his age is to the right of Anton’s) How much older? (2 years) Show how you can get from Anton’s age to Evan’s age with a jump forward of size 2 by tracing the jump with your finger. ASK: Do you know from this picture how old anyone is? (no, we just know how the ages compare)

NOTE: For the exercises below, tell students to make sure to leave lots of room on each side of the number line they draw, in case they have to add to it.

**Exercises:** Draw a number line to answer the questions.

a) Jake is 3 years older than Sandy. Rick is 5 years older than Sandy. Lily is 4 years younger than Rick. Who is older, Jake or Lily? How much older?

b) Rick, Marko, Sam, and Glen are identical quadruplets. Glen is 19 minutes older than Rick. Glen is 6 minutes older than Sam. Marko is 9 minutes younger than Sam. Who is older, Rick or Marko? How much older?

**Bonus:** Josh’s birthday is 3 weeks before Kyle’s. Sara’s birthday is 4 days after Kyle’s. Nina is 2 weeks younger than Sara. Whose birthday is first, Josh’s or Nina’s? By how many days? Hint: A week has 7 days.

**Answers:** a) Jake is 2 years older than Lily, b) Marko is 4 minutes older than Rick, Bonus: Josh’s birthday is 11 days before Nina’s

For extra practice, students can complete the following exercises.

**Exercises**

1. David plants four apple trees in a row. The nearest tree is 5 m from his house. The trees are 2 m apart. How far away from David’s house is the last tree? Hint: Put David’s house at zero on the number line.
   
   **Answer:** 11 m

2. A snail is at the bottom of a well on Monday morning. Every day, the snail climbs 3 m, and every night it slides back 1 m. The well is 7 m deep. On what day does the snail reach the top of the well?

   **Solution**

   ![Diagram showing the movement of the snail]
Problem Bank

1. When Ansel wakes up, he takes 4 minutes to get dressed, 3 minutes to brush his teeth, 8 minutes to have breakfast, 2 minutes to gather his books, 2 minutes to put on his shoes, and 4 minutes to bike to school. If Ansel needs to be at school by 8:30 a.m., what time should he set his alarm for?

   Answer: 8:07 a.m.

2. Marko gets up at 8:10 a.m. It takes him 8 minutes to shower and get dressed, 4 minutes to eat breakfast, 2 minutes to gather his books, and 1 minute to walk to the bus stop. If the bus leaves at 8:32 a.m., for how long can Marko play with his sister before he leaves?

   Answer: 7 minutes

3. Ella takes 10 minutes to have breakfast, and 10 minutes to get ready for school. It takes her 5 minutes to walk to school. If she has to be at school by 8:30 a.m., what time does Ella have to wake up?

   a) Draw a number line from 0 to 30. Use it to solve the problem.

   b) Solve the problem again using a number line that counts by 5.

   c) Did you get the same answer in parts a) and b)?

   d) Which way was faster?

   e) Glen tries to use a number line that counts by 10 to solve the same problem. What difficulty will he have?

   Answers: a) 8:05 a.m., b) 8:05 a.m., c) yes, d) part b), e) there won’t be a marking for when Ella gets to school—it will be halfway between two markings

4. Draw a number line that skip counts to solve the problem. Use as few markings as you can.

   a) Tina takes 6 minutes to have breakfast and 8 minutes to get ready for school. It takes her 12 minutes to bike to school. If Tina has to be at school by 8:45 a.m., what time does she have to wake up?

   b) Tina takes 6 minutes to get ready for school, 9 minutes to eat breakfast, and 3 minutes to walk to the bus stop. If the bus leaves at 8:30 a.m., what is the latest time Tina can wake up?

   Answers: a) skip count by 2s, she has to wake up by 8:19 a.m.;

   b) skip count by 3s, she can wake up as late as 8:12 a.m.

5. Zara is at soccer practice. She does dribbling exercises for 10 seconds, then rests for 5 seconds. Zara dribbles four times and rests three times. Does the drill take more or less than 1 minute (60 seconds)?

   Answer: less, it takes 55 seconds
6. For a school fundraiser, Marcel sold 5 more muffins than Eddy. Liz sold 3 fewer muffins than Eddy. Shelly sold 4 more muffins than Liz. Marcel sold 43 muffins. How many did the four students sell altogether?

Solution: Liz sold 35, Eddy sold 38, Shelly sold 39, and Marcel sold 43, so altogether they sold $35 + 38 + 39 + 43 = 155$ muffins.

NOTE: Problem 7 combines number lines with strategic searching from earlier problem-solving lessons. If students struggle, suggest that they decide who has the fewest marbles (Grace), and ask how many marbles the children have altogether if Grace has 1 marble, then 2 marbles, and so on.

7. Ethan has 8 more marbles than Grace. Grace has 5 fewer marbles than Cody. Jennifer has 7 more marbles than Cody. Altogether they have 41 marbles.

a) How many marbles does each friend have?

b) Check your answer to part a) by adding your answers together. Do you get 41? If not, find your mistake.

Answers: a) Grace has 4 marbles, Cody has 9 marbles, Ethan has 12 marbles, and Jennifer has 16 marbles; b) $4 + 9 + 12 + 16 = 41$

8. A painter’s ladder has 12 steps. The painter spills red paint on the ground, and on every second step. He spills blue paint on the ground and on every third step.

a) Which steps have red and blue paint on them?

b) Which steps will have no paint on them?

Answers: a) 6th and 12th; b) 1st, 5th, 7th, and 11th.
Number Line Word Problems

1. On Tuesday morning, Jin is camping 20 km away from the next town. He plans to walk 5 km each day towards the town. How far from the town will he be on Thursday evening? ________

2. Clara is playing baseball in a park 16 km from her home. She decides to jog home. She can jog 5 km every hour. How far from home will she be after 3 hours? ________

3. Draw a number line on the grid for the problem, then solve the problem. The first number line is started for you.
   a) Ren is 11 km from home. He is walking home. He can walk 3 km every hour. How far from home will he be after 3 hours? ________

   b) Mary is 12 blocks from home. She can walk 2 blocks in a minute. How long will it take her to walk home? ________________

   c) Ray walks 4 km in the first hour and 3 km each hour after that. When he starts walking home, he is 15 km from home. How far from home is he after 3 hours? ________________
Extended Problem: Clock Problems

**MATERIALS**

BLM Phone Rings (p. P-72)
BLM Clock Word Problems (pp. P-73–74)

**Preparation for the extended problem.** Have students complete BLM Phone Rings, in which they solve a multi-part problem in a contextual setting. This will prepare students for the extended problem and allow you to gauge students’ ability to do independent work in a multi-part contextual problem. If students are not ready to do such work, wait until later in the year when they have had a chance to build confidence. (a) 19, 16, 17; b) Tom’s; c) Matt’s; d) yes)

When students are ready to do the extended problem, tell students to look at the clock in the classroom, and ASK: What time does it say? Then ask if anyone has a watch that shows a different time. How much are they off by? Draw three clocks on the board, showing 3:21, 3:24, and 3:29. SAY: The correct time is 3:24. Have students point to the correct clock. Then point to the other two, and ASK: Is this clock ahead or behind? By how much? (the clock showing 3:21 is behind by 3 minutes and the clock showing 3:29 is ahead by 5 minutes)

**Extended Problem: Clock Word Problems.** Provide students with BLM Clock Word Problems. Question 6 provides an opportunity to apply the problem-solving strategy of using a number line. All students might find an answer to Question 6, but students who notice that the strategy can be used will find the problem easier.

**Answers:** 1. a) circle clock B, b) I circled clock B because it shows 2:19, which is 4 minutes ahead of the other clock; 2. a) 2:04, b) 1:09, c) 12:16; 3. a) 3:04, b) 1:11, c) 10:26; 4. 5:25; 5. a) 6, b) 9, c) after 20 years, d) $6 \times 7 = 42$; 6. 9:27
Phone Rings

I. Tom, Matt, and Kate each have a phone. Each phone has a different ring.

- On Tom’s phone, rings last for 3 seconds each with a 1-second pause between rings. His phone takes a message after 5 rings.
- On Matt’s phone, rings last for 4 seconds each with a 2-second pause between rings. His phone takes a message after 3 rings.
- On Kate’s phone, rings last for 2 seconds each with a 1-second pause between rings. Her phone takes a message after 6 rings.

a) Use a number line to show how long each person’s phone rings for from the beginning of the first ring to the end of the last ring.

Tom:

\[\begin{array}{cccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\
\end{array}\]

Tom’s phone takes a message after _______ seconds.

Matt:

\[\begin{array}{cccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\
\end{array}\]

Matt’s phone takes a message after _______ seconds.

Kate:

\[\begin{array}{cccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\
\end{array}\]

Kate’s phone takes a message after _______ seconds.

b) From the beginning of the first ring to the end of the last ring, whose phone ring lasts the longest? _________________________

c) From the beginning of the first ring to the end of the last ring, whose phone ring lasts the shortest? _________________________

d) When Kate is playing outside, it takes her 15 seconds from the time the first ring starts to get to her phone. Will she answer before the phone takes a message?
Clock Word Problems (I)

The clock in Mr. C’s classroom is always 4 minutes ahead.

1. One clock below is correct and the other one is the clock in Mr. C’s classroom.
   a) Circle the clock that is in Mr. C’s classroom.
      
      A. ![Clock A]  B. ![Clock B]

   b) Explain your choice.

   __________________________________________________________

2. The correct time is given. What time does Mr. C’s classroom clock show?
   a) 2:00 ______   b) 1:05 ______   c) 12:12 ______

3. Mr. C’s classroom clock shows the time given. What is the correct time?
   a) 3:08 ______   b) 1:15 ______   c) 10:30 ______

4. Mr. C’s classroom clock is shown below.

   ![Clock]

   What is the correct time? ______
Clock Word Problems (2)

5. Ms. K buys a clock for her classroom. She sets it to the correct time. After 1 year, it is 3 minutes ahead.
   a) How many minutes ahead will the clock be after 2 years? _______
   b) How many minutes ahead will the clock be after 3 years? _______
   c) When will the clock be 60 minutes ahead? _________________
   d) After two years, the principal says she will bring the class 7 balloons for every minute the clock is ahead. How many balloons will the principal bring?

6. Ray’s clock is 5 minutes ahead of Sally’s clock.
   Bill’s clock is 2 minutes behind Sally’s clock.
   Jun’s clock is 4 minutes ahead of Bill’s clock.
   What time does Jun’s clock say when Ray’s clock says 9:30?
Goals
Students will, given a problem, make a simpler problem and use the solution to the simpler problem to solve the harder problem.

Prior Knowledge Required
Can add and subtract two-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can represent fractions by using lengths and areas

Materials
Two sticks of different lengths and different colours
Scissors and BLM Fraction Strips and Circles (p. P-82, see Problem Bank 14)
BLM Planting a Flower Garden (pp. P-84–85, see Extended Problem)

Using a given simpler problem to help solve a harder problem. Write on the board:

There are 300 people in line. How many people are behind the 7th person?

Ask: What makes this problem hard? (students might say because 300 is a lot of people) Would it be easier if I asked how many people are behind the 299th person in line? (yes, there is only 1 person) Say: So, it is not how big 300 is that makes this problem hard. Ask: Can you say what exactly makes it hard? (7 and 300 are far apart) Write on the board:

There are 8 people in line. How many people are behind the 7th person?

Say: I’m going to draw the eight people. Draw on the board:

Front of line: ☺☺☺☺☺☺☺

Have a volunteer circle the seventh person in line. Ask: How many people are behind the seventh person? (1) Say: You don’t have to draw happy faces. You could just draw dots.

Draw on the board:

● ● ● ● ● ● ● ●

7th
Exercises: Draw a picture to show your answer.

a) There are 6 people in line. How many people are behind the 5\textsuperscript{th} person?

b) There are 5 people in line. How many people are behind the 3\textsuperscript{rd} person?

c) There are 9 people in line. How many people are behind the 4\textsuperscript{th} person?

Answers: a) 1, b) 2, c) 5

ASK: For exercises like the ones you just did, how can you get the answer from the two numbers given? (subtract) PROMPT: What operation can you do? (subtraction) Draw on the board:

\begin{align*}
\bullet & \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \\
\end{align*}

SAY: This picture shows there are nine people in line, and I want to know how many people are after the fifth person. ASK: Where are the dots that represent the people who are after the fifth person? (to the right of the line) SAY: So I have to subtract all the dots that are before the line. ASK: How many people are before the line? (5) SAY: So you subtract 9 − 5 to get how many people are after the fifth person. ASK: What is 9 − 5? (4) SAY: So there are four people after the fifth person. Now you know that you can do any problem like this with subtraction.

Exercises

a) There are 300 people in line. How many people are behind the 12\textsuperscript{th} person?

b) There are 487 people in line. How many people are behind the 30\textsuperscript{th} person?

Bonus: There are 3459 people in line. How many people are behind the 1459\textsuperscript{th} person?

Answers: a) 300 − 12 = 288; b) 487 − 30 = 457; Bonus: 3459 − 1459 = 2000

ASK: How did solving the easier problems make it easier to solve the harder problems? (doing so told me that the correct approach is to subtract: number of people in line − the position of the person in line)

Off-by-one errors. Tell students that you are waiting in line to get on a rollercoaster ride. You are 37\textsuperscript{th} in line and you see your friend who is 7\textsuperscript{th} in line. ASK: How many people are between my friend and me? Note various guesses. Most students will likely guess 37 − 7 = 30. If they do, SAY: That answer is close, but not quite correct. Let’s draw a simple picture using smaller numbers to see what is going on. Write on the board:

\begin{align*}
\text{Front of line:} & \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \\
\end{align*}

SAY: Each dot represents a person. ASK: How many dots did I draw? (9) SAY: For this simpler problem, suppose I am 9\textsuperscript{th} in line. Have a volunteer...
circle the last dot. SAY: My friend is 5th in line. Have another volunteer circle the 5th dot. Label the dots, as shown below:

Front of line: ● ● ● ● ● 5th ● ● ● ● 9th

ASK: How many people are between the 5th and 9th person? (3) Is that equal to 9 − 5? (no) SAY: It is close, but not quite equal.

Exercises: Draw a picture to decide how many people are between the given positions.

a) the 7th and 8th person
b) the 7th and 9th person
c) the 7th and 10th person
d) the 7th and 11th person
e) the 7th and 12th person
f) the 7th and 37th person

Answers: a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the correct answer? (no) Did it give close to the correct answer? (yes) How can you get the number of people between two people given their positions in line? (subtract the smaller position from the other and then subtract 1 from the difference) How many people are between the 37th person and the 7th person in the rollercoaster line? (29, because 37 − 7 = 30, 30 − 1 = 29) How did starting with smaller numbers help? (answers will vary) SAY: Sometimes, it is easier to start by using smaller numbers than the numbers given in the problem. Then you will see patterns and know how to solve the harder problem. Now that you know the pattern for finding the number of people between any two positions, you can use that method with any numbers.

Exercises: How many people are in line between the given positions?

a) the 8th and 78th person
b) the 314th and 1000th person
c) the 492nd and 613th person

Answers: a) 69, b) 685, c) 120

Making the problem easier by finding what is relevant. SAY: Sometimes making the problem easier has nothing to do with using smaller numbers and finding a pattern. Sometimes all you need to do is ignore information that you don’t need, and moving objects around can help with that.

Affix two sticks of different colours and different lengths to the board, end to end. Label one length and their combined length. The following is an example for 8 cm and 12 cm, but your sticks can be other lengths:

Tell students that all the measurements are in centimetres. SAY: How long is the second stick? (12 cm) SAY: It is easy to see with sticks, but now I’m
going to move these sticks around. Slide the grey stick down and draw the lines around it, as shown below:

\[ \text{Diagram:} \quad \begin{array}{c}
\phantom{8} \\
8 \quad ? \\
\end{array} \]

ASK: How did I move the sticks? (you slid one of them down) SAY: This now looks like a problem to do with shapes and the lengths of missing sides. There’s a lot of extra information in this second problem compared with the first problem, so it looks harder, but it actually has exactly the same answer as the other one. The total length of the two sticks is still 20 cm—I just slid one of the sticks down so that they are not side by side anymore.

**Exercises:** Find what the ? stands for by pretending the sticks are side by side.

\[ \begin{array}{ll}
a) & 11 \\
b) & 18 \\
c) & 25 \\
d) & 315 \\
e) & 12 \\
f) & 9 \\
\end{array} \]

**Answers:** a) 7, b) 9, c) 40, d) 368, Bonus: e) 17, f) 13

SAY: By pretending that the sticks were side by side, you turned the problem into an easier problem.

**Making the problem easier by emphasizing what is relevant.** SAY: We can look at a problem and focus on what we need to find the answer. For example, if you need to find a vertical edge—straight up and down—then
colour over all the vertical lines. If you need to find a horizontal edge, colour over all the horizontal lines.

**Exercises:** Find what the ? stands for by making the problem into an easier problem.

a) \[
\begin{array}{c}
13 \\
15 \\
\hline
3 \\
8 \\
\hline
? \\
12
\end{array}
\]

b) \[
\begin{array}{c}
7 \\
8 \\
\hline
5 \\
4 \\
\hline
? \\
12
\end{array}
\]

**Answers:** a) colour vertical, ? = 5; b) colour horizontal, ? = 12

Point out to students that by colouring the horizontal or vertical lines, they changed the problem into an easier problem.

**Finding perimeter by finding missing side lengths.** Remind students that to find the perimeter of a shape, they have to add up the lengths of all the outside edges.

**Exercises:** Find the perimeter by finding missing sides then adding all the sides.

a) \[
\begin{array}{c}
3 \\
5 \\
\hline
8 \\
13
\end{array}
\]

b) \[
\begin{array}{c}
7 \\
8 \\
\hline
3 \\
1
\end{array}
\]

**Answers:** a) 48, b) 30

**Problem Bank**

1. How many posts are needed to build the fence?
   a) A fence 38 m long made with posts 1 m apart.
   b) A fence 50 m long made with posts 5 m apart.
   c) A fence 60 m long made with posts 3 m apart.
   **Answers:** a) 39, b) 11, c) 21

2. Ken wants to cut a rope into 20 equal parts. How many cuts does he need to make?
   **Answer:** 19

3. A teacher tells his class to read pages for homework. How many pages do students read for the assignment?
   a) They are assigned pages 82 and 83.
   b) They are assigned pages 82 to 84.
3. They are assigned pages 82 to 85.

4. They are assigned pages 82 to 86.

**Answers:** a) 2, b) 3, c) 4, d) 5

4. Fill in the table below with your answers to Problem Bank 3. How can you get the number of pages read from the two page numbers given?

<table>
<thead>
<tr>
<th>Pages Assigned</th>
<th>Number of Pages Read</th>
<th>Result of Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 82 and 83</td>
<td>2</td>
<td>83 − 82 = 1</td>
</tr>
<tr>
<td>b) 82 to 84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 82 to 85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 82 to 86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** b) 3, 84 − 82 = 2; c) 4, 85 − 82 = 3; d) 5, 86 − 82; subtract and add 1

5. Students are assigned pages 87 to 104. How many pages do the students have to read for the assignment?

**Answer:** 18

6. Ben reads every night at home. How many pages does he read?

   a) from pages 352 to 386
   b) from pages 298 to 314
   c) from pages 408 to 451

**Answers:** a) 35, b) 17, c) 44

7. Ava reads pages 354 to 412 except for pages 363 to 389, which have illustrations only. How many pages does Ava read?

**Answer:** 32

8. How many whole numbers are greater than 11 and less than 45?

**Answer:** 33

9. When everyone in Ren’s class stands in line, Ren is 12th in line and 15th counting from the end of the line. How many people are in the class?

**Answer:** 26

10. Make several easier problems and solve them until you see a pattern in your answers to help you do the harder problem.

   a) A fence is made using 42 posts, each 1 metre apart, including a post at each end. How long is the fence?

   **Answers:** a) 41 metres, b) 66 metres
11. Iva builds a fence for a square garden with posts 1 m apart, including a post at each corner. How many posts does she need for a garden of the given size?
   a) 10 m by 10 m
      Hint: Start with a garden that is 1 m by 1 m and then move on to 2 m by 2 m, 3 m by 3 m, and so on.
   b) 20 m by 20 m
      **Answers:** a) 40, b) 80

12. Raj builds a fence around a square field that is 20 m by 20 m. He uses a post at each corner.
   a) How many posts does he need if the posts are 1 m apart?
   b) How many posts does he need if the posts are 2 m apart?
   c) How many posts does he need if the posts are 4 m apart?
   d) How many posts does he need if the posts are 5 m apart?
      **Answers:** a) 80, b) 40, c) 20, d) 16

13. Each line segment of the path below has a length of 1 metre. What is the path’s total length?
   ![Path Diagram]

   **Sample solution:** There are 3 groups of 6 vertical lines, so 18 vertical lines; there are 3 groups of 5 horizontal lines plus the 2 bottom horizontal lines, for a total of $15 + 2 = 17$ horizontal lines. The total length is $18 \text{ m} + 17 \text{ m} = 35 \text{ m}$.

14. Cut out the strips and circles from **BLM Fraction Strips and Circles** (you may cut one line down to the centre of the circles to make folding easier).
   a) Use folding to check that one fifth of strip A is shaded.
   b) Use folding to check that two fifths of strip B is shaded.
      Hint: Use your strategies from parts a) and b) to help you with parts c) and d).
   c) Use folding to check that one fifth of circle C is shaded.
   d) Use folding to check that two fifths of circle D is shaded.
Fraction Strips and Circles

A

B

C

D
Extended Problem: Planting a Flower Garden

**MATERIALS**

BLM Planting a Flower Garden (pp. P-84–85)

**Preparation for the extended problem.** Write on the board:

There are 7 trees in 4 rows. How many trees are there altogether?

ASK: How can you solve this problem? (multiply $4 \times 7$) Erase the 7 and write “18” in its place. ASK: How can you solve this problem? (multiply $4 \times 18$) SAY: But I don’t have the 18 times table memorized. What else can I do instead of multiply? ($18 + 18 + 18 + 18$) PROMPT: What is $4 \times 18$ short for? Write the addition vertically on the board and have a volunteer do the addition, as show below:

```
  3
 18
 18
+18
 72
```

SAY: Another way to make a problem easier is to turn it into one that you already know how to do. If you find one of the questions difficult, but you need the answer to do the next question, guess an answer and use it in the next question. You can still get the next question correct based on your answers to the other question, even if your answer to the first question is incorrect.

**Extended Problem: Planting a Flower Garden.** Give students BLM Planting a Flower Garden. Question 5 provides an opportunity to apply the problem-solving strategy of making an easier problem. All students might find an answer to Question 5, but students who notice that the strategy can be used will find the problem easier.

**Answers:** 1. 50 m; 2. $150; 3. $162; 4. 4; 5. 19, I subtracted 20 – 1; 6. 76
Planting a Flower Garden (I)

Ivan has a flower garden. His flower garden is 5 m wide by 20 m long.

1. Ivan wants to put a fence around his flower garden to keep his pet dog out. How much fencing does he need?

2. If fencing costs 3 dollars for each metre, how much will the fence cost?

3. Ivan will grow the flowers from seeds. He buys the seeds for 12 dollars. How much does Ivan spend on the fence and the seeds altogether?
Planting a Flower Garden (2)

4. To plant his garden, Ivan will:
   • plant each flower 1 m apart,
   • start planting 1 m from each edge of his garden.

   His garden is 5 m wide and 20 m long.

   How many rows of flowers can Ivan plant?

   20 m
   5 m

5. How many flowers can Ivan plant in each row? Explain how you got your answer.

6. How many flowers can he plant altogether? Hint: Use repeated addition.
Unit 14 Measurement: Capacity, Mass, and Temperature

Introduction

In this unit, students will learn about capacity, mass, and temperature. They will distinguish between liquid volume and capacity and will estimate and measure the capacity of containers in litres (and millilitres in extensions). They will estimate and measure the mass of an object in grams and kilograms and will use both balances and scales to explore mass. They will solve word problems involving capacity and mass. Students will estimate and measure temperature in degrees Celsius. Students will develop benchmarks for key capacities, masses, and temperatures.

Meeting Your Curriculum

Alberta—Lesson ME3-25 is recommended for Alberta students since it reviews the concept of mass that students studied in earlier grades. Lessons ME3-26 and ME3-27 are required to cover the curriculum. The rest of the lessons in this unit are optional.

British Columbia—Lessons ME3-23, ME3-26, ME3-27, and ME3-29, including Extensions 1 and 2 in Lesson ME3-23 and Extension 5 in Lesson ME3-29, are required to cover the curriculum. Lesson ME3-25 is recommended, since it reviews the concept of mass that students studied in earlier grades. The rest of the lessons in this unit are optional.

Manitoba—Lesson ME3-25 is recommended for Manitoba students since it reviews the concept of mass that students studied in earlier grades. Lessons ME3-26 and ME3-27 are required to cover the curriculum. The rest of the lessons in this unit are optional.

Ontario—Lesson ME3-25 is recommended for Ontario students since it reviews the concept of mass that students studied in earlier grades. All other lessons in this unit are required to cover the curriculum.

Materials. This unit requires many materials. Gathering and preparing them will take extra time.

For Lessons ME3-23 and ME3-24 involving capacity and liquid volume, the following list is an overview of materials needed. Specific items are listed in individual lessons.

• access to a sink and water
• funnel, tray, water jug
• many containers of various sizes (avoid containers with sharp edges)

For Lessons ME3-25, ME3-26, and ME3-28 involving mass, the following list is an overview of materials needed. Specific items are listed in individual lessons.

• balances to determine whether objects have equal mass or if one is heavier
• scales to measure mass in grams and in kilograms
• many objects of various masses
NOTE: Ensure that students understand how a balance differs from the more familiar scale. You might explain to students that a balance has two trays and resembles a see-saw. If the mass of the object on each tray is the same, the trays balance and will be level. If an object on one side has greater mass, that tray goes down and the other tray goes up.

For Lesson ME3-29 involving thermometers, students will require thermometers, a water jug, and access to a sink and water. We suggest students use trays to prevent spillage.

Fraction notation. In Lessons ME3-24 and ME3-28, students need to understand that 2 halves or 4 fourths make 1 whole. Students also need to understand that half is larger than a quarter when referring to the same whole. In these lessons, we also use fraction notation. We show fractions in two ways in our lesson plans:

Stacked: \( \frac{1}{2} \)  
Not stacked: \( \frac{1}{2} \)

We suggest that you only use the stacked form in the lessons.

Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>ME3-25 to 27</td>
<td>ME3-23, 25 to 27</td>
<td>ME3-25 to 27</td>
<td>ME3-23 to 27</td>
</tr>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>ME3-29</td>
<td>n/a</td>
<td>ME3-28, 29</td>
</tr>
<tr>
<td>Test</td>
<td>ME3-26, 27</td>
<td>ME3-23, 26, 27, 29</td>
<td>ME3-26, 27</td>
<td>ME3-23, 24, 26 to 29</td>
</tr>
</tbody>
</table>
Goals

Students will compare liquid volumes using both direct and indirect methods.
Students will distinguish between capacity and volume.
Students will use cups and litres to measure and estimate the volume of liquids and the capacity of containers.
Students will develop a sense of how much liquid is one litre.

PRIOR KNOWLEDGE REQUIRED

Can compare numbers or things using the words “more” or “less”
Understands the concept of measurement

MATERIALS

- ball
- access to a sink and water
- jugs for filling containers (optional)
- trays
- funnels
- markers
- containers that are clear or translucent
  - identical pairs in 4 different sizes and shapes, including a pair of thin bottles
  - 2 containers with the same shape and different heights and/or widths
  - 2 containers with very different shapes, such as a drinking glass and a bottle
  - large containers of different sizes, one per pair of students
  - 2 L jar
  - plastic cups of identical size
  - 5 L water jug
- several small containers (e.g., small empty milk carton, can, vase)
- empty 1 L milk cartons
- empty 2 L bottles, one per pair of students
- cups of two different sizes
- 50 mL container (see Extension 2)
- 500 mL container (see Extension 2)
- 500 mL measuring cup (see Extension 2)

Mental math minute. Give students subtraction problems involving subtraction of two-digit numbers. Toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer it. Occasionally ask volunteers to explain how they determined the answer. Encourage multiple strategies.
Comparing volumes directly with identical containers. SAY: A liquid is something that flows freely and can be poured. Milk and water are examples of liquids. The word volume describes how much liquid there is. Write both words on the board and have students read them together.

NOTE: When filling containers, you may want to use a jug of water and a funnel and then place the containers to be filled on a tray to avoid spilling.

Select two identical thin containers (such as water bottles) and place them at the front of the classroom. Pour water into one of the containers until it is about half full, as shown in the margin.

Have a volunteer use a marker to draw a horizontal line on the outside of the container to mark the level of the water. Mark the second container at the same height. Have a different volunteer pour water into the second container until it reaches the mark on that container (see margin).

ASK: What can you say about the volume of water in the two containers? (it is the same) How can you tell? (the lines are at the same height on the bottles)

Erase the line on the second container. Add more water to the second container until it is about three-quarters full. Have a volunteer redraw the line on the second container to indicate the new level of the water (see margin). SAY: The volume of water in the containers was the same, but then I poured more water into the second container. ASK: Which container has more liquid? (the second container) How can you tell from the lines on the outside of the container? (the mark for the liquid level on the second container is higher than the mark on the first container)

ASK: Which container has less liquid? (the first) Have students signal the answer by pointing their thumbs towards the correct container. ASK: How can you tell from the lines on the outside of the containers? (the mark for the liquid level on the first container is lower than the mark on the second container)

Repeat three times with pairs of identical containers of other sizes.

Comparing volumes directly with different-sized containers. Select two containers that have roughly the same shape, but have very different heights or widths. Place the containers at the front of the classroom on a tray. Pour water into the smaller container until it is about half full. Ask a volunteer to use a marker to mark the level of the water on the smaller container, as shown in the margin.

Mark the second, larger container at the same height as the first. Empty the contents of the first container into the second container, and refill the first container until the water reaches the level marked on it before. SAY: I poured the water from the first container into the second container and then refilled the first container, so we know that the second container has the same amount of liquid as the first container. Now we are going to pour more water into the second container until the water level reaches the same height as marked on the first container. Have a volunteer pour more water
into the second container until the water levels are at the same height, as shown in the margin.

ASK: Which container has more liquid? (the second one) How do you know? (because the first and second container had the same amount of liquid, and then we added lots more water to the second container)

SAY: For the first two containers, when the marks were at the same height, the containers had the same amount of liquid. ASK: What is different in this situation? (the containers were the same size before, and now the second container is much bigger)

Draw the pictures in the exercises below one at a time and ask students to point their thumbs towards the container that holds more. Alternatively, you could number the containers in each part “1” and “2” and have students signal their answer by holding up the corresponding number of fingers.

**Exercises:** Which container holds more?

a) ![A](image1.png) ![B](image2.png) 

b) ![A](image3.png) ![B](image4.png) 

c) ![A](image5.png) ![B](image6.png)

**Answers:** a) B, b) A, c) B

**Comparing volumes indirectly by using a pair of identical containers.**

In advance, select two containers that have completely different shapes. For example, one container might be a wide drinking glass and the other, a thin bottle.

SAY: It is difficult to tell which container has more liquid when the containers are not the same. Place the two containers at the front of the classroom on a tray. Pour some water into each container, as shown in the margin. The amount of water is not important.

Point to the containers and ASK: Are these two containers identical? (no) 

SAY: It would be easier to compare the amount of liquid in the containers if the containers were exactly the same. Since they are not, we will compare the liquid in these two containers by using two new containers that are exactly the same. Take two identical empty containers and place them between the two containers with water, as shown below.

Ask two volunteers to each pour the water from one container into the empty container beside it, as shown in the margin.
Ask two other volunteers to each mark the liquid level in the identical containers, as shown in the margin.

ASK: Which of the new containers has more liquid? (the left container) How can you tell? (the mark on the left container is higher than the mark on the right container) So which of the original containers had more liquid? (the one on the left)

Repeat this with several more pairs of small containers of different shapes, filling the containers completely (examples: cup, small empty carton of milk, can, small vase). Make sure that it is hard to tell which container has more liquid before emptying the containers into the identical containers. Present the containers and have students signal which container held less water by pointing to the correct container.

Comparing volumes indirectly by measuring the amount of liquid using non-standard units. In advance, select two large, empty containers that are not identical. Using identical clear plastic cups, fill the first container with five cups of water and the second container with three cups of water. Place the containers at the front of the classroom on a tray.

ASK: If you have two pencils and you want to determine which pencil is longer, what can you do? (place the pencils beside each other and see which is longer, measure the length of the pencils) PROMPT: If the pencils are not in the same place, and you cannot place them beside each other, what can you do? (measure the length of each pencil and compare the measurements) SAY: We use rulers to measure length. You also can measure length by placing cubes beside the object, or estimate length with a finger. I need a way to measure the amount of liquid, or liquid volume, using something similar to counting how many finger widths you can place beside a pencil.

ASK: Why would I need to measure liquid? Accept all answers. If nobody mentions recipes, show students a recipe and point out the different measurements. For example:

Pancakes
   2 cups of flour
   5 teaspoons of baking powder
   1/2 teaspoon of salt
   2 tablespoons of sugar
   2 cups of milk
   1 egg
   5 tablespoons of melted butter

ASK: What units are used to measure food in the recipe? (tablespoons, teaspoons, cups) Show students a pack of plastic cups and discuss how they could use the cups as tools to measure the water. Point out that you have to use the same cup, or identical cups, as a measuring tool. You cannot use small cups and large cups at the same time.
Emphasizing the need to fill cups completely. Use the container of five cups of water to completely fill five plastic cups. Take five more plastic cups and fill them about halfway using the container that had three cups of water. SAY: I have five cups of water here and five cups of water there. I think that I have the same amount of water. ASK: Is that correct? (no) How do you know? (one set of five cups is full and the other set of five cups is not full) How can we check? (fill all the cups the same way and compare the number of cups) SAY: Every cup of water must be the same. Refill the containers, then demonstrate filling cups the correct way (to the top, or, if the cups have a line inside, to that line). After demonstrating, refill the containers again.

SAY: We can compare volumes by finding how many cups of liquid are in each container. Place several empty cups beside each container, as shown below:

Have a volunteer pour all the contents of the first container into the adjacent cups. Have a different volunteer do the same with the second container, as shown below. You might first want to make a mark at the level of the water in each container in case the process needs to be repeated because of spills.

ASK: How many cups of liquid were in the container on the left? (5) How many were in the container on the right? (3) Which container had more liquid? (the one with 5, on the left)

Estimating and measuring capacity by filling a row of cups from the container. Show students a large see-through container, such as a 2 L jar full of water, and have students guess how many plastic cups of water this container holds. Write their estimates on the board. Fill 2 cups from the jar. Show students how much water is left in the jar. Let students adjust their estimates. Continue filling the cups, stopping every 2 cups to let students adjust their estimates, until you have emptied the container. Show several more containers and ask students to predict which containers will hold more water (filled to the brim) than the jar and which will hold less water. Ask students to estimate the volume of water each container can hold, in cups.

Hold up a container that students predicted would hold less water than the jar and SAY: The jar holds 10 cups of water. You predicted that this container can hold less than the jar. ASK: Should your estimate be
more than 10 cups or less than 10 cups? (less than 10 cups) How much less—a lot or a little? (answer depends on the size of the container)

**ACTIVITY**

Give each pair of students one container that can hold between 500 millilitres and 2 litres of water, a plastic cup, a funnel, a large (2 L) empty plastic bottle, and a marker. Make sure at least one pair has a 1 L milk carton as a container. Have students practise filling one cup correctly, then have them fill the container to the brim. Have students record the name of the container and their estimate for the amount of water in their container. Let students measure the volume of the liquid in their containers by pouring the water out, one cup at a time. When one partner has filled the cup from the container, the other partner pours the water from this cup into the 2 L bottle. Partners count to 10 to let the water in the bottle settle, then mark the level of the water with a marker and write on the bottle the number of the cup. (This will allow students to create individual measuring bottles to be used later in the lesson and in future lessons.) Demonstrate the whole process a few times if necessary. Have partners take turns filling the cups, checking that the cups are filled correctly, and pouring the water into the bottle. If the last cup is not completely full, have students stop and not pour it into the bottle.

ASK: Is the last cup nearly full, half-full, or almost empty? Have several students show how much water they have in their cups. Remind students that when they measured length, they had objects whose lengths were between exact numbers of centimetres. If the length was closer to the smaller measurement, they rounded down, and if it was closer to the next whole centimetre, they rounded up. Explain that they can do something similar with volume of water: if the cup is at least half-full, they count it as a full cup of water, and if there is only a little water in the cup, they do not count that cup. Explain that in this case they measured the amount of liquid to the nearest cup, and they can write the measurement as “about 5 cups,” for example.

**Introduce capacity.** Have students record the results of the activity (how much water each container held) in their notebooks. Write several results on the board. SAY: The amount of water the container can hold is called capacity. The capacity of the large jar we measured earlier is 10 cups. You have just estimated and measured the capacity of your containers. Ask students to order the containers listed on the board by capacity, from largest to smallest.

**Introduce measuring bottles.** Explain that the bottles students marked during the activity can be used as tools to measure capacity or volume of liquid. They are called measuring bottles. Pour one full cup of water into an empty measuring bottle and SAY: To find the volume of water in this measuring bottle, place the bottle on a flat surface and bend down so that your eye is at the same level as the top of the water. Bend down to
demonstrate. SAY: Look at the top of the water and see how the water level lines up to the measurement marked on the side. ASK: What is the volume of water in this measuring bottle? (1 cup)

Have students pour out some water from their bottles so that each bottle has several glasses of water but is not full to the topmost mark. Have pairs of students exchange bottles with other groups and practise reading the measurements.

Comparing volumes indirectly by using a measuring bottle. Select two different, large, non-identical containers. Use a measuring bottle to add 4 cups of water to the first container and 2 cups of water to the second container. Place a measuring bottle beside each container, as shown below:

SAY: We can compare volumes by using a measuring bottle. Have volunteers use a funnel to pour the contents of each container into the measuring bottle beside it. The water will reach the 4 mark on one bottle and the 2 mark on the other bottle.

SAY: Look at the measurements marked on the first measuring bottle. ASK: What number does the liquid line up to on the side? (4) How many cups of liquid were in the first container? (4) SAY: Look at the measurements marked on the second measuring bottle. ASK: What number does the liquid line up to on the side? (2) How many cups of liquid were in the second container? (2) Which container had more liquid? (the first) How can you tell? (4 is more than 2) Have pairs of students measure and compare the amount of water they currently have in their measuring bottles with several other pairs’ measuring bottles.

Introduce litres. Show students two cups of different sizes. ASK: Which of these cups is larger? Fill the larger cup with water, then, over a tray, demonstrate that when you try to pour the water from the larger cup to the smaller cup, it overflows. SAY: The capacity of a cup is how much it can hold. ASK: Which cup has a larger capacity? (the larger cup) How do you know? (there is more water in the larger cup than a smaller cup can hold, and the water overflows) SAY: Imagine that I measured the capacity of a pitcher by using small cups, and the pitcher can hold 5 cups of water. You then go and buy 5 large cups of juice, and I try to pour the juice into this pitcher. ASK: Will all the juice fit into the pitcher? (no) You bought 5 cups of juice, just as I asked, so why not? (5 large cups of juice have greater volume than 5 small cups of juice) SAY: This means we need a more reliable unit of measure than cups.
SAY: In many places in the world, the standard unit used to measure capacity and volume is a litre. Show students a 1 L carton of milk or juice and point out the label showing that it holds 1 litre. Write on the board:

1 litre = 1 L

SAY: The short form for litre is a capital L.

**Developing a sense of how much is 1 litre.** Write on the board:

<table>
<thead>
<tr>
<th>Less than 1 L</th>
<th>More than 1 L</th>
</tr>
</thead>
<tbody>
<tr>
<td>About 1 L</td>
<td></td>
</tr>
</tbody>
</table>

Show several of the containers students used during the activity, and hold one container at a time beside the 1 L carton. ASK: Is the capacity of this container more than 1 litre, about 1 litre, or less than 1 litre? Have students signal the answer by pointing their thumbs in the direction of the correct answer and place the containers in groups underneath the labels on the board.

Ask the pair of students who measured the capacity of the milk carton during the activity how many plastic cups are in 1 litre. (the answer will depend on the size of the cups used; for example, 5 cups) Go through each container and ask the students who measured the container what the capacity was. Verify the answers. For example, SAY: We placed this pitcher in the “more than 1 L” group. ASK: Do we expect its capacity to be less than 5 cups, about 5 cups, or more than 5 cups? (more than 5 cups) What is the capacity of the pitcher? (7 cups) Does this fit our expectation? (yes)

**Estimating and measuring capacities in litres.** Ask students to look at the measuring bottles they made and compare them to the 1 L carton. Ask students to guess what the capacity of these bottles is in litres, or how many litres of water these bottles can hold. PROMPT: Can these bottles hold 1 litre of water or more than 1 litre? (more than 1 L) Remind students that 1 litre is about 5 cups. ASK: Where on the bottles is the mark for 5 cups? (about the middle) How many litres do you think the bottle can hold? (about 2 L) Have students use 1 L cartons to pour exactly 1 litre of water into the bottles and make a mark for 1 litre on the opposite side from the cup marks. Have students label the mark as “1 L.” Then have students make the mark for 2 litres. ASK: Will a third litre fit? (no) What is the capacity of these bottles? (2 L)

Hold up an empty, clear 5 L water jug. SAY: The capacity of this container is larger than 1 litre but less than 10 litres. Ask students to estimate the capacity in litres by signalling their estimate with their fingers. SAY: Let’s find the capacity of the jug. Have six volunteers each pour out water from their measuring bottles into a sink until the water level in the bottles reaches the 1 litre mark. SAY: Each of these bottles now holds 1 litre of water. Ask the first volunteer to pour the contents of his or her bottle into the 5 L jug. Have another volunteer use a marker to mark the level of the water in the jug. Let students adjust their estimates of the capacity of the jug. Repeat with another volunteer pouring and one marking the level. Let students adjust
their estimates again. Have the other volunteers pour the water into the jug, one litre at a time, with different volunteers marking each level. The last volunteer will not be able to pour in any water, because there will be no room left in the jug. When the volunteers are done, the class will have a jug that can be used as a measuring jug, as shown below:

ASK: What is the capacity of the jug? (5 L) Pour out water from the jug until there are 2 litres left in it.

**Distinguishing between capacity and volume.** SAY: The capacity of this jug is 5 litres. This jug can hold at most 5 litres of liquid. The volume of the water is the actual amount of space the water takes up. ASK: What is the volume of the water in the jug? (2 L)

Draw on the board:

ASK: What is the capacity of the jug in the picture? (5 L) SAY: The jug is not full. ASK: What is the volume of the water in the jug? (4 L)

**Exercises:** Find the capacity of the container and the volume of the liquid.

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram a) 5L, 4L, 3L, 2L, 1L" /></td>
<td><img src="image" alt="Diagram b) 3L, 2L, 1L" /></td>
<td><img src="image" alt="Diagram c) 8L, 6L, 4L, 2L" /></td>
</tr>
</tbody>
</table>

**Answers:** a) capacity: 5 L, volume: 1 L; b) capacity: 3 L, volume: 2 L; c) capacity: 8 L, volume: 6 L

Students can use their measuring bottles to do **Question 9** in AP Book 3.2, p. 93.

**NOTE:** Students in British Columbia need to do whole-class Extensions 1 and 2 to cover the curriculum.
Extensions

1. **Millilitres.** SAY: Sometimes we need containers for volumes much less than 1 litre. A litre can be divided into 1000 smaller units. Each smaller unit is called one *millilitre*. Write on the board:

   $$1 \text{ millil litre} = 1 \text{ mL}$$

   SAY: The short form for millilitre is a small m and a capital L.

   Write on the board and read the sentence aloud:

   A teaspoon can hold about 5 mL of liquid.

   Write on the board:

   lake swimming pool tablespoon
   soup ladle bathtub small yogourt container

   Ask volunteers to come up to the board and circle the containers that would hold much less than 1 litre. (tablespoon, soup ladle, yogourt container)

2. **Estimating the capacities of containers in millilitres.** Select two containers that have capacities of 50 millilitres and 500 millilitres. Label the 50 mL container as Container A and the 500 mL container as Container B. Place the containers at the front of the classroom on a tray.

   ASK: Which container has a capacity that is much less than 1 litre? (Container A) Should we measure the capacity using millilitres or litres? (mL) SAY: The capacity is much less than 100 millilitres. Write the numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90 on the board and point to the numbers in turn as you ask students to estimate the capacity in millilitres. Fill Container A with water and pour it into a 500 mL measuring cup to find the capacity of the container. (50 mL)

   Repeat the process of estimating and then measuring with Container B. (500 mL)

3. Container C has more liquid than Container D. Container E has more liquid than Container C. What is the order of the containers from the smallest amount of liquid to the largest amount of liquid?

   **Answer:** D, C, E

4. Rob has 8 identical plastic cups filled with water and 2 containers, F and G. He pours one entire cup at a time and pours all 8 cups into the containers. Container F has more water than Container G. Use the table to list all the possible volumes the two containers could be holding.

<table>
<thead>
<tr>
<th>Container F</th>
<th>Container G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Container H has a capacity of 6 litres. Container I has a capacity of 2 litres. Ella fills Container H with water. Then she pours the water from Container H into Container I until Container I is full. How much water is left in Container H?

Answer: 4 L
**Goals**

Students will develop a sense of how much liquid is half a litre and a quarter of a litre.

Students will measure the capacity of containers that can hold less than one litre of liquid and describe the capacity in terms of fractions of a litre (1/4, 1/2, or 3/4).

**PRIOR KNOWLEDGE REQUIRED**

- Can compare numbers or things by using the words “more” or “less”
- Can measure capacity by using individual units or a measuring bottle
- Can identify containers that can hold about 1 L
- Knows that 2 halves make 1 whole, 4 quarters make 1 whole
- Is familiar with fraction notation

**MATERIALS**

- ball or relay race baton (optional)
- access to a sink and water
- jugs for filling containers (optional)
- trays
- funnels
- markers
- 1 L bottle
- 1 L carton
- 500 mL bottles
- 500 mL cartons
- 250 mL cup
- 250 mL cartons
- 2 L jar
- 2 L bottle
- four small yogourt containers
- 2 containers, approximately 750 mL
- four identical containers less than 1 L per pair of students
- 1 L container per pair of students
- measuring bottles made during Lesson ME3-23
- small pail (about 8 L)
- large empty paint can (about 4 L)

**Mental math minute.** Arrange students in a line and have them add two-digit numbers by adding tens and adding ones. For each addition problem, such as 35 + 46, students need to say three steps: adding the tens, 30 + 40 = 70; adding the ones, 5 + 6 = 11; and finishing the addition, 70 + 11 = 81, so 35 + 46 = 81. The next student in line gets a new problem. Students can pass a ball or a relay race baton to each other so that the person who receives the baton answers the next question.
Start with problems that do not require regrouping, such as $25 + 34$, and continue to questions that require regrouping the ones.

**Review capacity, volume, and litres.** Remind students that capacity is how much liquid a container can hold and that the amount of space taken by the liquid is its volume. **ASK:** What units did you measure capacity in during the last lesson? (cups, litres) Ahead of time prepare a few containers so that they can be paired up by capacity: for example, a 1 L bottle and a 1 L carton, a 500 mL bottle and a 500 mL carton, a 250 mL cup and a 250 mL carton, and a 2 L jar and a 2 L bottle. Show the containers in random order and ask students if each container has capacity more than 1 litre, about 1 litre, or less than 1 litre. Students can signal the answer by showing thumbs up for more, showing thumbs down for less, and making the equal sign with their thumbs for about 1 litre.

**Pairing containers by capacity.** Hold up one of the containers and **ASK:** Is there another container that has a capacity that is about the same as this one? (yes) Point to the containers one at a time and have students signal thumbs up if the containers have about the same capacity and thumbs down if they do not. Pair up all the containers.

**Review halves and quarters.** Draw on the board:

![Circle divided into halves](image)

**ASK:** How many parts is the circle divided into? (2) What fraction of the circle is shaded? (one half) How do you know it is half? (it is one of two equal parts) Draw another copy of the same picture beside it and **SAY:** Imagine I had two pizzas. We ate half of one pizza and half of the other pizza. **ASK:** If I transfer the leftover pieces onto one plate, will the pieces fit to make a whole pizza? (yes) Will there be any space left? (no) How many whole pizzas do I have left? (one) **SAY:** Two halves together make a whole.

**ASK:** How can we write one half using fractions? (1/2) **Invite a volunteer to write it on the board as a stacked fraction.** **ASK:** Which part of the fraction shows how many pieces the whole is divided into? (2, the bottom, the denominator) Which part shows that we only have 1 piece on each plate? (1, the top, the numerator) **SAY:** I can write an addition sentence to show that two halves make one whole. Write on the board underneath the picture:

$$\frac{1}{2} + \frac{1}{2} = 1$$

2 halves make 1 whole.

Draw on the board:
Point to one of the quarters and SAY: I have cut these pizzas into 4 equal parts. ASK: What is each part called? (one quarter, one fourth) How many fourths are shaded in each circle? (1) If the shaded parts are pieces of leftover pizza, how many fourths of pizza are leftover? (4) If I transfer all the pieces onto the same plate, what do I get? (a whole pizza) SAY: 4 fourths or 4 quarters together make one whole. ASK: How can I write an addition equation to show that 4 fourths make 1 whole? Invite a volunteer to write the equation, as shown below:

\[
\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1
\]

4 fourths make 1 whole.
4 quarters make 1 whole.

**Introduce half a litre.** Show students two 500 mL cartons. Hold them one above the other beside a 1 L carton. ASK: What do you think holds more, two of these containers or one large container? Could it be that they hold the same amount of water? Accept all answers. Invite volunteers to fill the small cartons, then have another volunteer pour the contents of the two cartons into the larger carton. Show students that the larger carton is now full. ASK: What is the capacity of the larger carton? (1 L) If students do not remember, invite a volunteer to find the label on the carton and read it. SAY: Two of these smaller cartons have the same total capacity as 1 litre. ASK: How can you describe the capacity of these cartons by using fractions and litres? (the small cartons each have a capacity of one half of a litre) Of the containers we sorted at the beginning of the lesson, is there another container that has a capacity of about half a litre? (yes) Which one? (small plastic bottle) Keep the water in the large carton for now.

Show students one half-litre carton and SAY: This carton has a capacity of one half of a litre. ASK: If I pour water from the large, 1 L container into this carton, will I be able to fill up the small container? (yes) Will there be any room left? (no) Will there be water left in the large carton? (yes) Add another half-litre carton and repeat the questioning with 2 cartons. ASK: How do you know that the cartons will be full and no water will be left in the large carton? (the large carton was filled from 2 full half-litre cartons) Add a third half-litre carton and repeat the questioning with 3 cartons. Then fill the three half-litre cartons with the water from the large carton and demonstrate that all three cartons are not full: room is left in at least one carton (in fact, it is empty) and no water is left in the large carton. Empty the small cartons back into the 1 L cartons and ensure it is full, as you will be using it later.

**Comparing containers to half a litre.** SAY: Half is always less than a whole, so half a litre is less than 1 litre. Hold up a container that students identified as having more than 1 litre, and ASK: Does this container hold more than half a litre or less than half a litre? (more) How do you know? (1 whole litre is larger than half a litre, so if a container holds more than 1 whole litre, it definitely holds more than half a litre) Write on the board:

<table>
<thead>
<tr>
<th>About half a litre</th>
<th>Less than half a litre</th>
<th>More than half a litre</th>
</tr>
</thead>
</table>

**Q-16**

**Teacher Resource for Grade 3**
SAY: If I want to decide if a container has a capacity of more than half a litre, about half a litre, or less than half a litre, where should I place all the containers that hold more than 1 litre? (to the right) Place all the containers that hold more than 1 litre there. Repeat with containers that can hold about 1 litre. Go through the containers that hold less than 1 litre one at a time and have students signal where to put each container by pointing their thumbs to the left or to the right, or holding both thumbs up for about half a litre.

**Comparing two containers to a litre.** Hold up two small yogourt containers. ASK: Do these containers have a capacity smaller than half a litre or larger than half a litre? (smaller) Hold up the full 1 L carton. SAY: I am going to fill these containers from the 1 L container. ASK: Predict, do I have enough water here to fill both containers? (yes) Will there be water left in the 1 litre container? (yes) What does this mean about the capacity of these containers: is it less than half a litre, about half a litre, or more than half a litre? (less than half a litre) Have a volunteer fill the containers to check. Emphasize that when the two small containers are full and you still have water left, you used less than 1 litre of water to fill the containers, and so each container is less than half a litre in capacity. ASK: How else can I check that the capacity of these containers is less than one half of a litre? (pour the water from one of the yogourt containers into a container that has a capacity of half a litre. If the container is not full, then these yogourt containers have a capacity smaller than half a litre) Invite a volunteer to demonstrate.

Repeat with two containers that have a capacity of about three quarters of a litre (750 mL). Students will see that 1 litre is not enough to fill both containers. The containers have room left in them, so the capacity of these containers is more than half a litre

**ACTIVITY 1**

1. Give each pair of students two identical containers that each have a capacity smaller than 1 litre, a funnel (especially if they use bottles), and a container with a capacity of 1 litre. Make sure several pairs have two 500 mL bottles as their two identical containers. If you do not have enough containers with a capacity of 1 litre, you can use the measuring bottles students created in Lesson ME3-23. Ensure that students pour in exactly 1 litre of water rather than fill the measuring bottle completely.

   Have students copy the table on the following page (without the sample answers shown in italics) in their notebooks. Have students fill in the prediction column. Students estimate the capacity of one of their smaller containers as less than half a litre, about half a litre, or more than half a litre and answer “yes” or “no” to the questions. In the completed table, the identical containers are 330 mL soda cans.
My container: Soda can

<table>
<thead>
<tr>
<th></th>
<th>Prediction</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>half a litre</td>
<td>less than half a litre</td>
</tr>
<tr>
<td>Both containers full?</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Room left in containers?</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Water left in 1 L container?</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Have students fill their larger containers with 1 litre of water and pour that water into the smaller containers to check their predictions. Then have them complete the actual column.

Have each pair exchange containers with a pair that has different containers and repeat the activity on a new table.

Discuss the results of Activity 1. Have students present their findings.
ASK: What did you get for containers that are less than half a litre in capacity? (both containers were full, no room left in them, water left in the 1 L container) What results did you get for containers that are more than half a litre in capacity? (both containers were not full, room left in containers, no water left in the 1 L container) Did anyone have containers that were full, but no water was left in the 1 L container? (yes) What does this tell you about the capacity of the smaller containers? (they each have a capacity of exactly half a litre)

**Introduce one fourth or a quarter of a litre.** Show students four 250 mL cartons and compare them to the 1 L carton. ASK: What do you think holds more, four of these cartons or one large carton? Could it be that they hold the same amount of liquid? Accept all answers. Invite volunteers to fill the small cartons with water, then have another volunteer pour the contents of the 4 small cartons into the large carton. Show students that the large carton is now full. ASK: What is the capacity of the larger carton? (1 L) SAY: Four of these smaller cartons have the same total capacity as 1 litre. ASK: How can you describe the capacity of these cartons by using fractions and litres? (the cartons have a capacity of one fourth of a litre) Which is larger, half of a litre or a quarter of a litre? (half a litre) Have students identify containers that have a capacity of about a quarter of a litre among the containers they have seen during the lesson.

**Comparing containers to a quarter of a litre.** Hold up 4 small yogourt containers. ASK: Do these containers have capacity smaller than one fourth of a litre or larger than one fourth of a litre? (smaller) How do you know? (they seem smaller than the small cartons) SAY: I am going to fill these containers from the 1 L carton. ASK: Predict, do I have enough water here to fill all four containers? (yes) Will there be water left in the 1 L container? (yes) What does this tell you about the capacity of one of these containers: is it less than one fourth of a litre, about one fourth of a litre, or more than one fourth of a litre? (less than one fourth of a litre) Have a volunteer fill the containers to check. ASK: How else can I check that the capacity of these
containers is less than one fourth of a litre? (pour the water from one of the yogourt containers into the small carton) SAY: We checked that the small carton has a capacity of exactly a quarter of a litre. If the carton is not full, then the yogourt container has a capacity smaller than a quarter of a litre. Invite a volunteer to demonstrate.

**ACTIVITY 2**

2. Repeat Activity 1, but this time provide each pair with 4 identical containers instead of 2. Discuss the results in a similar manner.

**Two quarters of a litre make one half of a litre.** Draw on the board:

![Circle divided into quarters](image)

ASK: What fraction of the circle is shaded? (one half) Divide the picture into quarters:

![Circle divided into quarters](image)

ASK: Did I shade any more of the picture? (no) Did the shaded amount change? (no) What fraction does it show now? (2/4) PROMPT: How many equal parts is the circle divided into? (4) How many parts are shaded? (2)

Show two 1/4 L cartons and one 1/2 L carton. ASK: What holds more, these two small cartons together, or the larger carton? Have students vote, though students are likely to predict that the cartons hold the same amount. Have volunteers fill the two small cartons and pour the contents into the larger carton. Emphasize that two quarters make one half in fractions, so in capacity two fourths of a litre equal half a litre.

**Ordering containers by capacity.** Show 4 containers of different capacities used during this lesson. For example, show a small milk carton, a 500 mL bottle, a 1 L carton, and a 2 L bottle. SAY: I want to order these containers from largest capacity to smallest capacity. ASK: Which is the largest container? (the large bottle) Place it off to the side, under the board if possible. ASK: What is the next largest container? Point to the containers one at a time and ask students to signal thumbs up if this is the next container in order or thumbs down if not. Place the containers in order, beside each other. Repeat until all four containers are sorted. Ask students who measured the containers in the activities what the capacity is for each container. Write the capacities on the board in order matching the containers. (2 L, 1 L, 1/2 L, 1/4 L) ASK: Are these numbers ordered from largest capacity to smallest capacity? (yes) SAY: Let’s check. ASK: Is 1 less than 2? (yes) Is a half less than 1? (yes) Is a quarter less than a half? (yes)

Add two more containers, such as an 8 L pail and a 4 L paint can and have volunteers find the labels showing the capacity of these containers.
Exercise: Order the containers by capacity from smallest to largest.

A. pail, 8 L  
B. small carton, \( \frac{1}{4} \) L  
C. large carton, 1 L  
D. paint can, 4 L  
E. small bottle, \( \frac{1}{2} \) L  
F. large bottle, 2 L  

Answer: B, E, C, F, D, A

Have a volunteer order the containers by size, and have students check their answers in the exercise.

Introduce three quarters of a litre. Draw on the board:

\[ \text{\( \frac{3}{4} \) of a litre} \]

Ask: What fraction of the circle is shaded? (1/4) Shade another part and ask: What fraction is shaded now? (2/4) How do you know? (there are 4 equal parts, 2 of them are shaded) Shade another part and ask: What fraction of the circle is shaded now? (3/4) Say: If this were a pizza, we started with one piece, a fourth, and added another piece of the same size, and got two fourths. Then we added another piece and it became three fourths of the pizza.

Show one of the measuring bottles students made during Lesson ME3-23 and ask: What is the capacity of this bottle? (2 L) Show 4 small (250 mL) cartons of milk and ask: What is the capacity of each carton? (1/4 L) What is the total capacity of these 4 cartons? (1 L) Prompt: How many of these cartons are needed to fill a 1 L carton? (4) How many quarters or fourths do you need to make a whole? (4)

Have four volunteers fill up one 250 mL carton each with water. Pour one carton into the bottle by using a funnel. Ask: How much water is in the bottle now? (1/4 L) Remind students that in this case we say that the volume of the water in the bottle is one quarter of a litre. Make a mark to show the level of the water and label it as “1/4 L.” Pour in the second carton and ask: How much water is in the bottle now? (2/4 of a litre) Make another mark and repeat with the third (3/4 L) and fourth carton. When the fourth carton is poured in, students should see that the water reached the mark for 1 L, confirming their earlier guess.

Identifying capacity and volume of water in the cup. Draw on the board:

\[ \text{1 L} \]

<table>
<thead>
<tr>
<th>1</th>
<th>L</th>
</tr>
</thead>
</table>
ASK: What is the capacity of the beaker? (1 L) How many parts is the beaker divided into? (4) How large should each part be? (1/4 of a litre) How many parts are shaded? (2 parts) What is the volume of the liquid in the beaker? (2/4 or 1/2 a litre)

Mark the container differently, as shown below.

[Diagram of a container divided into 4 parts, with 2 parts shaded, labeled 1 L and 2 L]

SAY: This is a different beaker. ASK: What is its capacity? (2 L) What is the volume of the liquid in the container? (1 L) How many parts is each litre broken into? (2) Erase the top half of the water and ASK: What is the volume of the water now? (1/2 L)

Exercises: Find the capacity of the container and the volume of the juice.

a)

b)

Bonus:

Answers: a) capacity: 2 L, volume 1/2 L; b) capacity 1 L, volume 3/4 L; Bonus: capacity 2 L, volume 3/2 L

Extensions

1. Container A has a capacity of 1 L. Container B has a capacity of \(1/4\) L. Ray fills Container A with water. He pours the water from Container A into Container B until Container B is full. How much water is left in Container A?

   Answer: 3/4 L

2. Jessica has a vase and a pitcher with a total capacity of 2 L. She uses a cup with a capacity of \(1/4\) L to fill the containers. She pours a whole number of cups into each container.

   a) How many cups in total does she pour into the vase and the pitcher?

   b) Use the table to list all the possible volumes the two containers could be holding. Extend the table as needed.

<table>
<thead>
<tr>
<th>Vase</th>
<th>Pitcher</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 L</td>
<td></td>
</tr>
<tr>
<td>2 L</td>
<td></td>
</tr>
<tr>
<td>3 L</td>
<td></td>
</tr>
</tbody>
</table>

   Answers

   a) 1 L = 4 cups, so 2 L = 8 cups
3. Tristan has a jar, a jug, and 2 identical bowls. The jar can hold 2 L. The jug can hold \( \frac{1}{2} \) L of water. The bowls can hold \( \frac{1}{4} \) L each. What is the total capacity of Tristan’s containers?

**Answer:** 3 L
Goals
Students will compare masses of objects by using a balance.

PRIOR KNOWLEDGE REQUIRED
Can compare numbers by using “more than” or “less than”

MATERIALS
- ball or relay race baton (optional)
- objects to compare mass (e.g., eraser, textbook, notebook, connecting cubes, pencil, scissors, etc.)
- balance

Mental math minute. Ask students to solve multiplication questions within the range of $0 \times 1$ to $5 \times 5$. For each number, first go through the questions in order, such as $0 \times 3$, $1 \times 3$, and so on to $5 \times 3$, then in reverse order, and after that go through the same questions out of order. Then progress to a different number. You can have students stand in a line and pass an object, such as a ball or a relay race baton, to each other so that the person who receives the ball or baton answers the next question.

Compare the masses of objects without using a balance or scale. Write “mass” on the board. SAY: Mass is the amount of matter in an object. The heavier the object, the greater the mass. The lighter the object, the less the mass. Place an eraser and a textbook at the front of the classroom on a desk. ASK: Which object is heavier? (the textbook) Which has more mass? (the textbook) Write on the board:

<table>
<thead>
<tr>
<th>plane</th>
<th>pencil</th>
</tr>
</thead>
<tbody>
<tr>
<td>desk</td>
<td>car</td>
</tr>
<tr>
<td>feather</td>
<td>house</td>
</tr>
<tr>
<td>bed</td>
<td>paper clip</td>
</tr>
<tr>
<td>Earth</td>
<td>building</td>
</tr>
</tbody>
</table>

For each pair of objects, have a volunteer come to the board and circle the object with more mass and underline the object with less mass. (objects with more mass: plane, car, house, bed, Earth)

Comparing the masses of objects by using a balance. Place a balance at the front of the classroom on a desk. SAY: We can use a balance to compare masses. When there is nothing on either side of the balance, the trays should be at the same height. Check that the trays are level.

Place an eraser on one side of the balance. ASK: What happened to the trays? (the tray with the eraser went down and the other tray went up) Hold a textbook in the air. Before you place it on the empty tray of the balance, ASK: What do you think will happen? (the tray with the textbook}
will go down and the tray with the eraser will go up) Place the textbook on the empty tray of the balance. ASK: Which object has more mass? (the textbook)

Comparing the masses of objects using a picture of a balance. Draw on the board:

ASK: Which is lighter, the truck or the pencil? (the pencil) How can you tell by looking at the balance? (the tray for the pencil is higher)

Exercises

1. Which object has more mass?
   a)  
   b)  
   c)  
   Answers: a) cat, b) scarf, c) glass of milk

2. Which object has less mass?
   a)  
   b)  
   Bonus:  
   Answers: a) tennis ball, b) orange, Bonus: large empty water bottle

Comparing the masses of objects when the balance is level. Place an eraser on one side of the balance. Place enough large connecting cubes on the other side of the balance to make the balance level. Suppose it takes three large connecting cubes until the balance is level. SAY: The balance is level. So one eraser has the same mass as three connecting cubes. ASK: Which object has less mass, one cube or one eraser? (one cube) How do you know? (it took 3 cubes to have the same mass as 1 eraser, so each cube has less mass than 1 eraser) Repeat with another item, such as a small pair of scissors.

Exercises:

a) A glue stick has the same mass as 5 connecting cubes. Which is heavier, a glue stick or a connecting cube?

b) A mitten has the same mass as 10 connecting cubes. Which is heavier, a mitten or a connecting cube?

c) Three erasers have the same mass as a granola bar. Which is heavier, an eraser or a granola bar?

Answers: a) glue stick, b) mitten, c) granola bar
Comparing the masses of two objects indirectly by using a third object. Point out to students that when they compared an object to several connecting cubes and the balance was level, they actually measured the mass of each object. Write the masses you measured on the board. Adjust the example below to show the items you used.

- Mass of eraser = 3 cubes
- Mass of scissors = 6 cubes

ASK: Which is heavier, the eraser or the scissors? (scissors) How do you know? (it has more mass, its mass is the same as 6 cubes, which is more than 3 cubes)

Exercises: Use the previous exercises to answer the questions.

a) Which is heavier, a mitten or a glue stick?

Bonus: An eraser has a mass of 3 cubes.

b) What is the mass of the granola bar in cubes?

c) Which is heavier, the mitten or the granola bar?

Answers: a) mitten, Bonus: b) 9, c) mitten

Extensions

1. Object A has the same mass as 2 erasers. Object B has the same mass as 6 erasers.
   a) If Object A and Object B are placed on the same balance, will the balance be level? Explain.
   b) How many of Object A will be needed to make the balance level?

   Answers: a) no, Object B has more mass so Object B’s side of the balance will be lower; b) 3

2. Object A has the same mass as 15 erasers and 20 new pencils. Object B has the same mass as 3 erasers and 4 new pencils.
   a) If Object A and Object B are placed on the same balance, will the balance be level? Explain.
   b) How many of Object B will be needed to make the balance level?

   Answers: a) no, Object A has more mass so Object A’s side of the balance will be lower; b) 5
**Goals**

Students will use a scale to measure the mass of objects in grams and kilograms.

**PRIOR KNOWLEDGE REQUIRED**

Understands the concept of measurement  
Understands the concept of mass  
Can compare masses of objects  
Understands the need for standard units of measurement

**MATERIALS**

- ball  
- large paper clips  
- nickels (real)  
- objects that weigh 100 g (e.g., chocolate bars, packs of nuts, dried fruit, etc.)  
- objects that weigh about 100 g (e.g., cans of fish, small potatoes, etc.)  
- tennis ball  
- stuffed toy or object that weighs between 50 g and 100 g  
- a variety of objects to measure in grams and kilograms  
- scale measuring in grams  
- 10 large connecting cubes per student  
- objects that weigh 1 kg (e.g., 1 L carton of milk, bag of flour, etc.)  
- objects that weigh 2 kg (e.g., bag of rice, etc.)  
- scale measuring in kilograms  
- a mug  
- 50 dry corn kernels (see Extension 2)  
- 50 pieces of popcorn (see Extension 2)

**NOTE:** For Questions 5 and 7 in AP Book 3.2, p. 100, students will need the following objects to measure mass in grams and kilograms: a quarter, large scissors, calculator, notebook, stack of books, backpack, and laptop. For each of these exercises, students will also need to select an object from their environment and measure its mass in grams and in kilograms.

**Mental math minute.** Give students subtraction problems involving subtraction of two-digit numbers. Toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer it. Occasionally ask volunteers to explain how they determined the answer. Encourage multiple strategies.

**Review mass.** Ask students how people can check which of two objects is heavier. If the idea of weighing the objects does not arise, ask how we check the weight of people. Remind students that we usually use the word “mass” to describe how heavy objects are, and we say that the heavier object has the greater mass.
Name some pairs of objects that have similar masses: a pen and a pencil, a full water bottle and a book, a truck and a bus, a grape and a dollar coin. Then name several everyday objects and have students name an object with a similar mass: What has a mass similar to an eraser? (e.g., a glue stick)

Introduce grams. Ask students in what units weight is measured. Students might be familiar with pounds or kilograms, or even grams from everyday life. SAY: Grams are the unit we use to measure the mass of small objects. We write g as a short form for grams. Write “gram” and “short form: g” on the board, and have a volunteer circle the letter g in the word “gram.”

Explain that a large paper clip and a very large chocolate chip weigh about 1 gram each. A nickel weighs about 4 grams. Give students each a nickel and a large paper clip, and have students hold one in each hand to compare the weights. ASK: Can you feel the difference? (no) Point out that grams are very small units, so if you hold a 1-gram weight in one hand and a 5-gram weight in the other hand, you would likely not be able to distinguish which one is heavier. You would need a good pan balance or a scale to check. Explain that most coins have a mass of more than 1 gram, and the smallest coin by size, a dime, has a mass close to 1 gram.

Point out that since 1 gram is such a small weight, few everyday items weigh less than that. For example, a grain of rice, a pill, a feather and most insects, such as ants, weigh less than 1 gram. Only very large insects weigh more than 1 gram.

Ask students to give examples of objects that weigh about 1 gram, other than dimes, large paper clips, and large chocolate chips. (paper tacks, large raisins, dried cherries, small nuts, such as pistachios or peanuts, tissue paper, play coins)

Developing a sense of 100 grams. Explain that since 1 gram is a small mass, it makes sense to use objects that weigh more, about 100 grams, to help estimate masses in grams correctly. Remind students that they used their fingers and hands to help estimate length. Knowing that an eraser is longer than the width of a finger, but shorter than a whole hand, they could say that its length is between 1 and 10 cm, so about 5 cm long. Mass can be estimated in the same way. Show objects that weigh exactly 100 grams (e.g., chocolate bar, some packages of instant noodles, nuts, dried fruit) and objects that weigh about 100 grams (e.g., portioned containers of yogourt or cottage cheese, some instant pudding, cans of fish, small potatoes). You might record the objects for each mass in a table on the board so students can refer to it as necessary.

Estimating mass in grams. Hold up a tennis ball and explain that it has a mass of about 50 g. Hold up a stuffed toy or another object that weighs between 50 grams and 100 grams, and SAY: I want to estimate the mass of this object. Have a volunteer check which object is heavier, the toy or the tennis ball. (toy) Have another volunteer compare it to a 100 gram
chocolate bar. (the chocolate bar is heavier) SAY: This toy is lighter than 100 grams and heavier than 50 grams. I estimate its mass is 80 grams.

Repeat with two more examples: A shoe is heavier than a chocolate bar. Is it a lot heavier than a chocolate bar? Is it heavier or lighter than two chocolate bars? (about the same) A pencil is lighter than a chocolate bar. Is a pencil closer to a small potato or a large paper clip in mass? (large paper clip)

Have students work in groups of five. Give students each an object and have them estimate its mass. Have students record their estimates, in a table, as shown below, then exchange objects with the others in their group and repeat. Students can check their estimates during Activity 1 below. Students should compare different objects to objects that have a mass of 100 grams (e.g., chocolate bar, small potato), 50 grams (e.g., tennis ball), and 1 gram (e.g., paper clip).

<table>
<thead>
<tr>
<th>Object</th>
<th>Estimated Mass</th>
<th>Actual Mass</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
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</table>

Measuring mass in grams. SAY: The tool we use to measure mass is called a scale. You can also use a pan balance, but to measure mass with a pan balance you need weights—some objects that have the mass of an exact number of grams. Show students the scale they will be using to measure the masses of objects in grams. Explain how to read the scale. If you are using digital scales, explain that they show mass in grams. If you are using a scale with a round face and a hand, point out the marks for kilograms, each hundred grams, and smaller marks, depending on the face of the scale. Point to different marks and have students tell you what mass the mark corresponds to.

**ACTIVITIES 1–2**

1. Students measure the mass of the objects they estimated. They record the mass in the table they copied from above.

2. Give each student 10 large connecting cubes. Have students make a shape from the connecting cubes and find its mass in grams. Have students make a different shape from the same number of cubes. Ask them to predict the mass of the new shape. Discuss how the shapes students made are different—some are taller, some are wider, some are longer. Have students measure the mass of the new shapes. ASK: Is the mass of the new shape different from the mass of the old shape? (no) Did everyone have the same shape? (no) Did everyone get the same mass? (yes) Does the arrangement of the cubes in the shape affect the mass of the shape? (no) Emphasize that the shape of the arrangement does not matter, only the number of cubes matters for the mass.
Introduce kilograms. Point out that even small objects, such as a shoe, a book, or an apple, are heavier than a chocolate bar, so their mass is more than 100 grams. ASK: Will 1 gram be a convenient unit to measure the mass of a laptop, a pile of books, or a human? (no) Why not? (the number of grams will be very large) SAY: We measure the mass of heavy objects in kilograms. The short form for kilogram is kg. Write “kilogram” and “short form: kg” on the board. Circle the letters “k” and “g” in the word “kilogram.”

Developing a sense of 1 kilogram. Present several objects that have a mass of exactly 1 kilogram (such as 1 L of milk in a carton, a bag of flour) and have students compare them with other objects. For example, ASK: Is a shoe heavier, lighter than, or about the same as 1 kilogram? (lighter) Include an object that weighs exactly 100 grams in a comparison, to ensure students understand that 100 grams is less than 1 kilogram. Point that out explicitly. NOTE: Students haven’t been taught that 1 kg = 1000 g yet.

Give students several objects and have them sort the objects into three groups: heavier than 1 kg, lighter than 1 kg, and about 1 kg. Then give students an object that has a mass of 2 kilograms, such as a bag of rice, and have students compare it with the object that has a mass of 1 kilogram. ASK: Why is it easier to decide which object has a mass of 1 kilogram and which has a mass of 2 kilograms than it was to decide which object has a mass of 1 gram and which has a mass of 4 grams, such as a large paper clip and a nickel? (kilograms are larger units, so the difference in the mass is much larger)

Estimating mass in kilograms. Have students work in groups of five. Give students each an object and have them estimate the mass. Have students record their estimates in the table they started for grams, then exchange objects with the others in their group and repeat. They can later check their estimates in Activity 3. Students should compare different objects to objects that have a mass of 1 kilogram (e.g., a bag of flour) or 2 kilograms (e.g., a bag of rice).

Measuring mass in kilograms. Show students the scale they will be using to measure the masses of objects in kilograms. Explain how to read the scale. Explain that when the mass is between 2 and 3 kilograms, the arrow will stop between the marks for 2 kilograms and 3 kilograms. If the mass is closer to 3 kilograms, the arrow will go past the half-kilogram mark. We say that the mass is about 3 kilograms. If the arrow does not go past the half-kilogram mark, we round down and say that the mass is about 2 kilograms. Draw a scale on the board and show several positions of the arrow, and have students decide what the mass is. Students can signal the answer by holding up the correct number of fingers. Then place several objects on the real scale and have students signal the mass to the nearest kilogram.

ACTIVITY 3

3. Students measure the mass of the objects they estimated earlier. They record the mass in kilograms in their table.
1 kilogram equals 1000 grams. Write on the board:

1 kilometre = _____ metres  
1 km = _____ m

ASK: What number goes in the blanks? (1000) Write “1000” in the blanks. Invite a volunteer to circle the common parts in the words “kilometre” and “metre.” Ask students to guess what the part “kilo” means. Explain that “kilo” means 1000 in Greek. When students see “kilo” in a measurement unit, they will know right away that there are 1000 smaller units in that unit. Write on the board:

1 kilogram = _____ grams  
1 kg = _____ g

ASK: What number goes in the blanks? (1000) Write “1000” in the blanks. Write on the board:

789  
1000

ASK: Which number is larger? (1000, students can signal the answer by pointing their thumbs to the correct side) How do you know? (answers may vary; example: 1000 has more digits) Write “grams” after both numbers. ASK: Which mass is larger, 789 grams or 1000 grams? (1000 g) Write on the board:

999 g  
1 kg

ASK: Which mass is larger? (1 kg) Why not 999 grams, since 999 is more than 1? (1 kg is 1000 g, and 1000 is more than 999) Write “= 1000 g” beside 1 kg. Then write:

3 kg  
285 g

ASK: Which mass is larger? (3 kg) How do you know? (3 kg is more than 1 kg, and 1 kg is 1000 g; 1000 is more than 285) Emphasize that any three-digit number of grams is less than 1 kilogram, so it is definitely less than any other whole number of kilograms.

Exercise: Order the masses from lightest to heaviest.

789 g  
1 kg  
999 g  
3 kg  
285 g

Answer: 285 g, 789 g, 999 g, 1 kg, 3 kg

Which measurement? Show students an empty mug. SAY: I want to know if the mass of a mug is 150 grams or 150 kilograms. Let’s think about objects that you compared to 1 gram, 100 grams, and 1 kilogram and predict objects that have a mass close to 150 grams (such as a potato) and close to 150 kilograms (such as 150 cartons of milk). ASK: Which of these objects is the mug closest to in mass? (a potato) Should we use 150 grams or 150 kilograms for a mug? (150 grams) Repeat with a cat: 2 grams or 2 kilograms, a berry: 5 grams or 5 kilograms, and a bear: 200 grams or 200 kilograms.
Choosing the best unit. ASK: Would it make sense to measure the mass of a car in grams? (no) PROMPT: Measuring a car in grams is the same as saying how many paper clips weigh the same as the car. Should you measure the mass of a car in grams or in kilograms? (kilograms) ASK: Would it make sense to ask how many kilograms a pen weighs? (no; the answer would be 0 kilograms) Explain that for objects that weigh about 1 kilogram or less, grams are the most appropriate unit, and for heavier objects, kilograms are usually better.

Exercises: What is the best unit, grams or kilograms, to measure the mass of the item?

a) a can of pop  
b) a box of crayons  
c) a bicycle  
d) a beaver  
e) a pair of glasses  
f) an adult human

Answers: a) grams, b) grams, c) kilograms, d) kilograms, e) grams, f) kilograms

Determining the missing mass. Remind students that when a pan balance is level, it means that the objects on both sides of the balance have the same mass. Draw on the board:

\[
\begin{align*}
1 \text{ kg} & \quad 1 \text{ kg} \\
2 \text{ kg} & \quad ?
\end{align*}
\]

ASK: What operation can we use to find the missing mass? (addition: \(2 + 2 + 1 = 5 \text{ g}\)) What single weight can we place on the right side to make the balance level? (5 g) Write “5 g” on the empty weight on the right side.

Draw on the board:

\[
\begin{align*}
1 \text{ kg} & \quad 5 \text{ kg} \\
1 \text{ kg} & \quad 3 \text{ kg}
\end{align*}
\]

ASK: What is the total mass on the left side of the balance? (7 kg) What operation can we perform to find the mass that needs to be added on the right side to make the balance level? (subtraction: \(7 - 3 = 4 \text{ kg}\)) Write “4 kg” on the empty weight.

Exercises: Find the missing mass.

a)
b) 25 g 25 g 30 g ?

**Answers:** a) 23 kg, b) 30 g

**Extensions**

1. The balance is level and each unknown weight has the same mass. Find the unknown mass.

   a) 1 kg 1 kg 1 kg
     1 kg 1 kg 1 kg

   **Answers:** a) 3 kg, b) 2 kg

2. Have students compare the mass of 50 dry corn kernels and 50 pieces of popcorn. Point out that they weigh about the same, even though one takes up much more space (volume) than another. Have students make more comparisons that show mass is not necessarily related to volume.
Goals
Students will solve word problems involving mass.
Students will solve problems involving different operations.

Prior Knowledge Required
Can multiply one-digit numbers and knows related division facts
Can multiply by 2, 4, and 8 by using doubling
Understands place value
Can add and subtract two-digit and three-digit numbers
Knows that mass is measured in grams and kilograms
Knows that capacity is measured in litres
Can solve simple word problems involving four operations

Materials
deck of cards without face cards

Mental Math Minute. Shuffle a deck of cards after removing the face cards. Divide students into pairs. Each student in the pair selects a random card. The students create two multiplication equations by using the cards selected. For example, if Student A selects a 7 and Student B selects an 8, they create the equations \(7 \times 8 = 56\) and \(8 \times 7 = 56\). If the card selected is an ace, treat it as having selected the number 1. One at a time, ask each pair of students to say the multiplication sentences they created. Then the pair does three jumping jacks and sits down. Continue until all students have had an opportunity to participate.

Review using multiplication and division to solve word problems. Write on the board:

A cherry weighs 5 g. What is the mass of 2 cherries?

ASK: What equations can you write to solve this problem? \((5 + 5 = 10, 2 \times 5 = 10)\) Write the equations on the board. ASK: Is the answer 10 cherries, 10 grams, or maybe 10 litres? (10 grams) Write “g” beside both 10s. ASK: How do you know? (each cherry has a mass of 5 g, we are adding together the masses of 2 cherries; we are looking for mass, and mass is measured in grams) Write “g” after 5 in both sentences to emphasize we are talking about grams. ASK: Why not litres? (mass is not measured in litres) What is measured in litres? (volume or capacity)

Write on the board:

What is the mass of 8 cherries?

ASK: What equation can you write to solve this problem? \((8 \times 5 g = 40 g)\) Write the equation on the board and emphasize the units again.
Write on the board:

Several cherries in a bag weigh 30 g.
How many cherries are in the bag?

ASK: What equation can you write to solve this problem? (30 ÷ 5 = 6)
Write the equation on the board. ASK: Is the answer 6 cherries, 6 grams, or something else? (6 cherries)

Repeat with the following: Mary has 7 identical coins. The total mass of the coins is 42 g. How many grams does each coin weigh? (42 g ÷ 7 = 6 g)

**Exercises:** Write an equation to solve the problem.

a) A pencil has a mass of 4 g. What is the mass of 6 pencils?
b) 8 crayons weigh 56 g. How much does each crayon weigh?

**Answers:** a) 6 × 4 g = 24 g, b) 56 g ÷ 8 = 7 g

**Solving problems that require doubling.** Tell students that a family is going on a plane trip. Write on the board:

Mrs. K has 2 bags that weigh 14 kg each.
What is the total mass of her luggage?

ASK: How much do 2 bags of 14 kilograms each weigh altogether? (28 kg)
How do you know? (14 + 14 = 28) Write the equation on the board.

SAY: Mr. K also has 2 bags of 14 kilograms each. ASK: How many bags do they have in total? (4) How much do the bags weigh together? (56 kg) How do you know? (each person has 28 kg of luggage, so they have 28 + 28 = 56 kg of luggage together) Write “14 + 14 = 28” on the board again. Ask a volunteer to explain how they doubled 28. (20 + 20 = 40, 8 + 8 = 16, 40 + 16 = 56)

Write on the board:

2 bags of 14 kg each: 2 × 14 = 14 + 14 = 28 kg
4 bags of 14 kg each: 4 × 14 = 28 + 28 = 56 kg
8 bags of 14 kg each:

Point out that you can find 4 times a number by doubling twice. ASK: What multiplication do we need to write to find the mass of 8 bags? (8 × 14) How can you find 8 times 14? (double 4 × 14) Have students help you finish finding the mass of 8 bags. (112 kg)

**Exercises:** There are 4 people in a family. Each person has 2 bags of 12 kg each.

a) What is the mass of one person’s luggage?
b) What is the total mass of the luggage for the family?

**Answers:** a) 24 kg, b) 96 kg
Solving problems that combine different operations. SAY: The airline allows each traveller 20 kilograms of luggage for free. If travellers bring more luggage, they need to pay extra. Write on the board:

Free luggage limit: 20 kg  
Extra charge: 15 dollars for each 1 kg over the limit

ASK: Will the family of four in the previous exercises need to pay extra? (yes) What is the mass of each person’s luggage? (24 kg) How much over the 20-kilogram limit is each person? (4 kg) How do you know? (24 kg − 20 kg = 4 kg) Write the equation on the board. SAY: 1 kilogram of extra luggage will cost 15 dollars. ASK: What equation do you need to do to find out how much the 4 extra kilograms of luggage will cost? (multiply 4 × 15) Write on the board:

4 × 15 =

ASK: How can you find 4 times 15? (add 15 four times, double 15 two times) Write on the board:

2 × 15 = 15 + 15 = ___  
4 × 15 = ___ + ___ = ___

Have students help you fill in the blanks. (30, 30, 30, 60) ASK: How much extra money does each person pay? (60 dollars) How much money will two family members pay? (120 dollars) How much money will the whole four-person family pay? (240 dollars)

Exercises: The family of 4 people from the previous exercise switches to a different airline. The first 20 kg of luggage are still free. Each extra kilogram of luggage now costs 11 dollars.

a) How much money does one person pay?  
b) How much money does the whole family of 4 pay?  
Answers: a) 44 dollars, b) 176 dollars

Extensions

1. a) A rabbit weighs 3 kg. A cat weighs twice as much as a rabbit. How much does the cat weigh?  
b) A dog weighs as much as three cats. How much does the dog weigh?  
c) How many times as much as the rabbit does the dog weigh?  
d) An adult beaver weighs as much as the dog and the rabbit together. How many rabbits does the beaver weigh?  
Bonus: How many cats do you need to balance two dogs and two rabbits on a scale?  
Answers: a) 6 kg, b) 18 kg, c) 6 times, d) 7 rabbits, Bonus: 7 cats
2. A barbell holds two 20 kg weights and two 10 kg weights. The barbell itself has a mass of 10 kg. What is the total mass of the barbell and the weights?

   **Answer:** The total mass of the weights is \((2 \times 20)\) kg + \((2 \times 10)\) kg = 40 kg + 20 kg = 60 kg, so the total mass of the barbell and weights is 60 kg + 10 kg = 70 kg.

3. The high school weight room has weights that have a mass of 2 kg, 5 kg, 10 kg, and 20 kg. The gym teacher counts the number of each weight and records it in a table:

<table>
<thead>
<tr>
<th>Mass of Each Weight (kg)</th>
<th>Number of Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

   a) Find the total mass of all the weights.
   b) The gym teacher wants to have a total of 500 kg in weights. How many more kilograms does he need?
   c) Which weights can the teacher order to get to 500 kg?

