Teacher Resources: Grade 5

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PA5-24
Finding Rules for T-tables—Part 1

Make a simple design from pattern blocks, like the one in the picture.

Ask your students: How many pentagons did I use? How many triangles? How many triangles and how many pentagons will I need for two such designs? For three designs? Remind your students that they previously used T-tables to solve this type of question. Invite volunteers to draw a T-table and to continue it to 5 rows. **ASK:** I want to make 20 such designs. Should I continue the table to check how many pentagons and triangles I need? Can you find a more efficient way to find the number of pentagons and triangles? How many triangles are needed for one pentagon in the design? What do you do to the number of pentagons to find the number of triangles in a set of designs?

Ask volunteers how to derive the number of triangles in a particular row of the T-table from the number of pentagons. Students should see that the procedure they follow to find the number of triangles can be expressed as a general rule: “The number of triangles is 5 times the number of pentagons”. Mathematicians often use letters instead of numbers to express this type of rule. For example, they would use “p” for the number of pentagons and “t” for the number of triangles, and get the rule “5 × p = t”. This rule is called a “formula” or “equation”. Write these terms on the board beside the formula itself.

Write another formula, such as 4 × s = t. Explain that “s” represents the number of squares and “t” is the number of triangles as before. What rule does the formula express? What do you have to do to the number of squares to get the number of triangles? You can make a table of values that matches this formula. All you have to do is to write the actual number of squares instead of “s” and do the multiplication. The result of the multiplication is the number of triangles. Draw a T-table:

<table>
<thead>
<tr>
<th>Number of Squares (s)</th>
<th>Formula (4 × s = t)</th>
<th>Number of Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 × 1 = 4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4 × 2 = 8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4 × 3 =</td>
<td>4</td>
</tr>
</tbody>
</table>

Ask volunteers to fill in the missing numbers and to add two more lines to the table. Ask them to find the number of triangles for 25 figures (25 squares). Let your students practice drawing T-tables for more formulas, like: 3 × t = s, 6 × s = t (t – number of triangles, s – number of squares).

Ask your students to create designs to go with the formulas above.
Students could make a design using concrete materials (as in Questions 3 and 4 in the worksheets) and predict how many of each element in their design they would need to make 8 copies.

**Extension**

Find the rules by which the following T-tables were made.

<table>
<thead>
<tr>
<th>a) Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b) Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c) Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>750</td>
</tr>
</tbody>
</table>

**PA5-25**

**Finding Rules for T-Tables—Part II**

*GOALS*

Students will find simple additive, multiplicative or subtractive rules for T-tables.

*PRIOR KNOWLEDGE REQUIRED*

Increasing sequences

*VOCABULARY*

T-table formula input output

Start with a simple problem: Rose invites some friends to a party. She needs one chair for each friend and one for herself. Can you provide Rose with a formula or equation for the number of chairs?

Ask your students to suggest a letter to use for the number of friends and a letter for the number of chairs. Given the number of friends how do you find the number of chairs? Ask your students to write a formula for the number of chairs. Suggest that your students make a T-table similar to the one they used in the last lesson for multiplicative rules.

Give your students several questions to practice writing rules and making T-tables, such as:

a) Lily and Rose invite some friends to a party. How many chairs do they need?

b) Rose, Lily and Pria invite some friends to a party. How many chairs they need? They invited 20 friends. How many chairs will they need?

c) A family invited several friends to a party. The number of chairs they need is \(6 + f = c\). How many people are in this family? If they invited 10 friends, how many chairs will they need?

Ask your students to write a problem for the formula \(4 + f = c\).

Explain to your students that the number that you put into a formula in place of a letter is often called the “input”. The result that the formula provides—
the number of chairs, for instance—is called the “output”. Write these terms on the board and ask volunteers to circle the input and underline the output in the formulas you have written on the board.

Draw several T-tables on the board, provide a rule for each and ask your students to fill in the tables. Start with simple inputs like 1, 2, 3 or 5, 6, 7 and continue to more complicated combinations like 6, 10, 14 and so on.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

The rules you provide should be additive (Add 4 to the input), multiplicative (Multiply the input by 5) and subtractive (Subtract 3 from the input).

Suggest that your students try a more complicated task—you write a table and they have to produce a rule for it. Ask them to think what was done to the input to get the output. Give them several simple tables, like:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

**Assessment**

1. Complete the tables:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td></td>
</tr>
</tbody>
</table>

Add 13 to the input

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Multiply the input by 11

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td></td>
</tr>
</tbody>
</table>

Subtract 5 from the input

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>21</td>
<td>28</td>
</tr>
<tr>
<td>23</td>
<td>30</td>
</tr>
</tbody>
</table>

2. Find the rules and the formulas (or equations) for the tables:
**Bonus**

Find the rule, the formula and the output for input = 7 and input = 10.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111</td>
</tr>
<tr>
<td>2</td>
<td>222</td>
</tr>
<tr>
<td>3</td>
<td>333</td>
</tr>
</tbody>
</table>

**ACTIVITY 2**

A Game for Two

Each pair of students will need a die and a spinner as in the previous activity. Player 1 spins a spinner and rolls the die as before, but so that Player 2 does not see the result. Player 1 writes the rule and the formula given by the spinner and the die and gives the other player 3 pairs of input and output numbers. Player 2 has to guess the formula.
PA5-26
Finding Rules for Patterns—Part I

Draw the following sequence of figures on the board and tell your students that the pictures show several stages in construction of a castle made of blocks.

How will they keep track of the number of blocks needed for the castle? Invite volunteers to make a T-table with two columns—the number of triangles and the number of squares.

Ask your students to find a verbal rule and a formula that tell how to get the number of squares from the number of triangles. Could they predict the formula before building a T-table? There are three squares for every triangle. So there are three times more squares than triangles, and we can write that as “Multiply the number of triangles by 3” or “3 × t = s”.

Repeat with the next block pattern:

Let your students practice with various patterns with multiplicative or additive rules.

Assessment

Make a T-table and write a formula and a rule for the patterns:

a) (use s for shaded squares and u for the un-shaded squares)

b) (use r for rhombuses and t for trapezoids)
Extension

A dragon has 44 teeth and 4 poisonous spikes on its tail. A dragon breeder uses 2 formulas: 
\[ t = 44 \times d, \quad s = 4 \times d. \] 
What does each formula describe? Can you write a formula that relates the quantities \( t \) and \( s \), and does not involve \( d \)? **HINT:** Make a T-table with columns: dragons, spikes, teeth. Students should derive the formula \( t = 11 \times s \). They should also recognize that not all inputs make sense. A dragon can’t have only one spike, for instance. Since a dragon has 4 spikes on its tail, only inputs that are multiples of 4 make sense.

PA5-27
Direct Variation

Show your students several sequences made of blocks with a multiplicative rule, such as:

\[
\begin{array}{c|c|c}
\text{Figure Number (f)} & \text{Number of Squares (s)} & \text{Number of Triangles (t)} \\
\hline
1 & 1 & 1 \\
2 & 2 & 2 \\
3 & 3 & 3 \\
\end{array}
\]

Invite volunteers to draw T-tables for the number of triangles and the number of squares:

<table>
<thead>
<tr>
<th>Figure Number (f)</th>
<th>Number of Squares (s)</th>
<th>Number of Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>s</td>
<td>t</td>
</tr>
</tbody>
</table>

Ask your students to write a rule and a formula for each table. Remind your students the meaning of the terms “input” and “output”.

Explain to your students that when the rule is “Multiply the input by ____”, we say that the output varies directly with the input. So in this pattern the number of squares varies directly with the Figure Number, and the Number of Triangles varies directly with both the Figure Number and the Number of Squares.

Draw several tables with columns “Input” and “Output” and ask your students to give examples of rules that will make the output vary directly with the input. (Multiply the input by ____) Invite volunteers to fill in the tables according to the rules. Vary the input from 1, 2, 3 to combinations like 5, 6, 7 and then to 4, 6, 8. Next pass to more complicated input combinations, such as 12, 8, 4 and even 2, 22, 7. After that write several rules such as “Multiply the input by 22”, “Add 7”, “Multiply by 5 and subtract 3” and ask your students to tell...
which rules make the output vary directly with the input.

Draw several tables on the board and ask your students to tell in which of these tables the output is a direct variation of the input. **EXAMPLES:**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>121</td>
</tr>
<tr>
<td>30</td>
<td>330</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>7</td>
<td>23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>121</td>
</tr>
<tr>
<td>30</td>
<td>330</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>11</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>11</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>11</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>

**PROMPTS:** What do you do to the input to get the output, when the output varies directly with the input? Is the output in the table a multiple of the input? (Check each row separately) What number did you multiply the input by in each row? Suggest that students write these numbers beside the table in the form if a multiplication statement. For instance, in the last table students could write:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>48</td>
</tr>
<tr>
<td>11</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>

6 × 8 = 48
11 × 8 = 88
4 × 8 = 32

The numbers in bold (the second factors) are the same, so the output varies directly with the input.

Draw a sequence of squares with sides 1, 2, 3, etc. Ask your students to find the areas and the perimeters of the squares. Ask them to make a T-table for both and to check which quantity varies directly with the side length (perimeter). Encourage them to write a formula not only for the perimeter, but also for the area of the square.

Present a problem:

The number of feet (f) varies directly with the number of people (p) (2 people—4 feet, 3 people—6 feet, 4 people—8 feet, f = 2 × p). Does the number of paws vary directly with the number of cats? What is the formula?

A cat has five claws on each front paw and four claws on each back paw. Make a T-table showing the number of cats and the number of claws and then another T-table showing the number of paws (add one paw at a time!) and the number of claws. Does the total number of claws vary directly with the number of paws or with the number of cats?
Assessment

Circle the tables where the input varies directly with the output:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

Extension

Find a sequence of rectangles so that the area will vary directly with length. Does the perimeter vary directly with the length?

For sequences where the number of blocks does not vary directly, ask students if they can figure out a rule for determining the number of blocks in a figure from the figure number. (A method for finding such rules is taught in the next three sections, but you might challenge your students to figure out how to derive such rules themselves. As a hint, you might tell the students that, in the case where the number of blocks does not vary directly with the figure number, their rule will involve multiplication and addition.)
PA5-28

Finding Rules for Patterns—Part II

Draw the following sequence of figures on the board and tell your students that the pictures show several stages in construction of a castle made of blocks.

How could they find a formula to keep track of the number of blocks needed for the castle? The castle has two towers and a gate between them. The gate does not change from figure to figure, but the towers grow. Ask your students to find the rule for the number of blocks in the towers (2 × Figure Number). They can use a T-table to find the rule if needed. After that ask them to find a T-table for the total number of blocks in the figures. In which T-table does the number of blocks in the output column vary directly with the figure number—the T-table showing the number of blocks in the towers or the one showing the total number of blocks?

Give your students several more block patterns, shade the part that varies directly with the figure number, as in QUESTION 1 of the worksheet, and ask them to find the rule for the number of shaded blocks.

Ask your students to create a sequence of figures that goes with the table and to shade the part of the figures that varies directly with the figure number. They should leave the part that does not change from figure to figure unshaded.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

Extension

A cab charges a $3.50 flat rate (that you pay just for using the cab) and $2 for every minute of the ride. Write a formula for the price of a cab ride. How much will you pay for a 4-minute cab ride? For a 5-minute ride?
PA5-29
Predicting the Gap Between Terms in a Pattern

Give your students a pair of dice of different colors each and ask them to try the following game. Roll the dice, and write a sequence according to the rule: to find each term multiply the term number by the result on the red die and add the result on the blue die. Ask your students to find the difference between the terms of their sequences each time. After they have created several sequences, ask them what they have noticed. Likely, the students have noticed that the difference (gap) in the sequence equals the result of the red die, which is the multiplicative factor.

Draw or make the following sequence:

![Figure 1](image1.png)  ![Figure 2](image2.png)  ![Figure 3](image3.png)

Ask students to describe what part of the pattern changes and what part stays the same? Draw a T-table for the number of blocks in each figure of the sequence as shown.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Ask students to predict what the gap between the terms in the output column (the “Number of Blocks” column) will be before you fill in the column. Students should see that the “gap” between terms in the T-table is simply the number of new blocks added to the pattern at each stage. **ASK:** What do you add to each figure to get the next one? How many blocks do you add? Students should also see that to find the number of shaded blocks in a particular figure they simply multiply the figure number by the “gap” (since the gap is the number of new shaded blocks added each time). In the next lesson, students will learn how to use this insight to find rules for patterns made by multiplication and addition (or multiplication and subtraction).

A game for pairs: The students will need pattern blocks and two dice of different colors. Player 1 rolls the dice so that player 2 does not see the result. He builds a pattern of figures according to the rule: The number of blocks = Figure number × Result on the red die + Result on the blue die. Player 2 has to guess what the results on the dice were.
PA5-30
Finding Rules for T-tables—Part III

Explain to your students that today they will continue to find rules for patterns but their task will be harder—you will not show them a particular block pattern, you will only give them the number of blocks used to build the pattern. Remind your students that the patterns they have worked with usually consisted of two parts: the part that stays the same and the part that varies with the number of the figure.

Draw the following sequence of figures on the board:


definition

Ask a volunteer to shade the part of each figure that varies directly with the figure number (the squares). Write the following T-table for the pattern on the board and ask students to fill in the gap between the numbers in the output column.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Figure Number × Gap</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

Then ask students to fill in the middle column of the table:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Figure Number × Gap</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 × 3 = 3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2 × 3 = 6</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3 × 3 = 9</td>
<td>11</td>
</tr>
</tbody>
</table>

Ask your students what the numbers in the middle column of the T-table correspond to in the pattern. Students should see that the “gap” is simply the number of squares that are added at each stage in the pattern. So to find the number of squares in each figure, you simply multiply the gap by the figure number. (In other words, the numbers in the middle column—3, 6, 9—simply gave the number of squares in the successive figures.)
Students should also see that to find the total number of blocks (the number in the output column), they just need to add an adjustment factor (the number of triangles) to the number in the middle column.

Hence the rule for the T-table is: Multiply the figure number by 3 (the gap) and add 2 (the adjustment factor). Students should notice that the adjustment factor corresponds to the number of triangles in each figure: this number is constant and does not change from figure to figure.

Give your students practice at finding rules for various T-tables.

**Example 1**

**STEP 1:** Multiply the figure number by 3 (the gap) and add 2 (the adjustment factor).

**STEP 2:** What must you add to each number in the second column to get the output? In the T-table above, the fixed amount of increase (gap) is 3. **NOTICE:** Multiplying the input 2 by 3 gives 6, one less than the output 7. Multiplying the input 3 by 3 gives 9, one less than the output 10. Multiplying the input 4 by 3 gives 12, one less than the output 13, etc.

**STEP 3:** The rule is: Multiply by 3 and add 1.

**Example 2**

**STEP 1:** Multiply the gap by the input.

**STEP 2:** What must you add to each number in the second column to get the output? In the T-table above, the fixed amount of increase (gap) is 4. **NOTICE:** Multiplying the input 2 by 4 gives 8, one more than the output 7. Multiplying the input 3 by 4 gives 12, one more than the output 11, etc.
STEP 4: The rule is: Multiply by 4 and subtract 1. If the input does not increase by 1 and the output does not increase by a fixed amount, your student should find the rule for the T-table by guessing and checking.

NOTE: It is extremely important that your students learn to find rules for T-tables by the method outlined above as all of the applications of T-tables in this unit involve finding such rules.

Let your students practice finding the rules for several patterns. In the first examples tell them whether you add or subtract something to get the output from the multiple input × gap, then use mixed questions. You may also use the activity below for practice.

After that you may draw (or build) several more complicated block patterns and ask the students to find the rules for them. Give them more practice questions, like:

Find the rules for the perimeter and the area of the following figures. Use your rules to predict the perimeter and the area of term 15.

For ASSESSMENT, you may ask students how many inner line segments the 20th figure has in this pattern.

For a more advanced activity, draw several block patterns like the one shown below (or use pattern blocks for each sequence) and ask your students to make a T-table for the number of blocks of each type, and for the total number of blocks in each figure.

Ask your students to find a rule for each T-table and to describe the parts of the pattern that change and the parts that stay the same.

Bonus

Can you draw or build a sequence of figures where the number of blocks in each figure is given by the rule

a) Number of Triangles = Figure Number + 3
b) Number of Squares = 3 × Figure Number – 2
c) Number of Squares = 2 × Figure Number – 1
A Game for Pairs

Each pair of students will need a coin with the signs + and – on its sides and two dice. Player 1 throws the dice and the coin so that Player 2 does not see the results. Suppose that the larger number rolled is L and the smaller is S. Player 1 writes the first three terms of the sequence of numbers according to the rule: The number of blocks = Figure number \(\times L\) \(+/-\) S. The coin defines the sign. Player 2 has to guess the results of the dice roll.

ADVANCED: Instead of the coin, students may use the spinner from ACTIVITY 1 of PA5-25: Finding Rules for T-tables. If the spinner reads “Add” or “Subtract”, the rule is as before. If the spinner says “Multiply”, the player multiplies the Figure Number (or Input) by the product of the two numbers rolled, so that the sequence rule will be multiplicative only.

Once students master the first two games, they may give each other 4 random terms (two of them adjacent, like 2, 3) of the sequence, as in the table below, rather than giving the first three terms.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
</tbody>
</table>

Extension

Can you draw or build a sequence of figures using triangles, squares and trapezoids where the number of each type of block will vary according to all of these rules?

- Number of triangles: Figure number + 2
- Number of squares: \(2 \times\) Figure number – 1
- Number of trapezoids: \(4 \times\) Figure number – 4

Write a formula that tells you how to find the total number of blocks in a figure from the figure number.
Let your students play a game in pairs. They will need a pair of dice and a spinner from Activity 1 of PA5-25: Finding Rules for T-tables. Player 1 rolls the dice and spins the spinner so that Player 2 does not see the result of the spinner. He adds, subtracts or multiplies the numbers rolled according to the instructions on the spinner, and presents both numbers and the result of the operation to Player 2. Player 2 has to tell what the spinner read. For example, if the dice give 2 and 6, and the spinner reads “Subtract”, Player 1 has to write: 6 \(\times\) 2 = 4, and Player 2 has to deduce that the sign is “–”.

After that you may write several sequences made by multiplication and ask your students to continue them. In the beginning tell them what the factor is:

- a) \(\times 2\) 5, 10, _, _,
- b) \(\times 4\) 2, 8, _, _,
- c) \(\times 3\) 3, 9, _, _,

Then tell your students that you are going to make the task harder: they will have to find the factor. Let them practice with questions like:

- a) \(\times\) 15, 30, 60, _, _,
- b) \(\times\) 2, 10, 50, _, _,
- c) \(\times\) 3, 12, 48, _, _

Explain to your students that since they are doing very well, you are sure they will succeed in the next task. You will give them several sequences made using one operation: addition, multiplication or subtraction. You will not tell them which operation was used, what number you added or subtracted from the term, or what number you multiplied the term by. They will have to continue the sequence and write a rule for each sequence. Remind your students of the way they write the rules for sequences: Start at ___ and ___. Discuss with the class what strategies can be used to decide which operation and numbers were used. ASK: If the sequence was made by addition, is it increasing or decreasing? By subtraction? By multiplication? If the first two numbers in the sequence are 2 and 4, how can I tell whether the sequence was made by addition or multiplication? What should I check? The students might reason as follows:

Look at the difference between the first two numbers. Check if the sequence is increasing or decreasing. If it is decreasing, subtraction was used and I have to check the difference between the numbers. If the sequence is increasing, it is either addition or multiplication, so I should
look at the difference between the first and the second terms, then at the difference between the second and the third terms. If the differences are the same, the sequence was made by addition. If not, it was made by multiplication.

Assessment
Continue the sequences and write the rule for each sequence.

\[
\begin{align*}
\text{a)} & \quad 15, \ 30, \ 45, \ \underline{\phantom{15}}, \ \underline{\phantom{30}} \\
\text{b)} & \quad 52, \ 39, \ 26, \ \underline{\phantom{52}}, \ \underline{\phantom{39}} \\
\text{c)} & \quad 3, \ 17, \ 31, \ \underline{\phantom{3}}, \ \underline{\phantom{17}}
\end{align*}
\]

Extensions
1. The patterns below were made by multiplying successive terms by a fixed number and then adding or subtracting a fixed number. Find the missing terms and state the rule for making the pattern. Include the word term in your answer. (For instance, the rule for the first pattern below is “Start at 1. Multiply each term by 2 and add 1.”)

\[
\begin{align*}
\text{a)} & \quad 1, \ 3, \ 7, \ 15, \ 31, \ \underline{\phantom{1}} \\
\text{b)} & \quad 1, \ 4, \ 13, \ 40, \ \underline{\phantom{1}} \\
\text{c)} & \quad 2, \ 7, \ 22, \ 67, \ \underline{\phantom{2}} \\
\text{d)} & \quad 2, \ 3, \ 5, \ 9, \ \underline{\phantom{2}} \\
\text{e)} & \quad 1, \ 2, \ 5, \ 14, \ \underline{\phantom{1}}
\end{align*}
\]

2. Put a dot in the centre of a polygon and draw a line from the centre to each vertex of the polygon. How many line segments are there in each figure? Predict how many line segments there would be in a hexagon and an octagon. Test your prediction.
PA5-32
Patterns with Increasing and Decreasing Steps

Draw the following pattern on the board or build it with blocks:

Ask a volunteer to build the next term of the sequence. Ask your students to fill the T-table. Then ASK: How many blocks are added each time? What are you adding to the structure? Another row. How many blocks are in the first row that you added? In the second row? This is the difference in the total number of blocks at each stage. Ask a volunteer to write the difference in the circles beside the table.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students if they can see a pattern in the differences. Ask a volunteer to add a term to the pattern of differences. After that ask another volunteer to fill in the next row of the table. Ask another volunteer to build another figure to check the result.

Give your students several questions to practice:

Find the differences between the terms of the sequences. Extend the sequence of the differences and then extend the sequence itself.

a) 5, 8, 12, 17, ___, ___

b) 3, 5, 11, 17, 25, ___, ___

c) 11, 15, 23, 39, ___, ___

d) 6, 8, 13, 21, 32, ___, ___

Let them also practice with decreasing sequences:

a) 65, 64, 62, 59, ___, ___

b) 73, 70, 64, 55, 43, ___, ___
Let your students build growing patterns of pattern blocks. For each sequence of patterns, find out how many blocks you need using a T-table.

**ACTIVITY**

Show the following geometrical pattern and ask how many triangles will be in the next design:

![Geometrical Pattern]

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Draw the T-table for the pattern. How many triangles do you add each time? We add a new row that is two triangles longer than the previous one. The new row is the difference. If your students have problems extending the sequence of differences, you can draw a second row of circles beside the first one to see the differences between the circles. Ask the volunteers to extend first the sequence of differences, then the sequence itself.

**Assessment**

a) 15, 18, 22, 27, ____, ____

b) 13, 16, 22, 31, 43, ____, ____

c) 101, 95, 87, 77, ____, ____

d) 88, 85, 78, 67, 52, ____, ____

**Extensions**

1. Janet is training for a marathon. On Monday she ran 5 km. Every day she ran 1 km more than on the previous day. How many kilometres did she run in the whole week?

   Draw the T-table. The first few entries should appear as follows:

   ![T-table for Janet's training]

   Each day Janet runs 1 km more.
2. The pattern that is created in **QUESTION 6 b)** of the worksheet is called The Sierpinski Triangle. If you take a part of this pattern and magnify it, the picture will look exactly the same as the original. A picture with this property is called a fractal. Here is another example of a fractal, called the H-fractal.

Count the line segments at each stage of the construction. Make a T-table and try to predict the total number of line segments at the 7th step.

**STEP 1:** Draw a horizontal line of 12 cm.

**STEP 2:** Draw two vertical lines of 8 cm each, as shown.

**STEP 3:** Draw four horizontal lines of length $\frac{1}{2}$ of the previous horizontal line (6 cm here), so that the middle of each segment is at the end of the previous vertical segments.

**STEP 4:** Draw eight vertical lines of length $\frac{1}{3}$ of the previous vertical line (4 cm here), so that the middle of each segment is at the end of the previous horizontal segments.

After 10 steps the picture will look as shown below.

3. A restaurant has tables shaped like trapezoids. Two people can sit along the longest side of a table, but only one person can sit along each shorter side:

a) Draw a picture to show how many people could sit at 4 and 5 tables. Then, fill in the T-table.

<table>
<thead>
<tr>
<th>Number of Tables</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Describe the pattern in the number of people. How does the step change?

c) Extend the pattern to find out how many people could sit at 8 tables.
PA5-33
Creating and Extending Patterns (Advanced)

Draw the following tree on the board and ask your students to count the number of branches in each level. Write the number of branches beside each level. Ask your students, “What happens to each branch?” It splits in two. This means that the number of branches doubles—multiplies by two—each time. So this sequence was made by multiplication.

Ask your students to find the sequence of differences between the numbers of branches in each level (1, 2, 4, 8…). What do they notice? They should see that the differences between terms in the sequence are the same as the numbers in the sequence.

Ask your students, “Which sequence grows faster—the one made by addition or the one made by multiplication?”

**FOR EXAMPLE:**

1, 11, 21, 31, … What is the rule for this sequence?

1, 2, 4, 8, … What is the rule for this sequence?

Start at ____ and multiply by ____.

You may call a vote: Which is larger—the 10th term of the first sequence or the 10th term of the second sequence? Invite a volunteer to make a tally chart of the vote results. Have volunteers extend both sequences to the 10th term.

If students are engaged, repeat the activity with the second sequence above and the sequence 1, 101, 201… Check the 10th and the 11th terms of both sequences.

Show another sequence: 3, 8, 6, 11, 9, 14, 12, 17, 15…

You might write the sequence in this form…

3  6  9  12  15

8  11  14  17

… and ask students to describe the patterns they see.

Ask students to continue the sequence and to explain the rule by which it was made. Here are some possible answers:

1. The top row is the sequence 3, 6, 9, 12… The rule is “Start at three and add three each time”. The bottom row is 8, 11, 14, 17, and the rule is “Start at eight and add three each time”.

**GOALS**
Students will extend increasing and decreasing sequences using addition, subtraction and multiplication and alphabet patterns.

**PRIOR KNOWLEDGE REQUIRED**
Increasing and decreasing sequences
Skip counting
T-tables
Alphabet

**VOCABULARY**
increasing sequence
decreasing sequence
difference
2. The rule for the whole pattern (looking at all terms) is “Start at three and add five or subtract two alternatively”.

**ANOTHER EXAMPLE:** 101, 97, 98, 94, 95, 91, 92, ...

Provide students with a copy of the alphabet and suggest that they extend several sequences based on the alphabet:

B, D, F, H, ... A, Z, B, Y, C, X, ...

CZ, DZ, EY, FY, GX, ... A, G, M, S, ...

**Assessment**

Continue the sequences:

2, 4, 8, 14, 22, 32, ____ , ____

2, 6, 18, 54, ____ , ____

B, f, J, n, R, ... Ab, De, Gh, Jk, ...

**Bonus**

A mighty and courageous but somewhat short-sighted knight assaulted a wicked 12-headed dragon. When the knight hews off a dragon head, two new heads spring in place of the hewn one. The knight can only hew one head in a minute. Our knight has ingloriously left the battle field after 15 minutes of battle. How many heads does the monster have now?

**Extensions**

1. “The Legend of the Chess Board.” The same doubling sequence is used in the beginning of the lesson.

   **POSSIBLE SOURCE:**
   
   http://britton.disted.camosun.bc.ca/jbchessgrain.htm

2. Write the differences for the patterns. Identify the rule for the sequence of differences. Extend first the sequence of differences, then the sequence itself.

   a) 7, 10, 14, 19
   b) 12, 15, 20, 27
   c) 57, 54, 50, 45
   d) 32, 30, 26, 20
   e) 15, 18, 22, 25, 29
   f) 77, 72, 69, 64, 61

**A Game for Pairs**

Player 1 writes a sequence, and Player 2 has to continue three next terms of it. Each correct term gives them one point as a team. If Player 2 has problems guessing the terms, Player 1 has to give him a hint (not a new term!), such as, “Look at the differences” or “I mixed two sequences.” Each hint takes one point from the team.
PA5-34
Patterns with Larger Numbers

Tell your students that they have done so well with patterns that today you are going to give them patterns with HUGE numbers.

Present several problems and call for volunteers to solve them, using T-tables.

A normal heartbeat rate is 72 times in a minute. How many times will your heart beat in five minutes?

There are 60 minutes in an hour. George sleeps for six hours. How many minutes does he sleep? He asks his friend to wake him after 500 minutes. About how many hours is this? (Skip count to find out.)

A sprinting ostrich’s stride is 700 cm long. A publicity-loving ostrich spots a photographer 3000 cm away and runs towards him. How far from the camera will it be after three strides?

Find the number of rhombuses in each of the following stars:

HINT: Count the number of rhombuses between the pair of heavy black lines and then multiply.

Assessment
An extinct elephant bird weighed about 499 kg. Make a T-table to show how much five birds would weigh. Do you see a pattern in the numbers (look at ones, tens and hundreds separately)? Can you write the weights of six, seven, and eight birds without actually adding?

Extensions
1. A regular year is 365 days long, a leap year (2000, 2004…) is 366 days long. Tom was born on Jan. 8, 2000. How many days old was he on January 8, 2003? January 8, 2005? When was he 2000 days old? (Finding the year, the month and the day are three different problems of increasing difficulty.)
2. Use the following pattern to figure out what $9 \times 999$ would be:

$2 \times 999$  $3 \times 999$  $4 \times 999$

(from the Atlantic Curriculum)

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**PA5-35**

**Introduction to Algebra**

Tell your students that today you will solve algebraic equations. Let them know that equations are like the scales people use to weigh objects (such as apples). But there’s a problem: A black box prevents you from seeing part of whatever object you are weighing. Solving an equation means figuring out what is inside the black box. Write the word “equation” on the board. Ask your students if they know any similar words (EXAMPLES: equal, equality, equivalence). So the word “equation” means “making the same,” or in other words, balancing the scales.

Draw a line down the middle of a desk. Put 7 apples on one side of the line and put 3 apples and a bag or box containing 4 more apples on the other side. Tell your students that there are the same number of apples on both sides of the line. (If you have a scale and a set of objects of equal weight, you might put the objects on the pans of the scale.) Students should be able to deduce that there are 4 apples in the box. They should also see that they can find the number of hidden apples either by counting up from 3 to 7, or by subtracting 3 from 7. Challenge students with several more examples.

Tell your students that it is easy to represent the problem they just solved with a picture:

```
+   +   =
 3   7
```

Invite a student to draw the missing apples in the box.

After students have had practice with this sort of problem, explain that it is inconvenient to draw the apples all the time. So people use numbers to represent the hidden quantities. Let them practice writing an equation that represents a picture, as on the worksheet. For instance, the equation for the picture above is:

```
+ 3 = 7
```

Students can create models for equations that involve addition using blocks. A square could stand for the unknown and a set of circles could be used to model the numbers in the equation.
For instance the equation:

\[
\square + 2 = 7
\]

has the model:

\[
\square \circ \circ \circ = \circ \circ \circ \circ \circ \circ \circ \circ \circ
\]

If the students think of the square as having a particular weight, then solving the equation becomes equivalent to finding the weight of the square in terms of the circles.

The square has the same weight as five circles.

How many circles are needed to balance the square?

You can find the answer by removing all the circles from the left hand scale and an equivalent number of circles from the other side.

Ask students to make a model with squares and circles to solve the following problems. (They should draw a picture of their model with a balance scale as in the figures above.) Then they should explain how many circles they would remove to find the weight of the square.

\[
\begin{align*}
7 + \square & = 11 \\
6 + \square & = 13 \\
4 + \square & = 10 \\
9 + \square & = 12
\end{align*}
\]

Write an equation and draw the picture:

\[
\circ \circ + \square = \circ \circ \circ \circ \circ \circ \circ \circ
\]

Ask if the picture makes sense. Why not? (You are a mathematician, not a magician. You cannot turn apples into bananas.) When you draw pictures like the ones students have worked with so far, the objects on both sides of the equation have to be the same. However, when you use numbers in an equation—rather than pictures—the numbers don’t always stand for the same thing.

For instance, the equation \(10 = 3 + \square\) could represent the problem: “There are ten pets in a store. Three are cats. How many are dogs?” Here the number in the box and the number outside the box on the right side of the equation stand for exactly different things. However, in all equations, the amounts on both sides of the equal sign will be in the same category; for instance the animals in the question above are all pets.

Explain that the square is often replaced by a letter, say “n”, to signify that this is “the unknown”. Ask students to make a model with squares and circles to solve the following problems. (They should draw a picture of their model with a balance scale as in the figures above.) Then they should explain how many circles they would remove to find the weight of the square.

a) \(n + 2 = 8\)  
b) \(n + 3 = 10\)  
c) \(n + 5 = 9\)
Ask students to translate the following pictures into equations using the letter “n” as the unknown.

a)  

b) 

Point out that the choice of the letter is arbitrary—both equations $n + 3 = 5$ and $a + 3 = 5$ have the same model ($\square\square\square = \square\square\square\square\square$) and the same solution ($n$ or $a = 2$).

Read several word problems. Invite volunteers to draw models, write equations using squares and numbers, and solve the equations:

There are ten trees in the garden. Three of them are apple trees. All the rest are cherry trees. How many cherry trees are in the garden?

Jane has 12 books. Three of them are fairy tales. All the rest are ghost stories. How many of her books are ghost stories?

Assessment
1. Solve the equations:
   
   $5 + \square = 11$  
   $8 + \square = 15$  
   $3 + n = 13$  
   $7 + a = 13$

2. Write an equation to solve the problem:

   There are 15 flowers in the flower-bed. Six are lilies. All the rest are peonies. How many peonies grow in the flower-bed?

Math Bingo—Addition only

Your students will each need a playing board (See: the BLM) and 16 tokens to mark the numbers. The teacher reads the card out loud. Players have to figure out the answer and then to place a token on the board, if they have the answer. The first player to fill a column, row or diagonal wins.

Bonus
1. There were 150 bloodthirsty pirates on a galleon and a schooner. 40 of them are on the schooner. How many pirates are on the galleon?

2. A multi-headed dragon has 15 heads. Some of them were cut off by a mighty and courageous knight. The dragon ran away from the knight with seven remaining heads. How many heads were hewn off by the knight?
Extensions

1. The same symbol in the equation means the same number. Solve the equations:
   
a) \[ \square + \square = 12 \]  
b) \[ 5 + \diamond + \diamond = 13 \]  
c) \[ \circ + \circ + \circ = 9 \]  
d) \[ 9 + \diamond + \diamond + \diamond = 15 \]  

Remind your students that in the last lesson they drew models and wrote equations for word problems. The equations they drew all had a + sign. Today they will learn to write other kinds of equations.

Present a word problem: Sindi has a box of apples. She took two apples from the box and four were left. How many apples were in the box before she removed the apples?

Draw the box. There are some apples inside, but we do not know how many. Draw two apples and cross them to show that they are taken away. Four apples were left in the box, so draw them too. How many were there from the beginning? (Six).

Explain that when we write an equation, we draw it differently. We draw the apples that we took out, outside the box with the “–” sign, to show that they were taken away:

\[ \square \square = \circ \circ \rightarrow \circ \circ \circ \circ \circ \circ \circ = \circ \circ \circ \circ \circ \circ \]  

So to solve the equation we have to put all the apples into the box—the ones that we took out and the ones that are outside.

Remind your students that they also learned to write equations in number form. Can they guess what the equation will look like:

\[ \square - 2 = 4? \]
Draw several models like the ones on the worksheets, and ask your students to write the equations for them. Ask volunteers to present the answers on the board.

Then ask your students to draw models for the equations and to solve them by drawing the original number of apples in the box:

- \[ -6 = 9 \]
- \[ -7 = 12 \]
- \[ -5 = 3 \]
- \[ -3 = 10 \]

Remind your students how they could use letters in equations and ask them to rewrite all the equations with letters.

Tell your students that they can also write equations for multiplication problems. Remind them that “2 ×” means that some quantity is taken two times.

**FOR EXAMPLE:**

\[ 2 \times \circ \circ \circ = \circ \circ \circ \circ \]

Present the problem below and ask your students to draw the appropriate number of circles in the box:

\[ 2 \times \bigcirc = \bigcirc \bigcirc \bigcirc \bigcirc \]

Present more problems and ask your students to write and solve numerical equations for these problems. Encourage them to use letters instead of squares. Then show your students how to write an equation for a word problem involving multiplication:

Tony has four boxes of pears. Each box holds the same number of pears. He has 12 pears in total. How many pears are in each box?

The students might reason in the following way: we usually represent the thing that we do not know by a square (the “unknown”). So four times the “unknown” constitutes 12, and we have the equation:

\[ 4 \times \square = 12. \]

**PRACTICE:**

Three identical evil dragons have 15 heads together. How many heads does each dragon have?

Five heffalumps have 10 tails. How many tails does each heffalump have?

A cat has four paws. 16 cat paws are scratching in the basket. How many kittens are inside?

Here is a mixture of problems students could try involving addition, subtraction, or multiplication:

Katie has several dogs. They have 20 paws together. How many dogs does Katie have?

Jenny used three eggs to bake muffins. Seven eggs remained in the carton. How many eggs were in the carton?

Bob has nine pets. Three of them are snakes. All the rest are iguanas. How many iguanas does Bob have?
Assessment
1. Solve the equations:
   \[3 + \square = 8\]
   \[3 \times \square = 15\]
   \[\square - 4 = 11\]
   \[2 \times \square = 14\]

2. Draw models to solve the problems:
   Hamide has 12 stamps. Four of them are Canadian. How many foreign stamps does she have?
   Joe has 15 stamps. Five of them are French and the rest are German. How many German stamps does Joe have?
   Marylyn has 18 stamps. They come in three identical sheets. How many stamps are in each sheet?

Bonus
Solve the equations:
\[243 + \square = 248\]
\[8 \times \square = 56\]
\[\square - 4 = 461\]
\[60 \times \square = 240\]

Math Bingo
(SEE: the BLM “Mixed Equations”): Your students will need a board each (SEE: the BLM) and 16 tokens to mark the numbers. The teacher reads the card out loud. Players have to figure out the answer and then to place a token on the board, if they have the answer. The first player to fill a column, row or diagonal wins.

Extensions
1. Ask students to translate each story problem below into an equation using a letter to stand for the unknown, rather than a box. They should also model problem a) with squares and circles.
   a) Carl has seven stickers. He has two more stickers than John. How many stickers does John have?

   **SOLUTION:** Let \(n\) stand for the number of stickers that John has. Carl has two more stickers, so you would have to add two to the number of stickers John has. So the correct equation is: \(n + 2 = 7\)

   b) Katie has ten stickers. She has three fewer stickers than Laura.

   **SOLUTION:** Let \(n\) be the number of stickers that Laura has. Katie has three fewer stickers than Laura so you need to subtract three from the number of stickers that Laura has. So the correct equation is: \(n - 3 = 10\)

2. Two birds laid the same number of eggs. Seven eggs hatched, and three did not. How many eggs did each bird lay?

3. 60 little crocodiles hatched from three crocodile nests of the same size. We know that only half of the eggs hatched. How many eggs were in each nest? (HINT: How many eggs were laid in total?)
Explain to your students that today they will learn to write equations the way mathematicians do it. In mathematics people usually use letters instead of drawing a square or a diamond for the unknown. Remind your students that they already used letters in formulas for patterns—for example, ask your students to tell how many triangles will be needed to make broaches using the design shown.

1 broach: 1 × 5 triangles
2 broaches: 2 × 5 triangles
7 broaches: ______

Remind your students that in previous lessons they used letters for the number of pentagons or triangles in a set of broaches.

Give them several problems, like the ones below, and ask them to write a formula or equation for the problem:

A boat travels at a speed of 10 km per hour. What distance will it cover in 2 hours? 5 hours? In h hours?
A house has 12 windows. How many windows do 3 houses have? 7 houses? X houses?

Ask a volunteer to write a formula/equation for a T-table:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Change the headings of the T-table to A and B and ask a volunteer to write the formula for the new table. Explain to your students, that even if the names of the columns change, the rule for the T-table will still have the same form. Previously the rule was Number of Blocks = Figure Number + 4, now it is B = A + 4.

Let your students practice finding rules for T-tables with letter headings. Start with simple rules, involving one operation and continue to more complicated rules like B = 2 × A + 3.

Explain to your students that when letters are used in an equation, the multiplication sign “×” is often omitted to avoid confusion with the letter X and to make the notation shorter. In this case instead of writing 2 × A people simply write 2A. Ask your students to rewrite the equations that they have written without the multiplication sign. After that give them several word problems, such as:
Ricardo made 12 sandwiches. Four of them are avocado sandwiches, and the rest are cheese sandwiches. How many cheese sandwiches does Ricardo have?

Ask your students to draw models for the problems, write equations with squares and rewrite the equations with a variable. Eventually students should find it easy to write the equation directly with a variable.

PA5-38
Equations

Give your students a mixed set of equations (like the ones on the worksheet) and ask them to solve the problems using guess and check. Tell them that sometimes they might see a way to solve the problem without guessing—for example, with a model. In this case they should draw the model and to solve the problem that way.

\[
\begin{align*}
a) \quad \square \times 6 &= 42 \\
b) \quad + 6 &= 8 \\
c) \quad \div 5 &= 10 \\
d) \quad - 4 &= 9 \\
e) \quad 9 - \square &= 7 \\
f) \quad + 2 &= 10 \\
g) \quad 17 - \square &= 14 \\
h) \quad 8 - \square &= 5 \\
i) \quad 2 \times \square &= 14 \\
j) \quad 20 + \square &= 4 \\
k) \quad 3 \times \square &= 24 \\
l) \quad \div = 9
\end{align*}
\]

Ask your students to rewrite some of the equations using letter variables. Present the following problem: There are several hats and gloves in a box. All left-hand gloves have a matching right-hand glove, and there are nine objects in the box. How many gloves and how many hats are there?

Draw an equation on the board. Say that the circle will represent the number of left-hand gloves. Gloves come in pairs, so there will be two circles—one for the right hands and one for the left hands. The square will represent the number of hats.

\[
\square + \square + \square = 9
\]

Ask volunteers to try to fit some numbers into the equation. Suggest starting with the circle. Draw a T-table of the results:

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>Rewrite the Equation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 + 1 + \square = 9</td>
<td>2 + \square = 9</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>2 + 2 + \square = 9</td>
<td>4 + \square = 9</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3 + 3 + \square = 9</td>
<td>6 + \square = 9</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4 + 4 + \square = 9</td>
<td>8 + \square = 9</td>
<td>1</td>
</tr>
</tbody>
</table>

Use as many volunteers to help filling the table as possible. **ASK:** Why did we have to stop after the 4th row?

Ask your students to write down the sequences in the third and the fourth columns and to give the rules for the sequences.
Present several problems such as the problem below and ask your students to write the equations using letter variables and to solve the equations.

Timur threw 3 darts and scored 12 points. The dart in the centre is worth twice as much as the dart on the outside. How much is each dart worth?

**SOLUTION:** Let \( n \) be the value of a dart on the outside. The dart in the centre is worth twice as much the dart on the outside, so its worth is \( 2 \times n \), or \( 2n \). The total value of the darts is \( n + n + 2n = 12 \), or \( 4n = 12 \).

**Assessment**
1. Ask your students to solve several equations such as:
   a) \( \square \times 6 = 48 \)
   b) \( \square + 6 = 28 \)
   c) \( \square ÷ 5 = 6 \)
   d) \( \square - 4 = 16 \)
   e) \( \square + \square + \bigcirc + \bigcirc = 10 \)
2. Sindi had 12 cookies. She shared 8 with her friends and ate the rest. How many cookies did she eat?

**Extensions**
1. In the magic trick below, the magician can always predict the result of a sequence of operations performed on any chosen number. Try the trick with students, then encourage them to figure out how it works using a block to stand in for the mystery number (give lots of hints).

<table>
<thead>
<tr>
<th>The Trick</th>
<th>The Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick any number</td>
<td>( \square )</td>
</tr>
<tr>
<td>Add 4</td>
<td>( \square \bigcirc \bigcirc \bigcirc \bigcirc )</td>
</tr>
<tr>
<td>Multiply by 2</td>
<td>( \square \bigcirc \bigcirc \bigcirc \bigcirc )</td>
</tr>
<tr>
<td>Subtract 2</td>
<td>( \square \bigcirc \bigcirc \bigcirc )</td>
</tr>
<tr>
<td>Divide by 2</td>
<td>( \square \bigcirc \bigcirc \bigcirc )</td>
</tr>
<tr>
<td>Subtract the mystery number</td>
<td>( \bigcirc \bigcirc \bigcirc )</td>
</tr>
</tbody>
</table>

The answer is 3!
No matter what number you choose, after performing the operations in the magic trick, you will always get the number 3. The model above shows why the trick works.

Encourage students to make up their own trick of the same type.

2. Give your students a copy of a times table. Ask them to write an equation that would allow them to find the numbers in a particular column of the times table given the row number. For instance, to find any number in the 5s column of the times table you multiply the row number by five: Each number in the 5s column is given by the algebraic expression $5 \times n$ where $n$ is the row number. Ask students to write an algebraic expression for the numbers in a given row.

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**PA5-39**

**Problems and Puzzles**

**PA5-39** is a review worksheet, which can be used for practice.

**Extension**

Consecutive numbers are numbers that follow each other on the number line. You can sum a set of consecutive numbers quickly, by grouping the numbers as follows:

**STEP 1:** Add the first and last number, the second and the second to last number and so on. What do you notice?

$$1 + 2 + 3 + 4 + 5 + 6 = 7 + 7 + 7$$

**STEP 2:** Rewrite the addition statement as a multiplication statement.

$$3 \times 7 = 21$$

Add the following sets of numbers by grouping pairs that add to the same number and multiplying that number by the number of pairs.

a) $$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$$

b) $$12 + 13 + 14 + 15 + 16 + 17 =$$
PA5 Part 2: BLM List

Math Bingo Game_______________________________________________________ 2
Math Bingo Game

Sample Boards

1  2  13  5
6  7  19  4
11 3  16 15
14 10  8 12

1  12  17  14
6  15  4  11
9  2  13  8
16 5  10  3

9  5  14  4
6  15  18 13
10 7  16  3
1  11  2  12

2  9  13 17
10 14  5  8
6  3  16 12
15 11  7  4

1  12  13  5
6  7  19  20
11 8  16 15
14 10  18 17

1  20  17  14
8  6  4  11
9  2  13  18
16 15  20  3

9  5  14  4
6  15  18 13
10 7  16  3
20 11  2  12

2  9  3  17
10 14  5  8
6  20  16 12
15 1  7  4

20  13  12  5
16  17  19  4
11  3  6  15
14 10  8  2

1  12  17  4
6  15  14 11
9  7  13  8
20 5  10  3

9  5  14 20
6  15  8  13
10 17  16  3
11 19  2  12

2  19  13 17
10 14  15  8
6  3  16  1
5  11  7  4
Math Bingo Game (continued)

Cards (Addition Only)

| 7 + □ = 11 | 6 + □ = 13 | 5 + □ = 10 | 9 + □ = 12 |
| □ + 9 = 11 | □ + 12 = 13 | □ + 4 = 10 | □ + 4 = 12 |
| 7 + □ = 16 | 6 + □ = 16 | 4 + □ = 15 | 5 + □ = 17 |
| □ + 7 = 21 | □ + 6 = 19 | □ + 3 = 18 | □ + 3 = 19 |
| 5 + □ = 22 | 4 + □ = 22 | 4 + □ = 23 | 2 + □ = 22 |

Cards (Mixed Equations)

| 7 + □ = 21 | 6 + □ = 13 | 5 + □ = 24 | 9 + □ = 20 |
| □ − 9 = 11 | □ − 1 = 11 | □ − 4 = 22 | □ − 4 = 16 |
| 7 × □ = 21 | 6 × □ = 30 | 4 × □ = 16 | 3 × □ = 18 |
| □ + 7 = 15 | □ + 6 = 21 | □ − 4 = 13 | □ − 5 = 11 |
| 9 × □ = 18 | 4 × □ = 36 | 8 × □ = 80 | 21 + □ = 22 |
ask your students if they have ever been given a fraction of something (like food) instead of the whole, and gather their responses. Bring a banana (or some easily broken piece of food) to class. Break it in two very unequal pieces. say: this is one of two pieces. Is this half the banana? Why not? Emphasize that the parts have to be equal for either of the two pieces to be a half.

draw numerous examples of shapes with one of two parts shaded, some that are equal and some that are not, and ask volunteers to mark the diagrams as correct or incorrect representations of one half.

ask: Which diagram illustrates one-fourth? What’s wrong with the other diagram? Isn’t one of the four pieces still shaded?

A sport played by witches and wizards on brooms regulates that the players must fly higher than 5 m above the ground over certain parts of the field (shown as shaded). Over what fraction of the field must the players fly higher than 5 m²?

explain that it’s not just shapes like circles and squares and triangles that can be divided into fractions, but anything that can be divided into equal parts. draw a line and ask if a line can be divided into equal parts. ask a volunteer to guess where the line would be divided in half. then ask the class to suggest a way of checking how close the volunteer’s guess is. Have a volunteer measure the length of each part. Is one part longer? How much longer? challenge students to discover a way to check that the two halves
are equal without using a ruler, only a pencil and paper. [On a separate sheet of paper, mark the length of one side of the divided line. Compare that length with the other side of the divided line by sliding the paper over. Are they the same length?]

Have students draw lines in their notebooks and then ask a partner to guess where the line would be divided in half. They can then check their partner’s work.

ASK: What fraction of this line is double?

SAY: The double line is one part of the line. How many equal parts are in the whole line, including the double line? [5, so the double line is \( \frac{1}{5} \) of the whole line.]

Mark the length of the double line on a separate sheet of paper. Compare that length to the entire line to determine how many of those lengths make up the whole line. Repeat with more examples.

Then ask students to express the fraction of shaded squares in each of the following rectangles.

![Rectangles with shaded squares]

Have them compare the top and bottom rows of rectangles.

ASK: Are the same fraction of the rectangles shaded in both rows? Explain. If you were given the rectangles without square divisions, how would you determine the shaded fraction? What could be used to mark the parts of the rectangle? What if you didn’t have a ruler? Have them work as partners to solve the problem. Suggest that they mark the length of one square unit on a separate sheet of paper, and then use that length to mark additional square units.

Prepare several strips of paper with one end shaded, and have students determine the shaded fraction without using a pencil or ruler. Only allow them to fold the paper.

Draw several rectangles with shaded decimetres.

![Rectangles with shaded decimetres]

Have students divide the rectangles into the respective number of equal parts (3, 4 and 5) with only a pencil and paper. Have them identify the fractions verbally and orally. Then draw a 50 cm rectangle with two shaded decimetres (i.e. two-fifths), and challenge your students to mark units half the length of the shaded decimetres. How can this be done with only a pencil and paper?

Fold the paper so that these markings meet. Draw a marking along the fold.
Draw a shaded square and ask students to extend it so the shaded part becomes half the size of the extended rectangle.

Repeat this exercise for squares becoming one-third and one-quarter the size of extended rectangles. ASK: How many equal parts are needed? [Three for one-third, four for one-quarter.] How many parts do you already have? [1.] So how many more equal parts are needed? [Two for one-third, three for one-quarter.]

**Bonus**
Extend the squares so that \( \frac{2}{5} \) of them are shaded. Repeat this exercise for \( \frac{1}{7}, \frac{3}{7}, \) etc.

Give your students rulers and ask them to solve the following puzzles.

- a) Draw a line 1 cm long. If the line represents \( \frac{1}{6} \) show what a whole line looks like.
- b) Line: 1 cm long. The line represents \( \frac{1}{3} \). Show the whole.
- c) Line: 2 cm long. The line represents \( \frac{1}{4} \). Show the whole.
- d) Line: 3 cm long. The line represents \( \frac{1}{2} \). Show the whole.
- e) Line: 1 \( \frac{1}{2} \) cm long. The line represents \( \frac{1}{4} \). Show the whole.
- f) Line: 1 \( \frac{1}{2} \) cm long. The line represents \( \frac{1}{3} \). Show the whole.
- g) Line: 3 cm long. The line represents \( \frac{1}{4} \). Show \( \frac{1}{2} \).
- h) Line: 2 cm long. The line represents \( \frac{1}{4} \). Show \( \frac{1}{4} \).

This is a good activity to do at the end of a day, so that students with extra time can play with the leftover play dough until the end of class.

Prepare enough small balls of coloured play dough for 3 for each student (they will only need two, but this allows students to choose their colours). Demonstrate to students how to make fractions using play dough. Tell them that you are going to roll one spoon of red play dough and three spoons of blue play dough into a ball. Explain the necessity of flattening the play dough on a spoon so that each spoonful is the same size. Demonstrate not leaving any play dough of the first colour on the spoon. Roll the play dough into a ball carefully mixing it so that it becomes a uniform colour. This has been tried with fresh play dough only; store-bought play dough may not produce the same effect and may be more difficult to mix thoroughly. For a recipe, SEE:

http://www.teachnet.com/lesson/art/playdoughrecipes/traditional.html

An alternative to play dough is to use small spoonfuls of food colouring.

ASK: How many spoons of play dough have I used altogether? How many spoons of red play dough have I used? What fraction of this ball is red? How many spoons of blue play dough have I used? What fraction of this ball is blue? Write the “recipe” on a triangular flag (see below) which can be made from a quarter of a regular sheet taped to a straw (insert the straw into your ball of play dough). The recipe is shown on the flag below:

(Continued on next page.)
Ask students to select two colours of play dough (red, blue, white or yellow). Then have them decide how many spoonfuls of each colour will be mixed into their recipe and recorded as fractions on their flag. The total number of spoons they use should be 2, 3, 4, 5 or 6 since they will need to compare the fractions later using the BLM “Fraction Strips”. (Advanced students can use three colours, but they should first make a ball with two colours, since they will need it for a later activity.) Have each student write their name on the back side of their flag.

After they have taped their flags to their straws, distribute one small measuring spoon to each student and have them prepare their play dough balls according to the recipes on their flags. The play dough balls should be mixed thoroughly to show only one colour.

Then have them insert their flags into their play dough balls and hand them in, as the flags will be needed later on when fractions are compared and ordered.

**Extensions**

1. The smaller angle is what fraction of the larger angle?

   ![Angle Diagram]

   **HINT:** Use tracing paper.

2. Draw the whole angle if the given angle is \(\frac{1}{2}\), \(\frac{1}{3}\), or \(\frac{1}{4}\) of the whole angle.

   a) \(\frac{1}{2}\)  
   b) \(\frac{1}{3}\)  
   c) \(\frac{1}{4}\)

3. a) Sketch a pie and cut it into fourths. How can it be cut into eighths?  
   b) Sketch a pie and cut it into thirds. How can it be cut into sixths?

   ![Pie Diagrams]

4. Ask students to make and identify as many fractions in the classroom as they can, for instance:
   
   - \(\frac{1}{4}\) of the blackboard is covered in writing.  
   - \(\frac{2}{3}\) of the counters are red.  
   - \(\frac{1}{2}\) the length of a 30 cm ruler is 15 cm.  
   - About \(\frac{1}{3}\) of the door is covered by a window.  
   - \(\frac{11}{25}\) of the class has black hair.  
   - The room is about \(\frac{4}{5}\) wide as it is long.
5. Ask students to make a model of a fraction from materials in the classroom.

6. (Adapted from Grade 4 Atlantic Curriculum)

   Ask students to divide the rectangle...

   a) into thirds two different ways.
   b) into quarters three different ways.

   **NOTE:** The division shown below may not be obviously divided into quarters until it is further divided into eighths.

   ![Diagram]

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**NS5-62**

**Equal Parts of a Set**

Review equal parts of a whole. Tell your students that the whole for a fraction might not be a shape like a circle or square. Tell them that the whole can be anything that can be divided into equal parts. Brainstorm with the class other things that the whole might be: a line, an angle, a container, apples, oranges, amounts of flour for a recipe. Tell them that the whole could even be a group of people. For example, the grade 5 students in this class is a whole set and I can ask questions like: what fraction of students in this class are girls? What fraction of students in this class are ten years old? What fraction of students wear glasses? What do I need to know to find the fraction of students who are girls? (The total number of students and the number of girls). Which number do I put on top: the total number of students or the number of girls? (the number of girls). Does anyone know what the top number is called? (the numerator) Does anyone know what the bottom number is called? (the denominator) What number is the denominator? (the total number of students). What fraction of students in this class are girls? (Ensure that they say the correct name for the fraction. For example, \( \frac{10}{25} \) is said “ten twenty fifths.”) Tell them that the girls and boys don’t have to be the same size; they are still equal parts of a set. Ask students to answer: What fraction of their family is older than 10? Younger than 10? Female? Male? Some of these fractions, for some students, will have numerator 0, and this should be pointed out. Avoid asking questions that will lead them to fractions with a denominator of 0 (For example, the question “What fraction of your siblings are male?” will lead some students to say 0/0).

Then draw pictures of shapes with two attributes changing:
a) \[ \triangle \square \bigcirc \bigcirc \bigcirc \]\n
**ASK:** What fraction of these shapes are shaded? What fraction are circles? What fraction of the circles are shaded?

b) \[ \square \bigtriangleup \square \square \bigtriangleup \]\n
**ASK:** What fraction of these shapes are shaded? What fraction are unshaded? What fraction are squares? Triangles? What fraction of the triangles are shaded? What fraction of the squares are shaded? What fraction of the squares are not shaded?

Then ask students for each picture below to write in their notebooks:

a) What fraction are circles?
b) What fraction are shaded?
c) What fraction are squares?
d) What fraction are triangles?

i) \[ \triangle \square \bigcirc \bigcirc \bigcirc \bigcirc \]\n
ii) \[ \triangle \bigcirc \bigcirc \bigcirc \square \bigtriangleup \bigtriangleup \bigtriangleup \]\n
iii) \[ \triangle \bigcirc \bigcirc \bigcirc \bigtriangleup \bigtriangleup \bigtriangleup \bigcirc \]\n
**Bonus**
(pictures with 3 attributes changing)

\[ \square \bigcirc \square \bigtriangleup \square \bigcirc \square \square \]\n
Have students answer the same questions as above and then more complicated questions like: What fraction of the triangles are shaded? (So the triangles are now the whole set). What fraction of the shaded shapes are triangles? **ASK:** Now what is the whole set?

Have students make up questions to ask each other.

Draw on the board:

c) \[ \bigcirc \bigtriangleup \square \square \bigtriangleup \bigcirc \square \square \]\n
Have students volunteer questions to ask and others volunteer answers.

Then have students write fraction statements in their notebooks for similar pictures.
Ask some word problems:

A basketball team played 5 games and won 2 of them. What fraction of the games did the team win?

A basketball team won 3 games and lost 1 game. How many games did they play altogether? What fraction of their games did they win?

A basketball team won 4 games, lost 1 game and tied 2 games. How many games did they play? What fraction of their games did they win?

Also give word problems that use words such as “and,” “or” and “not”: Sally has 4 red marbles, 2 blue marbles and 7 green marbles.

a) What fraction of her marbles are red?

b) What fraction of her marbles are blue or red?

c) What fraction of her marbles are not blue?

d) What fraction of her marbles are not green?

Bonus

Which two questions have the same answer? Why?

Challenge your students to write another question that uses “or” that will have the same answer as: What fraction of her marbles are not blue?

Look at the shapes below:

![Shapes](image)

a) What fraction of the shapes are shaded and circles?

b) What fraction of the shapes are shaded or circles?

c) What fraction of the shapes are not circles?

Write a question that has the same answer as c):

What fraction of the shapes are ______ or ______?

Have students create their own questions and challenge a partner to answer them. Find the number of each item given the fractions.

a) A team played 5 games. They won \( \frac{2}{5} \) of their games and lost \( \frac{3}{5} \) of their games. How many games did they win? Lose?

b) There are 7 marbles. \( \frac{2}{7} \) are red, \( \frac{4}{7} \) are blue and \( \frac{1}{7} \) are green. How many blue marbles are there? Red marbles? Green marbles?

Then tell your students that you have five squares and circles. Some are shaded and some are not. Have students draw shapes that fit the puzzles. (Students could use circles and sequences of two different colours to solve the problem.)

a) \( \frac{2}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the shapes are shaded. One circle is shaded.

SOLUTION:

![Squares and Circles](image)
b) \( \frac{3}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the shapes are shaded. No circle is shaded.

c) \( \frac{3}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the shapes are shaded. \( \frac{1}{5} \) of the squares are shaded.

**Extensions**

1. Have students investigate:
   a) What fraction of the squares on a Monopoly board are “Chance” squares? What fraction of the squares are properties that can be owned?
   b) On a Snakes and Ladders board, on what fraction of the board would you be forced to move down a snake?

   Have students make up their own fraction question about a board game they like and tell you the answer next class.

2. Draw a picture to solve the puzzle. There are 7 triangles and squares. \( \frac{3}{7} \) of the figures are triangles. \( \frac{2}{7} \) are shaded. 2 triangles are shaded.
3. Give your students harder puzzles by adding more attributes and more clues:

There are 5 squares and circles.

\( \frac{3}{5} \) of the shapes are squares.
\( \frac{3}{5} \) of the shapes are shaded.
\( \frac{2}{5} \) of the shapes are big.

\( \frac{2}{5} \) of the squares are big.
\( \frac{2}{5} \) of the squares are shaded.
No shaded shape is big.

SOLUTION:

4. Give your students red and blue counters (or any other pair of colours) and ask them to solve the following problems by making a model.

a) Half the counters are red. There are 10 red counters. How many are blue?

b) Two fifths of the counters are blue. There are 6 blue counters. How many are red?

c) \( \frac{3}{4} \) of the counters are red. 9 are red. How many are blue?

5. What fraction of the letters of the alphabet is...

a) in the word “fractions”

b) not in the word “fractions”

6. For the month of September, what fraction of all the days are...

a) Sundays
b) Wednesdays
c) School days
d) Not school days
e) What fraction of the days are divisible by 5?

7. What fraction of an hour has passed since 9:00? Reduce if possible.

a) 9:07
b) 9:15
c) 9:30
d) 9:40

8. a) There are 5 circles and triangles. Can you draw a set so that:

i) \( \frac{4}{5} \) are circles and \( \frac{2}{5} \) are striped?

Have volunteers show the different possibilities before moving on. Ask questions like:

How many are striped circles? How many different answers are there?

ii) \( \frac{3}{5} \) are circles and \( \frac{2}{5} \) are triangles?

What is the same about these two questions? (the numbers are the same in both) What is different? [one is possible, the other is not; one uses the same attribute (shape) in both, the other uses two different attributes (shape and shading)].

b) On a hockey line of 5 players, \( \frac{4}{5} \) are good at playing forward and \( \frac{2}{5} \) are good at playing defense. How many could be good at playing both positions? Is there only one answer? Which question from 4 a) is this similar to? What is similar about it?
NS5-63
Parts and Wholes

GOALS
Students will make divisions not already given to form equal-sized parts.

PRIOR KNOWLEDGE REQUIRED
A fraction of an area is a number of equal-sized parts out of a total number of equal-sized parts. The whole a fraction is based on can be anything.

VOCABULARY

<table>
<thead>
<tr>
<th>fraction</th>
<th>numerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole</td>
<td>denominator</td>
</tr>
<tr>
<td>part</td>
<td></td>
</tr>
</tbody>
</table>

Draw on the board the shaded strips from before:

ASK: Is the same amount shaded on each strip? Is the same fraction of the whole strip shaded in each case? How do you know? Then draw two hexagons as follows:

ASK: Is the same amount shaded on each hexagon? What fraction of each hexagon is shaded?

Then challenge students to find the fraction shaded by drawing their own lines to divide the shapes into equal parts:

Give your students a set of tangram pieces. Ask students what fraction of each of the tangram pieces a small triangle represents. ASK: How many small triangles cover a square? What fraction of the square is the small triangle? How many small triangles cover the large triangle? What fraction of the large triangle is the small triangle? What fraction of the medium triangle is the small triangle? What fraction of the parallelogram is the small triangle?

Ask students to make as many rectangles or trapezoids as they can using the pieces and determine the fraction of the shape that is covered by the small triangle. (For some sample rectangles and trapezoids, see questions 1 and 3 on the worksheet.)
Then draw shapes on the board and have students decide what fraction of the shape the small triangle is:

If \( \square = \text{red}, \) and \( \blacksquare = \text{blue}, \) approximately what fraction of each flag or banner is red and what fraction is blue:

For parts b) and c) students should subdivide the flag as shown below:

(From Atlantic Curriculum A3.6) Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.

**Extensions**

1. a) What fraction of a tens block is a ones block?
   b) What fraction of a tens block is 3 ones blocks?
   c) What fraction of a hundreds block is a tens block?
   d) What fraction of a hundreds block is 4 tens blocks?
   e) What fraction of a hundreds block is 32 ones blocks?
   f) What fraction of a hundreds block is 3 tens blocks and 2 ones blocks?
2. On a geoboard, show 3 different ways to divide the area of the board into 2 equal parts.

**EXAMPLES:**

3. Give each student a set of pattern blocks. Ask them to identify the whole of a figure given a part.

   a) If the pattern block triangle is \( \frac{1}{6} \) of a pattern block, what is the whole?
      **ANSWER:** The hexagon.

   b) If the pattern block triangle is \( \frac{1}{3} \) of a pattern block, what is the whole?
      **ANSWER:** The trapezoid.

   c) If the pattern block triangle is \( \frac{1}{2} \) of a pattern block, what is the whole?
      **ANSWER:** The rhombus.

   d) If the rhombus is \( \frac{1}{6} \) of a set of pattern blocks, what is the whole?
      **ANSWER:** 2 hexagons or 6 rhombuses or 12 triangles or 4 trapezoids.

4. Students can construct a figure using the pattern block shapes and then determine what fraction of the figure is covered by the pattern block triangle.

5. What fraction of the figure is covered by...

   a) The shaded triangle
   b) The small square

6. To prepare students for comparing and ordering fractions, ask them to guess what fraction with numerator 1 is closest to the answer to parts c) and d): one half, one third, one fourth or one fifth. Students can then check their estimates on an enlarged version of these drawings by using counters.

   **ASK:** How many counters cover the red part? How many counters cover the whole flag? About how many times more counters cover the whole flag than cover the red part? About what fraction of the flag is red? If students guess for example, that the fraction of the starred flag in the bonus that is red is one half, they could cover the red star with red counters, the blue boundary with (same sized) blue counters and see if the number of red counters is close to the number of blue counters.
NS5-64
Ordering and Comparing Fractions

**GOALS**
Students will understand that as the numerator increases and the denominator stays the same, the fraction increases and that as the numerator stays the same and the denominator increases, the fraction decreases.

**PRIOR KNOWLEDGE REQUIRED**
Naming fractions
Fractions show same-sized pieces

**VOCABULARY**

<table>
<thead>
<tr>
<th>part</th>
<th>numerator</th>
<th>whole</th>
<th>denominator</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw on the board:

Have students name the fractions shaded and then have them say which circle has more shaded. **ASK:** Which is more: one fourth of the circle or three fourths of the circle?

**ASK:** Is three quarters of something always more than one quarter of the same thing? Is three quarters of a metre longer or shorter than a quarter of a metre? Is three quarters of a dollar more money or less money than a quarter of a dollar? Is three fourths of an orange more or less than one fourth of the orange? Is three fourths of the class more or less people than one fourth of the class? If three fourths of the class have brown eyes and one quarter of the class have blue eyes, do more people have brown eyes or blue eyes?

Tell students that if you consider fractions of the same whole—no matter what whole you’re referring to—three quarters of the whole is always more than one quarter of that whole, so mathematicians say that the fraction $\frac{3}{4}$ is greater than the fraction $\frac{1}{4}$. Ask students if they remember what symbol goes in between:

$\frac{3}{4} \quad \square \quad \frac{1}{4}$

Remind them that the inequality sign is like the mouth of a hungry person who wants to eat more of the pasta but has to choose between three quarters of it or one quarter of it. The sign opens toward the bigger number:

$\frac{3}{4} > \frac{1}{4}$ or $\frac{1}{4} < \frac{3}{4}$.

Have students decide which is more and to write the appropriate inequality in between the numbers:

$\frac{2}{5} \quad \square \quad \frac{3}{5}$
Repeat with several examples, eventually having students name the fractions as well:

\[
\begin{array}{c}
\frac{2}{7} \quad \frac{3}{7} \\
\frac{5}{7} \quad \frac{4}{7} \\
\hline
\end{array}
\]

Draw the following pictures on the board:

\[
\begin{array}{c}
\frac{1}{4} \quad \frac{1}{2} \\
\frac{3}{4} \quad \frac{1}{3} \\
\hline
\end{array}
\]

ASK: Which is greater: one quarter or two quarters? One eighth or two eighths? One sixth or two sixths? Show students a pie cut into sixths on the board and ask: If Sally gets one sixth and Tony gets two sixths, who gets more? If Sally gets three sixths and Tony gets two sixths, who gets more? Which is greater:

\[
\begin{array}{c}
\frac{1}{9} \quad \frac{2}{9} \\
\frac{1}{12} \quad \frac{2}{12} \\
\frac{1}{100} \quad \frac{2}{100} \\
\frac{1}{807} \quad \frac{2}{807} \\
\frac{2}{9} \quad \frac{5}{9} \\
\frac{3}{11} \quad \frac{4}{11} \\
\frac{9}{11} \quad \frac{8}{11} \\
\frac{35}{87} \quad \frac{43}{87} \\
\frac{91}{102} \quad \frac{54}{102} \\
\end{array}
\]

Bonus

\[
\begin{array}{c}
7 \frac{432}{25401} \quad 8 \frac{69}{25401} \\
52 \frac{165}{4567341} \quad 54 \frac{154}{4567341} \\
\end{array}
\]

Then ask students to order a list of fractions with the same denominator (EXAMPLE: \(\frac{2}{7}, \frac{5}{7}, \frac{7}{7}\)) from least to greatest, eventually using bigger numerators and denominators and eventually using lists of 4 fractions.

**Bonus**

\[
\begin{array}{c}
\frac{4}{21}, \frac{11}{21}, \frac{8}{21}, \frac{19}{21}, \frac{6}{21}, \frac{12}{21}, \frac{5}{21} \\
\end{array}
\]

Then write on the board:

Ask students to think of numbers that are in between these two numbers. Allow several students to volunteer answers. Then ask students to write individually in their notebooks at least one fraction in between:

a) \(\frac{4}{11}, \frac{9}{11}\)  
 b) \(\frac{3}{12}, \frac{9}{12}\)  
 c) \(\frac{4}{16}, \frac{13}{16}\)  
 d) \(\frac{21}{48}, \frac{25}{48}\)  
 e) \(\frac{67}{131}, \frac{72}{131}\)

**Bonus**

\[
\begin{array}{c}
\frac{104}{18301} \quad \frac{140}{18301} \\
\end{array}
\]
Draw on the board:

\[
\begin{align*}
\frac{1}{2} & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| \\
\frac{1}{3} & \quad \bigg| & \quad \bigg| & \quad \bigg| \\
\frac{1}{4} & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg|
\end{align*}
\]

Have a volunteer colour the first part of each strip of paper and then ask students which fraction shows the most: \(\frac{1}{2}\), \(\frac{1}{3}\) or \(\frac{1}{4}\). **ASK:** Do you think one fifth of this fraction strip will be more or less than one quarter of it? Will one eighth be more or less than one tenth?

**ASK:** Is one half of something always more than one quarter of the same thing? Is half a metre longer or shorter than a quarter of a metre? Is half an hour more or less time than a quarter of an hour? Is half a dollar more money or less money than a quarter of a dollar? Is half an orange more or less than a fourth of the orange? Is half the class more or less than a quarter of the class? If half the class has brown eyes and a quarter of the class has green eyes, do more people have brown eyes or green eyes?

Tell students that no matter what quantity you have, half of the quantity is always more than a fourth of it, so mathematicians say that the fraction \(\frac{1}{2}\) is greater than the fraction \(\frac{1}{4}\). Ask students if they remember what symbol goes in between: \(\frac{1}{2} \, \bigg| \, \frac{1}{4} \) (< or >).

Tell your students that you are going to try to trick them with this next question so they will have to listen carefully. Then **ASK:** Is half a minute longer or shorter than a quarter of an hour? Is half a centimetre longer or shorter than a quarter of a metre? Is half of Stick A longer or shorter than a quarter of Stick B?

\[
\begin{align*}
\text{Stick A:} & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| \\
\text{Stick B:} & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg|
\end{align*}
\]

**ASK:** Is a half always bigger than a quarter?

Allow everyone who wishes to attempt to articulate an answer. Summarize by saying: A half of something is always more than a quarter of the same thing. But if we compare different things, a half of something might very well be less than a quarter of something else. When mathematicians say that \(\frac{1}{2} > \frac{1}{4}\), they mean that a half of something is always more than a quarter of the same thing; it doesn’t matter what you take as your whole, as long as it’s the same whole for both fractions.

Draw the following strips on the board:

\[
\begin{align*}
& \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg| \\
& \quad \bigg| & \quad \bigg| & \quad \bigg| & \quad \bigg|
\end{align*}
\]

Ask students to name the fractions and then to tell you which is more.

Have students draw the same fractions in their notebooks but with circles instead of strips. Is \(\frac{3}{4}\) still more than \(\frac{3}{8}\)? (yes, as long as the circles are the same size)
**Bonus**

Show the same fractions using a line of length 8 cm.

Ask students: If you cut the same strip into more and more pieces of the same size, what happens to the size of each piece?

Draw the following picture on the board to help them:

1 big piece

2 pieces in one whole

3 pieces in one whole

4 pieces in one whole

Many pieces in one whole

**ASK:** Do you think that 1 third of a pie is more or less pie than 1 fifth of the same pie? Would you rather have one piece when it's cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically ($\frac{1}{3} > \frac{1}{5}$).

Do you think 2 thirds of a pie is more or less than 2 fifths of the same pie? Would you rather have two pieces when the pie is cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically ($\frac{2}{3} > \frac{2}{5}$).

If you get 7 pieces, would you rather the pie be cut into 20 pieces or 30? Which way will you get more pie? How do we write that mathematically? ($\frac{7}{20} > \frac{7}{30}$).

Give students several problems similar to QUESTION 2 in the workbook. Use bigger numerators and denominators as bonus, always keeping the numerators the same.

**Extra Bonus**

Have students order the following list of numbers:

\[
\begin{array}{cccccccc}
\frac{21}{28} & \frac{21}{22} & \frac{8}{200} & \frac{19}{105} & \frac{13}{200} & \frac{19}{28} & \frac{13}{105} & \frac{19}{61}
\end{array}
\]

**SAY:** Two fractions have the same numerator and different denominators. How can you tell which fraction is bigger? Why? Summarize by saying that the same number of pieces gives more when the pieces are bigger. The numerator tells you the number of pieces, so when the numerator is the same, you just look at the denominator. The bigger the denominator, the more pieces you have to share between and the smaller the portion you get. So bigger denominators give smaller fractions when the numerators are the same.

**SAY:** If two fractions have the same denominator and different numerators, how can you tell which fraction is bigger? Why? Summarize by saying that if the denominators are the same, the size of the pieces are the same. So just as 2 pieces of the same size are more than 1 piece of that size, 84 pieces of the same size are more than 76 pieces of that size.

Emphasize that students can’t do this sort of comparison if the denominators are not the same.

**ASK:** Would you rather 2 fifths of a pie or 1 half? Draw the following picture to help them:
Tell your students that when the denominators and numerators of the fractions are different, they will have to compare the fractions by drawing a picture or by using other methods that they will learn later.

Have students draw several hundreds blocks on grid paper (or provide them with several already made) and ask them to show the following fractions: \(\frac{32}{100}, \frac{47}{100}, \frac{82}{100}, \frac{63}{100}\).

**ASK:** Which fractions are greater than \(\frac{1}{2}\)? Which fractions are less than \(\frac{1}{2}\)? Which fraction with denominator 100 would be exactly \(\frac{1}{2}\)?

Write the two fractions \(\frac{3}{4}\) and \(\frac{4}{5}\) on the board. **ASK:** Do these fractions have the same numerator? The same denominator? (no, neither) Explain that we cannot compare these fractions directly using the methods in this section, so we need to draw a picture.

Have a volunteer shade the fraction \(\frac{3}{4}\) on the first strip and the fraction \(\frac{4}{5}\) on the second strip. **ASK:** On which strip is a greater area shaded? On which strip is a smaller area unshaded? How many pieces are not shaded? What is the fraction of unshaded pieces in each strip? (\(\frac{1}{4}\) and \(\frac{1}{5}\)) Can you compare these fractions directly? (yes, they have the same numerator). Emphasize that if there is less left unshaded, then there is more shaded. So \(\frac{3}{4}\) is a greater fraction than \(\frac{4}{5}\) because the unshaded part of \(\frac{3}{4}\) (i.e. \(\frac{1}{4}\)) is smaller than the unshaded part of \(\frac{4}{5}\) (i.e. \(\frac{1}{5}\)).

Repeat for other such pairs of fractions:

\[
\frac{7}{8} \text{ and } \frac{6}{8}, \quad \frac{7}{9} \text{ and } \frac{6}{9}, \quad \frac{8}{9} \text{ and } \frac{7}{9}, \quad \frac{8}{9} \text{ and } \frac{7}{8}, \quad \frac{8}{9} \text{ and } \frac{7}{8}, \quad \frac{34}{36} \text{ and } \frac{56}{70}, \quad \frac{28}{38} \text{ and } \frac{34}{36}
\]

(draw the pictures when the numerators and denominators are small).

**ACTIVITY 1**

Give students their play dough flags made in a previous activity. Have them organize themselves into groups with people who chose the same two colours they did. For example, suppose 5 people chose red and blue as their two colours. Have those 5 students order their colours in terms of most red to least red and then order the fractions for red from the recipes in order from greatest to least. All students in the group should individually check their results by using fraction strips. Before distributing the BLM “Fraction Strips”, ask the class as a whole why they might expect slight disagreements with the fraction strip results and the play dough results. Which order of fractions do they think will be the correct order? What mistakes may have been made when making the play dough balls? (Some spoons may have had some red play dough still in them when making blue; the play dough may not have been completely flattened all the time.)
ACTIVITY 2

Give each student three strips of paper. Ask them to fold the strips to divide one strip into halves, one into quarters, one into eighths. Use the strips to find a fraction between

a) $\frac{3}{8}$ and $\frac{5}{8}$ (one answer is $\frac{1}{2}$)

b) $\frac{1}{3}$ and $\frac{2}{3}$ (one answer is $\frac{2}{5}$)

c) $\frac{5}{8}$ and $\frac{7}{8}$ (one answer is $\frac{3}{4}$)

ACTIVITY 3

Have students fold a strip of paper (the same length as they folded in ACTIVITY 2) into thirds by guessing and checking. Students should number their guesses.

EXAMPLE:

Is $\frac{1}{3}$ a good answer for any part of Activity 2? How about $\frac{2}{3}$?

Try folding here too short, so try a little further

Extensions

1. Write the following fractions in order from least to greatest. Explain how you found the order.

   $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{8}$

2. Why is $\frac{2}{3}$ greater than $\frac{2}{5}$? Explain.

3. Why is it easy to compare $\frac{2}{5}$ and $\frac{2}{12}$? Explain.

4. Have students compare $\frac{13}{87}$ and $\frac{14}{86}$ by finding a fraction with the same numerator as one of them and the same denominator as the other that is in between both fractions. For instance, $\frac{15}{86}$ is clearly smaller than $\frac{14}{86}$ and bigger than $\frac{13}{87}$. Another way to compare the two given fractions is to note that the second fraction has more pieces (14 instead of 13) and each piece is slightly bigger, so it must represent a bigger fraction.

5. (Atlantic Curriculum A10.4) If you know that $\frac{2}{2} > \frac{2}{7}$, what do you know about $\frac{2}{2}$?

6. (Atlantic Curriculum A10.6) Connection to probability: Have students conduct an experiment by rolling 2 coloured dice (one red and one blue) and making a fraction with numerator from the red die and denominator from the blue die) Have students predict whether the fraction will usually be less than half and to verify their prediction by rolling the dice several times.
NS5-65
Mixed Fractions

Introduce mixed fractions by drawing the following picture on the board:

![Diagram of mixed fractions]

Tell students that some friends ordered 3 pizzas with 4 pieces each. The shaded pieces show how much they have eaten. They ate two whole pizzas plus a quarter of another one. Draw the following pictures on the board.

a) ![Diagram of mixed fraction a]
b) ![Diagram of mixed fraction b]
c) ![Diagram of mixed fraction c]

**ASK:** How many whole pizzas are shaded? What fraction of the last pizza is shaded? Write the mixed fraction for the first picture and have volunteers write the fractions for the second and third pictures.

Draw models of several mixed fractions, asking students to name them in their notebooks. Use a variety of shapes for the whole piece, such as rectangles and triangles.

Write a fraction such as $3 \frac{1}{4}$ on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

**EXAMPLE:**

$3 \frac{1}{4}$

They should shade the first 3 circles and 1 part of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

**EXAMPLES:** $2 \frac{1}{5}$, $3 \frac{1}{8}$, $1 \frac{3}{4}$, $2 \frac{3}{5}$, $3 \frac{2}{5}$.

If students have trouble, give them practice drawing the whole number of pies drawn with the correct number of pieces.
Extensions

1. Teach students how to count forwards by halves, thirds, quarters, and tenths beyond 1.
   Ask students to complete the patterns:
   a) \( \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \__, \__, \__, \__, \__, \__ \)
   b) \( 2\frac{1}{4}, 2\frac{2}{4}, \__, \__, \__, \__, \__, \__ \)
   c) \( \frac{1}{3}, \frac{2}{3}, \__, \__, \__, \__, \__, \__ \)
   d) \( \frac{7}{10}, \frac{8}{10}, \__, \__, \__, \__, \__, \__ \)

2. What model represents \( 2\frac{3}{4} \)? How do you know?

   A. 
   B. 

3. ASK: Which fractions show more than a whole? How do you know?
   \( 2\frac{3}{5}, \frac{2}{7}, 1\frac{4}{5}, 2\frac{1}{3}, \frac{2}{9} \)

4. Ask students to order these fractions from least to greatest: \( 3\frac{1}{5}, 1\frac{5}{9}, 7\frac{1}{1} \)
   ASK: Did they need to look at the fractional parts at all or just the whole numbers? Why?
   Bonus
   Order the following list of numbers: \( 3\frac{1}{6}, 5\frac{7}{9}, 2\frac{1}{11}, 6\frac{3}{5}, 8\frac{7}{9}, 4\frac{3}{10} \)

5. Ask students to order mixed fractions where some of the whole numbers are the same, and the fractional parts have either the same numerator or the same denominator.
   EXAMPLES: a) \( 3\frac{1}{6}, 5\frac{2}{7}, 3\frac{1}{5} \)  b) \( 5\frac{3}{8}, 5\frac{2}{8}, 6\frac{1}{11} \)
   Bonus
   Give students longer lists of numbers that they can order using these strategies.
**NS5-66**

**Improper Fractions**

**GOALS**
Students will name improper fractions and fractions representing exactly one whole.

**PRIOR KNOWLEDGE REQUIRED**
Mixed fractions

**VOCABULARY**
mixed fraction  fraction  improper  proper fraction

Draw the following shapes on the board:

Have students name the fractions shaded. **ASK:** How many parts are shaded? How many parts are in one whole? Tell them that they are all 1 whole and write $1 = \frac{4}{4}$ and $1 = \frac{6}{6}$. Then have student volunteers fill in the blanks:

\[
\begin{align*}
1 &= \underline{9} \\
1 &= \frac{7}{\phantom{0}}
\end{align*}
\]

Then tell them that sometimes they might have more than 1 whole—they might have two whole pizzas, for **EXAMPLE:**

\[
\begin{align*}
2 &= \underline{2} \\
2 &= \underline{3}
\end{align*}
\]

**ASK:** Which number goes on top—the number of parts that are shaded or the number of parts in one whole? Tell them to look at the pictures. Ask them how many parts are in one whole circle and how many parts are shaded.

Then write:

\[
\begin{align*}
2 &= \underline{\phantom{2}} \\
2 &= \underline{\phantom{3}}
\end{align*}
\]

Then ask a volunteer to come and write the number of shaded pieces.

**ASK:** How are the numerator and denominator in each fraction related? (you double the denominator to get the numerator). Have students fill in the missing numbers:

\[
\begin{align*}
2 &= \underline{4} \\
2 &= \underline{28} \\
2 &= \underline{76}
\end{align*}
\]

(Always use an even number when giving the numerator.)

**ASK:** How are the fractions above different from the fractions we’ve seen so far? Tell them these fractions are called improper fractions because the numerator is larger than the denominator. Challenge students to guess what a fraction is called if its numerator is smaller than its denominator. (proper fractions)

Draw on the board:
ASK: How many pieces are shaded? (9)

SAY: I want to write a fraction for this picture. Should 9 be the numerator or the denominator? (numerator) Do I usually put the number of shaded parts on top or on bottom? (top) How many equal parts are in 1 whole? (4) Should this be the numerator or the denominator? (denominator) Do we usually put the number of parts in 1 whole on top or on bottom? (bottom) Tell your students that the fraction is written \( \frac{9}{4} \).

Have volunteers write improper fractions for these pictures.

a) 

b) 

c) 

ASK: How many parts are shaded? How many parts are in one whole?

Draw models of several improper fractions, asking students to name them in their notebooks. Use a variety of shapes such as rectangles and triangles for the whole.

Write a fraction such as \( \frac{15}{4} \) on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

EXAMPLE:

\[
\frac{15}{4}
\]

They should shade the first 3 circles and 3 parts of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

EXAMPLE: \( \frac{11}{4} \), \( \frac{15}{8} \), \( \frac{19}{8} \), \( \frac{10}{3} \), \( \frac{12}{5} \).

ASK: Which fractions show more than a whole? How do they know?

\( \frac{13}{5} \), \( \frac{2}{9} \), \( \frac{14}{15} \), \( \frac{8}{3} \), \( \frac{12}{7} \).
Mixed and Improper Fractions

**ASK:** What is a mixed fraction? What is an improper fraction? Have a volunteer write a mixed fraction for this picture and explain their answer:

![Mixed Fraction Picture]

Have another volunteer write an improper fraction for the same picture and explain their answer.

Draw several models of fractions larger than 1 on the board and have students write both the mixed fraction and the improper fraction. Use several different shapes other than circles.

Draw several more such models on the board and ask students to write an improper fraction if the model contains more than 2 whole pies, and a mixed fraction otherwise. This will allow you to see if students know the difference between the terms "mixed" and "improper".

Tell students to draw models for the following mixed fractions and to write the corresponding improper fraction that is equal to it (this may be done in 2 different steps if students need it broken down).

\[
\begin{align*}
&\text{a)} \ 2 \frac{3}{4} \\ &\text{b)} \ 3 \frac{1}{3} \\ &\text{c)} \ 2 \frac{1}{6} \\ &\text{d)} \ 1 \frac{5}{6} \\ &\text{e)} \ 3 \frac{2}{5} \\ &\text{f)} \ 2 \frac{7}{8}
\end{align*}
\]

Then tell students to draw models for the following improper fractions and then to write the mixed fraction that is equal to it.

\[
\begin{align*}
&\text{a)} \ \frac{13}{4} \\ &\text{b)} \ \frac{7}{3} \\ &\text{c)} \ \frac{11}{6} \\ &\text{d)} \ \frac{19}{5} \\ &\text{e)} \ \frac{27}{8}
\end{align*}
\]

**SAY:** You have written many numbers as both a mixed and an improper fraction. What is the same in both? What is different? (the denominators are the same because they tell you how many parts are in a whole, but the numerators will be different because the mixed fraction counts the pieces that make up the wholes separately from the pieces that only make up part of the whole; improper fractions count them all together)

**ACTIVITY**

Students can make models of the fractions on the worksheet by placing the smaller pattern blocks on top of the hexagon (or on top of whatever block is being used to represent the whole).
Extensions

1. Have students write their answers to these questions as both mixed and improper fractions.

   What fraction of a tens block is 7 ones blocks? 17 ones blocks? 32 ones blocks?
   What fraction of a hundreds block is 32 tens blocks? 43 tens blocks and 5 ones blocks?
   How many metres are in 230 cm? 571 cm?
   How many decimetres are in 54 centimetres? 98 cm?
   What fraction of a dime is a quarter?

2. Ask students to solve these problems with pattern blocks or by sketching their answers on triangular grid paper.
   a) Which two whole numbers is \(\frac{23}{6}\) between?
   b) What mixed fraction of a pie would you have if you took away \(\frac{1}{6}\) of a pie from 3 pies (and what would the improper fraction be)?

NS5-68
Mixed Fractions (Advanced)

**GOALS**
Students will use multiplication to find the improper fraction equivalent to a given mixed fraction.

**PRIOR KNOWLEDGE REQUIRED**
Mixed fractions
Improper fractions
Using pictures to see that mixed and improper fractions can represent the same amount
Multiplication

**VOCABULARY**
mixed fraction
improper fraction

Draw on the board:

![Fraction Diagram]

**SAY:** How many parts are in 1 pie? There are 4 quarters in one pie. How many quarters are in two pies? (8)

![Pie with Quarters]

What operation can we use to tell us the answer? (multiplication) How many quarters are there in 3 pies? \((4 \times 3 = 12)\)

![Pie with More Quarters]

**ASK:** How many quarters are in \(3 \frac{3}{4}\) pies?

\[\frac{12}{3} \quad \frac{3}{4} \quad 3 \text{ extra pieces}\]

\[\frac{3 \times 4}{(3 \times 4)}\]

So there are 15 pieces altogether.

How many halves are in 1 pie? (2) In 2 pies? (4) In 3 pies? (6) In 17 pies? (34)
How do you know? What operation did you use to find that? (17 \(\times\) 2)
**ASK:** How many halves are in 1 1/2 pies? Have a volunteer draw the picture on the board.

**ASK:** How many halves are in 2 1/2 pies? In 3 1/2 pies? In 4 1/2 pies? In 20 1/2 pies? What operations do you use to find the answer? (20 × 2 + 1)

40 pieces in 20 whole pies

1 extra piece

Draw 8 1/2 pies on the board and **ASK:** How many halves are in 8 1/2?

Emphasize that the extra half is just one more piece, so once they know how many halves are in 8 pies, they just add one to find how many are in 8 1/2.

Have students write in their notebooks how many halves are in...

a) 2 1/2
b) 5 1/2
c) 11
d) 11 1/2

**BONUS:** 49 1/2

**ASK:** How many thirds are in 1 pie? (3) Have a volunteer come to the board and divide a circle into thirds. **ASK:** How many thirds are in 2 pies? In 3 pies? In 10 pies? In 100 pies? In 1000 pies? How many thirds are in 1013 pies? In 1023 pies? In 523 pies?

Have students write in their notebooks how many thirds are in...

a) 2 1/3
b) 5 1/3
c) 11
d) 11 2/3

**BONUS:** 49 2/3

Include questions with denominator 4.

Then introduce problems with a context. **SAY:** I have boxes that will hold 4 cans each. What fraction of a box is each can? (one fourth) How many fourths are in 2 wholes? How many cans will 2 boxes hold? How are these questions the same? How are they different?

A box holds 4 cans. How many cans will:

a) 1 1/4 boxes hold.
b) 2 1/4 boxes hold.
c) 1 1/4 boxes hold.
d) 1 1/4 boxes hold.

To help your students, encourage them to rephrase the question in terms of fourths and wholes. For example, since a can is one fourth of a whole, a) becomes “How many fourths are in 1 1/4?”

Next, students will have to rephrase the question in terms of fractions other than fourths, depending on the number of items in each package.

a) A box holds 6 cans. How many cans will 1 5/6 boxes hold?
b) A box holds 8 cans. How many cans will 2 3/8 boxes hold?
c) **BONUS:** A box holds 326 cans. How many cans will 1 8/326 boxes hold?
d) Tennis balls come in cans of 3. How many balls will 7 1/3 cans hold?
e) A bottle holds 100 mL of water. How many mL of water will 7 89/100 bottles hold?

Teach students how to change mixed fractions to improper fractions: To change 2 3/8 to an improper fraction, start by calculating how many pieces are in the whole pies (2 × 8 = 16) and add on the remaining pieces (16 + 3 = 19), so 2 3/8 = 19/8. Give students several problems of this sort, where they convert from mixed to improper form.
Teach your students to explain how to turn a mixed fraction into an improper fraction, using a concrete model (for instance some pizzas or circles—cut into parts) as an example:

I multiply $3 \times 4$ because there are 3 whole pies which each have 4 pieces in them: this gives 12 pieces altogether.

\[ 3 \frac{1}{4} \]

\[ 3 \times 4 = 12 \]

I add 1 more piece because the remaining pie has 1 piece in it: this gives 13 pieces altogether.

\[ 3 \frac{1}{4} \]

\[ 12 + 1 = 13 \]

I keep the denominator the same because it tells the size of the pieces in each pie (which doesn’t change).

\[ \frac{13}{4} \]

Have students write the following mixed fractions as improper fractions and explain how they found the answer:

- a) $3 \frac{1}{7}$
- b) $5 \frac{1}{6}$
- c) $4 \frac{3}{9}$
- d) $7 \frac{5}{8}$
- e) $6 \frac{5}{6}$
**NS5-69**

**Mixed and Improper Fractions (Advanced)**

Have each student write in their notebooks the mixed and improper fractions for several pictures displaying area:

![Mixed and Improper Fractions](image)

Then provide examples involving length and capacity as well, as shown in QUESTION 4 of the worksheet.

How long is the line? How many litres are shown?

![Line and Litres](image)

Have students write each whole number below as an improper fraction with denominator 2 and show their answer with a picture and a multiplication statement:

a) $3 = \frac{6}{2}$

$b) 4$  $c) 2$  $d) 7$  $e) 10$

$3 \times 2 = 6$

**ASK:** If I have the improper fraction $\frac{10}{2}$, how could I find the number of whole pies it represents? Draw on the board:

![Pie Division](image)

There are 3 pies with 2 halves each.

Ask students how many whole pies 10 half-sized pieces would make? Show students that they simply divide 10 by 2 to find the answer:

5 whole pies. **ASK:** How many whole pies are in $\frac{9}{2}$ pies? In $\frac{12}{2}$ pies? In $\frac{20}{2}$ pies? In $\frac{15}{2}$ pies? In $\frac{25}{2}$? In $\frac{35}{2}$? In $\frac{42}{2}$?

Then have students write the mixed fractions below as improper fractions and show their answer with a picture. Students should also write a statement for the number of half-sized pieces in the pies.

a) $3 \frac{1}{2} = \frac{7}{2}$

b) $4 \frac{1}{2}$  c) $2 \frac{1}{2}$  d) $5 \frac{1}{2}$  e) $8 \frac{1}{2}$

$3 \times 2 + 1 = 7$ halves

There is one extra half-sized piece.

There are 3 pies with 2 halves each.

**GOALS**

Students will use division with remainders to find the mixed fraction given the corresponding improper fraction.

**PRIOR KNOWLEDGE REQUIRED**

- Reading the improper and mixed fractions from a picture
- Finding the improper fraction given the mixed fraction by using multiplication
- Division with remainders
- The relationship between multiplication and division.
**SAY:** If I have the improper fraction $\frac{15}{2}$, how can I know how many whole pies there are and how many pieces are left over? I want to divide 15 into sets of size 2 and I want to know how many full sets there are and then if there are any extra pieces. What operation should I use? (division) What is the leftover part called? (the remainder)

Write on the board: $15 ÷ 2 = 7$ Remainder 1, **SO:** $\frac{15}{2} = 7 \frac{1}{2}$.

Draw the following picture on the board with three number statements.

$\frac{15}{4} = 3 \frac{3}{4}$

$4 \times 3 + 3 = 15$

$15 ÷ 4 = 3$ Remainder 3

Have students discuss the three interpretations of the picture and what they mean. When we divide 15 into sets of size 4, we get 3 sets and then 3 extra pieces left over. This is the same as dividing pies into fourths and seeing that 15 fourths is the same as 3 whole pies (with 4 pieces each) and then 3 extra pieces.

Repeat for several pictures, having volunteers write the mixed and improper fractions as well as the multiplication and division statements. Then have students do similar problems individually in their notebooks.

Then give students improper fractions and have them draw the picture, write the mixed fraction, and the multiplication and division statements.

Then show students how to change an improper fraction into a mixed fraction:

$2 \frac{1}{4} = 2 \times 4 + 1$ quarters = $9$ quarters = $\frac{9}{4}$

Starting with $\frac{9}{4}$, we can find: $9 ÷ 4 = 2$ Remainder 1, **SO:**

$\frac{9}{4} = 2$ wholes and 1 more quarter = $2 \frac{1}{4}$

Have students change several improper fractions into mixed fractions without using pictures.

Tell them that $\frac{7}{2}$ pies is the same as 3 whole pies and another half a pie. **ASK:** Is this the same thing as 2 whole pies and three halves? Do we ever write $2 \frac{3}{2}$? **ASK:** When we find $7 ÷ 2$, do we write the answer as 3 Remainder 1 or 2 Remainder 3? Tell your students that as with division, we want to have the fewest number of pieces left over.

**Extension**

Write the following fractions in order: $3 \frac{1}{4}$, $\frac{27}{4}$, $\frac{11}{4}$, $2 \frac{1}{4}$, $\frac{36}{4}$, $4 \frac{3}{4}$
NS5-70
Investigating Mixed & Improper Fractions

GOALS
Students will make mixed and improper fractions using different shapes as the whole.

PRIOR KNOWLEDGE REQUIRED
Familiarity with pattern blocks
The relationship between mixed and improper fractions

VOCABULARY
mixed fraction
improper fraction

Give students pattern blocks or a copy of the pattern blocks BLM (and have them cut out the shapes).

Tell students that a hexagon represents one whole pie and ask them to show you a whole pie. If some students don’t know which piece is the hexagon, ASK: How many sides does a hexagon have? (6) When all students have shown you the hexagon, ask them how many triangles they would need to cover an entire hexagon. What fraction of the hexagon is a triangle? (1/6) Find a shape that is half of the hexagon. How many trapezoids would you need to make 1 1/2 hexagons? How many trapezoids would they need to make 7 1/2 hexagons? What fraction of a hexagon is a rhombus? How many rhombuses would you need to make 2 hexagons? What improper fraction is equal to 2 wholes? Tell them that there is no mixed fraction for 2 wholes—it is just a whole number.

NOTE: Students can make models of the fractions on the worksheet by placing the smaller pattern blocks on top of the hexagon block (or on top of whatever block is being used to represent the whole).

I want to make 1 1/6 hexagons. How many equal-sized pieces should the hexagon be divided into? What shape should I use for my equal-sized pieces? Why? (I should use triangles because 6 triangles can be put together to make a hexagon) Show how to make 1 1/6 hexagons using triangles.

What shape would you use to make 2 1/2 hexagons? Why? Show how to make 2 1/2 hexagons using trapezoids.

What shape would you use to make 13/3 hexagons? Why? Show how to make 13/3 hexagons using rhombuses.

SAY: So far, we have used only hexagons as a whole pie. Let’s use trapezoids as a whole pie. What shape would you use to make 2 1/3 trapezoids? Why? What piece is one third of the trapezoid? Show how to make 2 1/3 trapezoids using triangles.

Have students show how to make each fraction below using triangles and to draw pictures of their models in their notebooks:

a) 1 1/3
b) 4 1/3
c) 8/3
d) 11/3
e) 2 2/3
f) 3 1/3

Be sure everyone has done at least parts a) and b) above. ASK: Which 2 fractions from the list above are the same? How can you tell this from your pictures? Use your pictures to order the fractions above from least to greatest.

Write the following problem on the board.
Figure A is a model of 1 whole and Figure B is a model of $\frac{3}{2}$.

a) Ori says that the shaded area in Figure A is more than the shaded area in Figure B. Is he correct? How do you know?

b) Ori says that because the shaded area of Figure A is greater than the shaded area of Figure B, 1 whole must be more than $\frac{3}{2}$. What is wrong with his reasoning?

Extensions
1. If $\frac{4}{3}$ of a structure looks like this: 
   ![Diagram](image1)
   What could the whole look like?

2. If the triangle represents a whole, draw $2\frac{1}{2}$.

3. (Atlantic Mathematics Curriculum) Break egg cartons into sections (1 through 11), and use complete cartons as well. Distribute at least one of the sections to each of the students and say, “If this (whole carton) is one, what is $\frac{1}{3}$? If this (9 section piece) is one whole, show me one third. If this (2 sections) is one, show me $2\frac{1}{3}$,” etc. Students should realize that any one section can have many different names depending on the size of the whole. It is also beneficial for students to frame these types of questions for their classmates.

4. What fraction of a metre is a decimetre? 12 decimetres?

5. Ask questions of the form: Stick B is what fraction of Stick A? Stick A is what fraction of Stick B? Write your answers as proper or improper fractions; not as mixed fractions.

Demonstrate with the following example.

A: 
B: 

Stick B is $\frac{3}{5}$ of Stick A since putting it on top of Stick A will look like:

If Stick A is the whole, then the denominator is 5, because Stick A has 5 equal-sized parts. How many of those parts does Stick B take up? (3). So Stick B is $\frac{3}{5}$ of Stick A.

What fraction of Stick B is Stick A?

If Stick B is the whole, what is the denominator? (3) Why? (because Stick B has 3 equal-sized pieces) How many of those equal-sized pieces does Stick A take up? (5) So stick A is $\frac{5}{3}$ of Stick B.
As an improper fraction Stick A is $1 \frac{2}{3}$ of Stick B since it takes up one whole Stick B plus 2 more of those equal-sized pieces, but we will write our answers in terms of improper fractions instead of mixed fractions.

Let your students investigate with several examples. In this way, your students will discover reciprocals. If you know what fraction Stick A is of Stick B, what fraction is Stick B of Stick A? (just turn the fraction upside down!)

**REFLECT:** Why was it convenient to use improper fractions instead of mixed fractions?

---

**NS5-71**

**Equivalent Fractions**

**NS5-72**

**Models of Equivalent Fractions**

**GOALS**

Students will understand that different fractions can mean the same amount. Students will find equivalent fractions by using pictures.

**PRIOR KNOWLEDGE REQUIRED**

Comparing fractions
Fractions as area
Fractions of a set

**VOCABULARY**

equivalent fractions

Show several pairs of fractions on fraction strips and have students say which is larger, for **EXAMPLE:**

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{strip1.png}} \\
\text{\includegraphics[width=0.5\textwidth]{strip2.png}}
\end{array}
\]

\[
\frac{8}{9} \quad \frac{3}{4}
\]

Include many examples where the two fractions are equivalent. Tell your students that when two fractions look different but actually show the same amount, they are called equivalent fractions. Have students find pairs of equivalent fractions from the pictures you have on the board. Tell them that we have seen other examples of equivalent fractions from previous classes and ask if anyone knows where. (There are 2 possible answers here: fractions that represent 1 whole are all equivalent, and the same for fractions representing 2 wholes; also, mixed fractions have an equivalent improper fraction).

Then have students find equivalent fractions by shading the same amount in the second strip as in the first strip and writing the shaded amount as a fraction:

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{strip3.png}} \\
\text{\includegraphics[width=0.5\textwidth]{strip4.png}}
\end{array}
\]

\[
\frac{1}{2} = \frac{2}{4}
\]

Show students a fraction strip chart and have volunteers fill in the blank areas.
Shade the fraction $\frac{1}{2}$ and then ask: What other fractions can you see that are equivalent to $\frac{1}{2}$? ($\frac{2}{4}$ or $\frac{4}{8}$ or $\frac{5}{10}$). What other fraction from the chart is equivalent to $\frac{3}{4}$? Repeat with $\frac{6}{10}$, $\frac{4}{5}$, $\frac{1}{5}$, and $\frac{4}{10}$.

What fractions on the chart are equivalent to 1 whole?

Have students find as many fractions as they can that are all equivalent to the fraction shown in the picture.

Draw several copies of a square on the board with half shaded:

Have a volunteer draw a line to cut the square into 4 equal parts. Have another volunteer draw 2 lines to cut the square into 6 equal parts; have another cut the square into 8 equal parts and another into 10 equal parts.

Have volunteers name the equivalent fractions shown by the pictures ($\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$)

Then write on the board:

$$\frac{1}{2} \times \square = \frac{2}{4} \quad \frac{1}{2} \times \square = \frac{3}{6} \quad \frac{1}{2} \times \square = \frac{4}{8} \quad \frac{1}{2} \times \square = \frac{5}{10}$$

For each picture, ask: How many times more shaded pieces are there? How many times more pieces are there altogether? Emphasize that if each piece (shaded or unshaded) is divided into 4 pieces, then, in particular, each shaded piece is divided into 4 pieces. Hence if the number of pieces in the figure is multiplied by 4, the number of shaded pieces will also be multiplied by 4: that is why you multiply the top and bottom of a fraction by the same number to make an equivalent fraction.

For the pictures below, have students divide each piece into equal parts so that there are a total of 12 pieces. Then have them write the equivalent fractions with the multiplication statements for the numerators and denominators:
Ask students to draw 4 boxes of equal length on grid paper and shade 1 box.

Point out to students that \( \frac{1}{4} \) of the area of the boxes is shaded. Now ask students to draw the same set of boxes, but in each box to draw a line dividing the box into 2 parts.

Now \( \frac{2}{8} \) of the area is shaded. Repeat the exercise, dividing the boxes into 3 equal parts, (roughly: the sketch doesn’t have to be perfectly accurate), then 4 parts, then five parts.

Point out to your students that while the appearance of the fraction changes, the same amount of area is represented.

\( \frac{1}{4} \), \( \frac{2}{8} \), \( \frac{3}{12} \), \( \frac{4}{16} \), \( \frac{5}{20} \) all represent the same amount: They are equivalent fractions.

Ask students how each of the denominators in the fractions above can be generated from the initial fraction of \( \frac{1}{4} \). **ANSWER:** each denominator is a multiple of the denominator 4 in the original fraction:

\[ 8 = 2 \times 4; \quad 12 = 3 \times 4; \quad 16 = 4 \times 4; \quad 20 = 5 \times 4; \]

Then ask students how each fraction could be generated from the original fraction. **ANSWER:** multiplying the numerator and denominator of the original fraction by the same number:

\[
\begin{align*}
\frac{1}{4} \times \frac{2}{2} &= \frac{2}{8} \\
\frac{1}{4} \times \frac{3}{3} &= \frac{3}{12} \\
\frac{1}{4} \times \frac{4}{4} &= \frac{4}{16} \\
\frac{1}{4} \times \frac{5}{5} &= \frac{5}{20}
\end{align*}
\]

Point out that multiplying the top and bottom of the original fraction by any given number, say 5, corresponds to cutting each box into that number of pieces.

\[
\frac{1}{4} \times \frac{5}{5} \quad \text{there are 5 pieces in each box}
\]

\[
\frac{4}{5} \times \frac{5}{5} \quad \text{there are 4 \times 5 pieces}
\]

\[
4 \times 5 = 20 \text{ pieces altogether}
\]

The fractions \( \frac{1}{4} \), \( \frac{2}{8} \), \( \frac{3}{12} \), \( \frac{4}{16} \) … form a family of equivalent fractions. Notice that no whole number
ACTIVITY 1

Use the play dough activity described in NS4-71: Equal Parts and Models of Fractions. Have them investigate what happens when they use different sizes of spoons. Have them make a ball that is one third red and two thirds white using half a teaspoon and then another ball with the same fractions, but using a whole teaspoon. Did they get the same colour? (Yes.) What if they used 2 half teaspoons of red and 4 half teaspoons of white—how is this the same as using one teaspoon of red and 2 teaspoons of white? What pair of equivalent fractions does this show?

ACTIVITY 2

Ask your students to make a model (using concrete materials such as cubes or beads) of a fraction that can be described in two ways. Ask students to describe these fractions. For instance, if the student makes the following model:

They might say “I can say \(\frac{3}{6}\) of the counters are white or \(\frac{1}{2}\) of the counters are white.”

ACTIVITY 3

Give your students 10 counters of one colour and 10 counters of a different colour. Ask them to make a model of a fraction that can be described in at least 3 different ways.

Here are two solutions:

\[
\frac{1}{2} = \frac{2}{4} = \frac{4}{8} \quad \frac{6}{12} = \frac{2}{4} = \frac{1}{2}
\]

ACTIVITY 4

Give students blocks of 2 colours and have them make models of fractions of whole numbers using the method described in Exercise 5 of the Worksheet NS5-72: Models of Equivalent Fractions. Here are some fractions they might try:

a) \(\frac{3}{4}\) of 15  
b) \(\frac{3}{4}\) of 16  
c) \(\frac{3}{5}\) of 20  
d) \(\frac{2}{7}\) of 21
Extensions

1. Draw a ruler on the board divided into mm and cm, with a certain number of cm shaded, and write two fractions:

\[
\begin{array}{c|c}
\text{number of mm shaded} & \text{number of cm shaded} \\
\hline
\text{number of mm in total} & \text{number of cm in total}
\end{array}
\]

Are these fractions equivalent? They can also use the metre stick and write triples of equivalent fractions using cm, mm and dm.

2. Write as many equivalent fractions as you can for each picture.

a) ![Diagram](image1)

b) ![Diagram](image2)

3. List 3 fractions between \(\frac{1}{2}\) and 1. HINT: Change \(\frac{1}{2}\) to an equivalent fraction with a different denominator (EXAMPLE: \(\frac{4}{8}\)) and then increase the numerator or decrease the denominator. Show your answers on a number line (this part is easier if they increase the numerator instead of decrease the denominator).

4. (Atlantic Curriculum A3.1) Ask the students to use their fingers and hands to show that \(\frac{1}{2}\) and \(\frac{5}{10}\) are equivalent fractions.

5. Ask students to use the patterns in numerators and denominators of the equivalent fractions below to fill in the missing numbers.

a) \(\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{15}{?}\)

b) \(\frac{1}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{?}{?}\)

c) \(\frac{3}{8} = \frac{9}{12} = \frac{12}{16} = \frac{?}{?}\)

Ask students if the same patterning method will work to find equivalent fractions in these patterns:

a) \(\frac{2}{3} = \frac{3}{4} = \frac{4}{12} = \frac{15}{?}\)

b) \(\frac{1}{2} = \frac{3}{6} = \frac{5}{8} = \frac{?}{?}\)

Have students discuss what is different about these questions than the ones above. Ensure that students understand that all the fractions in a sequence of equivalent fractions must be obtained by multiplying the numerator and denominator of a particular fraction by the same number. In these examples, the patterns are only obtained through adding, not multiplying, so the pattern will not produce a sequence of equivalent fractions.

6. Divide students into groups. Give each group a fraction with numerator 1.

EXAMPLES: \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}\). Have each group find as many fractions as they can that are equivalent to their given fraction. Then ask each group to find, for each fraction, how many times more is the denominator than the numerator. Encourage students to notice that the multiplicative relationship between numerators and denominators is constant for equivalent fractions. This can also be explored for fractions with larger numerators (EXAMPLES: \(\frac{2}{3}, \frac{2}{5}\)) after students have seen how to describe multiplicative relationships using simple fractions (See Extension 4 of NS5-73: Fractions of Whole Numbers).
Brainstorm the types of things students can find fractions of (circles, squares, pies, pizzas, groups of people, angles, hours, minutes, years, lengths, areas, capacities, apples).

Brainstorm some types of situations in which it wouldn’t make sense to talk about fractions. For **Example**: Can you say \( \frac{3}{2} \) people went skiing? I folded the sheet of paper \( \frac{4}{4} \) times?

Explain to your students that it makes sense to talk about fractions of almost anything, even people and folds of paper, if the context is right. **Example**: Half of her is covered in blue paint; half the fold is covered in ink. Then teach them that they can take fractions of numbers as well. **Ask**: If I have 6 hats and keep half for myself and give half to a friend, how many do I keep? If I have 6 apples and half of them are red, how many are red? If I have a pie cut into 6 pieces and half the pieces are eaten, how many are eaten? Tell your students that no matter what you have 6 of, half is always 3. Tell them that mathematicians express this by saying that the number 3 is half of the number 6.

Tell your students that they can find \( \frac{1}{2} \) of 6 by drawing rows of dots. Put 2 dots in each row until you have placed 6 dots. Then circle one of the columns:

```
Step 1  Step 2  Step 3
● ●   ● ●   ● ●
● ●   ● ●   ● ●
● ●   ● ●   ● ●
```

The number of dots in one column is half of 6. Have students find \( \frac{1}{2} \) of each number using this method:

a) \( \frac{1}{2} \) of 4  b) \( \frac{1}{2} \) of 8  c) half of 10  d) half of 14

**Bonus**

Use this method to find \( \frac{1}{3} \) of each of the following numbers. **Hint**: Put 3 dots in each row.

a) \( \frac{1}{3} \) of 12  b) \( \frac{1}{3} \) of 15  c) one third of 18  d) one third of 3

**Ask**: If, you want to find \( \frac{1}{3} \) of 12, how many dots in a row would you draw? (4) You would draw 4 columns and circle the dots in 1 column. Now draw 4 sets and share the dots into the sets (1 dot to each of the sets, 4 in total, then another dot to each of the sets, continue till you get 12 dots in total). What fraction of the 12 dots does each set represent? What would you do if you needed \( \frac{5}{4} \) of 12? (You have to take 3 sets).
Invite volunteers to find \(\frac{3}{4}\) of 16 and \(\frac{2}{3}\) of 15 using this method. After that draw several pictures yourself and ask them to find the fractions they represent.

Teach your students to see the connection between the fact that 6 is 3 twos and the fact that \(\frac{1}{3}\) of 6 is 2. The exercise below will help with this:

Complete the number statement using the words “twos”, “threes”, “fours” or “fives”. Then draw a picture and complete the fraction statements. (The first one is done for you.)

<table>
<thead>
<tr>
<th>Number Statement</th>
<th>Picture</th>
<th>Fraction Statements</th>
</tr>
</thead>
</table>
| a) 6 = 3 twos    | ![Picture](image) | \(\frac{1}{3}\) of 6 = \______\  
\(\frac{2}{3}\) of 6 = \______\ |
| b) 12 = 4 ______ | ![Picture](image) | \(\frac{1}{4}\) of 12 = \______\  
\(\frac{2}{4}\) of 12 = \______\  
\(\frac{3}{4}\) of 12 = \______\ |
| c) 15 = 3 ______ | ![Picture](image) | \(\frac{1}{3}\) of 15 = \______\  
\(\frac{2}{3}\) of 15 = \______\ |

Then draw the following picture on the board:

![Picture](image)

Tell students that one person said the picture represented \(\frac{5}{4}\) and another person said the picture represented \(\frac{3}{4}\). Ask your students to explain what both people were thinking. (The picture could represent \(\frac{5}{4}\) because \(\frac{5}{8}\) of the squares are shaded. The picture could represent \(\frac{3}{4}\) because \(\frac{3}{8}\) of one whole block is shaded.) To say what fraction of a figure is shaded, you first have to know what part of the fraction is being taken as the whole.

