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Write a sequence on the board: 36, 31, 26, 21. Ask your students if they remember what this kind of sequence is called (a decreasing sequence). Ask students to tell you what the difference between successive terms in the sequence is. Ask a volunteer to continue the sequence and to explain to you how they continued the sequence. They may say that they counted backwards or used their number facts to subtract. Ask students what they would do if the numbers in the sequence were large, so that counting backwards or subtracting would be more difficult. In this case they can use a number line to solve the problem. That’s what you will teach them today.

Present the following EXAMPLE: Karen is driving from Vancouver to Edmonton, which is 1,200 km away. She can drive 300 km a day. She starts on Tuesday. How far from Edmonton is she on Thursday night?

Draw a number line:

Draw the arrows showing her progress every day. You could **ASK:**

- When will she reach Edmonton? (Friday evening).
- Kamloops is 800 km away from Edmonton. Invite a volunteer to mark Kamloops on the number line. When would Karen pass Kamloops?
- Jasper National park is 790 km from Vancouver. When will she enter Jasper National park?

Ask a volunteer to mark the approximate position of Jasper on the number line.

Let your students draw number lines in their notebooks: from 0 to 50 skip counting by 5s, from 0 to 30 skip counting by 2s, from 0 to 45 skip counting by 3s, from 0 to 80 skip counting by 4s. Here are some problems they could practice with:

- A magical post-owl flies 15 km in an hour. How long will it take this owl to reach a wizard that lives 50 km away from the sender?
- Jonathan has $30. He spends $8 a week for snacks. How much money will he have after three weeks? When will his money be completely spent?
- A snail crawls 6 cm in an hour. It is 45 cm away from the end of the branch. How far from the end of the branch it will be after four hours of crawling?
A dragonfly flies 8 m in a second (You can mention that it is one of the fastest insects, because it hunts other insects). It spots a fly on a tree 80 m away. How far is the dragonfly from its prey after seven seconds of flight?

Point out to students that for problems with large numbers, they may want to use a scale with intervals of 100, 1 000, or other multiples of 10. Which scale would be best for the next problem: 10, 100, 1 000?

A horse runs 30 km per hour. An errand-rider has to go to a fort that is 200 km from the King’s palace. How far is the rider from the fort after three-hour ride?

**Assessment**

Draw a number line from 0 to 15 and solve the problem:

Jane can walk two blocks in a minute. She is 15 blocks away from home. How far from home will she be in six minutes? How long will it take her to reach home?

**Bonus**

Draw a number line from 330 000 to 380 000 skip counting by 5 000s. A rocket travels from the Moon toward the Earth at the speed of about 5 000 km an hour. The Moon is 380 000 km from the Earth. How far from the Earth will the rocket ship be after five hours of flight? After seven hours?

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**PA4-16**

**Number Lines (Advanced)**

**GOALS**

Students will solve simple problems involving decreasing sequences and building their own number lines.

**PRIOR KNOWLEDGE REQUIRED**

Decreasing sequences
Division by skip counting
Number lines

**VOCABULARY**

decreasing sequence  term
number line  scale
hour hand  half

If students do not know how to divide by skip counting you might wait until you have covered **NUMBER SENSE PART 2** before you do this lesson.

Explain that today you are going to make the task your students have to do harder—not only will they need to solve the problem using a number line, but they will also have to decide which number line to use for a particular problem. Explain that the number they skip count by to mark the number line is called the “scale.”

Start with an easy problem: ask your students to choose a scale for a number line (say 5) and have a volunteer draw a number line (skip counting by 5s as they subdivide the line). Now ask your students to suppose that the distance involved in a problem is 22 km. Will their scale be convenient? Which scale would be more convenient? (A scale of 2, because two divides into 22 evenly.)

Present the following problem: John and Jane are 20 km from home. John walks at a speed of 5 km an hour. Jane rollerblades at 10 km an hour. How long will it take each of them to get home? Draw a number line without any subdivisions and ask students which scale would be convenient. Five will work for both speeds but 10 will not be a convenient scale for the distance John moves in an hour.
Suppose Jane rollerblades at a speed of 10 km an hour, but she has 200 km to travel. Ask students if they would use a scale of 5 for a number line showing how far she travels in a given time. They should see that a number line with a scale of 10 would have fewer subdivisions and be easier to draw.

Ask students to draw a number line for Rachel who bikes at a speed of 15 km an hour and has 75 km to go. Give them the choice of numbers for skip counting: 10, 15, or 30. Invite volunteers to draw the number lines for 10, 15 and 30. Then ask more volunteers to draw arrows to show how far Rachel travels each hour. Which scale is the most convenient? (15.) Why was 30 inconvenient? (It is larger than the speed.) Why was 10 inconvenient? (It does not divide the speed, so the first arrow points between the marks.) Students should see that a convenient scale should divide into the speed evenly, but be large enough so that you avoid long number lines.

It is often convenient to use the speed of the person who is travelling as the scale. But sometimes this approach will not work. If a person travels at 10 km per hour but is travelling 25 km, a scale of 10 on the number line won’t work well. Students should see that they will have to use a smaller scale that divides into both 10 and 25 (Example: 5) for their scale.

Present another problem: The speed is 60 km in an hour; the distance is 210 km. Which scale could students use? Ask which numbers divide into 60. If a student gives a number less than 10, ask them to approximate the number of subdivisions their number line will have. Students should eventually see that the most convenient scale is 30, as it is the largest number that it divides evenly into 210 and 60.

**Summarize**

In choosing a scale, start by checking to see if either condition A or B below holds:

- **A:** the speed divides evenly into the distance from home
- **B:** the speed does not divide evenly into the distance but there is a number greater than one that divides into both the speed and the distance from home.

If case A holds, then the student can label each segment on the number line with the speed. For instance, in **question a)** in the chart below, each segment stands for 3 km, because 3 (the speed) divides 27 (the distance).

![Number line example](image)

After five hours the cyclist will be 12 km from home.

If condition B holds, then the student can let each segment represent the greatest number that divides both the speed and the distance from home evenly. For example, five is the greatest number that divides 10 and 35 evenly. So each segment in the number line for **question b)** below can represent five km.

**NOTE:** In **question c)** below each segment can represent 2 km, and in **question d)**, 25 km. You can use these two questions for assessment as well.

For each question below ask students to determine **how far from home** the cyclist will be after travelling **towards home** for the given time.
Extensions

1. Yoko’s house is 20 m from the sidewalk. A dog is tied to a tree halfway between the house and the sidewalk. The dog’s leash is 8 m long. How close to the sidewalk can the dog come?

2. Ravi’s house is 15 m from the ocean. He is sitting in a chair 5 m away from his house. The tide rises 5 m each hour. How long will it take before his feet get wet?

3. Ask your students to colour blue all the numbers divisible by 4 on a number line, and to colour all numbers divisible by 7 red. Which number will have two colours? Explain that this number is called “the least common multiple” of 4 and 7. Ask students to find the least common multiple of 3 and 4 (LCM = 12) and 4 and 6 (LCM = 12)

PA4-17
Extending and Predicting Positions

Remind your students how to recognize the core of a pattern. Draw a sequence of blocks with a simple pattern like the shown one below (where B stands for blue and Y for yellow).

```
B Y Y B Y Y B Y Y
```

Then draw a rectangle around a set of blocks in the sequence and follow the steps below.

```
B Y Y B Y Y B Y Y
```

**STEP 1:**
Ask students to say how many blocks you have enclosed in the rectangle (in this case, three).

**STEP 2:**
Ask students to check if the pattern (BYY) inside the rectangle recurs exactly in the next three blocks of the sequence, and in each subsequent block of three, until they have reached the end of the sequence (if you had enclosed four blocks in the rectangle, then students would check sets of four, and so on). If the pattern in the rectangle recurs exactly in each set of three boxes, as in the diagram above, then it is the core
of the sequence. Otherwise students should erase your rectangle, guess another core and repeat steps 1 and 2. (Students should start by looking for a shorter core than the one you selected.)

You could also have your students do this exercise with coloured blocks, separating the blocks into sets, rather than drawing rectangles around them.

Draw several repeating patterns on the board. Draw a box around a group of numbers or symbols that is not the core for some of the patterns. For other patterns draw a box around the core. Ask your students to grade your work. If they think you have made a mistake, ask them to correct the mistake.

**EXAMPLES:**


Do a quick assessment to make sure everyone is able to find the core and its length in the pattern:

Find the core and the length of the core:

R E D D R E D D | D A D D A D D A | G R E G G G G

Ask you students if they remember what special word mathematicians use for the figures or numbers in patterns or sequences. Write the word “term” on the board.

Draw a pattern on the board:

Ask volunteers to circle the core of the pattern, find its length and continue the pattern. Ask which terms are circles. What will the 20th term be—a circle or a diamond? Write down the sequence of the term numbers for the circles (1, 3, 5, 7, …) and diamonds (2, 4, 6, 8, …), and ask which sequence the number 20 belongs to.

For more advanced work students could try predicting which elements of a pattern will occur in particular positions. For instance, you might **ASK:** "If the pattern below were continued, what would the colour of the 23rd block be?"

B B Y Y Y B B Y Y Y

A method of solving this sort of problem using a hundreds chart is outlined on the worksheet. Students could also use number lines, rather than the hundreds charts to solve the problem as shown below.

**STEP 1:**

Ask a volunteer to find the length of the core of the pattern:

B B Y Y Y B B Y Y Y

The core is five blocks long.
STEP 2:

Let another volunteer mark off every fifth position on a number line and write the colour of the last block in the core above the marked position.

The core ends on the 20th block and starts again on the 21st block. You might even group the core with an arc. Ask at which term the new core starts (21). Let another volunteer write the letters of the new core in order on the number line starting at 21: the 23rd block is yellow.

Eventually you should encourage students to solve problems like the one above by skip counting. The students might reason as follows:

“I know the core ends at every fifth block so I will skip count by 5s until I get close to 23. The core ends at 20, which is close to 23 so I’ll stop there and write out the core above the numbers 21, 22 and 23.”

To give practice predicting terms you might wish to use the activity games at this point.

Assessment
What is the 20th term of the pattern: A N A A N A A N A A N A A?
What is the 30th term?

Bonus
Find terms 20, 30, 40, 50, ..., 100 of the pattern:

R W W R R W W R W W R R W W R W W...
The students will need pattern blocks or coloured beads and one or two dice for all of the following activities:

**ACTIVITY 1**

Player 1 rolls the die so that the partner does not see the result. Then he builds a sequence of blocks or beads with the core of the length given by the die. Player 2 has to guess the length of the core and to continue the sequence.

**ACTIVITY 2**

Player 1 throws two dice (unseen by Player 2), and uses the larger number as the core length and the smaller as the number of different blocks or beads to use. For example, if the dice show three and four, the core length is four but only three types of beads might be used. **EXAMPLE:** RGYRYGYY.

**ACTIVITY 3**

Player 1 throws a die (unseen by Player 2), and uses the number as the core length for his sequence. Player 2 throws the die and multiplies the result by 10. He has to predict the bead for the term he got. For instance, if Player 2 got three on the die, he has to predict the 30th term of the sequence.

**Extensions**

1. Ask students to use the word “term” in their answers to questions 11 and 12; **EXAMPLE:** for question 12 a) they might write: “The 19th term is a penny.” Here are some questions using the word “term”:
   a) What is the 16th term in this sequence?
   
   
   R Y Y R Y Y R Y Y
   
   b) What is the 50th term in this sequence?
   
   
   □ △ △ △ □ △ △ △

2. Sometimes in a repeating pattern the repetition does not begin straight away. For example, the leaves of many plants have the same shape, which means they make a repeating pattern. But the first two leaves that spring from the seed often have a different shape! The pattern starts repeating after the first two “atypical” terms. **OR:** A B C B C B C B C is a repeating pattern with core B C.

Ask your students to find the core in these patterns:

- △ △ △ △ △ △ △ △
- ◊◊◊◊◊◊◊◊◊
GOALS
Students will identify increasing and decreasing sequences.

PRIOR KNOWLEDGE REQUIRED
Increasing and decreasing sequences

VOCABULARY
increasing sequence
decreasing sequence

Write a sequence on the board: 3, 5, 7, 9, … Ask your students what this sequence is called. (The numbers grow, or increase, therefore it’s called an “increasing sequence”). Write another sequence: 95, 92, 89, 86, … and ask students what this sequence is called. Write “decreasing sequence” on the board.

Ask your students: Are all the sequences in the world increasing or decreasing? For example, Jennifer saved $80. She receives a $20 allowance every two weeks. She spends money occasionally:

<table>
<thead>
<tr>
<th>Date</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep 1</td>
<td>80</td>
</tr>
<tr>
<td>Sep 5</td>
<td>100</td>
</tr>
<tr>
<td>Sep 7</td>
<td>89</td>
</tr>
<tr>
<td>Sep 12</td>
<td>79</td>
</tr>
<tr>
<td>Sep 19</td>
<td>99</td>
</tr>
</tbody>
</table>

Jennifer’s money pattern increases between the first two terms, decreases for the next two terms, and then increases again.

Write several sequences on the board and ask volunteers to put a “+” sign in any circle where the sequence increases, and the “−” sign where it decreases. Let your students practice this skill, and after that ask them to mark the sequences: “I” for increasing, “D” for decreasing, and “B” (for “both”) if the sequence sometimes increases and sometimes decreases.

SAMPLE QUESTIONS

a) 7, 8, 7, 10
c) 10, 7, 4, 2
e) 15, 23, 29, 28
b) 2, 4, 7, 9
d) 2, 5, 1, 17
f) 22, 52, 59, 72
**Assessment**

a) 12, 17, 14, 19  
b) 1, 5, 8, 3  
c) 18, 13, 17, 23  
d) 28, 26, 19, 12  
e) 17, 8, 29, 25  
f) 53, 44, 36, 38

**Bonus**

a) 257, 258, 257, 260  
b) 442, 444, 447, 449  
c) 310, 307, 304, 298  
d) 982, 952, 912, 972  
e) 815, 823, 829, 827  
f) 632, 652, 649, 572

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**PA4-19**

**Describing and Creating Patterns (Advanced)**

Explain to your students that in this lesson their task will be more difficult than in the last lesson. This time they will have to find not only if the sequence is increasing or decreasing, but also the magnitude of the difference between successive terms. Invite volunteers to find differences by counting forwards and backwards, both using their fingers and number lines. Use simple examples such as:

a) 7, 9, 11, 13  
b) 2, 5, 9, 12  
c) 10, 7, 4, 1  
d) 32, 36, 40, 44  
e) 15, 23, 31, 39  
f) 72, 52, 32, 12

Invite volunteers to identify the difference as follows:

Put a “+” sign in the circle where the sequence increases and a “−” sign when it decreases. Then write the difference between the terms:

a) 7, 10, 11, 13  
b) 12, 8, 7, 12  
c) 20, 17, 14, 21  
d) 32, 46, 50, 64  
e) 15, 24, 33, 42  
f) 77, 52, 37, 12

Give several possible descriptions of sequences, such as:

A: Increases by different amounts  
B: Decreases by different amounts  
C: Increases by constant amount  
D: Decreases by constant amount
Ask your students to match the descriptions with the sequences on the board. When the terms of the sequence increase or decrease by the same amount, you might ask students to write a more precise description. For example, the description of sequence e) could be "Increases by 9 each time." or "Start at 15 and add 9 each time."

Next let your students do the activity below.

**Assessment**

Match description to the pattern. Each description may fit more than one pattern.

- 17, 16, 14, 12, 11 __ 3, 5, 7, 3, 5, 7, 3 __ A: Increases by the same amount
- 10, 14, 18, 22, 26 __ 4, 8, 11, 15, 18, ___ B: Decreases by the same amount
- 54, 47, 40, 33, 26 __ 4, 5, 3, 2, 6, 2, 4 ___ C: Increases by different amounts
- 4, 8, 12, 8, 6, 4, 6 __ 11, 19, 27, 35, 43 ___ D: Decreases by different amounts
- 74, 69, 64, 59, 54 __ 12, 15, 18, 23, 29 ___ E: Repeating pattern
- 98, 95, 92, 86, 83 __ 67, 71, 75, 79, 83 ___ F: Increases and decreases

For patterns that increase or decrease by the same amount, write an exact rule.

**Bonus**

Match description to the pattern.

- 97, 96, 94, 92, 91 ____ A: Increases by the same amount
- 210, 214, 218, 222 ____ B: Decreases by the same amount
- 654, 647, 640, 633 ____ C: Increases by different amounts
- 3, 5, 8, 9, 3, 5, 8, 9 ____ D: Decreases by different amounts
- 444, 448, 451, 456 ____ E: Repeating pattern
- 741, 751, 731, 721 ____ F: Increases and decreases by different amounts

For patterns that increase or decrease by the same amount, write an exact rule.

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**A Game for Pairs**

Your students will need the spinner shown. Player 1 spins the spinner so that Player 2 does not see the result. Player 1 has to write a sequence of the type shown by the spinner.

Player 2 has to write the rule for the sequence or describe the pattern. For example, if the spinner shows, "Decreases by the same amount," Player 1 can write "31, 29, 27, 25." Player 2 has to write "Start at 31 and subtract 2 each time."
Extension

Write the differences for the patterns. Identify the rule for the sequence of differences.
Extend first the sequence of differences, then the sequence itself.

\[
\begin{align*}
a) \quad & 7, 10, 14, 19 \\
b) \quad & 12, 15, 20, 27 \\
c) \quad & 57, 54, 50, 45 \\
d) \quad & 32, 30, 26, 20 \\
e) \quad & 15, 18, 22, 25, 29 \\
f) \quad & 77, 72, 69, 64, 61 \\
\end{align*}
\]

PA4-20

2-Dimensional Patterns

Draw a 2-dimensional grid on the board. Remind your students that rows go vertically and columns go horizontally. Show the diagonals as well.

Draw the chart:

```
     V
    E
   R
  I
 T
 C
 A
 L
```

```
     D
    H
   O
  R
 I
 Z
 A
 N
 T
 A
 L
```

Explain that in 2-dimensional patterns you generally count columns from left to right but there are two common ways of counting rows. In coordinate systems, which they will learn later, you count from the bottom to the top. But in this lesson, you will count rows from the top to the bottom. Ask a volunteer to number the columns and rows on your grid.

Ask your students to draw a 4 × 4 grid in their notebooks and to fill it in:

```
<table>
<thead>
<tr>
<th>13</th>
<th>17</th>
<th>21</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>
```

Ask them to shade the first column, the first row, the third row, and to circle both diagonals. Let them describe the patterns that they see in the shaded rows and columns and in diagonals.

After that allow your students to do the activities below for more practice.
Shade in the multiples of nine on a hundreds chart. Describe the position of these numbers.

a) Add the ones digit and the tens digit of each multiple of nine. What do you notice?
b) What pattern do you see in the ones digits of the multiples of nine?
c) What pattern do you see in the tens digits of the multiples of nine?

Ask students to circle the multiples of 3 and 4 on a hundreds chart (They should know the multiples of 5 automatically)

Then give students a set of base ten blocks (ones and tens only). Ask them to build base ten representations of the following numbers.

a) You need three blocks to build me. I am a multiple of five. (ANSWER: 30)
b) You need six blocks to build me. I am a multiple of five. (There are two ANSWERS: 15 and 60).
c) You need five blocks to build me. I am a multiple of four. (ANSWER: 32)

CHALLENGING: Find all the possible answers:
d) You need six blocks to build me. I am a multiple of three. (There are six ANSWERS: 6, 15, 24, 33, 42, and 51)

Extensions

1. Students will need the hundreds chart BLM. Write the following patterns on the board and ask students to identify in which rows and columns in the hundreds chart the patterns occur:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>38</td>
<td>43</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>48</td>
<td>54</td>
<td>13</td>
</tr>
<tr>
<td>23</td>
<td>58</td>
<td>63</td>
<td>23</td>
</tr>
<tr>
<td>33</td>
<td>68</td>
<td>74</td>
<td>34</td>
</tr>
</tbody>
</table>

After students have had some practice at this, ask them to fill in the missing numbers in the patterns (challenge them to do this without looking at the hundreds chart).

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>57</td>
<td>7</td>
<td>45</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>18</td>
<td>56</td>
</tr>
<tr>
<td>32</td>
<td>77</td>
<td></td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>38</td>
<td></td>
</tr>
</tbody>
</table>

Make up more of these puzzles for students who finish their work early.

2. Add the digits of some 2-digit numbers on the hundreds chart. Can you find all the numbers that have digits that add to ten? Describe any pattern you see.

3. Sudoku is an increasingly popular mathematical game that is now a regular feature in many newspapers. In the BLM section of this guide (“Sudoku – Warm Up” and “Mini Sudoku”), you will find Sudoku suitable for children (with step-by-step instructions). Once students master this easier form of Sudoku, they can try the BLM “Sudoku – The Real Thing.”
Tell your students that today they are going to solve puzzles. Write a selection of puzzles like the ones below on the board. Give your students time to solve them and ask volunteers to present their solutions on the board.

1. Ask students to colour a $4 \times 4$ grid using four colours so that all colours in each row, column and diagonal are different. There are many solutions: ask several volunteers to present different solutions.

2. Colour a $4 \times 4$ grid using two colours so that in each row, column and diagonal there are exactly two squares of each colour (or make an array with blocks). When someone presents a solution, take two rows and exchange them. Does this produce a new solution? Encourage your students to present more solutions and to check if they can be obtained from the first combination by exchanging rows or columns.

3. Colour the grid according to a pattern:

Which row will be the same as the first one? (The fourth)

If you look at the rows as terms of sequence, can you circle the core? (The first three squares)

Can you predict the $15^{th}$ row? (Same as $3^{rd}$ row)

What will the $20^{th}$ row look like? (Skip count by 3s. Stop before 20—this means the core ends at $18^{th}$ row. So the $19^{th}$ row is like the first one and the $20^{th}$ row is like the second one.)
Draw a hundreds chart on the board (or use an overhead projector). Ask volunteers to shade and describe the patterns in the following parts of the hundreds chart:

a) 3rd row
b) 4th column
c) The diagonal starting at 1
d) Start at any number of the 1st column and continue downwards diagonally
e) Start at any number of the 1st column and continue upwards diagonally
f) Start at any number of the 10th column and continue downwards diagonally
g) Start at 91 and go one square up and two squares right.
h) Start at 99 and go one square up and four squares left.

Show several patterns:

3, 6, 9, 12, 15, …
4, 8, 12, 16, 20, …
5, 10, 15, 20, 25, …

Ask your students what the rule for each pattern is. Ask if they see any similarities between the patterns. **HINT:** All sequences were obtained by skip counting, starting from the number you skip count by. Explain that these numbers are called “the multiples of” 3, 4 or 5 respectively. Ask your students to explain the name. **(HINT:** what are you doing to the numbers 1, 2, 3, 4, 5 to get your sequence?) Another name for those numbers is “the numbers divisible by” 3, 4 or 5. Write both names on the board. Ask your students to explain the new name. Then ask a volunteer to find all numbers divisible by 6 between 1 and 30.

**More Puzzles**

Draw several number pyramids like the ones below on the board and ask the students to explain what the rule is.

```
9
4 5
```

```
5
2 3
```

```
6
4 2
```

In these number pyramids, every number above the bottom row is the sum of the two numbers directly below it. Draw an example with a missing number. Ask your students, how they could find the missing number.

```
2 3
```

```
5
2 3
```

Ask your students, how they could find the missing number in the next example. This time the missing number can be found using subtraction:

```
7
3
```

```
7
3 4
```

The number pyramids on the worksheets for this section can be solved directly by addition and subtraction (students should start by finding a part of each pyramid ( ), where two squares are already filled).

Here is a bonus question you can assign your students that can be solved by trial and error.
Students should guess the number in the middle square of the bottom row, then fill in the squares in the second row, checking to see if they add up to the number in the top square. A good way to do so is by using a T-table:

<table>
<thead>
<tr>
<th>The Number in the Middle of the Bottom Row</th>
<th>The Numbers in the Middle Row</th>
<th>The Number on the Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5, 6</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>6, 7</td>
<td>13</td>
</tr>
</tbody>
</table>

Even with larger numbers the students would soon be able to see that the numbers in the last column grow by two each time. This will allow them to reach the right number quickly.

Students might also notice the following trick: adding the two numbers in the bottom row, subtracting their sum from the number in the top row, then dividing the result by two gives the missing number in the bottom row. To see why this trick works, notice that the missing number in the bottom row appears in the second row twice, as in the following example, where the missing number is represented by the letter A:

As the number in the top row is the sum of the numbers in the middle row, we have:

\[ 15 = 4 + A + 5 + A \quad \text{OR:} \quad 15 = 2A + 9 \]

Hence we have the rule stated above: to find the missing number A, subtract the sum of the two numbers in the bottom row \((4 + 5 = 9)\) from the number in the top row \((15)\):

\[ 15 - 9 = 2A \]

Then divide the result by 2:

\[ 6 \div 2 = A \quad \text{SO:} \quad A = 3 \]

This exercise can be used to introduce more advanced students to algebra.

**Extensions**

1. Draw the charts below on the board and ask the students to explain what the rule is. (The sum of the numbers in the top row and in the bottom row is the same. You may ask them to look at the sums in the columns and ask if they see a pattern.)
ADVANCED: Adding the numbers in each row and each column gives the same number.

\[
\begin{array}{ccc}
4 & 7 & 9 \\
8 & 9 & 3 \\
5 & 1 & 5 \\
\end{array}
\]

Find the missing numbers. (HINT: Start with the row or column where only one number is missing.)

\[
\begin{array}{ccc}
7 & 2 & 5 \\
& & 3 \\
4 & & 6 \\
\end{array} \quad \begin{array}{ccc}
7 & 6 & 9 \\
& & 4 \\
8 & & 9 \\
\end{array}
\]

2. Find the missing numbers in these pyramids:

a) \[
\begin{array}{ccc}
35 & & \\
& 24 & \\
8 & 16 & \\
\end{array}
\]

b) \[
\begin{array}{ccc}
29 & & \\
& 32 & \\
4 & 24 & \\
\end{array}
\]
PA4-22

Calendars

GOALS
Students will identify and describe patterns in calendars.

PRIOR KNOWLEDGE REQUIRED
Identify increasing and decreasing sequences
Describe increasing and decreasing sequences
Identify repeating patterns
Days of the week
Months of the year

VOCABULARY
multiple row
diagonal column
row calendar
column

Give your students blank BLM calendars to work with and draw a large calendar on the board. You can use the overhead projector.

Remind your students which months have 30 days, 31 days, 28 days. Mention the leap year.

Solve the following problems as a class, so that a volunteer presents the solution on the board after the students have tried to solve the problem independently.

Fill in the calendar for August, so that August 1st is Tuesday.

Steven gets a pet snake on his birthday, August 8th. The snake should be fed every eight days. Mark the days when Steven has to feed his pet with a little snake. How are the snake-squares situated—vertically, horizontally or diagonally?

Steven plays the drums every Wednesday. Mark the days he drums on the calendar. Ask your students to describe the pattern of the drum days. Ask them to write out the drumming dates as a sequence, then ask them to write the rule for the sequence (Start at _____ and _____). Are there any days when Steven has to feed his pet snake and play the drums?

Steven bought his mother an orchid on August 6th. The orchid has to be watered every 6 days. Shade the days when the orchid has to be watered. Write out the sequence of the watering dates (6, 12, 18, 24, 30). Describe the pattern of the days. Give the rule for the pattern of numbers. On which date does Steven play the drums and water the orchid?

Are there any days when the snake needs feeding and the flower needs watering? (Aug 24th)

Ask a volunteer to write out the snake feeding dates beneath the pattern of watering dates (8, 16, 24). Ask your students: How can you generate these numbers? (Skip counting by 6s for the watering dates, skip counting by 8s for feeding dates) Which other names did you use for numbers that are obtained by skip counting? (Multiples of 6 or 8, divisible by 6 or 8). So 24 is a multiple of both 6 and 8.

Explain that it might be easier to draw the calendar than to use skip counting if the dates are NOT multiples of something. If students try to extend the schedules into September, they will find that the feeding and watering dates no longer fall on multiples of 6 and 8—they can use the calendar to help them count on by these numbers.
Extensions

1. Fill in a blank calendar, using any month and any day for the first day of the month.
   Draw a square around any four numbers. Add the pair of numbers on the diagonal.
   Then add the pair of numbers on the other diagonal. What do you notice about the sum?
   (The two diagonal sums are the same.) Will this always happen? (Yes.) Why?

2. **PROJECT:** Calendars of the World. Explain to your students that different calendars are used in other parts of the world to mark time. Students can learn more about any of these calendars.
   Questions to consider: How many months are in the year in your calendar? Is there a leap year?
   How often does a leap year occur? What is a leap year (additional day or additional month)?
   How long is the year? How long are the months? What defines the months and the year (movement of the Sun, the Moon, the Nile)? What patterns can be found in different calendars?
   (The Chinese calendar is particularly interesting in this respect.)

**POSSIBLE SOURCE:**

http://webexhibits.org/calendars/calendar.html
Part 2
Patterns & Algebra

Let your students do the activity on the worksheet. You might wish to explain where the name “even” comes from. Give your students 20 small objects, like markers or beads or ten base unit blocks to act as counters. Ask them to take 12 blocks and to divide the blocks into two equal groups. Does 12 divide into two groups evenly? Ask them to repeat the exercise with 13, 14, 15 and any other number of blocks. When the number of blocks is even, they divide into two groups evenly. If they do not divide evenly, the number is called “odd”. Write the on the board: “Even numbers: 2, 4, 6, 8, 10, 12, 14… All numbers that end with          ”. Ask volunteers to finish the sentence. After that write: “Odd numbers: 1, 3, 5, 7, 9, 11, 13… All numbers that end with _____ ” and ask another volunteer to finish that sentence. After that you might do the activities.

Assessment
Circle the even numbers. Cross the odd numbers.

23, 34, 45, 56, 789, 236, 98, 107, 3 211, 468 021.

Extension
The BLM “Colouring Exercise”. (ANSWER: the Quebec flag.)
Let your students do the activity on the worksheet. Then let them play the game below (SEE: Activities).

After students have played the game, write several numbers with blanks for the missing digits on the board and ask your students to fill in each blank so that the resulting number is divisible by five. Ask students to list all possible solutions for each number.

2__   32__   8__   56__   5__5   __35   8__0

Assessment
1. Circle the multiples of 5.
   75   89   5   134   890   40   234   78   4   99   100   205   301   45675
2. Fill in the blanks so that the numbers become a multiple of 5. In two cases it is impossible to do so. Put an "X" on them.
   3__   3__5   3__9   __0   70__   5__0   __3

Bonus
Fill in the blanks so that the numbers become multiples of five, (list all possible solutions). In two cases it is impossible to do so. Put an "X" beside these.

344__   34__25   136__9
7867__0   456770___   2345__6780
134600__3   154__0000   678345__

Extension

“Who am I?”

a) I am a number between 31 and 39. I am a multiple of five.
b) I am an even multiple of five between 43 and 56.
c) I am an odd multiple of five between 76 and 89.
d) I am the largest 2-digit multiple of five.
e) I am the smallest 3-digit multiple of five.
f) I am the smallest 2-digit odd multiple of five.

A Game for Pairs
Decide which one of the players will be “pro-5” person. The other will be the “anti-5” person. The anti-5 player starts by adding either 3 or 5 to the number 4. The pro-5 player is then allowed to add either 3 or 5 to the result. Each player then takes one more turn. If either player produces a multiple of 5, the pro-5 player gets a point. Otherwise the anti-5 player gets a point. The players exchange roles and start the game again. Students should quickly see that there is always a winning strategy for the pro-5 player.
PA4-25
Patterns in the Eight Times Tables

GOALS
Students will identify multiples of eight.

PRIOR KNOWLEDGE REQUIRED
Skip counting by 8s
Identify and extend the simplest increasing and decreasing sequences

VOCABULARY
column row multiple

Write the first five multiples of eight vertically—students should see that the ones digit of the multiples decreases by 2 as you move down the columns, and the tens digit increases by 1.

08 + 1
16 + 1
24 + 1

8 - 2
16 - 2
24
32
40

Invite students to explore this pattern by writing the next five multiples of eight in a column. Extension 2 explains how to check if a three-digit number is a multiple of 8.

Assessment
Continue the pattern up to five columns.

Extensions
1. “Who am I?”
   a) I am a number between 31 and 39. I am a multiple of eight.
   b) I am the largest 2-digit multiple of eight.
   c) I am the smallest 3-digit multiple of eight.
   d) I am a two digit number larger than 45. I am a multiple of eight and a multiple of five as well.

2. Check if a three-digit number is a multiple of 8. EXAMPLE: 266.

   STEP 1:
   Circle the stem of the three-digit number (the hundreds and the tens digit):

   26 6

   STEP 2:
   Skip count by 4s, stop just before 26: 4, 8, 12, 16, 20, 24.
   Write out the pattern for multiples of 8:

   24 8
   25 6
   26 4
   27 2
   28 0

   266 is not on the list, so it is not a multiple of 8.
More **Examples:** 273 and 256. 273 is odd, it cannot be a multiple of 8. For 256, do Steps 1 and 2 as above. 256 is on the list, so it is a multiple of 8. Ask students to figure out why this method works.

**“Eight-Boom” Game**

The players count up from one, each saying one number in turn. When a player has to say a multiple of eight, he says “Boom!” instead: 1, 2, 3, 4, 5, 6, 7, “Boom!”, 9, … If a player makes a mistake, he leaves the circle.

**Advanced Version:** When a number has “8” as one of its digits, the player says “Bang!” instead. If the number is a multiple of 8 and has 8 as a digit, one says both. **Example:** “6, 7, “Boom, Bang!”’, 9,” or “15, “Boom!”’, 17, “Bang!”’, 19”.

Both games are a fun way to learn the 8 times table, and can be used for multiples of any other number you would like to reinforce.
Review Venn diagrams (see: PDm4-2, 3).

Draw a Venn diagram with the properties:

1. Multiples of five
2. Multiples of two

Ask your students to sort the numbers between 20 and 30 in the diagram.

Draw another Venn diagram, this time with multiples of five in one circle and multiples of eight in the other. Ask your students to sort out the numbers between 30 and 41, then between 76 and 87 into this diagram. Ask volunteers to sort some 3-digit numbers into the diagram (for instance, 125 and 140).

Draw a more complicated Venn diagram:

1. Multiples of two
2. Multiples of five
3. Multiples of eight

Ask volunteers to sort into it the numbers:

5, 8, 14, 15, 27, 28, 56, 40, 25, 30, 99

Ask why there are parts that are empty. (There are no numbers that are multiples of eight and not multiples of two.) Ask your students to add two numbers to each part of the diagram that is not empty.

Assessment
Sort the following numbers into the last diagram:

5, 7, 10, 15, 26, 30, 42, 40, 50

Bonus
Add three 3-digit numbers to each non-empty part of the last diagram.

Extension
“Who am I?”

a) I am a number between 31 and 49. I am a multiple of five and a multiple of two.
b) I am an even multiple of five between 63 and 76.
c) I am an odd multiple of five between 96 and 109.
d) I am the largest 2-digit multiple of five and eight.
e) I am the smallest 3-digit multiple of five and two.
f) I am the smallest 2-digit even multiple of five.
g) I am a multiple of five and two. I am not divisible by eight. I am larger than 33 and smaller than 57.
PA4-27
Patterns with Increasing and Decreasing Steps

Draw the following pattern on the board or build it with blocks:

Ask a volunteer to build the next term of the sequence. Ask your students to fill the T-table. Then ask: How many blocks are added each time? What are you adding to the structure? Another row. How many blocks are in the first row that you added? In the second row? This is the difference in the total number of blocks at each stage. Ask a volunteer to write the difference in the circles beside the table.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students if they can see a pattern in the differences. Ask a volunteer to add a term to the pattern of differences. After that ask another volunteer to fill in the next row of the table. Ask another volunteer to build another figure to check the result.

Give your students several questions to practice:

Find the differences between the terms of the sequences. Extend the sequence of the differences and then extend the sequence itself.

a) 5, 8, 12, 17, ___, ___

b) 3, 5, 9, 15, 23, ___, ___

c) 11, 15, 23, 39, ___, ___

d) 6, 8, 13, 21, 32, ___, ___

Let them also practice with decreasing sequences:

a) 65, 64, 62, 59, ___, ___

b) 73, 70, 64, 55, 43, ___, ___

Show the following geometrical pattern and ask how many triangles will be in the next design:

△ △ △
Draw the T-table for the pattern. How many triangles do you add each time? We add a new row that is always two triangles longer than the previous one. The number of triangles in the new row is the difference between the total number of triangles in two successive figures. Hence if you fill in the right hand column of the T-table, you will find that the difference between successive terms grows by two. Ask volunteers to extend first the sequence of differences, then the sequence itself.

**Assessment**

a) 15, 18, 22, 27, ____, ____

b) 13, 16, 22, 31, 43, ____, ____

c) 101, 95, 87, 77, ____, ____

d) 88, 85, 78, 67, 52, ____, ____

**Extension**

Janet is training for a Marathon run. On Monday she ran 5 km. Every day she ran 1 km more than on the previous day. How many kilometres did she run in the whole week?

Draw the T-table. The first few entries should appear as follows:

<table>
<thead>
<tr>
<th>Day</th>
<th>Total km Run from the Beginning of the Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Monday</td>
<td>5</td>
</tr>
<tr>
<td>2. Tuesday</td>
<td>11</td>
</tr>
<tr>
<td>3. Wednesday</td>
<td>18</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Each day Janet runs 1 km more.
PA4-28
Advanced Patterns

Draw the following tree on the board and ask your students to count the number of branches in each level. Write the number of branches beside each level. Ask your students, “What happens to each branch?” It splits in two. This means that the number of branches doubles—multiplies by two—each time. So this sequence was made by multiplication.

Ask your students to find the sequence of differences between the numbers of branches in each level (1, 2, 4, 8...). What do they notice? They should see that the differences between terms in the sequence are the same as the numbers in the sequence.

Ask your students, “Which sequences tend to grow faster—ones made by addition or ones made by multiplication?”

FOR EXAMPLE:

1, 11, 21, 31, … What is the rule for this sequence?

1, 2, 4, 8, … What is the rule for this sequence?
Start at ____ and multiply by ____.

You may call a vote: Which will be larger—the 10th term of the first sequence or 10th term of the second sequence? Invite a volunteer to make a tally chart of the vote results. Have volunteers extend both sequences to the 10th term.

Show another sequence: 3, 8, 6, 11, 9, 14, 12, 17, 15…

You might write the sequence in this form…

3  6  9  12  15
8  11  14  17

… and ask students to describe the patterns they see.

Ask students to continue the sequence and to explain the rule by which it was made. Here are some possible answers:

1. The top row is the sequence 3, 6, 9, 12… The rule is “Start at three and add three each time”. The bottom row is 8, 11, 14, 17, and the rule is “Start at eight and add three each time”.

2. The general rule for the pattern (looking at all terms) is “Start at three and add five or subtract two alternatively”.

ANOTHER EXAMPLE: 101, 97, 98, 94, 95, 91, 92, …
Assessment
Continue the sequences:

2, 4, 8, 14, 22, 32, _____, _____  
10, 12, 9, 11, 8, 10, 7, _____, _____  
2, 6, 18, 54, _____, _____  
98, 95, 90, 83, 74, _____, _____

A Game for Pairs
Player 1 writes a sequence, and Player 2 has to continue three next terms of it. Each correct term gives them one point as a team. If Player 2 has problems guessing the terms, Player 1 has to give him a hint (not a new term!), such as, “Look at the differences” or “I mixed two sequences.” Each hint takes one point from the team.

Extension
“The Legend of the Chess Board.” The same doubling sequence is used in the beginning of the lesson.

POSSIBLE SOURCE:
http://britton.disted.camosun.bc.ca/jbchessgrain.htm
Tell your students that they have done so well with patterns that today you are going to give them patterns with huge numbers.

Present several problems and call for volunteers to solve them, using T-tables.

A normal heartbeat rate is 72 times in a minute. How many times will your heart beat in five minutes?

There are 60 minutes in an hour. George sleeps for six hours. How many minutes does he sleep? He asks his friend to wake him after 500 minutes. About how many hours is this? (Skip count to find out.)

A sprinting ostrich’s stride is 700 cm long. A publicity-loving ostrich spots a photographer 3,000 cm away and runs towards him. How far from the camera will it be after three strides?

Find the number of rhombuses in each of the following stars:

**Figure 1**

**Figure 2**

**Figure 3**

**Figure 4**

**HINT:** Count the number of rhombuses between the pair of heavy black lines and then multiply.

**Assessment**

An extinct elephant bird weighed about 499 kg. Make a T-table to show how much five birds would weigh. Do you see a pattern in the numbers (look at ones, tens and hundreds separately)? Can you write the weights of six, seven, and eight birds without actually adding?

**Extension**

A regular year is 365 days long, a leap year (2000, 2004…) is 366 days long. Tom was born on Jan. 8, 2000. How many days old was he on January 8, 2003? January 8, 2005? When was he 2000 days old? (Finding the year, the month and the day are three different problems of increasing difficulty.)
Tell your students that today they will solve algebraic equations. Let them know that equations are like the scales people use to weigh objects (such as apples). But there’s a problem: A black box prevents you from seeing part of whatever object you are weighing. Solving an equation means figuring out what is inside the black box. Write the word “equation” on the board. Ask your students if they know any similar words (EXAMPLES: equal, equality, equivalence). So the word “equation” means “making the same,” or in other words, balancing the scales.

Draw a line down the middle of a table. Put 7 apples on one side of the line and put 3 apples and a bag or box containing 4 more apples on the other side. Tell your students that there is the same number of apples on both sides of the line. (If you have a scale and a set of objects of equal weight, you might put the objects on the pans of the scale.) Students should be able to deduce that there are 4 apples in the box. They should also see that they can find the number of hidden apples either by counting up from 3 to 7, or by subtracting 3 from 7. Challenge students with several more examples.

Tell your students that it is easy to represent the problem they just solved with a picture:

\[
\begin{array}{c}
+ \\
\end{array} \quad 3 = 7
\]

Invite a student to draw the missing apples in the box.

After students have had practice with this sort of problem, explain that it is inconvenient to draw the apples all the time. So people use numbers to represent the visible quantities. Let them practice writing an equation that represents a picture, as on the worksheet. For instance, the equation for the picture above is:

\[
+ 3 = 7
\]

A student could solve an equation like the one above as follows:

**STEP 1:**
Say the number of apples beside the box with your fist closed.

**STEP 2:**
Count until you say the number of apples on the other side of the equal sign. The number of fingers you raised is the number of apples in the box.
Students can create models for equations that involve addition using blocks. A square could stand for the unknown and a set of circles could be used to model the numbers in the equation. For instance the equation:

\[ \square + 2 = 7 \]

has the model:

\[ \square \square \square = \square \square \square \square \square \square \square \square \]

If the students think of the square as having a particular weight, then solving the equation becomes equivalent to finding the weight of the square in terms of the circles.

Ask students to make a model with squares and circles to solve the following problems. (They should draw a picture of their model with a balance scale as in the figures above.) Then they should explain how many circles they would remove to find the weight of the square.

\[ 7 + \square = 11 \quad 6 + \square = 13 \quad 4 + \square = 10 \quad 9 + \square = 12 \]

Write an equation and draw the picture:

\[ \square + \square \square = \square \square \square \square \square \]

Ask if the picture makes sense. Why not? (You are a mathematician, not a magician. You cannot turn apples into bananas.) When you draw pictures like the ones students have worked with so far, the objects on both sides of the equation have to be the same. However, when you use numbers in an equation—rather than pictures—the numbers don’t always stand for the same thing.

For instance, the equation \[ 10 = 3 + \square \] could represent the problem: “There are ten pets in a store. Three are cats. How many are dogs?” Here the number in the box and the number outside the box on the right side of the equation stand for exactly different things. However, in all equations, the amounts on both sides of the equal sign will be in the same category; for instance the animals in the question above are all pets.
Read several word problems. Invite volunteers to draw models, write, and solve the equations:

There are ten trees in the garden. Three of them are apple trees. All the rest are cherry trees. How many cherry trees are in the garden?

Jane has 12 books. Three of them are fairy tales. All the rest are ghost stories. How many of her books are ghost stories?

Assessment
1. Solve the equations:
   \[ 5 + \square = 11 \quad 8 + \square = 15 \quad 3 + \square = 13 \quad 7 + \square = 13 \]
2. Write an equation to solve the problem:
   There are 15 flowers in the flower-bed. Six are lilies. All the rest are peonies. How many peonies grow in the flower-bed?

Bonus
1. There were 150 bloodthirsty pirates on a galleon. They seized a schooner and sent 40 of the pirates to sail it. How many pirates remained on the galleon?
2. A multi-headed dragon had 15 heads. Some of them were cut off by a mighty and courageous knight. The dragon ran away from the knight with seven remaining heads. How many heads were hewn off by the knight?

Extensions
1. The same symbol in the equation means the same number. Solve the equations:
   a) \[ \square + \square = 12 \]
   b) \[ 5 + \diamond + \diamond = 13 \]
   c) \[ \bigcirc + \bigcirc + \bigcirc = 9 \]
   d) \[ 9 + \bigcirc + \bigcirc + \bigcirc = 15 \]
2. Explain that the square is often replaced by a letter, say “n”, to signify that this is “the unknown”. Ask students to make a model with squares and circles to solve the following problems. (They should draw a picture of their model with a balance scale as in the figures above.) Then they should explain how many circles they would remove to find the weight of the square.
   a) \[ n + 2 = 8 \]
   b) \[ n + 3 = 10 \]
   c) \[ n + 5 = 9 \]
3. Ask students to translate the following pictures into equations using the letter “n” as the unknown.

a) \[ \begin{array}{c}
\text{\# of apples} \\
\end{array} \quad \begin{array}{c}
\text{\# of apples}
\end{array} \]

b) \[ \begin{array}{c}
\text{\# of apples} \\
\end{array} \quad \begin{array}{c}
\text{\# of apples}
\end{array} \]

Remind your students that in the last lesson they drew models and wrote equations for word problems. The equations they drew all had a + sign. Today they will learn to write other kinds of equations.

Present a word problem: Sindi has a box of apples. She took two apples from the box and four were left. How many apples were in the box before she removed the apples?

Draw the box. There are some apples inside, but we do not know how many. Draw two apples (or circles, if it is easier) and cross them to show that they are taken away. Four apples were left in the box, so draw them too. How many were there from the beginning? (Six).

Explain that when we write an equation, we draw it differently. We draw the apples that we took out, outside the box with the “ – ” sign, to show that they were taken away:

\[ \begin{array}{c}
\text{\# of apples} \\
\end{array} \quad \begin{array}{c}
\text{\# of apples}
\end{array} = \begin{array}{c}
\text{\# of apples}
\end{array} \]

So to solve the equation we have to put all the apples into the box—the ones that we took out and the ones that are outside.

Remind your students that they also learned to write equations in number form. Can they guess what the equation will look like? Show the answer:

\[ n - 2 = 4 \]

Draw several models like the ones on the worksheets, and ask your students to write the equations for them. Ask volunteers to present the answers on the board.
Then ask your students to draw models for the equations and to solve them by drawing the original number of apples in the box:

\[ \boxed{- 6 = 9} \quad \boxed{- 7 = 12} \quad \boxed{- 5 = 3} \quad \boxed{- 3 = 10} \]

Tell your students that they can also write equations for multiplication problems. Remind them that “\(2 \times\)” means that some quantity is taken two times.

**FOR EXAMPLE:**

\[
2 \times \square \square \square = \square \square \square \square \square
\]

Present the problem below and ask your students to draw the appropriate number of circles in the box:

\[
2 \times \boxed{} = \boxed{\square \square \square \square \square \square \square \square \square \square}
\]

Present more problems and ask your students to write and solve numerical equations for these problems. Then show your students how to write an equation for a word problem involving multiplication:

Tony has four boxes of pears. Each box holds the same number of pears. He has 12 pears in total. How many pears are in each box?

Students might reason in the following way: we usually represent the thing that we do not know by a square (the “unknown”). So four times the “unknown” constitutes 12, and we have the equation:

\[
4 \times \boxed{} = 12.
\]

**PRACTICE:**

Three identical evil dragons have 15 heads together. How many heads does each dragon have?

Five heffalumps have 10 tails. How many tails does each heffalump have?

A cat has four paws. 16 cat paws are scratching in the basket. How many kittens are inside?

Here is a mixture of problems students could try involving addition, subtraction, or multiplication:

Katie has several dogs. They have 20 paws together. How many dogs does Katie have?

Jenny used three eggs to bake muffins. Seven eggs remained in the carton. How many eggs were in the carton?

Bob has nine pets. Three of them are snakes. All the rest are iguanas. How many iguanas does Bob have?
Assessment
1. Solve the equations:
   \[3 + \square = 8\]
   \[3 \times \square = 15\]
   \[\square - 4 = 11\]
   \[2 \times \square = 14\]
2. Draw models to solve the problems:
   Hamide has 12 stamps. Four of them are Canadian. How many foreign stamps does she have?
   Joe has 15 stamps. Five of them are French and the rest are German. How many German stamps does Joe have?
   Marylyn has 18 stamps. They come in three identical sheets. How many stamps are in each sheet?

Bonus
Solve the equations:
\[243 + \square = 248\]
\[8 \times \square = 56\]
\[\square - 4 = 461\]
\[60 \times \square = 240\]

Math Bingo
(SEE: the BLM “Mixed Equations”): Your students will need a board each and 16 tokens to mark the numbers. The teacher reads the card out loud. Players have to figure out the answer and then to place a token on the board, if they have the answer. The first player to fill a column, row or diagonal wins.

Extensions
1. Ask students to translate each story problem below into an equation using a letter to stand for the unknown, rather than a box. They should also model problem a) with squares and circles.
   a) Carl has seven stickers. He has two more stickers than John. How many stickers does John have?
      SOLUTION: Let \( n \) stand for the number of stickers that John has. Carl has two more stickers, so you would have to add two to the number of stickers John has. So the correct equation is: \( n + 2 = 7 \)
   b) Katie has ten stickers. She has three fewer stickers than Laura.
      SOLUTION: Let \( n \) be the number of stickers that Laura has. Katie has three fewer stickers than Laura so you need to subtract three from the number of stickers that Laura has. So the correct equation is: \( n - 3 = 10 \)
2. Two birds laid the same number of eggs. Seven eggs hatched, and three did not. How many eggs did each bird lay?
3. 60 little crocodiles hatched from three crocodile nests of the same size. We know that only half of the eggs hatched. How many eggs were in each nest? (HINT: How many eggs were laid in total?)
PA4-32

Algebra (Advanced)

Give your students a variety of equations (like the ones on the worksheet) and ask them to solve the problems using guess and check. Tell them that sometimes they might see a way to solve the problem without guessing—for example, with a model. In this case they should draw the model and to solve the problem that way.

\[
\begin{align*}
\text{a)} & \quad \square + 3 = 7 \\
\text{b)} & \quad \square + 6 = 8 \\
\text{c)} & \quad \square + 5 = 10 \\
\text{d)} & \quad \square + 4 = 9 \\
\text{e)} & \quad 9 - \square = 7 \\
\text{f)} & \quad \square - 2 = 10 \\
\text{g)} & \quad 17 - \square = 14 \\
\text{h)} & \quad 8 - \square = 5 \\
\text{i)} & \quad 2 \times \square = 14 \\
\text{j)} & \quad 5 \times \square = 20 \\
\text{k)} & \quad 3 \times \square = 24
\end{align*}
\]

Present the following problem: There are several hats and gloves in a box. All left-hand gloves have a matching right-hand glove, and there are nine objects in the box. How many gloves and how many hats are there?

Draw an equation on the board. Say that the circle will represent the number of left-hand gloves. Gloves come in pairs, so there will be two circles—one for the right hands and one for the left hands. The square will represent the number of hats.

\[
\bigcirc + \bigcirc + \square = 9
\]

Ask volunteers to try to fit some numbers into the equation. Suggest starting with the circle. Draw a T-table of the results:

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<th></th>
<th>Equation</th>
<th>Rewrite the Equation</th>
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<td>1</td>
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Use as many volunteers to help filling the table as possible. **ASK:** Why did we have to stop after the 4th row?

Ask your students to write down the sequences in the third and the fourth columns and to give the rules for the sequences.

**Extension**

Give your students a copy of a times table. Ask them to write an equation that would allow them to find the numbers in a particular column of the times table given the row number. For instance, to find any number in the 5s column of the times table you multiply the row number by five: Each number in the 5s column is given by the algebraic expression \(5 \times n\) where \(n\) is the row number. Ask students to write an algebraic expression for the numbers in a given row.
PA4-33
Problems and Puzzles

PA4-33 is a review worksheet, which can be used for practice.
PA4 Partie 2 : Liste — Fiches reproductibles

Calendriers ................................................................. 2
Exercice de coloriage ..................................................... 3
Tableaux de centaines ..................................................... 4
Jeu de bingo mathématique ............................................. 5
Mini-Sudoku ................................................................. 7
Sudoku — Des vrais ....................................................... 9
Sudoku — Exercices de pratique .................................... 10
## Calendriers

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Exercice de coloriage

Colorie les nombres pairs en bleu, et laisse les nombres impairs en blanc.

Qu’est-ce que tu vois? _____________________________________________
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Jeux de bingo mathématique

Exemples de cartes

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### Jeu de bingo mathématique (suite)

#### Cartes (additions seulement)

| 7 + □ = 11 | 6 + □ = 13 | 5 + □ = 10 | 9 + □ = 12 |
| □ + 9 = 11 | □ + 12 = 13 | □ + 4 = 10 | □ + 4 = 12 |
| 7 + □ = 16 | 6 + □ = 16 | 4 + □ = 15 | 5 + □ = 17 |
| □ + 7 = 21 | □ + 6 = 19 | □ + 3 = 18 | □ + 3 = 19 |
| 5 + □ = 22 | 4 + □ = 22 | 4 + □ = 23 | 2 + □ = 22 |

#### Cartes (équations mélangées)

| 7 + □ = 21 | 6 + □ = 13 | 5 + □ = 24 | 9 + □ = 20 |
| □ − 9 = 11 | □ − 1 = 11 | □ − 4 = 22 | □ − 4 = 16 |
| 7 × □ = 21 | 6 × □ = 30 | 4 × □ = 16 | 3 × □ = 18 |
| □ + 7 = 15 | □ + 6 = 21 | □ − 4 = 13 | □ − 5 = 11 |
| 9 × □ = 18 | 4 × □ = 36 | 8 × □ = 80 | 21 + □ = 22 |
Mini-Sudokus

Dans ces mini-Sudokus, nous utilisons les nombres 1, 2, 3 et 4.
Chaque nombre doit paraître dans chaque rangée, chaque colonne et chaque boîte 2 × 2.
Quand tu résous les problèmes Sudoku :
1. Commence par une rangée ou un carré qui a plus d’un nombre déjà rempli.
2. Regarde ensuite toute la rangée, toute la colonne et dans les boîtes 2 × 2 pour résoudre.
3. Ne place un nombre dans une case que quand tu es certain(e) que c’est le bon nombre (utilise un crayon avec une gomme à effacer au cas où tu fais une erreur).

EXEMPLE :

Voici comment tu peux trouver les nombres dans la deuxième colonne :

Le 2 et le 4 sont déjà donnés, alors nous devons décider où placer le 1 et le 3.
Il y a déjà un 3 dans la troisième rangée, alors nous devons placer le 3 dans la première rangée de
la deuxième colonne, et le 1 dans la troisième rangée.
Continue de cette façon, en plaçant les nombres 1, 2, 3 et 4 dans les bonnes cases. Avant d’essayer les casse-têtes ci-dessous, fais d’abord les exercices de pratique à la page de travail suivante.

1. a) b) c)
Mini-Sudokus (suite)

Essaie ces problèmes en utilisant les nombres de 1 à 6. Les mêmes règles et stratégies s’appliquent!

Bonus

3.  

```
  1  4  
  6  4  5  1  
  2  3  6  
  4  1  6  2  3  
  5  1  2  
  2  5  3  4  6  
```

4.  

```
  5  4  2  1  
  1  5  6  
  3  6  4  5  
  6  4  
  2  3  1  
  2  3  
```

5.  

```
  4  3  6  4  5  2  
  6  
  5  1  
  1  5  2  3  
  2  
```

6.  

```
  2  3  2  3  4  1  2  
  6  5  1  
  4  5  1  3  6  
  3  5  1  
```
Sudokus — Des vrais

Essaie de faire ces Sudokus dans le format original de 9 × 9.

Tu dois placer les nombres de 1 à 9 dans chaque rangée, dans chaque colonne et dans chaque boîte. Bonne chance!

**Bonus**

```
<table>
<thead>
<tr>
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<th>2</th>
<th>1</th>
<th>8</th>
<th>3</th>
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<td>2</td>
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</table>
```