   **Answers**
   a) \((7 \times 2) + (8 \times 5) + (20 \times 10) + (10 \times 20) = 14 + 40 + 200 + 200 = 454\) kg
   b) 500 – 454 = 46 kg

   **Sample answer:** c) two 20 kg weights and three 2 kg weights
Goals

Students will use halves and quarters of a kilogram to estimate and measure the mass of objects using a scale.

PRIOR KNOWLEDGE REQUIRED

Can compare numbers and fractions (1/4, 1/2, 3/4)
Is familiar with grams and kilograms
Can read the display on a scale in grams
Knows that two halves make one whole, and four quarters make one whole
Can multiply and divide within 7 × 7
Understands the connection between addition, multiplication, and division
Is familiar with fraction notation

MATERIALS

deck of cards without face cards

two 500 g packs of food

1 kg pack of food

pan balance

a variety of objects to measure, including a soccer ball or a basketball

four 250 g packs of food

8 hundreds blocks and 20 tens blocks

scales measuring in grams

NOTE: Students will need the following objects to measure and estimate mass in Question 8 in AP Book 3.2, p. 105: a book (they can use a JUMP Math AP Book which has a mass close to 1/2 kg), an apple, a shoe, a bottle of water, and an object of their choice.

Mental math minute. Shuffle a deck of cards after removing the face cards. Divide students into pairs. Each student in the pair will select a random card. The students will create two division equations using the cards selected. For example, if Student A selects a 7 and Student B selects an 8, they will create the equations 56 ÷ 8 = 7 and 56 ÷ 7 = 8. If the card selected is an ace, treat it as having selected the number 1. One at a time, ask each pair of students to say the division sentences they created. Then the pair does three jumping jacks and sits down. Continue until all students have had an opportunity to participate.

Review halves and quarters. Draw the picture in the margin on the board. ASK: How many parts is the rectangle divided into? (2) What fraction of the rectangle is shaded? (one half) How do you know it is half? (it is one of two equal parts) How can you write one half using fractions? (1 over 2) Invite a volunteer to write the answer, 1/2, on the board. ASK: Which part of the
fraction shows how many pieces the whole is divided into? (2, the bottom, the denominator) Which part shows that we only have one part shaded? (1, the top, the numerator) How many halves do we need to make one whole? (2) How can you show that two halves make a whole with addition? ($\frac{1}{2} + \frac{1}{2} = 1$) Write on the board:

\[
\frac{1}{2} + \frac{1}{2} = 1
\]

2 halves make 1 whole.

Draw on the board:

\begin{array}{cccc}
\hline
& \square & \square & \square \\
\hline
& \square & \square & \square \\
\hline
& \square & \square & \square \\
\hline
\end{array}

Point to one of the quarters and SAY: Each rectangle is divided into 4 equal parts. ASK: What is each part called? (one quarter, one fourth) How many fourths are shaded in each rectangle? (one fourth) How many fourths or quarters together make a whole? (4) How can I write an addition equation to show that 4 fourths make 1 whole? Invite a volunteer to write the equation, as shown below.

\[
\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1
\]

4 fourths make 1 whole.

4 quarters make 1 whole.

**Introduce half a kilogram.** Show students a pack of food that weighs exactly 500 grams (e.g., scone mix, powdered milk, spaghetti, flour) and another pack that weighs exactly 1 kilogram. Have a volunteer hold the packs and say which one seems heavier. Have the volunteer examine the 1-kilogram pack and tell students its mass. (1 kg) Show a pan balance. ASK: If I place the packs on different sides of the pan balance, what will happen? (the side with the 1 kg pack will be lower) Check the prediction with the pan balance. Show another pack that weighs 500 grams and have students predict what will happen if you add it to the side of the balance that already has the 500-gram pack. Accept all answers. You may call a vote. Add the pack and show students that the pans are balanced.

SAY: I have here two equal parts that together make 1 kilogram. ASK: When we had two cartons that together had a capacity of 1 litre, what did we call the capacity of each carton? (one half of a litre) What is the mass of each of these packages? (one half of a kilogram)

One at a time, hold up several objects, including a soccer ball or a basketball, and have students say if the object weighs about half a kilogram. Students can signal the answer. If students think the object does not have a mass of about half a kilogram, ASK: Is the mass much less than half a kilogram or much more than half a kilogram? Students can show thumbs down for much less and thumbs up for much more.

Place each object on the balance to compare it with the half-kilogram benchmark package. Point out that if the pans are not balanced but are
close to being balanced, we can say that the mass of the object is about half of a kilogram.

**Introduce a quarter of a kilogram.** Show students a pack of food that weighs exactly 250 grams (e.g., sugar, marshmallows, cookies) and the 1-kilogram benchmark. Have a volunteer hold the packs and say which one seems heavier. Have the volunteer examine the 1-kilogram pack and tell students its mass. (1 kg) Repeat with the 250-gram pack and a half-kilogram benchmark.

SAY: I needed two half-kilogram packs to balance 1 kilogram. ASK: How many smaller packs do you think I will need to balance the 1-kilogram pack? Have students make guesses, and accept all answers. Check the prediction with the balance, starting with one 250-gram pack and adding one 250-gram pack at a time, until you have four packs on one side and the pans balance. Let students adjust their guesses after each pack.

SAY: I have four equal parts that together make 1 kilogram. ASK: When we had four cartons that together had a capacity of 1 litre, what did we call the capacity of the cartons? (one fourth or one quarter of a litre) ASK: What is the mass of each of these packages? (one fourth or one quarter of a kilogram)

Hold up several objects, one at a time, and have students say if the object weighs about 1/4 kg. Include several objects that have a mass close to 250 grams, such as an apple, pear, small orange, or tomato. Students can signal the answer. If students think the object does not have a mass of about 1/4 kg, ASK: Is the mass much less than a quarter of a kilogram or much more than a quarter of a kilogram? Students can show thumbs down for much less and thumbs up for much more.

Place each object on the balance to compare it with the quarter-kilogram benchmark. SAY: If the pans are not balanced but are close to being balanced, we can say that the mass of the object is about one quarter of a kilogram.

**Half equals two quarters.** Place two half-kilogram objects on one side of the balance and SAY: Two half-kilogram objects have the total mass of 1 kilogram. Four objects that have a mass of one fourth of a kilogram also have the total mass of 1 kilogram. Place four quarter-kilogram objects on the other side of the balance. Point to a quarter-kilogram benchmark, and ASK: How many objects that have the mass of one fourth of a kilogram will balance one half of a kilogram? (2, students can signal the answer) Remove one half-kilogram object and two quarter-kilogram objects from the other pan of the balance and have students see that the pans balance.

**Different ways to make 1 kilogram.** On the balance, replace the half-kilogram object with the 1 kilogram benchmark. ASK: What can we add to the other side of the balance to balance the pans? (two more quarter-kilogram objects or one half-kilogram object) PROMPT: What is the mass of two quarter-kilogram objects? (half a kilogram) If we only want to add one object on the other pan, what would it be? (a half-kilogram object)
Have a volunteer check the prediction. Draw the pictures in the exercises below on the board and have students show thumbs up or thumbs down to signal the answer. Check the answer with the actual weights on the pan balance.

Exercises: Will the balance be level?

a)

(b)

Answers: a) no, b) yes

Half of a number. ASK: What is double 3? (6) Invite volunteers to draw pictures that show that 6 is the double of 3. Examples:

Ask volunteers to explain how each picture shows that 6 is the double of 3. Point out that in each picture there is a group of 3 objects (dots in a row, stars in a circle, squares of the same colour) that is drawn two times.

ASK: What fraction of the dots is in the first row in the first model? (half) What fraction of the stars is in one circle in the second model? (half) What fraction of the squares is shaded in the third model? (half) SAY: All three models show us that half of 6 is 3.

Write on the board:

6 is double 3, so 3 is half of 6.

8 is double ___, so half of 8 is ___.

ASK: What number do you double to get 8? (4) Write “4” in the first blank. ASK: What is half of 8? (4) Invite volunteers to draw models that show that 4 is half of 8.

Exercises:

a) Fill in the blanks.

10 is double ___, so half of 10 is ___.

12 is double ___, so half of 12 is ___.

b) Draw a picture to show your answers in part a).
Answers: a) 5, 5, 6, 6

Sample answers:

b)  

Write on the board:

10 is double __, so half of 10 is __. __ + __ = 10 2 × __ = 10
100 is double __, so half of 100 is __. __ + __ = 100 2 × __ = 100
1000 is double __, so half of 1000 is __. __ + __ = 1000 2 × __ = 1000

ASK: What number do you double to get 10? (5) What number is half of 10? (5)
How do you show this with addition? (5 + 5 = 10) How do you show it with multiplication? (2 × 5 = 10) Fill in the blanks in the first row. Repeat the questioning for the other two rows. (50 in all blanks of the second row, 500 in all blanks of the third row)

Half of 1 kilogram equals 500 grams. Remind students that 1 kilogram equals 1000 grams. ASK: Have you ever seen any grocery item labelled as half a kilogram? (no) SAY: We know that half of 1000 is 500. ASK: How do you think a label shows the mass of an item that weighs half a kilogram? (500 grams) Invite volunteers to check the labels on the benchmark items to confirm the prediction.

SAY: Objects that weigh about half a kilogram often do not have the mass of exactly 500 grams. They have a mass that is close to 500 grams. ASK: What numbers can you think of that are close to 500 grams? Accept all reasonable answers. Explain that any object that has a mass above 450 grams but below 550 grams can be described as having a mass of about half a kilogram. Write the examples below on the board and have students signal thumbs up if the mass is about half a kilogram and thumbs down if it is not. Keep the exercises on the board as they will be modified slightly and used for the remainder of this lesson.

Exercises: Is the mass about \( \frac{1}{2} \) kg?

a) cucumber: 456 g  b) sunglasses: 150 g

c) apple: 254 g  d) melon: 690 g

e) rock: 540 g  f) knife: 237 g

g) basketball: 482 g  h) log: 501 kg

Answers: a) yes, b) no, c) no, d) no, e) yes, f) no, g) yes, h) no

Quarter of a number. Draw on the board:

ASK: What addition and what multiplication does this picture show?

(3 + 3 + 3 + 3 = 12, 4 × 3 = 12) Write both equations on the board.
SAY: So we have 12 stars, and they are grouped into 4 equal parts.
ASK: What fraction of the set of 12 stars does each circle show? (one fourth or one quarter) SAY: One fourth of 12 is 3. If you divide a set of 12 stars into 4 equal parts, you get 3 stars in each part. ASK: How can you write this with division? (12 ÷ 4 = 3) Have a volunteer write the division on the board.

SAY: Let’s find one fourth of 20. Write on the board:

\[
4 \times \_ = 20
\]

ASK: What number goes in the blank? (5) What division sentence tells you that the number in the blank is 5? (20 ÷ 4 = 5) Invite a volunteer to draw a model for this sentence. ASK: What is one fourth of 20? (5) Have another volunteer explain how the model shows that 1/4 of 20 is 5.

Exercises:

a) Find \(\frac{1}{4}\) of 8.
b) Draw a model to show your answer.
c) Write addition, multiplication, and division sentences to match your picture.

Answers: a) 2, c) \(2 + 2 + 2 + 2 = 8, 4 \times 2 = 8, 8 \div 4 = 2\)

Sample answer: b) \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)

Quarter of 1000. Show students 8 hundreds blocks and 20 tens blocks.
ASK: What number does this collection represent? (1000) How do you know? (800 in hundreds blocks, and 2 groups of 10 tens blocks, each group equals 100) Invite volunteers to divide the hundreds blocks and the tens blocks (separately) into four equal groups. Have another volunteer identify the number represented by each group. (250)

Have students identify addition, multiplication, and division sentences for the model and write them in their notebooks. Have volunteers write the sentences on the board, as shown below:

\[
250 + 250 + 250 + 250 = 1000 \\
4 \times 250 = 1000 \\
1000 \div 4 = 250
\]

ASK: What fraction of a thousand is 250? (one fourth) PROMPT: How many equal groups of 250 make 1000 together? (4) Write the sentences below on the board beside the corresponding equations.

\[
\frac{1}{4} \text{ of 1000 is 250.} \\
\frac{1}{4} \text{ of 1000 g is } \_ \text{ g.} \\
\frac{1}{4} \text{ of 1 kg is } \_ \text{ g.}
\]
Have students help you fill in the blanks (250 in both blanks). Invite volunteers to check the labels on the quarter-kilogram packages you used earlier in the lesson and verify that there is indeed 250 grams in each package.

SAY: Objects that weigh about a quarter of a kilogram do not have the mass of exactly 250 grams. They have a mass that is close to 250 grams. ASK: What numbers can you think of that are close to 250 grams? Accept all reasonable answers. Explain that any object that has a mass above 200 grams but below 300 grams can be described as having a mass of about one fourth of a kilogram. Return to the exercises about half a kilogram on the board and modify the question and part h) as shown below. Have students signal thumbs up if the mass is about a quarter of a kilogram and thumbs down if it is not. Keep the exercises on the board for use later in the lesson.

**Exercises:** Is the mass about \(\frac{1}{4}\) kg?

a) cucumber: 456 g  
b) sunglasses: 150 g  
c) apple: 254 g  
d) melon: 690 g  
e) rock: 540 g  
f) knife: 237 g  
g) basketball: 482 g  
h) bear: 249 kg

**Answers:** a) no, b) no, c) yes, d) no, e) no, f) yes, g) no, h) no

**Three quarters of a kilogram.** Draw the picture shown in the margin on the board. ASK: What fraction does this picture show? (3/4) What fraction of the large rectangle is each small rectangle? (1/4) How many quarters together make three fourths? (3) SAY: One quarter of a kilogram is 250 grams. ASK: What addition equation can you write to find three quarters of a kilogram? (250 g + 250 g + 250 g = 750 g) Write on the board:

\[
\frac{1}{4} \text{ of } 1 \text{ kg is } 250 \text{ g} \\
250 \text{ g} + 250 \text{ g} + 250 \text{ g} = 750 \text{ g} \\
\frac{3}{4} \text{ of } 1 \text{ kg is } 750 \text{ g}
\]

SAY: Objects that weigh about three fourths of a kilogram do not have a mass of exactly 750 grams. They have a mass that is close to 750 grams. For any object that has a mass above 700 grams but below 800 grams we can say that it has a mass of about three fourths of a kilogram. Draw students’ attention to the exercises on the board. Modify the question and parts d), e), and h) and have students signal thumbs up if the mass is about three fourths of a kilogram and thumbs down if it is not. Keep the exercises on the board for use later in the lesson.
Exercises: Is the mass about $\frac{3}{4}$ kg?

a) cucumber: 456 g  

b) sunglasses: 150 g

c) apple: 254 g  

d) melon: 790 g

e) rock: 670 g  

f) knife: 237 g

g) basketball: 482 g  

h) large moose: 700 kg

Answers: a) no, b) no, c) no, d) yes, e) yes, f) no, g) no, h) no

Add to the list of benchmark masses on the board “1/2 kg is 500 g” and “1 kg is 1000 g.” Modify the question on the board and parts b), d), and e). Delete part h) and replace it with the Bonus. Have students do the following exercises.

Exercises: About what fraction of a kilogram does the object weigh?

a) cucumber: 456 g  

b) ear of corn: 532 g

c) apple: 254 g  

d) melon: 783 g

e) rock: 739 g  

f) knife: 237 g

g) basketball: 482 g  

Bonus: bottle of water: 995 g

Answers: a) 1/2 kg, b) 1/2 kg, c) 1/4 kg, d) 3/4 kg, e) 3/4 kg, f) 1/4 kg, g) 1/2 kg, Bonus: 1 kg

ACTIVITY

Give students each an object with a mass close to 250, 500, or 750 grams. Have them estimate the mass in grams and then in fractions of a kilogram. Have students record their estimates, in a table, as shown below, then exchange objects with several other students and repeat.

<table>
<thead>
<tr>
<th>Object</th>
<th>Estimated Mass</th>
<th>Actual Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Grams</td>
<td>Fraction of Kilogram</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students use a scale to measure the mass of the objects they estimated and record the mass in the table.
Extensions

1. Fill in the blanks.
   a) 1 kg = ____ g, 1 L = ____ mL
   b) \( \frac{1}{4} \) kg = ____ g, \( \frac{1}{4} \) L = ____ mL
   c) \( \frac{3}{4} \) kg = ____ g, \( \frac{3}{4} \) L = ____ mL
   d) \( \frac{1}{2} \) kg = ____ g, \( \frac{1}{2} \) L = ____ mL
   e) Find three containers with a capacity of less than 1 L. Estimate and measure the capacity of each in millilitres. Then decide if the capacity of any of the containers is close to half, a quarter, or three quarters of a litre.

   **Selected answers:** a) 1000, 1000; b) 250, 250; c) 750, 750; d) 500, 500

2. Dory has 3 apples. One apple weighs 257 g, another weighs \( \frac{1}{4} \) kg, and a third apple weighs 234 g. What is the total mass of Dory’s apples in kilograms: about \( \frac{1}{4} \) kg, about \( \frac{1}{2} \) kg, about \( \frac{3}{4} \) kg, or about 1 kg?
   **Answer:** about \( \frac{3}{4} \) kg

3. Which is heavier?
   a) \( \frac{3}{4} \) kg or 754 g  b) \( \frac{3}{4} \) kg or 698 g  c) \( \frac{1}{4} \) kg or 255 g
   **Answers:** a) 754 g, b) \( \frac{3}{4} \) kg, c) 255 g
Goals

Students will estimate and measure positive temperatures in degrees Celsius.

Students will identify benchmarks for qualitative descriptions of temperatures of air and water.

Prior Knowledge Required

Can add two-digit numbers
Can locate numbers on a number line
Knows the multiples of 10
Can count by 10s and count on by 1s

Materials

ball or relay race baton (optional)
demonstration thermometer
access to sink with cold and hot water
3 cups per pair of students
thermometer per pair of students
ice cubes (optional)
Internet access (see Extensions 2, 3)
newspapers and magazines (see Extension 3)
large map of Canada (see Extension 3)

Mental math minute. Arrange students in a line and have them add two-digit numbers by adding tens and adding ones. For each addition problem, such as 35 + 46, students need to say three steps: adding the tens: 30 + 40 = 70; adding the ones, 5 + 6 = 11; and finishing the addition, 70 + 11 = 81, so 35 + 46 = 81. The next student in line gets a new problem. Students can pass a ball or a relay race baton to each other so that the person who receives the baton answers the next question. Start with problems that do not require regrouping, such as 25 + 34, and continue to questions that require regrouping the ones.

Introduce temperature and the thermometer. Discuss with students how they know what clothes they need to wear each day—how do they know if they need a short sleeved shirt or a sweater? Can they wear shorts or do they need long pants? Lead students to the idea that people check the temperature outside or on the TV or the Internet to decide what type of clothes to wear. Show a demonstration thermometer and SAY: We use a thermometer to check how hot or cold something is. A thermometer tells you the temperature of whatever it is put into. Ask students where they see thermometers used. Examples include a water thermometer to check the temperature of the water in a swimming pool, a cooking thermometer to check the temperature of food as it is cooking, and a thermometer that checks a person’s temperature.
Use a demonstration thermometer to show how the liquid in a thermometer goes up and down. Explain that as the temperature gets higher, the liquid in the thermometer goes up. If the temperature gets lower, the liquid goes down. You can demonstrate that by putting a thermometer into a glass with a little cold water and adding hot water to the glass in small portions. Students should be able to see the liquid in the thermometer going up as you add each portion of hot water.

**Introduce degrees Celsius.** SAY: In Canada, we measure temperature in units that are called degrees Celsius. Write "degrees Celsius" on the board and read it together as a class. Write °C on the board. Explain that the short form is °C, the small raised circle is the symbol for degrees, and the C means Celsius. Give an example of the temperature today. Write the temperature on the board in two ways: for example, 15 degrees Celsius and 15°C.

**Reading the scale.** Show the scale on the thermometer and explain that it is like a number line, but not all numbers are labelled on it. If the scale of your demonstration thermometer is large enough for all students to see clearly, work with it. If not, draw the pictures below and in exercises on the board.

![Temperature Scale](image)

Show students a part of the scale between two marked numbers, such as 20 and 30. Point out that only the multiples of 10 are written on the scale, and the ticks for them are the longest. Show the temperature going up to some of these marks and have students say what temperature the thermometer shows. Have students write the temperature by using the degree symbol and have volunteers write answers on the board.

**Exercises:** The picture shows part of a thermometer. What temperature does the thermometer show?

![Thermometer Pictures](image)

**Answers:** a) 20°C, b) 30°C, c) 60°C

Explain that numbers that are not multiples of 10 are shown with shorter tick marks. When you need to say what the temperature is, you need to count the degrees from the last multiple of 10 to the top of the coloured liquid. Show 12 degrees on a demonstration thermometer and have students determine the temperature.
Exercises: The picture shows part of a thermometer. What temperature does the thermometer show?

- **a)** ![Thermometer (20°C and -10°C)]
- **b)** ![Thermometer (30°C and -20°C)]
- **c)** ![Thermometer (60°C and -20°C)]

**Answers:** a) 14°C, b) 23°C, c) 57°C

Point out that one of the tick marks without numbers is longer than the others. ASK: How many degrees above the numbered tick mark is the longer tick mark? (5) Check several intervals. ASK: What is the ones digit of the numbers that have the longer tick mark? (5) Explain that the longer tick mark is there to make counting easier. You will know what number with a ones digit of 5 the tick mark shows based on the marked numbers below and above it. Then you can count forwards or backwards by ones from the longer tick mark. Use this strategy to determine the temperature in part c) of the previous exercise. PROMPT: What number with a ones digit of 5 is between 50 and 60? (55) SAY: Count by ones from 55, starting at the longer tick mark without a number: 56, 57. The temperature is 57°C.

Exercises: The picture shows part of a thermometer. What temperature does the thermometer show?

- **a)** ![Thermometer (-20°C and -10°C)]
- **b)** ![Thermometer (-30°C and -20°C)]
- **c)** ![Thermometer (50°C and -40°C)]

**Answers:** a) 19°C, b) 27°C, c) 48°C

**Benchmarks for air temperatures.** Explain that when people say, “It’s 15 degrees today,” they mean that the temperature of the air is 15 degrees Celsius. Ask students how they would describe the weather today: is it hot, cold, warm, or cool? Answers might vary. Explain that most people will describe temperatures above 25°C as hot, temperatures between 16°C and 25°C as warm, temperatures between 5°C and 15°C as cool, and temperatures below 5°C as cold.

Write the ranges on the board and show the thermometer in the margin.

Point out that temperatures also depend on the wind, rain, and other things, but the ranges on the board are good descriptions for most people.

SAY: It is hot enough to go to the beach. ASK: What could the outside temperature be? Accept all reasonable answers. You might want to point out that the hottest places on Earth are deserts, and the hottest temperatures that have been recorded there are no more than 58°C. This means that saying that the temperature today is 100°C is not a reasonable answer.
SAY: The air temperature outside is 12°C. I think it is good weather to go ice skating. ASK: Am I correct? (no) Why not? (12°C is cool weather; it is not cold enough to go skating) What is a good temperature to go skating? (anything below 5°C) You might want to point out that a good temperature for skating should be close to 0°C, because ice is frozen water and water freezes at 0°C. To maintain ice, the temperature should be close to 0°C.

**Exercises:**

a) The temperature is 7°C. Nora thinks she will wear sandals. Is she correct?
b) The temperature is 27°C. Ivan thinks he needs a jacket. Is he correct?
c) The temperature is 29°C. Shelly thinks she needs a hat and sunglasses. Is she correct?

**Bonus:** It is too warm to go skating, but it is not warm enough to go sunbathing. What could the temperature be?

**Answers:** a) no, b) no, c) yes, Bonus: any answer between 5°C and 25°C is correct

**Benchmarks for water temperatures.** Explain that water temperatures feel different from air temperatures. For example, it feels quite warm outside if the air temperature is 20°C, but if water is 20°C, it is rather cold water for swimming. Pour some room-temperature water into a glass and have a volunteer put a finger into the water and say how the water feels—does it feel cold, cool, or warm? Write the description on the board. Put a thermometer into the water and have a volunteer read the thermometer. Write the temperature on the board beside the description.

Add some hot water to the glass of room temperature water. Make sure the water is not too hot to touch. Have a volunteer check the water with a finger and describe it. Explain that water freezes at 0°C and boils at 100°C. The hot water in a tap usually has a temperature of about 50°C. Bath water is about the same temperature as a healthy human body, 37°C. Ask the volunteer who described the temperature of the water to estimate it and record the estimate on the board. Have another volunteer measure the temperature to check.

**ACTIVITY**

Each pair of students needs a cup for cold water, a cup for hot water, an empty cup, access to a sink with hot and cold water, and a thermometer. Partner 1 fills a cup with water. Partner 2 holds a hand over the cup to feel if the water is hot. If it is not, Partner 2 can put a finger in to check the water, describe its temperature (hot, warm, cool, or cold), and estimate its temperature in numbers. Students record the description and the estimate in a table, as shown below. Partner 1 measures the temperature. Partners record the temperature and switch roles.
Description | Temperature
-------------|--------------
| Estimated | Actual |

Have students try to create water of different temperatures. Have them use water from the cold or the hot tap only, use a mixture with more cold water or more hot water, or try to make a mixture with the same amount of hot and cold water. If ice cubes are available, have students add an ice cube to cold water as well.

**NOTE:** Extension 5 is required for the British Columbia curriculum.

**Extensions**

1. Discuss the consequences of temperature changes on humans, plants, and animals. Students can choose an animal or a plant and research what adaptations it makes to survive changes in temperature.

2. Choose three different provinces or territories that are not beside each other. Choose a place in each province or territory.
   a) Predict: Do these three places have the same or different temperatures in winter? In summer? Which place is the coldest? Which place is the warmest?
   b) Search the Internet for the average winter temperature and average summer temperature in each place.
   c) Predict what clothes people need in winter and in summer in all three places.

3. Assign a different Canadian town to each student. Assign at least one location from each province and territory, and try to pick locations in the same province or territory that are as far from each other as possible.

Have students check the temperatures at these locations on the same date. For each location, have students think of the activities people might do in this weather and what clothes they may need. Students can check their ideas by searching for forecasts on the Internet.

Students can draw a picture or use cut-outs from newspapers or magazines to show their answers. Have students find their assigned location on a large map of Canada. Display the students’ work and the map on a bulletin board titled: The Temperature in Canada on [date].

4. Discuss with students how reading a thermometer and determining the volume of liquid in a measuring cup is the same. When students notice the similarities, explain how a thermometer works. Explain that a thermometer is filled with liquid that expands, or increases in volume, when it heats up. When the liquid expands, it takes more space in the thermometer. The only room for it to grow is upward, so
it rises to higher levels in the thermometer. This means that reading the thermometer is similar to reading the volume of a liquid in a measuring cup.

5. **Temperature and Calendars.** Discuss with students what temperatures are typical in your area during different times of the year. Students can make a calendar showing the typical weather, including temperature and precipitation (such as rain or snow), events, celebrations, and activities based on a calendar of their choice: standard, traditional (such as the Haida or Salish calendars), referring to a specific culture (such as the Chinese, Persian, or Jewish calendars), or referring to an ancient civilization (such as the Incan or Aztec calendars).
Unit 15  Number Sense: Estimating

Introduction

In this unit, students will use their understanding of place value and the properties of operations to perform multi-digit arithmetic. They will round whole numbers to the nearest ten, and they will use rounding to estimate sums and differences within 1000 and evaluate the reasonableness of the answers to word problems. They will use a referent of 10 or 100 to estimate the number of objects in a collection. They will also use the standard algorithm to add three-digit numbers where the result exceeds 1000.

Meeting Your Curriculum

Alberta—Lessons NS3-71 to NS3-73 are required. We recommend teaching Lessons NS3-74 and NS3-75 to students in Alberta, since adding three-digit numbers often produces four-digit answers.

British Columbia—Lessons NS3-71 to NS3-73 are required. Extensions 2 and 3 in Lesson NS3-71 are also required. We recommend teaching Lessons NS3-74 and NS3-75 to students in British Columbia, since they need to count money amounts up to $100.00, which requires an understanding of four-digit numbers.

Manitoba—Lessons NS3-71 to NS3-73 are required. We recommend teaching Lessons NS3-74 and NS3-75 to students in Manitoba, since adding three-digit numbers often produces four-digit answers.

Ontario—Lessons NS3-71 and NS3-72 are required. All other lessons are optional—students will learn this material in Grade 4.

Materials. For Lesson NS3-73, you will need to prepare materials ahead of time. You will need containers, such as jars and bottles, and a large number of small objects, such as jelly beans, raisins, and plastic counters. When using a referent of 10, you will need fewer than 100 objects. When using a referent of 100, you will need fewer than 1000 objects.

In addition to the BLMs provided at the end of this unit, the following Generic BLM, found in section V, is used in Unit 15:

BLM Hundreds Chart  (p. V-9)

Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>NS3-71 to 73</td>
<td>NS3-71 to 73</td>
<td>NS3-71 to 73</td>
<td>NS3-71 to 72</td>
</tr>
<tr>
<td>Quiz</td>
<td>NS3-74 to 75</td>
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<td>NS3-74 to 75</td>
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<tr>
<td>Test</td>
<td>NS3-71 to 73</td>
<td>NS3-71 to 73</td>
<td>NS3-71 to 73</td>
<td>NS3-71 to 72</td>
</tr>
</tbody>
</table>
Goals
Students will round two-digit numbers to the nearest ten.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to the tens
Can count by tens to 100

MATERIALS
number line from 0 to 100 made from an enlarged copy of
BLM Hundreds Chart (p. V-9)
ball
BLM Number Cards (pp. R-27–28), a set of cards per pair of students

Mental math minute. Ask all students to stand up. One at a time, students say their first name and the number of letters in their first name, followed by their last name and the number of letters in their last name. They then multiply the number of letters in their first name by the number of letters in their last name. For example, if the student’s name is Jennifer Goncalves, the student will multiply $8 \times 9 = 72$. Then the student sits down. Repeat until all students have had a chance to participate.

NOTE: In this lesson students will round numbers to the nearest ten in two steps:

Step 1: Find the next and the previous multiple of 10.
Step 2: Decide which of them the number is nearest to.

Students will then be introduced to the rule for rounding based on the ones digit.

Review relevant vocabulary. Write “45” on the board. ASK: What digit shows the ones? (5) What digit shows the tens? (4) Students can signal their answers by raising the correct number of fingers. Remind students that a common expression that describes hundreds, tens, and ones is “place value.”

Remind students that when we skip count by a number starting from zero, the numbers we say are multiples of that number. Skip count by 10 as a class, recording the numbers on the board. (0, 10, 20, 30, and so on) SAY: These are the multiples of 10. Write “multiples of 10” under the numbers on the board.

Finding the multiples of 10 right before and after a number. Display a number line from 0 to 100. You can create one by cutting out each row from an enlarged copy of BLM Hundreds Chart and taping the rows
together into one long strip. Have students identify the multiples of 10 on
the number line. Then ask a volunteer to find the number 63 on the number
line. **ASK:** Which multiple of 10 comes right before 63? (60) Repeat with
47, 82, 35. (40, 80, 30) **ASK:** How can you tell what the multiple of 10 right
before the number is without looking at the number line? (pretend the ones
digit is zero)

**Exercises:** Find the multiple of 10 right before the number.

a) 51   b) 64   c) 29   d) 75  

e) 12 **Bonus:** 104

**Answers:** a) 50, b) 60, c) 20, d) 70, e) 10, **Bonus:** 100

**ACTIVITY 1**

1. Toss a ball to a student while saying a number. The student who
catches the ball says the multiple of 10 before the number and
tosses the ball back to you. Repeat with different numbers until all
students have had at least one turn.

Have volunteers find the multiple of 10 that is right after the numbers 63, 75,
82, 11, 9, and 27. (70, 80, 90, 20, 10, 30) Then discuss how students can
find the multiple of 10 right after a number without looking at the number
line. One way is to count up by ones from the number and see what
number you say when the tens digit changes. Another way is to find the
multiple of 10 immediately before the number and add 10. Have students
try both methods while solving the exercises below, and then discuss which
method they prefer.

**Exercises:** Find the multiple of 10 right after the number.

a) 51   b) 64   c) 29   d) 75

e) 12 **Bonus:** 94

**Answers:** a) 60, b) 70, c) 30, d) 80, e) 20, **Bonus:** 100

Repeat Activity 1 above, this time asking students for the multiple of 10
after the number.

Write on the board:

previous   next

Have students read the words aloud. Ask if anyone knows what the words
mean. If necessary, explain that previous means the one right before and
next means the one right after. Now ask students to find the two multiples of
10—the previous and the next—for 53. (50, 60)

Repeat Activity 1 again, this time asking students for both the previous and
the next multiples of 10 for each number.
Exercises: Find the previous and the next multiples of 10.

a) 34  b) 49  c) 38  d) 81  e) 92

Bonus
f) 104  g) 7

Answers: a) 30, 40; b) 40, 50; c) 30, 40; d) 80, 90; e) 90, 100;
Bonus: f) 100, 110; g) 0, 10

ACTIVITIES 2–3

2. Give each pair of students a set of cards from **BLM Number Cards** and have them sort the cards into multiples of 10 and not multiples of 10. Have students lay the multiples of 10 face up on the table in random order and place the remaining cards (that is, the not multiples of 10) face down in a pile.

   Player 1 picks a card from the face-down pile and places it face up on the table. Player 2 identifies the previous and the next multiples of 10 from the face-up cards and places them on either side of the card, in the correct order. Players return the multiples of 10 to the table and discard the middle card. Then the players switch roles. Play continues until all the cards in the face-down pile have been used.

   After the players have played several rounds, they can try the more complicated game below.

3. **Multiples of 10 Train**

   **Materials:** cards from **BLM Number Cards**

   **Objective:** To arrange all cards in a “train” from smallest to largest

   **Preparation:** Have students play in pairs. Have players sort the deck into multiples of 10 and not multiples of 10. They should spread out the multiples of 10 in random order and place the other cards face down in a pile. Player 1 picks a card from the face-down pile. Player 2 must find the two multiples of 10 the number is between and place them on either side of the card in the correct order, creating a train. If one of the multiples is part of an existing train, the player adds the card and the second multiple to the train. If both multiples are already in trains, Player 2 should either fit the card inside an existing train or combine two trains into one.

   **Variation:** To shorten the game, limit the number of cards that are not multiples of 10 to four cards.

   **Deciding which multiple of 10 is closest.** Draw a number line from 0 to 10 on the board. Circle the 2 and ask students if it is closer to the 0 or to the 10. (0) Students can signal the answer by making a circle with one hand or raising 10 fingers. Draw an arrow from the 2 to the 0 to show that 2 is
closer to 0 than to 10. Repeat with 7, 9, 4, and 1. (10, 10, 0, 0) ASK: Which numbers from 1 to 9 are closer to 0? (1, 2, 3, 4) Which numbers are closer to 10? (6, 7, 8, 9) What about 5? (it is right in the middle, the same distance from 0 as from 10)

Draw on the board:

```
  10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
```

Circle various numbers (avoiding 10, 15, 20, 25, and 30) and ask volunteers to draw arrows to the closest, or nearest, multiples of 10. SAY: The arrows point to the nearest multiples of 10. ASK: Which numbers have an arrow pointing left, to the previous multiple of 10? Which numbers have an arrow pointing right, to the next multiple of 10? Make lists on the board, as shown below:

<table>
<thead>
<tr>
<th>previous</th>
<th>next</th>
</tr>
</thead>
<tbody>
<tr>
<td>11, 12, 13, 14</td>
<td>16, 17, 18, 19</td>
</tr>
<tr>
<td>21, 22, 23, 24</td>
<td>26, 27, 28, 29</td>
</tr>
</tbody>
</table>

ASK: What are the ones digits of the numbers that are closer to the previous multiple of 10? (1, 2, 3, 4) What are the ones digits of the numbers that are closer to the next multiple of 10? (6, 7, 8, 9) Which two numbers that don’t have ones digit 0 are exactly the same distance from the previous and the next multiples of 10? (15 and 25) What do you notice about their ones digit? (it is 5)

SAY: I want you to try to predict which multiple of 10 is nearest without the number line. Try to use the pattern we found for numbers from 0 to 30. Repeat Activity 1, this time asking students for the nearest multiple of 10. Do not include numbers with ones digit 0 or 5.

**Introduce rounding.** Remind students that when measuring they do not always need to find an exact length but only a measurement that is close to the actual length. This is called an estimate of length. Explain that estimates are useful in other situations, too. For example, if you have 68 baseball cards and ten cards can be placed on one page of a binder, you may want to know how many pages are needed to store your baseball cards. One of the simplest ways to estimate is to use a multiple of 10 that is nearest to the number. This is called rounding the number. When you look for a multiple of 10 that is nearest to the number, you round to the nearest multiple of 10.

SAY: I want to round 24 to the nearest multiple of 10. ASK: What is the nearest multiple of 10? (20) SAY: 24 rounded to the nearest multiple of 10 is 20.

**The rule for rounding.** SAY: When we round to the nearest multiple of 10 and the answer is smaller than the number, we say that we round down. When we round to the nearest multiple of 10 and the answer is larger than the number, we say that we round up.
Write on the board:

**Step 1:** Write the previous and the next multiples of 10.

**Step 2:** If the ones digit is
- 1, 2, 3, or 4, round down (previous)
- 6, 7, 8, or 9, round up (next)

SAY: I want to round 32 to the nearest multiple of 10. ASK: What two multiples of 10 is 32 between? (30 and 40) What is the ones digit? (2) Which list is it in? (the first one) Will we round up or down? (down) What is 32 rounded to the nearest multiple of 10? (30) Repeat the questions for 58. (60)

Repeat Activity 1, this time asking students to round two-digit numbers to the nearest multiple of 10. Do not include numbers with ones digit 0 or 5.

ASK: When you round down, what happens to the tens digit? (it stays the same) What happens to the tens digit when you round up? (it goes up) Write on the board (leave space between “ones digit” and the list in each row):

When you round a number with:
- ones digit 1, 2, 3, 4 the tens digit stays the same
- ones digit 6, 7, 8, 9 the tens digit goes up

SAY: I want to round 20 to the nearest multiple of 10. But wait—it is already a multiple of 10! I do not need to go up or down. ASK: What happens with the tens digit? (it stays the same) Add 0 to the list of ones digits in the first row on the board, before the 1. ASK: What should we do with 5? Have students suggest options and explain why they prefer the option suggested. Remind students that mathematicians have agreed that a measurement that is exactly halfway between two numbers, say, 5 cm and 6 cm, is always rounded up, in this case to 6 cm. Explain that just as we round a measurement up to the next unit, we round numbers that have ones digit 5 up. SAY: So 25 rounded to the nearest ten is 30 and 15 rounded to the nearest ten is 20. ASK: Which list should we add 5 to? (the second) Add 5 to the second list on the board, before the 6. Point out that this gives both lists five digits.

Repeat Activity 1, this time asking students to round any two-digit numbers to the nearest multiple of 10.

**NOTE:** Extensions 2 and 3 are necessary to cover the British Columbia curriculum.
Extensions

1. Show students how to round a three-digit number to the nearest hundred. Write on the board:

   347

   ASK: What place value is the digit 3 under? (hundreds) What value does the digit 3 have? (300) SAY: To round this number to the nearest hundred, we need to ask if 347 is closer to 300 or to 400. Underline the tens and ones digits. ASK: What two-digit number does this make? (47) SAY: If the tens and ones digits make a number less than 50, round down to 300. If the tens and ones digits make number that is 50 or higher, round up to 400. Point to 47 and ASK: Do we round up or down? (down) SAY: So 347 rounded to the nearest hundred is 300.

   Have students round the number to the nearest hundred.
   a) 476  b) 627  c) 850  d) 975
   **Answers:** a) 500, b) 600, c) 900, d) 1000

2. Introduce another method of finding the next multiple of 10. Write on the board:

   **Step 1:** Change the ones digit to 0.
   **Step 2:** Add 1 to the tens digit.

   Have students use this method to find the next multiple of 10.
   a) 38  b) 69  c) 51  d) 43
   **Answers:** a) 40, b) 70, c) 60, d) 50

   Then ask students to try to use this method on 92. ASK: Why didn’t this work on 92? (the tens digit is 9, so when they add 1 it becomes 10) Explain that in this case you need to regroup. ASK: What are 10 tens equal to? (1 hundred) Explain that we need to add 1 to the hundreds digit. SAY: Right now, the hundreds digit is actually 0; we need to make it 1. So the next multiple of 10 after 92 is 100.

3. Show students how to round a three-digit number to the nearest ten. Write on the board:

   **Step 1:** Ignore the hundreds digit. Look only at the last two digits.
   **Step 2:** Round the two-digit number made by the last two digits to the nearest ten.
   **Step 3:** Write the hundreds digit at the beginning.

   Have students round the number to the nearest ten.
   a) 263  b) 738  c) 589  **Bonus:** 597
   **Answers:** a) 260, b) 740, c) 590, Bonus: 600
Goals
Students will estimate sums and differences by rounding numbers to the nearest ten.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to tens
Can round to the nearest ten
Can add and subtract using the standard algorithm

MATERIALS
counters

Mental math minute. Students work in pairs. Each student takes a small handful of counters and counts them. Then the pair uses the two numbers they got to create two addition sentences. For example, if Student A selects 23 counters and Student B selects 38 counters, they create the addition sentences $23 + 38 = 61$ and $38 + 23 = 61$. Ask all students to stand up. One at a time, have each pair say their addition sentences and then sit down. Repeat until all students have had a chance to participate.

Review rounding to the nearest ten. Remind students of the rule for rounding to the nearest ten. Explain that even though 5 is the same distance from 0 and 10, and any number with ones digit 5 is the same distance from the two nearest multiples of 10, we still round up.

Exercises: Round to the nearest ten.

a) 34 
   b) 59 
   c) 45 
   d) 98

Answers: a) 30, b) 60, c) 50, d) 100

Introduce estimating in calculations. Remind students that when measuring, sometimes they need an exact measurement, while other times they need only a sense of how large something is. For example, if I want to buy curtains for a window, it matters whether the window is 1 m wide or 4 m wide. But if the window is a little over 1 m, it does not really matter by how much, as long as the measurement is close. Explain that a similar situation can happen with calculations.

Explain to students that they can estimate the answer to a calculation by simplifying the numbers first. Their answer when they do this will be close to the exact answer of the original calculation. Estimating an answer before calculating can help them to catch mistakes because when they see the answer, they will already have an idea of what it should be. SAY: For example, when you use a calculator, you can sometimes press a wrong number by mistake. ASK: If I add 25 and 38 on a calculator, would 173 be a reasonable answer? Have students add the numbers. ($25 + 38 = 63$)
Point out that 173 is much larger than the correct answer, so it is unreasonable. Explain that students could have used an estimate instead of the exact answer to check if 173 was reasonable. The most common way to estimate a calculation is to round the numbers before calculating.

**Estimating a sum or a difference.** Write on the board:

\[
\begin{array}{c}
25 \\
+ 38
\end{array}
\]

SAY: When working with two-digit numbers, round to the nearest 10.

ASK: What is 25 rounded to the nearest ten? (30) What is 38 rounded to the nearest ten? (40) Have a volunteer write the rounded numbers in the boxes, then add them. (70) Point out that the estimate, 70, is very close to the actual answer, 63. If students had estimated the answer first, they would have known right away that 173 was unreasonable because it is much larger than 70.

**Exercises**

1. Round the numbers to the nearest ten, then add.

   a) \(41 + 38\)  
   b) \(52 + 11\)  
   c) \(73 + 19\)  
   d) \(84 + 13\)  
   e) \(92 + 37\)  
   f) \(83 + 24\)

   **Answers:** a) \(40 + 40 = 80\), b) \(50 + 10 = 60\), c) \(70 + 20 = 90\),
   d) \(80 + 10 = 90\), e) \(90 + 40 = 130\), f) \(80 + 20 = 100\)

2. Add the numbers in Exercise 1 without rounding them first, and compare your answers.

   **Answers:** a) 79, b) 63, c) 92, d) 97, e) 129, f) 107

ASK: How close are the answers? Is the difference between the estimate and the actual calculation ever larger than 10? (no) Explain that when we round to the nearest ten, getting an answer that is less than 10 away from the actual answer means that the estimate is good.

ASK: How would you estimate \(93 - 21\)? Write on the board:

\[
\begin{array}{c}
93 \\
- 21
\end{array}
\]

Ask students to round the numbers, then subtract. \((90 - 20 = 70)\) Then ask them to subtract the numbers without rounding. \((93 - 21 = 72)\) Again, have students compare the answers. ASK: Are they close? (yes) Which calculation is easier? \((90 - 20 = 70)\)
Exercises

1. Round the numbers to the nearest ten, then subtract.
   a) $53 - 21$  
   b) $72 - 29$  
   c) $68 - 53$  
   d) $48 - 17$  
   e) $63 - 12$  
   f) $74 - 37$

   **Answers:**
   a) $50 - 20 = 30$, b) $70 - 30 = 40$, c) $70 - 50 = 20$, d) $50 - 20 = 30$, e) $60 - 10 = 50$, f) $70 - 40 = 30$

2. Subtract the numbers in Exercise 1 without rounding them first, and compare your answers.

   **Answers:** a) 32, b) 43, c) 15, d) 31, e) 51, f) 37

   **ASK:** How close are the answers? Is the difference between the estimate and the actual calculation ever larger ever than 10? (no) Were our estimates good? (yes)

   **Using rounding in word problems.** SAY: A student I know added 43 and 45 and got the answer 78. **ASK:** Does this answer seem reasonable? Have students round each number to the nearest ten to check the answer. ($40 + 50 = 90$) **ASK:** Is the answer reasonable? (no, both 43 and 45 are greater than 40, so the sum must be greater than $40 + 40 = 80$)

   Write on the board:
   
   A store sold 58 red apples and 21 green apples.
   
   **How many apples did the store sell altogether?**
   
   SAY: It would be easier to add these numbers if they were multiples of 10.
   
   **ASK:** Are these numbers close to multiples of 10? (yes) What is the nearest multiple of 10 to 58? (60) To 21? (20) What is $60 + 20$? (80) Do you think that 80 is a good estimate for $58 + 21$? (yes) SAY: 58 is only 2 less than 60, and 21 is only 1 more than 20, so we changed the numbers very little and can expect a good estimate. **ASK:** What is the actual answer to $58 + 21$? (79) Was 80 a good estimate? (yes)

   **Exercises:** Estimate the total number of apples the store sold. Then find the actual answer. Was your estimate close?

   a) 27 red apples and 42 green apples
   b) 46 red apples and 78 yellow apples
   c) 52 Granny Smith apples and 31 Golden Delicious apples
   d) 42 Red Delicious apples and 29 Gala apples

   **Answers:** a) $30 + 40 = 70$, actual 69, yes; b) $50 + 80 = 130$, actual 124, yes; c) $50 + 30 = 80$, actual 83, yes; d) $40 + 30 = 70$, actual 71, yes

   Leave the exercises above on the board. Continue writing on the board:

   **About how many more green apples than red apples were sold in part a)?**
ASK: What word in this question tells you I only want an estimate? (about) Does the question ask for the sum of green apples and red apples or the difference between them? (the difference) How do you know? (the question asks for "how many more") What operation should I use to find the difference—addition or subtraction? (subtraction) Have students estimate the differences in all four exercises above. (a) about 10 more green than red, b) about 30 more yellow than red, c) about 20 more Granny Smith than Golden Delicious, d) about 10 more Red Delicious than Gala)

Sometimes estimation does not make sense. SAY: Jay is four years old. ASK: If you round 4 to the nearest ten, what do you get? (0) Does it make any sense to say that someone is about 0 years old? (no) Explain that estimation is a good thing in general, but there are some situations where it makes no sense.

Extensions

1. a) Jayden and Tasha are trying to estimate 46 + 25.
   
   Jayden says: I round both numbers to the nearest ten, so the answer is about 50 + 30 = 80.
   
   Tasha says: 25 is as close to 20 as it is to 30. If I round 46 up, I will get a bigger number than the actual answer. If I round 25 up too, the answer will be even bigger. I will round 25 down instead, so the answer should be close to 50 + 20 = 70.
   
   Who made a better estimate? Check against the actual answer.

b) Use both methods, Jayden’s and Tasha’s, to estimate the answer to each of the following additions. Which method gives a better estimate?

   29 + 55   87 + 35   15 + 98

   c) Marko thinks it could be better to round 25 down when estimating 22 + 25. Which method gives a better estimate—Jayden’s or Marko’s?

Answers

a) 46 + 25 = 71, so Tasha made a better estimate
b) 29 + 55: Jayden’s method: 30 + 60 = 90,
   Tasha’s method: 30 + 50 = 80, actual answer 84;
   87 + 35: Jayden’s method: 90 + 40 = 130,
   Tasha’s method: 90 + 30 = 120, actual answer 122;
   15 + 98: Jayden’s method: 20 + 100 = 120,
   Tasha’s method: 10 + 100 = 110, actual answer 113;
   so Tasha’s method gives a better estimate
c) Jayden’s method: 20 + 30 = 50, Marko’s method: 20 + 20 = 40, actual answer 47, so Jayden’s method gives a better estimate
2. 39 + 37 is about 40 + 40 = 80. Without adding the numbers, say if the actual answer is more than 80 or less than 80. Explain how you know.

Answer: 39 is less than 40, and 37 is less than 40. So 39 + 37 is less than 40 + 40 = 80.

3. Use estimation to find incorrect equations.

   a) 45 + 27 = 62    b) 48 + 28 = 76
   c) 52 – 13 = 29    d) 78 – 29 = 49

   Answers
   a) 50 + 30 = 80, so 62 is too low—the equation cannot be correct
   b) 50 + 30 = 80, about the same
   c) 50 – 10 = 40, so 29 is too low—the equation cannot be correct
   d) 80 – 30 = 50, about the same
Goals

Students will use a referent of 10 or 100 to estimate the number of items in a group.
Students will estimate a given quantity by choosing among three possible choices.
Students will select and justify a referent for making an estimate for a given quantity.

PRIOR KNOWLEDGE REQUIRED

Can count by tens to 100
Can count by hundreds to 1000

MATERIALS

counters
overhead projector
large numbers of various objects, such as beans, raisins, craft sticks, coins, and hole-punch confetti
plastic containers

BLM Number of Faces in a Crowd (p. R-29)
bag of rice and small cup (see Extension 1)
large basket and a tomato (see Extension 2)

Mental math minute. Students work in pairs. Each student takes a small handful of counters and counts them. Then the pair uses the two numbers they got to create two subtraction sentences. For example, if Student A selects 23 counters and Student B selects 38 counters, they create the subtraction sentences $61 - 38 = 23$ and $61 - 23 = 38$. Ask all students to stand up. One at a time, have each pair say their subtraction sentences and then sit down. Repeat until all students have had a chance to participate.

Review counting by tens and hundreds. Write on the board:

10, 20, 30, __, __, __, __, __, __,

ASK: What number are we skip counting by? (10) Ask a volunteer to write the missing numbers on the board. (40, 50, 60, 70, 80, 90, 100) ASK: How can we use counting by ones to help us count by tens? (to count by tens, put a zero after each number you would say when counting by ones)

Write on the board:

100, 200, 300, __, __, __, __, __, __,

ASK: What number are we skip counting by now? (100) Ask a volunteer to write the missing numbers on the board. (400, 500, 600, 700, 800, 900, 1000) ASK: How can we use counting by ones to help us count by hundreds? (to count by hundreds, put two zeros after each number you
Introduce the term “referent.” Using an overhead projector, project a large number (that is less than 100) of counters on the board. For example, you could display 78 counters. Place the counters so they are close together but not overlapping. SAY: I want an estimate of how many counters there are. We could take a guess, but I want a more reasonable estimate. We saw that counting by tens is easy. We can use this idea to help us find an estimate for the total number of counters. Count out 10 counters and move them to the side, bunched closely together but not overlapping. SAY: We are going to use these 10 counters to help us estimate the total number of counters. We call this group of counters the referent. Label the referent on the board, as shown below:

Have a volunteer come to the overhead projector and move aside a group of counters that is about the same size as the referent. Tell students that it doesn’t matter if the groups have exactly 10 counters. Repeat with more volunteers until all of the counters have been moved into groups of about 10. (see picture below)

SAY: Let’s pretend each group has exactly 10 counters. Let’s count by tens to estimate the number of counters. Point to each group, starting with the referent, and count aloud: 10, 20, 30, 40, 50, 60, 70, 80. SAY: We estimate there are about 80 counters altogether. ASK: What addition sentence can we use to estimate the total number? \(10 + 10 + 10 + 10 + 10 + 10 + 10 = 80\) Write the addition sentence on the board. ASK: What multiplication sentence can we use to represent this addition? \(8 \times 10 = 80\) Write the multiplication sentence on the board.

Remind students that each of the groups they put aside do not have to have exactly 10 counters. As long as there are about 10 counters in each group, the estimate will be reasonable.
Estimating the number of objects from a picture. Display the picture of 53 squares shown in the margin. SAY: If we can’t physically touch or move the objects, we have to imagine that we are putting aside groups of objects the same size as the referent. Count out 10 squares in the picture and colour them in. Circle the group and SAY: This is going to be our referent. (see sample picture below)

Have a volunteer circle another group of squares similar in size to the referent. Repeat with more volunteers until all the squares are in a circled group. (see sample picture below)

Count aloud by tens with the class as you point to each group: 10, 20, 30, 40, 50. SAY: So there are about 50 squares in the picture. Write on the board:

\[ \text{_____} \times 10 = \text{_____} \]

ASK: What multiplication sentence can we use to count the number of objects? Have a volunteer fill in the blanks on the board. \((5 \times 10 = 50)\)

**ACTIVITY 1**

1. This activity will require preparation ahead of time. Gather several plastic containers of different sizes and groups of different objects to place in each container. For example, one container might contain beans, another raisins, another craft sticks. Each container should contain fewer than 100 objects. Record the exact number of objects placed in each container for your reference. Ask each student to imagine counting out a referent group of 10 objects and estimate how many objects are in each container. Have them record their estimates for each container in their notebooks.

When all students have had an opportunity to estimate the number of objects in each container, assign a group of two or three students to each container to count the actual number of objects. Have students compare their estimates with the actual number of objects.
Using a referent of 100 instead of 10. SAY: When there are many more than 100 objects in a group, it is better to use a referent of 100 objects instead of 10. Draw about 30 dots on the board, as shown below:

Ask 10 volunteers to come to the board at the same time. Ask each volunteer to think of a number between 10 and 50 and write it on scrap paper. Then ask each volunteer to add that number of dots to the picture on the board. (see sample picture below)

ASK: If I want to estimate the total number of dots, why is it better to use a referent of 100 instead of 10? (the picture has a lot more than 100 dots)
SAY: I am going to circle about 100 dots. Point to small groups of about 10 and count aloud by tens until you get to 100. Circle the group of 100. SAY: There are not exactly 100 dots in the group, but there are about 100 dots. (see sample picture below)

ASK: What name do we give to this group of about 100 dots? (referent)
Have volunteers use the circle as an example and circle other groups of 100.
Tell volunteers that their groups should not overlap very much or they would be counting some dots twice. (see sample picture below)

ASK: How many groups did we circle altogether? (4) Write on the board:

\[ \_ \times 100 = \_ \]

Have a volunteer fill in the multiplication sentence to estimate the total number of dots. (\(4 \times 100 = 400\))

**ACTIVITY 2**

2. Repeat Activity 1 but with a much larger collection of objects. Each container should have fewer than 1000 objects. Have students write down their estimates in their notebooks. Then assign two or three students to each container. Rather than count the exact number in each container, ask students to physically separate the objects into groups of about 100 to come up with an estimate. Have students compare their estimates with the actual number of objects.

**Choosing between sizes of the referent.** SAY: Remember if there are fewer than 100 objects, it is better to use a referent of size 10. If there are many more than 100 objects, it is better to use a referent of size 100. Write on the board:

- pebbles on a beach
- books on a bookshelf
- kernels of popcorn in a bowl
- baseballs in an equipment bag
- dandelions on a front lawn
- coins in a cup

ASK: If we want an estimate of the number of objects in each situation, what referent should we use? Allow time for discussion of each situation. The answers may vary. Students should explain their choices. For example, for dandelions on a front lawn, if the lawn is very large and the dandelions are very close together, perhaps a referent of 100 is better.
SAY: At some outdoor concerts, chairs are not provided. People either stand or sit wherever they can. We can take a picture from the stage or a hill and estimate the size of the audience using an appropriate referent. If the concert is in a small park, there may be fewer than 100 people, and a referent of size 10 is appropriate. If the concert is in a large stadium, there may be many more than 100 people, and a referent of size 100 is appropriate.

Exercise: Choose a referent for the faces on BLM Number of Faces in a Crowd. Estimate the size of the crowd.

Sample answer: 900

Extensions

1. Show students a small bag of rice. Tell students that you want to estimate the number of grains of rice in the bag. Hold up a small cup and suggest that students use it as a referent.
   a) Have students estimate the number of grains of rice in the cup.
   b) Have students estimate the number of cups in the bag.
   c) Have students use their answers to parts a) and b) to estimate the number of grains of rice in the bag.

Sample answers: a) 500 grains of rice in a cup, b) 10 cups in the bag, c) 5000 grains of rice in the bag

2. Show students a tomato and a large empty basket. Have students estimate the number of tomatoes that can fit in the basket. ASK: How will the size of the tomato affect your answer?

Sample answer: If the tomato is smaller, more of them will fit in the basket.
Goals
Students will learn the place values in four-digit numbers.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to hundreds
Can read and write numbers to 1000

MATERIALS
BLM Place Value Cards (p. R-30)

Mental math minute. Students work in pairs. The first student creates a random multiplication sentence using single digit numbers. The second student creates two division sentences from the multiplication sentence. For example, if Student A says $9 \times 8 = 72$, Student B says $72 \div 9 = 8$ and $72 \div 8 = 9$. Ask all students to stand up. One at a time, ask pairs to say their multiplication sentence and division sentences. Then each student in the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Review place value to hundreds. Give each student four cards from BLM Place Value Cards (each student should get a ones card, a tens card, a hundreds card, and a thousands card). Remind students that the words on the cards are place values. Write “37” on the board and ASK: What is the place value of the 7? Students should raise the card with the correct place value (ones) simultaneously so that you can assess everyone at the same time. Repeat with the 3. (tens) Then have students identify the place value of each digit in 237 and 409.

Introduce thousands. Write “4156” on the board. ASK: How is this number different from most of the numbers we have worked with this year? (it has 4 digits) Underline the 4 and tell students that its place value is thousands. Remind students that there are 10 hundreds in a thousand.

Identifying place values. Write “4856” on the board. Affix the appropriate card from BLM Place Value Cards under each digit: thousands, hundreds, tens, ones. Repeat with 4237, asking students to tell you the correct place value as you point to each digit. Students can hold up the card with the correct place value as you name the digit. For example, ASK: What is the place value of 3 in 4237? (tens)

Write the numbers in the exercises below one at a time on the board and point to different digits, out of order. Students should raise the card with the correct place value as you point to the digit.

Exercises: Identify the place value of each digit.

a) 6198   b) 8739   c) 9065   d) 8402   e) 3760

VOCABULARY
digit
four-digit number
hundreds
ones
place value
tens
thousands
three-digit number
value
Selected answer: a) 6: thousands, 1: hundreds, 9: tens, 8: ones

Continue until students can identify place value correctly and confidently. Include examples where you ask for the place value of the digit 0. Point out that, although the digit 0 always has a value of 0, its place value changes with position the same as any other digit.

Explain that to read a four-digit number you say the number of thousands first, and then you read the three-digit number that is left. For example, 4856 is "four thousand, eight hundred fifty-six." Point out that just as we say "eight hundred" and not "eight hundreds," we also say "four thousand" and not "four thousands." Have students read the numbers in the exercises above.

Introduce the place value chart. Draw a place value chart on the board. Have students write the digits from the number 231 in the correct columns, as shown below:

```
<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>231</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
```

Add more numbers to the first column of the place value chart. Include numbers with one, two, three, and four digits, and have volunteers come to the board to write the digits in the correct columns. Include numbers with zeros in all places except the thousands.

Writing four-digit numbers. Explain that when numbers have more than three digits, it is hard to read them and hard to keep track of the place values. Read the numbers in the exercise below out loud and have students write them down as numerals.

Exercises: Write the number as a numeral.

a) seven thousand, one hundred sixty-eight
b) four thousand, two hundred thirty-nine
c) five thousand, four hundred sixty-five
d) one thousand, seven hundred thirty-two

Answers: a) 7168, b) 4239, c) 5465, d) 1732

ASK: How would you write the number four thousand? Have students think, then explain that since there are only thousands, and no additional hundreds, tens, or ones, the number is written with a 4 in the thousands place and zeros in all three other places. Have a volunteer write "4000" on the board. Then have different volunteers write the numbers eight thousand (8000) and one thousand (1000).

Repeat with the numbers eight thousand four hundred thirty (8430), five thousand one (5001), three thousand five hundred (3500), and four thousand eighty (4080).
The value of a digit. Remind students that there are 10 ones in a ten and 10 tens in a hundred. This means that each place value to the left is 10 times larger than the one to the right (and each place value to the right is 10 times smaller than the one to the left). For example, in the number 333, the first 3 stands for 300, the second 3 stands for 30, and the third 3 is just 3. Ask students how much each digit in 456 is worth. (4 stands for 400, 5 stands for 50, 6 stands for 6)

Write “2836” on the board. ASK: What is the place value of the digit 2? (thousands) SAY: The 2 is in the thousands place, so it stands for 2000. We also say that the value of 2 in 2836 is 2000. ASK: What does the digit 8 stand for? (800) What is the value of 3? (30) And the 6? (6) Repeat with 8902, 7431, and 9006.

Exercises: What does the digit 2 stand for in the number?

a) 3297  b) 2985  c) 7892  d) 2095
e) 8132  f) 9002  g) 3020  h) 8200

Answers: a) 200, b) 2000, c) 2, d) 2000, e) 2, f) 2, g) 20, h) 200

Extensions

1. Round four-digit numbers to all possible places.

Example: 1382
To the nearest thousand: 1000
To the nearest hundred: 1400
To the nearest ten: 1380

<table>
<thead>
<tr>
<th>Number</th>
<th>Nearest 1000</th>
<th>Nearest 100</th>
<th>Nearest 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 4562</td>
<td>5000</td>
<td>4600</td>
<td>4560</td>
</tr>
<tr>
<td>b) 6081</td>
<td>6000</td>
<td>6100</td>
<td>6080</td>
</tr>
<tr>
<td>c) 2345</td>
<td>2000</td>
<td>2300</td>
<td>2350</td>
</tr>
</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th>Number</th>
<th>Nearest 1000</th>
<th>Nearest 100</th>
<th>Nearest 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 4562</td>
<td>5000</td>
<td>4600</td>
<td>4560</td>
</tr>
<tr>
<td>b) 6081</td>
<td>6000</td>
<td>6100</td>
<td>6080</td>
</tr>
<tr>
<td>c) 2345</td>
<td>2000</td>
<td>2300</td>
<td>2350</td>
</tr>
</tbody>
</table>

2. Teach students the ancient Egyptian system for writing numerals to help them appreciate the utility of place value. Explain that Egyptians used the following symbols in their system:

1 =  | (stroke)  10 =  (arch)
100 =  | (coiled rope)  1000 =  (lotus leaf)
Write the following numbers on the board using both our system and the Egyptian system:

- 234
- 848
- 423

Invite students to study the numbers for a moment, then ASK: What is different about the Egyptian system for writing numbers? (You have to show the number of ones, tens, and so on individually—if you have 7 ones, you have to draw 7 strokes. In our system, a single digit, 7, tells you how many ones there are.)

Ask students to write a few numbers the Egyptian way and to translate those Egyptian numbers into our numerals. Have students write a number that is really long the Egyptian way (example: 798). ASK: How is our system more convenient? Why is it helpful to have a place value system, with the ones, tens, and so on always in the same place? (the number is much shorter) Tell students that the Babylonians, who lived at the same time as the ancient Egyptians, were the first people to use place value in their number system. Students might want to invent their own number system using the Egyptian system as a model.

3. Have students identify and write numbers given specific criteria and constraints. Examples:
   a) Write a number between 30 and 40.
   b) Write an even number with a 6 in the tens place.
   c) Write a number that ends with a zero.
   d) Write a two-digit number.
   e) Write an odd number greater than 70.
   f) Write a number with a tens digit one more than its ones digit.

   Sample answers: a) 39, b) 36, c) 80, d) 56, e) 75, f) 65

4. a) Which number has both digits the same: 34, 47, 88, 90?
   b) Write a number between 50 and 60 with both digits the same.

   Answers: a) 88, b) 55

5. a) Find the sum of the digits in each of these numbers: 37, 48, 531, 225, 444, 372.
   b) Write a three-digit number where the digits are the same and the sum of the digits is 15.

   Answers: a) 10, 12, 9, 9, 12, 12; b) 555
6. a) Which of these numbers has a tens digit one less than its ones digit: 34, 47, 88, 90?

b) Write a two-digit number with a tens digit eight less than its ones digit.

c) Write a three-digit number where all three digits are odd.

d) Write a three-digit number where the ones digit is equal to the sum of the hundreds digit and the tens digit.

Sample answers: d) 123, 347

Answers: a) 34; b) 19; c) 111, 333, 555, 777, or 999
Goals
Students will add three-digit numbers that require regrouping hundreds as a thousand.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to thousands
Can add three-digit numbers that require regrouping ones and tens

Mental math minute. Students work in pairs. The first student creates a random division sentence. The second student creates two multiplication sentences from the division sentence. For example, if Student A says $72 \div 9 = 8$, Student B will say $9 \times 8 = 72$ and $8 \times 9 = 72$. Ask all students to stand up. One at a time, ask pairs to say their division sentence and multiplication sentences. Then each student in the pair does three jumping jacks and sits down. Repeat until all students have had a chance to participate.

Review adding three-digit numbers using the standard algorithm.
Remind students that when they add large numbers it is convenient to write the numbers one above the other, aligning place values. Have a volunteer show on the board how to write the addition vertically for $235 + 341$. Remind students that they can add ones, tens, and hundreds separately. Have a volunteer perform the addition. (see answer below)

```
  235
+ 341

  576
```

Exercises: Write the numbers one above the other, then add.
a) $249 + 450$  
b) $502 + 374$  
c) $803 + 25$

Answers: a) 699, b) 876, c) 828

Review regrouping ones as tens and tens as hundreds. Write on the board:

```
  247
+ 225

  576
```

Start adding and have students tell you what to do. Remind them that when they add 5 and 7, they get 12, which is not a digit because it is greater than 9. They should tell you to replace the 10 ones in 12 with a ten, write the remaining 2 in the sum, and write the additional ten above the tens.
column in the addition, as shown below. Remind students that this is called regrouping.

\[
\begin{array}{ccc}
  & 1 \\
+ & 2 & 4 & 7 \\
\hline
+ & 2 & 2 & 5 \\
\hline
& 2 \\
\end{array}
\]

Finish the addition. (472) Repeat with 241 + 392, regrouping 10 tens as 1 hundred.

**Exercises:** Write the numbers one above the other, then add. You will need to regroup.

a) \(249 + 435\)  
b) \(504 + 376\)

c) \(673 + 152\)  
d) \(328 + 590\)

**Answers:** a) 684, b) 880, c) 825, d) 918

Remind students that sometimes they need to regroup twice. Have students tell you what to do to add 345 + 386. (731)

**Exercises:** Add. You will need to regroup twice.

a) \(275 + 455\)  
b) \(583 + 178\)  
c) \(898 + 25\)

**Answers:** a) 730, b) 761, c) 923

**Regrouping hundreds as thousands.** Remind students that 10 hundreds make a thousand. Write on the board:

\[
\begin{array}{ccc}
832 & \text{hundreds} & + \text{ tens} & + \text{ ones} \\
+ & 451 & \text{hundreds} & + \text{ tens} & + \text{ one} \\
\hline
\end{array}
\]

ASK: What does the digit 8 stand for in 832, or what is the value of 8? (800) Fill in the first blank in the first line. Repeat with the other digits in both numbers. Students can signal the number that should be written in each blank by holding up the appropriate number of fingers.

SAY: We add ones, tens, and hundreds separately. Add another row of “\(\_\) hundreds + \(\_\) tens + \(\_\) ones” and have students help you fill it in. The completed picture should look like this:

\[
\begin{array}{ccc}
832 & \text{8 hundreds} & + \text{3 tens} & + \text{2 ones} \\
+ & 451 & \text{4 hundreds} & + \text{5 tens} & + \text{1 one} \\
\hline
\text{12 hundreds} & + \text{8 tens} & + \text{3 ones} \\
\end{array}
\]

SAY: 10 hundreds is 1 thousand, so 12 hundreds is 1 thousand and 2 hundreds. This means that the sum has 1 thousand, 2 hundreds, 8 tens, and 3 ones. Have a volunteer write the number using only numerals in the vertical addition on the left. (1283)
Write another vertical addition, such as $743 + 825$, and have students tell you what to do to finish the calculation. Then have students complete the exercises below individually. Explain that they will need to do more regrouping starting from part e).

**Exercises:** Add. You will need to regroup.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>$849 + 430$</td>
<td>b</td>
</tr>
<tr>
<td>d</td>
<td>$828 + 540$</td>
<td>e</td>
</tr>
<tr>
<td>g</td>
<td>$873 + 869$</td>
<td>h</td>
</tr>
</tbody>
</table>

**Answers:** a) 1279, b) 1078, c) 1085, d) 1368, e) 1375, f) 1038, g) 1742, h) 1400

**ACTIVITY**

Player 1 writes a three-digit number. Player 2 writes another three-digit number so that the addition of the two will require regrouping in all three digits. Both players estimate the sum and then find the actual sum. They compare answers to check each other’s work before switching roles.

**Extensions**

1. If you taught students the ancient Egyptian system for writing numerals (see Extension 2 in Lesson NS3-74), you could ask them to show adding and regrouping using the same system. Examples:

   **Adding:**
   
   $\begin{array}{c}
   \underline{\phantom{000}} \\
   \phantom{000} + \underline{\phantom{000}} \\
   \underline{\phantom{000}} \\
   \end{array}$
   
   $= \underline{\phantom{000}}$
   
   $= \underline{\phantom{000}}$

   **Regrouping:**
   
   $\begin{array}{c}
   \underline{\phantom{000}} \\
   \phantom{000} \\
   \underline{\phantom{000}} \\
   \end{array}$
   
   $\rightarrow \begin{array}{c}
   \underline{\phantom{000}} \\
   \phantom{000} \\
   \underline{\phantom{000}} \\
   \end{array}$

2. Have students add more than 2 three-digit numbers at a time. For example, $427 + 382 + 975 + 211 = 1995$.

3. Fill in the missing numbers to make each addition correct.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$3\phantom{9}2$</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 361, b) $295 + 531 = 826$
Number Cards (I)

10  20  30
40  50  60
70  80  90
11  16  0
Number Cards (2)

27 35 3
44 100 52
59 61 67
73 99 84
Number of Faces in a Crowd
Place Value Cards

Ones

Tens

Hundreds

Thousands
Goals

Students will use number lines with only key points labelled to solve problems.

Prior Knowledge Required

Can count by 10s
Knows that numbers on number lines get bigger from left to right or bottom to top
Can compare numbers up to 1000
Can skip count by 2s, 3s, 4s, 5s, and 10s
Can multiply and divide up to $9 \times 9$
Can add and subtract within 1000
Knows that expressions within brackets are evaluated first (for Problem Bank 6)

Materials

BLM Feet (p. R-39, optional)
a small paper clip, a strip of opaque or coloured paper, and 3 strips from BLM Hidden Number Lines Game (p. R-40) for each pair of students

Review how grouping tens makes counting easy. Draw 10 pairs of feet on the board or copy BLM Feet five times, alternating the colours as shown below:

Point to the group of feet marked by the bracket and ASK: How many toes are in this group? (30) Verify by counting by 1s, but emphasize the multiples of 10 as you get to them. ASK: Do I need to count by 1s, or is there an easier way? (count by 5s or by 10s instead) Have students count by 10s to find how many toes are in other groups.

Estimating numbers on number lines. Underneath the row of toes, draw a number line with markings from 0 to 100 in multiples of 10, as shown below:
SAY: Not all the numbers from 0 to 100 are marked on this line, because it would be hard to write them in this small space. ASK: Where would you estimate 81 is on the number line? (right after the 80) PROMPTS: Which two tens is it between? (80 and 90) Is the number closer to 80 or to 90? (80) Is it a lot closer or a little closer to 80? (a lot closer) SAY: 81 is right next to 80. Have a volunteer mark an estimate for 81 on the number line. Repeat with similar numbers—for example, 41, 92, 22, 59, 38, 99—giving prompts as necessary. Repeat with numbers that are halfway between two tens—for example, 35, 75, 45, 95—and then with numbers that are only a little closer to one ten than the other—for example, 74, 86, 37, 13.

**ACTIVITY**

Provide each pair of students with the first third of BLM Hidden Number Lines Game and a small paper clip. Ask students to fold along the dotted line and put something opaque, such as a coloured strip of paper, between the two sides of paper so they cannot see through to the other number line. Player 2 holds the number line vertically so that Player 1 sees the side with only the endpoints 60 and 80 marked and Player 2 sees the side with all the numbers marked. Player 2 calls out a number from 60 to 80. Player 1 estimates where the number should be and puts a small paper clip on the number line. Player 2 tells Player 1 how to adjust the estimate by saying “too high” or “too low.” Play continues until Player 1’s estimate is accurate enough so that Player 2 can see the number inside the paper clip. Player 2 then shows Player 1 their side of the number line to verify the position of the paper clip. Players switch roles and repeat.

When students are comfortable estimating on the 60 to 80 number line, give each pair of students the number line from 0 to 100 from the BLM and repeat the activity. Finally, give each pair of students the number line from 0 to 1000 from the BLM and repeat. For the number line from 0 to 1000, Player 2 may need to estimate to check if they can see Player 1’s number inside the paper clip.

**Review counting by 10s starting from any number.** SAY: Mary has four stamps. Draw on the board:

```
  □ □ □ □
```

SAY: Stamps come in strips of 10. ASK: If Mary adds a strip of 10 stamps, how many stamps will she have now? (14) Continue drawing on the board:

```
  □ □ □ □ □ □ □ □ □ □
```

Verify that there are 14 stamps by counting them. Repeat with another strip of 10 stamps, again counting all the stamps, but this time emphasize the numbers 4, 14, and 24 as you count. As a class, count forwards by 10s from 4 to 94 (4, 14, 24, and so on), and then count backwards by 10s from 94 to 4 (94, 84, 74, and so on).
Exercises: Skip count by 10s, forwards or backwards.

a) from 3 to 43  b) from 37 to 87  c) from 59 to 99

d) from 74 to 44  e) from 69 to 39  f) from 81 to 31

Answers: a) 3, 13, 23, 33, 43; b) 37, 47, 57, 67, 77, 87; c) 59, 69, 79, 89, 99;
d) 74, 64, 54, 44; e) 69, 59, 49, 39; f) 81, 71, 61, 51, 41, 31

Remind students that when you skip count forwards by 10s, the ones digit stays the same and the number of tens gets bigger. ASK: If you were to make 134 using tens and ones blocks, and you didn’t have any hundreds blocks, how many tens blocks would you need? (13) SAY: You would need 10 tens blocks for the 1 hundred and 3 tens blocks for the 3 tens, so you would need 13 altogether, plus you would have 4 ones blocks. Draw on the board:

\[
\begin{align*}
10 \text{ tens} + 3 \text{ tens} &= 13 \text{ tens} \\
\end{align*}
\]

SAY: You can get the number of tens in any number by covering up the ones digit. Write on the board:

134  184

Cover up the 4 in 134 to show 13, the number of tens in 134. Ask a volunteer to cover up the 4 in 184 to show the number of tens in 184. (18) SAY: Let’s count forwards by 10s starting from 184. ASK: What comes next? (194) Write “194” on the board beside 184. ASK: What comes next? (204) Write “204” on the board. SAY: 184 has 18 tens, 194 has 19 tens, and 204 has 20 tens. You can keep going by keeping the ones digit the same and adding one more ten each time. Continue writing on the board:

184, 194, 204, 214, 224, …

Exercises

1. Skip count by 10s.

a) from 64 to 114  b) from 79 to 139  c) from 455 to 515

d) from 356 to 316  e) from 819 to 769  f) from 405 to 385

Answers: a) 64, 74, 84, 94, 104, 114; b) 79, 89, 99, 109, 119, 129, 139;
c) 455, 465, 475, 485, 495, 505, 515; d) 356, 346, 336, 326, 316;
e) 819, 809, 799, 789, 779, 769; f) 405, 395, 385

2. What number does Alexa end at?

a) She skip counts forwards by 10s three times starting at 43.
b) She skip counts forwards by 10s four times starting at 645.
c) She skip counts forwards by 10s five times starting at 87.

d) She skip counts forwards by 10s four times starting at 296.

**Bonus:** She skip counts backwards by 10s three times from 126.

**Answers:** a) 73, b) 685, c) 137, d) 336, Bonus: 96

SAY: Simon skip counts forwards by 10s three times and finishes at 56. I want to know what number he started at. Draw on the board:

```
   |   |   |  56
```

ASK: How do I know where to write 56? (Simon finishes at 56) How do I know to put three markings on the number line before 56? (Simon skip counts 3 times) Point to the third mark and ASK: Does he start here? (no) What number goes here? (46) Write “46” under the mark. Repeat for the next mark (no, 36), then the mark after that (yes, 26). SAY: So, Simon starts at 26.

**Exercises:** What number does Simon start at?

a) He skip counts forwards by 10s four times and ends at 83.

b) He skip counts forwards by 10s five times and ends at 72.

c) He skip counts forwards by 3s four times and ends at 30.

d) He skip counts forwards by 5s six times and ends at 42.

**Answers:** a) 43, b) 22, c) 18, d) 12

**Determining the number of jumps.** Write on the board:

```
Sharon skip counts by 5s from 35 to 50.
How many times does she count?
```

Read the problem aloud. SAY: You can solve this problem by drawing a number line that starts at 35, then skip counting by 5s. Draw on the board:

```
   |   |   |  35
```

Have a volunteer mark the numbers by counting by 5s from 35. (40, 45, 50, 55, 60) SAY: The problem says Sharon skip counts to 50. Circle “50” on the number line. ASK: How many times does she skip count? (3 times)

**Exercises:** How many times does Jayden skip count?

a) He skip counts by 6s from 30 to 72.

b) He skip counts by 3s from 16 to 40.

**Bonus:** He skip counts by 8s from 133 to 157.

**Answers:** a) 7 times, b) 8 times, Bonus: 3 times
Another way to determine the number of jumps. Refer back to the skip counting from 35 to 50 on the board. SAY: Sharon skip counts from 35 to 50. ASK: How much more than 35 is 50? (15) How did you get that? (subtracted 50 – 35) SAY: Sharon skip counts by 5s each time. ASK: How many jumps of five does she need to make 15? (3) SAY: Three jumps of five make 15. Write on the board:

\[
\_ \times 5 = 15
\]

SAY: You’re really trying to find the missing number in a multiplication sentence. ASK: What could you do instead of multiplication? (division) Write on the board:

\[
15 \div 5 = 
\]

SAY: These two questions have the same answer because a number multiplied by 5 to equal 15 is the same number that is 15 divided by 5.

Exercises: Write division sentences for your answers to the previous exercises.

Answers: a) 42 ÷ 6 = 7, b) 24 ÷ 3 = 8, Bonus: 24 ÷ 8 = 3

Determining the interval skip counted. Write on the board:

Anne skip counts by the same number three times.
She starts at 7 and ends at 25. What number does she skip count by?

Read the problem aloud. SAY: You can solve this problem by drawing a number line. Draw on the board:

\[
\begin{array}{c}
7 \\
\hline
25
\end{array}
\]

SAY: Anne starts at seven, so I wrote seven at the beginning of the number line. ASK: How many jumps should I draw? (3) How do you know? (Anne skip counts 3 times) Draw three tick marks on the number line, and ASK: What number does she end at? (25) Write “25” at the end, as shown below:

\[
\begin{array}{c}
7 \\
8 \\
9 \\
25
\end{array}
\]

SAY: We need to figure out what Anne skip counts by. Have students make predictions. ASK: There are a lot of possibilities—how can we try them in an organized way? (try the numbers in order, starting at 1) Continue writing on the board:

\[
\begin{array}{c}
7 \\
8 \\
9 \\
25
\end{array}
\]

ASK: Do I reach 25 by counting by 1s? (no) SAY: 25 isn’t 1 more than 9, so this doesn’t work. Have volunteers try skip counting on the number line by 2s (7, 9, 11, 25; doesn’t work), 3s (7, 10, 13, 25; doesn’t work), 4s (7, 11, 15, 25; doesn’t work), 5s (7, 12, 17, 25; doesn’t work), and 6s (7, 13, 19, ...
SAY: So, Anne skip counts by 6s three times, from 7 to 25. Leave the number line on the board for later use.

**Exercises:** Jax skip counts by the same number three times. What number does he skip count by?

a) from 13 to 25  
   b) from 456 to 462  
   c) from 394 to 409  
   d) back from 84 to 75

**Answers:** a) 4, b) 2, c) 5, d) 3

**Another way to solve the same problem.** Refer back to the example on the board and ASK: How many jumps does Anne have to do? (3) How do you know? (the problem says so) How far do we have to jump in total? (18) PROMPT: How far is it from 7 to 25? (18) How do you know? (25 is 18 more than 7, 25 − 7 is 18) SAY: So Anne needs to go 18 units in 3 equal jumps. ASK: How big should each jump be? (6) How did you get that? (18 ÷ 3 = 6) Ask a volunteer to check that the jumps are 6 units each. (See completed number line below.)

![Completed number line](image)

SAY: If you know that 3 jumps make 18, then each jump is 6.

**Exercises**

1. Write the answer to the previous exercises using a division sentence.

   **Answers:** a) 12 ÷ 3 = 4, b) 6 ÷ 3 = 2, c) 15 ÷ 3 = 5, d) 9 ÷ 3 = 3

2. Complete the skip counting on the number line.

   a)
   
   b)
   
   c)
   
   d)
   
   e)
Problem Bank

1. Show your answer to the question on a number line.

a) Zack has 140 dollars. He needs to buy a 200 dollar winter jacket before school starts in 3 weeks. How much money does Zack need to save each week for the 3 weeks?

b) Kim has 73 dollars. She wants to buy a 97 dollar gift for her mother’s birthday in 4 weeks. How much money does Kim need to save each week for the 4 weeks?

c) Anna has 68 dollars. She wants to buy a sweater for 100 dollars. She saves 4 dollars each week. How many weeks does Anna need to save money for?

d) A test has 7 questions. The test starts at 9:00 and ends at 9:30. Fred takes 2 minutes to read all the questions before starting to answer them. How much time does Fred have to answer each question?

e) On a worksheet full of simple questions, Lily takes 5 seconds to do each question. How many questions can she do in 1 minute? Remember: 1 minute = 60 seconds.

f) A television show ends at 9:25. The television channel then plays the same commercial 4 times. The next show starts at 9:33. How long is the commercial?

g) Ansel starts his homework at 7:15. He works on each subject’s homework for 10 minutes. He finishes at 5 minutes to 8. How many subjects did Ansel work on?

h) Gym class starts at 2:05. The class will do several 10-minute activities, one after the other. If the class ends at 2:45, how many activities can the class do?

i) Science class starts at 3:05. The class will do 5 different activities. Each activity is done for the same amount of time. If class ends at 3:55, how long does the class do each activity?

Selected solution: f) The commercial is 2 minutes long.

Answers: a) 20 dollars, b) 6 dollars, c) 8 weeks, d) 4 minutes, e) 12, g) 4, h) 4, i) 10 minutes
2. Which of the parts from Problem 1 require finding the number of jumps? Which parts require finding the length of each jump?

Answers: the number of jumps: c), e), g), h); the length of each jump: a), b), d), f), i)

3. A test has eight questions. Sally takes 5 minutes each for the first three questions and then 8 minutes each for the next five questions. Can Sally finish the test in 1 hour (60 minutes)?

Answer: yes, she finishes in 55 minutes

4. Draw a number line to answer the question.

a) \(3 \times 2 = \) __

b) \(4 \times 3 = \) __

c) \(2 \times 7 = \) __

Answers: a) 6, b) 12, c) 14

5. Write a multiplication equation for the skip counting.

a) \[
\begin{array}{cccccccc}
0 & 4 & 8 & 12 & 16 & 20 \\
\end{array}
\]

b) \[
\begin{array}{cccccccc}
0 & 3 & 6 & 9 & 12 & 15 & 18 \\
\end{array}
\]

Bonus: \[
\begin{array}{cccccccc}
0 & 40 & 80 & 120 & 160 & 200 & 240 \\
\end{array}
\]

Answers: a) \(5 \times 4 = 20\), b) \(6 \times 3 = 18\), Bonus: \(6 \times 40 = 240\)

6. Start a number line at 20 and use skip counting to find the answer.

a) \(20 + (5 \times 3) = \) __

b) \(20 + (3 \times 4) = \) __

Bonus: \(20 + (2 \times 10) = \) __

Selected solution: a) \[
\begin{array}{cccccccc}
20 & 23 & 26 & 29 & 32 & 35 \\
\end{array}
\]

So, \(20 + (5 \times 3) = 35\)

Answers: b) 32, Bonus: 40

Bonus: Write an addition and multiplication equation for the number line.

a) \[
\begin{array}{cccccccc}
14 & 17 & 20 & 23 & 26 & 29 \\
\end{array}
\]

b) \[
\begin{array}{cccccccc}
14 & 16 & 18 & 20 & 22 & 24 & 26 \\
\end{array}
\]

c) \[
\begin{array}{cccccccc}
14 & 44 & 74 & 104 & 134 & 164 & 194 \\
\end{array}
\]

Answers: a) \(14 + (5 \times 3) = 29\), b) \(14 + (6 \times 2) = 26\),
c) \(14 + (6 \times 30) = 194\)
Feet
Hidden Number Lines Game

Fold along the dotted line

R-40 Blackline Master — Problem-Solving Lessons — Teacher Resource for Grade 3
PS3-8 Creating Number Lines

Teach this lesson after:
Unit 15

VOCABULARY
empty number line
number line

Goals
Students will use an empty number line to add and subtract two- and three-digit numbers.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract within 1000
Can draw number lines for skip counting
Can add, subtract, and multiply one-digit numbers using a number line

MATERIALS
BLM Feet (p. R-48, optional)

Counting tens, then ones. Draw 10 pairs of feet on the board or copy BLM Feet five times, alternating the colours as shown below:

Point to the group of feet marked by the bracket and ASK: How many toes are in this group? (43) Have a volunteer fill in the blank. Point out that students can count by 10s first and then count by 1s. By counting as many 10s as possible, students are using the smallest number of jumps as they can. Continue with more numbers that are slightly more than multiples of 10, such as 31, 52, 61, 73, and 33.