Ask your students what they would do to find \(\frac{1}{3}\) of 15 dots? (Divide the dots into 3 columns and count how many were in each column). Ask them to find a division statement that would suit the model they drew. \((15 ÷ 3 = 5)\)

Write on the board: “\(\frac{1}{3}\) of 15 = 5 means 15 \(÷\) 3 = 5”. Give your students several more statements to rewrite as normal division statements, like \(\frac{1}{3}\) of 12, \(\frac{1}{5}\) of 15, \(\frac{1}{2}\) of 20, \(\frac{1}{8}\) of 16, \(\frac{1}{2}\) of 10, and so on.
Ask your students how they could use the exercise they finished to solve the following problem: circle \(\frac{1}{3}\) of a set of dots: ●●●●●●●●●●●●. Draw several sets of dots (such as 12 dots, 18 dots, 24 dots) in a line (not in two rows!) and ask them to circle half and then a third for each set.

Ask your students to tell you several ways to find \(\frac{2}{3}\) of 6 dots (or small circles). If the following two solutions do not arise, present them to your students:

1) To find two thirds of 6, you could find one third of six and multiply by 2
2) Circle two out of every 3 dots and count the total number of dots circled.

Have students find fractions of sets of objects (you can give the same number of objects arranged in different ways, for example, have your students find \(\frac{3}{4}\) of 16 boxes with the boxes arranged as \(4 \times 4, 8 \times 2\) or \(16 \times 1\) arrays.)

---

**ACTIVITY 1**

(Adapted from Atlantic Curriculum A4.3) Give students 18 counters and a large circle divided into 3 equal parts. Ask students to use the circle and the counters to find \(\frac{2}{3}\) of 18. Then ask them to explain how they would use the circle to find \(\frac{2}{3}\) of 33. Then have them draw a circle they would use to find \(\frac{3}{5}\) of 20.

**ACTIVITY 2**

Before doing this activity, ensure that students are comfortable finding fractions of numbers such as: \(\frac{1}{3}\) of 15. Teach this by saying: What is \(\frac{2}{3}\) of 4? If I divide 4 dots into 4 columns, how many do I have in each column? In 3 columns? What is \(\frac{2}{3}\) of 7? If I divide 7 dots into 7 columns, how many are in each column? In 5 columns? Repeat until students can tell you that \(\frac{3}{5}\) of 15 is 13 without having to divide 15 dots into 15 columns. Then use the **BLM** "Math Bingo Game (Sample boards)" with the **BLM** "Cards (Fractions of numbers)".
Extensions

1. How many months are in:
   a) \(\frac{1}{2}\) year?  
   b) \(\frac{2}{3}\)  year? 
   c) \(1\frac{1}{2}\) years?

2. How many minutes are in:
   a) \(\frac{2}{3}\) of an hour? 
   b) \(\frac{1}{4}\) of an hour? 
   c) \(1\frac{1}{10}\) of an hour?

3. \(\frac{5}{8}\) of a day is how many hours?

4. Teach students how to describe multiplicative relationships between quantities by using simple fractions. Explain to your students that if you have 4 marbles and I have 4 marbles, then I have as many marbles as you have and you have as many marbles as I have. **ASK:** If I have 4 marbles and you have 8 marbles, how many times more marbles do you have than I have? If I have 4 marbles and you have 3 times as many marbles as I have, how many marbles do you have? If I have 4 marbles and you have 10 marbles, how many times more marbles do you have than I have? Guide students by saying: Two times as many marbles would be 8 and three times as many would be 12. If you have 10, that is halfway between two times as many and three times as many. What number is halfway between two and three? (two and a half) So you have \(2\frac{1}{2}\) times as many marbles as I have.

   Ask students to decide how many times more marbles they have than you if:
   a) You have 4 marbles and they have 6 marbles. \((1\frac{1}{2})\)
   b) You have 3 marbles and they have 8 marbles. \((2\frac{2}{3})\)
   c) You have 10 marbles and they have 25 marbles. \((2\frac{1}{2})\)
   d) You have 8 marbles and they have 12 marbles. \((1\frac{1}{2})\)
   e) You have 6 marbles and they have 9 marbles. \((1\frac{1}{2})\)
   f) You have 6 marbles and they have 8 marbles. \((1\frac{1}{3})\)

5. By weight, about \(\frac{1}{5}\) of a human bone is water and \(\frac{3}{4}\) is living tissue. If bone weighs 120 grams, how much of the weight is water and how much is tissue?

6. Explain to your students that you can have a fraction of a fraction! Demonstrate finding half of a fraction by dividing it into a top half and a bottom half:

\[
\frac{1}{2} \text{ of } \frac{3}{5} \quad \text{is} \quad \frac{3}{10}
\]

Have them find:

a) \(\frac{1}{2} \text{ of } \frac{5}{9}\)

b) \(\frac{1}{3} \text{ of } \frac{5}{7}\)

Then have them draw their own pictures to find:

c) \(\frac{1}{2} \text{ of } \frac{3}{7}\)

d) \(\frac{1}{2} \text{ of } \frac{2}{6}\)

e) \(\frac{1}{2} \text{ of } \frac{5}{6}\)

f) \(\frac{1}{2} \text{ of } \frac{4}{7}\)

**Bonus**

Find two different ways of dividing the fraction \(\frac{3}{7}\) in half.
ANSWER:

\[
\begin{array}{c}
\text{\(\frac{1}{2}\) of \(\frac{4}{7}\) is } \frac{4}{14} \\
\text{Divide the 4 pieces in half from left to right.}
\end{array}
\]

\[
\begin{array}{c}
\frac{1}{2} \text{ of } \frac{5}{7} \text{ is } \frac{5}{14} \\
\end{array}
\]

ASK: Are the two fractions \(\frac{5}{7}\) and \(\frac{5}{14}\) equivalent? How do you know?

7. Revisit the worksheet NS5-63. Ask students if they remember how they estimated the red section in Question 4 and then to explain their thinking. Then ask them to estimate the fraction of each flag that is blue.

For instance, for the flag of Chile in Part a), a student might say:

Half the flag is not red.

About \(\frac{1}{3}\) of the part that is not red is covered by the blue square with the star in it.

If you take half of a flag and cut it into thirds, you are left with \(\frac{1}{6}\) of the flag (\(\frac{1}{3}\) of \(\frac{1}{2}\) is \(\frac{1}{6}\)).

So \(\frac{1}{6}\) of the flag is covered by the blue square with a star in it.

About half of that square is covered by a white star and the other half is blue. So \(\frac{1}{2}\) of \(\frac{1}{6}\) of the flag is blue, but \(\frac{1}{2}\) of \(\frac{1}{6}\) is \(\frac{1}{12}\).

So altogether about \(\frac{1}{12}\) of the flag is blue.

Ask your students to determine by drawing a picture (as shown above) what the following amounts would be: \(\frac{1}{3}\) of \(\frac{1}{2}\), \(\frac{1}{3}\) of \(\frac{1}{2}\), \(\frac{1}{3}\) of \(\frac{1}{2}\), \(\frac{1}{3}\) of \(\frac{1}{3}\), \(\frac{1}{3}\) of \(\frac{1}{3}\), \(\frac{1}{3}\) of \(\frac{1}{3}\), and so on.

Then have them find some flags in an atlas and estimate what fraction of each flag is covered by a particular colour.

Have students choose a sports team or other logo of their choice and decide what fraction of the logo is one colour by using counters. Students could revisit this exercise when they compare and order fractions.
GOALS
Students will solve problems involving fractions of whole numbers

PRIOR KNOWLEDGE REQUIRED
Fractions of a whole number

Remind your students of the problems they solved in NS5-73: Fractions of Whole Numbers.

Ask them to present as many methods to find the missing fraction in the statement 2 is _____ of 10 as they can. This time, they have to draw the picture themselves. They should keep making groups of 2 until they reach 10. Have them find the missing fraction for the following statements: 4 is _____ of 12, 4 is _____ of 20, 4 is _____ of 16, 5 is _____ of 20, etc.

ASK: How many months are in a year? How many months are in \( \frac{2}{3} \) of a year? Which is longer? 9 months or \( \frac{2}{3} \) of a year? Which is longer—21 months or 1 \( \frac{5}{6} \) years? HINT: How many months are in \( \frac{5}{6} \) years? In 1 \( \frac{5}{6} \) years?

How many minutes are in an hour? How many minutes are in \( \frac{2}{3} \) of an hour? If Rita studied for 35 minutes and Katie studied for \( \frac{2}{3} \) of an hour, who studied longer? Katie started studying at 7:48 PM. Her favourite television show starts at 8:30 PM. Did she finish on time?

Tell students that you want to change word problems about fractions into mathematical sentences. Ask what symbol they would replace each word or phrase by: more than (>), is (=), half \((\frac{1}{2})\), three quarters \((\frac{3}{4})\).

Write on the board:

Calli’s age is half of Ron’s age.
Ron is twelve years old.
How old is Calli?

Teach students to replace each word they do know with a math symbol and what they don’t know with a blank:

\[ \underline{\text{Calli’s age}} \quad \underline{=} \quad \frac{1}{2} \quad \underline{\text{of}} \quad \underline{12} \]

Have them do similar problems of this sort.

a) Mark gave away \( \frac{3}{4} \) of his 12 stamps. How many did he give away? \((\underline{= \frac{3}{4} \text{ of } 12})\)

b) There are 8 shapes. What fraction of the shapes are the 4 squares? \((\underline{\text{of } 8} = 4)\)

c) John won three fifths of his five sets of tennis. How many sets did he win? \((\underline{=} \frac{3}{5} \text{ of } 5)\)
Then have students change two sentences into one, replacing the underlined words with what they’re referring to:

a) Mark has 12 stamps. He gave away \(\frac{3}{4}\) of them. (Mark gave away \(\frac{3}{4}\) of his 12 stamps)

b) A team played 20 games. They won 11 of them.

Then have students solve several word problems, for **EXAMPLE**:  
Anna had 10 strawberries. She ate two of them. What fraction of her strawberries did she eat?

**Literacy Connection**

Read *Alice in Wonderland* with them and give them the following problems.

a) The gardeners coloured red \(\frac{2}{3}\) of 15 roses. How many roses are red now? How many roses remained blank?

b) Alice ate 12 of 20 shrinking cakes. Which fraction of the shrinking cakes did she eat? Is that more than a half?

**Extension**

The chart shows the times of day when the Eyed Lizard is active:

<table>
<thead>
<tr>
<th></th>
<th>Awake but inactive</th>
<th>Asleep</th>
<th>Awake and active</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) What fraction of the day is the lizard…</td>
<td>i) awake but inactive</td>
<td>ii) asleep</td>
<td>iii) awake and active</td>
</tr>
<tr>
<td></td>
<td>i) awake but inactive</td>
<td>ii) asleep</td>
<td>iii) awake and active</td>
</tr>
<tr>
<td>b) How many hours a day is the lizard…</td>
<td>i) awake but inactive</td>
<td>ii) asleep</td>
<td>iii) awake and active</td>
</tr>
</tbody>
</table>

![Chart showing times of day when the Eyed Lizard is active](chart.png)
**NS5-75**

**Comparing and Ordering Fractions**

Draw the following chart on the board.

<table>
<thead>
<tr>
<th></th>
<th>1 whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(\frac{1}{3})</td>
<td>(\frac{1}{3})</td>
</tr>
</tbody>
</table>

Have students fill in the remaining boxes. Then ask volunteers to name fractions that are:

- a) less than one third
- b) greater than two thirds
- c) between one half and two thirds
- d) between three fifths and four fifths
- e) between one quarter and one half
- f) equivalent to one half
- g) greater than three fifths
- h) equivalent to one whole

Have students write > (greater than) or < (less than) in between the fractions as appropriate:

- a) \(\frac{2}{3}\), \(\frac{3}{4}\)  
- b) \(\frac{2}{3}\), \(\frac{3}{5}\)
- c) \(\frac{1}{3}\), \(\frac{2}{5}\)  
- d) \(\frac{1}{2}\), \(\frac{2}{5}\)

**ASK:** Which fraction is larger: \(\frac{2}{7}\) or \(\frac{3}{7}\)? How do you know? What makes these fractions easy to compare?

Tell your students that you would like to compare \(\frac{1}{3}\) and \(\frac{5}{12}\). How can you turn this problem into a problem like the last one? Emphasize that turning one problem into a different one that they already know how to do is a tool that mathematicians use every day. Is \(\frac{1}{3}\) equivalent to a fraction with denominator 12? Which fraction? \(\frac{4}{12}\). Demonstrate this with a picture:

\[
\frac{1}{3} \times 4 = \frac{4}{12}
\]

Which fraction is larger—\(\frac{1}{12}\) or \(\frac{5}{12}\)? Which fraction is larger—\(\frac{1}{3}\) or \(\frac{5}{12}\)? How do you know?

Have students compare (and demonstrate using a picture):

- a) \(\frac{1}{2}\) and \(\frac{1}{5}\)  
- b) \(\frac{2}{3}\) and \(\frac{5}{6}\)
- c) \(\frac{3}{5}\) and \(\frac{5}{10}\)  
- d) \(\frac{1}{2}\) and \(\frac{7}{12}\)
Bonus
\[ \frac{4}{10} \text{ and } \frac{7}{12} \]. **HINT:** Use the answers to a) and d).

Then have students compare two fractions, where the denominator of the second divides evenly into the denominator of the first (**EXAMPLES:** \( \frac{3}{8} \text{ and } \frac{1}{4} \), \( \frac{7}{10} \text{ and } \frac{3}{5} \), \( \frac{2}{5} \text{ and } \frac{1}{2} \)).

Finally, mix up the order of the fractions, so that sometimes the larger denominator is first and other times, it is second. Always make sure that the smaller denominator divides evenly into the larger denominator. Challenge students not to need a picture.

**Extensions**

1. Have students place these fractions (0, \( \frac{1}{5} \), \( \frac{2}{5} \), \( \frac{3}{4} \), \( \frac{1}{4} \), \( \frac{1}{10} \), \( \frac{1}{40} \), \( \frac{1}{2} \), \( \frac{11}{40} \), \( \frac{19}{20} \)) on a number line by finding equivalent fractions over 40.

   \[
   0 \quad \frac{1}{5} \quad \frac{2}{5} \quad \frac{3}{4} \quad \frac{1}{4} \quad \frac{1}{10} \quad \frac{1}{40} \quad \frac{1}{2} \quad \frac{11}{40} \quad \frac{19}{20} \quad 1
   \]

2. (Adapted from Atlantic Curriculum A2) Ask students if they think that \( \frac{1}{40} \) is a small fraction or a large fraction. Then discuss how long \( \frac{1}{40} \) m is. It is 4 times smaller than a tenth of a m, so it is 4 times smaller than 10 cm. Have students hold their fingers apart the distance they think that represents. **ASK:** Is that a small distance or a large distance? Then ask students whether \( \frac{1}{40} \) of the Canadian population is a small number or a large number. **ASK:** If you take 1 out of every 40 people, how many of every 1 000 people is that? Write on the board:

   \[
   \frac{1}{40} \times \frac{1}{1000} = \ _ _ _ _
   \]

   Have students find the numerator. Then say: If we take 1 out of every 40 people in Canada, we are taking 25 out of every 1 000. How many out of every 10 000 people is that? (250) Out of every 100 000? (2 500) Out of every 1 000 000? (25 000) If Canada has about 30 000 000 people, how many people is 1 out of 40? (750 000) Tell students that this is about the number of people in all of New Brunswick; quite a large number even though it is a small fraction.
NS5-76
Lowest Common Denominator

GOALS
Students will compare fractions with different denominators by changing both fractions to have the same denominator.

PRIOR KNOWLEDGE REQUIRED
Comparing fractions with the same denominator
Changing fractions to an equivalent fraction with a different denominator
Lowest common multiples

VOCABULARY
lowest common multiple
lowest common denominator

Review finding lowest common multiples (see PA5-16) and drawing lines to cut pies into more pieces to make an equivalent fraction (see NS5-72).

Draw two pies on the board with a different fraction shaded in each.
(EXAMPLES: \(\frac{1}{3}\) and \(\frac{1}{2}\), or \(\frac{1}{4}\) and \(\frac{1}{2}\), or \(\frac{1}{2}\) and \(\frac{1}{6}\), or \(\frac{2}{3}\) and \(\frac{3}{4}\), or \(\frac{2}{5}\) and \(\frac{3}{5}\).)

Have volunteers find i) the number of pieces in each pie and ii) the LCM of these two numbers. Then have volunteers draw lines to divide the pies into that many pieces. Students should then individually write the new fractions in their notebooks and compare them. Which fraction is larger?

Teach students how they can compare two fractions with different denominators without using pictures:

STEP 1: Find the lowest common multiple of the two denominators

STEP 2: Change both fractions to equivalent fractions with that denominator

STEP 3: Compare the two fractions that have the same denominator (the fraction with the larger numerator will be larger)

Be sure to include examples where the larger denominator is first and examples where the larger denominator is second.

Bonus
Have students compare a list of 3 or 4 fractions by finding the lowest common multiple of all the denominators.

Extensions
1. Tom uses \(\frac{7}{9}\) of a cup of flour to make bread and Allan uses 2 \(\frac{1}{2}\) cups. Who uses more flour? (Encourage students to do this problem first by comparing improper fractions and then by comparing mixed fractions. Which method is easier?)

2. Fill in the missing numbers in the sequence:
   \(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, __, \frac{5}{6}, __\)

   HINT: Change all fractions to equivalent fractions with denominator 6.

3. Put the numbers into the correct boxes to complete the sequence:
   \(\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, __, __, __\)
   \(\frac{4}{10}, \frac{1}{2}, \frac{3}{5}, \frac{9}{10}, \frac{1}{5}, 1\)
NS5-77
Adding and Subtracting Fractions

GOALS
Students will add and subtract fractions with the same denominators.

PRIOR KNOWLEDGE REQUIRED
Naming fractions
Addition and subtraction of whole numbers
1 whole

VOCABULARY
fraction
regrouping
numerator
denominator

Draw two large circles representing pizzas (you can use paper pizzas as well) and divide them into 4 pieces each, shading them as shown.

Explain that these are two plates with several pizza pieces on each. How much pizza do you have on each plate? Write the fractions beneath the pictures. Tell them that you would like to combine all the pieces onto one plate, so put the “+” sign between the fractions and ask a volunteer to draw the results on a different plate. How much pizza do you have now?

Draw on the board:

\[
\begin{align*}
\frac{1}{4} + \frac{2}{4} &= \frac{3}{4}
\end{align*}
\]

Tell your students that you would like to regroup the shaded pieces so that they fit onto one circle. SAY: I shaded 2 fourths of one circle and 1 fourth of another circle. If I move the shaded pieces to one circle, what fraction of that circle will be shaded? How many pieces of the third circle do I need to shade? Tell them that mathematicians call this process adding fractions. Just like we can add numbers, we can add fractions too.

Do several examples of this, like \(\frac{1}{5} + \frac{3}{5}, \frac{1}{3} + \frac{1}{3}\), never extending past 1 whole circle. Ask your students: You are adding two fractions. Is the result a fraction too? Does the size of the piece change while we transport pieces from one plate to the other? What part of the fraction reflects the size of the piece—top or bottom? Numerator or denominator? When you add fractions, which part stays the same, the top or the bottom; the numerator or the denominator? What does the numerator of a fraction represent? (The number of shaded pieces) How do you find the total number of shaded pieces when you moved them to one pizza? What operation did you use?

Show a couple more examples using pizzas, and then have them add the fractions without pizzas. Assign lots of questions like \(\frac{3}{5} + \frac{1}{5} + \frac{2}{7} + \frac{3}{7} + \frac{2}{11} + \frac{4}{11}\), etc. Enlarge the denominators gradually.

**Bonus**

Add:

\[
\begin{align*}
\frac{12}{134} + \frac{45}{134} &+ \frac{67}{1567} + \frac{78}{1567} &+ \frac{67}{456} + \frac{49}{456} \\
\end{align*}
\]

**Bonus**

Add more fractions:

\[
\begin{align*}
\frac{3}{17} + \frac{1}{17} + \frac{5}{17} &+ \frac{5}{94} + \frac{4}{94} + \frac{7}{94} &+ \frac{3}{19} + \frac{5}{19} + \frac{5}{19} + \frac{1}{19} + \frac{3}{19} \\
\end{align*}
\]
Return to the pizzas and say that now you are taking pieces of pizza away. There was $\frac{3}{4}$ of a pizza on a plate. You took away $\frac{1}{4}$. Show on a model the one piece you took away:

![Pizza Model]

How much pizza is left? Repeat the sequence of exercises and questions you did for addition using subtraction.

**Extensions**

1. Tell your students that sometimes adding fractions can result in more than one whole. Draw on the board:

\[
\begin{align*}
\frac{3}{4} & \quad + \quad \frac{2}{4} \\
\hline
\end{align*}
\]

Ask how many parts are shaded in total and how many parts are in 1 whole circle. Tell your students that, when adding fractions, we like to regroup the pieces so that they all fit onto 1 circle. **ASK:** Can we do that in this case? Why not? Tell them that since there are more pieces shaded than in 1 whole circle, the next best thing we can do is to regroup them so that we fit as many parts onto the first circle as we can and then we put only the leftover parts onto the second circle.

Draw on the board:

\[
\begin{align*}
\frac{3}{4} & \quad + \quad \frac{2}{4} \\
\hline
\end{align*}
\]

Ask how many parts are shaded in the first circle and how many more parts do we need to shade in the second circle. Ask a volunteer to shade that many pieces and then tell them that mathematicians write this as:

\[
\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1 + \frac{1}{4}
\]

Do several examples of this where the sum of the fractions is more than 1 whole, using more complex shapes to make it look harder as students get used to the new concept and then continue with examples where the sum is more than 2 wholes.

2. Teach your students the role of 0 in adding and subtracting fractions.

a) $\frac{3}{5} - \frac{3}{5}$  

b) $\frac{5}{7} + \frac{0}{7}$  

c) $\frac{3}{8} - \frac{0}{8}$  

d) $\frac{6}{7} - 0$  

e) $1\frac{2}{3} + 0$

Tell them that 0 is the same number with any denominator. This is very different from fractions with any other numerator, where the denominator matters a lot.

3. Once students know how to change a fraction to an equivalent fraction, they can, as an enriched exercise, quickly learn to add fractions. See the JUMP fractions unit, available from our website www.jumpmath.org for material on adding fractions.
4. Revisit Exercise 4 d) of worksheet NS5-63. Have your students estimate what fraction of the flag is red and then do the subdividing necessary (the subdividing should reveal that \( \frac{12}{40} \) of the flag is red). Now that students know how to compare fractions, they should check how close their answer is to the real fraction. Have your students compare the fractions guesses such as \( \frac{1}{3} \), \( \frac{1}{4} \), and \( \frac{2}{5} \) to \( \frac{12}{40} \) by finding equivalent fractions with denominator 120: \( \frac{40}{120} \), \( \frac{30}{120} \), \( \frac{48}{120} \), and \( \frac{36}{120} \). **ASK:** Which of the three guesses (\( \frac{1}{3} \), \( \frac{1}{4} \), and \( \frac{2}{5} \)) are closest to \( \frac{36}{120} \)?

### NS5-78

**Fractions Review**

This worksheet is a review of the fractions section and can be used as consolidating homework.

**GOALS**

Students will compare fractions with the same numerators or same denominators and review equivalent fractions.

**Extensions**

1. **ASK:** Which is greater, eight thirds or twelve fifths? (2 and two thirds or 2 and two fifths) and have students write the fractions as improper fractions and then as mixed fractions. Which way makes it easier to compare the fractions? Why?

   **REFLECT:** Recall that when learning reciprocals in Extension 5 of NS5-70: Investigating Mixed and Improper Fractions, it was better to use improper fractions. When comparing 8 thirds to 12 fifths, it is better to change the fractions to mixed fractions. Emphasize that sometimes mixed fractions are more convenient and sometimes improper fractions are more convenient. As students gain more experience they will learn to predict which will be more convenient. It is important to understand both forms so that they can choose which one is more convenient for their purpose.

2. **Give pairs of students cards with the following fractions on them:**

   \[
   \frac{3}{6} \quad \frac{2}{3} \quad \frac{1}{4} \quad \frac{7}{8} \quad \frac{5}{12} \quad \frac{5}{6}
   \]

   Ask them to arrange the cards in order from least to greatest (\( \frac{1}{4} \), \( \frac{5}{12} \), \( \frac{3}{6} \), \( \frac{7}{8} \), \( \frac{5}{6} \)) and to give reasons for their arrangement.

   Possible reasons could include: \( \frac{5}{6} \) is only one step away from reaching the end of a number line divided into 6 parts. Similarly, \( \frac{7}{8} \) is only one step away from reaching a number line divided into 8 parts, but the steps in \( \frac{7}{8} \) are smaller, so the person who is \( \frac{7}{8} \) of the way across is closer to the end than the person who is only \( \frac{5}{6} \) of the way along the line; 6 steps out of 12 would be a half, and \( \frac{5}{12} \) is almost a half, so it is more than \( \frac{1}{2} \).
NS5-79
Place Value Decimals

**NOTE:** In this and subsequent lessons, blank hundreds charts/hundreds blocks are used extensively to illustrate decimals and decimal concepts. If you have an overhead projector or interactive white board, you can use the BLM “Blank Hundreds Charts” to project and work with hundreds blocks on a board, wall, or screen. Alternatively, you can (a) work with enlarged photocopies of the BLM or (b) draw an oversized hundreds block on chart paper and laminate it.

Remind students that there are 10 tenths in every whole—10 tenths in a whole pie, 10 tenths in a whole tens block, 10 tenths in a whole chocolate bar.

**ASK:** What is $\frac{1}{10}$ of a centimetre? (a millimetre) How many millimetres are in a centimetre? (10) What is $\frac{1}{10}$ of a dollar? (a dime or 10 cents) How many dimes do you need to make a dollar? (10) What is one tenth of 1000? (one hundred) How many hundreds do you need to make a thousand? What is one tenth of 100? (ten) What is one tenth of 10? (one) What is one tenth of 1? (one tenth)

**WRITE:**

$$1000 = 1 \text{ thousand} + 0 \text{ hundreds} + 0 \text{ tens} + 0 \text{ ones}$$
$$100 = 1 \text{ hundred} + 0 \text{ tens} + 0 \text{ ones}$$
$$10 = 1 \text{ ten} + 0 \text{ ones}$$
$$1 = 1 \text{ one}$$

Tell students that just as we have a place value for thousands, hundreds, tens, and ones, mathematicians have invented a place value for tenths. Write the number 47 on the board and **ASK:** If we read the digits from left to right, do we start at the largest place value or the smallest? Does the 4 mean 4 tens or 4 ones? So if the largest place value is on the left and the place values get smaller as we move right, where should we put the place value for the tenths—to the left of the 4 or to the right of the 7? If I have 47 pies and one tenth of a pie, I could try writing it like this:

$$\begin{array}{c}
\text{tens} & 4 & 7 & 1 \\
\text{ones} & \uparrow & \text{tenths}
\end{array}$$

Does anyone see a problem here? What number does this look like?

We need a way to tell the difference between “four hundred seventy-one” and “forty-seven and one tenth.” We need a way to separate the ones from the tenths so that we know how many whole units we have. We could actually use anything.

**DRAW:**

$$\begin{array}{c}
4 & 7 & 1 \\
\text{tens} & \text{ones} & \text{tenths}
\end{array}$$
**ASK:** How do we show the difference between whole dollars and tenths of a dollar, between dollars and dimes? What do we put in between the number of dollars and the number of dimes? Have a volunteer show four dollars and thirty cents on the board: $4.30 Tell them that the dot between the 4 and the 3 is called a **decimal point** and **ASK:** What if the decimal point wasn’t there—how much money would this look like?

Ask students to identify the place value of the underlined digits:

\[
\begin{align*}
2.5 & \quad 34.1 & \quad 6.3 & \quad 10.4 & \quad 192.4 & \quad 37.2 & \quad 8,073.2 & \quad 100.5
\end{align*}
\]

On a transparency or enlarged photocopy of a hundreds block, have a volunteer shade one tenth of the block. If the student shades a row or column, point out that the student chose an organized way of shading one tenth rather than randomly shading ten squares. If the student did the latter, have another volunteer show an organized way of shading one tenth. Then ask another volunteer to circle or otherwise highlight one tenth of the shaded part. **ASK:** What fraction of the hundreds block is the circled part? What coin is one tenth of a loonie? What coin is one tenth of a dime? What fraction of a loonie is that coin? What fraction of a whole anything is one tenth of a tenth? (one hundredth)

Write on the board: 4 183.25

Cover up all but the 4 and tell students that the 4 is the thousands digit. Uncover the 1 and **ASK:** What place value is the 1? How do you know? (PROMPT: What is one tenth of a thousand?) Continue in this way, uncovering each digit one at a time until you uncover the 5 and **SAY:** We know the 2 is the tenths digit. What is one tenth of a tenth? What place value is the 5?

Write on the board:

\[
\begin{align*}
\text{ones} & \quad \rightarrow \quad 8.04 & \quad \leftarrow \quad \text{hundredths} \\
\text{tenths} & \quad & \quad 
\end{align*}
\]

In their notebooks, have students write the place value of the underlined digit in these numbers:

\[
\begin{align*}
2.71 & \quad 3.42 & \quad 7.65 & \quad 8.46 & \quad 9.01 & \quad 0.83 & \quad 1.04 & \quad .97 & \quad .45 & \quad 4.5 & \quad 45
\end{align*}
\]

Students can refer to the board for the spelling of “tenths” and “hundredths.”

Have students identify the place value of the digit 5 in each of these numbers:

\[
\begin{align*}
5.03 & \quad 8.05 & \quad 50.03 & \quad 30.05 & \quad 74.35 & \quad 743.5 & \quad 7435
\end{align*}
\]

**Bonus**

432.15

34 521.08

**Extensions**

1. Remind students that a hundred is ten times more than ten. Now ask them to picture a square divided into ten parts and a square divided into a hundred parts. Each little part in the second square (each hundredth) is ten times smaller than a part in the first square (a tenth). Another way to describe this is to say that there are ten hundredths in each tenth. **ASK:** What is ten times a hundred? (a thousand) What is one tenth of a hundred? (a thousandth) What is ten times a thousand? (ten thousand) What is one tenth of a thousand? (one ten thousandth)
Have students write the place value of the digit 5 in each number:

41.015 32.6752 872.0105 54 321.02679 867 778.3415

How many times more is the first 5 worth than the second 5:

a) 385.5072 3 855.072 38.55072 .3855072
b) 525 52.5 5.25 .525 .0525
c) 5.5 51.5 512.5 5127.5 51270.5

How many times more is the 6 worth than the 2?

6.2 6.32 6.72 6.02 6.102 61.02 610.2 674.312

How many times more is the 2 worth than the 5?

2.5 2.35 2.75 2.05 2.105 21.05 210.5 274.315

2. Draw a decagon on the board and ask students to count the number of sides. Write “decagon” underneath it. Then write the word “decade” on the board. Ask if anyone can read the word and if anyone knows what the word means. ASK: Do these words have any letters in common? (yes; deca) What do the meanings of the words have in common? (Both words refer to 10 of something: the number of sides, the number of years.) What do you think “deca” means? Then ASK: How many events do you think are in a decathlon? Which month do you think was originally the tenth month when the calendar was first made? (December) Is it still the 10th month? (No, it’s now the 12th month.) Then write the word “decimal” on the board. Underline the first 3 letters and ask students to think about how 10 is important for decimal numbers. (Each digit has 10 times larger place value than the digit on the right. There are 10 single digits from 0 to 9.) Allow several students to answer in their own words.

3. Have students fill in the blanks in questions where you mix up the order of the words ones, tenths, and hundredths.

EXAMPLES:

5.02 ____ tenths ____ hundredths ____ ones

89.13 ____ hundredths ____ ones ____ tenths ____ tens
NS5-80
Decimal Hundredths

Draw the following pictures on the board and ask students to show the fraction \( \frac{4}{10} \) in each picture:

Tell students that mathematicians invented decimals as another way to write tenths: One tenth \( (\frac{1}{10}) \) is written as 0.1 or just .1. Two tenths \( (\frac{2}{10}) \) is written as 0.2 or just .2. Ask a volunteer to write \( \frac{7}{10} \) in decimal notation. (.7 or 0.7) Ask if there is another way to write it. (0.7 or .7) Then have students write the following fractions as decimals:

\[
\begin{align*}
a) \frac{3}{10} & \quad b) \frac{5}{10} & \quad c) \frac{9}{10} & \quad d) \frac{5}{10} & \quad e) \frac{6}{10} & \quad f) \frac{4}{10}
\end{align*}
\]

**Bonus**

Have students convert to an equivalent fraction with denominator 10 and then to a decimal:

\[
\begin{align*}
a) \frac{2}{5} & \quad b) \frac{1}{2} & \quad c) \frac{5}{10}
\end{align*}
\]

In their notebooks, have students rewrite each addition statement using decimal notation:

\[
\begin{align*}
a) \frac{3}{10} + \frac{1}{10} = \frac{4}{10} & \quad b) \frac{5}{10} + \frac{6}{10} = \frac{7}{10} & \quad c) \frac{5}{10} + \frac{3}{10} = \frac{8}{10} & \quad d) \frac{4}{10} + \frac{5}{10} = \frac{9}{10}
\end{align*}
\]

**Bonus**

\[
\begin{align*}
a) \frac{1}{2} + \frac{1}{10} = \frac{7}{10} & \quad b) \frac{1}{2} + \frac{3}{10} = \frac{8}{10}
\end{align*}
\]

Draw on the board:

**ASK:** What fraction does this show? \( \frac{3}{5} \) What decimal does this show? (0.4 or .4)

Repeat with the following pictures:

Have students write the fractions and decimals for similar pictures independently, in their notebooks.
Then ask students to convert the following decimals to fractions, and to draw models in their notebooks:

a) 0.3   b) .8   c) .9   d) 0.2

Demonstrate the first one for them:

\[ 0.3 = \frac{3}{10} \]

Tell students that we use 1 decimal place to write how many tenths there are, and we use 2 decimal places to write how many hundredths there are. Show this picture:

SAY: There are 13 hundredths shaded. We can write this as 0.13 or .13 or \( \frac{13}{100} \).

Have students write both a fraction and a decimal for each of the following pictures:

Tell your students that writing decimals is a little trickier when there are less than 10 hundredths. ASK: What if we have only 9 hundredths—would we write .9? If no one recognizes that .9 is 9 tenths, ASK: How do we write 9 tenths? Is 9 tenths the same as 9 hundredths? Which is larger?

Put up 2 hundreds blocks (with the grid drawn in) and have one volunteer shade 9 tenths and another volunteer shade 9 hundredths. Tell students that we write 9 tenths as .9 and 9 hundredths as .09; write each decimal beneath the corresponding picture.

Put up the following pictures:
Have volunteers write a decimal and a fraction for the first two and then have students do the same for the others independently, in their notebooks.

Put up several more hundreds blocks and tell the students that you are going to mix up the problems. Some pictures will show less than ten hundredths and some will show more than ten hundredths. Students should write the correct decimal and fraction for each picture in their notebooks.

**EXAMPLES:**

![Hundredths Blocks](image1)

One row and one column are shaded. How many squares are shaded? What fraction is shaded?

![Hundredths Blocks](image2)

Discuss various strategies students may have used to count the shaded squares. Did they count the squares to each side of the point at which the row and column overlap (2 + 7 in the row, 4 + 5 in the column, 1 for the square in the middle)? Did they add 10 + 10 for the row and column and then subtract 1 for the one square they counted twice? Did they count the squares in the column (10) and then add 2 + 7 for the squares not yet counted in the row? Did they add the 4 rectangles of white squares (8 + 10 + 28 + 35) and then subtract from 100? Did they push the 4 white rectangles together and see that it forms a 9 by 9 square?

Have students write decimals and fractions for the following pictures.

![Hundredths Blocks](image3)

Encourage students to count the shaded squares using different strategies. Students might count some squares twice and adjust their answers at the end; they might subtract the non-shaded squares from the total; they might divide the shaded parts into convenient pieces, count the squares in the pieces separately, and add the totals. Have students show and explain various solutions to their classmates, so that all students see different ways of counting the squares. Tell students that when they can solve a problem in two different ways and get the same answer, they can know that they’re right. They won’t need you to check the answer because they can check it themselves!
Extensions

1. Put the following sequence on the board: .1, .3, .5, _____

Ask students to describe the sequence (add .2 each time) and to identify the next number (.7). Even though the numbers are not in standard dollar notation, students can think of them in terms of dollars and dimes: .3 is 3 dimes, .5 is 5 dimes, .7 is 7 dimes, and so on. **ASK:** What is 1.3 in terms of dollars and dimes? (1 dollar and 3 dimes)

Have students complete the following sequences by thinking of the numbers in terms of dollars and dimes and counting out loud. This will help students to identify the missing terms, particularly in sequences such as h) and i). Students should also state the pattern rules for each sequence (**EXAMPLE:** start at .1 and add .3).

   a) .1, .4, .7, _____
   b) 3.1, 3.4, 3.7, _____
   c) 1, .9, .8, _____
   d) 1, .8, .6, _____
   e) 3, 2.9, 2.8, _____
   f) 4.4, 4.2, 4, _____
   g) .2, .3, .4, _____ , _____ , _____
   h) .7, .8, .9, _____ , _____ , _____
   i) 2.7, 2.8, 2.9, _____ , _____ , _____
   j) 1.4, 1.3, 1.2, _____ , _____ , _____

2. Have students draw a line 25 squares long on grid paper and mark the ends as shown:

   4.2                                 6.7

Have them mark the location of 4.8, 5.0, and 5.8.

Then repeat with endpoints 42 and 67 and have them mark the locations of 48, 50, and 58.

**ASK:** How are these two questions similar and how are they different?

Notice that 4.2 is just 42 tenths and 6.7 is 67 tenths, so the number line with endpoints 42 and 67 can be regarded as counting the number of tenths.
NS5-81
Tenths and Hundredths

Put up 2 blank hundreds blocks. Have one volunteer shade one tenth of one block, and have a second volunteer shade one hundredth of the other block. Invite other volunteers to write the corresponding decimals and fractions for each block:

0.1 = $\frac{1}{10}$

0.01 = $\frac{1}{100}$

Point to the block showing one tenth and say: How many hundredths does this block show? How else can we write that as a decimal? (We can write 0.10.) Emphasize that 0.1 is equal to 0.10, and tell students that 0.10 can be read as “10 hundredths” or “1 tenth and 0 hundredths.”

Draw on the board:

ASK: How many squares are shaded? (43) How many hundredths does this picture show? (43) How many full rows are shaded? (4) Since 1 full row is 1 tenth, how many tenths are shaded? (4) How many more squares are shaded? (3) SAY: There are 4 full rows and 3 more squares shaded. So there are 4 tenths and 3 hundredths. Tell students that we can write 4 tenths and 3 hundredths as follows:

<table>
<thead>
<tr>
<th>ones</th>
<th>0.43</th>
<th>hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>tenths</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can read this as “43 hundredths” or “4 tenths and 3 hundredths.”

ASK: How is this similar to the different ways we can read the 2-digit number 43? (we can read 43 as 4 tens and 3 ones or just as 43 ones)

On grid paper, have students draw a hundreds block, shade in a given fraction, and then write the fraction as a decimal. (Students can also work on a copy of the BLM “Blank Hundreds Charts.”) When students are done, have them read the decimal number in terms of hundredths only and then in terms of tenths and hundredths. Give students more fractions to illustrate and write as decimals:

a) $\frac{36}{100}$   b) $\frac{81}{100}$   c) $\frac{14}{100}$   d) $\frac{41}{100}$   e) $\frac{85}{100}$   f) $\frac{72}{100}$
Draw the following figure on the board:

\[
\begin{array}{cccccccccc}
\text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} \\
\end{array}
\]

**ASK:** How many full rows of ten are there? (none) So how many tenths do we have? (0) How many hundredths are there? (4). Write on the board:

\[
\begin{array}{c}
\text{ones} \\
\text{tenths} \\
\text{hundredths}
\end{array} \leftarrow 0.04
\]

**ASK:** How can we read this number? (4 hundredths or 0 tenths and 4 hundredths)

Have students draw the following fractions on grid paper and write the corresponding decimals:

\[
a) \ \frac{5}{100} \quad b) \ \frac{7}{100} \quad c) \ \frac{1}{100} \quad d) \ \frac{2}{100} \quad e) \ \frac{6}{100} \quad f) \ \frac{3}{100}
\]

Finally, draw the following figure on the board:

\[
\begin{array}{cccccccccc}
\text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} & \text{\_\_\_} \\
\end{array}
\]

**ASK:** How many full rows of ten are there? (4) So how many tenths do we have? (4) How many hundredths are in 4 tenths? (40) Are any other hundredths shaded? (no, just the 4 full rows)

Write on the board:

\[
\begin{array}{c}
\text{ones} \\
\text{tenths} \\
\text{hundredths}
\end{array} \leftarrow 0.40
\]

Tell students that we can read this as “4 tenths and 0 hundredths” or “40 hundredths.”

Have students draw the following fractions and write the corresponding decimals.

\[
a) \ \frac{50}{100} \quad b) \ \frac{70}{100} \quad c) \ \frac{80}{100} \quad d) \ \frac{30}{100} \quad e) \ \frac{50}{100} \quad f) \ \frac{10}{100}
\]

Now have students illustrate and rewrite fractions that have either 0 tenths or 0 hundredths:

\[
a) \ \frac{6}{100} \quad b) \ \frac{60}{100} \quad c) \ \frac{80}{100} \quad d) \ \frac{3}{100} \quad e) \ \frac{20}{100} \quad f) \ \frac{8}{100}
\]

Finally, give students a mix of all 3 types of fractions:

\[
a) \ \frac{36}{100} \quad b) \ \frac{40}{100} \quad c) \ \frac{5}{100} \quad d) \ \frac{18}{100} \quad e) \ \frac{46}{100} \quad f) \ \frac{9}{100}
\]

**Extension**

Decimals are not the only numbers that can be read in different ways. Show students how all numbers can be read according to place value. The number 34 can be read as “34 ones” or “3 tens and 4 ones.” Similarly, 7.3 can be read as “73 tenths” or “7 ones and 3 tenths.” Challenge students to find 2 ways of reading the following numbers:
a) 3 500 (3 thousands and 5 hundreds or 35 hundreds)
b) 320 (3 hundreds and 2 tens or 32 tens)
c) 5.7 (5 ones and 7 tenths or 57 tenths)
d) 1.93 (19 tenths and 3 hundredths or 193 hundredths)
e) 0.193 (19 hundredths and 3 thousandths or 193 thousandths)

---

**NS5-82**

**Changing Tenths to Hundredths**

**GOALS**

Students will understand that whole decimal tenths can be written with a 0 in the hundredths position to form an equivalent decimal.

**PRIOR KNOWLEDGE REQUIRED**

Equivalent fractions
Decimal notation
Tenths and hundredths

**VOCABULARY**

- equivalent fraction
- equivalent decimal

Draw the following figure on the board:

ASK: What fraction of the first square is shaded? What fraction of the second square is shaded? Are these equivalent fractions? How do you know?

Draw the following figure on the board:

ASK: What fraction of the square is shaded? How many of the 100 equal parts are shaded? How many of the 10 equal rows are shaded? Are these equivalent fractions?

Have students use the following pictures to find equivalent fractions with denominators 10 and 100:

a) \[
\frac{80}{100} = \frac{8}{10}
\]
b) \[
\frac{2}{10}
\]
c) \[
\frac{40}{100} = \frac{4}{10}
\]
d) \[
\frac{1}{10}
\]

Then have students find equivalent fractions without using pictures:

a) \[
\frac{80}{100} = \frac{8}{10}
\]
b) \[
\frac{100}{10} = \frac{10}{10}
\]
c) \[
\frac{40}{100} = \frac{4}{10}
\]
d) \[
\frac{100}{10} = \frac{1}{10}
\]
When students can do this confidently, ask them to describe how they are getting their answers. Then remind students that a fraction with denominator 100 can be written as a decimal with 2 decimal places. **ASK:** What decimal is equivalent to $\frac{80}{100}$? (0.80) Remind them that a fraction with denominator 10 can be written as a decimal with 1 decimal place and **ASK:** What decimal is equivalent to $\frac{8}{10}$? (0.8) Tell them that mathematicians call 0.80 and 0.8 equivalent decimals and ask if anyone can explain why they are equivalent. (They have the same value; the fractions they are equivalent to are equivalent).

Have students rewrite these equivalent fractions as equivalent decimals:

- a) $\frac{90}{100} = \frac{9}{10}$
- b) $\frac{20}{100} = \frac{2}{10}$
- c) $\frac{40}{100} = \frac{4}{10}$
- d) $\frac{10}{100} = \frac{1}{10}$

Tell students that saying “0.9 = 0.90” is the same as saying “9 tenths is equal to 90 hundredths or 9 tenths and 0 hundredths.” **ASK:** Is “3 tenths” the same as “3 tenths and 0 hundredths?” How many hundredths is that? Have a volunteer write the equivalent decimals on the board. (.3 = .30)

Have students fill in the blanks:

- a) .3 = $\frac{3}{10} = \frac{30}{100} = .03$
- b) .7 = $\frac{7}{10} = \frac{70}{100} = .70$
- c) .4 = $\frac{4}{10} = \frac{40}{100} = .40$
- d) .5 = $\frac{5}{10} = \frac{50}{100} = .50$
- e) .9 = $\frac{9}{10} = \frac{90}{100} = .90$
- f) .3 = $\frac{3}{10} = \frac{30}{100} = .03$

**Extension**

**ASK:** Do you think that 5 hundredths is the same as 5 hundredths and 0 thousandths? How would we write the decimals for those numbers? (.05 = .050) Do you think that 7 ones is the same as 7 ones and 0 tenths? How would you write that in decimal notation? (7 = 7.0) Have students circle the equivalent decimals in each group of three:

- a) 8 0.8 8.0
- b) 0.04 0.40 0.4
- c) 0.03 0.030 0.3
- d) 0.9 9.0 9
- e) 2.0 0.2 0.20
Decimals and Money

Tell students that a dime is one tenth of a dollar then **ASK:** Does this mean I can take a loonie and fit 10 dimes onto it? Does it mean 10 dimes weigh the same as a loonie? What does it mean? Make sure students understand that you are referring not to weight or area, but to value—a dime has one tenth the value of a dollar, it is worth one tenth the amount. **ASK:** What fraction of a dollar a penny is worth. (one hundredth)

Have students find different ways of making $0.54 using only dimes and pennies. (5 dimes and 4 pennies, 4 dimes and 14 pennies, and so on until 54 pennies)

**GOALS**

Students will relate tenths and hundredths of whole numbers to tenths and hundredths of dollars, that is, to dimes and pennies. Students will use this understanding to compare and order decimals having one and two decimal places.

**PRIOR KNOWLEDGE REQUIRED**

Pennies, dimes, and dollars
Decimal notation
Writing values such as
3 tenths and 5 hundredths as 35 hundredths

**EXAMPLES:**

.94 .04 .90 .27 .70 .03 .60 .58 .05

**ASK:** Which is worth more, 2 dimes and 3 pennies or 8 pennies? How many pennies are 2 dimes and 3 pennies worth? Which is more, 23 or 8? How would we write 2 dimes and 3 pennies as a decimal of a dollar? (.23) How would we write 8 pennies as a decimal? (.08) Which is more, .23 or .08?

**ASK:** How would you make $0.63 using pennies and dimes? How would you make .9 dollars using pennies and dimes? Which is more, .63 or .9?

Give students play-money dimes and pennies and ask them to decide which is more between:

a) .4 and .26 b) .4 and .62 c) .3 and .42 d) .3 and .24

Tell the class that you once had a student who said that .41 is more than .8 because 41 is more than 8. **ASK:** Is this correct? What do you think the student was thinking? Why is the student wrong? (The student was thinking
of the numbers after the decimal point as whole numbers. Since 41 is more than 8, the student thought that .41 would be more than .8. But .8 is 8 tenths and .41 is 41 hundredths. The tenths are 10 times greater than the hundredths, so comparing .8 to .41 is like comparing 8 tens blocks to 41 ones blocks, or 8 cm to 41 mm. There might be more ones blocks, but they’re worth a lot less than the tens blocks. Similarly, there are more mm than there are cm, but 8 cm is still longer than 41 mm.)

Students often make mistakes in comparing decimals with 1 and 2 decimal places. For instance, they will say that .17 is greater than .2. This activity will help students understand the relationship between tenths and hundredths.

Give each student play-money dimes and pennies. Remind them that a dime is a tenth of a dollar (which is why it is written as $0.10) and a penny is a hundredth of a dollar (which is why it is written as $0.01). Ask students to make models of the amounts in the left-hand column of the chart below and to express those amounts in as many different ways as possible by filling in the other columns. (Sample answers are shown in italics.) You might choose to fill out the first row together.

<table>
<thead>
<tr>
<th>Amount</th>
<th>Amount in Pennies</th>
<th>Decimal Names (in words)</th>
<th>Decimal Names (in numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 dimes</td>
<td>20 pennies</td>
<td>2 tenths (of a dollar)</td>
<td>.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 hundredths</td>
<td>.20</td>
</tr>
<tr>
<td>3 pennies</td>
<td>3 pennies</td>
<td>3 hundredths</td>
<td>.03</td>
</tr>
<tr>
<td>4 dimes and</td>
<td>43 pennies</td>
<td>4 tenths and</td>
<td>.43</td>
</tr>
<tr>
<td>3 pennies</td>
<td></td>
<td>3 hundredths</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>43 hundredths</td>
<td></td>
</tr>
</tbody>
</table>

When students have filled in the chart, write various amounts of money on the board in decimal notation and have students make models of the amounts. (EXAMPLES: .3 dollars, .27 dollars, .07 dollars) Challenge students to make models of amounts that have 2 different decimal representations (EXAMPLES: .2 dollars and .20 dollars both refer to 2 dimes).

When you feel students are able to translate between dollar amounts and decimal notation, ASK: Would you rather have .2 dollars or .17 dollars? In their answers, students should say exactly how many pennies each amount represents; they must articulate that .2 represents 20 pennies and so is actually the larger amount.

For extra practice, ask students to fill in the right-hand column of the following chart and then circle the greater amount in each column. (Create several such charts for your students.)

<table>
<thead>
<tr>
<th>Amount (in dollars)</th>
<th>Amount (in pennies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>.15</td>
<td></td>
</tr>
</tbody>
</table>
Extension

Have students create a hundredths chart by filling in a blank hundreds chart with hundredths, beginning with .01 in the top left corner and moving across and then down each row.

Ask students to find the following patterns in their charts. They should describe the patterns of where the numbers occur using the words “column” and “row.”

| .45 | .68 | .14 | .01 |
| .55 | .78 | .25 | .12 |
| .65 | .88 | .34 | .23 |
| .75 | .98 | .45 | .34 |

(Start in the fifth row and fifth column, move down one row and then repeat; Start in the seventh row and eighth column, move down one row and then repeat; Start in the second row and fourth column, move one row down and one column right, then one row down and one column left, and then repeat; Start in the first row and first column, move one row down and one column right, then repeat.)

Bonus

Fill in the missing numbers in the patterns below without looking at your chart.

| .65 | .33 |
| .75 | .53 |
| .95 | .64 |
NS5-84
Changing Notation: Fractions and Decimals

GOALS
Students will translate between fractional and decimal notation.

PRIOR KNOWLEDGE REQUIRED
Decimal notation
Fractions
Tenths and hundredths

Have students copy the following chart, with room for rows a) to e), in their notebooks:

<table>
<thead>
<tr>
<th></th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw on the board:

a) 

b) 

c) 

d) 

e) 

Have students fill in their charts by recording the number of tenths in the first column and the number of hundredths left over (after they’ve counted the tenths) in the second column. When students are done, have them write in their notebooks the fractions shown and the corresponding decimals. Remind them that hundredths are shown with 2 decimal places: 9 hundredths is written as .09, not .9, because .9 is how we write 9 tenths and we need to make 9 hundredths look different from 9 tenths. (More on the differences later in the lesson.)

Point out, or ask students to describe, the relationship between the chart and the decimal numbers: the tenths (in the first column) are recorded in the first decimal place; the extra hundredths are recorded in the second decimal place.

Have students make 5 (non-overlapping!) hundreds blocks (10 by 10) on grid paper and label them a) to e). (You could also hand out copies of the BLM “Blank Hundreds Charts.”)

While they are doing this, write on the board:

a) .17  b) .05  c) .20  d) .45  e) .03

Tell students to shade in the correct fraction of each square to show the decimal number. When they are done, students should translate the decimals to fractions.

Remind students that the first decimal place (to the right of the decimal point) counts the number of tenths and ASK: What does the second decimal place count? Then write .4 on the board and ASK: What does this number mean—is it 4 tenths, 4 hundredths, 4 ones, 4 hundreds—what? How do you know?
(It is 4 tenths, because the 4 is the first place after the decimal point). **ASK:** If you write .4 as a fraction, what will the denominator be? (You may need to remind students that the denominator is the bottom number.) What will the numerator be? Write \(\frac{4}{10}\) on the board.

Then write on the board: 0.28. **ASK:** How many tenths are in this number? How many hundredths are there in each tenth? How many more hundredths are there? How many hundredths are there altogether? What is the numerator of the fraction? The denominator? Write \(\frac{28}{100}\) on the board.

Have students fill in the numerator of each fraction:

\[
\begin{align*}
a) \quad .6 & = \frac{6}{10} & b) \quad .7 & = \frac{7}{10} & c) \quad .2 & = \frac{2}{10} & d) \quad .63 & = \frac{63}{100} & e) \quad .97 & = \frac{97}{100} & f) \quad .48 & = \frac{48}{100} \\
g) \quad .50 & = \frac{50}{100} & h) \quad .07 & = \frac{7}{100} & i) \quad .8 & = \frac{8}{10} & j) \quad .09 & = \frac{9}{100} & k) \quad .90 & = \frac{90}{100} & l) \quad .9 & = \frac{9}{10}
\end{align*}
\]

Then tell students that you are going to make the problems a bit harder. They will have to decide whether the denominator in each fraction is 10 or 100. Ask a volunteer to remind you how to decide whether the fraction should have denominator 10 or 100. Emphasize that if there is only 1 decimal place, it tells you the number of tenths, so the denominator is 10; if there are 2 decimal places, it tells you the number of hundredths, so the denominator is 100. Students should be aware that when we write \(\frac{.30}{10}\), we are saying that 30 hundredths are the same as 3 tenths.

The 2 decimal places in .30 tell us to count hundredths, whereas the denominator of 10 in \(\frac{3}{10}\) tells us to count tenths.

Have students write the fraction for each decimal in their notebooks:

\[
\begin{align*}
a) \quad .4 & \quad b) \quad .75 & \quad c) \quad .03 & \quad d) \quad .3 & \quad e) \quad .30 & \quad f) \quad .8 & \quad g) \quad .09 & \quad h) \quad .42 & \quad i) \quad .2 & \quad j) \quad .5 & \quad k) \quad .50
\end{align*}
\]

**Bonus**

\[
\begin{align*}
.789 & & .060 & & .007 & & .053 & & .301 & & .596 & & .0102507
\end{align*}
\]

Write on the board: \(\frac{38}{100}\). **ASK:** How would we change this to a decimal? How many places after the decimal point do we need? (2) How do you know? (Because the denominator is 100, so we’re counting the number of hundredths). Ask volunteers to write decimals for the following fractions:

\[
\begin{align*}
\frac{3}{100} & & \frac{24}{100} & & \frac{8}{10} & & \frac{80}{100} & & \frac{8}{10} & & \frac{7}{10}
\end{align*}
\]

Have them change the following fractions to decimals in their notebooks:

\[
\begin{align*}
a) \quad \frac{29}{100} & & b) \quad \frac{4}{10} & & c) \quad \frac{4}{100} & & d) \quad \frac{13}{100} & & e) \quad \frac{6}{10} & & f) \quad \frac{70}{100} & & g) \quad \frac{3}{10} & & h) \quad \frac{30}{100} & & i) \quad \frac{67}{100} & & j) \quad \frac{7}{100}
\end{align*}
\]

**Bonus**

\[
\begin{align*}
\frac{293}{1000} & & \frac{48}{1000} & & \frac{4}{1000}
\end{align*}
\]

**Bonus**

Have students rewrite the following addition statements using decimal notation:

\[
\begin{align*}
a) \quad \frac{3}{10} + \frac{4}{10} = \frac{34}{100} & & b) \quad \frac{40}{100} + \frac{5}{100} = \frac{45}{100} & & c) \quad \frac{21}{100} + \frac{31}{100} = \frac{52}{100} & & d) \quad \frac{22}{100} + \frac{7}{10} = \frac{92}{100}
\end{align*}
\]

Put the following equivalencies on the board and **SAY:** I asked some students to change decimals to fractions and these were their answers. Which ones are incorrect? Why are they incorrect?

\[
\begin{align*}
a) \quad .37 = \frac{37}{100} & & b) \quad .68 = \frac{68}{10} & & c) \quad .4 = \frac{4}{100} & & d) \quad .90 = \frac{90}{10} & & e) \quad .9 = \frac{9}{10} & & f) \quad .08 = \frac{8}{10}
\end{align*}
\]
Extensions

1. Cut the pie into more pieces to show $\frac{2}{5} = .4$

2. Draw a letter E covering more than .3 and less than .5 of a 10 $\times$ 10 grid.

3. **ASK:** When there is 1 decimal place, what is the denominator of the fraction? When there are 2 decimal places, what is the denominator of the fraction? What do you think the denominator of the fraction will be when there are 3 decimal places? How would you change .00426 to a fraction? How would you change $\frac{9823}{1000000}$ to a decimal?

4. Write the following decimals in order by changing the decimals to fractions with a denominator of 100 and then comparing the fractions:

   .8 , .4 , .07 , .17 , .32 , .85 , .3

---

**NS5-85**

**Decimals and Fractions Greater Than One**

**ASK:** If I use a hundreds block to represent a whole, what can I use to show one tenth? (a tens block) What fraction does a ones block show? (one hundredth)

Draw on the board:

2 wholes

4 tenths

3 hundredths

Tell students that this picture shows the decimal 2.43 or the fraction $2 \frac{43}{100}$.

Mixed fractions can be written as decimals, too! Have volunteers write the mixed fraction and the decimal shown by each picture:

a) 

b) 

c)
Have students draw base ten models for these mixed fractions in their notebooks:

a) $1 \frac{30}{100}$  

b) $2 \frac{8}{100}$  

c) $4 \frac{21}{100}$

**Bonus**

First change these fractions to mixed fractions and then draw models:

a) $1 \frac{103}{100}$  

b) $2 \frac{14}{100}$  

c) $3 \frac{90}{100}$

Then have students draw a model for each decimal:

a) 3.14  

b) 2.53  

c) 4.81

Put up the following pictures and have volunteers write a mixed fraction and a decimal for each. Remind students that each fully shaded hundreds block is a whole.

**EXCEPTIONS:**

a)  

b)  

ASK: How are these different from the base ten pictures? How are they the same? (The difference is that we haven’t pulled the rows (tenths) and squares (hundredths) apart; they’re both lumped together in a shaded square.)

Put up more pictures and have students write mixed fractions and decimals in their notebooks. Include pictures in which there are no tenths or no more hundredths after the tenths are counted.

**EXAMPLES:**

a)  

b)  

Have students draw pictures on grid paper and write decimals for each mixed fraction:

a) $1 \frac{95}{100}$  

b) $2 \frac{39}{100}$  

c) $2 \frac{4}{100}$  

d) $3 \frac{82}{100}$  

e) $1 \frac{9}{10}$  

f) $1 \frac{9}{100}$

Have students change these fractions to decimals without using a picture:

a) $3 \frac{18}{100}$  

b) $12 \frac{3}{10}$  

c) $25 \frac{4}{100}$  

d) $34 \frac{8}{10}$  

e) $11 \frac{98}{100}$  

f) $41 \frac{19}{100}$  

**BONUS:**

$5 \frac{138}{100}$

Then have students draw pictures on grid paper to show each decimal:

a) .53  

b) .03  

c) .30  

d) .19  

e) .8  

f) .08

Finally, have students draw pictures on grid paper to show each pair of decimals and then to decide which decimal is greater:

a) 3.04 or 3.17  

b) 1.29 or 1.7  

c) 1.05 or 1.50  

d) 5 tenths or 5 hundredths

**Bonus**

Have students draw pictures on grid paper to show each decimal and then to put the decimals
in order from smallest to largest. What word do the corresponding letters make?

E. 7.03  S. 7.30  I. 2.15  L. 2.8  M. 2.09

**NOTE:** The correct word is “miles.” (You might need to explain to students what a mile is.) If any students get “smile” or “slime,” they might be guessing at the correct answer (this is much more likely than having made a mistake in the ordering of the numbers). Encourage them to put the numbers in order before trying to unscramble the letters. Then they can be reasonably confident that their word is the right one. If the numbers are in order and the word makes sense, they don’t need you to confirm their answer. It is important to foster this type of independence.

**Extensions**

1. Students can make up their own puzzle like the one in the last Bonus questions. Partners can solve each other’s puzzles.

   **STEP 1:** Choose a 3- or 4-letter word whose letters can be used to create other words.

   **EXAMPLE:** The letters in the word “dare” can be rearranged to make “read” or “dear.”

   **NOTE:** This step is crucial. Be sure students understand why it is important for the letters they choose to be able to make at least 2 different words.

   **STEP 2:** Choose one decimal number for each letter and write them in order.

   **EXAMPLE:**
   - d. 1.47
   - a. 1.52
   - r. 2.06
   - e. 2.44

   **STEP 3:** Scramble the letters, keeping the corresponding numbers with them.

   **EXAMPLE:**
   - a. 1.52
   - e. 2.44
   - d. 1.47
   - r. 2.06

   **STEP 4:** Give the scrambled letters and numbers to a partner to put in the correct order. Did your partner find the word you started with?

2. Show .2 in each of these base ten blocks:

   a) 
   b) 
   c) 

   thousands block  hundreds block  tens block

3. (Adapted from Atlantic Curriculum A6.7 and A7)

   Teach your students that the decimal point is read as “and.” For example, 13.7 is read as “thirteen and seven tenths.”

   Have students list:

   a) whole numbers that take exactly 3 words to say

   **EXAMPLES:** 9 000 080, 600 000, 403

   b) decimal numbers that take exactly 6 words to say

   **EXAMPLES:** 403.08 (four hundred three and eight hundredths) 600 000.43 (six hundred thousand and forty-three hundredths) 9 000 080.09 (nine million eighty and nine hundredths)
If students are familiar with thousandths and ten thousandths, they might come up with:

3.542 (three and five hundred forty-two thousandths) or
500.000 1 (five hundred and one ten thousandth)

4. (Atlantic Curriculum A7) Teach students to interpret whole numbers written in decimal format
(EXAMPLE: 5.1 million as 5 100 000)

NS5-86
Decimals and Fractions on Number Lines

GOALS
Students will place decimal numbers and mixed fractions on number lines. Students will also write the words for decimals and fractions (proper, mixed, or improper).

PRIOR KNOWLEDGE REQUIRED
Decimal numbers with up to 2 decimal places and their equivalent fractions (proper or mixed)
Translating between mixed and improper fractions
Number lines

Draw on the board:

Have students count out loud with you from 0 to 1 by tenths: zero, one tenth, two tenths, … nine tenths, one.

Then have a volunteer write the equivalent decimal for \( \frac{1}{10} \) on top of the number line:

Continue in random order until all the equivalent decimals have been added to the number line.

Then have students write, in their notebooks, the equivalent decimals and fractions for the spots marked on these number lines:

a)  

b)  

c)  

d)  

Have volunteers mark the location of the following numbers on the number line with an X and the corresponding letter.

A. 0.7  
B. \( \frac{7}{10} \)  
C. 1.40  
D. \( \frac{8}{10} \)  
E. 1 \( \frac{9}{10} \)
Invite any students who don’t volunteer to participate. Help them with prompts and questions such as: Is the number more than 1 or less than 1? How do you know? Is the number between 1 and 2 or between 2 and 3? How do you know?

Review translating improper fractions to mixed fractions, then ask students to locate the following improper fractions on a number line from 0 to 3 after changing them to mixed fractions:

A. \( \frac{17}{10} \)  B. \( \frac{23}{10} \)  C. \( \frac{14}{10} \)  D. \( \frac{26}{10} \)  E. \( \frac{11}{10} \)

When students are done, **ASK**: When the denominator is 10, what is an easy way to tell whether the improper fraction is between 1 and 2 or between 2 and 3? (Look at the number of tens in the numerator—it tells you how many ones are in the number.)

**ASK**: How many tens are in 34? (3)  78? (7)  123? (12)  345? (34)

**ASK**: How many ones are in \( \frac{34}{10} \)? (3)  \( \frac{78}{10} \)? (7)  \( \frac{123}{10} \)? (12)  \( \frac{345}{10} \)? (34)

**ASK**: What two whole numbers is each fraction between?

a) \( \frac{29}{10} \)  b) 2 \( \frac{4}{10} \)  c) 12 \( \frac{7}{10} \)  d) \( \frac{81}{10} \)  e) \( \frac{127}{10} \)  f) \( \frac{318}{10} \)

Invite volunteers to answer a) and b) on the board, then have students do the rest in their notebooks. When students are finished, **ASK**: Which 2 fractions in this group are equivalent?

Tell students that there are 2 different ways of saying the number 12.4. We can say “twelve decimal four” or “twelve and four tenths”. Both are correct. (Note that “twelve point four” is also commonly used.) Point out the word “and” between the ones and the tenths, and tell students that we always include it when a number has both ones and tenths (and/or hundredths).

Have students place the following fractions on a number line from 0 to 3:

A. three tenths  B. two and five tenths  C. one and seven tenths
D. one decimal two  E. two decimal eight

Draw a number line from 0 to 3 on the board. Mark the following points with an X—no numbers—and have students write the number words for the points you marked:

1.3  .7  2.4  .1  2.8  2.1

Draw a line on the board with endpoints 0 and 1 marked:

0  1

Ask volunteers to mark the approximate position of each number with an X:

a) .4  b) \( \frac{6}{10} \)  c) 0.9

Then draw a number line from 0 to 3 with whole numbers marked:

0  1  2  3

Ask volunteers to mark the approximate position of these numbers with an X:

a) 2.1  b) 1 \( \frac{3}{10} \)  c) \( \frac{26}{10} \)  d) .4  e) 2 \( \frac{5}{10} \)
Continue with more numbers and number lines until all students have had a chance to participate.

(EXAMPLE: Draw a number line from 0 to 2 with whole numbers marked and have students mark the approximate position of 0.5, 1.25, and others.)

**Bonus**

Use larger whole numbers and/or longer number lines.

**Extensions**

1. Use a metre stick to draw a line that is 2 metres long and ask students to mark 1.76 metres.

2. Mark the given decimals on the number lines:
   a) Show .4
   b) Show 1.5

**NS5-87**

**Comparing & Ordering Fractions and Decimals**

**GOALS**

Students will use fractions (one half, one quarter, and three quarters) as benchmarks for decimals.

**PRIOR KNOWLEDGE REQUIRED**

Decimals and fractions on number lines

Draw on a transparency:

```
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1
```

**(NOTE:** If you don’t have an overhead projector, tape the transparency to the wall and invite students, in small groups if necessary, to gather around it as you go through the first part of the lesson.) Have a volunteer show where \( \frac{1}{2} \) is on the number line. Have another number line the same length divided into 2 equal parts on another transparency and superimpose it over this one, so that students see that \( \frac{1}{2} \) is exactly at the .5 mark. **ASK:** Which decimal is equal to \( \frac{1}{2} \)? Is 0.2 between 0 and \( \frac{1}{2} \)? Is between \( \frac{1}{2} \) and 1? Is 0.7 between 0 and \( \frac{1}{2} \) or between \( \frac{1}{2} \) and 1? What about 0.6? 0.4? 0.3? 0.9?

Go back to the decimal 0.2 and **ASK:** We know that 0.2 is between 0 and \( \frac{1}{2} \), but is it closer to 0 or to \( \frac{1}{2} \)? Draw on the board or the transparency:

```
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1
```

**ASK:** Is .6 between 0 and \( \frac{1}{2} \) or between \( \frac{1}{2} \) and 1? Which number is it closest to, \( \frac{1}{2} \) or 1? Have a volunteer show the distance to each number with arrows. Which arrow is shorter? Which number is .4 closest to, 0, \( \frac{1}{2} \) or 1? Which number is .8 closest to? Go through all of the remaining decimal tenths between 0 and 1.
On grid paper, have students draw a line 10 squares long. Then have them cut out the line—leaving space above and below for writing—and fold it in half so that the two endpoints meet. They should mark the points 0, $\frac{1}{2}$, and 1 on their line. Now have students fold the line in half again, and then fold it in half a second time. Have them unfold the line and look at the folds. **ASK:** What fraction is exactly halfway between 0 and $\frac{1}{2}$? How do you know?

($\frac{1}{4}$ because the sheet is folded into 4 equal parts so the first fold must be $\frac{1}{4}$ of the distance from 0 to 1) What fraction is halfway between $\frac{1}{2}$ and 1? How do you know? ($\frac{3}{4}$ because the sheet is folded into 4 equal parts so the third fold must be $\frac{3}{4}$ of the distance from 0 to 1).

Have students mark the fractions $\frac{1}{4}$ and $\frac{3}{4}$ on their number lines. Then have students write the decimal numbers from .1 to .9 in the correct places on their number lines (using the squares on the grid paper to help them).

Tell students to look at the number lines they’ve created and to fill in the blanks in the following questions by writing “less than” or “greater than” in their notebooks.

a) 0.4 is ________ $\frac{1}{4}$ b) 0.4 is ________ $\frac{1}{2}$ c) 0.8 is ________ $\frac{3}{4}$

d) 0.2 is ________ $\frac{1}{4}$ e) 0.3 is ________ $\frac{1}{2}$ f) 0.7 is ________ $\frac{3}{4}$

Have students rewrite each statement using the “greater than” and “less than” symbols: > and <.

**ASK:** Which whole number is each decimal, mixed fraction, or improper fraction closest to?

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
| & | & | & |
\end{array}
\]

a) 0.7 b) 1 $\frac{6}{10}$ c) 2.3 d) $\frac{18}{10}$ e) 2 $\frac{6}{10}$ f) 1.1

\[
\begin{array}{cccc}
15 & 16 & 17 & 18 \\
| & | & | & |
\end{array}
\]

a) 17.2 b) 16.8 c) 16 $\frac{3}{10}$ d) $\frac{174}{10}$ e) 15.9 f) 15.3

**ASK:** Which decimal is halfway between 1 and 2? Halfway between 17 and 18? Halfway between 31 and 32? Between 0 and 3? Between 15 and 18? Between 30 and 33? Between 25 and 28?

**Bonus**
Which whole number is each decimal closest to?

a) 23.4 b) 39.8 c) 314.1 d) 235.6 e) 981.1 f) 999.9

**Extension**
(Adapted from Atlantic Curriculum A6.3) Have students rearrange the following words to create different numbers. What are the smallest and largest numbers that can be made using these words:

- hundredths
- hundred
- thousand
- two
- five
- nine
- thirty-seven
- and

**ANSWER:**
Largest number: Thirty-seven thousand nine hundred five and two hundredths
Smallest number: Two thousand five hundred nine and thirty-seven hundredths
GOALS
Students will compare and order decimals and fractions by first changing them all to fractions with denominator 10 or 100.

PRIOR KNOWLEDGE REQUIRED
Ordering mixed fractions with the same denominator
Decimal place value
Translating between fractions with denominator 10 or 100 and decimals
Equivalent fractions

NS5-88
Ordering Fractions and Decimals

To ensure students have the prior knowledge required, ask them to do the following questions in their notebooks.

1. Write each decimal as a fraction or mixed fraction with denominator 10.
   a) 0.7   b) 0.4   c) 1.3   d) 2.9   e) 7.4   f) .6

2. Put the fractions in order from smallest to largest.
   a) 1 $\frac{7}{10}$ 2 $\frac{4}{10}$ 1 $\frac{3}{10}$ 2 $\frac{1}{10}$
   b) 1 $\frac{6}{10}$ 1 $\frac{3}{10}$ 2 $\frac{1}{10}$
   c) 13 $\frac{7}{10}$ 12 $\frac{4}{10}$ 12 $\frac{5}{10}$

Circulate while students work and assist individuals as required. You can also review these concepts by solving the problems together as a class. Think aloud as you work and invite students to help you. For example, SAY: I want to turn 1.3 into a mixed fraction. What should I do first? Why?

Then write on the board: 2 $\frac{4}{10}$ 2.3 3.7

Tell students you want to order these numbers from smallest to largest.
ASK: How is this problem different from the problems we just did? (Not all the numbers are fractions with denominator 10; some are decimals.) Can we change it into a problem that looks like the ones we just did? How? (Yes, by changing the decimals to fractions with denominator 10.) Have one volunteer change the decimals as described and another volunteer put all 3 fractions in the correct order.

Have students order the following numbers in their notebooks. Volunteers should do the first two or three problems on the board.

a) 0.8 0.4 $\frac{7}{10}$ b) 1.7 .9 1 $\frac{3}{10}$
   c) 8.4 5 $\frac{6}{10}$ 7.7
d) 2.8 1.5 $\frac{7}{10}$ e) 3.7 3.9 3 $\frac{3}{10}$
   f) 8.4 9 $\frac{6}{10}$ 9.7

Bonus
Provide problems where students have to change improper fractions to mixed fractions:

g) 3.7 2.9 $\frac{36}{10}$ h) 3.9 3.4 $\frac{38}{10}$
   i) 12.1 $\frac{116}{10}$ 12

Bonus
Provide problems where one of the fractions has denominator 2 or 5, so that students have to find an equivalent fraction with denominator 10:

j) 3.7 2.9 $\frac{5}{2}$ k) 3.9 3.4 $\frac{7}{2}$
   l) 12.3 12 $\frac{3}{5}$ 12

Have volunteers say how many hundredths are in each number:

a) .73 b) .41 c) .62 d) .69 e) .58 f) .50

For each pair of numbers, ask which number has more hundredths and then which number is larger:

a) .73 .68 b) .95 .99 c) .35 .42 d) .58 .57
Now have volunteers say how many hundredths are in each of these numbers:

\[ \begin{align*}
\text{a)} & \quad .4 & \quad \text{b)} & \quad .6 & \quad \text{c)} & \quad .5 & \quad \text{d)} & \quad .7 & \quad \text{e)} & \quad .1 & \quad \text{f)} & \quad .8 \\
\end{align*} \]

**ASK:** How can we write 0.4 as a number with 2 decimal places? Remind students that “4 tenths” is equivalent to “4 tenths and 0 hundredths” or “40 hundredths.” This means \(0.4 = 0.40\).

For each pair of numbers below, **ASK:** Which number has more hundredths? Which number is larger? Encourage students to change the number with 1 decimal place to a number with 2 decimal places.

\[ \begin{align*}
\text{a)} & \quad .48 & \quad \text{b)} & \quad .5 & \quad \text{c)} & \quad .73 & \quad \text{d)} & \quad .2 & \quad \text{e)} & \quad .17 \\
\end{align*} \]

Put on the board:

\[ \begin{align*}
\text{a)} & \quad \frac{4}{10} & \quad \text{b)} & \quad \frac{5}{10} & \quad \text{c)} & \quad \frac{7}{10} & \quad \text{d)} & \quad \frac{2}{10} & \quad \text{e)} & \quad \frac{1}{10} & \quad \text{f)} & \quad \frac{7}{10} \\
\end{align*} \]

Ask volunteers to write 2 different fractions for the amount shaded in the pictures. Have other volunteers change the fractions to decimals. **ASK:** Do these 4 numbers all have the same value? How do you know? What symbol do we use to show that different numbers have the same value? (the equal sign) Write on the board: \(0.9 = 0.90 = \frac{9}{10} = \frac{90}{100} \cdot \)

Have students change more decimals to fractions with denominator 100:

\[ \begin{align*}
\text{a)} & \quad .6 & \quad \text{b)} & \quad .1 & \quad \text{c)} & \quad .4 & \quad \text{d)} & \quad .8 & \quad \text{e)} & \quad .35 & \quad \text{f)} & \quad .42 \\
\end{align*} \]

Have students put each group of numbers in order by first changing all numbers to fractions with denominator 100:

\[ \begin{align*}
\text{a)} & \quad \frac{3}{10} & \quad \frac{48}{100} & \quad \frac{4}{10} & \quad 0.39 & \quad \text{b)} & \quad 2 \frac{17}{100} & \quad 2 \frac{3}{10} & \quad 2.2 \\
\end{align*} \]

Now show a hundreds block with half the squares shaded:

\[ \begin{align*}
\text{SAY:} & \quad \text{This hundreds block has 100 equal squares. How many of the squares are shaded? (50)} \\
& \quad \text{So what fraction of the block is shaded?} \left( \frac{50}{100} \right) \text{ Challenge students to give equivalent answers with different denominators, namely 10 and 2.} \\
& \quad \text{PROMPTS: If we want a fraction with denominator 10, how many equal parts do we have to divide the block into? (10) What are the equal parts in this case and how many of them are shaded? (the rows; 5)} \\
& \quad \text{What fraction of the block is shaded?} \left( \frac{5}{10} \right) \text{ What are the equal parts if we divide the block up into 2? (blocks of 50)} \text{ What fraction of the block is shaded now?} \left( \frac{1}{2} \right) \\
& \quad \text{Ask students to identify which fraction of each block is shaded:} \\
\end{align*} \]
Challenge them to find a suitable denominator by asking themselves: How many equal parts the size of the shaded area will make up the whole? Have students convert their fractions into equivalent fractions with denominator 100.

\( \frac{1}{5} = \frac{20}{100}, \frac{1}{4} = \frac{25}{100}, \frac{1}{20} = \frac{5}{100} \)

Write on the board: \( \frac{2}{4} = \frac{100}{100} \) and \( \frac{3}{4} = \frac{100}{100} \), and have volunteers fill in the blanks. Then have students copy the following questions in their notebooks and fill in the blanks.

\[
\begin{align*}
\text{a)} & \quad \frac{2}{5} = \frac{100}{100} & \quad \frac{3}{5} &= \frac{100}{100} & \quad \frac{4}{5} &= \frac{100}{100} & \quad \frac{5}{5} &= \frac{100}{100} \\
\text{b)} & \quad \frac{2}{20} = \frac{100}{100} & \quad \frac{3}{20} &= \frac{100}{100} & \quad \frac{4}{20} &= \frac{100}{100} \\
\end{align*}
\]

**BONUS:** 
\( \frac{17}{20} = \frac{100}{100} \)

Have students circle the larger number in each pair by first changing all numbers to fractions with denominator 100:

\[
\begin{align*}
\text{a)} & \quad \frac{1}{2} \quad \text{or} \quad .43 & \quad \text{b)} & \quad \frac{3}{2} \quad \text{or} \quad 1.6 & \quad \text{c)} & \quad 3.7 \quad \text{or} \quad 3 \frac{1}{2} & \quad \text{d)} & \quad \frac{1}{2} \quad \text{or} \quad .57 & \quad \text{e)} & \quad \frac{1}{4} \quad \text{or} \quad .23 & \quad \text{f)} & \quad \frac{3}{5} \quad \text{or} \quad .7
\end{align*}
\]

Give students groups of fractions and decimals to order from least to greatest by first changing all numbers to fractions with denominator 100. Include mixed, proper, and improper fractions. Start with groups of only 3 numbers and then move to groups of more numbers.

**Extensions**

1. Students can repeat Extension 1 from **NS5-85**: Decimals and Fractions Greater Than One using a combination of decimals and fractions. Remind students to start with a word whose letters can be used to create other words, and review why this is important. Students can now assign either a decimal or a fraction to each letter, scramble the letters and numbers, and invite a partner to order the numbers to find the original word.

2. Write a decimal for each fraction by first changing the fraction to an equivalent fraction with denominator 100.

\[
\begin{align*}
\text{a)} & \quad \frac{2}{5} \quad \text{b)} & \quad \frac{1}{2} \quad \text{c)} & \quad \frac{1}{4} \quad \text{d)} & \quad \frac{3}{5}
\end{align*}
\]

3. Compare without using pictures, and determine which number is larger:

\[
\begin{align*}
\text{a)} & \quad 2 \frac{3}{4} \quad \text{and} \quad 17 \text{ sevenths} & \quad \text{b)} & \quad 1.7 \quad \text{and} \quad 17 \text{ elevenths} \\
\text{c)} & \quad 1.5 \quad \text{and} \quad 15 \text{ ninths} & \quad \text{d)} & \quad 2.9 \quad \text{and} \quad 26 \text{ ninths}
\end{align*}
\]

Students will have to convert all of the numbers into fractions in order to compare them. Is it best to use improper or mixed fractions? You can invite students to try both and see which types of fractions are easier to work with in this case (mixed works better for parts a and d, improper works better for parts b and c).

4. Draw a picture to show a decimal between .3 and .4. Explain why more than one answer is possible.

5. Write digits in the boxes that will make the statement true.

\[
\begin{array}{c}
\square .5 \quad < \quad \square .3
\end{array}
\]

6. Teach students to describe multiplicative quantities by using simple decimals. For example, 6 is exactly halfway between 1 times 4 and 2 times 4, so 6 is 1.5 times 4. So if I have 4 marbles and you have 6 marbles, then you have 1.5 times as many marbles as I have.
NS5-89
Adding and Subtracting Tenths

GOALS
Students will add and subtract decimal numbers by using a number line and by lining up the numbers according to the decimal place.

PRIOR KNOWLEDGE REQUIRED
Adding whole numbers with regrouping
Knowing the number of tenths in a number with 1 decimal place (EXAMPLE: 4.7 has 47 tenths)
Knowing a decimal number given the number of tenths (EXAMPLE: 47 tenths is 4.7)

To ensure students have the prior knowledge required for the lesson, complete the blanks in the following questions as a class or have students complete them independently in their notebooks.

a) 5.3 = ____ tenths
b) .8 = ____ tenths
c) 1.5 = ____ tenths
d) ____ = 49 tenths
e) ____ = 78 tenths
f) ____ = 4 tenths

g) ____ = 897 tenths
h) ____ = 54301 tenths
i) ____ = 110 tenths

Review concepts with individual students or the whole class as required.

Now tell students that you want to add some decimals. Write on the board:

2.1
+ 1.0

ASK: How many tenths are in 2.1? (21) How many tenths are in 1.0? (10)
How many tenths are there altogether? (31) SAY: There are 31 tenths in the sum. What number is that? (3.1) Write the answer below the addends, being careful to state that you are lining up the tenths under the tenths and the ones under the ones:

2.1
+ 1.0
3.1

Do a second problem together. This time, write out the number of tenths and add them using the standard algorithm for addition.

<table>
<thead>
<tr>
<th>Numbers to add</th>
<th>Numbers of tenths in those numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>14 tenths</td>
</tr>
<tr>
<td>+ 2.3</td>
<td>+ 23 tenths</td>
</tr>
<tr>
<td>3.7</td>
<td>37 tenths</td>
</tr>
</tbody>
</table>

Have students add the following decimals in their notebooks using this method, that is, by adding the whole numbers of tenths first and then turning the answer into a decimal:

a) 3.4 + 1.5 = 4.9
b) 2.6 + 4.1 = 6.7
c) 8.5 + 1.2 = 9.7
d) 3.7 + 4.2 = 7.9
E) 134.3 + 245.5 = 379.8

When students are done, tell them that Sonia adds decimal numbers by lining up the ones digits with the ones digits, the tenths digits with the tenths digits, and then adding each digit separately. Show them an example by re-doing question a) this way. ASK: Does Sonia get the right answer with this method? Why do you think that is? How is what Sonia does similar to what you did? Did you line up the digits when you added the whole numbers? Did you add each digit separately?
Write on the board:

\[
\begin{array}{cccccc}
a) & 3.5 & b) & .7 & c) & 192.8 \\
+ & 4 & + & 3.5 & + & 154 \\
\end{array}
\]

\[
\begin{array}{cccccc}
d) & 4.8 & e) & 154.7 \\
+ & 12.1 & + & 16.3 \\
\end{array}
\]

**ASK:** In which questions are the digits lined up correctly? How can you tell? In all the questions where digits are lined up correctly, what else is also lined up? (the decimal point) Is the decimal point always going to be lined up if the digits are lined up correctly? (yes) How do you know? (The decimal point is always between the ones and the tenths, so if those digits are lined up correctly, then the decimal point will be as well.)

Demonstrate using this method to solve 12.1 + 4.8:

\[
\begin{array}{c}
12.1 \\
+ 4.8 \\
16.9
\end{array}
\]

Tell students that when you add the ones digits, you get the ones digit of the answer and when you add the tenths digits, you get the tenths digit of the answer. If the digits are lined up, then the decimal points are lined up, too—the decimal point in the answer must line up with the decimal points in the addends. Ask students why this makes sense and give several individuals a chance to articulate an answer.

Tell students that as long as they line up the numbers according to the decimal points, they can add decimals just like they add whole numbers. Demonstrate with a few examples, including some that require regrouping and carrying:

\[
\begin{array}{cccccc}
3.5 & 1 & 192.8 & 154.7 \\
+ & 4 & + & 154 & + & 16.3 \\
7.5 & 4.2 & 346.8 & 171.0
\end{array}
\]

Give students lots of practice adding decimals (with no more than 1 decimal place). Working on grid paper will help students to line up the digits and the decimal points. Include examples with regrouping. Bonus problems could include larger numbers (but still only 1 decimal place). Emphasize that the decimal point is always immediately after the ones digit, so a whole number can be assumed to have a decimal point (EXAMPLE: 43 = 43. = 43.0) It is also important that students line up the digits around the decimal point carefully, perhaps by using grid paper. To emphasize this, have students identify the mistake in:

\[
\begin{array}{c}
341.7 \\
+ 5216.2 \\
867.9
\end{array}
\]

The decimals are technically lined up properly, but the remaining digits are not.

**ASK:** What happened?

**ASK:** If we can add decimals the way we add whole numbers as long as we line up the decimal points, do you think we can subtract decimals the same way we subtract whole numbers? Use the following problem to check:

\[
\begin{array}{c}
4.5 \\
- 2.3
\end{array}
\]
Do the problem together two ways—first rewrite the decimals as tenths and subtract the whole numbers, then subtract using Sonia’s method—and compare the answers. Then subtract more numbers (with no more than 1 decimal place) together. **EXAMPLES:**

\[
\begin{array}{ccc}
6.7 & 4.9 & 8.12 \\
- 3 & - 1.7 & - 45.3 \\
3.7 & 3.2 & 347.3
\end{array}
\]

As with addition, give student lots of practise subtracting decimals. Remind them to line the numbers up according to the decimal point!

Then introduce adding decimals on a number line:

\[
\begin{array}{ccccccccc}
\hline
& & & & & & & & & \\
& & & & & & & & & \\
0 & 1 & 2 & 3 & 4 \\
\hline
\end{array}
\]

**SAY:** I want to add 1.6 + 0.9. How many tenths are in 1.6? Where is that on the number line? How many tenths are in 0.9? How can I show adding 1.6 to 0.9 on the number line? Draw arrows to illustrate the calculation:

\[
\begin{array}{ccc}
1.6 & + & 0.9 \\
\hline
& & = \\
0 & 1 & 2 & 3 & 4
\end{array}
\]

Have students use the number line to add:

a) 2.4 + 1.0  
 b) 1.3 + 1.0  
 c) 0.5 + 1.0  

d) 1.8 + 1.0  
 e) 2.6 + 1.0  
 f) 0.9 + 1.0  

Then have them add 1.0 to decimals without using the number line:

d) 1.8 + 1.0  
 e) 2.6 + 1.0  
 f) 0.9 + 1.0  

Invite students to use what they know about adding 1.0 to decimals to solve 1.4 + 1.5. **ASK:** What is 1.4 + 1.0? How can we use that to find 1.4 + 1.5? How would you find 1.1 + 2.3? (Students could find 1.1 + 2.0 and then add .3 or they could find 1.0 + 2.3 and then add .1. Both strategies should be discussed.)

Have students subtract using the number line as well.

**Extension**

Stick measuring tape to the board (the kind used by tailors) to make number lines (where each centimetre represents a hundredth). Students could draw arrows over the number lines to add and subtract decimal hundredths as shown on Worksheet **NS5-89** for decimal tenths (SEE: Question 4).
NS5-90
Adding Hundredths

GOALS
Students will add hundredths by lining up the decimal points.

PRIOR KNOWLEDGE REQUIRED
Adding whole numbers with regrouping
Adding tenths
Adding fractions with the same denominator
Knowing how many hundredths are in a number with 2 decimal places
Knowing which number has a given number of hundredths

Have students add these fractions:

\[
\begin{align*}
a) \left(\frac{36}{100}\right) + \left(\frac{24}{100}\right) & \quad b) \left(\frac{20}{100}\right) + \left(\frac{39}{100}\right) & \quad c) \left(\frac{32}{100}\right) + \left(\frac{16}{100}\right) & \quad d) \left(\frac{28}{100}\right) + \left(\frac{24}{100}\right)
\end{align*}
\]

Ask students to rewrite their addition statements in terms of decimals.

(EXAMPLE: \(0.3 + 0.24 = 0.54\))

Then give students addition problems that require regrouping. (EXAMPLE: \(\frac{36}{100} + \frac{47}{100}\)) Again, have students rewrite their addition statements in terms of decimals. (EXAMPLE: \(0.36 + 0.47 = 0.83\))

Now have students do the opposite: add decimals by first changing them to fractions with denominator 100. Invite volunteers to do some of the following problems on the board, then have students do the rest independently.

\[
\begin{align*}
a) \left(\frac{32}{100}\right) + \left(\frac{57}{100}\right) & \quad b) \left(\frac{43}{100}\right) + \left(\frac{16}{100}\right) & \quad c) \left(\frac{81}{100}\right) + \left(\frac{17}{100}\right) & \quad d) \left(\frac{44}{100}\right) + \left(\frac{44}{100}\right) & \quad e) \left(\frac{40}{100}\right) + \left(\frac{33}{100}\right)

f) \left(\frac{93}{100}\right) + \left(\frac{02}{100}\right) & \quad g) \left(\frac{05}{100}\right) + \left(\frac{43}{100}\right) & \quad h) \left(\frac{52}{100}\right) + \left(\frac{20}{100}\right) & \quad i) \left(\frac{83}{100}\right) + \left(\frac{24}{100}\right) & \quad j) \left(\frac{22}{100}\right) + \left(\frac{36}{100}\right)
\end{align*}
\]

Have a volunteer do a sum that requires regrouping \((0.54 + 0.28)\), then have students add the following independently:

\[
\begin{align*}
a) \left(\frac{37}{100}\right) + \left(\frac{26}{100}\right) & \quad b) \left(\frac{59}{100}\right) + \left(\frac{29}{100}\right) & \quad c) \left(\frac{39}{100}\right) + \left(\frac{46}{100}\right) & \quad d) \left(\frac{61}{100}\right) + \left(\frac{29}{100}\right)
\end{align*}
\]

Finally, add decimals whose sum is more than 1 by changing them to fractions first. Have volunteers do 2 examples on the board \((0.36 + 0.88 and 0.45 + 0.79)\). Students can then add the following independently:

\[
\begin{align*}
a) \left(\frac{75}{100}\right) + \left(\frac{68}{100}\right) & \quad b) \left(\frac{94}{100}\right) + \left(\frac{87}{100}\right) & \quad c) \left(\frac{35}{100}\right) + \left(\frac{99}{100}\right) & \quad d) \left(\frac{46}{100}\right) + \left(\frac{64}{100}\right) & \quad e) \left(\frac{85}{100}\right) + \left(\frac{67}{100}\right)

f) \left(\frac{75}{100}\right) + \left(\frac{50}{100}\right) & \quad g) \left(\frac{65}{100}\right) + \left(\frac{4}{100}\right) & \quad h) \left(\frac{7}{100}\right) + \left(\frac{38}{100}\right) & \quad i) \left(\frac{9}{100}\right) + \left(\frac{27}{100}\right)
\end{align*}
\]

ASK: How many hundredths are in .9? (90) How many hundredths are in .27? (27) How many hundredths is that altogether? (117) What number has 117 hundredths? (1.17) What number has 234 hundredths? 5682 hundredths? 48 hundredths? 901 hundredths? 800 hundredths? 80 hundredths? 8 hundredths?

Have students add more decimal numbers by identifying the number of hundredths in each number and then adding the whole numbers:

\[
\begin{align*}
a) \left(\frac{78}{100}\right) + \left(\frac{4}{100}\right) & \quad b) \left(\frac{37}{100}\right) + \left(\frac{49}{100}\right) & \quad c) \left(\frac{85}{100}\right) + \left(\frac{65}{100}\right) & \quad d) \left(\frac{43}{100}\right) + \left(\frac{34}{100}\right)

e) \left(\frac{25}{100}\right) + \left(\frac{52}{100}\right) & \quad f) \left(\frac{14}{100}\right) + \left(\frac{41}{100}\right) & \quad g) \left(\frac{76}{100}\right) + \left(\frac{67}{100}\right) & \quad h) \left(\frac{89}{100}\right) + \left(\frac{98}{100}\right)
i) \left(\frac{43}{100}\right) + \left(\frac{87}{100}\right) & \quad j) \left(\frac{55}{100}\right) + \left(\frac{55}{100}\right) & \quad k) \left(\frac{1.43}{100}\right) + \left(\frac{2.35}{100}\right) & \quad l) \left(\frac{3.5}{100}\right) + \left(\frac{2.71}{100}\right)
m) 4.85 + 3.09
\end{align*}
\]

Remind students that we were able to add tenths by lining up the digits and decimal points. ASK: Do you think we can add hundredths the same way? Do an example together:

\[
\begin{align*}
1 & \\
4.85 & + 3.09 \\
7.94 &
\end{align*}
\]
Invite students to help you add the numbers. Tell them to pretend the decimal point isn’t there and to add as though they are whole numbers. (PROMPTS: What do the hundredths digits add to? Where do I put the 4? the 1?) ASK: Why can we add as though the decimal points are not there? How many hundredths are in 4.85? (485) In 3.09? (309) In both numbers altogether? (485 + 309 = 794) How does this get the same answer? (794 hundredths = 7.94 ones, so we just line up the decimal points and proceed as though the decimal point is not there)

ASK: How can we check our answer? If no one suggests a method, invite a volunteer to add the numbers after rewriting them as fractions with denominator 100:

\[
\text{Method 1: } 4.85 + 3.09 = 4 \frac{85}{100} + 3 \frac{9}{100} = 7 \frac{84}{100} = 7.94
\]

\[
\text{Method 2: } 4.85 + 3.09 = \frac{485}{100} + \frac{309}{100} = \frac{794}{100} = 7.94
\]

Emphasize that all methods of addition reduce the problem to one students already know how to do: adding whole numbers. When they add using decimals, students know where to put the decimal point in the answer by lining up the decimal points in the addends. When they add using fractions, students know where to put the decimal point by looking at the denominator. If the denominator of the fraction is 10, they move the decimal point 1 place left. If the denominator of the fraction is 100, they move the decimal point 2 places left.

Give students lots of practice adding decimal hundredths by lining up decimal places. Include numbers that do and do not require regrouping.

EXAMPLES: \(.34 + .28\) \(.65 + .21\) \(.49 + .7\) \(1.3 + .45\) \(2.86 + .9\)

**Bonus**

\(12.3 + 1.23\) \(354.11 + 4 672.6\)

Give students base ten blocks and tell them to use the hundreds block as a whole. This makes the tens block a tenth and the ones block a hundredth. Assign the following problems to pairs or individuals and invite students to share their answers.

1. Start with these blocks:

   • What decimal does this model represent? (ANSWER: 2.3)

   • Add 2 blocks so that the sum, or total, is between 3.3 and 3.48. (ANSWER: add \[\square\] or add \[\square\])

   • Write a decimal for the amount you added. (ANSWER: 1.1 or 1.01)

2. Start with these blocks:

   • What decimal does this model represent? (ANSWER: 3.3)

   • Add 2 blocks so that the sum (or total) is between 3.4 and 3.48

   • Write a decimal for the amount you added (ANSWER: add 0.11).
3. Take the same number of blocks as number 1 above:

```
+ + |
```

• Add 2 blocks so that the sum is between 2.47 and 2.63. (ANSWER: 2 tens blocks)

• Write a decimal for the amount you added (ANSWER: 0.2).

4. Take these blocks:

```
a) or    b)
+ + +
```

• What decimal does this model represent? (ANSWER: a) 1.42  b) 2.42)

• Add 2 blocks so that the sum is between 2.51 and 2.6.

(ANSWER: a) add  

b) add  

• Write a decimal for the amount you added. (ANSWER: a) 1.1  b) 0.11)

Extensions

1. Write the numbers as decimals and add: 2 + \( \frac{3}{10} + \frac{7}{100} \).

2. **ASK**: What if the denominator of a fraction is 1 000—how do we know where to put the decimal point in the decimal number? What is .437 + .021? Turn the decimals into fractions to add them: \( \frac{437}{1000} + \frac{21}{1000} = \frac{458}{1000} \). **SAY**: The sum is 458 thousandths, or 0.458. Since the denominator of the fraction is 1000, we know to move the decimal point 3 places left. **ASK**: Can we add the decimals by lining up the decimal points? Do we get the same answer? Add the decimals this way to find out.
NS5-91
Subtracting Hundredths and
NS5-92
Adding and Subtracting Decimals (Review)

GOALS
Students will subtract hundredths by lining up the decimal points.

PRIOR KNOWLEDGE REQUIRED
Subtracting tenths
Subtracting fractions with the same denominator
Knowing how many hundredths are in a number with 2 decimal places
Knowing which number has a given number of hundredths

Tell your students that today they will learn to subtract hundredths. As in the last lesson, begin by subtracting hundredths written as fractions. (Start with problems that do not require regrouping and then move to problems that do.)

EXAMPLES:

a) 34/100 – 14/100   b) 58/100 – 11/100   c) 47/100 – 20/100   d) 43/100 – 10/100

Have students rewrite the subtraction statements in terms of decimals. (EXAMPLE: .34 – .14 = .20)

Then have students subtract decimals by first changing them into fractions (proper, mixed, or improper). Include numbers greater than 1.

EXAMPLES:

a) .45 – .14   b) .53 – .1   c) .85 – .3   d) 1.23 – .11

Tell students that we can subtract hundredths the same way we add them: by lining up the digits and decimal points. Solve 1.93 – .22 together and have students check the answer by rewriting the decimals as fractions.

```
1.93
– .22
1.71
```

Give students a chance to practice subtracting decimal hundredths. Include numbers that require regrouping and numbers greater than 1. EXAMPLES:

a) .98 – .42   b) 2.89 – .23   c) 3.49 – 1.99

Remind students of the relationship between missing addends and subtraction. Write on the board: 32 + 44 = 76. SAY: If 32 + 44 = 76, what is 76 – 44? What is 76 – 32? How can I find the missing addend in 32 + ___ = 76? (find 76 – 32) What is the missing addend in .32 + ____ = .76?

Have students find the missing addend in:

- .72 + ___ = .84
- .9 = ___ + .35
- .87 = ___ + .5
- .65 + ___ = .92

Have students demonstrate their answers using a model of a hundreds block. For example, students might fill in 72 hundredths with one colour and then fill in squares with another colour until they reach 84 to see that they need 12 more hundredths to make .84 from .72.
As in the previous lesson, give students base ten blocks and tell them to think of the hundreds block as a whole, the tens block as a tenth, and the ones block as a hundredth. Have pairs or individuals solve the following problems:

1. Take these blocks:

   • What decimal does this model represent?
   • Take away 2 blocks so the result (the difference) is between 1.21 and 1.35.
   • Write a decimal for the amount you took away.

2. Take these blocks:

   • What decimal does this model represent?
   • Take away 3 blocks so the result (the difference) is between 1.17 and 1.43.
   • Write a decimal for the amount you took away.
   • Repeat so that the difference is between 2.17 and 2.43.

Make up similar problems for students to solve independently or have students make up their own problems and exchange them with a partner.

Extensions

1. Show students how to subtract decimals from 1 by first subtracting from .99:

   \[ 1 - .74 = .01 + .99 - .74 = .01 + .25 = .26 \]

2. Teach students to estimate sums and differences of decimal numbers. Students can round to the nearest ten, one, tenth, hundredth and so on. (EXAMPLE: \$15.87 – \$11.02 is about \$16 – \$11 = \$5). Also teach students to reflect on whether their estimate will be higher or lower than the actual answer. In the example above, the actual difference will be just under \$5. You might also teach students to estimate sums by grouping numbers that add to about 10, 100 or 1000. For example, 3.96 + 6.1 + 98.4 + 63 + 35 is about 10 + 100 + 100 = 210.
NS5-93
Multiplying Decimals by 10 and

NS5-94
Multiplying Decimals by 100

Tell your students that a hundreds block represents 1 whole. **ASK:** What does a tens block represent? (one tenth) What does a ones block represent? (one hundredth)

**ASK:** How many tens blocks do we need to make a hundreds block? How many tenths do we need to make one whole? Can we write this as a multiplication statement? If we use four 3s to make 12, what multiplication statement do we have? (4 × 3 = 12) If we use ten 0.1s to make one whole, what multiplication statement do we have? (10 × 0.1 = 1) Show this using base ten materials (or a model of base ten materials):

```
<table>
<thead>
<tr>
<th>Ten of these:</th>
<th>equals</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 × 0.1</td>
<td>=</td>
</tr>
</tbody>
</table>
```

**ASK:** If 0.1 is represented by a tens block, how would you represent 0.2? How would you show 10 × 0.2? (2 hundreds blocks) What is 10 × 0.2? (2) Repeat for 10 × 0.3. Have students predict 10 × 0.7, 10 × 0.6 and 10 × 0.9. Students should then show their answers using base ten blocks.

**ASK:** If 0.1 is represented by a tens block, how would you represent 0.01? What block is ten times smaller than a tens block? (a ones block) How would you show 100 × 0.01? (a hundreds block) What is 100 × 0.01? (1) Repeat for 100 × 0.02 and 100 × 0.03. Have students predict 100 × 0.07, 100 × 0.06 and 100 × 0.09. Students should then show their answers using base ten blocks.

Repeat the exercise above with pennies, dimes and loonies in place of ones, tens and hundreds blocks.

Draw a ruler on the board that shows cm and mm:

```
0 10 20 30 40 50 (mm)
```

```
0 1 2 3 4 5 (cm)
```

**ASK:** If you know that the length of an object is 4 cm, how can you find the length of the object in mm? How can you obtain the number of mm from the number of cm? (multiply by 10) Why does this work? (because mm are ten times smaller than cm)
ASK: How many mm are in 0.7 cm? Then have a volunteer show this on the ruler you drew. Repeat for other numbers (0.9 cm, 0.3 cm, and so on)

Show a metre stick and repeat the exercise using cm and m.

Have students skip count by 0.3 to finish the pattern:

0.3, 0.6, 0.9, 1.2, ____, ____, ____, ____, ____, ____.  

ASK: What is $3 \times 0.3$? $5 \times 0.3$? $8 \times 0.3$? $10 \times 0.3$? Repeat with skip counting by 0.2 and 0.5.

Ask students to think about how they can multiply $325 \times 10$ by just adding a 0. **ASK:** Where do you add the zero when you multiply by 10? How does the values of each digit change when you multiply a number by 10?

- When you multiply by 10: 325 becomes 3 250
- 300, 20, and 5 become 3 000, 200, and 50

Write the number 346.51 and **ASK:** What is each digit worth? (300, 40, 6, 0.5 and 0.01) How can you move the decimal point so that each digit is worth ten times more (i.e., the ones digit becomes the tens digit, the tens digit becomes the hundreds digit, and so on)? The digits become worth 3000, 400, 60, 5 and 0.1, so the number becomes 3465.1. This is obtained by simply moving the decimal point one place to the right.

**ASK:** How would you find $32.5 \times 10$? Where should you move the decimal point to make every digit worth ten times more?

**ASK:** How is multiplying a decimal number by 10 different from multiplying a whole number by 10? Can we just add a zero to 32.5 to multiply by 10? What would we get if we added 0? (32.50) Does adding 0 to the decimal number change it at all? (No, but adding 0 to the end of a whole number changes the ones digit to 0, and every other digit becomes worth 10 times more.) Tell students that when they multiply a decimal number by 10, they move the decimal point one place to the right. Point out that they are, in fact, doing the same thing when they multiply whole numbers by 10. For example, $17 = 17.0$, so when you multiply by 10 by adding a 0 to the end, you’ve also moved the decimal point over to get 170.

Have students try these problems in their notebooks:

- $0.5 \times 10$
- $10 \times 0.8$
- $1.3 \times 10$
- $10 \times 2.4$
- $134.6 \times 10$
- $10 \times 12.45$

**ASK:** How would you find $32.5 \times 100$? Where should you move the decimal point to make every digit worth a hundred times more? How much is each digit worth right now? (30, 2 and 0.5) How much will each digit be worth when you multiply by 100? (3000, 200 and 50) What number is that? (3 250) Explain to students that this is like moving the decimal point two places to the right:

$$32.5 \rightarrow 3250.$$  

Have students try these problems in their notebooks:

- $0.54 \times 100$
- $0.92 \times 100$
- $0.3 \times 100$
- $1.4 \times 100$
- $432.789 \times 100$

Remind students that multiplying by 100 is the same as multiplying by 10 and then by 10 again. How do you do multiplying by 10 (move decimal point one place right). So what do you do to multiply by 10 and then by 10 again? (move decimal point two places right)
GOALS
Students will discover the connection between multiplying decimals by whole numbers and multiplying whole numbers by whole numbers.

PRIOR KNOWLEDGE REQUIRED
Place value up to hundredths
Multiplying 3- and 4-digit numbers by 1-digit numbers

VOCABULARY
divisor
dividend
quotient

Using the hundreds block as 1 whole, have volunteers show:

a) 1.23 and then 2 × 1.23 
   b) 4.01 and then 2 × 4.01 
   c) 3.12 and then 3 × 3.12

Have students individually solve the following problems by multiplying each digit separately:

a) 4.12 = ___ ones + ___ tenths + ___ hundredths
   2 × 4.12 = ___ ones + ___ tenths + ___ hundredths = ___

b) 3.11 = ___ ones + ___ tenths + ___ hundredths
   3 × 3.11 = ___ ones + ___ tenths + ___ hundredths = ___

c) 1.02 = ___ ones + ___ tenths + ___ hundredths
   4 × 1.02 = ___ ones + ___ tenths + ___ hundredths = ___

Have students multiply mentally:

a) 4 × 2.01 
   b) 3 × 2.31 
   c) 3 × 1.1213

Bonus

d) 2 × 1.114312 
   e) 3 × 1.1212231

Have students solve the following problems by regrouping when necessary:

a) 3 × 4.42 = ___ ones + ___ tenths + ___ hundredths
   = ___ ones + ___ tenths + ___ hundredths = ___

b) 4 × 3.32 = ___ ones + ___ tenths + ___ hundredths
   = ___ ones + ___ tenths + ___ hundredths = ___

c) 3 × 3.45 = ___ ones + ___ tenths + ___ hundredths
   = ___ ones + ___ tenths + ___ hundredths
   = ___ ones + ___ tenths + ___ hundredths
   = ___

Then have students solve the following problems by regrouping when necessary:

a) 3 × 442 = ___ hundreds + ___ tens + ___ ones
   = ___ hundreds + ___ tens + ___ ones = ___

b) 4 × 332 = ___ hundreds + ___ tens + ___ ones
   = ___ hundreds + ___ tens + ___ ones = ___
c) \(3 \times 345 = \) hundreds + tens + ones
\(= \) hundreds + tens + ones
\(= \)

Discuss with students the similarities and differences between these problems and solutions. Remind students about the standard algorithm for multiplying 3-digit by 1-digit numbers and ask students if they think they can use the standard algorithm for multiplying decimal numbers. Emphasize that none of the digits in the answer changes when the question has a decimal point; only the place value of the digits changes. The key to multiplying decimals, then, is to just pretend the decimal point isn’t there, and then add it back in at the end. The only tricky part is deciding where to put the decimal point at the end.

Demonstrate using \(442 \times 3\):

\[
\begin{array}{c}
1 \\
442 \\
\times 3
\end{array}
\quad
\begin{array}{c}
1 \\
4.42 \\
\times 3
\end{array}
\]

\[
\begin{array}{c}
1326 \\
\end{array}
\quad
\begin{array}{c}
1326 \\
\end{array}
\]

Tell students that now you need to know where to put the decimal point. Will the answer be closer to 1 or 13 or 132 or 1326? **ASK:** How many whole ones are in 4.42? How many ones are in 3? About how many ones should be in the answer? (4 \(\times\) 3 = 12) What is closest to 12: 1, 13, 132 or 1326? Have a volunteer guess where the decimal point should go and ask the class to explain why the volunteer chose the answer or to agree or disagree with the choice.

Repeat with several problems. (**EXAMPLES:** \(3.35 \times 6\); \(41.31 \times 2\); \(523.4 \times 5\); \(9.801 \times 3\))

**Bonus**

\(834\ 779.68 \times 2; \ 5\ 480.63 \times 7\)

**Extension**

(Adapted from Atlantic Curriculum A2.6) Teach students that just as they can take fractions of whole numbers, they can take decimals of whole numbers. Ask students whether they get the same answer when they take a quarter of a number (say 8) by dividing the number into 4 equal parts and when they take a quarter of the same number by multiplying the number by 0.25. Tell your students that \(\frac{1}{4}\) of the people in this class is the same as 0.25 of the people in this class. They can find 0.25 of a number by multiplying that number by 0.25.

Ask students to find a context in which 0.25 represents a small amount and one in which it represents a large amount.
GOALS

Students will discover the rule for dividing decimals by 10 by moving the decimal point one place left.

PRIOR KNOWLEDGE REQUIRED

Decimal tenths and hundredths
Base ten materials
Decimal place value

Tell students that a hundreds block represents one whole. ASK: When you divide one whole into 10 equal parts, what do you get? (a tens block or one tenth)

Write on the board:

\[
\begin{align*}
\text{÷ 10 = } & \quad 1.0 \div 10 = 0.1 \\
\end{align*}
\]

Then have a volunteer divide 0.1 by 10 using pictures:

\[
\begin{align*}
\text{÷ 10 = } & \quad 0.1 \div 10 = 0.01 \\
\end{align*}
\]

Draw several pictures on the board and have students write the corresponding division statements individually: EXAMPLES:

\[
\begin{align*}
\text{÷ 10 = } & \quad \text{EXAMPLES:} \\
\end{align*}
\]

Then give students the following problems and have them draw the models themselves:

a) 3 \(\div\) 10  
b) 0.2 \(\div\) 10  
c) 2 \(\div\) 10  
d) 0.4 \(\div\) 10  
e) 12 \(\div\) 10  
f) 1.2 \(\div\) 10

Write the number 43.5 on the board and ASK: How much is each digit worth? (40, 3 and 0.5). If we divide 43.5 by 10, how much does each digit become worth? (4, 0.3 and 0.05) What is 43.5 \(\div\) 10? (4.35) Repeat this exercise with several examples (567.8 \(\div\) 10, 43.57 \(\div\) 10, 89.312 \(\div\) 10, 41 325.5 \(\div\) 10)

ASK: How do you move the decimal point when dividing a number by 10? How can you make each digit worth ten times less? (move the decimal point one place to the left)

Teach students the connection between ten times more and the measurement units we use. For example, a dm is ten times larger than a cm. ASK: if an object is 3 dm long, how many cm long is it? What operation did you do? If an object is 40 cm long, how many dm long is it? What operation did you do? If an object is 7 cm long, how many dm long is it?
9 cm = _____ dm, 13 cm = _____ dm, 702 cm = _____ dm. Repeat the exercise with mm and cm.
(EXAMPLES: 8.4 mm = ____ cm, 39.2 mm = ____ cm)

**Extensions**

1. Teach students how to divide by 100 or 1 000 by moving the decimal point 2 or 3 places to the left. **ASK:** What is 4 531.2 ÷ 100? What is each digit worth? (4 000, 500, 30, 1 and 0.2) When we divide by 100, what will each digit be worth? (40, 5, 0.3, 0.01 and 0.002) What number is that? (45.312) How did the decimal point move when you divided by 100? Why does this make sense? How is dividing a number by 100 the same as dividing the number by 10 and then dividing it by 10 again? Emphasize that dividing something into 10 equal parts and then dividing those 10 equal parts into 10 parts again leaves the original object divided into 100 equal parts. Notice that moving the decimal point 2 places left is the same as moving the decimal point one place left and then another place left. Challenge students to predict how they would divide a decimal number by 1000.

**Bonus**

Find 31 498.76532 ÷ 1 000 000.

2. The wind speed in Vancouver was 26.7 km/h on Monday, 16.0 km/h on Tuesday and 2.4 km/h on Wednesday. What was the average wind speed over the 3 days?

3. A pentagonal box has a perimeter of 3.85 m. How long is each side?

4. (From Atlantic Curriculum A7) Draw a number line with only the endpoints 2 and 4 marked. Have students mark where the following numbers would be and to defend their positions: 2.3, 2.51, 2.999, 3.01, 3.75, 3.409, 3.490.

5. (Adapted from Atlantic Curriculum A7.1) Have students roll a die three times and ask them to use the digits to represent tenths, hundredths and thousandths. Have students make the smallest number they can and then to say how much would need to be added to make one whole.

6. (Atlantic Curriculum A7)
   a) If gas is priced at 56.9¢ per litre, what part of a dollar is this?
   b) If you drank 0.485 L of juice, how much more would you have to drink to equal 0.5 L?

7. (Atlantic Curriculum A7.9) Have students write a report on the use of 0.5 and ½. Students could survey adults and check newspapers and magazines to find when each is used.
NS5-97
Dividing Decimals by Whole Numbers

GOALS
Students will use long division to divide decimal numbers by single-digit whole numbers.

