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Patterns with Increasing & Decreasing Steps—Part I

Let your students play a game in pairs. They will need a pair of dice and a spinner from Activity 1 of PA6-15: Finding Rules for T-tables. Player 1 rolls the dice and spins the spinner so that Player 2 does not see the result of the spinner. He adds, subtracts or multiplies the numbers rolled according to the instructions on the spinner, and presents both numbers and the result of the operation to Player 2. Player 2 has to tell what the spinner read. For example, if the dice give 2 and 6, and the spinner reads “Subtract”, Player 1 has to write: 6 - 2 = 4, and Player 2 has to deduce that the sign is “-”.

After that you may write several sequences made by multiplication and ask your students to continue them. In the beginning tell them what the factor is:

a) $5 \times 2$, 10, ___, ___

b) $2 \times 4$, 8, ___, ___

c) $3 \times 3$, 9, ___, ___

Then tell your students that you are going to make the task harder: they will have to find the factor. Let them practice with questions like:

a) $\times 2$, 15, 30, 60, ___, ___

b) $\times 4$, 2, 10, 50, ___, ___

c) $\times 3$, 3, 12, 48, ___, ___

Explain to your students that since they are doing very well, you are sure they will succeed in the next task. You will give them several sequences made using one operation: addition, multiplication or subtraction. You will not tell them which operation was used, what number you added or subtracted from the term, or what number you multiplied the term by. They will have to continue the sequence and write a rule for each sequence. Remind your students of the way they write the rules for sequences: Start at ___ and ____. Discuss with the class what strategies can be used to decide which operation and numbers were used. **ASK**: If the sequence was made by addition, is it increasing or decreasing? By subtraction? By multiplication? If the first two numbers in the sequence are 2 and 4, how can I tell whether the sequence was made by addition or multiplication? What should I check? The students might reason as follows:

Look at the difference between the first two numbers. Check if the sequence is increasing or decreasing. If it is decreasing, subtraction was used and I have to check the difference between the numbers. If the sequence is increasing, it is either addition or multiplication, so I should look at the difference between the first and the second terms, then at the
difference between the second and the third terms. If the differences are the same, the sequence was made by addition. If not, it was made by multiplication.

a) 3, 6, 9, __, __

b) 3, 6, 12, __, __

c) 28, 24, 20, __, __

d) 105, 98, 92, __, __

e) 105, 98, 92, __, __

f) 5, 15, 45, __, __

g) 5, 15, 25, __, __

Bonus

h) 243, 81, 27, __, __

Assessment

Continue the sequences and write the rule for each sequence.

a) 15, 30, 45, __, __

b) 52, 39, 26, __, __

c) 3, 17, 31, __, __

Extensions

1. The patterns below were made by multiplying successive terms by a fixed number and then adding or subtracting a fixed number. Find the missing terms and state the rule for making the pattern. Include the word term in your answer. (For instance, the rule for the first pattern below is “Start at 1. Multiply each term by 2 and add 1.”)

a) 1, 3, 7, 15, 31, ___

b) 1, 4, 13, 40, ___

c) 2, 7, 22, 67, ___

d) 2, 3, 5, 9, ___

e) 1, 2, 5, 14, ___

2. Put a dot in the centre of a polygon and draw a line from the centre to each vertex of the polygon. How many line segments are there in each figure? Predict how many line segments there would be in a hexagon and an octagon. Test your prediction.
PA6-23
Patterns with Increasing & Decreasing Steps—Part II

GOALS
Students will extend increasing and decreasing sequences with increasing and decreasing differences.

PRIOR KNOWLEDGE REQUIRED
Increasing and decreasing sequences T-tables

VOCABULARY
increasing sequence decreasing sequence difference

Draw the following pattern on the board or build it with blocks:

Ask a volunteer to build the next term of the sequence. Ask your students to fill the T-table. Then ASK: How many blocks are added each time? What are you adding to the structure? Another row. How many blocks are in the first row that you added? In the second row? This is the difference in the total number of blocks at each stage. Ask a volunteer to write the difference in the circles beside the table.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students if they can see a pattern in the differences. Ask a volunteer to add a term to the pattern of differences. After that ask another volunteer to fill in the next row of the table. Ask another volunteer to build another figure to check the result.

Give your students several questions to practice:

Find the differences between the terms of the sequences. Extend the sequence of the differences and then extend the sequence itself.

a) 5, 8, 12, 17, ___, ___
   
   b) 3, 5, 11, 17, 25, ___, ___
   
   c) 11, 15, 23, 39, ___, ___
   
   d) 6, 8, 13, 21, 32, ___, ___

Let them also practice with decreasing sequences:

a) 65, 64, 62, 59, ___, ___
   
   b) 73, 70, 64, 55, 43, ___, ___

Show the following geometrical pattern and ask how many triangles will be in the next design:

△ △ △
Draw the T-table for the pattern. How many triangles do you add each time? We add a new row that is two triangles longer than the previous one. The new row is the difference. If your students have problems extending the sequence of differences, you can draw a second row of circles beside the first one to see the differences between the circles. Ask the volunteers to extend first the sequence of differences, then the sequence itself.

**Assessment**

a) 13, 16, 22, 31, 43, __, __

b) 13, 17, 24, 34, __, __

c) 101, 95, 87, 77, __, __

d) 188, 160, 136, 116, __, __

**Extensions**

1. Janet is training for a Marathon run. On Monday she ran 5 km. Every day she ran 1 km more than on the previous day. How many kilometres did she run in the whole week?

   Draw the T-table. The first few entries should appear as follows:

<table>
<thead>
<tr>
<th>Day</th>
<th>Total km Run from the Beginning of the Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Monday</td>
<td>5</td>
</tr>
<tr>
<td>2. Tuesday</td>
<td>11</td>
</tr>
<tr>
<td>3. Wednesday</td>
<td>18</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

   Each day Janet runs 1 km more.
2. The pattern that is created in QUESTION 6 b) of the worksheet is called The Sierpinski Triangle. If you take a part of this pattern and magnify it, the picture will look exactly the same as the original. A picture with this property is called a fractal. Here is another example of a fractal, called the H-fractal.

Count the line segments at each stage of the construction. Make a T-table and try to predict the total number of line segments at the 7th step.

**STEP 1:** Draw a horizontal line of 12 cm.

**STEP 2:** Draw two vertical lines of 8 cm each, as shown.

**STEP 3:** Draw four horizontal lines of length half of the previous horizontal line (6 cm here), so that the middle of each segment is at the end of the previous vertical segments.

**STEP 4:** Draw eight vertical lines of length half of the previous vertical line (4 cm here), so that the middle of each segment is at the end of the previous horizontal segments.

After 10 steps the picture will look as shown below.

PA6-24
Advanced Patterns

Review the previous lesson.

Write two rules on the board:

“Multiply the Term Number by 2 and add 7” and
“Start at 9 and add 2.”

Ask your students to write a sequence of numbers according to each rule. Did they get the same sequence? **ASK:** Suppose you know that the number 61 appears somewhere in the sequence. Which rule would you use to continue the sequence? Which rule would you use to find the 100th term of the sequence? Which rule would you use to find what is the term number of 61?

Explain to your students that the first type of rule is called **general** rule, it tells how to calculate any term from the term number. The second type of the rule is called **stepwise** rule, it tells you the first term of the sequence and what you need to do to a term in the sequence to get the next term.

If you taught your students about Pascal’s triangle (Extension 5 of PA6-14, ask them to find triangular numbers (**QUESTION 2 a**) of worksheet PA6-24) in Pascal’s triangle. (**ANSWER:** The triangular numbers are on the third diagonal of Pascal’s triangle.) Ask your students to write out the first 10 triangular and the first 10 square numbers. Ask them to find sum of several pairs of adjacent triangular numbers. What do they notice? (The sum of two consecutive triangular numbers is a square number.) Why? Students could use the picture below for explanation.

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**GOALS**
Students will write recursive rules for patterns with varying gaps.

**PRIOR KNOWLEDGE REQUIRED**
- T-tables
- Making rules for T-tables
- Increasing and decreasing sequences

**VOCABULARY**
- T-table
- increasing sequence
- decreasing sequence
- general rule
- stepwise rule
A Game in Pairs

Each pair of students will need a spinner as shown below:

Player 1 spins the spinner so that Player 2 does not see the result. Player 1 has to write the sequence of numbers with the gap that behaves as shown on the spinner. The increase or the decrease in the gap should be constant. Player 2 has to say what the spinner read, to continue the pattern for 3 more terms and to write the stepwise rule for the pattern and the general rule for the gap. For example, Player 1 writes the pattern 3, 5, 9, 15, … Player 2 has to say that the gap increases, the pattern is “Start at 3 and add 2, 4, 6, …” and the pattern in the gap is “Multiply the gap number by 2”. Player 2 also writes the next three terms of the sequence, which are 23, 33 and 45.

Extension

The patterns below were made by multiplying successive terms by a fixed number and then adding or subtracting a fixed number. Find the missing terms and state the rule for making the pattern. Include the word term in your answer. (For instance, the rule for the first pattern below is “Start at 1. Multiply each term by 2 and add 1.”)

a) 1, 3, 7, 15, 31, ____
b) 1, 4, 13, 40, ____
c) 2, 7, 22, 67, ____
d) 2, 3, 5, 9, ____
e) 1, 2, 5, 14, ____
PA6-25
Creating and Extending Patterns (Advanced)

GOALS
Students will extend increasing and decreasing sequences using addition, subtraction and multiplication and alphabet patterns.

PRIOR KNOWLEDGE REQUIRED
Increasing and decreasing sequences
Skip counting
T-tables
Alphabet

VOCABULARY
increasing sequence
decreasing sequence
difference
general rule
stepwise rule

Invite a volunteer to write a pattern according to the rule you provide. Start with a couple of simple rules, like "Start at 2 and add 7 each time", then give a more complicated rule: Start at 2, add 3, 5, 7, 9,... Each gap is 2 more than the previous one." Ask your volunteer to write 3 more terms for the sequence. Give a couple more examples, including decreasing difference. After that write several sequences and ask your students to write rules for them.

ASK: Are these rules stepwise or general rules?

SAMPLE SEQUENCES:

76, 74, 71, 67, 62, ... 89, 74, 62, 53, 47, ... 56, 58, 62, 70, 86, ...

Ask your students, "Which sequence grows faster—the one made by addition or the one made by multiplication?"

Write the sequences on the board and ASK: What is the rule for each of these sequences?

1, 11, 21, 31, ... 1, 2, 4, 8, ... 1, 2, 7, 16, 29, ...

You may call a vote: Which is larger—the 10th term of the first, second or third sequence? Invite a volunteer to make a tally chart of the vote results. Have volunteers extend both sequences to the 10th term.

Show another sequence: 3, 8, 6, 11, 9, 14, 12, 17, 15...

You might write the sequence in this form...

3
8
6
11
9
14
12
17
15

... and ask students to describe the patterns they see. Ask students to continue the sequence and to explain the rule by which it was made. Here are some possible answers:

1. The top row is the sequence 3, 6, 9, 12... The rule is “Start at three and add three each time”. The bottom row is 8, 11, 14, 17, and the rule is “Start at eight and add three each time”.

2. The stepwise rule for the pattern (looking at all terms) is “Start at three and add five or subtract two alternatively”.

ANOTHER EXAMPLE: 101, 97, 98, 94, 95, 91, 92, ...

Provide students with a copy of the alphabet and suggest that they extend several sequences based on the alphabet:

B, D, F, H, ...  A, Z, B, Y, C, X, ...

CZ, DZ, EY, FY, GX, ...  A, G, M, S, ...
Assessment
Continue the sequences:

2, 4, 8, 14, 22, 32, ____ , ____
10, 12, 9, 11, 8, 10, 7, ____ , ____

2, 6, 18, 54, ____ , ____
98, 95, 90, 83, 74, ____

B, f, J, n, R, ...
Ab, De, Gh, Jk, ...

Bonus
A mighty and courageous but somewhat short-sighted knight assaulted a wicked 12-headed
dragon. When the knight hews off a dragon head, three new heads spring in place of the hewn one.
The knight can only hew one head in a minute. Our knight has ingloriously left the battle field after
15 minutes of battle. How many heads does the monster have now?

The students will need a spinner as shown, and the list of pattern descriptions below. Player 1 spins a
spinner so that Player 2 does not see the result. Player 1 writes a pattern according to the rule given by
the spinner and the list. Player 2 has to write 2 more terms of the sequence and to tell what the rule is.

| ACTIVITY |
|-----------------|-----------------|
| 1. Increasing sequence | 5. Decreasing sequence |
| with the increasing gaps | with constant gap |
| 2. Increasing sequence | 6. Decreasing sequence |
| with constant gap | with decreasing gaps |
| 3. Increasing sequence | 7. Repeating pattern |
| with decreasing gaps | 8. Alphabetic pattern |

Extension
“The Legend of the Chess Board.” The same doubling sequence is used in the beginning
of the lesson.

POSSIBLE SOURCE:
http://britton.disted.camosun.bc.ca/jbchessgrain.htm
Tell your students that they have done so well with patterns that today you are going to give them patterns with **HUGE** numbers.

Present several problems and call for volunteers to solve them, using T-tables.

A normal heartbeat rate is 72 times in a minute. How many times will your heart beat in five minutes?

There are 60 minutes in an hour. George sleeps for six hours. How many minutes does he sleep? He asks his friend to wake him after 500 minutes. About how many hours is this? (Skip count to find out.)

A sprinting ostrich’s stride is 700 cm long. A publicity-loving ostrich spots a photographer 3 000 cm away and runs towards him. How far from the camera will it be after three strides?

Find the number of rhombuses in each of the following stars:

**HINT:** Count the number of rhombuses between the pair of heavy black lines and then multiply.

An extinct elephant bird weighed about 499 kg. Make a T-table to show how much five birds would weigh. Do you see a pattern in the numbers (look at ones, tens and hundreds separately)? Can you write the weights of six, seven, and eight birds without actually adding or multiplying?

**Extension**

A regular year is 365 days long, a leap year (2000, 2004…) is 366 days long. Tom was born on Jan. 8, 2000. How many days was he on January 8, 2003? January 8, 2005? When was he 2000 days old? (Finding the year, the month and the day are three different problems of increasing difficulty.)
PA6-27
Equations

Suggest your students to organize the data from the following problems in a table. Ask them to draw a box for the quantity they do not know.

There are ten apples. Three are outside the box. How many are in the box?

There are 17 plums. Some are on the table; the other 12 are in the container. How many plums are on the table?

Jen has some stamps. 34 are Canadian, the other 27 are Brazilian. How many stamps does he have?

After students have made a table showing the total amount of each quantity (and the parts that make up the total) ask students what equations they could write for the entries in each row of the table. For instance, the table for the last problem would look like:

<table>
<thead>
<tr>
<th>Canadian Stamps</th>
<th>Brazilian Stamps</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

The equation would be $37 + 27 = \Box$.

Remind your students that a fact family for the sentence $3 + 4 = 7$ is $4 + 3 = 7, 7 - 3 = 4, 7 - 4 = 3$. Ask your students to write the fact family (which is the family of equations in this case) for each of the equations that they have written for the problems. The new expressions are either equations that fit the same problem or solutions for the problem. For example, for the first problem above the equations will be:

$10 - \Box = 3, 3 + \Box = 10, \text{ and } \Box + 3 = 10 \text{ and the solution for the problem will be } 10 - 3 = \Box$.

Present the following problem: There are several hats and gloves in a box. All left-hand gloves have a matching right-hand glove, and there are nine objects in the box. How many gloves and how many hats are there?

Draw an equation on the board. Say that the circle will represent the number of left-hand gloves. Gloves come in pairs, so there will be two circles—one for the right hands and one for the left hands. The square will represent the number of hats.

$\bigcirc + \bigcirc + \Box = 9$

Ask volunteers to try to fit some numbers into the equation. Suggest starting with the circle. Draw a T-table of the results:
Use as many volunteers to help filling the table as possible. **ASK:** Why did we have to stop after the 4th row?

Ask your students to write down the sequences in the third and the fourth columns and to give the rules for the sequences.

Timur threw 3 darts and scored 12 points. The dart in the centre is worth twice as much as the dart on the outside. How much is each dart worth?

**SOLUTION:** Let \( n \) be the value of a dart on the outside. The dart in the centre is worth twice as much the dart on the outside, so its worth is \( 2 \times n \), or \( 2n \). The total value of the darts is \( n + n + 2n = 12 \), or \( 4n = 12 \). So \( n = 3 \), and \( 2n = 6 \).

**Assessment**

1. Ask your students to solve several equations such as:
   a) \( \square \times 6 = 48 \)
   b) \( \square + 6 = 28 \)
   c) \( \square \div 5 = 6 \)
   d) \( \square - 4 = 16 \)
   e) \( \square + \square + \bigcirc + \bigcirc = 10 \)

2. Sindi had 12 cookies. She shared 8 with her friends and ate the rest. How many cookies did she eat?

Math Bingo

(SEE: the BLM “Math Bingo Game”): Your students will need a board each (SEE: the BLM) and 16 tokens to mark the numbers. The teacher reads the card out loud. Players have to figure out the answer and then to place a token on the board, if they have the answer. The first player to fill a column, row or diagonal wins.
Extensions

1. Give your students a copy of a times table. Ask them to write an equation that would allow them to find the numbers in a particular column of the times table given the row number. For instance, to find any number in the 5s column of the times table you multiply the row number by five: Each number in the 5s column is given by the algebraic expression \( 5 \times n \) where \( n \) is the row number. Ask students to write an algebraic expression for the numbers in a given row.

2. In the magic trick below, the magician can always predict the result of a sequence of operations performed on any chosen number. Try the trick with students, then encourage them to figure out how it works using a block to stand in for the mystery number (give lots of hints).

<table>
<thead>
<tr>
<th>The Trick</th>
<th>The Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick any number</td>
<td>Use a square block to represent the mystery number</td>
</tr>
<tr>
<td>Add 4</td>
<td>Use 4 circles to represent the 4 ones that were added.</td>
</tr>
<tr>
<td>Multiply by 2</td>
<td>Create 2 sets of blocks to show the doubling</td>
</tr>
<tr>
<td>Subtract 2</td>
<td>Take away 2 circles to show the subtraction</td>
</tr>
<tr>
<td>Divide by 2</td>
<td>Remove one set of blocks to show the division.</td>
</tr>
<tr>
<td>Subtract the mystery number</td>
<td>Remove the square</td>
</tr>
</tbody>
</table>

The answer is 3!

No matter what number you choose, after performing the operations in the magic trick, you will always get the number 3. The model above shows why the trick works.

Encourage students to make up their own trick of the same type.
Explain to your students that today they will learn to write equations the way mathematicians do it. In mathematics people usually use letters instead of drawing a square or a diamond for the unknown.

Explain to your students that when letters are used in an equation, the multiplication sign “×” is often omitted to avoid confusion with the letter X and to make the notation shorter. In this case instead of writing $2 \times A$ people simply write $2A$.

Write several equations with figures and ask your students to replace the figures with letters of their choice. For example, the equation $6 + 4 = 20$ can be rewritten as $6 + n = 20$. Do not forget equations with two variables, such as $3 \times A - B = 15$. After that write several equations with letters and ask your students to rewrite them with boxes or diamonds.

Suggest that your students use the guess and check technique to solve the equations like: $2n + 4 = 12$, or $3 \times A - 4 = 20$. Show your students how to search systematically for a solution. For example, for the first equation, they can make a T-table:

<table>
<thead>
<tr>
<th>n</th>
<th>2n</th>
<th>2n + 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

If you continue the pattern, you will easily see that the solution is $n = 4$. 
PA6-29
Variables

Review the previous lesson.

Remind your students that when letters are used in an equation, the multiplication sign “×” is often omitted to avoid confusion with the letter X and to make the notation shorter. In this case instead of writing $2 \times A$ people simply write $2A$.

Remind your students that they already used letters in formulas for patterns—for example, ask your students to tell how many triangles will be needed to make broaches using the design shown.

1 broach: $1 \times 5$ triangles
2 broaches: $2 \times 5$ triangles
7 broaches: _____

Remind your students that in previous lessons they used letters for the number of pentagons or triangles in a set of broaches.

Give them several problems, like the ones below, and ask them to write a formula or equation for the problem:

A boat travels at a speed of 10 km per hour. What distance will it cover in 2 hours? 5 hours? In $h$ hours?

A house has 12 windows. How many windows do 3 houses have? 7 houses? $X$ houses?

Ask a volunteer to write a formula/equation for a T-table:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Change the headings of the T-table to A and B and ask a volunteer to write the formula for the new table. Explain to your students, that even if the names of the columns change, the rule for the T-table will still have the same form. Previously the rule was $\text{Number of Blocks} = \text{Figure Number} + 4$, now it is $B = A + 4$.

Let your students practice finding rules for T-tables with letter headings. Start with simple rules, involving one operation and continue to more complicated rules like $B = 2 \times A + 3$. 
Give your students several word problems, such as:

Ricardo made 12 sandwiches. Four of them are avocado sandwiches, and the rest are cheese sandwiches. How many cheese sandwiches does Ricardo have?

Ask your students to draw models for the problems, write equations with squares and rewrite the equations with a variable. Eventually students should find it easy to write the equation directly with a variable.

PA6-30
Algebraic Puzzles

GOALS
Students will solve equations in the form of models.

PRIOR KNOWLEDGE REQUIRED
T-tables
Making rules for T-tables

VOCABULARY
T-table
equation

Give your students two containers to represent the two balance scales in the problems that follow. Students will place blocks into each container according to the picture (e.g., circle = red block, triangle = blue block). Students must solve the problem by isolating the block that represents the solution (for instance, in the first EXAMPLE they must isolate the triangle). Students can add or remove blocks from the containers but they must follow two rules.

RULE 1: You may add or remove one or more blocks from one container as long as you add or remove the same number of blocks from the other container. (This mirrors the algebraic rule that whatever you add or subtract from one side of an equation you must add or subtract from the other side.)

EXAMPLE:

If 2 blocks are taken from the left side of the scale, then 2 blocks must be taken from the right side as well.

Let your students practice with problems such as:

Ask your students to write an equation that represents each scale. What did they do to solve the problem? Ask them to write down the solution. In the EXAMPLE above, the equation and the solution will look like:

\[ \triangle + 2 = 5 \]
remove (subtract) two from both sides:

\[ \triangle = 5 - 2 = 3 \]
Present the second rule:

**RULE 2:** If all the blocks in one container are of a particular type and all blocks in the other container are of a particular type, and if you can group the blocks in each container equally (and not exactly the same number of sets), you may remove all but one of the sets of blocks from each container.

**EXAMPLE:**

![Image of two sets of blocks, one with squares and one with triangles, each set being divided into three equal parts. Arrows indicate the removal of one set from each side to balance the scale.]

The blocks are placed into 3 equal sets on either side. So 2 sets can be removed from each side.

In the picture above, the blocks are grouped into three equal sets of squares (1 square in each set) and three equal sets of triangles (4 triangles in each set). Each square must weigh the same as 4 triangles. So you can remove all but one group of squares and triangles without unbalancing the scale.

![Image of a square and a triangle, with the square divided into three equal parts and the triangle divided into four equal parts. Arrows indicate the removal of one part from each to balance the scale.]

This rule mirrors the algebraic rule that you may divide both sides of an equation by the same number. The equation and the solution for the example above would be written as:

\[3 \times \square = 12, \text{ divide both sides by } 3, \text{ so } \square = 12 \div 3 = 4.\]

Let your students practice with problems such as:

![Image of two sets of blocks, one with squares and one with circles, each set being divided into three equal parts. Arrows indicate the removal of one set from each side to balance the scale.]

Encourage your students to replace the symbols in the equations with letters.

Students should write the equations and solutions for each problem. Now let students try more complicated problems that require the use of both rules, such as:

![Image of two sets of blocks, one with squares and one with circles, each set being divided into four equal parts. Arrows indicate the removal of one set from each side to balance the scale.]

When your students are comfortable finding and writing solutions from the models, you might present the sample problems on the next page and ask your students to draw pictures or models illustrating each balanced scale. Next, ask them to write and solve an equation for each unbalanced scale. Students should solve the problems in sequence, since information from some problems will be required to solve subsequent problems. For example, the first two balanced scales tell us that:

1 puppy = 1 cat + 3 mice

1 puppy = 8 mice

This information will be necessary to find the weight of the cat in mice.
SAMPLE PROBLEMS:

The scales on the left are balanced perfectly. Can you balance the scales on the right?

<table>
<thead>
<tr>
<th>Balanced Scales</th>
<th>Unbalanced Scales</th>
</tr>
</thead>
<tbody>
<tr>
<td>A puppy and 2 mice</td>
<td>A cat and 5 mice</td>
</tr>
<tr>
<td>A puppy and 2 mice</td>
<td>10 mice</td>
</tr>
<tr>
<td>A dog</td>
<td>5 puppies</td>
</tr>
<tr>
<td>Two cows</td>
<td>5 dogs</td>
</tr>
<tr>
<td>A puppy</td>
<td>How many cats and mice?</td>
</tr>
<tr>
<td>A puppy</td>
<td>How many mice?</td>
</tr>
<tr>
<td>A cat</td>
<td>How many mice?</td>
</tr>
<tr>
<td>A dog and a mouse</td>
<td>How many mice?</td>
</tr>
<tr>
<td>A cow</td>
<td>How many cats? Find a second solution involving dogs, puppies and mice.</td>
</tr>
</tbody>
</table>

Give your students several puzzles of the form: 36 + 2 □ = 61. Which digit is missing? Encourage students to use systematic search or to solve the equations by subtraction. (NOTE: The square here represents the missing digit, not a number as it did earlier.)

Extension

Use the digits 0, 1, 3, 5, 7, 9 instead of letters to solve: HILL + HILL = MICE. Note that the same letter should be replaced by the same digit.

SOLUTION: Since both addends are the same, MICE is an even number. This leaves only one choice for E – it is 0. This means L is 5, and we have HI55 + HI55 = MIC0, and addition proves that C = 1.

Looking at the hundreds digits, we see that either I + I + 1 = I or I + I + 1 = I + 10. (NOTE: On the left side, we have I + I + 1 because we know 55 + 55 = 110, so we have an extra hundred to account for. On the right side, the second option -- I+10 -- allows for the possibility that 10 hundreds have been regrouped as one thousand.) Only the second case is possible, which gives us I = 9. (Draw scales to see that.) We already have H955 + H955 = M910, with 3 and 7 to choose from. This gives us H = 3 and M = 7. ANSWER: 3955 + 3955 = 7910.
Remind your students of how the columns and the rows are numbered in coordinate grids (see the grid below). Draw a coordinate grid on the board and invite volunteers to locate points by their coordinates and to give the coordinates of various points on the grid. Remind them that the first number in the coordinate pair is the number of a column and the second number is the number of the row. You might use the activity for practice.

Connect several points that the students marked on the board so that the picture looks like a line graph. Ask your students what this picture looks like. Explain to your students that today they will start analyzing graphs in a new way—they would try to find a formula that goes with the graph.

Draw a line on the grid so that it passes through several grid vertices. (For example, take a line that passes through the points (1, 1) and (4, 7).) Invite volunteers to mark points on the line and to write their coordinates on the board. Ask your students: What is the second coordinate of a number with first coordinate 11? How can you find the answer to this question? (You can extend the line on the grid). What happens if you know only the second coordinate? What is the first coordinate of the point with the second coordinate 17? (Extend the line to find out). How would you find the second coordinate of a number with first coordinate 100? Suggest to your students that they might try to derive an equation or formula using the methods they learned for writing the rule or formula for T-tables. Ask your students how they could make a T-table from the graph. Your students might suggest making a T-table with headings: First number, Second number. Invite a volunteer to make such a table. Let your students practice making T-tables from several different graphs. They should mark points on the line, list their coordinates and make a T-table with headings as above.

**Assessment**

1. On grid paper, make a T-table and graph the rule “Multiply by 4 and subtract 1”.

2. Mark four points on the line segment. Write a list of ordered pairs, complete the T-table and write a rule that tells you how to calculate the output from the input.
Extension

1. Write a rule for each of the T-tables in QUESTIONS 1 and 2.

   HINT: 1 c) and 2 b) involve dividing and adding.

2. Ron has $30. For every book he reads his mother gives him $6.
   a) Draw a group showing the input as the number of books and the output as the amount of money.
   b) Write a rule for the amount of money he has after reading $n$ of books.
   c) How many books does he need to read to have $50? $100 $1000?
Review line graphs with your students. Draw a coordinate grid on the board and draw the following graph on it. Explain to your students that this graph represents the cost of parking a car in a lot. How much would it cost to leave the car for 1 hour? For 2 hours? For half an hour?

Ask your students: How much will you pay for entering the parking lot, even before you park the car there? How much does each additional hour cost? How do they know? Does the hourly charge vary? As a challenge, ask them to give a rule that allows you to calculate the cost of parking (For example, “$3 for parking, $2 each additional hour up to 3 hours, up to a maximum of 3 hours (or $9)” or, in a more mathematical way, “If you park for less than 3 hours, multiply the time by $2 and add $3. If you park for 3 hours or more, the price is $9.”)

Explain that a nearby parking lot charges $2.5 per hour. What is the mathematical rule for the parking cost there? Ask your students to write ordered pairs for time and cost for the second parking and ask them to plot the graph for the second rule. **ASK:** Which parking will be cheaper for 2 hours? For 4 hours?

For a challenge, your students might plot the cost of the third parking that charges $.5 dollars for any time less than 1 hour, and $4.5 per hour after that. Which of the three parking lots is better for parking times of 1, 2, 3 hours? Joe and Zoe left their cars at parking lots 2 and 3 respectively. They parked for the same time and paid the same amount. How long did they park?
Let your students analyze another graph that shows a race between two people, like the graph in QUESTION 3 of the worksheet. As an alternative, you may use the following question.

A boat leaves port at 9:00 am and travels at a steady speed. Some time later a man jumps into water and starts swimming in the same direction as the boat.

1. How many minutes passed between the time the boat left port until the man jumped into the water? When did the man jump into the water?
2. How far from the port was the man at 9:15?
3. When did the boat overtake the man?
4. How far can the boat travel in 1 hour?
5. How long does it take the man to swim 1 km?
6. How far did the man swim before he was taken aboard? How can you see from the graph that the man was taken to the boat?
7. Did the boat continue travelling when it met the man or did it stay at the same place for some time? How do you know?
8. The boat docked at another port 9 km from the starting point. At what time did this happen?
PA6-33
Concepts in T-tables (Advanced)

Review with your students how they can find a rule for a T-table—first find the gap, then multiply the gap by the input and look at the difference between the result and the output. Invite volunteers to find the rule for several examples, such as:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Present several more complicated patterns (in T-tables), such as patterns with growing gaps. For example, use the pattern 2, 3, 5, 8, 12 as output for input 1, 2, 3, 4, 5. Ask your students to write a verbal rule for this sequence. They can present the answer in the form: Start at 2 and add 1, 2, 3, ... Present at least one example with decimal numbers as gaps (like Start at 3.1 and add 2.4 each time). If your students are familiar with fractions, you may use fractions as gaps as well: \( \frac{1}{2} \), \( \frac{2}{3} \), \( \frac{3}{4} \), ... or even \( \frac{1}{2} \), \( \frac{1}{6} \), \( \frac{1}{2} \) ...

Show your students a T-table that has a pattern both in the input and in the output, like:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>10.5</td>
</tr>
<tr>
<td>7.0</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>12.5</td>
<td>37.5</td>
</tr>
<tr>
<td>14.5</td>
<td>43.5</td>
</tr>
</tbody>
</table>

Ask your students to describe first the patterns in the input and the output. How does the gap between the inputs change? (it is .5 smaller each time) How does the gap between the outputs change? (it is 1.5 smaller each time) By what number do you have to multiply .5 to get 1.5? What happens if you multiply the input by the same number? What is the relationship between the input and the output?
Draw a circle with 10 dots on the board. Explain to your students that a line that joins two dots on a circle is called a chord. Show your students how to create two figures in a circle by following the steps below:

**STEP 1:** Choose a dot and join it to every second dot on the circle. You get 4 chords.

**STEP 2:** Join the four dots on the ends of the chords you have drawn to each other. You get a pentagon with all the diagonals. How many chords do you have? (10)

**STEP 3:** Repeat the Steps 1 and 2 with the rest of the dots. Use different colour. You will get two pentagons with all diagonals so that each dot is joined to every second dot with a chord. The number of chords doubled, so you have 20 chords.

### CHALLENGING
Describe the patterns in the input, output and the relationship between the input and the output.

<table>
<thead>
<tr>
<th>Input</th>
<th>2.5</th>
<th>8.5</th>
<th>13</th>
<th>16</th>
<th>17.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>9</td>
<td>33</td>
<td>51</td>
<td>63</td>
<td>69</td>
</tr>
</tbody>
</table>

### Assessment
Describe the patterns in the input, output and the relationship between the input and the output.

<table>
<thead>
<tr>
<th>Input</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>14</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>35</td>
<td>55</td>
</tr>
</tbody>
</table>

### Extension
Rick wants to save $200. If the chart in QUESTION 1 f) represents his total savings each week, how many weeks will it take before he has saved half of his goal?
Suppose you would like to predict the number of chords you would get if you started with 18 dots. Explain to your students that a mathematician might draw a case with fewer dots and try to look at the pattern to make a prediction.

Suggest that your students draw a picture for 6 and 8 dots. Each dot must be connected with every second dot, so the number of dots is even. How many chords are in each figure? Summarize the results in a T-table:

<table>
<thead>
<tr>
<th>Number of Dots</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Chords</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

Do you have enough information to find a general rule relating the numbers of dots to the number of chords? If your students come up with a rule suggest that they check their rule for smaller numbers (i.e. for a figure with 4 dots). **ASK:** Why would you try smaller numbers and not larger numbers? (If your rule is correct, it will hold for smaller numbers and for larger numbers as well. However, it is easier to draw a smaller picture and there are fewer chances to miss a chord while counting.) Ask your students to extend the T-table for 2 more columns. (12 and 14 dots). Ask them to draw the picture for 12 dots and to check their predictions. After that ask your students to predict the number of chords in the figure with 18 dots.

**Extensions**

1. If the students enjoy what they did, you may add two rows to the table:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Dots</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Number of Chords</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

**ASK:** Does the Term Number always divide the number of chords? Ask your students to write the number of chords as a product so that the Term Number is one of the factors. Can they see a pattern? How many chords will there be in a figure with 100 dots? (The term number is 49, and the number of diagonals will be 50 × 49 = 2450).

2. Put a dot in the centre of a polygon and draw a line from the centre to each vertex of the polygon. How many line segments are there in each figure? Predict how many line segments there would be in a hexagon and an octagon. Test your prediction.
This is a review worksheet.

Extensions

1. Consecutive numbers are numbers that follow each other on the number line. You can sum a set of consecutive numbers quickly, by grouping the numbers as follows:
   - **STEP 1:** Add the first and last number, the second and the second to last number and so on. What do you notice?
     
     \[1 + 2 + 3 + 4 + 5 + 6 = 7 + 7 + 7\]
   - **STEP 2:** Rewrite the addition statement as a multiplication statement.
     \[3 \times 7 = 21\]

   Add the following sets of numbers:

   a) \[1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = \]
   b) \[12 + 13 + 14 + 15 + 16 + 17 = \]

2. Assign Extension 1 before this problem: A man ate 100 grapes in 5 days. Each day he ate 6 more than the previous day. How many grapes did he eat on the first day?
## PA6 Part 2: BLM List

<table>
<thead>
<tr>
<th>Item</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Paper</td>
<td>2</td>
</tr>
<tr>
<td>Math Bingo Game</td>
<td>3</td>
</tr>
</tbody>
</table>
Grid Paper (1 cm)
Math Bingo Game

Sample Boards

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<tbody>
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<tr>
<td>5</td>
<td>11</td>
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</table>
Math Bingo Game (continued)

**Cards (Addition Only)**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
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</table>

**Cards (Mixed Equations)**

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<tbody>
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<td>6 + □ = 13</td>
<td>5 + □ = 24</td>
<td>9 + □ = 20</td>
</tr>
<tr>
<td>□ − 9 = 11</td>
<td>□ − 1 = 11</td>
<td>□ − 4 = 22</td>
<td>□ − 4 = 16</td>
</tr>
<tr>
<td>7 × □ = 21</td>
<td>6 × □ = 30</td>
<td>4 × □ = 16</td>
<td>3 × □ = 18</td>
</tr>
<tr>
<td>□ + 7 = 15</td>
<td>□ + 6 = 21</td>
<td>□ − 4 = 13</td>
<td>□ − 5 = 11</td>
</tr>
<tr>
<td>9 × □ = 18</td>
<td>4 × □ = 36</td>
<td>8 × □ = 80</td>
<td>21 + □ = 22</td>
</tr>
</tbody>
</table>
NS6-54
Equal Parts and Models of Fractions

Ask your students if they have ever been given a fraction of something (like food) instead of the whole, and gather their responses. Bring a banana (or some easily broken piece of food) to class. Break it in two very unequal pieces. **SAY:** This is one of two pieces. Is this half the banana? Why not? Emphasize that the parts have to be equal for either of the two pieces to be a half.

Draw numerous examples of shapes with one of two parts shaded, some that are equal and some that are not, and ask volunteers to mark the diagrams as correct or incorrect representations of one half.

**ASK:** Which diagram illustrates one-fourth? What’s wrong with the other diagram? Isn’t one of the four pieces still shaded?

Explain that it’s not just shapes like circles and squares and triangles that can be divided into fractions, but anything that can be divided into equal parts. Draw a line and ask if a line can be divided into equal parts. Ask a volunteer to guess where the line would be divided in half. Then ask the class to suggest a way of checking how close the volunteer’s guess is. Have a volunteer measure the length of each part. Is one part longer? How much longer? Challenge students to discover a way to check that the two halves are equal without using a ruler, only a pencil and paper. [On a separate sheet of paper, mark the length of one side of the divided line. Compare that length with the other side of the divided line by sliding the paper over. Are they the same length?]

Have students draw lines in their notebooks and then ask a partner to guess where the line would be divided in half. They can then check their partner’s work.

**ASK:** What fraction of this line is double?

**SAY:** The double line is one part of the line. How many equal parts are in the whole line, including the double line? [5, so the double line is 1/5 of the whole line.]
Mark the length of the double line on a separate sheet of paper. Compare that length to the entire line to determine how many of those lengths make up the whole line. Repeat with more examples.

Then ask students to express the fraction of shaded squares in each of the following rectangles.

Have them compare the top and bottom rows of rectangles. **ASK:** Are the same fraction of the rectangles shaded in both rows? Explain. If you were given the rectangles without square divisions, how would you determine the shaded fraction? What could be used to mark the parts of the rectangle? What if you didn’t have a ruler? Have them work as partners to solve the problem. Suggest that they mark the length of one square unit on a separate sheet of paper, and then use that length to mark additional square units.

Prepare several strips of paper with one end shaded, and have students determine the shaded fraction without using a pencil or ruler. Only allow them to fold the paper.

Draw several rectangles with shaded decimetres.

Have students divide the rectangles into the respective number of equal parts (3, 4 and 5) with only a pencil and paper. Have them identify the fractions verbally and orally. Then draw a rectangle with two shaded decimetres (i.e. two-fifths), and challenge your students to mark units half the length of the shaded decimetres. How can this be done with only a pencil and paper?

*Fold the paper so that these markings meet. Draw a marking along the fold.*

Draw a shaded square and ask students to extend it so the shaded part becomes half the size of the extended rectangle.

Repeat this exercise for squares becoming one-third and one-quarter the size of extended rectangles. **ASK:** How many equal parts are needed? [Three for one-third, four for one-quarter.] How many parts do you already have? [1.] So how many more equal parts are needed? [Two for one-third, three for one-quarter.]
**Bonus**
Extend the squares so that $\frac{2}{5}$ of them are shaded. Repeat this exercise for $\frac{1}{7}$, $\frac{3}{7}$, etc.

Give your students rulers and ask them to solve the following puzzles.

- a) Draw a line 1 cm long. If the line represents $\frac{1}{6}$ show what a whole line looks like.
- b) Line: 1 cm long. The line represents $\frac{1}{3}$. Show the whole.
- c) Line: 2 cm long. The line represents $\frac{1}{2}$. Show the whole.
- d) Line: 3 cm long. The line represents $\frac{1}{2}$. Show the whole.
- e) Line: $1\frac{1}{2}$ cm long. The line represents $\frac{1}{4}$. Show the whole.
- f) Line: $1\frac{1}{2}$ cm long. The line represents $\frac{1}{3}$. Show the whole.
- g) Line: 3 cm long. The line represents $\frac{1}{3}$. Show $\frac{1}{2}$.
- h) Line: 2 cm long. The line represents $\frac{1}{6}$. Show $\frac{1}{4}$.

This is a good activity to do at the end of a day, so that students with extra time can play with the left-over play dough until the end of class.

Prepare enough small balls of coloured play dough for 3 for each student (they will only need two, but this allows students to choose their colours). Demonstrate to students how to make fractions using play dough. Tell them that you are going to roll one spoon of red play dough and three spoons of blue play dough into a ball. Explain the necessity of flattening the play dough on a spoon so that each spoonful is the same size. Demonstrate not leaving any play dough of the first colour on the spoon. Roll the play dough into a ball carefully mixing it so that it becomes a uniform colour. This has been tried with fresh play dough only; store-bought play dough may not produce the same effect and may be more difficult to mix thoroughly. For a recipe, **SEE:**

http://www.teachnet.com/lesson/art/playdoughrecipes/traditional.html

An alternative to play dough is to use small spoonfuls of food colouring.

**ASK:** How many spoons of play dough have I used altogether? How many spoons of red play dough have I used? What fraction of this ball is red? How many spoons of blue play dough have I used? What fraction of this ball is blue? Write the “recipe” on a triangular flag (see below) which can be made from a quarter of a regular sheet taped to a straw (insert the straw into your ball of play dough). The recipe is shown on the flag below:

$$\frac{1}{2} \text{ red, } \frac{3}{4} \text{ blue}$$
Extensions

1. The smaller angle is what fraction of the larger angle?

   HINT: Use tracing paper.

2. Draw the whole angle if the given angle is $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of the whole angle.
   
   a) $\frac{1}{2}$
   
   b) $\frac{1}{3}$
   
   c) $\frac{1}{4}$

3. a) Sketch a pie and cut it into fourths. How can it be cut into eighths?
   
   b) Sketch a pie and cut it into thirds. How can it be cut into sixths?

4. Ask students to make and identify as many fractions in the classroom as they can, for instance:
   
   $\frac{1}{4}$ of the blackboard is covered in writing.
   
   $\frac{2}{3}$ of the counters are red.
   
   $\frac{1}{2}$ the length of a 30 cm ruler is 15 cm.
   
   About $\frac{1}{3}$ of the door is covered by a window.
   
   $\frac{11}{25}$ of the class has black hair.
   
   The room is about $\frac{4}{5}$ wide as it is long.

5. Ask students to make a model of a fraction from materials in the classroom.
6. (Adapted from Grade 4 Atlantic Curriculum)

Ask students to divide the rectangle...

a) into thirds two different ways.
b) into quarters three different ways.

NOTE: The division shown below may not be obviously divided into quarters until it is further divided into eighths.

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**NS6-55**

**Equal Parts of a Set**

Review equal parts of a whole. Tell your students that the whole for a fraction might not be a shape like a circle or square. Tell them that the whole can be anything that can be divided into equal parts. Brainstorm with the class other things that the whole might be: a line, an angle, a container, apples, oranges, amounts of flour for a recipe. Tell them that the whole could even be a group of people. For example, the grade 6 students in this class is a whole set and I can ask questions like: what fraction of students in this class are girls? What fraction of students in this class are eleven years old? What fraction of students wear glasses? What do I need to know to find the fraction of students who are girls? (The total number of students and the number of girls). Which number do I put on top: the total number of students or the number of girls? (the number of girls). Does anyone know what the top number is called? (the numerator) Does anyone know what the bottom number is called? (the denominator) What number is the denominator? (the total number of students). What fraction of students in this class are girls? (Ensure that they say the correct name for the fraction. For example, \( \frac{10}{25} \) is said “ten twenty fifths”).) Tell them that the girls and boys don’t have to be the same size; they are still equal parts of a set. Ask students to answer: What fraction of their family is older than 10? Younger than 10? Female? Male? Some of these fractions, for some students, will have numerator 0, and this should be pointed out. Avoid asking questions that will lead them to fractions with a denominator of 0 (For example, the question “What fraction of your siblings are male?” will lead some students to say 0/0).

Then draw pictures of shapes with two attributes changing:
a)\[\text{ASK: What fraction of these shapes are shaded? What fraction are circles? What fraction of the circles are shaded?}\]

b)\[\text{ASK: What fraction of these shapes are shaded? What fraction are unshaded? What fraction are squares? Triangles? What fraction of the triangles are shaded? What fraction of the squares are shaded? What fraction of the squares are not shaded?}\]

**Bonus**
(pictures with 3 attributes changing)

Have students answer similar questions as above.
Have students make up questions to ask each other.

Draw on the board:

[c] Have students volunteer questions to ask and others volunteer answers.
Then have students write fraction statements in their notebooks for similar pictures.

Ask some word problems:

A basketball team played 5 games and won 2 of them. What fraction of the games did the team win?

A basketball team won 3 games and lost 1 game. How many games did they play altogether? What fraction of their games did they win?

A basketball team won 4 games, lost 1 game and tied 2 games. How many games did they play? What fraction of their games did they win?

Ensure that students understand that they must first determine the total number of members of a set before they can find the fraction made by each part. Also give word problems that use words such as “and,” “or” and “not”: Sally has 4 red marbles, 2 blue marbles and 7 green marbles.

a) What fraction of her marbles are red?
b) What fraction of her marbles are blue or red?
c) What fraction of her marbles are not blue?
d) What fraction of her marbles are not green?

**Bonus**
Which two questions have the same answer? Why?
Challenge your students to write another question that uses “or” that will have the same answer as:
What fraction of her marbles are not blue?

Look at the shapes below:

```

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a) What fraction of the shapes are shaded and circles?
b) What fraction of the shapes are shaded or circles?
c) What fraction of the shapes are not circles?

Write a question that has the same answer as c):
What fraction of the shapes are _____ or _____?

Have students create their own questions and challenge a partner to answer them. Find the number of each item given the fractions.

a) A team played 5 games. They won \( \frac{2}{5} \) of their games and lost \( \frac{3}{5} \) of their games. How many games did they win? Lose?
b) There are 7 marbles. \( \frac{2}{7} \) are red, \( \frac{4}{7} \) are blue and \( \frac{1}{7} \) are green. How many blue marbles are there? Red marbles? Green marbles?

Then tell your students that you have five squares and circles. Some are shaded and some are not. Have students draw shapes that fit the puzzles. (Students could use circles and sequences of two different colours to solve the problem.)

a) \( \frac{2}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the shapes are shaded. One circle is shaded.

SOLUTION:

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b) \( \frac{3}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the shapes are shaded. No circle is shaded.
c) \( \frac{3}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the shapes are shaded. \( \frac{1}{5} \) of the squares are shaded.

**ACTIVITY 1**

Students could use triangles and square blocks of two different colours to solve question 8.
Extensions

1. What word do you get when you combine...
   a) the first \( \frac{2}{3} \) of sun and the first \( \frac{1}{2} \) of person? (ANSWER: Super!)
   b) the first \( \frac{1}{2} \) of grease and the first \( \frac{1}{2} \) of ends?
   c) the first \( \frac{1}{3} \) of trance and the last \( \frac{3}{4} \) of luck?
   d) the first \( \frac{1}{2} \) of wood and the last \( \frac{3}{5} \) of arm?

   Try making up your own questions like this.

2. Draw a picture to solve the puzzle. There are 7 triangles and squares. \( \frac{2}{7} \) of the figures are triangles. \( \frac{3}{7} \) are shaded. 2 triangles are shaded.

3. Give your students harder puzzles by adding more attributes and more clues:
   There are 5 squares and circles. \( \frac{1}{3} \) of the squares are big.
   \( \frac{3}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the squares are shaded.
   \( \frac{3}{5} \) of the shapes are shaded. No shaded shape is big.
   \( \frac{2}{5} \) of the shapes are big.

   SOLUTION:
4. Give your students red and blue counters (or any other pair of colours) and ask them to solve the following problems by making a model.
   a) Half the counters are red. There are 10 red counters. How many are blue?
   b) Two fifths of the counters are blue. There are 6 blue counters. How many are red?
   c) \(\frac{3}{4}\) of the counters are red. 9 are red. How many are blue?

5. What fraction of the letters of the alphabet is…
   a) in the word “fractions”
   b) not in the word “fractions”

6. For the month of September, what fraction of all the days are…
   a) Sundays
   b) Wednesdays
   c) School days
   d) Not school days
   e) What fraction of the days are divisible by 5?

7. What fraction of an hour has passed since \(\textcolor{red}{9:00}\)? Reduce if possible.
   a) \(\textcolor{red}{9:07}\)
   b) \(\textcolor{red}{9:15}\)
   c) \(\textcolor{red}{9:30}\)
   d) \(\textcolor{red}{9:40}\)

8. a) There are 5 circles and triangles. Can you draw a set so that:
   i) \(\frac{3}{5}\) are circles and \(\frac{2}{5}\) are striped?
      Have volunteers show the different possibilities before moving on. Ask questions like:
      How many are striped circles? How many different answers are there?
   ii) \(\frac{4}{5}\) are circles and \(\frac{1}{5}\) are triangles?
      What is the same about these two questions? (the numbers are the same in both) What is different? [one is possible, the other is not; one uses the same attribute (shape) in both, the other uses two different attributes (shape and shading)].
   b) On a hockey line of 5 players, \(\frac{4}{5}\) are good at playing forward and \(\frac{2}{5}\) are good at playing defense. How many could be good at playing both positions? Is there only one answer? Which question from 4 a) is this similar to? What is similar about it?
GOALS
Students will make divisions not already given to form equal-sized parts.

PRIOR KNOWLEDGE REQUIRED
A fraction of an area is a number of equal-sized parts out of a total number of equal-sized parts. The whole a fraction is based on can be anything.

VOCABULARY
fraction numerator whole denominator part

Draw on the board the shaded strips from before:

ASK: Is the same amount shaded on each strip? Is the same fraction of the whole strip shaded in each case? How do you know? Then draw two hexagons as follows:

ASK: Is the same amount shaded on each hexagon? What fraction of each hexagon is shaded?

Then challenge students to find the fraction shaded by drawing their own lines to divide the shapes into equal parts:

Give your students a set of tangram pieces. Ask students what fraction of each of the tangram pieces a small triangle represents. ASK: How many small triangles cover a square? What fraction of the square is the small triangle? How many small triangles cover the large triangle? What fraction of the large triangle is the small triangle? What fraction of the medium triangle is the small triangle? What fraction of the parallelogram is the small triangle?

Ask students to make as many rectangles or trapezoids as they can using the pieces and determine the fraction of the shape that is covered by the small triangle. (For some sample rectangles and trapezoids, see questions 1 and 3 on the worksheet.)
Then draw shapes on the board and have students decide what fraction of the shape the small triangle is:

If $\square$ = red, and $\blacksquare$ = blue, approximately what fraction of each flag or banner is red and what fraction is blue:

For parts b) and c) students should subdivide the flag as shown below:

(From Atlantic Curriculum A3.6) Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.

**Extensions**

1. a) What fraction of a tens block is a ones block?
   b) What fraction of a tens block is 3 ones blocks?
   c) What fraction of a hundreds block is a tens block?
   d) What fraction of a hundreds block is 4 tens blocks?
   e) What fraction of a hundreds block is 32 ones blocks?
   f) What fraction of a hundreds block is 3 tens blocks and 2 ones blocks?
2. On a geoboard, show 3 different ways to divide the area of the board into 2 equal parts.

**EXAMPLES:**

![Geoboard Diagram]

3. Give each student a set of pattern blocks. Ask them to identify the whole of a figure given a part.

   a) If the pattern block triangle is $\frac{1}{6}$ of a pattern block, what is the whole?

   **ANSWER:** The hexagon.

   b) If the pattern block triangle is $\frac{1}{3}$ of a pattern block, what is the whole?

   **ANSWER:** The trapezoid.

   c) If the pattern block triangle is $\frac{1}{2}$ of a pattern block, what is the whole?

   **ANSWER:** The rhombus.

   d) If the rhombus is $\frac{1}{6}$ of a set of pattern blocks, what is the whole?

   **ANSWER:** 2 hexagons or 6 rhombuses or 12 triangles or 4 trapezoids.

4. Students can construct a figure using the pattern block shapes and then determine what fraction of the figure is covered by the pattern block triangle.

5. What fraction of the figure is covered by...

   a) The shaded triangle

   b) The small square

![Pattern Block Diagram]

6. To prepare students for comparing and ordering fractions, ask them to guess what fraction with numerator 1 is closest to the answer to parts c) and d): one half, one third, one fourth or one fifth. Students can then check their estimates on an enlarged version of these drawings by using counters.

**ASK:** How many counters cover the red part? How many counters cover the whole flag? About how many times more counters cover the whole flag than cover the red part? About what fraction of the flag is red? If students guess for example, that the fraction of the starred flag in the bonus that is red is one half, they could cover the red star with red counters, the blue boundary with (same sized) blue counters and see if the number of red counters is close to the number of blue counters.
NS6-57
Adding and Subtracting Fractions

GOALS
Students will add and subtract fractions with the same denominators.

PRIOR KNOWLEDGE REQUIRED
Naming fractions
Addition and subtraction of whole numbers
1 whole

VOCABULARY
fraction
regrouping
numerator
denominator

Draw two large circles representing pizzas (you can use paper pizzas as well) and divide them into 4 pieces each, shading them as shown.

Explain that these are two plates with several pizza pieces on each. How much pizza do you have on each plate? Write the fractions beneath the pictures. Tell them that you would like to combine all the pieces onto one plate, so put the “+” sign between the fractions and ask a volunteer to draw the results on a different plate. How much pizza do you have now?

Draw on the board:

\[
\frac{1}{4} + \frac{2}{4} =
\]

Tell your students that you would like to regroup the shaded pieces so that they fit onto one circle. SAY: I shaded 2 fourths of one circle and 1 fourth of another circle. If I move the shaded pieces to one circle, what fraction of that circle will be shaded? How many pieces of the third circle do I need to shade? Tell them that mathematicians call this process adding fractions. Just like we can add numbers, we can add fractions too.

Do several examples of this, like \(\frac{1}{5} + \frac{2}{5}\), never extending past 1 whole circle. Ask your students: You are adding two fractions. Is the result a fraction too? Does the size of the piece change while we transport pieces from one plate to the other? What part of the fraction reflects the size of the piece—top or bottom? Numerator or denominator? When you add fractions, which part stays the same, the top or the bottom; the numerator or the denominator?

What does the numerator of a fraction represent? (The number of shaded pieces) How do you find the total number of shaded pieces when you moved them to one pizza? What operation did you use?

Show a couple more examples using pizzas, and then have them add the fractions without pizzas. Assign lots of questions like \(\frac{2}{5} + \frac{1}{5} + \frac{2}{7} + \frac{3}{7} + \frac{2}{11} + \frac{4}{11}\), etc. Enlarge the denominators gradually.

**Bonus**
Add:

\[
\frac{12}{134} + \frac{45}{134} \quad \frac{67}{1567} + \frac{78}{1567} \quad \frac{67}{456} + \frac{49}{456}
\]

**Bonus**
Add more fractions:

\[
\frac{3}{17} + \frac{1}{17} + \frac{5}{17} \quad \frac{5}{34} + \frac{2}{34} + \frac{7}{34} \quad \frac{3}{19} + \frac{2}{19} + \frac{3}{19} + \frac{3}{19} + \frac{3}{19}
\]
Return to the pizzas and say that now you are taking pieces of pizza away. There was $\frac{3}{4}$ of a pizza on a plate. You took away $\frac{1}{4}$. Show on a model the one piece you took away:

How much pizza is left? Repeat the sequence of exercises and questions you did for addition using subtraction.

**Extensions**

1. Tell your students that sometimes adding fractions can result in more than one whole.

   Draw on the board:
   
   \[ \frac{3}{4} + \frac{2}{4} = \]  

   Ask how many parts are shaded in total and how many parts are in 1 whole circle. Tell your students that, when adding fractions, we like to regroup the pieces so that they all fit onto 1 circle. **ASK:** Can we do that in this case? Why not? Tell them that since there are more pieces shaded than in 1 whole circle, the next best thing we can do is to regroup them so that we fit as many parts onto the first circle as we can and then we put only the leftover parts onto the second circle.

   Draw on the board:
   
   

   Ask how many parts are shaded in the first circle and how many more parts do we need to shade in the second circle. Ask a volunteer to shade that many pieces and then tell them that mathematicians write this as:

   \[ \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1 + \frac{1}{4} \]

   Do several examples of this where the sum of the fractions is more than 1 whole, using more complex shapes to make it look harder as students get used to the new concept and then continue with examples where the sum is more than 2 wholes.

2. Teach your students the role of 0 in adding and subtracting fractions.

   a) $\frac{3}{5} - \frac{3}{5}$  
   b) $\frac{2}{7} + \frac{0}{7}$  
   c) $\frac{3}{8} - \frac{0}{8}$  
   d) $\frac{6}{7} - 0$  
   e) $1 \frac{2}{3} + 0$

   Tell them that 0 is the same number with any denominator. This is very different from fractions with any other numerator, where the denominator matters a lot.

3. Revisit Exercise 4 d) of worksheet **NS6-56**. Have your students estimate what fraction of the flag is red and then do the subdividing necessary (the subdividing should reveal that $\frac{13}{20}$ of the flag is red). Now that students know how to compare fractions, they should check how close their answer is to the real fraction. Have your students compare the fractions guesses such as $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{5}$ to $\frac{13}{20}$ by finding equivalent fractions with denominator 120: $\frac{40}{120}$, $\frac{60}{120}$, $\frac{48}{120}$ and $\frac{36}{120}$. **ASK:** Which of the three guesses ($\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{5}$) are closest to $\frac{13}{20}$?
NS6-58
Ordering and Comparing Fractions

Draw on the board:

Have students name the fractions shaded and then have them say which circle has more shaded. **ASK:** Which is more: one fourth of the circle or three fourths of the circle?

**ASK:** Is three quarters of something always more than one quarter of the same thing? Is three quarters of a metre longer or shorter than a quarter of a metre? Is three quarters of a dollar more money or less money than a quarter of a dollar? Is three fourths of an orange more or less than one fourth of the orange? Is three fourths of the class more or less people than one fourth of the class? If three fourths of the class have brown eyes and one quarter of the class have blue eyes, do more people have brown eyes or blue eyes?

Tell students that if you consider fractions of the same whole—no matter what whole you’re referring to—three quarters of the whole is always more than one quarter of that whole, so mathematicians say that the fraction \( \frac{3}{4} \) is greater than the fraction \( \frac{1}{4} \). Ask students if they remember what symbol goes in between:

\[
\frac{3}{4} \quad \text{or} \quad \frac{1}{4}
\]

Remind them that the inequality sign is like the mouth of a hungry person who wants to eat more of the pasta but has to choose between three quarters of it or one quarter of it. The sign opens toward the bigger number:

\[
\frac{3}{4} > \frac{1}{4} \quad \text{or} \quad \frac{1}{4} < \frac{3}{4}
\]

Have students decide which is more and to write the appropriate inequality in between the numbers:
Repeat with several examples, eventually having students name the fractions as well:

\[
\begin{array}{cccc}
\frac{2}{7} & \frac{3}{7} & \frac{5}{7} & \frac{4}{7} \\
\hline
\hline
\end{array}
\]

Draw the following pictures on the board:

\[
\begin{array}{cccc}
\hline
\hline
\end{array}
\]

**ASK:** Which is greater: one quarter or two quarters? One eighth or two eighths? One sixth or two sixths? Show students a pie cut into sixths on the board and ask: If Sally gets one sixth and Tony gets two sixths, who gets more? If Sally gets three sixths and Tony gets two sixths, who gets more? Which is greater:

\[
\begin{array}{cccc}
\frac{1}{9} & \frac{2}{9} & \frac{1}{12} & \frac{2}{12} \\
\frac{3}{11} & \frac{4}{11} & \frac{9}{11} & \frac{8}{11} \\
\frac{35}{87} & \frac{43}{87} & \frac{91}{102} & \frac{54}{102} \\
\frac{2}{9} & \frac{5}{9} & \frac{2}{11} & \frac{3}{11} \\
\frac{4}{11} & \frac{5}{11} & \frac{3}{16} & \frac{4}{16} \\
\frac{21}{48} & \frac{25}{48} & \frac{67}{131} & \frac{72}{131} \\
\frac{7}{432} & \frac{25}{401} & \frac{52}{567} & \frac{54}{567} \\
\end{array}
\]

**Bonus**

Then ask students to order a list of fractions with the same denominator (EXAMPLE: \(\frac{2}{7}, \frac{3}{7}, \frac{5}{7}\)) from least to greatest, eventually using bigger numerators and denominators and eventually using lists of 4 fractions.

**Bonus**

Then write on the board:

Ask students to think of numbers that are in between these two numbers. Allow several students to volunteer answers. Then ask students to write individually in their notebooks at least one fraction in between:

\[
\begin{array}{cccc}
a) \frac{4}{11} & b) \frac{3}{12} & c) \frac{4}{16} & d) \frac{21}{48} \\
\frac{9}{11} & \frac{9}{12} & \frac{13}{16} & \frac{25}{48} \\
\frac{104}{18301} & \frac{140}{18301} &
\end{array}
\]

**Bonus**
Draw on the board:

\[
\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4}
\]

Have a volunteer colour the first part of each strip of paper and then ask students which fraction shows the most: \(\frac{1}{2}\), \(\frac{1}{3}\) or \(\frac{1}{4}\). **ASK:** Do you think one fifth of this fraction strip will be more or less than one quarter of it? Will one eighth be more or less than one tenth?

**ASK:** Is one half of something always more than one quarter of the same thing? Is half a metre longer or shorter than a quarter of a metre? Is half an hour more or less time than a quarter of an hour? Is half a dollar more money or less money than a quarter of a dollar? Is half an orange more or less than a fourth of the orange? Is half the class more or less than a quarter of the class? If half the class has brown eyes and a quarter of the class has green eyes, do more people have brown eyes or green eyes?

Tell students that no matter what quantity you have, half of the quantity is always more than a fourth of it, so mathematicians say that the fraction \(\frac{1}{2}\) is greater than the fraction \(\frac{1}{4}\). Ask students if they remember what symbol goes in between: \(\frac{1}{2} \quad \frac{1}{4}\) (< or >).

Tell your students that you are going to try to trick them with this next question so they will have to listen carefully. Then **ASK:** Is half a minute longer or shorter than a quarter of an hour? Is half a centimetre longer or shorter than a quarter of a metre? Is half of Stick A longer or shorter than a quarter of Stick B?

- Stick A: 
- Stick B: 

**ASK:** Is a half always bigger than a quarter?

Allow everyone who wishes to attempt to articulate an answer. Summarize by saying: A half of something is always more than a quarter of the same thing. But if we compare different things, a half of something might very well be less than a quarter of something else. When mathematicians say that \(\frac{1}{2} > \frac{1}{4}\), they mean that a half of something is always more than a quarter of the same thing; it doesn’t matter what you take as your whole, as long as it’s the same whole for both fractions.

Draw the following strips on the board:

\[
\underline{\phantom{\frac{1}{2}}} \quad \underline{\phantom{\frac{1}{2}}} \quad \underline{\phantom{\frac{1}{2}}} \quad \underline{\phantom{\frac{1}{2}}}
\]

Ask students to name the fractions and then to tell you which is more.

Have students draw the same fractions in their notebooks but with circles instead of strips. Is \(\frac{3}{4}\) still more than \(\frac{3}{8}\) ? (yes, as long as the circles are the same size)
Bonuses

Show the same fractions using a line of length 8 cm.

Ask students: If you cut the same strip into more and more pieces of the same size, what happens to the size of each piece?

Draw the following picture on the board to help them:

- 1 big piece
- 2 pieces in one whole
- 3 pieces in one whole
- 4 pieces in one whole
- Many pieces in one whole

ASK: Do you think that 1 third of a pie is more or less pie than 1 fifth of the same pie? Would you rather have one piece when it’s cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically (\(\frac{1}{3} > \frac{1}{5}\)).

Do you think 2 thirds of a pie is more or less than 2 fifths of the same pie? Would you rather have two pieces when the pie is cut into 3 pieces or 5 pieces? Which way will you get more? Ask a volunteer to show how we write that mathematically (\(\frac{2}{3} > \frac{2}{5}\)).

If you get 7 pieces, would you rather the pie be cut into 20 pieces or 30? Which way will you get more pie? How do we write that mathematically? (\(\frac{7}{20} > \frac{7}{30}\)).

Give students several problems similar to QUESTION 2 in the workbook. Use bigger numerators and denominators as bonus, always keeping the numerators the same.

Extra Bonus

Have students order the following list of numbers:

\[
\begin{array}{cccccccccc}
21 & 21 & 8 & 19 & 13 & 19 & 13 & 19 & 61 \\
28 & 22 & 200 & 105 & 28 & 105 & 61 &
\end{array}
\]

SAY: Two fractions have the same numerator and different denominators. How can you tell which fraction is bigger? Why? Summarize by saying that the same number of pieces gives more when the pieces are bigger. The numerator tells you the number of pieces, so when the numerator is the same, you just look at the denominator. The bigger the denominator, the more pieces you have to share between and the smaller the portion you get. So bigger denominators give smaller fractions when the numerators are the same.

SAY: If two fractions have the same denominator and different numerators, how can you tell which fraction is bigger? Why? Summarize by saying that if the denominators are the same, the size of the pieces are the same. So just as 2 pieces of the same size are more than 1 piece of that size, 84 pieces of the same size are more than 76 pieces of that size.

Emphasize that students can’t do this sort of comparison if the denominators are not the same. ASK: Would you rather 2 fifths of a pie or 1 half? Draw the following picture to help them:
Tell your students that when the denominators and numerators of the fractions are different, they will have to compare the fractions by drawing a picture or by using other methods that they will learn later.

Have students draw several hundreds blocks on grid paper (or provide them with several already made) and ask them to show the following fractions: \( \frac{32}{100}, \frac{47}{100}, \frac{88}{100}, \frac{63}{100} \).

**ASK:** Which fractions are greater than \( \frac{1}{2} \)? Which fractions are less than \( \frac{1}{2} \)? Which fraction with denominator 100 would be exactly \( \frac{1}{2} \)?

Write the two fractions \( \frac{3}{4} \) and \( \frac{4}{5} \) on the board. **ASK:** Do these fractions have the same numerator? The same denominator? (no, neither) Explain that we cannot compare these fractions directly using the methods in this section, so we need to draw a picture.

![Drawing of fractions]

Have a volunteer shade the fraction 3 fourths on the first strip and the fraction 4 fifths on the second strip. **ASK:** On which strip is a greater area shaded? On which strip is a smaller area unshaded? How many pieces are not shaded? What is the fraction of unshaded pieces in each strip? (\( \frac{1}{4} \) and \( \frac{1}{5} \)) Can you compare these fractions directly? (yes, they have the same numerator). Emphasize that if there is less left unshaded, then there is more shaded. So \( \frac{3}{4} \) is a greater fraction than \( \frac{4}{5} \) because the unshaded part of \( \frac{3}{4} \) (i.e. \( \frac{1}{4} \)) is smaller than the unshaded part of \( \frac{4}{5} \) (i.e. \( \frac{1}{5} \)).

Repeat for other such pairs of fractions:

\[
\frac{7}{8} \text{ and } \frac{8}{9}, \quad \frac{7}{8} \text{ and } \frac{8}{9}, \quad \frac{89}{90} \text{ and } \frac{74}{75}, \quad \frac{3}{5} \text{ and } \frac{4}{6}, \quad \frac{72}{74} \text{ and } \frac{34}{36}, \quad \frac{56}{76} \text{ and } \frac{39}{59}
\]

(draw the pictures when the numerators and denominators are small).

**ACTIVITY 1**

Give students their play dough flags made in a previous activity. Have them organize themselves into groups with people who chose the same two colours they did. For example, suppose 5 people chose red and blue as their two colours. Have those 5 students order their colours in terms of most red to least red and then order the fractions for red from the recipes in order from greatest to least. All students in the group should individually check their results by using fraction strips. Before distributing the BLM “Fraction Strips”, ask the class as a whole why they might expect slight disagreements with the fraction strip results and the play dough results. Which order of fractions do they think will be the correct order? What mistakes may have been made when making the play dough balls? (Some spoons may have had some red play dough still in them when making blue; the play dough may not have been completely flattened all the time.)
ACTIVITY 2

Give each student three strips of paper. Ask them to fold the strips to divide one strip into halves, one into quarters, one into eighths. Use the strips to find a fraction between

a) \( \frac{3}{8} \) and \( \frac{5}{8} \) (one answer is \( \frac{1}{2} \))

b) \( \frac{1}{4} \) and \( \frac{3}{4} \) (one answer is \( \frac{3}{8} \))

c) \( \frac{5}{8} \) and \( \frac{7}{8} \) (one answer is \( \frac{3}{4} \))

ACTIVITY 3

Have students fold a strip of paper (the same length as they folded in ACTIVITY 2) into thirds by guessing and checking. Students should number their guesses.

EXAMPLE:

Is \( \frac{1}{3} \) a good answer for any part of Activity 2? How about \( \frac{2}{3} \)?

Extensions

1. Write the following fractions in order from least to greatest. Explain how you found the order.

   \( \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{8} \)

2. Why is \( \frac{2}{3} \) greater than \( \frac{3}{5} \)? Explain.

3. Why is it easy to compare \( \frac{2}{5} \) and \( \frac{2}{12} \)? Explain.

4. Have students compare \( \frac{13}{87} \) and \( \frac{14}{86} \) by finding a fraction with the same numerator as one of them and the same denominator as the other that is in between both fractions. For instance, \( \frac{13}{86} \) is clearly smaller than \( \frac{14}{86} \) and bigger than \( \frac{13}{87} \). Another way to compare the two given fractions is to note that the second fraction has more pieces (14 instead of 13) and each piece is slightly bigger, so it must represent a bigger fraction.

5. (Atlantic Curriculum A10.4 Grade 5) If you know that \( \frac{2}{5} > \frac{3}{7} \), what do you know about \( \square \)?

6. (Atlantic Curriculum A10.6 Grade 5) Connection to probability: Have students conduct an experiment by rolling 2 coloured dice (one red and one blue) and making a fraction with numerator from the red die and denominator from the blue die) Have students predict whether the fraction will usually be less than half and to verify their prediction by rolling the dice several times.
Mixed Fractions

Introduce mixed fractions by drawing the following picture on the board:

Tell students that some friends ordered 3 pizzas with 4 pieces each. The shaded pieces show how much they have eaten. They ate two whole pizzas plus a quarter of another one. Draw the following pictures on the board.

a)  

b)  

c)  

ASK: How many whole pizzas are shaded? What fraction of the last pizza is shaded? Write the mixed fraction for the first picture and have volunteers write the fractions for the second and third pictures.

Draw models of several mixed fractions, asking students to name them in their notebooks. Use a variety of shapes for the whole piece, such as rectangles and triangles.

Write a fraction such as $3 \frac{1}{4}$ on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

EXAMPLE:

They should shade the first 3 circles and 1 part of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

EXAMPLES: $2 \frac{1}{4}, \ 3 \frac{1}{2}, \ 1 \frac{3}{4}, \ 2 \frac{2}{3}, \ 3 \frac{5}{8}.$

If students have trouble, give them practice drawing the whole number of pies drawn with the correct number of pieces.
Extensions

1. Teach students how to count forwards by halves, thirds, quarters, and tenths beyond 1.
   Ask students to complete the patterns:
   
   a) \(\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \ldots, \ldots, \ldots, \ldots\)
   
   b) \(2\frac{1}{4}, 2\frac{2}{4}, \ldots, \ldots, \ldots, \ldots\)
   
   c) \(\frac{1}{3}, \frac{2}{3}, \ldots, \ldots, \ldots, \ldots\)
   
   d) \(\frac{7}{10}, \frac{8}{10}, \ldots, \ldots, \ldots, \ldots\)

2. What model represents \(2\frac{3}{4}\)? How do you know?

   - A.
   - B.

3. **ASK:** Which fractions show more than a whole? How do you know?
   
   \(2\frac{3}{8}, \frac{4}{7}, 1\frac{4}{5}, 2\frac{3}{8}, \frac{2}{6}\)

4. Ask students to order these fractions from least to greatest: \(\frac{3}{5}, \frac{1}{7}, \frac{7}{11}\)
   
   **ASK:** Did they need to look at the fractional parts at all or just the whole numbers? Why?

**Bonus**

Order the following list of numbers: \(\frac{3}{5}, \frac{5}{7}, \frac{2}{11}, \frac{6}{5}, \frac{8}{7}, \frac{3}{10}\)

5. Ask students to order mixed fractions where some of the whole numbers are the same, and the fractional parts have either the same numerator or the same denominator.

**EXAMPLES:**

   a) \(3\frac{1}{6}, 5\frac{2}{7}, 3\frac{1}{5}\)
   
   b) \(5\frac{3}{9}, 5\frac{3}{8}, 6\frac{1}{11}\)

**Bonus**

Give students longer lists of numbers that they can order using these strategies.
NS6-60
Improper Fractions

Draw the following shapes on the board:

Have students name the fractions shaded. **ASK:** How many parts are shaded? How many parts are in one whole? Tell them that they are all 1 whole and write $1 = \frac{4}{4}$ and $1 = \frac{6}{6}$. Then have student volunteers fill in the blanks:

$$1 = \frac{9}{9} \quad \quad 1 = \frac{7}{7}$$

Then tell them that sometimes they might have more than 1 whole—they might have two whole pizzas, for **EXAMPLE**:

**ASK:** Which number goes on top—the number of parts that are shaded or the number of parts in one whole? Tell them to look at the pictures. Ask them how many parts are in one whole circle and how many parts are shaded.

Then write:

$$2 = \frac{2}{2} \quad \quad 2 = \frac{3}{3}$$

Then ask a volunteer to come and write the number of shaded pieces.

**ASK:** How are the numerator and denominator in each fraction related? (you double the denominator to get the numerator). Have students fill in the missing numbers:

$$2 = \frac{4}{4} \quad 2 = \frac{28}{14} \quad 2 = \frac{76}{38} \quad 2 = \frac{10}{5} \quad 2 = \frac{62}{31}$$

(Always use an even number when giving the numerator.)

**ASK:** How are the fractions above different from the fractions we’ve seen so far? Tell them these fractions are called improper fractions because the numerator is larger than the denominator. Challenge students to guess what a fraction is called if its numerator is smaller than its denominator. (proper fractions)

Draw on the board:
ASK: How many pieces are shaded? (9)

SAY: I want to write a fraction for this picture. Should 9 be the numerator or the denominator? (numerator) Do I usually put the number of shaded parts on top or on bottom? (top) How many equal parts are in 1 whole? (4) Should this be the numerator or the denominator? (denominator) Do we usually put the number of parts in 1 whole on top or on bottom? (bottom) Tell your students that the fraction is written \( \frac{9}{4} \).

Have volunteers write improper fractions for these pictures.

a) 

b) 

c) 

ASK: How many parts are shaded? How many parts are in one whole?

Draw models of several improper fractions, asking students to name them in their notebooks. Use a variety of shapes such as rectangles and triangles for the whole.

Write a fraction such as \( \frac{15}{4} \) on the board. Draw a series of circles subdivided into the same number of parts, as given by the denominator of the fraction (since the denominator of the fraction in this example is 4, each pie has 4 pieces). Ask your students to shade the correct number of pieces in the pies to represent the fraction.

Tell your students that you have drawn more circles than they need so they have to know when to stop shading.

EXAMPLE:

\( \frac{15}{4} \)

They should shade the first 3 circles and 3 parts of the fourth circle.

Have students sketch the pies for given fractions in their notebooks.

EXAMPLE: \( \frac{11}{4} \), \( \frac{15}{8} \), \( \frac{19}{8} \), \( \frac{10}{3} \), \( \frac{12}{5} \).

ASK: Which fractions show more than a whole? How do they know?

\( \frac{13}{5} \), \( \frac{2}{9} \), \( \frac{14}{15} \), \( \frac{8}{3} \), \( \frac{12}{7} \).
Mixed and Improper Fractions

**ASK:** What is a mixed fraction? What is an improper fraction? Have a volunteer write a mixed fraction for this picture and explain their answer:

![Picture of fractions]

Have another volunteer write an improper fraction for the same picture and explain their answer.

**PRIOR KNOWLEDGE REQUIRED**

Mixed fractions
Improper fractions

**VOCABULARY**

mixed fraction
improper fraction

**GOALS**

Students will relate mixed and improper fractions.

**PRIOR KNOWLEDGE REQUIRED**

Draw several models of fractions larger than 1 on the board and have students write both the mixed fraction and the improper fraction. Use several different shapes other than circles.

Draw several more such models on the board and ask students to write an improper fraction if the model contains more than 2 whole pies, and a mixed fraction otherwise. This will allow you to see if students know the difference between the terms “mixed” and “improper”.

Tell students to draw models for the following mixed fractions and to write the corresponding improper fraction that is equal to it (this may be done in 2 different steps if students need it broken down).

- a) $2 \frac{1}{3}$
- b) $3 \frac{1}{2}$
- c) $2 \frac{1}{6}$
- d) $1 \frac{5}{6}$
- e) $3 \frac{2}{5}$
- f) $2 \frac{7}{9}$

Then tell students to draw models for the following improper fractions and then to write the mixed fraction that is equal to it.

- a) $\frac{13}{4}$
- b) $\frac{7}{3}$
- c) $\frac{11}{6}$
- d) $\frac{19}{5}$
- e) $\frac{27}{8}$

**SAY:** You have written many numbers as both a mixed and an improper fraction. What is the same in both? What is different? (the denominators are the same because they tell you how many parts are in a whole, but the numerators will be different because the mixed fraction counts the pieces that make up the wholes separately from the pieces that only make up part of the whole; improper fractions count them all together)

**Extensions**

1. Have students write their answers to these questions as both mixed and improper fractions.

   What fraction of a tens block is 7 ones blocks? 17 ones blocks? 32 ones blocks?

**ACTIVITY**

Students can make models of the fractions on the worksheet by placing the smaller pattern blocks on top of the hexagon (or on top of whatever block is being used to represent the whole).
What fraction of a hundreds block is 32 tens blocks? 43 tens blocks and 5 ones blocks?
How many metres are in 230 cm? 571 cm?
How many decimetres are in 54 centimetres? 98 cm?
What fraction of a dime is a quarter?

2. Ask students to solve these problems with pattern blocks or by sketching their answers on triangular grid paper.
   a) Which two whole numbers is $\frac{23}{6}$ between?
   b) What mixed fraction of a pie would you have if you took away $\frac{1}{6}$ of a pie from 3 pies (and what would the improper fraction be)?

3. Write the following fractions in order:
   $3\frac{1}{4}$, $\frac{27}{4}$, $\frac{11}{4}$, $2\frac{1}{4}$, $\frac{36}{4}$, $4\frac{3}{4}$

**GOALS**
Students will use multiplication to find the improper fraction equivalent to a given mixed fraction.

**PRIOR KNOWLEDGE REQUIRED**
Mixed fractions
Improper fractions
Using pictures to see that mixed and improper fractions can represent the same amount
Multiplication

**VOCABULARY**
mixed fraction
improper fraction

**NS6-62**
**Mixed Fractions (Advanced)**

Draw on the board:

SAY: How many parts are in 1 pie? There are 4 quarters in one pie. How many quarters are in 2 pies? (8)

What operation can we use to tell us the answer? (multiplication) How many quarters are there in 3 pies? (4 × 3 = 12)

ASK: How many quarters are in $3\frac{3}{4}$ pies?

12 pieces
$(3 \times 4)$

3 extra pieces

So there are 15 pieces altogether.

How many halves are in 1 pie? (2) In 2 pies? (4) In 3 pies (6) In 17 pies? (34)
How do you know? What operation did you use to find that? (17 × 2)
**ASK:** How many halves are in 1 \( \frac{1}{2} \) pies? Have a volunteer draw the picture on the board.

**ASK:** How many halves are in 2 \( \frac{1}{2} \) pies? In 3 \( \frac{1}{2} \) pies? In 4 \( \frac{1}{2} \) pies? In 20 \( \frac{1}{2} \) pies? What operations do you use to find the answer? \((20 \times 2 + 1)\)

40 pieces in 20 whole pies

1 extra piece

Draw 8 \( \frac{3}{2} \) pies on the board and **ASK:** How many halves are in 8 \( \frac{3}{2} \)?

Emphasize that the extra half is just one more piece, so once they know how many halves are in 8 pies, they just add one to find how many are in 8 \( \frac{3}{2} \).

Have students write in their notebooks how many halves are in...

- a) 2 \( \frac{1}{2} \)
- b) 5 \( \frac{1}{2} \)
- c) 11
- d) 11 \( \frac{1}{2} \)
- **BONUS:** 49 \( \frac{1}{2} \) 84 \( \frac{1}{2} \)

**ASK:** How many thirds are in 1 pie? (3) Have a volunteer come to the board and divide a circle into thirds. **ASK:** How many thirds are in 2 pies? In 3 pies? In 10 pies? In 100 pies? In 1000 pies? How many thirds are in 1013 pies? In 1023 pies? In 523 pies?

Have students write in their notebooks how many thirds are in...

- a) 2 \( \frac{1}{3} \)
- b) 5 \( \frac{1}{3} \)
- c) 11
- d) 11 \( \frac{2}{3} \)
- **BONUS:** 49 \( \frac{2}{3} \) 84 \( \frac{1}{3} \)

Include questions with denominator 4.

Then introduce problems with a context. **SAY:** I have boxes that will hold 4 cans each. What fraction of a box is each can? (one fourth) How many fourths are in 2 wholes? How many cans will 2 boxes hold? How are these questions the same? How are they different?

A box holds 4 cans. How many cans will:

- a) 1 \( \frac{1}{4} \) boxes hold.
- b) 2 \( \frac{1}{4} \) boxes hold.
- c) 1 \( \frac{1}{4} \) boxes hold.
- d) 1 \( \frac{3}{4} \) boxes hold.

To help your students, encourage them to rephrase the question in terms of fourths and wholes. For example, since a can is one fourth of a whole, a) becomes “How many fourths are in 1 \( \frac{1}{4} \)?”

Next, students will have to rephrase the question in terms of fractions other than fourths, depending on the number of items in each package.

- a) A box holds 6 cans. How many cans will 1 \( \frac{5}{6} \) boxes hold?
- b) A box holds 8 cans. How many cans will 2 \( \frac{3}{8} \) boxes hold?
- c) **BONUS:** A box holds 326 cans. How many cans will 1 \( \frac{5}{326} \) boxes hold?
- d) Tennis balls come in cans of 3. How many balls will 7 \( \frac{1}{3} \) cans hold?
- e) A bottle holds 100 mL of water. How many mL of water will 7 \( \frac{81}{100} \) bottles hold?

Teach students how to change mixed fractions to improper fractions: To change 2 \( \frac{3}{8} \) to an improper fraction, start by calculating how many pieces are in the whole pies \((2 \times 8 = 16)\) and add on the remaining pieces \((16 + 3 = 19)\), so \(2 \frac{3}{8} = \frac{19}{8}\). Give students several problems of this sort, where they convert from mixed to improper form.
Teach your students to explain how to turn a mixed fraction into an improper fraction, using a concrete model (for instance some pizzas or circles—cut into parts) as an example:

I multiply $3 \times 4$ because there are 3 whole pies which each have 4 pieces in them: this gives 12 pieces altogether.

$$3 \frac{1}{4} \quad \begin{array}{c} \includegraphics[height=2cm]{pie_4_parts.png} \\ 3 \times 4 = 12 \end{array}$$

I add 1 more piece because the remaining pie has 1 piece in it: this gives 13 pieces altogether.

$$3 \frac{1}{4} \quad \begin{array}{c} \includegraphics[height=2cm]{pie_4_parts.png} \\ 12 \quad + \quad 1 \quad = \quad 13 \, \text{pieces} \end{array}$$

I keep the denominator the same because it tells the size of the pieces in each pie (which doesn’t change).

$$3 \frac{1}{4} = \frac{13}{4} \quad \leftarrow 13 \, \text{equal parts shaded} \quad \text{4 equal parts in one whole}$$

Have students write the following mixed fractions as improper fractions and explain how they found the answer:

a) $3 \frac{1}{7}$  

b) $5 \frac{1}{6}$  

c) $4 \frac{3}{9}$  

d) $7 \frac{5}{8}$  

e) $6 \frac{5}{6}$
NS6-63
Mixed and Improper Fractions (Advanced)

GOALS
Students will use division with remainders to find the mixed fraction given the corresponding improper fraction.

PRIOR KNOWLEDGE REQUIRED
Reading the improper and mixed fractions from a picture
Finding the improper fraction given the mixed fraction by using multiplication
Division with remainders
The relationship between multiplication and division.

Have each student write in their notebooks the mixed and improper fractions for several pictures displaying area:

Then provide examples involving length and capacity as well, as shown in QUESTION 4 of the worksheet.

How long is the line? How many litres are shown?

Have students write each whole number below as an improper fraction with denominator 2 and show their answer with a picture and a multiplication statement:

\[ a) \ 3 = \frac{5}{2} \qquad b) \ 4 \qquad c) \ 2 \qquad d) \ 7 \qquad e) \ 10 \]

\[ 3 \times 2 = 6 \]

ASK: If I have the improper fraction \( \frac{10}{2} \), how could I find the number of whole pies it represents? Draw on the board:

\[ \text{whole pies} = \frac{10}{2} \text{ pies} \]

Ask students how many whole pies 10 half-sized pieces would make? Show students that they simply divide 10 by 2 to find the answer:
5 whole pies. ASK: How many whole pies are in \( \frac{3}{2} \) pies? In \( \frac{12}{2} \) pies?
In \( \frac{20}{2} \) pies? In \( \frac{18}{2} \) pies? \( \frac{15}{3} \)? \( \frac{20}{4} \)? \( \frac{35}{7} \)? \( \frac{42}{6} \)?

Then have students write the mixed fractions below as improper fractions and show their answer with a picture. Students should also write a statement for the number of half-sized pieces in the pies.

\[ a) \ 3 \frac{1}{2} = \frac{7}{2} \qquad b) \ 4 \frac{1}{2} \qquad c) \ 2 \frac{1}{2} \qquad d) \ 5 \frac{1}{2} \qquad e) \ 8 \frac{1}{2} \]

\[ 3 \times 2 + 1 = 7 \text{ halves} \]

There are 3 pies with 2 halves each.
**Say:** If I have the improper fraction $\frac{15}{2}$, how can I know how many whole pies there are and how many pieces are left over? I want to divide 15 into sets of size 2 and I want to know how many full sets there are and then if there are any extra pieces. What operation should I use? (division) What is the leftover part called? (the remainder)

Write on the board: $15 \div 2 = 7$ Remainder 1, **SO:** $\frac{15}{2} = 7 \frac{1}{2}$.

![Diagram of pies](image)

Draw the following picture on the board with three number statements.

- $\frac{15}{4} = 3 \frac{3}{4}$
- $4 \times 3 + 3 = 15$
- $15 \div 4 = 3$ Remainder 3

Have students discuss the three interpretations of the picture and what they mean. When we divide 15 into sets of size 4, we get 3 sets and then 3 extra pieces left over. This is the same as dividing pies into fourths and seeing that 15 fourths is the same as 3 whole pies (with 4 pieces each) and then 3 extra pieces.

Repeat for several pictures, having volunteers write the mixed and improper fractions as well as the multiplication and division statements. Then have students do similar problems individually in their notebooks.

Then give students improper fractions and have them draw the picture, write the mixed fraction, and the multiplication and division statements.

Then show students how to change an improper fraction into a mixed fraction:

$$2 \frac{1}{4} = 2 \times 4 + 1 \text{ quarters} = 9 \text{ quarters} = \frac{9}{4}$$

Starting with $\frac{9}{4}$, we can find: $9 \div 4 = 2$ Remainder 1, **SO:**

$$\frac{9}{4} = 2 \text{ wholes and 1 more quarter} = 2 \frac{1}{4}$$

Have students change several improper fractions into mixed fractions without using pictures.

Tell them that $\frac{5}{2}$ pies is the same as 3 whole pies and another half a pie. **Ask:** Is this the same thing as 2 whole pies and three halves? Do we ever write $2 \frac{3}{2}$? **Ask:** When we find $7 \div 2$, do we write the answer as 3 Remainder 1 or 2 Remainder 3? Tell your students that as with division, we want to have the fewest number of pieces left over.

**Extension**

Write the following fractions in order: $3 \frac{1}{4}, \frac{27}{4}, \frac{11}{4}, 2 \frac{1}{4}, \frac{36}{4}, 4 \frac{3}{4}$
NS6-64
Investigating Mixed & Improper Fractions

GOALS
Students will make mixed and improper fractions using different shapes as the whole.

PRIOR KNOWLEDGE REQUIRED
Familiarity with pattern blocks
The relationship between mixed and improper fractions

VOCABULARY
mixed fraction
improper fraction

give students pattern blocks or a copy of the pattern blocks BLM (and have them cut out the shapes).

Tell students that a hexagon represents one whole pie and ask them to show you a whole pie. If some students don’t know which piece is the hexagon, ASK: How many sides does a hexagon have? (6) When all students have shown you the hexagon, ask them how many triangles they would need to cover an entire hexagon. What fraction of the hexagon is a triangle? (1/6) Find a shape that is half of the hexagon. How many trapezoids would you need to make 1 1/2 hexagons? How many trapezoids would they need to make 7/2 hexagons? What fraction of a hexagon is a rhombus? How many rhombuses would you need to make 2 hexagons? What improper fraction is equal to 2 wholes? Tell them that there is no mixed fraction for 2 wholes—it is just a whole number.

NOTE: Students can make models of the fractions on the worksheet by placing the smaller pattern blocks on top of the hexagon block (or on top of whatever block is being used to represent the whole).

I want to make 1 1/6 hexagons. How many equal-sized pieces should the hexagon be divided into? What shape should I use for my equal-sized pieces? Why? (I should use triangles because 6 triangles can be put together to make a hexagon) Show how to make 1 1/6 hexagons using triangles.

What shape would you use to make 2 1/2 hexagons? Why? Show how to make 2 1/2 hexagons using trapezoids.

What shape would you use to make 1 1/3 hexagons? Why? Show how to make 1 1/3 hexagons using rhombuses.

SAY: So far, we have used only hexagons as a whole pie. Let’s use trapezoids as a whole pie. What shape would you use to make 2 1/3 trapezoids? Why? What piece is one third of the trapezoid? Show how to make 2 1/3 trapezoids using triangles.

Have students show how to make each fraction below using triangles and to draw pictures of their models in their notebooks:

a) 1 1/3  b) 4 1/3  c) 8/3
d) 11/3  e) 2 2/3  f) 3 1/3

Be sure everyone has done at least parts a) and b) above. ASK: Which 2 fractions from the list above are the same? How can you tell this from your pictures? Use your pictures to order the fractions above from least to greatest.

Write the following problem on the board.
Figure A is a model of 1 whole and Figure B is a model of \( \frac{5}{2} \).

a) Ori says that the shaded area in Figure A is more than the shaded area in Figure B. Is he correct? How do you know?

b) Ori says that because the shaded area of Figure A is greater than the shaded area of Figure B, 1 whole must be more than \( \frac{5}{2} \). What is wrong with his reasoning?

Extensions

1. If \( \frac{4}{5} \) of a structure looks like this: 

   What could the whole look like?

2. If the triangle represents a whole, draw \( 2 \frac{1}{2} \).

3. (Atlantic Mathematics Curriculum) Break egg cartons into sections (1 through 11), and use complete cartons as well. Distribute at least one of the sections to each of the students and say, "If this (whole carton) is one, what is \( \frac{1}{2} \)? If this (9 section piece) is one whole, show me one third. If this (2 sections) is one, show me \( 2 \frac{1}{2} \)," etc. Students should realize that any one section can have many different names depending on the size of the whole. It is also beneficial for students to frame these types of questions for their classmates.

4. What fraction of a metre is a decimetre? 12 decimetres?

5. Ask questions of the form: Stick B is what fraction of Stick A? Stick A is what fraction of Stick B? Write your answers as proper or improper fractions; not as mixed fractions.

Demonstrate with the following example.

A: 

B: 

Stick B is \( \frac{3}{5} \) of Stick A since putting it on top of Stick A will look like:

If Stick A is the whole, then the denominator is 5, because Stick A has 5 equal-sized parts. How many of those parts does Stick B take up? (3). So Stick B is \( \frac{3}{5} \) of Stick A.

What fraction of Stick B is Stick A?

If Stick B is the whole, what is the denominator? (3) Why? (because Stick B has 3 equal-sized pieces) How many of those equal-sized pieces does Stick A take up? (5) So stick A is \( \frac{5}{3} \) of Stick B.
As an improper fraction Stick A is 1 2/3 of Stick B since it takes up one whole Stick B plus 2 more of those equal-sized pieces, but we will write our answers in terms of improper fractions instead of mixed fractions.

Let your students investigate with several examples. In this way, your students will discover reciprocals. If you know what fraction Stick A is of Stick B, what fraction is Stick B of Stick A? (just turn the fraction upside down!)

REFLECT: Why was it convenient to use improper fractions instead of mixed fractions?

---

**NS6-65**

**Equivalent Fractions**

**NS6-66**

**Models of Equivalent Fractions**

**GOALS**

Students will understand that different fractions can mean the same amount. Students will find equivalent fractions by using pictures.

**PRIOR KNOWLEDGE REQUIRED**

Comparing fractions
Fractions as area
Fractions of a set

**VOCABULARY**

equivalent fractions

Show several pairs of fractions on fraction strips and have students say which is larger, for **EXAMPLE**:

\[
\begin{array}{c}
\text{8/9} \\
\text{3/4}
\end{array}
\]

Include many examples where the two fractions are equivalent. Tell your students that when two fractions look different but actually show the same amount, they are called equivalent fractions. Have students find pairs of equivalent fractions from the pictures you have on the board. Tell them that we have seen other examples of equivalent fractions from previous classes and ask if anyone knows where. (There are 2 possible answers here: fractions that represent 1 whole are all equivalent, and the same for fractions representing 2 wholes; also, mixed fractions have an equivalent improper fraction).

Then have students find equivalent fractions by shading the same amount in the second strip as in the first strip and writing the shaded amount as a fraction:

\[
\begin{array}{c}
\text{1/2} \\
\text{2/4}
\end{array}
\]

Show students a fraction strip chart and have volunteers fill in the blank areas.
Shade the fraction $\frac{1}{2}$ and then **ASK**: What other fractions can you see that are equivalent to $\frac{1}{2}$? ($\frac{2}{4}$ or $\frac{4}{8}$ or $\frac{5}{10}$). What other fraction from the chart is equivalent to $\frac{3}{4}$? Repeat with $\frac{8}{10}$, $\frac{3}{5}$, $\frac{1}{5}$ and $\frac{4}{10}$.

What fractions on the chart are equivalent to 1 whole?

Have students find as many fractions as they can that are all equivalent to the fraction shown in the picture.

Draw several copies of a square on the board with half shaded:

<p>| | | |</p>
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Have a volunteer draw a line to cut the square into 4 equal parts. Have another volunteer draw 2 lines to cut the square into 6 equal parts; have another cut the square into 8 equal parts and another into 10 equal parts.

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</table>

Have volunteers name the equivalent fractions shown by the pictures ($\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$)

Then write on the board:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
|\[\frac{1}{2} \times \square = \frac{2}{4}\] & \[\frac{1}{2} \times \square = \frac{3}{6}\] & \[\frac{1}{2} \times \square = \frac{4}{8}\] & \[\frac{1}{2} \times \square = \frac{5}{10}\]

For each picture, **ASK**: How many times more shaded pieces are there? How many times more pieces are there altogether? Emphasize that if each piece (shaded or unshaded) is divided into 4 pieces, then, in particular, each shaded piece is divided into 4 pieces. Hence if the number of pieces in the figure is multiplied by 4, the number of shaded pieces will also be multiplied by 4: that is why you multiply the top and bottom of a fraction by the same number to make an equivalent fraction.

For the pictures below, have students divide each piece into equal parts so that there are a total of 12 pieces. Then have them write the equivalent fractions with the multiplication statements for the numerators and denominators:
Ask students to draw 4 boxes of equal length on grid paper and shade 1 box.

Point out to students that $\frac{1}{4}$ of the area of the boxes is shaded. Now ask students to draw the same set of boxes, but in each box to draw a line dividing the box into 2 parts.

Now $\frac{2}{8}$ of the area is shaded. Repeat the exercise, dividing the boxes into 3 equal parts, (roughly: the sketch doesn’t have to be perfectly accurate), then 4 parts, then five parts.

Point out to your students that while the appearance of the fraction changes, the same amount of area is represented.

$\frac{1}{4}$, $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$ all represent the same amount: They are equivalent fractions.

Ask students how each of the denominators in the fractions above can be generated from the initial fraction of $\frac{1}{4}$. **ANSWER:** each denominator is a multiple of the denominator 4 in the original fraction:

$8 = 2 \times 4$; $12 = 3 \times 4$; $16 = 4 \times 4$; $20 = 5 \times 4$;

Then ask students how each fraction could be generated from the original fraction. **ANSWER:** multiplying the numerator and denominator of the original fraction by the same number:

$\frac{1 \times 2}{4 \times 2} = \frac{2}{8}$; $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$; $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$; $\frac{1 \times 5}{4 \times 5} = \frac{5}{20}$.

Point out that multiplying the top and bottom of the original fraction by any given number, say 5, corresponds to cutting each box into that number of pieces.

$\frac{1 \times 5}{4 \times 5} = \frac{5}{20}$; *there are 5 pieces in each box*

$\frac{5 \times 5}{4 \times 5} = \frac{25}{20}$; *there are 5 pieces in each box* *

$\frac{4 \times 5}{4 \times 5} = \frac{20}{20}$; *there are 4 \times 5 pieces in each box* *

The fractions $\frac{1}{4}$, $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$ … form a family of equivalent fractions. Notice that no whole number
ACTIVITY 1

Use the play dough activity described in NS4-71: Equal Parts and Models of Fractions. Have them investigate what happens when they use different sizes of spoons. Have them make a ball that is one third red and two thirds white using half a teaspoon and then another ball with the same fractions, but using a whole teaspoon. Did they get the same colour? (Yes.) What if they used 2 half teaspoons of red and 4 half teaspoons of white—how is this the same as using one teaspoon of red and 2 teaspoons of white? What pair of equivalent fractions does this show?

ACTIVITY 2

Ask your students to make a model (using concrete materials such as cubes or beads) of a fraction that can be described in two ways. Ask students to describe these fractions. For instance, if the student makes the following model:

They might say “I can say \( \frac{3}{6} \) of the counters are white or \( \frac{1}{2} \) of the counters are white.”

ACTIVITY 3

Give your students 10 counters of one colour and 10 counters of a different colour. Ask them to make a model of a fraction that can be described in at least 3 different ways.

Here are two solutions:

\[
\frac{1}{2} = \frac{2}{4} = \frac{4}{8}
\]

\[
\frac{6}{12} = \frac{2}{4} = \frac{1}{2}
\]

ACTIVITY 4

Give students blocks of 2 colours and have them make models of fractions of whole numbers using the method described in Exercise 5 of the Worksheet NS5-72: Models of Equivalent Fractions. Here are some fractions they might try:

a) \( \frac{3}{4} \) of 15
b) \( \frac{3}{4} \) of 16
c) \( \frac{3}{5} \) of 20
d) \( \frac{2}{7} \) of 21
Extensions

1. Draw a ruler on the board divided into mm and cm, with a certain number of cm shaded, and write two fractions:

\[
\begin{array}{ll}
\text{number of mm shaded} & \text{number of cm shaded} \\
\text{number of mm in total} & \text{number of cm in total}
\end{array}
\]

Are these fractions equivalent? They can also use the metre stick and write triples of equivalent fractions using cm, mm and dm.

2. Write as many equivalent fractions as you can for each picture.

a) ![Image of a ruler divided into mm and cm with shaded sections]

b) ![Image of a grid with shaded sections]

3. List 3 fractions between \(\frac{1}{2}\) and 1. HINT: Change \(\frac{1}{2}\) to an equivalent fraction with a different denominator (EXAMPLE: \(\frac{1}{3}\)) and then increase the numerator or decrease the denominator. Show your answers on a number line (this part is easier if they increase the numerator instead of decrease the denominator).

4. (Atlantic Curriculum A3.1 Grade 5) Ask the students to use their fingers and hands to show that \(\frac{1}{2}\) and \(\frac{5}{10}\) are equivalent fractions.

5. Ask students to use the patterns in numerators and denominators of the equivalent fractions below to fill in the missing numbers.

a) \(\frac{2}{6} = \frac{3}{9} = \frac{4}{15}\)

b) \(\frac{6}{10} = \frac{9}{15} = \frac{12}{20}\)

c) \(\frac{3}{8} = \frac{9}{12} = \frac{12}{16}\)

Ask students if the same patterning method will work to find equivalent fractions in these patterns:

a) \(\frac{2}{3} = \frac{3}{4} = \frac{4}{12} = \frac{15}{15}\)

b) \(\frac{1}{2} = \frac{3}{5} = \frac{5}{8} = \frac{15}{10}\)

Have students discuss what is different about these questions than the ones above. Ensure that students understand that all the fractions in a sequence of equivalent fractions must be obtained by multiplying the numerator and denominator of a particular fraction by the same number. In these examples, the patterns are only obtained through adding, not multiplying, so the pattern will not produce a sequence of equivalent fractions.

6. Once students know how to change a fraction to an equivalent fraction, they can, as an enriched exercise, quickly learn to add fractions. See the JUMP fractions unit, available from our website www.jumpmath.org for material on adding fractions.
NS6-67
Fractions of Whole Numbers

Brainstorm the types of things students can find fractions of (circles, squares, pies, pizzas, groups of people, angles, hours, minutes, years, lengths, areas, capacities, apples).

Brainstorm some types of situations in which it wouldn’t make sense to talk about fractions. For EXAMPLE: Can you say 3 1/2 people went skiing? I folded the sheet of paper 4 1/4 times?

Explain to your students that it makes sense to talk about fractions of almost anything, even people and folds of paper, if the context is right: EXAMPLE: Half of her is covered in blue paint; half the fold is covered in ink. Then teach them that they can take fractions of numbers as well. ASK: If I have 6 hats and keep half for myself and give half to a friend, how many do I keep? If I have 6 apples and half of them are red, how many are red? If I have a pie cut into 6 pieces and half the pieces are eaten, how many are eaten? If I have a rope 6 metres long and I cut it in half, how long is each piece?
Tell your students that no matter what you have 6 of, half is always 3. Tell them that mathematicians express this by saying that the number 3 is half of the number 6.

Tell your students that they can find 1/2 of 6 by drawing rows of dots. Put 2 dots in each row until you have placed 6 dots. Then circle one of the columns:

Step 1

\[
\begin{array}{c}
\bullet \\
\bullet \\
\end{array}
\]

Step 2

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\]

Step 3

\[
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\]

The number of dots in one column is half of 6. Have students find 1/2 of each number using this method:

a) 1/2 of 4  
 b) 1/2 of 8  
 c) half of 10  
 d) half of 14  

Bonus

Use this method to find 1/3 of each of the following numbers. HINT: Put 3 dots in each row.

a) 1/3 of 12  
 b) 1/3 of 15  
 c) one third of 18  
 d) one third of 3  

ASK: If you want to find 2/3 of 12, how many dots in a row would you draw?

4) You would draw 4 columns and circle the dots in 1 column. Now draw 4 sets and share the dots into the sets (1 dot to each of the sets, 4 in total, then another dot to each of the sets, continue till you get 12 dots in total). What fraction of the 12 dots does each set represent? What would you do if you needed 2/3 of 12? (You have to take 3 sets).
1 \frac{1}{4} of 12 = 3

\frac{3}{4} of 12 = 9

Invite volunteers to find \(\frac{3}{4}\) of 16 and \(\frac{2}{5}\) of 15 using this method. After that draw several pictures yourself and ask them to find the fractions they represent.

Teach your students to see the connection between the fact that 6 is 3 twos and the fact that \(\frac{1}{3}\) of 6 is 2. The exercise below will help with this:

Complete the number statement using the words “twos”, “threes”, “fours” or “fives”. Then draw a picture and complete the fraction statements. (The first one is done for you.)

<table>
<thead>
<tr>
<th>Number Statement</th>
<th>Picture</th>
<th>Fraction Statements</th>
</tr>
</thead>
</table>
| a) 6 = 3 twos    |         | \(\frac{1}{3}\) of 6 = ________
|                  |         | \(\frac{2}{3}\) of 6 = ________ |
| b) 12 = 4 ______ |         | \(\frac{1}{4}\) of 12 = ________
|                  |         | \(\frac{2}{4}\) of 12 = ________
|                  |         | \(\frac{3}{4}\) of 12 = ________ |
| c) 15 = 3 ______ |         | \(\frac{1}{3}\) of 15 = ________
|                  |         | \(\frac{2}{3}\) of 15 = ________ |

Then draw the following picture on the board:

Tell students that one person said the picture represented \(\frac{5}{8}\) and another person said the picture represented \(\frac{5}{4}\). Ask your students to explain what both people were thinking. (The picture could represent \(\frac{5}{8}\) because \(\frac{5}{8}\) of the squares are shaded. The picture could represent \(\frac{5}{4}\) because \(\frac{5}{4}\) of one whole block is shaded.) To say what fraction of a figure is shaded, you first have to know what part of the fraction is being taken as the whole.

Ask your students what they would do to find \(\frac{1}{3}\) of 15 dots? (Divide the dots into 3 columns and count how many were in each column). Ask them to find a division statement that would suit the model they drew. (15 \(\div\) 3 = 5)

Write on the board: “\(\frac{1}{3}\) of 15 = 5 means 15 \(\div\) 3 = 5”. Give your students several more statements to rewrite as normal division statements, like \(\frac{1}{6}\) of 12, \(\frac{1}{5}\) of 15, \(\frac{1}{4}\) of 20, \(\frac{1}{3}\) of 16, \(\frac{1}{2}\) of 10, and so on.
Ask your students how they could use the exercise they finished to solve the following problem: circle \( \frac{1}{2} \) of a set of dots: \( \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \). Draw several sets of dots in a line (not in two rows!) and ask them to circle half and then a third.

Ask your students to tell you several ways to find \( \frac{5}{6} \) of 6 dots (or small circles). If the following two solutions do not arise, present them to your students:

1) To find two thirds of 6, you could find one third of six and multiply by 2
2) Circle two out of every 3 dots and count the total number of dots circled.

Have students find fractions of sets of objects (you can give the same number of objects arranged in different ways, for example, have your students find \( \frac{3}{4} \) of 16 boxes with the boxes arranged as \( 4 \times 4 \), \( 8 \times 2 \) or \( 16 \times 1 \) arrays.)

Ask students to present as many methods to find the missing fraction in the statement 2 is ___ of 10 as they can. This time, they have to draw the picture themselves. They should keep making groups of 2 until they reach 10. Have them find the missing fraction for the following statements:

- 4 is ____ of 12, 4 is ____ of 20, 4 is ____ 16, 5 is ____ of 20, etc.

**ASK:** How many months are in a year? How many months are in \( \frac{2}{3} \) of a year? Which is longer? 9 months or \( \frac{2}{3} \) of a year? Which is longer—21 months or \( \frac{3}{5} \) years? **HINT:** How many months are in \( \frac{2}{5} \) years? In \( \frac{1}{5} \) years?

How many minutes are in an hour? How many minutes are in \( \frac{2}{3} \) of an hour? If Rita studied for 35 minutes and Katie studied for \( \frac{2}{3} \) of an hour, who studied longer? Katie started studying at 7:48 PM. Her favourite television show starts at 8:30 PM. Did she finish on time?

Tell students that you want to change word problems about fractions into mathematical sentences. Ask what symbol they would replace each word or phrase by: more than (>) is (=), half (\( \frac{1}{2} \)), three quarters (\( \frac{3}{4} \)).

Write on the board:

- Calli’s age is half of Ron’s age.
- Ron is twelve years old.
- How old is Calli?

Teach students to replace each word they do know with a math symbol and what they don’t know with a blank:

\[
\text{Calli’s age} = \frac{1}{2} \text{ of } 12
\]

Have them do similar problems of this sort.

a) Mark gave away \( \frac{3}{4} \) of his 12 stamps. How many did he give away? (____ = \( \frac{3}{4} \) of 12)

b) There are 8 shapes. What fraction of the shapes are the 4 squares? (____ = \( \frac{4}{8} \))

c) John won three fifths of his five sets of tennis. How many sets did he win? (____ = \( \frac{3}{5} \) of 5)

Then have students change two sentences into one, replacing the underlined words with what they’re referring to:

a) Mark has 12 stamps. He gave away \( \frac{3}{4} \) of them. (Mark gave away \( \frac{3}{4} \) of his 12 stamps)

b) A team played 20 games. They won 11 of them.
Then have students solve several word problems, for EXAMPLE:

Anna had 10 strawberries. She ate two of them. What fraction of her strawberries did she eat?

(Adapted from Atlantic Curriculum A4.3 Grade 5) Give students 18 counters and a large circle divided into 3 equal parts. Ask students to use the circle and the counters to find $\frac{2}{3}$ of 18. Then ask them to explain how they would use the circle to find $\frac{5}{3}$ of 33. Then have them draw a circle they would use to find $\frac{3}{5}$ of 20.

**Activity 1**

Before doing this activity, ensure that students are comfortable finding fractions of numbers such as: $\frac{13}{15}$ of 15. Teach this by saying: What is $\frac{3}{4}$ of 4? If I divide 4 dots into 4 columns, how many do I have in each column? In 3 columns? What is $\frac{3}{7}$ of 7? If I divide 7 dots into 7 columns, how many are in each column? In 5 columns? Repeat until students can tell you that $\frac{13}{15}$ of 15 is 13 without having to divide 15 dots into 15 columns. Then use the BLM “Math Bingo Game (Sample boards)” with the BLM “Cards (Fractions of numbers)”.

**Activity 2**

(Adapted from Atlantic Curriculum A4.3 Grade 5) Give students 18 counters and a large circle divided into 3 equal parts. Ask students to use the circle and the counters to find $\frac{2}{3}$ of 18. Then ask them to explain how they would use the circle to find $\frac{2}{3}$ of 33. Then have them draw a circle they would use to find $\frac{3}{5}$ of 20.

**Extensions**

1. How many months are in:
   a) $\frac{1}{2}$ year?
   b) $\frac{2}{3}$ year?
   c) $1\frac{1}{2}$ years?

2. How many minutes are in:
   a) $\frac{3}{5}$ of an hour?
   b) $\frac{1}{4}$ of an hour?
   c) $1\frac{1}{10}$ of an hour?

3. $\frac{5}{8}$ of a day is how many hours?

4. By weight, about $\frac{1}{5}$ of a human bone is water and $\frac{1}{4}$ is living tissue. If bone weighs 120 grams, how much of the weight is water and how much is tissue?

5. What number is…
   a) $1\frac{1}{2}$ times greater than 10?
   b) $2\frac{1}{2}$ times greater than 9?

6. Rona spent $\frac{2}{5}$ of her money. She had $60 left. How much money did she start with?

7. Explain to your students that you can have a fraction of a fraction! Demonstrate finding half of a fraction by dividing it into a top half and a bottom half:

   $\frac{1}{2}$ of $\frac{3}{5}$ is $\frac{3}{10}$

   Have them find:
   a) $\frac{1}{2}$ of $\frac{5}{6}$
   b) $\frac{1}{2}$ of $\frac{3}{5}$
Then have them draw their own pictures to find:

c) \( \frac{1}{2} \) of \( \frac{3}{7} \)  
d) \( \frac{1}{2} \) of \( \frac{2}{5} \)  
e) \( \frac{1}{2} \) of \( \frac{5}{6} \)  
f) \( \frac{1}{2} \) of \( \frac{4}{7} \)

**Bonus**

Find two different ways of dividing the fraction \( \frac{4}{7} \) in half.

**ANSWER:**

![Diagram of fraction division]

\[ \frac{1}{2} \text{ of } \frac{4}{7} \text{ is } \frac{4}{14} \]  

\[ \frac{1}{2} \text{ of } \frac{4}{7} \text{ is } \frac{2}{7} \]

**ASK:** Are the two fractions \( \frac{2}{7} \) and \( \frac{4}{14} \) equivalent? How do you know?

8. Revisit the worksheet **NS6-56**. Ask students if they remember how they estimated the red section in Question 4 and then to explain their thinking. Then ask them to estimate the fraction of each flag that is blue.

For instance, for the flag of Chile in Part a), a student might say:

Half the flag is not red.

About \( \frac{1}{3} \) of the part that is not red is covered by the blue square with the star in it.

If you take half of a flag and cut it into thirds, you are left with \( \frac{1}{6} \) of the flag (\( \frac{1}{3} \) of \( \frac{1}{2} \) is \( \frac{1}{6} \)).

![Diagram of fraction division]

So \( \frac{1}{6} \) of the flag is covered by the blue square with a star in it.

About half of that square is covered by a white star and the other half is blue. So \( \frac{1}{2} \) of \( \frac{1}{6} \) of the flag is blue, but \( \frac{1}{2} \) of \( \frac{1}{6} \) is \( \frac{1}{12} \).

So altogether about \( \frac{1}{12} \) of the flag is blue.

Ask your students to determine by drawing a picture (as shown above) what the following amounts would be: \( \frac{1}{3} \) of \( \frac{1}{2} \), \( \frac{1}{2} \) of \( \frac{1}{3} \), \( \frac{1}{3} \) of \( \frac{1}{2} \), \( \frac{1}{2} \) of \( \frac{1}{3} \), \( \frac{1}{3} \) of \( \frac{1}{2} \), \( \frac{1}{2} \) of \( \frac{1}{3} \), and so on. Students might compare \( \frac{1}{3} \) of \( \frac{1}{2} \) with \( \frac{1}{2} \) of \( \frac{1}{3} \), \( \frac{1}{2} \) of \( \frac{1}{3} \) with \( \frac{1}{2} \) of \( \frac{1}{3} \), and so on.

Then have them find some flags in an atlas and estimate what fraction of each flag is covered by a particular colour.

Have students choose a sports team or other logo of their choice and decide what fraction of the logo is one colour by using counters. Students could revisit this exercise when they compare and order fractions.
Introduce the phrase **lowest terms**: A fraction is in lowest terms when the only whole number that divides into its numerator and denominator is 1, i.e., 1 is the only factor of both the numerator and denominator.

Have volunteers decide if each fraction is in lowest terms:

\[
\frac{2}{6}, \frac{3}{5}, \frac{1}{4}, \frac{2}{4}
\]

Students should list the factors of each numerator and denominator and see if there are any in common.

Have students copy in their notebooks only those fractions that are in lowest terms:

\[
\frac{3}{6}, \frac{4}{7}, \frac{4}{8}, \frac{4}{9}, \frac{4}{10}, \frac{3}{7}, \frac{2}{8}, \frac{2}{9}, \frac{3}{9}
\]

**Bonus**

\[
\frac{12}{50}, \frac{42}{96}, \frac{36}{175}
\]

Tell students that mathematicians like to write fractions in lowest terms. There are many situations where it is most convenient to work with fractions in lowest terms. Tell students that the process of re-writing a fraction in lowest terms is called reducing the fraction to lowest terms. Tell your students that to reduce a fraction to lowest terms, the first step is to draw a model of the fraction. Draw the following models of \(\frac{2}{4}\) for your students:

![Models of \(\frac{2}{4}\)](image)

Tell students that, to make a smaller numerator and denominator, you have to group pieces so that the shaded pieces are together. **ASK:** In which models is it easiest to group the shaded pieces together? (the first and the fourth) Demonstrate how to group the parts in the first model (and have a volunteer do the fourth model) so that the shaded parts are all together and each part is still an equal part:

![Grouped Models](image)
Now, one of three equal parts are shaded, so $\frac{2}{6} = \frac{1}{3}$. Have students practise drawing models of various fractions and grouping so that only one equal part is shaded. **EXAMPLES:** $\frac{2}{4}$, $\frac{2}{8}$, $\frac{3}{6}$, $\frac{3}{9}$, $\frac{3}{12}$, $\frac{4}{20}$, $\frac{5}{30}$.

Then tell students that sometimes they cannot reduce the numerator to 1. For example, $\frac{4}{6}$ can only be reduced to $\frac{2}{3}$:

Point out to students that the parts in the reduced fraction are “bigger” (there are 2 dots in each part), but the parts are still equal and each one is either completely shaded or completely not shaded. Tell students you want to reduce $\frac{6}{9}$ and put up this array:

**ASK:** Why did I draw the dots in groups of 3? What are the factors of 9? (only 1, 3 and 9) Are any of these also factors of 6? How does drawing the dots in arrays make this easier to see? How many rows of 3 do we need to make 6? (2) Then show this on the board:

Tell students that you want to reduce $\frac{10}{12}$. **ASK:** What are the factors of 12? (1, 2, 3, 4, 6, 12) What possible arrays can we make with 12 dots? Have volunteers demonstrate on the board:

**ASK:** Which factor of 12 is also a factor of 10? (2) Which array uses that factor? Tell students that the array we should use depends on the factors that the numerator and denominator have in common. In this case, we should use the $2 \times 6$ array. **ASK:** How many groups of 2 do we need to make 10? (5) Demonstrate this on the board:

So $\frac{10}{12} = \frac{5}{6}$. 
Have students practise drawing models of various fractions and grouping the models so that each part is either completely shaded or completely not shaded. Students should find all possible arrays and then decide on the most appropriate one to use. **EXAMPLES:**

\[
\frac{6}{8}, \quad \frac{9}{12}, \quad \frac{8}{12}, \quad \frac{10}{15}, \quad \frac{12}{16},
\]

**Bonus**

\[
\begin{array}{cccc}
\frac{8}{20} & \frac{15}{20} & \frac{14}{21} & \frac{20}{35}
\end{array}
\]

When students are done, take up their answers. Start with the first fraction, \(\frac{6}{8}\):

\[
\frac{6}{8} = \frac{3}{4}
\]

Tell students that there are 8 circles divided into 4 groups of 2. **ASK:** What division statements can we write? (\(8 \div 4 = 2\) or \(8 \div 2 = 4\)) There are 6 circles divided into 3 groups of 2. What division statements can we write? (\(6 \div 3 = 2\) or \(6 \div 2 = 3\)). Repeat with the other examples above. Then write on the board:

\[
\begin{array}{cccc}
\frac{6}{8} \div = \frac{3}{4} & \frac{9}{12} \div = \frac{3}{4} & \frac{8}{12} \div = \frac{2}{3} & \frac{10}{15} \div = \frac{2}{3} & \frac{12}{16} \div = \frac{3}{4}
\end{array}
\]

Have students fill in the blanks. **ASK:** What do you notice about the numbers by which you divide the numerator and denominator in each fraction? Look at the pictures—why does this happen? If there are 2 dots in each group, you divide the total number of groups by 2 to get the total number of new groups. (the denominator) Having 2 dots in each group, also means there are 2 dots in each of the shaded groups, so you divide the number of shaded groups by 2 to get the number of new shaded groups. Repeat this reasoning for several examples, and then have volunteers attempt to explain it as well. Then relate this to finding equivalent fractions by multiplying the numerator and denominator by the same number. For example, to find a fraction equivalent to \(\frac{2}{3}\), you could multiply the numerator and denominator by the same number, say 4:

\[
\frac{2}{3} \times 4 = \frac{8}{12}
\]

To reduce \(\frac{8}{12}\), you could divide both the numerator and the denominator by 4:

\[
\frac{8}{12} \div 4 = \frac{2}{3}
\]

Notice that dividing by 4 just gets back the original fraction.

**NOTE:** You should complete the next lesson, **NS6-69**, before assigning Question 4 on worksheet **NS6-68**. (This problem may also be revisited after doing circle graphs in **NS6-107**, but with percents instead of fractions.) Then remind students that they can add fractions with the same denominator by adding the numerators. Challenge students to make these fractions have the same denominator so that they can add them:

\[
\frac{1}{2} + \frac{1}{3}
\]

Students should find the LCM (lowest common multiple) of the denominators 2 and 3, namely 6, and find the new fractions:

\[
\frac{1}{2} = \frac{3}{6} \quad \text{and} \quad \frac{1}{3} = \frac{2}{6}
\]

These new fractions are easy to add: \(\frac{3}{6} + \frac{2}{6} = \frac{5}{6}\), so in fact: \(\frac{1}{2} + \frac{1}{3} = \frac{5}{6}\).
Illustrate this with pictures. Tell students that you had 2 pizzas, but you only have $\frac{1}{2}$ of one left and $\frac{1}{3}$ of the other one left—what fraction of a pizza is left?

\[ \begin{array}{c}
\text{Divide each pie into 6 (LCM of 2 and 3) pieces.} \\
\text{Move them onto one pie.}
\end{array} \]

\[ \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6} \]

Have students find the LCM and use pictures to add pairs of fractions. (EXAMPLES: $\frac{1}{3} + \frac{1}{4}$, $\frac{1}{2} + \frac{1}{5} + \frac{1}{6}$, $\frac{1}{3} + \frac{1}{5} + \frac{4}{15}$, $\frac{1}{3} + \frac{3}{5} + \frac{2}{15}$) Students should reduce their answer whenever possible.

Have students find the missing number to make 1. (EXAMPLES: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$, $\frac{1}{3} + \frac{1}{4} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{15} = 1$, $\frac{1}{10} + \frac{3}{5} + \frac{1}{15} + \frac{1}{30}$)

**Extensions**

1. Teach students that when they have large numerators and denominators, they can reduce fractions in stages. Instead of looking for the largest common factor, they can look for any common factor and then, after dividing by that factor, look at the new smaller numerator and denominator and find another common factor. For EXAMPLE:

\[ \begin{array}{c}
42 \div 2 \quad 21 \div 3 \quad 7 \div 10 \\
60 \div 2 \quad 30 \div 3 \quad 60 \div 6 \\
\text{OR} \\
21 \div 3 \quad 7 \div 10 \\
30 \div 3 \quad 60 \div 6 \\
\end{array} \]

Whether you see right away that 6 is a factor of both doesn’t matter. It might be easier to work with smaller numbers. Have students use this method to reduce:

\[ \frac{12}{38} \quad (\text{ANSWER: } \frac{6}{19}) \quad \frac{42}{140} \quad (\text{ANSWER: } \frac{3}{10}) \quad \frac{45}{210} \quad (\text{ANSWER: } \frac{3}{14}) \]

**Bonus**

\[ \frac{420}{504} \quad (\text{ANSWER: } \frac{5}{7}) \]

2. Teach students to add fractions that add to more than 1: $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$. Draw the picture to show this, as in the lesson. Then have students add various pairs (and groups) of fractions that add to more than 1: $\frac{5}{6} + \frac{1}{4}$, $\frac{2}{3} + \frac{1}{2}$, $\frac{1}{3} + \frac{1}{4} + \frac{5}{6}$, $\frac{1}{2} + \frac{1}{4} + \frac{3}{8}$, $\frac{5}{8} + \frac{3}{4} + \frac{1}{2}$, $\frac{1}{3} + \frac{2}{5} + \frac{7}{15}$.
NS6-69
Lowest Common Multiples in Fractions and
NS6-70
Comparing and Ordering Fractions and
NS6-71
Comparing and Ordering Fractions (Advanced)

Draw the following chart on the board.

<table>
<thead>
<tr>
<th></th>
<th>1 whole</th>
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<tbody>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students fill in the remaining boxes. Then ask volunteers to name fractions that are:

a) less than one third  
b) greater than two thirds  
c) between one half and two thirds  
d) between three fifths and four fifths  
e) between one quarter and one half  
f) equivalent to one half  
g) greater than three fifths  
h) equivalent to one whole

Have students write > (greater than) or < (less than) in between the fractions as appropriate:

a) \( \frac{2}{3} \) \( \frac{3}{4} \)  
b) \( \frac{2}{3} \) \( \frac{3}{5} \)  
c) \( \frac{1}{3} \) \( \frac{2}{5} \)  
d) \( \frac{1}{2} \) \( \frac{5}{6} \)  

ASK: Which fraction is larger: \( \frac{2}{3} \) or \( \frac{3}{7} \)? How do you know? What makes these fractions easy to compare?

Tell your students that you would like to compare \( \frac{1}{3} \) and \( \frac{5}{12} \). How can you turn this problem into a problem like the last one? Emphasize that turning one problem into a different one that they already know how to do is a tool that mathematicians use every day. Is \( \frac{1}{3} \) equivalent to a fraction with denominator 12? Which fraction? \( \frac{4}{12} \). Demonstrate this with a picture:

\[
\frac{1}{3} \times 4 = \frac{4}{12}
\]
Which fraction is larger—$\frac{4}{12}$ or $\frac{5}{12}$? Which fraction is larger—$\frac{1}{3}$ or $\frac{5}{12}$? How do you know?

Have students compare (and demonstrate using a picture):

- a) $\frac{1}{2}$ and $\frac{4}{10}$
- b) $\frac{3}{5}$ and $\frac{5}{6}$
- c) $\frac{3}{5}$ and $\frac{6}{10}$
- d) $\frac{1}{3}$ and $\frac{7}{12}$

**Bonus**

$\frac{4}{10}$ and $\frac{7}{12}$. **Hint:** Use the answers to a) and d).

Then have students compare two fractions, where the denominator of the second divides evenly into the denominator of the first (**Examples:** $\frac{3}{8}$ and $\frac{4}{10}$, $\frac{7}{10}$ and $\frac{3}{5}$, $\frac{3}{10}$ and $\frac{3}{2}$).

Finally, mix up the order of the fractions, so that sometimes the larger denominator is first and other times, it is second. Always make sure that the smaller denominator divides evenly into the larger denominator. Challenge students not to need a picture.

Review finding lowest common multiples (see PA6-11).

Draw two pies on the board with a different fraction shaded in each. (**Examples:** $\frac{1}{3}$ and $\frac{1}{2}$, or $\frac{1}{4}$ and $\frac{1}{2}$, or $\frac{1}{2}$ and $\frac{1}{6}$, or $\frac{1}{2}$ and $\frac{3}{6}$, or $\frac{2}{3}$ and $\frac{3}{4}$, or $\frac{2}{3}$ and $\frac{5}{6}$.)

Have volunteers find i) the number of pieces in each pie and ii) the LCM of these two numbers. Then have volunteers draw lines to divide the pies into that many pieces. Students should then individually write the new fractions in their notebooks and compare them. Which fraction is larger?

Teach students how they can compare two fractions with different denominators without using pictures:

**STEP 1:** Find the lowest common multiple of the two denominators

**STEP 2:** Change both fractions to equivalent fractions with that denominator

**STEP 3:** Compare the two fractions that have the same denominator (the fraction with the larger numerator will be larger)

Be sure to include examples where the larger denominator is first and examples where the larger denominator is second.

Have students compare groups of 3 or 4 fractions by finding the lowest common multiple of all the denominators.

- a) $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{10}$
- b) $\frac{1}{3}$, $\frac{4}{6}$, $\frac{1}{2}$
- c) $\frac{5}{6}$, $\frac{2}{3}$, $\frac{3}{4}$
- d) $\frac{1}{2}$, $\frac{7}{8}$, $\frac{3}{4}$
- e) $\frac{3}{8}$, $\frac{9}{16}$, $\frac{3}{4}$
- f) $\frac{7}{12}$, $\frac{2}{3}$, $\frac{5}{8}$

Have students write the fractions in order from least to greatest:

- a)

  - \[\begin{array}{cccccc}
  & & & & \square & \\
  & & & \square & \square & \square \\
  \square & & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square \\
  \\end{array}\]

- b)

  - \[\begin{array}{cccccccc}
  & & & & & \square & & \square \\
  \square & & \square & & \square & & \square & \square \\
  \square & \square & \square & \square & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square & \square & \square \\
  \square & \square & \square & \square & \square & \square & \square & \square \\
  \\end{array}\]
Extensions

1. Have students place these fractions (0, \(\frac{1}{5}\), \(\frac{2}{5}\), \(\frac{3}{5}\), \(\frac{1}{10}\), \(\frac{1}{20}\), \(\frac{1}{40}\), \(\frac{1}{4}\), \(\frac{1}{12}\), \(\frac{1}{6}\), \(\frac{3}{12}\), \(\frac{3}{4}\), \(\frac{5}{6}\), \(\frac{1}{2}\), \(\frac{1}{4}\), \(\frac{11}{40}\), \(\frac{19}{20}\)) on a number line by finding equivalent fractions over 40.

(Adapted from Atlantic Curriculum A2 Grade 5) Ask students if they think that \(\frac{1}{40}\) is a small fraction or a large fraction. Then discuss how \(\frac{1}{40}\) can represent a small number or a large number. For example, discuss how long \(\frac{1}{40}\) m is. It is 4 times smaller than a tenth of a m, so it is 4 times smaller than 10 cm. Have students hold their fingers apart the distance they think that represents. ASK: Is that a small distance or a large distance? Then ask students whether \(\frac{1}{40}\) of the Canadian population is a small number or a large number. ASK: If you take 1 out of every 40 people, how many of every 1 000 people is that? Write on the board:

\[
\frac{1 \times \frac{1}{40}}{40 \times 1000} = \frac{1000}{1}
\]

Have students find the numerator. Then say: If we take 1 out of every 40 people in Canada, we are taking 25 out of every 1 000. How many out of every 10 000 people is that? (250) Out of every 100 000? (2 500) Out of every 1 000 000? (25 000) If Canada has about 30 000 000 people, how many people is 1 out of 40? (750 000) Tell students that this is about the number of people in all of New Brunswick; quite a large number even though it is a small fraction.

2. Find the next two numbers in each pattern:

a) \(\frac{1}{3}\), \(\frac{2}{6}\), \(\frac{1}{4}\), \(\frac{3}{12}\), \(\frac{1}{5}\), \(\frac{3}{20}\)

HINT: Can you find any equivalent pairs of fractions in the above? Reduce.

b) \(\frac{1}{2}\), \(\frac{2}{6}\), \(\frac{3}{12}\), \(\frac{4}{20}\)

3. Teach students to compare fractions by finding a common numerator instead of a common denominator. For example, to compare \(\frac{1}{3}\) and \(\frac{5}{12}\):

\[
\frac{1 \times 5}{3 \times 5} = \frac{5}{15} < \frac{5}{12}
\]

To compare \(\frac{2}{3}\) and \(\frac{5}{8}\):

\[
\frac{2 \times 5}{3 \times 5} = \frac{10}{15} > \frac{5 \times 2}{8 \times 2} = \frac{10}{16}
\]

The fifteenth-sized pieces are bigger than the sixteenth-sized pieces and there are 10 pieces in both fractions, so 10 fifteenth-sized pieces are more than 10 sixteenth-sized pieces. So, \(\frac{10}{15} > \frac{10}{16}\).

ASK: Is finding a common numerator any harder than finding a common denominator? (no, it’s just as easy) Explain to your students that in fact it is always the same two numbers you are comparing, whether you use common numerators or common denominators. For example,
to compare $\frac{2}{3}$ and $\frac{5}{8}$, you would compare either: $\frac{10}{24}$ and $\frac{15}{24}$ OR $\frac{12}{16}$ and $\frac{10}{16}$. In both cases, you are comparing the numbers 15 and 16 to decide which fraction is greater. There are situations where it may appear easier to use common numerators, simply because the numerators are nicer. For **EXAMPLE:** To compare $\frac{5}{43}$ and $\frac{2}{19}$, you could compare either:

$$\frac{95}{43 \times 19} \quad \text{and} \quad \frac{86}{43 \times 19} \quad \text{OR} \quad \frac{10}{86} \quad \text{and} \quad \frac{10}{95}.$$

In both cases, you need to compare the numbers 95 and 86.

**ASK:** Is finding a common numerator just as useful for adding fractions? Write on the board:

$$\frac{1}{4} + \frac{2}{3} = \frac{5}{6} + \frac{2}{3} \quad \text{OR} \quad \frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12}$$

Explain that both these statements are correct statements, but only one of them is useful. **ASK:** Which one is useful? Why isn’t the other one useful? Explain that most adults who remember anything about fractions (and maybe do reasonably well on the popular TV show “Are you Smarter than a 5th Grader?”) will likely compare fractions by using common denominators instead of common numerators, even though both are equally useful for that purpose. The difference is that common numerators are useless for adding fractions, so people just tend to use common denominators for everything.
NS6-72
Fractions Review

GOALS
Students will compare fractions with the same numerators or same denominators and review equivalent fractions.

PRIOR KNOWLEDGE REQUIRED
Representing proper and improper fractions as pictures
Fraction of a number or set

VOCABULARY
improper fractions
mixed fractions
equivalent fractions

This worksheet is a review of the fractions section and can be used as consolidating homework.

Extensions

1. **ASK:** Which is greater, eight thirds or twelve fifths? (2 and two thirds or 2 and two fifths) and have students write the fractions as improper fractions and then as mixed fractions. Which way makes it easier to compare the fractions? Why?

   **REFLECT:** Recall that when learning reciprocals in Extension 5 of NS6-64: Investigating Mixed and Improper Fractions, it was better to use improper fractions. When comparing 8 thirds to 12 fifths, it is better to change the fractions to mixed fractions. Emphasize that sometimes mixed fractions are more convenient and sometimes improper fractions are more convenient. As students gain more experience they will learn to predict which will be more convenient. It is important to understand both forms so that they can choose which one is more convenient for their purpose.

2. Give pairs of students cards with the following fractions on them:
   - 3/6
   - 2/3
   - 1/4
   - 7/8
   - 5/12
   - 5/6

   Ask them to arrange the cards in order from least to greatest (1/4, 5/12, 3/6, 2/3, 5/6, 7/8) and to give reasons for their arrangement.

   Possible reasons could include: 5/6 is only one step away from reaching the end of a number line divided into 6 parts. Similarly, 7/8 is only one step away from reaching a number line divided into 8 parts, but the steps in 7/8 are smaller, so the person who is 7/8 of the way across is closer to the end than the person who is only 5/6 of the way along the line; 6 steps out of 12 would be a half, and 5/12 is almost a half, so it is more than 1/4.

3. The **BLM "Always, Sometimes, or Never True (Numbers)"** provides a cumulative review of basic number sense, including multiplication, division, divisibility and fractions.
**NS6-73**

**Decimal Hundredths**

**GOALS**

Students will translate fractions with denominator 10 or 100 to decimals.

**PRIOR KNOWLEDGE REQUIRED**

- Fractions
- Thinking of different items (EXAMPLE: a pie, a hundreds block) as a whole
- Decimal place value
- 0 as a place holder

**VOCABULARY**

- decimal
- decimal tenth
- decimal point

---

Draw the following pictures on the board and ask students to show the fraction $\frac{4}{10}$ in each picture:

![Fraction Pictures]

Tell students that mathematicians invented decimals as another way to write tenths: One tenth ($\frac{1}{10}$) is written as 0.1 or just .1. Two tenths ($\frac{2}{10}$) is written as 0.2 or just .2. Ask a volunteer to write $\frac{7}{10}$ in decimal notation. (.7 or 0.7) Ask if there is another way to write it. (0.7 or .7) Then have students write the following fractions as decimals:

a) $\frac{3}{10}$  

b) $\frac{6}{10}$  

c) $\frac{9}{10}$  

d) $\frac{5}{10}$  

e) $\frac{6}{10}$  

f) $\frac{4}{10}$

**Bonus**

Have students convert to an equivalent fraction with denominator 10 and then to a decimal:

a) $\frac{2}{5}$  

b) $\frac{1}{2}$  

c) $\frac{4}{5}$

In their notebooks, have students rewrite each addition statement using decimal notation:

a) $\frac{3}{10} + \frac{1}{10} = \frac{4}{10}$  

b) $\frac{5}{10} + \frac{1}{10} = \frac{6}{10}$  

c) $\frac{6}{10} + \frac{3}{10} = \frac{9}{10}$  

d) $\frac{4}{10} + \frac{1}{10} = \frac{5}{10}$

**Bonus**

a) $\frac{1}{2} + \frac{1}{10} = \frac{7}{10}$  

b) $\frac{1}{2} + \frac{2}{10} = \frac{8}{10}$

Draw on the board:

![Addition Examples]

**ASK:** What fraction does this show? ($\frac{5}{10}$) What decimal does this show? (0.4 or .4)

Repeat with the following pictures:

![Additional Fractions and Decimals]

Have students write the fractions and decimals for similar pictures independently, in their notebooks.
Then ask students to convert the following decimals to fractions, and to draw models in their notebooks:

a) 0.3  

b) .8  

c) .9  

d) 0.2

Demonstrate the first one for them:

0.3 = \frac{3}{10}

Tell students that we use 1 decimal place to write how many tenths there are, and we use 2 decimal places to write how many hundredths there are. Show this picture:

SAY: There are 13 hundredths shaded. We can write this as 0.13 or .13 or \frac{13}{100}.

Have students write both a fraction and a decimal for each of the following pictures:

Tell your students that writing decimals is a little trickier when there are less than 10 hundredths. **ASK:** What if we have only 9 hundredths—would we write .9? If no one recognizes that .9 is 9 tenths, **ASK:** How do we write 9 tenths? Is 9 tenths the same as 9 hundredths? Which is larger?

Put up 2 hundreds blocks (with the grid drawn in) and have one volunteer shade 9 tenths and another volunteer shade 9 hundredths. Tell students that we write 9 tenths as .9 and 9 hundredths as .09; write each decimal beneath the corresponding picture.

Put up the following pictures:

a) 

b) 

c) 

d) 

e) 
f)
Have volunteers write a decimal and a fraction for the first two and then have students do the same for the others independently, in their notebooks.

Put up several more hundreds blocks and tell the students that you are going to mix up the problems. Some pictures will show less than ten hundredths and some will show more than ten hundredths. Students should write the correct decimal and fraction for each picture in their notebooks.

**EXAMPLES:**

One row and one column are shaded. How many squares are shaded? What fraction is shaded?

Discuss various strategies students may have used to count the shaded squares. Did they count the squares to each side of the point at which the row and column overlap (2 + 7 in the row, 4 + 5 in the column, 1 for the square in the middle)? Did they add 10 + 10 for the row and column and then subtract 1 for the one square they counted twice? Did they count the squares in the column (10) and then add 2 + 7 for the squares not yet counted in the row? Did they add the 4 rectangles of white squares (8 + 10 + 28 + 35) and then subtract from 100? Did they push the 4 white rectangles together and see that it forms a 9 by 9 square?

Have students write decimals and fractions for the following pictures.

Encourage students to count the shaded squares using different strategies. Students might count some squares twice and adjust their answers at the end; they might subtract the non-shaded squares from the total; they might divide the shaded parts into convenient pieces, count the squares in the pieces separately, and add the totals. Have students show and explain various solutions to their classmates, so that all students see different ways of counting the squares. Tell students that when they can solve a problem in two different ways and get the same answer, they can know that they’re right. They won’t need you to check the answer because they can check it themselves!
Extensions

1. Put the following sequence on the board: .1, .3, .5, _____

Ask students to describe the sequence (add .2 each time) and to identify the next number (.7). Even though the numbers are not in standard dollar notation, students can think of them in terms of dollars and dimes: .3 is 3 dimes, .5 is 5 dimes, .7 is 7 dimes, and so on. **ASK:** What is 1.3 in terms of dollars and dimes? (1 dollar and 3 dimes)

Have students complete the following sequences by thinking of the numbers in terms of dollars and dimes and counting out loud. This will help students to identify the missing terms, particularly in sequences such as h) and i). Students should also state the pattern rules for each sequence (**EXAMPLE:** start at .1 and add .3).

   a) .1, .4, .7, _____
   b) 3.1, 3.4, 3.7, _____
   c) 1, .9, .8, _____
   d) 1, .8, .6, _____
   e) 3, 2.9, 2.8, _____
   f) 4.4, 4.2, 4, _____
   g) .2, .3, .4, _____, _____, _____
   h) .7, .8, .9, _____, _____, _____
   i) 2.7, 2.8, 2.9, _____, _____, _____
   j) 1.4, 1.3, 1.2, _____, _____, _____

2. Have students draw a line 25 squares long on grid paper and mark the ends as shown:

```
| 4.2                             | 6.7 |
```

Have them mark the location of 4.8, 5.0, and 5.8.

Then repeat with endpoints 42 and 67 and have them mark the locations of 48, 50, and 58. **ASK:** How are these two questions similar and how are they different?

Notice that 4.2 is just 42 tenths and 6.7 is 67 tenths, so the number line with endpoints 42 and 67 can be regarded as counting the number of tenths.
GOALS
Students will practise identifying and distinguishing between tenths and hundredths.

PRIOR KNOWLEDGE REQUIRED
Tenths and hundredths
Decimal notation
Fractional notation
Reading 2-digit whole numbers such as 35 as 3 tens and 5 ones or 35 ones

Put up 2 blank hundreds blocks. Have one volunteer shade one tenth of one block, and have a second volunteer shade one hundredth of the other block. Invite other volunteers to write the corresponding decimals and fractions for each block:

\[
0.1 = \frac{1}{10}
\]

\[
0.01 = \frac{1}{100}
\]

Point to the block showing one tenth and **SAY:** How many hundredths does this block show? How else can we write that as a decimal? (We can write 0.10.) Emphasize that 0.1 is equal to 0.10, and tell students that 0.10 can be read as “10 hundredths” or “1 tenth and 0 hundredths.”

Draw on the board:

**ASK:** How many squares are shaded? (43) How many hundredths does this picture show? (43) How many full rows are shaded? (4) Since 1 full row is 1 tenth, how many tenths are shaded? (4) How many more squares are shaded? (3) **SAY:** There are 4 full rows and 3 more squares shaded. So there are 4 tenths and 3 hundredths. Tell students that we can write 4 tenths and 3 hundredths as follows:

\[
\begin{align*}
\text{ones} & \quad 0.43 & \quad \text{hundredths} \\
\text{tenths} & \quad & \quad \\
\end{align*}
\]

We can read this as “43 hundredths” or “4 tenths and 3 hundredths.”

**ASK:** How is this similar to the different ways we can read the 2-digit number 43? (we can read 43 as 4 tens and 3 ones or just as 43 ones)

On grid paper, have students draw a hundreds block, shade in a given fraction, and then write the fraction as a decimal. (Students can also work on a copy of the BLM “Blank Hundreds Charts.”) When students are done, have them read the decimal number in terms of hundredths only and then in terms of tenths and hundredths. Give students more fractions to illustrate and write as decimals:

\[
\begin{align*}
a) \quad & \frac{36}{100} & b) \quad & \frac{81}{100} & c) \quad & \frac{14}{100} & d) \quad & \frac{41}{100} & e) \quad & \frac{85}{100} & f) \quad & \frac{72}{100} \\
\end{align*}
\]
Draw the following figure on the board:

![Grid Figure]

**ASk:** How many full rows of ten are there? (none) So how many tenths do we have? (0) How many hundredths are there? (4). Write on the board:

\[
\begin{align*}
\text{ones} & \quad 0.04 & \quad \text{hundredths} \\
\text{tenths} & \\
\end{align*}
\]

**ASk:** How can we read this number? (4 hundredths or 0 tenths and 4 hundredths)

Have students draw the following fractions on grid paper and write the corresponding decimals:

a) \( \frac{5}{100} \)  

b) \( \frac{7}{100} \)  

c) \( \frac{1}{100} \)  

d) \( \frac{2}{100} \)  

e) \( \frac{6}{100} \)  

f) \( \frac{3}{100} \)

Finally, draw the following figure on the board:

![Grid Figure]

**ASk:** How many full rows of ten are there? (4) So how many tenths do we have? (4) How many hundredths are in 4 tenths? (40) Are any other hundredths shaded? (no, just the 4 full rows)

Write on the board:

\[
\begin{align*}
\text{ones} & \quad 0.40 & \quad \text{hundredths} \\
\text{tenths} & \\
\end{align*}
\]

Tell students that we can read this as “4 tenths and 0 hundredths” or “40 hundredths.”

Have students draw the following fractions and write the corresponding decimals.

a) \( \frac{60}{100} \)  

b) \( \frac{70}{100} \)  

c) \( \frac{80}{100} \)  

d) \( \frac{30}{100} \)  

e) \( \frac{50}{100} \)  

f) \( \frac{10}{100} \)

Now have students illustrate and rewrite fractions that have either 0 tenths or 0 hundredths:

a) \( \frac{6}{100} \)  

b) \( \frac{60}{100} \)  

c) \( \frac{90}{100} \)  

d) \( \frac{3}{100} \)  

e) \( \frac{20}{100} \)  

f) \( \frac{8}{100} \)

Finally, give students a mix of all 3 types of fractions:

a) \( \frac{36}{100} \)  

b) \( \frac{40}{100} \)  

c) \( \frac{3}{100} \)  

d) \( \frac{18}{100} \)  

e) \( \frac{46}{100} \)  

f) \( \frac{9}{100} \)

**Extension**

Decimals are not the only numbers that can be read in different ways. Show students how all numbers can be read according to place value. The number 34 can be read as “34 ones” or “3 tens and 4 ones.” Similarly, 7.3 can be read as “73 tenths” or “7 ones and 3 tenths.” Challenge students to find 2 ways of reading the following numbers:
a) 3 500 (3 thousands and 5 hundreds or 35 hundreds)
b) 320 (3 hundreds and 2 tens or 32 tens)
c) 5.7 (5 ones and 7 tenths or 57 tenths)
d) 1.93 (19 tenths and 3 hundredths or 193 hundredths)
e) 0.193 (19 hundredths and 3 thousandths or 193 thousandths)

**NS6-75**

**Changing Tenths to Hundredths**

**GOALS**

Students will understand that whole decimal tenths can be written with a 0 in the hundredths position to form an equivalent decimal.

**PRIOR KNOWLEDGE REQUIRED**

Equivalent fractions
Decimal notation
Tenths and hundredths

**VOCABULARY**

equivalent fraction
equivalent decimal

Draw the following figure on the board:

ASK: What fraction of the first square is shaded? What fraction of the second square is shaded? Are these equivalent fractions? How do you know?

Draw the following figure on the board:

ASK: What fraction of the square is shaded? How many of the 100 equal parts are shaded? How many of the 10 equal rows are shaded? Are these equivalent fractions?

Have students use the following pictures to find equivalent fractions with denominators 10 and 100.

\[
\begin{align*}
\text{a) } & \quad \frac{80}{100} = \frac{8}{10} \\
\text{b) } & \quad \frac{10}{100} = \frac{1}{10} \\
\text{c) } & \quad \frac{40}{100} = \frac{4}{10} \\
\end{align*}
\]

Then have students find equivalent fractions without using pictures:

\[
\begin{align*}
\text{a) } & \quad \frac{80}{100} = \frac{8}{10} \\
\text{b) } & \quad \frac{10}{100} = \frac{1}{10} \\
\text{c) } & \quad \frac{40}{100} = \frac{4}{10} \\
\end{align*}
\]
When students can do this confidently, ask them to describe how they are getting their answers. Then remind students that a fraction with denominator 100 can be written as a decimal with 2 decimal places. **ASK:** What decimal is equivalent to \( \frac{80}{100} \)? (0.80) Remind them that a fraction with denominator 10 can be written as a decimal with 1 decimal place and **ASK:** What decimal is equivalent to \( \frac{8}{10} \)? (0.8) Tell them that mathematicians call 0.80 and 0.8 equivalent decimals and ask if anyone can explain why they are equivalent. (They have the same value; the fractions they are equivalent to are equivalent).

Have students rewrite these equivalent fractions as equivalent decimals:

a) \( \frac{80}{100} = \frac{9}{10} \) 

b) \( \frac{20}{100} = \frac{2}{10} \)

c) \( \frac{40}{100} = \frac{4}{10} \)

d) \( \frac{10}{100} = \frac{1}{10} \)

Tell students that saying “0.9 = 0.90” is the same as saying “9 tenths is equal to 90 hundredths or 9 tenths and 0 hundredths.” **ASK:** Is “3 tenths” the same as “3 tenths and 0 hundredths?” How many hundredths is that? Have a volunteer write the equivalent decimals on the board. (.3 = .30)

Have students fill in the blanks:

a) .3 = \( \frac{3}{10} = \frac{30}{100} \) = .____ ____

b) .____ = \( \frac{7}{10} = \frac{70}{100} \) = .70

c) .4 = \( \frac{4}{10} = \frac{40}{100} \) = .____ ____

d) .____ = \( \frac{6}{10} = \frac{60}{100} \) = .____ ____

e) .____ = \( \frac{2}{10} = \frac{20}{100} \) = .____ ____

f) .9 = \( \frac{9}{10} = \frac{90}{100} \) = .____ ____

**Extension**

**ASK:** Do you think that 5 hundredths is the same as 5 hundredths and 0 thousandths? How would we write the decimals for those numbers? (.05 = .050) Do you think that 7 ones is the same as 7 ones and 0 tenths? How would you write that in decimal notation? (7 = 7.0) Have students circle the equivalent decimals in each group of three:

a) 8  0.8  8.0  

b) 0.04  0.40  0.4  

c) 0.03  0.030  0.3  

d) 0.9  9.0  9  

e) 2.0  0.2  0.20
GOALS

Students will relate tenths and hundredths of whole numbers to tenths and hundredths of dollars, that is, to dimes and pennies. Students will use this understanding to compare and order decimals having one and two decimal places.

PRIOR KNOWLEDGE REQUIRED

Pennies, dimes, and dollars
Decimal notation
Writing values such as 3 tenths and 5 hundredths as 35 hundredths

Tell students that a dime is one tenth of a dollar then ASK: Does this mean I can take a loonie and fit 10 dimes onto it? Does it mean 10 dimes weigh the same as a loonie? What does it mean? Make sure students understand that you are referring not to weight or area, but to value—a dime has one tenth the value of a dollar, it is worth one tenth the amount. Ask students what fraction of a dollar a penny is worth. (one hundredth)

Have students find different ways of making $0.54 using only dimes and pennies. (5 dimes and 4 pennies, 4 dimes and 14 pennies, and so on until 54 pennies)

Tell students that since a dime is worth a tenth of a dollar and a penny is worth a hundredth of a dollar, they can write 5 dimes and 4 pennies as 5 tenths and 4 hundredths. ASK: How else could you write 4 dimes and 14 pennies? (4 tenths and 14 hundredths) Continue rewriting all the combinations of dimes and pennies that make $0.54. Tell students that they have seen 2 of these combinations before. Do they remember which ones and why they are special? Prompt by asking: Which way uses the most tenths (or dimes)? The most hundredths (or pennies)?

Then say you have 6 tenths and 7 hundredths and ask students to say this in terms of just hundredths. Have students rewrite both expressions in terms of dimes and pennies. (6 dimes and 7 pennies; 67 pennies)

Then give students a decimal number (EXAMPLE: .18) and have students express it in hundredths only (18 hundredths), tenths and hundredths (1 tenth and 8 hundredths), pennies only (18 pennies), and dimes and pennies (1 dime and 8 pennies). Repeat with several decimals, including some that have no tenths or no hundredths.

EXAMPLES:

| .94 | .04 | .90 | .27 | .70 | .03 | .60 | .58 | .05 |

ASK: Which is worth more, 2 dimes and 3 pennies or 8 pennies? How many pennies are 2 dimes and 3 pennies worth? Which is more, 23 or 8? How would we write 2 dimes and 3 pennies as a decimal of a dollar? (.23) How would we write 8 pennies as a decimal? (.08) Which is more, .23 or .08?

ASK: How would you make $0.63 using pennies and dimes? How would you make .9 dollars using pennies and dimes? Which is more, .63 or .9?

Give students play-money dimes and pennies and ask them to decide which is more between:

a) .4 and .26  b) .4 and .62  c) .3 and .42  d) .3 and .24

Tell the class that you once had a student who said that .41 is more than .8 because 41 is more than 8. ASK: Is this correct? What do you think the student was thinking? Why is the student wrong? (The student was thinking
of the numbers after the decimal point as whole numbers. Since 41 is more than 8, the student thought that .41 would be more than .8. But .8 is 8 tenths and .41 is 41 hundredths. The tenths are 10 times greater than the hundredths, so comparing .8 to .41 is like comparing 8 tens blocks to 41 ones blocks, or 8 cm to 41 mm. There might be more ones blocks, but they’re worth a lot less than the tens blocks. Similarly, there are more mm than there are cm, but 8 cm is still longer than 41 mm.

Students often make mistakes in comparing decimals with 1 and 2 decimal places. For instance, they will say that .17 is greater than .2. This activity will help students understand the relationship between tenths and hundredths.

Give each student play-money dimes and pennies. Remind them that a dime is a tenth of a dollar (which is why it is written as $0.10) and a penny is a hundredth of a dollar (which is why it is written as $0.01). Ask students to make models of the amounts in the left-hand column of the chart below and to express those amounts in as many different ways as possible by filling in the other columns. (Sample answers are shown in italics.) You might choose to fill out the first row together.

<table>
<thead>
<tr>
<th>Amount</th>
<th>Amount in Pennies</th>
<th>Decimal Names (in words)</th>
<th>Decimal Names (in numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 dimes</td>
<td>20 pennies</td>
<td>2 tenths (of a dollar)</td>
<td>.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 hundredths</td>
<td>.20</td>
</tr>
<tr>
<td>3 pennies</td>
<td>3 pennies</td>
<td>3 hundredths</td>
<td>.03</td>
</tr>
<tr>
<td>4 dimes and 3 pennies</td>
<td>43 pennies</td>
<td>4 tenths and 3 hundredths</td>
<td>.43</td>
</tr>
</tbody>
</table>

When students have filled in the chart, write various amounts of money on the board in decimal notation and have students make models of the amounts. (EXAMPLES: .3 dollars, .27 dollars, .07 dollars) Challenge students to make models of amounts that have 2 different decimal representations (EXAMPLES: .2 dollars and .20 dollars both refer to 2 dimes).

When you feel students are able to translate between dollar amounts and decimal notation, ASK: Would you rather have .2 dollars or .17 dollars? In their answers, students should say exactly how many pennies each amount represents; they must articulate that .2 represents 20 pennies and so is actually the larger amount.

For extra practice, ask students to fill in the right-hand column of the following chart and then circle the greater amount in each column. (Create several such charts for your students.)

<table>
<thead>
<tr>
<th>Amount (in dollars)</th>
<th>Amount (in pennies)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>.15</td>
<td></td>
</tr>
</tbody>
</table>
Extension

Have students create a hundredths chart by filling in a blank hundreds chart with hundredths, beginning with .01 in the top left corner and moving across and then down each row.

Ask students to find the following patterns in their charts. They should describe the patterns of where the numbers occur using the words “column” and “row.”

\[
\begin{array}{cccc}
.45 & .68 & .14 & .01 \\
.55 & .78 & .25 & .12 \\
.65 & .88 & .34 & .23 \\
.75 & .98 & .45 & .34 \\
\end{array}
\]

(Start in the fifth row and fifth column, move down one row and then repeat; Start in the seventh row and eighth column, move down one row and then repeat; Start in the second row and fourth column, move one row down and one column right, then one row down and one column left, and then repeat; Start in the first row and first column, move one row down and one column right, then repeat.)

Bonus

Fill in the missing numbers in the patterns below without looking at your chart.

\[
\begin{array}{ccc}
.65 & .33 \\
.75 & .53 \\
.95 & .64 \\
\end{array}
\]
NS6-77
Changing Notation: Fractions and Decimals

GOALS
Students will translate between fractional and decimal notation.

PRIOR KNOWLEDGE REQUIRED
Decimal notation
Fractions
Tenths and hundredths

Have students copy the following chart, with room for rows a) to e), in their notebooks:

<table>
<thead>
<tr>
<th></th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw on the board:

a) ![Chart](image1)
   b) ![Chart](image2)
   c) ![Chart](image3)
   d) ![Chart](image4)
   e) ![Chart](image5)

Have students fill in their charts by recording the number of tenths in the first column and the number of hundredths left over (after they’ve counted the tenths) in the second column. When students are done, have them write in their notebooks the fractions shown and the corresponding decimals. Remind them that hundredths are shown with 2 decimal places: 9 hundredths is written as .09, not .9, because .9 is how we write 9 tenths and we need to make 9 hundredths look different from 9 tenths. (More on the differences later in the lesson.)

Point out, or ask students to describe, the relationship between the chart and the decimal numbers: the tenths (in the first column) are recorded in the first decimal place; the extra hundredths are recorded in the second decimal place.

Have students make 5 (non-overlapping!) hundreds blocks (10 by 10) on grid paper and label them a) to e). (You could also hand out copies of the BLM “Blank Hundreds Charts.”)

While they are doing this, write on the board:

a) .17   b) .05   c) .20   d) .45   e) .03

Tell students to shade in the correct fraction of each square to show the decimal number. When they are done, students should translate the decimals to fractions.

Remind students that the first decimal place (to the right of the decimal point) counts the number of tenths and ASK: What does the second decimal place count? Then write .4 on the board and ASK: What does this number mean—is it 4 tenths, 4 hundredths, 4 ones, 4 hundreds—what? How do you know?
(It is 4 tenths, because the 4 is the first place after the decimal point). **ASK:** If you write .4 as a fraction, what will the denominator be? (You may need to remind students that the denominator is the bottom number.) What will the numerator be? Write \( \frac{4}{10} \) on the board.

Then write on the board: 0.28. **ASK:** How many tenths are in this number? How many hundredths are there in each tenth? How many more hundredths are there? How many hundredths are there altogether? What is the numerator of the fraction? The denominator? Write \( \frac{28}{100} \) on the board.

Have students fill in the numerator of each fraction:

\[
\begin{array}{cccccc}
\text{a) } .6 &=& \frac{6}{10} & \text{b) } .9 &=& \frac{9}{10} & \text{c) } .2 &=& \frac{2}{10} \\
\text{d) } .63 &=& \frac{63}{100} & \text{e) } .97 &=& \frac{97}{100} & \text{f) } .48 &=& \frac{48}{100} \\
\text{g) } .50 &=& \frac{50}{100} & \text{h) } .07 &=& \frac{7}{100} & \text{i) } .8 &=& \frac{8}{10} & \text{j) } .09 &=& \frac{9}{100} \\
\text{k) } .9 &=& \frac{9}{10} 
\end{array}
\]

Then tell students that you are going to make the problems a bit harder. They will have to decide whether the denominator in each fraction is 10 or 100. Ask a volunteer to remind you how to decide whether the fraction should have denominator 10 or 100. Emphasize that if there is only 1 decimal place, it tells you the number of tenths, so the denominator is 10; if there are 2 decimal places, it tells you the number of hundredths, so the denominator is 100. Students should be aware that when we write \( .30 = \frac{3}{10} \), we are saying that 30 hundredths are the same as 3 tenths. The 2 decimal places in \( .30 \) tell us to count hundredths, whereas the denominator of 10 in \( \frac{3}{10} \) tells us to count tenths.

Have students write the fraction for each decimal in their notebooks:

\[
\begin{array}{cccccc}
\text{a) } .4 & \text{b) } .75 & \text{c) } .03 & \text{d) } .3 & \text{e) } .30 & \text{f) } .8 \\
\text{g) } .09 & \text{h) } .42 & \text{i) } .2 & \text{j) } .5 & \text{k) } .50 
\end{array}
\]

**Bonus**

\[
\begin{array}{cccccc}
\text{.789} & \text{.060} & .007 & .053 & .301 & .596 & .0102507 
\end{array}
\]

Write on the board: \( \frac{38}{100} \). **ASK:** How would we change this to a decimal? How many places after the decimal point do we need? (2) How do you know? (Because the denominator is 100, so we’re counting the number of hundredths). Ask volunteers to write decimals for the following fractions:

\[
\begin{array}{cccccc}
\text{a) } \frac{3}{10} & \text{b) } \frac{24}{100} & \text{c) } \frac{8}{10} & \text{d) } \frac{93}{100} & \text{e) } \frac{8}{10} & \text{f) } \frac{7}{10} \\
\text{g) } \frac{80}{100} & \text{h) } \frac{3}{10} & \text{i) } \frac{67}{100} & \text{j) } \frac{7}{100} 
\end{array}
\]

Have them change the following fractions to decimals in their notebooks:

\[
\begin{array}{cccccc}
\text{a) } \frac{29}{100} & \text{b) } \frac{4}{10} & \text{c) } \frac{4}{100} & \text{d) } \frac{13}{100} & \text{e) } \frac{6}{10} & \text{f) } \frac{70}{100} \\
\text{g) } \frac{3}{10} & \text{h) } \frac{30}{100} & \text{i) } \frac{67}{100} & \text{j) } \frac{7}{100} 
\end{array}
\]

**Bonus**

\[
\begin{array}{cccccc}
\text{a) } \frac{293}{1000} & \text{b) } \frac{48}{1000} & \text{c) } \frac{4}{1000} 
\end{array}
\]

**Bonus**

Have students rewrite the following addition statements using decimal notation:

\[
\begin{array}{cccccc}
\text{a) } \frac{3}{10} + \frac{4}{100} = \frac{34}{100} & \text{b) } \frac{40}{100} + \frac{5}{100} = \frac{45}{100} & \text{c) } \frac{21}{100} + \frac{31}{100} = \frac{52}{100} & \text{d) } \frac{22}{100} + \frac{7}{10} = \frac{92}{100} 
\end{array}
\]

Put the following equivalencies on the board and **SAY:** I asked some students to change decimals to fractions and these were their answers. Which ones are incorrect? Why are they incorrect?

\[
\begin{array}{cccccc}
\text{a) } .37 = \frac{37}{100} & \text{b) } .68 = \frac{68}{10} & \text{c) } .4 = \frac{4}{100} & \text{d) } .90 = \frac{90}{10} & \text{e) } .9 = \frac{9}{10} & \text{f) } .08 = \frac{8}{10} 
\end{array}
\]

Review reducing fractions to lowest terms. To reduce a fraction, find a number that is a factor of both the numerator and denominator (called a **common factor** because it is a factor they have in common) and divide both by this common factor. Do the new numerator and denominator have a common factor? Repeat until only 1 is a factor of both the numerator and denominator.
Have students change each decimal tenth (0.1, 0.2, 0.3, ..., 0.9) to a fraction in lowest terms
\( \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10} \). **HINT:** First change the decimal to a fraction with denominator 10.

Then have students change the following decimals to fractions, first with denominator 100, then reduced to lowest terms:

\[ 0.25 \quad 0.30 \quad 0.35 \quad 0.50 \quad 0.22 \quad 0.32 \quad 0.20 \quad 0.24 \quad 0.75 \quad 0.55 \]

**Extensions**

1. Cut the pie into more pieces to show \( \frac{2}{5} = 0.4 \)

2. Draw a letter E covering more than .3 and less than .5 of a 10 \( \times \) 10 grid.

3. **ASK:** When there is 1 decimal place, what is the denominator of the fraction? When there are 2 decimal places, what is the denominator of the fraction? What do you think the denominator of the fraction will be when there are 3 decimal places? How would you change 0.00426 to a fraction? How would you change \( \frac{9823}{1000000} \) to a decimal?

4. Write the following decimals in order by changing the decimals to fractions with denominator 100 and then comparing the fractions:

\[ .8 \quad .4 \quad .07 \quad .17 \quad .32 \quad .85 \quad .3 \]

5. (Adapted from Atlantic Curriculum A10)
Teach students to determine the greatest common factor of two numbers by listing the factors of each number and finding the largest factor that they have in common. Emphasise that the word “common” is used in the sense of “joint” rather than “ordinary” or “occurring often.”

**EXAMPLE:**

\[ 12 : 1 \quad 2 \quad 3 \quad 4 \quad 6 \quad 12 \\
15 : 1 \quad 3 \quad 5 \quad 15 \]

The largest number in both lists is 3, so the greatest common factor of 12 and 15 is 3.

Have students find the greatest common factor (or GCF) of:

a) 8, 12 (**ANSWER:** 4)  
b) 14, 15 (**ANSWER:** 1)

c) 20, 35 (**ANSWER:** 5)  
d) 6, 30 (**ANSWER:** 6)
GOALS
Students will write mixed fractions as decimals. Students will use pictures to compare and order decimals having 1 decimal place with decimals having 2 decimal places.

PRIOR KNOWLEDGE REQUIRED
Translating between decimal and fractional notation for numbers less than 1

VOCABULARY
mixed fraction

ASK: If I use a hundreds block to represent a whole, what can I use to show one tenth? (a tens block) What fraction does a ones block show? (one hundredth)

Draw on the board:

\[ \begin{array}{c}
\text{2 wholes} \\
\hline
\text{4 tenths} \\
\hline
\text{3 hundredths}
\end{array} \]

Tell students that this picture shows the decimal 2.43 or the fraction \( \frac{233}{100} \). Mixed fractions can be written as decimals, too! Have volunteers write the mixed fraction and the decimal shown by each picture:

\[ \begin{array}{c}
a) \quad \begin{array}{c}
\text{2 wholes} \\
\hline
\text{4 tenths}
\end{array} \\
\hline
\text{3 hundredths}
\end{array} \\
\begin{array}{c}
b) \\
\hline
\text{2 wholes} \\
\hline
\text{4 tenths} \\
\hline
\text{3 hundredths}
\end{array} \\
\begin{array}{c}
c) \\
\hline
\text{2 wholes} \\
\hline
\text{4 tenths} \\
\hline
\text{3 hundredths}
\end{array} \]

Have students draw base ten models for these mixed fractions in their notebooks:

\[ \begin{array}{c}
a) \ 1 \frac{30}{100} \\
b) \ 2 \frac{8}{100} \\
c) \ 4 \frac{41}{100}
\end{array} \]

Bonus
First change these fractions to mixed fractions and then draw models:

\[ \begin{array}{c}
a) \ \frac{103}{100} \\
b) \ \frac{314}{100} \\
c) \ \frac{290}{100}
\end{array} \]

Then have students draw a model for each decimal:

\[ \begin{array}{c}
a) \ 3.14 \\
b) \ 2.53 \\
c) \ 4.81
\end{array} \]
Put up the following pictures and have volunteers write a mixed fraction and a decimal for each. Remind students that each fully shaded hundreds block is a whole.

a)  

b)  

**ASK:** How are these different from the base ten pictures? How are they the same? (The difference is that we haven’t pulled the columns (tenths) and squares (hundredths) apart; they’re both lumped together in a shaded square.)

Put up more pictures and have students write mixed fractions and decimals in their notebooks. Include pictures in which there are no tenths or no more hundredths after the tenths are counted.

**EXAMPLES:**

a)  

b)  

Have students draw pictures on grid paper and write decimals for each mixed fraction:

a)  \( \frac{95}{100} \)  

b)  \( \frac{29}{100} \)  

c)  \( \frac{1}{10} \)  

d)  \( \frac{92}{100} \)  

e)  \( \frac{9}{10} \)  

f)  \( \frac{9}{100} \)

Have students change these fractions to decimals without using a picture:

a)  \( 3 \frac{18}{100} \)  

b)  \( 12 \frac{3}{10} \)  

c)  \( 25 \frac{4}{100} \)  

d)  \( 34 \frac{8}{10} \)  

e)  \( 11 \frac{98}{100} \)  

f)  \( 41 \frac{19}{100} \)  

**BONUS:**  \( \frac{5138}{100} \)

Then have students draw pictures on grid paper to show each decimal:

a)  .53  

b)  .03  

c)  .30  

d)  .19  

e)  .8  

f)  .08

Finally, have students draw pictures on grid paper to show each pair of decimals and then to decide which decimal is greater:

a) 3.04 or 3.17  

b) 1.29 or 1.7  

c) 1.05 or 1.50  

d) 5 tenths or 5 hundredths

**Bonus**

Have students draw pictures on grid paper to show each decimal and then to put the decimals in order from smallest to largest. What word do the corresponding letters make?

E. 7.03  

S. 7.30  

I. 2.15  

L. 2.8  

M. 2.09

**NOTE:** The correct word is “miles.” (You might need to explain to students what a mile is.) If any students get “smile” or “slime,” they might be guessing at the correct answer (this is much more likely than having made a mistake in the ordering of the numbers). Encourage them to put the numbers in order before trying to unscramble the letters. Then they can be reasonably confident that their word is the right one. If the numbers are in order and the word makes sense, they don’t need you to confirm their answer. It is important to foster this type of independence.
Extensions

1. Students can make up their own puzzle like the one in the last Bonus questions. Partners can solve each other’s puzzles.

   **STEP 1:** Choose a 3- or 4-letter word whose letters can be used to create other words.

   **EXAMPLE:** The letters in the word “dare” can be rearranged to make “read” or “dear.”

   **NOTE:** This step is crucial. Be sure students understand why it is important for the letters they choose to be able to make at least 2 different words.

   **STEP 2:** Choose one decimal number for each letter and write them in order.

   **EXAMPLE:**
   - d. 1.47
   - a. 1.52
   - r. 2.06
   - e. 2.44

   **STEP 3:** Scramble the letters, keeping the corresponding numbers with them.

   **EXAMPLE:**
   - a. 1.52
   - e. 2.44
   - d. 1.47
   - r. 2.06

   **STEP 4:** Give the scrambled letters and numbers to a partner to put in the correct order. Did your partner find the word you started with?

2. Show .2 in each of these base ten blocks:

   a) ![thousands block]
   b) ![hundreds block]
   c) ![tens block]

3. (Adapted from grade 5 Atlantic Curriculum A6.7 and A7)

   Teach your students that the decimal point is read as “and.” For example, 13.7 is read as “thirteen and seven tenths.”

   Have students list:

   a) whole numbers that take exactly 3 words to say

   **EXAMPLES:** 9 000 080, 600 000, 403

   b) decimal numbers that take exactly 6 words to say

   **EXAMPLES:**
   - 403.08 (four hundred three and eight hundredths)
   - 600 000.43 (six hundred thousand and forty-three hundredths)
   - 9 000 080.09 (nine million eighty and nine hundredths)

   If students are familiar with thousandths and ten thousandths, they might come up with:

   - 3.542 (three and five hundred forty-two thousandths)
   - 500.001 (five hundred and one ten thousandth)

4. (Grade 5 Atlantic Curriculum A7) Teach students to interpret whole numbers written in decimal format (**EXAMPLE:** 5.1 million as 5 100 000)
NS6-79

Decimals and Fractions on Number Lines

**GOALS**
Students will place decimal numbers and mixed fractions on number lines. Students will also write the words for decimals and fractions (proper, mixed, or improper).

**PRIOR KNOWLEDGE REQUIRED**
Decimal numbers with up to 2 decimal places and their equivalent fractions (proper or mixed)
Translating between mixed and improper fractions
Number lines

Draw on the board:

```
0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1
```

Have students count out loud with you from 0 to 1 by tenths: zero, one tenth, two tenths, … nine tenths, one.

Then have a volunteer write the equivalent decimal for 1/10 on top of the number line:

```
0 | 1/10 | 2/10 | 3/10 | 4/10 | 5/10 | 6/10 | 7/10 | 8/10 | 9/10 | 1
```

Continue in random order until all the equivalent decimals have been added to the number line.

Then have students write, in their notebooks, the equivalent decimals and fractions for the spots marked on these number lines:

```
a)  
  |
  |
  |
  |
  |
  |
  |
  |
  |
  | 1
b)  
  |
  |
  |
  |
  |
  |
  |
  |
  |
  |
  | 1
```

c)  
```
  |
  |
  |
  |
  |
  |
  |
  |
  |
  | 1
```

d)  
```
  |
  |
  |
  |
  |
  |
  |
  |
  |
  | 1
```

Have volunteers mark the location of the following numbers on the number line with an X and the corresponding letter.

A. 0.7  B. 2 7/10  C. 1.40  D. 8/10  E. 1 9/10

```
0 | 1 | 2 | 3
```

Invite any students who don’t volunteer to participate. Help them with prompts and questions such as: Is the number more than 1 or less than 1? How do you know? Is the number between 1 and 2 or between 2 and 3? How do you know?

Review translating improper fractions to mixed fractions, then ask students to locate the following improper fractions on a number line from 0 to 3 after changing them to mixed fractions:

```
A. 17/10  B. 23/10  C. 14/10  D. 28/10  E. 11/10
```

```
0 | 1 | 2 | 3
```
When students are done, **ASK:** When the denominator is 10, what is an easy way to tell whether the improper fraction is between 1 and 2 or between 2 and 3? (Look at the number of tens in the numerator—it tells you how many ones are in the number.)

**ASK:** How many tens are in 34? (3) 78? (7) 123? (12) 345? (34)

**ASK:** How many ones are in 34 _ _ _ 10 ? (3) 78 _ _ _ 10 ? (7) 123 _ _ _ 10 ? (12) 345 _ _ _ 10 ? (34)

**ASK:** What two whole numbers is each fraction between?

a) \( \frac{29}{10} \)  

b) 2 \( \frac{4}{10} \)  

c) 12 \( \frac{7}{10} \)  

d) \( \frac{81}{10} \)  

e) \( \frac{127}{10} \)  

f) \( \frac{318}{10} \)

Invite volunteers to answer a) and b) on the board, then have students do the rest in their notebooks. When students are finished, **ASK:** Which 2 fractions in this group are equivalent?

Tell students that there are 2 different ways of saying the number 12.4. We can say “twelve decimal four” or “twelve and four tenths”. Both are correct. (Note that “twelve point four” is also commonly used.) Point out the word “and” between the ones and the tenths, and tell students that we always include it when a number has both ones and tenths (and/or hundredths).

Have students place the following fractions on a number line from 0 to 3:

A. three tenths  
B. two and five tenths  
C. one and seven tenths  
D. one decimal two  
E. two decimal eight

Draw a number line from 0 to 3 on the board. Mark the following points with an X—no numbers—and have students write the number words for the points you marked:

1.3  
.7  
2.4  
.1  
2.8  
2.1

Draw a line on the board with endpoints 0 and 1 marked:

| 0 | 1 |

Ask volunteers to mark the approximate position of each number with an X:

a) .4  

b) \( \frac{6}{10} \)  

c) 0.9

Then draw a number line from 0 to 3 with whole numbers marked:

| 0 | 1 | 2 | 3 |

Ask volunteers to mark the approximate position of these numbers with an X:

a) 2.1  

b) 1 \( \frac{3}{10} \)  

c) \( \frac{29}{10} \)  

d) .4  

e) 2 \( \frac{2}{10} \)

Continue with more numbers and number lines until all students have had a chance to participate. (EXAMPLE: Draw a number line from 0 to 2 with whole numbers marked and have students mark the approximate position of 0.5, 1.25, and others.)

**Bonus**

Use larger whole numbers and/or longer number lines.
Extensions
1. Use a metre stick to draw a line that is 2 metres long and ask students to mark 1.76 metres.
2. Mark the given decimals on the number lines:
   a) Show 0.4
      0 .2
   b) Show 1.5
      0 .5

NS6-80
Comparing and Ordering Fractions and Decimals

GOALS
Students will use fractions (one half, one quarter, and three quarters) as benchmarks for decimals.

PRIOR KNOWLEDGE REQUIRED
Decimals and fractions on number lines

Draw on a transparency:

| 0 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | 1 |

(Note: If you don’t have an overhead projector, tape the transparency to the wall and invite students, in small groups if necessary, to gather around it as you go through the first part of the lesson.) Have a volunteer show where 1/2 is on the number line. Have another number line the same length divided into 2 equal parts on another transparency and superimpose it over this one, so that students see that 1/2 is exactly at the .5 mark. Ask: Which decimal is equal to 1/2? Is 0.2 between 0 and 1/2 or between 1/2 and 1? Is 0.7 between 0 and 1/2 or between 1/2 and 1? What about 0.6? 0.4? 0.3? 0.9?

Go back to the decimal 0.2 and Ask: We know that 0.2 is between 0 and 1/2, but is it closer to 0 or to 1/2? Draw on the board or the transparency:

![Number line with arrow]

Ask: Is .6 between 0 and 1/2 or between 1/2 and 1? Which number is it closest to, 1/2 or 1? Have a volunteer show the distance to each number with arrows. Which arrow is shorter? Which number is .4 closest to, 0, 1/2 or 1? Which number is .8 closest to? Go through all of the remaining decimal tenths between 0 and 1.

On grid paper, have students draw a line 10 squares long. Then have them cut out the line—leaving space above and below for writing—and fold it in half so that the two endpoints meet. They should mark the points 0, 1/2, and 1 on their line. Now have students fold the line in half again, and then fold it in half a second time. Have them unfold the line and look at the folds. Ask: What fraction is exactly halfway between 0 and 1/2? How do you know?
because the sheet is folded into 4 equal parts so the first fold must be \( \frac{1}{4} \) of the distance from 0 to 1) What fraction is halfway between \( \frac{1}{2} \) and 1? How do you know? (\( \frac{3}{4} \) because the sheet is folded into 4 equal parts so the third fold must be \( \frac{3}{4} \) of the distance from 0 to 1).

Have students mark the fractions \( \frac{1}{4} \) and \( \frac{3}{4} \) on their number lines. Then have students write the decimal numbers from .1 to .9 in the correct places on their number lines (using the squares on the grid paper to help them).

Tell students to look at the number lines they’ve created and to fill in the blanks in the following questions by writing “less than” or “greater than” in their notebooks.

a) 0.4 is ________ \( \frac{1}{4} \)  
b) 0.4 is ________ \( \frac{1}{2} \)  
c) 0.8 is ________ \( \frac{3}{4} \)  
d) 0.2 is ________ \( \frac{1}{4} \)  
e) 0.3 is ________ \( \frac{1}{2} \)  
f) 0.7 is ________ \( \frac{3}{4} \)

Have students rewrite each statement using the “greater than” and “less than” symbols: > and <.

**ASK:** Which whole number is each decimal, mixed fraction, or improper fraction closest to?

```
0   1   2   3
a) 0.7    b) 1 \( \frac{1}{10} \)    c) 2.3    d) \( \frac{18}{10} \)    e) 2 \( \frac{6}{10} \)    f) 1.1
```

```
15  16  17  18
a) 17.2    b) 16.8    c) 16 \( \frac{3}{10} \)    d) \( \frac{174}{10} \)    e) 15.9    f) 15.3
```

**ASK:** Which decimal is halfway between 1 and 2? Halfway between 17 and 18? Halfway between 31 and 32? Between 0 and 3? Between 15 and 18? Between 30 and 33? Between 25 and 28?

**Bonus**

Which whole number is each decimal closest to?

a) 23.4    b) 39.8    c) 314.1    d) 235.6    e) 981.1    f) 999.9

**Extension**

(Adapted from grade 5 Atlantic Curriculum A6.3) Have students rearrange the following words to create different numbers. What are the smallest and largest numbers that can be made using these words:

```
hundredths hundred thousand two five nine thirty-seven and
```

**ANSWER:**

Largest number: Thirty-seven thousand nine hundred five and two hundredths  
Smallest number: Two thousand five hundred nine and thirty-seven hundredths
NS6-81

Ordering Fractions and Decimals

To ensure students have the prior knowledge required, ask them to do the following questions in their notebooks.

1. Write each decimal as a fraction or mixed fraction with denominator 10.
   a) 0.7  b) 0.4  c) 1.3  d) 2.9  e) 7.4  f) .6

2. Put the fractions in order from smallest to largest.
   a) 1 $\frac{7}{10}$ 2 $\frac{4}{10}$ 1 $\frac{9}{10}$  b) 1 $\frac{3}{10}$ 1 $\frac{3}{10}$ 2 $\frac{1}{10}$  c) 13 $\frac{7}{10}$ 12 $\frac{5}{10}$ 12 $\frac{3}{10}$

Circulate while students work and assist individuals as required. You can also review these concepts by solving the problems together as a class. Think aloud as you work and invite students to help you. For example,

**SAY:** I want to turn 1.3 into a mixed fraction. What should I do first? Why?

Then write on the board: 2 $\frac{4}{10}$ 2.3 3.7

Tell students you want to order these numbers from smallest to largest.  

**ASK:** How is this problem different from the problems we just did? (Not all the numbers are fractions with denominator 10; some are decimals.) Can we change it into a problem that looks like the ones we just did? How? (Yes, by changing the decimals to fractions with denominator 10.) Have one volunteer change the decimals as described and another volunteer put all 3 fractions in the correct order.

Have students order the following numbers in their notebooks. Volunteers should do the first two or three problems on the board.

a) 0.8 0.4 $\frac{7}{10}$  b) 1.7 .9 $\frac{3}{10}$  c) 8.4 5 $\frac{6}{10}$ 7.7

d) 2.8 1.5 $\frac{7}{10}$  e) 3.7 3.9 $\frac{3}{10}$  f) 8.4 9 $\frac{6}{10}$ 9.7

**Bonus**

Provide problems where students have to change improper fractions to mixed fractions:

g) 3.7 2.9 $\frac{36}{10}$  h) 3.9 3.4 $\frac{38}{10}$  i) 12.1 $\frac{116}{10}$ 12

**Bonus**

Provide problems where one of the fractions has denominator 2 or 5, so that students have to find an equivalent fraction with denominator 10:

j) 3.7 2.9 $\frac{5}{2}$  k) 3.9 3.4 $\frac{7}{2}$  l) 12.3 12 $\frac{2}{5}$ 12

Have volunteers say how many hundredths are in each number:

a) .73  b) .41  c) .62  d) .69  e) .58  f) .50

For each pair of numbers, ask which number has more hundredths and then which number is larger:

a) .73 .68  b) .95 .99  c) .35 .42  d) .58 .57
Now have volunteers say how many hundredths are in each of these numbers:

a) .4  b) .6  c) .5  d) .7  e) .1  f) .8

**ASK:** How can we write 0.4 as a number with 2 decimal places? Remind students that “4 tenths” is equivalent to “4 tenths and 0 hundredths” or “40 hundredths.” This means $0.4 = 0.40$.

For each pair of numbers below, **ASK:** Which number has more hundredths? Which number is larger? Encourage students to change the number with 1 decimal place to a number with 2 decimal places.

a) .48 .5  b) .71 .6  c) .73 .9  d) .2 .17

Put on the board:

Ask volunteers to write 2 different fractions for the amount shaded in the pictures. Have other volunteers change the fractions to decimals.

**ASK:** Do these 4 numbers all have the same value? How do you know? What symbol do we use to show that different numbers have the same value? (the equal sign) Write on the board: $0.9 = 0.90 = \frac{9}{10} = \frac{90}{100}$.

Have students change more decimals to fractions with denominator 100:

a) .6  b) .1  c) .4  d) .8  e) .35  f) .42

Have students put each group of numbers in order by first changing all numbers to fractions with denominator 100:

a) .3 .7 .48  b) $\frac{38}{100}$ $\frac{4}{10}$ 0.39  c) $2 \frac{17}{100}$ 2 $\frac{3}{10}$ 2.2

Now show a hundreds block with half the squares shaded:

**SAY:** This hundreds block has 100 equal squares. How many of the squares are shaded? (50) So what fraction of the block is shaded? ($\frac{50}{100}$) Challenge students to give equivalent answers with different denominators, namely 10 and 2. **PROMPTS:** If we want a fraction with denominator 10, how many equal parts do we have to divide the block into? (10) What are the equal parts in this case and how many of them are shaded? (the rows; 5) What fraction of the block is shaded? ($\frac{5}{10}$) What are the equal parts if we divide the block up into 2? (blocks of 50) What fraction of the block is shaded now? ($\frac{1}{2}$)

Ask students to identify which fraction of the following blocks is shaded:
Challenge them to find a suitable denominator by asking themselves: How many equal parts the size of the shaded area will make up the whole? Have students convert their fractions into equivalent fractions with denominator 100.

\[
\left( \frac{1}{5} = \frac{20}{100}, \quad \frac{1}{4} = \frac{25}{100}, \quad \frac{1}{20} = \frac{5}{100} \right)
\]

Write on the board: \(\frac{2}{5} = \frac{100}{100}\) and \(\frac{3}{5} = \frac{100}{100}\), and have volunteers fill in the blanks. Then have students copy the following questions in their notebooks and fill in the blanks.

\[
\begin{align*}
a) & \quad \frac{2}{5} = \frac{100}{100} \quad \frac{3}{5} = \frac{100}{100} \quad \frac{4}{5} = \frac{100}{100} \quad \frac{5}{5} = \frac{100}{100} \\
b) & \quad \frac{2}{20} = \frac{100}{100} \quad \frac{3}{20} = \frac{100}{100} \quad \frac{4}{20} = \frac{100}{100}
\end{align*}
\]

**BONUS:** \(\frac{17}{20} = \frac{100}{100}\)

Have students circle the larger number in each pair by first changing all numbers to fractions with denominator 100:

\[
\begin{align*}
a) & \quad \frac{1}{2} \quad \text{or} \quad .43 \\
b) & \quad \frac{3}{5} \quad \text{or} \quad 1.6 \\
c) & \quad 3.7 \quad \text{or} \quad 3 \frac{1}{2} \\
d) & \quad \frac{1}{2} \quad \text{or} \quad .57 \\
e) & \quad \frac{1}{2} \quad \text{or} \quad .23 \\
f) & \quad \frac{3}{5} \quad \text{or} \quad .7
\end{align*}
\]

Give students groups of fractions and decimals to order from least to greatest by first changing all numbers to fractions with denominator 100. Include mixed, proper, and improper fractions. Start with groups of only 3 numbers and then move to groups of more numbers.

**Extensions**

1. Students can repeat Extension 1 from **NSS-85**: Decimals and Fractions Greater Than One using a combination of decimals and fractions. Remind students to start with a word whose letters can be used to create other words, and review why this is important. Students can now assign either a decimal or a fraction to each letter, scramble the letters and numbers, and invite a partner to order the numbers to find the original word.

2. Write a decimal for each fraction by first changing the fraction to an equivalent fraction with denominator 100.

\[
\begin{align*}
a) & \quad \frac{2}{5} \\
b) & \quad \frac{1}{2} \\
c) & \quad \frac{1}{4} \\
d) & \quad \frac{3}{5} \\
e) & \quad \frac{11}{20} \\
f) & \quad \frac{47}{50}
\end{align*}
\]

3. If you can walk 12 km in an hour, how many kilometres can you walk in a minute? Write your answer as a decimal number. (**HINT**: Reduce the fraction and then change to a fraction with denominator 100.)

4. Compare without using pictures, and determine which number is larger:

\[
\begin{align*}
a) & \quad 2 \frac{3}{5} \quad \text{and} \quad 17 \text{ sevenths} \\
b) & \quad 1.7 \quad \text{and} \quad 17 \text{ elevenths} \\
c) & \quad 1.5 \quad \text{and} \quad 15 \text{ ninths} \\
d) & \quad 2.9 \quad \text{and} \quad 26 \text{ ninths}
\end{align*}
\]

Students will have to convert all of the numbers into fractions in order to compare them. Is it best to use improper or mixed fractions? You can invite students to try both and see...
which types of fractions are easier to work with in this case (mixed works better for parts a and d, improper works better for parts b and c).

5. Write digits in the boxes that will make the statement true.

\[
\boxed{.5} < \boxed{.3}
\]

---

### NS6-82

#### Thousandths

Review place value up to hundredths, then write the number 6.142 on the board. Cover up all but the first 6 and the decimal point. **ASK:** What is the place value of the 6? How do you know? (The 6 is the ones digit because the decimal point is right next to it). Repeat the exercise, uncovering the 1 and then the 4. Emphasize that each digit is worth ten times less than the previous one. **ASK:** What is ten times less than a tenth? What is ten times less than a hundredth? Uncover the 2 and ask what its place value is.

Ensure that students can identify the place value of any given underlined digit. (EXAMPLES: 3.407, 6.015, 32.809)

Remind students of the connection between fractions and decimal numbers. Ask volunteers to write each decimal number as a fraction or mixed fraction with denominator 10 or 100: 0.7, 0.91, 0.09, 1.5, 1.07. Then **ASK:** How did you know whether the denominator was 10 or 100? (look at the number of decimal places) What if there are 3 decimal places? Then what will the denominator of the fraction be? Have students write each of these decimals as fractions or mixed fractions with denominator 10 or 100 or 1000: 0.9, 0.98, 0.987, 0.098, 0.009, 0.07, 0.652, 0.073.

Show students how to write a decimal number in expanded form by writing the place value of each digit: 3.241 = 3 ones + 2 tenths + 4 hundredths + 1 thousandth. Have students write the place value of each digit for various numbers (EXAMPLES: 2.4, 2.04, 2.004, 20.04, 200.4, 21.35, 2.135, 42.135).

Have students convert the following fractions to decimals:

\[
\begin{align*}
a) \quad \frac{32}{100} &= 0.32 \\
b) \quad \frac{3}{100} &= \boxed{.03} \\
c) \quad \frac{324}{1000} &= \boxed{.324} \\
d) \quad \frac{324}{100} &= \boxed{3.24} \\
e) \quad \frac{32}{1000} &= \boxed{.032} \\
f) \quad \frac{3}{1000} &= \boxed{.003}
\end{align*}
\]

Have students compare the decimals by first changing them to fractions with the same denominator:

\[
\begin{align*}
a) \quad .298 & \quad .32 \\
b) \quad .65 & \quad .541 \\
c) \quad .8 & \quad .312 \\
d) \quad .068 & \quad .07
\end{align*}
\]

Have students compare these same decimals by adding zeroes when necessary to make both decimals have the same number of digits. (EXAMPLE: compare .298 to .320) **ASK:** How is this the same as changing to fractions with the same denominator? How is this different?
Have students practice ordering decimals that have different numbers of decimal places.

**EXAMPLES:**

- a) 6.53 18.2
- b) 456.73 21.72006
- c) 85.7601 112.03
- d) 13.54 13.5

(Adapted from Atlantic Curriculum A2.3) Have students complete sentences such as:

- In 0.1 centuries from now, I could ...
- In 0.01 centuries from now, I could ...
- In 0.001 centuries from now, I could ...
- In 0.0001 centuries from now, I could ...

Repeat the questions with years instead of centuries.

**Extensions**

1. Have students compare numbers with more decimal places and with more digits before the decimal point. **(EXAMPLE: 32.4167 and 298.345)**

2. (From grade 5 Atlantic Curriculum A2.4) Show the students cards on which decimals have been written **(EXAMPLE: 0.75, and 0.265 m)**. Ask students to place the cards appropriately on a metre stick.

3. Challenge students to write decimals for the following fractions:
   - a) \( \frac{1}{10000} \)
   - b) \( \frac{1}{10000} \)
   - c) \( \frac{27}{10000} \)

4. (From the grade 5 Atlantic Curriculum)
   Have students use each of the digits from 0 to 9 once to fill in the 10 spaces and make these statements true.
   \[
   \boxed{.} \boxed{.} < \boxed{.} \boxed{.}
   \]
   \[
   \boxed{.} \boxed{.} \boxed{.} \boxed{.} > \boxed{.} \boxed{.} \boxed{.} \boxed{.}
   \]

5. (Atlantic Curriculum A2.5) Provide thousandths grids (25 cm by 40 cm or 20 by 50). Ask students to shade the grids, one at a time, to show the following decimals:
   - 0.004
   - 0.203
   - 0.023
   - 1.799

6. (Atlantic Curriculum A9.6) Use a calculator to find the decimal forms for a group of fractions, and make as many observations as possible about the fractions obtained. A sample group is:
   - \( \frac{1}{8} \)
   - \( \frac{2}{8} \)
   - \( \frac{3}{8} \)
   - \( \frac{4}{8} \)
   - \( \frac{5}{8} \)
   - \( \frac{6}{8} \)
   - \( \frac{7}{8} \)
   - \( \frac{8}{8} \)

7. Ask students to write the numerals for:
   - two thousandths
   - two thousand
   - twenty thousand
   - twenty thousandths

8. Relate the ordering of numbers to the alphabetical ordering of words. When we put words in alphabetical order, we compare first the leftmost letters, then the next letters over, and so on.
Ask students to put these pairs of words in alphabetical order by identifying the first letter that’s different:

- mouse, mice
- noun, none
- room, rope
- snap, snip
- trick, trim
- sun, fun
- pin, tin
- spin, shin

Students can write the words one above the other and circle the first letter that’s different:

- m i c e
- m o u s e

Tell students that a blank always comes first. Use the words “at” and “ate” to illustrate this. The first two letters in “at” and “ate” are the same. There is no third letter in “at”—there is nothing, or a blank, after the “t.” The blank comes before the “e” at the end of “ate,” so “at” comes before “ate.” Have students use this knowledge to put the following pairs in alphabetical order:

- mat, mate
- an, a
- no, noon
- bath, bat
- kit, kite

Point out the difference between lining up numbers and words. The numbers 61435 and 7384 would be lined up:

- June 14, 35
- July 3, 84

But the words “at” and “ate” would be lined up like this:

- at, not
- ate, not

When ordering words, you line up the leftmost letters. When ordering numbers, you line up the ones digits, whether they’re on the right, the left, or anywhere in between. For both words and numbers, you start comparing from the left.

Another important difference is that 5–digit whole numbers are always greater than 4-digit whole numbers, but 5-letter words can be before or after 4-letter words.

Point out that in numbers, as in words, a space is always less than a number. When we compare:

- 6.53
- 18.2

the blank is really a 0 and is less than, or comes before, any number.
NS6-83
Adding Hundredths

GOALS
Students will add tenths and hundredths by lining up the decimal points.

PRIOR KNOWLEDGE REQUIRED
Adding whole numbers with regrouping
Adding fractions with the same denominator
Knowing how many hundredths are in a number with 2 decimal places
Knowing which number has a given number of hundredths

To ensure students have the prior knowledge required for the lesson, complete the blanks in the following questions as a class or have students complete them independently in their notebooks.

a) 5.3 = ____ tenths  
   b) .8 = ___ tenths  
   c) 1.5 = ____ tenths 
   d) ____ = 49 tenths  
   e) ____ = 78 tenths  
   f) ____ = 4 tenths

Bonus
   ____ = 897 tenths  
   ____ = 54301 tenths  
   ____ = 110 tenths

Review concepts with individual students or the whole class as required.

Now tell students that you want to add some decimals. Write on the board:

2.1
+ 1.0

ASK: How many tenths are in 2.1? (21) How many tenths are in 1.0? (10) How many tenths are there altogether? (31) SAY: There are 31 tenths in the sum. What number is that? (3.1) Write the answer below the addends, being careful to line up the tenths under the tenths and the ones under the ones:

2.1
+ 1.0
3.1

Do a second problem together. This time, write out the number of tenths and add them using the standard algorithm for addition.

Numbers to add Numbers of tenths in those numbers
1.4 14 tenths
+ 2.3 + 23 tenths
3.7 37 tenths

Have students add the following decimals in their notebooks using this method, that is, by adding the whole numbers of tenths first and then turning the answer into a decimal:

a) 3.4  
   b) 2.6  
   c) 8.5  
   d) 3.7  
   BONUS: e) 134.3
   + 1.5  + 4.1  + 1.2  + 4.2  + 245.5

When students are done, tell them that Sonia adds decimal numbers by lining up the ones digits with the ones digits, the tenths digits with the tenths digits, and then adding each digit separately. Show them an example by re-doing question a) this way. ASK: Does Sonia get the right answer with this method? Why do you think that is? How is what Sonia does similar to what you did? Did you line up the digits when you added the whole numbers? Did you add each digit separately?
Write on the board:

\[
\begin{align*}
a) & \quad 3.5 \quad b) \quad .7 \quad c) \quad 192.8 \quad d) \quad 4.8 \quad e) \quad 154.7 \\
+ & \quad 4 \quad + \quad 3.5 \quad + \quad 154 \quad + \quad 12.1 \quad + \quad 16.3
\end{align*}
\]

**ASK:** In which questions are the digits lined up correctly? How can you tell? In all the questions where digits are lined up correctly, what else is also lined up? (the decimal point) Is the decimal point always going to be lined up if the digits are lined up correctly? (yes) How do you know? (The decimal point is always between the ones and the tenths, so if those digits are lined up correctly, then the decimal point will be as well.)

Demonstrate using this method to solve 12.1 + 4.8:

\[
\begin{align*}
12.1 \\
+ \quad 4.8 \\
\hline
16.9
\end{align*}
\]

Tell students that when you add the ones digits, you get the ones digit of the answer and when you add the tenths digits, you get the tenths digit of the answer. If the digits are lined up, then the decimal points are lined up, too—the decimal point in the answer must line up with the decimal points in the addends. Ask students why this makes sense and give several individuals a chance to articulate an answer.

Tell students that as long as they line up the numbers according to the decimal points, they can add decimals just like they add whole numbers. Demonstrate with a few examples, including some that require regrouping and carrying:

\[
\begin{align*}
3.5 & \quad + \quad 1.7 \\
+ & \quad 4 \\
\hline
7.5 & \quad + \quad 4.2 \\
\end{align*}
\]

\[
\begin{align*}
192.8 & \quad + \quad 154 \\
+ & \quad 16.3 \\
\hline
346.8 & \quad + \quad 171.0
\end{align*}
\]

Give students lots of practice adding decimals (with no more than 1 decimal place). Working on grid paper will help students to line up the digits and the decimal points. Include examples with regrouping. Bonus problems could include larger numbers (but still only 1 decimal place). Emphasize that the decimal point is always immediately after the ones digit, so a whole number can be assumed to have a decimal point (EXAMPLE: 43 = 43. = 43.0) It is also important that students line up the digits around the decimal point carefully, perhaps by using grid paper. To emphasize this, have students identify the mistake in:

\[
\begin{align*}
341.7 & \quad + \quad 5216.2 \\
\hline
867.9
\end{align*}
\]

The decimals are technically lined up properly, but the remaining digits are not.

**ASK:** What happened?
Then introduce adding decimals on a number line:

![Number Line](image)

**SAY:** I want to add 1.6 + 0.9. How many tenths are in 1.6? Where is that on the number line? How many tenths are in 0.9? How can I show adding 1.6 to 0.9 on the number line? Draw arrows to illustrate the calculation:

![Arrows](image)

Have students use the number line to add:

a) 2.4 + 1.0  

b) 1.3 + 1.0  

c) 0.5 + 1.0

d) 1.8 + 1.0  

e) 2.6 + 1.0  

f) 0.9 + 1.0

Invite students to use what they know about adding 1.0 to decimals to solve 1.4 + 1.5. **ASK:** What is 1.4 + 1.0? How can we use that to find 1.4 + 1.5? How would you find 1.1 + 2.3? (Students could find 1.1 + 2.0 and then add .3 or they could find 1.0 + 2.3 and then add .1. Both strategies should be discussed.)

Have students add these fractions:

a) \(\frac{35}{100} + \frac{24}{100}\)  
b) \(\frac{25}{100} + \frac{30}{100}\)  
c) \(\frac{25}{100} + \frac{16}{100}\)  
d) \(\frac{25}{100} + \frac{44}{100}\)

Ask students to rewrite their addition statements in terms of decimals. **(EXAMPLE: .3 + .24 = .54)**

Then give students addition problems that require regrouping. **(EXAMPLE: \(\frac{36}{100} + \frac{47}{100}\))** Again, have students rewrite their addition statements in terms of decimals. **(EXAMPLE: .36 + .47 = .83)**

Now have students do the opposite: add decimals by first changing them to fractions with denominator 100. Invite volunteers to do some of the following problems on the board, then have students do the rest independently.

a) .32 + .57  
b) .43 + .16  
c) .81 + .17  
d) .44 + .44  
e) .40 + .33  
f) .93 + .02  
g) .05 + .43  
h) .52 + .20  
i) .83 + .24  
j) .22 + .36

Have a volunteer do a sum that requires regrouping (.54 + .28), then have students add the following independently:

a) .37 + .26  
b) .59 + .29  
c) .39 + .46  
d) .61 + .29

Finally, add decimals whose sum is more than 1 by changing them to fractions first. Have volunteers do 2 examples on the board (.36 + .88 and .45 + .79). Students can then add the following independently:

a) .75 + .68  
b) .94 + .87  
c) .35 + .99  
d) .46 + .64  
e) .85 + .67  
f) .75 + .50  
g) .65 + .4  
h) .7 + .38  
i) .9 + .27

**ASK:** How many hundredths are in .9? (90) How many hundredths are in .27? (27) How many hundredths is that altogether? (117) What number has 117 hundredths? (1.17) What number has 234 hundredths? 5682 hundredths? 48 hundredths? 901 hundredths? 800 hundredths? 80 hundredths? 8 hundredths?
Have students add more decimal numbers by identifying the number of hundredths in each number and then adding the whole numbers:

a) .78 + .4  
  b) .37 + .49  
  c) .85 + .65  
  d) .43 + .34  
  
e) .25 + .52  
  f) .14 + .41  
  g) .76 + .67  
  h) .89 + .98  
  
i) .43 + .87  
  j) .55 + .55  
  k) 1.43 + 2.35  
  l) 3.5 + 2.71  
  
m) 4.85 + 3.09

Remind students that we were able to add tenths by lining up the digits and decimal points. **ASK:** Do you think we can add hundredths the same way? Do an example together:

\[
\begin{array}{c}
4.85 \\
+ 3.09 \\
\hline
7.94
\end{array}
\]

Invite students to help you add the numbers. Tell them to pretend the decimal point isn’t there and to add as though they are whole numbers. **PROMPTS:** What do the hundredths digits add to? Where do I put the 4? the 1? **ASK:** Why can we add as though the decimal points are not there? How many hundredths are in 4.85? (485) In 3.09? (309) In both numbers altogether? (485 + 309 = 794) How does this get the same answer? (794 hundredths = 7.94 ones, so we just line up the decimal points and proceed as though the decimal point is not there)

**ASK:** How can we check our answer? If no one suggests a method, invite a volunteer to add the numbers after rewriting them as fractions with denominator 100:

**Method 1:**
\[
4.85 + 3.09 = 4 \frac{85}{100} + 3 \frac{9}{100} = 7 \frac{94}{100} = 7.94
\]

**Method 2:**
\[
4.85 + 3.09 = \frac{485}{100} + \frac{309}{100} = \frac{794}{100} = 7.94
\]

Emphasize that all methods of addition reduce the problem to one students already know how to do: adding whole numbers. When they add using decimals, students know where to put the decimal point in the answer by lining up the decimal points in the addends. When they add using fractions, students know where to put the decimal point by looking at the denominator. If the denominator of the fraction is 10, they move the decimal point 1 place left. If the denominator of the fraction is 100, they move the decimal point 2 places left.
Give students lots of practice adding decimal hundredths by lining up decimal places. Include numbers that do and do not require regrouping.

EXAMPLES: \(.34 + .28\) \(.65 + .21\) \(.49 + .7\) \(1.3 + .45\) \(2.86 + .9\)

Bonus
\(12.3 + 1.23\) \(354.11 + 4672.6\)

Give students base ten blocks and tell them to use the hundreds block as a whole. This makes the tens block a tenth and the ones block a hundredth. Assign the following problems to pairs or individuals and invite students to share their answers.

1. Start with these blocks:

   \[
   \begin{array}{ccc}
   & & \\
   & & \\
   & & \\
   & & \\
   \end{array}
   \]

   • What decimal does this model represent? (\textbf{ANSWER}: 2.3)
   • Add 2 blocks so that the sum, or total, is between 3.3 and 3.48.
     (\textbf{ANSWER}: add \[
     \begin{array}{ccc}
     & & \\
     & & \\
     & & \\
     & & \\
     \end{array}
     \] or add \[
     \begin{array}{ccc}
     & & \\
     & & \\
     & & \\
     & & \\
     \end{array}
     \])

   • Write a decimal for the amount you added. (\textbf{ANSWER}: 1.1 or 1.01)

2. Start with these blocks:

   \[
   \begin{array}{ccc}
   & & \\
   & & \\
   & & \\
   \end{array}
   \]

   • What decimal does this model represent? (\textbf{ANSWER}: 3.3)
   • Add 2 blocks so that the sum (or total) is between 3.4 and 3.48
   • Write a decimal for the amount you added (\textbf{ANSWER}: add 0.11).

3. Take the same number of blocks as number 1 above:

   \[
   \begin{array}{ccc}
   & & \\
   & & \\
   & & \\
   & & \\
   \end{array}
   \]

   • Add 2 blocks so that the sum is between 2.47 and 2.63. (\textbf{ANSWER}: 2 tens blocks)
   • Write a decimal for the amount you added (\textbf{ANSWER}: 0.2).

4. Take these blocks:

   a) \[
   \begin{array}{ccc}
   & & \\
   & & \\
   & & \\
   & & \\
   \end{array}
   \] or b) \[
   \begin{array}{ccc}
   & & \\
   & & \\
   & & \\
   \end{array}
   \]

   • What decimal does this model represent? (\textbf{ANSWER}: a) 1.42 b) 2.42)
• Add 2 blocks so that the sum is between 2.51 and 2.6.
  (ANSWER: a) add \[\square\] b) add \[\square\]

• Write a decimal for the amount you added. (ANSWER: a) 1.1 b) 0.11)

**Extensions**

1. Write the numbers as decimals and add: \[2 + \frac{3}{10} + \frac{7}{100}\].

2. **ASK:** What if the denominator of a fraction is 1 000—how do we know where to put the decimal point in the decimal number? What is \(.437 + .021\)? Turn the decimals into fractions to add them: \(\frac{437}{1000} + \frac{21}{1000} = \frac{458}{1000}\). **SAY:** The sum is 458 thousandths, or 0.458. Since the denominator of the fraction is 1000, we know to move the decimal point 3 places left. **ASK:** Can we add the decimals by lining up the decimal points? Do we get the same answer? Add the decimals this way to find out.
NS6-84
Subtracting Hundredths

NS6-85
Adding and Subtracting Decimals

Begin by subtracting tenths and hundredths written as fractions. (Start with problems that do not require regrouping and then move to problems that do.)

**EXAMPLES:**

- a) \( \frac{5}{10} - \frac{3}{10} \)
- b) \( \frac{7}{10} - \frac{2}{10} \)
- c) \( \frac{5}{10} - \frac{4}{10} \)
- d) \( \frac{9}{10} - \frac{5}{10} \)
- e) \( \frac{34}{100} - \frac{14}{100} \)
- f) \( \frac{58}{100} - \frac{11}{100} \)
- g) \( \frac{47}{100} - \frac{29}{100} \)
- h) \( \frac{43}{100} - \frac{19}{100} \)

Have students rewrite the subtraction statements in terms of decimals. (Example: \( .34 - .14 = .20 \))

Then have students subtract decimals by first changing them into fractions (proper, mixed, or improper). Include numbers greater than 1.

**EXAMPLES:**

- a) \( .45 - .14 \)
- b) \( .53 - .1 \)
- c) \( .85 - .3 \)
- d) \( 1.23 - .11 \)

Tell students that we can subtract hundredths the same way we add them: by lining up the digits and decimal points. Solve \( 1.93 - .22 \) together and have students check the answer by rewriting the decimals as fractions.

\[
\begin{align*}
1.93 & - .22 \\
1.71 & \\
\end{align*}
\]

Give students a chance to practice subtracting decimal hundredths. Include numbers that require regrouping and numbers greater than 1. **EXAMPLES:**

- a) \( .98 - .42 \)
- b) \( 2.89 - .23 \)
- c) \( 3.49 - 1.99 \)

Remind students of the relationship between missing addends and subtraction. Write on the board: \( 32 + 44 = 76 \). SAY: If \( 32 + 44 = 76 \), what is \( 76 - 44 \)? What is \( 76 - 32 \)? How can I find the missing addend in \( 32 + ____ = 76 \)? (find \( 76 - 32 \)) What is the missing addend in \( .32 + ____ = .76 \)?

Have students find the missing addend in:

- \( .72 + ____ = .84 \)
- \( .9 = ____ + .35 \)
- \( .87 = ____ + .5 \)
- \( .65 + ____ = .92 \)

Have students demonstrate their answers using a model of a hundreds block. For example, students might fill in 72 hundredths with one colour and then fill in squares with another colour until they reach 84 to see that they need 12 more hundredths to make .84 from .72.
As in the previous lesson, give students base ten blocks and tell them to think of the hundreds block as a whole, the tens block as a tenth, and the ones block as a hundredth. Have pairs or individuals solve the following problems:

1. Take these blocks:

   - What decimal does this model represent?
   - Take away 2 blocks so the result (the difference) is between 1.21 and 1.35.
   - Write a decimal for the amount you took away.

2. Take these blocks:

   - What decimal does this model represent?
   - Take away 3 blocks so the result (the difference) is between 1.17 and 1.43.
   - Write a decimal for the amount you took away.
   - Repeat so that the difference is between 2.17 and 2.43.

Make up similar problems for students to solve independently or have students make up their own problems and exchange them with a partner.

**Extension**

Show students how to subtract decimals from 1 by first subtracting from .99:

\[
1 - .74 = .01 + .99 - .74 = .01 + .25 = .26
\]
NS6-86
Multiplying Decimals by 10 and

NS6-87
Multiplying Decimals by 100 and 1 000

Tell your students that a hundreds block represents 1 whole.

**ASK:** How many tens blocks do we need to make a hundreds block? How many tenths do we need to make one whole? Can we write this as a multiplication statement? If we use four 3s to make 12, what multiplication statement do we have? (4 \times 3 = 12) If we use ten 0.1s to make one whole, what multiplication statement do we have? (10 \times 0.1 = 1)

Ten of these: \[ \underline{\phantom{000}} \] equals one of these: \[ \underline{\phantom{000}} \]

**ASK:** If 0.1 is represented by a tens block, how would you represent 0.2? How would you show 10 \times 0.2? (2 hundreds blocks) What is 10 \times 0.2? (2) Repeat for 10 \times 0.3. Have students predict 10 \times 0.7, 10 \times 0.6 and 10 \times 0.9. Students should then show their answers using base ten blocks.

**ASK:** If 0.1 is represented by a tens block, how would you represent 0.01? What block is ten times smaller than a tens block? (a ones block) How would you show 100 \times 0.01? (a hundreds block) What is 100 \times 0.01? (1) Repeat for 100 \times 0.02 and 100 \times 0.03. Have students predict 100 \times 0.07, 100 \times 0.06 and 100 \times 0.09. Students should then show their answers using base ten blocks or models.

Have students use a ones block to represent .01 and make a model to show 10 \times .01 = 0.1

Repeat the exercises above with pennies, dimes and loonies in place of ones, tens and hundreds blocks.

Draw a ruler on the board that shows cm and mm:

\[
\begin{array}{ccccccc}
0 & 10 & 20 & 30 & 40 & 50 & \text{(mm)} \\
0 & 1 & 2 & 3 & 4 & 5 & \text{(cm)}
\end{array}
\]

**ASK:** If you know that the length of an object is 4 cm, how can you find the length of the object in mm? How can you obtain the number of mm from the number of cm? (multiply by 10) Why does this work? (because mm are ten times smaller than cm)

**ASK:** How many mm are in 0.7 cm? Then have a volunteer show this on the ruler you drew. Repeat for other numbers (0.9 cm, 0.3 cm, and so on)
Show a metre stick and repeat the exercise using cm and m.

Ask students to think about how they can multiply $325 \times 10$ by just adding a 0. **ASK:** Where do you add the zero when you multiply by 10? How does the values of each digit change when you multiply a number by 10?

When you multiply by 10: 325 becomes 3 250

300, 20, and 5 become 3 000, 200, and 50

Write the number 346.51 and **ASK:** What is each digit worth? (300, 40, 6, 0.5 and 0.01) How can you move the decimal point so that each digit is worth ten times more (i.e., the ones digit becomes the tens digit, the tens digit becomes the hundreds digit, and so on)? The digits become worth 3000, 400, 60, 5 and 0.1, so the number becomes 3465.1. This is obtained by simply moving the decimal point one place to the right.

**ASK:** How would you find $32.5 \times 10$? Where should you move the decimal point to make every digit worth ten times more?

**ASK:** How is multiplying a decimal number by 10 different from multiplying a whole number by 10? Can we just add a zero to 32.5 to multiply by 10? What would we get if we added 0? (32.50) Does adding 0 to the decimal number change it at all? (No, but adding 0 to the end of a whole number changes the ones digit to 0, and every other digit becomes worth 10 times more.) Tell students that when they multiply a decimal number by 10, they move the decimal point one place to the right. Point out that they are, in fact, doing the same thing when they multiply whole numbers by 10. For example, $17 = 17.0$, so when you multiply by 10 by adding a 0 to the end, you’ve also moved the decimal point over to get 170.

Have students try these problems in their notebooks:

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>a) 0.5 \times 10</td>
<td>b) 10 \times 0.8</td>
<td>c) 1.3 \times 10</td>
<td>d) 10 \times 2.4</td>
<td>e) 134.6 \times 10</td>
<td>f) 10 \times 12.45</td>
</tr>
</tbody>
</table>

**ASK:** How would you find $32.5 \times 100$? Where should you move the decimal point to make every digit worth a hundred times more? How much is each digit worth right now? (30, 2 and 0.5) How much will each digit be worth when you multiply by 100? (3000, 200 and 50) What number is that? (3 250) Explain to students that this is like moving the decimal point two places to the right:

32.5

\[ \rightarrow \]

3250.

Have students try these problems in their notebooks:

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</thead>
<tbody>
<tr>
<td>a) 0.54 \times 100</td>
<td>b) 0.92 \times 100</td>
<td>c) 0.3 \times 100</td>
<td>d) 1.4 \times 100</td>
<td></td>
</tr>
<tr>
<td>e) 432.789 \times 100</td>
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</table>

Remind students that multiplying by 100 is the same as multiplying by 10 and then by 10 again. How do you do multiplying by 100? (move decimal point one place right). So what do you do to multiply by 10 and then by 10 again? (move decimal point two places right) Then have students try multiplying decimals by 1000:

<p>| | | | |</p>
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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>a) 0.613 \times 1000</td>
<td>b) 0.54 \times 1000</td>
<td>c) 1.32 \times 1000</td>
<td>d) 41.6 \times 1000</td>
</tr>
</tbody>
</table>

Can they explain why their method works?

**Extension**

Can your students predict how to multiply by higher powers of 10?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 67 \times 10 000</td>
<td>b) 385 \times 100 000</td>
</tr>
</tbody>
</table>
NS6-88
Multiplying Decimals by Whole Numbers

Using the hundreds block as 1 whole, have volunteers show:

a) 1.23 and then 2 × 1.23
b) 4.01 and then 2 × 4.01
c) 3.12 and then 3 × 3.12

Have students individually solve the following problems by multiplying each digit separately:

a) 4.12 = _____ ones + _____ tenths + _____ hundredths
   2 × 4.12 = _____ ones + _____ tenths + _____ hundredths = _____

b) 3.11 = _____ ones + _____ tenths + _____ hundredths
   3 × 3.11 = _____ ones + _____ tenths + _____ hundredths = _____

c) 1.02 = _____ ones + _____ tenths + _____ hundredths
   4 × 1.02 = _____ ones + _____ tenths + _____ hundredths = _____

Have students multiply mentally:

a) 4 × 2.01
b) 3 × 2.31
c) 3 × 1.1213

d) 2 × 1.114312

Bonus
e) 3 × 1.1212231

Have students solve the following problems by regrouping when necessary:

a) 3 × 4.42 = _____ ones + _____ tenths + _____ hundredths
   = _____ ones + _____ tenths + _____ hundredths
   = _____

b) 4 × 3.32 = _____ ones + _____ tenths + _____ hundredths
   = _____ ones + _____ tenths + _____ hundredths
   = _____

c) 3 × 3.45 = _____ ones + _____ tenths + _____ hundredths
   = _____ ones + _____ tenths + _____ hundredths
   = _____ ones + _____ tenths + _____ hundredths
   = _____

Then have students solve the following problems by regrouping when necessary:

a) 3 × 442 = _____ hundreds + _____ tens + _____ ones
   = _____ hundreds + _____ tens + _____ ones
   = _____

b) 4 × 332 = _____ hundreds + _____ tens + _____ ones
   = _____ hundreds + _____ tens + _____ ones
   = _____
c) $3 \times 345 = \text{____ hundreds + ____ tens + ____ ones}$
   $= \text{____ hundreds + ____ tens + ____ ones}$
   $= \text{____}$

Discuss with students the similarities and differences between these problems and solutions.
Remind students about the standard algorithm for multiplying 3-digit by 1-digit numbers and
ask students if they think they can use the standard algorithm for multiplying decimal numbers.
Emphasize that none of the digits in the answer changes when the question has a decimal point;
only the place value of the digits changes. The key to multiplying decimals, then, is to just pretend
the decimal point isn’t there, and then add it back in at the end. The only tricky part is deciding
where to put the decimal point at the end.

Demonstrate using $442 \times 3$:

```
 1
442  4.42
× 3  × 3
 1326  1326
```

Tell students that now you need to know where to put the decimal point. Will the answer be closer
to 1 or 13 or 132 or 1326? **ASK:** How many whole ones are in 4.42? How many ones are in 3?
About how many ones should be in the answer? ($4 \times 3 = 12$) What is closest to 12: 1, 13, 132
or 1326? Have a volunteer guess where the decimal point should go and ask the class to explain
why the volunteer chose the answer or to agree or disagree with the choice.

Repeat with several problems. (**EXAMPLES:** $3.35 \times 6$; $41.31 \times 2$; $523.4 \times 5$; $9.801 \times 3$)

**Bonus**

$834 \times 779.68 \times 2$; $5480.63 \times 7$

**Extensions**

1. (Adapted from Atlantic Curriculum A2.6) Teach students that just as they can take fractions of
   whole numbers, they can take decimals of whole numbers. Ask students whether they get the
   same answer when they take a quarter of a number (say 8) by dividing the number into 4 equal
   parts and when they take a quarter of the same number by multiplying the number by 0.25. Tell
   your students that $\frac{1}{4}$ of the people in this class is the same as 0.25 of the people in this class.
   They can find 0.25 of a number by multiplying that number by 0.25.

   Ask students to find a context in which 0.25 represents a small amount and one in which it
   represents a large amount.

2. This extension will satisfy the demands of the Atlantic Curriculum.

   Have students find, using the standard algorithm:

   $32 \times 40$ and $\frac{32}{4} \times 0.4$

   Then have students find, again using the standard algorithm:

   $3.2 \times 40$ and $3.2 \times 0.4$
Discuss how the products in the same row compare to each other, and then the products in the same column. **ASK:** Which of these problems is unlike any that you have done so far? Why is it different? (The last one multiplies two decimal numbers instead of a whole number and a decimal).

Then show students another way to multiply $3.2 \times 0.4 = (3 + 0.2) \times 0.4 = 3 \times 0.4 + 0.2 \times 0.4$.

The first addend is easy to find. We can find $3 \times 0.4 = 1.2$ from previous methods ($3 \times 4 = 12$ and move the decimal point one place left). Notice that $3 \times 0.4$ is the same as adding $0.4$ three times: $0.4 + 0.4 + 0.4 = 1.2$. If we take a tens block as the whole, $0.4$ is $4$ tenths of a tens block, so $0.4 + 0.4 + 0.4$ is $4$ tenths of $3$ tens blocks, for $\frac{4}{10}$ of $3$. To find $0.2 \times 0.4$ is a little trickier, since it isn’t clear what “take $0.4$ and add it $0.2$ times” means. However, if instead of taking $\frac{4}{10}$ of $3$, we take $\frac{4}{10}$ of $0.2$, we can visualize what $0.2 \times 0.4$ means.

Draw a hundreds block and circle $\frac{2}{10}$ or $0.2$ of the block:

![Image of a hundreds block with 20 squares shaded]

Then shade $\frac{4}{10}$ or $0.4$ of the shaded block.

![Image of a hundreds block with 80 squares shaded]

Since $8$ out of $100$ squares are shaded, $0.2 \times 0.4 = 0.08$, and since $3 \times 0.4 = 1.2$, we find:

$$3.2 \times 0.4 = 3 \times 0.4 + 0.2 \times 0.4 = 1.2 + 0.08 = 1.28$$

Tell students that you want to find $4.5 \times 2.3$. Tell your students that a hundreds block represents a whole and have students model $4.5$ using base ten blocks.

![Image of a hundreds block with 45 squares shaded]

Then show students $4.5 \times 2$ and $4.5 \times 3$:

<table>
<thead>
<tr>
<th>$4.5 \times 2$</th>
<th>$4.5 \times 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image of a hundreds block with 90 squares shaded]</td>
<td>![Image of a hundreds block with 135 squares shaded]</td>
</tr>
</tbody>
</table>

Tell students that $4.5 \times 2.3$ will be between $4.5 \times 2$ and $4.5 \times 3$. Have a volunteer erase part of the model for $4.5 \times 3$ to show what $4.5 \times 2.3$ will look like. When they are done, divide the model into 4 regions and summarize the number of blocks in each region:
Show students how they can find $4.5 \times 2.3$ by adding the 4 regions together:

- $8$ ones + $12$ tenths + $10$ tenths + $15$ hundredths
- $= 8$ ones + $22$ tenths + $15$ hundredths
- $= 8$ ones + $2$ ones + $2$ tenths + $1$ tenth + $5$ hundredths
- $= 10$ ones + $3$ tenths + $5$ hundredths
- $= 10.35$

Then show the standard algorithm:

\[
\begin{align*}
& \phantom{0}4.5 \\
\times & \phantom{0}2.3 \\
\_ & \phantom{0}13.5 \quad ( = 15 \text{ hundredths} + 12 \text{ tenths}) \\
& \phantom{0}9.0 \quad ( = 10 \text{ tenths} + 8 \text{ ones}) \\
\_ & \phantom{0}10.35
\end{align*}
\]

Estimate $4.5 \times 2.3 \approx 5 \times 2 = 10.$

Discuss how this standard algorithm relates to each area of the picture.

3. Teach students to multiply whole numbers by 0.1, 0.01 and 0.001. Just as, to find $4 \times 3$, students would ask themselves: How much is 4 threes, to find $4 \times 0.1$, students should ask themselves: How much is 4 tenths? **ANSWER:** 0.4. Have students find:

   - a) $13 \times 0.1$ (**ANSWER:** 13 tenths = 1.3)
   - b) $27 \times 0.1$ (**ANSWER:** 27 tenths = 2.7)
   - c) $184 \times 0.1$ (**ANSWER:** 184 tenths = 18.4)
   - d) $184 \times 0.01$ (**ANSWER:** 184 hundredths = 1.84)
   - e) $184 \times 0.001$ (**ANSWER:** 184 thousandths = 0.184)

For practice with this skill, provide the BLM “Multiplying Whole Numbers by 0.1, 0.01 and 0.001.”
NS6-89
Dividing Decimals by 10 and 100

GOALS
Students will discover the rule for dividing decimals by 10 by moving the decimal point one place left.

PRIOR KNOWLEDGE REQUIRED
Decimal tenths and hundredths
Base ten materials
Decimal place value

Tell students that a hundreds block represents one whole. **ASK:** When you divide one whole into 10 equal parts, what do you get? (a tens block or one tenth)

Write on the board:

\[
\begin{array}{c}
\text{÷ 10} \\
1.0 ÷ 10 = 0.1
\end{array}
\]

Then have a volunteer divide 0.1 by 10 using pictures:

\[
\begin{array}{c}
\text{÷ 10} \\
0.1 ÷ 10 = 0.01
\end{array}
\]

Draw several pictures on the board and have students write the corresponding division statements individually: **EXAMPLES:**

\[
\begin{array}{c}
\text{÷ 10} \\
\text{÷ 10} \\
\text{÷ 10} \\
\text{÷ 10}
\end{array}
\]

Then give students the following problems and have them draw the models themselves:

a) \(3 ÷ 10\)  

b) \(0.2 ÷ 10\)  

c) \(2 ÷ 10\)  

d) \(0.4 ÷ 10\)  

e) \(12 ÷ 10\)  

f) \(1.2 ÷ 10\)

Write the number 43.5 on the board and **ASK:** How much is each digit worth? (40, 3 and 0.5). If we divide 43.5 by 10, how much does each digit become worth? (4, 0.3 and 0.05) What is 43.5 ÷ 10? (4.35)

Review the distributive property of multiplication and division.

\[
30 = 3 \times 10 = 3 \times (4 + 6) = 3 \times 4 + 3 \times 6 = 12 + 18
\]

\[
30 ÷ 3 = 10 = 4 + 6 = 12 ÷ 3 + 18 ÷ 3.
\]

Since \(30 = 12 + 18\), \(30 ÷ 3 = (12 + 18) ÷ 3 = 12 ÷ 3 + 18 ÷ 3\).

Ask students to check if this rule works for adding 3 or more numbers and dividing by the same number. For example, students could check that the following calculations result in the same answer:
a) \(8 \div 2 + 10 \div 2 + 4 \div 2 = (8 + 10 + 4) \div 2\)
b) \(15 \div 5 + 10 \div 5 + 5 \div 5 + 20 \div 5 = (15 + 10 + 5 + 20) \div 5\)
c) \(32 \div 4 + 20 \div 4 + 4 \div 4 = (32 + 20 + 4) \div 4\)

Then write on the board:

\[ 43.5 \div 10 = (40 + 3 + .5) \div 10 = 40 \div 10 + 3 \div 10 + .5 \div 10 = 4 + .3 + .05 = 4.35 \]

Explain that this distributive law is what allows students to divide each digit by 10 separately to divide the whole number by 10. Relate this to multiplication by 10:

\[ 4.35 \times 10 = (4 + .3 + .05) \times 10 = 4 \times 10 + .3 \times 10 + .05 \times 10 = 40 + 3 + .5 = 43.5 \]

Emphasize that this strategy of multiplying or dividing the digits separately by other numbers won’t work as nicely for numbers other than 10. Using the distributive property in the following problem does not make it easy to read the answer.

\[ 42 \div 8 = 40 \div 8 + 2 \div 8 \]

Our place value system makes it particularly easy to multiply and divide by 10. For example, \(42 \div 10 = 40 \div 10 + 2 \div 10 = 4 + .2 = 4.2\) makes it really easy to read the answer.

Repeat this exercise with several examples: \(567.8 \div 10, 43.57 \div 10, 89.312 \div 10, 41.325.5 \div 10\).

**Ask:** How do you move the decimal point when dividing a number by 10? How can you make each digit worth ten times less? (move the decimal point one place to the left)

Teach students the connection between ten times more and the measurement units we use. For example, a dm is ten times larger than a cm. **Ask:** if an object is 3 dm long, how many cm long is it? What operation did you do? If an object is 40 cm long, how many dm long is it? What operation did you do? If on object is 7 cm long, how many dm long is it? 9 cm = _____ dm, 13 cm = _____ dm, 702 cm = _____ dm. Repeat the exercise with mm and cm. (Examples: 8.4 mm = _____ cm, 39.2 mm = _____ cm)

Teach students how to divide by 100 or 1 000 by moving the decimal point 2 or 3 places to the left. **Ask:** What is 4 531.2 ÷ 100? What is each digit worth? (4 000, 500, 30, 1 and 0.2) When we divide by 100, what will each digit be worth? (40, 5, 0.3, 0.01 and 0.002) What number is that? (45.312) How did the decimal point move when you divided by 100? Why does this make sense? How is dividing a number by 100 the same as dividing the number by 10 and then dividing it by 10 again? Emphasize that dividing something into 10 equal parts and then dividing those 10 equal parts into 10 parts again leaves the original object divided into 100 equal parts. Notice that moving the decimal point 2 places left is the same as moving the decimal point one place left and then another place left. Challenge students to predict how they would divide a decimal number by 1000.

**Bonus**
Find \(31 \ 498.76532 \div 1 \ 000 \ 000\).

**Extensions**

1. Sarah has 2.7 m of ribbon. She wants to cut the ribbon into 10 equal parts. How long will each piece be?

**Bonus**
Can you give the measurement in m and cm?
2. The wind speed in Vancouver was 26.7 km/h on Monday, 16.0 km/h on Tuesday and 2.4 km/h on Wednesday. What was the average wind speed over the 3 days?

3. A pentagonal box has a perimeter of 3.85 m. How long is each side?

4. (From grade 5 Atlantic Curriculum A7) Draw a number line with only the endpoints 2 and 4 marked. Have students mark where the following numbers would be and to defend their positions: 2.3, 2.51, 2.999, 3.01, 3.75, 3.409, 3.490.

5. (Adapted from grade 5 Atlantic Curriculum A7.1) Have students roll a die three times and ask them to use the digits to represent tenths, hundredths and thousandths. Have students make the smallest number they can and then to say how much would need to be added to make one whole.

6. (grade 5 Atlantic Curriculum A7)
   a) If gas is priced at 56.9¢ per litre, what part of a dollar is this?
   b) If you drank 0.485 L of juice, how much more would you have to drink to equal 0.5 L?

7. (grade 5 Atlantic Curriculum A7.9) Have students write a report on the use of 0.5 and ½.

The following extensions satisfy the demands of the Atlantic Curriculum.

8. (Adapted from Atlantic Curriculum B10) Teach students how to divide decimals by 0.1. Just as 12 ÷ 3 asks how many 3s there are in 12, the question 2.6 ÷ 0.1 asks how many tenths there are in 2.6. If a tens block represents a whole, 2.6 can be represented by:

<table>
<thead>
<tr>
<th>Tiger</th>
<th></th>
<th></th>
<th></th>
<th>2</th>
<th>6</th>
</tr>
</thead>
</table>

There are 26 ones blocks or tenths in 2.6, so 2.6 ÷ 0.1 = 26.

ASK: How many tenths are there in 3? (30) What is 3 ÷ 0.1? (30) How many tenths are there in 3.4? (34) What is 3.4 ÷ 0.1? (34) What is 45 ÷ 0.1? (450 because there are 450 tenths in 45 ones) What is 0.7 ÷ 0.1? (7 because there are 7 tenths in 0.7)

ASK: How does dividing by 0.1 move the decimal point? (The decimal point moves one place to the right.) What else can we do to move the decimal point one place to the right? Does that remind you of anything? (multiplying by 10) What is 4 ÷ 0.1? (40) What is 4 × 10? (40) Emphasize that these will always be the same answer because both results were obtained by moving the decimal point one place right. ASK: Why does this make sense? Emphasize that the number of tenths in a number is 10 times the number of units in the number. If I divide 7 pies into tenths, I will have 70 tenths. If I divide 3.4 pies into tenths, I will have 34 tenths. If I divide 2 pies into hundredths, how many pieces will I have? What is 2 ÷ 0.01? What is 2.34 ÷ 0.01? 43.6 ÷ 0.01? What is dividing by 0.01 the same as? (multiplying by 100) What do you think dividing by 0.001 will be the same as? (multiplying by 1000)

Have students divide several numbers by 0.1, 0.01, and 0.001. **EXAMPLES:**

5 ÷ 0.01 23 ÷ 0.1 4.2 ÷ 0.001 9.78 ÷ 0.1 90.1 ÷ 0.01 704.3 ÷ 0.001

Then have students divide or multiply several numbers by 0.1, 0.01, and 0.001. **EXAMPLES:**

75 ÷ 0.01 42 × 0.1 4.7 × 0.01 8.01 ÷ 0.001 90.1 × 0.01 60.3 × 0.001
To give students practice with this concept, have them do the following questions:

a) What digits belong in the boxes? \[4 \square 6. \square \div 0.1 = 5. \square 3 \div 0.01\]

b) Which answer will have a 3 in the tens place?
\[
\begin{align*}
42345 \div 0.1 & \quad 42.345 \div 0.01 & \quad 42.345 \div 0.001 \\
\end{align*}
\]

c) What digit will be in the tens place after dividing 453.2 by 0.01. Why?

d) You have divided a decimal number by 0.001 and the answer is also a decimal number. What do you know about the original decimal number?

e) Explain why dividing by 0.01 produces the same result as multiplying by 100.

f) Why does multiplying a number by 0.1 usually give a lesser answer than dividing the same number by 0.1?

g) \[0.7834 \div 0.0001 = 0.7834 \times \square\]. Explain your answer.

9. Teach students how to divide by decimal numbers.

Show the following number line:

\[\begin{array}{cccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
\end{array}\]

ASK: How does this number line show \(18 \div 3\)? What is \(18 \div 3\)?

Then show the following number line and ask how it shows \(1.8 \div 0.3\):

\[\begin{array}{cccccccccccc}
0 & .1 & .2 & .3 & .4 & .5 & .6 & .7 & .8 & .9 & 1.0 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 & 1.7 & 1.8 \\
\end{array}\]

What is \(1.8 \div 0.3\)? Why are \(18 \div 3\) and \(1.8 \div 0.3\) the same?

Remind your students that, in division, you can multiply or divide both terms by the same number and keep the result the same. Have students verify this for very simple examples:

\[
\begin{align*}
6 \div 2 & \quad (6 \times 2) \div (2 \times 2) & \quad (6 \times 3) \div (2 \times 3) & \quad (6 \times 4) \div (2 \times 4) & \quad (6 \times 5) \div (2 \times 5) \\
\end{align*}
\]

To get from \(1.8 \div 0.3\) to \(18 \div 3\), what are you multiplying both numbers by? (10) So do they have the same answer? (yes) Since we know \(18 \div 3 = 6\), we also know that \(1.8 \div 0.3 = 6\).

Another way to see that these two questions will have the same answer is to ask: If you have a ruler that is 18 cm long and you divide it into parts that are 3 cm long, how many parts do you have? What if you have a ruler that is 1.8 dm long and you divide it into parts that are .3 dm long—then how many parts do you have? How are these questions the same? How do you know they will have the same answer?

Then ask students which of the following questions has an answer different from the others. Can they figure it out without doing any calculations?

\[
\begin{align*}
42.5 \div 0.5 & \quad 425 \div 5 & \quad 0.425 \div .05 & \quad 4.25 \div .05 \\
\end{align*}
\]
NS6-90
Dividing Decimals by Whole Numbers

**GOALS**

Students will use long division to divide decimal numbers by single-digit whole numbers.

**PRIOR KNOWLEDGE REQUIRED**

Long division of 3- and 4-digit whole numbers by single-digit whole numbers. Decimal place value up to hundredths

**VOCABULARY**

<table>
<thead>
<tr>
<th>tenths</th>
<th>hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>dividend</td>
<td>quotient</td>
</tr>
<tr>
<td>divisor</td>
<td></td>
</tr>
</tbody>
</table>

Tell students that a hundreds block represents one whole and draw on the board:

<table>
<thead>
<tr>
<th>1.0</th>
<th>0.1</th>
<th>0.01</th>
</tr>
</thead>
</table>

Have volunteers draw the base ten models for: 5.43, 8.01, 0.92. Then have students do similar problems in their notebooks.

Then tell students that they would like to find 6.24 ÷ 2. Have a volunteer draw the base ten model for 6.24. Then draw 2 circles on the board and have a volunteer show how to divide the base ten materials evenly among the 2 circles. What number is showing in each circle? What is 6.24 ÷ 2?

Repeat for other numbers where each digit is divisible by 2 (or 3).

**EXAMPLES:** 46.2 ÷ 2, 3.63 ÷ 3, 4.02 ÷ 2, 6.06 ÷ 2, 6.06 ÷ 3.

Then write on the board 3.54 ÷ 2. **ASK:** In what way is this problem different from the previous problems? (each digit is not divisible by 2). Have a volunteer draw the base ten model for 3.54.

Draw 2 circles on the board and ask why you chose to draw 2 circles rather than a different number of circles?

Then lead students through the steps of long division, as in **NS6-36: Long Division—3- and 4-Digit by 1-Digit.** Then do a comparison of the steps for 3.54 ÷ 2 and 354 ÷ 2. Emphasize that students can just pretend that the decimal point does not exist and then put the decimal point in the correct place at the end. For example, students will see that 354 ÷ 2 = 177. To find 3.54 ÷ 2, they can round 3.54 to 4 and estimate that 3.54 ÷ 2 is about 4 ÷ 2 = 2. Where should they put the decimal point so that the answer is close to 2? (**ANSWER:** 1.77) Have students do several problems where they figure out the answer this way, first by long division of whole numbers and then by estimating to find where to put the decimal place.

Then, to ensure students are doing this last step correctly, give problems where students do not need to do the long division, but only this last step of putting the decimal point in the correct place:

a) 856.1 ÷ 7 = 122.3 (**ANSWER:** 122.3 since the answer will be close to but more than 100)

b) 8922.06 ÷ 6 = 1487.01 (**ANSWER:** 1487.01 since the answer will be close to but more than 1000)

To ensure that students do not simply count decimal places, but actually estimate their answers, give some examples where you add an extra zero to the answer. For example, 23.28 ÷ 6 = 3.880 has answer: 3.880 or just 3.88.
Students should be encouraged to discover their own rule regarding where to put the decimal point when they have finished the long division (put the decimal point above the decimal point, as shown on the worksheet.) You can explain why you line up the decimal points when doing long division as follows: Compare the long division algorithms for \(342 \div 2\) and \(3.42 \div 2\): 

\[
\begin{array}{c|c}
342 & 171 \\
\hline
2 & 1.71
\end{array}
\]

If you divide 3.42 by 2 the answer will be 100 times smaller than the answer to \(342 \div 2\). Just as I need 100 times fewer 2s to make 8 as I need to make 800, I need 100 times fewer 2s to make 3.42 as I need to make 342. Hence the answer to \(3.42 \div 2\) by long division should look like:

Notice that when the decimal in the dividend (3.42) shifts 2 left, the decimal in the quotient (1.71) shifts 2 left, so the decimals in the dividend and the quotient line up.

**Extensions**

1. Teach students to divide decimals by single-digit decimals (EXAMPLE: 86.4 ÷ 0.9) by treating both the dividend and the divisor as whole numbers (864 ÷ 9 = 96) and then estimating by rounding each number to the nearest whole number (86 ÷ 1 is about 90) to decide where to put the decimal point.

These extensions satisfy the demands of the Atlantic Curriculum.

2. Remind students how to find fractions such as \(\frac{1}{4}\), \(\frac{1}{8}\), \(\frac{25}{100}\), and \(\frac{3}{5}\) as decimals—first change the fraction to an equivalent fraction with denominator 10 or 100, then write that fraction as a decimal. Take students through the process of changing \(\frac{1}{8}\) to a decimal: 8 does not divide into 10, so we can’t find an equivalent fraction with denominator 10; 8 does not divide into 100, so we can’t find an equivalent fraction with denominator 100. Does 8 divide into 1000? (yes) So we can find an equivalent fraction with denominator 1000.

\[
\frac{1}{8} \times \frac{125}{125} = \frac{125}{1000} = 0.125
\]

**ASK:** How can we find the number to multiply 8 by that will give 1000? (Find 1000 ÷ 8.)

Have students find 1000 ÷ 8 by whatever method they choose and then discuss strategies used (long division, dividing by 4 and then by 2, repeatedly dividing by 2, finding 200 ÷ 8 and then multiplying by 5, and so on). Then write on the board:

\[
\frac{1}{8} \times \frac{125}{125} = \frac{125}{1000} = 0.125
\]

Let students practise finding decimals for other fractions whose denominators divide into 1000, but not 100. **EXAMPLES:** \(\frac{7}{125}\), \(\frac{3}{8}\). **BONUS:** Write \(\frac{5}{16}\) as a decimal. (Answer has 4 decimal places.)

Show students how they can find decimals directly using division. Return to the example \(\frac{3}{8}\).

Tell students that when you find \(\frac{3}{8}\), you can think of it as trying to split 3 pizzas among 8 people. First, divide each pizza into 8 equal pieces:

Each person gets one piece from each pizza, and so gets \(\frac{3}{8}\) pizzas altogether. So another way of thinking about fractions is through division; \(\frac{3}{8}\) means 3 pizzas divided among 8 people.
Now find $\frac{3}{8}$ using base ten materials. Draw 8 circles to represent the 8 people you are sharing among. You have 3 wholes, so let 3 thousands blocks represent the 3 wholes. You will immediately need to replace the 3 thousands blocks with 30 hundreds blocks. Divide the 30 hundreds blocks into the 8 circles.

There are 3 hundreds blocks in each circle and 6 left over. Now you need to divide the 6 hundreds blocks into the 8 circles, so replace the 6 hundreds blocks with 60 tens blocks. We can place 7 tens blocks into each circle and have 4 left over. Replace the 4 tens blocks with 40 ones blocks and then place 5 ones blocks in each circle with none left over. So $\frac{3}{8} = 0.375$.

Show students what happens when you start with 3 hundreds blocks instead of 3 thousands blocks to represent the 3 wholes. When you get to the last step, you need to divide ones blocks into ten pieces. Since you can’t do that, you made a higher-level block the whole.

Have students write the following fractions as decimals by using base ten materials.

$\frac{7}{8}, \frac{3}{4}, \frac{5}{8}, \frac{6}{8}, \frac{3}{20}, \frac{4}{25}, \frac{7}{8}$

BONUS: $\frac{3}{50}$

Then show students what happens when you try to find $\frac{1}{3}$ as a decimal using this method. Start with the tens block as the whole. Immediately you need to trade for ten ones blocks. You put 3 in each circle and have one left over. Since you can’t divide the ones block into 3 pieces, you know that the decimal starts 0.3 but not how it continues. If you try using the hundreds block as a whole, you get 0.33 with a ones block left over. Then try using the thousands block as a whole, and get 0.333 with a ones block left over. Ask students what they think you would get for the decimal if you used a ten thousands block as the whole? (0.3333 with a ones block left over) Have students punch in $1 \div 3$ on their calculators. What do they get? Tell them that $\frac{1}{3}$ continues forever as 0.333…. Tell students that this decimal is called a repeating decimal because the 3 repeats. **ASK:** What do you think the decimal for $\frac{2}{3}$ will look like? Write on the board:

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

0.333…

+ 0.333…

Have a volunteer fill in the blank. Then have students find the repeating decimal for $\frac{2}{3}$ using base ten blocks. Do they get the same answer? Have students punch in $2 \div 3$ on their calculators. Discuss why the final number is a 7 (the calculator rounded the last digit).

Can your students predict the decimal for $\frac{1}{9}$ now that they know the decimal for $\frac{1}{3}$? What do they predict will be the decimal for $\frac{2}{9}$? Have students check their prediction directly.

3. Have students find several quotients by first pretending the numbers are whole numbers and then estimating. **EXAMPLES:** 9.2 ÷ 4, 2.38 ÷ 7, 364.5 ÷ 5, 27.65 ÷ 5, 369.2 ÷ 13.

**ASK:** What is the reminder when dividing 451 by 3? What is the remainder when dividing 45.1 by 3? Ensure that students understand how we use the term "remainder." Explain that 451 ÷ 3 = 150 R 1 because $3 \times 150 + 1 = 451$. However, it is not true that $3 \times 15 + 1 = 45.1$, so it is not true that 45.1 ÷ 3 = 15 R 1. In fact, 45.1 ÷ 3 = 15 R 0.1, so although the long division algorithm will make it appear as though the remainder is 1, the remainder is actually 0.1.

Then have students use long division to find the quotients and remainders for:

9.5 ÷ 4  37.01 ÷ 13  45.15 ÷ 8
NS6-91
Differences of 0.1, 0.01, and 0.001

Have your students add the following numbers in their notebooks:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>a)</td>
<td>.48 + .1</td>
<td>b) .48 + .01</td>
<td>c) .52 + .1</td>
<td>d) .52 + .01</td>
</tr>
<tr>
<td>e)</td>
<td>.63 + .1</td>
<td>f) .63 + .01</td>
<td>g) 4.32 + .01</td>
<td>h) 4.32 + 1</td>
</tr>
<tr>
<td>i)</td>
<td>4.32 + .1</td>
<td>j) 7.38 + .1</td>
<td>k) 7.38 + .01</td>
<td>l) 7.38 + 1</td>
</tr>
</tbody>
</table>

**ASK:** How do you add .1 to a number? (add 1 to the tenths digit) When you added .1 above, how many digits changed? (only 1, the tenths digit)

Then have students find .94 + .1. **PROMPTS:** How many hundredths are in .94? (94) How many hundredths are in .1? (10) How many hundredths is that altogether? (104) So the answer is 1.04. **ASK:** How is adding .1 to .94 different from adding .1 to the numbers above? What else changes besides the tenths digit? (the ones digit changes—from 0 to 1—because the answer is more than 100 hundredths)

Have students do the following problems and tell you when they just add 1 to the tenths digit and when they have to change the ones digit, too:

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<tbody>
<tr>
<td>.49 + .1</td>
<td>.86 + .1</td>
<td>.93 + .1</td>
<td></td>
<td></td>
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<tr>
<td>.97 + .1</td>
<td>.36 + .1</td>
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Now have students add .01 to various decimals:

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<tbody>
<tr>
<td>.49 + .01</td>
<td>.94 + .01</td>
<td>.86 + .01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.49 + .01</td>
<td>.94 + .01</td>
<td>.28 + .01</td>
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**ASK:** What number is .1 more than 9.3? Ask students to identify the number that is .1 more than: .7, 8.4, .6, .9, 5.9. Prompt students with questions such as: How many tenths are in 5.9? What is one more tenth? What number has 60 tenths?

What number is .01 more than:

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<tbody>
<tr>
<td>8.47</td>
<td>8.4</td>
<td>.3</td>
<td>.39</td>
<td>.86</td>
</tr>
<tr>
<td>.89</td>
<td></td>
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What number is 1 more than:

<p>| | | | | |</p>
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<tbody>
<tr>
<td>9.3</td>
<td>.7</td>
<td>1.43</td>
<td>9.8</td>
<td>9.09</td>
</tr>
<tr>
<td>9.99</td>
<td></td>
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What number is .001 more than:

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<tbody>
<tr>
<td>a) 8.569</td>
<td>b) 81.569</td>
<td>c) 8.56</td>
<td>d) 9.4</td>
<td></td>
</tr>
<tr>
<td>e) 7.02</td>
<td>f) 4.009</td>
<td>g) 634.165</td>
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</table>

To guide students, **ASK:** How many thousandths are in 8.569? (8 569)

What is 8 569 + 1? What number has 8 570 thousandths? (8 570 or 8.57)

Repeat with subtraction, asking students to find numbers that are .001, .01, .1, and 1 less than various numbers with 1, 2 and 3 decimal points. Include examples where students need to borrow/regroup.
Have a volunteer add the missing decimal numbers to this number line:

4.0 _______________ 5.0

Students can refer to the number line to complete these sequences in their notebooks:

4.3, 4.4, 4.5, ___  
4.0, 4.2, 4.4, ___

4.1, 4.4, 4.7, ___  
4.9, 4.7, 4.5, ___

Have students give the rule for each sequence. (EXAMPLE: start at 4.3 and add .1)

Have another volunteer add the missing decimal numbers to this number line:

7.3 _______________ 8.3

Students should complete and describe these sequences in their notebooks:

7.7, 7.8, 7.9, ___  
7.5, 7.7, 7.9, ___

7.2, 7.5, 7.8, ___  
8.3, 8.2, 8.1, ___, ___

Have students fill in the blanks in their notebooks:

5.9 + .1 = ___  
6.49 + .1 = ___  
8.93 + .1 = ___  
7.999 + .1 = ___

8.9 + .1 = ___  
6.49 + .01 = ___  
8.99 + .1 = ___  
7.999 + .01 = ___

.9 + .1 = ___  
6.49 + 1 = ___  
8.99 + .01 = ___  
7.999 + .001 = ___

Teach students that when adding numbers with 3 decimal places, they count how many thousandths each number has. Since 7.999 has 3 decimal places, they should think of .1 as .100, i.e., give it 3 decimal places, too. So, 7.999 + .100 is 7 999 thousandths + 100 thousandths = (7999 + 100) thousandths. To add 100 to any number, cover up the tens and ones digits (so that you just see 79). Then 79 hundreds + 1 hundred = 80 hundreds. The tens and ones digits do not change, so 7999 + 100 = 8099. **ASK:** What number has 8099 thousandths? (8.099)

**ASK:** How many thousandths are in .01? How would we write .01 with 3 decimal places? (.010)

There are 10 thousandths in .01 and 7999 thousandths in 7.999. How many thousandths is that altogether?

**Extension**

Ask students to count forward from the following numbers by tenths, orally.

a) 6  
b) 17.8  
c) 123.2
NS6-92
Place Value and Rounding

Review rounding.

**STEP 1:** Draw an arrow or point your pencil to the digit you want to round to (tens, ones, tenths, hundredths, etc.).

**EXAMPLE:** I want to round 362.5491 to the nearest hundredth:

\[
362.5971
\]

**STEP 2:** Look at the digit to the right. If the digit is 0, 1, 2, 3, or 4, round down. If the digit is 5, 6, 7, 8, or 9, round up.

**EXAMPLE:** The digit to the right is 7, so I round up.

**STEP 3:** Replace all digits to the right of the arrow with 0.

**EXAMPLE:** 362.5971 becomes 362.5900 or just 362.59

**STEP 4:** If rounding down, leave the number as is. If not, increase the number by the value of the place you are rounding to.

**EXAMPLE:** We are rounding to the nearest hundredth and rounding up, so we increase 362.59 to 362.60

Another way to round up is to just increase the digit you are rounding to by 1 and regroup if necessary:

\[
\begin{align*}
362.5971 \\
362.6000 \\
\end{align*}
\]

It is important to put the “0” in the hundredths position so that people know that the number was rounded to 2 decimal places. This is very different from 362.5731, which, when rounded to the nearest hundredth would be 362.57, but when rounded to the nearest tenth would be 362.6. Writing 362.60 tells you that the number is actually closer to 362.6 than if you had just written 362.6.

Give students practice with this skill.
**ASK:** What place value was 5.3 rounded to? (the tenths) What of the following numbers might have rounded to it?

5.26  52.6  5.31  5.36  5.42  5.35  5.25

What other numbers with 2 digits after the decimal point would round to 5.3?

Ensure that students understand that decimals are like fractions. Review this by changing decimals with 1 decimal place to fractions having denominator 10 (EXAMPLE: 5.3 = $\frac{53}{10}$) and decimals with 2 decimal places to fractions having denominator 100 (EXAMPLE: 3.41 = $\frac{341}{100}$). Then remind your students that how much a fraction represents depends on what unit is the whole. Brainstorm examples of what the whole could be. (The whole could be a hundreds block; a distance unit such as m, cm, or cm; a time measurement such as minute, hour, second, day, year; a money unit such as a dollar or a cent; numbers such as a hundred, a thousand, or a million; and so on.) Then **ASK:** What is a tenth of a km? (100 m) What does the tenths digit represent when we write: 4.35692 km? (300 m) What does the hundredths digit represent? (50 m) The thousandths digit? (6 m)

**Bonus**
Which digit in 4.35692 km represents cm? How many cm does that digit represent? (The hundred thousandths digit represents 2 cm.)

What does the hundredth digit represent in:

a) 5.723 million (20 000)  
b) $6.234$ thousand ($30$)  
c) 49.076 m (7 cm)

Teach students that when we write large numbers such as 52 342 103, it is often convenient to write the numbers in terms of larger units, in this case millions. We could technically write this number in many forms, for example: about 52 300 thousand, about 5.23 ten millions, or 523 hundred thousands. But we usually pick a thousand, a million, or a billion as the unit because of the way we space the numbers so that groups of 3 place values are together; our system makes it easier to read numbers with 3 zeroes, 6 zeroes, 9 zeroes, and so on. [**NOTE:** In some other countries, such as Japan, they group their numbers so that 4 place values are together.] In the example above, this number would be approximated as 52.3 million. Have students approximate other large numbers:

a) 32 948  
b) 432 176 893 321  
c) 34 576 009  
d) 321 005 042  
e) 324 631

Then show students how to estimate sums, differences, products, and quotients by rounding to the nearest whole number:

84.6 ÷ 4.7 ≈ 85 ÷ 5 = 17

Have students estimate, and then use a calculator to find, the actual answer for various questions. Students should round the calculator answer to the nearest hundredth if the original numbers had hundredths, the nearest tenth if the original numbers had tenths, and so on.

**EXAMPLES:** 3.87 × 4.96  
98.8 ÷ 11.3  
43.2 − 8.8  
162.34 + 16.234

**ASK:** Is the estimate close to the calculator answer? Do you think you punched in the numbers correctly on the calculator?

Tell your students that someone punched these numbers into a calculator and got these answers (rounded to the number of digits in the question). Are they reasonable?

a) 3.87 × 4.96 = 8.83  
b) 98.8 ÷ 11.3 = 8.7  
c) 43.2 ÷ 8.8 = 4.9  
d) 162.34 + 16.234 = 178.574
GOALS
Students will review concepts in decimals.

PRIOR KNOWLEDGE REQUIRED
Base ten materials
Units of measurement:
- metres (m), centimetres (cm), and millimetres (mm)
- Fractions and equivalent decimals

Extensions
1. (Atlantic Curriculum A1.4) Present this library information to students:
   Metropolitan Toronto Library 3 068 078 books
   Bibliotheque de Montreal 2 911 764 books
   North York Public Library 2 431 655 books
   Ask students to rewrite the numbers in a format such as _____ . ____ million books or
   ____ . ____ ____ million books. Then ask students to make comparison statements about
   the number of books.
2. (Atlantic Curriculum A2.4) Determine the number of whole numbers between 2.03 million
   and 2.35 million.

NS6-93
Decimals Review

This worksheet is a review and can be used as an assessment.

After students do QUESTION 3 on worksheet NS6-93, they could check their
answers by changing the number in the larger unit to a number in the smaller
unit and then changing the answer back to the larger unit. For example,
part a) says to find 5 cm + 7.3 dm by changing the smaller unit into a decimal
in the larger unit, so we find 5 cm + 7.3 dm = 0.5 dm + 7.3 dm = 7.8 dm.
Students could check their answer by changing both numbers to cm
(5 cm + 73 cm = 78 cm) and then converting back to dm (78 cm = 7.8 dm).
ACTIVITY

(From Atlantic Curriculum A9.2) Have students copy this template:

Roll a die. Call out the number rolled and ask each student to fill in a blank on his or her template. Roll the die 18 times. The students who end up with three true statements win a point. Repeat the process.

Extensions

Give each student or pair a set of base ten blocks. Tell students the hundreds block is the whole, so the tens block represents .1 and the ones block represents .01.

1. Ask students to show and write all the decimals they can make with these 3 blocks.

   (SOLUTION: 1.0, 1.1, 1.01, 1.11, 0.1, 0.01, 0.11)

   NOTE: One way to prepare students for this exercise is to hold up combinations of blocks and ask them to write the corresponding decimal in their notebook. For instance, if you hold up the hundreds block (which represents one unit) and the tens block (which represents a tenth) they should write 1.1.

2. Use base ten blocks to make a decimal
   a) greater than .7.
   b) less than 1.2.
   c) between 1 and 2.
   d) between 1.53 and 1.55.
   e) with tenths digit equal to its ones digit.
   f) with hundredths digit one more than its tenths digit.

3. Create models of 2 numbers such that
   a) one number is 4 tenths greater than the other.
   b) one number has tenths digit 4 and is twice as large as the other number.

4. One decimetre (1 dm) is 10 cm. Explain how you would change 3.2 dm into centimetres.

5. (Atlantic Curriculum A9.4) Ask students to explain why you cannot compare two decimal number by simply counting the number of digits in each.
6. Explain why it is important to keep track of the decimal place when finding sums like:

\[ 12.50 + 3.2 + 5.832 \]

7. Write in the decimal point by estimating (rather than carrying out the operation):

a) \[ 27.25 + 832.5 = 85975 \]  
b) \[ 57.23 \times 2.5 = 143075 \]  
c) \[ 989.2 \times 3.6 = 35612 \]  
d) \[ 74.2 \times 8.45 = 72699 \]

**NOTE:** For part d), students might incorrectly write \[ 74.2 \times 8.45 = 72.699 \]. This is an indication that they are counting decimal places in each factor and adding to find the number of decimal places instead of estimating that the answer should be about: \[ 74 \times 10 = 740 \]. The reason this approach doesn’t work in this case is because, when multiplying by hand, students would find \[ 742 \times 845 = 726990 \]. Counting the decimal places does tell you to correctly move the decimal point 3 places left. However, when punching \[ 74.2 \times 8.45 \] into a calculator, the calculator will not show the final 0. Other questions you might give your students to ensure that they are estimating and not using this incorrect trick include the following:

\[ 23.46 \times 127.5 = 2991.15 \]  
\[ 342.5 \times 247.32 = 84707.1 \]

8. Without adding, how can you tell whether the sum is greater than or less than 435?

\[ 9.5 + .37 + 407.63 \]

9. Find any pairs of numbers that add to a whole number:

\[ 6.3 \quad 2.3 \quad 1.4 \quad 3.6 \quad 2.9 \quad 4.7 \quad 1.1 \]

**Bonus**

Find groups of three numbers that add to a whole number.

10. Find \[ 100 \div 8 \] on a calculator. How do you know from the decimal that there is a remainder?
GOALS
Students will solve word problems outside the context of any particular lesson.

PRIOR KNOWLEDGE REQUIRED
Word problems
Adding and subtracting decimal numbers
Multiplying and dividing decimal numbers by 10 or 100
Multiplying and dividing decimal numbers by single-digit numbers
This lesson is a review of concepts in decimals

Review word problems with your students.

Extensions

1. The chart shows how many times stronger (or weaker) gravity is on the given planets than on earth.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Saturn</th>
<th>Jupiter</th>
<th>Mars</th>
<th>Mercury</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity Factor</td>
<td>1.15</td>
<td>2.34</td>
<td>0.83</td>
<td>.284</td>
</tr>
</tbody>
</table>

   a) On which planets is gravity less strong than on Earth?
   b) How much would a 7 kg infant weigh on each planet?
   c) How much more would the infant weigh on Jupiter than on Mars?
   d) John can jump 1 metre on Earth. How high can he jump on Mercury?

2. Tell students that information in a word problem can be redundant; sometimes you don’t need all the information given to solve the problem.

   EXAMPLE: Find the secret number:
   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My ones digit is greater than my tens digit.

   Have students try to figure out which 3 of the clues they need to answer the question.

   By changing only the last clue slightly, challenge your students to solve the puzzle using only two of the clues.

   A. I am an odd number between 20 and 30.
   B. My ones digit is not 4.
   C. I am divisible by 3.
   D. My tens digit is greater than my ones digit.

   Students can make up their own number puzzle with one piece of redundant information and have a partner solve the puzzle and identify the unneeded information.

3. Have students make up rhyming math poems as puzzles. EXAMPLE:

   A 2-digit number named Todd
   Has tens digit odd.
   But he’s even you see
   And his digits add to three.
   Which two numbers can he be?
NS6-95
Unit Rates

GOALS
Students will understand simple multiplicative relationships involving unit rates.

PRIOR KNOWLEDGE REQUIRED
Money (dollars and cents)
Distance (km, m and cm)
Time (weeks, hours)

VOCABULARY
rate
unit rate

Explain that a rate is the comparison of two quantities in different units. For example, “3 apples cost 50¢” is a rate. The units being compared are apples and cents. Have students identify the units being compared in the following rates.

- 5 pears cost $2.
- $1 for 3 kiwis.
- 4 tickets cost $7.
- 1 kiwi costs 35¢.
- Sally is driving at 50 km/hour.
- On a map, 1 cm represents 3 m.
- A student earns $6 an hour for babysitting.
- The recipe calls for 1 cup of flour for every teaspoon of salt. (NOTE: for this last example the units are not cups and teaspoons, they are cups of flour and teaspoons of salt.)

Explain that one of the quantities in a unit rate is always equal to one. Give several examples of unit rates, and have students identify the unit which makes it a unit rate.

- 1 kg of rice per 8 cups of water. (1 kg makes it a unit rate)
- 1 apple costs 30¢. (1 apple)
- $1 for 2 cans of juice. ($1)
- 1 can of juice costs 50¢. (1 can of juice)
- The speed limit is 40 km per hour. (1 hour)
- She runs 1 km in 15 minutes. (1 km)

Explain that knowing a unit rate can help to determine other rates. ASK: If one book costs $3, how much do two books cost? … three books? … four books?

Draw a map with two cities joined by a line. Assuming that 1 cm represents 10 km, have volunteers determine the actual distance between the cities by measuring the line with a metre stick. Then have them explain their calculation for the class.

If 1 cm on a map represents 2 km, how much does 3 cm represent? How much does 7 cm represent? 4.5 cm? Two schools are 6 km apart. How far apart should they be drawn on the map? Two buildings are 11 km apart. How far apart should they be drawn on the map?

ASK: If you know that two books cost $6, how can you determine the cost for three books? What makes this problem different from the other problems in this lesson? (Instead of starting with the cost of 1 book, we are now starting with the cost of 2 books; we are not given a unit rate) How does working with unit rates make it easier to calculate other rates?
Working in pairs, have your students change this problem into a unit rate and then share their procedures with the class. Explain that higher rates can be determined through multiplication of the given rate, but the single unit rate can only be determined through division:

If 1 peach costs 25¢, then 3 peaches cost 75¢ (3 × 25¢).
If 3 peaches cost 75¢, then 1 peach costs 25¢ (75¢ ÷ 3).

Assign several problems that require your students to determine unit rates. Be sure the answers are whole numbers.

a) 4 pears cost 80¢. How much does 1 pear cost?
b) 24 cans of juice cost $24. How much does 1 can of juice cost?
c) 2 books cost $14. How much does 1 book cost?
d) 3 teachers supervise 90 students on a field trip. How many students does each teacher supervise?

Extensions

1. Have students determine the unit rates and then solve the following problems.
   a) If 4 books cost $20, how much do 3 books cost?
   b) If 7 books cost $28, how much do 5 books cost?
   c) If 4 L of soy milk costs $8, how much do 5 L cost?

2. Bring in some flyers from a grocery store, and ask students to determine unit prices and calculate the cost of quantities greater than one. For instance, if the unit price is $2.75 per item, how much will three items cost? If they do not know how to multiply a decimal number with a single-digit number, challenge them to select and use an alternate unit to dollars—they should use cents. **ASK:** How many cents are in $2.75? If each item costs 275¢, how many cents will three items cost? What does that equal in dollars?

Ask students to calculate the unit price of an item using division. For instance, if three items cost $1.62, how much will one item cost? Again, have them convert dollars to cents and then back to dollars.
GOALS

Students will understand ratios as a way to compare one part of a whole to a different part of a whole.

PRIOR KNOWLEDGE REQUIRED

Fractions of a set

VOCABULARY

ratio
colon

Review fractions as a part-to-whole comparison. For example, a baseball player’s batting average is the number of hits over the total number of times at bat. Note that the successful hits are a part of the total times at bat. Brainstorm other examples in life that compare a part to a whole. EXAMPLES: the shooting average of a basketball player (shots made over shots taken), the save percentage of a goalie (saves made over shots taken on the goalie) number of questions answered correctly over number of questions on a test.

Introduce the concept of part-to-part comparison. Tell students that a team won 2 games, lost 1 game, and tied 4 games. ASK: How many games did the team play? What is the fraction of games won? (2/7) Based on that, does it look like the team is very good or not very good? Tell students that you’d like to compare games won to games lost but it’s not a fraction anymore because games won is not a part of the games lost. You want to compare a part of the games played to a different part of the games played. Explain that a part-to-part comparison is called a ratio and that ratios are written with a colon between them. So the ratio of games won to games lost is 2:1. Now how good do you think the team is? Point out that it makes just as much sense to compare games lost to games won (1:2), so it doesn’t matter which order you write the numbers in, as long as you’re clear about what you’re comparing. Ask students if this is the same as fractions or different. When writing fractions, did it matter whether you wrote the whole on the top or on the bottom?

Explain that a ratio is a comparison of two numbers. The ratio of the number of vowels to the consonants in the word “ratio” is 3:2 (or “3 to 2” or 3/2). Have students compare the number of vowels to the number of consonants in various words, and then the number of consonants to the number of vowels. Then compare nouns to verbs in sentences. And then adjectives to nouns or adverbs to verbs.

Draw several rectangles on the board and review the words “length” and “width”. Have students find the ratio of length to width in their notebooks. Then have students compare width to length instead of length to width. What do students notice about their ratios?

Bonus

Create your own rectangle and write the ratio of length to width. Investigate: What does the ratio look like for long and thin rectangles. For almost-square rectangles?

Extensions

1. Tell students that you can compare three things at a time—squares to circles to triangles.

Have students find the six possible ratios: squares to circles to triangles, squares to triangles to circles, and so on. How are these ratios the same
and how are they different? Ask if we can compare three numbers at a time using fractions. How are ratios better than fractions?

2. Remind students that area of rectangle = length × width. Investigate the area to perimeter ratio of various rectangles. What does the ratio look like for short and thin rectangles? What does the ratio look like for almost-square rectangles? Note that students will likely not be able to say at this point which ratio is “larger” as this has not been discussed yet.

3. Tell students that Toronto and Boston sports teams played each other in both hockey and basketball. The scores were 5 - 1 and 99 - 93. Which game was closer? Did you use the difference or the ratios to make your comparison?

**NS6-97**

**Equivalent Ratios**

Tell your students that you have a pancake recipe that calls for 3 cups of flour and 2 bananas. Remind them that this is called a rate because it compares 2 quantities in different units. **ASK:** What are the different units? (cups of flour and bananas) **ASK:** How many cups of flour do we need for 1 banana? (As an improper fraction: $\frac{3}{2}$ , and as a mixed fraction: $1 \frac{1}{2}$ ) **ASK:** How would you write $1 \frac{1}{2}$ as a decimal? (1.5) Tell students that they would need 1.5 cups of flour for every 1 banana. **ASK:** How does that make it easy to tell how many cups of flour you would need for 5 bananas? (multiply $5 \times 1.5$) Have students do this calculation. Emphasize that the number of cups of flour is always 1.5 times the number of bananas, so if they know how many bananas they have, they can deduce the number of cups of flour. Have students perform this calculation for various numbers of bananas (7, 6, 3).

Tell students that they’ve just found many equivalent ratios:

1.5:1 = 3:2 = 10.5:7 = 9:6 = 4.5:3

**ASK:** Why are these ratios called equivalent? Tell students that to say that the ratio of cups of flour to bananas is 3 to 2 is to say that for every 3 cups of flour, they need 2 bananas. Draw this on the board:

![Diagram showing 3 cups of flour and 2 bananas]

**ASK:** If you have 6 cups of flour, how many bananas do you have? Draw 3 cups of flour and 2 bananas and repeat until you have 6 cups of flour—how many bananas did you draw? (4)
**ASK:** How many cups of flour would you need for 10 bananas? Draw 3 cups of flour and 2 bananas and repeat until you have drawn 10 bananas—how many cups of flour did you draw? (15) So the ratio 15:10 is equivalent to the ratio 3:2. Emphasize that 15 is 1.5 × 10, so they can just multiply the number of bananas by 1.5 to get the number of cups of flour.

Emphasize that in the example above, students compared numbers through multiplication rather than through addition. If we had said: The recipe calls for 3 cups of flour and 2 bananas, so it calls for 1 more cup of flour than number of bananas, this would be comparing through addition and subtraction rather than through multiplication. **ASK:** If I want to make the recipe with 5 bananas, should I use 6 cups of flour since that is 1 more than 5? Could that mess up the recipe? Explain to students that in this situation, 3 (cups of flour) is 1.5 times 2 (bananas) and it’s this 3:2 ratio we want to preserve—the number of cups of flour should always be 1.5 times the number of bananas.

Then show students how they can make a series of equivalent ratios by repeatedly drawing 3 cups of flour and 2 bananas:

\[3:2 = 6:4 = 9:6 = 12:8 = 15:10\]

Have students predict the next ratio in this sequence of equivalent ratios. How did they do it?

Tell them that someone wrote: 5:2 = 7:3 = 9:4 = 11:5.

What were they thinking? Are they correct? (They are adding 2 to each first number and 1 to each second number. They thought this was right because the series above adds 3 to each first number and 2 to each second number. This, however, is incorrect. Ratios do not compare through addition, but through multiplication. You have to multiply both numbers by the same amount. \[3:2 = 3 \times 4 : 2 \times 4 = 12:8\].

Adding the same number repeatedly only works when you add the first number repeatedly and the second number repeatedly: \[3:2 = 3 + 3 + 3 + 3:2 + 2 + 2 + 2 = 12:8\]. This is just another way of saying that \[3:2 = 3 \times 4 : 2 \times 4 = 12:8\].

Ask students to write the first few equivalent ratios in these sequences:

\[
\begin{align*}
3:5 & = 6:10 = \_ : \_ = \_ : \_ = \_ : \_ \\
2:7 & = \\
4:5 & = \\
3:8 & =
\end{align*}
\]

Show students how to find the missing part of the ratio: \[1:4 = \_ : 12\]. (They should continue the sequence of equivalent ratios until 12 is the second number, so we have \[1:4 = 2:8 = 3:12\].) Ask a volunteer how they would find the missing part of the ratio \[5:6 = 15: \_ \]. (Continue the sequence of equivalent fractions until 15 is the first number.)

Have students find the missing term in each pair of equivalent ratios:

a) \[3:5 = \_ : 20\]  
  b) \[3:4 = \_ : 12\]  
  c) \[3:4 = 12: \_ \]  
  d) \[3:5 = 15: \_ \]  
  e) \[3:5 = \_ : 15\]
Ask students to find many real-life examples of ratios:

a) For every ____ months, there is 1 year, so the ratio of months to years is ____ : 1.

b) For every ____ days, there is ____ week, so the ratio of days to weeks is ____ : ____.

c) For every ____ dozen, there are ____ items, so the ratio of dozens to items is ____ : ____.

d) For every ____ mm, there are ____ cm, so the ratio of mm to cm is ____ : ____.

e) A recipe calls for 5 cups of flour and 2 cups of sugar.

   A double recipe calls for ____ cups of flour and ____ cups of sugar.

   The ratio of cups of flour to cups of sugar is ____ : ____ or ____ : ____.

Show students how to solve word problems using ratios. For EXAMPLE:

A recipe calls for 4 cups of oats for every 3 cups of flour. How many cups of oats are needed for 12 cups of flour? (Write the cups of oats to cups of flour ratio as 4:3. Continue the sequence of equivalent ratios until they see the second number (the cups of flour) is 12. So they have 4:3 = 8:6 = 12:9 = 16:12. So they would need 16 cups of oats for 12 cups of flour.)

Give students several similar word problems to practice:

a) A recipe calls for 6 cups of flour for every 5 cups of blueberries. How many cups of flour are needed for 30 cups of blueberries?

b) If 8 bus tickets cost $5, how much will 40 tickets cost?

c) Seven bus tickets cost $5. How many bus tickets can I buy with $20?

d) On a map, 3 cm represents 10 km. How many kilometres do 15 cm represent?

e) Three centimetres on a map represents 11 km. How far apart on the map will two places be if they are 44 km apart?

f) Tanya gets paid $25 for 3 hours worked. How much would she get paid for working 6 hours?
NS6-98
Finding Equivalent Ratios and
NS6-99
Word Problems (Advanced)

**GOALS**
Students will find equivalent fractions through multiplication rather than through repeated addition. Students will solve word problems using ratios.

**PRIOR KNOWLEDGE REQUIRED**
Equivalent ratios

**VOCABULARY**
equivalent ratios

Put the following problem on the board:

There are 3 boys for every 2 girls in a class. There are 12 girls in the class. How many boys are in the class?

\[
\begin{align*}
\text{3 boys} : & \quad 2 \text{ girls} \\
\end{align*}
\]

Have a volunteer write the first few terms of the sequence of equivalent ratios. **ASK:** Which ratio in the sequence are we looking for? Which number needs to be 12? (The second number) So continue the sequence: 3:2 = 6:4 = 9:6 = 12:8 = 15:10 = 18:12. So there are 18 boys if there are 12 girls.

Now change the question slightly: There are 3 boys for every 2 girls in a class. There are 12 boys in the class. How many girls are in the class? Which number has to be 12—the first or the second? (the first) When the first number is 12, what is the second number? (8) So there are 8 girls when there are 12 boys.

Now change the questions slightly again: There are 3 boys for every 2 girls in a class. There are 25 students in the class. How many boys are in the class? Now, students are looking for the term in the sequence where the two numbers add to 25. Write the sum of the two numbers under each ratio:

\[
\begin{align*}
3:2 & = 6:4 = 9:6 = 12:8 = 15:10 = 18:12 \\
5 & \quad 10 \quad 15 \quad 20 \quad 25 \quad 30 \\
\end{align*}
\]

So there are 15 boys in the class. (We know the first number is the number of boys because we are given the ratio of boys to girls, not girls to boys.)

Have students solve the following problems using a similar method:

a) There are 4 boys for every 7 girls in a class of 33 children. How many girls are in the class?

b) There are 6 boys for every 5 girls in a class of 22 children. How many boys are in the class?

c) There are 3 red marbles for every 4 blue marbles in a jar. If there are 28 marbles, how many of them are red?

Then take up these answers with the whole class. For example, write on the board:

\[
\begin{align*}
4:7 & = 8:14 = 12:21 \\
\end{align*}
\]
Have a volunteer circle the number in each ratio that represents the number of girls, and another volunteer write the total number of students in each case:

\[
\begin{align*}
\frac{4}{7} &= \frac{8}{14} = \frac{12}{21} \\
11 \text{ students} &= 22 \text{ students} = 33 \text{ students}
\end{align*}
\]

So with 33 students in the class, there are 21 girls.

Write on the board:

\[
\begin{align*}
4:7 &= 4 + 4 : 7 + 7 = 4 + 4 + 4 : 7 + 7 + 7 \\
4:7 &= 8:14 = 12:21 \\
4:7 &= 4 \times 2 : 7 \times 2 = 4 \times 3 : 7 \times 3
\end{align*}
\]

Total is 11  Total is 11 \times 2 = 22  Total is 11 \times 3 = 33

Notice that the ratio of girls to total number of students is 7:11 = 14:22 = 21:33.

Repeat with the other two examples. Then emphasize that the sequences can be obtained from repeated addition or from multiplication. Multiplication can be used as a short cut so that we don’t even need to find the whole sequence.

We can find 7:11 = \_ \_ :33 by finding the number to multiply 11 by to get 33. Since 11 \times 3 = 33, we multiply 7 \times 3 = 21.

Review finding equivalent fractions by multiplication and compare this to finding equivalent ratios. For example, to find 4:7 = 12: \_ \_ ,

\[
\begin{align*}
\frac{4}{7} \times 3 &= \frac{12}{7} \times 3 = \frac{12}{21}
\end{align*}
\]

Have students practice finding equivalent fractions:

\[
\begin{align*}
a) \frac{3}{5} &= \frac{20}{4} \\
b) \frac{2}{3} &= \frac{18}{4} \\
c) \frac{4}{9} &= \frac{16}{30} \\
d) \frac{5}{6} &= \frac{30}{6} \\
e) \frac{7}{25} &= \frac{100}{4}
\end{align*}
\]

Show students the following problem: \frac{18}{3} = \frac{3}{6}. \text{ ASK: How is this question different from ones above? Ask for possible solutions. One solution is to divide by the same number instead of multiplying.}

\[
\begin{align*}
a) \frac{15}{3} &= \frac{5}{4} \\
+ 5
\end{align*}
\]

\text{ ASK: What number divided by 5 gives 4? That’s the same as asking for 4 \times 5:}

\[
\begin{align*}
a) \frac{15}{3} &= \frac{3}{4} \\
\times 5
\end{align*}
\]

Have students add arrows and complete each question:

\[
\begin{align*}
a) \frac{15}{35} &= \frac{3}{5} \\
b) \frac{9}{6} &= \frac{3}{5} \\
c) \frac{5}{6} &= \frac{24}{100} \\
d) \frac{2}{5} &= \frac{100}{5}
\end{align*}
\]
Show students how to solve word problems using equivalent fractions. For example, if 5 bus tickets cost $9, how much would 20 tickets cost?

**STEP 1:** Write the ratio of bus tickets to dollars (5:9) as a fraction:

\[
\frac{5 \text{ tickets}}{9 \text{ dollars}}
\]

**NOTE:** It is important to write the words beside the numbers to keep track of what information is given.

**STEP 2:** On the other side of an equal sign, write the same words on the same levels:

\[
\frac{5 \text{ tickets}}{9 \text{ dollars}} = \frac{\text{tickets}}{\text{dollars}} \quad \text{NOT} \quad \frac{5 \text{ tickets}}{9 \text{ dollars}} = \frac{\text{dollars}}{\text{tickets}}
\]

**STEP 3:** Re-read the question to determine which quantity (i.e., tickets or dollars) has been given, and place that quantity on the proper level.

\[
\frac{5 \text{ tickets}}{9 \text{ dollars}} = \frac{20 \text{ tickets}}{\text{dollars}}
\]

**STEP 4:** Solve the ratio by finding the equivalent fraction:

\[
\frac{5 \times 4}{9 \times 4} = \frac{20}{36}
\]

Since 5:9 = 20:36, 20 tickets cost $36.

Have students practice with the following examples, allowing volunteers to do the first few:

a) If 5 bus tickets cost $4, how much will 15 bus tickets cost?

b) Five bus tickets cost $6. How many can you buy with $30?

c) There are 2 apples in a bowl for every 3 oranges. If there are 12 oranges, how many apples are there?

d) There are 3 apples in a bowl for every 4 oranges. If there are 12 apples, how many oranges are there?

e) A baseball player hit 2 out of every 3 times at bat. She was at bat 9 times. How many hits did she have?

f) A goalie stopped 18 out of every 19 shots. There were 38 shots. How many goals were scored? (i.e., how many did she not stop?)

**ASK:** The ratio of boys to girls in a class is 4:7. What is the ratio of boys to children? The ratio of girls to boys in a class is 5:3. Are there more girls or boys in the class?

Have students do the following problems in their notebooks:

a) The ratio of girls to boys in a school is 12:13. If the school has 200 students, how many girls are there?

b) The ratio of red marbles to green marbles in a jar is 4:11. If there are 60 marbles in the jar, how many green marbles are there?

c) Three centimetres on a map represents 20 km in real life. If a river is 80 km long, how long will it appear on the map?
d) Three centimetres on a map represents 20 km in real life. If a lake is 6 cm long on the map, what is its actual length?

Extensions

1. a) Sindi is reading on her way to work. She reads 3 pages on the 1 km bus ride. What is the ratio of pages to km on the bus? (3:1)

   b) Sindi gets off the bus and moves to the subway train. She reads 6 pages on the 6 km subway ride. What is the ratio of pages to km on the subway? (6:6 = 1:1)

   c) For each km she rides, what is the ratio of pages read on the bus to pages read on the train? (3:1)

   d) Sindi reads at the same rate on the bus as on the train. Which is faster—the bus or the train? How many times faster? (The train is three times as fast as the bus.)

   e) Anna is knitting on her way to work. She knits 120 stitches on the 2 km bus ride, switches to the train and then knits 450 stitches on the 15 km subway ride. How much faster is the subway train than Anna’s bus? What assumption did you need to make?

   f) Whose bus is faster—Anna’s or Sindi’s? (Anna’s bus is faster; the train is only twice as fast as her bus, but the train is three times as fast as Sindi’s bus.) What assumption do you need to make? (The subway is the same speed for both of them.)

2. There are 6 boys for every 10 girls on a school trip. If there are 35 girls, how many boys are there? (NOTE: To solve this question, you need to reduce the ratio given to lowest terms.)

3. Literacy Connection
   (From the Western Curriculum) In the book, Counting on Frank by Rod Clement, Frank’s master learns that the average ballpoint pen can produce a line twenty-one hundred metres long. The ratio of the lengths of a line drawn by a ballpoint pen compared to a pencil is about 1:18. About how many kilometres long would the pencil line be? Frank’s master imagines drawing lines on the walls. What process would you use to find about how many times you can draw a line around the perimeter of your classroom using a ballpoint pen and a pencil? Explain.
NS6-100
Advanced Ratios

GOALS
Students will solve problems involving ratios where one term is not a whole number.

PRIOR KNOWLEDGE REQUIRED
Equivalent ratios
Word problems

VOCABULARY
ratio

Tell your students that Tony can paint 3 walls in half an hour. Ask: What is the ratio of walls painted to hours? (3:6) Can you find an equivalent ratio where both numbers are whole numbers? (6:1) Tony has 5 hours this afternoon to paint walls. How many walls can he paint?

3 walls = 6 walls = ? walls
1/2 hour 1 hour 5 hours

Have students solve more such problems. Students should first change the ratio to an equivalent ratio in which both terms are whole numbers.

EXAMPLES:


b) A car uses 35 mL of gas for every 1/2 km. How far can the car go on 35 L of gas?

c) A newborn Siberian tiger cub gains 2 g every 1/2 hour. How much does it gain in a day?

For each ratio, find an equivalent ratio with one term equal to 10:

a) 2:7    b) 4:20    c) 5:16    d) 3:2    e) 3:5    f) 5:4    g) 30:9

(From Atlantic Curriculum A3.8) Have students collect and write about ratios found in the classroom. They could include such ratios as boys : girls; teacher : pupils; desks : students; tables : students; pencils : students; and square metres of classroom space : students.

(From Atlantic Curriculum A3.2) Ask students to find the following ratios for their bodies, and compare results with others:

- Length around wrist : length around ankle
- Length around wrist : length around neck
- Hand width : hand length
- Arm span : body height
NS6-101
Percents

GOALS
Students will write given fractions as percents, where the given fraction has a denominator that divides evenly into 100.

PRIOR KNOWLEDGE REQUIRED
Equivalent fractions
Reducing fractions to lowest terms

VOCABULARY
percent

Ask students what the word “per” means in these sentences:

Rita can type 60 words per minute.
Anna scores 3 goals per game.
John makes $10 per hour.
The car travels at a speed of up to 140 kilometres per hour.

Then write the word “percent” on the board. ASK: Has anyone seen the word “cent” before? What does it mean? Does anyone know a French word that is spelled the same way? What does that word mean? Explain that “percent” means “for every 100” or “out of 100.” For example, a score of 84% on a test would mean that you got 84 out of every 100 marks or points. Another example: if a survey reports that 72% of people read the newspaper every day, that means 72 out of every 100 people read the newspaper daily.

ASK: Sally got 84% on a test where there were 200 possible points. How many points did she get? Then rephrase the question: A test has 200 possible points. Sally got 84 points for every 100 possible points. How many points did she get?

\[
\frac{84 \text{ points}}{100 \text{ possible points}} = \frac{\text{_______}}{200 \text{ possible points}}
\]

Explain to your students that a percent is a ratio that compares a number to 100.

Have students rephrase the percents in these statements using the phrases “for every 100 ___________” or “out of 100 ___________.”

a) 52% of students in the school are girls (For every 100 students, 52 are girls OR 52 out of every 100 students in the school are girls.)
b) 40% of tickets sold were on sale (For every 100 tickets sold, 40 were on sale OR 40 out of every 100 tickets were on sale.)
c) Alejandra scored 95% on the test (For every 100 possible points, Alejandra scored 95 points on the test OR Alejandra got 95 out of every 100 points on the test.)
d) About 60% of your body weight is water (For every 100 kg of body weight, about 60 kg is water OR 60 kg out of every 100 kg of body weight is made up of water.)

Explain to students that a percent is just a short way of writing a fraction with denominator 100. Have students write each fraction as a percent:

\[
a) \frac{28}{100} \quad b) \frac{9}{100} \quad c) \frac{34}{100} \quad d) \frac{67}{100} \quad e) \frac{81}{100} \quad f) \frac{3}{100}
\]

Then have students write each percent as a fraction:

\[
a) 6\% \quad b) 19\% \quad c) 8\% \quad d) 54\% \quad e) 79\% \quad f) 97\%
\]
Now have students write each decimal as a percent by first changing the decimal into a fraction with denominator 100.

a) 0.74  b) 0.03  c) 0.12  d) 0.83  e) 0.91  f) 0.09

Write the fraction $\frac{3}{5}$ on the board and have a volunteer find an equivalent fraction with denominator 100. **ASK:** If 3 out of every 5 students at a school are girls, how many out of every 100 students are girls? (60%) Write on the board: $\frac{3}{5} = \frac{60}{100} = 60\%$.

Then have volunteers find the equivalent fraction with denominator 100 and then the equivalent percent for more fractions with denominator 5.

**EXAMPLES:** $\frac{2}{5}$, $\frac{4}{5}$ and $\frac{1}{5}$

Repeat for fractions with various denominators.

**EXAMPLES:** $\frac{4}{10}$, $\frac{8}{20}$, $\frac{3}{5}$, $\frac{1}{2}$, $\frac{29}{50}$, $\frac{21}{25}$, $\frac{17}{5}$

**Bonus**

Use the equivalent percents to put the above fractions in order from least to greatest.

Then have students write various decimal tenths as percents by first changing the decimal to a fraction with denominator 100.

**EXAMPLES:** $0.2 = \frac{2}{10} = \frac{20}{100} = 20\%$  0.3  0.9  0.7  0.5

Explain to students that they can find a percent of a figure just as they can find a fraction of a figure. Ask students to decide first what fraction and then what percent of each figure is shaded:

Now show students a fraction not in lowest terms, whose denominator does not divide evenly into 100 unless the fraction is reduced.

**EXAMPLE:** $\frac{6}{15}$

Tell your students that you want to find an equivalent fraction with denominator 100. **ASK:** How is this fraction different from previous fractions you have changed to percents? (The denominator does not divide evenly into 100.) Is there any way to find an equivalent fraction whose denominator does divide evenly into 100? (Reduce the fraction by dividing both the numerator and the denominator by 3.) Write on the board:

$\frac{3}{5} = \frac{90}{150} = 60\%$

Summarize the 3 steps for finding the equivalent percent of a fraction.

1. Reduce the fraction so that the denominator is divisible by 100.
2. Find an equivalent fraction with denominator 100.
3. Write the fraction with denominator 100 as a percent.

Have students write various fractions as percents:

a) $\frac{3}{12}$  b) $\frac{6}{30}$  c) $\frac{24}{30}$  d) $\frac{3}{75}$  e) $\frac{6}{15}$  f) $\frac{36}{48}$  g) $\frac{60}{75}$
Extension

**ASK:** How many degrees are in a circle? If I rotate an object 90° counter-clockwise, what fraction and what percent of a complete 360-degree turn has the object made? (\( \frac{90}{360} = \frac{1}{4} = \frac{25}{100} = 25\% \))

Repeat for various degrees: 180°, 18°, 126°, 270°, 72°, 216°.

---

**NS6-102**

**Visual Representations of Percents**

**GOALS**
Students will visualize various percentages of different shapes, including rectangles, squares, triangles, and lines.

**PRIOR KNOWLEDGE REQUIRED**
Equivalent fractions
The relationship between decimals with up to 2 decimal places and fractions with denominator 100

**VOCABULARY**
percent

Draw a hundreds block on the board and have students write what part of the block is shaded in three different ways:

**EXAMPLE:**

\( \left( \frac{39}{100}, 0.39, 39\% \right) \)

Have students find 25% of each shape in various ways:

Then draw shapes on the board and divide them into equal pieces, the number of which divide evenly into 100:

**ASK:** What fraction of each shape is shaded? Have students change each fraction to an equivalent fraction with denominator 100, and then to a decimal and a percent.
Show students a double number line with fractions on top and percents on the bottom.

\[
\begin{array}{cccccccccccc}
0 & \frac{1}{10} & \frac{2}{10} & \frac{3}{10} & \frac{4}{10} & \frac{5}{10} & \frac{6}{10} & \frac{7}{10} & \frac{8}{10} & \frac{9}{10} & 1 \\
0\% & 10\% & 20\% & 30\% & 40\% & 50\% & 60\% & 70\% & 80\% & 90\% & 100\%
\end{array}
\]

Then have volunteers add fractions and percents to each number line below:

\[
\begin{array}{cccccccccccc}
0 & & & & & & & & & & 1 \\
0 & & & & & & & & & & \frac{1}{2} \\
0 & & & & & & & & & & \frac{1}{5}
\end{array}
\]

Draw lines of varying lengths and have students mark a different percent on each one.

**EXAMPLES:**

\[
\begin{align*}
50\% & \quad \text{ANSWER:} \quad & 50\% \\
20\% & \\
75\% & \\
80\% & \\
60\%
\end{align*}
\]

Then have students draw two lines such that 20\% of the first line is longer than 50\% of the second line.

Now draw a line segment and identify what percent of a line the segment represents. Invite a volunteer to extend the line segment to its full length (i.e., to show 100\%).

**EXAMPLE:**

\[
\begin{align*}
50\% & \quad \text{ANSWER:} \quad & \text{Extend to full length} \\
\end{align*}
\]

Finally, have students estimate the percent of various marks on a number line (to the nearest 10\%), and then superimpose a number line of the same length divided into ten equal parts so that students can check their estimates.
NS6-103
Comparing Decimals, Fractions & Percents

GOALS
Students will compare and order fractions, percents, and decimals.

PRIOR KNOWLEDGE REQUIRED
Percents as fractions with denominator 100
Ordering fractions
Equivalent fractions
Signs for less than (<) and greater than (>)

VOCABULARY
percent

Review comparing and ordering

- fractions with the same denominator (7/10 is greater than 4/10).
- percents (30% is greater than 24% because 30/100 is greater than 24/100).
- fractions with different denominators (6/10 is greater than 3/20 = 3/10).
- fractions and decimals (3/5 is greater than 0.52 because 60/100 is greater than 52/100).

Remind students of the signs for less than (<) and greater than (>) and use them throughout the lesson.

Teach students how to compare fractions and percents by changing both to an equivalent fraction with denominator 100. (EXAMPLES: Have students decide which is larger between 1/2 and 38%, 3/5 and 70%, 9/10 and 84%, 7/25 and 30%, 9/20 and 46%.)

Then compare decimals and percents by changing both to an equivalent fraction with denominator 100. (EXAMPLES: 0.9 and 10%, 0.09 and 10%, 28% and 0.34, 4% and 0.3)

Now ask students, working independently, to put the following groups of numbers in order from least to greatest by first changing all three numbers to a fraction with denominator 100.

a) 0.28 42% 3/10
b) 14/50 23% 0.3

c) 19/25 0.72 7%
d) 1/4 4% 0.4

Bonus
13/20, 0.6, 66%, 0.7, 7%, 16/25, 3/50

Then teach students how to compare fractions and percents when a denominator does not divide evenly into 100. ASK: How can we compare 35% to 1/3? If we changed 35% to a fraction, what would it be? (35/100) Do we have a way to compare 1/3 to 35/100 or are we stuck? We have two fractions with different denominators, but 3 doesn't divide evenly into 100. How can we give both fractions the same denominator? (Use denominator 300.) Have volunteers change both fractions to equivalent fractions with denominator 300 and ask the class to identify which is greater, 35% or 1/3, and to explain how they know.

Repeat with various reduced fractions whose denominator does not divide evenly into 100. Then have students compare more fractions and percents independently, in their notebooks.

EXAMPLES: Compare 3/5 and 85%, 3/7 and 42%, 2/5 and 21%.
Bonus
Make up your own question and have a partner solve it.

Finally, have students compare lists of numbers (fractions, percents, and decimals) in which the fractions do not have denominators that divide evenly into 100.

EXAMPLES:

a) \( \frac{1}{6} \), 0.17, 13%  
b) 0.37, \( \frac{1}{3} \), 28%  
c) \( \frac{5}{7} \), 71%, 0.68

Bonus
\( \frac{7}{9} \), .8, \( \frac{4}{7} \), 51%, .78, 62%

Extensions

1. (From Atlantic Curriculum A5.6) Ask students to name percents that indicate
   • almost all of something,
   • very little of something,
   • a little less than half of something.

Ask students to explain their thinking.

2. (From Atlantic Curriculum A5.9) Ask students to look for percents in newspapers, flyers, magazines, and other printed materials, such as food packaging, trading cards, and order forms. What kind of information is expressed as a percent? Ask students to clip examples and to make a collage for a class display.
NS6-104
Finding Percents

GOALS
Students will find multiples of 10 percent of a number.

PRIOR KNOWLEDGE REQUIRED
Converting fractions to decimals and vice versa
The relationship between percents and fractions

VOCABULARY
percent

Tell your students that you will use one thousands block to represent one whole. Given this information, ask students to identify the fraction and the decimal each model represents:

Then make a model of the number 1.6 (again, using one thousands block as one whole). **ASK:** What do I need to make the model? (1 thousands block, 6 hundreds blocks) How do you know? How can I show \( \frac{1}{10} \) of 1.6? (one tenth of a thousands block is a hundreds block and one tenth of a hundreds block is a tenth block, so I need a hundreds block and 6 tenths blocks to make \( \frac{1}{10} \) of 1.6.) What number is \( \frac{1}{10} \) of 1.6? (0.16, since this is what the base ten materials show) Do a few more examples together.

Ask students to explain how they can find \( \frac{1}{10} \) of any number. Remind your students that when they move the decimal point one place left, each digit becomes worth \( \frac{1}{10} \) as much, so the whole number becomes \( \frac{1}{10} \) of what it was before they moved the decimal point. **EXAMPLE:** 4 is \( \frac{1}{10} \) of 40, and 0.1 is \( \frac{1}{10} \) of 1, so 4.1 is \( \frac{1}{10} \) of 41. **ASK:** What is this like dividing by? (10) Emphasize that to find \( \frac{1}{10} \) of anything, you divide it into 10 equal parts; to find \( \frac{1}{10} \) of a number, you divide the number by 10. **ASK:** What decimal is the same as \( \frac{1}{10} \)? (0.1 or .1) What percent is the same as \( \frac{1}{10} \)? (10%) Ask your students to find 10% of each number by just moving the decimal point.

Ask your students to find 10% of each number by just moving the decimal point.

Then show this number line:

Have a volunteer fill in the missing numbers on the number line. Then ask volunteers to look at the completed number line and identify: 10% of 30, 40% of 30, 90% of 30, 70% of 30.
Repeat the exercise for a number line from 0 to 21.

**ASK:** If you know 10% of a number, how can you find 30% of that number? (multiply 10% of the number by 3) Tell your students that you would like to find 70% of 12. **ASK:** What is 10% of 12? (1.2)
If I know that 10% of 12 is 1.2, how can I find 70% of 12? (multiply 1.2 \times 7)

Using this method, have students find:

a) 60% of 15  
b) 40% of 40  
c) 60% of 4  
d) 20% of 1.5  
e) 90% of 8.2  
f) 70% of 4.3  
g) 80% of 5.5

Remind students that 5% is half of 10%. Have them find 5% of the following numbers by first finding 10% then dividing by 2. (Students should use long division on a separate piece of paper.)

a) 80  
b) 16  
c) 72  
d) 50  
e) 3.2  
f) 2.34

Tell students to find 15% of the following numbers by finding 10% and 5%, and then adding. (Students should use a separate piece of paper for their rough work.)

a) 60  
b) 240  
c) 12  
d) 7.2  
e) 3.80  
f) 6.10

Explain to students that taking 1% of a number is the same as dividing the number by 100. (The decimal shifts 2 places to the left.) Have students find 1% of:

a) 27  
b) 3.2  
c) 773  
d) 12.3  
e) 68

**Extension**

Have students compare:

a) 20% of 60 and 60% of 20  
b) 30% of 50 and 50% of 30  
c) 40% of 20 and 20% of 40  
d) 70% of 90 and 90% of 70  
e) 80% of 60 and 60% of 80  
f) 50% of 40 and 40% of 50

What pattern do students see? Challenge them to figure out why this pattern holds.
NS6-105
Finding Percents (Advanced)

GOALS
Students will find any percentage of a number.

PRIOR KNOWLEDGE REQUIRED
Reducing fractions
Multiplying decimals
The standard algorithm for multiplying

VOCABULARY
percent

Ask the class how they would find 20% of 5.5. Someone will likely volunteer the following solution: 10% of 5.5 is 0.55, so 20% is $2 \times 0.55 = 1.1$. Now ask if anyone can think of an easier way to find 20% of 5.5. To guide students, ASK: What fraction is equivalent to 20%? If your students answer $20/100$, ask them if the fraction can be reduced. When they see that 20% is equivalent to $\frac{1}{5}$, ASK: How can you find $\frac{1}{5}$ of a number? Remind students that finding $\frac{1}{5}$ of a number is just like dividing that number into 5 equal parts, and 5.5 is particularly easy to divide by 5. WRITE: $5.5 \div 5 = 1.1$.

Tell students that Maria found 80% of 5.5 by first finding 10% of 5.5 and then multiplying by 8. Sean found 80% of 5.5 by first finding 20% and then multiplying by 4. Have volunteers write out both solutions. Do they produce the same answer? Can students explain why? (Both methods give the same answer because 20% is 2 × 10%, so 20% × 4 is the same as 10% × 2 × 4 = 10% × 8.)

ASK: How can you find 25% of a number easily? (divide by 4, since 25% is the same as $\frac{1}{4}$) If you know 25% of a number, how can you find 75%? (multiply by 3, since 75 = 3 × 25) Have students find:

a) 75% of 160 by first finding 25% of 160.

b) 75% of 160 by first finding 5% of 160.

Ask students what they need to multiply by in each case. (In a) they need to multiply 25% of 160 by 3 because 75% = $3 \times 25\%$. In b), they need to multiply by 15 because 75% = $15 \times 5\%$).

Remind students how to find a fraction of a whole number. For example, to find $\frac{3}{4}$ of 240, you can first find $\frac{1}{4}$ of 240 (240 ÷ 4 = 60) and then multiply by 3, since $\frac{3}{4}$ is 3 times as much as $\frac{1}{4}$. So, $\frac{3}{4}$ of 240 = 180.

Now tell students that you would like to find 53% of 12. First, ask if they can estimate the answer. To guide them, ASK: Is there a percentage of 12 that is close to 53% and easy to calculate? (yes, 50%) Will this estimate be lower or higher than the actual answer? (50% of 12 is 6, which is lower than the actual answer because 50% is less than 53%) Then ASK: What is 1% of 12? (0.12) How can we find 53% of 12 if we know 1% of 12? How many times greater than 1% is 53%? (53, so multiply 1% of 12 by 53 to find 53% of 12) Write on the board:

\[
53\% \text{ of } 12 = (12 \div 100) \times 53 \\
= 0.12 \times 53 \\
= 12 \times 53 \div 100 \\
= 636 \div 100 \\
= 6.36
\]

ASK: Is this answer reasonable? Why? Remind students that multiplying decimals is just like multiplying whole numbers except that they have to put the decimal point in the correct place.
**ASK:** Why is 1% of a number particularly easy to find? (Since 1% of a number is just the number divided by 100, you can move the decimal point 2 places to the left.) Have students find various percentages of different numbers using the strategy of first finding 1% and then multiplying by the correct amount. For example, to calculate 37% of 21, the student would first determine that 1% of 21 is 0.21 and then multiply $0.21 \times 37 = 21 \times 37 \div 100 = 777 \div 100 = 7.77$.

**EXAMPLES:** 13% of 85, 82% of 42, 33% of 33

Students can use the BLM "Percent Strips" to check their answers. They can also use the percent strip to check their estimates for various percentages of numbers. **(EXAMPLES:** 68% of 33, 91% of 33, 5% of 42, 76% of 85, 55% of 21)

**Extensions**

1. Sara says that to find 10% of a number, she divides the number by 10, so to find 5% of a number, she divides the number by 5. Is she right? Explain. (No—5% of a number is $\frac{5}{100}$ or $\frac{1}{20}$ of the number, so to find 5%, or $\frac{1}{20}$, of the number, she should divide it by 20.)

2. Continue the extension from NS6-104. Have students compare:
   - a) 36% of 24 and 24% of 36
   - b) 17% of 35 and 35% of 17
   - c) 29% of 78 and 78% of 29
   - d) 48% of 52 and 52% of 48

   Ask students to predict a rule and make up another example to check that the rule works.

3. **DISCUSS:** Does it make sense to talk about 140% of a number? What would it mean? Lead the discussion by referring to fractions greater than 1. Discuss what 100% and 40% of a number mean separately. Could 50% of a number be obtained by adding 20% and 30% of that number? Could 140% be obtained by adding 100% and 40% of the number?
NS6-106
Percents: Word Problems

GOALS
Students will solve word problems involving percents.

PRIOR KNOWLEDGE REQUIRED
Percents
Fractions and decimals
Calculating the percent of a number

VOCABULARY
percent

Tell students that Maria got \( \frac{17}{25} \) on her math test and \( \frac{14}{20} \) on her science test. What percent of the points, or marks, did she get on each test? (68% in math, 70% in science) On which test did she do better? Even though Maria got more marks on her math test than on her science test (17 instead of 14), she got a higher percentage of marks on the science test than the math test (70% instead of 68%). So she did better on the science test. **ASK:** Is it easier to compare test scores when they are given as fractions or when they are given as percents? What’s easier to compare — \( \frac{17}{25} \) to \( \frac{14}{20} \) or 70% to 68%? Why doesn’t the test have to have 100 marks in order for the result to be expressed as a percent? (We can convert any fraction to a percent by changing the denominator to 100.) Tell students that this is one application of percents—we can compare two test scores easily, even when the total number of marks in each test is different.

Have students convert Sally’s test scores to percents and decide which was her best test and which was her worst:

- Math: \( \frac{17}{20} \)
- Science: \( \frac{22}{25} \)
- Social Studies: \( \frac{36}{40} \)
- Language: \( \frac{43}{50} \)

**HINT:** One of the scores will need to be reduced before it can be expressed as a fraction with denominator 100.

Tell students that Olga has collected 50 stamps from various countries: 31 from Canada, 14 from the United States, and 5 from elsewhere. Ask students to calculate what percentage of Olga’s stamp collection is from Canada, what percentage is from the United States, and what percentage is from elsewhere.

Rita, on the other hand, has collected 3000 stamps: 1020 from Canada, 840 from the United States, and 1140 from elsewhere. Ask students to calculate what percentage of Rita’s stamp collection is from Canada, what percentage is from the United States, and what percentage is from elsewhere.

**ASK:** Who has more stamps from Canada? Who has a greater percentage of stamps from Canada? (Rita has more Canadian stamps than Olga, but Olga has a greater percentage of Canadian stamps in her collection—62% of Olga’s stamps are from Canada but only 34% of Rita’s are from Canada.) How do percentages help us to compare stamp collections even when one has many more stamps than the other?

Tell students that Anna’s stamp collection has this distribution: 41% from Canada, 26% from the United States, and an unknown percent from elsewhere. **ASK:** What percent of Anna’s collection is from somewhere other than Canada or the United States? Emphasize that percentages must add to 100 because the whole amount of anything is 100%.

Jennifer has stamps from all over the world. In her collection, \( \frac{5}{6} \) of the stamps are from Canada and 36% are from the United States. What percentage of Jennifer’s stamps are from neither Canada nor the United States?
Then have students find the missing percentages of other stamps in each collection:

- a) Canada: 40%  
  USA: \(\frac{1}{2}\)  
  Other:

- b) Mexico: 25%  
  USA: \(\frac{3}{5}\)  
  Other:

- c) Jamaica: \(\frac{17}{25}\)  
  Canada: 19%  
  Other:

- d) China: \(\frac{13}{30}\)  
  Japan: 0.31  
  Other:

Tell students that Sayaka has spent 500 days travelling the world. **ASK:** If she spent 60% of her days in Europe and \(\frac{3}{10}\) of her days in Africa, how many days did she spend in each place? How many days did she spend in neither Africa nor Europe? Draw a chart on the board and have volunteers complete the chart.

<table>
<thead>
<tr>
<th></th>
<th>Fraction of Trip</th>
<th>Percentage of Trip</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Extensions**

1. Five people—2 adults and 3 children—attend a hockey game. What percentage of the group do the children represent? Describe a group of a different size with the same percentage of children.

2. Mr. Bates buys
   - 5 single-scoop ice cream cones for $1.45 each
   - 3 double-scoop ice cream cones for $2.65 each

   A tax of 15% is added to the cost of the cones. Mr. Bates pays with a 20-dollar bill. How much change does he receive? Show your work.

3. The chart shows the fraction or percent of stamps that children have collected from various countries.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>England</th>
<th>Other Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brian’s Collection</td>
<td>23%</td>
<td>(\frac{3}{5})</td>
<td></td>
</tr>
<tr>
<td>Faith’s Collection</td>
<td>(\frac{3}{4})</td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td>Andrew’s Collection</td>
<td>(\frac{1}{2})</td>
<td>30%</td>
<td></td>
</tr>
</tbody>
</table>

Which child has the greatest percentage of stamps from Other Countries?
4. **DISCUSS:** Sally got $\frac{171}{200}$ on a national math test. Can this mark be written as a percent? The answer (yes, it can be written as a decimal percent) is not as important as the discussion that should arise. Leading questions you might use include: Is this mark better or worse than 80%? How do you know? Is it better or worse than 90%? Than 85%? Than 86%? Is it closer to 85% or to 86%? (It is halfway between them.) Is there a number halfway between 85 and 86? (yes, 85.5) Tell students that even though we said that percents are just fractions with denominator 100, percents are actually even better than fractions with denominator 100—you can’t write $\frac{85.5}{100}$ as a fraction, but you can write 85.5%. (You could tell students that they won’t learn about decimal percents until grade 8, but this class is smart enough to know about them in grade 6.)

5. Investigate on the Internet: What percentage of car passengers wear seat belts in Canada? In the United States? In other countries? (Emphasize that even though the United States has many more people than Canada, a meaningful comparison can still be made in terms of percentages.)

6. Which has a greater percentage of water by volume—your body or the Earth? (Emphasize that although the Earth has much more water than your body, your body has a greater percentage of water than does the Earth.)

7. Cross-curricular connection: Which has a greater percentage of water by surface area—Canada or the United States? Canada or Finland?

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**NS6-107 Circle Graphs**

**GOALS**

Students will use circle graphs to represent proportions.

**PRIOR KNOWLEDGE REQUIRED**

Comparing and ordering fractions
Equivalent fractions

**VOCABULARY**

circle graphs

Remind students that graphs are a way to visually represent data. Ask students what types of graphs they have learned about so far. Tell students that circle graphs can be used to represent percentages and that you will show them how to make a circle graph.

Show your students a strip of thick paper 140 cm long divided into 10 equal parts (14 cm each). Mark off the 10 equal parts on both sides of the paper.

Curl the strip around to form a ring, and tape or paste the two ends together. Place this ring against the board and trace the circle. Mark off the division points on the circle so that it, too, is divided into 10 equal parts. Now use opposite markings on the circle to draw 5 diameters through the circle—this will divide the circle into 10 equal pieces, or sections. **ASK:** What percentage of the circle is each piece? (10%) Then colour 3 sections red, 2 blue, and 5 yellow and ask your students what percentage of the circle is taken up by each colour. (30% red, 20% yellow, 50% blue)
Draw several circles of radius 22.3 cm using a large compass. Divide each circle into regions of different colours and ask students approximately what percentage of each circle is taken up by each colour. Use the ring of paper you used to trace the first circle (it should fit these circles closely) to check students’ estimates.

Tell students to make their own circle graph using a strip of thick paper 20 cm long divided into 10 equal parts (2 cm each). (They can also use the BLM “Large Circle Graph.”) Then have students transfer the following data to their circle graph:

**Favourite Colour**

![Diagram of favourite colours]

Ask students what percentage the whole circle represents. Give students more data to transfer to a circle graph. (They can use the BLM “Small Circle Graphs.”)

Then ask students to decide which combinations of percents are possible for a circle graph, and which are not.

a) 35%, 25%, 35%, 10%  
b) 20%, 20%, 30%, 30%  
c) 10%, 20%, 30%, 40%  
d) 15%, 15%, 40%, 40%  
e) 10%, 15%, 25%, 50%  
f) 15%, 18%, 26%, 31%

Tell students that the 21 members of a hockey team come from different provinces, as follows:

- Alberta: 2
- Manitoba: 1
- Ontario: 9
- Quebec: 7
- Saskatchewan: 2

Show your students how to present this data in a circle graph. Start with a strip of paper 147 cm long that is divided into 21 equal parts (7 cm each), one part for each person on the team:

![Diagram of hockey team members]

As before, curl the strip into a ring and trace the circle on the board. This time, draw a line from each marking on the circle to the centre of the circle. (You can’t join opposite pairs of markings this time because the number of markings is odd.)

Colour the number of pieces corresponding to the number of players from each province—2 for Alberta, 1 for Manitoba, and so on. Use a different colour for each province. Then ask the class to estimate the percent of hockey players from each province, and record the estimates. Check the students’ estimates using the first ring you made for the lesson (the one divided into 10 equal parts—it should fit just inside the circle). How close were the estimates? Were any within 5% or 10%?


Extensions

1. Sally surveys 20 families on her street to find out how many cars they have. She displays her result in both a frequency table and a circle graph.

<table>
<thead>
<tr>
<th># of Cars</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Sally uses the frequency table to find the mean:

\[(0 + 0 + 0 + 0 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2) ÷ 20\]

Sally's older sister Tina uses the circle graph to find the mean. She pretends there are only 5 families:

\[(0 + 1 + 1 + 2 + 2) ÷ 5\]

a) What answers do Sally and Tina get? (both get 1.2)

b) Can you explain why they get the same answer? (Because one data set is obtained from the other by repeating each data value the same number of times, so the mean isn’t changed. See PDM6-12 Extension 5.)

c) Whose method do you like better? Why?

d) If Sally accidentally divides 24 by 10 instead of by 20, she gets a mean of 2.4. How can she tell immediately that this answer must be wrong?

2. Have students write each degree turn as a percentage of a full rotation (see the Extension in NS6-101).

a) 180°  
b) 90°  
c) 18°  
d) 54°
Then have students convert various percentages to fractions over 360.

- a) 10%
- b) 30%
- c) 40%
- d) 5%
- e) 15%
- f) 35%
- g) 50%
- h) 70%
- i) 75%
- j) 85%

Discuss various methods of solving each question. For example, 40% could be calculated directly or by adding the answers to a) and b). You could find 5% directly or you could find it by dividing the answer to a) by 2. You could find 15% by dividing the answer to b) by 2, or by adding the answers to a) and d).

Give students a protractor and a circle with the centre marked and have students draw a circle graph for the following data.

**Favourite Sport**

- Hockey: 20%
- Baseball: 25%
- Basketball: 5%
- Soccer: 15%
- Other: 35%
NS6-108
Fractions, Ratios and Percents

GOALS
Students will solve word problems involving fractions, ratios, and percents.

PRIOR KNOWLEDGE REQUIRED
Fractions
Ratios
Percents

VOCABULARY
fractions
ratios
percents
circle graphs

To solve questions involving fractions, ratios, and percents, students need to be able to recognize the part and the whole (and to express the ratio of the part to the whole as a fraction). Use simple word problems to illustrate and work through different cases.

EXAMPLES:

a) There are 5 boys and 9 children. (In this case, you are given the whole and one of the parts.)

What fraction of the children are boys? \( \frac{5}{9} \)

What fraction of the children are girls? First, subtract the number of boys from the number of children to find the number of girls: \( 9 - 5 = 4 \). Therefore, \( \frac{4}{9} \) of the children are girls.

b) There are 5 boys and 6 girls. (In this case, you are given two parts).

What is the fraction of boys and girls?

First find out how many children there are (this is the whole):

\( 5 + 6 = 11 \).

Therefore, \( \frac{5}{11} \) of the children are boys and \( \frac{6}{11} \) are girls.

c) The ratio of boys to girls is 3:4. What is the fraction of boys and girls?

For every 3 boys there are 4 girls, so 7 children in total (i.e., add the parts of the ratio: \( 3 + 4 = 7 \)). So \( \frac{3}{7} \) of the children are boys and \( \frac{4}{7} \) are girls.

Ask students how many boys are in their class, how many girls, and how many children altogether:

\[ b: \quad \quad \quad g: \quad \quad \quad c: \quad \quad \quad \]

ASK: Did you count everyone one by one or was there an easier way once you found the number of boys and girls?

Ask students to fill in the numbers of boys, girls, and children given various pieces of information:

a) 7 girls and 8 boys

b) 6 girls in a class of 20

c) 12 boys in a class of 30

d) 17 girls in a class of 28

Then have students determine the numbers of boys, girls, and children, the fraction of girls, and the fraction of boys in these classes:

a) There are 6 boys and 5 girls.

b) There are 14 boys in a class of 23.
c) There are 15 girls in a class of 26.

Now have students write the fraction of girls and boys in these classes.

a) There are 3 boys and 4 girls in a class.
b) There are 7 boys and 13 children in a class.
c) There are 8 girls and 19 children in a class.
d) The ratio of boys to girls is 1:2.
e) The ratio of girls to boys is 2:3.
f) The ratio of boys to girls is 12:11.
g) The ratio of boys to girls is 11:12.
h) The ratio of girls to boys is 11:12.

Which 2 of the last 3 questions have the same answer? (parts f and h have the same answer)
Can you find a question that has the same answer as part g? (The ratio of girls to boys is 12:11.)

Then have students determine the number of girls and boys in each class.

a) There are 30 children in a class and \( \frac{3}{5} \) are girls.
b) There are 36 children in a class and \( \frac{4}{9} \) are girls.
c) There are 21 children in a class and \( \frac{5}{7} \) are boys.
d) There are 18 children in a class. The ratio of boys to girls is 7:2.
e) There are 18 children in a class. The ratio of girls to boys is 2:7.
f) There are 18 children in a class. The ratio of boys to girls is 2:7.

(Again, stop to discuss which 2 of the last 3 are the same (parts d and e) and how to find another question the same as part f.)

g) There are 30 children in the class and 60% are girls.
h) There are 45 children in the class and 40% are girls.

**Extension**

When you compare 2 numbers, you can estimate what fraction (or percent) the numbers make by changing one of the numbers slightly.

**EXAMPLE A:** 5 out of 11 is close to 5 out of 10, which is close to \( \frac{1}{2} \) or 50%.

**EXAMPLE B:** 9 out of 22 is close to 8 out of 24, which is \( \frac{1}{3} \) (\( \frac{8}{24} = \frac{1}{3} \)), which is close to 30%.

The chart shows the lengths of calves and adult whales (in feet). Approximately what fraction and what percent of the adult length is the calf’s length? Did you need to know how long a foot is to answer this question?

<table>
<thead>
<tr>
<th>Type of Whale</th>
<th>Killer</th>
<th>Humpback</th>
<th>Narwhal</th>
<th>Fin Backed</th>
<th>Sei</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calf Length (feet)</td>
<td>7</td>
<td>16</td>
<td>5</td>
<td>22</td>
<td>16</td>
</tr>
<tr>
<td>Adult Length (feet)</td>
<td>15</td>
<td>50</td>
<td>15</td>
<td>70</td>
<td>60</td>
</tr>
</tbody>
</table>
NS6-109
2-Digit Division

GOALS
Students will estimate the result of dividing 3-digit numbers by 2-digit numbers using rounding to the nearest ten.

PRIOR KNOWLEDGE REQUIRED
Skip counting and multiplication
Doubling and the 2 times tables
Rounding to the nearest ten

VOCABULARY
estimate divisor
dividend quotient
circle graphs

Write the following questions on the board:

\[
\begin{array}{cccccccc}
8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
\times 1 & \times 2 & \times 3 & \times 4 & \times 5 & \times 6 & \times 7 & \times 8 \\
\end{array}
\]

Have volunteers write the answers to these problems. Then have students solve the 28 times tables up to 9 independently, using the 8 times tables above:

\[
\begin{array}{cccccccc}
28 & 28 & 28 & 28 & 28 & 28 & 28 & 28 \\
\times 1 & \times 2 & \times 3 & \times 4 & \times 5 & \times 6 & \times 7 & \times 8 \\
\end{array}
\]

For example, to solve 28 × 3, students can add 8 × 3 (which they just calculated) and 20 × 3 (which is 3 × 2 tens).

Now go through an example of long division. Divide 28 into 149 using the 28 times tables:

\[
\begin{array}{c}
28 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
149 \\
\downarrow
\end{array}
\]

Since 149 is between 28 × 5 = 140 and 28 × 6 = 168 (from the list), the quotient is 5 and we subtract 149 – 140 = 9 to find the remainder:

\[
\begin{array}{c}
28 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
149 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
5 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
140 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
9 \text{ Remainder}
\end{array}
\]

Have students practise dividing 28 into several 3-digit numbers:

\[
\begin{array}{c}
28 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
149 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
237 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
70 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
261 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
170 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
220 \\
\downarrow
\end{array}
\]

Now tell students that they won’t always have multiplication tables to refer to when they are dividing, so they will have to guess how many times the divisor goes into the dividend.

Write on the board:

\[
\begin{array}{c}
41 \\
\downarrow
\end{array}
\]
\[
\begin{array}{c}
152 \\
\downarrow
\end{array}
\]

ASK: About how many times does 41 go into 152? What number is close to 41 and easy to skip count by? (40) How did you choose that number? (By rounding 41 to the nearest 10, because multiples of 10 are easy to skip count by.) Demonstrate skip counting by 40 to find the number of times that 40 goes into 152: 40, 80, 120. (Stop at 120 because the next number—160—is greater than 152.) So 40 goes into 152 three times. Explain that skip counting is a good way to find out how many times 40 goes into 152, and then ask if anyone can think of another way to estimate the quotient. Explain that finding out how many times 40 goes into 152 is almost like asking how many times 40 goes into 150, which, in turn, is like asking how many times 4 goes into 15. The answer, again, is 3.
Have students practise estimating the quotient for the following questions:

a) 186 ÷ 23  
b) 208 ÷ 29  
c) 293 ÷ 92  
d) 168 ÷ 19  
e) 223 ÷ 41  
f) 408 ÷ 71

Volunteers could do one or two questions on the board before students answer the rest independently in their notebooks. Students should leave a few blank lines beneath each question.

Then demonstrate multiplying the divisor by the quotient and writing the product underneath the dividend:

\[
\begin{array}{c}
8 \\
23 \overline{186} \\
184
\end{array}
\]

Have students do the same in their notebooks for parts b) to f) above.

Then have students complete both steps (estimating the quotient, then multiplying the divisor by the estimate and writing the product underneath the dividend) for various questions.

**EXAMPLES:**

a) 238 ÷ 51  
b) 183 ÷ 39  
c) 333 ÷ 67

**NOTE:** If you make up additional examples, choose your dividends and divisors so that the estimate obtained by rounding to the nearest ten always gives the correct quotient. For example, do not use 204 ÷ 53, since the estimate obtained by skip counting is 4 (50, 100, 150, 200 OR 5 goes into 20, 4 times), but 53 × 4 is too large. (Students will learn how to solve these types of questions in the next lesson.)

Then, for all the examples completed so far, have students subtract the product they found from the dividend to obtain the remainder. When they have done this for every question, tell them to finish the long division by writing the remainder beside the quotient.

\[
\begin{array}{c}
8 \quad \text{R} \quad 2 \\
23 \overline{186} \\
184 \\
2
\end{array}
\]

Finally, give students more problems and ask them to complete all 6 steps at once, as in **QUESTION 4** on the worksheet. **EXAMPLES:** 116 ÷ 38, 253 ÷ 62, 401 ÷ 58.
NS6-110

2-Digit Division—Correcting Your Estimate

GOALS
Students will do long division by estimating the quotient, deciding if the quotient is too high or too low, and revising their estimate accordingly.

PRIOR KNOWLEDGE REQUIRED
Long division by 1-digit numbers
Long division by 2-digit numbers when there is no need to correct the estimate

VOCABULARY
estimate
divisor
dividend
quotient
circle graphs

Review long division by a 1-digit divisor and use models to remind students that the remainder must be less than the divisor. For example, to find $17 \div 5$, students could draw 5 circles and place dots or counters in the circles, one at a time, until they can no longer place one more in every circle because there are too few left over. Only when the number of dots or counters is smaller than the number of circles can they stop. Mathematically, this means that the remainder is smaller than the divisor.

Since there are now fewer than 5 dots left over, we cannot place the remaining dots equally into the 5 circles, so the 2 dots are the remainder.

Now demonstrate how students can use this when dividing larger numbers, to determine if their estimate is too low. Begin solving $263 \div 37$ using the method students learned in the last lesson (round 37 to 40 and skip count by 40 to estimate the quotient, 6). Then multiply 37 by 6 and subtract the product from the dividend:

\[
\begin{array}{c|c|c}
6 & 37 & 263 \\
\hline
222 & & \\
41 & & \\
\end{array}
\]

Multiply 37 and 6:
37 \times 6 = 222
Subtract the product (222) from the dividend (263):
263 - 222 = 41

**ASK:** What is our remainder? (41) Point out that 41 is more than 37, so if we imagine 37 circles and 41 dots left over, we can actually place one more dot in every circle and have 4 left over. So, instead of 6 dots in each circle with 41 left over, we can fit 7 in each circle with only 4 left over. We know the estimate is too low because the remainder is larger than the divisor.

Have students decide whether the estimated quotient is too low or just right in these and other completed examples. (Do not include examples where the estimated quotient is too high.)

**EXAMPLES:**

\[
\begin{array}{c|c|c|c|c|c}
7 & 28 & 5 & 27 & 7 & 18 \\
\hline
227 & 304 & 241 & 231 & 164 \\
-196 & -285 & -216 & -189 & -144 \\
\end{array}
\]
Eventually, include only the estimate in the problem and have students calculate the product. **EXAMPLES:** $351 \div 38$ (estimate 8), $365 \div 57$ (estimate 7)

Then introduce an example where the estimated quotient is too high:

$$
\begin{array}{c}
\phantom{2}6
\end{array}
\begin{array}{c}
43
\end{array}
\begin{array}{c}
\underline{252}
\end{array}
\begin{array}{c}
-258
\end{array}
\begin{array}{c}
\phantom{-}43
\end{array}
\begin{array}{c}
\text{negative number!}
\end{array}
$$

Since 43 rounds to 40, skip counting by 40 leads to an estimated quotient of 6. But $6 \times 43$ is 258. To place 6 dots in each of 43 circles, we would need 258 dots, but we have only 252 dots. When we subtract the product from the dividend, we end up with a negative number, so we know our estimate is too high. We should try a lower estimate, say 5.

Have students practice solving problems in the following sequence:

- Decide if the estimate is too high or just right given the product. **EXAMPLES:**

  $$
  \begin{array}{c}
  8 \\
  31
  \end{array}
  \begin{array}{c}
  251 \\
  248
  \end{array}
  \begin{array}{c}
  32 \\
  100
  \end{array}
  \begin{array}{c}
  31
  \end{array}
  \begin{array}{c}
  152 \\
  155
  \end{array}
  \begin{array}{c}
  33
  \end{array}
  \begin{array}{c}
  191 \\
  198
  \end{array}
  $$

- Decide if the estimate is too high, just right, or too low given the product. **EXAMPLES:**

  $$
  200 \div 33 \quad (\text{estimate } 6) \quad 361 \div 54 \quad (\text{estimate } 7) \quad 361 \div 51 \quad (\text{estimate } 7)
  $$
  $$
  33 \times 6 = 198 \quad 54 \times 7 = 378 \quad 51 \times 7 = 357
  $$
  $$
  111 \div 26 \quad (\text{estimate } 3) \quad 161 \div 29 \quad (\text{estimate } 7) \quad 141 \div 17 \quad (\text{estimate } 7)
  $$
  $$
  26 \times 3 = 78 \quad 29 \times 5 = 145 \quad 17 \times 7 = 119
  $$

- Find the product of the given estimate with the divisor, and decide if the estimate is too high, too low, or just right.

- Estimate by rounding, find the product of the estimate with the divisor, and then decide if the estimate is too high, too low, or just right.

- Solve several long division problems (3-digit dividend, 2-digit divisor) and write the remainder beside the quotient.

Now teach students how to divide 4-digit numbers by 2-digit numbers. Start with the following **EXAMPLE:**

$$
\begin{array}{c}
74
\end{array}
\begin{array}{c}
\underline{5031}
\end{array}
$$

Tell students that instead of estimating the quotient (i.e., the number of dots in each circle), you will start by estimating the number of tens blocks in each circle. This is like estimating the tens digit of the quotient. **ASK:** About how many tens do we need to put in each of 74 circles to get as close to 5031 as we can? Let’s round the number of circles, too—round 74 to 70. Every time we place a tens block into every one of the 70 circles, we use up 70 tens or 700 ones, so skip count by 700 until we get close to 5031. The result is 7, so we estimate the tens digit to be 7. **ASK:** If we put the ones digit of the quotient above the ones digit of the dividend, where should we put the tens digit of the quotient? (above the tens digit of the dividend)

Demonstrate doing this:

$$
\begin{array}{c}
7
\end{array}
\begin{array}{c}
\underline{5031}
\end{array}
$$
ASK: If we put 7 tens blocks in each of the 74 circles, how many ones have we used altogether? (7 tens × 74 = 518 tens = 5180) Do we have 5180 ones to begin with? (No, we have only 5031.) Is 7 a good estimate? How should we change it—should we make it higher or lower? (lower, because we need to place fewer tens blocks in each circle) Replace the estimate of 7 with 6.

ASK: Now how many tens blocks are in each of the 74 circles? (6) How many ones does that make altogether? (60 × 74 = 4440) How many ones are left over? (5031 – 4440 = 591)

\[
\begin{array}{c}
6 \\
74 \overline{5031} \\
4440 \\
591 \\
\end{array}
\]

(74 × 6 tens blocks make 4440 ones altogether)

Now students have to place the 591 ones blocks left over into 74 circles. **ASK:** What problem can you solve that will tell you how to do this? (591 ÷ 74) Have students solve this by long division, using the method they have already learned:

\[
\begin{array}{c}
8 \\
74 \overline{591} \\
592 \\
74 \overline{591} \\
518 \\
518 \\
73 \\
\end{array}
\]

is the estimated quotient because 70 goes into 591 eight times.

Estimated quotient is too high, so try 7 instead.

Negative number!

is the revised estimated quotient.

Remainder

We can combine these two algorithms (the one for the tens digit and the one for the ones digit) as follows:

\[
\begin{array}{c}
67 \\
74 \overline{5031} \\
4440 \\
591 \\
518 \\
518 \\
73 \\
\end{array}
\]

We can combine these two algorithms (the one for the tens digit and the one for the ones digit)

Have students practice solving many more long division problems. **EXAMPLES:**

\[
\begin{array}{c}
2319 ÷ 44 \\
3416 ÷ 47 \\
3627 ÷ 53 \\
\end{array}
\]

**Extensions**

1. Have students make up their own long division questions where the estimate by rounding the divisor to the nearest ten would give a quotient that is a) too low, b) just right, and c) too high.

2. Teach students short division. Begin with 1-digit divisors, then move to 2-digit divisors.

In short division, students are expected to do some of the steps in long division mentally. For example, to find 57 ÷ 2 by long division, students would write out all the steps as follows:
To do the same problem by short division, students would solve \(2 \times 2\) in their heads, and then subtract the result from 5. They would note the result \((5 - 4 = 1)\) beside the 7 and raised, as follows:

\[
\begin{array}{c|c}
5 & 7 \\
\hline
2 & 8 \\
\end{array}
\]

Next, they would calculate \(8 \times 2\) mentally and subtract the result from 17 (reading the 1 that is beside the 7 as a 10 added to the 7). The result \((17 - 16 = 1)\) is the remainder.

\[
\begin{array}{c|c}
5 & 7 \\
\hline
2 & 8 \\
\end{array}
\]

Write several problems done by long division on the board and have students translate them to short division notation. **Examples:**

\[
\begin{array}{cccccccc}
79 & \div & 4 & & 94 & \div & 8 & & 93 & \div & 4 & & 82 & \div & 3 & & 71 & \div & 5 & & 247 & \div & 5 \\
378 & \div & 4 & & 742 & \div & 3 & & 7563 & \div & 25 & & 7523 & \div & 25 & & 7628 & \div & 25 & & 4367 & \div & 25 \\
\end{array}
\]

Then have your students do several problems by short division. **Examples:**

\[
\begin{array}{cccccccc}
95 & \div & 2 & & 77 & \div & 3 & & 83 & \div & 5 & & 194 & \div & 5 & & 3215 & \div & 2 & & 5791 & \div & 25 & & 7269 & \div & 25 & & 4725 & \div & 11 \\
\end{array}
\]

**Bonus**

\[
77103 \div 11
\]

3. (From the Atlantic Curriculum) Teach students how to divide decimals by decimals.

Tell students that just as they can multiply decimals by pretending they are whole numbers, they can also divide decimals by pretending they are whole numbers. For example, to calculate \(0.292 \times 0.3\), students can find \(292 \times 3 = 876\) and then move the decimal point \(4 = 3 + 1\) places left: 0.0876. In the same way, they can calculate 876 ÷ 3 (the answer is 292) and then move the decimal point the correct number of places. How can they determine the correct number of places to move the decimal? Estimation is a good technique.

To estimate 0.0876 ÷ 0.3, we first need to find:

\[
\begin{array}{cccc}
0.3 \times 100 = 30 & & 0.3 \times 10 = 3 & & 0.3 \times 1 = 0.3 \\
0.3 \times 0.1 = 0.03 & & 0.3 \times 0.01 = 0.003 & & 0.3 \times 0.001 = .0003 \\
\end{array}
\]

Between which two of the results does 0.0876 lie? (0.0876 is between 0.3 and 0.03)
So 0.0876 ÷ 0.3 is between 0.1 and 1. **Ask:** How many zeroes after the decimal point will 0.0876 ÷ 0.3 have? (none) How many zeroes after the decimal point does any number between 0.1 and 1 have? (none) Have students write down various numbers that they think are between 0.1 and 1. (**Examples:** 0.4, 0.306, 0.87)

Then write 0.0876 ÷ 0.3 = 2.92 and have a volunteer put the decimal in the correct place.
Have students use this technique to put the decimal point in the correct place in these quotients:

a) $0.42 \div 0.3 = 1.4 (1.4)$  
b) $18.3 \div 0.3 = 61 (61)$  
c) $0.00645 \div 0.3 = 2.15 (0.0215)$  
d) $0.0744 \div 0.3 = 2.48 (0.248)$

**Bonus**

e) $486 \div 0.3 = 1620 (1620)$  
f) $20.3 \div 0.07 = 290 (290)$

Then have students calculate several quotients themselves by first treating both numbers as whole numbers and then deciding where the decimal point goes by estimation.  
**EXAMPLES:** $3.43 \div 0.7$, $112 \div 1.6$

3. Have students solve word problems involving division.

a) 4 people taking tennis lessons are asked to pick up 50 tennis balls. How many should each player pick up?

b) 12 people taking tennis lessons are asked to pick up 185 tennis balls. How many should each player pick up?

c) A test has 10 questions worth 10 marks each. Sally has 1 hour to write the test. How long should she spend on each question?

d) A 3-hour test has 15 questions, worth 10 marks each. How long should be spent on each question?
These two pages form a review of concepts learned in number sense, including integers, percents, decimals, and fractions.

**Extensions**

1. Ron, Fatima and Chyann went to the store. One person had $20, one had $60, and one had $36. At the store, one person spent $\frac{1}{2}$ of their money, one spent $\frac{2}{3}$, and one spent 25%. Ron spent $10 and Fatima spent $9. Chyann had $20 left. How much money did each person go to the store with?

**NOTE:** Students will find it helpful to find $\frac{1}{2}$, $\frac{2}{3}$ and 25% of each given amount:

<table>
<thead>
<tr>
<th></th>
<th>$20$</th>
<th>$60$</th>
<th>$36$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$10$</td>
<td>$30$</td>
<td>$18$</td>
</tr>
<tr>
<td>$\frac{2}{3}$</td>
<td>$13.33$</td>
<td>$40$</td>
<td>$24$</td>
</tr>
<tr>
<td>25%</td>
<td>$5$</td>
<td>$15$</td>
<td>$9$</td>
</tr>
</tbody>
</table>

Ron Chiann Fatima

2. Ask students to perform the operations in the questions below in different orders: For instance, in part a) below, do the addition first, then the multiplication, then redo the calculation, multiplying then adding.

**EXAMPLE:**

\[
3 + 5 \times 2 \quad \quad \quad \quad 3 + 5 \times 2 \\
= 8 \times 2 \quad \quad \quad \quad = 3 + 10 \\
= 16 \quad \quad \quad \quad \quad = 13
\]

Students should notice that they get different answers depending on what order they did the calculating in.

\[a) \ 3 + 5 \times 2 \quad \quad \quad b) \ 4 \times 8 + 3 \quad \quad \quad c) \ 5 + 2 \times 3 + 7 \quad \quad \quad d) \ 4 + 12 \div 4\]

Tell your students that by convention operations with multiplication and division are done before operations with addition and subtraction.

Have students perform the following calculations using the correct order of operations:

\[3 \times 4 - 2 \quad \quad \quad 2 + 4 \times 3 \quad \quad \quad 4 \times 3 + 2 \quad \quad \quad 6 \times 3 - 2 \quad \quad \quad 6 - 2 \times 3\]

**Bonus**

\[3 + 5 \times 8 + 41 + 6 \times 3 + 5 + 2 \times 2\]
Sometimes an equation has only addition and subtraction. Have students perform the operations in different orders. Do they get different answers?

a) \(7 + 6 - 3\) (13 – 3 or 7 + 3, same answer)
b) \(8 - 5 + 1\) (3 + 1 or 8 - 6, different answers)
c) \(11 - 3 + 4\) (8 + 4 or 11 - 7, different answers)
d) \(9 + 8 - 5\) (17 – 5 or 9 + 3, same answer)

Explain that, instead of deciding to always do addition first, or always do subtraction first, mathematicians have decided to do addition and subtraction in order from left to right.

Have students find the correct answers for parts a) through d) above, and then for these problems.

\[
\begin{align*}
&8 - 7 + 4 \\
&7 + 4 - 3 + 6 - 8 \\
&7 + 4 - 3 - 2 + 8 - 5 - 4 + 3 - 7
\end{align*}
\]

Sometimes an equation has only multiplication and division. Have students perform the operations in different orders. Do they get different answers?

a) \(3 \times 4 \div 2\) (3 \times 2 or 12 ÷ 2, same answers)
b) \(12 \div 3 \times 2\) (4 \times 2 or 12 ÷ 6, different answers)
c) \(7 \times 8 \div 2\) (7 \times 4 or 56 ÷ 2, same answers)
d) \(30 \div 3 \times 2\) (10 \times 2 or 30 ÷ 6, different answers)

Explain that, instead of deciding to always do multiplication first, or always do division first, mathematicians have decided to do them in order from left to right. Have students find the correct answers for parts a) through d) above, and then for these problems.

\[
\begin{align*}
&20 \div 2 \times 5 \\
&84 \div 3 \times 8 \div 4 \\
&20 \times 5 \div 10 \times 3 \div 6 \\
&80 \div 2 \times 2 \div 4 \div 5 \\
&8 \times 3 \div 6 \div 2 \times 5 \times 3 \div 2
\end{align*}
\]

Then have students do problems requiring all four operations, remembering to do multiplication and division first in the order they occur and then the addition and subtraction (again, in the order they occur).

\[
\begin{align*}
&5 \times 3 + 2 - 6 \div 3 \\
&7 - 3 \times 2 + 8 \div 4 \\
&12 \div 2 \times 5 - 7 + 4 \\
&7 - 3 + 5 \times 8 \div 2
\end{align*}
\]

**Bonus**

\[
\begin{align*}
&5 \times 2 + 3 \times 2 - 4 \times 3 + 60 \div 2 \div 3 \times 4 - 18 \div 2 \times 3
\end{align*}
\]

Have students write a single expression to find how many marbles Sally has and then calculate the answer:

a) Sally had 8 marbles. She lost half of them and then bought two more.
\((8 \div 2 + 2 = 4 + 2 = 6)\)

b) Sally had 10 marbles in her left pocket and 14 marbles in her right pocket. Her right pocket had a hole in it, so she lost half of those marbles. \((10 + 14 \div 2 = 10 + 7 = 17)\)

c) Sally bought 3 packs of 12 marbles each. She kept half of the first pack, a third of the second pack and a quarter of the third pack. \((12 \div 2 + 12 \div 3 + 12 \div 4 = 6 + 4 + 3 = 13)\)

d) Sally bought 2 packs of 18 marbles each. She lost half of the first pack and a third of the second pack. \((2 \times 18 - 18 \div 2 - 18 \div 3 = 36 - 9 - 6 = 36 - 15 = 21\) OR \(18 + 18 - 18 \div 2 - 18 \div 3,\) if they know brackets, they might write: \(2 \times 18 - (18 \div 2 + 18 \div 3)\).)
Now introduce brackets. Tell students that brackets tell you to do everything inside the brackets before everything else. Demonstrate this. For example, $8 - 2 \times 3$ is calculated as $8 - 6 = 2$, but $(8 - 2) \times 3 = 6 \times 3 = 18$. Have volunteers demonstrate solving these problems:

a) $2 \times 3 + 5 \times 4 \div 2$ **ANSWER:** $6 + 10 = 16$

b) $(2 \times 3 + 5 \times 4) \div 2$ **ANSWER:** $(6 + 20) \div 2 = 26 \div 2 = 13$.

Then have students solve the following problems individually:

a) $20 - 5 \div 3$  
b) $5 \times (3 + 2)$  
c) $7 - (4 + 2)$  
d) $18 \div (2 + 4)$

**Bonus**

$(5 \times 3 - 7 + 9 \times 5 + 2) \div (5 + 2 \times 3)$

Then have students again write how many marbles Sally has:

a) Sally had 12 marbles. She lost 2 of them and then gave half of the rest to her brother. **ANSWER:** $(12 - 2) \div 2 = 5$

b) Sally had 20 marbles. She bought 4 more and then shared equally between herself and two friends. **ANSWER:** $(20 + 4) \div 3 = 8$

Then do the following problem together as a class.

An hour-long test has 12 questions. Questions 1-8 are worth 3 marks each and questions 9-12 are worth 4 marks each. Find a single expression which, when following the correct order of operations, will give you the answers:

a) How many marks are there on the test? **ANSWER:** $8 \times 3 + 4 \times 4$

b) How much time is there for each mark? **ANSWER:** $60 \div (8 \times 3 + 4 \times 4)$

c) How much time should be spent on each question? **ANSWER:** For questions 1-8: $60 \div (8 \times 3 + 4 \times 4) \times 3$

**ANSWER:** For questions 9-12: $60 \div (8 \times 3 + 4 \times 4) \times 4$

Students should then calculate these answers. (There are $24 + 16 = 40$ marks, $60 \div 40 = 1.5$ minutes for each mark and so 4.5 minutes for each question from 1 through 8 and 6 minutes for each question from 9 through 12.)
## NS6 Part 2: BLM List

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<tr>
<td>Small Circle Graphs</td>
<td>11</td>
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</tbody>
</table>
## Always/Sometimes/Never True (Numbers)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>If you multiply a 3-digit number by a one-digit number, the answer will be a three-digit number.</td>
<td><strong>B</strong></td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>If you divide a number by itself the answer will be 1.</td>
<td><strong>E</strong></td>
</tr>
<tr>
<td><strong>G</strong></td>
<td>The product of 2 even numbers is an even number.</td>
<td><strong>H</strong></td>
</tr>
<tr>
<td><strong>J</strong></td>
<td>When you round to the nearest thousands place, only the thousands digit changes.</td>
<td><strong>K</strong></td>
</tr>
<tr>
<td><strong>M</strong></td>
<td>The multiples of 5 are divisible by 2.</td>
<td><strong>N</strong></td>
</tr>
</tbody>
</table>

1. Choose a statement from the chart above and say whether it is **always** true, **sometimes** true, or **never** true. Give reasons for your answer.

   What statement did you choose? Statement Letter __________

   This statement is...

   **Always True**  
   **Sometimes True**  
   **Never True**

   Explain: ____________________________________________

   ____________________________________________

   ____________________________________________

2. Choose a statement that is sometimes true, and reword it so that it is always true.

   What statement did you choose? Statement Letter __________

   Your reworded statement: __________________________________

   ____________________________________________

3. Repeat the exercise with another statement.
Blank Hundreds Charts
## Cards (Fractions of Numbers)

<table>
<thead>
<tr>
<th>Fraction of 10</th>
<th>Fraction of 18</th>
<th>Fraction of 15</th>
<th>Fraction of 20</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$\frac{1}{9}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
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Fractions Strips
Large Circle Graph
Math Bingo Game

Sample Boards

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Multiplying Whole Numbers by 0.1, 0.01 & 0.001

Recall that multiplying a whole number by a decimal has the same effect as shifting the decimal.

For example: \[ .01 \times 5 = .05 \quad \text{OR} \quad .01 \times 5 = .01 \times 5 = .05 \]

Shift the decimal TWO places

\[ 6 \times 0.1 = .6 \quad \text{or} \quad 0.6 \] (decimal point moves ONE to the left)

1. Answer the following questions in your notebook:
   a) \[ 5 \times 0.1 = \]
   b) \[ 62 \times 0.1 = \]
   c) \[ 85 \times 0.1 = \]
   d) \[ 16 \times 0.1 = \]
   e) \[ 246 \times 0.1 = \]
   f) \[ 645 \times 0.1 = \]
   g) \[ 754 \times 0.1 = \]
   h) \[ 951 \times 0.1 = \]
   i) \[ 1\,154 \times 0.1 = \]
   j) \[ 187 \times 0.1 = \]
   k) \[ 3\,954 \times 0.1 = \]
   l) \[ 12\,784 \times 0.1 = \]

\[ 6 \times 0.01 = .06 \quad \text{or} \quad 0.06 \] (decimal point moves TWO to the left)

2. Answer the following questions in your notebook:
   a) \[ 4 \times 0.01 = \]
   b) \[ 45 \times 0.01 = \]
   c) \[ 26 \times 0.01 = \]
   d) \[ 78 \times 0.01 = \]
   e) \[ 264 \times 0.01 = \]
   f) \[ 856 \times 0.01 = \]
   g) \[ 776 \times 0.01 = \]
   h) \[ 422 \times 0.01 = \]
   i) \[ 1\,956 \times 0.01 = \]
   j) \[ 134 \times 0.01 = \]
   k) \[ 7\,584 \times 0.01 = \]
   l) \[ 12\,444 \times 0.01 = \]

\[ 6 \times 0.001 = .006 \quad \text{or} \quad 0.006 \] (decimal point moves THREE to the left)

3. Answer the following questions in your notebook:
   a) \[ 3 \times 0.001 = \]
   b) \[ 61 \times 0.001 = \]
   c) \[ 76 \times 0.001 = \]
   d) \[ 34 \times 0.001 = \]
   e) \[ 128 \times 0.001 = \]
   f) \[ 657 \times 0.001 = \]
   g) \[ 237 \times 0.001 = \]
   h) \[ 567 \times 0.001 = \]
   i) \[ 5\,647 \times 0.001 = \]
   j) \[ 654 \times 0.001 = \]
   k) \[ 2\,348 \times 0.001 = \]
   l) \[ 36\,559 \times 0.001 = \]

4. Answer the following questions in your notebook:
   a) \[ 8 \times 0.01 = \]
   b) \[ 65 \times 0.001 = \]
   c) \[ 27 \times 0.1 = \]
   d) \[ 82 \times 0.01 = \]
   e) \[ 645 \times 0.1 = \]
   f) \[ 872 \times 0.01 = \]
   g) \[ 364 \times 0.001 = \]
   h) \[ 229 \times 0.1 = \]
   i) \[ 6\,488 \times 0.01 = \]
   j) \[ 1\,599 \times 0.001 = \]
   k) \[ 7\,481 \times 0.001 = \]
   l) \[ 34\,122 \times 0.01 = \]
   m) \[ 2\,178 \times 0.01 = \]
   n) \[ 64\,788 \times 0.1 = \]
   o) \[ 26\,944 \times 0.01 = \]
   p) \[ 98\,756 \times 0.1 = \]
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
Percent Strips
Small Circle Graphs

[Blank circle graphs]

[Blank circle graphs]
PS6-3  Combining Systematic Search with Guess, Check, and Revise

Teach this lesson after: 6.2 Number Sense

Goals:
Students will search systematically and efficiently by skipping numbers and using the order of the numbers to inform when they have gone too far.
Students will learn to choose a starting point in order to find the answer quicker.

Prior Knowledge Required:
Can order and compare multi-digit whole numbers
Can substitute whole numbers for variables in expressions
Can multiply multi-digit whole numbers by one-digit whole numbers
Knows to evaluate expressions in brackets first
Can search systematically to find mystery numbers
Understands how guessing a middle number can make guessing efficient
Can multiply decimal hundredths by whole numbers (for Problem Bank 16)

Materials:
calculator

Review searching systematically to find mystery numbers. Write on the board:

If \( N \) is a whole number so that \( N \times N \times N \times N = 2401 \), what is \( N \)?

SAY: Remember that we can solve equations that look hard if we know that the answer is a whole number. Let’s try the whole numbers in order to see if we can find the answer quickly. Draw on the board:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1 \times 1 \times 1 \times 1 = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 2 \times 2 \times 2 = 16 )</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 3 \times 3 \times 3 = 81 )</td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 4 \times 4 \times 4 = 256 )</td>
</tr>
</tbody>
</table>

ASK: Are we getting closer to the answer? (yes) Are the numbers getting big quickly? (yes)
SAY: Let’s keep going because we might find the answer fairly quickly.

Allow students to use a calculator for the following exercises.

Exercises:
1. Continue the table until you get \( N \times N \times N \times N = 2401 \). What is \( N \)?
   **Answer:** \( N = 7 \)
2. \( N \) is a whole number so that \( N \times N \times N = 512 \). What is \( N \)?

**Answer:** \( N = 8 \)

**Searching faster by skipping numbers.** Write on the board:

If \( N \times N \times N \times N = 1048576 \), what is \( N \)?

Say: We just solved a problem like this. Ask: How did we do it? (we made a table and started at 1 and moved up the numbers in order) Ask: Would continuing the table be a good strategy for this question? (no) Why not? (it will take too long to get to the answer) Say: The answers are getting closer to the answer but not much closer; you still have a long way to go to find the answer. Maybe you need to take bigger steps to find the answer. Instead of trying 1, 2, 3, and so on, maybe we should start with 10, 20, 30, and so on.

**Exercises:**

a) Complete the table up to 50.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( 10 \times 10 \times 10 \times 10 = 10000 )</td>
</tr>
<tr>
<td>20</td>
<td>( 20 \times 20 \times 20 \times 20 = 160000 )</td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
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</tr>
</tbody>
</table>

b) Using \( N \times N \times N \times N = 1048576 \), what two 10s is \( N \) between? Explain how you know.

**Answers:** a) 810000, 2560000, 6250000; \( N \) is between 30 and 40 because \( N \times N \times N \times N \) is between 810000 and 2560000.

Say: Now we know that \( N \) is between 30 and 40. Write on the board:

\[
\begin{align*}
30 \times 30 \times 30 \times 30 & = 810000 \\
N \times N \times N \times N & = 1048576 \\
40 \times 40 \times 40 \times 40 & = 2560000
\end{align*}
\]

Say: Now we can move up by ones until we get the answer because we know that we are pretty close. Write on the board:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>( 30 \times 30 \times 30 \times 30 = 810000 )</td>
</tr>
<tr>
<td>31</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
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<tr>
<td>34</td>
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</tbody>
</table>

Ask a volunteer to use a calculator to complete each row of the table until you get the correct answer. (\( 31 \times 31 \times 31 \times 31 = 923521 \) and \( 32 \times 32 \times 32 \times 32 = 1048576 \), so \( N \) is 32)
Exercises:
1. Find \( N \) so that \( N \times N \times N \times N \) is …
   a) 10 556 001  b) 45 212 176
   **Bonus:** 1 698 181 681. **Hint:** Move up by hundreds, then by tens, then by ones.
   **Answers:** a) 57, b) 82, Bonus: 203

2. Find \( N \) so that \( N \times (N + 1) \times (N + 2) \) is …
   a) 24 360  b) 157 410
   **Bonus:** 70 444 584. **Hint:** Start moving up by hundreds, then by tens, and then by ones.
   **Answers:** a) 28, b) 53, Bonus: 412

**Searching from either direction.** **SAY:** Cameron and Avril went to a farm that has cows and chickens. Write on the board:

Cameron counts 36 legs.
Avril counts 10 heads.

**ASK:** How many animals are there altogether? (10) How do you know? (the number of heads is the same as the number of animals) Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
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<tbody>
<tr>
<td>0</td>
<td>10</td>
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</tr>
<tr>
<td>10</td>
<td>0</td>
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</tbody>
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**ASK:** If there are zero cows and 10 chickens, how many legs are there? (20) Write “20” in the first row of the third column. **ASK:** If there are 10 cows and zero chickens, how many legs are there? (40) Write “40” in the last row of the third column. **ASK:** Do you think the number of cows in our answer will be closer to zero or 10? (10) **Why?** (the number of legs is closer to 40 than to 20) **PROMPT:** Is the actual number of legs closer to 20 or 40? (40) **ASK:** So, is it better to start our search closer to zero or to 10? (10) **SAY:** We save ourselves a lot of work by starting at 10 cows and zero chickens instead of starting at zero cows and 10 chickens. Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td></td>
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<tr>
<td>8</td>
<td>2</td>
<td></td>
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<tr>
<td>6</td>
<td>4</td>
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</tbody>
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**ASK:** How many legs do nine cows have? (36) Write on the board:

36 +
ASK: How many legs does one chicken have? (2) Continue writing on the board:

\[ 36 + 2 = 38 \]

Write “38” as the total in the row for 9 cows and 1 chicken. Repeat for the row with 8 cows and 2 chickens. \((32 + 4 = 36)\)

SAY: So, 8 cows and 2 chickens have a total of 36 legs. Starting from 10 cows and searching is a lot less work than starting from zero cows and moving all the way up to 8 cows.

**Exercises:** If all the heads Avril counts belong to cows, how many legs are there? If all the heads Avril counts belong to chickens, how many legs are there?

- a) Avril counts 30 heads.
- b) Avril counts 37 heads.
- c) Avril counts 28 heads. **Bonus:** Avril counts 1000 heads.

**Answers:** a) 120, 60; b) 148, 74; c) 112, 56; Bonus: 4000, 2000

SAY: Once you know how many legs there are if all the animals are cows and if all the animals are chickens, you can compare those numbers with the total number of legs given. Then you can decide which option to use to start your search.

**Exercises:** How many cows and how many chickens are there?

- a) Cameron counts 22 legs. Avril counts 9 heads.
- b) Cameron counts 52 legs. Avril counts 14 heads.
- c) Cameron counts 114 legs. Avril counts 30 heads.
- d) Cameron counts 140 legs. Avril counts 37 heads.
- e) Cameron counts 60 legs. Avril counts 28 heads. **Bonus:** Cameron counts 3996 legs. Avril counts 1000 heads.

**Answers:** a) 2 cows, 7 chickens; b) 12 cows, 2 chickens; c) 27 cows, 3 chickens; d) 33 cows, 4 chickens; e) 2 cows, 26 chickens; Bonus: 998 cows, 2 chickens

SAY: Once you decide where to start, you might want to skip count by tens first and then by ones to get to the answer quickly.

**Exercises:** The are 100 cows and chickens altogether. How many cows and how many chickens are there?

- a) Cameron counts 240 legs.
- b) Cameron counts 372 legs.
- c) Cameron counts 300 legs. **Bonus:** There are 1000 cows and chickens altogether. Cameron counts 2366 legs. How many cows and how many chickens are there? Hint: Count by hundreds, then by tens, then by ones.

**Answers:** a) 20 cows, 80 chickens; b) 86 cows, 14 chickens; c) 50 cows, 50 chickens; Bonus: 183 cows, 817 chickens

**Using the guess-check-revise strategy when two quantities are changing.** SAY: Because the total number of legs increases as the number of cows increases and the number of chickens decreases, you can guess an answer and know right away whether your answer is too high or too low. This allows you to play a game like “too high” or “too low” when guessing numbers.
Write on the board:

Cameron counts 344 legs.
Avril counts 100 heads.
How many cows and how many chickens are there?

SAY: There are 100 heads. That means that there are 100 animals altogether, some of them are cows and the rest are chickens, but I don’t know how many of each there are. There might be more cows or there might be more chickens. For my first guess, I’m going to assume a middle situation—that there is the same number of each type of animal. ASK: How many cows am I assuming there are? (50) SAY: 50 is right in the middle, between zero and 100. ASK: Why is that a good starting guess? (it eliminates half the answers no matter what, whether 50 is too low or too high) And how will I know whether 50 cows is too high or too low? (If the number of legs with 50 cows is less than 344, then I need to add more cows and subtract chickens to get the number of legs up to 344. If the number of legs is more than 344, I need to add more chickens and subtract cows.)

SAY: Let’s see how many legs there are if there are 50 cows and 50 chickens. Write on the board:

50 cows have _____ legs altogether.
50 chickens have _____ legs altogether.

Have volunteers tell you what to put in the blanks. (200, 100) ASK: How many legs is that altogether? (300) Is that too many legs or too few? (too few) SAY: We need more legs. ASK: Does that mean we need more cows or more chickens? (more cows) Why do you say that? (cows have more legs than chickens) SAY: The number of cows we guessed was too low, so we need more cows. ASK: How many cows and chickens should we guess next? (take various answers) Would 51 cows be a good guess? (no) Why not? (that would only eliminate one more number if it doesn’t work; there are quite a few more legs than 300; adding one more cow won’t add that many more legs) Encourage students to pick a number that is somewhere in the middle of the numbers left to check, such as 75. Suppose students guess 75 cows. ASK: So, how many chickens will there be? (25) How do you know? (there are 100 animals altogether) Write on the board:

75 cows have _____ legs altogether.
25 chickens have _____ legs altogether.

Have volunteers tell you what to put in the blanks. (300, 50) ASK: How many legs is that altogether? (350) Is that too many legs or too few? (too many) SAY: So, we need fewer legs. ASK: Does that mean we need more cows or fewer cows? (fewer cows) SAY: Our first guess of 50 cows got us 300 legs and our next guess of 75 cows got us 350 legs in total. ASK: How many legs in total are we aiming for altogether? (344) Is that closer to 300 legs or to 350 legs? (350 legs) So, will the number of cows be closer to 50 or to 75? (75) SAY: We actually have more information than just that the number of cows is too high. We also have a sense that our guess is not too far off. That means we can make our next guess closer to 75 than to 50;
we don’t have to guess right in the middle. Continue in this way until the correct number of cows is guessed. (72 cows and, hence, 28 chickens)

**Exercises:** Cameron counts 272 legs. Avril counts 100 heads.

a) How many cows and how many chickens are there? Keep track of your guesses.

b) How many guesses did you use to answer part a)?

**Bonus:** Cameron counts 3166 legs. Avril counts 1000 heads. How many cows and how many chickens are there?

**Selected answers:** a) 36 cows and 64 chickens, Bonus: 583 cows and 417 chickens

**Problem Bank**

1. Use systematic search to find a whole number so that \(3 \times N + 5 = 29\).

**Answer:** 8

2. Answer all the following questions without using long division.

a) Is there a whole number \(N\) so that \(3 \times N = 414\)?

b) Is there a whole number \(N\) so that \(3 \times N = 415\)? Explain how you know.

c) Is there a whole number \(N\) so that \(3 \times N = 718\)? Explain how you know.

d) Is there a whole number \(N\) so that \(3 \times N + 5 = 2162\)? Explain how you know.

**Answers:** a) 138; b) no, sample explanation: because it is one more than 414; c) no, sample explanation: because it is 2 less than 720, which is \(3 \times 240\); d) yes, \(N = 719\)

3. Use a calculator to find \(N\) if \(N \times N = 1849\).

**Answer:** 43

4. Is there a whole number \(N\) so that \(N \times N = 7541\)? How do you know?

**Answer:** no, because \(86 \times 86 = 7396\), so 86 is too low, but \(87 \times 87 = 7569\), so 87 is too high

5. a) Jessica wants to find a whole number \(N\) so that \(N \times N \times N = 46656\). She starts by guessing \(N = 100\). Is that too high or too low? How do you know?

b) Find a whole number \(N\) so that \(N \times N \times N = 46656\).

c) Find a whole number \(N\) so that \(N \times N \times N \times N = 187388721\)

**Answers:** a) 100 \(\times 100 \times 100 = 1\ 000\ 000\), so 100 is too high; b) 36; c) 117

6. Is there a whole number \(N\) so that \(N \times N = 85\)? How do you know?

**Answer:** no, because \(9 \times 9 = 81\), so 9 is too low, but \(10 \times 10 = 100\), so 10 is too high

7. Find \(N\) so that ...

a) \((2 \times N) + 1 = 177\)

b) \((N \times 3) + N = 228\)

c) \((N \times 5) + 5 = 320\)

**Answers:** a) 88, b) 57, c) 63

8. Megan’s mom was 32 when she had Megan. Ten years from today, the sum of Megan’s age and her mother’s age will be 80. How old is Megan now?

**Answer:** 14
9. In 2037, Canada will be 153 more years old than it will be decades old. How old will Canada be in 2037?
**Answer:** 170 years, or 17 decades

10. Find the whole number \( A \).
   a) If \( \frac{A-1}{A+1} = \frac{4}{5} \), what is \( A \)?
   b) If \( \frac{A \times A}{A + A} = 4 \), what is \( A \)?
   c) If \( \frac{A + 2}{(A \times 2) + 1} = \frac{2}{3} \), what is \( A \)?
   d) If \( \frac{A + 4}{A \times A} = \frac{1}{2} \), what is \( A \)?
**Answers:** a) 9, b) 8, c) 4, d) 4

11. There are five-headed dragons and nine-headed dragons. Altogether, 100 dragons have 608 heads. How many of each kind of dragon are there?
**Answer:** 27 nine-headed dragons and 73 five-headed dragons

12. Matt built some bicycles and tricycles. Altogether, he made 100 vehicles. If he used 233 wheels altogether, how many bicycles and how many tricycles did he make?
**Answer:** 67 bicycles and 33 tricycles

13. A vending machine has quarters and dimes. Altogether, 100 coins have a value of $17.50 (that's 1750 cents). How many of the coins are quarters?
**Answer:** 50

14. What are the two numbers?
   a) The bigger number is seven times the smaller number. Their product is 252.
   b) The bigger number is seven times the smaller number. Their product is 11 200.
   c) The bigger number is seven times the smaller number. Their product is 47 068.
**Answers:** a) 6 and 42, b) 40 and 280, c) 82 and 574

15. A school fundraiser has a bake sale that sells muffins and cake. A muffin costs $2 and a piece of cake costs $3. The bake sale sold 30 items altogether and made $71. How many muffins and how many pieces of cake were sold?
**Answer:** 19 muffins and 11 pieces of cake

16. A school bake sale sells muffins and pieces of cake. A muffin costs $2.50 and a piece of cake costs $3.50. The bake sale sold 47 items and made $134.50 in total. How many muffins and how many pieces of cake were sold?
**Answer:** 30 muffins and 17 pieces of cake
17. Use a calculator to answer the question. Remember that two whole numbers are consecutive if there is no whole number between them.
   a) Calculate the product.
      i) 1 \times 2   
      ii) 2 \times 3
      iii) 3 \times 4
      iv) 4 \times 5
      v) 5 \times 6
   b) Is 14 the product of two consecutive whole numbers? Explain how you know.
   c) Can 160 be the product of two consecutive whole numbers? Explain how you know.
   d) Can 992 be the product of two consecutive whole numbers? Explain how you know.
   e) Write 6972 as a product of two consecutive whole numbers.
      **Answers:**
      a) i) 2, ii) 6, iii) 12, iv) 20, v) 30; b) no, it is between 3 \times 4 and 4 \times 5; c) no, it is between 12 \times 13 = 156 and 13 \times 14 = 182; d) yes, it is 31 \times 32; e) 83 \times 84

18. A perfect square is the product of a whole number with itself.
   a) Calculate the product.
      i) 1 \times 1   
      ii) 2 \times 2
      iii) 3 \times 3
      iv) 4 \times 4
      v) 5 \times 5
   b) Is 25 the product of two consecutive whole numbers? Explain how you know.
   c) Write 400 as a perfect square.
   d) Can you write 400 as the product of two consecutive whole numbers? Explain how you know.
   e) Explain why a perfect square cannot be the product of two consecutive whole numbers.
      **Answers:**
      a) i) 1, ii) 4, iii) 9, iv) 16, v) 25; b) no, because it is between 4 \times 5 = 20 and 5 \times 6 = 30 and there is no product of consecutive whole numbers between those two; c) 400 = 20 \times 20; d) no, because it is between 19 \times 20 = 380 and 20 \times 21 = 420; e) Any perfect square is between two consecutive products of consecutive whole numbers, so it cannot be the product of two consecutive whole numbers. For example, 15 \times 15 is in between 14 \times 15 and 15 \times 16.
PS6-4 Using Logical Reasoning

Teach this lesson after: 6.2 Number Sense

Goals:
Students will identify false statements of the form “all [of these] are [like this]” by using counter-examples.
Students will identify true statements of the same form by checking all examples or by using reasoning.

Prior Knowledge Required:
Can identify numbers divisible by 2, 5, and 10
Can identify even and odd numbers
Can order and compare multi-digit numbers
Can use long division to divide by one- and two-digit numbers
Can recognize multiples of 10

Vocabulary: counter-example, divisible, false, good, true

Introduce the term “counter-example.” Draw on the board:

All the circles are shaded.

Have a volunteer identify which circle shows that the statement is false. Repeat for the two different statements and picture below:

All the squares have a horizontal side.
All the squares are shaded.

Tell students that an example that proves a statement false is called a counter-example to the statement.

NOTE: Draw the triangles so that A and D are isosceles, B is right scalene, and C is equilateral but rotated.

Exercises: Which shape is the counter-example to the statement?

A.    B.             C.         D.
a) All triangles are striped.
b) All triangles have a horizontal side.
**Bonus:** All triangles have at least two equal sides.
**Answers:** a) D, b) C, Bonus: B

**Recognizing when a statement does not apply to all examples.** Draw on the board:

All the circles are shaded.

A.  B.  C.  D.  E.  F.

ASK: What is this statement about? (circles) Underline all the circles. Emphasize that the statement refers only to the circles; it doesn’t matter whether any of the other shapes are shaded or not. ASK: Are all circles shaded? (no) Have a volunteer circle the counter-example. (E) Erase the underlining and the circling and repeat with new statements (see below), underlining the relevant shapes first. Emphasize in each case that the sentence is only about the shapes you underline; the shapes that are not underlined don’t matter.
- All the squares are big. (D)
- All the squares are shaded. (D and F)
- All the big squares are shaded. (F)
- All the small circles are shaded. (E)

**Exercises:** Name the counter-example for the statement, using the same picture as above.

a) All the shaded shapes are circles.  
b) All the white shapes are small.  
c) All the shaded shapes are big.  
d) All the white shapes are squares.  
e) All the big shapes are squares.  
f) All the big shapes are shaded.  
g) All the small shapes are white.  
h) All the small white shapes are squares.

**Answers:** a) B, b) F, c) C, d) E, e) A, f) F, g) C, h) E

As students complete the exercises above, encourage them to first write down the shapes that the statement is talking about. (For example, the statement in part a) is about the shaded shapes: A, B, and C.) These are where the students should look for a counter-example. For students who need extra help, you can draw the all shapes in their notebook for them, and they can underline the shapes each question is referring to (and erase the underlining before starting each new question). Write on the board:

All words start with the letter b.

Ask if each of the examples below is a counter-example to the statement and have students explain why or why not:
- bat (no, because it does start with b)
- cat (yes, it is a word that does not start with b)
- boat (no, because it does start with b)
• bxcv (no, because it does start with b; or no, it is not a word)
• xcvb (no, because it is not a word, and the statement only talks about words, so something that is not a word cannot be a counter-example)

Exercises: Find the counter-example among the listed examples.
a) All nouns have the letter e.
   red   brown   truck   bike

b) All even numbers have a digit 2.
   23   32   34   43

c) All numbers divisible by 5 have a ones digit 5.
   35   40   52   55

Answers: a) truck, b) 34, c) 40

Proving a statement is true by checking all examples. Draw on the board:

All the squares are black.

A. □ B. △ C. △ D. △ E. △ F. ✷
G. ■ H. ○ J. ○ K. ⬠

Demonstrate checking all the squares to see whether they are black. They are, so the statement is true. Repeat with the statement “All triangles have a horizontal side” and have volunteers check all the triangles. (again, the statement is true) Repeat with “All squares have a horizontal side.” (this statement is false; I is a counter-example) Point out that in order to show that a statement is true, students need to check all examples. To show that a statement is false, students just need to identify any one counter-example.

Exercises: Decide whether the statement is true or false. If it is false, provide the counter-example.
a) All striped shapes are big.
b) All triangles are big.
c) All big circles are black.
d) All small squares have a horizontal side.
e) All small shapes have a horizontal side.
Bonus: All large black triangles are equilateral.
Answers: a) true; b) false, D; c) false, H or K; d) true; e) false, J; Bonus: false, E
**Using reasoning to prove a statement true.** Write on the board:

Whenever it is raining, there are clouds.

ASK: Is this statement true or false? (true) Do you have to check for clouds every time it rains to know that the statement is true? (no) How do you know without checking that it is true? (rain can only come from clouds) Tell students that there is often a reason why a statement is true. When there is, students don’t have to check all examples to prove it. Review the words “even” and “odd” as they apply to numbers. (even numbers are multiples of 2, odd numbers are not even)

Write on the board:

All even numbers have an even digit.
All even numbers have an odd digit.

Tell students that one of the statements is true and the other is false. ASK: Which statement is true? (all even numbers have an even digit) How do you know it’s true? (the ones digit is always even for any even number) Explain that if you had to check all even numbers, one by one, you would be checking forever! SAY: Because we know the reason this statement is true, we don’t have to check every example. Have a volunteer name a counter-example to the second statement. Again, point out that students don’t need to check every even number—one counter-example is enough to prove it’s false.

**Exercises:** Either explain why the statement is true or find a counter-example.

a) All three-digit numbers less than 200 have a digit 1.
b) All three-digit numbers more than 200 have a digit 1.
c) All three-digit numbers less than 900 have a digit 9.
d) All three-digit numbers more than 900 have a digit 9.

**Answers:** a) true, because all three-digit numbers less than 200 are in the hundreds, so their hundreds digit is 1; b) false, sample counter-example: 202; c) false, sample counter-example: 100; d) true, because all three-digit numbers more than 900 are in the 900s and so have hundreds digit 9

**Using systematic search to investigate conjectures.** Write on the board:

A two-digit number is called **good** if it is divisible by the sum of its digits.

SAY: We’ll call a two-digit number **good** if it is divisible by the sum of its digits. Ask volunteers to come to the board and divide various two-digit numbers by the sum of their digits: 36, 42, 43, 12, 19, 84, 55, 70, 90. ASK: Which of these numbers are good? (36, 42, 12, 84, 70, 90)

**Exercises:** Investigate if the statement is true by moving up in order through all possibilities. Write “true” if it is true; if it is false, write the first counter-example that you found.

a) All two-digit numbers that are multiples of 10 are good.
b) All two-digit numbers that are multiples of 3 are good.
c) All two-digit numbers that are multiples of 6 are good.
d) All two-digit numbers that are multiples of 9 are good.
Problem Bank
1. What is the smallest number that will make the statement true?
   “All three-digit numbers more than _____ have a digit 9.”
   Answer: 888, because 889 has ones digit 9, and any number in the 890s has tens digit 9, and any number in the 900s has hundreds digit 9

2. Remember that the letters a, e, i, o, u, and sometimes y are vowels.
a) For which of these statements is “Bob” a counter-example?
   A. All names have two vowels.
   B. All names have three letters.
   C. All names have four letters.
   D. All boys’ names start with D.
   E. All names are boys’ names.
   F. All names read the same backwards as they do forward.
b) Marcel wants to find a counter-example to each of the three statements for which “Bob” is not a counter-example (i.e., statements B, E, and F). Find one example that works as a counter-example to all three statements at the same time.
c) Explain why there cannot be a counter-example to all six statements at the same time.
   Hint: Look at statements D and E.
   Answers: a) A, C, and D; b) sample answer: Sara; c) To be a counter-example to D, the name would have to be a boy’s name. On the other hand, to be a counter-example to E, the name would have to not be a boy’s name. So, there cannot be a counter-example to both D and E at the same time. Therefore, there cannot be a counter-example to all six statements at the same time.

3. Make up a statement so that …
a) the word “run” is a counter-example.
b) the number 8 is a counter-example.

4. How many numbers do you have to check to show that the following statement is true?
   “When written out in words, no numbers less than one thousand have a letter A.”
   Solution: Number words to check: zero to twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred. That’s it! Every other number less than one thousand is written as a combination of these words, and so also will not have a letter A. Examples: three hundred forty-two; one hundred seventeen. NOTE: The word “and” is reserved for mixed numbers and decimals, such as writing 3.2 as “three and two tenths,” so 342 is not written as “three hundred and forty-two” as is commonly believed.
5. A three-digit number is called good if it is divisible by the sum of its digits. Are the three-digit numbers described always good?
   a) numbers that are multiples of 10
   b) numbers that are multiples of 100
   c) numbers whose sum of digits is 3
   d) numbers whose sum of digits is 6
   e) numbers whose sum of digits is 9
   **Answers:** a) no, b) yes, c) yes, d) no, e) yes

6. Make at least two statements about which four-digit numbers are always good. Verify your statement.
   a) Numbers that are multiples of ______ are always good.
   b) Numbers whose sum of digits is ______ are always good.
   **Answers:** a) 1000, b) 3 or 9

7. Show that the numbers in this sequence are all good: 42, 402, 4002, 40 002, ....
   Hint: Find a pattern in the quotients.

8. Provide students with scientific statements that can be proven true using logic or proven false using a counter-example. Examples:
   a) All solids expand when they melt.
   b) All ice cubes are colder than 10°C.
   **Answers:** a) ice is a counter-example; b) true, because all ice cubes have temperature at most 0°C, the freezing point of water

9. Have students decide whether statements of the form “all [of these] are not [like this]” are true or false. Example: For the shapes below, determine whether each statement is true or false.
   ![Shapes](image)
   a) All triangles are not equilateral.
   b) All squares are not black.
   c) All circles are not big.
   d) All circles are not white.
   e) All black shapes are not squares.
   f) All black shapes are not small
   **Answers:** a) false, B; b) true; c) true; d) false, F; e) true; f) false, D
10. Remember, a number is even if that many objects can be paired up without a remainder. For each statement, either explain why the statement is true or find a counter-example.
   a) The product of any two numbers is greater than their sum.
   b) The sum of any two even numbers is even.
   c) The sum of any two odd numbers is odd.
   d) The sum of any three even numbers is even.
   e) The sum of any three odd numbers is odd.

**NOTE:** A counter-example would consist of two numbers in parts a) to c) and three numbers in parts d) and e). Some students might need this pointed out to them.

**Answers:**
   a) sample counter-examples: 1 and 5 have product 5 and sum 6; 0 and 3 have product 0 and sum 3; b) true, because if you can pair up, for example, 8 objects without a remainder and you can pair up 10 objects without a remainder, then you can do the same to 10 + 8 = 18 objects by just combining your pairs; c) any pair of odd numbers is a counter-example, (e.g., 3 and 5 add to 8); d) true, because combining three sets of paired-up objects still leaves everything paired up; e) true, because combining three sets of paired-up objects, where one object from each set is not paired up, leaves three unpaired objects, two of which make a pair with one left over.

11. a) What are the ones digits of the multiples of 2?
   b) What are the ones digits of the multiples of 5?
   c) The numbers that are multiples of both 2 and 5 are the numbers that have ones digit _____.
   d) Explain why this statement is true: All numbers that are multiples of both 2 and 5 are multiples of 10.
   e) Find a counter-example for this statement: All numbers that are multiples of both 4 and 6 are multiples of 24.

**Answers:**
   a) 0, 2, 4, 6, or 8; b) 0 or 5; c) 0; d) true, because combining three sets of paired-up objects still leaves everything paired up; e) true, because combining three sets of paired-up objects, where one object from each set is not paired up, leaves three unpaired objects, two of which make a pair with one left over.

   a) 0, 2, 4, 6, or 8; b) 0 or 5; c) 0; d) The numbers that are multiples of both 2 and 5 must have ones digit 0 because that is the only number in both lists. But the numbers with ones digit 0 are exactly the multiples of 10; e) 12 is a multiple of both 4 and 6 but not a multiple of 24.
ME6-8
Millimetres and Centimetres

Set out a number of items that range in size from 1 mm to 5 cm. Ask students to select the items that are about 1 mm wide. Measure the selected items with a ruler to verify their widths, then add the items measuring 1 mm to the measurement box (SEE: Activity 1).

Remind your students that they can use the width of their little fingers as centimetre benchmarks. ASK: If there are 10 mm in a centimetre, how many millimetres are there in 2 cm?

Have students select several objects from their desks or backpacks (an eraser, a pencil, a pencil sharpener, etc.), measure three objects with their little fingers, and then complete the following sentence for both objects.

The _______ measures about ________ little fingers, so it is about ________ mm long.

Explain to your students that counting every millimetre in a measurement can take a long time, but there is a quick way to do it. Draw a ruler representing 30 mm and tell the class that you want to count 26 mm. Then demonstrate how to skip-count by 10s to the tens value preceding the amount, and continue to count by one (EXAMPLE: 10, 20, 21, 22, 23, 24, 25, 26).

Have volunteers demonstrate this shortcut method by counting to several different numbers.

Ask your students to sort the items they chose into piles of “greater than 50 mm” and “less than 50 mm.” Once they have sorted all of the items, have them measure each to verify the exact length in millimetres. Suggest that your students make a T-table with headings mm and cm. Ask them to record the lengths of the objects they have in millimetres. ASK: If an eraser is 30 mm long, how many centimetres long is it? What do you do to a measurement in millimetres to convert it into centimetres?

Suggest that your students make a T-table with headings “Objects,” “Length in mm” and “Length in cm.” Ask them to record the lengths of the objects they measured in millimetres. ASK: An eraser is 40 mm long. How many centimetres long is it? What do you do to a measurement in millimetres to convert it into centimetres? (divide by 10) A sharpener is 23 mm long. How many centimetres long is it? Review division by 10 with decimal results, such as 25\(\div\)10 = 2.5, 37\(\div\)10 = 3.7, and so on. Ask your students to convert the lengths of their objects from millimetres to centimetres. Give students more measurements to convert, such as: 2.4 cm, .6 cm, 80 cm, 34 mm, 307 mm, 307 cm.

Write several pairs of measurements on the board, like:

A: 8 cm and 9 cm   B: 9 cm and 10 cm   C: 10 cm and 11 cm

Ask your students to say which pair of measurements (A, B or C) the measurement 87 mm lies between. Repeat the question for 93 mm.
Can they find another measurement in millimetres between each pair?

Now write pairs of measurements in millimetres, and ask your students to find a measurement in centimetres that is between each pair:

56 mm and 63 mm, 78 mm and 86 mm, 102 mm and 114 mm.

Hold up a paper clip. Tell your students that a paper clip is 3 cm long. **ASK:** Which lengths can you measure exactly by lining up several paper clips end to end? Can you measure 1 cm? 2 cm? 6 cm? 12 cm? (You can measure any multiple of 3.) Now show your students a larger paper clip. This clip is 5 cm long. **ASK:** Which lengths can you measure with the large paper clip? How can you measure 8 cm? (with one large and one small paper clip) Ask your students to name three lengths that could be measured with a combination of small and large paper clips. Invite volunteers to draw how they would measure each length on the board and encourage multiple solutions (**EXAMPLE:** 23 cm = 4 large paper clips + 1 small paper clip OR 1 large paper clip + 6 small paper clips).

Ask your students what they did with the lengths of individual paper clips to measure the various given lengths. For example, if they measured 12 cm with 3 cm paper clips, what did they do with the 3? (added 4 times or multiplied by 4) **ASK:** If you measured 8 cm with one large paper clip and one small paper clip, what did you do with the 3 and 5? (added) Ask your students if there is a way to represent subtraction with paper clips. **PROMPTS:** In 5 – 3 = 2, what is 2? (the difference) What is the difference between two numbers? (How much one number is larger than the other number.) What is the difference between two clips? How could you measure that? Ask your students to show distances such as 2 cm, 1 cm, and 4 cm.

**Assessment**

Complete the following conversions.

<table>
<thead>
<tr>
<th>mm</th>
<th>230</th>
<th>134</th>
<th>800</th>
<th>45</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td>6</td>
<td>21.2</td>
<td></td>
<td>0.2</td>
<td>70</td>
</tr>
</tbody>
</table>

**Bonus**

1. Draw a line between 5 cm and 6 cm long that is...
   a) closer to 5 cm than to 6 cm.
   b) halfway between 5 cm and 6 cm.
   c) closer to 6 cm than to 5 cm.

   Give the lengths of the lines in millimetres.

2. Give a measurement in millimetres that is between 7 cm and 8 cm and is...
   a) closer to 7 cm than to 8 cm.
   b) halfway between 7 cm and 8 cm.
   c) closer to 8 cm than to 7 cm.

**ACTIVITY 1**

Create a box to collect benchmark items that correspond with each unit of measurement introduced. Everyone can then refer to and compare these items as the lessons progress.
Students could use non-standard units, such as the width of a penny (which is about 2 cm), to estimate or measure lengths. They can use play money pennies to measure the length of their hand, a notebook, a pencil, and other objects in the classroom. Students should convert their measurements into centimetres. (This is a good exercise in skip counting by 2s or doubling.)

Draw a triangle on grid paper and measure its sides to the nearest cm.

Draw a collection of long items.
   a) Draw a collection of alligators, each one being 1 cm longer than the previous one.
   b) A pencil shrinks when it is sharpened. Draw a collection of pencils, each one being 1 cm shorter than the previous one.
   c) Draw a sequence of toboggans where each one is 15 mm longer than the last.
   d) A carrot shrinks when it is eaten. Draw a collection of carrots, each one being 2 cm shorter than the previous one.

Write a story about one of the growing or shrinking items in Activity 4 (or invent your own!). Tell the story of how the item grows or shrinks. How does the carrot get eaten? Who wants to ride the toboggans? Write the story to go with the pictures that you have drawn.

Ask students to name an object in the classroom that they think would have length
   a) 60 mm
   b) 300 mm
   c) 100 cm
   d) 200 cm

Extensions
1. A toonie is about 2 mm thick. Josie has a stack of toonies 10 mm high. How much money does Josie have?
2. A nickel is about 1 1/2 mm thick. How many nickels make a stack 9 mm high?
3. How many millimetres are in… a) a metre? b) a decimetre?
4. Draw a line \( \frac{17}{100} \) of a metre long. How many cm long is the line?
5. Estimate the height of a classmate in cm. Then measure their height using your hand. (Your hand with fingers spread slightly should be about 10 cm wide.) Finally, use a metre stick to check your result. How close were you? **HINT:** Measure them against a wall to get an accurate result.

   Estimate ______ cm    Hand Measurement ______ cm    Actual Measurement ______ cm
Tell your students that today they will learn about another unit of measurement, the decimetre. Write the word “decimetre” on the board. Circle the letters “d” and “m” and explain that they form the abbreviation for decimetre, dm. Write the abbreviation next to the word.

Explain that a decimetre is equal to 10 cm. Remind your students that their hand with fingers spread slightly is about 10 cm wide. This is equal to approximately 1 dm. Ask your students to name 3 objects that are shorter than 1 dm, 3 objects that are longer than 1 dm, and 3 objects that are about 1 dm. Students can measure the objects that are about 1 dm to check their estimates.

This would be a good time to do the Activity (see below).

Remind students of their work in the previous lesson, converting centimetres to millimetres. **ASK:** How many millimetres are in 1 cm? If a paper clip is 3 cm long, how many millimetres is that? What did you do to the measure in centimetres to get the measure in millimetres? If an eraser is 40 mm long, how many centimetres is that? What do you do to the measure in millimetres to get the measure in centimetres?

Draw this diagram on the board:

```
  cm × 10
    \ 
  mm ÷ 10
```

**SAY:** I want to add decimetres to the diagram. Where should I write dm? (on the top step) How many centimetres are in 1 dm? If a pencil is 2 dm long, how many centimetres is that? If a book is 30 cm long, how many decimetres is that? Invite volunteers to add the dm and the corresponding arrows and operations (x 10, ÷ 10) to the diagram. Invite a volunteer to fill in the blanks: 1 dm = _____ cm = _____ mm.

**SAY:** There are 10 cm in 1 dm. Which fraction of a decimetre is 1 cm? (1/10) Which fraction of a centimetre is 1 mm? Which fraction of a decimetre is a millimetre? If your students are familiar with percentages, you might ask the same questions with percentages as well.

Have students convert more measures in centimetres to decimetres and vice versa. **EXAMPLES:**

12 dm = _____ cm  
50 cm = _____ dm  
102 dm = _____ cm  
200 cm = _____ dm  
100 dm = _____ cm

**ASK:** Can someone give me a measure in whole decimetres that is between 33 cm and 46 cm? Then ask for a measure in centimetres that is between
13 dm and 14 dm. (Several answers are possible—anything from 131 cm through 139 cm.)

**ASK:** Is the second measure more than 1 m or less than 1 m? How do you know?

**Assessment**

1. Write a measurement in centimetres that is between 5 dm and 6 dm.
2. Write a measurement in decimetres that is between 20 cm and 95 cm.
3. Complete the tables:

<table>
<thead>
<tr>
<th>cm</th>
<th>20</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>21.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>mm</th>
<th>300</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>dm</td>
<td>.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>

**Extensions**

1. If 1 dm is equal to 10 cm, how many decimetres are in 100 cm? Where would you add metres to the step diagram used in the lesson?

2. Write a measurement in decimetres that is between…
   a) 320 mm and 437 mm   b) 507 mm and 622 mm   c) 1 1/2 m and 1 3/4 m

3. John has a strip of paper 1 dm long. He folds the strip of paper so that it has a crease in its centre. What measurements can John make in centimetres using the strip?

**ACTIVITY**

To help students develop a sense of the size of 1 dm, have them search through the classroom to find objects that are 1 dm long. Ask students to estimate the measurement with their hand. If they find something they believe to be approximately 1 dm, ask them to carefully measure with a ruler to find its actual length. They can add the 1 dm objects to the class measurement box (see Activity 1 of ME6-8).
Remind your students that 1 km equals 1000 m. Ask your students to think of objects that can act as benchmarks for estimating large lengths, heights, and distances. You may measure the actual length or height of some of these objects if they are available. Here are some possible benchmarks:

- A (very tall) adult and a door are about 2 m tall.
- A level, or storey, in a building (viewed from outside the building) is about 2 doors tall, so about 4 m tall.
- A school bus is about 10 m long.
- A typical car is about 3 m long.
- You can walk about 1 km in 15 minutes.

Invite students to use these benchmarks to estimate greater lengths and distances, such as:

- A basketball field is about 9 cars long. How long is a basketball field in metres?
- Two minivans are as long as 1 school bus. How long is each minivan?
- A playground is about 10 cars long. How many minivans can be parked along the playground?
- Daniel lives in an apartment building with 18 storeys. About how many school buses standing end-to-end, one on top of the other, are as tall as Daniel’s building?
- How many minivans parked end-to-end would it take to form a line 100 m long? How many would you need to form a line 1 km long?
- How many school buses can be parked along 1 km?

Review number lines with your students. Draw a number line divided into 20 parts, and mark the ends 0 and 2 km. Ask your students to find and mark 1 km on the number line. Ask your students where the marking for 500 m should go. (PROMPT: Is it more than 1 km or less than 1 km?) Repeat with 1500 m. Ask students to explain how they know where to mark the distance. Point to an unmarked increment between 0 and 500 m and ask your students what number should go there. Have students identify the positions of 800 m, 1200, and 1900 m. Then ask your students to mark the approximate positions of 250 m, 660 m, 30 m, and 1772 m.

Extensions

1. How would you change a measurement in kilometres into centimetres?
2. Rita wants to estimate the height of a tree that grows near the school building. The tree is 3.5 storeys tall. Rita multiplies 3.5 by 4 m (the height of a school storey). How tall is the tree?

Cross-Curriculum Connection: ARTS

3. Remind your students that things in the distance appear smaller than they really are. Draw a line about 40 cm long on the blackboard. A pencil is far less than 40 cm long, but depending on where you stand, the line on the blackboard can appear to be the same length as the pencil! Ask your students to test this by holding a pencil in an outstretched hand. Ask them to move around the room, towards and away from the blackboard, until the line appears to be as long as the pencil. Can they move so that the line appears to be half as long as the pencil?

4. Rita wants to estimate the height of a tree growing in the middle of a park. Rita uses a pencil and a friend. Rita positions herself far enough from the tree so that the tree appears smaller than the pencil. She asks her friend Sindi to stand by the tree. Rita holds the pencil vertically in her outstretched hand (she keeps her hand outstretched at all times) and compares the size of the pencil and the tree. For example, the tree appears to be as long as three quarters of the pencil. She holds the pencil so that the point is level with the top of the tree and her fingers are level with the bottom of the tree. She turns the pencil horizontally (with her fingers as the center of rotation) and asks her friend Sindi to walk in the direction that is at a right angle to the line between Rita and the tree. Rita asks Sindi to stop when the distance between Sindi and the tree seems to her the same as the distance she marked on the pencil. (Rita holds the pencil so that her finger is still level with the bottom of the tree and the point of the pencil is level with Sindi’s feet.)

After that, Rita measures the distance between Sindi and the tree with giant steps. This is the height of the tree. Can you explain why Rita’s method works? HINT: Use congruent triangles, one vertical and the other horizontal. Discuss how Rita’s method of estimating and comparing lengths could be used in Arts.

EXPLANATION: There are two pairs of congruent right-angled triangles. In the first pair, one triangle has Rita, the bottom and the top of the tree as vertices, and the other triangle has Rita, Sindi and the bottom of the tree as vertices. The other pair of congruent triangles has Rita’s eye and arm in common and the pencil as the side opposite to Rita’s eye in both triangles. The smaller triangles are congruent, because two of their sides are same (arm and pencil) and
they both have a right angle. The larger triangles are also right-angled, and their angles are equal to the angles of the smaller triangles. This means that the angles of the larger triangles are equal, and they also have a common side, so the larger triangles have to be congruent.

ME6-11
Changing Units

Review the units of measurement: mm, cm, dm and m. Ask your students to name several objects that they would measure in each of these units.

Remind your students how to record measurements such as 1 metre and 25 centimetres, as 1 m and 25 cm and as 125 cm, then ask them to record their measurements in all three styles.

Show your students how to develop an estimate. First, estimate the blackboard’s length in whole metres (EXAMPLE: Estimate: 5 m). Then place a metre stick beside the blackboard and ask your students if they would like to change their estimate. Mark the points that are 1 m and 2 m from the end of the blackboard. Ask the students if they would like to change their estimate now. Continue until the remaining length is less than a metre. Ask your students to estimate the remaining distance in centimetres (EXAMPLE: Estimate: 5 m 25 cm).

Ask your students to estimate and measure other distances in the classroom, like the width of the classroom, the distance from the window to the blackboard, etc. You might also do Activity 3 at this point.

Let your students complete the first three questions of worksheet ME6-11.

Ask volunteers to convert, step by step, several measurements in metres and centimetres to centimetres. For example,

2 m 30 cm = 2 m + 30 cm;
2 m = 200 cm;
2 m + 30 cm = 200 cm + 30 cm = 230 cm.

For a measurement like 370 cm, reverse the step-by-step process. (Students could start by skip counting by hundreds to determine how many metres are in 370 cm.)

Complete several more examples, then allow your students to practise with questions like:

145 cm = ____ m ____ cm     354 cm = ____ m ____ cm
789 cm = ____ m ____ cm     3 m 78 cm = ____ cm
4 m 64 cm = ____ cm         9 m 40 cm = ____ cm

Ask your students how many metres are in 70 cm. Then ask them to convert to metres such measurements as 1 m and 40 cm, 2 m and 35 cm, and so on. Students might find it easier to convert the measurement to centimetres only first.
Assessment
1. 12 m = _____ cm = _____ dm = _____ mm  
   3 000 mm = _____ cm = _____ dm = _____ m  
   3 456 mm = _____ cm = _____ dm = _____ m  
   3 400 mm = _____ cm = _____ dm = _____ m
2. Convert the following units of measurements.
   a) 2 m 72 cm = _____ cm  
   b) 3 m 56 cm = _____ cm  
   c) 348 cm = _____ m _____ cm
Bonus
Convert the measurements:
123 456 789 mm into cm, dm, m and km  
987.654 m to dm, cm and mm  
0.987654321 km to m, dm, cm and mm

ACTIVITY 1
The Great Metre Hunt
Cut up a ball of string into numerous lengths, ranging from 5 to 20 centimetres. Hide these lengths of string around the classroom. Divide students into groups—pairs work well—and inform them that each group must find ten pieces of string. The goal is to find ten pieces of string that, when laid end to end, will come closest to measuring 1 metre. Explain that finding all of the long pieces will probably total much more than one metre in length. Don’t forget to hide or remove the metre sticks from the classroom, to prevent students from using them to measure their string during the hunt.
Allow students to roam around collecting string. Once every group has found ten pieces, have them lay out their string end to end. Use a metre stick to measure the lengths. The group with the combined length of string closest to one metre wins!
Then, ask students to trade pieces of string with other groups so that each group has collected a metre-long length of string pieces. Allow them to use metre sticks.

ACTIVITY 2
Ask students to measure and compare the lengths of various body parts using a string and a ruler.
EXAMPLE:
   a) Is your height greater than your arm span?  
   b) Is the distance around your waist greater than your height?  
   c) Is your leg longer than your arm?
Encourage your students to predict the answers before they perform the measurements. Ask your students to record the measurements in three ways: 1 metre and 25 cm, 1 m and 25 cm and 125 cm.
ACTIVITY 3
Estimate the width of the school corridor. First, try to compare the length to some familiar object. For example, a minivan is about 5 m long. Will a minivan fit across the school corridor? Then, estimate and measure the width with giant steps or let several students stand across the corridor with arms outstretched. Estimate the remaining length in centimetres. Measure the width of the corridor with a metre stick or measuring tape to check your estimate.

ACTIVITY 4
Hold up a metre stick and point to various positions on the stick. Ask students to say if the positions are closer to 0 metre, .5 metre, or 1 metre.

ACTIVITY 5
Measure your shoe. Use this natural benchmark to measure long lengths, such as the width of the classroom, the width of the hallway, or the length of the whole school!

ACTIVITY 6
Say whether each measurement is closer to 0 metres, $\frac{1}{2}$ metre, $\frac{3}{4}$ metre, or 1 metre.

a) 10 cm  b) 52 cm  c) 37 cm  d) 82 cm  e) 2 cm  f) 90 cm

Extension

Measure Your Stride and Create a Treasure Map
Draw a line on the floor. Position yourself with your toes on the line. Walk normally for ten strides (not giant steps!) and mark the place where your front leg’s toes are. Measure the distance you walked with a metre stick and divide it by 10 to get the average length of your stride.

Now that you have measured your stride, create a map that shows directions in strides, or paces, to a buried pirate treasure (EXAMPLE: 15 paces north, 3 paces east, etc.). Calculate the actual distance you need to walk to find the treasure.
Make sure your students are familiar with multiplication and division of decimals by 10, 100, and 1 000. They should have completed sections NS6-86, NS6-87, and NS6-89.

Draw a line on the board and ask students to measure it in decimetres and in centimetres. **ASK:** Which unit is larger, the centimetre or the decimetre? Will the same line hold more centimetres or more decimetres? If the line is 4 dm long, only 4 decimetres are needed to cover it, but it takes 40 centimetres to cover the same length. So the smaller the unit, the larger the number of the units you need. **ASK:** What do you do to a measurement in centimetres to get the measurement in decimetres? What do you do to measurements in decimetres to convert them to centimetres?

Draw this diagram on the board:

```
  dm  × 10
    ↓
  cm  ÷ 10
```

**ASK:** Where should I put millimetres? (on the bottom step) Where should I put metres? (on the top step) Where should I add arrows and how should I label them?

**ASK:** A line is 5 dm long. How long is it in millimetres? Encourage your students to show more than one solution to the problem. (For example, they could show 5 dm = 50 cm = 500 mm OR they could explain that since 1 dm = 100 mm, then 5 dm = 500 mm.)

Present a harder problem: change 4.7 cm to millimetres. **ASK:** Which unit is larger, the centimetre or the millimetre? Will a line 4.7 cm long hold more millimetres or more centimetres? Should you multiply or divide? By how much? Point out that the length in centimetres has 1 digit after the decimal point and the length in millimetres is a whole number.

Ask your students to perform several more complicated changes of measurements, such as:

- 6.7 dm to mm
- 345 mm to dm
- .54 cm to m
- .35 m to dm
- .24 m to mm
- 789 mm to m
- 35 mm to dm
- 5432 mm or 54.32 m
- 87 dm or 86543 mm

Do the Activity (see below) to give students more practise expressing measurements in different units.

Ask students to tell which measurement is longer:

- 324 mm or 3 dm
- 654 cm or 65 dm
- 5432 mm or 54.32 m
- 87 dm or 86543 mm
You may also give your students word problems that require changing units, such as:

- A fence board is 2 dm wide. Three boards cost $10. How much will 3 m of fence cost?
- One metre of ribbon costs 12 cents. How much would 900 mm of ribbon cost?

Review units of mass measurement with your students. Draw another diagram on the board:

```
  kg

   g

     mg
```

Ask your students to add arrows and multiplication or division signs to the diagram. Then ask them to convert several measurements, such as:

- 2 kg = _____ g
- 4 000 g = _____ kg
- 18 g = _____ mg
- 3500 mg = _____ g

They should also convert measurements involving decimals.

Ask your students to compare conversion between kg and g to conversion between km and m—how are they the same? (In both cases, you multiply by 1000.) **ASK:** How do we convert g to mg? How do we change a measurement in mg to kg?

**Assessment**

1. Fill in the blanks:
   a) .7 dm = _____ mm
   .35 m = _____ dm
   .24 m = _____ mm
   345 mm = _____ dm
   789 mm = _____ m
   .35 mm = _____ dm
   .54 cm = _____ m
   .789 mm = _____ m
   5.67 dm = _____ cm
   .05 dm = _____ m
   .27 cm = _____ mm
   364 cm = _____ dm
   b) 5.67 g = _____ mg
   .45 kg = _____ g = _____ mg
   36 mg = _____ g = _____ kg

2. Circle the largest length and underline the smallest length in each column above.

**Bonus**

What do you do to a measurement in millimetres to get a measurement in kilometres?

The largest snake in the world is 11362.5 mm long. The smallest snake in the world is .000108 km long. How much longer than the smallest snake is the largest snake? Give the difference in centimetres.

**ACTIVITY**

Ask your students to make a numbered list of all possible changes between the 4 units of measurement used in the lesson (cm to mm, cm to m, cm to dm, and so on—there are 12 combinations in total). Give each pair of students two dice. Player 1 rolls the dice. The sum of the numbers rolled gives him or her the number of the operation in the list. Player 2 provides Player 1 with a measurement to change accordingly. For example, if Player 1 rolled a sum of 8, and operation 8 on the list was “mm to m,” Player 2 might write 3.7, and Player 1 would have to convert 3.7 mm to metres.

**ADVANCED:** Player 1 may request a certain number of digits after the decimal point in the result. For example, if Player 1 rolls “mm to m” as above and asks for 1 digit after the decimal point, Player 2 might write 1700 mm, so that the result is 1.7 m.
Extension

Draw a line that is...

a) 0.11 m long   b) 1.7 dm long   c) 134 mm long   d) 0.003 m long   e) .15 dm long

ME6-13
Appropriate Units of Length

Present the items from the measurement box individually (see ME6-8) and have your students express the measurement that each item represents. Review the full name and abbreviation for each unit.

Explain that it is important to choose the appropriate unit of measurement for the length/distance being measured. Ask your students to tell you which unit of measurement would best express the distance from Halifax to Calgary. Which unit of measurement would best express the width of a piece of paper?

Display a variety of objects (a book, a stapler, a coin, etc.) and ask your students to tell you which unit of measurement will best express the length of the object. Is the length of a stapler best expressed in centimetres or metres? Is the thickness of a coin expressed best in millimetres or centimetres?

Have your students select five items in the classroom and guess which unit of measurement will best express each item’s height, width or length.

Have a volunteer measure one of the practice items (the stapler, for example), but do not specify a unit of measurement. After the volunteer relates the measurement to the class, identify the unit of measurement used and ask your students why the volunteer chose that unit of measurement. Students will likely respond that centimetres were used because the stapler is about the right size. It would be many millimetres long, and it’s much smaller than a metre.

Demonstrate that the stapler is, for example, 12 cm in length. That’s 120 mm, 0.12 m and 0.00012 km. Out of all those numbers, 12 is the nicest and the simplest.

Have your students measure their five objects. Which units of measurement did they use? Do these units of measurement lend themselves easily to the task? Would alternate units of measurement offer simpler measurements?

Ask your students to list in their notebooks five things that could be measured and are not in the room (EXAMPLE: a bicycle, a video game, a rocket ship, anything). Ask students to arrange the five things in order from smallest to largest. Then ask them to indicate which unit of measurement will give the simplest measurement for each item.

Review the relationships between units of length measurement. Ask your students: Which number is larger, 2 or 35? What length is larger, 2 m or 35 cm? Why?
Explain that the easiest way to compare measurements that are expressed in different units is to convert all measurements to the small unit. For instance, to compare 3 m and 250 cm, convert 3 m to 300 cm, and it becomes clear that 3 m is greater than 250 cm.

Let your students practice with questions like:

Circle the greater amount:
1 m or 80 cm  
6 m or 79 cm  
450 cm or 5 m  
230 cm or 2 m  
2 m or 38 cm

Ask students to order these lengths from shortest to longest and ask them to mark these lengths on a number line:

Assessment
1. Which unit of measurement would you use to measure the...
   Thickness of a book?
   Length of a chocolate bar?
   Height of the school?
   Height of a tree?
   Height of a mountain?
   Distance from a window to a door?
   Distance from your school to your home?
   Distance from your nose to your toes?
   Distance from the Earth to the Moon?
   Distance between your eyes?

2. Convert the distances to centimetres where necessary and then order them from greatest to smallest.
   a) 75 cm  
   b) 85 m  
   c) 230 cm  
   d) 7 m  
   e) 4 cm

On a school walking trip, ask students to say when they think they have walked half a kilometre or 1 kilometre.

Ask students if they have taken any trips to nearby towns. Ask them to estimate how many kilometres away the towns are and then have them check the actual distances by measuring the distances on a map with a ruler, and then converting their measurements to kilometres using the scale on the map.
## ACTIVITY 3

Divide your class into four groups and assign one unit of measurement (mm, cm, m or km) to each group. Give each group a sheet of chart paper, and ask them to write the full name of their unit of measurement and the abbreviation at the top of the page. Have them list as many things as they can that could be measured with that unit of measurement. Set a target quantity (maybe twenty) and ask students to try and list more than that quantity. Have each group share their ideas. Display the lists in the classroom until the instruction of ME6-17 is complete.

## ACTIVITY 4

Ask your students to list as many words as they can that start with the unit of measurement prefixes. Some examples include centipede (“hundred feet”), centurion (a commander in the Roman army in charge of 100 soldiers), million (one thousand thousands), decibel, decimal, decimate (historically meaning to kill or remove one in every ten). Students might suggest other metric units, such as kilograms or kilowatts, for “kilo.”

Ask your students what numbers they know in French or any of the other Romance languages.

Draw the following table and illustrate the patterns and similarities to your students.

<table>
<thead>
<tr>
<th>Language</th>
<th>10</th>
<th>100</th>
<th>1 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td>dix</td>
<td>cent</td>
<td>mille</td>
</tr>
<tr>
<td>Spanish</td>
<td>diez</td>
<td>cien</td>
<td>mil</td>
</tr>
<tr>
<td>Italian</td>
<td>dieci</td>
<td>cento</td>
<td>mille</td>
</tr>
<tr>
<td>Portuguese</td>
<td>dez</td>
<td>cem</td>
<td>mil</td>
</tr>
</tbody>
</table>

## ACTIVITY 5

Check out The Metric Song by Kathleen Carroll:

http://www.songsforteaching.com/kathleencarroll/metricsong.htm

This song reinforces the distinction between milli and kilo.

## Extensions

1. Introduce your students to the principle of metric prefixes. Explain to them that the metric system is comprised of many different units of measurement, all dependent on the dimensions being measured—distance is measured in metres, volume is measured in litres, weight is measured in kilograms.

   Then explain that each unit of measurement has a base unit (for these lessons, metres) that shares prefixes with the other base units. Write the word “metre” and ask your students to provide you with the prefixes.

   Write the prefixes and explain a bit about the etymology of each. For example, centi means one hundredth. That’s why there are 100 centimetres in a metre. Also, there are 100 cents in a dollar. There are 100 years in a century.
Milli means one thousandth. That’s why there are 1 000 millimetres in a metre and 1000 years in a millennium.

Deci means one tenth. That’s why there are 10 decimetres in a metre and 10 years in a decade.

Kilo means 1 000. That’s why there are 1 000 m in a kilometre.

2. Have your students research (and prepare reports or descriptive posters on) alternative systems of measurement, such as Ancient Egyptian, Chinese or Old English systems. How long were their respective units of measurement, and what were they called? Are they still in use?

3. Compare the lengths of non-metric units of measurement (EXAMPLE: inches, feet, yards) to the metric units of measurement. What is the difference between an inch and a centimetre or a yard and a metre?

4. Big Numbers. Here are some questions you could assign or discuss with students to give them practice calculating and estimating with large numbers. Your students will need to know how to multiply whole numbers by multiples of 10.

   a. (Warm Up): There are 60 seconds in 1 minute. How many seconds are there in 1 year?

      **SOLUTION:** Start with seconds:

      How many seconds in one minute? 60 seconds
      How many minutes in a day?

      60 minutes = 1 hour; 1 day = 24 hours
      24 × 60 = 1 440 minutes in one day
      1440 × 60 = 86 400 seconds in one day

      Finally, multiply the number of seconds in one day by the number of days in one year.

      86 400 seconds in one day × 365 days in one year = 31 536 000 seconds in one year!
      In one year there are more than 31 million seconds!

   b. About how many minutes have you been alive?

      **SOLUTION:** Start with the date the student was born and the current date.

      For example, a student was born August 29, 1996, and the current date is September 16, 2007.

      From August 29 to the end of the month we add three days. From September 1, 1996 to September 1, 2007 is 11 years. The current date is September 16, so we add: 3 days + 11 years + 16 days = 19 days + 11 years.

      However, 2000 and 2004 were leap years, so we have to add 2 more days. The student has been alive for about 11 years of 365 days and 21 days in total.

      How many minutes in one day?

      24 × 60 = 1440 minutes in one day

      How many minutes has the student been alive for?
1440 minutes in one day \times 365 \text{ days in one regular year} = 525 600 \text{ minutes in one regular year}

525 600 \text{ minutes in one year} \times 11 \text{ years} = 5 781 600 \text{ minutes in 11 years}

Add: \(1440 \times 21 \text{ (extra days)} = 30 240\)

\[
\begin{align*}
5 781 600 \text{ minutes in 11 years} \\
\quad + \quad 30 240 \text{ minutes in 21 days}
\end{align*}
\]

5 811 840 \text{ minutes in the student’s life so far!}

c. According to the Guinness Book of World Records, the tallest tree ever measured was an eucalyptus tree discovered in Watt’s River in Victoria, Australia in 1872 by a forester named William Ferguson. The eucalyptus tree was 132.6 m tall.

The Dyerville Giant, a coastal redwood tree, found in the Humboldt Redwoods State Park in California, USA is named as the tallest tree of modern times. This tree was 1 600 years old and 113.4 metres high when it fell in March 1991.

The height of a dime is 1 mm. How many dimes would need to be stacked to be as high as each of the tallest trees in history? How much would all these dimes be worth?

**SOLUTION:**

Convert each tree height from metres to millimetres.

\[
\begin{align*}
132.6 \text{ m} \times 1 000 &= 132 600 \\
113.4 \text{ m} \times 1 000 &= 113 400
\end{align*}
\]

The height of a dime is 1 mm. We can stack 132 600 dimes to be as high as the eucalyptus tree and 113 400 dimes to be as high as the redwood tree.

The total values are 1 326 000¢ and 1 134 000¢ or $13 260 and $11 340.

d. Ellen MacAurthur from France is the first woman to travel around the world on a sailboat, by herself. Her boat was named the Kingfisher and it could only carry one person. Ellen had dehydrated food and a few changes of clothing, as well as e-mail access on board her boat. It took her a total of 94 days, 4 hours, 25 minutes, and 40 seconds to complete her journey around the world.

Ellen had to be alert at all times so she could not take breaks to sleep. Amazingly, she would take 40 minute naps, 10 times a day during her trip.

About how many hours of Ellen’s trip sailing around the world was she sleeping?

**SOLUTION:**

\[
\begin{align*}
40 \text{ minutes} \times 10 \text{ naps per day} &= 400 \text{ minutes of sleep per day} \\
400 \text{ minutes of sleep} \div 60 \text{ minutes per hour} &= \text{approximately 7 hours of sleep each day} \\
7 \text{ hours} \times 94 \text{ days} &= 658 \text{ hours}
\end{align*}
\]

Ellen slept approximately 660 hours during her trip around the world.
e. “Wranny” is what Andy Martell from Toronto, Ontario, calls his creation made from Saran wrap. He is marked down in the Guinness Book of World Records for having created the largest ball made entirely from Saran wrap. Measured in February 2003, the ball had a circumference of 137 cm and weighed 20.4 kg.

A jelly bean weighs 0.5 grams. How many jelly beans do we need to equal the weight of “Wranny,” Andy’s famous Saran wrap ball?

**SOLUTION:**

The Saran wrap ball weighs 20.4 kg = 20.4 × 1000 = 20 400 grams

One jelly bean weighs 0.5 grams, so 2 jelly beans would weigh 1 gram. Therefore 20 400 × 2 = 40 800 jelly beans would weigh the same as the Saran wrap ball.

f. The largest flag created was flown in Brasilia, Brazil in August 1998. The flag has a width of 70 m and a height of 100 m. What is the area of the flag?

Measure the area of a regular piece of paper. How many regular pieces of paper would be needed to cover the entire flag?

**SOLUTION:**

The area of the largest flag is: 70 × 100 = 7 000 m²

The dimensions of a regular piece of paper are about 30 cm × 20 cm.

The area of a regular pieces of paper is: .3 m × .2 m = 0.06 m²

But 100 000 × .06 = 6 000 (which is fairly close to 7 000).

At least 100 000 pieces of regular paper would be needed to cover the largest flag in the world.

**NOTE:** All the data collected for questions c to f is from the Guinness Book of World Records website: www.guinnessworldrecords.com.
ME6-14
Speed

Explain to your students that speed measures the distance covered in a certain time. For example, you can walk 1 km in 15 minutes. However, we usually measure speed in kilometres per hour or metres per second.

**ASK:** Where have you seen or heard measurements of speed? *(EXAMPLE: traffic signs, weather reports)* What units were used in each case?

**ASK:** If I am travelling at a speed of 4 km per hour, how far will I get in 1 hour? (4 km) Invite volunteers to fill in the following chart:

<table>
<thead>
<tr>
<th>Speed (km per hour)</th>
<th>Time (hours)</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Ask students to add two more rows to the chart. **ASK:** How did you calculate the distance travelled in the last column? *(distance = 4 × time)* Repeat the exercise with a different speed (such as the speed of a car, 60 km per hour; a wind speed of 10 km per hour; or the speed of a hurricane, 90 m per second). Ask your students if they can tell a general rule for the distance travelled given the speed and the time. *(distance = speed x time)* Write the rule on the board. Invite volunteers to use the speeds given above to answer these questions:

- How far will the car travel in 5 hours?
- How far will a balloon be blown by the wind in 6 hours?
- A leaf is swept away by a hurricane. How far from the place the tree grew will it be after 30 seconds? After 1 minute?

Now ask your students to solve a different problem: A car travels at 90 km per hour for several hours. It covers a distance of 540 km. How long did it travel? Ask your students to write the data they know in the table above.

**ASK:** Which column is empty? What did you do with the time and the speed to get the distance? What should you do with the speed and the distance to get the time? As a challenge, you might ask your students to write an equation for the problem: 90 × T = 540, where T represents the time.

**ASK:** What should you do with 540 and 90 to find T?

Invite students to solve several problems, such as:

- John bikes at a speed of 15 km per hour. He biked 45 km. How long did it take him?
- Naima skateboards at a speed of 10 km per hour. How many hours will it take her to skateboard 50 km? 75 km?
How much further than Naima will John bike in 4 hours?

Present the following problem:

Michael is travelling from Calgary, AB to Prince George, BC. The total distance of the trip is 780 km. Michael makes some short stops along the way. His average speed is 80 km per hour. About how long will the trip take?

Suggest to students that they round the distance to the nearest 100 km to get an estimate of Michael’s travelling time. Then ask them to perform long division to find a more accurate answer. **ASK:** Does the more precise answer make more sense? Is that the time Michael actually spent behind the wheel? If his average speed was 80 km per hour, does this mean that he was driving at 80 km per hour all the time? No, 80 km per hour is his average speed; he might have driven faster for part of the time and slower for another part of the time. He also made some stops along the way—at those times, his speed would have been 0 km per hour!

Present the next problem and ask your students to tell whether precise answer makes more sense here.

Michael decided to make longer stops along the way to take some photos. His average speed is now only 50 km per hour. How long will his trip take at this speed?

Ask your students if rounding the distance to the nearest hundred is helpful in the next problem. **Why not?**

How long will the trip take if Michael is biking at a speed of 13 km per hour?

As a final challenge, present the following problem:

The distance from Calgary to Banff National Park is 108 km. Michael drives at 90 km per hour. Then Michael drives at a speed of 60 km per hour through Banff National Park and Jasper National Park. When he leaves Jasper National Park, he still has 345 km to go and he returns to 90 km per hour. How many kilometres did he travel through Banff and Jasper National Parks? How long was his trip through the parks? How long was his total trip?

To solve this problem, students might find it helpful to draw a line and mark the distances:

```
  108 km
Calgary
  Entrance to Banff
  Exit from Jasper
  Prince George

  345 km
```

A table like the one used above could be useful for each part of the trip. Encourage your students to present different ways of thinking about various parts of the problem. For example, the middle part of the trip is 327 km long (780 – 108 – 345 = 327), and the speed is 60 km per hour. To determine the time it takes to complete this part of the trip, students could divide 327 by 60 using long division, or they could say that 327 is almost 330, which gives 5 hours 30 minutes if divided by 60. They could also notice that there are 60 minutes in an hour, so if the speed is 60 km per hour, the car travels 1 km each minute. So it takes 27 minutes, or 5 hours 27 minutes, to cover 327 km. Outside the parks he had 108 + 345 = 453 km to go. Notice that 453 = 450 + 3, so the road took slightly more than 5 hours. How long would 3 km take at a speed of 90 km per hour? (90 km in an hour, 1.5 km in a minute, 3 km in 2 minutes) His total trip was 10 hours 29 minutes long.
Bonus
Sindi, Mona and Lisa plan a trip to the Moon. The Moon is 380 000 km away. Sindi wants to bike and her speed will be 15 km per hour. Lisa would take a car and go at a speed of 90 km per hour. Mona wants to go by space shuttle at a speed of 8 000 km per hour. How long will it take each of them to reach the Moon if they bike, drive or fly without rest? Do you need a precise answer or an estimate?

Extensions
1. Invite students to think about how distance affects the units used to measure speed. **ASK:** How far (approximately) can a car travel in a second? A minute? An hour? Why do you think we measure speed on a highway in km per hour instead of km per second or km per minute?

2. The graph shows the distance covered by a cat over a period of 12 seconds.

![Graph showing distance covered by a cat over time]

a) How fast was the cat running during the first 6 seconds?
b) What was the cat doing after 10 seconds?

3. Karla can run at a speed of 5 metres per second and Bonnie can cycle at a speed of 12 metres per second. If Bonnie passed Karla on her bike and both children were moving at top speed how far apart would they be after 3 seconds?
Project an image of the Mayan pyramid at Chichen-Itza, Mexico, on the blackboard. Ask your students to measure the height and the base of the pyramidal platform in the picture. (Ignore the square structure on top of the pyramid-shaped platform.) Explain that the platform has a staircase on each of its 4 sides, and each step in that staircase is 66 mm tall. There are 364 stairs in the staircase. How high is the pyramid? (66 mm × 364 stairs = 24 024 mm = about 24 m) So what is the scale of the picture? (If the projected pyramid is, say, 60 cm tall, then 24 m equals 60 cm, so 4 m equals 10 cm, and the scale is 40 to 1. This means each centimetre in the picture corresponds to 40 cm in real life.)

Ask your students to calculate the length of the pyramid base from their measurements and the scale. (It is about 60 m.) The students may also count the number of large “steps” in the pyramid (on either side of the staircases) to find the approximate height of each one.

Invite a volunteer to measure the square temple structure on top of the pyramid. Ask your students to use the scale they found to calculate the actual height of the structure. How much taller is the base than the temple?

ASK: Suppose a man 2 m tall was standing at the base of the pyramid. How tall would the man be in the picture? PROMPTS: One centimetre in the picture is 40 cm in real life. How many centimetres are in 2 m? How many times larger is 2 m than 40 cm? How many centimetres will be in the image of the man to scale? (5) Now invite a volunteer to measure the figure of a man on the right side of the picture. Is this figure the same height as our man? Why not? (The man in the picture is not standing at the base of the pyramid. He is closer to the viewer, so he appears larger than 5 cm.)
Extension

Over the last 40 years, the Pyramid at Giza has lost 1 decimetre from its original height. If the pyramid continues to erode at the same rate, how much shorter will it be in 12,000 years?

ME6-16
The Welland Canal

Students will need a ruler, a piece of string, and a large-scale, detailed map of Canada’s North that includes the Dempster highway. Ask your students to lay the string carefully along the line that represents the highway on the map, measure the length of the string, then figure out a scale for the map, knowing that the length they measured with the string (in centimetres) represents about 700 km (the approximate length of the actual highway). If you are using a large-scale map, use the actual length of 736 km.

Ask your students to use the string to measure the distance on the map from Inuvik to the place where the Dempster highway crosses the Arctic Circle.

ASK: How can you use the scale you found to find the actual distance?

PROMPTS: How many kilometres are in 1 cm on the map? How many centimetres on the map is the distance that you measured? How many kilometres is the actual distance?

ASK: If you drive along the Dempster highway at a speed of 100 km per hour, how many hours will it take you to get from Inuvik to the Arctic Circle? What should you do with the distance to get an estimate? (Round the distance to the nearest hundred, then divide by the speed.) How could you find a more accurate answer? (Divide without estimating, then convert parts of the hour to minutes.) What is more appropriate—the estimate or the precise answer?

Extensions

1. An earlier version of the Welland Canal had 40 locks instead of eight. Estimate the average distance between the locks.

2. An extension was built onto the canal called the Welland By-Pass. The By-Pass leads away from the canal about 15 km south of Lock 1 and rejoins it about 40 km south of Lock 1. Using the picture here as a guide, draw the Welland By-Pass on the map on the worksheet.
This is a review worksheet.

**ME6-17**
Changing Units of Measurement (Review)

Remind to your students that perimeter is the measurement around the outside of a shape. Illustrate the perimeters of some classroom items; run your hand along the perimeter of a desk, the blackboard or a chalkboard eraser. Write the phrase “the measurement around the outside of a shape.”

Demonstrate the method for calculating perimeter by counting the entire length of each side and creating an addition statement. Write the length of each side on the picture.

Draw several figures on a grid and ask your students to find the perimeter of the shapes. Include some shapes with sides one square long, like the shape in the assessment exercise, as students sometimes overlook these sides in calculating perimeter. Ask your students to draw several shapes of their own design on grid paper and exchange the shapes with a partner.

For the last exercise, suggest that students draw a letter, or simple word (like CAT), or their own names.

Explain that different shapes can have the same perimeter. Have your students draw as many shapes as they can with a given perimeter (say ten units).

Remind students that the opposite sides of a rectangle have equal length. Draw a rectangle on the board and write the lengths of two adjacent sides. Ask your students to find the perimeter of the rectangle. **ASK:** How many times did you use the length? The width? Why? Repeat with a couple of other rectangles.

**ASK:** Can you draw a rectangle with sides that measure a whole number of units and a perimeter of seven units? What about a rectangle with sides that measure a whole number of units and a perimeter that is an odd number of units? (Both are impossible.) Ask students to explain their answers.

**Assessment**
1. Write the length of each edge beside each edge and calculate the perimeter of this shape.
Do not miss any sides—there are ten!

\[
\begin{array}{|c|c|c|c|c|}
\hline
& & & & 2 \text{ cm} \\
\hline
\end{array}
\]

2. Draw two different rectangles with the same perimeter as this shape.

**Extensions**

1. Distribute pentomino pieces (a set of twelve shapes each made of five squares, see the BLM) to your students and have them calculate the perimeter of each shape. Create a table and order the perimeters from smallest to greatest. Have students also calculate the amount of square edges inside each shape. Can they see a pattern emerging in the table?

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Number of Inside Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. a) Review the notion of regular polygons with your students. (Regular polygons have all sides and all angles equal.) Ask your students to find the lengths of sides of a regular triangle, square, pentagon and hexagon that have perimeter of 1 m each.

b) Review sum of angles in polygons (G6-5). Ask your students to use protractors and rulers to construct the regular polygons with a perimeter of 1 m.

c) Ask your students to construct a rhombus that is not a square with a perimeter of 1 m. (The Ontario Curriculum)
ME6-19
Measuring Perimeter

GOALS
Students will estimate and measure perimeters with metric units. They will also solve simple word problems.

PRIOR KNOWLEDGE REQUIRED
Calculating perimeter
Measuring with a ruler
Different units of measurement
Selecting appropriate units of measurement
The ability to estimate
Ratios, solving equations
Conversion between m and cm

VOCABULARY
perimeter
millimetre
centimetre
decimetre
metre
kilometre

Draw this figure:

and have your students demonstrate the calculation of the perimeter by totalling the outside edges.

Then draw the same rectangle without the inside edges:

and ask them how they could calculate the perimeter again. Explain that it can be measured, and then have volunteers measure each side with a metre stick.

Draw an irregular figure with some edges that are neither horizontal nor vertical:

Invite a volunteer to estimate the lengths of the sides. Remind your students that their hands with fingers spread slightly are about 10 cm wide. Have a volunteer use a ruler or metre stick to measure each side, and then count the sides to confirm that they’ve all been measured.

Review appropriate units. Have students determine the best units of measurement for calculating the perimeters of the schoolyard, a chalkboard eraser, a sugar cube, etc.

Draw a rectangle on the board. Write the lengths of the sides on two adjacent sides and ask your students to write an addition statement for the perimeter. Ask your students if they can think of another way to find the perimeter. Ask them to think of a statement that will include addition and multiplication, or doubling. (Add the two sides and double, because each of the adjacent sides is equal to the opposite side.) Ask your students to find the perimeter of several more rectangles given the lengths of two sides. Increase the lengths of the sides to keep students engaged.

Draw two different rectangles with perimeter 10 units (1 × 4 and 2 × 3).
**ASK:** Are the shapes the same? Do they have the same perimeter? Ask your students to create two different rectangles with sides that measure a whole number of units and a perimeter of 14 units. If your students have trouble doing this, invite them to do a systematic search. **ASK:** What is the shortest
side that a rectangle with any perimeter can have? (1) Ask them to try different lengths for the other sides, ordering the results in a table, such as:

<table>
<thead>
<tr>
<th>Length of the Short Side</th>
<th>Length of the Long Side</th>
<th>Perimeter</th>
<th>More or Less Than 14?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>6</td>
<td>less</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>8</td>
<td>less</td>
</tr>
</tbody>
</table>

Students can try different lengths for the short side. They could also look for a pattern in the perimeter column.

Now draw a rectangle and mark the length of only one side; make it 2 cm. Tell your students that the perimeter of the rectangle is 16 cm. Ask your students to find the length of the other sides. Encourage them to find various solutions. If the following solution does not arise, show it:

Let \( n \) (or \( \square \)) be the length of each of the sides adjacent to the 2 cm side. The addition statement for the perimeter is then \( 2 + 2 + n + n = 16 \). This can be rewritten as \( 4 + n + n = 16 \) or even \( 4 + 2n = 16 \). The equation might be solved by guessing and checking or by subtracting 4 from both sides and dividing by 2 (if your students are familiar with the material from lesson PA6-28).

Repeat the exercise above with several rectangles, gradually enlarging the dimensions (EXAMPLE: side 10 cm, perimeter 60 cm; side 20 cm, perimeter 78 cm; side 20 cm, perimeter 1 m).

Review ratios with your students. Ask them to find the ratios of width to length and length to perimeter in some of the rectangles used in the lesson.

Ask your students to draw three different rectangles that are three times as long as they are wide. ASK: What would these rectangles be called in Geometry? (Similar rectangles). Ask your students to find the perimeters of the rectangles and the ratios of length to width, length to perimeter, and width to perimeter. Encourage your students to order the information in a T-chart. What do they notice? (The ratios are the same: length to width = 3:1, length to perimeter = 3:8, width to perimeter = 1:8.) ASK: Will the ratios for a rectangle that is not similar to these rectangles be the same? Students should check their prediction by drawing a dissimilar rectangle (EXAMPLE: a rectangle with length four times the width) and calculating the perimeter and ratios.

Assessment
The length of a minivan is 5 m. The house is a rectangle six minivans long and four minivans wide.

1. Sketch the house.
2. Mark the length of the edges (in minivans and in metres).
3. Calculate the perimeter of the house.

Extensions
1. The sides of a regular pentagon are all 5 cm long. What is the pentagon’s perimeter?
2. The perimeter of a regular hexagon is 42 cm. How can the length of its sides be determined?
3. Measure the perimeter of this star.

4. Sally wants to arrange eight square posters into a rectangle. How many different rectangles can she create? She plans to border the posters with a trim. For which arrangement would the border be least expensive? Explain how you know.

5. Estimate the perimeter of your school in toothpicks.

### ME6-20

**Investigating Perimeter**

Review the meaning of perimeter and the methods of finding it (counting the edges or adding the side lengths). Draw the following shape:

```
  
  
  
  
  
```

Ask a volunteer to find the perimeter. Then ask additional volunteers to add a square to all possible perimeter positions and identify how the perimeter measurement changes. Summarize the results in a table. (It is also good to mark congruent shapes. Do they have the same perimeter?) Why does the perimeter change the way it does? How many edges that had previously been on the outside are now inside?

**ADVANCED:** Try to guide your students toward developing a formula for the new perimeter (new perimeter = old perimeter + 4 – twice the number of sides that become inside edges).

Draw a rectangle on the board. Write a length on one of the longer sides. Ask your students what the length of the opposite side should be. Add the length of one of the shorter sides and ask your students what the length of the last side should be. What is the perimeter of the rectangle? Ask a volunteer to write the addition statement for the perimeter.

Then tell students you want to find all the rectangles with perimeter 16 cm. The only other requirement is that the lengths and widths are whole centimetres (i.e., 3 or 5, not 2.25 or 4 \( \frac{3}{4} \)). **SAY:** I’m going to start with a rectangle that has width 1 cm. That’s the smallest possible measurement I can have. Draw a rectangle and mark the width. **ASK:** If the width of the rectangle is 1 cm, what is the length? Invite the student who answers to explain how he or she came up with the answer. [If students need assistance use these **PROMPTS:** The perimeter is the distance all around the shape—if you took a rope that was 16 cm long, it would go all around the sides of our rectangle. If we know the width is 1 cm, how much of the rope have we used? (2 cm, for 2 sides)
How much of the rope is left? (14 cm) For how many sides? (2) So how long is each remaining side? (7 cm) Another way to solve the problem is to use the fact that 1 length and 1 width of the rectangle add up to half the perimeter, or 7. If $1 + \text{length} = 8$, the length must be 7.

Students can use a T-table with the headings Width and Length to systematically search for all the possible dimensions of a rectangle with perimeter 16. Ask them to start with the smallest possible side length, then go to the next possible length, and so on. Ask them how they know where to stop searching.

Draw another rectangle on the board and say that the length is $L$ and the width is 3 cm. What is the perimeter of this rectangle? What happens if the width is $W$ and the length is 4 cm? How would you express the perimeter of a rectangle with length $L$ and width $W$?

### Extensions

1. A parallelogram has one side that is 3 cm long and a perimeter of 16 cm. What are the lengths of the other sides?

2. An isosceles triangle has a perimeter of 15 units. The length of each side is a whole number. Find all possible triangles that meet these requirements. (Answer: 5, 5, 5; 7, 4, 4; 3, 6, 6; 1, 7, 7) Why can the length of the base not be an even number?

3. a) An isosceles triangle has perimeter 21 cm and all of its sides are a whole number of centimetres in length. One of its sides is 6 cm long. How long are the other sides?

   b) An isosceles triangle has perimeter 21 cm. One of its sides is 8 cm long. How long are the other sides? Give two different answers.

   c) An isosceles triangle has perimeter 21 cm. One of its sides is 5 cm long. How long are the other sides? How many answers can you find?
ME6-21
Circles and Irregular Polygons

Draw a square 10 cm by 10 cm on the board. Invite a volunteer to draw and to measure the diagonals of the square. (They will be about 14 cm long.) Ask your students to draw a square 1 cm by 1 cm in their notebooks. Ask them to predict the length of the diagonal in their squares. Then ask students to draw and to measure the diagonal. (It will be about 1.4 cm long.) Explain to your students that the length they have measured is an approximation, and they can use it to find the perimeter of the following shapes:

The horizontal and vertical distance between adjacent dots is 1 cm. The diagonal distance between dots is, as students just learned, about 1.4 cm. Ask your students to find the perimeter of the various shapes by adding together the lengths of the sides. Encourage them to use multiplication where possible, as a quicker way to add multiples of the same number. For example, in the diamond shape (second from the left), there are 4 equal sides. Instead of adding the length of each side 4 times, students could multiply the length by 4.

Now draw 3 circles—with radiuses of 15, 20, 25 cm—on the board, mark the centres, and explain that the distance around the outside of each circle is called the circumference. Write the term on the board. Then draw several horizontal lines through one of the circles such that one of them passes through the centre of the circle. Tell students that the width of the circle is measured by the longest line. **ASK:** Which line is the longest? (Another way to ask this: Suppose these circles are coins and you need to make slots, to push each coin through. Which line represents the minimal length of a slot for each “coin”?) Help students to identify the line that goes through the centre of the circle as the longest one. You may tell students that the width of a circle is also called the diameter. Now measure the diameters of the remaining circles and fill in the first column in this T-table:

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using a measuring tape (the type tailors use), string, or pieces of paper, have volunteers measure the circumference of each circle and fill in the second column in the table. Then look at the numbers and ask students if they can see a pattern. How does the circumference relate to the width?
How many times larger is the circumference than the width in each case? Students can either divide the numbers, or do the Activity, to find out. You might wish to explain that the circumference of a circle is always approximately 3 times greater than the width (or diameter). The exact number (3.14, rounded to 2 decimal places) is a very important number in mathematics and is called “pi.”

Before students try QUESTION 2 on the worksheets they could measure the diameter and circumference of several cylinders in the classroom (using string to find the circumference). Suggest that, for each of their measurements, they add the diameter repeatedly to see approximately how many diameters are needed to make the circumference. They will find that the answer is always about 3. (The ratio of the circumference to the diameter is called pi, and is about 3.14.)

Extensions

1. PROJECT: What were the ancient estimates of pi? POSSIBLE SOURCES:
   - http://www-history.mcs.st-and.ac.uk/HistTopics/Pi_through_the_ages.html
   - http://library.thinkquest.org/C0110195/history/history.html

2. Lee wants to arrange some shells around a circular flowerbed in her garden. The flowerbed is 1 m wide. Each shell is 5 cm long. About how many shells will Lee need? HINT: Use the width of the flowerbed (in centimetres or in number of shells) and the pattern from the T-table you made during the lesson to estimate the circumference.
Remind students that area is often measured in units called “centimetres squared” or cm². Show students an example of a square centimetre, that is, a square whose sides are all 1 cm long.

Draw several rectangles and other shapes (EXAMPLE: L-shape, E-shape) on the board and subdivide them into squares. Ask volunteers to count the number of squares in each shape and write the area in cm².

Then draw several more rectangles and mark their sides at regular intervals, as shown below.

Ask volunteers to divide the rectangles into squares by joining the marks using a metre stick. Ask more volunteers to calculate the area of these rectangles.

Ask students to draw their own shapes on grid paper and to find the area and perimeter for each one.
Extensions

1. Sketch the shape below (at left) on centimetre grid paper. What is its area in cm²? (16) Now calculate the area using a different unit: 2 cm × 2 cm square (see below right). What is the area in 2 cm × 2 cm squares? (4 cm²) What happens to your measurement of area when you double the length of the sides of the square you are measuring with? (The area measurement decreases by a factor of 4.)

   ![Grid Paper Shapes]

   The new unit:
   $2\,\text{cm} \times 2\,\text{cm} = 4\,\text{cm}^2$

2. If the area of a shape is 20 cm², what would its area be in 2 cm × 2 cm squares? Sketch a rectangle with area 20 cm² to check your answer.

3. Draw a rectangle that has the same area (in cm²) as it does perimeter (in cm).
   
   \[18 = 3 \times 6 \text{ (area)} = 2 \times 3 + 2 \times 6 \text{ (perimeter)}\]

4. Copy the figure onto grid paper. Draw a straight line to divide the shape into 2 parts of equal area.

5. Draw two shapes with area 15 so that one has perimeter twice as large as the other.
ME6-23
Area of Rectangles

GOALS
Students will find the area of rectangles in square units and in cm².

PRIOR KNOWLEDGE REQUIRED
Drawing lines with a ruler
Measuring sides with a ruler
Centimetres squared (cm²)
Multiplication

VOCABULARY
2-dimensional area
square centimeter
perimeter
rectangle
length
width

Draw an array of dots on the board. ASK: How many dots are in the array? Invite a volunteer to explain how he or she counted the dots. Ask another volunteer to write the corresponding multiplication statement on the board.

Now draw a rectangle on the board. (Draw it on a grid or subdivide it into squares.) Ask volunteers to write the length and width of the rectangle on the board, where length and width are measured in numbers of squares. Draw a dot in each square of the rectangle and ASK: How can a multiplication statement help us to find the area of the rectangle? Ask students to write and solve the multiplication statement for the area of the rectangle.

Draw several rectangles (again, subdivided into squares) and ask volunteers to write the length and the width and use them to calculate the area.

Draw rectangles with various lengths and widths (such as 20 cm × 30 cm, 40 cm × 35 cm, 33 cm × 55 cm) but don’t subdivide them into squares. Ask volunteers to measure the sides with a metre stick and calculate the area of the rectangles in cm².

Once students are comfortable finding the area of a rectangle by multiplying length and width, ask them to write the relationship as a formula using letters: \( A = L \times W \). You might also encourage students to develop a formula for perimeter \( P = 2 \times L + 2 \times W \).

Assessment
1. Calculate the area of the rectangle:

   \[
   \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|}
   \hline
   & & & & & & & & & & & \\
   \hline
   & & & & & & & & & & & \\
   \hline
   \end{array}
   \]

   Width: _____

   Length: ____

2. Measure the sides and calculate the area:

   Width: _____

   Length: ____
Ask students to construct rectangles of the same area, but with different lengths and widths. They can work on a geoboard or with square tiles on grid paper. How many different rectangles can they make with area 8 cm²?

Ask students to create various rectangles and record their length, width, and area. Ask partners to give each other the length and width of some of their rectangles so they can calculate the area and check each other’s work.

**Extensions**

1. Calculate the area of the figure by adding the area of the rectangles:

   ![Rectangle Diagram](image)

2. Divide the figure into rectangles and calculate the area:

   ![Rectangle Diagram](image)

3. Ask your students to draw several similar rectangles. (Similar rectangles have the same ratio of length to width. See lesson **ME6-19**.) Ask your students to order the rectangles from smallest to largest and to fill in the table:

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Area</th>
</tr>
</thead>
</table>

   Ask students to look for patterns and relationships. If the length of one rectangle is 3 times the length of another rectangle, what is the ratio of their widths? Their areas?

4. How would you change a measurement in square metres to one in square centimetres?
ME6-24
Exploring Area

**GOALS**
Students will find the area of rectangles with lengths and widths given in centimetres (cm), metres (m), and kilometres (km). Students will determine length (or width) given area and width (or length).

**PRIOR KNOWLEDGE REQUIRED**
Measuring sides with a ruler
Multiplication
Area of rectangle
Centimetres squared (cm²)

**VOCABULARY**
2-dimensional area
perimeter
rectangle
length
width
square centimetre and centimetre squared (cm²)
square kilometre and kilometre squared (km²)
square metre and metre squared (m²)

Draw a rectangle on the board and ASK: How can we calculate the area of this rectangle? Invite volunteers to help you solve the problem (one student could measure, another could write the measurements, a third could do the calculation and write the answer).

Tell students that a city block is 2 km long on every side. ASK: What shape is the block? What is its area? What units should we use for the area—is it square centimetres? Why not? If students do not infer the right answer and explanation, explain that a square kilometre is a square whose sides are 1 km long. When you multiply the length of the city block (in kilometres) by the width, you find out how many one-kilometre squares are in the block, so the area is in square kilometres, or km². ASK: What if the city block is 8 m long on every side. What is its area? What units do we use?

Then draw several rectangles on the board, write the length and width using different units of measurement, and ask volunteers to find the area.

**EXAMPLES:**

- 8 cm × 9 cm
- 3 m × 7 m
- 6 km × 10 km
- 20 cm × 7 cm
- 8 m × 7 m

ASK: Which rectangle has the greatest area? Which rectangle has the smallest area?

ASK: Which units would you use to measure the area of these objects or places—square centimetres, square metres, or square kilometres:

- Canada
- your classroom
- a book
- your city or town
- school yard
- a field
- a table

Draw another rectangle on the board and mark the length: 3 m. SAY: I know that the area of the rectangle is 6 m². How can I calculate the width of the rectangle? PROMPTS: If you knew the length and the width, how would you calculate the area? What do you have to multiply 3 (the length) by to get 6 (the area)? How do you know? What did you do to 6 to get 2?

Give several more problems of this kind. **EXAMPLES:**

- Length 4 cm, area 20 cm², find the width.
- Width 3 m, area 27 m², find the length.
- A square has area 16 km². What is its width? (What can you say about the length and the width of a square? Students can try various lengths—1 × 1, 2 × 2, and so on—until they find the answer.)

Ask students to draw a rectangle that has an area of 24 cm². How many rectangles of different proportions can they draw? Prompt them to start with a width of 1 cm, then try 2 cm, 3 cm, and so on. Does 5 cm work? Why not? What about 7 cm?
Tell your students the following story:

Once upon a time there lived a mighty King who loved feasts. He decided that his rectangular table (6 m by 8 m) was not large enough for all the dishes he liked to eat. So the King called for two carpenters and asked each of them to make him a larger table. The first carpenter made a table that was twice as long as the old table, but only half as wide. The second carpenter made a table that was twice as wide as the old table, but only half as long. But neither table pleased the King. Instead of thanking the carpenters, he threw them in jail. Why?

Ask your students to draw the three tables described in the story (6 m by 8 m, 3 m by 16 m, 12 m by 4 m). **ASK:** What do you think the King wanted when he asked for a larger table? Did he want a table with greater length, width, perimeter, or area? (He needed more room for the dishes, which means a greater area.) Why was the King angry with the carpenters? (Their tables have the same area as the old table.)

**ASK:** Do you think halving one dimension and doubling the other always creates rectangles with the same area? Invite students to draw different rectangles to confirm that this is a general rule. Encourage your students to explain why this happens geometrically. You can use the following diagram to illustrate what’s happening. (NOTE: Students will rearrange shapes like this when they learn to find the area of parallelogram.)

Assessment
1. What is the area of the rectangles?
   - A: Length 5 m, width 4 m.
   - B: Length 6 km, width 7 km.
   - C: Length 20 cm, width 15 cm.

   Order the rectangles from least to greatest.

2. A parking lot has area 800 ____. Its length is 40 ____. Fill in the appropriate units of measurement and calculate the width of the parking lot.

3. Draw 3 different rectangles that have area 20 cm².
Extensions

1. Find the area of the shape by dividing it into smaller rectangles.
   HINT: First you will have to fill in the missing side lengths.

   ![Diagram of a shape with side lengths 6 cm, 7 cm, 4 cm, and 10 cm]

2. A rectangle has perimeter 12 cm and length 4 cm. What is its area?

3. Find the area (all sides are 3 cm):

   ![Diagram of a plus-shaped figure with side lengths of 3 cm]

4. Olga is growing tulips. She plants 16 of them in every square metre of her field. How many tulips is she growing in total?

   ![Diagram of a field with dimensions 35 m x 10 m, with a shed of dimensions 4 m x 2 m]
ME6-25
Comparing Area and Perimeter

Draw several rectangles on a grid: $4 \times 6$, $5 \times 5$, $6 \times 3$, $7 \times 2$, $3 \times 8$. Label them and ask volunteers to find the area and the perimeter of each one. Ask students to list the rectangles from least to greatest by area. Then ask them to list the rectangles from least to greatest by perimeter. **ASK:** Are your lists the same? Does the rectangle with the greatest area also have the greatest perimeter? Does the rectangle with the smallest perimeter also have the smallest area? Are there rectangles with the same area? Do they have the same perimeter? Are there rectangles that have the same perimeter? Do they also have the same area?

What do you do to the length and width to calculate area? What do you do to the length and width to calculate perimeter?

**Assessment**
1. Draw 2 rectangles that have the same area—20 cm$^2$—but different perimeters. Calculate their perimeters.
2. Bob drew 2 shapes with the same perimeter but different areas. Is this possible? The sides of Bob’s shapes are whole centimetres. One shape is a square with area 9 cm$^2$. Can you draw this square? The other shape is a rectangle. Can you draw the rectangle?

**Extensions**
1. Mr. Green wants to make a rectangular flower bed with perimeter 24 m. Which dimensions of the flower bed will provide the greatest area?
2. Mr. Brown wants to make a rectangular flower bed with area 36 m$^2$. Which dimensions will give him the least perimeter?
3. Can you draw an irregular shape of area 9 cm$^2$ and perimeter more than 20 cm?
Give students several rectangles with lengths and widths that are whole centimeters and say you want to estimate both the area and the perimeter of each one. **ASK:** How can I do that? What can I use to help me? Invite students to share their suggestions and try them out. If necessary, remind students that the area of their thumbnail is approximately 1 cm². How can they use their thumbnail to estimate area and perimeter?

Have students check their estimates by measuring the length and width of the various rectangles and calculating the area and perimeter. Compare the estimates to the actual measurements.

Review with your students how they could find the missing sides of a rectangle given the length of one side and the perimeter. Then reverse the problem, and **ASK:** A rectangle has area 20 cm² and one side that is 4 cm long. What is its perimeter? **PROMPTS:** What do you need to know to find the perimeter of a rectangle? What do you know about this rectangle? (area and length of one side) How could you find the length of the missing side? How do we calculate area when we know the side lengths? So side lengths are factors, and the area is the multiple. You know the multiple and one of the factors. How can you find the other factor? What is the factor that you found? (length of missing side)

Now give your students a harder problem: A rectangle with sides that have lengths in whole centimetres (**EXAMPLE:** 3 cm, not 4.8 cm) has area 24 cm² and perimeter 22 cm. What are the dimensions of the rectangle? Students could use systematic search to find the answer. For instance, they can find all the factors of 24 and fill in the following table:

<table>
<thead>
<tr>
<th>Dimensions of Rectangle (L × W)</th>
<th>Perimeter (2 × L + 2 × W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 24</td>
<td>50</td>
</tr>
<tr>
<td>2 × 12</td>
<td>28</td>
</tr>
<tr>
<td>3 × 8</td>
<td>22</td>
</tr>
<tr>
<td>4 × 6</td>
<td>20</td>
</tr>
</tbody>
</table>

The table shows that the answer is a rectangle with side lengths 3 cm and 8 cm. Alternatively, students could search for all the side lengths that produce a perimeter of 22 cm (i.e., all pairs of numbers that add to 11) and an area of 24 cm².

Now draw a rectangle with width 2 cm and calculate the length. Repeat for a rectangle with width 3 cm. **ASK:** What is the length of the last rectangle? Do we need to continue making rectangles? Why not? (The last rectangle was 3 × 4. A rectangle with width 4 would have length 3—it would be the same rectangle, just rotated or turned on its side!) Do we need to make
rectangles with width 5 or 6? What about a rectangle with width 7? How many rectangles with perimeter 14 do we have in total? Ask students to find the area of the rectangles.

**Assessment**

1. Draw all possible rectangles with sides in whole cm with perimeter 16 cm. Which one has the greatest area? The least area?
2. The length and the width of a rectangle are in whole m. Its area is 30 m². Its perimeter is between 30 m and 40 m. What are the length and the width of the rectangle?

**Extensions**

1. Describe a situation in which you would have to measure area or perimeter, for instance, to cover a bulletin board or make a border for a picture. Make up a problem based on the situation.
2. A rectangle has area 20 cm² and length 5 cm. What is its perimeter?
3. The shape below has perimeter 24 cm. What is its area?

   ![Rectangle](image)

4. Review with your students how perimeter of a shape constructed from squares changes with the addition of each new square, depending on the position of the new square.

   Draw a 5 × 8 rectangle on a grid. Ask your students which of the following rectangles will fit inside this rectangle:

   $$2 \times 3, 5 \times 6, 10 \times 8, 10 \times 12, 6 \times 4, 4 \times 10$$

   Give your students a 15 cm × 10 cm sheet of grid paper and ask them to draw a rectangle with area 96 cm² on it. Point out that they should remain within the grid and that the side lengths should be in whole centimetres. Systematic search through factors of 96 would be helpful to solve this problem. (Actually, 8 cm × 12 cm rectangle is the only solution.) Ask the students find the perimeter of this rectangle.

   Ask your students to create inside the rectangle a shape constructed with squares that has perimeter of 40 cm. Is the area of the new shape the same as the area of the rectangle? Can they make area smaller keeping the perimeter? What is the smallest area that they can produce?

**BONUS:** How many shapes can you make with area 6 cm² and perimeter 10 cm?
Now ask your students to take the shape that has the smallest area and to add some squares to it so that the new shape has perimeter of 46 cm. The shape should stay inside the rectangle again. How many squares did they add? What is the smallest number of squares that they could add to create the shape with perimeter of 46 cm?

Enlarge the perimeter gradually. Finally ask the students whether they can create a polygon that has perimeter that is twice the perimeter of the outside rectangle (80 cm), or even 1 m.

POSSIBLE ANSWERS:

Smallest area – 19 cm²
Perimeter 40 cm

Perimeter 46 cm

Perimeter 80 cm

Perimeter 1 m
ME6-27
Area of Polygons and Irregular Shapes

GOALS
Students will find the area and perimeter of irregular shapes.

PRIOR KNOWLEDGE REQUIRED
Area, adding sequences of halves and whole numbers, comparing mixed fractions with the denominator 2, fractions of area, perimeter

VOCABULARY
area, length, perimeter, width

Draw a 1 × 2 rectangle on a grid on the board. Draw a diagonal and shade one of the two triangles formed. ASK: Which part of the rectangle is shaded? How do you know? What are triangles like the halves of this rectangle called? (congruent triangles) What is the area of the rectangle? What is the area of the shaded triangle? Repeat with rectangles 1 × 4, 1 × 6, then 1 × 3 and 2 × 3.

Draw several right-angled trapezoids and ask your students to divide them—by drawing a line—into a rectangle and a triangle. Then make right-angled trapezoids on a grid or geoboard and ask students to find the area of the trapezoids. Challenge them to find the area of a general, not-right-angled and not isosceles trapezoid, such as:

Make sure your students can add sequences of halves by grouping:

EXAMPLE:

\[
\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \frac{1}{2}
\]

3 wholes and a half left over

Also, make sure students understand that in order to find the area of the triangular parts of the figures, they simply have to view the triangle as covering half the area of a rectangle:

The triangle covers half the area of a 2 by 3 rectangle. The area of the rectangle is 2 × 3 = 6. So the area of the triangle is 3 (half of 6).

Draw a rectangle divided into two triangles on the board and tell your students that each of the triangles has area 3. What is the area of the rectangle? Ask them to draw a triangle of area 3 on grid paper.

Ask your students to draw two different rectangles of area 4. Then ask them to draw two different triangles of area 2. How many different triangles of area 6 can they produce?
Draw several squares on the board. Invite volunteers to shade half of each square. Encourage them to find as many different ways of doing this as possible, including:

Draw several shapes that include that include various types of half squares and ask students to find the area of each shape. Then ask students to draw designs for each other, and to find and to compare the shaded and unshaded areas of their designs.

**Bonus**

Have students draw their names using whole squares and different half squares, and calculate the area.

Draw a rectangle on a grid. Ask volunteers to shade half of each square in the rectangle using any design they like. **ASK:** What is the shaded area of the rectangle? Do you have to count all the half squares or there is a shortcut? If you know the total number of squares, what part of the total is shaded?

Draw a map of a lake on a grid (or use an overhead projector). Ask your students to find the area of the lake. **ASK:** Which squares contribute a whole $1\,\text{km}^2$ to the area? Which squares contribute about half a square kilometer? Which squares should not be counted at all because they contribute only a small fraction of a kilometre to the area?

![Lake on a grid](image)

Draw an irregular shape on a grid and tell students that you want to estimate the area of the shape. Enlarge some of the grid squares that are divided by the shape in unusual or irregular ways, such as:

Discuss with students how they can estimate the perimeter of the lines in these grid squares. In the two leftmost examples above, students can estimate the length of the line as 1 because it is close to being a straight line parallel to one side of the square. In the two examples in the middle, the line is about 1.5 units long. **ASK:** Why? What is the length of the diagonal if the square is 1 cm by 1 cm? If you straightened out the lines in the squares at right, they look like the might be about half the length of one side, so you could estimate their length as half. After discussing these and other estimates, ask students to estimate the perimeter of the shape you have drawn. Repeat with more irregular shapes.
Assessment
Find the area of the shapes:

Let your students create various irregular polygons on geoboards and find the area of the shapes they produce.
The worksheet ME6-28 is a review worksheet. To help your students with the bonus problems, ask them to find a pair of congruent triangles in the picture below and then determine the area of the shaded triangle:

Students might argue the following way:

1. Triangles A and C are congruent, so the shaded area is the same as the area of triangles A and B together, which is one half of a square.

2. Triangles B, C, and D form a triangle of area one square. Triangle D has area of one half of the square, so B and C together have area of half the square.

Let your students find the area of larger shapes containing triangles congruent to the shaded triangles, such as:

Students could investigate QUESTION 3 on the worksheet using a geoboard.

Ask students to make as many non-congruent shapes as they can by placing 5 squares edge to edge (these shapes are called Pentomino, and there are 12 different Pentomino altogether). Which Pentomino have the greatest perimeter? Which have the greatest number of lines of symmetry?
ME6-29
Area and Perimeter (Advanced)

Review conversion between measurements of length. Invite volunteers to draw two rectangles on the board, 20 cm × 15 cm and 40 cm × 30 cm. **ASK:** How many times longer is the larger rectangle? How many times wider it is? How many times larger than the area of the second rectangle is the area of the first rectangle? Ask your students to find the areas and calculate the difference.

Draw a 10 × 10 square on a grid. Ask your students how many small squares are in the large square. How do they know? Explain that each small square represents 1 cm. **ASK:** What is the length of the side of the large square? What does the large square represent? (a square decimetre) How many cm² are in one dm²? (100) There are 10 mm in 1 cm. How many mm² are in 1 cm²? Repeat with centimetres, decimetres, and metres.

Draw the following step diagram on the board and ask your students to add area units to the empty steps as well as arrows and multiplication and division signs (as was done earlier with a similar diagram for units of length).

```
  cm²
```

Have your students convert several area measurements, such as:

- \[ 12.34 \text{ m}² = \underline{1234} \text{ cm}² \]
- \[ 56.34 \text{ cm}² = \underline{0.5634} \text{ m}² \]
- \[ 12.78 \text{ cm}² = \underline{1278} \text{ mm}² \]
- \[ 789.34 \text{ cm}² = \underline{78.934} \text{ dm}² \]
- \[ 0.34 \text{ m}² = \underline{340} \text{ cm}² \]
- \[ 0.9834 \text{ m}² = \underline{98.34} \text{ dm}² \]

Review perimeter and the part of lesson ME6-19 about ratios before assigning the worksheet.

**Extensions**

1. Describe a situation in which you would have to measure area or perimeter (for instance, to cover a bulletin board or make a border for a picture). Make up a problem based on the situation.
2. Find the perimeter and area of this shape.

3. The perimeter of this shape is 40 cm. Find the lengths of the missing sides and the area of the shape.
ME6-30
Area of Parallelograms

GOALS
Students will find the area of parallelograms.

PRIOR KNOWLEDGE REQUIRED
Area
Perimeter
Create a straight angle
Use of ruler
Area of rectangle

VOCABULARY
area
perimeter
length
width
base
height (of a 2-D shape)

Draw several parallelograms as shown below on the board.

ASK: Can you find the length and width of these shapes? Discuss with the class how they might first identify the “length” and the “width.” Point out that the length of a side cannot be considered the length of the shape, since the shape is actually longer than the side itself. Is the longest diagonal the length of the shape? Should length be perpendicular to width? These questions, and the resulting discussion, should make it clear that parallelograms don’t have an easily identifiable length and width, as do rectangles. As a result, we use different concepts to define a parallelogram.

Have on hand a large, flexible rectangle made from geostrips or cardboard strips and brads. ASK: Do you think two parallelograms with the same side lengths have the same area? How could we find out? Take various answers, then hold up the rectangle and begin to deform it.

Point out that you aren’t changing the length of the sides, you’re just moving them and creating different parallelograms. ASK: Have we answered the question? How? (Students should see, qualitatively if not quantitatively, that parallelograms with the same side lengths can have different areas.) Emphasize that the length of adjacent sides does not define a parallelogram the way it does a rectangle.

Tell students that we define parallelograms by their “base” and “height.” Take any parallelogram above and mark the base and height. (If necessary, review with your students how to draw a line perpendicular to a given line. Students could use a triangle or a MIRA—see lesson G6-26 for instructions on using MIRA.) Point out that you can choose any side of a parallelogram to be the base (and the line opposite is also considered a base). Once a base is chosen, the distance between the bases (measured along the line perpendicular to the bases) is fixed.

Give your students several paper parallelograms and ask them to measure the sides, and to identify the base and height. Then ASK: How could you
cut your parallelogram into two pieces and rearrange them to create a rectangle? Ask students to think about this, to discuss with a partner, and to try out their ideas. Give everyone a chance to find the answer, and, if necessary, invite a student(s) to explain it to the class. Have students cut and rearrange one of their parallelograms accordingly and **ASK**: Are the sides of the rectangles the same length as the sides of the parallelogram? How many sides preserve their length after being rearranged? (2- the bases) What about the height of the parallelogram—where do you see that in the rectangle? (the height becomes the width of the rectangle)

Students will discover that if they know the height and the base of a parallelogram,

![Parallelogram Diagram](image)

\[
\begin{align*}
\text{height} &= 4 \text{ cm} \\
\text{base} &= 6 \text{ cm}
\end{align*}
\]

they can cut off a triangle at one end and attach it to the other end to form a rectangle.

![Triangle Cut](image)

**ASK**: Did the area of the parallelogram change when you cut and rearranged the pieces? How can we calculate the area of the parallelogram using what we’ve learned? (The width of the rectangle is the same as the height of the initial parallelogram, the length is the same as the length of the base, and the area did not change. The area of a rectangle is length × width. So the area of the parallelogram is found by multiplying the length of the base of the parallelogram by the height of the parallelogram, Area = base × height.)

Let your students practise finding the area of various parallelograms.

As a challenge, ask your students: I have two parallelograms; they have base 2 and height 3. Are they congruent? Are they similar? What is the same and what is different with these parallelograms? Invite volunteers to draw different parallelograms of the given dimensions on the board.

### ACTIVITY 1

Ask your students to create parallelograms with base 2 and height 2 on a geoboard. How many different parallelograms can they find? What is the same and what is different in these parallelograms?
Extensions

The next two extensions satisfy the Atlantic Curriculum expectation C3.

1. Anne wants to find the area of the parallelogram below:

She chooses the horizontal sides to be the bases. However, when she draws the height, it falls outside of the other base, so she says she cannot find the distance between the bases. And when she tries to cut off a triangle and to rearrange it, Anne does not get a rectangle:

How can Anne find the area of this parallelogram? Can you think of more than one solution?

HINTS:

a) Was the choice of base wise?

b) Recall the story of the King’s tables from ME6-24. Would cutting the parallelogram parallel to the base help?
2. a) Two parallelograms have the same height, but the base of one is three times larger than the base of the other. Draw two pairs of non-similar parallelograms that satisfy this description. What can you say about the area of these parallelograms?

b) Two parallelograms have the same base, but one has the height three times larger than the height of the other. Draw two pairs of non-similar parallelograms that satisfy this description. What can you say about the area of these parallelograms?

3. How many different parallelograms of area 12 can you create? Use dot paper to help you. Measure the sides of your parallelograms. Show your work in a table:

<table>
<thead>
<tr>
<th>Base</th>
<th>Height</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
</table>

Can you find two different parallelograms with the same base, height, area and perimeter?
Review with your students how to find the area of a right-angled triangle by arranging two congruent triangles in a rectangle. Review the previous lesson and write the formula for the area of a parallelogram (Area = base x height) on the board.

Ask your students to fold a sheet of paper in two, draw a triangle (without right angles) on it, and cut the triangle through both halves of the sheet to obtain two congruent triangles. Ask them if they can arrange these two triangles to create a rectangle. They should quickly realize that they can’t. **ASK:** Can you rearrange the triangles to make another kind of shape? What shape is it? Students should find that they can make parallelograms. **ASK:** Can you make different parallelograms with your triangles? Do these parallelograms have the same area? How could you find the area of the parallelogram? Which part, or fraction, of the parallelogram is the triangle? How could you find the area of the triangle from the area of the parallelogram? (divide by 2)

Show your students how to find the height of a triangle. Explain that since all sides in a triangle intersect, there is no sense talking about the distance between the lines, and so height is measured along the line that passes through a vertex and is perpendicular to the side opposite that vertex. Let your students practise drawing heights to various edges and measuring these heights.

Draw a triangle on a grid and ask your students to create a parallelogram, to find its area, and to use that area to find the area of the triangle. Then ask students if they can think of a rule, or formula, for the area of a triangle. Write the formula on the board: Area = (base × height) ÷ 2. (Use the Activity for further practice.)

As a final challenge, ask your students to find the unshaded area in these figures using what they know about the area of rectangles and right-angled triangles:

![Diagram of triangles](image)

**Bonus**

How many different ways can you find to solve this problem?
A Game for Two
Students will need grid paper or a geoboard. Player 1 draws a triangle. Player 2 has to draw a triangle congruent to the first one, so that both triangles together make a parallelogram. Both players could find the area of the parallelogram and deduce the area of the triangle.

Extensions

1. Copy this triangle onto grid paper and find the area of the triangle:

![Triangle on grid paper]

2. Tanya says she can use the formula for the area of a rectangle (length × width) to find a formula for the area of a triangle, even if the triangle is not a right triangle. She used this picture to do it:

![Diagram of rectangle with shaded triangle]

How would you express the area of the shaded triangle in this picture? (HINT: Which part of the rectangle is shaded? How do you know?)

3. Magic trick: Draw an 8 × 8 square on grid paper (the smaller the grid, the better the trick works). Cut it and rearrange the parts as shown:

![Rearranged parts of a square]

What is the area of the square? (64 square units) What is the length of the rectangle? What is its width? What is the area of the rectangle? (65 square units) How could that be?! (To understand where the additional square comes from, draw a 13 × 5 rectangle on grid paper. Carefully draw parts congruent to the parts of the square. What do you notice?)

4. a) Two triangles have the same height, but the base of one is three times larger than the base of the other. Draw two pairs of non-similar triangles that satisfy this description. What can you say about the area of these triangles?

b) Two triangles have the same base, but one has the height three times larger than the height of the other. Draw two pairs of non-similar triangles that satisfy this description. What can you say about the area of these triangles?

(The Atlantic Curriculum expectation C3)
ME6-32
Investigations

This worksheet is a review of perimeter and area.

ME6-33
Volume and Capacity of Rectangular Prisms

Review with your students the various units used to measure length and area. Point out that 1-dimensional objects, like strings and lines, have only 1 dimension, length, which we measure in centimetres, metres, kilometres, and so on. Objects that have area are 2-dimensional; they have length and width, and we measure the area with square units, such as m² (where the raised 2 reminds us that they are 2-dimensional). Objects that have length, width, and height are 3-dimensional, and we measure them in cubic units, such as cm³. Show your students a centimetre cube and point out that its sides are all 1 cm long. Ask your students what other measurement units for volume they know. How large are these units?

Remind your students that the third dimension in 3-D figures is called height. Identify the length, width, and height in the prism above. Then use the terms length, width, and height to label the multiplication statement that gives the volume:

\[
3 \times 2 \times 4 = 24 \text{ cm}^3
\]

Identify the height, width, and length (you can use different units for different prisms), and ask students to find the volume. SAMPLE PROBLEMS:

10 cm \times 6 cm \times 2 cm \quad 2 \text{ m} \times 3 \text{ m} \times 5 \text{ m} \quad 3 \text{ km} \times 4 \text{ km} \times 7 \text{ km}

ASK: What does the expression “width \times length” represent in this formula? What do you find when you multiply width by length? [the area of the bottom face (or the base) of the prism] Rewrite the formula as “height \times area of the base of prism.”

Remind students to include the right units in their answers.

Draw a parallelogram on the board and ask your students to write a multiplication statement to find its area. Now show your students a right prism with a parallelogram in the base (see the BLM “Right Prism with a Parallelogram Base” for a net, if necessary) and ask them what they think its volume should be. PROMPTS: What is the formula for finding the volume of rectangular right prisms? What is the base of this prism? (a parallelogram) You can use a concrete model to illustrate that prisms with a parallelogram...
in the base can be “cut” and rearranged to make rectangular prisms just as parallelograms can be turned into rectangles. Hold up a model clay prism with a parallelogram in the base, cut it perpendicularly to the base and replace the triangular prism:

Ask your students to find the volume of a prism with height 7 cm, and a parallelogram in the base that has base 5 cm and height 4 cm. Repeat with more prisms.

Now look at triangular prisms, that is, prisms with a triangle in the base. Ask students to think about how they could calculate the area of such prisms. If necessary, review finding the area of triangles. You might also show your students how two triangular prisms create a prism with a parallelogram base using two copies of the triangular prism from “Right Triangular Prism with a Scalene Base” in the BLM. Then ask your students to find the volumes of the following triangular prisms:

Capacity

Explain that the capacity of a container is how much it can hold. Write the term on the board. Explain that capacity is measured in litres (L) and millilitres (mL). ASK: Where have you seen the prefix “milli” before and what did it mean? (millimetre; one thousandth) How many millilitres are in 1 litre? In 2 litres? In 7 litres? What do you do to change litres to millilitres? (Multiply by 1 000.) Write on the board:

\[
\begin{align*}
1 \text{ metre} & = 1 000 \text{ millimetres} \\
1 \text{ litre} & = 1 000 \text{ millilitres}
\end{align*}
\]

Put out several containers (EXAMPLES: milk and juice boxes, medicine bottles, measuring cups, cans of paint, cans of pop) with capacities clearly marked on them. Invite students to help you separate the containers into 2 groups: those that can hold 1 or more litres and those that can hold less than 1 litre. Then ask students to help you order the containers by capacity, from least to greatest. The containers can also act as “capacity benchmarks” that you can keep in a class measurement box.

Write the following on the board and ask students whether they would measure the capacity of each container in millilitres or litres:

- a glass of juice
- a pot of soup
- a bowl of soup
- an aquarium
- a pail of water
- a backyard pool
Ask students to think of three more quantities that are measured in litres and three that are measured in millilitres.

Hold up a see-through measuring cup with some water in it. Drop a centicube into the cup. Ask your students if they can see how much liquid is displaced by the cube. If they have trouble seeing how much water is displaced by 1 cube, which is likely to happen unless your cup is very small, ask them how they would solve this problem. (One answer: They could drop 10 cubes in and divide the displacement by 10.) Invite volunteers to drop more centicubes into the cup and to measure the displacement. What is the capacity of 1 cm³ cube? (1 mL)

Present a small rectangular box and ask your students how they could measure its capacity. We know the capacity of 1 cm³. What is the capacity of 10 cm³? Of 20 cm³? Invite volunteers to measure the sides of the box and calculate its volume. What is the capacity of the box?

**ASK:** A cube has a capacity of 1 L. What are the dimensions of the cube? How many mL are in 1 L? How do you find the volume of the cube? (You multiply the side by itself 3 times.) Which number is multiplied by itself 3 times to get 1 000? So how long is the side of the cube? (10 cm). Do you know a specific term for this length? Write on the board:

\[ 1 \text{ dm}^3 = 1 000 \text{ cm}^3; \quad 1 \text{ L} = 1 000 \text{ mL} \]

Capacity of 1 cm³ is 1 mL.

Capacity of 1 dm³ is 1 L.

Draw a box on the board and write its dimensions: 30 cm × 40 cm × 50 cm. **ASK:** What is the capacity of the box? Let your students find the capacity in mL first, then ask them to convert it to L. Ask your students if they can solve the problem another way. (They can convert the dimensions to decimetres and get the result in litres: 3 × 4 × 5 = 60 L.)

**Assessment**

1. a) An aquarium holds two 8 L pails of water. What is its capacity? Write the capacity in litres and millilitres.
   
   b) A jar holds 500 mL of water. How many jars do you need to fill the aquarium?

2. A rectangular box has base 7 cm × 7 cm. It contains .5 L of milk. About how high is the box in centimetres? What is its exact height in millimetres?

**ACTIVITY 1**

Measure the capacity of several glasses or containers in your classroom. Estimate the capacity of the containers before you measure their capacity. Students should select and justify appropriate units to measure the capacity of a container. **NOTE:** Students will need a measuring cup and several containers for this activity.

**ACTIVITY 2**

Make two containers: 500 mL and 300 mL (you can use two empty 1 L cartons and cut them at the height of 10 cm and 6 cm respectively). Ask your students to use only these two containers, a tap, and a sink to measure 200 mL of water.

**CHALLENGING:** Measure 400 mL of water using only the same equipment.
ACTIVITY 3

Bring in some items from a grocery store. Ask students to look at the labels and sort the items into 2 groups: items where the amount of food is given as mass and items where the amount is given as capacity.

ACTIVITY 4

Collect 5 containers (cups, cans, bottles, pails) of different sizes on which capacities were covered or removed.

a) Estimate the capacity of each container.
b) Measure and record the capacity of each container.
c) Order the measurements from greatest to least.
d) Compare your measurements with your estimates.

ACTIVITY 5

Let your students find the volume of small objects like toys, coins, or apples by submerging them in water and measuring the displacement of the water, then converting the capacity of the water displaced to a volume.

Extensions

1. Find the volume of the shape:

2. Find the volume of the prisms:

   1 m × 1 km × 1 m  
   5 cm × 3 dm × 2 m  
   1 mm × 1 m × 1 km

3. JELLYBEAN JARS: Choose two straight-sided jars of similar size but different dimensions, for example, a tall, thin olive jar and a short, squat salsa jar. Fill both jars with jelly beans. Show students both jars and ASK: Which jar has a larger capacity? Why do you think that? How could we check our predictions? Discuss the students’ ideas for determining the answer, then choose one or more and try it! Here are some of the approaches you could investigate:

   a) Open the jars, dump out the contents, and count the jelly beans.

   b) Open the jars, dump out the contents of one jar, and pour the jelly beans from the other into it. Do the jelly beans fill the empty jar? Is there any empty space left over? Are there any jelly beans left in the first jar?
c) Have a student count the number of jelly beans visible through the bottom of the jar, and record the number. Have another student count the number of layers of jelly beans from the bottom to the top. Have students multiply the two numbers to estimate the total number of beans in each jar.

d) Fill a large glass container with enough water to submerge either jar. Ask your students to predict what will happen if you place the jar into the bowl. Why does the water level go up? Close both jars tightly. Submerge one jar in the water and mark the new water level with a piece of tape. Remove the first jar and point out what has happened to the water level. Place the second jar in the water and ask students to determine whether the water level is higher or lower than with the first jar. What does this tell us about the size of the jars?

4. **PROJECT:** The origins of SI and its usage throughout the world: When SI was introduced, which countries use it and which do not? Which countries are converting into SI? What are the advantages of SI?

**POSSIBLE SOURCE:**

http://lamar.colostate.edu/~hillger/#metric5

5. Daniela wants to find the volume of an apple. She puts the apple into a glass box with 600 mL of water. The box has a square base of 10 cm \( \times \) 10 cm. The water reaches a height of 9.8 cm. What is the volume of the apple?

6. A rectangular prism made of cubes has volume 10 cubic centimetres. Thirty-four faces of the cubes are on the inside of the prism and 26 are on the outside. What does the prism look like? Can you build it?

7. Valerie says that a triangular prism has volume that is half the volume of a rectangular prism of the same height. Is her statement always true, sometimes true, or never true? Draw two prisms that fit her statement and two prisms that show she is sometimes wrong. What would you change or add to make this statement always true?

8. A wealthy king has a treasure chest in the shape of a rectangular prism. He ordered his carpenters to create a larger chest for the kings treasure.

   a) The first carpenter doubled the length of the box and left the width and the height the same. The second carpenter doubled the width of the box and left the length and the height the same. The third carpenter doubled the height of the box and left the length and the width the same. Who made the largest chest for the king’s treasure?

   b) The fourth carpenter doubled the length, the width and the height of the king’s old treasure chest to create his new chest. How many times larger was the hew chest than the old one?

   c) The fifth carpenter, being jealous of the money the fourth carpenter got, decided to make a chest that will have the same volume as the chest of the fourth carpenter. He wants his chest to have the same height as the old king’s chest, but he decided that the length of his new chest will be two times more than the length of the old king’s chest. How many times wider than the old king’s chest should his chest be?

(The Atlantic Curriculum expectation C4)
**NOTE:** Mass is a measure of how much substance, or matter, is in a thing. Mass is measured in grams and kilograms. A more commonly used word for mass is weight: elevators list the maximum weight they can carry, package list the weight of their contents, and scales measure your weight. The word weight however, has another very different meaning. To a scientist, weight is a measure of the force of gravity on an object. An object’s mass is the same everywhere—on Earth, on the Moon, in space—but its weight changes according to the force of gravity. When we use the term weight in this and subsequent lessons, we use it as a synonym for mass.

Remind students that mass (which we often call weight) is measured in grams (g) and kilograms (kg). Give several examples of things that weigh about 1 gram or about 1 kilogram:

- **1 g:** a paper clip, a dime, a chocolate chip
- **1 kg:** 1 L bottle of water, a bag of 200 nickels, a squirrel.

Introduce the students to metric tonnes and explain that they are used for large masses: 1 tonne = 1 000 kg.

List several objects on the board and ask students to say which unit of measurement is most appropriate for each one—grams, kilograms, or tonnes:

- whale
- table
- napkin
- cup of tea
- workbook
- minivan

Ask students to match these masses to the objects above:

- 2 000 kg
- 50 000 kg
- 10 g
- 150 g
- 400 g
- 10 kg

Which of these weights will look better in tonnes? Have students order these objects from heaviest to lightest.

Ask students to think of 3 other objects that they would weigh in grams and 3 objects that would demand kilograms.

Review with your students multiplication and division of decimals by 1 000. Do not continue until all of your students pass the following diagnostic test:

**Multiply or divide:**

- \[1000 \times 2.3 = \quad \quad 1000 \times 34.56 = \quad \quad\]
- \[1000 \times .0056 = \quad \quad 7.006 \times 1000 = \quad \quad\]
- \[.08 \times 1000 = \quad \quad 0.008 \times 1000 = \quad \quad\]
- \[6.008 \div 1000 = \quad \quad .03 \div 1000 = \quad \quad\]
- \[80.845 \div 1000 = \quad \quad 7 \div 1000 = \quad \quad\]
- \[7 899.6 \div 1000 = \quad \quad 70.04 \div 1000 = \quad \quad\]
Ask your students what they should do to convert measurements from tonnes to kilograms:
If 1 tonne is 1 000 kg, should they multiply or divide by 1 000? (The new unit (kilogram) is 1 000 times smaller, so we need more units—multiply by 1 000.) Ask students to convert the following to kilograms: 5 tonnes, 45 tonnes, 3.4 tonnes, .9 tonnes, .08 tonnes, .007 tonnes. Then convert kilograms to tonnes, with masses such as 6 000 kg, 7 800 kg, 80 000 kg. Convert masses involving decimals, such as 800 kg, 90 kg, 25.3 kg, and so on.

Let your students solve simple problems involving tonnes, such as:

A newborn blue whale weighs about 2.7 tonnes. The calf gains about 90 kg every day during the first 7 months of its life. Does it gain more than a tonne or less than a tonne in a week? How long does it take the calf to put on a tonne? How much will a 30-day old calf weigh? About how much will a 7-month old calf weigh?

**ACTIVITY 1**

Students should use a scale to find the mass of several objects in the class, such as a (particularly heavy) textbook, a knapsack, or a box of tools or supplies. Ask your students to skip count using the rounded weight of the chosen object until they get to a thousand. For instance, if a knapsack weighs about 40 kg, skip count by 40s: 40, 80, 120, … 1 000. How many knapsacks will make a tonne?

**ESTIMATE:** Will the bags/knapsacks of all the students in your school weigh a tonne? How many JUMP workbooks weigh a tonne?

**ACTIVITY 2**

Jane wants to estimate the mass of one grain of rice. She weighs 100 grains of rice and divides the total by 100. Try to weigh 1 grain of rice. Explain why Jane uses this method above. Use Jane’s method to estimate the mass of a bean or a lentil.
## ME6 Part 2: BLM List

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Dot Paper
Grid Paper (1 cm)
Pentomino Pieces
Right Prism with a Parallelogram Base
Right Triangular Prism with a Scalene Base
PS6-5  Using Structure I: Multiplication Puzzles

Teach this lesson after: 6.2 Measurement

Goals:
Students will mentally compute the ones digit of a product of multi-digit numbers.
Students will solve multi-digit multiplication puzzles involving missing digits, where different letters stand for different digits and identical letters stand for identical digits.

Prior Knowledge Required:
Can use the guess-check-revise strategy
Can use systematic search
Can multiply 2 two-digit numbers
Can multiply a multi-digit number by a one-digit number
Can calculate the area of a rectangle given its side lengths (for Extended Problem)
Can calculate the volume of a rectangular prism given its dimensions (for Extended Problem)
Can apply the additive property of volume (for Extended Problem)
Can divide decimal tenths by whole numbers (for Extended Problem)
Can apply the distributive property

Vocabulary: area, product, thousands

Materials:
BLM Volume and Area (pp. 14–16, see Extended Problem)

Mentally determining the ones digits of products. Start with the following exercises.

Exercises: a) Multiply the number pairs.
4 × 3  14 × 3  34 × 3  24 × 3  74 × 3
b) Circle the ones digit in the answers from part a). What do you notice?
Answers: a) 12, 42, 102, 72, 222; b) the ones digit is always 2

ASK: Why do you think the ones digit is always the same? (the ones digits being multiplied are always the same; you are always multiplying 4 × 3 to get the ones digit) Write on the board:

\[
24 \times 3 = 20 \times 3 + 4 \times 3 \\
= 60 + 12 \\
= 72
\]

SAY: Adding 60 doesn’t change the ones digit, so the ones digit of 24 × 3 is the same as the ones digit of 4 × 3. You can do that with any number. You can break up the tens and ones. Multiplying the tens by the ones doesn’t contribute to the ones digit; multiplying the ones by the ones does.
Exercises: Mentally determine the ones digit of the product.
\[ 76 \times 3 \quad b) \ 87 \times 4 \quad c) \ 62 \times 9 \quad d) \ 54 \times 6 \]
Answers: a) 8, b) 8, c) 8, d) 4

SAY: You can do the same thing with multiplying multi-digit by one-digit numbers. You can break down a three-digit number into hundreds, tens, and ones. You can break down a four-digit number into thousands, hundreds, tens, and ones. The only part that contributes to the ones digit is when you multiply the ones.

Exercises: Mentally determine the ones digit of the product.
\[ 243 \times 7 \quad b) \ 182 \times 6 \quad c) \ 1435 \times 2 \quad d) \ 807\,431\,613 \times 3 \]
Answers: a) 1, b) 2, c) 0, d) 9

Write on the board:

\[
\begin{array}{c}
34 \\
\times \ 53 \\
\end{array}
\]

ASK: Without doing the full multiplication, how can you find the ones digit of the answer? (multiply 4 \times 3; the ones digit of 12 is 2, so the ones digit of the whole number is 2) Draw on the board:

\[
\begin{array}{c|c|c}
50 & & 3 \\
30 & & \\
4 & & \\
\end{array}
\]

ASK: How does this rectangle show the product 34 \times 53? (the area of the rectangle is 34 \times 53 because one side is 34 units long and the other side is 53 units long) Have volunteers write the area of each smaller rectangle in the diagram, as shown on the next page.
The whole area is $1500 + 90 + 200 + 12$. What is the only part that contributes to the ones digit? (12) SAY: The ones digit of $34 \times 53$ is the same as the ones digit of $4 \times 3$, which is easy to calculate.

Exercises: Mentally determine the ones digit of the product.

a) $27 \times 54$

b) $16 \times 32$

c) $85 \times 37$

d) $14 \times 29$

e) $243 \times 117$

f) $614 \times 516$

g) $235 \times 912$

h) $817 \times 4367$

Bonus:

i) $11 \times 22 \times 33 \times 44 \times 55$

j) $31 \times 71 \times 81 \times 21 \times 51$

k) $238 \times 342 \times 673 \times 501 \times 704$

Answers: a) 8, b) 2, c) 5, d) 6, e) 1, f) 4, g) 0, h) 9, Bonus: i) 0, j) 1, k) 2

Introduce missing digit puzzles. Write on the board:

\[ 7 \times A = B2 \]
\[ 6 \times C = 4C \]

SAY: The rule is that the same letters stand for the same digit and different letters stand for different digits. In the first puzzle, A and B stand for different digits. In the second puzzle, both Cs stand for the same digit. Pointing to the first puzzle, ASK: What number in the seven times table has ones digit 2? (42) SAY: If you don't have the times table memorized, you can skip count until you get ones digit 2: 7, 14, 21, 28, 35, 42. So, $B = 4$ in the first puzzle. ASK: What is A in the first puzzle? (6) How do you know? (7 \times 6 = 42) Pointing to the second puzzle, ASK: What numbers in the 6 times table are in the forties? (42 and 48) What is C? (8) PROMPTS: Does 6 \times 2 equal 42? (no) Does 6 \times 8 = 48? (yes) SAY: Remember that both Cs have to be the same, so both Cs stand for 8 in this case.

Exercises: Solve the puzzle.

a) $9 \times A = 4A$

b) $7 \times A = 5B$

c) $A \times A = 2A$

d) $A \times A = 4B$

Answers: a) $A = 5$; b) $A = 8, B = 6$; c) $A = 5$; d) $A = 7, B = 9$
Solving puzzles multiplying two digits by one digit. Write on the board:

\[
\begin{align*}
1A & \times 4 \\
\hline
48 & \\
\end{align*}
\]

Point to the first multiplication and ASK: When the ones were multiplied, was anything regrouped to the tens? (no) How do you know? (4 × 1 is 4; if something was regrouped, you would have had to add it to get the number of tens) Point to the second multiplication and ASK: When the ones were multiplied, was anything regrouped to the tens? (yes) How do you know? (you need to add 2 to 4 × 1 to get 6 tens) Keep these examples on the board.

**Exercises:** Was anything regrouped to the tens? If so, how many tens?

a) 3A \times 4 = 128

\[
\begin{align*}
3A & \times 4 \\
\hline
128 & \\
\end{align*}
\]

b) 3A \times 4 = 148

\[
\begin{align*}
3A & \times 4 \\
\hline
148 & \\
\end{align*}
\]

c) 7A \times 3 = 234

\[
\begin{align*}
7A & \times 3 \\
\hline
234 & \\
\end{align*}
\]

d) 7A \times 3 = 219

\[
\begin{align*}
7A & \times 3 \\
\hline
219 & \\
\end{align*}
\]

**Answers:** a) no; b) yes, 2 tens; c) yes, 2 tens; d) no

Refer students back to the examples on the board. SAY: In the first puzzle, there was no regrouping, but in the second puzzle, 2 tens were regrouped when you multiplied the ones. Write on the board:

\[
A \times 4 = 8
\]
\[
A \times 4 = 2 \text{ tens} + 8 \text{ ones} = 28
\]

SAY: In the first puzzle, A × 4 is just 8 because there is no regrouping. ASK: So, what is A? (2)

SAY: In the second puzzle, A × 4 is 28 because 2 tens were regrouped. ASK: So, what is A? (7)

**Exercises:** Solve the puzzle. Hint: Write the puzzle vertically.

a) 2A \times 6 = 126

b) 2A \times 6 = 156

c) 5A \times 3 = 171

d) 5A \times 3 = 156

**Answers:** a) A = 1, b) A = 6, c) A = 7, d) A = 2

SAY: You can also solve this type of puzzle by using long division because there is only one unknown digit. But solving this type of puzzle by writing it vertically will help you solve harder problems with more unknown digits.

**Missing tens digit with regrouping of ones.** Write on the board:

\[
\begin{align*}
A3 & \times 4 \\
\hline
252 & \\
\end{align*}
\]

SAY: I want to find A. If I do the multiplication, I start with the ones digits. ASK: What is 3 × 4? (12)
Show the regrouping on the board:

\[
\begin{array}{c}
1 \\
A3 \\
\times 4 \\
252
\end{array}
\]

SAY: By adding an extra ten, we get 25 tens. ASK: How many tens would we get without regrouping? (24) Write on the board:

\[A \times 4 = 24\]

ASK: What is A? (6) Write on the board:

\[
\begin{array}{c}
63 \\
\times 4
\end{array}
\]

Have a volunteer complete the multiplication to verify that the answer is 252.

**Exercises:** What is A × 7?

\[
\begin{array}{cccc}
& 3 & 1 & 5 \\
a) & A5 & b) & A1 & c) & A2 & d) & A8 \\
\times 7 & \times 7 & \times 7 & \times 7 \\
315 & 427 & 644 & 266
\end{array}
\]

**Answers:** a) 28, b) 42, c) 63, d) 21

SAY: You were able to determine what A × 7 is because you knew how much was regrouped when multiplying the ones. Now you will have to multiply the ones first to see how much was regrouped.

**Exercises:**

1. A7

\[
\begin{array}{c}
A7 \\
\times 6 \\
342
\end{array}
\]

a) Multiply the ones.
b) What is A × 6? Explain how you know.
c) What is A? Explain how you know.
d) Check your answer by doing the multiplication.

**Answers:** a) 42; b) 30, because 30 + 4 = 34; c) 5, because 5 × 6 = 30; d) check: 57 × 6 = 342

2. Find A. Check your answer by doing the multiplication.

\[
\begin{array}{ccccccc}
& 294 & & 260 & & 387 & & 224 & & 234 \\
a) & A8 & b) & A5 & c) & A3 & d) & A6 & e) & A8 \\
\times 3 & \times 4 & \times 9 & \times 4 & \times 3
\end{array}
\]

**Answers:** a) 9, b) 6, c) 4, d) 5, e) 7
3. Find B and then A. Check your answer by doing the multiplication.

a) \( A2 \times 7 \)  
\[ 8B \]

b) \( A5 \times 7 \)  
\[ 45B \]

c) \( A6 \times 9 \)  
\[ 32B \]

**Answers:** a) \( B = 4, A = 1 \); b) \( B = 5, A = 6 \); c) \( B = 4, A = 3 \)

**Solving multi-digit multiplication puzzles.** Write on the board:

\[
4A \\
\times B3 \\
2491
\]

**ASK:** What is the ones digit of the product? (1) Which digits in the puzzle multiply to give you ones digit 1? (\( A \times 3 \)) What does \( A \) have to be? (7)  
**Tell students that if they don’t have the three times table memorized, they can skip count through it to check for a number with 1 as the ones digit.** Write on the board:

\[
7 \times 3 = 21, \text{ so } A = 7
\]

Erase “A” in the first equation and write “7” on the board, as shown below:

\[
47 \\
\times B3 \\
2491
\]

**SAY:** Now we have to find B. We can try 1, 2, 3, and so on as B, but, instead of doing all the multiplying, let’s estimate to see which products are most likely to be close to 2491.  
**ASK:** 47 times what multiple of 10 is close to 2491? (50) **PROMPT:** 47 is close to 50, so 50 times what multiple of 10 is close to 2491? (50) **SAY:** By rounding and multiplying only multiples of 10, you are getting a good estimate. \( B = 5 \) is a good first guess and, even if it’s not right, you’ll know by multiplying whether to make the next guess higher or lower. Have a volunteer solve \( 47 \times 53 \) on the board, as shown below:

\[
47 \\
\times 53 \\
2491
\]

**SAY:** So, in the puzzle, \( A \) is 7 and \( B \) is 5.

**Exercises:** Solve the puzzle. Hint: Write the puzzle vertically.

a) \( 6A \times B7 = 6111 \)  
\[ \text{b) } A4 \times 6B = 4884 \]  
\[ \text{c) } A57 \times 3B = 17 \ 366 \]

**Answers:** a) \( A = 3, B = 9 \); b) \( A = 7, B = 6 \); c) \( A = 4, B = 8 \)

**Puzzles with more digits missing.** Write on the board:

\[
AB \times 7 = 13C
\]
ASK: How many digits are missing in this puzzle? (3) SAY: That might seem like a lot of missing digits, but if we can just find one of them, then we are down to only two missing digits. Let’s take this one step at a time. By telling us that the product is in the one hundred thirties, the puzzle clues are already telling us quite a bit. ASK: What happens if you multiply a number in the twenties by 7—what would you get? (at least 140) Write on the board:

\[ 20 \times 7 = 140 \]

ASK: Is that too high or too low? (too high) SAY: 2 is too high for A, and another rule for this type of puzzle is that no number can start with zero. ASK: What does that tell you about A? (it must be 1) SAY: You just reduced the problem to an easier one with only two unknown digits. Write on the board:

\[
\begin{array}{c}
1B \\
\times \quad 7 \\
13C \\
\end{array}
\]

SAY: \( 7 \times 1 \) is 7, but the answer says to write 13 tens. ASK: How many tens must have been regrouped from \( 7 \times B \)? (6) Continue writing on the board:

\[
\begin{array}{c}
6 \\
1B \\
\times \quad 7 \\
13C \\
\end{array} \quad B \times 7 = 6C \\
\]

SAY: B times 7 is sixty-C. ASK: What number times 7 is in the sixties? (9) So, what is C? (3) PROMPT: 9 \times 7 is sixty-what? SAY: So, A = 1, B = 9, and C = 3.

Exercises:
1. Multiply 19 \times 7. Do you get 133?
**Answer:** yes

2. Solve the puzzle. Hint: Write the puzzle vertically.
\[ a) \ AB \times 7 = 26C \quad b) \ AB \times 6 = 34C \quad c) \ AB \times 8 = 62C \quad d) \ AB \times 9 = 32C \]
**Answers:**
\[ a) \ A = 3, \ B = 8, \ C = 6; \ b) \ A = 5, \ B = 7, \ C = 2; \ c) \ A = 7, \ B = 8, \ C = 4; \ d) \ A = 3, \ B = 6, \ C = 4 \]

**Using structure to reduce the search required to solve a puzzle.** Write on the board:

\[ 4 \times AB = BBC \]

SAY: Remember the rules: the three Bs stand for the same digit, and A, B, and C all stand for different digits. Another rule for this kind of puzzle is that no number can start with zero. So AB is a two-digit number and BBC is a three-digit number. ASK: Can B equal zero? (no) Why not? (it starts the number BBC) SAY: AB is a two-digit number, so it’s less than 100.
ASK: What does that tell you about 4 times AB? (it is less than 400) What does that tell you about B? (it is 1, 2, or 3) SAY: Let's try B = 1, 2, and 3 in order. Write on the board:

B = 1

ASK: If B is 1, what is C? (4) To guide students, write on the board:

\[
\begin{array}{c}
\text{A}1 \\
\times \ 4 \\
\hline
114
\end{array}
\]

Point to the two 1s in 114 and ASK: How did I know these were 1s? (all the Bs are 1s) What does that tell us that A × 4 is? (11) SAY: To complete the multiplication, you start by multiplying the 1 and you get 1 × 4 = 4, then you multiply the tens and you get A × 4 = 11. ASK: Is there a whole number A that works here? (no) SAY: So, B = 1 doesn’t work.

**Exercise:** Try B = 2 and B = 3 in the puzzle on the board. Are there any possible values for A, B, and C?

**Solution:** If B = 2, then A2 × 4 = 22C, so C = 8 and A × 4 = 22, which again doesn’t work. If B = 3, then 4 × A3 = 33C, so C = 2 and 4 × A = 32 and so A = 8, which is the only answer because B cannot be greater than 3. So, the answer is A = 8, B = 3, C = 2.

When students finish the exercise, SAY: By using information about how big the product can be, you were able to reduce your work by a lot and check only three possibilities for B. That makes it a lot less overwhelming.

**Exercise:** Solve the puzzle: 5 × AB = BCC.

**Answer:** A = 3, B = 1, C = 5

**Problem Bank**

1. Fill in the blank.
   a) \((63 \times 2) + (63 \times 6) = 63 \times ____\)
   b) \((63 \times 41) + (41 \times 2) = 41 \times ____\)
   c) \((3 \times 5 \times 7 \times 9 \times 11) = 5 \times 7 \times 9 \times 11 \times ____\)
   d) \((579 \times 853) - (579 \times 852) = ____\)
   e) \((2 \times 7) + (2 \times 6) = (2 \times 10) + (2 \times ____\)
   f) \((2 \times 3 \times 8) + (2 \times 3 \times 5) = (2 \times 3 \times 10) + (2 \times 3 \times ____\)
   g) \((82 \times 41) + (82 \times 3) + (3 \times 39) + (3 \times 5) = 85 \times ____\)

**Answers:** a) 8, b) 65, c) 3, d) 579, e) 3, f) 3, g) 44

2. Evaluate.
   a) \((47 \times 8) + (8 \times 27) + (26 \times 8)\)
   b) \((35 \times 3) + (17 \times 3) - (3 \times 19)\)
   c) \((13 \times 19) + (25 \times 13) - (14 \times 13) - (29 \times 13)\)
   d) \((172 \times 27) - (27 \times 135) - 27 \times 26\)
Solutions: a) \((47 + 27 + 26) \times 8 = 800\), b) \((35 + 17 − 19) \times 3 = 99\), c) \((19 + 25 − 14 − 29) \times 13 = 13\), d) \((172 − 135 − 26) \times 27 = 297\)

3. The key with digit 5 on your calculator isn’t working. What could you press to find …
a) \(315 + 64\)  
b) \(351 + 64\)  
c) \(34 \times 15\)  
d) \(52 \times 8\)

Sample answers: a) \(310 + 69\), b) \(300 + 110 + 1 + 4\), c) \(34 \times 14 + 34\), d) \(42 \times 8 + 10 \times 8\)

4. Fill in the blanks.
a) \(2700 = 27 \times _____\), so \(2727 = 27 \times _____\)
b) \(272 700 = 27 \times _____\), so \(272 727 = 27 \times _____\)
c) \(27 272 727 = 27 \times _____\)
d) \(534 000 = 534 \times _____\), so \(534 534 = 534 \times _____\)
e) \(277 277 277 = 277 \times _____\)

Answers: a) 100, 101; b) 10 100, 10 101; c) 1 010 101; d) 1000, 1001; e) 1 001 001

5. a) \(805 805 = 1001 \times _____\)
b) Use \(7 \times 11 \times 13 = 1001\). Which of 7, 11, and 13 are factors of the given number?
   i) \(805 812\)    ii) \(805 805\)    iii) \(805 850\)
   iv) \(805 818\)    v) \(805 882\)

Answers: a) 805; b) i) 7 only; ii) 7, 11, and 13; iii) none; iv) 13 only; v) 7 and 11

6. Find the ones digit.
a) \(11 \times 21 \times 31 \times 41 \times 51 \times 61 \times 71 \times 81 \times 91\)
b) \(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\)

Answers: a) 1, b) 4

7. Find the ones digit of the sum of the numbers.
a) The numbers from 1 to 10.
b) The numbers from 1 to 100.
c) The numbers from 1 to 1000.
d) The numbers from 1 to 10 000.
e) The numbers from 1 to 1 000 000.

Answers: a) 5, b) 0, c) 0, d) 0, e) 0

8. A number is called a perfect square if you can write it as a product of a whole number times itself. The first five perfect squares are:

\[
1 \times 1 = 1 \quad 2 \times 2 = 4 \quad 3 \times 3 = 9 \quad 4 \times 4 = 16 \quad 5 \times 5 = 25
\]

What can be the ones digit of a perfect square?

Answers: 0, 1, 4, 5, 6, 9

9. Is there a whole number \(N\) with \(N \times N = 12 347\)? Decide using two methods. Which method is quicker?
a) Use systematic search.
b) Use the possible ones digit of a number times itself.
Answers: a) 12 347 is between $111 \times 111$ and $112 \times 112$, so there is no such $N$; b) no, because 7 cannot be the ones digit of a perfect square; using the possible ones digit of a perfect square was quicker

10. Is there a number $N$ where $N \times N = 342\,816\,517$? Explain how you know.
Answer: no, because 7 cannot be the ones digit of a perfect square

11. Draw an area model for $547 \times 613$ and use it to explain why its ones digit is the same as the ones digit for $7 \times 3$.
Answer: In the diagram below, the only region where the area is not a multiple of 10 is $7 \times 3$, so $7 \times 3$ provides the only contribution to the ones digit.

12. Check that $24 \times 63$ has the property in which reversing both numbers gets the same answer (i.e., $42 \times 36$ equals $24 \times 63$). Draw area models for both multiplications and compare them to explain why this is true. Then find more pairs of numbers that have the same property.
Solution: The four products in each case are:

- $24 \times 63$  
  - $20 \times 60$  
  - $4 \times 60$
- $42 \times 36$  
  - $40 \times 30$  
  - $4 \times 6$

The reason these products are equal is because $2 \times 6 = 4 \times 3$. Thus, this will also work for pairs such as: $64 \times 23$ because $6 \times 2 = 4 \times 3$, $13 \times 93$ because $1 \times 9 = 3 \times 3$, $26 \times 93$ because $2 \times 9 = 6 \times 3$, $23 \times 96$ because $2 \times 9 = 3 \times 6$, $14 \times 82$, $12 \times 84$, $12 \times 42$, $48 \times 63$, $43 \times 68$

13. a) Solve the puzzle.
   i) $9 \times B = AB$  
   ii) $9 \times A = BA$
   b) How are the puzzles the same? How are they different?
Answers: a) i) $A = 4$, $B = 5$; ii) $A = 5$, $B = 4$; b) they are the same puzzle but with A and B switched

14. Solve the puzzle. Hint: Write the puzzle vertically.
   a) $A7 \times 2B = 1482$
   b) $A8 \times 4B = 1786$
Bonus: $A34 \times 2B = 12\,586$
Answers: a) $A = 5$, $B = 6$; b) $A = 3$, $B = 7$; Bonus: $A = 4$, $B = 9$
15. Solve the puzzle \( A7 \times A2 = 4154 \).

**Answer:** \( A = 6 \)

**NOTE:** Solving some of the problems below will be easier when the problem is written vertically. Allow students to struggle before providing any hints.

16. Solve the puzzle.
   a) \( AAA \times 7 = 6216 \)  
   b) \( BAA \times 7 = 6916 \)  
   c) \( AAB \times 7 = 4655 \)  
   d) \( BAB \times 7 = 5159 \)
   e) \( BAB \times 9 = 5814 \)  
   f) \( AAA \times 6 = 4662 \)  
   g) \( BAA \times 3 = 2631 \)  
   h) \( AAB \times 5 = 2245 \)

**Answers:** a) \( A = 8 \); b) \( A = 8, B = 9 \); c) \( A = 6, B = 5 \); d) \( A = 3, B = 7 \); e) \( A = 4, B = 6 \); f) \( A = 7 \); g) \( A = 7, B = 8 \); h) \( A = 4, B = 9 \)

17. a) When Tasha multiplies 2 one-digit numbers, the answer has the ones digit 3. What might the two numbers be? List all possible answers.
   b) Solve the puzzle \( 6A \times 5B = 3933 \)
   c) A two-digit number \( AB \) is multiplied by its reverse \( BA \), with \( A < B \). The product is a four-digit number with ones digit 3. What are \( A \) and \( B \)?

**Answers:** a) 1 and 3, 7 and 9; b) \( A = 9 \) and \( B = 7 \); c) \( A = 7 \) and \( B = 9 \)

18. Solve the puzzle \( AB \times 5B = 4399 \).

**Solution:** Looking at the ones digit (9), \( B \) is either 3 or 7 because \( B \times B \) gives an answer with the ones digit 9. Check the two cases: \( A3 \times 53 = 4399 \) and \( A7 \times 57 = 4399 \). Now, \( A7 \times 57 = 4399 \) doesn’t have an answer because \( 87 \times 57 \) is too high (4959) and \( 77 \times 57 \) is too low (4389). If we check \( A3 \times 53 = 4399 \) with \( A = 8 \), we get \( 83 \times 53 = 4399 \), which is correct.

19. \( AB \) and \( BA \) are both two-digit numbers, so that neither \( A \) nor \( B \) is 0, and \( 5 \times AB = 6 \times BA \).
   a) Explain how you know that \( A \) must be 5.
   b) Explain how you know that \( B \) must be even.
   c) Use the information from parts a) and b) to solve the puzzle.

**Solutions:**
   a) Because \( 5 \times AB = 6 \times BA \), then \( 6 \times BA \) is a multiple of 5. It is even, so it is a multiple of 10. So, \( 6 \times BA \) has ones digit 0 and \( 6 \times A \) has ones digit 0. But \( A \) isn’t 0, so \( A \) is 5.
   b) \( 5 \times AB = 6 \times BA \), but \( 6 \times BA \) is even, so \( AB \) has to be even for it to be multiplied by 5 and come to an even number, so its ones digit \( B \) is even.
   c) We know \( A = 5 \) and \( B \) is even and not 0, so \( AB \) is 52, 54, 56, or 58. Trying each in turn, we find that \( 5 \times 54 = 6 \times 45 \) works, so \( A = 5 \) and \( B = 4 \).

20. Solve the puzzle. Hint: You need to solve an addition puzzle before you solve the multiplication puzzle.