Exercises: Count by 10s and then by 1s. Use the smallest number of jumps.

a) from 0 to 32
b) from 0 to 61
c) from 0 to 54

Answers: a) 0, 10, 20, 30, 31, 32; b) 0, 10, 20, 30, 40, 50, 60, 61; c) 0, 10, 20, 30, 40, 50, 51, 52, 53, 54

Sketching number lines. Show students how they can keep track of their counting by sketching a number line. Draw on the board:

SAY: This isn't a precise number line with, for example, numbers the same distance apart. On this line, 10 and 20 are closer together than 20 and 30, even though they are both 10 apart. But as a sketch of a number line, it is good enough to help us keep track of how we counted.
Exercises

1. Sketch a number line to show counting by 10s and 1s.
   a) from 0 to 41  
   b) from 0 to 62  
   c) from 0 to 71
   
   **Answers:**
   a) 0, 10, 20, 30, 40, 41; 
   b) 0, 10, 20, 30, 40, 50, 60, 61, 62; 
   c) 0, 10, 20, 30, 40, 50, 60, 70, 71

2. Sketch a number line to show counting by 100s, 10s, and 1s.
   a) from 0 to 341  
   b) from 0 to 152  
   c) from 0 to 333
   
   **Selected solution:**
   a) 0 100 200 300 310 320 330 340 341
   
   **Answers:**
   b) 0, 100, 110, 120, 130, 140, 150, 151, 152; 
   c) 0, 100, 200, 300, 310, 320, 330, 331, 332, 333

**Counting tens and then counting ones backwards.** Write on the board:

Count from 0 to 39 by jumping. Use only jumps of 10 or 1.
Use only 5 jumps.

Say: Here is a problem I want you to solve. Read the problem aloud. Give
students a few moments to think about the problem, then say: You are
allowed to use backward jumps. After allowing students another minute to
work, have a volunteer show the answer on the board, as shown below:

![Number line sketch]

Exercises

1. Use only 6 jumps. Use jumps of 1 or 10, forward or backward.
   a) Count from 0 to 51.  
   b) Count from 0 to 38.
   c) Count from 0 to 42.  
   d) Count from 0 to 49.
   e) Count from 0 to 27.  
   f) Count from 0 to 33.
   
   **Answers:**
   a) 0, 10, 20, 30, 40, 51; 
   b) 0, 10, 20, 30, 40, 39, 38; 
   c) 0, 10, 20, 30, 40, 41, 42; 
   d) 0, 10, 20, 30, 40, 50, 49; 
   e) 0, 10, 20, 30, 29, 28, 27; 
   f) 0, 10, 20, 30, 31, 32, 33

2. Use only 6 jumps. Use jumps of 1, 10, or 100, forward or backward.
   a) Count from 0 to 312.  
   b) Count from 0 to 114.
   c) Count from 0 to 499.  
   d) Count from 0 to 391.
   e) Count from 0 to 409.
   
   **Bonus:** Use jumps of 1, 10, 100, or 1000 to count from 0 to 2909.
**Answers:** a) 0, 100, 200, 300, 310, 311, 312; b) 0, 100, 110, 111, 112, 113, 114; c) 0, 100, 200, 300, 400, 499; d) 0, 100, 200, 300, 400, 390, 391; e) 0, 100, 200, 300, 400, 410, 409; Bonus: 0, 1000, 2000, 3000, 2900, 2910, 2909

**Review adding on a number line.** Remind students that they can add using a number line. Draw on the board:

![Number line with 2 + 3](image)

ASK: How would you show adding two plus three on the number line? (start at 2 and move 3 places right) Demonstrate doing so.

**Using a sketched number line, count on by 10s and 1s to add.** Draw on the board:

![Number line with 38 + 54](image)

SAY: It is a lot of work to draw a number line with 54 markings, so I'm just going to show the markings that I need to solve the problem. I mark 38 because I know that's my starting number. Instead of counting by 1s, I want to count by a bigger number so that I have less work to do. ASK: What's an easy number to skip count by? (10) If students say 2 or 5, PROMPT: Those are easy numbers to count by, but it would still be a lot of work. What's an easy number to skip count by that would be less work? (10) SAY: I'm going to start by adding 50. ASK: If I want to add 50 to 38, how many times do I have to add 10? (5) Show this on the board, as shown below:

![Number line with jumps](image)

SAY: I drew five markings because I want to add 10 five times. I'm not worried about making the jumps equally spaced, because I don't need them to be equally spaced to answer the question. Have a volunteer label the tick marks on the number line, as shown below:

![Number line with labeled jumps](image)

ASK: What is 38 + 50? (88) Now that I know 38 + 50, what do I need to do to get 38 + 54? (add 4 more) Add four jumps of 1 on the number line.
As shown below:

![Number line diagram]

ASK: What is $38 + 54$? Write “92” in the blank in the equation.

SAY: Using a number line like this, where you have to keep the numbers in order but you don’t have to make them the same distance apart, is sometimes called using an empty number line because it starts with no numbers; we could also call the number line a blank number line.

Exercises: Show how to add, starting with an empty number line.

a) $23 + 40$

b) $31 + 63$

c) $85 + 46$

**Bonus**

d) $128 + 30$

e) $128 + 314$

**Answers:**
a) 23, 33, 43, 53, 63; b) 31, 41, 51, 61, 71, 81, 91, 92, 93, 94; c) 85, 95, 105, 115, 125, 126, 127, 128, 129, 130, 131; Bonus: d) 128, 138, 148, 158; e) 128, 228, 328, 428, 438, 439, 440, 441, 442

Using backward jumps to add on a number line. Write on the board:

$27 + 39$

SAY: I want to solve this problem by sketching a number line. Draw a blank horizontal line on the board, and have a volunteer write the number to start with, as shown below:

![Number line diagram]

ASK: How can you add 39 using only five jumps of tens and ones? Have a volunteer show it on the sketched number line, as shown below:

![Number line diagram]

Circle “66.” SAY: Adding 39 is the same as adding 40 and then subtracting one. Write on the board:

$27 + 40 = 67$, so $27 + 39 = 66$

Leave this statement on the board.

**Exercises:** Add the second number using the smallest number of jumps as you can. Use only jumps of 10 or 1.

a) $16 + 48$

b) $38 + 51$

c) $17 + 48$
**Answers:** a) 16, 26, 36, 46, 56, 66, 65, 64; b) 38, 48, 58, 68, 78, 88, 89; c) 17, 27, 37, 47, 57, 67, 66, 65

**SAY:** You can add even faster by doing all the tens together and all the ones together. Instead of adding 10 four times, you can just add 40. Draw on the board:

![Diagram]

**Exercises:** Add 40.

a) 37 \(+ 40\)  b) 16 \(+ 40\)  c) 28 \(+ 40\)  d) 54 \(+ 40\)

**Answers:** a) 77, b) 56, c) 68, d) 94

Point to “27 \(+ 40\) = 67, so 27 \(+ 39\) = 66” on the board and **SAY:** If you can add 40, then you can add 39, because 39 is 1 less than 40. Draw on the board:

![Diagram]

**NOTE:** In the exercises below, some students might see how to answer parts a) and c) immediately by using 27 \(+ 39\) = 66.

**Exercises:** Add 39.

a) 37 \(+ 39\)  b) 16 \(+ 39\)  c) 28 \(+ 39\)  d) 54 \(+ 39\)

**Answers:** a) 76, b) 55, c) 67, d) 93

Write on the board:

\[ 54 + 28 \]

**ASK:** What number is close to 28 but easier to add? (30) **SAY:** So start by adding 30 on a number line. Draw on the board:

![Diagram]

**ASK:** What is 54 \(+ 30\)? (84) Write the sum on the number line. **ASK:** We added 30 instead of 28, so how do we adjust the answer? (subtract 2) Have a volunteer show that on the number line, as shown below.

![Diagram]
**Exercises:** Use a small number of easy jumps, forward or backward. Show your jumps by sketching a number line.

a) 57 + 24  
b) 36 + 48  
**Bonus:** 143 + 288

**Sample answers:** a) 57, 77, 81; b) 36, 86, 84; Bonus: 143, 443, 433, 431

**Counting on from numbers other than zero to subtract.** Write on the board:

\[ 61 - 39 \]

**SAY:** You can use jumps on a number line to see how far apart 39 and 61 are. That tells you the answer to 61 - 39. Draw on the board:

\[
\begin{align*}
39 & \quad 49 & \quad 59 & \quad 60 & \quad 61 \\
\hline
\end{align*}
\]

**SAY:** There are two jumps of 10 and two jumps of 1, so that’s 22 altogether. Write “= 22” on the board to the right of 61 - 39.

**Exercises:** Use jumps of 10 and 1 to subtract.

a) Count from 28 to 54 to find 54 - 28.

b) Count from 65 to 93 to find 93 - 65.

**Bonus:** Use jumps of 100, 10, and 1 to subtract 892 - 456.

**Selected solution**

b)

\[
\begin{align*}
65 & \quad 75 & \quad 85 & \quad 86 & \quad 87 & \quad 88 & \quad 89 & \quad 90 & \quad 91 & \quad 92 & \quad 93 \\
\hline
\end{align*}
\]

There are two jumps of 10 and eight jumps of 1, so 93 - 65 = 28.

**Answers:** a) 26, Bonus: 436

**Problem Bank**

1. Subtract 83 - 46 two ways using an empty number line. Make sure you get the same answer both ways.

a) Count up from 46 until you reach 83.

b) Count back 46 from 83.

**Answers:** a) 46, 56, 66, 76, 80, 83, so 83 - 46 is 10 + 10 + 10 + 4 + 3 = 37; b) 83, 73, 63, 53, 43, 42, 41, 40, 39, 38, 37, so 83 - 46 = 37

2. Without drawing a number line, picture the number line in your head to do these questions. Hint: Do the additions in order.

a) 26 + 50, 26 + 52, 26 + 49

b) 148 + 70, 148 + 270, 148 + 273, 148 + 268

c) 76 - 40, 76 - 39, 76 - 43
d) $95 - 35, 93 - 35, 193 - 35$

**Answers:** a) 76, 78, 75; b) 218, 418, 421, 416; c) 36, 37, 33; d) 60, 58, 158

3. Use counting forward to subtract, then use counting back to subtract. Which way was easier? Explain why it was easier.

a) $132 - 118$  
   b) $76 - 4$  
   c) $283 - 30$  
   d) $283 - 260$

e) $100 - 98$  
f) $100 - 12$  
g) $1000 - 372$

**Answers:** a) 14, counting forward; b) 72, counting back; c) 253, counting back; d) 23, counting forward; e) 2, counting forward; f) 88, counting back; g) 628, counting forward is slightly easier; in a) to f), numbers that are close together are easier by counting forward and when the number being subtracted is small, it is easier to count back, and in g) they are both about equal amounts of work, but some students may still find counting forward easier

4. Dory adds $57 + 28$ by adding the tens first.

![Diagram]

a) Add $57 + 28$ by adding 3 ones first, then adding the rest. Make sure you get the same answer.

b) Add $57 + 28$ by adding all the ones first, then all the tens. Make sure you get the same answer.

c) Add $57 + 28$ by adding 30, then subtracting 2. Make sure you get the same answer.

5. Add $28 + 66$ using a number line in two different ways.

**Sample answers:** 28, 88, 89, 90, 91, 92, 93, 94; or 28, 98, 94; or 28, 88, 94; or 28, 30, 90, 94; or 28, 30, 94

6. Add $544 + 388$ using a number line in as many ways as you can.

**Sample answers:** 544, 944, 934, 932; or 544, 550, 600, 900, 904, 924, 930, 932

7. Add $42 + 51$ two ways. Make sure you get the same answer both ways.

a) Start at 42 on a number line and add 51.

b) Start at 51 on a number line and add 42.

**Answers:** a) 93, b) 93
8. Add $43 + 59$ two ways. Make sure you get the same answer both ways.
   a) Start at 43 on a number line and add 59.
   b) Start at 59 on a number line and add 43.
   **Answers:** a) 102, b) 102

9. Add $743 + 8$ two ways.
   a) Start at 743 on a number line and add 8.
   b) Start at 8 on a number line and add 743.
   **Answers:** a) 751, b) 751

10. Which way of adding in Problem Bank 9 was faster? Why?
    **Answer:** a) was faster because I only had to add 10 and subtract 2; there were more steps to add 743.
Feet
Introduction

In this unit, students will learn how to count money, including coins and bills. They will learn the relationships among coins, including the penny. Although the penny is no longer used in Canada, the value of a penny as one cent is still important as the base value of all the other coins. One cent is also important in the calculation of taxes and still appears in some prices. Students will count money amounts involving coins starting with the largest coin value, with and without pictures. They will learn to represent money amounts using the least number of coins. They will learn how to make change by counting up to the nearest multiple of 10 and then counting up to the nearest dollar.

Students will learn how to count money, including loonies, toonies, and 5-dollar bills. Students using the British Columbia curriculum will also be introduced to 10-, 20-, 50-, and 100-dollar bills and count money involving those denominations. British Columbia and Ontario students will represent the amount of money in a collection of bills and coins using the standard dollars and cents notation.

Meeting Your Curriculum

Alberta—Lessons NS3-76 to NS3-83 are required. All other lessons are optional.

British Columbia—Lessons NS3-76 to NS3-85 and NS3-87 are required. Extensions 3 and 4 in Lesson NS3-85 are also required. Lesson NS3-86 is recommended. All other lessons are optional.

Manitoba—Lessons NS3-76 to NS3-83 are required. All other lessons are optional.

Ontario—Lessons NS3-76 to NS3-84, NS3-86, and NS3-89 are required. NS3-88 is dedicated to rounding to the nearest 5. This skill is essential for making cash purchases and giving change, since the penny is no longer in use in Canada. We recommend this lesson for Ontario students as they are required to engage in simulated purchases within $10. All other lessons are optional.

Materials. Many lessons in the unit require play money for the students. The play money should include pennies, nickels, dimes, quarters, loonies, toonies, 5-dollar bills, 10-dollar bills, 20-dollar bills, and 50-dollar bills.

An overhead projector is needed for several lessons. When asked to “display” a BLM, you will need to make a transparency and display it using the overhead projector.

In addition to the BLMs provided at the end of this unit, the following Generic BLM, found in section V, is used in Unit 16:

BLM Hundreds Chart (p. V-9)
Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

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<td>87</td>
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NS3-76 Counting by 5s and 25s
Pages 119–120

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
skip count

Goals
Students will count by 5s and 25s and recognize patterns in the ones digits.

PRIOR KNOWLEDGE REQUIRED
Can skip count by any number
Can add two 2-digit numbers

MATERIALS
pair of dice
transparency of BLM Hundreds Chart (p. V-9)
overhead projector
calculators

Mental math minute. Use a pair of dice to generate addition questions. Label one die with a red dot and the other with a blue dot. Choose two students to roll the dice. Create a two-digit number by first reading the blue die and then the red one. For example, if you roll a 2 with the blue die and a 6 with the red die, the number is 26. Ask the class to write down the two-digit number in their notebooks. Ask two other students to roll the pair of dice to create another two-digit number. Ask the class to write down the second two-digit number. Ask the class to add the 2 two-digit numbers. Repeat until all students have had a chance to roll the dice. Check answers as you go.

Skip counting by 5s using a numbers chart. Display BLM Hundreds Chart. Demonstrate how to count by 5s, starting at 1. Circle the number 1. Point to the numbers 2, 3, 4, 5, and 6 while counting out loud: 1, 2, 3, 4, 5. Circle the number 6, where you finished. Repeat by starting at 6 and pointing to the numbers 7, 8, 9, 10, and 11 while counting out loud: 1, 2, 3, 4, 5. You may need to remind students that you don’t start counting 1, 2, 3, 4, 5 until you point to the new number. Circle the number 11. ASK: What numbers will we circle if we continue counting by 5s? (16, 21, 26, 31, 36) Circle the numbers and then write all the circled numbers on the board:

1, 6, 11, 16, 21, 26, 31, 36

Ask volunteers to underline the ones digit in each number on the board. (1, 6, 1, 6, 1, 6, 1, 6) ASK: What pattern do you see in the ones digits? (1, 6, then repeat)

Erase the circles you displayed on the hundreds chart. Repeat counting by 5s seven times, but this time start by circling the number 2. The numbers you end up circling should be 2, 7, 12, 17, 22, 27, 32, and 37. Write the numbers on the board. Ask for volunteers to underline the ones digit in each number on the board. (2, 7, 2, 7, 2, 7, 2, 7) ASK: What pattern do you see in the ones digits this time? (2, 7, then repeat)
Repeat the instructions, starting at the numbers 3, 4, and 5. The numbers you end up circling and the patterns you see in the ones digits are shown below:

3, 8, 13, 18, 23, 28, 33, 38  Pattern: 3, 8, then repeat
4, 9, 14, 19, 24, 29, 34, 39  Pattern: 4, 9, then repeat
5, 10, 15, 20, 25, 30, 35, 40  Pattern: 5, 0, then repeat

Exercises

1. Count by 5s starting at the given number. Write the first seven numbers of the pattern.
   a) 6  b) 7  c) 8  d) 9  e) 10
   Answers: a) 6, 11, 16, 21, 26, 31, 36; b) 7, 12, 17, 22, 27, 32, 37;
           c) 8, 13, 18, 23, 28, 33, 38;  d) 9, 14, 19, 24, 29, 34, 39; e) 10, 15, 20,
           25, 30, 35, 40

2. What pattern is in the ones digits of the answers to Exercise 1?
   Answers: a) 6, 1, then repeat; b) 7, 2, then repeat; c) 8, 3, then repeat;
           d) 9, 4, then repeat; e) 0, 5, then repeat

3. a) If you count by 5s starting at 1, the pattern in the ones digits is 1, 6, then repeat. If you count by 5s starting at 6, the pattern in the ones digits is 6, 1, then repeat. How are the patterns different?
   Answers: a) instead of 1, 6, the pattern is 6, 1, which is the reverse;
   b) instead of 4, 9, the pattern is 9, 4, which is the reverse

Bonus: If you want to write out the multiples of 5, where should you start counting and how should you count?
   Answer: start at 0 and count by 5s

Skip counting by 5s without a chart. SAY: The patterns we found continue, even if we start at higher numbers. Write on the board:

   61, 66, 71, 76

SAY: Here we are also counting by 5s. While using your hand to count, SAY: 62, 63, 64, 65, 66. Point to the pattern you wrote on the board and ask for volunteers to write each of the next four numbers in the pattern. (81, 86, 91, 96)

Exercises: Skip count by 5s.
   a) 43, 48, __, __, __, __  b) 60, __, __, __, __
   c) 72, __, __, __, __  Bonus: 121, 126, __, __, __, __

Answers: a) 53, 58, 63, 68; b) 65, 70, 75, 80; c) 77, 82, 87, 92;
Bonus: 131, 136, 141, 146
Skip counting by 5s backwards using a numbers chart. Display BLM Hundreds Chart and erase any previous marks. Demonstrate how to count backwards by 5s, starting at 27. Circle the number 27. Starting at 27, point to the numbers 26, 25, 24, 23, and 22 while counting out loud: 1, 2, 3, 4, 5. Circle the number 22, where you finished. ASK: What numbers will we circle if we continue counting by 5s backwards? (17, 12, 7, 2) Write the circled numbers on the board:

27, 22, 17, 12, 7, 2

Have volunteers underline the ones digits in the answers on the board. (7, 2, 17, 12, 7, 2) ASK: What is the pattern in the ones digits? (7, 2, then repeat)

Exercises: Skip count backwards by 5s.

a) 39, 34, __, __, __, __

b) 26, 21, __, __, __, __

c) 63, 58, __, __, __, __

d) 90, 85, __, __, __, __

Bonus: 174, 169, __, __, __, __

Answers: a) 29, 24, 19, 14; b) 16, 11, 6, 1; c) 53, 48, 43, 38; d) 80, 75, 70, 65; Bonus: 164, 159, 154, 149

Skip counting by 25s. Draw on the board:

<table>
<thead>
<tr>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask for volunteers to continue counting by 5s to fill the chart. ASK: What is $5 \times 5$? (25) SAY: If you circle every fifth number in the chart, you are actually counting by 25s, because $5 \times 5 = 25$. Starting at 5, count 1, 2, 3, 4, 5 until you reach 25. Circle the 25. Ask for volunteers to continue the pattern of circling every fifth number. The numbers circled should be 25, 50, 75, and 100.

SAY: We can repeat the same process on a number line. Draw on the board a number line with increments of 5, from 25 to 150. ASK: What number are we counting by for this number line? (5) Circle the 25. Have volunteers circle every fifth number on the number line. The numbers circled should be 25, 50, 75, 100, 125, and 150. ASK: If you start counting by 25s at 200, what would be the next four numbers you circle? (225, 250, 275, 300) Write on the board:

225, 250, 275, 300, 325, 350, 375

Have a volunteer underline the tens and ones digits on the board. ASK: What is the pattern in the tens and ones digits when we count by 25s? (25, 50, 75, 00, then repeat) The board should look like this:

225, 250, 275, 300, 325, 350, 375

ASK: If we look at only the ones digits, what is the pattern? (5, 0, then repeat)
Exercises

1. Complete the pattern by skip counting by 25s.
   a) 300, 325, __, __, __, __
   b) 650, __, __, __, __
   Answers: a) 350, 375, 400, 425; b) 675, 700, 725, 750

2. Complete the pattern by skip counting backwards by 25s.
   a) 200, 175, __, __, __, __
   b) 450, __, __, __, __
   Answers: a) 150, 125, 100, 75; b) 425, 400, 375, 350

Skip counting by other numbers. Ask the class to skip count by 100s out loud five times, starting at 100. (100, 200, 300, 400, 500) ASK: What is the third number in the list? (300) What is the fourth number in the list? (400) What do you think will be the eighth number in the list if you keep counting? (800)

Ask the class to skip count by 200s out loud five times, starting at 200. (200, 400, 600, 800, 1000) ASK: What is the third number in the list? (600) What is the fourth number in the list? (800) What do you think will be the sixth number in the list? (1200)

Exercises: Skip count by 300 starting at 300.
   a) What are the first four numbers in the list?
   b) What will be the sixth number in the list?
   Answers: a) 300, 600, 900, 1200; b) 1800

ACTIVITY

Many calculators (both scientific calculators and those that are not) have the ability to repeatedly add or subtract the same number. For example, if you start at 10 and repeatedly add 5, you get this number pattern: 10, 15, 20, 25, 30, .... On some calculators, you can accomplish this by typing 10 + 5, and then repeatedly pressing the = sign.

On other calculators, you may have to type in 10, then press the + sign twice so that a “K” shows up on the screen. The “K” signals that it will repeat the operation. Then you skip count by repeatedly pressing the = sign. NOTE: Depending on the calculator, exact methods for skip counting may vary.

Use a calculator to skip count.
   a) 25, 30, 35, __, __, __, __
   b) 30, 40, 50, __, __, __, __
   c) 25, 50, 75, __, __, __, __
   d) 200, 300, 400, __, __, __, __
   Answers: a) 40, 45, 50, 55; b) 60, 70, 80, 90; c) 100, 125, 150, 175; d) 500, 600, 700
Extensions

1. Tess skip counts starting at an unknown number. The third, fourth, and fifth numbers in her list are 8, 11, and 14.
   a) What does Tess skip count by?
   b) What number does she start at?
   Solutions: a) $11 - 8 = 3$; b) $8 - 3 = 5$, $5 - 2 = 2$, she starts counting at 2

2. Jane also skip counts starting at an unknown number. The second and fourth numbers in her list are 7 and 11.
   a) What does Jane skip count by?
   b) What number does she start at?
   Solutions: a) $(11 - 7) \div 2 = 2$, b) $7 - 2 = 5$

3. Jack starts skip counting at 2. The sixth number in his list is 17. What number does Jack skip count by?
   Answer: 3 (2, 5, 8, 11, 14, 17)

4. Fred skip counts backwards starting at 33. The fifth number in his list is 9. What number does Fred skip count backwards by?
   Answer: 6 (33, 27, 21, 15, 9)

5. Skip count by 25s starting at 1.
   a) What are the first four numbers in the list?
   b) What will be the eighth number in the list?
   Answers: a) 1, 26, 51, 76; b) 176
**Goals**

Students will learn the value of a penny, nickel, dime, quarter, and loonie.

Students will count on by 5s, 10s, or 25s to find the total value of a collection of coins.

**PRIOR KNOWLEDGE REQUIRED**

Can skip count by 5s, 10s, and 25s
Can subtract two-digit numbers

**MATERIALS**

pair of dice
play money of different denominations
transparency of **BLM Money** (p. S-81)
overhead projector
tokens, a die, and **BLM Fake Money Game** (p. S-82)

**Mental math minute.** Use a pair of dice to generate subtraction questions. Label one die with a red dot and the other with a blue dot. Choose two students to roll the dice. Create a two-digit number by first reading the blue die and then the red one. For example, if you roll a 2 with the blue die and a 6 with the red die, the number is 26. Ask the class to write down the two-digit number in their notebooks. Ask two other students to roll the pair of dice to create another two-digit number. Ask the class to write down the second two-digit number so that the smaller number can be subtracted from the larger number. Ask the class to do the subtraction, and check answers as you go. Repeat until all students have had a chance to roll the dice.

**Comparing value.** Tell students that *value* refers to how much something is worth or how much money is needed to buy it. The greater the value of something, the more money is needed to buy it. Write on the board:

- carton of milk
- car
- house
- truck
- package of paper
- gold necklace

Ask volunteers to circle the item of greater value in each line. (car, house, gold necklace) SAY: We can compare the value of items by saying how much money they cost. We can use *dollars* to compare the values of different items. Have a discussion about how many dollars might be needed to buy the items in each line. (sample answers: carton of milk: $1, car: $20 000; house: $300 000, truck: $40 000; package of paper: $2, gold necklace: $200)
Write “one dollar,” “1 dollar,” and “$1” on the board. SAY: We can write “one dollar” in words (point to “one dollar”), numbers and words (point to “1 dollar”), or symbols and numbers (point to “$1”). Write “$” on the board. SAY: This is the symbol we use for the word “dollar,” which we call a dollar sign.

SAY: In Canada, we have both paper money and coins. Paper money in Canada used to be made of paper. Now it is made from plastic, but we still call it paper money. Coins are round and made of metal. Coins have two sides that some people call heads and tails. The side with the picture of a head on it is called heads. The other side is called tails. This side often has “Canada” written on it. Most countries print the name of their country on their money.

SAY: In Canada, the value of a dollar is broken up into 100 parts. Each part is called a cent. So, one dollar has the same value as 100 cents.

**Identifying coins.** Display BLM Money. Point to the word “Canada” on each coin. SAY: These pictures show the tails side of the coins.

Distribute play coins of every denomination (except the toonie) to each student. SAY: The coin that has a value of one cent is called a penny. Ask students to find a penny in their play money. ASK: How can we tell by looking what the value of a penny is? (it says “1 cent” on it) Ask students to look at the other coins and find all the different values of coins. (5 cents, 10 cents, 25 cents, 1 dollar) Draw on the board:

<table>
<thead>
<tr>
<th>Coin</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>penny</td>
<td>1 cent 1¢</td>
</tr>
<tr>
<td>nickel</td>
<td>5 cents 5¢</td>
</tr>
<tr>
<td>dime</td>
<td>10 cents 10¢</td>
</tr>
<tr>
<td>quarter</td>
<td>25 cents 25¢</td>
</tr>
<tr>
<td>loonie</td>
<td>1 dollar 100¢</td>
</tr>
</tbody>
</table>

SAY: Each coin is given a name. The coin that has a value of 5 cents is called a nickel, the coin that has a value of 10 cents is called a dime, the coin that has a value of 25 cents is called a quarter, and the coin that has a value of 100 cents or 1 dollar is called a loonie. Ask students to find each of the coins in their play money and check that the value matches the chart.

SAY: Like a dollar, we can write the value of a coin using numbers and words, or numbers and symbols. Write “¢” on the board. SAY: This is the symbol we use for the word “cent,” which we call a cent sign.

Write on the board:

- ship penny
- caribou nickel
- leaves dime
- loon quarter
- beaver loonie
SAY: On the tails side of the coins, there is often a picture. The pictures may change, but often the pictures on Canadian coins include a ship, a caribou, leaves, a loon, and a beaver. SAY: A **loon** is a bird that looks like a duck. Point out that the loonie was named after the picture of the loon on the coin. Ask for volunteers to draw lines to match the picture with the correct coin. (penny: leaves, nickel: beaver, dime: ship, quarter: caribou, loonie: loon)

**The value of a coin is not related to its size.** Display BLM Money. Point out to students that the table lists the coins in order by value. The coin with the smallest value is the penny. The coin with the next highest value is the nickel.

Ask students to place their play-money coins in order by size, with the smallest size first. After they have had time to place them in order, check that their order is correct. (smallest size to largest size coin: dime, penny, nickel, quarter, loonie) Ask students to compare this order of coins to the order on BLM Money. ASK: Which coin seems to be out of order when you place them in order by size? (the dime) What is the value of a dime? (10¢) What is the value of a nickel? (5¢) Which coin is bigger in size? (the nickel) SAY: So, even though a nickel is bigger than a dime, it actually has a smaller value. This may be true for coins of other countries as well. Just because a coin is larger does not mean it has a greater value. ASK: Where can we check the value of the coin? (written on the coin)

**ACTIVITY 1**

1. Have students work in pairs.
   a) Student A says the name of a picture on the tails side of a coin. Student B finds the coin with the picture. After several turns, students switch roles.
   b) Student A says the value of a coin (for example, 10 cents). Student B names the coin with that value (for example, dime). After several turns, students switch roles.
   c) Student A names a coin (for example, nickel). Student B says the value of the coin (for example, 5¢). After several turns, students switch roles.

**Explaining why we still need pennies.** SAY: In Canada, the government found it cost too much to make penny coins, so we stopped using them. But, a dollar is still broken into 100 parts called **pennies**. We still need to be able to calculate prices using pennies, because when taxes are calculated, they often involve pennies. Sometimes prices still have pennies in them.

**Finding the number of coins that are needed to make a different coin.** ASK: What is the value of a nickel? (5¢) What is the value of a penny? (1¢) How many pennies do I need to have the same value as one nickel? (5) Draw on the board:

\[
5\text{¢} \quad = \quad 1\text{¢} \quad 1\text{¢} \quad 1\text{¢} \quad 1\text{¢} \quad 1\text{¢}
\]
ASK: What is the value of a dime? (10¢) What is the value of a penny? (1¢)
How many pennies do I need to have the same value as one dime? (10)
Draw on the board:

\[
\begin{align*}
10¢ &= 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ \\
1¢ &+ 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢
\end{align*}
\]

ASK: What is the value of a nickel? (5¢) What is the value of a dime? (10¢)
How many nickels do I need to have the same value as a dime? (2) Ask students to explain using addition. (5¢ + 5¢ = 10¢) Draw on the board:

\[
\begin{align*}
10¢ &= 5¢ + 5¢ \\
5¢ &+ 5¢
\end{align*}
\]

ASK: What is the value of a nickel? (5¢) What is the value of a quarter? (25¢)
How many nickels do I need to have the same value as a quarter? (5) Ask students to explain using addition. (5¢ + 5¢ + 5¢ + 5¢ + 5¢ = 25¢) Ask students to explain using multiplication. (5 × 5¢ = 25¢) Draw on the board:

\[
\begin{align*}
25¢ &= 5¢ + 5¢ + 5¢ + 5¢ + 5¢ \\
5¢ &+ 5¢ + 5¢ + 5¢ + 5¢
\end{align*}
\]

ASK: What is the value of a quarter? (25¢) What is the value of a loonie in cents? (100¢) How many quarters do I need to have the same value as a loonie? (4) Ask students to explain using addition. (25¢ + 25¢ + 25¢ + 25¢ = 100¢) Draw on the board:

\[
\begin{align*}
100¢ &= 25¢ + 25¢ + 25¢ + 25¢ \\
25¢ &+ 25¢ + 25¢ + 25¢
\end{align*}
\]

SAY: We said before that a dollar can be broken into 100 parts, called cents.
ASK: So, how many pennies do we need to have the same value as a loonie? (100) Ask students to explain using addition. (1¢ + 1¢ + 1¢ + ... + 1¢ = 100¢) Ask students to explain using multiplication. (100 × 1¢ = 100¢)

**Finding the total value of a group of nickels.** Write on the board:

5, 10, 15, ____, ____, ____, ____, ____

ASK: What number are we counting on by? (5) Ask for volunteers to continue the pattern on the board. (20, 25, 30, 35, 40) SAY: We can count on by 5s to count nickels. Draw on the board:

\[
\begin{align*}
5¢ &+ 5¢ + 5¢ + 5¢ + 5¢ + 5¢ \\
5¢ &+ 5¢ + 5¢ + 5¢ + 5¢ + 5¢
\end{align*}
\]

SAY: If we are trying to find the total value of nickels, we count on by 5s. Point to the first nickel and SAY: 5. Point to the next nickel and ASK: If we count on by 5s, what is the next number? (10) SAY: Let’s continue counting on by 5s. Point to the nickels as you continue counting out loud: 15, 20, 25, 30.

Point back to the first nickel. SAY: Suppose we had already counted to 30 before we got to the first nickel. ASK: What is the next number after 30 if
We count by 5s? (35) Say: Let’s continue counting on by 5s. Point to the nickels as you continue counting out loud: 40, 45, 50, 55, 60.

Exercises: Count on by nickels from the given amount.

a) 

```
| 5¢ | 5¢ | 5¢ | 5¢ |
```

25, ___ , ___ , ___ , ___

b) 

```
| 5¢ | 5¢ | 5¢ |
```

50, ___, ___, ___, ___

Answers: a) 30, 35, 40, 45; b) 55, 60, 65, 70

Finding the total value of a group of dimes. Write on the board:

```
10, 20, 30, ___, ___, ___, ___, ___
```

Ask: What number are we counting on by? (10) Ask for volunteers to continue the pattern on the board. (40, 50, 60, 70, 80) Say: We can count on by 10s to count dimes. Draw on the board:

```
| 10¢ | 10¢ | 10¢ | 10¢ | 10¢ | 10¢ |
```

Say: If we are trying to find the total value of dimes, we count on by 10s. Point to the first dime and say: 10. Point to the next dime and ask: If we count on by 10s, what is the next number? (20) Say: Let’s continue counting on by 10s. Point to the dimes as you continue counting out loud: 30, 40, 50, 60.

Point to the first dime again. Say: Suppose we had already counted to 30 before we got to the first dime. Ask: What is the next number after 30 if we count by 10s? (40) Say: Let’s continue counting on by 10s. Point to the dimes as you continue counting out loud: 50, 60, 70, 80, 90.

Exercises: Count on by dimes from the given amount.

a) 

```
| 10¢ | 10¢ | 10¢ | 10¢ |
```

20, ___, ___, ___, ___

b) 

```
| 5¢ | 5¢ | 5¢ |
```

35, ___, ___, ___, ___

Answers: a) 30, 40, 50, 60; b) 45, 55, 65, 75

Finding the total value of a group of quarters. Write on the board:

```
25, 50, 75, ___, ___, ___, ___, ___
```

Ask: What number are we counting on by? (25) Ask for volunteers to continue the pattern on the board. (100, 125, 150, 175, 200) Say: We can count on by 25s to count quarters.
Draw on the board:

25¢ 25¢ 25¢ 25¢ 25¢

SAY: If we are trying to find the total value of quarters, we count on by 25s. Point to the first quarter and SAY: 25. Point to the next quarter and ASK: If we count on by 25s, what is the next number? (50) SAY: Let’s continue counting on by 25s. Point to the quarters as you continue counting out loud: 75, 100, 125.

Point back to the first quarter. SAY: Suppose we had already counted to 125 before we got to the first quarter. ASK: What is the next number after 125 if we count by 25s? (150) SAY: Let’s continue counting on by 25s. Point to the quarters as you continue counting out loud: 175, 200, 225, 250.

Exercises: Count on by quarters from the given amount.

a) 25¢ 25¢ 25¢ 25¢

75, ___, ___, ___, ___

b) 150, ___, ___, ___, ___

Answers: a) 100, 125, 150, 175; b) 175, 200, 225, 250

Counting on by coins of two different values. Write on the board:

5, 10, ___, ___ | ___, ___, ___, ___

Count by 5s. Continue counting by 1s.

SAY: This time, we are going to count on by two different numbers. The vertical line divides the question into two parts. In the first part, we are counting by 5s. Ask for volunteers to continue counting by 5s, starting from the 10. (15, 20) SAY: Now, we stop counting by 5s and start counting by 1s. ASK: What number comes after 20 if we count by 1s? (21) If we continue counting by 1s, what are the next three numbers after 21? (22, 23, 24) Write the answers on the board.

SAY: We can use this pattern to count a series of nickels and pennies. Draw on the board:

nickel nickel nickel nickel penny penny penny penny

Explain that you wrote “nickel” and “penny” because you would have difficulty drawing the real coins.

SAY: Here, we have two different coins: nickels and pennies. ASK: What should we count by if we are counting nickels? (5s) What should we count by if we are counting pennies? (1s) Point to the first nickel. SAY: This nickel is
worth 5 cents. ASK: If we count on by 5s, what are the next three numbers? (10, 15, 20) SAY: So, the total value of the nickels is 20 cents. Now, we find a penny. ASK: What should we count on by to count the pennies? (1s) If count on from 20 by 1s for the next four pennies, what are the next four numbers? (21, 22, 23, 24) SAY: So, the total value of the coins is 24 cents.

Exercises

1. Count on by 5s and then by 1s.
   a) 10, 15, __, __
   b) 25, 30, __, __

   **Answers:** a) 20, 25, 26, 27, 28; b) 35, 40, 41, 42, 43

2. Count on by the first coin value given and then by the second coin value.
   a)
   b)

   **Answers:** a) 5, 10, 11, 12, 13, 14; b) 5, 10, 15, 20, 21, 22

Write on the board:

10, 20, __, __

Count by 10s. Continue counting by 5s.

SAY: This question involves two parts: counting by 10s, then counting by 5s. ASK: If we count on by 10s, what are the next two numbers after 20? (30, 40) Write the answers on the board. SAY: Now, we reach the line so we continue counting by 5s instead of by 10s. ASK: If we continue counting by 5s after 40, what are the next four numbers? (45, 50, 55, 60) Write the answers on the board. Draw on the board:

```
dime
dime
dime
nickel
nickel
nickel
```

SAY: We want to find the total value of these dimes and nickels. ASK: What numbers should we count on by? (10, then 5) How do you know? (the value of a dime is 10¢, the value of a nickel is 5¢) How do you count on by 10s for the dimes? (10, 20, 30) How do you count on from 30 by 5s for the nickels? (35, 40, 45) SAY: So, the total value of the coins is 45¢.

**Exercises:** Count on by the value of the first coin, and then by the value of the second coin.

a) 

b)
Answers: a) 10, 20, 21, 22, 23, 24; b) 10, 20, 25, 30, 35, 40

Counting on using quarters and other coins. Write on the board:

25, 50, __, __, __, __, __, __

Have volunteers write the next numbers by counting on by 25s. (75, 100, 125, 150, 175, 200) Draw on the board:

\[ \text{quarter} \square \text{quarter} \square \text{quarter} \square \text{quarter} \]

75, __, __, __, __

ASK: If we count on from 75 cents, what are the next values if we count the quarters? (100, 125, 150, 175) Write the answers on the board.

Write on the board:

25, __, __

Count by 25s. | __, __, __

Continue counting by 5s.

ASK: What are the next numbers if we count on by 25s? (50, 75) Write the answers on the board. SAY: To get the next three numbers, we count on by 5s. ASK: What are the next three numbers? (80, 85, 90) Write the answers on the board.

Draw on the board:

\[ \text{quarter} \square \text{quarter} \square \text{dime} \square \text{dime} \square \text{dime} \]

ASK: If you want to find the value of these coins, what is the first number you count by? (25) How do you know? (the value of a quarter is 25¢) What is the second number you count on by? (10) How do you know? (the value of a dime is 10¢) What are the first two numbers when you count by 25s? (25, 50) Write the answers on the board. ASK: If you count on by 10s, what are the next three numbers? (60, 70, 80) Write the answers on the board.

Exercises: Count on by the first coin value given, and then by the second coin value.

a) 

b) 

Answers: a) 25, 50, 55, 60, 65; b) 25, 50, 75, 76, 77
ACTIVITY 2

2. Students can play a game using the board on BLM Fake Money Game, tokens, a die, and play money. Students can play individually or co-operatively in pairs. Each cell has the picture of either a real Canadian coin or a fake coin. The goal is to fill the board with real coins.

To start, each player places one token on any cell with a picture of a real coin and puts all the play money in the centre of the game board. Players take turns rolling a die and moving their tokens around the board according to the number rolled, in whichever direction they wish. Two tokens cannot be in the same cell at the same time, so if the number rolled takes a player to a cell that is already occupied, the player must go in the other direction. When a player’s token lands on a fake coin, the player puts any real coin in the cell.

Extension

In Canada, there is a coin called the 50-cent piece. It is not often used. The value of the coin is 50 cents. Count on by the first coin value given, and then by the second coin value.

a) 50, 55, 60, 65
b) 50, 60, 70, 80

c) 50, 100, 110, 120, 130

d) 50, 100, 125, 150

Answers: a) 50, 55, 60, 65; b) 50, 60, 70, 80; c) 50, 100, 110, 120, 130; d) 50, 100, 125, 150
**Goals**

Students will count money amounts starting from the largest coin value, with or without coin pictures.

**PRIOR KNOWLEDGE REQUIRED**

- Can skip count by 5s, 10s, and 25s
- Can identify coins and state their values
- Can count on by the appropriate amount to find the total value of a collection of coins with at most two denominations
- Can sort a list of two-digit numbers from greatest to least

**MATERIALS**

- deck of cards with face cards removed
- play money of different denominations

**Mental math minute.** Shuffle the deck of cards after removing the face cards. Divide students into pairs. One pair at a time, each student in the pair will select a random card. The students will create two multiplication equations using the cards selected. For example, if Student A selects a 7 and Student B selects an 8, they will create the equations $7 \times 8 = 56$ and $8 \times 7 = 56$. If the card selected is an ace, treat it as the number 1. Students will say the equations out loud, do three jumping jacks, and then sit down. Continue until all students have had an opportunity to participate.

**Reviewing less than and greater than.** Write on the board:

- $7 \, 10$

ASK: Which number is larger? (10) Which number is smaller? (7) SAY: We say that 7 is less than 10. Write on the board:

- $15 \, 3$

ASK: Which number is larger? (15) Which number is smaller? (3) SAY: We say that 15 is greater than 3.

**Exercises:** Circle the larger number.

- a) 14 8  b) 7 15  c) 1 25  d) 10 5

**Answers:** a) 14, b) 15, c) 25, d) 10

**Writing numbers from largest to smallest.** Write on the board:

- 8 10 15 3 7 10

ASK: Which number is the biggest? (15) Start a new line of numbers below the previous one, write “15,” and then SAY: We’ve used the number 15, and so we’ll cross it out so we don’t accidentally use it again. Cross out the 15.
ASK: Of the numbers that haven’t been crossed out, which number is the biggest? (10) SAY: There are two 10s. Let’s use one of them. Write “10” on the line below, and cross out one of the 10s. Continue until all the numbers have been crossed out. The board should look like this:

```
8  10  15  10  8  7  3
```

Exercises

1. Write the numbers in order from greatest to least.
   a) 10, 5, 25, 10, 5  
   b) 25, 10, 5, 5, 10  
   c) 10, 5, 25, 25, 5, 10

   **Answers:** a) 25, 10, 10, 5, 5; b) 25, 25, 10, 5, 5, 1; c) 25, 25, 25, 10, 10, 5, 5

2. Draw the coins in order from greatest value to least value.
   a) [Diagram of coins with values 10¢, 25¢, 5¢, 10¢, 25¢]
   b) [Diagram of coins with values 5¢, 10¢, 5¢, 25¢, 5¢]

   **Answers:** a) 25¢, 25¢, 10¢, 10¢, 5¢; b) 25¢, 10¢, 5¢, 5¢, 5¢

3. Draw the coins in order from greatest value to least value.
   a) [Diagram of coins with values quarter, quarter, dime, nickel, nickel]
   b) [Diagram of coins with values quarter, quarter, dime, nickel, nickel]

   **Answers:** a) quarter, quarter, dime, nickel, nickel; b) quarter, quarter, dime, nickel, nickel

**ACTIVITY 1**

1. Distribute play money coins (with loonies and toonies removed from the set). Students are to work in pairs. Student A takes a handful of coins. Student B places them in order from greatest value to least value. Student A checks the work. Students repeat several times, switching roles each time.

**Counting on by two or more numbers.** Write on the board:

```
25, ___, ___  
Count by 25s.  
___, ___, ___  
Count by 10s.  
___, ___, ___  
Count by 5s.
```

ASK: If we count by 25s, what are the next two numbers after 25? (50, 75) Write the answers on the board. SAY: Now we are at 75. ASK: If we count by 10s, what are the next three numbers after 75? (85, 95, 105) Write the answers on the board. SAY: Now we are at 105. ASK: If we count by 5s,
what are the next three numbers after 105? (110, 115, 120) Write the answers on the board.

**Exercises:** Count on to find the total.

a) 25,   
   Count by 25s.  
   |   ,    
   Count by 10s.  
   |   ,    
   Count by 5s.

b) 25,   ,   
   Count by 10s.  
   |   ,    ,   
   Count by 5s.  
   |   ,    ,   
   Count by 1s.

c) 25,   
   Count by 25s.  
   |   ,    
   Count by 10s.  
   |   ,    ,   
   Count by 5s.  
   |   ,    
   Count by 1s.

**Answers:** a) 50, 60, 70, 80, 85, 90; b) 35, 45, 50, 55, 60, 61, 62, 63; c) 50, 60, 70, 75, 80, 85, 86, 87

Finding the total value of coins that are in order from greatest value to least value. Draw on the board:

ASK: What is the value of each coin? (25¢, 25¢, 10¢, 5¢, 5¢) Write the answers on the board, above the coins. SAY: The first two coins tell us to count by 25s. ASK: What are the first two numbers if we count by 25s? (25, 50) Write “25¢” and “50¢” in the first two blanks. SAY: The dime tells us to count by 10s. ASK: What is the next number if we count by 10s after 50? (60) Write “60¢” in the next blank. SAY: The two nickels tell us to count by 5s. ASK: What are the next numbers if we count by 5s after 60? (65, 70) Write “65¢” and “70¢” in the last two blanks. SAY: So, the total value of the coins is 70 cents.

**Exercises:** Count on to find the total value.

a)  
   Count by 25s.  
   |   ,    
   Count by 10s.  
   |   ,    
   Count by 5s.

b)  
   Count by 25s.  
   |   ,    ,   
   Count by 5s.  
   |   ,    ,   
   Count by 1s.

c)  
   Count by 25s.  
   |   ,    
   Count by 10s.  
   |   ,    
   Count by 1s.

d)  
   Count by 25s.  
   |   ,    
   Count by 10s.  
   |   ,    
   Count by 1s.

**Answers:** a) 25¢, 50¢, 55¢, 60¢, 65¢; b) 25¢, 50¢, 75¢, 85¢, 95¢; c) 25¢, 50¢, 60¢, 70¢, 75¢; d) 25¢, 35¢, 45¢, 50¢, 55¢
Sorting the coins before finding the total value. SAY: It is easier to count money if you start with the highest values first. Draw on the board:

- dime
- quarter
- nickel
- nickel
- quarter

ASK: Which coin has the greatest value? (quarter) Which has the next highest value? (dime) Which has the least value? (nickel) SAY: Let’s draw them again but this time in order from greatest value to least value. Draw on the board:

- quarter
- quarter
- dime
- nickel
- nickel

ASK: What is the value of each coin? (25¢, 25¢, 10¢, 5¢, 5¢) Write the values above the coins. ASK: If we count by 25s, what are the first two numbers? (25, 50) Write “25¢” and “50¢” in the first two blanks. SAY: The next value is 10 cents. ASK: If we count on from 50 by 10, what is the next number? (60) Write “60¢” in the next blank. SAY: The next value is 5 cents. ASK: If we count on from 60 by 5s, what are the next two numbers? (65, 70) Write “65¢” and “70¢” in the last two blanks. SAY: So, the total value of the coins is 70 cents.

Exercises: What is the total amount in cents? Count by the greatest coin value first.

a)

b)

Answers: a) 25¢, 35¢, 45¢, 50¢, 55¢; b) 25¢, 50¢, 75¢, 85¢, 90¢

Replacing coins with a single coin. Draw on the board:

a)

b)

c)

S-20
Ask volunteers to come to the board to find the total value of each collection of coins. (a) 5¢, 10¢, 15¢, 20¢, 25¢; b) 1¢, 2¢, 3¢, 4¢, 5¢; c) 5¢, 10¢; d) 10¢, 20¢, 25¢; e) 25¢, 50¢, 75¢, 100¢)

SAY: In part a), the total value of the coins is 25 cents. ASK: What single coin has a value of 25 cents? (a quarter) SAY: In part b), the total value is 5 cents. ASK: What single coin has a value of 5 cents? (a nickel) SAY: In part c), the total value is 10 cents. ASK: What single coin has a value of 10 cents? (a dime) SAY: In part d), the total value is 25 cents. ASK: What single coin has a value of 25 cents? (a quarter) SAY: In part e), the total value is 100 cents. ASK: How many dollars does 100 cents make? ($1) What single coin has a value of one dollar? (loonie)

Finding coins with the same value as a quarter. Draw on the board:

![Diagram of coins: dime, dime, nickel]

ASK: What two values are the coins in the picture? (10¢, 1¢) Where should I draw a line to show when to change what we are counting by? (after the second dime) How do we count on for the first two dimes? (10, 20) How do we continue counting on for the four pennies? (21, 22, 23, 24) What is the total value of these coins? (24¢) What is the value of a quarter? (25¢) Do these coins have the same value as a quarter? (no) What coin could I add to this collection to have the same value as a quarter? (a penny)

ACTIVITY 2

2. Distribute play money dimes and nickels. Have students use only dimes and nickels to make as many different collections as possible that have the same value as a quarter. (2 dimes and 1 nickel, 1 dime and 3 nickels, 5 nickels)

Estimating then finding the actual value of a group of coins. Draw on the board:

![Diagram of coins: nickel, nickel, nickel, nickel, dime, dime, nickel, quarter, nickel, nickel, dime]
Circle the first four nickels. ASK: If we had one more nickel, what would be the value of the five nickels? (25¢) What single coin has a value of 25 cents? (a quarter) SAY: Let’s pretend that we replace these first few coins with a quarter. Circle the next three coins. ASK: What is the value of two dimes and a nickel? (25¢) SAY: Let’s pretend that we replace the two dimes and a nickel with a quarter. Circle the last three coins. ASK: What is the total value of two nickels and a dime? (20¢) SAY: That is almost the same as a quarter, so let’s pretend we replace the last three coins with a quarter. The final picture should look like this:

SAY: To estimate the total value, we can pretend we have four quarters. ASK: What is the total value of four quarters? (100¢) SAY: So, we can estimate the total value of the coins is 100 cents or 1 dollar.

ASK: How can we find the actual value? (add up the values of all the coins) What order should we add them to make it easier? (start with the coins of highest value) Ask students to find the actual value by adding the highest valued coins first. For students who need a prompt, draw the following

Ask a volunteer to fill in the blanks on the board as they count on. (25¢, 35¢, 45¢, 55¢, 60¢, 65¢, 70¢, 75¢, 80¢, 85¢, 90¢) SAY: So, the exact total value of the coins is 90 cents. Our estimate of 1 dollar was pretty close.

**ACTIVITY 3**

3. Distribute play money. Students work in pairs. Student A takes a collection of at least seven coins. Student B estimates the total value of the coins. Both students sort the coins from highest value to lowest value and then count to find the exact value. Repeat with the roles reversed. Play several times.
Extension

The 50-cent piece is a coin that is sometimes used in Canada. It has a value of 50 cents.

a) How many quarters do you need to have the same value as a 50-cent piece?

b) How many dimes do you need to have the same value as a 50-cent piece?

c) How many 50-cent pieces do you need to have the same value as a loonie?

Answers: a) 2, b) 5, c) 2
Goals
Students will find missing numbers in sequences counting by 5s and 10s.
Students will identify the missing coins to make the total value equal to a given amount.

PRIOR KNOWLEDGE REQUIRED
Can count by 5s, 10s, and 25s
Can subtract two-digit numbers
Can add two-digit numbers
Can find the total value of a collection of coins

MATERIALS
deck of cards with face cards removed
play money of different denominations

Mental math minute. Shuffle the deck of cards after removing the face cards. Divide students into pairs. One pair at a time, each student in the pair selects a random card, which they use to create two division equations. For example, if Student A selects a 7 and Student B select an 8, they will create the equations 56 ÷ 8 = 7 and 56 ÷ 7 = 8. If the card selected is an ace, treat it as the number 1. Students will say the equations out loud, do three jumping jacks, and then sit down. Continue until all students have had an opportunity to participate.

Filling in missing amounts in a sequence by counting on by 5s. Write on the board:

17, ___, 27, ___, 37

ASK: If we count on by 5s, what is the next number after 17? (22) Write “22” in the first blank. ASK: If we keep counting on by 5s, what comes after 22? (27) What comes after 27? (32) Write “32” in the second blank. ASK: What comes after 32? (37) Circle the ones digit for each number in the sequence on the board. (7, 2, 7, 2, 7) ASK: What is the pattern in the ones digits? (7, 2, then repeat)

Exercises: Fill in the missing amounts by counting on by 5s. Check your answers by circling the ones digits and noticing the pattern.

a) 31, ___, 41, ___, 51 b) 44, ___, 54, ___, 64
c) 70, ___, 80, ___, 90 d) 63, ___, 73, ___, 83

Answers: a) 30, 35, 40, 45, 50; pattern: 1, 5, then repeat; b) 40, 45, 50, 55, 60; pattern: 5, 5, then repeat; c) 70, 75, 80, 85, 90; pattern: 5, 5, then repeat; d) 60, 65, 70, 75, 80; pattern: 5, 5, then repeat
Finding the number of missing nickels. Draw on the board:

\[10\text{¢} \quad 10\text{¢} \quad 1\text{¢}\]

ASk: What is the total value of the coins? (21¢) How do you know? (if you count on by 10s, the number after 10 is 20, and if you count on by 1s after 20, the next number is 21)

Say: Suppose I need 31 cents. I want to find out how many nickels I need to add to the diagram to get to 31 cents. Draw a circle with “5¢” to the right of the diagram. ASk: If we count by 5s starting at 21, what is the next number? (26) Have we reached our total of 31 cents? (no) Say: Let’s add another nickel. Draw another circle with “5¢” to the right of the diagram. ASk: If we count by 5s starting at 26, what is the next number? (31) What is the total value of the coins now? (31¢) Say: We’ve reached our total, so we can stop adding nickels. ASk: How many nickels did we add? (2)

Exercises: Draw the extra nickels needed to make the total.

a) 45¢
   \[
   \quad \quad \quad \quad
   \]
   Answer: a) draw 2 nickels

b) 31¢
   \[
   \quad \quad \quad \quad
   \]
   Answer: b) draw 3 nickels

Filling in missing amounts in a sequence by counting on by 10s. Write on the board:

14, ___, 34, ___, 54

Ask: If we count on by 10s, what is the next number after 14? (24) Write “24” in the first blank. Ask: If we keep counting on by 10s, what comes after 24? (34) What comes after 34? (44) Write “44” in the second blank. Ask: What comes after 44? (54) Circle the ones digit for each number in the sequence on the board. Ask: What is the pattern in the ones digits? (4 repeats)

Exercises: Fill in the missing amounts by counting on by 10s. Check your answers by circling the ones digits and noticing the pattern.

a) 23, ___, 43, ___, 63
   b) 67, ___, 87, ___, 107
   c) 41, ___, 61, ___, 81
   d) 22, ___, 42, ___, 62

Answers: a) 20, 30, 40, 50, 60, pattern: 3 repeats; b) 60, 70, 80, 90, 100, pattern: 7 repeats; c) 40, 50, 60, 70, 80, pattern: 1 repeats; d) 20, 30, 40, 50, 60, pattern: 2 repeats

Finding the number of missing dimes. Draw on the board:

\[25\text{¢} \quad 10\text{¢}\]

Ask: What is the total value of the coins? (35¢) How do you know? (if you count on by 10s, the number after 25 is 35)
SAY: Suppose I need 65 cents. I want to find out how many dimes I need to add to the diagram to get to 65 cents. Draw a circle with “10¢” to the right of the diagram. ASK: If we count by 10s starting at 35, what is the next number? (45) Have we reached our total of 65 cents? (no) SAY: Let’s add another dime. Draw another circle with “10¢” to the right of the diagram. ASK: If we count by 10s starting at 45, what is the next number? (55) What is the total value of the coins now? (55¢) Have we reached our total of 65 cents? (no) SAY: Let’s add another dime. Draw a third circle with “10¢” to the right of the diagram. ASK: If we count by 10s starting at 55, what is the next number? (65) SAY: We’ve reached our total, so we can stop adding dimes. ASK: How many dimes did we add? (3)

Exercises: Draw the extra dimes needed to make the total.

a) 46¢   b) 45¢

Answers: a) draw 2 dimes, b) draw 3 dimes

Filling in missing amounts in a sequence by counting on by 25s. Write on the board:

25, __, 75, __, 125, __, __, __

ASK: If we count on by 25s, what is the next number after 25? (50) Write “50” in the first blank. ASK: If we keep counting on by 25, what comes after 50? (75) What comes after 75? (100) Write “100” in the second blank. ASK: What comes after 100? (125) Continue until all the blanks are filled. Circle both the tens and ones digits together for each number in the sequence on the board. ASK: What is the pattern in the tens and ones digits? (25, 50, 75, 00, then repeat)

Exercises: Fill in the missing amounts by counting on by 25s. Check your answers by circling the tens and ones digits and noticing the pattern.

a) 75, __, 125, __, 175   b) 250, __, 300, __, 350

Answers: a) 75, 100, 125, 150, 175; pattern: 75, 00, 25, 50, then repeat; b) 250, 275, 300, 325, 350; pattern: 50, 75, 00, 25, then repeat

Finding the number of missing quarters. Draw on the board:

25¢ 25¢

ASK: What is the total value of the coins? (50¢) How do you know? (if you count on by 25, the number after 25 is 50)

SAY: Suppose I need 125 cents. I want to find out how many quarters I need to add to the diagram to get to 125 cents. Draw a circle with “25¢” to the right of the diagram. ASK: If we count by 25s starting at 50, what is the next number? (75) Have we reached our total of 125 cents? (no) SAY: Let’s add another quarter. Draw another circle with “25¢” to the right of the diagram.
ASK: If we count by 25s starting at 75, what is the next number? (100)
What is the total value of the coins now? (100¢) Have we reached our total of 125 cents? (no)
SAY: Let’s add another quarter. Draw another circle with “25¢” to the right of the diagram. 
ASK: If we count by 25s starting at 100, what is the next number? (125) What is the total value of the coins now? (125¢)
SAY: We’ve reached our total, so we can stop adding quarters.
ASK: How many quarters did we add? (3)

Exercises: Draw the extra quarters needed to make the total.

a) 175¢ 
   b) 55¢

Answers: a) draw 3 quarters, b) draw 1 quarter

Finding two or more missing coin values. Draw on the board:

25¢ 25¢

ASK: What is the total value of the coins? (50¢) How do you know? (if you start at 25 and count by 25s, the next number is 50)

SAY: Suppose I need 85 cents. I want to find the coins I can add to make 85 cents. Let’s try to add coins beginning with the largest coin values first.

ASK: If I add a loonie, will the total value be more than 85 cents? (yes)
SAY: So I can’t add a loonie. ASK: What is the next highest coin value? (25¢)
SAY: Let’s add a quarter to my coins. Draw a quarter on the board.
SAY: We had 50 cents. ASK: If I count on by 25s from 50, what is the next number? (75)
SAY: So now we have 75 cents. ASK: Is 75 cents more than 85 cents? (no)
SAY: Let’s try to add another quarter. Draw another quarter on the board. ASK: If I count on by 25s from 75, what is the next number? (100)
SAY: So now we have 100 cents. ASK: Is that more than 85 cents? (yes)
SAY: We need to erase the quarter because the total value is too high. Erase the drawing of the last quarter on the board. ASK: What is the next highest coin value after the quarter? (10¢)
SAY: Draw a dime on the board.
SAY: We have 75 cents. We add a dime. ASK: What is the total value now? (85¢) That is exactly what we were looking for, so we can stop. The final diagram should look like this:

25¢ 25¢ 25¢ 10¢

SAY: When you add missing coins, start with the highest coin values first.

Exercises: Write the two coin values needed to make the total.

a) 50¢
   b) 80¢
**Bonus:** 190¢

Answers: a) 10¢, 5¢; b) 25¢, 5¢; Bonus: 10¢, 5¢

**Writing the names of missing coins.** Draw on the board:

- dime
- quarter
- nickel

SAY: I need 45 cents. ASK: Are the coins in order from greatest value to least value? (no) SAY: It’s easier to find the total value if the coins are in order. ASK: What is the order from greatest value to least value? (quarter, dime, nickel) Draw on the board:

- quarter
- dime
- nickel

ASK: What is the value of a quarter? (25¢) What is the value of a dime? (10¢) If we count on from 25 by 10, what is the next number? (35) What is the value of a nickel? (5¢) If we count on from 35 by 5, what is the next number? (40) SAY: We need 45 cents. We have 40 cents. ASK: Can we add a quarter or a dime? (no) Why? (the total value would be more than 45¢) ASK: What coin is the next lowest coin value? (5¢) If we count on from 40 by 5, what is the next number? (45) SAY: So now we have 45 cents. ASK: What coin has a value of 5 cents? (nickel) SAY: So the coin we should add is a nickel.

**Exercises:** Write the name of the missing coin needed to make the total.

a) 145¢

b) 65¢

Answers: a) dime, b) quarter

**Solving word problems involving money.** Write on the board:

Jennifer has 3 quarters, 2 nickels, and 1 dime. She needs 1 dollar to buy a pencil. What coin does she need?

ASK: How many cents are in one dollar? (100) Draw on the board:

- quarter
- quarter
- quarter
- nickel
- nickel
dime
ASK: Are the coins in order from greatest value to least value? (no) Ask for a volunteer to draw the coins in order, as shown below:

quarter quarter quarter dime nickel nickel

ASK: What is the value of a quarter? (25¢) If we start at 25 and count on by 25s, what are the next two numbers? (50, 75) What is the value of a dime? (10¢) If we start at 75 and count on by 10s, what is the next number? (85) What is the value of a nickel? (5¢) If we start at 85 and count on by 5s, what are the next two numbers? (90, 95) ASK: What coin can we add so that Jennifer will have 100 cents? (a nickel)

Exercise: Sam has 1 loonie, 2 dimes, and 2 quarters. What two coins is he missing if needs 2 dollars?

Solution: 100, 125, 150, 160, 170. Sam needs 30¢ to make 200¢: 1 quarter and 1 nickel.

Extensions

1. The 50-cent piece is a coin that is sometimes used in Canada. It has a value of 50 cents. Karen has three 50-cent pieces, three quarters, two dimes, and one nickel. She needs 260¢ to buy a sandwich.
   a) What is the total value in cents of the coins she has?
   b) What single coin can she use to reach 260¢?
   c) What two coins can she use to reach 260¢?
   Answers: a) 250¢, b) dime, c) two nickels

2. Draw the least number of coins to make the total. Use the 50-cent piece.
   a) 150¢  b) 80¢  c) 90¢
   Answers
   a) \(\text{50¢, 50¢, 50¢}\)  b) \(\text{50¢, 25¢, 5¢}\)  c) \(\text{50¢, 25¢, 10¢, 5¢}\)
Goals

Students will replace a collection of coins with a single coin of equal value.

Students will regroup coins to make the same total with the least number of coins.

Given a value, students will use the least number of coins to make the value.

PRIOR KNOWLEDGE REQUIRED

Knows the values of the different coins
Can find the total value of a collection of coins starting with the highest coin value
Can add two-digit numbers
Can subtract two-digit numbers
Can find the value of missing coins to make a given total

MATERIALS

deck of cards with face cards removed
play money of different denominations

Mental math minute. Shuffle a deck of cards after removing the face cards. Students work in pairs with one pair participating at a time. Student A selects two cards from the deck to create a two-digit number. The first card drawn is the tens digit and the second card drawn is the ones digit. Student B also selects two cards to form a two-digit number. The students then create two addition equations, by adding the two-digit numbers formed. The students say the equations out loud and then jump in the air as high as they can three times. Once they are done, they sit down. Continue until all students have had an opportunity to participate.

Replacing coins of equal value. Draw on the board:

\[
\text{penny} \quad \text{penny} \quad \text{penny} \quad \text{penny} \quad \text{penny}
\]

ASK: What is the value of each of the pennies? (1¢) What is total value of the coins? (5¢) How do you get a total of 5 using addition? (1 + 1 + 1 + 1 + 1 = 5) How do you get a total of 5 using multiplication? (5 x 1 = 5) What single coin has a value of 5 cents? (nickel) SAY: So, instead of five pennies, we could replace them with one nickel, and the total value would be the same.

Draw on the board:

\[
\text{nickel} \quad \text{nickel}
\]
ASK: What is the value of each of the nickels? (5¢) What is the total value of the coins? (10¢) How do you get a total of 10 using addition? (5 + 5 = 10) How do you get a total of 10 using multiplication? (2 x 5 = 10) What single coin has a value of 10 cents? (dime) SAY: So, instead of two nickels, we could replace them with one dime, and the total value would be the same.

Draw on the board:

![Dime, Dime, Nickel]

ASK: What is the value of each of the dimes? (10¢) What is the value of the nickel? (5¢) If we count on by 10s after 10, what is the next number? (20) If we count on by 5s from 20, what is the next number? (25) What is the total value of the coins? (25¢) What single coin has a value of 25 cents? (quarter) SAY: So, instead of two dimes and one nickel, we could replace them with one quarter, and the total value would be the same.

Draw on the board:

![Dime, Nickel, Nickel, Nickel]

ASK: What is the value of each of the nickels? (5¢) What is the value of the dime? (10¢) If we count on by 5s after 10, what are the next three numbers? (15, 20, 25) What is the total value of the coins? (25¢) What single coin has a value of 25 cents? (quarter) SAY: So, instead of one dime and three nickels, we could replace them with one quarter, and the total value would be the same.

Draw on the board:

![Nickel, Nickel, Nickel, Nickel, Nickel]

ASK: What is the value of each of the nickels? (5¢) If we count on by 5s after 5, what are the next four numbers? (10, 15, 20, 25) What is the total value of the coins? (25¢) What single coin has a value of 25 cents? (quarter) SAY: So, instead of five nickels, we could replace them with one quarter, and the total value would be the same.

Draw on the board:

![Quarter, Quarter, Quarter, Quarter]

ASK: What is the value of each of the quarters? (25¢) If we count on by 25s after 25, what are the next three numbers? (50, 75, 100) What is the total value of the coins? (100¢) What single coin has a value of 100 cents? (loonie) SAY: So, instead of four quarters, we could replace them with one loonie, and the total value would be the same.
Exercises: Fill in the blank with the correct number.

a) ___ nickels have the same value as a dime.

b) ___ quarter has the same value as 2 dimes and 1 nickel.

c) 3 nickels and ___ dime has the same value as 1 quarter.

d) 1 loonie has the same value as ___ quarters.

e) 5 nickels have the same value as ___ quarter.

Bonus

f) 2 loonies have the same value as ___ quarters.

g) 5 dimes have the same value as ___ quarters.

Answers: a) 2, b) 1, c) 1, d) 4, e) 1, Bonus: f) 8, g) 2

ACTIVITY

Trading coins. Give each student the same number of play pennies and nickels, as indicated below. Have pairs trade coins worth the same amount (e.g., 5 pennies for 1 nickel) so that each student’s amount of money does not change. Encourage students to count their money after each trade to verify that their amount is unchanged. Players each have a separate goal number of coins, but they will have to co-operate to achieve their goals.

a) Give each student 20 pennies and 8 nickels. Player 1’s goal: 20 coins; Player 2’s goal: 36 coins.

Answer: Player 2 gives Player 1 two nickels in exchange for 10 pennies

b) Give each student 17 pennies and 3 nickels. Player 1’s goal: 12 coins. Player 2’s goal: 28 coins.

Answer: Player 2 gives Player 1 two nickels in exchange for 10 pennies

Finding the total value by grouping coins without a diagram. Write on the board:

Jasmin has 3 quarters, 3 nickels, and 1 dime.

What is the total value of her money?

What single coin can replace her coins?

SAY: We can find the total value by counting on or grouping coins. Let’s count on to find the total value. ASK: What is the coin with the highest value? (quarter) What is the value of a quarter? (25¢) SAY: There are three quarters. ASK: If we start at 25 and count on by 25s, what are the next two numbers? (50, 75) SAY: So, the value of the three quarters is 75 cents.

ASK: What is the coin with the next highest value? (dime) SAY: We only have one dime. ASK: If we count on from 75 by 10, what is the next number? (85)
What is the coin with the next highest value? (nickel) If we start at 85 and count on by 5s, what are the next three numbers? (90, 95, 100) SAY: So, the total value of the coins is 100 cents. ASK: What single coin has a value of 100 cents? (loonie) SAY: So, a loonie has the same value as all her coins.

SAY: Let’s count Jasmin’s money again, but this time by grouping coins. ASK: What single coin can replace three nickels and one dime? (quarter) SAY: But Jasmin already has three quarters. ASK: If we replace the three nickels and a dime with one quarter, how many quarters will she have altogether? (four) What single coin can replace four quarters? (loonie) SAY: So, a loonie has the same value as all her coins.

Exercises: I have 2 quarters and 5 dimes.

a) What is the total value of my money?

b) What single coin has the same value?

Selected solution: a) By counting on: 25, 50, 60, 70, 80, 90, 100; By grouping: 5 dimes is the same as 2 quarters, so 4 quarters altogether, and total value = 100¢

Answer: b) loonie

Regrouping coins to make the same total with the least number of coins.

SAY: If we replace coins with a single coin of equal value, the total value stays the same. Draw on the board:

```
dime  nickel  nickel  nickel  nickel  nickel
```

ASK: What is the value of the first four coins? (25¢) What single coin has a value of 25 cents? (quarter) SAY: So, we can replace the first four coins with a quarter. Circle the first four coins and write “25¢” underneath. ASK: What is the value of the last two coins? (10¢) What single coin has a value of 10 cents? (dime) Circle the last two coins and write “10¢” underneath, as shown below:

```
dime  nickel  nickel  nickel  nickel  nickel
```

25¢  10¢

SAY: So, the collection of coins has the same value as a quarter and a dime. Have the class find the total value before and after they grouped the coins to check that the total value stayed the same. (35¢)
Exercises

1. Regroup coins to make the same total with the least number of coins.
   a) 
   b) 

Bonus: Regroup coins to make the same total. You may be able to regroup more than once.
   c) 
   d) 

Answers

a) 
   
   quarter dime dime nickel nickel nickel 
   25¢ 25¢ 10¢ 

b) 
   
   nickel nickel quarter quarter quarter quarter 
   10¢ 100¢ 

Bonus
   c) 
   
   quarter quarter quarter dime dime nickel 
   (25¢ 25¢ 25¢ 25¢) 100¢ 

d) 
   
   quarter loonie nickel dime dime dime 
   25¢ 100¢ 25¢ 10¢ 

Making an amount with the least number of coins. Write “70¢” on the board. SAY: Suppose we need exactly 70 cents for a drink. If we want to pay with the least number of coins, we should always start with the coins of highest value. ASK: What is the highest value of a coin that is less than 70 cents? (25¢) Draw a coin with “25¢” written on it on the board to the right of 70¢. SAY: Let's draw another quarter. Draw another coin with “25¢” written on it to the right. ASK: If we count by 25s, what are the first two numbers? (25, 50) SAY: So, we have 50 cents so far. Let's draw another quarter. Draw a third coin with “25¢” written on it to the right. ASK: If we
count by 25s, what is the next number? (75) SAY: 75 cents is more than we need, so we can’t use this last quarter. Cross out the drawing of the last 25-cent coin. SAY: So, the greatest amount we could pay in quarters is 50 cents, using two quarters. The picture should look like this:

\[
\begin{array}{c}
70\text{¢} \\
25\text{¢} \\
25\text{¢} \\
\times
\end{array}
\]

**Exercises**

1. How much of the total amount could you pay in quarters?
   a) 80¢
   b) 40¢
   c) 130¢

   **Answers:** a) 75¢, b) 25¢, c) 125¢

2. How much of the total amount could you pay in dimes?
   a) 45¢
   b) 75¢
   c) 35¢

   **Answers:** a) 40¢, b) 70¢, c) 30¢

   Draw on the board:

   \[
   \begin{array}{c}
   70\text{¢} \\
   25\text{¢} \\
   25\text{¢}
   \end{array}
   \]

   ASK: What is the total value of the quarters? (50¢) How much is left to make 70 cents? (20¢) How do you know? (70 − 50 = 20) How can you make 20 cents using the least number of coins? (2 dimes) Draw two coins with “10¢” written on each to the right of 70¢. Have the class check that the total value of the coins is indeed 70 cents.

   SAY: Cam has a different way of making 70 cents. Draw on the board:

   \[
   \begin{array}{c}
   70\text{¢} \\
   25\text{¢} \\
   25\text{¢} \\
   10\text{¢} \\
   5\text{¢} \\
   5\text{¢}
   \end{array}
   \]

   Have the class check that the total of Cam’s coins is also 70 cents. ASK: What is different about the answers? (Cam’s answer uses more coins)

**Exercises**

1. Write the greatest amount you can pay using quarters. Subtract to find the amount remaining. Then show the amount left over, using the least number of coins.

<table>
<thead>
<tr>
<th>Total Amount</th>
<th>Amount You Could Pay in Quarters</th>
<th>Amount Remaining</th>
<th>Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 85¢</td>
<td>75¢</td>
<td>85¢ − 75¢ = 10¢</td>
<td>10¢</td>
</tr>
<tr>
<td>b) 65¢</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 120¢</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **Answers:** b) 50¢, 65¢ − 50¢ = 15¢, dime and nickel; c) 100¢, 120¢ − 100¢ = 20¢, 2 dimes
2. Draw the least number of coins to make the total. Start by finding the greatest amount you can make with loonies.

   a) 110¢  
   b) 130¢  
   c) 165¢

   **Answers:** a) loonie, dime; b) loonie, quarter, nickel; c) loonie, two quarters, dime, nickel

**Extensions**

1. Find the missing coins.

   a) Megan uses 4 coins to make 120¢. One of the coins is a loonie. What are the other coins?

   **Answers:** a) 1 dime and 2 nickels

   b) Ed uses 5 coins to make 105¢. What are the coins?

   **Answers:** b) 4 quarters and 1 nickel

2. Sharon has 60 nickels in her piggy bank.

   a) How many dimes are needed to have the same total value?

   **Answers:** a) 30

   b) How many quarters are needed to have the same total value?

   **Answers:** b) 12

3. Using only nickels, dimes, and quarters, how many different ways are there to make 30¢? List the ways.

   **Answer:** 5 ways: 6 nickels; 4 nickels and 1 dime; 2 nickels and 2 dimes; 3 dimes; 1 quarter and 1 nickel
Goals
Students will calculate the difference between the amount paid and the cost.
Students will count up by 10s to find the difference owed from a dollar.

PRIOR KNOWLEDGE REQUIRED
Can count up by 10s
Can subtract two-digit numbers

MATERIALS
deck of cards with face cards removed
various items clearly labelled with prices under $1
play money

Mental math minute. Shuffle the deck of cards after removing the face cards. Students work in pairs with one pair participating at a time. Student A selects two cards from the deck to form the digits of a two-digit number. Student B does the same. The students create two subtraction equations using the numbers formed. For example, if Student A selects the numbers 7 and 2, forming the two-digit number 72, and Student B forms the two-digit number 35, they will create the equations $72 - 35 = 37$ and $72 - 37 = 35$. The students will say the equations out loud, and then jump in the air three times. Once they are done, they sit down. Continue until all students have had an opportunity to participate.

Calculating the difference owed when the difference is less than a dime. Write on the board:

Price of drink = 55¢
Amount paid = 60¢

SAY: When you pay more than you have to for something, the person you paid should give you some money back. The money you get back is the difference owed and is sometimes called the change. ASK: Did the person pay too much or not enough? (too much) How do you know? (60 > 50) SAY: When you pay too much for something at the store, the cashier, a person who handles payments for the store, will give you back money. You can count up from 55 cents to 60 cents to find the difference owed. Or, you can subtract 55 from 60. ASK: How much money should you get back? (5¢) What coin should the cashier give you? (a nickel) How do you know? (a nickel is worth 5¢)

Repeat the example but, this time, make the drink 62¢ and the amount paid 70¢. This time, the difference owed is 8 cents. ASK: If Canada was still using the penny coin, what coins should the cashier give? (1 nickel and 3 pennies)
Exercises: Calculate the difference owed.

a) Price of item = 75¢  
   Amount paid = 80¢

b) Price of item = 40¢  
   Amount paid = 50¢

c) Price of item = 95¢  
   Amount paid = 100¢

Answers: a) 5¢, b) 10¢, c) 5¢

Counting up by 10s to find the difference owed from a dollar. Write on the board:

   Price of item = 30¢  
   Amount paid = 100¢

SAY: A lot of times, people pay for products with a loonie. ASK: What is the value of a loonie? (100¢) If the product costs 30 cents, did the person pay too much or not enough? (too much) How do you know? (100 > 30) SAY: Here the person is owed the difference. You can count the difference by counting up from 30 or by subtracting. Let's count up from 30 first. Write on the board:

   30, __, __, __, __, __, __, __

Ask volunteers to write the next numbers after 30 if counting by 10s. Continue until the volunteers reach 100. ASK: How do we know we can stop counting? (we've reached 100, which is the amount paid) How many tens did we count up? (7) What is 7 × 10? (70) SAY: So, the person is owed the difference of 70 cents.

SAY: We can also find the difference by subtracting. Write on the board:

   \[ 100 - 30 \]

Ask a volunteer to do the subtraction on the board. (70) ASK: How could we use mental math to find the answer? (10 − 3 = 7 and then join a zero to the answer to get 70)

Exercises: Count on by 10s to find the difference owed from a dollar (100¢).

a) 40¢  
   b) 60¢  
   c) 80¢

Answers: a) 60¢, b) 40¢, c) 20¢

Finding the difference owed by counting to the next multiple of 10 and then counting by 10s to 100. Remind students that multiples of 10 are the numbers we get when we count up by 10s starting at 0.
Write on the board:

Multiples of 10: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

Price of item = 35¢
Amount paid = 100¢

ASK: What is the next multiple of 10 after 35? (40) Draw on the board:

\[
\begin{array}{c}
35 \\
\text{price} \\
\end{array} \quad \begin{array}{c}
40 \\
\text{next multiple of 10} \\
\end{array} \quad \begin{array}{c}
100 \\
\text{amount paid} \\
\end{array}
\]

ASK: If you count up from 35 to 40, what number do you get? (5) Write “5” in the first dotted box. ASK: If you count up by 10s from 40 to 100, what number do you get? (60) Write “60” in the second dotted box. SAY: To get from 35 to 100, we need to go from 35 to 40, which took 5 steps, and then from 40 to 100, which took 60 steps. ASK: What can we do with the numbers in the dotted boxes to find the difference owed from 100? (add) What is 5 + 60? (65) SAY: So, the difference owed here is 65 cents.

**Exercises:** Find the difference owed from a dollar.

a) \[
\begin{array}{c}
65 \\
\text{price} \\
\end{array} \quad \begin{array}{c}
\text{next multiple of 10} \\
\text{amount paid} \\
\end{array}
\]

b) \[
\begin{array}{c}
15 \\
\text{price} \\
\end{array} \quad \begin{array}{c}
\text{next multiple of 10} \\
\text{amount paid} \\
\end{array}
\]

**Answers:** a) 5 + 30 = 35¢, b) 5 + 80 = 85¢

**Finding the difference owed mentally.** Write on the board:

Price of item = 55¢
Amount paid = 100¢

ASK: What is the next multiple of 10 after 55? (60) If you count up from 55 to 60, what number do you get? (5) Write “5” in the first dotted box. ASK: If you count up by 10s from 60 to 100, what number do you get? (40) SAY: Remember that number. ASK: If you count up from 60 by 10s to 100, what number do you get? (40) SAY: Remember that number. Now, add 5 + 40. ASK: What is the answer? (45) SAY: So, the difference owed here is 45 cents.

**Exercises:** Find the difference owed from a dollar. Do the work mentally.

a) 25¢ b) 75¢ c) 15¢ **Bonus:** 36¢

**Answers**

a) \[
\begin{array}{c}
25 \\
\to \\
30 \\
\to \\
\text{100, } 5 + 70 = 75¢
\end{array}
\]

b) \[
\begin{array}{c}
75 \\
\to \\
80 \\
\to \\
\text{100, } 5 + 20 = 25¢
\end{array}
\]

(c) \[
\begin{array}{c}
15 \\
\to \\
20 \\
\to \\
\text{100, } 5 + 80 = 85¢
\end{array}
\]

**Bonus:** \[
\begin{array}{c}
36 \\
\to \\
40 \\
\to \\
\text{100, } 4 + 60 = 64¢
\end{array}
\]
ACTIVITY

Play Shop Keeper. Set up the classroom like a store, with items set out and their prices clearly marked. The prices should be under $1. Tell students that they will all take turns being the cashiers and the shoppers. Explain that making change (and checking you’ve got the right change!) is one of the most common uses of math that they will encounter in life. Allow students to explore the store and select items to “buy.” Give the shoppers play money to “spend,” and give the cashiers play money to make change with. Ask the shoppers to calculate the change in their heads at the same time as the cashiers when they are paying for the item. This way, students can double check and help each other out. Reaffirm that everyone needs to work together and encourage the success of all of their peers. Allow students a good amount of time in the store. Plan at least 30 minutes for this activity.

Extensions

1. Find the price of the product.
   a) Difference owed = 65¢
      Amount paid = 100¢
      \[ \text{Answer: } 100 \rightarrow 70 \rightarrow 65, \; 30 + 5 = 35¢; \]
      \[ + 5 = 35¢; \]
   b) Difference owed = 85¢
      Amount paid = 100¢
      \[ \text{Answer: } 100 \rightarrow 90 \rightarrow 85, \; 10 + 5 = 15¢ \]

2. Customers sometimes pay a greater amount so that the difference owed is made using fewer coins. For example:
   Price of item = 85¢
   Amount paid = 100¢
   \[ \begin{array}{c}
   85 \\
   90 \\
   100
   \end{array} \]
   Difference owed = 5 + 10 = 15¢
   Coins: dime, nickel
   Coin: quarter

   Price of item = 85¢
   Amount paid = 110¢
   \[ \begin{array}{c}
   85 \\
   90 \\
   100 \\
   110
   \end{array} \]
   Difference owed = 5 + 10 + 10 = 25¢
   \[ \text{Answer: } \]

Find the coins that can be used to pay the difference owed.

a) Price of item = 80¢
   Amount paid = 105¢
   \[ \text{Answer: } 80 \rightarrow 100 \rightarrow 105, \; 20 + 5 = 25¢, \; \text{quarter}; \]
   b) Price of item = 65¢
   Amount paid = 115¢
   \[ \text{Answer: } 65 \rightarrow 70 \rightarrow 100 \rightarrow 115, \; 5 + 30 + 15 = 50¢, \; \text{two quarters} \]
NS3-82  Counting Money with Dollars
Pages 136–138

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
5-dollar bill
cents
coins
dime
dollars
loonie
money
nickel
paper money
pennies
quarter
toonie
value

Goals
Students will find the number of cents in a loonie, toonie, and 5-dollar bill.
Students will find the number of loonies in a toonie and a 5-dollar bill.
Students will find the number of dollars and cents in a collection of coins.
Students will find the value of a missing coin to make a given total.
Students will regroup coins to make the same total with the least number of coins or bills.
Students will estimate the value of a collection of coins and bills.