PRIOR KNOWLEDGE REQUIRED
Long division of 3- and 4-digit whole numbers by single-digit whole numbers. Decimal place value up to hundredths

VOCABULARY
- tenths
- hundredths
- dividend
- divisor
- quotient

Tell students that a hundreds block represents one whole and draw on the board:

```
\[ \begin{array}{|c|c|c|}
\hline
1.0 & & \\
\hline
0.1 & & \\
\hline
0.01 & & \\
\hline
\end{array} \]
```

Have volunteers draw the base ten models for: 5.43, 8.01, 0.92. Then have students do similar problems in their notebooks.

Then tell students that you would like to find \( 6.24 \div 2 \). Have a volunteer draw the base ten model for 6.24. Then draw 2 circles on the board and have a volunteer show how to divide the base ten materials evenly among the 2 circles. What number is showing in each circle? What is \( 6.24 \div 2 \)?

Repeat for other numbers where each digit is divisible by 2 (or 3).

EXAMPLES: \( 46.2 \div 2, 3.63 \div 3, 4.02 \div 2, 6.06 \div 2, 6.06 \div 3 \).

Then write on the board \( 3.54 \div 2 \). **ASK:** In what way is this problem different from the previous problems? (each digit is not divisible by 2). Have a volunteer draw the base ten model for 3.54.

Draw 2 circles on the board and ask why you chose to draw 2 circles rather than a different number of circles?

Then lead students through the steps of long division, as in NS5-41: Long Division—3- and 4-Digit by 1-Digit. Then do a comparison of the steps for \( 3.54 \div 2 \) and \( 354 \div 2 \). Emphasize that students can just pretend that the decimal point does not exist and then put the decimal point in the correct place at the end. For example, students will see that \( 354 \div 2 = 177 \). To find \( 3.54 \div 2 \), they can round 3.54 to 4 and estimate that 3.54 ÷ 2 is about \( 4 \div 2 = 2 \). Where should they put the decimal point so that the answer is close to 2? (**ANSWER:** 1.77) Have students do several problems where they figure out the answer this way, first by long division of whole numbers and then by estimating to find where to put the decimal place.

Then, to ensure students are doing this last step correctly, give problems where students do not need to do the long division, but only this last step of putting the decimal point in the correct place:

a) \( 856.1 \div 7 = 122.3 \) (**ANSWER:** 122.3 since the answer will be close to but more than 100)

b) \( 8922.06 \div 6 = 1487.01 \) (**ANSWER:** 1487.01 since the answer will be close to but more than 1000)

To ensure that students do not simply count decimal places, but actually estimate their answers, give some examples where you add an extra zero to the answer. For example, \( 23.28 \div 6 = 3.880 \) has answer: 3.880 or just 3.88.
Students should be encouraged to discover their own rule regarding where to put the decimal point when they have finished the long division (put the decimal point above the decimal point, as shown on the worksheet.) You can explain why you line up the decimal points when doing long division as follows: When you divide 342 by 2 by long division, the answer is aligned as shown:

\[
\begin{array}{c}
171 \\
2 \overline{342}
\end{array}
\]

If you divide 3.42 by 2 the answer will be 100 times smaller than the answer to 342 ÷ 2. Just as I need 100 times fewer 2s to make 8 as I need to make 800, I need 100 times fewer 2s to make 3.42 as I need to make 342. Hence the answer to 3.42 ÷ 2 by long division should look like:

\[
\begin{array}{c}
1.71 \\
2 \overline{3.42}
\end{array}
\]

Notice that when the decimal in the dividend (3.42) shifts 2 left, the decimal in the quotient (1.71) shifts 2 left, so the decimals in the dividend and the quotient line up.

**Extensions**

1. Teach students to divide decimals by single-digit decimals (EXAMPLE: 86.4 ÷ 0.9) by treating both the dividend and the divisor as whole numbers (864 ÷ 9 = 96) and then estimating by rounding each number to the nearest whole number (86 ÷ 1 is about 90) to decide where to put the decimal point.

2. Compare multiplying and dividing decimals to adding and subtracting decimals. You can multiply 3 × 4.2 by removing the decimal point pretending they are whole numbers and then put the decimal point in the correct place. Can you add 3 + 4.2 using this same strategy? Why not? Discuss.
GOALS

Students will understand decimal place value up to thousandths

PRIOR KNOWLEDGE REQUIRED

Decimal place value up to hundredths

VOCABULARY

thousandths

NS5-98

Thousandths

Review place value up to hundredths, then write the number 6.142 on the board. Cover up all but the first 6 and the decimal point. **ASK:** What is the place value of the 6? How do you know? (The 6 is the ones digit because the decimal point is right next to it). Repeat the exercise, uncovering the 1 and then the 4. Emphasize that each digit is worth ten times less than the previous one. **ASK:** What is ten times less than a tenth? What is ten times less than a hundredth? Uncover the 2 and ask what its place value is.

Ensure that students can identify the place value of any given underlined digit. (**EXAMPLES:** 3.407, 6.015, 32.809)

Remind students of the connection between fractions and decimal numbers. Ask volunteers to write each decimal number as a fraction or mixed fraction with denominator 10 or 100: 0.7, 0.91, 0.09, 1.5, 1.07. Then **ASK:** How did you know whether the denominator was 10 or 100? (look at the number of decimal places) What if there are 3 decimal places? Then what will the denominator of the fraction be? Have students write each of these decimals as fractions or mixed fractions with denominator 10 or 100 or 1000: 0.9, 0.987, 0.998, 0.009, 0.07, 0.652, 0.073.

Show students how to write a decimal number in expanded form by writing the place value of each digit: 3.241 = 3 ones + 2 tenths + 4 hundredths + 1 thousandth. Have students write the place value of each digit for various numbers (**EXAMPLES:** 2.4, 2.04, 2.004, 20.04, 200.4, 21.35, 2.135, 42.135).

Have students convert the following fractions to decimals:

- a) $\frac{32}{100} = \boxed{0.32}$
- b) $\frac{3}{100} = \boxed{0.03}$
- c) $\frac{324}{1000} = \boxed{0.324}$
- d) $\frac{324}{100} = \boxed{3.24}$
- e) $\frac{32}{1000} = \boxed{0.032}$
- f) $\frac{3}{1000} = \boxed{0.003}$

Have students compare the decimals by first changing them to fractions with the same denominator:

- a) $\frac{32}{100}$
- b) $\frac{3}{100}$
- c) $\frac{324}{1000}$
- d) $\frac{324}{100}$

Have students compare these same decimals by adding zeroes when necessary to make both decimals have the same number of digits. (**EXAMPLE:** compare .298 to .320) **ASK:** How is this the same as changing to fractions with the same denominator? How is this different?

Have students practice ordering decimals that have different numbers of decimal places.

**EXAMPLES:**

- a) 6.53 18.2
- b) 456.73 21.72006
- c) 85.7601 112.03
- d) 13.54 13.5
Extensions

1. Have students compare numbers with more decimal places and with more digits before the decimal point. (EXAMPLE: 32.4167 and 298.345)

2. (From Atlantic Curriculum A2.4) Show the students cards on which decimals have been written (EXAMPLE: 0.75, and 0.265 m). Ask students to place the cards appropriately on a metre stick.

3. Challenge students to write decimals for the following fractions:
   a) \( \frac{1}{10000} \)
   b) \( \frac{1}{100000} \)
   c) \( \frac{77}{10000} \)

   and then fractions for the following decimals:
   a) 0.0003
   b) 0.12074
   c) 0.000981004

4. (From the Atlantic Curriculum)

   Have students use each of the digits from 0 to 9 once to fill in the 10 spaces and make these statements true.

   \[ \□ \cdot \□ < \□ \cdot \□ \]

   \[ \□□\□\cdot \□ > \□\cdot \□\□\□ \]

5. Have students add decimal thousandths by using these different methods:
   i) Count the number of thousandths, add the whole numbers and then change the number back to a decimal.
      EXAMPLE: \( 1.487 + 0.23 = 1 \text{ 487 thousandths} + 230 \text{ thousandths} = 1 \text{ 717 thousandths} = 1.717 \)

   ii) Change both decimals to fractions with denominator 1 000 and then add the fractions.
      EXAMPLE: \( 1.487 + 0.23 = \frac{1487}{1000} + \frac{230}{1000} = \frac{1717}{1000} = 1.717 \)

   iii) Line up the decimal points.
      EXAMPLE: 
      \[
      \begin{array}{c}
      1.487 \\
      +0.23 \\
      \hline
      1.717
      \end{array}
      \]

   SAMPLE EXERCISES:

   a) 0.81 + 0.081
   b) 5.7 + 4.32 + 5.014
   c) 0.23 + 6.3 + 306.158
   d) 19.41 + 173.2 + 6.471

6. Relate the ordering of numbers to the alphabetical ordering of words. When we put words in alphabetical order, we compare first the left most letters, then the next letters over, and so on. Ask students to put these pairs of words in alphabetical order by identifying the first letter that’s different:

   mouse, mice
   noun, none
   room, rope
   snap, snip
   
   trick, trim
   sun, fun
   pin, tin
   spin, shin

   Students can write the words one above the other and circle the first letter that’s different:

   mouse
   mice
Tell students that a blank always comes first. Use the words “at” and “ate” to illustrate this. The first two letters in “at” and “ate” are the same. There is no third letter in “at”—there is nothing, or a blank, after the “t.” The blank comes before the “e” at the end of “ate,” so “at” comes before “ate.” Have students use this knowledge to put the following pairs in alphabetical order:

mat mate an a no noon bath bat kit kite

Point out the difference between lining up numbers and words. The numbers 61435 and 7384 would be lined up

<table>
<thead>
<tr>
<th>61435</th>
<th>not</th>
<th>61435</th>
</tr>
</thead>
<tbody>
<tr>
<td>7384</td>
<td></td>
<td>7384</td>
</tr>
</tbody>
</table>

But the words “at” and “ate” would be lined up like this:

<table>
<thead>
<tr>
<th>ate</th>
<th>not</th>
<th>ate</th>
</tr>
</thead>
<tbody>
<tr>
<td>at</td>
<td></td>
<td>at</td>
</tr>
</tbody>
</table>

When ordering words, you line up the leftmost letters. When ordering numbers, you line up the ones digits, whether they’re on the right, the left, or anywhere in between. For both words and numbers, you start comparing from the left.

Another important difference is that 5-digit whole numbers are always greater than 4-digit whole numbers, but 5-letter words can be before or after 4-letter words.

Point out that in numbers, as in words, a space is always less than a number. When we compare:

\[
\begin{align*}
6.53 & \quad 17.1 \\
8.2 & \quad \text{OR: } 17.14
\end{align*}
\]

the blank is really a 0 and is less than, or comes before, any number.
NS5-99
Differences of 0.1 and 0.01

GOALS
Students will count by tenths and hundredths starting from any number with at most 2 decimal places.

PRIOR KNOWLEDGE REQUIRED
Adding and subtracting tenths and hundredths
Relationship between adding and “more than”
Number lines

Have your students add the following numbers in their notebooks:

\[
\begin{align*}
a) \quad & .48 + .1 \\
b) \quad & .48 + .01 \\
c) \quad & .52 + .1 \\
d) \quad & .52 + .01 \\
e) \quad & .63 + .1 \\
f) \quad & .63 + .01 \\
g) \quad & 4.32 + .01 \\
h) \quad & 4.32 + 1 \\
i) \quad & 4.32 + .1 \\
j) \quad & 7.38 + .1 \\
k) \quad & 7.38 + .01 \\
l) \quad & 7.38 + 1
\end{align*}
\]

ASK: How do you add .1 to a number? (add 1 to the tenths digit) When you added .1 above, how many digits changed? (only 1, the tenths digit)

Then have students find .94 + .1. PROMPTS: How many hundredths are in .94? (94) How many hundredths are in .1? (10) How many hundredths is that altogether? (104) So the answer is 1.04. ASK: How is adding .1 to .94 different from adding .1 to the numbers above? What else changes besides the tenths digit? (the ones digit changes—from 0 to 1—because the answer is more than 100 hundredths)

Have students do the following problems and tell you when they just add 1 to the tenths digit and when they have to change the ones digit, too:

\[
\begin{align*}
.49 + .1 & & .86 + .1 & & .93 + .1 & & .97 + .1 & & .36 + .1 \\
.49 + .01 & & .94 + .01 & & .86 + .01 & & .49 + .01 & & .94 + .01 \\
.28 + .01 & & & & & & & &
\end{align*}
\]

ASK: What number is .1 more than 9.3? Ask students to identify the number that is .1 more than: .7, 8.4, .6, .9, 5.9. Prompt students with questions such as: How many tenths are in 5.9? What is one more tenth? What number has 60 tenths?

What number is .01 more than:

\[
\begin{align*}
8.47 & & 8.4 & & .3 & & .39 & & .86 & & .89 \\
\end{align*}
\]

What number is 1 more than:

\[
\begin{align*}
\end{align*}
\]

Repeat with subtraction, asking students to find numbers that are .01, .1, and 1 less than various numbers with 1 and 2 decimal points. Include examples where students need to borrow/regroup.

Have a volunteer add the missing decimal numbers to this number line:

\[
\begin{array}{cccccccc}
4.0 & & & & & & & 5.0
\end{array}
\]

Students can refer to the number line to complete these sequences in their notebooks:

\[
\begin{align*}
4.3, 4.4, 4.5, & & 4.1, 4.4, 4.7, & & \\
4.0, 4.2, 4.4, & & 4.9, 4.7, 4.5, & &
\end{align*}
\]
Have students give the rule for each sequence. (EXAMPLE: start at 4.3 and add .1)

Have another volunteer add the missing decimal numbers to this number line:

```
7.3    |    |    |    |    |    | 8.3
```

Students should complete and describe these sequences in their notebooks:

- 7.7, 7.8, 7.9, ____  7.2, 7.5, 7.8, ____
- 7.5, 7.7, 7.9, ____  8.3, 8.2, 8.1, ____, ____

Have students fill in the blanks in their notebooks:

- 5.9 + .1 = ____   8.9 + .1 = ____   .9 + .1 = ____
- 6.49 + .1 = ____   6.49 + .01 = ____   6.49 + 1 = ____
- 8.93 + .1 = ____   8.99 + .1 = ____   8.99 + .01 = ____

**Extensions**

1. Ask students to count forward from the following numbers by tenths, orally.
   a) 6    b) 17.8    c) 123.2

2. a) Count forwards by hundredths from 0.96:
   - 0.96, _____, _____, _____, _____, _____

   b) Count forwards by cm from 96 cm:
   - 96 cm, _____ cm, _____ cm, _____ cm, _____ m, _____ m _____ cm,
     _____ m _____ cm

   c) Count forward by cents from 96 cents:
   - 96¢, _____¢, _____¢, _____¢, $_____, $_____, $_____

   d) How are questions a) to c) above the same? How are they different?

   e) What connections can you make between counting by hundredths and measuring lengths in centimeters and metres?

   f) What connections can you make between counting by hundredths and counting money in cents and dollars?
GOALS
Students will review concepts in decimals.

PRIOR KNOWLEDGE REQUIRED
Base ten materials
Units of measurement:
- metres (m), centimetres (cm), and millimetres (mm)
- Fractions and equivalent decimals

Draw on the board: 

Ask a volunteer to shade one tenth of the picture. Then have students draw one tenth of each of the following pictures in their notebooks:

Tell your students that the fraction of a measurement depends on which unit is the whole. **ASK:** What is one hundredth of a meter? (1 cm) How would you write this as a fraction? \( \frac{1}{100} \) m How would you write this as a decimal? (.01 m)

Have students write each of the following measurements as a fraction and a decimal in metres.

\[
\begin{align*}
2 \text{ cm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
20 \text{ cm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
25 \text{ cm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
132 \text{ cm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
\end{align*}
\]

Have your students write these measurements as a fraction and decimal in centimetres.

\[
\begin{align*}
8 \text{ mm} &= \underline{\underline{\text{cm}}} = \underline{\underline{\text{cm}}} \\
4 \text{ mm} &= \underline{\underline{\text{cm}}} = \underline{\underline{\text{cm}}} \\
17 \text{ mm} &= \underline{\underline{\text{cm}}} = \underline{\underline{\text{cm}}} \\
29 \text{ mm} &= \underline{\underline{\text{cm}}} = \underline{\underline{\text{cm}}} \\
\end{align*}
\]

**Bonus**

\[
\begin{align*}
54 \text{ mm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
135 \text{ mm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
6054 \text{ mm} &= \underline{\underline{\text{m}}} = \underline{\underline{\text{m}}} \\
\end{align*}
\]

Have your students add the measurements by changing the one in smaller units to a measurement in the larger units.

\[
\begin{align*}
a) \ 3 \text{ cm} + 9.46 \text{ m} & \quad b) \ 24 \text{ cm} + .12 \text{ m} \quad c) \ 584 \text{ cm} + 2.3 \text{ m} \\
\end{align*}
\]

Have students check their answers by changing the number in larger units to one in smaller units, adding, and then changing the answer back to the larger units. Did they get the same answer?
Extensions

Give each student or pair a set of base ten blocks. Tell students the hundreds block is the whole, so the tens block represents .1 and the ones block represents .01.

1. Ask students to show and write all the decimals they can make with these 3 blocks.

\[
\begin{array}{c|c|c|c}
\hline
& & & \\
\hline
\hline
\end{array}
\]

(SOLUTION: 1.0, 1.1, 1.01, 1.11, 0.1, 0.01, 0.11)

NOTE: One way to prepare students for this exercise is to hold up combinations of blocks and ask them to write the corresponding decimal in their notebook. For instance, if you hold up the hundreds block (which represents one unit) and the tens block (which represents a tenth) they should write 1.1.

2. Use base ten blocks to make a decimal
   a) greater than .7.
   b) less than 1.2.
   c) between 1 and 2.
   d) between 1.53 and 1.55.
   e) with tenths digit equal to its ones digit.
   f) with hundredths digit one more than its tenths digit.

3. Create models of 2 numbers such that
   a) one number is 4 tenths greater than the other.
   b) one number has tenths digit 4 and is twice as large as the other number.

4. One decimetre (1 dm) is 10 cm. Explain how you would change 3.2 dm into centimetres.

5. (Atlantic Curriculum A9.4) Ask students to explain why you cannot compare two decimal numbers by simply counting the number of digits of each.
NS5-101
Word Problems with Decimals

Review word problems with your students.

Extensions

1. The chart shows how many times stronger (or weaker) gravity is on the given planets than on earth.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity Factor</td>
<td>1.15</td>
<td>2.34</td>
<td>0.83</td>
<td>.284</td>
</tr>
</tbody>
</table>

   a) On which planets is gravity less strong than on Earth?
   b) How much would a 7 kg infant weigh on each planet?
   c) How much more would the infant weigh on Jupiter than on Mars?
   d) John can jump 1 metre on Earth. How high can he jump on Mercury?

2. Tell students that information in a word problem can be redundant; sometimes you don’t need all the information given to solve the problem.

   EXAMPLE: Find the secret number:
   
   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My ones digit is greater than my tens digit.

   Have students try to figure out which 3 of the clues they need to answer the question.

   By changing only the last clue slightly, challenge your students to solve the puzzle using only two of the clues.

   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My tens digit is greater than my ones digit.

   Students can make up their own number puzzle with one piece of redundant information and have a partner solve the puzzle and identify the unneeded information.

3. Have students make up rhyming math poems as puzzles. EXAMPLE:

   A 2-digit number named Todd
   Has tens digit odd.
   But he’s even you see
   And his digits add to three.
   Which two numbers can he be?
NS5-102
Unit Rates and
NS5-103
Scale Diagrams

GOALS
Students will understand simple multiplicative relationships involving unit rates.

PRIOR KNOWLEDGE REQUIRED
Money (dollars and cents)
Distance (km, m and cm)
Time (weeks, hours)

VOCABULARY
rate
unit rate

Explain that a rate is the comparison of two quantities in different units. For example, “3 apples cost 50¢” is a rate. The units being compared are apples and cents. Have students identify the units being compared in the following rates.

5 pears cost $2.
$1 for 3 kiwis.
4 tickets cost $7.
1 kiwi costs 35¢.
Sally is driving at 50 km/hour.
On a map, 1 cm represents 3 m.
She earns $6 an hour for babysitting.
The recipe calls for 1 cup of flour for every teaspoon of salt. (NOTE: for this last example the units are not merely cups and teaspoons, they are cups of flour and teaspoons of salt.)

Explain that one of the quantities in a unit rate is always equal to one. Give several examples of unit rates, and have students identify the unit which makes it a unit rate.

1 kg of rice per 8 cups of water. (1 kg makes it a unit rate)
1 apple costs 30¢. (1 apple)
$1 for 2 cans of juice. ($1)
1 can of juice costs 50¢. (1 can of juice)
The speed limit is 40 km per hour. (1 hour)
She runs 1 km in 15 minutes. (1 km)

Explain that knowing a unit rate can help to determine other rates. ASK: If one book costs $3, how much do two books cost? … three books? … four books?

Draw a map with two cities joined by a line. Assuming that 1 cm represents 10 km, have volunteers determine the actual distance between the cities by measuring the line with a metre stick. Then have them explain their calculation for the class.

If 1 cm on a map represents 2 km, how much does 3 cm represent? How much does 7 cm represent? 4.5 cm? Two schools are 6 km apart. How far apart should they be drawn on the map? Two buildings are 11 km apart. How far apart should they be drawn on the map?

ASK: If you know that two books cost $6, how can you determine the cost for three books? What makes this problem different from the other problems in this lesson? (Instead of starting with the cost of 1 book, we are now starting with the cost of 2 books; we are not given a unit rate) How does working with unit rates make it easier to calculate other rates?
Working in pairs, have your students change this problem into a unit rate and then share their procedures with the class. Explain that higher rates can be determined through multiplication of the given rate, but the single unit rate can only be determined through division:

If 1 peach costs 25¢, then 3 peaches cost 75¢ (3 × 25¢).
If 3 peaches cost 75¢, then 1 peach costs 25¢ (75¢ ÷ 3).

Assign several problems that require your students to determine unit rates. Be sure the answers are whole numbers.

a) 4 pears cost 80¢. How much does 1 pear cost?
b) 24 cans of juice cost $24. How much does 1 can of juice cost?
c) 2 books cost $14. How much does 1 book cost?
d) 3 teachers supervise 90 students on a field trip. How many students does each teacher supervise?

Extensions

1. Have students determine the unit rates and then solve the following problems.
   a) If 4 books cost $20, how much do 3 books cost?
   b) If 7 books cost $28, how much do 5 books cost?
   c) If 4 L of soy milk costs $8, how much do 5 L cost?

2. Bring in some flyers from a grocery store, and ask students to determine unit prices and calculate the cost of quantities greater than one. For instance, if the unit price is $2.75 per item, how much will three items cost? If they do not know how to multiply a decimal number with a single-digit number, challenge them to select and use an alternate unit to dollars—they should use cents. **ASK:** How many cents are in $2.75? If each item costs 275¢, how many cents will three items cost? What does that equal in dollars?

Ask students to calculate the unit price of an item using division. For instance, if three items cost $1.62, how much will one item cost? Again, have them convert dollars to cents and then back to dollars.
NS5-104
Proportions

GOALS
Students will use ratios to determine how many times greater one quantity is than another and will write this answer as a decimal.

PRIOR KNOWLEDGE REQUIRED
Comparing decimals and fractions Writing the fractions with denominator 4 as decimals (0.25, 0.5, 0.75, 1, 1.25 and so on) Equivalent fractions Unit rates

Tell your students that you have a pancake recipe that calls for 7 cups of flour and 4 cups of bananas. Remind them that this is called a rate because it compares 2 quantities in different units (cups of flour to cups of bananas). Remind them that last time, we found the unit rate and ask what a unit rate is. Then ASK: What makes the unit rate harder to find in this case than in the last lesson? How many cups of flour do we need for 1 cup of bananas? (As an improper fraction: \( \frac{7}{4} \), and as a mixed fraction: \( 1 \frac{3}{4} \)) ASK: How would you write \( 1 \frac{3}{4} \) as a decimal? (1.75) Tell students that they would need 1.75 cups of flour for every 1 cup of bananas. ASK: How does that make it easy to tell how many cups of flour they would need for 5 cups of bananas? (multiply 5 \( \times \) 1.75) Have students do this calculation. Emphasize that the number of cups of flour is always 1.75 times the number of cups of bananas, so if they know how many cups of bananas they have, then they can deduce the number of cups of flour. Have students perform this calculation for various amounts of bananas (2 cups, 7 cups, 6 cups, 3 cups).

Repeat with several problems.

Emphasize that in the examples above, they compared numbers through multiplication rather than through addition. If we had said: The recipe calls for 7 cups of flour and 4 cups of bananas, so it calls for 3 more cups of flour than cups of bananas, this would be comparing through addition and subtraction rather than through multiplication. ASK: If I want to make the same recipe, but with 5 cups of bananas instead, should I use 8 cups of flour since that is 3 more than 5? Could that mess up the recipe? Explain to students that in this situation, we want the ratio of flour to bananas to remain the same; that is, we want how many times more cups of flour than bananas to remain the same—in this case, the cups of flour is 1.75 times the cups of bananas.

Extensions
1. Ask students to find many real-life examples of ratios:
   a) For every ____ months, there is 1 year, so the ratio of months to years is ____ : 1.
   b) For every ____ days, there is ____ week, so the ratio of days to weeks is ____ : ____.
   c) For every ____ dozen, there are ____ items, so the ratio of dozens to items is ____ : ____.
   d) For every ____ mm, there are ____ cm, so the ratio of mm to cm is ____ : ____.
   e) A recipe calls for 5 cups of flour and 2 cups of sugar. A double recipe calls for ____ cups of flour and ____ cups of sugar. The ratio of cups of flour to cups of sugar is ____ : ____ or ____ : ____.
Encourage students to fill in the blanks in more than one way so that they can find equivalent ratios.

2. Someone gets paid $25 for 3 hours worked. What is the ratio of dollars earned to hours worked. How much would the person get paid for working 6 hours?

**NS5-105**

**Numbers in Careers and the Media**

This lesson combines concepts from number sense, measurement and data management and can be used as an assessment. No lesson plan is associated with this worksheet, although activities and extensions are described below.

**GOALS**

Students will see connections between numbers and real-life situations.

**PRIOR KNOWLEDGE REQUIRED**

Decimal numbers
Times as many
Multiplying decimals
Bar graphs

**ACTIVITY**

Ask students to find an advertisement or article in the paper in which numbers are a significant part of the content of the piece. Ask students to say which numbers in the piece are exact and which are estimates. Ask whether any numbers may have been used or represented in a misleading way. (For instance, an advertisement that says “Many items reduced to half price” doesn’t tell you what proportion of items in the store have been reduced to half price.) Have students make up a word problem using the numbers in the article or advertisement.

**Extensions**

1. Doctors study the body. Here are some facts a doctor might know:
   a) **FACT**: “The heart pumps about 0.06 L of blood with each beat.” How much blood would the heart pump in 3 beats?
   b) **FACT**: “The heart beats about 80 times a minute.” How long would it take the heart to beat 240 times?
   c) **FACT**: “All of the blood passes through the heart in a minute.” How many times would the blood pass through the heart in a day?
d) **FACT:** “$\frac{55}{100}$ of human blood is a pale yellow liquid called plasma.”

How much plasma would there be in 2 L of blood? (**ANSWER:** $\frac{55}{100} = \frac{50}{100} + \frac{5}{100} = \frac{1}{2} + \frac{1}{20}$.  

$\frac{1}{2}$ of 2 L is 1 L.  $\frac{1}{20}$ of 2 L is 1 tenth of $\frac{1}{2}$ or 0.1 L.  So $\frac{55}{100}$ of 2 L is 1 L + 0.1 L = 1.1 L).  

\[ \frac{1}{10} \text{ of } \frac{1}{2} \text{ is } \frac{1}{20}. \]

e) **FACT:** “Bones make up about $\frac{15}{100}$ of the weight of the body.” How much would the bones of a 62 kg person weigh? (**ANSWER:** $\frac{15}{100} = \frac{10}{100} + \frac{5}{100} = \frac{1}{10} + \frac{1}{20}$. But $\frac{1}{10}$ of 62 is 6.2 and $\frac{1}{20}$ of 62 is half that, or 3.1.  So $\frac{15}{100}$ of 62 kg is 6.2 kg + 3.1 kg = 9.3 kg)  

\[ \frac{1}{10} \text{ of } \frac{1}{10} \text{ is } \frac{1}{20}. \]

f) **FACT:** “The brain is $\frac{85}{100}$ water.” What fraction of the brain is not water? (**ANSWER:** $\frac{15}{100}$)  

g) **FACT:** “The most common type of blood is Type O blood. $\frac{45}{100}$ of people have Type O blood.” About how many children in a class of 24 kids would have Type O blood? (**ANSWER:** $\frac{45}{100} = \frac{1}{2} - \frac{1}{20}$, so $\frac{45}{100}$ of 24 is about $12 - 1 = 11$.)

2. Meteorologists study the weather. The world’s highest temperature in the shade was recorded in Libya in 1932. The temperature reached 58°C.

a) How long ago was this temperature recorded?

b) On an average summer day in Toronto, the temperature is 30°C. How much higher was the temperature recorded in Libya?

---

**NS5-106**

**Word Problems**

This worksheet is a review and can be used as an assessment.

**Extension**

Ask students to estimate their answers in QUESTIONS 1, 2, 3, and 4 to check the reasonableness of their answers. Here are some types of calculations they can practice estimating with:

- a) $271 \times 3.8$
- b) $278 + 77.2$
- c) $585 - 27.7$
- d) $38.5 \div 5$
**PRESENT THE FOLLOWING PROBLEM:** You need to program a machine that will sell candy bars for 45¢. Your machine accepts only dimes and nickels. It does not give change. But you have to teach your machine to recognize when it’s been given the correct change. The simplest way to do this is to give your machine a list of which coin combinations to accept and which to reject. So you need to list all combinations of dimes and nickels that add up to 45¢.

Explain that there are two quantities—the number of dimes and the number of nickels. The best way to find the possible combinations is to list one of the denominations (usually the larger—dimes) in increasing order. You start with no dimes, then with one dime, etc. Where do you stop? How many dimes will be too much? Then next to each number of dimes in the list, you write down the number of nickels needed to make 45¢. Make a list and ask volunteers to fill it in. You can also use a table, as shown below.

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Let students practice making lists of dimes and nickels (for totals such as 35¢, 75¢, 95¢), nickels and pennies (for totals such as 15¢, 34¢, 21¢), quarters and nickels (for totals such as 60¢, 85¢, 75¢).

**Bonus**

Make 135¢ and 175¢ using dimes and nickels.

**Assessment**

List all the combinations of dimes and nickels that make 55¢.

Now let your students solve a more complicated riddle:

Dragons come in two varieties: the Three-Headed Fearsome Forest Dragons and the Nine-Headed Horrible Hill Dragons. A mighty and courageous knight is fighting these dragons, and he has slain 5 of them. There are 27 heads in the pile after the battle. Which dragons did he slay?

Remind your students that it is convenient to solve such problems with a chart and to start with dragons that have the largest number of heads in the first column. How many nine-headed dragons could there be? Not more than 3—otherwise there are too many heads. How many
three-headed dragons could there be? There are 5 dragons in total. Make a list and ask the volunteers to fill in the numbers:

<table>
<thead>
<tr>
<th>Nine-Headed Horrible Hill Dragons</th>
<th>Three-Headed Fearsome Forest Dragons</th>
<th>Total Number of Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask another volunteer to pick out the right number from the table—there were 2 nine-headed dragons and 3 three-headed dragons slain.

Give your students additional practice, with more head numbers: 12 heads, 2 dragons; 20 heads, 4 dragons. What is the least and the most number of heads for 6 dragons?

Tell students that you want to find all pairs of numbers that multiply to 4.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Ask students how large can the first number be. Can I stop now? How do I know when I can stop? (the first number cannot be more than 4 because $5 \times 1$ is already more than 4, so $5 \times$ any number will also be more than 4). Then erase the 5 and 6 from the “First Number” column and complete the chart with your students, noting that 3 does not have a second number. There is no number that multiplies with 3 to give 4. Then list all the pairs: $1 \times 4$, $2 \times 2$, $4 \times 1$.

Have students create similar T-charts for pairs of numbers that multiply to 5, 6, 7, 8, 9 and 10 and then to list the pairs in order of increasing first numbers.

- $5: 1 \times 5$, $5 \times 1$
- $6: 1 \times 6$, $2 \times 3$, $3 \times 2$, $6 \times 1$
- $7: 1 \times 7$, $7 \times 1$
- $8: 1 \times 8$, $2 \times 4$, $4 \times 2$, $8 \times 1$
- $9: 1 \times 9$, $3 \times 3$, $9 \times 1$
- $10: 1 \times 10$, $2 \times 5$, $5 \times 2$, $10 \times 1$

Have students look for any patterns among the lists. Do they notice any symmetry? Why is there a symmetry in the list? (Multiplication is commutative, so if one pair multiplies to a given number, so does its “reverse” pair—the same numbers written in reverse order).
Then, as a whole class, find all the pairs of numbers that multiply to 36. Make a large T-chart with 36 numbers.

Check with the whole class if the numbers 1 through 8 are factors of 36 by using skip counting if necessary. (For example, skip count by 3s to 36 to see that $3 \times 12 = 36$, skip count by 5s to see that 5 does not have a “partner” that multiplies to 36—see NS5-35: Dividing by Skip Counting).

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
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<tbody>
<tr>
<td>1</td>
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Then, ask students how they can use the commutativity of multiplication to decide which numbers have “partners.” Does 9 have a partner number to make 36? How do you know? (yes, because we have already found that $4 \times 9 = 36$) Does 10? Tell your students that the partner for 9 is 4. If 10 has a partner, will it be more than 4 or less than 4? How do you know? (Since 9 and 4 multiply to 36, 10 and any number more than 4 will multiply to higher than 36, so 10’s partner must be less than 4). 

ASK: Is 10 the partner of any number less than 4? (no, the partners for 1, 2 and 3 are 36, 18 and 12, respectively). Does 11 have a partner? (no, same reason) Does 12 have a partner? (yes, $3 \times 12 = 36$ is already found) Does 13 have a partner? (no, its partner has to be less than 3, but 1 and 2 have different partners) In fact, the only numbers higher than 9 that have partners will already be found. Once we find a pair that is repeated, we can stop, so the pairs are:

we can stop

1, 36 2, 18 3, 12 4, 9 6, 6 9, 4 12, 3 18, 2 36, 1

Have students practise this skill of listing all the factors of a number until they see a pair of factors that is repeated.

When students finish the worksheet, ask them to explain why the charts in QUESTIONS 6 and 7 stop when they do.

**Extensions**

1. List all the combinations of quarters and dimes to make 60¢, 75¢.

2. Dragons come in three varieties: One-Headed Woe-of-Woods Dragons, Three-Headed Danger-of-Dale Dragons, and Ever-Quarrelling-Nine-Headed-Terror-of-Tundra Dragons. A mighty and courageous knight fought a mob of 13 dragons and cut off all of their heads. There were 27 heads in the pile after the battle (and none of them belonged to the knight). How many of each type of dragon did the knight slay? Find all possible solutions. 

(ANSWERS: 7 three-headed and 6 one-headed or 1 nine-headed, 3 three-headed and 9 one-headed).
3. The dragons from the previous question all have 4 legs each. When the knight next fought the dragons, he produced a pile with 48 legs and 86 heads. Which dragons were destroyed? **HINT:** There are many heads in the pile. Start with the largest number of nine-headed dragons possible.

4. a) One-headed dragons have 2 tails and four-headed dragons have 3 tails. The knight must cut off both the heads and the tails, as dragons with a tail can still breed other dragons, even without a head. After the battle, the knight counts 31 heads and 37 tails. How many dragons of each type did he slay? **HINT:** Students will need to be organized. Suggest the following chart to struggling students:

<table>
<thead>
<tr>
<th>Four-Headed Dragons</th>
<th>One-Headed Dragons</th>
<th>Total Dragons</th>
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</thead>
<tbody>
<tr>
<td>Dragons</td>
<td>Heads</td>
<td>Tails</td>
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<tr>
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Ask students the following question: if there are 31 heads, what is the maximum number of four-headed dragons? Why can't there be eight dragons of this sort? Challenge students to fill in the first column and then the second and third columns from the “Four-Headed Dragons” section of the chart. Can they fill in any columns from the “Total Dragons” section? How does this help with the columns for one-headed dragons? Students will see that there are 11 one-headed dragons and 5 two-headed dragons.

b) Change the question slightly, so that four-headed dragons have 5 tails and one-headed dragons still have 2 tails. Now, the knight counts 23 heads and 34 tails. From that, he knows immediately that there are 11 dragons altogether. How did he figure that out? **(ANSWER:** Each dragon has one more tail than head, so if there are 11 more tails than heads in the pile, and 34 is 11 more than 23, then there must have been 11 dragons.)

c) Using the information from b), find the number of each type of dragon slayed. Why is this question less work than part a) was? **SOLUTION:** Since we know the total number of dragons, the only possibilities are: 1 four-headed and 10 one-headed, 2 four-headed and 9 one-headed, and so on until 7 four-headed and 4 one-headed. Checking all possibilities shows that in fact there are 7 four-headed and 4 one-headed dragons.
NS5-108
Arrangements and Combinations

GOALS
Students will learn to problem solve.

PRIOR KNOWLEDGE REQUIRED
Comparing and ordering numbers
Concept of “closeness” for numbers
Counting by small numbers

VOCABULARY
product
difference
sum

Ask students to list in an organized way:

a) all the pairs of numbers that multiply to 10
b) all the pairs of numbers that add to 7

ASK: Can you find two numbers that:

a) multiply to 10 and add to 7
b) multiply to 12 and add to 7
c) multiply to 12 and add to 8

SAY: I want to make a two-digit number with the digits 1 and 2. How many different numbers can I make? [Two: 12 and 21.] Which two-digit numbers can I make with the digits 3 and 5? … 4 and 7? … 2 and 9?

Ask students to find a three-digit number with the digits 1, 2 and 3. Record their answers, stopping when they have listed all six. Have them write the six different numbers in an organized list. Repeat with the digits 2, 5 and 7, then with the digits 3, 4, and 8, and then with the digits 4, 6 and 7. Ask them to explain how their list is organized (for example, they might organize the numbers by the hundreds digit), and if organization makes it easy or not to know when all of the three-digit numbers have been listed.

Then write two sets of numbers: a) 4, 9, 7 and b) 6, 3, 10.

Have students find the product of all pairs of numbers, with one number taken from each set. ASK: Which pair of numbers has the smallest product? … the largest? … the product closest to 50? Did they need the entire set of products to find the pair with the largest product? … the smallest product? … the product closest to 50? The entire set of products might be needed to find the product closest to 50, but only the largest and smallest numbers in each set are needed to find the largest and smallest products. Repeat this exercise for sums and then differences.

Draw:

Show your students the two ways to arrange these circles in a row.

Then draw a third circle and have your students find all six ways to arrange three circles in a row.

Have volunteers illustrate, then ask if there is an organized way to find all six arrangements. Start by deciding which circle to place first in the arrangement.
—white, for example. The second circle in the arrangement can then be solid or striped. Have a volunteer illustrate these two arrangements. Have additional volunteers illustrate the arrangements with the solid circle and the striped circle placed first in the arrangement, respectively.

Have your students start exercise 3 of the worksheet, but first explain that mathematicians will often solve simple problems before they solve similar, more complicated problems, as a way to comprehend the problems. How can exercise 3 be simplified? Can fewer than four boxes be used to simplify the problem?

Tell students that tennis balls are sold in packages of 3 and 4. ASK: Can we buy exactly 5 tennis balls? … 6 tennis balls? … 7? … 8? … 9? … 10? Students might solve these by randomly guessing and finding solutions: e.g. 7 tennis balls can be bought by buying a can of 3 and a can of 4. Then ASK: Can we buy 42? How is this problem different from the previous problems? [The solution is harder to guess randomly.] Have them write an organized list of all of the possible ways of adding threes and fours to total 42. Explain that solving this problem for smaller numbers first will help them to solve the problem for larger numbers, like 42. ASK: Can you find ways to use only threes and fours to total 10? What is the greatest number of threes that can be used in the solution? Why can’t four threes be used in the solution? [4 × 3 = 12, which is greater than 10.] Then write

<table>
<thead>
<tr>
<th>No threes</th>
<th>1 three</th>
<th>2 threes</th>
<th>3 threes</th>
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</thead>
<tbody>
<tr>
<td>0 + 10</td>
<td>3 + 7</td>
<td>6 + 4</td>
<td>9 + 1</td>
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</tbody>
</table>

Point to the circled numbers and explain that these individual amounts have to be totalled with fours only. SAY: If a solution has exactly one three, and the solution is composed of only threes and fours, the remaining numbers have to be fours. Can you use only fours to total 7? Can you use only fours to total 10? Can you use only fours to total 4? To total 1? The only solution is to buy two cans containing three tennis balls and one can containing four tennis balls. Have students repeat this exercise for numbers such as 17 (see below), 35 or 42.

| 0 + 17    | 3 + 14   | 6 + 11   | 9 + 8    | 12 + 5   | 15 + 2   |

The only solution is three cans containing three tennis balls and two cans containing four tennis balls.

Then tell students that pencils are sold in packages of 5 and 6. Can exactly 13 pencils be bought? … 28 pencils? How can they systematically solve the problem? Repeat this exercise with pencils sold in packages of 4 and 6.

Note that there are various answers for the worksheet’s bonus questions.

Extensions

1. a) How many paths can be drawn from point A to point B using right and up directions only?

Have students extend the pattern and determine if the number pattern continues.
b) Repeat part a) for this pattern of pictures:

i) ![Pattern 1]
ii) ![Pattern 2]
iii) ![Pattern 3]
iv) ![Pattern 4]

2. How many four-digit numbers can be made with the digits 1, 2, 3 and 4 and a thousands digit of 1? ... a thousands digit of 2? ... a thousands digit of 3? ... a thousands digit of 4? How many four-digit numbers can be made altogether?

3. Find all triples of numbers that multiply to 24. Find three numbers that multiply to 24 and add to 10.

4. Show your students the following card trick. Take 6 cards and place them in pairs. Ask a volunteer to choose any pair without telling you. Then pick up the cards, keeping the pairs together. Then place the cards in the following order starting from the top of the deck:

1 2 3
4 5 6

Ask your students which rows the cards appear in. If both cards are in the first row, tell your students that the pair was cards 1 and 2. If both cards are in the second row, tell your students that the pair was 5 and 6 and if the cards are in rows 1 and 2, tell them that the pair was cards 3 and 4 (you can just pick the cards up and show them). Repeat the trick until students see the pattern. Allow pairs of students to try this trick out on each other.

Then show them the same trick, but using 12 cards instead. Again arrange the cards in pairs and have a volunteer choose a pair. Keeping the pairs together, pick up the cards and arrange them in this order:

1 2 3 5
4 7 8 9
6 10 11 12

Now, ask which row or rows the pair of cards appear in.

1st row only? cards 1 and 2
1st and 2nd row? cards 3 and 4
1st and 3rd row? cards 5 and 6
2nd row only? cards 7 and 8
2nd and 3rd row? cards 9 and 10
3rd row only? cards 11 and 12.

The only memorization this trick requires in the order you used to lay the cards down. Luckily, this is done in an organized way. Can students figure it out? Repeat the trick several times and then have students investigate to see if they can perform the trick themselves. The organization is by the row numbers that the pairs go into. The top two cards both go to the first row. The next pair goes to the first and second rows and so on.

Ask students if they think this trick can be performed with more cards. Have students investigate.
The trick can be performed with $6 = 2 \times 3$ cards or $12 = 3 \times 4$ cards or $20 = 4 \times 5$ cards, and so on. The number of pairs is then half of these numbers (3, 6, 10, and so on). Each pair of cards corresponds to a unique pair of rows and these pairs of rows can be listed in a very organized way:

For 3 pairs: (1, 1), (1, 2), (2, 2)

For 6 pairs: (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)

For 10 pairs: (1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)

For 15 pairs: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5)

Another pattern that your students might notice is how the gap in the number of pairs increases (3, 4, 5, and so on).

5. As a cumulative review of many number sense concepts, provide the BLM "Always, Sometimes, or Never True (Numbers)."
## NS5 Part 2: BLM List

<table>
<thead>
<tr>
<th>Resource</th>
<th>Page</th>
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<tbody>
<tr>
<td>Always, Sometimes or Never True (Numbers)</td>
<td>2</td>
</tr>
<tr>
<td>Blank Hundreds Charts</td>
<td>3</td>
</tr>
<tr>
<td>Cards (Fractions of Numbers)</td>
<td>4</td>
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<tr>
<td>Fraction Strips</td>
<td>5</td>
</tr>
<tr>
<td>Math Bingo Game</td>
<td>6</td>
</tr>
<tr>
<td>Pattern Blocks</td>
<td>7</td>
</tr>
</tbody>
</table>
Always, Sometimes or Never True (Numbers)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>If you multiply a 3-digit number by a one-digit number, the answer will be a three-digit number.</td>
<td>If you subtract a three-digit number from 999 you will not have to regroup.</td>
<td>The product of two numbers is greater than the sum.</td>
</tr>
<tr>
<td>D</td>
<td>If you divide a number by itself the answer will be 1.</td>
<td></td>
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<tr>
<td>G</td>
<td>The product of 2 even numbers is an even number</td>
<td>The product of 2 odd numbers is an odd number.</td>
<td>A number that ends with an even number is divisible by 4.</td>
</tr>
<tr>
<td>J</td>
<td>When you round to the nearest thousands place, only the thousands digit changes.</td>
<td>When you divide, the remainder is less than the number you are dividing by.</td>
<td>The sum of the digits of a multiple of 3 is divisible by 3.</td>
</tr>
<tr>
<td>M</td>
<td>The multiples of 5 are divisible by 2.</td>
<td>Improper fractions are greater than 1.</td>
<td>If you have two fractions the one with the smaller denominator is the larger fraction.</td>
</tr>
</tbody>
</table>

1. Choose a statement from the chart above and say whether it is **always** true, **sometimes** true, or **never** true. Give reasons for your answer.

What statement did you choose? Statement Letter __________

This statement is…

<table>
<thead>
<tr>
<th>Always True</th>
<th>Sometimes True</th>
<th>Never True</th>
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</thead>
<tbody>
<tr>
<td>Explain:</td>
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2. Choose a statement that is sometimes true, and reword it so that it is always true.

What statement did you choose? Statement Letter __________

Your reworded statement: __________________________________________________________________________

3. Repeat the exercise with another statement.
Blank Hundreds Charts
### Cards (Fractions of Numbers)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Number</th>
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<tbody>
<tr>
<td>$\frac{1}{10}$ of 10</td>
<td>$\frac{1}{9}$ of 18</td>
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<tr>
<td>$\frac{1}{4}$ of 20</td>
<td>$\frac{1}{3}$ of 18</td>
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<tr>
<td>$\frac{2}{3}$ of 12</td>
<td>$\frac{3}{5}$ of 15</td>
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<td>$\frac{1}{2}$ of 26</td>
<td>$\frac{7}{10}$ of 20</td>
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<tr>
<td>$\frac{17}{20}$ of 20</td>
<td>$\frac{3}{4}$ of 24</td>
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Fractions Strips
# Math Bingo Game

## Sample Boards

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</tr>
<tr>
<td>11</td>
<td>19</td>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>19</td>
<td>13</td>
<td>17</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
PS5-2  Searching Systematically

Teach this lesson after: 5.2 Number Sense

Goals:
Students will solve problems involving two or three related quantities (such as nickels, dimes, and quarters that together total a given value) by starting one of the quantities at zero and going up in order to find the other quantities.

Prior Knowledge Required:
Is familiar with T-tables
Can multiply a multi-digit number by a one-digit number using the standard algorithm

Vocabulary: categories, multiple, search systematically, table

Materials:
BLM Large Hundreds Chart (p. 10, see Problem Bank 4)

Review a context for organizing with two categories. Tell students that you need to program a machine that will sell snacks for 35 cents. The machine accepts only dimes and nickels and it does not give change. You have to teach the machine to recognize when it has been given the correct change. The simplest way to do this is to give the machine a list of which coin combinations to accept and which to reject. You need to list all combinations of dimes and nickels that add to 35 cents. SAY: Searching a list of all combinations in an organized way is called searching systematically.

SAY: The machine accepts two types of coins: dimes and nickels. The best way to find all the possible combinations is to list one of the coin types in increasing order. Let’s use dimes. Start with no dimes, then one dime, then two dimes, and so on. ASK: Where do you stop? (3 dimes) PROMPT: How many dimes will be too much? (4) Draw on the board:

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

SAY: If there are no dimes, I need seven nickels because seven nickels is 35 cents. Write “7” in the table. Tell students you want to know how many nickels you need if you have one dime. Demonstrate counting on from 10 by 5s until you reach 35. You will have five fingers up, so five nickels are required. Have volunteers demonstrate counting on for the remaining rows, and fill in the table as you go. (5, 3, 1)
**Exercises:** Complete the table to program a snack machine to make the total.

a) 55¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

b) 70¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:**

a) 55¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

b) 70¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

**Review reasons to start with the larger denomination.** Remind students that when finding all combinations of nickels and dimes that make 35 cents, you started with the dimes and listed all possibilities in order. SAY: You can also start by listing the number of nickels in increasing order. ASK: What is the biggest number of nickels I need to put in my table? (7) How do you know? (8 nickels would be too much money) Draw on the board:

<table>
<thead>
<tr>
<th>Nickels</th>
<th>Dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

SAY: In our table, it looks like there are more combinations now, but some of them won’t work. ASK: If there are no nickels, can I make 35 cents with just dimes? (no) Why not? (you can only make multiples of 10 cents with dimes) Write “X” in the table. Continue in this way to fill in the table, as shown on the following page.
Nickels | Dimes
---|---
0 | X
1 | 3
2 | X
3 | 2
4 | X
5 | 1
6 | X
7 | 0

SAY: So, we could start with nickels and get all the same answers, but it’s more work. It takes fewer dimes than nickels to make 35 cents because dimes are worth more than nickels.

**Exercises:** Start with the larger denomination. Make a list …

a) of dimes and nickels to make 75¢.

b) of quarters and dimes to make 95¢.

c) of quarters and nickels to make 85¢.

**Answers:**

a) | Dimes | Nickels |
---|---|---|
0 | 15 |
1 | 13 |
2 | 11 |
3 | 9 |
4 | 7 |
5 | 5 |
6 | 3 |
7 | 1 |

b) | Quarters | Dimes |
---|---|---|
0 | X |
1 | 7 |
2 | X |
3 | 2 |

Organizing with three categories. Tell students that this time you need to program a machine that accepts quarters, dimes, and nickels and sells snacks for 35 cents. As before, the machine does not give change. SAY: Just like with the previous problem, you need to list all combinations that add up to 35 cents. The best way to find the possible combinations is to list one of the coin types—the largest denomination—in increasing order. You start with no quarters, then one quarter, and so on. ASK: Where do you stop? (2 quarters is too much) Draw on the board:

| Quarters | Dimes | Nickels |
---|---|---|
0 |

SAY: If there are no quarters, the problem is like the problems that we solved before, with two categories. If there are no dimes, I need seven nickels. If there is one dime, I need five nickels, and so on. Fill in the table, as shown on the following page.
ASK: If I put one quarter in the machine, how much money do I have to pay with dimes and nickels? (10¢) PROMPT: The snack is 35 cents and a quarter is worth 25 cents. SAY: If there are no dimes, I need two nickels to make 10 cents. Add a row to the table, as shown below:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

ASK: If I put one quarter and one dime in the machine, how many nickels do I need to add to it? (0) Have a volunteer add another row to the table, as shown below:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

ASK: Can I continue with more dimes? (no) Can I continue with more quarters? (no) Point to the table and SAY: Based on the table, there are six different ways that you can pay 35 cents using quarters, dimes, and nickels.

**Exercises:** Complete the table to program a snack machine to make the total.

a) 40¢

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

b) 55¢
Answers:

a) 40¢

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

b) 55¢

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Calculating three categories for a constant number of coins. Tell students that you have three coins in your pocket, and each coin is a quarter, dime, or nickel. SAY: This time, the number of coins is constant, rather than their value. Tell students that you want to know the possible total values. ASK: If I have no quarters and no dimes, how many nickels must I have? (3) PROMPT: How many coins do I have in total? SAY: Now that you know how many of each coin there are, you can figure out the value of the coins. ASK: How much are zero quarters, zero dimes, and three nickels worth? (15¢) Draw on the board:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
<th>Total Value (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

Repeat for zero quarters and one dime (2 nickels), zero quarters and two dimes (1 nickel), and zero quarters and three dimes (no nickels). Fill in the nickels column and the value column as you go, as shown below:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
<th>Total Value (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

ASK: If I have one quarter, how many dimes and nickels must I have? (2) How much are one quarter, zero dimes, and two nickels worth? (35¢) Write the numbers in the last row of the table.
Have a volunteer continue the table, adding rows as needed, for other coin combinations with one quarter, as shown below:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
<th>Total Value (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>45</td>
</tr>
</tbody>
</table>

SAY: If I have two or three quarters, I can continue the table in the same way. Complete the table, as shown below:

<table>
<thead>
<tr>
<th>Quarters</th>
<th>Dimes</th>
<th>Nickels</th>
<th>Total Value (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>75</td>
</tr>
</tbody>
</table>

Point to the table and explain to students that since 50 cents is not in the total value column, it is impossible to have three coins in your pocket with a value of 50 cents. ASK: Is it possible to have three coins with a value of 70 cents? (no) Leave the table on the board for use in the following exercises.

**Exercises:**

a) I have 3 coins in my pocket worth 35¢. What coins do I have in my pocket?
b) I have 3 coins in my pocket worth 60¢. What coins do I have in my pocket?
c) If I have 3 different types of coins in my pocket, how much money do I have?
d) If I have 3 of the same type of coin in my pocket, how much money can I have?
e) I have 3 coins in my pocket. Is it possible for me to have 65¢?

**Answers:**

a) 1 quarter and 2 nickels, b) 2 quarters and 1 dime, c) 40¢, d) 15¢ or 30¢ or 75¢, e) no

**Using systematic search in different contexts.** Tell students that they can answer the same type of question in different contexts by using the same strategy. ASK: How many legs does a bird have? (2) How many legs does a cat have? (4) How many legs does a spider have? (8)

SAY: We have three categories of animals. There are 4 animals and 14 legs in total, and I want to know how many of each type of animal there are. ASK: What is the biggest number of spiders I need to put in my table? (1) How do you know? (2 spiders would have 16 legs, which is too many)
Demonstrate how to begin the table and then have students complete it individually (see completed table below).

<table>
<thead>
<tr>
<th>Spiders</th>
<th>Cats</th>
<th>Birds</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

Check the combinations and ASK: Can we have 4 animals with 14 legs? (yes) Ask a volunteer to mark the combinations that meet the requirements of 4 animals with 14 legs. (3 cats and 1 bird, 1 spider and 3 birds) SAY: Let’s look at the part of the table with 0 spiders. Point out that as you go down from one row to the next, you are replacing a bird with a cat. ASK: When you go down the table, does the number of legs increase or decrease? (increase) How do you know? (cats have more legs than birds) Point to the first combination of 14 legs and SAY: Once we get 14 legs with no spiders, we can go right away to checking for 1 spider because the number of legs is increasing. Once we get 14 legs with 1 spider, we can go right away to 2 spiders. But 2 spiders already have too many legs, so we can stop. Erase the rows of the table that you don’t need.

Before providing the exercises below, write on the board:

- Birds have 2 legs.
- Cats have 4 legs.
- Spiders have 8 legs.

**Exercises:**

a) Complete the table to find out how many legs each combination of two animals has.

<table>
<thead>
<tr>
<th>Spiders</th>
<th>Cats</th>
<th>Birds</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

b) Two animals have a total of 10 legs. How many of each animal are there?
c) Can two animals have 14 legs?

**Answers:**
a) 4, 6, 8, 10, 12, 16; b) 1 spider and 1 bird; c) no
Problem Bank

1. Some dragons have 3 heads, some have 5 heads, and some have 9 heads. Ron counts the heads of 4 dragons and gets 18 heads altogether. How many dragons of each kind are there?

Solution:

<table>
<thead>
<tr>
<th>Nine-headed Dragons</th>
<th>Five-headed Dragons</th>
<th>Three-headed Dragons</th>
<th>Total Number of Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

There are two possible combinations: 3 five-headed dragons and 1 three-headed dragon, or 1 nine-headed dragon and 3 three-headed dragons.

2. What number am I?
   a) I am a two-digit number. My digits add to 10 and my ones digit is 2 more than my tens digit.
   b) I am a two-digit number that is greater than 70. I am a multiple of 17.
   c) I am between 700 and 800. My digits add to 8. I am odd.
   d) I am a three-digit number. The sum of my digits is 10 and I am a multiple of 5, 7, and 8.
   e) I am a three-digit even number. I am a multiple of 11 and 37.
   f) I am a three-digit odd number. My digits add to 14 and I am a multiple of 5, 7, and 13.

Answers: a) 46, b) 85, c) 701, d) 280, e) 814, f) 455

3. a) Three cards have the numbers 3, 5, and 6 on them. Make a two-digit number and a one-digit number using each digit once, and then multiply the numbers to find the product (for example, $35 \times 6 = 210$). What numbers give you the biggest product?
   b) Four cards have the numbers 1, 4, 7, and 9 on them. Make 2 two-digit numbers using each digit once, and then multiply the numbers to find the product (for example, $14 \times 79 = 1106$). What 2 two-digit numbers give you the biggest product?

Selected solution: b) There are 24 products of 2 two-digit numbers:

- $14 \times 79$
- $14 \times 97$
- $41 \times 79$
- $41 \times 97$
- $79 \times 14$
- $97 \times 14$
- $79 \times 41$
- $97 \times 41$
- $17 \times 49$
- $17 \times 94$
- $71 \times 49$
- $71 \times 94$
- $49 \times 17$
- $94 \times 17$
- $49 \times 71$
- $94 \times 71$
- $19 \times 47$
- $19 \times 74$
- $91 \times 47$
- $91 \times 74$
- $47 \times 19$
- $74 \times 19$
- $47 \times 91$
- $74 \times 91$

By the commutative property, half of the products are the same as the other half, reducing the possibilities to 12. Also, in each group of four, there is a unique largest. This leaves three products to evaluate: $97 \times 41 = 3977$, $94 \times 71 = 6674$, and $74 \times 91 = 6734$. So, $74 \times 91$ is the biggest product.

Answer: a) $53 \times 6$
4. A two-digit number is divided by the sum of its digits. What two-digit number will result in the largest remainder? Solve this problem in steps.
a) Start by dividing the two-digit numbers by the sum of their digits, in increasing order:
10 ÷ 1 = ___ R ___
11 ÷ 2 = ___ R ___
12 ÷ ___ = ___ R ___
13 ÷ ___ = ___ R ___
14 ÷ ___ = ___ R ___
b) Is the strategy from part a) a good strategy to continue? Why or why not?
c) On BLM Large Hundreds Chart, calculate the sum of the digits of all the two-digit numbers. Write them on the hundreds chart squares.
d) What is the largest sum of digits a two-digit number can have?
e) The remainder must be smaller than the divisor, which is the sum of the digits. So, make a table starting with the largest sum of the digits.

<table>
<thead>
<tr>
<th>Sum of Digits</th>
<th>Two-Digit Number</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f) What is the largest remainder you found in part e)?
g) If you continue the chart, what is the next “sum of digits”? Can any new entry in the chart have a bigger remainder than the largest one you found in part e)? Explain.

Selected answers:
a) 10 ÷ 1 = 10 R 0, 11 ÷ 2 = 5 R 1, 12 ÷ 3 = 4 R 0, 13 ÷ 4 = 3 R 1, 14 ÷ 5 = 2 R 4
b) no, because the remainder can’t be bigger than the sum of the digits, so we should start with numbers that have bigger sums of digits
d) 18
e) | Sum of Digits | Two-Digit Number | Division |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>99</td>
<td>99 ÷ 18 = 5 R 9</td>
</tr>
<tr>
<td>17</td>
<td>98</td>
<td>98 ÷ 17 = 5 R 13</td>
</tr>
<tr>
<td>17</td>
<td>89</td>
<td>89 ÷ 17 = 5 R 4</td>
</tr>
<tr>
<td>16</td>
<td>97</td>
<td>97 ÷ 16 = 6 R 1</td>
</tr>
<tr>
<td>16</td>
<td>88</td>
<td>88 ÷ 16 = 5 R 8</td>
</tr>
<tr>
<td>16</td>
<td>79</td>
<td>79 ÷ 16 = 4 R 15</td>
</tr>
</tbody>
</table>
f) 15; g) 15, no, the biggest remainder you can get when dividing by 15 is 14, so 15 is the largest remainder possible
# Large Hundreds Chart

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
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<td>21</td>
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<td>40</td>
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<tr>
<td>41</td>
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<td>44</td>
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<td>48</td>
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</tr>
<tr>
<td>51</td>
<td>52</td>
<td>53</td>
<td>54</td>
<td>55</td>
<td>56</td>
<td>57</td>
<td>58</td>
<td>59</td>
<td>60</td>
</tr>
<tr>
<td>61</td>
<td>62</td>
<td>63</td>
<td>64</td>
<td>65</td>
<td>66</td>
<td>67</td>
<td>68</td>
<td>69</td>
<td>70</td>
</tr>
<tr>
<td>71</td>
<td>72</td>
<td>73</td>
<td>74</td>
<td>75</td>
<td>76</td>
<td>77</td>
<td>78</td>
<td>79</td>
<td>80</td>
</tr>
<tr>
<td>81</td>
<td>82</td>
<td>83</td>
<td>84</td>
<td>85</td>
<td>86</td>
<td>87</td>
<td>88</td>
<td>89</td>
<td>90</td>
</tr>
<tr>
<td>91</td>
<td>92</td>
<td>93</td>
<td>94</td>
<td>95</td>
<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
</tr>
</tbody>
</table>
PS5-3  Guessing, Checking, and Revising

Teach this lesson after: 5.2 Number Sense

Goals:
Students will make organized guesses and use the result of the previous guess to improve their next guess.

Prior Knowledge Required:
Can use the strategy of searching systematically
Can multiply up to three-digit numbers by single-digit numbers
Can round two-digit whole numbers to the nearest ten (for Problem Bank 2)
Can convert measurements in years to measurements in months (for Problem Bank 9)
Can convert measurements in centimetres to millimetres (for Problem Bank 10)
Can convert measurements in decades to measurements in years (for Problem Bank 12)
Can substitute numbers for variables in expressions involving fractions (for Problem Bank 14)
Can identify equivalent fractions (for Problem Bank 14)

Vocabulary: guess-check-revise, perfect square, search systematically, square

Materials:
calculators
books that are at least 150 pages long

Review the guess-check-revise strategy. Hide an object in the room and have a volunteer try to find the object. If the volunteer finds it quickly, play again until finding the object takes a while. When the volunteer finds the object, ASK: What strategy did you use? (guessed and tried again) Play again, but this time tell the volunteer whether they are “hot” or “cold” as they try to find the object. ASK: What strategy did you use this time? (guessed and tried again) When you tried again, was it easier than before? Why? Lead the discussion to the conclusion that, when students have more information about their wrong guess than just that it’s wrong, they can use that information to improve their next guess.

Write on the board:

guess-check-revise

SAY: When you play hide-and-seek, you are using a guess and check strategy, but when you are told whether you are hot or cold, you are using a three-step process: guess, check your guess, and then improve your next guess. This three-step strategy—guess-check-revise—is very useful in math.

Make sure students have a book that is at least 150 pages long. Have students try to open the book to page 60 on the first try. Have different volunteers tell you what page number they turned to on their first try. Point out how all the attempts are fairly close to 60. SAY: No one’s first try
was page 5 and no one’s first try was page 145. Everyone was pretty close to 60. Now have students use their first guess to make a second guess. ASK: Which way in the book should you turn? Should you turn a lot of pages or only a few? How close was your first guess?

**Using the guess-check-revise strategy to find a mystery number.** Write on the board:

\[ N \times N \times N = 343. \] What number is \( N \)?

SAY: Let’s start by checking the numbers in order. A table is a good way to do this. Draw on the board:

\[
\begin{array}{c|c}
N & N \times N \times N \\
\hline
1 & 1 \times 1 \times 1 = 1 \\
2 & 2 \times 2 \times 2 = 4 \times 2 = 8 \\
3 & \\
4 & \\
5 & \\
\end{array}
\]

Have volunteers continue filling out the last three rows of the table, as shown below:

\[
\begin{array}{c|c}
N & N \times N \times N \\
\hline
1 & 1 \times 1 \times 1 = 1 \\
2 & 2 \times 2 \times 2 = 4 \times 2 = 8 \\
3 & 3 \times 3 \times 3 = 9 \times 3 = 27 \\
4 & 4 \times 4 \times 4 = 16 \times 4 = 64 \\
5 & 5 \times 5 \times 5 = 25 \times 5 = 125 \\
6 & 6 \times 6 \times 6 = 36 \times 6 = 216 \\
7 & 7 \times 7 \times 7 = 49 \times 7 = 343 \\
\end{array}
\]

ASK: Are we getting closer to the answer? (yes) SAY: So, we can continue in this way. Leave the table on the board for use in the exercise below.

**Exercise:** Complete the table for \( N \times N \times N \). Stop when you get the answer 343. What is \( N \)?

**Solution:**

\[
\begin{array}{c|c}
N & N \times N \times N \\
\hline
1 & 1 \times 1 \times 1 = 1 \\
2 & 2 \times 2 \times 2 = 4 \times 2 = 8 \\
3 & 3 \times 3 \times 3 = 9 \times 3 = 27 \\
4 & 4 \times 4 \times 4 = 16 \times 4 = 64 \\
5 & 5 \times 5 \times 5 = 25 \times 5 = 125 \\
6 & 6 \times 6 \times 6 = 36 \times 6 = 216 \\
7 & 7 \times 7 \times 7 = 49 \times 7 = 343 \\
\end{array}
\]

\[ N = 7 \]

Write on the board:

If \( N \times N \times N = 46\,656 \), what is \( N \)?
Point to the table for $N \times N \times N$ on the board. ASK: Would continuing the chart be a good strategy for this question? (no) SAY: The answers are getting closer to the number, so you are getting warmer, but not much warmer, because you still have a long way to go to find the answer. Maybe we can take bigger steps to find the answer. Instead of trying 1, 2, 3, and so on, maybe we should start with 10, 20, 30, and so on.

**Exercises:** If $N \times N \times N = 46\ 656$, what is $N$?

a) Complete the chart up to 50.

<table>
<thead>
<tr>
<th></th>
<th>$N \times N \times N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>20</td>
<td>8000</td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

b) Which two tens is $N$ between? Explain how you know.

**Answers:** a) 27 000, 64 000, 125 000; b) $N$ is between 30 and 40 because $N \times N \times N$ is between 27 000 and 64 000

SAY: Now we know that $N$ is between 30 and 40. Write on the board:

\[
30 \times 30 \times 30 = 27\ 000 \\
N \times N \times N = 46\ 656 \\
40 \times 40 \times 40 = 64\ 000
\]

ASK: Do you think $N$ is a lot closer to 30 or to 40, or do you think $N$ is in the middle of 30 and 40? (in the middle) Why? (46 656 is in the middle between 27 000 and 64 000) SAY: Let’s try 35. Write on the board:

\[
35 \times 35 \times 35 = ______
\]

Have a volunteer do the calculation on a calculator and write the answer on the board. (42 875) ASK: Is 35 the right answer, too low, or too high? (too low) What should we try next? (36) Write on the board:

\[
36 \times 36 \times 36 = ______
\]

Again, have a volunteer do the calculation on a calculator and write the answer on the board. (46 656) ASK: Is 36 the right answer, too low, or too high? (the right answer) Write on the board:

So $N = 36$

Students may use a calculator for the following exercises.

**Exercises:** Find $N$ so that $N \times N \times N$ is …

a) 103 823  

b) 28 094 464

**Answers:** a) 47, b) 304
Review searching systematically when two related quantities are changing. SAY: A farmer has cows and chickens. Marko counts all the legs and Anna counts all the heads. Write on the board:

Marko counts 26 legs.
Anna counts 10 heads.

SAY: I want to know how many cows and how many chickens there are. Remember, to solve this type of problem, you can start by choosing one of the two quantities and systematically moving up in order through all the possibilities. Draw on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>26</td>
</tr>
</tbody>
</table>

Remind students that cows have four legs and chickens have two legs. SAY: Ten heads means there are 10 animals altogether. Point to the first row and ASK: If there are no cows, how many chickens are there? (10) If there is one cow, how many chickens are there? (9) Continue filling in the first four rows of the table, as shown below:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>26</td>
</tr>
</tbody>
</table>

ASK: Do I need to continue the table? (no) Why? (with 3 cows and 7 chickens, we now have a total of 10 animals and 26 legs) SAY: So there are three cows and seven chickens altogether. ASK: If you move down from one row to the next in the chart, how much does the total number of legs increase by? (2) SAY: When you start at the top of the table, you have 10 chickens. When you move down a row, you replace a chicken with a cow. But every time you replace a chicken with a cow, you replace two legs with four legs, so you have two more legs than before.
**Exercises:** How many cows and chickens are there if you have …

a) 34 legs and 12 animals  

b) 38 legs and 12 animals  

c) 48 legs and 12 animals  

**Answers:** a) 5 cows and 7 chickens, b) 7 cows and 5 chickens, c) 12 cows and 0 chickens

**Searching systematically to solve problems.** SAY: We often combine the guess-check-revise and searching systematically strategies. Pens and pencils both cost a whole number of dollars. Write on the board:

4 pens and 3 pencils cost $43.  
3 pens and 4 pencils cost $41.

SAY: Four pens and three pencils cost more than three pens and four pencils. ASK: Which costs more, a pen or a pencil? (pen) How do you know? (adding an extra pen adds more than adding an extra pencil) SAY: I would like to find out how much each pen and each pencil costs. I will start with four pens and three pencils costing $43, and with the cheaper item (pencils) costing $1 each. Draw on the board:

<table>
<thead>
<tr>
<th>1 Pencil</th>
<th>3 Pencils</th>
<th>4 Pens</th>
<th>1 Pen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$3</td>
<td>$43 - $3 = $40</td>
<td>$10</td>
</tr>
</tbody>
</table>

SAY: Now that you know how much one pencil and one pen cost, you can figure out if three pens and four pencils cost the right amount. ASK: How much are three pens and four pencils supposed to cost? ($41) How much do three pens cost if one pen costs $10? ($30) How much do four pencils cost if one pencil costs $1? ($4) So how much do three pens and four pencils cost? ($34) SAY: So this isn’t the right answer. We have to continue the chart. Have volunteers complete each row until you get the next possible value with a pen also costing a whole number of dollars:

<table>
<thead>
<tr>
<th>1 Pencil</th>
<th>3 Pencils</th>
<th>4 Pens</th>
<th>1 Pen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2</td>
<td>$6</td>
<td>$43 - $6 = $37</td>
<td>X</td>
</tr>
<tr>
<td>$3</td>
<td>$9</td>
<td>$43 - $9 = $34</td>
<td>X</td>
</tr>
<tr>
<td>$4</td>
<td>$12</td>
<td>$43 - $12 = $31</td>
<td>X</td>
</tr>
<tr>
<td>$5</td>
<td>$15</td>
<td>$43 - $15 = $28</td>
<td>$7</td>
</tr>
</tbody>
</table>

Ask a volunteer to calculate how much three pens and four pencils cost if one pencil costs $5 and one pen costs $7:

\[3 \times 7 + 4 \times 5 = 41\]

SAY: When the pens cost $7 each, both the first and second sentences are true, so this is the answer we are looking for. Write on the board:

Each pen costs $7 and each pencil costs $5.
Exercises: Notebooks and erasers each cost a whole number of dollars. Three notebooks and six erasers cost $30. Four notebooks and five erasers cost $34.

a) Which costs more, a notebook or an eraser?
b) How much does each notebook and each eraser cost?

Selected solution:

<table>
<thead>
<tr>
<th></th>
<th>1 Eraser</th>
<th>6 Erasers</th>
<th>3 Notebooks</th>
<th>1 Notebook</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$6</td>
<td>$30 - $6 = $24</td>
<td>$8</td>
<td></td>
</tr>
<tr>
<td>$2</td>
<td>$12</td>
<td>$30 - $12 = $18</td>
<td>$6</td>
<td></td>
</tr>
<tr>
<td>$3</td>
<td>$18</td>
<td>$30 - $18 = $12</td>
<td>$4</td>
<td></td>
</tr>
</tbody>
</table>

You don’t need to continue the table because in the next row, the eraser would be more expensive than the notebook. The answer is the second row because, with the eraser costing $2 and the notebook costing $6, the second sentence is true.

Answers: a) notebook

Problem Bank

1. When you multiply me by 9, the result is between 730 and 770. What numbers might I be? Hint: Evaluate 10 × 9, 20 × 9, 30 × 9, and so on until you get close to 700.

Answers: 82, 83, 84, 85

2. Multiply me by 9, then round to the nearest ten. The result is 370. What number am I? Use the table below, then make a new table that increases the numbers by 1 instead of by 10.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number × 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

Answer: 41

3. When you multiply me by 6, the result is less than 500. When you multiply me by 7, the result is more than 550. When you multiply me by 8, the result is more than 660. What number am I?

Answer: 83

4. What are the two numbers?
a) The bigger number is five times the smaller number. The product of the two numbers is 180. Use the table below.

<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>Bigger Number</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

b) The bigger number is five times as big as the smaller number. The product of the two numbers is 18 000.

c) The bigger number is five times as big as the smaller number. The product of the two numbers is 14 045.
Answers: a) 6 and 30, b) 60 and 300, c) 53 and 265

5. If \( N \times N \times N \times N = 187,388,721 \), what is \( N \)?
Answer: 117

6. A school bake sale sells muffins and pieces of cake. A muffin costs $2 and a piece of cake costs $3. The bake sale sold 47 items and made $111 in total. How many muffins and how many pieces of cake were sold?
Answers: 30 muffins and 17 pieces of cake

7. Use a calculator to answer these questions. Remember that two whole numbers are consecutive if there is no whole number between them.
   a) Calculate the product.
      i) \( 1 \times 2 \)   ii) \( 2 \times 3 \)   iii) \( 3 \times 4 \)   iv) \( 4 \times 5 \)   v) \( 5 \times 6 \)
   b) Is 14 the product of two consecutive whole numbers? Explain how you know.
   c) Can 160 be the product of two consecutive whole numbers? Explain how you know.
   d) Can 992 be the product of two consecutive whole numbers? Explain how you know.
   e) Write 6972 as a product of two consecutive whole numbers.
Answers: a) i) 2, ii) 6, iii) 12, iv) 20, v) 30; b) no, it is between 3 \times 4 and 4 \times 5; c) no, it is between \( 12 \times 13 = 156 \) and \( 13 \times 14 = 182 \); d) yes, it is \( 31 \times 32 \); e) \( 83 \times 84 \)

8. A perfect square is the product of a whole number with itself.
   a) Calculate the product.
      i) \( 1 \times 1 \)   ii) \( 2 \times 2 \)   iii) \( 3 \times 3 \)   iv) \( 4 \times 4 \)   v) \( 5 \times 5 \)
   b) Is 25 the product of two consecutive whole numbers? Explain how you know.
   c) Write 400 as a perfect square.
   d) Can you write 400 as the product of two consecutive whole numbers? Explain how you know.
   e) Explain why a perfect square cannot be the product of two consecutive whole numbers.
Answers: a) i) 1, ii) 4, iii) 9, iv) 16, v) 25
b) no, because it is between \( 4 \times 5 = 20 \) and \( 5 \times 6 = 30 \) and there is no product of consecutive whole numbers between those two
c) \( 400 = 20 \times 20 \)
d) no, because it is between \( 19 \times 20 = 380 \) and \( 20 \times 21 = 420 \)
e) Any perfect square is between two consecutive products of consecutive whole numbers, so it cannot be the product of two consecutive whole numbers. For example, \( 15 \times 15 \) is in between \( 14 \times 15 \) and \( 15 \times 16 \).

9. Today is Ben’s birthday. His age in years is 121 less than his age in months. How old is Ben? Hint: Use a chart:

<table>
<thead>
<tr>
<th>Age in Years</th>
<th>Age in Months</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

Answer: 11 years old
10. John measured his pencil in millimetres and centimetres. The length in centimetres is 63 less than it is in millimetres. How long is John’s pencil?

**Answer:** 7 cm or 70 mm

11. Lily calculated the number of weeks left until the summer holidays. Marko calculated the number of days (including weekends) left until the summer holidays. Marko’s answer is 42 more than Lily’s answer. How long until the summer holidays?

**Answer:** 7 weeks or 49 days

12. In 2047, Canada will be 162 more years old than it will be decades old. How old will Canada be in 2047?

**Answer:** 180 years or 18 decades

13. Three times a number is 20 more than half the number. What is the number?

**Answer:** 8

14. a) If \( \frac{A - 1}{A + 1} = \frac{4}{5} \), what is \( A \)?

b) If \( \frac{A \times A}{A + A} = 4 \), what is \( A \)?

c) If \( \frac{A + 2}{(A \times 2) + 1} = \frac{2}{3} \), what is \( A \)?

**Answers:** a) 9, b) 8, c) 4
PS5-4 Using Structure to Solve Multiplication Puzzles

Teach this lesson after: 5.2 Number Sense

Goals:
Students will use structure (place value and properties of operations) to reduce the work needed to solve a problem.
Students will solve multi-digit multiplication puzzles with missing digits, or with different letters representing different digits.

Prior Knowledge Required:
Can multiply a three-digit number by a one-digit number using the standard algorithm
Can multiply 2 two-digit numbers using the standard algorithm

Vocabulary: guess-check-revise, multiple

Materials:
BLM Secret Meeting Place with Multiplication (pp. 27–29)

NOTE: Many of the puzzles in this lesson can be done using long division. The challenge is for students to solve the puzzle without using long division. The techniques they learn will help with the remaining problems.

Using letters for missing digits. SAY: In math, sometimes we use letter for missing digits.
Write on the board:

\[ 3A \]

SAY: This means a number with tens digit 3 and ones digit A. You might read it as “three-A” but you really mean “thirty-A.” If A is 4, then 3A is 34.

Introduce the rules for solving puzzles with different or identical letters. Write on the board:

\[ 3 \times A = B1 \quad 9 \times A = 4A \]

SAY: A and B stand for different digits in the first puzzle, and in the second one, both As stand for the same digit. Point to the first puzzle and ASK: What number in the 3 times table has ones digit 1? (21) PROMPT: Let’s say the 3 times table together until we find the answer: 3, 6, 9, 12, 15, 18, 21. SAY: So B stands for 2 in the first puzzle. ASK: What is A? (7) How do you know? (3 \times 7 = 21)
Point to the second puzzle and ASK: What number in the 9 times table is in the forties? (45) PROMPT: Let’s say the 9 times table together until we find the answer: 9, 18, 27, 36, 45. ASK: So what is A? (5) SAY: That makes sense because 9 \times 5 is 45, so both As stand for 5.
**Exercises:** Solve the puzzle.

a) \(7 \times A = 3A\)  

b) \(8 \times A = 3B\)

**Bonus:**

- c) \(A \times A = 6B\)
- d) \(A \times A \times A = 6A\)

**Answers:**

a) \(A = 5\);  
b) \(A = 4, B = 2\);  
Bonus: c) \(A = 8, B = 4\);  
d) \(A = 4\)

**Solving puzzles multiplying two digits by one digit.** Write on the board:

\[
\begin{array}{c}
\text{A4} \\
\times 2 \\
68
\end{array}
\]

ASK: What do you multiply first, the ones or the tens? (the ones) Write on the board:

\[
\begin{array}{c|c}
\text{ones:} & \text{tens:} \\
4 \times 2 = 8 & A \times 2 = 6
\end{array}
\]

ASK: So what digit is \(A\)? (3)

Write on the board:

\[
\begin{array}{c|c}
2A & 2A \\
\times 4 & \times 4 \\
84 & 104
\end{array}
\]

ASK: How are these questions the same? (they both look like they are asking the same question: \(2A \times 4\)) How are they different? (they have different answers) SAY: Let’s look at the multiplication of the ones digit first. ASK: What is the ones digit of \(A \times 4\)? (4) Write on the board:

\[
A \times 4 = 4 \quad A \times 4 = 14 \quad A \times 4 = 24 \quad A \times 4 = 34 \quad A \times 4 = 44
\]

SAY: We can eliminate \(A \times 4 = 14\) and \(A \times 4 = 34\) because 14 and 34 are not multiples of 4. Cross out those two statements on the board. Pointing to each in turn, ASK: What is \(A\) if \(A \times 4 = 4\)? (1) What is \(A\) if \(A \times 4 = 24\)? (6) What is \(A\) if \(A \times 4 = 44\)? (11) SAY: But remember that \(A\) is a single digit. Refer students to the original question and point out that \(A\) is the ones digit of a number. SAY: So \(A\) is either 1 or 6. Write on the board:

\[
\begin{array}{c|c}
21 & 26 \\
\times 4 & \times 4
\end{array}
\]

Have volunteers do the multiplications on the board. (84 and 104) SAY: There is sometimes more than one possible answer for \(A\) when you just look at the ones digits. You have to finish the multiplication to see what the actual answer is. Pointing to the first multiplication, SAY: Four times two is eight. That’s how many tens are in the answer 84, so you know you don’t have to regroup the ones. That means that \(A \times 4 = 4\), not 24. So for the first multiplication, \(A = 1\). Pointing to the second multiplication, SAY: Four times two is eight, but there are 10 tens in the
answer of 104. That means there are two extra tens in $A \times 4$ and you are regrouping, so $A \times 4$ is 24, not 4. So for the second multiplication, $A$ is 6.

**Exercises:** Do you need to regroup the ones to solve? Hint: Write the question vertically.

- a) $5A \times 3 = 168$
- b) $2A \times 3 = 69$
- c) $3A \times 8 = 272$
- d) $4A \times 2 = 84$

**Answers:** a) yes, b) no, c) yes, d) no

SAY: Once you know if there is regrouping and how much, you can solve the problem. Write on the board:

```
    3A
x    7
```

ASK: Are any ones regrouped to the tens? (yes) How do you know? ($7 \times 3$ is 21, not 23) How many tens are needed? (2) Write this on top:

```
   2
7
```

SAY: So what is $A \times 7$? (21) Write on the board:

```
A \times 7 = 2 \text{ tens and 1 one}
= 21
```

ASK: So what is $A$? (3) Have a volunteer multiply 33 $\times$ 7 to verify that it is 231.

**Exercises:** Finish solving the problems from the previous exercises.

- a) $5A \times 3 = 168$
- b) $2A \times 3 = 69$
- c) $3A \times 8 = 272$
- d) $4A \times 2 = 84$

**Answers:** a) $A = 6$, b) $A = 3$, c) $A = 4$, d) $A = 2$

SAY: Even when there is more than one missing digit, you can solve the problem the same way.

**Exercises:** Solve the puzzle. Hint: Write the puzzle vertically.

- a) $B5A \times 7 = 2464$
- b) $B3A \times 9 = 1233$
- **Bonus:** $CB1A \times 4 = 7264$

**Answers:** a) $A = 2$, $B = 3$; b) $A = 7$, $B = 1$; Bonus: $A = 6$, $B = 8$, $C = 1$

**Using structure to reduce the amount of search required to solve the puzzle.** Write on the board:

```
5 \times AB = BCC
```

SAY: Remember the rules: the two Bs stand for the same digit, the two Cs stand for the same digit, and $A$, $B$, and $C$ all stand for different digits. Another rule for this kind of puzzle is that no
number can start with 0. So, AB is a two-digit number and BCC is a three-digit number.

ASK: What are the possibilities for B? (1, 2, 3, or 4, but students might say 1 to 9 or even 0 to 9)

How do you know that B cannot be 0? (it starts a number) How do you know that B can’t be 5 or higher? (5 times a two-digit number is less than 500)

PROMPT: What would AB have to be for 5 × AB to be in the 500s or greater? (at least 100)

SAY: AB is a two-digit number, so 5 times AB is at most in the 400s. That tells us that B is 1, 2, 3, or 4. Let’s try B = 1, 2, 3, and 4 in order.

Write on the board:

\[ B = 1 \]

ASK: If B = 1, what is C? (5) Write on the board:

\[
\begin{array}{c}
A1 \\
\times 5 \\
\end{array}
\]

SAY: We start by multiplying the ones digits. In this case, 1 × 5 is 5. Continue writing on the board:

\[
\begin{array}{c}
A1 \\
\times 5 \\
\hline
5 \\
\end{array}
\]

SAY: Now we know that if B = 1, then C = 5. Now that we have B and C, we can fill in the answer. Continue writing on the board:

\[
\begin{array}{c}
A1 \\
\times 5 \\
\hline
155 \\
\end{array}
\]

ASK: What does that tell us about A? (A × 5 = 15) PROMPT: What is A × 5? (15) Write on the board:

\[ A \times 5 = 15 \]

ASK: So what is A? (3) SAY: So when B = 1, then A = 3 and C = 5. Write on the board:

\[ A = 3, \ B = 1, \ C = 5 \]

SAY: But we still don’t know what happens if B = 2, 3, or 4. Let’s see if there are any solutions with B = 2. Write on the board:

\[
\begin{array}{c}
A2 \\
\times 5 \\
\hline
2CC \\
\end{array}
\]

ASK: What must C be? (0) How do you know? (I calculated 2 × 5)
Write on the board:

\[
\begin{array}{c}
1 \\
\times 5 \\
\hline
A2 \\
200
\end{array}
\]

ASK: Why did I write the “1” above the “A”? (because \(2 \times 5 = 10\), so you are regrouping 1 ten. SAY: A times 5, after adding the 1, gives 20. So before the regrouping, A times 5 must be 19. ASK: Is 19 a multiple of 5? (no) SAY: So there is no A for \(B = 2\). Leave the puzzle on the board for use in the following exercises.

**Exercises:** Try \(B = 3\) and 4 in the puzzle on the board. Are there any more possible values for A, B, and C?  
**Answers:** B = 3 doesn’t work because it gives \(5 \times A3 = 355\), which gives \(A \times 5 = 34\), which has no answer. B = 4 doesn’t work because it gives \(5 \times A4 = 400\), which gives \(A \times 5 = 38\), which has no answer.

Solving puzzles involving multiplying two-digit numbers by two-digit numbers. Write on the board:

\[
\begin{array}{c}
A4 \\
\times 61 \\
\hline
3294
\end{array}
\]

Explain to students that, if there is just one missing number, then they can find the missing number by checking possibilities for that digit. SAY: This is like the guess-check-revise strategy. Write on the board:

\[
\begin{array}{cccccccc}
14 & 24 & 34 & 44 & 54 & 64 & 74 & 84 & 94 \\
\times 61 & \times 61 & \times 61 & \times 61 & \times 61 & \times 61 & \times 61 & \times 61 & \times 61
\end{array}
\]

SAY: We can try A = 1, 2, 3, and so on, but, instead of doing all the multiplying, let’s estimate to see which products are most likely to be close to 3294. ASK: How can we estimate \(14 \times 61\) without doing a lot of work? (multiply 10 × 60 instead) SAY: By rounding both numbers to the nearest ten, you can get a good estimate. Point to each product in turn and ASK: What is your estimated product? Is that close to 3294? (10 × 60 = 600, no; 20 × 60 = 1200, no; 30 × 60 = 1800, no; 40 × 60 = 2400, no; 50 × 60 = 3000, yes; 60 × 60 = 3600, yes; 70 × 60 = 4200, no; 80 × 60 = 4800, no; 90 × 60 = 5400, no) SAY: Only \(54 \times 61\) and \(64 \times 61\) are close. But \(64 \times 61\) is more than 3600, and we’re looking for the product to be 3294, so that leaves us with only \(54 \times 61\) to try as our first guess. Have a volunteer perform the multiplication on the board, as shown below:

\[
\begin{array}{c}
54 \\
\times 61 \\
\hline
3294
\end{array}
\]
ASK: Did we make the right guess? (yes)

Write on the board:

\[
\begin{array}{c}
A3 \\
\times 34 \\
\hline
2822
\end{array}
\]

SAY: 34 is close to 30. ASK: What multiple of 10, when multiplied by 30, is close to the answer 2822? (90) SAY: 90 × 30 is 2700, so 93 × 34 is a reasonable guess. Write on the board:

\[
\begin{array}{c}
93 \\
\times 34
\end{array}
\]

Ask a volunteer to find the product. (3162) ASK: Is it too low or too high? (too high) Ask another volunteer to find 83 × 34. (2822) SAY: So A is 8.

**Exercises:** Solve the puzzle. Hint: Write the puzzle vertically.

a) A5 × 17 = 765  
b) A7 × 59 = 2183  
**Bonus:** A12 × 14 = 7168

**Answers:** a) A = 4, b) A = 3, Bonus: A = 5

**Solving puzzles with two missing digits, involving multiplying two-digit numbers by two-digit numbers.** Write on the board:

\[
\begin{array}{c}
A3 \\
\times 5B \\
\hline
2236
\end{array}
\]

Explain to students that the ones digit of the product is equal to the ones digit of B × 3. Write on the board:

\[
\begin{array}{c}
B \times 3 = 6 \\
B \times 3 = 16 \\
B \times 3 = 26 \\
B \times 3 = 36
\end{array}
\]

ASK: Can B × 3 be 16 or 26? (no) Why not? (16 and 26 are not multiples of 3) Can B × 3 be 36? (no) Why not? (36 is more than 9 × 3) Can B × 3 be anything greater than 36? (no) Why not? (it has to be at most 9 × 3 = 27) What can B × 3 be? (6) So what is B? (2) Erase B and write “2” in its place, as shown below:

\[
\begin{array}{c}
A3 \\
\times 52 \\
\hline
2236
\end{array}
\]

SAY: We need to know what equals 2236. ASK: Is it 52 × 13, 52 × 23, 52 × 33, or 52 × 43? Ask a volunteer to find A. (A is 4) SAY: You can use estimation to help you. 43 × 52 is close to 40 × 50 = 2000, so 4 is a good guess for A.
Exercises:
1. Solve the puzzle. Hint: Write the puzzle vertically.
   a) $A7 \times 2B = 1482$  
   b) $A8 \times 4B = 1786$
**Bonus:** $A34 \times 2B = 12586$
**Answers:** a) $A = 5$, $B = 6$; b) $A = 3$, $B = 7$; Bonus: $A = 4$, $B = 9$

2. Complete **BLM Secret Meeting Place with Multiplication**.
   **Answers:**
   1. a) $4, 8, 12, 16, 20, 24, 28, 32, 36, 40$; b) $B = 8$, because $4 \times 8 = 32$
   2. a) 3 or 8, b) $A = 3$
   3. a) $B = 8$; b) $A = 3$, $E = 1$; c) $U = 7$, $L = 2$; d) $N = 4$; e) $D = 5$, $Y = 9$; f) $N = 4$, $U = 7$, $R = 6$, $M = 0$
   4. a) $0 = M$, $1 = E$, $2 = L$, $3 = A$, $4 = N$, $5 = D$, $6 = R$, $7 = U$, $8 = B$, $9 = Y$; c) Mel and Ruby

**Problem Bank**
1. Solve the puzzle.
   a) $AAA \times 7 = 6216$  
   b) $BAA \times 7 = 6916$  
   c) $AAB \times 7 = 4655$  
   d) $BAB \times 7 = 5159$
   e) $BAB \times 9 = 5814$  
   f) $AAA \times 6 = 4662$  
   g) $BAA \times 3 = 2631$  
   h) $AAB \times 5 = 2245$
   **Answers:** a) $A = 8$; b) $A = 8$, $B = 9$; c) $A = 6$, $B = 5$; d) $A = 3$, $B = 7$; e) $A = 4$, $B = 6$; f) $A = 7$; g) $A = 7$, $B = 6$; h) $A = 4$, $B = 9$

2. When Tasha multiplies 2 one-digit numbers, the answer has ones digit 3. What might the two numbers be? List all possible answers.
   **Answers:** 1 and 3, 7 and 9

3. Solve the puzzle.
   i) $9 \times B = AB$  
   ii) $9 \times A = BA$
   **b)** How are the puzzles the same? How are they different?
   **Answers:** a) i) $A = 4$, $B = 5$, ii) $A = 5$, $B = 4$; b) they are the same puzzle, but with A and B switched

4. Solve the puzzle. $A3 \times A4 = 2862$
   **Answer:** $A = 5$

5. Solve the puzzle: $AB \times 4B = 2679$
   **Sample solution:** Looking at the ones digit, B is either 3 or 7 because $B \times B$ gives an answer with ones digit 9. We have to check the two cases: $A3 \times 43 = 2679$ and $A7 \times 47 = 2679$. $A3 \times 43 = 2679$ is not correct because $63 \times 43$ is too big (2709) and $53 \times 43$ is too small (2279). If we check $A7 \times 47 = 2679$ with $A = 5$, we get $57 \times 47 = 2679$, which is correct.

6. $AB$ and $BA$ are both two-digit numbers, so that neither $A$ nor $B$ is 0, and $5 \times AB = 6 \times BA$.
   a) $A$ must be 5. Why?  b) $B$ must be even. Why?
   c) Use the information from parts a) and b) to solve the puzzle.
Solutions:
a) Multiples of 5 have ones digit 0 or 5. We have 5 × AB = 6 × BA, so 6 × BA is a multiple of 5, but
the multiples of 6 that are multiples of 5 are every fifth multiple of 6: 6 × 5, 6 × 10, 6 × 15, and so
on, so BA must be a multiple of 5. In BA, A must be 0 or 5, but we are told it can’t be 0, so A
must be 5.
b) 6 × BA is even, but 5 × AB = 6 × BA, so 5 × AB is also even. But 5 is odd, so AB must be
even for it to be multiplied by 5 and result in an even number.
c) We know A = 5 and B is even and not 0, so AB is 52, 54, 56, or 58. Trying each in turn, we
find 5 × 54 = 6 × 45 works, so A = 5 and B = 4.

7. Solve the puzzle. Hint: You need to solve an addition puzzle before you solve the
multiplication puzzle.

\[
\begin{array}{ccc}
A & B & C \\
\times & D & 3 \\
\hline
6 & 5 & 4 \\
C & E & A & 0 \\
\hline
9 & 3 & 7 & 4 \\
\end{array}
\]

Answers: A = 2, B = 1, C = 8, D = 4, E = 7
Secret Meeting Place with Multiplication (1)

1. a) Complete the 4 times table.
   
   1 × 4 = _____  
   2 × 4 = _____  
   3 × 4 = _____  
   4 × 4 = _____  
   5 × 4 = _____  
   6 × 4 = _____  
   7 × 4 = _____  
   8 × 4 = _____  
   9 × 4 = _____  
   10 × 4 = _____  

   b) Look at the puzzle:
      
      4 × B = 32
      
      Which digit does B stand for? _____
      
      Explain how you know. ____________________________________________

2. Alex wants to solve this puzzle:
   
   3 A
   × 4
   _______
   1 3 2

   a) What could A be for the answer to have ones digit 2? _____ or _____
   
   b) Solve the puzzle.
Secret Meeting Place with Multiplication (2)

3. Solve the puzzle. The rules for the puzzles are:
   - The same letter stands for the same digit, even in different puzzles.
   - Two different letters stand for a different digit, even in different puzzles.
   - No factor begins with zero.

   Hint: Use your answers from Questions 1 and 2 to solve parts a) and b).

   a) \( B \times 4 = 32 \)  \( B = \) _____

   b) \[
   \begin{array}{c}
   3A \\
   \times 4 \\
   \hline
   E32
   \end{array}
   \]  \( A = \) _____  \( E = \) _____

   c) \[
   \begin{array}{c}
   L3U \\
   \times 9 \\
   \hline
   2133
   \end{array}
   \]  \( U = \) _____  \( L = \) _____

   d) \[
   \begin{array}{c}
   N5 \\
   \times 19 \\
   \hline
   855
   \end{array}
   \]  \( N = \) _____

   e) \[
   \begin{array}{c}
   D4 \\
   \times 2Y \\
   \hline
   1566
   \end{array}
   \]  \( D = \) _____  \( Y = \) _____

   f) \[
   \begin{array}{c}
   NN \\
   \times NR \\
   \hline
   2RN
   \end{array}
   \begin{array}{c}
   1UR \\
   \hline
   2M2N
   \end{array}
   \]  \( N = \) _____  \( U = \) _____  \( R = \) _____  \( M = \) _____

   Hint: Which letters do you already know from a previous puzzle?
Secret Meeting Place with Multiplication (3)

4. All the letters from Question 3 stand for a different digit from 0 to 9. When you put them in order, you can find the name of a secret meeting place.

a) Write the letters in order.

```
0 1 2 3 4 5 6 7 8 9
```

b) Is any number missing a letter? If so, find your mistake.

c) What two street names are in the message?

Meet at __________________________.

Mark the location on the map below.
ME5-8
Centimetres

**GOALS**
Students will measure and draw items to a specific length in centimetres.

**PRIOR KNOWLEDGE REQUIRED**
The ability to use a ruler

**VOCABULARY**
ruler
grid paper

Give your students several objects of standard length (paper clips, unused pencils, etc.) and ask them to measure the lengths.

Demonstrate how to draw a line 2 cm in length with a ruler. Have your students practice these steps—separately, if necessary—in their notebooks.

**STEP 1:** Find the zero mark on a ruler/number line. Draw a vertical line to mark the zero.

**STEP 2:** Count forward from zero by two hops.

**STEP 3:** Draw a vertical line to mark the two.

**STEP 4:** Draw a line connecting the two vertical marks.

Have your students draw lines of several lengths with a ruler. If the class does not have a standard set of rulers, have students trade rulers with their classmates so that each measurement is made using a different ruler. This reinforces the idea that rulers have equal measurement markings, even when they look different.

Ask your students to draw another line 2 cm in length, but to start it at 3 on the number line. Following the steps taught earlier in this lesson, but beginning at 3 and ending at 5 on the number line, have a volunteer demonstrate how the line is drawn. Have students reproduce the problem in their workbooks and solve several others using this method (**EXAMPLE:** a line 8 cm in length beginning at 3 on the number line, a line 11 cm in length beginning at 2 on the number line, a line 14 cm in length beginning at 1 on the number line, etc.).

**Assessment**
Draw a line 4 cm long and a pencil 8 cm long.

Distribute centimetre grid paper to your students and have them estimate the width of the squares. Then have them measure the width with their rulers. Demonstrate how to draw items of specific length by using the centimetre grid as a guide. On your grid, draw a pencil (or some other item) that is 5 cm long. Explain that you selected a starting point and then hopped forward to 5. Draw vertical marks at the start and end points, then draw the item to fill the length between the marks.
ACTIVITY 1
Create a box to collect benchmark items that correspond with each unit of measurement introduced. Everyone can then refer to and compare these items as the lessons progress.

ACTIVITY 2
Draw each object to the given measure.

a) A shoe 6 cm long  

b) A tree 5 cm high  

c) A glass 3 cm deep

ACTIVITY 3
Draw a triangle on grid paper and measure its sides to the nearest cm.

ACTIVITY 4
Draw a collection of long items.

a) Draw a collection of alligators, each one being 1 cm longer than the previous.

b) A pencil shrinks when it is sharpened. Draw a collection of pencils, each one being 1 cm shorter than the previous.

c) Draw a sequence of toboggans where each one is 2 cm longer than the last.

d) A carrot shrinks when it is eaten. Draw a collection of carrots, each one being 2 cm shorter than the previous.

ACTIVITY 5
Write a story about one of the growing or shrinking items in Activity 4 (or invent your own!). Tell the story of how the item grows or shrinks. How does the carrot get eaten? Who wants to ride the toboggans? Write the story to go with the pictures that you have drawn.

Extension
Estimate the height of a classmate in cm. Then measure their height using your hand. (Your hand with fingers spread slightly should be about 10 cm wide.) Finally, use a metre stick to check your result. How close were you?

HINT: Measure them against a wall to get an accurate result.

Estimate ______ cm    Hand Measurement ______ cm    Actual Measurement ______ cm
GOALS
Students will estimate measurements in millimetres and convert measurements between centimetres and millimetres.

PRIOR KNOWLEDGE REQUIRED
- Centimetres
- Skip-counting by ten
- Multiplying by ten
- Measuring with a ruler
- The values of dimes and toonies
- Decimals

VOCABULARY
- millimetre
- centimetre

Set out a number of items that range in size from 1 mm to 5 cm. Ask students to select the items that are about 1 mm wide. Measure the selected items with a ruler to verify their widths, then add the items measuring 1 mm to the measurement box (see Activity 1 of ME5-8).

Remind your students that they can use their index fingers as centimetre rulers. If there are 10 mm in a cm, how many mm are there in 2 cm?

Have students select several objects from their desks or backpacks (an eraser, a pencil, a pencil sharpener, etc.), measure three objects with their index fingers and then complete the following sentence for both objects.

The _______ measures about _______ index fingers, so it is about _______ mm long.

Explain to your students that counting every millimetre in a measurement can take a long time, but there is a quick way to do it. Draw a ruler representing 30 mm and tell the class that you want to count 26 mm. Then demonstrate how to skip-count by 10s to the tens value preceding the amount, and continue to count by one (EXAMPLE: 10, 20, 21, 22, 23, 24, 25, 26).

Have volunteers demonstrate this shortcut method by counting to several different numbers.

Ask your students to sort the items they chose into piles of “greater than 50 mm” and “less than 50 mm.” Once they have sorted all of the items, have them measure each to verify the exact length in millimetres. Suggest that your students make a T-table with headings mm and cm. Ask them to record the lengths of the objects they have in millimetres. ASK: If an eraser is 30 mm long, how many centimetres long is it? What do you do to measurement in millimetres to convert it into centimetres?

Suggest that your students make a T-table with headings “Objects”, “Length in mm” and “Length in cm”. Ask them to record the lengths of the objects they measured in millimetres. ASK: An eraser is 40 mm long. How many centimetres long is it? What do you do to a measurement in millimetres to convert it into centimetres? (divide by 10) A sharpener is 23 mm long. How many centimetres long is it? Review division by 10 with decimal results, such as $25 \div 10 = 2.5$, $37 \div 10 = 3.7$, and so on. Ask your students to convert the lengths of their objects from millimetres to centimetres. Add several more measurements to convert, such as: 2.4 cm, .6 cm, 80 cm, 34 mm, 307 mm, 307 cm.

Write several pairs of measurements on the board, like:

A: 8 cm and 9 cm  B: 9 cm and 10 cm  C: 10 cm and 11 cm

Ask your students to say which pair of centimetre measures (A, B or C) the measurement 87 mm lies between. Repeat the question for 93 mm. Can they find another millimetre measure between each pair of lengths?
Now write several measurements in mm, and ask your students to find a cm measure that is between the lengths in each pair:

56 mm and 63 mm, 78 mm and 86 mm, 102 mm and 114 mm.

**Assessment**

Complete the following conversions.

<table>
<thead>
<tr>
<th>mm</th>
<th>230</th>
<th>134</th>
<th>800</th>
<th>45</th>
<th>57</th>
<th>6</th>
<th>21.2</th>
<th>0.2</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Bonus**

1. Draw a line between 5 cm and 6 cm long and is...
   a) closer to 5 cm than to 6 cm.
   b) halfway between 5 cm and 6 cm.
   c) closer to 6 cm than to 5 cm.

   Give the lengths of the lines in mm.

2. Without using a ruler, give a measurement in mm that is between 7 cm and 8 cm and is...
   a) closer to 7 cm than to 8 cm.
   b) halfway between 7 cm and 8 cm.
   c) closer to 8 cm than to 7 cm.

**Extension**

Ask students to name an object in the classroom that they think would have length...

a) 60 mm  b) 300 mm  c) 100 cm  d) 200 cm

**Extensions**

1. A toonie is about 2 mm thick. Josie has a stack of toonies 10 mm high. How much money does Josie have?

2. A nickel is about 1 1/3 mm thick. How many stacked nickels equal 9 mm high?

3. Assume Carl has a pen that he can use to mark lengths on the sticks in **QUESTION 17** on the worksheet. Explain how he could use the sticks and the pen to measure.

   a) 2 cm  b) 4 cm  c) 1 cm

4. How many millimetres are in... a) a metre? b) a decimetre?

5. Draw a line \( \frac{17}{100} \) of a metre long. How many cm long is the line?
Tell your students that today they will learn about another unit of measurement, the decimetre. Write the word “decimetre” on the board. Circle the letters “d” and “m” and explain that they form the abbreviation for decimetre, dm. Write the abbreviation next to the word.

Explain that a decimetre is equal to 10 cm. Remind your students that the span of their hand is about 10 cm. This is equal to approximately 1 dm. Ask your students to name 3 objects that are shorter than 1 dm, 3 objects that are longer than 1 dm, and 3 objects that are about 1 dm. Students can measure the objects that are about 1 dm to check their estimates.

This would be a good time to do the Activity (see below).

Remind students of their work in the previous lesson, converting centimetres to millimetres.

**Ask:**
How many millimetres are in 1 cm? If a paper clip is 2 cm long, how many millimetres is that? What did you do to the measure in centimetres to get the measure in millimetres? If an eraser is 40 mm long, how many centimetres is that? What do you do to the measure in millimetres to get the measure in centimetres?

Draw this diagram on the board:

```
\[ \text{cm} \quad \times 10 \quad \text{mm} \quad \div 10 \]
```

**Say:** I want to add decimetres to the diagram. Where should I write dm? (on the top step) How many centimetres are in 1 dm? If a pencil is 2 dm long, how many centimetres is that? If a book is 30 cm long, how many decimetres is that? Invite volunteers to add the dm and the corresponding arrows and operations (x 10, ÷ 10) to the diagram. Invite a volunteer to fill in the blanks:

\[ 1 \text{ dm} = ___ \text{ cm} = _____ \text{ mm} \]

Have students convert more measures in centimetres to decimetres and vice versa. **Examples:**

\[
\begin{align*}
12 \text{ dm} &= ___ \text{ cm} \\
50 \text{ cm} &= ___ \text{ dm} \\
102 \text{ dm} &= _____ \text{ cm} \\
200 \text{ cm} &= ___ \text{ dm} \\
100 \text{ dm} &= _______ \text{ cm}
\end{align*}
\]

**Ask:** Can someone give me a measure in whole decimetres that is between 33 cm and 46 cm? Then ask for a measure in centimetres that is between 13 dm and 14 dm. (Several answers are possible—anything from 131 cm through 139 cm.) **Ask:** Is the second measure more than 1 m or less than 1 m? How do you know?
Assessment
1. Write a measurement in centimetres that is between 5 dm and 6 dm.
2. Write a measurement in decimetres that is between 20 cm and 95 cm.
3. Complete the table:

<table>
<thead>
<tr>
<th>cm</th>
<th>20</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm</td>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>21.2</td>
<td></td>
</tr>
<tr>
<td>mm</td>
<td>300</td>
<td>25</td>
</tr>
<tr>
<td>dm</td>
<td>.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Extensions
1. If 1 dm is equal to 10 cm, how many decimetres are in 100 cm? Where would you add metres to the step-diagram used in the lesson?
2. Write a measurement in decimetres that is between...
   a) 320 and 437 mm  
   b) 507 and 622 mm  
   c) 1 1/2 metres and 1 3/4 metres
3. John has a strip of paper 1 dm long. He folds the strip of paper so that it has a crease in its centre. What measurements can John make in centimetres using the strip?
ME5-11
Metres and Kilometres

Remind your students that 1 km equals 1000 m. Ask your students to think of objects that can act as benchmarks for estimating large lengths, heights, and distances. You may measure the actual length or height of some of these objects if they are available. Here are some possible benchmarks:

- A (very tall) adult and a door are about 2 m tall.
- A level, or storey, in a building (viewed from outside the building) is about 2 doors tall, so about 4 m tall.
- A school bus is about 10 m long.
- A typical car is about 3 m long.
- You can walk about 1 km in 15 minutes.

Invite students to use these benchmarks to estimate greater lengths and distances, such as:

- A basketball field is about 9 cars long. How long is a basketball field in metres?
- Two minivans are as long as 1 school bus. How long is each minivan?
- A playground is about 10 cars long. How many minivans can be parked along the playground?
- Daniel lives in an apartment building with 18 storeys. About how many school buses standing end-to-end, one on top of the other, are as tall as Daniel’s building?
- How many minivans parked end-to-end would it take to form a line 100 m long? How many would you need to form a line 1 km long?
- How many school buses can be parked along 1 km?

Extensions

1. How would you change a measurement in kilometres into centimetres?

2. Rita wants to estimate the height of a tree that grows near the school building. The tree is 3.5 storeys tall. Rita multiplies 3.5 by 4 m (the height of a school storey). How tall is the tree?

3. Remind your students that things in the distance appear smaller than they really are. Draw a line about 40 cm long on the blackboard. A pencil is far less than 40 cm long, but depending on where you stand, the line on the blackboard can appear to be the same length as the pencil! Ask your students to test this by holding a pencil in an outstretched hand. Ask them to move around the room, towards and away from the blackboard,
until the line appears to be as long as the pencil. Can they move so that the line appears to be half as long as the pencil?

4. Rita wants to estimate the height of a tree growing in the middle of a park. Rita uses a pencil and a friend. Rita positions herself far enough from the tree so that the tree appears smaller than the pencil. She asks her friend Sindi to stand by the tree. Rita holds the pencil vertically in her outstretched hand (she keeps her hand outstretched at all times) and compares the size of the pencil and the tree. For example, the tree appears to be as long as three quarters of the pencil. She holds the pencil so that the point is level with the top of the tree and her fingers are level with the bottom of the tree. She turns the pencil horizontally (with her fingers as the center of rotation) and asks her friend Sindi to walk in the direction that is at a right angle to the line between Rita and the tree. Rita asks Sindi to stop when the distance between Sindi and the tree seems to her the same as the distance she marked on the pencil. (Rita holds the pencil so that her finger is still level with the bottom of the tree and the point of the pencil is level with Sindi’s feet.)

After that, Rita measures the distance between Sindi and the tree with giant steps. This is the height of the tree. Can you explain why Rita’s method works? **HINT:** Use congruent triangles, one vertical and the other horizontal.

**EXPLANATION:** There are two pairs of congruent right-angled triangles. In the first pair, one triangle has Rita, the bottom and the top of the tree as vertices, and the other triangle has Rita, Sindi and the bottom of the tree as vertices. The other pair of congruent triangles has Rita’s eye and arm in common and the pencil as the side opposite to Rita’s eye in both triangles. The smaller triangles at the picture on the right are congruent, because two of their sides are same (arm and pencil) they both have a right angle. The larger triangles are also right-angled, and their angles are the equal to the angles of the smaller triangles. This means that the angles of the larger triangles are equal, and they also have a common side, so the larger triangles have to be congruent.
ME5-12
Speed

Work through this set of word problems with your students.

- Joanna can walk 1 km in 15 minutes. How far can she walk in 1 hour? (PROMPTS: How many minutes are in an hour? How many times will 15 minutes pass within an hour? So how many kilometres can Joanna walk in an hour?)

- Joanna’s school is 2 km from her home. How long will it take her to walk to school?

- This seems like a long walk, so Joanna bikes to school instead. It takes her 12 minutes to bike to school. How far can Joanna bike in an hour? In 3 hours? How long does it take Joanna to ride 1 km? 15 km? NOTE: If students have difficulty with the last two questions, ASK: How far is the school? (2 km) Which part of this distance is 1 km? ( 1/2 ) How much of the time interval will pass while she bikes 1 km? (it will take half the time to cover half the distance) What is half of 12 minutes? So how many minutes will pass while Joanna bikes 1 km?

- Joanna has to be at school at 8:45 a.m. When should she leave her house to be at school on time? School ends at 3:20 p.m. When does Joanna get home?

- After school Joanna bikes to a drawing class that is 3 km from school. When does she get to her drawing class?

To solve the last problem, break it into steps. ASK: What do you have to know to figure out when Joanna gets to the drawing class? Do you need to know when she gets out of school? When does that happen? Do you need to know how long she bikes? Do you know that? (Not yet.) If you knew how long she bikes, what would you do? (Add the time of the ride to the starting time, 3:20.)

So this is the last step of the problem. Suggest your students leave some space to fill in the unknown information (the time of the ride) and write: “3:20 + Time of the ride = Time Joanna gets to her drawing class.” ASK: How can you find the time of the ride? How far does Joanna ride? (3 km) What is her speed? Your students can present her speed in different ways: 2 km in 12 minutes, 1 km in 6 minutes, 10 km in an hour. ASK: Which way is the most common way to present speed? Which way is most convenient for this problem? (1 km in 6 minutes) If we know how long it takes Joanna to ride 1 km, what should we do to get the time of the ride? Write in the space left in the equation above: “Time of the ride = Time to ride 1 km x Distance.” Ask your students to calculate the time of the ride and the time Joanna will arrive at her drawing class.

MORE PROBLEMS:

- It takes an antelope 35 seconds to run 700 m. How many metres can it run in a second? In a minute? If an antelope were able to run an hour at
that speed (which never happens), how far would it run? How many kilometres is that?

• It takes an ostrich 5.5 minutes to run 5 km. How long does it take an ostrich to run 1 km?

**ASK:** How many seconds does it take an ostrich to run 5 km? How many metres is that? About how many metres does an ostrich run in 1 second? (Remind your students that 5 000 ÷ 330 = 500 ÷ 33, which is easier. Also, for an estimate, they can use the fact that 33 × 3 is about 100). Who runs faster—an antelope or an ostrich?

Students will need more practice with problems of the last type. Here are two more problems to try:

• Yesterday the wind speed was 500 metres in a minute. Today the wind speed is 5 metres per second. Was the wind yesterday stronger than today?

• The world record of train speed is 581 km per hour. The speed of sound is 340.3 metres in a second. Did the train achieving the record speed go faster than sound?

**Assessment**

1. It takes a cheetah 7 seconds to run 210 m. How many metres does a cheetah run in a second? In 10 seconds? If it were able to run that fast for a minute, how far would a cheetah run in 1 minute?

2. A car drives 90 km in 1 hour. How far will this car go in 1 minute? What has greater speed—a cheetah or a car? Explain.

**Extensions**

1. The graph shows the distance covered by a cat over a period of 12 seconds.

![Graph showing distance covered by a cat over 12 seconds]

   a) How fast was the cat running during the first 6 seconds?
   b) What was the cat doing after 10 seconds?

2. Karla can run at a speed of 5 metres per second and Bonnie can cycle at a speed of 12 metres per second. If Bonnie passed Karla on her bike and both children were moving at top speed how far apart would they be after 3 seconds?
ME5-13
Changing Units

GOALS
Students will take measurements in metres, and convert measurements between metres and centimetres.