**Super Bonus**

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<th>7</th>
<th>8</th>
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<tr>
<td>9</td>
<td></td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
```

Si tu veux faire plus de Sudokus, vérifie la section des casse-têtes dans ton journal local!
Sudokus — Exercices de pratique

1. Chaque rangée, colonne ou boîte doit contenir les nombres 1, 2, 3 et 4. Trouve le nombre qui manque dans chaque ensemble.

   a)  
   
   b)  
   
   c)  
   
   d)  
   
   e)  
   
   f)  

2. Encercle les paires d’ensembles dans lesquelles le même nombre manque.

   a)  
   
   b)  
   
   c)  

3. Trouve le nombre qui devrait être dans chaque case coloriée ci-dessous.  
   N’OUBLIE PAS : Dans les Sudokus, un nombre ne peut paraître qu’une seule fois dans chaque rangée, chaque colonne et chaque boîte.

   a)  
   
   b)  
   
   c)  
   
   d)  
   
   e)  
   
   f)  

4. Place un nombre dans la case coloriée. N’oublie pas que chaque rangée, chaque colonne et chaque boîte doit avoir les nombres 1, 2, 3 et 4.

   a)  
   
   b)  
   
   c)  
   
   d)  
   
   e)  
   
   f)  

Bonuses
Peux-tu trouver les nombres à placer dans les autres carrés vides (à part les carrés coloriés)?

5. Essaie de résoudre les casse-têtes suivants en utilisant les compétences que tu as acquises.


Maintenant tu peux retourner résoudre les mini-Sudokus!
Draw 12 glasses divided equally on 4 trays, as illustrated in the worksheet. Ask your students to identify the objects to be shared or divided into sets, the number of sets and the number of objects in each set. Repeat with 15 apples divided into 3 baskets and 12 plates placed on 3 tables. Have students complete the following examples in their notebook.

a) Draw 9 people divided into 3 teams (Team A, Team B, Team C)
b) Draw 15 flowers divided into 5 flowerpots.
c) Draw 10 fish divided into 2 fishbowls.

The objects to be divided are put into sets. What are the sets? [HINT: the noun immediately preceding “each” is the object, the noun immediately following “each” is the set.]

a) 5 boxes, 4 pencils in each box (pencils are being divided, boxes are sets)
b) 3 classrooms, 20 students in each classroom (students are being divided, classrooms are sets)
c) 4 teams, 5 people on each team (people are being divided, teams are the sets)
d) 5 trees, 30 apples on each tree (apples are being divided, trees are the sets)
e) 3 friends, 6 stickers for each friend (stickers are being divided, friends are the sets)

Then have students use circles to represent sets and dots to represent objects to be divided into sets.

a) 5 sets, 2 dots in each set
b) 7 groups, 2 dots in each group
c) 3 sets, 4 dots in each set
d) 4 groups, 6 dots in each group
e) 5 children, 2 toys for each child
f) 6 friends, 3 pencils for each friend
g) 3 fishbowls, 4 fish in each bowl, 12 fish altogether
h) 20 oranges, 5 boxes, 4 oranges in each box
i) 4 boxes, 12 pens, 3 pens in each box
j) 10 dollars for each hour of work, 4 hours of work, 40 dollars

**Bonus**

k) 12 objects altogether, 4 sets
l) 8 objects altogether, 2 objects in each set
m) 5 fish in each fishbowl, 3 fishbowls
n) 6 legs on each spider, 4 spiders
o) 3 sides on each triangle, 6 triangles
p) 3 sides on each triangle, 6 sides
q) 6 boxes, 2 oranges in each box
r) 6 oranges, 2 oranges in each box
s) 6 fish in each bowl, 2 fishbowls
t) 6 fish, 2 fishbowls
u) 8 boxes, 4 pencils in each box
v) 8 pencils, 4 pencils in each box

NS4-53
Sharing—Knowing the Number of Sets

Divide 12 volunteers into 4 teams, numbered 1-4, by assigning each of them a number in the following order: 1, 1, 2, 1, 1, 3, 1, 1, 4, 1, 1, 1. Separate the teams by their numbers, and then ask your students what they thought of the way you divided the teams. Was it fair? How can you reassign each of the volunteers a number and ensure that an equal amount of volunteers are on each team? An organized way of doing this is to assign the numbers in order: 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4. Demonstrate this method and then separate the teams by their numbers again. Does each team have the same amount of volunteers? Are the teams fair now?

Present your students with 4 see-through containers and ask them to pretend that the containers represent the 4 teams. Label the containers 1, 2, 3 and 4. To evenly divide 12 players (represented by a counter of some sort) into 4 teams, one counter is placed into one container until all 12 counters are placed into the 4 containers. This is like assigning each player a number. Should we randomly distribute the 12 counters and hope that each container is assigned an equal amount? Is there a better method? How is this similar to assigning each volunteer a number?

Now, suppose you want to share 12 cookies between yourself and 3 friends. How many people are sharing the cookies? [4.] How many containers are needed? [4.] How many counters are needed? [12.] What do the counters represent? [Cookies.] What do the containers represent? [People.] Instruct your students to draw circles for the containers and dots inside the circles for the counters. How many circles will you need to draw? [4.] How many dots will you need to draw inside the circles? [12.]

Draw 4 circles.

Counting the dots out loud as you place them in the circles, have your students yell “Stop” when you reach 12. Ask them how many dots are in each circle? If 4 people share 12 cookies, how many cookies does each person...
get? If 12 people are divided among 4 teams, how many people are on each team? Now what do the circles represent? Now what do the dots represent? If 12 people ride in 4 cars, how many people are in each car? What do the circles and dots represent? Have students suggest additional representations for circles and dots.

Assign your students practise exercises, drawing circles and the correct amount of dots. If it helps, allow them to first use counters and count them ahead of time so they know automatically when to stop.

a) 12 cookies, 3 people 

b) 15 cookies, 5 people 

c) 10 cookies, 2 people 

When students have mastered this, write the following word problem.

5 friends shared 20 strawberries. How many strawberries does each friend get?

Have a volunteer read the word problem out loud and then ask how many dots are needed. What is to be divided into groups? [Strawberries.] How many circles are needed? What will the circles represent? [Friends.] What will the amount of dots in each circle illustrate? [The number of strawberries that each friend will receive.] Have another volunteer solve the problem for the rest of the class. Then assign your students several word problems. Read all the word problems out loud, and remind students that they can use a dictionary if they don’t understand a word.

EXAMPLES:

a) 3 friends picked 15 cherries. How many cherries did each friend pick? 

b) Joanne shared 15 marbles among 4 friends and herself. How many marbles did each person receive? 

c) There are 18 plums on 6 trees. How many plums are on each tree? 

d) There are 16 apples on 2 trees. How many apples are on each tree? 

e) 20 children sit in 4 rows. How many children sit in each row? 

f) Lauren’s weekly allowance is $21. What is Lauren’s daily allowance?

Bonus

Allow students to use base ten materials for the following questions, if they wish.

a) 3 friends picked 69 cherries. How many cherries did each friend pick? 

b) Joanne shared 84 marbles among 3 friends and herself. How many marbles did each person receive? 

c) There are 63 plums on 3 trees. How many plums are on each tree? 

d) There are 68 apples on 2 trees. How many apples are on each tree?
NS4-54
Sharing—Knowing the Number in Each Set

Distribute 12 counters to each student and have them evenly divide the counters into 3 piles, then 2 piles, then 6 piles, and then 4 piles. Then have them divide the 12 counters first into piles of 2, then into piles of 3, then 6, then 4. Tell your students that, last class, the question always told them how many piles (or sets) to make. This class, the question will tell them how many to put in each pile (or in each set) and they will need to determine the number of sets.

Tell your students that Saud has 30 apples. Count out 30 counters and set them aside. Saud wants to share his apples so that each friend gets 5 apples. He wants to know how many people can get apples. What can be used to represent Saud’s friends? [Containers.] Do you know how many containers we need? [No, because we are not told how many friends Saud has.] How many counters are to be placed in each container? [5.]

Put 5 counters in one container. Are more containers needed for the counters? [Yes.] How many counters are to be placed in a second container? (5) How do they know? Will another container be needed? Continue until all the counters are evenly distributed. The 30 counters have been distributed and each friend received 5 apples. How many friends is Saud sharing with? How do they know? [6, because 6 containers were needed to evenly distribute the counters.]

Now draw dots and circles like in the last lesson. What will the circles represent? [Friends.] How many friends does Saud have? How many dots are to be placed in each circle? [5.] Place 5 dots in each circle and keep track of the amount used.

6 circles had to be drawn to evenly distribute 30 apples, meaning 6 friends can share the 30 apples. What is the difference between this problem and the problems in the previous lesson? [The previous lesson identified the number of sets, but not the number of objects in each set.]

Saud has 15 apples and wants to give each friend 3 apples. How many friends can he give apples to? How can we model this problem? What will the dots represent? What will the circles represent? Does the problem tell us how many dots or circles to draw? Or how many dots to draw in each circle? Draw 15 dots on the board and demonstrate distributing 3 dots in each circle:

There are 5 sets, so he can give apples to 5 people.
Have your students distribute 3 dots into each set, and then ask them to count the number of sets.

a) 

b) 

c) 

Repeat with 2 dots into each set:

a) 

b) 

Have students draw the dots to determine the correct amount of sets.

a) 15 dots, 5 dots in each set  
   b) 12 dots, 4 dots in each set  
   c) 16 dots, 2 dots in each set

Bonus

24 dots and

a) 3 dots in each set  
   b) 2 dots in each set  
   c) 4 dots in each set  
   d) 6 dots in each set  
   e) 8 dots in each set  
   f) 12 dots in each set?

Then explain that rows, instead of circles, can be used to represent friends.

Have your students explain what the number of dots, number of rows, and number of dots in each row tells them.

Then ask them to draw circles to divide these arrays into groups of 3.

Bonus
Samuel has 15 cookies. There are two ways that he can share or divide his cookies equally.

1. He can decide how many sets (or groups) of cookies he wants. If he wants to share his cookies with two friends, he will have to divide the cookies into 3 sets. He will draw 3 circles and place one cookie into each circle until all 15 cookies are placed in the circles.

2. He can decide that he wants each person to receive 5 cookies. He counts out sets of 5 cookies until he has counted all 15 cookies.

Then assign word problems with full sentences and have your students underline the important words. Demonstrate the first problem.

a) Sally has 10 apples. She puts 5 apples in each box. How many boxes?

b) Sally has 12 apples. She gives 4 apples to each of her siblings. How many siblings does she have?

c) Sally has 10 apples. She puts 2 apples on each plate. How many plates has she used?

d) Sally has 8 stamps. She puts 2 stamps on each envelope. How many envelopes does she have?

**Bonus**

e) Sally has 18 apples. She shares them among herself and some friends. Each person gets 6 apples. How many friends has she shared with?

---

**GOALS**

Students will recognize which information is provided and which is absent among: how many sets, how many in each set and how many altogether.

**PRIOR KNOWLEDGE REQUIRED**

Solving problems where either the number of sets or amount of objects in each set is given.

Samuel has 15 cookies. There are two ways that he can share or divide his cookies equally.

1. He can decide how many sets (or groups) of cookies he wants. If he wants to share his cookies with two friends, he will have to divide the cookies into 3 sets. He will draw 3 circles and place one cookie into each circle until all 15 cookies are placed in the circles.

2. He can decide that he wants each person to receive 5 cookies. He counts out sets of 5 cookies until he has counted all 15 cookies.

Draw 3 circles. How can 18 dots be equally placed into the 3 circles? Then equally place these triangles into 2 circles.
What is the difference between this problem and the previous problem? Can this problem be solved like the previous problem? (Count the triangles to determine how many need to be placed into the circles.)

Have a volunteer count the triangles. Then draw one triangle in each circle and repeat until all 20 triangles have been drawn. Another, more tedious, method is to draw one triangle in each circle and then cross out two of the 20 triangles, repeating this until all 20 triangles have been crossed out. This method allows students to not have to count the 20 triangles first.

Assign your students several similar problems.

   a) 2 rows of 6 squares into 3 circles
   b) 3 rows of 8 hearts into 6 circles
   c) 1 row of 18 dots into 2 circles
   d) 1 row of 12 vertical lines into 6 circles

**NOTE:** Some students may find it easier to draw dots instead of triangles, squares or hearts.

Then ask your students if they remember how to group the dots so that there are 4 dots in each set, and to explain how this is different from the previous problem.

   a) ●●●●●●●
   b) ●●●●●●●●●

Have students draw 12 dots and group them so that there are...

   a) 4 dots in each set
   b) 2 dots in each set
   c) 6 dots in each set
   d) 3 dots in each set

Sometimes the problem provides the number of sets, and sometimes the problem provides the number of objects in each set. Have your students tell you if they are provided with the number of sets or the number of objects in each set for the following problems.

   a) There are 15 children. There are 5 children in each canoe.
   b) There are 15 children in 5 canoes.
   c) Aza has 40 stickers. She gives 8 stickers to each of her friends.

Have your students draw and complete the following table for **PROBLEMS a)–i).** Insert a box for information that is not provided in the problem.

<table>
<thead>
<tr>
<th></th>
<th>What has Been Shared or Divided into Sets?</th>
<th>How Many Sets?</th>
<th>How Many in Each Set?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EXAMPLE</strong></td>
<td>18 strings</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>a)</td>
<td>24 strings</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE:** There are 6 strings on each guitar. There are 18 strings.

   a) There are 24 strings on 4 guitars.
   b) There are 3 hands on each clock. There are 15 hands altogether.
   c) There are 18 holes in 6 sheets of paper.
d) There are 15 rings on 5 binders.
e) 15 people sit on 5 couches.
f) There are 15 people. 5 people fit on each couch.
g) There are 4 cans of tennis balls. There are 3 tennis balls in each can.
h) There are 20 chairs in 4 rows.
i) There are 3 rows of chairs. There are 9 chairs altogether.

Ask your students to draw circles for the sets and dots for the objects being divided into sets.

a) 10 dots, 2 sets. How many in each set?
b) 10 dots, 2 in each set. How many sets are there?
c) 12 dots, 2 sets. How many in each set?
d) 12 dots, 3 in each set. How many sets are there?
e) There are 12 children in 2 boats. How many children are in each boat?
f) There are 15 children. 3 children can fit into each car. How many cars are needed?
g) Luybava has 12 pencils. She puts 4 pencils in each box. How many boxes does she have?
h) There are 12 strings on 2 guitars. How many strings are on each guitar?
i) There are 24 strings. There are 6 strings on each guitar. How many guitars are there?
j) Tina has 30 stickers. She wants to divide them among herself and 4 friends. How many stickers will each person receive?
k) Tina has 30 stickers. She wants each person to receive 3 stickers. How many people will receive stickers?
l) There are 24 toothpicks. Sally wants to make triangles with the toothpicks. How many triangles can Sally make?
m) There are 24 toothpicks. Sally wants to make squares with the toothpicks. How many squares can Sally make?
NS4-56
Division and Addition

GOALS
Students will learn the division symbol through repeated addition.

PRIOR KNOWLEDGE REQUIRED
Skip counting
Sharing or dividing into sets or groups

VOCABULARY
division (and its symbol ÷)
divided by
dividend
divisor

Distribute 15 counters to each of your students and then ask them to divide the counters into sets of 3. How many sets do they have? Explain that 15 objects divided into sets of 3 equals 5 sets. This is written as \(15 \div 3 = 5\), and read as “fifteen divided by 3 equals 5”.

The following website provides good worksheets for those having trouble with this basic definition of division:


With symbols similar to those below, have your students solve \(12 \div 2\), \(12 \div 3\), \(12 \div 4\) and \(12 \div 6\).

Have your students illustrate each of the following division statements.

\[
\begin{align*}
6 \div 3 &= 2 & 12 \div 3 &= 4 & 9 \div 3 &= 3 & 8 \div 2 &= 4 & 8 \div 4 &= 2 \\
\end{align*}
\]

Then have your students write the division statement for each of the following illustrations.

WRITE: \(15 \div 3 = 5\) (15 divided into sets 3 equals 5 sets).

\[
\begin{align*}
3 + 3 + 3 + 3 + 3 &= 15 \\
\end{align*}
\]
Explain that every division statement implies an addition statement. Ask your students to write the addition statements implied by each of the following division statements. Allow them to illustrate the statement first, if it helps.

\[
\begin{align*}
15 \div 5 &= 3 \\
10 \div 2 &= 5 \\
12 \div 2 &= 6 \\
12 \div 6 &= 2 \\
10 \div 5 &= 2 \\
6 \div 3 &= 2 \\
6 \div 2 &= 3 \\
\end{align*}
\]

WRITE: \(15 \div 3 = 5\) \(\text{this number \ many times}\)

Ask your students to write the following division statements as addition statements, without illustrating the statement this time.

\[
\begin{align*}
12 \div 4 &= 3 \\
12 \div 3 &= 4 \\
18 \div 6 &= 3 \\
18 \div 3 &= 6 \\
10 \div 2 &= 5 \\
6 \div 3 &= 2 \\
6 \div 2 &= 3 \\
\end{align*}
\]

**Bonus**

\[
\begin{align*}
132 \div 43 &= 3 \\
1700 \div 425 &= 4 \\
90 \div 30 &= 3 \\
1325 \div 265 &= 5 \\
\end{align*}
\]

Then have your students illustrate and write a division statement for each of the following addition statements.

\[
\begin{align*}
4 + 4 + 4 &= 12 \\
2 + 2 + 2 + 2 &= 10 \\
3 + 3 + 3 &= 9 \\
3 + 3 + 3 + 3 + 3 + 3 + 3 &= 21 \\
6 + 6 + 6 + 6 &= 24 \\
5 + 5 + 5 + 5 + 5 + 5 &= 30 \\
\end{align*}
\]

Then have your students write division statements for each of the following addition statements, without illustrating the statement.

\[
\begin{align*}
17 + 17 + 17 + 17 + 17 &= 85 \\
21 + 21 + 21 &= 63 \\
101 + 101 + 101 + 101 &= 404 \\
2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 &= 36 \\
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 &= 21 \\
\end{align*}
\]
Dividing by Skip Counting

Draw:

What division statement does this illustrate? What addition statement does this illustrate? Is the addition statement similar to skip counting? Which number could be used to skip count the statement?

Explain that the division statement 18 ÷ 3 = ? can be solved by skip counting on a number line.

How many skips of 3 does it take to reach 18? (6.) So what does 18 ÷ 3 equal? How does the number line illustrate this? (Count the number of arrows.)

Explain that the division statement expresses a solution to 18 ÷ 3 by skip counting by 3 to 18 and then counting the arrows.

Ask volunteers to find, using the number line: 12 ÷ 2; 12 ÷ 3; 12 ÷ 4; 12 ÷ 6;

SAY: If I want to find 10 ÷ 2, how could I use this number line?

What do I skip count by? How do I know when to stop? Have a volunteer demonstrate the solution.

Ask your students to solve the following division statements with a number line to 18.

8 ÷ 2, 12 ÷ 3, 15 ÷ 3, 15 ÷ 5, 14 ÷ 2, 16 ÷ 4, 16 ÷ 2, 18 ÷ 3, 18 ÷ 2

Then have your students solve the following division problems with number lines to 20 (see the BLM “Number Lines to Twenty”). Have them use the top and bottom of each number line so that each one number line can be used to solve two problems. For example, the solutions for 8 ÷ 2 and 8 ÷ 4 might look like this:
Then provide numerous solutions of division problems and have your students express the division statement and the addition statement. For example, students should answer with “20 ÷ 4 = 5” and “4 + 4 + 4 + 4 + 4 = 20” for the following number line.

Draw number lines for 16 ÷ 8, 10 ÷ 2, 12 ÷ 4, and 12 ÷ 3.

Explain that skip counting can be performed with fingers, in addition to on a number line. If your students need practice skip counting without a number line, see the MENTAL MATH section of this teacher’s guide. They might also enjoy the following interactive website:

http://members.learningplanet.com/act/count/free.asp

Begin with skip counting by 2. Have your students keep track of the count on their fingers.

To solve 15 ÷ 3, skip count by 3 to 15. The number of fingers it requires to count to 15 is the answer. [5.]

Perform several examples of this together as a class, ensuring that students know when to stop counting (and in turn, what the answer is) for any given division problem. Have your students complete the following problems by skip counting with their fingers.

Two hands will be needed to keep track of the count for the following questions.

**Bonus**

Then have your students express the division statement for each of the following word problems and determine the answers by skip counting. Ask questions like: What is to be divided into sets? How many sets are there and what are they?
a) 5 friends share 30 tickets to a sports game. How many tickets does each friend receive?
b) 20 friends sit in 2 rows at the movie theatre. How many friends sit in each row?
c) $50 is divided among 10 friends. How much money does each friend receive?

Have your students illustrate each of the following division statements and skip count to determine the answers.

\[
3 \div 3 \quad 5 \div 5 \quad 8 \div 8 \quad 11 \div 11
\]

Without illustrating or skip counting, have your students predict the answers for the following division statements.

\[
23 \div 23 \quad 180 \div 180 \quad 244 \div 244 \quad 1,896 \div 1,896
\]

Then have your students illustrate each of the following division statements and skip count to determine the answers.

\[
1 \div 1 \quad 2 \div 1 \quad 5 \div 1 \quad 12 \div 1
\]

Then without illustrating, have your students predict the answers for the following division statements.

\[
18 \div 1 \quad 27 \div 1 \quad 803 \div 1 \quad 6,692 \div 1
\]

---

**NS4-58**

**The Two Meanings of Division**

Draw 3 circles and ask your students to equally divide 12 dots into 3 sets by skip counting by 3. Instead of placing 3 dots in each circle, have them place one dot in each circle (3 dots altogether) and repeat this until all 12 dots have been placed in the circles. Skip counting will keep track of the number of placed dots, and the number of fingers it requires to count to 12 will equal the number of dots placed in each set.

\[
\begin{align*}
3 & \quad \ddots \\
& \text{3 dots altogether}.\\
6 & \quad \ddots \\
& \text{9 dots altogether}.\\
9 & \quad \ddots \\
& \text{9 dots altogether}.
\end{align*}
\]
I’ve placed 4 dots in each set (12 altogether).

Tell your students that so far when we skip counted by 3 to get 12, we built 4 sets of 3 things.

\[ 12 \div 3 = 4 \]

Now we have used skip counting to build 3 sets of 4 things. What did we do differently? Allow several students to explain it in their own words and then summarize: Instead of skip counting 3 objects for each circle, we have skip counted by 3 but placed 1 object in each of the 3 circles.

3 sets of 4 objects can always be expressed as 4 sets of 3 objects. Challenge your students to illustrate these 4 sets of 2 objects as 2 sets of 4 objects.

Distribute 4 pennies and 4 nickels to each student. Have them divide the money:

a) into a pile of pennies and a pile of nickels
b) into piles totalling 6¢

How many coins are in each pile? How many piles are there?

Explain that 18 ÷ 3 can be solved by assuming there are 3 objects in each set or by assuming there are 3 sets.

To divide the objects into 3 equal sets, count the objects (there are 18), and then determine how many sets there will be if there are 3 objects in each set. [There will be 6 sets of 3 objects.]

So how many objects will there be in 3 equal sets? [6.] Check this by drawing 18 objects and grouping 6 objects in each set.

Distribute 24 toothpicks, link-it cubes or counters to each of your students. Ask them to divide the objects into 3 equal sets by first grouping 3 objects into each set. That number of sets (8) will equal the number of objects in 3 equal sets. Repeat the exercise by asking your students to divide the same 24 objects into 2 equal sets, then 4 equal sets.

Draw 18 moons and ask your students to divide them into 3 equal sets. How many sets will there be if there are 3 moons in each set? What is 18 ÷ 3? So how many moons will there be in 3 equal sets? Repeat with several examples of pictures arranged linearly, as above.
Can this array be divided into 3 equal sets? How many dots are there? How many sets will there be if there are 3 dots in each group? So how many dots will there be in 3 equal sets? Is there an easier way to solve this problem? If the first row was divided into sets of 3 (demonstrate), can’t the whole array also be divided into sets of 3?

Repeat with...

Bonus
2 rows of 15 dots, 6 equal sets.

Ask your students to write 2 division statements for each illustration.

12 ÷ 3 = 4 and 12 ÷ 4 = 3

Have students solve the following problems with an illustration and division statement for each.

a) 20 triangles, 5 sets. How many triangles in each set?

b) 6 squares, 2 squares in each set. How many sets?

c) 18 triangles, 2 triangles in each set. How many sets?

d) 18 triangles, 6 sets. How many triangles in each set?

Then have students solve the following word problems with an illustration and division statement for each.

a) Joanne has 20 apples and 4 baskets. How many apples can she put in each basket?

b) Rita has 30 apples. Each basket can hold 6 apples. How many baskets does she need?

c) 5 chairs can fit in a row. There are 15 chairs. How many rows will there be?

Then ask students if there is another multiplication statement that can be obtained from the same picture as 3 × 4 = 12? How can the dots be grouped to express that 3 sets of 4 is equivalent to 4 sets of 3?
GOALS
Students will understand the relationship between division and multiplication

PRIOR KNOWLEDGE REQUIRED
Relationship between multiplication and skip counting
Relationship between division and skip counting

VOCABULARY
divided by

GOALS
Students will understand the relationship between division and multiplication.

PRIOR KNOWLEDGE REQUIRED
- Relationship between multiplication and skip counting
- Relationship between division and skip counting

VOCABULARY
- divided by

NS4-59
Division and Multiplication

Write “10 divided into sets of 2 results in 5 sets.” Have one volunteer read it out loud, another illustrate it, and another write its addition statement. Does the addition statement remind your students of multiplication? What is the multiplication statement?

\[
\begin{align*}
10 \div 2 &= 5 \\
2 + 2 + 2 + 2 + 2 &= 10 \\
5 \times 2 &= 10
\end{align*}
\]

Another way to express “10 divided into sets of 2 results in 5 sets” is to write “5 sets of 2 equals 10.”

Have volunteers illustrate the following division statements, write the division statements, and then rewrite them as multiplication statements.

\begin{align*}
a)\ 12 \div 4 &= 3 \quad \text{(12 ÷ 4 = 3 and 3 × 4 = 12)} \\
b)\ 10 \div 5 &= 2 \\
c)\ 9 \div 3 &= 3
\end{align*}

Assign the remaining questions to all students.

\begin{align*}
d)\ 15 \div 5 &= 3 \\
e)\ 18 \div 9 &= 2 \\
f)\ 6 \div 3 &= 2 \quad \text{(6 people divided into teams of 3 results in 2 teams.)} \\
g)\ 8 \div 2 &= 4 \quad \text{(8 fish divided so that each fishbowl has 4 fish results in 2 fishbowls.)} \\
h)\ 12 \div 4 &= 3 \quad \text{(12 people divided into 4 teams results in 3 people on each team.)} \\
i)\ 6 \div 2 &= 3 \quad \text{(6 fish divided into 3 fishbowls results in 2 fish in each fishbowl.)}
\end{align*}

Then ask students if there is another multiplication statement that can be obtained from the same picture as \(3 \times 4 = 12\)? How can the dots be grouped to express that 3 sets of 4 is equivalent to 4 sets of 3?

\[
\begin{array}{c}
\begin{array}{c}
\text{3 sets of 4}
\end{array} \\
\begin{array}{c}
\text{is equivalent to}
\end{array} \\
\begin{array}{c}
\text{4 sets of 3.}
\end{array}
\end{array}
\]

ANSWER: The second array of dots should be circled in columns of 3.

Then ask them what other division statement can be obtained from the same picture as \(12 \div 4 = 3\)? Can the 12 dots be divided into 3 sets of 4 instead of 4 sets of 3? (Place one dot of each colour into each circle.)
What division statement does this result in? \( [12 \div 3 = 4] \)

Draw similar illustrations and have your students write two multiplication statements and two division statements for each. Have a volunteer demonstrate the exercise for the class with the first illustration.

Demonstrate how multiplication can be used to help with division. For example, the division statement \( 20 \div 4 = _____ \) can be written as the multiplication statement \( 4 \times _____ = 20 \). To solve the problem, skip count by 4 to 20 and count the number of fingers it requires. Demonstrate this solution on a number line, as well.

Assign students the following problems.

a) \( 9 \times 3 = 27, \text{ SO: } 27 \div 9 = _____ \)
b) \( 2 \times 6 = 12, \text{ SO: } 12 \div 2 = _____ \)
c) \( 8 \times _____ = 40, \text{ SO: } 40 \div 8 = _____ \)
d) \( 10 \times _____ = 30, \text{ SO: } 30 \div 10 = _____ \)
e) \( 5 \times _____ = 30, \text{ SO: } 30 \div 5 = _____ \)
f) \( 4 \times _____ = 28, \text{ SO: } 28 \div 4 = _____ \)
NS4-60
Knowing When to Multiply or Divide (Introduction) and

NS4-61
Knowing When to Multiply or Divide

Have your students fill in the blanks.

1 1 1 1 1 1 1 1 1

sets
2 sets

objects per set
5 objects per set

objects altogether
10 objects altogether

When you are confident that your students are completely familiar with the terms “set,” “group,” “for every,” and “in each,” and you’re certain that they understand the difference between the phrases “objects in each set” and “objects” (or “objects altogether,” or “objects in total”), have them write descriptions of the diagrams.

1 1 1 1 1 1 1 1

2 groups

4 objects in each group

8 objects

Explain that a set or group expresses three pieces of information: the number of sets, the number of objects in each set, and the number of objects altogether. For the problems, have your students explain which piece of information isn’t expressed and what the values are for the information that is expressed.

a) There are 8 pencils in each box. There are 5 boxes. How many pencils are there altogether?

b) Each dog has 4 legs. There are 3 dogs. How many legs are there altogether?

c) Each cat has 2 eyes. There are 10 eyes. How many cats are there?

d) Each boat can fit 4 people. There are 20 people. How many boats are needed?

e) 30 people fit into 10 cars. How many people fit into each car?

f) Each apple costs 20¢. How many apples can be bought for 80¢?

g) There are 8 triangles divided into 2 sets. How many triangles are there in each set?

h) 4 polygons have a total of 12 sides. How many sides are on each polygon?
Introduce the following three problem types.

**TYPE 1:** Knowing the number of sets and the number of objects in each set, but not the total number of objects.

**ASK:** If you know the number of objects in each set and the number of sets, how can you find the total number of objects? What operation should you use—multiplication or division?

Write on the board:

\[ \text{Number of sets} \times \text{Number of objects in each set} = \text{Total number of objects} \]

If there are 3 sets and 2 objects in each set, how many objects are there in total? \[3 \times 2 = 6\]

If there are 5 sets and 4 objects in each set, how many objects are there in total?

If there are 3 objects in each set and 4 sets, how many objects are there in total?

If there are 2 objects in each set and 7 sets, how many objects are there in total?

If there are 100 objects in each set and 3 sets, how many objects are there in total?

If there are 100 sets and 3 objects in each set, how many objects are there in total?

**TYPE 2:** Knowing the total number of objects and the number of objects in each set, but not the number of sets.

\[ \text{Number of sets} \times \text{Number of objects in each set} = \text{Total number of objects} \]

If there are 4 objects in each set and 12 objects in total, how many sets are there?

\[ 12 \div 4 = 3 \text{. There are 3 sets.} \]

Elaborate on this problem type with several examples, as above.

**TYPE 3:** Knowing the total number of objects and the number of sets, but not the number of objects in each set.

\[ \text{Number of sets} \times \text{Number of objects in each set} = \text{Total number of objects} \]

If there are 6 sets and 12 objects in total, how many objects are in each set?

\[ 12 \div 6 = 2 \text{. There are 2 objects in each set.} \]

Elaborate on this problem type with several examples, as above.

If there are 5 sets and 40 objects altogether, how many objects are in each set? Vary the wording, as in: If there are 28 objects in 4 sets, how many objects are in each set?

Have students write multiplication statements for the following problems with the blank in the correct place.

a) 2 objects in each set. b) 2 objects in each set. c) 2 sets.

6 objects in total. 6 sets. 6 objects in total.

How many sets? How many objects in total? How many objects in each set?

\[ \boxed{\text{[ } \_ \times 2 = 6 \text{ ]}} \]

\[ \boxed{6 \times 2 = \_ \_ \_} \]

\[ \boxed{2 \times \_ \_ \_ = 6} \]

Which of these problems are division problems? [Multiplication is used to find the total number of objects, and division is used if the total number of objects is known.]
Assign several types of problems.

a) 5 sets, 4 objects in each set. How many objects altogether?

b) 8 objects in total, 2 sets. How many objects in each set?

c) 3 sets, 6 objects in total. How many objects in each set?

d) 3 sets, 6 objects in each set. How many objects in total?

e) 3 objects in each set, 9 objects altogether. How many sets?

When your students are comfortable with this lesson’s goal, introduce alternative contexts for objects and sets. Start with point-form problems (EXAMPLE: 5 tennis courts, 3 tennis balls on each court. How many tennis balls altogether?), and then move to complete sentence problems (EXAMPLE: If there are 5 tennis courts, and 3 balls on each court, how many tennis balls are on the tennis courts altogether?).

Emphasize that understanding the context of objects and sets within the word problem is unnecessary for answering the problem. Tell your students that you met someone from Mars last weekend, and they told you that there are 3 dulgs on each flut. If you count 15 dulgs, how many fluts are there? Explain the problem-solving strategy of replacing unknown words with words that are commonly used. For example, replace the object in the problem (dulgs) with students, and replace the set in the problem (flut) with bench. So, if there are 3 students on each bench and you count 15 students, how many benches are there? It wouldn’t make sense to replace the object (dulgs) with benches and the set (flut) with students, would it? If there are 3 benches on each student and you count 15 benches, how many students are there? A good strategy for replacing words is to replace the object and the set and then invert the replacement with the same two words. Only one of the two versions of the problem with the replacement words should make sense.

Students may wish to create their own science fiction word problems for their classmates. Encourage them to use words from another language, if they speak another language.

**Extension**

Have your students circle the given information in the following word problems, and then underline the information that isn’t given and needs to be determined. They should start by looking for the question mark; reading that sentence will tell them what needs to be determined. Then they should find the given information that they need to find the answer and circle it. It will be especially important to start by determining what the question is asking when irrelevant information is included. The two examples below contrast the situations where very little except the relevant information is given and where irrelevant information such as “are being used for tournaments” is included.

If there are 5 tennis courts and 3 balls on each court, how many tennis balls altogether?

If 5 tennis courts are being used for tournaments. There are 3 balls on each court. How many tennis balls are there altogether?

Start with problems that have very few extraneous words.

a) If there are 4 cans of tennis balls and 12 tennis balls altogether, how many tennis balls are in each can?

b) If there are 7 chairs in each row, and 3 rows of chairs, how many chairs are there altogether?

If 20 students ride in 4 vans, how many students ride in each van?
Constantly identify the information that is needed in a word problem in order to solve the word problem. For example, we need to know the number of students and vans to determine how many students will ride in each van.

For the following set of problems, write the interrogative clause first (for EXAMPLE: “How many hockey cards do they each have?”) and ask your students to identify the information that is needed to solve the problem. Then write the problem in its entirety and ask them to solve it.

   d) Three friends share 15 hockey cards. How many hockey cards do they each have?

   e) If 15 students are going to the zoo, and 5 students can ride in each van, how many vans are needed?

   f) Sally has 3 white binders with pretty flowers on the cover. Each binder holds 200 pages. How many pages will the binders hold altogether?

When your students are comfortable with circling the proper information, have them rewrite the following word problems in point-form with only the essential information (EXAMPLE: 3 binders. Each binder holds 200 pages. How many pages will the binders hold?).

   g) Sally has 3 white binders that she uses for different subjects. Each binder has 7 purple flowers on it. How many purple flowers are on her binders altogether?

   h) Bilal lives in a haunted house. On each window there are 6 spiders. Beside each creaky door there are 5 frogs. If there are 5 windows, how many spiders are there altogether?

   i) Sally has 3 blue binders. She uses each binder for 4 different subjects. Each binder has a picture of 5 basketballs on it. How many basketballs are pictured on her binders altogether?

Have your students create a word problem with extraneous information. Then have a partner rewrite the word problem in point-form with only the essential information.
Draw:

\[
\begin{align*}
6 \div 3 &= 2 \\
7 \div 3 &= 2 \text{ Remainder } 1 \\
8 \div 3 &= 2 \text{ Remainder } 2 \\
9 \div 3 &= 3 \\
10 \div 3 &= 3 \text{ Remainder } 1
\end{align*}
\]

Ask your students if they know what the word "remainder" means. Instead of responding with a definition, encourage them to only say the answers for the following problems. This will allow those students who don’t immediately see it a chance to detect the pattern.

\[
\begin{align*}
7 \div 2 &= 3 \text{ Remainder } \_ \_ \_ \\
11 \div 3 &= 3 \text{ Remainder } \_ \_ \_ \\
12 \div 5 &= 2 \text{ Remainder } \_ \_ \_ \_ \_ \\
14 \div 5 &= 2 \text{ Remainder } \_ \_ \_ \_ \_ \_ \_ \_ \\
\end{align*}
\]

Challenge volunteers to find the remainder by drawing a picture on the board. This way, students who do not yet see the pattern can see more and more examples of the rule being applied.

**SAMPLE PROBLEMS:**

\[
\begin{align*}
9 \div 2 & \quad 7 \div 3 & \quad 11 \div 3 & \quad 15 \div 4 \\
15 \div 6 & \quad 12 \div 4 & \quad 11 \div 2 & \quad 18 \div 5
\end{align*}
\]
What does “remainder” mean? Why are some dots left over? Why aren’t they included in the circles? What rule is being followed in the illustrations? [The same number of dots is placed in each circle, the remaining dots are left uncircled]. If there are fewer uncircled dots than circles then we can’t put one more in each circle and still have the same number in each circle, so we have to leave them uncircled. If there are no dots left over, what does the remainder equal? [Zero.]

Introduce your students to the word “quotient”: Remind your students that when subtracting two numbers, the answer is called the difference. ASK: When you add two numbers, what is the answer called? In 7 + 4 = 11, what is 11 called? (The sum). When you multiply two numbers, what is the answer called? In 2 × 5 = 10, what is 10 called? (The product). When you divide two numbers, does anyone know what the answer is called? There is a special word for it. If no-one suggests it, tell them that when you write 10 ÷ 2 = 5, the 5 is called the quotient.

Have your students determine the quotient and the remainder for the following statements.

a) 17 ÷ 3 = ____ Remainder ____

b) 23 ÷ 4 = ____ Remainder ____

c) 11 ÷ 3 = ____ Remainder ____

Write “2 friends want to share 7 apples.” What are the sets? [Friends.] What are the objects being divided? [Apples.] How many circles need to be drawn to model this problem? How many dots need to be drawn?

Draw 2 circles and 7 dots.

To divide 7 apples between 2 friends, place 1 dot (apple) in each circle.

Can another dot be placed in each circle? Are there more than 2 dots left over? So is there enough to put one more in each circle? Repeat this line of instruction until the diagram looks like this:

How many apples will each friend receive? Explain. [There are 3 dots in each circle.] How many apples will be left over? Explain. [Placing 1 more dot in either of the circles will make the compared amount of dots in both circles unequal.]

Repeat this exercise with “5 friends want to share 18 apples.” Emphasize that the process of division and placing apples (dots) into sets (circles) continues as long as there are at least 5 apples left to share. Count the number of apples remaining after each round of division to ensure that at least 5 apples remain.
Have your students illustrate each of the following division statements with a picture, and then determine the quotients and remainders.

a) \(11 \div 5 = \) ____ Remainder ____
b) \(18 \div 4 = \) ____ Remainder ____
c) \(20 \div 3 = \) ____ Remainder ____
d) \(22 \div 5 = \) ____ Remainder ____
e) \(11 \div 2 = \) ____ Remainder ____
f) \(8 \div 5 = \) ____ Remainder ____
g) \(19 \div 4 = \) ____ Remainder ____

Then have your students explain what the following three models illustrate.

Have them explain how these models are the same and how they are different. Have your students complete several division exercises using number lines, and then have them draw number lines for several division statements.

Can skip counting show that \(14 \div 3 = 4 \text{ Remainder 2}\)?

Why does the count stop at 12? [Continuing the count will lead to numbers greater than 14.]
How can the remainder be determined? [Subtract 12 from 14. \(14 - 12 = 2\).]

Assign the following exercise to students who have difficulties learning when to stop counting, when skip counting to solve a division statement.

Using a number line from 0 to 25, ask your student to skip count out loud by five and to stop counting before reaching 17. Have them point to the respective number on the number line as they count it. This should enable your student to see that their finger will next point to 20 if they don’t stop counting at 15, passing the target number of 17. You may need to put your finger on 17 to stop some students from counting further. Repeat this exercise with target numbers less than 25. After completing this exercise, most students will know when to stop counting before they reach a given...
target number, even if they are counting by numbers other than 5. With a few students, you will have to repeat the exercise with counting by 2s, 3s, etc.

**Extensions**

1. Which number is greater, the divisor (the number by which another is to be divided) or the remainder? Will this always be true? Have your students examine their illustrations to help explain. Emphasize that the divisor is equal to the number of circles (sets), and the remainder is equal to the number of dots left over. We stop putting dots in circles only when the number left over is smaller than the number of circles; otherwise, we would continue putting the dots in the circles. See the journal section below.

Which of the following division statements is correctly illustrated? Can one more dot be placed into each circle or not? Correct the two wrong statements.

\[
15 \div 3 = 4 \text{ Remainder } 3 \\
17 \div 4 = 3 \text{ Remainder } 5 \\
19 \div 4 = 4 \text{ Remainder } 3
\]

Without illustration, identify the incorrect division statements and correct them.

a) \( 16 \div 5 = 2 \text{ Remainder } 6 \)  
   b) \( 11 \div 2 = 4 \text{ Remainder } 3 \)  
   c) \( 19 \div 6 = 3 \text{ Remainder } 1 \)

2. Explain how a diagram can illustrate a division statement with a remainder and a multiplication statement with addition.

\[
14 \div 3 = 4 \text{ Remainder } 2 \\
3 \times 4 + 2 = 14
\]

Ask students to write a division statement with a remainder and a multiplication statement with addition for each of the following illustrations.

3. Compare mathematical division to normal sharing. Often if we share 5 things (say, marbles) among 2 people as equally as possible, we give 3 to one person and 2 to the other person. But in mathematics, if we divide 5 objects between 2 sets, 2 objects are placed in each set and the leftover object is designated as a remainder. Teach them that we can still use division to solve this type of problem; we just have to be careful in how we interpret the remainder.

Have students compare the answers to the real-life problem and to the mathematical problem:

a) 2 people share 5 marbles (groups of 2 and 3; \( 5 \div 2 = 2 \text{ R } 1 \))

b) 2 people share 7 marbles (groups of 3 and 4; \( 7 \div 2 = 3 \text{ R } 1 \))

  c) 2 people share 9 marbles (groups of 4 and 5; \( 9 \div 2 = 4 \text{ R } 1 \))
ASK: If $19 \div 2 = 9 \text{ R } 1$, how many marbles would each person get if 2 people shared 19 marbles? Emphasize that we can use the mathematical definition of sharing as equally as possible even when the answer isn’t exactly what we’re looking for. We just have to know exactly how to adapt it to what we need.

4. Find the mystery number. I am between 22 and 38. I am a multiple of 5. When I am divided by 7 the remainder is 2.

5. Have your students demonstrate two different ways of dividing...
   a) 7 counters so that the remainder equals 1.
   b) 17 counters so that the remainder equals 1.

6. As a guided class activity or assignment for very motivated students, have your students investigate the following division statements.

   \[ 17 \div 3 = 5 \text{ Remainder } 2, \text{ and } 17 \div 5 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]
   \[ 22 \div 3 = 7 \text{ Remainder } 1, \text{ and } 22 \div 7 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]
   \[ 29 \div 4 = 7 \text{ Remainder } 1, \text{ and } 29 \div 7 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]
   \[ 23 \div 4 = 5 \text{ Remainder } 3, \text{ and } 23 \div 5 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]
   \[ 27 \div 11 = 2 \text{ Remainder } 5, \text{ and } 27 \div 2 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]

What seems to be true in the first four statements but not the fifth?
Challenge your students to determine the conditions for switching the quotient with the divisor and having the remainder stay the same for both statements. Have students create and chart more of these problems to help them find a pattern. You could start a class chart where students write new problems that they have discovered belongs to one or the other category. As you get more belonging to one of the categories challenge them to find more examples that belong to the other category. Be sure that everyone has a chance to contribute. After students see the pattern, have them predict whether the quotient and divisor can be switched and keep the remainder the same.

\[ 27 \div 2 = 13 \text{ Remainder } 1, \text{ and } 27 \div 13 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]
\[ 40 \div 12 = 3 \text{ Remainder } 4, \text{ and } 40 \div 3 = \_\_\_\_ \text{ Remainder } \_\_\_\_ ? \]

**ANSWER:** If the remainder is smaller than the quotient, the quotient can be switched with the divisor and the remainder will stay the same for both statements. Note that $23 \div 4 = 5$ Remainder 3 is equivalent to $4 \times 5 + 3 = 23$. But this is equivalent to $5 \times 4 + 3 = 23$, which is equivalent to $23 \div 5 = 4$ Remainder 3. Note, however, that while $8 \times 6 + 7 = 55$ is equivalent to $55 \div 8 = 6$ Remainder 7, it is not true that $6 \times 8 + 7 = 55$ is equivalent to $55 \div 6 = 8$ Remainder 7, because in fact $55 \div 6 = 9$ Remainder 1.

**Journal**

The remainder is always smaller than the divisor because…
**GOALS**

Students will use long division to divide two-digit numbers by a one-digit number.

**PRIOR KNOWLEDGE REQUIRED**

- Tens and ones blocks
- Division as sharing

---

**NS4-65**

Long Division—Two-Digit by One-Digit

Write

\[
\begin{array}{cccc}
3 & \underline{\pm} & 6 & \\
5 & \underline{\pm} & 10 & \\
4 & \underline{\pm} & 12 & \\
5 & \underline{\pm} & 20 & \\
6 & \underline{\pm} & 18 & \\
9 & \underline{\pm} & 18 & \\
\end{array}
\]

Ask your students if they recognize this \( \underline{\pm} \) symbol. If they know what the symbol means, have them solve the problems. If they don’t recognize the symbol, have them guess its meaning from the other students’ answers to the problems. If none of your students recognize the symbol, solve the problems for them. Write more problems to increase the chances of students being able to predict the answers.

\[
\begin{array}{cccc}
2 & \underline{\div} & 8 & \\
2 & \underline{\div} & 6 & \\
4 & \underline{\div} & 16 & \\
5 & \underline{\div} & 15 & \\
4 & \underline{\div} & 8 & \\
6 & \underline{\div} & 12 & \\
\end{array}
\]

3

Explain that \( 2 \underline{\div} 6 \) is another way of expressing \( 6 \div 2 = 3 \), and that \( 2 \underline{\div} 7 \) is another way of expressing \( 7 \div 2 = 3 \) Remainder 1.

Ask your students to express the following statements using the new notation learned above.

a) \( 14 \div 3 = 4 \) Remainder 2
b) \( 26 \div 7 = 3 \) Remainder 5
c) \( 819 \div 4 = 204 \) Remainder 3

To ensure that they understand the long division symbol, ask them to solve and illustrate the following problems.

\[
\begin{array}{cccc}
2 & \underline{\div} & 11 & \\
4 & \underline{\div} & 18 & \\
5 & \underline{\div} & 17 & \\
4 & \underline{\div} & 21 & \\
3 & \underline{\div} & 16 & \\
\end{array}
\]

**Bonus**

\[
\begin{array}{cccc}
6 & \underline{\div} & 45 & \\
4 & \underline{\div} & 37 & \\
4 & \underline{\div} & 43 & \\
\end{array}
\]

Then demonstrate division using base ten materials.

\[
\begin{array}{cc}
3 & \underline{\div} 63 \\
\end{array}
\]

Have students solve the following problems using base ten materials.

\[
\begin{array}{cccc}
2 & \underline{\div} 84 & \\
2 & \underline{\div} 48 & \\
3 & \underline{\div} 96 & \\
4 & \underline{\div} 88 & \\
\end{array}
\]

---
Then challenge them to solve $3 \sqrt{72}$, again using base ten materials, but allow students to trade tens blocks for ones blocks as long as the value of the dividend (72) remains the same.

**ASK:** Can 7 tens blocks be equally placed into 3 circles? Can 6 of the 7 tens blocks be equally placed into 3 circles? What should be done with the leftover tens block? How many ones blocks can it be traded for? How many ones blocks will we then have altogether? Can 12 ones blocks be equally placed into 3 circles? Now, what is the total value of blocks in each circle? [24.] What is 72 divided by 3? Where do we write the answer?

Explain that the answer is always written above the dividend, with the tens digit above the tens digit and the ones digit above the ones digit.

$$\begin{array}{c|c|c|c|c|c|c|c|c|c} \hline & & & & & & & & & \hline \end{array}$$

Have your students solve several problems using base ten materials.

$$4 \quad \underline{92} \quad 4 \quad \underline{64} \quad 4 \quad \underline{72} \quad 3 \quad \underline{45} \quad 2 \quad \underline{78}$$

The following problems will have remainders.

$$4 \quad \underline{65} \quad 3 \quad \underline{82} \quad 5 \quad \underline{94} \quad 2 \quad \underline{35} \quad 4 \quad \underline{71}$$

Tell your students that they are going to learn to solve division problems without using base ten materials. The solutions to the following problems have been started using base ten materials. Can they determine how the solution is written?

$$\begin{array}{c|c|c|c|c|c|c|c|c|c} \hline & & & & & & & & & \hline \end{array}$$

As in the previous set, illustrate the following problems. Have your students determine how to write the solution for the last problem.

$$\begin{array}{c|c|c|c|c|c|c|c|c|c} \hline & & & & & & & & & \hline \end{array}$$

Challenge students to illustrate several more problems and to write the solution. When all students are writing the solution correctly, ask them how they determined where each number was written. Which number is written above the dividend? [The number of tens equally placed into each circle.] Which number is written below the dividend? [The number of tens placed altogether.]

Then, using the illustrations from the two previous sets of problems, ask your students what the circled numbers below express.
ASK: What does the number express in relation to its illustration? [The number of tens not equally placed into circles.] Why does the subtraction make sense? [The total number of tens minus the number of tens equally placed into circles results in the number of tens blocks left over.]

Teach your students to write algorithms without using base ten materials. Remind them that the number above the dividend’s tens digit is the number of tens placed in each circle. For example, if there are 4 circles and 9 tens, as in 4 94, the number 2 is written above the dividend to express that 2 tens are equally placed in each of the 4 circles. Explain that the number of tens placed altogether can be calculated by multiplying the number of tens in each circle (2) by the number of circles (4); the number of tens placed altogether is 2 × 4 = 8. Ask your students to explain if the following algorithms have been started correctly or not. Encourage them to illustrate the problems with base ten materials, if it helps.

Explain that the remaining number of tens blocks should always be less than the number of circles, otherwise more tens blocks need to be placed in each circle. The largest number of tens blocks possible should be equally placed in each circle.

Display the multiplication facts for 2 times 1 through 5 (i.e. 2 × 1 = 2, 2 × 2 = 4, etc.), so that students can refer to it for the following set of problems. Then write

2 \[ \frac{3}{85} \]

ASK: How many circles should be used? If 1 tens block is placed in each circle, how many tens blocks will be placed altogether? [2 × 1 = 2] What if 2 tens blocks are placed in each circle? [2 × 2 = 4] What if 3 tens blocks are placed in each circle? [2 × 3 = 6] And finally, what if 4 tens blocks are placed in each circle? [2 × 4 = 8] How many tens blocks need to be placed? [Seven.] Can 4 tens blocks be placed in each circle? [No, that will require 8 tens blocks.] Then explain that the greatest multiple of 2 not exceeding the number of tens is required. Have them perform these steps for the following problems.

\[ \frac{2}{75} \]

2 tens in each circle

3 tens block left over

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle

3 tens in each circle
Then display the multiplication facts for 3 times 1 through to 3 times 5 and repeat the exercise. Demonstrate the steps for the first problem.

\[ \begin{array}{c}
3 \left\lfloor \frac{75}{6} \right. \\
\hline
6 \quad \text{2 tens in each circle}
\end{array} \quad \begin{array}{c}
3 \left\lfloor \frac{65}{38} \right. \\
\hline
38 \quad \text{3 \times 2 = 6 tens place}
\end{array} \quad \begin{array}{c}
3 \left\lfloor \frac{81}{59} \right. \\
\hline
59 \quad \text{1 tens block left over}
\end{array} \]

Emphasize that the number above the dividend’s tens digit is the greatest multiple of 3 not exceeding the number of tens.

Then, using the illustrations already drawn to express leftover tens blocks (see the second page of this section), explain the next step in the algorithm. ASK: Now what do the circled numbers express?

\[ \begin{array}{c}
3 \left\lfloor \frac{63}{03} \right. \\
\hline
63 \quad \text{2 tens in each circle}
\end{array} \quad \begin{array}{c}
2 \left\lfloor \frac{64}{04} \right. \\
\hline
64 \quad \text{3 tens block left over}
\end{array} \quad \begin{array}{c}
4 \left\lfloor \frac{92}{12} \right. \\
\hline
92 \quad \text{4 tens block left over}
\end{array} \]

The circled number expresses the amount represented by the base ten materials not placed in the circles.

Using base ten materials, challenge students to start the process of long division for 85 ÷ 3 and to record the process (the algorithm) up to the point discussed so far. Then ask students to trade the remaining tens blocks for ones blocks, and to circle the step in the algorithm that expresses the total value of ones blocks.

Ensure that students understand the algorithm up to the step where the ones blocks are totalled with the remaining (if any) tens blocks.

\[ \begin{array}{c}
3 \left\lfloor \frac{75}{6} \right. \\
\hline
75 \quad \text{15 ones to be placed}
\end{array} \quad \begin{array}{c}
3 \left\lfloor \frac{65}{38} \right. \\
\hline
65 \quad \text{3 \times 2 = 6 tens place}
\end{array} \quad \begin{array}{c}
3 \left\lfloor \frac{81}{59} \right. \\
\hline
81 \quad \text{3 tens block left over}
\end{array} \]

Illustrate all of the placed tens and ones blocks and the finished algorithm, and then ask your students to explain the remaining steps in the algorithm. Perform this for the examples already started (63 ÷ 3, 64 ÷ 2, 92 ÷ 4, etc.). For EXAMPLE:

\[ \begin{array}{c}
4 \left\lfloor \frac{94}{8} \right. \\
\hline
94 \quad \text{23 Remainder 2.}
\end{array} \]

So, 94 ÷ 4 = 23 Remainder 2.
Ask your students to explain how the circled numbers are derived. How is the 3 derived? The 12? [Dividing the 14 ones blocks into 4 circles results in 3 blocks in each circle, for a total of 12.] 14 – 12 results in a remainder of 2.

Then challenge students to write the entire algorithm. Ask them why the second subtraction makes sense. [The total number of ones blocks subtracted by the number of ones blocks placed into circles equals the number of ones blocks left over.]

Using base ten materials, have students complete several problems and write the entire algorithms.

25
5 ones in each circle

3   75
6
15 – ones to be placed
15 – 5 × 3 ones placed
0 – Remainder (no ones left over)

Some students will need all previous steps done so that they can focus on this one.

If you prefer, you may use an example for a problem that has leftover ones. Have students finish the examples they have already started and then complete several more problems from the beginning and use base ten materials only to verify their answers.

2    39
3    39
5   79
6   87
4   57
6   85
7   94
8   94

With practice, students will learn to estimate the largest multiples that can be used to write the algorithms. When they are comfortable with moving forward in the lesson, introduce larger divisors.

4    69
5    79
6   87
4   57
6   85
7   94
8   94

Note that at this point in the lesson, the dividend’s tens digit is always greater than the divisor.

**Extension**

1. Teach students to check their answers with multiplication. For example:

   4    17
   2
   29
   28
   1R

   \[69 \times 4 = 68 + 1 = 69\]

2. Teach your students to use long division to divide three-digit numbers by one-digit numbers. Using base ten materials, explain why the standard algorithm for long division works.

**EXAMPLE:** Divide 726 into 3 equal groups.

**STEP 1.** Make a model of 726 units.
STEP 2. Divide the hundreds blocks into 3 equal groups.

Keep track of the number of units in each group, and the number remaining, by slightly modifying the long division algorithm.

\[
3 \overline{\div} 726
\]

2 hundred blocks, or 200 units, have been divided into each group

\[
3 \overline{\div} 726
\]

600 units (200 × 3) have been divided

\[
3 \overline{\div} 726
\]

126 units still need to be divided

NOTE: Step 2 is equivalent to the following steps in the standard long division algorithm.

\[
3 \overline{\div} 726
\]

\[
3 \overline{\div} 726
\]

\[
3 \overline{\div} 726
\]

STEP 3. Divide the remaining hundreds block and the 2 remaining tens blocks among the 3 groups equally.

There are 120 units, so 40 units can be divided into each group.

Keep track of this as follows:

\[
\frac{200}{3} \rightarrow \frac{2}{6} \leftarrow \frac{2}{12}
\]

40 new units have been divided into each group

120 units still need to be divided
NOTE: Step 3 is equivalent to the following steps in the standard long division algorithm.

\[
\begin{array}{c|c}
3 & 726 \\
\hline
6 & 12 \\
\hline
12 & 12 \\
\hline
12 & 12 \\
\hline
0 & 12 \\
\end{array}
\]

STEP 4. Divide the 6 remaining blocks among the 3 groups equally.

Group 1

Group 2

Group 3

242 units have been divided into each group; hence \( 726 \div 3 = 242 \).

\[
\begin{array}{c|c}
3 & 726 \\
\hline
6 & 12 \\
\hline
12 & 12 \\
\hline
12 & 12 \\
\hline
0 & 12 \\
\end{array}
\]

NOTE: Step 4 is equivalent to the following steps in the standard long division algorithm.

\[
\begin{array}{c|c}
3 & 726 \\
\hline
6 & 12 \\
\hline
12 & 12 \\
\hline
12 & 12 \\
\hline
0 & 12 \\
\end{array}
\]

Students should be encouraged to check their answer by multiplying 242 \( \times \) 3.

3. Assign students the following word problems.

a) An octagon has a perimeter 952 cm. How long is each side?

b) An octopus has 944 suckers. How many suckers are on each arm?
**NS4-66**  
**Further Division**

**GOALS**

Students will perform long division with a divisor that is greater in value than a dividend's tens digit or is the same as the dividend's tens digit and greater than the dividend's ones digit. They will divide by estimation, and solve word problems that require interpretation of the remainder.

**PRIOR KNOWLEDGE REQUIRED**

Long division  
Division with remainders  
Word problems

Review the long division from last class, where the divisor is no more than the dividend's tens digit. (EXAMPLE: 53 ÷ 4, 47 ÷ 4, 62 ÷ 5)

Write down a problem that has a divisor that is greater in value than the dividend's tens digit (i.e. there are fewer tens blocks available than the number of circles).

\[
5 \overline{)27}
\]

**ASK:** How many tens blocks are there in 27? Into how many circles do they need to be divided? Are there enough tens blocks to place one in each circle? How is this different from the problems in the previous lesson? Illustrate that there are no tens by writing a zero above the dividend's tens digit. Then **ASK:** What is \(5 \times 0\)? Write:

\[
\begin{array}{c}
5 \\
\hline
27 \\
\hline
0 \\
\hline
27 \quad \text{Number of ones blocks (traded from tens blocks) to be placed}
\end{array}
\]

Have a volunteer finish this problem, and then ask if the zero needs to be written at all. Explain that the algorithm can be started on the assumption that the tens blocks have already been traded for ones blocks.

\[
\begin{array}{c}
5 \\
\hline
27 \\
25 \\
\hline
2 \quad \text{Number of ones blocks left over}
\end{array}
\]

Emphasize that the answer is written above the dividend’s ones digit because it is the answer’s ones digit.

Have students complete several problems of this type.

\[
\begin{array}{c}
4 \overline{)}37 \\
5 \overline{)}39 \\
8 \overline{)}63 \\
8 \overline{)}71
\end{array}
\]

Then ask students to try to solve \(4 \overline{)}43\)

**ASK:** How many tens blocks are there in 43? Into how many circles do they need to be divided? [4.] Are there enough tens blocks to place one in each circle? Are there any tens blocks left over? How many ones blocks need to be placed? [3.] Are there enough ones blocks to place one in each circle? [No.] How is this recorded in the algorithm? Where is the answer for the number of ones blocks placed in each circle always written? How many ones blocks were placed? [Zero.] What number will be written for the answer’s ones digit? [0.]
Illustrate this

\[
\begin{array}{c}
4 \overline{) 43} \\
- 4 \\
\hline
03 \\
- 0 \\
\hline
3 \text{ Remainder}
\end{array}
\]

Tell your students that writing a zero above the dividend’s tens digit is unnecessary when the answer has no tens. **ASK:** Is it necessary to write a zero above the dividend’s ones digit when the answer has no ones? [Yes.] Why? If we didn’t write the 0 ones, it would look like 1 instead of 10 as the answer.

Have students complete several problems of this type.

\[
\begin{array}{c}
5 \overline{) 52} \\
- 5 \\
\hline
74 \\
- 7 \\
\hline
87 \\
- 8 \\
\hline
\end{array}
\]

Have students estimate the answers for division problems by rounding the divisors and dividends to the nearest ten. Have them check their estimates using long division.

a) \(86 \div 9\) Estimate: \(90 \div 10 = 9\) Calculation: \(9 \overline{) 86}\)

\[
\begin{array}{c}
9 \overline{) 86} \\
- 81 \\
\hline
5 \text{ Remainder}
\end{array}
\]

b) \(77 \div 9\) c) \(75 \div 8\) d) \(68 \div 9\) e) \(66 \div 8\)

Then explain that division can be used to solve word problems. **SAY:** Let’s start with a word problem that doesn’t require long division. A canoe can hold three kids. Four kids are going canoeing together. How many canoes are needed? (2) **ASK:** What happens when long division is used to solve this question? Does long division give us the same answer?

\[
\begin{array}{c}
3 \overline{) 4} \\
- 3 \\
\hline
1 \text{ Remainder}
\end{array}
\]

Long division suggests that three kids will fit in one canoe and one kid will be left behind. But one kid can’t be left behind because four kids are going canoeing together. So even though the quotient is one canoe, two canoes are actually needed.

How many canoes are needed if 8 kids are going canoeing together? …11 kids? …12 kids? …14 kids? Students should do these questions by using long division and by using common sense.

**BONUS:** … 26 kids? … 31 kids? … 85 kids?

How does long division help when we have larger numbers?

If Anna reads two pages from her book every day, and she has 13 pages left to read, how many days will it take Anna to finish her book?

If Rita reads three pages every day, how long will it take her to read …

a) 15 pages? b) 17 pages? c) 92 pages? d) 84 pages? e) 67 pages?
Extension

1. By multiplying the divisors by ten and checking if the products are greater or less than their respective dividends, have students decide if the answers to the following problems will have one or two digits.

\[
\begin{array}{c}
3 \cdot 72 \\
4 \cdot 38 \\
9 \cdot 74 \\
6 \cdot 82 \\
6 \cdot 34
\end{array}
\]

For example, multiplying the divisor in \(72 \div 3\) by ten results in a product less than 72 (\(3 \times 10 = 30\)), meaning the quotient for \(72 \div 3\) is greater than 10 and has two digits. On the other hand, multiplying the divisor in \(38 \div 4\) by ten results in a product greater than 38 (\(4 \times 10 = 40\)), meaning the quotient for \(38 \div 4\) is less than 10 and has one digit.

2. Write a word problem to express \(96 \div 4\).

3. (Adapted from Atlantic Curriculum) Students should understand that a remainder is always interpreted within the context of its respective word problem. Students should understand when a remainder...

- Needs to be divided further. For example, when 3 children share 7 licorice pieces, each child receives 2 pieces, and the remaining piece is further divided into thirds. So each child receives 2 and a third licorice pieces.

- Needs to be ignored. For example, $3.25 will buy four 75¢ notebooks. Since there is not enough money to buy five notebooks, the 25¢ is ignored.

- Needs to be rounded up. For example, five 4-passenger cars are needed to transport 17 children because none of the children can be left behind.

- Needs to be divided unequally among the groups. For example, if 91 students are to be transported in 3 buses, 30 students will ride in two buses, and 31 students will ride in the other.
4. Ask students to determine, without calculating, which quotient in each pair is higher, and to explain how they know.

a) $9 \div 3$ and $12 \div 3$

b) $14 \div 2$ and $8 \div 2$

c) $15 \div 3$ and $15 \div 5$

d) $20 \div 5$ and $20 \div 2$

e) $84 \div 3$ and $75 \div 3$

f) $84 \div 4$ and $92 \div 4$

g) $84 \div 3$ and $84 \div 2$

h) $84 \div 3$ and $84 \div 4$

i) $52 \div 4$ and $48 \div 5$

j) $52 \div 4$ and $60 \div 3$

Emphasize that if you have more objects in each group and/or fewer objects altogether, then you will need fewer groups, so the quotient will be smaller.

Now ask students to decide whether each estimate is too high or too low.

a) $87 \div 10$ is about $90 \div 10 = 9$ (too high, you need more groups of the same size to make $90$ than to make $87$)

b) $51 \div 10$ is about $50 \div 10 = 5$ (too low, you need fewer groups of the same size to make $50$ than to make $51$)

c) $70 \div 11$ is about $70 \div 10 = 7$ (too high, you need more groups of $10$ than groups of $11$ to make the same number)

d) $40 \div 9$ is about $40 \div 10 = 4$ (too low, you need fewer groups of $10$ than groups of $9$ to make the same number)

e) $71 \div 9$ is about $70 \div 10 = 7$ (too low, the fewer objects you have altogether and the more objects in each group, the smaller the quotient will be)

f) $57 \div 11$ is about $60 \div 10 = 6$ (too high, the more objects you have altogether and the fewer objects in each group, the higher the quotient will be)

Introduce problems where it is hard to tell if the estimate is too high or too low. **ASK:** Is $93 \div 11$ about $90 \div 10$? (yes, because $93$ is about $90$ and $11$ is about $10$) **ASK:** Is the estimate too high or too low? Explain that it is hard to tell because there are fewer in each group but also fewer objects altogether. This could go either way. For example, we know $20 \div 5 = 4$ but $18 \div 2 = 9$ (higher quotient) and $12 \div 4 = 3$ (lower quotient). In both cases, the dividend and the divisor are smaller than in $20 \div 5$.

Have students decide whether the estimate found by rounding both the dividend and divisor to the nearest ten will be too high, too low, or hard to evaluate (you can’t tell).

**EXAMPLES:**

$38 \div 11$  $57 \div 19$  $91 \div 28$

5. Teach students to use their times tables when estimating. Use $71 \div 9$ to illustrate. **ASK:** How could you estimate $71 \div 9$? Students will likely suggest rounding both numbers to the nearest $10$, as they did above: $70 \div 10 = 7$. **ASK:** What multiple of $9$ is close to $71$? ($72 = 9 \times 8$) How can we use this to find another estimate? ($72 \div 9 = 8$) Which estimate do you think is closer to $71 \div 9$: $70 \div 10$ or $72 \div 9$? Why? **ASK:** How does knowing the times tables by heart make it easier to estimate some quotients?

Have students use the times tables to find estimates for:

$63 \div 8$  $64 \div 9$  $723 \div 9$  $50 \div 7$  $89 \div 11$  $57 \div 11$

Are their estimates too high or too low?
NS4-67
Unit Rates

**GOALS**

Students will understand simple multiplicative relationships involving unit rates.

**PRIOR KNOWLEDGE REQUIRED**

- Money (dollars and cents)
- Distance (km, m and cm)
- Time (weeks, hours)

**VOCABULARY**

- rate
- unit rate

**Explain** that a rate is the comparison of two quantities in different units. For example, “3 apples cost 50¢” is a rate. The units being compared are apples and cents. Have students identify the units being compared in the following rates.

- 5 pears cost $2.
- $1 for 3 kiwis.
- 4 tickets cost $7.
- 1 kiwi costs 35¢.
- Sally is driving at 50 km/hour.
- On a map, 1 cm represents 3 m.
- She earns $6 an hour for babysitting.
- The recipe calls for 1 cup of flour for every teaspoon of salt. (NOTE: for this last example the units are not merely cups and teaspoons, they are cups of flour and teaspoons of salt.)

**Explain** that one of the quantities in a unit rate is always equal to one. Give several examples of unit rates, and have students identify the unit which makes it a unit rate.

- 1 kg of rice per 8 cups of water. (1 kg makes it a unit rate)
- 1 apple costs 30¢. (1 apple)
- $1 for 2 cans of juice. ($1)
- 1 can of juice costs 50¢. (1 can of juice)
- The speed limit is 40 km per hour. (1 hour)
- She runs 1 km in 15 minutes. (1 km)

**Explain** that knowing a unit rate can help to determine other rates. **ASK:** If one book costs $3, how much do two books cost? … three books? … four books?

**Draw a map** with two cities joined by a line. Assuming that 1 cm represents 10 km, have volunteers determine the actual distance between the cities by measuring the line with a metre stick. Then have them explain their calculation for the class.

**ASK:** If you know that two books cost $6, how can you determine the cost for three books? What makes this problem different from the other problems in this lesson? (Instead of starting with the cost of 1 book, we are now starting with the cost of 2 books; we are not given a unit rate) How does working with unit rates make it easier to calculate other rates?

**Working in pairs,** have your students change this problem into a unit rate and then share their procedures with the class. Explain that higher rates can be determined through multiplication of the given rate, but the single unit rate can only be determined through division:

- If 1 peach costs 25¢, then 3 peaches cost 75¢ (3 \times 25¢).
- If 3 peaches cost 75¢, then 1 peach costs 25¢ (75¢ \div 3).
Assign several problems that require your students to determine unit rates. Be sure the answers are whole numbers.

a) 4 pears cost 80¢. How much does 1 pear cost?

b) 24 cans of juice cost $24. How much does 1 can of juice cost?

c) 2 books cost $14. How much does 1 book cost?

d) 3 teachers supervise 90 students on a field trip. How many students does each teacher supervise?

**Extensions**

1. Have students determine the unit rates and then solve the following problems.
   
a) If 4 books cost $20, how much do 3 books cost?
   
b) If 7 books cost $28, how much do 5 books cost?
   
c) If 4 L of soy milk costs $8, how much do 5 L cost?

2. To complete this extension, students need to be able to multiply or divide a three-digit number by a one-digit number. (SEE: Extension 2 from **NS4-65**)

   Bring in some flyers from a grocery store, and ask students to determine unit prices and calculate the cost of quantities greater than one. For instance, if the unit price is $2.75 per item, how much will three items cost? If they do not know how to multiply a decimal number with a single-digit number, challenge them to select and use an alternate unit to dollars—they should use cents. **ASK:** How many cents are in $2.75? If each item costs 275¢, how many cents will three items cost? What does that equal in dollars?

   Ask students to calculate the unit price of an item using division. For instance, if three items cost $1.62, how much will one item cost? Again, have them convert dollars to cents and then back to dollars.
NS4-68

Concepts in Multiplication and Division

GOALS
Students will consolidate their knowledge of multiplication and division.

PRIOR KNOWLEDGE REQUIRED
Multiplication and division
Fact family of equations
Using multiplication to determine higher rates
Using division to determine unit rates
Word problems
Time (hours, minutes)

VOCABULARY
multiple of
divisible by
remainder
perimeter
rate

unit rate
fact family
equation
odd
even

Review the prior knowledge required and vocabulary as needed.

EXAMPLE: Numbers (greater than 0) that are multiples of or divisible by four are the numbers we note when skip counting by four: 4, 8, 12, 16, etc.
NS4-69

Systematic Search

**GOALS**
Students will learn to problem solve.

**PRIOR KNOWLEDGE REQUIRED**
Comparing and ordering numbers
Concept of “closeness” for numbers
Counting by small numbers

**VOCABULARY**
product
difference

**SAY:** I want to make a two-digit number with the digits 1 and 2. How many different numbers can I make? [Two: 12 and 21.] Which two-digit numbers can I make with the digits 3 and 5? … 4 and 7? … 2 and 9?

Ask students to find a three-digit number with the digits 1, 2 and 3. Record their answers, stopping when they have listed all six. Have them write the six different numbers in an organized list. Repeat with the digits 2, 5 and 7, then with the digits 3, 4, and 8, and then with the digits 4, 6 and 7. Ask them to explain how their list is organized (for example, they might organize the numbers by the hundreds digit), and if organization makes it easy or not to know when all of the three-digit numbers have been listed.

Then write two sets of numbers: a) 4, 9, 7 and b) 6, 3, 10.

Have students find the product of all pairs of numbers, with one number taken from each set. **ASK:** Which pair of numbers has the smallest product? … the largest? … the product closest to 50? Did they need the entire set of products to find the pair with the largest product? … the smallest product? … the product closest to 50? The entire set of products might be needed to find the product closest to 50, but only the largest and smallest numbers in each set are needed to find the largest and smallest products. Repeat this exercise for sums and then differences.

**Draw**

Show your students the two ways to arrange these circles in a row.

Then draw a third circle and have your students find all six ways to arrange three circles in a row.

Have volunteers illustrate, then ask if there is an organized way to find all six arrangements. Start by deciding which circle to place first in the arrangement—white, for example. The second circle in the arrangement can then be solid or striped. Have a volunteer illustrate these two arrangements. Have additional volunteers illustrate the arrangements with the solid circle and the striped circle placed first in the arrangement, respectively.

Have your students start exercise 3 of the worksheet, but first explain that mathematicians will often solve simple problems before they solve similar, more complicated problems, as a way to comprehend the problems. How can exercise 3 be simplified? Can fewer than four boxes be used to simplify the problem?
Tell students that tennis balls are sold in packages of 3 and 4. **ASK:** Can we buy exactly 5 tennis balls? … 6 tennis balls? … 7? … 8? … 9? … 10? Students might solve these by randomly guessing and finding solutions: e.g. 7 tennis balls can be bought by buying a can of 3 and a can of 4. Then **ASK:** Can we buy 42? How is this problem different from the previous problems? [The solution is harder to guess randomly.] Have them write an organized list of all of the possible ways of adding threes and fours to total 42. Explain that solving this problem for smaller numbers first will help them to solve the problem for larger numbers, like 42. **ASK:** Can you find ways to use only threes and fours to total 10? What is the greatest number of threes that can be used in the solution? Why can’t four threes be used in the solution? [4 × 3 = 12, which is greater than 10.] Then write

<table>
<thead>
<tr>
<th>No threes</th>
<th>1 three</th>
<th>2 threes</th>
<th>3 threes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0  +  10</td>
<td>3  +  7</td>
<td>6  +  4</td>
<td>9  +  1</td>
</tr>
</tbody>
</table>

Point to the circled numbers and explain that these individual amounts have to be totalled with fours only. **SAY:** If a solution has exactly one three, and the solution is composed of only threes and fours, the remaining numbers have to be fours. Can you use only fours to total 7? Can you use only fours to total 10? Can you use only fours to total 4? To total 1? The only solution is to buy two cans containing three tennis balls and one can containing four tennis balls. Have students repeat this exercise for numbers such as 17 (see below), 35 or 42.

| 0  +  17 | 3  +  14 | 6  +  11 | 9  +  8  | 12  +  5  | 15  +  2  |

The only solution is three cans containing three tennis balls and two cans containing four tennis balls.

Then tell students that pencils are sold in packages of 5 and 6. Can exactly 13 pencils be bought? … 28 pencils? How can they systematically solve the problem? Repeat this exercise with pencils sold in packages of 4 and 6.

Note that there are various answers for the worksheet’s bonus questions.

**Extensions**

1. **How many paths can be drawn from point A to point B using right and up directions only?**

   ![Diagram](image)

   Have students extend the pattern and determine if the number pattern continues.

2. **How many four-digit numbers can be made with the digits 1, 2, 3 and 4 and a thousands digit of 1? … a thousands digit of 2? … a thousands digit of 3? … a thousands digit of 4?** How many four-digit numbers can be made altogether?

3. **Provide the BLM “Elimination Game.”**
GOALS
Students will verbally express fractions given as diagrams or numerical notation.

PRIOR KNOWLEDGE REQUIRED
The ability to count Fraction of an area Ordinal numbers

VOCABULARY
| part | whole | fraction | numerator | denominator | area |

Draw:

Ask your students how many circles are shaded.

Draw __ then ask them again how many circles are shaded.

Explain that the whole circle is no longer shaded. Ask your students if they know the word for a number that is not a whole number, but is only part of a whole number. [Fraction.]

Explain that a fraction has a top and bottom number, then ask your students if they know what the numbers represent. Explain that the shaded fraction of the circle is written as $\frac{3}{4}$ (pronounce this as “three over four” for now).

What does the 3 represent? What does the 4 represent? Draw more fractions and ask students who understand the significance of the numbers to identify the fractions without explaining it to the rest of the class.

Draw examples with different denominators. **ASK:** What does the top number of the fraction represent? [The number of shaded parts.] What does the bottom number of the fraction represent? [The number of parts in a whole.]

If some students say that the bottom number is counting the number of parts altogether, tell them that from what they’ve seen so far, that’s a good answer, but later we will see improper fractions where we have more than one whole pie, so if each pie has 4 pieces and we have 2 pies, there are 8 pieces altogether, but we still write the bottom number as 4. For example, if you bought 2 pies and 3 pieces were eaten from each pie, you could say that $\frac{6}{4}$ of a pie was eaten. (You could also say that $\frac{6}{8}$ of what you bought was eaten, but that would change the whole to 2 pies instead of 1 pie). Explain that fractions aren’t generally pronounced as they are. Ask your students if they know the expressions for “three over four.” [Three-fourths or three-quarters.] Then ask if they know the expression for $\frac{1}{2}$. Draw a half-shaded circle to encourage answers.

Draw several diagrams and have students express the fraction for the shaded parts of each whole by writing the top and the bottom numbers.

Gradually increase the number of parts in each whole and shaded parts.
Ask them to explain their methods (skip counting or multiplication, for example) for determining the total number of squares. At first, shade parts orderly to facilitate a count. Then shade parts randomly, but never exceed more than 15 shaded parts, and start small.

Ask students if they know which number—the top or bottom—is called the numerator. [Top.]

**ASK:** Does anyone know what the bottom number is called? [The denominator.] Which number—the numerator or denominator—expresses the amount of equal parts in the whole?

Have students shade the correct number of parts to illustrate the following fractions.

\[
\begin{align*}
\frac{1}{6} & \quad \frac{2}{5} & \quad \frac{4}{7}
\end{align*}
\]

Then illustrate these fractions—\(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\)—with circles.

Ask students if they know the proper way to pronounce these fractions. Remind them of ordinal numbers. Say: If Sally is first in line, Tom is second in line and Rita is in line behind Tom, in what place is Rita? If Bilal is in line behind Rita, in what place is Bilal? Continue to the eighth position. Explain that most of the ordinal numbers—except for first and second—are also used for fractions. No one refers to half of a pie as one-second of a pie, but we do say one third, one fourth, one fifth, and so on.

Then ask your students to pronounce these fractions: \(\frac{1}{11}, \frac{1}{24}, \frac{1}{13}, \frac{1}{19}, \frac{1}{100}, \frac{1}{92}\).

If your students are comfortable with ordinal numbers up to a hundred, \(\frac{1}{92}\) could lead to some confusion, since “first” and “second” are usually unused when dealing with fractions. In this case, the fraction is expressed as “one ninety-second of a whole.” Explain that fractions with a numerator larger than one are expressed the same way, with the numerator followed by the ordinal number. For example, \(\frac{3}{11}\) is expressed as “three elevenths”. The ordinal number is pluralized when the numerator is greater than one, i.e., one eleventh, two elevenths, three elevenths, and so on. Some ESL students might find it helpful to contrast this with how we say \(200 = \) two hundred, not two hundreds.

Ask your students to pronounce these fractions: \(\frac{3}{14}, \frac{2}{95}, \frac{17}{100}, \frac{94}{95}, \frac{61}{83}, \frac{41}{51}, \frac{30}{52}\).

Include fractions on spelling tests by writing the numeric fraction on the board.

**Extensions**

1. Have students ask their French teacher if ordinal numbers are used for fractions in French, and have them tell you the answer next class.

2. Have students research the origin of the word “fraction” and write a summary in their journals or notebooks.
3. A sport played by witches and wizards on brooms regulates that the players must fly higher than 5 m above the ground over certain parts of the field (shown as shaded). Over what fraction of the field must the players fly higher than 5 m?

![Diagram of a grid representing parts of the field]

**NS4-71**

### Equal Parts and Models of Fractions

**GOALS**

Students will understand that parts of a whole must be equal to determine an entire measurement, when given only a fraction of the entire measurement.

**PRIOR KNOWLEDGE REQUIRED**

Expressing fractions
Fraction of an area
Fraction of a length

**VOCABULARY**

- **numerator**
- **denominator**
- **fraction**
- **part**
- **whole**

Ask your students if they have ever been given a fraction of something (like food) instead of the whole, and gather their responses. Bring a banana (or some easily broken piece of food) to class. Break it in two very unequal pieces. **SAY:** This is one of two pieces. Is this half the banana? Why not? Emphasize that the parts have to be equal for either of the two pieces to be a half. Then draw the following rectangle.

![Rectangle divided into two unequal parts]

Ask your students if they think the rectangle is divided in half. Explain that the fraction \( \frac{1}{2} \) not only expresses one of two parts, but it more specifically expresses one of two equal parts. Draw numerous examples of this fraction, some that are equal and some that are not, and ask volunteers to mark the diagrams as correct or incorrect.

![Examples of fractions]

**ASK:** Which diagram illustrates one-fourth? What’s wrong with the other diagram? Isn’t one of the four pieces still shaded?
Explain that it’s not just shapes like circles and squares and triangles that can be divided into fractions, but anything that can be divided into equal parts. Draw a line and ask if a line can be divided into equal parts. Ask a volunteer to guess where the line would be divided in half. Then ask the class to suggest a way of checking how close the volunteer’s guess is. Have a volunteer measure the length of each part. Is one part longer? How much longer? Challenge students to discover a way to check that the two halves are equal without using a ruler, only a pencil and paper. [On a separate sheet of paper, mark the length of one side of the divided line. Compare that length with the other side of the divided line by sliding the paper over. Are they the same length?]

Have students draw lines in their notebooks and then ask a partner to guess where the line would be divided in half. They can then check their partner’s work.

**ASK:** What fraction of this line is double?

**SAY:** The double line is one part of the line. How many equal parts are in the whole line, including the double line? [5, so the double line is $\frac{1}{5}$ of the whole line.]

Mark the length of the double line on a separate sheet of paper. Compare that length to the entire line to determine how many of those lengths make up the whole line. Repeat with more examples.

Then ask students to express the fraction of shaded squares in each of the following rectangles.

Have them compare the top and bottom rows of rectangles.

**ASK:** Are the same fraction of the rectangles shaded in both rows? Explain. If you were given the rectangles without square divisions, how would you determine the shaded fraction? What could be used to mark the parts of the rectangle? What if you didn’t have a ruler? Have them work as partners to solve the problem. Suggest that they mark the length of one square unit on a separate sheet of paper, and then use that length to mark additional square units.

Prepare several strips of paper with one end shaded, and have students determine the shaded fraction without using a pencil or ruler. Only allow them to fold the paper.

Draw several rectangles with shaded decimetres.
Have students divide the rectangles into the respective number of equal parts (3, 4 and 5) with only a pencil and paper. Have them identify the fractions verbally and orally. Then draw a rectangle with two shaded decimetres (i.e. two-fifths), and challenge your students to mark units half the length of the shaded decimetres. How can this be done with only a pencil and paper?

Fold the paper so that these markings meet. Draw a marking along the fold.

Draw a shaded square and ask students to extend it so the shaded part becomes half the size of the extended rectangle.

Repeat this exercise for squares becoming one-third and one-quarter the size of extended rectangles. ASK: How many equal parts are needed? [Three for one-third, four for one-quarter.] How many parts do you already have? [1.] So how many more equal parts are needed? [Two for one-third, three for one-quarter.]

This is a good activity to do at the end of a day, so that students with extra time can play with the left-over play dough until the end of class.

Prepare enough small balls of coloured play dough for 3 for each student (they will only need two, but this allows students to choose their colours). Demonstrate to students how to make fractions using play dough. Tell them that you are going to roll one spoon of red play dough and three spoons of blue play dough into a ball. Explain the necessity of flattening the play dough on a spoon so that each spoonful is the same size. Demonstrate not leaving any play dough of the first colour on the spoon. Roll the play dough into a ball carefully mixing it so that it becomes a uniform colour. This has been tried with fresh play dough only; store-bought play dough may not produce the same effect and may be more difficult to mix thoroughly. For a recipe, SEE:

http://www.teachnet.com/lesson/art/playdoughrecipes/traditional.html

An alternative to play dough is to use small spoonfuls of food colouring.

ASK: How many spoons of play dough have I used altogether? How many spoons of red play dough have I used? What fraction of this ball is red? How many spoons of blue play dough have I used? What fraction of this ball is blue? Write the “recipe” on a triangular flag (see below) which can be made from a quarter of a regular sheet taped to a straw (insert the straw into your ball of play dough). The recipe is shown on the flag below:

(Continued on next page.)
Extensions

1. Teach students that they can even take a fraction of an angle. Draw two angles on the board:

   A. \[ \hspace{2cm} \]
   B. \[ \hspace{2cm} \]

   **ASK:** Which angle is bigger? (angle A) Tell students that you think that angle B is about half the size of angle A. **ASK:** What do I mean by that? How can I check? Explain that, in order for angle B to be half the size of angle A, you would need two angles, the same size as B, to together make an angle the same size as A. Have students discuss in pairs a way to find out if angle B is half of angle A. Tell students that they are allowed to use paper and pencil and may need to come to the board to try out their solution once they have a solution in mind.

   **SOLUTION:** Trace angle B twice so that both copies are next to each other:

   \[ \hspace{2cm} \]

   Then line up this angle with angle A. Do the angles coincide? (yes)

   If students enjoy this exercise, have them determine whether a given angle is more than half, less than half, or exactly half of another given angle. Students may even wish to find the entire angle when half is given, or a third is given, or a quarter is given.

   The smaller angle is what fraction of the larger angle?

   \[ \hspace{2cm} \]

   **HINT:** Use tracing paper.
Draw the whole angle if the given angle is $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of the whole angle.

a) $\frac{1}{2}$  

b) $\frac{1}{3}$  

c) $\frac{1}{4}$

2. a) Sketch a pie and cut it into fourths. How can it be cut into eighths?
   b) Sketch a pie and cut it into thirds. How can it be cut into sixths?

3. a) Sketch a pie and cut it into fourths. How can it be cut into eighths?
   b) Sketch a pie and cut it into thirds. How can it be cut into sixths?

5. (Adapted from Atlantic Curriculum)
   Ask students to divide the rectangle…
   a) into thirds two different ways.
   b) into quarters three different ways.

**NOTE:** The division shown below may not be obviously divided into quarters until it is further divided into eighths.
NS4-72
Equal Parts of a Set

Review equal parts of a whole. Tell your students that the whole for a fraction might not be a shape like a circle or square. Tell them that the whole can be anything that can be divided into equal parts. Brainstorm with the class other things that the whole might be: a line, an angle, a container, apples, oranges, amounts of flour for a recipe. Tell them that the whole could even be a group of people. For example, the grade 4 students in this class is a whole set and I can ask questions like: what fraction of students in this class are girls? What fraction of students in this class are nine years old? What fraction of students wear glasses? What do I need to know to find the fraction of students who are girls? (The total number of students and the number of girls). Which number do I put on top: the total number of students or the number of girls? (the number of girls). Does anyone know what the top number is called? (the numerator) Does anyone know what the bottom number is called? (the denominator) What number is the denominator? (the total number of students). What fraction of students in this class are girls? (Ensure that they say the correct name for the fraction.) Tell them that the girls and boys don’t have to be the same size; they are still equal parts of a set. Ask students to answer: What fraction of their family is older than 9? Younger than 9? Female? Male? Some of these fractions, for some students, will have numerator 0, and this should be pointed out. Avoid asking questions that will lead them to fractions with a denominator of 0 (For example, the question “What fraction of your siblings are male?” will lead some students to say 0/0).

Then draw pictures of shapes with two attributes changing:

a)

ASK: What fraction of these shapes are shaded? What fraction are circles? What fraction of the circles are shaded?

b)

ASK: What fraction of these shapes are shaded? What fraction are unshaded? What fraction are squares? Triangles? What fraction of the triangles are shaded? What fraction of the squares are shaded? What fraction of the squares are not shaded?

Then ask students for each picture below to write in their notebooks:

a) What fraction are circles?
b) What fraction are shaded?
c) What fraction are squares?
d) What fraction are triangles?
Part 2
Number Sense 51
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i)  

ii)  

iii)  

Bonus  
(pictures with 3 attributes changing)  

Have students answer the same questions as above and then more complicated questions like:  
What fraction of the triangles are shaded? (So the triangles are now the whole set). What fraction of the shaded shapes are triangles? **ASK:** Now what is the whole set?  

Have students make up questions to ask each other.  

Draw on the board:  

c)  

Have students volunteer questions to ask and others volunteer answers.  
Then have students write fraction statements in their notebooks for similar pictures.  

Ask some word problems:  

A basketball team played 5 games and won 2 of them. What fraction of the games did the team win?  

A basketball team won 3 games and lost 1 game. How many games did they play altogether? What fraction of their games did they win?  

A basketball team won 4 games, lost 1 game and tied 2 games. How many games did they play? What fraction of their games did they win?  

Also give word problems that use words such as “and,” “or” and “not”:  

Sally has 4 red marbles, 2 blue marbles and 7 green marbles.  

a) What fraction of her marbles are red?  

b) What fraction of her marbles are blue or red?  

C) What fraction of her marbles are not blue?  

d) What fraction of her marbles are not green?  

**Bonus**  
Which two questions have the same answer? Why?  

Challenge your students to write another question that uses “or” that will have the same answer as:  
What fraction of her marbles are not blue?
Look at the shapes below:

a) What fraction of the shapes are shaded and circles?
b) What fraction of the shapes are shaded or circles?
c) What fraction of the shapes are not circles?

Write a question that has the same answer as c):

What fraction of the shapes are _____ or _____?

Have students go the other way—that is, have them find the number of each item given the fractions.

a) A team played 5 games. They won \(\frac{2}{5}\) of their games and lost \(\frac{3}{5}\) of their games. How many games did they win? Lose?
b) There are 7 marbles. \(\frac{2}{7}\) are red, \(\frac{4}{7}\) are blue and \(\frac{1}{7}\) are green. How many blue marbles are there? Red marbles? Green marbles?

Then tell your students that you have five squares and circles. Some are shaded and some are not. Have students draw shapes that fit the puzzles:

a) \(\frac{2}{5}\) of the shapes are squares. \(\frac{2}{5}\) of the shapes are shaded. One circle is shaded.

\[ \text{SOLUTION:} \]

\[ \bigcirc \bigcirc \bigcirc \square \square \]

b) \(\frac{3}{5}\) of the shapes are squares. \(\frac{3}{5}\) of the shapes are shaded. No circle is shaded.

c) \(\frac{3}{5}\) of the shapes are squares. \(\frac{3}{5}\) of the shapes are shaded. \(\frac{1}{3}\) of the squares are shaded.

On a geoboard, have students enclose a given fraction of the pegs with an elastic. For instance \(\frac{10}{25}\).

Ask students what fraction of the pegs are not enclosed? \(\frac{15}{25}\). They should see that the two numerators (in this case 10 and 15) always add up to the denominator (25).

(Adapted from Atlantic Curriculum) Have the students "shake and spill" a number of two-coloured counters and ask them to name the fraction that represents the red counters. What fraction represents the yellow counters? What do the two numerators add to? Why?
Extensions

1. What fraction of the letters in your full name are vowels?

2. Have students investigate:
   a) What fraction of the squares on a Monopoly board are "Chance" squares? What fraction of the squares are properties that can be owned?
   b) On a Snakes and Ladders board, on what fraction of the board would you be forced to move down a snake?

Have students make up their own fraction question about a board game they like and tell you the answer next class.

3. Draw a picture to solve the puzzle. There are 7 triangles and squares. \( \frac{2}{7} \) of the figures are triangles. \( \frac{3}{7} \) are shaded. 2 triangles are shaded.

4. Give your students harder puzzles by adding more attributes and more clues:

   There are 5 squares and circles. \( \frac{1}{3} \) of the squares are big.

   \( \frac{2}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the squares are shaded.

   \( \frac{1}{3} \) of the shapes are big. No shaded shape is big.

**SOLUTION:**

![Picture of shapes]

5. Ask students to make and identify as many fractions in the classroom as they can, for instance:

   \( \frac{1}{4} \) of the blackboard is covered in writing, \( \frac{2}{3} \) of the counters are red, \( \frac{1}{3} \) the length of a 30 cm ruler is 15 cm, about \( \frac{1}{3} \) of the door is covered by a window, \( \frac{11}{25} \) of the class has black hair.

6. a) There are 5 circles and triangles. Can you draw a set so that:

   i) \( \frac{4}{5} \) are circles and \( \frac{2}{5} \) are striped?

   Have volunteers show the different possibilities before moving on. Ask questions like: How many are striped circles? How many different answers are there?

   ii) \( \frac{1}{5} \) are circles and \( \frac{4}{5} \) are triangles?

   What is the same about these two questions? (the numbers are the same in both) What is different? [one is possible, the other is not; one uses the same attribute (shape) in both, the other uses two different attributes (shape and shading)].

   b) On a hockey line of 5 players, \( \frac{4}{5} \) are good at playing forward and \( \frac{3}{5} \) are good at playing defense. How many could be good at playing both positions? Is there only one answer? Which question from 4 a) is this similar to? What is similar about it?
NS4-73
Parts and Wholes

**GOALS**
Students will make divisions not already given to form equal-sized parts.

**PRIOR KNOWLEDGE REQUIRED**
A fraction of an area is a number of equal-sized parts out of a total number of equal-sized parts. The whole a fraction is based on can be anything.

**VOCABULARY**
- fraction
- numerator
- whole
- denominator
- part

---

Draw on the board the shaded strips from before:

ASK: Is the same amount shaded on each strip? Is the same fraction of the whole strip shaded in each case? How do you know? Then draw two hexagons as follows:

ASK: Is the same amount shaded on each hexagon? What fraction of each hexagon is shaded?

Then challenge students to find the fraction shaded by drawing their own lines to divide the shapes into equal parts:

Give your students pattern blocks. Ask them to make a rhombus from the triangles. How many triangles do they need? What fraction of a rhombus is a triangle? Challenge them to find:

a) What fraction of a hexagon is the rhombus? The trapezoid?
   The triangle?

b) What fraction of a trapezoid is the triangle?

c) **BONUS**: What fraction of a trapezoid is the rhombus?
Extensions

1. a) What fraction of a tens block is a ones block?
   b) What fraction of a tens block is 3 ones blocks?
   c) What fraction of a hundreds block is a tens block?
   d) What fraction of a hundreds block is 4 tens blocks?
   e) What fraction of a hundreds block is 32 ones blocks?
   f) What fraction of a hundreds block is 3 tens blocks and 2 ones blocks?

2. On a geoboard, show 3 different ways to divide the area of the board into 2 equal parts.
   
   EXAMPLES:

3. Give each student a set of pattern blocks. Ask them to identify the whole of a figure given a part.
   a) If the pattern block triangle is \( \frac{1}{6} \) of a pattern block, what is the whole?
      ANSWER: The hexagon.
   b) If the pattern block triangle is \( \frac{1}{3} \) of a pattern block, what is the whole?
      ANSWER: The trapezoid.
   c) If the pattern block triangle is \( \frac{1}{2} \) of a pattern block, what is the whole?
      ANSWER: The rhombus.
   d) If the rhombus is \( \frac{1}{6} \) of a set of pattern blocks, what is the whole?
      ANSWER: 2 hexagons or 6 rhombuses or 12 triangles or 4 trapezoids.

4. Students can construct a figure using the pattern block shapes and then determine what fraction of the figure is covered by the pattern block triangle.

5. What fraction of the figure is covered by...
   a) The shaded triangle
   b) The small square
NS4-74
Ordering and Comparing Fractions

GOALS
Students will understand that as the numerator increases and the denominator stays the same, the fraction increases.

PRIOR KNOWLEDGE REQUIRED
Naming fractions
Fractions show same-sized pieces

VOCABULARY
part
numerator
whole
denominator
fraction

Draw on the board:

Have students name the fractions shaded and then have them say which circle has more shaded. ASK: Which is more: one fourth of the circle or three fourths of the circle?

Then draw different shapes on the board:

Have volunteers show the fractions on the board by shading and ASK: Is three quarters of something always more than one quarter of the same thing? Is three quarters of a metre longer or shorter than a quarter of a metre? Is three quarters of a dollar more money or less money than a quarter of a dollar? Is three fourths of an orange more or less than one fourth of the orange? Is three fourths of the class more or less people than one fourth of the class? If three fourths of the class have brown eyes and one quarter of the class have blue eyes, do more people have brown eyes or blue eyes?

Tell students that if you consider fractions of the same whole—no matter what whole you’re referring to—three quarters of the whole is always more than one quarter of that whole, so mathematicians say that the fraction $\frac{3}{4}$ is greater than the fraction $\frac{1}{4}$. Ask students if they remember what symbol goes in between:

$\frac{3}{4} \quad \square \quad \frac{1}{4}$

(< or >).

Remind them that the inequality sign is like the mouth of a hungry person who wants to eat more of the pasta but has to choose between three quarters of it or one quarter of it. The sign opens toward the bigger number:

$\frac{3}{4} \quad > \quad \frac{1}{4}$ or $\frac{1}{4} \quad < \quad \frac{3}{4}$.

Then draw two circles divided into eighths and shade 3 parts on the first circle and 5 parts on the other one. ASK: Which circle has a greater area shaded?
Which is more: three eighths of the circle or five eighths of the circle? Do you think that 6 eighths of the circle will be more or less than 5 eighths of the circle? Will 2 eighths be more or less than 3 eighths?

Have students decide which is more and to write the appropriate inequality in between the numbers:

\[
\frac{2}{3} \quad \text{or} \quad \frac{3}{5}
\]

Repeat with several examples, eventually having students name the fractions as well:

\[
\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}
\]

Draw the following pictures on the board:

**ASK:** Which is greater: one quarter or two quarters? One eighth or two eighths? One sixth or two sixths? Show students a pie cut into sixths on the board and ask: If Sally gets one sixth and Tony gets two sixths, who gets more? If Sally gets three sixths and Tony gets two sixths, who gets more? Which is greater:

\[
\frac{1}{9} \quad \text{or} \quad \frac{2}{9} ? \quad \frac{1}{12} \quad \text{or} \quad \frac{2}{12} ? \quad \frac{1}{53} \quad \text{or} \quad \frac{2}{53} ? \quad \frac{1}{100} \quad \text{or} \quad \frac{2}{100} ? \quad \frac{1}{807} \quad \text{or} \quad \frac{2}{807} ?
\]

\[
\frac{2}{9} \quad \text{or} \quad \frac{5}{9} ? \quad \frac{3}{11} \quad \text{or} \quad \frac{4}{11} ? \quad \frac{9}{11} \quad \text{or} \quad \frac{8}{11} ? \quad \frac{35}{67} \quad \text{or} \quad \frac{43}{67} ? \quad \frac{91}{102} \quad \text{or} \quad \frac{54}{102} ?
\]

**Bonus**

\[
\frac{742}{25401} \quad \text{or} \quad \frac{809}{25401} ? \quad \frac{52645}{4567341} \quad \text{or} \quad \frac{54154}{4567341} ?
\]

Then ask students to order a list of fractions with the same denominator (EXAMPLE: \(\frac{2}{7}, \frac{5}{7}, \frac{4}{7}\)) from least to greatest, eventually using bigger numerators and denominators and eventually using lists of 4 fractions.

**Bonus**

\[
\frac{4}{21}, \frac{11}{21}, \frac{8}{21}, \frac{19}{21}, \frac{6}{21}, \frac{12}{21}, \frac{5}{21}
\]

Then write on the board:

Ask students to think of numbers that are in between these two numbers. Allow several students to volunteer answers. Then ask students to write individually in their notebooks at least one fraction in between:

\[
a) \quad \frac{4}{11}, \quad \frac{9}{11} \\
b) \quad \frac{3}{12}, \quad \frac{9}{12} \\
c) \quad \frac{4}{16}, \quad \frac{13}{16} \\
d) \quad \frac{21}{48}, \quad \frac{25}{48} \\
e) \quad \frac{67}{131}, \quad \frac{72}{131}
\]

**Bonus**

\[
\frac{104}{18301}, \quad \frac{140}{18301}
\]
**GOALS**

Students will understand that as the numerator stays the same and the denominator increases, the fraction decreases.

**PRIOR KNOWLEDGE REQUIRED**

Naming fractions
Fractions show same-sized pieces
As the numerator increases and the denominator stays the same, the fraction increases

**VOCABULARY**

part numerator
whole denominator
fraction

---

Draw on the board:

\[
\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4}
\]

Have a volunteer colour the first part of each strip of paper and then ask students which fraction shows the most: \(\frac{1}{2}\), \(\frac{1}{3}\) or \(\frac{1}{4}\). **ASK:** Do you think one fifth of this fraction strip will be more or less than one quarter of it? Will one eighth be more or less than one tenth? Then draw two circles the same size on the board:

![Circle diagram]

Have volunteers shade one part of each circle. **ASK:** What fractions are represented? Which fraction represents more? Repeat with different shapes:

![Different shapes diagram]

**ASK:** Is one half of something always more than one quarter of the same thing? Is half a metre longer or shorter than a quarter of a metre? Is half an hour more or less time than a quarter of an hour? Is half a dollar more money or less money than a quarter of a dollar? Is half an orange more or less than a fourth of the orange? Is half the class more or less than a quarter of the class? If half the class has brown eyes and a quarter of the class has green eyes, do more people have brown eyes or green eyes?

Tell students that no matter what quantity you have, half of the quantity is always more than a fourth of it, so mathematicians say that the fraction \(\frac{1}{2}\) is greater than the fraction \(\frac{1}{4}\). Ask students if they remember what symbol goes in between: \(\frac{1}{2} \quad \frac{1}{4}\) (< or >).
Tell your students that you are going to try to trick them with this next question so they will have to listen carefully. Then **ASK:** Is half a minute longer or shorter than a quarter of an hour? Is half a centimetre longer or shorter than a quarter of a metre? Is half of Stick A longer or shorter than a quarter of Stick B?

Stick A: 

Stick B: 

**ASK:** Is a half always bigger than a quarter?

Allow everyone who wishes to attempt to articulate an answer. Summarize by saying: A half of something is always more than a quarter of the same thing. But if we compare different things, a half of something might very well be less than a quarter of something else. When mathematicians say that $\frac{1}{2} > \frac{1}{4}$, they mean that a half of something is always more than a quarter of the same thing; it doesn’t matter what you take as your whole, as long as it’s the same whole for both fractions.

Draw the following strips on the board:

```
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Ask students to name the fractions and then to tell you which is more.

Have students draw the same fractions in their notebooks but with circles instead of strips. Is $\frac{3}{4}$ still more than $\frac{3}{8}$? (yes, as long as the circles are the same size)

**Bonus**

Show the same fractions using a line of length 8 cm.

Ask students: If you cut the same strip into more and more pieces of the same size, what happens to the size of each piece?

Draw the following picture on the board to help them:

```
<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

**ASK:** Do you think that 1 third of a pie is more or less pie than 1 fifth of the same pie? Would you rather have one piece when it’s cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically ($\frac{1}{3} > \frac{1}{5}$).

Do you think 2 thirds of a pie is more or less than 2 fifths of the same pie? Would you rather have two pieces when the pie is cut into 3 pieces or 5 pieces? Which way will get you more? Ask a volunteer to show how we write that mathematically ($\frac{2}{3} > \frac{2}{5}$).

If you get 7 pieces, would you rather the pie be cut into 20 pieces or 30? Which way will get you more pie? How do we write that mathematically? ($\frac{7}{20} > \frac{7}{30}$).
Give students several problems similar to QUESTION 2 in the workbook. Use bigger numerators and denominators as bonus, always keeping the numerators the same.

Extra Bonus
Have students order the following list of numbers:

\[
\frac{21}{28} \quad \frac{21}{22} \quad \frac{8}{105} \quad \frac{13}{200} \quad \frac{19}{28} \quad \frac{13}{105} \quad \frac{19}{61}
\]

SAY: Two fractions have the same numerator and different denominators. How can you tell which fraction is bigger? Why? Summarize by saying that the same number of pieces gives more when the pieces are bigger. The numerator tells you the number of pieces, so when the numerator is the same, you just look at the denominator. The bigger the denominator, the more pieces you have to share between and the smaller the portion you get. So bigger denominators give smaller fractions when the numerators are the same.

SAY: If two fractions have the same denominator and different numerators, how can you tell which fraction is bigger? Why? Summarize by saying that if the denominators are the same, the size of the pieces are the same. So just as 2 pieces of the same size are more than 1 piece of that size, 84 pieces of the same size are more than 76 pieces of that size.

Emphasize that students can’t do this sort of comparison if the denominators are not the same.

ASK: Would you rather 2 fifths of a pie or 1 half? Draw the following picture to help them:

Tell your students that when the denominators and numerators of the fractions are different, they will have to compare the fractions by drawing a picture or by using other methods that they will learn later.

Give students their play dough flags made in a previous activity. Have them organize themselves into groups with people who chose the same two colours they did. For example, suppose 5 people chose red and blue as their two colours. Have those 5 students order their colours in terms of most red to least red and then order the fractions for red from the recipes in order from greatest to least. All students in the group should individually check their results by using fraction strips. Before distributing the BLM “Fraction Strips”, ask the class as a whole why they might expect slight disagreements with the fraction strip results and the play dough results. Which order of fractions do they think will be the correct order? What mistakes may have been made when making the play dough balls? (Some spoons may have had some red play dough still in them when making blue; the play dough may not have been completely flattened all the time.)
Extensions

1. Compare the following fractions by comparing how much of a whole pie is left if the following amounts are eaten: $\frac{3}{4}$ or $\frac{4}{5}$. Emphasize that the fraction with a bigger piece left-over is the smaller fraction.

2. Write the following fractions in order from least to greatest. Explain how you found the order.

   \[
   \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{4}{5}, \frac{1}{8}
   \]

3. Give each student three strips of paper. Ask them to fold the strips to divide one strip into halves, one into quarters, one into eighths. Use the strips to find a fraction between

   a) $\frac{3}{8}$ and $\frac{5}{8}$ (one answer is $\frac{1}{2}$)
   
   b) $\frac{1}{2}$ and $\frac{3}{8}$ (one answer is $\frac{3}{8}$)
   
   c) $\frac{5}{8}$ and $\frac{7}{8}$ (one answer is $\frac{3}{4}$)

4. Have students fold a strip of paper (the same length as they folded in EXTENSION 3) into thirds by guessing and checking. Students should number their guesses.

   EXAMPLE:

   ![Fraction Strip Example](image)

   Is $\frac{1}{3}$ a good answer for any part of Extension 3? How about $\frac{2}{3}$?

5. Why is $\frac{2}{3}$ greater than $\frac{2}{5}$? Explain.

6. Why is it easy to compare $\frac{5}{8}$ and $\frac{1}{5}$? Explain.

7. Have students compare $\frac{13}{87}$ and $\frac{14}{86}$ by finding a fraction with the same numerator as one of them and the same denominator as the other that is in between both fractions. For instance, $\frac{13}{86}$ is clearly smaller than $\frac{16}{86}$ and bigger than $\frac{13}{87}$. Another way to compare the two given fractions is to note that the second fraction has more pieces (14 instead of 13) and each piece is slightly bigger, so it must represent a bigger fraction.

8. Have students use fraction strips to compare fractions to the benchmarks of 0, $\frac{1}{2}$ and 1. (e.g. $\frac{3}{8}$ is closer to $\frac{1}{2}$ than to 0; $\frac{4}{5}$ is closer to 1 than to $\frac{1}{2}$)
NS4-76
Parts and Wholes (Advanced)

Much of this worksheet is a review.

Have students copy the shapes below onto grid paper and tell them that these shapes are only half of the whole. Then have them draw the whole.

Draw this shape on the board:

Tell your students that a piece of paper was cut into three equal parts and this was one of the parts. Have a volunteer measure how long the part is. This part is one of three parts. How long are the other two parts? How long was the whole piece originally?

Have students copy the shapes below onto grid paper and tell them that these shapes are only one third of the whole. Then have them draw the whole.

Have a volunteer colour one third of the strip:

**ASK:** How much was not coloured? If one third of a pie was eaten, how much was not eaten? If one third of a box of crayons are red crayons, how much is not red? Tell them that one third and two thirds make one whole.

**ASK:** If one quarter of a pie was eaten, how much was not eaten? Write on the board: 

\[
\frac{1}{4} \quad \text{and} \quad \ \ 
\]

make one whole. Have a volunteer write the missing fraction in the box.
Introduce mixed fractions by drawing the following picture on the board:

```
  □□□□
```

**ASK:** What fraction is shaded? What fraction is not shaded? Then have a volunteer fill in the correct fraction: \(\frac{2}{5}\) and \(\square\) make one whole. Repeat with other fractions.

Review the concept that the same fraction of different-sized objects will be different sizes and that a larger fraction of a smaller object could be either smaller or larger than a smaller fraction of a larger object.

**GOALS**

Students will recognize and name mixed fractions.

**PRIOR KNOWLEDGE REQUIRED**

Fractions as area
Pies as models for fractions
Naming fractions
Doubling

Write a fraction such as \(3\frac{1}{4}\) on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.
Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

**EXAMPLE:**

![Pattern Image]

They should shade the first 3 circles and 1 part of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

**EXAMPLES:** $2 \frac{1}{4}$, $3 \frac{1}{6}$, $1 \frac{5}{6}$, $2 \frac{1}{3}$, $3 \frac{5}{8}$.

If students have trouble, give them practice drawing the whole number of pies drawn with the correct number of pieces.

Give students counters to make a model of the following problem:

Postcards come in packs of 4. How many packs would you need to buy to send 15 postcards? Write a mixed and improper fraction for the number of packs you would use.

Students could use a counter of a particular colour to represent the postcards they have used and a counter of a different colour to represent the cards left over. After they have made their model, students could fill in the following chart.

<table>
<thead>
<tr>
<th>Number of Postcards</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of packs of 4 postcards (improper fraction)</td>
<td>$\frac{15}{4}$</td>
</tr>
<tr>
<td>Number of packs of 4 postcards (mixed fraction)</td>
<td>$3 \frac{3}{4}$</td>
</tr>
</tbody>
</table>

**MODEL:**

4 postcards in each package

![Pie Model Image]

Here is another sample problem students could try:

Juice cans come in boxes of 6. How many boxes would you bring if you needed 20 cans? What fraction of the boxes would you use?

**Extensions**

1. Students should know how to count forwards by halves, thirds, quarters, and tenths beyond 1.

Ask students to complete the patterns:

- a) $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, $\frac{8}{4}$, $\frac{9}{4}$, $\frac{10}{4}$
- b) $2 \frac{1}{4}$, $3 \frac{1}{4}$, $4 \frac{1}{4}$, $5 \frac{1}{4}$, $6 \frac{1}{4}$, $7 \frac{1}{4}$, $8 \frac{1}{4}$, $9 \frac{1}{4}$, $10 \frac{1}{4}$
- c) $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$, $\frac{7}{3}$, $\frac{8}{3}$, $\frac{9}{3}$, $\frac{10}{3}$
- d) $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$, $\frac{9}{5}$, $\frac{10}{5}$
2. What model represents $2 \frac{3}{4}$? How do you know?

\[ \text{A.} \quad \text{B.} \]

3. **ASK:** Which fractions show more than a whole? How do you know?

\[ 2 \frac{3}{5}, \quad \frac{2}{7}, \quad 1 \frac{4}{5}, \quad 2 \frac{1}{3}, \quad \frac{2}{8} \]

4. Ask students to order these fractions from least to greatest: $3 \frac{1}{6}, \quad 1 \frac{5}{7}, \quad 7 \frac{1}{11}$

**ASK:** Did they need to look at the fractional parts at all or just the whole numbers? Why?

**Bonus**

Order the following list of numbers: $3 \frac{1}{6}, \quad 5 \frac{7}{9}, \quad 2 \frac{1}{11}, \quad 6 \frac{1}{5}, \quad 8 \frac{5}{9}, \quad 4 \frac{3}{10}$

5. Ask students to order mixed fractions where some of the whole numbers are the same, and the fractional parts have either the same numerator or the same denominator.

**EXAMPLES:**

a) $3 \frac{1}{6}, \quad 5 \frac{7}{9}, \quad 3 \frac{1}{9}$

b) $5 \frac{3}{9}, \quad 5 \frac{5}{9}, \quad 6 \frac{1}{11}$

**Bonus**

Give students longer lists of numbers that they can order using these strategies.

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**NS4-78**

**Improper Fractions**

**GOALS**
Students will name improper fractions and fractions representing exactly one whole.

**PRIOR KNOWLEDGE REQUIRED**
Mixed Fractions

**VOCABULARY**
mixed fraction fraction improper proper fraction

Draw the following shapes on the board:

\[ \text{Circle} \quad \text{Square} \quad \text{Triangle} \quad \text{Rectangle} \]

Have students name the fractions shaded. **ASK:** How many parts are shaded? How many parts are in one whole? Tell them that they are all 1 whole and write $1 = \frac{4}{2}$ and $1 = \frac{6}{6}$. Then have student volunteers fill in the blanks:

\[ 1 = \boxed{\frac{7}{9}} \quad 1 = \boxed{\frac{7}{9}} \]

Then tell them that sometimes they might have more than 1 whole—they might have two whole pizzas, for **EXAMPLE:**

\[ \text{Two circles} \quad \text{and} \quad \text{Two triangles} \]

**ASK:** Which number goes on top—the number of parts that are shaded or the number of parts in one whole? Tell them to look at the pictures. Ask them how many parts are in one whole circle and how many parts are shaded. Then
write:

\[ 2 = \frac{2}{2} \quad \text{and} \quad 2 = \frac{3}{3} \]

Then ask a volunteer to come and write the number of shaded pieces.

**ASK:** How are the numerator and denominator in each fraction related? (you double the denominator to get the numerator). Have students fill in the missing numbers:

\[ \frac{2}{4} \quad \frac{2}{28} \quad \frac{2}{76} \quad \frac{2}{10} \quad \frac{2}{62} \]

(Always use an even number when giving the numerator.)

**ASK:** How are the fractions above different from the fractions we’ve seen so far? Tell them these fractions are called improper fractions because the numerator is larger than the denominator. Challenge students to guess what a fraction is called if its numerator is smaller than its denominator. (proper fractions)

Draw on the board:

\[ \begin{array}{ccc} \circ & \circ & \circ \\ \circ & \circ \end{array} \]

**ASK:** How many pieces are shaded? (9)

**SAY:** I want to write a fraction for this picture. Should 9 be the numerator or the denominator? (numerator) Do I usually put the number of shaded parts on top or on bottom? (top) How many equal parts are in 1 whole? (4) Should this be the numerator or the denominator? (denominator) Do we usually put the number of parts in 1 whole on top or on bottom? (bottom) Tell your students that the fraction is written \( \frac{9}{4} \).

Have volunteers write improper fractions for these pictures.

a) 

\[ \begin{array}{ccc} \circ & \circ & \circ \\ \circ & \circ \end{array} \]

b) 

\[ \begin{array}{ccc} \circ & \circ & \circ \\ \circ & \circ \end{array} \]

c) 

\[ \begin{array}{cccc} \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \end{array} \]

**ASK:** How many parts are shaded? How many parts are in one whole?

Draw models of several improper fractions, asking students to name them in their notebooks. Use a variety of shapes such as rectangles and triangles for the whole.

Write a fraction such as \( \frac{15}{4} \) on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.
EXAMPLE:

\[
\frac{15}{4}
\]

They should shade the first 3 circles and 3 parts of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

EXAMPLE: \(\frac{11}{4}, \frac{15}{8}, \frac{19}{8}, \frac{10}{3}, \frac{12}{5}\).

ASK: Which fractions show more than a whole? How do they know?

\(\frac{13}{9}, \frac{2}{9}, \frac{14}{15}, \frac{8}{3}, \frac{12}{7}\)

NS4-79
Mixed and Improper Fractions

ASK: What is a mixed fraction? What is an improper fraction? Have a volunteer write a mixed fraction for this picture and explain their answer:

Have another volunteer write an improper fraction for the same picture and explain their answer.

Draw several models of fractions larger than 1 on the board and have students write both the mixed fraction and the improper fraction. Use several different shapes other than circles.

Draw several more such models on the board and ask students to write an improper fraction if the model contains more than 2 whole pies, and a mixed fraction otherwise. This will allow you to see if students know the difference between the terms “mixed” and “improper”.

Tell students to draw models for the following mixed fractions and to write the corresponding improper fraction that is equal to it (this may be done in 2 different steps if students need it broken down).

\[
\begin{align*}
a) \ 2 \frac{3}{4} & \quad b) \ 3 \frac{1}{3} & \quad c) \ 2 \frac{1}{6} & \quad d) \ 1 \frac{5}{6} & \quad e) \ 3 \frac{2}{5} & \quad f) \ 2 \frac{7}{8}
\end{align*}
\]

Then tell students to draw models for the following improper fractions and then to write the mixed fraction that is equal to it.

\[
\begin{align*}
a) \ 13 \frac{1}{4} & \quad b) \ 7 \frac{7}{8} & \quad c) \ 11 \frac{1}{6} & \quad d) \ 19 \frac{5}{6} & \quad e) \ 27 \frac{2}{6}
\end{align*}
\]

SAY: You have written many numbers as both a mixed and an improper fraction. What is the same in both? What is different? (the denominators are the same because they tell you how many parts are in a whole, but the numerators will be different because the mixed fraction counts the pieces that make up the wholes separately from the pieces that only make up part of the whole; improper fractions count them all together)
Extensions

1. Have students write their answers to these questions as both mixed and improper fractions.
   - What fraction of a tens block is 7 ones blocks? 17 ones blocks? 32 ones blocks?
   - What fraction of a hundreds block is 32 tens blocks? 43 tens blocks and 5 ones blocks?
   - How many metres are in 230 cm? 571 cm?
   - How many decimeters are in 54 centimetres? 98 cm?
   - What fraction of a dime is a quarter?

2. Ask students to solve these problems with pattern blocks or by sketching their answers on triangular grid paper.
   a) Which two whole numbers is \( \frac{23}{6} \) between?
   b) What mixed fraction of a pie would you have if you took away \( \frac{1}{6} \) of a pie from 3 pies (and what would the improper fraction be)?

NS4-80
Investigating Mixed & Improper Fractions

Give students pattern blocks or a copy of the pattern blocks BLM (and have them cut out the shapes).

Tell students that a hexagon represents one whole pie and ask them to show you a whole pie. If some students don’t know which piece is the hexagon, **ASK:** How many sides does a hexagon have? (6) When all students have shown you the hexagon, ask them how many triangles they would need to cover an entire hexagon. What fraction of the hexagon is a triangle? (\( \frac{1}{6} \)) Find a shape that is half of the hexagon. How many trapezoids would you need to make \( 1 \frac{1}{2} \) hexagons? How many trapezoids would they need to make \( 7 \frac{1}{2} \) hexagons? What fraction of a hexagon is a rhombus? How many rhombuses would you need to make 2 hexagons? What improper fraction is equal to 2 wholes? Tell them that there is no mixed fraction for 2 wholes—it is just a whole number.

I want to make \( 1 \frac{1}{2} \) hexagons. How many equal-sized pieces should the hexagon be divided into? What shape should I use for my equal-sized pieces? Why? (I should use triangles because 6 triangles can be put together to make a hexagon) Show how to make \( 1 \frac{1}{2} \) hexagons using triangles.

What shape would you use to make \( 2 \frac{1}{2} \) hexagons? Why? Show how to make \( 2 \frac{1}{2} \) hexagons using trapezoids.
What shape would you use to make \( \frac{13}{3} \) hexagons? Why? Show how to make \( \frac{13}{3} \) hexagons using rhombuses.

So far, we have used only hexagons as a whole pie. Let’s use trapezoids as a whole pie. What shape would you use to make \( 2 \frac{1}{3} \) trapezoids? Why? What piece is one third of the trapezoid? Show how to make \( 2 \frac{1}{3} \) trapezoids using triangles.

Show how to make each fraction below using triangles. Draw pictures of your models in your notebook:

- a) \( 1 \frac{1}{3} \)
- b) \( 4 \frac{1}{3} \)
- c) \( \frac{8}{3} \)
- d) \( \frac{8}{3} \)
- e) \( 2 \frac{2}{3} \)
- f) \( 3 \frac{1}{3} \)

Be sure everyone has done at least parts a) and b) above. **ASK:** Which 2 fractions from the list above are the same? How can you tell this from your pictures? Use your pictures to order the fractions above from least to greatest.

**Extensions**

1. If \( \frac{3}{5} \) of a structure looks like this:

   What could the whole look like?

2. If the triangle represents a whole, draw \( 2 \frac{1}{2} \).

3. (Atlantic Mathematics Curriculum) Break egg cartons into sections (1 through 11), and use complete cartons as well. Distribute at least one of the sections to each of the students and say, “If this (whole carton) is one, what is \( \frac{1}{3} \)? If this (9 section piece) is one whole, show me one third. If this (2 sections) is one, show me \( 2 \frac{1}{3} \)),” etc. Students should realize that any one section can have many different names depending on the size of the whole. It is also beneficial for students to frame these types of questions for their classmates.

4. What fraction of a metre is a decimetre? 12 decimetres?

5. Ask questions of the form: Stick B is what fraction of Stick A? Stick A is what fraction of Stick B? Write your answers as proper or improper fractions; not as mixed fractions.

Demonstrate with the following example.

A:

B:

Stick B is \( \frac{3}{5} \) of Stick A since putting it on top of Stick A will look like:

If Stick A is the whole, then the denominator is \( 5 \), because Stick A has \( 5 \) equal-sized parts. How many of those parts does Stick B take up? \( (3) \). So Stick B is \( \frac{3}{5} \) of Stick A.

What fraction of Stick B is Stick A?

If Stick B is the whole, what is the denominator? \( (3) \) Why? (because Stick B has \( 3 \) equal-sized pieces) How many of those equal-sized pieces does Stick B take up? \( (5) \) So stick A is \( \frac{5}{3} \) of Stick B.
As an improper fraction Stick A is $\frac{1}{2}$ of Stick B since it takes up one whole Stick B plus 2 more of those equal-sized pieces, but we will write our answers in terms of improper fractions instead of mixed fractions.

Let your students investigate with several examples. In this way, your students will discover reciprocals. If you know what fraction Stick A is of Stick B, what fraction is Stick B of Stick A? (just turn the fraction upside down!)

**REFLECT:** Why was it convenient to use improper fractions instead of mixed fractions?

---

**NS4-81**

**Mixed Fractions (Advanced)**

**GOALS**

Students will use multiplication to find the improper fraction equivalent to a given mixed fraction.

**PRIOR KNOWLEDGE REQUIRED**

Mixed fractions
Improper fractions
Using pictures to see that mixed and improper fractions can represent the same amount
Multiplication

**VOCABULARY**

mixed fraction
improper fraction

---

Draw on the board:

![Diagram of fractions]

**SAY:** How many parts are in 1 pie? There are 4 quarters in one pie. How many quarters are in 2 pies? (8)

![Diagram of fractions]

What operation can we use to tell us the answer? (multiplication) How many quarters are there in 3 pies? (4 × 3 = 12)

**ASK:** How many quarters are in $3\frac{3}{4}$ pies?

![Diagram of fractions]

12 pieces $\rightarrow$ $3\frac{3}{4}$ $\leftarrow$ 3 extra pieces

$(3 \times 4)$

So there are 15 pieces altogether.

How many halves are in 1 pie? (2) In 2 pies? (4) In 3 pies (6) In 17 pies? (34)

How do you know? What operation did you use to find that? (17 × 2)

**ASK:** How many halves are in $1\frac{1}{2}$ pies? Have a volunteer draw the picture on the board.

**ASK:** How many halves are in $2\frac{1}{2}$ pies? In $3\frac{1}{2}$ pies? In $4\frac{1}{2}$ pies? In $20\frac{1}{2}$ pies?

What operations do you use to find the answer? $(20 \times 2 + 1)$

40 pieces in 20 whole pies

1 extra piece
Draw $8\frac{1}{2}$ pies on the board and **ASK:** How many halves are in $8\frac{1}{2}$?

![Image of 8 pies]  
Emphasize that the extra half is just one more piece, so once they know how many halves are in 8 pies, they just add one to find how many are in $8\frac{1}{2}$.

Have students write in their notebooks how many halves are in...

- a) $2\frac{1}{2}$  
- b) $5\frac{1}{2}$  
- c) 11  
- d) $11\frac{1}{2}$  
**BONUS:** $49\frac{1}{2}$, $84\frac{1}{2}$

**ASK:** How many thirds are in 1 pie? (3) Have a volunteer come to the board and divide a circle into thirds. **ASK:** How many thirds are in 2 pies? In 3 pies? In 10 pies? In 100 pies? In 1000 pies? How many thirds are in 1013 pies? In 1023 pies? In 523 pies?

Have students write in their notebooks how many thirds are in...

- a) $2\frac{1}{3}$  
- b) $5\frac{1}{3}$  
- c) 11  
- d) $11\frac{2}{3}$  
**BONUS:** $49\frac{2}{3}$, $84\frac{1}{3}$

Include questions with denominator 4.

Then introduce problems with a context. **SAY:** I have boxes that will hold 4 cans each. What fraction of a box is each can? (one fourth) How many fourths are in 2 wholes? How many cans will 2 boxes hold? How are these questions the same? How are they different?

A box holds 4 cans. How many cans will:

- a) $1\frac{1}{4}$ boxes hold.  
- b) $2\frac{1}{4}$ boxes hold.  
- c) $1\frac{1}{2}$ boxes hold.  
- d) $1\frac{3}{4}$ boxes hold.

To help your students, encourage them to rephrase the question in terms of fourths and wholes. For example, since a can is one fourth of a whole, a) becomes “How many fourths are in $1\frac{1}{4}$?”

Next, students will have to rephrase the question in terms of fractions other than fourths, depending on the number of items in each package.

- a) A box holds 6 cans. How many cans will $1\frac{5}{6}$ boxes hold?  
- b) A box holds 8 cans. How many cans will $2\frac{3}{8}$ boxes hold?  
- c) **BONUS:** A box holds 326 cans. How many cans will $1\frac{5}{326}$ boxes hold?  
- d) Tennis balls come in cans of 3. How many balls will $7\frac{1}{3}$ cans hold?  
- e) A bottle holds 100 mL of water. How many mL of water will $7\frac{300}{300}$ bottles hold?

Teach students how to change mixed fractions to improper fractions: To change $2\frac{3}{4}$ to an improper fraction, start by calculating how many pieces are in the whole pies ($2 \times 4 = 8$) and add on the remaining pieces ($16 + 3 = 19$), so $2\frac{3}{4} = \frac{19}{8}$. Give students several problems of this sort, where they convert from mixed to improper form.
NS4-82
Mixed and Improper Fractions (Advanced)

GOALS
Students will use division with remainders to find the mixed fraction given the corresponding improper fraction.

PRIOR KNOWLEDGE REQUIRED
Reading the improper and mixed fractions from a picture
Finding the improper fraction given the mixed fraction by using multiplication
Division with remainders
The relationship between multiplication and division.

Have each student write in their notebooks the mixed and improper fractions for several pictures displaying area:

Then provide examples involving length and capacity as well, as shown in QUESTIONS 4 AND 5 of the worksheet.

How long is the line?

How many litres are shown?

Have students write each whole number below as an improper fraction with denominator 2 and show their answer with a picture and a multiplication statement:

a) 3 = \( \frac{6}{2} \)

b) 4
c) 2
d) 7
e) 10

\( 3 \times 2 = 6 \)

ASK: If I have the improper fraction \( \frac{10}{2} \), how could I find the number of whole pies it represents? Draw on the board:

whole pies = \( \frac{10}{2} \) pies

Ask students how many whole pies 10 half-sized pieces would make? Students should see that they simply divide 10 by 2 to find the answer: 5 whole pies. ASK: How many whole pies are in \( \frac{6}{2} \) pies? In \( \frac{12}{2} \) pies? In \( \frac{18}{2} \) pies? In \( \frac{20}{2} \)? In \( \frac{30}{2} \)? In \( \frac{42}{2} \)?

Then have students write the mixed fractions below as improper fractions and show their answer with a picture. Students should also write a statement for the number of half-sized pieces in the pie.

a) \( 3 \frac{1}{2} = \frac{7}{2} \)

b) \( 4 \frac{1}{2} \)
c) \( 2 \frac{1}{2} \)
d) \( 5 \frac{1}{2} \)
e) \( 8 \frac{1}{2} \)

\( 3 \times 2 + 1 = 7 \) halves

There are 3 pies with 2 halves each.
SAY: If I have the improper fraction $\frac{15}{2}$, how can I know how many whole pies there are and how many pieces are left over? I want to divide 15 into sets of size 2 and I want to know how many full sets there are and then if there are any extra pieces. What operation should I use? (division) What is the leftover part called? (the remainder)

Write on the board: $15 \div 2 = 7 \text{ Remainder } 1$, SO: $\frac{15}{2} = 7 \frac{1}{2}$.

Draw the following picture twice on the board with two explanations:

There are 15 fourths, or $\frac{15}{4}$ pies.

$4 \times 3 + 3 = 15$

There are 3 wholes and 1 quarter more, or $3 \frac{1}{4}$ pies, SO: $\frac{15}{4} = 3 \frac{1}{4}$.

Have students discuss both interpretations of the picture and what they mean. When we divide 15 into sets of size 4, we get 3 sets and then 3 extra pieces left over. This is the same as dividing pies into fourths and seeing that 15 fourths is the same as 3 whole pies (with 4 pieces each) and then 3 extra pieces.

Repeat for several pictures, having volunteers write the mixed and improper fractions as well as the multiplication and division statements. Then have students do similar problems individually in their notebooks.

Then give students improper fractions and have them draw the picture, write the mixed fraction, and the multiplication and division statements.

Then show students how to change an improper fraction into a mixed fraction:

$$2 \frac{1}{4} = 2 \times 4 + 1 \text{ quarters} = \frac{9}{4}$$

Starting with $\frac{9}{4}$, we can find: $9 \div 4 = 2 \text{ Remainder } 1$, SO:

$$\frac{9}{4} = 2 \text{ wholes and } 1 \text{ more quarter} = 2 \frac{1}{4}$$

Have students change several improper fractions into mixed fractions without using pictures.

Tell them that $\frac{7}{2}$ pies is the same as 3 whole pies and another half a pie. ASK: Is this the same thing as 2 whole pies and three halves? Do we ever write $2 \frac{3}{2}$? ASK: When we find $7 \div 2$, do we write the answer as 3 Remainder 1 or 2 Remainder 3? Tell your students that as with division, we want to have the fewest number of pieces left over.
Show several pairs of fractions on fraction strips and have students say which is larger, for **EXAMPLE**:

\[
\begin{array}{c}
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{8/9}}
\end{array}}} \\
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{3/4}}
\end{array}}}
\end{array}
\]

Include many examples where the two fractions are equivalent. Tell your students that when two fractions look different but actually show the same amount, they are called equivalent fractions. Have students find pairs of equivalent fractions from the pictures you have on the board. Tell them that we have seen other examples of equivalent fractions from previous classes and ask if anyone knows where. (There are 2 possible answers here: fractions that represent 1 whole are all equivalent, and the same for fractions representing 2 wholes; also, mixed fractions have an equivalent improper fraction).

Then have students find equivalent fractions by shading the same amount in the second strip as in the first strip and writing the shaded amount as a fraction:

\[
\begin{array}{c}
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{1/2}}
\end{array}}} \\
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{2/4}}
\end{array}}}
\end{array}
\]

Then show the chart from **QUESTION 3** on the Worksheet: **NS4-83**: Equivalent Fractions. Demonstrate using these charts how to find equivalent fractions and then give students similar questions.

**ASK**: What fraction of the squares are shaded?

\[
\begin{array}{c}
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{1/2}}
\end{array}}} \\
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{2/4}}
\end{array}}}
\end{array}
\]

Did I change the number of shaded squares by grouping them? Did I change the number of total squares by grouping them? Did I change the fraction of shaded squares by grouping them? Are 4/6 and 2/3 equivalent fractions? How do you know?

Have them look at this grouping and ask them if they can read the fraction of shaded squares:

\[
\begin{array}{c}
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{2/3}}
\end{array}}} \\
\text{\scalebox{1.5}{\begin{array}{c}
\text{\textcolor{red}{1/2}}
\end{array}}}
\end{array}
\]

**ASK**: Are there the same number of squares in each group? Can you read the fraction of shaded squares from this grouping? Tell them that we can’t say that 2 out of the 3 groups are shaded, so it is hard to see from this grouping that \(\frac{2}{3}\) of the squares are shaded.
Have students decide which groupings are good ways to find equivalent fractions:

```plaintext
\[\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{5}{10}\]
```

Summarize by reminding them that the whole group has to be shaded. Then give problems similar to 
**QUESTIONS 1 AND 2 (a, b, c)** on the Worksheet **NS4-84**. Alter it slightly, giving them only the numerator and let them find the denominator themselves. Ask them what is different from previous problems. Then give a problem where you give them neither the denominator nor the numerator. Give several problems of this type.

**Bonus**

Have students find as many equivalent fractions as they can in the picture:

```plaintext
\[\begin{array}{cccccccc}
\times & \times & \times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & \times & \times & \times & \times \\
\end{array}\]
```

Have students shade any \(\frac{3}{10}\) of the squares:

```plaintext
\[\begin{array}{cccccccc}
\times & \times & \times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & \times & \times & \times & \times \\
\end{array}\]
```

Then have them group the squares and make other equivalent fractions.

If you have two-colour cubes available, lay out a row of cubes:

```plaintext
\[\square \square \square \square \square \square \square \square \square \square \]
```

Ask a volunteer to rearrange the cubes in a rectangle, so that a number of whole rows is shaded. Ask how this makes it easy to determine the fraction that is shaded. Repeat with several examples where only one row is shaded and then move on to examples where the shaded cubes represent fractions like \(\frac{2}{8}\) or \(\frac{3}{8}\). After that, draw a model of the cubes on the board and ask volunteers to group the cubes (by circling them), so that they could tell which fraction of the cubes is shaded. Then assign several questions with pizza pieces, as in **QUESTION 2** of the Worksheet **NS4-85**: Further Equivalent Fractions.

Tell your students that now you want to give them a different problem. Before, they were grouping pieces together and now they will be dividing large pieces into smaller pieces. Show what you mean:

```
quarters can be cut into two pieces each.
```

Which fraction does the new picture represent? \(\frac{2}{8}\)

What fraction will appear if we cut each quarter into 3 pieces? Ask them to cut the pizza into pieces to represent a fraction with denominator 16.

Which fraction will that be?
Tell your students to write two fractions for several pictures, without telling them what the numerators and the denominators are. For **EXAMPLE:**

![Fraction Example](image)

Now ask them to do the opposite—you will give them the pair of equivalent fractions, and they will have to draw a picture that shows both fractions. Example pairs: \( \frac{6}{10} = \frac{3}{5} \) or \( \frac{9}{12} = \frac{3}{4} \). Include examples where the fractions you give them are written in words (**EXAMPLE**: three fifths is equivalent to six tenths).

**Extensions**

1. Draw a ruler on the board divided into mm and cm, with a certain number of cm shaded, and write two fractions:

   \[
   \frac{\text{number of mm shaded}}{\text{number of mm in total}} = \frac{\text{number of cm shaded}}{\text{number of cm in total}}
   \]

   Are these fractions equivalent? They can also use the metre stick and write triples of equivalent fractions using cm, mm and dm.
2. Write as many equivalent fractions as you can for each picture.

   a)  
   
   b)  

3. List 3 fractions between \( \frac{1}{2} \) and 1. **HINT**: Change \( \frac{1}{2} \) to an equivalent fraction with a different denominator (**EXAMPLE**: \( \frac{3}{6} \)) and then increase the numerator or decrease the denominator. Show your answers on a number line (this part is easier if they increase the numerator instead of decrease the denominator).

---

**NS4-86**

**Sharing and Fractions**

**GOALS**

Students will find fractions of numbers.

**PRIOR KNOWLEDGE REQUIRED**

Fractions as area
Fractions of a set

Brainstorm the types of things students can find fractions of (circles, squares, pies, pizzas, groups of people, angles, hours, minutes, years, lengths, areas, capacities, apples).

Brainstorm some types of situations in which it wouldn’t make sense to talk about fractions. For **EXAMPLE**: Can you say 3 \( \frac{1}{2} \) people went skiing? I folded the sheet of paper 4 \( \frac{1}{2} \) times?

Explain to your students that it makes sense to talk about fractions of almost anything, even people and folds of paper, if the context is right: **EXAMPLE**: Half of her is covered in blue paint; half the fold is covered in ink. Then teach them that they can take fractions of numbers as well. **ASK**: If I have 6 hats and keep half for myself and give half to a friend, how many do I keep? If I have 6 apples and half of them are red, how many are red? If I have a pie cut into 6 pieces and half the pieces are eaten, how many are eaten? If I have a rope 6 metres long and I cut it in half, how long is each piece? Tell your students that no matter what you have 6 of, half is always 3. Tell them that mathematicians express this by saying that the number 3 is half of the number 6.

Tell your students that they can find \( \frac{1}{2} \) of 6 by drawing rows of dots. Put 2 dots in each row until you have placed 6 dots. Then circle one of the columns:

- **Step 1**
- **Step 2**
- **Step 3**

The number of dots in one column is half of 6. Have students find \( \frac{1}{2} \) of each number using this method:

- a) \( \frac{1}{2} \) of 4
- b) \( \frac{1}{2} \) of 8
- c) half of 10
- d) half of 14
**Bonus**

Use this method to find $\frac{1}{3}$ of each of the following numbers. **HINT:** Put 3 dots in each row.

a) $\frac{1}{3}$ of 12  b) $\frac{1}{3}$ of 15  c) one third of 18  d) one third of 3

**ASK:** If, you want to find $\frac{1}{3}$ of 12, how many dots in a row would you draw? (4) You would draw 4 columns and circle the dots in 1 column. Now draw 4 sets and share the dots into the sets (1 dot to each of the sets, 4 in total, then another dot to each of the sets, continue till you get 12 dots in total). What fraction of the 12 dots does each set represent? What would you do if you needed $\frac{3}{4}$ of 12? (You have to take 3 sets).

$$\frac{\boxed{}}{\boxed{}}$$ of 8 = ____

$$\begin{array}{llll}
\bullet & \bullet & \bullet & \bullet \\
\end{array}$$

$$\frac{\boxed{}}{\boxed{}}$$ of ____ = ____

Invite volunteers to find $\frac{3}{4}$ of 16 and $\frac{2}{3}$ of 15 using this method. After that draw several pictures yourself and ask them to find the fractions they represent.

Teach your students to see the connection between the fact that 6 is 3 twos and the fact that $\frac{1}{3}$ of 6 is 2. The exercise below will help with this:

Complete the number statement using the words “twos”, “threes”, “fours” or “fives”. Then draw a picture and complete the fraction statements. (The first one is done for you.)

<table>
<thead>
<tr>
<th>Number Statement</th>
<th>Picture</th>
<th>Fraction Statements</th>
</tr>
</thead>
</table>
| a) 6 = 3 twos    | ![picture] | $\frac{1}{3}$ of 6 = _____  
|                  |         | $\frac{2}{3}$ of 6 = _____ |
| b) 12 = 4 ______ | ![picture] | $\frac{1}{4}$ of 12 = _____  
|                  |         | $\frac{2}{4}$ of 12 = _____  
|                  |         | $\frac{3}{4}$ of 12 = _____  |
| c) 15 = 3 ______ | ![picture] | $\frac{1}{3}$ of 15 = _____  
|                  |         | $\frac{2}{3}$ of 15 = _____  |

**Extension**

Explain to your students that you can have a fraction of a fraction! Demonstrate finding half of a fraction by dividing it into a top half and a bottom half:

$$\frac{1}{2} \text{ of } \frac{3}{5} \quad \text{is} \quad \frac{3}{10}$$
Have them find:

a) \( \frac{1}{2} \) of \( \frac{2}{9} \) 

b) \( \frac{1}{2} \) of \( \frac{5}{7} \) 

Then have them draw their own pictures to find:

c) \( \frac{1}{2} \) of \( \frac{3}{7} \)  
d) \( \frac{1}{2} \) of \( \frac{2}{5} \)  
e) \( \frac{1}{2} \) of \( \frac{5}{6} \)  
f) \( \frac{1}{2} \) of \( \frac{4}{7} \)  

**Bonus**

Find two different ways of dividing the fraction \( \frac{4}{7} \) in half.

**ANSWER:**

\[
\begin{align*}
\frac{1}{2} \text{ of } \frac{4}{7} & \quad \text{is} \quad \frac{4}{14} \\
\frac{1}{2} \text{ of } \frac{4}{7} & \quad \text{is} \quad \frac{2}{7}
\end{align*}
\]

ASK: Are the two fractions \( \frac{2}{7} \) and \( \frac{4}{14} \) equivalent? How do you know?

---

**NS4-87**

**More Sharing and Fractions**

Ask your students what they would do to find \( \frac{1}{3} \) of 15 dots? (Divide the dots into 3 columns and count how many were in each column). Ask them to find a division statement that would suit the model they drew. (15 ÷ 3 = 5)

Write on the board: “\( \frac{1}{3} \) of 15 = 5 means 15 ÷ 3 = 5”. Give your students several more statements to rewrite as normal division statements, like \( \frac{1}{6} \) of 12, \( \frac{1}{5} \) of 15, \( \frac{1}{2} \) of 20, \( \frac{1}{2} \) of 16, \( \frac{1}{2} \) of 10, and so on.

Ask your students how they could use the exercise they finished to solve the following problem: circle \( \frac{2}{3} \) of a set of dots: ● ● ● ● ● ● ● ● ● ● ● ●.

Draw several sets of dots in a line (not in two rows!) and ask them to circle half and then a third.

Ask your students to tell you several ways to find \( \frac{2}{3} \) of 6 dots (or small circles).

If the following two solutions do not arise, present them to your students:

1) To find two thirds of 6, you could find one third of six and multiply by 2
2) Circle two out of every 3 dots and count the total number of dots circled.

Have students find fractions of sets of objects (you can give the same number of objects arranged in different ways, for example, have your students find \( \frac{3}{4} \) of 16 boxes with the boxes arranged as \( 4 \times 4 \), \( 8 \times 2 \) or \( 16 \times 1 \) arrays.)
Before doing this activity, ensure that students are comfortable finding fractions of numbers such as: \( \frac{10}{15} \) of 15. Teach this by saying: What is \( \frac{3}{4} \) of 4? If I divide 4 dots into 4 columns, how many do I have in each column? In 3 columns? What is \( \frac{5}{7} \) of 7? If I divide 7 dots into 7 columns, how many are in each column? In 5 columns? Repeat until students can tell you that \( \frac{15}{10} \) of 15 is 15 without having to divide 15 dots into 15 columns. Then Use the BLM "Math Bingo Game (Sample boards)" with the BLM "Cards (Fractions of numbers)".

### Extensions

1. How many months are in:
   - a) \( \frac{1}{2} \) year?
   - b) \( \frac{2}{3} \) year?
   - c) 1 \( \frac{1}{2} \) years?
2. How many minutes are in:
   - a) \( \frac{2}{3} \) of an hour?
   - b) \( \frac{1}{4} \) of an hour?
   - c) 1 \( \frac{1}{10} \) of an hour?
3. \( \frac{5}{8} \) of a day is how many hours?

### NS4-88

**Sharing and Fractions (Advanced)**

Remind your students of the problems they solved in **NS4-86: Sharing and Fractions**.

- **Goal**: Students will solve problems involving fractions of sets.

- **Prior Knowledge Required**: Fractions of a set

- **Mathematical Sentence**

  \[ \square \text{ of } \blacksquare \text{ = } \blacksquare \]

  Ask them to present as many methods to find the missing fraction in the statement 2 is \( \square \) of 10 as they can. This time, they have to draw the picture themselves. They should keep making groups of 2 until they reach 10. Have them find the missing fraction for the following statements: 4 is \( \square \) of 12, 4 is \( \square \) of 20, 4 is \( \square \) 16, 5 is \( \square \) of 20, etc.

  Tell students that you want to change word problems about fractions into mathematical sentences. Ask what symbol they would replace each word or phrase by: more than (>), is (=), half \( (\frac{1}{2}) \), three quarters \( (\frac{3}{4}) \).

  Write on the board:

  Calli’s age is half of Ron’s age.
  Ron is twelve years old.
  How old is Calli?
Teach students to replace each word they do know with a math symbol and what they don’t know with a blank:

\[
\begin{align*}
\underline{\text{Calli’s age}} & \quad \underline{\text{is}} \quad \underline{\text{half}} \quad \underline{\text{of}} \quad \underline{\text{Ron’s age}}. \\
\underline{\text{\frac{1}{2}}} & \quad \underline{\text{of}} \quad \underline{\text{12}}
\end{align*}
\]

Have them do similar problems of this sort.

a) Mark gave away \(\frac{3}{4}\) of his 12 stamps. How many did he give away? ( \(\underline{\text{\frac{3}{4}}} \quad \underline{\text{of}} \quad \underline{\text{12}}\) )

b) There are 8 shapes. What fraction of the shapes are the 4 squares? ( \(\underline{\text{\frac{4}{8}}} = \underline{\text{1}}\) )

c) John won three fifths of his five sets of tennis. How many sets did he win? ( \(\underline{\text{\frac{3}{5}}} \quad \underline{\text{of}} \quad \underline{\text{5}}\) )

Then have students change two sentences into one, replacing the underlined words with what they’re referring to:

a) Mark has 12 stamps. He gave away \(\frac{3}{4}\) of them. (Mark gave away \(\frac{3}{4}\) of his 12 stamps)

b) A team played 20 games. They won 11 of them.

Then have students solve several word problems, for EXAMPLE:

Anna had 10 strawberries. She ate two of them. What fraction of her strawberries did she eat?

**Literacy Connection**

Read *Alice in Wonderland* with them and give them the following problems.

a) The gardeners coloured red \(\frac{2}{3}\) of 15 roses. How many roses are red now? How many roses remained blank?

b) Alice ate 12 of 20 shrinking cakes. Which fraction of the shrinking cakes did she eat? Is that more than a half?

**Extension**

A kilogram of nuts cost $8. How much would \(\frac{3}{4}\) of a kilogram of nuts cost?
NS4-89
More Mixed and Improper Fractions

GOALS
Students will fit fractions into number lines and learn to count by quarters, halves, thirds, and so on.

PRIOR KNOWLEDGE REQUIRED
Number lines
Finding the mixed fraction given the improper fraction
Equivalent fractions
Number words for fractions

VOCABULARY
mixed fraction
proper fraction
improper fraction

Draw a straight line on the board. Divide the line into two equal parts and ask your students which fraction of the line is before your marking. Divide each half again and ask what fraction of the whole is between each pair of markings. Mark one of the ends as 7 and the other as 8:

```
7   8
```

Ask what number is at the first mark. Emphasize that if it is $\frac{1}{4}$ of the way from 7 to 8, then the mark is at $7 \frac{1}{4}$. Ask a volunteer to continue writing the mixed fractions for each marking. Accept both $7 \frac{1}{2}$ and $7 \frac{2}{4}$ for the middle marking.

Draw several number lines on the board, divide the distances between the whole numbers into 2, 3, 4, or 5 parts and ask the volunteers to mark the lines. Include examples of longer number lines as well:

```
9   10   11
```

Ask students how they would extend the pattern: $\frac{1}{2}$, 1, $1 \frac{1}{2}$, 2, ... What is the pattern rule? How would they extend the pattern: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, ...

a) Using mixed fractions?

b) Using improper fractions?

How do they know when they reach a whole number. What are the pattern rules in each case? (They are both: Start at $\frac{1}{5}$ and add $\frac{1}{5}$ each time.)

Then write the following pattern on the board: $2 \frac{2}{7}$, $2 \frac{3}{7}$, $2 \frac{4}{7}$. Ask your students to continue the pattern. Then ASK: What if I continued the pattern in this way: $2 \frac{5}{7}$, $2 \frac{6}{7}$, $2 \frac{7}{7}$, $2 \frac{8}{7}$. Is this correct? What is wrong? The students should recall that mixed fractions must have a fractional part that is a proper fraction—not an improper one. Ask them how this requirement reminds them of division. (Just as we don’t write $\frac{22}{7}$, we also don’t write $22 \div 7 = 2$ Remainder 8. We write instead $\frac{22}{7} = 3 \frac{1}{7}$ and $22 \div 7 = 3$ Remainder 1. Give them several patterns with improper fractions to extend and then have them convert the patterns to mixed fractions. Then give several patterns with mixed fractions to extend and then have them convert the patterns to improper fractions.

Ask a student to draw a picture on the board representing $5 \frac{3}{4}$. How many quarters are there? You may prompt students to skip count by 4s until they reach 5 wholes and then to count by 1s until they have 3 more pieces (4, 8, 12, 16, 20, 21, 22, 23 quarters.) ASK them to write the result as an improper fraction. Repeat with several examples, and then give students similar problems to do individually.
Next ask your students to do the same exercise without the picture. What do they need to count up by? At what number do they start counting on by 1s? And how do they know when to stop counting up by 1s? Use more examples such as $3 \frac{3}{5} = \text{?} \text{ fifths}, 4 \frac{2}{3} = \text{?} \text{ thirds}$, etc. For extra practice, you may also ask students to mark the number lines they have drawn previously with improper fractions in addition to the mixed fractions.

**Extension**

(Adapted from the Atlantic Curriculum)

Ask your students to estimate how many steps it would take them to walk across the room. Then ask them to imagine a giant who can walk across the room in 8 steps.

a) Where would the giant be after 7 steps?

b) Ask the students to estimate where $\frac{7}{8}$ would be on a number line marked only with a 0 and a 1.

c) Ask the students to explain how questions a) and b) are similar and how they are different.

d) Ask the students to estimate where $\frac{5}{3}, \frac{1}{5}, \frac{1}{3}$, and $\frac{1}{2}$ go on the number line.

e) Have them make up word problems based on giants that are similar to the 3 questions from part d).

**Activity**

http://www.iknowthat.com/com/L3?Area=FractionGame

Go to “Improper fractions” and click “Begin Game”. The player has to change mixed fractions to improper fractions and to choose the right number from four possible answers.
NS4-90
Adding and Subtracting Fractions (Introduction)

GOALS
Students will add and subtract fractions with the same denominators.

PRIOR KNOWLEDGE REQUIRED
Naming Fractions
Addition and subtraction of whole numbers
1 whole

VOCABULARY
fraction
regrouping
numerator
denominator

Draw two large circles representing pizzas (you can use paper pizzas as well) and divide them into 4 pieces each, shading them as shown.

Explain that these are two plates with several pizza pieces on each. How much pizza do you have on each plate? Write the fractions beneath the pictures. Tell them that you would like to combine all the pieces onto one plate, so put the “+” sign between the fractions and ask a volunteer to draw the results on a different plate. How much pizza do you have now?

Draw on the board:

\[ \frac{1}{4} + \frac{2}{4} = \]

Tell your students that you would like to regroup the shaded pieces so that they fit onto one circle. SAY: I shaded 2 fourths of one circle and 1 fourth of another circle. If I move the shaded pieces to one circle, what fraction of that circle will be shaded? How many pieces of the third circle do I need to shade? Tell them that mathematicians call this process adding fractions. Just like we can add numbers, we can add fractions too.

Do several examples of this, like \( \frac{1}{5} + \frac{2}{5} \), never extending past 1 whole circle. Ask your students: You are adding two fractions. Is the result a fraction too? Does the size of the piece change while we transport pieces from one plate to the other? What part of the fraction reflects the size of the piece—top or bottom? Numerator or denominator? When you add fractions, which part stays the same, the top or the bottom; the numerator or the denominator?

What does the numerator of a fraction represent? (The number of shaded pieces) How do you find the total number of shaded pieces when you moved them to one pizza? What operation did you use?

Show a couple more examples using pizzas, and then have them add the fractions without pizzas. Assign lots of questions like \( \frac{3}{5} + \frac{1}{5} \), \( \frac{2}{7} + \frac{3}{7} \), \( \frac{2}{11} + \frac{4}{11} \), etc. Enlarge the denominators gradually.

Bonus
Add:

\[ \frac{12}{134} + \frac{45}{134} \hspace{1cm} \frac{67}{1567} + \frac{78}{1567} \hspace{1cm} \frac{67}{456} + \frac{49}{456} \]

Bonus
Add more fractions:

\[ \frac{3}{17} + \frac{1}{17} + \frac{2}{17} \hspace{1cm} \frac{5}{94} + \frac{4}{94} + \frac{7}{94} \hspace{1cm} \frac{3}{19} + \frac{5}{19} + \frac{7}{19} + \frac{4}{19} \hspace{1cm} \frac{2}{19} + \frac{5}{19} + \frac{7}{19} + \frac{3}{19} \]
Return to the pizzas and say that now you are taking pieces of pizza away. There was \(\frac{3}{4}\) of a pizza on a plate. You took away \(\frac{1}{4}\). Show on a model the one piece you took away:

![Pizza model](image)

How much pizza is left? Repeat the sequence of exercises and questions you did for addition using subtraction.

### Extensions

1. **Tell your students that sometimes adding fractions can result in more than one whole.**

   Draw on the board:

   \[
   \frac{3}{4} + \frac{2}{4} = \_
   \]

   Ask how many parts are shaded in total and how many parts are in 1 whole circle. Tell your students that, when adding fractions, we like to regroup the pieces so that they all fit onto 1 circle. **ASK:** Can we do that in this case? Why not? Tell them that since there are more pieces shaded than in 1 whole circle, the next best thing we can do is to regroup them so that we fit as many parts onto the first circle as we can and then we put only the leftover parts onto the second circle.

   Draw on the board:

   ![Regrouped pizza model](image)

   Ask how many parts are shaded in the first circle and how many more parts do we need to shade in the second circle. Ask a volunteer to shade that many pieces and then tell them that mathematicians write this as:

   \[
   \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1 + \frac{1}{4}
   \]

   Do several examples of this where the sum of the fractions is more than 1 whole, using more complex shapes to make it look harder as students get used to the new concept and then continue with examples where the sum is more than 2 wholes.

2. **Teach your students the role of 0 in adding and subtracting fractions.**

   a) \(\frac{3}{5} - \frac{3}{5}\)  
   b) \(\frac{2}{7} + \frac{0}{7}\)  
   c) \(\frac{3}{8} - \frac{0}{8}\)  
   d) \(\frac{5}{7} - 0\)  
   e) \(1\frac{2}{3} + 0\)

   Tell them that 0 is the same number with any denominator. This is very different from fractions with any other numerator, where the denominator matters a lot.

3. **(Atlantic Curriculum B6)**

   Have students use pattern blocks to add fractions.

   a) \((\frac{1}{2} + \frac{1}{2} = 1)\)
   b) \((\frac{1}{6} + \frac{1}{6} = \frac{5}{6})\)
   c) \((\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{2} = 1)\)
NS4-91
Fractions Review

This worksheet is a review of the fractions section and can be used as consolidating homework.

Extensions

1. **ASK:** Which is greater, eight thirds or twelve fifths? (2 and two thirds or 2 and two fifths) and have students write the fractions as improper fractions and then as mixed fractions. Which way makes it easier to compare the fractions? Why?

   **REFLECT:** Recall that when learning reciprocals in Extension 5 of NS4-80: Investigating Mixed and Improper Fractions, it was better to use improper fractions. When comparing 8 thirds to 12 fifths, it is better to change the fractions to mixed fractions. Emphasize that sometimes mixed fractions are more convenient and sometimes improper fractions are more convenient. As students gain more experience they will learn to predict which will be more convenient. It is important to understand both forms so that they can choose which one is more convenient for their purpose.

2. Give pairs of students cards with the following fractions on them:

   - \(\frac{3}{6}\)
   - \(\frac{2}{3}\)
   - \(\frac{1}{4}\)
   - \(\frac{7}{8}\)
   - \(\frac{5}{12}\)
   - \(\frac{5}{6}\)

   Ask them to arrange the cards in order from least to greatest (\(\frac{1}{4}\), \(\frac{5}{12}\), \(\frac{3}{6}\), \(\frac{2}{3}\), \(\frac{5}{6}\), \(\frac{7}{8}\)) and to give reasons for their arrangement.

   Possible reasons could include: \(\frac{5}{6}\) is only one step away from reaching the end of a number line divided into 6 parts. Similarly, \(\frac{7}{8}\) is only one step away from reaching a number line divided into 8 parts, but the steps in \(\frac{7}{8}\) are smaller, so the person who is \(\frac{7}{8}\) of the way across is closer to the end than the person who is only \(\frac{5}{6}\) of the way along the line; 6 steps out of 12 would be a half, and \(\frac{5}{12}\) is almost a half, so it is more than \(\frac{1}{2}\).
NS4-92
Dollar and Cent Notation

GOALS
Students will express monetary values less than a dollar in dollar and cent notation.

PRIOR KNOWLEDGE REQUIRED
Place value to tenths and hundredths
Canadian coins

VOCABULARY
- penny
- nickel
- dime
- quarter
- tenths
- hundredths
- loonie
- toonie
- cent
- dollar
- notation

Draw five quarters on the board. Ask students how much money you have in cents (or pennies). Write “125¢” on the board. Ask students if they know another way to write one hundred twenty-five cents.

Explain that there are two standard ways of representing money. The first is cent notation: you simply write the number of cents or pennies that you have, followed by the ¢ sign. In dollar notation, “one hundred twenty-five cents” is written $1.25. The number to the left of the decimal point tells how many dollars you have, the number to the right of the decimal represents the number of dimes and the next digit to the right represents the number of pennies.

Assessment
Write the total amount in dollar and cent notation:

\[ \text{Total amount} = _____ \, \text{¢} = \$_____ \]

\[ \text{Total amount} = _____ \, \text{¢} = \$_____ \]

ACTIVITY 1
Ask students to pretend that there is a vending machine which only takes loonies, dimes and pennies. Have them make amounts using only these coins (EXAMPLE: 453¢, 278¢, 102¢, etc).

ACTIVITY 2
A Game for Two: The Change Machine
One player makes an amount using nickels and quarters. The other (“the machine”) has to change the amount into loonies, dimes and pennies.
Extensions

1. Let students know that in decimal notation, the digit after the decimal point always stands for tenths and the next digit stands for hundredths. Ask students why this notation would be used for money. What is a dime a tenth of? And what is a penny a hundredth of?

2. Show how you would make these dollar amounts using exactly four coins:
   a) $4.30  
   b) $3.50  
   c) $5.10  
   d) $1.40

---

**NS4-93**

**Converting Between Dollar and Cent Notation**

Changing from dollar to cent notation is easy—all you need to do is to remove decimal point and dollar sign, and then add the ¢ sign to the right of the number.

Changing from cent notation to dollar notation is a little trickier. Make sure students know that, when an amount in cent notation has no tens digit (or a zero in the tens place), the corresponding amount in dollar notation must have a zero. **EXAMPLE:** $6¢ is written $.06 or $0.06, not $0.60.

Make a chart like the one shown below on the board and ask volunteers to help you fill it in.

<table>
<thead>
<tr>
<th>Amount</th>
<th>Amount in Cents</th>
<th>Dollars</th>
<th>Dimes</th>
<th>Pennies</th>
<th>Amount in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 quarters, 3 dimes, 2 nickels</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 toonies, 2 loonies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 quarters, 2 pennies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students could also practice skip counting by quarters, toonies and other coins (both in dollar and in cent notation).

$2.00, $4.00, ____ , ____  
$0.25, $0.50, ____ , ____ , ____ , ____

200¢, 400¢, ____ , ____  
25¢, 50¢, ____ , ____ , ____ , ____
**Bonus**
What coin is being used for skip counting?

$1.00, ___, ___, ___, $1.20

$2.00, ___, ___, ___, $3.00

You may wish to give your students some play money and to let them practice in pairs—pick several coins and write the amount in dollar and cent notation. Let your partner check your answer.

**Assessment**
1. Write the total amount in dollar and cent notation:
   a) 
   ![Coins](image)
   Total amount = ____ c = $ ___
   b)
   ![Coins](image)
   Total amount = ____ c = $ ___

2. Convert between dollar and cent notations:
   $ .24 = ____ c = 82c = 6c  $12.03 = ____  $120.30 = ____

**Bonus**
1. Change these numbers to dollar notation: 273258¢, 1234567890¢.

2. Change $245.56 to cent notation, and then either write or say the number of cents properly (twenty-four thousand five hundred fifty-six cents). Try these numbers: $76.34, $123.52, $3789.49.

---

**A Game in Pairs**
Player 1 hides several coins: for example, a loonie, a toonie and a dime or three loonies and a dime for $3.10. Player 1 tells her partner how many coins she has and the value of the coins in dollar notation. The partner has to guess which coins she has. There may be more than one possibility for the answer. In this case the partner has to give all the possibilities. For example, for three coins making $2.10, the possibilities are two loonies and a dime or a toonie and two nickels. Each correct possibility gives 5 points. If there is more than one answer, the Player 1 loses a point for each answer after the first one. In the example given, the Player 2 receives 10 points and Player 1 loses 1 point.

---

**Money Matching Memory Game.**
(See BLM) Students play this game in pairs. Each student takes a turn flipping over two cards. If the cards match, that player gets to keep the pair and takes another turn. Students will have to remember which cards are placed where and also match up identical amounts written in dollar and cent notation.
NS4-94
More Dollar and Cent Notation

GOALS
Students will express monetary values in dollar and cent notation.

PRIOR KNOWLEDGE REQUIRED
Place value to tenths and hundredths
Canadian coins

VOCABULARY
penny
dime
quarter
loonie
hundredths
toönie
cent
dollar
notation
tenths

Write two money amounts on the board: 23¢ and $2.15. Ask which represents more money. Ask a volunteer to change the amount in dollar notation into cent notation to compare. Ask why converting to cents is more convenient (no need to deal with decimal points and fractions). Give more practice questions, such as:

123¢ or $7.56
$9875 or 9875¢
9¢ or $0.08
40¢ or $0.04

Ask how these questions are similar to:

Which is longer: 1 234 cm or 7.56 m?
9 875 m or 98 756 cm?

Present a problem: Jane emptied her piggy bank. She asked her elder brother John to help her count the coins. He suggested she stack the coins of the same value in different stacks and count each stack separately. Jane has:

19 pennies
7 quarters
19 nickels
6 loonies
21 dimes
3 toonies

How much money is in each stack? Ask students to write the amount in dollar notation and in cent notation. Give them more practice questions, such as:

9 pennies
6 quarters
18 nickels
5 loonies
7 dimes
9 toonies

Assessment
1. Write each amount in dollar notation, and then circle the greater amount of money:

Ten dollars thirty-five cents or 1125¢
Three dollars fifty cents or 305¢

2. Write each amount in cent notation and in dollar notation:

7 pennies
21 nickels
5 dimes
2 quarters

ACTIVITY
Bring in several fliers from local businesses that would list many items under $10 (EXAMPLE: grocery stores, drug stores, department stores, hardware stores, etc.). In pairs, have students select an item and read the price aloud to their partner. They should read it as “x dollars and x cents.” Their partner should then write down that amount in both dollar and cent notation.
NS4-95
Canadian Bills and Coins

Remind students that when writing in dollar notation, the number of full dollars is written to the left of the decimal point. There is no limit to how many place value columns this can extend to. Demonstrate by writing $2.00, $22.00, $222.00 and $2222222222222.00 on the board.

However, there are only two place value columns to the right of the decimal place. Remind students that, if you have a number of cents which is only a single digit (say 3), in dollar notation a zero is placed in the dimes column.

Ask the class to invent some ways to write money amounts in incorrect notation. Allow them to come up with a wide variety of ideas and welcome silly answers. Something that looks like this: 54$, is incorrect. Add several examples yourself, for instance:

2.89$, $26.989, $67¢, ¢45, ¢576, 37.58¢, ¢67.89, $12.34¢

Review the names and values of Canadian coins and bills. Discuss the images depicted on the coins. Point out that the animal on the quarter is a caribou (not a moose) and that the dime shows the Bluenose, a type of sailing ship called a schooner.

Discuss with students the relationships between various coins and bills. Ask students how many...

a) dimes are in a loonie, toonie, or a 5 dollar bill.
b) nickels are in a loonie or toonie.
c) quarters are in a loonie, toonie, 5 or 10 dollar bill.
d) toonies are in a 10 or 20 dollar bill.

Students could use play money as manipulatives for the above problems.

As a class or in small groups, have students visit the Canadian Currency Museum website and their exhibit on Canada’s Coins. This has great information about the history and symbolism of Canada’s coins. You could have students write a short report about the different symbols on coins and why they are so important.

http://www.bankofcanada.ca/currencymuseum/eng/learning/canadascoins.php

Students could design their own coins. Ask them to explain what image they chose and why that could be an important symbol. You could design new coins for Canada or coins for your town or school.
Extensions

1. The Royal Canadian Mint has great resources available on their website. Their Currency Timeline might be of great interest to the students. This outlines the history of Canadian settlement and development and all the varieties of currency that have been used from the early 16th century to the present. Use this as a resource and have students research other kinds of coins (denominations, forms, images, etc.), that have existed in Canada's past.

   www.mint.ca/teach

Project Ideas

For each coin: what are the possible images and what do they commemorate, who drew the design for the coin, when was the coin introduced, were there changes in the metal, when and why were the changes made?

For each bill: who is the person on the bill (a short biography), when did the bill appear first and who designed the bill? Were any changes in the bill design, when and why did they happen?

2. If you have any students who have lived in or travelled to other countries, have them bring in samples of the other currency as a 'show and tell' for the class. Students will be very interested in the different images and shapes of the coins and bills.
NS4-96
Adding Money

Remind your students of how to add 2-digit numbers. Demonstrate the steps: line up the numbers correctly, add the digits in each column starting from the right. Start with some examples that would not require regrouping, and then move on to some which do, EXAMPLE: 15 + 23, 62 + 23, 38 + 46.

Use volunteers to pass to 3- and 4-digit addition questions, first without regrouping, then with regrouping. SAMPLE QUESTIONS:

545 + 123, 1 234 + 345, then 1 324 + 1 259, 2 345 + 5 567, 5 678 + 789, 3 456 + 3 987.

Model the steps for adding money. The difference is in the lining up—the decimal point is lined up over the decimal point. ASK: Are the ones still lined up over the ones? The tens over the tens? The dimes over the dimes? Tell them that if the decimal point is lined up, all the other digits must be lined up correctly too, since the decimal point is between the ones and the dimes. Students can model regrouping of terms using play money: for instance, in $2.33 + $2.74 they will have to group 10 dimes as a dollar.

Students should complete a number of problems in their workbooks. Some SAMPLE PROBLEMS:

a) $5.08 + $1.51 b) $3.13 + $2.98 c) $10.74 + $15.22
d) $23.95 + $42.28 e) $12.79 + $2.83

Use volunteers to solve several word problems, such as: Julie spent $14.98 for a T-shirt and $5.78 for a sandwich. How much did she spend in total?

Students should also practice adding coins and writing the amount in dollar notation, such as:

There are 19 pennies, 23 nickels and 7 quarters in Jane’s piggybank. How much money does she have?

Helen has a five-dollar bill, 6 toonies and 9 quarters in her pocket. How much money does she have?

Randy paid 2 twenty-dollar bills, 9 toonies, a loonie, 5 quarters and 7 dimes for a parrot. How much did his parrot cost?

A mango fruit costs 69¢. I have a toonie. How many mangos can I buy? If I add a dime, will it suffice for another one?

Assessment
1. Add:
   a) $18.25 + $71.64  b) $23.89 + $67.23  c) $45.08 + $8.87  d) $78.37 + $4.79
2. Sheila saved 6 toonies, 7 dimes and 8 pennies from babysitting. Her brother Noah saved a five-dollar bill, 2 toonies, a loonie and 6 quarters from mowing a lawn.
   
a) Who has saved more money?
   
b) They want to share money to buy a present for their mother. How much money do they have together?
   
c) They’ve chosen a teapot for $23.99. Do they have enough money?

Extension

Fill in the missing information in the story problem and then solve the problem.

a) Betty bought _____ pairs of shoes for _____ each. How much did she spend?

b) Una bought _____ apples for _____ each. How much did she spend?

c) Bertrand bought _____ brooms for _____ each. How much did he spend?

d) Blake bought _____ comic books for _____ each. How much did he spend?

ACTIVITY

Bring in fliers from local businesses. Ask students to select gifts to buy for a friend or relative as a birthday gift. They must choose at least two items. They have a $20.00 budget. What is the total cost of their gifts?
NS4-97
Subtracting Money

Begin by explaining that you will be learning how to subtract money today. Start with a review of 2-digit and 3-digit subtraction questions. Model a few on the board, or use volunteers. Start with some examples that would not require regrouping: 45 – 23, 78 – 67, 234 – 123, 678 – 354.

Show some examples on the board of numbers lined up correctly or incorrectly and have students decide which ones are done correctly.

Demonstrate the steps: line up the numbers correctly, subtract the digits in each column starting from the right. Move onto questions that require regrouping, EXAMPLE: 86 – 27, 567 – 38, 782 – 127, 673 – 185, 467 – 369.

Sample Problems
Subtract:

a) $98.89
b) $89.00
c) $45.00
d) $78.37

− $71.64
− $67.23
− $38.87
− $ 9.79

Next, teach your students the following fast way of subtracting from powers of 10 (such as 10, 100, 1 000 and so on) to help them avoid regrouping:

For example, you can subtract any money amount from a dollar by taking the amount away from 99¢ and then adding one cent to the result.

\[
1.00 - .57 = .43 = 43\text{¢}
\]

Another example, you can subtract any money amount from $10.00 by taking the amount away from $9.99 and adding one cent to the result.

\[
10.00 - 8.63 = 1.37
\]

**NOTE:** If students know how to subtract any one-digit number from 9, then they can easily perform the subtractions shown above mentally. To reinforce this skill have students play the Modified Go Fish game (in the MENTAL MATH section) using 9 as the target number.

Assessment

1. Subtract:

a) $96.85
b) $75.00
c) $55.00
d) $78.37

− $71.64
− $61.23
− $39.88
− $ 9.88
2. Alyson went to a grocery store with $15.00. She would buy buns for $1.69, ice cream for $6.99 and tomatoes for $2.50. Does she have enough money? If yes, how much change will she get?

**Literature Connection**


Last Sunday, Alexander’s grandparents gave him a dollar—and he was rich. There were so many things that he could do with all of that money!

He could buy as much gum as he wanted, or even a walkie-talkie, if he saved enough. But somehow the money began to disappear…

A great activity would be stopping to calculate how much money Alexander is left with every time he ends up spending money.

Students could even write their own stories about Alexander creating a subtraction problem of their own.

**Extension**

Fill in the missing information in the story problem and then solve the problem.

a) Kyle spent $4.90 for a notebook and pencils. He bought 5 pencils. How much did the notebook cost?

b) Sally spent $6.50 for a bottle of juice and 3 apples. How much did the juice cost?

c) Clarke spent $9.70 for 2 novels and a dictionary. How much did the dictionary cost?

d) Mary spent $16.40 for 2 movie tickets and a small bag of popcorn. How much did her popcorn cost?
NS4-98
Estimating

GOALS
Students will estimate monetary amounts to the nearest dollar using rounding.

PRIOR KNOWLEDGE REQUIRED
Rounding
Familiarity with Canadian coins
Adding money
Subtracting money

VOCABULARY
penny
toonie
dime
cent
quarter
dollar
loonie
estimate

Begin with a demonstration. Bring in a handful of change. Put it down on a table at the front of the class. Tell your students that you would like to buy a magazine that costs $2.50. Do they think that you have enough money? How could they find out?

Review rounding. Remind the class that rounding involves changing numbers to an amount that is close to the original, but which is easier to use in calculations. As an example, ask what is easier to add: 8 + 9 or 10 + 10.

Explain to the class that they will be doing two kinds of rounding—rounding to the nearest 10¢ and rounding to the nearest dollar. To round to the nearest 10¢ look at the digit in the ones column. If it is less than 5 (ask which numbers does this mean), you round down. If it is 5 or more (which numbers?), up. For example 33¢ would round down to 30¢, but 38¢ would round up to 40¢. Use volunteers to round: 39¢, 56¢, 52¢, 75¢, 60¢, 44¢.

Model rounding to the nearest dollar. In this case, the number to look at is in the tenths place (the dimes place). If an amount has 50¢ or more, round up. If it has less than 50¢, round down. So, $1.54 would round up to $2.00. $1.45 would round down to $1.00. Use volunteers to round: $1.39, $2.56¢, $3.50, $4.75, $0.60, $0.49.

Have students practice estimating by using play money coins. Place ten to fifteen play money coins on a table. Ask students to estimate the amount of money before they count the coins. (Students could play this game in pairs, taking turns placing the money and counting the money.)

Give several problems to show how rounding can be used for estimation:

Make an estimate and then find the exact amount:

a) Dana has $5.27. Tor has $2.38. How much more money does Dana have?

b) Mary has $18.74. Sheryl has $5.33. How much money do they have altogether?

c) Jason has saved $12.95. Does he have enough money to buy a book for $5.96 and a binder for $6.99? Why is rounding not helpful here?

Ask the students to explain why rounding to the nearest dollar isn’t helpful for the following question:

“Millicent has $8.15. Richard has $7.97. About how much more money does Millicent have than Richard?” (Both round to 8. Rounding to the nearest 10 cents helps.)

Make sure students understand that they can use rounding to check the reasonableness of answers. Ask students to explain how they know that $7.52 + $8.95 = $20.47 can’t be correct.
Assessment
1. Make an estimate and then find the exact amount: Molina has $7.89. Vinijaa has $5.79. How much more money does Molina have?

2. Benjamin spent $6.94 on pop, $12.85 on vegetables and dip and $2.15 on bagels. About how much did he spend altogether?

Extensions
1. What is the best way to round when you are adding two numbers: to round both to the nearest dollar, or to round one up and one down?

   Explore which method gives the best answer for the following amounts:
   
   $12.56 + $3.68  $34.55 + $54.57  $76.61 + $21.05

   Students should notice that, when two numbers have cent values that are both close to 50¢ and that are both greater than 50¢ or both less than 50¢, rounding one number up and one down gives a better result than the standard rounding technique. For instance, rounding $7.57 and $7.54 to the nearest dollar gives an estimated sum of $16.00 ($7.57 + $7.54 ≈ $8.00 + $8.00), whereas rounding one number up and the other down gives an estimated sum of $15.00, which is closer to the actual total.

2. Provide the BLM “Match the Numbers” to give students practice finding money amounts that add to $1 or $10.
NS4-99

Decimal Tenths

Draw the following pictures on the board and ask students to show the fraction $\frac{4}{10}$ in each picture:

Tell students that mathematicians invented decimals as another way to write tenths: One tenth ($\frac{1}{10}$) is written as 0.1 or just .1. Two tenths ($\frac{2}{10}$) is written as 0.2 or just .2. Ask a volunteer to write $\frac{7}{10}$ in decimal notation. (.7 or 0.7) Ask if there is another way to write it. (0.7 or .7) Then have students write the following fractions as decimals:

\[
a) \frac{3}{10} \\
b) \frac{8}{10} \\
c) \frac{9}{10} \\
d) \frac{5}{10} \\
e) \frac{6}{10} \\
f) \frac{4}{10}
\]

**Bonus**

Have students convert to an equivalent fraction with denominator 10 and then to a decimal:

\[
a) \frac{2}{5} \\
b) \frac{1}{2} \\
c) \frac{4}{5}
\]

In their notebooks, have students rewrite each addition statement using decimal notation:

\[
a) \frac{3}{10} + \frac{2}{10} = \frac{5}{10} \\
b) \frac{5}{10} + \frac{5}{10} = \frac{10}{10} \\
c) \frac{5}{10} + \frac{5}{10} = \frac{10}{10} \\
d) \frac{4}{10} + \frac{5}{10} = \frac{9}{10}
\]

**Bonus**

\[
a) \frac{1}{2} + \frac{1}{5} = \frac{7}{10} \\
b) \frac{1}{2} + \frac{5}{9} = \frac{9}{10}
\]

Draw on the board:

**ASK:** What fraction does this show? ($\frac{4}{10}$) What decimal does this show? (0.4 or .4)

Repeat with the following pictures:

Have students write the fractions and decimals for similar pictures independently, in their notebooks.
Then ask students to convert the following decimals to fractions, and to draw models in their notebooks:

a) 0.3       b) .8       c) .9       d) 0.2

Demonstrate the first one for them:

0.3 = \frac{3}{10}

Have students write addition statements, using fractions and decimals, for each picture:

\[ \begin{align*}
&\frac{1}{10} + \frac{3}{10} = \frac{4}{10} \\
&\frac{1}{10} + \frac{2}{10} = \frac{3}{10} \\
&\frac{2}{10} + \frac{3}{10} = \frac{5}{10}
\end{align*} \]

SAY: If I have 2 apples and add 3 more apples, how many apples do I have altogether? If I have 2 hockey cards and add 3 more hockey cards, how many hockey cards do I have altogether? If I have 2 tenths and add 3 more tenths, how many tenths do I have altogether? Write on the board: \(.2 + .3 = .5\)

ASK: What is \(.4 + .2\)? \(.3 + .4\)? \(.2 + .5\)? Have students rewrite each addition statement using fractions (EXAMPLE: \(\frac{2}{10} + \frac{3}{10} = \frac{5}{10}\)). SAY: I want to solve \(.2 + .8\). How would we write that in terms of fractions? What is \(\frac{2}{10} + \frac{8}{10}\)? Tell students that since 10 tenths is equal to one whole, we don’t need a decimal point. The answer is just 1. Write the complete addition statement on the board: \(.2 + .8 = 1\).

Have students add the following decimals:

\(.3 + .7 = \underline{1.0}\) \(.6 + .4 = \underline{1.0}\)

Then have them fill in the blanks:

\(.2 + \underline{.5} = 1\) \(.5 + \underline{.1} = 1\) \(.1 + \underline{.4} = 1\) \(.4 + \underline{.6} = 1\)

ASK: How is \(.2 + \underline{.4} = 1\) similar to \(2 + \underline{4} = 10\)? \(.2 + \underline{.4} = 1\) is just asking \(2\) tenths \(+\) \(4\) tenths = \(10\) tenths

Have students add the tenths and then shade the total number of tenths in the last tens block:

\[ \begin{align*}
&\frac{1}{10} + \frac{4}{10} = \frac{5}{10} \\
&\frac{2}{10} + \frac{3}{10} = \frac{5}{10} \\
&\frac{4}{10} + \frac{2}{10} = \frac{6}{10}
\end{align*} \]

Teach students to count forwards and backwards by decimal tenths using dimes. ASK: How many dimes make up a dollar? \(10\) What fraction of a dollar is a dime? \(\text{one tenth}\) Tell students that we can write the dime as \(.1\) dollars, since \(.1\) is just another way of writing \(\frac{1}{10}\). ASK: What fraction of a dollar are 2 dimes? How would we write that using decimal notation? What fraction of a dollar are…

a) 7 dimes? b) 3 dimes? c) 9 dimes? d) 6 dimes? e) 10 dimes?

Have students write all the fractions as decimals.
Extensions

1. Put the following sequence on the board: 0.1, 0.3, 0.5, _____
   
   Ask students to describe the sequence (add 0.2 each time) and to identify the next number (0.7). Even though the numbers are not in standard dollar notation, students can think of them in terms of dollars and dimes. 0.3 is 3 dimes, 0.5 is 5 dimes, 0.7 is 7 dimes, and so on. **ASK:** What is 1.3 in terms of dollars and dimes? (1 dollar and 3 dimes)

   Have students complete the following sequences by thinking of the numbers in terms of dollars and dimes and counting out loud. This will help students to identify the missing terms, particularly in sequences such as h) and i). Students should also state the pattern rules for each sequence (**EXAMPLE:** start at 0.1 and add 0.3).
   
   a) 0.1, 0.3, 0.5, _____
   b) 3.1, 3.4, 3.7, _____
   c) 1, 1.9, 1.8, _____
   d) 1, 1.8, 1.6, _____
   e) 3, 2.9, 2.8, _____
   f) 4.4, 4.2, 4, _____
   g) 0.2, 0.3, 0.4, _____, _____, _____
   h) 0.7, 0.8, 0.9, _____, _____
   i) 2.7, 2.8, 2.9, _____, _____, _____
   j) 1.4, 1.3, 1.2, _____, _____, _____

2. Have students draw a line 25 squares long on grid paper and mark the ends as shown:

   ![Number line diagram]

   Have them mark the location of 4.8, 5.0, and 5.8.

   Then repeat with endpoints 42 and 67 and have them mark the locations of 48, 50, and 58. **ASK:** How are these two questions similar and how are they different?

   Notice that 4.2 is just 42 tenths and 6.7 is 67 tenths, so the number line with end points 42 and 67 can be regarded as counting the number of tenths.
NOTE: In this and subsequent lessons, blank hundreds charts/hundreds blocks are used extensively to illustrate decimals and decimal concepts. If you have an overhead projector or interactive white board, you can use the BLM “Blank Hundreds Charts” to project and work with hundreds blocks on a board, wall, or screen. Alternatively, you can (a) work with enlarged photocopies of the BLM or (b) draw an oversized hundreds block on chart paper and laminate it.

Remind students that there are 10 tenths in every whole—10 tenths in a whole pie, 10 tenths in a whole tens block, 10 tenths in a whole chocolate bar.

ASK: What is \(\frac{1}{10}\) of a centimetre? (a millimetre) How many millimetres are in a centimetre? (10) What is \(\frac{1}{10}\) of a dollar? (a dime or 10 cents) How many dimes do you need to make a dollar? (10) What is one tenth of 1000? (one hundred) How many hundreds do you need to make a thousand? What is one tenth of 100? (ten) What is one tenth of 10? (one) What is one tenth of 1? (one tenth)

Write:

\[
1000 = 1 \text{ thousand} + 0 \text{ hundreds} + 0 \text{ tens} + 0 \text{ ones} \\
100 = 1 \text{ hundred} + 0 \text{ tens} + 0 \text{ ones} \\
10 = 1 \text{ ten} + 0 \text{ ones} \\
1 = 1 \text{ one}
\]

Tell students that just as we have a place value for thousands, hundreds, tens, and ones, mathematicians have invented a place value for tenths. Write the number 47 on the board and ASK: If we read the digits from left to right, do we start at the largest place value or the smallest? Does the 4 mean 4 tens or 4 ones? So if the largest place value is on the left and the place values get smaller as we move right, where should we put the place value for the tenths—to the left of the 4 or to the right of the 7? If I have 47 pies and one tenth of a pie, I could try writing it like this:

\[
\begin{array}{ccc}
\text{tens} & 4 & 7 \\
\text{ones} & 1 & 0
\end{array}
\]

Does anyone see a problem here? What number does this look like? We need a way to tell the difference between “four hundred seventy-one” and “forty-seven and one tenth.” We need a way to separate the ones from the tenths so that we know how many whole units we have. We could actually use anything.

Draw:

\[
\begin{array}{ccc}
4 & 7 & 1 \\
\text{tens} & \text{ones} & \text{tenths}
\end{array}
\]

\[
\begin{array}{c}
470 \\
47 \times 1
\end{array}
\]
**ASK:** How do we show the difference between whole dollars and tenths of a dollar, between dollars and dimes? What do we put in between the number of dollars and the number of dimes? Have a volunteer show four dollars and thirty cents on the board: $4.30 Tell them that the dot between the 4 and the 3 is called a decimal point and **ASK:** What if the decimal point wasn’t there—how much money would this look like?

Ask students to identify the place value of the underlined digits:

2.5  34.1  6.3  10.4  192.4  37.2  8,073.2  100.5

On a transparency or enlarged photocopy of a hundreds block, have a volunteer shade one tenth of the block. If the student shades a row or column, point out that the student chose an organized way of shading one tenth rather than randomly shading ten squares. If the student did the latter, have another volunteer show an organized way of shading one tenth. Then ask another volunteer to circle or otherwise highlight one tenth of the shaded part. **ASK:** What fraction of the hundreds block is the circled part? What coin is one tenth of a loonie? What coin is one tenth of a dime? What fraction of a loonie is that coin? What fraction of a whole anything is one tenth of a tenth? (one hundredth)

Write on the board: 4,183.25

Cover up all but the 4 and tell students that the 4 is the thousands digit. Uncover the 1 and **ASK:** What place value is the 1? How do you know? (PROMPT: What is one tenth of a thousand?) Continue in this way, uncovering each digit one at a time until you uncover the 5 and **SAY:** We know the 2 is the tenths digit. What is one tenth of a tenth? What place value is the 5?

Write on the board:

ones 8.0 4 hundredths
tenths

In their notebooks, have students write the place value of the underlined digit in these numbers:

2.71 3.42 7.65 8.46 9.01 0.83 0.14 0.97 0.45 4.5 45

Students can refer to the board for the spelling of “tenths” and “hundredths.”

Have students identify the place value of the digit 5 in each of these numbers:

5.03 8.05 50.03 30.05 74.35 743.5 7,435

**Bonus**

432.15 34,521.08

**Extensions**

1. Remind students that a hundred is ten times more than ten. Now ask them to picture a square divided into ten parts and a square divided into a hundred parts. Each little part in the second square (each hundredth) is ten times smaller than a part in the first square (a tenth). Another way to describe this is to say that there are ten hundredths in each tenth. **ASK:** What is ten times a hundred? (a thousand) What is one tenth of a hundredth? (a thousandth) What is ten times a thousand? (ten thousand) What is one tenth of a thousandth? (one ten thousandth)
Have students write the place value of the digit 5 in each number:

41.015       32.6752       872.0105       54 321.02679       867 778.3415

How many times more is the first 5 worth than the second 5:

a) 385.5072       3 855.072       38.55072       3.855072       .3855072
b) 525       52.5       5.25       .525       .0525
c) 5.5       51.5       512.5       5 127.5       51 270.5

How many times more is the 6 worth than the 2?

6.2       6.32       6.72       6.02       6.102       61.02       610.2       674.312

How many times more is the 2 worth than the 5?

2.5       2.35       2.75       2.05       2.105       21.05       210.5       274.315

2. Draw a decagon on the board and ask students to count the number of sides. Write “decagon” underneath it. Then write the word “decade” on the board. Ask if anyone can read the word and if anyone knows what the word means. **ASK:** Do these words have any letters in common? (yes; deca) What do the meanings of the words have in common? (Both words refer to 10 of something: the number of sides, the number of years.) What do you think “deca” means? Then **ASK:** How many events do you think are in a decathlon? Which month do you think was originally the tenth month when the calendar was first made? (December) Is it still the 10th month? (No, it’s now the 12th month.) Then write the word “decimal” on the board. Underline the first 3 letters and ask students to think about how 10 is important for decimal numbers. (Each digit has 10 times larger place value than the digit on the right. There are 10 single digits from 0 to 9.) Allow several students to answer in their own words.

3. Have students fill in the blanks in questions where you mix up the order of the words ones, tenths, and hundredths.

**EXAMPLES:**

5.02      _____ tenths _____ hundredths _____ ones

89.13      _____ hundredths _____ ones _____ tenths _____ tens
**NS4-101**

**Decimal Hundredths**

Have your students write the following fractions as decimals:

- \( \frac{7}{10} \)
- \( \frac{1}{10} \)
- \( \frac{3}{10} \)
- \( \frac{8}{10} \)
- \( \frac{6}{10} \)
- 9 tenths
- 5 tenths
- 6 tenths

Tell students that we use 1 decimal place to write how many tenths there are, and we use 2 decimal places to write how many hundredths there are. Show this picture:

**SAY:** There are 13 hundredths shaded. We can write this as 0.13 or .13 or \( \frac{13}{100} \).

Have students write both a fraction and a decimal for each of the following pictures:

Tell your students that writing decimals is a little trickier when there are less than 10 hundredths. **ASK:** What if we have only 9 hundredths—would we write .9? If no one recognizes that .9 is 9 tenths, **ASK:** How do we write 9 tenths? Is 9 tenths the same as 9 hundredths? Which is larger?

Put up 2 hundreds blocks (with the grid drawn in) and have one volunteer shade 9 tenths and another volunteer shade 9 hundredths. Tell students that we write 9 tenths as .9 and 9 hundredths as .09; write each decimal beneath the corresponding picture.

Put up the following pictures:

**a)**

**b)**

**c)**
d)  

e)  

f) 

Have volunteers write a decimal and a fraction for the first two and then have students do the same for the others independently, in their notebooks.

Put up several more hundreds blocks and tell the students that you are going to mix up the problems. Some pictures will show less than ten hundredths and some will show more than ten hundredths. Students should write the correct decimal and fraction for each picture in their notebooks.

**EXAMPLES:**

Extension

One row and one column are shaded. How many squares are shaded? What fraction is shaded?

Discuss various strategies students may have used to count the shaded squares. Did they count the squares to each side of the point at which the row and column overlap (2 + 7 in the row, 4 + 5 in the column, 1 for the square in the middle)? Did they add 10 + 10 for the row and column and then subtract 1 for the one square they counted twice? Did they count the squares in the column (10) and then add 2 + 7 for the squares not yet counted in the row? Did they add the 4 rectangles of white squares (8 + 10 + 28 + 35) and then subtract from 100? Did they push the 4 white rectangles together and see that it forms a 9 by 9 square?

Have students write decimals and fractions for the following pictures.

Encourage students to count the shaded squares using different strategies. Students might count some squares twice and adjust their answers at the end; they might subtract the non-shaded squares from the total; they might divide the shaded parts into convenient pieces, count the squares in the pieces separately, and add the totals. Have students show and explain various solutions to their classmates, so that all students see different ways of counting the squares. Tell students that when they can solve a problem in two different ways and get the same answer, they can know that they’re right. They won’t need you to check the answer because they can check it themselves!
GOALS
Students will practise identifying and distinguishing between tenths and hundredths.

PRIOR KNOWLEDGE REQUIRED
Tenths and hundredths
Decimal notation
Fractional notation
Reading 2-digit whole numbers such as 35 as 3 tens and 5 ones or 35 ones

Put up 2 blank hundreds blocks. Have one volunteer shade one tenth of one block, and have a second volunteer shade one hundredth of the other block. Invite other volunteers to write the corresponding decimals and fractions for each block:

\[
0.1 = \frac{1}{10} \quad 0.01 = \frac{1}{100}
\]

Point to the block showing one tenth and SAY: How many hundredths does this block show? How else can we write that as a decimal? (We can write 0.10.) Emphasize that 0.1 is equal to 0.10, and tell students that 0.10 can be read as “10 hundredths” or “1 tenth and 0 hundredths.”

Draw on the board:

ASK: How many squares are shaded? (43) How many hundredths does this picture show? (43) How many full rows are shaded? (4) Since 1 full row is 1 tenth, how many tenths are shaded? (4) How many more squares are shaded? (3) SAY: There are 4 full rows and 3 more squares shaded. So there are 4 tenths and 3 hundredths. Tell students that we can write 4 tenths and 3 hundredths as follows:

\[
\begin{array}{c}
\text{ones} \\
0.43 \\
\text{tenths}
\end{array}
\]

\[
\begin{array}{c}
\text{hundredths}
\end{array}
\]

We can read this as “43 hundredths” or “4 tenths and 3 hundredths.”

ASK: How is this similar to the different ways we can read the 2-digit number 43? (we can read 43 as 4 tens and 3 ones or just as 43 ones)

On grid paper, have students draw a hundreds block, shade in a given fraction, and then write the fraction as a decimal. (Students can also work on a copy of the BLM “Blank Hundreds Charts.”) When students are done, have them read the decimal number in terms of hundredths only and then in terms of tenths and hundredths. Give students more fractions to illustrate and write as decimals:

\[
\begin{align*}
a) \quad \frac{26}{100} & \quad b) \quad \frac{81}{100} & \quad c) \quad \frac{14}{100} & \quad d) \quad \frac{41}{100} & \quad e) \quad \frac{56}{100} & \quad f) \quad \frac{72}{100}
\end{align*}
\]
Draw the following figure on the board:

ASK: How many full rows of ten are there? (none) So how many tenths do we have? (0) How many hundredths are there? (4). Write on the board:

ones ⟷ 0.04 ⟷ hundredths

ASK: How can we read this number? (4 hundredths or 0 tenths and 4 hundredths)

Have students draw the following fractions on grid paper and write the corresponding decimals:

a) \( \frac{5}{100} \)  
b) \( \frac{7}{100} \)  
c) \( \frac{1}{100} \)  
d) \( \frac{2}{100} \)  
e) \( \frac{9}{100} \)  
f) \( \frac{3}{100} \)

Finally, draw the following figure on the board:

ASK: How many full rows of ten are there? (4) So how many tenths do we have? (4) How many hundredths are in 4 tenths? (40) Are any other hundredths shaded? (no, just the 4 full rows)

Write on the board:

ones ⟷ 0.40 ⟷ hundredths

tell students that we can read this as “4 tenths and 0 hundredths” or “40 hundredths.”

Have students draw the following fractions and write the corresponding decimals.

a) \( \frac{50}{100} \)  
b) \( \frac{70}{100} \)  
c) \( \frac{80}{100} \)  
d) \( \frac{30}{100} \)  
e) \( \frac{50}{100} \)  
f) \( \frac{10}{100} \)

Now have students illustrate and rewrite fractions that have either 0 tenths or 0 hundredths:

a) \( \frac{6}{100} \)  
b) \( \frac{60}{100} \)  
c) \( \frac{90}{100} \)  
d) \( \frac{3}{100} \)  
e) \( \frac{20}{100} \)  
f) \( \frac{8}{100} \)

Finally, give students a mix of all 3 types of fractions:

a) \( \frac{36}{100} \)  
b) \( \frac{40}{100} \)  
c) \( \frac{5}{100} \)  
d) \( \frac{18}{100} \)  
e) \( \frac{46}{100} \)  
f) \( \frac{8}{100} \)

Extension

Decimals are not the only numbers that can be read in different ways. Show students how all numbers can be read according to place value. The number 34 can be read as “34 ones” or “3 tens and 4 ones.” Similarly, 7.3 can be read as “73 tenths” or “7 ones and 3 tenths.” Challenge students to find 2 ways of reading the following numbers:
a) 3 500 (3 thousands and 5 hundreds or 35 hundreds)
b) 320 (3 hundreds and 2 tens or 32 tens)
c) 5.7 (5 ones and 7 tenths or 57 tenths)
d) 1.93 (19 tenths and 3 hundredths or 193 hundredths)
e) 0.193 (19 hundredths and 3 thousandths or 193 thousandths)

NS4-103
Changing Tenths to Hundredths

GOALS
Students will understand that whole decimal tenths can be written with a 0 in the hundredths position to form an equivalent decimal.

PRIOR KNOWLEDGE REQUIRED
Equivalent fractions
Decimal notation
Tenths and hundredths

VOCABULARY
equivalent fraction
equivalent decimal

Draw the following figure on the board:

**ASK:** What fraction of the first square is shaded? What fraction of the second square is shaded? Are these equivalent fractions? How do you know?

Draw the following figure on the board:

**ASK:** What fraction of the square is shaded? How many of the 100 equal parts are shaded? How many of the 10 equal rows are shaded? Are these equivalent fractions?

Have students use the following pictures to find equivalent fractions with denominators 10 and 100.

Then have students find equivalent fractions without using pictures:

\[
\begin{align*}
a) \quad & \frac{80}{100} = \frac{8}{10} \\
b) \quad & \frac{2}{100} = \frac{2}{10} \\
c) \quad & \frac{40}{100} = \frac{4}{10} \\
d) \quad & \frac{1}{100} = \frac{1}{10}
\end{align*}
\]
When students can do this confidently, ask them to describe how they are getting their answers. Then remind students that a fraction with denominator 100 can be written as a decimal with 2 decimal places. **ASK:** What decimal is equivalent to $\frac{80}{100}$? (0.80) Remind them that a fraction with denominator 10 can be written as a decimal with 1 decimal place and **ASK:** What decimal is equivalent to $\frac{8}{10}$? (0.8) Tell them that mathematicians call 0.80 and 0.8 equivalent decimals and ask if anyone can explain why they are equivalent. (They have the same value; the fractions they are equivalent to are equivalent).

Have students rewrite these equivalent fractions as equivalent decimals:

a) $\frac{90}{100} = \frac{9}{10}$  

b) $\frac{20}{100} = \frac{2}{10}$  

c) $\frac{40}{100} = \frac{4}{10}$  

d) $\frac{10}{100} = \frac{1}{10}$

Tell students that saying “0.9 = 0.90” is the same as saying “9 tenths is equal to 90 hundredths or 9 tenths and 0 hundredths.” **ASK:** Is “3 tenths” the same as “3 tenths and 0 hundredths?” How many hundredths is that? Have a volunteer write the equivalent decimals on the board. (.3 = .30)

Have students fill in the blanks:

a) $.3 = \frac{3}{10} = \frac{30}{100} = \_\_\_\_\_\_\_$  

de) $.7 = \frac{7}{10} = \frac{70}{100} = \_\_\_\_\_\_$

e) $.9 = \frac{9}{10} = \frac{90}{100} = \_\_\_\_\_\_$

**Extension**

**ASK:** Do you think that 5 hundredths is the same as 5 hundredths and 0 thousandths? How would we write the decimals for those numbers? (.05 = .050) Do you think that 7 ones is the same as 7 ones and 0 tenths? How would you write that in decimal notation? (7 = 7.0) Have students circle the equivalent decimals in each group of three:

a) 8  0.8  8.0
b) 0.04  0.40  0.4

c) 0.03  0.030  0.3

d) 0.9  9.0  9

e) 2.0  0.2  0.20
NS4-104
Decimals and Money

GOALS
Students will relate tenths and hundredths of whole numbers to tenths and hundredths of dollars, that is, to dimes and pennies. Students will use this understanding to compare and order decimals having one and two decimal places.

PRIOR KNOWLEDGE REQUIRED
Pennies, dimes, and dollars
Decimal notation
Writing values such as 3 tenths and 5 hundredths as 35 hundredths

Tell students that a dime is one tenth of a dollar then ASK: Does this mean I can take a loonie and fit 10 dimes onto it? Does it mean 10 dimes weigh the same as a loonie? What does it mean? Make sure students understand that you are referring not to weight or area, but to value—a dime has one tenth the value of a dollar, it is worth one tenth the amount. Ask students what fraction of a dollar a penny is worth. (one hundredth)

Have students find different ways of making $0.54 using only dimes and pennies. (5 dimes and 4 pennies, 4 dimes and 14 pennies, and so on until 54 pennies)

Tell students that since a dime is worth a tenth of a dollar and a penny is worth a hundredth of a dollar, they can write 5 dimes and 4 pennies as 5 tenths and 4 hundredths. ASK: How else could you write 4 dimes and 14 pennies? (4 tenths and 14 hundredths) Continue rewriting all the combinations of dimes and pennies that make $0.54. Tell students that they have seen 2 of these combinations before. Do they remember which ones and why they are special? Prompt by asking: Which way uses the most tenths (or dimes)? The most hundredths (or pennies)?

Then say you have 6 tenths and 7 hundredths and ask students to say this in terms of just hundredths. Have students rewrite both expressions in terms of dimes and pennies. (6 dimes and 7 pennies; 67 pennies)

Then give students a decimal number (EXAMPLE: .18) and have students express it in hundredths only (18 hundredths), tenths and hundredths (1 tenth and 8 hundredths), pennies only (18 pennies), and dimes and pennies (1 dime and 8 pennies). Repeat with several decimals, including some that have no tenths or no hundredths.

EXAMPLES:

| .94 | .04 | .90 | .27 | .70 | .03 | .60 | .58 | .05 |

ASK: Which is worth more, 2 dimes and 3 pennies or 8 pennies? How many pennies are 2 dimes and 3 pennies worth? Which is more, 23 or 8? How would we write 2 dimes and 3 pennies as a decimal of a dollar? (.23) How would we write 8 pennies as a decimal? (.08) Which is more, .23 or .08?

ASK: How would you make $0.63 using pennies and dimes? How would you make .9 dollars using pennies and dimes? Which is more, .63 or .9?

Give students play-money dimes and pennies and ask them to decide which is more between:

a) .4 and .26   b) .4 and .62   c) .3 and .42   d) .3 and .24

Tell the class that you once had a student who said that .41 is more than .8 because 41 is more than 8. ASK: Is this correct? What do you think the student was thinking? Why is the student wrong? (The student was thinking
of the numbers after the decimal point as whole numbers. Since 41 is more than 8, the student thought that .41 would be more than .8. But .8 is 8 tenths and .41 is 41 hundredths. The tenths are 10 times greater than the hundredths, so comparing .8 to .41 is like comparing 8 tens blocks to 41 ones blocks, or 8 cm to 41 mm. There might be more ones blocks, but they’re worth a lot less than the tens blocks. Similarly, there are more mm than there are cm, but 8 cm is still longer than 41 mm.)

Students often make mistakes in comparing decimals with 1 and 2 decimal places. For instance, they will say that .17 is greater than .2. This activity will help students understand the relationship between tenths and hundredths.

Give each student play-money dimes and pennies. Remind them that a dime is a tenth of a dollar (which is why it is written as $0.10) and a penny is a hundredth of a dollar (which is why it is written as $0.01). Ask students to make models of the amounts in the left-hand column of the chart below and to express those amounts in as many different ways as possible by filling in the other columns. (Sample answers are shown in italics.) You might choose to fill out the first row together.

<table>
<thead>
<tr>
<th>Amount</th>
<th>Amount in Pennies</th>
<th>Decimal Names (in words)</th>
<th>Decimal Names (in numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 dimes</td>
<td>20 pennies</td>
<td>2 tenths (of a dollar)</td>
<td>.2 20 hundredths</td>
</tr>
<tr>
<td>3 pennies</td>
<td>3 pennies</td>
<td>3 hundredths</td>
<td>.03</td>
</tr>
<tr>
<td>4 dimes and 3</td>
<td>43 pennies</td>
<td>4 tenths and 3</td>
<td>.43 43 hundredths</td>
</tr>
<tr>
<td>3 pennies</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When students have filled in the chart, write various amounts of money on the board in decimal notation and have students make models of the amounts. (EXAMPLES: .3 dollars, .27 dollars, .07 dollars) Challenge students to make models of amounts that have 2 different decimal representations (EXAMPLES: .2 dollars and .20 dollars both refer to 2 dimes).

When you feel students are able to translate between dollar amounts and decimal notation, **ASK:** Would you rather have .2 dollars or .17 dollars? In their answers, students should say exactly how many pennies each amount represents; they must articulate that .2 represents 20 pennies and so is actually the larger amount.

For extra practice, ask students to fill in the right-hand column of the following chart and then circle the greater amount in each column. (Create several such charts for your students.)

<table>
<thead>
<tr>
<th>Amount (in dollars)</th>
<th>Amount (in pennies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>.15</td>
<td></td>
</tr>
</tbody>
</table>
Extension

Have students create a hundredths chart by filling in a blank hundreds chart with hundredths, beginning with .01 in the top left corner and moving across and then down each row.

Ask students to find the following patterns in their charts. They should describe the patterns of where the numbers occur using the words “column” and “row.”

| .45 | .68 | .14 | .01 |
| .55 | .78 | .25 | .12 |
| .65 | .88 | .34 | .23 |
| .75 | .98 | .45 | .34 |

(Start in the fifth row and fifth column, move down one row and then repeat; Start in the seventh row and eighth column, move down one row and then repeat; Start in the second row and fourth column, move one row down and one column right, then one row down and one column left, and then repeat; Start in the first row and first column, move one row down and one column right, then repeat.)

Bonus

Fill in the missing numbers in the patterns below without looking at your chart.

| .65 | .33 |   |
| .75 |   | .53 |
| .95 |   | .64 |
NS4-105
Changing Notation: Fractions and Decimals

GOALS
Students will translate between fractional and decimal notation.

PRIOR KNOWLEDGE REQUIRED
Decimal notation
Fractions
Tenths and hundredths

Have students copy the following chart, with room for rows a) to e), in their notebooks:

<table>
<thead>
<tr>
<th></th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

draw on the board:

a)

b)

c)

d)

e)

Have students fill in their charts by recording the number of tenths in the first column and the number of hundredths left over (after they’ve counted the tenths) in the second column. When students are done, have them write in their notebooks the fractions shown and the corresponding decimals. Remind them that hundredths are shown with 2 decimal places: 9 hundredths is written as .09, not .9, because .9 is how we write 9 tenths and we need to make 9 hundredths look different from 9 tenths. (More on the differences later in the lesson.)

Point out, or ask students to describe, the relationship between the chart and the decimal numbers: the tenths (in the first column) are recorded in the first decimal place; the extra hundredths are recorded in the second decimal place.

Have students make 5 (non-overlapping!) hundreds blocks (10 by 10) on grid paper and label them a) to e). (You could also hand out copies of the BLM “Blank Hundreds Charts.”)

While they are doing this, write on the board:

a) .17  b) .05  c) .20  d) .45  e) .03

Tell students to shade in the correct fraction of each square to show the decimal number. When they are done, students should translate the decimals to fractions.

Remind students that the first decimal place (to the right of the decimal point) counts the number of tenths and ASK: What does the second decimal place count? Then write .4 on the board and ASK: What does this number mean—is it 4 tenths, 4 hundredths, 4 ones, 4 hundreds—what? How do you know?
(It is 4 tenths, because the 4 is the first place after the decimal point). **ASK:** If you write .4 as a fraction, what will the denominator be? (You may need to remind students that the denominator is the bottom number.) What will the numerator be? Write \( \frac{4}{10} \) on the board.

Then write on the board: 0.28. **ASK:** How many tenths are in this number? How many hundredths are there in each tenth? How many more hundredths are there? How many hundredths are there altogether? What is the numerator of the fraction? The denominator? Write \( \frac{28}{100} \) on the board.

Have students fill in the numerator of each fraction:

\[
\begin{align*}
a) \quad .6 &= \frac{6}{10} & b) \quad .9 &= \frac{9}{10} & c) \quad .2 &= \frac{2}{10} & d) \quad .63 &= \frac{63}{100} & e) \quad .97 &= \frac{97}{100} & f) \quad .48 &= \frac{48}{100} \\
g) \quad .50 &= \frac{50}{100} & h) \quad .07 &= \frac{7}{100} & i) \quad .8 &= \frac{8}{10} & j) \quad .09 &= \frac{9}{100} & k) \quad .90 &= \frac{90}{100} & l) \quad .9 &= \frac{9}{10}
\end{align*}
\]

Then tell students that you are going to make the problems a bit harder. They will have to decide whether the denominator in each fraction is 10 or 100. Ask a volunteer to remind you how to decide whether the fraction should have denominator 10 or 100. Emphasize that if there is only 1 decimal place, it tells you the number of tenths, so the denominator is 10; if there are 2 decimal places, it tells you the number of hundredths, so the denominator is 100. Students should be aware that when we write \( \frac{.30}{10} \), we are saying that 30 hundredths are the same as 3 tenths. The 2 decimal places in \( .30 \) tell us to count hundredths, whereas the denominator of \( \frac{3}{10} \) tells us to count tenths.

Have students write the fraction for each decimal in their notebooks:

\[
\begin{align*}
a) \quad .4 & \quad b) \quad .75 & \quad c) \quad .03 & \quad d) \quad .3 & \quad e) \quad .30 & \quad f) \quad .8 & \quad g) \quad .09 & \quad h) \quad .42 & \quad i) \quad .2 & \quad j) \quad .5 & \quad k) \quad .50
\end{align*}
\]

**Bonus**

\[
\begin{align*}
.789 & \quad .060 & \quad .007 & \quad .053 & \quad .301 & \quad .596 & \quad .0102507
\end{align*}
\]

Write on the board: \( \frac{38}{100} \). **ASK:** How would we change this to a decimal? How many places after the decimal point do we need? (2) How do you know? (Because the denominator is 100, so we’re counting the number of hundredths). Ask volunteers to write decimals for the following fractions:

\[
\begin{align*}
\frac{3}{100} & \quad \frac{24}{100} & \quad \frac{8}{10} & \quad \frac{85}{100} & \quad \frac{8}{10} & \quad \frac{7}{10}
\end{align*}
\]

Have them change the following fractions to decimals in their notebooks:

\[
\begin{align*}
a) \quad \frac{29}{100} & \quad b) \quad \frac{4}{10} & \quad c) \quad \frac{4}{10} & \quad d) \quad \frac{13}{100} & \quad e) \quad \frac{6}{10} & \quad f) \quad \frac{70}{100} & \quad g) \quad \frac{3}{10} & \quad h) \quad \frac{30}{100} & \quad i) \quad \frac{67}{100} & \quad j) \quad \frac{7}{100}
\end{align*}
\]

**Bonus**

\[
\begin{align*}
a) \quad \frac{293}{1000} & \quad b) \quad \frac{48}{1000} & \quad c) \quad \frac{4}{1000}
\end{align*}
\]

**Bonus**

Have students rewrite the following addition statements using decimal notation:

\[
\begin{align*}
a) \quad \frac{3}{10} + \frac{4}{100} &= \frac{36}{100} & b) \quad \frac{40}{100} + \frac{5}{100} &= \frac{45}{100} & c) \quad \frac{21}{100} + \frac{31}{100} &= \frac{52}{100} & d) \quad \frac{22}{100} + \frac{7}{10} &= \frac{92}{100}
\end{align*}
\]

Put the following equivalences on the board and **SAY:** I asked some students to change decimals to fractions and these were their answers. Which ones are incorrect? Why are they incorrect?

\[
\begin{align*}
a) \quad .37 &= \frac{37}{100} & b) \quad .68 &= \frac{68}{10} & c) \quad .4 &= \frac{4}{100} & d) \quad .90 &= \frac{90}{10} & e) \quad .9 &= \frac{9}{10} & f) \quad .08 &= \frac{8}{10}
\end{align*}
\]
Extensions

1. Cut the pie into more pieces to show $\frac{2}{5} = .4$

2. Draw a letter E covering more than .3 and less than .5 of a $10 \times 10$ grid.

3. **ASK:** When there is 1 decimal place, what is the denominator of the fraction? When there are 2 decimal places, what is the denominator of the fraction? What do you think the denominator of the fraction will be when there are 3 decimal places? How would you change .00426 to a fraction? How would you change $\frac{9,823}{10,000,000}$ to a decimal?

---

**NS4-106**

**Decimals and Fractions Greater Than One**

**GOALS**

Students will write mixed fractions as decimals. Students will use pictures to compare and order decimals having 1 decimal place with decimals having 2 decimal places.

**PRIOR KNOWLEDGE REQUIRED**

Translating between decimal and fractional notation for numbers less than 1

**VOCABULARY**

mixed fraction

**ASK:** If I use a hundreds block to represent a whole, what can I use to show one tenth? (a tens block) What fraction does a ones block show? (one hundredth)

Draw on the board:

<table>
<thead>
<tr>
<th>2 wholes</th>
<th>4 tenths</th>
<th>3 hundredths</th>
</tr>
</thead>
</table>

Tell students that this picture shows the decimal 2.43 or the fraction $\frac{233}{100}$. Mixed fractions can be written as decimals, too! Have volunteers write the mixed fraction and the decimal shown by each picture:

- a) [Diagram of mixed fraction]
- b) [Diagram of mixed fraction]
- c) [Diagram of mixed fraction]

Have students draw base ten models for these mixed fractions in their notebooks:

- a) $4 \frac{21}{100}$
- b) $2 \frac{8}{100}$
- c) $1 \frac{30}{100}$
**Bonus**

First change these fractions to mixed fractions and then draw models:

- a) \(\frac{103}{100}\)
- b) \(\frac{74}{100}\)
- c) \(\frac{290}{100}\)

Then have students draw a model for each decimal:

- a) 3.14
- b) 2.53
- c) 4.81

Put up the following pictures and have volunteers write a mixed fraction and a decimal for each. Remind students that each fully shaded hundreds block is a whole.

**ASK:** How are these different from the base ten pictures? How are they the same? (The difference is that we haven’t pulled the columns (tenths) and squares (hundredths) apart; they’re both lumped together in a shaded square.)

Put up more pictures and have students write mixed fractions and decimals in their notebooks. Include pictures in which there are no tenths or no more hundredths after the tenths are counted.

**EXAMPLES:**

Have students draw pictures on grid paper and write decimals for each mixed fraction:

- a) 1 \(\frac{95}{100}\)
- b) 2 \(\frac{39}{100}\)
- c) 2 \(\frac{4}{100}\)
- d) 3 \(\frac{82}{100}\)
- e) 1 \(\frac{9}{10}\)
- f) 1 \(\frac{9}{10}\)

Have students change these fractions to decimals without using a picture:

- a) 3 \(\frac{18}{100}\)
- b) 12 \(\frac{3}{10}\)
- c) 25 \(\frac{4}{100}\)
- d) 34 \(\frac{8}{10}\)
- e) 11 \(\frac{96}{100}\)
- f) 41 \(\frac{19}{100}\)  **BONUS:** \(\frac{5138}{100}\)

Then have students draw pictures on grid paper to show each decimal:

- a) .53
- b) .03
- c) .30
- d) .19
- e) .8
- f) .08

Finally, have students draw pictures on grid paper to show each pair of decimals and then to decide which decimal is greater:

- a) 3.04 or 3.17
- b) 1.29 or 1.7
- c) 1.05 or 1.50
- d) 5 tenths or 5 hundredths

**Bonus**

Have students draw pictures on grid paper to show each decimal and then to put the decimals in order from smallest to largest. What word do the corresponding letters make?

E. 7.03  S. 7.30  I. 2.15  L. 2.8  M. 2.09
NOTE: The correct word is “miles.” (You might need to explain to students what a mile is.) If any students get “smile” or “slime,” they might be guessing at the correct answer (this is much more likely than having made a mistake in the ordering of the numbers). Encourage them to put the numbers in order before trying to unscramble the letters. Then they can be reasonably confident that their word is the right one. If the numbers are in order and the word makes sense, they don’t need you to confirm their answer. It is important to foster this type of independence.

Extensions

1. Students can make up their own puzzle like the one in the last Bonus questions. Partners can solve each other’s puzzles.

   **STEP 1:** Choose a 3- or 4-letter word whose letters can be used to create other words.

   **EXAMPLE:** The letters in the word “dare” can be rearranged to make “read” or “dear.”

   **NOTE:** This step is crucial. Be sure students understand why it is important for the letters they choose to be able to make at least 2 different words.

   **STEP 2:** Choose one decimal number for each letter and write them in order.

   **EXAMPLE:**
   - d. 1.47
   - a. 1.52
   - r. 2.06
   - e. 2.44

   **STEP 3:** Scramble the letters, keeping the corresponding numbers with them.

   **EXAMPLE:**
   - a. 1.52
   - e. 2.44
   - d. 1.47
   - r. 2.06

   **STEP 4:** Give the scrambled letters and numbers to a partner to put in the correct order. Did your partner find the word you started with?

2. Show .2 in each of these base ten blocks:

   a) thousands block
   b) hundreds block
   c) tens block
NS4-107
Decimals and Fractions on Number Lines

GOALS
Students will place decimal numbers and mixed fractions on number lines. Students will also write the words for decimals and fractions (proper, mixed, or improper).

PRIOR KNOWLEDGE REQUIRED
Decimal numbers with up to 2 decimal places and their equivalent fractions (proper or mixed)
Translating between mixed and improper fractions
Number lines

Draw on the board:

| 0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1 |

Have students count out loud with you from 0 to 1 by tenths: zero, one tenth, two tenths, … nine tenths, one.

Then have a volunteer write the equivalent decimal for 1/10 on top of the number line:

| 0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1 |

Continue in random order until all the equivalent decimals have been added to the number line.

Then have students write, in their notebooks, the equivalent decimals and fractions for the spots marked on these number lines:

a) |
0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1 |

b) |
0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1 |

c) |
0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1 |

d) |
0 | A | 1 | B | C | 2 | D | 3 |

Have volunteers mark the location of the following numbers on the number line with an X and the corresponding letter.

A. 0.7  B. 2 7/10  C. 1.40  D. 8/10  E. 1 9/10

Invite any students who don’t volunteer to participate. Help them with prompts and questions such as: Is the number more than 1 or less than 1? How do you know? Is the number between 1 and 2 or between 2 and 3? How do you know?

Review translating improper fractions to mixed fractions, then ask students to locate the following improper fractions on a number line from 0 to 3 after changing them to mixed fractions:

A. 17/10  B. 23/10  C. 14/10  D. 28/10  E. 11/10
When students are done, **ASK:** When the denominator is 10, what is an easy way to tell whether the improper fraction is between 1 and 2 or between 2 and 3? (Look at the number of tens in the numerator—it tells you how many ones are in the number.)

**ASK:** How many tens are in 34? (3) 78? (7) 123? (12) 345? (34)

**ASK:** How many ones are in \(\frac{36}{10}\)? (3) \(\frac{78}{10}\)? (7) \(\frac{123}{10}\)? (12) \(\frac{345}{10}\)? (34)

**ASK:** What two whole numbers is each fraction between?

- a) \(\frac{29}{10}\)
- b) \(2 \frac{4}{10}\)
- c) \(12 \frac{7}{10}\)
- d) \(\frac{61}{10}\)
- e) \(12\frac{77}{10}\)
- f) \(\frac{318}{10}\)

Invite volunteers to answer a) and b) on the board, then have students do the rest in their notebooks. When students are finished, **ASK:** Which 2 fractions in this group are equivalent?

Tell students that there are 2 different ways of saying the number 12.4. We can say “twelve decimal four” or “twelve and four tenths”. Both are correct. (Note that “twelve point four” is also commonly used.) Point out the word “and” between the ones and the tenths, and tell students that we always include it when a number has both ones and tenths (and/or hundredths).

Have students place the following fractions on a number line from 0 to 3:

- A. three tenths
- B. two and five tenths
- C. one and seven tenths
- D. one decimal two
- E. two decimal eight

Draw a number line from 0 to 3 on the board. Mark the following points with an X—no numbers—and have students write the number words for the points you marked:

1.3  .7  2.4  .1  2.8  2.1

Draw a line on the board with endpoints 0 and 1 marked:

0   1

Ask volunteers to mark the approximate position of each number with an X:

- a) .4
- b) \(\frac{6}{10}\)
- c) 0.9

Then draw a number line from 0 to 3 with whole numbers marked:

0   1   2   3

Ask volunteers to mark the approximate position of these numbers with an X:

- a) 2.1
- b) 1 \(\frac{3}{10}\)
- c) \(\frac{39}{10}\)
- d) .4
- e) 2 \(\frac{2}{10}\)

Continue with more numbers and number lines until all students have had a chance to participate. (EXAMPLE: Draw a number line from 0 to 2 with whole numbers marked and have students mark the approximate position of 0.5, 1.25, and others.)

**Bonus**

Use larger whole numbers and/or longer number lines.
Extensions

1. Use a metre stick to draw a line that is 2 metres long and ask students to mark 1.76 metres.

2. Mark the given decimals on the number lines:
   a) Show .4
   
   0 .2

   b) Show 1.5
   
   0 .5

NS4-108
Comparing Fractions and Decimals

Draw on a transparency:

0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1

(NOTE: If you don’t have an overhead projector, tape the transparency to the wall and invite students, in small groups if necessary, to gather around it as you go through the first part of the lesson.) Have a volunteer show where $\frac{1}{2}$ is on the number line. Have another number line the same length divided into 2 equal parts on another transparency and superimpose it over this one, so that students see that $\frac{1}{2}$ is exactly at the .5 mark. **ASK:** Which decimal is equal to $\frac{1}{2}$? Is 0.2 between 0 and $\frac{1}{2}$ or between $\frac{1}{2}$ and 1? Is 0.7 between 0 and $\frac{1}{2}$ or between $\frac{1}{2}$ and 1? What about 0.6? 0.4? 0.3? 0.9?

Go back to the decimal 0.2 and **ASK:** We know that 0.2 is between 0 and $\frac{1}{2}$, but is it closer to 0 or to $\frac{1}{2}$? Draw on the board or the transparency:

0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1

**ASK:** Is .6 between 0 and $\frac{1}{2}$ or between $\frac{1}{2}$ and 1? Which number is it closest to, $\frac{1}{2}$ or 1? Have a volunteer show the distance to each number with arrows. Which arrow is shorter? Which number is .4 closest to, 0, $\frac{1}{2}$ or 1? Which number is .8 closest to? Go through all of the remaining decimal tenths between 0 and 1.

On grid paper, have students draw a line 10 squares long. Then have them cut out the line—leaving space above and below for writing—and fold it in half so that the two endpoints meet. They should mark the points 0, $\frac{1}{2}$, and 1 on their line. Now have students fold the line in half again, and then fold it in half a second time. have them unfold the line and look at the folds. **ASK:** What fraction is exactly halfway between 0 and $\frac{1}{2}$? How do you know?
(\frac{1}{4} \text{ because the sheet is folded into 4 equal parts so the first fold must be } \frac{1}{4} \text{ of the distance from 0 to 1}) \text{ What fraction is halfway between } \frac{1}{4} \text{ and 1? How do you know? (\frac{3}{4} \text{ because the sheet is folded into 4 equal parts so the third fold must be } \frac{3}{4} \text{ of the distance from 0 to 1}).}

Have students mark the fractions \(\frac{1}{4}\) and \(\frac{3}{4}\) on their number lines. Then have students write the decimal numbers from .1 to .9 in the correct places on their number lines (using the squares on the grid paper to help them).

Tell students to look at the number lines they’ve created and to fill in the blanks in the following questions by writing “less than” or “greater than” in their notebooks.

a) 0.4 is _______ \(\frac{1}{4}\)  
b) 0.4 is _______ \(\frac{1}{2}\)  
c) 0.8 is _______ \(\frac{3}{4}\)

d) 0.2 is _______ \(\frac{1}{4}\)  
e) 0.3 is _______ \(\frac{1}{2}\)  
f) 0.7 is _______ \(\frac{3}{4}\)

Have students rewrite each statement using the “greater than” and “less than” symbols: > and <.

ASK: Which whole number is each decimal, mixed fraction, or improper fraction closest to?

\[
\begin{array}{ccccccc}
& & & & & & \\
0 & & & & & & 1 & 2 & 3 \\
& & & & & & a) 0.7 & b) 1 \frac{4}{10} & c) 2.3 & d) \frac{18}{10} & e) 2 \frac{6}{10} & f) 1.1 \\
& & & & & & 15 & 16 & 17 & 18 \\
& & & & & & a) 17.2 & b) 16.8 & c) 16 \frac{3}{10} & d) \frac{174}{10} & e) 15.9 & f) 15.3 \\
\end{array}
\]

ASK: Which decimal is halfway between 1 and 2? Halfway between 17 and 18? Halfway between 31 and 32? Between 0 and 3? Between 15 and 18? Between 30 and 33? Between 25 and 28?

Bonus
Which whole number is each decimal closest to?

a) 23.4  
b) 39.8  
c) 314.1  
d) 235.6  
e) 981.1  
f) 999.9
NS4-109
Ordering Fractions and Decimals

GOALS
Students will compare and order decimals and fractions by first changing them all to fractions with denominator 10 or 100.

PRIOR KNOWLEDGE REQUIRED
Ordering mixed fractions with the same denominator
Decimal place value
Translating between fractions with denominator 10 or 100 and decimals
Equivalent fractions

To ensure students have the prior knowledge required, ask them to do the following questions in their notebooks.

1. Write each decimal as a fraction or mixed fraction with denominator 10.
   a) 0.7       b) 0.4       c) 1.3       d) 2.9       e) 7.4       f) .6
2. Put the fractions in order from smallest to largest.
   a) 1 \( \frac{7}{10} \) 2 \( \frac{4}{10} \) 1 \( \frac{9}{10} \)  b) 1 \( \frac{3}{10} \) 1 \( \frac{3}{10} \) 2 \( \frac{1}{10} \)  c) 13 \( \frac{7}{10} \) 12 \( \frac{5}{10} \) 12 \( \frac{3}{10} \)

Circulate while students work and assist individuals as required. You can also review these concepts by solving the problems together as a class. Think aloud as you work and invite students to help you. For example, SAY: I want to turn 1.3 into a mixed fraction. What should I do first? Why?

Then write on the board: 2 \( \frac{3}{10} \) 2.3 3.7

Tell students you want to order these numbers from smallest to largest.
ASK: How is this problem different from the problems we just did? (Not all the numbers are fractions with denominator 10; some are decimals.) Can we change it into a problem that looks like the ones we just did? How? (Yes, by changing the decimals to fractions with denominator 10.) Have one volunteer change the decimals as described and another volunteer put all 3 fractions in the correct order.

Have students order the following numbers in their notebooks. Volunteers should do the first two or three problems on the board.

a) 0.8 0.4 \( \frac{7}{10} \) b) 1.7 .9 1 \( \frac{3}{10} \) c) 8.4 5 \( \frac{6}{10} \) 7.7
   d) 2.8 1.5 \( \frac{7}{10} \) e) 3.7 3.9 3 \( \frac{3}{10} \) f) 8.4 9 \( \frac{6}{10} \) 9.7

Bonus
Provide problems where students have to change improper fractions to mixed fractions:

   g) 3.7 2.9 \( \frac{36}{10} \) h) 3.9 3.4 \( \frac{36}{10} \) i) 12.1 \( \frac{116}{10} \) 12

Bonus
Provide problems where one of the fractions has denominator 2 or 5, so that students have to find an equivalent fraction with denominator 10:

   j) 3.7 2.9 \( \frac{5}{2} \) k) 3.9 3.4 \( \frac{7}{2} \) l) 12.3 12 \( \frac{2}{5} \) 12

Have volunteers say how many hundredths are in each number:

a) .73 b) .41 c) .62 d) .69 e) .58 f) .50

For each pair of numbers, ask which number has more hundredths and then which number is larger:

a) .73 .68 b) .95 .99 c) .35 .42 d) .58 .57
Now have volunteers say how many hundredths are in each of these numbers:

\[ \begin{array}{cccc}
  a) \ .4 & b) \ .6 & c) \ .5 & d) \ .7 \\
  e) \ .1 & f) \ .8 \\
\end{array} \]

**ASK:** How can we write 0.4 as a number with 2 decimal places? Remind students that “4 tenths” is equivalent to “4 tenths and 0 hundredths” or “40 hundredths.” This means 0.4 = 0.40.

For each pair of numbers below, **ASK:** Which number has more hundredths? Which number is larger? Encourage students to change the number with 1 decimal place to a number with 2 decimal places.

\[ \begin{array}{cccc}
  a) \ .48 & b) \ .71 & c) \ .73 & d) \ .2 \\
  \text{or} & \ .6 & \text{or} & \ .9 \\
\end{array} \]

**Put on the board:**

- \[ \begin{array}{cccc}
  \text{Ask volunteers to write 2 different fractions for the amount shaded in the pictures. Have other volunteers change the fractions to decimals. **ASK:** Do these 4 numbers all have the same value? How do you know? What symbol do we use to show that different numbers have the same value? (the equal sign) Write on the board: .9 = \frac{90}{100} = \frac{9}{10} \] \\

Have students change more decimals to fractions with denominator 100:

\[ \begin{array}{cccc}
  a) \ .6 & b) \ .1 & c) \ .4 & d) \ .8 \\
  e) \ .35 & f) \ .42 \\
\end{array} \]

Have students put each group of numbers in order by first changing all numbers to fractions with denominator 100:

\[ \begin{array}{cccc}
  a) \ .3 & .7 & .48 & b) \frac{38}{100} \frac{4}{10} \ 0.39 \\
  c) 2 \frac{17}{100} & 2 \frac{3}{10} & 2.2 \\
\end{array} \]

Now show a hundreds block with half the squares shaded:

**SAY:** This hundreds block has 100 equal squares. How many of the squares are shaded? (50) So what fraction of the block is shaded? (\( \frac{50}{100} \)) Challenge students to give equivalent answers with different denominators, namely 10 and 2. **PROMPTS:** If we want a fraction with denominator 10, how many equal parts do we have to divide the block into? (10) What are the equal parts in this case and how many of them are shaded? (the rows; 5) What fraction of the block is shaded? (\( \frac{5}{10} \)) What are the equal parts if we divide the block up into 2? (blocks of 50) What fraction of the block is shaded now? (\( \frac{1}{2} \))

Ask students to identify which fraction of the following blocks is shaded:
Challenge them to find a suitable denominator by asking themselves: How many equal parts the size of the shaded area will make up the whole? Have students convert their fractions into equivalent fractions with denominator 100.

\[
\left( \frac{1}{5} = \frac{20}{100}, \quad \frac{1}{4} = \frac{25}{100}, \quad \frac{1}{20} = \frac{5}{100} \right)
\]

Write on the board: \(\frac{2}{5} = \frac{20}{100}\) and \(\frac{3}{4} = \frac{25}{100}\), and have volunteers fill in the blanks. Then have students copy the following questions in their notebooks and fill in the blanks.

\[
\begin{align*}
\text{a)} & \quad \frac{2}{5} = \frac{20}{100} \quad \frac{3}{5} = \frac{25}{100} \quad \frac{4}{5} = \frac{25}{100} \quad \frac{5}{5} = \frac{50}{100} \\
\text{b)} & \quad \frac{2}{5} = \frac{20}{100} \quad \frac{3}{20} = \frac{15}{100} \quad \frac{4}{5} = \frac{20}{100} \quad \frac{5}{20} = \frac{25}{100}
\end{align*}
\]

**BONUS:** \(\frac{17}{20} = \frac{85}{100}\)

Have students circle the larger number in each pair by first changing all numbers to fractions with denominator 100:

\[
\begin{align*}
\text{a)} & \quad \frac{1}{2} \quad .43 \quad \text{b)} \quad \frac{3}{5} \quad 1.6 \quad \text{c)} \quad 3.7 \quad \text{or} \quad \frac{17}{2} \quad \text{d)} \quad \frac{12}{5} \quad \text{or} \quad .57 \quad \text{e)} \quad \frac{1}{2} \quad \text{or} \quad .23 \quad \text{f)} \quad \frac{3}{5} \quad \text{or} \quad .7
\end{align*}
\]

Give students groups of fractions and decimals to order from least to greatest by first changing all numbers to fractions with denominator 100. Include mixed, proper, and improper fractions. Start with groups of only 3 numbers and then move to groups of more numbers.

**Extensions**

1. **Extensions**

   1. Students can repeat Extension 1 from **NS4-106**: Decimals and Fractions Greater Than One using a combination of decimals and fractions. Remind students to start with a word whose letters can be used to create other words, and review why this is important. Students can now assign either a decimal or a fraction to each letter, scramble the letters and numbers, and invite a partner to order the numbers to find the original word.

   2. Write a decimal for each fraction by first changing the fraction to an equivalent fraction with denominator 100.

   \[
   \begin{align*}
   \text{a)} \quad \frac{2}{5} & \quad \text{b)} \quad \frac{1}{2} & \quad \text{c)} \quad \frac{1}{4} & \quad \text{d)} \quad \frac{3}{5}
   \end{align*}
   \]

   3. Compare without using pictures, and determine which number is larger:

   \[
   \begin{align*}
   \text{a)} & \quad 2 \frac{2}{7} \quad \text{and} \quad 17 \text{sevenths} \quad \text{b)} \quad 1.7 \quad \text{and} \quad 17 \text{elevenths} \\
   \text{c)} & \quad 1.5 \quad \text{and} \quad 15 \text{ninths} \quad \text{d)} \quad 2.9 \quad \text{and} \quad 26 \text{ninths}
   \end{align*}
   \]

   Students will have to convert all of the numbers into fractions in order to compare them. Is it best to use improper or mixed fractions? You can invite students to try both and see which types of fractions are easier to work with in this case (mixed works better for parts a and d, improper works better for parts b and c).

   4. Which is greater?

   \[
   \begin{align*}
   \text{a)} \quad \frac{1}{2} \quad \text{or} \quad .57 & \quad \text{b)} \quad \frac{1}{2} \quad \text{or} \quad .23 \quad \text{c)} \quad \frac{3}{5} \quad \text{or} \quad .7
   \end{align*}
   \]

   5. Draw a picture to show a decimal between .3 and .4. Explain why more than one answer is possible.

   6. Write digits in the boxes that will make the statement true.

   \[
   \Box .5 \quad < \quad \Box .3
   \]

   **Extensions**

   1. Students can repeat Extension 1 from **NS4-106**: Decimals and Fractions Greater Than One using a combination of decimals and fractions. Remind students to start with a word whose letters can be used to create other words, and review why this is important. Students can now assign either a decimal or a fraction to each letter, scramble the letters and numbers, and invite a partner to order the numbers to find the original word.

   2. Write a decimal for each fraction by first changing the fraction to an equivalent fraction with denominator 100.

   \[
   \begin{align*}
   \text{a)} \quad \frac{2}{5} & \quad \text{b)} \quad \frac{1}{2} & \quad \text{c)} \quad \frac{1}{4} & \quad \text{d)} \quad \frac{3}{5}
   \end{align*}
   \]

   3. Compare without using pictures, and determine which number is larger:

   \[
   \begin{align*}
   \text{a)} & \quad 2 \frac{2}{7} \quad \text{and} \quad 17 \text{sevenths} \quad \text{b)} \quad 1.7 \quad \text{and} \quad 17 \text{elevenths} \\
   \text{c)} & \quad 1.5 \quad \text{and} \quad 15 \text{ninths} \quad \text{d)} \quad 2.9 \quad \text{and} \quad 26 \text{ninths}
   \end{align*}
   \]

   Students will have to convert all of the numbers into fractions in order to compare them. Is it best to use improper or mixed fractions? You can invite students to try both and see which types of fractions are easier to work with in this case (mixed works better for parts a and d, improper works better for parts b and c).

   4. Which is greater?

   \[
   \begin{align*}
   \text{a)} \quad \frac{1}{2} \quad \text{or} \quad .57 & \quad \text{b)} \quad \frac{1}{2} \quad \text{or} \quad .23 \quad \text{c)} \quad \frac{3}{5} \quad \text{or} \quad .7
   \end{align*}
   \]

   5. Draw a picture to show a decimal between .3 and .4. Explain why more than one answer is possible.

   6. Write digits in the boxes that will make the statement true.

   \[
   \Box .5 \quad < \quad \Box .3
   \]
NS4-110
Adding and Subtracting Tenths

GOALS
Students will add and subtract decimal numbers by using a number line and by lining up the numbers according to the decimal place.

PRIOR KNOWLEDGE REQUIRED
Adding whole numbers with regrouping
Knowing the number of tenths in a number with 1 decimal place (EXAMPLE: 4.7 has 47 tenths)
Knowing a decimal number given the number of tenths (EXAMPLE: 47 tenths is 4.7)

To ensure students have the prior knowledge required for the lesson, complete the blanks in the following questions as a class or have students complete them independently in their notebooks.

a) 5.3 = ____ tenths  
b) .8 = ____ tenths  
c) 1.5 = ____ tenths  
d) ____ = 49 tenths  
e) ____ = 78 tenths  
f) ____ = 4 tenths

Bonus

____ = 897 tenths  ____ = 54 301 tenths  ____ = 110 tenths

Review concepts with individual students or the whole class as required.

Now tell students that you want to add some decimals. Write on the board:

2.1
+ 1.0

ASK: How many tenths are in 2.1? (21) How many tenths are in 1.0? (10) How many tenths are there altogether? (31) SAY: There are 31 tenths in the sum. What number is that? (3.1) Write the answer below the addends, being careful to line up the tenths under the tenths and the ones under the ones:

2.1
+ 1.0
3.1

Do a second problem together. This time, write out the number of tenths and add them using the standard algorithm for addition.

Numbers to add  
1.4
+ 2.3
3.7

Numbers of tenths in those numbers  
14 tenths
+ 23 tenths
37 tenths

Have students add the following decimals in their notebooks using this method, that is, by adding the whole numbers of tenths first and then turning the answer into a decimal:

a) 3.4 + 1.5  
b) 2.6 + 4.1  
c) 8.5 + 1.2  
d) 3.7 + 4.2

BONUS: e) 134.3 + 245.5

When students are done, tell them that Sonia adds decimal numbers by lining up the ones digits with the ones digits, the tenths digits with the tenths digits, and then adding each digit separately. Show them an example by re-doing question a) this way. ASK: Does Sonia get the right answer with this method? Why do you think that is? How is what Sonia does similar to what you did? Did you line up the digits when you added the whole numbers? Did you add each digit separately?
Write on the board:

\[
\begin{array}{llllll}
\text{a)} & 3.5 & \text{b)} & .7 & \text{c)} & 192.8 & \text{d)} & 4.8 & \text{e)} & 154.7 \\
+ & 4 & + & 3.5 & + & 154 & + & 12.1 & + & 16.3 \\
\end{array}
\]

**ASK:** In which questions are the digits lined up correctly? How can you tell? In all the questions where digits are lined up correctly, what else is also lined up? (the decimal point) Is the decimal point always going to be lined up if the digits are lined up correctly? (yes) How do you know? (The decimal point is always between the ones and the tenths, so if those digits are lined up correctly, then the decimal point will be as well.)

Demonstrate using this method to solve 12.1 + 4.8:

\[
\begin{array}{c}
\text{12.1} \\
+ \text{4.8} \\
\hline \\
\text{16.9} \\
\end{array}
\]

Tell students that when you add the ones digits, you get the ones digit of the answer and when you add the tenths digits, you get the tenths digit of the answer. If the digits are lined up, then the decimal points are lined up, too—the decimal point in the answer must line up with the decimal points in the addends. Ask students why this makes sense and give several individuals a chance to articulate an answer.

Tell students that as long as they line up the numbers according to the decimal points, they can add decimals just like they add whole numbers. Demonstrate with a few examples, including some that require regrouping and carrying:

\[
\begin{array}{c}
\text{3.5} \\
+ \text{.7} \\
\hline \\
\text{4.2} \\
\end{array}
\]

\[
\begin{array}{c}
\text{192.8} \\
+ \text{154} \\
\hline \\
\text{346.8} \\
\end{array}
\]

\[
\begin{array}{c}
\text{154.7} \\
+ \text{16.3} \\
\hline \\
\text{171.0} \\
\end{array}
\]

Give students lots of practice adding decimals (with no more than 1 decimal place). Working on grid paper will help students to line up the digits and the decimal points. Include examples with regrouping. Bonus problems could include larger numbers (but still only 1 decimal place). Emphasize that the decimal point is always immediately after the ones digit, so a whole number can be assumed to have a decimal point (EXAMPLE: 43 = 43. = 43.0) It is also important that students line up the digits around the decimal point carefully, perhaps by using grid paper. To emphasize this, have students identify the mistake in:

\[
\begin{array}{c}
\text{341.7} \\
+ \text{5216.2} \\
\hline \\
\text{867.9} \\
\end{array}
\]

The decimals are technically lined up properly, but the remaining digits are not. **ASK:** What happened?

**ASK:** If we can add decimals the way we add whole numbers as long as we line up the decimal points, do you think we can subtract decimals the same way we subtract whole numbers? Use the following problem to check:

\[
\begin{array}{c}
\text{4.5} \\
- \text{2.3} \\
\hline \\
\end{array}
\]
Do the problem together two ways—first rewrite the decimals as tenths and subtract the whole numbers, then subtract using Sonia’s method—and compare the answers. Then subtract more numbers (with no more than 1 decimal place) together. **EXAMPLES:**

\[
\begin{array}{ccc}
6.7 & 4.9 & 8.12 \\
- 3 & -1.7 & -4.53 \\
\hline
3.7 & 3.2 & 347.3
\end{array}
\]

As with addition, give student lots of practise subtracting decimals. Remind them to line the numbers up according to the decimal point!

Then introduce adding decimals on a number line:

```
 0 1 2 3 4
```

**SAY:** I want to add 1.6 + 0.9. How many tenths are in 1.6? Where is that on the number line? How many tenths are in 0.9? How can I show adding 1.6 to 0.9 on the number line? Draw arrows to illustrate the calculation:

```
1.6 + 0.9 = 2.5
```

Have students use the number line to add:

- a) 2.4 + 1.0
- b) 1.3 + 1.0
- c) 0.5 + 1.0

Then have them add 1.0 to decimals without using the number line:

- d) 1.8 + 1.0
- e) 2.6 + 1.0
- f) 0.9 + 1.0

Invite students to use what they know about adding 1.0 to decimals to solve 1.4 + 1.5. **ASK:** What is 1.4 + 1.0? How can we use that to find 1.4 + 1.5? How would you find 1.1 + 2.3? (Students could find 1.1 + 2.0 and then add .3 or they could find 1.0 + 2.3 and then add .1. Both strategies should be discussed.)

Have students subtract using the number line as well.

**Extension**

Stick measuring tape to the board (the kind used by tailors) to make number lines (where each centimetre represents a hundredth). Students could draw arrows over the number lines to add and subtract decimal hundredths as shown on Worksheet **NS4-100** for decimal tenths (**SEE:** Question 4).
NS4-111
Adding Hundredths

GOALS
Students will add hundredths by lining up the decimal points.

PRIOR KNOWLEDGE REQUIRED
- Adding whole numbers with regrouping
- Adding tenths
- Adding fractions with the same denominator
- Knowing how many hundredths are in a number with 2 decimal places
- Knowing which number has a given number of hundredths

Have students add these fractions:

a) \(\frac{30}{100} + \frac{24}{100}\)  
b) \(\frac{30}{100} + \frac{36}{100}\)  
c) \(\frac{35}{100} + \frac{16}{100}\)  
d) \(\frac{35}{100} + \frac{24}{100}\)

Ask students to rewrite their addition statements in terms of decimals.

(EXAMPLE: \(.3 + .24 = .54\))

Then give students addition problems that require regrouping. (EXAMPLE: \(\frac{35}{100} + \frac{47}{100}\)) Again, have students rewrite their addition statements in terms of decimals. (EXAMPLE: \(.36 + .47 = .83\))

Now have students do the opposite: add decimals by first changing them to fractions with denominator 100. Invite volunteers to do some of the following problems on the board, then have students do the rest independently.

a) \(.32 + .57\)  
b) \(.43 + .16\)  
c) \(.81 + .17\)  
d) \(.44 + .44\)  
e) \(.40 + .33\)

f) \(.93 + .02\)  
g) \(.05 + .43\)  
h) \(.52 + .20\)  
i) \(.83 + .24\)  
j) \(.22 + .36\)

Have a volunteer do a sum that requires regrouping (\(.54 + .28\)), then have students add the following independently:

a) \(.37 + .26\)  
b) \(.59 + .29\)  
c) \(.39 + .46\)  
d) \(.61 + .29\)

Finally, add decimals whose sum is more than 1 by changing them to fractions first. Have volunteers do 2 examples on the board (.36 + .88 and .45 + .79). Students can then add the following independently:

a) \(.75 + .68\)  
b) \(.94 + .87\)  
c) \(.35 + .99\)  
d) \(.46 + .64\)  
e) \(.85 + .67\)

f) \(.75 + .50\)  
g) \(.65 + .4\)  
h) \(.7 + .38\)  
i) \(.9 + .27\)

ASK: How many hundredths are in .9? (90) How many hundredths are in .27? (27) How many hundredths is that altogether? (117) What number has 117 hundredths? (1.17) What number has 234 hundredths? 5682 hundredths? 48 hundredths? 901 hundredths? 800 hundredths? 8 hundredths?

Have students add more decimal numbers by identifying the number of hundredths in each number and then adding the whole numbers:

a) \(.78 + .4\)  
b) \(.37 + .49\)  
c) \(.85 + .65\)  
d) \(.43 + .34\)

e) \(.25 + .52\)  
f) \(.14 + .41\)  
g) \(.76 + .67\)  
h) \(.89 + .98\)

i) \(.43 + .87\)  
j) \(.55 + .55\)  
k) \(1.43 + 2.35\)  
l) \(3.5 + 2.71\)

m) \(4.85 + 3.09\)

Remind students that we were able to add tenths by lining up the digits and decimal points. **ASK:** Do you think we can add hundredths the same way? Do an example together:

\[
\begin{array}{c}
4.85 \\
+ \quad 3.09 \\
7.94
\end{array}
\]
Invite students to help you add the numbers. Tell them to pretend the decimal point isn’t there and to add as though they are whole numbers. (PROMPTS: What do the hundredths digits add to? Where do I put the 4? the 1?) ASK: Why can we add as though the decimal points are not there? How many hundredths are in 4.85? (485) In 3.09? (309) In both numbers altogether? (485 + 309 = 794) How does this get the same answer? (794 hundredths = 7.94 ones, so we just line up the decimal points and proceed as though the decimal point is not there)

ASK: How can we check our answer? If no one suggests a method, invite a volunteer to add the numbers after rewriting them as fractions with denominator 100:

**Method 1:** 4.85 + 3.09 = \( \frac{485}{100} + \frac{309}{100} = \frac{794}{100} = 7.94 \)

**Method 2:** 4.85 + 3.09 = \( \frac{485}{100} + \frac{309}{100} = \frac{794}{100} = 7.94 \)

Emphasize that all methods of addition reduce the problem to one students already know how to do: adding whole numbers. When they add using decimals, students know where to put the decimal point in the answer by lining up the decimal points in the addends. When they add using fractions, students know where to put the decimal point by looking at the denominator. If the denominator of the fraction is 10, they move the decimal point 1 place left. If the denominator of the fraction is 100, they move the decimal point 2 places left.

Give students lots of practice adding decimal hundredths by lining up decimal places. Include numbers that do and do not require regrouping.

**EXAMPLES:** .34 + .28  .65 + .21  .49 + .7  1.3 +.45  2.86 + .9

**Bonus**

12.3 + 1.23  354.11 + 4 672.6

Give students base ten blocks and tell them to use the hundreds block as a whole. This makes the tens block a tenth and the ones block a hundredth. Assign the following problems to pairs or individuals and invite students to share their answers.

1. **Start with these blocks:**

   ![Blocks](image)

   • What decimal does this model represent? (ANSWER: 2.3)

   • Add 2 blocks so that the sum, or total, is between 3.3 and 3.48. (ANSWER: add or add )

   • Write a decimal for the amount you added. (ANSWER: 1.1 or 1.01)

2. **Take the same number of blocks:**

   ![Blocks](image)

   • Add 2 blocks so that the sum is between 2.47 and 2.63. (ANSWER: 2 tens blocks)

   • Write a decimal for the amount you added (ANSWER: 0.2).
3. Take these blocks:

- What decimal does this model represent? (ANSWER: 1.42)
- Add 2 blocks so that the sum is between 1.51 and 1.6. (ANSWER: add □ □ □ □ □ □ □ □ □
- Write a decimal for the amount you added. (ANSWER: 0.11)

Extensions

1. Write the numbers as decimals and add: 2 + \( \frac{3}{10} + \frac{7}{100} \).

2. **ASK:** What if the denominator of a fraction is 1000—how do we know where to put the decimal point in the decimal number? What is .437 + .021? Turn the decimals into fractions to add them: \( \frac{437}{1000} + \frac{21}{1000} = \frac{458}{1000} \). **SAY:** The sum is 458 thousandths, or 0.458. Since the denominator of the fraction is 1000, we know to move the decimal point 3 places left. **ASK:** Can we add the decimals by lining up the decimal points? Do we get the same answer? Add the decimals this way to find out.

**NS4-112**

**Subtracting Hundredths** and

**NS4-113**

**Adding and Subtracting Decimals (Review)**

Tell your students that today they will learn to subtract hundredths. As in the last lesson, begin by subtracting hundredths written as fractions. (Start with problems that do not require regrouping and then move to problems that do.)

**EXAMPLES:**

- a) \( \frac{34}{100} - \frac{14}{100} \)  
- b) \( \frac{58}{100} - \frac{11}{100} \)  
- c) \( \frac{47}{100} - \frac{29}{100} \)  
- d) \( \frac{43}{100} - \frac{10}{100} \)

Have students rewrite the subtraction statements in terms of decimals. **(EXAMPLE: .34 – .14 = .20)**

Then have students subtract decimals by first changing them into fractions (proper, mixed, or improper). Include numbers greater than 1.

**EXAMPLES:**

- a) .45 – .14  
- b) .53 – .1  
- c) .85 – .3  
- d) 1.23 – .11

Tell students that we can subtract hundredths the same way we add them: by lining up the digits and decimal points. Solve 1.93 – .22 together and have students check the answer by rewriting the decimals as fractions.
Give students a chance to practice subtracting decimal hundredths. Include numbers that require regrouping and numbers greater than 1. **EXAMPLES:**

a) .98 – .42  
b) 2.89 – .23  
c) 3.49 – 1.99

Remind students of the relationship between missing addends and subtraction. Write on the board:

32 + 44 = 76. **SAY:** If 32 + 44 = 76, what is 76 – 44? What is 76 – 32? How can I find the missing addend in 32 + ___ = 76? (find 76 – 32) What is the missing addend in .32 + ____ = .76?

Have students find the missing addend in:

.72 + ___ = .84  
.9 = ____ + .35  
.87 = ____ + .5  
.65 + ___ = .92

As in the previous lesson, give students base ten blocks and tell them to think of the hundreds block as a whole, the tens block as a tenth, and the ones block as a hundredth. Have pairs or individuals solve the following problems:

1. Take these blocks:

   - What decimal does this model represent?  
   - Take away 2 blocks so the result (the difference) is between 1.21 and 1.35.  
   - Write a decimal for the amount you took away.

2. Take these blocks:

   - What decimal does this model represent?  
   - Take away 3 blocks so the result (the difference) is between 1.17 and 1.43.  
   - Write a decimal for the amount you took away.

Make up similar problems for students to solve independently or have students make up their own problems and exchange them with a partner.

**Extension**

Show students how to subtract decimals from 1 by first subtracting from .99:

\[ 1 - .74 = .01 + .99 - .74 = .01 + .25 = .26 \]
NS4-114

Differences of 0.1 and 0.01

Have your students add the following numbers in their notebooks:

a) .48 + .1  b) .48 + .01  c) .52 + .1  d) .52 + .01
e) .63 + .1  f) .63 + .01  g) 4.32 + .01  h) 4.32 + 1
i) 4.32 + .1  j) 7.38 + .1  k) 7.38 + .01  l) 7.38 + 1

**ASK:** How do you add .1 to a number? (add 1 to the tenths digit) When you added .1 above, how many digits changed? (only 1, the tenths digit) Then have students find .94 + .1. **PROMPTS:** How many hundredths are in .94? (94) How many hundredths are in .1? (10) How many hundredths is that altogether? (104) So the answer is 1.04. **ASK:** How is adding .1 to .94 different from adding .1 to the numbers above? What else changes besides the tenths digit? (the ones digit changes—from 0 to 1—because the answer is more than 100 hundredths)

Have students do the following problems and tell you when they just add 1 to the tenths digit and when they have to change the ones digit, too:

.49 + .1  .86 + .1  .93 + .1  .97 + .1  .36 + .1

Now have students add .01 to various decimals:

.49 + .01  .94 + .01  .86 + .01  .49 + .01  .94 + .01
.28 + .01

**ASK:** What number is .1 more than 9.3? Ask students to identify the number that is .1 more than: .7, 8.4, .6, .9, 5.9. Prompt students with questions such as: How many tenths are in 5.9? What is one more tenth? What number has 60 tenths?

What number is .01 more than:

8.47  8.4  .3  .39  .86  .89

What number is 1 more than:

9.3  .7  1.43  9.8  9.09  9.99

Repeat with subtraction, asking students to find numbers that are .01, .1, and 1 less than various numbers with 1 and 2 decimal points. Include examples where students need to borrow/regroup.

Have a volunteer add the missing decimal numbers to this number line:

| 4.0 | | | | | | 5.0 |

Students can refer to the number line to complete these sequences in their notebooks:

4.3, 4.4, 4.5, ___  4.1, 4.4, 4.7, ___
4.0, 4.2, 4.4, ___  4.9, 4.7, 4.5, ___
Have students give the rule for each sequence. (EXAMPLE: start at 4.3 and add .1)

Have another volunteer add the missing decimal numbers to this number line:

```
| | | | | | | | | |
7.3                         8.3
```

Students should complete and describe these sequences in their notebooks:

- 7.7, 7.8, 7.9, ____
- 7.2, 7.5, 7.8, ____
- 7.5, 7.7, 7.9, ____
- 8.3, 8.2, 8.1, ____, ___

Have students fill in the blanks in their notebooks:

- 5.9 + .1 = ____
- 8.9 + .1 = ____
- .9 + .1 = ____
- 6.49 + .1 = ____
- 6.49 + .01 = ____
- 6.49 + 1 = ____
- 8.93 + .1 = ____
- 8.99 + .1 = ____
- 8.99 + .01 = ____

**Extension**

Ask students to count forward from the following numbers by tenths, orally.

- a) 6
- b) 17.8
- c) 123.2
GOALS
Students will use place value to order decimal numbers. Students will round decimal numbers to the nearest whole number and tenth.

PRIOR KNOWLEDGE REQUIRED
Ordering whole numbers by place value
Decimal tenths and hundredths
Rounding up and down

VOCABULARY
rounding

GOALS
Write these numbers on the board: 345 247. **ASK:** Which number is larger? How do you know? **(PROMPTS:** Which number has a larger ones digit? A larger tens digit? A larger hundreds digit? Which digit should we look at first to determine which number is larger? Why?) Make sure students understand that a 3-digit number with a larger hundreds digit is always larger than a 3-digit number with a smaller hundreds digit, regardless of what the other digits are.

Write on the board:

\[
\begin{align*}
891 \\
690
\end{align*}
\]

**ASK:** What is the largest place value in which the numbers differ? Which number is larger? Show this on the board:

\[
\begin{align*}
891 & \quad this \ number \ is \ larger \\
690
\end{align*}
\]

Repeat with more pairs of 3-digit numbers:

\[
\begin{align*}
872 & \quad 543 & \quad 681 & \quad 872 \\
876 & \quad 571 & \quad 983 & \quad 805
\end{align*}
\]

Continue with decimal numbers. For each pair below, **ASK:** What is the largest place value in which the numbers differ? Which number is larger?

\[
\begin{align*}
87.3 & \quad 5.47 & \quad 5.13 & \quad 87.23 \\
86.4 & \quad 6.40 & \quad 6.4 & \quad 87.3
\end{align*}
\]

Then ask students if the following explanation is correct:

\[
\begin{align*}
5.47 & \quad this \ is \ the \ first \ place \ where \ the \ digits \ differ \ and \\
5.08 & \quad 4 \ is \ larger \ than \ 3, \ so \ 5.47 \ is \ larger \ than \ 53.8
\end{align*}
\]

**ASK:** How do the digits have to be lined up for us to compare the numbers? (The ones have to be lined up with the ones, the tenths with the tenths, and so on.) What else can we use to line the numbers up correctly? (the decimal point)

Line the above numbers up correctly on the board:

\[
\begin{align*}
5.47 & \quad 5.08
\end{align*}
\]

**ASK:** What is the largest place value in which these numbers are different? (tens) How many tens are in 5.47? (0) In 53.8? (5) Which number is larger? (53.8)

Have students practice lining up digits correctly and deciding which number is bigger with these pairs of decimal numbers:

\[
\begin{align*}
a) \ 27.8 & \quad 3.4 \quad & b) \ 3.97 & \quad 421.6 \quad & c) \ 512.25 & \quad 4134
\end{align*}
\]
Be sure students realize that when there is no decimal point in a number, they can still use place value to line the digits up. In a number with no decimal point, the rightmost digit is the ones digit.

Ask if anyone remembers the signs mathematicians use to show that a number is greater than or less than another number. Remind students how to use the > and < signs. Then have students put the correct sign between these pairs of numbers:

a) 8.46    8.19
b) 9.4    9.37
c) 18.2    7.99

Have students make any 3-digit number from the digits 1, 2, and 3. Write the various answers on the board. **ASK:** Which is the largest number? (321) Why? (because it has the most hundreds)

**SAY:** I want to make the largest possible 3-digit number from the digits 5, 7, and 8. Where should I put the largest digit? Why?

Write on the board:

```
  _  _
```

**ASK:** What is the largest number I can make in these boxes using the digits 5, 7, and 8? Where should I put the largest digit? Why? The next largest?

Move the decimal point one place to the left and repeat:

```
  . _
```

Repeat with different digits (**EXAMPLES:** 7, 8 and 3; 6, 9 and 0) and then with other arrangements of boxes and decimal points.

**EXAMPLES:**

```
  _  _  .  _
```

```
  _  _  _  .  _
```

```
  _  _  _  _  .
```

Have students round the following numbers to the nearest whole number.

1.8       12.3       51.4       19.9       0.6       14.5

Have students round the following numbers to the nearest tenth. Tell them that this is called “rounding to the nearest tenth” or “rounding to one decimal place.”

15.43     15.81     16.79     234.47     19.25

**Extensions**

1. Relate the ordering of numbers to the alphabetical ordering of words. When we put words in alphabetical order, we compare first the left most letters, then the next letters over, and so on. Ask students to put these pairs of words in alphabetical order by identifying the first letter that’s different:

mouse, mice

noun, none

room, rope

snap, snip

trick, trim

sun, fun

pin, tin

spin, shin
Students can write the words one above the other and circle the first letter that’s different:

```
mouse
mice
```

Tell students that a blank always comes first. Use the words “at” and “ate” to illustrate this. The first two letters in “at” and “ate” are the same. There is no third letter in “at”—there is nothing, or a blank, after the “t.” The blank comes before the “e” at the end of “ate,” so “at” comes before “ate.” Have students use this knowledge to put the following pairs in alphabetical order:

```
mat  mate  an  a  noon  bath  bat  kit  kite
```

Point out the difference between lining up numbers and words. The numbers 61 435 and 7 384 would be lined up

```
61 435   61 435
7 384     7 384
```

But the words “at” and “ate” would be lined up like this:

```
ate  not  ate
at   at
```

When ordering words, you line up the leftmost letters. When ordering numbers, you line up the ones digits, whether they’re on the right, the left, or anywhere in between. For both words and numbers, you start comparing from the left.

Another important difference is that 5–digit whole numbers are always greater than 4-digit whole numbers, but 5-letter words can be before or after 4-letter words.

Point out that in numbers, as in words, a space is always less than a number. When we compare:

```
6.53  18.1
18.2  OR:  17.14
```

the blank is really a 0 and is less than, or comes before, any number.

Have students practice ordering decimals that have different numbers of decimal places.

**EXAMPLES:**

a) 6.53   18.2  b) 456.73   21.72006  c) 85.7601   112.03  d) 13.54   13.5

2. (From the Atlantic Curriculum)

Have students use each of the digits from 0 to 9 once to fill in the 10 spaces and make these statements true.