PRIOR KNOWLEDGE REQUIRED
Knows the value of coins
Can find the value of a collection of coins

MATERIALS
BLM 3 × 3 Grid (p. S-83)
plastic coins
transparency of BLM 3 × 3 Grid (p. S-83)
overhead projector
transparency of BLM Money (p. S-81)
play money that includes paper bills

Mental math minute. Place a copy of BLM 3 × 3 Grid on the floor. Student A tosses a plastic coin onto the grid. Student B tosses another coin onto the grid. The closest numbers to the coins are the numbers selected. The students create two multiplication equations using the numbers selected. For example, if the coins land on 7 and 6, the students create the multiplication equations 7 × 6 = 42 and 6 × 7 = 42 and say them aloud. Repeat until all students have had a chance to select a number and say a multiplication equation. When the activity is over, display the BLM and ask students what is special about the numbers in the grid. PROMPT: What is the sum of the three numbers in every row, column, or diagonal? (the sum of each row, column, and diagonal is 15)

Introducing the value of a loonie, toonie, and 5-dollar bill. Display BLM Money. ASK: What is the highest value of a coin? ($2) What is the smallest value of paper money? ($5) SAY: Paper money that has a value of 5 dollars is called a 5-dollar bill. ASK: How many cents are in a dollar? (100¢) If we still used pennies, how many pennies would I need to have the same value as a loonie? (100) SAY: A toonie has the value of two dollars. ASK: If each dollar has the same value as 100 pennies, how many pennies would I need to have the same value as a toonie? (200) Using addition, explain why this is correct. (100 + 100 = 200)
Write on the board:

1 dollar = 100¢
2 dollars = 100¢ + 100¢ = 200¢
3 dollars = ___¢ + ___¢ + ___¢ = ___¢
4 dollars = ___¢ + ___¢ + ___¢ + ___¢ = ___¢
5 dollars = ___¢ + ___¢ + ___¢ + ___¢ + ___¢ = ___¢

Ask for volunteers to fill in the blanks to find the number of cents in 3, 4, and 5 dollars. (3 dollars = 100¢ + 100¢ + 100¢ = 300¢, 4 dollars = 100¢ + 100¢ + 100¢ + 100¢ = 400¢, 5 dollars = 100¢ + 100¢ + 100¢ + 100¢ + 100¢ = 500¢)

Exercises

1. Write the number of cents.
   a) $3  b) $5  c) $7
   Bonus: $25
   **Answers:** a) 300¢, b) 500¢, c) 700¢, Bonus: 2500¢

2. Write the number of dollars.
   a) 200¢  b) 600¢  c) 300¢  
   Bonus: 1400¢
   **Answers:** a) $2, b) $6, c) $3, Bonus: $14

**Combining loonies and toonies.** Write on the board:

2 = ___ + ___
5 = ___ + ___ + ___ + ___ + ___
5 = ___ + ___ + ___ + ___
5 = ___ + ___ + ___

ASK: Using only the numbers 1 and 2, how can we fill in the blanks to make the equations true? Have volunteers write the answers on the board. (2 = 1 + 1, 5 = 1 + 1 + 1 + 1 + 1, 5 = 2 + 1 + 1 + 1, 5 = 2 + 2 + 1)

SAY: Using the first addition equation, we know that a toonie has the same value as two loonies. Draw on the board:

\[
\begin{align*}
\text{\$2} &= \text{\$1} + \text{\$1} \\
1 \text{ toonie} &= 2 \text{ loonies}
\end{align*}
\]

ASK: Using the second addition equation, how many loonies are needed to make a 5-dollar bill? (5) Draw on the board:

\[
\begin{align*}
\text{\$5} &= \text{\$1} + \text{\$1} + \text{\$1} + \text{\$1} + \text{\$1} \\
1 \text{ \$5 bill} &= 5 \text{ loonies}
\end{align*}
\]
ASK: Using the third equation, how many toonies and loonies are needed to make a 5-dollar bill? (1 toonie and 3 loonies) Draw on the board:

\[
\begin{array}{c}
\$5 = 2 \quad 1 \quad 1 \quad 1 \\
\text{1 $5 bill} = \text{1 toonie} + \text{3 loonies}
\end{array}
\]

ASK: What coins can you regroup in the third equation to get the fourth equation? (2 loonies) How do you know? (2 loonies can be regrouped as 1 toonie) So, what is a different way of making 5 dollars? (2 toonies and 1 loonie) Draw on the board:

\[
\begin{array}{c}
\$5 = 2 \quad 2 \quad 1 \\
\text{1 $5 bill} = \text{2 toonies} + \text{1 loonie}
\end{array}
\]

**Writing the value of coins using dollars and cents.**

SAY: When we have money that includes paper money and loonies, toonies, and other coins, we say the number of dollars first and then the number of cents. Write “$3 and 15¢” on the board. SAY: We have 3 dollars and 15 cents. ASK: What is the least number of coins you can use to make 3 dollars? (2) Which coins are they? (loonie and toonie) How do you know? (a loonie is worth $1 and a toonie is worth $2, together they make $3) What is the least number of coins you can use to make 15 cents? (2) Which coins are they? (dime and nickel) How do you know? (a dime is worth 10¢ and a nickel is worth 5¢, together they make 15¢)

Draw on the board:

\[
\text{toonie} \quad \text{loonie} \quad \text{quarter} \quad \text{dime} \quad \text{nickel}
\]

ASK: Which coins are worth less than a dollar? (quarter, dime, nickel) Which coins are worth one dollar or more? (toonie, loonie) What is the value of the toonie? ($2) What is the value of the loonie? ($1) If you add these values, how many dollars do you have? (3) What is the value of a quarter? (25¢) What is the value of a dime? (10¢) What is the value of a nickel? (5¢) What is the total value of a quarter, dime, and nickel added together? (40¢) SAY: So, we have 3 dollars and 40 cents.

**Exercises**

1. Draw the least number of coins for the total.
   a) $2 and 60¢
   b) $3 and 30¢

**Answers**

a) \[
\text{toonie} \quad \text{quarter} \quad \text{quarter} \quad \text{dime}
\]

b) \[
\text{toonie} \quad \text{loonie} \quad \text{quarter} \quad \text{nickel}
\]
2. Write the number of dollars and cents.

a) [Image of coins and bills]

b) [Image of coins and bills]

c) [Image of coins and bills]

Answers: a) $2 and 45¢, b) $3 and 65¢, c) $6 and 60¢

Bonus: Write the number of dollars and cents. You will need to replace a group of coins with a loonie.

a) [Image of coins]

b) [Image of coins]

Answers: a) $2 and 15¢, b) $1 and 10¢

**ACTIVITIES 1–2**

1. Have students work in pairs. Student A selects coins and 5-dollar bills from a collection of play money. Student B finds the value of the money Student A selected. Student A checks the answer. Students repeat several times with roles reversed.

2. Have students work in pairs. Student A says aloud a number of dollars (less than 10) and a number of cents (less than 100). Student B creates a collection of coins and bills from play money that makes the total value. Student A checks the answer. Students repeat several times with roles reversed.

**Finding the value of the missing coin to make a given total.** Draw on the board:

$5 bill  toonie  quarter  dime  nickel

SAY: Abella needs 7 dollars and 50 cents. ASK: How many dollars does she have? (7) How do you know? (5 + 2 = 7) How many cents does she have? (40) How do you know? (25 + 10 + 5 = 40) Does she have enough dollars? (yes) Does she have enough cents? (no) How do you know? (she has 40¢ but she needs 50¢)
What single coin does she need to make 50 cents? (dime) How do you know? (a dime is worth 10¢, 50 – 10 = 40)

**Exercises:** Find the value of the missing coin to make the amount of money.

a) $3 and 70¢

![Image of coins]

b) $6 and 45¢

![Image of coins]

**Solutions:**
a) value = $2 + $1 + 25¢ + 25¢ + 10¢ + 5¢ = $3 and 65¢, missing 5¢: nickel;
b) value = $5 + 25¢ + 10¢ + 5¢ + 5¢ = $5 and 45¢, missing $1: loonie

**Regrouping coins to make the same total with the least number of coins.**

Review the coins that can be substituted to make a single coin with the same value. Write on the board:

2 ________ make a toonie.
4 ________ make a loonie.
5 ________ make a quarter.
2 ________ and 1 ________ make a quarter.
1 ________ and 3 ________ make a quarter.

Have volunteers fill in the answers on the board. (loonies, quarters, nickels, dimes, nickel, dime, nickels) Draw on the board:

![Image of coins]

ASK: What coin can replace the four quarters? (a loonie) Circle the quarters and draw a loonie below. Point to the remaining two dimes and a nickel. ASK: What single coin can replace two dimes and one nickel? (a quarter)

Circle the coins and draw a quarter, as shown below:

![Image of coins]

ASK: What is the total value of the coins? (125¢) Which drawing would you rather use to count the total value? (the second drawing)
Exercises: Regroup coins to make the same total with the least number of coins.

a) 

b) 

Bonus: 

Answers: a) replace 2 loonies with 1 toonie, replace 5 nickels with 1 quarter; b) replace 3 nickels and 1 dime with 1 quarter; Bonus: replace 5 dimes with 2 quarters, then replace all 4 quarters with 1 loonie

Estimating the value of a large collection of coins. SAY: Regrouping coins is a good way to estimate the value of a large collection of coins. Try to find groups of coins that can be replaced with a loonie or a quarter. Draw on the board:

ASK: What coins can be replaced by a loonie? (4 quarters) What coins remain? (3 dimes and 1 nickel) Do you think the remaining coins make a dollar? (no) SAY: So, we know there is about 1 dollar in the collection of coins. Ask students to find the actual value by counting the coins with the greatest value first. (25, 50, 75, 100, 110, 120, 130, 135) ASK: How many dollars and cents are in 135 cents? ($1 and 35¢) SAY: So, our estimate of 1 dollar was close to the actual value of 1 dollar and 35 cents.

Exercises: Estimate to the nearest dollar. Then find the actual value.

a) 

b) 

Answers: a) estimate: $3, actual: $3 and 20¢; b) estimate: $10, actual: $10 and 35¢
Extensions

1. Using only dimes and quarters, what are the different ways of making 1 dollar?

   Answer: 4 quarters, 2 quarters and 5 dimes, 10 dimes

2. Using only nickels and quarters, what are the different ways of making 1 dollar?

   Answer: 4 quarters, 3 quarters and 5 nickels, 2 quarters and 10 nickels, 1 quarter and 15 nickels, 20 nickels
Goals

- Students will represent amounts up to 10 dollars using loonies, toonies, and 5-dollar bills.
- Students will find the number of nickels, dimes, and quarters in a loonie or a toonie.
- Students will find the total value of a collection of money that includes loonies, toonies, 5-dollar bills, and smaller coins.
- Students will solve word problems involving finding the least number of coins to make change.

PRIOR KNOWLEDGE REQUIRED

- Knows the value of coins
- Can find the value of a collection of coins and 5-dollar bills

MATERIALS

BLM 3 × 3 Grid (p. S-83)
plastic coins

Mental math minute. Place a copy of BLM 3 × 3 Grid on the floor. Student A tosses a plastic coin onto the grid to select a number from 1 to 9. The closest number to the coin is the number selected. Student B does the same. The students create two division equations using the numbers. For example, if the numbers selected are 7 and 6, the students will create the division equations 42 ÷ 6 = 7 and 42 ÷ 7 = 6 and say them out loud. Repeat until all students have had a chance to select a number and say a division equation.

Review counting on to 100. Write on the board:

10, 20, __, __, __, __, __, __, __, __

Have a volunteer write the numbers as they count by 10s to 100. ASK: If we count by 10s with 10 being the first number, how many numbers are in the sequence? (10) What coin has a value of 10 cents? (a dime) SAY: Counting on by 10s is the same as finding the value of a collection of dimes. ASK: How many dimes will I need to make a value of 100 cents? (10) How many dimes are in a loonie? (10)

Write on the board:

5, 10, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __, __

Have a volunteer write the numbers as they count by 5s to 100. ASK: If we count by 5s with 5 being the first number, how many numbers are in the sequence? (20) What coin has a value of 5 cents? (a nickel) SAY: Counting on by 5s is the same as finding the value of a collection of nickels.

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

cents
change
coins
dime
division equation
dollar
loonie
money
nickel
quarter
toone
value

NS3-83 Representing Money to 10 Dollars
Pages 139–141
ASK: How many nickels will I need to make a value of 100 cents? (20) How many nickels are in a loonie? (20) Write on the board:

25, 50, __, __

Have a volunteer write the numbers as they count by 25s to 100. ASK: If we count by 25s with 25 being the first number, how many numbers are in the sequence? (4) What coin has a value of 25 cents? (a quarter) SAY: Counting on by 25s is the same as finding the value of a collection of quarters. ASK: How many quarters will I need to make a value of 100 cents? (4) How many quarters are in a loonie? (4)

Finding the number of nickels, dimes, and quarters in a toonie. ASK: How many loonies are in a toonie? (2) SAY: We can replace each loonie with 10 dimes because there are 10 dimes in a loonie. Write on the board:

1 toonie = 2 loonies
    = 1 loonie + 1 loonie
    = 10 dimes + 10 dimes

ASK: What is 10 + 10? (20) SAY: So, there are 20 dimes in a toonie. Continue writing on the board:

    = 20 dimes

ASK: How many nickels are in a loonie? (20) SAY: We can replace each loonie with 20 nickels because there 20 nickels in a loonie. Write on the board:

1 toonie = 2 loonies
    = 1 loonie + 1 loonie
    = 20 nickels + 20 nickels

ASK: What is 20 + 20? (40) SAY: So, there are 40 nickels in a toonie. Continue writing on the board:

    = 40 nickels

ASK: How many quarters are in a loonie? (4) SAY: We can replace each loonie with four quarters because there are four quarters in a loonie. Write on the board:

1 toonie = 2 loonies
    = 1 loonie + 1 loonie
    = 4 quarters + 4 quarters

ASK: What is 4 + 4? (8) SAY: So, there are eight quarters in a toonie. Continue writing on the board:

    = 8 quarters

Using loonies and toonies to make a 5-dollar bill. Write on the board:

5 = ___ + ___ + ___ + ___ + ___
5 = ___ + ___ + ___
5 = ___ + ___ + ___ + ___
Ask for volunteers to fill in the blanks, using only the numbers 1 and 2, to make each addition equation true. $(5 = 1 + 1 + 1 + 1, 5 = 2 + 2 + 1, 5 = 2 + 1 + 1 + 1)$

Point to the first equation and ASK: What coin has a value of 1 dollar? (loonie) How many loonies make a 5-dollar bill? (5) Point to the second equation and ASK: What coin has a value of 2 dollars? (toonie) How many loonies and toonies make a 5-dollar bill? (2 toonies and 1 loonie) Point to the third equation. ASK: How many loonies and toonies does this equation suggest make a 5-dollar bill? (1 toonie and 3 loonies)

**Finding the total value of 5-dollar bills, loonies, and toonies.** Draw on the board:

![Image of $5 bill and coins]

ASK: How much is a 5-dollar bill worth? ($5) How much is a toonie worth? ($2) How much is a loonie worth? ($1) What is $5 + 2 + 1$? (8) SAY: So, the total value of the money is 8 dollars.

**Exercises:** Find the total amount of money.

a) ![Image of coins]

b) ![Image of coins]

**Answers:** a) $5 + 2 + 1 + 2 = \$12$, b) $5 + 5 + 1 + 2 = \$14$

**Finding the total value of bills and coins.** Draw on the board:

![Image of $5 bill and coins]

SAY: We find the total amount of dollars and cents separately. ASK: What is the total value of the 5-dollar bill, toonie, and loonie? ($8$) How do you know? (a 5-dollar bill is worth $5$, a toonie is worth $2$, a loonie is worth $1$, $5 + 2 + 1 = 8$) What is the total value of the other coins? (40¢) How do you know? (a quarter is worth 25¢, a dime is worth 10¢, and a nickel is worth 5¢, $25 + 10 + 5 = 40$) SAY: So, the total value of the money is 8 dollars and 40 cents.
Exercises

1. Count the dollars and cents separately.
   a) [Image of 5-dollar bill and 50 cents]
   b) [Image of 5-dollar bill and 50 cents]

   **Answers:** a) $6 and 30¢, b) $7 and 80¢

2. Find the total number of dollars and cents.

<table>
<thead>
<tr>
<th>5-Dollar Bills</th>
<th>Toonies</th>
<th>Loonies</th>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
</table>
   a) 1 | 1 | 0 | 1 | 3 | 1 |
   b) 0 | 2 | 1 | 3 | 0 | 2 |

   **Answers:** a) $7 and 60¢, b) $5 and 85¢

**Word problems involving making change.** Write on the board:

Vicky pays for a book using a $5 bill and a toonie. The cost of the book is $5 and 70¢.
How much money should she get back? Use the least number of coins.

ASK: How much did Vicky pay altogether? ($7) How do you know? (a 5-dollar bill is worth $5, a toonie is worth $2, \(5 + 2 = 7\)) Did Vicky pay too little, exactly enough, or too much? (too much) How do you know? ($7 is more than $5 and 70¢)

Draw on the board:

\[\begin{array}{c}
\text{$5$} \\
\text{and} \\
\text{$70¢$} \\
\end{array}\]
\[\begin{array}{c}
\text{$6$} \\
\end{array}\]
\[\begin{array}{c}
\text{$7$} \\
\end{array}\]

SAY: Let’s determine the difference owed, or the change that Vicky should get back. ASK: What is the next dollar higher than 5 dollars? ($6) How much money do you need to get from 5 dollars and 70 cents to 6 dollars? (30¢)

PROMPT: How much money do you need to get to 100 cents from 70 cents?
Write “30¢” in the first dotted box. SAY: Now we have 5 dollars and 100 cents, which is the same as 6 dollars. ASK: How much money do you need to get from 6 dollars to 7 dollars? ($1) Write “$1” in the next dotted box. ASK: What do you get when you add the two dotted boxes? ($1 and 30¢) SAY: So, Vicky should get back 1 dollar and 30 cents. ASK: How can we make the amount using the least number of coins and bills? (loonie, quarter, nickel)

**Exercise:** Find the difference owed. Show how to make the amount using the least number of bills and coins.

Price of item = $3 and 60¢
Amount paid = $10
Answer

\[ \begin{array}{c}
40\text{¢} \\
6 \\
3 \text{ and } 60\text{¢} \\
4 \\
10
\end{array} \]

Difference owed = $6 and 40¢ ($5 bill, loonie, quarter, dime, nickel)

Extension

Find the difference owed. Show how to make the amount using the least number of bills and coins.

a) Price of item = $2 and 15¢
Amount paid = $5 and 25¢

Answers

\[ \begin{array}{c}
85\text{¢} \\
2 \\
25\text{¢}
\end{array} \]

Difference owed = $2 + 85¢ + 25¢ = $2 and 110¢ = $3 and 10¢ (toonie, loonie, dime)

b) Price of item = $8 and 30¢
Amount paid = $10 and 50¢

\[ \begin{array}{c}
70\text{¢} \\
1 \\
50\text{¢}
\end{array} \]

Difference owed = $1 + 70¢ + 50¢ = $1 and 120¢ = $2 and 20¢ (toonie, dime, dime)
Goals
Students will write money amounts using dollars and cents notation.
Students will represent a given amount of money written in dollars and cents notation using bills and coins.
Students will write the amount of a collection of bills and coins using dollars and cents notation.

PRIOR KNOWLEDGE REQUIRED
Knows the value of coins
Can find the value of a collection of coins and 5-dollar bills

Mental math minute. Teach students how to take their pulse by placing two fingers on the side of their neck and pressing gently. The class will need to be very quiet for them to be able to feel their pulse. Have them all count the beats for one minute. After they count their pulses, have students work in pairs to create addition equations. For example, if Student A has a pulse of 78 and Student B has a pulse of 83, they will create the addition equations 78 + 83 = 161 and 83 + 78 = 161. Repeat until all students have had an opportunity to create equations.

Introduce dollars and cents notation. Draw on the board:

$5 bill  loonie  quarter  nickel

ASK: How many dollars do we have? (6) How do you know? (a 5-dollar bill is worth $5 and a loonie is worth $1, 5 + 1 = 6) How many cents do we have? (30) How do you know? (a quarter is worth 25¢ and a nickel is worth 5¢, 25 + 5 = 30) SAY: So, the amount of money we have is 6 dollars and 30 cents. Write on the board:

6 dollars and 30 cents

SAY: We can use symbols to shorten this. Write “$6 and 30¢” on the board. SAY: There is an even shorter way of writing this called dollars and cents notation. Write “$6.30” on the board. Point to the last two lines and SAY: These notations refer to the same amount of money. Look at the dollars and cents notation. ASK: What do you think the number on the right side of the dot tells us? (the number of cents) Draw a box around the 30. ASK: What do you think the number on the left side of the dot tells us? (the number of dollars) Draw a box around the 6 and write the labels on the board, as shown below:

number of dollars

$6.30

dot

number of cents
SAY: We only use the dollar sign in dollars and cents notation. We do not include the cent sign because the dot tells us that the next two digits represent the number of cents.

Write "$8.45" on the board. ASK: How many dollars do we have in this amount? (8) How do you know? (the number on the left side of the dot tells us how many dollars) How many cents do we have? (45) How do you know? (the number on the right side of the dot tells us how many cents)

SAY: In dollars and cents notation, we always use two digits to show the number of cents. ASK: How do you think we should write 5 cents if we have to use two digits? (05) Write on the board:

$7.05

Write "$7.50" on the board. ASK: How is this different from the previous amount? (we have 50¢ instead of 5¢)

Exercises
1. Write the number of dollars and cents.
   a) $4.25    b) $8.95    c) $3.05
   **Answers:** a) $4 and 25¢, b) $8 and 95¢, c) $3 and 5¢

2. Write in dollars and cents notation.
   a) 3 dollars and 70 cents     b) $6 and 25¢
   c) $5 and 5¢
   **Answers:** a) $3.70, b) $6.25, c) $5.05

Writing the value of bills and coins using dollars and cents notation.
Draw on the board:

![Coins and bills diagram]

ASK: How many dollars do we have? (3) How do you know? (a toonie is worth $2 and a loonie is worth $1, 2 + 1 = 3) How many cents do we have? (65) How do you know? (a quarter is worth 25¢, two quarters are worth \(25 + 25 = 50¢\); a dime is worth 10¢; a nickel is worth 5¢; altogether, \(50 + 10 + 5 = 65\)) SAY: So, we have 3 dollars and 65 cents. Write on the board:

$3 and 65¢

ASK: How do we write 3 dollars and 65 cents in dollars and cents notation? ($3.65) Write "$3.65" on the board.
Exercises: Find the total number of dollars and cents. Write the answer in dollars and cents notation.

a)

b)

Answers: a) $5 and 45¢, $5.45; b) $8 and 5¢, $8.05

Collecting coins with a given value using the least number of bills and coins. Write “$8.65” on the board. SAY: Let’s find the bills and coins we need to make this amount of money. ASK: How many dollars are there? (8) How many cents are there? (65) What is the largest bill you can use without going over 8 dollars? (5-dollar bill) If you use a 5-dollar bill, how much more money do you need to get to 8 dollars? ($3) ASK: How do you know? (8 − 5 = 3) SAY: Now you need to make 3 dollars. ASK: What is the largest coin you can use without going over 3 dollars? (toonie) If you use a toonie, how much more do you need to get to 3 dollars? ($1) ASK: How do you know? (3 − 2 = 1) What coin can you use to make 1 dollar? (loonie) SAY: Let’s check: We’ve used a 5-dollar bill, a toonie, and a loonie. ASK: What is the total value? ($8) How do you know? (a 5-dollar bill is worth $5, a toonie is worth $2, and a loonie is worth $1, 5 + 2 + 1 = 8) How can you make 65 cents with the least number of coins? (2 quarters, 1 dime, and 1 nickel) How do you know? (each quarter is worth 25¢; two quarters are worth 50¢; a dime is worth 10¢; a nickel is worth 5¢; altogether, 50 + 10 + 5 = 65) Draw on the board:

Exercises

1. Draw the money for the value. Use the least number of bills and coins.

   a) $4.90  
   b) $6.30

Answers

   a)

   b)

2. Find two ways of using coins to make the value $4.05.

Sample answers
Extensions

1. Use exactly four coins to make the given amount.
   a) $3.25  
   b) $1.25
   
   **Answers:** a) loonie, loonie, loonie, quarter; b) loonie, dime, dime, nickel

2. Edmond has some nickels and dimes in his pocket. He has two more dimes than nickels. If he has 50¢ altogether, how many nickels and dimes does he have?

   **Answer:** 4 dimes and 2 nickels (10¢ + 10¢ + 10¢ + 10¢ + 5¢ + 5¢ = 50¢)

3. Rani has a 5-dollar bill, 5 quarters, and 2 dimes.
   a) How many cents does she have in coins?
   b) Rani writes down $5.145 for the total amount of money. Is she correct? Explain.
   c) How many dollars and cents does she have in coins?
   d) How many dollars does Rani have altogether?
   e) How many cents remain?
   f) How much money does she have using dollars and cents notation?

   **Answers:** a) 145¢ (25¢ + 25¢ + 25¢ + 25¢ + 10¢ + 10¢ = 145¢); b) no, she is not correct, in dollars and cents notation, we only use 2 digits for the number of cents; c) $1 and 45¢; d) $6 ($5 from the 5-dollar bill, $1 from the coins); e) 45 cents; f) $6.45
Goals
Students will find the total value of a collection of bills that includes 10-dollar bills, 20-dollar bills, 50-dollar bills, and 100-dollar bills.
Students will use paper bills to create a given value of money.
Students will find the value of a collection of paper money and coins.
Students will use paper bills and a collection of coins to create a given value of money using the least number of bills and coins.
Students will learn about bank accounts, deposits, withdrawals, and balances.

PRIOR KNOWLEDGE REQUIRED
Can find the total value of a collection of bills and coins
Can add and subtract multiples of 10 within 1000

MATERIALS
transparency of BLM Money (p. S-81)
overhead projector

Mental math minute. Teach students how to take their pulses by each placing two fingers on the side of their neck and pressing gently. The class will need to be very quiet for them to be able to feel their pulse. Have them all count their beats for one minute. After they count their pulses, have students work in pairs to create subtraction equations. For example, if Student A has a pulse of 78 and Student B has a pulse of 83, they will add their pulses and create the subtraction equations 161 - 83 = 78 and 161 - 78 = 83. Repeat until all students have had an opportunity to create equations.

Introducing more bills. Display BLM Money. Point to the paper money bills. SAY: The name of the bill tells you the value of the bill. A 10-dollar bill is worth 10 dollars. A 20-dollar bill is worth 20 dollars. A 50-dollar bill is worth 50 dollars. And a 100-dollar bill is worth 100 dollars. Write on the board:

10, 20, __, __, __, __, __, __

Have a volunteer fill in the blanks as they count by tens. ASK: If we count by 10s starting at 10, what is the second number in the list? (20) What is 10 + 10? (20) What is the fifth number in the list? (50) What is 10 + 10 + 10 + 10 + 10? (50) What is the tenth number in the list? (100) What is 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10? (100) Say the tens slowly so that students can count how many tens you have said.

ASK: How many 10-dollar bills do I need to make 20 dollars? (2) How do you know? (10 + 10 = 20) How many 10-dollar bills do I need to make 50 dollars? (5) How do you know? (10 + 10 + 10 + 10 + 10 = 50) ASK: How many 10-dollar bills do I need to make 100 dollars? (10) How do you know? (ten 10s is 100)
SAY: When we tried to make 5 dollars using only loonies and toonies, we found the following ways. Draw on the board:

$5 loonie loonie loonie loonie loonie

$5 toonie toonie loonie

$5 toonie toonie loonie loonie loonie

ASK: What are the different ways we can make 50 dollars using only 10-dollar bills and 20-dollar bills? Give time for the class to find as many ways as they can. Then, draw on the board:

$50 $10 bill

$50

$50

$50

Have volunteers fill in the answers. (five 10-dollar bills, two 20-dollar bills and one 10-dollar bill, one 20-dollar bill and three 10-dollar bills)

Finding the total amount of money using 10-dollar bills, 20-dollar bills, and 50-dollar bills. Draw on the board:

$50 bill $20 bill $10 bill $10 bill

ASK: What is the total value of the money here? ($90) How do you know? ($50 + 20 + 10 + 10 = 90)

Exercises

1. Find the total amount of money.

   a) 

   b)

   Answers: a) $70, b) $70
2. Marcel has a $50 bill, two $20 bills, and three $10 bills. What is the total value of his money?

Answer: $120, 50 + 20 + 20 + 10 + 10 + 10 = 120

Introduce terms involved in banking. SAY: Banks are buildings where people keep their money. Have a short discussion as to why people might keep their money in a bank instead of at home. Possible explanations include:

• your money won’t get lost or stolen
• you won’t be tempted to spend your money
• your money won’t get washed with your clothes or eaten by your dog
• your money is easy to get when you need it
• your boss can put your pay in the bank
• you can pay for things online without having to send money in the mail

SAY: The bank keeps people’s money in a vault or safe, where it is locked up and protected. When you need money, you can go to the bank and take some of it out. ASK: How does the bank know how much money you have in the bank? (they keep a record, they write it down) SAY: A bank account is a record of how much money you have and a list of every time you put money in the bank or take money out. When you put money in the bank, it is called a deposit. When you take money out of the bank, it is called a withdrawal. The amount of money you have in the bank account after each deposit or withdrawal is called the balance. Write on the board:

On Monday, Grace had $250 in her bank account.
On Tuesday, she took $30 out of her account.
On Wednesday, she put $50 in her account.
On Thursday, she had $270 left in her account.

ASK: Which amount represents a withdrawal? ($30) Which amount represents a deposit? ($50) Which amounts represent balances? ($250 on Monday, $270 on Thursday)

Solving problems with deposits, withdrawals, and balances. Write on the board:

Kyle has $100 in his bank account.
On Monday, he deposited $20 he had earned for babysitting.
On Tuesday, he withdrew $30 to buy his mom a gift.

ASK: What is the balance at the end of Monday? ($120) How do you know? (he had $100 in his account, then he deposited $20, 100 + 20 = 120) What is the balance at the end of Tuesday? ($90) How do you know? (at the end of Monday, his balance was $120 and he took out $30, 120 – 30 = 90)
Exercises: Tom has $70 in his bank account. Find the balance at the end of each day.

a) On Monday, Tom deposits $50.

b) On Tuesday, Tom deposits $30.

c) On Wednesday, Tom withdraws $40.

d) On Thursday, Tom withdraws $20.

Solutions: a) $70 + $50 = $120, b) $120 + $30 = $150, 
c) $150 − $40 = $110, d) $110 − $20 = $90

Finding the total value of bills and coins using 10-dollar bills, 20-dollar bills, and 50-dollar bills. Draw on the board:

$50 bill $10 bill toonie loonie quarter dime dime

ASK: Which of the coins and bills represent dollars? (50-dollar bill, 10-dollar bill, toonie, loonie) What is the total amount of dollars? ($63) How do you know? (a 50-dollar bill is worth $50, a 10-dollar bill is worth $10, a toonie is worth $2, and a loonie is worth $1, $50 + $10 + $2 + $1 = 63) Which coins represent cents? (quarter, dime) What is the total amount of cents? (45¢) How do you know? (a quarter is worth 25¢, a dime is worth 10¢, 25 + 10 + 10 = 45) Write on the board:

$63 and 45¢

ASK: How do we write this using dollars and cents notation? ($63.45) Write “$63.45” on the board.

Exercises: Count the number of dollars and the number of cents. Write your answer in dollars and cents notation.

a)

b)

Answers: a) $20 + $10 + $1 + $1 = $32, 25¢ + 25¢ + 10¢ = 60¢, so $32.60; 
b) $50 + $50 + $20 = $120, 25¢ + 10¢ + 5¢ = 40¢, so $120.40

Using the least number of bills and coins to make a given value. Write on the board:

$62.90

ASK: How many dollars do we have? (62) How many cents do we have? (90)

SAY: One way to make 60 dollars is using three 20-dollar bills. ASK: How can we make 60 dollars with two bills? (a 50-dollar bill and a 10-dollar bill) How can we make 2 dollars using the least number of coins? (a toonie)
Draw on the board:

$50$ bill  $10$ bill  toonie

SAY: So far, we made 62 dollars. ASK: How can we make 90 cents with the least number of coins? (3 quarters, 1 dime, 1 nickel) How do you know? ($25¢ + 25¢ + 25¢ + 10¢ + 5¢ = 90¢) Draw three quarters, one dime, and one nickel on the board to the right of the toonie.

**Exercises:** Use the least number of bills and coins to make the given value.

a) $90.45  
   b) $33.35

**Answers**

a) one $50 bill, two $20 bills, one quarter, two dimes  
   b) one $20 bill, one $10 bill, one toonie, one loonie, one quarter, one dime

**NOTE:** Extensions 3 and 4 are required to cover the British Columbia curriculum.

**Extensions**

1. Use exactly five bills to make the given amount.

   a) $90  
      b) $100  
      c) $55

   **Answers:** a) $20, $20, $20, $10, $10; b) $50, $20, $10, $10, $10 or $20, $20, $20, $20; c) $20, $20, $5, $5, $5 or $20, $10, $10, $10, $5

2. I have one bill and two different coins. If you give me a certain coin, I could replace all my money with a $10 bill. How much money do I have right now? What bill and coins do I have right now? What coin could I be given to make $10?

   **Answer:** $8; one $5 bill, one toonie, and one loonie; if I am given one toonie, I will have $10

3. Dentalium shells are a type of seashell that some Inuit and First Nations traditionally used as money for trading. The worth of a shell depended on its length. Suppose the shells in the picture below are worth 1 unit, 2 units, and 5 units.

   Find the total value of the string of dentalium shells.

   a)  
   b)  

   **Answers:** a) $2 + 6 + 5 = 13$, b) $1 + 4 + 15 = 20
4. Use the values of the dentalium shells in Extension 3. Suppose you wanted to trade for a canoe using dentalium shells and you have the following shells:

a) Draw the difference owed in dentalium shells if the canoe was worth 28 units.

b) Draw the difference owed in dentalium shells if the canoe was worth 27 units.

Answers

a) \(30 - 28 = 2: \) or

b) \(30 - 27 = 3: \)
Goals

Students will learn how to use multiplication and addition to determine the value of a collection of money that includes bills and coins.

PRIOR KNOWLEDGE REQUIRED

Can find the value of a collection of coins in cents by addition
Can find the value of a collection of bills in dollars by addition
Can find the total value of a collection of bills and coins in dollars and cents by addition
Can write the total value of a collection of bills and coins using dollars and cents notation

Mental math minute. Create random division questions by clapping your hands very quickly and having pairs of students count the number of claps. For example, if you clap quickly nine times for Student A and seven times for Student B, the students will use the product to create the division equations $63 \div 7 = 9$ and $63 \div 9 = 7$ and say them out loud. Once the pair is done, they touch their toes three times and sit down. Continue with new pairs until all students have had a chance to participate.

Review multiplication as repeated addition. Write “3 × 5” on the board.

ASK: How can you write the multiplication as addition? (5 + 5 + 5) What does the 3 tell you? (how many numbers are being added) What does the 5 tell you? (the number that is repeatedly added)

**Exercises**

1. Write the repeated addition as a multiplication.
   
   a) $6 + 6 + 6 + 6$
   b) $9 + 9 + 9 + 9 + 9$
   c) $2 + 2 + 2$

   **Answers:** a) $4 \times 6$, b) $5 \times 9$, c) $3 \times 2$

2. Multiply by using repeated addition.

   a) $4 \times 7$
   b) $3 \times 8$
   c) $2 \times 10$

   **Answers:** a) $7 + 7 + 7 + 7 = 28$, b) $8 + 8 + 8 = 24$, c) $10 + 10 = 20$

Review multiplication as skip counting. Write “$5 \times 3$” on the board.

ASK: How do we write this as repeated addition? ($3 + 3 + 3 + 3 + 3$)

SAY: Let’s add, saying the subtotals out loud together: 3, 6, 9, 12, 15.

Write on the board:

```
3  6  9  12  15
1st 2nd 3rd 4th 5th
```

SAY: $5 \times 3$ is the fifth number when you skip count by 3s.
Exercises: Multiply by skip counting.

a) \(6 \times 5\)  

b) \(7 \times 10\)  

c) \(3 \times 25\)

Answers: a) 5, 10, 15, 20, 25, 30; b) 10, 20, 30, 40, 50, 60, 70, 7 \(\times\) 10 = 70; c) 25, 50, 75; 3 \(\times\) 25 = 75

Finding the value of coins using multiplication. Draw on the board:

\[\text{dime } \text{dime } \text{dime } \text{dime}\]

ASK: What is the value of a dime? (10¢) How can you find the total by addition? \((10 + 10 + 10 + 10 = 40)\) Write the addition on the board.

ASK: How can you find the total by multiplication? \((4 \times 10)\) What is \(4 \times 10?\) (40) Write the multiplication on the board. SAY: So, the total value is 40 cents.

Exercises: Write a multiplication equation to find the total value of the coins.

a) 5 nickels  
b) 3 quarters  
c) 8 dimes

Answers: a) \(5 \times 5\text{c} = 25\text{c}\), b) \(3 \times 25\text{c} = 75\text{c}\), c) \(8 \times 10\text{c} = 80\text{c}\)

Finding the value of coins using multiplication and addition. Draw on the board:

\[\text{dime } \text{dime } \text{dime } \text{dime } \text{nickel } \text{nickel } \text{nickel}\]

ASK: How many dimes are there? (4) How can you use multiplication to find the value of the dimes? \((4 \times 10\text{c})\) How many nickels are there? (3) How can you use multiplication to find the value of the nickels? \((3 \times 5\text{c})\) How can you find the total value of the dimes and nickels? (add the value of the dimes and the value of the nickels) Write on the board:

\((4 \times 10\text{c}) + (3 \times 5\text{c})\)

ASK: What is \(4 \times 10\text{ cents}\)? (40¢) What is \(3 \times 5\text{ cents}\)? (15¢) What is 40 cents + 15 cents? (55¢) Write on the board:

\[= 40\text{c} + 15\text{c}\]  
\[= 55\text{c}\]

Exercises: Use multiplication and addition to find the total value.

a) \[
\]

b) \[
\]

Bonus: \[
\]
Answers: a) \((2 \times 25\text{¢}) + (3 \times 10\text{¢}) = 50\text{¢} + 30\text{¢} = 80\text{¢}\), b) \((3 \times 25\text{¢}) + (2 \times 5\text{¢}) = 75\text{¢} + 10\text{¢} = 85\text{¢}\), Bonus: \((3 \times 25\text{¢}) + (2 \times 10\text{¢}) + (3 \times 5\text{¢}) = 75\text{¢} + 20\text{¢} + 15\text{¢} = 110\text{¢}\)

Finding the values of bills, loonies, and toonies using multiplication.

Draw on the board:

\[
\begin{array}{ccc}
$5 \text{ bill} & $5 \text{ bill} & $5 \text{ bill} \\
\end{array}
\]

ASK: How can you find the total value of the bills using addition? \((5\text{+} + 5\text{+} + 5)\) How can you find the value using multiplication? \((3 \times 5)\) What is the total value? \((15)\)

Exercises: Use multiplication to the find the value.

a)

b)

Bonus:

Answers: a) \(4 \times 2 = 8\), b) \(5 \times 1 = 5\), Bonus: \(3 \times 10 = 30\)

Finding the value of bills, loonies, and toonies using multiplication and addition. Draw on the board:

\[
\begin{array}{ccc}
$5 \text{ bill} & $5 \text{ bill} & $5 \text{ bill} \\
\text{toonie} & \text{toonie} & \text{toonie} & \text{toonie} & \text{toonie} \\
\end{array}
\]

ASK: How many 5-dollar bills are there? \((3)\) How can you use multiplication to find the value of the bills? \((3 \times 5)\) How many toonies are there? \((5)\) How can you use multiplication to find the value of the coins? \((5 \times 2)\) How can you use addition to find the total value of the money? (add the values of the bills and coins) Write on the board:

\((3 \times 5) + (5 \times 2)\)

ASK: What is \(3 \times 5\)? \((15)\) What is \(5 \times 2\)? \((10)\) What is \(15 + 10\)? \((25)\) Continue writing on the board:

\(= 15 + 10\)

\(= 25\)
Exercises: Use multiplication and addition to find the value of the bills and coins.

a) 

b) 

Bonus

Answers: a) \((2 \times $5) + (3 \times $2) = $10 + $6 = $16\), b) \((3 \times $5) + (2 \times $2) = $15 + $4 = $19\), Bonus: \((3 \times $10) + (2 \times $5) = $30 + $10 = $40\)

Extensions

1. Use multiplication and addition to find the total value of three 50-dollar bills, two 20-dollar bills, two 10-dollar bills, three quarters, four dimes, and two nickels. Show the answer in dollars and cents notation.

   \[
   (3 \times $50) + (2 \times $20) + (2 \times $10) + (3 \times 25\text{¢}) + (4 \times 10\text{¢}) + (2 \times 5\text{¢}) = $150 + $40 + $20 + 75\text{¢} + 40\text{¢} + 10\text{¢} = $193.
   \]

   He needs \( $200 - $193 = $7\).

2. Jun has two 50-dollar bills, three 20-dollar bills, five 5-dollar bills, three toonies, and two loonies. He wants to buy a tablet that costs $200. How much more money does Jun need to make his purchase? What is the least number of bills and coins he can use?

   \[
   (2 \times $50) + (3 \times $20) + (5 \times $5) + (3 \times $2) + (2 \times $1) = $100 + $60 + $25 + $6 + $2 = $193.
   \]

   He needs \( $200 - $193 = $7\). He needs a $5 bill and a toonie.
Goals

Students will learn about different ways of withdrawing money from a bank account, including cash, cheques, and bank cards.

Students will find the balance in a bank account after a series of deposits and withdrawals.

Students will discuss different ways of earning money to make deposits.

PRIOR KNOWLEDGE REQUIRED

Can add and subtract multiples of 10 within 1000

MATERIALS

transparency of BLM Ways of Making Payments (p. S-84)
overhead projector

Mental math minute. Create random multiplication questions by clapping your hands very quickly and having pairs of students count the number of claps. For example, if you clap quickly nine times for Student A and seven times for Student B, the students will create the equations $9 \times 7 = 63$ and $7 \times 9 = 63$ and say them out loud. Once the pair is done, they touch their toes three times and sit down. Continue with new pairs until all students have had a chance to participate.

Different ways to make deposits or withdrawals. Display BLM Ways of Making Payments. Point to the bills and coins and SAY: We call bills and coins cash. You can use cash to pay for things at any store. It is usually not a good idea to send cash by mail to someone. ASK: Why not? (cash can be lost or stolen)

SAY: If you want to send money by mail, it is better to write a cheque. Point to the cheque on the display. SAY: A cheque is a note that gives permission to your bank to take money out of your bank account to pay the other person. They don’t even have to be in the same city as you. They can take the cheque to any bank in their city and deposit it into their bank account. Notice that the cheque has a space to write the name of the person who will be depositing the cheque. A cheque can only be used by the person whose name is written on the cheque. So, if it is stolen, a stranger cannot use the cheque to take money out of your account. Point to the line showing the amount. SAY: This is the line that tells the bank how much money to pay the person whose name appears on the cheque. The amount is written in dollars and cents notation as well as in words to be sure of the amount.

Point to the bank card on the displayed BLM. SAY: You can use a bank card at a bank machine to take money out of your account or to deposit money into your account. At many stores, you can even pay for purchases with your bank card.
Making deposits and withdrawals. SAY: Remember that making a withdrawal means you take money out of your bank account, and making a deposit means you put money in the account. The money left in your account is called the balance. Write on the board:

Zara’s father has $150 in his bank account.
a) On Monday, he went to the bank and deposited $50 cash.
b) On Tuesday, he wrote a cheque for $70 to pay for groceries.
c) On Wednesday, he used his bank card to withdraw $20 cash.

ASK: How much money did he have after Monday’s deposit? ($200) How do you know? (he had $150 in his account, and put $50 more into the account)
Write “$150 + $50 = $200” on the board beside part a). SAY: On Tuesday morning, he had 200 dollars in his account. ASK: When he wrote the cheque, did money go into his account or come out? (come out) How much money did he have left in the account after he wrote the cheque? ($130) How do you know? ($200 − $70 = $130) Write the subtraction beside part b). SAY: On Wednesday morning, he had 130 dollars in his account. ASK: After he used his bank card on Wednesday, how much money was left in his account? ($110) How do you know? ($130 − $20 = $110) Write the subtraction beside part c).

Recording deposits, withdrawals, and balances. SAY: The bank uses a table to keep track of the money in your account. Some people also record a table to check that the bank hasn’t made any mistakes. Write on the board:

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposit</th>
<th>Withdrawal</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td></td>
<td></td>
<td>$150</td>
</tr>
<tr>
<td>Monday</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tuesday</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wednesday</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: Let’s keep track of Zara’s father’s balance. ASK: How much money did he have in his account on Sunday? ($150) SAY: Remember the amount left in your account is called the balance. Write “$150” under Balance for Sunday.

ASK: On Monday, did he make a deposit or withdrawal? (deposit) How much was the deposit? ($50) Write “$50” under Deposit for Monday. ASK: When you make a deposit, should we add to or subtract from the previous balance? (add) What is $150 + $50? ($200) Write “$200” under Balance for Monday.

SAY: On Tuesday, he wrote a cheque for 70 dollars for groceries. ASK: Is that a deposit or a withdrawal for his account? (withdrawal) How do you know? (the cheque takes money out of his account to pay for the groceries) Write “$70” under Withdrawal for Tuesday. ASK: After the withdrawal on Tuesday, how much is left in the account? ($130) How do you know? ($200 − $70 = $130) Write “$130” under Balance for Tuesday.
SAY: On Wednesday, he used his bank card to take out 20 dollars. ASK: Is that a withdrawal or deposit? (withdrawal) Write “$20” under Withdrawal for Wednesday. ASK: How much will be in his account after the withdrawal? ($110) How do you know? ($130 − $20 = $110) The final table should look like this:

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposit</th>
<th>Withdrawal</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td></td>
<td>$50</td>
<td>$150</td>
</tr>
<tr>
<td>Monday</td>
<td>$50</td>
<td></td>
<td>$200</td>
</tr>
<tr>
<td>Tuesday</td>
<td></td>
<td>$70</td>
<td>$130</td>
</tr>
<tr>
<td>Wednesday</td>
<td>$20</td>
<td></td>
<td>$110</td>
</tr>
</tbody>
</table>

Point out to students that not all the cells will have numbers in them. In each row, Zara’s father is making a deposit or a withdrawal, but not both. You may want to shade in the empty cells to show students nothing is written in them.

**Exercise:** Record the deposits, withdrawals, and balances.

Glen has $200 in his account on February 1. On February 2, he uses his bank card to take out $50. On February 3, he writes a cheque for $100. On February 4, he deposits $60.

**Answer**

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposit</th>
<th>Withdrawal</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 1</td>
<td>$200</td>
<td></td>
<td>$200</td>
</tr>
<tr>
<td>February 2</td>
<td></td>
<td>$50</td>
<td>$150</td>
</tr>
<tr>
<td>February 3</td>
<td></td>
<td>$100</td>
<td>$50</td>
</tr>
<tr>
<td>February 4</td>
<td>$60</td>
<td></td>
<td>$110</td>
</tr>
</tbody>
</table>

**Earning money.** SAY: Some students have a bank account. They can use it to keep money they get as gifts. ASK: What other ways could students get money? (answers will vary) SAY: Some students, usually a little older than you, earn money by doing different jobs. Encourage a discussion of how students can earn money. Suggestions may include delivering newspapers, mowing lawns, running a lemonade stand, and raking leaves. Some students may earn money by doing chores at home while others may be expected to do chores without earning money.

**Exercises**

1. Alex earns $20 for each neighbour’s lawn that he mows. If he mowed 7 lawns last month, how much money did he earn? Use skip counting to find the answer.

   **Solution:** $20 × 7 = $140
2. Randi sells lemonade. She charges $1 for every 2 glasses of lemonade.
   a) If Luc has $1, how many glasses of lemonade can he buy from Randi?
   b) How much money do 10 glasses of lemonade cost?
   c) On Saturday, Randi sells 80 glasses of lemonade. The lemons cost $20. How much money does Randi earn after paying for the lemons?

   Answers: a) 2 glasses, b) $5, c) money collected = 80 \div 2 = $40, money earned = $40 - $20 = $20

3. Jax has $140 in his bank account on Friday. On Saturday, he shovels 5 driveways and earns $10 for each driveway. On Sunday, he shovels 4 larger driveways at $20 for each driveway. Jax deposits all the money in his bank account. Record the withdrawals, deposits, and balances for his bank account.

   Answer

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposit</th>
<th>Withdrawal</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friday</td>
<td>$140</td>
<td></td>
<td>$140</td>
</tr>
<tr>
<td>Saturday</td>
<td>$50</td>
<td></td>
<td>$190</td>
</tr>
<tr>
<td>Sunday</td>
<td>$80</td>
<td></td>
<td>$270</td>
</tr>
</tbody>
</table>

   Extensions

   1. On December 31, Raj has $100 in his account. Each month, Raj makes deposits of $75 and withdrawals of $50.
      a) How much money will be in his account after 1 month?
      b) How much money will be in his account after 2 months?
      c) Use skip counting to find how much money he will have after 6 months.

   Answers: a) $100 + (75 - 50) = $125, b) $125 + 25 = $150, c) $250 (see completed table below)

<table>
<thead>
<tr>
<th>Dec. 31</th>
<th>1 Month</th>
<th>2 Months</th>
<th>3 Months</th>
<th>4 Months</th>
<th>5 Months</th>
<th>6 Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$125</td>
<td>$150</td>
<td>$175</td>
<td>$200</td>
<td>$225</td>
<td>$250</td>
</tr>
</tbody>
</table>
2. Each time Liz tries to withdraw more money from the bank than she has, she is charged $5 by the bank. What problem will Liz have on January 30? Will she be charged $5 by the bank?

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposit</th>
<th>Withdrawal</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 5</td>
<td></td>
<td></td>
<td>$240</td>
</tr>
<tr>
<td>January 8</td>
<td></td>
<td>$75</td>
<td></td>
</tr>
<tr>
<td>January 10</td>
<td></td>
<td>$40</td>
<td></td>
</tr>
<tr>
<td>January 17</td>
<td></td>
<td></td>
<td>$255</td>
</tr>
<tr>
<td>January 21</td>
<td></td>
<td></td>
<td>$70</td>
</tr>
<tr>
<td>January 22</td>
<td></td>
<td>$20</td>
<td></td>
</tr>
<tr>
<td>January 30</td>
<td></td>
<td>$20</td>
<td></td>
</tr>
<tr>
<td>January 31</td>
<td>$300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answer:** Yes, she will be charged $5 by the bank. On January 22, the balance is $10. On January 30, she tries to take out $20 but only has $10 in her account.
Goals

Students will round a given number to the nearest 5.
Students will round a given amount of money to the nearest nickel.

PRIOR KNOWLEDGE REQUIRED

Can count by 5s to any multiple of 5
Can count by 10s to any multiple of 10
Can find the total value of a collection of money that includes bills and coins
Can find the difference owed when paying for an item
Can subtract two-digit numbers
Can label numbers on a number line

MATERIALS

transparency of BLM Counting by 5s on a Number Line (p. S-85)
overhead projector

Mental math minute. Have students calculate multiples of 5 and 10 by counting out loud. For example, if you have a student stand up and ask them to calculate \(7 \times 5\), the student will count the first seven multiples of 5 (5, 10, 15, 20, 25, 30, 35) out loud, and then say “\(7 \times 5 = 35\).” After the student answers, they do three jumping jacks and sit down. Alternate between asking multiples of 5 and 10 until all students have had a chance to participate.

Review counting by 5s on a number line. Ask the class to count by 5s out loud, starting at the number 0 and stopping at 100. Display BLM Counting by 5s on a Number Line. SAY: Notice the number line doesn’t have every tick mark labelled. It labels every fifth tick mark, starting at zero. Ask for a volunteer to come to the board and continue labelling the numbers by counting by 5s from 0 to 50. Have a different volunteer label the numbers by counting by 5s from 50 to 100.

Determining whether a multiple of 5 comes before or after a given number. Draw on the board or use BLM Counting by 5s on a Number Line to display:

```

30 35 45 50 65 70 25 30
```

Have volunteers label the number for each dot. (33, 46, 69, 27) SAY: On a number line, if a number is to the left of a given number, we say the multiple comes before the given number. If a number is to the right of a given number, the multiple comes after the given number. As an example, point to the 33. ASK: Is 30 to the left or right of 33? (left) SAY: So, we say 30 comes before 33. ASK: Is 35 to the left or right of 33? (right) SAY: So, we say 35 comes after 33. Remember, if we count by 5s starting at zero, the numbers...
we say out loud are called multiples of 5. Multiples of 5 end in either the digit 0 or the digit 5.

ASK: Which multiple of 5, 30 or 35, comes before 33? (30) Which comes after 33? (35) Repeat with the other examples on the board. (45 comes before 46 and 50 comes after 46, 65 comes before 69 and 70 comes after 69, 25 comes before 27 and 30 comes after 27)

**Exercises**

1. Name the multiple of 5 before the given number.
   a) 28  
   b) 41
   **Answers:** a) 25, b) 40

2. Name the multiple of 5 after the given number.
   a) 72  
   b) 89
   **Answers:** a) 75, b) 90

**Bonus:** Name the multiple of 5 before the given number.
   a) 762  
   b) 1989
   **Answers:** a) 760, b) 1985

**Labelling the multiple of 5 before and after a number.** Draw on the board or use BLM Counting by 5s on a Number Line to display:

```
  53  76
```

SAY: Remember that multiples of 5 end in the digit 0 or 5. Have volunteers label the multiple of 5 before and after 53. (50, 55) Repeat for 76. (75, 80)

SAY: We say that 53 is between 50 and 55. We say that 76 is between 75 and 80.

**Exercises:** Fill in the multiples of 5.
   a) 38 is between ___ and ___  
   b) 81 is between ___ and ___
   **Answers:** a) 35, 40; b) 80, 85

**Finding the closer multiple of 5.** Draw on the board or use BLM Counting by 5s on a Number Line to display:

```
  45  50
```

ASK: What number is shown by the dot? (48) What is the multiple of 5 before 48? (45) What is the multiple of 5 after 48? (50) How far from 45 is the number 48? (3) How far from 50 is the number 48? (2) Which multiple of 5 is 48 closer to: 45 or 50? (50)
Exercises: Circle the multiple of 5 that is closer to the number.

a) 30 b) 75

Answers: a) 30, b) 80

Rounding to the nearest multiple of 5. SAY: To round a number to the nearest multiple of 5, you need to find the multiples of 5 that the number is between and then pick the multiple of 5 that is closer. Write on the board:

Round 83 to the nearest multiple of 5.

Step 1: Find the multiple of 5 before 83.
Step 2: Find the multiple of 5 after 83.
Step 3: Pick the multiple of 5 that is closer to 83.

Draw on the board:

ASK: What is the multiple of 5 before 83? (80) What is the multiple of 5 after 83? (85) Label “80” and “85” on the number line. ASK: How far is 83 from 80? (3) How far is 83 from 85? (2) Which is closer to 83, 80 or 85? (85) SAY: So, when we round 83 to the nearest multiple of 5, we get 85.

Exercises: Use a number line to round to the nearest multiple of 5.

a) 61 b) 48

Answers:

da) 60 , b) 50

Rounding to the nearest multiple of 5 without a number line. Write “82” on the board. ASK: What is the multiple of 5 before 82? (80) What is the multiple of 5 after 85? (85) Label “80” and “85” on the number line. ASK: How far is 82 from 80? (3) How far is 82 from 85? (3) Which is closer to 82, 80 or 85? (80) SAY: So, 82 rounded to the nearest multiple of 5 is 80.

Exercises: Round to the nearest multiple of 5 without using a number line.

a) 74 b) 61 c) 93

Answers: a) 75, b) 60, c) 95

Rounding money to the nearest nickel. SAY: In Canada, we no longer use the penny. ASK: What is the value of a penny? (1¢) SAY: The smallest coin we use now is a nickel. ASK: What is the value of a nickel? (5¢) SAY: Governments often collect extra money when we buy items. This extra money is called tax. Sometimes, when tax is added, the total amount is not a multiple of 5. For example, the price including taxes might be 73 cents. ASK: How can you tell this amount is not a multiple of 5? (it doesn’t end in 0 or 5) SAY: If we still used the penny, what coins would we use to make 73 cents? (2 quarters, 2 dimes, 3 pennies)
Draw on the board:

![Image of coins: quarter, quarter, dime, dime, penny, penny, penny]

Verify with the class that the total value is 73 cents. SAY: Since we don’t use the penny, we need to round 73 to the nearest multiple of 5 because the value of our smallest coin, the nickel, is 5 cents.

ASK: What is the multiple of 5 before 73? (70) What is the multiple of 5 after 73? (75) Which is closer to 73, 70 or 75? (75) SAY: So, 73 cents is rounded to 75 cents. We call this rounding to the nearest nickel. So, if something costs 73 cents, you would pay 75 cents in coins.

**Exercises**

1. Use a number line to round to the nearest nickel.
   a)  54¢  
   b)  81¢  
   
   **Answers**
   a) 55¢  
   b) 80¢  

2. Round to the nearest nickel without a number line.
   a)  83¢  
   b)  41¢  
   c)  62¢  
   **Bonus:** 98¢
   
   **Answers:** a) 85¢, b) 40¢, c) 60¢, Bonus: 100¢ or $1

**Extensions**

1. If Canada stops using the nickel, we will have to round money to the nearest dime. Round the amount to the nearest 10¢.
   a)  73¢  
   b)  89¢  
   c)  $2.47  
   d)  $3.01  
   
   **Answers:** a) 70¢, b) 90¢, c) $2.50, d) $3.00

2. If Canada stops using the nickel and dime, we will need to round money to the nearest quarter.
   a)  What are the multiples of 25 to 100?
   b)  Round the given amounts to the nearest 25¢.
       73¢  89¢  $2.47  $3.01
   
   **Answers:** a) 25, 50, 75, 100; b) 75¢, 100¢, $2.50, $3.00
Giving Change (Advanced)

Goals
Students will find the difference owed for up to 10 dollars.

PRIOR KNOWLEDGE REQUIRED
- Can make change using mental math when the amounts are less than 1 dollar
- Can subtract two-digit numbers
- Can count up by 10s to subtract multiples of 10
- Can use dollars and cents notation

Mental math minute. Have students calculate multiples of 25 and 100 by counting out loud. For example, if you have a student stand up and calculate 25 \times 3, the student will count the first three multiples of 25 out loud (25, 50, 75) and then say “25 \times 3 = 75.” After the student answers, they do three jumping jacks and sit down. Alternate between asking for multiples of 25 and 100 until all students have had a chance to participate.

Making change for up to 10 dollars when prices are in whole dollars.
Write on the board:

Price of item = $7
Amount paid = $10

SAY: An item costs 7 dollars, but I paid 10 dollars. ASK: Did I pay too much or not enough? (too much) How do you know? ($10 > $7) How can I calculate the difference? (subtract the price from the amount paid) What is $10 − $7? ($3) SAY: So the difference owed, or change due, is 3 dollars.

Exercises: Find the difference owed.

a) Price of item = $2
   Amount paid = $10
   Answers: $10 − $2 = $8

b) Price of item = $4
   Amount paid = $10
   Answers: $10 − $4 = $6

Making change for up to 1 dollar when prices are in multiples of 10.
Write on the board:

Price of item = 40¢
Amount paid = 100¢

SAY: An item costs 40 cents, but I paid 1 dollar or 100 cents. ASK: Did I pay too much or not enough? (too much) How do you know? (100¢ > 40¢) How can I calculate the difference? (subtract the price from the amount paid) What is 100¢ − 40¢? (60¢) SAY: So the difference owed, or change due, is 60 cents. ASK: How can you subtract 100¢ − 40¢ using mental math? (subtract 10 − 4, and then put a 0 at the end)
Exercises: Find the difference owed.

a) Price of item = 60¢  
   Amount paid = 100¢  

b) Price of item = 30¢  
   Amount paid = $1

Answers: a) 100¢ − 60¢ = 40¢,  
b) $1 = 100¢, 100¢ − 30¢ = 70¢

Finding the difference to the next highest dollar. Write on the board:

$2.40

ASK: How many dollars do we have? (2) What is one greater than 2? (3)  
SAY: The next dollar after $2.40 is $3. We can write this as “$3.00.” Write  
on the board:

$2.40 → $3.00

ASK: How many cents are in $2.40? (40)  
SAY: If we get to 100 cents, we will  
have 3 dollars, which is the next whole dollar. ASK: What is the difference  
from 40 cents to 100 cents? (60¢)

Exercises: Find the difference to the next highest dollar.

a) $2.70 → $3.00  
b) $4.80 → $5.00  
c) $8.10 →

Answers: a) 30¢,  
b) 20¢,  
c) $9.00, 90¢

Finding the difference to $10.00. SAY: People often pay for items with  
a 10-dollar bill. We’re going to practise finding the difference owed when  
paying with a 10-dollar bill. Write on the board:

Price of item = $2.30  
Amount paid = $10.00

ASK: What is the next whole dollar after 2 dollars? ($3) Write on the board:

$2.30 → $3.00 → $10.00

ASK: How many cents do we need to get from $2.30 to the next dollar, $3? (70¢) Write “70¢” in the first dotted box. ASK: How many dollars do we  
need to get from 3 dollars to 10 dollars? ($7) To find the difference from  
$2.30 to $10, what can we do with the two dotted boxes? (add them) What  
is 70¢ + $7? ($7.70)

Repeat with “Price of item = $5.60” and “Amount paid = $10.00.”  
(Difference owed = $4.40)
Exercises: Find the difference owed from $10 for the given price.

a) $4.80  

\[\begin{align*}
\text{Price of item} &= 4.80 \\
\text{Amount paid} &= 10.00 \\
\text{Difference owed} &= 5\, \text{c} + 30\, \text{c} + 5 = 35\, \text{c} \\
\end{align*}\]

b) $6.30  

\[\begin{align*}
\text{Price of item} &= 6.30 \\
\text{Amount paid} &= 10.00 \\
\text{Difference owed} &= 70\, \text{c} + 3 = 73\, \text{c} \\
\end{align*}\]

c) $8.70  

\[\begin{align*}
\text{Price of item} &= 8.70 \\
\text{Amount paid} &= 10.00 \\
\text{Difference owed} &= 30\, \text{c} + 1 = 31\, \text{c} \\
\end{align*}\]

Finding the difference to the next highest dime. SAY: Sometimes, the price of the item is not a multiple of 10. It is easier to count up to the next dollar if we first count to the next highest multiple of 10 or dime. Write on the board:

Price of item = $4.65  
Amount paid = $10.00

ASK: How many cents are in $4.65? (65) What is next multiple of 10 after 65? (70) How many more cents do you need to get from $4.65 to $4.70? (5c) What is the next dollar after $4.65? (5) How many more cents do you need to get from $4.70 to $5? (30c) How many more dollars do you need to get from 5 dollars to 10 dollars? ($5) Write on the board:

\[\begin{align*}
\text{Price of item} &= 4.65 \\
\text{Amount paid} &= 10.00 \\
\text{Difference owed} &= 5\, \text{c} + 30\, \text{c} + 5 = 35\, \text{c} \\
\end{align*}\]

Repeat with “Amount paid = $10.00” and “Price of item = $3.55.” (Difference owed = $6.45)

Exercises: Find the difference owed from $10 for the given price.

a) $2.85  

\[\begin{align*}
\text{Price of item} &= 2.85 \\
\text{Amount paid} &= 10.00 \\
\text{Difference owed} &= 5\, \text{c} + 10\, \text{c} + 7 = 12\, \text{c} + 7 = 17\, \text{c} \\
\end{align*}\]

b) $8.15  

\[\begin{align*}
\text{Price of item} &= 8.15 \\
\text{Amount paid} &= 10.00 \\
\text{Difference owed} &= 5\, \text{c} + 80\, \text{c} + 1 = 85\, \text{c} + 1 = 86\, \text{c} \\
\end{align*}\]
Rounding money to the nearest nickel, and then finding the difference owed. SAY: Governments often collect extra money when we buy items. This extra money is called tax. In Canada, although we no longer use the penny, sometimes when tax is added we get numbers that would normally need a penny. Then, we round the money to nearest nickel before we find the difference owed. Write on the board:

\[
\begin{align*}
\text{Price of product including tax} &= \$3.73 \\
\text{Amount paid} &= \$10.00
\end{align*}
\]

ASK: How many cents are in $3.73? (73) What is the nearest multiple of 5 before 73? (70) What is the nearest multiple of 5 after 73? (75) SAY: We can pretend the price of the product is $3.75 instead of $3.73. Then we can find the difference owed, using $3.75. Write on the board:

\[
\begin{align*}
\$3.75 & \quad \rightarrow \quad \ldots \quad \rightarrow \quad \$10.00
\end{align*}
\]

ASK: What is the next multiple of 10 after 75? (80) Write "$3.80" in the first blank. ASK: What is the difference from $3.75 to $3.80? (5¢) Write "5¢" in the first dotted box. ASK: What is the next highest dollar after $3.80? (4) Write "$4.00" in the second blank. ASK: What is the difference from $3.80 to $4? (20¢) Write "20¢" in the next dotted box. ASK: How many more dollars do you need to get from 4 dollars to 10 dollars? ($6) Write "$6" in the last dotted box. ASK: How do you get the difference owed from $3.75 to $10.00? (add the dotted boxes) What is 5¢ + 20¢ + $6? ($6.25) Write on the board:

\[
\begin{align*}
\text{Difference owed} &= 5¢ + 20¢ + $6 = $6.25
\end{align*}
\]

**Exercises:** You need to pay the amount shown at the cash register. You have a 10-dollar bill. Round the amount to the nearest nickel. Then find the difference owed.

a) $2.31  b) $6.83

**Answers**

a)

\[
\begin{align*}
\$2.30 & \quad \rightarrow \quad \$3.00 \quad \rightarrow \quad \$10.00 \\
\text{Rounded price} &= \$2.30 \\
\text{Difference owed} &= 70¢ + $7 = $7.70
\end{align*}
\]

b)

\[
\begin{align*}
\$6.85 & \quad \rightarrow \quad \$6.90 \quad \rightarrow \quad \$7.00 \quad \rightarrow \quad \$10.00 \\
\text{Rounded price} &= $6.85 \\
\text{Difference owed} &= 5¢ + 10¢ + $3 = $3.15
\end{align*}
\]
Extensions

1. Find the difference owed when the amount paid is $100.00 and the price is $23.63.

   **Answer**

   \[
   \$23.65 \rightarrow \$23.70 \rightarrow \$24.00 \rightarrow \$30.00 \rightarrow \$100.00
   \]

   Rounded price = $23.65
   Difference owed = 5¢ + 30¢ + $6 + $70 = $76.35

2. The price of a toaster is $44.95. The tax is $5.84.

   a) Find the total price including tax.

   
   \[
   \text{Total price} = \$44.95 + \$5.84 = \$50.79
   \]

   b) Find the difference owed when the amount paid is $60.00

   **Answers:** a) Total price = $44.95 + $5.84 = $50.79; b) Price rounded to the nearest nickel = $50.80, difference owed = 20¢ + $9 = $9.20

3. Marko buys a book for $3.50, a notebook for $1.25, and a pack of pencils for 95¢. He has a 5-dollar bill and 3 quarters. There is no tax. Does he have enough money to buy what he needs? Explain.

   **Answer:** Yes, he does have enough money. The cost of the items is $3 + $1 + 50¢ + 25¢ + 95¢ = $4 and 170¢. 170¢ is the same as $1 + 70¢. So, altogether the total cost is $5 and 70¢, or $5.70. Marko has a 5-dollar bill and 3 quarters, which altogether is worth $5.75.
# Money

<table>
<thead>
<tr>
<th>Name</th>
<th>Picture</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny</td>
<td><img src="image" alt="Penny" /></td>
<td>1¢</td>
</tr>
<tr>
<td>Nickel</td>
<td><img src="image" alt="Nickel" /></td>
<td>5¢</td>
</tr>
<tr>
<td>Dime</td>
<td><img src="image" alt="Dime" /></td>
<td>10¢</td>
</tr>
<tr>
<td>Quarter</td>
<td><img src="image" alt="Quarter" /></td>
<td>25¢</td>
</tr>
<tr>
<td>Loonie</td>
<td><img src="image" alt="Loonie" /></td>
<td>$1 or 100¢</td>
</tr>
<tr>
<td>Toonie</td>
<td><img src="image" alt="Toonie" /></td>
<td>$2 or 200¢</td>
</tr>
<tr>
<td>5-dollar bill</td>
<td><img src="image" alt="5-dollar bill" /></td>
<td>$5 or 500¢</td>
</tr>
<tr>
<td>10-dollar bill</td>
<td><img src="image" alt="10-dollar bill" /></td>
<td>$10</td>
</tr>
<tr>
<td>20-dollar bill</td>
<td><img src="image" alt="20-dollar bill" /></td>
<td>$20</td>
</tr>
<tr>
<td>50-dollar bill</td>
<td><img src="image" alt="50-dollar bill" /></td>
<td>$50</td>
</tr>
<tr>
<td>100-dollar bill</td>
<td><img src="image" alt="100-dollar bill" /></td>
<td>$100</td>
</tr>
</tbody>
</table>
Fake Money Game

[Image of a grid of fake money images, including 1 cent, 5 cents, 2 dollars, 25 cents, 50 cents, and 1 dollar images]
3 × 3 Grid

<table>
<thead>
<tr>
<th>8</th>
<th>1</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>
Ways of Making Payments

Cash

Cheque

Bank Card
Counting by 5s on a Number Line
Goals

Students will solve problems involving two or more related quantities by systematically listing all possible options starting with one category.

PRIOR KNOWLEDGE REQUIRED

Is familiar with Canadian coins
Can find the area of a rectangle given its side lengths
Can multiply one-digit numbers
Can add several numbers together
Can find the perimeter of a rectangle given its side lengths
  (for Problem Banks 3–6)
Can recognize two-digit even and odd numbers
  (for Problem Bank 10)
Can divide within 25 (for Extended Problem)
Can read expressions with brackets (for Extended Problem)

MATERIALS

40 counters per student (see Problem Bank 9)
BLM Video Game Fun—Feeding the Dragons (pp. S-98–100, see Extended Problem)

Making sure no possibilities are missed. Tell students that you need to program a machine that will sell snacks for 45 cents. SAY: The machine only accepts dimes and nickels. It does not give change. You have to teach the machine to recognize when it has been given the correct change. The simplest way to do this is to give the machine a list of the only coin combinations to accept. So you need to list all combinations of dimes and nickels that add up to 45 cents.

SAY: The machine uses two types of coins, dimes and nickels. The best way to find all the possible combinations is to list one of the coin types in increasing order. Let’s use dimes. Start with no dimes, then one dime, then two dimes, and so on. ASK: Where do you stop? (4) PROMPT: How many dimes will be too much? (5)

Draw on the board:

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
SAY: If there are no dimes, I need to make 45 cents using only nickels. ASK: How many nickels do I need to make 45 cents if I use no dimes? (9) Verify by counting by 5s from zero, putting one finger up at a time until you have nine fingers up, one for each nickel used. Write “9” in the first row. Tell students you want to know how many nickels you need if you have one dime. Demonstrate counting on from 10 by 5s until you reach 45. ASK: How many fingers do I have up? (7) So how many nickels do I need? (7) Write “7” in the second row. Have volunteers demonstrate counting on for the remaining rows. The completed table is shown in the margin. Keep the table on the board for later use.

Exercises

1. Complete the table for a snack machine set to accept the price given.
   a) 35¢    b) 55¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

2. Decide how many rows to put in the table for dimes and nickels. Complete the table to make the total.
   a) 65¢    b) 85¢

   **Bonus:** Make 145¢ using dimes and nickels. Put dimes first.

**Selected answers**

1. b) 55¢
<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

2. b) 85¢
<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

**Reasons to start with the larger denomination.** Return to the dimes and nickels table on the board. Remind students that when finding all combinations of nickels and dimes that make 45 cents, you started with the dimes and listed all possibilities in order. Point out that you can also start by listing the number of nickels in increasing order.
Begin drawing a table with the “Nickels” heading first, then ASK: What is the biggest number of nickels I need to put in my table? (9) How do you know? (10 nickels would be too much money) Finish drawing the table with rows for zero to nine nickels, as shown in the margin.

SAY: It looks like there are more combinations now, but some of them won’t work. ASK: If there are no nickels, can I make 45 cents with just dimes? (no) Why not? (you can only make multiples of 10 cents with dimes) Put “X” in the first row under “Dimes.” ASK: If I have one nickel, can I make 40 cents with just dimes? (yes) How many dimes would I need? (4) Continue in this way to finish the chart, as shown below:

<table>
<thead>
<tr>
<th>Nickels</th>
<th>Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>X</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

SAY: These are all the same answers as before for the combinations of dimes and nickels that make 45 cents. Point to the second row and SAY: One nickel and four dimes is the same as four dimes and one nickel, which we got last time in the last row. We could start with nickels and get all the same answers, it’s just more work. It takes fewer dimes than nickels to make 45 cents because dimes are worth more than nickels, so you don’t have to try as many if you start with the one that’s worth more.

Exercises: Start with the coin that is worth more.

a) Make a list of dimes and nickels to make 75¢.

b) Make a list of dimes and nickels to make 80¢.

c) Make a list of quarters and nickels to make 85¢.
Answers

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Calculating values when the number of coins is constant instead of the value. Tell students that you have four coins in your pocket and each coin is a dime or a nickel. SAY: I want to know all the possible total values for the four coins. Draw on the board:

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
<th>Total Value (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: If I have no dimes, how many nickels must I have? (4) PROMPT: How many coins do I have in total? Write “4” in the first row under “Nickels.” Repeat for one dime, two dimes, three dimes, and four dimes. (3, 2, 1, 0) Fill in the nickels column as you go. SAY: Now that you know how many of each coin there are, you can figure out the value for each combination of four coins. ASK: How much are zero dimes and four nickels worth? (20¢) Write “20¢” in the in the first row under “Total Value (¢).” Repeat for one dime and three nickels, two dimes and two nickels, three dimes and one nickel, and four dimes and no nickels. (25¢, 30¢, 35¢, 40¢)

ASK: So, if I have four coins, each a dime or a nickel, worth 35 cents, which coins are they? (3 dimes and 1 nickel) If students need help, point to the row that shows a total value of 35 cents.

Exercises: Make a table to answer the question.

a) I have 5 coins, each a quarter or a dime, worth 95¢. Which coins are they?

b) I have 4 coins, each a quarter or a nickel, worth 40¢. Which coins are they?

Bonus: I have 5 coins, each a quarter, a dime, or a nickel, worth 60¢. Which coins are they? List all possibilities. Hint: List all the possible amounts of dimes and nickels when there are 0 quarters, then 1 quarter, then 2 quarters.
Selected solution
Bonus:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickles</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

So, the 5 coins worth 60¢ must be 1 quarter, 3 dimes, and 1 nickel.

**Answers:** a) 3 quarters and 2 dimes, b) 1 quarter and 3 nickels

**Using systematic search in different contexts.** Tell students that they can use the same method to answer the same type of question in different situations. ASK: How many legs does a bird have? (2) How many legs does a cat have? (4) SAY: There are four animals and 10 legs in total, and I want to know how many birds and how many cats there are. Demonstrate how to begin the table and then have students complete it individually as shown below:

<table>
<thead>
<tr>
<th>Birds (2 legs)</th>
<th>Cats (4 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

ASK: If there are four animals with 10 legs in total, how many of each type of animal are there? (3 birds and 1 cat)
Exercises

1. There are 3 animals. Complete the chart to find out how many legs each combination of animals has.

   a) Birds (2 legs) | Cats (4 legs) | Total Number of Legs
       0            | 3             |                
       1            | 2             |                
       2            | 1             |                
       3            | 0             |                

   b) Birds (2 legs) | Ants (6 legs) | Total Number of Legs
       0            | 3             |                
       1            | 2             |                
       2            | 1             |                
       3            | 0             |                

   c) Cats (4 legs) | Ants (6 legs) | Total Number of Legs
       0            | 3             |                
       1            | 2             |                
       2            | 1             |                
       3            | 0             |                

   Answers: a) 12, 10, 8, 6; b) 18, 14, 10, 6; c) 18, 16, 14, 12

2. Make a chart to find the solution to the problem.

   a) Two animals have a total of 8 legs. Each animal is either an ant or a bird. How many of each animal are there?

   b) Four pets have a total of 10 legs. Each animal is either a cat or a bird. How many of each animal are there?

   Answers: a) 1 ant and 1 bird, b) 1 cat and 3 birds

3. Two opposite sides of a die add to 7. The top side is 3 more than the bottom side. What number is on top?

   Answer: 5

4. A domino has a total of 12 dots. One side has two more dots than the other side. How many dots are on each side?

   Answer: 5 and 7

5. Edmond pays 25¢ for the first four minutes of a long-distance phone call, and then 3¢ for every minute after that. If he pays 40¢ altogether, how many minutes does he talk for?
Solution

<table>
<thead>
<tr>
<th>Length of Call (minutes)</th>
<th>Cost (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>6</td>
<td>31</td>
</tr>
<tr>
<td>7</td>
<td>34</td>
</tr>
<tr>
<td>8</td>
<td>37</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
</tr>
</tbody>
</table>

Edmond talks for 9 minutes.

6. Nora and Kyle have stickers. Nora has 5 more stickers than Kyle. Together, they have 19 stickers. How many stickers do they each have?

Answer: Nora has 12 stickers and Kyle has 7 stickers.

7. I roll 3 dice. Two of them show the same number. The total is 11. What did I roll? List all possibilities.

Answers: 3, 3, 5; 4, 4, 3; or 5, 5, 1

Problem Bank

1. The numbers 2 and 3 are factors of 6 because $2 \times 3 = 6$. Also, 1 and 6 are factors of 6 because $1 \times 6 = 6$.
   a) Check the numbers from 1 to 10 to find all the factors of 10.
   b) Check the numbers from 1 to 11 to find all the factors of 11.
   c) Check the numbers from 1 to 12 to find all the factors of 12.
   d) A number is prime if its only factors are 1 and itself. Which number is prime: 10, 11, or 12?

Answers: a) 1, 2, 5, 10; b) 1, 11; c) 1, 2, 3, 4, 6, 12; d) 11

2. Some dragons have 3 heads and some have 9. Rob counts the heads of 5 dragons and gets 27 heads altogether. How many dragons of each kind are there?

Solution

<table>
<thead>
<tr>
<th>Nine-headed Dragons</th>
<th>Three-headed Dragon</th>
<th>Number of Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>33</td>
</tr>
</tbody>
</table>

5 dragons with 27 heads must include 2 nine-headed dragons and 3 three-headed dragons.
3. The perimeter of a rectangle is the distance around the rectangle. Find the whole-number widths and lengths of a rectangle that has the given perimeter. Remember: The width of a rectangle is how long the shorter side is and the length is how long the longer side is.

a) 12 units  
b) 14 units  
c) 16 units  

**Bonus:** For each perimeter, which width and length gives the greatest area? Remember: \( \text{Area} = \text{length} \times \text{width}. \)

**Answers**

<table>
<thead>
<tr>
<th>Perimeter 12</th>
<th>Perimeter 14</th>
<th>Perimeter 16</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Width</strong></td>
<td><strong>Length</strong></td>
<td><strong>Width</strong></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bonus: The rectangle with the greatest area is the one with the greatest width. Perimeter 12: width = 3; Perimeter 14: width = 3; Perimeter 16: width = 4.

4. a) The perimeter of a rectangle is 16 cm. Find the other side length with the given first side length.

<table>
<thead>
<tr>
<th>First Side</th>
<th>Second Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td></td>
</tr>
<tr>
<td>2 cm</td>
<td></td>
</tr>
<tr>
<td>3 cm</td>
<td></td>
</tr>
<tr>
<td>4 cm</td>
<td></td>
</tr>
<tr>
<td>5 cm</td>
<td></td>
</tr>
<tr>
<td>6 cm</td>
<td></td>
</tr>
<tr>
<td>7 cm</td>
<td></td>
</tr>
</tbody>
</table>

b) The perimeter of a rectangle is 16 cm and the area is 15 cm². What are the side lengths? Remember: \( \text{Area} = \text{length} \times \text{width}. \)

**Answers:** a) 7 cm, 6 cm, 5 cm, 4 cm, 3 cm, 2 cm, 1 cm;  
b) 5 cm and 3 cm

5. The perimeter of a rectangle is 18 cm and the area is 20 cm². What are the side lengths?

**Answer:** 4 cm and 5 cm
6. The rectangle shown has a perimeter of 18 cm.

```
  5 cm
?        ?
  5 cm
```

a) What do the two unknown sides add to?
b) What are the two unknown sides?

**Answers:**
a) \(18 \text{ cm} - 5 \text{ cm} - 5 \text{ cm} = 8 \text{ cm}\),
b) \(8 \text{ cm} \div 2 = 4 \text{ cm}\)

7. There are four animals in total. Each animal is either a bird or a cat.

<table>
<thead>
<tr>
<th>Birds (2 legs)</th>
<th>Cats (4 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

a) As you replace a cat with a bird, does the total number of legs increase or decrease? By how much? Why?
b) Look at the pattern in the third column of the chart. What is the pattern rule? Why does this make sense?

**Answers:**
a) the total number of legs decreases by 2, because birds have two fewer legs than cats;
b) the pattern rule is start at 16 and subtract 2 each time, because when you move down a row, you are replacing a cat with a bird, so the total number of legs decreases by 2

8. Admission costs $2 for a child and $3 for an adult. 11 people pay $25 for admission. How many adults and how many children are there among the 11 people?

**Answer:** 8 children and 3 adults

**NOTE:** If students struggle with Problem Bank 9, suggest that they start with as small of an array as possible and increase the size by 1 until they find the solution.

9. Use 40 counters to solve the problem.

a) Jessica placed some counters in a square array. Then she added 11 more counters to arrange all the counters in an array with 1 more row and 1 more in each row. How many counters were in the original array?
b) Matt placed some counters in a square array. Then he removed 6 counters to arrange all the remaining counters in an array with
2 fewer rows but 1 more in each row. How many counters were in the original array?

c) Tasha placed some counters in a square array. Then she added 20 more counters to arrange all the counters in an array with 2 more rows and 2 more in each row. How many counters were in the original array?

**Answers:** a) 25, b) 16, c) 16

10. Birds have two legs. Lewis says he counted a total of 37 legs for the birds he saw. Did he miss any legs? Explain how you know.

**Answer:** Lewis must have missed at least one leg, because 37 is odd and the total number of legs has to be even.
Extended Problem: Video Game Fun—Feeding the Dragons

**MATERIALS**

BLM Video Game Fun—Feeding the Dragons (pp. S-98–100)

**Preparation for the extended problem.** Tell students that the problem is about a video game involving two kinds of dragons: dragons with two heads and dragons with five heads. Draw on the board:

Tell students that the video game has different levels with different numbers of dragons of each kind. Each dragon pops out one head at a time and the player needs to feed all the dragon heads in one minute. As you get to higher levels, there are more heads. When the player succeeds at one level, they go to the next level.

**Extended Problem: Video Game Fun—Feeding the Dragons.** Give students BLM Video Game Fun—Feeding the Dragons. Question 7 provides an opportunity for students to apply the problem-solving strategy of using systematic search. All students might find an answer to Question 7, but students who notice that the strategy can be used will find the problem easier.

**Answers:** 1. a) 5, b) 16; 2. b) 4, c) 17; 3. 30; 4. 9; 5. 5; 6. a) number of dragons, b) number of heads; 7. 4 two-headed dragons and 4 five-headed dragons
In a video game, there are two kinds of dragons:
- two-headed dragons
- five-headed dragons

In order for a dragon to survive, all the dragon’s heads must have food. It is not enough for only one head to eat.

Each dragon pops out one head at a time and the player needs to feed all the dragon heads in one minute. Once the player succeeds, the player moves to the next level.

1. In Level 1, there are 3 two-headed dragons and 2 five-headed dragons.
   ![Dragon Diagram]
   a) How many dragons are there in Level 1? ______
   b) How many heads need to be fed in Level 1? ______

2. In Level 2, there is 1 two-headed dragon and 3 five-headed dragons.
   a) Draw a picture to show the dragons and their heads.
   ![Dragon Diagram]
   b) How many dragons are there in Level 2? ______
   c) How many heads need to be fed in Level 2? ______
3. You get 5 points for feeding each head. How many points do you get for feeding 3 two-headed dragons?

4. In Level 3, there are 18 heads altogether. All the dragons are two-headed dragons. How many dragons are there?

5. In Level 4, there are 25 heads altogether. All the dragons are five-headed dragons. How many dragons are there?

6. In Level 5, there are 3 two-headed dragons and 4 five-headed dragons. Does the expression show the number of heads or the number of dragons?
   a) \(3 + 4\) ________________
   b) \((3 \times 2) + (4 \times 5)\) ________________
7. In Level 6, there are 8 dragons and 28 heads altogether. How many of each kind of dragon are there in Level 6?
Goals
Students will decide for one-step word problems what numbers make sense from a given choice.
Students will solve multistep word problems using the four operations, including in measurement contexts.
Students will solve problems and puzzles using any of the problem-solving strategies studied so far.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract within 1000
Can skip count by 2s, 3s, 4s, 5s, and 10s to multiply and divide
Can solve multistep problems
Can multiply simple multiples of 10 by one-digit numbers
Can round whole numbers to the nearest 10 or 100
(for Problem Bank 1)
Can find the perimeter of a rectangle with given side lengths
(for Problem Bank 7)

MATERIALS
BLM Number Chains (pp. S-108–110, see Problem Bank 6)

Finding numbers that make sense in a context. Write on the board:

___ people each eat ___ slices of pizza for lunch.
Altogether they eat ___ slices of pizza.

SAY: I’m going to fill in some numbers and you tell me if they make sense. Fill in the blanks as shown below:

3 people each eat 2 slices of pizza for lunch.
Altogether they eat 10 slices of pizza.

ASK: Do these numbers make sense? (no) Why not? (because 3 people eating 2 slices each would only be 6 slices altogether) Challenge students to change one number only to make it true. (change the 3 to a 5; change the 10 to a 6) Put the original numbers back in the blanks and ask if there is another way the original numbers can be changed to make it true. Leave this problem on the board for later use.

Exercises: Which two numbers make sense?

a) ___ people are in each car. Choose from: 3, 4, 5
   There are ___ cars.
   Altogether there are 12 people.
b) Three people go apple picking. Choose from: 50, 150, 200
   Each person picks ___ apples.
   Altogether they pick ___ apples.

**Answers:** a) 3 and 4 or 4 and 3, b) 50 and 150

Change the numbers in the blanks in the example on the board to make sense mathematically, but not contextually, as shown below:

- 7 people each eat 50 slices of pizza for lunch.
- Altogether they eat 350 slices of pizza.

ASK: Do these numbers make sense? (no) SAY: But $7 \times 50$ is 350.
ASK: Why don’t these numbers make sense? (because no one can eat 50 slices of pizza for lunch) PROMPT: Can someone eat 50 slices of pizza for lunch? SAY: So, the numbers multiply to the answer 350 but the numbers do not make sense in the problem.

**Exercises:** Which two numbers make the most sense from 5, 20, and 80?

a) ___ people are in each car.
   There are 4 cars.
   Altogether there are ___ people.

b) ___ people are in each bus.
   There are 4 buses.
   Altogether there are ___ people.

c) There are 4 floors in the building.
   Each floor needs ___ steps.
   Altogether, ___ steps are needed.

d) Morning recess lasts ___ minutes.
   Kate spends the recess skipping, talking to friends, playing tag, and playing soccer. She does each activity for ___ minutes.

**Answers:** a) 5 and 20, b) 20 and 80, c) 20 and 80, d) 20 and 5

**Choosing between problem-solving strategies.** Have students brainstorm the different types of problem-solving strategies they have learned so far this year. Write a list of strategies on the board, such as: use smaller numbers, use a number line, use organized search.

**NOTE:** The following examples reflect all the problem-solving strategies used in the problem-solving lessons for Grade 3. Choose among the examples based on which problem-solving lessons you have taught.
Write on the board:

Jack has 35 marbles and Tess has 63 marbles.
How many more marbles does Tess have than Jack?

ASK: How would you solve this problem? (use subtraction) Do you need to check using smaller numbers that subtraction is the correct thing to do here? (no) SAY: You can always do questions like “how many more” using subtraction, so you don’t need to use smaller numbers to check the strategy. ASK: Could you also use a number line to solve this problem? (yes)

Exercises: Sketch a number line to find how many more marbles Tess has than Jack.

a) Jack has 35 marbles and Tess has 63 marbles.
b) Jack has 288 marbles and Tess has 335 marbles.
c) Jack has 593 marbles and Tess has 818 marbles.