PRIOR KNOWLEDGE REQUIRED
Measuring with a ruler
Skip-counting
Concepts of millimetres, centimetres, metres, decimetres
Converting between cm and mm
Using tables to organize data
Decimals

VOCABULARY
metre conversion metre stick
centimetre

Review the units of measurement: mm, cm, dm and m. Ask your students to name several objects that they would measure in each of these units.

Remind your students how to record measurements as 1 metre and 25 centimetres, as 1 m and 25 cm, and as 125 cm, then ask them to record their measurements in all three styles.

Show your students how to develop an estimate: First, estimate the blackboard’s length in whole metres (EXAMPLE: estimate: 5 m). Then place a metre stick beside the blackboard and ask your students if they would like to change their estimate. Mark the points that are 1 m and 2 m from the end of the blackboard. Ask the students if they would change their estimate now. Continue until the remaining length is less than a metre. Ask your students to estimate the remaining distance in centimetres (EXAMPLE: Estimate: 5 m 25 cm).

Ask your students to estimate and measure other distances in the classroom, like the width of the classroom, the distance from the window to the blackboard, etc. You might also use the 3rd activity below at this point.

Let your students complete the QUESTIONS 1 and 2 of the worksheet ME5-13.

Ask volunteers to convert, step by step, several measurements in metres and centimetres to centimetres. For example,

\[ 2 \text{ m } 30 \text{ cm} = 2 \text{ m } + 30 \text{ cm}; \]
\[ 2 \text{ m } = 200 \text{ cm}; \]
\[ 2 \text{ m } + 30 \text{ cm} = 200 \text{ cm } + 30 \text{ cm} = 230 \text{ cm} \]

For a measurement like 370 cm, reverse the step-by-step process. (Students could start by skip counting by hundreds to determine how many metres are in 370 cm.)

Complete several more examples, then allow your students to practice with questions like:

\[ 145 \text{ cm} = \underline{\text{m}} \underline{\text{cm}} \]
\[ 354 \text{ cm} = \underline{\text{m}} \underline{\text{cm}} \]
\[ 789 \text{ cm} = \underline{\text{m}} \underline{\text{cm}} \]
\[ 3 \text{ m } 75 \text{ cm} = \underline{\text{cm}} \]
\[ 4 \text{ m } 64 \text{ cm} = \underline{\text{cm}} \]
\[ 9 \text{ m } 40 \text{ cm} = \underline{\text{cm}} \]

Ask your students how many metres are in 70 cm. Then ask them to convert to metres such measurements as 1 m and 40 cm, 2 m and 35 cm, and so on.
Assessment

1. \[ 12 \text{ m} = \underline{\_\_\_\_} \text{ cm} = \underline{\_\_\_\_} \text{ dm} = \underline{\_\_\_\_\_} \text{ mm} \]
   
   \[ 3 \ 000 \text{ mm} = \underline{\_\_\_\_} \text{ cm} = \underline{\_\_\_\_} \text{ dm} = \underline{\_\_}\text{ m} \]

   \[ 3 \ 456 \text{ mm} = \underline{\_\_\_\_} \text{ cm} = \underline{\_\_\_\_} \text{ dm} = \underline{\_\_}\text{ m} \]

   \[ 3 \ 400 \text{ mm} = \underline{\_\_\_\_} \text{ cm} = \underline{\_\_\_\_} \text{ dm} = \underline{\_\_}\text{ m} \]

2. Convert the following units of measurements.

   a) \[ 2 \text{ m} 72 \text{ cm} = \underline{\_\_\_\_} \text{ cm} \]
   b) \[ 3 \text{ m} 56 \text{ cm} = \underline{\_\_\_\_} \text{ cm} \]
   c) \[ 348 \text{ cm} = \underline{\_\_\_\_} \text{ m} \underline{\_\_}\text{ cm} \]

Bonus

Convert the measurements:

- \[ 123 \ 456 \ 789 \text{ mm} \text{ into cm}, \text{ dm}, \text{ m} \text{ and km} \]
- \[ 987.654 \text{ m} \text{ to dm}, \text{ cm} \text{ and mm} \]
- \[ 0.987654321 \text{ km} \text{ to m}, \text{ dm}, \text{ cm} \text{ and mm} \]

**ACTIVITY 1**

**The Great Metre Hunt**

Cut up a ball of string into numerous lengths, ranging from 5 to 20 centimetres. Hide these lengths of string around the classroom. Divide students into groups—pairs work well—and inform them that each group must find ten pieces of string. The goal is to find ten pieces of string that, when laid end to end, will come closest to measuring 1 metre. Explain that finding all of the long pieces will probably total much more than one metre in length. Don’t forget to hide or remove the metre sticks from the classroom, to prevent students from using them to measure their string during the hunt.

Allow students to roam around collecting string. Once every group has found ten pieces, have them lay out their string end to end. Use a metre stick to measure the lengths. The group with the combined length of string closest to one metre wins!

Then, ask students to trade pieces of string with other groups so that each group has collected a metre-long length of string pieces. Allow them to use metre sticks.

**ACTIVITY 2**

Ask students to measure and compare the lengths of various body parts using a string and a ruler.

**EXAMPLE:**

- a) Is your height greater than your arm span?
- b) Is the distance around your waist greater than your height?
- c) Is your leg longer than your arm?

Encourage your students to predict the answers before they perform the measurements. Ask your students to record the measurements in three ways: 1 metre and 25 cm, 1 m and 25 cm and 125 cm.
Estimate the width of the school corridor: First, try to compare the length to some familiar object. For example, a minivan is about 5 m long. Will a minivan fit across the school corridor? Then, estimate and measure the width with giant steps or let several students stand across the corridor with arms outstretched. Estimate the remaining length in centimetres. Measure the width of the corridor with a metre stick or measuring tape to check your estimate.

Hold up a metre stick and point to various positions on the stick. Ask students to express if the positions are closer to 0 metre, .5 metre, or 1 metre.

Measure your shoe. Use this natural benchmark to measure long lengths: the classroom, the width of the hallway, or the length of the whole school!

Say whether each measurement is closer to 0 metres, \( \frac{1}{4} \) metre, \( \frac{1}{2} \) metre, \( \frac{3}{4} \) metre or 1 metre.

a) 10 cm  b) 52 cm  c) 37 cm  d) 82 cm  e) 2 cm  f) 90 cm

**Extension**

**Measure your stride:** Draw a line on the floor. Position yourself with your toes on the line. Walk normally for ten strides (not giant steps!) from the line and mark the place where your front leg’s toes are. Measure the distance you walked with a metre stick and divide it by 10 to get the average length of your stride.

Now that you have measured your stride, create a map that shows directions in paces to a buried pirate treasure (EXAMPLE: 15 paces north, 3 paces east, etc.). Calculate the actual distance you need to walk to find the treasure.
ME5-14
Problem Solving with Kilometres

Present the following problem:

Michael is travelling from Calgary, AB to Prince George, BC. The total distance of the trip is 780 km. Michael makes some short stops along the way. His average speed is 80 km per hour. About how long will the trip take?

Suggest to students that they round the distance to the nearest 100 km to get an estimate of Michael’s travelling time. **ASK:** Is that the time Michael actually spent behind the wheel? If his average speed was 80 km per hour, does this mean that he was driving at 80 km per hour all the time? No, 80 km per hour is his average speed; he might have driven faster for part of the time and slower for another part of the time. He also made some stops along the way—at those times, his speed would have been 0 km per hour!

Michael decided to make longer stops along the way to take some photos. His average speed is now only 50 km per hour. How long will his trip take at this speed?

How long will the trip take if Michael is biking at a speed of 13 km per hour?

The distance from Calgary to Banff National Park is 108 km. Then Michael drives through Banff National Park and Jasper National Park. When he leaves Jasper National Park, he still has 346 km to go. How many kilometres did he travel through Banff and Jasper National Parks?

To solve this problem, students might find it helpful to draw a line and mark the distances:

Another way to solve this problem is to use a number line, as in the Geometry unit.

Michael turns into the Icefields Parkway that passes through the Banff and Jasper National Parks after driving 182 km from Calgary. The Parkway is 226 km long. How far from the end of his trip will Michael be when he gets to the end of the Icefields Parkway? How far is the beginning of the Parkway from the entrance to Banff National Park?

Invite students to make up their own problems based on the information in the previous problems.
Students will need a ruler, a piece of string, and the map on Worksheet **ME5-14** or a large-scale, detailed map of Canada’s North that includes the Dempster highway. Ask your students to lay the string carefully along the line that represents the highway on the map, measure the length of the string, then figure out a scale for the map, knowing that the length they measured with the string (in centimetres) represents about 700 km (the approximate length of the actual highway). If you are using a large-scale map, use the actual length of 736 km.

**Extensions**

1. Find two cities on a map and use a ruler to measure the distance between the cities. Use the scale on the map to change your measurement to kilometres. Estimate how long it would take to travel between the cities at a speed of 100 km per hour.

2. A car travels with a speed of 90 km per hour. How far will it travel in half an hour? In 10 minutes? In 2 minutes?

3. Michael drives from Calgary to the boundary of Banff National Park at a speed of 90 km per hour. He drives through Banff and Jasper National Parks at an average speed of 60 km per hour. How long does his trip (from Calgary through both parks) take?
NOTE: There is a typo in the Workbook in QUESTION 3 of ME5-15, parts a), b) and d). The last line of each question should read:

- a) 3.5 cm = ___35 mm  
- b) 2.7 cm = ___ mm  
- d) 3 m = ___ cm

Make sure your students are familiar with multiplication and division of decimals by 10, 100, and 1 000. They should have completed sections NS5-93 to NS5-100.

Draw a line on the board and ask students to measure it in decimetres and in centimetres. **ASK:** Which unit is larger, the centimetre or the decimetre? Will the same line hold more centimetres or more decimetres? If the line is 4 dm long, only 4 decimetres are needed to cover it, but it takes 40 centimetres to cover the same length. So the smaller the unit, the larger the number of the units you need. **ASK:** What do you do to a measurement in centimetres to get the measurement in decimetres? What do you do to measurements in decimetres to convert them to centimetres?

Draw this diagram on the board:

```
  dm  × 10
   cm  ÷ 10
```

**ASK:** Where should I put millimetres? (on the bottom step) Where should I put metres? (on the top step) Where should I add arrows and how should I label them?

**ASK:** A line is 5 dm long. How long is it in millimetres? Encourage your students to show more than one solution to the problem. (For example, they could show 5 dm = 50 cm = 500 mm OR they could explain that since 1 dm = 100 mm, then 5 dm = 500 mm.)

Present a harder problem: Change 4.7 cm to millimetres. **ASK:** Which unit is larger, the centimetre or the millimetre? Will a line 4.7 cm long hold more or fewer millimetres? Should you multiply or divide? By how much? Point out that the length in centimetres has 1 digit after the decimal point and the length in millimetres is a whole number.

Ask your students to perform several more complicated changes of measurements, such as:

- 6.7 dm to mm   
- 345 mm to dm   
- .54 cm to m

Do the Activity (see below) to give students more practise expressing measurements in different units.
Ask students to tell which measurement is longer:

- 324 mm or 3 dm
- 5432 mm or 54.32 m
- 654 cm or 65 dm
- 87 dm or 86543 mm

You may also give your students word problems that require changing units, such as:

- A fence board is 2 dm wide. Three boards cost $10. How much will 3 m of fence cost?
- One metre of ribbon costs 12 cents. How much would 900 mm of ribbon cost?

**Assessment**

1. Fill in the blanks:

   - .7 dm = ___ mm
   - .35 m = ___ dm
   - .24 m = ___ mm
   - 345 mm = ___ dm
   - 789 mm = ___ m
   - .35 mm = ___ dm
   - .54 cm = ___ m
   - .789 mm = ___ m
   - 5.67 dm = ___ cm
   - .05 dm = ___ m
   - .27 cm = ___ mm
   - 364 cm = ___ dm

2. Circle the largest length and underline the smallest length in each column above.

**Extensions**

1. What would you do to a measurement in millimetres to get a measurement in kilometres?
   - A snake is 0.0105 km long. How long is it in mm?

2. Draw a line that is...

   a) .011 m long
   b) 1.7 dm long
   c) 134 mm long
   d) 0.003 m long
   e) .15 dm long
ME5-16
Ordering and Assigning Appropriate Units

**GOALS**
Students will understand that particular units of measurement are more appropriate in certain contexts. They will also convert measurement units.

**PRIOR KNOWLEDGE REQUIRED**
Different units of measurement: mm, cm, m, km
Concept of centimetres and metres
Ability to convert metres to centimetres

**VOCABULARY**
- millimetre
- centimetre
- metre
- kilometre
- decimetre
- appropriate

Present the items from the measurement box individually (see ME5-8) and have your students express the measurement that each item represents. Review the full name and abbreviation for each unit.

Ask your students to tell you how far it is from Halifax to Calgary in millimetres. Then ask them to calculate the thickness of a coin in kilometres.

Explain that it is important to choose the appropriate unit of measurement for the length/distance being measured. Ask your students to tell you which unit of measurement will best express the distance from Halifax to Calgary. Which unit of measurement will best express the thickness of a coin?

Practice this by displaying a variety of objects (a book, a stapler, a coin, etc.) and asking your students to tell you which unit of measurement will best express the length, width, thickness or perimeter of the object. Is the length of a stapler expressed best by a centimetre or a metre? Is the thickness of a coin expressed best by a millimetre or a centimetre?

Have your students select five items in the classroom and guess which unit of measurement will best express each item’s height, width or length.

Have a volunteer measure one of the practice items (the stapler, for example), but do not specify a unit of measurement. After the volunteer relates the measurement to the class, identify the unit of measurement used and ask your students why the volunteer chose that unit of measurement. Students will likely respond that centimetres were used because the stapler is about the right size. It would be many millimetres long, and it’s much smaller than a metre.

Demonstrate that the stapler is, for example, 12 cm in length. That’s 120 mm, 0.12 m and 0.00012 km. Out of all those numbers, 12 is the nicest and the simplest.

Have your students measure their five objects. Which units of measurement do they automatically use? Do these units of measurement lend themselves easily to the task? Would alternate units of measurement offer simpler measurements?

Ask your students to list in their notebooks five things that could be measured and are not in the room (a bicycle, a video game, a rocket ship, anything). Ask students to arrange the five things in order from smallest to largest. Then ask them to indicate which unit of measurement will give the simplest measurement for each item.

Review the relationships between units of length measurement. Ask your students: Which number is larger, 2 or 35? What length is larger, 2 m or 35 cm? Why?

Explain that the easiest way to compare measurements that are expressed in different units is to convert all measurements to the small unit. For instance,
to compare 3 m and 250 cm, convert 3 m to 300 cm, and it becomes clear that 3 m is greater than 250 cm.

Let your students practice with questions like:

Circle the greater amount:

1 m or 80 cm   6 m or 79 cm   450 cm or 5 m   230 cm or 2 m   2 m or 38 cm

Ask students to order these lengths from shortest to longest and ask them to mark these lengths on a number line:

0 cm 100 cm 200 cm 300 cm 400 cm 500 cm 600 cm

Assessment

1. Which unit of measurement would you use to measure the…
   - Thickness of a book?
   - Length of a chocolate bar?
   - Height of the school?
   - Height of a tree?
   - Height of a mountain?
   - Distance from a window to a door?
   - Distance from your school to your home?
   - Distance from your nose to your toes?
   - Distance from the Earth to the Moon?
   - Distance between your eyes?

2. Convert the distances to cm and then order them from greatest to smallest.
   a) 75 cm   b) 85 m   c) 230 cm   d) 7 m   e) 4 cm

On a school walking trip, ask students to say when they think they have walked half a kilometre or 1 kilometre.

Ask students if they have taken any trips to nearby towns. Ask them to estimate how many kilometres away the towns are and then have them check the actual distances by measuring the distances on a map with a ruler, and then converting their measurements to kilometres using the scale on the map.

Divide your class into four groups and assign one unit of measurement (mm, cm, m or km) to each group. Give each group a sheet of chart paper, and ask them to write the full name of their unit of measurement and the abbreviation at the top of the page. Have them list as many things as they can that could be measured with that unit of measurement. Set a target quantity (maybe twenty) and ask students to try and list more than that quantity. Have each group share their ideas. Display the lists in the classroom until the instruction of ME5-18 is complete.
Ask your students to list as many words as they can that start with the unit of measurement prefixes. Some examples include centipede (“hundred feet”), centurion (a commander in the Roman army in charge of 100 soldiers), million (one thousand thousands), decibel, decimal, decimate (historically meaning to kill or remove one in every ten). Students might suggest other metric units, such as kilograms or kilowatts, for “kilo”.

Ask your students what numbers they know in French or any of the other Romance languages. Draw the following table and illustrate the patterns and similarities to your students.

<table>
<thead>
<tr>
<th>Language</th>
<th>10</th>
<th>100</th>
<th>1 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td>dix</td>
<td>cent</td>
<td>mille</td>
</tr>
<tr>
<td>Spanish</td>
<td>diez</td>
<td>cien</td>
<td>mil</td>
</tr>
<tr>
<td>Italian</td>
<td>dieci</td>
<td>cento</td>
<td>mille</td>
</tr>
<tr>
<td>Portuguese</td>
<td>dez</td>
<td>cem</td>
<td>mil</td>
</tr>
</tbody>
</table>

Check out The Metric Song by Kathleen Carroll:
http://www.songsforteaching.com/kathleencarroll/metricsong.htm

This song reinforces the distinction between milli and kilo.

Extensions

1. Introduce your students to the principle of metric prefixes. Explain to them that the metric system is comprised of many different units of measurement, all dependent on the dimensions being measured—distance is measured in metres, volume is measured in litres, weight is measured in kilograms.

Then explain that each unit of measurement has a base unit (for these lessons, metres) that shares prefixes with the other base units. Write the word “metre” and ask your students to provide you with the prefixes.

Write the prefixes and explain a bit about the etymology of each. For example, centi means one hundredth. That’s why there are 100 centimetres in a metre. Also, there are 100 cents in a dollar. There are 100 years in a century.

Milli means one thousandth. That’s why there are 1 000 millimetres in a metre. Also, there are 1000 years in a millennium.

Deci means one tenth. That’s why there are 10 decimetres in a metre. Also, there are 10 years in a decade.

Kilo means 1 000. That’s why there are 1 000 m in a kilometre.

2. Have your students research (and prepare reports or descriptive posters on) alternative systems of measurement, such as Ancient Egyptian, Chinese or Old English systems. How long were their respective units of measurement, and what were they called? Are they still in use?
3. Compare the lengths of non-metric units of measurement (Example: inches, feet, yards, etc.) to the metric units of measurement. What is the difference between an inch and a centimetre or a yard and a metre?

4. Big Numbers. Here are some questions you could assign or discuss with students to give them practice calculating and estimating with large numbers. Your students will need to know how to multiply whole numbers by multiples of 10.

   a. (Warm Up): There are 60 seconds in 1 minute. How many seconds are there in 1 year?

      SOLUTION: Start with seconds:

      How many seconds in one minute? 60 seconds
      How many minutes in a day?
      60 minutes = 1 hour; 1 day = 24 hours
      24 × 60 = 1 440 minutes in one day
      1440 × 60 = 86 400 seconds in one day

      Finally, multiply the number of seconds in one day by the number of days in one year.

      86 400 seconds in one day × 365 days in one year = 31 536 000 seconds in one year! In one year there are more than 31 million seconds!

   b. About how many minutes have you been alive?

      SOLUTION: Start with the date the student was born and the current date.

      Example: Born August 29, 1998. Current date is September 13, 2008. The student is 10 years and several days old. There are 365 days in a regular year.

      How many minutes in one day?
      24 × 60 = 1440 minutes in one day
      1440 minutes in one day × 365 days in one year = 525 600 minutes in one non-leap year.

      525 600 minutes in one year × 10 years; 5 256 000 minutes in 10 years

      How many days should we add to these 10 years? From August 29, 1998 to August 29, 2008 is 10 years. How many of them were leap years? 2000, 2004 and 2008 were leap years, so we have to add three days. From August 29 to the end of the month we add three more days. From September 1, 2008 to the current date (September 13, 2008) we add: 3 days + 3 days + 13 days = 19 days to add. How many minutes are in 19 days? 1440 × 19 = 27 360 minutes in 19 days

      How many minutes has the student been alive for?

      5 256 000 minutes in 10 non-leap years
      + 27 360 minutes in 19 additional days

      5 283 360 minutes in the student’s life so far!
c. According to the Guinness Book of World Records, the tallest tree ever measured was an Eucalyptus tree discovered in Watt’s River in Victoria, Australia in 1872 by a forester named William Ferguson. The Eucalyptus tree was 132.6 m tall.

The Dyerville Giant, a coastal redwood tree, found in the Humboldt Redwoods State Park in California, USA is named as the tallest tree of modern times. This tree was 1 600 years old and 113.4 metres high when it fell in March 1991.

The height of a dime is 1 mm. How many dimes would need to be stacked to be as high as each of the tallest trees in history? How much would all these dimes be worth?

**SOLUTION:**

Convert each tree height from metres to millimetres.

\[
132.6 \text{ m} \times 1000 = 132600 \\
113.4 \text{ m} \times 1000 = 113400
\]

The height of a dime is 1 mm. We can stack 132 600 dimes to be as high as the Eucalyptus tree and 113 400 dimes to be as high as the Redwood tree.

The total values are 1 326 000¢ and 1 134 000¢ or $13 260 and $11 340.

d. Ellen MacArthur from France is the first woman to travel around the world on a sailboat, by herself. Her boat was named the Kingfisher and it could only carry one person. Ellen had dehydrated food and a few changes of clothing, as well as e-mail access on board her boat. It took her a total of 94 days, 4 hours, 25 minutes, and 40 seconds to complete her journey around the world.

Ellen had to be alert at all times so she could not take breaks to sleep. Amazingly, she would take 40 minute long naps, 10 times a day during her trip.

About how many hours of Ellen’s trip sailing around the world was she sleeping?

**SOLUTION:**

40 minutes \times 10 \text{ naps per day} = 400 \text{ minutes of sleep per day}

400 \text{ minutes of sleep} \div 60 \text{ minutes per hour} = \text{approximately 7 hours of sleep each day}

7 \text{ hours} \times 94 \text{ days} = 658 \text{ hours}

Ellen slept approximately 660 hours during her trip around the world.

e. “Wranny” is what Andy Martell from Toronto, Ontario, Canada calls his creation made from Saran wrap. He is marked down in the Guinness Book of World Records for having created the largest ball made entirely from Saran wrap. Measured in February 2003, the ball had a circumference of 137 cm and weighed 20.4 kg.

A jelly bean weighs 0.5 grams. How many jelly beans do we need to equal the weight of “Wranny”, Andy’s famous Saran wrap ball?

**SOLUTION:**

The Saran wrap ball weighs 20.4 kg = 20.4 \times 1000 = 20 400 grams

One jelly bean weighs 0.5 grams, so 2 jelly beans would weigh 1 gram. Therefore 20 400 \times 2 = 40 800 jelly beans would weight the same as the Saran wrap ball.
f. The world’s longest pencil was created in Malaysia by the company Faber-Castell. The pencil is 19.75 m long, and has a diameter of 0.8 m.

   a) How many regular pencils do you need to stack end to end to be as high as the largest pencil in the world?

   b) Could you put your hand around the largest pencil in the world?

**SOLUTION:**

   a) Convert the height of the largest pencil to cm: \(19.75 \times 100 = 1975\) cm

      A regular pencil is about 20 cm long. \(1975 \div 20 = 198 \div 2 = 99\)

      It would take about 99 pencils to equal the length of the world’s longest pencil.

   b) Have your students brainstorm how to answer this question. Measure the length of the palm of their hand (this should be around 10 cm to 12 cm). Convert the diameter of the pencil to cm:

      \(0.8 \times 100 = 80\) cm

      Then approximate the circumference of a pencil. The circumference of a circle is approximately 3 times its diameter: \(3 \times 80 = 240\) cm

---

g. The largest flag created was flown in Brasilia, Brazil in August 1998. The flag has a width of 70 m and a height of 100 m.

   What is the area of the largest flag?

   Measure the area of a regular piece of paper. How many regular pieces of paper would be needed to cover the entire flag?

**SOLUTION:**

   The area of the largest flag is: \(70 \times 100 = 7000\) m\(^2\)

   The dimensions of a regular piece of paper are: about 30 cm \(\times\) 20 cm = \(.3\) m \(\times\) \(.2\) m = \(0.06\) m\(^2\)

   The area of a regular pieces of paper is: \(3\) m \(\times\) \(.2\) m = \(0.06\) m\(^2\)

   But \(100000 \times .06 = 6000\) (which is fairly close to 7000).

   At least 100 000 pieces of regular paper would be needed to cover the largest flag in the world.

**NOTE:** All the data collected for questions c to g is from the Guinness Book of World Records website: [www.guinnessworldrecords.com](http://www.guinnessworldrecords.com)
ME5-17
Mathematics and Architecture

GOALS
Students will use scale to determine the actual dimensions of historic buildings in pictures.

PRIOR KNOWLEDGE REQUIRED
m, mm, cm
Multiplication and division of decimals by 10, 100, and 1000
Converting between metres, millimetres, and centimetres

VOCABULARY
metre
millimetre
centimetre

Project an image of the Mayan pyramid at Chichen-Itza, Mexico, on the blackboard. Ask your students to measure the height and the base of the pyramid-shaped platform in the picture. (Ignore the square structure on top of the pyramid.) Explain that the platform has a staircase on each of its 4 sides, and each step in that staircase is 66 mm tall. There are 364 stairs in the staircase. How high is the pyramid? (66 mm × 364 stairs = 24024 mm = about 24 m) So what is the scale of the picture? (If the projected pyramid is, say, 60 cm tall, then 24 m equals 60 cm, so 4 m equals 10 cm, and the scale is 40 to 1. This means each centimetre in the picture corresponds to 40 cm in real life.)

Ask your students to calculate the length of the pyramid base from their measurements and the scale. (It is about 60 m.) The students may also count the number of large "steps" in the pyramid (on either side of the staircases) to find the approximate height of each one.

Students could look up the measurements (base and height) of another pyramid in a book or on the Internet and then draw a scale diagram of the pyramid (as in QUESTION 1 on the worksheet) with a scale of their choice.

Extension
Over the last 4000 years, the Pyramid at Giza has lost 10 metres from its original height. If the pyramid continues to erode at the same rate, how much shorter will it be in 12000 years?
Start with the diagnostic test below. Don’t continue until all your students have passed the test.

1. Multiply:
   a) $100 \times 80 = $ b) $17 \times 100 = $ c) $32 \times 1000 = $ d) $13.8 \times 1000 = $ 

2. Divide:
   a) $270 \div 100 = $ b) $3700 \div 100 = $ c) $1870 \div 100 = $ 
   d) $360 \div 1000 = $ e) $87000 \div 1000 = $ f) $6420 \div 1000 = $ 
   g) $5.45 \div 1000 = $ h) $923.47 \div 100 = $ 

Present this series of word problems:

- The Jacques Cartier Bridge over the St. Lawrence River in Montreal is 3.4 km long. How long is that in metres?
- The bridge actually consists of two smaller bridges with an island in the middle. The total bridge structure (excluding the support sections) is 2687.42 m long. How many centimetres is that?

• The river beneath the bridge is divided into 3 parts, one about 400 m wide, another about 200 m wide, and the third about 66 m wide. How wide is the total span of water beneath the bridge?

• The bridge goes across Sainte-Helene Island but passes right over Notre Dame Island. Sainte-Helene Island is about 600 m wide and Notre Dame Island is about 230 m wide at the point the bridge crosses them. What is the distance between the shores of the St. Lawrence River at that point?
• The main bridge is 590.35 m long. Its centre span is 33 435 cm long. There is a support structure leading to the centre span on each side of the main bridge. Both support structures have the same length. How long is each support structure?

• There are 5 lanes along the bridge. Suppose that there is a traffic jam, and about 5 cars are standing along every 20 metres in each lane of the bridge. How many cars are stuck in the jam along the whole length of the bridge (3.4 km)?

Extensions

1. Name two places near where you live that are approximately as far apart as the Confederation Bridge is long. Use a map if necessary.

2. About 27 000 tonnes of asphalt were used to pave the surface of the Confederation Bridge. Approximately how many tonnes of asphalt were used in each kilometre of the bridge?

   **HINT:** Round the length of the bridge to the nearest ten kilometres.
ME5-19
Perimeter

GOALS
Students will measure the perimeter of given and self-created shapes.

PRIOR KNOWLEDGE REQUIRED
Adding sequences of 1-digit numbers

VOCABULARY
perimeter
grid paper

GOALs
Write the word “perimeter” and explain to your students that perimeter is the measurement around the outside of a shape. Illustrate the perimeters of some classroom items; run your hand along the perimeter of a desk, the blackboard or a chalkboard eraser. Write the phrase “the measurement around the outside of a shape.”

Demonstrate the method for calculating perimeter by counting the entire length of each side and creating an addition statement. Write the length of each side on the picture.

Draw several figures on a grid and ask your students to find the perimeter of the shapes. Include some shapes with sides one square long, like the shape in the assessment exercise, as students sometimes overlook these sides in calculating perimeter. Ask your students to draw several shapes of their own design on grid paper and exchange the shapes with a partner.

For the last exercise, suggest that students draw a letter, or simple word (like CAT), or their own names.

Assessment
Write the length of each edge beside each edge and count the perimeter of this shape.

Do not miss any sides—there are ten!

Distribute one piece of string, about 30 cm long, and a geoboard to each student. Have them tie the ends of the string together to form a loop, and then create a variety of shapes on the geoboard with the string. Explain that the shapes will all have the same perimeter because the length of the string, which forms the outside edges, is fixed. How many different shapes can all have the same perimeter?
Extensions

1. Explain that different shapes can have the same perimeter. Have your students draw as many shapes as they can with a given perimeter (say ten units).

2. Can a rectangle be drawn with sides that measure a whole number of units and have a perimeter…
   a) of seven units?
   b) with an odd number of units?

[Both are impossible.]

Students can use a geoboard rather than grid paper, if preferred.

3. Create three rectangles with perimeters of 12 cm. (REMEMBER: A square is a rectangle.)

4. Distribute pentomino pieces (a set of twelve shapes each made of five squares, see the BLM) to your students and have them calculate the perimeter of each shape. Create a table and order the perimeters from smallest to greatest. Have students also calculate the amount of square edges inside each shape. Can they notice a pattern emerging in the table?

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Number of Inside Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
ME5-20
Problem Solving with Perimeter

GOALS
Students will estimate and measure perimeters with metric units.

PRIOR KNOWLEDGE REQUIRED
Calculating perimeter
Measuring with a ruler
Different units of measurement
Selecting appropriate units of measurement
The ability to estimate

VOCABULARY
perimeter
millimetre
centimetre
decimetre
metre
kilometre

Draw this figure:

and have your students demonstrate the calculation of the perimeter by totalling the outside edges.

Then draw the same rectangle without the inside edges:

and ask them how they could calculate the perimeter again. Explain that it can be measured, and then have volunteers measure each side with a metre stick.

Draw an irregular figure with some edges that are neither horizontal nor vertical:

Have a volunteer use a ruler or metre stick to measure each side, and then count the sides to confirm that they’ve all been measured.

Assign the first two exercises of the worksheet.

Assessment
Calculate the perimeter.

TEACHER: Draw the lengths of all sides in whole centimetres.

Ask your students to estimate the lengths of the shapes with their bodies (remind them that the widths of their hands are about 10 cm).

Review appropriate units. Have students determine the best units of measurement for calculating the perimeters of the schoolyard, a chalkboard eraser, a sugar cube, etc.

Have them complete the worksheet. As a possible homework assignment, have your students estimate then measure the perimeters of their bedrooms.
Assessment
The length of a minivan is 5 m. The house is a rectangle six minivans long and four minivans wide.
1. Sketch the house.
2. Measure the edges (in minivans and in metres).
3. Calculate the perimeter of the house.

Extensions
1. The sides of a regular pentagon are all 5 cm long. What is the pentagon’s perimeter?
2. The perimeter of a regular hexagon is 42 cm. How can the length of its sides be determined?
3. Measure the perimeter of this star.

4. Can 500 toothpicks line the entire perimeter of your school? Calculate an estimate.
5. Sally wants to arrange eight square posters into a rectangle. How many different rectangles can she create? She plans to border the posters with a trim. For which arrangement would the border be least expensive? Explain how you know.

Activity
Using metre sticks and rulers, challenge students to measure the perimeters of a variety of objects in the classroom, and to specify the best unit of measurement for each.
ME5-21
Exploring Perimeters

Review the meaning of perimeter and the methods of finding it (counting the edges or adding the side lengths). Draw the following shape:

```
  +---+
  |   |
  |   +---
  +---+   |
```

Ask a volunteer to find the perimeter. Then ask additional volunteers to add a square to all possible perimeter positions and identify how the perimeter measurement changes. Summarize the results in a table. (It is also good to mark congruent shapes. Do they have the same perimeter?) Why does the perimeter change the way it does? How many edges that had previously been on the outside are now inside?

**ADVANCED:** Try to guide your students toward developing a formula (new perimeter = old perimeter + 4 – twice the number of sides that become inside edges).

Draw a rectangle on the board. Write a length on one of the longer sides. Ask your students what the length of the opposite side should be. Add the length of one of the shorter sides and ask your students what the length of the last side should be. What is the perimeter of the rectangle? Ask a volunteer to write the addition statement for the perimeter.

Draw another rectangle and say that its perimeter is 14 m. Say that the length of one side is 3 m. Ask your students to find the lengths of the other sides. Encourage them to present more than one solution to the problem (For example, they could say that 14 – 3 – 3 = 8, so the sum of the other two sides is 8, so each side is 4 m. Another solution would be to use the fact that the sum of two adjacent sides is exactly half of the perimeter: half of 14 is 7, so 3 + something = 7, which means each of the other sides is 4 m.)

Ask your students to find all the rectangles with a perimeter of 14 units and sides that have length in whole units.

Draw another rectangle on the board and say that the length is L and the width is 3 cm. What is the perimeter of this rectangle? What happens if the width is W and the length is 4 cm? How would you express the perimeter of a rectangle with length L and width W?

**ACTIVITY 1**

Students could try **QUESTION 3** on the worksheet with a geoboard.
Students enjoy creating complex shapes and finding their perimeter (particularly if you are impressed with their work). You might suggest that they draw and find the perimeter of a letter of the alphabet such as L, H, C, and A, or even a simple word like “CAT.”.

Point out to your students that different shapes can have the same perimeter. Ask them to draw as many shapes with a given perimeter (e.g., 10 units) as they can.

**Extensions**

1. A parallelogram has one side that is 3 cm long, and a perimeter of 16 cm. What are the lengths of the other sides?

2. An isosceles triangle has a perimeter of 15 units. The length of each side is a whole number. Find all possible triangles that meet these requirements. (**Answer:** 5, 5, 5; 7, 4, 4; 3, 6, 6; 1, 7, 7) Why can the length of the base not be an even number?

3. A school bus is 10 m long and 2.5 m wide. Karen’s school is 25 m long and 11 m wide.
   a) How many school buses can park along the entire perimeter of the school, so that each bus has its entire length along a school wall?
   b) How many school buses can park along the entire perimeter of the school if a bus is allowed to park so that it has its length along a school wall or along the width of another bus?

   **Allowed in b) only**

   **Not allowed**

   **Solution:** a) Though the perimeter of the school is 72 m, only two buses will be able to park along its length, so the answer is 6 buses. In b), one can park 8 buses as shown below:
Circles and Irregular Polygons

**GOALS**
Students will estimate and measure perimeters with metric units.

**PRIOR KNOWLEDGE REQUIRED**
- Perimeter
- Adding sequences of decimal numbers
- Finding rules for patterns

**VOCABULARY**
- circle
- perimeter
- circumference
- width

Draw a square 10 cm by 10 cm on the board. Invite a volunteer to draw and to measure the diagonals of the square. (They will be about 14 cm long.) Ask your students to draw a square 1 cm by 1 cm in their notebooks. Ask them to predict the length of the diagonal in their squares. Then ask students to draw and to measure the diagonal. (It will be about 1.4 cm long.) Explain to your students that the length they have measured is an approximation, and they can use it to find the perimeter of the following shapes:

The horizontal and vertical distance between adjacent dots is 1 cm. The diagonal distance between dots is, as students just learned, about 1.4 cm. Ask your students to find the perimeter of the various shapes by adding together the lengths of the sides. Encourage them to use multiplication where possible, as a quicker way to add multiples of the same number. For example, in the diamond shape (second from the left), there are 4 equal sides. Instead of adding the length of each side 4 times, students could multiply the length by 4.

Now draw 3 circles—with radiuses of 15, 20, 25 cm—on the board, mark the centres, and explain that the distance around the outside of each circle is called the **circumference**. Write the term on the board. Then draw several horizontal lines through one of the circles such that one of them passes through the centre of the circle. Tell students that the width of the circle is measured by the longest line. **ASK:** Which line is the longest? (Another way to ask this: Suppose these circles are coins and you need to make slots, to push each coin through. Which line represents the minimal length of a slot for each “coin”?) Help students to identify the line that goes through the centre of the circle as the longest one. You may tell students that the width of a circle is also called the **diameter**. Now measure the diameters of the remaining circles and fill in the first column in this T-table:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Using a measuring tape (the type tailors use), string, or pieces of paper, have volunteers measure the circumference of each circle and fill in the second column in the table. Then look at the numbers and ask students if they can see a pattern. How does the circumference relate to the width?
How many times larger is the circumference than the width in each case? Students can either divide the numbers, or do the Activity, to find out. You might wish to explain that the circumference of a circle is always approximately 3 times greater than the width (or diameter). The exact number (3.14, rounded to 2 decimal places) is a very important number in mathematics and is called “pi.”

### Extensions

1. **PROJECT:** What were the ancient estimates of pi? **POSSIBLE SOURCES:**
   - http://www-history.mcs.st-and.ac.uk/HistTopics/Pi_through_the_ages.html
   - http://library.thinkquest.org/C0110195/history/history.html

2. Lee wants to arrange some shells around a circular flowerbed in her garden. The flowerbed is 1 m wide. Each shell is 5 cm long. About how many shells will Lee need? **HINT:** Use the width of the flowerbed (in centimetres or in number of shells) and the pattern from the T-table you made during the lesson to estimate the circumference.
ME5-23
Area in Square Centimetres

GOALS
Students will find the area in centimetres squared (cm²) of shapes drawn on grid paper.

PRIOR KNOWLEDGE REQUIRED
Drawing lines with a ruler
Measuring sides with a ruler
Perimeter

VOCABULARY
2-dimensional
area
square centimeter
centimeters squared (cm²)
perimeter
rectangle

Remind students that area is often measured in units called “centimetres squared” or cm². Show students an example of a square centimetre, that is, a square whose sides are all 1 cm long.

Draw several rectangles and other shapes (EXAMPLE: L-shape, E-shape) on the board and subdivide them into squares. Ask volunteers to count the number of squares in each shape and write the area in cm².

Then draw several more rectangles and mark their sides at regular intervals, as shown below.

Ask volunteers to divide the rectangles into squares by joining the marks using a metre stick. Ask more volunteers to calculate the area of these rectangles.

Ask students to draw their own shapes on grid paper and to find the area and perimeter for each one.

ACTIVITY 1
Students work in pairs. One student draws a shape on grid paper, and the other calculates the area and the perimeter. ADVANCED VARIATION: One student draws a rectangle so that his/her partner does not see it, calculates the perimeter and the area, and gives them to the partner. The partner has to draw the rectangle with the given area and perimeter.

ACTIVITY 2
Students could try to make as many shapes as possible with an area 6 units (or squares) on grid paper or a geoboard. For a challenge, students could try making shapes with half squares. For an extra challenge, require that the shapes have at least one line of symmetry. For instance, the shapes below have area 6 units and a single line of symmetry.
Extensions

1. Sketch the shape below (at left) on centimetre grid paper. What is its area in cm²? (16) Now calculate the area using a different unit: 2 cm × 2 cm square (see below right). What is the area in 2 cm × 2 cm squares? (4) What happens to your measurement of area when you double the length of the sides of the square you are measuring with? (The area measurement decreases by a factor of 4.)

![Grid paper with shapes]

The new unit:
2 cm × 2 cm = 4 cm²

2. If the area of a shape is 20 cm², what would its area be in 2 cm × 2 cm squares? Sketch a rectangle with area 20 cm² to check your answer.

3. Draw a rectangle that has the same area (in cm²) as it does perimeter (in cm).

[18 = 3 × 6 (area) = 2 × 3 + 2 × 6 (perimeter)]

4. Copy the figure onto grid paper. Draw a straight line to divide the shape into 2 parts of equal area.

![Grid paper with shapes]
ME5-24
Area of Rectangles

**GOALS**
Students will find the area of rectangles in square units and in cm².

**PRIOR KNOWLEDGE REQUIRED**
Drawing lines with a ruler
Measuring sides with a ruler
centimetres squared (cm²)
Multiplication

**VOCABULARY**
2-dimensional
area
square centimeter
perimeter
rectangle
length
width

Draw an array of dots on the board. **ASK:** How many dots are in the array? Invite a volunteer to explain how he or she counted the dots. Ask another volunteer to write the corresponding multiplication statement on the board.

Now draw a rectangle on the board. (Draw it on a grid or subdivide it into squares.) Ask volunteers to write the length and width of the rectangle on the board, where length and width are measured in numbers of squares. Draw a dot in each square of the rectangle and **ASK:** How can a multiplication statement help us to find the area of the rectangle? Ask students to write and solve the multiplication statement for the area of the rectangle.

Draw several rectangles (again, subdivided into squares) and ask volunteers to write the length and the width and use them to calculate the area.

Draw rectangles with various lengths and widths (such as 20 × 30 cm, 40 × 30 cm, 30 × 50 cm) but don’t subdivide them into squares. Ask volunteers to measure the sides with a metre stick and calculate the area of the rectangles in cm².

Once students are comfortable finding the area of a rectangle by multiplying length and width, ask them to write the relationship as a verbal rule or a formula using letters: Area = Length × Width, or \( A = L \times W \). You might also encourage students to develop a rule or a formula for perimeter (\( P = 2 \times L + 2 \times W \)).

**Assessment**
1. Calculate the area of the rectangle:

   |   |   |   |   |   |
   |   |   |   |   |   |
   |   |   |   |   |   |
   |   |   |   |   |   |

   **Width:** _____

   **Length:** ____

2. Measure the sides and calculate the area:

   **Width:** _____

   **Length:** ____
Ask students to construct rectangles of the same area, but with different lengths and widths. They can work on a geoboard or with square tiles on grid paper. How many different rectangles can they make with area 8 cm²?

ACTIVITY 2
Ask students to create various rectangles and record their length, width, and area. Ask partners to give each other the length and width of some of their rectangles so they can calculate the area and check each other’s work.

Extensions
1. Calculate the area of the figure by adding the area of the rectangles:

   ![Diagram of a figure composed of rectangles]

   2 cm
   4 cm
   3 cm

2. Divide the figure into rectangles and calculate the area:

   ![Diagram of a figure composed of rectangles]

   2 cm
   2 cm
   2 cm
   2 cm
   3 cm
   7 cm
   4 cm
ME5-25
Exploring Area

GOALS
Students will find the area of rectangles with lengths and widths given in centimetres (cm), metres (m), and kilometres (km). Students will determine length (or width) given area and width (or length).

PRIOR KNOWLEDGE REQUIRED
Measuring sides with a ruler
Multiplication
Area of rectangle
Centimetres squared (cm²)

VOCABULARY
2-dimensional area
perimeter rectangle
length
width
square centimetre and centimetre squared (cm²)
square kilometre and kilometre squared (km²)
square metre and metre squared (m²)

Draw a rectangle on the board and ASK: How can we calculate the area of this rectangle? Invite volunteers to help you solve the problem (one student could measure, another could write the measurements, a third could do the calculation and write the answer).

Tell students that a city block is 2 km long on every side. ASK: What shape is the block? What is its area? What units should we use for the area—is it square centimetres? Why not? If students do not infer the right answer and explanation, explain that a square kilometre is a square whose sides are 1 km long. When you multiply the length of the city block (in kilometres) by the width, you find out how many one-kilometre squares are in the block, so the area is in square kilometres, or km². ASK: What if the city block is 8 m long on every side? What is its area? What units do we use?

Then draw several rectangles on the board, write the length and the width using different units of measurement, and ask volunteers to find the area.

EXAMPLES:
- 8 cm × 9 cm
- 20 cm × 7 cm
- 3 m × 7 m
- 6 km × 10 km
- 8 m × 7 m

ASK: Which rectangle has the greatest area? Which rectangle has the smallest area?

ASK: Which units would you use to measure the area of these objects or places—square centimetres, square metres, or square kilometres:
- Canada
- your classroom
- a book
- your city or town
- school yard
- a field
- a table

Draw another rectangle on the board and mark the length: 3 m. SAY: I know that the area of the rectangle is 6 m². How can I calculate the width of the rectangle? (PROMPTS: If you knew the length and the width, how would you calculate the area? What do you have to multiply 3 (the length) by to get 6 (the area)? How do you know? What did you do to 6 to get 2?)

Give several more problems of this kind. EXAMPLES:
- Length 4 cm, area 20 cm², find the width.
- Width 3 m, area 27 m², find the length.
- A square has area 16 km². What is its width? (What can you say about the length and the width of a square? Students can try various lengths—1 × 1, 2 × 2, and so on—until they find the answer.)

Ask students to draw a rectangle that has an area of 24 cm². How many rectangles of different proportions can they draw? Prompt them to start with a width of 1 cm, then try 2 cm, 3 cm, and so on. Does 5 cm work? Why not? What about 7 cm?
Assessment
1. What is the area of the rectangles?
   A: Length 5 m, width 4 m.
   B: Length 6 km, width 7 km.
   C: Length 20 cm, width 15 cm.

   Order the rectangles from least to greatest.
2. A city square has area 800 ______. Its length is 40 ______. Fill in the appropriate units of measurement and calculate the width of the square.
3. Draw 3 different rectangles that have area 30 cm².

Extensions
1. Find the area of the shape by dividing it into smaller rectangles.
   
   HINT: First you will have to fill in the missing side lengths.

   2. A rectangle has perimeter 12 cm and length 4 cm. What is its area?
ME5-26
Area of Polygons

Draw a $1 \times 2$ rectangle on a grid on the board. Draw a diagonal and shade one of the two triangles formed. **ASK:** Which part of the rectangle is shaded? How do you know? What are triangles like the halves of this rectangle called? (congruent triangles) What is the area of the rectangle? What is the area of the shaded triangle? Repeat with rectangles $1 \times 4$, $1 \times 6$, then $1 \times 3$ and $2 \times 3$.

Draw several right-angled trapezoids and ask your students to divide them—by drawing a line—into a rectangle and a triangle. Then make right-angled trapezoids on a grid or geoboard and ask students to find the area of the trapezoids. Challenge them to find the area of a general, not-right-angled and not isosceles trapezoid, such as:

Make sure your students can add sequences of halves by grouping before you assign the worksheet:

**EXAMPLE:**

\[
\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \frac{1}{2}
\]

3 wholes and a half left over

Also, make sure students understand that in order to find the area of the triangular parts of the figures on the worksheet, they simply have to view the triangle as covering half the area of a rectangle:

The triangle covers half the area of a $2 \times 3$ rectangle.

The area of the rectangle is $2 \times 3 = 6$. So the area of the triangle is $3$ (half of 6).

Ask your students to find a pair of congruent triangles in the picture below:

What is the area of the shaded triangle then?
Students might argue the following way:

1. Triangles A and C are congruent, so the shaded area is the same as the area of triangles A and B together, which is one half of a square.

2. Triangles B, C, and D form a triangle of area one square. Triangle D has area of one half of the square, so B and C together have area of half the square.

Let your students find the area of larger shapes containing triangles congruent to the shaded triangles, such as:

Assessment
Find the area of the shapes:

Let your students create various irregular polygons on geoboards and find the area of the shapes they produce.
ME5-27

Area of Irregular Shapes and Polygons

GOALS
Students will find the area of irregular shapes.

PRIOR KNOWLEDGE REQUIRED
Area
Fractions of area
Adding sequences of halves and whole numbers
Comparing mixed fractions with denominator 2

VOCABULARY
area

Draw several squares on the board. Invite volunteers to shade half of each square. Encourage them to find as many different ways of doing this as possible, including:

Draw several shapes that include various types of half squares and ask students to find the area of each shape. Then ask students to draw designs for each other, and to find and to compare the shaded and unshaded areas of their designs.

Draw a rectangle on a grid. Ask volunteers to shade half of each square in the rectangle using any design they like. ASK: What is the shaded area of the rectangle? Do you have to count all the half squares or there is a shortcut? If you know the total number of squares, what part of the total is shaded?

Draw a map of a lake on a grid (or use an overhead projector). Ask your students to find the area of the lake. ASK: Which squares contribute a whole 1 km² to the area? Which squares contribute about half a square kilometre? Which squares should not be counted at all because they contribute only a small fraction of a kilometre to the area?

Bonus
Have students draw their names using whole squares and different half squares, and calculate the area.

Extensions
The next series of extensions fulfils the demands of the Atlantic Curriculum (Expectation E13).

1. If you know the length and height of a parallelogram, how can you find its area?
**ANSWER:** Cut off a triangle at one end and attach it to the other end to form a rectangle.

\[ \text{Area} = 4 \times 6 = 24 \text{ cm}^2 \]

2. Use the same method as in the previous question to find the area of an isosceles trapezoid.

3. How can you cut off and rearrange two triangles to find the area of an irregular trapezoid?

4. Jennifer wants to find the area of a triangle:
   She cuts through the midpoints of two sides of the triangle and rearranges the pieces:

5. Alex wants to cut a triangle with sides 4 cm, 5 cm and 6 cm once and to rearrange the pieces (slide, rotate or flip them) so that he obtains a quadrilateral or a different triangle. He cuts the triangle as shown below. Does this cut work? Explain why not. Draw at least four different cuts that would allow Alex to rearrange the pieces and to obtain a quadrilateral or a triangle. Copy the triangle onto tracing paper and check your solution by cutting and rearranging the pieces. How many different quadrilaterals or triangles can you produce?
ME5-28
More Area and Perimeter

GOALS
Students will find the area and perimeter of irregular shapes. Students will learn how the change in the length of a rectangle affects its area.

PRIOR KNOWLEDGE REQUIRED
Area
Perimeter
Fractions of area
Finding the area of irregular shapes
Multiplication and division
Direct variation
Finding rules for T-tables

VOCABULARY
perimeter area
width length
direct variation

Draw an irregular shape on a grid and tell students that you want to estimate the area of the shape. Enlarge some of the grid squares that are divided by the shape in unusual or irregular ways, such as:

Discuss with students how they can estimate the perimeter of the lines in these grid squares. In the two leftmost examples above, students can estimate the length of the line as 1 because it is close to being a straight line parallel to one side of the square. In the two examples in the middle, the line is about 1.5 units long. (ASK: Why? What is the length of the diagonal if the square is 1 cm by 1 cm?) If you straightened out the lines in the squares at right, they look like the might be about half the length of one side, so you could estimate their length as half. After discussing these and other estimates, ask students to estimate the perimeter of the shape you have drawn. Repeat with more irregular shapes.

Draw 3 squares on the grid: 1 × 1, 2 × 2, and 3 × 3. Ask your students to find the area and the perimeter for all 3 squares. Invite a volunteer to summarize the properties of the squares in a T-table:

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

Invite students to look for patterns in the numbers. ASK: Do the numbers in the second column vary directly with the numbers in the first column? How? (Perimeter is 4 times side length.) Do the numbers in the third column vary directly with the numbers in the first column? (No) How do you get the area from the length? (Area is side length squared.) What will be the perimeter and the area for the square with side length 4? 6? 8? Extend the T-table to include these quantities.

Present this problem: Paul says that if he doubles the side of a square, both the area and the perimeter double as well. Is that correct? Which part of Paul’s statement is correct? How would you change the incorrect part to make it correct? (The perimeter doubles, the area is multiplied by 4.)
Complete a similar T-table for a set of rectangles: $2 \times 3, 4 \times 6, 8 \times 12$. **ASK**: Bill multiplied both the length and the width of a rectangle by 3. How many times larger is the perimeter of the new rectangle? How many times larger is the area of the new rectangle? Let your students draw rectangles of various proportions to check their predictions.

Review the corrected version of Paul’s rule (above): When the sides of a square or rectangle are doubled, the perimeter doubles and the area is multiplied by 4. **ASK**: Is Paul’s rule a general rule or is it good only for squares and rectangles? Invite students to draw a simple shape (like a letter of the alphabet) on grid paper and to enlarge all the dimensions by 2. Ask your students to find the area and the perimeter of both shapes and to check if the rule holds.

**Assessment**

Rocco draws two shapes on grid paper. One of his shapes is 4 times longer and 4 times wider than the other. He found that the perimeter of the larger shape is 6 times larger than the perimeter of the smaller shape, and that the area is 16 times larger. Is he correct? What did he calculate correctly: area, perimeter, or both?
ME5-29
Comparing Area and Perimeter

GOALS
Students will understand that area and perimeter are independent.

PRIOR KNOWLEDGE REQUIRED
Area of rectangle
Perimeter

VOCABULARY
area
perimeter
rectangle

Draw several rectangles on a grid: $4 \times 6$, $5 \times 5$, $6 \times 3$, $7 \times 2$, $3 \times 8$. Label them and ask volunteers to find the area and the perimeter of each one. Ask students to list the rectangles from least to greatest by area. Then ask them to list the rectangles from least to greatest by perimeter.

ASK: Are your lists the same? Does the rectangle with the greatest area also have the greatest perimeter? Does the rectangle with the smallest perimeter also have the smallest area? Are there rectangles with the same area? Do they have the same perimeter? Are there rectangles that have the same perimeter? Do they also have the same area?

What do you do to the length and width to calculate area? What do you do to the length and width to calculate perimeter?

Assessment
1. Draw 2 rectangles that have the same area—20 cm$^2$—but different perimeters. Calculate their perimeters.
2. Bob drew 2 shapes with the same perimeter but different areas. Is this possible? The sides of Bob’s shapes are whole centimetres. One shape is a square with area 9 cm$^2$. Can you draw this square? The other shape is a rectangle. Can you draw the rectangle?

Extensions
1. Mr. Green wants to make a rectangular flower bed with perimeter 24 m. Which dimensions of the flower bed will provide the greatest area?
2. Mr. Brown wants to make a rectangular flower bed with area 36 m$^2$. Which dimensions will give him the least perimeter?
### ME5-30

#### Area and Perimeter

Give students several rectangles with lengths and widths that are whole centimetres and say you want to estimate both the area and the perimeter of each one. **ASK:** How can I do that? What can I use to help me? Invite students to share their suggestions and try them out. If necessary, remind students that the area of their thumbnail is approximately 1 cm². How can they use their thumbnail to estimate area and perimeter?

Have students check their estimates by measuring the length and width of the various rectangles and calculating the area and perimeter. Compare the estimates to the actual measurements.

Then tell students you want to find all the rectangles with perimeter 14 cm. The only other requirement is that the lengths and widths are whole centimetres (i.e. 3 or 5, not 2.25 or 4 \( \frac{3}{4} \)). **SAY:** I’m going to start with a rectangle that has width 1 cm. That’s the smallest possible measurement I can have. Draw a rectangle and mark the width. **ASK:** If the width of the rectangle is 1 cm, what is the length? Invite the student who answers to explain how he or she came up with the answer. If students need assistance use these **PROMPTS:**

- The perimeter is the distance all around the shape—if you took a rope that was 14 cm long, it would go all around the sides of our rectangle. If we know the width is 1 cm, how much of the rope have we used? (2 cm, for 2 sides) How much of the rope is left? (12 cm) For how many sides? (2) So how long is each remaining side? (6 cm) Another way to solve the problem is to use the fact that one length and one width of the rectangle add up to half the perimeter, or 7 cm. If 1 cm + length = 7 cm, the length must be 6 cm.

Now draw a rectangle with width 2 cm and calculate the length. Repeat for a rectangle with width 3 cm. **ASK:** What is the length of the last rectangle? Do we need to continue making rectangles? Why not? (The last rectangle was 3 \( \times \) 4. A rectangle with width 4 would have length 3—it would be the same rectangle, just rotated or turned on its side!) Do we need to make rectangles with width 5 or 6? What about a rectangle with width 7? How many rectangles with perimeter 14 do we have in total? Ask students to find the area of the rectangles.

**Assessment**

Draw all possible rectangles with perimeter 16 cm. Which one has the greatest area? The least area?

---

### GOALS

Students will estimate and find the area and perimeter of rectangles. Students will find the length of a rectangle given the area or perimeter and the width.

### PRIOR KNOWLEDGE REQUIRED

- Area of rectangle
- Perimeter

### VOCABULARY

- area
- rectangle
- width
- perimeter
- length

---

**ACTIVITY 1**

Students could try **QUESTIONS 3** and **4** on the worksheet using a geoboard.
Extensions

1. Describe a situation in which you would have to measure area or perimeter, for instance, to cover a bulletin board or make a border for a picture. Make up a problem based on the situation.

2. A rectangle has area 20 cm\(^2\) and length 5 cm. What is its perimeter?

3. The shape has perimeter 24 cm. What is its area?

ME5-31
Problems and Puzzles and
ME5-32
More Area and Perimeter

These are review worksheets that can be supplemented by Activities.

Students could investigate QUESTION 2 on worksheet ME5-31 using a geoboard.

Ask students to make as many non-congruent shapes as they can by placing 5 squares edge to edge. (These shapes are called pentominoes: there are 12 different pentominoes altogether.) Which pentominoes have the greatest perimeter? Which have the greatest number of lines of symmetry?

Extension
Find the perimeter and area of the shape.

```
4 cm  11 cm
8 cm
5 cm
```
Use concrete props to review the concepts of length and area, and to introduce the concept of volume. Start with a piece of string. It has only 1 dimension—length. **ASK:** What units do we measure length in? (cm, m, km)

Point to the top of a desk or table and explain that it has length and width—that’s 2 dimensions. Anything two-dimensional has area. The area of the desk or table is its surface, the space bound by, or “inside,” the length and width. **ASK:** What units do we measure area in? (cm², m², km²)

Now explain that a three-dimensional object, like a cupboard or a box, has length, width, and height. The space taken up by the box or cupboard or any three-dimensional object is called volume, and we can measure it, too. One of the units we use for volume is cubic centimetres, or cm³.

Draw a cube on the board and mark the length of all the sides as 1 cm. Tell students that this is a centimetre cube. It has length, height, and width 1 cm, and its volume is exactly 1 cm³. Tell students that they will be using centimetre cubes to calculate the volume of various shapes.

Ask students what they think larger volumes are measured in. (m³, km³)

Explain that these units are very large. For example, to fill an aquarium that has a volume of 1 m³, you would need about 100 pails of water!

Give students some centimeter cubes and ask them to build figures that have volume 4 cm³. Then ask them to build figures with volume 6 cm³. Assist any students who aren’t sure how many cubes to use for each figure, or who use the wrong number. Finally, invite students to build different figures with volume up to 8 cm³. They can work in pairs—one student makes a figure and the other calculates its volume.

Explain that for complicated figures, we can use a top view to calculate the volume. Make or draw the following 3-D figure:

Tell students to pretend that they are looking down on the figure from above. How many blocks would they see? (3) Draw 3 squares—the base, or bottom layer, of the figure—and invite volunteers to help you write the number of blocks stacked in each position:
Explain that this is the mat plan of the figure. You can make or draw a few more figures (like the one below) and invite volunteers to help you draw the mat plans. Start by drawing the shape of the figure’s bottom layer, then count the cubes in each position.

![Mat Plan and Figure]

Then do the opposite: start with the mat plan and build the figure. Show students this mat plan:

```
2 1 1 3
1
```

Ask one volunteer to build the bottom layer of the figure. Ask more volunteers to come and add cubes to the stacks that need more than 1 cube.

![Build Figure]

Then **ASK**: How can we calculate the volume of the figure? Do we need the figure or can we do it from the mat plan? (We can do it from the mat plan. Each cube has volume 1 cm³, so the total number of cubes gives you the total volume of the figure.)

Ask students to work in pairs. One partner builds a figure with volume no more than 12 cubes and height no more than 4 cubes and the other partner draws the top view for the figure. Partners should swap roles, so that each has a chance to do both the drawing and the building. After everyone has had a chance to do both at least once, have pairs do the reverse: one partner draws the top view for a figure and the other builds the figure accordingly.

**Assessment**
1. Build the figure according to the mat plan:

```
1
2 1 1 3
1
```

2. Fill in the mat plan of this figure.

![Fill Mat Plan]

3. Find the volume of both figures above.
Extensions

1. Draw the mat plan for this figure. How many cubes are “hidden” in the picture?

2. Give students the front, top, and side views of a structure and ask them to build it using interlocking cubes. As they work, encourage them to look at their structure from all sides, and to compare what they see to the pictures. Ask them to determine the volume of the completed structure.

**NOTE:** You might want to give students the front, top, and side views of a few simpler shapes before they tackle the ones above.

**Bonus**

Give students the front, top, and side view of a simple figure and ask them to calculate the volume without building the figure.
3. a) A structure has this front and side view and a volume of 6 cm³. Build the structure.

![Front and Top Views]

b) Suppose the structure in a) had volume 5 cm³. Can you build 2 structures consistent with the top and side views?

4. Build 3 structures with this front and side view.

![Front and Side Views]

**ME5-34**

**Volume of Rectangular Prisms**

Draw a rectangle on a grid (or subdivide a rectangle into equal squares). Ask students to write the addition and multiplication statements needed to calculate the area of this rectangle.

Show students a large rectangular box. Explain that you want to know the volume of the box. (You can say that you want to send something to someone, and the shipping company charges by volume. You need to know the volume to figure how much it will cost to ship this box.) You can fill the box with centimeter cubes, but that is not very practical. Tell students that today they will learn another method for calculating volume.

First, tell students that mathematicians have a fancy name for a rectangular box. They call it a “rectangular prism.” Write the term on the board.

Build a 2 × 4 rectangle using centimetre cubes and **ASK:** How many cubes are in this rectangular prism? What is the volume of this prism? Ask students to explain how they calculated the volume—did they count the cubes one by one or did they count them another way? Because each cube has volume 1 cm³, the total number of cubes gives you the total volume. You can use addition or multiplication to count the total number of cubes:

\[2 + 2 + 2 + 2 = 8 \text{ cm}^3 \text{ (2 centimetre cubes in each of 4 columns)}\]
\[2 \times 4 = 15 \text{ cm}^3 \text{ (2 rows of 4 centimetre cubes OR length \times width)}\]
Add 1 layer to the prism so that it is 2 cubes high. **ASK:** What is the volume of the new prism? Prompt students to use the fact that the prism has 2 horizontal layers. Remind them that they already know the volume of 1 layer. Ask volunteers to write addition and multiplication statements for the volume of the new prism using layers.

\[
8 + 8 = 16 \text{ cm}^3 \\
2 \times 8 = 16 \text{ cm}^3
\]

Add a third layer to the prism and repeat. Invite students to look at this last prism and to calculate the volume by adding vertical layers, or “walls,” instead of horizontal layers.

**ASK:** How many cubes are in the wall at the end of the prism? (3 × 2 = 6) What is the volume of the wall? (6) How many walls are in the prism? (4) Invite volunteers to write the addition and the multiplication statements for the volume of the prism using the volume of the “wall.” Does this method produce a different result than the previous method? (no, it’s the same answer)

Explain that the third dimension in 3-D figures is called height. Identify the length, width, and height in the prism above. Then use the terms length, width, and height to label the multiplication statement that gives the volume:

\[
3 \times 2 \times 4 = 24 \text{ cm}^3
\]

Draw several prisms on the board, mark the height, width, and length (you can use different units for different prisms), and ask students to find the volume. **SAMPLE PROBLEMS:**

\[
10 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm} \\
2 \text{ m} \times 3 \text{ m} \times 5 \text{ m} \\
3 \text{ km} \times 4 \text{ km} \times 7 \text{ km}
\]

Remind students to include the right units in their answers.

**ASK:** What does the top view for a rectangular prism look like? Is there any difference in heights above each square? (No, the height of a rectangular prism is the same everywhere.) Draw several top views on the board and ask students to identify which are rectangular prisms and which are not. Then ask them to write the length, width, and height for each prism, and to calculate the volume.

**SAMPLE PROBLEMS:**

\[
\begin{array}{ccc}
5 & 5 & 6 \\
5 & 5 & 5
\end{array} \\
\begin{array}{ccc}
7 & 7 & 7 \\
7 & 7 & 7
\end{array} \\
\begin{array}{ccc}
5 & 5 & 5 \\
5 & 5 & 5 \\
5 & 5 & 5
\end{array} \\
\begin{array}{ccc}
5 & 5 & 5 \\
5 & 5 & 5
\end{array}
\]

**Bonus**

Find the volumes of the figures that aren’t rectangular prisms.
Assessment
Find the volume of the prisms:

a) 6 × 6 × 6

b) 4 m × 5 m × 6 m

c) 3 cm × 3 cm × 3 cm

Extensions
1. Find the volume of the shape:

2. Find the volume of the prisms:

   1 m × 1 km × 1 m      5 cm × 3 dm × 2 m      1 mm × 1 m × 1 km
ME5-35
Mass

NOTE: Mass is a measure of how much substance, or matter, is in a thing. Mass is measured in grams and kilograms. A more commonly used word for mass is weight: elevators list the maximum weight they can carry, package list the weight of their contents, and scales measure your weight. The word weight however, has another very different meaning. To a scientist, weight is a measure of the force of gravity on an object. An object’s mass is the same everywhere—on Earth, on the Moon, in space—but it’s weight changes according to the force of gravity. When we use the term weight in this and subsequent lessons, we use it as a synonym for mass.

Remind students that mass (which we often call weight) is measured in grams (g) and kilograms (kg). Give several examples of things that weigh about 1 gram or about 1 kilogram:

1 g: a paper clip, a dime, a chocolate chip
1 kg: 1L bottle of water, a bag of 200 nickels, a squirrel.

List several objects on the board and ask students to say which unit of measurement is most appropriate for each one—grams or kilograms:

• A whale
• A table
• A napkin

• A cup of tea
• A workbook
• A minivan

Ask students to match these masses to the objects above:

2 000 kg  50 000 kg  10 g  150 g  400 g  10kg

Have students order these objects from heaviest to lightest.

Ask students to think of 3 other objects that they would weigh in grams and 3 objects that would demand kilograms.

Draw a set of scales on the board. Draw various weights on one scale (that is on one side) and ask volunteers to add weights to the other so that the scales are in balance. Then draw weights on both scales, but leave out the numbers on one side and have students fill them in. Include instances where students have to add and divide to find the right weights. SAMPLE PROBLEMS:
Work through the following set of problems as a class. Invite and encourage as many different students as possible to participate by contributing answers, explanations, and suggestions. Break some of the more complicated problems into smaller steps. Use prompts and questions to help students identify what they know and how they can use what they know to solve the problem.

You could keep a list of facts on the board and add to it as you go. For example, problem a) gives the weight of a rabbit (3 kg) and asks you to calculate the weight of a lazy cat (6 kg). Both of these facts could go on the list: Rabbit = 3 kg, Lazy Cat = 2 × Rabbit = 6 kg.

a) A rabbit weighs 3 kg. A lazy cat weighs twice as much as the rabbit. How much does the lazy cat weigh?

b) An angry dog weighs as much as 3 lazy cats. How many rabbits does this dog weigh?

c) What is the dog’s weight in kilograms?

d) An adult raccoon weighs as much as an angry dog and a rabbit. How many rabbits does the raccoon weigh? What is the raccoon’s weight in kilograms?

e) A beaver weighs 3 500 grams less than an adult raccoon. How much does the beaver weigh?

f) A female bear weighs 90 kg. How many rabbits is that equal to? How many lazy cats?

How many dogs?

g) How many lazy cats do you need to balance 2 angry dogs and 2 rabbits on a scale?

h) A newborn Siberian tiger weighs about 1 kg. It gains 700 g a week. How much weight does it gain in 4 weeks? How much does the tiger weigh after 4 weeks?

i) A 20-week old Siberian tiger cub is on one scale and an angry dog is on the other. What do you need to do to balance the scales? (Ask a volunteer to draw a model for this problem first.)

Let students feel the weight of objects that are close to 1 g (EXAMPLE: paperclip) or 1 kg (EXAMPLE: a 1 L bottle of water). They can use these referents to estimate the weight of the objects in the classroom, such as books, erasers, binders, games and calculators. (You could ask students to order the objects from lightest to heaviest.) Students should use scales to weight the objects and check their estimates.
ACTIVITY 2

Weigh an empty container, then weigh the container again with some water in it. Subtract 2 masses to find the mass of the water. Repeat this with a different container but the same amount of water. Point out to students that the mass of a substance doesn’t change even if its shape does.

ACTIVITY 3

Weigh a measuring cup. Pour 10 mL of water in the cup. Calculate the mass of the water by subtracting the weight of the cup from the weight of the water with the cup. How much does 10 mL of water weigh? (How much does 1 mL of water weigh?)

Extensions

1. Jane wants to estimate the mass of one grain of rice. She weighs 100 grains of rice and divides the total by 100. Try to weigh 1 grain of rice. Explain why Jane uses this method above. Use Jane’s method to estimate the mass of a bean or a lentil.

2. When Duncan stands on a scale the arrow points to 45 kg. When he stands on the scale and holds his cat, the arrow points to 50 kg. How could he use these two measurements to find the weight of his cat?
**GOALS**

Students will determine the capacity of various containers. Students will convert millilitres to litres and vice versa.

**PRIOR KNOWLEDGE REQUIRED**

Litres (L)  
Millilitres (mL)

**VOCABULARY**

capacity  
litre  
millilitre

Explain that the capacity of a container is how much it can hold. Write the term on the board. Explain that capacity is measured in litres (L) and millilitres (mL). **ASK:** Where have you seen the prefix “milli” before and what did it mean? (millimetre; one thousandth) How many millilitres are in 1 litre? In 2 litres? In 7 litres? What do you do to change litres to millilitres? (Multiply by 1 000.) Write on the board:

- 1 metre = 1 000 millimetres  
- 1 m = 1 000 mm  
- 1 litre = 1 000 millilitres  
- 1 L = 1 000 mL

Put out several containers (EXAMPLES: milk and juice boxes, medicine bottles, measuring cups, cans of paint, cans of pop) with capacities clearly marked on them. Invite students to help you separate the containers into 2 groups: those that can hold 1 or more litres and those that can hold less than 1 litre. Then ask students to help you order the containers by capacity, from least to greatest. The containers can also act as “capacity benchmarks” that you can keep in a class measurement box.

Write the following on the board and ask students whether they would measure the capacity of each container in millilitres or litres:

- a glass of juice  
- a bowl of soup  
- a pail of water  
- a pot of soup  
- an aquarium  
- a backyard pool

Ask students to think of three more quantities that are measured in litres and three that are measured in millilitres.

Show students a 1 L carton of milk or juice and a small (200 mL) glass. **ASK:** How can we determine the capacity of the glass? You may ask a volunteer to check how many glasses can be filled from the container. What is the capacity of the glass? Now bring out a larger glass (250 mL). How many glasses of this size can be filled from the carton? Ask a volunteer to check. *(NOTE: You will need a large bowl or pot in which to empty out the glasses as they are filled.)*

**Assessment**

a) An aquarium holds two 8 L pails of water. What is its capacity? Write the capacity in litres and millilitres.

b) A jar holds 500 mL of water. How many jars do you need to fill the aquarium?
Measure the capacity of several glasses or containers in your classroom. Estimate the capacity of the containers before you measure their capacity. Students should select and justify appropriate units to measure the capacity of a container. **NOTE:** Students will need a measuring cup and several containers for this activity.

### ACTIVITY 2
Make two containers: 500 mL and 300 mL (you can use two empty 1 L cartons and cut them at the height of 10 cm and 6 cm respectively). Ask your students to use only these two containers, a tap, and a sink to measure 200 mL of water.

**CHALLENGING:** Measure 400 mL of water using only the same equipment.

### ACTIVITY 3
Bring in some items from a grocery store. Ask students to look at the labels and sort the items into 2 groups: items where the amount of food is given as mass and items where the amount is given as capacity.

### ACTIVITY 4
Collect 5 containers (cups, cans, bottles, pails) of different sizes on which capacities were covered or removed.

a) Estimate the capacity of each container.
b) Measure and record the capacity of each container.
c) Order the measurements from greatest to least.
d) Compare your measurements with your estimates.

### Extensions
1. **JELLYBEAN JARS:** Choose two straight-sided jars of similar size but different dimensions, for example, a tall, thin olive jar and a short, squat salsa jar. Fill both jars with jelly beans. Show students both jars and **ASK:** Which jar has a larger capacity? Why do you think that? How could we check our predictions? Discuss the students’ ideas for determining the answer, then choose one or more and try it! Here are some of the approaches you could investigate:

   a) Open the jars, dump out the contents, and count the jelly beans.

   b) Open the jars, dump out the contents of one jar, and pour the jelly beans from the other into it. Do the jelly beans fill the empty jar? Is there any empty space left over? Are there any jelly beans left in the first jar?

   c) Have a student count the number of jelly beans visible through the bottom of the jar, and record the number. Have another student count the number of layers of jelly beans from the bottom to the top. Have students multiply the two numbers to estimate the total number of beans in each jar.
Fill a large glass container with enough water to submerge either jar. Ask your students to predict what will happen if you place the jar into the bowl. Why does the water level go up? Close both jars tightly. Submerge one jar in the water and mark the new water level with a piece of tape. Remove the first jar and point out what has happened to the water level. Place the second jar in the water and ask students to determine whether the water level is higher of lower than with the first jar. What does this tell us about the size of the jars?

2. **PROJECT:** The origins of SI and its usage throughout the world: When SI was introduced, which countries use it and which do not? Which countries are converting into SI? What are the advantages of SI?

---

### ME5-37 Volume and Capacity

**GOALS**

Students will use capacity to find volume and vice versa.

**PRIOR KNOWLEDGE REQUIRED**

- Volume
- Capacity
- Volume of rectangular prisms
- Converting between litres and millilitres

**VOCABULARY**

- volume
- capacity
- mL
- cm³

Hold up a see-through measuring cup with some water in it. Drop a centicube into the cup. Ask your students if they can see how much liquid is displaced by the cube. If they have trouble seeing how much water is displaced by 1 cube, which is likely to happen unless your cup is very small, ask them how they would solve this problem. (One answer: They could drop 10 cubes in and divide the displacement by 10.) Invite volunteers to drop more centicubes into the cup and to measure the displacement. What is the capacity of 1 cm³ cube? (1 mL)

Present a small rectangular box and ask your students how they could measure its capacity. We know the capacity of 1 cm³. What is the capacity of 10 cm³? Of 20 cm³? Invite volunteers to measure the sides of the box and calculate its volume. What is the capacity of the box?

**ASK:** A cube has a capacity of 1 L. What are the dimensions of the cube? How many mL are in 1 L? How do you find the volume of the cube? (You multiply the side by itself 3 times.) Which number is multiplied by itself 3 times to get 1 000? So how long is the side of the cube? (10 cm). Do you know a specific term for this length? Write on the board: Capacity of 1 dm³ = 1 L.

Draw a box on the board and write its dimensions: 30 cm × 40 cm × 50 cm. **ASK:** What is the capacity of the box? Let your students find the capacity in mL first, then ask them to convert it to L. Ask your students if they can solve the problem another way. (They can convert the dimensions to decimetres and get the result in litres: 3 × 4 × 5 = 60 L.)
Assessment
A rectangular box has base 7 cm × 7 cm. It contains 0.5 L of milk. About how high is the box in centimetres? What is its exact height in millimetres?

Let your students find the volume of small objects like toys, coins, or apples by submerging them in water and measuring the displacement of the water, then converting the capacity of the water displaced to a volume.

Extensions
1. Discuss the story of Archimedes and the Golden Wreath.
   **POSSIBLE SOURCE:**
   http://math.nyu.edu/~crorres/Archimedes/Crown/CrownIntro.html

2. Daniela wants to find the volume of an apple. She puts the apple into a glass box with 600 mL of water. The box has a square base of 10 × 10 cm. The water reaches a height of 9.8 cm. What is the volume of the apple?
ME5-38
Changing Units of Measurements

NOTE: Make sure your students understand that you need more of a smaller unit to fill the same amount of space as a larger unit. That’s why, in changing from a larger to a smaller unit, you multiply by a multiple of 10. The multiple depends on which unit you are in and which unit you are changing into. For instance, there are 10 mm in each centimetre, so to change .7 cm to millimetres you multiply by 10 (which, as demonstrated in section NS5-93, shifts the decimal one place right). When you change from a smaller unit to a larger unit you need fewer of the larger unit, so you divide.

Review the material in Lesson ME5-15 with the students. Review the dollar and cent notations, and conversion between units of time (e.g., 1 hour = 60 minutes). Give students a lot of problems requiring conversion between various units of measurement. With every problem, ASK: Is the new unit larger or smaller than the old unit? Do we need more units or less? Are we going to multiply or divide? How many smaller units fit into the larger unit? By how much are we going to multiply or divide?

GOALS
Students will practise changing various units of measurements.

PRIOR KNOWLEDGE REQUIRED
Volume
Capacity
Converting between various units of length, weight, time, volume, and capacity
Dollar and cent notations

VOCABULARY
L km
mL mm
m dollar
cm cent
dm conversion
ME5 Part 2: BLM List

Dot Paper _____________________________________________________________ 2
Grid Paper (1 cm) ______________________________________________________ 3
Pentomino Pieces ______________________________________________________ 4
Dot Paper
Grid Paper (1 cm)
Pentomino Pieces
PS5-5 Using Structure to Add Sequences

Teach this lesson after: 5.2 Measurement

Goals:
Students will use structure to understand the relationship between patterns.

Prior Knowledge Required:
Can multiply a multi-digit number by a one-digit number using the standard algorithm
Can add and subtract decimal tenths and hundredths (for Problem Banks 4, 5)
Can evaluate a fraction of a whole number (for Problem Bank 6)
Can divide whole numbers by 10 and get decimal tenths (for Problem Bank 8)

Vocabulary: denominator, equivalent fractions, exponents, numerator, powers of 10, sequence, term

Materials:
overhead projector
transparency of grid paper or BLM 1 cm Grid Paper (p. 8)
BLM 1 cm Grid Paper (p. 8)

Using area models to discover patterns. Write on the board:

1 = __________
1 + 2 = __________
1 + 2 + 3 = __________
1 + 2 + 3 + 4 = __________
1 + 2 + 3 + 4 + 5 = __________

Fill in the blanks as volunteers tell you the sums. (1, 3, 6, 10, 15) SAY: The gaps increase because that’s how we made the sequence, but I want to know if there is a way to get an expression that will help me find any term. One way to think of the sums is as an area. Project a transparency of grid paper or BLM 1 cm Grid Paper onto the board and draw the following shape:
SAY: Let’s count the squares inside the shape to find the area. ASK: How many squares are in the first row? (1) In the second row? (2) Third row? (4) Fourth row? (5) Fifth row? (5) SAY: So, we can add all these together to find the total. Write on the board:

\[ \text{Area} = 1 + 2 + 3 + 4 + 5 = 15 \text{ square units} \]

**Exercises:**
1. Write the area as an addition by adding the number of squares in each row.
   a) 
   b) 
   c) 

   ![Shapes](image)

   **Answers:** a) \(3 + 4 + 5 = 12 \text{ square units}\), b) \(1 + 2 + 3 + 4 = 10 \text{ square units}\), c) \(3 + 5 + 7 = 15 \text{ square units}\)

   **NOTE:** Provide students with grid paper or BLM 1 cm Grid Paper for the following exercises.

2. Draw an area model for the expression.
   a) \(2 + 3 + 4\) b) \(4 + 5 + 6 + 7\) c) \(2 + 5 + 8\)

   ![Shapes](image)

   **Answers:**

   a) 
   b) 
   c) 

   On grid paper, have students draw two identical shapes like the one on the board for \(1 + 2 + 3 + 4 + 5\) and then cut them out. Challenge students to arrange the shapes to make a rectangle, then ask them to find the area of each shape. When students are finished, draw on the board:

\[ (1 + 2 + 3 + 4 + 5) \times 2 = 5 \times 6 = 30 \]

So \(1 + 2 + 3 + 4 + 5 = 15\)
**Exercises:** Use two copies of a shape to make a rectangle. Then write the multiplication.

a) $(1 + 2 + 3) \times 2 = _____ \times _____$

b) $(3 + 5 + 7) \times 2 = _____ \times _____$

c) $(1 + 2 + 3 + 4) \times 2 = _____ \times _____$

d) $(2 + 5 + 8) \times 2 = _____ \times _____$

**Bonus:** Which two questions have the same answer? Why does that make sense?

**Answers:** a) $3 \times 4$; b) $3 \times 10$; c) $4 \times 5$; d) $3 \times 10$; Bonus: parts b) and d), $3 + 5 + 7 = 2 + 5 + 8$, so multiplying either by 2 gets the same answer.

**SAY:** Once you know the area of the rectangle, you can divide by 2 to find the area of the original shape. Draw on the board:

<table>
<thead>
<tr>
<th>Sum</th>
<th>Equivalent Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 + 2 + 3$</td>
<td>$(3 \times 4) \div 2$</td>
<td>6</td>
</tr>
<tr>
<td>$1 + 2 + 3 + 4$</td>
<td>$(4 \times 5) \div 2$</td>
<td>10</td>
</tr>
<tr>
<td>$1 + 2 + 3 + 4 + 5$</td>
<td>$(5 \times 6) \div 2$</td>
<td>15</td>
</tr>
</tbody>
</table>

**SAY:** From the exercises you just did, $(1 + 2 + 3) \times 2$ is $3 \times 4$, so that means you can divide $3 \times 4$ by 2 to get an expression that equals $1 + 2 + 3$. Write that in the table, and have volunteers write the equivalent expression for the next two rows. Then have volunteers write the value of each expression in the third column.

<table>
<thead>
<tr>
<th>Sum</th>
<th>Equivalent Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 + 2 + 3$</td>
<td>$(3 \times 4) \div 2$</td>
<td>6</td>
</tr>
<tr>
<td>$1 + 2 + 3 + 4$</td>
<td>$(4 \times 5) \div 2$</td>
<td>10</td>
</tr>
<tr>
<td>$1 + 2 + 3 + 4 + 5$</td>
<td>$(5 \times 6) \div 2$</td>
<td>15</td>
</tr>
</tbody>
</table>

**SAY:** There looks like a pattern here in the first two columns. Have volunteers complete the next two rows.

<table>
<thead>
<tr>
<th>Sum</th>
<th>Equivalent Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 + 2 + 3 + 4 + 5 + 6$</td>
<td>$(6 \times 7) \div 2$</td>
<td>21</td>
</tr>
<tr>
<td>$1 + 2 + 3 + 4 + 5 + 6 + 7$</td>
<td>$(7 \times 8) \div 2$</td>
<td>28</td>
</tr>
</tbody>
</table>

**ASK:** How can you get the equivalent expression from the sum? (start with the last number being added, multiply it by the next number, and then divide by 2) **SAY:** You can do that for larger numbers too. Write on the board:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$$

**ASK:** What would be the equivalent expression? $$((9 \times 10) \div 2)$$ **SAY:** If you want to add all nine numbers, you can use either expression because they will both get you the same answer.
students do it both ways, then ASK: Which way was faster? (answers may vary) (Using the multiplication will be faster for most students, but some students might use tricks to make the addition faster too, such as adding \((1 + 9) + (2 + 8)\) and so on to get 45.) SAY: When there are a lot of numbers to add, using the multiplication method will be faster.

**Exercises:** Evaluate.

a) \(1 + 2 + 3 + 4 + \ldots + 20\)

b) \(1 + 2 + 3 + 4 + \ldots + 30\)

**Bonus:** \(1 + 2 + 3 + 4 + \ldots + 100\)

**Answers:** a) \((20 \times 21) ÷ 2 = 210\), b) \((30 \times 31) ÷ 2 = 465\), Bonus: \((100 \times 101) ÷ 2 = 5050\)

When students have completed the exercises, point out how much time they saved by not having to do all that addition. Even with a calculator, adding 30 numbers takes a long time.

Write on the board:

\[21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30\]

SAY: I can add the numbers 1 to 20 and I can add the numbers 1 to 30. ASK: How can I add the numbers 21 to 30? \((465 - 210 = 255)\)

**Exercises:** Evaluate the sum.

a) \(31 + 32 + 33 + 34 + \ldots + 40\)

b) \(41 + 42 + 43 + 44 + \ldots + 50\)

c) \(51 + 52 + 53 + 54 + \ldots + 60\)

**Answers:** a) \((40 \times 41 ÷ 2) - (30 \times 31 ÷ 2) = 820 - 465 = 355\), b) \((50 \times 51 ÷ 2) - (40 \times 41 ÷ 2) = 1275 - 820 = 455\), c) \((60 \times 61 ÷ 2) - (50 \times 51 ÷ 2) = 1830 - 1275 = 555\)

When students finish, challenge them to look for a pattern in their answers. (each answer is 100 more than the previous one) ASK: Why does this happen? (each term is 10 more than in the previous sum, so you are adding ten 10s) Write on the board:

\[
\begin{align*}
21 & + 22 + 23 + \ldots + 30 \\
+ & 10 + 10 + 10 + \ldots + 10 \\
31 & + 32 + 33 + \ldots + 40
\end{align*}
\]

SAY: You are adding 10 to each term and there are ten 10s being added, so you are adding 100 in total. That’s why the answer is always 100 more.

**Exercises:** Add.

a) \(1 + 2 + 3 + 4 + 5 + \ldots + 50 = \) _____

b) \(1 + 2 + 3 + 4 + 5 + \ldots + 50 = \) _____

\[
\begin{align*}
&+ 1 + 2 + 3 + 4 + 5 + \ldots + 50 = \) _____ \\
&2 + 4 + 6 + 8 + 10 + \ldots + 100 = \) _____
\end{align*}
\]
c) \[2 + 4 + 6 + 8 + 10 + \ldots + 100 = \quad \]
\[- (1 + 1 + 1 + 1 + \ldots + 1) = \quad \]
\[1 + 3 + 5 + 7 + 9 + \ldots + 99 = \quad \]

d) \[1 + 2 + 3 + 4 + 5 + \ldots + 50 = \quad \]
\[+ (1 + 2 + 3 + 4 + \ldots + 49) = \quad \]
\[1 + 3 + 5 + 7 + 9 + \ldots + 99 = \quad \]

e) \[1 + 2 + 3 + 4 + 5 + \ldots + 50 = \quad \]
\[1 + 2 + 3 + 4 + 5 + \ldots + 50 = \quad \]
\[3 + 6 + 9 + 12 + 15 + \ldots + 150 = \quad \]

Answers: a) 1275, b) 1275 + 1275 = 2550, c) 2550 − 50 = 2500, d) 1275 + 1225 = 2500,
e) 1275 + 1275 + 1275 = 3825

When students finish, ASK: In which two parts did you add the same number in different ways? (parts c) and d)) Did you get the same answer both times? (yes) SAY: Look at parts b) and e). Instead of repeatedly adding the same sum, you can multiply instead. Write on the board:

\[2 + 4 + 6 + 8 + 10 + \ldots + 100 = 2 \times (1 + 2 + 3 + 4 + 5 + \ldots + 50)\]
\[3 + 6 + 9 + 12 + 15 + \ldots + 150 = 3 \times (1 + 2 + 3 + 4 + 5 + \ldots + 50)\]

SAY: Multiplying each term by the same number gets the same answer as multiplying the whole sum by that number.

Exercises: Fill in the blanks.
a) \[3 + 6 + 9 + 12 + 15 = \quad \times (1 + 2 + 3 + 4 + 5)\]
b) \[31 + 62 + 93 = \quad \times (1 + 2 + 3)\]
c) \[9 + 12 + 15 = 3 \times (\quad + \quad + \quad)\]

Answers: a) 3; b) 31; c) 3, 4, 5

SAY: You can use this strategy to evaluate sums that look hard to do. Write on the board:

\[11 + 22 + 33 + 44 + 55 + 66 + 77 + 88 + 99 = \quad \times (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)\]

Have a volunteer fill in the blank. (11) ASK: If you know that the sum of the first nine numbers is 45, what is the sum on the left? (11 × 45) How could you calculate 11 × 45 mentally? (use 10 × 45 = 450 and then add 45 to get 495)

Exercises:
a) Using \[3 \times (12924 + 208) = 39396\], determine what 12924 + 208 is without adding.
b) Using \[3 \times (394 + 367 + 442) = 3609\], determine what 394 + 367 + 442 is without adding.
c) Using \[2 + 5 + 8 = 15\], what is 6 + 15 + 24?
d) Using \[1 + 4 + 7 + 10 = 22\], what is 4 + 16 + 28 + 40?
e) Using \[5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 = 180\], what is \[1 + 2 + 3 + 4 + 5 + 6 + 7 + 8\]?

Bonus: If \[7 + 6 - 1 = 12\], what is \[21 + 18 - 3\]?
Answers: a) 13 132, b) 1203, c) 45, d) 88, e) 36, Bonus: 36

SAY: Sometimes, you just need to subtract some terms.

\[
\begin{align*}
1 + 2 + 3 + 4 + 5 + 6 + 7 &= 28 \\
- 1 + 2 + 3 &= 6 \\
\text{So} \quad 4 + 5 + 6 + 7 &= 22
\end{align*}
\]

Exercises: Use \(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55\) to evaluate the addition.

a) \(4 + 5 + 6 + 7 + 8 + 9 + 10\)
b) \(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9\)
c) i) \(3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30\)  
  ii) \(2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11\)

Bonus: Evaluate \(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19\) using the following:

\[
\begin{align*}
3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 \\
- 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 \\
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19
\end{align*}
\]

Answers: a) 49; b) 45; c) i) 165, ii) 65; Bonus: 100

Problem Bank

1. Use the first sum to evaluate the second sum. Explain your strategy.

a) Using \(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36\), what is \(4 + 5 + 6 + 7 + 8 + 9 + 10 + 11\)?

b) Using \(2 + 3 + 4 + 5 + 6 = 20\), what is \(34 + 51 + 68 + 85 + 102\)?

c) Using \(1 + 2 + 3 + 4 + 5 + \ldots + 15 = 120\), what is \(1 + 3 + 5 + 7 + 9 + \ldots + 29\)?

Sample answers:

a) \(36 + 8 \times 3 = 36 + 24 = 60\), because I added 3 to each of the eight terms

b) \(20 \times 17 = 340\), because I multiplied each term by 17

c) \(120 + (120 - 15) = 125\), because I added term by term the sums \((1 + 2 + 3 + \ldots + 15)\) and \((0 + 1 + 2 + \ldots + 14)\) and the second sum is 15 less than the first sum.

2. Evaluate \(46 + 47 + 48 + 49 + \ldots + 60\) in two ways. Make sure you get the same answer both ways.

a) \(1 + 2 + 3 + 4 + \ldots + 45 + 46 + 47 + \ldots + 60 = \phantom{000000000}
- (1 + 2 + 3 + 4 + \ldots + 45) = \phantom{000000000} \)

\[
46 + 47 + \ldots + 60 = \phantom{000000000}
\]

b) \(1 + 2 + 3 + \ldots + 15 = \phantom{000000000}
45 + 45 + 45 + \ldots + 45 = \phantom{000000000} \)

\[
46 + 47 + 48 + \ldots + 60 = \phantom{000000000}
\]

Answers: a) 1830 – 1035 = 795, b) 120 + 675 = 795

3. Evaluate \(24 + 26 + 28 + 30 + 32 + 34\) in two ways. Make sure you get the same answer both ways.

a) Use \(12 + 13 + 14 + 15 + 16 + 17\).

b) Use \(1 + 3 + 5 + 7 + 9 + 11\).
Answers: a) \(24 + 26 + 28 + 30 + 32 + 34 = 2 \times (12 + 13 + 14 + 15 + 16 + 17) = 2 \times 87 = 174\), 
b) \(24 + 26 + 28 + 30 + 32 + 34 = (1 + 3 + 5 + 7 + 9 + 11) + (23 + 23 + 23 + 23 + 23)\) 
\(= 36 + 6 \times 23 = 36 + 138 = 174\)

4. Using \(1 + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = 2.45\), what is \(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\)? Explain.
Answer: 2.45 - 1 = 1.45

5. a) Using \(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = 1.45\), what is \(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\)?

b) If 2.83 is a good estimate for \(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{9}\), what is a good estimate for \(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{10}\)?

c) Using \(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = 2.45\), what is \(2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6}\)?

Answers: a) 0.95, b) 2.93, c) 8.45

6. Using \(\frac{1}{3}\) of 24 is 8, what is \(\frac{1}{2}\) of \(\frac{1}{3}\) of 24?
Answer: 4

7. If \(4 \times (A + 5) = 32\), what is \(A + 5\)? What is \(A\)?
Answer: \(A + 5 = 8\), \(A = 3\)

8. a) Calculate.
\[
1 + 2 + 3 + 4 + 5 \\
+ 10 + 9 + 8 + 7 + 6
\]

b) Use your answer from part a) to calculate \(8 + 16 + 24 + 32 + 40 + 48 + 56 + 64 + 72 + 80\)
c) Use your answer from part b) to calculate \(0.8 + 1.6 + 2.4 + 3.2 + 4.0 + 4.8 + 5.6 + 6.4 + 7.2 + 8.0\)
Answers: a) 55, b) 440, c) 44
PS5-6 Predicting Repeating Patterns Using Division

Teach this lesson after: 5.2 Measurement

Goals:
Students will extend repeating patterns and predict terms in repeating patterns.

Prior Knowledge Required:
Can identify the core of a repeating pattern
Can extend a repeating pattern
Can divide with remainder up to three-digit by one-digit numbers
Knows that division means sharing objects into sets with leftovers

Materials:
BLM Patterns in Remainders (p. 15, see Problem Bank 6)

Vocabulary: core, remainder, repeating pattern, term

Review repeating patterns. Tell students you have red blocks and yellow blocks. Write on the board:

R Y Y R Y Y R Y Y

Remind students that if something is a pattern, it means you can predict what comes next.
ASK: Do the colours make a pattern? (yes) What colour comes next? (red) How can you tell? (the pattern is red, yellow, yellow, repeat) Remind students that when the terms of the pattern repeat, the pattern is called a repeating pattern. The part that repeats is called the core.

Exercises: The core is given. Write the next five terms.
a) RY   b) YRY   c) YRRY   d) RRY
Answers: a) RYRYR, b) YRYYR, c) YRRYY, d) RRYRR

Predicting terms. Draw on the board:

○ | ○ | ○ | ○ |

ASK: Do you see a pattern? (yes) What is the next term? (line) Continue having a few more volunteers predict the next term. (circle, line, circle) Then tell students that you would like to predict the 100th term. SAY: Let’s write numbers for what place each term is in. Do that on the board and SAY: This is the first term (as you write “1”), this is the second term (as you write “2”), and so on.

○ | ○ | ○ | ○ |

1 2 3 4 5 6 7
ASK: Which terms are circles? (the 1st, 3rd, 5th, 7th) Write on the board:

1, 3, 5, 7

ASK: Is this also a pattern? (yes) Can you predict the next term that will be a circle? (yes, the 9th) And the next circle after that? (the 11th term) Repeat the questions for lines (they are the 2nd, 4th, 6th, 8th, 10th, and 12th terms). Write on the board:

circles: 1, 3, 5, 7, 9, 11, ...
lines: 2, 4, 6, 8, 10, 12, ...

ASK: Will the 100th term be a circle or a line? (a line) How do you know? (all even numbered terms are lines, and 100 is an even number)

Exercises: Predict the 100th term of the pattern.

a) 4 7 4 7 4 7 4 7 b) B Y B Y B Y B Y
Answers: a) 7, b) Y

Point out that the 100th term is the same as the second term in each case. SAY: If we have a way to find out which member of the core the 100th term is equal to, then we only have to know the core to find out what the 100th term is. SAY: Let’s start with an easier problem and just find out which term of the core the 11th term is equal to.

Exercises:
1. The core is given. Continue the pattern until the 11th term.
a) R B  b) R B W  c) Y W R  d) B R G W
e) B R R  f) R W Y R  g) R B W Y G

2. Circle the core as many times as it happens.

Answers:
g) R B W Y G R B W Y G R

Point out that in each pattern, the core is always the same, so by circling the cores, students made equal groups. ASK: What kind of math equation can you write from equal groups? (multiplication or division) Point out that in each case, they are dividing 11 objects into groups the same size. Draw on the board:

R B R B B R B R B R B R  11 ÷ 2 = 5 R 1

Exercises: Write a division statement for the patterns in the previous exercises.

Answers: a) 11 ÷ 2 = 5 R 1, b) 11 ÷ 3 = 3 R 2, c) 11 ÷ 3 = 3 R 2, d) 11 ÷ 4 = 2 R 3, e) 11 ÷ 3 = 3 R 2, f) 11 ÷ 4 = 2 R 3, g) 11 ÷ 5 = 2 R 1
ASK: In part a) which term in the core is the 11th term equal to, the first or the second? (the first) SAY: It’s the next one after the last core is circled, so it’s the first term of the next core. Do you see the number 1 anywhere in the division equation? (yes, it’s the remainder) Repeat for part b). (this time, the 11th term is the same as the second term in the core, because it is the second one after the last core is circled; 2 is also the remainder in the division equation)

**Exercises:**
1. In the previous exercises, which term of the core is the 11th term equal to—the 1st, 2nd, 3rd, 4th, or 5th?
   **Answers:** a) 1st, b) 2nd, c) 2nd, d) 3rd, e) 2nd, f) 3rd, g) 1st

2. Circle the number in the division equation that shows which term in the core is equal to the 11th term—the 1st, 2nd, 3rd, 4th or 5th.
   **Answer:** Always circle the remainder.

ASK: What do you notice about the numbers that you circled? (it is always the remainder) Why does this make sense? After hearing students articulate their answers without judgment, summarize the reason by referring to the pictures on the board. SAY: You only drew the first 11 terms, so if there is one term outside the complete cores, the 11th term is the 1st term in the next core. If there are two terms outside the complete cores, the 11th term is the 2nd term in the next core.

**Exercises:** The core is given. Use division to decide what colour the 13th block is.
- a) Y W R
- b) R W Y
- c) Y W R Y

**Bonus:** What is the 100th term? Y R R

**Answers:** a) yellow, b) red, c) red, Bonus: Y

**Introduce the case where the term number is a multiple of the core length.**

**Exercises:** Continue the pattern to the 12th term.
- a) R W
- b) R W B
- c) R R Y W B G
- d) R W Y B


In the exercises above, ask students to underline the core, then circle the term in the core that is equal to the 12th term. ASK: Which term in the core is always equal to the 12th term? (the last one) SAY: When the term number is a multiple of the number of terms in the core, then the term is always the same as the last term in the core. Refer students’ attention to the pattern in part b):

R W B R W B R W B

SAY: The last term in the core is B. So B is the last term in every core: the 3rd, 6th, 9th, and 12th terms are all B. For any multiple of 3, that term will be B.

**Exercises:** Find the 100th term of the pattern with the given core.
- a) 8 3
- b) R W R Y
- c) 2 2 2 3 3
Answers: a) 3, b) Y, c) 3

Predicting terms in all cases. SAY: If you want to find a term, divide the term number by the length of the core. When the remainder is 0, the term is the last term of the core. When the remainder is any other number, the remainder tells you which term of the next core it is—remainder 1 says 1st term, remainder 2 says 2nd term, and so on.

Exercises:
1. Find the 35th term in the pattern with the given core. Justify your answer with a division equation.
   
   a) o △
   
   b) o □ △
   
   c) o □ □ □
   
   d) o □ □ □ △
   
   e) o □ □ △ △ □
   
   f) o □ □ □ △ o □
   
   g) o □ △ △ △ o □ □ △
   
   Answers: a) 35 ÷ 2 = 17 R 1, so circle; b) 35 ÷ 3 = 11 R 2, so square; c) 35 ÷ 4 = 8 R 3, so square; d) 35 ÷ 5 = 7 R 0, so triangle; e) 35 ÷ 6 = 5 R 5, so triangle; f) 35 ÷ 7 = 5 R 0, so square; g) 35 ÷ 8 = 4 R 3, so triangle

2. Predict the 100th term in the pattern.

   a) o △ △ o △ △ o △ △ o △ △
   
   b) R Y Y B R Y Y B R Y Y B
   
   c) R R Y Y R R Y Y Y R R
   
   Answers: a) circle, b) B, c) Y

NOTE: If students have trouble recognizing the core, encourage them to guess the core, starting with 2 terms, then 3, and so on, until they see that they are circling the same thing each time. For example, in Exercise 2.a) above, circling the first two doesn’t work because the first two circled groups of 2 are different.

Connect to earlier work on extending patterns by skip counting. When students finish the exercises, SAY: This is like what you did earlier with skip counting, but in this lesson, you wrote a division statement. Write on the board:

\[35 ÷ 3 = 11 R 2\]

SAY: Because the remainder was 2, you know the 35th term is the same as the 2nd term of the core. Before, to find the 35th term, you would have skip counted by 3s, perhaps on a hundreds
chart, until you got close to 35. So you would skip count to 33 and then count on, starting at a
new core. Beside the division, write a multiplication and addition statement:

\[
35 \div 3 = 11 \text{ R } 2 \quad 35 = 11 \times 3 + 2
\]

SAY: Because multiplication and division are related, you are really doing the same thing, so
you get the same answer.

**Exercises:** Find the 45th term in the sequence with core R B Y G in two ways. Make sure you
get the same answer both ways.
a) Skip count by 4s until you get close to 45, then count on.
b) Write a division statement and use remainders.

**Answers:** a) 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 45, all the multiples of 4 terms are G, so the
45th term begins a core, so it is R; b) \(45 \div 4 = 11 \text{ R } 1\), so the 45th term is the first term of the
core, so it is R

**Problem Bank**
1. What is the 1000th term of a pattern with core length 1?
   **Answer:** the same as the first term—every term is

2. Can R Y R Y be the core of a pattern? Why or why not?
   **Answer:** no, because if R Y R Y repeats, then so does R Y, so just R Y would be the core

3. Use long division to find the 431st term of the pattern with core A B C.
   **Answer:** \(431 \div 3 = 143 \text{ R } 2\), so the 431st term is the same as the second term of the core,
   which is B

4. What is the 100th term of the pattern?
   \[
   A \quad A \quad B \quad C \quad D \quad B \quad C \quad D \quad B \quad C \quad D
   \]
   **Answer:** The 100th term is the same as the 98th term of the pattern with core B C D; \(98 \div 3 = 32 \text{ R } 2\), so the 100th term is C.

5. What is the value of the first 20 coins in the pattern?

   ![Sample solution](image)

   **Sample solution:** \(20 \div 3 = 6 \text{ R } 2\). So, there are six full cores and two more terms. Each core is
   worth $2.10, so the six cores are worth $12.60 and the two extra coins are loonies, so
   altogether, the 20 coins are worth $14.60.
6. Have students do BLM Patterns in Remainders.  
Selected answers: 1. a) 1 R 0, b) 1 R 1, c) 1 R 2, d) 1 R 3, e) 2 R 0, f) 2 R 1, g) 2 R 2, h) 2 R 3, i) 3 R 0; 2. Increase the remainder when it is less than the 3 and increase the number of groups when the remainder is 3; 3 R 1, 3 R 2, 3 R 3, and 4 R 0  

7. Using $852 \div 7 = 121 \text{ R } 5$, what will be the remainder when dividing the number by 7?  
a) 853   b) 854  
Answers: a) 6, b) 0  

8. $1543 \div 12 = 328 \text{ R } 7$. What is the next smallest multiple of 12 that is greater than 1543?  
Answer: 1548  

9. A banner is made by using 100 triangles, which repeat six colours in order:  
blue, green, yellow, orange, red, purple  

![Flag Pattern](image)  

a) What colour is the 50th triangle?  
b) What colour is the 100th triangle?  
c) In which position is the last red triangle?  
Selected solution: c) The 100th term is orange, so use the order to find the last red triangle:  
red  purple  blue  green  yellow  orange  
95th  96th  97th  98th  99th  100th  
The 95th term is the last red triangle.  
Answers: a) 50 $\div$ 6 = 8 R 2, so green; b) 100 $\div$ 6 = 16 R 4, so orange  

10. January 1 is a Tuesday and the year has 365 days.  
a) What day of the week is the 100th day of the year?  
b) What day of the week is the last day of the year?  
c) What is the date of the last Friday of the year?  
Selected solution: c) the last day, December 31, is a Tuesday, December 30 is a Monday, December 29 is a Sunday, December 28 is a Saturday, and December 27 is a Friday, so December 27 is the last Friday of the year.  
Answers: a) Wednesday, b) Tuesday
Patterns in Remainders

REMINDER ▶ To divide 14 by 4, draw 14 dots and make as many groups of 4 as you can:

There are 3 groups of 4 dots and 2 dots left over, so 14 ÷ 4 = 3 R 2.

1. Divide using the picture. For parts f) to i), you will need to divide the dots into groups.
   a) 4 ÷ 4 = _____ R _____
   b) 5 ÷ 4 = _____ R _____
   c) 6 ÷ 4 = _____ R _____
   d) 7 ÷ 4 = _____ R _____
   e) 8 ÷ 4 = _____ R _____
   f) 9 ÷ 4 = _____ R _____
   g) 10 ÷ 4 = _____ R _____
   h) 11 ÷ 4 = _____ R _____
   i) 12 ÷ 4 = _____ R _____

2. Look at your answers to Question 1. How do you know when to increase the number of groups and when to increase the remainder?

3. Continue the pattern from Question 1 to find 13 ÷ 4, 14 ÷ 4, 15 ÷ 4, and 16 ÷ 4.

4. Use patterns to fill in the blanks as quickly as you can.
   5 ÷ 5 = _____ R _____
   6 ÷ 5 = _____ R _____
   7 ÷ 5 = _____ R _____
   8 ÷ 5 = _____ R _____
   9 ÷ 5 = _____ R _____
   10 ÷ 5 = _____ R _____
   11 ÷ 5 = _____ R _____
   12 ÷ 5 = _____ R _____
   13 ÷ 5 = _____ R _____
   14 ÷ 5 = _____ R _____
   15 ÷ 5 = _____ R _____
   16 ÷ 5 = _____ R _____
   17 ÷ 5 = _____ R _____
   18 ÷ 5 = _____ R _____
   19 ÷ 5 = _____ R _____
**PS5-7 Using Patterns in Sequences**

**Teach this lesson after:** 5.2 Measurement

**Goals:**
Students will look for patterns in sequences, find gaps, find remainders, write formulas, and solve problems with sequences.

**Prior Knowledge Required:**
- Can multiply a multi-digit number by a one-digit number using the standard algorithm
- Can perform long division
- Can do operations on decimals

**Vocabulary:** consecutive, gap, long division, remainder, sequence, term

**Review sequences with constant gap.** Remind students that the gap in a sequence is the difference between two consecutive terms of a sequence. Write on the board:

\[
4, 7, 10, 13, 16, \ldots
\]

**ASK:** What is the gap? (3) Draw the numbers on a number line, as shown below:

![Number line with gaps of 3](image)

Draw a new number line and ask a volunteer to show the positive multiples of 3 up to 15, as shown below:

![Number line with multiples of 3](image)

**SAY:** The two sequences have the same gap, but every term in the top sequence is moved over one unit to the right. **ASK:** What is the remainder if we divide 3, 6, 9, 12, and 15 by 3? (0) **Why?** (the terms are multiples of 3) What is the remainder if we divide 4 by 3? (1) If we divide 7 by 3? (1) If we divide 10 by 3? (1) Write on the board:

\[
\begin{align*}
4 \div 3 &= 1 \text{ R } 1 \\
7 \div 3 &= 2 \text{ R } 1 \\
10 \div 3 &= 3 \text{ R } 1 \\
13 \div 3 &= 4 \text{ R } 1 \\
\end{align*}
\]
Point to the sequence 4, 7, 10, 13, 16, … and SAY: The gap is 3, and if you divide each term by 3 you get the same remainder: 1. Write on the board:

20, 23, 26, 29, …

ASK: What is the gap? (3) Have a volunteer divide each term by 3 and find the remainder, as shown below:

\[
\begin{align*}
20 \div 3 &= 6 \text{ R } 2 \\
23 \div 3 &= 7 \text{ R } 2 \\
26 \div 3 &= 8 \text{ R } 2 \\
29 \div 3 &= 9 \text{ R } 2
\end{align*}
\]

Explain to students that because the gap is the same, when they divide each term by the gap, they get the same remainder.

**Exercises:**

1. Find the gap.
   a) 4, 6, 8, 10, … b) 6, 11, 16, 21, … c) 7, 11, 15, 19, …
   d) 12, 16, 20, 24, … e) 2, 8, 14, 20, … f) 11, 18, 25, 32, …
   **Answers:** a) 2, b) 5, c) 4, d) 4, e) 6, f) 7

2. For each part in Exercise 1, divide each term by the gap and find the remainder. Is the remainder the same for the whole sequence?
   **Answers:** a) 0, 0, 0, 0, yes; b) 1, 1, 1, 1, yes; c) 3, 3, 3, 3, yes; d) 0, 0, 0, 0, yes; e) 2, 2, 2, 2, yes; f) 4, 4, 4, 4, yes

**Finding a term using the remainder.** Write on the board:

3, 7, 11, 15, …

ASK: If I continue the sequence, will I see the number 35? Ask a volunteer to write the next terms of the sequence, as shown below:

3, 7, 11, 15, 19, 23, 27, 31, 35

SAY: Thirty-five is a term of the sequence and we could find it by continuing the terms. ASK: How can we find if 135 belongs to the sequence? Have students share their ideas. Guide students to see how they can use the property they’ve just learned. Point to the sequence and ASK: Is the gap always the same? (yes) What is the gap? (4) Ask a volunteer to divide 3, 7, and 11 by 4 and find the remainder, as shown below:

\[
\begin{align*}
3 \div 4 &= 0 \text{ R } 3 \\
7 \div 4 &= 1 \text{ R } 3 \\
11 \div 4 &= 2 \text{ R } 3
\end{align*}
\]
SAY: We know that 35 is in the sequence because when 35 is divided by 4, the remainder is 3. Write on the board:

\[ 35 \div 4 = 8 \text{ R } 3 \]

ASK: So how can we determine if 135 will belong to the sequence or not? (divide 135 by 4 and find the remainder) Ask a volunteer to divide 135 by 4 using long division, as shown below:

\[
\begin{array}{c|c}
4 & 135 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
33 & 135 \\
\hline
-12 & 15 \\
\hline
-12 & 3 \\
\hline
\end{array}
\]

So $135 \div 4 = 33 \text{ R } 3$

Point to the remainder and SAY: When we divide 135 by 4, the remainder is 3, just the way it is for the earlier numbers in the sequence. So we know that if we continue the sequence, we will see the number 135 in the sequence.

**Exercises:**
1. Is 500 in the sequence?
   a) 10, 16, 22, 28, …  
   b) 8, 14, 20, 26, …

**Solutions:**
   a) the gap is 6, and if you divide the first term by the gap, the remainder is 4; $500 \div 6 = 83 \text{ R } 2$, so 500 is not in the sequence
   b) the gap is 6, and $8 \div 6 = 1 \text{ R } 2$; $500 \div 6 = 83 \text{ R } 2$, so 500 is in the sequence

2. Consider the sequences below.
   A. 6, 10, 14, 18, …  
   B. 9, 13, 17, 21, …  
   C. 15, 19, 23, 27, …  
   D. 24, 28, 32, 36, …

Jen says 255 is in sequence A and Rani says 255 is in sequence B. Who is right?

**Solution:** Both are wrong. For all the sequences, the gap is 4 and the remainders are as follows: 2 for A, 1 for B, 3 for C, and 0 for D. $255 \div 4 = 63 \text{ R } 3$, so 255 is only in sequence C.

**Review writing rules for sequences.** Write on the board:

\[
\begin{array}{c}
9, \ 16, \ 23, \ 30, \ ...
\end{array}
\]

ASK: How could you describe this sequence without saying all of the individual terms? (start at 9 and add 7 each time) SAY: When you say where to start and how to get each term from the one before it, you are saying a rule for the sequence.
Exercises:
1. Write a rule for the sequence.
   a) 7, 11, 15, 19, …       b) 29, 23, 17, 11, …       c) 45, 53, 61, 69, …
   d) 104, 93, 82, 71, …    e) 1.3, 2.2, 3.1, 4, …
   **Answers:** a) start at 7 and add 4 each time, b) start at 29 and subtract 6 each time, c) start at 45 and add 8 each time, d) start at 104 and subtract 11 each time, e) start at 1.3 and add 0.9 each time

2. Consider the sequences below.
   A. Start at 6 and add 7.  B. Start at 9 and add 7.
   C. Start at 3 and add 7.  D. Start at 27 and add 7.

   Jen says 251 is in sequence A and Rani says 251 is in sequence D. Who is right?
   **Answer:** Both are right.

Problem Bank
1. Each shape is made from toothpicks.
   ![Shapes]
   a) Make a table to show the number of toothpicks in each shape.
   b) Find the gap and write a rule for the sequence.
   c) Determine how many toothpicks Shape 5 will have.
   d) Will any figure have 500 toothpicks? Explain how you know.
   **Selected answers:** b) the gap is 6 and the rule is start at 16 and add 6, c) Shape 5 has 40 toothpicks, d) no, because 500 has remainder 2 when divided by 6, but all terms in the pattern have remainder 4 when divided by 6

2. Is 1386 in the sequence?
   a) 1, 6, 11, 16, …       b) 2, 8, 14, 20, …       c) 1, 4, 7, 10, …
   d) 9, 18, 27, 36, …      e) 1, 3, 5, 7, …        f) 7, 14, 21, 28, …
   **Answers:** a) yes, b) no, c) no, d) yes, e) no, f) yes

3. Use 941 ÷ 7 = 134 R 3 to evaluate 1641 ÷ 7. Explain your answer.
   **Answer:** 1641 is 700 more than 941, so you need 100 more groups with the same number left over; 1641 ÷ 7 = 234 R 3

4. Which column is 400 in if the numbers continue as shown?
   1  2  3  4  5  6
   7  8  9 10 11 12
   13 14 15 16 17 18
   **Solution:** 400 ÷ 6 = 66 R 4, so 400 is in Column 4.
5. a) Which column is 365 in if the numbers continue as shown?

<table>
<thead>
<tr>
<th>Sun</th>
<th>Mon</th>
<th>Tues</th>
<th>Wed</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
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<td>8</td>
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<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

b) The year is not a leap year and January 1 is a Sunday. What day of the week is December 31?
c) Another year is a leap year and starts with January 1 on a Wednesday. What day of the week is January 1 of the following year?