```
.  .   <   .  .
.  .   >   .  .
```

```
Concepts in Decimals

GOALS
Students will use place value to order decimal numbers.
Students will round decimal numbers to the nearest whole number and tenth.

PRIOR KNOWLEDGE REQUIRED
Base ten materials
Units of measurement: metres (m), centimetres (cm), and millimetres (mm)
Fractions and equivalent decimals

Draw on the board: 

Ask a volunteer to shade one tenth of the picture. Then have students draw one tenth of each of the following pictures in their notebooks:

Tell your students that the fraction of a measurement depends on which unit is the whole. Ask: What is one hundredth of a meter? (1 cm) How would you write this as a fraction? (1/100 m) How would you write this as a decimal? (.01 m)

Have students write each of the following measurements as a fraction and a decimal in metres.

2 cm = ____ m = ____ m
20 cm = ____ m = ____ m
25 cm = ____ m = ____ m
132 cm = ____ m = ____ m

Have your students write these measurements as a fraction and decimal in centimetres.

8 mm = ____ cm = ____ cm
4 mm = ____ cm = ____ cm
17 mm = ____ cm = ____ cm
29 mm = ____ cm = ____ cm

Bonus

54 mm = ____ m = ____ m
135 mm = ____ m = ____ m
6 054 mm = ____ m = ____ m

Have your students add the measurements by changing the one in smaller units to a measurement in the larger units.

a) 3 cm + 9.46 m
b) 24 cm + .12 m
c) 584 cm + 2.3 m

Have students check their answers by changing the number in larger units to one in smaller units, adding, and then changing the answer back to the larger units. Did they get the same answer?
Extensions

Give each student or pair a set of base ten blocks. Tell students the hundreds block is the whole, so the tens block represents .1 and the ones block represents .01.

1. Ask students to show and write all the decimals they can make with these 3 blocks.

(SOLUTION: 1.0, 1.1, 1.01, 1.11, 0.1, 0.01, 0.11)

NOTE: One way to prepare students for this exercise is to hold up combinations of blocks and ask them to write the corresponding decimal in their notebook. For instance, if you hold up the hundreds block (which represents one unit) and the tens block (which represents a tenth) they should write 1.1.

2. Use base ten blocks to make a decimal
   a) greater than .7.
   b) less than 1.2.
   c) between 1 and 2.
   d) between 1.53 and 1.55.
   e) with tenths digit equal to its ones digit.
   f) with hundredths digit one more than its tenths digit.

3. Create models of 2 numbers such that
   a) one number is 4 tenths greater than the other.
   b) one number has tenths digit 4 and is twice as large as the other number.

4. One decimetre (1 dm) is 10 cm. Explain how you would change 3.2 dm into centimetres.
NS4-117
Dividing by 10 and 100

**GOALS**
Students will divide by 10 and 100 by removing zeroes or moving the decimal point to the left.

**PRIOR KNOWLEDGE REQUIRED**
Division
Decimal tenths

**NOTE:** Students will find it relatively easy to divide numbers with ones digit 0 by 10. It is much more difficult to divide whole numbers by 10 if the whole number has a ones digit other than 0. Please take the time to cover this topic thoroughly.

Draw 40 dots in an array on the board as shown below. **SAY:** I want to divide these 40 dots into 10 sets. What is a natural way to do that? (You could divide the array into columns.) How many dots are in each set? (4)

Then draw a number line on the board and **ASK:** If I divide the number line into steps of size 10, how many steps do I need to reach 40? (4) What is 40 ÷ 10? (4)

How many steps of size 10 would I need to reach 30? (3) What is 30 ÷ 10? (3) What is 70 ÷ 10? (7) What is 100 ÷ 10? (10) What is 110 ÷ 10? (11) If I need 10 steps of size 10 to reach 100, how many steps to do I need to reach 110? (11)

Now get students to think in terms of tens blocks. **ASK:** How many tens blocks do I need to make 50? (5) If I make the number 50 by dividing it into groups of 10, how many groups do I need? (5) If I used only tens blocks, how many would I need to make 240? (24).

Ask students to solve independently in their notebooks:

\[
\begin{array}{ccc}
20 \div 10 & 90 \div 10 & 200 \div 10 \\
530 \div 10 & 23000 \div 10 & 18400 \div 10 \\
\end{array}
\]

**SAY:** If I used only hundreds blocks, how many would I need to make 200? To make 700? To make 1800? To make 7000? Ask students to solve similar problems independently in their notebooks:

\[
\begin{array}{ccc}
300 \div 100 & 500 \div 100 & 1800 \div 100 \\
4000 \div 100 & 630000 \div 100 & \\
\end{array}
\]

**ASK:** What can you do to a number to divide it by 10? If students say “remove a zero” **ASK:** What is 5030 ÷ 10—503 or 530? Encourage students to describe precisely what they mean by “remove a zero.” Take several answers and summarize by saying that they have to remove the 0 in the ones digit.
Tell students that 53 doesn’t have a 0 in the ones digit but that you still want to divide 53 by 10.

**ASK:** What is the base ten model of 53? (5 tens blocks and 3 ones blocks) **SAY:** We have 5 full tens blocks but the leftover ones blocks make up only part of a tens block. What fraction of a tens block are the leftover ones? ( \( \frac{3}{10} \)) We have 5 full tens blocks and \( \frac{3}{10} \) of a tens block. How many tens blocks is that altogether? (5 \( \frac{3}{10} \)). Have a volunteer write that as a decimal on the board. (5.3)

Draw on the board:

```
  [Diagram of ten blocks and three ones blocks]
```

Tell students that each model has 3 tens blocks and 4 ones blocks, but you pushed the 4 ones blocks closer together in the model on the right so that you can see clearly what fraction of a tens block the ones blocks make up. Tell students that you are using a tens block as the whole and

**ASK:** What number do the blocks show? (34) Ask students to count the ones blocks as a fraction of a tens block. How many tens blocks are in the number 34? Have a volunteer write the answer on the board as both a mixed fraction (3 \( \frac{4}{10} \)) and a decimal (3.4).

Repeat with the following models:

```
  [Diagram of additional ten and one blocks]
```

Then have students write the mixed fraction and decimals for the number of tens blocks in the following pictures in their notebooks, independently. Remind them that they are counting the leftover ones as part of a tens block.

```
  a)  [Diagram of ten and three ones blocks]
  b)  [Diagram of ten and four ones blocks]
  c)  [Diagram of ten and five ones blocks]
  d)  [Diagram of ten and six ones blocks]
```

Now illustrate the same idea using number lines. Show a number line divided into steps of size 10 and tell students that you want to count the number of steps of size 10 to make different numbers.

```
  [Number line: 0 to 40, with steps of 10 labeled]
```

**ASK:** How many steps of size 10 would you need to make 30? (Show the 3 steps on the number line.) How many steps of size 10 would you need to make 3? Show the part of a step needed on the number line:

```
  [Number line with steps labeled and part of a step indicated]
```
**ASK:** What part, or fraction, of a step of size 10 do you need to make 3? \( \frac{3}{10} \) What decimal is that? (0.3) Repeat with other whole numbers between 1 and 9. Then move on to examples that require a mixed fraction of steps of size 10 (**EXAMPLES:** 13, 21, 34, 29, 35, 17). Always encourage students to write both the fraction and the decimal. Have them do a few independently in their notebooks.

Then show 2 number lines:

![Number Line 1](image1)

![Number Line 2](image2)

Tell students that when you divide 3 into steps of size 10, you only need 0.3 steps. **ASK:** When you divide 3 into 10 steps, how long is each step? (0.3 units long). What is \( 3 \div 10 \)? (0.3)

**ASK:** When you divide 4 into 10 equal steps, how big do you think each step will be? Will the steps need to be longer or shorter than 0.3? How do you know? On grid paper, have students copy this last number line and then guess how long each step must be so that 10 steps will get to the number 4. Have them try their guess on their number line. Was their guess correct? Or was their step too long or too short? Have them adjust their step, if necessary, and try again by drawing new steps underneath the number line. Students who finish quickly can move on to new problems, such as \( 1 \div 10 \) and \( 2 \div 10 \).

**NOTE:** Students will need to use 2 pages of grid paper to copy the number lines.

Now ask students to find \( 14 \div 10 \). **ASK:** How many tens blocks do I need to build 14? How many more ones blocks? What fraction of a tens block are the leftover ones blocks? Altogether, how many tens blocks do I need? Tell students to include the fraction in their answer. Then write \( 14 \div 10 = 1 \frac{4}{10} \) and ask students to explain why this is correct. (\( 1 \frac{4}{10} \) is the number of groups of size 10 you need to make 14) Have a volunteer change the answer to a decimal.

Have students do the following in their notebooks using the same method.

a) \( 21 \div 10 \)  b) \( 32 \div 10 \)  c) \( 35 \div 10 \)  d) \( 49 \div 10 \)  e) \( 149 \div 10 \)  f) \( 235 \div 10 \)

**Extensions**

1. **ASK:** If I have a piece of string that is 1 mm long, what fraction of a cm is that? If the string is 7 mm long, what fraction of a centimetre is that? How would I write that as a decimal? If a rope is 345 mm long, how many centimetres long is it? Tell students to be precise and to include fractional parts. They should write their answer in decimal notation.
2. (From the Atlantic Curriculum)

Tell students that Jasmine said she was supposed to divide 130 by 5 but found it easier to divide 260 by 10. Ask students to explain Jasmine’s method. Why did doubling both numbers make it easier to divide them? Why is 260 ÷ 10 the same as 130 ÷ 5?

3. Have students reflect back on the lesson. **ASK:** Which model of division (arrays divided into columns or number lines divided into steps) is better to show 4 ÷ 10? Why? (Number lines are better because it is easier to see what it means to divide 4 into 10 equal groups on a number line that is already divided into tenths. Dividing each of the 4 dots into tenths of a dot would be harder. Also, it is easy to see what fraction of a step of size 10 is needed to get to 4 on a number line; picturing what fraction of 10 dots you have in 4 dots is more abstract.)

4. Ask students if they see a short cut for dividing decimal numbers by 10. Write on the board 325 and tell them that they can’t write any numbers on the board but they have to show 325 ÷ 10 by putting a decimal point in the right place. How can they do it? Ask them to think about how they can multiply 325 × 10 by just adding a 0. **ASK:** Where do you add the zero when you multiply by 10? How does the values of each digit change when you multiply a number by 10?

When you multiply by 10:

- 325 → 3 250
- 300, 20, and 5 become 3 000, 200, and 50

And when you divide by 10:

- 325 → 32.5
- 300, 20, and 5 become 30, 2, and 0.5

Write the number 346.51 and **ASK:** What is each digit worth? (300, 40, 6, 0.5 and 0.01) How can you move the decimal point so that each digit is worth ten times more (i.e., the ones digit becomes the tens digit, the tens digit becomes the hundreds digit, and so on)? They should move the decimal point one place to the right. Then ask how they can move the decimal point so that each digit is worth ten times less. They should move the decimal point one place to the left.

**ASK:** How would you find 32.5 × 10? Where should you move the decimal point to make every digit worth ten times more?

**ASK:** How is multiplying a decimal number by 10 different from multiplying a whole number by 10? Can we just add a zero to 32.5 to multiply by 10? What would we get if we added 0? (32.50) Does adding 0 to the decimal number change it at all? (No, but adding 0 to the end of a whole number changes the ones digit to 0, and every other digit becomes worth 10 times more.) Tell students that when they multiply a decimal number by 10, they move the decimal point one place to the right. Point out that they are, in fact, doing the same thing when they multiply whole numbers by 10: 17 = 17.0, so when you multiply by 10 by adding a 0 to the end, you’ve also moved the decimal point over to get 170.

Have students try these problems in their notebooks:

- a) 0.5 × 10
- b) 0.8 × 10
- c) 1.3 × 10
- d) 2.4 × 10
- e) 134.6 × 10
- f) 12.45 × 10

Ask students to think about division. When they multiply by 10, they move the decimal point one place to the right. What do they do to the decimal point when they divide by 10? Why does that make sense? If you solve 1.4 × 10, you get 14. What do you get when you find 14 ÷ 10? (1.4) If you multiply a number by 10 and then divide the result by 10, do you always get back to the original number? (yes) **ASK:** If you move the decimal point one place to the right, what do you have to do to get back the original number? (move the decimal point one place to the left)
GOALS
Students will convert combined-unit measurements (EXAMPLE: 2 dollars 7 cents) to single-unit measurements (EXAMPLE: 207¢ or $2.07).

PRIOR KNOWLEDGE REQUIRED
Dollars ($) and cents (¢)
Hours (h) and minutes (min)
Kilometres (km), metres (m), and centimetres (cm)
Tens and hundreds
Tenths and hundredths

ASK: How many ones blocks are in 3 tens blocks? (30). If you have 3 tens blocks and 4 ones blocks, how many ones blocks would that be the same as? (34) Why do we use tens blocks at all—why not just use 34 ones blocks? If I have 3 dimes and 4 pennies, how many pennies is that? Why do you think we have dimes at all?

How many pennies, or cents, are in $1? If I have $1 and 14 more cents, how many cents do I have altogether? If I have $3 and 4 more cents, how many cents do I have altogether? Write similar problems on the board (EXAMPLES: 7 dollars 86 cents = _____ cents, 8 dollars 6 cents = _____ cents) and have volunteers fill in the blanks.

Continue with examples involving metres and centimetres. To start, **ASK:** If an ant walks 1 metre, how many centimetres did the ant walk? If the ant walks 2 metres and another 55 centimetres, how many centimetres did the ant walk altogether? Have students add various amounts (up to multiples of 100) to the ant’s journey. Repeat with kilometres and metres, this time having students add amounts to multiples of 1000. (EXAMPLE: 4 km and 130 m; 10 km and 2000 m)

Finally, repeat with hours and minutes. This is a bit trickier, since 1 hour has 60 minutes instead of 10, 100, or 1 000. Use only small numbers—1, 2, or 3 hours.

**Bonus**
Use larger numbers.

When students are very comfortable changing 2 measurements in 2 units to a single measurement in the smaller unit, show them how to change 2 measurements to a single measurement in the larger unit. Ask them to change 3 hundreds blocks and 7 ones blocks to hundreds blocks by looking at the 7 ones blocks as part of a hundreds block. In this case, 7 ones blocks is 7 hundredths of a hundreds block, **SO:**

3 hundreds blocks
and 7 ones blocks

**307 hundreds blocks**
or **307 ones blocks**

Do many questions involving dollars and cents and then metres and centimetres.

**Extensions**

1. How could you make a measurement such as 3 m 4 cm more precise?

   **(ANSWER:** In the measurement, the number of millimetres may have been ignored or rounded. You could make the measurement more precise by specifying how many millimetres were measured.)
2. Which measurement is more precise?
   a) 7 km or 7 km 540 m  
   b) 1 200 m or 1 km 1 m  
   c) 3 hours or 3 hours 5 min

**NOTE:** The word “precise” is different from “accurate”. A distance might be exactly 7 km, but it is still more precise to say 7.0 km than to say 7 km. Using a smaller unit of measurement always allows you to be more precise. For example, I might say that a hockey game lasted 3 hours, or 3 hours and 5 minutes or, to be even more precise, 3 hours, 5 minutes and 0 seconds. Although 3 hours and 5 minutes happens to be exactly accurate, it is less precise since it provides less information than saying 3 hours, 5 minutes and 0 seconds. A measurement of 3 hours and 5 minutes could mean anything from 3 hours, 4 minutes and 30 seconds to 3 hours, 5 minutes and 29 seconds.

3. Ask students to change times in hours and minutes to times in hours only:
   a) 3 hours 30 minutes = 3.5 hours  
   b) 4 hours 36 minutes  
   c) 2 hours 12 minutes  
   d) 5 hours 24 minutes  
   e) 6 hours 18 minutes  
   f) 9 hours 42 minutes

**HINT:** Students should first find the minutes as a fraction of an hour and then try to find an equivalent fraction with denominator 10. For example, 18 minutes is $\frac{18}{60}$ hours:

\[
\begin{array}{cccccccccccc}
\circ&\circ&\circ&\circ&\circ&\circ&\circ&\circ&\circ&\circ&\circ&\circ
\end{array}
\]

This is equivalent to 3 columns out of 10, so 18 minutes is $\frac{3}{10}$ or .3 hours.

**Bonus**

5 hours 15 minutes.
NS4-119
Exploring Numbers

GOALS
Students will explore the properties of multiplication.

PRIOR KNOWLEDGE REQUIRED
Multiplying by 10

Show your students a card with 2 rows of 4 dots. Write $4 + 4 = 8$ and ask students to tell you the corresponding multiplication statement. Then rotate the card so that it shows 4 rows of 2. Ask a volunteer to write the new addition statement and the new multiplication statement. **ASK:** Did I change the number of dots by rotating the card? How is this reflected in the 2 multiplication statements? (The product 8 is the same for both.) Ensure that students understand that you can rearrange the order of numbers in any multiplication statement.

Show 2 cards with 7 rows of 5 dots each:

Write $2 \times 7 \times 5$ on the board and ask students to explain how the picture shows the multiplication statement. Then turn the cards around horizontally:

**ASK:** What multiplication statement do the cards show now? ($2 \times 5 \times 7$). Did I change the total number of dots by turning the cards around? Then write on the board: $2 \times 7 \times 5 = 2 \times 5 \times 7$. Tell students that since $7 \times 5 = 5 \times 7$, having $7 \times 5$ twice is the same as having $5 \times 7$ twice. Have students draw models to illustrate this property of multiplication.

Give them multiplication statements with small numbers. (**EXAMPLE:** $3 \times 2 \times 5 = 3 \times 5 \times 2$).

Then **ASK:** Is $3 \times 4 \times 2 = 2 \times 3 \times 4$? How do you know? Have volunteers fill in the following blanks to see the equality: On the left side of the equation, $3 \times 4 \times 2 = \underline{\quad} \times 2$ and on the right side, $2 \times 3 \times 4 = 2 \times \underline{\quad}$.

**ASK:** How does this equation reflect that $2 \times 12 = 12 \times 2$? (Since $3 \times 4$ and $4 \times 3$ are both 12, then $2 \times 3 \times 4 = 3 \times 4 \times 2$ is saying the same thing as $2 \times 12 = 12 \times 2$.)
Have students write as many different but equivalent multiplication statements as they can think of for $3 \times 4 \times 2$. When students are comfortable doing this, show them how to group numbers to make multiplying easier. Ask them to solve $2 \times 7 \times 5$ by solving $14 \times 5$ or $2 \times 35$. Then have them rearrange the numbers so that the original multiplication statement becomes $2 \times 5 \times 7$. Now they can solve it by finding $10 \times 7$ or $2 \times 35$. **ASK:** Does any one of these combinations make it particularly easy to find the product? Which one? ($10 \times 7$) Why? (It’s easy to multiply by 10!) Have students group the 2s and 5s to solve these and other similar problems:

- a) $5 \times 4 \times 2$
- b) $2 \times 6 \times 5$
- c) $3 \times 5 \times 3 \times 2$
- d) $2 \times 5 \times 5 \times 2$

**Bonus**

$2 \times 3 \times 5 \times 5 \times 7 \times 2$

Encourage students to check their answers by multiplying the numbers in other orders, too. For example, in a) above, $5 \times 8 = 40$ and $4 \times 10 = 40$ and $20 \times 2 = 40$.

When students are comfortable grouping 2s and 5s to make 10, show them how to use these known products to multiply larger numbers easily: $4 \times 25 = 100$, $2 \times 50 = 100$, $4 \times 250 = 1000$, $2 \times 500 = 1000$. (**EXAMPLES:** $4 \times 9 \times 25$, $50 \times 17 \times 2$)

**Extension**

The following picture shows another way to group the dots in a $4 \times 4$ array.

```
1 + 2 + 3 + 4 + 3 + 2 + 1 = 4 \times 4
```

Have students draw similar pictures with multiplication and addition statements for arrays up to 5 by 5. Then challenge them to find $1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ without adding.
NS4-120
Word Problems

Review word problems with your students.

Extensions

1. Ask students to estimate their answers whenever possible, to check the reasonableness of their answers. They can practise estimating with calculations such as:
   a) $27 \times 38$
   b) $278 + 772$
   c) $585 - 277$
   d) $39 \div 5$

2. Tell students that information in a word problem can be redundant; sometimes you don’t need all the information given to solve the problem.

   EXAMPLE: Find the secret number:
   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My ones digit is greater than my tens digit.

   Have students try to figure out which 3 of the clues they need to answer the question.

   By changing only the last clue slightly, challenge your students to solve the puzzle using only two of the clues.

   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My tens digit is greater than my ones digit.

   Students can make up their own number puzzle with one piece of redundant information and have a partner solve the puzzle and identify the unneeded information.

3. Have students make up rhyming math poems as puzzles.

   EXAMPLE:
   A 2-digit number named Todd
   Has tens digit odd.
   But he’s even you see
   And his digits add to three.
   Which two numbers can he be?
4. (From Atlantic Curriculum B17) Teach students to use technology for computations involving many decimal places or large whole numbers. First, ensure that students are able to use calculators. Have them show the following numbers on a calculator in sequence without using the clear button: 443, 453, 452, 472, 412, 320, 160, 20, 50, 5 000, 5. Have students decide which of the following computations they would perform using a calculator, which they would perform mentally and which they would perform using paper and pencil:

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>785 × 28.3</td>
<td>56 × 480 × 90</td>
</tr>
<tr>
<td>100 − 39.5</td>
<td>1 000 − 395</td>
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<tr>
<td>70 − 43</td>
<td>8 × 400</td>
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<tr>
<td>699 ÷ 3</td>
<td>857.1 ÷ 3</td>
</tr>
</tbody>
</table>

Tell students that sometimes people press the wrong buttons on their calculator by mistake, but it can be annoying to go back to the beginning and press all the numbers again. Challenge students to find a way to not have to press clear and still find the answer they are looking for. They should try to press only one operation button and then a number.

a) You are trying to find 48 + 37, but actually pressed 48 + 27. (ANSWER: Press “+ 10”)

b) You are trying to find 4 572 + 2 987, but accidentally pressed 4 572 + 2 887 (ANSWER: “+ 100”)

c) You are trying to find 371 + 402, but accidentally pressed 381 + 402. (ANSWER: “− 10”)

d) You are trying to find 78 + 3, but accidentally pressed 78 − 3. (ANSWER: “+ 6”)

e) You are trying to find 4 872 − 7, but accidentally pressed 4 872 + 7. (ANSWER: “− 14”)

Bonus

f) You are trying to find 78 + 41, but accidentally pressed 78 − 41. (ANSWER: “+ 82”)

g) You are trying to find 1 238 ÷ 2, but accidentally pressed 1 238 × 2. (ANSWER: “÷ 4”)

h) You are trying to find 48 × 3, but accidentally pressed 48 ÷ 3. (ANSWER: “× 9”)

i) You are trying to find 67 + 296, but accidentally pressed 76 + 296 (ANSWER: “− 9”)


Tableaux de centaines vides ......................................................... 2
Cartes (fractions de nombres) ....................................................... 3
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Jeu de mémoire avec de l’argent .................................................... 8
Droites numériques jusqu’à vingt .................................................. 9
Blocs de régularités .................................................................... 10
Tableaux de centaines vides

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</tbody>
</table>
### Cartes (fractions de nombres)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{10} ) de 10</td>
<td>( \frac{1}{9} ) de 18</td>
<td>( \frac{1}{5} ) de 15</td>
<td>( \frac{1}{5} ) de 20</td>
</tr>
<tr>
<td>( \frac{1}{4} ) de 20</td>
<td>( \frac{1}{3} ) de 18</td>
<td>( \frac{7}{15} ) de 15</td>
<td>( \frac{1}{2} ) de 20</td>
</tr>
<tr>
<td>( \frac{2}{3} ) de 12</td>
<td>( \frac{3}{5} ) de 15</td>
<td>( \frac{1}{3} ) de 33</td>
<td>( \frac{3}{5} ) de 20</td>
</tr>
<tr>
<td>( \frac{1}{2} ) de 26</td>
<td>( \frac{7}{10} ) de 20</td>
<td>( \frac{3}{5} ) de 25</td>
<td>( \frac{4}{10} ) de 40</td>
</tr>
<tr>
<td>( \frac{17}{20} ) de 20</td>
<td>( \frac{3}{4} ) de 24</td>
<td>( \frac{1}{2} ) de 38</td>
<td>( \frac{4}{5} ) de 25</td>
</tr>
</tbody>
</table>
Jeu d’élimination

Trouve le nombre spécial dans chaque carré en lisant les indices et en éliminant.

N’OUBLIE PAS : Zéro est un nombre pair.

1. Raye les nombres qui sont …

   a)  
<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

   • Impairs
   • Plus petits que 4
   • Plus grands que 5

   Quel nombre reste-t-il?  

   b)  
<table>
<thead>
<tr>
<th>4</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

   • Pairs
   • Plus petits que 4
   • Plus grands que 6

   Quel nombre reste-t-il?  

   c)  
<table>
<thead>
<tr>
<th>4</th>
<th>20</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

   • Pairs
   • Multiples de 3
   • Plus grands que 10

   Quel nombre reste-t-il?  

   d)  
<table>
<thead>
<tr>
<th>4</th>
<th>20</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

   • Multiples de 2
   • Multiples de 3
   • Multiples de 4

   Quel nombre reste-t-il?  

Bonus

Raye les nombres avec …

   a)  
<table>
<thead>
<tr>
<th>42</th>
<th>31</th>
<th>92</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>99</td>
<td>88</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>21</td>
<td>66</td>
<td>52</td>
</tr>
</tbody>
</table>

   • Un 2 comme chiffre des unités
   • Un 2 comme chiffre des dizaines
   • Deux chiffres qui sont les mêmes

   Quel nombre reste-t-il?  

   b)  
<table>
<thead>
<tr>
<th>24</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>89</td>
<td>83</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>52</td>
</tr>
</tbody>
</table>

   • Un chiffre
   • Un 8 comme chiffre des dizaines
   • Un 2 comme chiffre des unités

   Quel nombre reste-t-il?  
Bandes de fractions
Associe les nombres

1. a) Trace une ligne entre toutes les paires de nombres dont le total est 10 :

   1 4 7 8 6 3 2

b) Trace une ligne entre toutes les paires de nombres dont le total est 10 :

   8 1 7 5 9 4 2

c) Trace une ligne entre toutes les paires de nombres dont le total est 100 :

   91 40 37 28 60 63 72

d) Trace une ligne entre toutes les paires de nombres dont le total est 100 :

   46 2 75 48 54 52 25 98

e) Trace une ligne entre toutes les paires de montants dont le total est 1 $ :

   0,71 $ 0,58 $ 0,37 $ 0,36 $ 0,63 $ 0,29 $ 0,42 $ 0,64 $ 0,28 $ 8,56 $ 9,72 $ 4,39 $

f) Trace une ligne entre toutes les paires de montants dont le total est 10 $ :

   1,73 $ 5,61 $ 4,39 $ 2,87 $ 1,44 $
Jeux de bingo mathématique

Exemples de cartes

1 2 13 5
6 7 19 4
11 3 16 15
14 10 8 12

1 12 17 14
6 15 4 11
9 2 13 8
16 5 10 3

9 5 14 4
6 15 18 13
10 7 16 3
1 11 2 12

2 9 13 17
10 14 5 8
6 3 16 12
15 11 7 4

1 12 13 5
6 7 19 20
11 8 16 15
14 10 18 17

1 20 17 14
8 6 4 11
9 2 13 18
16 15 20 3

9 5 14 4
6 15 18 13
10 7 16 3
20 11 2 12

2 9 3 17
10 14 5 8
6 20 16 12
15 1 7 4

20 13 12 5
16 17 19 4
11 3 6 15
14 10 8 2

1 12 17 4
6 15 14 11
9 7 13 8
20 5 10 3

9 5 14 20
6 15 8 13
10 17 16 3
11 19 2 12

2 19 13 17
10 14 15 8
6 3 16 1
Jeu de mémoire avec de l’argent

<table>
<thead>
<tr>
<th>0,75 $</th>
<th>75 ¢</th>
<th>7,50 $</th>
</tr>
</thead>
<tbody>
<tr>
<td>750 ¢</td>
<td>20 ¢</td>
<td>0,20 $</td>
</tr>
<tr>
<td>200 ¢</td>
<td>2 $</td>
<td>1 $</td>
</tr>
<tr>
<td>1 ¢</td>
<td>100 ¢</td>
<td>0,01 $</td>
</tr>
<tr>
<td>2,02 $</td>
<td>2,20 $</td>
<td>22 ¢</td>
</tr>
<tr>
<td>202 ¢</td>
<td>220 ¢</td>
<td>0,22 $</td>
</tr>
</tbody>
</table>
Droites numériques jusqu’à vingt
Bloc de régularités

Triangles

Carrés

Losanges

Trapèzes

Hexagones
PS4-6   Guessing, Checking, and Revising

Teach this lesson after: 4.2 Number Sense

Goals:
Students will make organized guesses and will use the result of the previous guess to revise their next guess.

Prior Knowledge Required:
Can use organized search
Can multiply two-digit numbers by one-digit numbers
Can compare two numbers up to 10 000 using place value
Can round whole numbers to the nearest ten, hundred, or thousand (for Problem Bank 2)
Understands that one number being eight times as big as another is the other number multiplied by 8 (for Problem Bank 3)
Can substitute a number for a variable in an expression (for Problem Bank 6)
Can divide with remainders (for Problem Bank 7)
Can divide without remainder (for Extended Problem)
Can multiply two-digit numbers by multiples of 10 up to 90 (for Extended Problem)

Vocabulary: column, divisor, factor, guess-check-revise, product, remainder, round, row

Materials:
an object to hide in the room
books that start on page 1
calculators
BLM Hockey Jerseys (pp. 10–11, see Extended Problem)

Introduce the guess-check-revise strategy. Hide an object in the room and have a volunteer try to find the object. If the student finds it quickly, play again until it takes a while. When the student finds it, ASK: What strategy did you use? (sample answer: I guessed and tried again) Play again, but this time tell the student whether he or she is hot or cold as the student tries to find the object. Use hints such as “freezing cold” for very far away from the object, “lukewarm” for getting close, and “burning hot” for very close. ASK: What strategy did you use? (sample answer: I guessed and tried again) When you tried again with a hint, was it easier than last time? Why? (answers may vary) SAY: When you have more information about your guess, you can use that information to revise your next guess. Write on the board:

    guess-check-revise

SAY: When you play hide-and-seek you are using a guess and check strategy, but when you play with hints such as “burning hot,” “lukewarm,” and “freezing cold,” you are guessing, checking, and revising the next guess. This guess-check-revise strategy is very useful in math.
Make sure everyone has a copy of a book that starts at page 1. Tell students to open their book to page 80 on their first try. Have different volunteers tell you what page number they turned to on their first try. Point out how all the attempts are fairly close to 80. SAY: No one’s first try was page 5 and no one’s first try was page 170. Everyone picked a page pretty close to 80. Now have students use their first guess to make a second guess. ASK: From the first page you turned to, which way in the book should you turn? Should you turn a lot of pages or only a few? (answers will vary) SAY: When you use your first guess to help you make your second guess, you are using the guess-check-revise strategy.

**Review systematic search when two related quantities are changing.** SAY: A farmer has cows and chickens. Jayden counts all the legs and Alice counts all the heads. Write on the board:

Jayden counts 16 legs.
Alice counts 6 heads.

ASK: Are there more heads or legs? (legs) Why does that make sense? (because each animal has more legs than heads) SAY: I want to know how many cows and how many chickens there are. Remember, to do this type of problem, you can start by choosing one of the two quantities and go up in order through all the possibilities. Draw on the board:

<table>
<thead>
<tr>
<th>Cows (4 legs)</th>
<th>Chickens (2 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
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</tbody>
</table>

ASK: How can you get the number of chickens from the number of cows? (they add to 6) SAY: There are six heads, so the total number of animals is six. Have a volunteer complete the second column. (6, 5, 4, 3, 2, 1, 0)

**Exercises:** Copy and complete the chart. How many cows and how many chickens are there if there are 16 legs in total?

**Answers:** 12, 14, 16, 18, 20, 22, 24; 2 cows and 4 chickens

Have a volunteer fill in the third column of the chart on the board. ASK: If you move down a row, does the total number of legs get larger or smaller? (larger) How much larger? (by 2) SAY: When you start at the top of the table, you have six chickens. When you move down a row, you replace a chicken with a cow, so now you have one cow and five chickens. Every time you replace a chicken with a cow, you replace two legs with four legs. In other words, you subtract two legs and add four legs, so you have two more legs than before.
**Searching from either direction.** SAY: Jayden and Alice went to another farm that has cows and chickens. Write on the board:

Jayden counts 36 legs.
Alice counts 10 heads.

ASK: How many animals are there altogether? (10) How do you know? (the number of heads)
Write on the board:

<table>
<thead>
<tr>
<th>Cows (4 legs)</th>
<th>Chickens (2 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td></td>
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<tr>
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<td></td>
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<td></td>
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<td>8</td>
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<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have a volunteer complete the second column. (10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0) ASK: If there are no cows and 10 chickens, how many legs are there? (20) Write “20” in the first row of the third column. ASK: If there are 10 cows and no chickens, how many legs are there? (40) Write “40” in the last row of the third column. ASK: Do you think the number of cows in our answer will be closer to zero or to 10? (10) Why? (the number of legs is closer to 40 than to 20) PROMPT: Is the actual number of legs closer to 20 or 40? (40) So, is it better to start our guess closer to zero or to 10? (10) SAY: We could save ourselves a lot of work by starting at 10 cows and zero chickens and moving up the chart instead of starting at zero cows and 10 chickens. ASK: How many legs do nine cows have? (36) Write on the board:

36 +

ASK: How many legs does one chicken have? (2) Continue writing on the board:

36 + 2 = 38

Write “38” as the total in the row for 9 cows and 1 chicken. Repeat for the row with 8 cows and 2 chickens. (32 + 4 = 36) SAY: So, eight cows and two chickens have a total of 36 legs. Starting from 10 cows and searching is a lot less work than starting from zero cows and going all the way to eight cows. Leave the chart on the board for later use.
**Exercises:** How many cows and how many chickens are there on the farm?

a) Jayden counts 22 legs. Alice counts 9 heads.
b) Jayden counts 26 legs. Alice counts 7 heads.
c) Jayden counts 32 legs. Alice counts 15 heads.
d) Jayden counts 52 legs. Alice counts 14 heads.

**Answers:** a) 2 cows, 7 chickens; b) 6 cows, 1 chicken; c) 1 cow, 14 chickens; d) 12 cows, 2 chickens

Refer students to the chart on the board. SAY: You don’t have to start the chart with zero cows and then move up the chart from the end. You can start with 10 cows and move down the chart instead. Draw on the board:

<table>
<thead>
<tr>
<th>Cows  (4 legs)</th>
<th>Chickens  (2 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>36</td>
</tr>
</tbody>
</table>

**Exercises:** If all the animals are cows, how many legs are there? If all the animals are chickens, how many legs are there?

a) Alice counts 30 heads.  
b) Alice counts 37 heads.  
c) Alice counts 28 heads.  
**Bonus:** Alice counts 1000 heads.

**Answers:** a) 120, 60; b) 148, 74; c) 112, 56; Bonus: 4000, 2000

SAY: Once you know how many legs there are if all the animals are cows and if all the animals are chickens, you can compare those numbers with the total number of legs given. Then you can decide which option to start your search with.

**Exercises:** How many cows and how many chickens are there?

a) Jayden counts 114 legs. Alice counts 30 heads.  
b) Jayden counts 140 legs. Alice counts 37 heads.  
c) Jayden counts 60 legs. Alice counts 28 heads.  
**Bonus:** Jayden counts 3996 legs. Alice counts 1000 heads.

**Answers:** a) 27 cows, 3 chickens; b) 33 cows, 4 chickens; c) 2 cows, 26 chickens; 
Bonus: 998 cows, 2 chickens

**Making guesses that increase or decrease by 10.** SAY: Now they visit a bigger farm with cows and chickens. Write on the board:

Jayden counts 344 legs  
Alice counts 100 heads.  
How many cows and chickens are there?

ASK: If all the animals are cows, how many legs are there? (400) If all the animals are chickens, how many legs are there? (200) Is 344 closer to 400 or to 200? (400) Do you think there are
more cows or chickens? (cows) SAY: Let’s start the search for the answer with 100 cows and zero chickens. Draw on the board:

<table>
<thead>
<tr>
<th>Cows (4 legs)</th>
<th>Chickens (2 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Have volunteers fill in the chart. (400, 398, 396, 394) SAY: We’re getting closer to 344 legs, but it’s going to take a while. ASK: How could I make the search go faster? Take students’ suggestions, then SAY: I am going to count by 10s instead of by 1s so that I can find the answer faster. Erase the chart on the board. Draw on the board:

<table>
<thead>
<tr>
<th>Cows (4 legs)</th>
<th>Chickens (2 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

**Exercises:**

a) Complete the chart.
b) Which 2 tens is the number of cows between? Explain how you know.

**Answers:**
a) 400, 380, 360, 340, 320; b) The number of cows is between 70 and 80, because 70 cows and 30 chickens have a total of 340 legs and 80 cows and 20 chickens have a total of 360 legs.

**Further narrowing the search.** Draw on the board:

<table>
<thead>
<tr>
<th>Cows (4 legs)</th>
<th>Chickens (2 legs)</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>30</td>
<td>340</td>
</tr>
</tbody>
</table>

ASK: Is the actual number of cows closer to 70 or to 80? (70) A lot closer or a little closer? (a lot closer) Why? (because 344 is a lot closer to 340 than to 360) What number should we try next? (71 or 72) SAY: Because 344 is a lot closer to 340 than it is to 360, you will get the answer faster by checking the number of legs counting up from 70 cows than by counting down from 80 cows, so the next guesses should be 71, 72, 73, until you find the answer.
Exercises:
Complete the chart until the total number of legs is 344.

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>30</td>
<td>340</td>
</tr>
<tr>
<td>71</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

Answer: 72 cows and 28 chickens

Problem Bank
NOTE: Students may need to use a calculator for many questions in this problem bank.

1. What numbers might I be?
   a) When you multiply me by 7, the result is between 500 and 550. Hint: Evaluate 10 × 7, 20 × 7, 30 × 7, and so on until you get close to 500.
   b) When you multiply me by 5, the result is less than 400. When you multiply me by 6, the result is more than 400.
   Answers: a) 72, 73, 74, 75, 76, 77, or 78; b) 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, or 79

2. What number am I?
   a) Multiply me by 7. Then round to the nearest 10. The result is 260. Use the table below, then make a new table that increases the number by 1 instead of by 10.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number × 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>70</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

b) Multiply me by 88. Then round to the nearest hundred. The result is 5000. Copy and continue the table below, then make a new table that increases the number by 1 instead of by 10.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number × 88</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>880</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

c) When you multiply me by 800 and then round to the nearest 1000, the result is 5000.
   Answers: a) 37, b) 57, c) 6
3. What are the two numbers?
   a) The bigger number is 8 times as big as the smaller number. The product of the two numbers is 200. Use the table below.
   
<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>Bigger Number</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
   
   b) The bigger number is 8 times as big as the smaller number. The product of the two numbers is 20 000.
   c) The bigger number is 8 times as big as the smaller number. The product of the two numbers is 10 952.
   **Answers:** a) 5 and 40, b) 50 and 400, c) 37 and 296

4. Students sell muffins and cake in a bake sale for a fundraiser. A muffin costs $2 and a piece of cake costs $3. The students sold 30 items altogether and made $71. How many muffins and how many pieces of cake did they sell?
   **Answer:** 19 muffins and 11 pieces of cake

5. Remember that two whole numbers are consecutive if there is no whole number between them. Examples: 4 and 5 are consecutive, but 4 and 6 are not because 5 is between them.
   a) Calculate the products.
      i) 1 × 2    ii) 2 × 3    iii) 3 × 4    iv) 4 × 5    v) 5 × 6
   b) Is 14 the product of two consecutive whole numbers? Explain how you know.
   c) Can 160 be the product of two consecutive whole numbers? Explain how you know.
   d) Can 992 be the product of two consecutive whole numbers? Explain how you know.
   e) Write 6972 as a product of two consecutive whole numbers.
   **Answers:** a) i) 2, ii) 6, iii) 12, iv) 20, v) 30; b) no, it is between 3 × 4 and 4 × 5; c) no, it is between 12 × 13 = 156 and 13 × 14 = 182; d) yes, it is 31 × 32; e) 83 × 84

6. Find \( N \) so that …
   a) \((2 \times N) + 1 = 177\)      b) \((N \times 3) + N = 228\)      c) \((N \times 5) + 5 = 320\)
   **Bonus:** Use a calculator to find \( N \) if \( N \times N = 1849\)
   **Answers:** a) 88, b) 57, c) 63, Bonus: 43
7. a) Fill in the blanks with a whole number where you can.

____ × 1 + 6 = 30
____ × 2 + 6 = 30
____ × 3 + 6 = 30
____ × 4 + 6 = 30
____ × 5 + 6 = 30
____ × 6 + 6 = 30

b) Which blanks have a whole number that works? Explain. Hint: Make sure the remainder is less than the divisor.

30 ÷ _____ = 1 R 6
30 ÷ _____ = 2 R 6
30 ÷ _____ = 3 R 6
30 ÷ _____ = 4 R 6
30 ÷ _____ = 5 R 6
30 ÷ _____ = 6 R 6

c) Glen divides 45 by a number and gets a remainder of 9. What numbers could he have divided by?

Selected solution: c) To fill in the blank in 45 ÷ ____ = ? R 9, you need ____ × ? + 9 = 45, so ____ is a factor of 36. ____ also must be bigger than 9, otherwise dividing by it can’t get a remainder of 9. So, the numbers he could have divided by are 12, 18, and 36.

Answers: a) 24, 12, 8, 6, no whole number possible, 4; b) 30 ÷ 24 = 1 R 6, 30 ÷ 12 = 2 R 6, and 30 ÷ 8 = 3 R 6 work. 6 R 6 and 4 R 6 don’t work because dividing by 6 or 4 can’t leave a remainder of 6. 5 R 6 doesn’t work because 5 is not a factor of 24 (30 − 6)
Extended Problem: Hockey Jerseys

Materials:
BLM Hockey Jerseys (pp. 10–11)

Preparation for the extended problem. Tell students that the extended problem is about hockey jerseys (team shirts) and how much it costs overall for a team to play ice hockey. Part of the total cost is for the jerseys and part is for buying hockey pucks. The players supply their own skates, hockey sticks, and other equipment. The team buys the jerseys, then sends them out to have numbers printed on them. The numbers start at 1 and go up in order to the number of players on the team. Numbers with two digits cost more than numbers with one digit, because the printing cost is per digit. Tell students that this will be part of what they investigate in the extended problem.

Extended Problem: Hockey Jerseys. Give students BLM Hockey Jerseys. Question 6 is a good opportunity to apply the problem-solving strategy learned in this lesson. Students who have not had the opportunity to do this lesson might find it difficult.
Answers: 1. $480, 2. 31, 3. $93, 4. $8, 5. $581, 6. 21
Hockey Jerseys (1)

In an ice hockey league, each team buys jerseys for their players before the season begins.

1. Jerseys cost $24 each. The Warriors team has 20 players. How much does the team need to pay for jerseys?

2. Each team puts numbers on the back of each jersey. Each team starts at 1 and numbers the jerseys in order. How many digits will the Warriors need in total?

3. Each digit costs $3 to put on. How much would a team of 20 players have to pay for the digits?

4. It costs the league $72 to buy hockey pucks for the year. There are 9 teams in the league. How much does each team pay for the pucks?
Hockey Jerseys (2)

5. What is the Warriors’s total cost, including hockey pucks, jerseys, and digits?

6. The Athenas is another team in the league. They paid $99 for the digits on their jerseys. How many players are on that team?
ME4-30
Area in Square Centimetres

GOALS
Students will find the area in centimetres squared (cm²) of shapes drawn on grid paper.

PRIOR KNOWLEDGE REQUIRED
Drawing lines with a ruler
Measuring sides with a ruler
Perimeter

VOCABULARY
2-dimensional
area
square centimeter
centimeters squared (cm²)
perimeter
rectangle

Remind students that area is often measured in units called “centimetres squared” or cm². Show students an example of a square centimetre, that is, a square whose sides are all 1 cm long.

Draw several rectangles and other shapes (EXAMPLE: L-shape, E-shape) on the board and subdivide them into squares. Ask volunteers to count the number of squares in each shape and write the area in cm².

Then draw several more rectangles and mark their sides at regular intervals, as shown below.

Ask volunteers to divide the rectangles into squares by joining the marks using a metre stick. Ask more volunteers to calculate the area of these rectangles.

Ask students to draw their own shapes on grid paper and to find the area and perimeter for each one.

ACTIVITY 1
Students work in pairs. One student draws a shape on grid paper, and the other calculates the area and the perimeter. ADVANCED VARIATION: One student draws a rectangle so that his/her partner does not see it, calculates the perimeter and the area, and gives them to the partner. The partner has to draw the rectangle with the given area and perimeter.

ACTIVITY 2
Students could try to make as many shapes as possible with an area 6 units (or squares) on grid paper or a geoboard. For a challenge, students could try making shapes with half squares. For an extra challenge, require that the shapes have at least one line of symmetry. For instance, the shapes below have area 6 units and a single line of symmetry.
Extensions

1. Sketch the shape below (at left) on centimetre grid paper. What is its area in cm²? (16) Now calculate the area using a different unit: 2 cm × 2 cm square (see below right). What is the area in 2 cm × 2 cm squares? (4) What happens to your measurement of area when you double the length of the sides of the square you are measuring with? (The area measurement decreases by a factor of 4.)

2. If the area of a shape is 20 cm², what would its area be in 2 cm × 2 cm squares? Sketch a rectangle with area 20 cm² to check your answer.

3. Draw a rectangle that has the same area (in cm²) as it does perimeter (in cm).

\[ \text{ANSWER:} \quad 18 = 3 \times 6 \quad (\text{area}) = 2 \times 3 + 2 \times 6 \quad (\text{perimeter}) \]
ME4-31
Area of Rectangles

**GOALS**
Students will find the area of rectangles in square units and in cm².

**PRIOR KNOWLEDGE REQUIRED**
Drawing lines with a ruler
Measuring sides with a ruler
centimetres squared (cm²)
Multiplication

**VOCABULARY**
2-dimensional
area
square centimeter
perimeter
rectangle
length
width

Draw an array of dots on the board. **ASK:** How many dots are in the array? Invite a volunteer to explain how he or she counted the dots. Ask another volunteer to write the corresponding multiplication statement on the board.

Now draw a rectangle on the board. (Draw it on a grid or subdivide it into squares.) Ask volunteers to write the length and width of the rectangle on the board, where length and width are measured in numbers of squares. Draw a dot in each square of the rectangle and **ASK:** How can a multiplication statement help us to find the area of the rectangle? Ask students to write and solve the multiplication statement for the area of the rectangle.

Draw several rectangles (again, subdivided into squares) and ask volunteers to write the length and the width and use them to calculate the area.

Draw rectangles with various lengths and widths (such as 20 × 30 cm, 40 × 30 cm, 30 × 50 cm) but don’t subdivide them into squares. Ask volunteers to measure the sides with a metre stick and calculate the area of the rectangles in cm².

Once students are comfortable finding the area of a rectangle by multiplying length and width, ask them to write the relationship as a rule: Area = Length × Width. You might also encourage students to write a similar rule for perimeter (Perimeter = Length + Length + Width + Width).

**Assessment**
1. Calculate the area of the rectangle:

   ![Diagram]

   Width: _____

   Length: _____

2. Measure the sides and calculate the area. Include units in your answer.

   ![Diagram]

   Width: _____

   Length: _____
Ask students to construct rectangles of the same area, but with different lengths and widths. They can work on a geoboard or with square tiles on grid paper. How many different rectangles can they make with area 8 cm²?

ACTIVITY 2
Ask students to create various rectangles and record their length, width, and area. Ask partners to give each other the length and width of some of their rectangles so they can calculate the area and check each other’s work.

Extensions
1. Calculate the area of the figure by adding the area of the rectangles:

```
2 cm  
|  
|  
|  
4 cm --
|  
3 cm  
|  
|  
4 cm  
```

2. Divide the figure into rectangles and calculate the area:

```
2 cm
|  
|  
|  
2 cm --
|  
2 cm  
|  
|  
3 cm  
|  
|  
|  
2 cm  
|  
7 cm  
```

4 cm
ME4-32
Exploring Area

**GOALS**

Students will find the area of rectangles with lengths and widths given in centimetres (cm), metres (m), and kilometres (km). Students will determine length (or width) given area and width (or length).

**PRIOR KNOWLEDGE REQUIRED**

Measuring sides with a ruler
Multiplication
Area of rectangle
Centimetres squared (cm²)

**VOCABULARY**

2-dimensional area
perimeter
rectangle
length
width
square centimetre and centimetre squared (cm²)
square kilometre and kilometre squared (km²)
square metre and metre squared (m²)

Draw a rectangle on the board and **ASK**: How can we calculate the area of this rectangle? Invite volunteers to help you solve the problem (one student could measure, another could write the measurements, a third could do the calculation and write the answer).

Tell students that a city block is 2 km long on every side. **ASK**: What shape is the block? What is its area? What units should we use for the area—is it square centimetres? Why not? If students do not infer the right answer and explanation, explain that a square kilometre is a square whose sides are 1 km long. When you multiply the length of the city block (in kilometres) by the width, you find out how many one-kilometre squares are in the block, so the area is in square kilometres, or km². **ASK**: A city block is 8 m long on every side. What is its area? What units do we use?

Then draw several rectangles on the board, write the length the width using different units of measurement, and ask volunteers to find the area.

**EXAMPLES:**

- 8 cm × 9 cm
- 3 m × 7 m
- 6 km × 10 km
- 20 cm × 7 cm
- 8 m × 7 m

**ASK**: Which rectangle has the greatest area? Which rectangle has the smallest area?

**ASK**: Which units would you use to measure the area of these objects or places—square centimetres, square metres, or square kilometres:

- Canada
- your classroom
- a book
- your city or town
- school yard
- a field
- a table

Draw another rectangle on the board and mark the length: 3 m. **SAY**: I know that the area of the rectangle is 6 m². How can I calculate the width of the rectangle? **(PROMPTS)**: If you knew the length and the width, how would you calculate the area? What do you have to multiply 3 (the length) by to get 6 (the area)? How do you know? What did you do to 6 to get 2?)

Give several more problems of this kind. **EXAMPLES**:

- Length 5 cm, area 20 cm², find the width.
- Width 4 m, area 24 m², find the length.
- A square has area 16 km². What is its width? (What can you say about the length and the width of a square? Students can try various lengths—1 × 1, 2 × 2, and so on—until they find the answer.)

Ask students to draw a rectangle that has an area of 18 cm². How many rectangles of different proportions can they draw? Prompt them to start with a width of 1 cm, then try 2 cm, 3 cm, and so on. Does 4 cm work? Why not? What about 5 cm?
Assessment
1. What is the area of the rectangles?
   A: Length 5 m, width 4 m.
   B: Length 6 km, width 7 km.
   C: Length 20 cm, width 15 cm.

   Order the rectangles from least to greatest.

2. A city square has area 800 _____. Its length is 40 _____. Fill in the appropriate units of measurement and calculate the width of the square.

3. Draw 3 different rectangles that have area 20 cm².

Extension
A rectangle has perimeter 12 cm and length 4 cm. What is its area?

Before the lesson, cut a square into two triangles by cutting across the diagonal. Now you have 2 congruent right triangles whose base and height are the same length. Hold up the two triangles and show students that they are congruent. Then demonstrate how you can put the triangles together to form a square. SAY: If I know that the base and height of the triangles is 2 cm, what is the area of the square? (2 × 2 = 4 cm²) What part of the square does each triangle cover? (half), or 2 cm². What area does it have?

Tell students the square is 1 cm². ASK: What is the area of the whole shape? How did you figure that out? Invite students to draw another shape with the same area.

Draw several shapes that include whole squares and an even number of half squares (i.e. triangles) on a grid and ask students to calculate the shapes’ area in terms of whole squares. Students might find it helpful to circle pairs of triangles that produce whole squares before finding the area. Then draw a shape with 3 half squares and ASK: What is the area of this shape? (one and a half squares) Ask students to draw shapes using both whole squares and
half squares. They should swap their drawings with a partner and calculate the area of their partners’ shapes. Students can then check each others’ area calculations.

**ASK:** If you have a shape built from 4 half squares, what is its area in whole squares? What did you do to calculate the area? (divided by 2) What is the area of a shape made of 10 half squares? 200 half squares? 100 half squares and 100 full squares?

Present this problem: Each student needs 4 sandwiches for lunch. Rita arranges the sandwiches on a tray like this:

![Image of sandwiches arranged on a tray](image)

If there are 20 students in the class, how many more trays does Rita need to make?

You might guide students to solve this problem in different ways:

1. Count the number of sandwiches on the tray. How many students will that be enough for? How many trays does Rita need in total? How many more trays should she make?
2. Each student needs 4 sandwiches. How many sandwiches does the whole class need? How many sandwiches are on the tray? How many more trays are needed?
3. Each student needs 4 sandwiches. That means 2 squares. How many squares does the class need? How many squares are on the tray? How many trays does the class need? How many more does Rita need to make?

You could also invite students to solve the problem in pairs or small groups and then share their solutions with the class. Guide the class to solve the problem using any approach(es) they don’t find and use on their own.

**Assessment**

1. Calculate the area of the shape.

![Image of a shape to calculate area](image)

2. If it takes Sandra two seconds to colour one triangle, or half square, how long will it take her to colour the whole shape?

**Bonus**

1. Calculate the area:
2. Write your name on grid paper using only squares and half squares. What is its total area? Compare your name with a partner’s. Whose name has the greater area? Who in the class has the name with the greatest area? The least area?

**Extension**

This triangle covers part of 2 squares:

Tell students you want to determine the area of this triangle. Use what students know about shapes and what they did earlier in the lesson to help them see that this triangle covers half the area of the 2 squares, or 1 square. **PROMPTS:** How did we figure out that the smaller triangles (from earlier in the lesson) had half the area of one square? Look at the two squares in this picture—do you see a second triangle? (yes, the unshaded area forms another triangle) Are the two triangles congruent? How do you know? (They have sides of the same lengths. If you cut the triangles out and lay them one on top of the other you’ll see that they’re the same.) If the triangles are the same, and there are 2 in the picture, what is the area of each triangle? (half of 2 squares, i.e. 1 square)

Now ask students to calculate the area of these shapes:

![Shape A](image1)

![Shape B](image2)

![Shape C](image3)
ME4-34
Finding and Estimating the Area

GOALS
Students will find area of shapes built from whole squares and half squares.

PRIOR KNOWLEDGE REQUIRED
Area of rectangle
Division by 2
half $\frac{1}{2} +$ half $\frac{1}{2} =$ whole (1)

VOCABULARY
area

Draw several squares on the board. Invite volunteers to shade half of each square. Encourage them to find as many different ways of doing this as possible, including:

Draw several shapes that include that include various types of half squares and ask students to find the area of each shape. Then ask students to draw designs for each other, and to find and to compare the shaded and unshaded areas of their designs.

Draw a rectangle on a grid. Ask volunteers to shade half of each square in the rectangle using any design they like. **ASK:** What is the shaded area of the rectangle? Do you have to count all the half squares or there is a shortcut? If you know the total number of squares, what part of the total is shaded?

Tell students that the distance between their wrists with arms spread out, is about 1 m. Ask 4 volunteers to hold out their arms and make a square. The area of that square (the space between their arms) is about 1 m$^2$. Ask the students to estimate if the area of their desks is more less than about 1 m$^2$. Can they find an object in the class that is closer to 1m$^2$?

Assessment
1. Find the shaded area of this square.

2. List 3 objects that have area more than 1m$^2$ and 3 objects with area less than 1m$^2$.

Bonus
Have students draw their names using whole squares and different half squares, and calculate the area.
Extensions

1. If $1 \text{ m} = 100 \text{ cm}$ and $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$, then $1 \text{ m}^2 = \underline{\hspace{2cm}} \text{ cm}^2$.

2. A workbook is about $33 \text{ cm}$ long and about $25 \text{ cm}$ wide. How many workbooks will you need to lay in a row for a total length of about $1 \text{ m}$? How many will you need to lay in a row for a total of about $1 \text{ m}$? How many workbooks do you need to cover an area of about $1 \text{ m}^2$?

3. Find the area of the workbook in Extension 2. Round to the nearest hundred. Divide your answer in Extension 1 (the total number of square centimetres in $1 \text{ m}^2$) by the rounded area of the workbook. Ignore the leftover. How many workbooks will fit in this area? Does your answer match the answer to Extension 2?

4. Make the design shown on an overhead geoboard and ask a volunteer to explain to the class how to find its area. Have students alter the shape on their geoboards to increase the area by $1 \text{ cm}^2$.
Comparing Area and Perimeter

Draw several rectangles on a grid: $4 \times 6, 5 \times 5, 6 \times 3, 7 \times 2, 3 \times 8$. Label them and ask volunteers to find the area and the perimeter of each one. Ask students to list the rectangles from least to greatest by area. Then ask them to list the rectangles from least to greatest by perimeter. **ASK:** Are your lists the same? Does the rectangle with the greatest area also have the greatest perimeter? Does the rectangle with the smallest perimeter also have the smallest area? Are there rectangles with the same area? Do they have the same perimeter? Are there rectangles that have the same perimeter? Do they also have the same area?

What do you do to the length and width to calculate area? What do you do to the length and width to calculate perimeter?

**Assessment**
1. Draw 2 rectangles that have the same area—20 cm$^2$—but different perimeters. Calculate their perimeters.
2. Bob drew 2 shapes with the same perimeter but different areas. Is this possible? The sides of Bob’s shapes are whole centimetres. One shape is a square with area 9 cm$^2$. Can you draw this square? The other shape is a rectangle. Can you draw the rectangle?

**Extensions**
1. Mr. Green wants to make a rectangular flower bed with perimeter 24 m. Which dimensions of the flower bed will provide the greatest area?
2. Mr. Brown wants to make a rectangular flower bed with area 36 m$^2$. Which dimensions will give him the least perimeter?
ME4-36
Area and Perimeter

GOALS
Students will estimate and find the area and perimeter of rectangles. Students will find the length of a rectangle given the area or perimeter and the width.

PRIOR KNOWLEDGE REQUIRED
Area of rectangle Perimeter of rectangle

VOCABULARY
area perimeter rectangle length width

Give students several paper rectangles with lengths and widths that are whole centimeters and say you want to estimate both the area and the perimeter of each one. ASK: How can I do that? What can I use to help me? Invite students to share their suggestions and try them out. If necessary, remind students that the area of their thumbnail is approximately 1 cm². How can they use their thumbnail to estimate area and perimeter?

Have students check their estimates by measuring the length and width of their rectangles and calculating the actual area and perimeter. Compare the estimates to the actual measurements.

Then tell students you want to find all the rectangles with perimeter 14 cm. The only other requirement is that the lengths and widths are whole centimetres (i.e. 3 or 5, not 2.25 or 4 3/8). SAY: I’m going to start with a rectangle that has width 1 cm. That’s the smallest possible measurement I can have. Draw a rectangle and mark the width. ASK: If the width of the rectangle is 1 cm, what is the length? Invite the student who answers to explain how he or she came up with the answer. If students need assistance use these PROMPTS: The perimeter is the distance all around the shape—if you took a rope that was 14 cm long, it would go all around the sides of our rectangle. If we know the width is 1 cm, how much of the rope have we used? (2 cm, for 2 sides) How much of the rope is left? (12 cm) For how many sides? (2) So how long is each remaining side? (6 cm) Another way to solve the problem is to use the fact that 1 length and 1 width of the rectangle add up to half the perimeter, or 7. If 1 + length = 7, the length must be 6.

Now draw a rectangle with width 2 cm and calculate the length. Repeat for a rectangle with width 3 cm. ASK: What is the length of the last rectangle? Do we need to continue making rectangles? Why not? (The last rectangle was 3 × 4. A rectangle with width 4 would have length 3—it would be the same rectangle, just rotated or turned on its side!) Do we need to make rectangles with width 5 or 6? What about a rectangle with width 7? How many rectangles with perimeter 14 do we have in total? Ask students to find the area of the rectangles.

Assessment
Draw all possible rectangles with perimeter 16 cm. Which one has the greatest area? The least area?

ACTIVITY1
Students could try QUESTIONS 4 and 5 on the worksheet using a geoboard.
Extensions

1. Describe a situation in which you would have to measure area or perimeter, for instance, to cover a bulletin board or make a border for a picture. Make up a problem based on the situation.

2. A rectangle has area 20 cm² and length 5 cm. What is its perimeter?

ME4-37
Problems and Puzzles

Worksheet ME4-37 serves as a review of the concepts of perimeter and area. It can be used as preparation for a test.

GOALS
Students will apply what they’ve learned to date about area and perimeter to solve word problems.

PRIOR KNOWLEDGE REQUIRED
Area of rectangle
Perimeter
Word problems

VOCABULARY
area
perimeter
length
width
Use concrete props to review the concepts of length and area, and to introduce the concept of volume. Start with a piece of string. It has only 1 dimension—length. **ASK:** What units do we measure length in? (cm, m, km)

Point to the top of a desk or table and explain that it has length and width—that’s 2 dimensions. Anything two-dimensional has area. The area of the desk or table is its surface, the space bound by, or “inside,” the length and width. **ASK:** What units do we measure area in? (cm², m², km²)

Now explain that a three-dimensional object, like a cupboard or a box, has length, width, and height. The space taken up by the box or cupboard or any three-dimensional object is called volume, and we can measure it, too. One of the units we use for volume is cubic centimetres, or cm³.

Draw a cube on the board and mark the length of all the sides as 1 cm. Tell students that this is a centimetre cube. It has length, height, and width 1 cm, and its volume is exactly 1 cm³. Tell students that they will be using centimetre cubes to calculate the volume of various shapes.

Ask students what they think larger volumes are measured in. (m³, km³)

Explain that these units are very large. For example, to fill an aquarium that has a volume of 1 m³, you would need about 100 pails of water!

Give students some centimetre cubes and ask them to build figures that have volume 4 cm³. Then ask them to build figures with volume 6 cm³. Assist any students who aren’t sure how many cubes to use for each figure, or who use the wrong number. Finally, invite students to build different figures with volume up to 8 cm³. They can work in pairs—one student makes a figure and the other calculates its volume.

Explain that for complicated figures, we can use a mat plan to calculate the volume. Make or draw the following 3-D figure:

Tell students to pretend that they are looking down on the figure from above. How many blocks would they see? (3) Draw 3 squares—the base, or bottom layer, of the figure—and invite volunteers to help you write the number of blocks stacked in each position:

```
 3 2 1
```
Explain that this is the mat plan of the figure. You can make or draw a few more figures (like the one below) and invite volunteers to help you draw the mat plans. Start by drawing the shape of the figure’s bottom layer, then count the cubes in each position.

Then do the opposite: start with the mat plan and build the figure. Show students this mat plan:

Then **ASK**: How can we calculate the volume of the figure? Do we need the figure or can we do it from the mat plan? (We can do it from the mat plan. Each cube has volume 1 cm$^3$, so the total number of cubes gives you the total volume of the figure.)

Ask students to work in pairs. One partner builds a figure with volume no more than 12 cubes and height no more than 4 cubes and the other partner draws the mat plan for the figure. Partners should swap roles, so that each has a chance to do both the drawing and the building. After everyone has had a chance to do both at least once, have pairs do the reverse: one partner draws the mat plan for a figure and the other builds the figure accordingly.

**Assessment**
1. Build the figure according to the mat plan:

2. Fill in the mat plan of this figure.

3. Find the volume of both figures above.
Extensions

1. Draw the mat plan for this figure. How many cubes are “hidden” in the picture?

![Diagram of a figure with hidden cubes]

2. Give students the front, top, and side views of a structure and ask them to build it using interlocking cubes. As they work, encourage them to look at their structure from all sides, and to compare what they see to the pictures. Ask them to determine the volume of the completed structure.

   a) ![Front, Top, Side, Possible Answer diagrams]

   ![Diagram of a structure with front, top, side, and possible answer views]

   **NOTE:** You might want to give students the front, top, and side views of a few simpler shapes before they tackle the ones above.

**Bonus**

Give students the front, top, and side view of a simple figure and ask them to calculate the volume without building the figure.
ME4-39
Volume of Rectangular Prisms

GOALS
Students will find the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED
Area of rectangle
Addition and multiplication
Mat plans, or top views

VOCABULARY
volume
length
width
height
column
row
layer
rectangular prism

Draw a rectangle on a grid (or subdivide a rectangle into equal squares). Ask students to write the addition and multiplication statements needed to calculate the area of this rectangle.

Show students a large rectangular box. Explain that you want to know the volume of the box. (You can say that you want to send something to someone, and the shipping company charges by volume. You need to know the volume to figure out how much it will cost to ship this box.) You can fill the box with centimeter cubes, but that is not very practical. Tell students that today they will learn another method for calculating volume.

First, tell students that mathematicians have a fancy name for a rectangular box. They call it a “rectangular prism.” Write the term on the board.

Build a 2 × 4 rectangle using centimetre cubes and ASK: How many cubes are in this rectangular prism? What is the volume of this prism? Ask students to explain how they calculated the volume—did they count the cubes one by one or did they count them another way? Because each cube has volume 1 cm³, the total number of cubes gives you the total volume. You can use addition or multiplication to count the total number of cubes:

\[2 + 2 + 2 + 2 = 8 \text{ cm}^3\] (2 centimetre cubes in each of 4 columns)
\[2 \times 4 = 15 \text{ cm}^3\] (2 rows of 4 centimetre cubes OR length × width)

Add 1 layer to the prism so that it is 2 cubes high. ASK: What is the volume of the new prism? Prompt students to use the fact that the prism has 2 horizontal layers. Remind them that they already know the volume of 1 layer. Ask volunteers to write addition and multiplication statements for the volume of the new prism using layers.

\[8 + 8 = 16 \text{ cm}^3\]
\[2 \times 8 = 16 \text{ cm}^3\]

Add a third layer to the prism and repeat. Invite students to look at this last prism and to calculate the volume by adding vertical layers, or “walls,” instead of horizontal layers.
ASK: How many cubes are in the wall at the end of the prism? (3 × 2 = 6) What is the volume of the wall? (6) How many “walls” are in the prism? (4) Invite volunteers to write the addition and the multiplication statements for the volume of the prism using the volume of the “wall.” Does this method produce a different result than the previous method? (no, it’s the same answer)

Explain that the third dimension in 3-D figures is called height. Identify the length, width, and height in the prism above. Then use the terms length, width, and height to label the multiplication statement that gives the volume:

\[3 \times 2 \times 4 = 24 \text{ cm}^3\]

height \hspace{0.5cm} width \hspace{0.5cm} length

Draw several prisms on the board, mark the height, width, and length (you can use different units for different prisms), and ask students to find the volume. **SAMPLE PROBLEMS:**

- 10 cm × 6 cm × 2 cm
- 2 m × 3 m × 5 m
- 3 km × 4 km × 7 km

Remind students to include the right units in their answers.

ASK: What does the top view for a rectangular prism look like? Is there any difference in heights above each square? (No, the height of a rectangular prism is the same everywhere.) Draw several top views on the board and ask students to identify which are rectangular prisms and which are not. Then ask them to write the length, width, and height for each prism, and to calculate the volume.

**SAMPLE PROBLEMS:**

- \[
\begin{array}{ccc}
5 & 5 & 6 \\
5 & 5 & 5 \\
\end{array}
\]
- \[
\begin{array}{ccc}
7 & 7 & 7 \\
7 & 7 & 7 \\
\end{array}
\]
- \[
\begin{array}{ccc}
5 & 5 & 5 \\
5 & 5 & 5 \\
5 & 5 & 5 \\
5 & 5 & 5 \\
\end{array}
\]
- \[
\begin{array}{ccc}
5 & 5 & 5 \\
5 & 5 \\
\end{array}
\]

**Bonus**
Find the volumes of the figures that aren’t rectangular prisms.

**Assessment**
Find the volume of the prisms:

- a) \[
\begin{array}{ccc}
6 & 6 & 6 \\
6 & 6 & 6 \\
\end{array}
\]
- b) 4 m × 5 m × 6 m
- c) ![Shape](image)

**Extension**
Find the volume of the shape:
Mass

GOALS
Students will assign appropriate units of mass to various objects.

PRIOR KNOWLEDGE REQUIRED
What is mass
Grams, kilograms
Ordering from least to greatest, from greatest to least

VOCABULARY
mass
gram (g)
kilogram (kg)

NOTE: Mass is a measure of how much substance, or matter, is in a thing. Mass is measured in grams and kilograms. A more commonly used word for mass is weight: elevators list the maximum weight they can carry, packages list the weight of their contents, and scales measure your weight. The word weight however, has another very different meaning. To a scientist, weight is a measure of the force of gravity on an object. An object’s mass is the same everywhere—on Earth, on the Moon, in space—but its weight changes according to the force of gravity. When we use the term weight in this and subsequent lessons, we use it as a synonym for mass.

Remind students that mass (which we often call weight) is measured in grams (g) and kilograms (kg). Give several examples of things that weigh about 1 gram or about 1 kilogram:

1 g: a paper clip, a dime, a chocolate chip
1 kg: 1 L bottle of water, a bag of 200 nickels, a squirrel.

List several objects on the board and ask students to say which unit of measurement is most appropriate for each one—grams or kilograms:

- A whale
- A table
- A napkin
- A cup of tea
- A workbook
- A minivan

Ask students to match these masses to the objects above:

2 000 kg  50 000 kg  10 g  150 g  400 g  10 kg

Have students order these objects from heaviest to lightest.

Ask students to think of 3 other objects that they would weigh in grams and 3 objects that would demand kilograms.

Draw a set of scales on the board. Draw various weights on one scale (that is on one side) and ask volunteers to add weights to the other so that the scales are in balance. Then draw weights on both scales, but leave out the numbers on one side and have students fill them in. Include instances where students have to add and divide to find the right weights. SAMPLE PROBLEMS:
### Extensions

1. A matchbox with 6 matches weighs 20 g. If the match box weighs 8 g, how much does each match weigh?

2. Jane wants to estimate the mass of one grain of rice. She weighs 100 grains of rice and divides the total by 100. Try to weigh 1 grain of rice. Explain why Jane uses this method above. Use Jane’s method to estimate the mass of a bean or a lentil.
ME4-41
Changing Units of Mass

GOALS
Students will convert kilograms to grams and vice versa.

PRIOR KNOWLEDGE REQUIRED
Grams, kilograms
Ordering from least to greatest and from greatest to least
Multiplying decimals by 1 000

VOCABULARY
mass
gram
kilogram

Write on the board:

1 Kilometre = _____ metres  1 km = _____ m
1 Kilogram = _____ grams  1 kg = _____ g

Invite volunteers to fill in the missing numbers.

ASK:
What does the prefix “kilo” mean? If 1 kg = 1 000 g, how many grams are in 2 kg? How did you get the answer? How many grams are in 5 kg? 17 kg? 457 kg?

Now ASK: If I have 3 000 g of peanuts, can I change the grams to kilograms? How? (divide by 1 000) How many kilograms is 6 000 g? 16 000 g? 78 000 g?

Draw a table on the board and ask students to fill in the missing masses:

<table>
<thead>
<tr>
<th>Animal</th>
<th>Mass in Kilograms</th>
<th>Mass in Grams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raccoon</td>
<td></td>
<td>21 000 g</td>
</tr>
<tr>
<td>Cat</td>
<td>5 kg</td>
<td></td>
</tr>
<tr>
<td>Newborn Gorilla</td>
<td></td>
<td>2 000 g</td>
</tr>
</tbody>
</table>

Add rows to the table to allow more practice. Include only whole numbers in the kilograms column and numbers rounded to a thousand in the grams column.

Ask students to circle the heavier object in the following pairs. In some cases, they will have to put the measurements into the same units before they can make the comparison.

- 30 g of ice cream or 55 g or salt
- 3 kg of bread or 30 kg of sugar
- 35 g of cinnamon or 3 kg of potatoes
- 3 010 g of cotton wool or 3 kg or iron
- 17 kg of wood or 1 700 g of paper

Ask students to order all the quantities above from least to greatest.

Finally, look at decimal quantities and review multiplying and dividing decimals by 1 000 (moving the decimal point right or left) to change grams to kilograms and vice versa.

EXAMPLES:

How many grams are in 5.5 kg? 3.91 kg?
How many kilograms is 7 500 g? What about 8 020 g?
Assessment
Order the quantities from greatest to least:

17 g  12 kg  3 000 kg  5 000 g  100 000 g  250 g  120 kg

Extension
A crate contains 3 kg of oranges. A package of oranges weighs 1 816 g. Three crates cost the same as 5 packages. To get the most oranges, should you buy 3 crates or 5 packages?

ME4-42
Problems Involving Mass

Work through the following set of problems as a class. Invite and encourage as many different students as possible to participate by contributing answers, explanations, and suggestions. Break some of the more complicated problems into smaller steps. Use prompts and questions to help students identify what they know and how they can use what they know to solve the problem.

You could keep a list of facts on the board and add to it as you go. For example, problem a) gives the weight of a rabbit (3 kg) and asks you to calculate the weight of a lazy cat (6 kg). Both of these facts could go on the list: Rabbit = 3 kg, Lazy Cat = 2 × Rabbit = 6 kg.

a) A rabbit weighs 3 kg. A lazy cat weighs twice as much as the rabbit. How much does the lazy cat weigh?

b) An angry dog weighs as much as 3 lazy cats. How many rabbits does this dog weigh?

c) What is the dog’s mass in kilograms?

d) An adult raccoon weighs as much as an angry dog and a rabbit. How many rabbits does the raccoon weigh? What is the raccoon’s mass in kilograms?

e) A beaver weighs 3 500 grams less than an adult raccoon. How much does the beaver weigh?

f) A female bear weighs 90 kg. How many rabbits it that equal to? How many lazy cats? How many dogs?

g) How many lazy cats do you need to balance 2 angry dogs and 2 rabbits on a scale?

h) A newborn Siberian tiger weighs about 1 kg. It gains 700 g a week. How much weight does it gain in 4 weeks? How much does the tiger weigh after 4 weeks?
i) A 20-week old Siberian tiger cub is on one scale and an angry dog is on the other. What do you need to do to balance the scales? (Ask a volunteer to draw a model for this problem first.)

**Extensions**

1. Describe how you could find the mass of 1 pack of a paper without weighing it directly. You can weigh many packs together or you can put it on a scale with other things, but you can’t weigh 1 pack by itself. (One method could be to calculate the difference between your weight with and without the pack. The other method could be to weigh many packs together and to divide the weight by the number of packs.)

2. Refer to the chart at the bottom of Worksheet ME4-40 to determine which is heavier:
   - 10 dimes or 1 loonie
   - 5 dimes or 2 quarters
   - 1 quarter or 2 dimes and 1 nickel

---

**ME4-43**

**Capacity**

Explain that the capacity of a container is how much it can hold. Write the term on the board. Explain that capacity is measured in litres (L) and millilitres (mL). **ASK:** Where have you seen the prefix “milli” before and what did it mean? (millimetre; one thousandth) How many millilitres are in 1 litre? In 2 litres? In 7 litres? What do you do to change litres to millilitres? (Multiply by 1 000.) Write on the board:

\[
\begin{align*}
1 \text{ metre} & = 1 000 \text{ millimetres} \\
1 \text{ litre} & = 1 000 \text{ millilitres}
\end{align*}
\]

Put out several containers (EXAMPLES: milk and juice boxes, medicine bottles, measuring cups, cans of paint, cans of pop) with capacities clearly marked on them. Invite students to help you separate the containers into two groups: those that can hold 1 or more litres and those that can hold less than 1 litre. Then ask students to help you order the containers by capacity, from least to greatest. The containers can also act as “capacity benchmarks” that you can keep in a class measurement box.

Write the following on the board and ask students whether they would measure the capacity of each container in millilitres or litres:

- a glass of juice
- a bowl of soup
- a pail of water
- a pot of soup
- an aquarium
- a backyard pool

Ask students to think of three more quantities that are measured in litres and three that are measured in millilitres.
Show students a 1 L carton of milk or juice and a small (200 mL) glass. **ASK:** How can we determine the capacity of the glass? You may ask a volunteer to check how many glasses can be filled from the container. What is the capacity of the glass? Now bring out a larger glass (250 mL). How many glasses of this size can be filled from the carton? Ask a volunteer to check. (**NOTE:** You will need a large bowl or pot in which to empty out the glasses as they are filled.)

**ASK:** How many glasses of each size would you need to fill a 2 L container? If such a container is available, allow the students to check their prediction.

**Assessment**

a) An aquarium holds two 8 L pails of water. What is its capacity? Write the capacity in litres and millilitres.

b) A jar holds 500 mL of water. How many jars do you need to fill the aquarium?

---

**ACTIVITY 1**

Measure the capacity of several glasses or containers in your classroom. Estimate the capacity of the containers before you measure their capacity. Students should select and justify appropriate units to measure the capacity of a container. **NOTE:** Students will need a measuring cup and several containers for this activity.

**ACTIVITY 2**

Ask students to fill a measuring cup marked in litres and estimate how many cups they would need to fill a 1 L container. Then, knowing the number of millilitres their cup holds, they should estimate how many millilitres are in a litre. Students could then test their prediction by filling up the litre container and keeping track of how many litres they need.

**ACTIVITY 3**

Give students several glasses of containers on which the capacities have been covered or removed. Ask them to estimate the capacity of each container and then check their answers. Remind students to include the appropriate units (mL or L) with their estimates.

**ACTIVITY 4**

Make 2 containers: 500 mL and 300 mL (you can use 2 empty 1 L cartons and cut them at the height of 10 cm and 6 cm respectively). Ask your students to use only these two containers, a tap, and a sink to measure 200 mL of water. **CHALLENGING:** Measure 400 mL of water using only the same equipment.
Extensions

1. **JELLYBEAN JARS:** Choose two straight-sided jars of similar size but different dimensions, for example, a tall, thin olive jar and a short, squat salsa jar. Fill both jars with jelly beans. Show students both jars and **ASK:** Which jar has a larger capacity? Why do you think that? How could we check our predictions? Discuss the students’ ideas for determining the answer, then choose one or more and try it! Here are some of the approaches you could investigate:

   a) Open the jars, dump out the contents, and count the jelly beans.
   
   b) Open the jars, dump out the contents of one jar, and pour the jelly beans from the other into it.

   Do the jelly beans fill the empty jar? Is there any empty space left over? Are there any jelly beans left in the first jar?

   c) Have a student count the number of jelly beans visible through the bottom of the jar, and record the number. Have another student count the number of layers of jelly beans from the bottom to the top. Have students multiply the 2 numbers to estimate the total number of beans in each jar.

   d) Fill a large glass container with enough water to submerge either jar. Ask your students to predict what will happen if you place the jar into the bowl. Why does the water level go up? Submerge one jar in the water and mark the new water level with a piece of tape. Remove the first jar and point out what has happened to the water level. Place the second jar in the water and ask students to determine whether the water level is higher of lower than with the first jar. What does this tell us about the size of the jars?

2. **PROJECT:** What are the connections between litres, cm$^3$ and grams? $1000$ cm$^3 = 1$ L. Base-ten cube representing $1000$ is a referent for $1$ L.

   **POSSIBLE SOURCES:**
   
   http://en.wikipedia.org/wiki/Liter
   http://lamar.colostate.edu/~hillger/faq.html#liter-defn

3. **PROJECT:** The origins of SI and its usage throughout the world: When SI was introduced, which countries use it and which do not? Which countries are converting into SI? What are the advantages of SI?

   **POSSIBLE SOURCE:**
   
   http://lamar.colostate.edu/~hillger/#metric
ME4-44
Mass and Capacity

Ask students which units (g, kg, L, mL) they would use to measure:
- the weight of a baby
- the amount of juice in a can
- the contents of a cup
- the amount of sugar in a bag
- the amount of milk in a pail
- the amount of meat in a frying pan
- the contents of a cement truck

Divide students into 4 groups, assign each group a unit of measurement, and ask the groups to make a list of at least 20 things that can be measured in their unit. They should also estimate the weight or capacity of each item on the list. What is the total estimated weight or capacity of the list? Each group should present their list to the rest of the class.

The activity will give students more practice in distinguishing between masses and capacities and adding like quantities.

Assessment
Total the masses and the capacities needed to make raisin pancakes:
250 mL of milk, 125 g of flour, 50 mL of oil, 3 eggs of 50 mL each, 50 g of sugar, 50 g of raisins.

Bonus
The ingredients to make apple pancakes are: 200 mL of milk, 125 g of flour, 50 mL of oil, 3 eggs, 50 g of sugar, 100 g of sliced apple. If 250 mL of oil weighs 225 g, an egg weighs 60 g and 100 mL of milk weighs 110 g, how much will your pancakes weigh?

Extension
Examine cookbooks, recipe cards, or recipes clipped from magazines and newspapers together. Ask students to choose a recipe and to check which products are presented as capacities and which are presented as weights. Then have them total the measures of mass and capacity for a chosen recipe.

Explain that some of the non-liquid ingredients in a recipe are often given as capacities because it is more convenient. Ask students to change the capacities in the recipe of their choice of sugar, cocoa or flour, rice, raisins or nuts into weight using the chart:

- 1 tsp = 5 mL = 4 mg sugar = 3 mg flour or cocoa
- 1 cup = 250 mL = 200 g sugar = 125 g flour or cocoa = 165 g raisins = 100 g nuts = 175 g rice
- 1 egg = 60 g = 50 mL
ME4-45
Temperature

GOALS
Students will read thermometer and estimate whether the given temperature is hot or cold.

PRIOR KNOWLEDGE REQUIRED
Thermometer
Number lines

VOCABULARY
thermometer
degree Celsius

Hold up a large thermometer and ask students to identify what it is. **ASK:**
What do we measure with a thermometer? (temperature) Where and when do people use thermometers? (to measure the temperature of a home, a greenhouse, the outdoors, the fridge or freezer, the body, and so on)

Explain how a thermometer works. The liquid inside the bulb goes up and down depending on the temperature. It goes up as the temperature gets warmer, and down as it gets colder. You can demonstrate this using cups of water at different temperatures. Fill one cup with hot water, one with water at room temperature, and another with cold water and ice. Invite a volunteer to touch the cups and to order them from coldest to hottest. Then insert the thermometer into each cup and let students observe the movement of the liquid in the thermometer.

Explain to your students that temperature is measured in units called degrees. In Canada, we use degrees Celsius, which can also be written like this: °C. Ask your students to observe the numbers alongside the liquid in thermometer. What do they notice? Explain that the zero on the thermometer indicates the freezing mark. This is the temperature at which water freezes. When the liquid in the thermometer drops below the zero, we say the temperature is “below freezing.” You might point out that water becomes ice below 0°C, but other liquids—like the liquid inside the thermometer—stay unfrozen.

As the liquid in the thermometer rises above zero, the temperature gets warmer, so we count up. The larger the number above zero, the warmer it is. As the liquid in the thermometer drops below zero, the temperature gets colder, so we put a minus sign in front of the number and count up. The larger the number below zero, with the minus sign, the colder it is.

Draw a large thermometer on the board. Ask students to help you add a scale to it. Ask them to count up by tens from zero, and mark the degrees on the thermometer. Write the descriptive words beside each range of temperatures: hot 30°C and up; warm 20°C to 30°C, cool 10°C to 20°C, cold 0°C to 10°C, and freezing cold 0° and below. Add the -10°C mark to the scale. Remind students that these ranges and descriptions refer to the temperature of air.

Point to different temperatures on the thermometer (between the ten degree increments marked) and **ASK:** If the liquid in the thermometer was here, how would you describe the temperature? Then ask students to think about the temperature that is appropriate or necessary for various outdoor activities. For each activity, draw two thermometers showing very different temperatures and ask students which one shows the temperature that is more appropriate for the activity. **EXAMPLES:** outdoor swimming: -10°C or 30°; berry picking: 0°C or 17°C; ice hockey: 15°C or -3°C.
Add the -5°C, 5°C, 15°C, 25°C marks to the scale. Explain to your students that on a real thermometer there is no room to write the other numbers, but you will teach them how to read the temperature even when these numbers are missing.

Invite a volunteer to count up by fives from 0 to 10 and by ones from 10 to 15. Then point at the mark showing 12°C and explain that, to read this temperature, people first count by fives and then continue by ones until they reach the mark. Invite students to count up with you until you reach 12°C. Repeat with several examples until all of your students are able to count first by fives then by ones to any mark you indicate. If necessary, separate the tasks—ask students who have trouble counting by both fives and then ones to count by fives only or to count by ones only from the nearest marked increment. Ask your students to tell whether the weather outside is hot, cold, warm, or cool for each of the temperatures you show.

Explain that water feels differently than air at different temperatures. For instance, what feels colder—the air on your face when it is 0°C outside or a piece of ice that you hold in your bare hand? Students can do the Activity 3 below to compare the temperatures of water and air.

**ASK:** Is -5°C hot or cold? Is water at this temperature a liquid or is it ice? Explain that if you have a piece of ice at -5°C, you have to heat it for some time to make it melt. How much do you have to heat it? (enough to raise its temperature by 5°C, to 0°C)

Write several temperatures on the board. Ask students to describe each temperature and to tie it to familiar events or activities, as well as states of water, such as:

- -10°C freezing cold; skating, skiing
- 0°C cold, water freezes
- 5°C cold, water is nearly frozen, ice has melted
- 20°C air is warm, water still cold; bike riding
- 36°C air is very hot, near to normal human body temperature; swimming, sitting in the shade
- 50°C very hot, desert (You can bake eggs in the sand!)

Add to this list the range of temperatures for each season where you live, to give students the information they need to complete the worksheet.

Have students solve several problems in which they have to add or subtract to find the temperature.

**SAMPLE PROBLEMS:**

The temperature today is 15°C. Yesterday it was 5°C higher. What was the temperature yesterday?

The average temperature in winter is -10°C. The average temperature in summer is 25°C. What is the difference between the average temperatures in winter and in summer? (Let students use a thermometer to solve this problem.)

**Assessment**

a) What was the temperature on Monday?
b) What was the temperature on Wednesday?
c) How much warmer was in on Monday than on Wednesday?
d) A jar of water was left outside. On which day did the water turn to ice?
Extension

Explain that the scale of the thermometer is based on the properties of water, so that anyone can build his or her own thermometer. Explain that 0°C is the temperature at which water freezes and 100°C is the temperature at which water boils. So to make a new thermometer, you need to freeze some water to find the 0°C mark and then boil some water to establish the 100°C mark. Then you divide the scale into 100 parts and your thermometer is ready. You can illustrate the process while drawing a thermometer on the board.
### ME4 Partie 2 : Liste — Fiches reproductibles

<table>
<thead>
<tr>
<th>Material</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Papier à points</td>
<td>2</td>
</tr>
<tr>
<td>Papier quadrillé (1 cm)</td>
<td>3</td>
</tr>
<tr>
<td>Pentominos</td>
<td>4</td>
</tr>
</tbody>
</table>
Papier à points
Papier quadrillé (1 cm)
Pentominos
PS4-7  Using Structure II

Teach this lesson after: 4.2 Measurement

Goals:
Students will use pictures to understand why two expressions are equal.

Prior Knowledge Required:
Can fluently add and subtract multi-digit whole numbers using the standard algorithm
Can multiply a whole number of up to two digits by a one-digit whole number
Can divide two-digit numbers by 2
Can apply the distributive property of multiplication over addition
Knows that expressions within brackets are evaluated first
Can find the area of a rectilinear shape drawn on grid paper
Can compute the area of a rectangle from its side lengths

Vocabulary: area, area model, increase, length, product, row, sequence, sum, term, width

Materials:
15 counters per student
transparency of grid paper or BLM 1 cm Grid Paper (p. 9)
grid paper or BLM 1 cm Grid Paper (p. 9)
scissors

Discovering patterns in sums. Start by having students do the exercises below.

Exercises: Add and then find a pattern. What will the next addition equation be?
1 + 2 + 3 = _____
2 + 3 + 4 = _____
3 + 4 + 5 = _____
4 + 5 + 6 = _____
Answers: 6, 9, 12, 15; 5 + 6 + 7 = 18

ASK: How did the answers in the exercises change? (they increased by 3) How did the additions change? (they started with a number 1 greater each time; each number increased by 1) Give students 12 counters each. Tell them to arrange the counters to show 3 + 4 + 5. Students should make three groups—a group of three, a group of four, and a group of five. SAY: Now I want you to show 4 + 5 + 6 instead of 3 + 4 + 5. ASK: How many more counters do I need to give you? (3) How do you know? (because the sum increases by 3 each time; because each of the three numbers increases by 1; because 4 + 5 is in both additions and 6 is 3 more than 3) Give students three more counters each. Challenge them to add the three counters, without moving the original counters, to show 4 + 5 + 6. (add 1 more to each group)

Draw on the board:

○ ○ ○ ○ ○ ○ ○ ○
● ● ● ● ● ● ● ●

Teacher's Guide for Grade 4 — Problem-Solving Lessons
SAY: The white dots show $3 + 4 + 5$. The black dots show adding one more dot to each group. The whole picture shows $4 + 5 + 6$.

**Exercises:**
a) If you have dots showing $8 + 9 + 10 + 11$, how many more dots would you need to show $9 + 10 + 11 + 12$?
b) If you have dots showing $5 + 7 + 9$, how many more dots would you need to show $6 + 8 + 10$?