Answers: a) 28, b) 47, c) 225

Write on the board:

Jack has 537 marbles and Tess has 545 marbles.

Tess wants to give some marbles to Jack so that they have the same number.

How many marbles should Tess give to Jack?

Read the problem aloud. ASK: Would using smaller numbers help here? (yes) Why? (because it is not obvious how to start the problem) SAY: When you’re not sure how to start the problem and one of the things making the problem difficult is the large numbers, you can try using smaller numbers. Leave this problem on the board.

Write on the board:

Jack has 3 marbles and Tess has 7 marbles.

SAY: Let’s do an organized search, starting with Tess giving Jack 1 marble, then 2 marbles, and so on. Suppose Tess gives Jack 1 marble. ASK: Now how many marbles does Tess have? (6) Now how many does Jack have? (4) Continue writing on the board:

Jack has 3 marbles and Tess has 7 marbles.

4  6

ASK: Do they have the same number of marbles yet? (no) SAY: So we have to keep going. Suppose Tess gives Jack another marble. ASK: Now how many does Tess have? (5) And Jack? (5) Continue writing on the board:

Jack has 3 marbles and Tess has 7 marbles.

4  6
5  5
ASK: Do they have the same number of marbles now? (yes) SAY: Now they have the same number of marbles.

Write on the board:

Tess has ___ more marbles than Jack.
She gives ___ marbles to Jack so they have the same amount.

Have volunteers fill in the blanks. (4, 2)

**Exercises:** How many more marbles does Tess have than Jack? How many should Tess give to Jack so they have the same amount?

a) Jack has 6 marbles and Tess has 14.

b) Jack has 5 marbles and Tess has 11.

c) Jack has 8 marbles and Tess has 12.

d) Jack has 9 marbles and Tess has 19.

**Answers:** a) 8 more, give 4; b) 6 more, give 3; c) 4 more, give 2; d) 10 more, give 5

ASK: Do you see a pattern in your answers? (yes) Challenge students to describe the pattern. (the number Tess needs to give Jack is always half the difference between what Jack and Tess have)

Have students solve the original problem on the board. (4)

**Exercise:** Yu has 352 marbles and Carl has 368 marbles. How many marbles does Carl have to give Yu so that they have the same number of marbles?

**Answer:** 8

**Multi-step word problems practice.**

**Exercises**

a) Mandy needs to be at school in one quarter of an hour. She bikes 1 block every 2 minutes. School is 8 blocks away. Will she be on time?

b) Lela buys 8 books that cost $8 each. Luc buys 9 books that cost $7 each. Who spends more money on books? How much more?

c) Rob swims 8 laps in 48 seconds. John swims 6 laps in 42 seconds. Who swims each lap faster?

d) Mandy has three dogs. All the dogs weigh the same amount. When Mandy weighs herself, the scale says 26 kg. When she holds all three dogs, the scale says 38 kg. How much does each dog weigh?

e) Ronin has six pails that hold 5 L of water each and three pails that hold 4 L of water each. He fills them all up and pours them into a 50 L pail. He wants to fill the 50 L pail completely. How much more water does he need to add?
f) Ed weighs 43 kg and his sister weighs 29 kg. Their dad weighs 72 kg. Ed and his sister go to one side of a seesaw and their dad sits on the other side. Will the seesaw be easy to balance? Explain.

g) Cam buys 3 large pizzas for $18. Sara buys 6 small pizzas for $24. Rani wants to buy 1 large pizza and 1 small pizza. How much will that cost?

h) A store sells 318 bananas, 396 apples, 203 oranges, 132 carrots, 516 potatoes, and 88 yams. Estimate how many more fruits (bananas, apples, and oranges) the store sold than vegetables (carrots, potatoes, and yams).

**Answers:**
a) no; b) Lela spends $1 more; c) Rob; d) 4 kg; e) 8 L; f) yes, because $43 \, \text{kg} + 29 \, \text{kg} = 72 \, \text{kg}$, so Ed and his sister weigh the same together as their dad weighs by himself; g) $10$; h) estimate as about $300 + 400 + 200 = 900$ fruits and about $100 + 500 + 100 = 700$ vegetables, so the store sold about 200 more fruits than vegetables.

**Problem Bank**

1. What number am I?

   a) When you round me to the nearest 10, you get 50. The sum of my digits is 11.

   b) When you round me to the nearest 10, you get 30. The sum of my digits is 8. I am even.

   **Answers:** a) 47, b) 26

2. Rayder has 85 marbles and Simon has 92 marbles. Can Simon give some marbles to Rayder so that they have the same number of marbles? Explain.

   **Answer:** No, Simon has 7 more marbles than Rayder, but 7 is not a multiple of 2, so Simon cannot give Rayder half of the extra marbles.

3. Nina has 5 apples. Alice has 8 apples. Jane has 11 apples.

   a) How many apples do they have altogether?

   b) They decide to share the apples equally. How many apples will each person have?

   c) Who does not need to give or receive any apples?

   d) How many apples do the other two people need to give or receive?

   **Answers:** a) 24, b) 8, c) Alice, d) Jane needs to give away 3 apples and Nina needs to receive 3 apples.
4. a) Add the numbers in each group.

<table>
<thead>
<tr>
<th>A.</th>
<th>B.</th>
<th>C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

b) Switch exactly two numbers so that all three groups have the same total.

**Answers:** a) A: 12, B: 20, C: 16; b) switch 5 and 9 between Groups A and B so all have 16

5. Use the 3s chart and the 9s chart to answer the question.

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>21</td>
<td>24</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>27</td>
</tr>
<tr>
<td>36</td>
<td>45</td>
<td>54</td>
</tr>
<tr>
<td>63</td>
<td>72</td>
<td>81</td>
</tr>
</tbody>
</table>

a) Use the 9s chart to divide.

- \(54 \div 9\)
- \(36 \div 9\)
- \(72 \div 9\)
- \(63 \div 9\)
- \(45 \div 9\)
- \(81 \div 9\)

b) Explain how you used the 9s chart to divide.

c) Look at the numbers in the first row of the 9s chart. Where do they appear in the 3s chart? Why does this make sense?

**Answers:** a) 6, 4, 8, 7, 5, 9; b) the position of the number in the 9s chart is the answer to the division, example: 54 is in the 6th position for \(6 \times 9 = 54\), so \(54 \div 9 = 6\); c) They are in the third column. This makes sense because 9 is \(1 \times 9\), which is \(3 \times 3\). All the numbers in the first row of the 9s chart increase by 9, as they do in each column of the 3s chart; alternatively, use the associative property: \(18 = 2 \times 9 = 2 \times (3 \times 3) = (2 \times 3) \times 3 = 6 \times 3\), so it is in the 6th position of the 3s chart.

6. Provide students with BLM Number Chains, which challenges students to discover a pattern and to make their own number chain that will have the same property. Students will need to recognize what stays the same and what changes from one number chain to the next in order to create their own. This task also allows students to practise the multiplication and division they did throughout the year in the context of an interesting puzzle.

**Answers**
1. a) 5, 25, 5, 1; b) 6, 30, 10, 2; c) 7, 35, 15, 3; d) 10, 50, 30, 6
2. The ending number always equals the starting number.
3. answers will vary
4. The ending number will always equal the starting number.
5. a) \(-10, \div 5\); b) subtraction, because all the other number chains did; c) 10, because that is \(2 \times 5\); d) division, because all the other number chains did; e) 5, because Step 2 used 5
6. a) 6, 30, 20, 4; b) yes, I got the number I started with, which is what I expected
7.  
   a) Add a square to the figure so that the perimeter increases.

   \[
   \begin{array}{|c|c|c|}
   \hline
   & & \\
   \hline
   & & \\
   \hline
   \end{array}
   \]

   b) Add a square to the figure in part a) so that the perimeter decreases.

   c) Add a square to the figure in part a) so that the perimeter stays the same.

   **Sample answers**

   a) \[
   \begin{array}{|c|c|c|}
   \hline
   & & \\
   \hline
   & & \\
   \hline
   \end{array}
   \]

   b) \[
   \begin{array}{|c|c|c|}
   \hline
   & & \\
   \hline
   & & \\
   \hline
   \end{array}
   \]

   c) \[
   \begin{array}{|c|c|c|}
   \hline
   & & \\
   \hline
   & & \\
   \hline
   \end{array}
   \]

8.  
   The capacities of the first two pails are shown. Predict the capacity of the third pail.

   a) \[
   \begin{array}{|c|c|c|}
   \hline
   10 \text{ L} & 20 \text{ L} & ? \\
   \hline
   \end{array}
   \]

   b) \[
   \begin{array}{|c|c|c|}
   \hline
   10 \text{ L} & 30 \text{ L} & ? \\
   \hline
   \end{array}
   \]

   c) \[
   \begin{array}{|c|c|c|}
   \hline
   100 \text{ L} & 300 \text{ L} & ? \\
   \hline
   \end{array}
   \]

   **Sample answers:** a) 50 L, b) 20 L, c) 400 L
Number Chains (I)

1. Look at the number chain.

   \[
   \text{starting number} \rightarrow +4 \rightarrow \times 5 \rightarrow -20 \rightarrow \div 5 \rightarrow \text{ending number}
   \]

   a) Start with 1. What number do you finish with?
   
   \[
   1 + 4 \rightarrow \times 5 \rightarrow -20 \rightarrow \div 5 \rightarrow \]

   b) Start with 2. What number do you finish with?
   
   \[
   2 + 4 \rightarrow \times 5 \rightarrow -20 \rightarrow \div 5 \rightarrow \]

   c) Start with 3. What number do you finish with?
   
   \[
   3 + 4 \rightarrow \times 5 \rightarrow -20 \rightarrow \div 5 \rightarrow \]

   d) Start with 6. What number do you finish with?
   
   \[
   6 + 4 \rightarrow \times 5 \rightarrow -20 \rightarrow \div 5 \rightarrow \]

2. Look at your answers to Question 1. How can you get the ending number from the starting number?
Number Chains (2)

3. Pick a starting number and find the ending number.
   a) \[ \text{starting number} \rightarrow +4 \rightarrow \times 6 \rightarrow -24 \rightarrow \div 6 \rightarrow \text{ending number} \]

   b) \[ \text{starting number} \rightarrow +3 \rightarrow \times 7 \rightarrow -21 \rightarrow \div 7 \rightarrow \text{ending number} \]

   c) \[ \text{starting number} \rightarrow +5 \rightarrow \times 6 \rightarrow -30 \rightarrow \div 6 \rightarrow \text{ending number} \]

4. Look at your answers to Question 3. How can you get the ending number from the starting number?
Number Chains (3)

5. a) Finish the chain so that the chain has the same property as all the other ones.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 2</td>
<td>× 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

starting number → ending number

b) What operation did you use for Step 3? _________________
Why? ____________________________________________________________________

c) What number did you use for Step 3? _____
Why? ____________________________________________________________________

d) What operation did you use for Step 4? _________________
Why? ____________________________________________________________________

e) What number did you use for Step 4? _____
Why? ____________________________________________________________________

6. a) Use the starting number 4 on the chain you created in Question 5.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 2</td>
<td>× 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

starting number → ending number

b) Did you get the ending number you expected? Explain.
Unit 17  Geometry: Transformations and 3-D Shapes

Introduction

In the first part of this unit, students will perform, identify, and describe translations on a grid, including simple maps. They will identify reflections and rotations of simple shapes, and connect reflections to symmetry. Students will also identify and create patterns using reflections, rotations, and/or translations.

In the second part of this unit, students will identify and describe 3-D shapes. They will build 3-D shapes from nets, and create skeletons of 3-D shapes. Students will use these concrete representations, as well as pictures, to identify vertices, edges, and faces of 3-D shapes and sort the shapes by the number of these elements and by the shape of the faces. Students will also describe pyramids and prisms by the shape of bases.

Meeting Your Curriculum

Alberta—Lessons G3-15 through G3-18 are optional for Alberta students. They will learn this material in later grades. Lessons G3-19 through G3-23 are required to cover the Alberta curriculum.

British Columbia—Lessons G3-15 through G3-18 are optional for British Columbia students. They will learn this material in later grades. Note that you can omit exercises involving Venn diagrams, which are also optional. Lessons G3-19 through G3-23 are required to cover the British Columbia curriculum, and so is Extension 3 of Lesson G3-22.

Manitoba—Lessons G3-15 through G3-18 are optional for Manitoba students. They will learn this material in later grades. Lessons G3-19 through G3-23 are required to cover the Manitoba curriculum.

Ontario—All of the lessons in this unit are required for students in Ontario, except Lesson G3 23, which is optional. In this lesson, students describe cones, cylinders, and spheres, which students in Ontario studied in the two previous grades.

Learning geometric terms. For the second part of this unit, students need to be fluent with identifying polygons. Review the shapes and display examples (such as the word wall cards used in Unit 5), along with their names, throughout the lessons. The key polygons for this unit are: triangle, square, rectangle, pentagon, hexagon, heptagon (if you are in Ontario), and octagon.

Recurring games. Variations on Picking Pairs and Memory are used several times throughout this unit. See the introduction to Unit 13 on p. P-1 for a full description of these games.
Materials. In the second part of this unit, students will use physical 3-D shapes. If a classroom set is unavailable, students can make the shapes using BLM Nets (1) to (13) (pp. T-60–72). Each student or pair of students will need one of each shape listed below:

<table>
<thead>
<tr>
<th>Pyramid</th>
<th>BLM Nets Page</th>
<th>Prism</th>
<th>BLM Nets Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>1</td>
<td>Triangular</td>
<td>4</td>
</tr>
<tr>
<td>Square</td>
<td>2</td>
<td>Square*</td>
<td>6</td>
</tr>
<tr>
<td>Rectangular*</td>
<td>13</td>
<td>Rectangular</td>
<td>5</td>
</tr>
<tr>
<td>Pentagonal</td>
<td>7</td>
<td>Pentagonal</td>
<td>8</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>9</td>
<td>Hexagonal</td>
<td>10</td>
</tr>
<tr>
<td>Octagonal*</td>
<td>11</td>
<td>Octagonal*</td>
<td>12</td>
</tr>
<tr>
<td>Cube</td>
<td>3</td>
<td>Cube</td>
<td>3</td>
</tr>
</tbody>
</table>

*One shape for each small group of students is sufficient.

Students in Alberta, British Columbia, and Manitoba will also need a cone and a cylinder (which can be found on BLM Nets (14) to (15) on pp. T-73–74), and a sphere—you can use any small ball, such as a ping-pong ball or tennis ball. Note that commercial pattern blocks are also examples of prisms, and can be used as such.

If students are not working in grid paper notebooks, provide them with grid paper or BLM 1 cm Grid Paper (p. V-8) in Lessons G3-15 to G3-17. You will also need a grid on the board. If your board does not have a grid section, photocopy BLM 1 cm Grid Paper onto a transparency and project it on the board. This will allow you to erase the pictures from the board without erasing the grid.

In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section V, are used in Unit 17:

BLM Empty Spinners (p. V-1)
BLM Multiplication Chain (pp. V-2–7)

Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>G3-15 to 18</td>
</tr>
<tr>
<td>Quiz</td>
<td>G3-19 to 23</td>
<td>G3-19 to 23</td>
<td>G3-19 to 23</td>
<td>G3-19 to 22</td>
</tr>
<tr>
<td>Test</td>
<td>G3-19 to 23</td>
<td>G3-19 to 23</td>
<td>G3-19 to 23</td>
<td>G3-15 to 22</td>
</tr>
</tbody>
</table>
**Goals**

Students will describe movement on a grid from one location to another in terms of translations. Students will translate shapes on a grid.

**PRIOR KNOWLEDGE REQUIRED**

- Can use strategies to mentally add numbers up to 100
- Can distinguish between right and left
- Can draw a shape congruent to a given simple shape

**MATERIALS**

- ball (optional)
- magnetic board (e.g., a baking sheet) with a grid on it for each student
- grid paper or BLM 1 cm Grid Paper (p. V-8)
- small circular magnets
- small figures of basketball players
- BLM Empty Spinners (p. V-1)
- paper clips
- sidewalk chalk (see Extension 1)

**Mental math minute.** Review different ways to add numbers mentally, such as using 10 (example: $4 + 8 = 4 + 6 + 2$), adding tens and ones separately then adding the result (example: to add $34 + 28$, add $30 + 20 = 50$, $4 + 8 = 12$, so $34 + 28 = 50 + 12 = 62$), and counting up to add small numbers (example: $68 + 4 = 72$). Give students more addition questions to practise. You can pass a ball to the student you want to answer the question, and have students pass the ball back to you as they answer. Have volunteers occasionally explain how they got their answers.

**Introduce slides in one direction.** Use a magnetic board, such as a baking sheet, with a grid drawn or taped to it (use grid paper or BLM 1 cm Grid Paper). Draw two points on the grid and label them, as shown below:

```
  A   B
```

SAY: Point A slides to point B. Demonstrate the slide using a small circular magnet. ASK: How many units right did the point slide? (3) Repeat the slide, counting each unit as a class. Write on the board:

```
  From A to B: slide 3 units right
```

Repeat with several other pictures, giving students the direction (right or left, up or down, but not a combination yet), and have students signal how many units the point moved from A to B.
NOTE: If students have difficulty distinguishing between left and right, write the letters “L” and “R” on the left and right sides of the board.

Sliding points in one direction. Draw a point on the grid and label it M. Demonstrate how to slide it 3 units left and label it N. Repeat with 5 units down (from M) and label the new point P.

Exercises

a) Draw a point on a grid. Label it E.

b) Slide point E 4 units right. Label the new point F.

c) Slide point E 2 units left. Label the new point G.

d) Slide point E 3 units up. Label the new point H.

e) Slide point E 2 units down. Label the new point I.

Answers

Identifying combined slides. Draw on the board:

SAY: I want to describe how the dot moved from point A to point B. ASK: Did the dot move right or left? (right) Did it move up or down? (down) Write on the board:

___ units right
___ units down

SAY: To describe the slide, you can draw arrows—first an arrow right, then an arrow down—so that the combination of arrows shows how the dot moved. Draw arrows, as shown below:

ASK: How many units right did the dot move? (3) PROMPT: How many units long is the arrow that points right? (3 units) ASK: How many units down did the dot move? (2) Students can signal the answers; fill in the blanks on the board. Repeat with the picture in the margin with the dot moving from point A to point B. (3 units left, 4 units up)
NOTE: We commonly list the horizontal component of the movement first. Some students might describe the movement using the vertical component first, such as 4 units up and 3 units left. This should not be considered a mistake.

**ACTIVITY 1**

1. Students work in pairs. Player 1 draws two dots that are not on the same grid line and label them 1 and 2. Player 2 describes how to slide from dot 1 to dot 2 along the grid lines.

**Sliding a dot in a combination of directions.** Draw a basketball court on a magnetic board with a grid on it and invite volunteers to move a small circular magnet as if they were passing a basketball. Start at the centre of the court. Have volunteers pass the ball according to the directions below, starting each new slide from the place where the ball ended up as the result of the previous pass.

   a) 3 units right
   b) 5 units left
   c) 7 units down
   d) 2 units up
   e) 2 units left and 5 units up
   f) 4 units right and 6 units down

Position several small figures of basketball players on grid intersections around the “court” and number them (as real-life athletes). Ask questions like the ones below, depending on the position of the players. You can move the players to vary the result. Students can signal the answer by raising the appropriate number of fingers. Sample questions:

   a) Player 1 passes the ball 5 units right and 2 units up. Who receives the pass?
   b) Player 2 wants to pass the ball to Player 3. How many units left and how many units down should the ball go?

**Bonus:** Player 4 sent the ball 3 units up. How many units, and in which direction(s), should Player 5 move to get the pass?

**ACTIVITIES 2–3**

2. Students work in pairs. Give each pair **BLM Empty Spinners** and a paper clip. Have students label the parts of the six-part spinner with the numbers 1 to 6. Have students label the parts of the eight-part spinner with the labels below. The order of segments on the spinner does not matter.

   ___ units left    ___ units left, ___ units up
   ___ units left, ___ units down    ___ units up
   ___ units right    ___ units right, ___ units up
   ___ units right, ___ units down    ___ units down

Have students draw a dot close to the centre of a grid.
Player 1 spins each spinner by spinning the paper clip around the tip of a pencil pressed to the centre of the spinner. Player 1 uses the resulting number and direction to produce a description of the slide. (If a two-direction slide is spun, Player 1 spins the numerical spinner again to obtain the second number of units.) Player 2 slides the dot according to the directions and draws a new dot there. Players switch roles, starting at the dot produced in the previous round. If there is not enough room for the dot to slide, players start at the centre again. **NOTE:** Students will use the same materials in Activity 4.

3. Have students stand in an array to mimic the points on a grid. Assign directions for up, down, right, and left. Give one student a ball and provide directions such as “Slide the ball 3 units to the right.” The student with the ball passes it to the correct “point on the grid.”

**Copying shapes.** For the following exercises, make sure that students leave plenty of space on the right side of each shape.

**Exercises:** Copy the shape. Draw a dot 3 units right from the dot on the shape. Copy the shape so that the dot is on the same vertex of the new shape.

a) ![Image](image1.png)

b) ![Image](image2.png)

c) ![Image](image3.png)

d) ![Image](image4.png)

**Bonus:**

![Image](image5.png)

**Answers**

a) ![Image](image6.png)

b) ![Image](image7.png)

c) ![Image](image8.png)

d) ![Image](image9.png)

**Bonus:**

Students who are struggling with the task will benefit from first working with squares and rectangles inside 2 by 2 grids, for both the resulting and final shape. Using small grids will allow students to position the shape using the sides of the block rather than the dot. See **Question 6.a) to d)** on AP Book 3.2 p. 158. Increase the grid size to 3 by 3, then to 4 by 4.

**Introduce translations.** SAY: When you move a shape without turning it or flipping it over, you translate it, or make a translation. For example, the shapes you just copied moved to the right without turning or flipping. The shape you drew was exactly the same as the first shape; it had the same shape, size, and direction. Mathematicians say that the second shape was a translation of the first shape, or that you translated the first shape. One way to show how many units the shape was translated is to draw a dot...
on the same vertex of both shapes and to write how the dot moved. In the previous exercises, the dot moved 3 units to the right, so we say that the shape was translated 3 units right.

Explain that in a translation, every point on a shape slides the same number of units in the same direction. Demonstrate this with the vertices in part a) of the previous exercises. Have students check parts b) through d) by sliding all of the vertices 3 units right. Emphasize to students that they can use this to help them sketch a translation or to check a translated drawing.

**NOTE:** Students may need to be reminded of this when they work on Questions 9–11 on AP Book 3.2 p. 159.

**Exercises:** Copy the shape. Translate the dot. Then copy the shape so that the dots are on the same vertex of both shapes.

a) 3 units down  

b) 2 units up  

c) 3 units left  

**Bonus:** 2 units right

**Answers**

a)  

b)  

c)  

**Translating shapes in a combination of directions.** Draw on the board:

Demonstrate how to translate the shape 3 units left and 1 unit up, writing the following steps on the board as you go.

**Step 1:** Draw a dot on a vertex of the shape.

**Step 2:** Draw an arrow 3 units left from the dot.

**Step 3:** Draw an arrow 1 unit up from the end of the first arrow. Draw a dot at the end of the arrow.

**Step 4:** Draw the new shape so that the dots are on the same vertex of both shapes.
The final picture should look like this:

![Image of a grid with dots and arrows]

ASK: Do you think it matters on which vertex we draw the first dot? (no)
Invite a volunteer to draw a dot on a different vertex of the original shape and another volunteer to draw the dot on the matching vertex of the second shape. Have another volunteer describe the translation. Repeat with a different vertex, so students see clearly that the choice of the vertex does not matter.

**ACTIVITY 4**

4. Have students repeat Activity 2 using the same labelled spinners and the following modification. Students start with a shape that has an area of 5 square units but is not a rectangle and translate it according to the directions produced by the spinners. Students who are struggling with copying shapes of area 5 square units can work with smaller shapes, such as shapes of area 3 square units, including rectangles. **NOTE:** Students will use the same materials in Extension 2.

**Extensions**

1. In the school yard, use sidewalk chalk to draw a grid on the ground and assign directions for up, down, right, and left. Have students move a certain number of units in various combinations of directions by hopping from point to point on the grid.

2. Have students repeat Activity 4 with the following modifications. Provide students with BLM Empty Spinners and grid paper. Have students label the six-part spinner with the names of special quadrilaterals and polygons, such as rhombus, rectangle, trapezoid, parallelogram, pentagon, and hexagon.

Player 1 spins all three spinners to produce the shape and directions for translation. Player 1 draws the shape that is given by the shape spinner. Player 2 translates the shape. Players can start by drawing the shape on a different location for translation every time, or translate an existing shape if the shape spun is already drawn on the grid.

3. Ella draws a triangle and labels it A. She translates the triangle 4 units right and 3 units up and labels it B. Which translation takes triangle B to triangle A? **Hint:** Draw a picture.

**Answer:** 4 units left and 3 units down
4. Rob draws a pentagon and labels it C. He translates the pentagon 5 units right and 2 units up and labels it D. Then he translates pentagon D 2 units right and 1 unit up. He labels this pentagon E. Which translation takes pentagon C to pentagon E? Hint: Draw a picture.

Answer: 7 units right and 3 units up

5. Zara draws a pentagon and labels it F. She translates the pentagon 3 units right and 2 units up and labels it G. Then she translates pentagon G 6 units left and 5 units down. She labels this pentagon H. Which translation takes pentagon F to pentagon H? Hint: Draw a picture.

Answer: 3 units left and 3 units down
Goals
Students will describe translations of dots and shapes on a grid.

PRIOR KNOWLEDGE REQUIRED
Can subtract two-digit numbers mentally, with and without regrouping
Can distinguish between right and left
Can translate points and shapes on a grid

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. V-8)

Mental math minute. Have students stand in a line. Give the first student a subtraction problem involving 2 two-digit numbers that does not need regrouping, such as $97 - 12$. Students in line repeatedly subtract a number, in this case 12, with each student saying one subtraction aloud. When a student says a subtraction that involves regrouping, emphasize that this subtraction was a bonus. Example: Student 1 says “$97 - 12 = 85$.” Student 2 says “$85 - 12 = 73$.” Student 3 says “$73 - 12 = 61$.” Student 4 says “$61 - 12 = 49$” (note that this is a bonus). Continue until Student 8 says “$13 - 12 = 1$,” then start a new chain.

Review describing translations. Remind students that to describe a translation, or a slide of a point on a grid, they can draw two arrows—one sideways, the other up or down—so that the two arrows together show how the point moves. Then they describe the direction and the length of each arrow. Show the example below, in which point $A$ moves 4 units left and 2 units up to point $B$:

![Diagram of point A moving to point B](image)

Draw another two points on the grid and have a volunteer describe the slide. Write the letters R and L on the right and the left side of the board for students who are struggling with distinguishing between right and left.

Exercises: Describe the slide from the star to each point.
**Answers:** A: 2 units right, B: 3 units up, C: 2 units right and 3 units up, D: 2 units left and 2 units down, E: 4 units left and 1 unit up

**ACTIVITIES 1–2**

1. Students play in pairs. Each player draws a polygon with at least eight vertices, so that all sides go along the grid lines. Players switch drawings and label the vertices with different letters. Each player then describes the path around the polygon, following the format below:

   From A to B: ___ units (up, down, right or left)

   Players check each other’s work.

2. Students play in pairs. Each player draws five points so that no two points are on the same horizontal or vertical lines. The player labels the points with letters in alphabetical order, starting from the letter of their choice. Players exchange drawings and describe the path from the first letter to the last letter in the sequence, then back again (five translations in total). Each translation should have two directions, horizontal (right or left) and vertical (up or down). Players check each other’s work.

**Identifying the starting point of a move.** Draw on the board:

![Diagram of a grid with points labeled E, F, G, C, and D.]

Point out that there are a few different ways to say how to get from one point to another. You can say “Move from A to B” or “Move to B from A.” You can use other words instead of “move,” such as “slide.” Have students suggest different ways to give a direction from one point to another and write them on the board, leaving room for the start and end points, as shown in the examples below:

- Go from ___ to ___.
- Start at ___ and go to ___.
- Proceed to ___ from ___.

Fill in the missing letters with letters from C to G, in any order. Point to the first sentence and ASK: Which word tells you what the starting point is? (from) Which letter should you start at? (the letter after “from”) Circle the starting letter. Discuss identifying the starting point in the rest of the sentences, and circle the starting point in each. Point out that the starting point might not be the first point in the sentence. For example, in the third sentence above, the starting point is mentioned after the finish point.
Have students describe the slide for each sentence you wrote on the board.

**Identifying the endpoint of a given slide.** ASK: If I start at point E and go 1 unit right and 3 units down, which point will I end at? (C) Have a volunteer show the slide with arrows on the board. ASK: If I move 1 unit left and 5 units down from point F, what point will I get to? (D) Again, have a volunteer show the slide with arrows on the board.

**Exercises:** Use the grid to answer the question.

![Grid with points A, B, C, D, and E]

a) Erica moves 2 units left and 3 units up from point A. Which point does she end at?

b) If you slide 2 units left and 3 units up from point D, where do you get?

c) Which point is 2 units right and 5 units up from point D?

d) Which point is 6 units right and 1 unit down from E?

**Bonus:** Which point is 2 units up and 4 units right from D?

**Answers:** a) B, b) E, c) B, d) A, Bonus: A

**Review cardinal directions.** Students might have learned the cardinal directions (west, north, east, south) in social studies in Grade 2. Remind students that people do not say “left,” “right,” “up,” or “down” when using maps. ASK: What directions do people use on maps? (west, north, east, south) Draw on the board:

![Cardinal directions diagram]

ASK: Where on a map is north? (at the top) What direction is opposite to north? (south) Where is south on a map? (at the bottom) Ask if anyone remembers where east and west should be labelled. If nobody remembers, explain that east is on the right, and west is on the left. Label the arrows as shown below. Keep the picture on the board for the rest of the lesson.
Finding places on a map. Draw on the board:

Explain that each square on the map is a city block, and you can only walk along the grid lines, which are the streets. Have a volunteer draw arrows to show how somebody would walk from Amir’s home to the playground. (3 blocks west and 2 blocks south) Have students describe the directions. ASK: If I walk two blocks west and three blocks north from the playground, where would I be? (the store) Have a volunteer draw the arrows to show the directions.

**Exercises:** Use the map on the board to answer the question.

a) What place is 3 blocks west and 1 block south of the school?
b) What place is 1 block west and 3 blocks south of the school?
c) Describe how to go from Amir’s home to the store.
d) Describe how to go to school from the store.

**Bonus:** It takes Amir 1 minute to walk each block. What takes longer, walking from Amir’s home to the school or to the playground? Explain.

**Answers:** a) the library; b) Amir’s home; c) go 5 blocks west and 1 block north; d) go 6 blocks east and 2 blocks north; Bonus: Amir’s home to school: 1 block east and 3 blocks north, 4 blocks in total, so it takes 4 minutes for Amir to walk from home to the school. Amir’s home to the playground: 3 blocks west and 2 blocks south, 5 blocks in total, so it takes Amir 5 minutes to walk from home to the playground. Walking to the playground takes longer.

**Review describing translations of shapes on a grid.** Draw on the board:

Remind students that when they move shapes without turning them or flipping them over, they perform translations. SAY: To know how a shape was translated, draw a dot on one vertex of the shape you start with and draw a dot on the matching vertex of the final shape. Then describe how the dot moves. ASK: How do the arrows help you describe how the shape moved? (the arrows show the slide in each direction: describe
the horizontal arrow first, then describe the vertical arrow) Have students describe the translation of the shape on the board. (3 units left and 1 unit up) ASK: If this were a map, how would you describe this translation? (3 units west and 1 unit north)

**Describing movement between squares on a grid.** Draw on the board:

```
meeting site
lake
village
forest
sports field
```

Explain that this is a map of a community. Each shaded square shows a site on the map. The sites are large, so they are shown as squares and not as dots on the grid. SAY: I want to know how to get from the village to the sports field. This is very similar to describing how to translate one square, such as the village, to another, such as the sports field. Invite volunteers to draw dots on a vertex of the village and the sports field and have other students signal if the dots are on the same vertex of each shaded square. Then have volunteers describe the translation. (3 units east and 1 unit south) SAY: Let’s imagine that each square is 100 m long. Label this on the map. ASK: How far east and how far south do you need to go from the village to the sports field? (300 m east and 100 m south)

**Exercises:** Use the map of the community to answer the question. One square is 100 m.

a) What is 200 m south and 300 m west of the village?

b) How can you get from the forest to the lake?

c) Ronin is at the lake. Describe his path to the meeting site.

**Bonus:** Sun walks 200 m east and 300 m north and arrives at the meeting site. Where did she start?

**Answers:** a) the forest, b) go 100 m east and 400 m north, c) 400 m east and 100 m north, Bonus: the village

**NOTE:** Students may not be familiar with the words “bog” and “swamp.” Before assigning the AP Book pages, make sure students understand that they would need to navigate around the bogs.
Extensions

**NOTE:** Extension 1 is suitable for students who know how to play chess.

1. a) A chess piece moves 2 squares up and 1 square left. What piece is it?
   b) A chess piece moves 3 squares down and 3 squares right. What pieces can move this way?
   c) A chess piece moves 5 squares right. What pieces can do this?
   d) A chess piece takes another piece by moving 1 square left and 1 square down. What piece could this be?

   **Answers:** a) knight; b) bishop or queen; c) rook or queen; d) king, queen, bishop, or pawn

2. Use the map in **Question 7** on AP Book 3.2 p. 162 to answer the question.
   a) Sara walks from the lake along this route: 2 km west, then 4 km south, then 1 km east. Where does she end?
   b) How long is Sara’s path?
   c) Is there a shorter path from the lake to the farm that still goes around the hill? How long is that path?

   **Answers:** a) the farm; b) 7 km; c) yes, 4 km south and then 1 km west, 5 km in total

3. Use the map of a museum below to answer the question. One square is 25 m long.

   ![Map of a museum](image)
   a) Luc is at the northwest corner of the dinosaur exhibit. He walks 25 m west and 100 m south. Where does he arrive?
   b) Describe a path from the northwest corner of the diamond display to the southeast corner of the exit.
   c) Describe a path from the southeast corner of the diamond display to the northwest corner of the exit.
d) Both parts b) and c) describe a path from the diamond display to the exit. Why do they have different answers?

e) Jane is at the centre of the bird exhibit. She walks 125 m east and 125 m north. Where does she end?

**Answers**

a) southeast corner of the bridges  
b) 150 m east and 100 m south  
c) 100 m east and 50 m south  
d) The paths start and end at different corners of the squares. In part b) the path is between the farthest corners and in part c) it is between the closest corners.  
e) centre of the dinosaur exhibit

4. Glen thinks that shape B is a translation of shape A. Is he correct? Explain.

![Diagram](image)

**Answer:** Glen is not correct. Translations do not change size, shape, or direction the shape is facing. Shapes A and B are facing different directions, so B cannot be a translation of A.
**G3-17 Reflections**

Pages 163–164

**CURRICULUM REQUIREMENT**

AB: optional  
BC: optional  
MB: optional  
ON: required  

**VOCABULARY**

attribute  
congruent  
core  
line of symmetry  
mirror image  
mirror line  
pattern  
reflect  
reflection  
term

**Goals**

Students will reflect shapes on a grid.  
Students will use reflections to make patterns.

**PRIOR KNOWLEDGE REQUIRED**

- Can multiply and divide numbers up to $5 \times 5$  
- Can identify a line of symmetry in a shape  
- Can identify shapes that have lines of symmetry  
- Can draw the missing half of a shape with a line of symmetry  
- Can identify the core of a pattern  
- Can extend a repeating pattern  
- Knows that a pattern is created by changing attributes

**MATERIALS**

- ball (optional)  
- grid paper or **BLM 1 cm Grid Paper** (p. V-8)  
- paper rectangle (e.g., sheet of paper)  
- mirror or **Mira**  
- overhead projector  
- blank transparency and markers  
- Mira for each student  
- pentominoes or **BLM Pentominoes** (p. T-53)

**Mental math minute.** Ask students to solve multiplication questions within the range of $1 \times 1$ to $5 \times 5$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 1$, $2 \times 3$, $6 \div 2$, and so on to $5 \times 3$ and $15 \div 5$. Then progress to a different number. Then, try questions out of order, but keep each multiplication and its corresponding division together. You can pass a ball to the student you want to answer the question, and have students pass the ball back to you as they answer.

**Review Identifying lines of symmetry.** Draw on the board:

\[ \begin{align*} & \uparrow \\ & \downarrow \end{align*} \]

SAY: The dashed line divides the arrow into two parts. ASK: What can you say about the two parts? (they are the same size and shape, if you fold the arrow they would match exactly) PROMPT: Are they the same shape? Are they the same size? What would happen if we could fold the arrow along the dashed line? ASK: What do we call shapes that are the same shape and...
size? (congruent shapes) What do we call a line that divides a shape into two congruent parts that match exactly when folded? (a line of symmetry)

Draw on the board:

\[ \text{(triangle)} \]

ASK: Is the dashed line a line of symmetry? (no) Why not? (the parts are not the same shape) Is there a way to draw another line so that it divides the triangle into parts that are congruent? (yes) Invite a volunteer to draw the new line. The line of symmetry should be a vertical line through the top vertex. ASK: Is this a line of symmetry? (yes)

**Exercises:** Copy the shape onto grid paper. Draw the line of symmetry.

a) ![Image](image1.png)  

b) ![Image](image2.png)  

c) ![Image](image3.png)

**Answers**

a) ![Answer Image](answer1.png)  

b) ![Answer Image](answer2.png)  

c) ![Answer Image](answer3.png)

Draw on the board:

\[ \text{(triangle)} \]

ASK: Is this a line of symmetry? (no) Can you draw a line of symmetry for this triangle? (no) If students suggest they can draw a line of symmetry, invite them to copy the shape and draw the line they think is a line of symmetry. Have them check by folding if this is indeed a line of symmetry. Students will see that there is no line of symmetry for this triangle.

Display the following pictures one at a time and have students signal if the shape has a line of symmetry using thumbs up and thumbs down. For the bonus, have students use their arms to show the direction of the line of symmetry.

**Exercises:** Does the shape have a line of symmetry?

a) ![Shape Image](shape1.png)  

b) ![Shape Image](shape2.png)  

c) ![Shape Image](shape3.png)  

d) ![Shape Image](shape4.png)

**Bonus:** What is the direction of the line of symmetry?
**Answers:** a) yes; b) no; c) yes; d) yes; Bonus: a) horizontal, c) diagonal (from bottom left to top right), d) vertical

Draw on the board:

![Diagram](attachment:image.png)

ASK: Is this a line of symmetry? (no) Why? (answers may vary) Does this line divide the rectangle into two congruent parts? (yes) PROMPT: Are the parts the same size? (yes) Are the parts the same shape? (yes) ASK: If you fold this rectangle along this line, will the parts match exactly? (no) Demonstrate with a paper rectangle, such as a regular sheet of paper. Show the folded rectangle from both sides so that students clearly see that the parts do not match exactly. SAY: The parts do not match exactly when folded, so this is not a line of symmetry. ASK: Does a rectangle have any lines of symmetry? (yes) PROMPT: Are there ways to fold the rectangle so that the parts will match exactly? (yes) Have volunteers come up and draw the lines of symmetry on the board (horizontal and vertical) and other volunteers show how to fold the rectangle to verify that the line is indeed a line of symmetry.

**Introduce mirror line and mirror image.** Explain that another way to check if the line is a line of symmetry is by using a mirror. Return to the rectangle on the board with the diagonal as the line of symmetry. Place a mirror along the diagonal line. If you do not have a mirror large enough, project the rectangle on a transparency and use a Mira. ASK: Is the triangle in the mirror the same as the triangle behind the mirror? (no) Repeat by placing the mirror along a true line of symmetry, so that students clearly see that a mirror indeed shows that there is a line of symmetry.

SAY: When two parts match exactly, as when there is a line of symmetry, we call the parts mirror images of each other. Another name for the line of symmetry is mirror line, because if you place a mirror along it, you will see that the halves of the shape are mirror images.

**Reflecting shapes.** Explain that you can make mirror images not only of parts of shapes, but of whole shapes as well. Project a transparency of the picture below on the board:

![Diagram](attachment:image.png)

Invite a volunteer to trace the triangle and the line on the board. Flip the transparency so that the line coincides with the line in the drawing and the projected triangle is a reflection of the triangle in the drawing, as shown below:
SAY: Let’s check if the triangles are mirror images of each other. Use a Mira to check. SAY: The movement I performed is called a flip or a reflection of the triangle over the line. I reflected the triangle in the mirror line. Remind students that when they try to draw a reflection, they are drawing congruent shapes, so the shapes need to be the same shape and size, only placed differently on a page.

Students will need Miras for the following exercises.

**Exercises**

1. Copy the picture onto grid paper. Place a Mira along the dashed line. Draw the reflection of the shapes.

   a) ![Reflection](image1.png)  
   b) ![Reflection](image2.png)  
   c) ![Reflection](image3.png)

   **Answers**
   a) ![Reflection](answer1.png)  
   b) ![Reflection](answer2.png)  
   c) ![Reflection](answer3.png)

2. The dashed line is the mirror line. Copy the picture onto grid paper, then draw the mirror image of the shapes.

   a) ![Reflection](image4.png)  
   b) ![Reflection](image5.png)  
   c) ![Reflection](image6.png)

   **Answers**
   a) ![Reflection](answer4.png)  
   b) ![Reflection](answer5.png)  
   c) ![Reflection](answer6.png)

Have students use Miras to check their work in Exercise 2.

**ACTIVITY 1**

1. Give each small group of students several different pentomino pieces. (Use commercial sets if available. The pieces can also be cut out from BLM Pentominoes.) Each student picks a piece.
Students trace the piece on grid paper, draw a mirror line along one side of the piece, then draw the reflection of the piece in the mirror line. Students check if they have drawn the image correctly by flipping the pentomino piece over the mirror line and seeing if it matches the image. Students then exchange pieces and repeat the activity.

**Using reflections to make patterns.** SAY: Size, colour, shape, and direction are all examples of attributes. ASK: When you look at a shape in a mirror or when you reflect the shape, does it change its size? (no) Does it change the shape? (no) Which attribute changes? (direction, or position on the page) Remind students that when they change attributes the same way repeatedly, say, change size or colour, they create patterns. Explain that they can use reflections to make patterns, too. Draw on the board:

Invite a volunteer to draw a reflection of the shape in the dashed line. Draw another dashed line, as shown in the picture on the left below, and invite another volunteer to perform another reflection. Repeat. The result is shown on the right below.

ASK: Can you predict what the next shape will look like? (the same as the first shape) SAY: The core of the pattern is the part that repeats. ASK: How many shapes are in the core of this pattern? (2) Have a volunteer circle the core, and a few more volunteers extend the pattern by one term each time.

**ACTIVITY 2**

2. Give each student one pentomino piece, either from a commercial set or from BLM Pentominoes. Students place the piece on grid paper and trace it. They draw a line through the rightmost or the lowest side of the shape and reflect the shape in the line. They check the answer by flipping the pentomino piece. Students produce a pattern with at least five terms. Then they exchange the patterns with a partner and draw the next three shapes in their partner’s pattern.
Extensions

1. Students practise reflecting shapes with a partner. Student 1 draws a shape made of 5 to 10 squares and chooses the mirror line, and Student 2 reflects the shape.

2. Player 1 draws a shape made of 5 to 10 squares and its reflection in a mirror line, but misplaces one of the squares in the reflection. Player 2 corrects the mistake.

3. Copy the shape. The slant line is the mirror line. Use a Mira to see what a reflection will look like. Then remove the Mira and draw the reflection.

Answers

a)  

b)  

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**Goals**

Students will identify rotations.
Students will distinguish between translations, reflections, and rotations.
Students will use translations, reflections, and rotations to extend patterns.

**PRIOR KNOWLEDGE REQUIRED**

Can add two-digit numbers
Can identify reflections and translations
Can translate a shape
Can reflect a shape
Can identify the core of a pattern
Can extend a repeating pattern
Knows that a pattern is created by changing attributes

**MATERIALS**

ball or relay race baton (optional)
blank transparencies and markers
overhead projector
BLM Shapes for Rotations and Reflections (p. T-54)
Mira for each student
transparencies of BLM Transparency Cards (p. T-55)
BLM Rotations—Advanced (pp. T-56–57, see Extension 1)

**Mental math minute.** Arrange students in a line and have them add two-digit numbers by adding tens and adding ones. For each addition problem, such as $35 + 46$, students need to say three steps: adding the tens: $30 + 40 = 70$, adding the ones, $5 + 6 = 11$, and finishing the addition, $70 + 11 = 81$, so $35 + 46 = 81$. Students can pass a ball or a relay race baton to the next person in line, who gets a new problem. Start with problems that do not require regrouping, such as $25 + 34$, and continue to problems that require regrouping ones.

**Review translations and reflections.** Draw a right scalene triangle on a transparency and project it on the board. Invite a volunteer to trace the triangle on the board. Slide the transparency without turning it and ASK: What do you call this movement? (translation) SAY: When you move a shape without flipping or turning it, you translate it. ASK: Do size, shape, or direction change when you translate a triangle? (no) SAY: When you make a translation, the new shape is exactly the same as the original shape; the attributes do not change. Sometimes we want to distinguish between the two shapes to show which one is original and which one is new, so we might use a different colour or design for the new shape, but the size, shape, and direction of the shapes are still the same.
Have a volunteer shade the triangle on the board. Flip the transparency over and place the triangle so that it shares a side with the triangle on the board, as shown below:

ASK: Is this a translation? (no) What did we call this change to the triangle in the last lesson? (reflection, the triangles are mirror images of each other) Did any attributes change? (yes) Which attribute changed when I reflected the triangle? (direction) Does this picture have a line of symmetry? (yes) SAY: Remember, the shading is only needed to tell apart the two triangles. When you reflect a shape, the shape and its mirror image together make a picture that has a line of symmetry. Have a volunteer trace the second triangle and draw the line of symmetry in the picture. Keep the picture on the board.

**Introduce rotations.** Flip the triangle back and have another volunteer trace the triangle on the board again. Draw a dot on one of the vertices of the triangle and place a pencil on the dot. Rotate the triangle, so that the picture on the board looks like the one shown below:

SAY: I turned the triangle a quarter-turn. In mathematics, we say that I rotated the triangle around the black dot or that I did a rotation of the triangle. ASK: Did the size, the shape, or the direction of the triangle change? (the size and the shape did not change, the direction did change) Does this picture have a line of symmetry? (no) Have a volunteer trace the second triangle.

**Rotating shapes.** Give each student one strip from BLM Shapes for Rotations and Reflections and have them cut out the shapes. Start with triangle A. Have students trace the triangle, draw a dot on one vertex of the tracing, and press the pencil on the dot and the corner of the triangle. Demonstrate how to do a half-turn and SAY: I turned the triangle a half-turn. Have students turn the triangle a half-turn and trace it again. Do the same by projecting the shape and tracing it on the board. Students’ pictures will depend on the vertex used to rotate the triangle. The three options are shown below:

ASK: Does your picture have a line of symmetry? If students say yes, have them try to fold the picture so that the parts match exactly. Students will see that there is no line of symmetry because the parts do not match.
Have students show their pictures on the board so that all students see the variations. ASK: Did you all produce the same picture? (no) Did anybody get a picture with a line of symmetry? (no) Students can also check for lines of symmetry using Miras.

Repeat with the rest of the shapes on the BLM. Assign some students a half-turn and some a quarter-turn, and have students produce a variety of pictures. Note that shape D can produce pictures with a line of symmetry because it has a line of symmetry itself.

**Identifying rotations.** In advance, photocopy **BLM Transparency Cards** onto transparencies and give each student three different cards. Display four cards as shown below, using the overhead projector.

A.  
B.  
C.  
D.  

Ask students to place the card with the same shape the same way shape A is placed. SAY: I want to know which of the shapes, B, C, or D, is a rotation of A. Write on the board:

Which shape is a rotation of card A?

ASK: Do you need to turn card A to get shape B? (no) What do you need to do? (slide the card over, translate it) Do you need to rotate card A to get shape C? (no) Invite a volunteer to turn the cards different ways without flipping it over so that they can see that C is not a rotation of A. ASK: Did you get shape C? (no) Did you get the shape D? (yes) Invite a volunteer to demonstrate how to turn shape A to get shape D.

NOTE: Students might notice that a full turn of card A returns it to the same direction, so they might claim that card B is a rotation of card A. This answer is not incorrect, but students would be expected to recognize this movement as a translation in external tests, such as standardized or provincial testing.

Repeat with other positions of the same card, so that only one option is a rotation, as shown in the picture below, where B is obtained from A using a rotation.

A.  
B.  
C.  
D.  

Repeat with other cards, including cases where two of the cards (B and C) are rotations, as shown below:

A.  
B.  
C.  
D.  

**Identifying reflections, rotations, and translations.** Use the same cards from BLM Transparency Cards, but display only two cards at a time. Ask students to identify if they need a translation, reflection, or rotation to make
one card from the other. Have students use the cards to try to get from one position to the other. Remind students that if they need to flip the card over, they are reflecting the shape. Point out and demonstrate that when cards show reflection, a reflection of either card takes one card to the other, and when cards show a rotation, you can rotate either card to get to the shape shown on the other card.

**ACTIVITY**

Students play in pairs, using cards from BLM Transparency Cards. Player 1 chooses a card and places it on the table. Player 2 places an identical card the same way and closes her eyes. Player 1 turns the card or flips it over. Player 2 opens her eyes and identifies if the card was rotated or reflected, by using her own card to check. Players switch roles and repeat.

**Extensions**

1. Have students complete BLM Rotations—Advanced.

   **Answers:**
   1. b) 1/2, c) 3/4, d) 3/4, e) 1, f) 1/4; 2. b) 1/2, c) 1/4, d) 3/4, e) 1/2, f) 1; 3. b) 1/4 CW, c) 3/4 CCW, d) 1/4 CCW, e) 1/4 CCW, f) 1 CW, 4. see answers below—the answer arrow is shown in grey:

   ![Diagram](image)

   ![Diagram](image)

   ![Diagram](image)

   ![Diagram](image)

2. Trace a triangle from BLM Shapes for Rotations and Reflections. Label it A.

   a) Rotate the triangle around one of the vertices a half-turn and trace it again. Label it B. Repeat with a half-turn in the opposite direction. What do you notice?

   b) Rotate the triangle around another vertex half turn and trace it. Label the result C. Do you need a reflection, rotation, or translation to get from B to C?

   c) Repeat parts a) and b) with a different shape from the same BLM. Did you get different answers?

   **Answers:**
   a) the result of the rotation is exactly the same for both directions, b) translation, c) the answers are the same

3. Use a shape from BLM Shapes for Rotations and Reflections to produce a pattern. Trace the shape, then translate it, reflect it, or rotate it to draw each next term. Use the same movement each time. Draw at least five shapes in the pattern. Have a partner extend your pattern.
Goals

Students will identify faces, vertices, and edges of 3-D shapes.

Students will count vertices and edges of 3-D shapes using actual shapes and pictures.

Students will sort 3-D shapes by the number of edges and vertices.

PRIOR KNOWLEDGE REQUIRED

Can subtract two-digit numbers
Can identify and count sides and vertices of 2-D shapes
Can identify and name polygons
Can fill in a Venn diagram

MATERIALS

large paper polygons, including a rectangle and a square
various 3-D shapes, including a cube, a square-based prism, and some everyday objects (e.g., boxes, a ball, a can, a hockey puck)
a cube, prism, and pyramid per student

BLM Matching 3-D Shapes (1) (p. T-58, see Extension 1)
flashlight or overhead projector (see Extension 3)

Mental math minute. Have students stand in a line. Give the first student a subtraction problem in which a small two-digit number is subtracted from a large two-digit number, such as 92 − 15. Students in line repeatedly subtract a number, in this case 15, by each student saying one subtraction aloud. Example: Student 1 says “92 − 15 = 77.” Student 2 says “77 − 15 = 62.” Student 3 says “62 − 15 = 47.” Continue until the last subtraction gets the answer smaller than 15, “17 − 15 = 2,” then start a new chain.

Introduce 3-D shapes. Hold up a large paper rectangle and square. Ask students to identify the shapes. ASK: How are these shapes different? (a rectangle has two longer sides and two shorter sides and a square has four equal sides) How can you check that? (measure the sides, fold to check) Invite volunteers to measure the sides of the rectangle and the square to confirm that they are indeed different shapes. Show students a large cube. Explain that squares, rectangles, circles, triangles, and other flat shapes are called two-dimensional or 2-D shapes. Shapes like cubes, that are not flat, are called three-dimensional or 3-D shapes.

Hold up various paper polygons and 3-D shapes, including everyday objects (e.g., a box, a ball, a cylindrical can, or a hockey puck). For each shape, have students signal two fingers if it is a 2-D shape and three fingers if it is a 3-D shape.

Introduce faces. Give each student a cube, a prism, and a pyramid—use a variety of different prisms and pyramids. Hold up a large cube and ask if
anyone remembers what this shape is called. Students should be familiar with cubes from earlier grades. Have students identify the cube in their collection and hold it up.

Explain that the flat sides of a 3-D shape are called faces. Point to the faces on the large cube, and ASK: What polygons are the faces of a cube? (squares) SAY: Hold up a different shape that has some faces that are not squares. Ask volunteers to show the shape to the class, point out the face that is not a square and identify its shape. Point out that some 3-D shapes have faces that are triangles, and some 3-D shapes have faces that are rectangles. ASK: Does anyone have a shape that has only triangles as faces? Have a student show the shape to the class, turning it around so that everyone can see that this shape has only triangular faces. Repeat with a shape that has only rectangles for faces. Remind students that squares are a special case of rectangles, so a cube qualifies. Ask students if anyone has a shape that has only rectangles as faces, but is not a cube. Again, have students show the shape if they have it.

**Counting faces.** Ask students to count the faces on their cubes. Discuss strategies to keep track of faces counted. If the following strategy does not arise, show it to students: Count the top and the bottom first, then look at the shape from the top. From the top, the cube looks like a square. But each side of that square is the side of another “hidden” square that you can see when you look at the cube from the side. Because we know that a square has four sides, there are four “hidden” squares. 4 squares (on the sides) + 2 squares (top and bottom) = 6 squares altogether, which means a cube has 6 faces.

Have students count faces on their other two shapes. Invite volunteers to show their shapes and explain how they kept track of the faces. Students who finish early can exchange shapes with a partner and count the faces on their partner’s shapes.

**Introduce edges of 3-D shapes.** Hold up a cube. Run your finger along one of the edges and explain that the place where two faces meet is called an edge. Ask students to show an edge on their cubes.

**Counting edges.** Hold up a square-based prism and explain that you want to count the edges of this shape. Place the prism on the desk and SAY: I see three groups of edges. There are edges on the bottom face, the edges that run along the desk. They are the sides of the bottom face. Lift up the prism and trace the edges of the bottom face. ASK: How many edges like this do we have? (4) Write “4” on the board. Place the prism on the desk again. SAY: There are edges that only touch the desk at one end, the longer, vertical edges. Trace these edges with a finger and have students show the same edges on their square-based prisms. ASK: How many edges like this do we have? (4) Write “+ 4” on the board. SAY: There are edges that do not touch the desk at all; they are the sides of the top face. Trace them with a finger. ASK: How many edges like this does the shape have? (4) Write “+ 4” on the board. ASK: Did we miss any edges? (no) How many edges are there in total? (12) Write “= 12” on the board to finish the calculation. SAY: The shape has 12 edges.
Have students count the edges of their other two shapes. Have them discuss strategies in pairs. Students who finish early can exchange shapes with a partner and count the edges of their partner’s shapes.

**Introduce vertices of 3-D shapes.** Show students a paper square. Invite a volunteer to identify the vertices. ASK: How do you know these are vertices? (they are the corners where sides meet) Do they feel different? (they are pointy, they are sharp turns, so they feel the sharpest) Ask students to hold up a cube. Ask them to feel their cubes and to show which places on the cubes feel pointy. Explain that these are also called vertices. Ask a volunteer to show the vertices of a cube. Have students show the vertices of the other 3-D shapes. Point out that edges of 3-D shapes meet at vertices. SAY: Just as sides of flat shapes meet at vertices, edges also meet at vertices.

**Counting vertices.** Discuss strategies for counting vertices of a cube. Guide students through the following strategy if no one suggests it: Set the cube on a desk. Point at two vertices, one on the bottom face and the other on the top face, and ASK: How are these two vertices different? (one is on the top and the other is on the bottom, or one touches the table and the other does not) Can we first count all of the vertices on the bottom, then all of the vertices on the top? (yes) How many vertices are on the bottom? (4) How many vertices are on the top? (4) Are there any vertices in the cube that we did not count? (no) Write on the board:

\[
4 + 4 = 8 \quad \text{A cube has 8 vertices.}
\]

SAY: By counting the bottom vertices separately from the top vertices, we solved two simpler problems and used them to solve the harder problem. ASK: Why was it so easy to find the number of vertices on the bottom? (the bottom is a square) How is the top face similar to the bottom? (it is also a square) How many vertices do two squares have? (8) How many vertices does a cube have? (8)

Ask students to count the vertices of their other two shapes. Students can problem-solve in pairs how to track the number of vertices. Let students share their strategies with the class. Students who finish early can exchange shapes and count the vertices on their partner’s shapes.

**Vertices and edges on pictures of 3-D shapes.** Draw on the board:

![cube diagram]

Ask students to identify the shape. (a cube) Invite a volunteer to place a dot on each vertex they can see in the picture. Count the vertices together (write the numbers beside the dots as you do so). ASK: How many vertices are in the picture? (7) How many vertices does a cube have? (8) Why did we get seven instead of eight? (there is a corner on the back that we do not see) Repeat with edges. Students will see that there are nine edges visible and three edges on the back.
**Introduce hidden edges.** Explain that in mathematics people often draw the parts of shapes that we cannot see (because they are hidden behind other parts) with dashed lines. The dashed lines are behind the solid lines and would only be seen if the shape were made of a clear, see-through material, such as glass. (Show the relative positions of a visible edge and a hidden edge in the cube that look like the intersect with your hands or with two pencils.) SAY: We call the edges we cannot see on a picture *hidden edges*. Add the dashed lines to the cube on the board, as shown below, and invite a volunteer to mark the hidden vertex.

![Diagram of a cube with hidden edges shown]

Draw on the board:

![Diagram of a cube with hidden edges shown]

SAY: I drew dots on the vertices of this box. ASK: Did I draw everything correctly? (no) Point to each dot one at a time and ASK: Is this a vertex? Have students signal the answer with thumbs up for yes and thumbs down for no. Remove (or cross out) the two incorrect vertices and have volunteers count the vertices (8) and the edges (12).

**Sorting shapes.** Display the shapes below:

A. ![Shape A]
B. ![Shape B]
C. ![Shape C]
D. ![Shape D]
E. ![Shape E]
F. ![Shape F]

Point to the shapes one at a time and have students raise the shape if they have it. Ask students to say how many vertices, edges, and faces the shape has. Record the information on the board, as summarized in the table below:

<table>
<thead>
<tr>
<th>Shape</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices</td>
<td>6</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Edges</td>
<td>9</td>
<td>6</td>
<td>12</td>
<td>15</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Number of Faces</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Draw a table on the board as shown below, but do not fill it in. Have students help you to fill in the table.

<table>
<thead>
<tr>
<th>6 or More Vertices</th>
<th>A, C, D, F</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 or Fewer Edges</td>
<td>A, B, E</td>
</tr>
<tr>
<td>Even Number of Faces</td>
<td>B, C, F</td>
</tr>
</tbody>
</table>
Keep both tables on the board for reference.

**NOTE:** The rest of the lesson deals with Venn diagrams, which are optional for British Columbia students.

**Review Venn diagrams.** Draw on the board:

```
3-D Shapes
6 Vertices 6 Faces
```

SAY: I want to sort the shapes into this diagram. ASK: What is this diagram called? (a Venn diagram) What do we call the ovals? (categories) SAY: The category on the left is “6 Vertices.” ASK: Which shapes have 6 vertices? (A and F) Point to the central region and ASK: Should I write the letter A in the central region? (no) Why not? (A has 5 faces, not 6 faces, only shapes with 6 vertices and 6 faces should be in the central region) Where should I write the letter A? (in the category on the left, but outside the central region) Write “A” in the left oval outside the central region. Repeat the discussion with shape F, which should be placed in the central region.

ASK: Does shape B belong in one of the categories? (no) Where should we write the letter B? (outside the ovals) Continue until all the shapes are sorted as shown in the completed diagram below:

```
3-D Shapes
   B, D, E
       6 or More Vertices
```

**Sorting 3-D shapes using a Venn diagram.** Re-label the Venn diagram as shown below. Point out that the categories now match the first two rows of the second table on the board. Remind students that shapes that appear in both categories are placed in the overlap region. Have a volunteer circle the shape that is in both categories. (A) Remind students that shapes that are in neither category should be placed outside both circles. ASK: Are there shapes like that in our collection? (no) Have students sort the shapes using the new categories. The completed diagram should look like this:

```
3-D Shapes
   C, D, F
       6 or More Vertices
```

```
3-D Shapes
   A, B, E
       9 or Fewer Edges
```
**Exercise:** Draw a Venn diagram for the shapes on the board. Use categories "9 or Fewer Edges" and "Even Number of Faces."

**Answer**

![Venn Diagram](image)

**NOTE:** Students who are struggling with counting the edges, vertices, and faces in the pictures in the AP Book can use actual shapes and stickers to keep track while counting.

**Extensions**

1. Have students play **Picking Pairs** and then Memory (see introduction to Unit 13, p. P-1) with the cards from **BLM Matching 3-D Shapes (1)**. The cards match if they show the same shape. For example, a long, thin rectangular prism on card 13 would not match a short, thick rectangular prism on card 11. Players can help each other by asking questions or making suggestions, but they should not tell each other where specific cards are. Example: “I think I know where both rectangular prisms are. Should I turn one of them over?”

2. Ask students to hold cubes in various positions and to look at them from different angles (on the table, on the floor and look at it directly from above, above the eye level, and so on). Ask students to describe what the faces look like when seen from different angles. The faces will look like squares, rectangles, parallelograms, or rhombuses, depending on the position.

3. Have students use a flashlight or an overhead projector and a cube. Tell students that by holding a cube in different positions, they can produce shadows of different shapes. Ask students what polygons they can produce as a shadow.

**Answers:** square, rectangle, hexagon, trapezoid, rhombus
Goals
Students will construct skeletons of pyramids and prisms.
Students will compare skeletons with actual shapes.
Students will name pyramids and prisms by the shape of base.
Students will look for patterns and relationships in the number of edges and vertices of pyramids and prisms.

PRIOR KNOWLEDGE REQUIRED
Can multiply and divide numbers up to $7 \times 7$
Recognizes and can name polygons and cubes
Can identify and extend a pattern in a table

MATERIALS
BLM Multiplication Chain (pp. V-2–7)
at least 80 short toothpicks and 10 long toothpicks per pair of students
modelling clay
pyramid per student (triangular, square, pentagonal, or hexagonal)
pictures of real-life structures that are pyramids
teabags in the shape of pyramids (optional)
paper triangle that is an enlarged copy of a triangular pattern block
triangular, square, and hexagonal pattern blocks
prism per student (triangular, square, pentagonal, or hexagonal)
square prism
rectangular prism
plastic knives (see Extension 2)
BLM Matching 3-D Shapes (pp. T-58–59, see Extension 4)

Mental math minute. Give each student a card from BLM Multiplication Chain. Use the cards up to $7 \times 7$. Call a volunteer to the front of the class. The volunteer reads the card, for example: "I have $3 \times 4$ and 25." Students who have 12 or $5 \times 5$ on their cards stand beside the volunteer and show their cards. If more than one student has a card that matches (e.g., 12 appears on multiple cards), you can pick who joins the chain at this moment and who will join the chain later. The students who just joined the chain read the second half of their cards, and new students join the chain. If the number called from one side of the chain matches the multiplication sentence on the other side of the chain, and there is no third student that can join either side of the chain, the chain is complete. The remaining students should try to make a new chain of their own. The game ends when everyone has come to the front.

Riddle. Present the following challenge: “You have 6 toothpicks. Make 4 triangles with them. The toothpicks must touch each other only at the ends.” Give students toothpicks and modelling clay to hold the
toothpicks together at the vertices of the triangles. Have students try to solve the riddle. After giving them a few minutes to think, suggest that they try to create a 3-D shape. The solution to the riddle is to create a triangular pyramid.

**Introduce pyramids.** Show several pyramids and place them base down in front of students. Point out how all these 3-D shapes are the same: they all have a polygon at the bottom, and they all have one vertex (point to it) opposite that polygon. Hold up the pyramids one at a time and show students that the bottom face is different in different pyramids. Explain that these shapes are all *pyramids*. Ask students if they have heard this word before. Write the term on the board and read it together. Show pictures of pyramids in real life, such as Egyptian pyramids, the entrance structure at the Louvre in Paris, France, and some tents. If available, show teabags in the shape of a pyramid.

**Building skeletons of pyramids.** Demonstrate making a rectangular (also called rectangle-based) pyramid using toothpicks and modelling clay. You will need six longer and two shorter toothpicks. Write the following steps on the board as you demonstrate them. When demonstrating the first step, explain that the polygon you start with is called the *base* of the pyramid.

**Step 1:** Make a polygon using clay balls for vertices and toothpicks for sides.

**Step 2:** Add a toothpick to each vertex of the polygon.

**Step 3:** Join the loose toothpicks to one vertex at the top.

Leave the steps on the board for students to refer to in the following activity.

### ACTIVITY 1

1. Students work in pairs using toothpicks and modelling clay to make skeletons of 3-D shapes. One partner makes pyramids that have triangular or hexagonal bases. The other partner makes pyramids that have square or pentagonal bases. Provide each pair of students with six toothpicks of longer length for the hexagonal pyramid (students need longer toothpicks for the edges in Step 2; otherwise the pyramid will have no height).

**Comparing skeletons and pyramids.** Give each student a triangular, square, pentagonal, or hexagonal pyramid. Discuss how the pyramid and the models students create are the same and how they are different. Explain that the models students create are called *skeletons* of pyramids. Discuss where students may have seen something that looks like these skeletons of pyramids or heard the word “skeleton.” Students might recall poles in a tent as an example of a skeleton for a pyramid and that the bones in a body make a skeleton.

**SAY:** Skeletons only have vertices and edges, there are no faces. You can see all the edges and vertices; nothing is hidden. Students might also
notice the different proportions for the pyramids—some are taller than others. For example, a square pyramid glued from the cut out on BLM Nets looks “thinner” than the skeletons students created. Have students match the pyramid they got to the skeleton of the pyramid with the same polygon in the base.

**Counting vertices and edges of pyramids.** Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>Triangle</th>
<th>Rectangle</th>
<th>Pentagon</th>
<th>Hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Vertices in the Pyramid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Edges in the Pyramid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students help you to fill in the table. Point to a relevant cell and have students count the vertices or the edges and signal the answers. (see completed table below)

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>Triangle</th>
<th>Rectangle</th>
<th>Pentagon</th>
<th>Hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Vertices in the Pyramid</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Number of Edges in the Pyramid</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Keep the table on the board for later use.

**Looking for patterns in the number of edges and the number of vertices.**

Add a column to the table and SAY: You did not make a skeleton for the next pyramid. ASK: How many vertices should be in the base of the next pyramid? (7) Remind students that a polygon with seven vertices is called a heptagon. Fill in the first two cells in the new column with “Heptagon” and “7.” ASK: Can you find the number of vertices in a pyramid with a heptagon as the base using the pattern in the second row of the table? (yes) How many vertices will the pyramid have? (8) What is the rule for the pattern in this row? (start at 4 and add 1 each time) Repeat for the edges. (14 edges, start at 6 and add 2 each time) Fill in the rest of the column. Then add a column for octagon and have volunteers fill it in. (8, 9, 16)

SAY: Imagine I have a pyramid with 15 sides in the base. ASK: How many vertices are in the base? (15) How do you know? (the number of vertices and the number of edges is the same in polygons) How many vertices will the pyramid have? (16) How do you know? Make sure both of the following explanations arise: the number in the second row of the table is always one more than the number in the first row; when you construct the skeleton, you start with the base which has 15 vertices, and then add one more vertex at the top, so you have 16 vertices in total. PROMPT: Imagine constructing the skeleton. How many vertices will be in the base? What will you do next?
(add edges to each vertex) What will you do next? (join the loose edges at the top vertex) How many vertices does that add? (1) How many vertices in total? (16)

ASK: How many edges will this pyramid that has 15 sides in the base have in total? (30) How do you know? Make sure both of the following explanations arise: 1. The number of edges in the pyramid is always double the number of vertices in the base, the number in the last row of the table we made is double the number in the first row; 2. We use 15 edges to make the base, and then add 15 more edges at Step 2, so we use 30 edges in total. PROMPT: Think of how you construct the skeleton. How many toothpicks would you use in Step 1? (15) How many toothpicks would you use in Step 2? (15) How do you know? (1 toothpick for each vertex, 15 vertices) Will you use any new toothpicks in Step 3? (no) How many toothpicks will you use in total? (30)

Exercise: Marla makes a pyramid with a polygon that has 11 sides in the base. How many vertices and how many edges does her pyramid have?

Bonus: Evan makes a pyramid with a polygon that has 50 sides in the base. How many vertices and how many edges does his pyramid have?

Answers: 12 vertices and 22 edges, Bonus: 51 vertices and 100 edges

Naming pyramids. Explain that pyramids are often named according to the shape of their base. For example, if the base is a rectangle, the pyramid is called a rectangular or rectangle-based pyramid. Write both terms on the board, then have students circle or underline the parts of “rectangle” and “rectangular” that are the same. Add a row to the table on the board, label it “Name of Pyramid,” and invite volunteers to write the name of each pyramid in both forms. (triangular or triangle-based pyramid, square or square-based pyramid, pentagonal or pentagon-based pyramid, hexagonal or hexagon-based pyramid, heptagonal or heptagon-based pyramid, octagonal or octagon-based pyramid)

Hold up pyramids one at a time and have students identify what the pyramid is called. Add the pyramid names to your word wall with pictures of both the pyramid and its base.

Introduce prisms. Show students a paper triangle that is an enlarged copy of a triangular pattern block. Ask them to identify the shape. Show students the pattern block and ask them how it is different from the paper triangle. (the paper triangle is larger and flat, or 2-D; the pattern block has thickness and is a 3-D shape) Have several volunteers stack different numbers of triangular pattern blocks one on top of the other, making sure that the sides are aligned. Discuss what the result looks like. (it has a triangle on top and the same triangle on the bottom, the sides are rectangles)

Explain that this shape is called a triangular or triangle-based prism. Then have students stack square pattern blocks and hexagonal pattern blocks, and introduce the terms square prism and hexagonal prism.
Building skeletons of prisms. Demonstrate making a skeleton of a triangular prism using toothpicks and modelling clay. Write the following steps on the board as you demonstrate them. When demonstrating Step 1, explain that the polygon you start with is called the base of the prism and that each prism has two bases, which are on opposite sides of the figure.

**Step 1:** Make two copies of the same polygon using clay balls for vertices and toothpicks for sides.

**Step 2:** Add one toothpick to each vertex of one of the polygons.

**Step 3:** Attach the other polygon on top of the loose toothpicks.

Leave the steps on the board for students to refer to in the following activity.

**ACTIVITY 2**

2. Students work in pairs using toothpicks and modelling clay. One partner makes prisms that have triangular and hexagonal bases. The other partner makes prisms that have square or pentagonal bases. Suggest that the partner who started with a triangle and a hexagon in Activity 1 now use a square and a pentagon. Provide each pair of students with four toothpicks of longer length—they should use these in Step 2 of creating the square prism.

Comparing skeletons and prisms. Give each student a triangular, square, pentagonal, or hexagonal prism. Discuss how the prism and the skeletons students created are the same and how they are different. Have students match the prism they got to the skeleton of the pyramid with the same polygon in the base.

**Cubes, square prisms, and rectangular prisms.** Show a cube and have students compare it to a square prism. **ASK:** How are a cube and a square-based prism the same? (they both have some faces that are squares) Students might also notice that they have the same number of faces, edges, and vertices, and that all of the faces have right angles. **ASK:** How are a cube and a square-based prism different? (the square prism can be taller or shorter; it has four faces that are rectangles, not squares) Explain that a cube is a special case of a square-based prism, just as a square is a special case of a rectangle.

Show a rectangular prism that has different height, length, and width. Have students compare it to a square prism. **ASK:** How are these two shapes the same? (they both have some faces that are rectangles) Students might also notice that the prisms have the same number of faces, edges, and vertices. **ASK:** How are these two shapes different? (the rectangular prism has no square faces) Explain that this new shape is called rectangular prism, and that a square prism is a special case of a rectangular prism—it has two faces that are special rectangles, meaning squares. You can use an analogy to explain: each student in this class is also part of a larger group, students in your school, which is in turn part of an even larger group, all
students in your province. Cubes are part of square prisms, which, in turn, are part of rectangular prisms. So every cube is also a rectangular prism; it is just a special one, where all the faces are squares.

**Counting vertices and edges of prisms.** Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>Triangle</th>
<th>Rectangle</th>
<th>Pentagon</th>
<th>Hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in Each Base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Name of Prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students help you to fill in the table. Point to a relevant cell, and have students count the vertices or the edges and then signal the answer for the numerical answers where possible. (see completed table below)

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>Triangle</th>
<th>Rectangle</th>
<th>Pentagon</th>
<th>Hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in Each Base</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Name of Prism</td>
<td>triangular or triangle-based</td>
<td>square or square-based</td>
<td>pentagonal or pentagon-based</td>
<td>hexagonal or hexagonal-based</td>
</tr>
</tbody>
</table>

**Finding patterns in the number of edges and the number of vertices.** Add a column to the table on the board and SAY: You did not make a skeleton for the next prism. ASK: How many sides should be in each base of the next prism? (7) What is the name of the shape of the base? (heptagon) How many vertices does each base have? (7) Fill in the first two cells of the new column. ASK: Can you find the number of vertices in the heptagon-based prism using the pattern in the third row of the table? (yes) How many vertices will the prism have? (14) What is the rule for the pattern in this row? (start at 6 and add 2 each time) Repeat with edges. (21 edges, start at 9 and add 3 each time) Add another column to the table and have volunteers fill it in for the next prism. (8, 16, 24, octagonal or octagon-based)

SAY: Imagine I have a prism with 15 sides in the base. ASK: Without extending the table, how many vertices are in the base? (15) How do you know? (the number of vertices and the number of edges is the same in polygons) How many vertices will the prism have? (30) How do you know? Make sure both of the following explanations arise: the number in the third row of the table is always double the number in the second row; when you construct the skeleton, you start with two bases, each with 15 vertices, and you never add more vertices. PROMPT: Imagine constructing the skeleton.
How many vertices will be in the base? What will you do next? (add edges to each vertex) Do you add any vertices at this point? (no) What will you do next? (place the second base on top) Does this create any new vertices? (no) How many vertices in total? (30)

ASK: How many edges will this prism with a 15 sided-polygon in the base have in total? (45) How do you know? Make sure both of the following explanations arise: the number of edges in the prism is always the sum of the numbers in the two rows above it or is 3 times as much as the number in the second row; we use 15 edges to make each base, and then add 15 more edges in Step 2, so we use $15 + 15 + 15 = 45$ edges in total.
PROMPT: Think of how you construct the skeleton. How many toothpicks would you use in Step 1? (30) How many toothpicks would you use in Step 2? (15) How do you know? (1 toothpick for each vertex in one base, 15 vertices) Will you use any new toothpicks in Step 3? (no) How many toothpicks will you use in total? (45)

**Exercise:** Rick makes a prism with a polygon with 12 sides in the base. How many vertices and how many edges does his prism have?

**Bonus:** Liz makes a prism with a 100-sided polygon as the base. How many vertices and how many edges does her prism have?

**Answers:** 24 vertices, 36 edges; Bonus: 200 vertices, 300 edges

**Extensions**

1. Give each student or pair of students blocks, including several pyramids and prisms, and have them create structures. Ask students to identify prisms and pyramids in their structures and then in structures built by a different pair of students.

2. Have students make pyramids and prisms out of modelling clay. They can use plastic knives to cut the clay to create flat surfaces.

3. **Project.** Ask students to learn about a pyramidal structure and give a presentation about it—what was the structure used for, when and where was it built? Have students include any other facts about the structure that they find interesting.

4. Have students play **Picking Pairs** and then **Memory** (see introduction to Unit 13, p. P-1) with cards 1 to 28 from **BLM Matching 3-D Shapes**. Two cards match if the shapes have the same name (e.g., rectangular prism). As a variation, allow cubes to be matched with rectangular or square prisms as well.
Goals

Students will identify shapes of faces of 3-D shapes on real shapes and on pictures.

Students will count faces of 3-D shapes.

Students will sort 3-D shapes by the number of faces and by the shape of the faces.

PRIOR KNOWLEDGE REQUIRED

Can multiply and divide numbers up to $7 \times 7$

Recognizes and can name polygons and cubes

Can identify edges, vertices, and faces on actual shapes and pictures

Can fill in a Venn diagram

Can name prisms and pyramids by the shape of the base(s)

MATERIALS

ball or relay race baton (optional)

set of 3-D shapes per pair of students (triangular, square, rectangular, pentagonal, and hexagonal pyramids; triangular, square, rectangular, pentagonal, and hexagonal prisms)

cube per student

blank paper and scissors

octagonal pyramid

octagonal prism

opaque bag per pair of students (see Extension 3)

Mental math minute. Ask students to solve multiplication questions within the range of $0 \times 1$ to $7 \times 7$. For each number, first go through the questions in order, such as $0 \times 3$, $1 \times 3$, and so on to $7 \times 3$, then in reverse order, then out of order. Then progress to a different number. You can have students stand in a line and pass a ball or a relay race baton to the next person in line, who answers the next question.

Identifying faces in pictures. Distribute a set of 3-D shapes (triangular, square, rectangular, pentagonal, and hexagonal pyramids; triangular, square, rectangular, pentagonal, and hexagonal prisms) to each pair of students, including a cube per student. Draw a picture of a cube on the board. ASK: What shape is this? (cube) Ask students to hold up a cube. ASK: What shape do the faces of a cube have? (square) Shade the front face of the cube on the board. ASK: Is this a square? (yes) Erase the shading and shade the top face of the cube. Ask students to find the same face on their cubes. ASK: What is the shape of the face on your cube? (square) Does it look like a square in the picture? Why not? (no; it has four sides, but it does not look like a square) Cover the rest of the picture to emphasize the shape of the top face of the cube on the board.
**ACTIVITY**

Have students trace one face of a cube on a separate piece of paper and cut it out. Have them check that the cutout matches the face of a cube. ASK: What shape is it? (square) Ask students to hold the square in front of them, so that it looks like a square. Ask students to tilt their square so that it lies horizontally at eye level. (If they hold the square too low, it is hard to see the change.) ASK: Does the square have any sides that look shorter? (yes) Which sides? (the sides that were vertical)

Present the shapes in the exercises below one at a time. Ask students whether the shaded face is a square. If the shape is not a square, have students identify the shape of the face. Have students find the 3-D shape among the shapes they have and find the face in the picture. Have students check that they identified the shape in the picture correctly. Sometimes students might be unsure if the shape is a square or a rectangle. Point out that sometimes it is indeed hard to tell whether a shape is a rectangle or a square.

**Exercises:** Is the shaded face a square? If not, what shape is it?

- a)  
- b)  
- c)  
- d)  
- e)  

**Bonus:**

Answers: a) yes; b) no, triangle; c) no, rectangle; d) no, rectangle; e) no, pentagon; Bonus: not sure, may be a square or a rectangle

**Review making skeletons of pyramids and prisms.** Remind students that they made skeletons of pyramids by making one base, then adding edges to each vertex, and then joining the loose edges to a single vertex. This created a shape with one base and a vertex opposite to it. Ask a volunteer to describe how they made skeletons of prisms. (make two identical bases, add edges to the vertices of one base, then add the second base on top) Summarize by explaining that this creates a shape with two bases opposite each other.

**Pyramids have one base and side faces that are triangles.** Ask students to look at the collection of the shapes they have and try to find all the pyramids. Have them place the pyramids so that the base rests on the desk. ASK: What shape are the faces that are not bases? (triangles) Explain that the faces that are not bases are called side faces. Side faces of pyramids are triangles.

ASK: Do you have a shape that has only triangles as faces? (yes) Have students find it and hold it up. ASK: What is it called? (triangular or triangle-based pyramid) Have students place the pyramid on different faces.
ASK: Do you still have a vertex on top? (yes) Are all the side faces triangles? (yes) SAY: The triangular pyramid is special—you can place it on any face, and you always have a vertex on top, and all the other sides are triangles. Any face of a triangular pyramid can be considered a base.

**Counting faces of a pyramid.** Ask students to count the side faces of different pyramids and to count the number of sides in the base of the pyramid. Draw the table below on the board and have students help you to fill it in. Have students make a guess for the last column, then show an octagonal pyramid and have volunteers check that their predictions are correct.

<table>
<thead>
<tr>
<th>Pyramid</th>
<th>Triangle</th>
<th>Square</th>
<th>Rectangle</th>
<th>Pentagon</th>
<th>Hexagon</th>
<th>Octagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Side Faces</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Number of Sides in the Base</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

ASK: What do you notice? (the number of side faces is the same as the number of sides in the base) SAY: So a pyramid has one base and as many side faces as the number of sides in the base. ASK: If your pyramid has 25 sides in the base, how many side faces does it have? (25) How many faces does it have in total? (26)

ASK: Which two pyramids in the table have the same number of faces? (the square and rectangular pyramids) Why do they have the same number of faces? (the bases of both are quadrilaterals, so they both have 4 side faces and a base, so 5 faces in total) How are these two shapes different? (the bases are different shapes; the side faces of a square pyramid are all the same, but only the opposite side faces of a rectangular pyramid are the same) Have students put all the pyramids to one side.

**Prisms have two bases and side faces that are rectangles.** Ask students to look at the rest of the shapes in their collection. ASK: Do all the other shapes in your collection have two faces that are the same and that are opposite each other? (yes) SAY: Think of making skeletons of these 3-D shapes. ASK: What polygon would you start with? Place a 3-D shape so that it stands on that polygon. ASK: Does your shape have the same polygon on the top too? (yes) Remind students that these top and bottom faces are the bases. ASK: What shape are the faces that are not bases? (rectangles) SAY: The faces that are not bases in prisms are also called side faces. Side faces of prisms are rectangles.

ASK: Do you have a shape that has only rectangles as faces? (yes) Have students find it and hold it up. ASK: What is it called? (a rectangular prism) Have students place the prism on a different face. ASK: Do you still have a face that is exactly the same on the top and on the bottom? (yes) Are all the side faces rectangles? (yes) Explain that rectangular prisms, including their special cases (square prisms and cubes) are special—you can place them on any face and you always have exactly the same face on top, and all the...
other sides are rectangles. Actually, any face of a rectangular prism can be considered a base.

**Counting faces of a prism.** Ask students to count the side faces of different prisms and to count the number of sides in the base of the prism. Draw the table below on the board and have students help you to fill it in. Have students make a guess for the last column, then show an octagonal prism and have volunteers check that their predictions are correct.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Triangle</th>
<th>Square</th>
<th>Rectangle</th>
<th>Pentagon</th>
<th>Hexagon</th>
<th>Octagonal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Side Faces</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Number of Sides in the Base</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

ASK: What do you notice? (the number of side faces is the same as the number of sides in the base) SAY: So a prism has two bases and as many side faces as the number of sides in the base. ASK: If your prism has 25 sides in the base, how many side faces does it have? (25) How many faces in total does it have? (27)

**Sorting shapes by the number of faces.** Have students use their entire collection of shapes again. Draw on the board:

Point to each empty cell and have students hold up the shapes with that number of faces. Have volunteers name the shapes. Ask students to organize the shapes on their desks in columns, so that shapes in each column have the same number of faces. Have a volunteer do the same for the collections of the shapes for demonstration, including the octagonal pyramid and prism. The table below shows the shapes that should be in each column:

<table>
<thead>
<tr>
<th>Number of Faces</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>more than 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D Shape</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Erase the leftmost column in the table and label the whole table Number of Faces in 3-D Shapes. Point out that students have created a graph with their shapes. ASK: What is the most common number of faces? (6) How does the graph make it easy to see the most common number of faces? (6 is the longest column)
**Sorting 3-D shapes by the shape of faces.** Draw on the board:

![3-D Shapes Diagram]

Hold up a cube and ASK: Which region should this shape be placed in? (the oval on the right) How do you know? (all faces of a cube are squares and squares are a special case of rectangles; there are no faces that are triangles, so it does not go in the central region) Repeat with triangular prism (central region) and pentagonal pyramid (oval on the left). Have students copy the diagram, giving as much room for the categories as the page allows, with a large central region. Then have them sort the shapes they have into the Venn diagram. The central region should have the triangular prism and rectangular and square pyramids; the oval on the left should have all other pyramids; and the oval on the right should have the cube and all other prisms. There should be no shapes outside the ovals.

ASK: Are there shapes in the outside region? (no) Can you think of a 3-D shape that should be placed in the outside region? (yes) What 3-D shapes can be placed in the outside region? (sample answers: spheres, cones, cylinders) PROMPT: Which shapes did you study in Grade 1 and Grade 2? Think of shapes that roll.

**Extensions**

1. Use the 3-D shapes in your collection to make a Venn diagram with categories “At least one face is a triangle” and “At least one face is a hexagon.”

2. I am a 3-D shape in the central region of a Venn diagram. The categories are “At least one face is a hexagon” and “At least one face is a rectangle.” What 3-D shape am I?

   **Answer:** hexagonal prism

3. Place a variety of pyramids and prisms into an opaque bag for each pair of students. Player 1 places a hand in the bag and picks a shape, but does not look at it or take it out. Player 1 describes the shape to the partner using sense of touch only, but does not say what the shape could be. (For example: I feel that the shape has two triangles opposite each other, like top and bottom faces.) Player 2 counts to 3 and then both players guess the name of the shape aloud. Player 1 takes the shape out of the bag to check. Partners take turns describing the shapes in the bag, then they switch roles and repeat.
**Goals**

Students will compare nets to actual 3-D shapes.
Students will make 3-D shapes from nets.
Students will count faces, edges, and vertices of 3-D shapes.
Students will identify 3-D shapes by the number of faces, edges, and vertices and the shape of the faces.

