NS8-82 Percents

The words “per cent” mean “out of 100.” A percent is a ratio that compares a number or amount to 100.
The symbol for a percent is %. Example: 45% = 45 : 100 = \frac{45}{100}

1. a) 40 out of 100 squares are shaded. The ratio of shaded squares to all squares is ____ : 100.
   So, ____% of the grid is shaded.

   b) 49 out of 100 letters are Bs. The ratio of Bs to all letters in the set is ____ : 100.
   So, ____% of the letters are Bs.

2. Write the ratio as a percent.
   a) 50 : 100 = ____%  b) 72 : 100 = ____%  c) 7 : 100 = ____%  d) 43 : 100 = ____%

3. Write the percent as a ratio.
   a) 70% = ____ : 100  b) 13% = ____ : ____  c) 38% = ____ : ____  d) 8% = ____ : ____

4. Write the ratio as a fraction and as a percent.
   a) 60 : 100 = \frac{60}{100} = ____%  b) 10 : 100 = \frac{10}{100} = ____%

5. Write the fraction as a percent.
   a) \frac{52}{100} = ____%  b) \frac{39}{100} = ____%  c) \frac{18}{100} =  d) \frac{2}{100} =  e) \frac{6}{100} =

6. Write the percent as a fraction.
   a) 12% = \frac{12}{100}  b) 7% = \frac{7}{100}  c) 49% =  d) 3% =  e) 100% =

7. Complete the chart.

<table>
<thead>
<tr>
<th>Drawing</th>
<th>Fraction</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\frac{34}{100}</td>
<td>34%</td>
</tr>
<tr>
<td></td>
<td>\frac{67}{100}</td>
<td>____%</td>
</tr>
<tr>
<td></td>
<td>\frac{52}{100}</td>
<td>52%</td>
</tr>
<tr>
<td></td>
<td>\frac{100}{100}</td>
<td>____%</td>
</tr>
<tr>
<td></td>
<td>\frac{100}{100}</td>
<td>____%</td>
</tr>
</tbody>
</table>
1. Write the fraction as a percent by changing it to a fraction over 100.
   a) \( \frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\% \)
   b) \( \frac{4}{5} \)
   c) \( \frac{3}{20} \)
   d) \( \frac{8}{25} \)

2. Two out of five friends, or \( \frac{2}{5} \), ordered pizza. What percent ordered pizza? ____

3. Change the fraction to a percent. Reduce the fraction to lowest terms if necessary.
   a) \( \frac{9}{15} = \frac{3}{5} = \frac{60}{100} = 60\% \)
   b) \( \frac{3}{15} \)
   c) \( \frac{9}{18} \)
   d) \( \frac{6}{24} \)
   e) \( \frac{2}{5} \)
   f) \( \frac{7}{10} \)
   g) \( \frac{6}{15} \)
   h) \( \frac{17}{20} \)
   i) \( \frac{12}{48} \)

4. Divide to change the fraction to a decimal. Then write the decimal as a percent.
   a) \( \frac{3}{4} = 3 \div 4 = 0.75 = 75\% \)
   b) \( \frac{4}{5} \)
   c) \( \frac{6}{15} \)
   d) \( \frac{15}{25} \)
   e) \( \frac{65}{500} \)

5. Write the percent as a decimal, then as a fraction, then in lowest terms.
   a) 30%  b) 84%  c) 55%  d) 4%  e) 90%

6. Is the fraction closest to 10%, 25%, 50%, 75%, or 100%?
   a) \( \frac{4}{5} \)
   b) \( \frac{2}{10} \)
   c) \( \frac{2}{5} \)
   d) \( \frac{9}{10} \)
   e) \( \frac{11}{20} \)
   f) \( \frac{16}{20} \)
   g) \( \frac{4}{25} \)

7. Estimate what percent the fraction is. Say what fraction you used to make your estimate. Then divide to change the fraction to a decimal. Was your estimate close?
   a) \( \frac{11}{40} \)
   b) \( \frac{23}{49} \)
   c) \( \frac{60}{84} \)
   d) \( \frac{14}{24} \)
   e) \( \frac{4}{42} \)
   f) \( \frac{21}{31} \)