**Answers:** a) 4, b) 3

**Patterns using the times tables.** Write on the board:

\[
\begin{array}{c}
1 + 2 + 3 = 6 = 3 \times \underline{2} \\
2 + 3 + 4 = 9 = 3 \times \underline{3} \\
3 + 4 + 5 = 12 = 3 \times \underline{4} \\
4 + 5 + 6 = 15 = 3 \times \underline{5}
\end{array}
\]

Have volunteers fill in the blanks. (2, 3, 4, 5) SAY: All these additions are three times a number. ASK: How can you know what number to multiply three by from just looking at the sum or addition? (it’s the middle number) PROMPT: Where can you see the underlined number in the addition? Have volunteers circle where the underlined number is in each addition. ASK: Is it always in the same place? (yes) Can you predict where the number will be for the next sum? (yes, the middle number) SAY: When you can write the numbers in the sequence as a product, you can predict any term without having to find all the terms in between. Draw on the board:

\[
\begin{array}{ccccccc}
1 + 2 + 3 & 2 + 3 + 4 & 3 + 4 + 5 & 4 + 5 + 6 & 9 + 10 + 11 & 99 + 100 + 101 \\
3 \times 2 = 6 & 3 \times 3 = 9 & 3 \times 4 = 12 & 3 \times 5 = 15 & 3 \times \underline{10} = \underline{30} & 3 \times \underline{100} = \underline{300}
\end{array}
\]

Have a volunteer fill in the blanks. (10, 30; 100, 300) Have students check the answers by doing the addition and the multiplication.

**Exercises:** Add the numbers and then multiply the middle number by 3. Did you get the same answer?
a) $16 + 17 + 18$  
b) $29 + 30 + 31$  
c) $42 + 43 + 44$  

**Bonus:** $999 + 1000 + 1001$

**Answers:** a) 51, 51, yes; b) 90, 90, yes; c) 129, 129, yes; Bonus: 3000, 3000, yes

**Using models to understand the pattern.** Have students use 12 counters to again show $3 + 4 + 5$. Challenge them to rearrange the counters to show $3 \times 4$ by moving only one counter. (move 1 counter from the group of 5 to the group of 3; this makes 3 groups of 4) When all students have found the answer, demonstrate it on the board as shown below:
Repeat with $2 + 3 + 4$ and have students change it to $3 \times 3$ by moving only one counter. (move 1 counter from the group of 4 to the group of 2)

**Exercises:** Draw a picture to show why the sum equals the product.

a) $5 + 6 + 7 = 3 \times 6$  

b) $7 + 8 + 9 = 3 \times 8$

**Answers:** a) moving a dot from the group of 7 to the group of 5 makes three groups of 6,  
b) moving a dot from the group of 9 to the group of 7 makes three groups of 8

SAY: When you can see a pattern and understand the reasons for it, you can sometimes see many other patterns.

**Exercises:** Draw a picture or use counters to show the addition. Then ...

a) move two dots to show that $2 + 4 + 6 = 3 \times 4$.  
b) move three dots to show that $2 + 5 + 8 = 3 \times 5$.  
c) move three dots to show that $4 + 5 + 6 + 7 + 8 = 5 \times 6$.  

**Bonus:** move six dots to show that $1 + 2 + 3 + 4 + 5 + 6 + 7 = 7 \times 4$.

**Answers:**  
a) moving two dots from the group of 6 to the group of 2 makes three groups of 4  
b) moving three dots from the group of 8 to the group of 2 makes three groups of 5  
c) moving one dot from the group of 7 to the group of 5 and moving two dots from the group of 8 to the group of 4 makes five groups of 6  

**Using area models to discover patterns.** Write on the board:

\begin{align*}
1 &= \_\_\_ \\
1 + 2 &= \_\_\_ \\
1 + 2 + 3 &= \_\_\_ \\
1 + 2 + 3 + 4 &= \_\_\_ \\
1 + 2 + 3 + 4 + 5 &= \_\_\_ \\
\end{align*}

Fill in the blanks as volunteers tell you the sums. (1, 3, 6, 10, 15) SAY: The gaps increase because that’s how we made the sequence, but I want to know if there is a way to get an expression that will help me find any term. One way to think of the sums is as an area.

Project a transparency of grid paper or **BLM 1 cm Grid Paper** onto the board and draw the following shape on the board:
SAY: Let’s count the squares inside the shape to find the area. ASK: How many squares are in the first row? (1) In the second row? (2) Third row? (4) Fourth row? (5) SAY: So, we can add all these together to find the total. Write on the board:

\[ \text{Area} = 1 + 2 + 3 + 4 + 5 = 15 \text{ square units} \]

**Exercises:**

1. Write the area as a sum by adding the number of squares in each row.

   a) [Diagram]
   
   b) [Diagram]
   
   c) [Diagram]

   **Answers:** a) 3 + 4 + 5, b) 1 + 2 + 3 + 4, c) 3 + 5 + 7

**NOTE:** Provide students with grid paper or BLM 1 cm Grid Paper for the following exercises.

2. Draw an area model for the expression.

   a) 2 + 3 + 4
   
   b) 4 + 5 + 6 + 7
   
   c) 2 + 5 + 8

   **Answers:**

   a) [Diagram]
   
   b) [Diagram]
   
   c) [Diagram]

Have students draw two identical shapes like the one on the board on grid paper for 1 + 2 + 3 + 4 + 5 and then cut them out. Challenge students to arrange the shapes to make a rectangle. Then ask them to determine the area of each shape.

When students are finished, draw on the board:

\[ (1 + 2 + 3 + 4 + 5) \times 2 = 5 \times 6 = 30 \]

So, \( 1 + 2 + 3 + 4 + 5 = 15 \)
SAY: Remember that you do the expression inside the brackets first, so you add to find the area of one shape, and then you multiply by 2 to get the area. ASK: How did I know to multiply $5 \times 6$ to get the area? (the area of a rectangle is length times width)

**Exercises:** Use two copies of a shape to make a rectangle. Then write the multiplication.

a) $(1 + 2 + 3) \times 2 = \text{____} \times \text{____}$  
b) $(3 + 5 + 7) \times 2 = \text{____} \times \text{____}$  
c) $(1 + 2 + 3 + 4) \times 2 = \text{____} \times \text{____}$  
d) $(2 + 5 + 8) \times 2 = \text{____} \times \text{____}$

**Bonus:** Which two questions have the same answer? Why does that make sense?

**Answers:** a) $3 \times 4$; b) $3 \times 10$; c) $4 \times 5$; d) $3 \times 10$; Bonus: parts b) and d), $3 + 5 + 7 = 2 + 5 + 8$, so multiplying both by 2 gets the same answer

**Using layers instead of rows in an area model.** Draw on the board:

![Area Model Diagram]

1 + 3 + 5 + 7 + 9

ASK: How does the picture show $1 + 3 + 5 + 7 + 9$? (the layers have 1, 3, 5, 7, and 9 squares)  
PROMPT: Where do you see the numbers 1, 3, 5, 7, and 9 in the picture? ASK: How does the picture show $5 \times 5$? (there are 5 rows of 5 squares) SAY: When you have two ways to show the number represented, you can write an equation. Continue writing on the board:

$$1 + 3 + 5 + 7 + 9 = 5 \times 5$$

**Exercises:**
1. Draw a picture using layers to write the sum as a product.
   a) $1 + 3$
   b) $1 + 3 + 5$
   c) $1 + 3 + 5 + 7$
   d) $1 + 3 + 5 + 7 + 9 + 11$

**Bonus:** Draw a picture to show that $2 + 4 + 6 + 8 + 10 = 5 \times 6$.

**Selected solution:** d) $6 \times 6$

**Answers:** a) $2 \times 2$, b) $3 \times 3$, c) $4 \times 4$

2. Draw a picture using rows instead of layers to show that $(1 + 3 + 5 + 7) \times 2 = 4 \times 8$. Does this match the answer you got in Exercise 1.c)? How do you know?

**Selected answer:** Yes, because $4 \times 8 = 4 \times 4 \times 2$. 
**Problem Bank**

1. a) Use grid paper to make a rectangle from two copies of a shape with area $8 + 9 + 10 + 11 + 12$.
   b) What is the area of the rectangle?
   c) What is the area of the original shape?
   d) What rectangle has the same area as two shapes with area $1 + 2 + 3 + \ldots + 7$?
   e) What rectangle has the same area as two shapes with area $1 + 2 + 3 + \ldots + 12$?
   f) How can you get your answer to part e) from your answers to parts b) and d)? Write an equation that shows this equality.
   g) Use your equation in part f) to mentally calculate $12 \times 13$.

   **Answers:**
   
   a) 100 square units
   
   b) 50 square units
   
   c) 7 × 8
   
   d) 12 × 13
   
   f) $(1 + 2 + 3 + 4 + 5 + 6 + 7) + (8 + 9 + 10 + 11 + 12) = 1 + 2 + 3 + \ldots + 12$, so $12 \times 13 = (7 \times 8) + (5 \times 20)$
   
   g) 156

2. a) Fill in the blanks.
   i) $1 + 2 + 3 = (\_ \_ \times \_ \_ \_) + 2$
   ii) $1 + 2 + 3 + 4 = (\_ \_ \times \_ \_ \_) + 2$
   iii) $1 + 2 + 3 + 4 + 5 = (\_ \_ \times \_ \_ \_) + 2$
   iv) $1 + 2 + 3 + 4 + 5 + 6 = (\_ \_ \times \_ \_ \_) + 2$
   v) $1 + 2 + 3 + 4 + 5 + 6 + 7 = (\_ \_ \times \_ \_ \_) + 2$

   b) Look for a pattern in your answers to part a). Then predict: $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$.

   **Answers:**
   
   a) i) 3, 4; ii) 4, 5; iii) 5, 6; iv) 6, 7; v) 7, 8
   
   b) $(9 \times 10) + 2 = 45$

3. a) Draw a picture with layers to show that $2 + 4 + 6 + 8 + 10 = 5 \times 6$.
   b) Draw a picture with rows to show that $2 + 4 + 6 + 8 + 10 = 5 \times 6$.
   c) Draw a picture with rows to show that $2 \times (1 + 2 + 3 + 4 + 5) = 5 \times 6$.
   d) Explain how you could have predicted that $2 + 4 + 6 + 8 + 10 = 2 \times (1 + 2 + 3 + 4 + 5)$.
   e) Calculate $1 + 2 + 3 + 4 + 5$ using $5 \times 6 = 30$. 

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Teacher's Guide for Grade 4 — Problem-Solving Lessons
Answers:
 
\[ a) \]
\[ \begin{array}{c}
\text{a) } \\
\text{b) } \\
\text{c) } \\
\text{d) When you double each term, you double the total} \\
\text{e) } 30 \div 2 = 15
\end{array} \]

4. Draw a picture to show that \(2 + 4 + 6 + 8 + 10 + 12 = 6 \times 7\).

Answer:

5. a) Draw a picture for the sum and move one dot to show that the sum is equal to the product.
   \[ i) \ 3 + 5 = 2 \times 4 \quad ii) \ 5 + 7 = 2 \times 6 \quad iii) \ 6 + 8 = 2 \times 7 \]
   b) Draw a picture for the sum and move two dots to show that the sum is equal to the product.
   \[ i) \ 1 + 5 = 2 \times 3 \quad ii) \ 2 + 6 = 2 \times 4 \quad iii) \ 3 + 7 = 2 \times 5 \]
   c) Use a number line to show that the number being doubled in part b) is halfway between the two numbers being added.
   d) Use a number line to find the number halfway between the two numbers. Then check that double that number is the sum of the two numbers.
   \[ i) \ 1 \text{ and } 7 \quad ii) \ 2 \text{ and } 7 \quad iii) \ 11 \text{ and } 16 \]

Selected answers:
 
\[ a) \]
\[ \begin{array}{c}
\text{a) } \\
\text{b) } \\
\text{d) } \\
\text{d) } iii)
\end{array} \]

\[ \begin{array}{c}
\text{13.5} \\
\text{13.5 + 13.5 = 27 and 11 + 16 = 27}
\end{array} \]
6. Draw a picture to show that $10 + 11 + 12 + 13$ is 4 more than $9 + 10 + 11 + 12$.

Answer:

\[
\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

7. If $33 + 158$ is 191, what is $34 + 159$?

Answer: 193

8. If $375 + 406 = 781$, what is $378 + 409$?

Answer: 787

9. If $74 + 75 + 76 + 77$ is 302, what is $75 + 76 + 77 + 78$? How do you know?

Answer: 306, because if you draw rows of 74, 75, 76, and 77, and then add 1 to each of the four rows, you get rows of 75, 76, 77, and 78.
1 cm Grid Paper
PS4-8 Using a Diagram

Teach this lesson after: 4.2 Measurement

Goals:
Students will create number lines to solve problems involving multiplication and division of numbers up to 10 000 by one-digit numbers.

Prior Knowledge Required:
Can apply the distributive property of multiplication
Can multiply up to 10 × 10
Can multiply whole numbers by 10, 100, and 1000
Can interpret products in terms of repeated addition
Can multiply one-digit whole numbers by multiples of 10 up to 90
Can add and subtract multi-digit numbers
Understands division as a missing factor problem
Knows that expressions in brackets are done first
Can find the perimeter of a rectangle given its side lengths (for Extended Problem)
Can multiply two-digit numbers by multiples of 10 up to 90 (for Extended Problem)
Can convert metres to centimetres (for Extended Problem)

Vocabulary: divide, division, multiplication, multiply, number line, product, sum

Materials:
calculators (optional, see Problem Bank 3)
BLM Posters (pp. 20–21, see Extended Problem)

Review the distributive property. Write on the board:

\[
\begin{array}{c}
\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \\
3 + 3 + 3 + 3 + 3 + 3 + 3
\end{array}
\]

\[
5 + 2 = 7
\]

\[
(5 \times 3) + (2 \times 3) = 7 \times 3
\]

SAY: Just like five plus two is seven, five 3s plus two 3s is seven 3s.

Exercises: Write 12 × 8 as a sum of smaller products.

a) \[
12 \times 8 = \underbrace{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8}_{6 + 6}
\]

\[
= \underline{_______} + \underline{_______}
\]

b) \[
12 \times 8 = \underbrace{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8}_{10 + 2}
\]

\[
= \underline{_______} + \underline{_______}
\]

c) \[
12 \times 8 = \underbrace{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8}_{12 + 0}
\]

\[
= \underline{_______} + \underline{_______}
\]
d) \(12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8\)
   \[= \underline{\ldots} + \underline{\ldots}\]

e) \(12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8\)
   \[= \underline{\ldots} + \underline{\ldots}\]

f) \(12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8\)
   \[= \underline{\ldots} + \underline{\ldots}\]

Answers: a) \((11 \times 8) + (1 \times 8)\), b) \((10 \times 8) + (2 \times 8)\), c) \((9 \times 8) + (3 \times 8)\), d) \((8 \times 8) + (4 \times 8)\),
     e) \((7 \times 8) + (5 \times 8)\), f) \((6 \times 8) + (6 \times 8)\)

ASK: Which way of separating 12 eights into two smaller numbers of eights would be easiest to use to calculate \(12 \times 8\) mentally? (split 12 into 10 and 2) SAY: Multiplying by 10 is easy to do, and the result is easy to add. Write on the board:

\[
10 \times 8 = 80 \quad 2 \times 8 = 16 \quad \text{so} \quad 12 \times 8 = 80 + 16 = 96
\]

SAY: You would get the same answer using any other way, but using the 10 times table is easiest.

**Showing the distributive property on a number line.** Draw on the board:

\[
\begin{array}{c}
0 & 10 \times 8 & + & 2 \times 8 & = 12 \times 8 \\
\hline
80 & & & 16 & = 96
\end{array}
\]

SAY: You can keep track of the multiplication in parts by sketching a number line. This isn’t a precise number line because I didn’t try to make the numbers the correct distance apart. But the sketch is good enough to help us keep track of the numbers that we are adding. You can use this method to help you multiply.

Draw on the board:

\[
\begin{array}{c}
0 & 10 \times 7 & + & \underline{\ldots} \times 7 & = 14 \times 7 \\
\hline
70 & & & & = \\
\hline
\end{array}
\]

SAY: For the multiplication \(14 \times 7\), I started with 10 sevens, because that’s easy to multiply. ASK: How many more sevens do I need? (4) PROMPT: I need 14 sevens altogether. Write “4” in the bottom blank. ASK: What is \(4 \times 7\)? (28) Write “28” in the blank above “\(4 \times 7\).” ASK: So, what is \(14 \times 7\)? (98) How do you know? (70 + 28 = 98) Write “98” in the final blank.
Exercises:
1. Use the diagram to multiply.
   a) $13 \times 6$
   \[
   \begin{array}{c}
   60 \\
   0 \\
   10 \times 6 \\
   + ____ \times 6 \\
   \hline
   = 13 \times 6
   \end{array}
   \]
   b) $14 \times 8$
   \[
   \begin{array}{c}
   80 \\
   0 \\
   10 \times 8 \\
   + ____ \times 8 \\
   \hline
   = 14 \times 8
   \end{array}
   \]
   c) $17 \times 8$
   \[
   \begin{array}{c}
   80 \\
   0 \\
   10 \times 8 \\
   + ____ \times 8 \\
   \hline
   = 17 \times 8
   \end{array}
   \]
   d) $16 \times 7$
   \[
   \begin{array}{c}
   70 \\
   0 \\
   10 \times 7 \\
   + ____ \times 7 \\
   \hline
   = 16 \times 7
   \end{array}
   \]
   Answers: a) $3 \times 6 = 18$, so $13 \times 6 = 78$; b) $4 \times 8 = 32$, so $14 \times 8 = 112$; c) $7 \times 8 = 56$, so $17 \times 8 = 136$; d) $6 \times 7 = 42$, so $16 \times 7 = 112$

2. Fill in the blanks to multiply.
   a) $13 \times 7 = (10 \times 7) + (____ \times 7)$
      \[
      = ____ + ____
      = ____
      \]
   b) $18 \times 4 = (10 \times 4) + (____ \times 4)$
      \[
      = ____ + ____
      = ____
      \]
   c) $16 \times 9 = (10 \times 9) + (____ \times 9)$
      \[
      = ____ + ____
      = ____
      \]
   Answers: a) 3, 70 + 21, 91; b) 8, 40 + 32, 72; c) 6, 90 + 54, 144

Review multiplying tens. SAY: Remember, if you can multiply $7 \times 3$, then you can multiply $70 \times 3$; the answer is just 10 times as much. Write on the board:

   $7 \times 3 = 21$, so $70 \times 3 = 210$

Exercises: Multiply.
   a) $60 \times 4$  b) $60 \times 8$  c) $90 \times 5$  d) $50 \times 4$
   Bonus: Multiply in order: $8 \times 4, 80 \times 4, 800 \times 4, 800 \times 40$
   Answers: a) 240; b) 480; c) 450; d) 200; Bonus: 32, 320, 3200, 32 000
Multiplying two-digit numbers by one-digit numbers using a number line. Write on the board:

\[ 78 \times 3 \]

SAY: You can calculate \( 78 \times 3 \) by multiplying parts of \( 78 \times 3 \) separately. Let’s split 78 into 70 + 8. Draw on the board:

\[ 0 \quad 70 \times 3 \quad 8 \times 3 \quad = \quad 78 \times 3 \]

ASK: What is \( 70 \times 3 \)? (210) Write “210” above “70 \times 3” on the number line. ASK: What is \( 8 \times 3 \)? (24) Write “+ 24 =” above “8 \times 3” on the number line, as shown below:

\[ \begin{array}{c}
0 \\
210 \\
70 \times 3 \\
8 \times 3 \\
= \quad 78 \times 3
\end{array} \]

ASK: So, what is \( 78 \times 3 \)? (234) Write “234” above “78 \times 3” on the number line. SAY: Keeping track of the numbers you’re adding on a number line picture means you don’t have to remember them mentally, and that makes adding the numbers easier.

Exercises: Sketch a number line to help you multiply.
a) 64 × 3  
b) 36 × 5  
c) 87 × 2  
d) 48 × 7

Selected solution:
a)                 180                       +     12    = 192
\[ 0 \quad 60 \times 3 \quad 4 \times 3 \quad = \quad 64 \times 3 \]

Answers: b) (30 × 5) + (6 × 5) = 150 + 30 = 180, c) 7 × 2 = (80 × 2) + (7 × 2) = 160 + 14 = 174, d) (40 × 7) + (8 × 7) = 280 + 56 = 336

Introduce the distributive property for division. SAY: Because multiplication and division are related, you can use the same idea to make dividing easier too. Draw on the board:

\[ 26 \div 2 \]

\[ 0 \quad 10 \times 2 \quad + \quad ____ \times 2 \quad = \quad 26 \]

SAY: I want to divide 26 by 2, so I need to find out how many twos I need to make 26. Point to the “___ × 2” under the 26 and SAY: I need to find out what times two is 26. I already have 20 from 10 × 2. ASK: How much more is 26 than 20? (6) Write “6” in the top blank. ASK: How many twos are in six? (3) Write “3” in the blank below “6.” SAY: We needed 10 twos and 3 more twos to make 26. ASK: How many twos did we need altogether? (13) Write “13” in the last blank.
The final picture should look like this:

\[ 26 \div 2 \]

\[ \begin{array}{c}
20 \\
10 \times 2 \\
+ \ 6 \\
= 26
\end{array} \]

\[ \begin{array}{c}
10 \times 2 \\
+ \ 3 \times 2 \\
= 13 \times 2
\end{array} \]

Then, underneath the picture, write on the board:

\[ 13 \times 2 = 26, \text{ so } 26 \div 2 = 13 \]

**Exercises:** Fill in the blanks and then use the number line to divide.

a) \( 24 \div 2 \)

\[ \begin{array}{c}
20 \\
10 \times 2 \\
+ \ ____ \\
= 24
\end{array} \]

\[ \begin{array}{c}
10 \times 2 \\
+ \ ____ \times 2 \\
= ____ \times 2
\end{array} \]

b) \( 28 \div 2 \)

\[ \begin{array}{c}
20 \\
10 \times 2 \\
+ \ ____ \\
= 28
\end{array} \]

\[ \begin{array}{c}
10 \times 2 \\
+ \ ____ \times 2 \\
= ____ \times 2
\end{array} \]

c) \( 48 \div 4 \)

\[ \begin{array}{c}
40 \\
10 \times 4 \\
+ \ ____ \\
= 48
\end{array} \]

\[ \begin{array}{c}
10 \times 4 \\
+ \ ____ \times 4 \\
= ____ \times 4
\end{array} \]

d) \( 39 \div 3 \)

\[ \begin{array}{c}
30 \\
10 \times 3 \\
+ \ ____ \\
= 39
\end{array} \]

\[ \begin{array}{c}
10 \times 3 \\
+ \ ____ \times 3 \\
= ____ \times 3
\end{array} \]

**Answers:**
a) 4, 2, 12, so \( 24 \div 2 = 12 \);
b) 8, 4, 14, so \( 28 \div 2 = 14 \);
c) 8, 2, 12, so \( 48 \div 4 = 12 \);
d) 9, 3, 13, so \( 39 \div 3 = 13 \)

*SAY:* You can divide bigger numbers this way, too. You will just have more to keep track of on the number line. Draw on the board:

\[ 76 \div 2 \]

\[ \begin{array}{c}
70 \\
10 \times 2 \\
+ \ 6 \\
= 76
\end{array} \]

*SAY:* When you’re dividing by 2, you can count by 20s until you get close to the number you’re dividing, in this case 76. You count by 20s because that’s ten times two.
Show the skip counting on the board:

\[
76 \div 2
\]

\[\begin{array}{cccc}
0 & 20 & 40 & 60 \\
\end{array}\]

SAY: Adding another 20 would be more than 76, so we can stop skip counting by 20s at 60.
ASK: From 60, how much more do we need to get 76? (16) 16 is what times 2? (8) Continue writing on the board:

\[
76 \div 2
\]

\[
\begin{array}{cccc}
10 \times 2 & 10 \times 2 & 10 \times 2 & 8 \times 2 = 16 \\
0 & 20 & 40 & 60 & 76 \\
\end{array}
\]

So \(\_\times 2 = 76\) and \(76 \div 2 = \_
\)

SAY: So, now it’s just a matter of totalling what we multiplied by. Circle the three 10s and the 8 above the number line. SAY: We added 10 twos, then 10 more, then 10 more, then 8 more.
ASK: So, how many twos did we add altogether? (38) Write “38” in the first blank. ASK: So, what is \(76 \div 2\)? (38) Write “38” in the second blank.

**Exercises:** Show counting by 20s, then by 2s, on a number line to divide.

a) 52 ÷ 2  
   b) 74 ÷ 2  
   c) 68 ÷ 2  
   d) 90 ÷ 2

**Answers:** a) 26, b) 37, c) 34, d) 45

Write on the board:

\[
285 \div 5
\]

ASK: What can you start skip counting by to divide by 5? (50s) Why? (50 is 10 × 5) Show the skip counting by 50s on a number line, as shown below:

\[\begin{array}{cccc}
0 & 50 & 100 & 150 & 200 & 250 \\
\end{array}\]

SAY: Skip counting one more time would pass 285, so we can stop skip counting at 250.
ASK: From 250, how much more do we need to get to 285? (35)
Continue writing on the board:

<table>
<thead>
<tr>
<th>10 × 5</th>
<th>10 × 5</th>
<th>10 × 5</th>
<th>10 × 5</th>
<th>10 × 5</th>
<th>7 × 5 = 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

So _____ × 5 = 285 and 285 ÷ 5 = _____

Circle the five 10s and the 7 above the number line. SAY: There are five 10s and a 7.

ASK: What number did we multiply by five altogether? (57) So, what is 285 ÷ 5? (57) Write “57” in both blanks. SAY: Counting by 50s allows you to count by 10s when you find the answer.

When you count by 10s to find the answer, the last product will only require multiplying by one-digit numbers. ASK: If you are dividing by 4, what would you count by on the number line? (40) If you are dividing by 7, what would you count by? (70) If you are dividing by 8, what would you count by? (80)

**Exercises:** Sketch a number line to divide. Check your answer using long division.

a) 190 ÷ 5  b) 136 ÷ 4  c) 364 ÷ 7

**Answers:** a) 38, b) 34, c) 52

**Problem Bank**

1. Find 678 ÷ 2 by counting by 200s and then by 20s. Sketch a number line to show your skip counting.

**Solution:**

<table>
<thead>
<tr>
<th>100 × 2</th>
<th>100 × 2</th>
<th>100 × 2</th>
<th>10 × 2</th>
<th>10 × 2</th>
<th>10 × 2</th>
<th>9 × 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>620</td>
<td>640</td>
<td>660</td>
</tr>
</tbody>
</table>

339 × 2 = 678, so 678 ÷ 2 = 339

2. a) How does 99 × 2 compare to 100 × 2? Explain.
b) Use 100 × 2 to calculate 99 × 2.
c) Use 100 × 3 to calculate 99 × 3.
d) Use 100 × 17 to calculate 99 × 17.
e) Use 1000 × 2 to calculate 999 × 2.
f) Use 1000 × 3 to calculate 999 × 3.
g) Use 1000 × 49 to calculate 999 × 49.

**Answers:** a) 99 × 2 is one fewer 2 than 100 × 2, so it is 2 less than 100 × 2; b) 200 − 2 = 198; c) 300 − 3 = 297; d) 1700 − 17 = 1683; e) 2000 − 2 = 1998; f) 3000 − 3 = 2997; g) 49 000 − 49 = 48 951

3. a) Calculate each product.

9 × 7 = _____
99 × 7 = _____
999 × 7 = _____
9999 × 7 = _____
b) Use the pattern in part a) to calculate 999 999 999 × 7.
c) Check your answer to part b) by doing the multiplication. You may use a calculator.
4. Multiply in order.
a) $7 \times 10, 7 \times 13, 7 \times 130, 7 \times 131$
b) $8 \times 10, 8 \times 20, 8 \times 30, 8 \times 32, 8 \times 320, 8 \times 321$

**Answers:**
a) 70, 91, 910, 917; b) 80, 160, 240, 256, 2560, 2568

5. a) Multiply $4126 \times 3$ using a number line.

\[
\begin{align*}
4000 \times 3 & + 100 \times 3 + 20 \times 3 + 6 \times 3 = \_\_\_\_\_\_ \\
\end{align*}
\]

b) Sketch a number line to multiply.

i) $852 \times 7$
ii) $613 \times 9$
iii) $4444 \times 4$

**Bonus:**

$312403 \times 2$

**Selected solution:**

b) i) $5600 + 350 + 14 = 5964$

\[
\begin{align*}
0 + 800 \times 7 + 50 \times 7 + 2 \times 7 = 852 \times 7 \\
\end{align*}
\]

**Answers:**

a) 12378; b) ii) 5517, iii) 17776; Bonus: 624806

6. Find the missing number two ways. Make sure you get the same answer both ways.
a) $? \times 6 = 192$

\[
\begin{align*}
10 \times 6 & + 10 \times 6 + 10 \times 6 + 2 \times 6 = ? \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ = \_\_\_\_\_\_ \\
0 & + 60 + 120 + 180 + 192 \\
\end{align*}
\]

\[
\begin{align*}
40 \times 6 = 240 & + 3 \times 6 + 5 \times 6 = ? \_\_\_\_\_\_ - \_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_ \\
0 & + 192 + 210 + 240 \\
\end{align*}
\]

b) $? \times 8 = 312$

\[
\begin{align*}
30 \times 8 & + 9 \times 8 = ? \_\_\_\_\_\_ + \_\_\_\_\_\_ = \_\_\_\_\_\_ \\
0 & + 240 + 312 \\
\end{align*}
\]

\[
\begin{align*}
40 \times 8 & + 1 \times 8 = ? \_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_ \\
0 & + 312 + 320 \\
\end{align*}
\]

**c) Use your answers to parts a) and b) to divide.**

i) $192 + 6$

ii) $312 + 8$

d) Sketch two different number lines to divide $294 \div 6$. Make sure you get the same answer both ways.

**Answers:**

a) $10 + 10 + 10 + 2 = 32$ or $40 - 5 - 3 = 32$; b) $30 + 9 = 39$ or $40 - 1 = 39$

c) i) 32, ii) 39; d) 49
Extended Problem: Posters

Materials:
BLM Posters (pp. 20–21)

Preparation for the extended problem. Tell students that the extended problem involves the following situation: students are making posters for parents to come and see in a poster show. Each student makes a poster on a large sheet of paper. Draw on the board:

![Diagram of a 20 cm by 27 cm poster]

Have students show with their hands about how wide and how tall the posters are. Attach some regular sheets of paper to the board, with some really close together and others really far apart. ASK: Is this a good way to display the posters? (no) Why not? (for example, it doesn’t look very good) Tell students that the extended problem is partly about decorating posters to make them look good and partly about how to display them to make them look good. Tell students that one of the things they will need to remember how to do is to convert metres to centimetres. ASK: How many centimetres are in 1 metre? (100) In 2 metres? (200) 3 metres? (300) SAY: You can always multiply by 100 to get the number of centimetres if you know the number of metres.

Extended Problem: Posters. Give students BLM Posters. Question 7 is an opportunity to apply the strategy of using a diagram in an unfamiliar context.

Answers: 1. 94 cm; 2. 2910 cm; 3. yes, because $100 \times 30 = 3000$ cm; 4. 14 cm; 5. from one edge of one poster to the next, 8 cm; 6. 7; 7. 9 cm (the poster itself is 6 cm from the door, so the pinhole is 9 cm from the door)
Posters (1)

Your class is holding a poster show for parents to come and see some of your class’s artwork. The posters will be hung in the school all along the hallway. Each poster is 20 cm wide by 27 cm tall. Each student can decorate the edges of their poster with yarn.

1. How much yarn do you need to decorate the edges of one poster?

2. Each student takes an extra 3 cm of yarn, to make sure they have enough. How much yarn, in centimetres, will a class of 30 students need?

3. A ball of yarn has 30 m of yarn. Is that enough for the whole class?
Posters (2)

4. You put two pins near the top of the poster, 3 cm from each edge. How far apart are the pins?

5. You decide to place all the pinholes the same distance apart. How far apart will the posters be?

6. Doors along the hallway are 2 m apart. How many posters can you put between each pair of doors?

7. You decide to centre the posters so that the posters closest to the doors are the same distance from the door. How far from the door should you make the first pinhole?
PS4-9  Making a Simpler Problem

Teach this lesson after: 4.2 Measurement

Goal:
Students will, when given a problem, make a simpler problem and use the solution to the simpler problem to help solve the original problem.

Prior Knowledge Required:
Can add and subtract numbers within 10 000
Can use long division to divide two-digit numbers by one-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can add and subtract decimal tenths
Can multiply one-digit numbers by two-digit numbers (for Extended Problem)
Can find the area of a rectangle given its side lengths (for Extended Problem)
Can interpret the remainder in division (for Extended Problem)
Can use long division to divide three-digit numbers by one-digit numbers (for Extended Problem)

Vocabulary: centimetre, horizontal, metre, perimeter, vertical

Materials:
2 sticks of different colours and lengths
chalk of two different colours
scissors and BLM Fraction Strips and Circles (p. 33, see Problem Bank 16)
BLM Flower Garden (pp. 36–37, see Extended Problem)

Using off-by-one patterns to solve problems. Tell students that you are waiting in line to get on a rollercoaster ride. You are 37th in line and you see your friend who is 7th in line. ASK: How many people are between my friend and me? Note various guesses. Most students will likely guess 37 – 7 = 30. If they do, SAY: That answer is close, but not exactly correct. Let’s draw a simple picture using smaller numbers to see what is going on. Write on the board:

Front of line: ● ● ● ● ● ● ● ● ●

SAY: Each dot represents a person. ASK: How many dots did I draw? (9) SAY: For this simpler problem, suppose I am 9th in line. Circle the last dot. SAY: My friend is 5th in line. Circle the 5th dot. Label the dots, as shown below:

Front of line: ● ● ● ● ● 5th ● ● ● 9th

ASK: How many people are between the 5th and 9th person? (3) Is that equal to 9 – 5? (no) SAY: It is close, but not quite equal.
Exercises: Draw a picture to decide how many people are between the given positions.

a) the 7th and 8th person  
 b) the 7th and 9th person  
 c) the 7th and 10th person  
 d) the 7th and 11th person  
 e) the 7th and 12th person  
 f) the 7th and 37th person  

Answers: a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the correct answer? (no) Did it give close to the correct answer? (yes) How can you get the number of people between two people given their positions in line? (subtract the smaller position from the other and then subtract 1 from the difference)

How many people are between the 37th person and the 7th person in the rollercoaster line? (29, because \(37 - 7 = 30\), \(30 - 1 = 29\)) How did starting with smaller numbers help? (answers will vary) SAY: Sometimes, it is easier to start by using smaller numbers than the numbers given in the problem. Then you will see patterns and learn how to solve the harder problem. Now that you know the pattern for finding the number of people between any two positions, you can use that method with any numbers.

Exercises: How many people are in line between the given positions?

a) the 8th and 78th person  
 b) the 314th and 1000th person  
 c) the 492nd and 613th person  

Answers: a) 69, b) 685, c) 120

Making the problem easier by finding what is relevant.  SAY: Sometimes making the problem easier has nothing to do with using smaller numbers and finding a pattern. Sometimes all you need to do is eliminate information that’s not relevant, and moving objects around can help with that.

Affix two pre-made sticks of different colours and different lengths to the board, end to end. Label one length and their combined length. The following is an example for 8 cm and 12 cm, but your sticks can be other lengths:

```
          8
          20
      ?
```

Tell students that all the measurements are in centimetres. ASK: How long is the second stick? (12 cm) SAY: It is easy to see with sticks, but now I’m going to move these sticks around. Slide the grey stick down and draw the lines around it, as shown below:

```
          8
          20
      ?
```
ASK: How did I move the sticks? (you slid one of them down) SAY: This now looks like a problem to do with shapes and the lengths of missing sides. There’s a lot of extra information in this second problem compared with the first problem, so it looks harder, but it actually has exactly the same answer as the other one. The total length of the two sticks is still 20 centimetres—I just slid one of the sticks down so that they are not side by side anymore.

**Exercises:** Find what the ? stands for by pretending the sticks are side by side.

a) ![Diagram](image1)

b) ![Diagram](image2)

c) ![Diagram](image3)

d) ![Diagram](image4)

e) ![Diagram](image5)

f) ![Diagram](image6)

**Answers:** a) 7, b) 9, c) 1240, d) 3368, e) 17, f) 13

SAY: By pretending that the sticks were side by side, you turned the problem into an easier problem.
Making the problem easier by emphasizing what is relevant. SAY: We can look at a problem and focus on what matters most. For example, if you need to find a vertical side—straight up and down—then colour along all the vertical lines. If you need to find a horizontal side—side to side—then colour along all the horizontal lines.

Exercises: Find what the ? stands for by making the problem into an easier problem.

a) \[ \text{Find what the } ? \text{ stands for.} \]

b) \[ \text{Find what the } ? \text{ stands for.} \]

c) \[ \text{Find what the } ? \text{ stands for.} \]

d) \[ \text{Find what the } ? \text{ stands for.} \]

Answers: a) colour vertical, ? = 5; b) colour horizontal, ? = 8659; c) colour horizontal, ? = 217; d) colour vertical, ? = 948

Point out to students that by colouring along the horizontal or vertical lines, they changed the problem into an easier problem.

Finding perimeter by finding missing side lengths. Remind students that to find the perimeter of a shape, they have to add up the lengths of all the sides.

Exercises: Find the perimeter by finding missing sides, then adding all the sides.

a) \[ \text{Find the perimeter.} \]

b) \[ \text{Find the perimeter.} \]
Answers: a) 3830, b) 44.4

Finding perimeter without knowing all the side lengths. Draw on the board:

```
  2
  1
  6
```

SAY: All measurements are in centimetres. Point to the side on the right and ASK: What is this length? (3 cm) Point to the bottom two horizontal sides and ASK: What might these be? (1 and 5, 2 and 4, or 3 and 3) SAY: We don’t know for sure what the missing horizontal sides are, but let’s find the perimeter using different possibilities.

Exercises: All lengths are in centimetres. Find the perimeter.

a) 
```
  2
  1
  1
  5
```

b) 
```
  2
  2
  1
  3
```

c) 
```
  2
  3
  1
  3
```

Answers: a) 18 cm, b) 18 cm, c) 18 cm

ASK: Did using different possibilities change the answer to the perimeter? (no) Why not? (because the two bottom sides always add to 6, so it didn’t change the total) SAY: The bottom numbers always add to 6 because they have to add to the same as the top side. So, the total perimeter didn’t change.

Using different colours for the horizontal and vertical sides, draw on the board:

```
  2
  1
  6
```
SAY: Let's go back to the original picture. There are two kinds of sides in this shape—horizontal and vertical. ASK: How long is the top side? (6 cm) How long are the two bottom sides put together? (6 cm) How do you know? (because if you lined up the two bottom sides together, they would match the top one) How long are the two vertical sides on the left side of the shape? (2 cm and 1 cm) How long is the side on the right? (3 cm) How do you know? (because it’s the same as the two other vertical sides put together) Write on the board:

Horizonal sides add to ____  Vertical sides add to ____

Perimeter is ____ + ____ = ____

Have volunteers fill in the blanks. (12, 6, 12 + 6 = 18)

**Exercises:** All measurements are in centimetres. Find the perimeter.

a) 

b) 

**Answers:** a) 8100 cm, b) 12 040 cm

**Problem Bank**

1. A teacher asks students to read some pages of a book for homework. Write the page numbers down, then count them to find out how many pages the students have to read in total.

a) Read pages 3 to 6.
b) Read pages 5 to 10.
c) Read pages 2 to 7.
d) Read pages 1 to 8.
e) Read pages 6 to 11.

**Answers:**
a) 3, 4, 5, 6; 4 pages
b) 5, 6, 7, 8, 9, 10; 6 pages;
c) 2, 3, 4, 5, 6, 7; 6 pages;
d) 1, 2, 3, 4, 5, 6, 7, 8; 8 pages;
e) 6, 7, 8, 9, 10, 11; 6 pages
2. a) Complete the chart.

<table>
<thead>
<tr>
<th>Pages to Read</th>
<th>How Many Pages</th>
<th>(Largest Page Number) – (Smallest Page Number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) 1, 2, 3, 4, 5</td>
<td>5</td>
<td>5 – 1 = 4</td>
</tr>
<tr>
<td>ii) 2, 3, 4, 5, 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii) 3, 4, 5, 6, 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) 4, 5, 6, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v) 5, 6, 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vi) 5, 6, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vii) 5, 6, 7, 8, 9</td>
<td>5</td>
<td>5 – 1 = 4</td>
</tr>
<tr>
<td>viii) 5, 6, 7, 8, 9, 10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) How can you get the number of pages to read from the result of the subtraction?

**Answers:** a) ii) 5, 6 – 2 = 4; iii) 5, 7 – 3 = 4; iv) 5, 8 – 4 = 4; v) 3, 7 – 5 = 2; vi) 4, 8 – 5 = 3; vii) 5, 9 – 5 = 4; viii) 6, 10 – 5 = 5; b) The number of pages is 1 more than the result of the subtraction.

3. A teacher tells his class to read the given pages in a textbook for homework. How many pages does the class have to read?

a) from 320 to 387  b) from 352 to 386  
c) from 298 to 314  d) from 408 to 451

**Answers:** a) 68, b) 35, c) 17, d) 44

4. What was the last page that Ray read?

a) Ray read 5 pages, starting at page 263.  
b) Ray read 156 pages, starting at page 24.

**Answers:** a) 267, b) 179

5. A teacher tells her class to read from page 354 to 412 but skip pages 363 to 389. How many pages does the class have to read?

**Answer:** 32

6. How many whole numbers are greater than 11 and less than 45?

**Answer:** 33

7. When everyone in Liz’s class stands in line, Liz is 12th in line and 15th from the end of the line. How many people are in Liz’s class?

**Answer:** 26

8. There are 800 people in line. How many people are behind the 12th person?

**Answer:** 788

9. There is a long line-up at a roller coaster. Edmond is 8th in line and Ava is 78th in line. How many people are between Edmond and Ava in the line-up?

**Answer:** 69
10. How long is the fence?
   a) A fence is made using 42 posts, each 1 metre apart.
   b) A fence is made using 34 posts, each 2 metres apart.
   **Answers:** a) 41 m, b) 66 m

11. How many posts are needed to make the fence?
   a) A fence is 38 metres long with posts 1 metre apart.
   b) A fence is 50 metres long with posts 2 metres apart.
   c) A fence is 63 metres long with posts 3 metres apart.
   **Answers:** a) 39, b) 26, c) 22

12. A fence for a square field is made with posts 1 metre apart, including a post at each corner. How many posts are needed for a field that is …
   a) 10 m by 10 m?  Hint: Start with 1 m by 1 m, then move on to 2 m by 2 m, 3 m by 3 m, etc.
   b) 20 m by 20 m?
   **Answers:** a) 40, b) 80

**NOTE:** For the following problems, encourage students to predict each answer before checking.

13. A field is a square 20 m by 20 m. How many posts are needed if the posts are ...
   a) 1 m apart?
   b) 2 m apart?
   c) 4 m apart?
   d) 5 m apart?
   **Bonus:** 40 cm apart?
   **Answers:** a) 80, b) 40, c) 20, d) 16, Bonus: 200

14. In 1993, an artist named Manfred Laber started a piece of public art called The Time Pyramid. Every 10 years, a cube is added to the pyramid. This will continue until 120 cubes are placed. In what year will the artwork be finished?
   **Answer:** 3183

15. The sides of a square are made of 76 dots (like in the sketch below, but with more dots). Each side has the same number of dots. How many dots are on each side?
   ![Square with dots]
   **Answer:** 20
16. Cut out the strips and circles from **BLM Fraction Strips and Circles** (you may cut the line down to the centre of the circles).

   a) Look at the strip of paper that is partly shaded. Sandy thinks that the amount shaded is one fifth. Is she correct? Use folding to check your answer.

   b) Estimate two fifths of the blank strip of paper. Colour to show your estimate. Use folding to check your estimate.

   c) Look at the circle that is partly shaded. Lewis thinks that the amount shaded is one fifth. Is he correct? Use folding to check your answer.

   d) Estimate two fifths of the blank circle. Colour to show your estimate. Use folding to check your estimate.

17. On this crooked path, each line segment is 1 metre long. What is the total length of the path? Look for a quick way to find the answer.

   ![Crooked path diagram]

   **Solution:** 18 vertical metres plus 17 horizontal metres = 35 metres altogether

18. All measurements are in centimetres.

   a) Add the horizontal edges and the vertical edges together to find the perimeter.

   ![Edge lengths diagram]

   b) Is there enough information to find the area of this shape? Explain.

   **Answers:** a) 34 cm; b) no, because to find the area you need the measurements of each part

19. Each shape was made by placing a small square on top of a large square. All measurements are in centimetres.

   a) Find the perimeter of each shape.

   ![Perimeter shapes]

   b) Make a table with headings “Size of Smaller Square,” and “Total Perimeter.” Use the pattern from part a) to solve the problems.

   i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?

   ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?
Answers: a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm

20. a) Convert the measurements in metres to centimetres. Hint: 1 m = 100 cm.
   i) 2 m = ____ cm   ii) 3 m = ____ cm  
   Bonus: 183 m = ____ cm
b) Find the perimeter, in centimetres.
   i) ii)

   \[ \text{Answers: a) i) 200, ii) 300, Bonus: 18000; b) i) 1040 cm, ii) 2050 cm} \]

21. Find the missing length.
   a) b)

   \[ \text{Answers: a) 1 cm, b) 8 cm} \]
Fraction Strips and Circles
Extended Problem: Flower Garden

Materials:
BLM Flower Garden (pp. 36–37)

Extended Problem: Flower Garden. Give students BLM Flower Garden. Question 6 is a good opportunity to apply the problem-solving strategy learned in this lesson. Students who have not had the opportunity to do this lesson may find that question difficult.

Answers: 1. 20 m; 2. $60; 3. $147; 4. 16 bags; 5. 60 + 147 + 16 = 223, so $223; 6. 6 rows of 14 flowers each, or 84 flowers
Flower Garden (1)

Kyle has a flower garden. His flower garden is 3 metres wide by 7 metres long.

1. He wants to put a fence around his garden to keep animals away from his plants. How many metres of fencing does Kyle need?

2. If fencing costs $3 per metre, how much will the fence cost?

3. Kyle needs to add more soil to his garden. The soil will cost $7 for each square metre. How much will it cost to cover his entire garden?
Flower Garden (2)

4. Kyle decides to plant 144 flowers in his garden. Each bag holds 9 flower seeds. How many bags does Kyle need to buy?

5. Flower seeds cost $1 per bag. What is Kyle’s total cost for his flower garden, including fencing, soil, and seeds?

6. After planting his garden this year, Kyle finds new instructions for planting a flower garden:
   - Plant each flower 50 cm apart.
   - Start planting 25 cm from each edge of the garden.
   If he follows these instructions next year and has the same size of garden, what is the greatest number of flowers he can plant?
Tell your students that they have learned a lot about collecting and classifying data. However, to understand what the data they have collected means, they have to analyze it. Today they will begin learning how to analyze data.

Sometimes knowing the highest and lowest values in a set, or group, of data can be helpful. **SAY:** Suppose you know that the temperature next week is predicted to be 15, 14, 12, 17, 11, 10 and 18 degrees. (Write the temperatures on the board.) **ASK:** What is the highest temperature? What is the lowest temperature? Do you need to know all the temperatures to decide if you will need snow pants next week, or is knowing the lowest temperature enough? Do you need all the temperatures to decide if you will need a pair of shorts, or is knowing the highest enough?

Write several unordered sets of numbers on the board and invite volunteers to circle the largest number and underline the smallest one. Explain that the range of a set is the difference between the highest and the lowest numbers. Calculate the range of the temperatures in the above example with students. (Range = highest – lowest = 18 – 10 = 8) Then ask volunteers to find the ranges for the other sets on the board.

**SAY:** Suppose you are packing for a trip and you know that the range of the temperature at your destination is usually about 5 degrees. This means that the temperature doesn’t change very much; it’s nearly the same every day. How can this help you decide what kinds of clothes to pack? (You might need either snow pants or shorts, but not both!) What if the range at your destination is 20 degrees? What does that tell you? What will you pack? (The temperature changes a lot. You will need clothes for very different temperatures.)

In addition to identifying highest and lowest values, and calculating ranges, students need to be able to order numbers from smallest to largest. Ask volunteers to order the numbers in the sets on the board from smallest to largest.

Tell your students that they will sometimes need to find the number in the middle of a set of data. This number is called the **median**. Find the median of the temperatures in the first example, by first ordering the temperatures and then circling the one in the middle:

10   11   12   14   15   17   18

Explain how you determined that 14 is the median. (It is in the middle of the set; there are 3 numbers before it and 3 after it.)

Now write this set of 6 temperatures:

10   11   12   14   15   21
SAY: When the number of terms in a set is even, there are two numbers in the middle and the median is the number halfway between them. The two numbers in the middle of this set are 12 and 14 (circle them) and the number halfway between 12 and 14 is 13. So the median is 13.

ASK: What number is halfway between 2 and 4? Between 5 and 7? 6 and 10? 12 and 16? 13 and 19? (If any students have difficulty finding the number in the middle, ask them to write all the numbers between the two in question and to count in from the sides: 13, 14, 15, 16, 17, 18, 19.)

Ask students to find the median of the sets they ordered previously. Then give them some unordered sets and ask them to find the median. (This would be a good time to do Activity 1, below.) Include sets in which numbers repeat. (SAMPLE DATA VALUES: 5 3 5 5 9 7 11)

Finally, calculate the range above and below the median for the temperatures in the original EXAMPLE:

range above median = highest temperature – median = 18 – 14 = 4
range below median = median – lowest temperature = 14 – 10 = 4

Look back at the range of the whole set of temperatures (8). Point out that the range of the whole set is the sum of the range above the median and the range below the median.

Explain to students that the range above and below the median is a measure of how much the data values are spread out. Use the first example to illustrate how this information can be useful. Let’s say you knew that the range of temperatures below the median was small. That means the lower temperatures don’t vary much—they’re nearly the same and close to the median. It also means that on half of the days, the temperature are at or only a little below the median. (Since, by definition, at least half of the data values are at or below the median.)

Give your students several sets (ordered, then unordered) and ask them to find the median and the range above and below the median. Include sets in which numbers repeat and in which the range above or below the median is 0.

SAMPLE DATA VALUES:

2, 3, 3, 4, 6, 7, 8 2, 2, 2, 2, 6 12, 15, 15, 15

Assessment

Find the median:

a) 2, 4, 6, 10  b) 3, 13, 5, 9, 11, 3, 4, 12

Find the median and the ranges above and below the median:

a) 2, 2, 2, 2, 2, 2, 28  b) 1, 3, 6, 9, 10

ACTIVITY 1

Invite 9 volunteers to be a set of data. Ask them to order themselves from shortest to tallest. Who is the median? Now have one of the volunteers sit down, and ask the class how they can find and represent the median. (Students may suggest making a mark on the board to represent the height halfway between the two middle students, or they may suggest choosing another student in the class whose height is approximately halfway between the two middle students to be the median.)
Part 2
Probability & Data Management

Extensions

1. Find the median and the range below and above the median for each set:
   a) 2, 3, 5, 7, 9, 10
   b) 12, 16, 19, 23, 26, 26, 26

2. Write 3 different sets that have median 9 and range 5.

3. Write 3 different sets that have median 10 with range above the median 0 and range below the median 5.

4. Find the highest and lowest temperatures. Find the range and the median of the temperatures.

<table>
<thead>
<tr>
<th>Days</th>
<th>Temperatures (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fri</td>
<td>10</td>
</tr>
<tr>
<td>Sat</td>
<td>15</td>
</tr>
<tr>
<td>Sun</td>
<td>25</td>
</tr>
<tr>
<td>Mon</td>
<td>10</td>
</tr>
<tr>
<td>Tue</td>
<td>15</td>
</tr>
</tbody>
</table>

Highest Temperatures in the Next Five Days
**Mean**

**SAY:** Suppose 5 people picked apples and wanted to share their apples so that everyone had the same number. They might pass apples to each other until everyone had the same number or they might put all of the apples in a bin and share one at a time. Either way, mathematicians would say that the people took the mean, or the average, of the numbers of apples picked. Today, we’re going to learn about the mean.

Lay out or draw 12 blocks in 4 groups:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Ask volunteers to come up and move blocks from one group to another until all the groups are equal. How many blocks are now in each group? (3) This is the mean of the set.

Demonstrate another way of finding the mean using the 12 blocks: Draw a circle for each group, put all of the blocks in a pile, and ask volunteers to distribute the blocks evenly into the 4 circles. Do a few more examples together using different groups and numbers of groups. **(SAMPLE PROBLEMS: 8, 5, 2; 7, 2, 5, 2; 1, 3, 9, 3)**

**ASK:** How could we use division to find the mean of a set of blocks? Students should see that they can count the total number of blocks and divide by the number of groups. Demonstrate using the first example, above:

\[
\text{mean} = \frac{\text{total}}{\text{number of groups}}
\]

\[
= \frac{12}{4}
\]

\[
= 3
\]

Write several sets of numbers on the board and have students practice finding the mean without using blocks. **(SAMPLE PROBLEMS: 4, 7, 2, 1, 1; 10, 12, 3, 7)** Invite volunteers to share their answers. Occasionally use the term data values to refer to the numbers in sets, so that students don’t think of them only as blocks.

Use the following sample test grades to illustrate how the mean helps us to analyze data:

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alix</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Sivan</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Marco</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Bryan</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Helen</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Parvati</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>
Have students find the mean grade for each test. **ASK:** On which test did students do better overall? Sivan got the same mark on both tests, but her teacher said she did better on English than Math. Why? (She is at the mean for English but below the mean for Math.)

Tell students that you want to find a set of 3 numbers with a mean of 5. Invite volunteers to explain how they might solve the problem. Invite students to use blocks to explain their thinking. If necessary, get students started by giving them one possible answer: the set 5, 5, 5. (There are 3 numbers in the set. The mean is 5, since 5 + 5 + 5 = 15 and 15 ÷ 3 = 5.) **ASK:** How can you use this set to find other solutions? (You can subtract 1 from one of the numbers and add it to another: 4, 5, 6. You can subtract 2 from one number and add it to another: 3, 5, 7. You can subtract 2 from one number and add 1 to another and 1 to the third: 3, 6, 6. And so on!)

**Assessment**
Find the mean: 2, 12, 3, 5, 8.

**Extensions**
1. The mean of a set is 10 and the data values are: 2, 19, 7, 4, 15 and _____. What is the missing number? Write on the board:

   \[
   \text{Sum of data values} \div \text{Number of data values} = \text{Mean}
   \]

   Ask students to fill in the numbers that they know. What’s missing? (the sum of the data) What number divided by 6 is 10? Ask students to write the equation for the sum of data and to solve it. (sum of data = 10 × 6 = 60)

2. Find the mean and the median of the sets created in the Activities. Did you get the same number? Find the ranges above and below the median.

3. Can you find a set so that the range below the median is 0, and the mean and the median are the same? Explain your thinking with blocks.
4. Have students make a chart comparing the mean and the median for data sets with only 2 values:

<table>
<thead>
<tr>
<th>Data Set with 2 Values</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 7</td>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>4, 10</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Other data sets they should add to the chart:

a) 0, 12  b) 5, 7  c) 9, 21  d) 11, 15

What do students notice? (the mean and the median are the same) Have students predict whether the same will be true for data sets with 3 or 4 data values and then check their predictions.

5. Create 3 different sets of data for which the mean is 6.

6. Create a set of data with the same mean as that for 36, 48, 52 and 67.

7. Find the mean of these test scores: 49, 49, 49, 50, 51, 52. Is it possible for the mean score on a test to be greater than 50 if more than half of the students have marks less than 50? Explain.

---

**PDM4-15**

**Stem and Leaf Plots**

**GOALS**

Students will build stem and leaf plots for data and find the mode of the data.

**PRIOR KNOWLEDGE REQUIRED**

Place value
Ordering numbers from least to greatest

**VOCABULARY**

stem  mode
leaf   range
stem and leaf plot

Introduce the words “stem” and “leaf” as they apply in mathematics: the leaf of a number is its rightmost digit and the stem is the number formed by all the remaining digits.

Put an example on the board: stem $\rightarrow$ 327 $\rightarrow$ leaf

Ask volunteers to underline the leaf and circle the stem for various 2-, 3-, and 4-digit numbers. (EXAMPLES: 25, 30, 230, 481, 643, 3 210, 5 403, 5 430)

Tell students that a 1-digit number has no stem (i.e., stem 0) because there are no digits except the rightmost one. They can underline the leaf but the 0 isn’t written, so there is nothing to circle. Have students identify the stems and leaves in more numbers, including 1-digit numbers.

Have students identify the pairs with the same stem:

a) 45  46  63  b) 79  80  81  c) 88  89  98
d) 435  475  431  e) 86  862  860  f) 701  70  707
g) 6 423  642  649

**Bonus**

h) 23  253  235  2  239  2 530  2 529
Ask students to circle the numbers with stem 4, underline the numbers with stem 42, and cross out the numbers with neither stem:

42  428  4280  4  43  438  420  46  40  4201

Have students order the numbers from smallest to largest, find the stems, and then order the stems:

a) 45  46  63  
b) 567  60  583  
c) 43  47  92  65  73  70

**Bonus**

Use larger numbers and/or longer lists of numbers.

**ASK:** When the numbers are in order from smallest to largest, are the stems in order, too? Do numbers with the same stem have the same number of digits? Do they have the same number of tens? Point out that the stem is just the number of tens and the leaf is the number of ones. That’s why 1-digit numbers have stem 0—they have 0 tens.

Have students put the following numbers in order:

5  19  23  90  107  86  21  45  98  102  43

Then demonstrate circling the stems and writing them in the order in which they appear:

0  1  2  9  10  8  4

(When a stem repeats, such as the 2 in 23 and 21, you don’t have to write it again.) Now have students put the stems in order. **ASK:** Which was easier, writing the numbers in order or writing the stems in order? (writing the stems in order) Why? (There are fewer numbers and the numbers are smaller.)

Draw a T-chart and show students how to make a stem and leaf plot of the above data in 3 steps:

**STEP 1:** Write the stems in order in the left column.

**STEP 2:** Write each leaf in the second column in the same row as its stem. Add the leaves in the order in which they appear. For numbers that have the same stem, put the second leaf next to the first. Think aloud as you do the first few numbers, then have students help you do the rest. **PROMPTS:** Which number is next? What’s the leaf of that number? Where should I write it?

**STEP 3:** Put the leaves in each row in order, from smallest to largest.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td></td>
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<td>8</td>
<td></td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td></td>
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<tr>
<td>9</td>
<td></td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>10</td>
<td>7</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Then have students read the numbers from the finished plot. **ASK:** What are the numbers with stem 0? (just 5) What are the numbers with stem 1? (just 19) With stem 2? (23 and 21) Are these two numbers in order? Were they in order in the original list of data values? What did
we do to make sure they would be in order in the stem and leaf plot? (We put the leaves in order. Because they have the same stem [the same number of tens], the number with the larger leaf [more ones] is larger.)

Continue reading and writing all the numbers from the plot. Compare the complete set of numbers (now in order) to the original set (unordered). Tell students that the numbers are now in order because we first put them in order according to the number of tens (stems) and then, within groups with the same number of tens, we ordered the ones. Ordering the numbers was easier because we did it in two smaller steps.

Have students make stem and leaf plots to order several data sets (include up to 8 numbers in each set). Start with problems where steps 1 and 2 are completed for them, and then move to examples where only step 1 is completed. Finally, have students do all three steps themselves. BONUS: Use up to 20 data values and/or larger numbers.

Ask students to tell you the smallest and largest numbers from each of the stem and leaf plots they have drawn, and the range. (If necessary, remind students that the range is the difference between the largest and smallest data values). ASK: How does making a stem and leaf plot help you find the range more quickly?

Write a stem and leaf plot where one number occurs twice. EXAMPLE:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>1 8 9</td>
</tr>
<tr>
<td>10</td>
<td>0 2 9 9</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

Have students identify the number that repeats. Then write a stem and leaf plot in which 1 number appears 2 times and another number appears 3 times. Have students identify the numbers that repeat. ASK: Which number appears most often? Explain that this number—the value that occurs most often—is called the mode. Ask if anyone knows the French phrase “a la mode.” Tell students that it means in style or popular. Things that are popular occur often. Similarly, the mode is the most popular number in a set. ASK: Does every set have to have a mode? Remind students of earlier sets in which every value occurred only once—those had no mode.

Ask students to identify the more stem and leaf plots, some of which have more than one repeated data value but only one most common data value. Then introduce data sets with more than one mode. Give them the stem and leaf plot at first and then have them build the stem and leaf plot first to find the mode or modes.

ACTIVITY

Have students measure their shoe size in centimetres and make a stem and leaf plot of the data. Find the mode, the range, and the median of the set. Is the data spread evenly? Does it bunch up around the median? Is the range wide or narrow? Why?

Extensions

1. Tell students that a stem and leaf plot doesn’t have to include only numbers. Here is a stem and leaf plot of friends’ birthdays—the stem is the month and the leaf is the date:
Have students draw a bar graph from this data, with bars for each month. The height of each bar corresponds to the number of birthdays in that month.

Ask the following questions and have students tell you which graph gives you the answer, the stem and leaf plot, the bar graph or both:

a) How many birthdays are in the first half of a month?
b) How many birthdays are in April?
c) Which month has the most birthdays? The fewest?
d) Which month has a birthday the closest to the beginning of the month? The end of the month?

ASK: If you were given only the bar graph of the birthdays, could you have built the stem and leaf plot? Why not? What is the advantage of the stem and leaf plot?

2. John counts the total number of pages in each of his favourite novels:

148      520      589      550      224      562      494      469

Have students draw a stem and leaf plot. What do they notice? (Each stem has only 1 leaf). Tell students that sometimes it is more useful to use the number of hundreds instead of the number of tens as the stem. Re-do the stem and leaf plot with the hundreds as the stem, and the remaining numbers as the leaf. Ask students to explain how using hundreds instead of tens as the stem tells them more about John's favourite novels (They can see at a glance that most of John's favourites are 400+ pages long!).

3. Create a set of data in which:

a) the largest number is 100.
b) the smallest number is 100.
c) the range is 100.
d) the median is 100.
e) the mean is 100.
f) one of the modes is 100.

Can you satisfy more than one requirement in the same set?

A Note About Terms in Probability: Outcomes and Events

A simple action such as rolling a die, flipping a coin, or spinning a spinner has various possible results. These results are called the outcomes of the action. If you flip a coin, the outcomes are: "You flip a head" and "You flip a tail." When you describe a specific outcome or set of outcomes, such as rolling a 6, rolling an even number, tossing a head, or spinning red, you identify an event.
In probability theory, the terms **outcome** and **event** have very precise meanings. But in elementary texts, outcome is occasionally misused.

This spinner has 3 coloured regions:

![Spinner Diagram](image)

There are 3 possible outcomes of spinning the spinner:

1. The pointer lands in the blue region.
2. The pointer lands in the red region.
3. The pointer lands in the green region.

Some textbooks will identify the outcomes as:

1. You spin blue.
2. You spin red.
3. You spin green.

What’s the difference? Identifying outcomes with only colours and not regions can cause confusion when 2 or more regions of a spinner are the same colour. Here is a spinner with 4 coloured regions:

![Spinner Diagram](image)

There are 4 outcomes of spinning the spinner:

1. The pointer lands in the blue region at top right.
2. The pointer lands in the blue region at bottom right.
3. The pointer lands in the blue region at bottom left.
4. The pointer lands in the red region.

This is clearly not the same as saying that the outcomes are:

1. You spin red.
2. You spin blue.

“You spin blue” and “You spin red” are events, not outcomes. To assess the relative likelihood of spinning red or blue, students must recognize that the pointer may land in 4 distinct regions of the spinner. In 3 of the 4 outcomes, the spinner lands in a blue region. Hence, the event “You spin blue” is more likely than the event “You spin red.”

The outcomes for **QUESTION 1 f)** on Worksheet **PDM4-16** are “The spinner lands in region 1,” “The spinner lands in region 2,” “The spinner lands in region 3,” and “The spinner lands in region 4.” Your student may write something more concise, such as “You spin a 1,” “You spin a 2,” “You spin a 3,” “You spin a 4.” Accept these answers, provided students understand that when different regions of a spinner have the same number or colour, each region must be counted as a distinct outcome.

To avoid confusion, we only use the term “outcome” on the worksheets when the regions of the spinner are uniquely coloured or labelled. When the same colour or label appears more than once on the spinner, we use phrases like “ways of spinning red” instead of “outcomes.”
Tell students that today they will start learning how to predict the future!

Hold up a die and ask students to predict what will happen when you roll it. Can it land on a vertex? On an edge? No, the die will land on one of its sides. Ask students to predict which number you will roll. Then roll the die (more than once, if necessary) to show that the prediction about landing on a side works, but the number they picked does not necessarily come. Explain that the possible results of rolling the die are called outcomes, and to predict the future students must learn to identify which outcomes of various actions are more likely to happen and which are not. But first, they must learn to identify outcomes correctly.

Hold up a coin and ASK: What are the possible outcomes of tossing a coin? How many outcomes are there? Show a spinner and a set of marbles. What are the possible outcomes of spinning the spinner or picking a marble with your eyes closed? Ask students to identify the possible outcomes of a soccer game. How many outcomes are there? (3 outcomes: team A wins, team B wins, a draw)

ASK: You have to make a spinner with 5 possible outcomes. How would you do this? Invite volunteers to draw possible spinners. Then draw the spinner below, and ASK: How many outcomes are there for this spinner?

Are all the outcomes bound to come equally, or is there an outcome that will happen more often than the other ones? Draw the second spinner and ASK: How many outcomes does the second spinner have? (4) Will the pointer ever be in the grey region? (no, never)

Extensions

1. Bill has a pentagonal pyramid with numbers on the sides. He puts a non-base side on the table and rolls the pyramid. How many outcomes are there? Make a pentagonal pyramid, number the sides, and check your answer. Are there sides the pyramid never lands on if you roll it in the manner described? (Yes—the base. It will not roll if it stands on a base.) Repeat with a hexagonal prism and an octagonal prism: predict the number of outcomes then make a model to check your prediction.

2. Ed and George are playing “Rock, paper, scissors.” Describe all the possible outcomes of the game. (NOTE: “Ed wins” is not an outcome, it is an event. “Ed has paper, George has rock” is an outcome.)
PDM4-17

Even Chances

Hold up a coin. **ASK:** What are the possible outcomes of flipping this coin? How many outcomes are there? Which is more likely—flipping a head or a tail? Explain that the chances are the same—you have even chances of flipping a head or a tail. Write the term “even” on the board and explain that the chances of an event are even when the event happens in exactly half of the outcomes. Flipping a tail is 1 out of 2 possible outcomes; 1 is half of 2. **ASK:** How many outcomes are there when you roll a die? (6) How many outcomes are even numbers? (3) Since half of the outcomes are even numbers, you have even chances of rolling an even number (and an even chance of rolling an odd number).

**SAY:** We have 8 marbles in a box. I take out 1 marble. How many outcomes are possible? (8, regardless of the colour of the marbles) What is half of 8? So if I want even chances to take out a green marble, exactly 4 marbles should be green. Does it matter what colour the other marbles are? (No, provided they’re not green.)

Draw 3 spinners:

Ask students to identify the outcomes for each spinner. **ASK:** In which spinners do you have even chances of spinning red? (the one on the left and the one on the right) Emphasize, if necessary, that the spinner on the left has 4 possible outcomes, because it has 4 different regions, but 2 of the 4—half—are red. Draw more spinners on the board and ask students to identify the spinners where you have even chances of spinning red.

Write several even numbers on the board and ask students to find half of each number. Then have volunteers help you solve several problems such as:

- I have 10 marbles. Half of them are red. How many marbles are red?
- I have 12 marbles and 6 of them are green. How many of my marbles are green: half, less than half, or more than half?
- I had 14 marbles. I lost 6 of them. Did I lose more than half, less than half, or exactly half?

Ask students to describe an outcome of rolling a die that has even chances. (Possible answers: roll a number that is 3 or less; roll an even number; roll 2, 3, or 5.) Ask students to describe another event with even outcomes. **EXAMPLE:** I pick a boot from a pair of boots without looking. It is either left or right. So “I pick a left boot” has even chances.)
Assessment
Circle the spinners where you have even chances to spin red.

Draw a combination of 6 marbles with even chances to pick a green marble.

Draw a combination of 8 marbles of at least 3 colours with even chances to pick a yellow marble.

Divide students into 3 groups. Let each group make one of the 3 spinners below, spin it 24 times, and record the results. Then have groups make a bar graph of the results.

Did groups spin red in more than half, less than half, or exactly half of the spins? Is this what they expected?

Chances are, students did not spin red in exactly half the spins. Make another series of 24 spins. Is the result of all 48 spins nearer to the expected result? Discuss with students the difference between theory and practice. Explain that the more data you have, the nearer the actual result will be to the theoretical prediction.

Extensions
1. Enlarge the number of data values in the Activity by having each student make 12 spins and draw a bar graph of the results on graph paper. Ask students to colour the bars in their graphs according to the colours they represent. Cut the bars out and glue them together in long strips, producing one large bar graph for the whole class. If the strips are too long, cut them into smaller strips of 10 squares, group them in hundreds to count the total, and draw a bar graph to scale on the board.

2. Students should be aware that half of an odd number will be a mixed fraction. For instance, if you have 7 pizzas, half would be $3 \frac{1}{2}$ pizzas. Ask students to find half of these numbers: 5, 9, 11, 17. (You could ask students to make a model of each number using toothpicks. To find half of the number they could divide the toothpicks into two even piles and break the left over toothpick in half).
PDM4-18
Even, Likely, and Unlikely

Have students make some predictions: ask them to tell you if the following events are likely or unlikely.

- The sun will rise tomorrow.
- The teacher will give the answers to the test before giving the test.
- An alien will walk into the class in the next minute.
- They will have lunch in half an hour.
- It will rain tomorrow.
- It will snow in June.

Invite students to name some events and have other students tell if the events are likely or unlikely. You could ask students to compare the likelihood of events. For example, it is unlikely to snow in June, but it is more unlikely that an alien will walk into the class!

Draw the spinner below and ask students if it is likely that the spinner will point to red. Conduct an experiment with 12 spins. Use a tally chart to describe the results. Explain to students that mathematicians call an event likely if it is expected to happen more than half the time and unlikely if it is expected to happen less than half the time. From the last lesson, they know that an event with even chances is expected to happen exactly half the time. Write all three terms on the board. Using the tally chart, describe the chances of spinning red, blue, and green in these terms.

Doing the Activity will help to reinforce the feeling that the results of the experiment are connected to the ratio between the possible outcomes.

Describe several events and ask students to count the total number of outcomes and the number of outcomes that suit the event, and to tell whether the event is likely, unlikely, or has even chances.

EXAMPLES: 4 pairs of boots in a dark closet; 2 black, 2 brown

- Pull out a right boot
- Pull out a brown left boot
- Pull out a boot that is not left brown
- Pull out a black boot
- Pull out a boot that is either black or right brown

Spinning green
Spinning purple

Spinning blue
Spinning purple

GOALS
Students will identify events as even, likely, or unlikely.

PRIOR KNOWLEDGE
Outcome
Half of a number

VOCABULARY
outcome likely even chances unlikely likely

Even, Likely, and Unlikely

Students will identify events as even, likely, or unlikely.

Prior Knowledge
Outcome
Half of a number

Vocabulary
outcome likely even chances unlikely likely
Divide students into 2 groups. Each group will conduct a different experiment and share the results with the second group afterwards.

**Group 1**

Students will each need a die. Ask them to list all the outcomes of rolling a die and to count the outcomes that suit each event listed below. Then ask them to describe each event as likely, unlikely, or having even chances:

1. Roll a number greater than 2. 
2. Roll a number greater than 5. 
3. Roll an odd number.

Each student rolls the die 12 times and tallies the results. Do the results match the predictions? Have students combine their results. (This group tally chart—how many times the group rolled 1, 2, and so on—may be useful during the next lessons.) How many rolls did the whole group make? How many times does the group expect to get an even number? A number greater than 5? A number greater than 2? Students should explain their answers. Do the group’s results match the predictions better than the individual results?

Students can create a table like this to summarize predictions and results as they conduct the experiment:

<table>
<thead>
<tr>
<th>Event</th>
<th>Suitable Outcomes</th>
<th>Individual Results (12 Rolls)</th>
<th>Group Results (___ Rolls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll a number &gt; 2</td>
<td>3, 4, 5, 6</td>
<td>Predicted</td>
<td>Actual</td>
</tr>
<tr>
<td>Roll a number &gt; 5</td>
<td>6</td>
<td>Predicted</td>
<td>Actual</td>
</tr>
<tr>
<td>Roll an odd number</td>
<td>1, 3, 5</td>
<td>Predicted</td>
<td>Actual</td>
</tr>
</tbody>
</table>

**Group 2**

Students will each need 3 red marbles, 2 yellow marbles, 1 green marble, and a non-transparent bag or box (e.g., a lunchbox).

Ask students to list all the outcomes of drawing a marble from the box and to count all the outcomes that suit each event from the list below. Ask students to describe each event as likely, unlikely, or having even chances:

1. Draw a red marble. 
2. Draw a green marble. 
3. Draw a marble that is not yellow.

Each student draws a marble 12 times (returning the marble after each draw and shaking the bag) and records the results in a tally chart. Do the individual results suit the prediction? Have students combine their results. Do the group’s combined results suit the predictions? Students can create a table similar to that used by Group 1 to summarize their results.

Ask the students to write a summary of the experiments. Prompts for Group 1:

I performed an experiment with a die. I rolled the die ___ times.
I expected that ___ of ___ rolls will give a number greater than 2. I got a number > 2 in ___ rolls.
I thought that it is __________ to get a number > 2, because ________________
I learned that ________________

Discuss the similarities and differences between the experiments of the two groups.
If the event matches more than half the outcomes, then that event is likely—chances are it will happen more than half the time. If the event matches less than half the outcomes, it is unlikely.

**Assessment**

1. Are the chances likely, unlikely, or even?
   a) Pull a black sock from a box with 6 green and 4 black socks?
   b) Pull a penny from a pocket with 5 pennies, 4 dimes and a nickel?
   c) Pull a penny from a pocket with 5 pennies and 4 dimes?

2. Harold rolls a die. Give an example of a likely event and an unlikely event for Harold. Both events should be possible.

---

**PDM4-19**

**Equal Likelihood**

Draw this spinner on the board:

![Spinner]

**ASK:** What is more likely, to spin red or to spin green? Green or blue? Red or something else? What is more likely to happen: rolling 2 or 3 on a die? (If students kept their tally charts from the Activity in the previous lesson, they can compare the group results for rolling 2 and 3.)

Use the following spinners to identify more events that are equally likely:

**ASK:** Spinning 1 is as likely as spinning 2. Spinning 2 is as likely as spinning 3. Spinning 3 is as likely as spinning 1. Spinning red is as likely as spinning blue. Spinning green is as likely as spinning yellow. Spinning red or blue is as likely as spinning green or yellow.

Point out that spinning any single number or color in itself on the spinners above is an unlikely event. Ask students to think of more events that are equally likely, using spinners, dice, marbles, or other objects.

Ask students which colour they are most likely to draw from the following collection of marbles:

```
R B G G Y G Y G B G
```

**ASK:** Which colour are you least likely to draw? Which colours are equally likely? Why?

Use a similar exercise for assessment.
Extension

Doug has a total of $95 in his wallet ($50, $20, $20, and $5). Which bill is he most likely to draw? Which bills are equally likely to be drawn?

PDM4-20
Describing Probability

Review the meaning of the terms likely and unlikely with students. Write the terms on the board. Ask your students which word people would use to describe an event like meeting a live dinosaur in the street. Can that happen at all? Add the word impossible to the list. Ask your students which words describe an event that will definitely happen, like rolling a number less than 7 on a die. Add the word certain to the list. Ask students to describe the following events as likely, unlikely, certain, or impossible:

- It will rain on September 12
- It will snow on July 15
- There will be a math test before the end of the month
- You will grow wings
- Pull a green sock from a drawer with 10 green socks and 2 red socks
- Pull a $5 bill from a wallet with three $20 bills and one $5 bill
- Roll a number greater than 0 on a die
- Meet a green panther

NOTE: Although students might use the word impossible to describe the likelihood of meeting a dinosaur, this event is not necessarily impossible (scientists might find a way to clone dinosaurs). The only events that are strictly impossible are events that are contradictory—like rolling a number greater than 6 on a die. You might discuss this idea with students.

Ask students to give examples of various events and explain whether they are likely, unlikely, certain, or impossible. Encourage students to think of events using marbles, dice, money, and other objects, as well events from daily life, such as meeting a tiger or an astronaut on the way to school.
Draw the spinner below on the board and ask students to describe the possibility of spinning each of the colors. Is it equally likely that they will spin green or that they will spin blue? Review the meaning of **equally likely**. Is it equally likely that they will spin yellow or blue? Why? (HINT: look at the angle at the centre)

![Spinner Diagram]

Ask students to draw a spinner to match this description:

1. It is likely you will spin yellow.
2. It is unlikely to get green.
3. It is equally likely to get green and red.
4. It is impossible to spin blue.

Now write this list of properties:

1. It is impossible to get green.
2. It is certain to get blue.
3. It is likely to get red.
4. It is unlikely to get white.
5. It is equally likely to get white and red.
6. It is equally likely to get red and purple.
7. It is equally likely to get white and yellow.
8. It is equally likely to get green and orange.

**ASK:** Do you think we can make a spinner that matches all of these? Do any of the properties contradict each other? Look at 3 and 6: can a spinner have property 3 and property 6? (No. If the spinner is likely to give red, more than half of the spinner should be red. Then only less than half of it can be purple. But if it is equally likely to give red and purple, the red and the purple should have the same area. But more than half is never equal to less than half.) Have students pick three of the properties on the list and create a spinner that will match them. If students don’t draw an all-blue spinner present that example yourself and ask students to identify the properties that describe it (1, 2 and 5 - 8). Which properties does the spinner below match? (Same as the all-blue spinner—the pointer never lands in any other region.)

![Spinner Diagram]

Show students a collection of marbles or coloured counters:

![Marbles Diagram]

**ASK:** Which colour are you most likely to pick? Which colour is less likely to be picked: yellow or red? So which colour is least likely to be picked? Which colours are equally likely?
Assessment

1. Make a spinner that matches this description:
   - It is most likely you will spin green.
   - It is unlikely you will spin red.
   - It is most unlikely you will spin purple.
   - It is equally likely to spin purple and to spin blue.
   - It is impossible to spin yellow.

2. Explain why it is impossible to make a collection of marbles to match this description:
   It is impossible you will pick red. It is certain to pick blue. It is very unlikely you will pick green.

3. Change one word in the description of the collection of marbles above to make it possible. Draw the collection that matches the new description.

Extensions

1. Write the numbers from 1 to 10 on ten cards. Ask students to say whether the following outcomes are certain, impossible, or possible, if you select 6 cards at random:
   - The sum of the numbers will be greater than 60.
   - All the numbers will be even.
   - Two numbers will be neighbours.
   - The sum of the numbers will be greater than 20.
   - No numbers will be neighbours.
   - The sum of the numbers will be less than 12.
   - Three cards will be odd.

2. Invent or describe a game where a certain player's chance of winning is very close to certain. What are the chances of the other player(s) to win?
3. Explain to your students that in mathematics probability is often described as a fraction. The more likely an event is to happen, the larger the fraction is. If an event is certain, the probability is said to be 1, and if an event is impossible, we say that its probability is 0. An event that has even chances to happen has probability \( \frac{1}{2} \). If an event has probability close to 1, is it certain, very likely, unlikely, very unlikely or impossible? If the probability is close to 0? Describe the probability of the following events as 0, close to 0, close to \( \frac{1}{2} \), close to 1 or 1:

- It will snow on Aug. 3
- You will eat ice cream three times today
- To roll a number greater than 15 on a regular die
- To flip heads on a coin
- To draw a sock of any colour from a box with 18 green socks and 2 red socks
- To draw a green sock from a box with 18 green socks and 2 red socks
- To draw a pink sock from a box with 18 green socks and 2 red socks
- To draw a red marble from a bag with three red marbles and three yellow marbles
- To draw a red marble from a bag with twenty three red marbles and twenty two yellow marbles
- To see a pink tiger at the schoolyard
- To get presents at Christmas

(The Atlantic Curriculum expectation G1)

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**PDM4-21**

**Fair Games**

**GOALS**

Students will use the concept of equal likelihood in games.

**PRIOR KNOWLEDGE REQUIRED**

Equally likely

**VOCABULARY**

equally likely

fair game

**SAY:** I would like to play a game with you. The rules of the game are simple. I will spin a spinner. If I get red, I win; if I get blue, the class wins. Ask students if they agree to play by these rules. Now show them the spinner. Do they still want to play? Why not?

![Image of spinner]

Write the term **fair game** on the board. Ask students to explain what they think this term might mean. Encourage students to use math vocabulary in their explanations. Point out that in a fair game, both players have equal chances, or are equally likely, to win. Does this mean there can never be a draw? No. Use this game to illustrate: Flip a nickel and a penny. If both give heads, you win. If both give tails, the class wins. If there is one head and one tail, it is a draw—no one wins. Explain the rules to the class, then **ASK:** Is the game fair? Why? (It might help to list all the possible outcomes.) Would the game stay fair if we added a third player who wins when one of the coins gives a head and the other a tail? Would the game be fair if the third player wins with head on the nickel and tail on the penny?

Give the rules for another game: There are 6 marbles in a box. If you draw a red marble, you win; if it is blue, the class wins. Otherwise, the game is a
draw. There are 2 red marbles in the box. **ASK:** To make the game fair, how many blue marbles should be in the box? What about the rest of the marbles—what colour can they be? Can you think of another combination of marbles that will make the game fair?

Vary the game: If you draw a red marble you win, but if you draw any other colour, the class wins. If 2 of the 6 marbles are red, who has more chances to win—you or the class? What if 5 marbles are red? What should be in the box to make the game fair?

**Assessment**

Two players are spinning this spinner. Invent two different rules of play to ensure that the game is fair.

![Spinner Diagram]

**Extensions**

1. How are the games in Activities 1 and 2 similar? Is the first game less fair than the second?
   
   (In terms of probability, the two games are identical. In both games, the second player has 5 out of 6 chances to win, so the probability of Player 2 winning is 5/6.)

2. Carl and Clara played a game based on luck. Carl won 15 times and Clara won 12 times. Does this mean the game is not fair?

3. Let the students keep track of 20 repetitions of the following game:
   
   Players take turns rolling a die. The person rolling the die wins if he rolled a 1. The other player wins otherwise. **ASK:** Is this game fair? Are the results what you expected?

   How are the results of 20 games in Activity 2 different from the results you’ve got? In the Activity 2 game, Player 2 is very likely to win—he wins in 5 out of 6 outcomes of any roll, regardless of who rolls the die, so he wins most of times. In the new game the player rolling the die is still unlikely to win, but the roles (and the chances to win) interchange after each roll, so the players win nearly the same number of times.
PDM4-22
Expectations

Show students this spinner and ask them if the chances of spinning red and spinning blue are equal. What about the chances of spinning green and spinning blue? Why? Write on the board:

Number of possible outcomes: 3 (1 green, 1 red, 1 blue)
Chances of spinning blue: 1 out of 3

SAY: I am going to spin the spinner 12 times. We’ve done this before. How many times do you expect me to spin blue? Why? Write the calculation on the board:

\[ \frac{1}{3} \text{ of } 12 = 4 \quad \text{ OR: } \quad 12 \div 3 = 4 \text{ times} \]

Remind students that actual results usually differ from expected results. You might not get blue 4 times every time you make 12 spins, but 4 times is the most likely number of times you will spin blue.

Review with students how to find a fraction of a set and a fraction of a number as well as visual representations of fractions. Practise by solving these and other questions:

- \( \frac{1}{2} \text{ of } 14 \)
- \( \frac{1}{3} \text{ of } 15 \)
- \( \frac{1}{5} \text{ of } 25 \)
- \( \frac{3}{4} \text{ of } 20 \)
- \( \frac{2}{3} \text{ of } 9 \)
- Quarter of 16
- Three eighths of 24

Which part of the set “R G R G G R Y Y G Y” is G?

ASK: Which part of this spinner is blue? What are the chances of spinning blue? (3 out of 4) In mathematics we say “The probability of spinning blue on this spinner is \( \frac{3}{4} \).” Write that on the board. Explain that you can determine the number of times you would expect to spin blue out of 20 spins as follows:

STEP 1: \( \frac{1}{4} \text{ of } 20 = 4 \). This is how many times you expect the spinner to land in each region.

STEP 2: Three regions are blue. You expect to spin blue \( \frac{3}{4} \) of all times. If you expect the spinner to land in each region 5 times, and 3 of the regions are blue, then the spinner will land in a blue region \( 3 \times 5 \) times:

\[ \frac{3}{4} \text{ of } 20 = 3 \times 5 = 15. \]
You can show this with a picture:

\[
\frac{1}{4} \text{ of } 20 = 20 \div 4 = 5
\]

\[
\begin{array}{c}
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I} \\
\text{I} & \text{I} & \text{I} & \text{I}
\end{array}
\]

Therefore, \(\frac{3}{4} \text{ of } 20 = 3 \times 5 = 15\)

Have students practice calculating expected outcomes with these and similar questions:

- If you flip a coin 16 times, how many times do you expect to get a tail?
- Hong wants to know how many times he is likely to spin green if he spins this spinner 24 times. He knows that \(\frac{1}{3} \text{ of } 24 = 8\) (24 ÷ 3 = 8). How can he use this information to find how many times he is likely to spin green?

\[
\begin{array}{c}
\text{R} & \text{G} & \text{B} \\
\text{R} & \text{G} & \text{B}
\end{array}
\]

- If you roll a die 18 times, how many times do you expect to get a 4? To get a 1?
- How many times would you expect to spin blue if you spin this spinner 50 times? How many times would you expect to spin green?

\[
\begin{array}{c}
\text{R} & \text{R} & \text{B} \ \\
\text{B} & \text{G} & \text{G} \\
\text{B} & \text{G}
\end{array}
\]

**MORE CHALLENGING:** If you roll a die 30 times, how many times do you expect to roll an even number? How many times do you expect to roll either 4 or 6?

**Assessment**

Rea spins this spinner 30 times. How many times is she likely to get blue?

\[
\begin{array}{c}
\text{R} & \text{B} & \text{B} \\
\text{B} & \text{G}
\end{array}
\]

**Bonus**

1. You flip two coins, a nickel and a dime, 12 times. List the possible outcomes. How many times do you expect to get one head and one tail?

2. Jack and Jill play “Rock, paper, scissors” 18 times. List all possible outcomes of the game. 
   (HINT: “Jack has rock and Jill has paper” is different from “Jack has paper, Jill has rock”! Why?) How many times do you expect to see a draw?
Extensions

1. Design an experiment with three possible outcomes in which one of the outcomes has a probability near $\frac{1}{3}$.

2. Choose a novel. Open it to any page and note whether or not the first letter is a “t”. Check 10 pages in this manner. Describe the probability that “t” is the first letter on a page of the book. On how many pages of the whole book do you expect to see “t” as the first letter?

PDM4-23
Problems and Puzzles

PDM4-23 is a review worksheet, which can be used for practice. After the students have solved QUESTION 4 of the worksheet, discuss what would happen if they used actual and not imaginary money. The dime is smaller and the quarter is larger than the nickel. Might they be able to pick 30¢ on the first try?
Introduction to Coordinate Systems

To illustrate the idea of a coordinate system you can start with the following card trick:

1. First, deal out nine cards—face up—in the arrangement shown in the picture below:

   Row 3
   Row 2
   Row 1

   Column 1 Column 2 Column 3

2. Next ask a student to select a card in the array and then tell you what column it’s in (but not the name of the card).

3. Gather up the cards, with the three cards in the column your student selected on the top of the deck. Show clearly how you do that.

4. Deal the cards face up in another 3 × 3 array making sure the top three cards of the deck end up in the top row of the array.

5. Ask your student to tell you what column their card is in now. The top card in that column is their card, which you can now identify!

6. Repeat the trick several times and ask your students to try to figure out how it works. You might give them hints by telling them to watch how you place the cards, or even by repeating the trick with a 2 × 2 array.

When your students understand how the trick works, you can ask the following questions:

- Would there be any point to the trick if the subject told the person performing the trick both the row and the column number of the card they had selected? Clearly there would be no trick if the performer knew
both numbers. Two pieces of information are enough to unambiguously identify a position in an array or graph. This is why graphs are such an efficient means of representation: two numbers can identify any location in two-dimensional space (in other words, on a flat sheet of paper). This discovery, made over 300 years ago by the French mathematician René Descartes, was one of the simplest and most revolutionary steps in the history of mathematics and science: his idea of representing position using numbers underlies virtually all modern mathematics, science, and technology.

You might ask your students how many numbers would be required to represent the position of an object relative to an origin in three-dimensional space. (The answer is three. Think of the origin as being situated on a plane or flat piece of paper that has a grid or graph on it. You need two numbers to tell you how to travel from the origin along the grid lines on the plane to situate yourself directly above or below the object, and one more number to tell you how far you have to travel up or down from the plane to reach the object.)

- Ask your students if the trick would work with a larger array. Have them try the trick with a 4 × 4 array. They should see that as long as the cards can be arranged in a square (with an equal number of rows and columns), the trick works. Ask your students to explain why this is so and why the trick doesn’t work if the array isn’t square (for instance, try it with 2 columns and 6 rows).

- Ask your students if the trick would work if the subject told the performer which row the card was in rather than which column. Have your students show you how the new trick would be performed. The fact that the trick works equally well in both cases illustrates a very deep principle of invariance in mathematics. In a square array, there is no real difference between the rows and columns. In fact, if you rotate the array by a quarter turn, the rows become columns and vice versa. More generally, once you fix an origin in space, it doesn’t matter how you set up your grid (the lines representing the rows and columns). In all cases you need only two numbers to identify a position.

Now draw an array of three columns and rows on the board and number the columns and rows:

```
3  •  •  •
2  •  •  •  C
1  •  •  •  R  O  W
1  2  3  L
    U
    M
    N
```

Point out a row and a column, and stress that we order rows from bottom to top, and columns from right to left. Ask several volunteers to locate the third column, second row, etc. Then ask your students to do the worksheets. The QUESTION 7 on the worksheets can be played as a game—one of the players gives the column and the row, the other has to mark the point according to the numbers. They might play “Hangman” hanging each other for incorrect answers.