**PRIOR KNOWLEDGE REQUIRED**

Can multiply and divide numbers up to $7 \times 7$
Can recognize and name polygons
Can identify edges, vertices, and faces on actual 3-D shapes and pictures
Can name prisms and pyramids by the shape of base(s)

**MATERIALS**

- ball (optional)
- square pyramid
- Polydron™ shapes or shapes made from cardstock
- modelling clay or tape
- BLM Nets (1) to (10), (13) (pp. T-60–69, 72) and a set of matching 3-D shapes per small group
- tape or glue
- scissors
- BLM Shape Table (p. T-75)
- pencil crayons (see Extension 1)

**Mental math minute.** Ask students to solve multiplication questions within the range of $1 \times 1$ to $7 \times 7$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3, 3 \div 3, 2 \times 3, 6 \div 3$, and so on to $7 \times 3$ and $21 \div 3$. Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can pass a ball to the student you want to answer the question, and have students pass the ball back to you as they answer.

**Introduce nets.** Hold up a square pyramid. ASK: What is this 3-D shape called? (a square pyramid or square-based pyramid) What are the shapes of the faces? (triangles and a square) How many triangle faces does it have? (4) How many square faces does it have? (1) If I cut out four identical triangles and one square, can I make a square pyramid from them? (yes) If Polydron™ shapes are available, invite a volunteer to make a square pyramid from four triangles and one square. If not, have a volunteer assemble a pyramid using four triangles and one square made from cardstock and modelling clay or tape to hold them together. You might
comment that it is not easy for any one person to assemble five shapes together by themselves.

Show students a net of a square pyramid from BLM Nets (2). ASK: How many triangles does this picture have? (4) How many squares does it have? (1) How is this picture different from the four triangles and one square that the volunteer used to make the pyramid? (the shapes are made of paper and they are joined together) Explain that you can fold this picture (demonstrate as you do so) and glue or tape it together to make a 3-D shape. ASK: What shape does this picture make? (a square pyramid) SAY: A picture that we can fold to make a 3-D shape is called a net of the 3-D shape. This was the net of a square pyramid.

Hold up a net of a pentagonal pyramid and a net of a pentagonal prism from BLM Nets (8). ASK: How are these two nets the same? (they both have pentagons in them) How are they different? (one has 2 pentagons and 5 rectangles; the other has 1 pentagon and 5 triangles) Ask students to try to guess which 3-D shapes these nets make. Have students explain their guesses.

**ACTIVITY**

Matching nets to shapes. Give each small group of students BLM Nets (1) to (10), (13) and a set of matching 3-D shapes. Have students cut out the nets. Have students divide the nets between themselves and match the nets they have to the shapes. Those who finish early can exchange nets and shapes with a partner and match up the new set. Then have students fold the nets and check that they have identified the shapes correctly.

Discuss how the shapes and the nets are the same. The nets fold into 3-D shapes, so they have the same faces as the 3-D shape. Demonstrate by showing a triangular prism and the net for a triangular prism.

Summarizing what we know about pyramids and prisms. Give students BLM Shape Table. Have students cut out the cards at the bottom of the BLM. Students find the card that shows the 3-D shape that matches the faces in the leftmost column of the table; then students fill in the table and glue the shape in the correct column. They can also use actual shapes. The completed table is shown on the following page.
<table>
<thead>
<tr>
<th>Sketch of the Faces</th>
<th>Shape</th>
<th>Name</th>
<th>Number of Faces</th>
<th>Edges</th>
<th>Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ △ △ △</td>
<td>triangular pyramid</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △</td>
<td>triangular prism</td>
<td>5</td>
<td>9</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △</td>
<td>square pyramid</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △</td>
<td>cube</td>
<td>6</td>
<td>12</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △ △</td>
<td>square prism</td>
<td>6</td>
<td>12</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △</td>
<td>pentagonal pyramid</td>
<td>6</td>
<td>10</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △</td>
<td>pentagonal prism</td>
<td>7</td>
<td>15</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △</td>
<td>hexagonal pyramid</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>△ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △</td>
<td>hexagonal prism</td>
<td>8</td>
<td>18</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

**Exercises:** Use the completed BLM Shape Table to answer the question.

a) Which shapes have 6 faces?

b) Which shape has 6 faces and 6 vertices?

c) Which two shapes have the same number of faces, edges, and vertices? Why does this happen?

**Bonus:** Shelly thinks that the number of edges in a pyramid is always an even number. Is she correct? Explain.

**Answers:** a) cube, square prism, pentagonal pyramid; b) pentagonal pyramid; c) cube and square prism, because both are special cases of rectangular prisms; Bonus: Shelly is correct. The number of edges in a pyramid is always double the number of edges in the base of the pyramid. Doubles are even numbers, so the number of edges in a pyramid is always even.
NOTE: Extension 3 is required in order to cover the British Columbia curriculum.

Extensions

1. Use a triangular prism and a net of a triangular prism to answer the question.
   a) How many edges does the prism have?
   b) How many edges does the net have?
   c) Are your answers to a) and b) the same? Explain.
   d) How many vertices does the prism have?
   e) How many vertices does the net of the prism have?
   f) Are your answers to parts d) and e) the same? Explain.
   g) Colour each pair of edges on the net that are glued together in a different colour. Colour each pair of vertices that are glued together in a different colour.

   Answers: a) 9; b) 14; c) no, some of the edges on the net need to be attached, which means they appear in two places on the net to make one edge on the prism; d) 6; e) 10; f) no, when some of the edges of the net are attached, the vertices overlap to make one vertex; g) see margin.

2. Give students several pyramids and prisms and ask them to create towers. ASK: Do prisms stack well? Do pyramids stack well? Ask students to identify various prisms and pyramids in their towers, and then in the towers of a partner. ASK: Does anybody have a tower with a pyramid in the middle or at the bottom? Can you put a pyramid in the middle or at the bottom? Why not? (pyramids do not stack well; you cannot place a pyramid so that it has a level surface on the top)

3. Jax thinks that shape A is different from shape B because shape A is 5 cm tall and has 8 vertices and 12 edges, while shape B is 2 cm tall and has 7 vertices and 9 edges. Is he correct? Explain.

   Answer: Jax is not correct. Shapes A and B are the same. They are both rectangular prisms, so both shapes have 8 vertices and 12 edges. The hidden edges are shown on shape A and not shown on shape B. Shape A and shape B have exactly the same dimensions, and you can get from one of them to the other by turning it on the side.
Goals

Students will describe cones, cylinders, and spheres.
Students will identify curved surfaces and flat faces of cones, cylinders, and spheres.
Students will compare 3-D shapes.

PRIOR KNOWLEDGE REQUIRED

Can add two-digit numbers mentally
Can recognize and name polygons
Can identify edges, vertices, and faces on actual shapes and pictures of shapes
Can name prisms and pyramids by the shape of bases

MATERIALS

cone, cylinder, and sphere
BLM Nets (14) to (15) (pp. T-73–74)
scissors
tape or glue
set of 3-D shapes per small group of students (triangular, square, rectangular, pentagonal, and hexagonal pyramids; triangular, square, pentagonal, and hexagonal prisms; a cube; two cones; two cylinders; and a sphere)
BLM Matching 3-D Shapes (pp. T-58–59, see Extension 3)
BLM Empty Spinners (p. V-1, See Extension 4)

Mental math minute. Have students stand in a line. Choose a starting number, such as 21, and have students repeatedly add a two-digit number, such as 14, with each student saying one addition aloud. Example: Student 1 says “21 + 14 = 35.” Student 2 says “35 + 14 = 49.” Student 3 says “49 + 14 = 63.” Continue until the sum passes 100, then start a new chain.

Review cones, cylinders, and spheres. Show students a cone, a cylinder, and a sphere. Ask students if anyone remembers what these shapes are called. Remind students of the names of the shapes and write them on the board. Ask students where they can find shapes like these in real life (many balls are spheres; hockey pucks, paper rolls, new round pencils, and some coins are cylinders; party hats, pylons, paper water cups, and some ice-cream cones look like cones)

Ask students to describe how cylinders, cones, and spheres are alike and how they are different. Draw on the board the table on the next page and have students help you fill it in.
Is the shape round? | Cylinder | Cone | Sphere
--- | --- | --- | ---
yes | yes | yes | yes
Can the shape roll? | yes | yes | yes
Is it easy to make the shape stand? | yes | yes | yes
Is it easy to place 3 of this shape side by side? | yes | yes | no
Can you build a tower from 3 of this shape? | yes | no | no

Introduce bases and curved surfaces. Explain that in mathematics, cylinders and cones have no holes at the ends, but have a flat circles. This circle is a flat face and is called the base of a cylinder and cone. A cylinder has two bases and a cone has one base. A sphere has no flat faces; it has no bases. Add a row to the table on board for the question “How many bases does the shape have?” and have volunteers fill it in.

Explain that the part of a cone or a cylinder that is not flat is called a curved surface. ASK: How many curved surfaces does a cylinder have? (1) How many curved surfaces are there on a cone? (1) How many curved surfaces are there on a sphere? (1) Add a row to the table for the question “How many curved surfaces does the shape have?” and have volunteers fill it in.

ASK: Do any of the prisms you saw in the previous lessons have a curved surface? (no) Do any of the pyramids you saw have a curved surface? (no) Point out that it is the curved surface that allows a shape to roll smoothly.

ACTIVITY

Give students BLM Nets (14) to (15). Have students try to guess what shape each net makes. Then have students cut out the nets and make the 3-D shapes.

Discuss how the cones that students produced during the activity are different. (one of the cones is lower and wider than the other; the circle at the base is much larger in one of the cones) Students might notice that the parts that make the curved surfaces were parts of a circle, and that it was a much larger circle than the base.

NOTE: Nets of cylinders are made from two circles and one rectangle. Note that the curved surface of a cylinder is not a rectangle, because a rectangle is a flat, 2-D shape. In the net of the cylinder on BLM Nets (14), the shorter sides of the rectangle are attached and so the edges disappear.

Discuss how geometric shapes appear in nature. For example, tree trunks are cylinders, and unwrapping the bark from a birch tree to make a canoe is the opposite to folding a net to make a cylinder. Encourage students to look for other geometric shapes around them.

Comparing shapes. Give each small group of students a set of 3-D shapes to use as a reference, including triangular, square, rectangular, pentagonal,
and hexagonal pyramids; triangular, square, pentagonal, and hexagonal prisms; a cube; two cones; two cylinders; and a sphere. Hold up a triangular pyramid and a triangular prism. SAY: Let’s compare these two shapes. ASK: What is the same about them? (they both have “triangular” in the name, they both have a triangle as a base, they both have no curved surfaces) Draw on the board the blank table below and have students copy it. Fill in the table as a class. (see completed table below)

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>Triangular Prism</th>
<th>Triangular Pyramid</th>
<th>Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bases</td>
<td>2</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>Number of Faces</td>
<td>5</td>
<td>4</td>
<td>no</td>
</tr>
<tr>
<td>Number of Triangular Faces</td>
<td>2</td>
<td>4</td>
<td>no</td>
</tr>
<tr>
<td>Number of Rectangular Faces</td>
<td>3</td>
<td>0</td>
<td>no</td>
</tr>
<tr>
<td>Number of Edges</td>
<td>9</td>
<td>6</td>
<td>no</td>
</tr>
<tr>
<td>Number of Vertices</td>
<td>6</td>
<td>4</td>
<td>no</td>
</tr>
</tbody>
</table>

Provide a table with the headings for students who are struggling with the comparison in the exercises below.

**Exercises:** Use the table to compare the shapes.

a) cube and square prism

b) rectangular pyramid and pentagonal pyramid

**Answers**

<table>
<thead>
<tr>
<th>Shape of Base</th>
<th>Cube</th>
<th>Square Prism</th>
<th>Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bases</td>
<td>2</td>
<td>2</td>
<td>yes</td>
</tr>
<tr>
<td>Number of Faces</td>
<td>6</td>
<td>6</td>
<td>yes</td>
</tr>
<tr>
<td>Number of Square Faces</td>
<td>6</td>
<td>2</td>
<td>no</td>
</tr>
<tr>
<td>Number of Rectangular Faces</td>
<td>0</td>
<td>4</td>
<td>no</td>
</tr>
<tr>
<td>Number of Edges</td>
<td>12</td>
<td>12</td>
<td>yes</td>
</tr>
<tr>
<td>Number of Vertices</td>
<td>8</td>
<td>8</td>
<td>yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shape of Base</th>
<th>Rectangular Pyramid</th>
<th>Pentagonal Pyramid</th>
<th>Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bases</td>
<td>1</td>
<td>1</td>
<td>yes</td>
</tr>
<tr>
<td>Number of Faces</td>
<td>5</td>
<td>6</td>
<td>no</td>
</tr>
<tr>
<td>Number of Triangular Faces</td>
<td>4</td>
<td>5</td>
<td>no</td>
</tr>
<tr>
<td>Number of Rectangular Faces</td>
<td>1</td>
<td>0</td>
<td>no</td>
</tr>
<tr>
<td>Number of Pentagonal Faces</td>
<td>0</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>Number of Edges</td>
<td>8</td>
<td>10</td>
<td>no</td>
</tr>
<tr>
<td>Number of Vertices</td>
<td>5</td>
<td>6</td>
<td>no</td>
</tr>
</tbody>
</table>
Extensions

1. Have students look at different pyramids, such as hexagonal or octagonal pyramids, and describe how they are the same as a cone and how they are different.

Sample answers: Pyramids and cones both have one base and one point opposite it. The bases are different shapes (a polygon in a pyramid and a circle in a cone). A cone has a curved surface and one flat face. A pyramid has many faces, one more than the number of sides in the base.

NOTE: Students are likely to call the apex a vertex in both cases. Technically, the apex of a cone is not a vertex—a vertex is a point where a finite number of edges meet, which is not true for the apex of a cone.

2. Have students build a city with a small number of 3-D shapes, then make a tally chart and count how many of each shape they used. Give each shape a monetary value, for example, cones cost 5 cents. How much will it cost to build the city?

3. Have students play Picking Pairs and then Memory (see introduction to Unit 13, p. P-1) with all of the cards from BLM Matching 3-D Shapes. Two cards match if the shapes have the same name (e.g., rectangular prism). As a variation, allow cubes to be matched with rectangular or square prisms as well.

4. Give each student a copy of BLM Empty Spinners. Point out that each spinner is a circle divided into equal parts, so they can shade part of a circle to make a fraction. Assign different fractions with denominators 6 and 8 to different students and have them shade these fractions and write the fraction on the shaded part. Have students cut out the shaded part of both circles and tape them into a cone so that only the shaded part is visible and without any overlap.

Compare the cones for different fractions. Students might notice that some fractions, namely 4/8 and 3/6, produce cones that are exactly the same. Explain that this happens because both fractions showed half of the circle. Since the circles are the same size, fractions that show the same part of a circle will produce the same cones. Students may also notice that the larger the part of the circle that was shaded, the wider the cone they produced. Explain that students can compare fractions the same way they compare numbers: the larger the shaded part, the bigger the fraction. So the larger the fraction, the wider the cone.
Pentominoes
Shapes for Rotations and Reflections

A

B

C

D

A

B

C

D

A

B

C

D

A

B

C

D
Transparency Cards

Blackline Master — Geometry — Teacher Resource for Grade 3

T-55
Rotations—Advanced (I)

1. What fraction of a circle is shaded?
   a) $\frac{1}{4}$
   b) 
   c) 
   d) 
   e) 
   f) 

Hands on a clock turn **clockwise**. The opposite direction is called **counter-clockwise**.

2. The arrow turns from “start” to “finish.” Shade the part of the circle the arrow turns across. How much does the arrow turn?
   a) $\frac{3}{4}$ turn clockwise
   b) turn clockwise
   c) turn clockwise
   d) turn counter-clockwise
   e) turn counter-clockwise
   f) turn counter-clockwise
Rotations—Advanced (2)

We write **CW** for clockwise and **CCW** for counter-clockwise.

3. How much and in what direction did the arrow turn from start to finish?

![Diagram of an arrow turning clockwise and counter-clockwise]

- a) \( \frac{1}{2} \) turn **CW**
- b) ____________
- c) ____________
- d) ____________
- e) ____________
- f) ____________

To turn or **rotate** an arrow a \( \frac{1}{4} \) turn clockwise (CW) around a dot:

**Step 1**: Draw a curved arrow to show how far the arrow should turn.

**Step 2**: Draw the final position of the arrow from the centre.

4. Rotate the arrow around the black dot.

- a) \( \frac{1}{4} \) turn CW
- b) \( \frac{1}{2} \) turn CW
- c) \( \frac{3}{4} \) turn CW
- d) 1 turn CW
- e) \( \frac{1}{4} \) turn CCW
- f) \( \frac{1}{2} \) turn CCW
- g) \( \frac{3}{4} \) turn CCW
- h) 1 turn CCW
Matching 3-D Shapes (I)

1. [Cube] 2. [Cube] 3. [Rectangular prism] 4. [Cube]
Matching 3-D Shapes (2)

17. 18. 19. 20.

21. 22. 23. 24.

25. 26. 27. 28.

29. 30. 31. 32.
Nets (I)
Nets (3)
Nets (6)
Nets (7)
Nets (9)
Nets (10)
Nets (I2)
Nets (14)
Nets (I5)
## Shape Table

<table>
<thead>
<tr>
<th>Sketch of the Faces</th>
<th>Shape</th>
<th>Name</th>
<th>Number of Faces</th>
<th>Edges</th>
<th>Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Triangle Sketch" /></td>
<td><img src="image2" alt="Triangle Shape" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Rectangle Sketch" /></td>
<td><img src="image4" alt="Rectangle Shape" /></td>
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</tr>
<tr>
<td><img src="image1" alt="Triangle Sketch" /></td>
<td><img src="image2" alt="Triangle Shape" /></td>
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<td><img src="image1" alt="Triangle Sketch" /></td>
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<td><img src="image1" alt="Triangle Sketch" /></td>
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</tr>
<tr>
<td><img src="image3" alt="Rectangle Sketch" /></td>
<td><img src="image4" alt="Rectangle Shape" /></td>
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<td></td>
</tr>
<tr>
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<td><img src="image2" alt="Triangle Shape" /></td>
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</tr>
<tr>
<td><img src="image3" alt="Rectangle Sketch" /></td>
<td><img src="image4" alt="Rectangle Shape" /></td>
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<tr>
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<tr>
<td><img src="image3" alt="Rectangle Sketch" /></td>
<td><img src="image4" alt="Rectangle Shape" /></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Introduction

In this unit, students will learn to read and interpret information presented in the form of pictographs and bar graphs. They will analyze the data presented in the graphs to solve one- and two-step word problems. Students will also explore scales and scaled pictographs and bar graphs. They will use multiplication and division to find the numbers represented on these scaled graphs.

Students will also learn to create their own scaled pictographs and bar graphs from given data, as well as from data collected by the students themselves. Finally, students will compare three different types of graphs: pictographs, bar graphs, and line plots (previously studied in Unit 9). They will examine the three types of graphs to see what information is represented and what is not, how easy it is to find information on each type, and the advantages of each type. In the probability section, students will learn about outcomes, events, even chances, more or less likely, and certain and impossible events. Students will learn about fair games and about expectation of events.

Meeting Your Curriculum

Alberta—Lessons PDM3-4 and PDM3-7 are required to cover the curriculum. The rest of the material in this unit will be studied in later grades.

British Columbia—Lessons PDM3-4, PDM3-7, PDM3-10, and PDM3-12 to PDM3-16 are required to cover the curriculum. The rest of the material in this unit will be studied in later grades.

Manitoba—Lessons PDM3-4 and PDM3-7 are required to cover the curriculum. The rest of the material in this unit will be studied in later grades.

Ontario—PDM3-10 will be studied in later grades. All other lessons are required.

Materials. In several lessons, you will need to show students graphs. Most of the graphs in this unit can be found as BLMs. You can photocopy the relevant BLMs onto transparencies and display them using an overhead projector.

Some of the extensions suggest drawing graphs from data students collect. Depending on the level of students who do the extension, they can use either BLM Pictograph Templates (p. U-76) or BLM Pictograph and Bar Graph Templates (p. U-78), or they can produce a graph on grid paper or using appropriate technology, such as math software or online resources. The last option takes more time and needs organizational skills on the part of the student.
In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section V, are used in Unit 18:

**BLM 1 cm Grid Paper** (p. V-8)
**BLM Multiplication Chain** (pp. V-2–7)

### Quizzes and Tests

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quiz</strong></td>
<td>PDM3-4, 7</td>
<td>PDM3-4, 7, 10</td>
<td>PDM3-4, 7</td>
<td>PDM3-4 to 9</td>
</tr>
<tr>
<td><strong>Quiz</strong></td>
<td>n/a</td>
<td>PDM3-12 to 16</td>
<td>n/a</td>
<td>PDM3-11 to 16</td>
</tr>
<tr>
<td><strong>Test</strong></td>
<td>PDM3-4, 7, 10, 12 to 16</td>
<td>PDM3-4, 7, 10, 12 to 16</td>
<td>PDM3-4, 7, 10, 12 to 16</td>
<td>PDM3-4 to 9, 11 to 16</td>
</tr>
</tbody>
</table>
Goals

Students will read and draw pictographs, each with one symbol representing one item.
Students will solve problems about the data presented in pictographs.

PRIOR KNOWLEDGE REQUIRED

Can subtract two-digit numbers
Can solve one- and two-step “how many more” or “how many less” word problems
Can read data from a table

MATERIALS

ball
20 connecting cubes in three different colours for each student
chalk in 3 different colours
BLM Pictograph Templates (p. U-76)
grid paper or BLM 1 cm Grid Paper (p. V-8, see Extensions 2–4)

Mental math minute. Give students subtraction problems involving subtraction of close two-digit numbers, such as 43 − 38. Toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer it.

Creating a concrete graph. Give each student 20 connecting cubes in three different colours. Students can have different combinations of the three colours and can have a few more or less than 20. Ask students to sort the cubes by colour. SAY: I want to see which colour you have the most of. ASK: How can you show me that quickly? Have students link the cubes into trains, or linked lines, of the same colour so that they can easily compare the lengths. Have students place the trains side by side, so that the difference in length is easy to see.

The need for matching. Use three different colours of chalk to draw on the board:

SAY: This is a picture of my cubes. ASK: Does this picture make it easy to see which colour of cubes I have the most of? (no) Why not? (sample answer: some of the cubes are spread out) Encourage different explanations. ASK: Is it easy to see how many more light grey cubes than dark grey cubes I have? (no) Why not? (the dark grey cubes are spread out)
Repeat with the dark grey and white cubes. **ASK:** How could we draw the cubes on grid paper so that we can see where there are more cubes? (put each cube in one grid square) **PROMPT:** Think of buddies. Buddies work in pairs. Can cubes go in pairs too? How could you order the cubes on grid paper so that we see the pairs?

Draw a 3 by 6 grid on the board and ask volunteers to draw the cubes on the grid, starting from the left with each colour. Ask them to make the squares that show the cubes smaller than the grid squares. The final picture should look like this:

![Grid with more white cubes](image)

**ASK:** Is it easy now to see which colour we have more of? (yes) Point out how putting each square inside a grid square helps to organize the picture.

**The need for a common starting line.** Show three trains of connecting cubes of different colours side by side without one common starting line, as shown in the margin. Compare each colour to another colour. **ASK:** Is it easy to see that there are more light grey cubes than white cubes? (yes) What about dark grey cubes and white cubes—can you compare them easily? (no) How should the trains be arranged so that we can clearly see the differences in length among all three colours? (the ends of all the trains should line up on one side) Invite a volunteer to rearrange your cube trains so they all line up on the left side. Have students rearrange their own cube trains if necessary.

**ASK:** Which colour do you have the most of? (for example, red) How do you know? (the train of red cubes sticks out the farthest)

**Introduce “graph.”** SAY: The picture you have created is called a graph. A *graph* is a way of ordering and showing information that is easy to see and compare. Write “graph” on the board and have students read the word. Draw their attention to the fact that the “f” sound at the end of the word is written as “ph.”

**Title, labels, and data.** Have students each draw a similar graph for their own collection of cubes. SAY: We sorted the cubes by colour. Add the title and labels to the graph on the board, as shown below:

<table>
<thead>
<tr>
<th>Colours of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
</tr>
<tr>
<td>Light grey</td>
</tr>
<tr>
<td>Dark grey</td>
</tr>
</tbody>
</table>

![Graph with different colours](image)
Explain that all graphs need a **title**, or name, so that everyone knows what the graph represents. **Labels** give more detail about parts of the graph—for example, what each row represents. Explain that, with the labels on the graph, you could draw all the cubes using the same colour, and you could still tell how many cubes of each colour there are.

Remind students that information in a graph is called **data**. SAY: This graph shows three facts: I have 5 white cubes, 6 light grey cubes, and 4 dark grey cubes. These facts are the data in this graph. We can say that the graph shows 3 **data values**, or 3 pieces of data.

**Graphs can be horizontal or vertical.** Explain that graphs can be created by arranging data in rows or columns. The columns need to have a common starting line, just like the trains and rows did—in other words, the columns need to line up at the bottom. Draw a 6 by 3 grid on the board. Invite a volunteer to rearrange the cubes so that it is set up like a graph with columns instead of rows, as shown below:

```
     O
     O
     O
     O
     O
     O
```

**Graphs can be used to display survey results.** Explain that sometimes people need to know what other people think or like to do. SAY: For example, you might want to know how many people are going to vote for somebody in an election. In this case, we ask a question and give some answers to choose from. For example, will you vote for Candidate A or for Candidate B—or have you not decided yet? Explain that asking many people a question with prepared answers is called a **survey**. Explain that you are going to conduct a survey. Your question is: What is your favourite season? SAY: There are four seasons, so my possible answers are winter, spring, summer, and fall. Write the four seasons on the board in a row. Have students suggest a title for the graph. (for example, Favourite Season) Write the title on the board above the seasons. Invite students to stand in a line in front of their favourite season.

**Introduce pictographs.** SAY: Suppose you want to show the results of this survey at home, but the whole class cannot line up every time you want to discuss the data. One way you could show the results would be to write down the names of all the people who voted for each season. But what if it does not really matter who likes which season the most? Suppose you only want to know the number of people who like each season. Explain that there is a way to show this information visually, without writing all the names. For example, you could draw a stick figure or a smiley face for each
person in the class. Draw lines on the board to help students organize the graph, as shown below. Extend the table to as many rows as needed.

**Favourite Season**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>Spring</td>
<td>Summer</td>
<td>Fall</td>
</tr>
</tbody>
</table>

Have the students draw a smiley face above the name of the season they chose earlier (they cannot change their answer). Ask them to draw only one smiley face in each box of the table, so that the picture is organized, the same as with the pictures of cubes. Explain that what they have created is called a **pictograph**. Write “pictograph” on the board.

**Analyzing a pictograph.** ASK: Which season is liked by the largest number of people? Which season is liked by the smallest number of people? How do you know? (answers will vary depending on class results) How does our graph make this easy to see? (the column for that season is the tallest) How many people like summer? How many people prefer spring? How can we see that? (count the number of smiley faces in the column for that season)

**How many more/fewer?** Help students compare the actual numbers for two of the seasons with “more.” **NOTE:** Your questions and the answers will vary depending on your class results.

For example, ASK: How many people voted for winter? How many people voted for spring? How many more people voted for spring than for winter? How can you find out? (subtract the number of smiley faces for winter from those for spring, or count the number of extra smiley faces after winter and spring are paired up) How can you see on the graph that more people voted for spring than for winter? (the spring column is taller than the winter column, or the spring column has more smiley faces) Have a volunteer draw circles around pairs of votes for spring and votes for winter. ASK: Are there any spring votes left that are not circled? How many? Is the number of spring votes that were left out after pairing the same as the result of the subtraction? How does the graph make the difference between votes for spring and winter easy to see? (the result of subtraction is the number of smiley faces that were not circled; it is also the number of smiley faces that go past the shorter row) Repeat this line of questioning for two other seasons, but this time ask for the number that is less than another—for example, how many fewer people voted for fall than for summer.

**NOTE:** As students use the phrases “more than” and “less than” and “fewer than,” they might use the last two interchangeably. “Fewer than” is typically used to compare things that can be counted (for example, votes, books, students, leaves), and “less than” is used to compare things that cannot
be counted (for example, water, sun). Model the distinction in language for students, but allow them to use the phrases interchangeably for now.

Remind students that the phrase most popular in surveys means that the largest number of people voted for that answer. Ask: What is the most popular season? How do you know? (It has the tallest column) What is the least popular season? How do you know? (It has the shortest column)

There should be no “gaps” in the data. Draw on the board:

<table>
<thead>
<tr>
<th>Students' Birthplace</th>
</tr>
</thead>
<tbody>
<tr>
<td>Born in Canada</td>
</tr>
<tr>
<td>Born outside of Canada</td>
</tr>
</tbody>
</table>

Explain that the pictograph shows in rows the number of students in a class that were born in Canada and outside Canada. Say: The faces to represent each person start at the same place, but the ones in the top row extend farther than the ones in the bottom row. So I think more students were born in Canada than outside Canada. Ask: Is that correct? (no) Have a volunteer explain your mistake. (There are gaps between the face pictures) Draw the graph correctly, with no gaps between the faces. Remind students that the rows must line up at one end and extend in a line of pictures without gaps.

Introduce symbols. Explain that people usually use very simple pictures of objects in pictographs. These simple pictures are called symbols and are very easy to draw. Write the word “symbol” on the board. Ask students to think about what symbols they could use to represent people, flowers, snacks, books, and other items in a pictograph. Explain that we usually write what the symbol means on the graph. Write underneath the pictograph:

\[
\text{= 1 student}
\]

Creating pictographs when given numerical data. Give students BLM Pictograph Templates. Have students create pictographs for the numerical data given in the exercises below. Students can use stick figures, smiley faces, circles, or another symbol of their choice to represent the data. Alternatively, ask a question, have students vote on the answers, and record the class data on the board. Before assigning the exercises, tell students to use the template with two rows for part a), the template with three rows for part b), and the template with five rows for part c).

Exercises: Use the data to draw a pictograph.

a) Soccer Players in Mr. L.’s Class

<table>
<thead>
<tr>
<th>Likes to Play Soccer</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>9</td>
</tr>
<tr>
<td>No</td>
<td>7</td>
</tr>
</tbody>
</table>
b) **Shoes We Are Wearing Today**

<table>
<thead>
<tr>
<th>Type of Shoe</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip on</td>
<td>7</td>
</tr>
<tr>
<td>With Laces</td>
<td>8</td>
</tr>
<tr>
<td>With Velcro</td>
<td>3</td>
</tr>
</tbody>
</table>

**Eye Colours in Ms. K.’s Class**

<table>
<thead>
<tr>
<th>Eye Colour</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>2</td>
</tr>
<tr>
<td>Brown</td>
<td>6</td>
</tr>
<tr>
<td>Grey</td>
<td>7</td>
</tr>
<tr>
<td>Green</td>
<td>2</td>
</tr>
</tbody>
</table>

Ask questions that require students to read their pictographs or to interpret and summarize them. Encourage students to solve the questions in different ways to check the answers. For example: How many more students in Ms. K.’s class have brown eyes than blue eyes? (4; there are 4 more symbols in the row for brown eyes than in the row for blue eyes, or $6 - 2 = 4$) How many fewer students wear shoes with Velcro fastenings than wear slip-on shoes? (3; $7 - 3 = 4$, or there are 3 more symbols in the slip on row) For which eye colours is the number of students the same? (blue and green) How do you know? (the rows are the same length)

Ask questions that require addition and subtraction, and have students write addition or subtraction sentences for the answer. For example: How many students in total were surveyed about shoes? ($7 + 8 + 3 = 18$) How many students have blue, green, or grey eyes? ($2 + 2 + 7 = 11$) How many more students have blue, green, or grey eyes than have brown eyes? ($11 - 6 = 5$)

**Adding missing information.** Present a more complicated situation with the graph from part c) in the exercises above. SAY: There are 22 students in Ms. K.’s class. The row for students with hazel eyes is missing. Hazel eyes are similar to brown eyes, but they are lighter, and they change colour a little when you wear different colours. Ask students to add a row to that graph for the students with hazel eyes. Have students write an addition equation for the total number of students who are in the graph. ($2 + 6 + 7 + 2 = 17$) ASK: How many students have hazel eyes? ($22 - 17 = 5$) Have them add five symbols to the row for hazel eyes.

Write on the board:

There are 25 students in Ron’s class. 5 students walk to school. 2 fewer students bike to school. 3 more students take the bus than walk. The rest of the students get a car ride to school.

Have students write an addition or a subtraction sentence to find the number of symbols that belong in each row. Then have them create a
pictograph to present the information. Remind students to give the graph a title and labels. (see answers below)

Ways to get to school:
- Walk: 5
- Bike: $5 - 2 = 3$
- Bus: $5 + 3 = 8$
- Bus, bike, or walk: $5 + 3 + 8 = 16$
- Car: $25 - 16 = 9$

**Ways to Get to School**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bike</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

 SOLUTION: $\text{Walk} = 5$ students,
$\text{Bike} = 3$ students,
$\text{Bus} = 8$ students,
$\text{Walk, bike, or bus} = 16$ students,
$\text{Car} = 9$ students

**Extensions**

1. Explain that sometimes people use the same symbol in all rows of the graph and sometimes they use different symbols. Draw the graph below on the board, and explain that it shows how many times during the week students have different after-school classes:

<table>
<thead>
<tr>
<th>After-School Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Art</td>
</tr>
<tr>
<td>Music</td>
</tr>
<tr>
<td>Soccer</td>
</tr>
</tbody>
</table>

SAY: I think that there are more art classes during the week than music classes or soccer classes. ASK: Is that correct? (no) Why not? (there are fewer symbols for art than for music and soccer) SAY: I think there are more soccer classes than music classes. ASK: Is that correct? (no) Why not? (there are fewer symbols for soccer than for music) Discuss with students why you might be making mistakes. (the paintbrushes are longer; the soccer balls are not lined up with the other symbols) ASK: How could we redraw the pictograph to make it easier to read? (make the symbols the same size and line them up; use different symbols that are all the same size; use the same symbol in every row) Have students redraw the pictograph using one or two of the suggestions made.

NOTE: If students do not have grid paper, they can use **BLM 1 cm Grid Paper** for Extensions 2–4.
2. Check what shoes 10 of your classmates are wearing today.
   a) How many people are wearing sneakers?
   b) How many people are wearing sandals?
   c) What other kinds of shoes are people wearing today?
   d) Use grid paper to create a pictograph showing the types of shoes
      the 10 students are wearing.

3. Change the Ways to Get to School pictograph from the lesson into a
   pictograph that shows the data in columns instead of rows.

   **Answer**

   **Ways to Get to School**

<table>
<thead>
<tr>
<th>Walk</th>
<th>Bike</th>
<th>Bus</th>
<th>Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>😊</td>
<td>😊</td>
<td>😊</td>
<td>😊</td>
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<tr>
<td>😊</td>
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<tr>
<td>😊</td>
<td>😊</td>
<td>😊</td>
<td>😊</td>
</tr>
</tbody>
</table>

   😊 = 1 student

4. In November, there was 1 day of snow. In December, there were 8 more
   days of snow than in November. In January, there were 2 fewer days of
   snow than in December. In February, there were 3 fewer days of snow
   than in December. Use grid paper to make a pictograph that shows the
   number of snow days from November to February.

   **Answer**

   **Number of Snow Days**

<table>
<thead>
<tr>
<th>November</th>
<th>December</th>
<th>January</th>
<th>February</th>
</tr>
</thead>
<tbody>
<tr>
<td>☃️</td>
<td>☃️ ☃️</td>
<td>☃️ ☃️ ☃️</td>
<td>☃️ ☃️</td>
</tr>
</tbody>
</table>

   ☃️ = 1 day
# PDM3-5 Pictographs

**Pages 185–186**

## Goals

Students will read and draw scaled pictographs.

### PRIOR KNOWLEDGE REQUIRED

- Can multiply and divide within $7 \times 7$
- Can solve one- and two-step “how many more” and “how many less” word problems
- Can read data from a table
- Can read and draw a pictograph with one symbol representing one item
- Can multiply one-digit numbers by a multiple of 10
- Can divide by one-digit numbers

### MATERIALS

- ball (optional)
- BLM Pictograph Templates (p. U-76)

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### Mental math minute.

Ask students to solve multiplication questions within the range of $1 \times 1$ to $7 \times 7$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, to $7 \times 3$ and $21 \div 3$. Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

### Introduce scaled pictographs.

Tell students that you have a garden and you made a pictograph to show how many flowers are in it. Draw on the board:

**Flowers in My Garden**

- Daffodils
  
- Buttercups
  
- Daisies

ASK: How many daisies are in my garden? (3) How do you know? (there are 3 flowers in the daisy row) SAY: But there is something I did not tell you. Each flower in the picture stands for more than one flower. Each flower could mean any number of actual flowers, but it is always the same number for each symbol on the graph. For example, if the first symbol means 2 flowers, then all the symbols on the graph mean 2 flowers.

ASK: If each symbol means only 1 flower, how many flowers do I have in my garden altogether? (12) How do you know? (add the flowers in each row) Have students write an addition sentence to show the answer.
(4 + 5 + 3 = 12) ASK: If each symbol means 2 flowers, how many daffodils do I have? (8) How do you know? Allow several students to explain how they found the answer, to illustrate the different strategies. (counting by 2s or multiplying $4 \times 2 = 8$) Repeat with buttercups and daisies. Have students count by 2s and write the multiplication equation for each row. (buttercups: $5 \times 2 = 10$, daisies: $3 \times 2 = 6$) ASK: If each symbol means 2 flowers, how many flowers do I have in total? (8 + 10 + 6 = 24) Repeat with each symbol meaning 3 flowers, then 5 flowers, and then 10 flowers. (12 daffodils, 15 buttercups, 9 daisies, 36 flowers altogether; 20 daffodils, 25 buttercups, 15 daisies, 60 flowers altogether; 40 daffodils, 50 buttercups, 30 daisies, 120 flowers altogether)

As a challenge, tell students that you have 20 buttercups in your garden and have them figure out what the symbol means. (4 flowers) Have them explain how they found the answer. Encourage students to think of multiple strategies. (strategies include dividing $20 \div 5 = 4$, and writing an equation with a missing number and solving for the missing number: $5 \times \_\_ = 20$)

Explain that, to avoid confusion about the number of items shown on the pictograph, people use a scale: they write what the symbol means on the graph. Add the scale below to the pictograph:

\[ \bigstar = 10 \text{ flowers} \]

**Drawing symbols to represent numbers when given a scale.** Remind students that symbols on a pictograph are usually simple so that it is easy to draw many symbols. SAY: A circle means 2 flowers. Write on the board:

\[ \bigcirc = 2 \text{ flowers} \]

ASK: How many circles do you need to draw for 6 flowers? (3) How do you know? Have volunteers explain the different methods. (counting by 2s and drawing 1 circle for each number until you reach 6; dividing $6 \div 2 = 3$; writing a multiplication equation with a missing number, $\_\_ \times 2 = 6$, and using your knowledge of the times table to fill in the missing number) Make sure all three of these methods are mentioned.

**Exercises:** Look at the scale. Draw symbols to show each number.

a) \(\bigcirc = 3 \text{ people}\)  
b) \(\bigcirc = 5 \text{ people}\)  
c) \(\bigcirc = 10 \text{ people}\)

12 people =  
10 people =  
20 people =  
15 people =  
20 people =  
40 people =

**Answers**

a) 12 people = \(\bigcirc \bigcirc \bigcirc \)  
15 people = \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)

b) 10 people = \(\bigcirc \bigcirc \)  
20 people = \(\bigcirc \bigcirc \bigcirc \bigcirc \)

c) 20 people = \(\bigcirc \bigcirc \)  
40 people = \(\bigcirc \bigcirc \bigcirc \)  

**Redrawing pictographs with a new scale.** If the answers from the previous exercises are not on the board, add them. Then circle the two ways 20 people are represented in parts b) and c). Point to the examples
and SAY: In part b), 20 people are shown with 4 symbols, and in part c), 20 people are shown with 2 symbols. ASK: Why is that? (the scales are different) SAY: This means that the same information can be shown on pictographs with different scales. The pictographs will look different, but they will mean the same thing. Draw on the board:

**Book Types Sold at a Book Fair**

<table>
<thead>
<tr>
<th>Type of Book</th>
<th>Number of Books</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fantasy</td>
<td></td>
</tr>
<tr>
<td>Mystery</td>
<td></td>
</tr>
<tr>
<td>Non-fiction</td>
<td></td>
</tr>
<tr>
<td>Picture</td>
<td></td>
</tr>
</tbody>
</table>

[Symbol] = 5 books

SAY: I see that all the numbers in the table are the numbers we say when we count by tens. ASK: What do we call such numbers? (multiples of 10) SAY: This means that the same information with a scale of 1 symbol meaning 5 books can also be shown in a new pictograph with 1 symbol meaning 10 books.

Distribute **BLM Pictograph Templates**. ASK: How many different types of books do we have in this pictograph? (4) Can you use the table at the top of the page for this pictograph? (no) Why not? (it has only 2 rows) Which table can you use? (the third or the fourth) SAY: The graph we are going to draw uses the same data as the graph on the board. So we can use the same title as in the graph on the board. Have students fill in the title. (Book Types Sold at a Book Fair)

Discuss what symbol students can use for their pictograph. Remind them that the symbol should be easy to draw, such as two rectangles together to represent the open pages of a book, or a square to represent the shape of a book. Have students write the scale below their pictograph (the symbol they chose and “= 10 books”).

Have students fill in the types of books on the graph, one type per row. ASK: If there were 20 fantasy books sold during the book fair and we are using a scale of 1 book symbol = 10, how many symbols should you draw to represent that number? (2) How do you know? (20 ÷ 10 = 2 or ___ × 10 = 20, the missing number is 2) Add a column to the table on the board.
called “Equation” and have volunteers fill in the division equation or the multiplication equation that helps them to find how many symbols they should draw on the pictograph. (see finished table below)

<table>
<thead>
<tr>
<th>Type of Book</th>
<th>Number of Books</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fantasy</td>
<td>20</td>
<td>$20 \div 10 = 2$</td>
</tr>
<tr>
<td>Mystery</td>
<td>30</td>
<td>$30 \div 10 = 3$</td>
</tr>
<tr>
<td>Non-fiction</td>
<td>10</td>
<td>$10 \div 10 = 1$</td>
</tr>
<tr>
<td>Picture</td>
<td>30</td>
<td>$30 \div 10 = 3$</td>
</tr>
</tbody>
</table>

After a volunteer fills in a row of the table on the board, have students draw the appropriate number of symbols in the proper row of their pictographs. Point out that the grid on the pictograph makes it easy to keep the pictograph organized; students need to draw each symbol in a separate box in each row without leaving empty boxes in between. The finished pictograph will look like this:

**Book Types Sold at a Book Fair**

<table>
<thead>
<tr>
<th>Fantasy</th>
<th>Mystery</th>
<th>Non-fiction</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Symbol for 10 books]</td>
<td>![Symbol for 10 books]</td>
<td>![Symbol for 10 books]</td>
<td>![Symbol for 10 books]</td>
</tr>
</tbody>
</table>

Comparing two pictographs. ASK: On the first pictograph, how many mystery books were sold at the fair? (30) On the second pictograph, how many mystery books were sold at the fair? (30) So, do the two graphs present the same data? (yes) Do they look exactly the same? (no) How are they different? (they have a different number of symbols in the same row; they have different scales) Emphasize that the different scales are the real reason the graphs look different. ASK: How many fantasy and mystery books were sold at the fair? (50) Discuss various ways to find the answer. (add the numbers from the table; add the number of symbols in the first two rows and then multiply by 5 or by 10, depending on the scale) Make sure both methods are mentioned.

Using a half symbol. SAY: Suppose there was another type of book at the fair, craft books. There were 5 craft books sold at the fair. I would like to add this information to both graphs. Have a volunteer come to the board and add a row for craft books to the graph (where the symbol means 5 books) and add 1 symbol in this new row. SAY: But on the second pictograph, where the symbol means 10 books, 5 books is fewer books than what one symbol means. Have students think of how they could represent 5 books in the second graph. (draw half of a symbol; use a smaller symbol; write something like a fraction, with the symbol as a numerator and 2 as the denominator) Explain that in such cases we can use half of a symbol.
Remind students that to find half of a pizza, they divide the pizza into 2 equal parts. ASK: Can we divide a group of 10 books into 2 equal parts? (yes) If possible, take 10 identical books (such as the JUMP Math AP Books) and have a volunteer divide them into 2 equal piles. ASK: How many books are in each pile? (5) How could you find out mathematically how many books should be in each pile? (divide 10 \( \div 2 = 5 \)) Have students write the division equation. Explain that, on the second pictograph, 1 whole symbol will still mean 10 books, and half of a book symbol will mean 5 books. Draw half of a book symbol on the board. Have students add a “Craft” row to the second pictograph and draw half a symbol in the row.

Draw on the board:

<table>
<thead>
<tr>
<th>☺</th>
<th>☼</th>
<th>Division Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>( 10 \div 2 = 5 )</td>
</tr>
</tbody>
</table>

SAY: Imagine that I am going to draw a pictograph that shows the results of a survey about people’s favourite colour. I am going to use a circle for a symbol. ASK: If a whole circle means 10 people, how much would a half circle mean? (5 people) How do you know? (10 \( \div 2 = 5 \)) PROMPT: This situation is very similar to the last pictograph, where one book symbol meant 10 books and half of the symbol meant 5 books, which we found by dividing 10 by 2. SAY: For our survey about favourite colours, let’s fill in the table. Fill in the first row of the table, as shown below:

<table>
<thead>
<tr>
<th>☺</th>
<th>☼</th>
<th>Division Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>( 10 \div 2 = 5 )</td>
</tr>
</tbody>
</table>

Emphasize that students need to divide the number the whole circle means by 2 to get the meaning of half a circle.

SAY: Now we are going to use a new scale. ASK: If a whole circle means 6 people, how many people does half of the circle mean? (3) How do you know? (6 \( \div 2 = 3 \)) Have a volunteer add a row to the table and fill in the row for 6. Add more rows to the table and repeat with other numbers. (see below for possible numbers and answers)

<table>
<thead>
<tr>
<th>☺</th>
<th>☼</th>
<th>Division Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
<td>( 6 \div 2 = 3 )</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>( 8 \div 2 = 4 )</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>( 16 \div 2 = 8 )</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>( 20 \div 2 = 10 )</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>( 12 \div 2 = 6 )</td>
</tr>
</tbody>
</table>

Students can signal the numerical answers, and volunteers can write the division sentence for each row. Leave the table on the board.

**Translating groups of symbols with a half symbol into numbers.** Draw on the board:

<table>
<thead>
<tr>
<th>☺</th>
<th>☼</th>
<th>☼</th>
<th>☼</th>
<th>☼</th>
<th>☼</th>
</tr>
</thead>
<tbody>
<tr>
<td>☺</td>
<td>☼</td>
<td>= 10 people</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Point to the 4 and a half symbols on the board. SAY: Sometimes you see something like this on pictographs. If one circle means 10 people, how can we find out how much this means? Cover the half circle so only the 4 circles show. ASK: How much does this mean? (40 people) Write “40” underneath the 4 circles. Then cover the 4 circles so only the half circle shows. ASK: How much does this mean? (5 people) What is 10 divided by 2? Write “5” underneath the half circle. SAY: So we have 4 circles and a half circle. ASK: How many people is that altogether? (45) Add the addition sign and finish the equation, as shown below:

\[
\begin{align*}
40 &+ 5 = 45 \\
\end{align*}
\]

Have students start a table as shown below:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 × 4 = 40</td>
<td>5</td>
<td>40 + 5 = 45</td>
</tr>
</tbody>
</table>

Change the scale to 6 people and repeat the exercise above. Have students record the numbers in the table. Then continue with the scales 8 and 20. The finished table is shown below:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 × 4 = 40</td>
<td>5</td>
<td>40 + 5 = 45</td>
</tr>
<tr>
<td>6</td>
<td>6 × 4 = 24</td>
<td>3</td>
<td>24 + 3 = 27</td>
</tr>
<tr>
<td>8</td>
<td>8 × 4 = 32</td>
<td>4</td>
<td>32 + 4 = 36</td>
</tr>
<tr>
<td>20</td>
<td>20 × 4 = 80</td>
<td>10</td>
<td>80 + 10 = 90</td>
</tr>
</tbody>
</table>

**Showing numbers using groups with a half symbol.** ASK: If one circle represents 10 people, how will you show 30 people? (3 circles) How do you know? (count by 10s until you reach 30; look for the missing number in \( \times 10 = 30 \); divide 30 \( \div 10 = 3 \)) How will you show 40 people? (4 circles)

How will you show 35 people? (3 and a half circles) How do you know? (35 is halfway between 30 and 40, or 3 circles means 30 and 4 circles means 40, so to show 35 we need half a circle) Draw 3 circles and a half circle and have students check the answer using the method of the table above.

**Exercises:** One circle means 10 people. Draw circles to represent the number.

a) 25  
b) 45  
c) 15

**Answers**

a)  
b)  
c) 

ASK: If a whole circle means 2 people, what does a half circle mean? (1 person)
**Exercises:** One circle means 2 people. Draw circles to represent the number.

a) 5  

b) 15  

c) 11  

**Answers**

a)  

b)  

c)  

**Extensions**

1. Discuss what is wrong with the following pictograph:

<table>
<thead>
<tr>
<th>Favourite Sports of Students in Class A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer</td>
</tr>
<tr>
<td>Ice Hockey</td>
</tr>
<tr>
<td>Basketball</td>
</tr>
</tbody>
</table>

Tell students that one happy face represents one student who picked that sport as their favourite. ASK: Which sport is the most popular? (ice hockey) Which sport has the longest row of faces? (soccer) Why is it easier to read the pictograph when all the faces are the same size? (look for the longest row) Have students redraw the pictograph correctly.

2. Ms. Smith’s class chose their favourite drinks. The possible answers were milk, orange juice, apple juice, and water. Draw a pictograph to show the data. Use 😊 for 2 people.

a) Apple juice was the most popular. 12 people chose it. Show this on the pictograph.  

b) 7 fewer people chose milk than apple juice. Show this on the pictograph.  

c) There are 25 students in the class. How many students chose orange juice or water?  

d) 2 more people chose orange juice than water. Show this on the pictograph.

**Answers**

<table>
<thead>
<tr>
<th>Favourite Drinks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple juice</td>
</tr>
<tr>
<td>Milk</td>
</tr>
<tr>
<td>Orange juice</td>
</tr>
<tr>
<td>Water</td>
</tr>
</tbody>
</table>

😊 = 2 people
Goals

Students will read and draw scaled pictographs.
Students will solve problems using data from pictographs.

PRIOR KNOWLEDGE REQUIRED

Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a scaled pictograph
Can multiply numbers
Can multiply one-digit numbers by a multiple of 10
Can divide by one-digit numbers (e.g., by finding the missing number in a times fact)

MATERIALS

ball (optional)
BLM Pictograph Templates (p. U-76)
30 connecting cubes of five different colours (blue, green, red, brown, yellow) per pair of students
large piece of cardboard (see Extension 2)
string (see Extension 2)
clothespins (see Extension 2)
world map (see Extension 2)
grid paper or BLM 1 cm Grid Paper (p. V-8, see Extension 2)

Mental math minute. Ask students to solve multiplication questions within the range of 1 × 1 to 7 × 7 and corresponding division questions. For each number, go through the questions in order, such as 1 × 3, 3 ÷ 3, 2 × 3, 6 ÷ 3, and so on, to 7 × 3 and 21 ÷ 3. Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

Review creating pictographs, including using a half symbol. Remind students that a symbol on a pictograph can mean more than one item. Write on the board:

😊 = 10 people
30 people =
40 people =
5 people =
35 people =

ASK: How can we show 30 on a pictograph with the scale given? (3 smiley faces) How do you know? (3 × 10 = 30, or 30 ÷ 10 = 3) Have a volunteer
fill in the answer on the board. ASK: How can we show 40? (4 smiley faces)
How do you know? (4 \times 10 = 40, or 40 \div 10 = 4) Have a volunteer fill in
the answer on the board. ASK: How can we show 5? (half of a smiley face)
How do you know? (10 \div 2 = 5, 5 is half of 10, two groups of 5 objects
make 10) Make sure all three methods are discussed. Have a volunteer fill
in the answer on the board. ASK: How can we show 35? (3 full faces and
1 half of a smiley face) How do you know? (35 is exactly half way between
30 and 40, 35 is 30 + 5) Have a volunteer fill in the answer on the board.
ASK: How can you show 15? (one full face and 1 half of a smiley face)

**ACTIVITY**

Give each student a copy of **BLM Pictograph Templates** and give
each pair of students a collection of 30 connecting cubes in any
combination of the following colours: blue, green, red, brown, and
yellow. **NOTE:** These should be random, unmatched collections, but
each student pair should have at least one cube of every colour. Have
students sort the cubes by colour and then work independently to
create a pictograph of their collection on the last pictograph template
on the BLM using the scale 1 square = 2 cubes. Each pair should
compare the two graphs for their cube collection. Keep these graphs
for the next lesson.

**Answering questions using a pictograph.** Write on the board:

a) For which colour are there the most cubes?
b) For which colour are there the least cubes? 
c) How many red and blue cubes do you have altogether?
d) How many cubes that are not green do you have?
e) How many cubes do you have in total?

Have students answer the questions about their own graphs. Remind
students that the mode is the most common data value. **SAY:** *Most common*
means the one that happens most often or has the greatest number. The
colour that has the most cubes is the mode colour.

Tell students that you also have a collection of cubes and have made a
graph from it. Draw on the board:

**Colours of Cubes**

<table>
<thead>
<tr>
<th>Blue</th>
<th>Green</th>
<th>Red</th>
<th>Brown</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SAY:** This pictograph is not finished, but let’s look at the data on it so far.
ASK: How many more red cubes than green cubes do I have? (6) How do
you know? (there are 6 green cubes and 11 red cubes, \(11 - 6 = 5\); the red row has 2 and a half symbols more, which represents 5 cubes) Make sure both answers are discussed.

ASK: How many fewer blue cubes than red cubes do I have? (2) How do you know? (there are 9 blue cubes and 11 red cubes, \(11 - 9 = 2\); there is one more square in the red column)

SAY: I have 3 more green cubes than brown cubes. ASK: How many brown cubes do I have? (3) How do you know? (6 \(-\) 3 \(=\) 3) How can I show this on the pictograph? (draw one full square and one half square in the row for brown) Have a volunteer add the squares on the board.

SAY: There are 38 cubes in my collection. ASK: How many yellow cubes do I have? (9) How do you know? (the total number of cubes shown on the pictograph is \(9 + 6 + 11 + 3 = 29\) cubes, and \(38 - 29 = 9\)) How many squares should I draw in my pictograph to show this? (4 full squares and one half square) Invite a volunteer to add this to the pictograph on the board. ASK: What is the mode? (red) How many times does the mode happen? (11) PROMPT: How many cubes are red? (11) Explain to students that the longer row in a pictograph is the mode because it has the most data values.

Choosing a scale for data. Explain that sometimes students will need to decide what scale to use for data. SAY: Suppose you surveyed 200 people about their favourite team sport and gave them three answers to choose from: baseball, soccer, and ice hockey. In the survey, 100 people chose baseball, 45 chose soccer, and 55 chose ice hockey. Write on the board:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball</td>
<td>100</td>
</tr>
<tr>
<td>Soccer</td>
<td>45</td>
</tr>
<tr>
<td>Ice Hockey</td>
<td>55</td>
</tr>
</tbody>
</table>

ASK: If I used a scale of one symbol representing two people, how many symbols would I need to show 100 people? (50) Have students try to count by 2s to reach 50. When they see that they do not have enough fingers on their hands to keep track, ASK: Does it make sense to use 2s, or should we count by a bigger number? Students can repeat the counting by 3s and by 5s. ASK: What number should we count by? (10s) Have students count by 10s to see that they need 10 symbols to represent the number of people who chose baseball.

ASK: Can we represent 45 or 55 people using the scale where one symbol is equal to 10 people? (yes) How would you represent these numbers? (4 and a half symbols for 45, 5 and a half for 55) SAY: So 10 seems like a reasonable scale in this case. Point out that if you had, say, 42 people and 58 people choosing soccer and ice hockey, you would have trouble using 10 because you would not be able to use halves to show 2 people and 8 people.
Change the numbers in the table to 15, 6, and 9. ASK: Is 10 a good scale for these new numbers? (no) Why not? (6 and 9 cannot be shown with a whole symbol or with half a symbol) SAY: Let’s try 1 symbol equals 2 people. Have students say how many symbols they would use for each number. (7 1/2, 3, 4 1/2) SAY: We say 15, 6, and 9 when we count by something other than 2. ASK: What number can we count by? (3) Have students say how many symbols they should draw for each of the numbers with the scale 1 symbol = 3 people. (5, 2, 3)

SAY: I am going to write groups of numbers. Each group of numbers is data. We need to show this data on a pictograph. Have students signal the answers for each part in the following exercises.

**Exercises:** Which scale should we use: 1 symbol equals 2, 3, or 5?

a) 6, 10, 8  

b) 6, 10, 7  

c) 15, 20, 10  

d) 9, 12, 21  

**Bonus:** 18, 15, 9, 21, 27, 30

**Answers:** a) 2, b) 2, c) 5, d) 3, Bonus: 3

**Review names of polygons and counting sides.** In order to complete Question 1 on AP Book 3.2, p. 187, students need to review names of polygons and counting sides. Write on the board:

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name of Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
</tr>
</tbody>
</table>

Have volunteers draw examples for each type of polygon above. Then count the sides in each shape as a class to make sure the examples are drawn correctly. Encourage students to create examples of shapes that look different, with sides of different lengths and indentations, as well as shapes with all equal sides.

**Extensions**

1. Make a pictograph for the letters in the city name Mississauga. Use each letter as its own symbol in the graph.

**Answer**

<table>
<thead>
<tr>
<th>Letters in Mississauga</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M M</td>
<td></td>
</tr>
<tr>
<td>I I</td>
<td></td>
</tr>
<tr>
<td>S S S S S</td>
<td></td>
</tr>
<tr>
<td>A A A</td>
<td></td>
</tr>
<tr>
<td>U U</td>
<td></td>
</tr>
<tr>
<td>G G</td>
<td></td>
</tr>
</tbody>
</table>
2. In advance, prepare materials to conduct a survey and record the results. On a large piece of cardboard, write the title and labels shown below and attach strings that hang down from each label. In the survey, have students attach a clothespin to the appropriate string to show their answers. Ensure that students hang their clothespins the same distance apart or touching.

**Our Birthplaces**

<table>
<thead>
<tr>
<th>Canada</th>
<th>North America (not Canada)</th>
<th>South America</th>
<th>Africa</th>
<th>Asia</th>
<th>Europe</th>
<th>Australia</th>
</tr>
</thead>
</table>

Tell students you are going to conduct a survey to find out where they were born. Distribute a clothespin to each student. Show a world map, point out each continent, show where Canada is, and show the areas of North America that are not Canada. Point out that Australia is both a country and a continent and that the continent of Antarctica is missing because nobody lives there permanently. ASK: Where were you born? Have students record their answers by each attaching one clothespin to the appropriate string. Help students who know the country of their birth but are not sure which continent it is on. Have volunteers count the clothespins and record the answers in a table. Have students make a pictograph on grid paper or **BLM 1 cm Grid Paper** to show the results.

ASK: Where was the largest number of people in our class born? Where was the smallest number of people in our class born? How do you know? How many people were born in Canada? Make a comparison based on your class results. For example, how many people were born in Asia? How many more people were born in Canada than in Asia?
**Goals**

Students will read bar graphs and solve problems using information presented in bar graphs.

**PRIOR KNOWLEDGE REQUIRED**

- Can subtract two-digit numbers
- Can solve one- and two-step "how many more" and "how many less" word problems
- Can read data from a table
- Can read and draw a pictograph with one symbol representing one item
- Can multiply numbers
- Can divide by one-digit numbers (e.g., by finding the missing number in a times fact)

**MATERIALS**

- ball
- overhead projector
- transparency of BLM Colours of Cubes (p. U-77)
- transparency of BLM Snacks Bar Graphs (p. U-78)
- erasable markers of different colours (blue, green, red, black)

**Mental math minute.** Give students subtraction problems involving subtraction of close two-digit numbers, such as 43 − 38. Toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer it.

**Review pictographs.** Remind students what features pictographs have: They all have titles, symbols, labels, a grid on which the results are shown, and a scale. SAY: Sometimes, in place of more complicated pictures, we use squares as symbols to represent our data.

**Introduce bar graphs.** Explain that, to make graphs simple, you can join the squares on graphs into columns or rows of squares. These columns or rows of squares, joined together, are called bars. Explain that you will use one grid square for one cube. Project a transparency of BLM Colours of Cubes on the board. The graphs on the BLM show the following data:

<table>
<thead>
<tr>
<th>Colours of Cubes</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>4</td>
</tr>
<tr>
<td>Green</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>5</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
</tr>
</tbody>
</table>
Explain that both graphs show the same data, or the same collection of cubes. SAY: The graph on the left is a pictograph, like the ones you have looked at and created before. The graph on the right is called a bar graph, because it shows the data in bars. The blocks in each bar are usually squares, but sometimes there is not enough space to make the blocks square, so people use rectangles as well. In this bar graph, each square block in each bar means 1 cube. The bar for blue cubes is 4 blocks long, so we know that there are 4 blue cubes in the collection. ASK: How many green cubes are there? (3) How do you know? (the bar is 3 blocks long) How many red cubes are there? (5) How many yellow cubes? (4)

Introduce vocabulary. Ask students what the two graphs have that is the same, besides showing the same data. (the title, the labels) Circle the title in both graphs using a red marker and write “title” beside the graphs in red. Circle the labels that are shared in both graphs using a blue marker and write “labels” in blue. Explain that a bar graph has more labels than a pictograph. All other markings in words (not numbers) on a bar graph are also called labels. Circle the rest of the labels in blue.

Point out the general organization of the bar graph. Trace the axes and explain that these two lines make an L-shape and the lines are called axes. Explain that when we talk about one axis, we call it an axis but when there are two of them, we call them axes. Write both words on the board and trace the axes on the bar graph with a black marker. Have a volunteer underline the part that is different in the two words. (the third letter)

Explain that axes and their labels help us understand what we are looking at on bar graphs. SAY: One of the lines has numbers. Cover the rest of the graph so that only the bottom axis and the numbers are visible. ASK: What do you call a line and numbers in counting order under it? (number line) What other type of graph has a number line? (line plot) SAY: The numbers on the number line are called a scale. Point out that a pictograph also has a scale, something that tells you how many pieces of data each symbol means. Explain that a scale in a bar graph plays a similar role, and that you will talk about that more in the next lesson. Circle the scales on both graphs with a green marker and write the word “scale” in green beside the graphs.

Draw students’ attention to the fact that the bars on a bar graph usually have spaces between them. Explain that this makes a bar graph easier to read. Space is usually added on either side of each bar, including the bar closest to the number line.

Reading a bar graph. Project the Favourite Snacks graph from BLM Snacks Bar Graphs on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>5</td>
</tr>
<tr>
<td>Bagels</td>
<td>4</td>
</tr>
<tr>
<td>Fruit</td>
<td>7</td>
</tr>
<tr>
<td>Cheese</td>
<td>0</td>
</tr>
</tbody>
</table>
Explain that the graph shows most of the results of students voting for their favourite out of four snacks. Look at the parts of this bar graph. Point out to students that the title tells us what the graph shows, the label at the bottom (Snack Type) describes what the bars show, and the labels for each bar give detail about what that bar shows.

Have students signal the numerical answers for the next questions.
ASK: How many students chose muffins as their favourite snack? (5)
How many students chose fruit? (7) How many students voted for baked snacks? (9) How do you know? (muffins and bagels are both baked snacks, 5 + 4 = 9)

SAY: We have some more information to add. Four more students voted for fruit than for cheese. How many students voted for cheese? (3) How do you know? (7 − 4 = 3) Invite a volunteer to shade the blocks of the bar for cheese and have the class signal if they agree with what was done. Emphasize that the blocks should be shaded together and that the bar should start at the bottom of the graph, where the rest of the bars start, and above the label “Cheese.”

ASK: How many students voted in total? (19) How do you know? (5 + 4 + 7 + 3 = 19) What is the most popular snack of the four, or the snack that was chosen the most number of times? (fruit) What is the least popular snack, or the snack that was chosen the smallest number of times? (cheese)

Most popular and least popular, most common and least common.
Point out that you probably eat your favourite snack many days but you probably eat other snacks as well. For example, if your favourite snack is fruit, you still might have a muffin for a snack sometimes. If you survey the same people about what snack they had today, you may get a different graph. Project the Snacks Eaten Today graph from BLM Snacks Bar Graphs on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>2</td>
</tr>
<tr>
<td>Bagels</td>
<td>7</td>
</tr>
<tr>
<td>Fruit</td>
<td>6</td>
</tr>
<tr>
<td>Vegetables</td>
<td>3</td>
</tr>
</tbody>
</table>

ASK: Which bar on the bar graph is the tallest now? (bagels) Is that the favourite snack shown on the other snack graph? (no) Remind students that the phrase “most common” means the one that happens most often or has the greatest number. ASK: So, was the bagel the most common snack today for this group of people? (yes) ASK: What is the least common snack on this graph? (muffins)

Explain that we describe something as most common or least common when there is no choice involved, such as when we are not picking favourites. SAY: We can choose our favourite snack, but the snack we have today does not have to be the favourite one. We might even not have any
choice about the snack we had today. ASK: What other things do we have no choice about? (sample answers: hair colour, eye colour, height, types of flowers that grow where we live, the weather where we live)

**Exercises:** Write “most popular” or “most common” to complete the sentence.

a) The Ottawa Senators is the _____ sports team in Ottawa.

b) The _____ leaves in our garden are maple leaves.

c) I have to buy ice cream for a class party. I buy the _____ flavour.

d) Most students in the class have black hair. Black is the _____ hair colour in the class.

**Sample answers:** a) most popular, b) most common, c) most popular, d) most common

**Solving problems using multiplication with information from a bar graph.** Explain that the snacks in the bar graph are the options that the school council plans to sell during a school fair. Project the Favourite Snacks graph from BLM Snacks Bar Graphs on the board. Write on the board:

- Muffin: 50¢
- Bagel: 50¢
- Fruit: 25¢
- Cheese stick: 20¢

SAY: The school council is going to sell the snacks at these prices at the fair. Remind students that the symbol after the numbers means that the price is given in cents. ASK: If the same numbers of snacks are sold at the school fair as are shown on the Favourite Snacks bar graph, how many cents will the muffins for the whole class be sold for? (250¢) How do you know? (by skip counting by 50 five times) Write the multiplication equation on the board and remind students that, to multiply 5 × 50, they can think of the multiplication as skip counting.

SAY: I want to know what will sell for more money, all the fruit or all the bagels. ASK: How can we find out? (find the total price for bagels and the total price for fruit, then compare) Have students write multiplication sentences to find for how much each type of snack will sell. (bagels: 4 × 50 = 200¢, fruit: 7 × 25 = 175¢, cheese: 3 × 20 = 60¢)

ASK: What will sell for more, the fruit or the bagels? (the fruit) How much will the snacks sell for in total? (250 + 200 + 175 + 60 = 685¢) Have students write the addition sentence. SAY: A dollar is 100 cents. ASK: How many cents are in 2 dollars? (200) In 3 dollars? (300) In 4 dollars? (400) SAY: I want to know how many dollars I would need to buy all the snacks at the fair. ASK: Whole dollars always give me multiples of what number? (100) SAY: If I round down, I will get the number that is smaller than what I need. Then I will not be able to pay for all the food. ASK: What is the next multiple of 100 after 685? (700) How many dollars is that? (7 dollars) SAY: This means I need 7 dollars to pay for all the food.
ASK: What if each bagel sells for 55¢? I do not know how to multiply 4 × 55! How else can I find the answer? (repeated addition, or double twice) PROMPT: Could I use doubling? Have students double twice to find the price of the 4 bagels if they cost 55¢ each. (double of 55 is 110, double of 110 is 220)

Drawing bar graphs from tally charts. Draw on the board:

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Tally</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>UTTT</td>
<td>8</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>UTTTT</td>
<td>14</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>UTTT</td>
<td>10</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Explain that the table shows the times when students in a Grade 3 class go to bed. A student who goes to bed at 8:45 p.m. is counted with the students in the row that says 8:30 p.m.–8:59 p.m. Ask a volunteer to complete the count column, as shown below:

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Tally</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>UTTT</td>
<td>8</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>UTTTT</td>
<td>14</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>UTTT</td>
<td>10</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Point to the Count column and SAY: Now we have all the data values, so we can draw a bar graph to show the data.

Draw the bar graph for the data values on the board, as shown below:
NOTE: In the next lessons, Ontario students will draw a scaled bar graph for the same data values.

Extensions

1. Present the graph below:

![Bar Graph Image]

**Our Favourite Breakfast**

<table>
<thead>
<tr>
<th>Breakfast</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>0</td>
</tr>
<tr>
<td>Bagels</td>
<td>9</td>
</tr>
<tr>
<td>Eggs</td>
<td>3</td>
</tr>
<tr>
<td>Cereal</td>
<td>10</td>
</tr>
<tr>
<td>Other</td>
<td>5</td>
</tr>
</tbody>
</table>

**Breakfast**

ASK: How many students picked eggs as their favourite breakfast? (0) Point out that an empty or missing bar is not a mistake, it just shows that zero people chose that answer. SAY: My favourite breakfast is fruit. ASK: How can we mark my answer on the graph? If the following answers do not arise, explain that you can either add a bar for fruit or add your answer in the column labelled “Other.” Point out that the “Other” bar is used when there are many answers that few people give; it is easier to group all these answers together.

2. The bar graph shows a word we use in math. The height of the bar tells you how many times to use the letter that labels the bar. Write the letters you will use to make the word. Then rearrange the letters to make a word.

**Example:**

<table>
<thead>
<tr>
<th>Letter</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>4</td>
</tr>
<tr>
<td>l</td>
<td>3</td>
</tr>
<tr>
<td>n</td>
<td>2</td>
</tr>
<tr>
<td>v</td>
<td>1</td>
</tr>
</tbody>
</table>

Letters: e e e l n v  Word: eleven
Hint: Parts a) and c) are number words.

3. Redraw the Snacks Eaten Today graph from BLM Snacks Bar Graphs with horizontal bars instead of vertical bars.
**Goals**

Students will read and draw scaled bar graphs and solve problems using information presented in scaled bar graphs.

**PRIOR KNOWLEDGE REQUIRED**

Can solve one- and two-step “how many more” and “how many less” word problems

Can read data from a table

Can read and draw a bar graph with one symbol representing one item

Can read and draw a scaled pictograph

Can skip count by 2s, 3s, 4s, 5s, and 10s

Can multiply numbers

Can divide by one-digit numbers (e.g., by finding the missing number in a multiplication fact)

**MATERIALS**

ball (optional)

overhead projector

transparency of BLM Pictograph and Bar Graph Templates (p. U-79)

BLM Pictograph and Bar Graph Templates (p. U-79)

transparency of BLM Bar Graphs for Display (p. U-80)

erasable markers

**Mental math minute.** Ask students to solve multiplication questions within the range of $1 \times 1$ to $7 \times 7$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3, 3 \div 3, 2 \times 3, 6 \div 3,$ and so on, to $7 \times 3$ and $21 \div 3.$ Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

**Review scaled pictographs.** Draw on the board:

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td>2</td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>8</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>14</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>10</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td>4</td>
</tr>
</tbody>
</table>
Explain that the table shows the times when students in a Grade 3 class go to bed. A student who goes to bed at 8:45 p.m. is counted with the students in the row that says 8:30 p.m.–8:59 p.m. SAY: I would like to draw a pictograph to show this data. I would like to use rows of symbols for this graph. Project the top part of BLM Pictograph and Bar Graph Templates on the board and give each student a copy of the BLM so that they can draw the same graph. Have students suggest a title for the graph (such as Bedtimes of Grade 3 Students) and write it at the top of the graph. Together with students, fill in the labels on the pictograph. (Bedtimes, Before 8:00 p.m., 8:00 p.m.–8:29 p.m., 8:30 p.m.–8:59 p.m., 9:00 p.m.–9:29 p.m., 9:30 p.m. or later) NOTE: Leave the line below the pictograph empty; students will use it later to note the scale.

ASK: What is the most common time for students to go to bed? (between 8:30 p.m. and 8:59 p.m.) How many students go to bed in that time period? (14) How many grid squares do we have in each row of the table? (8) Is that enough for 14 symbols? (no) What can we do to show 14 students? (use a scale, use 1 symbol for more than 1 student) What symbol can we use to show students? (sample answers: smiley face, pillow) Record the options students suggest and have the class vote on a symbol.

ASK: What is a scale on a pictograph? (the part that says how many things one symbol represents) SAY: We need to decide what scale we are going to use. ASK: Are all the numbers in the table multiples of the same number? (yes, 2) PROMPT: Is there a number that we can skip count by and we will say all the numbers? ASK: How many symbols would we use for the largest number? (7) How do you know? (14 ÷ 2 = 7) Is there enough space to show 7 symbols? (yes, there are 8 grid squares, and 8 is more than 7) SAY: 2 students for 1 symbol is a good scale for this data. Add the scale to the template on the empty line below the graph, and have students do the same on their graphs.

SAY: To find how many symbols to draw for each number, we need to divide the number of students by the number each symbol means. So we divide each number by 2 to find how many symbols we will use. Add a "Number of Symbols" column to the table on the board. Have students write division sentences for each number of students per bedtime and have volunteers write the division sentences in the new column on the board. Then have students draw the symbols for each row on their pictographs. The completed table and pictograph are shown below and on the next page. Leave the table on the board for later use.

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Number of Students</th>
<th>Number of Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td>2</td>
<td>2 ÷ 2 = 1</td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>8</td>
<td>8 ÷ 2 = 4</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>14</td>
<td>14 ÷ 2 = 7</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>10</td>
<td>10 ÷ 2 = 5</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td>4</td>
<td>4 ÷ 2 = 2</td>
</tr>
</tbody>
</table>
**Bedtimes for Grade 3 Students**

<table>
<thead>
<tr>
<th>Bedtimes</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td>2 students</td>
</tr>
<tr>
<td>8:00 p.m. – 8:29 p.m.</td>
<td></td>
</tr>
<tr>
<td>8:30 p.m. – 8:59 p.m.</td>
<td></td>
</tr>
<tr>
<td>9:00 p.m. – 9:29 p.m.</td>
<td></td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td></td>
</tr>
</tbody>
</table>

**Introduce scaled bar graphs.** Project the bottom part of BLM Pictograph and Bar Graph Templates on the board. Explain that you would like to draw a bar graph for the same data as in the pictograph above. Point to the bar graph template. SAY: I would like to use this table but I do not have enough room to use 1 grid square, or 1 block, for 1 student. Explain that you can do with bar graphs exactly the same thing you did for pictographs: you can use 1 block to mean more than 1 person. SAY: In this graph, we will use 1 block for 2 people. To show this, we will mark the number line using skip counting by 2. Demonstrate how to mark the number line using the template on the bottom part of BLM Pictograph and Bar Graph Templates, and have students do the same on their copies of the template.

ASK: What else should be marked on a bar graph? (title, labels) Have students mark the title and the labels on their own graphs, and have volunteers add them to the template on the board. Then explain that the division sentences in the table you created earlier actually give you the length of each bar in blocks. Change the header in the third column of the table from “Number of Symbols” to “Length of Bar.” For example, the top bar, showing students who go to bed “Before 8:00 p.m.,” will be 1 block long. Shade the “Before 8:00 p.m.” bar on the board, and have students do the same on their copies. Then have them shade the rest of the bars on their graphs and have volunteers do the same on the board. The finished graph should look like this:
Analyzing a bar graph with horizontal bars. Remove the data table from the board and ask students to fold the BLM so that they cannot see the pictograph they created. ASK: What is the most common time for students to go to bed? (8:30 p.m.–8:59 p.m.) How can you see that from the bar graph? (the bar for that time is the longest) What is the least common time? (before 8:00 p.m.) How can you see that from the graph? (the bar is the shortest) How many more students go to bed between 8:00 p.m. and 8:29 p.m. than go to bed before 8:00 p.m.? (6 more) Discuss different ways to find the answer. (the longer of the two bars is 3 blocks longer than the shorter bar, and $3 \times 2 = 6$; count the blocks of each bar, subtract, and multiply the difference using the scale: 7 blocks – 1 block = 6 blocks; $6 \times 2 = 12$; find the number each bar represents and subtract the numbers: $14 - 2 = 12$) Ask students to find the total number of students in the bar graph, and again discuss with them different ways to find the answer. (add the number of blocks in the bars: $1 + 4 + 7 + 5 + 2 = 19$ and then double: $2 \times 19 = 38$; find all the numbers for the individual bars and add them: $2 + 8 + 14 + 10 + 4 = 38$) Ask students to find the answer both ways and to check that the answers are the same.

Deciding what number a scale skip counts by. SAY: Just as a symbol can mean different numbers on pictographs, people use different numbers to skip count on scales of bar graphs. Draw on the board:

```
0 4 8 12 16 20
```

SAY: Imagine this is the scale of a bar graph. ASK: What number does the scale skip count by? (4) How do you know? (it is the first number after zero) As a class, skip count by 4s to check that the numbers you say when skip counting by 4s are the numbers on the scale.

Exercises: What number does the scale skip count by?

a)  

```
0 5 10 15 20 25 30
```

b)  

```
0 10 20 30 40 50
```

c)  

```
0 3 6 9 12 15 18
```

d)  

```
0 20 40 60 80 100
```

Answers: a) 5, b) 10, c) 3, d) 20

Some bar graphs use shortcuts in labels. Project the Number of Snow Days in Calgary, AB, graph from BLM Bar Graphs for Display on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Months</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct–Nov</td>
<td>9</td>
</tr>
<tr>
<td>Dec–Jan</td>
<td>21</td>
</tr>
<tr>
<td>Feb–Mar</td>
<td>27</td>
</tr>
<tr>
<td>Apr–May</td>
<td>6</td>
</tr>
</tbody>
</table>
Point to the horizontal axis and SAY: The horizontal axis shows the time in periods of 2 months so that we do not have too many bars on the graph. So the first bar shows how many snow days there were in October and November together. ASK: What does the second bar show? (the number of snow days in December and January together) The third bar? (the number of snow days in February and March together)

Now examine the vertical axis. Point out that the longest bar goes to the top of the grid, but the scale does not have a number there. ASK: What is the last number that is written on the scale? (24) What number does the scale skip count by? (3) If you skip count by 3s, what is the next number after 24? (27) SAY: Sometimes there is no space to write all the numbers on a graph, but if you know what number the scale skip counts by, you can always figure out the missing numbers.

**Exercises:** Fill in the missing numbers on the scale.

a) 

| 0 | 5 | 10 | 15 | 20 | 25 | 30 |

b) 

| 0 | 4 | 8 | 12 | 16 |

**Bonus:** 

| 0 | 40 | 80 | 120 |

**Answers:** a) 10, 25; b) 8, 12, 20; Bonus: 20, 60, 100

**Analyzing a bar graph with vertical bars.** ASK: On the bar graph for snow days in Calgary, what does each block in the bars mean? (3 days of snow) How can we find out how many days of snow each bar shows? (multiply the number of blocks in the bar by 3) Draw on the board:

<table>
<thead>
<tr>
<th>Months</th>
<th>Height of Bar (blocks)</th>
<th>Multiplication</th>
<th>Number of Snow Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct–Nov</td>
<td>3</td>
<td>$3 \times 3 = 9$</td>
<td>9</td>
</tr>
</tbody>
</table>

Have students copy the table and fill it in for the other three bars. Have volunteers write the answers on the board. The completed table should look like this:

<table>
<thead>
<tr>
<th>Months</th>
<th>Height of Bar (blocks)</th>
<th>Multiplication</th>
<th>Number of Snow Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct–Nov</td>
<td>3</td>
<td>$3 \times 3 = 9$</td>
<td>9</td>
</tr>
<tr>
<td>Dec–Jan</td>
<td>7</td>
<td>$7 \times 3 = 21$</td>
<td>21</td>
</tr>
<tr>
<td>Feb–Mar</td>
<td>9</td>
<td>$9 \times 3 = 27$</td>
<td>27</td>
</tr>
<tr>
<td>Apr–May</td>
<td>2</td>
<td>$2 \times 3 = 6$</td>
<td>6</td>
</tr>
</tbody>
</table>

ASK: How many more snow days are there in December and January than in October and November? (12) How do you know? ($21 - 9 = 12$) How can you see that on the bar graph? (the bar for December and January is 4 blocks longer, and $4 \times 3 = 12$) How many fewer snow days are there in April and May than in February and March? (21) How do you know?
(27 – 6 = 21) Again, have students check that they get the same answer working directly from the bar graph.

ASK: Are there more snow days from the beginning of October to the end of January or from the beginning of February to the end of May? (from February to May)  How do you know? (9 + 21 = 30 is less than 27 + 6 = 33; or 3 + 7 = 10 blocks is less than 9 + 2 = 11 blocks) Have students check the answer both ways. ASK: How many more? (3 days more) Discuss which way students prefer to find the answers and why. Some students might prefer to work with the numbers because it is more familiar; others might prefer to work with the bar graph because the numbers are smaller and the multiplication at the end is easier.

ASK: If we had a bar in this graph for June and July or for August and September, what would the bars show? (there would be no bar) Why would the bars have height 0? (it does not usually snow in Calgary in the summer)

**Extensions**

1. The bar graph shows the coins in Jayden's pocket.

**Coins in Jayden's Pocket**

- **Nickels**
  - 0
- **Dimes**
  - 2
- **Quarters**
  - 4
- **Loonies**
  - 6

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickels</td>
<td>0</td>
</tr>
<tr>
<td>Dimes</td>
<td>2</td>
</tr>
<tr>
<td>Quarters</td>
<td>4</td>
</tr>
<tr>
<td>Loonies</td>
<td>6</td>
</tr>
</tbody>
</table>

**a)** For each type of coin, how much money does Jayden have?

**b)** How much money does Jayden have in total?

**Answers**

**a)** 50¢ in nickels, 80¢ in dimes, 200¢ in quarters, and 200¢ in loonies

**b)** 530¢ or $5.30
2. Sharon made a bar graph showing coins in her piggy bank, but she forgot to label the scale. She has 25 nickels.

**Coins in Sharon’s Piggy Bank**

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickels</td>
<td></td>
</tr>
<tr>
<td>Dimes</td>
<td></td>
</tr>
<tr>
<td>Quarters</td>
<td></td>
</tr>
<tr>
<td>Loonies</td>
<td></td>
</tr>
</tbody>
</table>

a) What number did she skip count by for the scale?
b) How many coins of each type does she have?
c) How much money does she have in her piggy bank in total?

**Answers**
a) 5s
b) 25 nickels, 10 dimes, 10 quarters, and 5 loonies
c) $9.75

3. Marko drew a bar graph of the coins in his piggy bank, but he forgot to label the scale. He has 24 coins in total.

**Coins in Marco’s Piggy Bank**

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickels</td>
<td></td>
</tr>
<tr>
<td>Dimes</td>
<td></td>
</tr>
<tr>
<td>Quarters</td>
<td></td>
</tr>
<tr>
<td>Loonies</td>
<td></td>
</tr>
</tbody>
</table>

a) How many coins of each type does he have?
b) What number did he skip count by for the scale?
c) How much money does he have in his piggy bank in total?

**Answers**
a) Marko has 6 coins of each of the 4 types
b) 3s
c) $8.40
Goals
Students will read and draw scaled bar graphs, including graphs where the bars end between grid lines. Students will solve problems using data presented in scaled bar graphs.

PRIOR KNOWLEDGE REQUIRED
Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a scaled bar graph
Can read and draw a scaled pictograph with half symbols
Can multiply numbers
Can divide by one-digit numbers (e.g., by finding the missing number in a multiplication fact)
Can find half of a number by dividing by 2

MATERIALS
ball (optional)
BLM Winter Graphs (pp. U-81–82)
overhead projector
transparency of BLM Favourite Winter Activities (p. U-83)
pictures of bar graphs (e.g., from magazines) that are not drawn on grids (optional)
transparency of BLM Bar Graphs for Display (p. U-80)
text paragraphs (see Extension 1)
selection of books popular in the class (see Extension 2)

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $7 \times 7$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, to $9 \times 3$ and $27 \div 3$. Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

Graphs with bars that end between grid lines. Distribute BLM Winter Graphs (1). Project Graph 1 from BLM Favourite Winter Activities on the board. The graph shows the data on the following page.
SAY: A nature park asked 200 visitors to vote on their favourite activity that the park offers during winter. The graph shows some of the results of the vote. ASK: What number does the scale on the graph skip count by? (10)

Draw on the board:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building snow forts</td>
<td>0</td>
</tr>
<tr>
<td>Building snowmen</td>
<td>30</td>
</tr>
<tr>
<td>Skiing</td>
<td>60</td>
</tr>
<tr>
<td>Sledding</td>
<td>50</td>
</tr>
<tr>
<td>Snowshoeing</td>
<td>0</td>
</tr>
</tbody>
</table>

Have students copy the table and fill in the rows for Building snowmen, Skiing, and Sledding using the data from the bar graph on the BLM. (30, 60, 50) Have a volunteer fill in these rows on the board. SAY: Two hundred people answered the question. ASK: How many answers does the graph show? (140) How do you know? (30 + 60 + 50 = 140) How many answers does the graph not show? (60) How do you know? (200 − 140 = 60)

SAY: We need to show information for two more activities on the bar graph. In the survey, 45 people chose building snow forts as their favourite activity. Remind students that, on pictographs, they sometimes had to show a number that was not a multiple of the number each symbol meant (for example, 1 symbol meant 10, and they needed to represent 45). Have students explain what they did in such a case. (used whole and half symbols) ASK: How many symbols do you need to mean 40? (4 symbols) 50? (5 symbols) 45? (4 full symbols and one half symbol) How do you know? (45 is exactly halfway between 40 and 50)

ASK: Does the number 45 appear on the scale of the bar graph? (no) Where on the number line should 45 be? (between 40 and 50) Ask a volunteer to show where 45 is on the number line. Explain that, similar to symbols on pictographs, the bars on a bar graph do not have to end exactly on a grid line to show a full block. For example, we can draw a bar that ends where the number 45 should be on the number line. Add 45 to the table for building snow forts and add a bar that ends at 45 for building snow forts on the bar graph. Have students write the number in the table and draw the bar on their own graphs.
ASK: How many more people chose sledding than building snow forts? (5)
How can you see that from the graph? (the bar for sledding is a half block longer than the bar for snow forts)
How many fewer people voted for building snowmen than for building snow forts? (15 fewer)
Discuss which way makes it easier to find the answer to the second question. (subtract the numbers on the table: $45 - 30 = 15$; count the blocks on the bar graph)
The bar for building snow forts is one whole and one half block longer. One whole block means 10 people and a half block means 5 people, so together the difference is $10 + 5 = 15$ people.