8. Write the fraction as a decimal. Round to two decimal places. Write the approximate percent.
   a) \( \frac{5}{12} = 5 \div 12 = 0.416 \approx 0.42 = 42\% \)
   b) \( \frac{1}{3} \)
   c) \( \frac{2}{3} \)
   d) \( \frac{2}{9} \)
   e) \( \frac{5}{6} \)
   f) \( \frac{1}{7} \)
NS8-86 Visual Representations of Percents

1. What percent of the figure is shaded?
   a) [Diagram] _____ %
   b) [Diagram] _____ %
   c) [Diagram] _____ %
   d) [Diagram] _____ %
   e) [Diagram] _____ %
   f) [Diagram] _____ %
   g) [Diagram] _____ %

2. Shade 50% of each figure.
   a) [Diagram]
   b) [Diagram]

3. Write different expressions for the shaded area.
   \[ \frac{20}{100} = 0.2 = 20\% \]

4. Write the percents that are equivalent to the fractions.
   \[
   \begin{array}{cccc}
   0 & 2 & 1 & 7 \\
   10 & 5 & 2 & 10 \\
   \end{array}
   \]
   _____ %  _____ %  _____ %  _____ %

5. Measure the line segment. Extend the segment to show 100%.
   a) [Segment] 50%
   b) [Segment] 20%
   c) [Segment] 75%

6. Estimate the percent of the line segment to the left of the mark.
   a) [Segment] 0% — 100%
   b) [Segment] 0% — 100%
   about ___ %  about ___ %

7. 25 out of 50 squares in a grid are shaded. What fraction and percent of the squares are shaded?

8. Alice must do 40 hours of community service. She has completed 10 hours. What fraction and percent of the hours has she completed? What percent of the hours must still be completed?

9. When would you use the measurement to describe the amount, and when would you use the percent (if ever)? Write a sentence using each expression.
   a) 3 h of the school day or 50% of the school day
   b) 12 kg of berries or 40% of the berries
### NS8-87 Comparing Fractions, Decimals, and Percents

1. Complete the chart.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>$\frac{1}{4}$</th>
<th>$\frac{3}{20}$</th>
<th>$\frac{6}{15}$</th>
<th>$\frac{23}{25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal</td>
<td>0.25</td>
<td>0.30</td>
<td>0.40</td>
<td>0.92</td>
</tr>
<tr>
<td>Percent</td>
<td>25%</td>
<td>30%</td>
<td>40%</td>
<td>75%</td>
</tr>
</tbody>
</table>

2. Write $<$ or $>$ or $=$ between each pair of numbers. First change the numbers to a pair of decimal fractions with the same denominator.

a) $\frac{1}{2}$ $47\%$  
   b) $\frac{1}{2}$ $57\%$  
   c) $\frac{1}{5}$ $22\%$  
   d) $\frac{3}{5}$ $80\%$  

\[
\begin{align*}
\frac{1 \times 50}{2 \times 50} & = \frac{47}{100} \\
\frac{50}{100} & > \frac{47}{100} \\
\end{align*}
\]

\[
\begin{align*}
e) \frac{3}{4} & = 67\% \\
f) 0.26 & = 42\% \\
g) 0.05 & = 7\% \\
h) \frac{3}{10} & = 30\% \\
i) \frac{21}{25} & = 18\% \\
j) \frac{39}{50} & = 76\% \\
k) 0.8 & = 15\% \\
l) \frac{16}{20} & = 32\% \\
\end{align*}
\]

3. Change the numbers in each set to decimals. Then order the decimals from least to greatest.

a) $\frac{3}{5}$, $42\%$, 0.73  
   b) $\frac{1}{2}$, 0.73, 80\%  
   c) $\frac{1}{4}$, 0.09, 15\%

4. a) In Abeed’s school, $\frac{3}{5}$ of students like gym and 65% like drama. Which class is more popular?

b) In Rachel’s class, 0.45 of the students like pepperoni pizza best, 35% like cheese, and $\frac{1}{5}$ like vegetarian. Which type of pizza do the most students like best?
If you use a thousands cube to represent 1 whole, you can see that taking \( \frac{1}{10} \) of a number is the same as dividing by 10 (the decimal shifts one place left):

\[
\begin{align*}
\frac{1}{10} \text{ of } 1 &= 0.1 \\
\frac{1}{10} \text{ of } 0.1 &= 0.01 \\
\frac{1}{10} \text{ of } 0.01 &= 0.001
\end{align*}
\]