**Assessment**

1. Join the dots in the given column and row:
   a) Column 3, Row 2
   b) Column 1, Row 3
   c) Column 3, Row 3

```
  •  •  •
  •  •  •
  •  •  •
```

```
2. Circle the dot where the two lines meet:
   a) Column 2, Row 3
   b) Column 3, Row 1
   c) Column 1, Row 1

3. Identify the proper column and row for the circled dot:
   a) Column _____  Row _____
   b) Column _____  Row _____
   c) Column _____  Row _____
   d) Column _____  Row _____

Bonus
Which letters of the alphabet can be written on the grid and described in terms of rows and columns only? (See QUESTION 3 of the worksheet.) Which numbers can be written this way?

ACTIVITY
Ball Game
The students are the points in a coordinate system. Ask each student to say which column and row they are in. Then throw a ball according to directions—start by giving a student a ball, and a column and row number. The student holding the ball has to throw it to the student sitting at the given point.

Extension
The card trick can be modified for non-square arrays if one allows one extra rearrangement. Deal out an array of 3 columns, 9 rows. Have a student select a card and tell you what column it’s in. Re-deal the cards so that all of the nine cards from the chosen column land in the top three rows of the new array. Ask the student to tell you what column their card is in now, and re-deal the top three cards in that column into the top row of a new array. Once the student tells you what column their card is in, you can identify the top card in that column as the one they selected.

This version of the trick illustrates a powerful general principle in science and mathematics: when you are looking for a solution to a problem, it is often possible to eliminate a great many possibilities by asking a well-formulated question. In the card trick one is able to single out one of 27 possibilities by asking only three questions. Repeat the trick, asking your students how many possibilities were eliminated by the first question (18), by the second question (6), and by the third (2).
Introduction to Slides

For this lesson, a magnetic board with a grid on it (or an overhead projector with a grid drawn on a transparent slide) would be helpful. Let your students practice sliding dots in the form of a small circular magnet right and left, then up and down. Students should be able to identify how far a dot slid in a particular direction and also be able to slide a dot a given distance. If any students have difficulty in distinguishing between right and left, write the letters L and R on the left and right sides of the board.

After students can slide a dot in given direction, show them how to slide a dot in a combination of directions.

Assessment

Slide the dot:

(a) 3 units right; 3 units up
(b) 6 units left; 3 units down
(c) 7 units left; 2 units up

Ball Game

The students are points on the grid, and you give directions such as: "The ball slides three units to the right"; the student with the ball has to throw it to the right place in the grid.

In the schoolyard, draw a grid on the ground. Ask your students to move a certain number of units in various combinations of directions by hopping from point to point in the grid.
ACTIVITY 3

Memory Game
Students will need a grid and several (1 to 4) small objects (play money of different values or beads of different colours could be used). The objects are placed on the intersections of the grid. Player 1 slides one of the objects while Player 2’s back is turned, and Player 2 then has to guess which object was moved and describe the slide. This game will become much easier when coordinates are placed on the grid and the students are familiar with the coordinate system. When students learn coordinate systems (in section G4-25), they will be able to memorize the coordinates of the objects. They can then compare the coordinates after the objects were moved with the coordinates before the objects were moved to determine exactly which object moved and how.

G4-22
Slides

Tell your students the following story. You might use two actual figures to demonstrate the movements in the story.

Suppose you have a pair of two-dimensional figures and you wish to place one of the figures on top of the other. But the figures are very heavy and very hot sheets of metal. You need to program a robot to move the sheets: to write the program you have to divide the process into very simple steps. It is always possible to move a figure into any position in space by using some combination of the following three movements:

1. You may slide the figure in a straight line (without allowing the object to turn at all):

   SLIDE

2. You may turn the figure around some fixed point (usually on the figure):

   TURN or ROTATION
3. And you may flip the figure over:

**FLIP or REFLECTION**

Two figures are congruent if the figures can be made to coincide by some sequence of flips, slides and turns. For instance, the figures in the picture below can be brought into alignment by rotating the right hand figure counter clockwise a quarter turn around the indicated point, then sliding it to the left.

It is not always possible to align two figure using only slides and turns. To align the figures below you must, at some point, flip one of the figures:

One way to flip a figure is to reflect the figure through a line that passes through an edge or a vertex of the figure. Tell your students that today you are going to teach them about slides.

Show students the following picture and ask them how far the rectangle slid to the right. Ask for several answers and record them on the board. You may even call a vote.

Students might say the shape moved anywhere between one and seven units right. Take a rectangular block and perform the actual slide, counting the units with the students. The correct answer is 4.
Show another picture:

![Figure 1]

This figure has a dot on its corner. How much did it slide? This time it is easier to describe the slide—just use the benchmark dot on the corner. Check with the block.

Show a third picture.

![Figure 2]

Is this a slide? The answer is NO, this is a slide together with a rotation. You cannot slide this block from one position to the other, without turning it.

**ACTIVITY**

Give your students a set of pattern blocks or Pentomino pieces and ask them to trace a shape on dot paper so that at least one of the corners of the shape touches a dot. Ask students to slide the shape a given combination of directions. After the slide, trace the pattern block again.
G4-23
Slides (Advanced)

GOALS
Students will slide shapes on a grid, and describe the slide.

PRIOR KNOWLEDGE REQUIRED
Slide a dot on a grid
Distinguish between right and left

VOCABULARY
slide, translation
translation arrow

Draw a shape on a grid on the board and perform a slide, say three units right and two units up. Draw a translation arrow as shown on the worksheet. Ask your students if they can describe the slide you’ve made. If they have trouble, suggest that they look at how the vertex of the figure moved (as shown by the transition arrow). To help students describe the slide, you might tell them that the grid lines represent streets and they have to explain to a truck driver how to get from the location at the tail of the arrow to the location at the tip of the arrow. The arrow shows the direction as the crow flies, but the truck has to follow the streets.

Make sure your students know that a slide is also called a “translation”. Students should also understand that a shape and its image under a translation are congruent.

Extensions
1. Slide the figures however you want, and then describe the slide:

2. Describe a move made by a chess knight as a slide. Describe some typical moves of other pieces such as a pawn or a rook (castle).
G4-24

Grids and Maps

Assign a letter to each row of desks in your class and a number to each column. Ask your students to give the coordinates of their desks. Then play “postman”—a student writes a short message to another student and writes the student’s “address” in coordinates. A volunteer postman then delivers the letter. The postman has to describe how the letter moved (two to the front and one to the left, for example).

GOALS
Students will describe and perform a slide on a grid, and find a point given by coordinates on a map.

PRIOR KNOWLEDGE REQUIRED
Slides
Coordinate systems

VOCABULARY
slide translation
row column
coordinates
G4-25
Games and Activities with Maps and Grids

GOALS
Students will describe (and find on a grid) a point given by coordinates on a map.

PRIOR KNOWLEDGE REQUIRED
Coordinate systems

VOCABULARY
row
column
coordinates

Place a slide with a map of Saskatchewan on the overhead projector (see the BLM). Ask volunteers to find the cities on the map and to answer the questions:

- What are the coordinates of Saskatoon?
- What are the coordinates of Regina?
- What are the coordinates of Uranium City?
- What are the coordinates of Prince Albert?
- What can you find in the square A4? D5? D1?

As a warm-up for the Secret Squares game on worksheet G4-25, have the class as a whole try to guess the locations of each hidden square from the information given in the grids below. Start with showing an example. Use volunteers for each step.

STEP 1: Shade all the squares that are of distance 1 from the square 1.

STEP 2: Cross the squares you can reach by two steps from the square 2.

STEP 3: Check for each of the squares that are both shaded and crossed, if they can be reached by three steps from square 3.

Let your students practice:

In 2 grids, not enough information is given (have students marked all possible locations for the hidden square) and in one grid too much information is given (have your students identify one piece of redundant information):
Once the students understand the game, they can either play it in pairs or you might choose to play the game with the entire class.

**Battleship Game**

This game may be played in pairs or a teacher can play against the whole class, with the class is guessing the teacher’s ships.

*Sample Placement:*

Player 1 and Player 2 each draw a grid as shown. Each player shades:

- 1 battleship
- 2 cruisers
- 2 destroyers
- 1 submarine

(See grid for an example. No square of a ship may be adjacent to a square of another ship, including diagonally.)

Players try to sink all their partner’s battleships by guessing their coordinates. If a player’s ship is in a square that is called out, the player must say “hit.” Otherwise they say “miss.”

Each player should keep track of the squares they have guessed on a blank grid by marking hits with X’s and misses with ✓’s. The game ends when all of one player’s ships are sunk. A ship is sunk when all its squares are hit, and the owner of the ship must indicate that to the partner.

**HINTS/PROMPTS:** When a player hits something, where should he or she look for the other squares of the ship? If you sink a ship, which squares do you know are empty?
**Solitaire Battleship Game**

Once again, it is good to give an example, reasoning out loud.

One cruiser, one destroyer and one submarine are hidden in each grid [see the description of Battleship above for sizes]. The row number tells the number of shaded squares in the row. The column number tells the number of shaded squares in that column. Find the 3 ships in each grid. **HINT:** Start by putting an 'X' in all the squares where you know there can’t be a ship.

There is more than one solution in two of the three grids.

![Grids](image-url)
G4-26
Reflections

GOALS
Students will perform reflections of simple shapes through a line.

PRIOR KNOWLEDGE REQUIRED
Symmetry

VOCABULARY
reflection
symmetry
symmetry line
mirror line

Give your students an assortment of Pentomino pieces. Ask them to trace each piece on grid paper, draw a mirror line through a side of the piece and then draw the reflection of the piece in the mirror line. Students could check if they have drawn the image correctly by flipping the grouping of Pentomino pieces over the mirror line and seeing if it matches the image. Let your students know that a “flip” is also called a “reflection.” Students should notice that each vertex on the original shape is the same distance from the mirror line as the corresponding vertex of the image. Let your students practice reflecting shapes with partners: Each student draws a shape of no more than 10 squares, and chooses the mirror line. The partner has to reflect the shape over the given mirror line.

Advanced game: One student draws two shapes of no more than 10 squares so that the shapes are symmetric in a line but one square is misplaced. The partner has to correct the mistake.

Extensions

1. Ask students to find 5 letters of the alphabet that look the same after a reflection in a mirror line (as in QUESTION 7 on the worksheet) and 5 letters that look different after a reflection.

2. Students could try to copy and reflect a shape in a slant line: for EXAMPLE:

3. Judy makes patterns with pattern blocks. She reflects each shape through one of the sides.
   a) Draw mirror lines that she used in her designs.
   b) Build her designs from pattern blocks and add three more pattern blocks to each design.
   c) Create your own pattern from pattern blocks using reflections. Ask a partner to extend it.

i) ii) iii) iv)
G4-27
Reflections (Advanced)

GOALS
Students will perform reflections of points and simple geometric shapes through a line.

PRIOR KNOWLEDGE REQUIRED
Symmetry

VOCABULARY
reflection
symmetry
symmetry line
mirror line

Draw a grid on the board or use a grid on an overhead projector. Then draw a vertical mirror line and mark a point on a grid. Ask a volunteer to find the reflection of the point. Ask him/her to explain how they found the reflection. Your students should be aware that a point and its image are always the same distance from the mirror line.

Draw four points as shown and explain that two of these points are reflections of the other two. Challenge students to draw the mirror line. How do they know that the line they have drawn is the mirror line? Which point is reflection of which?

Write several words, such as MOODY CAT IN A WOODEN BOX, on a transparency sheet and project it onto the board in an incorrect way (flipped horizontally or vertically). Ask your students if they can read the text. Which words are still readable? Which letters look normal? Which transformation should be performed to make the text look completely normal? (A reflection.) Ask your students to draw the mirror line. Show them that a reflection in a mirror and a flip of the transparency sheet both achieve the goal.

Bonus
Sort all the capital letters of the alphabet into a Venn diagram:
1. Letters that look the same after a reflection in a horizontal line.
2. Letters that look the same after a reflection in a vertical line.

Extensions
1. Which letters of the alphabet look different after a horizontal or vertical reflection, but look the same after two reflections? EXAMPLE: If you reflect the letter E or L through a vertical line, the image faces backwards. If you reflect the image through a second vertical line, you produce the original letter. What happens if you reflect a letter first through a horizontal line, then through a vertical line?

2. Draw an equilateral triangle. If you reflect it through one of the sides and look at two shapes together, what shape will you get? Write down your prediction and check it. Is the result different for the other sides? Repeat with an isosceles triangle and a triangle with a right angle. Are the results different for different sides? Check all sides.
G4-28
Rotations

GOALS
Students will describe and perform rotations that are multiples of a quarter turn.

PRIOR KNOWLEDGE REQUIRED
Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
Clockwise
Counter clockwise

VOCABULARY
clockwise
counter clockwise
rotation

Review the meaning of the terms “clockwise” and “counter clockwise” using a large clock or by drawing arrows on the board. If you have a large clock, ask volunteers to rotate the minute hand—clockwise and counter clockwise—a full turn, half turn, and a quarter of turn. You might also ask your students to be the clocks: each student stands with a hand forward and turns clockwise (CW) or counter clockwise (CCW) according to your commands.

Draw several clocks on the board as shown below and ask your students to tell you how far and in which direction each hand moved from start to finish:

![Clocks showing rotations](image)

Then draw examples with only one arrow and ask students to turn the arrow:

a) $\frac{1}{4}$ turn CW  
   b) $\frac{1}{2}$ turn CCW  
   c) $\frac{3}{4}$ turn CW  
   d) $\frac{3}{4}$ turn CCW

Assessment
1. Describe the rotation of an arrow:

2. Show the position of the arrow after each turn:

   a) $\frac{1}{4}$ turn CW  
   b) $\frac{1}{2}$ turn CW  
   c) $\frac{3}{4}$ turn CCW  
   d) $\frac{3}{4}$ turn CW
G4-29
Rotations (Advanced)

GOALS
Students will describe and perform rotations of shapes that are multiples of a quarter turn.

PRIOR KNOWLEDGE REQUIRED
Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
Clockwise
Counter clockwise

VOCABULARY
clockwise
counter clockwise
rotation

Review the previous lesson by drawing several arrows or clock hands. To help your students visualize the effect of a rotation of a shape, have them make a small flag (as in QUESTION 1 on the worksheet) by taping a triangular piece of paper to a straw. Ask students to rotate the flag and trace its image after the rotation. Students could also cut out shapes similar to the other ones on the worksheet and trace the images of these shapes after a rotation. Then ask your students to trace or draw a figure, decide on rotation and draw the new shape without the prop. Students could also practice rotating pattern blocks or Pentomino shapes (around vertices of the shapes) on a grid.

Assessment
Draw the shape after each turn:

a) $\frac{1}{4}$ turn CW  

b) $\frac{1}{2}$ turn CW  

c) $\frac{3}{4}$ turn CCW  

d) $\frac{3}{4}$ turn CW

Extensions
1. Using pattern blocks or cardboard polygons, trace the figure on a sheet of paper. Then choose a vertex and rotate the shape around the vertex $\frac{1}{2}$ turn. Trace the figure again. Would you get the same result if you had reflected the figure? Shapes that are particularly interesting in this case are right-angled trapezoids.

2. Trace the flower onto tracing paper and cut it out. Try to reflect it vertically, then rotate it $\frac{1}{2}$ turn CW. Draw the result. Now rotate and reflect the flower vertically. Is the result the same? Try various combinations of $\frac{1}{4}$ turn rotations and reflections. Which combinations give the same results? Why?

3. What happens to the line of symmetry of a figure after a rotation of $\frac{1}{4}$ turn? $\frac{1}{2}$ turn? If the shape has a vertical line of symmetry, will it have a vertical line of symmetry after either a $\frac{1}{4}$ or $\frac{1}{2}$ turn?
Building Pyramids

GOALS
Students will build a skeleton of a pyramid and describe the properties of pyramids.

PRIOR KNOWLEDGE REQUIRED
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon

VOCABULARY
edge
vertices
triangular
hexagonal
base
vertex
pyramid
pentagonal
skeleton

Start with a riddle: “You have 6 toothpicks. Make 4 triangles with them. The toothpicks must touch each other only at the ends.” Let your students try to solve the riddle using toothpicks and modelling clay to hold the toothpicks together at the vertices of the triangles. The answer, of course, is the triangular pyramid. You might give your students the hint that the solution is three-dimensional.

Sketch a rectangular pyramid on the board and shade the base. Ask volunteers to mark the edges and the vertices (counting them, and making a tally chart). Write the words “base”, “edges”, “vertex” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a pyramid—first a base, then add an edge to each vertex of the base and join the edges at a point. The students should make triangular, square and pentagonal pyramids. Then let them fill in the chart and answer the questions on the worksheet.

After finishing the worksheet, they may check their prediction for the hexagonal pyramid by making one.

Tell your students that the shapes they have built are called “skeletons” of pyramids. You might write on the board the “equation”: “SKELETON = Edges + Vertices”. As animal skeletons are covered with flesh and skin, the skeleton of a pyramid can be covered with paper or glass or other substances and will have faces. Show a pyramid (with faces) and write the word “faces” on the board as well.

Assessment
Create and fill in the fifth row of the chart on the worksheet—for the heptagonal (7-sided) based pyramid.

ACTIVITY
On a class picnic build skeletons of pyramids and prisms from marshmallows and toothpicks or straws.

Extensions
1. How many faces, edges, and vertices would a pyramid with a ten-sided base have?
2. Give a rule for calculating the number of edges in a pyramid that has a base with n sides. (Use n in your answer.)

SOLUTION: A pyramid with n edges in the base also has n vertices in the base. But attached to each vertex in the base there is one non-base edge.
Hence there are \( n \) non-base edges and \( n \) base edges. Therefore there are \( 2 \times n \) edges altogether in a pyramid with \( n \) base edges. (So, for example, a pyramid with 5 base edges would have \( 2 \times 5 = 10 \) edges altogether.)

3. Ask your students to bring to class pyramids or pictures of pyramids (Egypt, Mexico, Japan, entrance to Louver, Paris, France and any others) that they can find at home. You can use the pyramids they brought in the lessons G4-33 and G4-34.

4. **PROJECT:** Ask your students to find a picture of a pyramidal structure and give a presentation about it—what was the structure used for, when and where was it built, why does it have the pyramidal form.

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**G4-31**

**Building Prisms**

**GOALS**

Students will build a skeleton of a prism and describe the properties of prisms.

**PRIOR KNOWLEDGE REQUIRED**

Geometrical shapes: triangle, square, rectangle, pentagon, hexagon

**VOCABULARY**

| face      | edge     |
| vertex    | vertices |
| prism     | cube     |
| triangular| pentagonal|
| hexagonal | base     |

Sketch a prism on the board, shade the bases. Ask volunteers to mark the edges and the vertices (counting them and making a tally chart). Write the words “base”, “edges”, “vertex” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a prism—first make two copies of the base, and then join each vertex on one base to a vertex on the other base with an edge. Students should make triangular and pentagonal prisms and a cube. Let them fill in the chart and answer the questions on the worksheet.

After finishing the worksheet, they may check their prediction for the hexagonal prism by making one.

Ask your students what they have built. (Skeletons of prisms.) What are the “bones”? (The edges.) What do the skeletons need to become prisms? (Write the word “faces” on the board.)

**Assessment**

Create and fill in a fifth row of the chart—for the heptagonal (7-sided) based prism.

**Extensions**

The following two extensions were adapted from the Atlantic Curriculum.

1. Give each student two boxes that are both rectangular prisms (you may ask them in advance to bring various boxes from home. The length, width and height of the boxes should be different). Ask your students to tell you the proper mathematical name for the boxes (rectangular prisms) and ask them if they can find any faces that are congruent. Your students might find the congruent faces by tracing the faces on paper. How many congruent faces does each prism have? Ask your students to label the faces, so that the congruent faces are marked with the same letter. Then ask them to measure the edges and to write down the dimensions for...
each face. What happens if they join a pair of prisms along congruent faces? Ask your students to predict the dimensions of the new prisms and to check their predictions. How many faces of the prism double in size? (4) How many dimensions of the prism double? (1)

2. Hold up a triangular or pentagonal prism. Join it to the board by one of the faces and ask your students: Suppose the board is a mirror. Let’s look at the solid that is composed of both the prism and the reflection. Describe the solid and name it if you can. Use only prisms with equilateral bases for this activity. (More complicated bases could be used for very advanced students.) Let your students practice with a mirror. You can use a large mirror at the front of the class or smaller mirrors and smaller shapes that students could work with in groups. Are the results different if you attach the side face or a base of the prism to the mirror? (Yes—the base gives a prism of double height, and the side face creates a prism with an irregular base.

For instance, if you use a triangular prism with an equilateral base, the resulting prism will have a rhombus as the base, and a pentagonal prism will produce an octagonal prism with an irregular base: (▱). What happens if you repeat the process with a pyramid with an equilateral base? Will you still get a pyramid? (Generally, no). Is it a prism? (No as well)

3. Ask students to build the following shapes using interlocking cubes:
   a) a rectangular prism with a square base and side faces that are not square.
   b) a rectangular prism (with a square base) that is 3 times as high as the length of its base.

4. Take two identical triangular prisms with scalene bases. (See the BLM "Triangular Prism with a Scalene Base" for a net.) Can you predict the shape of the base when you join the prism along identical faces (not bases)? What happens if you turn one of the prisms over? (The result is a prism with either a kite or a parallelogram in the base.)
G4-32

Vertices, Edges and Faces

Remind your students that there are lots of 3-D shapes in the world around us that are either pyramids or prisms. As an example you might show them a photo of the pyramids in Egypt.

Hold up a 3-D shape and draw a picture of the shape on the board. Write the words “3-D shape”, ask volunteers to show the edges, the faces and the vertices, both on the shape itself and on the drawing, and write the terms “edge”, “face” and “vertex” on the board. Remind your students that the plural of “vertex” is “vertices”.

Your students will need the skeletons of the cubes they made during the last two lessons. Give each student 2 squares made of paper and 4 squares made of transparent material. Ask them to add:

- The non-transparent squares as the bottom face and the back face
- The transparent ones—as the top, the front and the side faces

Add the faces step by step, one face at a time, emphasizing the positions and names of the faces. It is a good idea to show the students how to add faces on a larger model.

Ask students to identify the edges of the cube that they see only through the transparent paper. If the transparent faces were made of paper, would they see these edges? No. The edges that would be invisible if all the faces were non-transparent are called the “hidden edges”. On a two-dimensional drawing of a cube these hidden edges are marked with dotted lines.

Extension

Ask students to hold their cubes in various positions (on the table, on the floor looked at from above, slightly above them and so on.) Ask students to describe what the faces look like when seen from different angles (they look like square, parallelogram, etc). The outline of the shape itself can look like a square, a rectangle, a hexagon, a trapezoid and a rhombus.
Prisms and Pyramids

Divide your students into groups. Give each group several 3-D shapes, so that each group has some rectangular and triangular pyramids, rectangular and triangular prisms and a cube. Ask your students to count the faces of the shapes. If some students are having trouble keeping track of the number of faces, they might mark each face with a chalk dot or a small sticker. Ask your students to count the edges and vertices on the 3-D shapes as well (they also might shade edges with chalk and mark vertices with stickers.) Ask them to write the results of their count in the table on the worksheet.

Draw a pentagonal pyramid and a triangular prism on the board and let volunteers count the edges, faces and vertices of these figures. Ask them to mark the edges and circle the vertices as they count.

**Extensions**

1. Ask your students to try and add the number of faces and vertices of a cube and subtract the number of edges. The result is 2. What happens if you do that to another solid? (It will be 2 as well. This fact is known as Euler’s formula and was discovered by the great Swiss mathematician Leonard Euler in the 18th century.)

2. Let your students construct shapes from Polydrons—from regular 3-D shapes like prisms and pyramids to animals and castles. Count the faces, edges and vertices. Check if the Euler’s formula holds.

**ACTIVITY 1**

Show your students an example of a cone and a cylinder. Explain that a cone has one curved surface and one flat surface, while a cylinder has two flat surfaces and one curved surface. Ask students to find as many examples of pyramids, prisms, cones, and cylinders in the classroom as they can.

**ACTIVITY 2**

Ask your students to bring objects that are prisms, pyramids, cubes, cylinders, cones. Create a collection of such shapes for further use in class. **EXAMPLES:** paper cylinder, glasses, boxes—sometimes you can find a cylinder or a hexagonal prism—juice or milk cartons are pentagonal prisms.
G4-34
Prism and Pyramid Bases

GOALS
Students will distinguish between prisms and pyramids, and identify their bases.

PRIOR KNOWLEDGE REQUIRED
Pyramid
Prism
Geometric shapes: triangle, rectangle, quadrilateral, pentagon, hexagon

VOCABULARY
triangle   rectangle
quadrilateral   pentagon
hexagon   prism
pyramid   base
point

Hold up a pentagonal pyramid. Ask a volunteer to count the faces. What shapes are they? (A pentagon and five triangles.) Ask a volunteer to draw a pentagon and a triangle on the board. Ask another volunteer to write the number of faces that have each shape inside the shape itself.

Tell your students that in the pentagonal pyramid the face that is not the same shape as the others is the base. Hence in this pyramid a pentagon is the base.

In a pyramid there is always one base, unless the pyramid has all triangular faces (in which case it must be a triangular pyramid). The shape of the base of a pyramid gives the pyramid its name. Hence if a pyramid has a base with 5 sides, it is called a “pentagonal pyramid”. A prism has two bases. The non-base sides of the prism are always rectangles.

Give your students a set of 3-D shapes (you can use the shapes from your collection or ask students to construct some shapes using the nets from the BLM). Ask your students to place each shape on a piece of paper and trace the faces. Ask them to write the number of faces of the traced shape inside the shape the way you did on the board. Students should also write the name of the figure beside the base. If your students have trouble spelling the names of the figures, write them on the board. Later include them into a spelling test.

Assessment
1. Use three shapes for each student, each set should include a pyramid and a prism, and one shape with all faces congruent, like a cube or a triangular pyramid with equilateral faces. A good set: a pentagonal pyramid, a triangular prism and a cube.

   a) Place each shape—base downward—on a piece of paper and trace the base. (That way you can verify that each student knows how to find the base.)

   b) Write the name of the figure beside the base and indicate whether the figure has one or two bases.

   c) If all faces of the figure are congruent, indicate this.

2. Look at the shapes in QUESTION 3 of the worksheet. Which shapes are pyramids? Which shapes are prisms? Can there be a shape that is both prism and pyramid? Why not?
**G4-35**
Properties of Pyramids and Prisms

Give your students a collection of 3-D shapes or have them make a set of shapes using the nets from the BLM. Ask your students to order the shapes (pyramids and prisms separately) so that the number of edges in the base increases. After that they should fill in the chart:

<table>
<thead>
<tr>
<th>Picture of Base</th>
<th>Number of...</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>edges</td>
<td>vertices</td>
</tr>
</tbody>
</table>

Ask the students if they can see any patterns in the number of edges, vertices and faces of pyramids and prisms.

Look for the pattern rules in the chart not only vertically, but horizontally:

**FOR PYRAMIDS:** What do you have to do to the number of vertices in the base to get the number of vertices in the whole pyramid? (Add 1.) What do you have to do to the number of edges in the base to get the number of edges in the whole pyramid? (Multiply by 2.)

**FOR PRISMS:** What do you have to do to the number of vertices in the base to get the number of vertices in the whole prism? (Multiply by 2.) What do you have to do to the number of edges in the base to get the number of edges in the whole prism? (Multiply by 3.) What do you have to do to the number of edges in the base to get the number of faces in the whole prism? (Add 2.) Why? (The number of edges in the base equals the number of side faces of the prism. Add two bases to get the total number of faces.)

Ask your students to add a row to each chart and to fill it in so you can see if they can extend the patterns in the chart.

**ASK:** If you have a prism with 100-gon in the base, how many vertices does this prism have?

How many faces does a pyramid with 200-sided base have? And how many vertices?

**Bonus**
Fill in a row of your chart (no need to draw the base) for 1 000-gon pyramid and prism.
Ask your students to draw the following chart in their notebooks and to fill it in, using the shapes they used for the previous exercise:

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangular Pyramid</th>
<th>Triangular Pyramid</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show your students how you can make a cone from a piece of paper. Ask them where they have seen this shape (cone of ice-cream, clown hat, etc). Does a cone remind some other geometric shape that they have studied in the lesson? (a pyramid) Let volunteers fill in the following chart on the board:

<table>
<thead>
<tr>
<th>Property</th>
<th>Pyramid</th>
<th>Cone</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>two</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>1, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Has a vertex opposite to the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of vertices = # of vertices in the base + 1</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>
Ask your students to imagine a pyramid with 1 000-sided base. Does it look like a cone? Write a comparison paragraph on the board:

**SIMILARITIES:** Both shapes have one base and a vertex at the opposite end. A cone is like a pyramid with a circular base. The more sides the base of a pyramid has, the nearer it is to a circle and so the nearer the pyramid is to a cone.

**DIFFERENCES:** Pyramid has many faces, one polygonal (base), and the other triangular. It has many edges that are straight lines. It has many vertices, not just the one at the point. A cone has only one flat face, that is a circle, and one curved “face”. It has only one curved “edge”.

Show your students how to make a cylinder from a piece of paper. Ask volunteers to fill in the comparison chart:

<table>
<thead>
<tr>
<th>Property</th>
<th>Prism</th>
<th>Cylinder</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>3</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of vertices in the base</td>
<td>many</td>
<td>0</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>2</td>
<td>2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>2, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>0</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td># of vertices = 2 x # of vertices in the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of faces = # of edges in the base + 2</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students to imagine a prism with 100-sided base. Does it look similar to a cylinder?

Write a comparison paragraph on the board:

**SIMILARITIES:** The cylinder and the prism both have two bases. A cylinder is like a prism with a circular base. The more sides the base of a prism has, the nearer the base is to a circle. The nearer the base is to a circle, the nearer the prism is to a cylinder.

**DIFFERENCES:** A prism has many faces, two polygonal (bases), and the other rectangular. It has many edges that are straight lines. It has many vertices. A cylinder has only two flat faces, that are circles, and one curved “face”. It has only two curved “edges” and no vertices at all.
After doing the exercise of comparing cones and cylinders to pyramids and prisms your students should have a better idea how to answer the question on the worksheet that asks them to compare two 3-D shapes.

Draw several shapes on the board and ask your students which 3-D shape they make. **PROMPTS:** circle the bases. How many bases are there? Is it a pyramid or a prism? What is the shape of the base? Are the other faces rectangles or triangles?

**SAMPLES:**

Students may try the activity after that.

**Assessment**
1. Make a property chart for the rectangular and triangular prisms.
2. Write a paragraph comparing rectangular and triangular prisms using the chart.
3. Who am I?
   
   a) I have only rectangular faces.
   b) I have 8 faces, 6 of them are rectangles.
   c) I have 6 edges. No shape has fewer vertices than I do!
   d) I am a prism with 9 edges.
   e) I have one circular base.

**A Game in Pairs**

One player gives a description of a shape; the other has to name the shape. **ADVANCED:** the player gives only two numbers of the three possibilities: number of edges, vertices and faces. **EXAMPLE:** 12 edges and 6 faces: a rectangular prism. 12 edges and 7 faces: a hexagonal pyramid.

Your students might need a table of the number of faces, edges and vertices they made building prisms and pyramids.

**Extension**

3-D Shapes Word Search Puzzle: Please refer to the BLM.
G4-36
Nets

Hold up a rectangular or a pentagonal pyramid. Ask your students to tell you which two types of faces it has. How many bases does it have? And what is the shape of the side faces? How many side faces does it have? If you need to make a net for this pyramid, the easiest way would be to start with a base (draw it on the board and write “base” on it) and to add a side face along each edge of the base (draw one side face and ask volunteers to draw the rest).

Ask students to cut out the nets of the pyramids from the BLM in this manual—they can use the shapes later to help find the answers to QUESTION 1 on the worksheet.

Ask your students, what happens if you cut off one of the side faces and try to re-glue it at some other place. They might actually cut off one of the triangles and try to fit the triangle to some other edge. You might draw an example on the board. Try the following positions: will the shape fold into a pyramid if the face is glued in this position?

At this point, you may let your students do the first activity. When students have finished the activity, draw the picture below on the board and ask if this will work as the pentagonal prism net:

Invite volunteers to come up to the board and draw pictures that will not make a net for a pyramid (not necessary pentagonal). For each drawing, ask your students to explain why this picture is not a net of a pyramid. Invite volunteers to change the drawing so that it will make a net.

Hold up a triangular or a pentagonal prism. Ask your students, which two types of faces it has. How many bases does it have? What is the shape of the side faces? How many side faces does it have? If you wanted to make a net for this prism, the easiest way would be to start with the band of rectangles for side faces and to add the bases. Illustrate this on the board.
Repeat the exercise of cutting off a face of the prism and attaching it in some other place. Do that separately for a base and a side face. Ask students to cut out the nets of the pyramids from the BLM. Let them cut off the faces and try to rearrange them at other places.

Draw several examples of “nets” of triangular prism on the board and ask volunteers to explain why these drawings cannot serve as nets for prisms:

- The bases are not the same
- The middle face is too short
- The bottom base is flipped
- Side face is missing

Invite volunteers to draw more pictures that will not work as prism nets, and ask the class to guess why these drawings cannot be prism nets. For a more challenging task, ask volunteers to draw pictures that might or might not work as nets, and let the class guess if these are nets of prisms.

ACTIVITY 1

Give students square or pentagonal pyramids and ask them to trace the faces on a piece of paper, so that they create a net. Ask them to cut out the nets they have drawn. Let them cut off faces of the net and re-attach the faces at different places. Will the new net fold into the same pyramid? Which edges are places where you would want to re-glue the faces and which are not? Repeat this exercise with a prism. This activity is important—students will explore various ways to create nets for the same solid, rather than memorizing a single net shape.

ACTIVITY 2

Allow your students to play the following game: one partner (or group) draws a net, the other has to guess what shape the net can be folded into. (Use the nets they have created in the previous activity.)

ACTIVITY 3

Give your students Pentomino pieces made of paper or bristol board. Which ones can be folded into a square box without a lid? Let them first try to predict the result, then check it. This activity is a good preparation for QUESTION 4 on the worksheet.
Extensions

1. On grid paper draw as many different nets for cubes as you can (after you try QUESTION 4 on the worksheet). Compare the results with a partner.

2. Describe the nets for different shapes – describe the shape of each face and number of faces of a given shape. Draw a free hand sketch of all the faces that make up a particular 3-D shape. For example, the parts of a square based prism are:

3. Give your students many toothpicks of various lengths and some modelling clay. Ask them to make several triangular pyramids with different bases. For example, students might make acute-angled, right-angled and obtuse-angled triangle bases. Then ask your students to choose two different triangular pyramids and trace their faces to obtain nets for them. Ask your students to compare the nets. Students might also measure the angles of the nets with protractors, and the sides with rulers. Ask your students to make a triangular pyramid with…
   a) …more than one right angle.
   b) ...at least two obtuse angles.

   ADVANCED:
   c) …two right angles and one obtuse angle.
   d) …two right angles and two obtuse angles.

Build-a-Net Game:
Divide your class into groups of 3 to 4 students. Give each group a set of polygons from the BLM Build-a-Net Game. Each student receives 6 shapes, the rest are left in a pile on the side. If a student has a set of polygons that form the net of a pyramid or a prism, she is allowed to turn the set in for a point and take an equal number of new cards from the side pile. Students take turns placing polygons in the central pile. Each time a student places a polygon in the central pile, they pick up a new shape from the side pile. As soon as a student places a polygon in the central pile that can be combined with other polygons in the pile to make a net, that player is allowed to remove the net and scores a point. The game ends when there are no cards left in the side pile. The winner is the player who has created the greatest number of nets.

NOTE: Review the nets of prisms and pyramids that can be built from the shapes in the BLM before starting the game. Ask your students: Which shapes can only be used for one or two nets? (pentagon and hexagon) Which shapes can be used for more than two nets?
G4-37
Sorting 3-D Shapes

GOALS
Students will sort 3-D shapes according to their properties.

PRIOR KNOWLEDGE REQUIRED
Properties of pyramids and prisms
Venn diagrams

VOCABULARY
- cylinder
- cone
- prism
- pyramid
- base
- vertex
- vertices
- edge
- face

Give each student (or team of students) a deck of shape cards and a deck of property cards. These cards are in the BLM section. (If you have enough 3-D shapes have students use 3-D shapes instead of the cards.) Let them play the following games:

3-D Shape Sorting Game: Each student flips over a property card and then sorts the shapes onto two piles according to whether a shape on a card has the property or not.

Students get a point for each card that is on the correct pile. (If you prefer, you could choose a single property for the class and have everyone sort the shapes using that property.)

Once students have mastered this sorting game they can play the next game.

3-D Venn Diagram Game: Give each student a copy of the Venn diagram sheet in the BLM section (or have students create their own Venn diagram on a sheet of construction paper or bristol board). Ask students to choose two property cards and place one beside each circle of the Venn diagram. Students should then sort their shape cards using the Venn diagrams. Give 1 point for each shape that is placed in the correct region of the Venn diagram.

Assessment

Use the shapes below to complete the following Venn diagram:

1. One or more rectangular faces    2. One or more triangular faces

Extension

Draw a Venn diagram to sort the shapes on the worksheet according to the properties:

1. Pyramid    2. One or more triangular faces.

What do you notice about your Venn diagram? Explain why part of one of the circles is empty.
Isoparametric Drawings

GOALS
Students will draw simple shapes built from interlocking cubes on isometric dot paper.

PRIOR KNOWLEDGE REQUIRED
Understand a drawing on isometric dot paper

VOCABULARY
isometric dot paper
top view
interlocking cubes

Project a sheet of isometric dot paper (use BLM, if necessary) onto the board using the overhead projector. Show students how to draw a cube using the dots. Start from the top face, then draw the vertical edges (no hidden ones!), and then draw the visible bottom edges.

Step 1

Step 2

Step 3

Explain that to create an isoparametric drawing, it helps to start from the top. Look at the topmost layer and draw the top face or faces first. Then draw the vertical edges that are part of the topmost layer as you did with the single cube.

Hold up a shape made with three cubes:

Invite a volunteer to draw the top layer (a single cube). What does the next layer look like? It consists of two cubes. Take two cubes locked together and compare this shape to the original shape: Which edges of the new shape are hidden (by the top cube) in the original shape? Which visible edges of the original shape are already drawn (because they are the bottom edges of the cube)? Ask a volunteer to draw the remaining visible edges of the second layer.

You may wish to do the worksheets as a class—so that volunteers draw pictures from the worksheet on the board. Students may find it easier to copy a shape onto isoparametric dot paper if they start by shading the top layer of the shape.

Give your students some interlocking cubes and ask them to build the figures from the second exercise on the worksheet.
Isometric Drawings

**GOALS**
Students will build shapes from interlocking cubes and draw top views of shapes drawn on isometric dot paper.

**PRIOR KNOWLEDGE REQUIRED**
Understand a drawing on isometric dot paper

**VOCABULARY**
isometric dot paper
top view
interlocking cubes

Explain that sometimes it is hard to read the drawing on the isoparametric dot paper, because some cubes are hidden and some edges overlap. Project the drawing given here on the board as an example:

Explain that a very convenient way to understand the drawing is to try to construct the “mat plan” of the shape. A mat plan represents the bottom level of the shape. In this case the mat plan would look like this:

Shade one square on the mat plan, as shown. Invite a volunteer to shade the column that stands above this square in the isoparametric diagram. How many cubes are in it? (3) Ask the volunteer to write “3” in the shaded square. Ask more volunteers to finish the mat plan. Remove the drawing and ask another volunteer to construct the shape using the mat plan. Give your students more examples like those in QUESTION 2 of the worksheet.

**Assessment:**
Draw the mat plan and build the shape from interlocking cubes:
Extension

Draw the figures from the worksheets on regular dot paper. For example:

![Figure](image)

G4-40

Geometry in World

The worksheet **G4-40: Geometry in World** is an extension and review worksheet. It can complement the presentation of cross-curricular projects. Here are some project ideas:

**Symmetry:**

1. Flags and Coats of Arms of Canadian provinces/cities. Which ones have lines of symmetry? Which ones have more than one line of symmetry? (None!)
2. Flags of the world. Make a list of world countries with flags that have 2 lines of symmetry.
3. Coats of Arms of Soccer/Baseball/Hockey clubs. Which ones have lines of symmetry? Which ones have more than one line of symmetry?
4. Cultural diversity: Alphabets. Use a non-Latin alphabet to find letters that have lines of symmetry. Are there symbols that have more than one line of symmetry?
5. Make several designs of snowflakes. How many lines of symmetry do your snowflakes have?

**Geometry in everyday life:**

1. Bee hives and hexagons—Research why bees build hexagonal shapes in the hive: Why not rectangular or triangular shapes?
2. Bridges—Which geometric shapes are used in bridges design? Try to build a bridge with rectangles. Put a heavy weight on it and watch the bridge collapse. Triangles are rigid—you cannot change the shape of the triangle without altering the side lengths. If you add diagonals to the rectangles, the bridge will stand, but then it is built with triangles.
3. Geometrical floor patterns—Which shapes can be used to make a floor pattern? Design a pattern using only regular shapes. Design a pattern using only equilateral shapes of two kinds: triangles and rhombuses; octagons and squares; dodecagons (12 sides) and triangles.

**GOALS**

Students will see applications of geometry in real life.

**PRIOR KNOWLEDGE REQUIRED**

Symmetry
Transformations
3-D Shapes

**VOCABULARY**

- line of symmetry
- rotation
- reflection
- slide
- pyramid
- prism
- cone
- cylinder
- square
- rectangle
- pentagon
- hexagon
- octagon
- circle
- triangle
- polygon
4. Floor patterns—Use reflections and rotations to create a pattern design.

5. Which geometric shapes are used in various traffic signs?

6. A robot is used to draw letters. The robot understands the following commands:
   - Draw a line from the current position to the point ___ units up/down and ___ units right/left
   - Move ___ units up/down and ___ units right/left without drawing a line

   Give the robot directions to draw various letters of the alphabet. Can you make the robot write your name?

Geometry and history:

1. Ancient Egypt—Which geometric shapes were used in ancient Egyptian buildings?

2. Ancient Egypt – the Pyramid of Khufu—how many right angles can you find there? Find more interesting facts about Egyptian Pyramids.

3. Ancient Maya—Which geometric shapes were used in ancient Mayan temples?

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**G4-41**

**Problems and Puzzles**

The worksheet **G4-41**: Problems and Puzzles is a review worksheet and may be used for extra practice.
G4 Partie 2 : Liste — Fiches reproductibles

Jeu de tri des formes en 3-D ................................................................. 2
Jeu — Construis un développement .............................................. 6
Papier à points ................................................................................. 8
Papier quadrillé .............................................................................. 9
Papier à points isométrique ............................................................. 10
Carte de la Saskatchewan ................................................................. 11
Développements pour les formes en 3-D ..................................... 12
 Blocs de régularités ....................................................................... 15
Pentominoes .................................................................................... 16
Prisme triangulaire avec base scalène ........................................... 17
Diagramme de Venn ......................................................................... 18
Casse-tête — Mots cachés (formes en 3-D) .................................. 19
Jeu de tri des formes en 3-D
Jeu de tri des formes en 3-D (suite)
### Jeu de tri des formes en 3-D (suite)

<table>
<thead>
<tr>
<th>Plus de quatre faces</th>
<th>Base de forme carrée</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base de forme triangulaire</td>
<td>Moins de six faces</td>
</tr>
<tr>
<td>Deux faces de forme carrée ou plus</td>
<td>Quatre faces de forme triangulaire ou plus</td>
</tr>
</tbody>
</table>
### Jeu de tri des formes en 3-D (suite)

<table>
<thead>
<tr>
<th>Dix arêtes ou plus</th>
<th>Six sommets ou moins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quatre sommets ou plus</td>
<td>Exactement douze arêtes</td>
</tr>
<tr>
<td>Pyramides</td>
<td>Prismes</td>
</tr>
</tbody>
</table>
Jeu — Construis un développement
Jeu — Construis un développement (suite)
Papier à points
Papier quadrillé (1 cm)
Papier à points isométrique
Carte de la Saskatchewan

- A: Uranium City
- B: Parc provincial de la rivière Clearwater
- C: Lac Wollaston
- D: Prince Albert

- 1: Maple Creek
- 2: Swift Current
- 3: Saskatoon
- 4: 224 km, 260 km, 76 km, 110 km
- 5: Regina, Moose Jaw, Weyburn

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Développements pour les formes en 3-D

Pyramide carrée

Pyramide triangulaire
Développements pour les formes en 3-D (suite)
Développements pour les formes en 3-D (suite)

Pyramide pentagonale

Prisme pentagonal
Blocs de régularités

Triangles

Carrés

Losanges

Trapèzes

Hexagones
Pentominos
Prisme droit triangulaire avec base scalène
Diagramme de Venn
Casse-tête — Mots cachés (formes en 3-D)

MOTS À CHERCHER :

- base
- arête
- face
- hexagonal
- développement
- pentagonal
- prisme
- pyramide
- rectangle
- squelette
- triangle
- carré
- sommets
PS4-10  Choosing Strategies

Teach this lesson after: 4.2 Geometry

Goals:
Students will solve problems and puzzles using any of the problem-solving strategies studied so far in the Grade 4 problem-solving lessons.

Prior Knowledge Required:
Can compare numbers using place value up to decimal tenths (for Problem Bank 1)
Can fluently add and subtract within 1000 (for Problem Banks 2–7, 19)
Can multiply two-digit numbers by one-digit numbers (for Problem Banks 3–5, 9, 20, 21)
Can round four-digit whole numbers to the nearest ten, hundred, or thousand (for Problem Bank 8)
Can divide two-digit numbers by one-digit numbers (for Problem Banks 20, 21)
Can find the area of rectilinear shapes on grid paper (for Problem Bank 22)
Can evaluate a simple fraction of a whole number (for Problem Bank 23)

Vocabulary: decimal, decimal point, pentomino, reflection

Materials:
scissors (see Problem Bank 22)
grid paper or BLM 1 cm Grid Paper (p. 7, see Problem Bank 22)

NOTE: The following Problem Bank questions reflect a selection of the problem-solving strategies used in the problem-solving lessons for Grade 4. Students will need to choose among all the strategies they have learned this year to solve the problems.

Problem Bank
1. I am a decimal with two digits after the decimal point. What number am I?
a) I am less than 0.1. My hundredths digit is 7.
b) I am between 0.4 and 0.5. My digits add to 8.
c) I am less than 1. My tenths digits and hundredths digits are equal. My digits add to 12.
d) I am between 1 and 10. All my digits are equal. My digits add to 9.
Answers: a) 0.07, b) 0.44, c) 0.66, d) 3.33

2. Add mentally: 1 + 11 + 111 + 1111.
   Solution: In the ones place, there are four 1s, in the tens place, there are three 1s, in the hundreds, there are two 1s, and in the thousands, there is one 1, so the sum is 1234.

3. Kathy has exactly 7 loonies and an unknown number of 5 dollar bills, 10 dollar bills, and 20 dollar bills. Which of these can be the total value of the money: $90, $91, $92, or $93?
   Answer: $92

4. On your calculator, the key with the digit 4 isn't working. What could you press instead to find ...
a) 214 + 63    b) 241 + 63   c) 841 + 34   d) 34 × 15   e) 42 × 8
Sample answers: a) 210 + 67, b) 200 + 100 + 3 + 1, c) 800 + 70 + 5, d) 33 × 15 + 15, e) 32 × 8 + 10 × 8

5. Fill in the blank.
a) (83 × 2) + (83 × 4) = 83 × ____
b) (83 × 41) + (2 × 41) = ____ × 41
c) (72 × 41) + (72 × 3) + (3 × 39) + (3 × 5) = 75 × ____
Answers: a) 6, b) 85, c) 44

6. Add: 98 + 98 + 98 + 98 + 98.
Answer: 490

Answer: 600

8. A number rounds to 500 when rounded to the nearest hundred and 450 when rounded to the nearest ten. The digits add to 12. What number is it?
Answer: 453

9. Fill in the blank: 3131 = 31 × ____.
Answer: 101

10. Fill in the blank: 2 × 3 × 4 × 5 × 6 = 3 × 4 × 5 × 6 × ____.
Answer: 2

11. a) Jin and Vicky’s ages add to 32. What will they add to 3 years from now?
b) Jin, Vicky, and Tessa’s ages add to 32. What will they add to 3 years from now?
Answer: a) 38, b) 41

12. Jasmin has 85 marbles and Don has 92 marbles. Can Don give some marbles to Jasmin so that they have the same number of marbles? Explain.
Answer: No. Don has 7 more marbles than Jasmin, and since 7 is not a multiple of 2, Don can’t give Jasmin half of the extra marbles.

13. Ethan has 5 apples. Hanna has 8 apples. Sally has 11 apples.
a) How many apples do they have altogether?
b) They decide to share the apples equally. How many apples should each person get?
c) Who doesn’t need to give or receive any apples?
d) How many apples do the other two people need to give or receive?
Answers: a) 24, b) 8, c) Hanna, d) Sally needs to give away 3 apples and Ethan needs to receive 3 apples
14. There are three sets of numbers.
Set A: 2, 3, 5  Set B: 4, 5, 6  Set C: 1, 8, 11
a) What is the sum of the numbers in each set?
b) Jane traded exactly two numbers between sets. When she was done, all sets had the same sum.
   i) Which set did she leave alone? How do you know?
   ii) What two numbers did she trade?

**Answers:**
a) A: 10, B: 15, C: 20
b) i) Jane must have left Set B alone because it has the number in the middle; if one sum increased and another decreased, it must be the sum in the middle that stayed the same;
   ii) 3 and 8

**NOTE:** Problem Banks 15 to 18 should be done in order.

15. Clara and Tom play a game. The rules are that Player 1 rolls three dice and then Player 2 rolls three dice. They both win when they get the same total. To help them get the same total, players are allowed to trade exactly one die for one die. For example: Player 1 rolls 2, 3, 5 and Player 2 rolls 1, 5, 6. Player 1’s total is 10 and Player 2’s total is 12. Player 1 can trade the 5 for Player 2’s 6 so that they each get the same total of 11.
Clara rolls 2, 3, 6. Tom rolls 4, 5, 6.
a) What do Clara’s dice add to and what do Tom’s dice add to?
b) How far apart are their totals?
c) When they trade, who should give away a bigger number? Why?
d) Clara and Tom make the following trades. Do they win?
   i) They trade Clara’s 3 for Tom’s 4.
   ii) They trade Clara’s 3 for Tom’s 5.
   iii) They trade Clara’s 2 for Tom’s 4.
   iv) They trade Clara’s 2 for Tom’s 6.
e) When they won, how far apart were the numbers they traded?

**Answers:**
a) Clara’s dice add to 11 and Tom’s dice add to 15; b) 4; c) Tom, because his total is greater; d) i) Clara has 12 and Tom has 14, ii) Clara has 13 and Tom has 13, iii) Clara has 13 and Tom has 13, iv) Clara has 15 and Tom has 11; e) 2

16. Clara and Tom roll the given numbers. Help them win.
a) Clara: 4, 6, 6  Tom: 1, 2, 5  b) Clara: 3, 3, 5  Tom: 1, 2, 4
b) Clara: 2, 3, 6  Tom: 4, 4, 5

**Answers:**
a) trade Clara’s 6 for Tom’s 2, b) trade Clara’s 3 for Tom’s 1, c) trade Clara’s 3 for Tom’s 4

17. Clara rolls 1, 4, 5. Tom rolls 2, 3, 6.
a) What do Clara’s dice add to and what do Tom’s dice add to?
b) How far apart are their totals?
c) When they trade, who should give away a bigger number? Why?
d) Clara and Tom make the following trades. Do they win?
   i) They trade Clara’s 1 for Tom’s 2.
   ii) They trade Clara’s 5 for Tom’s 6.
   iii) They trade Clara’s 1 for Tom’s 3.
e) Can Clara and Tom win? Explain.
Answers:
a) Clara’s dice add to 10 and Tom’s dice add to 11; b) 1; c) Tom, because his dice add to more; 
d) i) no, ii) no, iii) no; e) no, because no matter what they trade, if Tom gives away a bigger 
number than Clara, Clara’s total will be bigger than Tom’s because they were only 1 apart to 
begin with

a) Clara rolls 1, 2, 6. Tom rolls 1, 6, 6. b) Clara rolls 1, 1, 3. Tom rolls 1, 6, 6. 
Answers: a) no, because their totals are 4 apart, so they would have to trade numbers that are 
2 apart to win, but there are no such numbers; b) no, because their totals are 8 apart, so they 
would have to trade numbers that are 4 apart to win, but there are no such numbers

19. You can find the reflection of a number by placing a mirror to the right of the number. 
a) The dashed line is a mirror. Draw the reflection of each digital clock digit:

b) Which digits, when written like on a digital clock, have a reflection that is also a digit? 
c) What is the reflection of the number? 

i) 202 ii) 11 iii) 55 iv) 218

d) What can the two numbers be if a number and its reflection … 

i) add to 7 ii) add to 2 iii) multiply to 10 iv) add to 99

v) add to 909 vi) add to 9009 vii) add to 9999 viii) add to 50

Bonus: 
ix) add to 5000 x) are the same

Answers:
a) 

b) 0, 1, 2, 5, and 8

c) i) 505, ii) 11, iii) 22, iv) 815
d) i) 2 and 5, ii) 1 and 1, iii) 2 and 5, iv) 18 and 81, v) 108 and 801, vi) 1008 and 8001, vii) 1818 
and 8181 or 1188 and 8811, viii) 25 and 25, Bonus: ix) 2185 and 2815, x) 0, 1, 8, 25, 52, 205, 
502, 215, 512, 285, 582, 2255, 5522, and many more

20. Pens and markers each cost a whole number of dollars. Three pens and two markers cost 
$34. Two pens and three markers cost $31. 
a) If three pens and two markers cost more than two pens and three markers, what costs more, 
a pen or a marker? Explain.
b) How much does each pen and each marker cost? Use the information from part a) to make 
sure your answer makes sense.
Sample solution: b) Start the cost of markers at $1 each in the equation 3 pens and 2 markers cost $34, and continue raising the cost of each marker until the cost of a pen becomes less than the cost of a marker.

<table>
<thead>
<tr>
<th>Cost of 1 marker</th>
<th>Cost of 2 markers</th>
<th>Cost of 3 pens</th>
<th>Cost of 1 pen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$2</td>
<td>$34 − $2 = $32</td>
<td>X</td>
</tr>
<tr>
<td>$2</td>
<td>$4</td>
<td>$34 − $4 = $30</td>
<td>$10</td>
</tr>
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<td>$6</td>
<td>$34 − $6 = $28</td>
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<td>X</td>
</tr>
<tr>
<td>$8</td>
<td>$16</td>
<td>$34 − $16 = $18</td>
<td>$6</td>
</tr>
</tbody>
</table>

We can stop here because we are at the point where markers cost $8 each and pens cost $6 each, but we need a marker to cost less than a pen. So, there are only two possibilities to check with the second given sentence (2 pens and 3 markers cost $31). Looking at the first possibility, if 1 marker costs $2 and 1 pen costs $10, then 2 pens and 3 markers cost $26; in the second possibility, if 1 marker costs $5 and 1 pen costs $8, then 2 pens and 3 markers cost $31. So 1 marker costs $5 and 1 pen costs $8.

Answer: a) a pen costs more because, compared to buying two pens and three markers, an extra pen adds a greater cost than an extra marker.

21. Shirts and crayons each cost a whole number of dollars. Anton pays $30 for 3 shirts and 2 crayons. Lily pays $23 for 1 shirt and 5 crayons. How much does each shirt and each crayon cost?
Answer: each crayon costs $3 and each shirt costs $8

22. A pentomino is made of 5 squares in the same way a domino is made of 2 squares. The picture below shows all 12 pentominoes. Using grid paper or BLM 1 cm Grid Paper, create and cut out the pentominoes.

a) What is the total area of all 12 pentominoes?
b) For your answer to part a), what pairs of numbers multiply to the number?
c) For which pairs of numbers from part b) can you make the 12 pentominoes into a rectangle with those dimensions? You will need to use the pentominoes you created and cut out.
**Answers:**  a) 60 units²; b) 1 × 60, 2 × 30, 3 × 20, 4 × 15, 5 × 12, 6 × 10; c) 3 × 20, 4 × 15, 5 × 12, 6 × 10

23. Solve this problem by working backwards: \(\frac{1}{2}\) of \(\frac{2}{3}\) of \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is ______.

   a) \(\frac{4}{5}\) of 30 is ______.
   b) \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is ______.
   c) \(\frac{2}{3}\) of \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is ______.
   d) \(\frac{1}{2}\) of \(\frac{2}{3}\) of \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is ______.

**Solutions:** a) 24, b) 18, because \(\frac{3}{4}\) of 24 is 18, c) 12, because \(\frac{2}{3}\) of 18 is 12, d) 6, because \(\frac{1}{2}\) of 12 is 6
1 cm Grid Paper
Contents

Patterns & Algebra – Part 1
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Patterns & Algebra 1 – AP Book 4.1

AP Book PA4-1
page 1

1. a) 3
   b) 5
   c) 2
   d) 5
   e) 4
   f) 4
   g) 5
   h) 2
   i) 3
   j) 5
   k) 5
   l) 3
   m) 4
   n) 3
   o) 4
   p) 5
   q) 2
   r) 9
   s) 2
   t) 4
   BONUS: u) 5
   v) 4
   w) 6
   x) 5
   y) 4
   z) 5
   aa) 9
   bb) 8

2. a) 3
   b) 4
   c) 3
   d) 7
   e) 22
   f) 22
   g) 22
   h) 25
   i) 38
   j) 36
   k) 41
   l) 37
   BONUS: m) 69
   n) 90
   o) 45
   p) 72
   q) 60
   r) 68
   s) 102
   t) 101

3. a) 2
   b) 4
   c) 3
   d) 2
   e) 7
   f) 3

AP Book PA4-2
page 3

1. a) 9
   b) 10
   c) 10
   d) 7
   e) 22
   f) 22
   g) 22
   h) 25
   i) 38
   j) 36
   k) 41
   l) 37
   BONUS: m) 69
   n) 90
   o) 45
   p) 72
   q) 60
   r) 68
   s) 102
   t) 101

2. a) 9
   b) 10
   c) 10
   d) 7
   e) 22
   f) 22
   g) 22
   h) 25
   i) 38
   j) 36
   k) 41
   l) 37
   BONUS: m) 69
   n) 90
   o) 45
   p) 72
   q) 60
   r) 68
   s) 102
   t) 101

2. a) 10
   b) 11
   c) 12
   d) 20
   e) 40
   f) 23
   g) 37
   h) 31
   i) 45
   j) 24

AP Book PA4-3
page 4

1. a) Gap = 2; 7, 9, 11
   b) Gap = 2; 6, 8, 10
   c) Gap = 4; 15, 19, 23
   d) Gap = 4; 14, 18, 22
   e) Gap = 3; 10, 13, 16
   f) Gap = 4; 17, 21, 25
   BONUS: g) Gap = 10; 31, 41, 51
   h) Gap = 7; 26, 33, 40
   i) Gap = 3; 30, 33, 36
   j) Gap = 2; 92, 94, 96

2. a) 77 tonight;
   b) 82 tomorrow night.
   2
   16 on Tuesday;
   20 on Wednesday;
   28 on Friday.

3. 16 on Tuesday;
   20 on Wednesday;
   28 on Friday.

AP Book PA4-4
page 5

1. a) – 3
   b) – 5
   c) – 5
   d) – 7
   e) – 7
   f) – 3
   g) – 6
   h) – 6
   BONUS: i) – 9
   j) – 6
   k) – 7
   l) – 8
   m) – 11
   n) – 7
   o) – 2
   p) – 4
   q) – 4
   r) – 5
   s) – 4
   t) – 7
   u) – 6
   v) – 6
   w) – 7
   x) – 7
   y) – 8
   z) – 9

2. a) – 4
   b) – 2
   c) – 5
   d) – 1
   e) – 4
   f) – 3
   g) – 6
   h) – 6
   BONUS: i) – 9
   j) – 6
   k) – 7
   l) – 8
   m) – 11
   n) – 7
   o) – 2
   p) – 4
   q) – 4
   r) – 5
   s) – 4
   t) – 7
   u) – 6
   v) – 6
   w) – 7
   x) – 7
   y) – 8
   z) – 9

3. a) – 5
   b) – 6
   c) – 2
   d) – 2
   e) – 5
   f) – 4
   BONUS: g) – 9
   h) – 6
   i) – 7
   j) – 4
   k) – 6
   l) – 8
   m) – 11
   n) – 7
   o) – 2
   p) – 4
   q) – 4
   r) – 5
   s) – 4
   t) – 7
   u) – 6
   v) – 6
   w) – 7
   x) – 7
   y) – 8
   z) – 9

4. a) 3
   b) 3
   c) 2
   d) 4
   e) 4
   f) 4
   g) 5
   h) 2
   i) 3
   j) 5
   k) 5
   l) 3
   m) 4
   n) 3
   o) 4
   p) 5
   q) 2
   r) 9
   s) 2
   t) 4
   BONUS: u) 5
   v) 4
   w) 6
   x) 5
   y) 4
   z) 5
   aa) 9
   bb) 8

5. a) – 3
   b) – 2
   c) – 3
   d) – 11
   e) – 7
   f) – 5
   g) – 7
   h) – 9

AP Book PA4-5
page 7

1. a) 1
   b) 9
   c) 4
   d) 8
   e) 3
   f) 6
   g) 4
   h) 7
   BONUS: i) 24
Patterns & Algebra 1 – AP Book 4.1 (continued)

Answer Key for AP Book 4.1

1. a) 3
b) 7
c) 5
d) 12
e) 16
f) 19
g) 21
h) 28
i) 35

2. a) 3
b) 7
c) 5
d) 12
e) 16
f) 19
g) 21
h) 28
i) 35

3. a) Gap = – 1; 7, 6, 5
b) Gap = – 2; 8, 6, 4
c) Gap = – 1; 20, 19, 18
d) Gap = – 3; 15, 12, 9
e) Gap = – 10; 60, 50, 40
f) Gap = – 5; 30, 25, 20

4. $33

5. 9 pears

BONUS:

3. Answers will vary.

4. c) is made by adding 3

5. Faruq is right: the gap between each pair of numbers is 8.

1. a) Start at 3, add 4.
b) Start at 2, add 4.
c) Start at 2, add 2.
d) Start at 1, add 5.
e) Start at 5, add 4.
f) Start at 12, add 6.
g) Start at 2, add 8.
h) Start at 3, add 3.
i) Start at 6, add 7.

BONUS:

2. a) 17, 22, 27; Need 27 blocks for the 6th figure.
b) 12, 15, 18; Need 18 blocks for the 6th figure.
c) 18, 23, 28; Need 28 blocks for the 6th figure.

3. a) No, she would need 15 blocks.
b) No, she would need 16 blocks.
c) Yes, she would need 13 blocks.

4. a) 12
b) 18

AP Book PA4-12

1. a) 7
b) 10
c) 8
d) 9
e) 10
f) 14

2. Teacher to check drawings.
   Figure 5 would have 20 line segments.

3. Teacher to check drawings.
   Figure 5 would have 15 line segments.

4. Teacher to check drawings.
   a) 16
   b) 19
   c) 22

5. Teacher to check drawings
   a) 13
   b) 15
   c) 17

6. a) 3 – 15
   4 – 20
   5 – 25
   5 fox would have 25 cubs

b) 3 – 12
   4 – 16
   5 – 20
   5 Woodchucks would have 20 pups

c) 3 - 6
   4 – 8
   5 – 10
   5 deer would have 10 fawns.

d) 3 - 9
   4 – 12
   5 – 15
   5 osprey would have 15 eggs.

7. a) The candle burns down 3 cm each hour.
   b) It will be 15 cm high at 11pm.

8. $14
9. $20
10. $39
11. 11 L

AP Book PA4-14

1. $27
2. No.
3. 39 cm
4. Amanda $58
   Jacob $54
5. Answers will vary.
6. Edith – 19 cm
   Ron – 13 cm
7. Chloe – 8 cm
   Dora – 6 cm

8. No. Peter would need 10 triangles.
9. Hanna would need 15 triangles

AP Book PA4-13

1. 16 bikes
2. $50
3. 14 cm
4. 21 cm
5. 46 pages
6. 15 trees
Number Sense 1 – AP Book 4.1

AP Book NS4-1
page 22
1. a) Tens
   b) Hundreds
   c) Ones
   d) Thousands
   e) Thousands
   f) Hundreds
   g) Tens
   h) Hundreds
   i) Ones
   j) Thousands

2. a) Thousands
   b) Hundreds
   c) Tens
   d) Ones
   e) Tens
   f) Thousands
   g) Hundreds
   h) Tens
   i) Ones

3. a) 5, 2, 3, 1
   b) 8, 0, 5, 3
   c) 0, 4, 8, 9
   d) 0, 0, 2, 7
   e) 9, 1, 0, 4
   f) 4, 6, 8, 7

AP Book NS4-2
page 23
1. a) 7, 40, 500, 6000
   b) 1, 30, 200, 8000
   c) 5, 0, 200, 3000

2. a) 30
   b) 30
   c) 300
   d) 3000
   e) 300
   f) 30
   g) 3000
   h) 3
   i) 3000
   j) 3
   k) 300
   l) 30

3. a) 500

AP Book NS4-3
page 24
1. a) 7
   b) 6
   c) 8
   d) 23
   e) 32
   f) 95
   g) 270
   h) 479
   i) 9217
   j) 5391

2. a) One
   b) Seven
   c) Nine
   d) Six
   e) Two
   f) Three
   g) Twenty-one
   h) Sixty-seven
   i) Forty-three
   j) Fifty-five

3. a) 5, 2, 3, 1
   b) 8, 0, 5, 3
   c) 0, 4, 8, 9
   d) 0, 0, 2, 7
   e) 9, 1, 0, 4
   f) 4, 6, 8, 7

AP Book NS4-4
page 26
1. a) 2 hundreds + 5 tens + 2 ones = 252
   b) 3 hundreds + 3 tens + 6 ones = 336
   c) 1 hundred + 8 tens + 5 ones = 185
   d) 5 hundreds + 0 tens + 7 ones = 507
   e) 6 hundreds + 2 tens + 3 ones = 623

2. Teacher to check.

3. Teacher to check.

4. a) 2 thousands + 3 hundreds + 3 tens + 2 ones = 2332
   b) 3 thousands + 2 hundreds + 0 tens + 6 ones = 3206
   c) 1 thousand + 2 hundreds + 3 tens + 9 ones = 1239

AP Book NS4-5
page 28
1. Teacher to check
2. a) Two thousand two hundred thirty-four (2234)
   b) One thousand three hundred sixty-eight (1368)

3. a) 2, 4, 2, 7
   b) 4, 5, 6, 9
   c) 3 thousands + 8 hundreds + 7 tens + 5 ones
   d) 7 thousands + 2 hundreds + 11 tens
   e) 6 hundreds + 2 tens + 3 ones

4. a) 2000 + 600 + 10 + 3
   b) 20 + 7
   c) 40 + 8
   d) 1000 + 200 + 30 + 2
   e) 6000 + 100 + 3
   f) 3000 + 500 + 70
   g) 500 + 90 + 8
   h) 2000 + 900 + 1

5. a) 36
   b) 52
   c) 65
   d) 468
   e) 523
   f) 3253
   g) 5721
   h) 645
   i) 8972

BONUS:
   j) 607
   k) 906
   l) 870

Answer Key for AP Book 4.1
m) 5 100  
 n) 5 020  
 o) 6 002  
 p) 8 013  
 q) 9 004  
 r) 4 105  
 s) 6 320  
 t) 8 200  
 u) 3 010  

6. a) 3  
 b) 80  
 c) 20  
 d) 70  
 e) 800  
 f) 200  

BONUS:  
 g) 2  
 h) 20  
 i) 7 000  
 j) 80  
 k) 200, 60  
 l) 700, 3  
 m) 20, 1  
 n) 900  

7. Teacher to check base ten models.  
 a) 2 000 + 300 + 10 + 7  
 b) 1 000 + 400 + 40 + 6  

BONUS:  
 8. 1 239  

AP Book NS4-6  
page 31  
1. a) 1 hundred + 2 tens + 5 ones = 125  
 b) 2 hundreds + 3 tens + 4 ones = 234  
 c) 3 hundreds + 7 ones = 307  
 d) 100 + 20 + 5 = 125  
 e) 200 + 30 + 4 = 234  
 f) 300 + 7 = 307  

2. a) 41  
 b) 29  
 c) 731  
 d) 190  
 e) 65  

3. Teacher to check base ten models.  
 a) 460  
 b) 1 300  

AP Book NS4-7  
page 32  
1. a) (i) 268;  
(Two hundred sixty-eight)  
 (ii) 354;  
(Three hundred fifty-four)  
 (ii) is larger  
 b) (i) 2 362;  
(Two thousand three hundred sixty-two)  
 (ii) 1 350;  
(One thousand three hundred fifty)  
 (i) is larger  
 c) Compare the number of blocks of the largest size  

2. a) (i) 424  
 (ii) 224  
 (i) is larger  
 b) (i) 1 232  
 (ii) 1 230  
 (i) is larger  

3. Teacher to check base ten models.  
 a) 97 is greater than 87  
 b) 7, 20, 300  
 (427 is greater than 327)  
 c) 800  
 d) 300  
 e) 200  
 f) 100  

AP Book NS4-8  
page 33  
1. a) 7, 80  
 b) 7, 90  
 c) Four thousand seven hundred  
 d) Six thousand forty  
 e) Two thousand nine hundred eighty-one  
 f) Five thousand eight hundred sixty-two  

2. a) Circle tens, 2 475  
 b) Circle hundreds, 1 360  
 c) Circle ones, 4 858  
 d) Circle thousands, 7 325  
 e) Circle hundreds, 584  
 f) Circle ones, 2 906  
 g) Tens, 875  
 h) Ones, 238  

3. a) Circle tens, 1 597  
 b) Circle hundreds, 6 542  
 c) Circle thousands, 6 034  
 d) Circle ones, 9 432  

4. Underline Circle  
 a) Tens 2 351  
 b) Tens 5 275  
 c) Hundreds 6 327  
 d) Hundreds 7 923  
 e) Ones 5 542  
 f) Thousands 9 234  
 g) Ones 3 502  
 h) Thousands 7 254  
 i) Tens 2 145  

5. a) 3 603  
 b) 5 012  
 c) 6 726  
 d) 3 729  
 e) 8 175  
 f) 6 000  
 g) 389  

AP Book NS4-9  
page 35  
1. a) “10 more”  
 b) “10 less”  
 c) “10 less”  
 d) “10 more”  

2. a) “100 more”  
 b) “100 less”  
 c) “100 more”  
 d) “100 less”  

3. a) “1 000 more”  
 b) “1 000 less”  

4. a) 3 20, 800  
 b) 3, 10, 800  
 c) 823 is 10 more than 813  
 d) 8, 40, 200  
 e) 8, 40, 300  
 f) 248 is 100 less than 348
5. b) Circle the hundreds.
1 382 is 100 less than 1 482
c) Circle the thousands.
6 830 is 1 000 less than 7 830
d) Circle the thousands.
3 621 is 1 000 more than 2 621
e) Circle the tens.
8 405 is 10 less than 8 415
f) Circle the ones.
5871 is 1 less than 5872.

AP Book NS4-10

page 36
1. a) 297
   b) 353
c) 1 972
d) 3 613
e) 692
 f) 4 035
g) 6 921
 h) 3 195
 i) 7 305
 j) 5 253
 2. a) 753
    b) 2 392
c) 9 045
d) 1 370
e) 2 052
 f) 9 321
g) 347
 h) 673
 i) 832
 j) 2 387
 k) 1 801
 l) 4 316
 3. a) 10
    b) 100
c) 10
d) 100
e) 1 000
 f) 100

AP Book NS4-11

page 37
1. a) 60, 70, 80
   b) 40, 50, 60
c) 80, 90, 100
d) 53, 63
e) 57, 67, 77
 f) 45, 55, 65
g) 79, 89, 99
 h) 31, 41
 i) 130, 140, 150
 j) 190, 200, 210
 2. a) 400, 500, 600
    b) 900, 1 000, 1 100
c) 600, 700, 800
d) 1300, 1 400, 1 500
 3. a) 200
    b) 400
c) 500

AP Book NS4-13

page 39
1. a) 3, 12 = 4, 2
    b) 2, 15 = 3, 5
c) 2, 13 = 3, 3
 d) 4, 19 = 5, 9
 2. a) 6 + 2 = 8, 5 = 85
    b) 3 + 8 = 11, 2 = 112
c) 5 + 3 = 8, 1 = 81
 d) 7 + 1 = 8, 7 = 87
 e) 6 + 2 = 8, 9 = 89
 f) 1 + 5 = 6, 2 = 62
 3. a) 2, 3
    b) 5, 6
c) 8, 6
d) 5, 8
 e) 1, 8
 f) 7, 2
g) 8, 0
 h) 0, 7
 i) 9, 8
 4. a) 5 + 1 = 6, 1
    b) 2 + 1 = 3, 5
c) 6 + 1 = 7, 7
 d) 6 + 1 = 7, 2
 e) 2 + 1 = 3, 7
 f) 5 + 1 = 6, 0
 5. a) 4, 3, 4
    b) 7, 1, 1
c) 4 hundreds + 5 ones
 d) 4 hundreds + 4 tens + 7 ones
 6. a) 4 hundreds + 3 tens + 9 ones
    b) 9 hundreds + 5 tens + 2 ones
c) 6 hundreds + 3 tens + 6 ones
d) 7 hundreds + 7 tens + 1 one
 e) 2 hundreds + 6 tens + 3 ones
 7. a) 3 + 1 = 4, 2
    b) 4 + 1 = 5, 3
c) 7 + 1 = 8, 4
 8. a) 6 thousands + 2 hundreds + 3 tens + 1 one
### Number Sense 1 – AP Book 4.1 (continued)

#### 9. No – Roger needs 88 more ones cubes to build the model.

**AP Book NS4-14**

**page 42**

Teacher to check drawings.

1. \[\begin{array}{|c|c|}
   \hline
   \text{tens} & \text{ones} \\
   \hline
   2 & 5 \\
   2 & 2 \\
   4 & 7 \\
   3 & 1 \\
   2 & 7 \\
   5 & 8 \\
   \hline
   \end{array}\]

2. \[\begin{array}{|c|c|}
   \hline
   \text{ones} & \text{tens} \\
   \hline
   3 & 1 \\
   0 & 1 \\
   4 & 1 \\
   3 & 1 \\
   3 & 1 \\
   2 & 1 \\
   2 & 1 \\
   6 & 1 \\
   4 & 1 \\
   3 & \\
   2 & \\
   3 & \\
   \hline
   \end{array}\]

**AP Book NS4-15**

**page 43**

Teacher to check drawings.

1. \[\begin{array}{|c|c|}
   \hline
   \text{tens} & \text{ones} \\
   \hline
   2 & 5 \\
   3 & 7 \\
   5 & 12 \\
   6 & 2 \\
   2 & 9 \\
   3 & 6 \\
   5 & 15 \\
   6 & 5 \\
   1 & 7 \\
   3 & 5 \\
   4 & 12 \\
   5 & 2 \\
   \hline
   \end{array}\]

2. \[\begin{array}{|c|c|}
   \hline
   \text{ones} & \text{tens} \\
   \hline
   5 & 11 \\
   6 & 1 \\
   5 & 12 \\
   6 & 2 \\
   7 & 11 \\
   8 & 1 \\
   7 & 12 \\
   8 & 2 \\
   \hline
   \end{array}\]

**AP Book NS4-16**

**page 45**

1. a) 5, 1 
   b) 2, 3 
   c) 6, 7 
   d) 9, 2 
   e) 8, 4 
   f) 7, 0 
   g) 0, 2 
   h) 0, 5 
   i) 3, 2 
   j) 6, 3 
   k) 8, 7 
   l) 5, 8 

2. a) 819 
   b) 828 
   c) 836 
   d) 959 
   e) 879 

3. a) 342 
   b) 763 
   c) 960 
   d) 551 
   e) 566 

4. a) 717 
   b) 672 
   c) 874 
   d) 836 
   e) 653 
   f) 990 

5. a) 483 
   b) 485 
   c) 1361 
   d) 962 
   e) 419 
   f) 1741 
   g) 488 
   h) 1757 
   i) 590 
   j) 1800 
   k) 159 
   l) 595 

**AP Book NS4-17**

**page 46**

1. a) 3 hundreds + 5 tens + 3 ones 
   b) 1 hundred + 6 tens + 4 ones 
   c) 4 hundreds + 11 tens + 7 ones 
   d) 5 hundreds + 1 tens + 7 ones 

2. a) 18 
   b) 198 
   c) 1998 
   d) 19998 
   e) 199998 

3. a) 79 
   b) 858 
   c) 666
Number Sense 1 – AP Book 4.1 (continued)

AP Book NS4-18

1. a) 7,076
   b) 8,114
   c) 8,066
   d) 3,130

7. a) 85,893
   b) 77,940
   c) 521,811

AP Book NS4-19

1. a) 39 – 18 = 21
   b) 25 – 11 = 14
   c) 43 – 21 = 22
   d) 45 – 32 = 13

2. a) 817
   b) 8,287
   c) 6,368
   d) 5,934
   e) 7,597

3. a) 4,817
   b) 4,829
   c) 6,617
   d) 9,836
   e) 6,887

4. a) 3,561
   b) 9,870
   c) 9,696
   d) 7,772
   e) 8,893

AP Book NS4-20

1. a) 37
   b) 25
   c) 23
   d) 8

2. a) 16
   b) 38
   c) 25
   d) 48

3. a) 36 = 30 + 6
   b) 84 = 80 + 4
   c) 98 = 90 + 8

5. a) 5,185
   b) 6,425
   c) 8,664
   d) 8,058
   e) 6,567
   f) 8,368
   g) 9,225
   h) 9,352
   i) 4,676
   j) 6,676

4. a) 3,411
   b) 21,111
   c) 432,752

5. a) Teacher to check drawings. (answer: 122)
   b) 543

6. a) HELP! 4 < 9, 35
   b) OK, 21
   c) OK, 32

7. a) 73 = 70 + 3
   b) 26 = 20 + 6
   c) 88 = 80 + 8

8. a) 5,778
   b) 2,888
   c) 3,847
   d) 2,579
   e) 1,942
   f) 1,494
   g) 3,488
   h) 4,729

9. a) 543
   b) 25
   c) 367
   d) 111

10. a) 7,274
   b) 1,973
   c) 2,579
   d) 231
Number Sense 1 – AP Book 4.1 (continued)

AP Book NS4-21

1. a) Red, green
   Difference: 2
   Total: 8
b) Red, green
   Difference: 2
   Total: 10
c) Green, red
   Difference: 3
   Total: 11
d) Red, green
   Difference: 4
   Total: 10
2. 5, two more green apples than red; 7, 5 more red than green; 3, 7.
3. Teacher to check pictures;

   RA | GA | Diff | Ttl
   ---|----|------|-----
   4  | 8  | 4    | 12
b) 5 | 7  | 2    | 12
c) 6 | 4  | 2    | 10

BONUS:

   GG | PG | Total
   ---|----|-----
   7  | 2  | 9
   6  | 4  | 10
   2  | 9  | 11
   9  | 5  | 14

AP Book NS4-22

1. a) 3 + 4 = 7; 4 + 3 = 7;
   7 – 4 = 3; 7 – 3 = 4
b) 5 + 4 = 9; 4 + 5 = 9;
   9 – 4 = 5; 9 – 5 = 4
   GG | PG | Total
   ---|----|-----
   7  | 2  | 9
   6  | 4  | 10
   2  | 9  | 11
   9  | 5  | 14
(answers for the remaining two columns appear in the next column)
2. a) 4 × 3 = 12
   b) 4 × 4 = 16
   c) 5 × 3 = 15
   d) 7 × 2 = 14
3. a) +
   b) –
   c) –
   d) +
4. a) 19 stickers
   b) 4 cats
   c) 4 km

AP Book NS4-23

1. 510 mL
2. a) Alice’s class raised more money:
   $312 > $287
   $599

AP Book NS4-24

1. a) Thousands
   b) Ten thousands
   c) Hundreds
   d) Thousands
2. a) 22 544
   b) 1 420
   c) 63 936
   d) 99 901
3. a) Sixty-one thousand, one hundred forty-five
   2 more green than purple
   7 more purple than green
4. a) 10 000 + 7 000 + 300 + 50 + 9
   b) 10 000 + 4 000 + 900 + 70 + 2
   c) 70 000 + 2 000 + 600 + 60 + 4
   d) 90 000 + 2 000 + 400 + 20 + 5
   e) 50 000 + 100 + 30 + 7
   f) 20 000 + 1
5. a) 25 848
   b) 32 166
   c) 98 400
6. a) 86 597
   b) 89 999
   c) 90 963
   d) 41 235
   e) 40 424
   f) 2 838

AP Book NS4-25

1. There 62 – 17 = 45 girls.
   Addition check:
   45 + 17 = 62
2. a) Ontario – 193 km
   Huron – 206 km
   Erie – 241 km
   Michigan – 307 km
   Superior – 350 km
   b) 101 km
   c) 157 km
3. a) 3 rows
   4 dots in each row
   3 × 4 = 12
   b) 4 rows
   4 dots in each row
   4 × 4 = 16
   c) 4 rows
   5 dots in each row
   4 × 5 = 20
4. Teacher to check arrays.
   Total number of dots:
   a) 25
   b) 15
   c) 4
   d) 12
   e) 6
   f) 0
5. Teacher to check arrays;
   a) 3 × 5 = 15
   b) 6 × 4 = 24
   c) 4 × 8 = 32
5. Teacher to check arrays;
   a) Same
   b) Yes, the products are both 24 (e.g. there are 24 dots in both arrays).

AP Book NS4-27
page 62
1. a) 4 + 4 + 4
   b) 8 + 8
   c) 6 + 6 + 6 + 6 + 6
   d) 2 + 2 + 2 + 2
   e) 5 + 5 + 5
   f) 3 + 3 + 3 + 3 + 3 + 3
   g) 7 + 7 + 7 + 7 + 7
   h) 1 + 1
   i) 1 + 1 + 1 + 1 + 1 + 1
   + 1 + 1
2. a) 3 × 4
    b) 3 × 5
    c) 2 × 4
    d) 4 × 7
    e) 2 × 9
    f) 3 × 8
    g) 3 × 2
    h) 4 × 9
    i) 3 × 1
    j) 5 × 6
    k) 6 × 8
    l) 4 × 3
3. a) 2 + 2 + 2
    3 × 2
    b) 4 + 4 + 4
    3 × 4
    c) 3 + 3 + 3 + 3
    4 × 3
    d) 5 + 5
    2 × 5
    e) 3 + 3 + 3 + 3 + 3
    3 × 5
    f) 2 + 2 + 2 + 2
    2 × 4
4. a) Subtotal = 5
    Total = 10
   b) Subtotal = 6
    Total = 13

AP Book NS4-28
page 64
1. Teacher to check
2. a) 4, 8, 12, 16, 20
   b) 6, 12, 18, 24, 30
   c) 7, 14, 21, 28, 35
3. a) 20
   b) 10
   c) 16
   d) 12
   e) 7
   f) 21
   g) 9
   h) 6
   i) 14
   j) 25
   k) 4
   l) 21
   m) 2
   n) 24
   o) 18
5. Teacher to check.

AP Book NS4-29
page 65
1. a) 4 × 3
   b) 7 × 2
   c) 5 × 4
   d) 8 × 3
   e) 5 × 3, 4 × 3, 3
   f) 8 × 2 + 2 + 2 + 2
   g) 16 = 4 + 4 + 4 + 4
   h) 15 = 5 + 5 + 5
6. Teacher to check as answers will vary.
   Example:
   a) Addition:
      3 + 3 + 3 + 3 + 3 + 3 + 3 = 21 or
      7 + 7 + 7 + 7 = 21
      Multiplication:
      3 × 7 = 21 or
      7 × 3 = 21
7. Teacher to check; refer to Question 6 for example.

AP Book NS4-30
page 67
1. a) 4 × 20 = 4 × 2 tens = 80 tens = 80
    b) 2 × 30 = 2 × 3 tens = 6 tens = 60
2. a) 3 × 7 tens = 21 tens
    b) 3 × 5 tens = 15 tens
    c) 5 × 5 tens = 25 tens
    d) 4 × 6 tens = 24 tens
3. a) 4, 40, 400
    b) 5, 50, 500
    c) 8, 80, 800
    d) 9, 90, 900
4. a) 120
    b) 150
    c) 160
    d) 100
    e) 300
    f) 2000
    g) 180
    h) 2400
    i) 1400
    j) 420
    k) 320
    l) 1800
6. 3 × 20 = 60

3 × 200 = 600

3 × 2 000 = 6 000

AP Book NS4-31
page 68
1. a) 3 × 20
b) 4 × 10
c) 5 × 20
d) 5 × 10

2. Answers are top to bottom, left to right:
   a) 3 × 24, 3 × 20, 3 × 4
   b) 4 × 13, 4 × 10, 4 × 3
   c) 2 × 25, 2 × 20, 2 × 5
d) 3 × 14, 3 × 10, 3 × 4

3. a) 2 × 24
   = 2 × 20 + 2 × 4
   b) 4 × 12
   = 4 × 10 + 4 × 2
c) 4 × 25
   = 4 × 20 + 4 × 5
d) 3 × 13
   = 3 × 10 + 3 × 3

AP Book NS4-32
page 69
1. a) 2 × 24
   = 2 × 20 + 2 × 4
   b) 2 × 23
   = 2 × 20 + 2 × 3
c) 3 × 32
   = 3 × 30 + 3 × 2
d) 4 × 12
   = 4 × 10 + 4 × 2

2. a) 3 × 10 + 3 × 3
   = 30 + 9 = 39
   b) 3 × 20 + 3 × 1
   = 60 + 3
   = 63
c) 2 × 10 + 2 × 4
   = 20 + 8
   = 28
d) 3 × 200 + 3 × 10 + 3 × 3 = 600 + 30 + 9 = 639
e) 2 × 200 + 2 × 30 + 2 × 1 = 400 + 60 + 2 = 462

AP Book NS4-33
page 70
1. 2, 4, 6, 8, 10, 12, 14, 16, 18
2. 48, 28, 24, 64, 128, 44, 26
   164, 102, 68, 108, 184, 148, 142
3. 32, 30, 50, 74, 56, 36, 96
   34, 90, 132, 70, 92, 58, 110
4. 28, 42, 56, 24
   36, 48, 32, 64
   36, 54, 72, 48

5. a) 84¢
   b) 74¢
   c) 96¢
   d) 70¢

AP Book NS4-34
page 71
1. a) 124
   b) 106
c) 164
   d) 126
e) 93
   f) 142
g) 186
   h) 168
   i) 208
   j) 44
   k) 105
   l) 159
   m) 168
   n) 129
   o) 128
   p) 219
   q) 108
   r) 248
   s) 216
   t) 182
   u) 189
   v) 162
   w) 255
   x) 288
   y) 305
   z) 144
   aa) 249
   bb) 819
   cc) 246
   dd) 488
   ee) 368
   ff) 85
   gg) 86
   hh) 427
   ii) 568

2. a) 186
   b) 148
   c) 105
d) 248
e) 205
   f) 147

AP Book NS4-35
page 72
1. a) 0 carry the 2
   b) 2 carry the 1
   c) 5 carry the 1
d) 2 carry the 1
e) 0 carry the 2

2. a) 96
   b) 60
c) 70
d) 84
e) 75
   f) 70
   g) 94
   h) 72
   i) 81
   j) 80

3. a) 50
   b) 96
c) 140
d) 105
e) 102
   f) 160
g) 222
   h) 410
   i) 161

AP Book NS4-36
page 73
1. a) 300 × 3
  + 20 × 3
  +1 × 3
  = 900 + 60 + 3
  = 963
   b) 400 × 2
  + 30 × 2
  +2 × 2
  = 800 + 60 + 4
  = 864

2. a) 248
   b) 639
c) 488
d) 969
e) 826

3. a) 492
   b) 975
c) 570
d) 632
e) 672

4. a) 964
   b) 755
c) 726
d) 456
5. a) 968
   b) 1 560
   c) 861
   d) 2 512
   e) 2 277
   f) 1 446
6. Teacher to check drawing.
   a) 369
   b) 636
   c) 963

AP Book NS4-37
page 74
1. a) \(3 \times 2 + 3 \times 1 = 3 \times (2 + 1) = 3 \times 3\)
   b) \(3 \times 2 + 3 \times 4 = 3 \times (2 + 4) = 3 \times 6\)
   c) \(3 \times 3 + 3 \times 4 = 3 \times (3 + 4) = 3 \times 7\)
   d) \(3 \times (5 + 4) = 3 \times 9\)
   e) \(3 \times (2 + 6) = 3 \times 8\)
   f) \(7 \times (4 + 3) = 7 \times 7\)
2. a) 2
   b) 4
3. a) \(35 \times 4 = 140\)
   b) \(34 \times 3 = 102\)
   c) \(87 \times 5 = 435\)
   d) \(84 \times 6 = 504\)
4. a) \(43 \times 5 = 215\)
   b) \(45 \times 3 = 135\)
5. a) 0
   b) 0
   c) 0
   d) 0
6. The number was 0 (zero).

AP Book NS4-38
page 75
1. 1 920 suckers
2. 1 056 mL
3. a) \(1 680 \text{ litres}\)
   b) 960 litres
4. The product is greater.
5. No, the product is not always greater than the sum:
   \(2 \times 2 = 2 + 2\)
   \(0 \times 0 = 0 + 0\)
   but…
   \(0 \times 1 < 0 + 1\)
   \(0 \times 2 < 0 + 2\)
   \(1 \times 1 < 1 + 1\)
   \(1 \times 2 < 1 + 2\)
6. 1
7. Multiply to 20:
   a) 1, 20
   b) 2, 10
   c) 4, 5
   Multiply to 40:
   a) 1, 40
   b) 2, 20
   c) 4, 10
   d) 5, 8
8. a) \(12 \times 10 = 120\)
   b) \(120 \times 2 = 240\)
9. a) 3 ways:
   \(4 \times 1\)
   \(2 \times 2\)
   \(1 \times 4\)
   b) 4 ways:
   \(1 \times 8\)
   \(2 \times 4\)
   \(8 \times 1\)
   \(4 \times 2\)
   c) 6 ways:
   \(1 \times 12\)
   \(2 \times 6\)
   \(3 \times 4\)
   \(12 \times 1\)
   \(6 \times 2\)
   \(4 \times 3\)
   d) 7 ways:
   \(1 \times 16\)
   \(2 \times 8\)
   \(3 \times 6 \quad 4 \times 4\)
   \(16 \times 1\)
   \(8 \times 2\)
   \(6 \times 3\)
10. \(325 \times 6 = 1 950 \text{ m (since a hexagon has 6 sides)}\)

AP Book NS4-39
page 76
1. a) 0
2. 1 056 mL
3. a) 1 680 litres
   b) 960 litres
4. The product is greater.
5. No, the product is not always greater than the sum:
   \(2 \times 2 = 2 + 2\)
   \(0 \times 0 = 0 + 0\)
   but…
   \(0 \times 1 < 0 + 1\)
   \(0 \times 2 < 0 + 2\)
   \(1 \times 1 < 1 + 1\)
   \(1 \times 2 < 1 + 2\)
6. 1
7. Multiply to 20:
   a) 1, 20
   b) 2, 10
   c) 4, 5
   Multiply to 40:
   a) 1, 40
   b) 2, 20
   c) 4, 10
   d) 5, 8
8. a) \(12 \times 10 = 120\)
   b) \(120 \times 2 = 240\)
9. a) 3 ways:
   \(4 \times 1\)
   \(2 \times 2\)
   \(1 \times 4\)
   b) 4 ways:
   \(1 \times 8\)
   \(2 \times 4\)
   \(8 \times 1\)
   \(4 \times 2\)
   c) 6 ways:
   \(1 \times 12\)
   \(2 \times 6\)
   \(3 \times 4\)
   \(12 \times 1\)
   \(6 \times 2\)
   \(4 \times 3\)
   d) 7 ways:
   \(1 \times 16\)
   \(2 \times 8\)
   \(3 \times 6 \quad 4 \times 4\)
   \(16 \times 1\)
   \(8 \times 2\)
   \(6 \times 3\)
10. \(325 \times 6 = 1 950 \text{ m (since a hexagon has 6 sides)}\)

AP Book NS4-40
page 78
1. a) 0
2. 50 is in the middle
3. a) 100
4. a) 137 round to 100
   b) 182 round to 200
   c) 315 round to 300
   d) 363 round to 400
5. a) 200
6. Teacher to check number line.
   a) 518 round to 500
   b) 576 round to 600
   c) 687 round to 700
   d) 629 round to 600

AP Book NS4-41
page 79
1. a) 0
   b) 1 000
   c) 1 000
   d) 0
2. 500 is in the middle
3. a) 0
   b) 1 000
   c) 1 000
   d) 0
4. a) 1217 round to 1 000
   b) 1 847 round to 2 000
   c) 6 348 round to 6 000
   d) 6 865 round to 7 000
5. a) 2 000
   b) 6 000
   c) 6 000
   d) 3 000
6. If the hundreds number is 500 or greater, you round **up** to the nearest thousand. If the hundreds number is less than 500, you round **down** to the nearest thousand.

### AP Book NS4-42

**Page 80**

1. a) 20  
   b) 20  
   c) 70  
   d) 70  
   e) 80  
   f) 90  
   g) 10  
   h) 60  
   i) 70  
   j) 40  
   k) 50  

2. a) 150  
   b) 170  
   c) 320  
   d) 260  
   e) 780  
   f) 670  
   g) 440  
   h) 940  
   i) 320  
   j) 520  
   k) 990  
   l) 530  
   m) 760  
   n) 850  
   o) 290  

3. a) 2 200  
   b) 4 400  
   c) 3 200  
   d) 1 900  
   e) 5 300  
   f) 9 100  
   g) 6 500  
   h) 8 800  
   i) 7 300  
   j) 1 100  
   k) 3 900  
   l) 4 600  
   m) 8 100  
   n) 6 400  
   o) 9 600  
   p) 2 600  
   q) 5 900  

**BONUS:**  
   r) 3 000  
   s) 1 000  
   t) 4 000  

4. a) 2 200  
   b) 4 400  
   c) 3 200  
   d) 1 900  
   e) 5 300  
   f) 9 100  
   g) 6 500  
   h) 8 800  
   i) 7 300  
   j) 1 100  
   k) 3 900  
   l) 4 600  
   m) 8 100  
   n) 6 400  
   o) 9 600  
   p) 2 600  
   q) 5 900  

### AP Book NS4-43

**Page 82**

1. a) 3 - Round down  
   b) 5 - Round up  
   c) 8 - Round up  
   d) 3 - Round up  
   e) 7 - Round up  
   f) 7 - Round up  

2. a) Rd: 3 000  
   b) Ru: 7 000  
   c) Rd: 4 300  
   d) Ru: 8 700  
   e) Ru: 5 240  
   f) Rd: 3 920  
   g) Ru: 2 900  
   h) Rd: 6 310  
   i) Rd: 5 000  

3. a) 2 200  
   b) 4 000  
   c) 10 000  
   d) 13 290  
   e) 4 900  
   f) 7 000  
   g) 1 240  
   h) 7 900  

### AP Book NS4-44

**Page 83**

1. a) 50 + 30 = 80  
   b) 20 + 70 = 90  
   c) 50 − 10 = 40  
   d) 100 − 60 = 40  
   e) 30 + 10 = 40  
   f) 70 + 30 = 100  
   g) 40 + 30 = 70  
   h) 80 + 30 = 110  
   i) 30 + 40 = 70  
   j) 30 − 10 = 20  

2. a) 40 + 20 = 60  
   b) 90 − 20 = 70  
   c) 200 + 400 = 600  
   d) 200 + 700 = 900  
   e) 500 − 100 = 400  
   f) 1000 − 600 = 400  
   g) 500 + 200 = 700  
   h) 600 + 300 = 900  
   i) 200 + 800 = 1000  
   j) 800 + 200 = 1000  
   k) 400 + 400 = 800  
   l) 900 − 500 = 400  
   m) 500 − 300 = 200  
   n) 700 + 200 = 900  

3. a) 1 000 + 4 000  
   b) 5 000 − 3 000  
   c) 3 000 + 1 000  
   d) 9 000 − 5 000  
   e) 3 300 + 1 200  
   f) 3 600 − 1 800  
   g) 4 800 − 3 700  

### AP Book NS4-45

**Page 84**

1. a) 50  
   b) 100  
   c) 90  

2. a) 700  
   b) 520  
   c) 1480  

3. Teacher to check, answers will vary.  

4. a) Est: 600  
   b) Est: 600  
   c) Est: 300  

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**Answer Key for AP Book 4.1**
Number Sense 1 – AP Book 4.1 (continued)

AP Book NS4-46
page 85
1. Teacher to check estimates.
   a) 7 794
   b) 6 954
   c) 5 548
2. a) 700 600 697
   b) 1100 1000 1099
   c) 500 500 531
Front-end estimation does not provide a better result for addition.
3. a) 698
   b) 738
   c) 699
4. Answers will vary.
5. About 3 (2.82 exactly)
6. About 8200 (8192 exactly)
BONUS:
7. Teacher to check.
   Sample answer: 749 - 699

AP Book NS4-47
page 86
1. a) Name: Penny
   Value: $0.01
   b) Name: Nickel
   Value: $0.05
   c) Name: Dime
   Value: $0.10
   d) Name: Quarter
   Value: $0.25
2. a) 5
   b) 10
   c) 2
   d) 5
   e) 25
   f) 2
3. a) 80, 85, 90, 95
   b) 40, 45, 50, 55
   c) 60, 65, 70, 75
   d) 70, 75, 80, 85
   e) 105, 110, 115, 120
   f) 120, 125, 130, 135
4. a) 55, 60, 65, 70, 75
   b) 75, 80, 85, 90, 95
   c) 85, 90, 95, 100, 105
   d) 100, 110, 120, 130
   e) 120, 130, 140, 150
5. a) 30, 40, 50, 60
   b) 60, 70, 80, 90
   c) 80, 90, 100, 110
   d) 70, 80, 90, 100
   e) 100, 110, 120, 130
   f) 120, 130, 140, 150
6. a) 55, 65, 75, 85, 95
   b) 70, 80, 90, 100, 110
   c) 85, 95, 105, 115, 125
7. a) 5, 10, 15, 20, 25 |
     26, 27, 28
   b) 5, 10, 15, 20 | 21, 22, 23
8. a) 5, 15, 25 | 30, 35, 40,
     45, 50
   b) 10, 20, 30 | 35, 40, 45, 50
   c) 25, 50, 75 | 85, 95
   d) 25, 50, 75 | 80, 85
9. a) 25, 50, 75 | 85, 85 |
   b) 25, 50, 60, 70, 75,
     80, 85
   c) 25, 50 | 60, 65, 70, 75,
     95, 96
   d) 25, 50, 75 | 85, 95 |
     125
BONUS:
  e) 25, 50 | 60, 70, 80, 85,
     90, 91, 92
10. a) 10, 20, 30, 35, 40, 41
    b) 5, 10, 15, 25, 35, 36
BONUS:
    c) 25, 50, 75, 100, 110,
     120, 125, 130, 135,
     136, 137, 138, 139
11. a) 10, 20, 30 | 35, 40, 41
    b) 25, 50, 60, 70, 71, 72, 73
    c) 25, 50, 55, 60, 61, 62
    d) 25, 50, 75 | 85, 95 |
     100, 105
BONUS:
    e) 25, 50, 60, 70, 80 |
     85, 90, 91, 92
12. a) 39 cents
    b) 120 cents
    c) 180 cents
BONUS:
  d) 82 cents