**Solutions:**
a) $365 \div 7 = 52 \ R 1$, so 365 is in the Sunday column
b) Sunday, because December 31 is the 365th day of the year
c) If January 1 is a Wednesday and the year is a leap year, the year will end on the 366th day—a Thursday—and the next year will start on Friday, January 1.

6. January 1 is a Thursday and this year is not a leap year. Braden plays soccer every Monday. His birthday is December 1. Will he play soccer on his birthday?

**Solution:** There are 365 days in the year, and 30 of them are after Braden’s birthday, so his birthday is the 335th day of the year. Since $335 \div 7 = 47 \ R 6$, his birthday is the sixth day of the core. The core starts Thursday, and the sixth term is Tuesday. No, Braden won’t play soccer on his birthday.

7. Hanna’s birthday is April 21 and Simon’s birthday is October 13. Are their birthdays on the same day of the week?

**Solution:** How many days after April 21 is October 13? 9 in April, 31 in May, 30 in June, 31 in July, 31 in August, 30 in September, and 13 in October, for a total of $9 + 31 + 30 + 31 + 31 + 30 + 13 = 175$ and $175 \div 7 = 25 \ R 0$, so there are 25 full weeks from April 21 to October 13. Yes, their birthdays are on the same day of the week.

8. Ken used one kind of block to build a structure. He added the same number of blocks to his structure at each stage of its construction. But he made a mistake when he copied the number of blocks at each stage into the table below. Can you find his error and correct it?

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
</tbody>
</table>

**Answer:** The number of blocks in Stage 2 should be 11, not 12, because the gap is 7.
9. The table shows the number of line segments in a figure, but some information is missing. The same number of line segments was added at each stage. Fill in the missing numbers. Hint: First find the gap.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Line Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

Solution: There are four gaps between Figure 1 and Figure 5. The number of line segments increased by $20 - 8 = 12$, so the gap is $12 \div 4 = 3$. The missing numbers are 11, 14, 17.

10. The table shows the number of blocks used to build a structure. The same number of blocks was added at each stage. Fill in the missing numbers.

a)  

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>23</td>
</tr>
</tbody>
</table>

b)  

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>39</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
</tbody>
</table>

Answers: a) 11, 13; b) 17, 28; c) 17, 29

11. Suppose you want to construct a block wall following the steps shown below. You would like the finished wall to be 11 blocks long. Each block costs 7¢ and you have $1.25 in total. Do you have enough money to buy all the blocks you need? Hint: Make a table with four columns: Step, Number of Blocks Used, Number of Blocks Long, and Cost (¢).

Step 1 Step 2 Step 3

Solution:  

<table>
<thead>
<tr>
<th>Step</th>
<th>Number of Blocks Used</th>
<th>Number of Blocks Long</th>
<th>Cost (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>7</td>
<td>77</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>9</td>
<td>98</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>11</td>
<td>119</td>
</tr>
</tbody>
</table>

In Step 5, the wall will be 11 blocks long and will need 17 blocks. Since 17 blocks costs $1.19, there is enough money to buy all the blocks needed.
12. Make a table and extend it to predict the number of line segments in Figure 10. Hint: First find a pattern for the gap because the gap is not the same.

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Figure 2</th>
<th>Figure 3</th>
<th>Figure 4</th>
<th>Figure 5</th>
</tr>
</thead>
</table>

**Solution:** The sequence begins: 0, 1, 3, 6, 10, … , with gaps 1, 2, 3, 4, …. The sequence of gaps continues 5, 6, 7, 8, … , so the sequence itself continues: 15, 21, 28, 36, 45. Figure 10 has 45 line segments.
PS5-8 Making a Simpler Problem

Teach this lesson after: 5.2 Measurement

Goals:
Students will, when given a problem, make a simpler problem and use the solution to the simpler problem to help solve the original problem.

Prior Knowledge Required:
Can add and subtract decimals up to hundredths
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can write an expression for a given term in a pattern
Can convert measurements expressed in metres to centimetres (for Extended Problem)

Vocabulary: area, horizontal, length, perimeter, vertical

Materials:
BLM Fraction Strips and Circles (p. 35, see Problem Bank 7)
BLM Class Art Show (pp. 38–39, see Extended Problem)

Using a given simpler problem to help solve a harder problem. Write on the board:

There are 300 people in line. How many people are after the 12th person?

ASK: What makes this problem hard? (students will likely say that 300 is a big number) Would it be easier if I asked how many people are after the 299th person in line? (yes) SAY: So, it’s not exactly how big 300 is that makes this problem hard. ASK: Can you find a more precise way to say what makes it hard? (12 and 300 are far apart)

Exercises: Answer the question.
a) There are 13 people in line. How many people are after the 12th person?
b) There are 5 people in line. How many people are after the 3rd person?
c) There are 300 people in line. How many people are after the 12th person?
Bonus: There are 3459 people in line. How many people are after the 1459th person?
Answers: a) 1, b) 2, c) 288, Bonus: 2000

ASK: How did solving the easier problems make it easier to solve the harder problems? (it told me that the right approach was to subtract: number of people in line – position of the person in line)

Using off-by-one patterns to solve problems. Tell students that you are waiting in line to get on a roller coaster. You are 37th in line and you see your friend, who is 7th in line. ASK: How many people are between us? (students will likely say 30)
Draw on the board:

```
0                     7                                                                 37
```

SAY: Thirty is a good guess because, on a number line, the length of the part of the line between 7 and 37 is 30. Add 8 to the number line. ASK: If you are 7th in line and I’m 8th in line, how many people are between us? (none) SAY: But 8 – 7 is 1, not 0, so although subtracting gets us close to the right answer, it’s not exactly right. Let’s try to figure out what is going on.

**Exercises:** How many people are in between? Hint: Use a number line to solve.

a) the 7th and 8th person  
b) the 7th and 9th person  
c) the 7th and 10th person  
d) the 7th and 11th person  
e) the 7th and 12th person  
f) the 7th and 37th person

**Answers:** a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the right answer? (no) Did it give close to the right answer? (yes) How can you get the number of people between two people, given their positions in line? (the number is off by 1, so I can find the difference between the positions and then subtract 1 more)

How many people are between the 37th person and the 7th person? (29) How did starting with smaller numbers help? (it made the problem clearer) SAY: Sometimes, it’s easier to start by using smaller numbers rather than using what is given in the problem. Then you will see things that help you solve the harder problem. Now that you know the pattern, you can find the number of people between any two positions.

**Exercises:** How many people are in between?

a) the 8th and 78th person  
b) the 314th and 1000th person  
c) the 492nd and 613th person

**Answers:** a) 69, b) 685, c) 120

Write on the board:

```
A teacher tells her class to read pages 287 to 304 for homework.  
How many pages is that?
```

Have volunteers give you similar, simpler problems that you can solve first (for example, make the numbers smaller). Write down all the volunteers’ suggestions on the board.
**Exercises:** Solve all the simpler problems on the board. Do you see a pattern in your answers?

**Answers:** In all cases, you can find the number of pages by subtracting the smaller number from the bigger number and then adding 1. Have a volunteer tell you the pattern. (subtract the numbers and add 1) SAY: Now that you know the pattern, you can solve any problem of the same type.

**Exercises:** A teacher tells her class to read pages in a textbook for homework. How many pages of reading do the students need to do?

a) from 352 to 386  
b) from 298 to 314  
c) from 408 to 451

**Answers:** a) 35, b) 17, c) 44

Tell students that it can be helpful to examine the simpler problems in an organized way. Refer students to the problem about reading from pages 287 to 304. Write on the board:

<table>
<thead>
<tr>
<th>Read Pages</th>
<th>How Many Pages?</th>
</tr>
</thead>
<tbody>
<tr>
<td>287 to 288</td>
<td>2</td>
</tr>
<tr>
<td>287 to 289</td>
<td>3</td>
</tr>
<tr>
<td>287 to 290</td>
<td>4</td>
</tr>
<tr>
<td>287 to 291</td>
<td>5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>287 to 304</td>
<td>?</td>
</tr>
</tbody>
</table>

SAY: By being organized, you might find the pattern sooner. Patterns can be easier to see when you have something organized to look at, like a table or a diagram.

**Exercises:** Make several simpler problems until you see the pattern to do the harder problem. Organize the simpler problems.

a) A fence is made using 42 posts, each 1 m apart. How long is the fence?  
b) A fence is made using 34 posts, each 2 m apart. How long is the fence?

**Answers:** a) 41 m, b) 66 m

SAY: You can extend this type of problem to fences that go all the way around a field.

**Exercises:** A fence for a square field is made with posts 1 m apart, including a post at each corner. How many posts are needed for a field that is …

a) 10 m by 10 m? Hint: Start with a field that is 1 m by 1 m and then move on to 2 m by 2 m, 3 m by 3 m, and so on.  
b) 20 m by 20 m?

**Answers:** a) 40, b) 80

**NOTE:** For the following exercises, encourage students to predict the answer before checking.
Exercises: A square field is 20 m by 20 m. How many posts are needed if the posts are ...  
a) 1 m apart?  
b) 2 m apart?  
c) 4 m apart?  
d) 5 m apart?  
Bonus: 40 cm apart?  
Answers: a) 80, b) 40, c) 20, d) 16, Bonus: 200  

Using whole numbers instead of decimals to make a problem easier. Draw on the board:  

Tell students that you have two sticks. You know one stick’s length and the total length, but you want to know the length of the second stick. ASK: What makes this problem hard? (there are decimals) SAY: Let’s solve the problem approximately with whole numbers first. Erase the decimal part of the numbers on the board, as shown below:  

ASK: Approximately how long is the second stick? (11) How did you get that? (17 - 6 = 11)  
SAY: So you can do the problem the same way when the lengths include decimals. Instead of subtracting 6 from 17, you subtract 6.7 from 17.6. Write on the board:  

17.6  
- 6.7  

Have a volunteer complete the subtraction. (10.9) SAY: When we estimated with whole numbers, the answer was 11. ASK: Is the actual answer close to 11? (yes)  

Exercises: Find the missing length.  
a)  

b)
**Bonus:**

![Diagram of two sticks with lengths 4.2 and 6.5, and a gap of ? between them.]

**Answers:** a) 8.88, b) 2.58, Bonus: 2.7

**Focusing only on relevant information to make a problem simpler.** Remind students of the example of 6.7 + ? = 17.6 from earlier. SAY: I'm going to move these sticks around. Draw on the board:

![Diagram of two sticks with total length 17.6, one stick of length 6.7, and a gap of ? between them.]

ASK: How did I move the sticks? (slid one of them down) SAY: This second problem has a lot of extra information so it looks harder, but it actually has exactly the same answer as the other one, so you might as well do the easier one. The total length of the two sticks at the bottom is still 17.6, they are just not side by side anymore.

**Exercises:** Find what the ? stands for by making the problem into a simpler problem.

a) ![Diagram of a rectangle with dimensions 11 by 18 and a gap of ? in the middle.]

b) ![Diagram of a rectangle with dimensions 15.7 by 6.5 and a gap of ? in the middle.]

c) ![Diagram of a rectangle with dimensions 1.27 by 3.46 and a gap of ? in the middle.]

d) ![Diagram of a rectangle with dimensions 3 by 5.75 and a gap of ? in the middle.]

**Answers:** a) 7, b) 9.2, c) 4.73, d) 1.05
SAY: By pretending that the sticks are side by side, you turn the problem into an easier problem.

**Making a problem easier by emphasizing what is relevant.** SAY: We can look at a problem and focus on what matters most. For example, if you need to find a vertical side—straight up and down—then colour over all the vertical lines. If you need to find a horizontal side, colour over all the horizontal lines.

**Exercises:** Find what the ? stands for by making the problem into an easier problem.

a)          b) 

Exercises:

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<tbody>
<tr>
<td>13</td>
<td>15</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>

b)          

Exercises:

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<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>4.73</td>
<td>3.53</td>
<td>8.54</td>
<td>?</td>
</tr>
<tr>
<td>3.81</td>
<td>3.53</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Exercises:

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<tbody>
<tr>
<td>631</td>
<td>383</td>
<td>258</td>
<td>?</td>
</tr>
<tr>
<td>522</td>
<td>4450</td>
<td>1163</td>
<td></td>
</tr>
</tbody>
</table>

Exercises:

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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.56</td>
<td>1.83</td>
<td>2.53</td>
<td>?</td>
</tr>
<tr>
<td>3.41</td>
<td>2.88</td>
<td>2.16</td>
<td>1.51</td>
</tr>
</tbody>
</table>

Exercises:

<p>| | |</p>
<table>
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<th></th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Answers: a) colour vertical, ? = 5; b) colour horizontal, ? = 8.65; c) colour horizontal, ? = 2567; d) colour vertical, ? = 1.28

Point out to students that by colouring over all the horizontal or vertical lines, they changed the problem into an easier problem.

Finding perimeter without knowing all the side lengths. Remind students that to find the perimeter of a shape, we add up the lengths of all the sides. Draw on the board:

SAY: I want to find the perimeter of this shape. It looks like a hard problem, because there are a lot of missing side lengths. Ask a volunteer to mark three sides that are missing lengths. (the two bottom horizontal sides and the right side) SAY: There are two kinds of sides in this shape: horizontal sides and vertical sides.
ASK: How long is the top side? (20) How long are the two bottom sides put together? (20)
How do you know? (put together they are the same length as the top side) How long are the
two sides on the left of the shape? (5 and 3) How long is the side on the right? (8) How do you
know? (it’s the same as the two left sides put together) Write on the board:

Horizontal edges add to _____   Vertical edges add to _____

Perimeter is _____ + _____ = _____

Have volunteers fill in the blanks. (40, 16, 40 + 16 = 56)

Exercises: Find the perimeter of the shape. All measurements are in centimetres.

a)  

b)  

Answers: a) 48 cm, b) 560 cm

SAY: You can find the area of these shapes by taking away rectangles, finding the area of each
rectangle you took away, and subtracting the results from the area of the big rectangle.

Exercises:
1. Find the area of each shape from the previous exercises.
   Answers: a) 128 cm², b) 13 262 cm²

2. a) Find the perimeter.

Bonus: Is there enough information to find the area of this shape? Explain.
   Answers: a) 34; Bonus: no, because to find the area you need the length of each part

Problem Bank
1. When everyone in Tom’s class stands in line, Tom is 14th in line and 11th from the end of the
line. How many people are in the class?
   Answer: 24

2. There are 126 people in line. How many people are behind the 94th person?
   Answer: 32
3. Make several simpler problems until you see how to do the harder problem.
   a) A fence is made using 53 posts, each 3 m apart. How long is the fence?
   b) A fence is made using 61 posts, each 2.5 m apart. How long is the fence?
   **Answers:** a) 156 m, b) 150 m

4. How many posts are needed to make the fence?
   a) A fence is 47 m long, with posts at 1 m intervals.
   b) A fence is 100 m long, with posts at 2.5 m intervals.
   c) A fence is 84 m long, with posts at 3.5 m intervals.
   **Answers:** a) 48, b) 41, c) 25

5. A fence for a square garden is made with posts 1.5 m apart, including a post at each corner.
   How many posts are needed for the garden? Hint: Start with a garden that is 1.5 m by 1.5 m and then move on to 3 m by 3 m, 4.5 m by 4.5 m, and so on.
   a) The garden is 12 m by 12 m.
   b) The garden is 21 m by 21 m.
   **Answers:** a) 32, b) 56

6. Predict each answer before checking. A square field is 30 m by 30 m. How many posts are needed if the posts are ...
   a) 1 m apart? b) 2 m apart?
   c) 1.5 m apart? d) 2.5 m apart?
   **Bonus:** 60 cm apart?
   **Answers:** a) 120, b) 60, c) 80, d) 48, Bonus: 200

7. Cut out the strips and circles from **BLM Fraction Strips and Circles** (you may cut the line down to the centre of the circles). Estimate to colour the given amount. Use folding to check your estimate.
   a) one fifth of a strip of paper, starting from the left
   b) two fifths of a strip of paper, starting from the left
   **Hint:** Use your answers to parts a) and b) to help you determine a strategy for parts c) and d).
   Hold the circle so that the cut line is at the top.
   c) one fifth of a circle, going clockwise starting from the top
   d) two fifths of a circle, going clockwise starting from the top

8. Each line segment is 1 m long. What is the total length of this path? Look for a quick way to answer.

   ![Path Diagram]

   **Answer:** 18 vertical metres plus 17 horizontal metres = 35 metres altogether
9. a) Find the perimeter.

```
3.6
8.2
1.1
5.4
```

b) Is there enough information to find the area of this shape? Explain.

**Answers:** a) 36.6; b) no, we don’t have the side length for the small rectangles

10. Each shape was made by placing a small square on top of a large square. All measurements are in centimetres.
   a) Find the perimeter of each shape.

   i) 11
   ii) 11
   iii) 11
   iv) 11

b) Make a table with the headings “Size of Smaller Square” and “Total Perimeter.” Use the pattern from part a) to solve the problems.

   i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?
   ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?

**Answers:** a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm

11. a) Convert the measurements in metres to centimetres. Hint: 1 m = 100 cm.

   i) 2 m = _____ cm   ii) 3 m = ____ cm   Bonus: 183 m = _____ cm

b) Find the perimeter in centimetres.

   i) 320 cm
   ii) 75 cm

**Answers:** a) i) 200, ii) 300, Bonus: 18 300; b) i) 1040 cm, ii) 2050 cm

12. Find the missing length.

```
38 m
? 20 m
50 m
```
Answer: 8 m

13. In the figures below, each square has a side length of 1 m.
   a) Complete the table for the figures.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
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<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   Figure 1 Figure 2 Figure 3

   b) What is the perimeter of the 10th figure?
   c) Which figure has perimeter 48 m?

   **Answers:** a) 8, 12, 16; b) 40 m; c) 12th figure
Fraction Strips and Circles
Extended Problem: Class Art Show

Materials:
BLM Class Art Show (pp. 38–39)

Extended Problem: Class Art Show. Give students BLM Class Art Show. Tell students the context for the extended problem: You are holding an event for parents to view posters created by your class. The posters will be hung in the school hallway. Tell students to use diagrams in their answers.

Answers: 1. a) 10 cm, b) 155 cm; 2. a) 104 cm, b) 12 posters
Class Art Show (1)

1. Each student makes a poster that is 22 cm tall and 17 cm wide. You put two pins in the poster 1 cm below the top of the poster and 1 cm from each edge.

![Diagram of a poster with two pins placed 1 cm below the top and 1 cm from each edge.]

The centre of the posters should be at eye level so that they can be seen easily. For adults, eye level is about 145 cm above the floor.

a) How high is each pinhole above the level of the centre of the poster?

b) How high above the floor is each pinhole?
Class Art Show (2)

2. Two doors along the school hallway are 3 m apart. Posters on the wall between the doors should be 8 cm apart from each other. The posters should also be centred so that the posters nearest to the doors are each the same distance from a door.

   a) If you put 4 posters between the two doors, how far will the posters nearest to the doors be from the door’s edge?

   b) What is the maximum number of posters you can hang between the two doors?
PS5-9 Using Tape Diagrams

Teach this lesson after: 5.2 Measurement

Goals:
Students will use tape diagrams to solve multistep word problems involving all four operations and fractions.

Prior Knowledge Required:
Can interpret statements involving “times as many”
Can represent fractions using fraction bars
Can divide to find a fraction (with numerator 1) of a whole number amount
Can determine the volume of a right rectangular prism given its dimensions (for Extended Problem)
Knows that 1 litre converts to 1000 millilitres (for Extended Problem)
Can recognize “for each” as recognizing a multiplicative relationship (for Extended Problem)

Vocabulary: tape diagram

Materials:
BLM Swimming Pool (pp. 52–53, see Extended Problem)

Drawing a diagram for a “times as many” situation. Tell students that Kim and Rob have some stickers. Write on the board:

Kim has three times as many stickers as Rob.

SAY: I want to draw a diagram to represent this situation. ASK: Who has more stickers, Kim or Rob? (Kim) Draw a small rectangle on the board and explain that this rectangle represents Rob’s stickers. Label the bar as shown below:

Rob’s stickers

ASK: How can we show that Kim has three times as many stickers as Rob? Accept all reasonable answers. Then explain that you are going to use a specific way to draw a diagram. You will make a bar that contains three of Rob’s bar of stickers. Finish the picture on the board as shown below and keep it for future reference:

Rob’s stickers

Kim’s stickers

Explain that this type of diagram is called a tape diagram.
SAY: Tristan has four times as many nickels as dimes. Draw on the board:

A. dimes       B. dimes
nickels

C. nickels     D. nickels
dimes               dimes

ASK: Which of these tape diagrams fit Tristan’s situation? (A and C both work, but B and D do not) Have volunteers explain why B and D don’t work. (B shows more dimes than nickels; D shows five times as many nickels as dimes) ASK: How do you know that the short bar should be the number of dimes? (because there are more nickels than dimes) If you want to show that David is twice as old as Karen, whose age would be the shorter bar? (Karen’s) Why? (because David is older, so his bar will be longer)

Exercises: Draw a tape diagram for the situation.

a) Bethany is three times as tall as her baby brother.
b) Pria’s full name (including her last name) is four times as long as Joshua’s full name.
c) There are eight times as many students in the school as in our class.
d) The library is three times as far from my home as the school is.
e) A book is twice as thick as a notebook.

Answers:

a) Bethany's height       b) Pria's name
Brother's height           Joshua's name

c) Students in class       d) Distance to library
Students in school         Distance to school

e) Thickness of book       Thickness of notebook

Finding the length of the bars when the smaller part is given. Write on the board:

Ivan has five times as many crayons as pencils. Ivan has four pencils.

crayons                pencils

ASK: What information do we have that is not shown on the tape diagram? (Ivan has 4 pencils)
SAY: Let's show that on the tape diagram. Write “4” in Ivan’s “pencils” bar, as shown below:

<table>
<thead>
<tr>
<th>crayons</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>pencils</td>
<td>4</td>
</tr>
</tbody>
</table>

SAY: Each of the crayon blocks is also four, so let's write that in the diagram. The final picture should look like this:

<table>
<thead>
<tr>
<th>crayons</th>
<th>4</th>
<th>4</th>
<th>4</th>
<th>4</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>pencils</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: How many crayons does Ivan have? (20) SAY: Ivan has five blocks of four crayons. Write on the board:

\[5 \times 4 = 20\]

**Exercises:** Draw a tape diagram and find the length of the bars.

a) There are six apples on the table. There are twice as many bananas as apples.
b) A car holds five people. A van holds three times as many people.
c) Dan’s apartment building is three storeys high. Elona’s building is five times as high as Dan’s.
d) Christoph is five years old. Sasha is four times as old as Christoph.

**Bonus:** A sparrow has four eggs in its nest. A duck has three times as many eggs in its nest as the sparrow. An ostrich has five times as many eggs in its nest as the sparrow.

**Answers:**

| a) apples | 6 | 6 | b) car | 5 | 5 |
| bananas  | 6 | 6 | 12     | van  | 5 | 5 | 5 | 15 |
| c) Dan’s building | 3 | 3 | d) Christoph | 5 | 5 |
| Elona’s building | 3 | 3 | 3 | 3 | 3 | 15 |
| Sasha | 5 | 5 | 5 | 5 | 20 |

**Bonus**

| Sparrow’s nest | 4 | 4 |
| Duck’s nest    | 4 | 4 | 4 | 12 |
| Ostrich’s nest | 4 | 4 | 4 | 4 | 4 | 20 |

**Using tape diagrams to solve problems when the larger part is given.** Write on the board:

Marla has four times as many stickers as Amir.
Have a volunteer draw the bars for the situation, as shown below:

Marla has four times as many stickers as Amir.

Amir

Marla

ASK: How many blocks are in Marla’s bar? (4) Write “Marla has 20 stickers.” on the board. SAY: Marla’s bar has four blocks and together the blocks represent 20 stickers. Show this on the picture:

Marla has four times as many stickers as Amir. Marla has 20 stickers.

Amir

Marla

20

ASK: How many stickers does each bar represent? (5) SAY: Four bars represent 20 stickers, so one bar represents five stickers. ASK: How many stickers does Amir have? (5) How do you know? (because Amir has 1 bar) Write “5” in each bar in the diagram to emphasize this.

Exercises: Draw bars and find the length of each block for the situation.

a) There are six apples on the table. There are twice as many apples as pears. How many pears are there?
b) A mini-bus holds 16 people. The mini-bus holds twice as many apples as a van. How many people can the van hold?
c) Jun’s apartment building is 30 storeys high. Jun’s building is five times as high as Cathy’s building. How tall is Cathy’s building?
d) Edmond is 14 years old. Edmond is seven times as old as Nina. How old is Nina?

Bonus: A sugar pine cone is 45 cm long. It is three times as long as an eastern white pine cone. The sugar pine cone is nine times as long as a jack pine cone. How long is the eastern white pine cone? How long is the jack pine cone?

Answers: a) 3 pears; b) 8 people; c) 6 storeys; d) 2 years old; Bonus: eastern white pine cone = 15 cm long, jack pine cone = 5 cm long

Finding the size of a single block when the difference is given. Write on the board:

Rick has four times as many white shirts as black shirts.

Have a volunteer show the situation using a tape diagram, then continue writing on the board:

He has 18 more white shirts than black shirts.
ASK: What does this mean on the tape diagram? (the longer bar represents 18 more than the shorter bar) Point to the extra part and SAY: This extra part here represents 18. Show this on the tape diagram:

White shirts

Black shirts 18

ASK: How many extra blocks are there in the longer bar? (3) How many shirts do those three bars represent altogether? (18) So how many shirts does each block represent? (6) What division sentence can you write to show that? (18 ÷ 3 = 6) Write “6” in each block. SAY: All the blocks represent six shirts. ASK: How many black shirts does Rick have? (6) How many white shirts does he have? (24) How did you get that? (4 × 6 = 24)

Exercises: What is the size of each block?

a) b) c) d)

Answers: a) 5, b) 8, c) 7, d) 11

Finding the size of the block when the total is given. SAY: Sometimes you are given the total instead. For example, Lewis might have 20 shirts altogether. Draw on the board:

White shirts Black shirts 20

ASK: How many blocks represent 20 shirts? (5) So how many shirts does one block represent if five blocks represent 20 shirts? (4) Write “4” in each block. SAY: When each block represents four shirts, five blocks represent 20 shirts.

Exercises:

1. What is the size of one block?

a) b) c) d)

Answers: a) 3, b) 4, c) 4, d) 12
2. Either the total is given or the difference is given. What is the size of one block?

   a) \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \quad 55
   \]
   b) \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \quad 28
   \]
   c) \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \quad 48
   \]
   d) \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \quad 130
   \]

   **Answers:** a) 11, b) 14, c) 12, d) 10

**Solving problems with the difference, the total, or one part given.** SAY: Sometimes you are given the difference, or the total, or one part. Write on the board:

Raj has five times as many t-shirts as sweaters.

A. Raj has 60 t-shirts.
B. Raj has 60 t-shirts and sweaters altogether.
C. Raj has 60 more t-shirts than sweaters.

<table>
<thead>
<tr>
<th>t-shirts</th>
<th>sweaters</th>
</tr>
</thead>
</table>
| \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] | \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] |
| t-shirts | sweaters |
| \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] | \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] |

Point out that each diagram shows five times as many t-shirts as sweaters. Have volunteers show what the 60 is representing in each situation, as shown below:

<table>
<thead>
<tr>
<th>t-shirts</th>
<th>sweaters</th>
</tr>
</thead>
</table>
| \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] | \[60\] |
| t-shirts | sweaters |
| \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] | \[60\] |
| t-shirts | sweaters |
| \[
   \begin{array}{c}
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \] | \[60\] |

ASK: Which picture shows being given one part? (A) Which picture shows being give the difference? (C) Which picture shows being given the total? (B) SAY: Now that you know what 60 represents, you can decide what each bar represents. Pointing to each situation (A, B, and C) in turn, ASK: What does each bar represent? (12 in A, 10 in B, and 15 in C) Work through the first two exercises below as a class, then have students work individually.
Exercises: Solve.

a) Mary saved three times as much money as Sun. Sun saved $18 less than Mary. How much money did they save together?

b) Cam and Amy put all their money together to buy a gift for their grandmother. They have $60 together. Cam has twice as much money as Amy has. How much money does each of them have?

c) Abella is three times as tall as her baby brother Jax. She is 80 cm taller than Jax. How tall is Jax? How tall is Abella?

d) Zara’s full name is four times as long as Ronin’s. Ronin’s full name is 54 letters shorter than Zara’s. How long is each full name? Hint: Take the full name into account; Ronin’s name is not five letters long.

e) The number of students in the school who are not in Grade 5 is eight times as large as the number of students in Grade 5. There are 248 students in the school who are not in Grade 5. How many students altogether are in the school?

Answers: a) $36; b) Amy had $20, Cam had $40; c) Jax is 40 cm tall, Abella is 120 cm tall; d) Ronin’s full name is 18 letters long, Zara’s full name is 72 letters long; e) 279 students in the school in total

Problem Bank

1. The library is four times as close to Emma’s home as the school. Emma walks from school to her home, then goes to the library. This makes a walk of 15 blocks. How far from Emma’s home are the school and the library?

Answers: The library is 3 blocks away from Emma’s home and the school is 12 blocks away

2. A number is five times as large as a smaller number. If you add the two numbers together, you get 54. What are the numbers?

Answers: 9 and 45

3. Armand reads the same number of pages every school day and twice as many pages every weekend day. He finished a book of 108 pages in a week. How many pages does he read on Monday? How many pages does he read on Sunday?

Answers: 12 pages on Monday, 24 pages on Sunday

4. Kim has \( \frac{3}{5} \) as many blue shirts as black shirts. Which diagram shows the situation correctly? What is wrong with the other two?

\[
\begin{array}{ccc}
\text{Blue shirts} & A. & B. & C. \\
\text{Black shirts} & & &
\end{array}
\]

Answers: C shows the situation correctly. A shows 3/4 as many blue shirts as black shirts, and B shows 3/5 as many black shirts as blue shirts.
5. Draw a diagram to solve the problem.
   a) Jane had $36. She spent $\frac{3}{4}$ of her money on a pair of shoes. How much money does she have left?
   b) Kyle spent $\frac{2}{5}$ of his money on a toy. He has $15 left. How much did the toy cost?
   c) Alice spent $\frac{2}{5}$ of her money on a poster that cost $8. How much money did she have before she bought the poster?
   d) Kate spent $\frac{2}{5}$ of her money on a shirt and a hat. The shirt cost $18 and the hat cost $6. How much money did Kate have at first?

**Solutions:**

\begin{itemize}
  \item \textbf{a)} $36$  
    \begin{center}
      \framebox(2cm){$9$} \hspace{1cm} \framebox(2cm){$27$}
    \end{center}
    shoes \hspace{1cm} left over = $9$
  
  \item \textbf{b)} $15$
    \begin{center}
      \framebox(2cm){$5$} \hspace{1cm} \framebox(2cm){$10$}
    \end{center}
    toy = $10$
  
  \item \textbf{c)} total before = $20$
    \begin{center}
      \framebox(2cm){$4$} \hspace{1cm} \framebox(2cm){$16$}
    \end{center}
    Poster = $8$
  
  \item \textbf{d)} The shirt and hat together cost $24$, so each block is $24 \div 2 = 12$. Five blocks together is $5 \times 12 = 60$
    \begin{center}
      \framebox(2cm){$12$} \hspace{1cm} \framebox(2cm){$48$}
    \end{center}
    total = $60$
    shirt + hat = $24$
\end{itemize}

6. Eric has some eggs. He uses $\frac{3}{7}$ of them to make pancakes and $\frac{1}{2}$ of the remainder to make sandwiches. Now Eric has six eggs left. How many eggs did he use to make pancakes? How many eggs did he have at first?

**Answer:** Eric used 9 eggs to make pancakes and he had 21 eggs at first.

7. Ray had 30 stickers. He gave $\frac{2}{5}$ of his stickers to his brother and $\frac{1}{2}$ of the rest to his friend. How many stickers did Ray’s brother get? How many stickers are left?

**Answer:** Ray’s brother got 12 stickers. At the end, there are 9 stickers left.
8. The next time Ray has stickers, he decides to give \( \frac{2}{5} \) of his 30 stickers to his brother and \( \frac{5}{6} \) of the remainder to his friend.
a) How many stickers did Ray’s brother get?
b) How many stickers did Ray’s friend get?
c) How many stickers are left?

**Solution:**

\[
\begin{align*}
\text{total stickers} &= 30 \\
\text{remainder} &= 18 \\
\text{brother} &= 12 \\
\text{friend} &= 15 \\
\text{left} &= 3
\end{align*}
\]

9. Nora has some stickers. She colours \( \frac{1}{4} \) of them red and \( \frac{2}{5} \) of the remainder green.

If Nora doesn’t colour 9 stickers, how many stickers does she have in total?

**Answer:** 20

10. Evan spent \( \frac{3}{5} \) of his money on a book and \( \frac{3}{4} \) of the remainder on some music. Evan has $4 after he paid for the book and the music. How much money did Evan have initially?

**Solution:**

\[
\begin{align*}
\text{initial money} &= $40 \\
\text{left after book} &= $16 \\
\text{book} &= $24 \\
\text{left after book} &= $16 \\
\text{music} &= $12 \\
\text{left after music} &= $4
\end{align*}
\]

11. Shelly received some money for her birthday. She donated \( \frac{1}{5} \) to charity, and she saved \( \frac{2}{3} \) of the remainder in her savings. Of what was left, \( \frac{1}{4} \) was used to buy a gift card for an ice cream store. She used the rest of the money to buy 3 books for $7 each, a T-shirt for $10, and a basketball for $8. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each other part (donation, savings, gift card)?

**Answers:**

- the books are $7 each, the T-shirt is $10, the basketball is $8, so she spent $39 for purchases, plus $13 for the ice cream store gift card; $104 went to savings; $39 was given to charity; total birthday gift was $195.

12. Randi reads 10 pages of a book on Saturday and she reads \( \frac{3}{4} \) of the rest of the book on Sunday. She still has 17 pages to read. How many pages are in the book?

**Answer:** 78 pages
13. A convenience store has some ice cream treats. It sells \( \frac{2}{5} \) of them on Friday, \( \frac{1}{4} \) of the remainder of Saturday, and \( \frac{2}{3} \) of the rest on Sunday. The store has 30 ice cream treats left by the end of the day on Sunday.

a) How many ice cream treats did the store have initially?

b) On which day did the store sell the most ice cream treats?

**Answers:** a) 200; b) Friday, 80
Extended Problem: Swimming Pool

Materials:
BLM Swimming Pool (pp. 52–53)

Extended Problem: Swimming Pool. Give students BLM Swimming Pool. Tell students that they will be doing an extended problem involving a swimming pool and volume.

Answers: 1. 96 m³; 2. 480 mL; 3. about $4, because 480 mL is about half of 1 L; 4. a) $12, b) $40, c) $16, d) 2 boxes, e) 4 times
Swimming Pool (1)

Jay's pool is in the shape of a rectangular prism. It is 8 m long, 6 m wide, and 2 m deep.

1. Find the capacity of the pool in cubic metres.

2. Jay disinfects the water in the pool every day by adding 5 mL of chlorine for each cubic metre of water in the pool. How many millilitres of chlorine does Jay need to put in the pool when the pool is full?

3. Each 1 L bottle of chlorine costs $8.00. About how much does it cost each time Jay adds chlorine to a full pool? Explain your estimate.
Swimming Pool (2)

4. Jay spent $\frac{2}{5}$ of his money at a pool store to buy some chlorine and half of the remainder on a pool toy. Jay has $12.00 after he paid for the chlorine and the pool toy.
   a) How much did Jay pay for the pool toy?

b) How much money did Jay have when he went to the store?

c) How much did Jay pay for the chlorine?


e) How many times can Jay disinfect the water in the pool with the amount of chlorine he bought?
PDM5-13
The Mean

SAY: Suppose 5 people picked apples and wanted to share their apples so that everyone had the same number. They might pass apples to each other until everyone had the same number or they might put all of the apples in a bin and hand them out one at a time. Either way, mathematicians would say that the people took the mean, or average, of the number of apples picked.

Suppose that Andrew picked 6 apples, Brian picked 2 apples, Cindy picked 4 apples, Dara picked 1 apple and Erin picked 2 apples. In total, the 5 friends picked 15 apples. Lay out or draw 15 blocks (to represent the 15 apples) in 5 groups (to represent the 5 people):

Ask a volunteer to move blocks from one group to another until all the groups are equal in number. How many blocks are now in each group? (3) The mean of the set of numbers 6, 2, 4, 1, 2, is 3.

Demonstrate another way of finding the mean using the same 15 blocks. Draw a circle for each of the 5 friends, put all the blocks in a pile, and ask a volunteer to distribute the blocks evenly into the 5 circles. Find the mean of a few more sets using either method above. (EXAMPLES: 8, 5, 2; 7, 2, 5, 2; 1, 3, 9, 3)

ASK: How can we find the mean of a set of numbers without using pictures or blocks? How many groups did we divide the blocks into? How did we find that number from the set of numbers? (The number of groups is the number of data values.) How many blocks do we have altogether? How can we find that total from the data values? (Add them together.) How can we find the number of blocks in each group if we know the total number of blocks and the number of groups? (Divide.) Demonstrate with some of the data sets used in the lesson. For EXAMPLE:

\[
\text{Mean} = \frac{\text{Total}}{\text{Number of Groups}} = \frac{(6 + 2 + 4 + 1 + 2)}{5} = \frac{15}{5} = 3
\]

Write several sets of numbers on the board and have students practise finding the mean without using pictures. EXAMPLES: 4, 7, 2, 1, 1; 10, 12, 3, 7
Bonus
0, 8, 15, 16, 16, 19, 24

Invite volunteers to share their answers. Occasionally use the term data values when referring to
the numbers so that students don’t think of them only as blocks.

Extensions
1. Find the mean. Write your answer as a mixed fraction.
   a) 3, 4, 6, 8  
   \[ \text{mean} = \frac{21}{4} = 5 \frac{1}{4} \]
   \[ \text{sum of data values} = 3 + 4 + 6 + 8 = 21 \]
   \[ \text{number of data values} \]
   b) 4, 7, 8
   c) 0, 2, 3, 6, 7

2. a) The data set 1, 4, 10 has mean $15 \div 3 = 5$. Add the following data values to this set and
decide if the mean increased, decreased, or stayed the same.
   i) New data value: 3  
   New mean: $18 \div 4 = 4 \frac{1}{2}$  
   The mean decreased.
   ii) New data value: 7  
   New mean: ______  
   The mean ______
   iii) New data value: 5  
   New mean: ______  
   The mean ______

   b) Find the mean of each set, then add a data value so that the mean stays the same.
   HINT: If your first guess increases the mean, try a lower value; if your first guess decreases
   the mean, try a higher value.
   i) 2, 6  
   Mean: ______  
   Added value: ______
   ii) 2, 6, 7  
   Mean: ______  
   Added value: ______
   iii) 3, 4, 5, 12  
   Mean: ______  
   Added value: ______

c) Look at your answers to part b). How can you know what data value to add if you want
to keep the mean the same?

d) Find the mean.
   i) 1, 4, 10  
   ii) 1, 4, 5, 10  
   iii) 1, 4, 5, 5, 10  
   iv) 1, 4, 5, 5, 10
PDM5-14
Finding the Mean

GOALS
Students will understand that the differences between the mean and the values below the mean balance out the differences between the mean and the values above the mean.

PRIOR KNOWLEDGE REQUIRED
Addition
Subtraction
Multiplication
Division
The mean as the sum of the data values divided by the number of data values.

VOCABULARY
sum
data values
mean

Ask students to find the mean of the data set 2, 7, 6 in two different ways:

Using division:

\[
\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}} = \frac{15}{3}
\]

Using a picture:

\[\begin{array}{c}
2 \quad 7 \quad 6 \\
\end{array}\]

Add a horizontal line in the picture to show where the mean is. **ASK:** When you moved some blocks to other piles, which piles did you take the blocks from—the piles with more blocks than the mean or fewer blocks than the mean? Which piles did you move them to—the piles with more blocks than the mean or the piles with fewer blocks than the mean? **ASK:** Why does that make sense? Explain to the students that if all the piles are going to have the same number of blocks, they need to add blocks to the piles that have less than the mean and take blocks away from the piles that have more than the mean. **ASK:** Is the number of blocks you took away from some piles equal to the number of blocks you added to other piles? How do you know? Did you change the total number of blocks by moving some of them?

Have students calculate the mean for the following set using division: 3, 0, 4, 6, 2. Then draw a picture for the set and add a horizontal line to show the mean:

\[\begin{array}{c}
3 \quad 0 \quad 4 \quad 6 \quad 2 \\
\end{array}\]

**ASK:** How many blocks are above the horizontal line? How many spaces are below the line? If you moved the blocks above the mean to the spaces below the mean, how many would you have in each pile? (3, the mean)
Then draw a picture in which the horizontal line representing the mean is in the wrong place.

**EXAMPLE:**

```
2 0 4 8 1
```

**ASK:** Is the line in the right place—does it show the mean? How do you know? Right now, there are more spaces below the line than blocks above the line. How can we move the line so that there are fewer spaces below the line and more blocks above the line—should we move the line up or down? (down) How far? (one “row”) Have a volunteer do this and then ask the class: Is the line in the right place now? (yes) How do you know? (There are 5 blocks above the line and 5 spaces below the line, so moving the 5 blocks to the 5 spaces will make all the piles have the same number of blocks.)

Draw more pictures on the board. Have volunteers guess where the mean is by drawing a line, and then have students check the volunteer’s guess independently by counting blocks above the line and spaces below the line. Are these numbers the same? Adjust the line accordingly when necessary.

**Extensions**

1. The data set is 3, 4, 7, 8, 8.
   a) Find the mean.
   b) Is the data value 7 above or below the mean?
   c) Will removing 7 from the data set increase or decrease the mean? Find the mean of the data set 3, 4, 8, 8 to check your prediction.

2. For each data set below:
   a) Find the mean.
   b) Remove the circled data value and predict whether the mean will increase, decrease or stay the same.
   c) Check your answer by calculating the new mean.

   i) 0, 0, 1, 8, 8, 8, 9, 10

   ii) 4, 4, 4, 5, 8

   iii) 1, 1, 2, 3, 9, 20, 28

3. Sonia’s math test scores (out of 10) were: 2, 5, 5, 6, 7, 8, 9, 9, 10.
   a) What is the mean of her test scores? (68 ÷ 10 = 6.8)
   b) Sonia’s teacher decides not to count her lowest score. Predict how removing this data value will change her mean and explain your answer. (This change will increase her average because we are removing a data value that is lower than the average.)
   c) What is the new mean of Sonia’s test scores? Write your answer as a mixed fraction in lowest terms. (66 ÷ 9 = 7 2/3 = 7 1/3)
d) Sonia's dad said he would buy her tickets to see her favourite rock group if she got an average of at least 7 on her math tests. Will Sonia get her tickets? (Yes.)

4. (Grade 5 Atlantic Curriculum)

a) Ten people work in an office. They get paid different salaries depending on what job they do.

- Sales person: $25 000 per year
- Secretary: $20 000 per year
- Clerk: $17 500 per year

If there are 5 salespeople, 3 secretaries and 2 clerks, find the mean salary.

Predict whether the mean will increase or decrease if:

i) a secretary retires
ii) a clerk retires
iii) 2 salespeople are hired

Then check your predictions.

b) Eleven people work in an office. They get paid different salaries for different jobs.

- Salesperson: $18 per hour
- Secretary: $17 per hour
- Bookkeeper: $16 per hour
- Clerk: $12 per hour

If there are 5 salespeople, 2 secretaries, 1 bookkeeper and 3 clerks, find the mean salary.

Predict whether the mean will increase or decrease if:

i) a secretary retires
ii) a salesperson retires
iii) a bookkeeper is hired
iv) 2 new clerks are hired
v) a clerk retires
vi) a secretary is hired
vii) BONUS: 2 secretaries and a clerk are hired.

Then check your prediction.
PDM5-15
Mean (Advanced)

Show students the data set: 80, 86, 82, 86, 86. Tell students that you want to find the mean without actually adding the data values. Tell them that you guess the mean to be 83 because all the data values are between 80 and 86.

**ASK:** Do you think my guess is reasonable? Is my guess too high or too low? How can I check without actually calculating the mean?

Take a few suggestions. If necessary, guide students to the answer with these **PROMPTS:** If 83 is the mean, how many spaces below or how many blocks above it would each data value have if we were to pile blocks? (Reading the 5 data values in the order given above: 3 spaces below, 3 blocks above, 1 space below, 3 blocks above, and 3 blocks above, for a total of 3 + 1 = 4 spaces below and 3 + 3 + 3 = 9 blocks above.) Are there more spaces below or blocks above the predicted mean? (There are more blocks above the predicted mean.) Is the mean too high or too low? How should we move the mean—up or down? (We need more blocks below the line, so we should move the line up.) Have students find the spaces below and the blocks above the new mean, 84, for each data value. (4 below, 2 above, 2 below, 2 above, 2 above, for a total of 4 + 2 = 6 below and 2 + 2 + 2 = 6 above) Since the number below is equal to the number above, the mean is indeed 84. Ask students to add the 5 data values and divide by 5 to check their answer. Did they get the same answer both ways? Ask students think about which is easier: adding 4 + 2 and 2 + 2 + 2 or adding the 5 data values and dividing by 5.

Find the mean of another set using a similar method. First make an educated guess, then write the difference between each data value and the mean above or below each number (depending on whether the data value is greater than or less than the mean).

**EXAMPLE:**
Set: 173, 180, 172, 176, 174
Mean (guess): 175

<table>
<thead>
<tr>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>173</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Total above = 5 + 1 = 6
Total below = 2 + 3 + 1 = 6

The total differences above and below the mean are equal, so the mean is indeed 175.

Have your students practise finding the mean of various data sets by guessing and checking rather than calculating. They should write addition statements to verify their guesses.

**EXAMPLES:**

a) 56, 68, 69, 59, 67, 65 (Mean = 64 because 8 + 5 = 4 + 5 + 3 + 1)
b) 45, 46, 47, 45, 45, 52, 46, 50 (Mean = 47 because $2 + 1 + 2 + 2 + 1 = 5 + 3$)

c) 128, 131, 127, 129, 136, 131, 128 (Mean = 130 because $2 + 3 + 1 + 2 = 1 + 6 + 1$)

Students can double-check their answers by adding the data values (with a calculator) and then dividing the sum by the number of data values.

Now show students how they can use addition statements to find sets with a given mean. Write the addition statement $1 + 1 = 2$ and tell students that you would like to build a data set with mean 4 using this addition statement. Now show on the board:

```
Spaces below the mean = Blocks above the mean
1 + 1 = 2
```

Draw a line to indicate where the mean is, then put in the spaces below the mean and the blocks above the mean. Then fill in the rest of the blocks below the line:

```
|   |   |   |
+---+---+---
|   |   |   |
```

**ASK:** What data set did I create? (3, 3, 6) How did I know the first data value was 3? How did I know the last data value was 6? Ask students to add the data values independently and divide by the number of data values. Is the mean 4?

Have a volunteer draw a picture to find the set with the addition statement $2 = 1 + 1$ and mean 5:

```
Spaces below the mean = Blocks above the mean
2 = 1 + 1
```

The data set is 3, 6, 6. Students should check that this data set does indeed have mean 5.

Then ask the class to find other data sets with mean 5 using the following number sentences, where the numbers to the left of the equal sign represent “spaces below” and the numbers to the right represent “blocks above.”

```
a) 2 + 3 = 1 + 4
b) 1 + 4 = 2 + 3
c) 1 + 1 + 2 = 4
d) 4 = 1 + 1 + 2
```

**ASK:** How many data values are in each data set? (4) Why did that happen? (Because there are 4 numbers in each addition statement and each number corresponds to a data value.) How many data values would there be in the data set for this number statement: $1 + 1 + 1 + 1 + 1 + 2 = 3 + 2 + 2$? (9)

**ASK:** Could you use the addition statement $4 + 7 = 3 + 3 + 3 + 2$ to build a data set with mean 5? (No—you can’t have 7 spaces below the 5 unless you use negative numbers.) Can you use $3 + 3 + 3 + 2 = 4 + 7$ to build a data set with mean 5? (Yes, the data values can be greater than the mean by any amount.)

Challenge students to build their own data sets with mean 5 by coming up with their own addition statements.

**Bonus**

Specify the number of data values to make it harder.
Use the following sample test grades to illustrate how the mean helps us to analyze data:

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alix</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Sivan</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Marco</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Bryan</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Helen</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Parvati</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

Have students find the mean grade for each test. **ASK:** On which test did students do better overall? Sivan got the same mark on both tests, but her teacher said she did better on English than Math. Why? (She is at the mean for English but below the mean for Math.)

**Extensions**

1. Teach students another way to find a set of 3 numbers with a mean of 5. For example, they might start with the data set 5, 5, 5, and then add and subtract the same number from the data values. Every time you add 1 to a data value, you must subtract 1 from a different data value. One sequence of data sets with mean 5 obtained in this way is:
   - 5, 5, 5
   - 4, 5, 6
   - 3, 6, 6
   - 2, 7, 6
   - 2, 8, 5
   - 3, 8, 4

2. The mean of a set is 10 and the data values are 2, 19, 7, 4, 15 and ____. What is the missing number? Invite students to solve the problem two different ways.

   **Solution A:**
   
<table>
<thead>
<tr>
<th>Data Set:</th>
<th>2, 19, 7, 4, 15</th>
<th>Mean = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>5</td>
<td>Total value above = 9 + 5 = 14</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>Total value below = 8 + 3 + 6 = 17</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

   Since 9 + 5 = 14, there are 14 blocks above 10. Since 8 + 3 + 6 = 17, there are 17 spaces below 10. We need 3 more spaces above 10, so the final data value must be 13.
Solution B:
Mean = sum of data values ÷ number of data values
= (sum of data values) ÷ 6 = 10

So the sum of data values must be 60. The sum of the given data values is
2 + 19 + 7 + 4 + 15 = 47, so we need 13 to make 60.

3. Create 3 different sets of data for which the mean is 6.

4. Create a set of data with the same mean as that for 36, 48, 52 and 67.

5. Find the mean of these test scores: 49, 49, 49, 50, 51, 52. Is it possible for the mean score on
a test to be greater than 50 if more than half of the students have marks less than 50? Explain.

6. Use the number sentence 4 + 3 = 2 + 5 to find data sets with:
   a) mean 5   b) mean 6   c) mean 7   d) mean 8

   What do you notice about your data sets? How do they compare to each other? (You can add
   1 to each data value in part a) to obtain the data set for part b), and so on.) Remind students of
   the discovery they already made in QUESTION 3 of PDM 5-13—when you add 1 to each data
   value, you increase the mean by 1.
PDM5-16
Stem and Leaf Plots

Introduce the words “stem” and “leaf” as they apply in mathematics: the leaf of a number is its rightmost digit and the stem is the number formed by all the remaining digits.

Put an example on the board: stem $\rightarrow$ 327 leaf

Ask volunteers to underline the leaf and circle the stem for various 2-, 3-, and 4-digit numbers, such as 25, 30, 230, 481, 643, 3210, 5403, 5430.

Tell students that a 1-digit number has no stem (or alternately stem 0) because there are no digits except the rightmost one. They can underline the leaf but the 0 isn’t written, so there is nothing to circle. Have students identify the stems and leaves in more numbers, including 1-digit numbers.

Have students identify the pairs with the same stem:

a) 45 46 63
b) 79 80 81
c) 88 89 98
d) 435 475 431
e) 86 862 860
f) 701 70 707
g) 6423 642 649

**Bonus**
h) 23 253 235 2 239 2530 2529

Ask your students to circle the numbers with stem 4, underline the numbers with stem 42, and cross out the numbers with neither stem:

42 428 4280 4 43 438 420 46 40 4201

**Bonus**
Use larger numbers and/or longer lists of numbers.

**ASK:** When the numbers are in order from smallest to largest, are the stems in order, too? Do numbers with the same stem have the same number of digits? Do they have the same number of tens? Point out that the stem is just the number of tens and the leaf is the number of ones. That’s why 1-digit numbers have stem 0—they have 0 tens.

Have students put the following numbers in order:

5 19 23 90 107 86 21 45 98 102 43

Then demonstrate circling the stems and writing them in the order in which they appear:

0 1 2 9 10 8 4

(When a stem repeats, such as the 2 in 23 and 21, you don’t have to write it again.) Now have students put the stems in order. **ASK:** Which was easier, writing the numbers in order or writing the stems in order? (writing the stems in order) Why? (There are fewer numbers and the numbers are smaller.)
Have students find the stems of the numbers, order the stems from smallest to largest, and then write the numbers from smallest to largest:

a) 45 46 63  
b) 567 60 583  
c) 43 47 92 65 73 70

Draw a T-chart and show students how to make a stem and leaf plot of the above data in 3 steps:

**STEP 1:** Write the stems in order in the left column.

**STEP 2:** Write each leaf in the second column in the same row as its stem. Add the leaves in the order in which they appear. For numbers that have the same stem, put the second leaf next to the first. Think aloud as you do the first few numbers, then have students help you do the rest. **PROMPTS:** Which number is next? What’s the leaf of that number? Where should I write it?

**STEP 3:** Put the leaves in each row in order, from smallest to largest.

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>STEP 2</th>
<th>STEP 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>stem</td>
<td>leaf</td>
<td>stem</td>
</tr>
<tr>
<td>0</td>
<td>0-5</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1-9</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2-3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4-8</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>8-9</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9-0</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>10-7</td>
<td>10</td>
</tr>
</tbody>
</table>

Then have students read the numbers from the finished plot. **ASK:** What are the numbers with stem 0? (just 5) What are the numbers with stem 1? (just 19) With stem 2? (21 and 23) Are these 2 numbers in order? Were they in order in the original list of data values? What did we do to make sure they would be in order in the stem and leaf plot? (We put the leaves in order. Because they have the same stem [the same number of tens], the number with the larger leaf [more ones] is larger.)

Continue reading and writing all the numbers from the plot. Compare the complete set of numbers (now in order) to the original set (unordered). Tell them that the numbers are now in order because we first put them in order according to the number of tens (stems) and then, within groups with the same number of tens, we ordered the ones. Ordering the numbers was easier because we did it in 2 smaller steps.

Have students make stem and leaf plots to order several data sets (include up to 8 numbers in each set). Start with problems where steps 1 and 2 are completed for them, and then move to examples where only step 1 is completed. Finally, have students do all 3 steps themselves. **BONUS:** Use up to 20 data values and/or larger numbers.

Explain to your students that the range is the difference between the largest and the smallest data values. Ask students to tell you the smallest and largest numbers from each of the stem and leaf plots they have drawn, and the range. **ASK:** How does making a stem and leaf plot help you find the range more quickly?
Extensions

1. Tell students that a stem and leaf plot doesn’t have to include only numbers. Here is a stem and leaf plot of friends’ birthdays—the stem is the month and the leaf is the date:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>21</td>
</tr>
<tr>
<td>April</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>May</td>
<td>8</td>
</tr>
<tr>
<td>June</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>27</td>
</tr>
</tbody>
</table>

Have students draw a bar graph from this data, with bars for each month. The height of each bar corresponds to the number of birthdays in that month.

Ask the following questions and have students tell you which graph gives you the answer, the stem and leaf plot, the bar graph or both:

a) How many birthdays are in the first half of a month?
b) How many birthdays are in April?
c) Which month has the most birthdays? The fewest?
d) Which month has a birthday the closest to the beginning of the month? The end of the month?

ASK: If you were given only the bar graph of the birthdays, could you have built the stem and leaf plot? Why not? What is the advantage of the stem and leaf plot?

2. John counts the total number of pages in each of his favourite novels:

<table>
<thead>
<tr>
<th></th>
<th>148</th>
<th>520</th>
<th>589</th>
<th>550</th>
<th>224</th>
<th>562</th>
<th>494</th>
<th>469</th>
</tr>
</thead>
</table>

Have students draw a stem and leaf plot. What do they notice? (Each stem has only 1 leaf). Tell students that sometimes it is more useful to use the number of hundreds instead of the number of tens as the stem. Re-do the stem and leaf plot with the hundreds as the stem, and the remaining numbers as the leaf. Ask students to explain how using hundreds instead of tens as the stem tells them more about John’s favourite novels (They can see at a glance that most of John’s favourites are 400+ pages long!).
Tell students that they have learned a lot about collecting and classifying data. However, to understand what the data they have collected means, they have to analyze it. Today they will begin learning how to analyze data.

Sometimes knowing the highest and lowest values in a set, or group, of data can be helpful. **SAY:** Suppose you know that the temperature next week is predicted to be 15, 14, 12, 17, 11, 10 and 18 degrees. (Write the temperatures on the board.) **ASK:** What is the highest temperature? What is the lowest temperature? Do you need to know all the temperatures to decide if you need snow pants next week, or is knowing the lowest temperature enough? Do you need all the temperatures to decide if you need a pair of shorts, or is knowing the highest enough?

Write several unordered sets of numbers on the board and invite volunteers to circle the largest number and underline the smallest one. Explain that the **range** of a set is the difference between the highest and the lowest numbers. Calculate the range of the temperatures in the above example with students. (Range = highest – lowest = 18 – 10 = 8) Then ask volunteers to find the ranges for the other sets on the board.

**SAY:** Suppose you are packing for a trip and you know that the range of the temperature at your destination is usually about 5 degrees. This means that the temperature doesn’t change very much; it’s nearly the same every day. **ASK:** How can this help you decide what kinds of clothes to pack? (You might need either snow pants or shorts, but not both!) What if the range at your destination is 20 degrees? What does that tell you? What will you pack? (The temperature changes a lot. You will need clothes for very different temperatures.)

In addition to identifying highest and lowest values, and calculating ranges, students need to be able to order numbers from smallest to largest. Ask volunteers to order the numbers in the sets on the board.

Tell your students that they will sometimes need to find the number in the middle of a set of data. This number is called the **median**. Find the median of the temperatures in the first example, by first ordering the temperatures and then circling the one in the middle:

10          11          12          14          15          17          18

Explain how you determined that 14 is the median. (It is in the middle of the set; there are 3 numbers before it and 3 after it.)

Now write this set of 6 temperatures:

10          11          12          14          15          21
SAY: When the number of terms in a set is even, there are two numbers in the middle and the median is the number halfway between them. The two numbers in the middle of this set are 12 and 14 (circle them) and the number halfway between 12 and 14 is 13. So the median is 13.

ASK: What number is halfway between 2 and 4? Between 5 and 7? 6 and 10? 12 and 16? 13 and 19? (If any students have difficulty finding the number in the middle, ask them to write all the numbers between the two in question and to count in from the sides: 13, 14, 15, 16, 17, 18, 19.)

Ask students to find the median of the sets they ordered previously. Then give them some unordered sets and ask them to find the median. (This would be a good time to do Activity 1, below.) Include sets in which numbers repeat. (SAMPLE SET OF DATA: 5 3 5 5 9 7 11)

Finally, calculate the range above and below the median for the temperatures in the original example:

\[
\text{range above median} = \text{highest temperature} - \text{median} = 18 - 14 = 4
\]
\[
\text{range below median} = \text{median} - \text{lowest temperature} = 14 - 10 = 4
\]

Look back at the range of the whole set of temperatures (8). Point out that the range of the whole set is the sum of the range above the median and the range below the median.

Explain to students that the range above and below the median is a measure of how much the data values are spread out. Use the first example to illustrate how this information can be useful. Let’s say you knew that the range of temperatures below the median was small. That means the lower temperatures don’t vary much—they’re nearly the same and close to the median. It also means that on nearly half of the days, the temperature will be at or only a little below the median. (Since, by definition, at least half of the data values are at or below the median.)

Give your students several sets (ordered, then unordered) and ask them to find the median and the range above and below the median. Include sets in which numbers repeat and in which the range above or below the median is 0.

SAMPLE SETS OF DATA:

\[
2, 3, 3, 4, 6, 7, 8 \quad 2, 2, 2, 2, 6 \quad 12, 15, 15, 15
\]

Write a stem and leaf plot where one number occurs twice. EXAMPLE:

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>1 8 9</td>
</tr>
<tr>
<td>10</td>
<td>0 2 9 9</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

Have students identify the number that repeats. Then write a stem and leaf plot in which 1 number appears 2 times and another number appears 3 times. Have students identify the numbers that repeat. ASK: Which number appears most often? Explain that this number—the value that occurs most often—is called the **mode**. Ask if anyone knows the French phrase “a la mode.” Tell students that it means in style or popular. Things that are popular occur often. Similarly, the mode is the most popular number in a set. ASK: Does every set have to have a mode? Remind students of earlier sets in which every value occurred only once—those had no mode.

Ask students to identify the mode in several stem and leaf plots, some of which have more than one repeated data value but only one most common data value. Then introduce data sets with more than one mode. Give them the stem and leaf plot at first and then have them build the stem and leaf plot first to find the mode or modes.
Present the two following sets of students’ heights rounded to the nearest centimetre:

Set 1: 148, 152, 153, 155, 155, 156, 156, 155, 163

Explain that one of the sets shows the heights of a grade 5 basketball team, and the other shows the heights of a grade 7 chess team. **ASK:** Which set is which?

Brainstorm with your students what they expect about the heights of basketball players in grade 5 and chess players in grade 7. Are grade 7 students taller on average than grade 5 students? If necessary, explain that basketball players tend to be taller than other people. Are chess players taller or shorter than other people or average? **ASK:** For any set, which is larger, the range or the range above the mean? The range or the range above the median? If you choose 3 data values from the whole set and 3 from the set of values above the mean, which three values might be spread out more? (The values from the whole set. You can choose values from both ends of the whole set, and not from the middle and up only.) If the basketball players are taller than average, do you expect their heights to be more, or less, spread out than the heights of the chess players?

Ask your students to draw bar graphs for both sets using the axes shown below:

```
Number of students
146-150 151-155 156-160 161-165

Height of students (cm)
```

**ASK:** How are the graphs different? (In Set 1 the data is grouped around the middle, whereas in Set 2 the data is grouped toward the edges.) Can you decide which set belongs to which team by looking at the graphs? (The heights of the basketball players form Set 1. They are taller than other children of the same age, and most of them are about the same height, so the data is grouped around the centre. The chess players are older, so there is more difference in the heights of the boys and the girls, so their heights make Set 2.) Explain that this property of the graphs that they noticed would be called the shape of the sets.

Ask your students to find the mean, median and range of each set. What do they notice? (The mean and the median are 155 and the range is 15 for both sets.) Could the mean, the median and the range alone help them answer the question which set is which?

**ACTIVITY 1**

Invite 9 volunteers to be a set of data. Ask them to order themselves from shortest to tallest. Who is the median? Now have one of the volunteers sit down, and ask the class how they can find and represent the median. (Students may suggest making a mark on the board to represent the height halfway between the two middle students, or they may suggest choosing another student in the class whose height is approximately halfway between the two middle students to be the median.)
Students may use Excel and its built-in statistical functions to investigate the result of changes in values on sets of data. For example, let the students create a table with the following values:

<table>
<thead>
<tr>
<th>75</th>
<th>77</th>
<th>69</th>
<th>75</th>
<th>90</th>
<th>75</th>
<th>73</th>
<th>65</th>
<th>68</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>73</td>
<td>71</td>
<td>75</td>
<td>70</td>
<td>95</td>
<td>97</td>
<td>65</td>
<td>72</td>
<td>86</td>
</tr>
</tbody>
</table>

The students can find the mean (average), median and mode (all found among statistical functions) of the set. They can change or add data values to see how the changing a value or adding a new data value affects the mean, the median and the mode.
Extensions

1. Find the median and the range below and above the median for each set:
   a) 2, 3, 5, 7, 9, 10
   b) 12, 16, 19, 23, 26, 26, 26

2. Write 3 different sets that have median 9 and range 5.

3. Write 3 different sets that have median 10 with range above the median 0 and range below the median 5.

4. Write a set of data in which:
   a) the largest number is 100.
   b) the smallest number is 100.
   c) the range is 100.
   d) the median is 100.
   e) the mean is 100.
   f) one of the modes is 100.

Can you satisfy more than one requirement in the same set?

5. Ron counted the number of floors of the buildings in his block: 5, 3, 3, 1, 13
   a) Find the mean, median and modes of the set of data.
   b) The five story building is replaced by a skyscraper of 50 floors. Find the mean, median and the mode of the new data set.
   c) Ron says: The increase in any data value increases the mean. The increase in any data value also increases the median. Are both his statements correct?
   d) The number 50 is much greater than the others. (It is called an outlier). Which value changed the most when you added the outlier, the mean, or the median?
   e) Look at 4 sets of data: 3, 3, 1, 13; 5, 3, 1, 13; 5, 3, 3, 13; 5, 3, 3, 1
      What is the difference between the initial set and these 4 sets? Find the mean for each set. Which of the following statements are always correct?
      • The mean increases if a piece of data below the mean is removed.
      • The mean increases if the piece of data below the median is removed.
      • The mean decreases if a piece of data below the mean is removed.
      • The mean decreases if a piece of data above the mean is removed.
      • The mean increases if a piece of data above the mean is removed.
      • The mean decreases if a piece of data above the median is removed.

6. Can you find a set so that the range below the median is 0, and the mean and the median are the same? Explain your thinking with blocks.

7. Have students make a chart comparing the mean and the median for data sets with only 2 values:

<table>
<thead>
<tr>
<th>Data Set with 2 Values</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 7</td>
<td>10 ÷ 2 = 5</td>
<td>3 4 5 6 7</td>
</tr>
<tr>
<td>4, 10</td>
<td></td>
<td>4 5 6 7 8 9 10</td>
</tr>
</tbody>
</table>

Other data sets they should add to the chart:
   a) 0, 12          b) 5, 7          c) 9, 21          d) 11, 15
What do students notice? (the mean and the median are the same) Have students predict whether the same will be true for data sets with 3 or 4 data values and then check their predictions.

8. In a game as in the Activity 2 above, there are 9 envelopes. The minimal amount is $200, the range is $800, the mean and the median are both $600. There is no mode. You pick an envelope with $600. Are you trading or not?

Discuss with your students how the shape of the set might affect the choice using the sets:

Set 1: 200, 300, 400, 500, 600, 700, 800, 900, 1000
Set 2: 200, 597, 598, 599, 600, 601, 602, 603, 1000.

With the first set, the game is very risky – the data is spread evenly, and the gain (or the loss) is large with any trade. In the second case the data bunches up around the median and the mean, and in most cases the gain (or the loss) is small. Presenting the data in bar graphs might be helpful.

### PDM5-18
#### Choices in Data Representation

Give your students a set of data values and invite volunteers to create a bar graph, a line graph and a stem-and-leaf plot for the values. Sample data set:

Katie found some data on the sightings of whales in the Gulf of Maine in the summer of 2007:

<table>
<thead>
<tr>
<th>Date</th>
<th>Jun 1</th>
<th>Jun 8</th>
<th>Jun 16</th>
<th>Jul 3</th>
<th>Jul 4</th>
<th>Jul 13</th>
<th>Jul 21</th>
<th>Aug 1</th>
<th>Aug 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Whales Seen</td>
<td>27</td>
<td>34</td>
<td>17</td>
<td>15</td>
<td>21</td>
<td>17</td>
<td>52</td>
<td>31</td>
<td>27</td>
</tr>
</tbody>
</table>

Discuss with your students what the scale for their bar graph should be and what range they should use. On the horizontal axis of their line graph, should the distance between the successive pairs of dates be the same for all pairs or should some dates have more (or less) space between them than other dates?

On which graph is it easiest to see the range of the values? On which graph is it easiest to see the median? What does the median mean for this set of data? (On half of the dates more whales were seen, and on half of the dates fewer whales were seen.) What information is lost in the stem and leaf plot?

Ask your students to find the mean. What does it mean for this set of data? (The average number of whales seen on a trip.) Is it possible to see this
number of whales? (No, it is not a whole number.) Which graph is most convenient to use to find the mean? On which graph is it easiest to see how far the various values are above and below the mean?

Which trends can be seen in this set of data? On which graph can you see the trends most clearly? If you wanted to estimate how many whales might be seen on a trip on June 23, which graph would you use?

**Extensions**

1. Determine the values of the other bars on the graphs:

   a) ![Graph a]
   
   b) ![Graph b]
   
   c) ![Graph c]

2. Students may use Excel to plot the graphs of larger sets of data. Let them compare various graphs as they did during the lesson.

---

### A Note About Terms in Probability

**Outcomes and Events**

A simple action such as rolling a die, flipping a coin, or spinning a spinner has various possible results. These results are called the **outcomes** of the action. If you flip a coin, the outcomes are: “You flip a head” and “You flip a tail.” When you describe a specific outcome or set of outcomes, such as rolling a 6, rolling an even number, tossing a head, or spinning red, you identify an **event**.

In probability theory, the terms **outcome** and **event** have very precise meanings. But in elementary texts, outcome is occasionally misused.

This spinner has 3 coloured regions:

![Spinner]

There are 3 possible outcomes of spinning the spinner:

1. The pointer lands in the blue region.
2. The pointer lands in the red region.
3. The pointer lands in the green region.
Some textbooks will identify the outcomes as:

1. You spin blue.
2. You spin red.
3. You spin green.

What’s the difference? Identifying outcomes with only colours and not regions can cause confusion when 2 or more regions of a spinner are the same colour. Here is a spinner with 4 coloured regions:

![Spinner Diagram]

There are 4 outcomes of spinning the spinner:

1. The pointer lands in the blue region at top right.
2. The pointer lands in the blue region at bottom right.
3. The pointer lands in the blue region at bottom left.
4. The pointer lands in the red region.

This is clearly not the same as saying that the outcomes are:

1. You spin red.
2. You spin blue.

“You spin blue” and “You spin red” are events, not outcomes. To assess the relative likelihood of spinning red or blue, students must recognize that the pointer may land in 4 distinct regions of the spinner. In 3 of the 4 outcomes, the spinner lands in a blue region. Hence, the event “You spin blue” is more likely than the event “You spin red.”

The outcomes for QUESTION 1 f) on Worksheet PDM5-19 are “The spinner lands in region 1,” “The spinner lands in region 2,” “The spinner lands in region 3,” and “The spinner lands in region 4.” Your student may write something more concise, such as “You spin a 1,” “You spin a 2,” “You spin a 3,” “You spin a 4.” Accept these answers, provided students understand that when different regions of a spinner have the same number or colour, each region must be counted as a distinct outcome.

To avoid confusion, we only use the term “outcome” on the worksheets when the regions of the spinner are uniquely coloured or labelled. When the same colour or label appears more than once on the spinner, we use phrases like “ways of spinning red” instead of “outcomes”.

**Probability and Outcomes**

In probability theory, an “event” is a particular “subset” of a set of outcomes. For instance, on the spinner above, the event “You spin blue” is the subset of outcomes in which the pointer lands in a blue region. On a die, the event “You roll an even number” is the subset of outcomes: {“You roll a 2,” “You roll a 4,” “You roll a 6”}. Your student needn’t learn the technical meaning of the term “event” until they are older. (For now, tell them an event is any particular result of an action or occurrence they wish to assess the likelihood of.) But they should know that the “outcomes” of an occurrence are all the ways the occurrence could happen.

The probability or likelihood of an event may be expressed as a fraction: given a set of outcomes (where each outcome is equally likely), the denominator of the fraction is the total number of outcomes and the numerator is the number of ways the event could happen.
Probability = \frac{\# \text{ of ways the event can happen}}{\# \text{ of outcomes}}

If you flip a coin, the probability of flipping a head is \( \frac{1}{2} \), since there are two outcomes of flipping the coin, but only one way to flip a head.

If you roll a die, the probability of rolling a five is \( \frac{1}{6} \), since there are six outcomes but only one way to roll a five.

On the spinner below, the probability of spinning red is \( \frac{3}{8} \), since there are eight regions (all of the same size) in which the pointer might land, but only three of the regions are red.

\[ \text{R R Y Y R G B G} \]

**Expectation**

The probability of spinning blue on the spinner below is \( \frac{1}{3} \).

\[ \text{R B G} \]

If you were to spin the spinner 12 times you would expect to spin blue \( \frac{1}{3} \) of the time, or \( 12 \div 3 = 4 \) times.
Tell students that today they will start learning how to predict the future! Hold up a die and ask students to predict what will happen when you roll it. Can it land on a vertex? On an edge? No, the die will land on one of its sides. Ask students to predict which number you will roll. Then roll the die (more than once, if necessary) to show that the prediction about landing on a side works, but the number they picked does not necessary come. Explain that the possible results of rolling the die are called outcomes, and to predict the future students must learn to identify which outcomes of various actions are more likely to happen and which are not. But first, they must learn to identify outcomes correctly.

Hold up a coin and ask: What are the possible outcomes of tossing a coin? How many outcomes are there? Show a spinner and a set of marbles. What are the possible outcomes of spinning the spinner or picking a marble with your eyes closed? Ask students to identify the possible outcomes of a soccer game. How many outcomes are there? (3 outcomes: team A wins, team B wins, a draw)

Ask: You have to make a spinner with 5 possible outcomes. How would you do this? Invite volunteers to draw possible spinners. If the spinner in the picture at right does not arise, draw it and ask: How many outcomes are there for this spinner?

Are all the outcomes bound to come equally, or is there an outcome that they think will happen more often than the other ones? Draw the second spinner and ask: How many outcomes does the second spinner have? (4) Will the pointer ever be in the grey region? (no, never)

**Extensions**

1. Bill has a pentagonal pyramid with numbers on the sides. He puts a non-base side on the table and rolls the pyramid. How many outcomes are there? Make a pentagonal pyramid, number the sides, and check your answer. Are there sides the pyramid never lands on if you roll it in the manner described? (Yes—the base. It will not roll if it stands on a base.) Repeat with a hexagonal prism and an octagonal prism: predict the number of outcomes then make a model to check your prediction.

2. Ed and George are playing “Rock, paper, scissors.” Describe all the possible outcomes of the game. (Note: “Ed wins” is not an outcome, it is an event. “Ed has paper, George has rock” is an outcome.)
**PDM5-20**  
**Describing Probability**

Have students make some predictions. Ask them to tell you if the following events are likely or unlikely:

- The sun will rise tomorrow.
- The teacher will give the answers to the test before giving the test.
- An alien will walk into the class in the next minute.
- They will have lunch in half an hour.
- It will rain tomorrow.
- It will snow in June.

Invite students to name some events and have other students tell if the events are likely or unlikely. You could ask students to compare the likelihood of events. For example, it is unlikely to snow in June, but it is more unlikely that an alien will walk into the class!

Explain to your students that mathematicians call an event **likely** if it is expected to happen more than half the time and **unlikely** if it is expected to happen less than half the time.

Ask your students which word people would use to describe an event like meeting a live dinosaur in the street. Can that happen at all? Write the terms **likely**, **unlikely** and **impossible** on the board. Ask your students which words describe an event that will definitely happen, like rolling a number less than 7 on a die. Add the word **certain** to the list.

**NOTE:** Although students might use the word impossible to describe the likelihood of meeting a dinosaur, this event is not necessarily impossible (scientists might find a way to clone dinosaurs). The only events that are strictly impossible are events that are contradictory—like rolling a number greater than 6 on a die.

Ask students to give examples of various events and explain whether they are likely, unlikely, certain, or impossible. Encourage students to think of events using marbles, dice, money, and other objects, as well events from daily life, such as meeting a tiger or an astronaut on the way to school.

Hold up a coin. **ASK:** What are the possible outcomes of flipping this coin? How many outcomes are there? What is more likely—to flip a head or a tail? Explain that the chances are the same—you have **even chances** of flipping a head or a tail. Add the term to the list and explain that the chances of an event are even when the event happens in exactly half of the outcomes. Flipping a tail is 1 out of 2 possible outcomes; 1 is half of 2. **ASK:** How many outcomes are there when you roll a die? (6) How many outcomes are even numbers? (3) Since half of the outcomes are even numbers, you have even chances of rolling an even number (and an even chance of rolling an odd number).

**SAY:** We have 8 marbles in a box. I take out a marble. How many outcomes are possible? (8, regardless of the colour of the marbles) What is half of...
8? So if I want even chances to take out a green marble, exactly 4 marbles should be green. Does it matter what colour the other marbles are? (No, provided they’re not green.)

Draw the spinner below on the board and ask students to describe the possibility of spinning each of the colors. Is it equally likely that they will spin green or that they will spin blue? Is it equally likely that they will spin yellow or blue? Why? (HINT: Look at the angle at the centre)

Ask students to draw a spinner to match this description:

1. It is likely you will spin on the yellow.
2. It is unlikely to get green.
3. It is equally likely to get green and red.
4. It is impossible to spin blue.

Now write this list of properties:

1. It is impossible to get green.
2. It is certain to get blue.
3. It is likely to get red.
4. It is unlikely to get white.
5. It is equally likely to get white and red.
6. It is equally likely to get red and purple.
7. It is equally likely to get white and yellow.

ASK: Do you think we can make a spinner that matches all of these? Do any of the properties contradict each other? Look at 3 and 6: can a spinner match 3 and 6? (No. If the spinner is likely to give red, more than half of the spinner should be red. Then only less than half of it can be purple. But if it is equally likely to give red and purple, the red and the purple should have the same area. But more than half is never equal to less than half!) Have students pick 3 of the properties on the list and create a spinner that will match them. If the all-blue spinner does not arise, show that example yourself and ask students to identify the properties that describe it (1, 2, 5, 6 and 7).

Which properties does this spinner match? (Same as the all-blue spinner: the pointer never reaches any other region)

Show students a collection of marbles or coloured counters:

G Y G B G
R B G G Y

ASK: Which colour are you most likely to pick? Which colour is less likely to be picked: yellow or red? So which colour is least likely to be picked? Which colours are equally likely?
Draw a line on the board:

```
impossible          certain
```

Ask your students to mark on the line the probability of picking…

- a green marble
- a red marble
- a blue marble
- a pink marble
- a yellow marble
- a marble of any colour

**Assessment**

1. Make a spinner that matches this description:
   - It is most likely to spin green.
   - It is unlikely to spin red.
   - It is most unlikely to spin purple.
   - It is equally likely to spin purple and blue.
   - It is impossible to spin yellow.

2. Explain why it is impossible to make a collection of marbles to match this description:
   - It is impossible to pick red. It is certain to pick blue. It is very unlikely to pick green.

3. Change one word in the description of the collection of marbles above to make it possible. Draw the collection that matches the new description.

---

**A Game for Two**

Give students several marbles or counters of various colours. One player chooses a set of 12 counters or marbles, unseen to the partner. The player describes the set to the partner, using probability terms (certain, likely, unlikely, and so on). The second player has to reconstruct the set from the first player’s description.
Divide students into 2 groups. Each group will conduct a different experiment and share the results with the second group afterwards.

**Group 1**
Students will each need a die. Ask them to list all the outcomes of rolling a die and to count the outcomes that suit each event listed below. Then ask them to describe each event as likely, unlikely, or having even chances:

1. Roll a number greater than 2.
2. Roll a number greater than 5.
3. Roll an odd number.

Each student rolls the die 12 times and tallies the results. Do the results match the predictions? Have students combine their results. (This group tally chart—how many times the group rolled 1, 2, and so on—may be useful during the next lessons.) How many rolls did the whole group make? How many times does the group expect to get an even number? A number greater than 5? A number greater than 2? Students should explain their answers. Do the group’s results match the predictions better than the individual results?

Students can create a table like this to summarize predictions and results as they conduct the experiment:

**TOTAL OUTCOMES:** 1, 2, 3, 4, 5, 6  
**NUMBER OF POSSIBLE OUTCOMES:** 6

<table>
<thead>
<tr>
<th>Event</th>
<th>Suitable Outcomes</th>
<th>Individual Results (12 Rolls)</th>
<th>Group Results (___ Rolls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll a number &gt; 2</td>
<td>3 4 5 6</td>
<td>___ out of 12</td>
<td></td>
</tr>
<tr>
<td>Roll a number &gt; 5</td>
<td>6</td>
<td>___ out of 12</td>
<td></td>
</tr>
<tr>
<td>Roll an odd number</td>
<td>1 3 5</td>
<td>___ out of 12</td>
<td></td>
</tr>
</tbody>
</table>

**Group 2**
Students will each need 3 red marbles, 2 yellow marbles, 1 green marble, and a non-transparent bag or box (e.g., a lunchbox).

Ask students to list all the outcomes of drawing a marble from the box and to count all the outcomes that suit each event from the list below. Ask students to describe each event as likely, unlikely, or having even chances:

1. Draw a red marble.
2. Draw a green marble.
3. Draw a marble that is not yellow.

Each student draws a marble 12 times (returning the marble after each draw and shaking the bag) and records the results in a tally chart. Do the individual results suit the prediction? Have students combine their results. Do the group’s combined results suit the predictions? Students can create a table similar to that used by Group 1 to summarize their results.

Ask the students to write a summary of the experiments. Prompts for Group 1:

I performed an experiment with a die. I rolled the die ___ times.
I expected that ___ of ___ rolls will give a number greater than 2. I got a number > 2 in ___ rolls.
I thought that it is _____________ to get a number > 2, because ________________
I learned that ________________

Discuss the similarities and differences between the experiments of the two groups.
Extensions

1. If you roll a die, are your chances of rolling a number greater than 2: unlikely, even or likely? Explain your answer.

2. Write the numbers from 1 to 10 on ten cards, one number on each card. Ask students to select 6 cards at random. Let them check which of the following statements hold. Repeat the experiment 10 times. Are these events certain, impossible, likely or unlikely? Have the students explain their answers.
   - The sum of the numbers will be greater than 60.
   - All the numbers will be even.
   - Two numbers will be neighbours.
   - The sum of the numbers will be greater than 20.
   - No numbers will be neighbours.
   - The sum of the numbers will be less than 12.
   - Three cards will be odd.

3. Invent or describe a game where a certain player’s chance of winning is very close to certain. What are the chances of the other player(s) to win?

4. Doug has a total of $95 in his wallet ($50, $20, $20, and $5). Which bill is he most likely to draw? Which bills are equally likely to be drawn?

5. Match the spinner with the correct statement:

   A
   B
   C
   D

   _____ Spinning blue is three times as likely as spinning white
   _____ Spinning blue and white are equally likely
   _____ Spinning any colour is equally likely
   _____ Spinning one colour is twice as likely as spinning any other colour
PDM5-21
Probability

GOALS
Students will describe probability of simple events using fractions.

PRIOR KNOWLEDGE REQUIRED
Fractions of sets, numbers Visual representations of fractions Reduction of fractions

VOCABULARY
probability fraction outcome

Show your students two pencils of different length. Ask them how they could determine which pencil is longer. Then show two objects where direct comparison is impossible, such as a ruler and the circumference of a cup. Students might suggest measuring the length with a measuring tape. Then ask your students how they could compare the weight of two objects, say a book and a cup. What could they do to compare the temperature in two different places? Point out that in all cases they tried to attach numbers to each object and to compare the numbers. They used different tools for that purpose—measurement tape, scale or thermometer. Each tool provided a number, a measurement. What would they do to compare the likelihood of two events, such as the likelihood of rolling 8 on a pair of dice and the likelihood that their favourite hockey team wins 5 to 3 in the next game? Is there a tool to measure likelihood? No. Explain that probability is the branch of mathematics that studies likelihood of events and expresses the likelihood of various events in numbers. The measure of likelihood of an event is called probability.

Draw the following spinner on the board:

ASK: How many different regions does the spinner have? How many different ways can you spin red? (Only one.) How many different ways can you spin blue? (Two ways) Since the regions of the spinner are all the same size, it is equally likely that the spinner will land in any of the regions. Since two regions are coloured blue and only one is coloured red, students should see that it is twice as likely that they will spin blue as it is that they will spin red.

Explain that in mathematics we describe probability as a fraction:

Probability = \frac{\# \text{ of ways the event can happen}}{\# \text{ of possibilities}}

For our spinner, there are three possible outcomes. (Even though there are only two colours there are three regions). So the probability of spinning red is \(\frac{1}{3}\) and the probability of spinning blue is \(\frac{2}{3}\). ASK: Which fraction of the spinner is coloured red? Which fraction of the spinner is coloured blue?

Give your students several exercises such as:

How many ways can you draw a yellow marble?

How many ways can you draw a marble of any colour?
What is the probability of drawing a yellow marble?

How many ways can you draw a white marble? What is the probability of drawing a white marble?

Show your students the following spinner:

```
R
B
B
B
W
```

**ASK:** Which colour are you more likely to spin: red or white? Why? How many regions does this spinner have? How many are coloured red? How many are coloured white?

What do we get if we write a fraction where the numerator is just the number of white regions and the denominator is the number of all regions? (\( \frac{1}{5} \)) Repeat with the red colour. Did we get the same fractions? (yes) However, we want these fractions to measure the likelihood of spinning red and spinning white. If one of them is more likely to happen than the other, the fractions should be different. This means that we made a mistake in our calculations. Ask your students if they can guess what was wrong.

Explain that the number of possibilities in the definition of probability refers to equal possibilities. The red part of the spinner is smaller than the white part. Can we divide the spinner into equal parts so that each part is coloured in one colour? (The spinner now is divided into quarters, but one of the quarters is coloured in two colours.) Invite a volunteer to cut the spinner into equal parts.

**ASK:** How many regions does the spinner have now? (8) How many regions are coloured red? What is the probability of spinning red? (\( \frac{1}{8} \)) White? (\( \frac{2}{8} \)) Blue? (\( \frac{5}{8} \))

Review the concept of reducing fractions with the students and ask them to write the probability of spinning white on the spinner above as a reduced fraction.

Let your students solve several more complicated questions like:

Write a fraction that gives the probability of rolling a multiple of 3 on a die.

To solve this question, ask your students to write out all the possible outcomes of rolling a die. Then ask them to circle the numbers that are multiples of 3. How many are there? Ask students to write the probability as a fraction and to reduce the fraction to lowest terms.

More questions to practice:

- What is the probability of a letter in the alphabet to be a vowel?
- What is the probability to roll a number greater than 4 on a die?

**ACTIVITY 1**

Give your students two dice. Ask them to roll the dice and to write a fraction where the numerator is given by the least number rolled, and the denominator is given by the larger number rolled. The students then have to draw a spinner or a collection of marbles so that the probability of spinning blue or drawing a blue marble equals the fraction.
Extension

Match the net for a tetrahedron (\(\text{[Diagram]}\)) to the correct statement:

A  \[
\begin{array}{ccc}
2 & 3 & 4 \\
4 & 3 & 4 \\
\end{array}
\]  
B  \[
\begin{array}{ccc}
2 & 3 & 4 \\
1 & 3 & 4 \\
\end{array}
\]  
C  \[
\begin{array}{ccc}
2 & 3 & 1 \\
1 & 3 & 1 \\
\end{array}
\]  
D  \[
\begin{array}{ccc}
2 & 3 & 4 \\
3 & 3 & 4 \\
\end{array}
\]  

- The probability of rolling a 1 is \(\frac{1}{4}\)
- The probability of rolling an even number is \(\frac{3}{4}\)
- The probability of rolling an odd number is \(\frac{3}{4}\)
- The probability of rolling a 3 is \(\frac{1}{2}\)

ACTIVITY 2

Give each pair of students 6 to 12 different beads. Ask them to list all the attributes of the beads that they can think of. Player 1 picks a bead, chooses an attribute and Player 2 has to write the probability of picking a bead that has this attribute. For example, the attribute is colour and Player 1 holds up a red bead. Player 2 has to tell the probability of picking a red bead from the set.

ADVANCED: Player 1 picks a bead, thinks of an attribute without telling Player 2, shows the bead to his partner and gives the probability to pick a bead from the set that shares the chosen attribute. Player 2 has to guess which attribute was chosen.
PDM5-22
Advanced Probability

GOALS
Students will describe probability of simple events using fractions.

PRIOR KNOWLEDGE REQUIRED
Fractions of sets, numbers Visual representations of fractions Reduction of fractions

VOCABULARY
probability fraction outcome fair game equally likely

SAY: I would like to play a game with you. The rules of the game are simple. I will spin a spinner. If I get red, I win; if I get blue, the class wins. Ask students if they agree to play by these rules. Now show them the spinner. Do they still want to play? Why not?

Write the term fair game on the board. Ask students to explain what they think this term might mean. Encourage students to use math vocabulary in their explanations. Point out that in a fair game, both players have equal chances, or are equally likely, to win. Does this mean there can never be a draw? No. Use this game to illustrate: Flip a nickel and a penny. If both give heads, you win. If both give tails, the class wins. If there is one head and one tail, it is a draw—no one wins. Explain the rules to the class, then ask a volunteer to list all the four outcomes. ASK: Is the game fair? (Yes) Why? (Each player wins in one case only, so they have even chances to win.) Would the game stay fair if we added a third player who wins when one of the coins gives a head and the other a tail? (No - the third player will be more likely to win than the other two - he will have two winning outcomes.) Would the game be fair if the third player wins with head on the nickel and tail on the penny? (yes)

Give the rules for another game: There are 6 marbles in a box. If you draw a red marble, you win; if it is blue, the class wins. Otherwise, the game is a draw. There are 2 red marbles in the box. ASK: To make the game fair, how many blue marbles should be in the box? What about the rest of the marbles—what colour can they be? Can you think of another combination of marbles that will make the game fair?

Vary the game: If you draw a red marble you win, but if you draw any other colour, the class wins. If 2 of the 6 marbles are red, who has more chances to win—you or the class? What if 5 marbles are red? What should be in the box to make the game fair?

Let your students describe the probability of throwing a dart (assume the dart always hits the board) and landing on each of the colours:
You may also ask some more complicated questions, such as: What is the probability of getting red or yellow? What is the probability of getting any colour other than blue? What is the probability of getting a colour that is on the flag of Canada? On the flag of your province? Which two colours have the same probability of occurring? Find a combination of colours such that the probability of the dart landing on one of the colours is $\frac{5}{8}$. Ask your students to make up their own problems using this chart.

**Assessment**

1. Two players are spinning this spinner. Invent two different rules of play to ensure that the game is fair.

![Spinner Diagram]

2. What is the probability to draw a red marble from the box with 3 red marbles, 2 white marbles and 3 green marbles?

**Extensions**

1. Emily has a bag of 8 marbles. 5 are blue. Peter has a bag of 7 marbles. 3 are blue.
   a) What is the probability of drawing a blue marble from Emily’s bag?
   b) What is the probability of drawing a blue marble from Peter’s bag?
   c) Emily and Peter pour all of their marbles into one bag. What is the probability of drawing a blue marble from the bag?

2. Mark draws all possible rectangles with a perimeter of 12 cm (with whole number sides). If he picks one of the rectangles at random, what is the probability that it will have length of 3 cm?

3. Carl and Clara played a game based on luck. Carl won 15 times and Clara won 12 times. Does this mean the game is not fair?

4. Carmel wants to express probability by drawing rectangles. For example, he draws a rectangle to represent the probability of tossing a head on a coin. The probability of tossing a head is $\frac{1}{2}$, so he shades $\frac{1}{2}$ of the rectangle.

Use Carmel’s method to show the probability of:

   a) Drawing a red marble from a box with 4 red marbles and 8 green marbles
   b) Picking a consonant from the word “rectangle”
   c) Spinning blue on the spinner below.

![Spinner Diagram]
PDM5-23
Expectations

Show students this spinner and ask them if the chances of spinning red and spinning blue are equal. What about the chances of spinning green and spinning blue? Why? Write on the board:

Number of possible outcomes: 3 (1 green, 1 red, 1 blue)
Chances of spinning blue: 1 out of 3

SAY: I am going to spin the spinner 12 times. How many times do you expect me to spin blue? Why? Write the calculation on the board:

1/3 of 12 = 4 OR: 12 ÷ 3 = 4 times

Remind students that actual outcomes usually differ from expected outcomes. You might not get blue 4 times every time you make 12 spins, but 4 times is the most likely number of times you will spin blue.

Review with students how to find a fraction of a set and a fraction of a number as well as visual representation of fractions. Practise by solving these and other questions:

1/2 of 14
1/3 of 15
1/5 of 25

2/3 of 20
3/5 of 9
Quarter of 16

Three eighths of 24

Which part of the set “R G R G G R Y Y G Y” is G?

ASK: Which part of this spinner is blue? What are the chances of spinning blue? (3 out of 4) In mathematics we say “The probability of spinning blue on this spinner is 3/4.” Write that on the board. Explain that you can determine the number of times you would expect to spin blue out of 20 spins as follows:

STEP 1: 1/2 of 20 is 20 ÷ 4 = 5.
This is how many times you expect the spinner to land in each region.

STEP 2: Three regions are blue. You expect to spin blue 3/4 of all times.
If you expect the spinner to land in each region 5 times, and 3 of the regions are blue, then the spinner will land in a blue region 3 × 5 times:

3/4 of 20 is 3 × 5 = 15.
You can show this with a picture:

\[
\frac{1}{4} \text{ of } 20 = 20 \div 4 = 5
\]

Therefore, \( \frac{1}{4} \text{ of } 20 = 3 \times 5 = 15 \)

Have students practice calculating expected outcomes with these and similar questions:

- If you flip a coin 16 times, how many times do you expect to get a tail?
- Hong wants to know how many times he is likely to spin green if he spins this spinner 24 times. He knows that \( \frac{1}{3} \) of 24 is 8 \((24 \div 3 = 8)\). How can he use this information to find how many times he is likely to spin green?

- If you roll a die 18 times, how many times do you expect to get a 4? To get a 1?
- How many times would you expect to spin blue if you spin this spinner 50 times? How many times would you expect to spin green?

**CHALLENGING:** If you roll a die 30 times, how many times do you expect to roll an even number? How many times do you expect to roll either 4 or 6?

You should review long division with your students before you assign the worksheets on expectation.

**Assessment**

Rea spins this spinner 30 times. How many times is she likely to get blue?

**Bonus**

1. You flip two coins, a nickel and a dime, 12 times. List the possible outcomes. How many times do you expect to get one head and one tail?

2. Jack and Jill play “Rock, paper, scissors” 18 times. List all possible outcomes of the game. (HINT: “Jack has rock and Jill has paper” is different from “Jack has paper, Jill has rock”! Why?) How many times do you expect to see a draw? What is the probability of a draw?
Extensions

1. Design an experiment with three possible outcomes in which one of the outcomes has a probability near $\frac{1}{2}$.

2. Choose a novel. Open it to any page and note whether or not the first letter is a “t”. Check 10 pages in this manner. Describe the probability that “t” is the first letter on a page of the book.

3. Give your students two dice of different colors, say red and blue. The red die will give the numerator of a fraction and the blue die will give the denominator. Roll the dice 40 times and record the fractions. How many times did you get a fraction that was reduced to lowest terms? What is the experimental probability of getting a reduced fraction? Pool the results of the whole class. Are they all the same? What is the experimental probability of getting a reduced fraction from the results of the whole class? Which result is more likely to match the theoretical probability?

List all the possible combinations of the two dice as fractions (there are 6 different results of the red die for every result of the blue die, so the total number of combinations is 36). Count the reduced fractions. What is the theoretical probability of getting a reduced fraction? Compare with the experimental results and write a report about what you’ve learnt.

Have students flip a coin ten times and keep a tally of the number of heads. Point out that (unless a miracle occurred in your class) that not every student flipped heads exactly half the time. Ask the students to identify the result that was furthest from the expected number of 5 heads. Then combine all the results of the entire class (i.e. add up the total number of heads from all the tallies). The overall proportion of heads should be closer to half of the total number of tosses. Explain to your students that the more trials you conduct (i.e. the more times you repeat an experiment) the more closely the actual result will match the expected result for the event.
PDM5-24
Games and Expectation

PDM5-24 is a review worksheet, which can be used for practice. Here is an additional activity:

Pair up students and give each pair a container with 1 red counter and 5 blue counters inside. Player 1 wins if they draw a red counter and Player 2 wins if they draw a blue counter. Ask students to explain whether the game is fair or not. Have students play the game 20 times (replacing the counter each time) and keep a tally of who wins each time. Ask students if the results were what they expected.

Then ask students to keep track of who wins and loses in 20 repetitions of the following game:

Players roll a die. Player 1 wins if a 1 is rolled. Player 2 wins otherwise.

Ask students the following questions: Is the second game fair? Are the results what you expected?

How are the games in questions 1 and 2 similar? Is the first game less fair than the second?
(When looked at from the point of view of probability, the two games are identical. In both games the second player has 5 out of 6 chances to win: the probability of Player 2 winning is \(\frac{5}{6}\).)

Students could predict the number of reds they would expect to spin for a given number of trials (i.e. for 15 trials, 30 trials, etc.) They could keep a tally of their results and compare them to the expected number of reds.

Extension

a) Look at the second game in the activity. How many times do you expect Player 1 to win, if the game is played 24 times?

b) Compare the second game in the activity with the following game:

Players take turns rolling a die. If a player rolls a 1, he gets 1 point. If he rolls any other number, his partner gets 1 point. The die is rolled 24 times. Player with more points wins.

Is this game fair? Explain.

c) How many points is each player expected to get in the game in part b)?
To illustrate the idea of a coordinate system you can start with the following card trick:

1. First, deal out nine cards—face up—in the arrangement shown below:

   Row 3
   Column 1 Column 2 Column 3
   Row 2
   Column 1 Column 2 Column 3
   Row 1

2. Next ask a student to select a card in the array and then tell you what column it’s in (but not the name of the card).

3. Gather up the cards, with the three cards in the column your student selected on the top of the deck. Show clearly how you do that.

4. Deal the cards face up in another 3 × 3 array making sure the top three cards of the deck end up in the top row of the array.

5. Ask your student to tell you what column their card is in now. The top card in that column is their card, which you can now identify!

6. Repeat the trick several times and ask your students to try to figure out how it works. You might give them hints by telling them to watch how you place the cards, or even by repeating the trick with a 2 × 2 array.

When your students understand how the trick works, you can ask the following questions:

- Would there be any point to the trick if the subject told the person performing the trick both the row and the column number of the card they had selected? Clearly there would be no trick if the performer knew
both numbers. Two pieces of information are enough to unambiguously identify a position in an array or graph. This is why graphs are such an efficient means of representation: two numbers can identify any location in two-dimensional space (in other words, on a flat sheet of paper). This discovery, made over 300 years ago by the French mathematician René Descartes, was one of the simplest and most revolutionary steps in the history of mathematics and science: his idea of representing position using numbers underlies virtually all modern mathematics, science, and technology.

You might ask your students how many numbers would be required to represent the position of an object relative to an origin in three-dimensional space. (The answer is three. Think of the origin as being situated on a plane or flat piece of paper that has a grid or graph on it. You need two numbers to tell you how to travel from the origin along the grid lines on the plane to situate yourself directly above or below the object, and one more number to tell you how far you have to travel up or down from the plane to reach the object.)

• Ask your students if the trick would work with a larger array. Have them try the trick with a 4 × 4 array. They should see that as long as the array is square (with an equal number of rows and columns), the trick works for any number of cards. Ask your students to explain why this is so and why the trick doesn’t work if the array isn’t square (for instance, try it with 2 columns and 6 rows).

• Ask your students if the original trick (i.e. with a square array) would work if the subject told the performer which row the card was in rather than which column. Have your students show you how the new trick would be performed. The fact that the trick works equally well in both cases illustrates a very deep principle of invariance in mathematics. In a square array, there is no real difference between the rows and columns. In fact, if you rotate the array by a quarter turn, the rows become columns and vice versa. More generally, once you fix an origin in space, it doesn’t matter how you set up your grid (the lines representing the rows and columns). In all cases you need only two numbers to identify a position.

Now draw an array of three columns and rows on the board and number the columns and rows:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>R</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>U</td>
</tr>
<tr>
<td></td>
<td>W</td>
<td>M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
</tr>
</tbody>
</table>

Point out a row and a column, and stress that we order rows from bottom to top, and columns from right to left. Ask several volunteers to locate the third column, second row, etc. Then ask your students to do the worksheets. The QUESTION 7 on the worksheets can be played as a game—one of the players gives the column and the row, the other has to mark the point according to the numbers. They might play “Hangman” hanging each other for incorrect answers.

**Assessment**

1. Join the dots in the given column and row:
   
   a) Column 3, Row 2  
   b) Column 1, Row 3  
   c) Column 3, Row 3  

   ```
   . . .  
   . . .  
   . . .  
   . . .  
   . . .  
   ```
2. Circle the dot where the two lines meet:
   a) Column 2, Row 3
   b) Column 3, Row 1
   c) Column 1, Row 1

3. Identify the proper column and row for the circled dot:
   a) ● ● ●
      ● ● ●
   b) ● ● ●
      ○ ● ●
   c) ● ● ●
      ● ● ●
   d) ● ● ●
      ● ● ●

   Column ___  Column ___  Column ___  Column ___
   Row ___    Row ___    Row ___    Row ___

Bonus
Which letters of the alphabet can be written on the grid and described in terms of rows and columns only? (See QUESTION 3 of the worksheet.) Which numbers can be written this way?

Extension
The card trick can be modified for non-square arrays if one allows one extra rearrangement. Deal out an array of 3 columns, 9 rows. Have a student select a card and tell you what column it’s in. Re-deal the cards so that all of the nine cards from the chosen column land in the top three rows of the new array. Ask the student to tell you what column their card is in now, and re-deal the top three cards in that column into the top row of a new array. Once the student tells you what column their card is in, you can identify the top card in that column as the one they selected.

This version of the trick illustrates a powerful general principle in science and mathematics: when you are looking for a solution to a problem, it is often possible to eliminate a great many possibilities by asking a well-formulated question. In the card trick one is able to single out one of 27 possibilities by asking only three questions. Repeat the trick, asking your students how many possibilities were eliminated by the first question (18), by the second question (6), and by the third (2).
Review the previous lesson. Draw an array of dots on the board, circle a point and ask your students to write the coordinates of the point: Column___, Row___. Ask your students: Imagine you have to write the coordinates of 100 points. Would you like to write the words “column” and “row” 100 times? What could you do to shorten the notation? Students might suggest making a T-table or even writing a pair of numbers, because the column is always the first number. **ASK:** How do you know, which is first, column or row? What if you have to ask a partner to find a dot given a pair of numbers, without telling them which number belongs to the column and which number is the row number? Give your students a pair of numbers, such as 2, 3 and do not tell them which one is the column number and which one is the row number. How many points can they find that could go with these two numbers? Suppose your partner does not speak English and you cannot show him, that you use column number first. How would they overcome this difficulty?

Explain to your students that mathematicians found the way out – they made an international convention: The place of the dot will be given by two numbers, put in parenthesis, and the column number is always on the left, and the row number is always on the right: (column, row). Give your students several pairs of numbers and ask them to identify the points on the grid. Explain that the pair of numbers is called "coordinates of the point". Include this term into the spelling test.

Draw an array of dots and label the columns by letters and the rows by numbers. Ask your students to identify some points, such as (A, 4), (B, 2), (C, 3). Mark the points (B, 3) and (C, 2) and ask your students to identify them. Repeat the exercise with a grid instead of an array. As a variation, you might mark both rows and columns with letters.

Students need lots of practice.

Review the names of special quadrilaterals and triangles before assigning these questions:

1. Graph the vertices A (1,2), B (2,4), C (4,4), D (5,2). Draw lines to join the vertices. What kind of polygon did you draw? How many lines of symmetry does the shape have? How many pairs of parallel sides does it have?

2. On grid paper, draw a coordinate grid. Graph the vertices of the triangle A (1,2), B (1,5), C (4,2). Draw lines to join the vertices. What kind of triangle did you make?

Ask your students if they have seen any of these methods of marking points with letters or numbers in real life. You might show them a map with a grid on it.
Assessment
1. Mark the positions: (3, 2), (4, 1).
2. Write the coordinates of the marked positions: _____, _____

Bonus
Draw the points below on a grid, then join the points in the order you’ve drawn them. Join the first and the last points. What shape did you make? Find the area of the shape.

a) (A, 4), (A, 5), (B, 5), (C, 4), (D, 4), (E, 3), (D, 3), (C, 2), (C, 1), (B, 2), (B, 3).
b) (0, 1), (2, 3), (2, 6), (3, 7), (4, 6), (4, 3), (6, 1), (6, 0), (5, 0), (4, 1), (3, 0), (2, 1), (1, 0), (0, 0).

Students will need a pair of dice of different colours. The player rolls the dice and records the results as a pair of coordinates: (the number on the red die, the number on the blue die). He plots a point that has this pair of coordinates on grid paper. He rolls the dice for the second time and obtains the second point in the same way. The player joins the points with a line. After that he has to draw a rectangle so that the line he drew is a diagonal of the rectangle.

There could be several rectangles drawn this way. If the line is neither vertical, nor horizontal, the simplest solution is to make the sides of the rectangle horizontal and vertical. In this case, ask your students if they see a pattern in the coordinates of the vertices.

ADVANCED: Students will need a pair of dice of different colours and a spinner below.

Player rolls the dice twice and plots the points the same way he did in the previous activity. He also spins a spinner. He has to draw a quadrilateral of the type the spinner shows, so that the line he drew is the diagonal of the quadrilateral.
G5-20
Introduction to Slides

For this lesson, a magnetic board with a grid on it (or an overhead projector with a grid drawn on a transparent slide) would be helpful. Let your students practice sliding dots in the form of a small circular magnet right and left, then up and down. Students should be able to identify how far a dot slid in a particular direction and also be able to slide a dot a given distance. If any students have difficulty in distinguishing between right and left, write the letters L and R on the left and right sides of the board.

After students can slide a dot in given direction, show them how to slide a dot in a combination of directions.

You might draw a hockey rink on a magnetic grid and invite volunteers to move a small circular magnet as if they were passing a puck. Sample questions and tasks:

Pass the puck three units right; five units left; seven units down; two units up.
Pass the puck two units left and five units up.
Position several small figures of players on grid intersections in various points of the rink and ask your students such questions as:

Player 3 passes the puck 5 units right and two units up. Who receives the pass?
Player 5 wants to pass the puck to Player 7. How many units left and how many units down should the puck go?

Bonus
Player 4 sent the puck 3 units up. How many units left should Player 7 move to get the pass?

Assessment
Sliding the dot:

a) 3 units right; 3 units up

b) 6 units left; 3 units down

c) 7 units left; 2 units up
ACTIVITY 1

**Ball Game**
The students are points on the grid, and you give directions such as: “The ball slides three units to the right”; the student with the ball has to throw it to the right place in the grid.

ACTIVITY 2

In the school yard, draw a grid on the ground. Ask your students to move a certain number of units in various combinations of directions by hopping from point to point in the grid.

ACTIVITY 3

**Memory Game**
Students will need a grid and several (1 to 4) small objects (play money of different values or beads of different colours could be used). The objects are placed on the intersections of the grid. Player 1 slides one of the objects while Player 2’s back is turned, and Player 2 then has to guess which object was moved and describe the slide. This game will become much easier when coordinates are placed on the grid and the students are familiar with the coordinate system. When students learn coordinate systems (in section G5-23), they will be able to memorize the coordinates of the objects. They can then compare the coordinates after the objects were moved with the coordinates before the objects were moved to determine exactly which object moved and how.
Tell your students the following story. You might use two actual figures to demonstrate the movements in the story.

Suppose you have a pair of two-dimensional figures and you wish to place one of the figures on top of the other. But the figures are very heavy and very hot sheets of metal. You need to program a robot to move the sheets: to write the program you have to divide the process into very simple steps. It is always possible to move a figure into any position in space by using some combination of the following three movements:

1. You may slide the figure in a straight line (without allowing the object to turn at all):

   ![Slide](image)

   **SLIDE**

2. You may turn the figure around some fixed point (usually on the figure):

   ![Turn or Rotation](image)

   **TURN or ROTATION**

3. And you may flip the figure over:

   ![Flip or Reflection](image)

   **FLIP or REFLECTION**
Two figures are congruent if the figures can be made to coincide by some sequence of flips, slides and turns. For instance, the figures in the picture below can be brought into alignment by rotating the right hand figure counter clockwise a quarter turn around the indicated point, then sliding it to the left.

![Diagram of congruent figures]

It is not always possible to align two figure using only slides and turns. To align the figures below you must, at some point, flip one of the figures:

![Diagram of non-congruent figures]

One way to flip a figure is to reflect the figure through a line that passes through an edge or a vertex of the figure. Tell your students that today you are going to teach them about slides.

Show students the following picture and ask them how far the rectangle slid to the right. Ask for several answers and record them on the board. You may even call a vote.

![Rectangle slide diagram]

Students might say the shape moved anywhere between one and seven units right. Take a rectangular block and perform the actual slide, counting the units with the students. The correct answer is 4.

Show another picture:

![Another rectangle slide diagram]
ACTIVITY

Give your students a set of pattern blocks or Pentomino pieces and ask them to trace a shape on dot paper so that at least one of the corners of the shape touches a dot. Ask students to slide the shape a given combination of directions. After the slide, trace the pattern block again.

This figure has a dot on its corner. How much did it slide? This time it is easier to describe the slide—just use the benchmark dot on the corner. Check with the block.

Show a third picture.

Is this a slide? The answer is NO, this is a slide together with a rotation. You cannot slide this block from one position to the other, without turning it.
G5-22
Slides (Advanced)

**GOALS**
Students will slide shapes on a grid, and describe the slide.

**PRIOR KNOWLEDGE REQUIRED**
Slide a dot on a grid
Distinguish between right and left

**VOCABULARY**
slide, translation
translation arrow

Draw a shape on a grid on the board and perform a slide, say three units right and two units up. Draw a translation arrow as shown on the worksheet. Ask your students if they can describe the slide you’ve made. If they have trouble, suggest that they look at how the vertex of the figure moved (as shown by the transition arrow). To help students describe the slide, you might tell them that the grid lines represent streets and they have to explain to a truck driver how to get from the location at the tail of the arrow to the location at the tip of the arrow. The arrow shows the direction as the crow flies, but the truck has to follow the streets.

Make sure your students know that a slide is also called a “translation”. Students should also understand that a shape and its image under a translation are congruent.

**Extensions**
1. Slide the figures however you want, and then describe the slide:

![Image of a grid with a slide example]

2. Describe a move made by a chess knight as a slide. Describe some typical moves of other pieces such as a pawn or a rook (castle).
Assign a letter to each row of desks in your class and a number to each column. Ask your students to give the coordinates of their desks. Then play “postman”—a student writes a short message to another student and writes the student’s “address” in coordinates. A volunteer postman then delivers the letter. The postman has to describe how the letter moved (two to the front and one to the left, for example).

Place a slide with a map of Saskatchewan on the overhead projector (see the BLM). Ask volunteers to find the cities on the map and to answer the questions:

- What are the coordinates of Saskatoon?
- What are the coordinates of Regina?
- What are the coordinates of Uranium City?
- What are the coordinates of Prince Albert?
- What can you find in the square A4? D5? D1?

Battleship Game
This game may be either played in pairs or a teacher can play against the whole class, when the class is guessing the teacher’s ships. You might also give some tips—when a player hits something, where can the other squares of the ship be? When the ship is sunk, where there are no ships?

Sample Placement:
ACTIVITY 2

ACTIVITY

Let your students draw their own map, possibly based on a book they are reading. Ask them to make their own questions of the same kind as on the worksheet and ask their partners to answer them.

G5-24

Reflections

GOALS

Students will perform reflections of points and shapes through a line.

PRIOR KNOWLEDGE REQUIRED

Symmetry

VOCABULARY

reflection symmetry mirror line symmetry line

Give your students an assortment of Pentomino pieces. Ask them to trace each piece on grid paper, draw a mirror line through a side of the piece and then draw the reflection of the piece in the mirror line. Students could check if they have drawn the image correctly by flipping the grouping of Pentomino pieces over the mirror line and seeing if it matches the image. Let your students know that a “flip” is also called a “reflection.” Students should notice that each vertex on the original shape is the same distance from the mirror line as the corresponding vertex of the image. Let your students practice reflecting shapes with partners: Each student draws a shape of no more than 10 squares, and chooses the mirror line. The partner has to reflect the shape over the given mirror line.

Advanced game: One student draws two shapes of no more than 10 squares so that the shapes are symmetric in a line but one square is misplaced. The partner has to correct the mistake.

Draw four points as shown and explain that two of these points are reflections of the other two. Challenge students to draw the mirror line. How do they know that the line they have drawn is the mirror line? Which point is reflection of which?
Write several words, such as **MOODY CAT IN A WOODEN BOX**, on a transparency sheet and project it onto the board in an incorrect way (flipped horizontally or vertically). Ask your students if they can read the text. Which words are still readable? Which letters look normal? Which transformation should be performed to make the text look completely normal? (A reflection.) Ask your students to draw the mirror line. Show them that a reflection in a mirror and a flip of the transparency sheet both achieve the goal.

**Bonus**

1. Students could try to copy and reflect a shape in a slant line, for example:

   ![Shape Reflections](image)

2. Sort all the capital letters of the alphabet into a Venn diagram:
   1. Letters that look the same after a reflection in a horizontal line.
   2. Letters that look the same after a reflection in a vertical line.

**Extensions**

1. Which letters of the alphabet look different after a horizontal or vertical reflection, but look the same after two reflections? **EXAMPLE:** If you reflect the letter E or L through a vertical line, the image faces backwards. If you reflect the image through a second vertical line, you produce the original letter. What happens if you reflect a letter first through a horizontal line, then through a vertical line?

2. Draw an equilateral triangle. If you reflect it through one of the sides and look at two shapes together, what shape will you get? Write down your prediction and check it. Is the result different for the other sides? Repeat with an isosceles triangle and a triangle with a right angle. Are the results different for different sides? Check all sides.

The following five extensions answer the demands of The Atlantic Curriculum for Grade 5.

3. Cyril experiments with a mirror and a straight line. He draws a straight line and puts the mirror across it. He looks at the angle between the line he drew and the mirror and at the angle between the reflection of the line and the mirror. He thinks that these angles are the same. Is he correct?

   Cyril turns the mirror and looks at the angle between the line he drew and its reflection in the mirror. The angle between the line and the mirror is 20°. How large is the angle between the line and its reflection? Cyril wants to put a mirror so that it is at a right angle with the line. What is the degree measure of a right angle? How large is the angle between the line and its reflection be when the mirror is at a right angle to the line? What does Cyril see in the mirror?
4. Boris experiments with a Mira and an angle. He draws an angle and places the Mira so that it touches the vertex on his angle and divides the angle in two. He rotates the Mira around the vertex until the angle on one side of the Mira and its reflection are the same. Using the mirror as a ruler he draws a line through the angle (starting at the vertex of the angle). He says that the line cuts the angle into two equal parts. Is he correct? The line that divides an angle into two equal parts is called a bisector.