1. Find \( \frac{1}{10} \) of each number by shifting the decimal. Write your answers in the boxes provided.
   
   a) 7  
   b) 10  
   c) 35  
   d) 210  
   e) 6.4  
   f) 50.6

2. 10% is short for \( \frac{10}{100} \) or \( \frac{1}{10} \). Find 10% of each number.
   
   a) 1  
   b) 3.9  
   c) 4.05  
   d) 6.74  
   e) 0.09  
   f) 60.08

How to Find Percents That Are Multiples of 10

**Step 1:** Find 10% of the number.

**Step 2:** Multiply the result by the number of tens in the percent.

Example: Find 30% of 21.

10% of 21 = \( 2.1 \)

There are 3 tens in 30 (30 = 3 \times 10).

\[ 3 \times 2.1 = 6.3 \]

So 30% of 21 = 6.3.

3. Find the percent using the method above.

   a) 30% of 15
   
   \[
   \begin{align*}
   10\% \text{ of } 15 &= \underline{\hspace{2cm}} \\
   3 \times \underline{\hspace{2cm}} &= \underline{\hspace{2cm}}
   \end{align*}
   \]

   b) 40% of 35
   
   \[
   \begin{align*}
   10\% \text{ of } 35 &= \underline{\hspace{2cm}} \\
   \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} &= \underline{\hspace{2cm}}
   \end{align*}
   \]

   c) 20% of 2.7
   
   \[
   \begin{align*}
   10\% \text{ of } 2.7 &= \underline{\hspace{2cm}} \\
   \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} &= \underline{\hspace{2cm}}
   \end{align*}
   \]

   d) 50% of 62
   
   \[
   \begin{align*}
   10\% \text{ of } 62 &= \underline{\hspace{2cm}} \\
   \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} &= \underline{\hspace{2cm}}
   \end{align*}
   \]

   e) 80% of 17
   
   \[
   \begin{align*}
   10\% \text{ of } 17 &= \underline{\hspace{2cm}} \\
   \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} &= \underline{\hspace{2cm}}
   \end{align*}
   \]

   f) 30% of 0.7
   
   \[
   \begin{align*}
   10\% \text{ of } 0.7 &= \underline{\hspace{2cm}} \\
   \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} &= \underline{\hspace{2cm}}
   \end{align*}
   \]
4. If you know 10% of a number \( n \), then 5% of \( n \) is 10% divided by 2. Complete the chart.

<table>
<thead>
<tr>
<th>5%</th>
<th>3</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>6</td>
<td>20</td>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>100%</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use these steps to find 1% of a number:

**Step 1:** Change the percent to a decimal and replace “of” with “x.”

**Step 2:** Multiply by 0.01 by shifting the decimal two places left.

5. Fill in the blanks.

a) 1% of 300 = \( 0.01 \times 300 \) = ______  
b) 1% of 2000 = ______ \( \times \) ______ = ______

c) 1% of 15 = ______ \( \times \) ______ = ______  
d) 1% of 60 = ______ \( \times \) ______ = ______

6. Find 1% of 200 and use your answer to calculate each percent.

a) 2% of 200 = ______  
b) 3% of 200 = ______  
c) 12% of 200 = ______

7. Use the method of Question 6 to calculate...

a) 4% of 800  
b) 2% of 50  
c) 11% of 60  
d) 2% of 4  
e) 7% of 45

8. Fill in the missing numbers. (Hint: 8% = 4% + 4%.)

<table>
<thead>
<tr>
<th>2%</th>
<th>4%</th>
<th>8%</th>
<th>10%</th>
<th>20%</th>
<th>50%</th>
<th>25%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

9. a) If 45% is 9, what is 90%?  
b) If 3% is 12, what is 1%?  
c) If 40% is 64, what is 100%?  
d) If 20% is 13, what is 100%?