SAY: The rest of the people in this survey voted for snowshoeing as their favourite activity. ASK: How many people voted for snowshoeing? (15) How do you know? ($60 - 45 = 15$; add all the numbers for the other columns and subtract from 200, so $200 - 185 = 15$) Add the number 15 to the table, add the bar for snowshoeing on the board, and have students do the same to their tables and graphs. Remind students that the mode is the most common data value. ASK: What is the mode? (skiing) Explain to students that the mode activity is skiing because it happens 60 times.

Order of categories in a graph. Project Graph 2 from BLM Favourite Winter Activities on the board. ASK: Does this graph have the same title as the other graph? (yes) The same scale? (yes) The same labels? (yes) What is different between the two graphs? (the order in which the labels for activities are written and the order of the bars on the graph) Have students check that the graphs show the same number of people for each activity. For example, has the number of people who like skiing changed from the other bar graph or stayed the same? (stayed the same) ASK: Do the graphs show the same data? (yes) Why do they look different? (in the second bar graph, the bars have changed order so that they go from longest bars at the top to shortest bars at the bottom)

ASK: Which activity was the most popular? (skiing) Which activity was the least popular? (snowshoeing) Which graph makes that easier to see? (Graph 2) SAY: We say that, on Graph 2, the activities were sorted from most popular to least popular. On Graph 1, the activities were also organized in a special way, but the order has nothing to do with numbers. ASK: Can you guess what the order is? (the activities are in alphabetical order)
PROMPT: Look at the first two letters of each word. What do you notice?

Redrawing a bar graph with a different scale. Project Graph 1 from BLM Favourite Winter Activities on the board. SAY: I would like to show the same data using a different scale. Distribute BLM Winter Graphs (2). Have students fill in the title and labels (students can choose the order in which to write the labels). SAY: I want to use counting by 5s on this graph.
ASK: What number do we start counting with? (0) Point out that 0 should be in the first blank from the left, the blank that is under the vertical axis. Have students fill in the zero and the rest of the numbers on the scale. ASK: What is the last number you filled in? (60)

SAY: There are 15 people who chose snowshoeing. ASK: How long should the bar be? (3 blocks) How do you know? ($15 \div 5 = 3$) Have students fill
in the bar on their graphs. SAY: Let's repeat this with the bar for building snowmen. ASK: Do you need to count the blocks when you fill in the bar or is there an easier way to find where the bar should end? (when you know the number the bar should extend to, look to the scale to find the correct number and then look directly above to where the bar should end) Trace your finger along the scale to find 30 and then up from 30, along a grid line, to show where the bar must end. ASK: How many people answered that they like sledding? (50) Have students trace their fingers from 50 on the scale and along the line on their new graphs to mark the place where the bar for sledding should end. Have them draw the bar for sledding. ASK: How many blocks long is the bar you drew? (10 blocks) What division sentence would you use to find the length of the bar? (50 ÷ 5 = 10) Do we get the same length both ways? (yes) Repeat with the bar for building snow forts. (45 people, 9 blocks)

ASK: Can we use division to find the length of the bar for skiing? (no) Why not? (we don’t know how to do 60 ÷ 5) Can we use the other method, following grid lines from the scale? (yes) Have students draw the bar for skiing on their graphs. ASK: How many blocks long is the bar? (12 blocks) SAY: If we drew the bar correctly, 12 × 5 should be equal to 60. ASK: How can we find out how much 12 × 5 is? Encourage students to come up with different methods to do the calculation or to explain how they know 12 × 5 = 60. (use repeated addition; use the distributive property: 12 × 5 = (10 × 5) + (2 × 5); double 6 × 5; start at 5 × 10 and add on 5 two additional times; use the fact that there are 60 minutes in an hour and 12 divisions of 5 minutes on a clock; make the bar for skiing two blocks longer than the bar for sledding)

Have students compare the two graphs. ASK: Which graph takes more space? (the graph with the scale counting by 5s) Which graph has no bars that end between the grid lines? (the graph with the scale counting by 5s) On which graph was it easier to draw the bar for skiing? On which graph was it easier to draw the bar for building snowmen? On which graph is it easier to see how many more people voted for skiing than for sledding? How many fewer people voted for snowshoeing than for building snowmen? Have students explain their opinions.

**Reading graphs that are not on a grid.** Explain that sometimes graphs are not drawn on a grid. If possible, show pictures from magazines or from the Internet that show bar graphs without a grid. Explain that, in this case, we often do not see the blocks of the bars and have to imagine the lines that go from the ends of the bars to the scale. Project the Sam's Reading on Weekdays graph from **BLM Bar Graphs for Display** on the board. The graph shows the data on the following page.
Have students explain what the graph shows. (how much Sam reads during weekdays) Point out that the days of the week are too long to write in full, so we use short forms for the labels. ASK: What is the short form for Tuesday? (Tue) What is the short form for Thursday? (Thu) Explain that sometimes we use just the first letter as a label. ASK: Why are we not using just the first letter to show days of the week? (Tuesday and Thursday would both be shown with a T, and it would be hard to see which bar belongs to which day)

ASK: What number does the scale skip count by? (4) How many pages did Sam read on Monday? (12) How do you know? (the bar for Monday ends right at the line that we can trace directly to 12) Repeat with Wednesday and Friday. (4, 24) ASK: Is there a bar that does not end at a line that extends to one of the numbers on the scale? (yes, the bar for Tuesday) ASK: Between which two lines does the bar end? (between 8 and 12) Does it end closer to one of the lines? (no) Does the bar end at a point halfway between 8 and 12? (yes) SAY: So the bar is showing that the number is halfway between 8 and 12. ASK: What number is that? (10)

To prompt students, draw a number line from 8 to 12, and make hops from both sides at the same time, as shown below:

\[
\begin{array}{c}
8 \quad 9 \quad 10 \quad 11 \quad 12 \\
\end{array}
\]

\[
\begin{array}{c}
8 \quad 9 \quad 10 \quad 11 \quad 12 \\
\end{array}
\]

Exercises

a) How many more pages did Sam read on Friday than on Monday?

b) How many fewer pages did Sam read on Wednesday than on Tuesday?

c) How many pages in total did Sam read during the weekdays?

**Bonus:** Sam’s teacher says that Sam should read at least 10 pages every day. On which days did Sam read enough?

**Answers:** a) 12; b) 6; c) 58; **Bonus:** Monday, Tuesday, Friday

Have students do Questions 1–5 on AP Book 3.2, pp. 195–198. **NOTE:** For students who struggle with using only the first letters of the dog breeds in Question 3, tell them they can write the full names of the breeds vertically or diagonally underneath the graphs.
Extensions

1. Give students a paragraph of text (for example, from a favourite story read in class or from another subject) and ask them to each create a bar graph that shows the number of words on each line.

2. Ask students to name some of their favourite authors. To prompt students, have a selection of books that are popular during read-alouds and independent reading time on hand to refer to. Then select the top three authors as well as “Other,” for a total of four categories. Explain to students that they are to each vote for only one favourite author, and then ask students to vote by raising their hand as you say the categories one at a time. Record the votes. Have students make a bar graph to show the results.
Goals
Students will review and compare pictographs, bar graphs, and line plots.

PRIOR KNOWLEDGE REQUIRED
Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a scaled bar graph and pictograph
Can read and draw a line plot with a number line divided into quarters
Can multiply numbers
Can divide by one-digit numbers (e.g., by finding the missing number in a multiplication fact)
Can find half of a number by dividing by 2

MATERIALS
ball (optional)
overhead projector
transparency of BLM Tree Cone Graphs (p. U-84)
leaves from different trees, 5 per student
BLM Pictograph and Bar Graph Templates (p. U-79, optional)
poster of trees and leaves (optional)
BLM Comparing Graphs (p. U-85, optional)
BLM Tree Cone Graphs (p. U-84, see Extension 1)
coins (see Extension 2)
shoeboxes (see Extensions 2 and 3)
dice (see Extension 3)

Mental math minute. Ask students to solve multiplication questions within the range of 1 × 1 to 5 × 5 and corresponding division questions. For each number, go through the questions in order, such as 1 × 3, 3 ÷ 3, 2 × 3, 6 ÷ 3, and so on, to 5 × 3 and 15 ÷ 3. Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

Comparing visual features of pictographs, bar graphs, and line plots. Explain that some students went to a park and collected some cones from different kinds of trees and that then the students made graphs showing their findings. Project BLM Tree Cone Graphs on the board.
Have students signal their answers for the next questions by raising the number of fingers that corresponds to the number of the graph, or thumbs up for “yes” and thumbs down for “no.” ASK: Which graph is a line plot? (3) Which graph is a pictograph? (1) Which graph is a bar graph? (2) Do all
three graphs have a title? (yes) Do all three graphs have labels? (yes) Do all three graphs have a number line? (no) Which graph does not have a number line? (1) Do all three graphs have a vertical axis? (no) Which graph has a vertical axis? (2) Do all three graphs use symbols? (no) Point out that line plots use Xs, which are easy-to-draw symbols. This means that the pictograph and the line plot use symbols, and the bar graph is the only one of the three graph types that does not use symbols. ASK: What symbols do the other two graphs use? (the pictograph uses circles, the line plot uses Xs) Point out to students how, on the bar graph, the label for eastern white pine had to be shortened, just as other labels have been shortened in previous lessons.

Ask students to explain how the number lines are different in the bar graph and the line plot. (in a bar graph, the number line can be horizontal or vertical, and it has whole numbers: in this bar graph, the number line is vertical and skip counts by 2s; in a line plot, the number line is always horizontal: this line plot shows numbers from 5 to 13) Make sure all of the points are raised.

**Comparing ways the data are presented in the graphs.** ASK: What does the pictograph show? (what types of cones students collected and the number of each) Can we tell from the bar graph what types of cones students collected? (yes) Can we tell that from the line plot? (no) Why not? (the line plot shows the lengths of the cones and the numbers of cones with those lengths, not the types) How else could students have sorted the cones? (sample answers: by who found what number, by what number of cones is short vs long or wide vs narrow) What type of graph could they use to present the data? (answers will vary)

ASK: How many cones did students collect in total? (16) How can you see that from each type of graph? Have students share different ways to answer this question for different graphs. (count the Xs on the line plot; find how many cones of each type were collected from the bar graph or the pictograph and add the numbers) Students can also count the symbols in the pictograph and multiply by the scale (2), but they need to figure out what to do with half symbols. To prompt them, remind them that two halves make one whole, so they can count the two half symbols as one whole symbol and then conclude that there are 8 full symbols in the pictograph, $8 \times 2 = 16$, so students collected 16 cones. Students can also use a similar method with the bar graph.

**Answering questions about the data in graphs.** ASK: How many more red spruce cones than eastern white pine cones did students collect? (4) Which graph can you use to answer the question? (pictograph or bar graph) Have students explain the different ways to find the difference from the bar graph and the pictograph. ASK: How many fewer 9 cm long cones than 5 cm long cones did students collect? (5) Which graph did you use to answer the question? (line plot) How can you find the answer on the graph? (there are 6 Xs in the 5 cm column and 1 X in the 9 cm column, $6 - 1 = 5$)
ACTIVITY

Bring in five leaves from different trees per student or, if possible, have students collect them. Divide students into groups of three. Have each group put the leaves together, look at and discuss the leaves, and then make graphs showing their findings. In each group, one student should make a pictograph, another should make a bar graph, and a third should make a line plot, similar to the graphs on AP Book 3.2, p. 199. Provide templates for graphs from BLM Pictograph and Bar Graph Templates to students who struggle with organizing their graphs. If available, display a poster about local trees and their leaves and help students who struggle with identifying the leaves. Then ask the same kinds of questions as in Question 2 in AP Book 3.2, p. 200, and have students answer them using the different graphs they created.

Have students do Questions 1–3 on AP Book 3.2, pp. 199–200. For Questions 1 and 2, students must compare the three types of graphs; to make comparing easier, and so that students can annotate the graphs, you could give students copies of BLM Comparing Graphs. For Question 3, you might remind students that sometimes there is not enough space on a graph to put a full name (for example, for a type of book, a breed of dog, or the name of a weekday or a month). Ask students if they know what “sci-fi” means. Prompt them to understand that it means “science fiction,” which is spelled out in full in Question 3, part b). Write “science fiction” and “sci-fi” on the board and have a volunteer circle the common parts.

Extensions

1. Project or distribute BLM Tree Cone Graphs. The table shows the length of cones for the types of trees discussed in the lesson:

<table>
<thead>
<tr>
<th>Type of Tree</th>
<th>Length of Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red pine</td>
<td>from 4 cm to 6 cm</td>
</tr>
<tr>
<td>Eastern white pine</td>
<td>from 13 cm to 20 cm</td>
</tr>
<tr>
<td>Red spruce</td>
<td>from 3 cm to 5 cm</td>
</tr>
<tr>
<td>Balsam fir</td>
<td>from 4 cm to 13 cm</td>
</tr>
</tbody>
</table>

Use the line plot and one of the other two graphs shown in the lesson to answer the questions.

a) What type of tree has the longest cones?
b) One of the cones is 9 cm long. What type of tree could it belong to?
c) How many eastern white pine cones did the students collect? Which X shows that cone on the line plot? How long is that cone?
d) Which column contains all the red spruce cones?
e) How many balsam fir cones did the students collect? How long are these cones?
**Answers**

a) Eastern white pine

b) Balsam fir

c) Students collected only 1 eastern white pine cone. Its length is a number between 13 cm and 20 cm. The line plot shows only cones that are up to 13 cm long, so the single X in the column for 13 cm has to be the X for the eastern white pine cone. The cone is 13 cm long.

d) Red spruce cones are up to 5 cm long. All 5 of the red spruce cones have to be in the column for cones that are 5 cm long.

e) Students collected 4 balsam fir cones, and these cones vary in length. The 13 cm long cone is an eastern white pine cone, so the rest of the cones are shorter than 13 cm. Only balsam fir cones can be from 9 cm to 12 cm long, and there are 4 Xs for these cones on the graph. So these 4 marks belong to the balsam fir cones. The lengths of the balsam fir cones are as follows: one cone 9 cm long, two cones 10 cm long, and one cone 12 cm long.

2. Give each student a coin and a shoebox. Have them toss the coin 20 times to find if the coin turns heads or tails. Students should toss the coin over the shoebox to keep it from rolling away. Have students tally the results for the tosses using a table with headings “Heads” and “Tails.” Have them make a bar graph or a pictograph to show the results of the tossing. Students who struggle with organizing graphs can use BLM Pictograph and Bar Graph Templates.

3. Give students two dice each and a shoebox. Have them conduct this experiment: toss the dice at the same time and add the results. Students should toss the dice over the shoebox to keep them from rolling away. Then have students draw a line plot to show the sums on the dice using a number line from 2 to 12. For example, they will plot an X above 8 on the number line for a roll of 3 and 5, plot an X above 10 for a roll of 4 and 6, and so on. Students can work in pairs using two pairs of dice and draw only one graph. Each student should toss the dice 10 to 15 times, so that the line plot contains 20 to 30 Xs. Both students in a pair should have two dice to save time.

Compare the most common results for different pairs of students. Explain that 6, 7, and 8 come out more often than other numbers.

ASK: Why could that be? PROMPT: How many addition sentences can you write that add to 2? (three: 0 + 2, 1 + 1, 2 + 0) Which of them can be rolled on a pair of dice? (only 1 + 1) How many ways can we roll 2? (1) Repeat with 6, 7, and 8 so students can see how there are many more combinations that can make up those numbers.
Goals

Students will conduct a survey and then display and analyze the results. Students will create a survey question, tally the data, and present their data in a bar graph.

PRIOR KNOWLEDGE REQUIRED

Can read data from a table
Can distinguish between a statement and a question
Can transfer data from a tally chart to a pictograph
Can read and draw a scaled bar graph and pictograph

MATERIALS

ball or relay race baton (optional)
pictures of people with cats and/or dogs or neither (optional)
BLM My Survey (p. U-86)

Mental math minute. Arrange students in a line and have them add two-digit numbers by adding tens and adding ones. For each addition, such as 35 + 46, students need to say three steps: adding the tens (30 + 40 = 70), adding the ones (5 + 6 = 11), and finishing the addition (70 + 11 = 81, so 35 + 46 = 81). The next student in line gets a new problem. Students can pass a ball or a relay race baton to each other, so that the person who receives the baton answers the next question. Start with problems that do not require regrouping, such as 25 + 34, and continue to questions that require regrouping ones.

Introduce first-hand and second-hand data. SAY: Data you collect by yourself is called first-hand data. Some ways you can collect first-hand data are by measuring items, conducting an experiment, conducting a survey. Explain to students that data collected by someone else is called second-hand data. SAY: You can find second-hand data in books and magazines, and on the Internet or in commercials.

Good survey questions. Tell students that you want to conduct a survey about how many siblings (brothers or sisters) they have. Explain that to be successful at conducting surveys, students must learn how to ask good questions. The question should be clear and simple and allow for all possible answers. Explain to the class that the quality of a survey question determines the quality of the data collected. Read and discuss the following potential survey questions with the class. Have students describe the possible answers in each case, and identify problems with the questions.

• “Do you have a brother?” (answers: yes, no) ASK: But what if somebody has a sister? Point out that the question is limiting.
• “Do you have a brother or a sister?” (answers: brother, sister) ASK: But
what if I somebody has neither? But what if somebody has both? Should they circle both words? What if they have two brothers, or three sisters? Point out how many situations are not reflected in the question.

• “How many brothers and sisters do you have?” (answers should be numbers starting from zero) ASK: How many possible answers do we want? Will it be hard to create a chart with 12 columns? Do we want to limit the number of answers? Point out that the question may produce too many answers.

Write on the board:

How many siblings do you have? 0, 1, 2, 3, more than 3

ASK: Is this question easy to understand? (yes) Do we have all possible situations in the answers? (yes) Do we have too many answers? (no)

Have students signal the answers to your questions using thumbs up or thumbs down.

Explain the reason for limiting answer choices. Conduct a survey with students by asking each student what their favourite flavour of ice cream is—do not limit their choices at this point.

Tally the answers, then ask students how many bars will be needed to display the results on a bar graph. Point out that too many bars would be needed to display all the results. ASK: How can the question be changed to reduce the number of bars needed to display the results? How can the choices be limited? Should choices be limited to the most popular flavours? Why is it important to offer an "other" choice?

Choosing the possible answers to a survey question. Explain to students that it is helpful to predict the most popular answers to a survey question before a survey is conducted. ASK: Why is it important to predict the most popular answers? Could the three most popular flavours of ice cream have been predicted?

Have volunteers predict the most popular answers for the following survey questions:

• What is your favourite colour?
• What is your favourite vegetable?
• What is your favourite fruit?
• What is your hair colour?
• What is your favourite animal?

Students may disagree on the choices. Explain to them that a good way to predict the most popular choices for a survey question is to ask the survey question of a few people before asking everyone.

Wording the question to receive only one answer. Emphasize that the question has to be worded so that each person can give only one answer.
Exercises: Will the question receive one answer or multiple answers from each person you ask?

a) What is your favourite ice cream flavour?
b) What flavours of ice cream do you like?
c) Whom will you vote for in the election?
d) Which of the candidates do you like in the election?
e) What is your favourite colour?
f) Which colours do you like?

Answers: a) one, b) multiple, c) one, d) multiple, e) one, f) multiple

Including the “other” category. Have students think about whether or not an “other” category is needed for the following questions. Write on the board:

What is your favourite food group?
- ☐ Vegetables and Fruits
- ☐ Meat and Alternatives
- ☐ Milk and Alternatives
- ☐ Grain Products

What is your favourite food?
- ☐ Soup
- ☐ Spaghetti
- ☐ Tacos
- ☐ Salad

ASK: How do you know when an “other” category is needed? (when there are too many answers) For the following exercises, write each question on the board one by one and discuss whether the question would require an “other” category and why.

Exercises: Does the question require an “other” category? Why?

a) What is your favourite day of the week?
   Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday

b) What is your favourite day of the week?
   Friday, Saturday, Sunday

c) What is your favourite animal?
   horse, cow, dog, pig, cat

d) How many siblings do you have?
   0, 1, 2, 3, 4 or more

e) Whom will you vote for in the student council election?
   Student A, Student B, Student C

Selected sample answers: a) the category “other” is not necessary because all days are listed, c) the category “other” is necessary because some people like other animals

Point out that an “other” category may not be an option. For example, if the teacher wants to bring two movies to show on the last day before the...
December holidays and she has only five movies at home, she would give only those five movies as choices and bring the two most popular movies to class.

**ACTIVITY 1**

**NOTE:** This activity may be advanced for some students.

1. Discuss the difference between “and” and “or” in a question and what these words mean. For example, the questions “Do you have a cat and a dog?” and “Do you have a cat or a dog?” will produce different answers. For the first question, respondents can answer no (I don’t have a cat AND a dog) or yes (I have both a cat AND a dog). For the second question, respondents can answer yes and no, but the meaning changes: “yes” means I have a cat OR a dog; “no” means I have neither.

If possible, bring in pictures (perhaps cut from magazines) of people with cats and/or dogs: some with a cat and a dog, some with only a cat, some with only a dog, and some with neither. Hold up the pictures one at a time and ask students how each person would answer the questions “Do you have a cat or a dog?” and “Do you have a cat and a dog?” Tally the results. For example, your tallies might look like this:

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have a cat or a dog</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I have a cat and a dog</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students summarize the answers in separate bar graphs for each question.

**Writing clear survey questions.** Ask students what they think most people in the class would prefer to do for a party: go to a movie or go on a skating trip. Ask students what kind of question you should ask to gather this data. Record all of their suggestions on the board. Once you have done this, review all the questions and determine all the possible answers people might give for each question on the board.

Questions and answers might include the following:

1. Do you want to have a party? (answers: yes, no)
2. If you had a party, would you like to go to a movie? (answers: yes, no)
3. If you had a party, would you like to go on a skating trip? (answers: yes, no)
4. If you had a party, would you like to go to a movie or go on a skating trip? (answers: movie, skating trip, neither, both)
5. If you had a party, which one of these things would you rather do: go to a movie, go on a skating trip? (answers: movie, skating trip)

Discuss with students which is the best question to ask. For example, questions 1, 2, and 3 are limiting and will not capture all data because people might say yes to a party, yes to a skating trip, and yes to a movie; therefore the data won’t tell you which one people would rather do. Question 4 is not clearly worded and might get different types of answers, making it difficult to gather the data and make a decision (i.e., determine which is the most popular choice). Question 5 makes it clear which activity is preferred.

**Graphing first-hand data.** Tell students you want to do a survey to find out what they will be doing on their summer holidays. Present the question “What will you be doing over the summer holidays?” and the choices: camp, family trip, summer school, staying at home, other. (Modify the choices according to students’ interests and activities.)

Ask students to each identify their main summer activity by raising their hands as you call out each choice. Record the data in a tally chart with the choices listed. Count the tallies for each category to determine how much space you will need for the bar graph. Draw a grid with a fixed number of markings, for example, 8. ASK: What scale should we use?

Remind students that there is a space between the bars in a bar graph and have students independently create a bar graph for the data collected. Remind them to include a title and labels on their graphs.

Analyze the data together. Ask students what the data is telling them. Record those statements.

**ACTIVITY 2**

2. Ask students to name some of their favourite authors. (Have on hand a selection of popular books for students to refer to.) Select the top three authors as well as “other” for categories. Ask students to identify their favourite author (explain that they can only raise their hand once when “voting”) and collect the data from the class by recording the data using tally marks.

Then ask students how they can tell if everyone voted. PROMPT: Do they think anyone voted twice? Does the number of votes equal the number of students in the class? What would happen if someone didn’t vote? What if someone voted twice?

On the board, draw a graph with five markings on the vertical axis. Label the horizontal axis with the names of the three favourite authors and “other.” ASK: If we want no more than five markings on our graph, what scale should we use? Would it make sense to have only one marking on the graph? How would that make the graph hard to read? Decide on the number of markings and the scale for the graph. Then complete the graph.
Provide each student with a copy of **BLM My Survey**. Tell students that they will be designing their own survey and then surveying their classmates. Everyone can ask a different question, so suggest several topics if they have trouble getting started. (sample questions: How do you get to school? What is your favourite colour? How many people are in your family? What time do you wake up on weekdays? How long do you take to get ready for school? Does your jacket have a hood? What pizza toppings do you like? What is your favourite meal? What is your favourite cereal? What is your favourite season? Who is your favourite person? What type of home do you live in? What is your favourite summer or winter activity?) After students create their surveys and survey their classmates, have students create a pictograph to show the data from their surveys.

**Extensions**

1. Discuss a graph where there is an “other” category with a bar higher than at least one of the other bars. Draw on the board:

   ![Graph](image)

   **Favourite Pizza Topping**

   - Olives
   - Cheese
   - Pepperoni
   - Other

   ASK: If we ask “What is the least favourite pizza topping shown on this graph?”, what will the answer be? (olives) SAY: Olives has the fewest votes, but the “other” bar may contain three different answers—in which case, each one of them is the least favourite. We do not know what answers are included in the “other” bar. Also, there might be toppings that had no votes and so are not recorded at all. This means we cannot say what the least favourite topping is.

2. Have students use the data from the surveys they created at the end of the lesson to create a graph on a computer using either math software or an online resource.

3. Have students work in pairs or small groups. Ask them how they could find out the favourite colour of every teacher in the school. ASK: How would you collect the data? How would you organize the data? Groups should come up with an action plan and then follow through with it. After collecting and representing the data, students can report their findings orally.
Goals
Students will identify the possible outcomes of various events.

PRIOR KNOWLEDGE REQUIRED
Experience using a spinner and rolling a die

MATERIALS
die
coin
5 marbles of different colours
spinner per pair of students

Mental math minute. Have students stand in a line. Give the first student a subtraction that does not need regrouping, such as 97 – 12. Have subsequent students in line repeatedly subtract a number, in this case 12, by having each student say one subtraction aloud. When a student says a subtraction that involves regrouping, emphasize that this answer was a bonus. Example: Student 1 says, “97 – 12 = 85.” Student 2 says, “85 – 12 = 73.” Student 3 says, “73 – 12 = 61.” Student 4 says, “61 – 12 = 49.” (Student 4’s answer is a bonus because it involves regrouping.) Continue until Student 8 says, “13 – 12 = 1,” and then start a new chain.

Introduce outcomes. Tell students that today they will start learning how to predict the future! Hold up a die and ask students to predict what will happen when you roll it. ASK: Can it land on a vertex? (no) On an edge? (no) What do you predict will happen? (the die will land on one of its faces) Have students predict which number you will roll. Then roll the die (more than once, if necessary) to show that the prediction about the die landing on a face happened, but the number students picked might not be rolled. SAY: The possible results of rolling the die are called outcomes. To predict the future, you must learn to identify which outcomes of various actions are more likely to happen and which are not. But first, you must learn to identify outcomes correctly.

Explain that rolling a die, tossing a coin, and spinning a spinner are all examples of experiments. Hold up a coin. ASK: What are the possible outcomes of tossing a coin? (coming up heads, coming up tails) How many outcomes are there? (2) Ask students to identify the possible outcomes of a soccer game. ASK: How many outcomes are there? (3 outcomes: team A wins, team B wins, it is a draw)

Outcomes of spinning a spinner. Draw the spinner in the margin on the board. SAY: This is a spinner. I can press a pencil to the centre of the circle and put a paper clip around the pencil. Demonstrate how to do that. SAY: Now I can spin the paper clip and it will point to one of the regions. If
it points to the line between the regions, the spin does not count and I will have to spin again.

ASK: How many outcomes does this spinner have? (4) SAY: When the spinner lands in each of the regions, it is a different outcome. Number the regions 1 to 4 and list the outcomes on the board, as shown below:

1. The pointer lands in region 1.
2. The pointer lands in region 2.
3. The pointer lands in region 3.
4. The pointer lands in region 4.

SAY: The spinner has four possible outcomes.

Outcomes of a spinner with several regions coloured the same colour. Colour regions 1, 2, and 3 blue. Colour region 4 red. SAY: When I spin the spinner now, the pointer will still land in one of the regions. The number of regions on the spinner did not change. This means that the number of outcomes did not change either. The outcomes of spinning this spinner are still the same.

Introduce events. SAY: Imagine I am going to play a game with you using this spinner. If the spinner lands on blue, you win. If the spinner lands on red, I win. ASK: How many different ways can the spinner land on blue? (3 ways, regions 1, 2, and 3) How many different ways can the spinner land on red? (1 way, region 4) SAY: This game can have two different results, spinning blue or spinning red. We call each result of this game an event. Each event can be created by one or more outcomes. ASK: How many outcomes make the event “spinning blue”? (3 outcomes) Which outcomes are these? (pointer lands in region 1, pointer lands in region 2, pointer lands in region 3) How many outcomes make the event “spinning red”? (1 outcome) Which outcome? (pointer lands in region 4)

Exercises: How many outcomes does the spinner have? How many outcomes make the event “spinning white”?

a) b) c) d)

Answers: a) 4, 2; b) 8, 4; c) 6, 3; d) 4, 4

Outcomes of taking a marble out of a collection of marbles without looking. Show a set of five marbles of different colours, but of the same size. SAY: I am going to close my eyes and pick a marble. ASK: What are the possible outcomes of picking a marble with your eyes closed? (picking each marble is a separate outcome) How many outcomes are there when picking a marble out of five marbles? (5) Emphasize that, as with a spinner, the colour of the marbles does not affect the number of outcomes. The number of outcomes is the total number of marbles. If two marbles are red, then there are two outcomes that make the event “picking a red marble.”
Exercises

1. You roll a die. How many outcomes are there?

   Answer: 6

2. You pick a ball from the box. How many outcomes are there?

   \[
   \begin{array}{ccc}
   & R & B \\
   B & & \\
   \end{array}
   \quad
   \begin{array}{ccc}
   & R & B \\
   B & R & \\
   \end{array}
   \quad
   \begin{array}{ccc}
   & R & B \\
   B & R & \\
   \end{array}
   \]

   Answers: a) 2 outcomes, b) 3 outcomes, c) 3 outcomes

3. You pick a marble out of a box without looking. How many outcomes are there? How many outcomes make the event “picking a red marble”?

   \[
   \begin{array}{ccc}
   & R & B \\
   Y & & \\
   \end{array}
   \quad
   \begin{array}{ccc}
   & R & Y \\
   R & R & \\
   \end{array}
   \quad
   \begin{array}{ccc}
   & R & B \\
   B & R & Y \\
   \end{array}
   \]

   Answers: a) 3 outcomes in total, 1 outcome is picking a red marble; b) 3 outcomes in total, 2 outcomes are picking a red marble; c) 5 outcomes in total, 3 outcomes are picking a red marble

Unequal outcomes. SAY: You have to make a spinner with five possible outcomes. ASK: How would you do this? Invite volunteers to draw possible spinners. Draw on the board:

\[
\begin{array}{c}
\text{Spinner with five outcomes.}
\end{array}
\]

Shade each region with a different colour, and ASK: How many outcomes are there for this spinner? (5) Are all the outcomes expected to happen equally, or is there an outcome that might happen more often than the other ones? (one happens more often) Draw another spinner on the board with a short pointer, as shown in the margin. ASK: How many outcomes does the second spinner have? (4) Will the pointer ever be in the grey region? (no)
**ACTIVITY**

In advance, prepare for each pair of students a spinner like the one below, with all regions coloured differently.

![Spinner diagram](image)

One partner spins the spinner 10 times and the other partner tallies the results. Then the partners switch roles. Have students determine which colour occurs the greatest number of times. (R, red)

**Extensions**

1. You flip two coins at the same time. Find how many outcomes there are.

   **Answer:** there are 4 outcomes: heads and heads, heads and tails, tails and heads, tails and tails

2. You pick a ball out of the box. Find how many outcomes there are.

   ![Box with balls](image)

   **Answer:** 3 outcomes, all outcomes are picking a blue ball

3. Design a spinner that has 6 outcomes, with 2 outcomes of spinning white.

   **Sample answer**

   ![Sample spinner](image)
Goals
Students will identify situations in which the chances of an event are even.

PRIOR KNOWLEDGE REQUIRED
- Can identify outcomes of an experiment
- Can find half of a number
- Can identify visual representations of fractions

MATERIALS
- BLM Multiplication Chain (pp. V-2–7)
- Coin
- Supplies to make spinners (red, blue, and green pencil crayons, paper, and paper clips)

Mental math minute. Give each student a card from BLM Multiplication Chain. Call a volunteer to the front of the class. The volunteer reads the card (e.g., I have 3 \times 4 and 25). Students who have, for example, 12 or 5 \times 5 on their cards come to the front of the class and stand beside the volunteer, showing their cards. If there is more than one student with a card that matches (for example, 12 appears on multiple cards), choose one student to join the chain now, and have the others join the chain later. The students who just joined the chain read the second half of their cards, and new students join the chain. If the number called from one side of the chain matches the multiplication sentence on the other side of the chain and there is no third student who can join either side of the chain, the chain is complete. The remaining students should try to make a new chain of their own. The game ends when everyone is part of a chain.

Finding half of a number. Remind students that finding half is dividing by 2. Review with students how they can find half of a number, half of a pie, and half of a set of objects. Draw six squares on the board. Invite a volunteer to circle half of the squares, as shown below:

Have the volunteer explain their answer. ASK: What is half of 8? (4) How do you know? (because double of 4 is 8) If I need a number that is less than half of eight, which numbers fit this description? (0, 1, 2, or 3) Repeat these questions for different even numbers. Then draw even numbers of shapes and ask students to find several ways to shade more than half of the shapes.
Draw a different set of six squares and ask a volunteer to shade less than half of the squares red. (the volunteer can shade 0, 1, or 2 squares) Draw another set of six squares and ask another volunteer if they can shade a different number of squares blue, so that the number of blue squares will be still less than half of six.

**Exercises**

1. I have 10 marbles, and half of them are red. How many marbles are red?
   
   **Answer:** 5

2. I have 12 marbles, and 6 of them are green. How many of my marbles are green: half, less than half, or more than half?
   
   **Answer:** half

3. I had 14 marbles, and I lost 6 of them. Did I lose more than half, less than half, or exactly half?
   
   **Answer:** less than half

4. Complete the statement by writing “more than half,” “half,” or “less than half.”
   
   a) 2 is _____ of 4.  
   
   b) 3 is _____ of 6.  
   
   c) 6 is _____ of 14.  
   
   d) 7 is _____ of 12.  
   
   e) 5 is _____ of 8.  
   
   f) 5 is _____ of 12.  
   
   **Answers:** a) half, b) half, c) less than half, d) more than half, e) more than half, f) less than half

5. Shade half of the pieces.
   
   ![Sample answers]

   **Sample answers**
   
   a)  
   
   b)  
   
   c)  

   **Even chances in events.** Hold up a coin. ASK: What are the possible outcomes of flipping this coin? (coming up heads, coming up tails) How many outcomes are there? (2) What happens more often, flipping heads or flipping tails? (the outcomes occur the same number of times) SAY: The chances are the same. In mathematics, we say that you have an even chance of flipping heads or tails. Write “even chance” on the board and explain that the chances of an event are even when the event happens in...
exactly half of the outcomes. Flipping tails is 1 out of 2 possible outcomes, and 1 is half of 2. ASK: How many outcomes are there when you roll a die? (6) How many of those outcomes are numbers that are more than 3? (3) What are those numbers? (4, 5, and 6) SAY: Since half of the outcomes are numbers greater than 3, you have an even chance of rolling a number greater than 3. You also have an even chance of rolling a number that is 3 or less.

SAY: There are eight marbles in a box. I take out one marble without looking. ASK: How many outcomes are possible? (8, regardless of the colour of the marbles) What is half of 8? (4) SAY: I would like to have an even chance of taking out a green marble. ASK: How many marbles should be green? (4) Does it matter what colour the other marbles are? (no, as long as they’re not green) Invite a volunteer to draw on the board a collection of eight marbles that gives an even chance of taking a green marble. If the collection uses only two colours, ask another volunteer to make a collection that uses at least three colours but still gives an even chance of taking a green marble.

**Exercises**

a) Draw a collection of six marbles that gives an even chance of picking a green marble.

b) Draw a collection of eight marbles using at least three colours that gives an even chance of picking a yellow marble.

**Sample answers**

a) , b)

Draw on the board:

ASK: Which part of the spinner is shaded green? (the right half) What are the possible outcomes for this spinner? (the spinner lands on the green region, the spinner lands on the red region) How many outcomes are there? (2) What fraction of the outcomes will the pointer land on the green region? (half) Are the chances of spinning green even? (yes)

**Exercises:** Is there an even chance of spinning green? Why?
Answer: yes, this spinner has 4 possible outcomes because it has 4 regions, but 2 of the 4 (half of 4) are green

Draw on the board:

- ![Spinner 1](image1.png)
- ![Spinner 2](image2.png)
- ![Spinner 3](image3.png)

Have students identify the outcomes for each spinner. ASK: In which spinners do you have an even chance of spinning red? (the one on the left and the one on the right)

**Exercises:** Is there an even chance of spinning red?

- a) ![Spinner a](image4.png)
- b) ![Spinner b](image5.png)
- c) ![Spinner c](image6.png)
- d) ![Spinner d](image7.png)

**Answers:** a) no, b) no, c) yes, d) yes, Bonus: yes

Ask students to describe several events that have even chances of happening when rolling a die. (possible answers: roll a number that is 3 or less; roll an even number; roll 2, 3, or 5; roll an odd number) Encourage students to think of examples that do not involve rolling dice, spinning spinners, or drawing marbles. (For example, several pairs of boots are in a closet. I pick a boot without looking. It is either a left boot or a right boot. So, “I pick a left boot” has an even chance of happening.)

**ACTIVITY**

Divide students into three groups. Have each group make one of the three spinners below (groups should colour their spinners as shown), spin it 24 times, and record the results. Then have groups make a bar graph of the results.

- ![Spinner e](image8.png)
- ![Spinner f](image9.png)
- ![Spinner g](image10.png)

ASK: Did your group spin red in more than half, less than half, or exactly half of the spins? Is this what you expected? Discuss as a class.

**NOTE:** Extension 3 is required to cover the British Columbia curriculum.
Extensions

1. Complete each statement by writing “more than half” or “less than half.”
   a) 2 is _____ of 5.          b) 3 is _____ of 7.
   c) 6 is _____ of 13.         d) 7 is _____ of 11.
   e) 11 is _____ of 15.        f) 5 is _____ of 11.

   **Answers:** a) less than half, b) less than half, c) less than half, d) more than half, e) more than half, f) less than half

2. A spinner has five regions and two of them are red. If you spin the spinner 20 times, do you expect to spin red more than 10 times or fewer than 10 times? Explain.

   **Answer:** Fewer than 10 times. Two is less than half of 5, so of 20 spins we expect less than half of the 20 spins to be red.

3. **Snowsnake game.** A snowsnake is a traditional hunting tool for catching small wild animals. The snowsnake game, which helps hunters learn to hunt with spears, is played by sliding a long spear, the snowsnake, along a smooth field of snow or a special runway. The object of the game is to make the spear slide as far as possible along the snow or runway using an underhand throw.

   Yura and Nita played snowsnake 10 times. Yura won 3 games and Nita won 7 games. Based on the last 10 games, does Yura have an even chance of winning the next game? Why?

   **Answer:** no, because Yura won less than half of the games
Goals

Students will describe the chances of events as even, likely, and unlikely. Students will describe the likelihood of events.

PRIOR KNOWLEDGE REQUIRED

Can identify outcomes of an experiment
Can find half of a number
Can identify visual representations of fractions

MATERIALS

ball (optional)
supplies to create a spinner (red, yellow, and blue pencil crayons, paper, and paper clip)
collection of marbles or counters of different colours (red, blue, green, and yellow)

Mental math minute. Ask students to solve multiplication questions within the range of \(1 \times 1\) to \(7 \times 7\) and corresponding division questions. For each number, go through the questions in order, such as \(1 \times 3, 3 \div 3, 2 \times 3, 6 \div 3,\) and so on, to \(7 \times 3\) and \(21 \div 3\). Then progress to a different number. Next try questions out of order, but keep each multiplication and its corresponding division together. You can toss a ball to the student you want to answer the question, and have students toss the ball back to you as they answer.

Likely and unlikely events. Explain that when events happen often, but not always, we say that such events are likely. For example, you are likely to go to school on a Wednesday in October. You might get sick and not go to school, but most Wednesdays during the school year you go to school, so the event is likely. Ask students to give you more examples of likely events.

Explain that when an event happens not so often, but still happens sometimes or at least can happen in theory, we call such events unlikely. Have students make some predictions and tell you if the following events are likely or unlikely.

a) The sun will rise tomorrow. (likely)
b) The teacher will give the answers to the test before giving the test. (unlikely)
c) An alien will walk into the class in the next minute. (unlikely)
d) You are taller than last year. (likely)
e) It will snow in July. (unlikely)

Invite students to name some events and have other students say if the events are likely or unlikely. You could ask students to compare the
likelihood of events. For example, it is unlikely to snow in July, but it is even more unlikely that an alien will walk into the class!

**Likely and unlikely events in probability experiments.** In advance, create the spinner below and colour the parts marked R red, Y yellow, and B blue:

![Spinner diagram](image)

ASK: How many outcomes are there? (6) How many parts are coloured red? (4) Is that more than half? (yes) SAY: More than half of the spinner is red. Mathematicians call an event *likely* if it is expected to happen more than half the time. Point to the spinner. SAY: Red is likely to happen because more than half of the spinner is coloured red. ASK: How many parts are coloured yellow? (1) Is that more than half or less than half? (less) SAY: Less than half of the spinner is yellow, so yellow is *unlikely* to happen.

Conduct an experiment with 12 spins. Use a tally chart on the board to record the results. From the last lesson, students know that an event with an even chance is expected to happen exactly half the time. Write all three terms (“likely,” “unlikely,” and “even chance”) on the board. Using the tally chart, describe the chances of spinning red, blue, and yellow in these terms. (red is likely, blue and yellow are both unlikely)

**Exercises**

1. There are two pairs of black boots and two pairs of brown boots in a dark closet. Count the total number of outcomes if you pull a boot from the closet. Count the number of outcomes for the given event. Say whether the event is likely, unlikely, or has an even chance.
   a) Pull out a right boot
   b) Pull out a black boot
   c) Pull out a brown left boot
   d) Pull out a boot that is either black or brown and right
   **Bonus**
   e) Pull out a boot that is not brown or left
   f) Pull out a boot that is not black

   **Answers:** total outcomes = 8; a) 4, even; b) 4, even; c) 2, unlikely; d) 6, likely; Bonus: e) 2, unlikely; f) 4, even
2. Describe the given events as even, likely, or unlikely.

   a) Events: Spin green. Spin pink.

   b) Events: Spin blue. Spin pink.

**Answers:** a) spinning green is even, spinning pink is unlikely; b) spinning blue is unlikely, spinning pink is likely

3. Are the chances likely, unlikely, or even?

   a) Pull a black sock from a box with 6 green socks and 4 black socks.

   b) Pull a nickel from a pocket with 5 nickels, 4 dimes, and 1 quarter.

**Answers:** a) unlikely, b) even

4. Give an example of a likely event and an unlikely event when rolling a die. Both events should be possible.

**Sample answer:** rolling 3 or more is likely, rolling less than 3 is unlikely

**Certain and impossible events.** Write “unlikely,” “even,” and “likely” in a row on the board. ASK: Which word would you use to describe an event like meeting a live dinosaur in the street when you walk home from school today? Can that happen at all? (no) Write the word “impossible” to the left of the other terms. SAY: Impossible describes an event that cannot happen. Ask students which words describe an event that will definitely happen, such as rolling a number less than 7 on a die. (certain, definite, sure, absolute) SAY: We call an event that will definitely happen certain. Write the word “certain” to the right of the list, as shown below:

   impossible unlikely even likely certain

**NOTE:** Although students might use the word impossible to describe the likelihood of meeting a dinosaur, this event is not necessarily impossible (scientists might find a way to clone dinosaurs in the future). The only events that are strictly impossible are events that are contradictory—like rolling a number greater than 6 on a regular die. You might wish to discuss this distinction with students.

**Exercises:** Describe the event as impossible, unlikely, even, likely, or certain.

   a) Pull a green sock from a drawer with 10 green socks and 2 red socks.

   b) Pull a $5 bill from a wallet with three $20 bills and one $5 bill.

   c) You will be older than your mother.
d) Roll a number greater than 0 on a die.
e) Meet a green panther.
f) Roll an odd number on a die.

Answers: a) likely, b) unlikely, c) impossible, d) certain, e) unlikely, f) even

Ask students to give examples of various events and explain whether they are likely, unlikely, certain, impossible, or even. Encourage students to think of events using marbles, dice, money, and other objects, as well as events from daily life, such as meeting an astronaut on the way to school.

Show students the following collection of marbles or coloured counters:

ASK: What are your chances of picking green? (even) Which colour are you most likely to pick? (green) Which colour is less likely to be picked, yellow or red? (red) So, which colour is least likely to be picked? (red)

Exercises: For each spinner, describe the events in the table below as certain, likely, even, unlikely, or impossible.

<table>
<thead>
<tr>
<th>Event</th>
<th>Spinner A</th>
<th>Spinner B</th>
<th>Spinner C</th>
<th>Spinner D</th>
<th>Spinner E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spinning green</td>
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<tr>
<td>Spinning red</td>
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<tr>
<td>Spinning blue</td>
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<tr>
<td>Spinning yellow</td>
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</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th>Event</th>
<th>Spinner A</th>
<th>Spinner B</th>
<th>Spinner C</th>
<th>Spinner D</th>
<th>Spinner E</th>
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<tbody>
<tr>
<td>Spinning green</td>
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<tr>
<td>Spinning red</td>
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<td>impossible</td>
<td>likely</td>
<td>impossible</td>
<td>even</td>
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<tr>
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<td>unlikely</td>
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<td>certain</td>
<td>unlikely</td>
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<tr>
<td>Spinning yellow</td>
<td>impossible</td>
<td>even</td>
<td>impossible</td>
<td>impossible</td>
<td>impossible</td>
</tr>
</tbody>
</table>
Extensions

1. Draw a spinner to match the description.
   a) You are likely to spin yellow.
   b) You are unlikely to spin green.
   c) It is impossible to spin blue.
   Sample answers: a–c)  
   ![Spinner Diagram]

2. Draw a collection of marbles of at least three different colours to match the description.
   a) You are likely to draw a green marble.
   b) You are unlikely to draw a yellow marble.
   c) It is impossible to draw a purple marble.
   Sample answers: a–c)  
   ![Marbles Diagram]

3. Invent or describe a game in which one player’s chance of winning is very close to certain. What are the chances of the other player(s) winning?
   Sample answer: Two players roll a die three times. Player 1 wins if the number rolled is greater than 1 at least one time. Player 2 wins when the number rolled is 1 all three times. Players 2’s chance of winning is very close to impossible (1/216, or less than half a percent).
Goals

Students will use the concept of even chances in games.

PRIOR KNOWLEDGE REQUIRED

- Can identify outcomes of an experiment
- Can find half of a number
- Can identify visual representations of fractions
- Can identify likely and unlikely events

MATERIALS

- deck of cards without face cards
- collection of marbles (optional)
- container with 2 red and 2 blue counters per pair of students
- 1 die per pair of students

Mental math minute. Shuffle a deck of cards after removing the face cards. Divide students into pairs. One pair at a time, each student selects a random card. The students create two multiplication equations using the cards selected. For example, if Student A selects a 7 and Student B selects an 8, they create the equations $7 \times 8 = 56$ and $8 \times 7 = 56$. If the card selected is an ace, treat it as number 1. The students say the equations aloud, do three jumping jacks, and then sit down. Continue until all students have had a chance to participate.

Introduce fair games. SAY: I would like to play a game with you. The rules of the game are simple. I spin a spinner. If I get red, I win; if I get blue, the class wins. Ask students if they agree to play by these rules. Draw the spinner in the margin on the board. ASK: Do you still want to play? (no) Why not? (because you are more likely to spin red than blue) How do you know? (the part that is coloured red is more than half the spinner, and the part that is coloured blue is less than half the spinner; there is more red than blue) Write “fair game” on the board. Ask students to explain what they think this term might mean. Encourage students to use math vocabulary in their explanations. SAY: In mathematics, a fair game means that all players have an equal chance of winning, or are equally likely to win.

Exercises: Suppose players spin the spinner shown. If they spin grey, Player 1 wins. If they spin white, Player 2 wins. Who has a better chance of winning, Player 1 or Player 2? Or is it a fair game?

a)  

b)  

c)
Answers: a) Player 2, b) fair game, c) Player 1

Fair games for experiments with equally likely outcomes. ASK: In all the spinners that I have shown you today, how many outcomes were there? (2) SAY: There were two outcomes, but they were very different outcomes for each spinner. ASK: What happens if we have equal outcomes, as when we have marbles in a box? Draw on the board (or show a collection of marbles, if available):

ASK: If I take one marble out of this box, without looking, how many outcomes are there? (7) Are the outcomes equal, or am I more likely to pull out one marble in particular? PROMPT: No matter what colour, is there one specific marble that I might want? (the marbles are the same, if you do not see the colour, you can pull out any of them, so the outcomes are equal) ASK: What colour do you think I am most likely to pull out? (white) Why? (there are more white marbles than any other colour) If I win when I pull out a white marble and you win if you pull out a grey marble, is this a fair game? (no) Why not? (there are more white marbles than grey marbles) SAY: When an outcome brings me a win, we call it a winning outcome. So if I have more winning outcomes than you do, the game is not fair.

SAY: Let’s check another game with the same collection of marbles. If I pull a black marble, I win. If you pull a grey marble, you win. ASK: Is this game fair? (yes) Why? (we both have only one marble that will bring us a winning outcome) SAY: I have only one winning outcome, and so do you. The number of winning outcomes is the same for both players, so it is a fair game.

Present the steps below to check if a game is fair. Write on the board:

Step 1: Check that all outcomes are equal.
Step 2: Count how many winning outcomes each player has.

Explain to students that if the number of winning outcomes is the same, the game is fair. If not, the player with more winning outcomes has a better chance of winning.

Exercises: Is the game fair? If not, which player has a better chance of winning?

a) Player 1 spins red to win. Player 2 spins green to win.

b) Player 1 spins red to win. Player 2 spins green to win.
c) 

![Spinner diagram with 6 regions: Y W W Y Y W]

Player 1 picks white to win.
Player 2 picks yellow to win.

**Answers:** a) the game is fair, b) Player 2 has a better chance of winning, c) Player 1 has a better chance of winning

**Games with more than two players.** Draw on the board:

![Spinner diagram with 6 regions: R B G R B G]

SAY: The spinner has six regions. ASK: Are the outcomes equal? (yes)
SAY: We have three players. Player 1 must spin red to win, Player 2 must spin blue to win, and Player 3 must spin green to win. ASK: To make the game fair, how many regions should be red? (2) How many should be blue? (2) How many should be green? (2) Why? (because spinning red, blue, and green would have an equal number of winning outcomes) Emphasize that we divide the regions between the players equally. SAY: If there are six regions in total, and we want to give the same number of regions to each player, we need to divide 6 by 3. Since $6 \div 3 = 2$, each player gets 2 regions. Have a volunteer write two Rs and two Bs and two Gs in the regions of the spinner, as shown in the margin, or colour the spinner.

**Exercises:** Is the game fair? If not, who has a better chance of winning, Player 1, Player 2, or Player 3?

a) 

![Spinner diagram with 3 regions: B R B]

Player 1 spins red to win. Player 2 spins blue to win.

b) 

![Spinner diagram with 3 regions: B G B]

Player 1 spins red to win. Player 2 spins green to win. Player 3 spins blue to win.

c) 

![Spinner diagram with 4 regions: Y W R W Y Y]

Player 1 picks white to win. Player 2 picks yellow to win. Player 3 picks red to win.
Answers: a) Player 2, b) Player 1, c) the game is fair

Designing fair games. Tell students that for a game to be fair, all players have an even chance of winning. SAY: A box has six marbles; they are either red or blue. If I pick a red marble, I win; if I pick a blue marble, the class wins. ASK: To make the game fair, how many blue marbles should be in the box? (3) How many red marbles? (3) Why? (because picking red and blue marbles must be even) Vary the game. ASK: If two of the six marbles are red, who has a better chance of winning, me or the class? (the class) Is it a fair game? (no) What if five marbles are red, is it a fair game? (no) Who has a better chance to win? (teacher) What should be in the box to make the game fair? (the same number of red and blue marbles)

Draw on the board:

SAY: There are six regions in the spinner. If I spin red, I win; if I spin blue, the class wins. ASK: To make the game fair, how many regions should be blue? (3) How many should be red? (3) How did you figure that out? (there are 6 regions and 2 players, $6 \div 2 = 3$, so each player gets 3 regions) Have a volunteer write three Rs and three Bs in the regions of the spinner, as shown in the margin.

Explain to students that it is important to have 3 red and 3 blue regions to make the game fair, but the position of the red and blue regions is not important. Draw on the board:

SAY: This is another spinner that makes the game fair.

Exercises: Design a two-player game with a spinner that has at least 4 regions so that the two players each have an equal chance of winning.

Sample answer: Player 1 must spin red to win, and Player 2 must spin green to win
ACTIVITIES 1–2

1. Pair up students and give each pair a container with 2 red counters and 4 blue counters. Either student can pick counters from the container without peeking. Player 1 wins if the counter drawn is red, and Player 2 wins if the counter drawn is blue. Ask students to explain whether the game is fair or not. Have students play the game 20 times (returning the counter to the container each time) and keep a tally of who wins each time. Ask students if the results are what they expected.

2. Ask pairs to keep track of who wins and who loses in 20 repetitions of the following game: Players take turns rolling a die. Player 1 wins if the number rolled is a 1 or a 6; Player 2 wins otherwise. Ask students to explain whether the game is fair or not. Ask if the results are what they expected.

Extensions

1. Suppose players spin the spinner below.

   ![Spinner with colors: R, G, B]

   If they spin green, Player 1 wins. If they spin blue, Player 2 wins. If they spin red, it is a draw. Is the game fair?

   Answer: The game is fair, since both players have equal chances of winning (the fractions coloured green and blue are the same).

2. Three players are spinning the spinner below. Invent rules of play to ensure that the game is fair.

   ![Spinner with colors: R, Y, W, B]

   Sample answer: If they spin red, Player 1 wins. If they spin yellow, Player 2 wins. If they spin blue, Player 3 wins. If they spin white, it is a draw.
Goals

Students will compare theoretical and experimental probability.

PRIOR KNOWLEDGE REQUIRED

- Can identify outcomes of an experiment
- Can find half of a number
- Can identify visual representations of fractions
- Can identify likely and unlikely events

MATERIALS

- deck of cards without face cards
- spinners
- one coin, one die, and one paper clip per student
- BLM Shape Spinner (p. U-87, see Extension 1)

Mental math minute. Shuffle a deck of cards after removing the face cards. Divide students into pairs. One pair at a time, each student selects a random card. The students create two division equations using the cards selected. For example, if Student A selects a 7 and Student B selects an 8, they create the equations $56 \div 8 = 7$ and $56 \div 7 = 8$. If the card selected is an ace, treat it as number 1. The students say the equations aloud, do three jumping jacks, and then sit down. Continue until all students have had an opportunity to participate.

Introduce expectations. In advance, prepare spinners like the one shown below. Draw on the board:

```
G  B  R
R  B  G
```

ASK: How many outcomes does this spinner have? (4) How many parts of the spinner are coloured red? (2) How would you describe the chances of spinning red? (even) What fraction of the outcomes is “spinning red”? (2 out of 4, or half) If I spin the spinner four times, how many times would I expect to spin red? (2 times) What if I spin it eight times? (4 times) How do you know? (half of the times would be red) Give students the spinners you prepared, and ask them to spin the spinner 10 times and tally the results. ASK: Did everybody get 5 reds? (no) What should be the most common number of red spins? (5)

Combine the results for the whole class. Compare the class results with the individual results using graphs or other visual representations. For example, individuals could make bar graphs of their results (showing the number
of times they spun red, blue, and green) and you could put all the results together in a class bar graph.

Discuss with students the difference between the class results and the individual results. ASK: Which results are closer to the prediction, the individual results or the class result? (class result) What data do you think is more reliable, the individual data or the group data? (group data) Point out that the more results you have, the closer you get to the expected outcome. The number of reds spun by the class will be closer to half of the total (the expected outcome) than the number of reds spun in many of the individual results.

SAY: You flip a coin. ASK: How many times would you expect to get heads if you flip the coin four times? (2 times) Six times? (3 times) 10 times? (5 times) Point out that the answer is always half the given number because there are two possible outcomes.

Exercises:

1. Does the event have an even chance of happening?
   a) Flipping a coin and getting heads
   b) Rolling an even number (2, 4, or 6) on a die
   c) Taking a blue marble from a box that has 3 blue and 3 green marbles
   d) Taking a blue marble from a box that has 3 blue, 1 yellow, and 2 green marbles
   
   **Answers**: yes, the chances are even for all

2. How many times do you expect the outcome?
   a) You flip a coin 10 times. How many times do you expect to get heads?
   b) You roll a die 20 times. How many times do you expect to roll an even number (2, 4, or 6)?
   c) A box has 3 blue marbles and 3 green marbles. You take out one marble, then return it to the box, and you repeat this 12 times. How many times do you expect to take out a blue marble?
   d) A box of marbles has 3 blue, 1 yellow, and 2 green marbles. You take out one marble, then return it to the box, and you repeat this 12 times. How many times do you expect to take out a blue marble?

   **Answers**: a) 5 times, b) 10 times, c) 6 times, d) 6 times

Designing games for expected results. Remind students that for a game to be fair, all players have even chances of winning. SAY: There are eight marbles in a box, four red and four blue. Player 1 must pick a red marble to win, and Player 2 must pick a blue marble to win. ASK: Is it a fair game? (yes) How do you know? (the number of red and blue marbles are equal)
SAY: In a fair game with two players when there is no tie, each player expects to win half of the times that they play.

Draw on the board:

SAY: There are six regions in the spinner. If I spin red, I win; if I spin blue, the class wins. In a fair game, the number of red regions and the number of blue regions must be equal. Write three Rs and three Bs in the regions of the spinner, as shown below:

SAY: I expect to win half of the times that we play. ASK: If I spin the spinner 10 times, how many times would you expect that I spin red? (5 times) How many times blue? (5 times)

Exercises: Design a game with 8 marbles in which you would expect to get a certain result 10 times in 20 turns (in other words, the result has even chances).

Sample answer: Put 4 red and 4 blue marbles in a bag. Have players pair up and take turns picking marbles from the bag. Player 1 wins if a red marble is picked and Player 2 wins if a blue marble is picked. After each round, the players have to put the marbles back in the bag.

Extensions

1. Complete BLM Shape Spinner in pairs.

2. Imagine there are 10 cards with the numbers from 1 to 10 on them (one number per card). You select a card at random, write down the number you see on it, and put the card back. Predict whether, if you repeat this six times, each event is certain, impossible, likely, unlikely, or has even chances.

   a) The sum of the numbers will be greater than 60.
   b) All the numbers will be even.
   c) The sum of the numbers will be less than 12.
   d) The sum of the numbers will be greater than 20.

Answers: a) impossible, b) even chances, c) unlikely, d) likely
3. Here are the favourite colours of four students:

Anna: red, John: blue, Amir: green, Sandy: yellow

Draw a spinner divided into 8 parts. Colour the spinner so each student has the same chance of spinning their favourite colour.

Sample answer

![Sample spinner diagram with eight parts of red, blue, green, and yellow]
# Pictograph Templates

**Title:**

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</table>
Colours of Cubes

![Bar graph showing the distribution of colours on cubes.]

- **Blue**: 1 cube
- **Green**: 2 cubes
- **Red**: 4 cubes
- **Yellow**: 5 cubes

**Legend**
- Blue
- Green
- Red
- Yellow

Each small square represents 1 cube.
Snacks Bar Graphs

**Snacks Eaten Today**

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<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
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<tbody>
<tr>
<td>Bagels</td>
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</tr>
<tr>
<td>Muffins</td>
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</tr>
<tr>
<td>Fruit</td>
<td>4</td>
</tr>
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<td>Vegetables</td>
<td>3</td>
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<td>Cheese</td>
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**Favourite Snacks**

<table>
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<tr>
<th>Snack Type</th>
<th>Number of Students</th>
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<tr>
<td>Bagels</td>
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Pictograph and Bar Graph Templates
Bar Graphs for Display

**Sam's Reading on Weekdays**

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**Number of Snow Days in Calgary, AB**

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<th>Nov-Dec</th>
<th>Dec-Jan</th>
<th>Jan-Feb</th>
<th>Feb-Mar</th>
<th>Mar-Apr</th>
<th>Apr-May</th>
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<tbody>
<tr>
<td>Number of Days</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

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Winter Graphs (I)

Favourite Winter Activities

- Building snow forts
- Building snowmen
- Skiing
- Sledding
- Snowshoeing

Number of People

0 10 20 30 40 50
Winter Graphs (2)
Favourite Winter Activities

Graph 1

Graph 2


des


des


Blackline Master — Probability and Data Management — Teacher Resource for Grade 3
## Tree Cone Graphs

### Graph 1

<table>
<thead>
<tr>
<th>Types of Cones</th>
<th>Red pine</th>
<th>E. wh. pine</th>
<th>Red spruce</th>
<th>Balsam fir</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>○</td>
<td>●</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td></td>
<td>○</td>
<td>●</td>
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<td></td>
<td>○</td>
<td>●</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

○ = 1 cone

### Graph 2

#### Types of Cones

<table>
<thead>
<tr>
<th>Tree</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red pine</td>
<td>5, 6, 7, 8, 9, 10, 11, 12, 13</td>
</tr>
<tr>
<td>E. wh. pine</td>
<td>2</td>
</tr>
<tr>
<td>Red spruce</td>
<td>4</td>
</tr>
<tr>
<td>Balsam fir</td>
<td>5</td>
</tr>
</tbody>
</table>

### Graph 3

#### Lengths of Cones

<table>
<thead>
<tr>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 6, 7, 8, 9, 10, 11, 12, 13</td>
</tr>
</tbody>
</table>

○ = 1 cone
Comparing Graphs

Karen's graph:

Leaves Collected

- Beech
- Elm
- Willow

Sal's graph:

Number of Leaves

- 0
- 1
- 2
- 3
- 4
- 5
- 6

Type of Leaf

- Beech
- Elm
- Willow

Yu's graph:

Lengths of Leaves

Length (cm)

- 10
- 11
- 12
- 13
- 14

Blackline Master — Probability and Data Management — Teacher Resource for Grade 3  U-85
My Survey

Question: ____________________________

I. Write the answers in the Answers column. Tally your results.

Title: ____________________________

<table>
<thead>
<tr>
<th>Answers</th>
<th>Tally</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Make a pictograph to show your data.

Title: ____________________________

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Shape Spinner

1. Create a spinner like the one below.
   Spin the spinner until it lands on the same shape 10 times.

![Diagram of a spinner with four sections: triangle, square, rectangle, and circle.]

2. Predict which shape the spinner will land on 10 times.

<table>
<thead>
<tr>
<th>Player 1’s Prediction</th>
<th>Player 2’s Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Test your predictions. Take turns spinning with your partner.
   The first player with 10 tallies in one column wins! Record the data in these charts.

<table>
<thead>
<tr>
<th>Player 1’s Chart</th>
<th>Player 2’s Chart</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>□</td>
</tr>
<tr>
<td>△</td>
<td>□</td>
</tr>
<tr>
<td>△</td>
<td>□</td>
</tr>
</tbody>
</table>
Empty Spinners
## Multiplication Chain (I)

<table>
<thead>
<tr>
<th>1 × 5</th>
<th>9</th>
<th>2 × 5</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 3</td>
<td>3</td>
<td>2 × 3</td>
<td>4</td>
</tr>
<tr>
<td>1 × 1</td>
<td>12</td>
<td>2 × 1</td>
<td>16</td>
</tr>
<tr>
<td>1 × 4</td>
<td>6</td>
<td>2 × 4</td>
<td>8</td>
</tr>
<tr>
<td>1 × 2</td>
<td>15</td>
<td>2 × 2</td>
<td>20</td>
</tr>
</tbody>
</table>
## Multiplication Chain (2)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$3 \times 5$</td>
<td>15</td>
<td>$4 \times 5$</td>
</tr>
<tr>
<td>$3 \times 3$</td>
<td>5</td>
<td>$4 \times 3$</td>
</tr>
<tr>
<td>$3 \times 1$</td>
<td>20</td>
<td>$4 \times 1$</td>
</tr>
<tr>
<td>$3 \times 4$</td>
<td>10</td>
<td>$4 \times 4$</td>
</tr>
<tr>
<td>$3 \times 2$</td>
<td>25</td>
<td>$4 \times 2$</td>
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</tbody>
</table>
## Multiplication Chain (3)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$5 \times 5$</td>
<td>$4$</td>
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<tr>
<td>$5 \times 3$</td>
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<td>$5 \times 1$</td>
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<td></td>
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<tr>
<td>$5 \times 4$</td>
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<td></td>
</tr>
<tr>
<td>$5 \times 2$</td>
<td>$8$</td>
<td></td>
</tr>
</tbody>
</table>
## Multiplication Chain (4)

<table>
<thead>
<tr>
<th>1 × 6</th>
<th>42</th>
<th>2 × 6</th>
<th>35</th>
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<tbody>
<tr>
<td>6 × 6</td>
<td>7</td>
<td>6 × 5</td>
<td>7</td>
</tr>
<tr>
<td>1 × 7</td>
<td>35</td>
<td>2 × 7</td>
<td>42</td>
</tr>
<tr>
<td>6 × 7</td>
<td>24</td>
<td>7 × 7</td>
<td>30</td>
</tr>
<tr>
<td>7 × 6</td>
<td>6</td>
<td>7 × 5</td>
<td>12</td>
</tr>
</tbody>
</table>
## Multiplication Chain (5)

| 3 \times 6 | 28 | 4 \times 6 | 21 |
| 6 \times 4 | 14 | 6 \times 3 | 21 |
| 3 \times 7 | 6  | 4 \times 7 | 12 |
| 6 \times 1 | 36 | 7 \times 1 | 49 |
| 7 \times 4 | 18 | 7 \times 3 | 24 |
## Multiplication Chain (6)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$5 \times 6$</td>
<td>$14$</td>
<td></td>
</tr>
<tr>
<td>$6 \times 2$</td>
<td>$28$</td>
<td></td>
</tr>
<tr>
<td>$5 \times 7$</td>
<td>$18$</td>
<td></td>
</tr>
<tr>
<td>$7 \times 2$</td>
<td>$30$</td>
<td></td>
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</tbody>
</table>
1 cm Grid Paper
# Hundreds Chart

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</table>
Number Sense: Division – AP Book 3.2: Unit 10

AP Book NS3-48

1. b) 5
   c) 3
   d) 3
   e) 5
   f) 3
   g) 5
   h) 3
2. a) 5
   b) 3
   c) 5
   d) 3
3. a) 2
   b) 4
   c) 4
   d) 4
   e) 5

AP Book NS3-50

1. a) apples
   b) stars
2. a) 3
   b) 5
   c) 3
   d) 3
3. a) cookies, 5, 4
   b) oranges, 6, 3
   c) spots, 4, 5
   d) stamps, 7, 5
   e) swings, 3, 4
   f) people, 2, 5
   g) chairs, 5, 4
4. Teacher to check pictures.
   a) 4
   b) 3

AP Book NS3-51

1. Teacher to check pictures.
   a) 4
   b) 3
   c) 3
   d) 3
   e) 5
   f) 4
   g) 5
   h) 4
   i) 4
   j) 2
   k) 6
   l) 1
   m) 1
   n) 6
   o) 5
   p) 5
   q) 8
   r) 1
   s) 10
   t) 3
   u) 5
   v) 12, 18, 24, 30
   w) 14, 21, 28, 35
   x) 15 + 3 = 5
   y) 20 + 5 = 4
   z) 12 + 2 = 6
   {) 20 + 5 = 4
   |) 5
   ) 4
   @) 2
   #) 3
   $) 3
   %) 5
   ^) 4
   _) 5
   `) 5

Answer Keys for AP Book 3.2
h) 1  
i) 1

6. a) 4 stickers  
b) 4 students

AP Book NS3-55  
page 15

1. Teacher to check circling.  
a) 6  
   3  
   5  
   6  
   8  
   4  

b) 9  
   3  
   9 + 3 = 12  
   9 + 3 = 12  
   10  
   5  

2. b) 3  
c) 2  
d) 1  

3. a) 10  
   2  
   5  
   10 + 2 = 5  
   10 + 5 = 2  
   10 + 2 = 5  
   10 + 5 = 2  
   10 + 2 = 5  
   10 + 5 = 2  
   10 + 2 = 5  
   10 + 5 = 2  

b) 8  
   2  
   4  
   8 + 2 = 4  
   8 + 4 = 2  

b) 18  
   3  
   6  
   18 + 3 = 6  
   18 + 6 = 3  
   12 + 3 = 4  
   12 + 4 = 3  
   12 + 3 = 4  
   12 + 4 = 3  
   12 + 3 = 4  

4. a) 8  
   2  
   4  
   8 + 2 = 4  
   8 + 4 = 2  

b) 18  
   3  
   6  
   18 + 3 = 6  
   18 + 6 = 3  
   12 + 3 = 4  
   12 + 4 = 3  
   12 + 3 = 4  
   12 + 4 = 3  
   12 + 3 = 4  

b) 18  
   3  
   6  
   18 + 3 = 6  
   18 + 6 = 3  
   12 + 3 = 4  
   12 + 4 = 3  
   12 + 3 = 4  
   12 + 4 = 3  
   12 + 3 = 4  

5. Teacher to check pictures.  
a) 9 + 3 = 3  
b) 12 + 4 = 3  
c) 30 + 5 = 6

AP Book NS3-56  
page 17

1. a) 4 × 4 = 16  
   4 × 4 = 16  
   16 + 4 = 20  
   16 + 4 = 20  
   20 + 4 = 24  
   20 + 4 = 24  
   24 + 6 = 30  
   24 + 6 = 30  
   30 + 6 = 36  
   30 + 6 = 36  
   36 + 6 = 42  
   36 + 6 = 42  

b) 4 × 5 = 20  
   5 × 4 = 20  
   20 + 4 = 24  
   20 + 4 = 24  
   24 + 6 = 30  
   24 + 6 = 30  
   30 + 6 = 36  
   30 + 6 = 36  
   36 + 6 = 42  
   36 + 6 = 42  

2. a) 6  
   2  
   3  
   9 × 2 = 18  
   9 × 2 = 18  
   18 + 2 = 20  
   18 + 2 = 20  
   20 + 4 = 24  
   20 + 4 = 24  
   24 + 6 = 30  
   24 + 6 = 30  

b) 20  
   4  
   5  
   20 + 4 = 24  
   20 + 4 = 24  
   24 + 6 = 30  
   24 + 6 = 30  
   30 + 6 = 36  
   30 + 6 = 36  
   36 + 6 = 42  
   36 + 6 = 42  

AP Book NS3-57  
page 19

1. c) 20 + 5 = 25  
   20 + 5 = 25  
   25 ÷ 5 = 5  
   25 ÷ 5 = 5  
   5 × 5 = 25  
   5 × 5 = 25  
   25 ÷ 5 = 5  
   25 ÷ 5 = 5  
   5 × 5 = 25  
   5 × 5 = 25  

b) 10 + 5 = 15  
   10 + 5 = 15  
   15 ÷ 5 = 3  
   15 ÷ 5 = 3  
   5 × 3 = 15  
   5 × 3 = 15  

2. a) 15 + 5 = 20  
   3  
   5  
   20 ÷ 5 = 4  
   20 ÷ 5 = 4  
   4 × 5 = 20  
   4 × 5 = 20  
   20 + 4 = 24  
   20 + 4 = 24  
   24 ÷ 4 = 6  
   24 ÷ 4 = 6  

b) 20 ÷ 4 = 5  
   5  
   15 ÷ 3 = 5  
   15 ÷ 3 = 5  
   5 × 3 = 15  
   5 × 3 = 15  

3. Answers will vary.  
   Teacher to check.

AP Book NS3-58  
page 21

1. c) 15, 5, ?, 15 + 5 = ?  
   ? = 10  
   10 ÷ 2 = 5  
   5 × 2 = 10  
   5 ÷ 5 = 1  
   1 × 5 = 5  
   5 ÷ 5 = 1  
   1 × 5 = 5  
   5 ÷ 5 = 1  
   1 × 5 = 5  
   5 ÷ 5 = 1  

b) 3, 2, 4, 2 × 4 = ?  
   8  
   4 × 2 = 8  
   8 ÷ 4 = 2  
   2 × 4 = 8  
   8 ÷ 4 = 2  

b) 18, ?, 9, 18 + 9 = ?  
   27  
   9 × 3 = 27  
   27 ÷ 3 = 9  
   9 × 3 = 27  
   27 ÷ 3 = 9  

i) 15, ?, 15 + 3 = ?  
   18  
   3 × 6 = 18  
   18 ÷ 3 = 6  
   6 × 3 = 18  
   18 ÷ 3 = 6  

3. Answers will vary.  
   Teacher to check.
7. a) 9  
b) 24  
**BONUS**  
   2

AP Book NS3-59  
**page 23**
1. 3 × 2 = 6  
   6 ÷ 2 = 3  
   6 ÷ 3 = 2
2. a) 12  
   b) 18
3. 42 times
4. a) 28  
   b) Answers will vary. Teacher will check.
5. yes
6. 12  
   3  
   4  
   12 ÷ 3 = 4  
   12 ÷ 4 = 3  
   3 × 4 = 12
7. a) 8  
   b) 10  
   c) 5
8. 6 sets of 4 equals 2 sets of 4 plus 4 sets of 4
9. a) 2 × 4 + 3 = 11  
   b) 4 + 2 + 3 = 5
10. Answers will vary. Teacher to check.
11. Answers will vary. Teacher to check.

**BONUS**  
a) 9  
b) 15

AP Book NS3-60  
**page 25**
1. Teacher to check numbering.  
   b) 5  
   2  
   5 × 2 = 10  
   2 × 5 = 10
2. b) 3  
   c) 4  
   d) 7  
   e) 8  
   f) 10  
   g) 12  
   h) 15
3. 6 ÷ 2 = 3  
   6 ÷ 3 = 2  
   3 × 2 = 6
4. 4 × 2 = 8  
   8 ÷ 2 = 4  
   4 × 2 = 8
5. 16 ÷ 2 = 8  
   8 ÷ 2 = 4
6. 4 × 2 = 8  
   8 ÷ 2 = 4
7. 7 rows
8. 4 flowers
9. 63 beads

**BONUS**  
a) 8  
b) 7

AP Book NS3-61  
**page 28**
1. b) 2  
   5  
   10  
   5 × 2 = 10  
   5 ÷ 2 = 10
2. b) 9 × 5 = 45  
   5 × 9 = 45  
   45 ÷ 5 = 9
3. b) 6, 4, ?, ? = 6 × 4  
   c) ?, 5, 35, ? = 35 ÷ 5
4. 16 hamsters
5. 3 packs
6. 4 in each box
7. a) 40  
   b) 36
8. Ansel
9. 2
10. 3
11. 75 players

**BONUS**  
a) 4  
b) 96

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Patterns and Algebra: Patterns and Equations – AP Book 3.2: Unit 11

1. a) Pattern A: gap: +2
   6, 8, 10, 12
Pattern B: gap: +3
   3, 6, 9, 12, 15
b) A: 4, 6, 8, 10, 12
   Start at 4 and add 2 each time.
B: 3, 6, 9, 12, 15
   Start at 3 and add 3 each time.
c) A: yes; B: no

2. a) Pattern A: gap: −4
   24, 20, 16
Pattern B: gap: −1
   10, 9, 8
b) A: 24, 20, 16
   Start at 24 and subtract 4 each time.
B: 10, 9, 8
   Start at 10 and subtract 1 each time.
c) A: 12, 8, 4
   B: 7, 6, 5

3. a) gap: −2
   Start at 10 and subtract 2 each time.
b) gap: +3
   Start at 16 and subtract 3 each time.
c) Start at 1 and add 4 each time.

4. a) Start at 3 and add 2 each time.
b) Start at 16 and subtract 3 each time.
c) Start at 1 and add 4 each time.

5. Explanations may vary. Teacher to check.
a) 24
b) 13
c) 26

6. a)  

<table>
<thead>
<tr>
<th>Step Number</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

b) 6 squares or 6 toothpicks

BONUS

14

7. a)  

<table>
<thead>
<tr>
<th>Step Number</th>
<th>Number of Toothpicks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>

b) 5 squares or 5 toothpicks

BONUS

8. Answers will vary. Teacher to check.

AP Book PA3-14

1. a) 11, 1
b) 30, 32, 34, 36, 38
   49, 47, 45
d) 80, 77, 74, 71

2. Teacher to check dots on number lines.

b) Start at 61. Add 2 each time.
c) Start at 90. Subtract 10 each time.
d) Start at 105. Add 10 each time.
e) Start at 525. Add 15 each time.
f) Start at 775. Subtract 25 each time.

BONUS

Start at 225. Add 25 each time.

3. a) Teacher to check.
b) column 4 and column 9
c) 4, 9, 4, 9, 4, 9, 4, 9, 4, 9, 4, 9, 4, 9
   Yes, no
   4, 9, then repeat.
e) diagonal

4. a) 2, 7, 12, 17, 22, 27
   2, 7, 2, 7, 2, 7
   2, 7, then repeat.
d) 0, 0, 1, 1, 2, 2
e) 32, 37, 42, 47
f) column 2 and column 7

BONUS

Start at 225. Add 25 each time.

3. The following equations should be circled.
   a) 31, 33, 35 37
   b) 90, 87, 84, 81
c) 105, 110, 115, 120
d) 325, 320, 315, 310
e) 100, 96, 92, 88

f) 99, 94, 89, 84
g) 73, 68, 63, 58

BONUS

Yes

5. Answers will vary. Teacher to check.

6. a) A: 3, 5, 7
   B: 1, 4, 7
b) Teacher to check.

7. Teacher to check number line.

19

AP Book PA3-15

1. a) 11, 1
b) Start at 5 and add 10 each time
c) Answers will vary. Teacher to check.
d) Teacher to check.

2. a) Start at 10 and add 9 each time.
b) Start at 3 and add 11 each time.
c) 9, 18, 27, 36, 45, 54, 63, 72, 81
d) Teacher to check.

e) diagonal

3. a) Teacher to check.
b) column 4 and column 9
c) 4, 9, 4, 9, 4, 9, 4, 9, 4, 9, 4, 9, 4, 9
   4, 9, 4, 9
   4, 9, then repeat.
e) yes, no

4. a) 2, 7, 12, 17, 22, 27
   2, 7, 2, 7, 2, 7
   2, 7, then repeat.
d) 0, 0, 1, 1, 2, 2
e) 32, 37, 42, 47
f) column 2 and column 7

BONUS

Start at 225. Add 25 each time.

3. The following equations should be circled.
   a) 31, 33, 35 37
   b) 90, 87, 84, 81
c) 105, 110, 115, 120
d) 325, 320, 315, 310
e) 100, 96, 92, 88

f) 99, 94, 89, 84
g) 73, 68, 63, 58

BONUS

Yes

5. Answers will vary. Teacher to check.

6. a) 7
   14
   21
   28
b) Teacher to check.
c) They are all in the Wednesday column.
d) Answers will vary. Teacher to check.
e) Answers will vary. Teacher to check.

7. a) 6
   12
   18
   24
   30
b) Teacher to check.
c) On a diagonal starting at 6 and ending at 30.

AP Book PA3-16

1. b) 3 = 3
c) 5 = 5
d) 3 ≠ 5

2. a) =
b) 2 + 3 = 5
c) 6 ≠ 2 + 3
d) 7 ≠ 3 + 5
e) 3 + 4 = 7
f) 2 + 5 ≠ 5
g) 4 + 4 ≠ 9
h) 6 = 4 + 2

3. The following equations should be circled.
   a) 8 = 6 + 2
d) 5 ≠ 3 + 1
e) 11 + 5 = 16
f) 12 + 3 = 15

4. C and D should be circled.

5. c) F
Patterns and Algebra:
Patterns and Equations – AP Book 3.2: Unit 11

(continued)

Answer Keys for AP Book 3.2
c) $15 - 7 = 8$
   $8$
d) $28 + 10 = 38$
   $38$
e) $24 - 6 = 18$
   $18$
f) $35 + 7 = 42$
   $42$

4. 1 number, because $12 - 5 = 7$.

5. b) $28 + n = 70$
   $70 - 28 = 42$
   $n = 42$
c) $x - 10 = 39$
   $39 + 10 = 49$
   $x = 49$
d) $25 = b - 15$
   $25 + 15 = 40$
   $b = 40$
e) $p + 12 = 28$
   $28 - 12 = 16$
   $p = 16$

BONUS
   $40 - a = 20$
   $40 - 20 = 20$
   $a = 20$

6. a) $44 - \bigstar = 20$
   b) $25 - 6 = \bigstar$
   c) $35 = 7 + \bigstar$

7. a) $? = 8 + 10$
   $18$
   b) $\bigstar = 13 - 8$
   $5$
   c) $\bigstar = 11 + 7$
   $18$
   d) $? = 29 - 19$
   $10$
   e) $\bigstar = 50 - 25$
   $25$

BONUS
   $\bigstar = 75 - 75$
   $0$

BONUS
Yes. Any number plus zero is equal to that number.
1. b) 4
   one fourth

c) 2
   one half
d) 4
   one fourth
e) 2
   one half
f) 4
   one fourth

2. a) 2
   one half
b) 4
   one fourth
c) 3
   one third
d) 6
   one sixth
e) 6
   one sixth
f) 4
   one fourth

BONUS

8
one eighth

3. No. Each part needs to be of equal size.

AP Book NS3-63

page 52

1. b) 4
   one fourth, \( \frac{1}{4} \)
c) 6
   one sixth, \( \frac{1}{6} \)
d) 5
   one fifth, \( \frac{1}{5} \)
e) 8
   one eighth, \( \frac{1}{8} \)
f) 12
   one twelfth, \( \frac{1}{12} \)

2. b) \( \frac{1}{6} \)

AP Book NS3-64

page 54

1. b) 5
   \( \frac{8}{5} \)
c) 5
   \( \frac{5}{6} \)
d) 3
   \( \frac{4}{3} \)

2. b) \( \frac{1}{2} \)
c) \( \frac{5}{6} \)

3. Circle the second and third pictures.

b) The number on top is greater than 1.

5. a) Circle the first, second, and fourth pictures.

b) In the first picture the parts are not equal size. The second picture has five equal parts instead of four. The fourth picture has three equal parts instead of four.

AP Book NS3-65

page 57

1. a) hexagon

b) triangle
5. a) no for all parts
b) shaded

6. a)
   - squares
   - triangles or not shaded
b)
   - circles
   - circles

c) 5/8

d) 1/4

7. No
The parts are not of equal size.

BONUS
Sample answers:
- Triangles
- Squares
- Circles

AP Book NS3-67
page 61
1. a) 2/5
   - Teacher to check.
   - Teacher to check.
   - Teacher to check.
   - Teacher to check.
2. a) squares
   - Teacher to check shading.
   - Teacher to check shading.
   - Teacher to check shading.
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c) \[
\frac{1}{5}
\]
\[
\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}
\]

d) \[
\frac{1}{2}
\]
\[
\frac{1}{2}, \frac{1}{2}, \frac{2}{2}, \frac{2}{2}
\]

3. a) \[
\frac{1}{4}
\]
\[
\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}
\]

b) \[
\frac{1}{4}
\]
\[
\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}
\]

c) \[
\frac{1}{2}
\]
\[
\frac{1}{2}, \frac{1}{2}, \frac{2}{2}, \frac{2}{2}
\]

d) \[
\frac{1}{2}
\]
\[
\frac{1}{2}, \frac{1}{2}, \frac{2}{2}, \frac{2}{2}
\]

4. Teacher to check circling.

a) 3
b) 4
c) 2
d) 3

5. a) 2
b) 3
c) 2
d) 5

6. a) 6
b) 7

BONUS

\[
15 \frac{1}{2}
\]
1. b) 12:20  
   c) 1:03
2. b) 20 minutes past 10  
   c) 35 minutes past 8  
   d) 40 minutes past 2  
   f) 9 minutes past 6
3. b) 04:15  
   c) 03:08  
   e) 12:12  
   f) 08:15  
   g) 02:23  
   h) 06:30  
   i) 02:01
4. b) hour  
   c) hour  
   d) minute  
   e) hour  
   f) minute  
   g) minute  
   h) hour  
   i) minute  
   j) minute
5. Teacher to check lines.  
   b) 3  
   c) 5
6. Teacher to check circling.  
   a) 2  
   b) 6  
   c) 10  
   d) 1  
   e) 8  
   f) 9
BONUS  
   a) A  
   b) B
1. b) 15  
   c) 50  
   d) 35  
   e) 30  
   f) 45
2. When the minute hand points to 5, you need to skip count by 5s five times to get 25 minutes. The time is 9:25.
3. Teacher to check circling.  
   a) 15  
   b) 10  
   c) 00  
   d) 40  
   e) 45  
   f) 20  
   g) 35  
   i) 55
4. a) 25  
   b) 5  
   c) 50
5. Teacher to check arrows.  
   b) 4 \times 5 = 20  
   c) 8 \times 5 = 40  
   d) 5 \times 5 = 25  
   e) 9 \times 5 = 45  
   f) 10 \times 5 = 50
6. b) 2 \times 5 = 10  
   c) 1 \times 5 = 5
BONUS  
   60 minutes passed.  
   12 \times 5 = 60
### Measurement: Time – AP Book 3.2: Unit 13

#### AP Book ME3-19

**Page 82**

1. b) 4:00  
   4 o'clock  

c) 9:15  
   09:15  
   quarter past nine  
   quarter past 9  
   fifteen minutes past nine  

6. a) twelve minutes past six  
   forty-eight minutes to seven  

b) fifteen minutes past seven  
   quarter past seven  
   forty-five minutes to eight  

c) three o'clock  

d) forty-five minutes past twelve  
   fifteen minutes to one  
   quarter to one  

e) thirty minutes past ten  
   half past ten  
   thirty minutes to eleven  

f) thirty-five minutes past two  
   twenty-five minutes to three  

**AP Book ME3-20**

**Page 85**

1. a) a.m.  
   b) a.m.  
   c) p.m.  
   d) p.m.  
   e) p.m.  
   f) a.m.  