  e) 95 cents
  f) 76 cents
  g) 184 cents
13. a) 47 cents
    b) 72 cents
    c) 76 cents
    d) 51 cents
BONUS:
  e) 113 cents

AP Book NS4-48
page 89
1. a) 19
    b) 35, 40
    c) 72, 77
    d) 23, 28
    e) 76, 81
    f) 50, 55
2. a) 73
    b) 34, 44
    c) 49, 59
3. a) 1¢
    b) 5¢
    c) 1¢
    d) 25¢
    e) 5¢
    f) 25¢
4. a) 2 additional nickels needed
    b) 2 additional nickels needed
    c) 2 additional nickels needed
    d) 2 additional nickels needed
    e) 3 additional nickels needed
    f) 6 additional nickels needed
5. a) 3 additional dimes needed
    b) 1 additional dime needed
    c) 3 additional dimes needed
    d) 2 additional dimes needed
    e) 3 additional dimes needed
    f) 4 additional dimes needed
    g) 2 additional dimes needed
    h) 1 additional dime needed
    i) 3 additional dimes needed
6. a) 3 additional dimes needed
    b) 2 additional nickels needed
    c) 2 additional dimes needed
    d) 2 additional quarters needed
BONUS:
  e) 1 dime, 1 penny
  f) 2 dimes
  g) 1 dime, 1 nickel
  h) 2 dimes
  i) 1 dime, 1 nickel
8. a) 1 toonie, 1 loonie
    b) 1 toonie, 1 loonie
    c) 2 loonies
    d) 1 toonie, 1 loonie
    e) 2 toonies
    f) 1 toonie, 1 loonie
9. a) Tashi needs 2 dimes.
    b) Zoltan needs 1 nickel and 3 pennies.
    c) Marzuk needs 1 toonie and 1 quarter.
10. a) 80¢ = 2 quarters + 3 dimes
    b) 80¢ = 3 quarters + 1 nickel;
      2 quarters + 6 nickels;
      1 quarter + 11 nickels
11. Answers will vary.
Number Sense 1 – AP Book 4.1 (continued)

AP Book NS4-49
page 91

1. a) One dime, two pennies
   b) One dime, one nickel, one penny
   c) Two dimes, two pennies
   d) One nickel, three pennies
   e) One dime, one nickel
   f) Two dimes
   g) One dime, one nickel, two pennies
   h) Two dimes, four pennies
   i) One dime, one penny
   j) One dime, one nickel
   k) One dime, one nickel, four pennies
   l) Two dimes, three pennies

2. a) 50¢
   b) 75¢
   c) 100¢

3. a) 25¢
   b) 50¢
   c) 75¢
   d) 75¢
   e) 50¢
   f) 50¢
   g) 25¢
   h) 25¢
   i) 75¢
   j) 75¢
   k) 25¢
   l) 50¢

4. a) 75¢; 8¢ = one dime + three pennies
   b) 50¢; 6¢ = one nickel + one penny
   c) 25¢; 8¢ = one nickel + three pennies
   d) 75¢; 10¢ = one dime
   e) 75¢; 22¢ = two dimes + two pennies

5. a) One quarter and one nickel
   b) Three quarters and one penny

6. 55¢ = two quarters + one nickel

7. a) One quarter
   b) One loonie
   c) One toonie, one dime
   d) One loonie, one quarter
   e) One toonie, one loonie, one quarter
   f) Two toonies, one quarter, one nickel

8. a) One loonie, one quarter
   b) Two quarters
   c) Three toonies
   d) Three toonies, one loonie, two quarters
   e) Four toonies, one loonie, three quarters

AP Book NS4-50
page 93

1. a) 50¢ – 42¢ = 8¢
   b) 50¢ – 34¢ = 16¢
   c) 90¢ – 81¢ = 9¢
   d) 60¢ – 56¢ = 4¢
   e) 80¢ – 78¢ = 2¢
   f) 70¢ – 63¢ = 7¢

2. a) 100¢ – 90¢ = 10¢
   b) 30¢
   c) 50¢
   d) 60¢
   e) 90¢
   f) 70¢
   g) 80¢
   h) 40¢
   i) 20¢

3. a) 50¢
   b) 40¢
   c) 70¢
   d) 40¢
   e) 20¢
   f) 80¢
   g) 90¢
   h) 60¢
   i) 30¢

4. a) 80
   b) 60
   c) 50
   d) 30
   e) 60
   f) 10

5. a) 2¢; 70¢; 30¢; 32¢
   b) 8¢; 80¢; 20¢; 28¢
   c) 7¢; 60¢; 40¢; 47¢
   d) 7¢; 30¢; 70¢; 77¢
   e) 2¢; 50¢; 50¢; 52¢
   f) 6¢; 90¢; 10¢; 16¢

6. a) 42¢
   b) 36¢
   c) 73¢
   d) 64¢
   e) 48¢
   f) 71¢
   g) 3¢
   h) 86¢
   i) 11¢
   j) 9¢

BONUS:
7. a) Change required: 13¢
   b) Change required: 17¢

AP Book NS4-51
page 95

1. a) Nickels Pennies
   0 19
   1 14
   2 9

b) Dimes Nickels
   0 9
   1 7
   2 5
   3 3

2. Two quarters make 50 cents, three make 75 cents, so he only needs 2 quarters and one nickel.

3. a) Dimes Pennies
   0 13
   1 3

b) Quarters Nickels
   0 16
   1 11
   2 6
   3 1

4. a) 8 legs
   b) 12 legs
   c) 18 legs

5. a) 1 bird + 1 dog = 6 legs
   b) 2 birds + 1 cat = 8 legs
1.  a) Answers may vary. 
b) Answers may vary. 
2. Objects measured and lengths may vary. 
3. a) 5 toonies  
b) 6 toonies  
c) 10 toonies  
4. a) Answers may vary.  
b) Answers may vary.  
5. Classroom objects chosen for this question vary as will their lengths (in cm).

1. a) 2 cm  
b) 4 cm  
2. a) 3 cm  
b) 1 cm  
3. a) 3 cm  
b) 2 cm  
4. a) 4 cm  
b) 3 cm  
c) 3 cm  
d) 4 cm  
5. a) 3 cm  
b) 5 cm  

1. a) 24 mm  
b) 38 mm  
2. a) 38 mm  
b) 18 mm  
3. a) Teacher to check.  
b) Teacher to check.  
4. Less than More than 
30 mm 30 mm  
a)  

4. To change from mm to cm, you have to divide by 10. 
a) 4  
b) 6 

1. b) 3.5; 35 mm  
c) 5; 50 mm  
d) 1; 10 mm  
2. a) 20  
b) 30  
c) 50  
d) 100  
3. Jamelia has 4 stacks of dimes, each about 1 cm high. We’ve learned that 1 dime is about 1 mm thick. We also know that there are 10 mm in one centimetre. Therefore, Jamelia has 10 x 4 (stacks) = 40 dimes = $4.00.
2. Teacher to check
3. Teacher to check
4. Teacher to check
5. a) Line must be 50 mm (5 cm) long – teacher to check.
   b) Line must be 70 mm (7 cm) long – teacher to check.
   c) Line must be 20 mm (2 cm) long – teacher to check.
6. Peter is wrong. If you convert each length to the same unit, to mm for example, then the two lengths become 5 mm and 20 mm (2 cm). In this way, it is easy to see that 20 mm is greater than 5 mm.

2. a) 1 cm = 10 mm
   b) 1 m = 100 cm
   c) 1 m = 1 000 mm
3. a) \[ \begin{array}{c|c|c}
   m & \text{cm} \\
   \hline
   1 & 100 \\
   14 & 1400 \\
   80 & 8 000 \\
   \end{array} \]
   b) \[ \begin{array}{c|c|c}
   m & \text{mm} \\
   \hline
   2 & 2000 \\
   19 & 19 000 \\
   21 & 21 000 \\
   \end{array} \]
   c) \[ \begin{array}{c|c|c}
   \text{cm} & \text{mm} \\
   \hline
   3 & 30 \\
   65 & 650 \\
   106 & 1 060 \\
   \end{array} \]
4. There was no difference in Sheena’s measurements since 215 cm = 2 m 15 cm.
5. a) 513 cm = 5 m 13 cm
   b) 217 cm = 2 m 17 cm
   c) 367 cm = 3 m 67 cm
   d) 481 cm = 4 m 81 cm
   e) 706 cm = 7 m 6 cm
   f) 303 cm = 3 m 3 cm
6. a) 3 m 71 cm = 371 cm
   b) 4 m 51 cm = 451 cm
   c) 3 m 45 cm = 345 cm
   d) 8 m 2 cm = 802 cm
   e) 9 m 7 cm = 907 cm
   f) 7 m 50 cm = 750 cm

1. a) 220 km
   b) 360 km
   c) 490 km
   d) 330 km
2. a) 710 km
   b) 820 km
   c) 690 km
3. 1. Bloodvein – 200 km
   2. Clearwater – 187 km
   3. Athabasca – 168 km
   4. Jacques Cartier – 128 km
   5. Kicking Horse – 67 km
4. a) 800 meters
   b) No
   c) About 4
   d) 5 times

1. There are 100 centimetres in a 1 metre.
2. a) 4 m = 400 cm
   b) 6 m = 600 cm
   c) 100 cm = 1 m
   d) 3 m = 300 cm
3. a) 1 m = 100 cm, which is greater than 60 cm
   b) 7 m = 700 cm, which is greater than 82 cm
   c) 5 m = 500 cm, which is greater than 410 cm
   d) 340 cm
   e) 4 meters = 400 cm
   f) 7 meters = 700 cm
4. Teacher to check.
5. Teacher to check.
1. a) cm  
   b) m  
   c) km  

2. a) mm  
   b) cm  
   c) m  
   d) km  
   e) m  
   f) cm  
   g) m  

3. a) metres  
   b) metres or kilometres  
   c) kilometres  
   d) metres  

4. a) km  
   b) cm  
   c) m  
   d) m  
   e) m; m  
   f) cm  
   g) m  
   h) km  

5. a) mm  
   b) km  
   c) m  
   d) cm or mm, depending on your hair’s length!  
   e) km  

6. Answers will vary.  

AP Book ME4-17  

1. a) 2 + 3 + 2 + 3 = 10 cm  
   b) 2 + 2 + 5 + 5 = 14 cm  
   c) 2 + 1 + 1 + 4 + 1 + 5 = 14 cm  
   d) 1 + 5 + 2 + 2 + 1 + 3 = 14 cm  

2. A: 12 units  
   B: 16 units  
   C: 24 units  

3. Answers may vary.  

4. Answers may vary.  

AP Book ME4-18  

1. a) 10 cm  
   b) 8 cm  
   c) 14 cm  

2. C: 6 km  
   A: 24 m  
   D: 30 cm  
   B: 28 cm  
   C, A, D, B  

3. a) Actual: 18 cm  
   b) Actual: 16 cm  

4. Dimensions of the JUMP Math workbook:  
   21.5 cm by 28 cm  
   Actual Perimeter = 21.5 + 21.5 + 28 + 28 = 99 cm  

5. Answers will vary.  

6. a) 3.5 × 2 = 7 m  
   b) 7 × 4 = 28 m  

7. a) m  
   b) cm  
   c) m  
   d) km  
   e) cm  
   f) km  
   g) m  
   h) km  

8. Answers will vary.  

9. 5 + 5 + 5 + 5 = 20 cm  

10. Perimeter = 10 m.  
    If each metre of border costs 15¢, it will cost Sally 15 × 10 = 150¢ = $1.50 altogether.  

11. To measure the perimeter of a round object using a string, wrap the string tightly around the outside edge of the object. Mark the point where the string meets itself. Then hold the string up to a straight ruler and measure the length from the end to the mark you made. This is the perimeter of your round object!  

12. Perimeter is the measurement around an object.  

13. Yes, different shapes can have the same perimeter. (Teacher to check explanations.)  

AP Book ME4-19  

1. a) 5, 10, 15  
   b) 5, 10, 15, 20, 25  
   c) 5, 10, 15, 20  
   d) 5, 10, 15, 20, 25, 30  

2. a) 1  
   b) 6  
   c) 11  
   d) 7  
   e) 1  
   f) 7  
   g) 4  
   h) 9  
   i) 12  
   j) 4  
   k) 3  
   l) 9  

3. a) 12:30  
    Thirty minutes after twelve  
    OR  
    Half past the hour  

4. a) 11:10  
    Ten minutes after 11  

5. Teacher to check.  

AP Book ME4-20  

1. a) Half past  
   b) quarter to  
   c) half past  

Answer Key for AP Book 4.1
d) quarter past

e) quarter to

f) half past
g) quarter past

h) quarter to

2. a) 8
b) 10
c) 11
d) 2
e) 2
f) 4
g) 5
h) 9

AP Book ME4-21

Page 122

1. Teacher to check

2. a) 12
b) 10
c) 6
d) 9
e) 5
f) 12
g) 10
h) 4
i) 2
j) 7

3. a) 8, 9
b) 6, 7
c) 4, 5
d) 10, 11

AP Book ME4-22

Page 123

1. a) 24 minutes past
b) 19 minutes past
c) 11 minutes past
d) 56 minutes past
e) 33 minutes past
f) 47 minutes past

2. a) 6:24
b) 12:40
c) 7:27
d) 5:04
e) 3:42
f) 9:33

AP Book ME4-23

Page 125

1. a) 20 minutes
b) 30 minutes
c) 35 minutes
d) 25 minutes
e) 20 minutes
f) 35 minutes

2. a) 6:50, 6:55, 7:00, 7:05, 7:10, 7:15, 7:20, 7:25

Time elapsed: 35 minutes

b) 4:45, 4:50, 4:55, 5:00, 5:05

Time elapsed: 20 minutes

c) 12:35, 12:40, 12:45, 12:50, 12:55, 1:00, 1:05

Time elapsed: 30 minutes

d) 1:55, 2:00, 2:05, 2:10, 2:15, 2:20, 2:25, 2:30

Time elapsed: 35 minutes

3. 45 minutes

4. 8:15

5. 5:30

AP Book ME4-24

Page 126

1. a) 2 hours 25 minutes
b) 3 hours 15 minutes
c) 1 hour 35 minutes

2. a) 3:45, 3:50, 3:55, 4:00, 5:00, 6:00, 6:05

Time elapsed: 2 hours 20 minutes

b) 7:50, 7:55, 8:00, 9:00, 9:05, 9:10

Time elapsed: 1 hour 20 minutes

AP Book ME4-25

Page 127

1. a) a.m.
b) p.m.
c) p.m.
d) p.m.
e) p.m.
f) a.m.

BONUS:

g) a.m.
h) p.m.
i) p.m.
j) a.m.

2. Teacher to check, answers will vary.

3. a) 7:45 am
b) 8:30 am
c) 1:00 pm
d) 12:00 am
e) 6:00 pm
f) 5:00 pm
g) 6:00 am
h) 11:00 pm

AP Book ME4-26

Page 128

1.  12-hr clock | 24-hr clock

| 12:00 am | 00:00 |
| 1:00 am | 01:00 |
| 2:00 am | 02:00 |
| 3:00 am | 03:00 |
| 4:00 am | 04:00 |
| 5:00 am | 05:00 |
| 6:00 am | 06:00 |
| 7:00 am | 07:00 |
| 8:00 am | 08:00 |

5. Time finished:
Dinosaur – 11:30
Reptiles – 13:30
Lunch – 14:00
Ancient Egypt – 15:00
Bat Cave – 15:30
3. How old are you? - Years
   How long does recess last? - Minutes
   How long do you sleep each night? - Hours
   How long is March break? - Weeks
   How long is summer vacation? - Months
4. a) Teacher to check
   b) 5 years old
   c) 47 years

AP Book ME4-29

1. January
   February
   March
   April
   May
   June
   July
   August
   September
   October
   November
   December

2. April = 4
   February = 2
   December = 12

3. a) 1963 – 06 – 18
   b) 1976 – 04 – 09
   c) 2001 – 05 – 24
   d) 1987 – 12 – 25
   e) 1942 – 09 – 29
   f) 1867 – 07 – 01
   g) 1973 – 03 – 14
4. a) July 25, 1982
   b) December 31, 1999
   c) June 01, 2001
   d) May 07, 1963
   e) May 17, 1977
   f) May 08, 1981
5. a) 4
   b) 6
   c) 9
   d) 2

6. Not possible – month and date are switched around (there is no 24th month).

7. a) Yes
   b) Teacher to check. (As of 2013: roughly 15 decades)
1. a) Hawk, sparrow, robin
   b) Tuna, shark
   c) Birds 3
      Fish 2
      Insects 3

2. Fruit 4
   Meat 2
   Dairy Products 3

3. C. Pizza toppings
   E. Coins
   D. Trees
   A. Cities in Canada
   B. Sports

4. a) Greater than four
    b) Odd numbers
    c) Fractions

5. Odd numbers
   Even numbers

1. a) Inside the circle: A and C
    b) Inside the circle: B and C
    c) Inside the circle: B and E

2. a) C is inside both circles
    b) D and E are outside both circles

3. a) 
   b) 

4. Intersection of “has feathers” and “has wings”: C, F, G
   “Has wings”: H, E, A
   Outside circles: B, D

5. a) 
   b) 
   c) 
   d) In July, there were 5 days (31 – 26) that weren’t sunny.
   e) Answers will vary.

2. Rose = 50 seeds
   Dandelion = 20 seeds
   Pansy = 10 seeds

<table>
<thead>
<tr>
<th>Colour</th>
<th>Tally</th>
<th>Pictograph</th>
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<tbody>
<tr>
<td>Blue</td>
<td>1 1 2</td>
<td></td>
</tr>
<tr>
<td>Green</td>
<td>1 1</td>
<td></td>
</tr>
<tr>
<td>Yellow</td>
<td>1 1 1</td>
<td></td>
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</tbody>
</table>

1. a) 
   b) 

2. a) 6
    b) 4
    c) 5
    d) 11
    e) 13
    f) 20
    g) 30
    h) 25
    i) 45
    j) 75
    k) 8
    l) 12
    m) 6
    n) 18
    o) 22

<table>
<thead>
<tr>
<th>Material</th>
<th># of Students</th>
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<tbody>
<tr>
<td>Mystery</td>
<td>4</td>
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<tr>
<td>Adventure</td>
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<td>4</td>
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<tr>
<td>Comics</td>
<td>6</td>
</tr>
<tr>
<td>Magazines</td>
<td>2</td>
</tr>
</tbody>
</table>

2. If there were 40 students in Theresa’s class, each book would represent 4 students (since there are 10 books in the graph and 40 students, and 40 ÷ 10 = 4 students per book).

1. a) 8 green, 12 brown
    b) 16
    c) Teacher to check
    d) 24
    e) 33

2. a) Count:
    b) Teacher to check
    c) Teacher to check.

1. a) 2, 12
    b) 5, 30
    c) 25, 175

2. a) The height of the bars are disproportional to the data.
    b) Scale between 1-6 is missing.
    c) Teacher to check.

3. Teacher to check.
   a) Teacher to check
   b) Teacher to check.
4. a) A = 12.5
   B = 20
   C = 10
b) A = 12
   B = 9
c) A = 9
   B = 12
   C = 3

AP Book PDM4-9
page 143
1. a) □ Boys
    □ Girls
b) Girls: Novel
   Overall: Novel
c) Comics
d) 1 (Facts)
e) Girls – 18
   Boys - 17
2. Many of the numbers in the data are not multiples of three, which makes it hard to display on the scale of 3. AND there is a number (25) that does not fit on the scale.
3. Answers will vary.

AP Book PDM4-10
page 144
1. a) Answers will vary.
2. a) C - Measurement
   b) A - Survey
   c) C – Measurement
   d) B – Observation
   e) B - Observation
3. a) Teacher to check
    b) Answers will vary.
       Sample answer:
       Pictograph

AP Book PDM4-11
page 145
1. a) Other
b) -
   c) Other
d) -
2. Answers will vary. Teacher to check.
AP Book G4-1

1. a) 4 sides
   4 vertices
b) 3 sides
   3 vertices
c) 4 sides
   4 vertices
d) 5 sides
   5 vertices
e) 6 sides
   6 vertices
f) 8 sides
   8 vertices
g) 3 sides
   3 vertices
BONUS:
h) 8 sides
   8 vertices
i) 12 sides
   12 vertices
j) 10 sides
   10 vertices
k) 8 sides
   8 vertices
l) 8 sides
   8 vertices
m) 12 sides
   12 vertices
2. a) 2 curved sides
   2 straight sides
b) 4 curved sides
   4 straight sides
3. a) 3 sides
   b) 4 sides
   c) 5 sides
   d) 6 sides
4. a) 3
   b) 4
   c) 4
   d) 3

<table>
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<tr>
<th>Shapes</th>
<th>Letters</th>
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<tr>
<td>Triangles</td>
<td>A, D</td>
</tr>
<tr>
<td>Quadrilaterals</td>
<td>B, C</td>
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</tr>
<tr>
<td>Quadrilaterals</td>
<td>B, F, G</td>
</tr>
<tr>
<td>Pentagons</td>
<td>C, D</td>
</tr>
<tr>
<td>Hexagons</td>
<td>E, H</td>
</tr>
</tbody>
</table>

6. a) Answers will vary.
   b) Answers will vary
7. Answers will vary but, no, you can’t draw a polygon in which the number of sides does not equal the number of vertices.
8. 23 sides

AP Book G4-2

1. a) less than
   b) greater than
   c) greater than
   d) greater than
   e) less than
   f) greater than
   g) greater than
   h) greater than
2. The angles in (b) and (d) are right angles.
3. Teacher to check.
4. 
5. a) 
   b) No right angles
   c) 
   d) 
   e) 
   f) 

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</tr>
<tr>
<td>Hexagons</td>
<td>E, H</td>
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</table>

6. a) Answers will vary.
   b) B, E, H and I each have 4 right angles
7. a) Answers will vary.
   b) K and R have both a right angle and an acute angle.

8. 

AP Book G4-3

1. b) and d) are half right angles
2. Teacher to check
3. Teacher to check
4. Teacher to check

AP Book G4-4

1. a) less than 90°
   b) greater than 90°
   c) less than 90°
   d) greater than 90°
   e) greater than 90°
   f) less than 90°
   g) greater than 90°
   h) less than 90°
2. a) less than 90°
   b) greater than 90°
   c) less than 90°
   d) greater than 90°
3. a) less than 90°
   b) less than 90°
   c) greater than 90°
   d) less than 90°
   e) greater than 90°
   f) greater than 90°
   g) less than 90°
   h) greater than 90°
4. In the angles provided, some of the lines are too short to be measured by a standard protractor. Please show your students how to use a ruler to extend the lines if necessary.
   Answers may vary by a few degrees.
a) 43°
b) 30°
c) 129°
d) 124°
e) 104°
f) 57°

AP Book G4-5

1. a) N
   b) Y
   c) N
   d) Y
   e) Y
   f) Y
   g) Y
   h) N
BONUS:
Answers will vary
2. Teacher to check.
3. Teacher to check – in all cases, the original lines will still be parallel.
4. b) 
   c) 
   d) 
   e) 
   f) 

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<tr>
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<td>C, D</td>
</tr>
<tr>
<td>Hexagons</td>
<td>E, H</td>
</tr>
</tbody>
</table>
f)  

5. a) 1 pair  
b) 2 pairs  
c) 1 pair  
d) 2 pairs  

AP Book G4-6  
page 158  

1. Property | Shape  
---|---  
Quadrilateral | A, C, E, G, J, L  
Non-Quadrilateral | B, D, F, H, I  

2. a) A, B, D, E, F, G  
b) A, E, G  
c) C  
d) 4 sides  
e) 5 sides  
f) F (not equilateral)  
g) Answers will vary.  

AP Book G4-7  
page 159  

1. A - 2 pairs  
B - 2 pairs  
C - 1 pair  
D - 2 pairs  
E - 0 pairs  
F - 2 pairs  
G - 1 pair  
H - 2 pairs  

2. None | 1 pair | 2 pairs  
---|---|---  
E | C, G | A, B, D, F, H  

3. a) Property | Shape  
---|---  
No right | B, G, I, K  
1 right | J  
2 right | D, E, F  
4 right | A, C, H  

b) Property | Shape  
---|---  
No parallel lines | B, J  
1 pair | D, E, F, I  
2 pairs | A, C, G, H, K  

4. a) Each side of the triangle is 2.5 cm - should be circled  
b) Each side of the pentagon is 2 cm - should be circled  
c) The two horizontal sides of the rectangle are 4 cm, and the two vertical sides are 1 cm - not equilateral  
d) Each side of the rhombus is 1.5 cm - should be circled  

5. a) Property | Shape  
---|---  
Equilateral | B, G, H, J  
Not Equilateral | A, C, D, E, F, I  

b) Property | Shape  
---|---  
No right | A, B, F, H, J  
1 right | I  
2 right | D, E  
4 right | C, G  

AP Book G4-8  
page 161  

1. a) 4 right angles  
   - top and bottom: 3 cm  
   - left and right: 2 cm  
   - Shape - rectangle  

b) 0 right angles  
   - all sides: 2 cm  
   - Shape - rhombus  

2. a) rectangle  
b) parallelogram  
c) square  
d) rhombus  

3. a) 4 right angles; rectangle  
b) 4 right angles; square  
c) 0 right angles; parallelogram  

4. Square: 
   - A parallelogram with 4 right angles and 4 equal sides  
   - Rectangle: 
     - A parallelogram with 4 right angles  
   - Rhombus: 
     - A parallelogram with 4 equal sides  

5. a) 2 pairs – rectangle  
b) 1 pair – trapezoid  
c) 2 pairs – parallelogram  

6. Answers will vary.  
7. Answers will vary.  
8. a) all  
b) no  
c) no  
d) some  

9. square or rectangle  
10. square or rhombus  

11. Answers should include three of the following: 
   - quadrilateral – since it has 4 sides; 
   - parallelogram – since it has 2 pairs of parallel sides; 
   - rectangle – since it's a parallelogram with 4 right angles; 
   - rhombus – since it's a parallelogram with 4 equal sides. 

12. a) Similarities: 
   - Both have 2 pairs of parallel sides. 
   - Differences: 
     - A rhombus has 4 equal sides.  

b) Similarities: 
   - Both have 4 equal sides. 
   - Differences: 
     - A square contains 4 right angles. 

c) Similarities: 
   - Both have 4 sides; 
   - both have at least 1 pair of parallel sides. 
   - Differences: 
     - A parallelogram has 2 pairs of parallel sides; a trapezoid has only 1 pair. 

AP Book G4-9  
page 163  

1. a) 4, 6  
b) 1, 2, 3, 5, 7  
c) 4, 6  

2. a)  

b)  

c)  

d)  

e)  

3. Answers will vary. 

AP Book G4-10  
page 164  

1. a) not congruent  
b) congruent  
c) not congruent  

2. a) 2 and 3  
b) 1 and 3  

3. Answers will vary.
c) $2^{nd}$ and $3^{rd}$
d) $1^{st}$ and $2^{nd}$
e) $1^{st}$ and $3^{rd}$
f) $1^{st}$ and $2^{nd}$
g) $1^{st}$ and $3^{rd}$
h) $2^{nd}$ and $4^{th}$
i) $1^{st}$ and $3^{rd}$
j) $2^{nd}$ and $3^{rd}$

3. From left to right – A; B; shape not congruent to others; B; A; shape not congruent to others

4. Teacher to check.

AP Book G4-11
page 165
1. Teacher to check.
2. Teacher to check.
3. Two A’s:

Three B’s:

Two C’s:

Two D’s:

** remaining shapes aren’t congruent with anything

4. Teacher to check.
5. a) Yes, because they are the same size and shape (colour doesn’t matter)
b) No, because they are not the same shape

6. a)

b) 

or

AP Book G4-12
page 166
1. Teacher to check.
2. Teacher to check.
3. a)

b)

4. a)

b)

5. a) 1 line of symmetry:

b) 2 lines of symmetry:

c) 2 lines of symmetry

d) 1 line of symmetry:

e) no lines of symmetry

AP Book G4-14
page 168
1. a) Teacher to check.

Example:

b) If a polygon is regular, the number of edges it has will equal the number of lines of symmetry it has.

2. Answers may vary.

Example:

AP Book G4-15
page 167
1. a) Answers will vary.
b) Teacher to check lines of symmetry drawn on page.

AP Book G4-16
page 170
1. a)
3. Students’ answers should include the following information:

<table>
<thead>
<tr>
<th>Property</th>
<th>F1</th>
<th>F2</th>
<th>S?</th>
<th>D?</th>
</tr>
</thead>
<tbody>
<tr>
<td># vertices</td>
<td>5</td>
<td>5</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td># edges</td>
<td>5</td>
<td>5</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>but F1 has all straight sides and F2 includes a curved side!</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of parallel sides</td>
<td>0</td>
<td>1</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td># of right angles</td>
<td>0</td>
<td>0</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>lines of symmetry?</td>
<td>Y</td>
<td>Y</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td># of lines of symmetry</td>
<td>5</td>
<td>1</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>equilateral?</td>
<td>Y</td>
<td>N</td>
<td>√</td>
<td></td>
</tr>
</tbody>
</table>

AP Book G4-17
page 171

1. a) D & H have both properties:

   A
   B  C  D  E  F
   G

b) Property 1: C, D, H
   Property 2: B, C, F, H
   C & H have both properties:

   A  D  E
   G  B

   C  A  G  E  F
   B

c) Property 1: A, C, D, E, H
   Property 2: D, E, G, H
   D, E & H have both properties:

   C  A  G  E  F
   B

2. Answers will vary.

AP Book G4-18
page 173

1. a) T - 4 vertices
   F - 2 pairs of parallel sides
   T - at least 2 right angles
   b) F - 3 vertices
   F - 5 sides
   T - no right angles
   T - equilateral
   c) T - quadrilateral
   F - 2 pairs of parallel sides
   F - at least one right angle
   d) F - 6 vertices
   T - no right angles
   T - at least 2 pairs of parallel sides

2. a) T - 4 vertices
   F - 2 pairs of parallel sides
   T - at least 2 right angles
   b) F - 3 vertices
   F - 5 sides
   T - no right angles
   T - equilateral
   c) T - quadrilateral
   F - 2 pairs of parallel sides
   F - at least one right angle
   d) F - 6 vertices
   T - no right angles
   T - at least 2 pairs of parallel sides

AP Book G4-19
page 174

1. 16

2. a) Trapezoid
   B: Trapezoid
   C: Square
   D: Square
   E: Rectangle
   F: Rectangle

3. A: 4 names (square, rectangle, rhombus, parallelogram)
   B: 1 name (parallelogram)
   C: 2 names (rectangle, parallelogram)
   D: 2 names (rhombus, parallelogram)
   E: 1 name (trapezoid)

4. A & H, D & E and B & G; each pair is the same size and shape

5. a) Teacher to check.
   Example:

   b) Both shapes have two parallel sides

6. Teacher to check.
   Example answers:
   a) Because all four angles are right angles.
   b) All four sides are not always equal
   c) Only 2 sides are parallel – there are no 2 pairs of parallel sides.

7. a) Parallelogram or rhombus
   b) rectangle
   c) trapezoid
1. 8 km
   (6 km + 6 km = 12 km; 20 km – 12 km = 8 km)
2. 5 km
   (15 km × 3 hours = 45 km; 50 km – 45 km = 5 km)
Teacher to check number lines for Questions 3 & 4.
3. 4 blocks
   (After 3 minutes she’ll have travelled 4 blocks × 3 min = 12 blocks, so she will be 16 – 12 = 4 blocks from home.)
4. 6 minutes
   (12 blocks ÷ 2 blocks per minute = 6 minutes)

Teacher to check number lines for all questions.
1. 25 km
   (After three days James will have travelled 75 + 75 + 75 km = 225 km, so he will be 250 – 225 = 25 km from the finish)
2. 10 minutes
   (250 words ÷ 25 words per minute = 10 minutes)
3. 200 metres
   (100 + 50 + 50 = 200 m from start)
4. 30 metres
   (10 + 5 + 5 + 5 + 5 = 30 metres)
5. The 6th and 12th steps have both red and blue paint on them (since 6 and 12 are multiples of both 2 and 3).

1. a) RYRYRYRY
   b) YRYRYRYR
   c) RYRYYRYRYYR
   d) RYRYRRYRYR
   e) YRYRYRYRYR

2. a) +, –, +
   b) +, +, +
   c) –, –, –
   d) +, +, +
   e) +, +, +

3. a) +, +, –
   b) +, –, +
   c) –, –, +
   d) +, +, +

4. a) Start at 8, add 3
   b) Start at 14, subtract 4
   c) No rule
   d) Start at 61, add 4

5. a) Increasing
   b) Repeating
   c) Decreasing
   d) Increasing
   e) Repeating
   f) Decreasing

6. a) 6, 9, 12, 15, 18
   b) 26, 22, 18, 14, 10
   c) 39, 44, 49, 54, 59

7. Answers will vary.
8. Answers will vary.
9. Answers will vary.

BONUS:
   Value of core: 7¢ so the value of first 15 coins = 7¢ × 5 sets of 3 coins = 35¢
2. a) 

b) 

c) 

d) 

3. a) 2 4 6 8 10 12 14 16 18 

b) 2 4 6 8 10 12 14 16 18 

c) 2 4 6 8 10 12 14 16 18 

d) 2 4 6 8 10 12 14 16 18 

4. a) Start at 8; add 2 

b) Start at 2; add 6 

c) Start at 6; add 6 

d) 1st diagonal (given): Start at 8; add 8 

2nd diagonal: Start at 6; add 4 

5. Both rows and columns are increasing by 2. 

One diagonal increases by 4; the other repeats. 


Each row increases by 3. 

One diagonal repeats; the other diagonal decreases by 6. 

7. 

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

a) Corresponding rows and columns are equal. 

Each row & column increases by its row / column number (for example, the 5th column increases by 5). 

The main diagonal increases by 3, 5, 7, 9, 11. Each diagonal above and below the main increases by patterns of increasing numbers (for example: the diagonal 1, 2, 6, 12, 20, 30 increases by 4, 6, 8, 10).

NOTE: Students may observe other patterns – teacher to check. 

b) Answers will vary – teacher to check. 

8. Answers may vary – teacher to check. 

Sample: 

<table>
<thead>
<tr>
<th>A</th>
<th>A</th>
<th>B</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

9. 

h) 5 

i) 15 

j) 5 

k) 5 

l) 5 

m) 70 

n) 8 

o) 6 

BONUS: 

p) 13 

q) 17 

r) 15 

s) 45 

t) 14 

u) 123 

AP Book PA4-22 

page 187 

1. 

2. If April 1st is a Sunday, there are 4 Tuesdays in the month: Huyan will have $20. 

3. Alex and Dan both have piano lessons on the 18th of October (a Friday). 

December 

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| S | M | T | W | T | F | S |

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>31</td>
<td>32</td>
</tr>
</tbody>
</table>

b) The shaded boxes show the day when Rona has guitar lessons.
4. Teacher to check that calendars are completed properly.
   a) Start at “x”, add 7.
   b) Again, the pattern will be an increase of 7. This occurs because there are 7 days in a week.

AP Book PA4-23
page 188
1. Teacher to check:
   2, 4, 6, 8, 10, 12, …, 100
2. The 2nd, 4th, 6th, 8th and 10th columns are shaded. Each row has 5 multiples of 2 evenly spaced apart.
3. All the ones digits for the multiples of two will be one of 0, 2, 4, 6, or 8, and they will cycle through in order.
4. If the ones digit of a number is 2, 4, 6, 8 or 0 then the number is a multiple of two.
5. Circle: 18, 32, 76, 30, 94, 82
6. Circle: 5, 75, 37, 83
7. When you add two even numbers, the sum will always be an even number.

AP Book PA4-24
page 189
1. Teacher to check:
   5, 10, 15, 20, 25, …, 100
2. In every row there are two shaded numbers. There are two shaded columns in the whole chart – the 5th and the 10th.
3. The ones digits of the multiples of 5 will always be 5 or 0, and will alternate between these two numbers.
4. For any number, if the ones digit is a 5 or a 0, the number is a multiple of 5.
5. Circle: 45, 60, 90, 85, 25, 50
6. No – the multiples end in 0, even but multiples that end at 5 are odd.
7. Circle: 205, 225, 385, 755

AP Book PA4-25
page 190
1. Teacher to check:
   8, 16, 24, 32, 40, …, 96
2. 0 8 4 8 16 5 6 24 6 4 7 2 40 8 0
   The ones digits start at 8 and decrease by 2 down each column: 8, 6, 4, 2, 0.
3. The tens digits increase by one down each column: 0, 1, 2, 3, 4.
4. 88 128 96 136 104 144 112 152 120 160

AP Book PA4-26
page 191
1. 
   b) Answers will vary, though will be two of 60, 70, 80 or 90.
   c) Answers will vary – teacher to check.

AP Book PA4-27
page 192
1. a) Gaps:
   + 2, + 3, + 4, + 5, + 6
   Pattern: 2, 4, 7, 11, 16, 22
   b) Gaps:
   + 1, + 2, + 3, + 4, + 5, + 6
   Pattern: 3, 4, 6, 9, 13, 18, 24
   c) Gaps:
   + 3, + 5, + 7, + 9, + 11
   Pattern: 11, 14, 19, 26, 35, 46
   d) Gaps:
   + 2, + 4, + 6, + 8, + 10, + 12
   Pattern: 6, 8, 12, 18, 26, 36, 48
   e) Gaps:
   – 1, – 2, – 3, – 4, – 5
   Pattern: 17, 16, 14, 11, 7, 2
   f) Gaps:
   – 2, – 4, – 6, – 8, – 10
   Pattern: 32, 30, 26, 20, 12, 2
   g) Gaps:
   – 1, – 3, – 5, – 7, – 9
   Pattern: 31, 30, 27, 22, 15, 6
   h) Gaps:
   Pattern: 110, 105, 95, 80, 60, 35, 5

2. 
   Fig # | # of Squares | # Added
   1 | 1           |
   2 | 4           | 3
   3 | 9           | 5
   4 | 16          | 7
   5 | 25          | 9
   6 | 36          | 11

3. 
   Fig # | # of Squares | # Added
   1 | 1
   2 | 5
   3 | 11
   4 | 19
   5 | 29

AP Book PA4-28
page 193
1. a) 2
   b) 32, 64, 128, 256
   c) In each case, the gap is equal to the most recent number in the pattern.
2. a) Start at $15 and add $5 each week.
   b) Start at $1 and multiply by 2 each week.
   c) Answers will vary.
   d) W | O | K
   1 | $1 | $15
   2 | $2 | $20
   3 | $4 | $25
   4 | $8 | $30
   5 | $16 | $35
   6 | $32 | $40
   7 | $64 | $45
3. a) The gap alternates between +3 and –2.
   b) 6, 9, 7

4. Figure # | Dots | Gap
---|---|---
1 | 3 | -
2 | 6 | 3
3 | 10 | 4
4 | 15 | 5
5 | 21 | 6
6 | 28 | 7

5. Monday: 10 min
   Tuesday: 12 min
   Wednesday: 14 min
   Thursday: 16 min
   And 16 + 14 + 12 + 10 = 52, so Jane trained for a total of 52 minutes in the first four days.

AP Book PA4-29
page 194
1. a) Min Sec
   1 | 60
   2 | 120
   3 | 180
   4 | 240
   5 | 300

b) Years Weeks
   1 | 52
   2 | 104
   3 | 156
   4 | 208

c) Years Days
   1 | 365
   2 | 730
   3 | 1095
   4 | 1460

2. There are 48 months in 4 years: 12 x 4 = 48.

AP Book PA4-30
page 195
For Questions 1 & 2, teacher to check that apples are drawn properly.
1. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

2. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

BONUS:
8. Answers will vary – teacher to check.

AP Book PA4-31
page 196
For Questions 1 & 2, teacher to check that apples are drawn properly.
1. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

2. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

AP Book PA4-32
page 197
1. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

2. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

3. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

5. The core of the pattern consists of 5 shapes. We know that 5 x 4 = 20, so a complete pattern ends on the 20th shape. A new pattern starts on the 21st shape, so to find the 23rd shape in the pattern look at the third element in the core: a square.

6. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

7. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

8. a) The 12th step will have both.
b) His left foot will land on the 6th and 12th steps.

9. The 6th and 12th people receive both a free pen and book

10. Emma will need 21 blocks to build a stairway 6 steps high: 1 + 2 + 3 + 4 + 5 + 6 = 21.
AP Book NS4-52
page 200

1. a) bikes; 4 sets; 3 bikes in each set
   b) birds; 3 sets; 4 birds in each set
2. a) 4 circles with 6 dots in each circle
    b) 6 circles with 3 dots in each circle
    c) 6 circles with 2 dots in each circle
    d) 4 circles with 5 dots in each circle
3. a) 20 toys; 5 sets; 4 in each set
    b) 21 pencils; 7 sets; 3 in each set
    c) 16 students; 4 sets; 4 in each set
    d) 24 flowers; 8 sets; 3 in each set
    e) 42 grapefruits; 7 sets; 6 in each set
    f) 30 kids; 3 sets; 10 in each set
    g) 36 puppies; 6 sets; 6 in each set
BONUS:
4. a) 5 circles with 4 dots in each circle
    b) 7 circles with 3 dots in each circle
    c) 4 circles with 4 dots in each circle

AP Book NS4-53
page 202
1. Teacher to check.
2. a) 4 people per van
    b) 3 stickers per kid
    c) 4 flowers per plant
    d) 2 grapefruits per box
3. 4 cherries
4. 3 stickers
5. 2 apples

AP Book NS4-54
page 203

1. a) 2 sets
    b) 4 sets
2. a) 3 sets of 3
    b) 3 sets of 4
    c) 5 sets of 3
    d) 4 sets of 4
3. a) 3 boxes
    b) 5 kids
4. 6 siblings
5. 7 envelopes

AP Book NS4-55
page 204

1. a) 5 dots in each set
    b) 4 dots in each set
2. a) 6 triangles in each circle
    b) 3 triangles in each circle
3. 2 squares in each circle
4. a) 3 sets
    b) 4 sets
    c) 2 sets
5. a) 2 circles with 6 dots in each circle
    b) 3 circles with 4 dots in each circle
6. a) 25 pencils; There are 5 sets; 5 in each set
    b) 30 children; 10 sets of boats; 3 children per boat
    c) 36 stickers; There are 4 sets. 9 in each set
    d) 12 books; There are 4 sets; There are 3 books in each set.
    e) 15 girls; There are 3 sets; 5 girls at each table.
7. a) 3 dots in each set
    b) 2 sets
    c) 3 sets
    d) 2 dots in each set
8. a) 4 circles with 2 dots in each circle
    b) 2 circles with 5 dots in each circle
    c) 3 circles with 4 dots in each circle
    d) 2 circles with 8 dots in each circle
   e) 4 circles with 3 dots
   f) 2 circles with 9 dots

AP Book NS4-56
page 207
1. a) 4
    b) 2
2. a) 20
    b) 6
3. a) 7
    b) 3
    c) 3
    d) 3
    e) 7
    f) 9
    g) 8
    h) 8
4. 28
    7 = 4
5. 30
    6 = 5

AP Book NS4-57
page 208
1. a) 4
    b) 2
2. a) 20
    b) 6
3. a) 7
    b) 3
    c) 3
    d) 3
    e) 7
    f) 9
    g) 8
    h) 8
4. 8
5. 8

AP Book NS4-58
page 209
1. a) 3 circles with 4 lines in each circle;
    b) 5 circles with 2 hearts in each circle;
    c) 2 circles with 4 stars in each circle;
    d) 4 circles with 3 stars in each circle;
    e) 7 circles with 2 dots in each circle;
    f) 2 circles with 8 cylinders in each circle.
g) 3 circles with 5 diamonds in each
h) 6 circles with 2 ovals in each

BONUS:
i) 3 circles with 8 dots in each
j) 5 circles with 6 dots in each
k) 4 circles with 12 dots in each

2. $16 \div 4 = 4$

3. a) 12 lines; 3 sets; 4 lines in each set; $12 \div 3 = 4$; $12 \div 4 = 3$
b) 16 lines; 4 sets; 4 lines in each set; $16 \div 4 = 4$; $16 \div 4 = 4$
c) 12 lines; 4 sets; 3 lines in each set; $12 \div 4 = 3$; $12 \div 3 = 4$

4. a) 3 sets; 4 squares in each set; $12 \div 3 = 4$; $12 \div 4 = 3$
b) 2 sets; 6 triangles in each set; $12 \div 2 = 6$; $12 \div 6 = 2$
c) 3 sets; 3 stars in each set; $9 \div 3 = 3$; $9 \div 3 = 3$

5. Teacher to check drawings;
a) 4 circles with 3 triangles in each circle; $12 \div 4 = 3$; 3 triangles in each set
b) 2 circles with 3 squares in each circle; $6 \div 3 = 2$; 2 sets

6. Teacher to check drawings;
a) 5 circles with 4 dots in each circle; $20 \div 5 = 4$ (4 people in each car)
b) 3 circles with 4 dots in each circle; $12 \div 3 = 4$ (4 children in each boat)

AP Book NS4-59

1. a) $3 \times 5 = 15$; $5 \times 3 = 15$; $15 \div 5 = 3$; $15 \div 3 = 5$
b) $6 \times 4 = 24$; $4 \times 6 = 24$; $24 \div 4 = 6$; $24 \div 6 = 4$
c) $2 \times 3 = 6$; $3 \times 2 = 6$; $6 \div 3 = 2$; $6 \div 2 = 3$; 6 fish; 2 sets;
   3 fish in each set
d) $6 \times 2 = 12$; $2 \times 6 = 12$; $12 \div 2 = 6$; $12 \div 6 = 2$; $12$ snails; 6 sets;
   2 snails in each set

2. Teacher to Check

AP Book NS4-60

1. a) 12 lines; 4 lines in each set; 3 sets

AP Book NS4-61

1. a) $6 \times 3 = 18$
b) $20 \div 4 = 5$
c) $15 \div 5 = 3$
d) $10 \div 2 = 5$
e) $24 \div 6 = 4$
f) $21 \div 7 = 3$

2. a) $18 \div 3 = 6$; 6 sets
   b) $5 \times 4 = 20$; 20 things
c) $8 \times 3 = 24$; 24 things
d) $12 \div 6 = 2$; 2 sets
e) $10 \div 5 = 2$; 2 things

AP Book NS4-62

1. No: 2 circles with 2 dots in each circle and 1 dot outside of all circles.

2. Teacher to check diagrams.
a) 3 dots in each circle; 1 dot remaining
b) 3 dots in each circle; 1 dot remaining
c) 2 dots in each circle; 0 dots remaining
d) 2 dots in each circle; 1 dot remaining

e) 2 dots in each circle; 2 dots remaining

f) 3 dots in each circle; 1 dot remaining

3. Teacher to check diagrams.

b) \(11 \div 3 = 3 \text{ R}2\)

c) \(14 \div 3 = 4 \text{ R}2\)

d) \(10 \div 6 = 1 \text{ R}4\)

e) \(10 \div 4 = 2 \text{ R}2\)

4. Each friend will receive 2 cherries, 1 will be left:

\(7 \div 3 = 2 \text{ R}1\)

5. 3 circles with 4 granola bars in each circle; 1 bar remaining

\((13 \div 3 = 4 \text{ R}1);\)

OR

4 circles with 3 granola bars in each circle; 1 bar remaining

\((13 \div 4 = 3 \text{ R}1)\)

6. There are 2 answers:

\(9 \div 3 = 3 \text{ oranges};\)

\(6 \div 3 = 2 \text{ oranges}\)

AP Book NS4-63

page 217

1. a) 3 (size); 2 (number); 1 (remainder)

b) 5 (size); 1 (number); 2 (remainder)

c) 5 (size); 2 (number); 4 (remainder)

d) 3 (size); 3 (number); 1 (remainder); \(10 \div 3 = 3 \text{ R}1\)

e) 2 (size); 3 (number); 1 (remainder); \(7 \div 2 = 3 \text{ R}1\)

2. 2 bags; 3 oranges left over

AP Book NS4-64

page 218

1. a) 3 R3

b) 4 R3

c) 5 R1

d) 5 R3

e) 3 R1

f) 4 R0

k) 2 R1

l) 4 R1

m) 2 R2

n) 3 R1

o) 3 R1

p) \(13 + 2 = 6 \text{ R}1\)

q) \(45 + 8 = 5 \text{ R}5\)

r) \(63 + 7 = 9 \text{ R}0\)

2. 3 each; 1 left over

3. 3 friends; 2 left over

AP Book NS4-65

page 219

1. b) Groups – 4;

Tens blocks – 9;

Ones – 2

c) Groups – 5;

Tens block – 8;

Ones – 6

d) Groups – 2;

Tens block – 8;

Ones – 7

2. b) 2

c) 1

d) 3

e) 2

f) 4

g) 2

h) 1

i) 1

j) 1

3. b) Groups – 2;

Number of tens in each group – 4

c) Groups – 4;

Number of tens in each group – 2

d) Groups – 2;

Number of tens in each group – 3

4. a) Groups – 3;

Tens – 8;

Number of tens in each group – 2;

Number of tens placed – 6

b) Groups – 4;

Tens – 9;

Number of tens in each group – 2;

Number of tens placed – 8

5. a) Number of tens that can be placed in each group – 3;

Total number of tens placed – 6

b) 2; 6

c) 3; 6

d) 2; 8

e) 2; 6

f) 1; 4; 1

g) 1; 8; 1

h) 4; 8; 1

i) 2; 6; 1

j) 2; 8; 0

6. a) Number of tens that can be placed in each group – 1;

Total number of tens placed – 6;

Number of tens left over – 3

b) 2; 6; 1

c) 2; 4; 0

d) 2; 8; 0

e) 2; 6; 2

f) 1; 4; 1

g) 1; 8; 1

h) 4; 8; 1

i) 2; 6; 1

j) 2; 8; 0

7. a) Number of tens that can be placed in each group – 2;

Total number of tens placed – 6;

Number of ones to place – 15

b) 2; 4; 17

c) 4; 8; 13

d) 2; 8; (0)3

e) 1; 6; 21

f) 1; 4; 23

g) 1; 2; 15

h) 1; 7; 18

i) 1; 8; 11

j) 1; 9; 3

8. a) Number of tens that can be placed in each group – 2;

Total number of tens placed – 6;

Number of ones to place – 16;

Total number of ones placed in each group – 5

b) 1; 5; 25; 5

c) 2; 4; 15; 7

d) 1; 4; 11; 2

e) 1; 3; 12; 4

f) 1; 7; (0)5; 0
Answer Key for AP Book 4.2

9. a) 3; 9; (0)2; 0
b) 1; 9; (0)6; 2
c) 3; 6; 13; 6
d) 4; 8; 11; 5
e) 1; 9; (0)2; 0
f) 3; 6; 13; 6

h) 3; 9; (0)6; 2
i) 1; 9; (0)2; 0
j) 3; 6; 13; 6

10. 2 tomatoes
11. 12 weeks
12. 19 cm
13. 12 days
14. 14 boats

15. Mike had 2 apples left over, and Alexa had 1 left over. So Mike had more left over.

AP Book NS4-66

1. a) 13 ÷ 4 = 3 R1
b) 54 ÷ 2 = 22 R0

2. a) 4
b) 8

c) 9

3. a) 9, 9 R6
b) 8, 9 R6
c) 9, 11 R3
d) 13, 14

4. The result is the number itself.

5. 15 canoes
6. 13 nights
7. 16 cards
8. 11 pages

AP Book NS4-67

1. a) 4, 8, 12, 16
b) 5, 10, 15, 20
c) 20, 40, 60, 80
d) 60 km

2. Killer Whale - 2 cm : 6 m
   Baleen Whale - 6.5 cm : 19.5 m
   Blue Whale - 11 cm : 33 m

3. $32
4. $80

5. a) $5
b) $3
c) $2

AP Book NS4-68

1. a) 8 × 6 = 48
   48 ÷ 6 = 8
   48 ÷ 8 = 6

2. a) 28
b) 24

3. a) 23
b) $138 ($6 × 23)
4. Answers may vary.  
   Possible Answers: 
   4 groups of 3 each;  
   3 groups of 4 each;  
   2 groups of 6 each;  
   6 groups of 2 each.  

5. Answer may vary.  
   Sample Answers: 
   - 12, 15, and 18 all have a remainder of 0 when divided by 3.  
   - 10, 13, and 16 all have a remainder of 1 when divided by 3.  

4. Answers may vary.  
   Possible Answers: 
   - 4 groups of 3 each;  
   - 3 groups of 4 each;  
   - 2 groups of 6 each;  
   - 6 groups of 2 each.  

5. Answer may vary.  
   Sample Answers: 
   - 12, 15, and 18 all have a remainder of 0 when divided by 3.  
   - 10, 13, and 16 all have a remainder of 1 when divided by 3.  

6. a) 6  
   b) 12  
   c) 360  

7. 16 books  

8. 21  

9. 170 m  

10. 944 m  

11. 15 cm  

12. a) 9 eggs  
    b) 18 eggs  
    c) Teacher to check.  

AP Book NS4-69  
page 227  
1. a) 1 × 3 = 3  
    b) 7 × 10 = 70  
    c) 7 × 3 = 21  
    d) 2 – 1 = 1  

2. Teacher to check.  
   Maximum of 6 ways: 
   RGB, RBG, GRB, GBR, 
   BRG, BGR  

3. a) 2  
    b) 3  
    c) 4  
    d) 4  
    e) 4  
    f) 3  
    g) 2  
    h) 1  

4. a) Yes – 2 boxes of 4  
    b) Yes – 2 boxes of 5  
    c) No  
    d) Yes – 2 boxes of 5, 1 box of 4  
    e) Yes – 1 box of 5, 3 boxes of 4  
    f) Yes – 2 boxes of 5, 2 boxes of 4  
    g) Yes – 3 boxes of 5, 1 box of 4  

2. Teacher to check.  

AP Book NS4-70  
page 228  
1. a) 3/8  
    b) 3/4  
    c) 1/4  
    d) 5/8  
    e) 5/9  
    f) 8/12  
    g) 1/8  
    h) 3/10  

2. Teacher to check shading.  

3. a) Sixths  
    b) Sixths  
    c) Fifths  
    d) Thirds  
    e) Ninths  
    f) Halves  

AP Book NS4-69  
page 227  
1. 1 × 2 = 2  
   7 × 10 = 70  
   7 × 3 = 21  
   2 – 1 = 1  

2. Teacher to check.  
   Maximum of 6 ways: 
   RGB, RBG, GRB, GBR, 
   BRG, BGR  

3. a) 2  
    b) 3  
    c) 4  
    d) 4  
    e) 4  
    f) 3  
    g) 2  
    h) 1  

4. a) Yes – 2 boxes of 4  
    b) Yes – 2 boxes of 5  
    c) No  
    d) Yes – 2 boxes of 5, 1 box of 4  
    e) Yes – 1 box of 5, 3 boxes of 4  
    f) Yes – 2 boxes of 5, 2 boxes of 4  
    g) Yes – 3 boxes of 5, 1 box of 4  

3. a) Sixths  
    b) Sixths  
    c) Filths  
    d) Thirds  
    e) Ninths  
    f) Halves  

AP Book NS4-71  
page 229  
1. Teacher to check.  

2. Teacher to check.  

3. a) 1/3 is shaded  
    b) 1/5 is shaded  
    c) 2/5 is shaded  
    d) 1/5 is shaded  

4. Teacher to check  

5. Teacher to check  

6. a) That the pie was cut into 5 pieces.  
    b) That you have 3 of those pieces.  

7. a) NO: Shaded area is smaller than the 3 non-shaded areas – parts are not the same size.  
    b) NO: Pie is divided into thirds, not quarters.  
    c) YES: All four parts are equal in size and one is shaded.  
    d) YES: Each triangle is half the size of the left-hand square; each rectangle is half the size of the right-hand square. 
   Both squares are the same size.  

AP Book NS4-72  
page 230  
1. a) 2/3  
    b) 3/5  
    c) 2/8 = 1/4  
    d) 4/8 = 1/2  
   8/16 = 1/2  

2. Teacher to check shading.  

3. a) Sixths  
    b) Sixths  
    c) 5/6  
    d) 3/9  
    e) 1/8  

4. a) 2/8  
    b) 1/8  
    c) 3/9  
    d) 1/12  

5. 1/3  

6. 1/2  

7. 1/3  

8. 1/12  

9. 1/6  

AP Book NS4-74  
page 233  
1. 3/4 has the greater numerator and is the greater fraction  
   Explanations will vary – teacher to check.  
   Sample Answer: 
   Since both squares are equal and have been divided into fourths, 
   the fraction with more pieces shaded (e.g. with the greater numerator) 
   will be the greater fraction.
2. a) \( \frac{5}{14} \)
b) \( \frac{7}{12} \)
c) \( \frac{5}{9} \)
d) \( \frac{5}{7} \)
e) \( \frac{7}{27} \)
f) \( \frac{20}{98} \)
g) \( \frac{47}{125} \)
h) \( \frac{88}{267} \)

3. a) \( \frac{1}{3} \), \( \frac{2}{3} \), \( \frac{3}{3} \)
b) \( \frac{1}{10} \), \( \frac{2}{10} \), \( \frac{7}{10} \), \( \frac{9}{10} \)
c) \( \frac{5}{17} \), \( \frac{8}{17} \), \( \frac{9}{17} \), \( \frac{16}{17} \)

4. Teacher to check.

5. If two fractions have the same denominator, you know that the one with the larger numerator will be greater.

### AP Book NS4-76 page 235

1. Teacher to check
2. Teacher to check
3. Teacher to check
4. a) \( \frac{1}{2} \)
b) \( \frac{2}{3} \)
c) \( \frac{4}{5} \)
d) \( \frac{4}{7} \)

5. No: since Figure 2 is larger than Figure 1, \( \frac{1}{4} \) of it will also be larger!

6. Yes – depending on the size of the figures:
   e.g. \( \bigcirc \bigcirc \)

7. Ken ate more pie than Karen:
   \( \frac{3}{5} \) (Ken) > \( \frac{2}{5} \) (Karen)

### AP Book NS4-78 page 237

1. a) \( \frac{5}{4} \)
b) \( \frac{5}{3} \)
c) \( \frac{5}{2} \)
d) \( \frac{15}{8} \)
e) \( \frac{19}{8} \)
f) \( \frac{10}{6} \)
g) \( \frac{15}{4} \)
h) \( \frac{9}{2} \)

2. a) \( \bigcirc \bigcirc \bigcirc \bigcirc \)

### AP Book NS4-79 page 238

1. a) \( \frac{3}{2} = \frac{7}{2} \)
b) \( \frac{3}{4} = \frac{11}{4} \)
c) \( \frac{2}{3} = \frac{8}{3} \)
d) \( \frac{3}{8} = \frac{25}{8} \)
e) \( \frac{3}{4} = \frac{13}{4} \)
f) \( \frac{1}{2} = \frac{3}{2} \)

2. Teacher to check shading.
   a) \( \frac{5}{2} \)
b) \( \frac{13}{4} \)
c) \( \frac{13}{6} \)
d) \( \frac{21}{8} \)

3. a) \( \frac{2}{3} \)
b) \( \frac{2}{6} \)
c) \( \frac{1}{4} \)
d) \( \frac{2}{5} \)
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<td>1. a) [Diagram]</td>
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<td>2. a) Mixed: ( \frac{1}{2} )</td>
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<td>3. b) [Diagram]</td>
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<td>4. a) [Diagram]</td>
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<td>5. b) [Diagram]</td>
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<td>6. Teacher to check accompanying sketches.</td>
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<tr>
<td>a) ( \frac{5}{6} ) is greater</td>
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<td>b) ( \frac{3}{6} ) is greater</td>
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<tr>
<td>1. a) 2 halves</td>
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<td>2. a) 3 thirds</td>
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<td>3. a) 4 quarters</td>
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<td>4. a) 8 cans</td>
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<td>5. a) 13 cans</td>
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<td>6. 13 pens</td>
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<td>7. 15 bottles</td>
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<td>1. a) 2 whole pies</td>
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<tr>
<td>2. a) 2; 1; 2 ( \frac{1}{2} ) pies</td>
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<td>3. a) ( \frac{1}{2} )</td>
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<td>1. a) ( \frac{7}{8} )</td>
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<td>2. a) ( \frac{2}{4} )</td>
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<td>3. a) ( \frac{2}{8} )</td>
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<td>4. a) ( \frac{2}{10} )</td>
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<td>2. Teacher to check grouping</td>
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<tr>
<td>a) ( \frac{1}{2} )</td>
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<td>2. [Diagram]</td>
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**Answer Key for AP Book 4.2**
b) \(\frac{2}{6} = \frac{1}{3}\)

c) \(\frac{2}{10} = \frac{1}{5}\)

d) \(\frac{3}{4}\)

e) \(\frac{2}{3}\)

f) \(\frac{2}{5}\)

3. a) \(\frac{2}{3} = \frac{4}{6}\)

b) \(\frac{2}{3} = \frac{6}{9}\)

c) \(\frac{1}{2} = \frac{2}{4}\)

4. a) \(\frac{1}{2} = \frac{3}{6}\)

b) \(\frac{6}{8} = \frac{3}{4}\)

c) \(\frac{2}{3} = \frac{6}{9}\)

d) \(\frac{3}{4} = \frac{6}{8}\)

e) \(\frac{1}{2} = \frac{4}{8}\)

f) \(\frac{5}{10} = \frac{1}{2}\)

5. a) \(\frac{1}{3}\)

b) \(\frac{3}{3}\)

c) \(\frac{2}{3}\)

6. Yes, Dan is right because \(\frac{1}{2}\) and \(\frac{2}{4}\) are equivalent fractions.

AP Book NS4-86

page 246

1. b) \(\frac{4}{5}\) of 15

c) \(\frac{2}{3}\) of 9

d) \(\frac{3}{5}\) of 10

2. a) \(\frac{1}{3}\) of 6 = 2

b) \(\frac{1}{4}\) of 8 = 2

c) \(\frac{1}{3}\) of 9 = 3

b) \(\frac{1}{4}\) of 8 = 2

c) \(\frac{1}{3}\) of 9 = 3

d) \(\frac{2}{3}\) of 9 = 6

e) \(\frac{3}{4}\) of 10 = 7

e) \(\frac{3}{4}\) of 12 = 9

3. a) Circle 2 of 3 sets

b) Circle 3 of 4 sets

AP Book NS4-87

page 248

1. a) \(8 + 2 = 4\)

b) \(10 + 2 = 5\)

c) \(16 + 2 = 8\)

d) \(20 + 2 = 10\)

e) \(9 + 3 = 3\)

f) \(15 + 3 = 5\)

g) \(12 + 4 = 3\)

h) \(18 + 6 = 3\)

2. a) Circle 3 of 6 lines

b) Circle 5 of 10 lines

c) Circle 2 of 4 lines

d) Circle 6 of 12 lines

e) Circle 7 of 14 lines

3. a) \(\frac{1}{3}\) Shade 2 circles

b) \(\frac{1}{3}\) Shade 4 circles

c) \(\frac{1}{3}\) Shade 1 circles

d) \(\frac{1}{3}\) Shade 5 circles

e) \(\frac{2}{3}\) Circle 4 circles

f) \(\frac{2}{3}\) Circle 8 circles

g) \(\frac{2}{3}\) Circle 2 circles

h) \(\frac{2}{3}\) Circle 10 circles

4. a) \(\frac{1}{4}\) : Circle 1 triangle

b) \(\frac{1}{4}\) : Circle 3 triangles

c) \(\frac{1}{4}\) : Circle 4 triangles

5. a) Shade 6 of the 10 boxes

b) Shade 12 of the 20 boxes

c) Shade 9 of the 15 boxes

AP Book NS4-88

page 249

1. b) \(\frac{3}{5}\) + \(\frac{2}{5}\) = \(\frac{3}{4}\)

2. a) \(\frac{1}{7}\) + \(\frac{2}{7}\) = \(\frac{3}{7}\)

b) \(\frac{1}{3}\) + \(\frac{1}{3}\) = \(\frac{2}{3}\)

3. a) \(\frac{4}{5}\)

b) \(\frac{3}{4}\)

c) \(\frac{5}{7}\)

d) \(\frac{7}{8}\)

e) \(\frac{10}{11}\)

f) \(\frac{14}{17}\)

g) \(\frac{21}{24}\)

h) \(\frac{31}{57}\)

4. a) \(\frac{2}{4}\)

b) \(\frac{1}{5}\)

5. a) \(\frac{1}{3}\)

b) \(\frac{1}{5}\)

c) \(\frac{3}{7}\)

d) \(\frac{3}{8}\)

e) \(\frac{7}{12}\)

f) \(\frac{2}{19}\)

g) \(\frac{6}{28}\)

h) \(\frac{5}{57}\)

AP Book NS4-90

page 251

1. a) \(\frac{1}{4}\) + \(\frac{2}{4}\) = \(\frac{3}{4}\)

2. a) \(\frac{1}{5}\) + \(\frac{2}{5}\) = \(\frac{3}{5}\)

b) \(\frac{1}{3}\) + \(\frac{1}{3}\) = \(\frac{2}{3}\)

3. a) \(\frac{4}{5}\)

b) \(\frac{3}{4}\)

c) \(\frac{5}{7}\)

d) \(\frac{7}{8}\)

e) \(\frac{10}{11}\)

f) \(\frac{14}{17}\)

g) \(\frac{21}{24}\)

h) \(\frac{31}{57}\)

4. a) \(\frac{2}{4}\)

b) \(\frac{1}{5}\)

5. a) \(\frac{1}{3}\)

b) \(\frac{1}{5}\)

c) \(\frac{3}{7}\)

d) \(\frac{3}{8}\)

e) \(\frac{7}{12}\)

f) \(\frac{2}{19}\)

g) \(\frac{6}{28}\)

h) \(\frac{5}{57}\)

Answer Key for AP Book 4.2
1. Teacher to check the groupings of the squares. The fractions are all the same since the drawings given can be grouped in each way.

2. \[ \frac{1}{3} = \frac{8}{24} = \frac{2}{6} = \frac{4}{12} \]

3. \( \frac{7}{2} > \frac{5}{2} \)

Seven half-pies are more than five half-pies.

4. Teacher to check the drawings.
   a) \(3 \frac{1}{2}\)
   b) \(7\)
   c) \(3 \frac{1}{2}\)
   d) \(6\)

5. a) \(9\)
   b) \(11\)
   c) \(13\)
   d) \(9\)

6. \[ \frac{7}{3} = \frac{14}{6}, \frac{5}{2} = \frac{15}{6}; \]
   \(\frac{15}{6} > \frac{14}{6}\)

   is the greater fraction.

   Teacher to check model.

7. \(\frac{7}{4} = \frac{3}{4}\)

   which is between 1 and 2.

8. There will be 6 black squares in the finished quilt.

AP Book NS4-93

1. a) \$3 \quad 35¢ \quad $3.35
   b) $6 \quad 11¢ \quad $6.11
   c) $4 \quad 25¢ \quad $4.25
   d) $6 \quad 55¢ \quad $6.55
   e) $10 \quad 7¢ \quad $10.07
   f) $20 \quad 11¢ \quad $20.11

2. a) 105¢ $1.05
   b) 96¢ $0.96
   c) 150¢ $1.50
   d) 110¢ $1.10

3. a) 7¢ = $0.07
   b) 20¢ = $0.20
   c) 60¢ = $0.60
   d) 4¢ = $0.04
   e) 13¢ = $0.13
   f) 25¢ = $0.25
   g) 25¢ = $0.25
   h) 75¢ = $0.75
   i) 80¢ = $0.80
   j) 1200¢ = $12
   k) 400¢ = $4
   l) 700¢ = $7

4. 168¢ is greater. \$1.65 = 165¢ which is less than 168¢.

AP Book NS4-94

1. a) $3 \quad 35¢ \quad $3.35
   b) $6 \quad 11¢ \quad $6.11
   c) $4 \quad 25¢ \quad $4.25
   d) $6 \quad 55¢ \quad $6.55
   e) $10 \quad 7¢ \quad $10.07
   f) $20 \quad 11¢ \quad $20.11

2. a) 105¢ $1.05
   b) 96¢ $0.96
   c) 150¢ $1.50
   d) 110¢ $1.10

3. Quarters
4. 1 toonie, 1 loonie and 2 quarters
5. 2 toonies, 1 loonie and 1 quarter

AP Book NS4-95

1. 1. Circle: $10.05, $84.32, $0.95, $0.17, $25.30, 36¢, $18.50, $95.99.
   2. From left to right - $2.00, $1.00, 25¢, 10¢, $0.05, 1¢
   3. From left to right - $100.00, $50.00, $20.00, $10.00, $5.00

AP Book NS4-96

1. a) 95
   b) 77
   c) 54
   d) 89
   e) 79

2. a) $8.68
   b) $37.49
   c) $38.48

3. a) $39.15
   b) $71.37
   c) $51.75
   d) $60.60
   e) $34.85
   f) $48.75

Answer Key for AP Book 4.2
4. 58¢ or $0.58
5. $24.39
7. a) Yes
   b) No
8. Dog A – Sandor
   Dog B – Tory
   Dog E – Mike
   Dog F – Anthony
9. a) $3.75
    b) 3 lemons
    c) 4

AP Book NS4-97
page 260
1. a) $1.53
    b) $3.24
    c) $5.41
    d) $2.63
    e) $2.20
2. a) $2.55
    b) $5.74
    c) $0.09
    d) $30.69
    e) $9.00
    f) $13.82
3. $0.40
4. $0.77
5. No. $1 short
6. $6.75

AP Book NS4-98
page 261
1. a) 15 $15.36
    b) 31 $30.85
    c) 27 $27.21
    d) 23 $22.50

2. a) 50¢
    b) 40¢
    c) 80¢
    d) 70¢
    e) 50¢
    f) 70¢
    g) 20¢

3. a) Circle
    b) Don’t circle
    c) Circle
    d) Don’t circle
    e) Circle
    f) Don’t circle
    g) Circle
    h) Don’t circle
    i) Circle
    j) Circle
    k) Circle
    l) Don’t circle

4. a) $6.00
    b) $13
    c) $23
    d) $7
    e) $37
    f) $12
    g) $48
    h) $412
    i) $4
    j) $35
    k) $30
    l) $46

5. a) $8.00
    b) $8.00
    c) $9.00
    d) $5.00
    e) $16.00
    f) $40.00
    g) $42.00
8. $29.00
9. $42.00
10. a) Estimate: $2
    Actual: $1.92
    b) Estimate: $27
    Actual: $27.06
11. Yes

12. Both numbers will round to $7.00, which will make them seem equal when they are not.

AP Book NS4-99
page 263
1. a) \( \frac{2}{10} \)
    b) \( \frac{5}{10} \) or \( \frac{1}{2} \)
    c) \( \frac{7}{10} \)
    d) \( \frac{1}{10} \)

2. a) \( \frac{4}{10} = 0.4 \)
    b) \( \frac{3}{10} = 0.3 \)
    c) \( \frac{8}{10} = 0.8 \)
    d) \( \frac{2}{10} = 0.2 \)

3. a) \( \frac{0.2 + 0.2}{0.4} \)
    b) \( \frac{0.4 + 0.3}{0.7} \)
    c) \( \frac{0.5 + 0.2}{0.7} \)
    d) \( \frac{0.1 + 0.6}{0.7} \)
    e) \( \frac{0.5 + 0.5}{1} \)
    f) \( \frac{0.8 + 0.2}{1} \)
    g) \( \frac{0.3 + 0.1}{0.4} \)
    h) \( \frac{0.2 + 0.7}{0.9} \)
    i) \( \frac{0.2 + 0.4}{0.6} \)

4. 0.8, 1

AP Book NS4-100
page 264
1. a) hundredths
    b) tenths
    c) ones
    d) tenths
    e) hundredths
    f) tenths
    g) ones
    h) hundredths
    i) hundredths

2. Teacher to Check Shading.
   a) 0.38
   b) 0.45
   c) 0.05

3. \( \frac{4}{10} = 0.4 \)
   \( \frac{6}{100} = 0.06 \)
   \( \frac{9}{100} = 0.09 \)

4. Answers will vary.

AP Book NS4-102
page 266
1. a) \( \frac{32}{100} = 0.32 \)
    b) \( \frac{47}{100} = 0.47 \)
    c) \( \frac{76}{100} = 0.76 \)
    d) \( \frac{87}{100} = 0.87 \)

2. a) 7 tenths
    1 hundredths
b) 2 tenths
8 hundredths;
\( \frac{28}{100} = 0.28 \)

c) 4 tenths
1 hundredths;
\( \frac{41}{100} = 0.41 \)

d) 6 tenths
0 hundredths;
\( \frac{60}{100} = 0.60 \)

e) 0 tenths
8 hundredths;
\( \frac{8}{100} = 0.08 \)

f) 0 tenths
2 hundredths;
\( \frac{2}{100} = 0.02 \)

3. a) 5 tenths
2 hundredths;
\( \frac{52}{100} \)

b) 8 tenths
3 hundredths;
\( \frac{83}{100} \)

c) 2 tenths
4 hundredths;
\( \frac{24}{100} \)

d) 7 tenths
0 hundredths;
\( \frac{70}{100} \)

e) 0 tenths
7 hundredths;
\( \frac{7}{100} \)

f) 0 tenths
2 hundredths;
\( \frac{2}{100} \)

AP Book NS4-103
page 267

1. Teacher to check drawings.

<table>
<thead>
<tr>
<th>F</th>
<th>D</th>
<th>ED</th>
<th>EF</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{10} )</td>
<td>0.3</td>
<td>0.30</td>
<td>( \frac{30}{100} )</td>
</tr>
<tr>
<td>( \frac{7}{10} )</td>
<td>0.7</td>
<td>0.70</td>
<td>( \frac{70}{100} )</td>
</tr>
<tr>
<td>( \frac{1}{1} )</td>
<td>1.0</td>
<td>1.00</td>
<td>( \frac{100}{100} )</td>
</tr>
</tbody>
</table>

2. a) \( \frac{50}{100} = \frac{5}{10} \)

b) \( \frac{60}{100} = \frac{6}{10} \)

AP Book NS4-104
page 268

1. a) 7 dimes 3 pennies
7 tenths
3 hundredths
73 pennies
73 hundredths

b) 6 dimes 2 pennies
6 tenths
2 hundredths
62 pennies
62 hundredths

c) 4 dimes 8 pennies
4 tenths
8 hundredths
48 pennies
48 hundredths

d) 0 dimes 3 pennies
0 tenths
3 hundredths
3 pennies
3 hundredths

e) 0 dimes 9 pennies
0 tenths
9 hundredths
9 pennies
9 hundredths

f) 1 dimes 9 pennies
1 tenth
9 hundredths
9 pennies
9 hundredths

g) 48 hundredths
48 hundredths

2. a) 6 dimes 0 pennies
6 tenths
0 hundredths
0 pennies
0 hundredths

b) 8 dimes 0 pennies
8 tenths
0 hundredths
80 pennies
80 hundredths

AP Book NS4-105
page 269

1. a) \( \frac{4}{10} \)

b) \( \frac{8}{10} \)

2. a) \( \frac{7}{10} \)

b) \( \frac{3}{10} \)

3. a) \( \frac{6}{10} \)

b) \( \frac{3}{10} \)
### Number Sense 2 – AP Book 4.2 (continued)

#### AP Book NS4-106

**page 270**

1. a) $\frac{23}{100} = 1.23$
   
   b) $\frac{34}{100} = 1.34$
   
   c) $\frac{47}{100} = .47$
   
   d) $\frac{30}{100} = 2.30$
   
   e) $\frac{454}{100} = 4.54$

2. Teacher to check.

3. a) $\frac{23}{100} = 2.35$
   
   b) $\frac{3}{100} = 0.03$

4. a) 1.32
   
   b) 2.71
   
   c) 8.7
   
   d) 4.27
   
   e) 3.07
   
   f) 17.8
   
   g) 27.1
   
   h) 38.05

5. Teacher to check pictures.
   
   a) 6 tenths
   
   b) .8
   
   c) 1.20

#### AP Book NS4-107

**page 271**

1.  
   
   B: $\frac{3}{10} = 1.3$
   
   C: $\frac{8}{10} = 1.8$
   
   D: $\frac{9}{10} = 2.9$
   
   E: $\frac{4}{10} = .4$
   
   F: $\frac{5}{10} = 1.5$

#### AP Book NS4-108

**page 272**

1. a) From left to right:
   
   .1, .2, .3, .4, .5, .6, .7, .8, .9
   
   b) $\frac{1}{2} = 0.5$

2. a) Zero
   
   b) A half
   
   c) One
   
   d) A half
   
   e) One
   
   f) Zero

3. a) Less than
   
   b) Greater than
   
   c) Greater than
   
   d) Greater than
   
   e) Less than
   
   f) Less than

4. a) One
   
   b) Two
   
   c) Zero
   
   d) Three
   
   e) One
   
   f) Three

#### AP Book NS4-109

**page 273**

1. Teacher to check fraction conversions.
   
   a) $\frac{3}{10} \cdot \frac{5}{10} = \frac{1}{2}$
   
   b) $\frac{1}{3} \cdot \frac{3}{10} = \frac{3}{10}$
   
   c) $\frac{2}{10} \cdot \frac{3}{10} = \frac{6}{10}$
   
   d) $\frac{12}{10} \cdot \frac{3}{10} = \frac{36}{100}$
   
   e) $\frac{1}{10} \cdot \frac{5}{10} = \frac{5}{100}$
   
   f) $\frac{7}{10} \cdot \frac{1}{10} = \frac{35}{100}$

#### AP Book NS4-110

**page 275**

1. 13 tenths altogether

2. a) 47 tenths
   
   b) 71 tenths
   
   c) 30 tenths
   
   d) 3.8
   
   e) 4.2
   
   f) 7.0

3. a) 3.1
   
   b) 2.4; 13 tenths

   + 11 tenths = 24 tenths
   
   c) 8.7; 14 tenths
   
   + 73 tenths = 87 tenths
   
   d) 1.5; 25 tenths
   
   - 10 tenths = 15 tenths
   
   e) 3.4; 76 tenths
   
   - 42 tenths = 34 tenths
   
   f) 7.5; 89 tenths
   
   - 14 tenths = 75 tenths

4. a) 1.7
   
   b) 1.2

   Teacher to check arrows for next two questions:
   
   c) 3.7
   
   d) 0.8

5. a) 2.3
   
   b) 7.8
   
   c) 7.1
   
   d) 4.3
   
   e) 8.6
   
   f) 0.9

#### AP Book NS4-111

**page 276**

1. Teacher to check shading.
   
   a) $\frac{20}{100} + \frac{55}{100} = \frac{75}{100}$
   
   b) $\frac{30}{100} + \frac{69}{100} = \frac{99}{100}$
   
   c) $\frac{45}{100} + \frac{23}{100} = \frac{68}{100}$
   
   d) $\frac{32}{100} + \frac{26}{100} = \frac{58}{100}$

2. a) $\frac{20}{100} + \frac{55}{100} = \frac{75}{100}$

   Teacher to check fraction shading.
   
   a) $\frac{32}{100} + \frac{50}{100} = \frac{82}{100}$
   
   b) $\frac{25}{100} + \frac{60}{100} = \frac{85}{100}$
   
   c) $\frac{35}{100} + \frac{40}{100} = \frac{75}{100}$

   Teacher to check fraction shading convert.
b) 0.3 + 0.69 = 0.99

c) 0.45 + 0.23 = 0.68

d) 0.32 + 0.26 = 0.58

3.

a) 0.89
b) 0.97
c) 0.81
d) 0.97
e) 0.95
f) 1.17
g) 1.03
h) 0.90

4.

a) .49
b) .87
c) .58
d) .89
e) .97
f) .80
g) .87
h) .79

AP Book NS4-112

page 277

1. Teacher to check crossing out.
   a) \(\frac{30}{100}\)
   b) \(\frac{13}{100}\)
   c) \(\frac{27}{100}\)

2.

a) .50 - .20 = .30
b) .38 - .25 = .13
c) .69 - .42 = .27

d) .32
b) .62
c) .61
d) .28
e) .08
f) .04
g) .15
h) .75
i) .54
j) .74
k) .43
l) .11

4.

a) .09
b) .62
c) .28
d) .03
f) .39

3.

a) .01
b) .1
f) .1
h) .33
g) .82
e) .08
f) .33
g) .82
h) .33

AP Book NS4-113

page 278

1. Teacher to check base ten models.
   a) 2.35
   b) 2.35

2. Teacher to check base ten model.
   b) 1.13

3.

a) 3.39
b) 4.47
c) 4.13
d) 6.80
e) 8.51
f) 2.13
g) 1.11
h) 2.43
i) 6.79
j) 2.59

4. 1397 kg

5. 7.13 meters

AP Book NS4-114

page 279

1.

a) .63
b) .33
c) .17
d) .69
e) .85
f) .31
g) 3.76
h) .473
i) 5.99
j) .43
l) .11

2.

a) .9
b) .38
c) .4
d) .62
e) .8
f) 10
g) 5
h) 1

6. a) .2, .4, .6, .8, 1
   b) .3, .6, .9, 1.2, 1.5

7. The numbers are not lined up properly – the decimal places should all line up.

8. \(1.02 = 1 \frac{2}{100}\)
   \(1.20 = 1 \frac{20}{100}\)

AP Book NS4-116

page 281

1. Teacher to check.

2. 

   a) \(\frac{4}{100} m = 0.04 m\)
   b) \(\frac{75}{100} m = 0.75 m\)
   c) \(\frac{17}{100} m = 0.17 m\)
   d) \(\frac{8}{10} cm = 0.8 cm\)
   f) \(\frac{7}{10} cm = 0.7 cm\)
   g) \(\frac{5}{10} cm = 0.5 cm\)
   h) \(\frac{4}{10} cm = 0.4 cm\)

3.

   a) .18 + .24 = 2.58 m
   b) .06 + 8.2 = 8.26 m
   d) .26 + 1.52 = 1.78 m
   e) 4.23 + 1.75 = 5.98 m

4. a) From front to back:

Yellow Sorrel - 0.5 m
Field Birdwell - 1 m
Canada Golden - 1.5 m
White Sweet Clover - 300 cm

b) 2.5 m = 250 cm

5. Decimal notation is used for money because there is still a monetary value to money less than a full dollar (cents).

   A dime is a tenth of a dollar.

   A penny is a hundredth of a dollar.
AP Book NS4-117

1. Teacher to check groupings.
   a) \( 30 \div 10 = 3 \)
   b) \( 20 \div 10 = 2 \)

2. a) 7
   b) 4
   c) 6
   d) 9
   e) 28
   f) 36
   g) 72
   h) 125

3. a) \( 300 \div 100 = 3 \)
   b) \( 400 \div 100 = 4 \)
   c) \( 900 \div 100 = 9 \)

4. a) 7
   b) 8
   c) 6
   d) 18
   e) 200
   f) 20
   g) 910
   h) 100

5. a) 0.4
   b) 0.6
   c) 1.5

AP Book NS4-118

1. a) 207 cents
   b) 521 cents
   c) 604 cents
   d) 805 cents

2. a) 302 cm
   b) 409 cm
   c) 219 cm
   d) 810 cm
   e) 1730 cm
   f) 101 cm

BONUS:
3. a) 7002 m
   b) 2036 m
   c) 8007 m
   d) 6003 m
   e) 4125 m

AP Book NS4-119

1. a) Teacher to check diagrams.
   b) Yes

2. a) 2700
   b) 75000
   c) 97000
   d) 37200
   e) 17000
   f) 10000

3. a) 0 A B C F G E
    1 C F
    2 A C D E G
    3 A C D F G
    4 B D C F
    5 A B D F G
    6 B E G F D
    7 A C F
    8 A B D C E F G
    9 A B D C F
   b) F: this trapezoid appears in 9 out of 10 numbers.

4. a) \( 1 + 3 + 5 + 7 = 16 \)
   b) Teacher to check drawing.
      \( 1 + 3 + 5 + 7 + 9 = 25; 5 \times 5 = 25; \) the numbers are all odd.
      \( 6 \times 6 = 36 \)

AP Book NS4-120

1. Box of 2:
   \( 3\text{ boxes} \times 10\text{¢} = 30\text{¢} \)
   Box of 3:
   \( 2\text{ boxes} \times 12\text{¢} = 24\text{¢} \)
   The cheapest way is to buy 2 boxes of 3 crayons.

2. Diary:
   \( $10 + 2 \times 5 = $5 \)
   Carol has $3.75 left ($10 – $5 – $1.25).

3. Tray of 4:
   \( 6\text{ trays} \times 60\text{¢} = $3.60 \)
   Tray of 6:
   \( 4\text{ trays} \times 80\text{¢} = $3.20 \)
   The cheapest way is to buy 4 trays of 6 plants.

4. Henry is 8 years old.

5. a) 70 students were girls (150 – 80).
   b) There are 6 classes:
      \( 150 \div 25 = 6 \)
   c) Aside:
      6 classes = 6 teachers
      In total, 9 adults work at the school (6 teachers + 1 principal + 1 vice-principal + 1 secretary).
   d) 12 students were missing that day:
      \( 2 \times 6 = 12 \)
   e) 138 students were at school:
      \( 150 – 12 = 138 \)

6. 330 shirts were left at the end of the week:
   \( 500 – 20 – 50 – 100 = 330 \)

7. \( .09 \text{ m} \)

8. \( 1.02 \text{ km} \)

9. \( 1.8 \text{ m} = 180 \text{ cm} \)
**Answer Key for AP Book 4.2**

### AP Book ME4-30
**Page 286**

1. a) 12 cm²  
   b) 7 cm²  
   c) 14 cm²  
2. Teacher to check that rectangles have been divided properly.  
   a) 4 cm²  
   b) 8 cm²  
   c) 3 cm²  
3. Answers to “how” question will vary – teacher to check.  
   a) 4 cm²  
   b) 8 cm²  
   c) 3 cm²

### AP Book ME4-31
**Page 287**

1. a) 3 × 5 = 15  
   b) 2 × 4 = 8  
   c) 3 × 2 = 6  
2. b) 4 × 4 = 16  
   c) 4 × 2 = 8  
   d) 2 × 6 = 12  
3. a) L = 7;  
   W = 2;  
   7 × 2 = 14 sq units  
   b) L = 3;  
   W = 3;  
   3 × 3 = 9 sq units

### AP Book ME4-32
**Page 288**

1. a) 5 cm × 3 cm  
   = 15 cm²  
   b) 1 cm × 2 cm  
   = 2 cm²  
   c) 5 cm × 2 cm  
   = 10 cm²  
2. a) Area of A = 40 m²  
   Area of B = 54 cm²  
   Area of C = 50 m²  
   Area of D = 60 km²  
   b) D – 60 km²  
   C - 50 m²  
   A - 40 m²  
   B - 54 cm²  
3. a) 35 m²  
   b) 18 m²  
   c) 48 cm²

### AP Book ME4-33
**Page 289**

1. a) 3 whole squares  
   b) 2 whole squares  
   c) 3 whole squares  
   d) 2 whole squares  
   e) 4 whole squares  
   f) 8 whole squares  
   g) 5 whole squares  
2. a) 3 whole squares  
   b) 4 whole squares  
   c) 6 whole squares  
3. a) Equal –  
   There are 4 unshaded and 2 + (4 halves)  
   = 2 + 2  
   = 4 shaded  
   b) Less than –  
   There are 3 + (2 halves)  
   = 3 + 1  
   = 4 unshaded and 1 + (4 halves)  
   = 1+ 2  
   = 3 shaded

### AP Book ME4-34
**Page 290**

1. a) 6 half squares;  
   3 whole squares  
   b) 8 half squares;  
   4 whole squares  
   c) 14 half squares;  
   7 whole squares  
2. b) 3 full squares;  
   4 half squares  
   = 2 full squares;  
   Area = 3 + 2 = 5  
   c) 11 full squares;  
   4 half squares  
   = 2 full squares;  
   Area = 11 + 2 = 13  
3. Estimates will vary.  
   Actual area = 6 cm²

### AP Book ME4-35
**Page 291**

1. | P | A |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12 cm</td>
</tr>
<tr>
<td>B</td>
<td>20 cm</td>
</tr>
<tr>
<td>C</td>
<td>26 cm</td>
</tr>
<tr>
<td>D</td>
<td>18 cm</td>
</tr>
<tr>
<td>E</td>
<td>30 cm</td>
</tr>
<tr>
<td>F</td>
<td>16 cm</td>
</tr>
<tr>
<td>G</td>
<td>16 cm</td>
</tr>
</tbody>
</table>

2. No
3. Answers will vary – teacher to check.
4. E - 30 cm  
   C – 26 cm  
   B – 20 cm  
   D – 18 cm  
   F & G – 16 cm  
   A – 12 cm
5. E – 44 cm²  
   B – 24 cm²  
   C – 22 cm²  
   F – 16 cm²  
   D – 14 cm²  
   A - 8 cm²  
   G - 7 cm²

6. No
7. The perimeter is the measure of the distance around the outside edge of a shape (given in linear units); area is the measure of the space taken up by the shape (given in square units).
1. This table gives the shapes’ actual measurements only (units not included). Students’ estimates will vary – teacher to check.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>W</th>
<th>P</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>3</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>2</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>7</td>
<td>2</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>2</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>3</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>2</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

2. a) Answers will vary – teacher to check.
   b) No – non congruent shapes can have the same area.

3. a) Length = (10 cm – 4 cm) ÷ 2 = 3 cm
   So…
   Area = 3 cm × 2 cm = 6 cm²
   b) Length = (18 cm – 8 cm) ÷ 2 = 5 cm
   So…
   Area = 5 cm × 4 cm = 20 cm²

4. 
   a) 
   Perimeter = 12 cm
   So…
   Length and width can be found:
   = 12 cm ÷ 4
   (since a square has four equal sides)
   = 3 cm
   So…
   Area = 3 cm × 3 cm = 9 cm²
   b) 
   The 2 × 2 square has the smallest perimeter (8 units)

5. a) Students will be able to create 5 different figures:

   
   b) 
   Perimeter = 20 cm
   So…
   Length and width can be found:
   = 20 cm + 4
   = 5 cm
   So…
   Area
   = 5 cm × 5 cm
   = 25 cm²

AP Book ME4-38

1. a) 3 cubes
   b) 6 cubes
   c) 6 cubes
   d) 10 cubes
   e) 8 cubes
   f) 12 cubes

2. a) 
   
   b) 
   c) 
   d) 

AP Book ME4-39

1. b) 
   
   c) 
   d) 

AP Book ME4-40

1. a) 2 g
   b) 8 g
   c) Answers will vary – teacher to check.
   d) Answers will vary – teacher to check.

2. a) 3 g
   b) 5 g
   c) 8 g

3. Answers will vary – teacher to check.
4. Answers will vary – teacher to check.

5. a) 12 g
   b) 18 g
   c) 16 g
   d) 14 g
   e) 6 quarters
   f) Answers will vary – but a toonie would weight slightly more than a loonie.
Measurement 2 – AP Book 4.2

6. Elephant → Whale
   Easel → Chair
   Tissue box → Glue
7. Balloon → Grams
   Key → Grams
   Truck → Kilograms

8. a) B – Ant
    C – Horse
    A – Blue whale
b) C – Elephant
    A – 10-year-old human
    B – House cat
9. a) Moose - Kg
    b) Desk - Kg
    c) Piece of cheese - G
    d) Tiny bird - G
    e) Pencil - G
    f) Yourself - Kg
10. a) 22 kilograms
    b) 13 kilograms
11. a) 4 g
    b) 2 g, 2 g, 2 g

AP Book ME4-41

1. thousand
2. 1 000
3. a) 3 000
   b) 9 000
   c) 17 000
   d) 25 000
4. Answers will vary – teacher to check.
5. a) 3 kg
    b) 1 kg = 1 000 g
6. a) 35 g
    b) 20 g
    c) 5 kg
    d) 2 kg
    e) 1 kg
    f) 2 000 g
7. Answers will vary – teacher to check.

AP Book ME4-42

1. a) 6 kg
    b) 14 kg
2. Answers will vary – teacher to check.
3. 3.8 kg or 3 800 g
4. 529 kg less
5. Tomato:
   12 × 2 g = 24 g
   Eggplant:
   8 × 2 g = 16 g
   Zucchini:
   5 × 3 g = 15 g
   24 g + 16 g + 15 g = 55 g
6. a) $24
    b) 360 g
    c) 30 kg
7. 6 000 g or 6 kg

AP Book ME4-43

1. a) mL
    b) mL
    c) g
    d) L
    e) L
    f) mL
2. 10 mL
3. a) 4
    b) 4
    c) 4B
4. a) 10 containers
   (1 000 ÷ 100 = 10)
    b) 5 containers
   (1 000 ÷ 200 = 5)
    c) 2 containers
   (1 000 ÷ 500 = 2)
    d) 4 containers
   (1 000 ÷ 250 = 4)
5. 1 000 mL or 1 L

AP Book ME4-44

1. 2 700 mL
2. a) False
    b) False
    c) False
   d) True
3. a) mL
    b) kg
    c) g
    d) L
4. a) Ice Cream
   Circle: fresh fruit,
   lemon juice, heavy
   cream, light cream
   Underline: sugar
   Tomato Sauce
   Circle: Olive oil, can
   of tomatoes, tomato
   paste
   Underline: fresh
   oregano, fresh basil
   Birthday Cake
   Circle: milk
   Underline: butter,
   sugar, flour
b) Ice cream:
   150 g
   Tomato Sauce:
   7 g
   Birthday Cake:
   695 g
   c) Ice cream:
   1 550 mL
   Tomato Sauce:
   860 mL
   Birthday Cake:
   150 mL

AP Book ME4-45

1. a) 5°C
    b) 10°C
    c) 0°C
    d) – 5°C
    e) 15°C
2. Answers will vary – teacher to check.
3. Kyle’s temperature is
   1°C higher than normal.
4. The temperature must
   rise another 25°C.
BONUS:
5. The temperature rose
   15°C altogether.
1. a) $12 - 4 = 8$
   b) $11 - 4 = 7$
   c) $42 - 36 = 6$

2. a) $6$
   b) $13$
   c) $30$
   d) $13$
   e) $15$
   f) $8$

3. a) $6$
   b) $3$
   c) $13$
   d) $8$

4. a) $25 - 13 = 12$
   b) $30 - 25 = 5$
   c) $5 - 2 = 3$
   d) $13 - 7 = 6$
   e) $455 - 430 = 25$
   f) $540 - 455 = 85$

5. a) $3$
   b) $3$
   c) $5$
   d) $4$
   e) $8$
   f) $0$
   g) $1$
   h) $4$
   i) $2$
   j) $4$

6. Mean $= 70 ÷ 10 = 7$

7. True

8. a) 
   Stem | Leaf
   --- | ---
   2   | 1
   3   | 4
   5   | 2 3

   Data: 21, 24, 35, 36, 38, 52, 53

   b) 
   Stem | Leaf
   --- | ---
   0   | 4
   1   | 5
   2   | 3 8

   Data: 4, 15, 19, 20, 23, 28

   c) 
   Stem | Leaf
   --- | ---
   8   | 0
   9   | 2 7

   Data: 80, 83, 90, 92, 97, 106

   d) 
   Stem | Leaf
   --- | ---
   9   | 1
   10  | 2 4 4
   11  | 0 5

   Data: 91, 92, 98, 102, 104, 106, 110, 115

9. a) 
   Stem | Leaf
   --- | ---
   0   | 7
   1   | 0 2 9

   b) 
   Stem | Leaf
   --- | ---
   9   | 8
   10  | 2

   Data: 98, 99, 102, 104, 106, 110, 115

10. a) 
   b) 
   Stem | Leaf
   --- | ---
   2   | 6
   3   | 2
   4   | 0

11. a) S: 82
   L: 104
   Range: 22

   b) S: 5
   L: 23
   Range: 18

   c) S: 95
   L: 120
   Range: 25

   d) 108
   4

   e) 30
   26

   f) 100
   32

   g) 53

   a) 
   Stem | Leaf
   --- | ---
   5   | 4
   6   | 8
   7   | 5
   8   | 4 4
   9   | 1

   b) 70-79
   c) The most common mark range is the one with the most leaves.

12. a) 108
   b) 4
   c) 26

13. a) 100
   b) 32
   c) 53

14. a) 
   Stem | Leaf
   --- | ---
   0   | 3
   1   | 2

   b) 70-79
   c) The most common mark range is the one with the most leaves.
Answer Key for AP Book 4.2

1. a) 2 in half; 4 in pie
   b) 3 in half; 6 in pie
   c) 4 in half; 8 in pie
2. a) 5
   b) 6
   c) 9
   d) 10
   e) 8
   f) 4
   g) 2
   h) 7
   i) 3
   j) 11
3. # Half the # Sum
   8 4 4 + 4 = 8
   14 7 7 + 7 = 14
   16 8 8 + 8 = 16
   20 10 10 + 10 = 20
4. a) 2 sets of 2
   b) 2 sets of 3
   c) 2 sets of 4
5. 6 marbles are red
6. 3 pieces are half
7. b) Less than half
c) Half
d) More than half
e) Half
f) Less than half
g) More than half
h) More than half
i) More than half
j) More than half
k) More than half
l) More than half
8. a) 2, 4 - circle
   b) 3, 6 - circle
c) 4, 6
d) 4, 8 – circle
e) 5, 8
9. Circle Circle – Circle X Circle Circle X
10. 2, 4; 3, 6; 4, 8
11. Teacher to check.
1. a)
   b)
   c)
   d)
   e)
   f)
   g)
   h)
   i)
   j)
   k)

2. a)
   b)
   c)
   d)
   e)
   f)
   g)
   h)
   i)

3. a) T
   b) L
   c) F

4. a) Column 3 Row 2
   b) Column 3 Row 1
   c) Column 2 Row 1
   d) Column 2 Row 2
   e) Column 2 Row 3

5. a)
   b)
   c)
   d)
   e)
   f)
   g)
   h)

6. a) Column 1 Row 2
   b) Column 2 Row 3
   c) Column 3 Row 1
   d) Column 1 Row 1

7. Answers will vary – teacher to check.
8. Answers will vary – teacher to check.

1. a) 2 units right
   b) 3 units right
   c) 4 units right

2. a) 3 units left
   b) 5 units left
   c) 3 units left

3. a)
   b)
   c)

4. a) 6 units right 3 units down
   b) 4 units right 2 units down
   c) 2 units right 3 units down

5. a)
   b)
   c)

6. a)
   b)
   c)
   d)

7. Answers will vary – teacher to check.

BONUS:
4. Since a translation arrow cannot be drawn in this figure, Shape B is not a slide or translation of Shape A. Therefore, Marco is incorrect.
1. A: right 3, up 1  
   B: right 6, down 2  
   C: left 2, down 1  
   D: right 5, up 4  
   E: up 4  
2. a) 3 left, 2 down  
    b) 1 down, 3 left  
    c) 4 right, 1 down  
    d) 2 up, 1 left  
    e) 1 right, 3 down  
    f) 4 down, 2 left  
3. a) Start at A  
    b) Start at C  
    c) Start at D  
    d) Start at A  
    e) Start at A  
    f) Start at A  
4. a) D  
    b) C  
    c) B  
    d) 3 down, 1 right  
    e) 2 right, 4 up  
    f) 5 down, 1 left  
5. a) School  
    b) Gym  
    c) Pool  
    d) 3 blocks east and 3 blocks north  
    e) 3 blocks west  
    f) 1 block west and 4 blocks south  
6. a) Bear  
    b) Pig  
    c) (E,3)  
    d) 3 squares south and 2 squares east or 30 m south and 20 m east (using key provided)  
    e) 3 squares east and 1 square north or 30 m east and 10 m north (using key provided)  
7. | Clara | John | Mary |  
    | Ed | George | Tom |  
    | Erin | Jane | Abdul |  
8. a) 6 units north  
    b) 3 units east  
    c) 2 units south  
    d) 5 units east  
    e) 4 units north  
9. Answers will vary – teacher to check.  
   BONUS:  
   If the pair of shapes on either side of the mirror are the same size and shape, the pair of shapes are congruent.
1. Teacher should check that all students construct their shapes properly.

<table>
<thead>
<tr>
<th>Base</th>
<th>#S</th>
<th>#E</th>
<th>#V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.P.</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>S.P.</td>
<td>4</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>P.P.</td>
<td>5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>H.P.</td>
<td>6</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

2. The patterns in the columns are:
   • The number of edges of a prism is three times larger than the number of sides of its base.
   • The number of vertices of a prism is double the number of sides of its base.

3. See Question 1.
4. See Question 2.

AP Book G4-33
page 338

1. a) 6
   b) 6
   c) 6
   d) 4
   e) 5
   f) 8
   g) 5
   h) 5

BONUS:
Teacher to check.
2. a) 

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td># F</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td># E</td>
<td>8</td>
<td>6</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

b) C and D have the same number of faces, vertices and edges. Both are quadrilateral prisms.

AP Book G4-34

page 339

1. a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

3. a) 

b) 

c) 

d) 

e) 

f) 

g) 

h) 

2. a) 

b) 

c) 

d) 

4. a) 

<table>
<thead>
<tr>
<th>Base</th>
<th>E</th>
<th>V</th>
<th>F</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>4</td>
<td>4</td>
<td>Triangle Pyramid</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>12</td>
<td>8</td>
<td>Rectangular Prism</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>8</td>
<td>5</td>
<td>Rectangular Prism</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>8</td>
<td>5</td>
<td>Square Pyramid</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>18</td>
<td>12</td>
<td>Hexagonal Prism</td>
<td></td>
</tr>
</tbody>
</table>

BONUS:

4. Circle: a), f) and h)

AP Book G4-35

page 341

1. From left to right:
   X X O O X O X O

2. From left to right:
   rectangular prism
   square pyramid
   cone
   cylinder
   triangular pyramid
   triangular prism

3. a) 

<table>
<thead>
<tr>
<th>RP</th>
<th>SP</th>
<th>S?</th>
<th>D?</th>
</tr>
</thead>
<tbody>
<tr>
<td># faces</td>
<td>6</td>
<td>5</td>
<td>✓</td>
</tr>
<tr>
<td>shape of base</td>
<td>□</td>
<td>□</td>
<td>✓</td>
</tr>
<tr>
<td># bases</td>
<td>2</td>
<td>1</td>
<td>✓</td>
</tr>
<tr>
<td># of faces that aren’t bases</td>
<td>4</td>
<td>4</td>
<td>✓</td>
</tr>
<tr>
<td>Shape of faces that aren’t bases</td>
<td>□</td>
<td>△</td>
<td>✓</td>
</tr>
<tr>
<td># edges</td>
<td>12</td>
<td>8</td>
<td>✓</td>
</tr>
<tr>
<td># vertices</td>
<td>8</td>
<td>5</td>
<td>✓</td>
</tr>
</tbody>
</table>

b) A rectangular prism and a square pyramid are the same in that both shapes have 4 faces that are not bases.

A rectangular prism and a square pyramid are different in many ways: number of faces, bases, edges and vertices.

6. 

7. 

8. Answers will vary – teacher to check.

9. a) Rectangular prism

b) Triangular prism

c) Square pyramid

10. Answers will vary – teacher to check.

11. a) Face 3

b) Face 3

12. a)
Answer Key for AP Book 4.2

**AP Book G4-36**

**page 344**

1. | Name                  | #F | #E | #V |
---|-----------------------|----|----|----|
   | Triangular Pyramid    | 4  | 6  | 4  |
   | Square Pyramid        | 5  | 8  | 5  |
   | Pentagonal Pyramid    | 6  | 10 | 6  |
   | Triangular Prism      | 5  | 9  | 6  |
   | Cube                  | 6  | 12 | 8  |
   | Pentagonal Prism      | 7  | 15 | 10 |

2. Teacher to check the missing faces are drawn properly.
   a) Triangle
   b) Pyramids – each net has only one base, and the remaining faces are all triangles.

3. Teacher to check the missing faces are drawn properly.
   a) Quadrilaterals
   b) Prism – each net has two bases, and the remaining faces are all quadrilaterals.

4. Teacher to check student constructions and predictions.
   a) Yes
   b) No
   c) Yes
   d) No
   e) No
   f) Yes

**AP Book G4-37**

**page 345**

<table>
<thead>
<tr>
<th>Property</th>
<th>Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>B, C, D and E</td>
</tr>
<tr>
<td>2.</td>
<td>A, C and D</td>
</tr>
</tbody>
</table>

a) C, D
b) 

**AP Book G4-38**

**page 346**

1. Teacher to check

2. Teacher to check

**AP Book G4-39**

**page 347**

1. Teacher to check.

2. b) 

3. Teacher to check.

**AP Book G4-40**

**page 348**

1. a) Two: horizontal and vertical
   b) Two: horizontal and vertical
   c) One: horizontal

2. a) Pentagon
   b) Hexagon
   c) Octagon

3. Teacher to check.

4. 4 blocks South, 2 blocks West, 2 blocks South, 4 blocks East, 2 blocks North

5. a) Square pyramid
   b) Teacher to check.
   c) Teacher to check.

**AP Book G4-41**

**page 349**

1. a) B
   b) C
   c) A

2. Answers will vary – teacher to check.

3. From left to right: X O - X O X -
   b) – is a slide and c) does not have congruent shapes

5. Answers will vary – teacher to check.

6. This net is for a pentagonal prism: It has two pentagon faces (which would be the bases) surrounded by 5 rectangular faces.

7. a) A triangular prism has 2 triangular bases joined by 3 rectangular faces. It has 6 vertices and 9 edges.
   b) A rectangular prism has 2 rectangular bases joined by 4 rectangular faces. It has 8 vertices and 12 edges.

8. Answers will vary – teacher to check.
Contents

Patterns & Algebra – Part 1
Answer Key for Patterns & Algebra – Part 1
Number Sense – Part 1
Answer Key for Number Sense – Part 1
Measurement – Part 1
Answer Key for Measurement – Part 1
Probability & Data Management – Part 1
Answer Key for Probability & Data Management – Part 1
Geometry – Part 1
Answer Key for Geometry – Part 1
Patterns & Algebra – Part 2
Answer Key for Patterns & Algebra – Part 2
Number Sense – Part 2
Answer Key for Number Sense – Part 2
Measurement – Part 2
Answer Key for Measurement – Part 2
Probability & Data Management – Part 2
Answer Key for Probability & Data Management – Part 2
Geometry – Part 2
Answer Key for Geometry – Part 2
Les régularités et l’algèbre
Nom : _____________________________
Date : _________________

Section A

1. Remplis les nombres qui manquent :
   a) _____ est 3 de plus que 5      b) 13 est _____ de plus que 7      c) _____ est 9 de moins que 14

2. Trouve l’intervalle entre les nombres et prolonge ensuite la régularité :
   NOTE : N’oublie pas de vérifier si l’intervalle est le même entre chaque paire de nombres!
   a) 2 , 5 , 8 , _____ , _____ , ____
   b) 21 , 19 , 17 , _____ , _____ , ____
   c) 0 , 4 , 8 , _____ , _____ , ____
   d) 46 , 41 , 36 , _____ , _____ , ____

3. Écris la règle pour chacune des régularités suivantes :
   a) 65, 75, 85, 95, 105         additionne _____
   b) 39, 33, 27, 21, 15         soustrais _____
   c) 200, 191, 182, 173, 164
      Commence à ____et ________________
   d) 55, 66, 77, 88, 99
      ___________________________________

4. Crée ta propre régularité. Donne ensuite la règle que tu as utilisée :
   Ma régularité : ____ ,  ____ ,  ____ ,  ____ ,  ____  Ma règle : ______________________________

5. Josephine lit 7 pages de son livre chaque soir. Hier soir, elle était à la page 64. Quelle page atteindra-t-elle ce soir? Et demain soir?

6. 3, 9, 14, 20...
   Philip dit que la régularité ci-dessus a été créée en additionnant 6 à chaque fois. A-t-il raison? Explique :
Les régularités et l’algèbre

Test sur l’unité

Section B

7. Encercle le cœur des régularités suivantes :
   a)  
   b) 3 1 5 3 1 5 3 1 5 3 1 5
   c) C D B D C D B D C D B D
   d)  
   e) 2 2 5 5 2 2 5 5 2 2 5 5
   f) 

8. Encercle le cœur de la régularité. Continue ensuite la régularité :
   a)  
   b) A C E A C E A  
   c) 1 8 7 4 1 8 7 4  
   d) 

9. Écris l’attribut (un seul) qui change dans chaque régularité :
   a)  
   b)  

10. Écris les deux ou trois attributs qui changent dans chaque régularité :
   a)  
   b)  
   c)  
   d)  
Les régularités et l’algèbre

Test sur l’unité

Section C

11. Prolonge les régularités numériques suivantes :

<table>
<thead>
<tr>
<th>Figure</th>
<th>Nombre de blocs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Nombre de blocs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure</th>
<th>Nombre de blocs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>

d) Combien de blocs y aurait-il dans la figure 7 de la question a) ci-dessus? Explique comment tu le sais :

12. Complète chaque tableau en T pour trouver combien d’argent Daniella gagnerait en 4 heures :

<table>
<thead>
<tr>
<th>Heures travaillées</th>
<th>Dollars gagnés en une heure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11 $</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Heures travaillées</th>
<th>Dollars gagnés en une heure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15 $</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Heures travaillées</th>
<th>Dollars gagnés en une heure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17 $</td>
</tr>
</tbody>
</table>

13. Crée un tableau en T pour résoudre le problème suivant :

La journée 1, Remi a planté 12 plantes dans son jardin. Chaque jour par la suite, il a planté 7 plantes. Combien de plantes Remi a-t-il plantées à la fin de la journée 4?
14. Christian commence à travailler le jeudi matin. Il tond 7 pelouses chaque jour. Combien de pelouses aura-t-il tondues en arrivant au dimanche soir?

15. Jake a 56 $ dans son compte d’épargne à la fin avril. Il dépense 6 $ chaque mois par la suite. Combien d’argent lui restera-t-il dans son compte à la fin juillet?

<table>
<thead>
<tr>
<th>Mois</th>
<th>Épargnes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avril</td>
<td>56 $</td>
</tr>
</tbody>
</table>

18. Clare fabrique un ornement en utilisant 1 rectangle et 3 triangles. Elle a 6 rectangles. Combien de triangles lui faudra-t-il si elle veut utiliser les 6 rectangles?
### Section A

1. a) 8
   b) 6
   c) 5
2. a) Gap = + 3; 11, 14, 17
   b) Gap = - 2; 15, 13, 11
   c) Gap = + 4; 12, 16, 20
   d) Gap = - 5; 31, 26, 21
3. a) Add 10
   b) Subtract 6
   c) Start at 200 and subtract 9
   d) Start at 55 and add 11
4. Answers will vary.
5. Tonight: pg. 71
   Tomorrow Night: pg. 78
6. No, Philip is not correct. The "gap" between 3 & 9 and 14 & 20 is 6, but the gap between 9 & 14 is 5.

### Section B

7. a) 3 1 5
   b) C D B D
   c) Y R R
   d) 2 2 5 5
8. a) Core = □ △ □;
     □ △ □ △ □
   b) Core = A C E;
     C E A C E
   c) Core = 1 8 7 4;
     1 8 7 4 1 8
     7 4
   d) The core (which is □ □ □ □ ) repeats 3 more times.

### Section C

11. a) Gap = + 3;
    b) Gap = + 6;
    c) Gap = + 4;
    d) Figure 7 would have 21 blocks – continue to add 3 blocks.

### Section D

14. Christian has mowed 28 lawns by Sunday evening:

<table>
<thead>
<tr>
<th>Day</th>
<th># Lawns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thursday</td>
<td>7</td>
</tr>
<tr>
<td>Friday</td>
<td>14</td>
</tr>
<tr>
<td>Saturday</td>
<td>21</td>
</tr>
<tr>
<td>Sunday</td>
<td>28</td>
</tr>
</tbody>
</table>

15. By the end of July, Jake has $38:

<table>
<thead>
<tr>
<th>Month</th>
<th># Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>$56</td>
</tr>
<tr>
<td>May</td>
<td>$50</td>
</tr>
<tr>
<td>June</td>
<td>$44</td>
</tr>
<tr>
<td>July</td>
<td>$38</td>
</tr>
</tbody>
</table>

16. To use all 6 rectangles, Claire will need 18 triangles:

<table>
<thead>
<tr>
<th>Rectangles</th>
<th>Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

### Section D

13. By the end of Day 4, Remi had 33 plants:

<table>
<thead>
<tr>
<th>Day</th>
<th>Plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
</tr>
</tbody>
</table>
Logique numérique

Test sur l’unité

Nom : ____________________________________
Date : _________________

Section A

1. À côté de chaque nombre, écris la valeur de position du chiffre souligné :
   a) 382
   b) 726
   c) 9 453
   d) 3 107
   e) 2 168
   f) 5 381

2. Écris les nombres pour les adjectifs numéraux suivants :
   a) quatre cent vingt-six __________
   b) mille six cent trente-sept __________
   c) huit mille cinq cent dix __________
   d) trois mille deux cent quatre __________

3. Écris les adjectifs numéraux pour les nombres suivants :
   a) 562
   b) 1 319
   c) 4 308

4. Pour chaque question ci-dessous, écris le nombre représenté par l’illustration. Écris chaque nombre en forme décomposée (chiffres et mots) en premier :
   a)  
      ___ milliers + ___ centaines + ___ dizaines + ___ unités = 

   b)  
      ___ milliers + ___ centaines + ___ dizaines + ___ unités = 
Logique numérale
Test sur l’unité

Nom : _____________________________
Date : _________________

Section A (suite)

5. Écris les nombres représentés par les blocs de base dix :

<table>
<thead>
<tr>
<th>Milliers</th>
<th>Centaines</th>
<th>Dizaines</th>
<th>Unités</th>
<th>Nombre</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Représente les nombres donnés par les blocs de base dix dans le tableau de valeur de position :

<table>
<thead>
<tr>
<th>Nombre</th>
<th>Milliers</th>
<th>Centaines</th>
<th>Dizaines</th>
<th>Unités</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 1 263</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 3 195</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 2 304</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section A (suite)

7. Décompose les nombres suivants en chiffres et en mots :
   a) \(5276 = \) ____ milliers + ____ centaines + ____ dizaines + ____ unités
   b) \(3014 = \) ____ milliers + ____ centaines + ____ dizaines + ____ unités
   c) \(1938 = \) ______________________________________________________________________
   d) \(6460 = \) ______________________________________________________________________

8. Écris le nombre en forme décomposée (en utilisant des chiffres seulement) :
   a) \(253 = \) ______________________________
   b) \(2657 = \) ______________________________

9. Écris le nombre dans la boîte. Encercle ensuite le plus grand nombre de chaque paire :
   Indice : S’il y a le même nombre de milliers, compte le nombre de centaines ou de dizaines.
   a) (i) [Diagramme] (ii) [Diagramme]
   b) (i) [Diagramme] (ii) [Diagramme]

10. Encercle le plus grand nombre de chaque paire :
    a) 646 ou 664  b) 327 ou 237  c) 5688 ou 5788  d) 3612 ou 3610

11. Fais une liste de tous les nombres à trois chiffres que tu peux faire avec les chiffres 4, 7 et 6. Encercle le plus grand nombre :

Tests sur les unités – Cahier 4, Partie I
Logique numérale
Test sur l’unité

Nom : _____________________________
Date : __________________________

Section B

12. Complète les tableaux ci-dessous en échangeant 10 dizaines pour 1 centaine :

<table>
<thead>
<tr>
<th>centaines</th>
<th>diz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>centaines</th>
<th>diz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

13. Complète les tableaux ci-dessous en échangeant 10 centaines pour 1 millier :

<table>
<thead>
<tr>
<th>milliers</th>
<th>centaines</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>milliers</th>
<th>centaines</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

14. Échange des centaines pour des milliers, ou des dizaines pour des centaines :
   a) 2 milliers + 13 centaines + 4 dizaines + 6 unités = ____ milliers + ____ centaines + ____ dizaines + ____ unités
   b) 4 milliers + 7 centaines + 28 dizaines + 5 unités = ________________________________________________

15. Additionne (en regroupant si nécessaire) :
   a) 4 3 + 2 9 = __________
   b) 5 1 7 + 1 9 2 = __________
   c) 7 2 5 + 6 8 3 = __________
   d) 2 4 9 0 + 1 3 5 3 = __________
   e) 5 8 3 1 + 2 1 7 6 = __________

16. Soustrais (en regroupant si nécessaire) :
   a) 5 4 – 2 7 = __________
   b) 7 2 6 – 3 1 3 = __________
   c) 9 2 1 – 1 5 6 = __________
   d) 6 0 6 5 – 3 4 1 2 = __________
   e) 9 5 7 2 – 2 8 4 6 = __________

17. Pour répondre à ces questions, tu vas devoir regrouper deux ou trois fois :
   a) 1 0 0
       – 8 1

   b) 1 0 0 0
       – 3 4 7

18. Georgia a gagné 2 418 $ durant ses vacances d’été. Emma a gagné 1 345 $. Combien d’argent de plus Georgia a-t-elle gagné qu’Emma?
Logique numérale
Test sur l’unité

Nom : ____________________________
Date : ______________

Section C

19. Dessine deux matrices pour chaque énoncé de multiplication (ou produit) :

a) \(2 \times 3\)  

b) \(3 \times 5\)  

c) \(4 \times 6\)

20. Dessine une matrice pour répondre aux questions suivantes. Écris un énoncé de multiplication pour chaque réponse :

a) Jenny a planté 5 graines dans chaque rangée. Il y a 7 rangées de graines. Combien de graines Jenny a-t-elle plantées?

b) Dans une salle, il y a 8 tables. Chaque table peut accommoder 4 personnes. Combien de personnes peuvent s’assembler en même temps dans la salle?

21. Multiplie en regroupant les unités en dizaines ou les dizaines en centaines :

a) \[
\begin{array}{c}
3 & 2 & 5 \\
\times & 3
\end{array}
\]

b) \[
\begin{array}{c}
1 & 1 & 4 \\
\times & 5
\end{array}
\]

c) \[
\begin{array}{c}
1 & 5 & 1 \\
\times & 5
\end{array}
\]

d) \[
\begin{array}{c}
2 & 4 & 2 \\
\times & 3
\end{array}
\]

e) \[
\begin{array}{c}
1 & 5 & 2 \\
\times & 3
\end{array}
\]

22. Jacob a multiplié deux nombres. Le produit était le même que l’un des nombres. Quel était l’autre nombre? Comment le sais-tu?

23. Florence a multiplié 5 par un nombre. Le produit était zéro. Par quel nombre l’a-t-elle multiplié? Comment le sais-tu?
Section C (suite)

24. Arrondis les nombres suivants à la dizaine près. **INDICE : Souligne le chiffre des dizaines en premier.**
   
   a) 16
   b) 81
   c) 255

25. Arrondis à la centaine près. **INDICE : Souligne le chiffre des centaines en premier.**
   
   a) 178
   b) 236
   c) 419
   d) 975
   e) 1 477
   f) 2 831

26. Arrondis au millier près (souligne le chiffre des milliers en premier) :
   
   a) 2 457
   b) 8 193
   c) 3 524

27. Un magasin a les articles suivants à vendre :

   A. Sofa – 525 $  
   B. Fauteuil – 216 $  
   C. Table – 219 $  
   D. Bureau – 354 $  
   E. Lampe – 97 $

   a) Que pourrais-tu acheter si tu avais 750 $ à débourser? Fais une estimation pour trouver la réponse. Additionne ensuite les prix pour vérifier :

   b) Que pourrais-tu acheter si tu avais 1 000 $ à dépenser? Fais de nouveau une estimation, et puis additionne les prix pour trouver le montant exact :
Logique numérale
Test sur l'unité

Section D