10. Arri wants to leave a 15% tip on a meal that cost $60. How much tip should she leave? (Hint: 15% = 10% + 5%.)

11. a) A shirt that usually costs $40 is on sale for 25% off. What is 25% of $40? What is $40 – (25% of $40)? What is the sale price of the shirt?

b) How would you estimate the price if a shirt that usually costs $32.99 is on sale for 25% off?
NS8-90  Writing Equivalent Statements for Proportions

These are equivalent statements:

\[
\begin{align*}
\text{6} & \text{ of the circles are shaded.} \\
\text{2} & \text{ of the circles are shaded.} \\
\text{6} & \text{ is } \frac{2}{3} \text{ of 9.} \\
\text{6 : 9} & = \text{2 : 3}
\end{align*}
\]

1. Write four equivalent statements for each picture.

a) \[
\begin{align*}
\text{4} & \text{ are shaded} \\
\text{2} & \text{ are shaded} \\
\text{4} & \text{ is } \frac{2}{3} \text{ of 6} \\
\text{4 : 6} & = \text{2 : 3}
\end{align*}
\]

b) \[
\begin{align*}
\text{6} & \text{ are shaded} \\
\text{4} & \text{ are shaded} \\
\text{6} & \text{ is } \frac{2}{3} \text{ of 9} \\
\text{6 : 9} & = \text{2 : 3}
\end{align*}
\]

c) \[
\begin{align*}
\text{4} & \text{ are shaded} \\
\text{2} & \text{ are shaded} \\
\text{4} & \text{ is } \frac{2}{3} \text{ of 6} \\
\text{4 : 6} & = \text{2 : 3}
\end{align*}
\]

d) \[
\begin{align*}
\text{6} & \text{ are shaded} \\
\text{4} & \text{ are shaded} \\
\text{6} & \text{ is } \frac{2}{3} \text{ of 9} \\
\text{6 : 9} & = \text{2 : 3}
\end{align*}
\]

2. For each picture, write a pair of equivalent ratios.

a) \[
\begin{align*}
4 \text{ is } \frac{1}{2} \text{ of 8} \\
\frac{4}{8} & = \frac{1}{2}
\end{align*}
\]

b) \[
\begin{align*}
6 \text{ is } \frac{3}{5} \text{ of 10} \\
\frac{6}{10} & = \frac{3}{5}
\end{align*}
\]

c) \[
\begin{align*}
2 \text{ is } \frac{1}{4} \text{ of 8} \\
\frac{2}{8} & = \frac{1}{4}
\end{align*}
\]

3. For each statement, write a pair of equivalent ratios and equivalent fractions.

a) \[
\begin{align*}
15 \text{ is } \frac{3}{4} \text{ of 20} \\
\frac{15}{20} & = \frac{3}{4}
\end{align*}
\]

b) \[
\begin{align*}
18 \text{ is } \frac{9}{10} \text{ of 20} \\
\frac{18}{20} & = \frac{9}{10}
\end{align*}
\]
4. Write a question mark where you are missing a piece of information.
   a) 12 is $\frac{4}{5}$ of what number? 
      \[ \frac{12}{?} = \frac{4}{5} \]
      \[
      \text{part} : \text{whole} = 12 : \_ \quad \text{part} \quad \frac{12}{?} = \frac{4}{5}
      \]
   b) 6 is how many quarters of 8? 
      \[ \frac{6}{8} = \_ : 4 \]
      \[
      \text{part} : \text{whole} = \_ : 4 \quad \text{part} \quad \frac{6}{8} = \_ : 4
      \]
   c) What is $\frac{3}{4}$ of 16? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
   d) 20 is how many thirds of 30? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]

5. For each statement, write a pair of equivalent ratios and a pair of equivalent fractions.
   a) 15 is what percent of 20? 
      \[ \frac{15}{20} = \_ : 100 \]
      \[
      \text{part} : \text{whole} = 15 : 20 \quad \text{part} \quad \frac{15}{20} = \_ : 100
      \]
   b) What is 25% of 80? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
   c) 9 is what percent of 12? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
   d) 18 is 3% of what number? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]

6. Write the two pieces of information you are given and what you need to find (?). Then write an equation for the problem.
   a) What percent of 30 is 5? 
      \[ \frac{5}{30} = \_ : 100 \]
      \[
      \text{part} : \text{whole} = 5 : 30 \quad \text{part} \quad \frac{5}{30} = \_ : 100
      \]
   b) If 7 is 20%, what is 100%? 
      \[
      \frac{7}{?} = \_ : 100 \quad \text{part} \quad \frac{7}{?} = \_ : 100 
      \]
   c) What is 6% of 24? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
   d) If 3 is 12%, what is 100%? 
      \[
      \frac{3}{\_} = \_ : 100 \quad \text{part} \quad \frac{3}{\_} = \_ : 100 
      \]
   e) What percent of 90 is 4? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
   f) What is 52% of 18? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
   g) 7 is what percent of 25? 
      \[
      \frac{\_}{\_} = \_ : \_ \quad \text{part} \quad \frac{\_}{\_} = \_ : \_ 
      \]
NS8-91 Using Proportions to Solve Percent Problems

If 5 subway tickets cost $4, how much do 20 tickets cost? Write the ratio of tickets to dollars as a fraction, then find an equivalent fraction by multiplying.