5. Which of the points on the line L is closest to the point A? Estimate, then measure the distances between the points to check your prediction. Connect the points on the line with the point A. Measure the angles between the lines you drew and the line L. What is the angle at the point that is nearest to A?

6. Gleb wants to find the midpoint of a line segment AB using symmetry. He knows that a point and its image in a mirror are the same distance from the mirror. He puts a Mira across the segment AB and looks at the point A' (the mirror image of the point A). He also sees the point B through the Mira. He makes sure that the mirror is perpendicular to the line and he moves the mirror between the points A and B. What does he see in the Mira when it is in the middle of the line segment? Why does this happen exactly at the midpoint of the segment?

**Answer:** When the Mira is at the midpoint of the segment, Gleb sees that the points A and B coincide. This happens because the distance between the mirror and the point A (which is the same as the distance between the mirror and A') is now the same as the distance between the Mira and the point B.

7. A line that is both perpendicular to the given segment and passes through its middle is called a perpendicular bisector of a segment. How can Gleb use Mira to draw a perpendicular bisector of a segment? **(Hint:** Gleb and Cyril are friends.)
**GOALS**
Students will describe and perform rotations that are multiples of a quarter turn.

**PRIOR KNOWLEDGE REQUIRED**
Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
Clockwise
Counter clockwise

**VOCABULARY**
clockwise
counter clockwise
rotation

Review the meaning of the terms “clockwise” and “counter clockwise” using a large clock or by drawing arrows on the board. If you have a large clock, ask volunteers to rotate the minute hand—clockwise and counter clockwise—a full turn, half turn, and a quarter of turn. You might also ask your students to be the clocks: each student stands with a hand forward and turns clockwise (CW) or counter clockwise (CCW) according to your commands.

Draw several clocks on the board as shown below and ask your students to tell you how far and in which direction each hand moved from start to finish:

Then draw examples with only one arrow and ask students to turn the arrow:

a) $\frac{1}{4}$ turn CCW
b) $\frac{1}{2}$ turn CCW
c) $\frac{3}{4}$ turn CW
d) $\frac{3}{4}$ turn CCW

**Assessment**
1. Describe the rotation of an arrow:

2. Show the position of the arrow after each turn:

a) $\frac{1}{4}$ turn CW
b) $\frac{1}{2}$ turn CW
c) $\frac{3}{4}$ turn CCW
d) $\frac{3}{4}$ turn CW
G5-26
Rotations (Advanced)

GOALS
Students will describe and perform rotations of shapes that are multiples of a quarter turn.

PRIOR KNOWLEDGE REQUIRED
Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
Clockwise
Counter clockwise

VOCABULARY
clockwise
counter clockwise
rotation

Review the previous lesson by drawing several arrows or clock hands. To help your students visualize the effect of a rotation of a shape, have them make a small flag (as in QUESTION 1 on the worksheet) by taping a triangular piece of paper to a straw. Ask students to rotate the flag and trace its image after the rotation. Students could also cut out shapes similar to the other ones on the worksheet and trace the images of these shapes after a rotation. Then ask your students to trace or draw a figure, decide on rotation and draw the new shape without the prop. Students could also practice rotating pattern blocks or Pentomino shapes (around vertices of the shapes) on a grid.

Assessment
1. Draw the shape after each turn:
   a) $\frac{1}{4}$ turn CW
   b) $\frac{1}{2}$ turn CW
   c) $\frac{3}{4}$ turn CCW
   d) $\frac{3}{4}$ turn CW

Extensions
1. Using pattern blocks or cardboard polygons, trace the figure on a sheet of paper. Then choose a vertex and rotate the shape around the vertex $\frac{1}{2}$ turn. Trace the figure again. Would you get the same result if you had reflected the figure? Shapes that are particularly interesting in this case are right-angled trapezoids.

2. Trace the flower onto tracing paper and cut it out. Try to reflect it vertically, then rotate it $\frac{1}{4}$ turn CW. Draw the result. Now rotate and reflect the flower vertically. Is the result the same? Try various combinations of $\frac{1}{4}$ turn rotations and reflections. Which combinations give the same results? Why?

3. What happens to the line of symmetry of a figure after a rotation of $\frac{1}{4}$ turn? $\frac{1}{2}$ turn? If the shape has a vertical line of symmetry, will it have a vertical line of symmetry after either a $\frac{1}{4}$ or $\frac{1}{2}$ turn?
The following three extensions satisfy the demands of The Atlantic Curriculum (Expectation E11).

4. Hold up a large paper rectangle. Pin it to a piece of bristol board with a single pin (not in the middle of the rectangle) and trace the rectangle. Explain to your students that you want to turn the rectangle so that it returns to its original position. Explain that the place where the pin is stuck is called the **centre of rotation**. Ask a volunteer to trace the position of the rectangle after a \( \frac{1}{4} \) clockwise turn. Does the image look exactly the same as the original? No—the image is oriented differently. Make another \( \frac{1}{4} \) clockwise turn. Trace the image again and compare it with the original again. This time the rectangle is displaced. Repeat this exercise several times. Students should see that the rectangle will not return to its original position after a rotation that is less than 360° unless the centre of rotation is the centre of the rectangle.

Stick the pin in the centre of the rectangle. **ASK:** How much should I rotate the rectangle so that the image is exactly the same as the original? I rotate the rectangle and stop every time the position of the shape coincides with the original position. How many times do I stop until I make the full 360° turn? (2 times: once for a 180° turn and once for a 360° turn)

Explain to your students that a shape that returns to the original position before it completes the full turn is said to have **rotational symmetry**. Explain also that the number of times we stop during the complete rotation around the centre of the shape is called the **order of the rotational symmetry**. So a rectangle has rotational symmetry of order 2: we stop once after a half-turn and a second time after the full turn. Ask your students to predict the order of rotational symmetry for a square. Repeat the rotational exercise above (with the pin in the middle of the square) and check their prediction. The square has rotational symmetry of order 4, because we stop 4 times during the full turn. As a challenge, let your students find the order of rotational symmetry for an equilateral triangle (3) and a regular pentagon (5) and regular hexagon (6). Can they see a pattern?

5. What is the order of rotational symmetry of the flower in Extension 2? (**ANSWER:** One. This shape has no centre, and rotation around any point brings the flower to its original position only after a whole 360° turn.)

6. What is the order of rotational symmetry for other special quadrilaterals? (**ANSWER:** Parallelogram and rhombus have rotational symmetry of order 2. The trapezoid, kite and any other quadrilateral do not have rotational symmetry, so the order is 1).
Rotations and Reflections

Write on the board:

- Half-turn 270° $\frac{3}{4}$ turn
- Quarter of turn 180° $\frac{1}{4}$ turn
- Three quarters of turn 90° $\frac{1}{2}$ turn

Invite volunteers to join the degrees to the verbal descriptions of turns.

Explain to your students that today their task will be harder than the one you assigned in the last lesson—they will have to rotate and reflect shapes using grid paper. Draw a simple shape on a grid on the board and mark one of the corners with a dot. Highlight one of the sides passing through the dot. Highlight one of the sides passing through the dot. ASK: where will the side be located after a 180° rotation? Invite a volunteer to draw the rotated side. Highlight another side and ask another volunteer to draw its image after the rotation. Ask your students to finish the rotation of the shape.

Ask your students to describe the shape before and after the rotation. (EXAMPLE: The shape is three squares long and two squares wide. It is longer horizontally than vertically. It has an indentation of 1 square at the top-right corner.) Repeat the exercise with several more complicated shapes, including triangles (and with quarter turns).

Let your students practice combinations of rotations and reflections (only two transformations at a time). After some practice, draw a shape like the one above and ask your students the following question: Pam performed a reflection of a shape in a horizontal line through the top side and then turned the shape 90° clockwise around the marked corner. Padma first turned the shape 90° clockwise around the same corner and then reflected it in a horizontal line through the top side. Did they get the same result? You may call a vote, and after that check the results. ASK: Which transformation should Padma perform after the $\frac{1}{4}$ turn to get the same result as Pam? Why? (She should use a reflection through the right side, because it is the image of the top side after rotation.)
Assessment

Rotate the shape $\frac{1}{4}$ turn clockwise around the bottom corner and reflect it through the left side.

BONUS: Which single transformation is the same as a reflection in a vertical line and then in a horizontal line?

Extensions

1. Draw a coordinate grid on grid paper. Draw a triangle with vertices A (1,5), B(4,5), C(4,7). Draw a vertical mirror line through the points (5,0) and (5,7). Reflect the triangle in the mirror line. Then rotate the triangle $\frac{1}{4}$ turn counter clockwise around the image of vertex C. Write the coordinates of the vertices of the new triangle.

2. To rotate a shape $\frac{1}{4}$ turn clockwise around a point that is not a vertex:

   STEPS 1: Highlight the side that contains the point (the point is called the rotation centre).

   STEP 2: Rotate the side around the point.

   STEP 3: Draw an arrow from the rotation centre to one of the vertices of the shape.
STEP 4: Rotate the arrow to get the image of the vertex.

Repeat steps 3 and 4 if necessary for the other vertices.

STEP 5: Draw the image of the shape using the images of the vertices.

Rotate the figures 90° clockwise and counter clockwise around the shown points:
G5-28
Slides, Rotations and Reflections

GOALS
Students will perform and identify combinations of two transformations.

PRIOR KNOWLEDGE REQUIRED
Perform and identify a slide, a rotation or a reflection of a shape

VOCABULARY
rotation, centre of rotation
reflection, mirror line
slide, translation arrow

Draw a triangle in a square as shown and several results of transformations, such as:

Ask your students to identify the transformation used to get from the left triangle to the right triangle. If the transformation used was a reflection, the students should also identify the line of reflection, and if it was a rotation—its centre. Show a slide as well and remind your students how you identify the slide using a marked corner. Students could cut out copies of the triangle and use it to test their guesses.

Repeat the exercise above with two transformations made one after the other. These questions often have more than one answer, so encourage your students to find as many answers as they can.

A Game for Pairs
Ask your students to draw a non-symmetric shape on a piece of grid paper. The original shape is visible to both players at all times. Player 1 performs two transformations of her choice, and shows the result to her partner. Player 2 has to identify the transformations performed by her partner. She may present several answers, and each answer that gives the same result grants her a point. She has to guess the exact transformations that Player 1 used as well. For example, suppose Player 1 used the transformations “Rotate clockwise $\frac{1}{4}$ turn, then reflect through the right side.” If Player 2 says: “Reflect through the top side, rotate $\frac{1}{4}$ turn clockwise,” she gets 1 point but has to continue guessing.

Extension
Draw a coordinate grid on grid paper. Draw a trapezoid with vertices A (1,5), B(4,5), C(3,6), D(2,6). Rotate the trapezoid 90° counter clockwise around point D. Then slide the image 3 units right and 1 down. Write the coordinates of the vertices of the new trapezoid.
G5-29

Slides, Rotations and Reflections (Advanced)

**GOALS**
Students will perform and identify combinations of two transformations.

**PRIOR KNOWLEDGE REQUIRED**
Perform and identify a slide, a rotation or a reflection of a shape.

**VOCABULARY**
- rotation
- reflection
- centre of rotation
- mirror line
- slide
- translation arrow

Draw the shapes shown below on the board or on an overhead. Ask your students to identify the single transformation that takes each shape into the others.

Draw the shapes shown below on the board or on an overhead. Ask your students to find a pair of shapes that can be transformed into one another by a reflection, (A and B, for instance) and ask a volunteer to draw the mirror line. Draw a vertical mirror line through the left-hand side of A and **ASK:** What should I do to the image of A (after the reflection) to move it onto shape B?

Draw a translation arrow between the shapes C and D. Ask your students to identify the slide. Mark another vertex on shape C and ask your students to identify the vertex of D which is the image of the marked vertex. Ask them to draw the translation arrow between the new vertices. Students should notice that the second translation arrow is the same length and points in the same direction as the first.

Ask your students to find a pair of shapes that can be transformed into one another by a reflection, (A and B, for instance) and ask a volunteer to draw the mirror line. Draw a vertical mirror line through the left-hand side of A and **ASK:** What should I do to the image of A (after the reflection) to move it onto shape B?

Return to the shapes A and B. **ASK:** Can I pass from A to B using only reflections through the sides? How many reflections will I need? Can you tell which way the letter L (shape A) will point after each reflection without actually making the reflections? I reflected shape A seven times through vertical lines. Which way does it point now? I slid shape A in some direction. Which way does it point now? I turned it 90° clockwise. Which way does it point now? A harder question: I slid it, turned it 180° and reflected it through a vertical line. Which way does it point now?
Assessment
1. Describe a series of transformations that could be used to get shape A onto shape C. Give at least two answers.

2. Which transformations were used to move shape B onto shape D? How many answers can you find?

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G5-30
Building Pyramids

**GOALS**
Students will build a skeleton of a pyramid and describe the properties of pyramids.

**PRIOR KNOWLEDGE REQUIRED**
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon

**VOCABULARY**
- edge
- vertices
- triangular
- hexagonal
- base
- vertex
- pyramid
- pentagonal
- skeleton

Start with a riddle: “You have 6 toothpicks. Make 4 triangles with them. The toothpicks must touch each other only at the ends.” Let your students try to solve the riddle using toothpicks and modelling clay to hold the toothpicks together at the vertices of the triangles. The answer, of course, is the triangular pyramid. You might give your students the hint that the solution is three-dimensional.

Sketch a rectangular pyramid on the board and shade the base. Ask volunteers to mark the edges and the vertices (counting them, and making a tally chart). Write the words “base”, “edges”, “vertex” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a pyramid—first a base, then add an edge to each vertex of the base and join the edges at a point. The students should make triangular, square and pentagonal pyramids. Then let them fill in the chart and answer the questions on the worksheet.

After finishing the worksheet, they may check their prediction for the hexagonal pyramid by making one.

Tell your students that the shapes they have built are called “skeletons” of pyramids. You might write on the board the “equation”: “SKELETON = Edges + Vertices”. As animal skeletons are covered with flesh and skin, the skeleton of a pyramid can be covered with paper or glass or other substances and will have faces. Show a pyramid (with faces) and write the word “faces” on the board as well.

**Assessment**
Create and fill in the fifth row of the chart on the worksheet—for the heptagonal (7-sided) based pyramid.
Extensions

1. How many faces, edges, and vertices would a pyramid with a ten-sided base have?

2. Give a rule for calculating the number of edges in a pyramid that has a base with $n$ sides. (Use $n$ in your answer.)

   **SOLUTION:** A pyramid with $n$ edges in the base also has $n$ vertices in the base. But attached to each vertex in the base there is one non-base edge.

   Hence there are $n$ non-base edges and $n$ base edges. Therefore there are $2 \times n$ edges altogether in a pyramid with $n$ base edges. (So, for example, a pyramid with 5 base edges would have $2 \times 5 = 10$ edges altogether.)

3. Ask your students to bring to class pyramids or pictures of pyramids (Egypt, Mexico, Japan, entrance to Louvre, Paris, France and any others) that they can find at home. You can use the pyramids they brought in the lessons G5-33 and G5-34.

4. **HOMEWORK PROJECT:** Ask your students to find a picture of a pyramidal structure and give a presentation about it—what was the structure used for, when and where was it built, why does it have the pyramidal form.

**ACTIVITY**

On a class picnic build skeletons of pyramids and prisms from marshmallows and toothpicks or straws.
G5-31
Building Prisms

Sketch a prism on the board, shade the bases. Ask volunteers to mark the edges and the vertices (counting them and making a tally chart). Write the words “base”, “edges”, “vertex” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a prism—first make two copies of the base, and then join each vertex on one base to a vertex on the other base with an edge. Students should make triangular and pentagonal prisms and a cube. Let them fill in the chart and answer the questions on the worksheet.

After finishing the worksheet, students may check their prediction for the hexagonal prism by making one.

Ask your students what they have built. (Skeletons of prisms.) What are the “bones”? (The edges.) What do the skeletons need to become prisms? (Write the word “faces” on the board.)

Assessment
Create and fill in a fifth row of the chart—for the heptagonal (7-sided) based prism.

Extensions
The following two extensions were adapted from The Atlantic Curriculum.

1. Give each student two boxes that are both rectangular prisms (you may ask them in advance to bring various boxes from home. The length, width and height of the boxes should be different). Ask your students to tell you the proper mathematical name for the boxes (rectangular prisms) and ask them if they can find any faces that are congruent. Your students might find the congruent faces by tracing the faces on paper. How many congruent faces does each prism have? Ask your students to label the faces, so that the congruent faces are marked with the same letter. Then ask them to measure the edges and to write down the dimensions for each face. What happens if they join a pair of prisms along congruent faces? Ask your students to predict the dimensions of the new prisms and to check their predictions. How many faces of the prism double in size? (4) How many dimensions of the prism double? (1)

2. Hold up a triangular or pentagonal prism. Join it to the board by one of the faces and ask your students: Suppose the board is a mirror. Let’s look at the solid that is composed of both the prism and the reflection. Describe the solid and name it if you can. Use only prisms with equilateral bases for this activity. (More complicated bases could be used for very advanced students.) Let your students practice with a mirror. You can use a large mirror at the front of the class or smaller mirrors and smaller shapes that students could work with in groups. Are the results different if
you attach the side face or a base of the prism to the mirror? (Yes—the base gives a prism of double height, and the side face creates a prism with an irregular base.

For instance, if you use a triangular prism with an equilateral base, the resulting prism will have a rhombus as the base, and a pentagonal prism will produce an octagonal prism with an irregular base: . What happens if you repeat the process with a pyramid with an equilateral base? Will you still get a pyramid? (Generally, no.) Is it a prism? (No as well.)

3. Ask students to build the following shapes using interlocking cubes:
   a) a rectangular prism with a square base and side faces that are not square.
   b) a rectangular prism (with a square base) that is 3 times as high as the length of its base.

4. Take two copies of the Triangular Prism with a Scalene Base (see “Right Prisms” BLM). Join one of the non-base faces of one prism to a congruent face of the other prism. What 3-D figure do you get? What is the shape of its base? Turn one of the prisms upside down and repeat the exercise. Is the result different? Students can make predictions of the results they get with other non-base faces of the same prism and then check their predictions.

G5-32 Vertices, Edges and Faces

Remind your students that there are lots of 3-D shapes in the world around us that are either pyramids or prisms. As an example you might show them a photo of the pyramids in Egypt.

Hold up a 3-D shape and draw a picture of the shape on the board. Write the words “3-D shape”, ask volunteers to show the edges, the faces and the vertices, both on the shape itself and on the drawing, and write the terms “edge”, “face” and “vertex” on the board. Remind your students that the plural of “vertex” is “vertices”.

Your students will need the skeletons of the cubes they made during the last two lessons. Give each student 2 squares made of paper and 4 squares made of transparent material. Ask them to add:

• The non-transparent squares as the bottom face and the back face
• The transparent ones—as the top, the front and the side faces
Add the faces step by step, one face at a time, emphasizing the positions and names of the faces. It is a good idea to show the students how to add faces on a larger model.

Ask students to identify the edges of the cube that they see only through the transparent paper. If the transparent faces were made of paper, would they see these edges? No. The edges that would be invisible if all the faces were non-transparent are called the “hidden edges”. On a two-dimensional drawing of a cube these hidden edges are marked with dotted lines.

Ask students to hold their cubes in various positions (on the table, on the floor looked at from above, slightly above them and so on.) Ask students to describe what the faces look like when seen from different angles (they look like square, parallelogram, etc). The outline of the shape itself can look like a square, a rectangle, a hexagon, a trapezoid and a rhombus.

**Extension**

The following exercise was adapted from the Atlantic Curriculum.

Suggest that your students make models of various prisms and pyramids with play dough or modelling clay. Start with a model of a cube. Ask your students to cut off a single vertex of the model with piano wire or a plastic cutter.

![Cube with a vertex cut off](image)

Explain that the new face that appeared is called the **cross-section**. What shape does the cross section have? Ask your students: How many faces meet at a vertex of a cube? How many edges? (3) How many sides does the cross-section have? How many vertices? Why? (Each face of the cube produces an edge of the cross-section. Each edge of the cube produces a vertex of the cross-section.)

Ask your students to predict the shape of the cross-sections (resulting from cutting off vertices) for other shapes, first prisms, and then pyramids. For pyramids, first ask your students if they expect that all vertices will produce the same cross section, or there might be exceptions? (The point of the pyramid produces the shape with the same number of sides as the base, whereas the other vertices produce triangles.)
Prisms and Pyramids

Divide your students into groups. Give each group several 3-D shapes, so that each group has some rectangular and triangular pyramids, rectangular and triangular prisms and a cube. Ask your students to count the faces of the shapes. If some students are having trouble keeping track of the number of faces, they might mark each face with a chalk dot or a small sticker. Ask your students to count the edges and vertices on the 3-D shapes as well (they also might shade edges with chalk and mark vertices with stickers.) Ask them to write the results of their count in the table on the worksheet.

Draw a pentagonal pyramid and a triangular prism on the board and let volunteers count the edges, faces and vertices of these figures. Ask them to mark the edges and circle the vertices as they count.

Extensions

1. a) Pick 3 shapes and trace each of their faces (e.g. if you picked a square-based pyramid, trace all 5 of the faces on the pyramid—even if some of them are congruent). Compare the number of faces in your tracings with those in your chart from QUESTION 1: do you have the correct number of faces? Be sure to organize your work neatly so you can tell which faces go with which shapes.

   b) Underneath the faces for each shape, answer the following questions:

   (i) How many different-shaped faces does this 3-D shape have? What are they?

   (ii) Circle the face (or faces) that form the base of the 3-D shape. Copy and complete this sentence:

   “The base of a __________________ is a _______________."

2. Ask your students to try and add the number of faces and vertices of a cube and subtract the number of edges. The result is 2. What happens if you do that to another solid? (It will be 2 as well. This fact is known as Euler’s formula and was discovered by the great Swiss mathematician Leonard Euler in the 18th century.)

ACTIVITY 1

Show your students an example of a cone and a cylinder. Explain that a cone has one curved surface and one flat surface, while a cylinder has two flat surfaces and one curved surface. Ask students to find as many examples of pyramids, prisms, cones, and cylinders in the classroom as they can.
3. Let your students construct shapes from Polydrons—from regular 3-D shapes like prisms and pyramids to animals and castles. Count the faces, edges and vertices. Check if the Euler’s formula holds.

4. This exercise was adapted from the Atlantic Curriculum.

Suggest that your students make models of various prisms and pyramids with play dough or model clay. Start with a model of a cube. **SAY:** I want to make a cross-section of a cube, but this time I will cut off a single edge of the model. Ask your students to predict the shape of the cross-section that is parallel to an edge. Let them check the prediction by actual cutting their models with piano wire or plastic cutters.

![Diagram of a cube with edges labeled a, b, c, and d]

Ask your students to explain why the cross section has four sides. **CHALLENGING:** Why is it rectangular? (In the diagram, the edge that was cut off was perpendicular to face E. If the cross-section is parallel to the edge that was cut off then edges a and b will also be perpendicular to face E, and also to any line that lies in face E, such as line c. Hence lines a and b are perpendicular to line c (and by the same argument to line d).

5. Ask your students to predict what shape the cross-section of a cube will have if it is made by a plane parallel to one of the faces of the cube. Let your students check their predictions. Invite them to make cross-sections using various other planes, such as planes through various diagonals of the cube. Do these planes produce different shapes? What different quadrilaterals can they produce? (Rectangles, squares, trapezoids, parallelograms and rhombuses.)
**GOALS**
Students will distinguish between prisms and pyramids, and identify their bases.

**PRIOR KNOWLEDGE REQUIRED**
- Pyramid
- Prism
- Geometric shapes: triangle, rectangle, quadrilateral, pentagon, hexagon

**VOCABULARY**
- triangle
- rectangle
- quadrilateral
- pentagon
- hexagon
- prism
- pyramid
- base
- point

Hold up a pentagonal pyramid. Ask a volunteer to count the faces. What shapes are they? (A pentagon and five triangles.) Ask a volunteer to draw a pentagon and a triangle on the board. Ask another volunteer to write the number of faces that have each shape inside the shape itself.

Tell your students that in the pentagonal pyramid the face that is not the same shape as the others is the base. Hence in this pyramid a pentagon is the base.

In a pyramid there is always one base, unless the pyramid has all triangular faces (in which case it must be a triangular pyramid). The shape of the base of a pyramid gives the pyramid its name. Hence if a pyramid has a base with 5 sides, it is called a "pentagonal pyramid". A prism has two bases. The non-base sides of the prism are always rectangles.

Give your students a set of 3-D shapes (you can use the shapes from your collection or ask students to construct shapes using the nets from the "Nets for 3-D Shapes" BLM). Ask your students to place each shape on a piece of paper and trace the faces. Ask them to write the number of faces of the traced shape inside the shape the way you did on the board. Students should also write the name of the figure beside the base. If your students have trouble spelling the names of the figures, write them on the board. Later include them into a spelling test.

**Assessment**
1. Use three shapes for each student, each set should include a pyramid and a prism, and one shape with all faces congruent, like a cube or a triangular pyramid with equilateral faces. A good set: a pentagonal pyramid, a triangular prism and a cube.
   a) Place each shape—base downward—on a piece of paper and trace the base. (That way you can verify that each student knows how to find the base.)
   b) Write the name of the figure beside the base and indicate whether the figure has one or two bases.
   c) If all faces of the figure are congruent, indicate this.

2. Look at the shapes in QUESTION 3 of the worksheet. Which shapes are pyramids? Which shapes are prisms? Can there be a shape that is both prism and pyramid? Why not?
G5-35
Properties of Pyramids and Prisms

GOALS
Students will compare pyramids and prisms systematically. They will also describe the similarities and differences between polyhedra, cylinders and cones.

PRIOR KNOWLEDGE REQUIRED
Prism and pyramid
Counting faces, edges, vertices of polyhedron
Bases of prism and pyramid

VOCABULARY
- cylinder
- prism
- vertex
- edge
- base
- cone
- pyramid
- vertices
- face
- net

Give your students a collection of 3-D shapes or have them make a set of shapes using the “Nets for 3-D Shapes” BLM. Ask your students to order the shapes (pyramids and prisms separately) so that the number of edges in the base increases. After that they should fill in the chart:

<table>
<thead>
<tr>
<th>Picture of Base</th>
<th>Number of...</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>edges</td>
<td>vertices</td>
</tr>
</tbody>
</table>

Ask the students if they can see any patterns in the number of edges, vertices and faces of pyramids and prisms.

Look for the pattern rules in the chart not only vertically, but horizontally:

FOR PYRAMIDS: What do you have to do to the number of vertices in the base to get the number of vertices in the whole pyramid? (Add 1.) What do you have to do to the number of edges in the base to get the number of edges in the whole pyramid? (Multiply by 2.)

FOR PRISMS: What do you have to do to the number of vertices in the base to get the number of vertices in the whole prism? (Multiply by 2.) What do you have to do to the number of edges in the base to get the number of edges in the whole prism? (Multiply by 3.) What do you have to do to the number of edges in the base to get the number of faces in the whole prism? (Add 2.)

Ask your students to add a row to each chart and to fill it in so you can see if they can extend the patterns in the chart.

ASK: If you have a prism with 100-gon in the base, how many vertices does this prism have?

How many faces does a pyramid with 200-sided base have? And how many vertices?

Bonus
Fill in a row of your chart (no need to draw the base) for 1000-gon pyramid and prism.
Ask your students to draw the following chart in their notebooks and to fill it in, using the shapes they used for the previous exercise:

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangular Pyramid</th>
<th>Triangular Pyramid</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show your students how you can make a cone from a piece of paper. Ask them where they have seen this shape (cone of ice-cream, clown hat, etc). Does a cone remind some other geometric shape that they have studied in the lesson? (a pyramid) Let volunteers fill in the following chart on the board:

<table>
<thead>
<tr>
<th>Property</th>
<th>Pyramid</th>
<th>Cone</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>two</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>1</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Has a vertex opposite to the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of vertices = # of vertices in the base + 1</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>
Ask your students to imagine a pyramid with 1,000-sided base. Does it look like a cone? Write a comparison paragraph on the board:

- **SIMILARITIES**: Both shapes have one base and a vertex at the opposite end. A cone is like a pyramid with a circular base. The more sides the base of a pyramid has, the nearer it is to a circle and so the nearer the pyramid is to a cone.

- **DIFFERENCES**: Pyramid has many faces, one polygonal (base), and the other triangular. It has many edges that are straight lines. It has many vertices, not just the one at the point. A cone has only one flat face, that is a circle, and one curved “face”. It has only one curved “edge”.

Show your students how to make a cylinder from a piece of paper. Ask volunteers to fill in the comparison chart:

<table>
<thead>
<tr>
<th>Property</th>
<th>Prism</th>
<th>Cylinder</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>3</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of vertices in the base</td>
<td>many</td>
<td>0</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>2</td>
<td>2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>2, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>0</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of vertices = 2 x # of vertices in the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of faces = # of edges in the base + 2</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students to imagine a prism with 100-sided base. Does it look similar to a cylinder? Write a comparison paragraph on the board:

**SIMILARITIES**: The cylinder and the prism both have two bases. A cylinder is like a prism with a circular base. The more sides the base of a prism has, the nearer the base is to a circle. The nearer the base is to a circle, the nearer the prism is to a cylinder.

**DIFFERENCES**: A prism has many faces, two polygonal (bases), and the other rectangular. It has many edges that are straight lines. It has many vertices. A cylinder has only two flat faces, that are circles, and one curved “face”. It has only two curved “edges” and no vertices at all.
After doing the exercise of comparing cones and cylinders to pyramids and prisms your students should have a better idea how to answer the question on the worksheet that asks them to compare two 3-D shapes.

Students may try the first activity after that.

**Assessment**

1. Make a property chart for the rectangular and triangular prisms.
2. Write a paragraph comparing rectangular and triangular prisms using the chart.
3. Who am I?
   a) I have only rectangular faces.
   b) I have 8 faces, 6 of them are rectangles.
   c) I have 6 edges. No pyramid or prism has fewer vertices than I do!
   d) I am a prism with 9 edges.
   e) I have one circular base.
4. Which of these are right prisms?
Extensions

1. Word Search Puzzle (3-D Shapes): Please refer to the BLM.

2. Sketch a skeleton of a prism or pyramid. For EXAMPLE:

   ![Diagram of a prism and pyramid](image)

   Ask students to answer true or false to the following questions:
   
   • My bases are all triangles.
   • I have more vertices than edges.
   • All of my faces are congruent

   Ask more questions of this sort.

3. Select a prism and say how many pairs of parallel edges each face has.

4. Repeat the exercise from Extension 5 of the lesson G5-33 with a prism that is not a right prism. What is the shape of the cross-section parallel to an edge of the prism that joins the bases? Do different edges produce different cross-sections?
G5-36
Nets and Faces

GOALS
Students will create nets for pyramids, prisms and cubes.

PRIOR KNOWLEDGE REQUIRED
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon
Faces, edges, vertices of a 3-D shape

VOCABULARY
net vertices
face pyramid
edge prism
vertex cube
triangular pentagonal hexagonal

Draw a copy of the charts your students made in the previous lesson. Ask students if they see any relationships between the number of vertices, edges or faces in a prism or pyramid and the shape of the base. Ask your students what patterns they see, and ask them to give geometrical reasons for these patterns. For EXAMPLE, the number of vertices in a pyramid is one more than the number of vertices in the base, and the number of edges in a pyramid is twice the number of edges in the base. You might prompt your students to think about the construction of a pyramid to see if they can explain these relationships.

Pyramids are constructed by connecting all vertices in the base to an additional vertex at the top of the pyramid. Hence the number of vertices in a pyramid is one more than the number of vertices in the base. Each pyramid has two types of edges—the base edges and the non-base edges. The number on non-base edges is equal to the number of base vertices (since each base vertex is connected to the vertex at the top of the pyramid by a non-base edge). But the number of base vertices is equal to the number of base edges. Hence the total number of edges in the pyramid is twice the number of edges in the base.

The number of vertices in a prism is twice the number of vertices in the base, because all the vertices belong either to the top base or to the bottom base. Each prism has three types of edges—the edges in the top base, the edges in the bottom base and the edges that connect the vertices in the top with the vertices in the bottom. The top and bottom bases are congruent, so they have the same number of edges. The connecting edges join each vertex in the bottom with a single vertex in the top. Hence the number of connecting edges is equal to the number of edges in each base (in either the top or bottom base), so the total number of edges in the prism is three times the number of edges in the base. The number of faces in the prism is two more than the number of edges in the base, because the number of side faces of the prism is the same as the number of edges in the base, (then you add two for the top and bottom base).

Draw several shapes on the board and ask your students which 3-D shape they make.

SAMPLES:
Hold up a rectangular or a pentagonal pyramid. Ask your students to tell you which two types of faces it has. How many bases does it have? And what is the shape of the side faces? How many side faces does it have? If you need to make a net for this pyramid, the easiest way would be to start with a base (draw it on the board and write “base” on it) and to add a side face along each edge of the base (draw one side face and ask volunteers to draw the rest). Ask students to cut out the nets of the pyramids from the BLM “Nets for 3-D Shapes” in this manual—they can use the shapes later to help find the answers to QUESTION 1 on the worksheet.

Ask your students, what happens if you cut off one of the side faces and try to re-glue it at some other place. They might actually cut off one of the triangles and try to fit the triangle to some other edge. You might draw an example on the board. Try the following positions: will the shape fold into a pyramid if the face is glued in this position?

- (no)
- (yes)
- (no)

At this point, you may let your students do the first activity. When students have finished the activity, draw the picture below on the board and ask if this will work as the pentagonal pyramid net:

- (yes)

Invite volunteers to come up to the board and draw pictures that will not make a net for a pyramid (not necessarily pentagonal). For each drawing, ask your students to explain why this picture is not a net of a pyramid. Invite volunteers to change the drawing so that it will make a net.

Hold up a triangular or a pentagonal prism. Ask your students, which two types of faces it has. How many bases does it have? What is the shape of the side faces? How many side faces does it have? If you wanted to make a net for this prism, the easiest way would be to start with the band of rectangles for side faces and to add the bases. Illustrate this on the board.
Repeat the exercise of cutting off a face of the prism and attaching it in some other place. Do that separately for a base and a side face. Ask students to cut out the nets of the prisms from the BLM “Nets for 3-D Shapes”. Let them cut off the faces and try to rearrange them at other places.

Draw several examples of “nets” of triangular prism on the board and ask volunteers to explain why these drawings cannot serve as nets for prisms:

- The bases are not the same
- The middle face is too short
- The bottom base is flipped
- Side face is missing

Invite volunteers to draw more pictures that will not work as prism nets, and ask the class to guess why these drawings cannot be prism nets. For a more challenging task, ask volunteers to draw pictures that might or might not work as nets, and let the class guess if these are nets of prisms.

**ACTIVITY 1**

Give students square or pentagonal pyramids and ask them to trace the faces on a piece of paper, so that they create a net. Ask them to cut out the nets they have drawn. Let them cut off faces of the net and re-attach the faces at different places. Will the new net fold into the same pyramid? Which edges are places where you would want to re-glue the faces and which are not? Repeat this exercise with a prism. This activity is important—students will explore various ways to create nets for the same solid, rather than memorizing a single net shape.

**ACTIVITY 2**

Allow your students to play the following game: one partner (or group) draws a net, the other has to guess what shape the net can be folded into. (Use the nets they have created in the previous activity.)

**ACTIVITY 3**

Give your students Pentomino pieces made of paper or bristol board. Which ones can be folded into a square box without a lid? Let them first try to predict the result, then check it. This activity is a good preparation for Extension 1.
**ACTIVITY 4**

Build-a-Net Game:
Divide your class into groups of 3 to 4 students. Give each group a set of polygons from the BLM “Build-a-Net Game”. Each student receives 6 shapes, the rest are left in a pile on the side. If a student has a set of polygons that form the net of a pyramid or a prism, she is allowed to turn the set in for a point and take an equal number of new cards from the side pile. Students take turns placing polygons in the central pile. Each time a student places a polygon in the central pile, they pick up a new shape from the side pile. As soon as a student places a polygon in the central pile that can be combined with other polygons in the pile to make a net, that player is allowed to remove the net and scores a point. The game ends when there are no cards left in the side pile. The winner is the player who has created the greatest number of nets.

**NOTE:** Review the nets of prisms and pyramids that can be built from the shapes in the “Build-a-Net Game” BLM before starting the game. Ask your students: Which shapes can only be used for one or two nets? (pentagon and hexagon) Which shapes can be used for more than two nets?

**ACTIVITY 5**

After students have constructed the pyramids and prisms from the BLM “Nets for 3-D Shapes”, ask them to sketch the nets from memory. You might also ask them to sketch what they think the net for a hexagonal pyramid would look like.

**ACTIVITY 6**

Predict whether the net shown will make a pyramid. Copy the shape onto grid paper. Cut the shape out and try to construct a pyramid. Was your prediction correct?

**ACTIVITY 7**

Sketch the net for a prism by rolling a 3-D prism on paper and tracing its faces. For reference, use the steps shown (at right) for a pentagonal prism:

1. **Step 1:** Trace one of the non-base faces.
2. **Step 2:** Roll the shape onto each of its bases and trace the bases.
3. **Step 3:** Roll the shape onto each of its remaining rectangular faces and trace each face.
ACTIVITY 8
Find a prism or pyramid at home or at school and sketch its faces.

The next two activities fulfil the demands of the Ontario Curriculum.

ACTIVITY 9
Does this make a net of a 3-D shape? Cut out and check your prediction.

(no)

ACTIVITY 10
What shapes do these nets make? Cut them out and check.

Extensions

1. Copy the following nets onto centimetre grid paper (use 4 grid squares for each face).
   Predict which nets will make cubes. Cut out each net and fold it to check your predictions.

   After that draw as many different nets for a cube as you can.
2. Give your students many toothpicks of various lengths and some modelling clay. Ask them to make several triangular pyramids with different bases. For example, students might make acute-angled, right-angled and obtuse-angled triangle bases. Then ask your students to choose two different triangular pyramids and trace their faces to obtain nets for them. Ask your students to compare the nets. Students might also measure the angles of the faces with protractors, and the sides with rulers. Ask your students to make a triangular pyramid with…

   a) …more than one right angle.
   b) …at least two obtuse angles.

ADVANCED:

c) …two right angles and one obtuse angle.
d) …two right angles and two obtuse angles.

3. Copy the shapes into your notebook. How many hidden faces does each shape have? How many hidden edges does each shape have? Draw in the missing edges with dotted lines.

![Shapes](image)

4. How many faces of the little cubes are on…

   ![Cube](image)

   a) the outside of the figure? (including the hidden faces)
   b) the interior of the figure?

5. On a cube draw a line from vertex A to B, then a line from B to C. What is the measure of ABC?

   ![Cube](image)

   HINT: Imagine drawing a line from vertex A to C on the hidden face. What kind of triangle is \( \triangle ABC \)?

   ANSWER: \( \triangle ABC \) is an equilateral triangle. So \( \angle ABC = 60° \)

6. Each edge of the cube was made with 5 cm of wire.

   ![Cube](image)

   How much wire was needed to make the cube? (Don’t forget the hidden edges.)
G5-37

Sorting 3-D Shapes

GOALS

Students will sort 3-D shapes according to their properties.

PRIOR KNOWLEDGE REQUIRED

Properties of pyramids and prisms
Venn diagrams

VOCABULARY

cylinder
cone
prism
pyramid
base
vertex
vertices
edge
face

Give each student (or team of students) a deck of shape cards and a deck of property cards. These cards are in the “3-D Shape Sorting Game” BLM. (If you have enough 3-D shapes have students use 3-D shapes instead of the cards.) Let them play the following games:

3-D Shape Sorting Game: Each student flips over a property card and then sorts the shapes onto two piles according to whether a shape on a card has the property or not.

Students get a point for each card that is on the correct pile. (If you prefer, you could choose a single property for the class and have everyone sort the shapes using that property.)

Once students have mastered this sorting game they can play the next game.

3-D Venn Diagram Game: Give each student a copy of the Venn diagram sheet in the BLM section (or have students create their own Venn diagram on a sheet of construction paper or bristol board). Ask students to choose two property cards and place one beside each circle of the Venn diagram.

Students should then sort their shape cards using the Venn diagrams. Give 1 point for each shape that is placed in the correct region of the Venn diagram.

Assessment

Use the shapes below to complete the following Venn diagram:

1. One or more rectangular faces
2. One or more triangular faces

Extension

Draw a Venn diagram to sort the shapes on the worksheet according to the properties:

1. Pyramid
2. One or more triangular faces.

What do you notice about your Venn Diagram? Explain why part of one of the circles is empty.
GOALS
Students will identify polygons that tessellate. They will create tessellation patterns using a variety of shapes.

PRIOR KNOWLEDGE REQUIRED
Degrees
Sum of angles in a triangle is 180°
A straight angle has measure 180°
Polygons

VOCABULARY
polygon vertex
vertices quadrilateral
pentagon hexagon
triangle octagon
tessellate degree

tessellation

Explain to your students that a tessellation is a pattern made up of one or more shapes that completely covers a surface, without any gaps or overlaps. One example of a tessellation is a floor tiling with polygons (or other shapes). If you can tile the floor with one shape, this shape is said to tessellate. Ask your students: Do you know of any shape that tessellates? Your students will almost certainly say “square”, but they might also name other shapes, such as rectangles (ask them to draw a picture: there is more than one way to tessellate with rectangles), diamonds, other parallelograms, triangles or hexagons. Show a picture of a beehive. Is it a tessellation? If there are any interesting tessellation patterns around the schools, mention them as well. Use the first two activities.

Present a problem:

Joshua says that he can tile the floor of the bathroom using only regular octagons. Is this correct?

Josef says that he can tile the floor using only regular pentagons. Is this correct? Do they encounter the same problem?

Let your students use paper pentagons, hexagons and octagons to check whether these figures tessellate. Ask volunteers to sketch tessellations with triangles, squares and hexagons on the board.

Ask your students: what is the degree measure of a straight angle? Draw a straight angle and ask what the degree measure around the vertex is. There are two straight angles, so there are 360° around the point. Draw a picture of three line segments shaped like the letter Y and ask your students what the sum of the angles around the vertex should be. If the three angles in the Y are the same, what is the measure of each angle?

Draw a table on the board with the following headings (first five columns) and the shapes below (add the rightmost column later, as part of the discussion outlined below). Use as many volunteers as possible to fill in the table:
<table>
<thead>
<tr>
<th>Regular polygon</th>
<th>Number of vertices</th>
<th>Number of triangles</th>
<th>Sum of angles</th>
<th>Degree measure of one angle</th>
<th>6 shapes tessellate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>180°</td>
<td>180° ÷ 3 = 60°</td>
<td>6 × 60° = 360°, 6 shapes tessellate</td>
</tr>
<tr>
<td>Square</td>
<td>4</td>
<td>2</td>
<td>360°</td>
<td>360° ÷ 4 = 90°</td>
<td>4 × 90° = 360°, 4 shapes tessellate</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>540°</td>
<td>540° ÷ 5 = 108°</td>
<td>3 × 108° = 324°, 4 × 108° = 432°, does not tessellate</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>720°</td>
<td>720° ÷ 6 = 120°</td>
<td>3 × 120° = 360°, 3 shapes tessellate</td>
</tr>
<tr>
<td>Regular octagon</td>
<td>8</td>
<td>6</td>
<td>1080°</td>
<td>1080° ÷ 8 = 135°</td>
<td>2 × 135° = 270°, 3 × 135° = 405°, does not tessellate</td>
</tr>
</tbody>
</table>

Ask your students to identify the rules for the sequences they see in the columns of the table. You might wish to ask them to write a formula for the sequences in the second, third and fourth columns. Ask them to use the formulas to add a line for the regular octagon to the table.

Ask your students which polygons tessellate and how many polygons meet at a vertex in each tessellation. **ASK:** Could you predict that 6 equilateral triangles are needed to fill the space around a vertex from the data in the table? Which column would you use? Can you write a multiplication statement that explains why 6 equilateral triangles around a vertex leave no gaps and do not overlap? Add the sixth column and fill in the information about triangles. Repeat with squares and hexagons.

**ASK:** What happens when you try to arrange a set of pentagons around a vertex? Ask students what will happen if they try to place three or four pentagons around a vertex. Students should see that the sum of the interior angles of three pentagons arranged around a vertex is 324°. Since this is less than 360° the three pentagons will not completely fill the space around the vertex. Four pentagons will not tessellate the space around the vertex either, since the sum of the four interior angles is 432°, which is greater than 360°, so one of the pentagons will overlap the others.

Ask your students: Have you ever seen floor patterns made with shapes that are not regular polygons? Or patterns made with several kinds of tiles?

Draw a grid and the following pattern on the board (or use an overhead projector) and ask your students to extend the design:
Is this the only way to tessellate with this shape? (No, you can also use a reflection, a rotation or a combination of two or three transformations.)

Suggest your students to draw various letters of alphabet on grid paper and to check whether they tessellate.

**Bonus**

Suggest that your students create a design using regular octagons and squares with equal sides on grid paper. **HINT:** Two regular octagons fill 270° around a vertex. How many degrees do you need to make the full 360°? What shape has this angle?

---

**ACTIVITY 1**

John says that he can tile the floor of the bathroom with quadrilaterals. Any quadrilaterals!

Fold a sheet of paper three times, so that there are eight layers; draw a quadrilateral on it and cut it out, cutting through all the 8 layers. Try to arrange the 8 quadrilaterals so that they do not have gaps and do not overlap to check if John is right. Check various quadrilaterals, not only the special ones!

---

**ACTIVITY 2**

Repeat the activity above with triangles. Note that two congruent triangles create a quadrilateral.

---

**ACTIVITY 3**

Divide your students into groups and give each group one of the sets of shapes listed below. Ask them to create as many different tessellations as possible using one or more shapes from their set. Let the students share their designs when they are finished. You might ask them to construct the shapes using a ruler and a protractor as well.

- Set 1: A square and a regular octagon. Side length 5 cm for both.
- Set 2: A rhombus with angles 60° and 120° and an equilateral triangle. Side length 5 cm for both.

(Continued on next page.)
(Continued from previous page.)

- Set 3: An isosceles trapezoid with sides and smaller base of 5 cm and base angles of 135° and a square with side of 5 cm.
- Set 4: An isosceles trapezoid with sides and smaller base of 5 cm and base angles of 60° and an equilateral triangle with side of 5 cm.
- Set 5: A regular hexagon and an equilateral triangle. Side length 5 cm for both.
- Set 6: A rectangle with sides of 5 and 10 cm, and a square with side length of 5 cm.

**Extension**

**PROJECT:** Research the designs by M. C. Escher, choose a drawing and check:

- Which polygons were used to create a tessellation?
- Which transformations were used to create the tessellation?

**POSSIBLE SOURCE:**

http://www.mcescher.com/
G5-39
Making Patterns with Transformations

GOALS
Students will perform simple transformations on patterns. They will create patterns that look the same after various transformations.

PRIOR KNOWLEDGE REQUIRED
Clockwise, counter clockwise 1/2, 1/4, 3/4 turn Reflection Mirror line Lines of symmetry

VOCABULARY
pattern clockwise counter clockwise 1/2, 1/4, 3/4 turn

Review the meaning of the terms clockwise and counter clockwise. Draw several arrows at various positions and ask your students to draw the final position after 1/2 turn clockwise, 1/4 turn counter clockwise, 3/4 turn clockwise, 3/4 turn counter clockwise.

Draw a 2 x 2 grid on the board and make a simple pattern on the grid, such as:

Ask your students to draw this pattern on grid paper and to cut it out. Ask them to turn the design 1/2 turn clockwise around a given vertex and to draw the result in a separate grid. Repeat with several other turns.

Show your students a method to draw the result without the prop. Start with a simple 2 x 2 grid like the one shown, mark the top-left corner with a dot and shade the top side. ASK: Where will the dot be after 1/4 clockwise turn? Invite a volunteer to draw the dot on a separate grid, then ask where the shaded line will go and invite another volunteer to draw it. After that ask a volunteer to shade the squares after the turn. Draw several simple designs on 2 x 2 and 3 x 3 grids and ask your students to draw the results of various turns using this method.

An alternative method to form mental pictures of rotations of figures:

STEP 1: Shade some of the squares on a 3 by 3 grid.

STEP 2: Imagine rotating the grid by a certain amount around any of the vertices; say, 1/4 turn counter clockwise. Mark the side of the grid that would be on top after the rotation. (Students should notice that the same side will be on top no matter which of the vertices they use as the centre of rotation.)

If students have trouble with this step, have them practice it with different directions and sizes of rotations.

STEP 3: Show what the figure would look like if the side marked with an × was on top.
(You can turn your head when you look at the original shape to see what it would look like if the side marked with an × was on top.)

Here is a set of **SAMPLE QUESTIONS** you could assign to help with this step. In each question, show what the grid would look like if the side marked with an × was on top.

a) 

b) 

c) 

d) 

e) 

f) 

For variation you could ask what the shape would look like if the marked side was rotated to various positions, for instance:

Draw a 3 × 3 design, like the one in the picture below and ask your students to reflect it in each of the sides.

Which results look the same? Repeat with several designs, including ones with horizontal, vertical or both lines of symmetry. Include also a design that does not have a line of symmetry.

Ask your students to draw lines of symmetry in each of the designs they used that have lines of symmetry. You can suggest that your students make a table with headings:

<table>
<thead>
<tr>
<th>Design</th>
<th>Has a horizontal line of symmetry</th>
<th>Has a vertical line of symmetry</th>
<th>Does not change after reflection in a horizontal line</th>
<th>Does not change after reflection in a vertical line</th>
</tr>
</thead>
</table>

Ask your students if they see any relations in the columns. Then ask them to draw a design that would look the same after a reflection in both a horizontal line and a vertical line.

For a challenge, ask them to draw a design that looks different in a vertical or a horizontal reflection, but such that the results of both reflections are the same. You might suggest that students should look at shapes that have lines of symmetry that are not horizontal or vertical. (**ANSWER:** A design that does not have a vertical or a horizontal line of symmetry, but has both diagonals as lines of symmetry.)
Ask your students to make a design that will look the same after a $\frac{1}{2}$ turn. Invite volunteers to draw their designs on the board. Ask your students to draw lines of symmetry in their designs.

Then ask the following question: Suppose a design does not change after a $\frac{1}{2}$ turn. Does it mean it has a line of symmetry? If students cannot find an example of a shape that looks the same after a half turn, but does not also have a line of symmetry, draw the following **EXAMPLE:**

```
+
+
+
+
```

Challenge students to find a pair of squares such that the first square will be rotated onto the second and the second onto the first. Students can create more complicated solutions by finding other pairs of squares that are the images of one another under rotation.

Ask your students to make some $5 \times 5$ designs that stay the same after a $\frac{1}{2}$ turn, where not every square is shaded. (Note that it does not matter whether the turn is clockwise or counter clockwise. If the pattern stays the same after a $\frac{1}{4}$ turn in one direction, it will stay the same after a $\frac{1}{4}$ turn in the other direction.)

Students might also shade a corner square and shade successive images of the square after a $\frac{1}{4}$ turn. After they have done this three times they will have shaded all the corner squares and will have a pattern that does not change.

If students have trouble finding a solution you might suggest the following method:

Start by shading a square and ask where it will be located in the $5 \times 5$ grid after a $\frac{1}{4}$ turn. If we want our design to stay the same, then the square that is the image of the original square should be shaded in the pattern as well. However, this square will not stay in its place—it will move. Where to? Ask a volunteer to shade the place where the second square is located after the rotation. Repeat shading squares until you get the original square shaded.

**Assessment**

1. Circle the designs that do not change after $\frac{1}{2}$ turn. Put an $\times$ through the designs that do not change after $\frac{1}{2}$ turn.

```
+
+
+
+
```

2. Choose a design that was not circled or crossed and reflect it through a horizontal line. Then turn the result $\frac{1}{4}$ turn clockwise.

3. Create your own $3 \times 3$ design that stays the same after a reflection in a vertical line.

4. Create your own $4 \times 4$ design that stays the same after $\frac{1}{2}$ turn.

**Extension**

How many lines of symmetry can be in a square pattern that does not change after $\frac{1}{2}$ turn? (0, 2 or 4). Try to create a $3 \times 3$ pattern with 1 line of symmetry that does not change after $\frac{1}{2}$ turn. Shade a square on one side of the line. Shade its image after the rotation. Shade the images of both after reflection through the line of symmetry. How many lines of symmetry does the resulting shape have? Repeat with $5 \times 5$ grid, start by shading the square with coordinates (4, 5).
G5-40
Patterns with Transformations (Advanced)

GOALS
Students will create patterns with transformations.

PRIOR KNOWLEDGE REQUIRED
Clockwise, counter clockwise
$\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ turn
Reflection
Mirror line

VOCABULARY
pattern
clockwise
counter clockwise
$\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ turn

Hold up a square grid (with a pattern of shaded squares) like one of those you used last lesson. Pin the square to a piece of bristol board with a single pin (not in the middle) and trace it. Explain to your students that you want to turn the square $\frac{1}{4}$ turn so that it returns to its original position. Remind your students that the place where the pin is stuck is called the centre of rotation. Ask a volunteer to trace the position of the square after the rotation. Repeat this exercise several times. Students should see that the square will not return to its original position after a rotation unless the centre of rotation is the centre of the square.

Now place the pin in the corner of the square and ask students to imagine the image after a $\frac{1}{4}$ turn. Shade one of the sides of the square passing through the centre of rotation and invite a volunteer to shade its image after rotation. What is the angle between the shaded lines? (90°) Then invite volunteers to draw the image of the rotated square. Repeat with more complicated designs, with triangles and various colours or even striped patterns.

Ask your students to draw a $3 \times 3$ square on grid paper with an asymmetric design. Ask them to create a larger design by rotating the square repeatedly counter clockwise around the top-right corner and drawing the result.

Draw a simple asymmetric shape, like the one below:

Ask your students to create a pattern by: sliding the shape repeatedly 2 units right, reflecting the shape repeatedly in a line through the right side or rotating it half-turn (how many degrees is that?) around the marked point.

As a bonus question, ask your students to create a pattern using two transformations, say, first a reflection, then a $\frac{1}{4}$ turn, then repeat.

ACTIVITY 1
Have students answer questions 1, 2 and 4 by making designs on grid paper, cutting them out and turning them.
Extensions

1. Mark the centres of rotation to get:

- Shape B from Shape A (Note that the centre will be in the middle of the bottom edge of A, not on a vertex)
- Shape C from Shape A
- Shape D from Shape B
- Shape D from Shape C

2. Which transformation was used to get Shape D from Shape A?

3. Vinijaa moved shape A to Shape C without using a rotation. Describe what she did. If she used reflection—draw the mirror line, and if she used a slide—show the translation arrow and describe the slide.

4. Draw reflection of shape A through the right (slanted) side.
Isoparametric Drawings

GOALS
Students will draw simple shapes built from interlocking cubes on isometric dot paper.

PRIOR KNOWLEDGE REQUIRED
Understand a drawing on isometric dot paper

VOCABULARY
isometric dot paper
top view
interlocking cubes

Project a sheet of isometric dot paper onto the board using the overhead projector. Show students how to draw a cube using the dots. Start from the top face, then draw the vertical edges (no hidden ones!), and then draw the visible bottom edges.

Step 1

Step 2

Step 3

Explain that to create an isoparametric drawing, it helps to start from the top. Look at the topmost layer and draw the top face or faces first. Then draw the vertical edges that are part of the topmost layer as you did with the single cube.

Hold up a shape made with three cubes:

Invite a volunteer to draw the top layer (a single cube). What does the next layer look like? It consists of two cubes. Take two cubes locked together and compare this shape to the original shape: Which edges of the new shape are hidden (by the top cube) in the original shape? Which visible edges of the original shape are already drawn (because they are the bottom edges of the cube)? Ask a volunteer to draw the remaining visible edges of the second layer.

You may wish to do the worksheets as a class—so that volunteers draw pictures from the worksheet on the board. Students may find it easier to copy a shape onto isoparametric dot paper if they start by shading the top layer of the shape.

Give your students some interlocking cubes and ask them to build the figures from QUESTION 2 on the worksheet.
Building and Drawing Figures

GOALS
Students will build shapes from interlocking cubes and draw top views of shapes drawn on isometric dot paper.

PRIOR KNOWLEDGE REQUIRED
Understand a drawing on isometric dot paper

VOCABULARY
isometric dot paper
top view
interlocking cubes

Explain that sometimes it is hard to read the drawing on the isometric dot paper, because some cubes are hidden and some edges overlap. Project the drawing given here on the board as an example:

Explain that a very convenient way to understand the drawing is to try to construct the “mat plan” of the shape. A mat plan represents the bottom level of the shape. In this case the mat plan would look like this:

Shade one square on the mat plan, as shown. Invite a volunteer to shade the column that stands above this square in the isoparametric diagram. How many cubes are in it? (3) Ask the volunteer to write “3” in the shaded square. Ask more volunteers to finish the mat plan. Remove the drawing and ask another volunteer to construct the shape using the mat plan. Give your students more examples like those in QUESTION 2 of the worksheet.

Assessment
Draw the mat plan and build the shape from interlocking cubes:
Extension

Draw the figures from the worksheets on regular dot paper. For example:

![Figure example]

G5-43

Geometry in the World

The worksheet G5-43: Geometry in the World is an extension and review worksheet. It can complement the presentation of cross-curricular projects. Here are some project ideas:

**Symmetry:**
1. Flags and Coats of Arms of Canadian provinces/cities. Which ones have lines of symmetry? Which ones have more than one line of symmetry? (None!)
2. Flags of the world. Make a list of world countries with flags that have 2 lines of symmetry.
3. Coats of Arms of Soccer/Baseball/Hockey clubs. Which ones have lines of symmetry? Which ones have more than one line of symmetry?
4. Cultural diversity: Alphabets. Use a non-Latin alphabet to find letters that have lines of symmetry. Are there symbols that have more than one line of symmetry?
5. Make several designs of snowflakes. How many lines of symmetry do your snowflakes have?

**Geometry in everyday life:**
1. Bee hives and hexagons—Research why bees build hexagonal shapes in the hive: Why not rectangular or triangular shapes?
2. Bridges—Which geometric shapes are used in bridges design? Try to build a bridge with rectangles. Put a heavy weight on it and watch the bridge collapse. Triangles are rigid—you cannot change the shape of the triangle without altering the side lengths. If you add diagonals to the rectangles, the bridge will stand, but then it is built with triangles.
3. Floor patterns: Make your own floor pattern. Choose a tessellation made by regular polygons using slides only (no rotations!) Cut the...
tessellating polygon along a line of your choice, slide and re-glue the pieces, as in Activity 4 of G5-38. Create a picture based on the shape that you’ve got and create a floor pattern with that picture. **EXAMPLE:**

**STEP 1:** Take a tessellation with rhombuses:

![Rhombus tessellation diagram]

**STEP 2:** Cut off and slide several times:

![Rhombus cutting and sliding diagram]

**STEP 3:** Add drawing to a shape:

![Rhombus drawing diagram]

**STEP 4:** Arrange copies of your shape in a floor pattern!

![Rhombus floor pattern diagram]
G5-44

Problems and Puzzles

The worksheet G5-44: Problems and Puzzles is a review worksheet and may be used for extra practice.
# G5 Part 2: BLM List

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3-D Shape Sorting Game
3-D Shape Sorting Game (continued)
3-D Shape Sorting Game (continued)
3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>More than four faces</th>
<th>Square-shaped base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular-shaped base</td>
<td>Fewer than six faces</td>
</tr>
<tr>
<td>Two or more square-shaped faces</td>
<td>Four or more triangular-shaped faces</td>
</tr>
</tbody>
</table>
### 3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>Ten or more edges</th>
<th>Six or fewer vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four or more vertices</td>
<td>Exactly twelve edges</td>
</tr>
</tbody>
</table>

- Pyramids
- Prisms
### 3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>All faces congruent</th>
<th>No rectangular faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular base</td>
<td>Exactly one pair of parallel faces</td>
</tr>
<tr>
<td>At least 3 pairs of parallel faces</td>
<td>No parallel edges</td>
</tr>
</tbody>
</table>
Build-a-Net Game
Build-a-Net Game (continued)
Dot Paper
Grid Paper (1 cm)
Isometric Dot Paper
Nets for 3-D Shapes

Square Pyramid

Triangular Pyramid
Nets for 3-D Shapes (continued)
Nets for 3-D Shapes (continued)

Pentagonal Pyramid

Pentagonal Prism
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
Pentomino Pieces
Right Prisms

Right Prism with a Parallelogram Base
Right Prisms  (continued)

Triangular Prism with a Scalene Base
Skew Prisms

Parallelepiped with Three Different Pairs of Parallelogram Faces
Skew Prisms (continued)

Skew Rectangular Prism
Skew Prisms (continued)

Skew Square Prism
Skew Prisms (continued)
Venn Diagram
Word Search Puzzle (3-D Shapes)

WORDS TO SEARCH:

- base
- edge
- face
- hexagonal
- net
- pentagonal
- prism
- pyramid
- rectangular
- skeleton
- triangular
- vertex
- vertices
PS5-10 Choosing Strategies

Use this lesson after: 5.2 Geometry

Goals:
Students will solve applied problems across the Grade 5 curriculum using a variety of problem-solving strategies, such as using structure, making diagrams, and making a simpler problem.

Prior Knowledge Required:
Can multiply multi-digit numbers by multiples of 10 (see Problem Bank 3)
Can evaluate a fraction of a whole number (for Problem Bank 4, 5)
Can use decimal notation for tenths and hundredths (for Problem Bank 8)
Can identify and extend patterns (for Problem Bank 9)
Can add and subtract decimals to hundredths (for Problem Bank 10)
Can write a rule to get the value of each term in a pattern with constant gaps from the term number (for Problem Bank 11)
Can evaluate expressions at a given value for the variable (for Problem Bank 11)
Can determine the coordinates of points in the first quadrant (for Problem Bank 11)

Materials:
counters
BLM Banquet Hall (pp. 8–10, see Extended Problem)
BLM Electric! (pp. 13–15, see Extended Problem)

Introduce the buckets problem. Write on the board:

You have a 5 L bucket, a 2 L bucket, and you are beside a river.

\[
\begin{array}{cc}
5 \text{ L} & 2 \text{ L} \\
\end{array}
\]

ASK: If you have a 5 L bucket and a 2 L bucket, how can you fill the 5 L bucket with 3 L of water? (fill the 5 L bucket and pour as much as possible into the 2 L bucket; when the 2 L bucket is full, the 5 L bucket has 3 L in it) Model this on the board with counters representing litres, as shown below:

\[
\begin{array}{cccc}
5 \text{ L} & 2 \text{ L} & 5 \text{ L} & 2 \text{ L} \\
\end{array}
\]

SAY: Start with the two empty buckets again. ASK: How can you fill the 5 L bucket with 4 L? (fill the 2 L bucket and pour the water into the 5 L bucket and then repeat) Have volunteers show this process on the board with counters representing litres. ASK: When the 5 L bucket has 4 L in
it, how can you fill the 2 L bucket with 1 L? (fill the 2 L bucket with water and pour all that you can into the 5 L bucket until the 5 L bucket is full; the amount left in the 2 L bucket is 1 L) Again, have volunteers show this process on the board using counters.

Write on the board:

You have a 4 L bucket, a 9 L bucket, and you are beside a river.

SAY: Suppose you want to fill the 9 L bucket with 6 L. ASK: Why would it help to start with 1 L in the 4 L bucket? (you can fill the 9 L bucket and then pour into the 4 L bucket until the 4 L bucket is full; that means you poured 3 L from the 9 L bucket into the 4 L bucket to leave 6 L in the 9 L bucket)

**Exercises:**
a) How would you fill the 4 L bucket with 1 L so that you can still follow the steps to get 6 L in the 9 L bucket?
b) You have a 3 L bucket, a 5 L bucket, and you are beside a river. How can you get exactly 4 L of water from the river?

**Solutions:**
a) Fill the 9 L bucket and then pour 4 L into the 4 L bucket. Empty the 4 L bucket into the river and pour another 4 L from the 9 L bucket into the 4 L bucket. The 9 L bucket now has 1 L. Empty the 4 L bucket and pour the 1 L from the 9 L bucket into the 4 L bucket. The 4 L bucket now has 1 L. Now you can follow the steps to get 6 L in the 9 L bucket.
b) Fill the 5 L bucket and use it to fill the 3 L bucket. Empty the 3 L bucket and pour the remaining 2 L from the 5 L bucket into the 3 L bucket. There is now 2 L in the 3 L bucket and the 5 L bucket is empty. Fill the 5 L bucket and pour 1 L into the 3 L bucket so that the 3 L bucket is full. The 5 L bucket now has 4 L in it.

**Visual representation of solution:**

![Visual representation of solution](image)

**NOTE:** The following Problem Bank questions reflect a selection of the problem-solving strategies used in the problem-solving lessons for Grade 5. Students will need to choose among all the strategies they have learned this year to solve the problems.

**Problem Bank**

1. You have an 8 L bucket, a 5 L bucket, and a 3 L bucket. There is no river nearby. The 8 L bucket is full and the other two buckets are empty. How can you pour half of the water from the 8 L bucket into the 5 L bucket, so that 4 L is in the 5 L bucket and 4 L is in the 8 L bucket?
Solution: Fill the 5 L bucket with water from the 8 L bucket, leaving 3 L in the 8 L bucket.

\[
\begin{align*}
8 \text{ L} & \quad 0 \text{ L} & \quad 0 \text{ L} \\
\rightarrow & \quad 3 \text{ L} & \quad 5 \text{ L} & \quad 0 \text{ L}
\end{align*}
\]

Then fill the 3 L bucket with water from the 5 L bucket, leaving 2 L in the 5 L bucket.

\[
\begin{align*}
3 \text{ L} & \quad 2 \text{ L} & \quad 3 \text{ L} \\
\rightarrow & \quad 6 \text{ L} & \quad 2 \text{ L} & \quad 0 \text{ L} \\
\rightarrow & \quad 6 \text{ L} & \quad 0 \text{ L} & \quad 2 \text{ L}
\end{align*}
\]

Fill the 8 L bucket with water from the 3 L bucket, and then pour the 2 L from the 5 L bucket into the 3 L bucket.

\[
\begin{align*}
1 \text{ L} & \quad 5 \text{ L} & \quad 2 \text{ L} \\
\rightarrow & \quad 1 \text{ L} & \quad 4 \text{ L} & \quad 3 \text{ L}
\end{align*}
\]

Pour the water from the 3 L bucket into the 8 L bucket.

\[
\begin{align*}
4 \text{ L} & \quad 4 \text{ L} & \quad 0 \text{ L}
\end{align*}
\]

2. It is 7:34 a.m. on Wednesday. What day and what hour was it 9 hours and 45 minutes ago?
Solution: 9 hours ago it was 10:34 p.m. on Tuesday. 45 minutes before that it was 9:49 p.m. on Tuesday.

3. The heartbeat of an adult is around 70 beats per minute. How many times does the heart beat in a solar year? Hint: A solar year to the nearest minute is 365 days, 5 hours, and 49 minutes.
Solution: The number of minutes in a solar year is \((365 \times 24 \times 60) + (5 \times 60) + 49 = 525,949\), so the number of beats per year is about \(70 \times 525,949 = 36,816,430\).

4. Colour the shape to show the fraction of the whole number and then evaluate the fraction of the whole number.

\[
\begin{align*}
a) & \quad \frac{1}{2} \text{ of } 8 \\
b) & \quad \frac{2}{3} \text{ of } 12
\end{align*}
\]
Answers: a) 4, b) 8

5. Aputik reads $\frac{3}{4}$ of a book and Megan reads $\frac{1}{2}$ of another book. Megan says she read more pages than Aputik. Is that possible? Explain with an example.
Sample answer: Yes, it is possible. For example, if Megan’s book was 200 pages long and Aputik’s book was only 100 pages long, then Megan read 100 pages and Aputik only read 75 pages.

6. Billy wants to add 345 + 687 on his calculator, but he accidentally presses “345 x” instead of “345 +.” What could he press next so that he doesn’t have to re-enter 345?
Answer: 1 (345 × 1 + 687)

7. Put the decimal point in the appropriate position to make the data realistic.
a) height of a room: 27 m  b) gas in a car tank: 245 L
c) length of a pencil: 145 cm  d) patient’s body temperature: 385°C
Answers: a) 2.7 m, b) 24.5 L, c) 14.5 cm, d) 38.5°C

8. What number am I?
a) I am a two-digit number. My digits add to 13 and my ones digit is 3 less than my tens digit.
b) I am a two-digit number that is greater than 60. I am a multiple of 26.
c) I am an odd three-digit number. The sum of my digits is 18 and I am a multiple of 5, 7, and 9.
d) I am a two-digit decimal between 0.20 and 0.30. My hundredths digit is 4 times my tenths digit.
e) I am a three-digit decimal between 5 and 6. My tenths digit and hundredths digit are equal. My digits add to 13.
f) I am a three-digit decimal between 4 and 5. My digits add to 9 and my hundredths digit is 3 more than my tenths digit.
Answers: a) 85, b) 78, c) 945, d) 0.28, e) 5.44, f) 4.14

9. How many shaded and unshaded triangles would be in Figure 10? Hint: First look at the number of shaded triangles only.

Figure 1  Figure 2    Figure 3       Figure 4

Solution: The sequence for shaded triangles is: 1, 3, 6, 10, .... The sequence does not have a common gap, but the tenth term is $1 + 2 + 3 + 4 + \ldots + 10$. Find the answer by adding:
$$
1 + 2 + 3 + 4 + 5 \\
+ 10 + 9 + 8 + 7 + 6 \\
11 + 11 + 11 + 11 + 11
$$
The total for shaded triangles in Figure 10 is 55. The sequence for the unshaded triangles is 0, 1, 3, 6, 10, .... Figure 10 has the same number of unshaded triangles as Figure 9 has shaded triangles, which is 10 less than 55, so the total for unshaded triangles in Figure 10 is 45.
10. The number on top is equal to the sum of two numbers on the bottom. Find the missing number.

a) \[
\begin{array}{c}
5 \\
3
\end{array}
\]  

b) \[
\begin{array}{c}
9 \\
2
\end{array}
\]

c) \[
\begin{array}{c}
0.37 \\
0.14
\end{array}
\]

d) \[
\begin{array}{c}
5.2 \\
3.4
\end{array}
\]

**Answers:** a) 8, b) 7, c) 0.51, d) 1.8

11. A number is equal to the sum of the two numbers below it. Find the top number.

a) \[
\begin{array}{c}
4 \\
2 \\
2 \\
4
\end{array}
\]

b) \[
\begin{array}{c}
2.2 \\
1.1 \\
1.1 \\
1.1
\end{array}
\]

**Answers:** a) 20, b) 8.8

12. What will be the coordinates of the centre of the 100th rectangle in the pattern? Each grid mark represents one unit on the coordinate plane.

**Solution:** The coordinates of the terms are: (2, 2), (5, 5), (8, 8), and so on. Each coordinate is equal to \(3 \times \text{term number} - 1\), so the coordinates of the centre of the 100th rectangle are (299, 299).
Extended Problem: Banquet Hall

Materials:
BLM Banquet Hall (pp. 8–10)

Extended Problem: Banquet Hall. Give students BLM Banquet Hall. Tell students that this extended problem involves hosting an event at a banquet hall.

Selected answers:
1. a) 6, 10, 14; b) 22; c) 7
2. a) i) 13.5 m and 28 people, ii) 16.5 m and 32 people
   b) i) the arrangement requires exactly 22 m, including the 1 m space at the ends of the banquet hall, and it seats 38 people; ii) the arrangement would require 24 m, which is too long; iii) the arrangement would require 25 m, which is too long
Bonus: a) 40; b) the arrangement would require 23.5 m, which is too long; c) make any two groups—such as a group of 4 and a group of 5, a group of 3 and a group of 6, or a group of 2 and a group of 7—or 1 single table and a group of 8 (1 long table will fit but will only seat 38 people, while splitting the tables into 3 groups will not fit because it will need 23 m)
Banquet Hall (1)

1. Sara is preparing for a banquet. She puts tables together in three different ways:

   a) Use the picture to fill in the table below and find how many people can fit around one, two, or three joined tables.

<table>
<thead>
<tr>
<th>Number of Joined Tables</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

   b) If Sara puts five tables in a row, how many people will fit around the table?

   c) Sara wants to make one long table for 30 people. How many tables will she need to join together to fit that many people?
2. Each table is 2 m long and there needs to be 1.5 m of space between the table groupings.
   
   a) How much space is required for the arrangement of six tables? How many people can sit in this arrangement?

   i) \[ \begin{array}{c}
   \text{2 m} \\
   \text{1.5 m}
   \end{array} \]

   ii) \[ \begin{array}{c}
   \text{2 m} \\
   \text{1.5 m} \\
   \text{1.5 m} \\
   \text{1.5 m}
   \end{array} \]

   b) The banquet hall is 22 m long and only fits one row of tables. There needs to be 1 m of space from the ends of the table to the walls of the banquet hall.

   i) Draw a picture to show how four separate tables and three joined tables (seven tables in total) can fit in the banquet hall. How many people can sit in this arrangement?

   ii) Draw a picture to show how one long table made from 11 joined tables cannot fit in the banquet hall.

   iii) Draw a picture to show how seven separate tables cannot fit in the banquet hall.
Banquet Hall (3)

BONUS ▶ Sara has a party for 40 people. She wants to have her party in the 22 m long banquet hall that only fits one row of tables. Remember: Each table is 2 m long, there needs to be 1.5 m of space between the table groupings, and there needs to be 1 m of space from the ends of the table to the walls of the banquet hall.

a) How many people can sit around five separate tables and two joined tables (seven tables in total)?

b) Draw a picture to show that the arrangement in part a) cannot fit in the banquet hall.

c) Draw a picture to show how to arrange nine tables in the banquet hall and seat 40 people.
Extended Problem: Electric!

Materials:
BLM Electric! (pp. 13–15)

Preparation for the extended problem. Write on the board:

\[ 3 \times 4 = \quad \quad 30 \times 4 = \quad \]

Have volunteers fill in the blanks. (12, 120) ASK: How can you use \(3 \times 4\) to get \(30 \times 4\)? (multiply the answer by 10)

SAY: You can do the same thing with two-digit numbers. Write on the board:

\[
\begin{array}{c}
31 \\
\times 14 \\
\end{array} \quad \quad \text{so } 31 \times 140 = \quad \]

Have one volunteer complete the first multiplication and another volunteer complete the second. (434, 4340) ASK: How can you use \(31 \times 14\) to get \(31 \times 140\)? (multiply the answer by 10)

Tell students they are going to do an extended problem about the amount of power that we use at home and they will need to use this multiplication strategy. SAY: Different appliances use different amounts of power. ASK: Which do you think uses more power, a ceiling fan or an air conditioner? (air conditioner)

SAY: Just like length can be measured using centimetres and metres, electricity can be measured using watts or amps. Write on the board:

\[ 1 \text{ metre} = 100 \text{ centimetres} \\
\text{A } 1 \text{ amp appliance in a home uses } 120 \text{ watts} \]

ASK: How can you convert five metres to centimetres? (multiply 5 by 100) Write on the board:

\[ 5 \times 100 = 500, \text{ so } 5 \text{ m} = 500 \text{ cm} \]

ASK: How can you convert five amps to watts? (multiply 5 by 120) Write on the board:

\[ 5 \times 120 = 600, \text{ so a } 5 \text{ amp appliance uses } 600 \text{ watts} \]

SAY: \(5 \times 12\) is 60, so \(5 \times 120\) is 600.
Draw on the board:

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blender</td>
<td>360</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1200</td>
</tr>
<tr>
<td>Electric can opener</td>
<td>150</td>
</tr>
<tr>
<td>Kettle</td>
<td>1800</td>
</tr>
<tr>
<td>Microwave</td>
<td>1080</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>720</td>
</tr>
<tr>
<td>Coffee maker</td>
<td>960</td>
</tr>
</tbody>
</table>

SAY: These are some typical appliances in a home along with how much power they might use. ASK: Which appliance uses the most power? (the kettle) SAY: You don’t use a kettle for a long time, but when you do use it, it uses a lot of power. A kettle will use less power in a year than a microwave, because you use it for less time. ASK: Which appliance uses the least amount of power? (the can opener) Which appliance do you think will use the most power in a year? (the refrigerator) PROMPTS: Which one gets used the most? Which one is almost always running?

Tell students that the way power works in a house is that each outlet has a maximum number of watts you can use at one time. For example, in some houses, if you are using a kettle, you can’t use a microwave plugged into the same outlet at the same time, because you will blow the fuse, and the power will go out. Extension cords allow you to plug in more than two appliances to the same outlet, but you have to be careful not to go over the number of watts allowed. The extended problem that follows investigates that kind of situation.

NOTE: The word “outlet” is being used imprecisely here; the precise word that should be used is “circuit,” but we use outlet here since students will be more familiar with the word and also because the concept of a circuit is not necessary for this task.

Extended Problem: Electric! Provide students with BLM Electric! On the BLM, Question 4 and the Bonus question are good opportunities for students to apply the guess-check-revise and using structure problem-solving strategies learned this year. If students have not done these problem-solving lessons, they will likely find these questions difficult.

Answers:
1. a) kettle; b) i) yes, ii) no, iii) yes
2. a) 1560 watts; b) 600 watts; c) television only, lamp only, stereo only, the television and lamp
3. 120 more watts because 1 amp is the same as 120 watts
4. 9 amps
Bonus: 1001 amps
Electric! (1)

Electricity is needed to power an appliance. The amount of power an appliance uses is given in watts.

These are some typical appliances in a kitchen and how much power they might use.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric can opener</td>
<td>150</td>
</tr>
<tr>
<td>Blender</td>
<td>360</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>720</td>
</tr>
<tr>
<td>Coffee maker</td>
<td>960</td>
</tr>
<tr>
<td>Microwave</td>
<td>1080</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1200</td>
</tr>
<tr>
<td>Kettle</td>
<td>1800</td>
</tr>
</tbody>
</table>

1. The number of watts used by an outlet should not pass the outlet's capacity. One outlet in the kitchen has a capacity of 1620 watts.

   a) Which appliance can never be used in that outlet?

   b) Can the given appliances be used in the outlet at the same time?

   i) the blender and dishwasher

   ii) the microwave and refrigerator

   iii) can opener, blender, and coffee maker
Electric! (2)

2. Sometimes the capacity of an outlet is given in amps. In a typical home, to get the number of watts from the number of amps, multiply by 120.

These are some typical appliances in a living room and how much power they might use.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp</td>
<td>72</td>
</tr>
<tr>
<td>Television</td>
<td>240</td>
</tr>
<tr>
<td>Stereo</td>
<td>360</td>
</tr>
<tr>
<td>Window air conditioner</td>
<td>960</td>
</tr>
<tr>
<td>Heater</td>
<td>1200</td>
</tr>
</tbody>
</table>

a) An outlet in the living room has a capacity of 13 amps. How many watts can be used at the outlet?

b) The outlet from part a) is already being used by a window air conditioner. How many additional watts can be used at the outlet?

c) In the winter, a heater is plugged into the living room outlet instead of the air conditioner. Which of these items can be used in that outlet at the same time as the heater? List all possible combinations.

A. Television  B. Lamp  C. Stereo
Electric! (3)

3. How many more watts could an outlet with a capacity of 10 amps use than an outlet with a capacity of 9 amps? Explain how you know.

4. An outlet has a capacity that is a whole number of amps. The number of watts it can use, to the nearest 100, is 1100. What is the outlet’s capacity in amps?

BONUS ► In a building, 120 120 watts are being used at the same time. How many amps is that?
Answer Keys: Workbook 5

JUMP Math

Contents

Patterns & Algebra – Part 1
Number Sense – Part 1
Measurement – Part 1
Probability & Data Management – Part 1
Geometry – Part 1
Patterns & Algebra – Part 2
Number Sense – Part 2
Measurement – Part 2
Probability & Data Management – Part 2
Geometry – Part 2
Patterns & Algebra – AP Book 5.1

AP Book PA5-1

1. a) 5
   b) 3
   c) 6
   d) 4
   e) 6
   f) 6
   g) 5
   h) 6
   i) 2
   j) 9
   k) 5
   l) 4
   m) 3
   n) 6
   o) 5

2. a) 8
   b) 12
   c) 12
   d) 19
   e) 20
   f) 34
   g) 41
   h) 65
   i) 87
   j) 97

3. a) 11
   b) 33
   c) 25
   d) 34
   e) 42
   f) 74

AP Book PA5-2

1. a) Gap = 3: 10, 13, 16
   b) Gap = 4: 13, 17, 21
   c) Gap = 5: 18, 23, 28
   d) Gap = 3: 12, 15, 18
   e) Gap = 5: 16, 21, 26
   f) Gap = 6: 22, 28, 34
   g) Gap = 10: 32, 42, 52
   h) Gap = 6: 25, 31, 37
   i) Gap = 3: 40, 43, 46
   j) Gap = 6: 100, 106, 112
   k) Gap = 11: 35, 46, 57
   l) Gap = 9: 35, 44, 53
   m) Gap = 6: 23, 29, 35
   n) Gap = 4: 12, 16, 20

AP Book PA5-3

1. a) – 3
   b) – 5
   c) – 5
   d) – 7
   e) – 7
   f) – 3
   g) – 5
   h) – 6
   i) 82
   j) 97

2. a) – 4
   b) – 2
   c) – 5
   d) – 7
   e) – 4
   f) – 4
   g) – 7
   h) – 8

3. a) – 5
   b) – 9
   c) – 2
   d) – 2
   e) – 5
   f) – 4
   g) – 8
   h) – 6
   i) – 9
   j) – 5
   k) – 6
   l) – 10

AP Book PA5-4

1. a) 4
   b) 10
   c) 5
   d) 16
   e) 11
   f) 15
   g) 24
   h) 27
   i) 34
   j) 39
   k) 55
   l) 68

2. a) 12
   b) 16
   c) 16
   d) 20
   e) 12
   f) 25
   g) 28
   h) 33
   i) 82

3. a) Gap = – 2: 7, 5, 3
   b) Gap = – 5: 18, 13, 8
   c) Gap = – 3: 55, 52, 49
   d) Gap = – 9: 28, 19, 10
   e) Gap = – 20: 50, 30, 10

4. She will save $66.

AP Book PA5-5

1. a) 17, 23, 29
   b) 9, 13, 17
   c) 11, 15, 19
   d) 12, 15, 18
   e) 26, 21, 16
   f) 24, 31, 38
   g) 9, 5, 1
   h) 11, 7, 3
   i) 17
   j) 23
   k) 29
   l) 35

2. a) Gap = + 4: 16, 20
   b) Gap = + 7: 24, 31
   c) Gap = + 3: 10, 13
   d) Gap = + 4: 33, 37
   e) Gap = + 5: 26, 31
   f) Gap = – 2: 49, 47
   g) Gap = – 6: 61, 55

3. 28 stamps are left.

AP Book PA5-6

1. a) 47, 50, 53
   b) 70, 75, 80
   c) 78, 80, 82
   d) 40, 50, 60
   e) 69, 73, 77
   f) 49, 58, 67
   g) 32, 38, 44
   h) 20, 18, 16
   i) 19, 16, 13
   j) 75, 70, 65
   k) 50, 40, 30
   l) 48, 44, 40
   m) 42, 35, 28
   n) 119, 108, 97

BONUS:

3. Answers will vary.
4. b)
5. Hyun is correct.

AP Book PA5-7

1. a) 17, 23, 29
   b) 9, 13, 17
   c) 11, 15, 19
   d) 12, 15, 18
   e) 26, 21, 16
   f) 24, 31, 38
   g) 9, 5, 1
   h) 11, 7, 3

2. a) 2
   b) 5
   c) 1
   d) 3
   e) 3
   f) 2
3. a) subtract 7
   b) add 8
   c) add 4
   d) subtract 12
   e) subtract 11
   f) add 9
   g) add 8
   h) subtract 6

4. a) 37, 42, 47: Start at 22 and add 5 each time
   b) 59, 66, 73: Start at 38 and add 7 each time
   c) 160, 172, 184: Start at 124 and add 12 each time

5. a) Genevieve’s rule is correct.
   b) Jonah said to subtract (instead of add) 4.
   Pria said to add 5 (instead of 4).

AP Book PA5-8

page 8

1. a) Start at 2 and add 5.
   b) Start at 2 and add 7.
   c) Start at 1 and add 3.
   d) Start at 1 and add 6.
   e) Start at 5 and add 7.
   f) Start at 13 and add 8.
   g) Start at 3 and add 8.
   h) Start at 7 and add 4.
   i) Start at 8 and add 6.
   j) Start at 6 and add 2.
   k) Start at 2 and add 2.
   l) Start at 6 and add 2.
   m) Start at 2 and add 2.
   n) Start at 6 and add 2.

2. a) Figure # of Shapes
    1 3
    2 5
    3 7
    4 9
    5 11

3. Figure # of Line Seg
   1 4
   2 7
   3 10
   5 16
   7 22

4. a) 13
   b) 6
   c) 15
   d) 10 squares
   e) She would need 14 triangles.
   f) She would need 16 triangles.
   g) She would need 18 triangles.
   h) 28 trapezoids

AP Book PA5-9

page 10

1. a) 7
   b) 10
   c) 8
   d) 9
   e) 7
   f) 13

2. Figure # of Line Seg
   1 3
   2 6
   3 9
   4 12
   5 15

AP Book PA5-11

page 14

1. Term # Term
   1 3
   2 5
   3 7
   4 9
   5 11

2. a) 17
   b) 51
   c) No, it’s 15.
   d) Teacher to check.
   e) $31
   f) Adrian
   g) 7 kg
   h) 77 L
   i) 77 days

AP Book PA5-12

page 15

1. Circle:
   a) First 4 figures
   b) First 4 figures
   c) First 4 figures
   d) First 3 figures
   e) First 3 figures
   f) First 4 figures
   g) First 3 figures
   h) First 3 figures
   i) First 3 figures
   j) First 3 figures
   k) First 3 figures
   l) First 3 figures
   m) First 4 figures
   n) First 3 figures

2. Teacher to check continuation of pattern
   a) First 3 figures
   b) First 3 figures
   c) A B C
   d) 2 8 9 6
   e) 3 0 4

3. Teacher to check.

AP Book PA5-13

page 16

1. a) BBYBBY
b) BYBYBYB

c) BBYBBYBYB

d) YBBYBBYBB

e) YBYYBYBYY

4. 3 blocks

AP Book PA5-15

page 20

1. 25 km
   (from the finish line)
2. 250 m
3. 250 m
4. 17 m
5. 2 m

AP Book PA5-16

page 21

Teacher to check number lines.

1. a) LCM = 12
   b) LCM = 12
2. b) LCM = 20
   c) LCM = 6
   d) LCM = 10
   e) LCM = 6
   f) LCM = 12
   g) LCM = 8
   h) LCM = 40
   i) LCM = 15
   j) LCM = 30
   k) LCM = 30
   l) LCM = 24
   m) LCM = 18
BONUS:
   e) C
   f) A

AP Book PA5-17

page 22

1. b) +, +, −
   c) −, +, +
   d) +, −, +
   e) −, +, −
   f) +, −, +
   g) +, +, +
   h) −, +, +
   i) +, +, +
   j) +, +, +
   k) −, +, +
   l) +, +, +

2. a) C; A; B
   b) C; B; A

3. a) + 4, −2, + 7, −4
   b) + 5, −3, + 6, −5
   c) + 3, + 4, + 4, + 6
   d) −2, + 6, −8, + 9
   e) + 2, + 4, −3, + 5
   f) −6, + 3, −6, −5
   g) −7, + 5, + 4, −5
   h) + 3, −7, + 8, + 6

4. a) B
   b) A

AP Book PA5-18

page 25

1. a) 
   b) 
   c) 
   d) 
2. a) 
   b) 
   c) 
   d) 
3. a) 
   b) 
   c) 
   d) 
4. a) 
   b) 
   c) 
   d) 

5. b) Start at 3 and add 6.
   c) Start at 18 and subtract 3.
   d) Start at 43 and subtract 5.

6. a) Start at 8 and add 5.
   b) Start at 26 and subtract 7.
   c) No rule
   d) Start at 71 and add 4.

7. a) Increasing
   b) Repeating
   c) Decreasing
   d) Increasing
   e) Repeating
   f) Decreasing

8. a) 8, 12, 16, 20, 24
   b) 37, 31, 25, 19, 13
   c) 99, 106, 113, 120, 127

9. Answers will vary.
10. Answers will vary.
11. Answers will vary.
4. a) Across rows: add 2
    Down columns: add 2
    Diagonal (l to r): add 4
    Diagonal (r to l): stay the same
b) Across rows: add 5
    Down columns: add 5
    Diagonal (l to r): add 10
    Diagonal (r to l): stay the same
c) Across rows: add 3
    Down columns: subtract 3
    Diagonal (l to r): stay the same
    Diagonal (r to l): subtract 6

5. Answers will vary.

6. a) Row 3
   b) Column 5
   c) Column 1 – Start at 0 and add 6.
      Column 2 – Start at 5 and add 2.
   d) Column 3 = 2 × Column 4
   e) Answers will vary
   f) Row 2, Row 4, Row 5

7. Answers will vary but one option would be:

   | A | B | A | B |
   | A | B | A | B |
   | B | A | B | A |

8. | 2 | 7 | 6 |
  | 9 | 5 | 1 |
  | 4 | 3 | 8 |

Answer Key for AP Book 5.1
### AP Book NS5-1

#### page 32

1. a) tens  
   b) thousands  
   c) thousands  
   d) thousands  
   e) ten thousands  
   f) ten thousands  
   g) hundreds  
   h) tens  
   i) hundreds  
   j) ones  
   k) ten thousands  
   l) ones  

2. a) thousands  
   b) tens  
   c) thousands  
   d) ones  
   e) ten thousands  
   f) tens  
   g) hundreds  
   h) ones  
   i) thousands  

3. a) 5, 2, 9, 5, 3  
   b) 4, 2, 0, 0, 1  
   c) 0, 3, 6, 9, 9  
   d) 1, 9, 0, 5, 3  
   e) 0, 0, 5, 4, 6  
   f) 2, 0, 1, 2, 7  

4. 2, 60, 300, 4,000, 50,000  
   7, 30, 500, 8,000, 20,000  
   5, 70, 200, 3,000, 10,000  

5. a) 40  
   b) 40  
   c) 400  
   d) 40,000  
   e) 4,000  
   f) 40  
   g) 40,000  
   h) 4,000  

6. a) 500  
   b) 30  
   c) 60,000  
   d) 8,000  

### AP Book NS5-2

#### page 34

1. a) 23  
   b) 32  
   c) 95  
   d) 270  
   e) 479  
   f) 19217  
   g) 47509  

2. a) Two hundred forty-five  
   b) Four hundred fifty-one  
   c) Three hundred seventy-eight  
   d) One hundred nine  

3. a) Thirty-six thousand  
   b) Four thousand  
   c) Twenty-five thousand  
   d) Nineteen thousand  

4. 7,163  
   19,789  
   43,567  
   1,987  
   38,527  
   70,144  

5. a) Twenty-six thousand  
   b) Three thousand  
   c) Thirty-seven thousand  
   d) Nineteen thousand  
   e) Two hundred thirty-four  
   f) Six hundred ninety-seven  
   g) One hundred twenty-one  
   h) Four thousand six hundred twenty-one  

### AP Book NS5-3

#### page 36

1. a) 3, 4, 5, 345  
   b) 2, 5, 3, 253  
   c) 1, 7, 8, 178  
   d) 5, 0, 6, 506  
   e) 2  
   f) 20  

2. Teacher to check.  

3. a) 2, 4, 4, 6 = 2446  
   b) 3, 2, 2, 4 = 3224  
   c) 2 thousands + 2 hundreds + 2 tens + 9 ones = 2229  

4. Teacher to check the base ten model.  

5. a) 2 354  
   b) 1 266  

### AP Book NS5-4

#### page 39

1. b) 2, 5, 3, 1, 2  
   c) 2 ten thousands + 8 thousands + 5 hundreds + 4 tens + 7 ones  

2. b) 20 + 7  
   c) 40 + 8  
   d) 1000 + 200 + 30 + 2  
   e) 30 000 + 6 000 + 200 + 70 + 3  
   f) 10 000 + 9 000 + 300 + 80 + 4  
   g) 40 000 + 9 000 + 800 + 5  

3. a) 4 953  
   b) 2 032  
   c) 63 997  
   d) 50 034  
   e) 92 588  
   f) 41 500  
   g) 13 607  
   h) 80 302  
   i) 70 496  
   j) 90 005  
   k) 80 808  
   l) 35 001  
   m) 23 000  

4. a) 20  
   b) 70  
   c) 800  
   d) 200  
   e) 2  
   f) 20  
   g) 7 000  
   h) 80  
   i) 200 + 60  
   j) 7 000 + 3  

5. Teacher to check base ten models.  

a) 5 000 + 800 + 30 + 2
6. Teacher to check base ten model. 
one thousand three hundred sixty-five 
1 365 = 
1 thousands 
+ 3 hundreds 
+ 6 tens 
+ 5 ones 
1 365 = 1 000 + 300 + 60 + 5 
7. 1 000 (100 000 ÷ 100 = 1 000)

AP Book NS5-5
page 41

1. a) 6, 40, 800; 6, 50, 800; 856 > 846 
b) 7, 20, 300; 7, 20, 400; 427 > 327

c) 2, 3; 134 
d) 2, 3; 374 
c) 7, 6; 875 
d) 3, 2; 238 
e) 8, 9; 41 597 
f) 2, 5; 28 542 
g) 5, 6; 60 347 
h) 9, 8; 62 149

2. a) 10 more 
b) 100 less 
c) 10 less 
d) 10 more 
e) 100 less 
f) 10 less 

3. a) 335 
b) 1 552 
c) 692 
d) 4 035 
e) 6 921 
f) 3 195 
g) 7 305 
h) 5 253 
i) 63 528 
j) 61 381

4. a) Thirty-five 
b) 392 
c) 81 
d) 1 232 
e) Fifty thousand three hundred eighty-five 

7. Teacher to check number line. Circle A.

8. Teacher to check, answers will vary.

9. A 5-digit number is greater than a 4-digit number, except if the first digit is zero.

10. Nine

11. Concepción (9106 > 9001)

AP Book NS5-6
page 43

1. a) (i) 438 
   (ii) 422 
   438 is greater 
   b) (i) 2 224 
   (ii) 1 108 
   2 224 is greater 

2. a) 78, 79, 87, 89, 97, 98; 98 is greatest
   b) NOTE: 03 and 04 are equal to 3 and 4, which are 1-digit numbers (so not included in list)
   30, 34, 40, 43; 43 is greatest

3. 999 

4. a) 999 
   b) 9 999 
   c) 99 999

5. a) < 
   b) > 
   c) > 
   d) < 
   e) > 
   f) <

6. a) 6 432 
   b) 9 874 
   c) 4 210

7. a) 84 321 
   b) 98 521 
   c) 65 431

8. a) 87 521, answers will vary, 12 578 
   b) 95 321, answers will vary, 12 359 
   c) 53 310, answers will vary. (0)1 335.

9. a) 3 183, 3 257, 3 352 
   b) 17 251, 17 256, 17 385

AP Book NS5-7
page 45

1. a) (i) 438 
   (ii) 422 
   438 is greater 
   b) (i) 2 224 
   (ii) 1 108 
   2 224 is greater 

2. a) 78, 79, 87, 89, 97, 98; 98 is greatest
   b) NOTE: 03 and 04 are equal to 3 and 4, which are 1-digit numbers (so not included in list)
   30, 34, 40, 43; 43 is greatest

3. 999 

4. a) 999 
   b) 9 999 
   c) 99 999

5. a) < 
   b) > 
   c) > 
   d) < 
   e) > 
   f) <

6. a) 6 432 
   b) 9 874 
   c) 4 210

7. a) 84 321 
   b) 98 521 
   c) 65 431

8. a) 87 521, answers will vary, 12 578 
   b) 95 321, answers will vary, 12 359 
   c) 53 310, answers will vary. (0)1 335.

9. a) 3 183, 3 257, 3 352 
   b) 17 251, 17 256, 17 385

Answer Key for AP Book 5.1
8. Yes; by regrouping, he has 4 thousands, 3 hundreds, 4 tens and 10 ones – which is 4 more ones than he needs.

AP Book NS5-9

1. b) 
   \[
   \begin{array}{c|c}
   \text{tens} & \text{ones} \\
   \hline
   3 & 5 \\
   2 & 7 \\
   5 & 12 \\
   6 & 2 \\
   \end{array}
   \]

2. b) 5 ones, 1 ten; sum = 85
c) 2 ones, 1 ten; sum = 92
d) 0 ones, 1 ten; sum = 80
e) 5 ones, 1 ten; sum = 45

AP Book NS5-10

1. = 5, 7, 2
   \[
   \frac{= 2, 5, 1}{= 7, 12, 3}
   \]
   \[
   = 8, 2, 3
   \]

2. a) 629
   b) 857
c) 919
d) 1549
e) 907

3. a) 653
   b) 713
c) 831
d) 649
e) 962

4. a) 825
   b) 783
c) 773
d) 348

AP Book NS5-11

1. = 6, 8, 2, 6
   \[
   = 2, 5, 4, 3
   \]
   \[
   = 8, 13, 6, 9
   \]
   \[
   = 9, 3, 6, 9
   \]

2. a) 6 199
   b) 9 399
c) 9 179
d) 7 477

3. a) 6 728
   b) 8 847
c) 5 929
d) 9 659
e) 5 707

AP Book NS5-12

1. a) 29
   b) 6, 15; 29
c) 2, 14; 16
d) 6, 17; 48

2. a) 6 186
   b) 9 186
c) 8 949
d) 7 857
e) 6 852

3. b) Help! 2 is less than 6
c) OK
d) Help! 2 is less than 9
e) OK

4. a) 2 579
   b) 2 531
c) 9 216
d) 6 852

5. a) 9 816
   b) 5 279
c) 2 531
d) 9 216

6. a) 6 815
   b) 62 829
c) 71 890
d) 69 631

7. a) 216 + 612
   \[
   = 828
   \]
   \[
   605 + 506
   \]
   \[
   = 1111
   \]

8. Yes; by regrouping, he has 4 thousands, 3 hundreds, 4 tens and 10 ones – which is 4 more ones than he needs.

AP Book NS5-9

1. b) 
   \[
   \begin{array}{c|c}
   \text{tens} & \text{ones} \\
   \hline
   3 & 5 \\
   2 & 7 \\
   5 & 12 \\
   6 & 2 \\
   \end{array}
   \]

2. b) 5 ones, 1 ten; sum = 85
c) 2 ones, 1 ten; sum = 92
d) 0 ones, 1 ten; sum = 80
e) 5 ones, 1 ten; sum = 45

AP Book NS5-10

1. = 5, 7, 2
   \[
   \frac{= 2, 5, 1}{= 7, 12, 3}
   \]
   \[
   = 8, 2, 3
   \]

2. a) 629
   b) 857
c) 919
d) 1549
e) 907

3. a) 653
   b) 713
c) 831
d) 649
e) 962

4. a) 825
   b) 783
c) 773
d) 348

AP Book NS5-11

1. = 6, 8, 2, 6
   \[
   = 2, 5, 4, 3
   \]
   \[
   = 8, 13, 6, 9
   \]
   \[
   = 9, 3, 6, 9
   \]

2. a) 6 199
   b) 9 399
c) 9 179
d) 7 477

3. a) 6 728
   b) 8 847
c) 5 929
d) 9 659
e) 5 707

AP Book NS5-12

1. a) 29
   b) 6, 15; 29
c) 2, 14; 16
d) 6, 17; 48

2. a) 6 186
   b) 9 186
c) 8 949
d) 7 857
e) 6 852

3. b) Help! 2 is less than 6
c) OK
d) Help! 2 is less than 9
e) OK

4. a) 2 579
   b) 2 531
c) 9 216
d) 6 852

5. a) 9 816
   b) 5 279
c) 2 531
d) 9 216

6. a) 6 815
   b) 62 829
c) 71 890
d) 69 631

7. a) 216 + 612
   \[
   = 828
   \]
   \[
   605 + 506
   \]
   \[
   = 1111
   \]

8. Yes; by regrouping, he has 4 thousands, 3 hundreds, 4 tens and 10 ones – which is 4 more ones than he needs.
d) 8, 15; 193

5. a) 127
b) 6; 12; 133
c) 5; 14; 326
d) 8; 10; 474

6. a) 6; 14; 12; 389
b) 7; 11; 13; 648
c) 2; 9; 14; 277
d) 8; 17; 13; 399

7. b) 3; 12; 2 432
c) 8; 16; 3 921
d) 5; 15; 2 722

8. a) 127
b) 6; 12; 133
c) 5; 14; 326
d) 8; 10; 474

9. a) 6; 15; 14; 12; 5 757
b) 7; 12; 11; 14; 4 459
c) 3; 14; 16; 11; 2 687
d) 8; 9; 15; 18; 7 489

10. a) 0; 9; 9; 10; 642
b) 0; 9; 10; 52
c) 0; 9; 9; 10; 238
d) 0; 9; 9; 10; 741

AP Book NS5-13

1. b) 9 green
   6 red
   Diff: 3 apples
   Total: 15 apples
c) 6 red
   8 green
   Diff: 2 apples
   Total: 14 apples

d) 5 red
   4 green
   Diff: 1
   Total: 9 apples

c) Green – 3
   Purple – 6
   Total = 9
   3 + 6 = 9,
   6 + 3 = 9,
   9 – 3 = 6,
   9 – 6 = 3
   Three more purple than green

2.

<table>
<thead>
<tr>
<th>R</th>
<th>G</th>
<th>T</th>
<th>How many more?</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>9</td>
<td>1 more green than red</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>8</td>
<td>6 more red than green</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>13</td>
<td>3 more red than green</td>
</tr>
</tbody>
</table>

3. a) Teacher to check.
   b) Teacher to check.
   BONUS:
   c) Teacher to check.

AP Book NS5-14

1. a) 2 + 4 = 6
   4 + 2 = 6
   6 – 2 = 4
   6 – 4 = 2
   b) 3 + 7 = 10
   7 + 3 = 10
   10 – 7 = 3
   10 – 3 = 7
   c) 12 + 5 = 17
   5 + 12 = 17
   17 – 5 = 12
   17 – 12 = 5

2. b) Green – 5
   Purple – 4
   Total = 9
   5 + 4 = 9,
   4 + 5 = 9,
   9 – 5 = 4,
   9 – 4 = 5
   One more green than purple.

3. a) +
   b) –
   c) –
   d) +

4. Teacher to check.
   a) 17 stickers
   b) 4 dogs
   c) 3 km

AP Book NS5-15

1. $57 + $12 = $69
2. 2 375 + 5 753 = 8 128 km
3. 406 – 244 = 162 m
4. 5 895 – 3 776 = 2 119 m
5. 39 666 + 39 666 = 79 332 km
6. 12 475 + 14 832 = 27 307
7. Answers will vary depending on current year (As of 2014 : 147 years ago)
8. Answers will vary.
   BONUS: The difference is always 198 because, in a number with this property, the digit in the hundreds place is always 2 greater than the digit in the ones place.

AP Book NS5-16

1. a) tens
   b) millions
   c) hundred
   d) hundreds
   e) ones
2. a) 5 647 110
   b) 7 823 925
3. a) two million
   three hundred twenty-five thousand
   eight hundred fifty-three
   b) nine million
   three hundred seven thousand
   two hundred eleven

4. b) 5 000 000 + 200
   000 + 10 000 + 8
   000 + 900 + 60
   + 7
5. a) 3 215 139
   b) 4 238 537
6. a) 4 007 507
   b) 5 527 998
   c) 2 721 162

AP Book NS5-17

1. a) 58 020 – 25 690 = 32 330
   b) Erie – 25 690
   Great Slave - 28 570
   Great Bear – 31 340
   Michigan – 58 020
   Superior – 82 100
c) \(82 100 - 25 690 = 56 410\)

\[\text{d) } 370 990 - 82 100 = 288 890\]

2. 330

3. Order of numbers may vary but additions are:
   \(8 + 1, \ 7 + 2, \ 6 + 3, \ 5 + 4\)

4. a) \(87 645\)
   b) \(56 748\) or \(56 784\) or \(56 847\) or \(56 874\)
   c) Exact answers will vary but possible combinations include:
      \(7+ 5 \text{ with } 4, 6, \text{ or } 8 \text{ as the ones digit}\)
      \(4 + 8 \text{ with } 5, 6, \text{ or } 7 \text{ as the ones digit}\)
   d) Exact answers will vary but the thousands digit will be 8 and the hundreds digit will be 4.

5. Answers will vary – teacher to check.

6. 25 746

---

AP Book NS5-19
page 63

1. a) \(0 \rightarrow 4 \rightarrow 8; \ 4 + 4 = 8\)
   b) \(0 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 8; \ 2 + 2 + 2 + 2 = 8\)

2. a) 8, 12, 16, 20
   b) 12, 18, 24, 30
   c) 14, 21, 28, 35

3. a) 7 × 4
   b) 7 × 7
   c) 9 × 6

AP Book NS5-20
page 64

1. b) \(5 \times 6; \ 4 \times 6, 6; \ 5 \times 6 = 4 \times 6 + 6\)
   c) \(3 \times 7; \ 2 \times 7, 7; \ 3 \times 7 = 2 \times 7 + 7\)

---

AP Book NS5-21
page 65

1. a) 3, 12, 120
   b) Teacher to check.

2. b) 3 × 40 + 3 × 3
   c) 4 × 20 + 4 × 2

3. a) 6, 60, 600
   b) 5, 50, 500
   c) 20, 200, 2 000

---

AP Book NS5-22
page 66

1. a) 3 × 20
   b) 4 × 10
   c) 5 × 20
   d) 5 × 10

2. Answers are top to bottom, left to right:
   b) \(4 \times 13, \ 4 \times 10, \ 4 \times 3\)
   c) \(2 \times 25, \ 2 \times 20, \ 2 \times 5\)
   d) \(3 \times 14, \ 3 \times 10, \ 3 \times 4\)

3. a) \(4 \times 12\)
   b) \(4 \times 10 + 4 \times 2\)
   c) \(4 \times 25\)

---

AP Book NS5-23
page 67

1. a) 2; 4; 2 × 4 = 8
   b) 4; 5; 4 × 5 = 20

2. b) 4 × 2
   c) 5 × 4
   d) 7 × 3

3. a) 6 × 5 = 30
   b) 3 × 7 = 21

---

AP Book NS4-22
page 66

1. a) \(3 \times 20\)
   b) \(4 \times 10\)
   c) \(5 \times 20\)
   d) \(5 \times 10\)

2. Answers are top to bottom, left to right:
   b) \(4 \times 13, \ 4 \times 10, \ 4 \times 3\)
   c) \(2 \times 25, \ 2 \times 20, \ 2 \times 5\)
   d) \(3 \times 14, \ 3 \times 10, \ 3 \times 4\)

3. a) \(4 \times 12\)
   b) \(4 \times 10 + 4 \times 2\)
   c) \(4 \times 25\)

---

AP Book NS5-21
page 65

1. a) 3, 12, 120
   b) Teacher to check.

2. b) 3 × 40 + 3 × 3
   c) 4 × 20 + 4 × 2

3. a) 6, 60, 600
   b) 5, 50, 500
   c) 20, 200, 2 000

4. a) 150
   b) 120
   c) 160
   d) 150
   e) 1 500
   f) 3 000
   g) 240
   h) 2 500
   i) 1 800
   j) 420
   k) 320
   l) 2 700

5. Teacher to check.

6. \(4 \times 2 \text{ thousands} = 8 \text{ thousands} = 8 000\)
3. a) 24  
b) 84  
c) 36  
d) 44  
e) 84  
f) 123  
g) 64  
h) 69  
i) 336  
j) 466  
k) 696  
l) 888  
m) 999  

4. a) Atilla planted 996 trees altogether.
    b) 960

AP Book NS5-24  
page 68

1. a) 204
    b) 126
    c) 284
    d) 126
    e) 273
    f) 162
    g) 216
    h) 188
    i) 168
    j) 184
    k) 405
    l) 146
    m) 66
    n) 219
    o) 148
    p) 249
    q) 128
    r) 128
    s) 369
    t) 455
    u) 189
    v) 729
    w) 355
    x) 144

2. a)  288
      b) 567
      c) 728

3. a) 1; 70
    b) 3; 156
    c) 2; 180
    d) 1; 165
    e) 1; 102
    f) 1; 160
    g) 4; 282
    h) 1; 460
    i) 2; 231

4. a) Atilla planted 996 trees altogether.
    b) 960

AP Book NS5-25  
page 69

1. a) 2; 0
    b) 1; 6
    c) 2; 5
    d) 3; 6
    e) 3; 5

2. a) 9
    b) 7
    c) 7
    d) 9
    e) 7
    f) 9
    g) 9
    h) 4
    i) 8
    j) 9

3. a) 1; 70
    b) 3; 156
    c) 2; 180
    d) 1; 165
    e) 1; 102
    f) 1; 160
    g) 4; 282
    h) 1; 460
    i) 2; 231

4. a) 1; 756
    b) 3; 805
    c) 1; 759
    d) 1; 568
    e) 2; 1; 822

5. a) 568
    b) 1 866
    c) 1 561
    d) 2 592
    e) 12 888
    f) 15 222

6. a) 252
    b) 639
    c) 1288

AP Book NS5-26  
page 70

1. a) 400, 10, 2; 1 200, 30, 6; 1 236
    b) 300, 20, 3; 600, 40, 6; 646

2. a) 68
    b) 936
    c) 848
    d) 969
    e) 639

3. a) 1; 456
    b) 1; 678
    c) 1; 896
    d) 1; 648
    e) 2; 684

4. a) 1; 756
    b) 3; 805
    c) 1; 759
    d) 1; 568
    e) 2; 1; 822

5. a) 568
    b) 1 866
    c) 1 561
    d) 2 592
    e) 12 888
    f) 15 222

6. a) 252
    b) 639
    c) 1288

AP Book NS5-28  
page 72

1. b) 2; 00
    c) 1; 50
    d) 0; 80
    e) 2; 50

2. a) 1; 1 100
    b) 1; 1 360
    c) 2; 1 000
    d) 1; 2 150
    e) 1; 780
    f) 840
    g) 720
    h) 1 080
    i) 1 380
    j) 3 010

3. b) 20 × 40 + 20 × 2 = 800 + 40 = 840
    c) 30 × 20 + 30 × 3 = 600 + 90 = 690

AP Book NS5-29  
page 73

1. a) 1; 115
    b) 1; 72
    c) 1; 52
    d) 0; 86
    e) 2; 70
    f) 1; 50
    g) 1; 72
h) 2; 120
i) 2; 144
j) 1; 102
2. b) 1; 720
c) 2; 1400
d) 1; 2150
e) 3; 2100
3. a) 1; 3; 210; 700
   b) 1; 3; 175; 750
c) 1; 92; 690
d) 1; 2; 75; 450
e) 3; 1; 112; 2 800
f) 1; 1; 90; 1 350
g) 1; 1; 132; 1 320
h) 2; 3; 90; 600
i) 1; 0; 46; 920
j) 2; 1; 195; 2 600
4. b) 3 591
c) 2 835
d) 1 161
e) 456
5. a) 1; 1; 75; 750; 825
   b) 1; 0; 86; 2150; 2236
c) 1; 2; 441; 1 260; 1 701
d) 1; 4; 405; 900; 1 305
e) 2; 3; 112; 840; 952
6. a) 864
   b) 4 088
c) 5 440
d) 1 767
e) 6 364
   f) 6 272

AP Book NS5-30
page 75
1. 46, 88, 24, 62, 86, 108, 166, 184, 142
2. 50, 90, 32, 56, 36, 34, 70, 110, 78
3. a) 68¢
b) 96¢
4. Yes – teacher to check the explanation.
5. a) 200

AP Book NS5-31
page 76
1. b) 2, 4; = 3 × 6
c) 3, 4; = 3 × 7
d) 2, 2; = 3 × 4
e) 5, 4; = 3 × 9
f) 2, 6; = 3 × 8
g) 4, 3; = 7 × 7
h) 3, 2; = 9 × 5
2. b) 4 × 10 000 +
   5 × 1 000 +
   3 × 100 +
   2 × 10 +
   6
c) 7 × 10 000 +
   2 × 1 000 +
   2 × 10 +
   3
3. Teacher to check the grouping, which is based on these combinations:
   1 × 24 or 2 × 12 or 3 × 8 or 4 × 6.
4. a) Sometimes
   b) Always
   c) Sometimes
   d) Always
   e) Always
   f) Never
5. The smallest 2-digit number is 10. Thus the product of 10 × 10 = 100. Everything that is greater than 10 will produce a greater product.
6. a) 4 × 321 = 1284
   b) 1 × 234 = 234

AP Book NS5-32
page 77
1. 1 950
2. 744
3. 1 044
4. 5 880
5. 564
6. a) 333
   b) 444
   c) 555
   d) 666
   e) 777
   f) 888
7. 900
8. Answers will vary depending on how students approximate – teacher to check.

AP Book NS5-33
page 78
1. a) hats, 2, 5
   b) ducks, 4, 3
2. Teacher to check.
3. b) 32 crackers, 8, 4
c) 18 flowers, 3, 6
d) 45 oranges, 9, 5

AP Book NS5-34
page 80
1. a) 3
   b) 4
2. a) 5 \( \Delta \) per set
   b) 3 \( \Delta \) per set
3. 2 \( \Delta \) per set
4. a) 2
   b) 4
   c) 3
5. a) 2 sets
   b) 4 sets
6. a) 30 stickers, 5, 6
   b) 24 children, 6, 4
   c) 14 apples, 7, 2
   d) 24 comic books, 3, 8
   e) 35 children, 7, 5
   f) 24 people, 2, 12
   g) 12 books, 4, 3
   h) 10 flowers, 2, 5
   i) 8 hamsters, 4, 2
7. a) 2
   b) 3
   c) 3
   d) 2
   e) 4
   f) 2
   g) 2
   h) 3
   i) 8
   j) 5

AP Book NS5-35
page 83
1. Teacher to check representation/picture.
   b) 2 + 2 + 2 + 2 = 8
   c) 5 + 5 + 5 + 5 = 20
2. a) 24 + 6 = 4

Answer Key for AP Book 5.1
b) 24 ÷ 4 = 6

c) 21 ÷ 7 = 3

d) 15 ÷ 3 = 5

e) 16 ÷ 4 = 4

f) 24 ÷ 8 = 3

3. a) 18 + 2 = 9

b) 9 ÷ 3 = 3

AP Book NS5-37

page 86

1. a) 16, 4, 4

b) 15, 3, 5

c) 6, 4, 24

d) 3, 3, 9

e) 12, 4, 3

f) 12, 6, 2

2. Teacher to check.

3. Teacher to check picture.

a) 21 + 3 = 7; 21 + 7 = 3; 7 × 3 = 21

b) 14 + 2 = 7; 2 × 7 = 14

AP Book NS5-38

page 89

1. No; since 7 ÷ 2 = 3 R1

– teacher to check any diagrams provided.