2. b) 9:55 p.m.  
   c) 12:20 p.m.  
   d) 8:40 a.m.  
   e) 8:30 a.m.  
   f) 5:15 p.m.  

**BONUS**

   g) 9:00 a.m.  
   h) 12:12 a.m.  

3. a) 30, 8:45 a.m.  
   b) 3:15 p.m., 3:30 p.m., 3:45 p.m.  

4. a) 3:15 p.m.  
   3:45 p.m.  
   4:30 p.m.  

**AP Book ME3-21**

**Page 87**

1. a) 7  
   b) 14  
   c) 21  
   d) 24  
   e) 48  
   f) 72  
   g) 60  
   h) 120  
   i) 180  

2. a) 14  
   b) 48  

3. between 3 and 4 days  

**AP Book ME3-22**

**Page 89**

1. a) 120, 180, 240, 300, 360  
   b) 180  

2. b) 180 + 40  
   220  

**BONUS**

   c) 240 + 23  
   263  
   d) 60 + 57  
   117  
   e) 360 + 10  
   370  
   f) 300 + 5  
   305  

3. Ethan  

4. a) minutes  
   b) years  
   c) hours  
   d) seconds  

---

**Answer Keys for AP Book 3.2**
Measurement: Time – AP Book 3.2: Unit 13

(continued)

e) days
f) months
g) years
h) seconds
i) minutes

5. a) more
   b) less
c) more
d) less
e) more
f) more

6. a) less
   b) more
c) less
d) less
e) less
f) more

7. 31, 28, 31, 30;
   31, 30, 31, 31;
   30, 31, 30, 31

8. a) August 2
   b) May 4
   c) March 30

BONUS

   a) more than 1 hour
   b) 4
Measurement: Capacity, Mass, and Temperature – AP Book 3.2: Unit 14

AP Book ME3-23
page 91
1. Circle the following:
   a) bottle on the right
   b) bottle on the left
   c) bottle on the right

2. Circle the following:
   a) glass on the right
   b) glass on the left
   c) glass on the left

3. Circle the following:
   a) glass
   b) wider glass
   BONUS
   wide bottle

4. Circle the following:
   a) bottle
   b) vase

5. Circle the following:
   a) thermos
   b) can
   BONUS
   tall kettle

6. Circle the following:
   a) milk carton
   b) mug
   c) bathtub

7. a) 1
   b) 2
   c) 4

8. a) 4
   b) 3
   c) 5

9. Teacher to check.

AP Book ME3-24
page 94
1. glass to small carton
   soda bottle to medium carton
   soda can to mug

2. | no | yes |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

3. a) □
   □
   □
   □
   □
   □

4. Circle glass, milk carton, soda bottle

5. a) 1st
   3rd
   2nd
   b) 3rd
   1st
   2nd

6. a) B
   b) A
   c) C

7. 5th, 4th, 2nd, 6th, 3rd, 1st

8. a) □
   □
   □
   □
   □
   □

9. Circle glass, mug, milk carton, soda can

10. a) 1 \frac{2}{4}
    b) 1 L, 1 L
    c) \frac{2}{4} L, \frac{3}{4} L

11. a) 1 L
    b) \frac{2}{4} L
    c) \frac{1}{2} L
    d) 1 L
    e) \frac{1}{4} L
    f) \frac{3}{4} L

f) 2 L
   \frac{3}{2} L

12. Answers will vary. Teacher to check.

AP Book ME3-25
page 97
1. Circle the following:
   a) elephant
   b) car
   c) book

2. Circle the following:
   a) scissors
   b) calculator
   c) chair

3. Circle the following:
   a) watermelon
   b) grapes
   c) phone
   d) football
   e) book
   f) orange

4. Circle the following:
   a) push pin
   b) nail
   c) baseball

5. Circle the following:
   a) eraser
   b) scissors
   c) sunglasses
   d) granola bar

6. Circle the following:
   a) cylinder
   BONUS
   pyramid

7. Circle:
   banana, basketball, chair
   X: acorn, shuttlecock, marker

8. Circle the following:
   a) 3 kg
   b) 1 kg
   c) 10 kg

9. Circle the following:
   a) kg
   b) g
   c) g
   d) kg

10. a) 15 kg
    b) 6 kg

11. Circle the following:
    a) 100 g
    b) 800 kg
    c) 10 g
    d) 3 kg
    e) 1 g
    f) 1 kg
    g) 1 kg
    h) 1 g

12. Black bear, Pacific salmon, Puffin, Chipmunk

13. a) No. Each tower is made from the same number of cubes, so each tower has the same mass.
    b) Lewis’s tower weighs 18 g. The two towers are made from the same number of cubes, so they weigh the same.

AP Book ME3-26
page 99
1. Circle dime, ticket, push pin
    b) book to laptop
    straw to feather
    loonie to cherry

2. a) 250
   b) 400
   c) 100

3. Circle: banana, basketball, chair
   X: acorn, shuttlecock, marker

4. a) 250
   b) 400
   c) 100

5. Answers will vary. Teacher to check.

6. Circle book, baseball bat, juice carton

7. Answers will vary. Teacher to check.

8. Circle the following:
   a) 3 kg
   b) 1 kg
   c) 10 kg

9. Circle the following:
   a) kg
   b) g
   c) g
   d) kg

10. a) 15 kg
    b) 6 kg

11. Circle the following:
    a) 100 g
    b) 800 kg
    c) 10 g
    d) 3 kg
    e) 1 g
    f) 1 kg
    g) 1 kg
    h) 1 g

12. Black bear, Pacific salmon, Puffin, Chipmunk

13. a) No. Each tower is made from the same number of cubes, so each tower has the same mass.
    b) Lewis’s tower weighs 18 g. The two towers are made from the same number of cubes, so they weigh the same.

AP Book ME3-27
page 102
1. 18 g
2. a) 24 g  
b) 60 g  
**BONUS**

600 g

3. 7 kg

4. a) 187 kg  
b) 13 kg

5. 28 + 23 = 27 + Y  
51 = 27 + Y  
51 – 27 = Y  
24 = Y

Yu weighs 24 kg

6. a) 4 kg  
b) $80

7. a) 50 kg  
b) 100 kg  
c) 200 kg

8. a) 16 g  
b) 4 g  
c) 4  
**BONUS**

400

9. a) 400  
b) 600

**BONUS**

4 g

10. a) 60 g  
b) 70 g  
c) Yes. 5 x 5 g = 25 g  
d) A gerbil weighs 41 g more than a vole. A vole eats 28 g more food in two days.  
e) A mouse weighs 40 g. It will take 8 days for a mouse to eat 40 g of food.

11. No. Liters measure volume or capacity, not mass.

AP Book ME3-28

**page 104**

1. a) no  
b) yes  
c) yes

2. Circle: soccer ball, basketball  
X: bicycle, elephant

AP Book ME3-29

**page 106**

1. Circle the following:  
a) glass on the left  
b) glass on the right  
c) glass on the right

2. Circle the following:  
a) thermometer on the right  
b) thermometer on the left

**BONUS**

thermometer on the right

3. a) 20°C  
b) 40°C  
c) 10°C

4. b) 5°C  
c) 15°C  
d) 36°C  
e) 24°C  
f) 49°C

5. Answers will vary. Teacher to check.
1. b) 70
c) 80
BONUS
110
2. b) 80
c) 20
BONUS
100
3. b) 60, 70
c) 10, 20
d) 70, 80
e) 0, 10
f) 30, 40
4. b) arrow pointing right
c) arrow pointing right
d) arrow pointing left
e) arrow pointing left
f) arrow pointing right
5. a) 2, 3, 4
b) 6, 7, 8
c) They are halfway between the previous and next multiples of 10.
6. Circle the following:
   b) 30
c) 60
d) 20
e) 80
f) 30
7. b) 40, 50
c) 80, 90
d) 60, 70
e) 10, 20
f) 0, 10
8. b) 50
c) 40
d) 20
9. 90
10. 40
11. 80

1. a) 10
b) 30
c) 70
d) 40
e) 20
f) 50
d) 70, 90
c) 50
d) 30
f) 20
BONUS
100
2. b) 70 - 30 = 40
c) 40 + 30 = 70
d) 60 - 40 = 20
e) 40 + 30 = 70
f) 70 - 30 = 40
BONUS
20 + 40 + 20 = 80
60 - 20 + 50 = 90
50 - 20 - 10 = 20
3. a) 30 + 20 = 50
b) 90 + 20 = 110
5. a) 60 - 40 = 20
b) 80 - 70 = 10
6. 50 + 40 = 90
7. 80 - 40 = 40
8. 20 + 20 + 10 + 30 + 10 = 90
9. 20 + 20 + 20 + 20 + 20 + 20 = 100
BONUS
5 × 20 = 100
1. b) hundreds
c) ones
d) thousands
e) thousands
f) hundreds
BONUS
200
200
3. b) 0
0
4. b) 1
3
0
2
0
0
8
0
0
0
5. b) 300
c) 30
d) 300
e) 30
f) 3000
g) 300
h) 3
i) 30
6. a) 500
b) 30
c) 8000
d) 8
e) 700
f) 9
7. It looks like there are less than 100 triangles.
8. Edmond should use 100 as a referent because there are far more than 100 people in the race.
   There are about 900 people in the race because 9 × 100 = 900.
9. Dory should use 10 as a referent because there are less than 100 people.
   There are about 70 people in the stands because 7 × 10 = 70.

1. Teacher to check outlining.
2. Teacher to check underlining.
3. Teacher to check circling.
4. Teacher to check outlining.
5. Teacher to check circling.

Answer Keys for AP Book 3.2
c)  

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BONUS  

a) 5996  
b) 8095  
c) 3611  
d) 1414  
e) 8591  
f) 2698
1. a) 16, 21, 26, 31, 36
   b) Teacher to check circling.

2. Teacher to check underlining.
   a) 2, 7, 2, 7
   b) 4, 9, 4, 9
   c) 3, 8, 3, 8

3. a) 38, 43, 48
   b) 55, 60, 65, 70
   c) 26, 31, 36, 41

4. a) 33, 28, 23
   b) 65, 60, 55, 50
   c) 26, 21, 16, 11
   d) 44, 39, 34, 29

5. 15 20 25 30 35 40 45 50
   65 70 75 80 85 90 95 100

6. Teacher to check circling.

7. Teacher to check circling.

8. a) 100, 125, 150, 175, 200, 225, 250
   b) 25, 50, 75, 100

9. a) dime
   b) quarter
   c) quarter
   d) loonie

10. a) 10¢, 5¢
    b) 10¢
    c) 10¢, 5¢
    d) 10¢, 5¢

11. a) nickel
    b) dime
    c) quarter
    d) dime
    e) loonie
    f) quarter

12. Teacher to check for a combination of coins that adds to 30¢.
1. e) 25¢  
f) 100¢

2. a) dime  
b) 2, 1  
c) 5  
d) 4, loonie  
e) 5  

BONUS  
10

3. a) 100¢  
b) loonie

4. a) 100¢  
b) loonie

5.  b) 10¢  
   c) 25¢  
   d) 25¢, 25¢, 25¢  
   e) 100¢

BONUS  
10

6. b) 25¢, 25¢, 25¢  
c) 25¢  
d) 25¢, 25¢, 25¢

7. b) 10¢  
c) 10¢, 10¢, 10¢, 10¢

d) 10¢, 10¢, 10¢

8. b) 25¢  
   45¢ - 25¢ = 20¢  
   10¢, 10¢

c) 50¢  
   65¢ - 50¢ = 15¢  
   10¢, 5¢

d) 75¢  
   95¢ - 75¢ = 20¢  
   10¢, 10¢

e) 5  
   96¢ - 75¢ = 21¢  
   10¢, 10¢, 1¢

5.  a) 5, 30  
   b) 5, 80, 20  
   c) 5, 60, 40  
   d) 7, 50, 50  
   e) 57¢

6.  a) 5, 30, 70  
   b) 5, 9, 10  
   c) 5, 40, 60  
   d) 2, 70, 30  
   e) 3, 60, 40  
   f) 8, 70, 30  
   g) 3, 50, 50  
   h) 1, 80, 20  
   i) 15¢

BONUS  
15¢

7. a) 25¢, 25¢, 10¢, 5¢  
   b) 25¢, 25¢, 10¢, 10¢  
   c) 25¢, 25¢, 10¢, 25¢  
   d) 25¢, 25¢, 10¢, 5¢

BONUS  
10¢

8. b) 100¢, 25¢  
   c) 100¢, 25¢, 25¢, 10¢

9. a) 5¢  
   b) 5¢  
   c) 8¢  
   d) 6¢  
   e) 10¢, 10¢  
   f) 4¢

2. a) 30¢  
   80¢

c) 60¢  
   40¢  
   70¢

d) 50¢  
   90¢

3. a) 50¢  
   b) 20¢  
   c) 80¢  
   d) 40¢

4. a) 30¢  
   b) 70¢  
   c) 90¢

AP Book NS3-81

page 133

5. a) 35¢  
   b) 25¢

6. a) 15¢  
   b) 35¢  
   c) 75¢  
   d) 55¢  
   e) 65¢  
   f) 5¢
   g) 45¢
   h) 30¢
   i) 25¢
   j) 87¢

BONUS  
a) 40¢

7. b) $2
   c) $1
   d) $2

8. a) 3  
   b) 2, 95  
   c) 3  
   d) 4, 50
c) 6, 5, 95
9. $2.25

AP Book NS3-83
page 139
1. a) 30, 40, 50, 60, 70, 80, 90, 100
b) 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

c) 50, 75, 100
2. a) 10
b) 20
c) 4

BONUS
100
3. b) 20, 20, 40
c) 4, 4, 8
d) 100, 100, 200

BONUS
200, 100, 300
4. b) 2, 2, 1
c) 2, 1, 1, 1
5. b) 2, 1
c) 1, 3
6. a) 8
b) 9
c) 8

d) 9
7. a) 8, 40
b) 6, 60
c) 4, 45
8. b) 6, 80
c) 5, 50
d) 7, 95
9. 1 toonie, 1 quarter
10. No.
$10.00 − $7.25 = $2.75
The cashier owes Jake another 25¢.

AP Book NS3-84
page 144
1. 20, 30, 40, 50, 60, 70, 80, 90, 100
2. a) 2
b) 5
c) 10
3. a) 10, 10, 10, 10, 10
b) 20, 20, 10
c) 20, 10, 10, 10
4. a) 5
b) 2, 1
c) 1, 3

AP Book NS3-85
page 144
1. 20, 30, 40, 50, 60, 70, 80, 90, 100
2. a) 2
b) 5
c) 10
3. a) 10, 10, 10, 10, 10
b) 20, 20, 10
c) 20, 10, 10, 10
4. a) 5
b) 2, 1
c) 1, 3

AP Book NS3-86
page 147
1. a) 10 + 10 + 10 + 10
   40
b) 25 + 25 + 25
   75
c) 5 + 5 + 5 + 5 + 5 + 5
   30
d) 20 + 20 + 20
   60
e) 50 + 50 + 50
   150
f) 100 + 100
   200
2. b) 25, 50, 75
   100
c) 5, 10, 15, 20, 25
   30
d) 20, 40, 60, 80
   100
e) 50
   100
f) 100, 200
   300
g) 5, 10, 15

5. Sample answers:
   $50, $20, $20, $10
   $50, $20, $10, $10
   $50, $10, $10, $10
   $10
6. a) 90
   b) 100
c) 70
7. a) $330
   b) $20

8. $20, 330
   $330
   330 − 35 = 295
   $295
   295 + 27 = 322
   $322

9. a) $330 − $40 = $290
   b) $290 + $30 = $320
c) No. He needs $30 more.

10. b) 30, 75, $30.75
c) $50, $10, $10, $10, $10, $10

BONUS
112, 15, $112.15

11. b) $20, $10, $5, $2
   25¢, 5¢
c) $50, $10, $1, $1, 25¢
    25¢, 25¢, 10¢, 10¢

BONUS
$185.80

AP Book NS3-87
page 150
1. a) $630
2. $890
3. $340  
   $410  
   $260
4. a) $125 + $50 + $75  
    + $250 = $500  
   b) $500 + $50 = $550
5. a) $20  
   b) $270
6. a) $60  
   b) $40
7. a) $200  
   b) $550
8. a) $20  
    b) $75, $60  
    c) $95
9. 3 × $5 = $15  
   $515

**AP Book NS3-88**  
*page 152*
1. a) 15, 20, 25, 30, 35, 40, 45  
   b) 65, 70, 75, 80, 85, 90, 95
2. c) 70  
    d) 15  
    e) 90  
    f) 25
3. c) 75  
    d) 20  
    e) 95  
    f) 30
4. b) 30  
   c) 40, 45  
   d) 75, 80  
   e) 55, 60  
   f) 15, 20
5. Circle the following:  
   c) 70  
   d) 90  
   e) 10  
   f) 100  
   g) 35  
   h) 60

**AP Book NS3-89**  
*page 154*
1.  
   $10 − $4  
   $6  
   $10 − $7  
   $3  
   $10 − $3  
   $7  
   $10 − $6  
   $4
2.  
   100¢ − 40¢  
   60¢  
   100¢ − 60¢  
   40¢  
   100¢ − 90¢  
   10¢  
   100¢ − 50¢  
   50¢
3. b) 20¢  
   c) 40¢, $5.00  
   d) 10¢, $2.00  
   e) 90¢, $8.00  
   f) 80¢, $1.00  
   g) 60¢, $7.00  
   h) 20¢, $8.00
4. a) 40¢, $6, $6.40  
    b) 20¢, $3, $7, $7.20  
    c) 90¢, $8, $2, $10, $2.90  
    d) 20¢, $1, $9, $10, $9.20
5. Circle the following:  
   c) 70  
   d) 90  
   e) 10  
   f) 100  
   g) 35  
   h) 60

**BONUS**

- e) 10¢, $9, $11, $20, $11.10
- f) 40¢, $4, $16, $20, $16.40
5. b) 5¢, 30¢, $8, $2, $2.35
   c) 5¢, $2.20, 80¢, $3, $7, $10, $7.85
6. $2, 21¢, 20¢, $2.20  
   $9, 78¢, 80¢, $9.80  
   $3, 7¢, 5¢, $3.05
7. a) 20¢, $5, $5.25  
    b) $6.15, 5¢, $6.20  
    c) $7.10, 90¢, $8, $2, $3.85
    c) $7.10, 90¢, $8, $2, $10, $2.90

30                35  
65                70  
20                25  
10                15  
70                75  
90                95
1. a) 2
   b) 4
   c) 2

2. a) 2
   b) 2
   c) 3

3. a) 
   b) 
   c) 

4. a) 
   b) 2
   c) 6
   d) 3
   e) 4
   f) 2

5. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

6. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

7. a) 
   b) 
   c) 
   d) 
   e) 

8. a) 
   b) 
   c) 

9. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

10. a) 
    b) 
    c) 
    d) 
    e) 
    f) 

11. a) 
     b) 
     c) 

12. a) 
    b) 
    c) 

BONUS

1. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

2. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

3. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

4. a) 
   b) 
   c) 

5. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

6. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

7. a) 
   b) 
   c) 
   d) 

8. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

9. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

10. a) 
    b) 
    c) 
    d) 
    e) 
    f) 

11. a) 
     b) 
     c) 

12. a) 
    b) 
    c) 

BONUS

1. a) library
   b) library
   c) Iva’s home
   d) 1 block west, 3 blocks south
   e) 2 blocks east, 2 blocks south
   f) 3 blocks west, 1 block south

2. a) 5, right
   b) 2, up
   c) 4, right
   d) 1, down

3. a) 
   b) 
   c) 
   d) 
   e) 
   f) 

BONUS

1st row: Sally, Tasha
2nd row: Zack, Lela
3rd row: Fred, Jin

7. a) city
   b) hill
   c) 1 km east, 3 km south
   d) 1 km east, 2 km north

BONUS

1 km east, 4 km north

Answer Keys for AP Book 3.2
2. a) yes  
b) yes  
c) no  
3. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]
   d) 
   ![Diagram]
   e) 
   ![Diagram]
   f) 
   ![Diagram]
4. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]
   d) 
   ![Diagram]
   e) 
   ![Diagram]
   f) 
   ![Diagram]
5. a) Answers will vary. Teacher to check.  
b) Yes. Sample explanation: They are the same size and shape.  
6. a) 
   ![Diagram]
   b) 
   ![Diagram]
7. Answers will vary. Teacher to check.  
8. b) reflect  
c) rotate  
d) rotate  
e) rotate  
f) reflect  
g) rotate  
h) reflect  
i) rotate  
9. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]
   d) 
   ![Diagram]
   e) 
   ![Diagram]
   f) 
   ![Diagram]
10. reflection, rotation, translation, reflection, rotation, translation  

AP Book G3-19
page 169
1. Circle the following:  
a) rectangular prism, square pyramid, cone, pentagonal prism, cylinder.  
b) rectangle  
c) triangle  
d) hexagon  
2. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]
3. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]
9. a) A. 6, 9  
    B. 8, 12  
    C. 7, 12  
    D. 9, 16  
    E. 5, 8  
    F. 12, 18  

   b) 7 or Fewer Vertices  
      10 or More Edges  
      A, C, E  
      B, C, D, F

5. a) hexagonal pyramid  
      b) octagon  
6. b) triangular pyramid  
      c) square pyramid  
7. a) rectangular prism  
      b) pentagonal prism  
      c) octagonal prism  
8. Dory is correct. Prisms have two identical bases. Each base has the same number of vertices.

AP Book G3-20  
page 172
1. a) 3 4 5 6  
       4 5 6 7  
       6 8 10 12  

   b) Start at 4 and add 1 each time.  
   c) Start at 6 and add 2 each time.  

2. a) 8 9 10 11  
       14 16 18 20  

   b) Add 1  
   c) 21  

   BONUS  
   40
3. a) 6 8 10 12  
       9 12 15 18  

   b) Start at 6 and add 2 each time.  
   c) Start at 9 and add 2 each time.  

4. a)  

   B, E  
   C, D, G  
   A, F, H  

   b) Circle B, D, G, H triangles  
   c) Circle A, C, E, F rectangles  
   d) hexagonal prism, hexagonal pyramid  

   BONUS  
   Multiply by 2.  

   c) 200  

   BONUS  
   300

AP Book G3-21  
page 175
1. b) square  
   c) rectangle  
   d) square  
   e) square or rectangle  
   f) rectangle  

   BONUS  
   40

2. Teacher to check.  

3. a) 6  
   b) 4  
   c) 5  
   d) 5  
   e) 8  
   f) 6

4. a)  

   B, E  
   C, D, G  
   A, F, H  

   b) 1 2 no  
   c) circle circle yes  
   d) 1 1 yes

   BONUS  
   No. The total number of vertices for a prism is always twice the number of vertices in the base. The double of any number is an even number.
6.  a) 

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b) 

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c) 

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<td>No. of Vertices</td>
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<td>N</td>
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<tr>
<td>Shape of Faces</td>
<td>Δ</td>
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7.  a) sphere
b) rectangular pyramid

**BONUS**
cube

**BONUS**
hexagonal prism
Probability and Data Management: Graphs and Probability – AP Book 3.2: Unit 18

AP Book PDM3-4
page 182
1. a) 4, 5, 4
   b) July
   c) June, August
   d) 4
   e) 30 − 4 = 26
   f) Teacher to check.
   g) April
   h) July
2. a) at home, 4
   b) no
   c) Teacher to check
   d) Teacher to check
   e) Teacher to check
3. a) Teacher to check.
   b) Ice Hockey
   Ice hockey has the greatest number of circles.
   c) 2
   d) 12
   BONUS
   6
4. a) R
   P
   M
   b) 8
   d) 24
   4 3 10 7 6 100
   5
   3
   2
   5
4. a) Teacher to check.
   b) Teacher to check
   c) Grade 3
   It is the tallest column.
   1 2 3 4 5
5. a) 4
   b) 20
   5
   5
   5
   b) 8
   16
   40
   1
   2
   5
   9
   18
   45
   b) Teacher to check.
   c) Grade
   d) It is the tallest column.
   e) 4
   f) 3
   g) 4
   h) 4
   i) 4
   4
   × 3 = 12
   12
   3
   3
   × 3 = 9
   9
   AP Book PDM3-5
page 185
1. a) 25
   b) 4, 8
   c) 9, 15
   d) 18, 27
   BONUS
   100
2. a) □
   b) □ □, □ □ □ □ □ □ □ □
   c) □
   d) 12
   8
   4
   12
AP Book PDM3-6
page 187
1. a) 10
   5
   3
   2
   5
   b) Teacher to check.
   c) T
   Q
   H
   C
   ● = 2 shapes
   d) triangle
   hexagon
   e) 20
   f) 15
2. Circle the following:
   b) ♦ = 10
   c) ♦ = 3
   d) ♦ = 5
3. a) O
   O
   O
   O
   O
   O
   O
   O
   O
   1 2 3 4 5
   Grade
   b) catfish
   c) perch
   d) 12
   e) 6
   25
   b) 4
   c) 6
   3
   × 3 = 9
   9
   AP Book PDM3-7
page 189
1. a) 4
   b) 2
   c) 3
   b) 10
   c) 25
   d) 12
   e) 4
   f) 5
   d) 25
   25 ÷ 5 = 5
   5
   15
   15 ÷ 5 = 3
   3
   50
   50 ÷ 5 = 10
   10
   e) Teacher to check.
4. a) Teacher to check
   b) 3
   c) 5
   5
   b) 4
   c) 6
   d) 12
   e) 18
   25
   25 ÷ 5 = 5
   5
   15
   15 ÷ 5 = 3
   3
   50
   50 ÷ 5 = 10
   10
   e) Teacher to check.
5. a) 40 cm
   b) 170 cm
   c) July–September
   have no bar. It rarely snows in the summer in Ottawa.
   AP Book PDM3-8
page 192
1. a) apple
   b) mango
   c) apple, orange
   d) 2
   e) 6
   4
   8
2. a) 3
   b) 5
   3
   c) 4
   4 × 3 = 12
   12
   3
   3 × 3 = 9
   9
   AP Book PDM3-9
page 195
1. a) 4
   b) Teacher to check.
   c) Ice Hockey
   Ice hockey has the longest bar.
   d) Answers will vary.
   Teacher to check.
Probability and Data Management:
Graphs and Probability – AP Book 3.2: Unit 18

W-26 Answer Keys for AP Book 3.2

AP Book PDM3-10

1. a) pictograph, bar graph, line plot
   b) F P BG LP
   T ✓ ✓ ✓
   L ✓ ✓ ✓
   N × × ✓
   V × ✓ ×
   Sc × ✓ ✓
   Sy ✓ ✓ ×
   12 pictures of leaves
   12 blocks in the bars
   12 X's

2. b) 1: pictograph, bar graph
   c) 3: line plot
   d) 1: line plot
   e) willow; pictograph, bar graph

3. a) Teacher to check.
   b) 2
   c) 1

AP Book PDM3-11

1. a) No. There are four seasons and Megan only gave three as choices.
   b) Fall

2. a) no
   b) more than 3

3. Answers will vary. Teacher to check.

AP Book PDM3-12

1. b) spin 1, spin 2, spin 3
   c) spin R, spin B, spin G, spin Y
   d) spin G
   1

2. b) 1, 2, 3, 4, 5, 6; 6
   c) win, lose, tie; 3

3. a) 4
   b) 2
   c) 1
   d) 3

4. a) 3
   b) 3
   c) 4

5. a) 5
   b) 3
   c) 85

6. 4, 7, 8

7. Shade the following:
   a) 4 squares
   b) 4 squares
   c) 6 squares
   d) 5 squares

8. b) less than half
   c) less than half
   d) more than half
   e) half
   f) less than half
   g) more than half
   h) more than half
   i) more than half
   j) more than half
   k) half
   l) more than half

9. a) 2
   b) 3
   c) 5

10. Circle the following:
    a) 1st
    b) no

11. a) yes
    b) yes

12. a) no
    b) no

13. Yes. 6 is half of 12 and 7 is greater than 6.
BONUS

No. The team would have an even chance of winning if they had won half of their games (3 out of 6).

AP Book PDM3-14

page 209

1. a) half the time
   b) more than half the time
   c) half the time
   d) less than half the time
   e) half the time
   f) more than half the time

2. a) unlikely
    b) likely
    c) likely

3. a) even
    b) likely
    c) likely
    d) unlikely

4. a) likely
    b) unlikely
    c) impossible
    d) likely
    e) certain
    f) likely
    g) unlikely
    h) unlikely
    i) likely

5. Red. There are more red marbles (4) than blue marbles (2).

AP Book PDM3-15

page 211

1. a) 6
    b) 6
    Equal
    d) 6
    Equal

2. b) 1
   c) 1
   1
   3

3. part b) True

4. b) True
    True
    True
    d) True
    Not true
    Not true

5. a) The game is fair.
    b) Player 2
    c) Player 1
    d) The game is fair.

6. Answers will vary. Sample answers:
   a) 
   b) 
   c) 

BONUS

7. No. Player 1 has 3 winning outcomes while Player 2 has only 2 winning outcomes.

Answer Keys for AP Book 3.2
Grade 3 JUMP Math Correlation to the Alberta Curriculum

NOTES:

Underlined JUMP Math lessons are review from a previous grade.

Italicized JUMP Math lessons contain prerequisite material required to meet the learning standard.

An asterisk (*) indicates that a JUMP Math lesson covers a curriculum requirement primarily in the Teacher’s Guide.

JUMP Math strands are represented by:

- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

<table>
<thead>
<tr>
<th>General Outcome</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Develop number sense.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Say the number sequence 0 to 1000 forward and backward by:  • 5s, 10s or 100s, using any starting point  • 3s, using starting points that are multiples of 3  • 4s, using starting points that are multiples of 4  • 25s, using starting points that are multiples of 25.</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[C, CN, ME]</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>2. Represent and describe numbers to 1000, concretely, pictorially and symbolically.</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Compare and order numbers to 1000.</td>
<td>Part Unit Lessons</td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Estimate quantities less than 1000, using referents.</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>Illustration/Description</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>5.</td>
<td>Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.</td>
</tr>
<tr>
<td></td>
<td>[C, CN, R, V]</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Describe and apply mental mathematics strategies for adding two 2-digit numerals.</td>
</tr>
<tr>
<td></td>
<td>[C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Describe and apply mental mathematics strategies for subtracting two 2-digit numerals.</td>
</tr>
<tr>
<td></td>
<td>[C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td>8.</td>
<td>Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context.</td>
</tr>
<tr>
<td></td>
<td>[C, ME, PS, R]</td>
</tr>
<tr>
<td>9.</td>
<td>Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by:</td>
</tr>
<tr>
<td></td>
<td>• using personal strategies for adding and subtracting with and without the support of manipulatives</td>
</tr>
<tr>
<td></td>
<td>• creating and solving problems in context that involve addition and subtraction of numbers.</td>
</tr>
<tr>
<td></td>
<td>[C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td>Note: Students investigate a variety of strategies, including standard/traditional algorithms, to become proficient in at least one appropriate and efficient strategy that they understand.</td>
</tr>
<tr>
<td>10.</td>
<td>Apply mental mathematics strategies and number properties in order to understand and recall basic addition facts and related subtraction facts to 18.</td>
</tr>
<tr>
<td></td>
<td>[C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td>Understand, recall and apply addition facts up to and including 9 + 9 and related subtraction facts.</td>
</tr>
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</tbody>
</table>
### Number

**11.** Demonstrate an understanding of multiplication to \(5 \times 5\) by:
- representing and explaining multiplication using equal grouping and arrays
- creating and solving problems in context that involve multiplication
- modelling multiplication using concrete and visual representations, and recording the process symbolically
- relating multiplication to repeated addition
- relating multiplication to division.

[C, CN, PS, R]

Understand and recall multiplication facts to \(5 \times 5\).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 1    | 6    | NS3-28  
     |       | NS3-32 to 38 |
| 1    | 7    | NS3-39 to 41, 44, 46, 47 |
| 2    | 10   | NS3-60, 61 |
| 2    | 14   | ME3-27 |

**12.** Demonstrate an understanding of division (limited to division related to multiplication facts up to \(5 \times 5\)) by:
- representing and explaining division using equal sharing and equal grouping
- creating and solving problems in context that involve equal sharing and equal grouping
- modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically
- relating division to repeated subtraction
- relating division to multiplication.

[C, CN, PS, R]

Understand and recall division facts related to multiplication facts to \(5 \times 5\).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>NS3-48 to 53, 54*, 55*, 56 to 61</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>ME3-27</td>
</tr>
</tbody>
</table>

**13.** Demonstrate an understanding of fractions by:
- explaining that a fraction represents a part of a whole
- describing situations in which fractions are used
- comparing fractions of the same whole that have like denominators.

[C, CN, ME, R, V]

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12</td>
<td>NS3-62 to 69</td>
</tr>
</tbody>
</table>
### Patterns & Relations — Patterns

**General Outcome**

Use patterns to describe the world and to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
</table>
| **1.** Demonstrate an understanding of increasing patterns by:  
  • describing  
  • extending  
  • comparing  
  • creating numerical (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds and actions.  
  [C, CN, PS, R, V] | Part | Unit | Lessons |
|                  | 1    | 1    | \(PA3-1, 7\)  
  PA3-2, 5, 6, 8, 9 |      | 6    | NS3-27 | 2 11 | PA3-13, 15 |
| **2.** Demonstrate an understanding of decreasing patterns by:  
  • describing  
  • extending  
  • comparing  
  • creating numerical (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds and actions.  
  [C, CN, PS, R, V] | Part | Unit | Lessons |
|                  | 1    | 1    | \(PA3-7\)  
  PA3-4 to 6, 9 |      | 6    | NS3-27 | 2 11 | PA3-13, 15 |
| **3.** Sort objects or numbers, using one or more than one attribute.  
  [C, CN, R, V] | Part | Unit | Lessons |
|                  | 1    | 1    | \(PA3-10\) | 1 5 | G3-1, 2, 4 | 1 6 | NS3-27* |

### Patterns & Relations — Variables and Equations

**General Outcome**

Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
</table>
| **4.** Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.  
  [C, CN, PS, R, V] | Part | Unit | Lessons |
|                  | 1    | 3    | NS3-24 | 2 11 | PA3-16 to 19 |
### Shape & Space — Measurement

**General Outcome**

Use direct and indirect measurement to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Relate the passage of time to common activities, using nonstandard and standard</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>units (minutes, hours, days, weeks, months, years).</td>
<td>2 13 ME3-14, 22</td>
</tr>
<tr>
<td>[CN, ME, R]</td>
<td></td>
</tr>
<tr>
<td>2. Relate the number of seconds to a minute, the number of minutes to an hour and</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>the number of days to a month in a problem-solving context.</td>
<td>2 13 ME3-21</td>
</tr>
<tr>
<td>[C, CN, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td>3. Demonstrate an understanding of measuring length (cm, m) by:</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>• selecting and justifying referents for the units cm and m</td>
<td>1 4 ME3-1 to 4, 6</td>
</tr>
<tr>
<td>• modelling and describing the relationship between the units cm and m</td>
<td></td>
</tr>
<tr>
<td>• estimating length, using referents</td>
<td></td>
</tr>
<tr>
<td>• measuring and recording length, width and height.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td>4. Demonstrate an understanding of measuring mass (g, kg) by:</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>• selecting and justifying referents for the units g and kg</td>
<td>2 14 ME3-25, 26</td>
</tr>
<tr>
<td>• modelling and describing the relationship between the units g and kg</td>
<td></td>
</tr>
<tr>
<td>• estimating mass, using referents</td>
<td></td>
</tr>
<tr>
<td>• measuring and recording mass.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td>5. Demonstrate an understanding of perimeter of regular and irregular shapes by:</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>• estimating perimeter, using referents for cm or m</td>
<td>1 4 ME3-7, 8</td>
</tr>
<tr>
<td>• measuring and recording perimeter (cm, m)</td>
<td></td>
</tr>
<tr>
<td>• constructing different shapes for a given perimeter (cm, m) to demonstrate that</td>
<td>1 6 NS3-38</td>
</tr>
<tr>
<td>many shapes are possible for a perimeter.</td>
<td></td>
</tr>
</tbody>
</table>
### Shape & Space — 3-D Objects and 2-D Shapes

**General Outcome**

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]</td>
<td>Part 2 Unit 17 Lessons G3-19 to 23</td>
</tr>
<tr>
<td>7. Sort regular and irregular polygons, including: • triangles • quadrilaterals • pentagons • hexagons • octagons according to the number of sides. [C, CN, R, V]</td>
<td>Part 1 Unit 5 Lessons G3-3, 4</td>
</tr>
</tbody>
</table>

### Statistics & Probability — Data Analysis

**General Outcome**

Collect, display and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Collect first-hand data and organize it using: • tally marks • line plots • charts • lists to answer questions. [C, CN, PS, V] [ICT: C4-1.3]</td>
<td>Part 1 Unit 5 Lessons G3-2</td>
</tr>
<tr>
<td>2. Construct, label and interpret bar graphs to solve problems. [C, PS, R, V] [ICT: C4-1.3, C7-1.3, C7-1.40]</td>
<td>Part 2 Unit 18 Lessons PDM3-7</td>
</tr>
</tbody>
</table>
Grade 3 JUMP Math Correlation to the New BC Curriculum

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JUMP Math strands are represented by:

- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

### Big Ideas

Fractions are a type of number that can represent quantities.

Development of computational fluency in addition, subtraction, multiplication, and division of whole numbers requires flexible decomposing and composing.

Regular increases and decreases in patterns can be identified and used to make generalizations.

Standard units are used to describe, measure, and compare attributes of objects’ shapes.

The likelihood of possible outcomes can be examined, compared, and interpreted.

### Content

<table>
<thead>
<tr>
<th>number concepts to 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS3-1 to 11</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NS3-28 NS3-27, 29 to 32, 34, 38</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>NS3-74</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>counting:</th>
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<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<tr>
<th>Part</th>
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<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS3-10</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NS3-28 NS3-27, 29 to 32, 34, 38</td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
<td></td>
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<tr>
<td>------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>° skip-counting by any number from any starting point, increasing and</td>
<td>Part 1 2 NS3-10</td>
<td></td>
</tr>
<tr>
<td>decreasing (i.e., forward and backward)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>° Skip-counting is related to multiplication.</td>
<td>Part 1 6 NS3-27, 29 to 31</td>
<td></td>
</tr>
<tr>
<td>° investigating place-value based counting patterns (e.g., counting</td>
<td>Part 2 16 NS3-76</td>
<td></td>
</tr>
<tr>
<td>by 10s, 100s; bridging over a century; noticing the role of zero as</td>
<td></td>
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<tr>
<td>a placeholder 698, 699, 700, 701; noticing the predictability of our</td>
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<td></td>
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<tr>
<td>number system)</td>
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</tr>
<tr>
<td>• Numbers to 1000 can be arranged and recognized:</td>
<td>Part 1 2 NS3-7 to 10</td>
<td></td>
</tr>
<tr>
<td>° comparing and ordering numbers</td>
<td>Part 2 15 NS3-74</td>
<td></td>
</tr>
<tr>
<td>° estimating large quantities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• place value:</td>
<td>Part 1 2 NS3-1 to 6, 11</td>
<td></td>
</tr>
<tr>
<td>° 100s, 10s, and 1s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>° understanding the relationship between digit places and their values,</td>
<td>Part 1 2 NS3-1 to 3, 6</td>
<td></td>
</tr>
<tr>
<td>to 1000 (e.g., the digit 4 in 342 has the value of 40 or 4 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>° understanding the importance of 0 as a place holder (e.g., in the</td>
<td>Part 1 2 NS3-1 to 3, 6</td>
<td></td>
</tr>
<tr>
<td>number 408, the zero indicates that there are 0 tens)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• instructional resource: Math in a Cultural Context, by Jerry Lipka</td>
<td>Not addressed</td>
<td></td>
</tr>
<tr>
<td>fraction concepts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>° Fractions are numbers that represent an amount or quantity.</td>
<td>Part 2 10 NS3-48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part 2 12 NS3-62 to 70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part 2 12 NS3-69</td>
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</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
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<tr>
<td>------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>• Fractions can represent parts of a region, set, or linear model.</td>
<td>Part  Unit  Lessons</td>
<td></td>
</tr>
<tr>
<td>2 12 NS3-62 to 70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Fraction parts are equal shares or equal-sized portions of a whole or unit.</td>
<td>Part  Unit  Lessons</td>
<td></td>
</tr>
<tr>
<td>2 12 NS3-62 to 70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Provide opportunities to explore and create fractions with concrete materials.</td>
<td>Part  Unit  Lessons</td>
<td></td>
</tr>
<tr>
<td>2 12 NS3-62, 65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• recording pictorial representations of fraction models and connecting to symbolic notation</td>
<td>Part  Unit  Lessons</td>
<td></td>
</tr>
<tr>
<td>2 12 NS3-62 to 70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• equal partitioning</td>
<td>Part  Unit  Lessons</td>
<td></td>
</tr>
<tr>
<td>2 10 NS3-48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 12 NS3-62 to 67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• equal sharing, pole ratios as visual parts, medicine wheel, seasons</td>
<td>Part  Unit  Lessons</td>
<td></td>
</tr>
<tr>
<td>2 12 NS3-64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**addition and subtraction to 1000**

<table>
<thead>
<tr>
<th>Part  Unit  Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 PA3-3</td>
</tr>
<tr>
<td>1 2 \frac{NS3-12}{NS3-10^*}, 13 to 17</td>
</tr>
<tr>
<td>1 3 \frac{NS3-24, 25}{NS3-21 to 23, 26}</td>
</tr>
<tr>
<td>2 14 ME3-27</td>
</tr>
<tr>
<td>2 15 NS3-71, 72</td>
</tr>
</tbody>
</table>

• using flexible computation strategies, involving taking apart (e.g., decomposing using friendly numbers and compensating) and combining numbers in a variety of ways

• estimating sums and differences of all operations to 1000

• using addition and subtraction in real-life contexts and problem-based situations

• whole-class number talks

<table>
<thead>
<tr>
<th>Part  Unit  Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 NS3-14, 16</td>
</tr>
<tr>
<td>1 3 NS3-21 to 23</td>
</tr>
<tr>
<td>2 15 NS3-71, 72</td>
</tr>
<tr>
<td>1 2 NS3-16^*, 17</td>
</tr>
<tr>
<td>1 3 \frac{NS3-24, 25}{NS3-23^*, 26}</td>
</tr>
<tr>
<td>2 14 ME3-27</td>
</tr>
<tr>
<td>1 2 NS3-10^<em>, 17^</em></td>
</tr>
<tr>
<td>Content</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>addition and subtraction facts to 20 (emerging computational fluency)</td>
</tr>
<tr>
<td>• adding and subtracting of numbers to 20</td>
</tr>
<tr>
<td>• demonstrating fluency with math strategies for addition and subtraction (e.g., decomposing, making and bridging ten, related doubles, and commutative property)</td>
</tr>
<tr>
<td>• Addition and subtraction are related.</td>
</tr>
<tr>
<td>• At the end of Grade 3, most students should be able to recall addition facts to 20.</td>
</tr>
<tr>
<td>multiplication and division concepts</td>
</tr>
<tr>
<td>• understanding concepts of multiplication (e.g., groups of, arrays, repeated addition)</td>
</tr>
<tr>
<td>• understanding concepts of division (e.g., sharing, grouping, repeated subtraction)</td>
</tr>
<tr>
<td>• Multiplication and division are related.</td>
</tr>
<tr>
<td>• Provide opportunities for concrete and pictorial representations of multiplication.</td>
</tr>
<tr>
<td>• Use games to develop opportunities for authentic practice of multiplication computations.</td>
</tr>
<tr>
<td>• looking for patterns in numbers, such as in a hundred chart, to further develop understanding of multiplication computation</td>
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<tr>
<td>Content</td>
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<tr>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>• Connect multiplication to skip-counting.</td>
</tr>
<tr>
<td>Part</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
</tr>
<tr>
<td>• Connect multiplication to division and repeated addition.</td>
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<tr>
<td>Part</td>
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<td>1</td>
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<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>• Memorization of facts is not intended for this level.</td>
</tr>
<tr>
<td>• fish drying on rack; sharing of food resources in First Peoples communities</td>
</tr>
<tr>
<td>increasing and decreasing patterns</td>
</tr>
<tr>
<td>Part</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>• creating patterns using concrete, pictorial, and numerical representations</td>
</tr>
<tr>
<td>Part</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>• representing increasing and decreasing patterns in multiple ways</td>
</tr>
<tr>
<td>Part</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>• generalizing what makes the pattern increase or decrease (e.g., doubling, adding 2)</td>
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<tr>
<td>Part</td>
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<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>pattern rules using words and numbers based on concrete experiences</td>
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<tr>
<td>Part</td>
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<tr>
<td>1</td>
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<td>1</td>
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<td>2</td>
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<tr>
<td>Content</td>
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<td>------------------------------------------------------------------------</td>
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<tr>
<td>from a concrete pattern, describing the pattern rule using words and</td>
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<tr>
<td>numbers</td>
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<td></td>
</tr>
<tr>
<td>predictability in song rhythm and patterns</td>
</tr>
<tr>
<td>Share examples of local First Peoples art with the class, and ask</td>
</tr>
<tr>
<td>students to notice patterns in the artwork.</td>
</tr>
<tr>
<td>one-step addition and subtraction <strong>equations</strong> with an unknown number</td>
</tr>
<tr>
<td>start unknown (e.g., $n + 15 = 20$ or $\Box + 15 = 20$)</td>
</tr>
<tr>
<td>change unknown (e.g., $12 + n = 20$ or $12 + \Box = 20$)</td>
</tr>
<tr>
<td>result unknown (e.g., $6 + 13 = n$ or $6 + 13 = \Box$)</td>
</tr>
<tr>
<td>investigate even and odd numbers</td>
</tr>
<tr>
<td>measurement using <strong>standard units</strong> (linear, mass, and capacity)</td>
</tr>
<tr>
<td>linear measurements using standard units (e.g., centimetre, metre,</td>
</tr>
<tr>
<td>kilometre)</td>
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<tr>
<td>capacity measurements using standard units (e.g., millilitre, litre)</td>
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</tr>
<tr>
<td>Introduce concepts of perimeter, area, and circumference (the</td>
</tr>
<tr>
<td>distance around); use of formula and pi to calculate not intended —</td>
</tr>
<tr>
<td>the focus is on the concepts.</td>
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<tr>
<td>Content</td>
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<tr>
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</tr>
<tr>
<td>• area measurement, using square units (standard and non-standard)</td>
</tr>
<tr>
<td>• mass measurements using standard units (e.g., gram, kilogram)</td>
</tr>
<tr>
<td>• estimation of measurements using standard referents (e.g., If this cup holds 100 millilitres, about how much does this jug hold?)</td>
</tr>
<tr>
<td>time concepts</td>
</tr>
<tr>
<td>• understanding concepts of time (e.g., second, minute, hour, day, week, month, year)</td>
</tr>
<tr>
<td>• understanding the relationships between units of time</td>
</tr>
<tr>
<td>• Telling time is not expected at this level.</td>
</tr>
<tr>
<td>• estimating time, using environmental references and natural daily/seasonal cycles, temperatures based on weather systems, traditional calendar</td>
</tr>
<tr>
<td>construction of 3D objects</td>
</tr>
<tr>
<td>• identifying 3D objects according to the 2D shapes of the faces and the number of edges and vertices (e.g., construction of nets, skeletons)</td>
</tr>
<tr>
<td>• describing the attributes of 3D objects (e.g., faces, edges, vertices)</td>
</tr>
<tr>
<td>• identifying 3D objects by their mathematical terms (e.g., sphere, cube, prism, cone, cylinder)</td>
</tr>
<tr>
<td>• comparing 3D objects (e.g., How are rectangular prisms and cubes the same or different?)</td>
</tr>
<tr>
<td>• understanding the preservation of shape (e.g., the orientation of a shape will not change its properties)</td>
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<tr>
<td>• jingle dress bells, bentwood box, birch bark baskets, pithouses</td>
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<tr>
<td>Content</td>
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<tr>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>one-to-one correspondence</strong> with bar graphs, pictographs, charts,</td>
</tr>
<tr>
<td>and tables</td>
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<tr>
<td>• collecting data, creating a graph, and describing, comparing, and</td>
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<tr>
<td>discussing the results</td>
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<tr>
<td>• choosing a suitable representation</td>
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<tr>
<td><strong>likelihood of simulated events</strong>, using comparative language</td>
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<tr>
<td></td>
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<tr>
<td>• using comparative language (e.g., certain, uncertain; more, less,</td>
</tr>
<tr>
<td>or equally likely)</td>
</tr>
<tr>
<td>• developing an understanding of chance (e.g., tossing a coin</td>
</tr>
<tr>
<td>creates a 50-50 chance of landing a head or tail; drawing from a</td>
</tr>
<tr>
<td>bag, using spinners, and rolling dice all simulate probability events)</td>
</tr>
<tr>
<td>• story: <em>The Snowsnake Game</em></td>
</tr>
<tr>
<td></td>
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<tr>
<td><strong>financial literacy</strong> — fluency with coins and bills to 100 dollars,</td>
</tr>
<tr>
<td>and earning and payment</td>
</tr>
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<tr>
<td>• counting mixed combinations of coins and bills up to $100:</td>
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<tr>
<td>° totalling up a set of coins and bills</td>
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<tr>
<td>Content</td>
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<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>o using different combinations of coins and bills to make the same amount</td>
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<tr>
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<tr>
<td>• understanding that payments can be made in flexible ways</td>
</tr>
<tr>
<td>(e.g., cash, cheques, credit, electronic transactions, goods and services)</td>
</tr>
<tr>
<td>• understanding that there are different ways of earning money to</td>
</tr>
<tr>
<td>reach a financial goal (e.g., recycling, holding bake sales, selling</td>
</tr>
<tr>
<td>items, walking a neighbour’s dog)</td>
</tr>
<tr>
<td>• Using pictures of First Peoples trade items (e.g., dentalium shells,</td>
</tr>
<tr>
<td>dried fish, or tools when available) with the values indicated on the</td>
</tr>
<tr>
<td>back, have students play a trading game.</td>
</tr>
</tbody>
</table>
# Grade 3 JUMP Math Exemplar Lessons for Curricular Competencies

The Curricular Competencies in the new BC Mathematics curriculum are addressed throughout JUMP Math’s Grade 3 resource. The following table lists a selection of JUMP Math lessons that provide effective illustrations of how each Curricular Competency is addressed.

<table>
<thead>
<tr>
<th>Curricular Competencies</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reasoning and analyzing</strong></td>
<td></td>
</tr>
<tr>
<td>• Use reasoning to explore and make connections</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 7 NS3-42</td>
<td></td>
</tr>
<tr>
<td>1 8 ME3-10</td>
<td></td>
</tr>
<tr>
<td>2 17 G3-20</td>
<td></td>
</tr>
<tr>
<td>• Estimate reasonably</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 4 ME3-3</td>
<td></td>
</tr>
<tr>
<td>2 14 ME3-23</td>
<td></td>
</tr>
<tr>
<td>• Develop <strong>mental math strategies</strong> and abilities to make sense of quantities</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 3 NS3-19, 20</td>
<td></td>
</tr>
<tr>
<td>2 15 NS3-72</td>
<td></td>
</tr>
<tr>
<td>• Use <strong>technology</strong> to explore mathematics</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 2 NS3-10</td>
<td></td>
</tr>
<tr>
<td>2 16 NS3-76</td>
<td></td>
</tr>
<tr>
<td>• <strong>Model</strong> mathematics in contextualized experiences</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 6 NS3-33</td>
<td></td>
</tr>
<tr>
<td>2 10 NS3-50</td>
<td></td>
</tr>
<tr>
<td><strong>Understanding and solving</strong></td>
<td></td>
</tr>
<tr>
<td>• Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 7 NS3-47</td>
<td></td>
</tr>
<tr>
<td>2 13 ME3-22</td>
<td></td>
</tr>
<tr>
<td>2 18 PDM3-15</td>
<td></td>
</tr>
<tr>
<td>• Visualize to explore mathematical concepts</td>
<td></td>
</tr>
<tr>
<td>Part Unit Lessons</td>
<td></td>
</tr>
<tr>
<td>1 4 ME3-5</td>
<td></td>
</tr>
<tr>
<td>2 17 G3-22</td>
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</tbody>
</table>
### Curricular Competencies

- **Develop and use multiple strategies to engage in problem solving**
  - Part 1: Unit 6, Lessons NS3-30
  - Part 2: Unit 11, Lessons PA3-17
  - Part 2: Unit 17, Lessons G3-19

- **Engage in problem-solving experiences that are connected to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures**
  - Part 1: Unit 4, Lessons ME3-3, 5
  - Part 2: Unit 12, Lessons NS3-64
  - Part 2: Unit 18, Lessons PDM3-13

### Communicating and representing

- **Communicate mathematical thinking in many ways**
  - Part 1: Unit 6, Lessons NS3-32
  - Part 2: Unit 11, Lessons PA3-15
  - Part 2: Unit 16, Lessons NS3-87

- **Use mathematical vocabulary and language to contribute to mathematical discussions**
  - Part 1: Unit 3, Lessons NS3-18
  - Part 2: Unit 17, Lessons G3-20

- **Explain and justify mathematical ideas and decisions**
  - Part 1: Unit 1, Lessons PA3-6
  - Part 2: Unit 11, Lessons PA3-13

- **Represent mathematical ideas in concrete, pictorial, and symbolic forms**
  - Part 1: Unit 7, Lessons NS3-42
  - Part 2: Unit 18, Lessons PDM3-10

### Connecting and reflecting

- **Reflect on mathematical thinking**
  - Part 1: Unit 4, Lessons ME3-8
  - Part 1: Unit 6, Lessons NS3-29, 32
  - Part 2: Unit 12, Lessons NS3-65
### Curricular Competencies

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>ME3-5, 8</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>PA3-16</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>G3-20</td>
</tr>
</tbody>
</table>

- Connect mathematical concepts to each other and to **other areas and personal interests**

- **Incorporate** First Peoples worldviews and perspectives to **make connections** to mathematical concepts

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>PA3-11</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>ME3-22</td>
</tr>
</tbody>
</table>
Grade 3 JUMP Math Correlation to the Manitoba Curriculum

NOTES:

Underlined JUMP Math lessons are review from a previous grade.

Italicized JUMP Math lessons contain prerequisite material required to meet the learning standard.

An asterisk (*) indicates that a JUMP Math lesson covers a curriculum requirement primarily in the Teacher’s Guide.

JUMP Math strands are represented by:

- **NS** Number Sense
- **ME** Measurement
- **G** Geometry
- **PA** Patterns and Algebra
- **PDM** Probability and Data Management

### Number

#### General Learning Outcome

Develop number sense.

#### Specific Learning Outcomes

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.N.1 Say the number sequence between any two given numbers forward and backward</td>
<td>Part</td>
</tr>
<tr>
<td>• from 0 to 1000 by</td>
<td>1</td>
</tr>
<tr>
<td>◦ 10s or 100s, using any starting point</td>
<td>1</td>
</tr>
<tr>
<td>◦ 5s, using starting points that are multiples of 5</td>
<td>2</td>
</tr>
<tr>
<td>◦ 25s, using starting points that are multiples of 25</td>
<td>2</td>
</tr>
<tr>
<td>• from 0 to 100 by</td>
<td>2</td>
</tr>
<tr>
<td>◦ 3s, using starting points that are multiples of 3</td>
<td>1</td>
</tr>
<tr>
<td>◦ 4s, using starting points that are multiples of 4</td>
<td>2</td>
</tr>
<tr>
<td>3.N.2 Represent and describe numbers to 1000, concretely, pictorially, and symbolically.</td>
<td>Part</td>
</tr>
<tr>
<td>[C, CN, V]</td>
<td>1</td>
</tr>
<tr>
<td>3.N.3 Compare and order numbers to 1000.</td>
<td>Part</td>
</tr>
<tr>
<td>[CN, R, V]</td>
<td>1</td>
</tr>
<tr>
<td>3.N.4 Estimate quantities less than 1000 using referents.</td>
<td>Part</td>
</tr>
<tr>
<td>[ME, PS, R, V]</td>
<td>2</td>
</tr>
<tr>
<td>Number</td>
<td>3.N.5 Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]</td>
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<td>-------------------------------------------------------------------------------------------------</td>
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<td></td>
<td>3.N.6 Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles [C, ME, PS, R, V]</td>
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<td>3.N.7 Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles [C, ME, PS, R, V]</td>
</tr>
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<td></td>
<td>3.N.8 Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context. [C, ME, PS, R]</td>
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<td></td>
<td>3.N.9 Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems in contexts that involve addition and subtraction of numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS, R]</td>
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<td></td>
<td>3.N.10 Apply mental math strategies to determine addition facts and related subtraction facts to 18 (9 + 9). [C, CN, ME, R, V]</td>
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</tbody>
</table>
| Number | 3.N.11 | Demonstrate an understanding of multiplication to $5 \times 5$ by  
• representing and explaining multiplication using equal grouping and arrays  
• creating and solving problems in context that involve multiplication  
• modelling multiplication using concrete and visual representations, and recording the process symbolically  
• relating multiplication to repeated addition  
• relating multiplication to division  
[C, CN, PS, R] | Part | Unit | Lessons |
<table>
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<td></td>
<td>1</td>
<td>6</td>
<td>NS3-28, 32 to 38</td>
<td></td>
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<td></td>
<td></td>
<td>1</td>
<td>7</td>
<td>NS3-39 to 41, 44, 46, 47</td>
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<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>NS3-60, 61</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>2</td>
<td>14</td>
<td>ME3-27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 3.N.12 | Demonstrate an understanding of division by  
• representing and explaining division using equal sharing and equal grouping  
• creating and solving problems in context that involve equal sharing and equal grouping  
• modelling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically  
• relating division to repeated subtraction  
• relating division to multiplication  
• (limited to division related to multiplication facts up to $5 \times 5$)  
[C, CN, PS, R] | Part | Unit | Lessons |
| | | 2 | 10 | NS3-48 to 53, 54*, 55*, 56 to 61 |
| | | 2 | 14 | ME3-27 |
| 3.N.13 | Demonstrate an understanding of fractions by  
• explaining that a fraction represents a portion of a whole divided into equal parts  
• describing situations in which fractions are used  
• comparing fractions of the same whole with like denominators  
[C, CN, ME, R, V] | Part | Unit | Lessons |
| | | 2 | 12 | NS3-62 to 69 |
### Patterns and Relations (Patterns)

**General Learning Outcome**

Use patterns to describe the world and solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.PR.1</strong> Demonstrate an understanding of increasing patterns by • describing • extending • comparing • creating patterns using manipulatives, diagrams, and numbers (to 1000). [C, CN, PS, R, V]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 1 PA3-1, 7 PA3-2, 5, 6, 8, 9</td>
</tr>
<tr>
<td></td>
<td>1 6 NS3-27</td>
</tr>
<tr>
<td></td>
<td>2 11 PA3-13, 15</td>
</tr>
<tr>
<td><strong>3.PR.2</strong> Demonstrate an understanding of decreasing patterns by • describing • extending • comparing • creating patterns using manipulatives, diagrams, and numbers (starting from 1000 or less). [C, CN, PS, R, V]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 1 PA3-7 PA3-4 to 6, 9</td>
</tr>
<tr>
<td></td>
<td>1 6 NS3-27</td>
</tr>
<tr>
<td></td>
<td>2 11 PA3-13, 15</td>
</tr>
</tbody>
</table>

### Patterns and Relations (Variables and Equations)

**General Learning Outcome**

Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.PR.3</strong> Solve one-step addition and subtraction equations involving symbols representing an unknown number. [C, CN, PS, R, V]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 3 NS3-24</td>
</tr>
<tr>
<td></td>
<td>2 11 PA3-16 to 19</td>
</tr>
</tbody>
</table>
# Shape and Space (Measurement)

## General Learning Outcome

Use direct or indirect measurement to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.SS.1 Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]</td>
<td>Part 2  Unit 13  Lessons ME3-14, 22</td>
</tr>
<tr>
<td>3.SS.2 Relate the number of seconds to a minute, the number of minutes to an hour, and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]</td>
<td>Part 2  Unit 13  Lessons ME3-22</td>
</tr>
<tr>
<td>3.SS.3 Demonstrate an understanding of measuring length (cm, m) by • selecting and justifying referents for the units cm and m • modelling and describing the relationship between the units cm and m • estimating length using referents • measuring and recording length, width, and height [C, CN, ME, PS, R, V]</td>
<td>Part 1  Unit 4  Lessons ME3-1 to 4, 6</td>
</tr>
<tr>
<td>3.SS.4 Demonstrate an understanding of measuring mass (g, kg) by • selecting and justifying referents for the units g and kg • modelling and describing the relationship between the units g and kg • estimating mass using referents • measuring and recording mass [C, CN, ME, PS, R, V]</td>
<td>Part 2  Unit 14  Lessons ME3-25, 26</td>
</tr>
</tbody>
</table>
| 3.SS.5 Demonstrate an understanding of perimeter of regular and irregular shapes by • estimating perimeter using referents for centimetre or metre • measuring and recording perimeter (cm, m) • constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter [C, ME, PS, R, V] | Part 1  Unit 4  Lessons ME3-7, 8  
Part 1  Unit 6  Lessons NS3-38 |
### Shape and Space (3-D Objects and 2-D Shapes)

**General Learning Outcome**
Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.SS.6</strong> Describe 3-D objects according to the shape of the faces and the number of edges and vertices. [C, CN, PS, R, V]</td>
<td>Part: 2, Unit: 17, Lessons: G3-19 to 23</td>
</tr>
<tr>
<td><strong>3.SS.7</strong> Sort regular and irregular polygons, including: - triangles - quadrilaterals - pentagons - hexagons - octagons according to the number of sides. [C, CN, R, V]</td>
<td>Part: 1, Unit: 5, Lessons: G3-3, 4</td>
</tr>
</tbody>
</table>

### Statistics and Probability (Data Analysis)

**General Learning Outcome**
Collect, display, and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.SP.1</strong> Collect first-hand data and organize it using: - tally marks - line plots - charts - lists to answer questions. [C, CN, V]</td>
<td>Part: 1, Unit: 5, Lessons: G3-1, G3-2</td>
</tr>
<tr>
<td><strong>3.SP.2</strong> Construct, label, and interpret bar graphs to solve problems. [PS, R, V]</td>
<td>Part: 2, Unit: 18, Lessons: PDM3-7</td>
</tr>
</tbody>
</table>
Grade 3 JUMP Math Correlation to the Ontario Curriculum

NOTES:

- Underlined JUMP Math lessons are review from a previous grade.
- *Italicized* JUMP Math lessons contain prerequisite material required to meet the learning standard.
- An asterisk (*) indicates that a JUMP Math lesson covers a curriculum requirement primarily in the Teacher’s Guide.

Expectation codes source: Ontario Curriculum Unit Planner

JUMP Math strands are represented by:

- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

### Number Sense and Numeration

#### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m8</td>
<td>read, represent, compare, and order whole numbers to 1000, and use concrete materials to represent fractions and money amounts to $10;</td>
</tr>
<tr>
<td>3m9</td>
<td>demonstrate an understanding of magnitude by counting forward and backwards by various numbers and from various starting points;</td>
</tr>
<tr>
<td>3m10</td>
<td>solve problems involving the addition and subtraction of single- and multi-digit whole numbers, using a variety of strategies, and demonstrate an understanding of multiplication and division.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

<table>
<thead>
<tr>
<th>Quantity Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m11</td>
<td>represent, compare, and order whole numbers to 1000, using a variety of tools (e.g., base ten materials or drawings of them, number lines with increments of 100 or other appropriate amounts);</td>
</tr>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3m12</td>
<td>read and print in words whole numbers to one hundred, using meaningful contexts (e.g., books, speed limit signs);</td>
</tr>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Number Sense and Numeration</td>
<td>Part</td>
</tr>
<tr>
<td>----------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3m13 identify and represent the value of a digit in a number according to its position in the number (e.g., use base ten materials to show that the 3 in 324 represents 3 hundreds);</td>
<td>1</td>
</tr>
<tr>
<td>3m14 compose and decompose three-digit numbers into hundreds, tens, and ones in a variety of ways, using concrete materials (e.g., use base ten materials to decompose 327 into 3 hundreds, 2 tens, and 7 ones, or into 2 hundreds, 12 tens, and 7 ones);</td>
<td>1</td>
</tr>
<tr>
<td>3m15 round two-digit numbers to the nearest ten, in problems arising from real-life situations;</td>
<td>2</td>
</tr>
<tr>
<td>3m16 represent and explain, using concrete materials, the relationship among the numbers 1, 10, 100, and 1000, (e.g., use base ten materials to represent the relationship between a decade and a century, or a century and a millennium);</td>
<td>1</td>
</tr>
<tr>
<td>3m17 divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation;</td>
<td>2</td>
</tr>
<tr>
<td>3m18 represent and describe the relationships between coins and bills up to $10 (e.g., “There are eight quarters in a toonie and ten dimes in a loonie.”);</td>
<td>2</td>
</tr>
<tr>
<td>3m19 estimate, count, and represent (using the $ symbol) the value of a collection of coins and bills with a maximum value of $10;</td>
<td>2</td>
</tr>
<tr>
<td>3m20 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1000 (Sample problem: Do you know anyone who has lived for close to 1000 days? Explain your reasoning.).</td>
<td>1</td>
</tr>
<tr>
<td>3m21 count forward by 1’s, 2’s, 5’s, 10’s, and 100’s to 1000 from various starting points, and by 25’s to 1000 starting from multiples of 25, using a variety of tools and strategies (e.g., skip count with and without the aid of a calculator; skip count by 10’s using dimes);</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
### Number Sense and Numeration

<table>
<thead>
<tr>
<th>3m22</th>
<th>count backwards by 2’s, 5’s, and 10’s from 100 using multiples of 2, 5, and 10 as starting points, and count backwards by 100’s from 1000 and any number less than 1000, using a variety of tools (e.g., number lines, calculators, coins) and strategies.</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>NS3-10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>6</td>
<td>NS3-27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>NS3-14</td>
</tr>
</tbody>
</table>

### Operational Sense

<table>
<thead>
<tr>
<th>3m23</th>
<th>solve problems involving the addition and subtraction of two-digit numbers, using a variety of mental strategies (e.g., to add 37 + 26, add the tens, add the ones, then combine the tens and ones, like this: 30 + 20 = 50, 7 + 6 = 13, 50 + 13 = 63);</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>NS3-18, 24 NS3-25, 26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3m24</th>
<th>add and subtract three-digit numbers, using concrete materials, student-generated algorithms, and standard algorithms;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>NS3-12 NS3-13 to 17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3m25</th>
<th>use estimation when solving problems involving addition and subtraction, to help judge the reasonableness of a solution;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>15</td>
<td>NS3-72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3m26</th>
<th>add and subtract money amounts, using a variety of tools (e.g., currency manipulatives, drawings), to make simulated purchases and change for amounts up to $10 <em>(Sample problem: You spent 5 dollars and 75 cents on one item and 10 cents on another item. How much did you spend in total?)</em>;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>16</td>
<td>NS3-81, 86, 89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3m27</th>
<th>relate multiplication of one-digit numbers and division by one-digit divisors to real-life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences) <em>(Sample problem: Give a real-life example of when you might need to know that 3 groups of 2 is 3 \times 2).</em>;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>6</td>
<td>NS3-32 to 38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>7</td>
<td>NS3-39 to 42, 47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>NS3-48 to 61</td>
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<td></td>
<td>2</td>
<td>11</td>
<td>PA3-16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>16</td>
<td>NS3-86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3m28</th>
<th>multiply to 7 \times 7 and divide to 49 \div 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting).</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>6</td>
<td>NS3-32 to 38</td>
</tr>
<tr>
<td></td>
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<td>1</td>
<td>7</td>
<td>NS3-39 to 42, 44 to 47</td>
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<tr>
<td></td>
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<td>2</td>
<td>10</td>
<td>NS3-53 to 61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>11</td>
<td>PA3-16</td>
</tr>
</tbody>
</table>
# Measurement

## Overall Expectations

3m29 estimate, measure, and record length, perimeter, area, mass, capacity, time, and temperature, using standard units;

3m30 compare, describe, and order objects, using attributes measured in standard units.

## Specific Expectations

### Attributes, Units, and Measurement Sense

<table>
<thead>
<tr>
<th>Specific Expectation</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3m31</strong> estimate, measure, and record length, height, and distance, using standard units (i.e., centimetre, metre, kilometre) (<strong>Sample problem:</strong> While walking with your class, stop when you think you have travelled one kilometre.);</td>
<td>Part Unit Lessons&lt;br&gt;1 4 ME3-1, 3 to 5</td>
</tr>
<tr>
<td><strong>3m32</strong> draw items using a ruler, given specific lengths in centimetres (<strong>Sample problem:</strong> Draw a pencil that is 5 cm long);</td>
<td>Part Unit Lessons&lt;br&gt;1 4 ME3-2</td>
</tr>
<tr>
<td><strong>3m33</strong> read time using analogue clocks, to the nearest five minutes, and using digital clocks (e.g., 1:23 means twenty-three minutes after one o’clock), and represent time in 12-hour notation;</td>
<td>Part Unit Lessons&lt;br&gt;2 13 ME3-15&lt;br&gt;ME3-14, 16 to 21</td>
</tr>
<tr>
<td><strong>3m34</strong> estimate, read (i.e., using a thermometer), and record positive temperatures to the nearest degree Celsius (i.e., using a number line; using appropriate notation) (<strong>Sample problem:</strong> Record the temperature outside each day using a thermometer, and compare your measurements with those reported in the daily news.);</td>
<td>Part Unit Lessons&lt;br&gt;2 14 ME3-29</td>
</tr>
<tr>
<td><strong>3m35</strong> identify benchmarks for freezing, cold, cool, warm, hot, and boiling temperatures as they relate to water and for cold, cool, warm, and hot temperatures as they relate to air (e.g., water freezes at 0°C; the air temperature on a warm day is about 20°C, but water at 20°C feels cool);</td>
<td>Part Unit Lessons&lt;br&gt;2 14 ME3-29</td>
</tr>
<tr>
<td><strong>3m36</strong> estimate, measure, and record the perimeter of two-dimensional shapes, through investigation using standard units (<strong>Sample problem:</strong> Estimate, measure, and record the perimeter of your notebook.);</td>
<td>Part Unit Lessons&lt;br&gt;1 4 ME3-7, 8</td>
</tr>
<tr>
<td><strong>3m37</strong> estimate, measure (i.e., using centimetre grid paper, arrays), and record area (e.g., if a row of 10 connecting cubes is approximately the width of a book, skip counting down the cover of the book with the row of cubes [i.e., counting 10, 20, 30, ...] is one way to determine the area of the book cover);</td>
<td>Part Unit Lessons&lt;br&gt;1 8 ME3-9 to 13&lt;br&gt;2 12 NS3-70</td>
</tr>
<tr>
<td><strong>3m38</strong> choose benchmarks for a kilogram and a litre to help them perform measurement tasks;</td>
<td>Part Unit Lessons&lt;br&gt;2 14 ME3-25, 26</td>
</tr>
</tbody>
</table>
### Measurement

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m39 estimate, measure, and record the mass of objects (e.g., can of apple juice, bag of oranges, bag of sand), using the standard unit of the kilogram or parts of a kilogram (e.g., half, quarter);</td>
<td>2</td>
<td>14</td>
<td>ME3-24 to 26, 28</td>
</tr>
<tr>
<td>3m40 estimate, measure, and record the capacity of containers (e.g., juice can, milk bag), using the standard unit of the litre or parts of a litre (e.g., half, quarter).</td>
<td>2</td>
<td>14</td>
<td>ME3-23</td>
</tr>
</tbody>
</table>

### Measurement Relationships

<table>
<thead>
<tr>
<th>Measurement Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m41 compare standard units of length (i.e., centimetre, metre, kilometre) (e.g., centimetres are smaller than metres), and select and justify the most appropriate standard unit to measure length;</td>
<td>Part</td>
</tr>
<tr>
<td>3m42 compare and order objects on the basis of linear measurements in centimetres and/or metres (e.g., compare a 3 cm object with a 5 cm object; compare a 50 cm object with a 1 m object) in problem-solving contexts;</td>
<td>1</td>
</tr>
<tr>
<td>3m43 compare and order various shapes by area, using congruent shapes (e.g., from a set of pattern blocks or Power Polygons) and grid paper for measuring</td>
<td>1</td>
</tr>
<tr>
<td>3m44 describe, through investigation using grid paper, the relationship between the size of a unit of area and the number of units needed to cover a surface</td>
<td>1</td>
</tr>
<tr>
<td>3m45 compare and order a collection of objects, using standard units of mass (i.e., kilogram) and/or capacity (i.e., litre);</td>
<td>2</td>
</tr>
<tr>
<td>3m46 solve problems involving the relationships between minutes and hours, hours and days, days and weeks, and weeks and years, using a variety of tools (e.g., clocks, calendars, calculators).</td>
<td>2</td>
</tr>
</tbody>
</table>
## Geometry and Spatial Sense

### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m47</td>
<td>compare two-dimensional shapes and three-dimensional figures and sort them by their geometric properties;</td>
</tr>
<tr>
<td>3m48</td>
<td>describe relationships between two-dimensional shapes, and between two-dimensional shapes and three-dimensional figures;</td>
</tr>
<tr>
<td>3m49</td>
<td>identify and describe the locations and movements of shapes and objects.</td>
</tr>
</tbody>
</table>

### Specific Expectations

#### Geometric Properties

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m50</td>
<td>use a reference tool (e.g., paper corner, pattern block, carpenter’s square) to identify right angles and to describe angles as greater than, equal to, or less than a right angle (Sample problem: Which pattern blocks have angles bigger than a right angle?);</td>
<td>Part: 1, Unit: 5, Lessons: G3-5</td>
</tr>
<tr>
<td>3m51</td>
<td>identify and compare various polygons (i.e., triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons) and sort them by their geometric properties (i.e., number of sides; side lengths; number of interior angles; number of right angles);</td>
<td>Part: 1, Unit: 5, Lessons: G3-3, 5, 11, 12</td>
</tr>
<tr>
<td>3m52</td>
<td>compare various angles, using concrete materials and pictorial representations, and describe angles as bigger than, smaller than, or about the same as other angles (e.g., “Two of the angles on the red pattern block are bigger than all the angles on the green pattern block.”);</td>
<td>Part: 1, Unit: 5, Lessons: G3-12</td>
</tr>
<tr>
<td>3m53</td>
<td>compare and sort prisms and pyramids by geometric properties (i.e., number and shape of faces, number of edges, number of vertices), using concrete materials;</td>
<td>Part: 2, Unit: 17, Lessons: G3-19, 21, 22</td>
</tr>
<tr>
<td>3m54</td>
<td>construct rectangular prisms (e.g., using given paper nets; using Polydrons), and describe geometric properties (i.e., number and shape of faces, number of edges, number of vertices) of the prisms.</td>
<td>Part: 2, Unit: 17, Lessons: G3-20 to 22</td>
</tr>
</tbody>
</table>

#### Geometric Relationships

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m55</td>
<td>solve problems requiring the greatest or least number of two-dimensional shapes (e.g., pattern blocks) needed to compose a larger shape in a variety of ways (e.g., to cover an outline puzzle) (Sample problem: Compose a hexagon using different numbers of smaller shapes.);</td>
<td>Part: 1, Unit: 5, Lessons: G3-12*</td>
</tr>
<tr>
<td>3m56</td>
<td>explain the relationships between different types of quadrilaterals (e.g., a square is a rectangle because a square has four sides and four right angles; a rhombus is a parallelogram because opposite sides of a rhombus are parallel);</td>
<td>Part: 1, Unit: 5, Lessons: G3-6 to 10</td>
</tr>
</tbody>
</table>
### Geometry and Spatial Sense

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m57</td>
<td>Identify and describe the two-dimensional shapes that can be found in a three-dimensional figure. <em>(Sample problem:)</em> Build a structure from blocks, toothpicks, or other concrete materials, and describe it using geometric terms, so that your partner will be able to build your structure without seeing it.</td>
<td>2</td>
<td>17</td>
<td>G3-20, 21</td>
</tr>
<tr>
<td>3m58</td>
<td>Describe and name prisms and pyramids by the shape of their base (e.g., rectangular prism, square-based pyramid).</td>
<td>2</td>
<td>17</td>
<td>G3-20</td>
</tr>
<tr>
<td>3m59</td>
<td>Identify congruent two-dimensional shapes by manipulating and matching concrete materials (e.g., by translating, reflecting, or rotating pattern blocks).</td>
<td>1</td>
<td>5</td>
<td>G3-13</td>
</tr>
</tbody>
</table>

### Location and Movement

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m60</td>
<td>Describe movement from one location to another using a grid map (e.g., to get from the swings to the sandbox, move three squares to the right and two squares down).</td>
<td>2</td>
<td>17</td>
<td>G3-15, 16</td>
</tr>
<tr>
<td>3m61</td>
<td>Identify flips, slides, and turns, through investigation using concrete materials and physical motion, and name flips, slides, and turns as reflections, translations, and rotations (e.g., a slide to the right is a translation; a turn is a rotation).</td>
<td>2</td>
<td>17</td>
<td>G3-17, 18</td>
</tr>
<tr>
<td>3m62</td>
<td>Complete and describe designs and pictures of images that have a vertical, horizontal, or diagonal line of symmetry. <em>(Sample problem:)</em> Draw the missing portion of the given butterfly on grid paper.</td>
<td>1</td>
<td>5</td>
<td>G3-14</td>
</tr>
</tbody>
</table>
## Patterning and Algebra

### Overall Expectations

- **3m63** describe, extend, and create a variety of numeric patterns and geometric patterns;
- **3m64** demonstrate an understanding of equality between pairs of expressions, using addition and subtraction of one- and two-digit numbers.

### Specific Expectations

#### Patterns and Relationships

<table>
<thead>
<tr>
<th>Expectation</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3m65</strong> identify, extend, and create a repeating pattern involving two attributes (e.g., size, colour, orientation, number), using a variety of tools (e.g., pattern blocks, attribute blocks, drawings) (<strong>Sample problem:</strong> Create a repeating pattern using three colours and two shapes.);</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 1  <strong>Lessons</strong> PA3-10 to 12</td>
</tr>
<tr>
<td><strong>3m66</strong> identify and describe, through investigation, number patterns involving addition, subtraction, and multiplication, represented on a number line, on a calendar, and on a hundreds chart (e.g., the multiples of 9 appear diagonally in a hundreds chart);</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 1  <strong>Lessons</strong> PA3-7, 8  <strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> PA3-14, 15</td>
</tr>
<tr>
<td><strong>3m67</strong> extend repeating, growing, and shrinking number patterns (<strong>Sample problem:</strong> Write the next three terms in the pattern 4, 8, 12, 16, ...);</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 1  <strong>Lessons</strong> PA3-1  <strong>Part</strong> 1  <strong>Unit</strong> 6  <strong>Lessons</strong> NS3-29, 30, 31*  <strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> PA3-14, 15</td>
</tr>
<tr>
<td><strong>3m68</strong> create a number pattern involving addition or subtraction, given a pattern represented on a number line or a pattern rule expressed in words (<strong>Sample problem:</strong> Make a number pattern that starts at 0 and grows by adding 7 each time.);</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 1  <strong>Lessons</strong> PA3-6  <strong>Part</strong> 1  <strong>Unit</strong> 6  <strong>Lessons</strong> NS3-29, 30, 31*  <strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> PA3-14, 15</td>
</tr>
<tr>
<td><strong>3m69</strong> represent simple geometric patterns using a number sequence, a number line, or a bar graph (e.g., the given growing pattern of toothpick squares can be represented numerically by the sequence 4, 7, 10, ..., which represents the number of toothpicks used to make each figure);</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> PA3-13, 14</td>
</tr>
<tr>
<td><strong>3m70</strong> demonstrate, through investigation, an understanding that a pattern results from repeating an action (e.g., clapping, taking a step forward every second), repeating an operation (e.g., addition, subtraction), using a transformation (e.g., slide, flip, turn), or making some other repeated change to an attribute (e.g., colour, orientation).</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 1  <strong>Lessons</strong> PA3-5*, 6, 12  <strong>Part</strong> 2  <strong>Unit</strong> 17  <strong>Lessons</strong> G3-17, 18</td>
</tr>
</tbody>
</table>
## Expressions and Equality

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m71 determine, through investigation, the inverse relationship between addition and subtraction (e.g., since $4 + 5 = 9$, then $9 - 5 = 4$; since $16 - 9 = 7$, then $7 + 9 = 16$);</td>
<td>1 1</td>
<td></td>
<td>PA3-3*</td>
</tr>
<tr>
<td>3m72 determine, the missing number in equations involving addition and subtraction of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) <em>(Sample problem:)</em> What is the missing number in the equation $25 - 4 = 15 + \underline{\text{?}}$;</td>
<td>2 11</td>
<td></td>
<td>PA3-17 to 19</td>
</tr>
<tr>
<td>3m73 identify, through investigation, the properties of zero and one in multiplication (i.e., any number multiplied by zero equals zero; any number multiplied by 1 equals the original number) <em>(Sample problem:)</em> Use tiles to create arrays that represent $3 \times 3$, $3 \times 2$, $3 \times 1$, and $3 \times 0$. Explain what you think will happen when you multiply any number by 1, and when you multiply any number by 0;</td>
<td>1 7</td>
<td></td>
<td>NS3-43</td>
</tr>
<tr>
<td>3m74 identify, through investigation, and use the associative property of addition to facilitate computation with whole numbers (e.g., &quot;I know that $17 + 16$ equals $17 + 3 + 13$. This is easier to add in my head because I get $20 + 13 = 33.&quot;.&quot;)</td>
<td>1 3</td>
<td></td>
<td>NS3-19</td>
</tr>
</tbody>
</table>
### Data Management and Probability

#### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m75</td>
<td>collect and organize categorical or discrete primary data and display the data using charts and graphs, including vertical and horizontal bar graphs, with labels ordered appropriately along horizontal axes, as needed;</td>
</tr>
<tr>
<td>3m76</td>
<td>read, describe, and interpret primary data presented in charts and graphs, including vertical and horizontal bar graphs;</td>
</tr>
<tr>
<td>3m77</td>
<td>predict and investigate the frequency of a specific outcome in a simple probability experiment.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

**Collection and Organization of Data**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>3m78</td>
<td>demonstrate an ability to organize objects into categories, by sorting and classifying objects using two or more attributes simultaneously (<em>Sample problem:</em> Sort a collection of buttons by size, colour, and number of holes.);</td>
</tr>
<tr>
<td>3m79</td>
<td>collect data by conducting a simple survey about themselves, their environment, issues in their school or community, or content from another subject;</td>
</tr>
<tr>
<td>3m80</td>
<td>collect and organize categorical or discrete primary data and display the data in charts, tables, and graphs (including vertical and horizontal bar graphs), with appropriate titles and labels and with labels ordered appropriately along horizontal axes, as needed, using many-to-one correspondence (e.g., in a pictograph, one car sticker represents 3 cars; on a bar graph, one square represents 2 students) (<em>Sample problem:</em> Graph data related to the eye colour of students in the class, using a vertical bar graph. Why does the scale on the vertical axis include values that are not in the set of data?).</td>
</tr>
</tbody>
</table>

**JUMP Math Lessons**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>PA3-10</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>G3-1 to 3</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>PDM3-11</td>
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</tbody>
</table>

**Data Relationships**

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>3m81</td>
<td>read primary data presented in charts, tables, and graphs (including vertical and horizontal bar graphs), then describe the data using comparative language, and describe the shape of the data (e.g., &quot;Most of the data are at the high end.&quot;); &quot;All of the data values are different.&quot;);</td>
</tr>
<tr>
<td>3m82</td>
<td>interpret and draw conclusions from data presented in charts, tables, and graphs;</td>
</tr>
</tbody>
</table>

**JUMP Math Lessons**

<table>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>PDM3-2*, 3*</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>PDM3-3</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>PDM3-4 to 9</td>
</tr>
<tr>
<td>Data Management and Probability</td>
<td>Probability</td>
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<td>--------------------------------</td>
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</tbody>
</table>
| **3m83** demonstrate an understanding of mode (e.g., "The mode is the value that shows up most often on a graph.")
and identify the mode in a set of data. | **JUMP Math Lessons** |
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>PDM3-3</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>PDM3-6, 9</td>
</tr>
</tbody>
</table>

**Probability**

| **3m84** predict the frequency of an outcome in a simple probability experiment or game (e.g., "I predict that an even number will come up 5 times and an odd number will come up 5 times when I roll a number cube 10 times.")
then perform the experiment, and compare the results with the predictions, using mathematical language; | |
<table>
<thead>
<tr>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
<td>PDM3-12, 14 to 16</td>
</tr>
</tbody>
</table>

<p>| <strong>3m85</strong> demonstrate, through investigation, an understanding of fairness in a game and relate this to the occurrence of equally likely outcomes. | |</p>
<table>
<thead>
<tr>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
<td>PDM3-13</td>
</tr>
</tbody>
</table>
**Grade 3 Essential Lessons for EQAO Test Preparation**

EQAO test questions cover the majority of Ontario math curriculum topics. However, if you find that your class has been progressing too slowly and you are unable to cover the complete curriculum before the EQAO test, make sure to cover the most crucial topics.

The list below includes lessons that are essential for preparing for the EQAO test. Teach as many of these lessons as possible prior to the test. After the test, cover the remaining curriculum in the normal order.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Unit</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Division</td>
<td>10</td>
<td>NS3-48 to 56</td>
</tr>
<tr>
<td>Patterns</td>
<td>11</td>
<td>PA3-13, 15</td>
</tr>
<tr>
<td>Fractions</td>
<td>12</td>
<td>NS3-63, 65, 66, 68, 70</td>
</tr>
<tr>
<td>Time</td>
<td>13</td>
<td>ME3-14 to 17, 21</td>
</tr>
<tr>
<td>Estimation</td>
<td>15</td>
<td>NS3-71, 72</td>
</tr>
<tr>
<td>Money</td>
<td>16</td>
<td>NS3-76 to 80, NS3-82 to 84</td>
</tr>
<tr>
<td>Probability</td>
<td>18</td>
<td>PDM3-12, 13, 15, 16</td>
</tr>
<tr>
<td>Transformations</td>
<td>17</td>
<td>G3-15 to 18</td>
</tr>
<tr>
<td>3-D Shapes</td>
<td>17</td>
<td>G3-19 to 21</td>
</tr>
<tr>
<td>Graphs</td>
<td>18</td>
<td>PDM3-4 to 8</td>
</tr>
</tbody>
</table>

If you have not yet started Unit 16 on Money by the middle of April, teach that unit next and then teach the part of Unit 18 on Probability. If you have not yet started the Unit 18 lessons on Graphs and Probability by the end of April, teach the lessons on Probability first.