<table>
<thead>
<tr>
<th>Step 1: ( \frac{4}{5} = \frac{?}{20} )</th>
<th>Step 2: ( \frac{4}{5} \times \frac{4}{4} = \frac{16}{20} )</th>
<th>Step 3: ( \frac{4}{5} \times \frac{4}{4} = \frac{16}{20} )</th>
</tr>
</thead>
</table>

1. Solve the ratio. Draw arrows and show what you multiply by.
   a) \( \frac{3}{4} = \frac{20}{x} \)  
   b) \( \frac{1}{5} = \frac{15}{y} \)  
   c) \( \frac{3}{5} = \frac{35}{z} \)  
   d) \( \frac{4}{7} = \frac{49}{w} \)  
   e) \( \frac{3}{8} = \frac{24}{v} \)  
   f) \( \frac{2}{3} = \frac{18}{u} \)  
   g) \( \frac{13}{20} = \frac{100}{w} \)  
   h) \( \frac{5}{9} = \frac{72}{x} \)

2. Solve the ratio as you did in Question 1. Note: The arrows will point from right to left.
   a) \( \frac{15}{4} = \frac{3}{1} \)  
   b) \( \frac{12}{5} = \frac{2}{3} \)  
   c) \( \frac{15}{7} = \frac{3}{1} \)  
   d) \( \frac{12}{18} = \frac{3}{3} \)

3. For each question, you will have to reduce the fraction given before you can find the equivalent fraction. The first one has been started for you.
   a) \( \frac{8}{10} = \frac{4}{5} = \frac{15}{x} \)  
   b) \( \frac{4}{6} = \frac{2}{3} = \frac{15}{y} \)  
   c) \( \frac{40}{100} = \frac{4}{10} = \frac{45}{z} \)
   d) \( \frac{15}{18} = \frac{5}{6} = \frac{30}{x} \)  
   e) \( \frac{70}{100} = \frac{7}{10} = \frac{90}{y} \)  
   f) \( \frac{50}{75} = \frac{2}{3} = \frac{36}{z} \)

4. Write a proportion to represent the percent problem. Solve the proportion.
   a) What percent of 20 is 4?  
      part ____  whole ____  percent ____  ____ = ____ 100
   b) If 6 is 25%, what is 100%?  
      part ____  whole ____  percent ____  ____ = ____ 100
   c) What is 17% of 10?  
      part ____  whole ____  percent ____  ____ = ____ 100
   d) What is 17% of 50?  
      part ____  whole ____  percent ____  ____ = ____ 100
   e) 4 is what percent of 5?  
   f) 6 is 25% of what number?  
   g) 24 is 80% of what number?
5. Explain why the proportion \( \frac{3}{25} = \frac{x}{100} \) will be easy to solve.

6. Write a proportion \( \frac{a}{b} = \frac{x}{100} \) to represent each problem. Solve by first writing \( \frac{a}{b} \) in lowest terms.
   a) What percent of 15 is 3?  
   b) What percent of 24 is 6?  
   c) What percent of 30 is 12?

7. Write a proportion to represent the percent problem. Find an equivalent ratio to rewrite the proportion.
   a) If 6 is 40%, what is 100%?  
      part 6 whole ? percent 40  
      Hint: Start by writing \( \frac{40}{100} \) as an equivalent ratio with numerator 2.
   b) What is 75% of 48?  
      part ____ whole ____ percent ____  
      Hint: Start by writing 75% as an equivalent ratio with denominator 4.
   c) What percent of 60 is 45?  
      part ____ whole ____ percent ____  
      Hint: Start by writing \( \frac{45}{60} \) as an equivalent ratio with denominator 20.
   d) What is 64% of 15?  
      part ____ whole ____ percent ____  
      Hint: Start by writing \( \frac{64}{100} \) as an equivalent ratio with denominator 5.