2. a) 2, 2

b) 3, 1

c) 4, 9, 2

d) 5, 8, 6

3. Teacher to check drawings.

a) 14 ÷ 4 = 3 R2

b) 18 ÷ 6 = 3

c) 17 ÷ 4 = 4 R1

d) 22 ÷ 3 = 7 R1

4. Each child will receive 4 sea shells, and 2 will be left over.

5. Answers will vary.

Possible answers:

7 groups of 4, R1

4 groups of 7, R1

2 groups of 14, R1

14 groups of 2, R1

6. There are two answers: 8 or 12 stickers.

AP Book NS5-39

page 90

1. a) 3, 7, 6

b) 4, 9, 5

c) 4, 9, 2

d) 5, 8, 6

2. a) 1

b) 1

c) 1

d) 2

e) 2

f) 1

g) 1

h) 2

i) 1

j) 1

3. Teacher to check drawings.

a) 14 ÷ 4 = 3 R2

b) 18 ÷ 6 = 3

c) 17 ÷ 4 = 4 R1

d) 22 ÷ 3 = 7 R1

4. Each child will receive 4 sea shells, and 2 will be left over.

5. Answers will vary.

Possible answers:

7 groups of 4, R1

4 groups of 7, R1

2 groups of 14, R1

14 groups of 2, R1

6. There are two answers: 8 or 12 stickers.

AP Book NS5-40

page 91

1. a) 6 × 4 = 24; 4 × 6 = 24; 24 + 4 = 6; 24 + 6 = 4

c) 3 × 4 = 12; 4 × 3 = 12; 12 + 3 = 4; 12 + 4 = 3; 12 + 4 = 3; 12 + 4 = 3; 12 + 4 = 3; 12 + 4 = 3

d) 2 × 4 = 8; 4 × 2 = 8; 8 + 2 = 4; 8 + 4 = 2; 8 + 4 = 2; 8 + 4 = 2

2. a) 3, 3

b) 8, 8

c) 3, 3

d) 5, 5

e) 5, 5

3. Teacher to check.

AP Book NS5-37

page 86

1. a) 16, 4, 4

b) 15, 3, 5

c) 6, 4, 24

d) 3, 3, 9

e) 12, 4, 3

f) 12, 6, 2

2. Teacher to check.

3. Teacher to check picture.

a) 21 + 3 = 7; 21 + 7 = 3; 7 × 3 = 21

b) 14 + 7 = 7; 14 + 2 = 7; 2 × 7 = 14

AP Book NS5-38

page 89

1. No; since 7 ÷ 2 = 3 R1

– teacher to check any diagrams provided.

2. a) 2, 2

b) 3, 1

c) 4, 9, 2

d) 5, 8, 6

3. Teacher to check drawings.

a) 14 ÷ 4 = 3 R2

b) 18 ÷ 6 = 3

c) 17 ÷ 4 = 4 R1

d) 22 ÷ 3 = 7 R1

4. Each child will receive 4 sea shells, and 2 will be left over.

5. Answers will vary.

Possible answers:

7 groups of 4, R1

4 groups of 7, R1

2 groups of 14, R1

14 groups of 2, R1

6. There are two answers: 8 or 12 stickers.

Answer Key for AP Book 5.1
l) 1, 8
m) 1, 7
n) 3, 9
o) 2, 6
p) 2, 8
q) 1, 9
r) 1, 7
s) 2, 6
t) 4, 8

6. a) 1, 8, 1
b) 3, 6, 1
c) 1, 4, 2
d) 2, 6, 2
e) 1, 3, 1
f) 1, 5, 3
g) 1, 6, 3
h) 2, 6, 2
i) 1, 5, 2
j) 2, 8, 0

7. a) 1, 5, 25
b) 1, 3, 27
c) 2, 8, 13
d) 2, 6, 2
e) 1, 3, 1
f) 1, 5, 3
g) 1, 6, 3
h) 2, 6, 2
i) 1, 5, 2
j) 2, 8, 0

8. a) 24, 8, 16
b) 17, 5, 35
c) 37, 6, 15
d) 17, 3, 21
e) 14, 5, 22
f) 12, 7, 15
g) 47, 8, 15
h) 12, 8, 16
i) 30, 9, 2
j) 46, 8, 13

9. a) 14, 5, 24, 20, 4
b) 25, 6, 17, 15, 2
c) 33, 6, 7, 6, 1
d) 17, 4, 30, 28, 2
e) 22, 8, 10, 8, 2
f) 16, 5, 31, 30, 1
g) 21, 8, 4, 4, 0

10. 2 are left over (since 98 ÷ 8 = 12 R2).

11. There are 13 weeks in 93 days (since 93 ÷ 7 = 13 R2).

12. 15 days

13. 12 m

14. Guerdy uses 15 boxes (rounded up from 14 R1).
Tyree uses 17 boxes (rounded up from 16 R3).
Tyree uses more boxes.

AP Book NS5-42
page 99

1. 30 children
2. a) 105, 150, 501, 510
   b) 150, 510
   c) 105, 150, 510
   d) 150, 510
   e) 105, 150, 501, 510
3. 14
4. 8 packets
5. 6 cars
6. 13 days
7. 8 plums
8. 11 pages

AP Book NS5-43
page 100

1. 900 students
2. 18, 240
3. 2 pencils in a set; 4 sets in total; cost is 17¢ per set and 68¢ in total.
4. $133
5. 4,500 m
6. Answers will vary.
7. 12 apples
8. a) 25 years old
   b) 49 years old
9. a) 1800 m
   b) 200 m
   c) 3 laps
10. 12 + 3 = 4
So there are 4 groups of 3 in 12 CDs:
4 × $23 = $92
11. 2. 6 × 400 = 2400

AP Book NS5-44
page 101

1. a) 0 ←
   b) → 10
   c) 0 ←
   d) → 10
2. a) i) 1, 2, 3, 4
   ii) 6, 7, 8, 9
   b) The number 5 is special because it is 5 units from both 0 and 10
3. a) 10 ←, 20 ←, → 30
   b) 60 ←, → 70, → 80
   c) 250 ←, → 260, → 270
4. a) 30
   b) 10
   c) 40
   d) 70
   e) 250
   f) 490
5. a) → 100
   b) 0 ←
6. 50 is in between 0 and 100.
7. a) 100
   b) 0
   c) 0
   d) 100
8. Teacher to check number line.
   a) 600
   b) 700
   c) 800
   d) 700
9. a) 200
   b) 600
   c) 900

AP Book NS5-42
page 99

1. 30 children
2. a) 105, 150, 501, 510
   b) 150, 510
   c) 105, 150, 510
   d) 150, 510
   e) 105, 150, 501, 510
3. 14
4. 8 packets
5. 6 cars
6. 13 days
7. 8 plums
8. 11 pages

AP Book NS5-43
page 100

1. 900 students
2. 18, 240
3. 2 pencils in a set; 4 sets in total; cost is 17¢ per set and 68¢ in total.
4. $133
5. 4,500 m
6. Answers will vary.
7. 12 apples
8. a) 25 years old
   b) 49 years old
9. a) 1800 m
   b) 200 m
   c) 3 laps
10. 12 + 3 = 4
So there are 4 groups of 3 in 12 CDs:
4 × $23 = $92
11. 2. 6 × 400 = 2400

AP Book NS5-44
page 101

1. a) 0 ←
   b) → 10
   c) 0 ←
   d) → 10
2. a) i) 1, 2, 3, 4
   ii) 6, 7, 8, 9
   b) The number 5 is special because it is 5 units from both 0 and 10
3. a) 10 ←, 20 ←, → 30
   b) 60 ←, → 70, → 80
   c) 250 ←, → 260, → 270
4. a) 30
   b) 10
   c) 40
   d) 70
   e) 250
   f) 490
5. a) → 100
   b) 0 ←
6. 50 is in between 0 and 100.
7. a) 100
   b) 0
   c) 0
   d) 100
8. Teacher to check number line.
   a) 600
   b) 700
   c) 800
   d) 700
9. a) 200
   b) 600
   c) 900
14. If the number represented by the hundreds column is 5 or greater, you round up to the nearest thousand. If the number is less than 5, you round down to the nearest thousand.
c) 800, 20 - high
300, 40 - low
1 100, 20 - low
d) 700, 20 - low
200, 10 - high
900, 10 - low
2. a) Too low: 955
b) Too high: 697
3. a) 500
b) 1 000
c) 1 200
4. a) Nearest hundred
b) 1 up, 1 down
c) Front end
d) 1 up, 1 down
e) Nearest hundred
5. Teacher to check.

AP Book NS5-51

1. Teacher to check explanations.
2. a) 50
b) 50
c) 18 000
d) 30
e) 8 000
f) 40 000
g) 100
h) 10
i) 9 000
3. a) 60
b) 40
c) 150
d) 70
e) 60
f) 30
g) 70
h) 90
i) 40
4. Teacher to check.
5. a) 46.8 = 47
b) 85
c) 462.4 = 462
6. a) 300 + 100 + 600
   + 200 + 100 + 100
   = 1 400
b) 200 + 300 + 300
   + 500 + 100 + 100
   = 1 500

7. Answers will vary.
8. Answers may vary.
   Possible answer:
   Count the numbers on a page, then multiply that number by the number of pages in the book.

AP Book NS5-52

1. a) 5, 10, 15, 20, 25 | 26, 27, 28
b) 5, 10, 15, 20 | 21, 22, 23
2. a) 10, 20, 30 | 35, 40, 45, 50, 55
b) 10, 20, 30 | 35, 40, 45, 50
c) 25, 50, 75 | 80, 85
d) 25, 50, 75 | 85, 95
3. a) 25, 50, 60, 70 | 71, 72, 73
b) 25, 50, 60, 70 | 75, 80
c) 25, 50, 75 | 85, 95 | 96, 97
BONUS:
25, 50 | 60, 70, 80 | 85, 90 | 91, 92, 93, 94
4. b) 25, 50, 60, 70, 80, 81
BONUS:
25, 50, 75, 100, 125, 135, 145, 150, 155, 160, 161, 162, 163
5. b) 25, 50 | 55, 60 | 61, 62, 63
b) 25, 50 | 60, 70 | 71, 72
d) 25, 50, 75 | 85, 95 | 100, 105
BONUS:
25, 50, 75, 100, 125, 135, 145, 150, 155, 160, 161, 162, 163

AP Book NS5-53

1. a) + 10¢ + 10¢
b) + 25¢ + 25¢
c) + 10¢ + 10¢
d) + 25¢ + 25¢
2. a) + 5¢ + 1¢
b) + 10¢ + 5¢
c) + 10¢ + 5¢
d) + 25¢ + 10¢
e) + 10¢ + 10¢
f) + 10¢ + 5¢
g) + 25¢ + 5¢
h) + 10¢ + 10¢
i) + $2 + $1
j) + $2 + $1
k) + $1 + $1
l) + $2 + $1
m) + 25¢ + 1¢
3. a) + 25¢ + 5¢
b) + 5¢ + 1¢ + 1¢
c) + $2 + $2
d) + $2 + 25¢ + 10¢ + 10¢
4. a) $1 + 25¢ + 25¢
b) 25¢ + 25¢ + 10¢ + 10¢
c) $2 + $2 + $2 + $2
d) $2 + $2 + $2 + $2 + $1 + 25¢ + 25¢
e) $2 + $2 + $2 + $2 + $2 + 25¢ + 25¢

AP Book NS5-54

1. a) 25¢
b) 25¢

AP Book NS5-55

1. b) 4, 6, 5, $4.65
c) 0, 6, 2, $0.62
d) 0, 0, 2, $0.02
e) $0.00
2. b) 20¢, $0.20
c) 60¢, $0.60
d) 4¢, $0.04
e) 13¢, $0.13
f) 25¢, $0.25
g) 25¢, $0.25
h) 75¢, $0.75
i) $0.80
j) $12.00

Answer Key for AP Book 5.1
3. a) $5.00, 55¢, $5.55
b) $15.00, 36¢, $15.36
c) $20.00, 51¢, $20.51

4. b) 110¢, $1.10

5. a) $3.25
b) $0.20
c) $0.06
d) $2.83
e) $2.05

6. a) 299¢
b) 343¢
c) 141¢
d) 8¢

7. a) $1.96
b) 103¢
c) 840¢

8. a) Seven dollars and seventy cents
b) Nine dollars and eighty-three cents
c) Fifteen dollars and eighty cents

9. a) 1, 1, 2, 1, 1, 1, 1, 1, 1, 1; $43.42
b) 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1; $60.40
c) 1, 2, 1, 1, 1, 2, 0, 1, 2; $48.57

10. $2.62; 200¢ = $2.00 and $0.62 > 56¢

11. 1 toonie, 1 loonie, 2 quarters

12. 5 loonies, 1 quarter

13. a) Three dollars and fifty-seven cents
b) Twelve dollars and twenty-three cents

k) 400¢, $4.00
l) 700¢, $7.00

AP Book NS5-56

1. b) 2, 50¢, 0, 50¢, 0, 50¢, 2 = 52¢
c) 3, 75¢, 2, 95¢, 0, 59¢, 2 = 97¢
d) 0, 0¢, 2, 20¢, 0, 20¢, 3 = 42¢
e) 1, 25¢, 1, 35¢, 1, 40¢, 4 = 42¢
f) 3, 75¢, 1, 85¢, 1, 90¢, 4 = 94¢

2. a) $40
b) $20
c) $20
d) $40
e) $20

3. Answer may vary – teacher to check.
b) 0, 1, 1, 0, 0, 0
c) 1, 0, 0, 0, 2, 0
d) 1, 1, 1, 1, 0, 0
e) 1, 0, 1, 0, 2, 0

4. a) 25¢ + 25¢ + 10¢ + 10¢ + 1¢ + 1¢
b) 25¢ + 25¢ + 25¢ + 10¢ + 5¢ + 1¢ + 1¢ + 1¢
c) 25¢ + 25¢ + 25¢ + 5¢ + 1¢ + 1¢
d) 25¢ + 25¢ + 1¢ + 1¢

5. a) $50 + $5
b) $50 + $10 + $5 + $2
c) $50 + $10 + $2 + $2

AP Book NS5-57

1. a) 6¢
b) 9¢
c) 6¢
d) 30¢
e) 60¢
f) 40¢
g) 90¢
h) 60¢
i) 40¢
j) 50¢
k) 70¢

2. a) 10¢
b) 30¢
c) 50¢
d) 60¢
e) 90¢
f) 70¢
g) 80¢
h) 40¢
i) 20¢

3. a) 20¢
b) 30¢
c) 80¢
d) 40¢

4. b) 60

5. a) 4, 60, 40, 44¢
b) 7, 90, 10, 17¢
c) 6, 60, 46¢
d) 5, 30, 70, 75¢
e) 3, 50, 50, 53¢
f) 9, 40, 60, 69¢

6. a) 26¢
b) 33¢
c) 64¢
d) 47¢
e) 28¢
f) 65¢
g) 3¢
h) 41¢
i) 11¢
j) 8¢

7. a) 13¢
b) 17¢

8. 25¢ + 25¢ + 5¢ + 1¢ + 1¢ + 1¢ = 58¢

9. a) $8.00
b) $6.00
c) $6.00
d) $3.00

10. a) 3, 30, 20, $23.00
b) 2, 40, 60, $62.00
c) 7, 60, 40, $47.00
d) 6, 20, 30, $36.00

11. a) $16
b) $75
c) $54

d) $12

e) $48
BONUS:

12. $2, $70, $72.43
13. a) $15 + $7 + $60 = $67.15
   b) 73¢ + $3 + $10 = $13.73
   c) 81¢ + $7 + $40 = $47.81
   d) 57¢ + $3 + $30 = $33.57

AP Book NS-58

120
1. a) $8.68
   b) $58.38
   c) $49.89
2. a) $40.35
   b) $72.57
   c) $93.95
   d) $60.60
   e) $82.65
   f) $64.77
3. $30.78
4. $660.07
5. $36.90
6. Yes
7. a) $64.55
   b) $52.75
   c) $73.25
   d) $64.60
8. a) $55.19
   b) Pants & soccer ball
   c) Yes
   d) $128.31
   e) Answers will vary.
9. a) $9.20
   b) 7
   c) 8
   d) No
   e) 3 oranges

AP Book NS-60

123
1. a) 1, 1, 2, 2, 1, 1, 1, 1; Estimates may vary; Total = $45.41
   b) 2, 2, 1, 3, 1, 2, 1, 1; Estimates may vary; Total = $72.66
   c) 2, 3, 1, 1, 1, 2, 1, 3; Estimates may vary; Total = $78.68
2. b) $90¢
   c) $50¢
   d) $20¢
   e) $50¢
   f) $80¢
   g) $30¢
   h) $10¢
   i) $30¢
3. b), e)
4. b) $13.00
   c) $26.00
   d) $7.00
   e) $45.00
   f) $12.00
   g) $53.00
   h) $65.00
   i) $12.00
   j) $78.00

5. b) Estimate: $5.00
   Answer: $4.40
   c) Estimate: $9.00
   Answer: $9.18
   d) Estimate: $7.00
   Answer: $7.18
   e) Estimate: $11.00
   Answer: $11.27
   f) Estimate: $78.00
   Answer: $77.99
   g) Estimate: $92.00
   Answer: $92.69

6. $21.00
7. $8.00
8. $37.00
9. $36.00
10. a) $31, $30.96
    b) $62, $61.80
11. Rounding in this scenario would provide the same number. Less than a dollar more.
AP Book ME5-1

1. a) 15
   b) 35
   c) 25
   d) 30

2. a) 9:12
   b) 12:34
   c) 3:57

3. Teacher to check.

AP Book ME5-2

1. a) 5:00:37
   b) 8:23:55
   c) 7:15:18
   d) 10:40:08

2. a) 1 min 10 s = 70 s
   b) 1 min 29 s = 89 s
   c) 1 min 57 s = 117 s

3. Teacher to check.

AP Book ME5-3

1. a) 5:20
   b) 11:15
   c) 3:56
   d) 8:30
   e) 7:41
   f) 8:45

2. a) 35 minutes after 11;
    25 minutes to 12
   b) 40 minutes after 7;
    20 minutes to 8
   c) 57 minutes after 4;
    3 minutes to 5
   d) 34 minutes after 1;
    26 minutes to 2

3. Teacher to check.

AP Book ME5-4

1. a) 20 minutes
   b) 30 minutes
   c) 35 minutes
   d) 20 minutes
   e) 20 minutes
   f) 35 minutes

2. a) 35 minutes
   b) 30 minutes
   c) 40 minutes
   d) 40 minutes

3. 45 minutes

4. 8:15

5. 4:35

BONUS:

5. a) 12
   b) Any number after
      12:00 p.m

3. a) 06:00
   b) 19:00
   c) 16:00
   d) 20:00
   e) 21:00
   f) 00:00
   g) 12:00
   h) 17:00
   i) 10:00

4. a) 8:00 a.m.
   b) 2:00 p.m.
   c) 16:00
   d) 12:00 p.m.
   e) 12:00 a.m.
   f) 7:00 p.m.
   g) 4:00 p.m.

5. Teacher to check digital
   and analog forms.

AP Book ME5-5

1. a) 12-hour 24-hour
   12:00 a.m. 00:00
   1:00 a.m. 01:00
   2:00 a.m. 02:00
   3:00 a.m. 03:00
   4:00 a.m. 04:00
   5:00 a.m. 05:00
   6:00 a.m. 06:00
   7:00 a.m. 07:00
   8:00 a.m. 08:00
   9:00 a.m. 09:00
   10:00 a.m. 10:00
   11:00 a.m. 11:00
   12:00 p.m. 12:00
   1:00 p.m. 13:00
   2:00 p.m. 14:00
   3:00 p.m. 15:00
   4:00 p.m. 16:00
   5:00 p.m. 17:00
   6:00 p.m. 18:00
   7:00 p.m. 19:00
   8:00 p.m. 20:00
   9:00 p.m. 21:00
   10:00 p.m. 22:00
   11:00 p.m. 23:00

2. a) 12
   b) Any number after
      12:00 p.m

3. a) 06:00
   b) 19:00
   c) 16:00
   d) 20:00
   e) 21:00
   f) 00:00
   g) 12:00
   h) 17:00
   i) 10:00

BONUS:

5. Spent Finished
   Start 10:30
   Reptiles 2 h 12:30
   Monkeys 1 h 30 m 14:00
   Lunch 30 m 14:30
   Polar Bears 45 m 15:15
   Lions 20 m 15:35

6. a) In the morning:
   For 12:00 a.m.,
   the 24-hr time is
   00:00.
   For single-digit
   a.m. times, there
   is a 0 in front of
   the hour
   (e.g. 1:00 a.m.
   is 01:00) and
   remove the a.m.

   b) In the afternoon
      or evening:
   For the hours after
   12:00 p.m., you
   need to add 12
to the hour to
get the proper
24-hr time
(e.g. 1:00 p.m. is
13:00) and
remove the p.m.

AP Book ME5-6

1. a) Days Hours
    1 24
    2 48
    3 72

   b) Weeks Days
    1 7
    2 14
    3 21

Answer Key for AP Book 5.1
c) | Years | Weeks |
<table>
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<th></th>
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<tbody>
<tr>
<td>1</td>
<td>52</td>
</tr>
<tr>
<td>2</td>
<td>104</td>
</tr>
<tr>
<td>3</td>
<td>156</td>
</tr>
</tbody>
</table>

d) | Years | Days |
<table>
<thead>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>365</td>
</tr>
<tr>
<td>2</td>
<td>730</td>
</tr>
<tr>
<td>3</td>
<td>1095</td>
</tr>
</tbody>
</table>

2. a) 4
b) 6
c) 9
d) 2
e) 8
f) 15
g) 20
h) 300
i) 4

3. This was roughly 5 centuries ago.

4. a) 1 hr 20 minutes (= 80 minutes) is longer.
   b) 3 hrs 10 minutes (= 190 minutes) is longer.

5. A cheetah can run 1800 (30 × 60) metres in a minute, which is 1200 metres further than a human.

6. Clara worked for 2 hours and 15 minutes.

7. | Time  | A     | B     |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>14:00</td>
<td>0 km</td>
<td>0 km</td>
</tr>
<tr>
<td>15:00</td>
<td>0 km</td>
<td>4 km</td>
</tr>
<tr>
<td>16:00</td>
<td>5 km</td>
<td>8 km</td>
</tr>
<tr>
<td>17:00</td>
<td>10 km</td>
<td>12 km</td>
</tr>
<tr>
<td>18:00</td>
<td>15 km</td>
<td>16 km</td>
</tr>
<tr>
<td>19:00</td>
<td>20 km</td>
<td>20 km</td>
</tr>
<tr>
<td>20:00</td>
<td>25 km</td>
<td>24 km</td>
</tr>
</tbody>
</table>

   a) 2 km apart
   b) 19:00

AP Book ME5-7

1. a) 5°C
   b) 10°C

2. Answers will vary – teacher to check.

3. It is 1°C lower than normal.

4. a) 2°C
   b) Rattlesnake (15°C - 37°C)
   c) Lizard - 31°C - 35°C
       Salmon - 5°C - 17°C
       Rattlesnake - 15°C - 37°C

BONUS:

5. 20°C
Answer Key for AP Book 5.1

AP Book PDM5-1

1. a) Wood – 2
   Glass – 1
   Metal – 3
   Green – 3
   Blue – 1
   Red – 4
b) Green – 3
   Blue – 1
   Red – 4
2. a) C, D
   b) A, C, F
   c) A, B, E, F
3. a) Intersection of Dark and Triangles: E
   b) Neither Light nor Polygons: D
4. a) Dark (only): B, D
   Triangles (only): A, F
   Both Dark and Triangles: E
   Neither: C, G
   b) Light (only): C
   Polygons (only): B, E
   Both Light and Polygons: G, F, A
   Neither: D
5. Lakes in NA (only): I
   Lakes larger than 50,000 km (only): A, C, E
   Intersection: B, D, F
   Neither: G, H

AP Book PDM5-2

1. a) E, F, G
   b) A, B, C, D, E
   c) E
   d) H
   e) 

AP Book PDM5-3

1. a) 15, 3, 27
   b) 500, 25, 600
   c) 700, 5, 720

AP Book PDM5-4

1. a) 8
   b) Fruits
   c) Vegetables
   d) 20
   e) November, because the amount of healthy food consumed went up in December.
2. a) Teacher to check.
   b) Naoko - 65
   c) Matias - 95
   d) 1 person (Matias) – teacher to check explanation.
   e) 3 girls voted for the winner (Matias).

AP Book PDM5-5

1. a) 0, 5, 15
   b) Tuesday
   c) Tuesday
   d) Thursday
   e) Sunday and Friday
2. a) August
   b) (i) 6
   (ii) 14
   c) May through October (inclusive)
   d) Summer - 40
   Spring - 20
   Fall - 12
   Winter - 4
3. a) The second graph makes it easier – teacher to check explanation.
   b) Gisela

AP Book PDM5-6

1. a) Yes, Continuous
   b) No, Discrete
   c) No, Discrete
   d) No, Discrete
2. a) Kilometre, Continuous
   b) Fold, Discrete
   c) Runner, Discrete
   d) Millilitres, Continuous
3. a) H: Discrete
   V: Discrete
   b) H: Discrete
   V: Continuous
   c) H: Continuous
   V: Continuous

AP Book PDM5-7

1. a) 2 km
   b) 6 km
   c) 7 km
2. Teacher to check graphs.
   a) $35
   b) 35
   c) 250 metres
3. Teacher to check graphs.
   a) 15
   b) 10
   c) 9

AP Book PDM5-8

1. a) Broken line
   b) Continuous line
   c) Continuous line
   d) Broken line
2. a) Teacher to check
   b) Months and number of birds are not continuous.
   c) There is a 3 hour difference.
   Flying squirrels become active just before it starts to get dark; it gets dark earlier in the winter.
3. a) Bar, as it compares the # of muffins sold by each child.
   b) Line, as it shows the decrease in price with the increase in number of CDs.

AP Book PDM5-9
page 146

1. a) A - Survey or B - Observation
   b) C - Measurement
   c) A - Survey
   d) A - Survey
   e) C - Measurement

2. a) A - Primary
    b) B - Secondary
    c) A - Primary
    d) B - Secondary

3. Teacher to check – answers will vary.
4. Teacher to check – answers will vary.

AP Book PDM5-10
page 147

1. a) \( \frac{1}{2} \)
    b) \( \frac{40}{160} = \frac{1}{4} \)
    c) Yes she could, but it would not be accurate.
    d) Looking at 20 plants is better as it gives a more accurate sense of how many pods each plant has, and how many of these pods are ripe.
    e) (i) 160 pods per 20 plants \( \times 5 \) sets of twenty plants = 800 pods in the garden
        (ii) 40 ripe pods per 20 plants \( \times 5 \) sets of twenty plants = 200 ripe pods in the garden

2. a) sample
    b) whole
    c) whole
    d) sample

AP Book PDM5-11
page 148

1. C is the best question since it allows for variety in the amount of time students can devote to reading, gives different choices beyond yes/no and doesn't rely on people remembering the precise number of books they read.

2. Teacher to check – answers will vary.
3. Teacher to check – answers will vary.
4. Teacher to check – answers will vary.

AP Book PDM5-12
page 149

1. Teacher to check – answers and explanations will vary.
2. Teacher to check – answers and explanations will vary.
AP Book G5-1
page 151
1. a) 6; 6  
   b) 5; 5  
   c) 12; 12  
   d) 8; 8  
   e) 8; 8  
   f) 10; 10
2. a) 3  
   b) 4  
   c) 5  
   d) 6
3. Triangles: B  
   Quads: A, D, F, G  
   Pentagons: C, H  
   Hexagons: E, I
4. a) Teacher to check.  
   b) Teacher to check.
5. 3 × 4 sides + 5 × 5 sides  
   = 12 + 25 sides  
   = 37 sides in total

AP Book G5-2
page 152
1. a) Less than  
    b) Greater than  
    c) Right angle  
    d) Less than
2. A, C, D, E have at least one right angle.  
   A, C have two right angles.
3. A: 2 obtuse, 2 acute  
   B: 3 right, 2 obtuse  
   C: 2 acute, 4 obtuse  
   D: 3 right, 2 obtuse
4. a) Answers will vary, for example: E, H, T, L, P  
    b) Answers could vary, for example: E (4), H (4)
5. a) Answers will vary  
    b) Answers will vary

AP Book G5-3
page 153
1. a) Acute
2. a) Acute-angled  
    b) Obtuse-angled  
    c) Right-angled  
    d) Obtuse-angled  
    e) Acute-angled
3. a) 30°  
    b) i) 120°  
       ii) 150°

AP Book G5-4
page 156
1. Teacher to check.
2. Teacher to check.
3. a) 30°  
    b) i) 120°  
       ii) 150°

AP Book G5-5
page 157
1. a) Acute-angled  
    b) Obtuse-angled  
    c) Right-angled  
    d) Obtuse-angled  
    e) Acute-angled
2. a) 90°, 45°, 45° – right-angled  
    b) 40°, 110°, 30° – obtuse-angled
3. a) 50°, 60°, 70° – acute-angled
   b) 60°  
   c) 90°  
   d) 110°  
   e) 120°
3. a) i) 60°  
    b) 90°  
    c) 110°  
    d) 120°
4. a) Acute only: E  
    b) Equilateral only: none  
    c) Both acute and equilateral: A  
    d) Outside: B, C, D
5. Answers may vary.

AP Book G5-6
page 158
NOTE: Side lengths and angles recorded clockwise from left to right.
1. A △ 60° angles with 3 cm sides  
   B △ 90°, 53°, 37° with 3 cm, 5 cm, 4 cm side lengths  
   C △ 35°, 110°, 35° with side lengths 4 cm, 4 cm, 6.5 cm  
   D △ 25°, 120°, 35° with side lengths 4 cm, 3.2 cm, 6.2 cm  
   E △ 70°, 40°, 70° with side lengths 2 cm, 3 cm, 3 cm
2. a) Acute only: E  
    b) Equilateral only: none  
    c) Both acute and equilateral: A  
    d) Outside: B, C, D
3. Answers may vary.

AP Book G5-7
page 159
1. There is only one way to draw both a) and b).  
   Using a protractor with the given angles, the appropriate triangles can be sketched.
2. a) First a 5 cm base should be drawn, followed by a 45° angle to create the left vertices, a 50° angle to create the right vertices and create the triangle.  
   b) First a 5 cm side should be drawn. Then, using that side, measure a 40° angle and draw a 7 cm side at this angle to the 5 cm side. Join to the sides to complete the triangle.
3. a) See above.  
    b) Each of the three triangles have a pair of equal sides.  
    c) All three triangles are isosceles.
4. a) △  
    b) △  
    c) △  
5. All the angles where the diagonals meet are 90°.
1. a) Parallel  
b) Non-parallel  
c) Parallel  
d) Non-parallel  
e) Parallel  
f) Parallel  
g) Parallel  
h) Non-parallel  

BONUS:  
i) The pair of lines that are not parallel are b, d and h.  
j) If the pairs b or d are extended, the lines will intersect. By definition, h cannot be parallel as the pair are curved lines.

2. a) Teacher to check  
b) Teacher to check  
c) Teacher to check  
d) Teacher to check  
e) Yes

3. a) Vertical sides  
b) Vertical sides  
c) Vertical sides  
d) Vertical sides  
e) Horizontal sides  
f) Vertical sides  
g) Vertical sides

4. a) 1 pair  
b) 2 pairs  
c) 1 pair  
d) 2 pairs

5. Teacher to check.

E has the greatest number of parallel lines

---

AP Book G5-9  
page 162

1. A - 2 pairs  
B - 2 pairs  
C - 1 pair  
D - 1 pair  
E - 2 pairs

---

AP Book G5-10  
page 164

1. a) No right angles.  
Side lengths: 2 cm and 3 cm  
Parallelogram  
b) 4 right angles.  
Side lengths: all 2 cm  
Square

2. Square →  
A parallelogram with 4 right angles and 4 equal sides.  
Rectangle →  
A parallelogram with 4 right angles.  
Rhombus →  
A parallelogram with 4 equal sides.

3. a) Rectangle  
b) Parallelogram  
c) Square  
d) Rhombus

4. a) 0 right angles;  
Parallelogram  
b) 4 right angles;  
Square  
c) 0 right angles;  
Rhombus  
d) 4 right angles;  
Rectangle

5. a) 2 pairs;  
Rectangle  
b) 2 pairs;  
Parallelogram  
c) 2 pairs;  
Square

6. Answers may vary, however, a shape with only one pair of parallel sides must be drawn.

7. a) All  
b) No  
c) No  
d) Some

8. a) Rhombus  
b) Rectangle  
c) Trapezoid

---

AP Book G5-11  
page 166

1. a) Not congruent  
b) Congruent  
c) Congruent

2. There are two pairs of congruent shapes: from the left, the 3rd and 6th shapes are congruent, and the 2nd and 4th are congruent.

3. a) No; they are not the same size.  
b) Yes because they are the same in shape and size.

4. a) Answer may vary  
b) Trapezoid has to be drawn vertically

5. Shapes are numbered from left to right:  
A, B, C, D, E

---

AP Book G5-12  
page 167

1. a) 2 non-congruent shapes
b) 3 non-congruent shapes
c) 4 non-congruent shapes

2. a) 4 non-congruent shapes
b) 4 non-congruent shapes

3. a) 3
b) 1
c) 11

AP Book G5-13
page 168
1–4 Teacher to check
5. a) Answer may vary, teacher to check.
b) OR

AP Book G5-14
page 170
1. a) F1 F2 S? D?
   4 4 √
   4 4 √
   2 1 √
   0 0 √
   2 2 √
   2 2 √
   2 1 √
   Y N √

b) S? D?
   √
   √
   √
   √
   √
   √
   √

   c) P1 A, C, E, F
   P2 A, D
   Share both properties: A
   Equilateral only: C, E, F
   More than one right angle only: D
   Both equilateral and more than one right angle: A
   Outside: B, G

3. Figure 1 2
   Vertices 4 3
   Edges 4 3
   # of Parallel Sides 0 0
   # of Right Angles 2 1
   # of Lines of Symmetry 0 0
   Equilateral? No No

AP Book G5-15
page 171
1. a) Share both properties: A, C
   Quadrilateral only: D
   Equilateral only: E, F
   Both quadrilateral and equilateral: A, C
   Outside: B, G
b) P1 C, E, F, G
   P2 A, C, D, E, F
   Share both properties: C, E, F
   No right angles only: G
   Four or more sides only: A, D.
   Both no right angles and four or more sides: C, E, F
   Outside: B

c) Right-angled triangle
b) Isosceles triangle
4. Descriptions should include the following:

<table>
<thead>
<tr>
<th>Sides</th>
<th>Vertices</th>
<th># of Parallel Sides</th>
<th># of Right Angles</th>
<th># of Acute Angles</th>
<th># of Obtuse Angles</th>
<th># of L of S</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b)</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>c)</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

5. a) Common Properties:
Number of sides, vertices, and pairs of parallel sides
Differences:
Equilateral, number of right, acute and obtuse angles, and lines of symmetry.
b) Common Properties:
Number of sides and vertices
Differences:
Equilateral, number of right, acute and obtuse angles, lines of symmetry and pairs of parallel sides.

AP Book G5-16
page 174
1. a) 3 vertices – T
   No right angles – T
   2 parallel pairs – T
   5 vertices – F
   Equilateral – T
   1 parallel pair – F
   3 parallel pairs – T
   No acute angles – T
   b) 4 vertices – F
   No parallel sides – T
   4 sides – F
   1 right angle – T
   3 vertices – F
   5 sides – T
   No right angles – F
   Equilateral – F
   c) Quadrilateral – T
   1+ right angles – F
   1+ pairs of parallel sides – T
   5 vertices – T
   5 edges – T
   No right angles – T
   Equilateral – F

d) B – Rhombus
2. | S | Re | Rh | P |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

2 2 2 2

4 2 2 0

3. a) Equilateral triangle
   b) Obtuse-angled triangle
   c) Right-angled triangle
   d) Acute-angled triangle

4. a) 6
   b) 4

5. Teacher to check.
Patterns & Algebra – AP Book 5.2

AP Book PA5-24

1. a) 
\[
\begin{array}{c|c}
\text{s} & 3\times s = t \\
1 & 3 \times 3 = 9 \\
2 & 3 \times 2 = 6 \\
3 & 3 \times 3 = 9 \\
\end{array}
\]

b) 
\[
\begin{array}{c|c}
\text{s} & 4\times s = t \\
1 & 4 \times 1 = 4 \\
2 & 4 \times 2 = 8 \\
3 & 4 \times 3 = 12 \\
\end{array}
\]

2. a) Multiply by 4: 
\[4 \times s = t\]

b) Multiply by 5: 
\[5 \times s = t\]

c) Multiply by 2: 
\[2 \times s = t\]

d) Multiply by 6: 
\[6 \times s = t\]

3. a) 
\[
\begin{array}{c|c|c}
\text{Squares} & \text{Triangles} \\
1 & 4 \\
2 & 8 \\
3 & 12 \\
\end{array}
\]

AP Book PA5-25

1. a) 
\[
\begin{array}{c|c|c}
\text{Row} & \text{r + 2 = c} & \text{Chairs} \\
1 & 1 + 2 = 3 & 3 \\
2 & 2 + 2 = 4 & 4 \\
3 & 3 + 2 = 5 & 5 \\
\end{array}
\]

b) 
\[
\begin{array}{c|c|c}
\text{Row} & \text{r + 5 = c} & \text{Chairs} \\
1 & 1 + 5 = 6 & 6 \\
2 & 2 + 5 = 7 & 7 \\
3 & 3 + 5 = 8 & 8 \\
\end{array}
\]

2. a) Add 3; \(r + 3 = c\)

b) Add 5; \(r + 5 = c\)

c) Add 4; \(r + 4 = c\)

d) Add 3; \(r + 3 = c\)

3. a) 
\[
\begin{array}{c|c|c}
\text{Row} & \text{Chairs} \\
1 & 4 \\
2 & 5 \\
3 & 6 \\
\end{array}
\]

4. No. For the design shown, an equation that shows how to calculate the number of triangles from the number of squares is: \(5 \times s = t\). For 8 brooches, \(5 \times 8 = 40\). Therefore, Wendy does not have enough triangles to make 8 brooches using this design.

5. a) Answers will vary.

b) Answers will vary.

c) Multiply by 5

d) Multiply by 3

e) Multiply by 4

f) Subtract 2

AP Book PA5-26

1. a) 
\[
\begin{array}{c|c|c}
\text{V. lines} & \text{H. lines} \\
1 & 3 \\
2 & 6 \\
3 & 9 \\
\end{array}
\]

b) 
\[
\begin{array}{c|c}
\text{Row} & \text{Chairs} \\
1 & 1 \\
2 & 2 \\
3 & 3 \\
\end{array}
\]

2. a) Circle a) and c)

b) Circle b)

c) Circle c)

d) Circle d)

AP Book PA5-27

1. a) 
\[
\begin{array}{c|c|c}
\text{Fig.} & \text{# Blocks} \\
1 & 1 \\
2 & 4 \\
3 & 6 \\
\end{array}
\]

b) 
\[
\begin{array}{c|c}
\text{Fig.} & \text{# Blocks} \\
1 & 3 \\
2 & 6 \\
3 & 9 \\
\end{array}
\]

c) 
\[
\begin{array}{c|c|c}
\text{Fig.} & \text{# Blocks} \\
1 & 1 \\
2 & 2 \\
3 & 3 \\
\end{array}
\]

d) 
\[
\begin{array}{c|c|c}
\text{Fig.} & \text{# Blocks} \\
1 & 3 \\
2 & 6 \\
3 & 9 \\
\end{array}
\]

2. a) Circle a) and c)
AP Book PA5-28 (continued) page 181

1. a) \(2 \times FN;\) 
   \(2 \times FN + 1\) 
   \(2 \times FN + 1\) 
   \(2 \times FN + 2\) 
   \(2 \times FN + 1\) 
   \(2 \times FN + 3\)

2. Teacher to check.

AP Book PA5-29 page 182

1. a) 1 5 
    2 8 
    3 11 
    Gap = 3

b) 1 4 
   2 7 
   3 10 
   Gap = 3

c) 1 6 
   2 8 
   3 10 
   Gap = 4

d) 1 7 
   2 9 
   3 11 
   Gap = 2

e) The gap and number to multiply the input of the rule are equal.

2. a) 1 4 
    2 7 
    3 10 
    Gap: 3
    RULE: 
    \(3 \times Figure \ Number + 1\)

AP Book PA5-30 page 183

5. a) Multiply by 5, subtract 3 
   b) Multiply by 6, subtract 3 
   c) Add 4 
   d) Multiply by 2, add 3 
   e) Multiply by 3 
   f) Multiply by 2, add 1

6. a) 

   \begin{align*} 
   &\text{Figure} \quad \text{# of Tri} \\
   &1 \quad 2 \\
   &2 \quad 4 \\
   &3 \quad 6 \\
   &4 \quad 8 \\
   \end{align*}

   RULE: Multiply by 2 
   Figure 9 will have 18 triangles.

AP Book PA5-31 page 187

1. a) + 
   b) × 
   c) + 
   d) × 
   e) + 
   f) + 
   g) × 
   h) × 
   i) × 
   j) + 
   k) + 
   l) x

2. a) × 
   b) x 
   c) x 
   d) – 
   e) x 
   f) + 
   g) – 
   h) + 
   i) – 
   j) + 
   k) x 
   l) +

3. a) 18, 54, 162 
   b) 9, 27, 81 
   c) 12, 24, 48 
   d) 36, 216, 1296 

4. a) Gap = \times 2; 
   16, 32 
   b) Gap = \times 3; 
   81, 243 
   c) Gap = \times 5; 
   125, 625 
   d) Gap = \times 5; 
   250, 1250 

5. a) 8, 16 
   b) 14, 17 
   c) 6, 2 
   d) 24, 48 
   e) 26, 30 
   f) 27, 81 

6. a) Start at 1, multiply by 2. 
   b) Start at 5, add 3.
c) Start at 18, subtract 4.
d) Start at 3, multiply by 2.
e) Start at 14, add 4.
f) Start at 1, multiply by 3.

AP Book PA5-32
page 188
1. a) Gaps: + 2, + 3, + 4, + 5, + 6
   Rest of Pattern: 17, 23  
   b) Gaps: + 1, + 2, + 3, + 4, + 5, + 6
   Rest of Pattern: 19, 25  
   c) Gaps: + 3, + 5, + 7, + 9, + 11
   Rest of Pattern: 37, 48
   d) Gaps: + 2, + 4, + 6, + 8, + 10, + 12
   Rest of Pattern: 37, 49
   e) Gaps: – 6, – 5, – 4, – 3, – 2
   Rest of Pattern: 10, 8
   f) Gaps: – 10, – 8, – 6, – 4, – 2
   Rest of Pattern: 24, 22
   g) Gaps: – 9, – 7, – 5, – 3, – 1
   Rest of Pattern: 38, 37
   Rest of Pattern: 210, 205

2. Figure # of Sq
   1 2
   2 5
   3 9
   4 14
   5 20
   6 27

3. Figure # of Tri
   1 1
   2 4
   3 9
   4 16
   5 25
   6 36

   Figure 6 will need 36 triangles.

4. b) Start at 7. Add 4, then subtract 2. Repeat.
c) Start at 1. Add 1, 2, 3, … (The step increases by 1.)
d) Start at 6. Add 2, then subtract 3. Repeat.
e) Start at 24. Subtract 1, 3, 5… (The step increases by 2.)
f) Start at 17. Add 3, 5, 7, … (The step increases by 2.)

5. a) Start at 0. Add 3, 5, 7, … (The step increases by 2.)
   5th term = 24
   b) Start at 1. Multiply by 3. Repeat.
   5th term = 81

AP Book PA5-33
page 190
1. a) 382, 387, 392; Start at 357. Add 5.
   b) 9, 13, 10;
   Start at 6. Add 4, then subtract 3. Repeat.
   c) 28, 22, 15;
   Start at 42. Subtract 2, 3, 4, … (The step increases by 1.)

AP Book PA5-35
page 192
1. a) 5
   b) 3
   c) 3

2. a) 5 = 3 + 2
   b) 8 = 4 + 4
   c) 2 + 3 = 5
   d) 4 + 3 = 7

3. a) 9 = 5 + 4
   b) 7 = 3 + 4
   c) 8 – 3 = 5
   d) 10 – 2 = 8
   e) 7 – 2 = 5
   f) 13 – 5 = 8
   g) 15 – 9 = 6
   h) 9 – 5 = 4
d) 2
  e) 2
  f) 4
4. a) $3 + 2 = 5$
   b) $3 + 5 = 8$
   c) $15 + 3 = 5$
5. a) $3 + 2 = 5$
   b) $10 - 4 = 6$
   c) $3 \times 4 = 12$

---

AP Book PA5-37
page 194
1. a) $3 \times 2$
   b) $5 \times 3$
   c) $6 \times 3$
2. a) $60 \times 2$
   b) $80 \times 3$
   c) $70 \text{ h or } 70 \times \text{ h}$
3. a) $5h \text{ or } 5 \times h$
   b) $5t \text{ or } 5 \times t$
   c) $5x \text{ or } 5 \times x$
   d) $5n \text{ or } 5 \times n$
4. a) $A + 3 = B$
   b) $2 \times A = B$
   c) $A + 2 = B$
   d) $A \times 3 = B$
   e) $5 \times A = B$
5. a) $10 - 4 = x$
   b) $12 - 7 = x$

---

AP Book PA5-38
page 195
1. a) 6
   b) 3
   c) 6
   d) 2
   e) 4
   f) 3
   g) 4
   h) 4
   i) 4
   j) 6
   k) 15
   l) 10
   BONUS:
   m) 4
   n) 5
2. a) $2 (\text{ if both numbers are the same})$
   OR
   $1, 3$
   p) 6
   q) 13
   r) 1
   s) 4
   t) 6
   u) 30
2. a) 0 + 0 + 10 = 10;
   1 + 1 + 8 = 10;
   2 + 2 + 6 = 10;
   3 + 3 + 4 = 10;
   4 + 4 + 2 = 10;
   5 + 5 + 0 = 10
   b) 0 + 0 + 8 = 8;
   1 + 1 + 6 = 8;
   2 + 2 + 4 = 8;
   3 + 3 + 2 = 8;
   4 + 4 + 0 = 8
   c) 1 + 1 + 3 + 3 = 8;
   4 + 4 + 0 + 0 = 8
   d) 1 + 2 + 6 = 9;
   1 + 3 + 5 = 9;
   1 + 8 + 0 = 9;
   2 + 3 + 4 = 9;
   2 + 7 + 0 = 9;
   3 + 6 + 0 = 9;
   4 + 5 + 0 = 9
3. Answers are any two of the following:
   0 + 0 + 7 = 7;
   1 + 1 + 5 = 7;
   3 + 3 + 1 = 7;
   2 + 2 + 3 = 7
4. 2
5. 1; 2; 3
6. a) 2; 3
   b) 2; 4
   c) 2; 5
   d) All numbers but 1.
   e) 2; 2; 3
   f) 3; 3; 2
   g) 3; 5
7. a) $10 + 1 = 11$
   $10 + 2 = 12$
   $10 + 3 = 13$
   $10 + 4 = 14$
   b) $10 - 1 = 9$
   $10 - 2 = 8$
   $10 - 3 = 7$
   $10 - 4 = 6$
   c) $10 \times 1 = 10$
   $10 \times 2 = 20$
   $10 \times 3 = 30$
   $10 \times 4 = 40$
8. The number in a circle increases by 1 as the number in a square increases by 1.
9. The product doubles:
   $5 \times 16 = 80$
10. $7 \times 3 \times 2 = 7 \times 6 = 42$
11. Outer darts = 3 points; Inner darts = 4 points; $3 + 3 + 4 = 10.$

---

AP Book PA5-39
page 197
1. a)

<table>
<thead>
<tr>
<th># of Tables</th>
<th># of Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

RULE:
Multiply the number of tables by 2, then add 2 ($2 \times \text{ number of tables} + 2$).

b) 32 chairs
2. 36 triangles
3. 140 km
4. a) Yes (subtract 3)
   b) No (add 4)
   c) Yes (subtract 2, add 3, repeat)
5. 24 cups of water
6. a) 24
   b) 36
7. 24th and 48th
8. Start with 1 shaded square. Add light squares to the perimeter of the previous figure, then add shaded squares to that perimeter. Repeat this pattern.
9. The pattern repeats after 5 shapes. Finding the largest multiple of 5 without exceeding 63, we find $63 + 5 = 12$ remainder 3. Then, the 3rd shape of the pattern core is a hexagon.
10. Day 1: 2
   Day 2: 5
   Day 3: 8
   Day 4: 11
   Total: 26
11. The first method to rent a sailboat is less expensive. The first hour is $8.50. The additional 4 hrs cost $18. In total, $8.50 + $18 = $26.50. The second method costs 5 hrs $\times$ $6.00 = $30.
12. a) Decrease
   b) 2,000 m
   c) 0.8 cm
   d) 0.8 cm = 2,000 m
   e) Yes
   f) 6°C, 3°C
13. No. Marlene will need 28 blocks for Figure 7.
1. a) \( \frac{7}{9} \)
   b) \( \frac{2}{4} \)
   c) \( \frac{3}{4} \)
   d) \( \frac{4}{6} \)

2. a) The total number of pieces in the pie
   b) The number of pieces in the pie that you have

3. a) 
   b) 

4. a) \( \frac{2}{3} \)
   b) \( \frac{1}{5} \)

5. a) \( \frac{1}{3} \)
   b) \( \frac{1}{2} \)

6. a) Teacher to check.
    – Final line should be 3 cm long.
   b) Teacher to check.
    – Final line should be 6 cm long.

7. a) Doesn’t show \( \frac{1}{3} \):
    there are three pieces but they’re not the same size.
   b) Does show \( \frac{1}{3} \):
    the circle is cut into three equal pieces and one piece is shaded.
   c) Doesn’t show \( \frac{1}{3} \):
    there are four pieces in the pie.
   d) Does show \( \frac{1}{3} \):
    the circle is cut into three equal pieces and one piece is shaded.

8. a) \( \frac{3}{6} = \frac{1}{2} \)
   b) \( \frac{2}{6} = \frac{1}{3} \)
   c) \( \frac{1}{6} \)
   d) \( \frac{2}{6} - \frac{1}{3} \)

9. Teacher to check.

10. NOTE:
    In the following two answers, the sequence of shapes is irrelevant.
    a) 
   b) 

3. The fraction with the larger numerator is greater.

4. a) \( \frac{1}{3} \)
   b) \( \frac{1}{2} \)
   c) \( \frac{5}{125} \)

5. The fraction with the smaller denominator is greater.

6. a) \( \frac{1}{3} \cdot \frac{2}{3} = \frac{3}{9} \)
   b) \( \frac{1}{10} \cdot \frac{2}{10} \cdot \frac{5}{10} = \frac{9}{100} \)
   c) \( \frac{1}{13} \cdot \frac{1}{3} \)
   d) \( \frac{2}{16} \cdot \frac{2}{11} \cdot \frac{7}{5} \)

7. a) \( \frac{2}{3} \)
   b) \( \frac{11}{17} \)
   c) \( \frac{6}{18} \)

8. \( \frac{1}{2} \cdot \frac{50}{100} > \frac{45}{100} \)

9. Yes, since the pies can be very different sizes:

   ![Pie Diagram]

   \( \frac{3}{6} \)
4. Fractions (b) and (c) are greater than a whole since the numerator is greater than its denominator.

AP Book NS5-67

1. a) \( \frac{3}{3} \cdot \frac{10}{3} \)
b) \( \frac{2}{4} \cdot \frac{11}{4} \)
c) \( \frac{2}{3} \cdot \frac{8}{3} \)
d) \( \frac{6}{8} \cdot \frac{30}{8} \)
e) \( \frac{2}{4} \cdot \frac{14}{4} \)
f) \( \frac{1}{8} \cdot \frac{25}{8} \)

2. Teacher to check shading.
   a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

3. Teacher to check shading.
   a) \( \frac{3}{3} \)
b) \( \frac{4}{6} = \frac{2}{3} \)
c) \( \frac{2}{4} \)
d) \( \frac{3}{5} \)

4. Teacher to check pictures.
   a) \( \frac{3}{2} \) is greater
   b) \( \frac{2}{5} \) is greater
   c) \( \frac{14}{3} \) is greater

AP Book NS5-68

1. a) 2 halves 
b) 4 halves 
c) 6 halves 
d) 5 halves 
e) 7 halves 
f) 9 halves 

2. a) 3 thirds

b) 6 thirds 
c) 9 thirds 
d) 5 thirds 
e) 7 thirds 
f) 14 thirds 
g) 4 quarters 
h) 8 quarters 
i) 20 quarters 
j) 11 quarters 
k) 21 quarters 
l) 23 quarters

AP Book NS5-69

1. a) \( \frac{3}{3} \cdot \frac{10}{3} \)
b) \( \frac{2}{4} \cdot \frac{11}{4} \)
c) \( \frac{2}{3} \cdot \frac{8}{3} \)
d) \( \frac{6}{8} \cdot \frac{30}{8} \)
e) \( \frac{2}{4} \cdot \frac{14}{4} \)
f) \( \frac{1}{8} \cdot \frac{25}{8} \)

2. Teacher to check pictures.
   a) \( \frac{3}{3} \cdot \frac{10}{3} \)
b) \( \frac{2}{4} \cdot \frac{11}{4} \)
c) \( \frac{2}{3} \cdot \frac{8}{3} \)
d) \( \frac{6}{8} \cdot \frac{30}{8} \)
e) \( \frac{2}{4} \cdot \frac{14}{4} \)
f) \( \frac{1}{8} \cdot \frac{25}{8} \)

3. Teacher to check shading.
   a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

4. Teacher to check shading.
   a) \( \frac{3}{3} \)
b) \( \frac{4}{6} = \frac{2}{3} \)
c) \( \frac{2}{4} \)
d) \( \frac{3}{5} \)

5. a) \( \frac{3}{7} \)
b) \( \frac{1}{5} \)
c) \( \frac{1}{3} \)

6. Teacher to check shading.
   a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

7. Teacher to check shading.
   a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

8. Teacher to check shading.
   a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

9. Teacher to check shading.
   a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

10. Teacher to check shading.
    a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

11. Teacher to check shading.
    a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)

12. Teacher to check shading.
    a) \( \frac{7}{2} \)
b) \( \frac{11}{4} \)
c) \( \frac{12}{5} \)
d) \( \frac{27}{6} \)
10. \( \frac{8}{3} = 2 \frac{2}{3} \)

11. Ahmed

12. 11 pieces

AP Book NS5-71 page 211

1. a) \( \frac{1}{4} \)
   b) \( \frac{3}{5} \)
   c) \( \frac{1}{3} \)

2. a) [Diagram]
   b) [Diagram]

3. Teacher to check groupings.
   a) \( \frac{4}{5} \)
   b) \( \frac{1}{2} \)
   c) \( \frac{1}{3} \)
   d) \( \frac{3}{5} \)
   e) \( \frac{2}{7} \)
   f) \( \frac{2}{3} \)

4. Answers may vary. Teacher to check.
   Sample Answer:
   \[
   \frac{6}{12} \cdot \frac{3}{6} = \frac{2}{4} \cdot \frac{1}{2}
   \]

5. [Diagram]

6. a) \( \frac{2}{8} = 1 \frac{1}{4} \)
   b) \( \frac{2}{8} = 1 \frac{1}{3} \)

AP Book NS5-72 page 213

Teachers to check drawings.

1. a) \( \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} \)
   b) \( \frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30} \)

2. a) \( 2; 2 \frac{2}{3}, 6, 4 \)
   b) \( 1 \frac{4}{3} \cdot 2; 3 \frac{4}{3}, 8, 6 \)
   c) \( 1 \frac{3}{3} \cdot 3; 2 \frac{3}{3} \cdot 9, 6 \)
   d) \( 3 \frac{3}{4} \cdot 10, 6 \)
   e) \( 3 \frac{3}{4} \cdot 12, 9 \)

3. a) [Diagram]
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

4. a) [Diagram]
   b) [Diagram]

5. Teacher to check drawings.
   a) 2
   b) 5
   c) 4
   d) 9

6. a) 6
   b) 6
   c) 10
   d) 4
   e) 15
   f) 4

AP Book NS5-73 page 214

1. a) \( \frac{3}{4} \)
   b) \( \frac{4}{5} \)

2. a) 2; 2 \frac{2}{3}, 6, 4
   b) \( \frac{1}{4} \cdot 2; 3 \frac{4}{3}, 8, 6 \)
   c) 1 \frac{3}{3} \cdot 3; 2 \frac{3}{3} \cdot 9, 6
   d) \( \frac{3}{2} \cdot 10, 6 \)
   e) \( \frac{3}{4} \cdot 12, 9 \)

9. One (1) piece has both olives and mushrooms:

[Diagram]

AP Book NS5-74 page 217

1. \$6

2. a) 6 oranges
   b) 4 oranges

3. Teacher to check the drawing.
   6 squares are blank
   7 are green
   5 \( \frac{2}{3} \) of a year, or 21 months
   6 months
   7:45 minutes
   8:4 stickers
   9. \( \frac{4}{10} \) and \( \frac{2}{5} \) are equivalent fractions
   10. 7:51
   11. No, she has 7 left, which is more than half.
   12. 12 marbles are blue.

AP Book NS5-75 page 218

1. a) <
   b) >
   c) <
   d) >

2. \( \frac{2}{5} \cdot \frac{1}{2} \)

3. \( \frac{3}{5} \cdot \frac{3}{4} \)

4. a) \( \frac{2}{4} \cdot \frac{2}{3} = \frac{1}{4} \)
b) $\frac{4}{6} < \frac{5}{6}
5. a) $\frac{3}{6} < \frac{4}{6}$
b) $\frac{4}{8} < \frac{5}{8}$
c) $\frac{2}{4} < \frac{3}{4}$
d) $\frac{5}{10} > \frac{4}{10}$
e) $\frac{6}{12} > \frac{3}{12}$
f) $\frac{3}{9} < \frac{4}{9}$
g) $\frac{2}{10} < \frac{7}{10}$
h) $\frac{2}{10} < \frac{4}{10}$
i) $\frac{4}{16} < \frac{7}{16}$

AP Book NS5-76

page 219
1. a) $\frac{2}{4}$
b) $\frac{3}{6}$
c) $\frac{2}{6}$
d) $\frac{3}{9}$
e) $\frac{3}{12}$
2. a) $\frac{3}{6} \div \frac{2}{6}$
b) $\frac{2}{4} \div \frac{1}{4}$
c) $\frac{8}{12} \div \frac{3}{12}$
d) $\frac{5}{10} \div \frac{4}{10}$
3. a) $\frac{3}{6} \div \frac{2}{6}$
b) $\frac{4}{12} \div \frac{3}{12}$
c) $\frac{5}{10} \div \frac{2}{10}$

AP Book NS5-77

page 220
1. $\frac{3}{4}$
2. a) $\frac{3}{5}$
b) $\frac{2}{3}$
3. a) $\frac{4}{5}$
b) $\frac{3}{4}$
c) $\frac{5}{7}$

AP Book NS5-78

page 221
1. Teacher to check.
2. a) $\frac{3}{5} \div \frac{1}{5}$
b) $\frac{7}{5} \div \frac{8}{5}$
3. a) 11 quarters
b) 17 fifths
c) 13 thirds
4. a) $\frac{3}{10} \div \frac{2}{10} \div \frac{1}{2}$
b) $\frac{1}{3} \div \frac{1}{2} \div \frac{5}{6}$
c) $\frac{1}{2} \div \frac{5}{8} \div \frac{3}{4}$
5. a) $\frac{2}{5}$
b) $\frac{5}{2}$
c) $\frac{2}{4}$
d) $\frac{3}{2}$
6. (c) $\frac{10}{4}$

7. 4 divided by 13 is 3
   Remainder 1.
   3 is the whole number, which in this case represents whole pies.

AP Book NS5-79

page 222
1. a) hundredths
b) tenths
c) ones
d) tenths
e) hundredths
f) ones
g) ones
h) hundredths
i) tenths
2. a) tenths
b) hundredths
c) tenths
d) hundredths
e) hundredths
f) tenths
g) tenths
h) ones
i) hundredths

AP Book NS5-80

page 223
1. a) $\frac{64}{100} \div 0.64$
b) $\frac{23}{100} \div 0.23$
c) $\frac{38}{100} \div 0.38$
d) $\frac{64}{100} \div 0.64$
e) $\frac{39}{100} \div 0.39$
f) $\frac{76}{100} \div 0.76$
2. Teacher to check shading.
a) 0.38
b) 0.45
c) 0.05

AP Book NS5-81

page 224
1. a) $\frac{32}{100} \div 0.32$
b) $\frac{63}{100} \div 0.63$
c) $\frac{83}{100} \div 0.83$
d) $\frac{67}{100} \div 0.67$
2. a) $\frac{53}{100} \div 0.53$
b) $\frac{27}{100} \div 0.27$
c) $\frac{65}{100} \div 0.65$
d) $\frac{90}{100} \div 0.90$
e) $\frac{6}{100} \div 0.06$
f) $\frac{3}{100} \div 0.03$
3. a) $\frac{52}{100} \div 0.52$
b) $\frac{44}{100} \div 0.44$
c) $\frac{30}{100} \div 0.30$

Answer Key for AP Book 5.2
d) 2 ; 3 ;
   23 hundredths

e) 0 ; 5 ;
   5 hundredths

f) 0 ; 8 ;
   8 hundredths

AP Book 5.2

1. Teacher to check.

2. a) \( \frac{70}{100} = 0.7 \)
   b) \( \frac{40}{100} = 0.4 \)
   c) \( \frac{20}{100} = 0.2 \)
   d) \( \frac{90}{100} = 0.9 \)

3. a) \( \frac{80}{100} = 0.8 \)
    b) \( \frac{20}{100} = 0.2 \)
    c) \( \frac{60}{100} = 0.6 \)
    d) \( \frac{70}{100} = 0.7 \)
    e) \( \frac{30}{100} = 0.3 \)
    f) \( \frac{40}{100} = 0.4 \)
    g) \( \frac{90}{100} = 0.9 \)
    h) \( \frac{40}{100} = 0.4 \)

AP Book NS5-83

226

1. b) 6 dimes 2 pennies
  6 tenths
  2 hundredths
  62 pennies
  62 hundredths

c) 5 dimes 7 pennies
  5 tenths
  7 hundredths
  57 pennies
  57 hundredths

d) 0 dimes 5 pennies
  0 tenths
  5 hundredths
  5 pennies
  5 hundredths

AP Book NS5-84

227

1. a) 5
   b) 6
   c) 2
   d) 0

2. a) \( \frac{5}{10} \)
   b) \( \frac{3}{10} \)
   c) \( \frac{6}{10} \)
   d) \( \frac{2}{10} \)
   e) \( \frac{1}{10} \)
   f) \( \frac{34}{100} \)
   g) \( \frac{59}{100} \)
   h) \( \frac{77}{100} \)
   i) \( \frac{84}{100} \)
   j) \( \frac{31}{100} \)
   k) \( \frac{8}{100} \)
   l) \( \frac{3}{100} \)
   m) \( \frac{9}{100} \)
   n) \( \frac{5}{100} \)
   o) \( \frac{1}{100} \)
   p) \( \frac{7}{100} \)
   q) \( \frac{3}{100} \)
   r) \( \frac{6}{100} \)
   s) \( \frac{8}{100} \)
   t) \( \frac{6}{100} \)
   u) \( \frac{46}{100} \)
   v) \( \frac{5}{100} \)
   w) \( \frac{9}{100} \)
   x) \( \frac{6}{100} \)
   y) \( \frac{6}{100} \)

3. a) 0.50
   b) 0.40
   c) 0.60
   d) 0.90

AP Book NS5-85

228

1. a) \( \frac{21}{100} = 0.21 \)
   b) \( \frac{38}{100} = 0.38 \)
   c) \( \frac{59}{100} = 0.59 \)
   d) \( \frac{240}{100} = 2.40 \)
   e) \( \frac{564}{100} = 5.64 \)

2. Teacher to check.

3. a) \( 2.98 = 2 \frac{98}{100} \)
   b) \( 1.08 = 1 \frac{8}{100} \)

4. a) 2.57
   b) 3.17
   c) 5.30
   d) 1.03
   e) 2.07
   f) 19.90
   g) 35.01
   h) 87.06

5. Teacher to check pictures
   a) 3 tenths
   b) 0.7
   c) 1.80
Number Sense – AP Book 5.2 (continued)  

AP Book NS5-87  
page 230  
1. a) \(0.1, 0.2, 0.3, 0.4, 0.5, 0.6, \ldots\)  
   b) \(0.5\)  
   c) 0  
2. a) Zero  
   b) Half  
   c) One  
   d) Half  
   e) One  
   f) Zero  
3. a) <  
   b) >  
   c) >  
   d) >  
   e) <  
   f) <  
4. a) 1  
   b) 2  
   c) 2  

AP Book NS5-88  
page 231  
1. a) \(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\)  
   b) \(\frac{1}{10}, \frac{3}{10}, \frac{9}{10}\)  
   c) \(\frac{2}{10}, \frac{3}{10}, \frac{6}{10}\)  
   d) \(\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\)  
   e) \(\frac{1}{10}, \frac{2}{10}, \frac{3}{10}\)  
   f) \(\frac{1}{10}, \frac{1}{10}, \frac{3}{10}\)  
   g) \(\frac{1}{10}, \frac{2}{10}, \frac{3}{10}\)  
   h) \(\frac{1}{10}, \frac{2}{10}, \frac{4}{10}\)  

2. a) \(\frac{3}{10}, \frac{5}{10}, \frac{9}{10}\)  
   b) \(\frac{2}{10}, \frac{3}{10}, \frac{4}{10}\)  

AP Book NS5-89  
page 233  
1. a) 47  
2. a) 47  

AP Book NS5-90  
page 234  
1. Teacher to check shading.  
2. Teacher to check.  
3. Teacher to check shading.  
4. Teacher to check arrows for next two questions:  
5. Teacher to check fraction conversions.  
6. Teacher to check fraction conversions.  

AP Book NS5-91  
page 235  
1. Teacher to check crossing out.  
2. Teacher to check shading.  
3. Teacher to check shading.  
4. Teacher to check explanations.
AP Book NS5-92  
**page 236**

1. Teacher to check base ten models.
   a) 2.47
   b) 2.79
2. Teacher to check base ten model.
   b) 1.21
3. a) 7.69
   b) 7.23
   c) 1.71
   d) 2.04
   e) 5.32
   f) 11.90
   g) 16.09
   h) 2.47
   i) 8.57
   j) 13.61
4. 0.9 m
5. 16.78 m
6. a) .2, .4, .6, .8, 1.0, 1.2
   b) .3, .6, .9, 1.2, 1.5, 1.8

AP Book NS5-93  
**page 237**

1. a) 2
   b) 3
   c) 6
2. a) 5
   b) 7
   c) 14
   d) 9
   e) 17
   f) 16
   g) 182
   h) 173
   i) 235
   j) 17.2
   k) 426
   l) 53.6
3. a) .6 dm = 6 cm
   b) .8 dm = 8 cm
   c) 1.6 dm = 16 cm
4. 3
5. Teacher to check.

AP Book NS5-94  
**page 238**

1. a) 100 × 0.02 = 2
   b) 100 × 0.03 = 3
2. a) 70
   b) 180
   c) 460
   d) 3
   e) 625
   f) 307
   g) 7
   h) 6
   i) 6.7
   j) 95
   k) 182
   l) 407
   n) 50
   n) 70
   o) 18
   p) 190
   q) 60
   r) 170
3. a) 15.2 cm
   b) 375 mm
   c) 5 mm
   d) 0.8 cm
   e) 60 mm
   f) 12.3 cm
4. a) When multiplying a number by 100, the decimal shifts two places to the right (as shown in question 2).
   b) See a)
5. The decimal moves two places to the right because, one decimal shift represents x 10 thus two decimal shifts represents x 100.

AP Book NS5-95  
**page 239**

1. a) 2.86
   b) 3.6
   c) 5.05
   d) 8.4
   e) 10.68
f) 8.4
   g) 9.36
   h) 12.96

AP Book NS5-96  
**page 240**

1. Teacher to check pictures.
   b) 3.0 + 10 = .3
   c) .3 + 10 = .03

AP Book NS5-97  
**page 241**

2. a) 1.44
   b) 1.56 R 0.01
   c) 1.24 R 0.03
   d) 1.66
3. a) 0.18
   b) 1.34 R 0.01
   c) 0.34
   d) 0.68 R 0.01
   e) 4.1
4. $0.55
5. 15.6 km
6. $7.29
7. Since $4.98 + 6 = $0.83 and $6.96 + 8 = $0.87, 6 is a better deal.

AP Book NS5-98  
**page 243**

1. a) tenths
   b) thousandths
c) tenths  
d) thousandths  
e) ones  
f) hundredths

2.  
a) | o | t | h | th |
---|---|---|---|---|
b) | 6 | 5 | 1 | 2 |
c) | 4 | 0 | 8 | 1 |
d) | 2 | 8 | 3 | 0 |
e) | 1 | 3 | 0 | 6 |
f) | 0 | 5 | 3 | 0 |
g) | 9 | 0 | 2 | 0 |
h) | 8 | 0 | 0 | 0 |
i) | 4 | 0 | 8 | 1 |
j) | 2 | 0 | 1 | 1 |

3.  
a) 0.74  
b) 0.45  
c) 0.16  
d) 0.99  
e) 0.74  
f) 0.41  
g) 4.24  
h) 2.97  
i) 11.96

2.  
a) 0.8  
b) 2.7  
c) 1.42  
d) 0.73  
e) 0.36  
f) 0.21  
g) 0.01  
h) 0.1  
i) 1.0  
j) 0.1

3.  
Teacher to check.

5.  
a) .5, .6, .7  
b) 6.9, 7.0, 7.1  
c) 3.8, 3.9, 4.0  
d) 9.9, 10.0, 10.1  
e) 4.74, 4.75, 4.76  
f) 5.99, 6.00, 6.01

6.  
a) 4.0  
b) 5.0  
c) 9.03  
d) 3.80  
e) 6.10  
f) 8.00

AP Book NS5-100
page 245

1.  Teacher to check.

2.  
a) 1 cm = \frac{10}{100} \text{ dm} 
    = .1 \text{ dm}  
b) 100 cm = \frac{100}{10} \text{ dm} 
    = 10 \text{ dm}  
c) 1 \text{ mm} = \frac{10}{100} \text{ cm} 
    = .1 \text{ cm}  
d) 16 \text{ mm} = \frac{16}{100} \text{ cm} 
    = 1.6 \text{ cm}  
e) 77 \text{ mm} = \frac{77}{100} \text{ dm} 
    = .77 \text{ dm}  
f) 83 \text{ cm} = \frac{83}{100} \text{ m} 
    = .83 \text{ m}

3.  
a) 9.6 \text{ dm}  
b) 3.5 \text{ dm}  
c) 9.3 \text{ cm}  
d) 1.97 \text{ m}

4.  
a) 5 \text{ dm} = 50 \text{ cm} = 500 \text{ mm}  
b) 20\text{¢}  
c) 30 \text{ cm} = 3 \text{ dm}  
d) 700,000  
e) 800  
f) 30 \text{ cm} = 3 \text{ dm}

5.  
a) .30  
b) .3  
c) .70  
d) 1.40  
e) 3.0  
f) 4

AP Book NS5-101
page 247

1.  7 × 0.67 = 4.69 \text{ m}

2.  Yes (1.02 \text{ cm})

3.  $62.93

4.  4 pens for $2.96

5.  34.5 \text{ km}

6.  Tim must have calculated 6.42 + 71.9.

7.  Decimal notation is used for money because not every monetary amount is a whole number.

Dollar

8.  a) 26.1 \text{ m}  
b) About 4 times longer  
c) About 9 \text{ m}

AP Book NS5-102
page 248

1.  a) $5, $10, $15, $20  
b) 6\text{¢}, 12\text{¢}, 18\text{¢}, 24\text{¢}  
c) 20\text{¢}, 40\text{¢}, 60\text{¢}, 80\text{¢}  
d) 90

e) $60  
f) 72 \text{ students}  
g) 60 \text{ cups of water}

2.  a) 1 \text{ cm}; 60 \text{ m}  
b) About 5.5 \text{ cm}; 330 \text{ m}  
c) 8.8 \text{ cm}; 528 \text{ m}

3.  $44

4.  $120

5.  a) $5  
b) $3  
c) $2

AP Book NS5-103
page 249

1.  a) 9 \text{ cm} : 18 \text{ km}  
b) 8.4 \text{ cm} : 16.8 \text{ km}  
c) 2.5 \text{ cm} : 5 \text{ km}  
d) 8 \text{ cm} + 0.6 \text{ cm} + 3.4 \text{ cm} 
    = 12 \text{ cm} : 24 \text{ km}

2.  a) 200 \text{ km}  
b) 250 \text{ km}  
c) 25 \text{ km}
d) 125 km  
e) 225 km

**AP Book NS5-104**  
*page 250*

1.  

<table>
<thead>
<tr>
<th># of pies</th>
<th>Cups of Flour</th>
<th>Cups of Cherries</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

b) Multiply # of pies by 3  
c) Multiply # of pies by 2  
d) 15  
e) No. Andy needs 10 cups of flour to make pies with 15 cups of cherries.  
f) He can make 4 pies and will have 1 cup of flour left over.

2.  

b) 6 to 4;  
  \( \frac{6}{4} \) to 1 or \( \frac{3}{2} \) to 1;  
  \( \frac{1}{2} \) to 1;  
  1.5 to 1;  
  1.5  
c) 13 to 4;  
  \( \frac{13}{4} \) to 1;  
  \( \frac{3}{2} \) to 1;  
  3.25 to 1;  
  3.25  
d) 20 to 8;  
  \( \frac{20}{8} \) to 1;  
  \( \frac{5}{2} \) to 1;  
  \( \frac{12}{2} \) to 1;  
  2.5 to 1;  
  2.5

**AP Book NS5-105**  
*page 251*

1. a) i) 2.1 kg per day;  
  \[ 2.07 \text{ kg per day} \]  
  ii) 2.07 kg  
  iii) 2.1 kg  
  iv) 2.1 – 2.07 = 0.03 kg  
  b) 90 kg / 2. kg  
  = 45 times more kilograms  
  c) 30 x 2.07 kg + 3 kg  
  = 65.1 kg  
  2. a) 25  
  b) 2000, 2001, 2002  
  c) 1998 – 2004

**AP Book NS5-106**  
*page 252*

1. a) 75.35 km  
  b) 25.12 km  
  c) 25.12 / 6 = 4.19 km  
  = 4 km  
  d) 30 kg  
  2. 9 beads  
  3. $2.99  
  4. Since there are 144 students (\( 6 \times 24 = 144 \)), and there is room for 120 students on four buses (\( 30 \times 4 = 120 \)), there is not enough room for all six classes to ride on four buses.  
  5. $21.40  
  6. $8.90  
  7. 1.25 m  
  8. 9.1 kg  
  9. 33.6 km  
  10. 1799

**AP Book NS5-107**  
*page 253*

3. a)  

<table>
<thead>
<tr>
<th>nickels</th>
<th>pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>22</td>
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<tr>
<td>2</td>
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<td>3</td>
<td>12</td>
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<td>7</td>
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<tr>
<td>5</td>
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</tr>
</tbody>
</table>

**AP Book NS5-105**  
*page 251*

1. a)  

<table>
<thead>
<tr>
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<tbody>
<tr>
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<td>3</td>
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</table>

2.  

<table>
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3. a)  

<table>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>9</td>
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<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

**AP Book NS5-105**  
*page 251*

1. a)  

<table>
<thead>
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<td>1</td>
<td>22</td>
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<td>12</td>
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<tr>
<td>3</td>
<td>2</td>
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</table>

2.  

<table>
<thead>
<tr>
<th>dimes</th>
<th>pennies</th>
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<tbody>
<tr>
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<td>1</td>
<td>8</td>
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<tr>
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<td>3</td>
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</table>

3. a)  

<table>
<thead>
<tr>
<th>quarters</th>
<th>nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**AP Book NS5-105**  
*page 251*

1. a)  

<table>
<thead>
<tr>
<th>quarters</th>
<th>nickels</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
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</table>

2.  

<table>
<thead>
<tr>
<th>quarters</th>
<th>nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
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<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

He stops at 2 quarters because 3 quarters is 75c (larger than 60c).

4. a)  

<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

**AP Book NS5-105**  
*page 251*

1. a)  

<table>
<thead>
<tr>
<th>quarters</th>
<th>nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

2.  

<table>
<thead>
<tr>
<th>dimes</th>
<th>pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
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</table>

3. a)  

<table>
<thead>
<tr>
<th>quarters</th>
<th>dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
</tr>
</tbody>
</table>

**AP Book NS5-105**  
*page 251*

1. a)  

<table>
<thead>
<tr>
<th>quarters</th>
<th>dimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>
b) quarters dimes
   0 -
   1 8
   2 -
   3 3

7. W L
   1 16
   2 8
   3 -
   4 4

8. a) First Second
    1 12
    2 6
    3 4
    4 3
    6 2
    12 1

b) First Second
   1 14
   2 7
   7 2
   14 1

c) First Second
   1 20
   2 10
   4 5
   5 4
   10 2
   20 1

d) First Second
   1 24
   2 12
   3 8
   4 6
   6 4
   8 3
   12 2
   24 1

9. 6 (6x6x1x1)
   5 (5x5x2x2)
   4 (4x4x3x3)
10. 5 (5x2) 10 (10x1)
Answer Key for AP Book 5.2

AP Book ME5-8
page 256
1. a) 9 cm
    b) 10 cm
c) 13 cm
2. a) 5 cm
    b) 3 cm
3. a) Top and bottom: 4 cm
     Sides: 2 cm
    b) Base: 3 cm
     Height: 4 cm
     Hypotenuse: 5 cm
11. Teacher to check.

AP Book ME5-9
page 257
1. a) About 4 index fingers
    = About 40 mm
    b) About 6 index fingers
    = About 60 mm
2. a) 38 mm
    b) 47 mm
3. Top and bottom: 5 cm
     Sides: 1.5 cm
     Diagonal: 5.2 cm = 52 mm
4. Answers will vary.
5. Teacher to check.
6. Answers will vary.
7. Answers will vary.

AP Book ME5-10
page 260
1. My arm – More
   A paperclip – Less
   My pencil – Answers will vary
   Height of door - More
2. 10
3. 10
4. a)

<table>
<thead>
<tr>
<th>cm</th>
<th>dm</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>15</td>
</tr>
<tr>
<td>230</td>
<td>23</td>
</tr>
<tr>
<td>320</td>
<td>32</td>
</tr>
</tbody>
</table>

b)

<table>
<thead>
<tr>
<th>cm</th>
<th>dm</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>9</td>
</tr>
<tr>
<td>5100</td>
<td>510</td>
</tr>
<tr>
<td>400</td>
<td>40</td>
</tr>
</tbody>
</table>

c)

<table>
<thead>
<tr>
<th>cm</th>
<th>dm</th>
</tr>
</thead>
<tbody>
<tr>
<td>610</td>
<td>61</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>780</td>
<td>78</td>
</tr>
</tbody>
</table>

5. Teacher to check.
6. Answers will vary – teacher to check.
7. Answers will vary – teacher to check.
8. 1 dm = 100 mm

AP Book ME5-11
page 261
1. Answers will vary.
2. Teacher to check.
3. Answers will vary.
4. a) 3000 m
   b) 6000 m
   c) 7000 m
d) 12000 m
5. Teacher to check.
6. Answers will vary.
7. 2

AP Book ME5-12
page 262
1. Streetcar
   4
   5
   6
   7:45  8:00
   7:50  8:05
   7:55  8:10
   8:00  8:15
   a) Dunn – 5 min
      Dufferin – 10 min
      Strachan – 15 min
   b) 7:25
      8:00
      10:55
2. a) 6 km
    b) 9 km
    c) 10 km
    d) 8 km
3. a) 24 km
    b) 84 km
    c) 30 km
    d) 3 km
4. No

AP Book ME5-13
page 263
1. 3 4 5 6
   30 40 50 60
   300 400 500 600
   3000 4000 5000 6000
2. a) 100
    b) 1000
    c) 10
3. m mm
   8  800
   70  7000
   5  5000
   17 17 000
   cm mm
   4  40
   121 1210
4. Since 2 m with 25 cm is equal to 225 cm there is no difference in measurements.
5. a) 4 m 23 cm
   b) 5 m 14 cm
   c) 6 m 27 cm
   d) 6 m 73 cm
   e) 3 m 81 cm
   f) 2 m 3 cm
6. a) 283 cm
   b) 365 cm
   c) 485 cm
   d) 947 cm
   e) 704 cm
   f) 640 cm
7. a) 5 m 46 cm = 5.46 m
   b) = 2 m 17 cm = 2.17 m
   c) = 7 m 83 cm = 7.83 m
   d) = 6 m 48 cm = 6.48 m
8. A dollar is 100 cents and a metre is 100 cm.
b) 15 000 m
2. 12 900 m
3. 1 290 buses
4. 11 000 m
5. 115 m
6. 17

AP Book ME5-19
page 271
1. a) \(2 + 2 + 5 + 3 + 1 = 14\) cm
b) 5 + 2 + 1 + 1 + 3 + 1 + 1 + 2 = 16 cm
c) 3 + 1 + 1 + 2 + 4 + 3 = 14 cm
d) 6 + 3 + 2 + 1 + 2 + 1 + 2 + 3 = 20 cm
2. A: 16 units
   B: 22 units
   C: 22 units
3. Answers will vary.
   Teacher to check.
4. Teacher to check.

AP Book ME5-20
page 272
1. 10 cm; 8 cm; 14 cm
2. a) 24 m
    b) 28 cm
    c) 6 km
    d) 30 cm
    e) C – 6 km
       A – 24 m
       D – 30 cm
       B – 28 cm
3. Estimations will vary.
   a) P = 18 cm
   b) P = 16 cm
4. a) 10 \times 1; 2 \times 5
    b) 12 \times 1; 2 \times 6; 3 \times 4
    c) 1 \times 7
    d) 12 \times 1; P = 26
5. a) $4.20
b) Sally could make two different rectangles: 2 \times 4 and 1 \times 8. The border would be least expensive for the 2 \times 4 one since its perimeter is less (12 m < 18 m).

AP Book ME5-21
page 273
1. a) 6 m
    b) 6 m
    c) 3 m
2. a) Length = 4 cm
    b) Width = 4 cm
    c) Length = 3 cm
    d) Width = 1 m
3. a) W = 1, L = 2
    b) W = 1, L = 5;
       W = 2, L = 4;
       W = 3, L = 3.
    c) W = 1, L = 7;
       W = 2, L = 6;
       W = 3, L = 5;
       W = 4, L = 4.
    d) W = 1, L = 8;
       W = 2, L = 7;
       W = 3, L = 6;
       W = 4, L = 5.
4. To find the perimeter of a rectangle, add its length and width, and then multiply by 2.
5. a) Input | Output
    1     | 6
    2     | 8
    3     | 10
    4     | 12
    5     | 14
b) Multiply the INPUT by 2 and add 4.
c) P = 24
6. a) [Diagram]
    Original Perimeter = 8 units
b) [Diagram]
    Original Perimeter = 10 units
    c) [Diagram]
    Original Perimeter = 12 units
7. a) [Diagram]
    OR [Diagram]
    Original Perimeter = 5 units
b) [Diagram]
    Original Perimeter = 7 units
8. a) Input | Output
    1     | 8
    2     | 10
    3     | 12
   ii) Multiply the Input by 2 and add 6
   iii) 10\textsuperscript{th} figure = 26
b) Input | Output
    1     | 8
    2     | 10
    3     | 12
   ii) Multiply the Input by 2 and add 6
   iii) 10 figure = 26
9. Yes.

AP Book ME5-22
page 275
1. a) 4 \times 3 = 12
    b) 2 \times 3 = 6
    c) 3 \times 2 = 6
    d) 5 \times 3 = 15
2. a) 3 \times 7 = 21
    b) 3 \times 4 = 12
    c) 3 \times 2 = 6
    d) 4 \times 6 = 24
3. a) Length = 6 units
    Width = 2 units
    Area = 12 sq units
   b) Length = 3 units
    Width = 2 units
    Area = 8 sq units
   c) Length = 4 units
    Width = 3 units
    Area = 12 sq units
4. a) 2 cm × 3 cm
   = 6 cm²
b) 1 cm × 3 cm
   = 3 cm²
c) 2 cm × 4 cm
   = 8 cm²
d) 4 cm × 3 cm
   = 12 cm²
e) 2 cm × 5 cm
   = 10 cm²

5. Area = Length × Width

AP Book ME5-25
page 278

1. a) 2 cm × 5 cm
   = 10 cm²
b) 1 cm × 3 cm
   = 3 cm²
c) 3 cm × 5 cm
   = 15 cm²
d) 4 cm × 8 cm
   = 32 cm²

2. a) 6 m × 7 m
   = 42 m²
b) 3 m × 7 m
   = 21 m²
c) 4 cm × 8 cm
   = 32 cm²

3. a) A: 3 m × 7 m
     = 21 m²
     B: 4 cm × 5 cm
     = 20 cm²
     C: 11 m × 6 m
     = 66 m²
     D: 3 km × 2 km
     = 6 km²
b) D, C, A, B

4. Width = 3 cm
   (since you can find the width by dividing the area by the length, and 18 cm² / 6 cm = 3 cm)

5. 3 cm
6. 5 cm

7. Answers may vary.

Box 1: 2 × 2 = 4
Box 2: 3 × 6 = 18
Total Area: 4 + 18 = 22

OR
Box 1: 2 × 5 = 10
Box 2: 3 × 4 = 12
Total Area: 10 + 12 = 22

8. 3 possible rectangles:
   1 × 20, 2 × 10, 4 × 5

AP Book ME5-26
page 279

NOTE:
For estimation questions, consider the diagonal of the square unit equals 1.5 cm.

1. a) 3 whole squares
   b) 2 whole squares
   c) 3 whole squares
   d) 3 whole squares
   e) 8 whole squares
   f) 8 whole squares
   g) 4 whole squares
   h) 5 whole squares
   i) 11 whole squares
   j) 13 whole squares
   k) 10 whole squares

2. a) 8 square units
   b) 6 square units
   c) 7.5 square units

3. a) More:
     shaded = 7
     unshaded = 5
     and 7 > 5
   b) Equal:
     shaded = 4
     unshaded = 4
   c) Less:
     shaded = 3
     unshaded = 4
     and 3 < 4

4. a) \( \frac{1}{2} \)
   b) 4 square units
   c) 2 square units

5. a) 1 square units
   b) 3 square units
   c) 3 square units
d) 5 square units

6. Teacher to check lines.

   a) Triangle: 1 square unit
   Rectangle: 2 square units

7. a) Rectangle and Triangle - 5 square units
   b) Triangle and Triangle - 2 square units
   c) Triangle and Square - 8 square units
d) Triangle and Rectangle - 9 square units

8. a) 6 square units
   b) 8.5 square units
c) 8.5 square units

9. a) Area:
     5 square units
     Fraction: \( \frac{5}{9} \)
   b) Area:
     1 square units
     Fraction: \( \frac{1}{4} \)
c) Area:
     2 square units
     Fraction: \( \frac{2}{8} \)

AP Book ME5-27
page 281

1. a) \( 7 \) half squares = 3.5 total squares
   b) \( 9 \) half squares = 4.5 total squares
c) \( 14 \) half squares = 7 total squares

2. a) Area = 3 + 3 = 6
   b) Area = 3 + 4 = 7
c) Area = 9 + 3 = 12
d) Area = 5 + 3 = 8

3. NOTE:
   Answers may vary since they are estimates.
   a) 10 total squares
   b) 7 total squares

AP Book ME5-28
page 282

NOTE:
Answers may vary since they are estimates.

1. a) Area:
     5 square units
     Perimeter: 14 units
   b) Area:
     13 square units
     Perimeter: 15 units
c) Area:
     6 square units
     Perimeter: 11 units

2. a) Teacher to check shapes drawn by students.
   b) Shape A
     Old Area: 1 units²
     Old Per: 4 units
     New Area: 4 units²
     New Per: 8 units
   Shape B
     Old Area: 2 units²
     Old Per: 6 units
     New Area: 8 units²
     New Per: 12 units
   Shape C
     Old Area: .5 units²
     Old Per: 3.4 units
     New Area: 2 units²
     New Per: 6.8 units
   Shape D
     Old Area: 1 units²
     Old Per: 4.8 units
     New Area: 4 units²
     New Per: 9.6 units
   c) The area of the shape quadruples (is multiplied by 4).
d) The perimeter doubles.
Measurement – AP Book 5.2

Answer Key for AP Book 5.2

AP Book ME5-29
page 283
1. **Shape** | **P** | **A**
--- | --- | ---
A | 12 cm² | 8 cm²
B | 22 cm | 30 cm²
C | 22 cm | 18 cm²
D | 20 cm | 21 cm²
E | 26 cm | 30 cm²
F | 14 cm | 10 cm²
G | 22 cm | 10 cm²

2. No
3. D & G
4. E, B, C & G; D; F; A
5. E & B; D; C; F & G; A
6. No
7. Perimeter is the length along the outside edge of a shape. Area is the measure of the space contained within the edges of a shape.

AP Book ME5-30
page 284
1. This table gives actual measurements only:

| R | P | A |
--- | --- | ---
A | 14 cm | 10 cm²
B | 16 cm | 12 cm²
C | 16 cm | 15 cm²
D | 10 cm | 6 cm²
E | 18 cm | 14 cm²
F | 12 cm | 8 cm²
G | 14 cm | 12 cm²

2. a) 6 cm²
   b) 20 cm²
3. Teacher to check.
4. a) 5 cm × 2 cm
   b) 4 cm × 2 cm
5. The perimeter is unchanged.

AP Book ME5-31
page 285
1. a) 4 m
   b) Teacher to check.
   c) $168
   d) $9.60

2. Length | Width
--- | ---
8 cm² | 1 | 8
   | 2 | 4
   | 4 | 2
   | 8 | 1
14 cm² | 1 | 14
   | 2 | 7
   | 7 | 2
   | 14 | 1
18 cm² | 1 | 18
   | 2 | 9
   | 6 | 3
   | 9 | 2
   | 18 | 1
3. Answers will vary.
4. 3 × 3
5. 5 × 4
6. Since length = 2 × width, then P = 3 widths × 2 = 6 width

AP Book ME5-32
page 286
1. a) 280 cm
   b) 175 cm
   c) 160 cm
   d) 120 cm
2. a) 15 cm
   b) 24 cm
   c) 64 cm
3. a) 0.5 m
   b) 2 m
   c) 6 m
   d) Answers will vary.
4. a) Perimeter = 22 m
   Area = 30 m²
   b) 12 m²
   c) Teacher to check.
5. 12
6. Teacher to check.
7. Teacher to check.

AP Book ME5-34
page 288
1. a) \(3 + 3 + 3 + 3 = 12\)
   \(3 \times 4 = 12\)
   b) \(2 + 2 + 2 + 2 + 2 = 10\)
   \(2 \times 5 = 10\)
   c) \(3 + 3 + 3 + 3 + 3 = 21\)
   \(3 \times 7 = 21\)
2. 3 in each
3. a) \(3 + 3 + 3 + 3 = 12\)
   b) \(3 \times 4 = 12\)
4. a) 6
   b) \(6 + 6 + 6 + 6 = 24\)
   c) \(6 \times 4 = 24\)
5. a) \(2 + 2 + 2 = 6\)
   \(2 \times 3 = 6\)
   b) \(10 + 10 + 10 + 10 = 40\)
   \(10 \times 4 = 40\)
   c) \(9 + 9 + 9 + 9 + 9\)
   \(9 \times 5 = 45\)
6. a) 3; 6
   b) 2; 12
   c) 3; 18
   d) 4; 24
7. a) \(3 \times 4 \times 2 = 24\)
   b) \(5 \times 4 \times 3 = 60\)
   c) \(4 \times 4 \times 3 = 48\)
   d) \(4 \times 5 \times 2 = 40\)
8. Yes
9. a) A: 12 cm²
   B: 12 cm²
   C: 18 cm²
   b) A: 12 cm³
   B: 12 cm³
   C: 18 cm³
   c) A: 24 cm³
   B: 60 cm³
   C: 36 cm³
10. a) \(3 \times 2 = 6\)
   b) \(4 \times 2 = 8\)
   c) \(4 \times 3 = 12\)
11. a) \(3 \times 2 \times 4 = 24\)
   b) \(4 \times 2 \times 3 = 24\)
   c) \(4 \times 3 \times 2 = 24\)
12. a) Width: 2
   Length: 2
   Height: 2
   Volume = 8
   b) Width: 2
   Length: 3
   Height: 2
   Volume = 12
   c) Width: 2
   Length: 4
   Height: 5
   Volume = 30
13. a) Width: 2
   Length: 3
   Height: 5
   Volume = 30
b) Width: 2
   Length: 2
   Height: 3
   Volume = 12

c) Width: 2
   Length: 5
   Height: 10
   Volume = 100

AP Book ME5-35
page 291
1. Answers will vary.
2. Answers will vary.
3. a) 28 g
   b) 24 g
   c) 18 g
   d) 350 g
   e) About 11 quarters
   f) About 3 pennies
4. Answers will vary.
5. Balloon = grams
   Key = grams
   Truck = kilograms
6. Scissors = g
   Computer = kg
   Tape = g
7. C, A, B
8. a) kg
   b) kg
   c) g
   d) g
   e) g
   f) g
9. a) 4 kg
   b) 2 g, 2 g, 2 g or 1 g, 2 g, 3 g
10. Thousand
11. a) $42
    b) 300 g
    c) 520 kg
12. 1 000
13. Teacher to check.
14. 3.5 kg
15. 2 g

AP Book ME5-36
page 293
1. a) mL
   b) mL
   c) L
   d) mL or L
   e) L
   f) mL
2. a) L
   b) Answers may vary.
   c) mL
   d) Answers may vary.
3. a) 4
   b) 6
   c) 3 containers of C
4. a) 10
   b) 5
   c) 2
   d) 4
5. 4 000 mL

AP Book ME5-37
page 294
1. a) 1 mL
   b) 1 mL
   c) 1 mL
2. a) 12
   b) 36
   c) 36 cm³
   d) 36 mL
3. a) Water displaced: 300 mL
   Volume of toy: 300 cm³
   b) Water displaced: 100 mL
   Volume of toy: 100 cm³
   c) Water displaced: 600 mL
   Volume of toy: 600 cm³
   d) Water displaced: 700 mL
   Volume of toy: 700 cm³
4. Since a cylinder has a circular base, measuring its volume using centicubes would be difficult.
5. 300 cm³

AP Book ME5-38
page 295
1. a) 327 cents
   b) 816 cents
   c) 902 cents
   d) 307 cents
2. a) 501 cm
   b) 308 cm
   c) 714 cm
   d) 948 cm
   e) 1 610 cm
   f) 102 cm
3. a) 8 005 m
   b) 3 062 m
   c) 9 006 m
   d) 5 007 m
   e) 12 327 m
   f) 19 001 m
4. a) 122 min
   b) 65 min
   c) 130 min
   d) 195 min
   e) 228 min
   f) 265 min
BONUS:
5. a) $9.02
   b) $18.03
   c) $57.02
   d) 9.07 m
   e) 5.27 m
   f) 5.2 cm
   g) 6.1 dm
   h) 7.002 L
   i) 8.027 kg
6. 3–4 months
1. a) 3
   b) 3
   c) 5

2. Teacher to check shading.
   a) 3
   b) 5
   c) 4

3. a) \( \frac{20}{5} = 4 \)
   b) \( \frac{25}{5} = 5 \)
   c) \( \frac{30}{5} = 6 \)
   d) \( \frac{35}{5} = 7 \)
   e) When you add 1 to each data value, the mean increases by a value of 1.

1. a) 15
   b) 12
   c) 16

2. a) 4 spaces below mean; 4 blocks above mean
   b) 5 spaces below mean; 5 blocks above mean
   c) 4 spaces below mean; 4 blocks above mean

3. The number of blocks below and above the mean are equal in value.

4. a) i) too low
    ii) too high
    iii) too low
   b) i) 4; 4
      The mean is 3.
     ii) 2; 2
      The mean is 3.
     iii) 5; 5
      The mean is 4.

1. a) 4 = 3 + 1
   b) 2 + 1 + 0 = 2 + 1
   c) 4 + 1 = 2 + 3
   Teacher to check the blocks.
   d) Mean – 6
      3 = 1 + 2
   e) Mean – 6
      3 + 2 = 1 + 1 + 3
   f) Mean – 5
      2 = 1 + 1

2. Teacher to check the blocks.
   a) 3, 3, 4, 7, 8
   b) 5 spaces below mean; 5 blocks above mean
   c) 4 spaces below mean; 4 blocks above mean

3. The number of blocks below and above the mean are equal in value.

4. a) i) circle 6; median is 6
    ii) circle 3 & 3; median is 3
    iii) circle 13; median is 13
   b) circle 6 & 10; median is 8

5. a) circle 6; median is 6
   b) circle 3 & 3; median is 3
   c) circle 13; median is 13
   d) circle 6 & 10; median is 8

6. Since a leaf of a number is its right-most digit, and hence only 1-digit, and if two numbers have the same stem, then the total number of digits of each number must be the same. In other words, the statement that numbers with the same stem must have the same number of digits is true.

1. a) circle 6; median is 6
   b) circle 3 & 3; median is 3
   c) circle 13; median is 13
   d) circle 6 & 10; median is 8

2. a) Smallest: 82
   Largest: 104
   Range: 22
   b) Smallest: 5
   Largest: 23
   Range: 18
   c) Smallest: 95
   Largest: 122
   Range: 27

3. a) range below median:
   4 – 3 = 1; above median:
   11 – 4 = 7; above median:
   b) range below median:
   25 – 13 = 12; above median:
   30 – 25 = 5; below median:

4. a) Stem Leaf
   0 4 6 6 6 6
   1 1 2 4
   b) Mean. However answers may vary.
   c) Mean = 8
      i) more spread out below the mean
      ii) more spread out above the median

5. a) Stem Leaf
   0 7
   1 0 2 9
   b) Stem Leaf
   9 8 9 9
   10 1 2

6. Since a leaf of a number is its right-most digit, and hence only 1-digit, and if two numbers have the same stem, then the total number of digits of each number must be the same. In other words, the statement that numbers with the same stem must have the same number of digits is true.

1. a) Stem Leaf
   0 7
   1 0 2 9
   b) Stem Leaf
   9 8 9 9
   10 1 2

2. a) Smallest: 82
   Largest: 104
   Range: 22
   b) Smallest: 5
   Largest: 23
   Range: 18
   c) Smallest: 95
   Largest: 122
   Range: 27

3. a) range below median:
   4 – 3 = 1; range above median:
   11 – 4 = 7; above median:
   b) range below median:
   25 – 13 = 12; range above median:
   30 – 25 = 5; below median:

4. a) Stem Leaf
   0 4 6 6 6 6
   1 1 2 4
   b) Mean. However answers may vary.
   c) Mean = 8
      i) more spread out below the mean
      ii) more spread out above the median

5. a) Stem Leaf
   0 7
   1 0 2 9
   b) Stem Leaf
   9 8 9 9
   10 1 2

6. Since a leaf of a number is its right-most digit, and hence only 1-digit, and if two numbers have the same stem, then the total number of digits of each number must be the same. In other words, the statement that numbers with the same stem must have the same number of digits is true.
v) A: stem & leaf
B: broken line graph
C: stem & leaf

AP Book PDM5-19
page 302
1. a) You spin at a 1, 2, 3 or 4; 4 outcomes
b) You flip heads or tails; 2 outcomes
c) You win or lose; 2 outcomes
   NOTE: The final game of the Stanley Cup must have a winner, so a tie is not possible.
d) You roll a 1, 2, 3, 4, 5 or 6; 6 outcomes
e) You spin a 6 or 9; 2 outcomes
f) You spin a 6; 1 outcome
2. a) 3 outcomes
   b) 4 outcomes
3. a) 2, 4, 12
   b) 1, 3, 5, 7, 9
   c) 7, 9, 12

AP Book PDM5-20
page 303
1. a) less than half
   b) more than half
   c) more than half
   d) half
   e) less than half
   f) less than half
   g) less than half
   h) more than half
2. a) less than half
   b) more than half
   c) half
   d) less than half
3. a) unlikely

Answer Key for AP Book 5.2

1. a) You spin at a 1, 2, 3 or 4; 4 outcomes
b) You flip heads or tails; 2 outcomes
c) You win or lose; 2 outcomes
   NOTE: The final game of the Stanley Cup must have a winner, so a tie is not possible.
d) You roll a 1, 2, 3, 4, 5 or 6; 6 outcomes
e) You spin a 6 or 9; 2 outcomes
f) You spin a 6; 1 outcome
2. a) 3 outcomes
   b) 4 outcomes
3. a) 2, 4, 12
   b) 1, 3, 5, 7, 9
   c) 7, 9, 12

AP Book PDM5-21
page 306
1. a) 1; 3
   b) 0; 4
   c) 2; 4
   d) 3; 8
2. a) \( \frac{2}{5} \)
   b) \( \frac{1}{2} \)
   c) \( \frac{1}{4} \)
   d) \( \frac{1}{3} \)
3. a) \( \frac{2}{4} = \frac{1}{2} \)
   b) \( \frac{1}{3} \)
   c) \( \frac{2}{6} = \frac{1}{3} \)
4. a) \( \frac{2}{6} = \frac{1}{3} \)
   b) \( \frac{3}{4} \)
   c) \( \frac{1}{4} \)
   d) \( \frac{3}{8} \)
5. a) 1, 2, 3, 4, 5, 6
   b) 6
6. a) 2, 4, 6
   b) 3
   c) \( \frac{3}{6} = \frac{1}{2} \)
7. a) 5, 6
   b) 2
   c) \( \frac{2}{6} = \frac{1}{3} \)
8. a) i) 1, 2
   ii) \( \frac{2}{6} = \frac{1}{3} \)
   b) i) 1, 3, 5
   ii) \( \frac{3}{6} = \frac{1}{2} \)
   c) i) 3, 6
   ii) \( \frac{2}{6} = \frac{1}{3} \)
9. a) \( \frac{2}{8} = \frac{1}{4} \)
   b) \( \frac{1}{8} \)
   c) \( \frac{2}{8} = \frac{1}{4} \)
   d) \( \frac{6}{8} = \frac{3}{4} \)
   e) \( \frac{4}{8} = \frac{1}{2} \)
   f) \( \frac{3}{8} \)
10. a) \( \frac{2}{5} \)
    b) \( \frac{1}{5} \)
    c) \( \frac{1}{5} \)
    d) \( \frac{3}{5} \)
    e) \( \frac{2}{5} \)
    f) \( \frac{3}{5} \)
11. Teacher to check.

AP Book PDM5-22
page 308
1. a) No. Player 2 has a better chance

12. Teacher to check.

AP Book PDM5-23
page 309
1. a) 2; 4
   b) 3; 6
   c) 4; 8
2. # pieces # in half
   a) 4 2
   b) 8 4
   c) 12 6
3. Circle b) and d).
   a) 3 pieces shaded; 4 pieces
b) 3 pieces shaded; 6 pieces
c) 4 pieces shaded; 6 pieces
d) 4 pieces shaded; 8 pieces
e) 5 pieces shaded; 8 pieces
4. Circle figures 2, 3, 4, 5
Cross out figures 1 & 6
5. a) 5
   b) 12
   c) 24
   d) 26
6. a) \(\frac{1}{2}\) or half
   b) 10
7. Flipping a coin has 2 outcomes, heads or tails, so you would expect to flip heads half the time. Since the coin was flipped 40 times, you would expect to flip heads 20 \((40 \div 2)\) times.
8. a) 20
    b) 14
    c) 13
    d) 21
9. a) \(\frac{2}{3}\)
   b) I would expect \(\frac{1}{4}\) of the spins to be red.
10. a) 22
    b) 24
11. Teacher to check.
12. For the given spinner, you would expect to spin yellow 3 out of 4 times. So, if you spun the spinner 100 times, you would expect to spin yellow 75 times.

**Answer Key for AP Book 5.2**

1. a) 5 times
   b) Since the outcome for spinning red is equal to the outcome of spinning yellow, then the chance of spinning red and yellow are equally likely and the game is fair. Therefore, Daniel is not right.
2. a) 10 times
   b) Teacher to check.
3. Teacher to check spinners.
   NOTE: the order in which the numbers are placed on the spinner do not matter.
   a) 3 should be on one section of the spinner
   b) Even numbers should be on five sections of the spinner.
   c) A number that is a multiple of 3 should be on two sections of the spinner
   d) 2 should be on three sections of the spinner
AP Book G5-18  
page 312

1. a)  
b)  
c)  
d)  
e)  

4. a) Column 2, Row 2  
b) Column 3, Row 1  
c) Column 3, Row 2  
d) Column 2, Row 3  
e) Column 2, Row 1  

5. a)  
b)  
c)  
d)  
e)  

AP Book G5-19  
page 314

1. a)  
b)  
c)  

2. a)  
b)  
c)  
d)  
e)  

4. a) Column 2, Row 2  
b) Column 3, Row 1  
c) Column 3, Row 2  
d) Column 2, Row 3  
e) Column 2, Row 1  

5. a)  
b)  
c)  
d)  
e)  

6. A (2, 1)  
   B (7, 2)  
   C (9, 4)  
   D (3, 3)  
   E (0, 0)  
   F (1, 2)  
   G (0, 5)  
   H (5, 0)  

7. a) This polygon is a triangle.  
   b) This polygon is a square.  
   c) This polygon is a rectangle.
BONUS:
8. Answers will vary – teacher to check.

AP Book G5-20
page 317
1. a) 4 units right
   b) 3 units right
   c) 2 units right
2. a) 3 units left
   b) 5 units left
   c) 2 units left
3. a) 4 units right
   b) 3 units right
   c) 2 units right
4. a) 4 units right
   b) 2 units down
   c) 2 units right
   d) 4 units down
5. a) In this translation, Figure A moved 3 up and two left.
   b) 5 units left
   c) 2 units left
   d) 3 units left
   e) 5 units left
6. a) In this translation, Figure A moved 3 up and two left.
   b) 5 units left
   c) 2 units left
   d) 3 units left
   e) 5 units left

AP Book G5-22
page 319
1. NOTE:
   Corner selected for dot will vary but shapes and their positions will be as follows:
   a) 4 units right
   b) 3 units right
   c) 2 units right
   d) 4 units right
2. a) 4 units right
   b) 3 units right
   c) 2 units right
   d) 4 units right
   e) 2 squares up
   f) Mizar
3. a) Teacher to check.
   b) Red Rock and Tall Fir
   c) Treasure
   d) The Fang Cliff
4. a) Bear Cave
   b) The Fort
   c) Treasure
   d) The Fang Cliff
5. a) (7, 4)
   b) (6, 1)
   c) (5, 5)
6. a) Swamp
   b) Mouth Bay
   c) Ear Wood
   d) Swamp
6. a) Swamp
   b) Mouth Bay
   c) Ear Wood
7. a) 4.5 km west
   b) 2 km west
   c) 3 km south
   d) 3 km east and 1 km south
   e) 1.5 km west and 1 km north
   f) 2 km west and 3 km south
   g) 3 km west and 4 km north
8. Answers will vary – teacher to check.
b)  

c)  

d)  

5. a)  

b)  

c)  

d)  

6. a)  

b)  

7. a) Yes  
b) No: this is a slide  
c) No: the shapes are not congruent  

8. Answers will vary – teacher to check.

AP Book G5-25  
page 324  
1. a) $\frac{1}{4}$  
b) $\frac{1}{2}$  
c) $\frac{3}{4}$  
d) 1 (whole)  
e) $\frac{1}{2}$  
f) $\frac{1}{4}$  
g) $\frac{1}{4}$  
h) $\frac{3}{4}$

2. a) $\frac{1}{4}$ turn clockwise  
b) $\frac{1}{2}$ turn clockwise  
c) $\frac{3}{4}$ turn clockwise  
d) 1 turn clockwise  
e) $\frac{1}{2}$ turn clockwise  
f) $\frac{1}{4}$ turn clockwise  
g) 1 turn counter cw  
h) $\frac{1}{2}$ turn clockwise  
i) $\frac{1}{2}$ turn counter cw  
j) $\frac{1}{4}$ turn clockwise  
k) $\frac{1}{4}$ turn counter cw  
l) $\frac{1}{2}$ turn counter cw

3. a) $\frac{1}{4}$ turn clockwise  
b) $\frac{3}{4}$ turn clockwise  
c) $\frac{1}{2}$ turn clockwise

AP Book G5-26  
page 326  
1. a)  

b)  

c)  

d)  

e)  

f)  

g)  

h)  

BONUS  
i)  

j)  

k)  

l)  

M
2. Answers will vary –
   teacher to check.

BONUS:
3. Teacher to check.

AP Book G5-27

page 327
1. a)
   
   
   b)
   
   
   c)
   
   
   d)
   
   2. Answers will vary –
      teacher to check.

AP Book G5-28

page 328
1. a) Reflection in M
   b) \( \frac{1}{4} \) turn counter cw
      around P
   c) Slide 1 unit left
   d) Reflection in M
   e) Slide 1 unit right
   f) \( \frac{1}{4} \) turn counter cw
      around P

2. Answers will vary.

3. a) Reflection in Line 1
   b) \( \frac{1}{2} \) turn clockwise
      around P
   c) Slide 1 unit right
   d) \( \frac{1}{4} \) turn clockwise
      around P

4. Answers will vary –
   teacher to check.

AP Book G5-29

page 330
1. a) i) Slide
   ii) Reflection

3. Answers may vary –
   teacher to check.

   Possible answer:
The number of edges of a pyramid is a double the number of sides of its base. The number of vertices of a pyramid is one more than the number of sides of its base.

4. An octagonal pyramid would have 16 edges and 9 vertices.

AP Book G5-31

page 332
1. Teacher to check shape construction.

<table>
<thead>
<tr>
<th>Base</th>
<th>#S</th>
<th>#E</th>
<th>#V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.P</td>
<td>3</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>P.P</td>
<td>5</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>H.P</td>
<td>6</td>
<td>18</td>
<td>12</td>
</tr>
</tbody>
</table>

2. # sides increases by one each time
   # edges increases by three each time
   # vertices increases by two each time

   See Question 1 for answer, re: hexagonal prism.

3. Answers may vary.
   Teacher to check.

   Possible answer:
The number of edges of a pyramid is double the number of sides of its base. The number of vertices of a pyramid is one more than the number of sides of its base.

4. An octagonal pyramid

   has 24 edges and
   16 vertices.
Answer Key for AP Book 5.2

**AP Book G5-32**  
*page 333*

1. Teacher to check.
2. a) 12  
   b) 6  
   c) 8  
   d) 15  
   e) 10  
   f) 9  
   g) 12  
   h) 18
3. a) 8  
   b) 4  
   c) 5  
   d) 10
4. Teacher to check.
5. Teacher to check.

**BONUS:**
6. Teacher to check.

**AP Book G5-33**  
*page 335*

1. a) 6  
   b) 6  
   c) 6  
   d) 4  
   e) 5  
   f) 8  
   g) 5  
   h) 5
2. a)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td># F</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td># V</td>
<td>5</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td># E</td>
<td>8</td>
<td>6</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

b) Shapes C and D have the same number of faces, vertices and edges. Both shapes are quadrilateral prisms.

**AP Book G5-34**  
*page 336*

1. a)

2. a)

3. Circle: (a), (e) and (g)

**AP Book G5-35**  
*page 338*

1. From left to right:  
   XVXOXXOXXO
2. From left to right:  
   rectangular prism  
   square pyramid  
   cone  
   cylinder  
   triangular pyramid  
   triangular prism

<table>
<thead>
<tr>
<th></th>
<th>TP</th>
<th>SP</th>
<th>S?</th>
<th>D?</th>
</tr>
</thead>
<tbody>
<tr>
<td># faces</td>
<td>5</td>
<td>5</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>shape of base</td>
<td>△</td>
<td>□</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td># bases</td>
<td>2</td>
<td>1</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td># faces that are not bases</td>
<td>3</td>
<td>4</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>Shape of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># edges</td>
<td>9</td>
<td>8</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td># vertices</td>
<td>6</td>
<td>5</td>
<td>√</td>
<td></td>
</tr>
</tbody>
</table>

b) A triangular prism and a square pyramid are the same in that both shapes have 5 faces.  
A triangle prism and a square pyramid are different in their number of edges, vertices and bases.

4. a)

<table>
<thead>
<tr>
<th>Name</th>
<th>E</th>
<th>V</th>
<th>F</th>
<th>Faces</th>
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<tbody>
<tr>
<td>T.P</td>
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<td>6</td>
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<td>△△△△△</td>
</tr>
<tr>
<td>S.P</td>
<td>8</td>
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<tr>
<td>H.P</td>
<td>18</td>
<td>12</td>
<td>8</td>
<td>□□□□□□□</td>
</tr>
<tr>
<td>O.P</td>
<td>16</td>
<td>9</td>
<td>9</td>
<td>O O O O O</td>
</tr>
</tbody>
</table>

5. Hexagonal prism, Hexagonal pyramid
b) The number of vertices in each pyramid is one greater than its number of sides.

c) The number of vertices in each prism is twice the number of sides in its base.

5. Teacher to check.

BONUS:

6. Teacher to check.

AP Book G5-36

1. | Name               | Base | # F | # E |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular Pyramid</td>
<td>△</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Square Pyramid</td>
<td>□</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Pentagonal Pyramid</td>
<td>□</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Triangular Prism</td>
<td>△</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Cube</td>
<td>□</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Pentagonal Prism</td>
<td>□</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

2. Teacher to check.
   a) Triangle
   b) Since each has one base surrounded by triangles, we know that the nets are pyramids.

3. Teacher to check.
   a) Quadrilateral (square or rectangle)
   b) Since there are two congruent faces (bases) edged with rectangles attached, the nets are prisms.