```
  A B C
×    D 3
  6 5 4
C E A 0
  9 3 7 4
```

**Answers:** \( A = 2, B = 1, C = 8, D = 4, E = 7 \)
21. A four-digit number ABCD has all different digits. When it is multiplied by 9, the answer is the reverse, also a four-digit number: DCBA. What is the original four-digit number? Hint: Write the multiplication vertically and determine one digit at a time. Use the fact that the answer to multiplying a number by 9 is not a five-digit number.

**Solution:** A must be 1 because a number in the two thousands multiplied by 9 would be at least 18,000, which has five digits. But then D × 9 has ones digit 1, so D must be 9. So far, we have:

\[
\begin{align*}
\text{8} & \quad \text{1BC9} \\
\times & \quad \text{9} \\
\hline
\text{9CB1} & 
\end{align*}
\]

B must be 0 or 1 because 2 or greater would carry over to the thousands, but there is no regrouping, and B isn’t 1 because A is 1, so B is 0. Then, since 9 × C + 8 has ones digit 0, then 9 × C has ones digit 2, and that makes C = 8. Check: 1089 × 9 = 9801.
Extended Problem: Volume and Area

Materials:
BLM Volume and Area (pp. 14–16)

Extended Problem: Volume and Area. Give students BLM Volume and Area. In this Extended Problem, students calculate the dimensions of a storage unit, including the height, the area of the rectangular sides, and the volume. Students use a given painter’s rate to determine the cost of having the storage unit painted.

Answers: 1. a) 8 m, b) 41.6 m², c) 291.2 m³; 2. a) 41.6 m³, b) 20.8 m³; 3. 312 m³; 4. 184.8 m²; 5. $974.00; Bonus: Jen is right because a perfect square cannot have ones digit 3.
Volume and Area (1)

A storage unit has a rectangular base and a slant roof with dimensions shown.

1. a) What is the height of the highest part of the storage unit roof from the ground?

b) Find the area of the base of the storage unit.

c) Find the volume of the rectangular prism part of the storage unit.

---

[Diagram of a storage unit with dimensions: 5.2 m x 7 m x 8 m, 1 m at the top]
Volume and Area (2)

2. To get the volume of the top part of the storage unit, you can find half of the volume of the rectangular prism below.

\[ \text{Volume of rectangular prism} = 5.2 \times 8 \times 1 \]

a) What is the volume of the rectangular prism shown? 

b) What is the volume of the top part of the storage unit?

3. Find the total volume of the storage unit.

4. Find the total area of the four side faces of the rectangular prism part of the storage unit.
Volume and Area (3)

5. A painter charges $5.00 per square metre, plus $50.00 for paint. How much will it cost to have all four side faces of the rectangular prism part painted, not including the slant roof?

BONUS ▶ Rick said he built a storage unit that had a square base with the length of each side a whole number of centimetres. He said the area of the base was 45 293 cm². Jen replied, “That's not possible!” Who is right? Explain how you know.
PS6-6 Using Structure II: Division Puzzles

Teach this lesson after: 6.2 Measurement

Goals:
Students will solve problems involving patterns in division.
Students will solve division puzzles in which different letters stand for different digits and identical letters stand for identical digits.

Prior Knowledge Required:
Can divide with remainders
Can recognize repeating patterns
Understands multiplication as skip counting
Can use long division to divide three-digit numbers by one-digit numbers
Can use long division to divide by two-digit numbers (for Problem Banks 11–13)
Can mentally calculate the ones digit of a multi-digit multiplication

Vocabulary: divisor, multiple, quotient, remainder

Noticing patterns in division. Write on the board:

30 ÷ 6 = 5 R 0
31 ÷ 6 = 5 R 1
32 ÷ 6 = 5 R 2
33 ÷ 6 = 5 R 3
34 ÷ 6 = 5 R 4
35 ÷ 6 = 5 R 5
36 ÷ 6 = 6 R 0
37 ÷ 6 = 6 R 1
38 ÷ 6 = 6 R 2
39 ÷ 6 = 6 R 3
40 ÷ 6 = 6 R 4

ASK: What patterns do you notice? (the quotient repeats for six numbers and then increases by 1; the remainders increase from 0 to 5 and then repeat) How can you tell from the remainder if the quotient will increase? (when the remainder is 5) SAY: If you know the quotient and remainder when dividing one number by 6, you can determine the quotient and remainder when dividing the next number by 6. Keep these divisions on the board throughout the lesson. Write on the board:

73 ÷ 6 = 12 R 1

ASK: What is 74 ÷ 6? (12 R 2) How did you know the quotient would stay the same? (the remainder isn’t 5) SAY: When the quotient stays the same, the remainder increases by 1.
Write on the board:

\[ 77 \div 6 = 12 R 5 \]

ASK: What is \( 78 \div 6 \)? (13 R 0) How did you know the quotient would increase? (the remainder is 5, and you can’t have remainder 6 when dividing by 6) SAY: Imagine 12 groups of 6 with 5 left over. Adding one more object would make 6 left over, but then that’s another group of 6, so really you have 13 groups of 6 and 0 left over.

**Exercises:** Find the next quotient and remainder.

a) \( 92 \div 6 = 15 R 2 \). What is \( 93 \div 6 \)?

b) \( 83 \div 6 = 13 R 5 \). What is \( 84 \div 6 \)?

c) \( 2534 \div 6 = 422 R 2 \). What is \( 2535 \div 6 \)?

d) \( 89418 \div 6 = 14903 R 0 \). What is \( 89419 \div 6 \)?

e) \( 724253 \div 6 = 120708 R 5 \). What is \( 724254 \div 6 \)?

**Answers:** a) 15 R 3, b) 14 R 0, c) 422 R 3, d) 14903 R 1, e) 210709 R 0

SAY: You can do the same thing with dividing by any number. If you want to find the next quotient and remainder, you have to add 1 to the leftover part unless doing that will make another group. Then you add 1 to the quotient and the remainder starts over at 0.

**Exercises:** Find the next quotient and remainder. Check one of your answers with long division.

a) \( 85 \div 3 = 28 R 1 \). What is \( 86 \div 3 \)?

b) \( 341 \div 9 = 37 R 8 \). What is \( 342 \div 9 \)?

c) \( 344 \div 15 = 22 R 14 \). What is \( 345 \div 15 \)?

d) \( 39560 \div 54 = 732 R 32 \). What is \( 39561 \div 54 \)?

**Answers:** a) 28 R 2, b) 38 R 0, c) 23 R 0, d) 732 R 33

**Finding shortcuts to solve division problems.** Write on the board:

\[ 343 \div 6 = 57 R 1 \]. What is \( 353 \div 6 \)?

SAY: Suppose you calculated \( 343 \div 6 \), but you meant to calculate \( 353 \div 6 \). You could complete the long division again, but I think there might be a shorter way. ASK: How much more is \( 353 \) than \( 343 \)? (it is 10 more) How do you know? (its tens digit is 1 greater) How many groups of 6 do you need to add? (1) How many do you need to add to the leftover part? (4) SAY: Right now, there are 57 groups of 6 and 1 left over. After adding 10, there will be 58 groups of 6 and 5 left over. Write on the board:

\[ 353 \div 6 = 58 R 5 \]

**Exercises:** Use the given division to solve the other division.

a) \( 742 \div 9 = 82 R 4 \). What is \( 752 \div 9 \)?

b) \( 344 \div 7 = 49 R 1 \). What is \( 354 \div 7 \)?

**Bonus:** \( 384269 \div 4 = 96067 R 1 \). What is \( 384279 \div 4 \)?

**Answers:** a) 83 R 5, b) 50 R 4, Bonus: 96069 R 3
Write on the board:

\[ 287 \div 6 = 47 \text{ R } 5. \text{ What is } 297 \div 6? \]

SAY: This looks a lot like the other problems we just did, but it’s a little different. Give students some time to try the problem and figure out what is different about it. (you can still add 1 group of 6 and 4 to the leftover part, but when you do, you get 48 R 9, but there can’t be 9 left over)

Write on the board:

\[ 287 \div 6 = 47 \text{ R } 5 \]
\[ 297 \div 6 = 48 \text{ R } 9 \]

Pointing to 48, SAY: I added one more group of 6. Pointing to 9, SAY: I added 4 more to the leftover part. ASK: Can this be right? (no) Why not? (the remainder can’t be 9 when dividing by 6) SAY: Having 9 left over means you can make another group of 6 and only have 3 left over.

Write on the board:

\[ 297 \div 6 = 49 \text{ R } 3 \]

Pointing to 48 R 9, SAY: It’s helpful to write the middle division equation even though it’s wrong because it helps you to find the correct division equation.

**Exercises:** Use the given division to solve the other division.

a) \( 516 \div 7 = 73 \text{ R } 5. \text{ What is } 526 \div 7? \)
b) \( 638 \div 9 = 70 \text{ R } 8. \text{ What is } 648 \div 9? \)
c) \( 795 \div 4 = 198 \text{ R } 3. \text{ What is } 805 \div 4? \)

**Bonus:** \( 237 \div 7 = 33 \text{ R } 6. \text{ What is } 267 \div 7? \)

**Selected solutions:** a) add 1 group of 7 and 3 to the leftover part to get 74 R 8, but now I can make another group of 7 from the leftover part, so that becomes 75 R 1; Bonus: add 30, so add 4 groups of 7 and 2 to the leftover part, so that becomes 37 R 8, which then becomes 38 R 1

**Answers:** b) 72 R 0, c) 201 R 1

**Adding a multiple of the divisor doesn’t change the remainder.** Write on the board:

\[ 287 \div 6 = 47 \text{ R } 5. \text{ What is } 887 \div 6? \]

ASK: How much more am I adding to 287 to get 887? (600) So, how many groups of 6 do I need to add? (100) SAY: I just need to add 100 groups of 6; I don’t need to add anything to the leftover part. So, the answer is just 100 more with the same remainder. Write on the board:

\[ 887 \div 6 = 147 \text{ R } 5 \]

SAY: When you add a multiple of 6, the remainder when dividing by 6 doesn’t change; only the number of groups changes.
Exercises: Use the given division to solve the other division. Check one of your answers with long division.

a) \( 345 \div 6 = 57 \text{ R } 3 \). What is \( 945 \div 6 \)?

b) \( 214 \div 6 = 35 \text{ R } 4 \). What is \( 274 \div 6 \)?

c) \( 9541 \div 6 = 1590 \text{ R } 1 \). What is \( 15541 \div 6 \)?

Answers: a) \( 157 \text{ R } 3 \), b) \( 45 \text{ R } 4 \), c) \( 2590 \text{ R } 1 \)

SAY: In general, adding a multiple of the number you’re dividing by doesn’t change the remainder. That’s because you’re only adding full groups, not anything to the leftover part.

Exercises: Use the given division to solve the other division.

a) \( 3458 \div 9 = 384 \text{ R } 2 \). What is \( 4358 \div 9 \)?

b) \( 2653 \div 3 = 884 \text{ R } 1 \). What is \( 2683 \div 3 \)?

Answers: a) \( 484 \text{ R } 2 \), b) \( 894 \text{ R } 1 \)

Finding the next multiple of a number. ASK: If you know one multiple of 6, how can you get the next multiple of 6? (add 6 to the number) SAY: Remember that the multiples of 6 are the numbers you say when skip counting by 6: 0, 6, 12, 18, 24, and so on.

Exercises: a) 276 is a multiple of 6. What is the next multiple of 6?

b) 9276 is a multiple of 6. What is the next multiple of 6?

c) 429276 is a multiple of 6. What is the next multiple of 6?

Answers: a) 282, b) 9282, c) 429282

ASK: How can you tell whether or not a number is a multiple of 6 by using division? (multiples of 6 are numbers that have remainder 0 when you divide them by 6) Write on the board:

\[ 782 \div 6 = 130 \text{ R } 2 \]

SAY: I want to use this division to find the next multiple of 6. ASK: How far apart are the multiples of 6? (6 apart) SAY: The multiples of 6 are 6 apart: 0, 6, 12, 18, 24, and so on. Draw on the board:

\[
\text{Remainder: } 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 0
\]

\[
\text{one multiple of 6} \quad 782 \quad \text{the next multiple of 6}
\]

ASK: How did I know where 782 was, relative to a multiple of 6? (you used the remainder) SAY: Because the remainder is 2, it is 2 more than a multiple of 6. Write on the board:

\[ 782 = 130 \times 6 + 2 \]
ASK: How much do I need to add to 782 to get the next multiple of 6? (4) Show this on the number line:

\[ \text{782} \]

SAY: I know the multiples of 6 are 6 apart and 782 is two more than one multiple of 6, so it is four less than the next multiple of 6. ASK: So, what is the next multiple of 6? (786) Write on the board:

\[ 782 + 4 = 786 \]

**Exercises:** Use the given division to answer the question.

a) \(345 ÷ 6 = 57 \text{ R } 3\). What is the next multiple of 6 after 345?
b) \(345 ÷ 7 = 49 \text{ R } 2\). What is the next multiple of 7 after 345?
c) \(345 ÷ 8 = 43 \text{ R } 1\). What is the next multiple of 8 after 345?
d) \(345 ÷ 9 = 38 \text{ R } 3\). What is the next multiple of 9 after 345?

**Answers:** a) 348, b) 350, c) 352, d) 351

**Introduce division puzzles.** Write on the board:

\[ \text{92 A} \]

SAY: Some multiple of 7 is in the one hundred thirties, and I want to know which number in the one hundred thirties that is. Brainstorm and have students express their ideas. Then suggest trying to search systematically, and write on the board:

\[ 7 \times 1 = 7 \quad 7 \times 2 = 14 \quad 7 \times 3 = 21 \]

Tell students that you want the remainder to be 0, so the number being divided is a multiple of 9 and starts with 2. ASK: What number must the unknown digit be? (7) SAY: If you have the times tables memorized, you can answer this type of question quickly. If you don’t, you will need to look at the times table charts to find the answers.

**Exercises:** Find the unknown digit so there is no remainder.

a) \(9\overline{4A}\)  b) \(8\overline{5B}\)  c) \(7\overline{6C}\)  d) \(6\overline{2D}\)  e) \(9\overline{7E}\)  f) \(8\overline{6F}\)

**Answers:** a) A = 5, b) B = 6, c) C = 3, d) D = 4, e) E = 2, f) F = 4
SAY: This might take a while. ASK: How can I make the search quicker? (skip some numbers)
SAY: Let's skip count by 10. Write on the board:

\[ 7 \times 10 = \_ \quad 7 \times 20 = \_ \quad 7 \times 30 = \_ \]

Have volunteers fill in the blanks. SAY: We are looking for something in between \( 7 \times 10 \) and \( 7 \times 20 \). ASK: Is one hundred thirty-something closer to \( 7 \times 10 \) or \( 7 \times 20 \)? \( 7 \times 20 \) SAY: one hundred thirty-something is really close to one hundred forty, so let's try multiplying 7 by 19.

Have a volunteer solve the equation. \( 133 \) ASK: Is that in the one hundred thirties? (yes) SAY: So, A is 3. Tell students that there is another way to do this type of problem too, based on what they have been doing in this class. SAY: Let's suppose A was 0 and then complete the division, check what the remainder is, and decide what we have to add in order to get the next multiple. Write on the board:

\[
\begin{array}{c}
7 \overline{) 130} \\
\end{array}
\]

Have a volunteer solve the division. \( 18 \ R 4 \) SAY: 130 is 4 more than a multiple of 7.

ASK: What do we have to add to get the next multiple of 7? (3) Show this on a number line:

\[ \ldots \times 130 \ldots \]

SAY: The endpoints show two multiples of 7. I know where 130 is because it has remainder 4, so the next multiple of 7 is three more than 130, so 7 divides evenly into 133, and A is 3.

**Exercises:** Find the unknown digit so there is no remainder.

a) \( 7 \overline{) 66\_} \)  

b) \( 8 \overline{) 42\_} \)  

c) \( 6 \overline{) 65\_} \)  

d) \( 9 \overline{) 75\_} \)

**Answers:** a) D = 5, b) A = 4, c) M = 4, d) X = 6

**Division puzzles with an unknown middle digit.** Write on the board:

\[
\begin{array}{c}
9 \overline{) 8A2} \\
\end{array}
\]

Tell students that you want to find a middle digit so there is no remainder. ASK: What makes starting the long division harder than before? (we don’t know what we are dividing 9 into)

SAY: We don’t know what we are dividing 9 into, but we do know that we are dividing it into eighty-something. ASK: What are the possible answers to eighty-something divided by 9? (8 or 9) Prompt students by writing on the board:

\[
\begin{array}{cccc}
9 \overline{) 80} & 9 \overline{) 81} & 9 \overline{) 82} & \ldots & 9 \overline{) 89} \\
\end{array}
\]
Have volunteers fill in the quotients, as shown below:

\[
\begin{array}{cccc}
8 & 9 & 9 & 9 \\
\hline
9)80 & 9)81 & 9)82 & \ldots & 9)89
\end{array}
\]

SAY: There are two possible quotients. We will need to check them both. Write on the board:

\[
\begin{array}{ccc}
8 & 9 \\
\hline
9)82 & 9)82 \\
72 & 72 \\
72 & 72
\end{array}
\]

SAY: Let's work backwards. If 9 divides into something-two, what must the something be? (70) Erase both question marks and write “7” in their places, as shown below:

\[
\begin{array}{ccc}
8 & 9 \\
\hline
9)82 & 9)82 \\
72 & 72 \\
72 & 72
\end{array}
\]

Point to the first long division and SAY: If something minus 72 is 7, what is the something? (79) SAY: But our something starts with 8, so that's not possible. Draw a big X through the first example. If something minus 81 is 7, what is the something? (88) Write “8” as the middle digit. SAY: So, the middle digit is 8. Have a volunteer check by dividing 9 into 882.

### Exercises:

1. The square represents a single digit. What are the possible quotients?
   a) 84□
   b) 72□
   c) 65□
   d) 79□
   e) 37□

   **Answers:** a) 5, 6; b) 2, 3, 4; c) 8, 9; d) 12, 13, 14; e) 23, 24, 25, 26

2. Find the missing digit so there is no remainder.
   a) 75□6
   b) 93□4
   c) 78□8

   **Bonus:** Find all possible missing digits so there is no remainder.
   d) 83□8
   e) 74□6
   f) 87□2

   **Answers:** a) 4, b) 2, c) 6, Bonus: d) 2 and 6, e) 0 and 7, f) 1, 5, and 9

### Problem Bank

1. Use 98 ÷ 6 = 16 R 2 to solve the division.
   a) 99 ÷ 6
   b) 104 ÷ 6
   c) 158 ÷ 6
   d) 698 ÷ 6

   **Answers:** a) 16 R 3, b) 17 R 2, c) 26 R 2, d) 116 R 2
2. Use the given division to do the other division.
   a) \(9823 \div 7 = 1403 \text{ R } 2\). What is \(9822 \div 7\)?
   b) \(1071 \div 7 = 153 \text{ R } 0\). What is \(1070 \div 7\)?
   c) \(3425 \div 7 = 489 \text{ R } 2\). What is \(3415 \div 7\)?
   d) \(3425 \div 7 = 489 \text{ R } 2\). What is \(2025 \div 7\)?
   \textbf{Answers:} a) \(1403 \text{ R } 1\), b) \(152 \text{ R } 6\), c) \(487 \text{ R } 6\), d) \(289 \text{ R } 2\)

3. Lela writes:
   \(30 \div 7 = 4 \text{ R } 2\)
   \(60 \div 7 = 8 \text{ R } 4\)
   \(90 \div 7 = 12 \text{ R } 6\)
   \(120 \div 7 = 16 \text{ R } 8 = 17 \text{ R } 1\)
   a) Explain Lela’s thinking.
   b) Write the next four terms in Lela’s pattern.
   \textbf{Answers:} a) Lela is adding 4 groups of 7 and 2 to the leftover pile each time; at the last step, she has 8 in the leftover group, so she makes another group of 7, leaving only 1 in the leftover group; b) \(150 \div 7 = 21 \text{ R } 3\), \(180 \div 7 = 25 \text{ R } 5\), \(210 \div 7 = 29 \text{ R } 7 = 30 \text{ R } 0\), \(240 \div 7 = 34 \text{ R } 2\)

4. \(45 \div 7 = 6 \text{ R } 3\). What is \(180 \div 7\)?
   \textbf{Answer:} \(24 \text{ R } 12 = 25 \text{ R } 5\)

5. \(40 \div 6 = 6 \text{ R } 4\). What is \(200 \div 6\)?
   \textbf{Answer:} \(30 \text{ R } 20 = 33 \text{ R } 2\)

6. \(90 \div 8 = 11 \text{ R } 2\). What is \(360 \div 8\)?
   \textbf{Answer:} \(44 \text{ R } 8 = 45 \text{ R } 0\)

7. \(11 \div 8 = 1 \text{ R } 3\). What is \(110 \div 8\)?
   \textbf{Answer:} \(10 \text{ R } 30 = 13 \text{ R } 6\)

8. a) Use \(10 \div 9 = 1 \text{ R } 1\) to find \(40 \div 9\).
   b) Use \(10 \div 9 = 1 \text{ R } 1\) to find \(100 \div 9\).
   c) Use your answer to \(100 \div 9\) to find \(700 \div 9\).
   d) Use your answers to \(700 \div 9\) and \(40 \div 9\) to find \(740 \div 9\).
   e) Use your answer to \(740 \div 9\) to find \(742 \div 9\).
   f) Check your answer to part e) by using long division.
   \textbf{Answers:} a) \(4 \text{ R } 4\), b) \(10 \text{ R } 10 = 11 \text{ R } 1\), c) \(77 \text{ R } 7\), d) \(81 \text{ R } 11 = 82 \text{ R } 2\), e) \(82 \text{ R } 4\)

9. a) Fill in the blanks with a whole number when you can. Explain why you cannot in the other cases.
   \(____ \times 1 + 6 = 30\) \(____ \times 2 + 6 = 30\) \(____ \times 3 + 6 = 30\)
   \(____ \times 4 + 6 = 30\) \(____ \times 5 + 6 = 30\) \(____ \times 6 + 6 = 30\)
   b) Fill in the blanks with a whole number when you can. Explain why you cannot in the other cases. Hint: Make sure the remainder is less than the divisor.
   \(30 + ____ = 1 \text{ R } 6\) \(30 + ____ = 2 \text{ R } 6\) \(30 + ____ = 3 \text{ R } 6\)
   \(30 + ____ = 4 \text{ R } 6\) \(30 + ____ = 5 \text{ R } 6\) \(30 + ____ = 6 \text{ R } 6\)
c) Simon divides 45 by a number and gets a remainder of 9. What numbers could he have divided by?

**Answers:**

a) 24, 12, 8, 6, the fifth blank cannot be filled (the whole number being multiplied has to be a factor of 24 because 24 is 30 – 6. Since 5 is not a factor of 24, no whole number works), 4;
b) 30 ÷ 24 = 1 R 6, 30 ÷ 12 = 2 R 6, and 30 ÷ 8 = 3 R 6 work here; 30 ÷ 6 = 4 R 6 and 30 ÷ 4 = 6 R 6 don’t work because dividing by 6 or 4 can’t leave a remainder of 6
c) To fill in the blank in 45 ÷ ____ = ? R 9, you need ____ × ? + 9 = 45, so ____ is a factor of 36. ____ also must be bigger than 9; otherwise, dividing by it can’t get a remainder of 9. So, the numbers he could have divided by are 12, 18, and 36.

10. Find the missing digit so there is no remainder.

a) 8\(\overline{42A}\)
b) 9\(\overline{B2}\)
c) 7\(\overline{8C4}\)

**Answers:** a) A = 4, b) B = 7, c) C = 5

11. Find the digit M if 13\(\overline{3M8}\) has remainder 0.

**Answer:** M = 3

**NOTE:** Parts a) to g) in Problem Banks 12 and 13 guide students to solve the puzzle. Some students may appreciate the opportunity to solve the puzzle without such guidance.

12. Look at the puzzle. The goal is to find M.

\[
\begin{array}{c}
27)M64 \\
\overline{MC} \\
\overline{D4} \\
\overline{D4} \\
\hline
0
\end{array}
\]

a) Fill in the blanks using what is given in the long division.

27 × A = ____ ____ 27 × B = ____ __
b) What is the ones digit of 27 × B?
c) What is the ones digit of 7 × B?
d) What is B?
e) Use B to find D.
f) Use D to find C.
g) Use C to find A and M.
h) Check your answer by doing the long division

**Answers:** a) MC, D4; b) 4; c) 4; d) 2; e) 5; f) 1; g) since C = 1), 27 × A = M1, so A = 3 and M = 8
13. In the puzzle, each question mark represents a missing digit. AB and BC are two-digit numbers, and none of A, B, and C is zero.

\[
\begin{array}{c}
BC \\
AB) \ ? \ ? \ ? \ ? \\
\hline
? 6 \\
? ? \\
? 0 \\
0 \\
\end{array}
\]

a) What is the ones digit of the partial product AB × B?
b) What is the ones digit of B × B?
c) Which two values of B satisfy your observation from part b)?
d) How many digits does the partial product AB × C have?
e) What is the ones digit of the partial product AB × C?
f) What is the ones digit of B × C?
g) How many digits does AB × BC have?
h) What are A, B, and C? How do you know?

\textbf{Solution:} h) C must be 5 because it is not 0 and C multiplied by 4 or 6 has ones digit 0; A must be 1 because B is either 4 or 6 and the product 24 × 5 is already greater than 100, but AB × C has only two digits; so the product is either 14 × 45 or 16 × 65, but 14 × 45 = 630 has only three digits and 16 × 65 = 1040, so B is 6

\textbf{Answers:} a) 6, b) 6, c) 4 and 6, d) 2, e) 0, f) 0, g) 4
PS6-7  Looking for a Pattern I

Teach this lesson after: 6.2 Measurement

Goals:
Students will predict terms in a repeating pattern using division with remainders and use the result to solve problems.
Students will discover repeating patterns in unexpected places.

Prior Knowledge Required:
Can translate between a division with remainder equation and the corresponding multiplication with addition equation
Can extend repeating patterns
Can mentally compute the ones digit of products (for Problem Bank 5)

Vocabulary: core, pattern, remainder, repeating pattern, term

Predicting terms in repeating patterns when the term number is a multiple of the core length. Write on the board:

1, 2, 8, 1, 2, 8, 1, 2, 8, 1, 2, 8, ...

ASK: Can you predict the next term? (yes, it is 2) How do you know? (the pattern repeats 1, 2, 8) SAY: This is a repeating pattern with three terms in the core. Let’s say I want to know the 12th term. One way to do that is to list the first 12 terms. ASK: How many terms did I list so far? (10) SAY: I want to make the number of terms easier to see, so I am going to separate out the cores. Write on the board:

1, 2, 8, 1, 2, 8, 1, 2, 8, 1, 2, 8

SAY: By grouping it like this, it is easy to write 12 terms—I just write four groups of three. ASK: What is the 12th term equal to? (8) What is the next term after the 12th that will also be equal to 8? (the 15th term) And what is the next term after that, which will also be equal to 8? (the 18th) How can you tell by the term number whether or not the term will equal 8? (when the term number is a multiple of 3, the term will equal 8) SAY: The core is three terms long, so every third term, starting at the third, will be equal to the last term in the core.

Exercises: The core is given. For which term numbers are the terms in the pattern equal to 8?

a) 1, 3, 4, 8   b) 2, 8   c) 4, 5, 6, 7, 8

Answers: a) the multiples of 4, b) the multiples of 2, c) the multiples of 5

SAY: Because there are equal groups, you can write a multiplication and a division statement. Write on the board:

4 × 3 = 12 and 12 ÷ 3 = 4
Point to the pattern on the board (1, 2, 8, …) and SAY: Each core is a group of three, and four groups of three is 12 terms. You can show that as a multiplication or a division. If you want the 12th term, you can see that the 12th term ends a core because 12 is a multiple of 3.

**Exercises:** How many cores do you need to write the 20th term for each of the previous exercises? Write a multiplication and a division statement to show why.

**Answers:**

a) $5 \times 4 = 20$ and $20 \div 4 = 5$, so the 20th term ends the fifth repetition of the core;
b) $10 \times 2 = 20$ and $20 \div 2 = 10$, so the 20th term ends the 10th repetition of the core;
c) $4 \times 5 = 20$ and $20 \div 5 = 4$, so the 20th term ends the fourth repetition of the core.

**Predicting terms in repeating patterns when the term number is not a multiple of the core length.** Write on the board:

1, 2, 8, 1, 2, 8, 1, 2, 8, 1, …

SAY: Let’s say I want to know the 14th term. ASK: How many times will I have to write the core 1 2 8? (4 or 5 times) How do you know? (there are 3 terms in the core, so 4 cores would be 12 terms and 5 cores would be 15 terms) Finish writing the first four cores, or 12 terms, displaying the groups of three, as shown below:

1, 2, 8, 1, 2, 8, 1, 2, 8

SAY: I have four cores, so I have 12 terms so far. ASK: How many more terms do I need? (2 more) Continue the pattern with two more terms, as shown below:

1, 2, 8, 1, 2, 8, 1, 2, 8, 1, 2

Leave this pattern on the board. SAY: Now I have 14 terms. All the cores are equal groups. ASK: How many are in each group? (3) How many groups? (4) What kind of math equation can I write from equal groups? (multiplication or division) Write on the board:

$$\_\_\_ \times 3 + \_\_\_ = 14 \quad 14 \div 3 = \_\_\_ \text{ R } \_\_\_$$

SAY: You can write a multiplication and a division for the four groups of 3 and 2 left over. Have volunteers fill in the blanks to do so. ($4 \times 3 + 2 = 14$, $14 \div 3 = 4 \text{ R } 2$)

**Exercises:**

1. Write out the first 14 terms. What is the 14th term in the pattern with the given core?
   a) A, B, C
   b) A, B, B, C
   c) A, B, B, C, A
   **Answers:**
   a) A, B, C, A, B, C, A, B, A, B, the 14th term is B;

2. For each pattern in Exercise 1, write a multiplication and a division equation based on the cores being equal groups and some terms being left over to make 14.
   **Answers:**
   a) $4 \times 3 + 2 = 14$ and $14 \div 3 = 4 \text{ R } 2$,
   b) $3 \times 4 + 2 = 14$ and $14 \div 4 = 3 \text{ R } 2$,
   c) $5 \times 2 + 4 = 14$ and $14 \div 5 = 2 \text{ R } 4$
When students finish the exercises, SAY: The multiplication without the leftover part shows the complete cores. Then the added number tells how many terms to add in the new core to get the term number. Refer students to the example of the pattern with core “1, 2, 8” on the board. SAY: There are four complete cores of three terms and then you write the next two terms, starting a new core. Circle the 2 in the addition sentence, as shown below:

\[ 4 \times 3 + 2 = 14 \]

SAY: The number you add tells you how many terms in the core you need to write after completing full cores. So, this tells you which term the 14th term is equal to. ASK: Where is that number in the division statement? (it’s the remainder) SAY: The remainder tells you how many terms are left out of the groups. That’s the same as the number of terms you write after you finish complete cores.

**Exercises:** For each pattern in Exercise 1, what position in the core is the same as the 14th term? How does the division statement show this?

**Answers:**
- a) the second, because the remainder is 2; b) the second, because the remainder is 2; c) the fourth, because the remainder is 4

SAY: You can use division to find terms that are very far in a sequence. Again, refer students to the example of the pattern with core “1 2 8” on the board, and tell them that you would like to find the 22nd term. ASK: If you were to write the first 22 terms, how many whole groups of three would you need to write? (7) SAY: So, you would write the core seven times. ASK: How many are left over after you finish the seven groups of three? (1) Write on the board:

\[ 22 \div 3 = 7 \text{ R } 1 \]

ASK: So, what is the 22nd term? (1) How do you know? (it is the first term of the next core)

**Exercises:** The core is given. Use division to decide what colour the 37th block is, red (R), white (W), or yellow (Y).

**Bonus:** What colour is the 100th term in the pattern with core Y, R, R?

**Answers:**
- a) \[ 37 \div 3 = 12 \text{ R } 1, \text{ so Y} \]; b) \[ 37 \div 4 = 9 \text{ R } 1, \text{ so R} \]; c) \[ 37 \div 5 = 7 \text{ R } 2, \text{ so W} \]; Bonus: Y

SAY: Now imagine the 30th term in the pattern with core “1 2 8.” ASK: How many groups of three do you need? (10) How many are left over? (none) SAY: Here, the remainder is zero, so the 30th term completes a core. That means the term is equal to the last term in the core. When the remainder is zero, the term equals the last term. When the remainder is not zero, the remainder tells you which term in the core it is equal to.

**Exercises:** Find the 35th term of the pattern.
- a) 1, 2, then repeat  b) 1, 2, 3, then repeat  c) 1, 2, 3, 4, then repeat  d) 1, 2, 3, 4, 5, then repeat  e) 1, 2, 3, 4, 5, 6, then repeat  f) 1, 2, 3, 4, 5, 6, 7, then repeat

**Answers:**
- a) 1, b) 2, c) 3, d) 5, e) 5, f) 7
Predicting terms to solve problems. Draw on the board:

CONGRATСS! CONGRATСS! CONGRATСS!

SAY: A banner was hung across a wall. ASK: Is this a repeating pattern? (yes) What is repeating? (the word “congrats” and an exclamation mark) How many characters are repeating? (9) SAY: 100 characters fit along the wall. ASK: If the first character is C, what is the last character? (C) How do you know? (100 ÷ 9 = 11 R 1, so the 100th character is the same as the first character)

Exercises: A banner is made using 200 triangles, which repeat six colours in the following order: blue, green, yellow, orange, red, purple

a) What colour is the 50th banner?
b) What colour is the 100th banner?
c) What colour is the 200th banner?
d) In which position is the last red banner?

Selected solution: d) The last banner is green, from part c). So, working backwards from the 200th banner, draw the diagram below. So, the last red banner is in the 197th position.

197 198 199 200
R P B G

Answers: a) green, b) orange, c) green

Finding repeating patterns in unexpected places. Write on the board:

2, 3, 1, ____

SAY: A sequence starts 2, 3, 1. After the first three terms, the next term is found by adding the previous three terms and subtracting the result from 10. Write on the board:

2 + 3 + 1 = 6 and 10 − 6 = 4

SAY: 2 + 3 + 1 is 6 and 10 − 6 is 4, so the next term is 4.

Exercises: Find the next four terms in the pattern on the board.

Answers: 2, 3, 1, 4

ASK: What kind of pattern does this look like it is? (a repeating pattern) SAY: Once you know that the pattern repeats one core, then it has to keep repeating because you are doing the same operations on the same numbers to get the next terms. Write on the board:

2, 3, 1, 4, 2, 3, 1, 4, …
SAY: Just like I used $2 + 3 + 1 = 6$ to get the first 4, I used $2 + 3 + 1 = 6$ to get the next 4. Circle the second 2 (the fifth term) and ASK: What did I do to get this 2? (added $3 + 1 + 4 = 8$ and subtracted 8 from 10) What will I do to get the next term after the 4? (add $3 + 1 + 4$ and subtract the result from 10) Without doing the adding and subtracting, will you get the same answer? (yes) How do you know? (I am adding the same numbers and the subtracting the result from 10, just as before) SAY: When you do the same operations to the same numbers, you will always get the same answer. So, this pattern repeats. Now that you know it repeats, you can find any term in the pattern.

**Exercises:**

a) Find the 20th term in the pattern.

b) Find the 50th term in the pattern.

c) Find the 85th term in the pattern.

**Answers:**

a) 4, b) 3, c) 2

SAY: You can find repeating patterns in places where you don’t expect them.

**Exercises:**

a) A sequence starts 2, 3. After the first two terms, the next term is found by multiplying the previous two terms and dividing 24 by the result, so the next term is $24 ÷ (2 × 3) = 24 ÷ 6 = 4$.

a) Find the next three terms in the pattern.

b) Is the pattern a repeating pattern? Explain how you know.

c) Predict the 56th term in the pattern.

**Bonus:** Find another repeating pattern that is made using …

i) addition and subtraction ii) multiplication and division

**Answers:**

a) 2, 3, 4; b) repeating, because you are doing the same operations to the same numbers every three times, so you always get the same answer every three terms; c) 3; Bonus: sample answers: i) Start 3, 4, then, after the first two terms, the next term is found by adding the previous two terms and subtracting the result from 8. The pattern becomes: 3, 4, 1, 3, 4, 1, …; ii) Start 3, 4. After the first two terms, the next term is found by multiplying the previous two terms and dividing 60 by the result. Then the pattern is 3, 4, 5, 3, 4, 5, …

**Problem Bank**

1. Look at the sequence: $1 \times 1$, $2 \times 2$, $3 \times 3$, $4 \times 4$, $5 \times 5$, $6 \times 6$, …

a) Evaluate each of the first six terms of the sequence.

b) Predict the expression for the seventh term of the sequence and evaluate it.

c) Find the gaps between the terms in the sequence.

d) Use the gaps to predict the seventh term of the sequence.

e) Were your predictions in parts b) and d) the same?

**Answers:**

a) 1, 4, 9, 16, 25, 36; b) $7 \times 7 = 49$; c) 3, 5, 7, 9, 11, … (add 2 each time); d) 36 + 13 = 49; e) yes, both were 49

2. Look at the sequence from Problem Bank 1.

a) For the first 20 terms in the sequence, write the ones digit of each term.

b) Does the sequence of ones digits look like a repeating pattern?

c) Explain how you know that the sequence of ones digits will repeat.
Answers: a) 1, 4, 9, 6, 5, 6, 9, 4, 1, 0, 1, 4, 9, 6, 5, 6, 9, 4, 1, 0; b) yes; c) Because the sequence of gaps increases by 2, the gaps have a repeating pattern in the ones digits: 3, 5, 7, 9, 1, 3, 5, 7, 9, 1, ...; since the 11th term is 121 and has the same ones digit as the first term, adding gaps with the same ones digit will repeat the ones digits again.

3. Look at the sequence from Problem Bank 1. Find the remainder of each term when dividing by 8. Do you see a repeating pattern?
Answer: 1, 4, 1, 0, 1, 4, 1, 0, 1, 4, 1, 0, ...; yes, the pattern repeats

4. Find the remainder when dividing 9475 ÷ 8. Can 9475 be equal to \( N \times N \) for some whole number \( N \)? How do you know?
Answer: The remainder is 3, which is not in the sequence of remainders from Problem Bank 3, so 9475 is not in the sequence. Therefore, it cannot be equal to \( N \times N \) for any whole number \( N \).

5. Look at the sequence: 2, 2 × 2, 2 × 2 × 2, 2 × 2 × 2 × 2, 2 × 2 × 2 × 2 × 2, ...  
a) Evaluate each of the first five terms of the sequence.  
b) Predict the expression for the sixth term of the sequence and evaluate it.  
c) How can you get the seventh term of the sequence if you know the sixth term?  
d) The ones digit of the 90th term is 4. What is the ones digit of the 91st term?  
e) The ones digit of the 35th term is 8. What is the ones digit of the 36th term?  
f) What is the ones digit of the 100th term?
Answers: a) 2, 4, 8, 16, 32; b) 2 × 2 × 2 × 2 × 2 × 2 = 64; c) multiply by 2; d) 8; e) 6; f) 6

6. Tess multiplied one hundred 3s. What is the ones digit of her answer?
Answer: 1

7. To find the digital root of a number, add its digits, then add the digits of the resulting number until you reach a single digit. Example: 395 → 17 → 8.
   a) Find the digital root of ...
      i) 25     ii) 384     iii) 670     iv) 36     v) 198
   b) Use a pattern to predict what could be the digital root of ...
      i) a multiple of 3     ii) a multiple of 2     iii) a multiple of 9
      iv) a number in the sequence 1, 4, 9 16, 25, 36, ...
      v) a number in the sequence 1, 2, 4, 8, 16, 32, ...
   c) Can zero ever be the digital root of a number? Explain how you know.
Answers: a) i) 7, ii) 6, iii) 4, iv) 9, v) 9; b) i) 3, 6, or 9; ii) any digit; iii) only 9; iv) 1, 4, 7, or 9; v) 1, 2, 4, 5, 7, or 8; c) no, because the sum of a number’s digits can never be zero.
8. a) Find the remainder when dividing by 9.
   i) 25     ii) 384     iii) 670     iv) 36     v) 198
b) Look at your answers to Problem Bank 7. Predict when the digital root is equal to the remainder when dividing by 9 and when it is not.
c) Explain why the number of shaded squares is a multiple of 9.
   i) 25
   [Diagram]
   ii) 384
   [Diagram]
   d) In part c), how does the diagram show that you can subtract a multiple of 9 from the number to get the sum of its digits?
e) Draw a diagram to show that you can subtract a multiple of 9 from each number to get the sum of its digits.
   i) 670     ii) 36     iii) 198
f) Explain why a number and the sum of its digits always have the same remainder when dividing by 9.

Selected answers: a) i) 7, ii) 6, iii) 4, iv) 0, v) 0; b) When the number is not a multiple of 9, the digital root is equal to the remainder when the number is divided by 9. When the number is a multiple of 9, the digital root is 9 and the remainder is 0; c) i) each tens block has nine shaded squares, so there are $2 \times 9 = 18$ shaded squares; ii) each hundreds block has 99 shaded squares and each tens block has 9 shaded squares, so the total number of shaded squares is also a multiple of 9: $99 + 99 + 9 + 9 + 9 + 9 + 9 + 9 + 9 = 9 \times (11 + 11 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)$; d) taking away the shaded squares leaves the unshaded squares, and the number of these is the sum of the digits; f) subtracting a multiple of 9 doesn’t change the remainder when dividing by 9.
PS6-8 Looking for a Pattern II

Teach this lesson after: 6.2 Measurement

Goals:
Students will recognize that sequences with constant gaps have the same remainder when dividing by that gap and will then apply the result to solve problems.
Students will find a term given the term number and the term number given the term in a sequence with constant gaps.

Prior Knowledge Required:
Can multiply multi-digit numbers
Can divide with remainders using long division
Can relate a division with remainder equation to its corresponding multiplication with addition equation

Vocabulary: core, gap, multiple, pattern, remainder, repeating pattern, sequence, term

Comparing sequences. Write on the board:

3, 6, 9, 12, …

ASK: What is the gap? (3) SAY: Every gap is 3. ASK: What is the next term? (15) How did you get that? (I added 3 to 12) What is the 10th term? (30) PROMPT: These terms are all multiples of 3. ASK: How do you know the 10th term is 30? (3 × 10 = 30) Write on the board:

4, 7, 10, 13, …

ASK: What is the gap? (3) SAY: Every gap is 3, just like the other sequence. Explain to students that to find the 10th term of this sequence, they can compare each term of this sequence with the previous sequence using a number line. Draw on the board:

ASK: If the 10th term of the top sequence is 30, what is the 10th term of the bottom sequence? (31) How did you get that? (I added 1) Leave the number lines on the board for the following exercises.

Exercises: a) What is the 100th term of the top sequence?
b) What is the 100th term of the bottom sequence?
Answers: a) 300, b) 301
SAY: Both sequences have the same gap, but the top sequence is the multiples of the gap, so it’s easier to work with.

**Exercises:**
1. Decide how the two sequences are related. Then find the 100th term of each sequence.
   a) A. 2, 4, 6, 8, 10, …   b) A. 4, 8, 12, 16, 20, …
   B. 7, 9, 11, 13, 15, …   B. 3, 7, 11, 15, 19, …
   c) A. 3, 6, 9, 12, 15, …   d) A. 5, 10, 15, 20, 25, …
   B. 1, 4, 7, 10, 13, …   B. 1, 6, 11, 16, 21, …
   **Answers:** a) A. 200, B. 205; b) A. 400, B. 399; c) A. 300, B. 298; d) A. 500, B. 496

2. Compare to another sequence to find the 100th term. Hint: Find the gap and use the multiples of the gap.
   a) 5, 9, 13, 17, 21, …   b) 3, 9, 15, 21, 27, …
   **Selected solution:** a) The gap is 4. The sequence of multiples of 4 is 4, 8, 12, … and it has 100th term 400, so the given sequence has 100th term 401.
   **Answer:** b) 597

**Sequences with a constant gap have the same remainder when dividing by the gap.** Write on the board:

9 ÷ 4    10 ÷ 4    11 ÷ 4    12 ÷ 4    13 ÷ 4    14 ÷ 4    15 ÷ 4    16 ÷ 4

Have a volunteer write the sequence of answers, including the remainder, below the sequence of divisions. (see below)

2 R 1    2 R 2    2 R 3    3 R 0    3 R 1    3 R 2    3 R 3    4 R 0

ASK: What is the sequence of remainders? (1, 2, 3, 0, then repeat; or 1, 2, 3, 0, 1, 2, 3, 0)
Is that a repeating pattern? (yes) What is the core? (1, 2, 3, 0) SAY: Every four terms has the same remainder. That’s because when you increase a number by 4, it doesn’t change the remainder when you divide by 4: you make one more group of 4 and have the same number left over.

**Exercises:**
1. Find the gap.
   a) 4, 6, 8, 10, …   b) 6, 11, 16, 21, …   c) 7, 11, 15, 19, …
   d) 12, 16, 20, 24, …   e) 2, 8, 14, 20, …   f) 11, 18, 25, 32, …
   **Answers:** a) 2, b) 5, c) 4, d) 4, e) 6, f) 7

2. For each part in Exercise 1, divide each term by the gap and find the remainder. Is the remainder the same for the whole sequence?
   **Answers:** a) 0, 0, 0, 0, yes; b) 1, 1, 1, 1, yes; c) 3, 3, 3, 3, yes; d) 0, 0, 0, 0, yes; e) 2, 2, 2, 2, yes; f) 4, 4, 4, 4, yes
Deciding if a given number is in a given sequence with constant gap. Write on the board:

3, 7, 11, 15, ...

ASK: If I continue the sequence, will I see the number 35? Ask a volunteer to write the next terms of the sequence until they either see or pass 35, as shown below:

3, 7, 11, 15, 19, 23, 27, 31, 35

SAY: 35 is a term of the sequence, and we could find it by continuing the terms. ASK: How can we find if 135 belongs to the sequence? Have students share their ideas. Guide students to see how they can use the property they just learned. Point to the sequence and ASK: Is the gap always the same between consecutive terms? (yes) What is the gap? (4) Ask a volunteer to divide 3, 7, and 11 by 4 and find the remainder. (see below)

\[
\begin{align*}
3 \div 4 &= 0 \text{ R } 3 \\
7 \div 4 &= 1 \text{ R } 3 \\
11 \div 4 &= 2 \text{ R } 3
\end{align*}
\]

SAY: We know that 35 is in the sequence because when you divide 35 by 4, the remainder is 3. Write on the board:

35 ÷ 4 = 8 R 3

ASK: So, how can we determine if 135 will belong to the sequence or not? (divide 135 by 4 and find the remainder) Ask a volunteer to divide 135 by 4 using long division. (see below)

\[
\begin{align*}
33 \\
4 \longdiv{135} \\
-12 \\
\underline{15} \\
-12 \\
\underline{3}
\end{align*}
\]

So, \(135 \div 4 = 33 \text{ R } 3\).

Point to the remainder and SAY: When we divide 135 by 4, the remainder is 3, just the way it is for the earlier numbers in the sequence. So, we know that if we continue the sequence, we will see the number 135 in the sequence.

**Exercises:** Is 500 in the sequence?

a) 10, 16, 22, 28, ...

b) 8, 14, 20, 26, ...

**Solutions:**

a) The gap is 6 and if you divide the first term by the gap, the remainder is 4. But \(500 \div 6 = 83 \text{ R } 2\), so 500 is not in the sequence.

b) The gap is 6 and \(8 \div 6 = 1 \text{ R } 2\). Since \(500 \div 6 = 83 \text{ R } 2\), 500 is in the sequence.
Deciding which sequence a given number is in. SAY: Sometimes on contest problems, you have to decide which sequence a given number is in.

Write on the board:

A  B  C  D  E  F
1  2  3  4  5  6
7  8  9 10 11 12
13 14 15 16 17 18
…

SAY: I want to know which column 250 is in. ASK: What does every number in column A have in common? (remainder 1 when dividing by 6) What does every number in column B have in common? (remainder 2 when dividing by 6) What do we have to know about 250 in order to decide what column it is in? (its remainder when divided by 6) Have a volunteer complete the long division on the board. (see below)

\[
\begin{array}{c}
41 \\
6 \overline{)250} \\
-24 \\
10 \\
-6 \\
-4 \\
\end{array}
\]

So, \(250 \div 6 = 41 R 4\).

SAY: The remainder is 4. ASK: So, which column is 250 in? (column D) SAY: Every number in D has remainder 4 when dividing by 6, and all numbers with remainder 4 are in that column, so 250 is also in column D.

Exercises: Which column is 310 in?

a) A  B  C
1  2  3
4  5  6
7  8  9
…

b) A  B  C  D
1  2  3  4
5  6  7  8
9 10 11 12
…

c) A  B  C  D  E  F  G  H
1  2  3  4  5  6  7  8
9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24
…

Selected solution: c) \(310 \div 8 = 38 R 6\), so 310 is in column F

Answers: a) column A, b) column B
SAY: On a calendar, the columns can stand for days of the week. So, you can use this to solve problems about days of the week. Draw on the board:

<table>
<thead>
<tr>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: What do all the numbers in the Monday column have in common? (they all have remainder 6 when dividing by 7) How could you use this to find which day of the week is the 100th day of the year? (divide 100 by 7 and find the remainder) Have a volunteer do this. (100 ÷ 7 = 14 R 2) SAY: 100 has the remainder 2 when dividing by 7. ASK: Which day of the week has all the numbers with remainder 2? (Thursday) If we continued the numbers past 31 and wrote 32 for Saturday (which is the 32nd day of the year) and 33 for Sunday (the 33rd day of the year), which column would 100 be in? (the Thursday column) Keep the calendar on the board.

**Exercises:** Assume this year is not a leap year. January 1 is a Wednesday.

a) How many days are in this year?
b) What day of the week is December 31?
c) What day of the week is the first day of next year?
d) How many days are in January and February altogether?
e) What day of the week is March 1?
f) Nina plays soccer every Tuesday. Her birthday is April 15. Will she play soccer on her birthday?

**Answers:** a) 365, b) Wednesday, c) Thursday, d) 59, e) Saturday, f) yes

Now have students look at the calendar again. ASK: Are January 3 and January 31 on the same day of the week? (yes, they are both Friday) Is there a way to know that they are on the same day without knowing which day, just from the numbers? (they have the same remainder when dividing by 7) SAY: One way is to find the remainders and see if they are the same. There is another way too. Write on the board:

\[ 31 - 3 = 28 \]

SAY: When I perform the subtraction, I get 28. January 3 is a Friday. ASK: What is the date of the next Friday? (January 10) SAY: When you add 7 to the day, you don’t change the day of the week, because that’s a full week, and you can keep adding as many sevens as you want. So another way to check whether two days are on the same day of the week is to subtract the two numbers and see if the difference is a multiple of 7.
Exercises: Assume this year is a leap year.
a) How many days are in this year?
b) How many days are in February?
c) Fill in the blank: March 18 is the ______th day of the year.
d) Fill in the blank: September 22 is the _____th day of the year.
e) Are March 18 and September 22 on the same day of the year?
f) Next year is not a leap year. Will March 18 and September 22 be on the same day of the week, next year?
g) Are January 1 and December 31 on the same day of the week, this year?
h) Are January 1 and December 31 on the same day of the week, next year?

Selected solutions: e) 266 − 78 = 188 and 188 ÷ 7 = 26 R 6, so no, they are not on the same day of the year; f) no, because both days decrease by 1, and 265 − 77 gives exactly the same answer as 266 − 78

Answers: a) 366, b) 29, c) 78, d) 266, g) no, h) yes

Finding a given term in a sequence with constant gaps. Write on the board:

2, 5, 8, 11, 14, 17, ...

ASK: What does every term in the sequence have in common? (the remainder when dividing by 3 is 2) Write on the board:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Quotient when divided by 3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Point to each term in turn and ASK: What is the quotient when dividing by 3? Fill in the table as students tell you the answers:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Quotient when divided by 3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

SAY: The first term has quotient 0, the second term has quotient 1, the third term has quotient 2. ASK: What is the quotient when the 100th term is divided by 3? (99) How did you get that? (the quotient is always 1 less than the term number) Write on the board:

100th term ÷ 3 = 99 R 2

ASK: How can you use this to find the 100th term? (multiply 99 by 3 and add 2, so the 100th term is 299) Keep the table on the board.

Exercises:
1. Find the 45th term of the sequence on the board.
   Solution: 44 × 3 + 2 = 134
2. Look at the sequence: 11, 16, 21, 26, 31, 36, 41, 46, ….  
   a) What is the gap?  
   b) What are all the remainders when dividing by the gap?  
   c) What are all the quotients when dividing by the gap?  
   d) What is the quotient of the 100th term when dividing by the gap?  
   e) What is the 100th term?  
   **Answers:** a) 5; b) 1; c) 2, 3, 4, 5, … or term number plus 1; d) 101; e) 101 × 5 + 1 = 506  

3. Find the 200th term.  
   a) 1, 3, 5, 7, …  
   b) 24, 27, 30, 33, …  
   c) 18, 23, 28, 33, …  
   d) 5, 9, 13, 17, …  
   **Answers:** a) 399; b) 621; c) 1013; d) 801  

**Finding which term a given number is in a sequence.** SAY: You can use this method to find which term a given number is in a sequence. Write on the board:  

\[2, 5, 8, 11, 14, \ldots\]  

ASK: Is 752 in this sequence? (yes) How do you know? (752 ÷ 3 = 250 R 2) Write on the board:  

\[752 ÷ 3 = 250 R 2\]  

SAY: You know from the remainder that 752 is in the sequence and you can tell from the quotient where it is in the sequence. Continue the table to term 752 by drawing on the board:  

<table>
<thead>
<tr>
<th>Term Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>…</th>
<th>752</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>…</td>
<td>752</td>
</tr>
<tr>
<td>Quotient when divided by 3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>…</td>
<td>250</td>
</tr>
</tbody>
</table>

ASK: When the quotient is 250, what is the term number? (251) How do you know? (it's always 1 greater) SAY: So, 752 is the 251st term.  

**Exercises:** Is 500 in the sequence? If so, which term is it?  
   a) 2, 5, 8, 11, 14, …  
   b) 2, 6, 10,14, 18, 22, …  
   c) 20, 26, 32, 38, …  
   **Bonus:** 143, 150, 157, 164, 171, …  
   **Answers:** a) yes, the 167th term; b) no; c) yes, the 81st term; Bonus: yes, the 52nd term  

**Problem Bank**  
1. January 1 is a Tuesday and the year has 365 days.  
   a) What day of the week is the 100th day of the year?  
   b) What day of the week is the last day of the year?  
   c) What is the date of the last Friday of the year?  
   **Selected solution:** c) The last day, December 31, is a Tuesday, December 30 is a Monday, December 29 is a Sunday, December 28 is a Saturday, and December 27 is a Friday, so December 27 is the last Friday of the year.  
   **Answers:** a) Wednesday, b) Tuesday
2. Decide how the three sequences are related. Then find the 100th term of each.
   a) A. 1, 2, 3, 4, 5, …  
      B. 4, 5, 6, 7, 8, …  
      C. 4, 10, 18, 28, 40, …  
   b) A. 2, 4, 6, 8, 10, …  
      B. 1, 3, 5, 7, 9, …  
      C. 1, 6, 15, 28, 45, …  
   **Answers:** a) A. 100, B. 103, C. 10 300; b) A. 200, B. 199; C. 19 900

3. Create other sequences to help you find the 100th term of the following sequence:
   4, 14, 30, 52, 80, …  
   **Solution:** Make the sequence 4, 7, 10, 13, 16, … by dividing each term by the term number, then make the sequence 3, 6, 9, 12, 15, … by subtracting 1 from each term. The sequence 3, 6, 9, 12, 15, … has 100th term 300; the sequence 4, 7, 10, 13, 16, … has 100th term 301, and so the sequence 4, 14, 30, 52, 80, … has 100th term 30 100.

4. a) Which column is 500 in?
   A  B  C  D  E  
   1  2  3  4  5  
   6  7  8  9 10  
   11 12 13 14 15  
   …
   b) Which column is 500 in? Hint: Where do all the numbers from column E in part a) fit in this picture?
   A  B  C  
   1  2  3  
   5  4  
   6  7  8  
   10 9  
   11 12 13  
   15 14  
   …
   c) Why was part a) easier than part b)? How did using the easier problem help with part b)?
   **Answers:** a) E, b) B, c) it was easier because all the multiples of 5 were the only numbers in column E

5. Which column is 568 in?
   a) A  B  C  D  E  F  G  
      1  2  3  4  5  6  7  
      8  9 10 11 12 13 14  
      15 16 17 18 19 20 21  
      …
   b) A  B  C  D  E  F  G  
      1  2  3  4  5  6  7  
      14 13 12 11 10 9  8  
      15 16 17 18 19 20 21  
      …
   **Answers:** a) A, b) G
6. Look at the pattern.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<td>5</td>
<td>6</td>
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<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Complete the table.

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>A</td>
<td>B</td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

b) What is the core for the pattern of columns?

c) Which column is 455 in? Explain how you know.

**Answers:** a) C, D, B, C, A, B, C, D, B, C; b) A, B, C, D, B, C; c) 455 ÷ 6 = 75 R 5, so 455 is in column B

7. What column is 455 in?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
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<tr>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answer:** C

8. This year is a leap year. January 1 is a Tuesday.

a) What day will January 1 be next year?

<table>
<thead>
<tr>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
<th>Monday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
</tbody>
</table>

b) Mary’s birthday is December 1. What day of the week was that last year? Hint: Put December 1 of last year as Day 1. What number is January 1 of this year?

c) What day was January 1 last year? Hint: Since this year is a leap year, last year wasn’t.

d) Randi’s birthday is September 9. What day of the week will that be this year?

**Solutions:**

a) January 1 next year is the 367th day, since there are 366 days in a leap year. Now 367 ÷ 7 = 52 R 3, so it is in the Thursday column;

b) Put December 1 of last year as Day 1. Then the last day of December is Day 31 and January 1 of this year is Day 32. Day 32 is a Tuesday, so all the Tuesdays have remainder 4 when dividing by 7. We need to know which days have remainder 1 when dividing by 7 because that will tell us which day is Day 1. The Wednesdays have remainder 5, the Thursdays have remainder 6, the Fridays have remainder 0, and the Saturdays have remainder 1. So, Day 1, or December 1 of last year, is a Saturday.

c) January 1 this year is Day 366 because last year wasn’t a leap year. So, this year starts

<table>
<thead>
<tr>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
<th>Monday</th>
</tr>
</thead>
<tbody>
<tr>
<td>366</td>
<td>367</td>
<td>368</td>
<td>369</td>
<td>370</td>
<td>371</td>
<td>372</td>
</tr>
</tbody>
</table>
These numbers, when divided by 7, have these remainders:

Tuesday  Wednesday  Thursday  Friday  Saturday  Sunday  Monday
   2         3         4         5         6          0          1

Day 1, which also has remainder 1 when dividing by 7 (because \(1 \div 7 = 0 \, R \, 1\)), must be in the Monday column.

d) September 9 is day \(31 + 29 + 31 + 30 + 31 + 30 + 31 + 31 + 9 = 253\). Now, \(253 \div 7 = 36 \, R \, 1\), so September 9 is the same day of the week as January 1, which is a Tuesday.

9. Anna’s birthday is June 27 and Fred’s birthday is September 14. Are their birthdays on the same day of the week?

**Solution:** The number of days after Anna’s birthday and up to and including Fred’s birthday is \(3 + 31 + 31 + 14 = 79\). This is not a whole number of weeks because 79 is not a multiple of 7, so their birthdays are not on the same day of the week.

10. If it just turned 2:15 p.m. on Monday now, what time and day will it be in …
   a) 1000 seconds?  b) 1000 minutes?  c) 1000 days?

**Solutions:**
   a) 1000 seconds = 16 minutes and 40 seconds, so it will be 2:31 p.m. on Monday
   b) 1000 minutes = 16 hours and 40 minutes, so it will be 6:55 a.m. on Tuesday
   c) The time will still be 2:15 p.m., so just find out what day it is. Put Monday as Day 1, then Tuesday is Day 2, next Monday is Day 8, the Monday after that is Day 15, and so on. All the Mondays are equal to days that have remainder 1 when divided by 7. Tuesdays have remainder 2, Wednesdays have remainder 3, Thursdays have remainder 4, Fridays have remainder 5, Saturdays have remainder 6, and Sundays have remainder 0. So, calculate \(1000 \div 7 = 142 \, R \, 6\). Day 1000 has remainder 6, so it must be a Saturday.

11. Look at the grid below and explain how it shows the expression.

   a) 1 + 3 + 5 + 7 + 9
   b) 5 \times 5

**Answers:** a) count the squares in each alternating layer and add them, b) there are 5 rows of 5

12. Draw a grid to show that 1 + 3 + 5 + 7 + 9 + 11 = 6 \times 6.

**Answer:**
13. a) Find a shortcut to add $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$.
b) Use your answer to part a) to add $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20$.
c) Use your answer to part b) to add $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$.
d) Find the sum in part c) a different way:

\[ x = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 \]
\[ + x = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 \]
\[ 2x = \phantom{100} = \phantom{100} \]

So, \( x = \phantom{55} \)
e) Use the method in part d) to evaluate the sum of the first 20 whole numbers.

**Answers:** a) $10 \times 10 = 100$; b) 110; c) 55; d) $2x = 110$, so $x = 55$; e) $21 \times 20 + 2 = 210$
PS6-9 Using a Diagram

Teach this lesson after: 6.2 Measurement

Goals:
Students will use diagrams to solve multistep word problems.

Prior Knowledge Required:
Can represent fractions with a model
Can determine half of a whole number
Can multiply and divide decimal tenths by whole numbers (for Problem Banks 1-4)
Can add decimals, up to hundredths (for Problem Bank 7)
Can multiply and divide decimal hundredths by whole numbers (for Problem Bank 7 and Extended Problem)
Can compare fractions (for Extended Problem)
Can solve proportions (for Extended Problem)
Can calculate a fraction of a whole number (for Extended Problem)
Can divide by a two-digit whole number (for Extended Problem)

Materials:
BLM Making Punch (pp. 56–58, see Extended Problem)

Identifying parts of a diagram and solving a problem given the diagram. SAY: Marla had $35 and spent three fifths of her money on a shirt. We are going to represent this problem with a diagram. Draw on the board:

SAY: I want to draw a diagram to show three fifths, so I drew a rectangle divided into five parts. Ask a volunteer to shade the part of the diagram that represents three fifths. (3 blocks)
SAY: This diagram shows the total amount of money that Marla had and the shaded part shows the money spent on a shirt. Label the diagram, as shown below:

$35

shirt

ASK: If all five blocks represent $35, how much money does each block represent? ($7) How do you know? (35 ÷ 5 = 7)

Finish the diagram, as shown below:

$35

$7

shirt
ASK: How much money did Marla spend on the shirt? ($21) How do you know? (3 × 7 = $21)
How much money does she have left? ($14) How do you know? (35 − 21 = $14; or 2 unshaded blocks, 2 × 7 = $14) Point out that, even though the first solution is correct, the second solution shows that you can find the leftover in the diagram without calculating how much money was spent on the shirt.

**Exercises:** Draw a diagram to solve the problem.

a) Jane had $36. She spent \( \frac{3}{4} \) of her money on a pair of shoes. How much money does she have left?

b) Tristan spent \( \frac{2}{5} \) of his money on a toy. He has $15 left. How much did the toy cost?

c) Nora spent \( \frac{2}{5} \) of her money on a poster that cost $8. How much money did she have before she bought the poster?

**Solutions:**

\[
\begin{align*}
\text{a)} & \quad \hspace{1cm} \text{b)} & \quad \hspace{1cm} \text{c)} \\
\text{shoes} \hspace{1cm} \text{leftover} = \$9 & \quad \text{toys} \hspace{1cm} \text{leftover} = \$9 & \quad \text{total before} = \$20 \\
\text{\hspace{1cm} $36$} & \quad \text{\hspace{1cm} $15$} & \quad \text{\hspace{1cm} $4$} \\
\end{align*}
\]

**Solving problems where an amount is divided and then further divided into unequal parts.** Write on the board:

Eric has some eggs. He uses \( \frac{3}{7} \) of them to make pancakes and \( \frac{1}{2} \) of the remainder to make sandwiches. Now Eric has 6 eggs left. How many eggs did Eric use to make pancakes? How many eggs did Eric have at first?

Point out that the first two sentences are similar to the problem you just did, so to solve this problem we can start with the same type of diagram as earlier. Draw on the board:

\[
\begin{align*}
\text{pancakes} \quad \text{sandwiches}
\end{align*}
\]

ASK: Which part of the diagram shows the leftover? (the unshaded squares) Cover up the shaded part and ASK: Which part is half of the leftover? (2 squares) Mark that on the diagram, as shown below:

\[
\begin{align*}
\text{pancakes} \quad \text{sandwiches}
\end{align*}
\]
ASK: How many eggs are left after Eric made the pancakes and sandwiches? (6) So how many eggs does each block represent? (3) How do you know? (6 ÷ 2 = 3) How many eggs did he use for the pancakes? (9) How many eggs did Eric have initially? (21) How do you know? (7 blocks, 7 × 3 = 21)

**Exercises:**

a) Tristan spent \(\frac{2}{5}\) of his money on a toy and \(\frac{2}{3}\) of the remainder on a gift for his sister. He has $8 left. How much did he spend altogether?

b) Marla spent \(\frac{2}{7}\) of her money on a shirt. She spent \(\frac{3}{5}\) of the remainder on a CD. She has $10 left. How much did the CD cost?

**Answers:** a) $32, b) $15

Write on the board:

Raj had 30 stickers. He gave \(\frac{2}{5}\) of his stickers to his brother and \(\frac{1}{2}\) of the rest to his friend. How many stickers did Raj’s brother get? How many stickers are left?

ASK: How many blocks are there in total? (5) How many stickers does each block represent? (6) Explain how you know. (30 ÷ 5 = 6) How many blocks did Raj’s brother get? (2) So how many stickers did Raj’s brother get? (12) SAY: Raj’s friend got half of the rest. Draw a dashed line to show Raj’s friend’s stickers, as shown below:

**Explain to students that it is sometimes easier to take away the first part of the question and then continue with the rest of the problem.** SAY: At the beginning of the question, we know Raj has 30 stickers and he gave two fifths of his stickers to his brother.
Draw the original diagram on the board again:

\[
\begin{align*}
\text{total stickers} &= 30 \\
\text{brother} &\quad \text{stickers left} = 18 \\
\text{friend} &\quad \text{rest} = 9
\end{align*}
\]

ASK: How many stickers does each block represent? (6) How many are left? (18) What fraction of the leftover goes to Raj’s friend? (half) Draw on the board:

\[
\begin{align*}
\text{stickers left} &= 18 \\
\text{friend} &\quad \text{not coloured} = 9
\end{align*}
\]

Point to the new diagram and ASK: How many stickers does each block represent? (9) How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

**Exercises:** The next time Raj has stickers, he decides to give \( \frac{2}{5} \) of his 30 stickers to his brother and \( \frac{5}{6} \) of the remainder to his friend.

a) How many stickers did Raj’s friend get?
b) How many stickers are left?

**Solution:**

\[
\begin{align*}
\text{total stickers} &= 30 \\
\text{remainder} &= 18 \\
\text{brother} &= 12 \\
\text{friend} &= 15 \\
\text{left} &= 3
\end{align*}
\]

**Solving problems backwards.** Write on the board:

Emma has some stickers. She colours \( \frac{1}{4} \) of them red and \( \frac{2}{5} \) of the remainder green.

If Emma doesn’t colour 9 stickers, how many stickers does Emma have in total?

Explain to students that, like in the previous problem, they can draw diagrams, one for each step of the problem. Draw on the board:

\[
\begin{align*}
\text{total stickers} &= \? \\
\text{stickers left} &= \? \\
\text{red} &= \? \\
\text{stickers left} &= \? \\
\text{green} &= \? \\
\text{not coloured} &= 9
\end{align*}
\]
SAY: In the diagram on the left, all parts are unknown. ASK: Can I start with the diagram on the left? (no) SAY: Look at the diagram on the right. ASK: How many stickers are not coloured? (9) How many stickers does each block represent? (3) How do you know? (9 ÷ 3 = 3) How many blocks are green? (2 × 3 = 6)

Erase the question mark beside “green” and write “6” in its place, as shown below:

Point to the diagram on the right and ASK: How many stickers are shown here in total? (9 + 6 = 15) SAY: So, how many are left after Emma colours some red? (15) Erase the question mark beside “stickers left” in both diagrams and write “15” in its place, as shown below:

Ask a volunteer to solve the diagram on the left and find the total numbers of stickers. (20)

Exercise: Marko spent \( \frac{3}{5} \) of his money on a book and \( \frac{3}{4} \) of the remainder on some music. Marko has $4 after he paid for the book and the music. How much money did he have initially?

Solution:

\[
\text{initial money} = \$40 \\
\text{left after book} = \$16 \\
\text{book} = \$24 \\
\text{left after book} = \$16 \\
\text{music} = \$12 \\
\text{left after music} = \$4
\]

Problem Bank
1. Remember, 60 minutes = 1 hour. How many minutes are in each decimal number of hours?
   a) 0.5 hours  
   b) 0.1 hours  
   c) 0.3 hours  
   d) 0.7 hours  
   e) 1.2 hours  
   f) 1.5 hours  
   g) 1.6 hours  
   h) 3.4 hours

Answers: a) 30, b) 6, c) 18, d) 42, e) 72, f) 90, g) 96, h) 204

2. Since 0.2 hours is 12 minutes, you can write 1.2 hours as “1 hour 12 minutes.” Write the decimal number of hours as hours and minutes.
   a) 2.1 hours  
   b) 1.4 hours  
   c) 3.5 hours  
   d) 8.9 hours

Answers: a) 2 hours 6 minutes, b) 1 hour 24 minutes, c) 3 hours 30 minutes, d) 8 hours 54 minutes
3. The number of hours is often given in decimal numbers. Convert the times shown in these situations to hours and minutes.
   a) Listen to 1.6 hours of music.
   b) In a flight course, you can use a flight training device for 1.2 hours.
   c) A group of people surveyed said they use social media networks an average of 2.7 hours per day.
   d) The average person surveyed watches 4.8 hours of television a week.
   **Answers:** a) 1 hour 36 minutes, b) 1 hour 12 minutes, c) 2 hours 42 minutes, d) 4 hours 48 minutes

4. a) This week, Glen spent $\frac{3}{5}$ of his time doing homework on math and science. He spent 2 hours on math and 4 hours on science. How much time did he spend doing homework altogether?
   b) This week, Kate spent $\frac{2}{5}$ of her time doing homework on math and science. She spent 1.6 hours on math and 2 hours on science. How many hours of homework did she do altogether?
   c) Did you do part b) by converting the number of hours to minutes and dividing the whole number of minutes by 2 or by dividing the decimal number of hours by 2? Do the question again the other way, and make sure you get the same answer.
   **Selected solutions:**
   a) He spent 6 hours on math and science altogether, so each block represents 2 hours. Five blocks together is 10 hours:
      
      \[
      \begin{array}{c}
      \text{math + science} = 6 \\
      \text{total} = 10
      \end{array}
      \]

   b) She spent 3.6 hours on math and science altogether, so each block is 3.6 hours ÷ 2 = 1.8 hours. Five blocks together is 5 × 1.8 hours = 9 hours.
      
      \[
      \begin{array}{c}
      \text{math + science} = 3.6 \\
      \text{total} = 9
      \end{array}
      \]

5. Rani reads 10 pages of a book on Saturday and she reads $\frac{3}{4}$ of the rest of the book on Sunday. She still has 17 pages to read. How many pages are in the book?
   **Answer:** 78 pages
6. A convenience store has some ice cream treats. It sells $\frac{2}{5}$ of them on Friday, $\frac{1}{4}$ of the remainder on Saturday, and $\frac{2}{3}$ of the rest on Sunday. The store has 30 ice cream treats left by the end of Sunday.

a) How many ice cream treats did the store have initially?
b) On which day did the store sell the most ice cream treats?

**Answers:** a) 200, b) Friday, 80

7. Zara received some money for her birthday. She donated $\frac{1}{5}$ to charity, and she saved $\frac{2}{3}$ of the remainder in her savings. Of what was left, $\frac{1}{4}$ was a gift card to an ice cream store. She used the rest of the money to buy three books for $6.99 each, a T-shirt for $5.99, and a basketball for $7.99. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each part (donation, savings, gift card)?

**Answers:** the books were $6.99 each, the T-shirt was $5.99, and the basketball was $7.99, so she spent $34.95 on purchases; she was given $174.75 altogether; she gave $34.95 to charity; she put $93.20 in savings; and she had $11.65 on the ice cream store gift card.
Extended Problem: Making Punch

Materials:
BLM Making Punch (pp. 56–58)

Extended Problem: Making Punch. Provide students with BLM Making Punch. Explain to students who are not familiar with punch that it is a drink made from fruit juices and soda.

NOTE: The Bonus question provides students with an opportunity to use diagrams to solve a word problem. As students apply this problem-solving strategy, they will need to work backwards using the information given in the problem.

Answers:
1. a) Recipe A: 3/5, Recipe B: 5/8, Recipe C: 7/12; b) 5/8 is the largest fraction, so Recipe B has the strongest ginger ale taste; c) 7/12 is the smallest fraction, so Recipe C has the strongest cranberry juice taste
2. a) 360 cups, b) 120 cups, c) 72 cups ginger ale and 48 cups cranberry juice; d) Recipe B: 75 cups ginger ale and 45 cups cranberry juice, Recipe C: 70 cups ginger ale and 50 cups cranberry juice, in total: 217 cups ginger ale and 143 cups cranberry juice; Bonus: a) the initial budget was $146.25, b) the committee paid $48.75 for ginger ale
Making Punch (1)

A graduation committee has three recipes to make cranberry juice/ginger ale punch for a graduation party.

**Recipe A:** 3 cups of ginger ale for every 2 cups of cranberry juice
**Recipe B:** 5 cups of ginger ale for every 3 cups of cranberry juice
**Recipe C:** 7 cups of ginger ale for every 5 cups of cranberry juice

1. a) What fraction of each recipe is ginger ale?
   
   Recipe A: ________  
   Recipe B: ________  
   Recipe C: ________

b) Find the recipe that has the strongest ginger ale taste. Explain how you found your answer.

c) Find the recipe that has the strongest cranberry juice taste. Explain how you found your answer.
Making Punch (2)

2. There will be 194 students and 256 parents at the graduation party. The graduation committee decides to make enough punch so that everybody can have one glass. Each plastic glass holds 200 mL of punch.

a) How many cups of punch in total are needed for the party? (Use 1 cup = 250 mL.)

b) The committee decides to make an equal amount of each type of punch. How many cups of each recipe are needed?

c) How many cups of ginger ale are needed for each recipe? How many cups of cranberry juice are needed for each recipe?

d) How many cups of ginger ale and how many cups of cranberry juice are needed for the party in total?
Making Punch (3)

BONUS The committee spends $\frac{1}{3}$ of its budget buying ginger ale and $\frac{2}{5}$ of the leftover money buying cranberry juice.

a) After buying cranberry juice, the committee has $58.50 left to buy plastic glasses. How much was the initial budget?

b) How much did the committee pay for ginger ale?
PS6-10  Making a Simpler Problem

Teach this lesson after: 6.2 Measurement

Goals:
Students will learn a variety of strategies to make a problem easier or clearer.

Prior Knowledge Required:
Can add decimal tenths
Can multiply decimal tenths by a whole number

Materials:
grid paper (e.g., from BLM 1 cm Grid Paper, p. 68)
BLM Fraction Strips and Circles (p. 69, see Problem Bank 7)

Using smaller numbers to make a simpler problem. Write on the board:

A teacher tells her students to read pages 287 to 354 for homework.
How many pages is that?

ASK: What makes this problem hard? (sample answer: 287 and 354 are big numbers) Would it be easier to know how many pages the students have to read if the teacher tells them to read pages 353 to 355 for homework? (yes, you could just count the pages: 353, 354, and 355 are three pages) SAY: So, it’s not exactly how big the numbers are that makes this problem hard. ASK: Can you find a more precise way to say what makes this problem hard? (the numbers are far apart) Have volunteers give you similar, simpler problems that you could solve first. (for example, make the numbers smaller and closer together) Write all the suggestions on the board.

Exercise: Solve all the simpler problems on the board. Do you see a pattern in your answers?
Answer: In all cases, you can find the number of pages by subtracting the smaller number from the bigger number and then adding 1.

Have a volunteer tell you the pattern in the exercise. (subtract the numbers and add 1) SAY: Now that you know the pattern, you can solve any problem of the same type.

Exercises: A teacher tells her students to read pages in a textbook for homework. How many pages do the students need to read?
a) from 352 to 386
b) from 298 to 314
c) from 408 to 451
Answers: a) 35, b) 17, c) 44
Listing similar problems in an organized way. Tell students that it can be helpful to examine the simpler problems in an organized way. Refer students to the problem about reading from pages 287 to 354. Write on the board:

<table>
<thead>
<tr>
<th>Pages Read</th>
<th>How Many Pages?</th>
</tr>
</thead>
<tbody>
<tr>
<td>287 to 288</td>
<td>2</td>
</tr>
<tr>
<td>287 to 289</td>
<td>3</td>
</tr>
<tr>
<td>287 to 290</td>
<td>4</td>
</tr>
<tr>
<td>287 to 291</td>
<td>5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>287 to 354</td>
<td>?</td>
</tr>
</tbody>
</table>

SAY: By being organized, you might find the pattern quicker. Patterns can be easier to see when you have something organized to look at, like a table.

**Exercises:** Make several simpler problems until you see the pattern to complete the harder problem. Organize the simpler problems.

a) A fence is made using 42 posts, each 1 m apart. How long is the fence?
b) A fence is made using 34 posts, each 2 m apart. How long is the fence?

**Answers:** a) 41 m, b) 66 m

The importance of seeing given information visually. Tell students that you want them to think of a word that has the letters l, t, and r (sample answers: letter, later, trail, rattle, teller, retell, retail, trailer, relent, relate). After some students tell you an answer, ASK: Did anyone write the letters down so that you didn't have to remember them? SAY: There are many ways to make a problem easier. One of them is to write down details so you don't have to keep everything in your head.

**Drawing a diagram to solve a problem.** Write on the board:

Kyle decided to go for a walk in his neighbourhood. He started by going 1 block east. Then he turned left and went 2 more blocks. Then he turned left again and went 3 more blocks. He kept turning left and going 1 more block than the previous turn. His school is 5 blocks east of his home. How many blocks did he walk when he passed his school?

Give students time to read the problem, then ASK: What makes this problem hard? (there are a lot of words, it's hard to picture what is happening) Tell students you are going to read the problem aloud again, but this time you want students to close their eyes and imagine the diagram as you read it. Remind students that when facing north, east is on the right.
Read the problem aloud, and then have a volunteer draw a map on the board of the first few turns of Kyle’s walk. (see example)

If “Home” is not already labelled, have a volunteer mark where it is. SAY: When you have a diagram drawn, you don’t have to keep everything in your head. That means you can focus on solving the problem. ASK: Where is the school? (5 blocks east of home) Have different volunteers estimate where it is on the map. (see example below)

Tell students that it is hard to estimate because it is hard to see exactly how far from home each vertical line is. SAY: One tool we can use to make this easier is grid paper. Draw a grid on the board or project BLM 1 cm Grid Paper, and redraw the diagram, as shown below. Point out how much easier it is to say for sure how far each point is from home in each direction.

Point to various corners and have volunteers tell how far north or south and east or west of home the point is. ASK: Does the diagram show walking all the way to the school yet, or do we still have to draw more? (there will likely be more to draw; if so, have a volunteer do so)
Then label all the number of blocks on the diagram as one greater, except for the last block, as shown below:

Pointing to the last turn, ASK: How many blocks north did he walk on his last turn? (4) How does the grid make it easy to see this? (I can just count the squares) SAY: Without the grid, you would have to calculate how many blocks south he went altogether, so you would have to look at all the times he went north and south and see how they cancel each other out. You can still do it, but it would take more work.

SAY: If you are ever taking a test, and you don’t have grid paper, you can draw the grid yourself. Show students a rough drawing of the grid on the board. SAY: Now that you know all the distances, you can find the total distance. Write on the board:

\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 4 = ____ \]

Ask volunteers to find and explain a quick way to add this long list of numbers. If necessary, remind students that there is an easy way to add many numbers together: look for pairs that make 10 and add those first: \((1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 5 + 4 = 49\). Write “49” in the blank.

**Exercises:** Draw a diagram to solve the problem.

a) Yu walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, then turns right again and walks 4 blocks. She then turns right again and walks 4 blocks, turns right again and walks 3 blocks, then turns right again and walks 2 blocks, and then turns right again and walks 1 block. Where does she end up, relative to home?

b) Yu follows the same pattern as in part a) but goes 10 blocks before starting to count down. Where does she end up, relative to her home?

c) Yu walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, then turns right again and walks 4 blocks. She then turns left and walks 4 blocks, turns left again and walks 3 blocks, then turns left again and walks 2 blocks, and then turns left again and walks 1 block. How far does she end up from home, and in which direction?

**Bonus:** Yu follows the same pattern as in part c) but changes the direction of turns from right to left after \(4n\) blocks instead of after 4 blocks. How far does she end up from home and in what direction?

**Answers:**
a) at home, b) 1 block west and 1 block south, c) 4 blocks west,
Bonus: \(4n\) blocks west
Focusing only on relevant information to make a problem simpler. Draw on the board:

![Diagrams](image)

Point to the first diagram and ASK: What is this problem asking you to do? (find the length of the thicker stick) What are the other two problems asking you to do? (find the length of the thicker stick) What makes the first problem look easier to do than the other two? (the numbers are whole numbers; the third problem looks harder because the sticks are not right next to each other) SAY: There’s a lot of extra information in this third problem, so it looks harder, but it actually has exactly the same answer as the other one, so you might as well complete the easier one. The total length of the two sticks at the bottom is still 17.6, they are just not side by side anymore.

**Exercises:** All measurements are in centimetres. Find what the question mark stands for by making the problem into a simpler problem.

a) ![Diagram](image)

b) ![Diagram](image)
Answers: a) 7 cm, b) 9.2 cm, c) 4.73 cm, d) 1.05 cm

SAY: If you need to find a vertical edge—straight up and down—then colour over all the vertical lines. If you need to find a horizontal edge, colour over all the horizontal lines.

Exercises: Find what the question mark stands for by making the problem into an easier problem.

Answers: a) coloured vertical, ? = 5 cm; b) coloured horizontal, ? = 8.65 cm; c) coloured horizontal, ? = 2.567 cm; d) coloured vertical, ? = 1.28 cm

Point out to students that by colouring over the horizontal or vertical lines, they changed the problem into an easier problem.
**Finding perimeter without knowing all the side lengths.** Remind students that to find the perimeter of a shape, they add up the lengths of all the sides. Draw on the board:

```
  +---+---+
 |   |   |
  +   +---+
    5      20
    |      |
    |      |
  +---+---+
    3      |
```

SAY: I want to find the perimeter of this shape. It looks like a hard problem because there are a lot of missing side lengths. Ask a volunteer to mark three sides that you do not know the length of. (the two bottom horizontal sides and the right side) SAY: There are two kinds of sides in this shape: horizontal sides and vertical sides.

ASK: How long is the top side? (20) How long are the two bottom sides put together? (20) How do you know? (put together, they are the same length as the top side) How long are the two sides on the left of the shape? (5 and 3) How long is the side on the right? (8) How do you know? (it’s the same as the two left sides put together) Write on the board:

Horizontal edges add to _______  Vertical edges add to _______

Perimeter is ______ + ______ = _______

Have volunteers fill in the blanks. (40, 16, 40 + 16 = 56)

**Exercises:** Find the perimeter of the shape.

a)  
```
  +---+---+
 |   |   |
  +---+---+
    13      8
    |      |
    |      |
```

b)  
```
  +---+---+
 |   |   |
  +   +---+
    4.6    4
    |      |
    |      |
    +---+---+
      8
```

c)  
```
  +---+---+
 |   |   |
  +   +---+
    3      5
    |      |
    |      |
    +---+---+
      1
```

**Answers:** a) 48, b) 56.2, c) 34
Problem Bank
1. When everyone in Tom’s class stands in line, Tom is 14th in line and 11th from the end of the line. How many people are in the class?
   Answer: 24

2. There are 126 people in line. How many people are behind the 94th person?
   Answer: 32

3. Make several simpler problems until you see how to complete the harder problem.
   a) A fence is made using 53 posts, each 3 m apart. How long is the fence?
   b) A fence is made using 61 posts, each 2.5 m apart. How long is the fence?
   Answers: a) 156 m, b) 150 m

4. How many posts are needed to make the fence?
   a) A fence is 47 m long with posts at 1 m intervals.
   b) A fence is 100 m long with posts at 2.5 m intervals.
   c) A fence is 84 m long with posts at 3.5 m intervals.
   Answers: a) 48, b) 41, c) 25

5. A fence for a square garden is made with posts 1.5 m apart, including a post at each corner. How many posts are needed for the garden? Hint: Start with a garden that is 1.5 m by 1.5 m and then move on to 3 m by 3 m, 4.5 m by 4.5 m, and so on.
   a) The garden is 12 m by 12 m.
   b) The garden is 21 m by 21 m.
   Answers: a) 32, b) 56

6. Predict each answer before checking. A field is a square 30 m by 30 m. How many posts are needed if the posts are ...
   a) 1 m apart? b) 2 m apart?
   c) 1.5 m apart? d) 2.5 m apart?
   Bonus: 60 cm apart?
   Answers: a) 120, b) 60, c) 80, d) 48, Bonus: 200

7. Cut out the strips and circles from BLM Fraction Strips and Circles (you may cut the line down to the centre of the circles). Estimate to colour the given amount. Use folding to check your estimate.
   a) one fifth of a strip of paper, starting from the left
   b) two fifths of a strip of paper, starting from the left
   Hint: Use your answers to parts a) and b) to help you determine a strategy for parts c) and d).
   Hold the circle so that the cut line is at the top.
   c) one fifth of a circle, starting from the top
   d) two fifths of a circle, starting from the top
8. a) Find the perimeter.

b) Is there enough information to find the area of this shape? Explain how you know.

Answers: a) 36.6; b) no, we don’t have the side length for the small rectangles

9. What is the length of the thick-line path from A to B?

Solution: 7 + 7 + 15 = 29, so 29 m

10. Each shape was made by placing a small square on top of a large square. All measurements are in centimetres.
   a) Find the perimeter of each shape.

   i) ![Perimeter 1](image1)
   ii) ![Perimeter 2](image2)
   iii) ![Perimeter 3](image3)
   iv) ![Perimeter 4](image4)

b) Make a table with headings “Size of Smaller Square” and “Total Perimeter.” Use the pattern from part a) to solve the problems.

   i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?

   ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?

Answers: a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm
1 cm Grid Paper
Fraction Strips and Circles
A Note About Terms in Probability

Outcomes and Events

A simple action such as rolling a die, flipping a coin, or spinning a spinner has various possible results. These results are called the **outcomes** of the action. If you flip a coin, the outcomes are: “You flip a head” and “You flip a tail.” When you describe a specific outcome or set of outcomes, such as rolling a 6, rolling an even number, tossing a head, or spinning red, you identify an **event**.

In probability theory, the terms **outcome** and **event** have very precise meanings. But in elementary texts, outcome is occasionally misused.

This spinner has 3 coloured regions:

![Spinner with R, B, G]

There are 3 possible outcomes of spinning the spinner:

1. The pointer lands in the blue region.
2. The pointer lands in the red region.
3. The pointer lands in the green region.

Some textbooks will identify the outcomes as:

1. You spin blue.
2. You spin red.
3. You spin green.

What’s the difference? Identifying outcomes with only colours and not regions can cause confusion when 2 or more regions of a spinner are the same colour. Here is a spinner with 4 coloured regions:

![Spinner with R, B, B]

There are 4 outcomes of spinning the spinner:

1. The pointer lands in the blue region at top right.
2. The pointer lands in the blue region at bottom right.
3. The pointer lands in the blue region at bottom left.
4. The pointer lands in the red region.

This is clearly not the same as saying that the outcomes are:

1. You spin red.
2. You spin blue.

“You spin blue” and “You spin red” are events, not outcomes. To assess the relative likelihood of spinning red or blue, students must recognize that the pointer may land in 4 distinct regions of the spinner. In 3 of the 4 outcomes, the spinner lands in a blue region. Hence, the event “You spin blue” is more likely than the event “You spin red.”
The outcomes for QUESTION 3 a) on Worksheet PDM6-21 are “The spinner lands in region 1,” “The spinner lands in region 2,” “The spinner lands in region 3,” and “The spinner lands in region 4.” Your students may write something more concise, such as “You spin a 1,” “You spin a 2,” “You spin a 3,” “You spin a 4.” Accept these answers, provided students understand that when different regions of a spinner have the same number or colour, each region must be counted as a distinct outcome.

To avoid confusion, we only use the term “outcome” on the worksheets when the regions of the spinner are uniquely coloured or labelled. When the same colour or label appears more than once on the spinner, we use phrases like “ways of spinning red” instead of “outcomes”.

**Probability and Outcomes**

In probability theory, an “event” is a particular “subset” of a set of outcomes. For instance, on the spinner above, the event “You spin blue” is the subset of outcomes in which the pointer lands in a blue region. On a die, the event “You roll an even number” is the subset of outcomes: {“You roll a 2,” “You roll a 4,” “You roll a 6”}. Your students needn’t learn the technical meaning of the term “event” until they are older. (For now, tell them an event is any particular result of an action or occurrence they wish to assess the likelihood of.) But they should know that the “outcomes” of an occurrence are all the ways the occurrence could happen.

The probability or likelihood of an event may be expressed as a fraction: given a set of outcomes (where each outcome is equally likely), the denominator of the fraction is the total number of outcomes and the numerator is the number of ways the event could happen.

\[
\text{Probability} = \frac{\# \text{ of ways the event can happen}}{\# \text{ of outcomes}}
\]

If you flip a coin, the probability of flipping a head is \(\frac{1}{2}\), since there are two outcomes of flipping the coin, but only one way to flip a head.

If you roll a die, the probability of rolling a five is \(\frac{1}{6}\), since there are six outcomes but only one way to roll a five.

On the spinner below, the probability of spinning red is \(\frac{3}{8}\), since there are eight regions (all of the same size) in which the pointer might land, but only three of the regions are red.

**Expectation**

The probability of spinning blue on the spinner below is \(\frac{1}{3}\).

If you were to spin the spinner 12 times you would expect to spin blue \(\frac{1}{3}\) of the time, or \(12 \div 3 = 4\) times.
Tell students that today they will start learning how to predict the future!

Hold up a die and ask students to predict what will happen when you roll it. Can it land on a vertex? On an edge? No, the die will land on one of its sides. Ask students to predict which number you will roll. Then roll the die (more than once, if necessary) to show that the prediction about landing on a side works, but the number they picked does not necessary come. Explain that the possible results of rolling the die are called outcomes, and to predict the future students must learn to identify which outcomes of various actions are more likely to happen and which are not. But first, they must learn to identify outcomes correctly.

Hold up a coin and **ASK:** What are the possible outcomes of tossing a coin? How many outcomes are there? Show a spinner and a set of marbles. What are the possible outcomes of spinning the spinner or picking a marble with your eyes closed? Ask students to identify the possible outcomes of a soccer game. How many outcomes are there? (3 outcomes: team A wins, team B wins, a draw)

**ASK:** You have to make a spinner with 5 possible outcomes. How would you do this? Invite volunteers to draw possible spinners. If the spinner in the picture at below does not arise, draw it and **ASK:** How many outcomes are there for this spinner?

Are all the outcomes bound to come equally, or is there an outcome that they think will happen more often than the other ones? Draw the second spinner and **ASK:** How many outcomes does the second spinner have? (4) Will the pointer ever be in the grey region? (no, never)
Tell students that you’re going to flip a coin. **ASK:** What are the outcomes of flipping a coin? (heads and tails) Is one outcome more likely than the other? (no) If we flip the coin a few times, how often do you think we’ll get tails? How often do you think we’ll get heads? Explain that since there are only two outcomes and each is equally likely, you would expect the coin to land on heads half the time and tails the other half of the time.

Now test this expectation.

Have students flip a coin ten times and keep a tally of the number of heads. Point out that (unless a miracle occurred in your class) not every student flipped heads exactly half the time. Ask the students to identify the result that was furthest from the expected number of 5 heads. Then combine all the results of the entire class (i.e., add up the total number of heads from all the tallies). The overall proportion of heads should be closer to half of the total number of tosses. Explain to your students that the more trials you conduct (i.e., the more times you repeat an experiment) the more closely the actual result will match the expected result for the event.

**Extensions**

1. Bill has a pentagonal pyramid with numbers on the sides. He puts a non-base side on the table and rolls the pyramid. How many outcomes are there? Make a pentagonal pyramid, number the sides, and check your answer. Are there sides the pyramid never lands on if you roll it in the manner described? (Yes—the base. It will not roll if it stands on a base.) Repeat with a hexagonal prism and an octagonal prism: predict the number of outcomes then make a model to check your prediction.

2. Ed and George are playing “Rock, paper, scissors.” Describe all the possible outcomes of the game. **(NOTE:** “Ed wins” is not an outcome, it is an event. “Ed has paper, George has rock” is an outcome.)
PDM6-22
Probability

Show your students two pencils of different length. Ask them how they could determine which pencil is longer. Then show two objects where direct comparison is impossible, such as a ruler and the circumference of a cup. Students might suggest measuring the length with a measuring tape. Then ask your students how they could compare the weight of two objects, say a book and a cup. What could they do to compare the temperature in two different places? Point out that in all cases they tried to attach numbers to each object and to compare the numbers. They used different tools for that purpose—measurement tape, scale or thermometer. Each tool provided a number, i.e., a measurement. What would they do to compare the likelihood of two events, such as the likelihood of rolling 8 on a pair of dice and the likelihood that their favourite hockey team wins 5 to 3 in the next game? Is there a tool to measure likelihood? No. Explain that probability is the branch of mathematics that studies likelihood of events and expresses the likelihood of various events in numbers. The measure of likelihood of an event is called probability.

Give each student a spinner divided into three equal parts marked by 1, 2 and 3 respectively. Ask your students to identify the possible outcomes of spinning the spinner. ASK: Is the spinner equally divided? Is there any part that the spinner should land on more than other parts? You are going to spin the spinner 15 times. How many times do you think the spinner should land on each part? Why? Ask them to spin the spinner 15 times and tally the results. Are the results exactly as they expected? Ask groups of 4–5 students to combine their results. How many spins did they do altogether? How many times do they expect to spin each number? How many times did they spin each number in fact? Is that result nearer to the prediction? Pool the results of the whole class and repeat.

Now draw on the board a spinner equally divided into three regions, coloured into three different colours, say red, blue and green. Ask your students to compare this spinner with the spinner they used. If they were to spin this spinner, would the results be different from the first time? (No—the spinners are essentially the same, they have three different regions of the same size, and each region is expected to come the same number of times.) ASK: If you spin this spinner 150 times, how many times would you expect to spin each colour?

Change the colouring of the spinner to be as shown below.

![Spinner diagram]

ASK: How is this spinner different from the previous one?
How many different regions does the spinner have? How many different ways can you spin red? (Only one.) How many different ways can you spin blue? (Two ways) Since the regions of the spinner are all the same size, it is equally likely that the spinner will land in any of the regions. If you spin this spinner 150 times, about how many times do you expect the spinner to land in each region? How many times do you expect to spin red? How many times do you expect to spin blue? Point out that two regions are coloured blue and only one is coloured red, students should see that it is twice as likely that they will spin blue as it is that they will spin red.

Explain that in mathematics we describe probability as a fraction:

\[
\text{Probability} = \frac{\# \text{ of ways the event can happen}}{\# \text{ of possibilities}}
\]

For our spinner, there are three possible outcomes. (Even though there are only two colours there are three regions). So the probability of spinning red is \( \frac{1}{3} \) and the probability of spinning blue is \( \frac{2}{3} \). **ASK:** Which fraction of the spinner is coloured red? Which fraction of the spinner is coloured blue?

Draw the following collection of marbles on the board:

\[\text{R, W, R, Y, R, W}\]

**ASK:** How many yellow marbles are in the collection? How many marbles are there altogether? What fraction of the marbles is yellow? Write both the questions and the answers on the board.

Then write and **ASK:** How many ways can you draw a yellow marble from the collection? How many ways can you draw a marble of any colour? What is the probability of drawing a yellow marble?

Ask students to compare the two sets of questions. How are they different? How are they the same? The answers to each set of questions are the same, but the first set deals with fractions and the second set deals with probability. Explain that both sets of questions are asking for the same information, but the first is asking in terms of fractions and the second is asking in terms of probability. This illustrates the application of fractions to probability.

Let your students practise determining and expressing probability with more questions about this collection of marbles:

- How many ways can you draw a white marble? What is the probability of drawing a white marble?
- How many ways can you draw a red marble? What is the probability of drawing a red marble?
- Is there any way to draw a green marble? What is the probability of drawing a green marble?

Review visual representation of fractions with your students. Show your students a spinner below.

\[\text{R, B, B, W}\]

Ask your students to predict which colour is most likely to come out if you spin this spinner. Which colour is least likely to come out? **ASK:** Do you expect the spinner to land on red as often as on white? Invite volunteers to spin the spinner several times and tally the results. Do the results fit the expectation? If not, ask your students what they could do to make the experiment results nearer to the expectation. From their experience with probability experiments in the previous grades the
students should suggest to conduct an additional series of experiments. Do so to convince the students that white comes out more often than red.

**ASK:** What do we get if we write a fraction where the numerator is just the number of white regions and the denominator is the number of all regions? \( \frac{1}{5} \) Repeat with the red colour. Did we get the same fractions? (yes) However, we want these fractions to measure the likelihood of spinning red and spinning white. If one of them is more likely to happen than the other, the fractions should be different. This means that we made a mistake in our calculations. Ask your students if they can guess what was wrong.

Explain that the number of possibilities in the definition of probability refers to equal possibilities. The red part of the spinner is smaller than the white part. Which part of the spinner is white? Which part of the spinner is red? How can we make sure that the spinner is \( \frac{1}{8} \) red? Can we divide the spinner into equal parts so that each part is coloured in one colour? (The spinner now is divided into quarters, but one of the quarters is coloured in two colours.) Invite a volunteer to cut the spinner into equal parts. **ASK:** How many regions does the spinner have now? (8) How many regions are coloured red? What is the probability of spinning red? \( \frac{1}{8} \) White? \( \frac{2}{8} \) Blue? \( \frac{5}{8} \)

Review the concept of reducing fractions with the students and ask them to write the probability of spinning white on the spinner above as a reduced fraction.

Ask your students to write a fraction that gives the probability of rolling a multiple of 3 on a die.

To solve this question, ask your students to write out all the possible outcomes of rolling a die. Then ask them to circle the numbers that are multiples of 3. How many are there? Ask students to write the probability as a fraction and to reduce the fraction to lowest terms. Students need more practice with similar problems.

Show your students the following collection of marbles:

Give your students two dice. Ask them to roll the dice and to write a fraction where the numerator is given by the least number rolled, and the denominator is given by the larger number rolled. The students then have to draw a spinner or a collection of marbles so that the probability of spinning blue or drawing a blue marble equals the fraction.

**ACTIVITY 2**

Give each pair of students 6 to 12 different beads. Ask them to list all the attributes of the beads that they can think of. Player 1 picks a bead and chooses an attribute, and Player 2 has to write the probability of picking a bead that has this attribute. For example, the attribute is colour and Player 1 holds up a red bead. Player 2 has to tell the probability of picking a red bead from the set.

**ADVANCED:** Player 1 picks a bead, thinks of an attribute without telling Player 2, shows the bead to his partner and gives the probability of picking a bead from the set that shares the chosen attribute. Player 2 has to guess which attribute was chosen.
GOALS
Students will describe probability of simple events using fractions.

PRIOR KNOWLEDGE REQUIRED
Fractions of sets, numbers
Visual representations of fractions
Reduction of fractions
Percentages as fractions and ratios

VOCABULARY
probability fraction
outcome fair game equally likely

SAY: I would like to play a game with you. The rules of the game are simple.
I will spin a spinner. If I get red, I win; if I get blue, the class wins. Ask students
if they agree to play by these rules. Now show them the spinner.
Do they still want to play? Why not?

Write the term fair game on the board. Ask students to explain what they
think this term might mean. Encourage students to use math vocabulary
in their explanations. Point out that in a fair game, both players have equal
chances, or are equally likely, to win. In other words, the probability of
winning is the same for all players. Does this mean there can never be a
draw? No. Use this game to illustrate: Flip a nickel and a penny. If both give
heads, you win. If both give tails, the class wins. If there is one head and
one tail, it is a draw—no one wins. Explain the rules to the class, then invite
volunteers to list all the possible outcomes and to find the probability of
winning for both players.

ASK: Is the game fair? Why? Would the game stay
fair if we added a third player who wins when one of the coins gives a head
and the other a tail? Would the game be fair if the third player wins with head
on the nickel and tail on the penny?

ASK: Jane says that a game is fair if each player’s chance of winning is 50%.
Is she correct? (Yes. If each player’s chance of winning is 50%, their chances
are equal and the game is fair.) Is this the only way a game can be fair?
In other words, are there other percentages that make a game fair? Use the
game with the penny and the nickel, above, to illustrate that what matters
isn’t the exact percentage, it’s only that the chance of winning be the same
for each player. By the first set of rules (2 heads, I win; 2 tails, you win; 1 head
and 1 tail, draw) each player’s chance of winning is only 25%, but it’s the
same for each player, so the game is fair. When the third player who wins
with a head and a tail is added, his chances of winning are 50%, but the
other two players still have a 25% chance of winning. This game is not fair
because one player has a greater chance of winning than the others. In the
third version of the game, when the third player wins only with a head on the
nickel and a tail on the penny, each player’s chance of winning is 25% and
the game is fair again.

Give the rules for another game: There are 6 marbles in a box. If you draw
a red marble, you win; if it is blue, the class wins. Otherwise, the game
is a draw. There are 2 red marbles in the box. ASK: To make the game fair,
how many blue marbles should be in the box? What about the rest of the
marbles—what colour can they be? Can you think of another combination
of marbles that will make the game fair?
Vary the game: If you draw a red marble you win, but if you draw any other colour, the class wins. If 2 of the 6 marbles are red, who has more chances to win—you or the class? What if 5 marbles are red? What should be in the box to make the game fair?

Let your students describe the probability of throwing a dart (assume the dart always hits the board) and landing on each of the colours:

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You may also ask some more complicated questions, such as: what is the probability of getting red or yellow? What is the probability of getting any colour other than blue? What is the probability of getting a colour that is on the flag of Canada? On the flag of your province? Which two colours have the same probability of occurring? Find a combination of colours such that the probability of the dart landing on one of the colours is \( \frac{5}{8} \). Ask your students to make up their own problems using this chart.

**Assessment**

1. Two players are spinning this spinner. Invent two different rules of play to ensure that the game is fair.

2. What is the probability of drawing a red marble from the box with 3 red marbles, 2 white marbles and 3 green marbles?

Pair up students and give each pair a container with 1 red counter and 5 blue counters. Player 1 wins if they draw a red counter and Player 2 wins if they draw a blue counter. Ask students to explain whether the game is fair or not. Have students play the game 20 times (replacing the counter each time) and keep a tally of who wins each time.

Ask your students to summarize the results of the experiment using the following prompts:

I played a game of chance with 1 red and 5 blue counters. The rules of the game were:

_____________________________________________________________________________________

I expected to win about ____ of the 20 games. The probability that I will win is ____ because

_____________________________________________________________________________________

I actually drew a red/blue counter ____ times.

I think that the game is fair/unfair, because _____________________________________________

I learned that _____________________________________________
Ask students to keep track of who wins and loses in 20 repetitions of the following game:
Players roll a die. Player 1 wins if a 1 is rolled. Player 2 wins otherwise.

**ASK:** Is this game fair? Are the results what you expected? What is the theoretical probability of Player 1 winning? Are the experimental results what you expected? What should you do to get the results nearer to the expectation?

### Extensions

1. Emily has a bag of 8 marbles. 5 are blue. Peter has a bag of 7 marbles. 3 are blue.
   
   a) What is the probability of drawing a blue marble from Emily’s bag?
   
   b) What is the probability of drawing a blue marble from Peter’s bag?
   
   c) Emily and Peter pour all of their marbles into one bag. What is the probability of drawing a blue marble from the bag?

2. Mark draws all possible rectangles with a perimeter of 12 cm (with whole number sides). If he picks one of the rectangles at random, what is the probability that it will have length of 3 cm?

3. How are the games in Activities 1 and 2 similar? Is the first game less fair than the second? (In terms of probability, the two games are identical. In both games, the second player has 5 out of 6 chances to win, so the probability of Player 2 winning is \(\frac{5}{6}\).) Create a spinner and invent rules for a game that will have the same probability of winning as in these activities.

4. Carl and Clara played a game based on luck. Carl won 15 times and Clara won 12 times. Does this mean the game is not fair?

5. Carmel wants to express probability by drawing rectangles. For example, he draws a rectangle to represent the probability of tossing a head on a coin. The probability of tossing a head is \(\frac{1}{2}\), so he shades \(\frac{1}{2}\) of the rectangle.

   Use Carmel’s method to show the probability of:
   
   a) Drawing a red marble from a box with 4 red marbles and 8 green marbles
   
   b) Picking a consonant from the word “rectangle”
   
   c) Spinning blue on the spinner below.

[Image of a spinner with B and W]

6. Draw a spinner so that the probability of spinning red will be the same as the probability of rolling a number less than 3 on a die. Make up your own question like this one: ask the reader to design an action or object (EXAMPLE: spinner, collection of items) so that a specific event happens with the same probability as another given event.

7. What is the probability of a child being born a boy? Can you name 3 different events that will happen with the same probability? (Possible answers: Tossing tails on a coin, drawing a red marble from a box with 6 red and 6 white marbles, choosing a left boot from a regular pair of boots.)
PDM6-24
Expectations

Show your students this spinner and ask them which part of it is coloured in each colour. What are the possible outcomes of spinning this spinner? How many outcomes are there? (3) What is the probability of spinning each of the colours? Are the chances of spinning blue the same as the chances of spinning red? Why? What about the chances of spinning green and the chances of spinning blue?

**SAY:** I am going to spin the spinner 12 times. How many times do you expect me to spin blue? Why? Write the calculation on the board:

\[
\frac{1}{3} \text{ of } 12 = 4 \quad \text{OR: } \quad 12 \div 3 = 4 \text{ times}
\]

Remind students that actual outcomes usually differ from expected outcomes. You might not get blue 4 times every time you make 12 spins, but 4 is the most likely number of times you will spin blue.

Remind students that when we say “the chances of spinning blue are 1 out of 3,” we mean that out of 3 spins 1, on average, is blue. What does this mean? Suppose 4 people each spin this spinner 3 times. They write the number of times they got blue. We expect the mean of this set of data to be 1. Invite 4 volunteers to conduct the experiment and to write down the number of times they spun blue. Ask another volunteer to find the mean. Is the experimental mean the same as the expected mean?

Review with students how to find a fraction of a set and a fraction of a number. Practise by solving these and other questions:

- \(\frac{1}{2} \) of 14
- \(\frac{1}{3} \) of 15
- \(\frac{1}{5} \) of 25
- \(\frac{3}{4} \) of 20
- \(\frac{2}{3} \) of 9
- Quarter of 16
- Three eighths of 24

Which part of the set “R G R G G R Y Y G Y” is G?

**ASK:** Which part of this spinner is blue? What are the chances of spinning blue? (3 out of 4) What is the probability of spinning blue on this spinner?

Explain that you can determine the number of times you would expect to spin blue out of 20 spins as follows:
STEP 1: \( \frac{1}{4} \) of 20 is \( 20 \div 4 = 5 \).
This is how many times you expect the spinner to land in each region.

STEP 2: Three regions are blue. You expect to spin blue \( \frac{3}{4} \) of all times. If you expect the spinner to land in each region 5 times, and 3 of the regions are blue, then the spinner will land in a blue region \( 3 \times 5 \) times:
\[
\frac{3}{4} \text{ of } 20 = 3 \times 5 = 15.
\]
You can show this with a picture:

\[
\frac{1}{4} \text{ of } 20 = 20 \div 4 = 5
\]

Therefore, \( \frac{3}{4} \text{ of } 20 = 3 \times 5 = 15 \)

Have students practice calculating expected outcomes with these and similar questions:

- If you flip a coin 16 times, how many times do you expect to get a tail?
- Hong wants to know how many times he is likely to spin green if he spins this spinner 24 times. He knows that \( \frac{1}{3} \) of 24 is 8 (24 ÷ 3 = 8). How can he use this information to find how many times he is likely to spin green?
- If you roll a die 18 times, how many times do you expect to get a 4? To get a 1?
- How many times would you expect to spin blue if you spin this spinner 50 times? How many times would you expect to spin green?

CHALLENGING: If you roll a die 30 times, how many times do you expect to roll an even number? How many times do you expect to roll either 4 or 6?

You should review long division with your students before you assign the worksheets for this lesson.

Assessment
Rea spins this spinner 30 times. How many times is she likely to get blue?
Divide students into 2 groups. Each group will conduct a different experiment and share the results with the second group afterwards.

**Group 1**

Students will each need a die. Ask them to list all the outcomes of rolling a die and to count the outcomes that suit each event listed below. Then ask them to describe each event as likely, unlikely, or having even chances:

1. Roll a number greater than 2.  
2. Roll a number greater than 5.  
3. Roll an odd number.

Each student rolls the die 12 times and tallies the results. Do the results match the predictions? Have students combine their results. (This group tally chart—how many times the group rolled 1, 2, and so on—may be useful during the next lessons.) How many rolls did the whole group make? How many times does the group expect to get an even number? A number greater than 5? A number greater than 2? Students should explain their answers. Do the group’s results match the predictions better than the individual results?

Students can create a table like this to summarize predictions and results as they conduct the experiment:  
**TOTAL OUTCOMES:** 1, 2, 3, 4, 5, 6  
**NUMBER OF POSSIBLE OUTCOMES:** 6

<table>
<thead>
<tr>
<th>Event</th>
<th>Suitable Outcomes</th>
<th>Individual Results (12 Rolls)</th>
<th>Group Results (___ Rolls)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll a number &gt; 2</td>
<td>3 4 5 6</td>
<td>Predicted out of 12</td>
<td>Predicted Actual</td>
</tr>
<tr>
<td>Roll a number &gt; 5</td>
<td>6 1  out of 6</td>
<td>Predicted out of 12</td>
<td>Predicted Actual</td>
</tr>
<tr>
<td>Roll an odd number</td>
<td>1 3 5 3 out of 6</td>
<td>Predicted out of 12</td>
<td>Predicted Actual</td>
</tr>
</tbody>
</table>

**Group 2**

Students will each need 3 red marbles, 2 yellow marbles, 1 green marble, and a non-transparent bag or box (i.e., a lunch box).

Ask students to list all the outcomes of drawing a marble from the box and to count all the outcomes that suit each event from the list below. Ask students to describe each event as likely, unlikely, or having even chances:

1. Draw a red marble.  
2. Draw a green marble.  
3. Draw a marble that is not yellow.

Each student draws a marble 12 times (returning the marble after each draw and shaking the bag) and records the results in a tally chart. Do the individual results suit the prediction? Have students combine their results. Do the group’s combined results suit the predictions? Students can create a table similar to that used by Group 1 to summarize their results.

Ask the students to write a summary of the experiments. Prompts for Group 1:

I performed an experiment with a die. I rolled the die ___ times.
I expected that ___ of ___ rolls would give a number greater than 2. I got a number > 2 in ___ rolls.
I thought that it was _____________ to get a number > 2, because _____________
I learned that _____________

Discuss the similarities and differences between the experiments of the 2 groups.
Bonus
1. You flip two coins, a nickel and a dime 12 times. List the possible outcomes. How many times do you expect to get one head and one tail?

2. Jack and Jill play “Rock, paper, scissors” 18 times. List all possible outcomes of the game. (HINT: “Jack has rock and Jill has paper” is different from “Jack has paper, Jill has rock”! Why?) How many times do you expect to see a draw? What is the probability of a draw?

Extensions
1. Design an experiment with three possible outcomes in which one of the outcomes has a probability near \( \frac{1}{2} \).

2. Choose a novel. Open it to any page and note whether or not the first letter is a “t”. Check 10 pages in this manner. Describe the probability that “t” is the first letter on a page of the book.

3. Give your students two dice of different colours, say red and blue. The red die will give the numerator of a fraction and the blue die will give the denominator. Roll the dice 40 times and record the fractions. How many times did you get a fraction that was reduced to lowest terms? What is the experimental probability of getting a reduced fraction? Pool the results of the whole class. Are they all the same? What is the experimental probability of getting a reduced fraction from the results of the whole class? Which result is more likely to match the theoretical probability?

List all the possible combinations of the two dice as fractions (there are 6 different results of the red die for every result of the blue die, so the total number is 36 combinations). Count the reduced fractions. What is the theoretical probability of getting a reduced fraction? Compare with the experimental results and write a report about what you’ve learned.
PDM6-25
Describing Probability

Have students make some predictions. Ask them to tell you if the following events are likely or unlikely:

- The sun will rise tomorrow.
- The teacher will give the answers to the test before giving the test.
- An alien will walk into the class in the next minute.
- Students will have lunch in half an hour.
- It will rain tomorrow.
- It will snow in June.

Invite students to name some events and have other students tell if the events are likely or unlikely. You could ask students to compare the likelihood of events. For example, it is unlikely to snow in June, but it is more unlikely that an alien will walk into the class!

Explain to your students that mathematicians call an event likely if it is expected to happen more than half the time and unlikely if it is expected to happen less than half the time.

SAY: The probability of rolling a number less than 5 on a die is 4 out of 6, or \( \frac{4}{6} \). Is it a likely or an unlikely event? (Likely) How do you know? Give more examples, such as:

- The probability of spinning red is \( \frac{1}{4} \). Is it likely or unlikely you will spin red? How do you know?
- The probability of pulling a black sock out of a dark closet is 5/8. Are you likely or unlikely to pull out a black sock? How do you know?

Ask your students which word people would use to describe an event like meeting a live dinosaur in the street. Can that happen at all? Write the terms likely, unlikely and impossible on the board. Ask your students which words describe an event that will definitely happen, like rolling a number less than 7 on a die. Add the word certain to the list.

NOTE: Although students might use the word impossible to describe the likelihood of meeting a dinosaur, this event is not necessarily impossible (scientists might find a way to clone dinosaurs). The only events that are strictly impossible are events that are contradictory—like rolling a number greater than 6 on a die.

Ask students to give examples of various events and explain whether they are likely, unlikely, certain, or impossible. Encourage students to think of events using marbles, dice, money, and other objects, as well events from daily life, such as meeting a tiger or an astronaut on the way to school.

Ask your students to describe the probability of getting a number less than 17 when rolling a die. (Certain) Ask them to list all the possible outcomes of rolling a die, and ask them which outcomes are suitable (i.e., less than 17).
So what is the probability of a certain event, like rolling a number less than 17 on a die? (\( \frac{6}{6} \) or 1).
Repeat with an impossible event, such as rolling a number less than 1 on a die. (The probability of an impossible event is 0, since there are no suitable outcomes.)

Hold up a coin. **ASK:** What are the possible outcomes of flipping this coin? How many outcomes are there? What is more likely—to flip a head or a tail? Explain that the chances are the same—you have even chances of flipping a head or a tail. Add the term to the list and explain that the chances of an event are even when the event happens in exactly half of the outcomes. Flipping a tail is 1 out of 2 possible outcomes; 1 is half of 2. **ASK:** How many outcomes are there when you roll a die? (6)
How many outcomes are even numbers? (3) Since half of the outcomes are even numbers, you have even chances of rolling an even number (and an even chance of rolling an odd number).
What is the probability of rolling an even number on a die? (\( \frac{1}{2} \))

**SAY:** We have 8 marbles in a box. I take out a marble. How many outcomes are possible? (8, regardless of the colour of the marbles) What is half of 8? So if I want even chances to take out a green marble, exactly 4 marbles should be green. Does it matter what colour the other marbles are? (No, provided they’re not green.)

Draw the spinner below on the board and ask students to describe the possibility of spinning each of the colours. Is it equally likely that they will spin green or that they will spin blue? Is it equally likely that they will spin yellow or blue? Why? (**HINT:** Look at the angle at the centre.)

![Spinner Diagram]

Ask students to draw a spinner to match this description:

1. It is likely to get yellow.
2. It is unlikely to get green.
3. It is equally likely to get green and red.
4. It is impossible to spin blue.

Now write this list of properties:

1. It is impossible to get green.
2. It is certain to get blue.
3. It is likely to get red.
4. It is unlikely to get white.
5. It is equally likely to get white and red.
6. It is equally likely to get red and purple.
7. It is equally likely to get white and yellow.
8. It is equally likely to get green and orange.

**ASK:** Do you think we can make a spinner that matches all of these? Do any of the properties contradict each other? Look at 3 and 6: can a spinner match 3 and 6? (No. If the spinner is likely to give red, more than half of the spinner should be red. Then only less than half of it can be purple. But if it is equally likely to give red and purple, the red and the purple should have the same area. But more than half is never equal to less than half!) Have students pick 3 of the properties on the list and create a spinner that will match them. If the all-blue spinner does not arise, show that example yourself and ask students to identify the properties that describe it (1, 2, 5, 6, 7 and 8).
Which properties does this spinner match? (Same as the all-blue spinner—the pointer never lands on anything else.

Show students a collection of marbles or coloured counters:

G Y G B G
R B G G Y

ASK: Which colour are you most likely to pick? Which colour is less likely to be picked: yellow or red? So which colour is least likely to be picked? Which colours are equally likely?

Draw a line on the board:

impossible certain
0 1

Ask your students to mark on the line the probability of picking…

• a green marble • a blue marble • a yellow marble
• a red marble • a pink marble • a marble of any colour

Ask your students to find the numerical probability of each event and to mark the fractions on the line as well.

Assessment

1. Make a spinner that matches this description:

   • It is most likely you will spin green.
   • It is unlikely you will spin red.
   • It is most unlikely you will spin purple.
   • It is equally likely you will spin purple and blue.
   • It is impossible to spin yellow.

2. Explain why it is impossible to make a collection of marbles to match this description:

   It is impossible you will pick red. It is certain to pick blue. It is very unlikely you will pick green.

3. Change one word in the description of the collection of marbles above to make it possible. Draw the collection that matches the new description.
A Game for Two

Give students several marbles or counters of various colours. One player chooses a set of 12 counters or marbles, unseen to the partner. The player describes the set to the partner, using probability terms (certain, likely, unlikely, and so on). The second player has to reconstruct the set from the first player’s description.

Extensions

1. If you roll a die, are your chances of rolling a number greater than 2 unlikely, even or likely? Explain your answer.

2. Write the numbers from 1 to 10 on ten cards, one number on each card. Ask your students to draw 6 cards at random and check which of the following statements are true. Let students repeat the experiment several times, then ask them to say whether the following events are certain, impossible, or possible, if you select 6 cards at random:
   - The sum of the numbers will be greater than 60.
   - All the numbers will be even.
   - Two numbers will be neighbours.
   - The sum of the numbers will be greater than 20.
   - No numbers will be neighbours.
   - The sum of the numbers will be less than 12.
   - Three cards will be odd.

   Ask students to explain the answers using their number sense. For example, it is impossible to draw 6 cards with the sum that is greater than 60, because even if you draw the largest numbers, their sum is $10 + 9 + 8 + 7 + 6 + 5 = 45 < 60$.

3. Invent or describe a game where a certain player’s chance of winning is very close to certain. What are the other player(s) chances of winning?

4. Doug has a total of $95 in his wallet ($50, $20, $20, and $5). Which bill is he most likely to draw? Which bills are equally likely to be drawn?

5. Tell your students that the possibility of rain on Saturday is 60%, and the chances of rain on Sunday are 40%. Richard says that the chances of rain during this weekend are 100%. Is he correct? (Adapted from the Atlantic Curriculum).
PDM6-26
Games and Expectation

Present the following game of chance: There are 8 marbles in a box—3 green, 3 red, and 2 blue. If a red marble is taken out, Player 1 wins, and if a green marble is taken out, Player 2 wins. (Taking out a blue marble results in a draw.) Ask your students if the game is fair and why. (Both players have the same probability of winning: $\frac{3}{8}$.)

Tell students that Jack and Jill played this game 24 times, replacing the marble after each game. **ASK:** How many times do you expect Jack to win? How do you know? (We expect Jack to win 9 times because $\frac{3}{8}$ of 24 is 9.) Jack and Jill tallied the results. Which one of the following tables is most likely to be theirs?

<table>
<thead>
<tr>
<th></th>
<th>Jack</th>
<th>Jill</th>
<th>Draw</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Jack</th>
<th>Jill</th>
<th>Draw</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Jack</th>
<th>Jill</th>
<th>Draw</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Let your students explain their choice. Then give your students marbles and let them play the same game 24 times in pairs. Have them tally their results in tables and compare them to Jack and Jill’s. Which tables were nearest to the expected, or theoretical, results? Which tables were furthest from the expected results? Ask your students to add the results for all groups. Is the group experimental probability nearer to $\frac{3}{8}$? Discuss with students the differences between the theoretical and experimental results and between the individual and the group results.

Students can create bar graphs to show the results of the class’s experiment. Ask your students to make bars of fixed width and to colour them according to the colours of the marbles. The height of each bar will depend on each pair’s results (i.e., how many times that colour was chosen). Ask your students to cut the bar graphs and to glue the individual bars into longer bars representing the group results. Discuss with your students which bar graph is nearer to the expectation: the individual graph or the group graph.
Extensions

1. Students can make a scatter plot of the data they collected during the lesson. Each pair of students will be represented by a point with the coordinates (number of times Player 1 wins, number of times Player 2 wins). For example, because Jack wins when a red marble is drawn and Jill wins when a green marble is drawn, the first table above will be represented by the point (11, 8), as shown:

```
Green
8
6
4
2

Red
2 4 6 8 10 12 14
```

Have each pair add their point to the graph. Then ask students if they see any trends in the positions of the points. Most likely, the points will be grouped around point (9, 9), because that is the theoretical expectation of the result of each set of 24 games.

2. Give groups of students boxes with 6 red, 3 green, and 3 blue marbles but do not tell them how many of each colour are in the boxes! Tell them only there are 12 marbles of 3 colours and that all the boxes have the same contents. Students could conduct 12 or 24 experiments, drawing and replacing a marble each time. Let them plot the group results in a scatter plot. Can they infer the contents of the box from the graph? (Instead of being grouped around point (9, 9), the results will be grouped around point (6, 3) for 12 draws and around point (12, 6) for 24 draws.) Let students open the boxes and check their inferences.
3. Compare the following two games:

**GAME 1:** Players roll a die 30 times. Player 1 gets a point if a 1 or a 6 is rolled. Player 2 gets a point otherwise. The player with the greater number of points wins.

**GAME 2:** Players take turns rolling the die. The player rolling a die gets a point if a 1 or a 6 is rolled. His partner gets a point otherwise. The die is rolled 30 times. The player with the greater number of points wins.

a) How many points do you expect each player to get in each game?

b) Is each of these games fair? Why or why not?

c) What is the difference between these games and the second game of the activity?

**ANSWER:** The second game in the activity is a single event, whereas the games above are combinations of 30 events. The game in the activity is not a fair game, because the probability of Player 2 winning the game is \( \frac{2}{3} \), twice more than that of Player 1. In Game 1 this experiment is repeated 30 times without changing the roles, so the probability of the players to win is the same as in a single die roll. Player 1 is expected to get 10 points, and Player 2 is expected to get 20 points. Hence Game 1 is not a fair game. In Game 2 the roles of the players interchange. Each rolls the die 15 times and is expected to win in \( \frac{1}{3} \) of these cases, getting 5 points from these rolls. In each of the other 15 rolls his chances to win are \( \frac{2}{3} \), and he is expected to get 10 points more. This means each player is expected to get 15 points. They have even chances to win the total game, and Game 2 is a fair game.
PDM6-27
Puzzles and Problems

PDM6-27 is a review worksheet, which can be used for practice.

NOTE: The next three lessons are enriched units, beyond the curriculum expectations.

PDM6-28
Tree Diagrams

GOALS
Students will draw tree diagrams and describe outcomes of compound events.

PRIOR KNOWLEDGE REQUIRED
Probability as a fraction
Describe events as likely, unlikely, certain, impossible
Outcomes

VOCABULARY
probability
outcome
tree diagram

Explain to your students that mathematicians often use tree diagrams when they have to make choices and want to keep track of all the possible combinations. For example, Katie has 3 pairs of mittens and 2 hats. How many different outfits can she wear? Show your students how Katie can build a tree diagram to keep track of her choices:

<table>
<thead>
<tr>
<th>Hat</th>
<th>Mittens</th>
</tr>
</thead>
<tbody>
<tr>
<td>White hat</td>
<td>White mittens</td>
</tr>
<tr>
<td></td>
<td>Blue mittens</td>
</tr>
<tr>
<td></td>
<td>Green mittens</td>
</tr>
<tr>
<td>Blue hat</td>
<td>White mittens</td>
</tr>
<tr>
<td></td>
<td>Blue mittens</td>
</tr>
<tr>
<td></td>
<td>Green mittens</td>
</tr>
</tbody>
</table>

The first two branches lead to the first choice: white hat or blue hat. Once she’s chosen a hat, Katie can choose from 3 pairs of mittens: green, white, or blue. Each path along the diagram is a different outfit. For example, the highlighted path shows a combination of a white hat and blue mittens. The number of endpoints, or “leaves,” is the total number of possible combinations. In this case, Katie has a total of 6 different outfits.

Expand the tree diagram by adding a third choice: a scarf. Katie has 2 scarves, a white scarf and a green scarf. First she chooses the hat, then the mittens, and then the scarf. Add 2 new leaves to each endpoint in the current diagram to show her third choice. ASK: How many different combinations does Katie have? (12) How many of these combinations have 3 colours? (3) How many of the combinations have only 1 colour? (1) Two colours? (12 – 1 – 3 = 8) Let your students count the different paths to find the answers.
Explain to your students that tree diagrams make it easy to see all the different outcomes of an action or series of actions, such as putting together an outfit. When you know the total number of outcomes, you can quickly identify the number of suitable outcomes for a particular problem or question. **ASK:** Is it likely or unlikely that Katie will wear an outfit in two colours? Is it likely or unlikely that Katie’s outfit will be all white?

**SAMPLE PROBLEMS**

Draw a tree diagram to show the results of tossing a pair of dice. Is it likely or unlikely that one of the numbers is twice as large as the other?

A restaurant offers 3 main courses (chicken, fish, turkey) and 5 desserts (cake, ice cream, cookies, fruit, pie). Draw a tree diagram to show all the possible dining combinations.
Review the previous lesson. Tell students that another way to keep track of the outcomes for two or more actions is by using a T-table.

Show students these spinners:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>G</td>
</tr>
<tr>
<td>B</td>
<td>Y</td>
</tr>
</tbody>
</table>

Draw a T-table with these headings:

<table>
<thead>
<tr>
<th>Colour Spinner</th>
<th>Number Spinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>1</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
</tr>
<tr>
<td>Red</td>
<td>3</td>
</tr>
</tbody>
</table>

SAY: I am going to spin the colour spinner again. Suppose I get a different colour. What are the possible outcomes of the number spinner? Are they different from the previous spin? (No, we have the same three outcomes: 1, 2, 3.) Continue filling in the table for different outcomes. How many outcomes are there in total? (12)

ASK: How many times did we write each outcome of the first spinner? (3) Why? (one for each possible outcome of the second spinner) If the second spinner had 5 numbers, how many times would you write each outcome of the first spinner? (5)

Review the steps for systematically listing all of the different outcomes for 2 actions:

**STEP 1:** Determine all the outcomes for the second action, in this case spinning the number spinner. (There are 3 outcomes: 1, 2, 3.)
STEP 2: List each of the outcomes for the first action as many times as there are outcomes for the second action (in this case 3 times).

R
R
R
B
B
B
Y
Y
Y
G
G
G

STEP 3: Beside each outcome for the first action, list all of the outcomes for the second action.

R, 1
R, 2
R, 3
B, 1
B, 2
B, 3
Y, 1
Y, 2
Y, 3
G, 1
G, 2
G, 3

Students can create more T-tables to answer questions such as:

- Find all possible outcomes of rolling a die and spinning a spinner with three colours.
- Find all possible outcomes of rolling two dice.
- Find all possible outcomes of flipping two coins.
- Find all possible outcomes of “Rock, paper, scissors” for 2 players.

(NOTE: “I win” is not an outcome, it is an event! It can happen in 3 different ways: I have rock, you have scissors; I have scissors, you have paper; I have paper, you have rock.)
Remind students that systematically listing all possible outcomes makes it easier to answer probability questions. Then review some familiar probabilities. **ASK:** If I flip a coin, what is the probably of getting heads? (\( \frac{1}{2} \)) If I roll a die, what is the probability of rolling an odd number? (\( \frac{3}{6} \) or \( \frac{1}{2} \))

Now present the following problem:

If you flip a coin and then roll a die, what is the probability of getting heads and an odd number?

Let your students first guess what the probability is. Since the probability of each separate event is \( \frac{1}{2} \), students might think that the probability of the compound event (heads AND an odd number) is also \( \frac{1}{2} \). This is not the case. Have students build a tree diagram to see all the possible combinations and find the answer (\( \frac{1}{4} \)).

As a challenge, ask students to find the probability of getting heads OR an odd number. The answer is \( \frac{3}{4} \). Encourage your students to express the probability both in fractions and in percents.

Explain that in probability, an event that is a combination of two events is called a compound event. For example, getting heads when you flip a coin and rolling a 4 on a die is a compound event. Look at the tree diagram the class made above. **ASK:** How many outcomes were there for the first action (flipping a coin)? (2) How many outcomes were there for the second action (rolling a die)? (6) How many outcomes were there in total? (12) Students likely found the total number by counting all the leaves on the tree diagram. **ASK:** Do you think there is another, faster, way to calculate the total number of outcomes? Point out that 12 = 6 \times 2. In other words, the total number of outcomes can be found by multiplying the outcomes for each separate action. Students can check for themselves that this rule works by predicting the total number of outcomes for other sets of actions and drawing tree diagrams to check their answers. (Students could also look back at the tree diagrams and T-charts from previous lessons.)

Students can practise calculating the probabilities of compound events with these **SAMPLE PROBLEMS:**

1. Jennifer rolls two dice, one red and one blue. She uses the result of the red die as the denominator and the result of the blue die as the numerator of a fraction. List all possible combinations and find:
   a) the probability of rolling a fraction already reduced to lowest terms.
   b) the probability of rolling a fraction greater than 1.
   c) the probability of rolling a fraction less than 1.
2. Sindi flips 3 coins: 2 pennies and 1 nickel. Use a tree diagram to find:
   a) the probability of flipping 2 heads and 1 tail.
   b) the probability of flipping 2 tails and 1 head. What do you notice?
   c) the probability of flipping tails on 2 coins with a total value of 6 cents.

3. Sara draws 2 marbles from a box that contains 6 marbles in total: 3 red, 2 purple, and 1 brown. List all the possible combinations and find:
   a) the probability that both marbles are of the same colour.
   b) the probability that both marbles are not purple.
G6-21
Coordinate Systems

To illustrate the idea of a coordinate system you can start with the following card trick:

1. First, deal out nine cards—face up—in this arrangement:

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Row 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Row 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Row 3</td>
<td></td>
</tr>
</tbody>
</table>

2. Next, ask a student to select a card in the array and then tell you what column it’s in (but not the name of the card).

3. Gather up the cards, with the three cards in the column your student selected on the top of the deck. Show clearly how you do that.

4. Deal the cards face up in another 3 × 3 array making sure the top three cards of the deck end up in the top row of the array.

5. Ask your student to tell you what column their card is in now. The top card in that column is their card, which you can now identify!

6. Repeat the trick several times and ask your students to try to figure out how it works. You might give them hints by telling them to watch how you place the cards, or even by repeating the trick with a 2 × 2 array.

When your students understand how the trick works, you can ask the following questions:

- Would there be any point to the trick if the subject told the person performing the trick both the row and the column number of the card they had selected? Clearly there would be no trick if the performer knew both numbers. Two pieces of information are enough to unambiguously identify a position in an array or graph. This is why graphs are such an...
efficient means of representation: two numbers can identify any location in two-dimensional space (in other words, on a flat sheet of paper). This discovery, made over 300 years ago by the French mathematician René Descartes, was one of the simplest and most revolutionary steps in the history of mathematics and science: his idea of representing position using numbers underlies virtually all modern mathematics, science, and technology.

You might ask your students how many numbers would be required to represent the position of an object relative to an origin in three-dimensional space. (The answer is three. Think of the origin as being situated on a plane or flat piece of paper that has a grid or graph on it. You need two numbers to tell you how to travel from the origin along the grid lines on the plane to situate yourself directly above or below the object, and one more number to tell you how far you have to travel up or down from the plane to reach the object.)

• Ask your students if the trick would work with a larger array. Have them try the trick with a 4 × 4 array. They should see that as long as the array is square (with an equal number of rows and columns), the trick works for any number of cards. Ask your students to explain why this is so and why the trick doesn’t work if the array isn’t square (for instance, try it with 2 columns and 6 rows).

• Ask your students if the original trick (i.e., with a square array) would work if the subject told the performer which row the card was in rather than which column. Have your students show you how the new trick would be performed. The fact that the trick works equally well in both cases illustrates a very deep principle of invariance in mathematics. In a square array, there is no real difference between the rows and columns. In fact, if you rotate the array by a quarter turn, the rows become columns and vice versa. More generally, once you fix an origin in space, it doesn’t matter how you set up your grid (the lines representing the rows and columns). In all cases you need only two numbers to identify a position.

Now draw an array of three columns and rows on the board and number the columns and rows:

```
  3  •  •  •  •  x  C
  2  •  •  •  •  x  R
  1  •  •  •  •  x  O
  1  2  3  x  W
```

Point out a row and a column, and stress that we order rows from bottom to top, and columns from right to left.

Ask several volunteers to locate the second column, the third row, the point that is in the third row and the first column, etc. Students that have trouble locating any particular dot could join the points in the given row and column prior to circling the dot itself. (The dot they are looking for will be where the lines intersect.)

Draw an array of dots on the board, circle a point, and ask your students to write the coordinates of the point: Column ____, Row ____. Ask your students: Imagine you have to write the coordinates of 100 points. Would you like to write the words “column” and “row” 100 times? What could you do to shorten the notation? Students might suggest making a T-table or even writing a pair of numbers, because the column is always the first number. **ASK:** How do you know which is first, column or row? What if you have to ask a partner to find a dot given a pair of numbers, without telling them which number belongs to the column and which number is the row number? Give your students a pair of numbers, such as 2, 3 and do not tell them which one is the column number and which one
is the row number. How many points can they find that could go with these two numbers?

Explain to your students that mathematicians have made an international convention: The place of the dot is given by two numbers, in parenthesis, and the column number is always on the left, and the row number is always on the right: (column, row). Give your students several pairs of numbers and ask them to identify the corresponding points on the grid. Explain that the pair of numbers is called “coordinates of the point.”

Draw an array of dots and label the columns with letters and the rows with numbers. Ask your students to identify some points, such as (A, 4), (B, 2), (C, 3). Mark the points (B, 3) and (C, 2) and ask your students to identify them. Repeat the exercise with a grid instead of an array. As a variation, you might label both rows and columns with letters.

Students need lots of practice.

Review the names of special quadrilaterals and triangles before assigning these questions:

1. Graph the vertices A(1,2), B(2,4), C(4,4), D(5,2). Draw lines to join the vertices. What kind of polygon did you draw? How many lines of symmetry does the shape have? How many pairs of parallel sides does it have?

2. On grid paper, draw a coordinate grid. Graph the vertices of the triangle A(1,2), B(1,5), C(4,2). Draw lines to join the vertices. What kind of triangle did you make?

Ask your students if they have seen any of these methods of marking points with letters or numbers in real life. You might show them a map with a grid on it.

**Assessment**

1. Identify the proper column and row for the circled dot:

   a)  
   b)  
   c)  
   d)  

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</table>

2. Mark the positions: (3, 2), (4, 1).

3. Write the coordinates of the marked positions: _____, _____

**Bonus**

Draw the points below on a grid, then join the points in the order you’ve drawn them. Join the first point to the last point. What shape did you make? Find the area of the shape.

   a) (A, 4), (A, 5), (B, 5), (C, 4), (D, 4), (E, 3), (D, 3), (C, 2), (C, 1), (B, 2), (B, 3).
   b) (0, 1), (2, 3), (2, 6), (3, 7), (4, 6), (4, 3), (6, 1), (6, 0), (5, 0), (4, 1), (3, 0), (2, 1), (1, 0), (0, 0).
ACTIVITY 1

 Invite students to sit or stand in an array. Identify the “columns” and the “rows” in the array, and make sure each student knows which row and column they are in. Give one of the students a ball and identify a point on the array with a column number and a row number. The student with the ball has to toss it to the student at the given point. Students can continue tossing the ball around by calling out coordinates rather than names.

ACTIVITY 2

Students will need a pair of dice of different colours. The player rolls the dice and records the results as a pair of coordinates: (the number on the red die, the number on the blue die). He plots a point that has this pair of coordinates on grid paper. He rolls the dice a second time and obtains a second point in the same way. The player joins the points with a line, then has to draw a rectangle so that the line he drew is a diagonal of the rectangle.

There could be several rectangles drawn this way. If the line is neither vertical nor horizontal, the simplest solution is to make the sides of the rectangle horizontal and vertical. In this case, ask your students if they see a pattern in the coordinates of the vertices.

ADVANCED: Students will need a pair of dice of different colours and a spinner below.

The player rolls the dice twice and plots the points the same way he did in the previous activity. He also spins a spinner. He has to draw a quadrilateral of the type the spinner shows, so that the line he drew is the diagonal of the quadrilateral.

Extension

The card trick can be modified for non-square arrays if one allows one extra rearrangement. Deal out an array of 3 columns, 9 rows. Have a student select a card and tell you what column it’s in. Re-deal the cards so that all of the nine cards from the chosen column land in the top three rows of the new array. Ask the student to tell you what column their card is in now, and re-deal the top three cards in that column into the top row of a new array. Once the student tells you what column their card is in, you can identify the top card in that column as the one they selected.

This version of the trick illustrates a powerful general principle in science and mathematics: when you are looking for a solution to a problem, it is often possible to eliminate a great many possibilities by asking a well-formulated question. In the card trick one is able to single out one of 27 possibilities by asking only three questions. Repeat the trick, asking your students how many possibilities were eliminated by the first question (18), by the second question (6), and by the third (2).
Review negative numbers on a number line. Remind your students that number lines can be extended in both directions.

Draw a coordinate grid on the board (or use the overhead). Make sure the axes are centered on the grid so that all four quadrants are visible. Tell students that each line in this coordinate system is called an axis. (You might mention that the plural of axis is axes.) The horizontal line is called the x-axis, and the vertical line is called the y-axis. Tell students that the point at which the 2 axes intersect is called the origin. Label the axes and the origin. Point out that the axes separate the grid into four parts. Explain that these are called quadrants, and ask your students which words that they know have the same beginning. (quadrilateral, quadruple) Draw several points in various quadrants and ask your students to tell which quadrant the points are in.

Remind your students that in a coordinate pair such as (3, 6), 3 is the column number and 6 is the row number. Tell students that you are going to plot, or place, this pair on the coordinate system. Add numbers to the positive halves of each axis (as you would on a number line) and ask your students how they might find the point (3, 6). Where are the columns and the rows? Point out that numbers on the x-axis correspond to the numbers of columns, and numbers on the y-axis correspond to the numbers of rows. This means that the first number in the coordinate pair should be counted on the x-axis. Ask your students to plot points such as (1, 2), (4, 0), (0, 3), (0, 0).

Now make a mark at point (-1, -1) and ask your students what the coordinates of this point might be. Invite volunteers to add numbers to the negative parts of each axis. Mark a point in the second quadrant, such as (-3, 4), and show students how to find the signs of its coordinates—the point is on the negative side of the x-axis and the positive side of the y-axis, so the signs of the point will be (-, +). Repeat with a point in the fourth quadrant. Ask the students to tell the signs of various points on the coordinate system. Students might mark the signs for each quadrant. Ask your students in which quadrants different points appear. EXAMPLES: (-3, 2), (4, -6), (-2, -2).

Show your students how to find the complete coordinates of a point in any quadrant (number and sign). Then have students determine the coordinates of points in various parts of the grid. For example, ask your students to name the coordinate pairs for five points in the second quadrant. What do these points have in common? (They have the same signs.) Then do the reverse: give students various coordinates and ask them to mark the points. Start with easier points, such as (2, 3), (5, 4), (-3, -3), then try slightly harder points, such as (-4, -5), and then harder points, such as (-3, 3) and (3, -3). As a last
challenge, give your students points such as (-4, 7) or (3, -6). Students need lots of practice plotting points in different quadrants.

As an additional exercise, let your students plot the following points, join them, and tell what type of quadrilateral they have drawn.

a) (1, 2), (-1, -1), (2, -3), (4, 0)

b) (-2, -2), (-1, 1), (-2, 4), (-3, 1)

Repeat Activity 2 from G6-21 using both dice and the spinner shown. Students should spin the spinner with each roll of the dice. The spinner will indicate the quadrant that the point is plotted to.
**GOALS**
Students will slide dots and shapes on a grid.

**PRIOR KNOWLEDGE REQUIRED**
Slide a dot on a grid
Distinguish between right and left

**VOCABULARY**
slide
translation

For this lesson, a magnetic board with a grid on it (or an overhead projector with a grid drawn on a transparency) would be helpful. Let your students practice sliding dots in the form of a small circular magnet right and left, then up and down. Students should be able to identify how far a dot slid in a particular direction and also be able to slide a dot a given distance. If any students have difficulty distinguishing between right and left, write the letters L and R on the left and right sides of the board.

Once students have demonstrated that they slide a dot in a given direction, show them how to slide a dot in a combination of directions.

You might draw a hockey rink on a magnetic grid and invite volunteers to move a small circular magnet as if they were passing a puck. **SAMPLE QUESTIONS AND TASKS:**

- Pass the puck three units right; five units left; seven units down; two units up.
- Pass the puck two units left and five units up.

Position several small figures of players on grid intersections in various points of the rink and ask your students:

- Player 3 passes the puck 5 units right and 2 units up. Who receives the pass?
- Player 5 wants to pass the puck to Player 7. How many units left and how many units down should the puck go?

**Bonus**
Player 4 sent the puck 3 units up. How many units, and in which direction(s), should Player 7 move to get the pass?

Tell your students the following story. You might use two actual figures to demonstrate the movements in the story.

Suppose you have a pair of two-dimensional figures and you wish to place one of the figures on top of the other. But the figures are very heavy and very hot sheets of metal. You need to program a robot to move the sheets. To write the program you have to divide the process into very simple steps. It is always possible to move a figure into any position in space by using some combination of the following three movements:
1. You may slide the figure in a straight line (without allowing the object to turn at all):

SLIDE

2. You may turn the figure around some fixed point (usually on the figure):

TURN or ROTATION

3. And you may flip the figure over:

FLIP or REFLECTION

Two figures are congruent if the figures can be made to coincide by some sequence of flips, slides and turns. For instance, the figures in the picture below can be brought into alignment by rotating the right hand figure counter-clockwise a quarter turn around the indicated point, then sliding it to the left.
It is not always possible to align two figures using only slides and turns. To align the figures below you must, at some point, flip one of the figures:

One way to flip a figure is to reflect the figure through a line that passes through an edge or a vertex of the figure. Tell your students that today you are going to teach them about slides.

Show students the following picture and ask them how far the rectangle slid to the right. Ask for several answers and record them on the board. You may even call a vote.

Students might say the shape moved anywhere between one and seven units right. Take a rectangular block and perform the actual slide, counting the units with the students. The correct answer is 4.

Show another picture:

This figure has a dot on its corner. How much did it slide? This time it is easier to describe the slide—just use the benchmark dot on the corner. Check with the block.

Show a third picture.

Is this a slide? The answer is NO, this is a slide together with a rotation. You cannot slide this block from one position to the other without also turning it.
Assessment
1. Slide the dot 6 units left, 3 units down.

2. Slide the shape 5 units left, 2 units up.

**Ball Game**
The students are points on the grid, and you give directions such as: The ball slides three units to the right. The student with the ball has to throw it to the right point on the grid.

**Memory Game**
Students will need a grid and several (1 to 4) small objects (play money of different values or beads of different colours could be used). The objects are placed on the intersections of the grid. Player 1 slides one of the objects while Player 2’s back is turned, and Player 2 then has to guess which object was moved and describe the slide. This game will become much easier when coordinates are placed on the grid. Students might memorize the coordinates of the objects. They can then compare the coordinates after the objects were moved with the coordinates before the objects were moved to determine exactly which object moved and how.

**Activity 4**
Give your students a set of pattern blocks or Pentomino pieces and ask them to trace a shape on dot paper so that at least one of the corners of the shape touches a dot. Ask students to slide the shape a given combination of directions. After the slide, trace the pattern block again.
G6-24

Slides (Advanced)

GOALS
Students will slide shapes on a grid, and describe the slide.

PRIOR KNOWLEDGE REQUIRED
Slide a dot on a grid
Distinguish between right and left

VOCABULARY
slide, translation
translation arrow

Draw a shape on a grid on the board and perform a slide, say three units right and two units up. Draw a translation arrow as shown on the worksheet. Ask your students if they can describe the slide you’ve made. If they have trouble, suggest that they look at how the vertex of the figure moved (as shown by the transition arrow). To help students describe the slide, you might tell them that the grid lines represent streets and they have to explain to a truck driver how to get from the location at the tail of the arrow to the location at the tip of the arrow. The arrow shows the direction as the crow flies, but the truck has to follow the streets.

Make sure your students know that a slide is also called a “translation.” Students should also understand that a shape and its image under a translation are congruent.

Draw an isosceles right-angled triangle and its image under translation on a grid, and mark the vertices as shown below:

Ask your students to use the translation arrow to describe the slide of the triangle. Then ask them to mark the unlabelled vertices of the second triangle and to check if these vertices moved the same way (the slide is the same).

Show your students the pair of triangles below and tell them that one triangle was taken to the other so that vertex A went to A’, vertex B went to B’, and vertex C went to C’. Paul says that this was a slide. Is he correct? Ask your students to explain their thinking.

ANSWER: Paul is wrong. A slide would translate each vertex the same way. Vertex C slid four units left, but vertex A moved only two units left! This means the transformation was not a slide. In fact, it was a reflection. These two triangles can be taken from one to the other by a slide (4 units left), but the order of vertices in the image would be different. In a slide, the vertex now marked B’ would be A’, or the image of A.
Bonus
Translate the figures however you want, and then describe the translation:

![Translation of figures]

Extensions

1. Describe a move made by a chess knight as a slide. Describe some typical moves of other pieces such as a pawn or a rook (castle).

2. A shape has coordinates A(2,1), B(6,1), C(3,5), D(5,5). Under a translation, vertex A moved to position (7,8). Give the coordinates of the other vertices under the translation.

Table Hockey
Draw a 12 × 8 grid on grid paper. Let each player place 5 “players” and a “goalie” on the grid (see a sample placement below). Players will need a pair of dice of different colours and a counter to represent the puck.

The puck starts at centre ice (the circle in the centre). Player 1 rolls the dice and slides the puck according to the numbers rolled on the dice. The red die represents the horizontal size of the slide, and the blue die represents the vertical size. Player 1 can choose the direction (up or down, left or right). If the pass is “successful,” Player 1 rolls the dice again. (A pass is successful if it lands on a vertex of the same square as a player on Player 1’s team.) If not, the puck is in Player 2’s possession, regardless of its distance to any of Player 2’s players. Player 2 now rolls the dice and moves the puck. The object of the game is to get the puck to one of the points in the goal zone of the rival “team.”

The puck is in a player’s possession if the puck and player are on vertices of the same square.
GOALS
Students will describe and perform a slide on a grid, and find a point given by coordinates on a map.

PRIOR KNOWLEDGE REQUIRED
Slides
Coordinate systems

VOCABULARY
slide translation
row column
coordinates

Assign a letter to each row of desks in your class and a number to each column. Ask your students to give the coordinates of their desks. Then play “postman”—a student writes a short message to another student and writes the student’s “address” in coordinates. A volunteer postman then delivers the letter. The postman has to describe how the letter moved (two to the front and one to the left, for example).

Place a slide with a map of Saskatchewan on the overhead projector (see the BLM “Map of Saskatchewan”). Ask volunteers to find the cities on the map and to answer the questions:

- What are the coordinates of Saskatoon?
- What are the coordinates of Regina?
- What are the coordinates of Uranium City?
- What are the coordinates of Prince Albert?
- What can you find in the square A4? D5? D1?

As a warm-up for the Secret Squares game in Activity 3, have the class as a whole try to guess the locations of each hidden square from the information given in the grids below. Start by showing an example. Use volunteers for each step.

STEP 1: Shade all the squares that are one step away from square 1.

STEP 2: Cross the squares you can reach by two steps from the square 2.

STEP 3: Check each of the squares that are both shaded and crossed, to see if they can be reached by three steps from square 3.

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Let your students practice:

In two of the following grids, not enough information is given (have students mark all possible locations for the hidden square) and in one grid too much information is given (have your students identify one piece of redundant information):
Once students understand the game, they can either play it in pairs or you might choose to play the game with the entire class.

Battleship Game
This game may be played in pairs or a teacher can play against the whole class, with the class is guessing the teacher’s ships.

Sample Placement:

Player 1 and Player 2 each draw a grid as shown. Each player shades:

1 battleship  2 cruisers   2 destroyers  1 submarine

(See grid for an example. No square of a ship may be adjacent to a square of another ship, including diagonally.)

(Continued on next page)
(Continued from previous page)

Players try to sink all their partner’s battleships by guessing their coordinates. If a player’s ship is in a square that is called out, the player must say “hit.” Otherwise they say “miss.”

Each player should keep track of the squares they have guessed on a blank grid by marking hits with X’s and misses with ✓’s. The game ends when all of one player’s ships are sunk. A ship is sunk when all its squares are hit, and the owner of the ship must indicate that to the partner.

HINTS/PROMPTS: When a player hits something, where should he or she look for the other squares of the ship? If you sink a ship, which squares do you know are empty?

Let your students draw their own map, possibly based on a book they are reading. Ask them to make questions about the map (like those on the worksheet) and ask a partner to answer them.

Secret Squares Game
Player 1 draws a 4 × 4 grid as shown and picks a square. Player 2 tries to guess the square by giving its coordinates.

Each time Player 2 guesses, Player 1 writes the distance between the guessed square and the hidden square.

For instance, if Player 1 has chosen square B2 (✓) and Player 2 guesses C4, Player 1 writes 3 in the guessed square. (Distances on the grid are counted horizontally and vertically, never diagonally.)

The game ends when Player 2 guesses the correct square.

---

Extension
These maps got turned around. If town A is east of town B on all of these maps, what direction is north?

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G6-26
Reflections

**GOALS**

Students will perform reflections of points and shapes through a line.

**PRIOR KNOWLEDGE REQUIRED**

Symmetry

**VOCABULARY**

reflection  mirror line
symmetry    symmetry line

---

Give your students an assortment of Pentomino pieces. Ask them to trace each piece on grid paper, draw a mirror line through a side of the piece and then draw the reflection of the piece in the mirror line. Students could check if they have drawn the image correctly by flipping the grouping of Pentomino pieces over the mirror line and seeing if it matches the image. Let your students know that a “flip” is also called a “reflection.” Students should notice that each vertex on the original shape is the same distance from the mirror line as the corresponding vertex of the image. Let your students practice reflecting shapes with partners: Each student draws a shape of no more than 10 squares, and chooses the mirror line. The partner has to reflect the shape over the given mirror line.

**Advanced game:** One student draws two shapes of no more than 10 squares so that the shapes are symmetric in a mirror line but one square is misplaced. The partner has to correct the mistake.

Draw four points as shown and explain that two of these points are reflections of the other two. Challenge students to draw the mirror line. How do they know that the line they have drawn is the mirror line? Which points are reflections of each other?

![Diagram of points A, B, C, D arranged in a 4x4 grid]

Write several words, such as MOODY CAT IN A WOODEN BOX, on a transparency sheet and project it onto the board in an incorrect way (flipped horizontally or vertically). Ask your students if they can read the text. Which words are still readable? Which letters look normal? Which transformation should be performed to make the text look completely normal? (A reflection.) Ask your students to draw the mirror line. Show them that a reflection in a mirror and a flip of the transparency sheet both achieve the goal.

**Bonus**

1. Students could try to copy and reflect a shape in a slanted line, for example:

![Diagram of shapes reflected in a slanted line]
2. Sort all the capital letters of the alphabet into a Venn diagram:

1. Letters that look the same after a reflection in a horizontal line.
2. Letters that look the same after a reflection in a vertical line.

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**Find-a-Flip Game**

Divide your students into groups of 2-5 players each. Each group will need two copies of the “Find-a-Flip Game” BLM. Let the students cut out the cards (they will have 48 in total). Players shuffle the deck, deal out 4 cards for each player, and lay one card face up on the table. If a player has a card that is a reflection of the card on the table over one of its sides, he or she can add the card to the table and pick a new card from the deck.

**EXAMPLE:**

Players take turns placing the cards until they have formed a square.

**EXAMPLE:**

The player that placed the last card in the square obtains 1 point.

If a player does not have a card that is a reflection of one of the cards on the table, the player picks cards from the deck until he or she can either add a card to the table or create a square of shapes from his or her own cards. A player that does not have any more cards in hand picks 4 cards from the deck. If a player runs out of cards and there are no more cards in the deck, the player exits the game and obtains 2 points. The player with the greatest number of points wins.
ACTIVITY 2

Triangle Transformations Game

Divide your students into groups of 2-5 players each. Each group will need a copy of the “Triangle Transformations” BLM. Let the students cut out the cards. Each deck contains 3 groups of 12 cards: 6 cards with the same shape and 6 cards with a reflection of this shape. Players shuffle the deck, deal out 4 cards for each player, and lay a card face up on the table. If a player has a card that is a reflection of the card on the table over one of its sides, he or she can add the card to the table and pick a new card from the deck.

EXAMPLE:

Players take turns placing the cards until they have a hexagon.

EXAMPLE:

The player that placed the last card in the hexagon obtains 1 point.

The remaining rules of play are the same as for the previous game: if a player does not have a card to add, he or she picks from deck; players can add to the hexagon on the table or put down a hexagon using their own cards; a player who runs out of cards picks 4 from the deck; a player who exits the game gets 2 points; the player with the most points wins.

This game involves reflections through slanted lines, which makes it much harder than the previous game. Students might also notice that unlike the previous game, the cards here that have a common corner but do not have a common side are not a 180° rotation of each other. Also, the shapes opposite each other in the hexagon are reflections along a line through the centre of the hexagon. Challenge your students to find all the pairs of shapes in hexagons that differ by reflections as well as the mirror lines for those reflections.

Extensions

1. Draw a triangle with vertices A(1,1), B(4,2), and C(3,3). Draw a vertical mirror line through the points (5,0) and (5,5). Reflect the triangle in the mirror line and write the coordinates of the vertices of the image.

2. Draw an equilateral triangle. If you reflect it through one of the sides and look at two shapes together, what shape will you get? Write down your prediction and check it. Is the result different for the other sides? Repeat with an isosceles triangle and a triangle with a right angle. Are the results different for different sides? Check all sides.
3. Cyril experiments with a mirror and a straight line. He draws a straight line and puts the mirror across it. He looks at the angle between the line he drew and the mirror and at the angle between the reflection of the line and the mirror. He thinks that these angles are the same. Is he correct?

Cyril turns the mirror and looks at the angle between the line he drew and its reflection in the mirror. The angle between the line and the mirror is 20°. How large is the angle between the line and its reflection? Cyril wants to put a mirror so that it is at a right angle with the line. What is the degree measure of a right angle? How large is the angle between the line and its reflection when the mirror is at a right angle to the line? What does Cyril see in the mirror?

4. Boris experiments with a Mira and an angle. He draws an angle and places the Mira so that it touches the vertex on his angle and divides the angle in two. He rotates the Mira around the vertex until the angle on one side of the Mira and its reflection are the same. Using the mirror as a ruler he draws a line through the angle (starting at the vertex of the angle). He says that the line cuts the angle into two equal parts. Is he correct? The line that divides an angle into two equal parts is called a bisector.

5. Which of the points on the line L is closest to the point A? Estimate, then measure the distances between the points to check your prediction. Connect the points on the line with the point A. Measure the angles between the lines you drew and the line L. What is the angle at the point that is nearest to A?

\[ \text{A} \quad \text{B} \quad \text{C} \quad \text{D} \quad \text{E} \quad \text{F} \quad \text{L} \]

Angela measures the acute angles. She says: The further the point from A, the less the angle between the line I drew from the point to A and the line L. Is she correct? Can you draw a point on L that is nearer to A than D? The distance from a point to a line is the shortest among the distances from the point to any point on the line. What is the distance from point A to line L?

**CHALLENGING:** How can you use Cyril’s method to find a point on a line that is nearest to a given point? **ANSWER:** Put a mirror across the line so that it touches the given point (which we will call point A). Turn the mirror (around the point A) until you see that the reflection of the line continues the line itself. At that point you know that the mirror is perpendicular to the line. Draw a line through A using the mirror as a ruler. The point where your line meets the given line is the point nearest to A.

6. Gleb wants to find the midpoint of a line segment AB using symmetry. He knows that a point and its image in a mirror are the same distance from the mirror. He puts a Mira across the segment AB and looks at the point A' (the mirror image of the point A). He also sees the point B through the Mira. He makes sure that the mirror is perpendicular to the line and he moves the mirror between the points A and B. What does he see in the Mira when it is in the middle of the line segment? Why does this happen exactly at the midpoint of the segment?

**ANSWER:** When the Mira is at the midpoint of the segment, Gleb sees that the points A and B coincide. This happens because the distance between the mirror and the point A (which is the same as the distance between the mirror and A') is now the same as the distance between the Mira and the point B.
7. A line that is both perpendicular to the given segment and passes through its middle is called a perpendicular bisector of a segment. How can Gleb use Mira to draw a perpendicular bisector of a segment? (HINT: Gleb and Cyril are friends.)

8. Draw the reflections of the lines through the mirror line shown. Use the dots to help you. What do you notice?

   ![Mirror Line Diagram]

   **ANSWERS:** The top line and its reflection create a right angle. The middle line and its reflection create a straight line. The bottom line and its reflection create an obtuse angle. The angle between the line and its reflection is twice the angle between the line and the mirror line; in other words, the mirror line bisects the angle between the line and its reflection.

9. **NOTE:** Assign Extension 3 before this exercise.

   Draw a pair of intersecting lines and measure the angle between them. Choose one line to be a mirror line. How can you use Extension 3 to draw the reflection of the other line through your mirror line? **ANSWER:** The new line (the reflection) and the mirror line intersect at the same angle as the first two lines, so that the mirror line bisects the angle between the other two lines. Create an angle equal to the angle between the first two lines to the other side of the mirror line. You can do so as follows: Place a Mira along the mirror line so that the line you want to reflect is behind the Mira and you see the line through the Mira. Move your pencil perpendicularly to the Mira. Stop when you see that the reflection of the pencil reached the line you are reflecting. Join the point you stopped at with the intersection of the lines using the Mira as a straight edge.

10. Draw an equilateral triangle. How many lines of symmetry does it have? (3) What is the angle between the lines of symmetry? Label the vertices of the triangle ABC. Draw the lines of symmetry and label the lines of symmetry by LA, LB, and LC so that the line that passes through vertex A is LA. Reflect the triangle through LA. What are the images of the vertices A, B, and C? Which vertex remained fixed? What is the image of the line LA after the reflection of the triangle through LA? What is the image of the line LC after the same reflection? (Students should see the line LA reflected into the line LB and vice versa—the lines of symmetry reflected into each other.)

   Repeat the exercise (i.e., check what happens to the lines of symmetry under a reflection) with a square, a regular pentagon, and two other shapes, one with two lines of symmetry and the other with at least three lines of symmetry. What is the image of a line of symmetry under a reflection? **ANSWER:** Always a line of symmetry.

11. **NOTE:** Assign Extensions 3, 8, 9, and 10 before this exercise.

   a) Haled says he can draw a shape with exactly two lines of symmetry so that an angle between these lines is 45°. Is he correct? Explain. **ANSWER:** Haled is mistaken. A reflection in one line of symmetry should take a line of symmetry into a line of symmetry. If the angle between the lines is 45°, reflection in one of the lines takes the other line into a third line of symmetry. So the shape has more than two lines of symmetry.

   b) If a shape has exactly two lines of symmetry, what is the angle between the two lines? **ANSWER:** 90°.

   c) A shape has n lines of symmetry. What is the angle between the lines? **ANSWER:** 180°/n.
G6-27
Rotations

GOALS
Students will describe and perform rotations that are multiples of a quarter turn.

PRIOR KNOWLEDGE REQUIRED
Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$
Clockwise
Counter-clockwise

VOCABULARY
clockwise
counter-clockwise
rotation

Review the meaning of the terms “clockwise” and “counter-clockwise” using a large clock or by drawing arrows on the board. If you have a large clock, ask volunteers to rotate the minute hand—clockwise and counter-clockwise—a full turn, half turn, and a quarter turn. You might also ask your students to be the clocks: each student stands with a hand out and turns clockwise (CW) or counter-clockwise (CCW) according to your commands.

Draw several clocks on the board as shown below and ask your students to tell you how far and in which direction each hand moved from start to finish:

Then draw examples with only one arrow and ask students to turn the arrow:

a) $\frac{1}{4}$ turn CCW  
b) $\frac{1}{2}$ turn CCW  
c) $\frac{3}{4}$ turn CW  
d) $\frac{3}{4}$ turn CCW

Assessment
1. Describe the rotation of the arrow:

2. Show the position of the arrow after each turn:

a) $\frac{1}{4}$ turn CW  
b) $\frac{1}{2}$ turn CW  
c) $\frac{3}{4}$ turn CCW  
d) $\frac{3}{4}$ turn CW
**ACTIVITY**

Let your students play the Find-a-Flip Game as in the Activity in G6-26 with one change: each card placed should be a quarter turn rotation of the adjacent cards. Players must indicate around which vertex the rotation was made.

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**G6-28**

**More Rotations**

**GOALS**

Students will describe and perform rotations of shapes.

**PRIOR KNOWLEDGE REQUIRED**

Fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$

Clockwise

Counter-clockwise

Drawing angles with protractors

Construct polygons using a ruler and a protractor

**VOCABULARY**

clockwise

counter-clockwise

rotation

rotation centre

---

**Rotations Around Vertices**

Review the previous lesson by drawing several arrows or clock hands. To help your students visualize the effect of a rotation on a shape, have them make a small flag (as in QUESTION 1 on the worksheet) by taping a rectangular piece of paper to a straw. Ask students to rotate the flag and trace its image after the rotation. Students could also cut out shapes similar to the other ones on the worksheet and trace the images of these shapes after a rotation. Then ask your students to trace or draw a figure, decide on a rotation, and draw the new shape without the prop. Students could also practice rotating pattern blocks or Pentomino shapes (around vertices of the shapes) on a grid.

**Assessment**

Draw the shape after each turn:

- a) $\frac{1}{4}$ turn CW
- b) $\frac{1}{2}$ turn CW
- c) $\frac{3}{4}$ turn CCW
- d) $\frac{3}{4}$ turn CW

---

**Rotations Around Points That Are Not Vertices**

When your students are comfortable rotating shapes around vertices, show them the following method for rotating a shape around a point that is not a vertex. Students can use this same method to rotate shapes around points inside the shape.

To rotate a shape $\frac{1}{4}$ turn clockwise around a point that is not a vertex:

**STEP 1:** Highlight the line that contains the point (the point is called the rotation centre).
STEP 2: Rotate the line around the point.

STEP 3: Draw an arrow from the rotation centre to one of the vertices of the shape.

STEP 4: Rotate the arrow to get the image of the vertex.

Repeat steps 3 and 4, if necessary, for the other vertices.

STEP 5: Draw the image of the shape using the images of the vertices.

Ask your students to rotate the figures 90° clockwise and counter-clockwise around the points shown:

Performing Rotations by a Free Angle Around Vertices
The same method can be used to rotate shapes by any angle. First, review how to use protractors to build angles. You might ask the students to draw a triangle given two sides and an angle between them.

Then show your students how they can rotate a shape 70° counter-clockwise around one of its vertices:

STEP 1: Highlight one of the sides passing through the centre of rotation.
**STEP 2:** Rotate the highlighted side 70° counter-clockwise around the point using a protractor.

**STEP 3:** Draw an arrow from the centre of rotation to a vertex of the shape. Measure the arrow.

**STEP 4:** Rotate the arrow 70° counter-clockwise around the point using a protractor.

Repeat steps 3 and 4 until you have rotated all the vertices of the shape.

Then join the vertices to get the rotated image.

A slightly different method for rotating a shape by any angle is outlined on the worksheets (see QUESTION 3 on G6-28). There, instead of rotating each point and joining the points at the end, students can rotate one side of a shape and then rebuild the shape.

### Identifying Rotations by a Free Angle Around Vertices

Draw two right pentagons sharing a common side as shown below (do not label the vertices yet). Ask a volunteer to measure the angles of the pentagon and review with your students the way to verify the measurement from the sum of the angles of the pentagon. **SAY:** Anne says that the pentagon on the right was rotated clockwise to get the pentagon on the left. How much was it rotated and around which point? Invite volunteers to show various centres of rotation and let them check the angle of rotation. Use a paper pentagon if needed. (There are at least three possible rotation centres: both mutual vertices and the midpoint of the common side. The angles are 108° for the upper common vertex, 180° for the midpoint, and 252° for the lower common vertex.) Repeat with rotation counter-clockwise and reflection.

After this exercise label the vertices as shown and **ASK:** How do the labels help you to identify the centre of rotation? (The centre of rotation is a fixed point. D coincides with D', so this is the centre of rotation. The rotation was 108° counter-clockwise which is the same as a rotation 252° clockwise.)
You will need the “Triangle Transformations” BLM. Give each student one group of 12 cards from the deck, that is, 6 copies of one shape and 6 copies of its reflection. Ask your students to place two identical cards as shown below. **ASK:** How can you get the left card from the right card? What is the centre of rotation? How much do you have to rotate the right card to get the left card?

**ANSWER:** The card on the right is rotated 60° counter-clockwise around its top vertex to get the card on the left.

Add another card, so that the third card is obtained by the same rotation from the second (left) card. What is the transformation that takes the first card to the third card? (Rotation by 120° (60° + 60°) in the same direction) Repeat with more cards until you get a full turn (360°). **ASK:** What pattern can you see? What is the rule for finding the angle of rotation for any card, if the rotations are performed in the same direction around the same point? (Add the angles of rotation) What if you performed rotations in different directions, say, three rotations clockwise and then one rotation counter-clockwise, how would you find the angle of rotation for the last shape? Does the order of the rotations matter? Let your students check the answer with the triangles.

Take a card with a reflected shape. Do any of the cards in the hexagon of 60° turns show the same shape as the reflected one? (No, a combination of rotations cannot produce a reflection if the shape is non-symmetric.)

Show the two cards placed below and **ASK:** Which transformation takes one card to the other? (180° rotation) How is this rotation different from the rotation between two opposite cards in a hexagon? (Different centre of rotation) What is the centre of rotation? (The midpoint of the common side)

**Extensions**

1. A shape has coordinates A(1,2), B(1,4), and C(4,2). Under a rotation around one of the vertices, point A moves to position (4, 6). Which vertex in the shape was used as the centre of rotation and what was the rotation?

2. Take four identical cards from the “Triangle Transformations” BLM. Place them on the table as shown.

Which rotation took card 1 to card 2? What is the centre of rotation? Which rotation took card 3 to card 4? What is the centre of rotation? Cards 2 and 3 are identical and have identical position. However, card 1 and card 4 are placed differently. Place cards 4 and 3 on top of cards 1 and 2, so that cards 2 and 3 coincide. Which transformation takes card 4 onto card 1?
What is the centre of rotation? **Answer:** Card 2 is obtained from card 1 by a counter-clockwise rotation of 60° around the top vertex. Card 4 is obtained from card 3 by a counter-clockwise rotation of 60° around the bottom-left vertex. Cards 1 and 4 differ by a rotation of $60° + 60° = 120°$ counter-clockwise. When cards 1 and 4 are placed on top of each other, their outlines coincide, which means they differ by rotation around the centre. Indeed, we know that the order of rotational symmetry of an equilateral triangle is 3 (see lesson **G6-16**), so the angle should be $\frac{1}{3}$ of 360°, which is 120°.

3. Create a right-angled trapezoid from sturdy paper. Trace it on a sheet of paper. Then choose a vertex and rotate the shape around the vertex $\frac{1}{2}$ turn. Trace the figure again. Would you get the same result if you had reflected the figure?

4. Trace the flower onto tracing paper and cut it out. Try to reflect it vertically, then rotate it $\frac{1}{4}$ turn CW. Draw the result. Now rotate and reflect the flower vertically. Is the result the same? Try various combinations of $\frac{1}{4}$ turn rotations and reflections. Which combinations give the same results? Why?

5. What happens to the line of symmetry of a figure after a rotation of $\frac{1}{4}$ turn? $\frac{1}{2}$ turn? If the shape has a vertical line of symmetry, will it have a vertical line of symmetry after either a $\frac{1}{4}$ or $\frac{1}{2}$ turn?
Review with your students various ways to describe rotations (90°, quarter of turn, \( \frac{1}{4} \) turn, the terms clockwise and counter clockwise, and so on.)

Explain to your students that today their task will be harder than the one you assigned in the last lesson—they will have to rotate and reflect shapes without protractors. Draw a simple shape on a grid on the board and mark one of the corners with a dot. Highlight one of the sides passing through the dot. **ASK:** Where will the side be located after a 180° rotation? Invite a volunteer to draw the rotated side. Highlight another side and ask another volunteer to draw its image after the rotation. Ask your students to finish the rotation of the shape.

Ask your students to describe the shape before and after the rotation. (EXAMPLE: The shape is three squares long and two squares wide. It is longer horizontally than vertically. It has an indentation of 1 square at the top-right corner.) Repeat the exercise with several more complicated shapes, including triangles (and with quarter turns).

Draw two pairs of triangles as shown. In the first pair, ask your students to mark the vertices of the bottom triangle that correspond to the vertices A, B, and C under a half turn around the middle of the base AB. Repeat for a reflection through AB in the second pair.

**ASK:** Which vertices of ABC changed position in a rotation? Which points stayed fixed (did not move at all)? Repeat with reflection.

Draw the first pair of squares and ask students to identify the transformation that takes square ABCD onto square A'B'C'D'. How much was the rotation and around which point? Repeat with the other pictures. In the third, fourth, and fifth squares, the transformation is a reflection, so ask your students to identify the mirror lines.
**ACTIVITY 1**

**A Game for Pairs**

Ask your students to draw a non-symmetric shape on a piece of grid paper. The original shape is visible to both players at all times. Player 1 rotates the shape $\frac{1}{2}$ or $\frac{1}{4}$ turn (in any direction) or reflects it through a vertical or a horizontal line (the player can only use a single transformation!), so that his partner does not see the result. Then Player 1 describes the position and orientation of the shape to Player 2. Player 2 has to draw the shape and identify the transformation performed by Player 1.

**ANSWERS:**

- Rotation around C. 90° counter-clockwise.

- Rotation around the midpoint of CD. 180° CW or CCW.

- Reflection through CD.

- Reflection through a horizontal line through the centre of the square.

As challenge, ask your students to show which points stay the same on the square in each of these cases.
ACTIVITY 2

Have students play another version of the “Find-a-Flip Game”. Each group of 2-4 will need two copies of any 3 rows of shapes on the BLM. Players shuffle their deck, deal out 4 cards to each player, and lay a card face up on the table. If a player has a card that is a) a quarter turn around a common vertex of the card on the table or b) a reflection of the card on the table over one of its sides, he or she can add the card to the table (and pick a new card from the deck) and name the transformation.

EXAMPLES:

- Reflection through the right side
- 90° turn CW around the bottom-right vertex
- 90° turn CCW around the bottom-right vertex

Players take turns placing cards until they have a rectangle of area 4 (or more) cards. The player that placed the last card in the rectangle obtains a number of points equal to the area of the rectangle. Check with the students which rectangles are possible (4 × 1, 2 × 2, 2 × 3, 2 × 4; 5 × 1 seems possible but isn’t because the previous player will have claimed the 4 × 1 rectangle). A player is not allowed to place a card so that there will not be enough cards to make a rectangle. (There are only 8 cards with shapes that can go into same rectangle.)

If a player does not have a card that is a reflection or a rotation of one of the cards on the table, the player picks cards from the deck until he or she can either add a card to the shape on the table or make a rectangle from the shapes on his or her own cards. A player that does not have any more cards picks 4 cards from the deck. If there are no more cards left in the deck, the player exits the game and obtains two points. The player with the greatest number of points wins.

ACTIVITY 3

Play the game in Activity 2 with the cards from the “Triangle Transformations” BLM. Students make regular triangles (of area 4) or hexagons instead of rectangles.
Extensions

1. Investigate various shapes and fill in the T-table:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Number of Lines of Symmetry</th>
<th>Order of Rotational Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Shapes to use: regular triangle, pentagon, hexagon, octagon, and all special quadrilaterals. Do you see a pattern?

2. Look at the shapes you used in Extension 1. Draw lines of symmetry and mark the centre of rotation of each shape. Where is the centre of rotation situated? ** ANSWER: It is always at the intersection of the lines of symmetry.**

3. Draw a coordinate grid on grid paper. Draw a triangle with vertices A (1,5), B (4,5), and C (4,7). Draw a vertical mirror line through the points (5,0) and (5,7). Reflect the triangle in the mirror line. Then rotate the triangle \( \frac{1}{4} \) turn counter-clockwise around the image of vertex C. Write the coordinates of the vertices of the new triangle.
Transformations on Grid and Dot Paper

**GOALS**
Students will perform and identify combinations of two transformations.

**PRIOR KNOWLEDGE REQUIRED**
Perform and identify a slide, a rotation or a reflection of a shape.

**VOCABULARY**
- rotation
- reflection
- centre of rotation
- mirror line
- slide
- translation arrow

Draw a simple non-symmetric shape on the board, such as the shape below, and review with your students how they can slide this shape, rotate it around one of its vertices, and reflect it through sides AF and FE. Then ask your students to perform reflections through a line parallel to EF and through the line DE. As a challenge, ask them to reflect the shape in the slanted line shown, to rotate it 90° clockwise around the point D, and to perform a quarter turn counter-clockwise around the midpoint of the side DE.

Now list the following combinations of two transformations:

- slide the shape two squares up and reflect it through the left side
- reflect the shape through the left side and slide it two squares up
- slide the shape two squares to the right and rotate it 90° clockwise around the bottom-right corner
- rotate the shape 90° counter-clockwise around the bottom-left corner and reflect it through the bottom side
- reflect the shape through the bottom side and rotate the shape 90° counter-clockwise around the bottom-left corner
- rotate the shape 90° clockwise around the bottom-right corner and slide the shape two squares to the right
- reflect the shape through the bottom side and rotate the shape 90° clockwise around the top-left corner

Ask your students to predict which pairs of transformations will produce the same results, then let students check their predictions. Which combination of two transformations does not have a pair? (the fifth one) Ask students to think of a pair of transformations that will produce the same result as this combination. Encourage them to find more than one answer. (Possible pairs could be: reflect through the left side and rotate 90° clockwise around the
point on the bottom side; rotate counter-clockwise around the point on the side AF that is 2 units right from A, then reflect through the left side; reflect through the line AD then slide two units down and two units left. If some of these answers are not given, suggest that your students perform these pairs of transformations as well.)

### Extension

Draw a coordinate grid on grid paper. Draw a trapezoid with vertices A(1,5), B(4,5), C(3,6), D(2,6). Rotate the trapezoid 90° counter clockwise around point D. Then slide the image 3 units right and 1 down. Write the coordinates of the vertices of the new trapezoid.
G6-31
Transformations (Advanced)

GOALS
Students will perform and identify combinations of two transformations of shapes on a (coordinate) grid. Students will perform rotations of shapes on a grid around points outside the shapes.

PRIOR KNOWLEDGE REQUIRED
Coordinate systems
Plot points given their coordinates
Identify coordinates of points in the first quadrant
Identify and describe slides, rotations, reflections, and combinations of two transformations
Perform slides, reflections, and rotations around vertices

VOCABULARY
coordinates axis/axes
quadrant origin
rotation reflection
slide translation

Review coordinate systems with your students. Draw a simple shape, such as a parallelogram, on a grid. Ask your students to rotate the parallelogram around one of the vertices. Add coordinate axes to the grid and ask your students to identify the coordinates of the vertices of both parallelograms. Ask your students to perform slides, rotations, and reflections on various shapes on the grid and to find the coordinates of both the original shapes and the new shapes. Include reflections through lines parallel to the sides of a shape (but not passing through them) and rotations around points on the side of a shape (that are not vertices).

Next, give your students coordinates of vertices and ask your students to plot the shape and to perform some transformations on it.

SAMPLE PROBLEM
Draw a trapezoid with vertices A(1,5), B(4,5), C(3,6), and D(2,6). Rotate the trapezoid 90° counter-clockwise around point D. Then slide the image 3 units right and 1 unit down. Write the coordinates of the vertices of the new trapezoid.

Ask your students to plot three triangles:
- A (3,1), B (4,4), C (5,1)
- A'(0,1), B'(1,4), C'(2,1)
- C''(7,1), B''(8,4), A''(9,1)

Tell your students that someone says the second and third triangles (A'B'C' and A''B''C'') were obtained by translation of ABC. Is that correct? Ask the students to identify the transformations that took triangle ABC to each of the other two triangles. (HINT: One triangle is obtained by a slide and the other by a flip.) Ask students to make up a similar problem (draw three shapes such that two are obtained through transformations of the first). Students could exchange problems with a partner.

Teach your students to rotate shapes around points outside the shape. They should connect the centre of rotation to a vertex of the shape and use the line between the point and the vertex as a prop. An alternative method using arrows would be similar to the method used in G6-28.

As a final challenge, ask your students to find three different ways (one flip, one rotation, and one slide) to transform square 1 to square 2. They should label the vertices in both squares according to each transformation.
Extensions

1. Rotate the point (5, -3) around the origin and write the coordinate pair of the image:
   a) 180° clockwise  
   b) 90° clockwise  
   c) 90° counter-clockwise

To perform the rotation, you might draw an arrow from the origin to the point (5, -3) and rotate the arrow.

Repeat with a different point. Which quadrant is the image located in? What do you notice about the coordinates of the original point and each of the images? Which quadrilateral do the point and its three images produce? You could ask students to try and describe why the resulting shape is a square. (ANSWER: The arrows are halves of the diagonals. Since all the arrows are of the same length, and the angles between them are all 90°, the diagonals are equal, perpendicular, and bisect each other. This means the shape is a square.)

2. Draw a triangle in the first quadrant. Reflect the triangle through the horizontal axis (the x-axis) and write the coordinates of the vertices. What do you notice? Which coordinates in each coordinate pair changed and which remained the same? Repeat with the vertical axis. (Adapted from the Atlantic Curriculum)

G6-32

Slides, Rotations and Reflections Review

GOALS
Students will perform and identify combinations of two transformations.

PRIOR KNOWLEDGE REQUIRED
Perform and identify a slide, a rotation or a reflection of a shape

VOCABULARY
rotation reflection slide centre of rotation mirror line translation arrow

Draw the shapes shown below on the board or on an overhead. Ask your students to identify the single transformation that takes each shape into the others.

Draw the shapes shown below on the board or on an overhead.
**ASK:** Which transformation takes shape A onto shape B? Draw a translation arrow between the shapes A and B. Ask your students to identify the slide. Mark a vertex on shape A and ask your students to identify the vertex of B which is the image of the marked vertex. Ask them to draw the translation arrow between the new vertices. Students should notice that the second translation arrow is the same length and points in the same direction as the first.

Ask your students to find a pair of shapes that can be transformed into one another by a reflection (C and D, for instance), and ask a volunteer to draw the mirror line. Draw a vertical mirror line through the left-hand side of C and **ASK:** What should I do to the image of C (after the reflection) to move it onto shape D?

Repeat the exercise above with rotation (shapes D and E, 90° rotation around a common vertex; shapes E and F, 180° rotation around the midpoint of the common horizontal edge, shapes G and A, 90° rotation around a common vertex). Then ask your students to identify a pair of transformations that would take shape C onto shape E. Encourage your students to find multiple answers to this question. **(SAMPLE ANSWERS:** reflect C through the vertical mirror line in the centre of the grid to obtain D, and rotate the image (D) a quarter turn counter-clockwise; reflect C through the right side and rotate around the point on the right side of the image one unit down from the top; rotate C 90° clockwise around the vertex with the reflexive angle (i.e., the “crook” in the L), then reflect through the right side of the shape.)

Ask your students to describe shape C without using the words “L-shape.” (A rectangle with base 1 and height 3, with an additional 1 × 1 square at the bottom of the right side, for instance.) Then ask them to describe shape D.

**SAY:** I reflect shape C through a vertical line. Where is the additional square attached now? (At the bottom of the left side) I reflect shape C through a horizontal line. Where is the additional square attached? (Top of the right side) Invite volunteers to check the predictions.

Can I pass from C to D using only reflections through the sides? How many reflections will I need? Can you tell where the additional square will be attached after each reflection without actually making the reflections? I reflected shape C seven times through vertical lines. Where is the square? I slid shape C in some direction. Where is the square now? I turned it 90° clockwise. Which way does it point now? A harder question: I slid it, turned it 180° and reflected it through a vertical line. Where is the additional square after all those transformations?

**Assessment**

1. Describe a series of transformations that could be used to get shape A onto shape C. Give at least two answers.

2. Which transformations were used to move shape B onto shape D?

3. Corinne says she can get shape D from shape A using two transformations. Is she correct? Explain.
Extensions

1. Reflect the shape in a slanted line. Describe the move as a combination of a reflection in a horizontal or a vertical line and a rotation. How many different descriptions can you find?

2. Look at the picture from the assessment. Corinne says that she can reflect shape A in a slanted line and slide it to get shape D. Draw the mirror line and describe the slide.

3. Use the cards from BLM “Triangle Transformations”. Ask your students to choose one shape and arrange copies of that shape and its reflection in a hexagon, such that each card is a reflection of each adjacent card through their common side:

   ![Hexagon diagram]

   This is a reflection of the card on either side.

   Ask your students to tell which transformation takes each card onto each other. Repeat the activity with cards from the BLM “Find-a-Flip Game”, arranging them first into a square and then into a 1 × 4 rectangle. Discuss the similarities and the differences in these three activities as a class. (In all three activities the only transformation performed was a reflection. However, two reflections produced a 120° rotation in the first case, a 180° rotation in the second case and a slide in the third case. In the first activity a pair of opposite shapes differs by reflection, and in the second activity they differ by a rotation. One can also notice that reflections over intersecting lines produce alternating reflections and rotations, as in the hexagon and in a 2 × 2 square, but reflections in parallel lines as in a 1 × 4 rectangle produce alternating reflections and slides.)

   Draw a triangle with angles 90°, 60°, and 30°. Invite a volunteer to mark the angles. Ask your students to predict the result of reflecting the triangle in two lines in each of the following situations:
Can students describe each set of reflections as a single transformation? What is that transformation? (Each of the shapes above can be obtained through one transformation.)

**ANSWERS:**
- Clockwise turn (less than half) around the intersection of the lines of symmetry
- Half turn around the intersection of the lines of symmetry
- Slide right twice the distance between the lines of symmetry

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**G6-33 Building Pyramids**

**GOALS**
Students will build a skeleton of a pyramid and describe the properties of pyramids.

**PRIOR KNOWLEDGE REQUIRED**
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon

**VOCABULARY**
- edge
- vertices
- triangular pyramid
- hexagonal base
- vertex
- pyramid
- pentagonal skeleton
- face

Start with a riddle: “You have 6 toothpicks. Make 4 triangles with them. The toothpicks must touch each other only at the ends.” Let your students try to solve the riddle using toothpicks and modelling clay to hold the toothpicks together at the vertices of the triangles. The answer, of course, is the triangular pyramid. You might give your students the hint that the solution is three-dimensional.

Sketch a rectangular pyramid on the board and shade the base. Ask volunteers to mark the edges and the vertices (counting them, and making a tally chart). Write the words “base,” “edges,” “vertex,” and “vertices” on the board.

Give your students modelling clay and toothpicks. Show them how to make a pyramid—first make a base, then add an edge to each vertex of the base and join the edges at a point. The students should make triangular, square, and pentagonal pyramids. Then let them fill in the chart on the worksheet.

After finishing the worksheet, they may check their prediction for the hexagonal pyramid by making one.

Tell your students that the shapes they have built are called “skeletons” of pyramids. You might write on the board: “Skeleton = Edges + Vertices.” As animal skeletons are covered with flesh and skin, the skeleton of a pyramid can be covered with paper or glass or other substances and will have faces. Show a pyramid (with faces) and write the word “faces” on the board as well.
Ask your students to look for relationships between the columns of the chart. Encourage them to write a rule to obtain the number of vertices and the number of edges of a pyramid from the number of sides of the base. Ask them to provide geometrical explanations for any patterns or rules they see, such as “The number of vertices in a pyramid is one more than the number of sides in the base, so the rule is Add 1 to the number of sides in the base.” To help students articulate a geometrical explanation, you might ask: How did we create a pyramid? What did we build first? Does the base have more vertices than edges or vice versa? What did we add to the base to get a pyramid?

As a challenge, ask your students to record the rules as formulas, using n for the number of vertices.

**SOLUTION:** A pyramid with n edges in the base also has n vertices in the base. But attached to each vertex in the base there is one non-base edge. Hence there are n non-base edges and n base edges. Therefore there are $2 \times n$ edges altogether in a pyramid with n base edges. (So, for example, a pyramid with 5 base edges would have $2 \times 5 = 10$ edges altogether.)

**Assessment**

How many faces, edges, and vertices would a pyramid with a ten-sided base have?

**Extensions**

1. Ask your students to bring to class pyramids or pictures of pyramids (Egypt, Mexico, Japan, entrance to Louvre, Paris, France and any others) that they can find at home. You can use the pyramids they brought in the lessons G6-36 and G6-37.

2. **HOMEWORK PROJECT:** Ask your students to find a picture of a pyramidal structure and give a presentation about it—what was the structure used for, when and where was it built, why does it have the pyramidal form.

**ACTIVITY**

On a class picnic build skeletons of pyramids and prisms from marshmallows and toothpicks or straws.
Repeat the previous lesson for prisms instead of pyramids. During the building activity, ask your students to build several triangular prisms with different bases. If you have toothpicks of different lengths, students could make prisms with different triangular bases: isosceles, right-angled isosceles, scalene.

Hold up a triangular prism. Join it to the board by one of the faces and ask your students: Suppose the board is a mirror. Let’s look at the solid that is composed of both the prism and the reflection. Ask students to describe the solid and name it.

Let your students practice with a mirror and the triangular prism skeletons they made. You can use a large mirror at the front of the class or smaller mirrors and smaller shapes that students could work with in groups. Are the results different if you attach the side face or a base of the prism to the mirror? (Yes—the base gives a prism of double height, and the side face creates a prism with a quadrilateral base.)

For instance, if you use a triangular prism with an equilateral base, the resulting prism will have a rhombus as the base. What happens if you use a prism with an isosceles base? What other base do you get? Does the base change if you use a different side face as a mirror plane? What happens if you repeat the process with a pyramid with an equilateral base? Will you still get a pyramid? (Generally, no) Is it a prism? (No as well.)

**GOALS**
Students will build a skeleton of a prism and describe the properties of prisms.

**PRIOR KNOWLEDGE REQUIRED**
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon
Special quadrilaterals
Symmetry

**VOCABULARY**
face edge
vertex vertices
prism cube
triangular pentagonal base
hexagonal
mirror plane

**ACTIVITY 1**
Take two copies of the Triangular Prism with a Scalene Base (see “Right Prisms” BLM). Join one of the non-base faces of one prism to a congruent face of the other prism. What 3-D figure do you get? What is the shape of its base? Turn one of the prisms upside down and repeat the exercise. Is the result different? Students can make predictions of the results they get with other non-base faces of the same prism and then check their predictions.
This activity satisfies one of the demands of the Ontario and Atlantic Curricula.

Place two skeletons of triangular prisms on a table. Push the sides of one of the prisms so that the entire prism is tilted to one side. Ask students to compare the number of faces, edges and vertices of the two prisms. Students should see that some of the faces of the new prisms are parallelograms rather than rectangles. Explain to your students that the original figure is called a **right prism** because the angles between the base and the vertical edges are right angles, and the new prism is called a **skew prism**. Students can also create skew prisms using the nets provided in the BLM. For assessment, provide your students with a collection of prisms, some of them right prisms and some of them not, and ask your students to sort the shapes into right prisms and skew prisms.

**Assessment**
Which of these are right prisms?

![Images of prisms]

**ACTIVITY 2**

This activity satisfies one of the demands of the Ontario and Atlantic Curricula.

Place two skeletons of triangular prisms on a table. Push the sides of one of the prisms so that the entire prism is tilted to one side. Ask students to compare the number of faces, edges and vertices of the two prisms. Students should see that some of the faces of the new prisms are parallelograms rather than rectangles. Explain to your students that the original figure is called a **right prism** because the angles between the base and the vertical edges are right angles, and the new prism is called a **skew prism**. Students can also create skew prisms using the nets provided in the BLM. For assessment, provide your students with a collection of prisms, some of them right prisms and some of them not, and ask your students to sort the shapes into right prisms and skew prisms.

**Assessment**
Which of these are right prisms?

![Images of prisms]

**ACTIVITY 3**

Give each student two boxes that are both rectangular prisms (You may ask them in advance to bring various boxes from home. The length, width, and height of the boxes should be different.) Ask your students to tell you the proper mathematical name for the boxes (rectangular prisms) and ask them if they can find any faces that are congruent. Your students might find the congruent faces by tracing the faces on paper. How many congruent faces does each prism have? Ask your students to label the faces, so that the congruent faces are marked with the same letter. Then ask them to measure the edges and to write down the dimensions for each face. What happens if they join a pair of prisms along congruent faces? Ask your students to predict the dimensions of the new prisms and to check their predictions. How many faces of the prism double in size? (4) How many dimensions of the prism double? (1) What happened to the volume of the prism?

**Extensions**

The next series of extensions satisfies one of the demands of the Western Curriculum.

1. Ask your students to explain what parallel lines are. Students should say that these are straight lines that never meet, even if extended. Explain that things are more complicated in 3-dimensional space, but what they just said works well for flat surfaces or faces. For example, the floor and the ceiling are (usually) parallel surfaces, because they never meet, even if you extend them. What other parallel surfaces can students see around them? (Possible answers: opposite walls; the top of a desk is usually parallel to the floor, and so on.)

2. **Parallel Faces**
   a) Before doing this extension, prepare ahead by taking an empty milk carton and colouring the bases (the front and the back faces in the picture) red and the shaded faces blue.
Start by showing your students a prism with a regular pentagon in the base. Which faces are parallel? If students have trouble noticing the parallel bases, place the prism face down on the table. How many pairs of parallel faces does this prism have? Ask a volunteer to draw the shape of the base on the board and to record the number of pairs of parallel faces. Now show your students the milk carton you prepared. Ask them to find the bases of this shape. What 3-D shape is it? (pentagonal prism) Ask students to look at the blue faces of the carton. What do they notice? (The faces are parallel.) Are these bases? (no) Are the bases parallel? Ask your students to draw the shape of the base and to write the number of pairs of parallel faces. Compare the results with the regular pentagonal prism. **ASK:** How are the prisms different? How are the shapes of the bases different?

Ask your students to pick a pair of parallel faces and to find a face that intersects (has common edges with) both faces. **ASK:** What do you notice about edges that meet the parallel faces? (The edges are parallel.) Repeat with a different face. Then choose a different pair of parallel faces and repeat again.

b) Show your students a triangular prism and ask them to find parallel faces. Repeat with a cube and other prisms. Fill in the table:

<table>
<thead>
<tr>
<th>Name of Prism</th>
<th>Shape of Base</th>
<th># of Pairs of Parallel Sides in the Base</th>
<th># of Pairs of Parallel Faces</th>
</tr>
</thead>
</table>

Include some prisms with irregular bases and skew prisms into this activity. (Nets for several right prisms with irregular bases and skew prisms are included in BLMs.)

Repeat the above with pyramids.

c) Ask your students to look for relationships between the two last columns of the tables. **ANSWER:** Pyramids do not have parallel faces. Prisms have at least one pair of parallel faces (the bases). If a prism has a pair of parallel sides (edges) in its base, the faces that intersect the base in these edges are parallel as well. Hence, the number of pairs of parallel faces in a prism (including the bases) is one more than the number of pairs of parallel sides in the bases.

3. In two-dimensional space (**EXAMPLE:** on a piece of a paper) parallel lines are lines that never meet. In three-dimensional spaces, however, this definition is not sufficient. Point to the line where the floor meets a wall and the line where the ceiling meets an adjacent wall. These lines never meet, but they are not parallel. (They are called skew lines.) **ASK:** How can we describe parallel lines in three-dimensional space? (You can say that parallel lines are lines that never meet and that travel in the same direction.)

Place a triangular prism on the table so that it stands on one of its bases. Point at one of the vertical edges and ask your students to show edges parallel to that one. How many vertical edges are there? Choose a horizontal edge and repeat. This time there is only one edge parallel to the given edge, and these edges belong to the same rectangular side face. Repeat with a pentagonal prism with a regular base and a rectangular prism.
4. Ask students to think about the following statements. They can use models to determine, and explain, their answers.

   a) Anna says that if two edges of a prism are parallel, they always belong to the same face. Is she correct? (No. Side edges of a prism are all parallel, but they do not always belong to the same face.)

   b) Dimitra says that if two edges of a prism are parallel, they belong to parallel faces. Is she correct? (No. Side edges of a regular pentagonal prism are all parallel, but there are no parallel side faces. These edges do not always belong to the same face either.)

   c) Evan says that if two faces of a prism are parallel and they both have edges that belong to the same third face, then these edges are parallel and equal in length. Is he correct? (Yes)

G6-35
Edges, Faces and Vertices

Remind your students that there are lots of 3-D shapes in the world around us that are either pyramids or prisms. As an example you might show them a photo of the pyramids in Egypt.

Hold up a 3-D shape and draw a picture of the shape on the board. Write the words “3-D shape;” ask volunteers to show the edges, the faces, and the vertices, both on the shape itself and on the drawing; and write the terms “edge,” “face,” and “vertex” on the board. Remind your students that the plural of “vertex” is “vertices.”

Your students will need the skeletons of the cubes they made during the last two lessons. Give each student 2 squares made of paper and 4 squares made of transparent material. Ask them to add:

- The non-transparent squares as the bottom face and the back face
- The transparent ones—as the top, the front, and the side faces

Add the faces step by step, one face at a time, emphasizing the positions and names of the faces. It is a good idea to show the students how to add faces on a larger model.
Ask students to identify the edges of the cube that they see only through the transparent paper. If the transparent faces were made of paper, would they see these edges? No. The edges that would be invisible if all the faces were non-transparent are called the “hidden edges.” On a two-dimensional drawing of a cube these hidden edges are marked with dotted lines.

Suggest that your students make models of various prisms and pyramids with play dough or modelling clay. Start with a model of a cube. Ask your students to cut off a single vertex of the model with a plastic knife.

![Cube model with a vertex cut off]

Explain that the new face that appeared is called the **cross-section**. What shape does the cross section have? **ASK:** How many faces meet at a vertex of a cube? How many edges? (3) How many sides does the cross-section have? How many vertices? Why? (Each face of the cube produces an edge of the cross-section. Each edge of the cube produces a vertex of the cross-section.)

Ask your students to predict the shape of the cross-sections (resulting from cutting off vertices) for other shapes, first prisms and then pyramids. For pyramids, first ask your students if they expect that all vertices will produce the same cross section. Might there be exceptions? To help your students visualize the cross-section before cutting, use elastic bands stretched on shapes. (The point of the pyramid produces the shape with the same number of sides as the base, whereas the other vertices produce triangles.)

**Extension**

Let the students use cubes made of modelling for this extension. Pick one vertex of a cube and mark points on the three edges that meet at this vertex at the same distance from the vertex. What will the shape of the cross-section be? What shape did you cut off? Use elastic bands, or cut and replace the shape, to visualize the cut and check the prediction. Repeat with different distances from the vertex, such 2 cm, 2 cm, and 3 cm, or 1 cm, 2 cm, and 3 cm.

**ACTIVITY**

Ask students to hold their cubes in various positions (on the table, on the floor looked at from above, slightly above them and so on.) Ask students to describe what the faces look like when seen from different angles (they look like square, parallelogram, etc). The outline of the shape itself can look like a square, a rectangle, a hexagon, a trapezoid and a rhombus.
G6-36
Prisms and Pyramids

Divide your students into groups. Give each group several 3-D shapes, so that each group has some rectangular and triangular pyramids, rectangular and triangular prisms, and a cube. Ask your students to count the faces of the shapes. If some students are having trouble keeping track of the number of faces, they might mark each face with a chalk dot or a small sticker. Ask your students to count the edges and vertices on the 3-D shapes as well (they also might shade edges with chalk and mark vertices with stickers.) Ask them to write the results of their count in the table on the worksheet (QUESTION 2).

Draw a pentagonal pyramid and a triangular prism on the board and let volunteers count the edges, faces, and vertices of these figures. Ask them to mark the edges and circle the vertices as they count.

Review the rules that students found for the number of edges and vertices of pyramids and prisms in G6-33 and G6-34. Then add a row for the number of vertices in the base of each shape. ASK: What do you notice about the number of vertices in a prism? (It is even, and it is twice the number of vertices in the base.) What do you notice about the number of vertices in a pyramid? (It is one more than the number of vertices in the base.) How can you get the number of faces in each shape from the number of edges in the base? How can you get the number of edges in each shape from the number of edges in the base? Summarize these rules on the board:

<table>
<thead>
<tr>
<th></th>
<th>Prism</th>
<th>Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices</td>
<td>vertices in base × 2 (number is even)</td>
<td>vertices in base + 1</td>
</tr>
<tr>
<td>Faces</td>
<td>edges in base + 2</td>
<td>edges in base + 1</td>
</tr>
<tr>
<td>Edges</td>
<td>edges in base × 3</td>
<td>edges in base × 2</td>
</tr>
</tbody>
</table>

SAY: I am thinking of a shape. It is either a pyramid or a prism. It has 15 vertices. What is it? Can it be a prism? (No, because the number of vertices is odd.) So my shape is a pyramid. How can I get the number of vertices in a pyramid from the number of vertices in the base? (add 1) What do you have to do to get the number of vertices in the base from the total number of vertices? (subtract 1) How many vertices are in the base of my pyramid? (14)

SAY: I have a shape with 70 vertices. Can it be a pyramid? Can it be a prism? (The number is even, so it could be either one.) How many vertices are in the base if it is a pyramid? (69) If it is a prism? (35) What else do you need to know to decide whether the shape is a prism or a pyramid? (the number of faces or number of edges) What number of edges would these shapes have? What number of faces? (Edges: prism 35 × 3 = 105, pyramid 69 × 2 = 138. Faces: prism 35 + 2 = 37, pyramid 69 + 1 = 70.)

Review area for rectangles, parallelograms, and triangles. Tell students that the surface area of a prism is the sum of the areas of all faces of the shape. Ask your students when they might need to know the surface area of a prism. (One EXAMPLE: to calculate the amount of paper needed to wrap a present.) Present a rectangular prism. Invite volunteers to measure the sides of the
prism. Ask your students to draw the faces of the prism and to mark the dimensions of each face. Ask your students to check how many faces the prism has. Did they draw all the faces? For each face, ask your students to write a multiplication statement for the area of the face and to add the results for all the faces. For example, for a $2 \times 3 \times 4$ prism students should draw and write:

- $2 \times 3 = 6 \text{ cm}^2$
- $2 \times 4 = 8 \text{ cm}^2$
- $3 \times 4 = 12 \text{ cm}^2$

Surface area: $6 + 6 + 8 + 8 + 12 + 12 = 52 \text{ cm}^2$.

Point out to the students that because they are measuring area, the measurement units are cm$^2$ and not cm$^3$, even though this is a 3-D shape. Repeat the exercise with several prisms, including a triangular prism.

As a challenge, you might present the following question:

Pam needs 5 mL of paint for each square decimetre of area. How much paint does she need to paint the outside of a box with a hexagonal base and lid and rectangular sides? The box is 40 cm tall and its base (which is the same as the lid) is shown below:

Students should sketch the faces of the box, mark the dimensions, find the area of each face and then the surface area of the box, and determine the amount of paint needed.

(\textbf{ANSWER:} The surface area of the box is 52.48 dm$^2$ and this will require 262.4 mL of paint.)

\textbf{Extensions}

1. Ask your students to try and add the number of faces and vertices of a cube and subtract the number of edges. The result is 2. What happens if you do that to another solid? (It will be 2 as well. This fact is known as Euler's formula and was discovered by the great Swiss mathematician Leonard Euler in the 18th century.)
2. Let your students construct shapes from Polydrons—from regular 3-D shapes like prisms and pyramids to animals and castles. Count the faces, edges and vertices. Check if the Euler’s formula holds.

3. This exercise was adapted from the Atlantic Curriculum.

Suggest that your students make models of various prisms and pyramids with modelling clay. Start with a model of a cube. **SAY:** I want to make a cross section of a cube, but this time I will cut off a single edge of the model. Ask your students to predict the shape of the cross section that is parallel to an edge. Let them check the prediction by actual cutting their models with fishing wire or a plastic knife.

![Cube Diagram]

Ask your students to predict what shape the cross section of a cube will have if it is made by a plane parallel to one of the faces of the cube. Let your students check their predictions. Invite them to make cross-sections using various other planes, such as planes through various diagonals of the cube. Do these planes produce different shapes? What different quadrilaterals can they produce?

**ASK:** How many edges are affected when you cut off one edge? (only 1) How many faces were affected by cutting off one edge? (4) challenging: What shape is the cross section? (a rectangle) invite students to try and explain why the shape is a rectangle. **ANSWER:** The edge that was cut off was perpendicular to face E and parallel to a and b. Therefore, edges a and b are perpendicular to face E and to line c, which is on face E. If a and b are perpendicular to c (and by the same argument to d), then abdc is a rectangle.

Explore what happens when you cut the edge off a pyramid. Let students cut a base edge first. When they see that the resulting cross section is always a trapezoid, you might challenge them to explain why again. Next, let them predict the shape of the cross section parallel to an edge that joins a vertex in the base to the point. Let them check their predictions, first with an elastic band and then by cutting the actual shape. Does the shape of the cross section depend on the shape of the base? (Yes. The cross section parallel to an edge joining the point will cut through all faces of the pyramid, so the cross section will have one side more than the number of sides of the base.)

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**ACTIVITY 1**

Show your students an example of a cone and a cylinder. Explain that a cone has one curved surface and one flat surface, while a cylinder has two flat surfaces and one curved surface. Ask students to find as many examples of pyramids, prisms, cones, and cylinders as they can in the classroom.

**ACTIVITY 2**

Ask your students to bring from home objects that are prisms, pyramids, cubes, cylinders, and cones. Create a collection of such shapes for reference and future use in the class. **EXAMPLES:** paper towel rolls, drinking glasses, boxes and tins (some boxes are in the shape of cylinders or hexagonal prisms), juice or milk cartons (these are pentagonal prisms).
## ACTIVITY 3

### A Game for Pairs

One player gives a description of a shape; the other has to name the shape. **ADVANCED:** the player gives only two numbers of the three possibilities: number of edges, vertices and faces. **EXAMPLE:** 12 edges and 6 faces: a rectangular prism. 12 edges and 7 faces: a hexagonal pyramid.

### GOALS

Students will distinguish between prisms and pyramids, and identify their bases.

### PRIOR KNOWLEDGE REQUIRED

- Pyramid
- Prism
- Geometric shapes: triangle, rectangle, quadrilateral, pentagon, hexagon

### VOCABULARY

<table>
<thead>
<tr>
<th>triangle</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadrilateral</td>
<td>pentagon</td>
</tr>
<tr>
<td>hexagon</td>
<td>prism</td>
</tr>
<tr>
<td>pyramid</td>
<td>base</td>
</tr>
<tr>
<td>point</td>
<td></td>
</tr>
</tbody>
</table>

Hold up a pentagonal pyramid. Ask a volunteer to count the faces. What shapes are they? (A pentagon and five triangles.) Ask a volunteer to draw a pentagon and a triangle on the board. Write the number of times each shape appears in the pentagonal pyramid as a face.

Tell your students that in the pentagonal pyramid the face that is not the same shape as the others is the base. Hence in this pyramid a pentagon is the base.

In a pyramid there is always one base, unless the pyramid has all triangular faces (in which case it must be a triangular pyramid). The shape of the base of a pyramid gives the pyramid its name. Hence if a pyramid has a base with 5 sides, it is called a “pentagonal pyramid.” A prism has two bases. The non-base sides of the prism are always rectangles.

Give your students a set of 3-D shapes (you can use the shapes from your collection or ask students to construct shapes using the nets from the “Nets for 3-D Shapes” BLM). Ask your students to place each shape on a piece of paper and trace the faces. Ask them to write the number of times the shape appears as a face inside the shape, the way you did on the board. Students should also write the name of the figure beside the base. If your students have trouble spelling the names of the figures, write them on the board.

### Assessment

1. Give each student a pentagonal pyramid, a triangular prism, and a cube.
   - a) Place each shape—base down—on a piece of paper and trace the base. (That way you can verify that each student knows how to find the base.)
   - b) Write the name of the figure beside the base and indicate whether the figure has one or two bases.
   - c) If all faces of the figure are congruent, indicate this.

2. Look at the shapes in QUESTION 3 of the worksheet. Which shapes are pyramids? Which shapes are prisms? Can a shape be both a prism and a pyramid? Why not?
Extensions

These extensions fulfil the demands of the Atlantic Curriculum. Use only right pyramids (pyramids with the apex, or point, directly above the centre of the base).

1. a) Ask each student to create a right prism with modelling clay, and encourage them to use different bases. **ASK:** If you cut your prisms with a plane parallel to the bases, what shape does the cross section have? Let the students compare the shape of the base and the shape of the cross section. (the shapes will be the same) Ask your students to look at the two 3-D shapes that they obtained by cutting the prism in two parts. What are they? Do cross sections made at different heights produce different shapes? (They produce prisms of different height with the same bases.) Ask students to cut prisms through the midpoints of the edges joining the bases. What do they notice? (the shapes are congruent)

Draw a rectangle on the board and mark one of the lines of symmetry. **ASK:** What happens if you cut this rectangle along the line? What is this line called? (line of symmetry) How is the last cut that you made on the prism similar to the cut on the rectangle? What do you think the plane that you used to cut the prism into two congruent prisms should be called? (plane of symmetry) Explain that a plane of symmetry cuts a shape into two congruent shapes that are mirror images of each other. Brainstorm with the students how they could check the congruency of 3-D shapes.

b) Draw a rectangle. What are its lines of symmetry? Draw or hold up a right pyramid with a rectangle in the base and ask students to find its planes of symmetry. How many planes of symmetry does this pyramid have? (2) Is a plane parallel to the base also a plane of symmetry? (no) Why not?

Repeat for square pyramids and triangular pyramids with equilateral bases.

Complete this table as you answer the same questions for each shape:

<table>
<thead>
<tr>
<th>Pyramid</th>
<th>Draw Shape of Base and its Lines of Symmetry</th>
<th>Number of Lines of Symmetry in the Base</th>
<th>Number of Planes of Symmetry in the Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular Pyramid</td>
<td><img src="image" alt="Triangular Pyramid" /></td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Add rows for pyramids with any or all of the following bases: regular pentagon, regular hexagon, square, rhombus and rectangle. Do your students see a relationship between the last two columns?

2. Repeat Extension 1 for prisms, but do not use rectangular prisms with any faces that are squares.

3. Ask your students to draw the lines of symmetry on each face of a triangular prism with equilateral base. What do they notice? Repeat with a square prism with rectangular side faces.

4. Ask your students to draw symmetry lines on each face of a cube. What do they notice? How many planes of symmetry does a cube have? (9) Let your students check their answer with a cube made of modelling clay.
5. Students will need the BLM “Triangular Pyramid with Three Right-Angled Faces” and two mirrors (e.g., Mira).

   a) How many lines of symmetry does each face of this pyramid have? (**Answer:** The right-angled faces have 1 line of symmetry, and the fourth face has 3 lines of symmetry, since it is an equilateral triangle.)

   b) How many planes of symmetry does this pyramid have? (3)

   c) Suppose you attach one of the faces of the pyramid to a mirror. What shape will the pyramid and its reflection make together? Do you expect different results with different faces? Check with a mirror. (**Answer:** If you attach the equilateral face of the pyramid to a mirror, the result will be neither a pyramid, nor a prism. However, if you attach a right-angled face to a mirror, you get a larger triangular pyramid, with two equilateral faces and two right-angled faces.)

   d) Attach one of the right-angled faces of the pyramid to a mirror. Which faces of the original pyramid are extended in the mirror so that the face and its reflection become part of the same new face? (The right-angled faces.) Explain to the students that this happens because the right-angled faces that are not attached to the mirror are both perpendicular to it. Point out that the original pyramid can be obtained from a cube by cutting off a vertex (through three adjacent vertices, for instance). Also, if you attach a cube to a mirror, you see the faces perpendicular to the mirror “double,” extending into the mirror. Compare this to the process of reflecting a square through one of its sides in two dimensions.

   e) How many copies of the pyramid do you need to make a square pyramid? (4) How can you check that using two mirrors? (**Answer:** Place two mirrors intersecting at a right angle. Place the pyramid at the intersection so that two of its right-angled faces are attached to the mirrors. You will see three reflections of the pyramid, creating a new square pyramid.)
G6-38
Properties of Pyramids and Prisms

Review lesson G6-36 using a collection of 3-D shapes. Review the connections between the number of vertices and edges in the base and the number of vertices, edges, and faces in pyramids and prisms:

**FOR PYRAMIDS:** What do you have to do to the number of vertices in the base to get the number of vertices in the whole pyramid? (Add 1.)
What do you have to do to the number of edges in the base to get the number of edges in the whole pyramid? (Multiply by 2.)

**FOR PRISMS:** What do you have to do to the number of vertices in the base to get the number of vertices in the whole prism? (Multiply by 2.)
What do you have to do to the number of edges in the base to get the number of edges in the whole prism? (Multiply by 3.) What do you have to do to the number of edges in the base to get the number of faces in the whole prism? (Add 2.)

**ASK:** If you have a prism with 100 sides in the base, how many vertices does this prism have? How many edges? How many faces?
How many faces does a pyramid with a 200-sided base have? And how many vertices? How many edges?

**Bonus**
How many edges, faces, and vertices would a pyramid and a prism with a 1000-sided base have?

Ask your students to draw the following chart in their notebooks and to fill it in, using the actual shapes:

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangular Pyramid</th>
<th>Triangular Pyramid</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shape of faces that are not bases</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Show your students how you can make a cone from a piece of paper. Ask them where they have seen this shape (ice cream cone, clown hat, etc). Does a cone remind them of any other geometric shape that they have studied in the lesson? (a pyramid) Let volunteers fill in the following chart on the board:

<table>
<thead>
<tr>
<th>Property</th>
<th>Pyramid</th>
<th>Cone</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>two</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of bases</td>
<td>1</td>
<td>1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>1, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Has a vertex opposite to the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of vertices = # of vertices in the base + 1</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Ask your students to imagine a pyramid with 1 000-sided base. Does it look like a cone? Write a comparison paragraph on the board:

- **SIMILARITIES:** Both shapes have one base and a vertex at the opposite end. A cone is like a pyramid with a circular base. The more sides the base of a pyramid has, the nearer it is to a circle and so the nearer the pyramid is to a cone.

- **DIFFERENCES:** The pyramid has many faces, one polygonal (base) and the others triangular. It has many edges that are straight lines. It has many vertices, not just the one at the point. A cone has only one flat face, that is a circle, and one curved “face.” It has only one curved “edge.”

Show your students how to make a cylinder from a piece of paper. Ask your students to imagine a prism with 100-sided base. Does it look like to a cylinder? Ask volunteers to fill in the comparison chart:
<table>
<thead>
<tr>
<th>Property</th>
<th>Prism</th>
<th>Cylinder</th>
<th>Same?</th>
<th>Different?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td>many</td>
<td>3</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Shape of base</td>
<td>polygon</td>
<td>circle</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of vertices in the base</td>
<td>many</td>
<td>0</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Number of bases</td>
<td>2</td>
<td>2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of faces that are not bases</td>
<td>many</td>
<td>1, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of edges</td>
<td>many</td>
<td>2, curved</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Number of vertices</td>
<td>many</td>
<td>0</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of vertices = 2 x # of vertices in the base</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td># of faces = # of edges in the base + 2</td>
<td>yes</td>
<td>yes</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Write a comparison paragraph on the board:

- **SIMILARITIES**: The cylinder and the prism both have two bases. A cylinder is like a prism with a circular base. The more sides the base of a prism has, the nearer the base is to a circle. The nearer the base is to a circle, the nearer the prism is to a cylinder.

- **DIFFERENCES**: A prism has many faces, two polygonal (bases) and the others rectangular. It has many edges that are straight lines. It has many vertices. A cylinder has only two flat faces, that are circles, and one curved “face.” It has only two curved “edges” and no vertices at all.

After comparing cones and cylinders to pyramids and prisms your students should have a better idea how to answer the question on the worksheet that asks them to compare two 3-D shapes.

**Assessment**

1. a) Make a property chart for rectangular and triangular prisms.
   b) Write a paragraph comparing rectangular and triangular prisms using the chart.

2. Who am I?
   a) I have only rectangular faces.
   b) I have 8 faces, 6 of them are rectangles.
   c) I have 6 edges. No pyramid or prism has fewer vertices than I do!
   d) I am a prism with 9 edges.
   e) I have one circular base.
3-D Shapes Bingo
Students will need 3-D Shapes Bingo boards (see BLMs) and counters. The teacher reads the cards and players place counters on shapes that fit the description. Some descriptions apply to more than one shape, in which case the player can decide on which shape to place the counter. Once a counter is placed, it should not be moved, even if a new description applies to a shape already covered by a counter. If a description does not fit any free shapes, no counter is placed.

1. Word Search Puzzle (3-D Shapes): Please refer to the BLM.
2. Sketch a skeleton of a pyramid or prism. For EXAMPLE:

   ![Example Sketch]

   Ask students to say whether the following statements are true or false:
   - My bases are all triangles.
   - I have more vertices than edges.
   - All of my faces are congruent

   Ask more questions of this sort.

3. Select a prism and say how many pairs of parallel edges each face has.

4. Show students a collection of shapes. Include prisms, pyramids, and shapes that are neither. Ask students to circle the pyramids and cross out the prisms.

EXAMPLES:
ANSWERS: Shapes B and D are prisms, and shapes C and E are pyramids. Pyramid C has a trapezoid in the base and is laying on its side, and the bases of D are its front and back faces. (NOTE: A is composed of two pyramids but is not itself a pyramid. It has 8 congruent triangular faces and is called an octahedron. F is a truncated pyramid, that is, a pyramid with its top cut off. It has two bases, but they are not congruent, one is a dilation of the other. Shape G has two congruent pentagonal bases, but it has ten side faces which are triangles. It is called a pentagonal antiprism.)

5. This extension satisfies the demands of the Atlantic Curriculum.

Let your students make a pyramid with a regular shape in the base using modelling clay. Encourage different students to produce different shapes, so that results can be compared later. Let your students cut the pyramid parallel to the base. What is the shape of the cross section? Compare the cross section and the base. How can we describe such a pair of shapes? (similar) Repeat with a cut that is not parallel to the base, but does not touch the point or the base of the pyramid. Does the cross section have the same number of vertices and edges as the base? (yes) Is the cross section similar to the base? (no) What shape does a cross section have if the cut is performed vertically through the point? (triangle)

Let your students predict and then check the results of similar cuts performed with a cone. What will the shape of the cross section that is not parallel to the base be? Students might feel intuitively that the shape has to be curved but not similar to a circle. Explain that the shape that is obtained in a diagonal cut of the cone is called an ellipse.

Repeat the exercise with a prism and a cylinder.
G6-39
Nets and Faces

GOALS
Students will create nets for pyramids, prisms and cubes.

PRIOR KNOWLEDGE REQUIRED
Geometrical shapes: triangle, square, rectangle, pentagon, hexagon
Faces, edges, vertices of a 3-D shape

VOCABULARY
net vertices
face pyramid
edge prism
vertex cube
triangular pentagonal
hexagonal

Review the connections between the number of vertices and edges in the base and the number of vertices, edges, and faces in pyramids and prisms.

Draw several shapes on the board and ask your students which 3-D shape they make. If students have trouble identifying the shapes, ask them to circle the base(s) first.

SAMPLES:

Hold up a rectangular or a pentagonal pyramid. Ask your students to tell you which two types of faces it has. How many bases does it have? And what is the shape of the side faces? How many side faces does it have? If you need to make a net for this pyramid, the easiest way would be to start with a base (draw it on the board and write “base” on it) and to add a side face along each edge of the base (draw one side face and ask volunteers to draw the rest).

Ask students to cut out the nets of the pyramids from the BLM “Nets for 3-D Shapes” in this guide—they can use the nets later to help find the answers to QUESTION 1 on the worksheet.

ASK: What happens if you cut off one of the side faces and try to re-glue it at some other place. They might actually cut off one of the triangles and try to fit the triangle to some other edge. You might draw an example on the board. Try the following positions. Will the shape fold into a pyramid if the face is glued in this position?

(no) (yes) (no)

At this point, you may let your students do Activity 1. When students have finished the activity, draw the picture below on the board and ask if this will work as the net for a pentagonal pyramid:

(yes)
Invite volunteers to come up to the board and draw pictures that will not make a net for a pyramid (not necessarily pentagonal). For each drawing, ask your students to explain why this picture is not a net of a pyramid. Invite volunteers to change the drawing so that it will make a net.

Hold up a triangular or a pentagonal prism. Ask your students which two types of faces it has. How many bases does it have? What is the shape of the side faces? How many side faces does it have? If you wanted to make a net for this prism, the easiest way would be to start with the band of rectangles for side faces and to add the bases. Illustrate this on the board.

Repeat the exercise of cutting off a face of the prism and attaching it in some other place. Do that separately for a base and a side face. Ask students to cut out the nets of the prisms from the BLM “Nets for 3-D Shapes”. Let them cut off the faces and try to rearrange them at other places.

Draw several incorrect examples of nets of triangular prism on the board and ask volunteers to explain why these drawings cannot serve as nets for prisms:

- The bases are not the same
- The middle face is too short
- The bottom base is flipped
- Side face is missing

Invite volunteers to draw more pictures that will not work as prism nets, and ask the class to explain why these drawings cannot be prism nets. For a more challenging task, ask volunteers to draw pictures that might or might not work as nets, and let the class guess if these are nets of prisms.

**ACTIVITY 1**

Give students square or pentagonal pyramids and ask them to trace the faces on a piece of paper, so that they create a net. Ask them to cut out the nets they have drawn. Let them cut off faces of the net and re-attach the faces at different places. Will the new net fold into the same pyramid? Which edges are places where you would want to re-glue the faces and which are not? Repeat this exercise with a prism. This activity is important—students will explore various ways to create nets for the same solid, rather than memorizing a single net shape.
ACTIVITY 2
Allow your students to play the following game: one partner (or group) draws a net, the other has to guess what shape the net can be folded into. (Use the nets students created in the previous activity.)

ACTIVITY 3
Give your students Pentomino pieces made of paper or bristol board. Which pieces can be folded into a square box without a lid? Let them first try to predict the result, then check it. This activity is a good preparation for Extension 1.

Build-a-Net Game
Divide your class into groups of 3 to 4 students. Give each group a set of polygons from the BLM “Build-a-Net Game”. Each student receives 6 shapes, the rest are left in a pile on the side. If a student has a set of polygons that form the net of a pyramid or a prism, she is allowed to turn the set in for a point and take an equal number of new cards from the side pile. Students take turns placing polygons in the central pile. Each time a student places a polygon in the central pile, they pick up a new shape from the side pile. As soon as a student places a polygon in the central pile that can be combined with other polygons in the pile to make a net, that player is allowed to remove the net and scores a point. The game ends when there are no cards left in the side pile. The winner is the player who has created the greatest number of nets.

NOTE: Review the nets of prisms and pyramids that can be built from the shapes in the “Build-a-Net Game” BLM before starting. Ask your students: Which shapes can only be used for one or two nets? (pentagon and hexagon) Which shapes can be used for more than two nets?

ACTIVITY 5
After students have constructed the pyramids and prisms from the BLM “Nets for 3-D Shapes”, ask them to sketch the nets from memory. You might also ask them to sketch what they think the net for a hexagonal pyramid would look like.

ACTIVITY 6
Predict whether each of the nets shown will make a pyramid. Copy the shapes onto dot or grid paper. Cut the shapes out and try to construct a pyramid. Was your prediction correct?
Extensions

1. Perform Activity 3 before assigning this exercise. Copy the following nets onto centimetre grid paper (use 4 grid squares for each face).
   
   Predict which nets will make cubes. Cut out each net and fold it to check your predictions.

<table>
<thead>
<tr>
<th>ACTIVITY 7</th>
<th>ACTIVITY 8</th>
<th>ACTIVITY 9</th>
<th>ACTIVITY 10</th>
</tr>
</thead>
</table>
| Sketch the net for a prism by rolling a 3-D prism on paper and tracing its faces. For reference, use the steps shown for a pentagonal prism:

Step 1: Trace one of the non-base faces.

Step 2: Roll the shape onto each of its bases and trace the bases.

Step 3: Roll the shape onto each of its remaining rectangular faces and trace each face.

Find a prism or pyramid at home or at school and sketch its faces.

Does this make a net of a 3-D shape? Cut out and check your prediction.

(no) (no)

What shapes do these nets make? Cut them out and check.
After that draw as many different nets for a cube as you can.

2. Give your students many toothpicks of various lengths and some modelling clay. Ask them to make several triangular pyramids with different bases. For example, students might make acute-angled, right-angled and obtuse-angled triangle bases. Then ask your students to choose two different triangular pyramids and trace their faces to obtain nets for them. Ask your students to compare the nets. Students might also measure the angles of the faces with protractors, and the sides with rulers. Ask your students to make a triangular pyramid with…
   a) …more than one right angle.
   b) …at least two obtuse angles.

ADVANCED:
   c) …two right angles and one obtuse angle.
   d) …two right angles and two obtuse angles.

3. Copy the shapes into your notebook. How many hidden faces does each shape have? How many hidden edges does each shape have? Draw in the missing edges with dotted lines.

4. How many faces of the little cubes are on…

   a) the outside of the figure? (including the hidden faces)
   b) the interior of the figure?

5. On a cube draw a line from vertex A to B, then a line from B to C. What is the measure of ABC?

   HINT: Imagine drawing a line from vertex A to C on the hidden face. What kind of triangle is $\triangle ABC$?

   ANSWER: $\triangle ABC$ is an equilateral triangle. So $\angle ABC = 60^\circ$.

6. Each edge of the cube was made with 5 cm of wire.

   How much wire was needed to make the cube? (Don’t forget the hidden edges.)
G6-40
Sorting 3-D Shapes

Give each student (or team of students) a deck of 3-D shape cards and a deck of 3-D property cards. These cards are in the BLM section. (If you have enough 3-D shapes have students use 3-D shapes instead of the cards.) Let them play the following games:

3-D Shape Sorting Game
Each student flips over a property card and then sorts the shapes into two piles according to whether a shape on a card has the property or not. Students get a point for each card that is in the correct pile. (If you prefer, you could choose a single property for the class and have everyone sort the shapes using that property.) Once students have mastered this sorting game they can play the next game.

3-D Venn Diagram Game
Give each student a copy of the Venn diagram BLM (or have students create their own Venn diagram on a sheet of construction paper or bristol board). Ask students to choose two property cards and place one beside each circle of the Venn diagram. Students should then sort their shape cards into the Venn diagram. Give 1 point for each shape that is placed in the correct region of the Venn diagram.

Draw a Venn diagram to sort the shapes on the worksheet according to the properties:

1. Pyramid 2. One or more triangular faces.

What do you notice about your Venn diagram? Explain why part of one of the circles is empty.

NOTE: Some property cards involve planes of symmetry. Exclude these cards if you have not taught this material (Extensions in G6-37).

Assessment
Use the shapes below to complete the following Venn diagram:

1. One or more rectangular faces 2. One or more triangular faces
G6-41
Patterns with Transformations (Advanced)

GOALS
Students will create patterns with transformations.

PRIOR KNOWLEDGE REQUIRED
Clockwise, counter clockwise
$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ turn
Reflection
Mirror line
Patterning down
Core
Repeating pattern

VOCABULARY
pattern
clockwise
counter clockwise
$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ turn

Draw a simple asymmetric shape, like the one below:

Ask your students to create a pattern by: sliding the shape repeatedly 2 units right, reflecting the shape repeatedly in a line through the right side, or rotating it half a turn (how many degrees is that?) around the marked point.

Ask your students to create a pattern using two transformations, say, first a reflection, then a $\frac{1}{4}$ turn, then repeat.

Use pattern blocks to create patterns made by various transformations. (Describe how you made the pattern.) EXAMPLE:

a) slide

b) reflection

c) rotation

Each pair of students will need a die. Player 1 draws a simple shape or design on a $3 \times 3$ grid. Player 2 rolls the die and creates a repeating pattern of shapes by performing different transformations (one at a time) on the design provided by Player 1. The number of transformations (and the number of terms in the core of the pattern) is determined by the number rolled on the die. Player 1 has to identify the transformations. If there is more than one transformation taking the shape to the next one, the player has to identify all of them. Give Player 1 a point for each correctly identified transformation.
Extensions

1. a) Mark the centres of rotation to get:

- Shape B from Shape A (Note that the centre will be in the middle of the bottom edge of A, not on a vertex)
- Shape C from Shape A
- Shape D from Shape B
- Shape D from Shape C

b) Which transformation was used to get Shape D from Shape A?

c) Vinijaa moved Shape A to Shape C without using a rotation. Describe what she did. If she used reflection, draw the mirror line, and if she used a slide, show the translation arrow and describe the slide.

d) Draw reflection of shape A through the right (slanted) side.

2. Majd makes a pattern of floor tiles. He starts with this tile:

To make a row, he rotates the tile 180°:

Mark the centre of each rotation on the grid. This kind of pattern—when one or more shapes covers a surface with no gaps or overlaps—is called a tessellation.

Majd adds a column to his tessellation:

Which transformation does he use in the column?

Majd likes his tessellation rules. He wants to use the rule from the top row in each row and the rule from the left column in each column. He places an additional tile to check whether this works:
Continue Majd’s pattern to fill the $5 \times 5$ grid.

3. Shams wants to use the same tile as Majd. However, he uses a different pattern. Which transformation does he use in the row?

![Pattern](image1)

Which transformation does he use in the first column?

![Pattern](image2)

Shams wants to use the rule from the top row in each row and the rule from the left column in each column, as Majd did. Does this work?

4. Nabeel uses the same tile as Majd and Shams. He has his own pattern. Which transformations does he use in the first row and column?

![Pattern](image3)

Can Nabeel complete the rows and columns using the same rules, like Majd and Shams? Continue the pattern to fill the $5 \times 5$ grid.

Circle the tiles in one of the diagonals. Which transformation takes the tiles in the diagonal into each other?
**GOALS**

Students will draw simple shapes built from interlocking cubes on isometric dot paper.

**PRIOR KNOWLEDGE REQUIRED**

Understand a drawing on isometric dot paper

**VOCABULARY**

isometric dot paper

top view

interlocking cubes

---

Project a sheet of isometric dot paper onto the board using the overhead projector. Show students how to draw a cube using the dots. Start from the top face, then draw the vertical edges (no hidden ones!), and then draw the visible bottom edges.

![Step 1: Top Layer](image1)

![Step 2: Vertical Edges](image2)

![Step 3: Bottom Edges](image3)

Step 1  Step 2  Step 3

Explain that to create an isoparametric drawing, it helps to start from the top. Look at the topmost layer and draw the top face or faces first. Then draw the vertical edges that are part of the topmost layer as you did with the single cube.

Hold up a shape made with three cubes:

![Three Cubes](image4)

Invite a volunteer to draw the top layer (a single cube). What does the next layer look like? It consists of two cubes. Take two cubes locked together and compare this shape to the original shape: Which edges of the new shape are hidden (by the top cube) in the original shape? Which visible edges of the original shape are already drawn (because they are the bottom edges of the cube)? Ask a volunteer to draw the remaining visible edges of the second layer.

You may wish to do the worksheets as a class, so that volunteers draw pictures from the worksheet on the board. Students may find it easier to copy a shape onto isoparametric dot paper if they start by shading the top layer of the shape.

Give your students some interlocking cubes and ask them to build the figures from **QUESTION 2** on the worksheet.
GOALS
Students will build shapes from interlocking cubes and draw top views of shapes drawn on isometric dot paper.

PRIOR KNOWLEDGE REQUIRED
Understand a drawing on isometric dot paper

VOCABULARY
isometric dot paper
top view
interlocking cubes
front view
side view

Explain that it is sometimes hard to read the drawing on the isoparametric dot paper because some cubes are hidden and some edges overlap. Project the drawing given here on the board as an example:

Explain that a very convenient way to understand the drawing is to try to construct the “mat plan” of the shape. A mat plan represents the bottom level of the shape. In this case the mat plan would look like this:

Shade one square on the mat plan, as shown. Invite a volunteer to shade the column that stands above this square in the isoparametric diagram. How many cubes are in it? (3) Ask the volunteer to write “3” in the shaded square. Ask more volunteers to finish the mat plan. Remove the drawing and ask another volunteer to construct the shape using the mat plan. Give your students more examples like those in QUESTION 2 of the worksheet.

As a challenge, ask your students to identify the difference between the figure above and the figure below.

As a challenge, ask your students to identify the difference between the figure above and the figure below.
To help your students see the difference between the shapes, draw a cube and **SAY:** Each face of the cube is a square. However, in the picture each face looks like a parallelogram.

```
Top
```
```
Front View   Side View
```

Ask your students to describe each parallelogram in the drawing of the cube. (For example, the front face is a rhombus with two vertical sides—the left vertical edge is situated above the right vertical edge—and the non-vertical edges go from top left to bottom right.) Label the front, top, and sides as shown. Point to some of the parallelograms on one of the more complicated isoparametric drawings and **ASK:** Where does this side face? Then ask your students to shade the sides facing the front side on both drawings. This will help your students to see stacks of cubes in a clear way.

The mat plan for the second shape could be:

```
2 3
2 1 1 1
```
```
2 1 3
2 1 1 1
```
```
or
```
```
2 1 1 1
```
```
2 1 1 1
```

**Assessment**

Draw the mat plan and build the shape from interlocking cubes:

```
```

**ACTIVITY**

Have students make simple shapes with interlocking cubes and copy them onto isometric dot paper (see BLM).

**Extension**

Draw the figures from the worksheet on regular dot paper. For example:
GOALS
Students will draw side, front, and right views of shapes built from linking cubes. Students will draw side, front, and right views of shapes from their mat plans and isoparametric drawings.

PRIOR KNOWLEDGE REQUIRED
Mat plans
Understand isoparametric drawings
Distinguish between the top, front, and right sides of isoparametric drawings

VOCABULARY
isometric dot paper
top view
side view
front view
interlocking cubes

Review the previous lesson. Ask your students what was easier to draw, a picture of the whole figure or a mat plan. Show them the following mat plan and invite a volunteer to build the shape. Invite a different volunteer to check the shape. Thank both volunteers, but explain that the shape they built, though fitting the mat plan, is not the same as the shape the mat plan was made for! Show the shape below and ask why the mat plan was misleading. Ask students what could be done to overcome the problem.

Explain to your students that instead of mat plans, people have agreed to record and describe complicated shapes like this one by drawing the front and the right-side views of the shapes as well as the top view. These are called orthographic views of a shape. Let the students make a copy of this shape with linking cubes and ask them to hold it level with their faces, so that they see only the front face. Ask your students to draw the front view of the shape. Ask them to identify the right side of the shape and to turn the shape so that they see only the right side of it. Let them draw the right side view of the shape as well. Repeat with simpler shapes and then more complicated shapes, such as:

Reverse the task—give your students top, right, and front views of simple shapes and ask them to create the shapes with linking cubes.
EXAMPLES:

Gradually raise the bar by giving students pairs of shapes that have the same top and front view, but different side views. Ask your students to draw mat plans for the shapes they created.

You might also give your students several simple mat plans and ask them to create models and to draw the orthographic views of the shapes.

**Bonus**

1. Create this monster with interlocking cubes:

2. Create the shape with interlocking cubes using the side view and the mat plan:
**G6-45**

Problems and Puzzles

The worksheet **G6-45**: Problems and Puzzles is a review worksheet. It can complement the presentation of cross-curricular projects. Here are some project ideas:

### Symmetry:
1. Flags and Coats of Arms of Canadian provinces/cities. Which ones have lines of symmetry? Which ones have more than one line of symmetry? (None!)
2. Flags of the world. Make a list of world countries with flags that have 2 lines of symmetry.
3. Coats of Arms of Soccer/Baseball/Hockey clubs. Which ones have lines of symmetry? Which ones have more than one line of symmetry?
4. Cultural diversity: Alphabets. Find letters in a non-Latin alphabet that have lines of symmetry. Are there symbols that have more than one line of symmetry?
5. Make several designs of snowflakes. How many lines of symmetry do your snowflakes have?

### Geometry in Everyday Life:
1. Bee hives and hexagons—Research why bees build hexagonal shapes in the hive: Why not rectangular or triangular shapes?
2. Cross Curriculum Connection: Bridges—Which geometric shapes are used in bridges design? Try to build a bridge with rectangles. Put a heavy weight on it and watch the bridge collapse. Triangles are rigid—you cannot change the shape of the triangle without altering the side lengths. If you add diagonals to the rectangles, the bridge will stand, but then it is built with triangles.
3. Floor patterns—Make your own floor pattern. Choose a tessellation made by regular polygons using slides only (no rotations!). Cut the tessellating polygon along a line of your choice, then slide and re-glue the pieces. Create a picture based on the shape that you’ve got and create a floor pattern with that picture. **EXAMPLE:**

   **STEP 1:** Take a tessellation with rhombuses:

<table>
<thead>
<tr>
<th>GOALS</th>
<th>PRIOR KNOWLEDGE REQUIRED</th>
<th>VOCABULARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students will see applications of geometry in real life.</td>
<td>Symmetry Transformations 3-D Shapes</td>
<td>line of symmetry rotation reflection slide pyramid cone cylinder square rectangle pentagon hexagon octagon circle triangle polygon</td>
</tr>
</tbody>
</table>
STEP 2: Cut off and slide several times:

STEP 3: Add drawing to a shape:

STEP 4: Arrange copies of your shape in a floor pattern!
G6 Part 2: BLM List

3-D Shape Sorting Game _________________________________________________ 2
3-D Shapes Bingo ______________________________________________________ 8
Build-a-Net Game ______________________________________________________ 13
Dot Paper ___________________________________________________________ 15
Find-a-Flip Game ______________________________________________________ 16
Grid Paper ___________________________________________________________ 17
Isometric Dot Paper ____________________________________________________ 18
Map of Saskatchewan __________________________________________________ 19
Nets for 3-D Shapes ____________________________________________________ 20
Nets for Irregular Pyramids _____________________________________________ 23
Pattern Blocks _________________________________________________________ 25
Pentomino Pieces ______________________________________________________ 26
Right Prisms __________________________________________________________ 27
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Triangular Pyramid with Three Right-Angled Faces ___________________________ 36
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Word Search Puzzle (3-D Shapes) _________________________________________ 38
3-D Shape Sorting Game
3-D Shape Sorting Game (continued)
3-D Shape Sorting Game (continued)
3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>More than four faces</th>
<th>Square-shaped base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular-shaped base</td>
<td>Fewer than six faces</td>
</tr>
<tr>
<td>Two or more square-shaped faces</td>
<td>Four or more triangular-shaped faces</td>
</tr>
</tbody>
</table>
### 3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>Ten or more edges</th>
<th>Six or fewer vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four or more vertices</td>
<td>Exactly twelve edges</td>
</tr>
<tr>
<td>Pyramids</td>
<td>Prisms</td>
</tr>
</tbody>
</table>
3-D Shape Sorting Game (continued)

<table>
<thead>
<tr>
<th>At least four planes of symmetry</th>
<th>All faces congruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of planes of symmetry equals the number of edges in the base</td>
<td>No rectangular faces</td>
</tr>
<tr>
<td>Number of planes of symmetry is larger than number of edges in the base</td>
<td>Circular base</td>
</tr>
</tbody>
</table>
### 3-D Shapes Bingo

<table>
<thead>
<tr>
<th>Right hexagonal prism</th>
<th>Triangular prism</th>
<th>8 vertices</th>
<th>9 vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 edges</td>
<td>A pyramid with a quadrilateral in the base</td>
<td>15 edges</td>
<td>9 faces</td>
</tr>
<tr>
<td>7 faces</td>
<td>1 curved face, 2 curved edges</td>
<td>1 curved face, 1 flat face</td>
<td>Four of its faces are congruent triangles</td>
</tr>
<tr>
<td>9 planes of symmetry</td>
<td>3 planes of symmetry</td>
<td>6 faces, all parallelograms; no planes of symmetry</td>
<td>6 edges</td>
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</tbody>
</table>
### 3-D Shapes Bingo (continued)

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<tr>
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<tbody>
<tr>
<td><img src="image1.png" alt="Pyramid" /></td>
<td><img src="image2.png" alt="Cuboid" /></td>
<td><img src="image3.png" alt="Cone" /></td>
</tr>
<tr>
<td>45°</td>
<td>100°</td>
<td>60°</td>
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<tr>
<td><img src="image4.png" alt="Prism" /></td>
<td><img src="image5.png" alt="Hexagonal Prism" /></td>
<td><img src="image6.png" alt="Triangular Pyramid" /></td>
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<tr>
<td><img src="image7.png" alt="Triangular Prism" /></td>
<td><img src="image8.png" alt="Cylinder" /></td>
<td><img src="image9.png" alt="Rectangular Pyramid" /></td>
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<tr>
<td><img src="image10.png" alt="Cone" /></td>
<td><img src="image11.png" alt="Trapezoidal Prism" /></td>
<td><img src="image12.png" alt="Rectangular Prism" /></td>
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<tr>
<td><img src="image13.png" alt="Octagonal Prism" /></td>
<td><img src="image14.png" alt="Cube" /></td>
<td><img src="image15.png" alt="Cylindrical Shape" /></td>
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**BLACKLINE MASTERS**

Workbook 6 - Geometry, Part 2

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3-D Shapes Bingo (continued)
### 3-D Shapes Bingo (continued)

<table>
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<tr>
<th>45°</th>
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### 3-D Shapes Bingo (continued)
3-D Shapes Bingo (continued)

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<tbody>
<tr>
<td><img src="image" alt="Dodecahedron" /></td>
<td><img src="image" alt="Right Circular Cone" /></td>
<td><img src="image" alt="Triangular Prism" /></td>
</tr>
<tr>
<td><img src="image" alt="Cube" /></td>
<td><img src="image" alt="Rectangular Prism" /></td>
<td><img src="image" alt="Square Pyramid" /></td>
</tr>
<tr>
<td><img src="image" alt="Tetrahedron" /></td>
<td><img src="image" alt="Rectangular Pyramid" /></td>
<td><img src="image" alt="Cylinder" /></td>
</tr>
</tbody>
</table>

Note: Angles are approximate.
Build-a-Net Game
Build-a-Net Game (continued)
Dot Paper
Find-a-Flip Game
Grid Paper (1 cm)
Isometric Dot Paper

...
Map of Saskatchewan

- Uranium City
- Clearwater River Provincial Park
- Wollaston Lake
- Prince Albert
- Saskatoon
- Regina
- Swift Current
- Moose Jaw
- Weyburn
- Maple Creek
Nets for 3-D Shapes

Square Pyramid

Triangular Pyramid
Nets for 3-D Shapes (continued)
Nets for 3-D Shapes (continued)
Nets for Irregular Pyramids

Triangular Pyramid with Scalene Faces
Nets for Irregular Pyramids (continued)
Pattern Blocks

Triangles

Squares

Rhombuses

Trapezoids

Hexagons
Right Prisms

Right Prism with an Irregular Hexagonal Base
Right Prisms (continued)

Right Prism with a Parallelogram Base
Right Prisms (continued)
Right Prisms (continued)

Triangular Prism with a Scalene Base
Skew Prisms

Parallelepiped with Three Different Pairs of Parallelogram Faces
Skew Prisms (continued)
Skew Prisms (continued)
Skew Prisms (continued)
Triangle Transformations
Triangular Pyramid with Three Right-Angled Faces
Venn Diagram
Word Search Puzzle (3-D Shapes)

WORDS TO SEARCH:

- base
- edge
- face
- hexagonal
- net
- pentagonal
- prism
- pyramid
- rectangular
- skeleton
- triangular
- vertex
- vertices

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PS6-11 Choosing Strategies

Teach this lesson after: 6.2 Geometry

Goals:
Students will choose among the strategies learned this year to solve problems.

Prior Knowledge Required:
Can solve problems using all the strategies learned
Can create ratios equivalent to a given ratio (for Problem Banks 1, 2, 3)
Can compute the area of a parallelogram (for Problem Bank 1)
Can multiply two-digit numbers by two-digit numbers (for Problem Banks 5, 11, 17)
Can add decimal tenths (for Problem Bank 6)
Can evaluate the mean of a set of numbers (for Problem Bank 8)
Can determine the coordinates of points in the first quadrant (for Problem Bank 13)
Can multiply a decimal by a whole number (for Problem Bank 14)
Can evaluate the LCM and GCF of small numbers (for Problem Bank 20)
Can graph sequences on a coordinate grid with the first coordinate representing the term number and the second coordinate representing the term (for Extended Problem)
Can write a rule for obtaining the term from the term number in a sequence with constant gaps (for Extended Problem)

Vocabulary: angle, area, base, consecutive, coordinate, factor, GCF, height, LCM, length, mean, parallelogram, perfect square, perimeter, ratio, rectangle, remainder, width

Materials:
BLM Hundreds Chart (p. 8)
BLM International Text Cost (pp. 10–12, see Extended Problem)

NOTE: The following Problem Bank questions reflect a selection of the problem-solving strategies used in the problem-solving lessons for Grade 6. Students will need to choose among all the strategies they have learned this year to solve the problems.

Problem Bank
1. The ratio of base to height in a parallelogram is 1 : 2. The area is 50 cm². What is the height of the parallelogram?
Solution: Make a table for base, height, and area with the ratio 1 : 2 between base and height.

<table>
<thead>
<tr>
<th>Base (cm)</th>
<th>Height (cm)</th>
<th>Area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

The height of the parallelogram is 10 cm.
2. a) The ratio of length to width of a rectangle is 4 : 3. The perimeter is 70 cm. What is the area?
b) The ratio of length to width of a rectangle is 7 : 4 and its perimeter is 44 mm. What is the area?
   **Answers:** a) 300 cm², b) 112 mm²

3. The ratio of girls to boys is 5 : 3. There are 8 more girls than boys. How many girls and how many boys are there?
   **Answer:** 20 girls and 12 boys

4. How many multiples of 9 are there from …
   a) 1 to 900?  
   b) 1 to 1000?  
   c) 901 to 1000?
   **Answers:** a) 100, b) 111, c) 11

5. How many perfect squares (1 = 1 × 1, 4 = 2 × 2, 9 = 3 × 3, and so on) are there from …
   a) 1 to 900?  
   b) 1 to 1000?  
   c) 901 to 1000?
   **Answers:** a) 30, b) 31, c) 1

6. Add: 1.1 + 2.2 + 3.3 + 4.4 + … + 9.9.
   **Answer:** 49.5

7. A regular hexagon and a regular triangle have the same perimeter. How do their areas compare?
   **Solution:** To have the same perimeter, the triangle must have side lengths twice those of the hexagon because the hexagon has twice as many sides. In the picture below, each of the small equilateral triangles has the same side length and area.

   ![Hexagon and Triangle](image)

   So, the area of the triangle is 4/6, or 2/3, of the area of the hexagon.

8. The mean of two numbers is 30, and the mean of three other numbers is 40. What is the mean of all five numbers?
   **Answer:** 36

9. The reciprocal of a whole number is the number that you need to multiply it by to get 1. For example, the reciprocal of 2 is \(\frac{1}{2}\) and the reciprocal of 3 is \(\frac{1}{3}\). How many whole numbers have a reciprocal between (and including) …
   a) 0.3 and 0.5?  
   b) \(\frac{11}{91}\) and \(\frac{58}{91}\) ?
   **Answers:** a) 2, b) 7
   **Selected solution:** a) reciprocal of 0.3 is 10/3 (or 3.33) and reciprocal of 0.5 is 2, so the integers are 2 and 3
10. a) What is the pattern on the left side of the equations? What is the pattern on the right side of the equations?
2 = 1 × 2
2 + 4 = 2 × 3
2 + 4 + 6 = 3 × 4
2 + 4 + 6 + 8 = 4 × 5
b) Find the sum of the first 15 even numbers using the 15th row.
c) What is the sum of the first 15 whole numbers? Hint: Divide each even number by 2. What happens?
**Answers:** a) The left side is the sum of the even numbers starting from 2 and increasing the number of even numbers by 1 each row. The right side is the product of two consecutive numbers and increasing the factors by 1 each row; b) 15 × 16 = 240; c) 120 (by dividing each even number by 2, you get a whole number)

11. a) Complete the table.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>(2 × 2) − (1 × 1) = _____</td>
</tr>
<tr>
<td>2</td>
<td>(3 × 3) − (2 × 2) = _____</td>
</tr>
<tr>
<td>3</td>
<td>(4 × 4) − (3 × 3) = _____</td>
</tr>
<tr>
<td>4</td>
<td>(5 × 5) − (4 × 4) = _____</td>
</tr>
<tr>
<td>5</td>
<td>(6 × 6) − (5 × 5) = _____</td>
</tr>
<tr>
<td>6</td>
<td>(7 × 7) − (6 × 6) = _____</td>
</tr>
</tbody>
</table>
b) How can you get your answers in each row from the row number?
c) Will the answer to (23 × 23) − (22 × 22) be in the 22nd row or the 23rd row? Explain how you know.
d) Use your rule in part b) to solve (23 × 23) − (22 × 22).
e) Check your answer.
**Answers:** a) 3, 5, 7, 9, 11, 13; b) multiply the row number by 2 and add 1; c) the 22nd row because the product being subtracted is the one with the row number in it; d) 2 × 22 + 1 = 44 + 1 = 45; e) 23 × 23 = 529, 22 × 22 = 484, and so 23 × 23 − 22 × 22 = 529 − 484 = 45

12. Using 44 + 45 + 46 + 47 = 182, what is \( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \)?
**Answer:** 184

13. What will be the coordinates of the centre of the 100th rectangle in the pattern?
Solution: The coordinates of the terms are (2, 2), (5, 5), (8, 8), and so on. Each coordinate is equal to $3 \times \text{term number} - 1$, so the coordinates of the centre of the 100th rectangle are (299, 299).

14. In the figures below, each square has a side length of 1.5 m.

![Figures 1, 2, 3]

a) Complete the table for the figure pattern.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Perimeter (m)</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
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<td>3</td>
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<tr>
<td>4</td>
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</table>

b) What is the perimeter of the 10th figure?
c) Which figure has perimeter 72 m?

Answers: a) 12, 18, 24; b) 60; c) 12th figure

15. Bowl A has 5 spoonfuls of red paint and 2 spoonfuls of white paint. Bowl B has 1 spoonful of red paint and 1 spoonful of white paint. All the spoons are the same size.
a) Which bowl has paint that is darker red? Explain how you know using fractions.
b) If you pour the contents of Bowl B into the contents of Bowl A, will it make the paint in Bowl A darker or lighter red?
c) What is the new fraction of red paint in the bowl in part b)? Is that fraction greater than or less than $\frac{5}{7}$? How can you tell without doing any calculations?
d) Is $\frac{35}{69}$ more or less than one half?
e) Without doing any calculations, use your answer to part d) to decide if $\frac{36}{71}$ is more or less than $\frac{35}{69}$.

Selected solution: e) Suppose I have a mixture of red and white paint with 35 spoonfuls of red paint and 34 spoonfuls of white paint. The mixture is 35/69 red. If I add 1 spoonful of red paint and 1 spoonful of white paint, the mixture becomes 36/71 red. However, doing this will make the paint a lighter colour of red because $1/2 < 35/69$, so 36/71 must be less than 35/69.

Answers: a) Bowl A is darker red because the fraction of red paint is 5/7, while Bowl B is 1/2, and 5/7 is greater than half; b) lighter red because you are adding paint that is lighter red to Bowl A; c) 6/9, or 2/3, which is less than 5/7 because the paint is lighter red; d) more
16. How many factors does 1 000 000 000 have?
    **Solution:** Look for a pattern: 10 has 4 factors, 100 has 9 factors, 1000 has 16 factors. These numbers are the perfect squares: add 1 to the number of zeros in the number and multiply the result by itself. There are 9 zeros in the given number, so the number of factors is \(10 \times 10 = 100\).

17. A path continues spiralling, as shown below. Each arrow shows one unit along the x- or y-axis. What is the length of the path from (0,0) to (0,10)?

   ![Diagram of spiral path]

   **Answer:** 120 (one less than \(11 \times 11 = 121\))

18. How much greater is \(\frac{2003}{25} + 25\) than \(\frac{2003}{25}\)?

   **Answer:** 24

19. Lewis divides 48 by a number and gets a remainder of 6. What could he have divided by?
    **Solution:** If \(48 \div A = B\ R\ 6\), then \(A \times B + 6 = 48\), so \(A \times B = 42\). Then, \(A\) is a factor of 42 and \(A\) is at least 7, so \(A\) can be 7, 14, 21, or 42. Indeed, \(48 \div 7 = 6\ R\ 6\), \(48 \div 14 = 3\ R\ 6\), \(48 \div 21 = 2\ R\ 6\), and \(48 \div 42 = 1\ R\ 6\).

20. How can you get the LCM and GCF of 2000 and 3000 from the LCM and GCF of 2 and 3?
    **Solution:** Make a table as follows:

    |        | LCM | GCF |
    |--------|-----|-----|
    | 2 and 3| 6   | 1   |
    | 4 and 6| 12  | 2   |
    | 6 and 9| 18  | 3   |
    | 8 and 12| 21  | 4   |

    The pair 2000 and 3000 would be in the thousandth row, so the LCM and GCF of 2000 and 3000 can be obtained from the LCM of 2 and 3 by multiplying both by 1000. The LCM of 2 and 3 is 6, so the LCM of 2000 and 3000 is 6000. The GCF of 2 and 3 is 1, so the GCF of 2000 and 3000 is 1000.

21. a) Sharon walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, then turns right again and walks 4 blocks. She continues this pattern until she goes 100 blocks. Then she turns right again and goes another 100 blocks, turns right again and goes 99 blocks, turns right again and walks 98 blocks, and so on until walking 1 block. Where does she end up, relative to home?

    b) What if she did the same pattern as in part a) but with starting to count down after 97 blocks instead of 100 blocks; now where would she end up relative to home?

    **Answers:** a) at home, b) 1 block east and 1 block south
22. A two-digit number is divided by the sum of its digits. What two-digit number will result in the largest remainder? Solve this problem in steps.

a) Start by dividing the two-digit numbers by the sum of their digits, in order:
   \[ 10 ÷ 1 = \_ \_ R \_ \_ \]
   \[ 11 ÷ 2 = \_ \_ R \_ \_ \]
   \[ 12 ÷ \_ \_ = \_ \_ R \_ \_ \]
   \[ 13 ÷ \_ \_ = \_ \_ R \_ \_ \]
   \[ 14 ÷ \_ \_ = \_ \_ R \_ \_ \]

b) Is the strategy from part a) a good strategy to continue? Why or why not?

c) On a hundreds chart, e.g., BLM Hundreds Chart, calculate the sum of the digits of all the two-digit numbers. Write them on the hundreds chart squares.

d) What is the largest sum of digits a two-digit number can have?

e) The remainder must be smaller than the divisor, which is the sum of the digits. So, make a table starting with the largest sum of the digits.

<table>
<thead>
<tr>
<th>Sum of Digits</th>
<th>Two-Digit Number</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f) What is the largest remainder you found in part c)?

g) All the two-digit numbers not in the table so far have the sum of the digits at 15 at the most. Can they get a higher remainder than you found in part c)? Explain how you know.

**Answers:**
a) \[ 10 ÷ 1 = 10 R 0, \]
\[ 11 ÷ 2 = 5 R 1, \]
\[ 12 ÷ 3 = 4 R 0, \]
\[ 13 ÷ 4 = 3 R 1, \]
\[ 14 ÷ 5 = 2 R 4; \]
b) no, because the remainder can’t be bigger than the sum of the digits, so we should start with numbers that have bigger sums of digits; d) 18,

e)

<table>
<thead>
<tr>
<th>Sum of Digits</th>
<th>Two-Digit Number</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>99</td>
<td>99 ÷ 18 = 5 R 9</td>
</tr>
<tr>
<td>17</td>
<td>98</td>
<td>98 ÷ 17 = 5 R 13</td>
</tr>
<tr>
<td>17</td>
<td>89</td>
<td>89 ÷ 17 = 5 R 4</td>
</tr>
<tr>
<td>16</td>
<td>97</td>
<td>97 ÷ 16 = 6 R 1</td>
</tr>
<tr>
<td>16</td>
<td>88</td>
<td>88 ÷ 16 = 5 R 8</td>
</tr>
<tr>
<td>16</td>
<td>79</td>
<td>79 ÷ 16 = 4 R 15</td>
</tr>
</tbody>
</table>

f) 15; g) no, the highest remainder you can get when dividing by 15 is 14, so 15 is the largest remainder possible
23. Ivan divides a three-digit number by the sum of its digits. What is the largest possible remainder he can get?

**Solution:** Make a table starting with the largest possible sum of digits.

<table>
<thead>
<tr>
<th>Number</th>
<th>Sum of Digits</th>
<th>Number ÷ Sum of Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>999</td>
<td>27</td>
<td>37 R 0</td>
</tr>
<tr>
<td>998</td>
<td>26</td>
<td>38 R 10</td>
</tr>
<tr>
<td>989</td>
<td>26</td>
<td>38 R 1</td>
</tr>
<tr>
<td>899</td>
<td>26</td>
<td>34 R 15</td>
</tr>
<tr>
<td>988</td>
<td>25</td>
<td>39 R 13</td>
</tr>
<tr>
<td>898</td>
<td>25</td>
<td>35 R 23</td>
</tr>
<tr>
<td>889</td>
<td>25</td>
<td>35 R 14</td>
</tr>
<tr>
<td>799</td>
<td>25</td>
<td>31 R 24</td>
</tr>
<tr>
<td>979</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>997</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

The largest possible remainder is 24, since dividing by 25 or less cannot result in a remainder that is larger than 24, so we can stop here at 799 ÷ 25 = 31 R 24.

**NOTE:** In Problem Bank 24, parts a) to d) guide students to solve the puzzle. Some students may appreciate the opportunity to solve the puzzle without

24. In a “very right” polygon, all angles are either 90° or 270°. Here are some very right polygons.

A. ![Polygon A]

B. ![Polygon B]

C. ![Polygon C]

D. ![Polygon D]

Investigate the problem: A very right polygon has 100 edges. How many 90° angles does the polygon have?

a) Count the number of 90° angles, 270° angles, and edges in each shape above.

b) Draw three different very right polygons that have 10 edges. How many 90° angles and how many 270° angles does each of your shapes have?

c) Complete the table.

<table>
<thead>
<tr>
<th>Number of 90° angles</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d) Compare the sequences in part c) to predict the number of 90° angles in a right polygon that has 100 edges.

**Answers:** a) A: 4, 0, 4, B: 5, 1, 6, C: 6, 2, 8, D: 6, 2, 8; b) each shape has seven 90° angles and three 270° angles; c) 4, 6, 8, 10; d) divide each term in the bottom sequence by 2: 2, 3, 4, 5, … , then the result is 2 less than the corresponding term in the top row, so when the bottom row is 100, the top row is 2 more than 50 (i.e., 52), so the number of 90° angles in a very right polygon with 100 edges is 52.
# Hundreds Chart

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
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<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
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<td>18</td>
<td>19</td>
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<td>29</td>
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<td>31</td>
<td>32</td>
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<td>66</td>
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<td></td>
</tr>
<tr>
<td>71</td>
<td>72</td>
<td>73</td>
<td>74</td>
<td>75</td>
<td>76</td>
<td>77</td>
<td>78</td>
<td>79</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>82</td>
<td>83</td>
<td>84</td>
<td>85</td>
<td>86</td>
<td>87</td>
<td>88</td>
<td>89</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>92</td>
<td>93</td>
<td>94</td>
<td>95</td>
<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Extended Problem: International Text Cost

Materials:
BLM International Text Cost (pp. 10–12)

Extended Problem: International Text Cost. Provide students with BLM International Text Cost. In this Extended Problem, students plot data from a T-table and find the rule for how to get the output from the input. Students interpret the results in the context of sending international text messages.

Selected answers: 1. a) (2, 0.40), (3, 0.60), (4, 0.80); c) divide by 5 (or multiply by 0.2); d) $4;
2. a) (2, 5.20), (3, 5.30), (4, 5.40); c) divide by 10 (or multiply by 0.1), then add 5; d) $7;
e) Company A; f) Company B, because Company A will charge $20 and Company B will only charge $15; Bonus: 50 text messages
1. Phone Company A charges $0.20 for each international text message.
   a) Complete the table for the cost of sending one, two, three, and four international text messages. Write a list of ordered pairs for the table.

<table>
<thead>
<tr>
<th>Text Messages</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   (1, 0.20) ( , ) ( , ) ( , )

   b) Plot the ordered pairs from the table of values in part a) on the grid below.

   c) Write a rule that tells you how to calculate the cost ($) from the number of international text messages sent.

   d) Edmond sends 20 international text messages each month. How much will it cost him if he uses Company A?
2. Phone Company B charges $5 per month, plus $0.10 for each international text message sent.

   a) Create a table for the cost of sending zero, one, two, three, and four international text messages. Write a list of ordered pairs for the table.

<table>
<thead>
<tr>
<th>Text Messages</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.00</td>
</tr>
<tr>
<td>1</td>
<td>5.10</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   b) Plot the ordered pairs from the table of values in part a) on the grid below.

   c) Write a rule that tells you how to calculate the cost ($) from the number of international text messages sent.
**International Text Cost (3)**

d) Edmond sends 20 international text messages each month. How much will it cost him if he uses Company B?

e) Which company gives Edmond the lower rate, Company A or Company B?

f) Lily sends 100 international text messages each month. Which company will give her the lower rate?

**BONUS** ► For how many text messages is the cost the same for both companies?
Contents

Patterns & Algebra – Part 1
Number Sense – Part 1
Measurement – Part 1
Probability & Data Management – Part 1
Geometry – Part 1
Patterns & Algebra – Part 2
Number Sense – Part 2
Measurement – Part 2
Probability & Data Management – Part 2
Geometry – Part 2
Patterns & Algebra – AP Book 6.1

AP Book PA6-1

1. a) Gap = 3; 11, 14, 17
   b) Gap = 6; 19, 25, 31
   c) Gap = 5; 17, 22, 27
   d) Gap = 4; 16, 20, 24
   e) Gap = 5; 16, 21, 26
   f) Gap = 6; 22, 28, 34
   g) Gap = 10; 32, 42, 52
   h) Gap = 8; 31, 39, 47
   i) Gap = 3; 40, 43, 46
   j) Gap = 6; 110, 116, 122
   k) Gap = 11; 45, 56, 67
   l) Gap = 8; 24, 32, 40

2. a) 23 cm
   b) 5 days

AP Book PA6-2

1. a) Gap = 3; 9, 6, 3
   b) Gap = 6; 14, 8, 2
   c) Gap = 5; 37, 32, 27
   d) Gap = 4; 22, 18, 14
   e) Gap = 5; 36, 31, 26
   f) Gap = 4; 72, 68, 64
   g) Gap = 11; 29, 18, 7
   h) Gap = 8; 73, 65, 57
   i) Gap = 7; 50, 43, 36
   j) Gap = 4; 50, 46, 42

AP Book PA6-3

1. a) 49, 53, 57
   b) 76, 84, 92
   c) 80, 83, 86
   d) 42, 53, 64
   e) 77, 85, 93
   f) 53, 64, 75
   2. a) 19, 16, 13
   b) 30, 28, 26
   c) 73, 67, 61
   d) 65, 53, 41
   e) 41, 33, 25
   f) 43, 36, 29

AP Book PA6-4

1. a) 5
   b) 3
   c) 2
   d) 7
   e) 4
   f) 3
   2. a) 2
   b) 5
   c) 1
   d) 3
   e) 6
   f) 2

3. a) Subtract 7
   b) Add 8

AP Book PA6-5

1. a) Start at 2 and add 6 each time.
   b) Start at 3 and add 6 each time.
   c) Start at 1 and add 5 each time.
   d) Start at 1 and add 7 each time.
   e) Start at 5 and add 8 each time.
   f) Start at 11 and add 11 each time.
   g) Start at 3 and add 9 each time.
   h) Start at 6 and add 7 each time.
   i) Start at 7 and add 6 each time.

2. a) 19 shapes needed
   b) 11 shapes needed
   c) Yes (31)
   d) No (39)
   e) Yes (33)

AP Book PA6-6

1. a) 8
   b) 13
   c) 9

AP Book PA6-7

1. a) 23
   b) 66

2. No, the 6th term is 37

3. a) 16
   b) 16, 21
   c) 18, 21, 27
   d) 62, 65, 68

4. Table 1: Rule – Add 2.
   Term 3 should be = 17
   Table 2: Rule – Add 4.
   Term 3 should be = 33

5. a) 42 pentagons
   b) 72 triangles and 36 pentagons
   c) 72 triangles

Answer Key for AP Book 6.1
6. a) i) a baby  
   ii) tiger  
 b) 4 weeks  = 28 days. They will both weigh 4100 g

AP Book PA6-8  
page 10  
1. a) Circle first 4  
 b) Circle first 4  
 c) Circle first 4  
 d) Circle first 4  
 e) Z G H H U  
 f) 1 2 4 8  
 g) 9 3 3 9 8  
 h) Circle first 5  
 i) Circle first 4  
 j) Z Y Z

2. Teacher to check the continuation.  
 a) Circle first 3  
 b) Circle first 3  
 c) 4 5 4 6 1  
 d) 2 2 0  
 e) A A C  
 f) 2 6 2

3. Teacher to check, answers will vary.

AP Book PA6-9  
page 11  
1. Teacher to check.

2. a) Yes  
 b) Yes  
 c) Yes  
 d) No  
 e) No  
 f) Yes

3. a) Yes  
 b) Yes  
 c) No (Core is 1st four blocks)  
 d) No (Core is 1st four blocks)  
 e) Yes  
 f) No (Core is 1st five blocks)

AP Book PA6-10  
page 14  
1. 3 km  
2. 100 km  
3. 15 L  
4. 25 km  
5. 25 km  
6. 3 m  
7. 3 hours  
8. 30 cm  
9. 12 m forward

AP Book PA6-11  
page 16  
1. a) 12  
 b) 12  
2. b) 20  
 c) 9

4. a) Core is 1st five blocks.  
 b) Core is 1st three blocks.  
 c) Core is 1st four blocks.  
 d) Core is 1st four blocks.  
 e) Core is 1st four blocks.  
 f) Core is 1st four blocks.

5. a) Core is 1st five blocks.  
 b) Core is 1st three blocks.  
 c) Core is 1st four blocks.  
 d) Core is 1st four blocks.  
 e) Core is 1st four blocks.

6. a) 38, 42, 46, 50, 54  
 b) 67, 61, 55, 49, 43  
 c) 98, 105, 112, 119, 126

7. Teacher to check, answers will vary.

8. Teacher to check, answers will vary.

9. Teacher to check, answers will vary.

AP Book PA6-12  
page 17  
1. a) + 4, – 2, + 7, – 4  
 b) + 4, – 1, + 4, – 5  
 c) + 4, + 3, + 10, + 6  
 d) + 4, – 1, – 6, + 9  
 e) + 2, + 2, – 1, + 5  
 f) – 1, + 3, – 9, + 1  
 g) – 7, + 5, – 6, – 6  
 h) + 3, – 7, + 8, – 5

2. a) B; A  
 b) A; B  
 c) C; B; A  
 d) C; A; D; B

3. a) Start at 4 and add 3  
 b) Start at 23 and add 6  
 c) Start at 28 and subtract 3  
 d) Start at 53 and subtract 5

4. a) Start at 9 and add 5.  
 b) Start at 27 and subtract 8.  
 c) This sequence has no rule.  
 d) Start at 81 and add 4.

5. a) Increasing  
 b) Repeating  
 c) Decreasing  
 d) Increasing  
 e) Repeating  
 f) Decreasing

6. a) 3rd row  
 b) 5th column  
 c) Start at 0 and add 5; Start at 4 and add 2
Patterns & Algebra – AP Book 6.1 (continued)

AP Book PA6-14
page 20
1. 15
2. a) 2  7  6
    9  5  1
    4  3  8
b)  6 13  8
   11  9  7
   10  5  12
c)  6 20 10
   16 12  8
   14  4 18
3. The number at the top of the pyramid is the sum of the two numbers beneath it.
4. a) 6
    b) 8
    c) 11
    d) 3
    e) 4
    f) 13
    5  8
    2  3  5
g) 17
   10  7
   6  4  3
h) 15
   7  8
   2  5  3
i) 45
   26 19
   17  9 10
j) 32
   14 18
   11  3  15
k) 75
   35 40
   19 16 24
   11  8  8 16
l) 73
   29 44
   17 12 32
   13  4  8 24

AP Book PA6-15
page 21
1. a) 4 x 1 = 4
   Triangles: 4
   4 x 2 = 8
   Triangles: 8
   4 x 3 = 12
   Triangles: 12
b) 3 x 1 = 3
   Triangles: 3
   3 x 2 = 6
   Triangles: 6
   3 x 3 = 9
   Triangles: 9
2. a) 4 x s = t
    b) 5 x s = t
    c) 2 x s = t
    d) 6 x s = t
3. a) squares rectangles
    1   4
    2   8
    3  12
    4 x s = r
b) rectangles triangles
   1   6
   2  12
   3  18
   6 x r = t
4. No, Wendy would need 42 triangles (skip count by 6s seven times).
5–6 Teacher to check, answers will vary.
7. Row Chairs
   a) 1   7
    2   8
    3  12
   b) 1  10
    2  11
    3  12
8. a) Add 7;  r + 7 = c
    b) Add 8;  r + 8 = c
    c) Add 5;  r + 5 = c
9. a) Vertical Lines
    1   3
    2   6
    3   9
   Horizontal Lines
    r + 4 = c
   Multiply the input by 3.
b) Crosses Triangles
   1   2
   2   4
   3   6
   Multiply the input by 2.
c) Suns Moons
   1   2
   2   3
   3   4
   Add 1 to the input.
Patterns & Algebra – AP Book 6.1 (continued)

Page 4

AP Book PA6-19

1.  

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>b)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
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<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>c)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
</tr>
</tbody>
</table>

d)  

Multiply the input by 2.

e)  

Add 1 to the input.

AP Book PA6-17

page 26

1. a)  $3 \times \text{Figure Number}$
   b)  $2 \times \text{Figure Number}$
   c)  $4 \times \text{Figure Number}$
   d)  $3 \times \text{Figure Number}$

2. Circle a) and c).

AP Book PA6-18

page 27

1. a)  
   i)  $2 \times \text{FN}$
   ii) $2 \times \text{FN} + 1$
   b)  
   i)  $4 \times \text{FN}$
   ii) $4 \times \text{FN} + 2$
   c)  
   i)  $2 \times \text{FN}$
   ii) $2 \times \text{FN} + 1$
   d)  
   i)  $3 \times \text{FN}$
   ii) $3 \times \text{FN} + 2$
   e)  
   i)  $3 \times \text{FN}$
   ii) $3 \times \text{FN} + 1$

2. Teacher to check.

AP Book PA6-20

page 29

1. a)  Multiply by 4 and add 5
   b)  Multiply by 2 and add 1
   c)  Multiply by 3 and add 4
   d)  Multiply by 2 and add 4

2. a)  Multiply by 5 and add 4
   b)  Multiply by 6 and add 6

AP Book PA6-21

page 32

1. a)  $2 \times \text{FN} + 2$
   Figure 9: 20 triangles
   b)  $2 \times \text{FN} + 1$
   Figure 11: 23 line segments
   c)  $2 \times \text{FN} + 1$
   Figure 10: 21 squares
   d)  $4 \times \text{FN} + 2$
   Figure 23: perimeter of 94
### AP Book NS6-1

**page 33**

1. a) Tens  
   b) Millions  
   c) Hundred thousands  
   d) Hundreds  
   e) Ones  
   f) Ten thousands  
   g) Thousands  

2. a) Thousands  
   b) Millions  
   c) Ones  
   d) Ones  
   e) Hundreds  
   f) Ten thousands  
   g) Tens  
   h) Ten thousands  
   i) Hundred thousands  

3. a) | 2 | 3 | 1 | 6 | 9 | 5 | 3 |
    b) | 6 | 2 | 5 | 0 | 7 |
    c) | 6 | 0 | 4 | 8 | 9 | 1 |
    d) | 1 | 3 | 9 | 9 |
    e) | 1 | 7 |
    f) | 9 | 9 | 8 | 2 | 6 | 0 |

### AP Book NS6-2

**page 34**

1. a) 2; 70; 800; 4000; 50000; 600000  
    b) 7; 30; 500; 8000; 20000; 100000  

2. a) 70  
    b) 700  
    c) 700  
    d) 700000  
    e) 7000  
    f) 7  
    g) 70  
    h) 7  

3. a) 500  
    b) 30000  
    c) 80  
    d) 70000  
    e) 2  

### AP Book NS6-3

**page 35**

1. a) millions  
    b) thousands  
    c) thousands  
    d) millions  

2. a) Three hundred seventy-five million  
    b) Thirty-six thousand  
    c) Seventy-nine million  
    d) Seven hundred seventy thousand  

3. a) 2 3 1 6 9 5 3  
    b) 6 2 5 0 7  
    c) 5 6 0 4 8 9 1  
    d) 1 3 9 9  
    e) 1 7  
    f) 9 9 8 2 6 0  

### AP Book NS6-4

**page 36**

1. a) 2 435  
    b) 3 316  
    c) 2 thousands + 3 hundreds + 8 ones = 2818  

2. a) 3 thousands, 4 hundreds, 6 tens, 8 ones  
    b) 1 thousand, 5 hundreds, 4 tens, 2 ones  
    c) 2 thousands, 6 hundreds, 0 tens, 9 ones  

3. a) 4 438  
    b) 2 490  

### AP Book NS6-5

**page 38**

1. a) | 2 millions + 5 hundred thousands + 3 ten thousands + 6 thousands + 7 hundreds + 8 tens + 4 ones  
    b) | 4000 + 300 + 50 + 4  
    c) | 8  
    d) | 80  
    e) | 700  
    f) | 200  
    g) | 80  
    h) | 7  

2. Teacher to check.  

### Answer Key for AP Book 6.1
AP Book NS6-6
page 40

1. a) 5; 20; 700; 5; 30; 700; 735 > 725
b) 7; 20; 400; 7; 20; 500; 527 > 427

5. a) 10 000 less
b) 100 less
c) 10 000 more
d) 10 000 less
e) 10 000 less
f) 10 000 less

2. a) 83 762
b) 273 605
c) 614 858
d) 483 250
e) 813 349
f) 579 274
g) 324
h) 196 385

3. a) 641 796
b) 38
3 605
c) 614 858
d) 250 83
3 274
e) 32 74
f) 19
6 385

6. a) 3 792
b) 39 827
c) 3 882
d) 14 023
e) 297 532
f) 22 685
g) 18 305
h) 104 253
i) 173 528
j) 168 253

AP Book NS6-8
page 43

1. a) 254, 416
b) 3 128, 2 209

2. a) Forty-eight
b) 3 508
c) Ninety-four
d) 662
e) Sixty thousand
two hundred and twenty-five

3. a) 67, 68, 76, 78, 86, 87
b) 24, 29, 42, 49, 92, 94
c) 20, 25, 50, 52

4. a) 6 432
b) 9 874
c) 4 210

5. a) 84 321
b) 98 521
c) 65 431

6. Greatest Least

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td>a) 10</td>
<td>737</td>
</tr>
<tr>
<td>b) 31 487</td>
<td>81 801</td>
</tr>
<tr>
<td>c) 15 836</td>
<td>39 045</td>
</tr>
<tr>
<td>d) 42 227</td>
<td>28 583</td>
</tr>
<tr>
<td>e) 64 283</td>
<td>39 827</td>
</tr>
<tr>
<td>f) 68 372</td>
<td>3 922</td>
</tr>
<tr>
<td>g) 28 63</td>
<td>475</td>
</tr>
<tr>
<td>h) 168 253</td>
<td>14 023</td>
</tr>
<tr>
<td>i) 20</td>
<td>10</td>
</tr>
</tbody>
</table>
| j) 173 528 | 20 |}

7. a) 67, 68, 76, 78, 86, 87
b) 24, 29, 42, 49, 92, 94
c) 20, 25, 50, 52

8. a) 10
b) 100
c) 10
d) 100
e) 1000
f) 100
g) 100
h) 10
i) 100
j) 10 000
k) 1 261 053

9. a) 6 437, 6 447
b) 49 640, 50 640
c) 624 843
d) 28 383
e) 827 325 is 10 less than 827 335
f) 482 305 is 100 000 greater than 382 305
g) 915 778 is 10 000 less than 925 778

AP Book NS6-7
page 41

1. a) 10 more
b) 100 less
c) 10 more
d) 10 more

2. a) 1 000 less
b) 1 000 less
c) 100 less
d) 1 000 more

3. a) 1 000 more
b) 1 000 less
c) 10 000 less
d) 10 000 more
e) 1 000 more
f) 1 000 more

4. a) 10 000 more

5. a) 10 000 less
b) 100 less
c) 10 000 more
d) 10 000 less
e) 10 000 less
f) 10 000 less

6. a) 10
b) 100
c) 10
d) 100
e) 1000
f) 100
g) 100
h) 10
i) 100
j) 10 000
k) 1 261 053

BONUS:

9. a) 6 437, 6 447
b) 49 640, 50 640
c) 624 843

10. a) Ottawa
b) 414 284, 662 401, 774 072

11. a) 999
b) 9 999
c) 99 999

12. There are 2 correct answers, example: 42 310 and 42 130.

13. Answers will vary – number will begin with 6 digits and end with either the 5 or 7 digit; the second digit can be 4, 5 or 7.

14. a) 4
b) 2

AP Book NS6-9
page 45

1. a) 4 tens + 12 ones = 5 tens + 2 ones
b) 2 tens + 18 ones = 3 tens + 8 ones

2. a) 5 tens + 3 ones
b) 8 tens + 5 ones
c) 1 tens + 4 ones
d) 2 tens + 7 ones
e) 3 tens + 2 ones
f) 1 tens + 6 ones
g) 1 tens + 1 ones
h) 8 tens + 2 ones
i) 9 tens + 3 ones

3. Hundreds Tens

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>a) 87 521</td>
<td>12 578</td>
</tr>
<tr>
<td>b) 95 321</td>
<td>12 359</td>
</tr>
<tr>
<td>c) 53 310</td>
<td>01 335</td>
</tr>
</tbody>
</table>

The students’ “in between” numbers will vary – teacher to check.

7. a) 683 759, 693 231, 693 238
b) 42 380, 47 832, 473 259

10. a) 385 290, 532 135, 928 381
b) 195, 2 575, 38 258

9. a) >
b) <
c) >
d) <

11. a) 999
b) 9 999
c) 99 999

12. There will be 2 correct answers, example: 42 310 and 42 130.

13. Answers will vary – number will begin with 6 digits and end with either the 5 or 7 digit; the second digit can be 4, 5 or 7.

14. a) 4
b) 2
f) 2 + 2 = 4

4. a) 6 hundreds + 8 tens + 9 ones
b) 2 hundreds + 7 tens + 5 ones
c) 10 hundreds + 8 tens + 9 ones

5. T H
   a) 3 + 1 = 4 2
b) 8 + 2 = 10 0

6. a) 7 thousands + 3 hundreds + 2 tens + 5 ones
b) 6 thousands + 4 hundreds + 2 tens + 6 ones
c) 9 thousands + 5 hundreds + 3 tens

7. a) 3 thousands + 3 hundreds + 2 tens + 5 ones
b) 5 thousands + 2 hundreds + 8 tens + 6 ones
c) 5 ten thousands + 7 thousands + 5 hundreds + 7 tens + 8 ones

8. Yes: Teresa needs 6 590 blocks to build her model, and she has 6 700 blocks.

AP Book NS6-10

1. a) tens ones
   2 6
   3 6
   5 12
   6 2
b) tens ones
   5 7
   2 7
   7 14
   8 4

2. a) 1, 3
   Final answer: 33
b) 1, 0
   Final answer: 60

AP Book NS6-11

1. 4 hundreds + 8 tens + 3 ones;
   2 hundreds + 4 tens + 5 ones;
   6 hundreds + 12 tens + 8 ones;
   7 hundreds + 2 tens + 8 ones.
   a) 617
   b) 826
   c) 746
   d) 846
   e) 619

3. a) 491
   b) 617
c) 418
   d) 624
e) 760
   f) 729

4. a) 795
   b) 729
c) 941
   d) 419

5. a) 9 899
   b) 9 831
c) 8 848
d) 8 407

6. a) 6 728
   b) 91 628
c) 474 917
d) 748 188
e) 13 322
   f) 14 535
g) 4 400

7. a) 828
   b) 1 111
c) 6 666
d) 58 285
e) 989

AP Book NS6-12

1. 5 thousands + 4 hundreds + 8 tens + 6 ones;
   3 thousands + 7 hundreds + 1 tens + 3 ones;
   8 thousands + 11 hundreds + 9 tens + 9 ones;
   9 thousands + 1 hundreds + 9 tens + 9 ones.
   a) 7 395
   b) 7 158
c) 9 378
d) 8 097
e) 8 378

3. a) 9 914
   b) 6 638
c) 6 815
d) 2 749
e) 9 845

4. a) 6 981
   b) 6 377
c) 9 828
d) 9 917
e) 8 378
   f) 9 086
g) 8 716
   h) 6 598
   i) 9 718
   j) 6 029

5. a) 536
   b) 1 221
c) 989
d) 47 674

b) 478  
c) 473  
d) 397  

7. a) 2 832  
b) 2 721  
c) 5 081  
d) 28 211  

8. a) 1 714  
b) 3 062  
c) 5 081  
d) 6 361  

9. a) 7 779  
b) 3 759  
c) 2 768  
d) 2 978  

10. a) 532  
b) 68  
c) 3 514  
d) 4 889  

1. 70 are girls  
2. 453 stamps  
3. 8 110 km  
4. 1 007 509 people  
5. 168 cans  
6. 85 years  
7. Answers will vary: 
The difference between a three-digit number and its reverse number is always 198 because regardless of the numbers, there will always be 198 numbers in-between. 

8. 256 km  
9. 250 km  

AP Book NS6-14  
page 55  
1. 70 are girls  
2. 453 stamps  
3. 8 110 km  
4. 1 007 509 people  
5. 168 cans  
6. 85 years  
7. Answers will vary: 
The difference between a three-digit number and its reverse number is always 198 because regardless of the numbers, there will always be 198 numbers in-between.  
8. 256 km  
9. 250 km  

AP Book NS6-15  
page 56  
1. a) Vancouver Island - 31 290  
   Newfoundland – 108 860  
   Ellesmere Island – 196 240  
   Baffin Island – 507 450  
   b) 476 160 km²  
   c) 87 380 km²  
   d) No.  
   507 450 + 31 290 = 538 740. Which is less than 2 166 086  

2. a) 87 645  
   b) 56 748; 56 784  
   c) Teacher to check – there are 16 correct answers.  
   Example: 86 754  
   d) 68 754  

3. 844 400  
4 a)  1 4  
   + 2 2  
   3 6  
   OR  
   1 2  
   + 2 4  
   3 6  

b) 2 4  
   - 1 3  
   1 1  
   OR  
   4 2  
   - 3 1  
   1 1  

c) 3 4  
   + 2 1  
   5 5  
   OR  
   4 3  
   + 1 2  
   5 5  

5. a) Answer depends on the year (currently 471 yrs).  
   b) Copernicus to Galileo: 67 years;  
   Galileo to Newton: 57 years;  
   Copernicus to Newton: 124 years  
   6. 1, 2, 3, 6  
   8: 1, 2, 4, 8  
   9: 1, 3, 9  
   10: 1, 2, 5, 10  
   12: 1, 2, 3, 4, 6, 12  

AP Book NS6-16  
page 57  
1. b) 3 rows; 5 dots in each row;  
   c) 4 rows; 5 dots in each row;  

2. a) 4 × 3  
   b) 2 × 5  
   c) 5 × 3  
   d) 7 × 2  

3. a) • • • • •  
   b) • • • • • • •  
   c) Teacher to check – there are 16 correct answers.  
   Example: 86 754  
   d) 68 754  

4. a) 1 × 6;  
   2 × 3;  
   3 × 2;  
   6 × 1  
   b) 1 × 8;  
   2 × 4;  
   4 × 2;  
   8 × 1  
   c) 1 × 9;  
   3 × 3;  
   9 × 1  
   d) 1 × 10;  
   2 × 5;  
   5 × 2;  
   10 × 1  
   e) 1 × 12;  
   2 × 6;  
   3 × 4;  
   6 × 2;  
   12 × 1  

AP Book NS6-17  
page 58  
1. a) 1  
   b) No  
2. 2, 3, 5, 7  
3. 10, 12, 14, 15, 16, 18, 20  
4. 29  
5. 13, 17, 29  
6. Primes:  
7. 11, 13;  
   17, 19;  
   5, 7.  

AP Book NS6-18  
page 59  
1. a) 1, 5, 25  
   b) 1, 2, 4, 8  
   c) 1, 2, 3, 4, 6, 12  
   d) 1, 2, 4, 8, 16  
   e) 1, 3, 9  
   f) 1, 2, 3, 6, 9, 18  
   g) 1, 2, 5, 10, 25, 50  
   h) 1, 3, 5, 9, 15, 45  
   i) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60  
   j) 1, 2, 3, 6, 7, 14, 21, 42  
2. Composite numbers:  
   30, 32, 33, 34, 35, 36  
3. a) Any primes less than 20  
   b) 6, 8, 10, 14  
   c) 16  
4. Cross out: 19, 34, 50  
5. 10, 20, 30  
6. Answers will vary.  
   Example: 8, 9, 10  
7. 15, 21, 27, 33, 39  
8. 3  
9. 1  
10. 4 + 6 + 8 + 9 + 10 = 37  
11. There are 5 prime numbers between 30 and 50.
### Answer Key for AP Book 6.1

#### AP Book NS6-19

**Page 60**

1. a) Bottom row: 2, 3, 2
   b) Bottom row: 2, 2, 2
   c) Bottom row: 3, 2, 2
2. a) $5 \times 2 \times 3$
   b) $3 \times 3 \times 2$
   c) $2 \times 2 \times 2$
   d) $7 \times 2$
3. a) $5 \times 3 \times 2$
   b) $3 \times 3 \times 2 \times 2$
   c) $3 \times 3 \times 3$
   d) $7 \times 2 \times 2$
   e) $5 \times 5 \times 3$
4. Answers will vary.
   Examples:
   1st branch – 24
   2nd branch – 8 x 3
   3rd branch – 4 x 2 x 3
   4th branch – 2 x 2 x 2 x 3
   OR
   1st branch 24
   2nd branch 12 x 2
   3rd branch 3 x 4 x 2
   4th branch 3 x 2 x 2 x 2

#### AP Book NS6-20

**Page 61**

1. a) $5 \times 3$ tens
   = 15 tens
   = 150
   b) $3 \times 4$ tens
   = 12 tens
   = 120
2. a) $3 \times 6$ tens
   = 18 tens
   = 180
   b) $6 \times 5$ tens
   = 30 tens
   = 300
   c) $4 \times 5$ tens
   = 20 tens
   = 200
   d) $5 \times 4$ tens
   = 20 tens
   = 200

#### AP Book NS6-21

**Page 62**

1. a) $2 \times 20 + 2 \times 5$
   = 50 + 10
   = 60
   b) $3 \times 10 + 3 \times 5$
   = 30 + 15
   = 45
2. a) $5 \times 10 + 5 \times 3$
   = 50 + 15
   = 65
   b) $4 \times 20 + 4 \times 1$
   = 80 + 4
   = 84
   c) $3 \times 40 + 3 \times 3$
   = 120 + 9
   = 129
   d) $2 \times 400 + 2 \times 30$
   + $2 \times 2$
   = 800 + 60 + 4
   = 864
   e) $3 \times 300 + 3 \times 10$
   + $3 \times 2$
   = 900 + 30 + 6
   = 936
   f) $4 \times 300 + 4 \times 20$
   + $4 \times 1$
   = 1200 + 80 + 4
   = 1284
3. a) 15; 150; 1 500
   b) 6; 60; 600
   c) 12; 120; 1 200
   d) 20; 200; 2 000
4. a) $1 \times 15$
   b) 0
   c) 2
   d) 3
   e) 30
   f) 300
   g) 208
   h) 186
   i) 20
   j) 30
   k) 135
   l) 20
   m) 30
   n) 120
   o) 20
   p) 20
5. Teacher to check.
6. $18 000$. With three extra zeros after the three (3000), you need to move the decimal point over three places to the right.

#### AP Book NS6-22

**Page 63**

1. a) 153
   b) 246
   c) 124
   d) 204
   e) 255
   f) 366
   g) 249
   h) 148
   i) 188
   j) 168
   k) 168
   l) 205
   m) 217
   n) 128
   o) 126
   p) 189
   q) 88
   r) 279
   s) 205
   t) 549
   u) 587
   v) 276
   w) 368
   x) 156
   y) 208

#### AP Book NS6-23

**Page 64**

1. a) 1, 2
   b) 1, 5
   c) 2, 0
   d) 3, 6
   e) 1, 8
2. a) 96
   b) 105
   c) 75
   d) 78
   e) 64
   f) 92
   g) 144
   h) 75
   i) 87
   j) 96
3. a) 70
   b) 90
   c) 90
   d) 75
   e) 96
   f) 135
   g) 256
   h) 210
   i) 182
   j) 368
1. a) 200 + 30 + 4
   \[ \times 2 \]
   = 400 + 60 + 8
   = 468
   b) 100 + 30 + 3
   \[ \times 3 \]
   = 300 + 90 + 9
   = 399

2. a) 164
   b) 868
   c) 936
   d) 248
   e) 969

3. a) 454
   b) 868
   c) 672
   d) 872
   e) 696

4. a) 728
   b) 906
   c) 968
   d) 855
   e) 768

5. a) 670
   b) 2 947
   c) 792
   d) 1 206
   e) 992
   f) 810

6. Teacher to check drawings.
   a) 228
   b) 888
   c) 969

AP Book NS6-25
page 66
1. a) 150
   b) 150
   c) 1 500
   d) 1 500
2. a) 1
   b) 2
   c) 3

3. a) 60;
   600;
   6 000;
   60 000
   b) 360;
   3600;
   36 000;
   360 000
   c) 850;
   8 500;
   85 000;
   850 000

4. a) 190
   b) 560
   c) 830
   d) 4 200
   e) 8 000
   f) 1 300
   g) 4 000
   h) 230
   i) 6 000
   j) 5 720
   k) 28 000
   l) 93 000

5. a) 10 \times 30 = 300
   b) 10 \times 20 = 200
   c) 10 \times 60 = 600
   d) 10 \times 70 = 700
   e) 70 \times 100 = 7000
   f) 60 \times 100 = 6000

6. 12 \times 38 = 456. Al's weekly income is $456.

7. a) 1 000
   b) 10 000
   1 000 000 + 100 = 10 000

8. 25 723 dimes =
   $2 572.30
   231 524 pennies =
   $2 315.24
   25 723 dimes is greater.

AP Book NS6-26
page 67
1. a) 3 \times 10
   b) 4 \times 10
   c) 7 \times 10
   d) 5 \times 10

2. a) 2 \times 10 \times 33
   b) 2 \times 10 \times 21
   c) 3 \times 10 \times 17

3. a) 2 \times 240 = 480
   b) 3 \times 320 = 960
   c) 4 \times 120 = 480
   d) 5 \times 410 = 2 050

4. a) 990
   b) 1 200
   c) 1 600
   d) 1 360
   e) 840
   f) 2 490
   g) 1 280
   h) 2 220
   i) 1 680
   j) 1 590
   k) 3 060
   l) 4 550
   m) 1 800
   n) 3 200
   o) 4 680
   p) 540

5. a) 40 \times 60 = 2 400
   b) 30 \times 70 = 2 100
   c) 30 \times 80 = 2 400
   d) 60 \times 50 = 3 000
   e) 70 \times 30 = 2 100
   f) 20 \times 20 = 400

AP Book NS6-27
page 68
1. a) 60 (carry 1)
   b) 20 (carry 4)
   c) 60 (carry 1)
   d) 90
   e) 50 (carry 3)

2. a) 810
   b) 1 200
   c) 1 380
   d) 1 840
   e) 910
   f) 1 080
   g) 840
   h) 1 040
   i) 720
   j) 2 240

3. b) 30 \times 20 + 30 \times 3
   = 600 + 90
   = 690
   c) 40 \times 30 + 40 \times 2
   = 1 200 + 80
   = 1 280

AP Book NS6-28
page 69
1. a) 72
   b) 108
   c) 198
   d) 248
   e) 80
   f) 75
   g) 108
   h) 144
   i) 204
   j) 296

2. a) 1 360
   b) 900
   c) 4 140
   d) 1 680
   e) 1 340

3. a) 210; 700
   b) 91; 390
   c) 128; 1 600
   d) 225; 1 350
   e) 32; 640
   f) 180; 1 350
   g) 115; 920
   h) 108; 360
   i) 184; 2 760
   j) 225; 4 500

4. a) 728
   b) 3 672
   c) 3 268
   d) 1 701
   e) 1 026
   f) 1 530
   g) 1 216
   h) 3 848
   i) 1 836
   j) 2 784

6. a) 805
   b) 5 184
c) 1 075
d) 3 654
e) 1 222
f) 1 036

AP Book NS6-29
page 71
1. 50, 90, 32, 56, 36, 34, 70,
   110, 78
2. a) Teacher to check.
   b) Yes they will.
   c) Teacher to check.
3. a) 350
   b) 8 400
   c) 29 000
   d) 47 500
   e) 36 000
   f) 10 000
   g) 15 000
   h) 853 000
   i) 95 200
4. C is the fastest.
   Converting to the same
   units (pages/second):
   A – 0.5 pages a second.
   30 pages a minute
   B – 1.5 pages a second.
   90 pages a minute
   C – 2 pages a second.
   120 pages a minute
   D – 1.33 pages a second.
   80 pages a minute
5. a) $4.80
   b) $6.40 + $2.50
      = $8.90
   c) $8.50

AP Book NS6-30
page 72
1. a) 19 822
   b) 10 787
   c) 18 495
2. Cross out: 13, 50, 2, 27
3. 21, 35, 49, 63, 77
4. 11
5. a) 19 346
   b) 68 904
   c) 88 896
   d) 461 384
6. 1 000 cm
7. 15 322
8. 12
9. No (952 minutes)
10. a) 2
    b) 7
    c) 4

AP Book NS6-31
page 73
1. a) Cups; 2; 4
    b) Orange; 3; 2
2. a) Books; 4; 8
    b) Flowers; 6; 4
    c) Apples; 4; 5
    d) Trees; 7; 3
3. Teacher to check.

AP Book NS6-32
page 74
1. a) 6
    b) 4
2. a) 4 ∆ per set
    b) 3 ∆ per set
3. 3 □ per set
4. a) 3 sets
    b) 7 sets
    c) 4 sets
5. a) 2 sets of 9
    b) 3 sets of 6
    c) Stickers; 2 sets
    d) Pictures; 8 per set
    e) Children; 3 sets
    f) Flowers; 5 sets
6. a) 7
    b) 2
    c) 4 sets
    d) 5 dots in each set

AP Book NS6-33
page 76
1. a) 4
    b) 2
2. a) 15 + 3 = 5
    b) 10 + 2 = 5

3. a) 7
    b) 2
    c) 4
    d) 3
    e) 9
    f) 9
    g) 5
    h) 7
    i) 9
    j) 8
    k) 6
    l) 5
    m) 6
    n) 7
    o) 5
4. 4
5. 4
6. 4

AP Book NS6-34
page 77
1. No. 4 pancakes on each
   plate. (1 remainder)
2. a) 3; 1
    b) 4; 1
3. a) 13 + 3 = 4 R1
    b) 19 + 3 = 6 R1
    c) 36 + 5 = 7 R1
    d) 33 + 4 = 8 R1
    e) 43 + 7 = 6 R1
4. 25 + 8 = 3 R1. Each
   friend will get 3 apples.
   There will be one left over
5. 3 groups of 2;
    3 tens in each group;
    8 tens altogether
   a) 1; 5
   b) 2; 6
   c) 2; 8
   d) 1; 5
   e) 1; 9
   f) 1; 8
   g) 1; 5
   h) 4; 8
5. a) 4 groups;
    2 tens in each group;
    8 tens altogether
   b) 4 groups;
    9 tens;
    2 tens in each group;
    8 tens altogether
5. a) 1; 5
    b) 2; 6
    c) 2; 8
   d) 1; 5
   e) 1; 9
   f) 1; 8
   g) 1; 5
   h) 4; 8
6. a) 3 groups;
    8 tens;
    2 tens in each group;
    6 tens altogether
6. a) 3 groups;
    8 tens;
    2 tens in each group;
    6 tens altogether
7. a) 2
    b) 6
    c) 4 sets
    d) 8 sets
    e) Children; 3 sets
    f) Flowers; 5 sets
8. a) 2 sets of 9
    b) 3 sets of 6
    c) Stickers; 2 sets
    d) Pictures; 8 per set
    e) Children; 3 sets
    f) Flowers; 5 sets
9. No (952 minutes)
10. a) 2
    b) 7
    c) 4
    d) 2
    e) 2
    f) 1
    g) 1
    h) 2
    i) 1
    j) 1

AP Book NS6-35
page 78
1. a) 2; 5; 3
    b) 5; 7; 1
    c) 4; 9; 5
2. a) 1; 7; 2
   b) 4; 8
   c) 1; 6
   d) 5; 8; 8
   e) 2; 6
   f) 1; 7
b) 2; 6; 1
c) 3; 6; 0
d) 1; 4; 3
e) 1; 6; 2
f) 1; 7; 1
g) 1; 7; 1
h) 2; 6; 2
i) 1; 5; 2
j) 1; 4; 1

7. a) 1; 5; 25
b) 1; 7; 17
c) 2; 8; 13
d) 3; 6; 13
e) 3; 6; 14
f) 1; 8; 17
g) 1; 4; 36
h) 3; 9; 04
i) 1; 7; 21
j) 1; 9; 04

8. a) 18; 5; 44
b) 21; 8; 07
c) 37; 6; 15
d) 17; 3; 21
e) 14; 5; 22
f) 12; 7; 15
g) 47; 8; 15
h) 12; 8; 16
i) 30; 9; 02
j) 46; 8; 13

9. a) 16 R1
b) 13
c) 28
d) 25
e) 32
f) 17 R3
g) 16 R4
h) 12
i) 12 R1
j) 10 R5
k) 11 R3
l) 19 R1
m) 10 R4
n) 13 R3
o) 16 R3

10. $99 \div 8 = 12 \text{ R}3$. There are 3 left over

11. 12 weeks
12. 13 rows; 6 books left over
13. $13
14. 14 cherries each; Wendy has 4 left over, while Saran has only 1 left over.

AP Book NS6-36 page 83

1. Teacher to check.
2. a) 157 R1
   b) 278 R1
   c) 145 R5
   d) 124 R3
3. b) 94 R2
c) 33 R2
d) 52 R3
4. a) 38 R1
   b) 85 R1
c) 53 R1
d) 63 R1
e) 73 R1
f) 305
g) 1 504 R3
h) 1 737 R2
i) 446 R1
j) 316 R1
5. 36 m
6. 122 km

AP Book NS6-37 page 85

1. a) 300
   b) 30
   c) 3
2. a) 300 000
   b) $2 016
   c) Yes (prime number 3); otherwise, no.

AP Book NS6-38 page 86

1. 3 120
2. $2\text{.}07$ or $2.07$
3. a) 25
   b) 49
4. $1 016
5. 198
6. 100
7. 2
8. 150
9. 560
10. $42$
   b) $50$
   c) $50$

AP Book NS6-39 page 87

1. a) 3 000
   b) 4 000
   c) 3 000
   d) 1 000
2. a) $0$
   b) $10$
   c) $70$
   d) $80$

Answer Key for AP Book 6.1
14. If the number in the hundreds place is 500 or greater, you round up to the nearest thousand. If the number in the hundreds place is 499 or less, you round down to the nearest thousand.

AP Book NS6-40

1. a) 40
   b) 50
   c) 20
   d) 60
   e) 80
   f) 80
   g) 30
   h) 40
   i) 90
2. a) 660
   b) 270
   c) 150
   d) 360
   e) 420
   f) 570
   g) 130
   h) 470
   i) 340
3. a) 300
   b) 500
   c) 600
   d) 300
   e) 200
   f) 400
   g) 500
   h) 800
   i) 1 000
4. a) 200
   b) 300
   c) 600
   d) 300
   e) 900
   f) 300
5. a) 5 000
   b) 3 000
   c) 8 000
   d) 5 000
   e) 3 000
   f) 9 000

6. a) 3. r.d.
   b) 1. r.d.
   c) 6. r.d.
   d) 3. r.u.
   e) 4. r.u.
   f) 1. r.d.

7. a) r.d. 72 000
    b) r.d. 90 000
    c) r.d. 84 200
    d) r.d. 27 500
    e) r.d. 461 270
    f) r.d. 140 000

8. a) 3 290
   b) 5 900
   c) 10 000
   d) 13 980
   e) 23 200
   f) 1 000 000
   g) 400 000

AP Book NS6-41

1. a) 40 + 20 = 60
   b) 30 + 50 = 80
   c) 60 – 20 = 40
   d) 90 – 60 = 30
   e) 70 + 20 = 90
   f) 90 – 50 = 40
   g) 20 + 30 = 50
   h) 60 + 30 = 90
   i) 80 + 50 = 130
   j) 50 – 20 = 30
   k) 50 + 30 = 80
   l) 80 + 10 = 90
   m) 90 – 40 = 50

2. a) 400 + 500 = 900
   b) 800 – 600 = 200
   c) 700 + 200 = 900
   d) 500 – 200 = 300
   e) 600 + 200 = 800
   f) 800 + 200 = 1000
   g) 700 + 300 = 1000

AP Book NS6-42

1. 250 000
2. 660 000
3. a) 628 320
   b) 628 300
   c) 628 000
   d) 630 000
4. a) 30 × 80 = 2 400
   b) 500 × 80 = 40 000
   c) 300 × 10 = 3 000
   d) 3 000 × 800 = 2 400 000
5. a) 6 × $5 = $30
   b) 5 × $3 = $15
   c) 8 × $8 = $64
6. a) Answers will vary.
   Correct example: 1392
   b) Answers will vary.
   Correct example: 5874
   c) Hundreds
   d) 60 × 30 = 1800; so her estimate is a little high
   e) 2 289
   f) Answers will vary.
   g) Rounding to the nearest hundreds gives a better estimate.

AP Book NS6-43

1. Predictions may vary.
   a) D
   b) B
   c) B
   d) E
2. a) Rounding
   b) Front-end estimation
   c) Round one up and the other down
   d) Answers may vary.
   None of the listed methods would be overly effective for d).
3. a) Too high
   b) Too low
   c) Correct
4. Teacher to check.
   a) 2 550
   b) 747
   c) 7 884
   d) 17 380
   e) 1 596
   f) 1 700
   g) 600
5. a) Answers will vary.
   Correct example: 1392
   b) Answers will vary.
   Correct example: 5874
6. a) Answers will vary.
   Correct example: measuring the length of one loonie, then multiplying it by 10 000.
   b) Answers will vary.
   Correct example: calculating the number of seconds in a day, then multiplying it by 365.

AP Book NS-44

1. a) 10, 20, 30, 35, 40, 41
   b) 25, 50, 55, 60, 61, 62, 63
4. a) 1 toonie, 1 loonie, 3 quarters, 5 nickels or 3 loonies, 2 quarters, 5 dimes

5. Answers will vary.

AP Book NS-46
page 96
1. a) 50¢
   b) 50¢
   c) 75¢
   d) 75¢
   e) 25¢
   f) 50¢
   g) 25¢
   h) 0¢
   i) 75¢
   j) 75¢

2. a) 75¢, 9¢
    b) 50¢, 17¢
    c) 75¢, 13¢
    d) 75¢, 16¢

3. a) 25¢, 5¢
    b) $1
    c) $1, 10¢, 5¢
    d) $1, 25¢, 5¢
    e) $2, 25¢
    f) $2, $2, 25¢, 25¢, 5¢, 1¢, 1¢
    g) $50, $5, $2, 25¢, 25¢, 1¢
    h) $50, $20, $1, 10¢, 1¢
    i) $50, $10, $2, $1, 5¢, 1¢
    j) $100, $50, $5, $2, $1, 25¢, 25¢
    k) $50, $20, $20, $1, 25¢, 25¢
    l) $20, $10, $5, 10¢, 1¢, 1¢

4. a) $1, 25¢, 25¢, 25¢
    b) 25¢, 25¢, 10¢, 10¢
    c) $2, $2, $2, $2, $2, 25¢, 25¢
    d) $2, $2, $2, $2, $2, $1, 25¢, 25¢
    e) $2, $2, $2, $2, $2, 25¢, 25¢

5. a) 25¢, 10¢, 10¢, 5¢
    b) 25¢, 25¢, 25¢
    c) 25¢, 25¢, 25¢, 25¢
    d) 25¢, 25¢, 25¢, 25¢, 25¢
    e) 25¢, 25¢, 25¢, 25¢, 25¢

6. a) $40
    b) $20
    c) $20
    d) $40
    e) $60

7. a) 50¢, $10, $5, $2, $1
    b) $50, $20, $2
    c) $50, $10, $2, $25¢, 10¢
    d) $50, $5, $2, 25¢, 25¢, 1¢
    e) $50, $20, $1, 10¢, 1¢
    f) $20, $20, $2, $1, 10¢, 1¢, 1¢
    g) $50, $5, $2, 25¢, 25¢, 1¢
    h) $50, $20, $1, 10¢, 1¢
    i) $50, $10, $2, $1, 5¢, 1¢
    j) $100, $50, $5, $2, $1, 25¢, 25¢
    k) $50, $20, $20, $1, 25¢, 25¢
    l) $20, $10, $5, 10¢, 1¢, 1¢, 1¢

AP Book NS-47
page 98
1. a) $1, 10¢, 1¢
    b) $1
    c) $1, 10¢, 5¢
    d) $1, 25¢, 5¢
    e) $2, 25¢
    f) $2, $2, 25¢, 25¢, 5¢, 1¢, 1¢
    g) $50, $5, $2, 25¢, 25¢, 5¢, 1¢
    h) $50, $20, $1, 10¢, 1¢
    i) $50, $10, $2, $1, 5¢, 1¢
    j) $100, $50, $5, $2, $1, 25¢, 25¢
    k) $50, $20, $20, $2, 25¢, 25¢, 5¢, 1¢, 1¢
    l) $20, $10, $5, 10¢, 1¢, 1¢, 1¢

2. a) 7¢ = $0.07
    b) 7¢ = $0.07
    c) 7¢ = $0.07
    d) 7¢ = $0.07
    e) 7¢ = $0.07

3. a) $4, 5¢, $4.55
    b) $25, 40¢, $25.40
    c) $20, 51¢, $20.51

4. a) 105¢, $1.05
    b) 106¢, $1.06

5. a) $4.37
    b) $0.40
    c) $0.05
    d) $3.48
    e) $3.06

6. a) 239¢
    b) 553¢
    c) 641¢
    d) 6¢

7. a) $2.96
    b) 107¢
    c) $0.70
    d) 686¢
    e) $40¢
    f) 122¢

8. a) Seven dollars and seventy cents
    b) Nine dollars and eighty-three cents
    c) Fifteen dollars and forty cents

9. a) $48.51
    b) $55.40
    c) $48.75

10. $427
11. 25¢, 25¢, 25¢, 5¢, 5¢,
12. $2, $2, 25¢, 25¢
13. \$2, \$2, \$2, \$2, \$2, 5¢, 10¢, 10¢ OR \$2, \$2, \$1, \$1, \$1, \$1, 25¢

14. a) Four dollars and eighty-five cents
b) Thirteen dollars and twenty-four cents
c) Eight dollars and twenty-five cents
d) Four hundred sixty one dollars and ninety-nine cents
e) Three hundred eighty five dollars and ninety nine cents
f) Four thousand five hundred twenty three dollars and two cents

Answer Key for AP Book 6.1

AP Book NS-48

1. a) 6¢
b) 9¢
c) 6¢
d) 7¢
e) 4¢
f) 5¢
g) 2¢
h) 1¢
i) 2¢

2. a) 10¢
b) 60¢
c) 80¢
d) 30¢
e) 90¢
f) 40¢
g) 50¢
h) 70¢
i) 20¢

3. a) 20¢
b) 30¢
c) 80¢
d) 40¢
e) 50¢
f) 70¢
g) 90¢

4. a) 80
b) 60
c) 50
d) 30
e) 60
f) 10

5. a) 44¢
b) 17¢
c) 46¢
d) 75¢
e) 53¢
f) 69¢

6. a) 26¢
b) 53¢
c) 64¢
d) 47¢
e) 28¢
f) 65¢
g) 3¢
h) 11¢
i) 8¢

7. a) 13¢
b) 17¢

8. 58¢ change: 25¢, 25¢, 5¢, 1¢, 1¢, 1¢

9. a) $8.00
b) $6.00
c) $6.00
d) $3.00
e) $8.00

10. a) $23
b) $62
c) $47
d) $36

11. a) $16
b) $75
c) $54
d) $12
e) $48

12. $72.43

13. a) $67.15
b) $13.73
c) $47.81
d) $33.57

AP Book NS-49

1. a) $9.61
b) $78.57
c) $39.97

2. a) $62.36
b) $92.55
c) $105.75
d) $126.44
e) $148.48
f) $115.53

3. a) $105.97
b) $1 159.82
c) $51.75

4. a) $64.90
b) Paint set & pallette
c) No. You would need $31.44.
d) $107.23
– $100.00 = $7.23 more
e) Answers will vary.

5. a) Yes: the total would be $20.75.
b) Yes: the total would be $24.95.

6. a) $12.80
b) 8
c) 6
d) No. It would cost $100.12.
e) 3 snowboards

7. a) 100
b) 200
c) 500

8. Teacher to check.

AP Book NS-50

1. a) Total = $51.26
b) Total = $74.76
c) Total = $78.68

2. a) 70¢
b) 50¢
c) 90¢
d) 30¢
e) 30¢
f) 10¢
g) 90¢
h) 20¢
i) 50¢

3. Circle: a), d), e)

4. a) $7
b) $38
c) $4
d) $100
e) $26
f) $59
g) $365
h) $17
i) $124
j) $128

5. b) Estimate: $5.00
Answer: $4.40
c) Estimate: $9.00
Answer: $9.18
d) Estimate: $7.00
Answer: $7.18
e) Estimate: $11.00
Answer: $11.27
f) Estimate: $78.00
Answer: $77.99
Number Sense – AP Book 6.1 (continued)

6. $40
7. $5
8. About $90
9. Rounding to the nearest dollar in this question, would suggest that Hannah and Ali have the same amount of money. This would be inaccurate because Hannah has about 80 cents more.

10. a) Estimate: $10
    Answer: $8.77
b) Estimate: $1 900
    Answer: $1 877.95

AP Book NS-52
page 108

1. a) M  Tu  W  Th  F
    - 5  15  5  - 10  - 20
    20  - 10  - 15  - 10

2. a) Teacher to check.
b) i) 1
tii) 5
iii) 2
c) Three

3. a) -7 degrees
b) -5 degrees
c) +8 degrees

4. a) Uranus
b) About 100 degrees
c) About 580 degrees

5. a) Dog
b) 5 000 years
c) 4 000 years
d) Teacher to check, answers will vary.

6. Mackerel: -200m
    Gulper: -1 000 m
    Gulper live 800 meters below the Mackerel

7. Five
8. Because they are closer to zero.

AP Book NS-53
page 110

1. a) nickels  pennies
    0  17
    1  12
    2  7
    3  2

b) dimes  nickels
    0  9
    1  7
    2  5
    3  3
    4  1

c) nickels  pennies
    0  23
    1  18
    2  13
    3  8
    4  3

d) dimes  pennies
    0  32
    1  22
    2  12
    3  2

e) quarters  nickels
    0  13
    1  8
    2  3

f) quarters  nickels
    0  17
    1  12
    2  7
    3  2

2. a) quarters  nickels
    0  12
    1  7
    2  2

He stops at 2 quarters because 3 quarters is 75¢ (and that’s larger than 60¢).

b) quarters  dimes
    0  -
    1  8
    2  -
    3  3
    4  -

3. a) dimes  nickels
    0  18
    1  16
    2  14
    3  12
    4  10

b) dimes  quarters
    1  10
    2  5
    3  0
    5

4. a) 1
    b) 2
    c) 3
    d) 4

5. a) quarters  dimes
    0  7
    1  -
    2  2

b) quarters  dimes
    0  8
    1  -
    2  3

6. a) quarters  dimes
    0  12
    1  7
    2  2

b) quarters  dimes
    0  12
    1  7
    2  2

7. Width  Length
    1  5
    2  4
    3  3
    4  2
    5  1

8. a) 1
    b) 2
    c) 3
    d) 4

Answer Key for AP Book 6.1
Answer Key for AP Book 6.1

AP Book ME6-1 page 112

1. First column: grams; grams; kilograms
   Second column: grams, kilograms, grams

2. Teacher to check.
   a) Answers will vary.
   b) Answers will vary.
   c) Answers will vary.

3. Answers will vary, teacher to check.

4. Teacher to check.
   a) Answers will vary.
   b) Answers will vary.
   c) Answers will vary.

5. a) i) 60 g
   ii) 30 g
   iii) 36 g
   iv) 1400 g
  b) 40 quarters
  c) 8 pennies
  d) Answers will vary – teacher to check.

6. 1000

7. a) Nickel: 4000 mg
   Loonie: 7000 mg
   b) A loonie (7000 mg) is 3000 mg heavier than a nickel (4000 mg).

8. Two monarch butterflies would weigh a gram; 2000 butterflies would weigh a kilogram.

AP Book ME6-2 page 114

1. a) 4 cubes
   b) 12 cubes
   c) 13 cubes

2. a) 2 1 2
   b) 1 1 2
   c) 2 1 2
   d) 3 2 1

3. a) 2 1 1
   b) 1 2 2 2 1
   c) 3 3 3 2 1
   d) 1 1 1 1 1

AP Book ME6-3 page 115

1. b) 2 + 2 + 2 + 2 + 2 = 10
   2 × 5 = 10
   c) 3 + 3 + 3 + 3 + 3
   + 3 + 3 = 21
   3 × 7 = 21

2. 3 blocks are shaded in each case

3. a) 3 + 3 + 3 + 3
   = 12 cm³
   b) 3 × 4 = 12 cm³

4. a) 6
   b) 6 + 6 + 6 + 6
   = 24 cm³

5. a) 2 + 2 + 2 = 6 cm³
   2 × 3 = 6 cm³
   b) 10 + 10 + 10 + 10
   = 40 cm³
   10 × 4 = 40 cm³
   c) 9 + 9 + 9 + 9 + 9
   = 45 cm³
   9 × 5 = 45 cm³

6. b) 2; 12
   c) 3; 18
   d) 4; 24
   e) 5; 15
   f) 6; 18

7. a) 12 × 2 = 24
   b) 20 × 3 = 60
   c) 16 × 3 = 48
   d) 20 × 2 = 40

8. Yes

AP Book ME6-4 page 118

1. a) 20 minutes
   b) 30 minutes
   c) 35 minutes

2. a) 35 minutes
   b) 30 minutes

3. a) 2 hrs and 25 min
   b) 3 hrs and 20 min

4. a) 2 hrs and 30 min
   b) 4 hrs and 25 min

5. a) 0:23
   = 23 min
   b) 1:01
   = 1 hr and 1 min
   c) 6:18
   = 6 hr and 18 min
   d) 4:15
   = 4 hr and 15 min
   e) 2:37
   = 2 hr and 37 min

6. Teacher to check students’ time lines.
   a) 5 hours 15 minutes
   b) 1 hour 30 minutes
   c) 1 hour 35 minutes

7. Karl studied for 1 hour and 45 minutes.
1. a) 1:41
   b) 3:32
   c) 0:51
   d) 0:50
   e) 3:43
   f) 5:21
   g) 4:47
   h) 1:47
   i) 1:44
   j) 1:16
   k) 3:11
   l) 2:17
   m) 4:51
   n) 4:33
   o) 3:33
   p) 6:27

2. a) 6:15
   b) 13:29
   c) 19:07

3. a) 05:00
   b) 23:00
   c) 18:00
   d) 02:00
   e) 15:00
   f) 00:00

4. a) 7:00 a.m.
   b) 3:00 p.m.
   c) 1:00 p.m.
   d) 12:00 a.m.
   e) 6:00 p.m.
   f) 5:00 p.m.
   g) 6:00 a.m.
   h) 11:00 p.m.

5. a) 10:45
   b) 8:26
   c) 20:11
   d) 02:00
   e) 15:00
   f) 00:00

6. a) 2 km
   b) 18:00
   c) Answers may vary – teacher to check.

7. The speed of sound is faster (≈ 20 km/minute).
   Answers may vary – teacher to check.

Sample answer:
There are about 1 500 minutes in a day, so about 1 500 mL (or 1.5 L) of water leaks each day.

Therefore, 365 × 1.5 = 550 L of water would leak in a year.
**AP Book PDM6-1 page 122**

1. a) E
   
   b) C

2. a) Dark (only): A, C
   
   Triangles (only): B, G
   
   Both: E

   b) Light (only): D
   
   Polygons (only): E, A
   
   Both: B, F, G

   c) In Norway (only): H, I, J
   
   Higher than 750 m (only): A, B, E
   
   In Norway and higher than 750 m: C, D

**AP Book PDM6-2 page 123**

1. a) Teacher to check.
   
   b) The bar graph had a scale of 5 and ranged from 0-40. A scale of 3 with a range of 0-15 would have been more appropriate.

   c) Answers will vary.

2. a) Graph A: Start at 0, count by 10 000, stop at 50 000
   
   Graph B: Start at 47 000, count by 100, stop at 47 300.
   
   b) Graph B
   
   c) Gisela

3. Teacher to check.

4. Teacher to check graphs.
   
   a) Graph A
   
   b) Graph B
   
   c) Each graph has its limitations, as students should notice in parts a) and b).

5. a) Start at 0,
   
   Count by 1
   
   Stop at 10.
   
   b) Start at 0,
   
   Count by 2,
   
   Stop at 20.
   
   c) Start at 0,
   
   Count by 250,
   
   Stop at 2 000.
   
   d) Start at 11 000,
   
   Count by 500,
   
   Stop at 13 000.

**AP Book PDM6-3 page 125**

1. a) Class A
   
   b) Class B
   
   c) Scale starts at 100
   
   d) Teacher to check.
   
   e) April
   
   f) February
   
   (or possibly Jan)

2. Teacher to check.

**AP Book PDM6-4 page 126**

1. b) Circle 3.
   
   Underline 7.
   
   c) Circle 12.
   
   Underline 4.
   
   d) Circle 5.
   
   Underline 1.
   
   e) Circle 900.
   
   Underline 0.
   
   f) Underline 7.

2. a) 0 2 3 6
   
   b) 0 3 4 5
   
   c) 7 8 9 10

3. Plot 1:
   
   21, 25, 41, 45, 48, 52, 56
   
   Plot 2:
   
   7, 13, 19, 20, 25, 28

4. a) 

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7 8</td>
</tr>
<tr>
<td>1</td>
<td>0 3 8</td>
</tr>
</tbody>
</table>

   b) 

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>7 9 9</td>
</tr>
<tr>
<td>10</td>
<td>1 3</td>
</tr>
</tbody>
</table>

5. Q3. Plot 1: Range- 35
   
   Plot 2: Range- 21

   Q4. a) Range- 11
   
   b) Range- 6
   
   c) Range- 37

**AP Book PDM6-5 page 127**

1. a) Start at 15,
   
   Count by 5,
   
   Stop at 30.
   
   b) “The Temperature in my Backyard this Week”
   
   c) Monday
   
   d) Thursday
   
   e) About 8°C

2. a) Largest profit: July
   
   Smallest profit: February
   
   b) January: $2 000
   
   May: $3 000
   
   c) July, August, September
   
   d) July 1st

3. a) Decrease.
   
   b) Reversed the timeline.

**AP Book PDM6-6 page 128**

1. a) $15
   
   b) $20
   
   c) $30
   
   2. a) 3 CDs
   
   b) 5 CDs
   
   c) 6 CDs

3. a) i) $30
   
   ii) $40
   
   b) $35
   
   c) i) $60
   
   ii) $5
   
   d) It is easier to read values between data points on the line graph.

**AP Book PDM6-7 page 129**

1. a) About 130 times
   
   b) About 200 times
   
   c) About 290 times

   NOTE:
   
   For each 5° increase in temperature there is an increase of about 40 chirps – so at 35°C there would be about 290 chirps.

   d) About 28°

2. a) Broken line graph.
   
   Next week’s temperature should be cooler, as the temperatures dropped throughout the week.

   b) Bar graph.
   
   Toronto was warmest.

   c) Broken line graph.
   
   The trend suggests that profits will increase next year.

3. a) Decrease.
   
   b) Reversed the timeline.

**AP Book PDM6-8 page 130**

1. a) Discrete
   
   b) Continuous
   
   c) Discrete
   
   d) Continuous
   
   e) Discrete

2. a) Horizontal-axis: Discrete

   Vertical-axis: Continuous

   b) Horizontal-axis: Discrete

   Vertical-axis: Continuous

   c) Horizontal axis: Continuous

   Vertical-axis: Continuous
1. a) 2 km 
   b) 6 km 
   c) 7 km 
2. Teacher to check graphs. 
   a) $35 
   b) 250 meters 
3. a) The more time the student spent studying, the higher the mark they received. 
   b) Mark received. 
   c) Yes, according to the trend in the graph, the student would score over 90% if they studied for more than an hour. 

AP Book PDM6-12 
page 134 
1. a) $15 + 3 = 5$ 
   b) $12 + 3 = 4$ 
   c) $16 + 4 = 4$ 
2. b) 5 below, 5 above 
   c) 4 below, 4 above 
3. The number of blocks above the mean is always equal to the number below. 
4. a) i) low 
    ii) high 
    iii) low 
   b) i) Move line up. 4 below, 4 above. The mean is 3. 
    ii) Move line down. 3 below, 3 above. The mean is 3. 

AP Book PDM6-13 
page 135 
1. a) $4 = 3 + 1$ 
   b) $2 + 2 + 0 = 1 + 3$ 
   c) $4 + 1 = 3 + 2$ 
   Teacher to check drawings. 
   d) 3 = 1 + 3 
   e) $3 + 2 = 2 + 3$ 
   f) $2 + 0 = 1 + 1$ 
2. Answers will vary: 
   a) 1 2 6 7 
   b) 2 2 3 6 7 
   c) 3 3 3 3 8 
3. a) 9 cm 
   b) 3 eggs 
   c) Albatross - 1 
   Emperor Penguin - 1 
   Flamingo - 1 
   Common Loon - 2 

AP Book PDM6-14 
page 136 
1. a) 6, median: 6 
   b) 3 and 3, median 3 
   c) 13, median 13 
   d) 6 and 10, median 8 
2. a) Median: 3 
   b) Median: 15 
   a) Median: 3 
   b) Median: 15 
   Mean: 5 
   Range below median: 1 
   Range above median: 13 
   Range below mean: 3 
   Range above mean: 11 
   b) Median: 15 
   Mean: 11 
   Range below median: 1 
   Range above median: 13 
   Range below mean: 3 
   Range above mean: 11 
3. Teacher to check the changes to the values; answers will vary. 
   a) Median: 4 
   Mean: 4 
   b) Median: 9 
   Mean: 9 
   c) Median: 8 
   Mean: 8 
4. a) Data is spread out more above the median. 
   b) Data is spread out more below the median. 
   c) Data is spread out equally above and below the median. 
5. Teacher to check. 

AP Book PDM6-15 
page 137 
1. a) 
   b) 
   c) 
   d) 
   e) 

Answer Key for AP Book 6.1
iii) Although it is the mode, only three other students got the same grade – this isn’t a lot.
iv) Only five students did better.
v) Yes, his mark is the mode.

2. Median:
   10 - No;
   13 - Yes (number <13)
   14 - Yes (14)
   15 - Yes (number > 15)
   20 – No

Mean:
   10 - No
   14 - Yes (14)
   20 - Yes (38)

Mode:
   10 - No
   12 - Yes (12)
   14 - Yes (14)
   20 - No

3. a) Median: 3
    Mean: 5
    Mode: 3
b) Median: 3
    Mean: 3
    Mode: 3

3. a) Median: 3
    Mean: 5
    Mode: 3
b) Median: 3
    Mean: 3
    Mode: 3

c) The mean changed the most.

4. Answers will vary.
   Sample Answer:
   2, 2, 2, 2, 5, 7, 10, 11
   Mean: 4.8
   Mode: 2

AP Book PDM6-16
page 138

1. a)  

<table>
<thead>
<tr>
<th>stem</th>
<th>leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>3 5 5 8 88</td>
</tr>
<tr>
<td>8</td>
<td>2 3 6</td>
</tr>
<tr>
<td>9</td>
<td>1 3</td>
</tr>
</tbody>
</table>

Teacher to check broken line graph.

b) i) 6; stem and leaf
    ii) 78; stem and leaf
    iii) Increase; broken line graph
    iv) After the 4th, 5th and 11th; broken line graph
    v) 91; stem and leaf

2. a) 60
    b) 15
    c) 40
    d) 40 x 5 = 200
       20 x 2 = 40
       200 + 40 = $240
    e) i) $60
       ii) 12 tickets

3. Stem and Leaf Plot →
   Makes it easy to see the largest, smallest and most common data values.
   Double Bar Graph →
   Compares two sets of data.
   Bar Graph →
   Shows the frequency of results and trends clearly.
   Scatter Plot →
   Shows whether one type of data increases, decreases or neither when another type of data increases.

4. Teacher to check.

5. b) Most likely b)
6. a) 
   A: Rainforest
   B: Coniferous Forest
   C: Grasslands
   D: Tundra
b) Teacher to check.
c) Teacher to check.

7. a) 3 months
    b) 12 weeks
    c) The guinea pig’s weight increases quickly in first 6 months, and slower after that.
    d) July
    e) January and March

8. a) Mean: 6.7
    Median: 6
    Mode: 3
    Range: 14
b) 1984 – 8
   1988 – 9
   1996 – 15
   2000 – 12
   2004 - 10
    c) Answers may vary.
        Broken line graph would probably be the best choice.

AP Book PDM6-17
page 141

1. a) Survey
    b) Measurement
    c) Observation/Survey
    d) Survey
    e) Observation

2. a) A. Primary
    b) B. Secondary
    c) B. Secondary
    d) A. Primary
    e) B. Secondary

3. a) Teacher to check
    b) Teacher to check.

AP Book PDM6-18
page 142

1. a) 
   
   | 11 | 59 |
   | 10 | 60 |
   | 1  | 6 |

   | 63 | 321 |
   | 60 | 300 |
   | 1  | 5 |

   b) You could, but it would not give a good estimate.
   c) Yes it does (provides a larger sample).
   d) Higher

2. a) B – a sample
    b) A – every player

3. Answers will vary. Teacher to check.

AP Book PDM6-19
page 143

1. a) Because the earlier students are biased toward arriving earlier.
    b) B
    c) About 200

2. a) A – Biased
    b) B - Representative
    c) A – Biased
       B - Representative

3. A – the people present at this location would most likely want a new baseball stadium
   B - the people present at this location would most likely want a new library.
   C - the people present at this location would most likely want a new baseball stadium (due to the sport connection).
   D - the people present at this location would probably represent the younger population, who may not be as interested in libraries.

AP Book PDM6-20
page 144

Teacher to check.
AP Book G6-1
page 145
1. a) 5; 5  
   b) 6; 6  
   c) 7; 7  
   d) 5; 5  
   e) 8; 8  
   f) 10; 10  
2. a) 3  
   b) 4  
   c) 5  
   d) 6  
3. Shapes Letters  
   Triangles   C  
   Quadrilaterals   B, D, F, G, H  
   Pentagons   A  
   Hexagons   E, I  
4. Teacher to check.  
5. 37 sides  

AP Book G6-2
page 146
1. a) Less than  
   b) Greater than  
   c) Right angle  
   d) Less than  
2. a) 2 right angles  
   b) No right angles  
   c) 2 right angles  
   d) 1 right angle  
   e) 2 right angles  
3. Teacher to check.  
4. Teacher to check.  

AP Book G6-3
page 147
1. a) Acute  
   b) Obtuse  
   c) Acute  
   d) Acute  
   e) Obtuse  
   f) Obtuse  
   g) Obtuse  
   h) Acute  
   i) Obtuse  
2. a) Acute; 60º  
   b) Obtuse; 120º  
   c) Acute; 30º  
   d) Obtuse; 150º  
3. a) Acute; 60º  
   b) Obtuse; 135º  
   c) Obtuse; 115º  
   d) Acute; 30º  
   e) Acute; 40º  
   f) Obtuse; 137º  
   g) Obtuse; 125º  
   h) Acute; 20º  
4. Teacher to check.  

AP Book G6-4
page 150
1. Teacher to check.  
2. Teacher to check.  
3. a) 30º  
   b) 130º  
   c) 45º  
   d) 45º  
   e) 105º  
   f) 90º  

AP Book G6-5
page 151
1. a) Acute-angled  
   b) Right-angled  
   c) Obtuse-angled  
   d) Obtuse-angled  
2. a) 110º, 30º, 40º; Obtuse-angled  
   b) 90º, 45º, 45º; Right-angled  
   c) 60º, 50º, 70º; Acute-angled  
3. a) About 110º, 110º, 110º, 110º; Obtuse-angled  
   b) 60º, 60º, 60º; Acute-angled  

AP Book G6-6
page 152
1. Teacher to check.  
2. Teacher to check.  
3. a) Teacher to check.  
   b) In each triangle, two sides are the same length.  
   c) (i) Obtuse-angled (or Isosceles – see G6-8)  
      (ii) Acute-angled (or Equilateral – see G6-8)  
      (iii) Right-angled (or Isosceles – see G6-8)  
4. Teacher to check.  
5. Two congruent right-angled triangles.  
6. Teacher to check.  

AP Book G6-7
page 153
1. Teacher to check.  
2. a) Right-angled; Isosceles  
   b) Obtuse-angled; Scalene  
3. Teacher to check.  

AP Book G6-8
page 154
NOTE:  
Side lengths and angles are recorded clockwise, from left to right.  
1. A ± 52º, 90º, 38º  
   with 3 cm, 4 cm, 5 cm side lengths  
   B ± 110º, 35º, 35º  
   with 4 cm, 6.5 cm, 4 cm side lengths  
   C ± 60º angles with 3 cm side lengths  
   D ± 22º, 79º, 79º  
   with 5.2 cm, 5.2 cm, 2.1 cm side lengths  
   E ± 120º, 35º, 25º  
   with 3.1 cm, 6 cm, 4 cm side lengths  

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute-angled</td>
<td>C, D</td>
</tr>
<tr>
<td>Obtuse-angled</td>
<td>B, E</td>
</tr>
<tr>
<td>Right-angled</td>
<td>A</td>
</tr>
<tr>
<td>Equilateral</td>
<td>C</td>
</tr>
<tr>
<td>Isosceles</td>
<td>B, D</td>
</tr>
<tr>
<td>Scalene</td>
<td>A, E</td>
</tr>
</tbody>
</table>

2. a) Scalene: A, E  
   Right-angled: A  
   Both: A  
   b) Obtuse: E, B  
   Isosceles: D, B  
   Both: B  

3. Teacher to check.  

AP Book G6-9
page 155
1. Teacher to check.  
2. a) Right-angled; Isosceles  
   b) Obtuse-angled; Scalene
3. The angles of every triangle must add up to 180°. We know the top angle is 120° and the base angles are always the same on an isosceles triangle, therefore the two base angles must be 30°. 120 + 30 +30 = 180.

AP Book G6-10
page 156
1. a) and c) should be marked as parallel

BONUS:
e) If the pair of lines in b) are extended, they will intersect so aren’t parallel.

By definition, the lines in d) cannot be parallel as they are curved.

2. Teacher to check.

3. a) 1
b) 2
c) 3
d) 1
e) 2
f) 2
g) 0
h) 0

4. Teacher to check.

5. Teacher to check.

6. E has three lines that are parallel; H and M each have one pair of parallel lines; W and K have no parallel lines.

AP Book G6-11
page 158
1. A 2 pairs
B 2 pairs
C 1 pair
D 1 pair
E 2 pairs
F No pairs
G 1 pair

H 2 pairs
2. No Pairs: F
   One Pair: C, D, G
   Two Pairs: A, B, E, H

3. | Property | Shapes with Property |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) No right angles</td>
<td>D, E, I, J</td>
</tr>
<tr>
<td>1 right angle</td>
<td>C</td>
</tr>
<tr>
<td>2 right angles</td>
<td>A, B, G, K</td>
</tr>
<tr>
<td>4 right angles</td>
<td>F, H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Shapes with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) No right angles</td>
<td>D, E, I, J</td>
</tr>
<tr>
<td>1 pair</td>
<td>A, B, D, G, K</td>
</tr>
<tr>
<td>2 pairs</td>
<td>E, F, H</td>
</tr>
</tbody>
</table>

4. a) 4 cm, 2.5 cm, 4 cm, 2.5 cm; (not equilateral)
   b) 2.7 cm, 2.7 cm, 2.7 cm, 2.7 cm; (equilateral)
   c) 1.7 cm, 1.7 cm, 1.7 cm, 1.7 cm; (equilateral)
   d) 2 cm, 2 cm, 2 cm, 2 cm; (equilateral)

5. | Property | Shapes with Property |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Equilateral</td>
<td>A, B, D, F, I</td>
</tr>
<tr>
<td>Not Equilateral</td>
<td>C, E, G, H, J</td>
</tr>
<tr>
<td>b) No right</td>
<td>A, D, F, G, I</td>
</tr>
<tr>
<td>1 right</td>
<td>E</td>
</tr>
<tr>
<td>2 right</td>
<td>H</td>
</tr>
<tr>
<td>3 right</td>
<td>J</td>
</tr>
<tr>
<td>4 right</td>
<td>B, C</td>
</tr>
<tr>
<td>c) No obtuse</td>
<td>A, B, C, E, G</td>
</tr>
<tr>
<td>1 or more obtuse</td>
<td>D, F, H, I, J</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Shapes with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) No parallel sides</td>
<td>A, E, F</td>
</tr>
<tr>
<td>1 pair of parallel sides</td>
<td>G, H</td>
</tr>
<tr>
<td>2 pairs of parallel sides</td>
<td>B, C, D, J</td>
</tr>
<tr>
<td>3 pairs of parallel sides</td>
<td>I</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Shapes with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Triangles</td>
<td>A, E</td>
</tr>
<tr>
<td>Quadrilaterals</td>
<td>B, C, D</td>
</tr>
<tr>
<td>Pentagons</td>
<td>F, H</td>
</tr>
<tr>
<td>Hexagons</td>
<td>I, J</td>
</tr>
</tbody>
</table>

AP Book G6-12
page 160
1. a) 3 cm, 2 cm, 3 cm; parallelogram
   b) 2 cm, 2 cm, 2 cm, 2 cm; square

2. Square – A parallelogram with 4 right angles and 4 equal sides.
   Rectangle – A parallelogram with 4 right angles.
   Rhombus – A parallelogram with 4 equal sides.

3. a) Rectangle
   b) Parallellogram
   c) Square
   d) Rhombus

4. a) Parallelogram
   b) Square
   c) Rhombus
   d) Rectangle

5. a) 2 pairs; rectangle
   b) 2 pairs; parallelogram
   c) 2 pairs; square
   d) 1 pair; trapezoid

6. Teacher to check.

7. a) All

AP Book G6-13
page 162
1. a) yes
   b) no
   c) no
   d) no

2. Teacher to check.
3. The intersection of the diagonals is perpendicular.
4. a) B, D
   b) The diagonals bisect at 90º.
   One of the two diagonals is the line of symmetry.
5. The angles opposite the line of symmetry are equal.
6. Diagonals meet at right angles (only): none
   Two pairs of equal adjacent sides (only): none
   Inside: A, E, F
   Outside: B, D, C

7. a) If rounding to the nearest cm, all triangles will have sides of length 3 cm, 5 cm and 6 cm.
   b) ΔABC:
      26º, 50º, 105º
      ΔDEF:
      30º, 60º, 90º
      ΔHIJ:
      30º, 60º, 90º
      ΔKLM:
      26º, 50º, 105º
   c) Answers will vary.
   d) Answers will vary.
8. a) ΔABC and ΔKLM;
     ΔDEF and ΔHIJ
   b) Answers will vary.
   c) Answers will vary.
9. Teacher to check drawings.
   a) Sides: 3cm, 4cm, 5cm
      Angles: 90º, 36º, 125º
   b) Sides: 4cm, 4cm, 4cm
      Angles: 60º, 120º, 60º, 120º

AP Book G6-16
page 167
1. Teacher to check.
2. Teacher to check.
3. Teacher to check.
4. a) 3
   b) 5
   c) 2
   d) 6
   e) 5
   f) 4
   g) 2
   h) 1
5. The 2nd and 5th should be shaded.
6. a) Less than two lines of symmetry:
     B, C, D, F, G
     More than two lines of symmetry: A, E
   b) C, B
7. a) Number of edges; number of lines of symmetry:
     Equilateral triangle – 3; 3
     Square – 4; 4
     Pentagon – 5; 5
     Hexagon – 6; 6
   b) The number of edges is equal to the number of lines of symmetry.
   c) Order of rotation symmetry:
     Triangle – 3
     Square – 4
     Regular Pentagon – 5
     Regular Hexagon – 6
     A shape has an order of rotation symmetry equal to the number of lines of symmetry.
     E.g., an equilateral triangle has an order of rotation symmetry of 3.
8. No, the shapes are not mirror images.
9. Answers will vary.

AP Book G6-17
page 170
1. a) 3 4 √
   b) 0 1 √
   c) 0 2 √
   d) 3 1 √
   e) 0 1 √
   f) 3 0 √
   g) Y N √
2. Answers will vary.
3. Teacher to check.

**AP Book G6-18**  
*page 171*

1. a) C, H
    b) Prop | Figures |
         1 | B, C, F, H |
         2 | B, C, F |
     B, C & F have both properties.
    c) Prop | Figures |
         1 | A, C, D, E, F, H |
         2 | C, F |
     C & F have both properties.

2. **Q E 2+ 90º A O**
   A Y N Y N Y Y
   B Y N N N Y Y
   C Y N Y Y N N
   D N Y Y N N Y Y
   E N Y N N N Y
3. Answers will vary.

**AP Book G6-19**  
*page 173*

1. a) F; F; F; F;
    b) F; F; F; F;
    c) F; T; T; T;
    d) T; T; T; T;
    e) T; F; T; F;
    f) F; T; T; F
2. a) Equilateral triangle
    b) Isosceles triangle
    c) Parallelogram
    d) Trapezoid
3. a) Teacher to check.

b) Teacher to check.

c) Teacher to check.

4. a) Teacher to check.
    b) Teacher to check.

**AP Book G6-20**  
*page 174*

1. Teacher to check.
2. Teacher to check.
3. a) ∠ABC
   b) If line is perfectly diagonal and bisects a 90º angle, the angle will be 45º.
4. Teacher to check.
5. 1 name:
   B (parallelogram),
   E (trapezoid)
   2 names:
   C (rectangle, parallelogram),
   D (rhombus, parallelogram)
   4 names:
   A (parallelogram, rectangle square, rhombus)
6. a) 3 acute angles
    b) 2 acute angles and 1 obtuse angle
    c) 2 acute angles and 1 right angle
7. a) Equilateral triangle
    b) Obtuse-angled triangle
    c) Right-angled triangle
    d) Acute-angled triangle
Patterns & Algebra – AP Book 6.2

AP Book PA6-22

page 175

1. a) +
   b) ×
   c) +
   d) ×
   e) +
   f) +
   g) ×
   h) +
   i) ×
   j) +
   k) +
   l) ×

2. a) ×
   b) ×
   c) ×
   d) –
   e) +
   f) ×
   g) –
   h) +
   i) +
   j) +
   k) ×
   l) +

3. a) 27, 81, 243
    b) 9, 27, 81
    c) 16, 32, 64
    d) 49, 343, 2401

4. a) × 4; 128, 512
    b) × 2; 24, 48
    c) × 5; 125, 625
    d) × 5; 250, 1250

    b) Start at 5. Add 3.
    c) Start at 18. Subtract 4

6. a) Start at 3, multiply by 2
    b) Start at 14, add 4
    c) Start at 1, multiply by 3

AP Book PA6-23

page 176

1. a) Gap: 2, 3, 4, 5, 6
    Pattern: 16, 22
   b) Gap: 1, 2, 3, 4, 5, 6
    Pattern: 18, 24
   c) Gap: 3, 5, 7, 9, 11
    Pattern: 36, 47
   d) Gap: 2, 4, 6, 8, 10
    Pattern: 36, 48
   e) Gap: – 5, – 4, – 3, – 2, – 1
    Pattern: 4, 3
   f) Gap: – 10, – 8, – 6, – 4, – 2
    Pattern: 14, 12
   g) Gap: – 9, – 7, – 5, – 3, – 1
    Pattern: 28, 27
    Pattern: 110, 105

2. Fig # Tri # Added
   1 1
   2 4 3
   3 9 5
   4 16 7
   5 25 9
   6 36 11

AP Book PA6-24

page 178

1. a) Table Chairs
   1 5
   2 6
   3 9
   4 10
   5 13
b) Start at 5. Add 1, then add 3. Repeat.

2. a) i) Teacher to check pictures.
    5th: 15
    6th: 21
   ii) Start at 1. Add 2, 3, 4, ...
        (The step increases by 1.)
   iii) 36
   b) i) Teacher to check pictures.
    5th: 25
    6th: 36
   ii) Option 1:
        Start at 1.
        1 × 1, 2 × 2, 3 × 3, 4 × 4, ...
        Option 2:
        Start at 1, add 3, 5, 7, ...
        (The step increases by 2.)
iii) 64

3. a) Gap: 0, 1, 2, 3, 5, 8, 13, 21
   Pattern: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55
   (Each subsequent number is the sum of the previous two)
   b) Odd, odd, even. Repeat.
   c) 1 + 1 + 3 + 5 = 10;
      2 + 8 = 10
      The sum of the first 4 odd Fibonacci numbers equals the first 2 even numbers.
   d) 1 + 1 + 3 + 5 + 13 + 21 + 34 + 55 = 125

AP Book PA6-26
page 180
1. a) Years Weeks
    1 52
    2 104
    3 156
    4 208

b) Years Days
   1 365
   2 730
   3 1095
   4 1460

c) Hours Seconds
   1 3600
   2 7200
   3 10800
   4 14400

2. Tank 1 drains at a constant rate of 40 L per minute.
   Tank 2 drains at a variable rate per minute:
   10 L, 20 L, 30 L, ... (the step increases by 10 L each time).
   After 10 min, 30 sec (or approximately 11 min), Tank 2 will be empty.
   Therefore, Tank 2 will empty first.

3. a) After 25 minutes, the airplane will have 950 litres of fuel.
    b) After 30 minutes, the airplane will be 75 km from the airport.

c) When the plane reaches the airport, it will be carrying 850 litres of fuel.

AP Book PA6-27
page 181
1. a) 6
    b) 3
    c) 5
    d) 3
    e) 7
    f) 5

2. a) 6
    b) 3
    c) 5
    d) 3
    e) 7
    f) 5

3. a) 6
    b) 3
    c) 5

4. a) 0 + 6 = 6
    b) 1 × 6 = 6
    c) 6 – 0 = 6
    d) 2 × 3 = 6
    e) 6 – 1 = 5
    f) 6 – 2 = 4
    g) 6 – 4 = 2
    h) 6 – 5 = 1
    i) 6 – 6 = 0

5. The only thing that changes between the two equations is variables; 'a' and 'b' and because the equation that has 'b' in it (2b + 6 = 14) provides a larger answer, 'b' must be greater than 'a'.

6. Answers will vary.

7. a) x = 4
    b) x = 4
    c) x = 8

4. a) 13 times
    b) 2014
    c) n = 6
    d) x = 3
    e) y = 12
    f) n = 5
    g) b = 16
    h) x = 5
    i) z = 25
    j) m = 27

2. a) □ = 3
    b) □ = 2
    c) □ = 3
    d) □ = 4
    e) □ = 2
    f) n = 9
    g) n = 10
    h) n = 3
    i) n = 5

3. a) 6
    b) 3
    c) 5
    d) 3
    e) 7
    f) 5

4. a) 0 + 6 = 6
    b) 1 × 6 = 6
    c) 6 – 0 = 6
    d) 2 × 3 = 6
    e) 6 – 1 = 5
    f) 6 – 2 = 4
    g) 6 – 4 = 2
    h) 6 – 5 = 1
    i) 6 – 6 = 0

5. The only thing that changes between the two equations is variables; 'a' and 'b' and because the equation that has 'b' in it (2b + 6 = 14) provides a larger answer, 'b' must be greater than 'a'.

6. Answers will vary.

7. a) x = 4
    b) x = 5
    c) x = 1
1. a) \(5 \times 2 = 10\)
   b) \(5 \times 4 = 20\)
   c) \(7 \times 5 = 35\)
2. a) \(70 \times 3 = 210\)
   b) \(40 \times 2 = 80\)
   c) \(h \times 100\) OR \(100h\)
3. a) \(7 \times h = 7h\)
   b) \(7 \times t = 7t\)
   c) \(7 \times x = 7x\)
   d) \(7 \times n = 7n\)
4. a) \(A + 4 = B\)
   b) \(3 \times A = B\)
   c) \(A + 7 = B\)
   d) \(5 \times A = B\)
   e) \(8 \times A = B\)
5. Since the letter or symbol used as the variable is arbitrary, the equation gives the same solution.
6. Answers may vary.
   a) \(28 - 15 = n,\) \(n\) represents the number of boys. (13 boys)
   b) \(48 - 24 = s,\) \(s\) represents the number of stamps that were given away. (24 stamps)

AP Book PA6-30

1. Two circles
2. Two circles
3. Two triangles
4. Three circles
BONUS:
5. Three circles
6. Two circles
7. a) 4
   b) 7
   c) 3
   d) 3
   e) 2
   f) 6
   g) 4

8. Teacher to check.

AP Book PA6-31

1. a) 8345 - 2878 = 5467
2. Teacher to check graphs.
   "Ordered pairs:"
   (3, 1), (4, 3), (5, 5), (6, 7)
3. Teacher to check graphs.
4. "Sample answer:"
5. Graphs will vary.
6. Teacher to check graphs.
   NOTE: Sample T-tables are given but answers may vary.
7. A: | Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

RULE: Begin with an input of 1. To find the output, multiply the input number by 3, then subtract 1. Repeat this rule for each ordered pair. (Increasing the input number by 1)

B: | Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

RULE: Begin with an input of 1. To find the output, add 2 to the input number. Repeat this rule for each ordered pair. (Increasing the input number by 1)

C: | Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

RULE: Begin with an input of 1. The output is equal to the input. Repeat this rule for each ordered pair (increasing the input number by 2).

8. Answers may vary.

Patterns & Algebra – AP Book 6.2 (continued)

2. a) 20 km
b) 40 km
c) Yes. The difference in distance from 3 to 4 hours is 0. Therefore, Kathy had not travelled during this time interval and likely rested.
d) No. Prior to Kathy’s rest break, she was travelling at a speed of 10 km/h. After her break she travelled at 5 km/h.

e) Input increases by 0.5, 1.0, 1.5... (the step increases by 0.5 each time); Output increases by 1, 2, 3... (the step increases by 1 each time); Multiply input by 2 to get the output.

3. a) 40 metres
b) 60 metres
c) Tom won the race by 40 metres.
d) Ben had a 40 metre head start.
e) 15 seconds

4. a) i) $8.00
ii) $10.00
iii) $9.00
b) $4.00
c) For a 3-hour rental, Dave’s store costs $10.50 whereas Mike’s store costs $9.00. Mike’s store is less expensive.

c) Input increases by 1 each time; Output increases by 1 each time; Multiply input by 2 to get the output.

d) Input increases by 1 each time; Output increases by 7 each time; Multiply input by 7 to get the output.
e) Input increases by 0.5, 1.0, 1.5... (the step increases by 0.5 each time); Output increases by 1, 2, 3... (the step increases by 1 each time); Multiply input by 2 to get the output.

f) Input increases by 1 each time; Output increases by 3, 5, 7... (the step increases by 2); Multiply the input by the input to get the output.

g) Input increases by 1 each time; Output increases by 2.1 each time; Multiply the input by 2.1 to get the output.

1. a) Input increases by 1 each time; Output increases by 3 each time; Multiply input by 3 and add 3 to get the output.
b) Input increases by 1 each time; Output increases by 1 each time; Add 20 to the input to get the output.

c) Input increases by 1 each time; Output increases by 4 each time; Multiply input by 4 and add 2 to get the output.

d) Input increases by 1 each time; Output increases by 7 each time; Multiply input by 7 to get the output.

e) Input increases by 0.5, 1.0, 1.5... (the step increases by 0.5 each time); Output increases by 1, 2, 3... (the step increases by 1 each time); Multiply input by 2 to get the output.

2. Gaps: 1, 2, 3
Sequence: 0, 1, 3, 6

3. Gaps: 1, 2, 3, 4, 5
Sequence: 0, 1, 3, 6, 10, 15

4. Yes.
5. a) 28
b) 45

AP Book PA6-35

1. a) Tables Chairs

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

RULE: Multiply the number of tables by 4 and add 2.

2. $260.00
3. 30 cups of water
4. 140 km
5. The 24th and the 48th receive both a book and a calendar.
6. Counting by 24s and 30s, you see that 120 is divisible by both numbers (and is less than 150). So they each collected 120 apples: Anna collected 5 baskets of 24, and Emily collected 4 baskets of 30.
7. a) There will be 44 shaded squares on the perimeter, and we know this since # shaded pieces = 4 x figure # + 4 (or, in this specific case, 4 x 10 + 4).

AP Book PA6-33

1. a) 0
b) 1
c) 3
d) 6

AP Book PA6-34

1. | Dist | Sec | Min | Hr |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3 km</td>
<td>900</td>
<td>15</td>
<td>1/4</td>
</tr>
<tr>
<td>4.6 km</td>
<td>1800</td>
<td>30</td>
<td>1/2</td>
</tr>
<tr>
<td>6.9 km</td>
<td>2700</td>
<td>45</td>
<td>3/4</td>
</tr>
</tbody>
</table>

Answer Key for AP Book 6.2
b) There will be 49 white squares:
32 – 4 = 28
28 ÷ 4 = 7
(this gives us the figure # we need)
And, in Figure 7, there are 7 × 7 = 49 white squares.
(Figure # x Figure # = # of white squares)

8. i) This offer will cost Gerome $235 (5 × $35 + $60)
ii) This offer will cost Gerome $270 (6 × $45).
So offer i) is cheaper.

9. The core pattern is 5 shapes in length. So we know that every multiple of 5 is where the core ends. The closest multiple of 5 to 72 – without going over – is 70 (14 × 5), with remainder 2.
So, to find the 72nd shape, look at the 2nd shape of the core pattern: it is a pentagon.

10. Paul shovelled the following # of sidewalks each day:
Day 1: 3
Day 2: 6
Day 3: 9
Day 4: 12

11. | # E | # T | P |
--- | --- | --- |
1 | 1 | 1 | 3 |
2 | 2 | 4 | 6 |
3 | 3 | 9 | 9 |
4 | 4 | 16 | 12 |

Patterns in columns:
# of edges increases by 1 each time;
# of triangles increases by 3, then by 5, then by 7, etc… (step increases by 2 each time);
Perimeter increases by 3 each time.

Relations in rows:
# of edges multiplied by itself gives # of triangles
# of edges multiplied by 3 gives perimeter

12. a) Decreases
b) 500 metres
c) 1 cm = 500 m
d) Yes (decreases by 2.5° each time)
e) i) 7.0°C
ii) 2.0°C

13. No, Marlene isn’t right. She will need 28 blocks to make Figure 7.
Starting at 1, the # of blocks increases by 2, 3, 4…(step increases by 1)

| Figure # | # of Blocks |
--- | --- |
1 | 1 |
2 | 3 |
3 | 6 |
4 | 10 |
5 | 15 |
6 | 21 |
7 | 28 |
Answer Key for AP Book 6.2

AP Book NS6-54

page 193

1. a) \(\frac{6}{9}\)
b) \(\frac{4}{6}\)
c) \(\frac{8}{16}\)
d) \(\frac{3}{9}\)
2. a) [Diagram: three bars]
b) [Diagram: four bars]

3. a) \(\frac{3}{5}\) are shaded;
b) \(\frac{3}{5}\) are pentagons
4. a) \(\frac{6}{11}\)
b) \(\frac{4}{11}\)
c) \(\frac{1}{11}\)
3. Divide the trapezoid into equal-sized triangles:

A single small triangle = \(\frac{1}{16}\) of the figure.

5. a) \(\frac{7}{9}\)
6. a) \(\frac{1}{14}\)
b) \(\frac{5}{14}\)
c) \(\frac{3}{14}\)
d) \(\frac{5}{14}\)
7. a) A [Diagram: A and P]

<table>
<thead>
<tr>
<th>A</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
</tr>
</tbody>
</table>

b) \(\frac{12}{23} \times \frac{11}{23}\)
c) \(\frac{5}{12} \times \frac{7}{12}\)
8. \(\frac{8}{9}\)
9. a) \(\frac{2}{7}\)
b) \(\frac{4}{7}\)
c) \(\frac{3}{7}\)
d) \(\frac{4}{7}\)
10. Teacher to check.

AP Book NS6-55

page 194

1. a) \(\frac{3}{5} \cdot \frac{3}{5}\)
b) \(\frac{\frac{4}{5}}{\frac{5}{5}}\)
2. a) pentagons

AP Book NS6-56

page 196

1. a) \(\frac{1}{3}\)
b) \(\frac{1}{4}\)
c) \(\frac{1}{5}\)
d) \(\frac{1}{12}\)
e) \(\frac{1}{12}\)
f) \(\frac{1}{12}\)
2. a) \(\frac{2}{6}\)
b) \(\frac{8}{16}\)
c) \(\frac{1}{16}\)
3. Divide the trapezoid into equal-sized triangles:

A single small triangle = \(\frac{1}{16}\) of the figure.

4. a) \(\frac{1}{2}\)
b) \(\frac{2}{3}\)
c) \(\frac{1}{3}\)
d) \(\frac{20}{25} \div \frac{4}{5}\)
5. a) \(\frac{1}{6}\)
b) \(\frac{8}{18}\)
c) \(\frac{7}{200}\)
6. Since both fractions have the same denominator, the fraction with the smaller numerator is the greater fraction.

AP Book NS6-57

page 197

1. \(\frac{3}{4}\)
2. a) \(\frac{1}{5} \div \frac{2}{5} = \frac{3}{5}\)
b) \(\frac{1}{3} \div \frac{2}{3} = \frac{2}{9}\)
3. a) \(\frac{4}{5}\)
b) \(\frac{3}{4}\)
c) \(\frac{5}{7}\)
d) \(\frac{7}{8}\)
e) \(\frac{10}{17}\)
f) \(\frac{14}{17}\)
g) \(\frac{21}{24}\)
h) \(\frac{31}{57}\)
4. a) \(\frac{1}{5} \cdot \frac{3}{5} \cdot \frac{4}{5}\)
b) \(\frac{1}{10} \cdot \frac{2}{10} \cdot \frac{5}{10} \cdot \frac{9}{10}\)
5. a) \(\frac{1}{6}\)
b) \(\frac{8}{9}\)
c) \(\frac{7}{200}\)
6. Since both fractions have the same numerator, the fraction with the smaller denominator is the greater fraction.

AP Book NS6-58

page 198

1. \(\frac{5}{6}\) has the greater numerator and is also the greater fraction.

Sample Explanation:
Since both fractions have the same denominator, the fraction with the greater numerator is the greater fraction.

2. a) \(\frac{11}{17}\)
b) \(\frac{4}{17}\)
c) \(\frac{11}{25}\)
d) \(\frac{57}{17}\)
3. Since both fractions have the same denominator, the fraction with the greater numerator is the greater fraction.

4. a) \(\frac{1}{5} \cdot \frac{3}{5} \cdot \frac{4}{5}\)
b) \(\frac{1}{10} \cdot \frac{2}{10} \cdot \frac{5}{10} \cdot \frac{9}{10}\)
5. a) \(\frac{1}{6}\)
b) \(\frac{8}{9}\)
c) \(\frac{7}{200}\)
6. Since both fractions have the same numerator, the fraction with the smaller denominator is the greater fraction.
9. \( \frac{1}{2} = \frac{50}{100} > \frac{1}{100} \)

10. Yes, since the pies can be very different sizes:

AP Book NS6-59
page 199

1. a) 2  
   b) 3  
   c) 1

2. a) 2 \frac{1}{2} 
   b) 1 \frac{2}{3} 
   c) 2 \frac{5}{6} 
   d) 4 \frac{2}{3} = 4 \frac{1}{2} 
   e) 1 \frac{7}{8} 
   f) 3 \frac{5}{6} 
   g) 2 \frac{1}{4} 

3. a)  
   b)  
   c)  
   d)  

4. a)  
   b)  
   c)  
   d)  

5. Comparing the whole numbers of each fraction, we see that 4 pies > 3 pies. 
Thus, 4 \frac{1}{2} represents more pie than 3 \frac{2}{5}, regardless of the denominator of the remaining pieces of pie.

AP Book NS6-60
page 200

1. a) \frac{5}{2} 
   b) \frac{5}{4} 
   c) \frac{4}{3} 
   d) \frac{15}{8} 
   e) \frac{19}{8} 
   f) \frac{16}{9} 
   g) \frac{5}{3} 
   h) \frac{18}{5} 

2. a)  
   b)  
   c)  
   d)  

AP Book NS6-61
page 201

1. a) \frac{3}{1} \cdot \frac{10}{3} 
   b) \frac{2}{3} \cdot \frac{11}{4} 
   c) \frac{2}{3} \cdot \frac{8}{3} 
   d) \frac{3}{5} \cdot \frac{30}{8} 
   e) \frac{3}{4} \cdot \frac{15}{4} 
   f) \frac{2}{5} \cdot \frac{23}{9} 

2. Teacher to check shading. 
   a) \frac{7}{2} 
   b) \frac{15}{4} 

3. Teacher to check shading 
   a) \frac{2}{3} 
   b) \frac{3}{6} 
   c) \frac{3}{4} 
   d) \frac{2}{5} 
   e) \frac{3}{8} 
   f) \frac{4}{3} 

4. a) \frac{2}{1} 
   b) \frac{2}{1} 
   c) \frac{2}{1} 
   d) \frac{2}{1} 
   e) \frac{2}{1} 
   f) \frac{2}{1} 

5. Comparing the whole numbers of each fraction, we see that 4 pies > 3 pies. 
Thus, 4 \frac{1}{2} represents more pie than 3 \frac{2}{5}, regardless of the denominator of the remaining pieces of pie.

AP Book NS6-62
page 202

1. a) 2 halves 
   b) 4 halves 
   c) 8 halves 
   d) 7 halves 
   e) 9 halves 
   f) 11 halves 

2. a) 3 thirds 
   b) 6 thirds 
   c) 12 thirds 
   d) 4 thirds 
   e) 8 thirds 
   f) 17 thirds 

AP Book NS6-63
page 203

1. a) 2  
   b) 3 
   c) 6 
   d) 2 
   e) 5 
   f) 2 

2. a) 2; 1; 2 \frac{1}{2} 
   b) 3; 2; 3 \frac{2}{5} 
   c) 3; 1; 3 \frac{1}{3} 
   d) 4; 1; 4 \frac{1}{2} 

AP Book NS6-64
page 204

1. a) \frac{2}{3} 
   b) \frac{4}{5} 
   c) \frac{11}{8} 
   d) \frac{4}{7} 
   e) \frac{12}{7} 

2. a) \frac{3}{1} 
   b) \frac{5}{2} 
   c) \frac{7}{3} 
   d) \frac{9}{4} 
   e) \frac{11}{5} 
   f) \frac{13}{6} 

3. a) \frac{1}{2} 
   b) \frac{1}{3} 
   c) \frac{2}{5} 
   d) \frac{3}{7} 
   e) \frac{4}{9} 
   f) \frac{5}{11} 

4. a) \frac{8}{3} 
   b) \frac{9}{4} 
   c) \frac{15}{8} 
   d) \frac{19}{12} 
   e) \frac{22}{15} 

5. a) \frac{10}{3} 
   b) \frac{11}{4} 
   c) \frac{12}{5} 

Answer Key for AP Book 6.2
6. \( \frac{5}{2} \) is greater – you can tell by converting both to mixed fraction:
\[
\frac{5}{2} = 2 \frac{1}{2} : \frac{7}{2} = 2 \frac{1}{3} ; \quad 2 \frac{1}{2} > 2 \frac{1}{3}
\]
7. Between 1 and 2
8. a) \( \frac{3}{7} \)
b) \( \frac{1}{5} \)
c) \( \frac{1}{3} \)
d) \( \frac{1}{10} \)

**AP Book NS6-64 page 204**
1. a) 
   ![Fraction Image]
   b) 17
   c) \( \frac{17}{6} \)
2. a) \( \frac{1}{2} \cdot \frac{3}{2} \)
b) \( \frac{1}{3} \cdot \frac{5}{3} \)
c) \( \frac{1}{5} \cdot \frac{11}{6} \)
3. a) 
   ![Fraction Image]
   b) 
   ![Fraction Image]
   c) 
   ![Fraction Image]
   d) 
   ![Fraction Image]
4. a) 
   ![Fraction Image]
   b) 
   ![Fraction Image]
   c) 
   ![Fraction Image]
   d) 
   ![Fraction Image]
5. a) 
   ![Fraction Image]
   b) 
   ![Fraction Image]
   c) 
   ![Fraction Image]
6. \( \frac{5}{6} \)
7. \( \frac{11}{3} \)
8. \( \frac{11}{6} \)
9. \( \frac{5}{6} \)
10. \( \frac{1}{3} \)
11. Yes
13. 8

**AP Book NS6-65 page 206**
1. a) \( \frac{3}{5} \)
b) \( \frac{2}{3} \)
c) \( \frac{5}{6} \)
2. a) \( \frac{4}{5} \cdot \frac{1}{2} \)
b) \( \frac{2}{4} \cdot \frac{1}{2} \)
3. a) \( \frac{2}{3} \)
b) \( \frac{1}{2} \)
c) \( \frac{1}{3} \)
d) \( \frac{2}{3} \)
e) \( \frac{3}{5} \)
4. a) \( \frac{4}{6} \)
b) \( \frac{6}{9} \)
c) \( \frac{9}{12} \)
5. a) Teacher to check.
b) \( \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} \)
6. a) ![Fraction Image]
b) ![Fraction Image]

**AP Book NS6-66 page 207**
1. Teacher to check.
2. Teacher to check.
3. Teacher to check.
4. One (1) piece has both olives and mushrooms:
   ![Circle Diagram]
5. 4 pieces

**AP Book NS6-67 page 208**
1. a) \( \frac{3}{4} \)
b) \( \frac{4}{5} \)
2. a) \( \frac{1}{2} \) of 6 = \( \frac{2}{3} \)
of 6 = \( \frac{4}{5} \)
b) \( \frac{1}{4} \) of 8 = \( \frac{2}{3} \)
of 8 = \( \frac{3}{4} \)
c) \( \frac{1}{3} \) of 9 = \( \frac{2}{3} \)
of 9 = \( \frac{6}{5} \)
d) \( \frac{3}{5} \) of 10 = \( \frac{6}{5} \)
e) \( \frac{3}{4} \) of 12 = \( \frac{9}{5} \)
3. a) ![Dotted Circles]
b) ![Dotted Circles]
c) ![Dotted Circles]
d) ![Dotted Circles]
4. a) ![Dotted Circles]
b) ![Dotted Circles]
5. Teacher to check drawings.
a) 2
b) 5
c) 4
d) 9
6. a) 6
b) 6

c) 10
d) 4
e) 15
f) 4
g) 3
h) 6
i) 9
j) 14
k) 6
l) 9

**AP Book NS6-68 page 211**
1. a) \( \frac{1}{2} \)
b) \( \frac{1}{3} \)

c) \( \frac{2}{3} \)

d) \( \frac{1}{2} \)

e) \( \frac{3}{4} \)

2. a) \( \frac{1}{2} \) (+2)

b) \( \frac{1}{3} \) (+3)

c) \( \frac{2}{3} \) (+2)

d) \( \frac{1}{6} \) (+2)

e) \( \frac{1}{3} \) (+3)

3. a) \( \frac{1}{5} \) (+2)

b) \( \frac{1}{3} \) (+2)

c) \( \frac{1}{4} \) (+2)

d) \( \frac{1}{6} \) (+2)

e) \( \frac{1}{3} \) (+3)

4. \( \frac{5}{3} \cdot \frac{1}{2} \) (continued)

AP Book NS6-69

Page 220

1. a) 66

b) 46

c) white

d) red

Page 221

4. a) 6 cans

AP Book NS6-71

Page 225

1. a) \( \frac{3}{10} \cdot \frac{5}{6} \cdot \frac{1}{2} \)

b) \( \frac{3}{4} \) and \( \frac{5}{6} \)

c) \( \frac{7}{10} \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{3} \cdot \frac{5}{6} \cdot \frac{9}{10} \)

3. Answers may vary.

Sample Answers:

a) \( \frac{3}{6} \cdot \frac{4}{8} \)

b) \( \frac{3}{9} \cdot \frac{7}{12} \)

c) \( \frac{6}{9} \cdot \frac{9}{12} \)

d) \( \frac{4}{9} \) and \( \frac{6}{10} \)

AP Book NS6-73

Page 227

1. a) \( \frac{66}{100} \) : 0.66

b) \( \frac{4}{100} \) : 0.04

c) \( \frac{60}{100} \) : 0.60

2. Teacher to check shadings.

a) 0.39

b) 0.65

c) 0.07

AP Book NS6-74

Page 228

3. All possible answers:

11 10 9 8 7

22 20 18 16 14

AP Book NS6-75

Page 229

3. 10, 6, 2

5. 6, 3, 2

7. 6, 5, 4

9. Spaghetti sauce
3. \[
\begin{align*}
11 & : 0.11 \\
12 & : 0.12 \\
54 & : 0.54
\end{align*}
\]

4. Answers will vary. Teacher to check picture.

AP Book NS6-74

page 218

1. a) 47 hundredths = 4 tenths 7 hundredths 
   \[\frac{47}{100} = 0.47\]

b) 68 hundredths = 6 tenths 8 hundredths 
   \[\frac{68}{100} = 0.68\]

c) 86 hundredths = 8 tenths 6 hundredths 
   \[\frac{86}{100} = 0.86\]

d) 74 hundredths = 7 tenths 4 hundredths 
   \[\frac{74}{100} = 0.74\]

2. a) \(\frac{43}{100} = 0.43\)

b) \(\frac{28}{100} = 0.28\)

c) \(\frac{66}{100} = 0.66\)

d) \(\frac{64}{100} = 0.64\)

e) \(\frac{9}{100} = 0.09\)

f) \(\frac{30}{100} = 0.30\)

3. a) \(5 \div 2 = 2.5\) hundredths

b) \(5 \div 5 = 1\) hundredths

c) \(4 \div 0 = 0\) hundredths

d) \(2 \div 3 = 0.66\) hundredths

e) \(0 \div 2 = 0\) hundredths

f) \(1 \div 8 = 0.125\) hundredths

AP Book NS6-75

page 219

1. Teacher to check drawings.

\[
\begin{align*}
4 \div 0.4 & , 0.40 & , 40 \div 100 \\
5 \div 0.5 & , 0.50 & , 50 \div 100 \\
8 \div 0.8 & , 0.80 & , 80 \div 100 \\
10 \div 1.0 & , 1.00 & , 100 \div 100
\end{align*}
\]

b) .72

7 dimes 2 pennies
7 tenths 2 hundredths
72 pennies
72 hundredths

c) .43

4 dimes 3 pennies
4 tenths 3 hundredths
43 pennies
43 hundredths

AP Book NS6-76

page 220

1. a) .64

6 dimes 4 pennies
6 tenths 4 hundredths
64 pennies
64 hundredths

AP Book NS6-77

page 221

1. \[
\begin{array}{|c|c|}
\hline
\text{Tents} & \text{Hund} \\
\hline
5 & 6 \\
6 & 10 \\
3 & 1 \\
0 & 9 \\
\hline
\end{array}
\]

b) 3 dimes 0 pennies
3 tenths 0 hundredths
30 pennies
30 hundredths

.3 > .27

5. George most likely forgot to add a zero to .8 (.8 = .80).

.80 > .63
Number Sense – AP Book 6.2 (continued)

q) \(\frac{46}{100}\)

r) \(\frac{25}{100}\)

s) \(\frac{93}{100}\)

t) \(\frac{6}{100}\)

3. a) .2

b) .4

c) .3

d) .9

e) .93

f) .78

g) .66

h) .05

4. a) .2

b) .4

c) .3

d) .9

e) .93

f) .78

g) .66

h) .05

3. a) .2

b) .4

c) .3

d) .9

e) .93

f) .78

g) .66

h) .05

4. a) .2

b) .4

c) .3

d) .9

e) .93

f) .78

g) .66

h) .05

4. Circle: b), f), g), and j)

5. a) 0.82

b) 0.09

6. \(\frac{46}{100} = \frac{23}{50}\)

AP Book NS6-78 page 222

1. a) \(\frac{21}{100} , 1.21\)

b) \(\frac{38}{100} , 1.38\)

c) \(\frac{59}{100} , .59\)

d) \(\frac{2.40}{100} , 2.40\)

e) \(\frac{5.64}{100} , 5.64\)

2. Teacher to check.

3. a) \(\frac{2.35}{100} , 2.35\)

b) \(\frac{1.03}{100} , 1.03\)

4. a) 1.32

b) 2.71

c) 8.7

d) 4.27

e) 3.07

f) 17.8

g) 27.1

h) 38.05

5. a) 6 tenths because it is equivalent to 60 hundredths which is greater than 6 hundredths

AP Book NS6-80 page 224

1. a) \(.1 , .2 , .3 , .4 , .5 , .6 , .7 , .8 , .9\)

b) \(.5\)

2. a) a half

b) a half

c) one

d) one

e) zero

f) zero

3. a) <

b) <

c) >

4. a) two

b) three

c) one

AP Book NS6-81 page 225

1. a) \(\frac{4}{10} , \frac{6}{10} , \frac{7}{10}\)

b) \(\frac{1}{2} , \frac{3}{5} , \frac{7}{10}\)

c) \(\frac{4}{5} , \frac{4}{5} , \frac{7}{10}\)

2. a) .40

b) .80

c) .32

3. a) .7 = .70 = \(\frac{70}{100}\)

b) .9 = .90 = \(\frac{90}{100}\)

c) .1 = .10 = \(\frac{10}{100}\)

4. a) \(\frac{30}{100} , \frac{45}{100} , \frac{90}{100}\)

b) \(\frac{37}{100} , \frac{39}{100} , \frac{80}{100}\)

c) \(\frac{1.34}{100} , \frac{1.35}{100} , \frac{1.40}{100}\)

5. Teacher to check shading.

6. a) \(\frac{2.5}{10}\)

b) \(\frac{3.7}{10}\)

C) \(\frac{8.6}{10}\)

d) 6

E) \(\frac{1.86}{100}\)

f) \(\frac{1.78}{100}\)

7. a) \(\frac{3.5}{10} = 3.5\)

b) \(\frac{3.8}{10} = 3.8\)

C) \(\frac{7.8}{10} = 8.7\)

d) \(\frac{5.3}{10} = 5.3\)

e) \(\frac{1.53}{100} = 1.53\)

8. \(\frac{23}{10} = 2.3; 2.4 > 2.3\)

9. Possible answers: 1.33, 1.34, 1.35, 1.36, 1.37, 1.38, 1.39

10. Teacher to check shading.

AP Book NS6-82
16. Since the fraction was multiplied by three, if the decimal were to be multiplied by three, it would produce the correct answer. 
\[ 3 \times \frac{1}{4} = 3 \times 0.25 = 0.75 \]

17. \( \frac{1}{2} = 0.5 < 0.65 \)

**AP Book NS6-82 page 227**

1. a) hundredths 
b) thousandths 
c) tenths 
d) thousandths 
e) ones 
f) tenths 
2. a) 6, 5, 1, 2 
b) 6, 3, 5, 4 
c) 7, 0, 3, 0 
d) 1, 3, 0, 5 
e) 1, 7, 6, 3 
f) 0, 5, 3, 6 
g) 6, 3, 8, 0 
h) 8, 0, 0, 0 
i) 5, 8, 1, 3 
j) 0, 1, 3, 0 
3. a) \( \frac{652}{1000} \) 
b) \( \frac{372}{1000} \) 
c) \( \frac{20}{100} \) 
d) \( \frac{2}{1000} \) 
4. a) 2 tenths + 3 hundredths + 7 thousandths 
b) 3 tenths + 2 hundredths + 5 thousandths 
c) 6 ones + 3 tenths + 3 hundredths + 6 thousandths 
5. a) \( 0.49 \) 
b) \( 0.50 \) 
c) \( 0.758 \) 
d) \( 0.025 \) 
6. a) \( < \)

**AP Book NS6-83 page 228**

1. a) \( \frac{25}{100} + \frac{50}{100} = \frac{75}{100} \) 
b) \( \frac{30}{100} + \frac{37}{100} = \frac{67}{100} \) 
c) \( \frac{62}{100} + \frac{31}{100} = \frac{93}{100} \) 
d) \( \frac{44}{100} + \frac{37}{100} = \frac{81}{100} \) 
2. a) \( 0.25 + 0.50 = 0.75 \) 
b) \( 0.30 + 0.37 = 0.67 \) 
c) \( 0.62 + 0.31 = 0.93 \) 
d) \( 0.44 + 0.37 = 0.81 \) 
3. a) \( 0.89 \) 
b) \( 0.95 \) 
c) \( 1.14 \) 
d) \( 0.79 \) 
e) \( 0.66 \) 
f) \( 0.98 \) 
g) \( 1.39 \) 
h) \( 0.68 \) 
4. a) \( 6.49 \) 
b) \( 8.87 \) 
c) \( 12.58 \) 
d) \( 0.89 \) 
e) \( 5.11 \) 
5. a) \( 0.65 \) 
b) \( 0.28 \) 
c) \( 0.59 \) 

**AP Book NS6-85 page 230**

1. Teacher to check drawings and charts. 
   a) \( 2.35 \) 
b) \( 2.79 \) 
2. Teacher to check drawings. 
   a) \( 1.23 \) 
b) \( 1.13 \) 
3. a) \( 7.69 \) 
b) \( 7.23 \) 
c) \( 1.71 \) 
d) \( 2.04 \) 
e) \( 5.32 \) 
4. a) \( 3.84 \) 
b) \( 3.39 \) 
c) \( 2.47 \) 
d) \( 27.31 \) 
e) \( 13.61 \) 
5. \( 18.43°C \) 
6. \( 91.04 \) million kilometres 
7. a) \( 8, 1.0, 1.2 \) 
b) \( 1.2, 1.5, 1.8 \) 

**AP Book NS6-84 page 229**

1. a) \( \frac{20}{100} \) 
b) \( \frac{26}{100} \) 
c) \( \frac{35}{100} \) 
2. a) \( 0.50 - 0.30 = 0.20 \) 
b) \( 0.38 - 0.12 = 0.26 \) 
c) \( 0.69 - 0.34 = 0.35 \) 
3. a) \( 0.32 \) 
b) \( 0.54 \) 
c) \( 0.23 \) 
4. a) \( 0.54 \) 
b) \( 0.16 \) 
c) \( 0.26 \) 
d) \( 0.33 \) 
e) \( 0.27 \) 
f) \( 0.18 \) 
g) \( 0.64 \) 
h) \( 0.56 \) 
i) \( 0.71 \) 
j) \( 0.37 \) 
k) \( 0.59 \) 
l) \( 0.08 \) 
m) \( 0.14 \) 
5. a) \( 0.28 \) 
b) \( 0.02 \) 
c) \( 0.95 \) 
d) \( 0.44 \) 
e) \( 0.65 \) 
f) \( 0.28 \) 
g) \( 0.59 \) 
h) \( 0.68 \) 
i) \( 0.98 \) 
j) \( 1.39 \) 
k) \( 0.79 \) 
l) \( 0.66 \) 
m) \( 1.14 \) 
6. \( 7.90 \) cm 
7. \( 0.99 \) L 

**AP Book NS6-86 page 231**

1. a) \( 2 \) 
b) \( 3 \) 
c) \( 6 \) 
2. a) \( 5 \) 
b) \( 7 \) 
c) \( 14 \) 
d) \( 9 \) 
e) \( 17 \) 
f) \( 16 \) 
g) \( 182 \) 
h) \( 173 \) 
i) \( 235 \) 
j) \( 17.2 \) 
k) \( 426 \) 
l) \( 53.6 \) 
3. a) \( 6 \) 
b) \( 8 \) 
c) \( 16 \) 
4. \( .3 + .3 + .3 + .3 + .3 + .3 + .3 + .3 + .3 + .3 + .3 + .3 = 3 \) 
5. Multiplying by 10 exchanges tenths for ones. 

**AP Book NS6-87 page 232**

1. a) \( 2 \) 
b) \( 100 \times .03 = 3 \) 
2. a) \( 70 \) 
b) \( 180 \) 
c) \( 460 \) 
d) \( 590 \) 
e) \( 230 \) 
f) \( 400 \) 
g) \( 16 \) 
h) \( 69 \) 
i) \( 7 \) 
j) \( 8 \) 
k) \( 67 \) 
d) \( 95 \) 
3. a) \( 7 \) 
b) \( 6 \) 
c) \( 67 \) 
d) \( 95 \) 
e) \( 182 \) 
f) \( 407 \) 
g) \( 50 \) 
h) \( 70 \)
Number Sense – AP Book 6.2 (continued)

4. a) 1
b) 1

5. Shift 3 decimal places to the right.

6. a) 860
b) 325
c) 1329
d) 760
e) 8250
f) 7500

AP Book NS6-88
page 233

1. a) 2.86
b) 3.6
c) 5.05
d) 8.4
e) 10.68
f) 8.4
g) 9.36
h) 12.96

2. a) 6; 24; 8; 4; 8.4
b) 6; 15; 7; 5; 7.5
c) 6; 21; 8; 1; 8.1
d) 8 ones + 24 tenths = 10 ones + 4 tenths = 10.4

3. a) 6; 15; 3 = 7; 5; 3 = 7.53
b) 8, 4, 16 = 8, 5, 6 = 8.56
c) 5, 20, 5 = 7, 0, 5 = 7.05

4. a) 1; 1; 10.35
b) 2; 0; 30.48
c) 1; 0; 25.86
d) 0; 1; 25.86

5. a) 10.5
b) 24.9
c) 37.5
d) 25.29
e) 25.2

AP Book NS6-89
page 234

1. a) 2.0 ÷ 10 = .2
b) 3.0 ÷ 10 = .3
c) 4.0 ÷ 10 = .4
d) 5.0 ÷ 10 = .5

2. a) 6.2 ÷ 10 = .62
b) 7.3 ÷ 10 = .73

3. a) .03
b) .05
c) .07
d) .13
e) .76
f) 1.2
g) .9
h) .6
i) 4.2
j) 1.7
k) .9
l) 2.73
m) .03
n) .062
do) .007
p) .172

4. Dividing by 100, shifts the decimal two places to the left. A dollar (1.00) divided 100 ways yields a penny (0.01).

5. .035 m
6. 1.0 + 10 = .1

AP Book NS6-90
page 235

1. 2.56

2. a) 1.44

AP Book NS6-91
page 237

1. a) .84
b) .33
c) .19
d) .89
e) .51
f) 2.80
g) 3.057
h) .010
i) 2.382

2. a) .5
b) 1.0
c) 3.35
d) .76
e) .80
f) .373

3. a) 0.01
b) 0.10
c) 0.01
d) 0.10
e) 0.010
f) .001

4. Teacher to check.

5. a) .3, .4, .5, .6, 7, 8
b) 9.6, 9.7, 9.8, 9.9, 10.0, 10.1
c) 2.5, 2.6, 2.7, 2.8, 2.9, 3.0

6. Any number from 7.15 to 7.24

7. Circle b) Distance shot put is thrown in Olympic Games

Answer Key for AP Book 6.2
8. a) 50 cm
   b) 200 m
   c) 6 cm
   d) 600 000
   e) 40 g
   f) 4 mm

AP Book NS6-93
page 239
1. Teacher to check.
2. a) \(\frac{1}{10} = 0.1\)
   b) \(\frac{1}{10} = 0.1\)
   c) \(\frac{1}{10} = 0.1\)
   d) \(1\frac{6}{10} = 1.6\text{ cm}\)
   e) \(\frac{77}{100} = 0.77\)
   f) \(\frac{39}{90} = 0.43\)

3. a) 0.5 dm + 7.3 dm = 7.8 dm
   b) 0.5 dm + 3.2 dm = 3.7 dm
   c) 0.8 cm + 5.7 cm = 6.5 cm
   d) \(\frac{33}{100} = 0.33\text{ m} + 1.64\text{ m} = 1.97\text{ m}\)
   e) 6.85 m + 12.3 m = 19.15 m
   f) 9.82 m + 1.5 m = 11.32 m

4. a) Teacher to check.
   b) Teacher to check.
   c) Teacher to check.
   d) Teacher to check.
   e) 5.33
   f) 3.99

5. a) 4 307.01
   b) 20 330.06
   c) 325 070.196

6. a) >
   b) >
   c) >
   d) >

7. a) 5.72
   b) 10.7
   c) 28.759
   d) .7

8. a) \(.5706\)
   b) \(5.670\) or \(.5706\)

9. Any number from .31 to .49 for numbers in the hundredths

10. a) +
    b) ×
    c) +

11. a) \(.275, .37, .371\)
    b) \(.007, .07, .7\)
    c) \(1.29, 1.3, 2.001\)

    When you expand each number so that each number has the same amount of digits, it is easier to distinguish the smallest/greatest decimals. (1.290, 1.300, 2.001)

12. \(\frac{1}{2} - C\)
    \(\frac{1}{3} - B\)
    \(\frac{3}{4} - D\)
    \(\frac{1}{10} - A\)

13. Yes; it is the product rounded to the nearest unit.

14. Multiplying by 10 moves the decimal one place to the right.

15. \(\frac{1}{4}\) hours = 75 minutes

AP Book NS6-94
page 241
1. To convert 5.47 m into centimetres, multiple 5.47 by 100 (547).

2. Canadian currency requires decimal notation because, not every monetary value is a whole number.
   A dime is a tenth of a dollar;
   A penny is a hundredth of a dollar.

3. 15.03 km/h
4. 2.1 m

AP Book NS6-95
page 242
1. a) 51
   b) \$19.80
   c) 225
   d) \$0.99
   e) \$3
   f) \$0.82

2. a) 15
   b) 59
   c) \$0.33

3. a) 3.7 cm; 1.85 m
   b) 3.4 cm; 1.70 m
   c) 7.3 cm; 3.65 m

4. \$15/hr
5. \$15/hr

AP Book NS6-96
page 243
1. a) 2:6
   b) \(\frac{1}{2}\)
   c) \(\frac{3}{3}\)
   d) \(\frac{3}{6}\)
   e) \(\frac{3}{2}\)
   f) \(\frac{3}{15}\)

2. a) \(\frac{1}{2}\)
   b) \(\frac{3}{3}\)
   c) \(\frac{3}{3}\)
   d) \(\frac{2}{2}\)

3. a) \(\frac{3}{6}\)
   b) \(\frac{2}{6}\)
   c) \(\frac{4}{1}\)
   d) \(\frac{4}{2}\)
   e) \(\frac{3}{5}\)
   f) \(\frac{6}{1}\)

4. 4:9

5. a) ratio of triangles to circles
   b) ratio of squares to all figures

6. Teacher to check.

AP Book NS6-97
page 244
1. a) 6 apples, 4 bananas;
   b) 10 apples, 8 bananas;
   c) 5 apples, 4 bananas

2. Triangles △△△△
   Squares □□□□□□□□
   Ratio 4:6, 6:9, 8:12

3. If going in numerical order:
   a) \(\frac{3}{4} = 6:8 = 9:12 = 12:16 = 15:20\)
   b) \(\frac{2}{5} = 4:10 = 6:15 = 8:20 = 10:25\)

4. a) 6
   b) 14
   c) 10

5. a) \(\frac{5}{3} = 10:6 = 15:9 = 20:12\)
   20 cups of oats
   b) \(\frac{2}{11} = 4:22 = 6:33 = 8:44 = 44\text{ km}\)
   c) \(\frac{6}{5} = 12:10 = 18:15 = 24:20 = 30:25\)

   \$15

AP Book NS6-98
page 245
1. a) \(\frac{5}{4} = 10:8 = 15:12\)
   15 + 12 = 27
   12 girls
   b) \(\frac{3}{5} = 6:10 = 9:15\)
   9 + 15 = 24
   15 blue fish
   c) \(\frac{3}{2} = 6:4 = 9:6\)
   9 + 6 = 15
   9 L of orange juice
**Number Sense – AP Book 6.2 (continued)**

2. a) \(\frac{15}{20}\)  
   b) \(\frac{5}{25}\)  
   c) \(\frac{8}{20}\)  
   d) \(\frac{30}{35}\)  
   e) \(\frac{12}{16}\)  
   f) \(\frac{8}{12}\)  
   g) \(\frac{60}{100}\)  
   h) \(\frac{25}{45}\)  

**BONUS:**  
3. a) \(\frac{15}{20}\)  
   b) \(\frac{10}{25}\)  
   c) \(\frac{9}{18}\)  
   d) \(\frac{12}{24}\)  
   f) \(\frac{6}{12}\)  
   g) \(\frac{18}{36}\)  
   h) \(\frac{14}{28}\)  

---

**AP Book NS6-99**  
**page 246**  
1. 6 apples  
2. 15 bus tickets  
3. 10 games  
4. 6 L of pineapple juice  
5. 9 laps  
6. 25 girls  
7. 15 km  

**AP Book NS6-100**  
**page 247**  
1. a) 1:4  
   b) 1:8  
   c) 1:9  
   d) 1:6  
   e) 3:10  
   f) 17:3  
2. a) 24 km  
   b) 8 days  

---

**AP Book NS6-101**  
**page 248**  
1. a) \(\frac{7}{100}\)  
   b) \(\frac{92}{100}\)  
   c) \(\frac{5}{100}\)  
   d) \(\frac{15}{100}\)  
   e) \(\frac{50}{100}\)  
   f) \(\frac{100}{100}\)  
   g) \(\frac{2}{100}\)  
   h) \(\frac{7}{100}\)  
2. a) 2%  
   b) 31%  
   c) 52%  
   d) 100%  
   e) 17%  
   f) 88%  
   g) 2%  
   h) 1%  
3. a) \(\frac{72}{100} = 72\%\)  
   b) \(\frac{27}{100} = 27\%\)  
   c) \(\frac{4}{100} = 4\%\)  
4. a) \(\frac{80}{100} = 60\%\)  
   b) \(\frac{40}{100} = 40\%\)  
   c) \(\frac{80}{100} = 80\%\)  

---

**AP Book NS6-102**  
**page 250**  
1. Teacher to check drawings.  
2. \(\frac{24}{100} = 0.24\), 24%  
3. \(\frac{63}{100} = 0.63\)  
4. \(0.45, 45\%\)  
5. \(\frac{81}{100} = 81\%\)  
6. Teacher to check.  
7. Teacher to check.  
8. Teacher to check.  
9. Teacher to check.  
10. Estimates may vary.  
   a) 63%  
   b) 30%  
11. Teacher to check.  
12. ~5%  

**AP Book NS6-103**  
**page 252**  
1. a) 50%  
   b) 75%  
   c) 50%  
   d) 25%  
   e) 10%  
   f) 50%  
   g) 100%  
   h) 10%  
   i) 50%  
   j) 50%  
   k) 75%  
   l) 100%  
   m) 10%  
2. a) \(\frac{50}{100} = 0.5\) > \(\frac{47}{100}\)  
   b) \(\frac{50}{100} < \frac{53}{100}\)
c) \[ \frac{25}{100} = \frac{23}{100} \]
d) \[ \frac{75}{100} = \frac{70}{100} \]
e) \[ \frac{40}{100} = \frac{32}{100} \]
f) \[ \frac{27}{100} < \frac{62}{100} \]
g) \[ \frac{2}{100} < \frac{11}{100} \]
h) \[ \frac{10}{100} = \frac{10}{100} \]
i) \[ \frac{76}{100} < \frac{93}{100} \]
j) \[ \frac{46}{100} > \frac{46}{100} \]
k) \[ \frac{90}{100} > \frac{10}{100} \]
l) \[ \frac{56}{100} = \frac{19}{100} \]

3. a) \[ \frac{42}{100} = \frac{3}{5} > \frac{73}{100} \]
b) \[ \frac{1}{2} > \frac{73}{100} \]
c) \[ \frac{9}{100} < \frac{15}{100} < \frac{25}{100} \]
d) \[ \frac{57}{100} > \frac{62}{100} > \frac{2}{3} \]

AP Book NS6-104

page 253

1. a) .4
   
   b) .7
   
   c) 3.2
   
   d) 12
   
   e) .38
   
   f) .25
   
   2. a) .9
   
   b) .57
   
   c) .405
   
   d) .635
   
   e) .006
   
   f) 2.11
   
   3. a) i) 1.5
        
        ii) 1.5; 6
        
   b) i) 25; 2.5
        
        ii) 6; 2.5; 15
        
   c) i) 2.3; .23
        
        ii) 9; 23; 2.07
        
   d) i) 35; 3.5
        
        ii) 6; 3.5; 21
        
   e) i) 24; 2.4
        
        ii) 4; 2.4; 9.6
        
   f) i) 1.3; .13

AP Book NS6-105

page 254

1. a) \[
\begin{array}{c}
1 \\
3 \\
2 \\
\end{array}
\times
\begin{array}{c}
4 \\
5 \\
1 \\
6 \\
0 \\
1 \\
2 \\
8 \\
0 \\
1 \\
4 \\
4 \\
0 \\
\end{array}
\]

\[ 1440 ÷ 100 = 14.4 \]

So 45% of 32 is 14.4.

b) \[
\begin{array}{c}
2 \\
6 \\
3 \\
\end{array}
\times
\begin{array}{c}
2 \\
8 \\
5 \\
0 \\
1 \\
2 \\
6 \\
0 \\
\end{array}
\]

\[ 1764 ÷ 100 = 17.64 \]

So 28% of 63 is 17.64.

2. a) 1.17
   
   b) 3.64
   
   c) 5.2
   
   d) 7.02
   
   e) 9.66
   
   f) 11.56
   
   g) 29.6
   
   h) 46.5
   
   3. a) 20
   
   b) 70
   
   c) 3
   
   d) 15
   
   e) 240

AP Book NS6-106

page 255

1. a) 40%, 50%, 10%
   
   b) 80%, 10%, 10%
   
   c) 50%, 40%, 10%
   
   d) 22%, 60%, 18%
   
   e) 75%, 15%, 10%
   
   f) 75%, 10%, 15%
   
   2. Brushes: \[ \frac{1}{10} \cdot 10\% \]
   
   Paint: 40%, $200
   
   Canvas: \[ \frac{5}{10} \text{ or } \frac{1}{2} \cdot \frac{250}{100} \]

AP Book NS6-107

page 256

1. a) City A:

\[ 20, 40, 30, 10, 100 \]

City B:

\[ 38, 19, 34, 9, 100 \]

b) 100% - this makes sense because the sum of the % of the answers must add to a whole = 100%

2. The total sum of percentages does not equal 100%. (104%)

3. a) Calli – 200 students
   
   Bilal - 50 students
   
   b) Calli: \[ \frac{10}{200} = 5\% \]
   
   \[ \frac{20}{200} = 10\% \]
   
   \[ \frac{140}{200} = 70\% \]
   
   \[ \frac{30}{200} = 15\% \]

   Bilal: \[ \frac{20}{50} = 40\% \]
   
   \[ \frac{15}{50} = 30\% \]
   
   \[ \frac{5}{50} = 10\% \]
   
   \[ \frac{10}{50} = 20\% \]

   c) Teacher to check.

AP Book NS6-108

page 258

1. a) 8, 5, 13
   
   b) 4, 7, 11
   
   c) 12, 15, 27
   
   d) 11, 9, 20
   
   e) 7, 3, 10
   
   2. a) \[ \frac{5}{12} \text{ boys} \]
   
   \[ \frac{11}{12} \text{ girls} \]
   
   b) \[ \frac{3}{4} \text{ boys} \]
   
   \[ \frac{7}{4} \text{ girls} \]
   
   c) \[ \frac{11}{20} \text{ boys} \]
   
   \[ \frac{9}{20} \text{ girls} \]
   
   d) \[ \frac{5}{14} \text{ boys} \]
   
   \[ \frac{9}{14} \text{ girls} \]
   
   e) \[ \frac{8}{15} \text{ boys} \]
   
   \[ \frac{7}{15} \text{ girls} \]
   
   f) \[ \frac{10}{21} \text{ boys} \]
   
   \[ \frac{11}{21} \text{ girls} \]
   
   3. a) 8 boys, 12 girls
   
   b) 24 boys, 18 girls
   
   c) 6 boys, 9 girls
   
   d) 15 boys, 9 girls
   
   5. a) 10 boys, 15 girls
   
   b) 12 boys, 16 girls
   
   c) 10 boys, 20 girls
   
   6. a) A (24 > 20)
   
   b) B (18 > 8)
   
   7. a) 4.6 : 2:3
   
   b) \[ \frac{4}{10} = \frac{2}{5} \]
   
   c) 60%
8. Answers may vary.
   Dog - $\frac{40}{100} \cdot 40\%$
   Cat - $\frac{27}{100} \cdot 27\%$
   Bird - $\frac{23}{100} \cdot 23\%$
   Other - $\frac{10}{100} \cdot 10\%$

9. $\frac{1}{20} \cdot 0.2, 20\%$ (0.2 and 20\% are equivalent, so the order in the sequence can be reversed).

10. Maple Leafs: 108 cards
    Canadiens: 180 cards
    Canucks: 72 cards

11. 37 cm is 37\% of a metre, since $37 \text{ cm} = \frac{37}{100} \text{ m} = 37\%$

---

**Answers Key for AP Book 6.2**

---

12. a) $24, 23$
    b) $57$

13. 0.77 m

14. $2.46$

---

15. Chair: $\frac{3}{10}, 30\%, $150
    Table: $\frac{1}{10}, 10\%, $50
    Sofa: $\frac{3}{5}, 60\%, $300
### Measurement – AP Book 6.2

**AP Book ME6-8**

**page 266**

1. a) 5 fingers: 50 mm
   b) 7 fingers: 70 mm
2. a) 38 mm
   b) 47 mm
3. a) Teacher to check.
   b) Teacher to check.
4. a) i) Same length
   ii) Same length
   b) i) 30 mm
   ii) 20 mm
5. Estimates will vary – teacher to check.
   Actual Lengths:
   a) 38 mm
   b) 15 mm
   c) 60 mm
6. Diagonal: 5.2 cm, 52 mm
   Sides: 5 cm, 1.5 cm
7. 10
8. 10
9. | mm | cm |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>13</td>
</tr>
<tr>
<td>320</td>
<td>32</td>
</tr>
</tbody>
</table>
   a) 5
   b) 8
   c) 320
   d) 43
   e) 46 cm
   f) 6 cm
   g) 58 cm
11. a) 40 mm
   b) 180 mm
   c) 13 cm
12. a) 70 mm
   b) 910 mm
   c) 45 cm
   d) 2 cm
   e) 6 200 mm
   f) 72 cm
13. Teacher to check lines:
   a) 30 mm
   b) 8 cm
   c) 70 mm
14. Teacher to check.
15. Teacher to check.
16. Teacher to check.
17. Teacher to check.
18. No, Rebecca is incorrect. When 3 cm is converted in millimetres it becomes 30 mm, 30 mm is greater than 7 mm.
19. a) 4 cm + 4 cm
   b) 7 cm + 4 cm
   c) 7 cm + 7 cm + 4 cm + 4 cm
   d) 7 cm + 7 cm + 4 cm + 4 cm + 4 cm
   e) 7 cm + 7 cm + 7 cm + 7 cm + 4 cm
20. Possible solutions:
   a) 7 cm – 4 cm
   b) 4 cm + 4 cm – 7 cm
   c) 7 cm + 7 cm – 4 cm + 4 cm + 4 cm (14 – 12)
   d) 7 cm + 7 cm + 7 cm – 4 cm + 4 cm + 4 cm (21 – 16)
   e) 7 cm + 7 cm + 7 cm – 4 cm (21 – 4)
   BONUS:
   f) Yes.
   Teacher to check.

**AP Book ME6-9**

**page 269**

1. 10 cm
   2. \( \frac{1}{10} \)
   3. 10
   4. 10
   5. 10
   6. | cm | dm |
      |---|---|
      | 120 | 12 |
      | 310 | 31 |
      | 420 | 42 |
   7. Teacher to check.
   8. Teacher to check.
   9. Teacher to check.
10. 10

**AP Book ME6-10**

**page 270**

1. Answers will vary.
2. About 28 m
3. Teacher to check.
4. Teacher to check.
5. Answers will vary.
6. | cm | dm |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>8</td>
</tr>
<tr>
<td>6 200</td>
<td>620</td>
</tr>
<tr>
<td>300</td>
<td>30</td>
</tr>
</tbody>
</table>
   7. Teacher to check.
   8. Teacher to check.
   9. Teacher to check.
10. 10
11. 100

**AP Book ME6-11**

**page 271**

1. Answers listed as columns:
   - 3 m, 30 dm, 300 cm, 3 000 mm
   - 4 m, 40 dm, 400 cm, 4 000 mm
   - 5 m, 50 dm, 500 cm, 5 000 mm
2. 6 m, 60 dm, 600 dm, 6 000 mm.
3. 100
4. | m | cm |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>800</td>
</tr>
<tr>
<td>70</td>
<td>7 000</td>
</tr>
<tr>
<td>m</td>
<td>mm</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>5 000</td>
</tr>
<tr>
<td>17</td>
<td>17 000</td>
</tr>
</tbody>
</table>
   5. a) 4 m 23 cm
   b) 5 m 14 cm
   c) 6 m 27 cm
   d) 6 m 73 cm
   e) 3 m 81 cm
   f) 2 m 3 cm
6. a) 283 cm
   b) 365 cm
   c) 485 cm
   d) 947 cm
   e) 704 cm
   f) 640 cm
7. a) 546 cm
   \( = 5 \text{ m } 46 \text{ cm} \)
   \( = 5.46 \text{ m} \)
   b) 2 m 17 cm
   \( = 2.17 \text{ m} \)
   c) 7 m 83 cm
   \( = 7.83 \text{ m} \)
   d) 6 m 8 cm
   \( = 6.08 \text{ m} \)
   e) 72 cm = 0.72 m
   f) 7 cm = 0.07 m
8. 100 cents = 1 dollar
   and 100 cm = 1 m
9. Yes, Michelle is correct.
   She multiplies 6 m by 100 in order to convert the measure to cm (since there are 100 cm in a metre).
1. a) 100 mm = 10 cm = 1 dm
   Largest = dm
   Smallest = mm
b) smaller
c) more

2. a) 10 mm
   b) 10 cm
   c) 100 mm
   d) 10 dm
e) 100 cm
f) 1 000 mm

3. a) i) 10 times smaller
   ii) 10 times more
   iii) Divide by 10
   3.5 = 35 mm
b) i) 10 times smaller
   ii) 10 times more
   iii) Divide by 10
   3.5 = 35 mm
c) i) 10 times smaller
   ii) 10 times more
   iii) Divide by 10
   7.2 = 72 mm
d) i) 10 times smaller
   ii) 10 times more
   iii) Divide by 10
   2.6 dm = 26 cm
e) i) 10 times larger
   ii) 10 times more
   iii) Divide by 10
   7.53 cm = 75.3 mm
e) i) 10 times larger
   ii) 10 times fewer

4. a) 40 dm
   b) 130 mm
c) 200 mm

5. 45¢

6. Since the total weight of Emily's book is 3.703 kg and is less than her maximum weight of 4 kg, she can carry all her books in her backpack.

7. The perimeter is 244 cm (2.44 m), and is greater than 2.4 m.

8. Both relations are 1 000 times larger units
   (1 kg = 1000 g; 1 km = 1000 m).

9. Both relations are 1 000 times smaller in units
   (1 mg = 0.001 g; 1 mm = 0.001 m).

   a) mm = millimetre
   → length of bee's antenna
   cm = centimetre
   → diameter of a drum
   m = metre
   → width of a swimming pool
   km = kilometre
   → distance of a marathon
   b) km = kilometre
   → diameter of the moon
   cm = centimetre
   → length of a ruler
   m = metre
   → length of a soccer field
   mm = millimetre
   → thickness of a nail

2. a) m
   b) dm
c) cm

3. a) cm
   b) m
   c) cm
d) cm
e) m

4. a) km
   b) m
c) m
d) cm
e) km
f) m
g) cm

5. 1. Western Red Cedar
   – 5 900 cm
   2. Lodgepole Pine
   – 3 050 cm
   3. Red Oak – 2 400 cm
   4. White Birch
   – 2 000 cm

   a) 47 mm, 329 m
   b) 33 mm, 231 m
c) 28 mm, 196 m
d) 19 mm, 133 m
e) 16 mm, 112 m
f) 11.5 mm, 80.5 m
g) 8 mm, 56 m
h) 7 mm, 49 m
i) 6.5 mm, 45.5 m
j) 8 mm, 56 m

2. a) About 500 m
   b) About 10

3. 42 mm

4. a) Since 2 buildings of the Wall Centre has 96 floors and is still shorter than the First Canadian Place, the 76 floors in the First Canadian Place are greater in height than two times the floors in the Wall Centre.
   
   b) Wall Centre:
   \[ 19 \text{ mm} \times 7 \text{ m/mm} = 133 \text{ m} \]
   \[ 133 \text{ m} \div 48 \text{ floors} = 2.77 \text{ m/floor} \]
   FCP:
   \[ 47 \text{ mm} \times 7 \text{ m/mm} = 329 \text{ m} \]
   \[ 329 \text{ m} \div 72 \text{ floors} = 4.57 \text{ m/floor} \]

AP Book ME6-16

NOTE:
Answers may vary, depending on how students place their rulers. The following distances assume that students measure from (and to) the middle of each lock.

1. Lock 1 and Lock 2
   Distance on map = 13 mm
   Actual distance = 13 × 200
   = 2600 m = 2.6 km

2. Lock 2 and Lock 3
   Distance on map = 20 mm
   Actual distance = 20 × 200
   = 4000 m = 4 km

3. Lock 7 and Lock 8
   Distance on map = 128 mm
   Actual distance = 128 × 200
   = 25600 m = 25.6 km

   2. At 10 km/h, the boat would travel 20 km in two hours and would not reach Lock 8.

   3. Time spent stopped
      = 8 locks × 0.5 h/lock
      = 4 hours
      Time spent travelling
      = 44 km ÷ 10 km/h
      = 4.4 hours
      Total time spent
      = 8 hours and 24 minutes

4. Teacher to check line.
   Line should be placed 9 mm south of Lock 2

5. Twice the length of the Welland Canal
   = 2 × 44 km
   = 88 km
   Rounding both lengths (the Welland Canal to 90 km and the Grand Canal to 1800 km), we see that you would have to travel up and down the Welland Canal 20 times (1800 + 90) to travel the distance of the Grand Canal.

AP Book ME6-17

NOTE:
Estimations will vary.

1. a) 528 cents
   b) 714 cents
   c) 1003 cents
   d) 408 cents

2. a) 602 cm
   b) 409 cm
   c) 613 cm
   d) 1153 cm
   e) 1420 cm
   f) 301 cm

3. a) 9002 m

AP Book ME6-18

NOTE:
Answers will vary.

1. a) 10 cm
   b) 8 cm
   c) 14 cm

2. a) 183 min
   b) 124 min
   c) 191 min
   d) 250 m
   e) 232 m
   f) 326 min

AP Book ME6-20

NOTE:
Answers will vary.

1. a) 14 units
   b) 20 units
   c) 28 units

2. a) Original P = 10 units
    New P = 12 units
   
   b) Input × 2 + 6
   c) 26

3. Answers will vary.

AP Book ME6-21

NOTE:
Answers will vary.

1. a) Original P = 14 units
    New P = 12 units

2. a) 12 × 1; 2 × 6;
   3 × 4
   c) Yes; 1 × 7
   d) 12 × 1; 26 units

3. Answers will vary.
### AP Book ME6-21

**Page 283**

1. A: 10.8 cm  
   B: 8.8 cm  
   C: 15.6 cm  
   D: 15.6 cm  
   E: 16.8 cm  
   F: 19.2 cm  
   \[ P = 12 \text{ units} \]

<table>
<thead>
<tr>
<th>W</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ P = 16 \text{ units} \]

<table>
<thead>
<tr>
<th>W</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ P = 18 \text{ units} \]

2. a) \[ \begin{array}{c|c} W & C \\ \hline 2 & 6 \\ 3 & 9 \\ 4 & 12 \\ \end{array} \]

b) The circumference is about 3 times greater than the width.

### AP Book ME6-22

**Page 284**

1. a) 8 cm²  
   b) 8 cm²  
   c) 9 cm²  
   d) 4 cm²  

2. a) 8 cm²  
   b) 3 cm²  
   c) 4 cm²  

3. To find the area, extend the lines through the shape.
   - Area of A = 6 cm²
   - Area of B = 4 cm²
   - Area of C = 8 cm²

4. Answers will vary – teacher to check.
5. Answers will vary – teacher to check.
6. 3 cm × 4 cm rectangle:

   OR
   4 cm × 3 cm rectangle:

### Answer Key for AP Book 6.2

- **Page 285**

   1. a) 4 × 3 = 12
      b) 2 × 3 = 6
      c) 3 × 2 = 6
      d) 5 × 3 = 15

   2. a) 3 × 7 = 21
      b) 3 × 4 = 12
      c) 3 × 2 = 6
      d) 4 × 6 = 24

   3. a) Length = 6 units  
      Width = 2 units  
      \[ 6 × 2 = 12 \]
      b) Length = 3 units  
      Width = 2 units  
      \[ 3 × 2 = 6 \]
      c) Length = 4 units  
      Width = 3 units  
      \[ 4 × 3 = 12 \]

### Page 287

1. Shape | P | A |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12 cm</td>
<td>8 cm²</td>
</tr>
<tr>
<td>B</td>
<td>22 cm</td>
<td>30 cm²</td>
</tr>
<tr>
<td>C</td>
<td>22 cm</td>
<td>18 cm²</td>
</tr>
<tr>
<td>D</td>
<td>20 cm</td>
<td>21 cm²</td>
</tr>
<tr>
<td>E</td>
<td>26 cm</td>
<td>30 cm²</td>
</tr>
<tr>
<td>F</td>
<td>14 cm</td>
<td>10 cm²</td>
</tr>
<tr>
<td>G</td>
<td>22 cm</td>
<td>10 cm²</td>
</tr>
</tbody>
</table>

2. No
3. D & G
4. E – 26 cm  
   B, C & G – 22 cm  
   D – 20 cm  
   F – 14 cm  
   A – 12 cm
5. E & B – 30 cm²  
   D – 21 cm²  
   C – 18 cm²  
   F & G – 10 cm²  
   A – 8 cm²  

6. No  

7. Perimeter is the measure of the length along the outside edge of a shape. Area is the measure of the space contained within the edges of a shape.

AP Book ME6-26  
page 288

1. This table gives the shapes’ actual measurements only:  

<table>
<thead>
<tr>
<th>R</th>
<th>P</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14 cm</td>
<td>10 cm²</td>
</tr>
<tr>
<td>B</td>
<td>16 cm</td>
<td>12 cm²</td>
</tr>
<tr>
<td>C</td>
<td>16 cm</td>
<td>15 cm²</td>
</tr>
<tr>
<td>D</td>
<td>10 cm</td>
<td>6 cm²</td>
</tr>
<tr>
<td>E</td>
<td>18 cm</td>
<td>14 cm²</td>
</tr>
<tr>
<td>F</td>
<td>12 cm</td>
<td>8 cm²</td>
</tr>
<tr>
<td>G</td>
<td>14 cm</td>
<td>12 cm²</td>
</tr>
</tbody>
</table>

2. a) P = 14 cm; A = 10 cm²  
   b) P = 8 cm; A = 3 cm²  
   c) P = 10 cm; A = 6 cm²  

3. a) More: shaded = 7 and 7 > 5  
   b) Equal: shaded = 4 and unshaded = 4  
   c) Less: shaded = 3 and 3 < 4  

4. a) 1 \( \frac{1}{2} \)  
   b) 4 square units  
   c) 2 square units  

5. a) 1 square units  
   b) 3 square units  
   c) 3 square units  
   d) 5 square units  

6. a) Triangle: 3 square units  
   b) Rectangle: 2 square units  

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NOTE:  
For estimation questions, consider the diagonal of the square unit equals 1.5 cm.  

1. a) 3 whole squares  
   b) 2 whole squares  
   c) 3 whole squares  
   d) 3 whole squares  
   e) 8 whole squares  
   f) 8 whole squares  
   g) 4 whole squares  
   h) 5 whole squares  
   i) 11 whole squares  
   j) 10 whole squares  
   k) 10 whole squares  

2. a) P = 7.5 square units  
   b) A = 2 square units  

3. a) 7 half squares = 3.5 total squares  
   b) 9 half squares = 4.5 total squares  
   c) 14 half squares = 7 total squares  

4. a) Area = 3 + 3 = 6  
   b) Area = 3 + 4 = 7  
   c) 4 half squares  
   d) 2 half squares  
   e) 6 full squares  
   f) 7 total squares  

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1. a) 20 square units  
   b) 48 – 20 Shaded = 28 Unshaded  
   c) 600 cm²  
   d) 30 cm  
   e) 40 berries  
   f) $10.00 – $5.98 = $4.02  

3. a) 30 : 5  
   b) 60 : 10  
   c) 36 : 6  
   d) 48 : 8  

4. Patti is incorrect because 100 cm = 1 m, then 100 cm × 100 cm = 10 000 cm² and 1 m × 1 m = 1 m².

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1. a) 10 000 cm²  
   b) 10 000 times smaller;  
   c) 10 000 times more;  
   d) 146 500 cm²  
   e) 146 500 cm²  
   f) 100 cm²  

2. a) 3 760 cm²  
   b) 72 000 cm²  

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1. a) Yes. The part of the shape that was taken away from the first parallelogram, is the exact same that was added to the second parallelogram.  
   b) Both have a base of 4  
   c) Both have a height of 5  
   d) Area = B × H  

2. a) 4 × 3 = 12 cm²
b) \(7 \times 2 = 14\) cm

3. a) 35 cm²

b) 12 cm²
c) 48 cm²
d) 22.2 cm²

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page 295

1. a) Teacher to check that lines are drawn correctly.

<table>
<thead>
<tr>
<th>Base</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
</tr>
</tbody>
</table>

b) A:

Area of 1st triangle
\(= (2 \times 4) ÷ 2\)
\(= 4\) cm²

Area of 2nd triangle
\(= (1 \times 4) ÷ 2\)
\(= 2\) cm²

Total Area of D
\(= 4\) cm² + 2 cm²
\(= 6\) cm²

2. Area of Triangle A = Area of Parallelogram B ÷ 2

3. Teacher to check drawing.

Area of parallelogram
\(= 4 \times 4 = 16\) units²

The area of the triangle is
\(8\) cm² (half of 16, or 8 units²).

4. \(A = (\text{Base} \times \text{Height}) ÷ 2\)

5. Area of Triangle A
\(= (4 \times 4) ÷ 2\)
\(= 16 ÷ 2\)
\(= 8\) cm²

AP Book ME6-32

page 296

1. a) 6 cm²

b) 6 cm²
c) 12 cm²
d) 12.8 cm²

2. a) 35 cm²

b) 170 cm²
c) 31.5 cm²
d) 22 cm²

3. a) 6 cm²

b) 4 cm²
c) 6 cm²

4. A
\(= 9\) units²

B
\(= 18\) units²

C
\(= 24\) units²

D
\(= 20\) units²

5. Area = 44 units²

6. a) \(P = 24\) units

\(A = 28\) units²

b) \(P = 34\) units

\(A = 57\) units²

7. a) Teacher to check drawings.

Each shape should have a height of two square units (double the height).

b)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area</th>
<th>New Shape Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>.5</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

c) The area is quadrupled (multiplied by 4).

8. 5 cm; 20 cm

9. 2 cm; 16 cm

10. 6 cm

11. 2 times

12. Teacher to check.

13. A - 150 cm²

B - 200 cm²

C - 225 cm²

14. Less (P = 10 cm)

15. 16 m². The triangle has the same base and height as the parallelogram. Therefore the area is half because when calculating the area of a triangle you divide by 2 \((A = B \times H ÷ 2)\).

16. \(1\frac{1}{2}\) m². See Q.16 for explanation