8. Solve the proportions in Question 7. Explain why the proportions in Question 7 were more challenging to solve than those in Question 4.

   a) 8 is 40% of what number?  
   b) What is 60% of 30?  
   c) 15 is 75% of what number?  
   d) What percent of 240 is 60?

10. If 4 of 25 fish are blue, what percent of the fish are blue? What percent are not blue?

11. If 45% of 180 students voted for Kendra for student council, how many of the students voted for Kendra?

12. 12 students in a class (60% of the class) are fluent in French. How many students are in the class?
NS8-92 Solving Percent Problems — Advanced

\[
\frac{3}{4} = 0.75 \text{ means the same thing as } 3 \div 4 = 0.75.
\]

1. a) Write \( \frac{a}{b} = c \) as a division statement. \( \_ \_ \_ \_ \div \_ \_ \_ \_ = \_ \_ \_ \_ \)
b) Use the information from part a) to write \( a \) as a product. \( a = \_ \_ \_ \_ \times \_ \_ \_ \_ \)

2. Change the equation to a multiplication statement.
   a) \( \frac{9}{x} = 2 \) \hspace{1cm} b) \( \frac{7}{5} = x \) \hspace{1cm} c) \( \frac{x}{3} = 11 \) \hspace{1cm} d) \( \frac{3}{x} = 21 \)
   \[
   \begin{align*}
   9 &= 2x \\
   7 \times 5 &= x \\
   \end{align*}
   \]
   e) \( \frac{12}{x} = 11 \) \hspace{1cm} f) \( \frac{x}{9} = 7 \) \hspace{1cm} g) \( \frac{24}{x} = 8 \) \hspace{1cm} h) \( 6 = \frac{x}{7} \)

3. Write the equation as a multiplication statement. Then solve for \( x \).
   a) \( \frac{7}{x} = 3 \) \hspace{1cm} b) \( \frac{8}{5} = x \) \hspace{1cm} c) \( \frac{2}{x} = 5 \) \hspace{1cm} d) \( \frac{x}{3} = 10 \)
   \[
   \begin{align*}
   7 &= 3x \\
   7 &= 3x \\
   \frac{7}{3} &= x \\
   \end{align*}
   \]
   e) \( \frac{5}{20} = x \) \hspace{1cm} f) \( \frac{9}{8} = x \) \hspace{1cm} g) \( \frac{x}{5} = 11 \) \hspace{1cm} h) \( \frac{36}{x} = 4 \)

\[
\frac{3}{4} = \frac{9}{12} \quad \text{so} \quad 3 \div 4 = 9 \div 12
\]
\[
12 \times 3 \div 4 = 12 \times 9 \div 12 \quad \text{Multiply both sides by 12.}
\]
\[
12 \times 3 \div 4 = 9\quad \text{Rewrite the right side.}
\]
\[
12 \times 3 \div 4 \times 4 = 9 \times 4 \quad \text{Multiply both sides by 4.}
\]
\[
12 \times 3 = 9 \times 4 \quad \text{Rewrite the left side.}
\]
To rewrite \( \frac{3}{4} = \frac{9}{12} \) as \( 12 \times 3 = 9 \times 4 \) is called **cross-multiplying** because the products can be obtained from an "X":

4. Check that cross-multiplying works for these equivalent fractions.
   a) \( \frac{2}{5} = \frac{6}{15} \) \hspace{1cm} b) \( \frac{3}{4} = \frac{6}{8} \) \hspace{1cm} c) \( \frac{1}{2} = \frac{5}{10} \) \hspace{1cm} d) \( \frac{2}{3} = \frac{8}{12} \) \hspace{1cm} e) make your own

\[
2 \times 15 = 5 \times 6
\]
\[
30 = 30 \ Check
\]

Number Sense 8-92
5. Cross-multiply and write $=$ (equal) or $\neq$ (not equal) in the box. Then decide if the fractions are equivalent.

a) $\frac{3}{4}$ and $\frac{10}{13}$

\[
\begin{array}{c}
3 \\ \times \\
\hline
13 \\
\end{array} \\
\begin{array}{c}
4 \\ \times \\
\hline
10 \\
\end{array}
\]

Are $\frac{3}{4}$ and $\frac{10}{13}$ equivalent? __________

b) $\frac{2}{5}$ and $\frac{10}{25}$

\[
\begin{array}{c}
2 \\ \times \\
\hline
5 \\
\end{array} \\
\begin{array}{c}
10 \\ \times \\
\hline
25 \\
\end{array}
\]