28. Compte les pièces données et écris le montant total :
   Indice : Compte le plus grand montant en premier.
   a) Montant total = ________ ¢
   b) Montant total = ________
   c) Montant total = ______

29. Pour chaque question, dessine deux pièces de monnaie additionnelles pour obtenir le total exact :
   a) 20 ¢
   b) 41 ¢
   c) 4 $
   d) 7 $

30. Utilise le moins de pièces de monnaie possible pour obtenir le total :
   Indice : Commence en essayant de voir combien de pièces de dix cents il te faut (s'il y a lieu), ensuite combien de pièces de cinq cents, et enfin combien de pièces de un cent.
   a) 16 ¢
   b) 23 ¢

31. Erik a vendu des biscuits pour recueillir de l’argent pour une sortie avec sa classe. Il a recueilli 4 pièces de 2 $, 6 pièces de 1 $, 2 pièces de vingt-cinq cents, 7 pièces de dix cents, 3 pièces de cinq cents et 9 pièces d’un cent. Combien d’argent a-t-il recueilli en tout?

32. Cathy a dépensé 73 ¢ pour acheter une gomme à effacer. Elle a donné 1 $ à la caissière. Calcule la monnaie qu’elle recevra en retour :
## Section A

1. a) Tens  
   b) Hundreds  
   d) Ones  
   d) Thousands  
   e) Hundreds  
   f) Tens

2. a) 426  
   b) 1 637  
   c) 8 510  
   d) 3 204

3. a) Five hundred sixty-two  
   b) One thousand three hundred nineteen  
   c) Four thousand three hundred eight

4. a) 1 346  
   b) 3 209

5. a) 2 438  
   b) 4 361

6. Teacher to check.

7. a) Five hundred sixty-two  
   b) One thousand three hundred nineteen  
   c) Four thousand three hundred eight

8. a) 200 + 50 + 3  
   b) 2000 + 600 + 50 + 7

9. a) i) 424  
   b) i) 1232  
   ii) 1 132

10. a) 664  
    b) 327  
    c) 5 788  
    d) 3 612

11. 476, 467, 647, 674, 746, 764

## Section B

### Table 12

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

### Table 13

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

### Table 14

| 1. a) 3 thousands + 3 hundreds + 4 tens + 6 ones |
| 12       |
| 13       |
| 14       |
| 15       |
| 16       |
| 17       |
| 18       |
| 19       |
| 20       |

## Section C

19. a) 72  
   b) 1408  
   c) 3 843  
   d) 8 007

20. a) 27  
   b) 413  
   c) 765  
   d) 2 653  
   e) 6 726

21. a) 19  
   b) 653

22. a) 19  
   b) 653

23. Georgia earned $1073 more than Emma ($2418 – $1345).
Section D

28. a) 67¢
    b) 62¢
    c) 104¢

29. a) 5¢, 10¢
    b) 10¢, 1¢
    c) $1, $1
    d) $2, $2

30. a) 10¢, 5¢, 1¢
    b) 10¢, 10¢, 1¢, 1¢, 1¢

31. Eric collected $15.44 in total.

32. Cathy’s change would be 27¢.
La mesure
Test sur l’unité

Nom : _____________________________ Date : ______________

Section A

1. Mesure tous les côtés de chaque figure :
   a) _____ cm _____ cm _____ cm
   b) _____ cm _____ cm _____ cm

2. Mesure les lignes suivantes en centimètres et en millimètres :
   _____ cm _____ mm _____ cm _____ mm _____ cm _____ mm _____ cm _____ mm

3. Écris les unités suivantes par ordre, de la plus petite à la plus grande :
   cm m km mm

4. Complète les équations suivantes :
   1 cm = ______ mm 1 m = ______ cm 1 km = ______ m

5. Remplis les nombres qui manquent dans les tableaux suivants. Fais bien attention aux en-têtes!

<table>
<thead>
<tr>
<th>cm</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>cm</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>m</th>
<th>km</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>5000</td>
<td></td>
</tr>
</tbody>
</table>

6. Convertis les mesures données en cm suivantes en unités de mesure multiples :
   a) 427 cm = _____ m _____ cm
   b) 259 cm = _____ m _____ cm
   c) 619 cm = _____ m _____ cm
   d) 504 cm = _____ m _____ cm
La mesure

Test sur l’unité

Nom : _____________________________  
Date : __________________

Section A (suite)

7. Gustav fait partie de l’équipe d’athlétisme sur piste de son école. La piste mesure 300 m de long :
   
a) Si Gustav fait 3 fois le tour de la piste, combien de mètres aura-t-il parcouru? Montre ton travail.

   b) Gustav veut participer à une course de 2 000 m à la finale régionale. Environ combien de fois doit-il faire le tour de la piste pour courir 2 000 m? Explique ta réponse.

8. Mets les objets suivants par ordre, du plus petit au plus grand, en leur donnant un numéro (1 = plus petit, 2 = moyen, 3 = plus grand). Quelle unité de mesure utiliserais-tu pour mesurer la hauteur ou la longueur de chaque objet? Écris l’unité en-dessous de chaque illustration :

   a) b)

   _______  _______  _______
   _______  _______  _______

9. Quelle unité de mesure utiliserais-tu pour mesurer :
   a) La longueur d’une coccinelle : __________
   b) La hauteur de ton école : __________
   c) La longueur de ton bras : __________ Explique ton raisonnement :
   d) La distance voyagée par avion entre Halifax et Winnipeg : _______ Explique ton raisonnement :
La mesure
Test sur l’unité

Nom : _____________________________
Date : __________________

Section B

10. Chaque arête mesure 1 unité de long. Écris la longueur de chaque côté à côté de la figure (assure-toi de ne pas manquer d’arêtes!). Utilise ensuite la longueur des côtés pour trouver le périmètre. Montre ton travail :

a)  
\[\text{Périmètre} = \]

b)  
\[\text{Périmètre} = \]

11. Trouve le périmètre de chaque figure. N’oublie pas d’inclure l’unité appropriée dans ta réponse :

a)  
\[\text{Périmètre} = \]  6 m  
\[\text{Périmètre} = \]  4 m

b)  
\[\text{Périmètre} = \]  3 km

\[\text{Périmètre} = \]  3 km


Tests sur les unités – Cahier 4, Partie I
Section C

13. Pour chaque horloge, écris l’heure au complet (c’est-à-dire, l’heure et la minute exacte) :

a) 

b) 

c) 

d) 

e) 

f) 

14. Écris l’heure sur l’horloge numérique. Écris ensuite l’heure en mots :

a) 

b) 

c) 

Nom : _____________________________
Date : _________________
Section C (suite)

15. Combien de temps s’est écoulé entre 10 h 55 et 12 h 20?

16. Combien y a-t-il ...
   a) de mois dans une année? _______
   b) de semaines dans un mois? _______
   c) de jours dans une année? _______
   d) de secondes dans une minute? _______

17. Combien de mois y a-t-il dans 3 années?

18. Trace des lignes pour relier le temps écoulé indiqué dans la première colonne à une période de temps égale dans la deuxième colonne. Assure-toi de bien convertir!

<table>
<thead>
<tr>
<th>1 an</th>
<th>400 ans</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 ans</td>
<td>24 heures</td>
</tr>
<tr>
<td>1 jour</td>
<td>3 décennies</td>
</tr>
<tr>
<td>4 siècles</td>
<td>2 heures</td>
</tr>
<tr>
<td>120 minutes</td>
<td>365 jours</td>
</tr>
</tbody>
</table>

19. Dans chaque cas, associe la question avec l’unité de temps que tu utiliserais pour trouver la réponse :

<table>
<thead>
<tr>
<th>Quel âge a ton ami?</th>
<th>Années</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combien de temps te faut-il pour faire le tour de ton quartier?</td>
<td>Semaines</td>
</tr>
<tr>
<td>Quelle est la durée d’un jour d’école?</td>
<td>Mois</td>
</tr>
<tr>
<td>Quelle est la longueur de la période de vacances en mars?</td>
<td>Minutes</td>
</tr>
<tr>
<td>Quelle est la longueur des vacances d’été?</td>
<td>Heures</td>
</tr>
</tbody>
</table>

20. Convertis les heures suivantes de la notation 24 heures à la notation 12 heures, et indique a.m. ou p.m. :
   a) 13 h 00 ____________
   b) 7 h 30 ____________
Section A

1. a) 
   \[ \begin{array}{c}
   4 \text{ cm} \\
   2 \text{ cm} \\
   4 \text{ cm} \\
   2 \text{ cm}
   \end{array} \]
   b) Specific answers will vary, but the estimate should be 6 or 7 times around the track.

2. a) 1.5 cm; 15 mm
   b) 4 cm; 40 mm
   c) 3 cm; 30 mm
   d) 2.5 cm; 25 mm

3. mm; cm; m; km

4. 1 cm = 10 mm
   1 m = 100 cm
   1 km = 1000 m

5. | cm | mm |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
</tr>
<tr>
<td>14</td>
<td>140</td>
</tr>
</tbody>
</table>

6. a) 4 m 27 cm
   b) 2 m 59 cm
   c) 6 m 19 cm
   d) 5 m 4 cm

7. a) Gustav would have travelled 900 m (3 x 300).

8. a) (2) cm; (3) m; (1) mm
   b) (3) km; (1) mm; (2) cm

9. a) mm
   b) m
   c) cm
   (explanations will vary)
   d) km
   (explanations will vary)

10. a) 4 + 2 + 4 + 2 = 12 units
    b) 3 + 2 + 1 + 3 + 2 + 1 + 4 + 6 = 22 units

11. a) 20 m
    b) 9 km
    c) 38 cm
    d) 24 m
    e) B, D, A, C

12. Yes, they will get the same answer, since the 4 sides on a square are all equal.

Section B

13. a) 6:24
    b) 12:40
    c) 7:27
    d) 5:04
    e) 3:42
    f) 9:33

14. a) 01:11
    eleven minutes after one
    b) 04:29
    twenty-nine minutes after four
    c) 11:47
    Answers will vary: thirteen minutes before twelve; forty-seven minutes after eleven.
    d) $2, $2

15. 1 hr 25 min

16. a) 12
    b) About 4
    c) 365
    d) 60

17. \( 3 \times 12 = 36 \text{ months} \)

18. 1 year = 365 days
    30 years = 3 decades
    1 day = 24 hours
    4 centuries = 400 years
    120 minutes = 2 hours

19. Teacher to check.

20. a) 1:00 p.m.
    b) 7:30 a.m.
1. Il y a trois couleurs différentes de billes dans un sac : bleu (B), vert (V) et jaune (J).

   a) Utilise le tableau ci-dessous pour compter la fréquence des billes. Crée ensuite un pictogramme en utilisant la clé fournie :

   **CLÉ :**  = 2 billes

<table>
<thead>
<tr>
<th>Couleur</th>
<th>Fréquence</th>
<th>Pictogramme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bleu</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vert</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jaune</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b) Suppose plutôt qu’il y avait 9 billes vertes dans le sac. Comment utiliserais-tu la clé ci-dessus pour dessiner le pictogramme pour représenter 9 billes?

2. Complète le tableau de fréquence ci-dessous :

   **Question** : Quel est ton animal préféré ?

<table>
<thead>
<tr>
<th>Animal</th>
<th>Fréquence parmi les élèves</th>
<th>Compte</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chien</td>
<td>///// // // //</td>
<td>_____</td>
</tr>
<tr>
<td>Chat</td>
<td>///// // //</td>
<td>_____</td>
</tr>
<tr>
<td>Cheval</td>
<td>// // // // // //</td>
<td>_____</td>
</tr>
<tr>
<td>Lapin</td>
<td>///// // // // // // // //</td>
<td>_____</td>
</tr>
</tbody>
</table>

   Ensuite, complète le diagramme à bandes pour afficher les données du tableau.

   Assure-toi de donner un nom aux axes et aussi d’inclure un titre. Pense aussi à une échelle qui représenterait bien les données.
3. Peux-tu aider Luke à lire le diagramme à bandes doubles suivant?

**Livres préférés de mes amis**

- **Filles**
- **Garçons**

<table>
<thead>
<tr>
<th>Livre</th>
<th>Filles</th>
<th>Garçons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roman</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>B.D.</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>Faits</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

a) Quel type de livre est populaire à égalité parmi les garçons et les filles?

b) Quel type de livre est le plus populaire parmi les filles? Et parmi les garçons?

c) Au total, combien de filles ont voté? Et combien de garçons?

4. Détermine les valeurs des autres bandes dans les diagrammes :

a) ![Diagramme a](image_a)

b) ![Diagramme b](image_b)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

4. Détermine les valeurs des autres bandes dans les diagrammes :

b) ![Diagramme b](image_b)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

4. Détermine les valeurs des autres bandes dans les diagrammes :

b) ![Diagramme c](image_c)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Ivan a trouvé les informations suivantes sur Internet :

a) Ivan doit créer un diagramme pour afficher les données qu'il a recueillies. D'après toi, quel type de diagramme devrait-il utiliser? Pourquoi?

b) Affiche les données d'Ivan sur le type de diagramme que tu as nommé ci-dessus. N’oublie pas d’inclure les noms.

<table>
<thead>
<tr>
<th>Ville</th>
<th>Moyenne annuelle des chutes de neige (en cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellowknife, T.N.-O.</td>
<td>143</td>
</tr>
<tr>
<td>Regina, SK</td>
<td>107</td>
</tr>
<tr>
<td>Winnipeg, MB</td>
<td>114</td>
</tr>
<tr>
<td>Halifax, N.-É.</td>
<td>261</td>
</tr>
</tbody>
</table>

6. Examine les figures suivantes :

Remplis le tableau suivant (utilise des crochets), et puis utilise le tableau comme base pour compléter le diagramme de Venn :

<table>
<thead>
<tr>
<th>Figure</th>
<th>Côtés droits seulement</th>
<th>Côtés courbés seulement</th>
<th>Côtés droits et courbés</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tests sur les unités – Cahier 4, Partie I
**Section A**

1. a) Colour Tally Pictograph
   
<table>
<thead>
<tr>
<th>Colour</th>
<th>Tally</th>
<th>Pictograph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>12</td>
<td>⬤⬤⬤⬤⬤⬤⬤⬤⬤⬤⬤⬤</td>
</tr>
<tr>
<td>Green</td>
<td>8</td>
<td>⬤⬤⬤⬤⬤⬤⬤⬤</td>
</tr>
<tr>
<td>Yellow</td>
<td>6</td>
<td>⬤⬤⬤⬤⬤⬤</td>
</tr>
</tbody>
</table>

   b) 9 would be drawn as:
   
   ⬤⬤⬤⬤⬤⬤⬤⬤

2. Animal Count
   
<table>
<thead>
<tr>
<th>Animal</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog</td>
<td>12</td>
</tr>
<tr>
<td>Cat</td>
<td>9</td>
</tr>
<tr>
<td>Horse</td>
<td>3</td>
</tr>
<tr>
<td>Rabbit</td>
<td>6</td>
</tr>
</tbody>
</table>

   Bar graphs will vary, but here is a sample:

   **Our Favourite Animals**

<table>
<thead>
<tr>
<th># of Students</th>
<th>Dog</th>
<th>Cat</th>
<th>Horse</th>
<th>Rabbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. a) Comics
   
   b) Girls prefer novels; boys prefer comics and facts books.
   
   c) $15 + 9 + 3 = 27$ girls
      
      $7 + 9 + 9 = 25$ boys

4. a) $A = 25$
   
   b) $B = 40$
   
   c) $B = 11$
   
   c) $B = 24$
   
   C = 12

5. a) Teacher to check.

   b) Answers will vary; teacher to check.

**Section B**

6. | Shape | Straight Only | Curved Only | Both |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>B</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

**Shapes**

- Straight sides: B, D
- Curved sides: C, A, E, F

---

*Answer Keys – Workbook 4 Unit Tests*
La géométrie
Test sur l’unité

Section A

1. a) Complète le tableau :

   b) En regardant les formes suivantes, quelle relation peux-tu voir entre le nombre de côtés et le nombre de sommets?

<table>
<thead>
<tr>
<th>Nom de la forme</th>
<th># de côtés</th>
<th># de sommets</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Triangle" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Pentagone" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Hexagone" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Marque (d’un petit carré) tous les angles droits dans les formes suivantes. Encercle ensuite les formes qui ont exactement deux angles droits :

   a) ![Image](image4.png)
   b) ![Image](image5.png)
   c) ![Image](image6.png)
   d) ![Image](image7.png)
   e) ![Image](image8.png)
   f) ![Image](image9.png)

3. Avec des flèches, marque les paires de lignes parallèles dans les formes ci-dessous :

   a) ![Image](image10.png)
   b) ![Image](image11.png)
   c) ![Image](image12.png)
   d) ![Image](image13.png)

   ____ paires  ____ paires  ____ paires  ____ paires
Test sur l’unité

Section A (suite)

4. a) Écris le nom des formes suivantes :
   INDICE : Utilise les mots losange, carré, parallélogramme et rectangle. Fais attention à l’orthographe!
   (i) ___________________
   (ii) ___________________
   (iii) ___________________
   (iv) ___________________

b) Comment as-tu décidé quelle forme est un losange? Explique.

5. Mesure les angles suivants avec ton rapporteur. N’oublie pas les unités!
   a) 
   b) 

6. Combien de degrés mesure un angle droit? _______

7. Quand Vanessa a mesuré l’angle dans le diagramme, elle a pensé qu’il mesurait 70°. Quelle erreur a-t-elle faite? Quelle est la mesure exacte de l’angle?
La géométrie
Test sur l’unité

Section B

8. Encercle les paires de formes qui sont congruentes :

a)  

b)  

c)  

d)  

9. Trouve les formes qui sont congruentes avec la forme A, et donne-leur aussi la lettre A. Si tu peux trouver d’autres formes congruentes, identifie-les en leur donnant la même lettre (par ex., B, C …) :

A

10. Dans le tableau ci-dessous, trace DEUX formes : (i) une forme qui est congruente avec la forme donnée mais tournée sur son côté, et (ii) une forme qui n’est pas congruente avec la forme donnée. Identifie-les clairement :

11. Qu’est-ce que ça veut dire si une forme est « équilatérale »?
La géométrie
Test sur l’unité

Section B (suite)
12. En te basant sur les formes suivantes, réponds aux questions ci-dessous :

a) Quelles formes ci-dessus sont équilatérales (identifie-les par lettre)? _________________________

b) Classe les formes par type :

<table>
<thead>
<tr>
<th>Formes</th>
<th>Lettre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td>Quadrilatères</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formes</th>
<th>Lettre</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagones</td>
<td></td>
</tr>
<tr>
<td>Octogones</td>
<td></td>
</tr>
</tbody>
</table>

c) Quelles formes (par lettre) ne correspondaient à aucun des noms de formes donnés? Pourquoi?

d) Complète le tableau suivant. Ensuite, en utilisant l’information dans ton tableau, place les formes dans le diagramme de Venn ci-dessous, en te basant sur leurs propriétés :


<table>
<thead>
<tr>
<th>Propriété</th>
<th>Formes qui ont cette propriété</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. J’ai au moins 1 angle droit.</td>
<td></td>
</tr>
</tbody>
</table>

INDICE : Quelles formes partagent les deux propriétés? Quelles formes n’ont ni l’une ni l’autre des propriétés?
### Section A

1. **a)**

<table>
<thead>
<tr>
<th>Name</th>
<th># S</th>
<th># V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

**b)** The number of sides equals the number of vertices.

2. **a)**

**b)** No right angles

**c)**

3. **a)**

**b)** 1 pair

**c)** 2 pairs

**d)** 0 pairs

4. **a)**

   (i) square
   (ii) parallelogram
   (iii) rectangle
   (iv) rhombus

**b)** Although both a square and a rhombus are equilateral and have 2 pairs of parallel sides, a rhombus does require right angles!

5. **a)** 105°

**b)** 46°

6. 90°

7. Since the angle given is larger than a right angle (90°), Vanessa should have read the measure from the inner row of numbers. The correct measurement is 110°.

### Section B

8. **a)**

**b)**

**c)**

**d)**

9. **NOTE:**

   Letters used may vary.

10. Answers will vary; teacher to check.

11. It means that all sides are the same length.

12. **a)** B, G, I, J

    **b)**

<table>
<thead>
<tr>
<th>Shapes</th>
<th>Letter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangles</td>
<td>B, H</td>
</tr>
<tr>
<td>Quadrilaterals</td>
<td>C, F, G</td>
</tr>
<tr>
<td>Pentagons</td>
<td>D, E, I</td>
</tr>
<tr>
<td>Octagons</td>
<td>J</td>
</tr>
</tbody>
</table>

c) ‘A’ doesn’t fit any of the shapes given: it is not a polygon / quadrilateral since it contains sides that are curved.

d) Property | Figures |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>D, E, I, J</td>
</tr>
<tr>
<td>#2</td>
<td>C, D, E, G, H</td>
</tr>
</tbody>
</table>
Les régularités et l’algèbre

Test sur l’unité

Nom : _____________________________

Date : _________________

Section A

1. Décris chaque régularité en indiquant si elle est croissante, décroissante ou si elle se répète :

   a) 1, 4, 7, 10, 13, 16 _______________

   b) 1, 5, 8, 1, 5, 8 _______________

   c) 9, 8, 7, 6, 5, 4 _______________

   d) 2, 4, 6, 8, 10, 12 _______________

   e) 21, 16, 10, 7, 5, 1 _______________

   f) 3, 8, 3, 8, 3, 8 _______________

2. Un jardinier plante des roses (R), des lys (L) et des tulipes (T) dans des rangées, selon la régularité indiquée ci-contre à droite :

   a) Complète le tableau.

   b) Dans quelle rangée est-ce que la régularité dans la deuxième rangée sera répétée? _______

3. a) Dans le tableau ci-contre, encercle chaque 11° nombre (c.-à-d., encercle les nombres que tu dirais si tu comptais par bonds de 11 : 11, 22, 33, …).

   Les nombres que tu encercles sont les multiples de 11 (jusqu’à 132).

   b) Quelles régularités peux-tu voir dans le chiffre des unités et le chiffre des dizaines des multiples de 11?
Les régularités et l’algèbre
Test sur l’unité

Nom : _____________________________
Date : _________________

Section A (suite)


a) 2 , 4 , 7 , 11 , ___ , ___

b) 3 , 4 , 6 , 9 , 13 , ___ , ___

c) 10 , 22 , 32 , 40 , ___ , ___

d) 6 , 8 , 12 , 18 , 26 , ___ , ___

e) 99 , 78 , 60 , 45 , ___ , ___

f) 110 , 105 , 95 , 80 , 60 , ___ , ___

5. Roger et Eve ont économisé les montants indiqués dans le tableau.

a) Quelle est la règle de la régularité pour le montant économisé par Roger?

b) Quelle est la règle de la régularité pour le montant économisé par Eve?

c) D’après toi, qui aura économisé le plus d’argent à la fin des sept semaines?

d) Continue les régularités dans le tableau. Avais-tu raison?
Les régularités et l’algèbre

Test sur l’unité

Nom : _____________________________

Date : _________________

Section B

6. Combien de triangles y aura-t-il dans la figure 6? Comment le sais-tu?

7. Quelle sera la 23e forme dans cette régularité? Explique comment tu le sais.

8. Regarde les nombres ci-dessous et encercle ceux qui sont des multiples de 5. Comment sais-tu que les nombres encerclés sont des multiples de 5?

75 125 132 270 382 597 670

9. Prolonge chaque régularité :

   a) 3427  3527  3627  __________  __________  __________
   b) 4234  5235  6236  __________  __________  __________
   c) 1234  2345  3456  __________  __________  __________
Les régularités et l’algèbre

Test sur l’unité

Section C

10. Trouve le nombre qui manque pour rendre l’équation vraie, et écris-le dans la boîte :

a) \[ \square + 2 = 5 \]

b) \[ 3 + \square = 9 \]

c) \[ \square + 2 = 11 \]

d) \[ 9 - \square = 4 \]

e) \[ 17 - \square = 12 \]

f) \[ 8 - \square = 6 \]

g) \[ 2 \times \square = 8 \]

h) \[ \square \times 5 = 15 \]

i) \[ 3 \times \square = 12 \]

j) \[ \square \div 3 = 4 \]

k) \[ \square \div 5 = 2 \]

l) \[ \square \div 2 = 4 \]

m) \[ 9 + 3 = 6 + \square \]

n) \[ 10 - 3 = \square + 4 \]

**NOTE :** Dans ces questions, tu dois placer le même nombre dans chacune des deux boîtes.

o) \[ \square + \square = 8 \]

p) \[ \square + \square + 3 = 13 \]

11. Trouve 3 ensembles de nombres pour rendre l’équation vraie :

**NOTE :** Dans chaque équation, les formes congruentes représentent le même nombre.

\[ \square + \square + \square = 7 \]

\[ \square + \square + \square = 7 \]

\[ \square + \square + \square = 7 \]

12. Raegan a jeté 3 fléchettes et obtenu 5 points. La fléchette au centre vaut plus que les autres. Combien vaut chaque fléchette? Montre ton travail :

13. Trouve le nombre mystère :

« Je suis plus grand que 17 et plus petit que 24. Je suis un multiple de 4. Quel nombre suis-je? »
Unit Test: Patterns & Algebra – Workbook 4, Part 2

Section A

1. a) increasing
   b) repeating
   c) decreasing
   d) increasing
   e) decreasing
   f) repeating

2. a)
   1
   R  L  T  R  L
   2
   T  R  L  T
   3
   L  T  R  L  T
   4
   R  L  T  R  L
   5
   T  R  L  T
   6
   L  T  R  L  T
   7
   R  L  T  R  L

   b) Row 5

3. a) Teacher to check.
   b) For the two-digit numbers, the ones and tens digits are the same.
      For the three-digit numbers, the ones digit is 1 less than the tens digit.

4. a) Gaps:
    2, 3, 4, 5, 6
    Continued Pattern:
    16, 22

   b) Gaps:
    1, 2, 3, 4, 5, 6
    Continued Pattern:
    18, 24

   c) Gaps:
    12, 10, 8, 6, 4
    Continued Pattern:
    46, 50

   d) Gaps:
    2, 4, 6, 8, 10, 12
    Continued Pattern:
    36, 48

   e) Gaps:
    –21, –18, –15, –12, –9
    Continued Pattern:
    33, 24

   f) Gaps:
    Continued Pattern:
    35, 5

5. a) Each week, Roger saves twice as much as he did the previous week.
   b) Each week, Eve saves $4 more than she did the previous week.
   c) Answers may vary.

   d) Wk | R | E
       1 | $1 | $17
       2 | $2 | $21
       3 | $4 | $25
       4 | $8 | $29
       5 | $16 | $33
       6 | $32 | $37
       7 | $64 | $41

   Roger will save more by the end of seven weeks ($64 vs $41).

Section B

6. a) Fig # | □ | △
    1 | 2 | 4
    2 | 3 | 6
    3 | 4 | 8
    4 | 5 | 10
    5 | 6 | 12
    6 | 7 | 14

   For Figure 6, you will need 14 triangles.

7. Skip count by 4s:
   4, 8, 12, 16, 20.
   20th term is a, the core starts anew at 21st term.

   Term: 21 23 23
   Shape:

8. Multiples of 5:
   75, 125, 270, 670
   You can tell by looking at the ones digit: those with a 5 or 0 in the ones digit are divisible by 5.

   a) 3 727, 3 827, 3 927
   b) 7 237, 8 238, 9 239
   c) 4 567, 5 678, 6 789

Section C

10. a) 3
    b) 6
    c) 9
    d) 5
    e) 5
    f) 2
    g) 4
    h) 3
    i) 4
    j) 12
    k) 10
    l) 8
    m) 6
    n) 3

   NOTE: For the following questions, you must put the same number in both boxes.

   o) 4
   p) 5

11. 1 + 1 + 5 = 7
    2 + 2 + 3 = 7
    3 + 3 + 1 = 7

12. You can rewrite this question as an equation:
    □ + □ + O = 5
    The possible solutions are:
    1 + 1 + 3 = 5
    2 + 2 + 1 = 5
    But, since the centre ring is worth more, O > □ so the first solution is correct.
    Outside dart = 1 point
    Centre dart = 3 points

13. The mystery number is 20.
Logique numérale

Test sur l’unité

Section A

1. Mary Anne a 12 biscuits. Elle donne 3 biscuits à chacune de ses amies. À combien d’amies donne-t-elle des biscuits?

2. Aidan a 14 timbres. Il met 2 timbres sur chaque enveloppe. Combien d’enveloppes utilise-t-il?

3. a) 6 pamplemousses dans chaque boîte; 42 pamplemousses; 7 boîtes.
   Qu’est-ce qui est partagé ou divisé en ensembles? ___________ _________
   Combien d’ensembles? _______ Combien dans chaque ensemble? _______

   b) 3 autobus scolaires; 30 enfants; 10 enfants dans chaque autobus scolaire.
   Qu’est-ce qui est partagé ou divisé en ensembles? ___________ _________
   Combien d’ensembles? _______ Combien dans chaque ensemble? _______

4. 5 amis partagent 15 billets. Combien de billets reçoit chaque ami? Montre ton travail.

5. Écris un énoncé de division et un énoncé d’addition pour l’illustration ci-contre :
Logique numérale
Test sur l’unité

Nom : ___________________________________________
Date : ____________________

Section B

6. Pour chaque question, écris un énoncé de multiplication ou de division pour résoudre le problème :

a) 18 objets en tout
   3 objets dans chaque ensemble
   ___________________
   Combien d’ensembles? _____

b) 5 ensembles
   4 objets dans chaque ensemble
   ___________________
   Combien d’objets en tout? _____

c) 8 ensembles
   3 objets dans chaque ensemble
   ___________________
   Combien d’objets en tout? _____

d) 6 objets dans chaque ensemble
   12 objets en tout
   ___________________
   Combien d’ensembles? _____

7. Résous les problèmes suivants dans l’espace prévu. Montre ton travail :

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 20 personnes; 4 fourgonnettes.</td>
<td>b) 3 billes dans chaque bocal; 6 bocaux.</td>
</tr>
<tr>
<td>Combien de personnes dans chaque fourgonnette?</td>
<td>Combien de billes?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 15 fleurs; 5 pots.</td>
<td>d) 4 chaises à chaque table; 2 tables.</td>
</tr>
<tr>
<td>Combien de fleurs dans chaque pot?</td>
<td>Combien de chaises?</td>
</tr>
</tbody>
</table>

8. Trouve deux façons différentes de partager 9 pommes également pour qu’il reste une pomme :
Logique numérale
Test sur l’unité

Section B (suite)
9. Pour chaque question, effectue les étapes de la longue division :
   a) $\begin{array}{c}
   5 \overline{)24} \\
   -
   \end{array}$
   b) $\begin{array}{c}
   3 \overline{)13} \\
   -
   \end{array}$
   c) $\begin{array}{c}
   5 \overline{)19} \\
   -
   \end{array}$
   d) $\begin{array}{c}
   2 \overline{)17} \\
   -
   \end{array}$

10. Effectue toutes les étapes de la longue division :
   a) $\begin{array}{c}
   3 \overline{)74} \\
   -
   \end{array}$
   b) $\begin{array}{c}
   4 \overline{)54} \\
   -
   \end{array}$
   c) $\begin{array}{c}
   2 \overline{)27} \\
   -
   \end{array}$
   d) $\begin{array}{c}
   5 \overline{)70} \\
   -
   \end{array}$
   e) $\begin{array}{c}
   5 \overline{)84} \\
   -
   \end{array}$
   f) $\begin{array}{c}
   4 \overline{)64} \\
   -
   \end{array}$
   g) $\begin{array}{c}
   3 \overline{)96} \\
   -
   \end{array}$
   h) $\begin{array}{c}
   6 \overline{)89} \\
   -
   \end{array}$

11. Un canot peut contenir 3 enfants. Combien de canots faudrait-il pour 44 enfants?

12. Alexa met 73 pommes dans des sacs de 6. Mike met 46 pommes dans des sacs de 4. Qui aura le plus de pommes qui restent?
Logique numérique
Test sur l'unité

Section B (suite)

13. Divise :

\[
\begin{array}{c|cc}
& 6 & 2 \\
4 & | & \\
- & & \\
\end{array}
\]

14. Un triangle équilatéral a un périmètre de 531 cm. Quelle est la longueur de chaque côté?

\[
\begin{array}{c|cc}
& 5 & 2 \\
5 & | & \\
- & & \\
\end{array}
\]

15. Trouve le nombre mystère :
   
   a) « Je suis un multiple de 6. Je suis plus grand que 21 et plus petit que 27. »

   b) « Je suis divisible par 7. Je suis plus petit que 25 et je suis un nombre pair. »

16. Trouve deux nombres plus petits que 20 qui donnent un reste de 1 quand ils sont divisés par 4 :

Logique numérique
Test sur l'unité

Nom : __________________________
Date : ________________

Section C
18. Nomme les fractions suivantes :
   a) \[ \frac{2}{10} \]
   b) \[ \frac{1}{10} \]
   c) \[ \frac{7}{10} \]
   d) \[ \frac{9}{10} \]
   e) \[ \frac{5}{10} \]

19. Quelle fraction est représentée par la partie coloriée de chaque figure?
   a) 
   b) 
   c) 
   d) 

20. Écris les fractions par ordre, de la plus petite à la plus grande :
   a) \( \frac{2}{10} \), \( \frac{1}{10} \), \( \frac{7}{10} \), \( \frac{9}{10} \), \( \frac{5}{10} \)
   b) \( \frac{1}{5} \), \( \frac{1}{2} \), \( \frac{1}{4} \)
   c) \( \frac{2}{3} \), \( \frac{2}{5} \), \( \frac{2}{7} \)

21. Colorie un morceau à la fois jusqu'à ce que le nombre de tartes indiqué soit colorié. Il y a peut-être plus de tartes que ce dont tu as besoin :
   a) \( 2 \frac{1}{2} \)
   b) \( 3 \frac{1}{2} \)
   c) \( 1 \frac{3}{4} \)
   d) \( 2 \frac{2}{3} \)
   e) \( \frac{8}{3} \)
   f) \( \frac{13}{4} \)
Logique numérique
Test sur l’unité

Nom : _______________________________
Date : __________________________

Section C (suite)

22. Écris ces fractions sous forme de nombre fractionnaire et de fraction impropre :
   a)  
   b) 
   \[ \frac{\text{_______}}{\text{_______}} = \frac{\text{_______}}{\text{_______}} \]

23. Trouve la fraction de chacun des nombres suivants en écrivant un énoncé de division équivalent. Ensuite compte par bonds pour trouver la réponse :
   a) \( \frac{1}{2} \) de 8  
   b) \( \frac{1}{2} \) de 10  
   c) \( \frac{1}{3} \) de 9  
   d) \( \frac{1}{4} \) de 12  

24. Est-ce que \( \frac{2}{3} \) est plus grand qu’une tarte entière, ou plus petit? Comment le sais-tu?

   Combien de bouteilles y a-t-il dans 3 \( \frac{1}{2} \) caisses?

26. Fais un dessin (avec des points) pour représenter \( \frac{4}{5} \) de 10.

27. Quelle fraction est plus grande : \( \frac{2}{4} \) ou \( \frac{5}{2} \)? Fais un dessin pour montrer ta réponse :

28. Additionne ou soustrais :
   a) \( \frac{9}{15} - \frac{3}{15} = \)  
   b) \( \frac{3}{7} - \frac{2}{7} = \)  
   c) \( \frac{7}{9} - \frac{3}{9} = \)
Section D

29. Écris une fraction qui représente le nombre de centaines. Écris ensuite une fraction qui représente le nombre de dizaines :

a) \[ \frac{8}{100} = \frac{0}{10} \]

b) \[ \frac{7}{100} = \frac{0}{10} \]

c) \[ \frac{7}{100} = \frac{0}{10} \]

d) \[ \frac{7}{100} = \frac{0}{10} \]

30. Remplis le tableau ci-dessous :

<table>
<thead>
<tr>
<th>Dessin</th>
<th>Fraction</th>
<th>Décimale</th>
<th>Décimale équivalente</th>
<th>Fraction équivalente</th>
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31. Écris les décimales suivantes sous forme de fraction :

a) \(0,2 =\) b) \(0,35 =\) c) \(0,04 =\) d) \(0,8 =\) e) \(0,6 =\)

f) \(0,02\) g) \(0,72 =\) h) \(0,4 =\) i) \(0,23 =\) j) \(0,25 =\)

32. Change les fractions suivantes à des décimales :

a) \(\frac{82}{100} = \) b) \(\frac{7}{100} = \) c) \(\frac{77}{100} = \) d) \(\frac{7}{10} = \)
Logique numérale
Test sur l’unité

Section D (suite)

33. Écris une décimale pour chacun des nombres fractionnaires ci-dessous :
   a) \( \frac{23}{100} = \)
   b) \( \frac{71}{100} = \)
   c) \( \frac{7}{10} = \)
   d) \( \frac{27}{100} = \)
   e) \( \frac{7}{100} = \)
   f) \( \frac{8}{10} = \)
   g) \( \frac{1}{10} = \)
   h) \( \frac{5}{100} = \)

34. Écris les nombres par ordre (du plus petit au plus grand) en changeant en premier chaque décimale en une fraction avec un dénominateur de 10 :
   a) 0,7 , 0,3 , 0,5
   b) \( \frac{1}{10} , 0,3 , 0,9 \)
   c) \( \frac{7}{10} , 0,3 , \frac{4}{10} \)
   d) 0,7 , 0,8 , \( \frac{2}{10} \)

35. Aligne et additionne ou soustrais les décimales suivantes :
   a) 0,32 + 0,17
   b) 0,64 – 0,23
   c) 0,67 – 0,2

36. Marque chaque point avec un ‘X’ et écris la lettre à laquelle il correspond :

   \begin{align*}
   &A. \quad 1,1 \quad \text{B.} \quad 2,5 \quad \text{C.} \quad 0,60 \quad \text{D.} \quad 1,9
   \end{align*}

37. Lequel est plus grand : \( \frac{23}{10} \) ou 2,4? Explique.

38. Une couleuvre mesure 0,36 mètres de long.
   a) Quelle fraction d’un mètre est la longueur de la couleuvre?
   b) Combien de cm de long mesureraient 2 couleuvres si elles étaient placées l’une derrière l’autre?
Logique numérique

Test sur l’unité

Nom : ____________________________________ Date : _________________

Section E

39. Sarah a 4,67 $ et Uma a 5,24 $. Combien d’argent Uma a-t-elle de plus que Sarah?

40. Ash a 25,62 $. Il veut acheter un cadeau pour son père pour 17,38 $ et un livre pour lui-même pour 5,97 $. A-t-il assez d’argent pour acheter le cadeau et le livre?

41. Estime en arrondissant chaque montant au dollar près avant de faire le calcul :
   a) 34,21 $ – 26,57 $
      Estimation : ______
      Actuel : ____________________________
   b) 47,93 $ + 12,44 $
      Estimation : ______
      Actuel : ____________________________

42. Erika avait 10,00 $. Elle a acheté un ensemble de crayons pour 7,89 $. Estime la monnaie qu’elle a reçue en retour :

43. Remplis les espaces vides :
   a) _______ est 0,1 de plus que 0,8    b) _______ est 0,1 de moins que 0,6      c) 2,3 + _______ = 2,4
   d) 3,71 – _______ = 3,61            e) 3,48 – _______ = 3,47             f) 4,53 + _______ = 4,54
Section A
1. 4 friends
2. 7 envelopes
3. a) Grapefruits; 7; 6
   b) Kids; 3; 10
4. 15 + 5 = 3;
   Each friend gets 3 tickets.
5. 15 + 5 = 3;
   5 + 5 + 5 = 15

Section B
6. a) 18 ÷ 3 = 6; 6
   b) 5 × 4 = 20; 20
   c) 8 × 3 = 24; 24
   d) 12 + 6 = 2; 2
7. a) 20 + 4 = 5;
   5 people in each van
   b) 6 × 3 = 18;
   18 marbles
   c) 15 + 5 = 3;
   3 flowers in each pot
   d) 4 × 2 = 8;
   8 chairs
8. 2 groups of 4 apples
   OR
   4 groups of 2 apples
9. a) 4 R4
   b) 4 R1
   c) 3 R4
   d) 8 R1
10. a) 24 R2
    b) 13 R2
    c) 13 R1
    d) 14
    e) 16 R4
    f) 16
    g) 32
    h) 14 R5
11. 14 R2;
    They will need 15 canoes.
12. Alexa: 73 ÷ 6 = 12 R1
    Mike: 46 ÷ 4 = 11 R2
    So, Mike had more apples left over.
13. 156 R1
14. Each side of the triangle is 177 cm long (531 + 3).
15. a) 24
    b) 14
16. Four possibilities so answers may vary:
    5, 9, 13 and 17
17. Five boxes will hold 30 bottles.
    Approach may vary – for example:
    24 + 4 = 6 bottles per box
    6 × 5 = 30 bottles

Section C
18. a) \(\frac{1}{2}\)
    b) \(\frac{1}{4}\)
    c) \(\frac{5}{9}\)
    d) \(\frac{3}{10}\)
19. a) \(\frac{2}{8} = \frac{1}{4}\)
    b) \(\frac{1}{8}\)
    c) \(\frac{3}{9} = \frac{1}{3}\)
    d) \(\frac{1}{12}\)
20. a) \(\frac{1}{10} \cdot \frac{2}{10} \cdot \frac{5}{10} \cdot \frac{7}{10} \cdot \frac{9}{10}\)
    b) \(\frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{2}\)
    c) \(\frac{2}{7} \cdot \frac{2}{5} \cdot \frac{3}{7}\)
21. a)
    b)
    c)
    d)
    e)
    f)
22. a) \(2 \frac{1}{3} ÷ \frac{7}{3}\)
    b) \(3 \frac{1}{8} ÷ \frac{25}{8}\)
23. a) \(8 ÷ 2 = 4\)
    b) \(10 + 2 = 5\)
    c) \(9 + 3 = 3\)
    d) \(12 + 4 = 3\)
24. \(\frac{2}{5} < 1\). Teacher to check explanation.
25. 21 bottles
26. ☐ ☐ ☐ ☐ ☐ ☐
34. a) \( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \)
   b) \( \frac{1}{10}, \frac{3}{10}, \frac{9}{10} \)
   c) \( \frac{3}{10}, \frac{4}{10}, \frac{7}{10} \)
   d) \( \frac{2}{10}, \frac{7}{10}, \frac{8}{10} \)

35. a) 0.49
   b) 0.41
   c) 0.47

36. C A D B

37. \( \frac{23}{10} = 2 \frac{3}{10} = 2.3 < 2.4 \)
   So 2.4 is greater.

38. a) \( \frac{36}{100} \)
   b) 72 cm

39. Uma has 57¢ more.

40. Yes, he needs $23.35 (< $25.62)

41. a) Estimate:
    
    $34 - $27 = $7
    
    Actual:
    
    $34.21
    
    $7.64
    
    b) Estimate:
    
    $48 + $12 = $60
    
    Actual:
    
    $47.93
    
    + $12.44
    
    $60.37

42. Estimate $2
   (Actual $2.11)

43. a) .9
   b) .5
   c) .1
   d) .1
   e) .01
   f) .01
La mesure
Test sur l’unité

Section A

1. Trouve l’aire (en unités carrées) de chacune des formes :

Aire de A = ______ unités²
Aire de B = ______ unités²
Aire de C = ______ unités²

2. Calcule l’aire de chaque rectangle (assure-toi d’inclure les unités). Ensuite, crée une liste des rectangles, par ordre (en les identifiant par lettre), de celui avec la plus grande aire à celui avec la plus petite aire. Fais attention aux unités de mesure!

a)  

\[
\begin{array}{c}
5 \text{ m} \\
7 \text{ m}
\end{array}
\]

b)  

\[
\begin{array}{c}
9 \text{ cm} \\
4 \text{ cm}
\end{array}
\]

c)  

\[
\begin{array}{c}
10 \text{ m} \\
5 \text{ m}
\end{array}
\]

d)  

\[
\begin{array}{c}
14 \text{ km} \\
4 \text{ km}
\end{array}
\]

Liste des aires (par lettre, de la plus grande à la plus petite) : ______ ______ ______ ______

3. Trouve l’aire des rectangles qui ont les mesures suivantes :

a) largeur : 5 m  longueur : 7 m   b) largeur : 2 m  longueur : 9 m   c) largeur : 6 cm  longueur : 8 cm

4. Si tu connais la longueur et la largeur d’un rectangle, comment peux-tu trouver son aire?

5. Un rectangle a une aire de 10 cm² et une longueur de 5 cm. Quelle est sa largeur? Explique comment tu as trouvé ta réponse :
La mesure
Test sur l’unité

Nom : _____________________________
Date : ___________________

Section A (suite)

6. Deux demi-carrés □ □ occupent la même aire qu’un carré entier □ .

Compte chaque paire de demi-carrés comme un carré entier pour trouver l’aire de la surface coloriée :

a)  

b)  

c)  

Aire = ____ carrés entiers  
Aire = ____ carrés entiers  
Aire = ____ carrés entiers

7. Pour chaque rectangle, fais une estimation et puis mesure la longueur et la largeur avec ta règle. Écris tes réponses dans le tableau :

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Périmètre estimé</th>
<th>Aire estimée</th>
<th>Longueur</th>
<th>Largeur</th>
<th>Périmètre actuel</th>
<th>Aire actuelle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cm</td>
<td>cm²</td>
<td>cm</td>
<td>cm</td>
<td>cm</td>
<td>cm²</td>
</tr>
<tr>
<td>B</td>
<td></td>
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<tr>
<td>C</td>
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</tbody>
</table>

8. Karen veut construire un lit de fleurs rectangulaire qui mesure 2 m de large et 3 m de long.

a) Quel est le périmètre du lit de fleurs?

b) Une clôture coûte 2 $ le mètre. Combien cela coûterait pour construire une clôture autour du lit de fleurs?

c) Karen veut planter 3 fleurs dans chaque mètre carré du lit de fleur. Combien de fleurs devra-t-elle acheter?
La mesure
Test sur l'unité

Nom : _____________________________
Date : _________________

Section B

9. Change les mesures suivantes en grammes :
   a) 3 kg = ________
   b) 9 kg = ________
   c) 17 kg = ________

10. Un chat domestique pèse environ 5 kg. Quelle est sa masse en grammes? _____________

11. Une pièce d'un cent pèse 2 grammes, une pièce de cinq cents pèse 4 grammes, et une pièce de 1 $ pèse 7 grammes :
   a) Combien de pièces d'un cent pèsent autant que 4 pièces de cinq cents?

   b) Combien de pièces de 1 $ pèsent autant que 7 pièces de cinq cents?

12. Quelle unité est la plus appropriée pour mesurer chaque objet? Encercle l'unité appropriée :
   grammes ou kilogrammes?

13. Coche la case appropriée. Utiliserais-tu des grammes ou des kilogrammes pour peser ...
   a) un orignal?  
      □ g        □ kg  
   b) un bureau?  
      □ g        □ kg
   c) un bout de fromage?  
      □ g        □ kg  
   d) un petit oiseau? □ g  
   f) toi-même?  
      □ g        □ kg

14. a) Un bébé éléphant pesait 160 kilogrammes à la naissance. Il a pris du poids au rythme de 8 kilogrammes par semaine. Combien le bébé éléphant pesait-il à 4 semaines?

   b) Les graines de concombre et de pois pèsent 2 grammes chacune, et les graines de radis pèsent 3 grammes chacune. Joel a acheté 8 graines de concombre, 12 graines de pois et 3 graines de radis. Combien pèsent toutes ces graines en tout?
La mesure

Test sur l’unité

Name: _____________________________

Date: _________________

Section B (suite)

15. Change les mesures suivantes en millilitres :
   a) 5 L = ________
   b) 2 L = ________
   c) 12 L = ________
   d) 47 L = ________

16. Encercle l’unité appropriée pour mesurer la capacité de chaque contenant. En litres (L) ou en millilitres (ml)?
   a) L ou ml?
   b) L ou ml?
   c) L ou ml?

17. Lequel des ensembles de contenants suivants a la plus grande capacité? Comment le sais-tu?
   a) OU
   b) OU

18. Pour chacune des capacités suivantes, combien de contenants te faudrait-il pour faire un litre? Explique comment tu le sais :
   a) 100 ml
   b) 500 ml
   c) 250 ml

19. Trouve le volume des formes ci-dessous. Chaque cube mesure 1 cm³. Explique comment tu as compté les cubes que tu ne peux pas voir :
   a) Volume : ____ cm³
   b) Volume : ____ cm³
   c) Volume : ____ cm³

20. Pour la recette de boisson fouettée suivante ...
   a) Encercle les mesures de la capacité et souligne les mesures de la masse.
   b) Calcule le total de toutes les mesures de la masse :
   c) Calcule le total de toutes les mesures de la capacité :

Boissons fouettées pour 10 personnes :
300 grammes de fraises
2 L de lait de soja
200 ml de yogourt
1 kg de bananes
Section A
1. Area of A = 8 units²
   Area of B = 4 units²
   Area of C = 12 units²
2. a) 35 m²
    b) 36 cm²
    c) 50 m²
    d) 56 km²
3. a) 35 m²
    b) 18 m²
    c) 48 cm²
4. To get the area, multiply the length by the width (A = l × w).
5. Width = 2 cm
   To find, skip count by 5’s until you ‘hit’ 10 or divide 10 by 5.
6. a) 6 whole squares
    b) 6 whole squares
    c) 8 whole squares
7. Actual measurements:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>W</th>
<th>P</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3 cm</td>
<td>5 cm</td>
<td>16 cm</td>
<td>15 cm²</td>
</tr>
<tr>
<td>B</td>
<td>2 cm</td>
<td>4 cm</td>
<td>12 cm</td>
<td>8 cm²</td>
</tr>
<tr>
<td>C</td>
<td>5 cm</td>
<td>2 cm</td>
<td>14 cm</td>
<td>10 cm²</td>
</tr>
</tbody>
</table>
8. a) 10 m
    b) $20
    c) 18 flowers

Section B
9. a) 3 000 g
    b) 9 000 g
    c) 17 000 g
10. 5 000 g
11. a) 8 pennies
    b) 4 loonies
12. kilograms; grams; kilograms
13. a) kg
    b) kg
    c) g
    d) g
    e) g
    f) kg
14. a) 192 kg
    b) 49 g
15. a) 5 000 mL
    b) 2 000 mL
    c) 12 000 mL
    d) 47 000 mL
16. a) L
    b) mL
    c) L
17. Set a) has the greatest capacity – after you cross out the ‘shared’ containers, you are left with 2 large containers in a) and 2 small containers in b); 2 large are greater in capacity than 2 small.
18. a) 10 containers
    (1000 ÷ 100 = 10)
    b) 2 containers
    (1000 ÷ 500 = 2)
    c) 4 containers
    (1000 ÷ 250 = 4)
19. a) 8 cubes
    b) 12 cubes
    c) 27 cubes
    To count the boxes you can’t see, you might count the “front” and multiply by the number of layers.
20. a) Capacity:
    2 L soy milk;
    200 mL yogurt
    Mass:
    300 g strawberries;
    1 kg bananas
    b) 1.3 kg / 1 300 g
    c) 2.2 L / 2 200 mL
Probabilité et traitement des données

Test sur l’unité

Section A

1. Trouve l’étendue des ensembles de données suivants :
   a) 45, 23, 14, 95, 44, 7
   Étendue : De ____ à ____
   b) 123, 46, 35, 70, 21, 354
   Étendue : De ____ à ____

2. Trouve la moyenne des ensembles de données suivants :
   a) 3, 5, 7, 11, 14
   Moyenne : ________
   b) 16, 5, 11, 3, 20
   Moyenne : ________

3. Trouve le mode des ensembles de données suivants :
   a) 3, 8, 8
   Mode : ________
   c) Tige  Feuille
       3       227
       4       3344
       5      18889
       6     0344
   Mode : ________
   b) 7, 7, 4, 5, 7, 4, 4, 7
   Mode : ________

4. Trouve la médiane des ensembles de données suivants :
   a) 3, 4, 10, 12, 17
   Médiane : ________
   b) 3, 11, 8, 10, 4
   Médiane : ________
   c) 18, 5, 18, 76, 10, 92
   Médiane : ________
   d) 27, 3, 1, 85, 553, 23
   Médiane : ________
5. Mme Lynch a donné un test d’orthographe à ses élèves (sur 20) et elle a inscrit les notes obtenues dans le tableau. Fais un diagramme à tige et à feuilles pour illustrer ces données.

<table>
<thead>
<tr>
<th>Tige</th>
<th>Feuilles</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>16</td>
<td>19</td>
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<tr>
<td>29</td>
<td>20</td>
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<tr>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>
Probabilité et traitement des données
Test sur l’unité

Section B

6. Quels sont les résultats possibles pour ces roulettes? (Le premier est fait pour toi.)
   a) ________________________________
   b) ________________________________
   c) ________________________________

Tu obtiens un 5, un 6 ou un 7

6. Quels sont les résultats possibles pour ces roulettes? (Le premier est fait pour toi.)
   a) ________________________________
   b) ________________________________
   c) ________________________________

Tu obtiens un 5, un 6 ou un 7

7. Tu retires une balle d’une boîte. Combien de résultats différents peut-il y avoir dans chacun des cas suivants?
   a) ______ résultats  b) ______ résultats  c) ______ résultats  d) ______ résultats

8. Remplis les nombres qui manquent :
   a) \(\frac{1}{3}\) de 39 est ____
   b) \(\frac{1}{3}\) de 42 est ____
   c) \(\frac{1}{3}\) de 75 est ____
   d) \(\frac{1}{4}\) de 8 est ____
   e) \(\frac{1}{4}\) de 12 est ____
   f) \(\frac{1}{4}\) de 36 est ____
   g) \(\frac{1}{4}\) de 52 est ____
   h) \(\frac{1}{4}\) de 84 est ____

9. Sur chaque roulette ci-dessous, quelle fraction de tes tours s’arrêtera sur le rouge?
   a) Je m’attends à ce que ______ des tours de roulette s’arrêtent sur le rouge.
   b) ________________________________

10. Si tu faisais tourner la roulette à la question 9 a) douze fois, combien de fois t’attendrais-tu à ce qu’elle s’arrête sur le rouge? Explique.
Probabilité et traitement des données
Test sur l’unité

Section B (suite)

11. En utilisant les mots « certain », « probable », « improbable » ou « impossible », décris la probabilité …

   a) d’obtenir le rouge
   b) d’obtenir le vert
   c) d’obtenir le jaune
   d) d’obtenir le rouge

12. Utilise les mots « impossible », « improbable », « probable » ou « certain » pour décrire la probabilité :

   a) d’obtenir le vert
   b) d’obtenir le rouge
   c) d’obtenir le jaune
   d) d’obtenir le jaune

13. Explique ta réponse à la question 12 c) :

14. Nomme un événement qui est ...
   a) certain : ___________________________
   b) impossible : ___________________________

15. Dennis et Kevin font tourner la roulette ci-dessous. Si elle s’arrête sur le rouge, Kevin gagne. Si elle s’arrête sur le jaune, Dennis gagne. Le jeu est-il juste? Si non, qui a la meilleure chance de gagner? Explique ta réponse.
### Section A

1. a) 7 to 95  
   b) 21 to 354  
2. a) 8  
   b) 11  
3. a) 8  
   b) 7  
   c) 58  
4. a) 10  
   b) 8  
   c) 18  
   d) 25 (the average of 23 and 27)  
5. | Stem | Leaves |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>59</td>
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<td>1</td>
<td>0025678999</td>
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<tr>
<td>2</td>
<td>0059</td>
</tr>
<tr>
<td>3</td>
<td>00</td>
</tr>
</tbody>
</table>

### Section B

6. b) You spin a 1, 2, 3 or 4.  
   c) You spin a 2.  
7. a) 3 outcomes  
   b) 2 outcomes  
   c) 4 outcomes  
   d) 6 outcomes  
8. a) 13  
   b) 14  
   c) 25  
   d) 2  
   e) 3  
   f) 9  
   g) 13  
   h) 21  
9. a) \( \frac{2}{3} \)  
   b) I would expect \( \frac{1}{4} \) of the spins to be red.  
10. \( \frac{2}{3} \) of the spinner is Red, so \( \frac{2}{3} \) of 12 spins will be red. \( \frac{2}{3} \) of 12 is 8, so 8 times should produce red.  
11. a) Likely  
   b) Unlikely  
   c) Impossible  
   d) Certain  
12. a) Unlikely  
   b) Unlikely  
   c) Likely  
   d) Impossible  
13. Answers will vary. Sample: More than half a spinner is yellow, so it is likely to spin yellow.  
14. Answers will vary. Teacher to check.  
15. The game is not fair, Kevin has a better chance of winning, since 3 out of 8 possible outcomes are red, and only 2 are yellow.
La géométrie
Test sur l'unité

Section A

1. Trace des lignes sur la colonne et la rangée données. Encercle ensuite le point où les deux lignes se rencontrent :
   a) Colonne 1 Rangée 3
   b) Colonne 2 Rangée 3
   c) Colonne 1 Rangée 2
   d) Colonne 3 Rangée 1

2. Identifie la colonne et la rangée du point encerclé :
   a) Colonne _____ Rangée _____
   b) Colonne _____ Rangée _____
   c) Colonne _____ Rangée _____
   d) Colonne _____ Rangée _____

3. Fais glisser le point de …
   a) 3 unités vers le bas
   b) 5 unités vers la droite
   c) 6 unités vers la gauche; 4 unités vers le bas
   d) 3 unités vers la droite; 1 unité vers le haut

4. Fais glisser chaque forme de 5 cases vers la droite et de 2 cases vers le bas :
   a) 
   b) 

5. Encercle les illustrations qui ne représentent pas des réflexions :
   a) b) c) d)

   d) Comment sais-tu que les formes que tu as encerclées ne sont pas des réflexions?
Section A (suite)

6. Réponds aux questions suivantes en utilisant le système de coordonnées :

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>ville</th>
<th>vallée</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>lac</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>colline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>

a) Qu’est-ce que tu trouverais dans le carré (A,3)?

b) Qu’est-ce que tu trouverais si tu te déplaçais de 2 carrés vers l’ouest de la vallée?

c) Donne les coordonnées de la ville :

d) Décris comment aller de la ville au lac :

e) Décris comment aller de la colline à la ville :

7. Utilise les indices suivants pour trouver où sont assis tous les enfants :

- Avance de 2 pupitres vers le bas et de 1 pupitre vers la droite d’Eric pour trouver le pupitre de John.
- Samir est à 1 pupitre à la gauche d’Eric.
- Sally est entre Lars et Indra.
- Avance de 2 pupitres vers la droite et de 1 pupitre vers le haut du pupitre de Lars pour trouver le pupitre de Mary.
- Emma est à 2 pupitres en haut de Peter.
- Avance de 1 pupitre vers le haut et de 1 pupitre vers la gauche d’Anne pour trouver Janet.
Section A (suite)

8. Montre où serait la flèche ou la figure après chaque rotation :

a) ¼ de tour dans le sens des aiguilles

b) ½ tour dans le sens des aiguilles

c) ¾ de tour dans le sens des aiguilles

d) 1 tour complet dans le sens des aiguilles

e) ¼ de tour dans le sens inverse des aiguilles

f) ½ tour dans le sens inverse des aiguilles

g) ¾ de tour dans le sens inverse des aiguilles

h) 1 tour complet dans le sens inverse des aiguilles
Section B

9. Relie chaque forme à son nom :

- pyramide à base carrée
- cylindre
- prisme triangulaire
- cône
- prisme rectangulaire
- pyramide à base triangulaire

10. Remplis le tableau pour chaque forme :

<table>
<thead>
<tr>
<th>Forme</th>
<th>Nombre de faces</th>
<th>Nombre de sommets</th>
<th>Nombre d’arêtes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyramide pentagonale</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pyramide à base carrée</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. a) Remplis le tableau ci-dessous :

<table>
<thead>
<tr>
<th>Propriété</th>
<th>Pyramide pentagonale</th>
<th>Pyramide à base carrée</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nombre de faces</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nombre d’arêtes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nombre de sommets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nombre de bases</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forme de la base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forme des faces qui ne sont pas des bases</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Utilise ton travail dans la partie a) ci-dessus pour décrire comment les formes sont pareilles et comment elles sont différentes.
12. À partir de chaque liste ci-dessous, choisis une propriété qui s’applique aux formes en 3-D et utilise-la pour classer les formes ci-dessus :

Liste 1
Prismes
Pyramides
Ont 1 base
Ont 2 bases

Liste 2
A au moins 1 face triangulaire
A au moins 1 face rectangulaire
8 arêtes ou plus
6 sommets ou plus

<table>
<thead>
<tr>
<th>Propriété</th>
<th>Formes qui ont cette propriété</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
</tr>
</tbody>
</table>

13. Fais le lien entre la description de la forme et son nom :

- _____ cône  
  A. J’ai 6 faces congruentes.
- _____ prisme triangulaire  
  B. J’ai 5 faces : 2 triangles et 3 rectangles.
- _____ cube  
  C. J’ai 4 faces. Chaque face est un triangle.
- _____ cylindre  
  D. J’ai 2 bases circulaires et une face courbe.
- _____ pyramide à base triangulaire  
  E. J’ai 1 base circulaire et une face courbe.

a)

Ce développement est ______________________________

b)

Ce développement est ______________________________
Section A

1. a) 
   ![Diagram](image1.png)

   b) 
   ![Diagram](image2.png)

   c) 
   ![Diagram](image3.png)

   d) 
   ![Diagram](image4.png)

2. a) Column 1 Row 2

   b) Column 2 Row 3

   c) Column 3 Row 1

   d) Column 1 Row 1

3. a) 
   ![Diagram](image5.png)

   b) 
   ![Diagram](image6.png)

   c) 
   ![Diagram](image7.png)

   d) 
   ![Diagram](image8.png)

4. a) 
   ![Diagram](image9.png)

   b) 
   ![Diagram](image10.png)

5. a) Not circled

   b) Circled

   c) Circled

6. a) Lake

   b) Hill

   c) (D, 4)

   d) 1 square south, then 3 squares west (or in reverse order)

   e) 2 squares north, then 1 square east (or in reverse order)

7. | Emma | Samir | Eric | Mary |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Janet</td>
<td>Lars</td>
<td>Sally</td>
<td>Indra</td>
</tr>
<tr>
<td>Peter</td>
<td>Anne</td>
<td>Yen</td>
<td>John</td>
</tr>
</tbody>
</table>

8. a) 
   ![Diagram](image11.png)

   b) 
   ![Diagram](image12.png)

   c) 
   ![Diagram](image13.png)

   d) 
   ![Diagram](image14.png)

   e) 
   ![Diagram](image15.png)

   f) 
   ![Diagram](image16.png)

9. Shapes, from left to right:
   - Rectangular (or square) prism
   - Square pyramid
   - Cone
   - Cylinder
   - Triangular pyramid
   - Triangular prism

10. | Faces | Vertices | Edges |
    |------|---------|------|
    | 5    | 4       | 6    |
    | 4    | 8       | 6    |
    | 6    | 8       | 12   |
    | 5    | 6       | 9    |

11. a) PP

    SP

   b) Answers will vary. Description should include:
      - Same:
        - pyramids
        - both have 1 base,
        - non-base faces are triangles in both shapes
      - Different:
        - # of faces, vertices, edges
        - Shape of base
Section C

12. Answers will vary. Teacher to check.

13. E Cone
    B Triangular Prism
    A Cube
    D Cylinder
    C Triangular Pyramid

14. Pictures may vary.
    a) Triangular pyramid
    b) or Hexagonal prism
Contents

Number Sense and Numeration 3
Measurement 6
Geometry and Spatial Sense 8
Patterning and Algebra 10
Data Management and Probability 12
Notes

To ensure that the curriculum is fully covered, use the worksheets with the lessons plans in the Teacher’s Guide.

Starred lesson numbers (*) indicate that the curriculum requirement is covered primarily in the lesson plan (possibly in the activities or extensions).

OCUP: Ontario Curriculum Unit Planner

JUMP Math workbook units are represented by:

- NS  Number Sense
- PA  Patterns and Algebra
- ME  Measurement
- G   Geometry
- PDM Probability and Data Management
Number Sense and Numeration

Overall Expectations
By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m8</td>
<td>read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100;</td>
</tr>
<tr>
<td>4m9</td>
<td>demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts;</td>
</tr>
<tr>
<td>4m10</td>
<td>solve problems involving the addition, subtraction, multiplication, and division of single- and multi-digit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies;</td>
</tr>
<tr>
<td>4m11</td>
<td>demonstrate an understanding of proportional reasoning by investigating whole-number unit rates.</td>
</tr>
</tbody>
</table>

Quantity Relationships
By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------</td>
</tr>
<tr>
<td>4m12</td>
<td>represent, compare, and order whole numbers to 10 000, using a variety of tools;</td>
</tr>
<tr>
<td>4m13</td>
<td>demonstrate an understanding of place value in whole numbers and decimal numbers from 0.1 to 10 000, using a variety of tools and strategies;</td>
</tr>
<tr>
<td>4m14</td>
<td>read and print in words whole numbers to one thousand, using meaningful contexts;</td>
</tr>
<tr>
<td>4m15</td>
<td>found four-digit whole numbers to the nearest ten, hundred, and thousand, in problems arising from real-life situations;</td>
</tr>
<tr>
<td>4m16</td>
<td>represent, compare, and order decimal numbers to tenths, using a variety of tools and using standard decimal notation;</td>
</tr>
<tr>
<td>4m17</td>
<td>represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered;</td>
</tr>
<tr>
<td>4m18</td>
<td>compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional parts;</td>
</tr>
</tbody>
</table>
### Quantity Relationships (continued)

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Expectations</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>4m19 compare fractions to the benchmarks of 0, 1, 2, and 1;</td>
<td>2</td>
</tr>
<tr>
<td>4m20 demonstrate and explain the relationship between equivalent fractions, using concrete materials and drawings;</td>
<td>2</td>
</tr>
<tr>
<td>4m21 read and represent money amounts to $100;</td>
<td>2</td>
</tr>
<tr>
<td>4m22 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 10,000.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

### Counting

By the end of Grade 4, students will:

<table>
<thead>
<tr>
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<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Expectations</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>4m23 count forward by halves, thirds, fourths, and tenths to beyond one whole, using concrete materials and number lines;</td>
<td>2</td>
</tr>
<tr>
<td>4m24 count forward by tenths from any decimal number expressed to one decimal place, using concrete materials and number lines.</td>
<td>2</td>
</tr>
</tbody>
</table>

### Operational Sense

By the end of Grade 4, students will:

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Expectations</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>4m25 add and subtract two-digit numbers, using a variety of mental strategies;</td>
<td>1</td>
</tr>
<tr>
<td>4m26 solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard algorithms;</td>
<td>1</td>
</tr>
<tr>
<td>4m27 add and subtract decimal numbers to tenths, using concrete materials and student-generated algorithms;</td>
<td>2</td>
</tr>
</tbody>
</table>
## Operational Sense (continued)

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>4m28</td>
<td>add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools;</td>
</tr>
<tr>
<td>4m29</td>
<td>multiply to $9 \times 9$ and divide to $81 \div 9$, using a variety of mental strategies;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4m30</td>
<td>solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies;</td>
</tr>
<tr>
<td>4m31</td>
<td>multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4m32</td>
<td>multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools, student-generated algorithms, and standard algorithms;</td>
</tr>
<tr>
<td>4m33</td>
<td>divide two-digit whole numbers by one-digit whole numbers, using a variety of tools and student-generated algorithms;</td>
</tr>
<tr>
<td>4m34</td>
<td>use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution.</td>
</tr>
</tbody>
</table>

## Proportional Relationships

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>4m35</td>
<td>describe relationships that involve simple whole-number multiplication;</td>
</tr>
<tr>
<td>4m36</td>
<td>determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools and strategies;</td>
</tr>
<tr>
<td>4m37</td>
<td>demonstrate an understanding of simple multiplicative relationships involving unit rates, through investigation using concrete materials and drawings.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Measurement

## Overall Expectations

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m38</td>
<td>estimate, measure, and record length, perimeter, area, mass, capacity, volume, and elapsed time, using a variety of strategies;</td>
</tr>
<tr>
<td>4m39</td>
<td>determine the relationships among units and measurable attributes, including the area and perimeter of rectangles.</td>
</tr>
</tbody>
</table>

## Attributes, Units and Measurement Sense

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------</td>
</tr>
<tr>
<td>4m40</td>
<td>estimate, measure, and record length, height, and distance, using standard units (i.e., millimetre, centimetre, metre, kilometre);</td>
</tr>
<tr>
<td>4m41</td>
<td>draw items using a ruler, given specific lengths in millimetres or centimeters;</td>
</tr>
<tr>
<td>4m42</td>
<td>estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest minute;</td>
</tr>
<tr>
<td>4m43</td>
<td>estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in five-minute intervals, hours, days, weeks, months, or years;</td>
</tr>
<tr>
<td>4m44</td>
<td>estimate, measure using a variety of tools and strategies, and record the perimeter and area of polygons;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4m45</td>
<td>estimate, measure, and record the mass of objects, using the standard units of the kilogram and the gram;</td>
</tr>
<tr>
<td>4m46</td>
<td>estimate, measure, and record the capacity of containers, using the standard units of the litre and the millilitre;</td>
</tr>
<tr>
<td>4m47</td>
<td>estimate, measure using concrete materials, and record volume, and relate volume to the space taken up by an object.</td>
</tr>
</tbody>
</table>
### Measurement Relationships

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>4m48</td>
<td>describe, through investigation, the relationship between various units of length (i.e., millimetre, centimetre, decimetre, metre, kilometre);</td>
</tr>
<tr>
<td>4m49</td>
<td>select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure the side lengths and perimeters of various polygons;</td>
</tr>
<tr>
<td>4m50</td>
<td>determine, through investigation, the relationship between the side lengths of a rectangle and its perimeter and area;</td>
</tr>
<tr>
<td>4m51</td>
<td>pose and solve meaningful problems that require the ability to distinguish perimeter and area;</td>
</tr>
<tr>
<td>4m52</td>
<td>compare and order a collection of objects, using standard units of mass (i.e., gram, kilogram) and/or capacity (i.e., millilitre, litre);</td>
</tr>
<tr>
<td>4m53</td>
<td>determine, through investigation, the relationship between grams and kilograms;</td>
</tr>
<tr>
<td>4m54</td>
<td>determine, through investigation, the relationship between millilitres and litres;</td>
</tr>
<tr>
<td>4m55</td>
<td>select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram) and the most appropriate standard unit to measure the capacity of a container (i.e., millilitre, litre);</td>
</tr>
<tr>
<td>4m56</td>
<td>solve problems involving the relationship between years and decades, and between decades and centuries;</td>
</tr>
<tr>
<td>4m57</td>
<td>compare, using a variety of tools, two-dimensional shapes that have the same perimeter or the same area.</td>
</tr>
<tr>
<td>4m57</td>
<td></td>
</tr>
</tbody>
</table>
Geometry and Spatial Sense

Overall Expectations
By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m58</td>
<td>identify quadrilaterals and three-dimensional figures and classify them by their geometric properties, and compare various angles to benchmarks;</td>
</tr>
<tr>
<td>4m59</td>
<td>construct three-dimensional figures, using two-dimensional shapes;</td>
</tr>
<tr>
<td>4m60</td>
<td>identify and describe the location of an object, using a grid map, and reflect two-dimensional shapes.</td>
</tr>
</tbody>
</table>

Geometric Properties
By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------</td>
</tr>
<tr>
<td>4m61</td>
<td>draw the lines of symmetry of two-dimensional shapes, through investigation using a variety of tools and strategies;</td>
</tr>
<tr>
<td>4m62</td>
<td>identify and compare different types of quadrilaterals (i.e., rectangle, square, trapezoid, parallelogram, rhombus) and sort and classify them by their geometric properties;</td>
</tr>
<tr>
<td>4m63</td>
<td>identify benchmark angles (i.e., straight angle, right angle, half a right angle), using a reference tool, and compare other angles to these benchmarks;</td>
</tr>
<tr>
<td>4m64</td>
<td>relate the names of the benchmark angles to their measures in degrees;</td>
</tr>
<tr>
<td>4m65</td>
<td>identify and describe prisms and pyramids, and classify them by their geometric properties (i.e., shape of faces, number of edges, number of vertices), using concrete materials.</td>
</tr>
</tbody>
</table>
### Geometric Relationships

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part Unit Lesson</td>
</tr>
<tr>
<td>4m66 construct a three-dimensional figure from a picture or model of the figure, using connecting cubes;</td>
<td>2 G 38, 39</td>
</tr>
<tr>
<td>4m67 construct skeletons of three-dimensional figures, using a variety of tools, and sketch the skeletons;</td>
<td>2 G 30, 31</td>
</tr>
<tr>
<td>4m68 draw and describe nets of rectangular and triangular prisms;</td>
<td>2 G 36</td>
</tr>
<tr>
<td>4m69 construct prisms and pyramids from given nets;</td>
<td>2 G 36</td>
</tr>
<tr>
<td>4m70 construct three-dimensional figures, using only congruent shapes.</td>
<td>2 G 32</td>
</tr>
</tbody>
</table>

### Location and Movement

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part Unit Lesson</td>
</tr>
<tr>
<td>4m71 identify and describe the general location of an object using a grid system;</td>
<td>2 G 20–25</td>
</tr>
<tr>
<td>4m72 identify, perform, and describe reflections using a variety of tools;</td>
<td>2 G 26, 27</td>
</tr>
<tr>
<td>4m73 create and analyze symmetrical designs by reflecting a shape, or shapes, using a variety of tools, and identify the congruent shapes in the designs.</td>
<td>1 G 10 26, 40</td>
</tr>
<tr>
<td>4m73</td>
<td>2 G 26, 40</td>
</tr>
</tbody>
</table>
# Patterning and Algebra

## Overall Expectations

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m74</td>
<td>describe, extend, and create a variety of numeric and geometric patterns, make predictions related to the patterns, and investigate repeating patterns involving reflections;</td>
</tr>
<tr>
<td>4m75</td>
<td>demonstrate an understanding of equality between pairs of expressions, using addition, subtraction, and multiplication.</td>
</tr>
</tbody>
</table>

## Patterns and Relationships

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>4m76</td>
<td>extend, describe, and create repeating, growing, and shrinking number patterns;</td>
</tr>
<tr>
<td>4m77</td>
<td>connect each term in a growing or shrinking pattern with its term number, and record the patterns in a table of values that shows the term number and the term;</td>
</tr>
<tr>
<td>4m78</td>
<td>create a number pattern involving addition, subtraction, or multiplication, given a pattern rule expressed in words;</td>
</tr>
<tr>
<td>4m79</td>
<td>make predictions related to repeating geometric and numeric patterns;</td>
</tr>
<tr>
<td>4m80</td>
<td>extend and create repeating patterns that result from reflections, through investigation using a variety of tools.</td>
</tr>
</tbody>
</table>

## Expressions and Equality

By the end of Grade 4, students will:

<table>
<thead>
<tr>
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<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>4m81</td>
<td>determine, through investigation, the inverse relationship between multiplication and division;</td>
</tr>
<tr>
<td>4m82</td>
<td>determine the missing number in equations involving multiplication of one- and two-digit numbers, using a variety of tools and strategies;</td>
</tr>
<tr>
<td>4m83</td>
<td>identify, through investigation, and use the commutative property of multiplication to facilitate computation with whole numbers;</td>
</tr>
</tbody>
</table>
Expressions and Equality (continued)

By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>----------</td>
<td>----------------------</td>
</tr>
<tr>
<td>4m84</td>
<td>identify, through investigation, and use the distributive property of multiplication over addition to facilitate computation with whole numbers.</td>
</tr>
</tbody>
</table>
Data Management and Probability

Overall Expectations
By the end of Grade 4, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m85</td>
<td>collect and organize discrete primary data and display the data using charts and graphs, including stem-and-leaf plots and double bar graphs;</td>
</tr>
<tr>
<td>4m86</td>
<td>read, describe, and interpret primary data and secondary data presented in charts and graphs, including stem-and-leaf plots and double bar graphs;</td>
</tr>
<tr>
<td>4m87</td>
<td>predict the results of a simple probability experiment, then conduct the experiment and compare the prediction to the results.</td>
</tr>
</tbody>
</table>

Collection and Organization of Data
By the end of Grade 4, students will:

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>4m88 collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or the community, or content from another subject, and record observations or measurements;</td>
<td>1</td>
</tr>
<tr>
<td>4m89 collect and organize discrete primary data and display the data in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Data Relationships
By the end of Grade 4, students will:

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>4m90 read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs);</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>4m91 demonstrate, through investigation, an understanding of median, and determine the median of a set of data;</td>
<td>2</td>
</tr>
<tr>
<td>4m92 describe the shape of a set of data across its range of values, using charts, tables, and graphs;</td>
<td>2</td>
</tr>
<tr>
<td>4m93 compare similarities and differences between two related sets of data, using a variety of strategies.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
Probability
By the end of Grade 4, students will:

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part Unit Lesson</td>
</tr>
<tr>
<td>4m94 predict the frequency of an outcome in a simple probability experiment, explaining their reasoning; conduct the experiment; and compare the result with the prediction;</td>
<td>2 PDM 16, 18*, 22, 23</td>
</tr>
<tr>
<td>4m95 determine, through investigation, how the number of repetitions of a probability experiment can affect the conclusions drawn.</td>
<td>2 PDM 17*, 18*, 20</td>
</tr>
</tbody>
</table>
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>3</td>
</tr>
<tr>
<td>Patterns and Relations</td>
<td>10</td>
</tr>
<tr>
<td>Shape and Space</td>
<td>13</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>17</td>
</tr>
</tbody>
</table>
Notes

To ensure that the curriculum is fully covered, use the worksheets with the lessons plans in the Teacher’s Guide.

Starred lesson numbers (*) indicate that the curriculum requirement is covered primarily in the lesson plan (possibly in the activities or extensions).

Underlined lesson numbers indicate relevant preparatory exercises.

WNCP Abbreviations:

[C] Communication
[CN] Connections
[ME] Mental Mathematics and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

JUMP Math workbook units are represented by:

NS Number Sense
PA Patterns and Algebra
ME Measurement
G Geometry
PDM Probability and Data Management
# Number

## General Outcome

- Develop number sense.

## Develop Number Sense

It is expected that students will:

### 1. Specific Outcome

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>Part</td>
</tr>
<tr>
<td>Represent and describe whole numbers to 10 000, pictorially and symbolically. [C, CN, V]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Read a given four-digit numeral without using the word “and,” e.g., 5321 is five thousand three hundred twenty one, NOT five thousand three hundred AND twenty one.

- Write a given numeral using proper spacing without commas, e.g., 4567 or 4 567, 10 000.

- Write a given numeral 0 – 10 000 in words.

- Represent a given numeral using a place value chart or diagrams.

- Describe the meaning of each digit in a given numeral.

- Express a given numeral in expanded notation, e.g., 321 = 300 + 20 + 1.

- Write the numeral represented by a given expanded notation.

- Explain and show the meaning of each digit in a given 4-digit numeral with all digits the same, e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens and the fourth digit two ones.

### 2. Specific Outcome

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>Part</td>
</tr>
<tr>
<td>Compare and order numbers to 10 000. [C, CN]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Order a given set of numbers in ascending or descending order and explain the order by making references to place value.

- Create and order three different 4-digit numerals.

- Identify the missing numbers in an ordered sequence or on a number line.

- Identify incorrectly placed numbers in an ordered sequence or on a number line.
3. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:</td>
<td>1</td>
</tr>
<tr>
<td>• using personal strategies for adding and subtracting</td>
<td>2</td>
</tr>
<tr>
<td>• estimating sums and differences</td>
<td>1</td>
</tr>
<tr>
<td>• solving problems involving addition and subtraction. [C, CN, ME, PS, R]</td>
<td></td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Explain how to keep track of digits that have the same place value when adding numbers, limited to 3- and 4-digit numerals.
- Explain how to keep track of digits that have the same place value when subtracting numbers, limited to 3- and 4-digit numerals.
- Describe a situation in which an estimate rather than an exact answer is sufficient.
- Estimate sums and differences using different strategies, e.g., front-end estimation and compensation.
- Solve problems that involve addition and subtraction of more than 2 numbers.

4. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Explain the properties of 0 and 1 for multiplication, and the property of 1 for division. [C, CN, R]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Explain the property for determining the answer when multiplying numbers by one.
- Explain the property for determining the answer when multiplying numbers by zero.
- Explain the property for determining the answer when dividing numbers by one.
## 5. WNCP CURRICULUM

### Specific Outcome

Describe and apply mental mathematics strategies, such as:
- skip counting from a known fact
- using doubling or halving
- using doubling or halving and adding or subtracting one more group
- using patterns in the 9s facts
- using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R]

### Achievement Indicators

Provide examples for applying mental mathematics strategies:
- doubling, e.g., for 4 × 3, think 2 × 3 = 6, and 4 × 3 = 6 + 6
- doubling and adding one more group, e.g., for 3 × 7, think 2 × 7 = 14, and 14 + 7 = 21
- use ten facts when multiplying by 9, e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63
- halving, e.g., if 4 × 6 is equal to 24, then 2 × 6 is equal to 12
- relating division to multiplication, e.g., for 64 ÷ 8, think 8 × □ = 64.

### JUMP MATH LESSONS

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>PA</td>
<td>23–25</td>
</tr>
<tr>
<td>1</td>
<td>NS</td>
<td>27–29, 33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See the Mental Math section of the Guide.</td>
</tr>
</tbody>
</table>

## 6. WNCP CURRICULUM

### Specific Outcome

Demonstrate an understanding of multiplication (2 or 3-digit by 1-digit), to solve problems by:
- using personal strategies for multiplication with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products. [C, CN, ME, PS, R, V]

### Achievement Indicators

Model a given multiplication problem using the distributive property, e.g., \( 8 \times 365 = (8 \times 300) + (8 \times 60) + (8 \times 5) \).

Use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically.

Create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit.

### JUMP MATH LESSONS

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NS</td>
<td>30–36, 38 119, 120</td>
</tr>
<tr>
<td>2</td>
<td>NS</td>
<td>See the Mental Math section of the Guide.</td>
</tr>
</tbody>
</table>
6. **Achievement Indicators**

Estimate a product using a personal strategy, e.g., $2 \times 243$ is close to or a little more than $2 \times 200$, or close to or a little less than $2 \times 250$.

Model and solve a given multiplication problem using an array and record the process.

Solve a given multiplication problem and record the process.

---

7. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend), to solve problems by:</td>
<td>2</td>
</tr>
<tr>
<td>• using personal strategies for dividing with and without concrete materials</td>
<td></td>
</tr>
<tr>
<td>• estimating quotients</td>
<td></td>
</tr>
<tr>
<td>• relating division to multiplication.</td>
<td></td>
</tr>
<tr>
<td><em>It is not intended that remainders be expressed as decimals or fractions.</em></td>
<td></td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Solve a given division problem without a remainder using arrays or base ten materials.

Solve a given division problem with a remainder using arrays or base ten materials.

Solve a given division problem using a personal strategy and record the process.

Create and solve a word problem involving a 1- or 2-digit dividend.

Estimate a quotient using a personal strategy, e.g., $86 \div 4$ is close to $80 \div 4$ or close to $80 \div 5$.

---

8. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:</td>
<td>2</td>
</tr>
<tr>
<td>• name and record fractions for the parts of a whole or a set</td>
<td></td>
</tr>
<tr>
<td>• compare and order fractions</td>
<td></td>
</tr>
<tr>
<td>• model and explain that for different wholes, two identical fractions may not represent the same quantity</td>
<td></td>
</tr>
<tr>
<td>• provide examples of where fractions are used. [C, CN, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>
8. **Achievement Indicators**

- Represent a given fraction using concrete materials.
- Identify a fraction from its given concrete representation.
- Name and record the shaded and non-shaded parts of a given set.
- Name and record the shaded and non-shaded parts of a given whole.
- Represent a given fraction pictorially by shading parts of a given set.
- Represent a given fraction pictorially by shading parts of a given whole.
- Explain how denominators can be used to compare two given unit fractions with numerator 1.
- Order a given set of fractions that have the same numerator and explain the ordering.
- Order a given set of fractions that have the same denominator and explain the ordering.
- Identify which of the benchmarks 0, 1 2 or 1 is closer to a given fraction.
- Name fractions between two given benchmarks on a number line.
- Order a given set of fractions by placing them on a number line with given benchmarks.
- Provide examples of when two identical fractions may not represent the same quantity, e.g., half of a large apple is not equivalent to half of a small apple; half of ten cloudberries is not equivalent to half of sixteen cloudberries.
- Provide an example of a fraction that represents part of a set and a fraction that represents part of a whole from everyday contexts.

9. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WNCP Curriculum</strong></td>
<td><strong>JUMP MATH LESSONS</strong></td>
</tr>
<tr>
<td>Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically. [C, CN, R, V]</td>
<td>2 NS 92–95, 99–105, 116 (omit decimals greater than one)</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relate decimals to fractions (to hundredths). [CN, R, V]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Explain the meaning of each digit in a given decimal with all digits the same.
- Represent a given decimal using money values (dimes and pennies).
- Record a given money value using decimals.
- Provide examples of everyday contexts in which tenths and hundredths are used.
- Model, using manipulatives or pictures, that a given tenth can be expressed as hundredths, e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies.

- Read decimals as fractions, e.g., 0.5 is zero and five tenths.
- Express orally and in written form a given decimal in fractional form.
- Express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal.
- Express a given pictorial or concrete representation as a fraction or decimal, e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or 15/100.
- Express orally and in written form the decimal equivalent for a given fraction, e.g., 50/100 be expressed as 0.50.
### WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:</td>
<td>1</td>
</tr>
<tr>
<td>• using compatible numbers</td>
<td>2</td>
</tr>
<tr>
<td>• estimating sums and differences</td>
<td></td>
</tr>
<tr>
<td>• using mental math strategies to solve problems. [C, ME, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>

#### Achievement Indicators

- Predict sums and differences of decimals using estimation strategies.
- Solve problems, including money problems, which involve addition and subtraction of decimals, limited to hundredths.
- Determine the approximate solution of a given problem not requiring an exact answer.
- Estimate a sum or difference using compatible numbers.
- Count back change for a given purchase.
## Patterns and Relations

### General Outcomes
- Patterns: Use patterns to describe the world and solve problems.
- Variables and Equations: Represent algebraic expressions in multiple ways.

### Patterns
It is expected that students will:

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>Part</td>
</tr>
<tr>
<td>Identify and describe patterns found in tables and charts, including a multiplication chart. [C, CN, PS, V]</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Achievement Indicators</td>
<td></td>
</tr>
<tr>
<td>Identify and describe a variety of patterns in a multiplication chart.</td>
<td></td>
</tr>
<tr>
<td>Determine the missing element(s) in a given table or chart.</td>
<td></td>
</tr>
<tr>
<td>Identify error(s) in a given table or chart.</td>
<td></td>
</tr>
<tr>
<td>Describe the pattern found in a given table or chart.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>Part</td>
</tr>
<tr>
<td>Reproduce a pattern shown in a table or chart using concrete materials. [C, CN, V]</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Achievement Indicators</td>
<td></td>
</tr>
<tr>
<td>Create a concrete representation of a given pattern displayed in a table or chart.</td>
<td></td>
</tr>
<tr>
<td>Explain why the same relationship exists between the pattern in a table and its concrete representation.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>Part</td>
</tr>
<tr>
<td>Represent and describe patterns and relationships using charts and tables to solve problems. [C, CN, PS, R, V]</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Achievement Indicators</td>
<td></td>
</tr>
<tr>
<td>Extend patterns found in a table or chart to solve a given problem.</td>
<td></td>
</tr>
</tbody>
</table>
3. **Achievement Indicators**
   Translate the information provided in a given problem into a table or chart.
   Identify and extend the patterns in a table or chart to solve a given problem.

4. **WNCP CURRICULUM**
<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Identify and explain mathematical relationships using charts and diagrams to solve problems. [CN, PS, R, V]</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

4. **Variables and Equations**
   It is expected that students will:

5. **WNCP CURRICULUM**
<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Express a given problem as an equation in which a symbol is used to represent an unknown number. [CN, PS, R]</td>
<td>2</td>
</tr>
</tbody>
</table>

   **Achievement Indicators**
   Explain the purpose of the symbol, such as a triangle or circle, in a given addition, subtraction, multiplication or division equation with one unknown, e.g. $36 \div \square = 6$
5. **Achievement Indicators**

Express a given pictorial or concrete representation of an equation in symbolic form.

Identify the unknown in a story problem, represent the problem with an equation and solve the problem concretely, pictorially or symbolically.

Create a problem in context for a given equation with one unknown.

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Outcome</td>
<td>Part</td>
</tr>
<tr>
<td>Solve one-step equations involving a variable to represent an unknown number. [C, CN, PS, R, V]</td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Solve a given one-step equation using manipulatives.

Solve a given one-step equation using guess and test.

Describe, orally, the meaning of a given one-step equation with one unknown.

Solve a given equation when the unknown is on the left or right side of the equation.

Represent and solve a given addition or subtraction problem involving a “part-part-whole” or comparison context using a symbol to represent the unknown.

Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.
Shape and Space

**General Outcomes**
- Measurement: Use direct or indirect measurement to solve problems.
- 3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations: Describe and analyze position and motion.

**Measurement**
It is expected that students will:

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcome</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>Read and record time using digital and analog clocks, including 24-hour clocks. [C, CN, V]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**
- State the number of hours in a day.
- Express the time orally and numerically from a 12-hour analog clock.
- Express the time orally and numerically from a 24-hour analog clock.
- Express the time orally and numerically from a 12-hour digital clock.
- Describe time orally and numerically from a 24-hour digital clock.
- Describe time orally as “minutes to” or “minutes after” the hour.
- Explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM.

<table>
<thead>
<tr>
<th>WNCP CURRICULUM</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcome</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>Read and record calendar dates in a variety of formats. [C, V]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**
- Write dates in a variety of formats, e.g., yyyy/mm/dd, dd/mm/yyyy, March 21, 2006, dd/mm/yy.
- Relate dates written in the format yyyy/mm/dd to dates on a calendar.
- Identify possible interpretations of a given date, e.g., 06/03/04.
### WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of area of regular and irregular 2-D shapes by:</td>
<td>Part</td>
</tr>
<tr>
<td>• recognizing that area is measured in square units</td>
<td>1</td>
</tr>
<tr>
<td>• selecting and justifying referents for the units cm$^2$ or m$^2$</td>
<td>2</td>
</tr>
<tr>
<td>• estimating area by using referents for cm$^2$ or m$^2$</td>
<td></td>
</tr>
<tr>
<td>• determining and recording area (cm$^2$ or m$^2$)</td>
<td></td>
</tr>
<tr>
<td>• constructing different rectangles for a given area (cm$^2$ or m$^2$) in order to demonstrate that many different rectangles may have the same area. [C, CN, ME, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>

#### Achievement Indicators

- Describe area as the measure of surface recorded in square units.
- Identify and explain why the square is the most efficient unit for measuring area.
- Provide a referent for a square centimetre and explain the choice.
- Provide a referent for a square metre and explain the choice.
- Determine which standard square unit is represented by a given referent.
- Estimate the area of a given 2-D shape using personal referents.
- Determine the area of a regular 2-D shape and explain the strategy.
- Determine the area of an irregular 2-D shape and explain the strategy.
- Construct a rectangle for a given area.
- Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

### 3-D Objects and 2-D Shapes

It is expected that students will:

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe and construct rectangular and triangular prisms. [C, CN, R, V]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

#### Achievement Indicators

- Identify and name common attributes of rectangular prisms from given sets of rectangular prisms.
4. **Achievement Indicators**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify and name common attributes of triangular prisms from given sets of</td>
<td></td>
</tr>
<tr>
<td>triangular prisms.</td>
<td></td>
</tr>
<tr>
<td>Sort a given set of rectangular and triangular prisms using the shape of the</td>
<td></td>
</tr>
<tr>
<td>base.</td>
<td></td>
</tr>
<tr>
<td>Construct and describe a model of rectangular and triangular prisms using</td>
<td></td>
</tr>
<tr>
<td>materials, such as pattern blocks or modelling clay.</td>
<td></td>
</tr>
<tr>
<td>Construct rectangular prisms from their nets.</td>
<td></td>
</tr>
<tr>
<td>Construct triangular prisms from their nets.</td>
<td></td>
</tr>
<tr>
<td>Identify examples of rectangular and triangular prisms found in the environment.</td>
<td></td>
</tr>
</tbody>
</table>

5. **Transformations**

It is expected that students will:

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of line symmetry by:</td>
<td></td>
</tr>
<tr>
<td>• identifying symmetrical 2-D shapes</td>
<td></td>
</tr>
<tr>
<td>• creating symmetrical 2-D shapes</td>
<td></td>
</tr>
<tr>
<td>• drawing one or more lines of symmetry in a 2-D shape.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, V]</td>
<td></td>
</tr>
<tr>
<td>(in 16 and 17, students sort shapes according to various properties, including</td>
<td></td>
</tr>
<tr>
<td>symmetry)</td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Identify the characteristics of given symmetrical and non-symmetrical 2-D</td>
<td></td>
</tr>
<tr>
<td>shapes.</td>
<td></td>
</tr>
<tr>
<td>Sort a given set of 2-D shapes as symmetrical and non-symmetrical.</td>
<td></td>
</tr>
<tr>
<td>Complete a symmetrical 2-D shape given half the shape and its line of symmetry.</td>
<td></td>
</tr>
<tr>
<td>Identify lines of symmetry of a given set of 2-D shapes and explain why each</td>
<td></td>
</tr>
<tr>
<td>shape is symmetrical.</td>
<td></td>
</tr>
<tr>
<td>Determine whether or not a given 2-D shape is symmetrical by using a Mira or</td>
<td></td>
</tr>
<tr>
<td>by folding and superimposing.</td>
<td></td>
</tr>
<tr>
<td>Create a symmetrical shape with and without manipulatives.</td>
<td></td>
</tr>
<tr>
<td>Provide examples of symmetrical shapes found in the environment and identify</td>
<td></td>
</tr>
<tr>
<td>the line(s) of symmetry.</td>
<td></td>
</tr>
</tbody>
</table>
5. **Achievement Indicators**

Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry or more than one line of symmetry.
Statistics and Probability

General Outcomes
• Data Analysis: Collect, display and analyze data to solve problems.

Data Analysis
It is expected that students will:

1. **WNCP CURRICULUM**
   **Specific Outcome**
   Demonstrate an understanding of many-to-one correspondence. [C, R, T, V]
   **Achievement Indicators**
   - Compare graphs in which different intervals or correspondences are used and explain why the interval or correspondence was used.
   - Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.
   - Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.
   - Find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines and the Internet, and describe the correspondence used.

2. **WNCP CURRICULUM**
   **Specific Outcome**
   Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]
   **Achievement Indicators**
   - Identify an interval and correspondence for displaying a given set of data in a graph and justify the choice.
   - Create and label (with categories, title and legend) a pictograph to display a given set of data using many-to-one correspondence, and justify the choice of correspondence used.
   - Create and label (with axes and title) a bar graph to display a given set of data using many-to one correspondence, and justify the choice of interval used.
   - Answer a given question using a given graph in which data is displayed using many-to-one correspondence.