17. 16 m. The shaded triangle is half of the square, therefore the area of the square is 16 m². Because the shape is a square, we know all sides must be even \((16 = 4 \times 4)\). When all 4 sides are added, the result is 16 cm.

18. 4 cm. You can check your answer by inserting the numbers into the formula \((A = B \times H ÷ 2)\).

20. Teacher to check.
c) Answers will vary – roughly 20 (1 000 kg = 50 kg \times 20)

4. No, the total mass is about 1 250 kg or 1.25 tonnes and exceeds the 1 tonne limit.

5. 5 000 kg = 5 tonnes
   More. An elephant would eat about 1 050 kg in one week.

6. Yes, Merinda family’s furniture weighs 1 000 kg, and is less than the maximum weight of the truck.
1. a) Heads or tails
   b) 1, 2, 3, 4, 5 or 6
   c) Win or lose
2. a) 6
   b) 2
   c) 3
3. a) You spin a 1, 2, 3 or 4; 4 outcomes
   b) You spin a 6; 1 outcome
   c) You spin a 2, 3 or 4; 3 outcomes
   d) You spin a 6 or 9; 2 outcomes
4. a) 3
   b) 4
5. a) 2, 4, 12
   b) 1, 3, 5, 7, 9
   c) 12
6. a) 3 pieces shaded; 4 pieces
   b) 3 pieces shaded; 6 pieces
   c) 4 pieces shaded; 6 pieces
   d) 4 pieces shaded; 8 pieces
   e) 5 pieces shaded; 8 pieces
2. Circle figures b), c), d), e)
   Cross out figures a), f)
3. a) 5
   b) 12
   c) 24
   d) 26
4. a) \( \frac{1}{2} \)
   b) 10
5. Flipping a coin has 2 outcomes, heads or tails, so you would expect to flip heads half the time.
   Since the coin was flipped 40 times, you would expect to flip heads 20 \( (40 \div 2) \) times.
6. a) 14
   b) 25
   c) 13
   d) 21
7. a) 5
   b) 12
   c) 22
8. a) 4
   b) 11
   c) 24
9. The answers below give the number of marbles of each colour.
   a) Red: 3 Green: 3
   b) Red: 6 Green: 3
   c) Red: 4 Green: 6
   d) Red: 12 Green: 4
   e) Red: 8 Green: 4
   f) Red: 6 Green: 2
10. Teacher to check spinner.
   Red needs to be on 3 of 4 sections of the spinner.
11. The probability of spinning a colour that is not yellow is \( \frac{1}{3} \).
   Teacher to check picture.
12. \( \frac{20}{50} = \frac{2}{5} \): You could expect to spin blue 20 times.

13. \( \frac{75}{100} = \frac{3}{4} \): You could expect to spin yellow 75 times.

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**page 307**

1. a) unlikely  
   b) likely  
   c) unlikely  
   d) likely  
2. a) likely  
   b) unlikely  
3. a) unlikely  
   b) impossible  
   c) likely  
   d) unlikely  
4. A. the event is impossible  
   B. the event is certain  
   C. the event is unlikely  
   D. the event is likely  
5. Teacher to check probability line.  
   A. certain  
   B. unlikely  
   C. even  
   D. likely  
6. Teacher to check.  
7. green  
   yellow, blue  
   unlikely  
   even  
8. Since the space taken up by red is greater than the space taken up by blue or green, the outcome on the spinner is not equally likely.

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**page 309**

1. a) 5 times  
   b) Since the outcome for spinning red is equal to the outcome of spinning yellow, the chance of spinning red and yellow are equally likely and the game is fair. Therefore, Daniel is not right.  
   c) a multiple of 3 needs to be on two sections  
   d) 2 needs to be on three sections  
2. a) 10 times  
   b) Chart B  
3. a) 6 times  
   b) Chart A  
   c) Chart C  
4. a) 10 times, \( \frac{3}{5} \times \frac{10}{30} \)  
   b) Teacher to check.  
5. Since rolling a die will have the possible outcomes of the numbers 1, 2, 3, 4, 5 or 6, the fraction of the time you would expect to roll the number 6 is 1 out of 6, or one-sixth (\( \frac{1}{6} \)) of the time.  
6. a) $5 and $10  
   or $5 and $20  
   or $10 and $20  
   b) Since the combination of one $10 and one $20 bill is the only combination that adds up to $30, you would expect to pull out this combination only 1 out of 3 times. Since this chance is less than half, the chances are considered to be "unlikely."  
   c) Teacher to check.  
7. Teacher to check spinners.  
   a) 3 needs to be on one section  
   b) an even number needs to be on five sections  
   c) Teacher to check.  
8. Teacher to check spinners.  
   a) 3 needs to be on three of the sections  
   b) an even number needs to be on 5 of the sections  
   c) a multiple of 3 needs to be on two sections  
   d) 2 needs to be on three of the sections.

**AP Book PDM6-28**  
**page 313**

1. From left to right:  
   #1 – baseball, football  
   #2 – baseball, swimming  
   #3 – basketball, football  
   #4 – basketball, swimming  
2. H                   T  
   H    T            H      T  
3. cricket       rowing  
   tkd     judo            tkd     judo  
4. a) There are 4 paths through the maze:  
   Path #1 – UU  
   Path #2 – UD  
   Path #3 – DU  
   Path #4 – DD  
   b) The probability of meeting the dragon is unlikely: only \( \frac{1}{4} \).  
5. H T  
   R G Y     R  G  Y  
   HR  HG  HY   TR  TG  TY  
6. 2 3 4  
   1  2  1  2  1  2  
7. E               P  
   A   O   G    A   O   G  
   EA   EO   EG    PA   PO   PG  
8. 1 2 3  
   1   2   3  1  2  3  1   2  3  
   Only 2 of 9 combinations add to five: 2, 3 and 3, 2.
1. a) 2
   b) Abdul should write Blue twice on his list. Also, he should write Red twice on his list.
   c) 4

<table>
<thead>
<tr>
<th>1st sp.</th>
<th>2nd sp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>2</td>
</tr>
<tr>
<td>Blue</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
</tr>
<tr>
<td>Red</td>
<td>3</td>
</tr>
</tbody>
</table>

2. a) 4
   b) On his list, Abdul should write Yellow 4 times and Green 4 times.
   c) 8

<table>
<thead>
<tr>
<th>1st sp.</th>
<th>2nd sp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow</td>
<td>1</td>
</tr>
<tr>
<td>Yellow</td>
<td>2</td>
</tr>
<tr>
<td>Yellow</td>
<td>3</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
</tr>
<tr>
<td>Green</td>
<td>1</td>
</tr>
<tr>
<td>Green</td>
<td>2</td>
</tr>
<tr>
<td>Green</td>
<td>3</td>
</tr>
<tr>
<td>Green</td>
<td>4</td>
</tr>
</tbody>
</table>

3. a) 2
   b) On his list, Abdul should write Blue two times. He should also write Yellow 2 times.
   c) 4

<table>
<thead>
<tr>
<th>1st sp.</th>
<th>2nd sp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>1</td>
</tr>
<tr>
<td>Blue</td>
<td>2</td>
</tr>
<tr>
<td>Yellow</td>
<td>1</td>
</tr>
<tr>
<td>Yellow</td>
<td>2</td>
</tr>
</tbody>
</table>

4. a) 3
   b) On his list, Abdul should write Green, Blue and Yellow three times.
   c) 9

5. | Coin | Spinner |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td>R</td>
</tr>
<tr>
<td>Heads</td>
<td>G</td>
</tr>
<tr>
<td>Tails</td>
<td>R</td>
</tr>
<tr>
<td>Tails</td>
<td>G</td>
</tr>
</tbody>
</table>

6. | Right | Left | Value |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10¢</td>
<td>25¢</td>
<td>35¢</td>
</tr>
<tr>
<td>10¢</td>
<td>10¢</td>
<td>20¢</td>
</tr>
<tr>
<td>5¢</td>
<td>25¢</td>
<td>30¢</td>
</tr>
<tr>
<td>5¢</td>
<td>10¢</td>
<td>15¢</td>
</tr>
</tbody>
</table>

7. a) | AM | PM |
      |----|----|
      | Painting | Drama |
      | Painting | Pottery |
      | Painting | Dance  |
      | Music    | Drama  |
      | Music    | Pottery |
      | Music    | Dance  |

8. | AM | PM |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>Canoeing</td>
</tr>
<tr>
<td>Swimming</td>
<td>Baseball</td>
</tr>
<tr>
<td>Swimming</td>
<td>Hiking</td>
</tr>
<tr>
<td>Tennis</td>
<td>Canoeing</td>
</tr>
<tr>
<td>Tennis</td>
<td>Baseball</td>
</tr>
<tr>
<td>Tennis</td>
<td>Hiking</td>
</tr>
</tbody>
</table>

9. a) | 1st | 2nd | Total |
     |-----|-----|-------|
     | 2   | 2   | 4     |
     | 2   | 4   | 6     |
     | 2   | 6   | 8     |
     | 4   | 2   | 6     |
     | 4   | 4   | 8     |
     | 4   | 6   | 10    |
     | 6   | 2   | 8     |
     | 6   | 4   | 10    |
     | 6   | 6   | 12    |

4. a) | R  | L  | V  |
     |-----|----|----|
     | $5  | $5 | $10 |
     | $5  | $10| $15 |
     | $10 | $5 | $15 |
     | $10 | $5 | $20 |
     | $2  | $1 | $3 |

b) Yes, there are several combinations that give the same score.
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1. a) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
b) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
c) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
d) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
e) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
f) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
g) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3
h) $\begin{array}{c}
3 \bullet \\
2 \bullet \\
1 \circ \\
\end{array}$
1 2 3

2. a) $\begin{array}{c}
3 \circ \\
2 \circ \\
1 \circ \\
\end{array}$
A B C
b) $\begin{array}{c}
3 \circ \\
2 \circ \\
1 \circ \\
\end{array}$
A B C
c) $\begin{array}{c}
2 \circ \\
1 \circ \\
0 \circ \\
\end{array}$
0 1 2
d) $\begin{array}{c}
2 \circ \\
1 \circ \\
0 \circ \\
\end{array}$
0 1 2

3. $\begin{array}{c}
5 E \\
4 C \\
3 I \\
2 B \\
1 A \\
0 M \\
\end{array}$
0 1 2 3 4 5 6 7 8 9 10

4. a) $\begin{array}{c}
\bullet \\
\bullet \\
\circ \\
\circ \\
\end{array}$
1 2 3 4 5
b) $\begin{array}{c}
\circ \\
\bullet \\
\bullet \\
\circ \\
\end{array}$
1 2 3 4
c) $\begin{array}{c}
\bullet \\
\bullet \\
\circ \\
\circ \\
\end{array}$
1 2 3 4
d) rhombus

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page 321

1. a) 4 units right
b) 3 units left
c) 2 units right
2. a) 4 units right 2 units down
b) 4 units left 2 units up
c) 5 units right 1 unit down

3. a) $\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array}$
b) $\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array}$
c) $\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array}$
d) $\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array}$

5. a) $\begin{array}{c}
A' (4,5) \\
B' (4,7) \\
C' (6,7) \\
D' (6,5) \\
\end{array}$
b) $\begin{array}{c}
A' (7,4) \\
B' (6,2) \\
C' (9,1) \\
\end{array}$

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page 323

1. a) C
b) A
c) B
d) 1 unit left 2 units down
e) 4 units right 1 unit up
f) 1 unit right 4 units down

2. a) hill
b) cliff
c) (A,1)
d) 2 km west 1 km south
e) 4 km east 2 km north

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page 324

1. a) $\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}$

BONUS
10. Answers will vary – teacher to check.
2. a) 
   ![Diagram 2a]

   b) 
   ![Diagram 2b]

   c) 
   ![Diagram 2c]

   BONUS:

   3. Answers will vary – teacher to check.

4. a) 
   ![Diagram 4a]

   b) 
   ![Diagram 4b]

   c) 
   ![Diagram 4c]

   d) 
   ![Diagram 4d]

5. a) 
   ![Diagram 5a]

   b) 
   ![Diagram 5b]

   c) 
   ![Diagram 5c]

   d) 
   ![Diagram 5d]

6. a) 
   ![Diagram 6a]

   b) 
   ![Diagram 6b]

   c) 
   ![Diagram 6c]

7. a) 
   ![Diagram 7a]

   b) 
   ![Diagram 7b]

   BONUS:

   1. a) $\frac{1}{4}$

   b) $\frac{3}{4}$

   c) $\frac{1}{2}$

   d) $\frac{1}{4}$

2. a) $\frac{1}{4}$

   b) $\frac{3}{4}$

   c) $\frac{3}{4}$

   d) $\frac{1}{2}$

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1. a) 
   ![Diagram 1a]

   b) 
   ![Diagram 1b]

   c) 
   ![Diagram 1c]

   d) 
   ![Diagram 1d]

   e) 
   ![Diagram 1e]

   f) 
   ![Diagram 1f]

   g) 
   ![Diagram 1g]

   h) 
   ![Diagram 1h]

   BONUS:

   i) 
   ![Diagram Bi]

   j) 
   ![Diagram Bj]

   k) 
   ![Diagram Bk]
Answer Key for AP Book 6.2

3. Answers will vary – teacher to check.

4. Answers will vary – teacher to check.

5. The two shapes collectively, create a rhombus.

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page 328

1.

2.

3.

4.

5. a) 1: 180°
   2: 90°
   b) 1: 90°
   2: 180°
   c) 1: 180°
   2: 90°
6. a) 1: 180°
   2: R
   b) 1: 180°
   2: R
   c) 1: R
   2: 180°

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page 329

1. a)

2.

3.

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page 331

1. a) (i) slide
   (ii) reflection
   (iii) slide
   b) Yes
   c) Reflect Shape C across the Line AC, the slide down.
2. Slide and reflection
3. A: reflection
   B: 90° rotation counter clockwise
   C: slide 5 units to the left, or reflection
4. Teacher to check predictions.
   - The letter will be identical to the original letter
5. a) D & F
   C & J
   E & G
   B & H
   A & I
   b) F & D
   B & H
   c) (5,2), (8,2), (8,4)
   (4,2), (7,2)
   (6,3), (5,3)
   (4,2), (7,2)
   (6,3), (5,3)

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1. Teacher to check shape construction.

<table>
<thead>
<tr>
<th>Base</th>
<th>#S</th>
<th>#E</th>
<th>#V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.P.</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>S.P.</td>
<td>4</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>P.P.</td>
<td>5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>H.P.</td>
<td>6</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

2. # sides increases by one each time
   # edges increases by two each time
   # vertices increases by one each time

See Question 1 for answer, re: hexagonal pyramid.

3. Answers may vary – teacher to check.
   Possible answer:
   The number of edges of a pyramid is a double the number of sides of its base. The number of vertices of a pyramid is one more than the number of sides of its base.

4. An octagonal pyramid would have 16 edges and 9 vertices.

AP Book G6-34
page 333

1. Teacher to check shape construction.

<table>
<thead>
<tr>
<th>Base</th>
<th>#S</th>
<th>#E</th>
<th>#V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.P.</td>
<td>3</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>C.</td>
<td>4</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>P.P.</td>
<td>5</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>H.P.</td>
<td>6</td>
<td>18</td>
<td>12</td>
</tr>
</tbody>
</table>
2. # sides increases by one each time
   # edges increases by three each time
   # vertices increases by two each time

   See Question 1 for answer, re: hexagonal prism.

3. Answers may vary – teacher to check.

   **Possible answer:**
   The number of edges of a prism is three times larger than the number of sides of its base. The number of vertices of a prism is double the number of sides of its base.

4. An octagonal prism has 24 edges and 16 vertices.

**AP Book G6-35**

1. Teacher to check.
2. a) 12
   b) 6
   c) 8
   d) 15
   e) 10
   f) 9
   g) 12
   h) 18
3. a) 8
   b) 4
   c) 5
   d) 10
4. Teacher to check.
5. Teacher to check.

**AP Book G3-36**

1. a) 6
   b) 6
   c) 6
   d) 4

2. a) 6
   b) 6
   c) 6

**AP Book G6-37**

1. a) 
2. a) 
   b) 
   c) 
   d) 
   e) 
   f) 
   g) 
   h) 

**AP Book G6-38**

1. From left to right:
   X X O O X O X O

2. From left to right:
   rectangular prism
   square pyramid
   cone
   cylinder
   triangular pyramid
   triangular prism

3. | TP | SP | S? | D? |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td># faces</td>
<td>5</td>
<td>5</td>
<td>√</td>
</tr>
<tr>
<td>shape of base</td>
<td></td>
<td></td>
<td>√</td>
</tr>
<tr>
<td># bases</td>
<td>2</td>
<td>1</td>
<td>√</td>
</tr>
<tr>
<td># faces that are not bases</td>
<td>3</td>
<td>4</td>
<td>√</td>
</tr>
<tr>
<td># edges</td>
<td>9</td>
<td>8</td>
<td>√</td>
</tr>
<tr>
<td># vertices</td>
<td>6</td>
<td>5</td>
<td>√</td>
</tr>
</tbody>
</table>
Answer Key for AP Book 6.2

1. a) Triangular pyramid, Triangular prism.  
   **Similarities:**  
   Triangle as a base  
   3 faces that are not bases  
   **Differences:**  
   # of edges, faces, vertices and bases  
   
b) Pentagonal prism, Rectangular prism  
   **Similarities:**  
   Shapes that are not bases (rectangles)  
   # of bases  
   **Differences:**  
   # of edges, vertices and faces  
   Base shape

2. Teacher to check.

3. b) Since each has one base surrounded by triangles, we know that the nets are pyramids.

4. a) Rectangular Prism  
   b) Triangular Prism  
   c) Square Pyramid

5. a) Triangular pyramid, Triangular prism.  
   **Similarities:**  
   Triangle as a base  
   3 faces that are not bases  
   **Differences:**  
   # of edges, faces, vertices and bases  
   
b) Pentagonal prism, Rectangular prism  
   **Similarities:**  
   Shapes that are not bases (rectangles)  
   # of bases  
   **Differences:**  
   # of edges, vertices and faces  
   Base shape

6. The number of sides on the base and the number of triangular faces are equal.

7. a) 4; 1  
   b) 2; 4  
   c) 6; 2

---

**AP Book G6-39 page 341**

1. a) Name F E  
   Triangular Pyramid 4 6  
   Square Pyramid 5 8  
   Pentagonal Pyramid 6 10  
   Triangular Prism 5 9  
   Cube 6 12  
   Pentagonal Prism 7 15  

   **Shape of Base:**  
   Triangular Pyramid: Triangle  
   Square Pyramid: Square  
   Pentagonal Pyramid: Pentagon  
   Triangular Prism: Triangle  
   Cube: Square  
   Pentagonal Prism: Pentagon

   b) Teacher to check.

2. a) i) Triangle  
   ii) Triangle  
   iii) Triangle
2. a) ![Diagram](image)
b) ![Diagram](image)

3. a) ![Diagram](image)
b) ![Diagram](image)

4. a) Slide 1 unit right and reflection
b) ![Diagram](image)
c) 90° rotation clockwise and reflection

5. a) ![Diagram](image)
b) ![Diagram](image)
c) ![Diagram](image)

6. a) i) 1 to 2: reflection
   ii) 2 to 3: reflection
   iii) 3 to 4: reflection
   iv) 4 to 5: reflection
b) i) 1 to 2: 180° rotation
   ii) 2 to 3: reflection
   iii) 3 to 4: 180° rotation

7. Answers will vary – teacher to check.

AP Book G6-42 page 346
1. Teacher to check.
BONUS: Teacher to check.

AP Book G6-43 page 347
1. Teacher to check.

AP Book G6-44 page 348
1. a) Front:
   Top:
   Side:
   b) Front:
   Top:
   Side:
   c) Front:
   Top:
   Side:
   d) Front:
   Top:
   Side:

5. There are 5 different rectangular prisms that can be made with 8 cubes.

AP Book G6-45 page 349
1. a) Front:
   Top:
   Side:
   b) (1,1) (5,2) (5,4) (1,3)
   c) Front:
   Top:
   Side:
   d) Front:
   Top:
   Side:
   e) Front:
   Top:
   Side:
   f)–o) Answers will vary – teacher to check
3. E: cone  
   B: triangular prism  
   A: cube  
   D: cylinder  
   C: triangular pyramid  

4. Answers will vary – teacher to check.  

5. **Similarities:**  
   Base shape  
   3 faces that are not bases  

**Differences:**  
# of edges, vertices, faces, bases  

6. Answers will vary – teacher to check.
## Contents

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Answer Key for Patterns & Algebra – Part 1  
Number Sense – Part 1  
Answer Key for Number Sense – Part 1  
Measurement – Part 1  
Answer Key for Measurement – Part 1  
Probability & Data Management – Part 1  
Answer Key for Probability & Data Management – Part 1  
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Answer Key for Probability & Data Management – Part 2  
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Answer Key for Geometry – Part 2
Patterns & Algebra

Unit Test

Name: _____________________________  Date: __________________

Section A

1. For the following pattern, use the first three numbers in the pattern to find the rule. Then continue the pattern by filling in the blanks:

   a) 22, 27, 32, _____, _____, _____  The rule is: _________________________________

   b) 48, 45, 42, _____, _____, _____  The rule is: _________________________________

   c) 1028, 1019, 1010, _____, _____, _____  The rule is: _________________________________

2. Extend the number pattern. How many squares would be used in the 6th figure?

   a)  
   
   b)  
   
   c)  

   3. The snow is 11 cm deep at 3 p.m. 6 cm of snow falls each hour. How deep is the snow at 7 p.m.?

   4. Una’s candle is 28 cm high when she lights it at 7 p.m. It burns down 3 cm every hour. Mona’s candle is 30 cm high when she lights it at 7 p.m. It burns down 4 cm every hour. Whose candle is taller at 11 p.m.?
Patterns & Algebra
Unit Test

Name: _____________________________  Date: _________________

Section A (continued)

5. Circle the core of the pattern. Then continue the pattern:
   a) O △ O O △ O ______    ______    ______    ______    ______
   b) 3 5 3 7 1 3 5 7 1 ______    ______    ______    ______    ______

6. Draw a rectangle around the core of the pattern:
   R Y Y R R Y Y R R Y Y R

7. Megan plants a row of daisies (the first flower) and pansies in the pattern shown here:
   a) How long is the core of the pattern?

   b) Is the 45th flower a daisy or a pansy?

8. Y R R Y R R Explain how you could find the colour of the 37th block in this pattern without using a hundreds chart:
   HINT: How could skip counting help?

9. 25¢ 10¢ 10¢ 10¢ 25¢ 10¢ 10¢ 10¢ What is the 21st coin in this pattern?
   Explain how you know.
Patterns & Algebra

Unit Test

Section A (continued)

10. Find the lowest common multiple of each pair of numbers:

<table>
<thead>
<tr>
<th>a) 6 and 10</th>
<th>b) 5 and 15</th>
<th>c) 2 and 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCM = _____</td>
<td>LCM = _____</td>
<td>LCM = _____</td>
</tr>
</tbody>
</table>

11. Every 6th person who arrives at a book sale receives a free calendar and every 8th person receives a free book. Which of the first 50 people receive a book and a calendar?

12. Find the amount by which the sequence increases or decreases. At each step, write your answer in the circles with a + or – sign:

a) 8, 2, 14, 16, 1

b) 16, 23, 4, 90, 2

13. a) Which row of the chart has a decreasing pattern? (Looking left to right.)

| 0 | 5 | 10 | 5 | 0 |
| 6 | 7 | 8 | 4 | 10 |

b) Which column has a repeating pattern?

| 12 | 9 | 6 | 3 | 0 |

| 18 | 11 | 4 | 2 | 10 |

| 24 | 13 | 2 | 1 | 0 |

c) Write pattern rules for the first and second column:

d) Describe the relationship between the numbers in the third and fourth columns:

e) Describe one other pattern in the chart:

f) Name a row or column that does not appear to have any pattern:
Patterns & Algebra

Section B

14. Create an increasing number pattern. Give the rule for your pattern:

15. Create a repeating pattern using...
   a) letters:
   b) shapes
   c) numbers

16. Wendy makes key chains using squares (s), grey rectangles (r), and triangles (t). Complete the chart. Write a formula (like 4 \times s = t) for each design:

   a)  
   \[
   \begin{array}{c|c}
   \text{Squares (s)} & \text{Rectangles (r)} \\
   \hline
   1 & \\
   2 & \\
   3 & \\
   \end{array}
   \]

   b)  
   \[
   \begin{array}{c|c}
   \text{Squares (s)} & \text{Triangles (t)} \\
   \hline
   1 & \\
   2 & \\
   3 & \\
   \end{array}
   \]

17. For each chart, give a rule that tells you how to make the OUTPUT numbers from the INPUT numbers:

   a)  
   \[
   \begin{array}{c|c}
   \text{INPUT} & \text{OUTPUT} \\
   \hline
   4 & 11 \\
   5 & 12 \\
   6 & 13 \\
   \end{array}
   \]
   Rule:

   b)  
   \[
   \begin{array}{c|c}
   \text{INPUT} & \text{OUTPUT} \\
   \hline
   3 & 12 \\
   5 & 14 \\
   7 & 16 \\
   \end{array}
   \]
   Rule:

   c)  
   \[
   \begin{array}{c|c}
   \text{INPUT} & \text{OUTPUT} \\
   \hline
   19 & 6 \\
   15 & 2 \\
   21 & 8 \\
   \end{array}
   \]
   Rule:
Patterns & Algebra

Unit Test

Section B (continued)

18. Complete the T-Table for the following pattern. Then write a rule that tells you how to calculate the output numbers from the input number:

NOTE: Use the word INPUT in your answer: For instance, “multiply the INPUT by 3.”

<table>
<thead>
<tr>
<th>Number of White Hexagons</th>
<th>Number of Shaded Hexagons</th>
<th>Rule:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

19. Write the rule that tells you how to make the OUTPUT from the INPUT:

NOTE: Each rule involves two operations: either multiplication and addition, or multiplication and subtraction.

a) INPUT | OUTPUT  
1  | 9   
2  | 12  
3  | 15  

b) INPUT | OUTPUT  
1  | 3   
2  | 8   
3  | 13  

c) INPUT | OUTPUT  
1  | 10  
2  | 12  
3  | 14  

Rule: _________________________________________________________________________

Rule: _________________________________________________________________________

Rule: _________________________________________________________________________

20. Draw Figure 4 and fill in the T-table. Write a rule for calculating the Number of Squares from the Figure Number.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Rule for T-table: _________________________________________________________________________

Use your rule to predict the number of squares needed for Figure 10: _________________
Section A

1. a) 37, 42, 47; Start at 22 and add 5.
   b) 39, 36, 33; Start at 48 and subtract 3.
   c) 1001, 992, 983; Start at 1028 and subtract 9.

2. Fig. # of Squares
   a) 4 24
      5 31
      6 38
   b) 4 17
      5 21
      6 25
   c) 4 17
      5 22
      6 27

3. The snow will be 35 cm deep at 7 p.m.

4. Una’s candle will be taller at 11 p.m. (16 cm vs 14 cm).

5. a) Core = ○ △ ○ ○
    ○ △ ○ ○ ○
    ○ ○
   b) Core = 3 5 3 7 1; 3 5 3 7 1

6. Core = Ｒ Ｙ Ｙ Ｒ

7. a) 3 flowers long
   b) Pansy

8. Sample Answer:
The core is 3 blocks long. I could skip count by 3’s until I got closest to 37 without going over (36). The first block in the core (37 – 36) is yellow, so the 37th block would be yellow.

9. The 21st coin would be a quarter (using a similar process to #8 above).

10. a) LCM = 30
    b) LCM = 15
    c) LCM = 18

11. The 24th and 48th people will receive both a book and a calendar.

12. a) – 6, + 12, + 2, – 15
    b) + 7, – 19, + 86, – 86

13. a) Row 3 or Row 5
    b) Column 5
    c) Column 1: Start at 0 and add 6.
    Column 2: Start at 5 and add 2.
    d) If you divide each number in Column 3 by 2, you will get the corresponding number in Column 4.
    e) Answers will vary.
    (e.g. Column 4: Start at 5 and subtract 1.)
    f) Rows 2, 4, 5 do not follow a clear pattern

Section B

14. Answers will vary.

15. a) Answers will vary.
    b) Answers will vary.
    c) Answers will vary.

16. a) S R
    1 4
    2 8
    3 12

    4 × s = r

    b) S T
    1 6
    2 12
    3 18
    6 × s = t

17. a) Input + 7 = Output
    b) Input + 9 = Output
    c) Input – 13 = Output

18. White Hexagons Shaded Hexagons

| RULE: Multiply the INPUT (# of white hexagons) by 2. |

<table>
<thead>
<tr>
<th>Figure</th>
<th># of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>
Number Sense

Unit Test

Section A

1. Write the name of the place value of each underlined digit:
   a) 1 278 930 _________________________
   b) 842 208 _________________________
   c) 2 007 217 _________________________
   d) 42 600 _________________________
   e) 842 _________________________
   f) 9 000 460 _________________________

2. Write numerals for the following number words:
   a) twenty-nine thousand, six hundred forty-three _______________
   b) eighty thousand, two hundred four _______________
   c) fifty-one thousand thirty-nine _______________

3. Write number words for the following numerals:
   a) 2 180 __________________________________________________
   b) 13 008 __________________________________________________
   c) 1 019 800 __________________________________________________

4. Expand the following numbers using numerals and words:
   a) 18 060    = ______________________________________________________________________
   b) 819     =  ______________________________________________________________________
   c) 38 349    = ______________________________________________________________________

5. Sketch a base ten model of each number, then write the number in expanded form using number words and using numerals:
   a) 3 622

   3 622 = ____________________________
   __________________________________
   3 622 = ____________________________

   b) 4 387

   4 387 = ____________________________
   __________________________________
   4 387 = ____________________________
Section A (continued)

6. Write an inequality to show which number is greater:
   a) 8 643 ___________ 8 786  
   b) 6 267 ___________ 8 232  
   c) 8 000 ___________ 6 999  
   d) 3 979 ___________ 6 001  
   e) 37 855 ___________ 37 122  
   f) 87 226 ___________ 87 934  
   g) 153 002 ___________ 177 244

7. In the questions below, you will have to regroup two or three times:
   a)  
      \[ \begin{array}{c|c|c|c} 
      & 1 & 0 & 0 \\
      \hline 
      - & 5 & 7 & 3 \\
      \hline 
      \end{array} \]
   b)  
      \[ \begin{array}{c|c|c|c|c} 
      & 1 & 0 & 0 & 0 \\
      \hline 
      - & 3 & 1 \\
      \hline 
      \end{array} \]
   c)  
      \[ \begin{array}{c|c|c|c|c} 
      & 1 & 0 & 4 & 0 & 0 \\
      \hline 
      - & 4 & 5 & 8 & 9 \\
      \hline 
      \end{array} \]

8. This chart gives the area of some of the largest lakes in North America:
   a) How much more area does the largest lake cover than the smallest lake?
   b) How much more area does Lake Michigan cover than Lake Erie?
   c) Write the areas of the lakes in order from least to greatest:
   d) The largest lake in the world is the Caspian Sea in Russia. Its area is 370 990 km². How much smaller than the area of the Caspian Sea is the area of Lake Superior?

<table>
<thead>
<tr>
<th>Lake</th>
<th>Area (in km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erie</td>
<td>25 693</td>
</tr>
<tr>
<td>Great Slave</td>
<td>28 568</td>
</tr>
<tr>
<td>Michigan</td>
<td>58 016</td>
</tr>
<tr>
<td>Great Bear</td>
<td>31 339</td>
</tr>
<tr>
<td>Superior</td>
<td>82 103</td>
</tr>
</tbody>
</table>

9. Write 10, 100, 1 000 or 10 000 in the box to make the statement true:
   a) 256 + \[ \_ \_ \_ \_ \_ \_ \_ \] = 266
   b) 5 673 + \[ \_ \_ \_ \_ \_ \_ \_ \] = 5 773
   c) 9 328 + \[ \_ \_ \_ \_ \_ \_ \_ \] = 10 328
   d) 57 264 + \[ \_ \_ \_ \_ \_ \_ \_ \] = 67 264
   e) 85 043 − \[ \_ \_ \_ \_ \_ \_ \_ \] = 84 943
   f) 81 263 − \[ \_ \_ \_ \_ \_ \_ \_ \] = 80 263
Number Sense

Section A (continued)

10. Use each of the digits 4, 5, 6, 7, 8 once to create…
   a) The greatest odd number possible:  b) a number between 57,000 and 56,700:
   c) An even number whose tens digit and hundreds digit add to 12:
   d) An odd number whose thousands digit is twice its hundreds digit:

11. Circle the prime numbers:
   a) 11  25  14  13  17  20  
      b) 27  15  12  18  29  33

12. Draw a factor tree for the following numbers:
   a) 18  
      b) 24

13. Find two pairs of prime numbers less than 20 that differ by 4:

14. Multiply:
   a) 
      b) 
      c) 

15. A hummingbird flaps its wings 15 times per second. How many times does it flap its wings in a minute?
Section B

16. Find two different ways to share 29 pens into equal groups so that one pen is left over:

17. Divide:
   
   |   |   |   |
   - |   |   |   |
   - |   |   |   |
   - |   |   |   |

17. a) 3

17. b) 4

17. c) 5

17. d) 25

18. Jason eats 8 almonds a day. How many days will he take to eat 104 almonds?

19. What is the least number of whole apples that can be shared equally among 2, 3, or 4 people?

20. Nandita ran 24 laps of her school track. The track is 75 metres long.
   
   a) How far has she run?

   b) How much further must she run if she wants to run 2000 metres?

   c) About how many extra laps must she run?
Number Sense

Section B (continued)

21. Sterling packs 59 books into boxes of 5, and Philip packs 47 books into boxes of 6. Who uses more boxes? Who has more left over?

22. Share the squares equally among the sets:

   a) How many cards did he put in the book? __________
   b) If each page held only 5 cards, how many pages would he need to place the cards? __________
   c) How many cards did he have before he gave half of his collection away? __________

24. These thermometers show the temperatures on Thursday and Friday.
   a) In the blanks, write an integer for each temperature.
   b) How much did the temperature change?

   Thursday:  _____ °C

   Friday: _____ °C
Number Sense

Unit Test

Section B (continued)

25. How many negative integers are greater than -6?

26. Round to the nearest thousands place:
   a) 4 787 b) 93 092 c) 723 871

27. Round to the nearest ten thousands place:
   a) 82 839 b) 43 003 c) 397 603

28. Round 81 649 to the nearest...

   __________   __________   __________   __________   __________
   tens       hundreds      thousands      ten thousands

29. Estimate the products by rounding to the leading digits:
   a) 45 \times 75 = b) 427 \times 56 = c) 306 \times 17 = d) 81 \times 819 =

30. The population of New Brunswick and Nova Scotia are listed in an almanac as 750 000 and 936 900. What digit do you think these numbers have been rounded to? Explain.

31. The population of Newfoundland is 520 200 and the population of Prince Edward Island is 137 900. Estimate the difference in the two populations. Explain how you estimated the difference.
Number Sense

Section C

32. How much money would you have if you had the following coins? Write your answer in cent notation then in dollar notation:
   a) 35 pennies = _____ = ______
   b) 7 nickels = _____ = ______
   c) 8 dimes = _____ = ______
   d) 28 pennies = _____ = ______
   e) 6 toonies = _____ = ______
   f) 3 quarters = _____ = ______

33. Circle the greater amount of money in each pair:
   a) 183¢ or $1.86
   b) $1.41 or 143¢
   c) 7¢ or $0.70

34.

a) If you bought a watch and a soccer ball, how much would you pay?

b) Which costs more: a watch and a cap or a pair of pants and a soccer ball?

c) Could you buy a soccer ball, a pair of tennis rackets and a pair of pants for $100?

d) What would be the total cost of the three most expensive things shown in the pictures above?

e) Danny paid 2 $20 bills for the watch. Estimate his change.
**Number Sense**

*Unit Test*

Section C  (continued)

35. Tanya’s weekly allowance is $4.50. Her mom gave her four coins. Which coins did she use?

36. Mera has $12.16 and Wendy has $13.47. How much more money does Wendy have than Mera?

37. First estimate the amount of money shown. Then tally the amount of each denomination and use the space provided to calculate the actual total:

**Estimated Total:** ______________

<table>
<thead>
<tr>
<th>$2</th>
<th>25¢</th>
<th>5¢</th>
<th>5¢</th>
<th>1¢</th>
<th>25¢</th>
<th>25¢</th>
</tr>
</thead>
<tbody>
<tr>
<td>___ × $20</td>
<td>___ × $10</td>
<td>___ × $5</td>
<td>___ × $2</td>
<td>___ × $1</td>
<td>___ × 25¢</td>
<td>___ × 10¢</td>
</tr>
</tbody>
</table>

**Actual Total:** ______________

38. Draw a picture to show how to make the following amounts with the least number of coins and bills:

<table>
<thead>
<tr>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $64</td>
</tr>
<tr>
<td>b) $97</td>
</tr>
<tr>
<td>c) $78.73</td>
</tr>
</tbody>
</table>
## Section A

1. a) Hundred thousands  
   b) Ten thousands  
   c) Hundreds  
   d) Tens  
   e) Ones  
   f) Millions

2. a) 29 643  
   b) 80 204  
   c) 51 039

3. a) Two thousand one hundred eighty  
   b) Thirteen thousand eight  
   c) One million nineteen thousand eight hundred

4. a) 1 ten thousands + 8 thousands + (0 hundreds +)  
   6 tens + (0 ones)  
   b) 8 hundreds + 1 ten + 9 ones  
   c) 3 ten thousands + 8 thousands + 3 hundreds + 4 tens + 9 ones

5. a) Teacher to check base ten model.  
   3 622 = 3 thousands + 6 hundreds + 2 tens + 2 ones  
   3 622 = 3 000 + 600 + 20 + 2  
   b) Teacher to check base ten model.  
   4 387 = 4 thousands + 3 hundreds + 8 tens + 7 ones  
   4 387 = 4 000 + 300 + 80 + 7

6. a) <  
   b) <  
   c) >  
   d) <  
   e) >  
   f) <

7. a) 427  
   b) 69  
   c) 5811

8. a) 56 410 km²  
   Largest: Lake Superior  
   Smallest: Lake Erie

   b) 32 323 km²  
   c) 25 693 km² (Erie);  
   28 568 km² (Great Slave);  
   31 339 km² (Great Bear);  
   58 016 km² (Michigan);  
   82 103 km² (Superior)

9. a) 427  
   b) 69  
   c) 5811

10. a) 87 645  
   b) Answers will vary:  
   56 748; 56 784; 56 847; 56 874  
   c) Teacher to check.  
   2 cases:  
   - tens digit is 4 and hundreds digit is 8;  
   ones digit is 6  
   OR  
   - tens digit is 8 and hundreds digit is 4;  
   ones digit is 6

11. a) 11, 13, 17  
   b) 29

12. a) Answers will vary:  
   18  
   9 × 2  
   3 × 3 × 2  
   OR  
   18  
   3 × 6  
   3 × 2 × 3

13. Answers will vary:  
   3, 7;  
   13, 17; 7, 11

14. a) 18 088  
   b) 14 170  
   c) 11 780

15. 15 × 60 = 900 times

16. Answers will vary:  
   4 groups of 7;  
   7 groups of 4;  
   2 groups of 14;  
   14 groups of 2

17. a) 27  
   b) 21  
   c) 129 R3  
   d) 282 R5

18. 13 days

19. 12 apples

20. a) 1 800 m  
   b) 200 m  
   c) 2 laps + 50 m OR  
   2 1 3 laps

21. Sterling uses more boxes (11);  
   Philip has more boxes left over (5).

22. 4 squares in each of 6 sets

23. a) 405  
   b) 81  
   c) 810

24. a) Thursday: – 10ºC  
   Friday: 25 ºC  
   b) 35ºC

25. 5 (– 5, – 4, – 3, – 2, – 1)

26. a) 5 000  
   b) 93 000  
   c) 724 000

27. a) 80 000  
   b) 40 000  
   c) 400 000

28. 10s: 81 650  
   100s: 81 600  
   1000s: 82 000  
   10 000s: 80 000

29. a) 50 × 80  
   = 4 000  
   b) 400 × 60  
   = 24 000  
   c) 300 × 20  
   = 6 000  
   d) 80 × 800  
   = 64 000
30. Looking at the populations, it seems that one is rounded to the nearest ten thousands (750 000) and the other, to the nearest hundreds (936 900). However, in an almanac both numbers would be rounded to the same digit. If they’re both rounded to the same digit, they must both be rounded to the nearest hundreds.

31. Answers will vary as students may round the numbers to different digits (e.g. to the nearest hundred thousands, ten thousands or thousands). Teacher to check.

Section C

32. a) 35¢ = $0.35  
   b) 35¢ = $0.35  
   c) 80¢ = $0.80  
   d) 28¢ = $0.28  
   e) 1200¢ = $12.00  
   f) 75¢ = $0.75

33. a) $1.86  
   b) 143¢  
   c) $0.70

34. a) $32.89 + $10.30 = $43.19  
   b) Watch and cap ($58.53 vs $50.25)  
   c) Yes – the total is $95.72.  
   d) $123.92 (for the tennis rackets, pants and shoes)  
   e) About $7.00 ($7.11)

35. 2 × $2  
   2 × 25¢

36. $1.31

37. Actual Total = $72.86

38. a) 3 × $20  
   2 × $2  
   b) 1 × $50  
   2 × $20  
   1 × $5  
   1 × $2  
   c) 1 × $50  
   1 × $20  
   1 × $5  
   1 × $2  
   1 × $1  
   2 × 25¢  
   2 × 10¢  
   3 × 1¢
Measurement
Unit Test

**Section A**

1. Answer the questions based on the given information on the weight of Canadian coins. Do your work in the space provided below.

   a) How much would 20 dimes weigh? ______________

   b) How much would 65¢ in nickels weigh? ______________

   c) How much would $1.50 in quarters weigh? ______________

   d) How much would 40 loonies weigh? ______________

   e) How many dimes weigh as much as 6 loonies? ______________

   f) How many pennies would weigh as much as 10 nickels? ______________

2. Check off the appropriate box. Would you use grams or kilograms to weigh...
   a) a computer? □ g    □ kg  
   b) a bed?    □ g    □ kg  
   c) a piece of bread? □ g    □ kg  
   d) a frog?    □ g    □ kg  
   e) a pen?    □ g    □ kg  
   f) an apple? □ g    □ kg

3. A dog weighs 4 kg. A cat weighs 2570 grams. How much more does the dog weigh? Show your work:

4. An insect weighs 250 mg.
   a) How many insects weigh 1 gram? Show your work:

   b) How many insects weigh 1 kg? Show your work:
Measurement
Unit Test

Section B

5. Find the volume of each box with the indicated dimensions (assume all units are in metres):

a) Width _______
   Length _______
   Height _______
   Volume _______

b) Width _______
   Length _______
   Height _______
   Volume _______

c) Width _______
   Length _______
   Height _______
   Volume _______

d) Width _______
   Length _______
   Height _______
   Volume _______

6. Marcus is building a pyramid with cubic centimetre blocks:

a) Fill in the volumes in each layer:

   Volume of top layer: _______
   Volume of second layer: _______
   Volume of bottom layer: _______

b) If Marcus added another row to his pyramid (following the same pattern), what would the total volume of the pyramid be? Explain.

7. A structure made of cubes each with volume 1 cm³ has this mat plan. What is the volume of the structure?

   3 1 1
   2 5 3

Unit Tests – Workbook 6, Part I
Measurement

Unit Test

Section B (continued)

8. This picture shows the top view of a cube built with cubic centimetres. What is the volume of the cube? Explain how you know:

9. How many millilitres are in a litre? _____________

10. Circle the greater measure in each pair:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 25 g</td>
<td>35 mg</td>
<td>b) 20 g</td>
<td>17 kg</td>
<td>c) 3 L</td>
</tr>
<tr>
<td>d) 50 g</td>
<td>2 kg</td>
<td>e) 400 mL</td>
<td>1 L</td>
<td>f) 2000 mL</td>
</tr>
</tbody>
</table>

11. Explain how you found the answers to Questions 10 d) and 10 f):

12. Circle True or False for each statement below:

a) You would measure the mass of a car in litres. True    False
b) A gram is used to measure volume. True    False
c) The contents of a can of pop are usually measured in kilograms. True    False
d) Grams are used to measure the mass of objects. True    False
Measurement

Unit Test

Section C

13. How much time passed?

a) From 8:45 a.m. to 9:20 a.m.: ______________________________________

b) From 11:20 a.m. to 4:35 p.m.: ______________________________________

c) From 6:52 a.m. to 8:21 p.m.: ______________________________________

d) From 11:25 a.m. to 5:43 a.m.: ______________________________________

e) From 23:00 to 7:00: ______________________________________

f) From 22:51 to 14:43: ______________________________________

14. Boat A left Halifax at 13:00, 1 hour before Boat B. Both boats travelled at a steady speed in the same direction.

<table>
<thead>
<tr>
<th>Time</th>
<th>13:00</th>
<th>14:00</th>
<th>15:00</th>
<th>16:00</th>
<th>17:00</th>
<th>18:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Halifax</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boat A</td>
<td>0 km</td>
<td>5 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boat B</td>
<td>0 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>28 km</td>
</tr>
</tbody>
</table>

a) How far apart were the boats at 18:00?

b) How far from Halifax were both boats at 15:30? At 16:30?

c) When did Boat B overtake Boat A?
**Section A**

1. a) 40 g  
   b) 52 g  
   c) 27 g  
   d) 280 g  
   e) 21 dimes  
   f) 16 pennies  

2. a) kg  
   b) kg  
   c) g  
   d) g  
   e) g  
   f) g  

3. \(4000 - 2570 = 1430\) g  

4. a) \(1000 + 250 = 4\) insects  
   b) 1 kg = 1 000 g, so 1 kg = \(4 \times 1000 = 4000\) insects  

**Section B**

5. a) \(W = 2\) m  
    \(L = 2\) m  
    \(H = 2\) m  
    \(V = 8\) m\(^3\)  
   b) \(W = 4\) m  
    \(L = 5\) m  
    \(H = 4\) m  
    \(V = 80\) m\(^3\)  
   c) \(W = 2\) m  
    \(L = 4\) m  
    \(H = 1\) m  
    \(V = 8\) m\(^3\)  
   d) \(W = 4\) m  
    \(L = 4\) m  
    \(H = 8\) m  
    \(V = 128\) m\(^3\)  
   
6. a) Top – 1 cm\(^3\)  
   
   Second – 9 cm\(^3\)  
   
   Bottom – 25 cm\(^3\)  
   
   b) Following the pattern, the volume of the new layer would be: \(7 \times 7 = 49\) cm\(^3\);  
   
   So total volume \(= 1 + 9 + 25 + 49 = 84\) cm\(^3\)  

7. 15 cm\(^3\)  

8. 27 cm\(^3\)  
   (length = 3 and width = 3  
   so, as a cube, height = 3  
   and \(3 \times 3 \times 3 = 27\))  

9. \(1000 \text{ mL} = 1\) L  

10. a) 25 g  
    b) 17 kg  
    c) 3 L  
    d) 2 kg  
    e) 1 L  
    f) 2 000 mL  

11. For 10 d), convert both measures to grams (2 kg = 2 000 g, which is greater than 50 g).  
   For 10 f), convert both to millilitres (1 L = 1 000 mL is less than 2 000 mL).  

12. a) False  
   b) False  
   c) False  
   d) True  

**Section C**

13. a) 35 min  
    b) 5 hrs 15 min  
    c) 13 hrs 29 min  
    d) 18 hrs 18 min  
    e) 8 hrs  
    f) 15 hrs 52 min  

14.  

<table>
<thead>
<tr>
<th>Time</th>
<th>Boat A (km)</th>
<th>Boat B (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14:00</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>15:00</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>16:00</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>17:00</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>18:00</td>
<td>25</td>
<td>28</td>
</tr>
</tbody>
</table>

   a) 3 km apart  
   b) Distance from Halifax:  

<table>
<thead>
<tr>
<th>Time</th>
<th>Boat A (km)</th>
<th>Boat B (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15:30</td>
<td>12.5</td>
<td>10.5</td>
</tr>
<tr>
<td>16:30</td>
<td>17.5</td>
<td>17.5</td>
</tr>
</tbody>
</table>

   c) At 16:30
Section A

1. Rene’s class has a fish tank. It contains a variety of small fish, each with different characteristics:

Complete the table. Then sort fish into Venn diagram.

<table>
<thead>
<tr>
<th>Category</th>
<th>Fish (by letter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish with a pattern</td>
<td>A, B,</td>
</tr>
<tr>
<td>Dark fish</td>
<td></td>
</tr>
</tbody>
</table>

2. 

Use the above line graph to answer the following questions:

a) In which month did Anne drink: (i) the most smoothies? (ii) the least smoothies?

b) How many smoothies did Anne drink: (i) in March? (ii) in October?

c) In which months did Anne drink more than 6 smoothies?
3. In order to identify how their fellow students got to school, a Grade 5 class at Baldwin Public School designed a short survey and gave it to every student in the school.

a) Using the final results (below), complete the bar graph provided:

<table>
<thead>
<tr>
<th>Transportation Used to Get to School</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bike</td>
<td>66</td>
</tr>
<tr>
<td>Subway</td>
<td>33</td>
</tr>
<tr>
<td>Walk</td>
<td>138</td>
</tr>
<tr>
<td>Bus</td>
<td>156</td>
</tr>
<tr>
<td>Car</td>
<td>22</td>
</tr>
</tbody>
</table>

HINT: In this bar graph, the bars will run horizontally. The first one has been done for you.

Next, answer the following questions:

b) Identify the scale used in the bar graph (e.g. what it counts by and when it stops). Do you think it was a good choice? Why or why not?

c) How do the students at your school get to school? Would you predict similar or different results than those at found at Baldwin PS? Explain.

4. Melanie surveyed her friends about their favourite authors. Here are her results:

<table>
<thead>
<tr>
<th>J.K. Rowling</th>
<th>Lemony Snicket</th>
<th>Tamora Pierce</th>
<th>Cornelia Funke</th>
<th>Louis Sachar</th>
<th>Kenneth Opel</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

If you were Melanie, how would you choose to display your data? Why?
Section A (continued)

5. Ms Young’s Grade 5 class carried out an experiment: each day (for 12 days) they dropped 10 pennies on the ground. They counted the number of pennies that came up “heads” and created the following scatter plot graph:

Read the scatter plot carefully and complete the result chart below (the first day has been done for you):

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pennies</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. The following graph from a newspaper article shows how many new planets have been discovered by astronomers:

a) How many more planets were discovered in 2002 than in 2004?

b) In which years were more than 15 planets discovered?

c) Between which years were at least 5 planets discovered each year?
Section B

7. Find the range of the following data sets:
   HINT: Don’t forget to re-write the list in order from lowest to highest first!
   a) 45, 27, 14, 95, 44, 8
   b) 124, 46, 34, 71, 24, 355
   c) 56, 37, 7, 44, 28, 422, 80
   ______________________
   ______________________
   ______________________
   Range: _____ to _____
   Range: _____ to _____
   Range: _____ to _____

8. Find the mean of the following data sets:
   a) 5, 1, 7, 2, 8, 7
   b) 6, 3, 7, 15, 11, 10, 11
   c) 17, 6, 12, 4, 21
   ______________________
   ______________________
   ______________________
   Mean: _______
   Mean: _______
   Mean: _______

9. Find the mode of the following data sets:
   a) 3, 8, 8
   b) 30, 22, 52, 30
   c) 7, 7, 4, 5, 7, 4, 7, 9
   Mode: _______
   Mode: _______
   Mode: _______
   d) 53, 57, 35, 57, 75, 58
   e) 18, 88, 81, 8, 88, 88, 18
   f) 17, 17, 4
   Mode: _______
   Mode: _______
   Mode: _______

10. Find the median of the following data sets:
    HINT: Don’t forget to rewrite the list in order from lowest to highest first!
    a) 10, 18, 4, 13, 5
    b) 32, 33, 63, 16, 8, 13, 19
    c) 72, 22, 43, 6, 61, 77, 18
    ______________________
    ______________________
    ______________________
    Median: _______
    Median: _______
    Median: _______
Section B (continued)

11. Mrs. Gatlin gave her students a spelling test (marked out of 20) and entered all the marks in the chart:

<table>
<thead>
<tr>
<th>Mark</th>
<th>11</th>
<th>14</th>
<th>18</th>
<th>10</th>
<th>11</th>
<th>15</th>
<th>10</th>
<th>16</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19</td>
<td>15</td>
<td>19</td>
<td>19</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

a) Create a stem and leaf plot of the data.

b) Find the range, mode, median, and mean of the data. Which is hardest to read from the stem and leaf plot?

range: ________
mode: ________
median: ________
mean: ________

11. Mrs. Gatlin gave her students a spelling test (marked out of 20) and entered all the marks in the chart:

<table>
<thead>
<tr>
<th>Mark</th>
<th>11</th>
<th>14</th>
<th>18</th>
<th>10</th>
<th>11</th>
<th>15</th>
<th>10</th>
<th>16</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19</td>
<td>15</td>
<td>19</td>
<td>19</td>
<td>20</td>
<td>20</td>
<td>19</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

12. Marisa made a scatter plot of her pet rabbits' weight:

a) How many months does the interval shown by the arrow represent?

b) How many weeks does the interval represent?

c) Describe any trends you see in the graph.

d) Circle on the graph the point that shows a 6 month old rabbit with weight of 500g.
### Section A

**1. With a pattern:** A, D, E, G  
Dark fish: B, D, H  

![Pattern Diagram]

**2. a)** (i) August  
(ii) April  

**b)** (i) 10 smoothies  
(ii) 15 smoothies  

**c)** January, March, July, August, September, October, November  

**3. a)** Teacher to check.  

**b)** The scale starts at 0, “counts” by 20 and ends at 160.  
Yes, this is a good scale because it covers the entire range of data (22 to 156) in the space given.  

**c)** Answers will vary – teacher to check.  

**4. Answers will vary.**  
NOTE: A pie graph or bar graph should be used, rather than a line graph or a scatter plot.  

**5.**  
<table>
<thead>
<tr>
<th>Day</th>
<th># of Pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>

**6. a)** 25 more (31 – 6)  


**c)** From 1998 to 2004  

### Section B

**7. a)** 8 to 95  

**b)** 24 to 355  

**c)** 7 to 422  

**8. a)** 5  

**b)** 9  

**c)** 12  

**9. a)** 8  

**b)** 30  

**c)** 7  

**d)** 57  

**e)** 88  

**f)** 17  

**10. a)** 10  

**b)** 19  

**c)** 43  

**11. a)**  

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>59</td>
</tr>
<tr>
<td>1</td>
<td>001455689999</td>
</tr>
<tr>
<td>2</td>
<td>000</td>
</tr>
</tbody>
</table>

**b)** Range: from 5 to 20  
Mode: 19  
Median: 15.5  
Mean: 15  
Mean is hardest to read from the stem and leaf plot.  

**c)** i) is True, the average grade is the mean.  
Tom’s grade is below the median, so ii) is false, and the most common mark is the mode, 19, so iii) is also false.  

**12. a)** 1.5 months (3 ÷ 2)  

**b)** Approximately 6 weeks (1.5 × 4)  

**c)** The rabbits’ weights increase over time (but the incremental amount of weight gained gets smaller as time goes along).  

**d)** Teacher to check
Geometry
Unit Test

Section A

1. Complete the chart. Find as many shapes as you can for each shape name:

<table>
<thead>
<tr>
<th>Shapes</th>
<th>Letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangles</td>
<td></td>
</tr>
<tr>
<td>Quadrilaterals</td>
<td></td>
</tr>
</tbody>
</table>

2. Without using a protractor, identify each angle as “acute” or “obtuse”:
   a) 
   b) 
   c) 

3. Use the charts to classify the triangles below. NOTE: Triangles are not drawn to scale.

   a) Classify the triangles by their angles:
   b) Classify the triangles by their sides:

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute-angled</td>
<td></td>
</tr>
<tr>
<td>Obtuse-angled</td>
<td></td>
</tr>
<tr>
<td>Right-angled</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>Triangles with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral</td>
<td></td>
</tr>
<tr>
<td>Isosceles</td>
<td></td>
</tr>
<tr>
<td>Scalene</td>
<td></td>
</tr>
</tbody>
</table>
Section A (continued)

4. Measure all of the angles in each triangle and write your measurement in the triangle. Then say whether the triangle is acute, obtuse or right angled:

a) ___________________  
b) ___________________  
c) ___________________

5. Can a triangle be equilateral and obtuse? Explain.

6. Using arrows, mark all the pairs of parallel lines in the figures below:

a) ______ pairs  
b) ______ pairs  
c) ______ pairs  
d) ______ pairs

7. (i) Mark the angles that are right angles in the quadrilaterals below.

(ii) Measure the length of each side with a ruler and write it onto the pictures. Use this to help you decide on the best (or most specific) name for each quadrilateral.

a) ______ cm ______ cm  
   ______ cm ______ cm  
   ______ cm ______ cm

b) ______ cm ______ cm  
   ______ cm ______ cm

Name: __________________________  Name: ______________________________

8. Match the name of the quadrilateral to the best description:

Square  
Rectangle  
Rhombus  
A parallelogram with 4 right angles.
A parallelogram with 4 equal sides.
A parallelogram with 4 right angles and 4 equal sides.
Section A (continued)

9. Name the shapes: HINT: Use the words rhombus, square, parallelogram and rectangle.
   a) __________  
   b) __________  
   c) __________  
   d) __________  

10. For each quadrilateral, say how many **pairs** of sides are parallel. Then identify each quadrilateral as a square, a rectangle, a parallelogram or a trapezoid:
   a) __________  
   b) __________  
   c) __________  
   d) __________  

11. Which special quadrilaterals have diagonals that intersect at a right angle? List all names that apply.

12. a) Why is a rhombus a parallelogram?
   
   b) Why are some parallelograms not rhombi?

13. a) Draw a quadrilateral that has two right angles and one pair of parallel sides.
   b) What is the name of the shape you drew?
Geometry

Unit Test

Section B

14. a) Draw a triangle that is not congruent to the one shown:

b) Draw a trapezoid congruent to the one shown, but turned on its side:

15. Some of the shapes below are congruent. Find any shapes that are congruent to Shape A and label them with the letter A. If you can find any other shapes that are congruent to each other, label them all with the same letter.

HINT: You will need to use the letters A, B, C and D.

16. Which shapes are congruent? Which are similar? Explain how you know:
Section B (continued)

17. Complete the picture so that the dotted line is a line of symmetry:

a)  

b)  

c)  

d)  

18. a) Using the line provided, use a protractor to construct a triangle with two 60° angles:

   

   60°  60°

b) Measure the sides of the triangle. (Write the measurements on the sides.) What kind of triangle did you draw?

c) What is the order of rotational symmetry of this triangle?

19. a) Draw a trapezoid with one line of symmetry and a trapezoid with no lines of symmetry and no right angles:

   

   

b) Draw a parallelogram:
Section B (continued)

20. Record the properties of each shape. Write “yes” in the column if the shape has the given property. Otherwise, write “no”:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Quadrilateral</th>
<th>Equilateral</th>
<th>Two or more pairs of parallel sides</th>
<th>At least one right angle</th>
<th>At least one acute angle</th>
<th>At least one obtuse angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

21. Describe this figure completely. In your description you should mention the following properties:

- Number of sides
- Number of vertices
- Number of pairs of parallel sides
- Is the figure equilateral?
- Number of right, obtuse and acute angles
- Number of lines of symmetry
- Order of rotational symmetry

22. I have three sides. Two of my sides are the same length. What am I?
Section A

1. Shapes | Letters
---|---
Triangles | B
Quadrilaterals | A, D, F, G, H
Pentagons | C, I
Hexagons | E, J

2. a) acute
   b) obtuse
   c) acute

3. a) Acute-angled | A
   Obtuse-angled | D
   Right-angled | B, C

8. Square:
   A parallelogram with 4 right angles and 4 equal sides.
   Name: square

   Rectangle:
   A parallelogram with 4 right angles.

   Rhombus:
   A parallelogram with 4 equal sides.

9. a) rectangle
   b) parallelogram
   c) square
   d) rhombus

10. a) 2 pairs; rectangle
     b) 2 pairs; parallelogram
     c) 2 pairs; square
     d) 1 pair; trapezoid

11. Kite, rhombus, square

12. a) Because it has 2 pairs of parallel sides
     b) It depends on the shape’s adjacent sides – if they’re not equal, the shape is a parallelogram, not a rhombus.

13. a) Answers will vary.
     b) Trapezoid

Section B

14. a) Answers will vary – teacher to check.
     b) Answers will vary – teacher to check.

15. Two A’s:

Three B’s:

Two C’s:

Two D’s:

**remaining shapes aren’t congruent with anything**

16. Congruent: A & H
   Similar: A & F and H & F.
   Teacher to check explanation.

17. a) Answers will vary.
     b) Answers will vary.

18. a) Teacher to check.
     b) Equilateral
     c) Rotational symmetry of order 3.

19. a) Answers will vary.
     Examples:

   One line of symmetry -

   No lines of symmetry and no right angles -

b) Answers will vary.

20. Q | E | 2+ | 90º | Ac | Obt
---|---|---|---|---|---
A | N | Y | N | N | Y
B | Y | N | N | Y | Y
C | N | Y | N | N | N
D | Y | N | Y | Y | N

21. Description should include the following details:
   ✓ 6 sides
   ✓ 6 vertices
   ✓ 3 pairs of parallel sides
   ✓ Equilateral
   ✓ No right angles
   ✓ No acute angles
   ✓ 6 obtuse angles
   ✓ 6 lines of symmetry
   ✓ Rotational symmetry of order 6

22. Isosceles triangle

---

Answer Keys – Workbook 6 Unit Tests
Patterns & Algebra

Unit Test

Section A

1. Find the gap between the numbers, then write a rule for the pattern:

   a) 2, 3, 5, 8, 12

      Rule: _______________________________________________________________________

   b) 5, 7, 4, 6, 3

      Rule: _______________________________________________________________________

   c) 34, 33, 30, 25, 18

      Rule: _______________________________________________________________________

   d) 18, 21, 26, 33, 42

      Rule: _______________________________________________________________________

2. Extend each pattern for the next three terms. Then write a rule for the pattern.

   a) 237, 243, 249, 255, 261, _____, _____, _____

      Rule: _______________________________________________________________________

   b) 6, 10, 7, 11, 8, 12, _____, _____, _____

      Rule: _______________________________________________________________________

   c) 47, 45, 42, 38, _____, _____, _____

      Rule: _______________________________________________________________________

3. Use the letters of the alphabet to continue the following patterns:

   A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

   a) A, D, G, J, _____, _____

   b) Z, Y, W, T, _____, _____

   c) Z, X, V, T, _____, _____

   d) A, C, F, J, O, _____

Patterns & Algebra

Unit Test

Section A (continued)

4. Figure out how each of the patterns below was made, and then find the missing terms:
   a) 7, 12, 17, 22, 27, _____, _____
   b) 23, 25, 28, 30, 33, _____, _____
   c) 1, 5, 13, 29, 61, _____, _____
   d) 53, 55, 59, 65, 73, _____, _____
   e) 1, 3, 6, 10, 15, _____, _____
   f) 1, 2, 4, 8, 16, _____, _____
   g) 55, 51, 47, 43, 39, _____, _____
   h) 67, 69, 64, 66, 61, _____, _____
   i) 210, 220, 230, 240, 250, _____, _____
   j) .3, .9, 1.5, 2.1, 2.7, _____, _____

5. Solve each equation.
   a) n + 2 = 5
   b) n – 3 = 8
   c) 5n = 20
   d) 4 + x = 15
   e) 12 – n = 10
   f) 12 ÷ A = 3

6. Write an algebraic equation that tells you the relationship between the numbers in Column A and Column B.

   a) | A | B |
      | 1 | 7 |
      | 2 | 14 |
      | 3 | 21 |

   b) | A | B |
      | 2 | 5 |
      | 3 | 6 |
      | 4 | 7 |
Patterns & Algebra

Unit Test

Section B

7. Draw a graph for each T-table below:

\[\begin{array}{c|c}
\text{Input} & \text{Output} \\
2 & 5 \\
4 & 6 \\
6 & 7 \\
8 & 8 \\
\end{array}\]

\[\begin{array}{c|c}
\text{Input} & \text{Output} \\
1 & 6 \\
3 & 8 \\
5 & 10 \\
7 & 12 \\
\end{array}\]

8. Ben and Tom run a 120 m race.
   a) How far from the start was Tom after 20 seconds?
   b) How far from the start was Ben after 30 seconds?
   c) How many seconds from the start did Tom catch up to Ben?

9. The graph shows the cost of renting a bike from Mike’s store.
   a) How much would you pay to rent the bike for 6 hours?
   b) Dave’s store charges $3.00 an hour for a bike. Whose store would you rent from if you wanted the bike for 5 hours?
Patterns & Algebra
Unit Test

Section B (continued)

10. The picture shows how many chairs can be placed at each arrangement of tables:
   
   a) Make a T-table and state a rule that tells you how to calculate the number of chairs from and the number of tables:

   b) How many chairs can be placed at 12 tables?

11. Andy has $30 in his bank account. He saves 25 dollars each month. How much does he have in his account after 10 months?

12. A recipe calls for 5 cups of flour for every 6 cups of water. How many cups of water will be needed for 25 cups of flour? Show your work:

13. Jo-Leigh’s basket holds 24 apples and Emily’s basket holds 36 apples. They each collected less than 100 apples. How many baskets did they collect if they collected the same number of apples?
Patterns & Algebra
Unit Test

Section B (continued)

14. Find the mystery numbers:
   a) I am a two-digit number divisible by 6 and 8. My ones digit is 4. I am less than 40.
   b) I am between 20 and 40. I am a multiple of 7. My tens digit is two less than my units digit.

15. 

   △ □ □ △ □ □ △ △ △

   What is the 63rd term in this pattern? Explain how you know.

16. A camp offers two ways to rent a canoe: you can either pay $7.50 for the first hour and $3.50 for every hour after that OR you can pay $5.00 for every hour. If you wanted to rent a canoe for 5 hours, which way would you choose to pay? Show your work:
17. The picture below shows how the temperature inside a cloud changes at different heights:

a) Does the temperature increase or decrease at greater heights?

b) What distance does the arrow represent in real life? Show your work:

![Temperature Chart](image)

11.5°C  ▉  14.0°C  ▉  16.5°C  ▉  19.0°C  ▉  21.5°C  ▉  1000 m

earth

c) Measure the length of the arrow. What is the scale of the picture?

________ cm = ______________ m

d) Do the numbers in the sequence of temperatures decrease by the same amount each time?

e) If the pattern in the temperature continued, what would the temperature be at 1400 m?

18. Marlene says she will need 27 blocks to make Figure 7. Is she right? Explain:

Figure 1  Figure 2  Figure 3
Section A

1. a) Gaps: 1, 2, 3, 4
   Rule: Start at 2. Add 1, then 2, then 3...
   (Each step you are adding one more than the step before.)
   b) Gaps: 2, –3, 2, –3
   Rule: Start at 5. Add 2 and then subtract 3.
   Repeat.
   c) Gaps: –1, –3, –5, –7
   Rule: Start at 34. Subtract 1, then 3, then 5...
   (Each step you are subtracting two more than the step before.)
   d) Gaps: 3, 5, 7, 9;
   Rule: Start at 18. Add 3, then 5, then 7...
   (Each step you are adding two more than the step before.)

2. a) 267, 273, 279
   b) 9, 13, 10;
   Rule: Start at 6. Add 4 and then subtract 3.
   Repeat.
   c) 33, 27, 20;
   Rule: Start at 47. Subtract 2, then 3, then 4...
   (Each step you are subtracting one more than the step before.)

3. a) M, P
   b) P, K
   c) R, P
   d) U

4. a) 32, 37
   b) 35, 38
   c) 125, 253
   d) 83, 95
   e) 21, 28
   f) 32, 64
   g) 35, 31
   h) 63, 58
   i) 260, 270
   j) 3.3, 3.9

5. a) n = 3
   b) n = 11
   c) n = 4
   d) x = 11
   e) n = 2
   f) A = 4

6. a) B = 7 \times A or B = 7A
   b) B = A + 3

Section B

7. a) [Graph]

8. a) 80 m
   b) 80 m
   c) 15 seconds

9. a) $16.00
   b) Renting from Dave’s store would cost:
   \[ \frac{3.00}{hour} \times 5\ hours = $15.00 \]

   From the graph, we can see that renting from Mike’s store would cost $14.00.

   So… I would choose Mike’s store because it is $1.00 cheaper.

10. a) [Table]

    | Tables | Chairs |
    |-------|--------|
    | 1     | 6      |
    | 2     | 10     |
    | 3     | 14     |

    Multiply the numbers of chairs by 4 and add 2.
    b) \[ 4 \times 12 + 2 = 48 + 2 = 50 \text{ chairs} \]

11. After 10 months, Andy would have:
    \[ $30 + (25 \times 10) = $30 + $250 = $280 \]

   NOTE: Students may also choose to use a T-table.

12. [Table]

    | Flour | Water |
    |-------|-------|
    | 5     | 6     |
    | 5 \times 2 = 10 | 6 \times 2 = 12 |
    | 5 \times 3 = 15 | 6 \times 3 = 18 |
    | 5 \times 4 = 20 | 6 \times 4 = 24 |
    | 5 \times 5 = 25 | 6 \times 5 = 30 |

   So 30 cups of water will be needed for 25 cups of flour.

13. Since Jo-Leigh and Emily both collected the same number of apples, we are looking for a shared number in the following T-tables – that is also less than 100:

    | Jo-Leigh |
    |---------|
    | baskets | # of apples |
    | 1       | 24         |
    | 2       | 24 \times 2 = 48 |
    | 3       | 24 \times 3 = 72 |
    | 4       | 24 \times 4 = 96 |

    | Emily |
    |-------|
    | baskets | # of apples |
    | 1       | 36         |
    | 2       | 36 \times 2 = 72 |
    | 3       | 36 \times 3 = 108 |

   So both girls collected 72 apples, which means that Jo-Leigh collected 3 baskets and Emily collected 2.

14. a) 24
   b) 35

15. The pattern repeats itself every 5 shapes (i.e. the core has a length of 5) and \[ 63 + 5 = 12 \text{ R3}. \]

   This means that the pattern would repeat itself 12 times fully, and then you would need to go 3 more shapes to get to the 63rd one.

   So the 63rd shape is the same as the 3rd shape in the core, which is a square.
16. **Option 1:**
\[ 7.50 + (3.50 \times 4) \]
\[ = 7.50 + 14.00 \]
\[ = 21.50 \]

**Option 2:**
\[ 5.00 \times 5 \]
\[ = 25.00 \]

Therefore, students should choose Option 1 since it is cheaper.

17. a) The temperature decreases at greater heights.

b) The top height is 1 000 m above earth and, using the dotted lines, we see that the arrow represents \( \frac{1}{5} \) of that height:

\[ \frac{1}{5} \text{ of } 1000 \]
\[ = 1000 \div 5 \]
\[ = 200 \text{ m} \]

c) 1 cm = 200 m

d) Yes, each 200 m height increase results in a temperature drop of 2.5˚C.

e) At 1 000 m, the temperature is 11.5˚C – using the information from parts c) and d), we know that:

The temperature at 1 200 m is:
\[ = 11.5˚ – 2.5˚ \]
\[ = 9.0˚ \text{C} \]

And the temperature at 1 400 m is:
\[ = 9.0˚ – 2.5˚ \]
\[ = 6.5˚ \text{C} \]

18. | Figure | Blocks |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
</tr>
</tbody>
</table>

No, Marlene is not right – from the T-table we see that Figure 7 requires 28 blocks.
Section A

1. A field hockey team wins 8 games and loses 5 games:
   a) How many games did the team play?  
   b) What fraction of the games did the team win?  
   c) Did the team win more than half its games? Explain how you know.

2. The following chart shows the number of walls in a house that were painted a particular colour:

<table>
<thead>
<tr>
<th>Colour</th>
<th>Number of Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>6</td>
</tr>
<tr>
<td>Yellow</td>
<td>3</td>
</tr>
<tr>
<td>Blue</td>
<td>2</td>
</tr>
<tr>
<td>Green</td>
<td>1</td>
</tr>
</tbody>
</table>

   a) What fraction of the walls were painted green? ________  
   b) What colour was used to paint one fourth of the walls? ________  
   c) What colour was used to paint one half of the walls? ________

3. Write the fractions in order from least to greatest:
   a) \(\frac{2}{7}, \frac{1}{7}, \frac{5}{7}\)  
   b) \(\frac{2}{12}, \frac{2}{6}, \frac{2}{7}, \frac{2}{3}, \frac{2}{14}\)  
   c) \(\frac{9}{18}, \frac{9}{11}, \frac{9}{19}\)

4. Shade one piece at a time until you have shaded the amount of pie given in bold. There may be more pies than you need:
   a) \(1 \frac{1}{2}\)  
   b) \(2 \frac{1}{4}\)

5. Shade one piece at a time until you have shaded the amount of pie given in bold. There may be more pies than you need:
   a) \(\frac{10}{3}\)  
   b) \(\frac{9}{4}\)
Number Sense

Unit Test

Section A (continued)

6. Cut each pie into smaller pieces to make an equivalent fraction:
   a) \( \frac{2}{3} = \frac{6}{9} \)
   b) \( \frac{2}{3} = \frac{9}{9} \)
   c) \( \frac{1}{2} = \frac{4}{4} \)

7. A pizza is cut into 8 pieces. Each piece has at least one topping: hot peppers, mushrooms or both. \( \frac{3}{4} \) of the pizza is covered in hot peppers. \( \frac{5}{8} \) of the pizza is covered in mushrooms. Draw a picture to show how many pieces have both hot peppers and mushrooms on them:

8. Find the fraction of the whole amount by sharing the cookies equally:
   HINT: draw the correct number of plates then place the cookies one at a time. Then circle the correct amount.
   a) Find \( \frac{2}{3} \) of 6 cookies.
   b) Find \( \frac{3}{4} \) of 12 cookies.

\( \frac{2}{3} \) of 6 is _______  \( \frac{3}{4} \) of 12 is _______

9. Find the fraction of the whole number:
   a) \( \frac{2}{3} \) of 9 = _______  b) \( \frac{3}{4} \) of 8 = _______  c) \( \frac{3}{4} \) of 12 = _______  d) \( \frac{2}{3} \) of 15 = _______
   e) \( \frac{3}{5} \) of 25 = _______  f) \( \frac{2}{7} \) of 14 = _______  g) \( \frac{3}{4} \) of 100 = _______  h) \( \frac{3}{7} \) of 21 = _______
Number Sense

Unit Test

Name: _____________________________
Date: _________________

Section A (continued)

10. Write the fractions in order from least to greatest by first changing the fractions to fractions with the same denominator:
   a) \(\frac{1}{2}, \frac{2}{5}, \frac{7}{10}\)
   b) \(\frac{1}{3}, \frac{1}{2}, \frac{5}{6}\)
   c) \(\frac{1}{2}, \frac{3}{4}, \frac{5}{8}\)

11. Draw a picture to show which fraction is greater:
   a) \(2\frac{1}{2}\) or \(\frac{5}{3}\)

12. Bagels come in bags of eight. How many bagels are in \(2\frac{3}{4}\) bags?

13. Shade \(\frac{2}{5}\) of the squares.
    Draw stripes in \(\frac{1}{4}\) of the squares.

14. Twelve children had drinks for lunch. \(\frac{2}{3}\) had juice. \(\frac{1}{4}\) had water.
   a) How many children had juice?
   b) How many had water?
   c) How many did not have either drink?
Section B

15. Write a fraction and a decimal for each shaded part:

16. Fill in the missing numbers:
   a) .94 = ____ tenths _____ hundredths
   b) .37 = ____ tenths _____ hundredths
   c) .41 = ____ tenths _____ hundredths
   d) .05 = ____ tenths _____ hundredths

17. Write as a decimal:
   a) 8 tenths 3 hundredths =
   b) 0 tenths 7 hundredths =
   c) 3 tenths 2 hundredths =
   d) 0 tenths 5 hundredths =

18. Write the following decimals as fractions. Reduce your answers where possible:
   a) .6 =
   b) .53 =
   c) .04 =
   d) .1 =
   e) .48 =

19. Change the following fractions to decimals:
   a) \( \frac{76}{100} = \)
   b) \( \frac{6}{100} = \)
   c) \( \frac{46}{100} = \)
   d) \( \frac{8}{100} = \)

20. Using numbers and words, write the amount of tenths and hundredths in each of the following decimals:
   a) .3
   b) .05
   c) .97
   _______ tenths
   _______ hundredths
Number Sense

Unit Test

Section B (continued)

21. Write the numbers in order from least to greatest by first changing each decimal or fraction to a fraction with a denominator of 10:
   a) 0.8 , 0.3 , 0.4   b) \( \frac{7}{10} , 0.2 , \frac{1}{10} \)   c) 0.3 , 0.6 , \( \frac{2}{5} \)   d) 1.39 , 1 \( \frac{30}{100} \) , 1 \( \frac{49}{100} \)

22. Write the following fractions as decimals:
   a) \( \frac{875}{1000} \)   b) \( \frac{25}{1000} \)

23. Compare each pair of decimals by writing < or > in the box:
   HINT: Add zeroes wherever necessary to give each number the same number of digits.
   a) .275 \( \square \) .273   b) .27 \( \square \) .123   c) .596 \( \square \) .7   d) 1.7 \( \square \) 1.6

24. Line up the decimals and add or subtract the following decimals:
   a) 0.32 + 0.97 =   b) 0.64 – 0.23 =   c) 0.94 + 0.3 =

25. Find the products:
   a) 3 \times 8.3 =   b) 8 \times 2.63 =   c) 7 \times .207 =

26. Divide:
   a) 0.3 ÷ 10 =   b) 0.5 ÷ 100 =
   c) 17 ÷ 10 =   d) 27 ÷ 100 =
   e) 6.2 ÷ 100 =   f) .03 ÷ 10 =
Section B (continued)

27. Divide:

\[
8 \overline{1.44}
\]

28. Karen cycled 62.4 km in 4 hours. How many km did she cycle in an hour? Show your work:

29. Which is a better deal: 6 pens for $4.99 or 8 pens for $6.99? Show your work:

30. Round each decimal to the nearest tenth. Underline the hundredths digit first:
   
   a) 0.25
   b) 0.32
   c) 0.68
   d) 1.35

31. Round each decimal to the nearest whole number. Underline the tenths digit first:
   
   a) 3.25
   b) 4.13
   c) 2.95
   d) 68.7

32. Add:

   a) \(3000 + 200 + 7 + 0.02\) = 
   b) \(10000 + 500 + 20 + 0.1 + .05\) =

33. Which is greater: 3.70 or 3.07? Explain.

34. Write a decimal…

   a) between 4.257 and 4.253:
   b) One thousandth greater than 4.270:
Number Sense

Unit Test

Name: _____________________________  Date: _________________

Section C

35. □ □ □ □ □ □ □ □ □

a) What does the ratio 2 : 3 describe (i.e. what shapes are being compared)?

b) What does the ratio 5 : 10 describe?

36. Solve the following ratios. Draw arrows to show what you multiply by:

a) \( \frac{3}{4} = \frac{20}{\text{?}} \)  
   b) \( \frac{2}{3} = \frac{12}{\text{?}} \)  
   c) \( \frac{6}{7} = \frac{35}{\text{?}} \)

   d) \( \frac{15}{25} = \frac{100}{\text{?}} \)  
   e) \( \frac{12}{20} = \frac{80}{\text{?}} \)  
   f) \( \frac{21}{30} = \frac{90}{\text{?}} \)

37. Write the following percents as fractions:

a) 7% =  \( \frac{?}{100} \)  
   b) 92% =  \( \frac{?}{100} \)  
   c) 5% =  \( \frac{?}{100} \)  
   d) 50% =  \( \frac{?}{100} \)  
   e) 100% =  \( \frac{?}{100} \)

38. Write the following fractions as percents:

a) \( \frac{2}{100} = \)  
   b) \( \frac{31}{100} = \)  
   c) \( \frac{52}{100} = \)  
   d) \( \frac{100}{100} = \)  
   e) \( \frac{88}{100} = \)

39. Write each fraction as a percent by changing it to a fraction over 100:

a) \( \frac{2}{5} = \)  
   b) \( \frac{3}{4} = \)  
   c) \( \frac{1}{2} = \)

40. Write the following decimals as a percents. Show your work:

a) .2  
   b) .9

41. Change the following fractions to percents by first reducing them to lowest terms:

a) \( \frac{9}{15} = \)  
   b) \( \frac{3}{6} = \)  
   c) \( \frac{10}{40} = \)
Section C (continued)

42. Write each set of numbers in order from least to greatest. (Change all of the numbers into fractions with denominator 100.)

   a) \( \frac{3}{5} , \quad 42\% , \quad .73 \)
   b) \( \frac{1}{2} , \quad .73 , \quad 80\% \)

43. Find the following percents by first finding 10% of each number:

   a) 60% of 35
   b) 40% of 24
   c) 20% of 1.3

44. Find 15% of the following numbers by finding 10% and 5%.

   a) 60
   b) 240
   c) 12

45. The top of a pentagonal box has a perimeter of 3.85 m. How long is each side?

46. A family travelled in a car for 105 days. Gas cost $72 each week. How much money did they spend on gas?

47. Tony bought a book for $17.25 and a pen for $2.35. He paid 15% more in taxes. How much change did he receive from $25.00?
Section C (continued)

48. It took Cindy 20 minutes to finish her homework. She spent \( \frac{2}{5} \) of the time on math and \( \frac{1}{4} \) of the time on history.
   a) How many minutes did she spend on math and history?
   b) How many minutes did she spend on other subjects?
   c) What percent of the time did she spend on other subjects?

49. In Angela’s class there are 30 children. 60% are girls. In Steven’s class there are 27 children. The ratio of boys to girls is 5:4. Which class has more boys?

50. Dianne copied the following data from a circle graph she saw on the web.

<table>
<thead>
<tr>
<th>Favourite Subjects of Grade 6 Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science</td>
</tr>
<tr>
<td>31%</td>
</tr>
</tbody>
</table>

a) How can you tell that she made a mistake?

b) Her mistake was in the last column. What should the percentage of students who prefer arts be?

c) Draw the circle graph Dianne saw on the web.
Section A

1. a) 13
   b) \(\frac{8}{13}\)
   c) Yes – 13 + 2 = 6.5 and 8 > 6.5, so the team won more than half its games.

2. From the chart, we can see that the house has a total of 12 walls.
   a) \(\frac{1}{12}\)
   b) Yellow, since \(\frac{1}{4} = \frac{3}{12}\)
   c) White, since \(\frac{1}{2} = \frac{6}{12}\)

3. a) \(\frac{1}{7} + \frac{2}{7} = \frac{3}{7}\)
   b) \(\frac{2}{14} \div \frac{2}{7} = \frac{2}{6} \div \frac{2}{3}\)
   c) \(\frac{9}{19} \div \frac{9}{18} = \frac{9}{9} \div \frac{9}{11}\)

4. a)
   b)

5. a)
   b)

6. Teacher to check that students have cut pies properly:
   a) \(\frac{2}{3} = \frac{4}{6}\)
   b) \(\frac{2}{3} = \frac{6}{9}\)
   c) \(\frac{1}{2} = \frac{2}{4}\)

7. Sample Answer:

   Exact pictures may vary but, in all cases, 3 pieces will have both toppings.

8. a)
   b) \(\frac{2}{3}\) of 6 is 4
   c) \(\frac{3}{4}\) of 12 is 9

9. a) 6
   b) 6
   c) 9
   d) 10
   e) 15
   f) 4
   g) 75
   h) 9

10. a) \(\frac{2}{5} \div \frac{1}{7} = \frac{7}{10}\)
    * first change fractions so denominator = 10
    b) \(\frac{1}{3} \div \frac{1}{5} = \frac{5}{6}\)
    * first change fractions so denominator = 6
    c) \(\frac{1}{2} \div \frac{5}{8} = \frac{3}{4}\)
    * first change fractions so denominator = 8

11. a) \(2\frac{1}{2} > \frac{5}{3}\)
    \(2\frac{1}{2} = \frac{5}{3}\)

12. 2 full bags will have:
   \(8 \times 2 = 16\) bagels
   and:
   \(\frac{3}{4}\) of 8 = 6 bagels
   So there will be 22 bagels (16 + 6) in total.

13. Exact answers will vary – teacher to check.

14. a) \(\frac{2}{3}\) of 12 = 8
    b) \(\frac{1}{4}\) of 12 = 3
    c) One child (12 – 8 – 3) had neither drink.

Section B

15. a) \(\frac{43}{100} = 0.43\)
    b) \(\frac{34}{100} = 0.34\)
    c) \(\frac{23}{100} = 0.23\)

16. a) 9 tenths
    b) 4 hundredths
    c) 3 tenths
    d) 7 hundredths
    e) 4 tenths
    f) 1 hundredths
    g) 0 tenths
    h) 5 hundredths

17. a) 0.83
    b) 0.07
    c) 0.32
    d) 0.05

18. a) \(\frac{6}{10} = \frac{3}{5}\)
    b) \(\frac{5}{10}\)
    c) \(\frac{4}{10} = \frac{2}{5} = \frac{1}{25}\)
    d) \(\frac{1}{10}\)
    e) \(\frac{48}{100} = \frac{24}{50} = \frac{12}{25}\)

19. a) 0.76
    b) 0.06
    c) 0.46
    d) 0.08

20. a) 3 tenths
    b) 0 hundredths
    c) 9 tenths

21. a) 0.3, 0.4, 0.8
    b) \(\frac{1}{10} \div \frac{0.2}{10}\)
    c) \(\frac{0.3}{2} \div 0.6\)
    d) \(\frac{30}{100} \div 1.39, \frac{49}{100}\)

22. a) 0.875
    b) 0.025

23. a) >
    b) >
    c) <
    d) >

24. a) 1.29
Section C

35. a) triangles : circles  
b) squares : all shapes

36. a) \( \frac{3}{4} = \frac{15}{20} \) 
   b) \( \frac{2}{3} = \frac{8}{12} \) 
   c) \( \frac{6}{7} = \frac{30}{35} \) 
   d) \( \frac{15}{25} = \frac{60}{100} \) 
   e) \( \frac{12}{25} = \frac{48}{80} \) 
   f) \( \frac{24}{30} = \frac{63}{90} \)

37. a) \( \frac{7}{100} \) 
   b) \( \frac{92}{100} = \frac{46}{50} = \frac{23}{25} \) 
   c) \( \frac{5}{100} \) 
   d) \( \frac{50}{100} = \frac{1}{2} \) 
   e) \( \frac{100}{100} = \frac{1}{1} = 1 \)

38. a) 2%  
   b) 31%  
   c) 52%  
   d) 100%  
   e) 88%

39. a) 40%  
   b) 75%  
   c) 50%

40. a) \( \frac{0.2}{100} = \frac{20}{100} = 20\% \) 
   b) \( \frac{0.9}{100} = \frac{90}{100} = 90\% \)

41. a) 60%  
   b) 50%  
   c) 25%

42. a) 42\% \cdot \frac{3}{5} = 0.73 
   b) \( \frac{1}{2}, 0.73, 80\% \)

43. a) 10% of 35 = 3.5 and 3.5 \times 6 = 21 
   So: 60% of 35 is 21 
   b) 10% of 24 = 2.4 and 2.4 \times 4 = 9.6 
   So: 40% of 24 is 9.6

44. a) 10% of 60 = 6 and 5% of 60 = 3 
   So: 15% of 60 = 9 + 3 = 12 
   b) 10% of 240 = 24 and 5% of 240 = 12 
   So: 15% of 240 = 24 + 12 = 36 
   c) 10% of 12 = 1.2 and 5% of 12 = 0.6 
   So: 15% of 12 = 1.2 + 0.6 = 1.8

45. \( 3.85 + 5 = 0.77 \) m

46. There are 7 days in a week so, to find the number of weeks the family travelled, we use division: 
   \( 105 \div 7 = 15 \) weeks 
   Price of gas: 
   \$72 per week \times 15 weeks = \$1080 
   So the family spent \$1080 on gas.

47. The total of Tony's purchases was: 
   \$17.25 + \$2.35 = \$19.60 
   Taxes paid would then be 15% of \$19.60: 
   10% of \$19.60 = \$1.96 
   5% of \$19.60 = \$0.98 
   15% of \$19.60 = \$2.94 
   In total, Tony spent: 
   \$19.60 + \$2.94 = \$22.54. 
   So his change would be \$2.46 (\$25.00 – \$22.54).

48. a) Cindy spent 8 minutes \( \frac{2}{5} \) of 20 minutes) on math. 
   She spent 5 minutes \( \frac{1}{4} \) of 20 minutes) on history. 
   So, in total, she spent 13 minutes (8 + 5) on these two subjects.
**Measurement**

*Unit Test*

**Section A**

1. Write a measurement in a whole number of cm that is between ...
   
   a) 83 mm and 75 mm: _____ cm  
   b) 36 mm and 66 mm: ________  
   c) 34 mm and 5 cm: ________

2. Find the numbers missing from the following charts:

<table>
<thead>
<tr>
<th>mm</th>
<th>cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
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<tr>
<td>444</td>
<td>600</td>
</tr>
<tr>
<td>70</td>
<td>420</td>
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</table>

<table>
<thead>
<tr>
<th>cm</th>
<th>dm</th>
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<tbody>
<tr>
<td>70</td>
<td>240</td>
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<tr>
<td>600</td>
<td>100</td>
</tr>
<tr>
<td>420</td>
<td>35</td>
</tr>
</tbody>
</table>

3. Write a measurement in a whole number of dm that is between ...

   a) 51 and 61 cm: ________  
   b) 25 and 41 cm: ________  
   c) 68 and 74 cm: ________

4. The Sky Tower in New Zealand is 328 m high. About how many Sky Towers, laid end to end, would make a kilometre? Show your work:

5. Clare can cycle at a speed of 21 km/hr and Erin can cycle at a speed of 15 km/hr. How much further can Clare cycle in 3 hours than Erin? Show your work:

6. Helen walked 3 km in the first hour and then cycled 13 km in the second hour. How far did she travel? What was her average speed?
Measurement

Unit Test

7. Fill in the missing numbers:
   a) 10 cm = ___________ mm  
   b) 10 dm = ___________ cm  
   c) 10 dm = ___________ mm

8. Convert the measurement given in cm to a measurement using multiple units:
   a) 407 cm = _____ m _____ cm  
   b) 823 cm = _____ m _____ cm

9. Is 492 mm longer or shorter than 20 cm? Explain how you know:

10. For the questions below, you will need to multiply or divide by 10 or 100. Look at the units carefully and fill in the missing numbers and words in each step.
   a) Change 14 m to a measure in dm:
      i) The new units are _____ times _________
      ii) So I need _____ times _________ units
      iii) So I _______________ by _______
      14 m = _______ dm
   b) Change 23 cm to a measure in m:
      i) The new units are _____ times _________
      ii) So I need _____ times _________ units
      iii) So I _______________ by _______
      23 cm = _______ m

11. Change the units using the same steps as in Question 10.
   a) 3.5 mm = ___________ cm  
   b) 2.31 kg = _____________ g  
   c) 7 cm = _____________ m
   d) 14.62 mm = _________ dm  
   e) 2.05 cm = ___________ dm  
   f) 152 mg = ___________ g
   g) 37 mL = _____________ L  
   h) 2.75 L = ____________ mL  
   i) 305 g = ______________ mg

12. Name any object in your classroom. Write down a unit of measurement that would be best for measuring it. Explain why it would be the best unit of measurement:
Measurement
Unit Test

Section B
13. Use a ruler to measure the perimeter of each figure (in cm):
   a)  
   b)  
   c)  

14. Find the perimeter of each shape. Be sure to include the units in your answer:
   a)  
   b)  
   c)  
   d)  
   e) Write the letters of the shapes in order from greatest perimeter to least perimeter. (Make sure you look at the units!)

15. Find the area of these figures in square centimetres:
   a)  
   b)  
   c)  
   Area = _____ cm²  
   Area = _____ cm²  
   Area = _____ cm²
Measurement
Unit Test

Section B (continued)

16. Find the area (in cm$^2$) of each of the given shapes:

- Area of A = _________________
- Area of B = _________________
- Area of C = _________________

17. Find the area of the rectangles with the following dimensions:
   a) width: 6 m length: 7 m   b) width: 3 m length: 7 m   c) width: 4 cm length: 8 cm

18. A rectangle has an area of 18 cm$^2$ and a length of 6 cm. What is its width?

19. Measure the length and width of each rectangle, then calculate its perimeter and area:

   a) Perimeter = _____ cm
      Area = _____ cm$^2$
   b) Perimeter = _____ cm
      Area = _____ cm$^2$
   c) Perimeter = _____ cm
      Area = _____ cm$^2$
20. Show all the ways you can make a rectangle with a perimeter of 12 units:

21. A rectangle has sides whose lengths are whole number of cm. Its area is 24 cm$^2$. Find all the possible rectangles of this sort:

22. Sally says she can find the area of a rectangle if she knows the perimeter of the rectangle and the length of one side. Is she correct? Explain with an example.
23. Calculate the area of each shape. Show your work:

a) \[ \text{Area} = \]  
b) \[ \text{Area} = \]  
c) \[ \text{Area} = \]  

24. Find the area of the following parallelograms:

a) Base = 6 cm \hspace{1cm} b) Base = 2 cm \hspace{1cm} c) Base = 7 cm \hspace{1cm} d) Base = 5 cm  
   \hspace{1cm} Height = 8 cm \hspace{1cm} Height = 6 cm \hspace{1cm} Height = 2 cm \hspace{1cm} Height = 3 cm  
   \hspace{1cm} Area = \hspace{1cm} Area = \hspace{1cm} Area = \hspace{1cm} Area =  

25. Measure the base and height of the triangle using a ruler. Then find the area of the triangle:

a) \[ \text{Area} = \]  
b) \[ \text{Area} = \]  
c) \[ \text{Area} = \]  

26. A parallelogram has base 8 cm and area 24 cm$^2$. How high is the parallelogram?

27. Each edge on the grid represents .5 cm. Is the perimeter of the rectangle greater than or less than .145 m? How do you know?
**Section A**

1. a) 8 cm  
   b) Answers may vary: 4, 5 or 6 cm  
   c) 4 cm

2. | mm | cm |
<table>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>60</td>
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<table>
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<tr>
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<th>dm</th>
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</thead>
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<td>7</td>
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<tr>
<td>4200</td>
<td>420</td>
</tr>
<tr>
<td>600</td>
<td>60</td>
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</table>

<table>
<thead>
<tr>
<th>m</th>
<th>dm</th>
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<tbody>
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<td>240</td>
<td>2400</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>35</td>
<td>350</td>
</tr>
</tbody>
</table>

3. a) 6 dm  
   b) Answers may vary: 3 or 4 dm  
   c) 7 dm

4. 1 km = 1 000 m, and: 1 000 ÷ 328 = 3 R16  
   So it would take about 3 Sky Towers, laid end to end, to make a kilometre.

5. In 3 hours, Clare can travel 21 km × 3 = 63 km  
   In 3 hours, Erin can travel 15 km × 3 = 45 km  
   Difference: 63 km – 45 km = 18 km  
   In 3 hours, Clare can cycle 18 km more than Erin.

6. Total distance travelled: 3 km + 13 km = 16 km  
   Average Speed: 16 km ÷ 2 hours = 8 km/hr  
   Helen travelled 16 km, at an average speed of 8 km/hr.

7. a) 100  
    b) 100  
    c) 1 000

8. a) 4 m 7 cm  
    b) 8 m 23 cm

9. 492 mm is longer than 20 cm = 200 mm.

10. a) i) 10 times smaller  
      ii) 10 times more units  
      iii) multiply by 10  
      14 m = 140 dm  
   b) i) 100 times larger  
      ii) 100 times less units  
      iii) divide by 100  
      23 cm = 0.23 m

11. a) 0.35 cm  
      b) 2 310 g  
      c) 0.07 m  
      d) .1462 dm  
      e) .205 dm  
      f) 0.152 g  
      g) 0.037 L  
      h) 2750 mL  
      i) 305 000 mg

12. Answers will vary.  
    Teacher to check.

13. a) 10 cm  
      b) 10 cm  
      c) 14 cm

14. a) 28 m  
      b) 56 cm  
      c) 9 km  
      d) 36 cm  
      e) C, A, B, D

15. a) 8 cm²  
      b) 8 cm²  
      c) 9 cm²  
      d) 23 cm²  
      e) 32 cm²  
      f) 18 + 6 = 3  
      So the width of the rectangle is 3 cm.

16. Area of A = 6 cm²  
    Area of B = 8 cm²  
    Area of C = 12 cm²

17. a) 42 m²  
      b) 21 m²  
      c) 32 cm²

18. 18 ÷ 6 = 3  
    So the width of the rectangle is 3 cm.

19. a) Length = 5 cm  
      Width = 2 cm  
      Perimeter = 14 cm  
      Area = 10 cm²  
   b) Length = 2 cm  
      Width = 1 cm  
      Perimeter = 6 cm  
      Area = 2 cm²  
   c) Length = 3 cm  
      Width = 2 cm  
      Perimeter = 10 cm  
      Area = 6 cm²

20. Rectangle dimensions (teacher to check student diagrams):  
    1 × 5  
    2 × 4  
    3 × 3 – since a square is also a rectangle

21. 1 × 24  
    2 × 12  
    3 × 8  
    4 × 6

22. Yes, Sally is correct.  
    Explanation will vary.  
    Perimeter = (length + width) × 2, so to get the width divide the perimeter by 2 and subtract the length.  
    Multiply the width by the length to get the area.

23. a) 6 square units  
      b) 8.5 square units  
      c) 8.5 square units

24. a) Area = 48 cm²  
      b) Area = 12 cm²  
      c) Area = 14 cm²  
      d) Area = 15 cm²

25. a) Base = 6 cm  
      Height = 2 cm  
      Area = 6 cm²  
   b) Base = 4 cm  
      Height = 2 cm  
      Area = 4 cm²  
   c) Base = 4 cm  
      Height = 3 cm  
      Area = 6 cm²

26. 24 ÷ 8 = 3  
    So the parallelogram is 3 cm high.

27. The grey rectangle has a perimeter of 20 squares.  
    Since the edge of each square is 0.5 cm long, the perimeter of the rectangle is:  
    20 × 0.5 cm = 10 cm  
    And 10 cm = 0.1 m, which is less than .145 m.
Probability & Data Management

Unit Test

Name: _____________________________

Date: _________________

Section A

1. What are the possible outcomes for these spinners?

a) ____________
   ____________
   ______ outcomes

b) ____________
   ____________
   ______ outcomes

c) ____________
   ____________
   ______ outcomes

2. For each spinner, write the probability of spinning red. Reduce your answer if possible:

a) b) c) d)

3. Write a fraction that gives the probability of spinning:

   a) the number 4
   b) the number 5
   c) an even number
   d) an odd number
   e) a number less than 7
   f) a number greater than 3

4. Imogen throws a dart at this board. The dart can only land on the board. Write the probability of the dart landing on each colour:
Section A (continued)

5. For each spinner below, what fraction of your spins would you expect to be red?
   a) I would expect \[ \frac{\text{number of red}}{\text{total spins}} \] of my spins to be red.
   b) 

6. Label the balls red (R) or green (G) to match the probability of drawing a ball of the given colour:
   a) \[ P(\text{Green}) = \frac{2}{3} \]
   b) \[ P(\text{Red}) = \frac{1}{2} \]
      \[ P(\text{Green}) = \frac{1}{4} \]

7. Use the words impossible, likely, unlikely or certain to describe the following events:
   a) If you flip a coin once, you will get a head and a tail: ________________
   b) If you roll a die once, you will get a number less than six: ________________
   c) Eight metres of snow will fall today: ________________

8. Write numbers on the spinners to match the probabilities:
   a) The probability of spinning a 3 is \[ \frac{1}{4} \].
   b) The probability of spinning an even number is \[ \frac{5}{6} \].
   c) The probability of spinning a multiple of 3 is \[ \frac{2}{5} \].
   d) The probability of spinning a 2 is \[ \frac{1}{2} \].

9. If you spun the following spinners 50 times, how many times would you expect to spin yellow? Show your work:
   a) \[ \text{times} \]
   b) \[ \text{times} \]
Section A (continued)

10. If you spun this spinner 21 times…

   a) How many of your spins would you expect to be green? Show your work.

   b) Which of these charts shows a result you’d be most likely to get? Explain.

   c) Which result would surprise you? Why?

11. The probability of spinning blue on a spinner is \( \frac{1}{3} \). If you used the spinner 100 times about how many times would you expect to spin blue?

12. Sketch a spinner on which the probability of spinning red is \( \frac{3}{4} \):
Section B

13. If you flip a coin there are two outcomes: heads (H) and tails (T). Using the chart provided, list all the outcomes for flipping a coin and spinning the spinner given below:

<table>
<thead>
<tr>
<th>Coin</th>
<th>Spinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>G</td>
</tr>
</tbody>
</table>

14. Draw a tree diagram to show all the combinations of numbers you could spin on these two spinners:

- [Diagram of two spinners showing combinations 1-4]

a) How many of the combinations add to four? _______

b) How many of the combinations have a product of four? _______

15. You have three coins in your pocket: a penny (P), a nickel (N) and a dime (D).

a) What are all the possible combinations of two coins you could pull out?
   HINT: Use alphabetical order to organize your answer.

b) Would you expect to pull a pair of coins that add to 6 cents? Are the chances likely or unlikely? Explain.
Section A

1. a) 1, 3, 5, 7; 4 outcomes
   b) 8; 1 outcome
   c) 2, 4, 6; 3 outcomes

2. a) \( P(R) = \frac{1}{4} \)
   b) \( P(R) = \frac{2}{5} \)
   c) \( P(R) = \frac{3}{8} = \frac{1}{2} \)
   d) \( P(R) = 0 \)

3. a) \( \frac{1}{8} \)
   b) \( \frac{2}{8} = \frac{1}{4} \)
   c) \( \frac{3}{8} \)
   d) \( \frac{4}{8} \)
   e) \( \frac{6}{8} = \frac{3}{4} \)
   f) \( \frac{5}{8} \)

4. Students should think of the board like this:

   B  B  R  R
   B  B  R  R
   G  B  R  R

   \( P(B) = \frac{3}{5} \)
   \( P(G) = \frac{1}{5} \)
   \( P(R) = \frac{3}{5} = \frac{1}{3} \)

5. a) I would expect \( \frac{2}{3} \) of my spins to be red.
   b) I would expect \( \frac{1}{4} \) of my spins to be red.

6. a) Teacher to check answer.
   4 red balls;
   8 green balls.
   b) Teacher to check answer.
   4 red balls;
   2 green balls.

7. a) Impossible
   b) Likely
   c) Unlikely

8. Answers will vary.
   Teacher to check.

9. a) 10 times since:
   \[ P(Y) = \frac{1}{5} \]
   and \( \frac{1}{5} \) of 50 = 10
   b) 30 times since:
   \[ P(Y) = \frac{3}{5} \]
   and \( \frac{3}{5} \) of 50 = 30

10. a) I would expect 7 of 21 spins to be green since:
    \[ P(G) = \frac{1}{3} \]
    and \( \frac{1}{3} \) of 21 = 7
    b) Chart B.
    Reasons will vary.
    Teacher to check.
    c) Charts C and A are unexpected.
    Reasons will vary.
    Teacher to check.

11. The spinner is expected to be blue about 33 times out of 100 –
    \[ P(B) = \frac{1}{3} \]
    and \( \frac{1}{3} \) of 100 = 33
    NOTE:
    Students have to round 33 R1 to 33 since the “number of times” has to be a whole number.

12. Answers will vary.
    Teacher to check.

Section B

13. Coin  | Spinner
         | H  B
         | H  G
         | T  B
         | T  G

14. Two possible answers:

OR

15. a) 2 combinations
      add 4:
      2, 2 and 1, 3
      b) 2 combinations
      have a product of 4:
      2, 2 and 4, 1

b) The chance of pulling a pair of coins that adds to 6¢ are unlikely:
   \[ P = \frac{2}{6} = \frac{1}{3} \]
   and \( \frac{1}{3} < \frac{1}{2} \)
Section A

1. Circle the points in the following positions (connecting the dots first, if necessary):
   a) 3 3 3
       2 2 2
       1 1 1
       1 2 3 1 2 3
   Column 1 Column 2
   Row 2 Row 3
   (1,3) (3,2)

2. Circle the points in the following positions:
   a) 3 3 3
       2 2 2
       1 1 1
       A B C
   (B,2)
   b) C 3 3 3
       B B B
       A A A
       X Y Z
   (X,C)
   c) 2 2 2
       1 1 1
       0 0 0
       0 1 2
   (0,1)
   d) 2 2 2
       1 1 1
       0 0 0
       0 1 2
   (2,0)

3. Graph each set of ordered pairs and join the dots to form a polygon. Identify the polygon drawn:
   a) A (0,2) B (0,4) C (4,4) D (4,2)
      This polygon is a ____________________.
   b) A (2,0) B (1,3) C (3,3) D (4,0)
      This polygon is a ____________________.

4. Write the coordinates of the following points:
   A ( , ) B ( , )
   C ( , ) D ( , )
   E ( , ) F ( , )
   G ( , ) H ( , )
Section B

5. Slide each shape 4 boxes to the right. (Start by putting a dot on one of the corners of the figure.
   Slide the dot four boxes right, then draw the new figure.)

   a) 
   
   b) 

6. Slide each figure 5 boxes to the right and 2 boxes down:

   a) 
   
   b) 

7. Draw the reflection (or flip) of the shapes below:

   a) 
   b) 
   c) 

8. Give two reasons why this picture does not show a reflection:
Geometry
Unit Test

Section B (continued)

9. Show where the arrow would be after each turn:
   a)  b)  c)  d)

   ¼ turn clockwise  ½ turn clockwise  ¼ turn counter clockwise  ½ turn counter clockwise

10. Show what the figure would look like after the rotation. First rotate the dark line, then draw the rest of the figure:
   a)  b)  c)  d)

   ¼ turn clockwise  ½ turn clockwise  ¾ turn counter clockwise  ¼ turn counter clockwise

11. Colour or shade in the sections of the left-hand square using at least 3 colours or shadings. Then create a border design by rotating the square ¼ turn clockwise around the bottom right corner.
Section B (continued)

12. Shapes B, C and D were obtained from shape A by using two transformations.
   Write the correct letter in the blank, and describe each transformation.
   For rotations, mark the centre of the rotation, for reflections, draw the mirror line.

   ____ : Reflection and rotation

   B: ______

   ____ : Rotation and slide

   C: ______

   ____ : Reflection and slide

   D: ______
13. Compare the sets of shapes below. **Name** the shapes first, and then write a paragraph outlining how they are the **same** and how they are **different**:

a)

![Shapes diagram]

<table>
<thead>
<tr>
<th>Name</th>
<th>i –</th>
<th>ii –</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Section C (continued)

b) 

![Diagram of geometric shapes](image)

<table>
<thead>
<tr>
<th>Name</th>
<th>i –</th>
<th>ii –</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

14. Complete the following property chart:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Number of...</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of shapes" /></td>
<td></td>
<td>edges</td>
<td>vertices</td>
</tr>
</tbody>
</table>

| ![Diagram of shapes](image) | | |
| ![Diagram of shapes](image) | | |
| ![Diagram of shapes](image) | | |
Geometry
Unit Test

Section C  (continued)

15. If you know how many sides the base of a prism has, how can you tell how many vertices the prism has? Explain.

16. Draw the front, top and side view of the figure given by this mat plan.


Section A

1. a) The polygon is a rectangle.
b) The polygon is a parallelogram.
c) The polygon is a parallelogram.
d) The polygon is a parallelogram.

2. a) 3 2 1 A B C
b) C B A X Y Z
c) 2 1 0
   0 1 2
d) 2 1 0
   0 1 2

3. a) The polygon is a rectangle.
b) The polygon is a parallelogram.

4. A (3, 2) B (9, 1)
   C (8, 4) D (6, 3)
   E (1, 1) F (4, 4)
   G (0, 5) H (5, 0)

Section B

5. NOTE: Location of dots may vary.
a) b) c) d) e) f)

6. a) The two shapes are not the same size and both shapes are facing the same direction (which is NOT a reflection).
   Exact answers may vary. Teacher to check.
   b) c) d) e) f)

7. a) b) c) d) e) f)

8. The two shapes are not the same size and both shapes are facing the same direction (which is NOT a reflection).
   Exact answers may vary. Teacher to check.

9. a) b) c) d) e) f)

10. a) b) c) d) e) f)

11. Answers will vary. Teacher to check.

12. From top to bottom: C, D, B
   Descriptions will vary, teacher to check.
Section C

13. a) Name:
   i) Rectangular Prism
   ii) Triangular Prism
   Answers will vary, but should include:
   
   Similarities:
   • both are prisms
   • both have 2 bases
   • Non-base faces are rectangles
   
   Differences:
   • i) has 2 rectangular bases
   • ii) has triangular bases
   • Any pair of opposite faces can be considered bases in i) not so for ii)
   • # of edges, faces, vertices

b) Name:
   i) Rectangular Pyramid
   ii) Rectangular Prism
   Answers will vary, but should include:
   
   Similarities:
   • both have a rectangular base
   
   Differences:
   • i) has 1 base, ii) has 2 bases
   • # of edges, faces, vertices
   • The non-base faces are triangles in i) and rectangles in ii).
   • i) has a vertex opposite to the base, ii) doesn’t.

14. Hexagonal Pyramid
   edges – 12
   vertices – 7
   faces – 7

15. Answers may vary.
   Teacher to check.
   Sample Answer:
   Each vertex of a prism belongs either to the top or the bottom base.
   The bases have the same number of vertices, so the number of vertices of a prism is twice the number of vertices in the base.

16. Top view
   
   Front view
   
   Side view
# Contents

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<tr>
<td>Measurement</td>
<td>5</td>
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<td>Geometry and Spatial Sense</td>
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<td>Patterning and Algebra</td>
<td>9</td>
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<tr>
<td>Data Management and Probability</td>
<td>11</td>
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</tbody>
</table>
Notes

To ensure that the curriculum is fully covered, use the worksheets with the lessons plans in the Teacher’s Guide.

Starred lesson numbers (*) indicate that the curriculum requirement is covered primarily in the lesson plan (possibly in the activities or extensions).

OCUP: Ontario Curriculum Unit Planner

JUMP Math workbook units are represented by:

- **NS** Number Sense
- **PA** Patterns and Algebra
- **ME** Measurement
- **G** Geometry
- **PDM** Probability and Data Management
# Number Sense and Numeration

## Overall Expectations

By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m8</td>
<td>read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;</td>
</tr>
<tr>
<td>6m9</td>
<td>solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;</td>
</tr>
<tr>
<td>6m10</td>
<td>demonstrate an understanding of relationships involving percent, ratio, and unit rate.</td>
</tr>
</tbody>
</table>

## Quantity Relationships

By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>6m11 represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools;</td>
<td>1</td>
</tr>
<tr>
<td>6m12 demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools and strategies;</td>
<td>1</td>
</tr>
<tr>
<td>6m13 read and print in words whole numbers to one hundred thousand, using meaningful contexts;</td>
<td>1</td>
</tr>
<tr>
<td>6m14 represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools and using standard fractional notation;</td>
<td>2</td>
</tr>
<tr>
<td>6m15 estimate quantities using benchmarks of 10%, 25%, 50%, 75%, and 100%;</td>
<td>2</td>
</tr>
<tr>
<td>6m16 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1 000 000;</td>
<td>1</td>
</tr>
<tr>
<td>6m17 identify composite numbers and prime numbers, and explain the relationship between them.</td>
<td>1</td>
</tr>
</tbody>
</table>
### Operational Sense

By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
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</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>6m18 use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers;</td>
<td>1</td>
</tr>
<tr>
<td>6m19 solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools and strategies;</td>
<td>1</td>
</tr>
<tr>
<td>6m20 add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators;</td>
<td>2</td>
</tr>
<tr>
<td>6m21 multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators;</td>
<td>2</td>
</tr>
<tr>
<td>6m22 multiply whole numbers by 0.1, 0.01, and 0.001 using mental strategies;</td>
<td>2</td>
</tr>
<tr>
<td>6m23 multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies;</td>
<td>2</td>
</tr>
<tr>
<td>6m24 use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution;</td>
<td>1</td>
</tr>
<tr>
<td>6m25 explain the need for a standard order for performing operations, by investigating the impact that changing the order has when performing a series of operations.</td>
<td>2</td>
</tr>
</tbody>
</table>

### Proportional Relationships

By the end of Grade 6, students will:

<table>
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<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>6m26 represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation;</td>
<td>2</td>
</tr>
<tr>
<td>6m27 determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions, decimal numbers, and percents;</td>
<td>2</td>
</tr>
<tr>
<td>6m28 represent relationships using unit rates.</td>
<td>2</td>
</tr>
</tbody>
</table>
### Measurement

**Overall Expectations**
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m29</td>
<td>estimate, measure, and record quantities, using the metric measurement system;</td>
</tr>
<tr>
<td>6m30</td>
<td>determine the relationships among units and measurable attributes, including the area of a parallelogram, the area of a triangle, and the volume of a triangular prism.</td>
</tr>
</tbody>
</table>

### Attributes, Units and Measurement Sense
By the end of Grade 6, students will:

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<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------</td>
</tr>
<tr>
<td>6m31</td>
<td>demonstrate an understanding of the relationship between estimated and precise measurements, and determine and justify when each kind is appropriate;</td>
</tr>
<tr>
<td>6m32</td>
<td>estimate, measure, and record length, area, mass, capacity, and volume, using the metric measurement system.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Measurement Relationships
By the end of Grade 6, students will:

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------------</td>
</tr>
<tr>
<td>6m33</td>
<td>select and justify the appropriate metric unit (i.e., millimetre, centimetre, decimetre, metre, decametre, kilometre) to measure length or distance in a given real-life situation;</td>
</tr>
<tr>
<td>6m34</td>
<td>solve problems requiring conversion from larger to smaller metric units;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m35</td>
<td>construct a rectangle, a square, a triangle, and a parallelogram, using a variety of tools, given the area and/or perimeter;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m36</td>
<td>determine, through investigation using a variety of tools and strategies, the relationship between the area of a rectangle and the areas of parallelograms and triangles, by decomposing and composing;</td>
</tr>
<tr>
<td>6m37</td>
<td>develop the formulas for the area of a parallelogram and the area of a triangle, using the area relationships among rectangles, parallelograms, and triangles;</td>
</tr>
</tbody>
</table>
## Measurement Relationships (continued)

By the end of Grade 6, students will:

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<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
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</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>6m38</td>
<td>solve problems involving the estimation and calculation of the areas of triangles and the areas of parallelograms;</td>
</tr>
<tr>
<td>6m39</td>
<td>determine, using concrete materials, the relationship between units used to measure area (i.e., square centimetre, square metre), and apply the relationship to solve problems that involve conversions from square metres to square centimetres;</td>
</tr>
<tr>
<td>6m40</td>
<td>determine, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula;</td>
</tr>
<tr>
<td>6m41</td>
<td>determine, through investigation using a variety of tools and strategies, the surface area of rectangular and triangular prisms;</td>
</tr>
<tr>
<td>6m42</td>
<td>solve problems involving the estimation and calculation of the surface area and volume of triangular and rectangular prisms.</td>
</tr>
</tbody>
</table>
# Geometry and Spatial Sense

## Overall Expectations

By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m43</td>
<td>classify and construct polygons and angles;</td>
</tr>
<tr>
<td>6m44</td>
<td>sketch three-dimensional figures, and construct three-dimensional figures from drawings;</td>
</tr>
<tr>
<td>6m45</td>
<td>describe location in the first quadrant of a coordinate system, and rotate two-dimensional shapes.</td>
</tr>
</tbody>
</table>

## Geometric Properties

By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------</td>
</tr>
<tr>
<td>6m46</td>
<td>sort and classify quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools and strategies;</td>
</tr>
<tr>
<td>6m47</td>
<td>sort polygons according to the number of lines of symmetry and the order of rotational symmetry, through investigation using a variety of tools;</td>
</tr>
<tr>
<td>6m48</td>
<td>measure and construct angles up to 180° using a protractor, and classify them as acute, right, obtuse, or straight angles;</td>
</tr>
<tr>
<td>6m49</td>
<td>construct polygons using a variety of tools, given angle and side measurements.</td>
</tr>
</tbody>
</table>

## Geometric Relationships

By the end of Grade 6, students will:

<table>
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<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
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</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------</td>
</tr>
<tr>
<td>6m50</td>
<td>build three-dimensional models using connecting cubes, given isometric sketches or different views (i.e., top, side, front) of the structure;</td>
</tr>
<tr>
<td>6m51</td>
<td>sketch, using a variety of tools, isometric perspectives and different views (i.e., top, side, front) of three-dimensional figures built with interlocking cubes.</td>
</tr>
</tbody>
</table>
Location and Movement
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>6m52</td>
<td>explain how a coordinate system represents location, and plot points in the first quadrant of a Cartesian coordinate plane;</td>
</tr>
<tr>
<td>6m53</td>
<td>identify, perform, and describe, through investigation using a variety of tools, rotations of 180° and clockwise and counterclockwise rotations of 90°, with the centre of rotation inside or outside the shape;</td>
</tr>
<tr>
<td>6m54</td>
<td>create and analyze designs made by reflecting, translating, and/or rotating a shape, or shapes, by 90° or 180°.</td>
</tr>
</tbody>
</table>
Patterning and Algebra

Overall Expectations
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m55</td>
<td>describe and represent relationships in growing and shrinking patterns (where the terms are whole numbers), and investigate repeating patterns involving rotations;</td>
</tr>
<tr>
<td>6m56</td>
<td>use variables in simple algebraic expressions and equations to describe relationships.</td>
</tr>
</tbody>
</table>

Patterns and Relationships
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>6m57</td>
<td>identify geometric patterns, through investigation using concrete materials or drawings, and represent them numerically;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m58</td>
<td>make tables of values, for growing patterns given pattern rules, in words, then list the ordered pairs (with the first coordinate representing the term number and the second coordinate representing the term) and plot the points in the first quadrant, using a variety of tools;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m59</td>
<td>determine the term number of a given term in a growing pattern that is represented by a pattern rule in words, a table of values, or a graph;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m60</td>
<td>describe pattern rules (in words) that generate patterns by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term, then distinguish such pattern rules from pattern rules, given in words, that describe the general term by referring to the term number;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m61</td>
<td>determine a term, given its term number, by extending growing and shrinking patterns that are generated by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term;</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6m62</td>
<td>extend and create repeating patterns that result from rotations, through investigation using a variety of tools.</td>
</tr>
</tbody>
</table>
### Variables, Expressions, and Equations

By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OCUP Code</strong></td>
<td><strong>Specific Expectation</strong></td>
</tr>
<tr>
<td>6m63</td>
<td>demonstrate an understanding of different ways in which variables are used;</td>
</tr>
<tr>
<td> </td>
<td> </td>
</tr>
<tr>
<td>6m64</td>
<td>identify, through investigation, the quantities in an equation that vary and those that remain constant;</td>
</tr>
<tr>
<td>6m65</td>
<td>solve problems that use two or three symbols or letters as variables to represent different unknown quantities;</td>
</tr>
<tr>
<td> </td>
<td> </td>
</tr>
<tr>
<td>6m66</td>
<td>determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies.</td>
</tr>
</tbody>
</table>
Data Management and Probability

Overall Expectations
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>OCUP Code</th>
<th>Overall Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m67</td>
<td>collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including continuous line graphs;</td>
</tr>
<tr>
<td>6m68</td>
<td>read, describe, and interpret data, and explain relationships between sets of data;</td>
</tr>
<tr>
<td>6m69</td>
<td>determine the theoretical probability of an outcome in a probability experiment, and use it to predict the frequency of the outcome.</td>
</tr>
</tbody>
</table>

Collection and Organization of Data
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code</td>
<td>Specific Expectation</td>
</tr>
<tr>
<td>6m70</td>
<td>collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;</td>
</tr>
<tr>
<td>6m71</td>
<td>collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including continuous line graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools;</td>
</tr>
<tr>
<td>6m72</td>
<td>select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph; (i.e., from types of graphs already studied, such as pictographs, horizontal or vertical bar graphs, stem-and-leaf plots, double bar graphs, stem-and-leaf plots, broken-line graphs, and continuous line graphs)</td>
</tr>
<tr>
<td>6m73</td>
<td>determine, through investigation, how well a set of data represents a population, on the basis of the method that was used to collect the data.</td>
</tr>
</tbody>
</table>
Data Relationships
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>6m74 read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including continuous line graphs);</td>
<td>1</td>
</tr>
<tr>
<td>6m75 compare, through investigation, different graphical representations of the same data;</td>
<td>1</td>
</tr>
<tr>
<td>6m76 explain how different scales used on graphs can influence conclusions drawn from the data;</td>
<td>1</td>
</tr>
<tr>
<td>6m77 demonstrate an understanding of mean, and use the mean to compare two sets of related data, with and without the use of technology;</td>
<td>1</td>
</tr>
<tr>
<td>6m78 demonstrate, through investigation, an understanding of how data from charts, tables, and graphs can be used to make inferences and convincing arguments.</td>
<td>1</td>
</tr>
</tbody>
</table>

Probability
By the end of Grade 6, students will:

<table>
<thead>
<tr>
<th>ONTARIO CURRICULUM EXPECTATION</th>
<th>JUMP MATH WORKBOOK</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCUP Code Specific Expectation</td>
<td>Part</td>
</tr>
<tr>
<td>6m79 express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely;</td>
<td>2</td>
</tr>
<tr>
<td>6m80 represent the probability of an event (i.e., the likelihood that the event will occur), using a value from the range of 0 (never happens or impossible) to 1 (always happens or certain);</td>
<td>2</td>
</tr>
<tr>
<td>6m81 predict the frequency of an outcome of a simple probability experiment or game, by calculating and using the theoretical probability of that outcome.</td>
<td>2</td>
</tr>
</tbody>
</table>
WNCP Curriculum Correlation: Grade 6

JUMP Math

Contents

Number 3
Patterns and Relations 7
Shape and Space 10
Statistics and Probability 16
Notes

To ensure that the curriculum is fully covered, use the worksheets with the lessons plans in the Teacher’s Guide.

Starred lesson numbers (*) indicate that the curriculum requirement is covered primarily in the lesson plan (possibly in the activities or extensions).

Underlined lesson numbers indicate relevant preparatory exercises.

WNCP Abbreviations:

[C] Communication  
[CN] Connections  
[ME] Mental Mathematics and Estimation  
[PS] Problem Solving  
[R] Reasoning  
[T] Technology  
[V] Visualization

JUMP Math workbook units are represented by:

NS  Number Sense  
PA  Patterns and Algebra  
ME  Measurement  
G  Geometry  
PDM  Probability and Data Management
# Number

## General Outcome

- Develop number sense.

## Develop Number Sense

It is expected that students will:

### 1. WNPC CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of place value for numbers:</td>
<td>Part</td>
</tr>
<tr>
<td>• greater than one million</td>
<td>1</td>
</tr>
<tr>
<td>• less than one thousandth. [C, CN, R, T]</td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Explain how the pattern of the place value system, e.g., the repetition of ones, tens and hundreds, makes it possible to read and write numerals for numbers of any magnitude.
- Provide examples of where large numbers and small decimals are used, e.g., media, science, medicine, technology.

### 2. WNPC CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve problems involving large numbers, using technology. [ME, PS, T]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Identify which operation is necessary to solve a given problem and solve it.
- Determine the reasonableness of an answer.
- Estimate the solution and solve a given problem.

### 3. WNPC CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of factors and multiples by:</td>
<td>Part</td>
</tr>
<tr>
<td>• determining multiples and factors of numbers less than 100</td>
<td>1</td>
</tr>
<tr>
<td>• identifying prime and composite numbers</td>
<td></td>
</tr>
<tr>
<td>• solving problems involving multiples. [PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>
3. **Achievement Indicators**

Identify multiples for a given number and explain the strategy used to identify them.

Determine all the whole number factors of a given number using arrays.

Identify the factors for a given number and explain the strategy used, e.g., concrete or visual representations, repeated division by prime numbers or factor trees.

Provide an example of a prime number and explain why it is a prime number.

Provide an example of a composite number and explain why it is a composite number.

Sort a given set of numbers as prime and composite.

Solve a given problem involving factors or multiples.

Explain why 0 and 1 are neither prime nor composite.

---

4. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relate improper fractions to mixed numbers. [CN, ME, R, V]</td>
<td>2 NS 59, 60, 61–64, 79, 81</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Demonstrate using models that a given improper fraction represents a number greater than 1.

Express improper fractions as mixed numbers.

Express mixed numbers as improper fractions.

Place a given set of fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position.

---

5. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of ratio, concretely, pictorially and symbolically. [C, CN, PS, R, V]</td>
<td>2 NS 95–100</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Provide a concrete or pictorial representation for a given ratio.

Write a ratio from a given concrete or pictorial representation.
5. **Achievement Indicators**

Express a given ratio in multiple forms, such as 3:5, \( \frac{3}{5} \), 3 per 5 or 3 to 5.

Identify and describe ratios from real-life contexts and record them symbolically.

Explain the part/whole and part/part ratios of a set, e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8 and 5:8.

Solve given problems involving ratio.

---

6. **WNCP CURRICULUM**

**Specific Outcome**

Demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically. [C, CN, PS, R, V]

**Achievement Indicators**

Explain that “percent” means “out of 100.”

Explain that percent is a ratio out of 100.

Use concrete materials and pictorial representations to illustrate a given percent.

Record the percent displayed in a given concrete or pictorial representation.

Express a given percent as a fraction and a decimal.

Identify and describe percents from real-life contexts, and record them symbolically.

Solve a given problem involving percents.

**JUMP MATH LESSONS**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>NS</td>
<td>101–108</td>
</tr>
</tbody>
</table>

7. **WNCP CURRICULUM**

**Specific Outcome**

Demonstrate an understanding of integers, concretely, pictorially and symbolically. [C, CN, R, V]

**Achievement Indicators**

Extend a given number line by adding numbers less than zero and explain the pattern on each side of zero.

Place given integers on a number line and explain how integers are ordered.

Describe contexts in which integers are used, e.g., on a thermometer.

**JUMP MATH LESSONS**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NS</td>
<td>52</td>
</tr>
</tbody>
</table>
7. **Achievement Indicators**

- Compare two integers, represent their relationship using the symbols $<, >$ and $=$, and verify using a number line.
- Order given integers in ascending or descending order.

8. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Place the decimal point in a product using front-end estimation, e.g., for $15.205 \text{ m} \times 4$, think $15 \text{ m} \times 4$, so the product is greater than $60 \text{ m}$.</td>
<td></td>
</tr>
<tr>
<td>Place the decimal point in a quotient using front-end estimation, e.g., for $26.83 \div 4$, think $24 \div 4$, so the quotient is greater than $6$.</td>
<td></td>
</tr>
<tr>
<td>Correct errors of decimal point placement in a given product or quotient without using paper and pencil.</td>
<td></td>
</tr>
<tr>
<td>Predict products and quotients of decimals using estimation strategies.</td>
<td></td>
</tr>
<tr>
<td>Solve a given problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.</td>
<td></td>
</tr>
</tbody>
</table>

9. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers). [CN, ME, PS, T]</td>
<td>2</td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Demonstrate and explain with examples why there is a need to have a standardized order of operations.</td>
<td></td>
</tr>
<tr>
<td>Apply the order of operations to solve multi-step problems with or without technology, e.g., computer, calculator.</td>
<td></td>
</tr>
</tbody>
</table>
Patterns and Relations

General Outcomes

• Patterns: Use patterns to describe the world and solve problems.
• Variables and Equations: Represent algebraic expressions in multiple ways.

Patterns

It is expected that students will:

1. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R]</td>
<td>PA 1–4, 5–7, 15–21</td>
</tr>
<tr>
<td></td>
<td>PA 22, 23, 24, 26, 33, 35</td>
</tr>
<tr>
<td></td>
<td>ME 20</td>
</tr>
</tbody>
</table>

Achievement Indicators

Generate values in one column of a table of values, given values in the other column and a pattern rule.

State, using mathematical language, the relationship in a given table of values.

Create a concrete or pictorial representation of the relationship shown in a table of values.

Predict the value of an unknown term using the relationship in a table of values and verify the prediction.

Formulate a rule to describe the relationship between two columns of numbers in a table of values.

Identify missing elements in a given table of values.

Identify errors in a given table of values.

Describe the pattern within each column of a given table of values.

Create a table of values to record and reveal a pattern to solve a given problem.
### Variables and Equations

It is expected that students will:

3. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Outcome</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>Represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]</td>
<td>2</td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td>2</td>
</tr>
<tr>
<td>Write and explain the formula for finding the perimeter of any given rectangle.</td>
<td></td>
</tr>
<tr>
<td>Write and explain the formula for finding the area of any given rectangle.</td>
<td></td>
</tr>
<tr>
<td>Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication, e.g., (a + b = b + a) or (a \times b = b \times a).</td>
<td></td>
</tr>
<tr>
<td>Describe the relationship in a given table using a mathematical expression.</td>
<td></td>
</tr>
<tr>
<td>Represent a pattern rule using a simple mathematical expression, such as (4d) or (2n + 1).</td>
<td></td>
</tr>
</tbody>
</table>
4. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically. [C, CN, PS, R, V]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations and orally explain the process.
- Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials, e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$. 


Shape and Space

General Outcomes

• Measurement: Use direct or indirect measurement to solve problems.
• 3-D Objects and 2-D Shapes: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
• Transformations: Describe and analyze position and motion.

Measurement

It is expected that students will:

1. WNCP CURRICULUM | JUMP MATH LESSONS

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>Part</th>
<th>Unit</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of angles by:</td>
<td>1</td>
<td>G</td>
<td>2–5, 7–9</td>
</tr>
<tr>
<td>• identifying examples of angles in the environment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• classifying angles according to their measure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• estimating the measure of angles using 45°, 90° and 180° as reference angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• determining angle measures in degrees</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• drawing and labelling angles when the measure is specified.</td>
<td></td>
<td></td>
<td>[C, CN, ME, V]</td>
</tr>
</tbody>
</table>

Achievement Indicators

Provide examples of angles found in the environment.

Classify a given set of angles according to their measure, e.g., acute, right, obtuse, straight, reflex.

Sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them.

Estimate the measure of an angle using 45°, 90° and 180° as reference angles.

Measure, using a protractor, given angles in various orientations.

Draw and label a specified angle in various orientations using a protractor.

Describe the measure of an angle as the measure of rotation of one of its sides.

Describe the measure of angles as the measure of an interior angle of a polygon.
### WNCP Curriculum Correlation for Grade 6

#### 2. WNCP Curriculum

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Demonstrate that the sum of interior angles is:</td>
<td>1</td>
</tr>
<tr>
<td>• 180° in a triangle</td>
<td></td>
</tr>
<tr>
<td>• 360° in a quadrilateral. [C, R]</td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Explain, using models, that the sum of the interior angles of a triangle is the same for all triangles.</td>
<td></td>
</tr>
<tr>
<td>Explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals.</td>
<td></td>
</tr>
</tbody>
</table>

#### 3. WNCP Curriculum

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Develop and apply a formula for determining the:</td>
<td>1</td>
</tr>
<tr>
<td>• perimeter of polygons</td>
<td>2</td>
</tr>
<tr>
<td>• area of rectangles</td>
<td></td>
</tr>
<tr>
<td>• volume of right rectangular prisms. [C, CN, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Explain, using models, how the perimeter of any polygon can be determined.</td>
<td></td>
</tr>
<tr>
<td>Generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares.</td>
<td></td>
</tr>
<tr>
<td>Explain, using models, how the area of any rectangle can be determined.</td>
<td></td>
</tr>
<tr>
<td>Generalize a rule (formula) for determining the area of rectangles.</td>
<td></td>
</tr>
<tr>
<td>Explain, using models, how the volume of any right rectangular prism can be determined.</td>
<td></td>
</tr>
<tr>
<td>Generalize a rule (formula) for determining the volume of right rectangular prisms.</td>
<td></td>
</tr>
<tr>
<td>Solve a given problem involving the perimeter of polygons, the area of rectangles and/or the volume of right rectangular prisms.</td>
<td></td>
</tr>
</tbody>
</table>
3-D Objects and 2-D Shapes

It is expected that students will:

4. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct and compare triangles, including:</td>
<td></td>
</tr>
<tr>
<td>• scalene</td>
<td>Part 1, Unit G</td>
</tr>
<tr>
<td>• isosceles</td>
<td>5–9</td>
</tr>
<tr>
<td>• equilateral</td>
<td></td>
</tr>
<tr>
<td>• right</td>
<td></td>
</tr>
<tr>
<td>• obtuse</td>
<td></td>
</tr>
<tr>
<td>• acute in different orientations. [C, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Sort a given set of triangles according to the length of the sides.
- Sort a given set of triangles according to the measures of the interior angles.
- Identify the characteristics of a given set of triangles according to their sides and/or their interior angles.
- Sort a given set of triangles and explain the sorting rule.
- Draw a specified triangle, e.g., scalene.
- Replicate a given triangle in a different orientation and show that the two are congruent.

5. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe and compare the sides and angles of regular and irregular polygons. [C, PS, R, V]</td>
<td>Part 1, Unit G</td>
</tr>
<tr>
<td>NOTE: See Question 7 on G6-16 for a definition of the term “regular.”</td>
<td>11, 12, 14, 17–19</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Sort a given set of 2-D shapes into polygons and non-polygons, and explain the sorting rule.
- Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing.
- Demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring.
5. **Achievement Indicators**

- Demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure.
- Sort a given set of polygons as regular or irregular and justify the sorting.
- Identify and describe regular and irregular polygons in the environment.

### Transformations

It is expected that students will:

6. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V]</td>
<td>Part 1 2 16* 24*, 26, 27, 28, 29, 30, 31, 32, 41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Achievement Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate that a 2-D shape and its transformation image are congruent.</td>
</tr>
<tr>
<td>Model a given set of successive translations, successive rotations or successive reflections of a 2-D shape.</td>
</tr>
<tr>
<td>Model a given combination of two different types of transformations of a 2-D shape.</td>
</tr>
<tr>
<td>Draw and describe a 2-D shape and its image, given a combination of transformations.</td>
</tr>
<tr>
<td>Describe the transformations performed on a 2-D shape to produce a given image.</td>
</tr>
<tr>
<td>Model a given set of successive transformations (translation, rotation and/or reflection) of a 2-D shape.</td>
</tr>
<tr>
<td>Perform and record one or more transformations of a 2-D shape that will result in a given image.</td>
</tr>
</tbody>
</table>
### 8. WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identification</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>Identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs. [C, CN, V]</td>
<td>2</td>
</tr>
<tr>
<td><strong>Achievement Indicators</strong></td>
<td></td>
</tr>
<tr>
<td>Label the axes of the first quadrant of a Cartesian plane and identify the origin.</td>
<td></td>
</tr>
<tr>
<td>Plot a point in the first quadrant of a Cartesian plane given its ordered pair.</td>
<td></td>
</tr>
<tr>
<td>Match points in the first quadrant of a Cartesian plane with their corresponding ordered pair.</td>
<td></td>
</tr>
<tr>
<td>Plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5 or 10 on its axes, given whole number ordered pairs.</td>
<td></td>
</tr>
<tr>
<td>Draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane.</td>
<td></td>
</tr>
<tr>
<td>Determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane.</td>
<td></td>
</tr>
<tr>
<td>Draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them.</td>
<td></td>
</tr>
</tbody>
</table>
### WNCP CURRICULUM

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V]</td>
<td>Part  Unit Lesson</td>
</tr>
<tr>
<td></td>
<td>G    23, 24, 29, 30, 31</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

- Identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane).
- Perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant).
- Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant).
Statistics and Probability

General Outcomes

• Data Analysis: Collect, display and analyze data to solve problems.

• Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Data Analysis

It is expected that students will:

1. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Create, label and interpret line graphs to draw conclusions. [C, CN, PS, R, V]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Determine the common attributes (title, axes and intervals) of line graphs by comparing a given set of line graphs.

Determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why.

Create a line graph from a given table of values or set of data.

Interpret a given line graph to draw conclusions.

2. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>Select, justify and use appropriate methods of collecting data, including: • questionnaires • experiments • databases • electronic media. [C, PS, T]</td>
<td>1</td>
</tr>
</tbody>
</table>

**Achievement Indicators**

Select a method for collecting data to answer a given question and justify the choice.

Design and administer a questionnaire for collecting data to answer a given question, and record the results.

Answer a given question by performing an experiment, recording the results and drawing a conclusion.

Explain when it is appropriate to use a database as a source of data.
2. **Achievement Indicators**

   Gather data for a given question by using electronic media including selecting data from databases.

3. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph collected data and analyze the graph to solve problems. [C, CN, PS]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

   **Achievement Indicators**

   Determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph.

   Solve a given problem by graphing data and interpreting the resulting graph.

---

**Chance and Uncertainty**

It is expected that students will:

4. **WNCP CURRICULUM**

<table>
<thead>
<tr>
<th>Specific Outcome</th>
<th>JUMP MATH LESSONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrate an understanding of probability by:</td>
<td>Part</td>
</tr>
<tr>
<td>• identifying all possible outcomes of a probability experiment</td>
<td>2</td>
</tr>
<tr>
<td>• differentiating between experimental and theoretical probability</td>
<td></td>
</tr>
<tr>
<td>• determining the theoretical probability of outcomes in a probability experiment</td>
<td></td>
</tr>
<tr>
<td>• determining the experimental probability of outcomes in a probability experiment</td>
<td></td>
</tr>
<tr>
<td>• comparing experimental results with the theoretical probability for an experiment. [C, ME, PS, T]</td>
<td></td>
</tr>
</tbody>
</table>

   **Achievement Indicators**

   List the possible outcomes of a probability experiment, such as:

   • tossing a coin
   • rolling a die with a given number of sides
   • spinning a spinner with a given number of sectors.

   Determine the theoretical probability of an outcome occurring for a given probability experiment.
<table>
<thead>
<tr>
<th>4. <strong>Achievement Indicators</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability.</td>
</tr>
<tr>
<td>Conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability.</td>
</tr>
<tr>
<td>Explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome.</td>
</tr>
<tr>
<td>Distinguish between theoretical probability and experimental probability, and explain the differences.</td>
</tr>
</tbody>
</table>