Are $\frac{2}{5}$ and $\frac{10}{25}$ equivalent? __________

c) $\frac{9}{10}$ and $\frac{81}{100}$

\[
\begin{array}{c}
9 \\ \times \\
\hline
10 \\
\end{array} \\
\begin{array}{c}
81 \\ \times \\
\hline
100 \\
\end{array}
\]

Are $\frac{9}{10}$ and $\frac{81}{100}$ equivalent? __________

d) $\frac{5}{7}$ and $\frac{28}{35}$

\[
\begin{array}{c}
5 \\ \times \\
\hline
7 \\
\end{array} \\
\begin{array}{c}
28 \\ \times \\
\hline
35 \\
\end{array}
\]

Are $\frac{5}{7}$ and $\frac{28}{35}$ equivalent? __________

e) $\frac{3}{4}$ and $\frac{15}{20}$

f) $\frac{5}{6}$ and $\frac{35}{42}$

g) $\frac{91}{105}$ and $\frac{104}{120}$

h) $\frac{14}{21}$ and $\frac{30}{48}$

6. Cross-multiply to write an equation for $x$. (Do not solve.)

a) $\frac{7}{x} = \frac{3}{5}$

\[
7 \times 5 = 3x
\]

b) $\frac{x}{9} = \frac{2}{5}$

\[
5x = 2 \times 9
\]

c) $\frac{11}{x} = \frac{5}{2}$

\[
11 \times 2 = 5x
\]

d) $\frac{4}{9} = \frac{x}{3}$

e) $\frac{5}{21} = \frac{3}{x}$

f) $\frac{x}{52} = \frac{4}{8}$

g) $\frac{20}{x} = \frac{12}{25}$

h) $\frac{12}{x} = \frac{3}{10}$

7. Solve for $x$.

a) $\frac{9}{6} = \frac{x}{3}$

b) $\frac{4}{x} = \frac{2}{3}$

c) $\frac{3}{4} = \frac{6}{x}$

d) $\frac{100}{7} = \frac{9}{x}$

e) $\frac{2}{x} = \frac{10}{4}$

You can solve percent problems by first writing a proportion and then cross-multiplying.

Example: What is 70% of 9?

\[
\frac{x}{9} = \frac{70}{100}
\]

\[
100x = 70 \times 9
\]

\[
100x = 630
\]

\[
\frac{100x}{100} = \frac{630}{100}
\]

\[
x = 6.3
\]

8. Solve the problem by first writing a proportion.

a) What is 90% of 6?

b) 9 is 2% of what number?

c) 5 is what percent of 8?
Write an equation for each of the problems below and solve the equation. Use a calculator.

9. a) What percent of 32 is 8?  
   b) What percent of 125 is 5?  
   c) What percent of 128 is 32?  
   d) What percent of 15 is 0.6?  

10. Round the solution to the nearest one.  
    a) 5 is about what percent of 24?  
    b) About what percent of 17 is 9?  
    c) 4 is about what percent of 9?  
    d) About what percent of 7,560 is 3,000?  
    e) 1.3 is about what percent of 27?  

11. If Yvonne has read 54 of the 297 pages in her library book, about what percent of the book has she read so far?

12. Find the amounts. Include units in your answers.  
    a) 26% of 130 g  
    b) 11% of 407 m  
    c) 32% of 11 mL  
    d) 99% of 8 m²  
    e) 40% of 2,222 min  

13. About 3% of 592 students are vegans. About how many of the students are vegans?

14. A basketball team won 60% of the 25 games it played this year.  
    a) What percent of the games played did the team lose?  
    b) How many games did the team lose?

15. Find 100% if…  
    a) 25% is 30  
    b) 15% is 30  
    c) 3% is 12

16. Round the solution to the nearest one.  
    a) 10 is 7%. About what is 100%?  
    b) 74 is 32%. About what is 100%?  
    c) 2 is 9%. About what is 100%?  

17. In a Grade 8 class, 6 students, or about 27%, were on the honour roll. How many students were in the class?

18. Kai bought a new computer at a 15% discount. He paid $1,020.  
    a) What percent of the original price did he pay?  
    b) What was the original price?  
    c) How many dollars did Kai save by buying the computer at a discount?