4. | A | B | C |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

   The numbers in both rows increase by one

5. a) 4; 1
   b) 2; 4
   c) 6; 2

6. a) Rectangular prism
   b) Triangular prism
   c) Square pyramid

7. Teacher to check.

AP Book G5-37

1. a) A, C, D, E, F
   b) A, C, E, G

2. a) A, C, E
   b) B, F
   c) A, C, E, F, G

3. Answers will vary – teacher to check.

AP Book G5-38

1. a) B, F
   b) A, C, E, F, G

2. a) F
   b) B, F
   c) A, C, E, G

3. Answers will vary – teacher to check.

AP Book G5-39

1. a) Rectangular prism
   b) Triangular prism
   c) Square pyramid

2. a) The numbers in both rows increase by one

3. Teacher to check.

4. Teacher to check.

5. Teacher to check.

6. Answers will vary – teacher to check.

AP Book G5-40

1. a) Rectangular prism
   b) Triangular prism
   c) Square pyramid

2. Answers will vary – teacher to check.

3. a) B, F
   b) A, C, E, F, G

4. The numbers in both rows increase by one
4. a) 

b) 

c) 

AP Book G5-41
page 346
1. Teacher to check.
BONUS:
2. Teacher to check.

AP Book G5-42
page 347
1. Teacher to check.
2. a) 

b) 

c) 

AP Book G5-43
page 348
1. A to B: 50 km north
   B to C: 175 km west
   C to D: 50 km south
   D to E: 50 km west
   E to F: 200 km south
   F to G: 50 km east
2. Teacher to check.
3. Teacher to check.
4. A → B = Reflection
   A → C = Translation
   A → D = Translation
   A → E = Rotation CW
   A → F = Rotation CW

AP Book G5-44
page 349
1. a) C. a reflection
   b) A. a slide
   c) B. a rotation
2. a)–e) 

f) Teacher to check.
3. Teacher to check.
4. From left to right:
   X O - X O X -
5. E = cone
   B = triangular prism
   A = cube
   D = cylinder
   C = triangular pyramid
6. Answers may vary.
   a) Triangular prism
      2 bases (triangles)
      6 vertices
      9 edges
      5 faces
   b) Rectangular prism
      2 bases (rectangles)
      8 vertices
      12 edges
      6 faces
7. Answers will vary, teacher to check.
8. Answers may vary.
   Similarities:
   Triangular base
   Differences:
   # of edges, faces, vertices
Contents

Patterns & Algebra – Part 1
Answer Key for Patterns & Algebra – Part 1
Number Sense – Part 1
Answer Key for Number Sense – Part 1
Measurement – Part 1
Answer Key for Measurement – Part 1
Probability & Data Management – Part 1
Answer Key for Probability & Data Management – Part 1
Geometry – Part 1
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Number Sense – Part 2
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Probability & Data Management – Part 2
Answer Key for Probability & Data Management – Part 2
Geometry – Part 2
Answer Key for Geometry – Part 2
Patterns & Algebra

Unit Test

Name: _____________________________

Date: _________________

Section A

1. Find the amount by which the sequence increases or decreases. In the circle, write a number with a + sign if the sequence increases, and a – sign if it decreases.:

   a) 4 , 12 , 25 , 7 , 9
   b) 2 , 9 , 7 , 11 , 8

2. Continue the following sequences by adding the number given:

   a) (add 4) 22, 26, ____, ____, ____
   b) (add 9) 21, 30, ____, ____, ____
   c) (add 6) 52, 58, ____, ____, ____
   d) (add 7) 63, 70, ____, ____, ____

3. Continue the following sequences by subtracting the number given:

   a) (subtract 4) 72, 68, ___, ___, ___
   b) (subtract 7) 71, 64, ___, ___, ___
   c) (subtract 3) 88, 85, ___, ___, ___
   d) (subtract 11) 127, 116, ____, ____, ____

4. For the following pattern, use the first three numbers in the pattern to find the rule. Then continue the pattern by filling in the blanks:

   a) 62, 57, 52, _____, _____, _____
      The rule is: Start at____ and__________________
   b) 68, 75, 82, _____, _____, _____
      The rule is: ___________________________________
   c) 274, 286, 298, _____, _____, _____
      The rule is: ___________________________________
   d) 528, 519, 510, _____, _____, _____
      The rule is: ___________________________________

5. Match each sequence with the sentence that describes it. This sequence....

   a) A ... increases by 5 each time.
      B ... decreases by different amounts.
      C ... increases by different amounts.

   ___ 17 , 22 , 28 , 32 , 34
   ___ 17 , 14 , 10 , 9 , 6
   ___ 14 , 19 , 24 , 29 , 34

   b) A ... increases and decreases.
      B ... increases by the same amount.
      C ... decreases by different amounts.
      D ... decreases by the same amount.

   ___ 21 , 19 , 15 , 13 , 9
   ___ 10 , 13 , 9 , 7 , 5
   ___ 19 , 17 , 15 , 13 , 11
   ___ 9 , 12 , 15 , 18 , 21
Patterns & Algebra

Unit Test

Section A (continued)

6. Write a rule for each pattern:
   NOTE: One sequence doesn't have a rule – see if you can find it.
   a) 18 , 23 , 28 , 33
   ______________________________________________________
   b) 41 , 34 , 27 , 20
   ______________________________________________________
   c) 29 , 21 , 17 , 14 , 9
   ______________________________________________________
   d) 51 , 55 , 59 , 63
   ______________________________________________________

7. Extend the number pattern:

   a) b) c)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

8. Continue the pattern below, then complete the chart:

   Figure 1
   Figure 2
   Figure 3
   Figure 4
   Figure 5

<table>
<thead>
<tr>
<th>Number of Triangles</th>
<th>Number of Line Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a) How many line segments would Figure 6 have? _______________
   b) How many line segments would you need to make a figure with 15 triangles? __________
Patterns & Algebra

Unit Test

Section B

9. The snow is 17 cm deep at 4 pm. 4 cm of snow falls each hour. How deep is the snow at 9 pm?

10. Zoe saves $55 in August. She saves $8 each month after that. Adrian saves $66 in August. He saves $6 each month after that. Who has saved the most money by the end of November?

11. Explain how you could find the colour of the 43rd block in this pattern without using a hundreds chart:

12. What is the 24th coin in this pattern? Explain how you know:

### Table

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
</tbody>
</table>
Patterns & Algebra

Unit Test

Section B (continued)

13. Frances is on a bicycle tour 425 km from her home. She can cycle 75 km each day. If she starts riding towards home on Monday morning, how far from home will she be on Thursday evening?

Show your work:

14. Find the lowest common multiple of each pair of numbers:
   Hint: count up by the largest number until you find a number that both numbers divide into.
   
   a) 2 and 8
   LCM = ______

   b) 4 and 12
   LCM = ______

   c) 5 and 4
   LCM = ______

   d) 6 and 9
   LCM = ______

15. Jones has piano lessons at 4 o’clock every 4th day. Piotr has guitar lessons at 4 o’clock every 6th day. They both had a lesson on September 30th. When will they have their next lesson at the same time?

16. You can make a volcano with baking soda, food colouring and vinegar. For every 2 tablespoons of red food colouring, you need 4 tablespoons of baking soda and 5 tablespoons of vinegar.
   
   a) If you had 15 tablespoons of vinegar, how many tablespoons each of food colouring and baking soda would you need?
   
   b) If you had 16 tablespoons of baking soda, how many tablespoons each of food colouring and vinegar would you need?
Section B (continued)

17. a) Which row of the chart has a decreasing pattern (looking left to right)?

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>12</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

b) Which column has a repeating pattern?

c) Write pattern rules for the first and second columns:


d) Describe the relationship between the numbers in the third and fourth columns:

e) Describe one other pattern in the chart:

f) Name a row or column that does not appear to have any pattern:

18. Hannah makes a Christmas ornament using a trapezoid (the shaded shape) and 3 triangles. She has 5 trapezoids. How many triangles will she need if she plans to use all 5 trapezoids to make ornaments?
Section A

1. a) +8, +13, −18, +2
   b) +7, −2, +4, −3

2. a) 30, 34, 38
   b) 39, 48, 57
   c) 64, 70, 76
   d) 77, 84, 91

3. a) 64, 60, 56
   b) 57, 50, 43
   c) 82, 79, 76
   d) 105, 94, 83

4. a) 47, 42, 37; Start at 62 and subtract 5.
   b) 89, 96, 103; Start at 68 and add 7.
   c) 310, 322, 334; Start at 274 and add 12.
   d) 501, 492, 483; Start at 528 and subtract 9.

5. a) C
    b) A

6. a) Start at 18 and add 5.
   b) Start at 41 and subtract 7.
   c) No rule.
   d) Start at 51 and add 4.

7. | Gap | Fig. | # of Squs |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>39</td>
</tr>
<tr>
<td>b)</td>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>27</td>
</tr>
<tr>
<td>c)</td>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>34</td>
</tr>
</tbody>
</table>

8. | Fig. | Tri. | L.S. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>23</td>
</tr>
</tbody>
</table>

9. a) 27
   b) 31

Section B

9. The snow will be 37 cm deep at 9 p.m.

10. By the end of November, Adrian ($84) has saved more than Zoe ($79).

11. Sample Answer:
    The core is 5 blocks long. I could skip count by 5’s until I got closest to 43 without going over (40). The third block in the core (43 – 40) is yellow, so the 43rd block would be yellow.

12. The 24th coin would be a dime (using a similar process to #11 above).

13. She will be 125 km from home on Thursday evening.

14. a) LCM = 8
    b) LCM = 12
    c) LCM = 20
    d) LCM = 18

15. They will have lessons at the same time on the 12th of October.

16. a) F.C. – 6 tbsp
    b) F.C. – 8 tbsp
    c) B.S. – 12 tbsp
    d) V. – 20 tbsp

17. a) Row 3
    b) Column 5
    c) Column 1: Start at 0 and add 5.
       Column 2: Start at 4 and add 2.
    d) If you subtract 2 from each number in Column 3, it will give you the corresponding number in Column 4.
    e) Answers will vary.
    (e.g. Column 4: Start at 6 and subtract 1.)

18. Hannah will need 15 triangles.
### Section A

1. Write the following numbers into the place value chart:

<table>
<thead>
<tr>
<th></th>
<th>ten thousands</th>
<th>thousands</th>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 14 124</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 5 205</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 894</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write numerals for the following number words:
   a) twenty-three thousand, seven hundred, forty-one
   b) sixty thousand, eight hundred, six
   c) fifty-three thousand, fifty-three

3. Write number words for the following numerals:
   a) 8 320
   b) 2 246
   c) 45 700

4. Write the number for the given base ten blocks:

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Base Ten Blocks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Sketch a base ten model of the number given:

<table>
<thead>
<tr>
<th>Number</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 432</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section A (continued)

6. Write the number in expanded form (using numerals):
   a) 2 886 = _____________________________  
   b) 229 = _____________________________

7. Write 10, 100, 1000 or 10 000 in the box to make each equation true:
   a) 6 356 + _____ = 7 356  
   b) 6 686 - _____ = 6 586  
   c) 23 487 + _____ = 33 487  
   d) 8 570 - _____ = 8 560  
   e) 29 554 + _____ = 30 554  
   f) 19 789 - _____ = 18 789

8. Create the greatest possible number using these numbers. (Only use each number once!)
   a) 3, 2, 7, 5 _________  
   b) 0, 5, 3, 2 _________  
   c) 1, 9, 4, 8 _________

9. What is the greatest number less than 10 000 whose digits are all the same?

10. Use < or > to identify the greater number:
   a) 6 223  6 233  
   b) 47 748  37 197  
   c) 14 847  8 923  
   d) 73 325  73 324

11. Using the digits 4, 5, 6, 7 and 8, create an even number greater than 62 000 and less than 65 000:

12. Write the following numbers in order (from least to greatest):
   54 879  12 325  34 389  34 341  6 372

13. Exchange tens for hundreds, or ones for tens:
   a) 5 hundreds + 4 tens + 24 ones = _____________________________  
   b) 3 hundreds + 57 tens + 8 ones = _____________________________
Number Sense

Unit Test

Section B

14. Add, regrouping where necessary:

a) \[ 3 \ 3 \ 6 \ 5 \]
   \[ + 4 \ 3 \ 8 \ 4 \]
   \[ - \]

b) \[ 7 \ 3 \ 3 \ 8 \]
   \[ + 1 \ 5 \ 4 \ 4 \]
   \[ - \]

c) \[ 3 \ 6 \ 7 \ 2 \]
   \[ + 4 \ 2 \ 6 \ 6 \]
   \[ - \]

d) \[ 1 \ 7 \ 9 \ 1 \]
   \[ + 5 \ 2 \ 2 \ 4 \]
   \[ - \]

15. In the questions below you will have to regroup two or three times:

a) \[ 1 \ 0 \ 0 \ 0 \]
   \[ - 4 \ 8 \ 3 \]
   \[ - \]

b) \[ 1 \ 0 \ 0 \ 0 \]
   \[ - 3 \ 7 \]
   \[ - \]

c) \[ 1 \ 0 \ 0 \ 0 \]
   \[ - 5 \ 2 \ 1 \]
   \[ - \]

16. Here are some important dates in the history of science:
   - In 1543, Copernicus published a book claiming the sun is the center of our solar system.
   - In 1610, Galileo Galilei used his newly invented telescope to discover the moons of Jupiter.
   - In 1667, Issac Newton announced his law of gravity.

   a) How long ago did Copernicus publish his book? Show your work:

   b) How many years passed between each pair of dates given? Show your work:

17. At a summer camp, 324 children are enrolled in baseball. There are 128 more children enrolled in swimming than in baseball.

   a) How many children are enrolled in swimming?

   b) How many children are enrolled in lessons altogether?
18. Use counters or draw arrays (of dots or squares) to solve each question. Write a multiplication statement for each question:

a) In a garden, there are 6 rows of plants. There are 5 plants in each row. How many plants are there altogether?

b) Paul lines up 7 chairs in each row. There are 3 rows of chairs. How many chairs are there altogether?

c) Jenny planted 8 seeds in each row. There are 4 rows of seeds. How many seeds did Jenny plant?

19. Multiply:

a) \[
\begin{array}{c}
2 \\
8 \\
1 \\
\hline
5 \\
\end{array}
\]

b) \[
\begin{array}{c}
5 \\
5 \\
2 \\
\hline
3 \\
\end{array}
\]

c) \[
\begin{array}{c}
2 \\
4 \\
1 \\
\hline
6 \\
\end{array}
\]

d) \[
\begin{array}{c}
5 \\
2 \\
3 \\
\hline
7 \\
\end{array}
\]

20. Multiply:

a) \[
\begin{array}{c}
4 \\
3 \\
\hline
2 \\
3 \\
\end{array}
\]

b) \[
\begin{array}{c}
2 \\
7 \\
\hline
4 \\
1 \\
\end{array}
\]

c) \[
\begin{array}{c}
3 \\
1 \\
\hline
2 \\
4 \\
\end{array}
\]

d) \[
\begin{array}{c}
7 \\
2 \\
\hline
1 \\
2 \\
\end{array}
\]

21. A hamster can make an exercise wheel turn 45 times in a minute. At that rate, how many times will the wheel turn in an hour?
22. Find three different ways to share 19 cookies into equal groups so that one is left over.

23. Three siblings have more than 9 and less than 13 marbles. They share the marbles evenly. How many marbles do they have?

24. Hassim plays basketball every week for 136 minutes. He needs 1350 minutes to get a job at the summer camp. If he plays for 7 weeks, will he have enough hours?

25. Divide:
   a) \[ \frac{84}{5} \]
   b) \[ \frac{710}{8} \]
   c) \[ \frac{964}{8} \]
   d) \[ \frac{951}{9} \]

26. Guerdy packs 78 books into boxes of 5 and Tyree packs 83 books into boxes of 9. Who uses more boxes? Who has more books left over?
Number Sense

Unit Test

Section C

27. Round to the nearest thousands place:
   a) 4 225  
   b) 3 732  
   c) 9 849  

28. Round to the nearest ten thousands place:
   a) 81 284  
   b) 37 786  
   c) 11 899  

29. Round 37 184 to the nearest ... 

   _______   _______   _______   _______  
   tens     hundreds  thousands   ten thousands

30. A store has the following items for sale:
   A. Sofa - $397   B. Arm Chair - $163   C. Table - $124   D. Desk - $527   E. Lamp - $94

   a) What could you buy if you had $800 to spend? Estimate to find out. Then add the actual prices to check. Show your work:

   b) List a different set of items you could buy for the same amount of money:

31. Estimate the products by rounding to the leading digits:
   a) 37 \times 51 =  
   b) 454 \times 81 =  
   c) 388 \times 19 = 
Number Sense

Unit Test

Name: _____________________________

Date: _________________

Section D

32. Round the money amounts to the nearest dollar:
   a) $21.85   ___________  b) $3.07   ___________  c) $127.42   ___________

33. How much money would you have if you had the following coins? Write your answer in cent notation then in dollar notation:
   a) 35 pennies = _____ = ______
   b) 7 nickels = _____ = ______
   c) 8 dimes = _____ = ______
   d) 8 pennies = _____ = ______
   e) 6 toonies = _____ = ______
   f) 3 quarters = _____ = ______

34.

   $10.30  
   $39.95  
   $2.74   

   $38.50  
   $6.26   
   $25.64  
   $32.89

   a) If you bought a watch and a soccer ball, how much would you pay?

   b) Which costs more: a watch and a cap or a pair of pants and a soccer ball?

   c) Could you buy a soccer ball, a pair of tennis rackets and a pair of pants for $100?

   d) What would be the total cost of the three least expensive things shown in the pictures above?
Number Sense

Unit Test

Section D (continued)

35. Circle the greater amount of money in each pair:
   a) 183¢ or $1.86   b) $1.41 or 143¢   c) 7¢ or $0.70

36. Tanya’s weekly allowance is $4.50. Her mom gave her five coins. Which coins did she use?

37. Mera has $12.16 and Wendy has $13.47. How much more money does Wendy have than Mera?

38. Estimate the amount of money shown. Then tally the amount of each denomination and use the space provided to calculate the actual total:
   a) Estimated Total: __________

   
   
   
   
   
   
   
   
   
   
   
   
   Actual Total: __________

   b) Estimated Total: __________

   
   
   
   
   
   
   
   
   
   
   
   
   Actual Total: __________
Section A

1. | Ten Thousands | Thousands | Hundreds | Tens | Ones |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14 124</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5 205</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>894</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>92</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

10. a) <
   b) >
   c) >
   d) >

11. Answers may vary: 64 578 or 64 758

12. 6 372; 12 325; 34 341; 34 389; 54 879

13. a) 5 hundreds + 6 tens + 4 ones
   b) 8 hundreds + 7 tens + 8 ones

Section B

14. a) 7 749
   b) 8 882
   c) 7 938
   d) 7 015

15. a) 517
   b) 63
   c) 479

    b) Between: Copernicus & Galileo – 67 years (1610 – 1543);
       Galileo & Newton – 57 years (1667 – 1610);
       Copernicus & Newton – 124 years (1667 – 1543)

17. a) There are 452 children enrolled in swimming (324 + 128).
    b) Altogether, there are 776 children enrolled in lessons (452 + 324).

18. a) 
   
   6 × 5 = 30, so altogether there are 30 plants.
   
   b) 
   
   7 × 3 = 21, so altogether there are 21 chairs.
   
   c) 
   
   8 × 4 = 32, so altogether there are 32 seeds.

19. a) 1 405
    b) 1 656
    c) 1 446
    d) 3 661

20. a) 989
    b) 1 107
    c) 744
    d) 864

21. The wheel will turn 2 700 times in an hour (60 × 45).

22. Answers may vary:
   - 3 groups of 6;
   - 6 groups of 3;
   - 2 groups of 9;
   - 9 groups of 2

23. 12

24. No; 7 weeks = 952 minutes.

25. a) 16 R4
    b) 88 R6
    c) 120 R4
    d) 105 R6

26. i) Guerdy will use more boxes than Tyree (15 > 9).
    ii) Guerdy will also have more books left over (3 > 2).
Section C

27. a) 4 000
   b) 4 000
   c) 10 000

28. a) 80 000
    b) 40 000
    c) 10 000

29. Rounded to the nearest:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tens</td>
<td>37 180</td>
</tr>
<tr>
<td>Hundreds</td>
<td>37 200</td>
</tr>
<tr>
<td>Thousands</td>
<td>37 000</td>
</tr>
<tr>
<td>Ten thousands</td>
<td>40 000</td>
</tr>
</tbody>
</table>

30. a) Answers will vary.
    b) Answers will vary.

31. a) \(40 \times 50 = 2 000\)
    b) \(500 \times 80 = 40 000\)
    c) \(400 \times 20 = 8 000\)

Section D

32. a) $22.00
    b) $3.00
    c) $127.00

33. a) 35¢ = $0.35
    b) 35¢ = $0.35
    c) 80¢ = $0.80
    d) 8¢ = $0.08
    e) 1200¢ = $12.00
    f) 75¢ = $0.75

34. a) You would pay $43.19 altogether ($32.89 + $10.30).
    b) A watch and a cap would cost more ($58.53 versus $50.25).
    c) Yes you could – the total would be $95.72.
    d) The total cost $19.30 would be ice cream hot dog and soccer ball.

35. a) $1.86
    b) 143¢
    c) $0.70

36. She used 1 toonie, and 2 loonies and 2 quarters.

37. Wendy has $1.31 more than Mera.

38. a) Estimates will vary.
    Actual = $60.40
    b) Estimates will vary.
    Actual = $72.86
Measurement

Unit Test

1. In each case, how much time has passed? Show your work:
   a) 10:15 a.m. to 12 noon: ________________
   b) 11:45 a.m. to 2:30 p.m.: ________________

2. For each question below, convert the given time into the units required:
   a) 2 days = ______ hours
   b) 200 years = ______ centuries

3. In each case, write the time in numbers:
   a) twenty five minutes after four  _____:_____
   b) quarter to eleven  _____:_____
   c) three forty seven  _____:_____
   d) eight thirty eight  _____:_____

4. For each question below, use words to write the time in two different ways:
   a) \(10:25\) ____________________________  ____________________________
   b) \(4:40\) ____________________________  ____________________________

5. Can you think of three different ways of writing the following time?
   i) ___________________________________________________________________
   ii) ___________________________________________________________________
   iii) ___________________________________________________________________
Measurement

Unit Test

Name: _____________________________  Date: _________________

6. Write the time in 24-hour notation.
   a) 6:00 a.m. ______________________________________
   b) 7:30 p.m. ______________________________________

7. Write the time using a.m. or p.m.
   a) 12:30 ______________________________________
   b) 08:45 ______________________________________
   c) 15:15 ______________________________________

8. The temperature on Wednesday was -10°C. The temperature on Saturday was 10°C. How much did the temperature raise from Wednesday to Saturday?

9. A cheetah can run 30 m in a second.
   A car passes 60 km in an hour.
   Will a cheetah travel farther in 1 minute than a car? Show your work.
1. a) 1 hour, 45 minutes  
   b) 2 hours, 45 minutes  
2. a) 48 hours  
   b) 2 centuries  
3. a) 4:25  
   b) 10:45  
   c) 3:47  
   d) 8:38  
4. a) Answers may vary:  
   - twenty five minutes after ten;  
   - ten twenty five;  
   - thirty five minutes to eleven  
   b) Answers may vary:  
   - forty minutes after four;  
   - four forty;  
   - twenty minutes to five  
5. i) quarter after six  
   ii) fifteen minutes after six  
   iii) six fifteen  
6. a) 6:00  
   b) 19:30  
7. a) 12:30 p.m.  
   b) 8:45 a.m.  
   c) 3:15 p.m.  
8. 20°C  
9. Cheetah:  
   30 metres in 1 second =  
   = 1 800 metres in 1 minute  
   Car: 60 kilometres in 1 hour  
   = 1 kilometre in 1 minute  
   Cheetah will cover longer distance in 1 minute.
Probability & Data Management

Unit Test

Section A

1. Rene’s class has a fish tank. It contains a variety of small fish, each with different characteristics:

![Fish images]

a) Rene designed the following table to classify the fish. Can you help him complete it?

<table>
<thead>
<tr>
<th>Category</th>
<th>Fish (by letter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish with a pattern</td>
<td></td>
</tr>
<tr>
<td>Light fish</td>
<td></td>
</tr>
</tbody>
</table>

b) Complete a Venn diagram to show your results.

![Venn diagram]

2. Complete the bar graph to display the following data:

<table>
<thead>
<tr>
<th>Average Annual Rainfall by City (in cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capetown, South Africa</td>
</tr>
<tr>
<td>Quebec City, Canada</td>
</tr>
<tr>
<td>Melbourne, Australia</td>
</tr>
<tr>
<td>Rio de Janeiro, Brazil</td>
</tr>
<tr>
<td>Denver, USA</td>
</tr>
</tbody>
</table>
Use the above line graph to answer the following questions:

a) In which month did John visit a library ...
   (i) the greatest number of times? _______  (ii) the fewest number of times? _______

b) How many times did John visit a library in ...
   (i) in October? ____________  (ii) in February? ____________

c) In which months did John visit a library more than 10 times? ___________________________
   ________________________________________________________________________________

4. Look at the following chart:

<table>
<thead>
<tr>
<th>Grade</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>40</td>
<td>30</td>
<td>50</td>
<td>50</td>
<td>60</td>
<td>30</td>
</tr>
</tbody>
</table>

a) Which of the bar graphs below best represents the data above? What errors were made in the other two graphs?

A. Number of Students in Each Grade

B. Number of Students in Each Grade

C. Number of Students in Each Grade
b) Draw a line graph to show the data in the chart.

c) Is the data continuous or discrete on each axis? Did you use continuous (——) or broken (----) line for your graph? Explain your choice.

5.

Body Temperature of a Giant Toad during Day

a) Describe any trends you see in the data.

b) What is the body temperature of a toad around 5 p.m.?

c) List two periods when the body temperature of the toad does not change.

From _______ to _______ and from _______ to _______.

d) Is the data on each axis continuous or discrete?
1. a) With a pattern: A, D, E, G
   Light fish: A, C, E, F, G

   b) B, H

   with pattern light

2. **Average Annual Rainfall by City (in cm)**

3. a) i) September
   ii) December

   b) i) 9 times
   ii) 8 times

   c) July, August, September

4. a) Graph B represents data correctly.
   Graph A has the axes labelled incorrectly.
   Graph C contains the wrong data.

   b)

5. a) Exact answers will vary.
   The temperature rises from 6 a.m. to 2 p.m. then falls back gradually.

   b) 27.5°C

   c) From 8 p.m. to 10 p.m. and from 2 a.m. to 4 a.m.

   d) The data is continuous on both axes.
Geometry
Unit Test

Section A

1. Complete the chart. Find as many shapes as you can for each shape name:

<table>
<thead>
<tr>
<th>Shapes</th>
<th>Letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td>Quadrilaterals</td>
<td></td>
</tr>
</tbody>
</table>

2. Without using a protractor, identify each angle as “acute” or “obtuse”:

a) ___________________

b) ___________________

c) ___________________

3. Use the charts to classify the triangles below. NOTE: Triangles are not drawn to scale.

A.  

<table>
<thead>
<tr>
<th>5 m</th>
<th>60°</th>
<th>5 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 m</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>5 m</td>
<td>60°</td>
<td>5 m</td>
</tr>
</tbody>
</table>

B.  

<table>
<thead>
<tr>
<th>3 m</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 m</td>
<td>30°</td>
</tr>
</tbody>
</table>

C.  

<table>
<thead>
<tr>
<th>1.4 m</th>
<th>145°</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 m</td>
<td>45°</td>
</tr>
<tr>
<td>1.4 m</td>
<td></td>
</tr>
</tbody>
</table>

D.  

<table>
<thead>
<tr>
<th>5.2 m</th>
<th>105°</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 m</td>
<td>45°</td>
</tr>
<tr>
<td>5.2 m</td>
<td></td>
</tr>
</tbody>
</table>

a) Classify the triangles by their angles:

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute-angled</td>
<td></td>
</tr>
<tr>
<td>Obtuse-angled</td>
<td></td>
</tr>
<tr>
<td>Right-angled</td>
<td></td>
</tr>
</tbody>
</table>

b) Classify the triangles by their sides:

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral</td>
<td></td>
</tr>
<tr>
<td>Isosceles</td>
<td></td>
</tr>
<tr>
<td>Scalene</td>
<td></td>
</tr>
</tbody>
</table>
Section A (continued)

4. Measure all of the angles in each triangle and write your measurement in the triangle. Then say whether the triangle is acute, obtuse or right angled:
   a)  
   b)  
   c)  

5. Can a triangle be equilateral and right-angled? Explain.

6. Using arrows, mark all the pairs of parallel lines in the figures below.
   a)  
   b)  
   c)  
   d)  

   ____ pairs  ____ pairs  ____ pairs  ____ pairs

7. (i) Mark the angles that are right angles in the quadrilaterals below.
   (ii) Measure the length of each side with a ruler and write it onto the pictures. Use this to help you decide on the best (or most specific) name for each quadrilateral.
   a)  
   b)  

   ____ cm  ____ cm  ____ cm  ____ cm
   ____ cm  ____ cm  ____ cm  ____ cm

8. Match the name of the quadrilateral to the best description:

<table>
<thead>
<tr>
<th>Square</th>
<th>Rectangle</th>
<th>Rhombus</th>
</tr>
</thead>
<tbody>
<tr>
<td>A parallelogram with 4 right angles.</td>
<td>A parallelogram with 4 equal sides.</td>
<td>A parallelogram with 4 right angles and 4 equal sides.</td>
</tr>
</tbody>
</table>
Section A (continued)

9. Name the shapes: HINT: Use the words rhombus, square, parallelogram and rectangle.
   a) __________________
   b) __________________
   c) __________________
   d) __________________

10. For each quadrilateral, say how many pairs of sides are parallel. Then identify each quadrilateral as a square, a rectangle, a parallelogram or a trapezoid:
   a) __________________
   __________________
   b) __________________
   __________________
   c) __________________
   __________________
   d) __________________
   __________________

11. Describe any similarities between a square and a rhombus:

12. a) Why is a rectangle a parallelogram?

   b) Why are some parallelograms not rectangles?

13. a) Draw a quadrilateral that has two right angles and one pair of parallel sides.
   b) What is the name of the shape you drew?
Section B

14. Are these pairs of shapes congruent?
   a)  ___________ because _______________________________________
   b)  ___________ because _______________________________________ 

15. a) Draw a triangle that is not congruent to the one shown:

   

   b) Draw a trapezoid congruent to the one shown, but turned on its side:

   

16. Some of the shapes below are congruent. Find any shapes that are congruent to Shape A and label them with the letter A. If you can find any other shapes that are congruent to each other, label them all with the same letter. 
   HINT: You will need to use the letters A, B, C and D.

   

17. Starting with the shape on the left, add a square to each shape in the position shown by the arrow. If you create a shape that is congruent to a shape you have already made, cross it out. How many non-congruent shapes did you make?

   

   non-congruent shapes
Section B (continued)

18. Complete the picture so that the dotted line is a line of symmetry:
   a) ![Image]
   b) ![Image]
   c) ![Image]
   d) ![Image]

19. a) Using the line provided, use a protractor to construct a triangle with two 60° angles.
   b) Measure the sides of the triangle. (Write the measurements on the sides.) What kind of triangle did you draw?
   c) How many lines of symmetry does your triangle have? (Sketch in the lines of symmetry.)

20. a) Draw a trapezoid with one line of symmetry and a trapezoid with no lines of symmetry:
   b) Draw a parallelogram:
# Geometry

## Unit Test

Name: _____________________________

Date: _________________

## Section B (continued)

21. Record the properties of each shape. Write “yes” in the column if the shape has the given property. Otherwise, write “no”:

![Shapes A, B, C, D](image)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Quadrilateral</th>
<th>Equilateral</th>
<th>Two or more pairs of parallel sides</th>
<th>At least one right angle</th>
<th>At least one acute angle</th>
<th>At least one obtuse angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

22. Describe this figure completely. In your description you should mention the following properties:

- Number of sides
- Number of vertices
- Number of pairs of parallel sides
- Is the figure equilateral?
- Number of right angles
- Number of acute angles
- Number of obtuse angles
- Number of lines of symmetry

23. I have three sides. Two of my sides are the same length. What am I?
Section A

1. Shapes

<table>
<thead>
<tr>
<th>Shapes</th>
<th>Letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangles</td>
<td>B</td>
</tr>
<tr>
<td>Quadrilaterals</td>
<td>A, D, F, G, H</td>
</tr>
<tr>
<td>Pentagons</td>
<td>C, I</td>
</tr>
<tr>
<td>Hexagons</td>
<td>E, J</td>
</tr>
</tbody>
</table>

2. a) acute
   b) obtuse
   c) acute

3. a)

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute-angled</td>
<td>A</td>
</tr>
<tr>
<td>Obtuse-angled</td>
<td>D</td>
</tr>
<tr>
<td>Right-angled</td>
<td>B, C</td>
</tr>
</tbody>
</table>

b)

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral</td>
<td>A</td>
</tr>
<tr>
<td>Isosceles</td>
<td>C</td>
</tr>
<tr>
<td>Scalene</td>
<td>B, D</td>
</tr>
</tbody>
</table>

4. a) acute
   b) obtuse
   c) right

5. No – if you draw an equilateral triangle, you can see that all the angles will be acute:

6. a) 1 pair
   b) 2 pairs
   c) 1 pair
   d) 2 pairs

7. a)

![Diagram](image)

Name: parallelogram

b)

![Diagram](image)

Name: square

8. **Square**
   A parallelogram with 4 right angles and 4 equal sides.

**Rectangle**
   A parallelogram with 4 right angles.

**Rhombus**
   A parallelogram with 4 equal sides.

9. a) rectangle
   b) parallelogram
   c) square
   d) rhombus

10. a) 2 pairs; rectangle

b)

![Diagram](image)

2 pairs; parallelogram

c)

![Diagram](image)

2 pairs; square

d)

![Diagram](image)

1 pair; trapezoid

11. Both a square and a rhombus have 2 pairs of parallel sides, and 4 equal sides.

12. a) Because it has 2 pairs of parallel sides
   b) It depends on the shape’s angles – if they’re not right angles, the shape is a parallelogram, not a rectangle.

13. a) Answers will vary.
   b) trapezoid

   **Example:**

Section B

14. a) No; different sizes
   b) Yes; same shape and size (and colour, direction don’t matter)

15. a) Answers will vary – teacher to check.
   b) Answers will vary – teacher to check.

16. Two A’s:

   ![Diagram](image)

Three B’s:

   ![Diagram](image)

Two C’s:

   ![Diagram](image)

Two D’s:

   ![Diagram](image)

   **remaining shapes aren’t congruent with anything**

17. 4 non-congruent shapes

18. a)  

   ![Diagram](image)

b)  

   ![Diagram](image)

c)  

   ![Diagram](image)

d)  

   ![Diagram](image)
19. a) Teacher to check.
   b) equilateral
   c) 3 lines of symmetry:

\[ \begin{array}{c}
\text{\includegraphics[width=0.3\textwidth]{triangle.png}} \\
\end{array} \]

20. a) Answers will vary.  
   Examples:
   - One line of symmetry -

\[ \begin{array}{c}
\text{\includegraphics[width=0.3\textwidth]{triangle.png}} \\
\end{array} \]

   - No lines of symmetry -

   b) Answers will vary.

21.

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>E</th>
<th>2+</th>
<th>90°</th>
<th>Ac</th>
<th>Obt</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>B</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>C</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>D</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

22. Description should include the following details:
   - ✔ 6 sides
   - ✔ 6 vertices
   - ✔ 3 pairs of parallel sides
   - ✔ equilateral
   - ✔ no right angles
   - ✔ no acute angles
   - ✔ 6 obtuse angles
   - ✔ 6 lines of symmetry

23. Isosceles triangle
Section A

1. For each chart, write a verbal rule that tells you how to calculate the number of chairs from the row number:

   a)  
   
<table>
<thead>
<tr>
<th>Row</th>
<th>Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

   Rule:

   b)  
   
<table>
<thead>
<tr>
<th>Row</th>
<th>Chairs</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

   Rule:

   c)  
   
<table>
<thead>
<tr>
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<tr>
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<td>14</td>
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<tr>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

   Rule:

2. Complete the charts, and write a formula for each arrangement of chairs. Use r for the number of rows and C for the number of chairs.

   a)  
   
<table>
<thead>
<tr>
<th>Row</th>
<th>Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
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   Rule:

   b)  
   
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>4</td>
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<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
</tr>
</tbody>
</table>

   Rule:

3. For each chart, give a rule and a formula that tells you how to make the OUTPUT numbers from the INPUT numbers:

   a)  
   
<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

   Rule:

   b)  
   
<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

   Rule:

   c)  
   
<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
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<tbody>
<tr>
<td>3</td>
<td>9</td>
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<tr>
<td>5</td>
<td>11</td>
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<tr>
<td>7</td>
<td>13</td>
</tr>
</tbody>
</table>

   Rule:
**Patterns & Algebra**

*Unit Test*

**Section A (continued)**

4. Write the rule or the formula that tells you how to make the OUTPUT numbers from the INPUT numbers.

<table>
<thead>
<tr>
<th>a) Input</th>
<th>Gap × Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>16</td>
</tr>
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</table>

Rule:

<table>
<thead>
<tr>
<th>b) Input</th>
<th>Gap × Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
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<td></td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>7</td>
</tr>
</tbody>
</table>

Rule:

c) Input | Output
---|---
1 | 3
2 | 9
3 | 15
4 | 21

Rule:

d) Input | Output
---|---
1 | 5
2 | 6
3 | 7
4 | 8

Rule:

5. This picture shows how many chairs can be placed at each arrangement of tables. Make a T-table and state a rule that tells the relationship between the number of tables and the number of chairs:
Patterns & Algebra
Unit Test

Section B

6. In each sequence below, the step changes in a regular way: it either increases, decreases or increases and decreases. Write a rule for each pattern:

a) 34, 33, 30, 25, 18
   Rule: _______________________________________________________________________

b) 27, 30, 35, 42, 51
   Rule: _______________________________________________________________________

7. Extend this pattern for the next three terms. Then write a rule for the pattern:

52, 50, 47, 43, ______, ______, ______
   Rule: _______________________________________________________________________

8. Find the number that makes each equation true and write it in the box:

a) 8 – [ ] = 6   b) 3 × [ ] = 12   c) [ ] + [ ] + 3 = 7

9. Find two answers for the following equation:
   Note: In the question, congruent shapes represent the same number.

   [ ] + [ ] + [ ] = 5   [ ] + [ ] + [ ] = 5

10. Continue the patterns:

a) G g G, G g g G, G g g g G, ____________

b) ZA2, ZB4, ZC6, ZD8, ____________

c) ____________

d) ____________
Patterns & Algebra
Unit Test

Section B (continued)

11. 20 millimetres of rain fell in the first hour, and 14 millimetres fell each hour after that. How many millimetres fell in 4 hours? Show your work.

12. Trina saves $35 Monday. She saves $12 each day after that. How much has she saved by Friday? Show your work.

13. How many triangles will you need to make ornaments with all 20 squares? Show your work.
Patterns & Algebra

Unit Test

Section B (continued)

14. Marla says she will need 29 blocks to make Figure 7. Is she right? Explain how you found your answer.

15. The picture shows how the temperature changes at different heights over a mountain:

   a) Does the temperature increase or decrease at greater heights?

   b) What distance does the arrow represent in real life?

   c) Measure the length of the arrow. What is the scale of the picture?

       _______ cm = _____________ m

   d) Do the numbers in the sequence of temperatures decrease by the same amount each time?

   e) If the pattern in the temperature continued, what would the temperature be at:

      i) 3000 m?       ii) 4000 m?
Patterns & Algebra
Unit Test

Section B (continued)

16. Write an algebraic expression for the cost of renting a boat for…

a) 3 hours: ___________

b) x hours: _____ or ______

c) n hours: _____ or ______

17. Write an equation that tells you the relationship between the numbers in the column A and Column B.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
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<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
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<tr>
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<table>
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<th>B</th>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>19</td>
<td></td>
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</table>

Boat Renting
1 hour = $7
Section A

1. a) # chairs  
   = row # + 4  
   # chairs  
   = row # × 4  
   c) # chairs  
   = row # + 6  
2. a)  
   Row Chairs  
   1 3  
   2 4  
   3 5  
   Formula: r + 2 = c  
   b)  
   Row Chairs  
   1 2  
   2 3  
   3 4  
   Formula: r + 1 = c  
3. a) Multiply the input by 5 to get the output.  
   I × 5 = O  
   b) Add 5 to the input to get the output  
   I + 5 = O  
   c) Add 6 to the input to get the output  
   I + 6 = O  
4. a) I × 5 + 1 = O  
   b) I × 3 – 2 = O  
   c) I × 6 – 3 = O  
   d) I + 4 = O  
5. Tables Chairs  
   2 6  
   3 8  
   4 10  
   RULE: t × 2 + 2 = c  

Section B

6. a) Start at 34.  
   Subtract 1, then 3, then 5… (each step, subtract 2 more than you did the step before)  
   b) Start at 27.  
   Add 3, then 5, then 7… (each step, add 2 more than you did the step before)  
7. Rest of Pattern:  
   38, 32, 25  
   Rule:  
   Start at 52.  
   Subtract 2, then 3, then 4… (each step, subtract 1 more than you did the step before)  
8. a) 2  
   b) 4  
   c) 2, 2  
9. 2 + 2 + 1 = 5  
   AND  
   1 + 1 + 3 = 5  
10. a) GggggG  
    b) ZE10  
    c) From left to right:  
    5 6 7 8 9 8 7 6 5  
    d) From left to right:  
    3 6 9 12 15 12 9 6 3  
11. 62 millimetres of rain fell in 4 hours.  
   14 × 3 + 20 = 42 + 20 = 62  
   Students might also use a T-table.  
12. Trina has saved $83 by Friday.  
13. 28 triangles.  
   Explanations will vary.  
14. Figure Blocks  
   1 1  
   2 3  
   3 6  
   4 10  
   5 15  
   6 21  
   7 28  
   No, Marla is not right – from the T-table we see that Figure 7 requires 28 blocks (not 29).  
15. a) The temperature decreases at greater heights.  
   b) The top height is 2000 m above earth and, using the dotted lines, we see that the arrow represents 1/4 of that height:  
   1/4 of 2000  
   = 2000 ÷ 4  
   = 500 m  
   c) 1 cm = 500 m  
   d) Yes, each 500 m height increase results in a temperature drop of 2.5°C.  
   e) At 2000 m, the temperature is 12.0°C – using the information from parts c) and d), we can see that:  
   i) At 3000 m, the temperature will be:  
    = 12.0°C – 2.5°  
    = 12.0°C – 5.0°  
    = 7.0°C  
   ii) At 4000 m, the temperature will be:  
    = 7.0°C – 2.5°  
    = 7.0°C – 5.0°  
    = 2.0°C
Section A

1. A basketball team wins 9 games, loses 6 games and ties 3 games. What fractions of the games did the team:
   a) win? _________  
   b) lose? _________  
   c) tie? _________

2. What fraction of each figure is the shaded part?
   a) _________  
   b) _________  
   c) _________  
   d) _________

3. What fraction of each figure is the shaded part?
   a) _________  
   b) _________  
   c) _________  
   d) _________

4. Write the fractions in order from least to greatest:
   a) \( \frac{2}{4}, \frac{1}{4}, \frac{3}{4} \)  
   b) \( \frac{2}{11}, \frac{1}{11}, \frac{8}{11} \)  
   c) \( \frac{5}{18}, \frac{2}{18}, \frac{9}{18} \)

5. Write the fractions in order from least to greatest:
   a) \( \frac{1}{7}, \frac{1}{5}, \frac{1}{13} \)  
   b) \( \frac{2}{19}, \frac{2}{9}, \frac{2}{17} \)  
   c) \( \frac{9}{28}, \frac{9}{16}, \frac{9}{23} \)
Section A (continued)

6. a) Is \( \frac{1}{4} \) of Figure 1 the same amount as \( \frac{1}{4} \) of Figure 2?

b) Explain why or why not:

7. Shade one piece at a time until you have shaded the amount of pie given in bold. Then write an improper fraction for the amount of pie:
   a) \( 3 \frac{1}{2} \)
   b) \( 4 \frac{3}{4} \)

8. Shade one piece at a time until you have shaded the amount of pie given in bold. Then write a mixed fraction for the amount of pie:
   a) \( \frac{7}{3} \)
   b) \( \frac{21}{6} \)

9. How could you use division to find out how many whole pies are in \( \frac{25}{3} \) of a pie? Explain your answer:
Section B

10. Group the buttons to make an equivalent fraction:
   a) \[ \frac{4}{6} = \]
   b) \[ \frac{8}{10} = \]

11. Cut each pie into smaller pieces to make an equivalent fraction:
   a) \[ \frac{2}{3} = \]
   b) \[ \frac{2}{3} = \]
   c) \[ \frac{1}{2} = \]

12. A pizza is cut into 6 pieces. Each piece has at least one topping: green peppers, tomatoes or both. \[ \frac{1}{3} \] of the pizza is covered in green peppers. \[ \frac{5}{6} \] of the pizza is covered in tomatoes. Draw a picture to show how many pieces have both green peppers and tomatoes on them:

13. Solve:
   a) \( \frac{1}{6} \) of 18 =
   b) \( \frac{3}{5} \) of 25 =
   c) \( \frac{2}{3} \) of 15 =
   d) \( \frac{2}{5} \) of 10 =

14. Gerald has 12 oranges. He gives away \( \frac{5}{6} \) of the oranges.
   a) How many oranges did he give away? Show your work.
   b) How many oranges did he keep? Show your work.
15. Mina has a collection of 30 sports cards. \( \frac{1}{3} \) of the cards are baseball cards. \( \frac{2}{5} \) of the cards are hockey cards. The rest of the cards are soccer cards. How many of Mina’s cards are soccer cards?

16. Write the fractions in order from least to greatest. (First, change the fractions in each set so they share the same denominator.)

   a) \( \frac{1}{2} \), \( \frac{3}{5} \), \( \frac{3}{10} \)
   b) \( \frac{5}{6} \), \( \frac{1}{2} \), \( \frac{5}{12} \)
   c) \( \frac{1}{2} \), \( \frac{1}{4} \), \( \frac{7}{8} \)

17. A recipe for chilli calls for \( \frac{3}{8} \) of a can of beans and a recipe for soup calls for \( \frac{1}{4} \) of a can. Which recipe uses more beans? Explain.

18. Draw a picture to show which fraction is greater: \( 2\frac{1}{2} \) or \( \frac{9}{4} \).
Number Sense

Unit Test

Section C

19. Write the decimal that is given in words in decimal notation:
   a) 44 hundredths = ________  b) 68 hundredths = ________  c) 7 hundredths = ________

20. Write a decimal for the fraction:
   a) \( \frac{86}{100} = \)  b) \( \frac{48}{100} = \)  c) \( \frac{36}{100} = \)  d) \( \frac{7}{100} = \)
   e) \( \frac{65}{100} = \)  f) \( \frac{52}{100} = \)  g) \( \frac{89}{100} = \)  h) \( \frac{3}{100} = \)

21. Write a fraction and a decimal for each shaded part:

22. Write a fraction for the number of hundredths. Then draw a heavy line around each column and write a fraction for the number of tenths:

   a)  
   b)  
   c)  
   d)  

23. Rita says .26 is greater than .8 because 26 is greater than 8. Is she right? Explain why or why not:
Number Sense

Unit Test

Section C (continued)

24. Answer the following questions. REMEMBER: \( \frac{10}{100} = \frac{1}{10} \)

a) \( \frac{8}{10} = \) \( \frac{_____}{100} \) b) \( \frac{9}{10} = \) \( \frac{_____}{100} \) c) \( \frac{3}{10} = \) \( \frac{_____}{100} \) d) \( \frac{50}{100} = \) \( \frac{_____}{10} \) e) \( \frac{_____}{10} = \frac{30}{100} \)

25. Fill in the missing numbers:

a)

b)

c)

d)

<table>
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<tr>
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<td>( . )</td>
<td>100</td>
<td>( . )</td>
<td>100</td>
<td>( . )</td>
<td>100</td>
<td>( . )</td>
</tr>
</tbody>
</table>

26. Write the following decimals as fractions:

a) \( .23 = \frac{23}{100} \) b) \( .48 = \frac{48}{100} \) c) \( .61 = \frac{61}{100} \) d) \( .05 = \frac{5}{100} \) e) \( .30 = \frac{30}{100} \)

27. Write the following decimals as fractions:

a) \( .3 = \) b) \( .53 = \) c) \( .09 = \) d) \( .26 = \)

28. Change the following fractions to decimals:

a) \( \frac{9}{10} = \) b) \( \frac{6}{10} = \) c) \( \frac{9}{100} = \) d) \( \frac{68}{100} = \)

29. Circle the equalities that are incorrect:

\( .15 = \frac{15}{100} \) \( .3 = \frac{3}{10} \) \( .9 = \frac{9}{10} \) \( \frac{48}{100} = .48 \) \( .2 = \frac{2}{100} \)

30. Write words for the following decimals:

a) \( .6 \) b) \( .07 \) c) \( .27 \)

______________________  __________________  __________________
Section C (continued)

31. Write a decimal and a mixed fraction for each of the pictures below:
   a) [grid image]
   b) [grid image]

32. Write a decimal for each of the mixed fractions below:
   a) \[ \frac{2}{10} = \]
   b) \[ \frac{3}{100} = \]
   c) \[ \frac{21}{10} = \]
   d) \[ \frac{89}{100} = \]

33. Write the numbers in order from least to greatest by first changing all of the decimals to fractions with denominator 100:
   a) \[ \frac{3}{2} = \]
   b) \[ \frac{5}{100} = \]
   c) \[ \frac{37}{100} = \]
   d) \[ \frac{4}{10} = \]
   e) \[ \frac{68}{100} = \]

34. Line up the decimals and add or subtract the following decimals:
   a) \[ 0.32 + 0.17 = \]
   b) \[ 0.64 – 0.23 = \]
   c) \[ 0.87 – 0.02 = \]

35. Subtract 3.51 – 1.34 two ways: (i) by drawing a base ten model and (ii) by lining up the decimal points.
   (i) 
   (ii)

36. Bamboo can grow up to 0.3 m in a single day in ideal conditions. How high could it grow in 3 days?

37. Lisa bought a slice of pizza for $3.91 and a video game for $14.27.
   How much change did she get from $25.00?
Number Sense

Unit Test

Section D

38. Multiply:
   a) \(10 \times .5 = \_\_\_\_\_\_\_\_\;
   b) \(1.6 \times 10 = \_\_\_\_\_\_\_\_\;
   c) \(2.75 \times 10 = \_\_\_\_\_\_\_\_\;

39. Multiply:
   a) \(100 \times .07 = \_\_\_\_\_\_\_\_\_\;
   b) \(10 \times .67 = \_\_\_\_\_\_\_\_\;
   c) \(100 \times 1.82 = \_\_\_\_\_\_\_\_\;
   d) \(.50 \times 100 = \_\_\_\_\_\_\_\_\;
   e) \(1.8 \times 10 = \_\_\_\_\_\_\_\_\;
   f) \(.64 \times 10 = \_\_\_\_\_\_\_\_\;

40. To change a measurement from dm to cm, you multiply by 10. To change from dm to mm, you multiply by 100. Find the answers:
   a) \(1.38 \text{ dm} = \_\_\_\_\_\_\_\_\_\_\text{ cm}\)
   b) \(4.21 \text{ dm} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{ mm}\)
   c) \(.08 \text{ dm} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\text{ mm}\)

41. Find the products:
   a) \(5 \times 7.5\)
   b) \(5 \times 6.35\)
   c) \(8 \times 2.63\)

42. Divide:
   a) \(0.4 \div 10 = \_\_\_\_\_\_\_\_\;
   b) \(0.9 \div 10 = \_\_\_\_\_\_\_\_\;
   c) \(7.2 \div 10 = \_\_\_\_\_\_\_\_\;
   d) \(7 \div 10 = \_\_\_\_\_\_\_\_\;
   e) \(39 \div 10 = \_\_\_\_\_\_\_\_\;
   f) \(.7 \div 10 = \_\_\_\_\_\_\_\_\;

43. Dana has 2.7 m of rope. She wants to cut the rope into 10 equal lengths. How long will each piece be?

44. a) Write a decimal and a mixed fraction for each of the pictures below:

   A: ______________________
   B: ______________________
   C: ______________________
   D: ______________________

   b) Mark each point with an ‘X’ and label the point with the correct letter:

   A. 1.6
   B. 2.1
   C. .90
   D. 2.5
Number Sense

Unit Test

Section D (continued)

45. Divide:

a) \( 8 \div 1.44 \)  
   b) \( 7 \div 9.1 \)  
   c) \( 8 \div 2.72 \)  
   d) \( 9 \div 6.12 \)

46. Five apples cost $2.65. How much does each apple cost?

47. Ayca cycled 56.4 km in 4 hours. How many kilometres did she cycle in an hour?

48. Add:

   a) \( 5000 + 300 + 8 + 0.01 = \)  
   b) \( 10000 + 400 + 30 + 0.2 + 0.06 = \)

49. Write < or > to show which decimal is greater:

   a) \( 4.7 \) \( < \) \( 4.5 \)  
   b) \( 6.42 \) \( < \) \( 6.47 \)  
   c) \( 1.8 \) \( < \) \( 1.79 \)  
   d) \( 0.3 \) \( > \) \( 0.33 \)

50. Giant Kelp is the fastest growing ocean plant. It can grow 0.67 m in a day. How much could it grow in four days? Show your work:

51. Mariam travelled 62.5 m in 100 steps. How many metres was each step?
52. You need 1 cup of blueberries and 2 cups of flour to make 12 blueberry muffins.
   a) How many cups of blueberries do you need to make 72 muffins?
   
   b) How many cups of flour do you need to make 30 muffins?
   
   c) Tegan has 5 cups of blueberries and 9 cups of flour. How many muffins will she be able to make?
   Show your work.
Section A

1. a) \( \frac{9}{18} = \frac{1}{2} \)
b) \( \frac{6}{18} = \frac{1}{3} \)
c) \( \frac{3}{18} = \frac{1}{6} \)

2. a) 1
b) \( \frac{1}{2} \)
c) \( \frac{5}{6} \)
d) \( \frac{1}{6} \)

3. a) \( \frac{3}{8} \)
b) \( \frac{1}{4} \)
c) \( \frac{4}{9} \)
d) \( \frac{1}{12} \)

4. a) \( \frac{1}{4} \cdot \frac{2}{3} = \frac{3}{4} \)
b) \( \frac{1}{11} \cdot \frac{2}{11} = \frac{6}{11} \)
c) \( \frac{2}{18} \cdot \frac{5}{18} = \frac{9}{18} \)

5. a) \( \frac{1}{10} \cdot \frac{1}{7} = \frac{1}{70} \)
b) \( \frac{2}{19} \cdot \frac{2}{17} = \frac{2}{323} \)
c) \( \frac{9}{28} \cdot \frac{9}{23} = \frac{9}{68} \)

6. a) No: \( \frac{1}{4} \) of Figure 1 is smaller than \( \frac{1}{4} \) of Figure 2.
b) Although the fraction of each figure is the same, the size of the two figures differs. Therefore, the size of a quarter piece in Figure 2 is different (and – in this case – larger) than a quarter piece in Figure 1.

7. a) \[ \begin{array}{c}
\text{Improper fraction: } \frac{7}{4}
\end{array} \]
b) \[ \begin{array}{c}
\text{Improper fraction: } \frac{10}{4}
\end{array} \]

8. a) \( \begin{array}{c}
\text{Mixed fraction: } 4 \frac{1}{3}
\end{array} \)
b) \( \begin{array}{c}
\text{Mixed fraction: } 3 \frac{3}{6} = 3 \frac{1}{2}
\end{array} \)

9. Using division, we know:
\[ 25 \div 3 = 8 \text{ R}1 \]
So \( \frac{25}{3} \) – as a mixed fraction – would be \( 8 \frac{1}{3} \).
and this means that it has 8 whole pies (and one third of another pie).

10. a) Teacher to check grouping:
\[ \frac{4}{6} = \frac{2}{3} \]
b) Teacher to check grouping:
\[ \frac{8}{10} = \frac{4}{5} \]

11. a) Teacher to check “cutting”:
\[ \frac{2}{3} = \frac{4}{6} \]
b) Teacher to check “cutting”:
\[ \frac{2}{3} = \frac{6}{9} \]
c) Teacher to check “cutting”:
\[ \frac{1}{2} = \frac{4}{8} \]

12. Sample Answer:

Exact pictures may vary but, in all cases, only 1 of the 6 pieces will have both toppings.

13. a) 3
b) 15
c) 10
d) 4

14. a) \( \frac{5}{8} \) of 12 = 10
b) Gerald kept 2 oranges.

15. Number of baseball cards:
\[ \frac{1}{3} \text{ of } 30 = 10 \]
Number of hockey cards:
\[ \frac{2}{5} \text{ of } 30 = 12 \]
So the total number of cards that are not soccer cards is 10 + 12 = 22.
This means that 8 of the Mina’s cards (30 – 22) are soccer cards.

16. a) \( \frac{3}{10} \cdot \frac{1}{2} \cdot \frac{3}{5} \)
b) \( \frac{5}{12} \cdot \frac{1}{2} \cdot \frac{5}{6} \)
c) \( \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{7}{8} \)

17. \( \frac{1}{4} = \frac{2}{8} \) and \( \frac{2}{8} < \frac{3}{8} \)
So the chilli recipe uses more beans than the soup recipe.

So we see that \( 2 \frac{1}{2} > \frac{9}{4} \).
### Section C

19. a) 0.44
d) 0.68
b) 0.07
c) 0.36
d) 0.07
e) 0.65
f) 0.52
g) 0.89
h) 0.03

20. a) 0.86
d) 0.07
c) 0.36
e) 0.65
f) 0.52
g) 0.68
h) 0.03

21. a) 0.51
b) 0.30
c) 0.19

22. a) Teacher to check lines.
   b) 0.70
c) 0.90
d) 0.60
e) 0.30

23. Student explanations may vary. Teacher to check.
   Sample Answer:
   No, Rita is not right. You can tell by writing each decimal as a fraction over 100:
   0.26 = 26/100; 0.8 = 80/100

24. a) 8/10 = 80/100
d) 5/100 = 1/20

25. | 10ths | 100ths |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 5</td>
<td>5 59/100 = 0.59</td>
</tr>
<tr>
<td>b) 0</td>
<td>0 7/100 = 0.07</td>
</tr>
<tr>
<td>c) 8</td>
<td>0 80/100 = 0.8</td>
</tr>
<tr>
<td>d) 4</td>
<td>5 45/100 = 0.45</td>
</tr>
</tbody>
</table>

### Section D

38. a) 5
   b) 16
   c) 27.5

39. a) 7
   b) 6.7
   c) 182
d) 50
e) 18
f) 6.4

g) 0.27 m (or 27 cm) long.

40. a) 6 cups
   b) 421 mm
c) 8 mm

41. a) 37.5
   b) 31.75
c) 21.04

42. a) 0.04
   b) 0.09
   c) 0.72
d) 0.7
   e) 3.9
   f) 0.07

43. $0.53 each ($2.65 ÷ 5).

44. a) A: 0.4 = 4/10
   B: 1.2 = 1 2/10 = 1 1/5
   C: 1.6 = 1 6/10 = 1 3/5
   D: 2.7 = 2 7/10

45. a) 0.18
   b) 1.3
c) 0.34
d) 0.68

46. 14.1 km (56.4 + 4)

48. a) 5308.01
   b) 10430.26

49. a) 4.7 > 4.5
   b) 6.42 < 4.67
c) 1.8 > 1.79
d) 0.3 < 0.33

50. 2.68 m (0.67 × 4)

51. 0.625 m (62.5 ÷ 100) or 62.5 m

52. a) 6 cups
   b) 5 cups
c) 5 cups of blueberries will require 10 cups of flour. Tegan has only 9 cups of flour, so she can only use 9 ÷ 2 = 4.5 cups of blueberries. This gives 4.5 × 12 = 54 muffins.
Measurement

Unit Test

Section A

1. Fill in the numbers missing from the following charts:

<table>
<thead>
<tr>
<th>mm</th>
<th>cm</th>
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</thead>
<tbody>
<tr>
<td>14</td>
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</tr>
<tr>
<td>90</td>
<td>70</td>
</tr>
<tr>
<td>170</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>m</th>
<th>cm</th>
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</thead>
<tbody>
<tr>
<td>900</td>
<td>263</td>
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<tr>
<td>180</td>
<td>7200</td>
</tr>
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</table>

<table>
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<tr>
<th>dm</th>
<th>cm</th>
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<tbody>
<tr>
<td>263</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
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<th>dm</th>
<th>m</th>
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<tbody>
<tr>
<td>4</td>
<td>367</td>
</tr>
<tr>
<td>80</td>
<td>6</td>
</tr>
</tbody>
</table>

2. Circle the greater measurement in each pair of measurements. Do this by first converting one of the measurements so that both units are the same.

   a) 6 cm 80 mm  
   b) 93 cm 510 mm  
   c) 47 cm 63 mm

3. Write a measurement in mm that is between:

   a) 9 and 10 cm: ______ mm  
   b) 15 and 16 cm: ___________  
   c) 37 and 38 cm: ___________

4. Change these measurements into metres:

   a) 3 km  
   b) 6 km  
   c) 1 km  
   d) 12 km  
   e) 19 km

5. The Minolta Tower in Niagara Falls is 160 metres high. About how many Minolta Towers, laid end to end, would make a kilometre?

6. Convert the measurement given in cm to a measurement using multiple units:

   a) 623 cm = _____ m _____ cm  
   b) 594 cm = _____ m _____ cm  
   c) 477 cm = _____ m _____ cm  
   d) 823 cm = _____ m _____ cm
Measurement

Unit Test

Section A (continued)

7. Is 492 mm longer or shorter than 20 cm? Explain how you know:

8. Some BIG and SMALL facts about Canada! Choose which unit (km, m or cm) is needed to complete each sentence. Read the statements carefully.
   a) The highest point is Mount Logan. It is 5,959 ______ high.
   b) Niagara Falls is 52 ______ high.
   c) The St. Lawrence River is 3058 ______ long.
   d) The border between Canada and the United States is 8893 ______ long.
   e) A Sugar Maple tree can grow to a height of 30 ______.
   f) A chipmunk is 20 ______ long.

9. Write the words “larger” or “smaller” in line i); “more” or “fewer” in line ii); “multiply” or “divide” in line iii).
   a) Change 14 m to a measure in dm:
      i) The new units are _____ times __________
      ii) So I need ___ times ___________ units
      iii) So I _______________ by _______
          14 m = _______ dm
   b) Change 23 cm to a measure in m:
      i) The new units are _____ times __________
      ii) So I need ___ times ___________ units
      iii) So I _______________ by _______
          23 cm = _______ m

10. Change each amount to a decimal in the larger unit:
    a) 9 m 7 cm
    b) 5 m 70 cm
    c) 9 m 5 dm

11. Name any object in your classroom. Write down a unit of measurement that would be best for measuring it. Explain why it would be the best unit of measurement:
Measurement

Unit Test

Name: ___________________  Date: ________________

Section B

12. Change the following measurements to grams:
   a) 4 kg = ________  b) 8 kg = ________  c) 16 kg = ________  d) 22 kg = ________

13. Change the following measurements to millilitres:
   a) 4 L = ___________  b) 5 L = ___________  c) 11 L = ___________  d) 42 L = __________

14. Which will be more shampoo: five 250 mL bottles or two 1000 mL bottles? ___________________
   Explain how you know:

15. Change each amount to a decimal in the larger unit:
   a) 8 kg 370 g   b) 3 L 2 mL   c) 5 kg 23 g

16. For each figure below, fill in the missing numbers in the mat plan to indicate the number of
    stacked cubes:
   a)  b)  c)  d)

17. Calculate the volume (in cubes) of each shape in Question 16:
   a) __________  b) __________  c) __________  d) __________
Measurement
Unit Test

Section C
18. Use a ruler to measure the perimeter of each figure (in cm):

a)

b)

c)

19. Find the perimeter of each shape. Be sure to include the units in your answer:

a) Perimeter __________
b) Perimeter __________
c) Perimeter __________
d) Perimeter __________

e) Write the letters of the shapes in order from greatest perimeter to least perimeter. (Make sure you look at the units!)

_______, ________, ________, ________

20. Find the area of these figures in square centimetres:

a) Area = _______ cm²  

b) Area = _______ cm²  

c) Area = _______ cm²
Section C (continued)

21. Find the area (in cm²) of each of the given shapes:

   Area of A = _________________
   Area of B = _________________
   Area of C = _________________

22. Measure the length and width of the following figures, then find the area:

   a) ______________________  
   b) ______________________  
   c) ______________________  

23. Find the area of the rectangles with the following dimensions:

   a) width: 6 m   length: 7 m  
   b) width: 3 m   length: 7 m  
   c) width: 4 cm   length: 8 cm

24. A rectangle has an area of 18 cm² and a length of 6 cm. What is its width?

25. Measure the length and width of each rectangle, then calculate its perimeter and area:

   a) 
   Perimeter = _____ cm
   Area = _____ cm²

   b) 
   Perimeter = _____ cm
   Area = _____ cm²

   c) 
   Perimeter = _____ cm
   Area = _____ cm²
Section C (continued)

26. Show all the ways you can make a rectangle with a perimeter of 12 units:

27. How many different rectangles with sides whose lengths are whole numbers of cm have area 18cm²? Show your work.

28. Sally says she can find the area of a rectangle if she knows the perimeter of the rectangle and the length of one side. Is she correct? Explain with an example.
Section A

1.  

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<td>7 000</td>
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<td>7 200</td>
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<th>m</th>
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</thead>
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<tr>
<td>3 670</td>
<td>367</td>
</tr>
<tr>
<td>80</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

2.  
   a) 80 mm  
   b) 93 cm  
   c) 47 cm

3.  
   Answers will vary. Teacher to check.

4.  
   a) 3 000 m  
   b) 6 000 m  
   c) 1 000 m  
   d) 12 000 m  
   e) 19 000 m

5.  
   It would take about six Minolta Towers, laid end to end, to make a kilometre.

   NOTE:
   Answers will vary slightly, depending on the method used.

6.  
   a) 6 m 23 cm  
   b) 5 m 94 cm  
   c) 4 m 77 cm  
   d) 8 m 23 cm

7.  
   492 mm > 20 cm  
   (since 20 cm = 200 mm)

8.  a) m  
    b) m  
    c) km  
    d) km  
    e) m  
    f) cm

9.  a)  
    i) 10 times smaller  
    ii) 10 times more  
    iii) multiply by 10

   14 m = 140 dm

   b)  
    i) 100 times larger  
    ii) 100 times fewer  
    iii) divide by 100

   23 cm = 0.23 m

10.  a) 9.07 m  
     b) 5.7 m  
     c) 9.5 m

11.  Answers will vary – teacher to check.

12.  a) 4 000 g  
     b) 8 000 g  
     c) 16 000 g  
     d) 22 000 g

13.  a) 4 000 mL  
     b) 5 000 mL  
     c) 11 000 mL  
     d) 42 000 mL

14.  Sample explanation:

   Option 1:  
   5 × 250 = 1 250

   Option 2:  
   2 × 1 000 = 2 000

   So two 1 000 mL bottles will hold more shampoo.

15.  a) 8.37 kg  
     b) 3.002 L  
     c) 5.023 kg

16.  a)  
     b)  
     c)  
     d)  
     e)  
     f)  

17.  a) 5 cubes  
     b) 8 cubes  
     c) 7 cubes  
     d) 7 cubes

18.  a) 10 cm  
     b) 10 cm  
     c) 14 cm

19.  a) 28 m  
     b) 56 cm  
     c) 9 km  
     d) 36 cm  
     e) C, A, B, D

20.  a) 8 cm²  
     b) 8 cm²  
     c) 9 cm²

21.  
   Area of A = 6 cm²  
   Area of B = 8 cm²  
   Area of C = 12 cm²

22.  a) 6 cm²  
     b) 9 cm²  
     c) 15 cm²

23.  a) 42 cm²  
     b) 21 cm²  
     c) 32 cm²

24.  3 cm (18 ÷ 6 = 3)

25.  a) Perimeter = 14 cm  
     Area = 10 cm²  
     b) Perimeter = 8 cm  
     Area = 3 cm²  
     c) Perimeter = 10 cm  
     Area = 6 cm²

26.  3 rectangles are possible:

   i) 1 × 5  
   ii) 2 × 4  
   iii) 3 × 3 (since a square is also a rectangle)

27.  3 rectangles.

<table>
<thead>
<tr>
<th>Area = 18 cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
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<td>----</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

Rectangles 1 and 6, 2 and 5, 3 and 4 are congruent, so there are only 3 different rectangles.
28. Yes, Sally is correct:
   If we’re given the perimeter of a rectangle and the length of one of its sides, it is possible to find the width of the rectangle.
   Then, once we know the width, we can multiply it by the length to find the area.
   Examples will vary.
Section A

1. Find the range of the following data sets:
   a) 45, 27, 14, 95, 44, 8
      _____________________________
      Range: _____ to _____
   b) 124, 46, 34, 71, 24, 355
      _____________________________
      Range: _____ to _____

2. Find the mean of the following data sets:
   a) 2, 4, 6, 10, 13
      ______________________________
      ______________________________
      Mean: ________
   b) 17, 6, 12, 4, 21
      ______________________________
      ______________________________
      Mean: ________

3. Find the mode of the following data sets:
   a) 3, 8, 8
      Mode: ________
   b) 30, 22, 52, 30
      Mode: ________
   c) 7, 7, 4, 5, 7, 4, 4, 7
      Mode: ________
   d) 18, 88, 81, 8, 88, 88, 18
      Mode: ________

4. Find the median of the following data sets:
   a) 10, 18, 4, 13, 5
      _____________________________
      Median: ________
   b) 2, 10, 7, 9, 3
      _____________________________
      Median: ________
   c) 24, 3, 16, 74, 10, 92
      _____________________________
      Median: ________
   d) 27, 25, 1, 85, 553, 3
      _____________________________
      Median: ________
Section A (continued)

5. Mrs. Lynch gave her students a spelling test (marked out of 20) and entered the marks in the chart below.

<table>
<thead>
<tr>
<th>11</th>
<th>8</th>
<th>18</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>10</th>
<th>16</th>
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</thead>
<tbody>
<tr>
<td>19</td>
<td>15</td>
<td>19</td>
<td>19</td>
<td>9</td>
<td>20</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

a) Make a stem and leaf plot of the data.

c) In the space provided, find the mean, mode and median of the class marks:
NOTE: You may use a calculator to find the mean.

c) Which measurement – the mean or the mode – do you think gives better sense of the class’s performance on the test? Explain below:

d) Describe the data. Is it spread out more above the median or below the median?
Section B

6. What are the possible outcomes for these spinners?

a)  
   1  3  5  7  
   ______ outcomes

b)  
   ______ outcomes

c)  
   ______ outcomes

d)  
   ______ outcomes

7. For each spinner, write the probability of the given events. \textbf{Hint: Cut the spinners into equal parts.}

a)  
   \begin{tikzpicture}
   
   \fill[green] (0,0) circle (0.3cm);
   \fill[red] (1,0) circle (0.3cm);
   \fill[yellow] (2,0) circle (0.3cm);
   \end{tikzpicture}

   P(Red) =

b)  
   \begin{tikzpicture}
   
   \fill[blue] (0,0) circle (0.3cm);
   \fill[red] (1,0) circle (0.3cm);
   \fill[yellow] (2,0) circle (0.3cm);
   \end{tikzpicture}

   P(Blue) =

c)  
   \begin{tikzpicture}
   
   \fill[red] (0,0) circle (0.3cm);
   \fill[green] (1,0) circle (0.3cm);
   \fill[yellow] (2,0) circle (0.3cm);
   \end{tikzpicture}

   P(Green) =

d)  
   \begin{tikzpicture}
   
   \fill[yellow] (0,0) circle (0.3cm);
   \fill[red] (1,0) circle (0.3cm);
   \fill[blue] (2,0) circle (0.3cm);
   \end{tikzpicture}

   P(Green) =

8. Write a fraction that gives the probability of spinning:

a)  the number 1

b)  the number 3

c)  an even number

d)  an odd number

e)  a number less than 5

f)  a number greater than 5

9. Sketch a spinner on which the probability of spinning red is $\frac{3}{4}$:
Section B (continued)

For some questions, you will need to do long division. (Show your work on a separate piece of paper.)

10. Fill in the missing numbers:
   a) \( \frac{1}{3} \) of 9 is _____
   b) \( \frac{1}{3} \) of 12 is _____
   c) \( \frac{1}{3} \) of 15 is _____
   d) \( \frac{1}{3} \) of 18 is _____
   e) \( \frac{1}{3} \) of 39 is _____
   f) \( \frac{1}{3} \) of 42 is _____
   g) \( \frac{1}{3} \) of 75 is _____
   h) \( \frac{1}{4} \) of 52 is _____

11. How many times would you expect to spin yellow if you spun this spinner 20 times? Explain.

12. Draw a line to connect each spinner to its matching probability statement:

   A. The probability of spinning a 3 is \( \frac{1}{4} \).
   B. The probability of spinning an even number is \( \frac{5}{6} \).
   C. The probability of spinning a multiple of 3 is \( \frac{2}{5} \).
   D. The probability of spinning a 2 is \( \frac{1}{2} \).

13. Use the words “certain”, likely”, “unlikely” or “impossible” to describe the following events:
   a) The chance of rolling a number less than 10 on a die: __________________________
   b) The chance of rolling a number less than 2 on a die: __________________________
   c) The chance of rolling a number less than 5 on a die: __________________________
### Section A
1. a) 8 to 95  
   b) 24 to 355  
2. a) 7  
   b) 12  
3. a) 8  
   b) 30  
   c) 7  
   d) 88  
4. a) 10  
   b) 7  
   c) 20 (the average of 24 and 16)  
   d) 26 (the average of 25 and 27)  
5. a)  
   Stem | Leaves  
   --- | ---  
   0 | 89  
   1 | 001255689999  
   2 | 00  
   b) Mean: 15  
   Median: 15.5  
   Mode: 19  
   c) The mean gives the better sense.  
   Explanations will vary.  
   d) Answers will vary.  
   The data bunches up at the ends.  
   It is spread out more below the median (the range below the median is 7, the range above the median is 5)  

### Section B
6. a) 1, 3, 5, 7, 4 outcomes  
   b) 3, 1 outcome  
   c) 4, 5, 6, 3 outcomes  
   d) 1, 7, 2 outcomes  
7. a) P(\text{Red}) = \frac{1}{4}  
   b) P(\text{Blue}) = \frac{3}{8}  
   c) P(\text{Green}) = \frac{2}{6} = \frac{1}{3}  
   d) P(\text{Green}) = 0  
8. a) \frac{3}{8}  
   b) \frac{1}{8}  
   c) \frac{1}{8}  
   d) \frac{7}{8}  
   e) \frac{4}{8} = \frac{1}{2}  
   f) \frac{3}{8}  
9. Answers will vary slightly. Teacher to check.  
   Sample answer:  
   ![Sample answer diagram]  
10. a) 3  
    b) 4  
    c) 5  
    d) 6  
    e) 13  
    f) 14  
    g) 25  
    h) 13  
11. In this spinner,  
   \( P(\text{yellow}) = \frac{1}{4} \)  
   So, if you spun the spinner 20 times, you would expect to spin yellow 5 times \((20 \div 4)\).  
12. A. Spinner 3  
    B. Spinner 4  
    C. Spinner 1  
    D. Spinner 2  
13. a) certain  
    b) unlikely  
    c) likely
Geometry

Unit Test

Section A

1. Circle the points in the following positions (connecting the dots first, if necessary):
   a) 3 • • •
       2 • • •
       1 • • •
       1 2 3
   b) 3 • • •
       2 • • •
       1 • • •
       1 2 3
   c) 3 • • •
       2 • • •
       1 • • •
       1 2 3
   d) 3 • • •
       2 • • •
       1 • • •
       1 2 3

   Column 1
   Row 2
   (1,1)

   Column 2
   Row 3
   (3,3)

2. Circle the points in the following positions:
   a) 3 • • •
       2 • • •
       1 • • •
       A B C
   b) C • • •
       B • • •
       A • • •
       X Y Z
   c) 2 • • •
       1 • • •
       0 • • •
       0 1 2
   d) 2 • • •
       1 • • •
       0 • • •
       0 1 2

   (B,2)
   (X,C)
   (0,2)
   (2,0)

3. Graph each set of ordered pairs and join the dots to form a polygon. Identify the polygon drawn:
   a) A (0,2) B (0,4) C (4,4) D (4,2)
      This polygon is a ____________________.

   b) A (1,1) B (1,3) C (3,3) D (3,1)
      This polygon is a ____________________.

4. Write the coordinates of the following points:
   A ( , ) B ( , )
   C ( , ) D ( , )
   E ( , ) F ( , )
   G ( , ) H ( , )
Section B

5. Slide each shape 4 boxes to the right. (Start by putting a dot on one of the corners of the figure. Slide the dot four boxes right, then draw the new figure.)

a)  

b)  

6. Slide each figure 5 boxes to the right and 2 boxes down:

a)  

b)  

7. Draw the reflection (or flip) of the shapes below:

a)  

b)  

c)  

7. Are the shapes on both sides of M congruent? Explain your answer.

8. Draw a shape on the grid paper. Translate the shape and draw a translation arrow between a point on the shape and a point on the image. Describe how far the shape moved (right / left and up / down).
9. Show where the arrow would be after each turn:

a) $\frac{1}{4}$ turn clockwise

b) $\frac{1}{2}$ turn clockwise

c) $\frac{3}{4}$ turn clockwise

d) $\frac{3}{4}$ turn counter clockwise

10. Show what the figure would look like after the rotation. First rotate the dark line, then draw the rest of the figure:

a) $\frac{1}{4}$ turn clockwise

b) $\frac{1}{2}$ turn clockwise

c) $\frac{3}{4}$ turn counter clockwise

d) $\frac{1}{4}$ turn counter clockwise

11. Colour or shade in the sections of the left-hand square using at least 3 colours or shadings. Then create a border design by rotating the square:

12. Give two reasons why this picture does not show a reflection:
Section B  (continued)

13. Describe how the figure moved from Position 1 to Position 2 by using two transformations:

a) 

b) 

14. For each question below, you will need to copy the given figure onto the grids below:

Pick any point on the figure as a centre of rotation and turn the figure \( \frac{1}{4} \) or \( \frac{1}{2} \) turn around the point. Then reflect the figure through any side.

Draw the initial and final positions of the figure on the grid.

Describe the transformations you used.
Geometry
Unit Test

Section C
15. Complete the following property chart:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Number of...</th>
<th>Pictures of Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>edges</td>
<td>vertices</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. Match the description of the figure with its name:

- _____ cone          A. I have 6 congruent faces.
- _____ triangular prism B. I have 5 faces: 2 triangles and 3 rectangles.
- _____ cube           C. I have 4 faces. Each face is a triangle.
- _____ cylinder       D. I have 2 circular bases and a curved face.
- _____ triangular pyramid E. I have 1 circular base and a curved face.
17. Compare the shapes below. Name the shapes first, and then write a paragraph outlining how they are the same and how they are different:

<table>
<thead>
<tr>
<th>Name</th>
<th>i –</th>
<th>ii –</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Geometry
Unit Test

Section C (continued)

b)

18. Draw two different ways to tessellate a region with rhombi.

19. Fill in the numbers in the mat plan.
Section A
1. a) 3
   2
   1
   1 2 3

   b) 3
      2
      1
      1 2 3

c) 3
   2
   1
   1 2 3

d) 3
   2
   1
   1 2 3

2. a) 3
   2
   1
   A B C

   b) C
      B
      A
      X Y Z

c) 2
   1
   0
   0 1 2

d) 2
   1
   0
   0 1 2

3. a) The polygon is a rectangle.

   b) The polygon is a square.

4. A (3, 2) B (9, 1)
   C (8, 4) D (6, 3)
   E (1, 1) F (4, 4)
   G (0, 5) H (5, 0)

Section B
5. NOTE:
   Location of dots may vary.
   a) 
      b) 

6. a) 
    b) 

7. a) 
    b) 
    c) 
    d) Yes. Explanations will vary.

8. Answers will vary.
   Teacher to check.

9. a) 
    b) 
    c) 
    d) 

10. a) 
    b) 
    c) 
    d) 

11. Answers may vary.
    Teacher to check.

12. The two shapes are not the same size and both shapes are facing the same direction (which is NOT a reflection).
    Exact answers may vary.
    Teacher to check.
13. a) Answers may vary. Teacher to check.
   Sample answer:
   Transformation #1: Rotation 90° counter – clockwise
   Transformation #2: Reflection in Line 2

b) Answers may vary. Teacher to check.
   Sample answer:
   Transformation #1: Slide down one unit
   Transformation #2: Reflection in Line 2

14. Answer will vary. Teacher to check.

15. Triangular Pyramid
   - edges: 6
   - vertices: 4
   - faces: 4

   Pentagonal Prism
   - edges: 15
   - vertices: 10
   - faces: 7

   Triangular Prism
   - edges: 9
   - vertices: 6
   - faces: 5

   Section C

17. a) Name:
   i) Triangular Pyramid
   ii) Triangular Prism

   Similarities:
   - both have a triangular base

   Differences:
   - i) has 1 base,
   - ii) has 2 bases
   - i) has 4 faces,
   - ii) has 5
   - i) has 6 edges,
   - ii) has 9
   - i) has 4 vertices,
   - ii) has 6
   - faces that are not bases are:
     - i) triangles
     - ii) rectangles

b) Name:
   i) Rectangular Pyramid
   ii) Triangular Pyramid

   Similarities:
   - pyramids
   - have 1 base
   - have a point opposite to base
   - faces that are not bases are triangles.

   Differences:
   - i) has rectangular base,
   - ii) has triangular bases
   - i) has 5 faces,
   - ii) has 4
   - i) has 8 edges,
   - ii) has 6
   - i) has 5 vertices,
   - ii) has 4
   - any face of ii) can be considered a base, not so for i).

18. Answers may vary. Teacher to check.

19. 2
    3
    1
    1
## Contents

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<tr>
<th>Topic</th>
<th>Page</th>
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<td>8</td>
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<tr>
<td>Patterning and Algebra</td>
<td>10</td>
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<tr>
<td>Data Management and Probability</td>
<td>12</td>
</tr>
</tbody>
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Ontario Curriculum Correlation: Grade 5

JUMP Math

MULTIPPLYING POTENTIAL.

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Notes

To ensure that the curriculum is fully covered, use the worksheets with the lessons plans in the Teacher’s Guide.

Starred lesson numbers (*) indicate that the curriculum requirement is covered primarily in the lesson plan (possibly in the activities or extensions).

OCUP: Ontario Curriculum Unit Planner

JUMP Math workbook units are represented by:

- **NS** Number Sense
- **PA** Patterns and Algebra
- **ME** Measurement
- **G** Geometry
- **PDM** Probability and Data Management
Number Sense and Numeration

Overall Expectations
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m8</td>
<td>read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;</td>
</tr>
<tr>
<td>5m9</td>
<td>demonstrate an understanding of magnitude by counting forward and backwards by 0.01;</td>
</tr>
<tr>
<td>5m10</td>
<td>solve problems involving the multiplication and division of multi-digit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies;</td>
</tr>
<tr>
<td>5m11</td>
<td>demonstrate an understanding of proportional reasoning by investigating whole-number rates.</td>
</tr>
</tbody>
</table>

Quantity Relationships
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>5m12</td>
<td>represent, compare, and order whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools;</td>
</tr>
<tr>
<td>5m13</td>
<td>demonstrate an understanding of place value in whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools and strategies;</td>
</tr>
<tr>
<td>5m14</td>
<td>read and print in words whole numbers to ten thousand, using meaningful contexts;</td>
</tr>
<tr>
<td>5m15</td>
<td>round decimal numbers to the nearest tenth, in problems arising from real-life situations;</td>
</tr>
<tr>
<td>5m16</td>
<td>represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation;</td>
</tr>
<tr>
<td>5m17</td>
<td>demonstrate and explain the concept of equivalent fractions, using concrete materials;</td>
</tr>
<tr>
<td>5m18</td>
<td>demonstrate and explain equivalent representations of a decimal number, using concrete materials and drawings;</td>
</tr>
<tr>
<td>5m19</td>
<td>read and write money amounts to $1000;</td>
</tr>
<tr>
<td>5m20</td>
<td>solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 100 000.</td>
</tr>
</tbody>
</table>
### Counting
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>5m21</td>
<td>count forward by hundredths from any decimal number expressed to two decimal places, using concrete materials and number lines.</td>
</tr>
</tbody>
</table>

### Operational Sense
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>5m22</td>
<td>solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies;</td>
</tr>
<tr>
<td>5m23</td>
<td>add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms;</td>
</tr>
<tr>
<td>5m24</td>
<td>multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms;</td>
</tr>
<tr>
<td>5m25</td>
<td>divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms;</td>
</tr>
<tr>
<td>5m26</td>
<td>multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies;</td>
</tr>
<tr>
<td>5m27</td>
<td>use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.</td>
</tr>
</tbody>
</table>
## Proportional Relationships

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>5m28</td>
<td>describe multiplicative relationships between quantities by using simple fractions and decimals;</td>
</tr>
<tr>
<td>5m29</td>
<td>determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms;</td>
</tr>
<tr>
<td>5m30</td>
<td>demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation using concrete materials and drawings.</td>
</tr>
</tbody>
</table>
# Measurement

## Overall Expectations

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Specific Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m31</td>
<td>estimate, measure, and record perimeter, area, temperature change, and elapsed time, using a variety of strategies;</td>
</tr>
<tr>
<td>5m32</td>
<td>determine the relationships among units and measurable attributes, including the area of a rectangle and the volume of a rectangular prism.</td>
</tr>
</tbody>
</table>

## Attributes, Units and Measurement Sense

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>5m33 estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest second;</td>
<td>1</td>
</tr>
<tr>
<td>5m34 estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in minutes, hours, days, weeks, months, or years;</td>
<td>1</td>
</tr>
<tr>
<td>5m35 measure and record temperatures to determine and represent temperature changes over time;</td>
<td>1</td>
</tr>
<tr>
<td>5m36 estimate and measure the perimeter and area of regular and irregular polygons, using a variety of tools and strategies.</td>
<td>2</td>
</tr>
</tbody>
</table>

## Measurement Relationships

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>5m37 select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure length, height, width, and distance, and to measure the perimeter of various polygons;</td>
<td>2</td>
</tr>
<tr>
<td>5m38 solve problems requiring conversion from metres to centimetres and from kilometers to metres;</td>
<td>2</td>
</tr>
<tr>
<td>5m39 solve problems involving the relationship between a 12-hour clock and a 24-hour clock;</td>
<td>1</td>
</tr>
<tr>
<td>5m40 create, through investigation using a variety of tools and strategies, two-dimensional shapes with the same perimeter or the same area;</td>
<td>2</td>
</tr>
</tbody>
</table>
## Measurement Relationships (continued)

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>5m41</td>
<td>determine, through investigation using a variety of tools and strategies, the relationships between the length and width of a rectangle and its area and perimeter, and generalize to develop the formulas [i.e., Area = length \times width; Perimeter = (2 \times length) + (2 \times width)]:</td>
</tr>
<tr>
<td>5m42</td>
<td>solve problems requiring the estimation and calculation of perimeters and areas of rectangles;</td>
</tr>
<tr>
<td>5m43</td>
<td>determine, through investigation, the relationship between capacity (i.e., the amount a container can hold) and volume (i.e., the amount of space taken up by an object), by comparing the volume of an object with the amount of liquid it can contain or displace;</td>
</tr>
<tr>
<td>5m44</td>
<td>determine, through investigation using stacked congruent rectangular layers of concrete materials, the relationship between the height, the area of the base, and the volume of a rectangular prism, and generalize to develop the formula [i.e., Volume = area of base \times height]:</td>
</tr>
<tr>
<td>5m45</td>
<td>select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram, tonne).</td>
</tr>
</tbody>
</table>
## Geometry and Spatial Sense

### Overall Expectations
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m46</td>
<td>identify and classify two-dimensional shapes by side and angle properties, and compare and sort three-dimensional figures;</td>
</tr>
<tr>
<td>5m47</td>
<td>identify and construct nets of prisms and pyramids;</td>
</tr>
<tr>
<td>5m48</td>
<td>identify and describe the location of an object, using the cardinal directions, and translate two-dimensional shapes.</td>
</tr>
</tbody>
</table>

### Geometric Properties
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>5m49</td>
<td>distinguish among polygons, regular polygons, and other two-dimensional shapes;</td>
</tr>
<tr>
<td>5m50</td>
<td>distinguish among prisms, right prisms, pyramids, and other three-dimensional figures;</td>
</tr>
<tr>
<td>5m51</td>
<td>identify and classify acute, right, obtuse, and straight angles;</td>
</tr>
<tr>
<td>5m52</td>
<td>measure and construct angles up to 90°, using a protractor;</td>
</tr>
<tr>
<td>5m53</td>
<td>identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral), and classify them according to angle and side properties;</td>
</tr>
<tr>
<td>5m54</td>
<td>construct triangles, using a variety of tools given acute or right angles and side measurements.</td>
</tr>
</tbody>
</table>

### Geometric Relationships
By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>5m55</td>
<td>identify prisms and pyramids from their nets;</td>
</tr>
<tr>
<td>5m56</td>
<td>construct nets of prisms and pyramids, using a variety of tools.</td>
</tr>
</tbody>
</table>
Location and Movement

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>5m57</td>
<td>locate an object using the cardinal directions (i.e., north, south, east, west) and a coordinate system;</td>
</tr>
<tr>
<td>5m58</td>
<td>compare grid systems commonly used on maps (i.e., the use of numbers and letters to identify an area; the use of a coordinate system based on the cardinal directions to describe a specific location);</td>
</tr>
<tr>
<td>5m59</td>
<td>identify, perform, and describe translations, using a variety of tools;</td>
</tr>
<tr>
<td>5m60</td>
<td>create and analyse designs by translating and/or reflecting a shape, or shapes, using a variety of tools.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Patterning and Algebra

## Overall Expectations

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m61</td>
<td>determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations;</td>
</tr>
<tr>
<td>5m62</td>
<td>demonstrate, through investigation, an understanding of the use of variables in equations.</td>
</tr>
</tbody>
</table>

## Patterns and Relationships

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>5m63</td>
<td>create, identify, and extend numeric and geometric patterns, using a variety of tools;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5m64</td>
<td>build a model to represent a number pattern presented in a table of values that shows the term number and the term;</td>
</tr>
<tr>
<td>5m65</td>
<td>make a table of values for a pattern that is generated by adding or subtracting a number (i.e., a constant) to get the next term, or by multiplying or dividing by a constant to get the next term, given either the sequence or the pattern rule in words;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5m66</td>
<td>make predictions related to growing and shrinking geometric and numeric patterns;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5m67</td>
<td>extend and create repeating patterns that result from translations, through investigation using a variety of tools.</td>
</tr>
</tbody>
</table>
Variables, Expressions, and Equations

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part Unit Lesson</td>
</tr>
<tr>
<td>5m68 demonstrate, through investigation, an understanding of variables as changing quantities, given equations with letters or other symbols that describe relationships involving simple rates;</td>
<td>2 PA 24, 26, 27, 38</td>
</tr>
<tr>
<td>5m69 demonstrate, through investigation, an understanding of variables as unknown quantities represented by a letter or other symbol;</td>
<td>2 PA 35–38</td>
</tr>
<tr>
<td>5m70 determine the missing number in equations involving addition, subtraction, multiplication, or division and one- or two-digit numbers, using a variety of tools and strategies.</td>
<td>2 PA 35, 36, 38</td>
</tr>
</tbody>
</table>
# Data Management and Probability

## Overall Expectations

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m71</td>
<td>collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including broken-line graphs;</td>
</tr>
<tr>
<td>5m72</td>
<td>read, describe, and interpret primary data and secondary data presented in charts and graphs, including broken-line graphs;</td>
</tr>
<tr>
<td>5m73</td>
<td>represent as a fraction the probability that a specific outcome will occur in a simple probability experiment, using systematic lists and area models.</td>
</tr>
</tbody>
</table>

## Collection and Organization of Data

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------</td>
</tr>
<tr>
<td>5m74</td>
<td>distinguish between discrete data (i.e., data organized using numbers that have gaps between them, such as whole numbers, and often used to represent a count, such as the number of times a word is used) and continuous data (i.e., data organized using all numbers on a number line that fall within the range of the data, and used to represent measurements such as heights or ages of trees);</td>
</tr>
<tr>
<td>5m75</td>
<td>collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;</td>
</tr>
<tr>
<td>5m76</td>
<td>collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including broken-line graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools;</td>
</tr>
<tr>
<td>5m77</td>
<td>demonstrate an understanding that sets of data can be samples of larger populations;</td>
</tr>
<tr>
<td>5m78</td>
<td>describe, through investigation, how a set of data is collected and explain whether the collection method is appropriate.</td>
</tr>
</tbody>
</table>
### Data Relationships

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5m79</strong> read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including broken-line graphs);</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td><strong>5m80</strong> calculate the mean for a small set of data and use it to describe the shape of the data set across its range of values, using charts, tables, and graphs;</td>
<td>2</td>
</tr>
<tr>
<td><strong>5m81</strong> compare similarities and differences between two related sets of data, using a variety of strategies.</td>
<td>2</td>
</tr>
</tbody>
</table>

### Probability

By the end of Grade 5, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5m82</strong> determine and represent all the possible outcomes in a simple probability experiment; using systematic lists and area models;</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td><strong>5m83</strong> represent, using a common fraction, the probability that an event will occur in simple games and probability experiments;</td>
<td>2</td>
</tr>
<tr>
<td><strong>5m84</strong> pose and solve simple probability problems, and solve them by conducting probability experiments and selecting appropriate methods of recording the results.</td>
<td>2</td>
</tr>
</tbody>
</table>
## Contents

<table>
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<td>Patterns and Relations</td>
<td>9</td>
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<tr>
<td>Shape and Space</td>
<td>11</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>16</td>
</tr>
</tbody>
</table>
Notes

To ensure that the curriculum is fully covered, use the worksheets with the lessons plans in the Teacher’s Guide.

Starred lesson numbers (*) indicate that the curriculum requirement is covered primarily in the lesson plan (possibly in the activities or extensions).

Underlined lesson numbers indicate relevant preparatory exercises.

WNCP Abbreviations:

[C] Communication
[CN] Connections
[ME] Mental Mathematics and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

JUMP Math workbook units are represented by:

NS  Number Sense
PA Patterns and Algebra
ME Measurement
G  Geometry
PDM Probability and Data Management
## General Outcome

- Develop number sense.

## Develop Number Sense

It is expected that students will:

### 1. WNCP CURRICULUM

**Specific Outcome**

Represent and describe whole numbers to 1 000 000.

<table>
<thead>
<tr>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Write a given numeral using proper spacing without commas, e.g., 934 567.
- Describe the patterns of adjacent place positions moving from right to left.
- Describe the meaning of each digit in a given numeral.
- Provide examples of large numbers used in print or electronic media.
- Express a given numeral in expanded notation, e.g., 45 321 = (4 × 10 000) + (5 × 1 000) + (3 × 100) + (2 × 10) + (1 × 1) or 40 000 + 5 000 + 300 + 20 + 1.
- Write the numeral represented by a given expanded notation.

### 2. WNCP CURRICULUM

**Specific Outcome**

Use estimation strategies including:
- front-end rounding
- compensation
- compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]

<table>
<thead>
<tr>
<th>JUMP MATH LESSONS</th>
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</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
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<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Provide a context for when estimation is used to:
  - make predictions
  - check reasonableness of an answer
  - determine approximate answers.
- Describe contexts in which overestimating is important.
- Determine the approximate solution to a given problem not requiring an exact answer.
2. **Achievement Indicators**

Estimate a sum or product using compatible numbers.

Estimate the solution to a given problem using compensation and explain the reason for compensation.

Select and use an estimation strategy for a given problem.

Apply front-end rounding to estimate:
- sums, e.g., 253 + 615 is more than 200 + 600 = 800
- differences, e.g., 974 – 250 is close to 900 – 200 = 700
- products, e.g., the product of 23 × 24 is greater than 20 × 20 (400) and less than 25 × 25 (625)
- quotients, e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 (200).

### WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apply mental mathematics strategies and number properties, such as:</td>
<td>Part</td>
</tr>
<tr>
<td>• skip counting from a known fact</td>
<td>1</td>
</tr>
<tr>
<td>• using doubling or halving</td>
<td></td>
</tr>
<tr>
<td>• using patterns in the 9s facts</td>
<td></td>
</tr>
<tr>
<td>• using repeated doubling or halving to determine answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V]</td>
<td></td>
</tr>
</tbody>
</table>

### Achievement Indicators

Describe the mental mathematics strategy used to determine a given basic fact, such as:

- skip count up by one or two groups from a known fact, e.g., if 5 × 7 = 35, then 6 × 7 is equal to 35 + 7 and 7 × 7 is equal to 35 + 7 + 7
- skip count down by one or two groups from a known fact, e.g., if 8 × 8 = 64, then 7 × 8 is equal to 64 – 8 and 6 × 8 is equal to 64 – 8 – 8
- doubling, e.g., for 8 × 3 think 4 × 3 = 12, and 8 × 3 = 12 + 12
- patterns when multiplying by 9, e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63
- repeated doubling, e.g., if 2 × 6 is equal to 12, then 4 × 6 is equal to 24 and 8 × 6 is equal to 48
- repeated halving, e.g., for 60 ÷ 4, think 60 ÷ 2 = 30 and 30 ÷ 2 = 15.

Explain why multiplying by zero produces a product of zero.

Explain why division by zero is not possible or undefined, e.g., 8 ÷ 0.

Recall multiplication facts to 81 and related division facts.
### 4. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td><strong>Apply mental mathematics strategies for multiplication, such as:</strong></td>
<td>1</td>
</tr>
<tr>
<td>• annexing then adding zero</td>
<td></td>
</tr>
<tr>
<td>• halving and doubling</td>
<td></td>
</tr>
<tr>
<td>• using the distributive property. [C, ME, R]</td>
<td></td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Determine the products when one factor is a multiple of 10, 100 or 1000 by annexing zero or adding zeros, e.g., for 3 × 200 think 3 × 2 and then add two zeros.
- Apply halving and doubling when determining a given product, e.g., 32 × 5 is the same as 16 × 10.
- Apply the distributive property to determine a given product involving multiplying factors that are close to multiples of 10, e.g., 98 × 7 = (100 × 7) − (2 × 7).

### 5. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td><strong>Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems. [C, CN, PS, V]</strong></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Illustrate partial products in expanded notation for both factors, e.g., for 36 × 42, determine the partial products for (30 + 6) × (40 + 2).
- Represent both 2-digit factors in expanded notation to illustrate the distributive property, e.g., to determine the partial products of 36 × 42: (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512.
- Model the steps for multiplying 2-digit factors using an array and base ten blocks, and record the process symbolically.
- Describe a solution procedure for determining the product of two given 2-digit factors using a pictorial representation, such as an area model.
- Solve a given multiplication problem in context using personal strategies and record the process.
### 6. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems. [C, CN, PS]</td>
<td>Part  Unit  Lesson</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Model the division process as equal sharing using base ten blocks and record it symbolically.</td>
<td></td>
</tr>
<tr>
<td>Explain that the interpretation of a remainder depends on the context:</td>
<td></td>
</tr>
<tr>
<td>• ignore the remainder, e.g., making teams of 4 from 22 people</td>
<td></td>
</tr>
<tr>
<td>• round up the quotient, e.g., the number of five passenger cars required to transport 13 people</td>
<td></td>
</tr>
<tr>
<td>• express remainders as fractions, e.g., five apples shared by two people</td>
<td></td>
</tr>
<tr>
<td>• express remainders as decimals, e.g., measurement and money.</td>
<td></td>
</tr>
<tr>
<td>Solve a given division problem in context using personal strategies and record the process.</td>
<td></td>
</tr>
</tbody>
</table>

### 7. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of fractions by using concrete and pictorial representations to:</td>
<td>Part  Unit  Lesson</td>
</tr>
<tr>
<td>• create sets of equivalent fractions</td>
<td>2  NS  61–63, 64, 65, 66, 67–72, 75, 76, 78, 86–88</td>
</tr>
<tr>
<td>• compare fractions with like and unlike denominators. [C, CN, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Create a set of equivalent fractions and explain why there are many equivalent fractions for any given fraction using concrete materials.</td>
<td></td>
</tr>
<tr>
<td>Model and explain that equivalent fractions represent the same quantity.</td>
<td></td>
</tr>
<tr>
<td>Determine if two given fractions are equivalent using concrete materials or pictorial representations.</td>
<td></td>
</tr>
<tr>
<td>Formulate and verify a rule for developing a set of equivalent fractions.</td>
<td></td>
</tr>
<tr>
<td>Identify equivalent fractions for a given fraction.</td>
<td></td>
</tr>
<tr>
<td>Compare two given fractions with unlike denominators by creating equivalent fractions.</td>
<td></td>
</tr>
<tr>
<td>Position a given set of fractions with like and unlike denominators on a number line and explain strategies used to determine the order.</td>
<td></td>
</tr>
</tbody>
</table>
8. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically. [C, CN, R, V]</td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
- Represent an equivalent tenth, hundredth or thousandth for a given decimal using a grid.
- Express a given tenth as an equivalent hundredth and thousandth.
- Express a given hundredth as an equivalent thousandth.
- Describe the value of each digit in a given decimal.

9. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relate decimals to fractions (to thousandths). [CN, R, V]</td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Write a given decimal in fractional form.
- Write a given fraction with a denominator of 10, 100 or 1000 as a decimal.
- Express a given pictorial or concrete representation as a fraction or decimal, e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or 250/1000.

10. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare and order decimals (to thousandths) by using:</td>
<td>2</td>
</tr>
<tr>
<td>• benchmarks</td>
<td>NS</td>
</tr>
<tr>
<td>• place value</td>
<td>83, 86–88, 98–101</td>
</tr>
<tr>
<td>• equivalent decimals. [CN, R, V]</td>
<td></td>
</tr>
</tbody>
</table>
10. **Achievement Indicators**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order a given set of decimals by placing them on a number line that contains benchmarks, 0.0, 0.5, 1.0.</td>
<td></td>
</tr>
<tr>
<td>Order a given set of decimals including only tenths using place value.</td>
<td></td>
</tr>
<tr>
<td>Order a given set of decimals including only hundredths using place value.</td>
<td></td>
</tr>
<tr>
<td>Order a given set of decimals including only thousandths using place value.</td>
<td></td>
</tr>
<tr>
<td>Explain what is the same and what is different about 0.2, 0.20 and 0.200.</td>
<td></td>
</tr>
<tr>
<td>Order a given set of decimals including tenths, hundredths and thousandths using equivalent decimals.</td>
<td></td>
</tr>
</tbody>
</table>

11. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). [C, CN, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td>Place the decimal point in a sum or difference using front-end estimation, e.g., 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312.</td>
<td></td>
</tr>
<tr>
<td>Correct errors of decimal point placements in sums and differences without using paper and pencil.</td>
<td></td>
</tr>
<tr>
<td>Explain why keeping track of place value positions is important when adding and subtracting decimals.</td>
<td></td>
</tr>
<tr>
<td>Predict sums and differences of decimals using estimation strategies.</td>
<td></td>
</tr>
<tr>
<td>Solve a given problem that involves addition and subtraction of decimals, limited to thousandths.</td>
<td></td>
</tr>
</tbody>
</table>
Patterns and Relations

General Outcomes

• Patterns: Use patterns to describe the world and solve problems.

• Variables and Equations: Represent algebraic expressions in multiple ways.

Patterns

It is expected that students will:

1. WNPC CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Achievement Indicators

- Extend a given pattern with and without concrete materials, and explain how each element differs from the proceeding one.
- Describe, orally or in writing, a given pattern using mathematical language, such as one more, one less, five more.
- Write a mathematical expression to represent a given pattern, such as $r + 1$, $r - 1$, $r + 5$.
- Describe the relationship in a given table or chart using a mathematical expression.
- Determine and explain why a given number is or is not the next element in a pattern.
- Predict subsequent elements in a given pattern.
- Solve a given problem by using a pattern rule to determine subsequent elements.
- Represent a given pattern visually to verify predictions.

Variables and Equations

It is expected that students will:

2. WNPC CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]</td>
<td>2</td>
</tr>
</tbody>
</table>
2. **Achievement Indicators**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Express a given problem in context as an equation where the unknown is represented by a letter variable.</td>
<td>( n + 2 = 5 ), ( 4 + a = 7 ), ( 6 = r - 2 ), ( 10 = 2c ).</td>
</tr>
<tr>
<td>Solve a given single-variable equation with the unknown in any of the terms, e.g., ( n + 2 = 5 ), ( 4 + a = 7 ), ( 6 = r - 2 ), ( 10 = 2c ).</td>
<td></td>
</tr>
<tr>
<td>Create a problem in context for a given equation.</td>
<td></td>
</tr>
</tbody>
</table>
Shape and Space

General Outcomes

- Measurement: Use direct or indirect measurement to solve problems.
- 3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations: Describe and analyze position and motion.

Measurement

It is expected that students will:

1. **WNCP CURRICULUM**
   
<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design and construct different rectangles given either perimeter or area, or both (whole numbers) and draw conclusions. [C, CN, PS, R, V]</td>
<td>2 ME 19–21, 23–25, 29–32</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   **Achievement Indicators**
   
   - Construct or draw two or more rectangles for a given perimeter in a problem-solving context.
   - Construct or draw two or more rectangles for a given area in a problem-solving context.
   - Illustrate that for any given perimeter, the square or shape closest to a square will result in the greatest area.
   - Illustrate that for any given perimeter, the rectangle with the smallest possible width will result in the least area.
   - Provide a real-life context for when it is important to consider the relationship between area and perimeter.

2. **WNCP CURRICULUM**
   
<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of measuring length (mm) by:</td>
<td>2 ME 8, 9, 11, 13, 15–17</td>
</tr>
<tr>
<td>• selecting and justifying referents for the unit mm</td>
<td></td>
</tr>
<tr>
<td>• modelling and describing the relationship between mm and cm units, and between mm and m units. [C, CN, ME, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   **Achievement Indicators**
   
   - Provide a referent for one millimetre and explain the choice.
   - Provide a referent for one centimetre and explain the choice.
   - Provide a referent for one metre and explain the choice.
### Achievement Indicators

Show that 10 millimetres is equivalent to 1 centimetre using concrete materials, e.g., ruler.

Show that 1000 millimetres is equivalent to 1 metre using concrete materials, e.g., metre stick.

Provide examples of when millimetres are used as the unit of measure.

### WNCP CURRICULUM

#### Specific Outcome

Demonstrate an understanding of volume by:
- selecting and justifying referents for cm\(^3\) or m\(^3\) units
- estimating volume by using referents for cm\(^3\) or m\(^3\)
- measuring and recording volume (cm\(^3\) or m\(^3\))
- constructing rectangular prisms for a given volume. [C, CN, ME, PS, R, V]

**Achievement Indicators**

Identify the cube as the most efficient unit for measuring volume and explain why.

Provide a referent for a cubic centimetre and explain the choice.

Provide a referent for a cubic metre and explain the choice.

Determine which standard cubic unit is represented by a given referent.

Estimate the volume of a given 3-D object using personal referents.

Determine the volume of a given 3-D object using manipulatives and explain the strategy.

Construct a rectangular prism for a given volume.

Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same given volume.

### WNCP CURRICULUM

#### Specific Outcome

Demonstrate an understanding of capacity by:
- describing the relationship between mL and L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity (mL or L). [C, CN, ME, PS, R, V]

**Achievement Indicators**

Show that 10 millimetres is equivalent to 1 centimetre using concrete materials, e.g., ruler.

Show that 1000 millimetres is equivalent to 1 metre using concrete materials, e.g., metre stick.

Provide examples of when millimetres are used as the unit of measure.
4. **Achievement Indicators**

Demonstrate that 1000 millilitres is equivalent to 1 litre by filling a 1 litre container using a combination of smaller containers.

Provide a referent for a litre and explain the choice.

Provide a referent for a millilitre and explain the choice.

Determine which capacity unit is represented by a given referent.

Estimate the capacity of a given container using personal referents.

Determine the capacity of a given container using materials that take the shape of the inside of the container, e.g., a liquid, rice, sand, beads, and explain the strategy.

---

3-D Objects and 2-D Shapes

It is expected that students will:

5. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are:</td>
<td>Part</td>
</tr>
<tr>
<td>• parallel</td>
<td>1</td>
</tr>
<tr>
<td>• intersecting</td>
<td></td>
</tr>
<tr>
<td>• perpendicular</td>
<td>2</td>
</tr>
<tr>
<td>• vertical</td>
<td></td>
</tr>
<tr>
<td>• horizontal. [C, CN, R, T, V]</td>
<td></td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Identify parallel, intersecting, perpendicular, vertical and horizontal edges and faces on 3-D objects.

Identify parallel, intersecting, perpendicular, vertical and horizontal sides on 2-D shapes.

Provide examples from the environment that show parallel, intersecting, perpendicular, vertical and horizontal line segments.

Find examples of edges, faces and sides that are parallel, intersecting, perpendicular, vertical and horizontal in print and electronic media, such as newspapers, magazines and the Internet.

Draw 2-D shapes or 3-D objects that have edges, faces and sides that are parallel, intersecting, perpendicular, vertical or horizontal.
5. **Achievement Indicators**

Describe the faces and edges of a given 3-D object using terms, such as parallel, intersecting, perpendicular, vertical or horizontal.

Describe the sides of a given 2-D shape using terms, such as parallel, intersecting, perpendicular, vertical or horizontal.

6. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify and sort quadrilaterals, including:</td>
<td>1</td>
<td>G</td>
<td>1, 2, 8, 9, 10, 13, 14–17</td>
</tr>
<tr>
<td>• rectangles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• squares</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• trapezoids</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• parallelograms</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• rhombuses according to their attributes. [C, R, V]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Identify and describe the characteristics of a pre-sorted set of quadrilaterals.

Sort a given set of quadrilaterals and explain the sorting rule.

Sort a given set of quadrilaterals according to the lengths of the sides.

Sort a given set of quadrilaterals according to whether or not opposite sides are parallel.

---

**Transformations**

It is expected that students will:

7. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. [C, CN, T, V]</td>
<td>2</td>
<td>G</td>
<td>20–29, 38–40, 44</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Translate a given 2-D shape horizontally, vertically or diagonally, and describe the position and orientation of the image.

Rotate a given 2-D shape about a point, and describe the position and orientation of the image.
### 7. Achievement Indicators

- Reflect a given 2-D shape in a line of reflection, and describe the position and orientation of the image.
- Perform a transformation of a given 2-D shape by following instructions.
- Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement.
- Draw a 2-D shape, rotate the shape and describe the direction of the turn (clockwise or counterclockwise), the fraction of the turn and point of rotation.
- Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.
- Predict the result of a single transformation of a 2-D shape and verify the prediction.

### 8. WNCP Curriculum | JUMP MATH Lessons

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify a single transformation, including a translation, rotation and reflection of 2-D shapes. [C, T, V]</td>
<td>1, 2</td>
<td>G, G</td>
<td>11, 13, 18, 19, 20–29, 39, 40, 43, 44</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Provide an example of a translation, a rotation and a reflection.
- Identify a given single transformation as a translation, rotation or reflection.
- Describe a given rotation by the direction of the turn (clockwise or counterclockwise).
# Statistics and Probability

## General Outcomes

- **Data Analysis:** Collect, display and analyze data to solve problems.
- **Chance and Uncertainty:** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

## Data Analysis

It is expected that students will:

1. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Differentiate between first-hand and second-hand data.</td>
<td>Part</td>
</tr>
<tr>
<td>1 PDM</td>
<td>1, 9–12</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Explain the difference between first-hand and second-hand data.
- Formulate a question that can best be answered using first-hand data and explain why.
- Formulate a question that can best be answered using second-hand data and explain why.
- Find examples of second-hand data in print and electronic media, such as newspapers, magazines and the Internet.

2. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct and interpret double bar graphs to draw conclusions.</td>
<td>Part</td>
</tr>
<tr>
<td>1 PDM</td>
<td>1, 3, 4, 11</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Determine the attributes (title, axes, intervals and legend) of double bar graphs by comparing a given set of double bar graphs.
- Represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology.
- Draw conclusions from a given double bar graph to answer questions.
- Provide examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines and the Internet.
- Solve a given problem by constructing and interpreting a double bar graph.
### Chance and Uncertainty

It is expected that students will:

<table>
<thead>
<tr>
<th>Specific Outcome</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>3.</td>
<td></td>
</tr>
<tr>
<td><strong>WNCP CURRICULUM</strong></td>
<td>2</td>
</tr>
<tr>
<td><strong>JUMP MATH LESSONS</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Describe the likelihood of a single outcome occurring using words such as:</td>
<td></td>
</tr>
<tr>
<td>• impossible</td>
<td></td>
</tr>
<tr>
<td>• possible</td>
<td></td>
</tr>
<tr>
<td>• certain</td>
<td></td>
</tr>
</tbody>
</table>

### Chance and Uncertainty

It is expected that students will:

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>4.</td>
<td></td>
</tr>
<tr>
<td><strong>WNCP CURRICULUM</strong></td>
<td>2</td>
</tr>
<tr>
<td><strong>JUMP MATH LESSONS</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Provide examples of events that are impossible, possible or certain from personal contexts.</td>
<td></td>
</tr>
<tr>
<td>Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible or certain.</td>
<td></td>
</tr>
<tr>
<td>Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible or certain.</td>
<td></td>
</tr>
<tr>
<td>Conduct a given probability experiment a number of times, record the outcomes and explain the results.</td>
<td></td>
</tr>
<tr>
<td>Compare the likelihood of two possible outcomes occurring using words such as:</td>
<td></td>
</tr>
<tr>
<td>• less likely</td>
<td></td>
</tr>
<tr>
<td>• equally likely</td>
<td></td>
</tr>
<tr>
<td>• more likely</td>
<td></td>
</tr>
</tbody>
</table>

Identify outcomes from a given probability experiment which are less likely, equally likely or more likely to occur than other outcomes.

Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.

Design and conduct a probability experiment in which one outcome is equally as likely to occur as the other outcome.

Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.