19. A computer costs $1,000 plus 15% tax. Which of these is the best deal?  
    A: The store offers a 15% discount on the $1,000 purchase price, then adds the tax onto the sale price.  
    B: The store will pay the tax.  
    C: The store offers a 15% discount, calculated after the tax is added.
1. Determine the total percent of the grids that is shaded as a fraction, decimal, and percent.
   a) 
   
   Fraction: \( \frac{100}{100} + \frac{100}{100} = \frac{1}{100} \)
   Decimal: \( \frac{1}{10} + 0.\frac{1}{10} = \frac{1}{10} \)
   Percent: \( \frac{1}{10} \% + \frac{1}{10} \% = \frac{1}{10} \% \)
   b) 
   
   Fraction: _____ + _____ + _____ = _____
   Decimal: _____ + _____ + _____ = _____
   Percent: _____ \% + _____ \% + _____ \% = _____ \%

2. a) Shade the grids to represent 134%.
    b) Shade the grids to represent 273%.

3. Add the percents.
   a) \( 125\% + 240\% = _____ \% \)
   b) \( 80\% + 60\% = _____ \% \)
   c) \( 150\% + 75\% = _____ \% \)

4. Subtract the percents.
   a) \( 117\% - 17\% = _____ \% \)
   b) \( 125\% - 40\% = _____ \% \)
   c) \( 675\% - 50\% = _____ \% \)

5. Measure the line segment. Extend the segment to show 150%.
   a) \( \frac{50\%}{\text{mark}} \)
   b) \( \frac{75\%}{\text{mark}} \)

6. Estimate the percent of the line segment to the left of the mark.
   a) \( \frac{\text{mark}}{\text{mark}} \)
   b) \( \frac{\text{mark}}{\text{mark}} \)

7. Write the ratio as an improper fraction and as a percent.
   a) \( 110 : 100 = \frac{110}{100} = _____ \% \)
   b) \( 350 : 100 = \frac{350}{100} = _____ \% \)
   c) \( 261 : 100 = \frac{261}{100} = _____ \% \)

8. Write the percent as a mixed number with the fractional part in lowest terms.
   a) \( 130\% \)
   b) \( 275\% \)
   c) \( 308\% \)
   d) \( 1505\% \)
   e) \( 785\% \)
9. Complete the chart. Hint: If a decimal with one decimal place is given, add a zero to make two decimal places.

<table>
<thead>
<tr>
<th>Percent</th>
<th>190%</th>
<th>535%</th>
<th>( \frac{176}{100} )</th>
<th>( \frac{18}{100} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mixed Number</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decimal</td>
<td>9.2 = 9.20</td>
<td>2.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.4 = _____</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. About what percent does the decimal represent? Example: 4.715 \( \approx 4.72 = 472\% \)
   a) 4.382 \( \approx \) _____\%  b) 5.925 \( \approx \) _____\%  c) 5.007 \( \approx \) d) 2.999 \( \approx \)

11. Write the percent as a decimal, then as a mixed number, then in lowest terms.
   a) 350\%  b) 540\%  c) 275\%  d) 360\%  e) 515\%

12. Write the mixed number as a percent.
    a) \( \frac{2}{\frac{1}{2}} \)  b) \( \frac{3}{\frac{3}{4}} \)  c) \( \frac{8}{\frac{3}{10}} \)  d) \( \frac{1}{\frac{1}{5}} \)  e) \( \frac{20}{\frac{3}{20}} \)  f) \( \frac{17.9}{\frac{25}{25}} \)

13. Write the mixed number as a decimal. Round the decimal to two places. Then write the approximate percent.
    a) \( \frac{3}{\frac{5}{12}} = 3 + 0.41\overline{6} \approx 3.42 = \) _____\%  b) \( \frac{3}{\frac{1}{3}} \)  c) \( \frac{4}{\frac{2}{3}} \)  d) \( \frac{1}{\frac{2}{9}} \)  e) \( \frac{1}{\frac{2}{7}} \)

14. Change the numbers in each set to decimals. Then order the numbers from greatest to least.
    a) \( \frac{1}{\frac{1}{2}} \)  1.73  180\%  b) \( \frac{1}{\frac{6}{10}} \)  157\%  1.62  c) \( \frac{6}{\frac{1}{4}} \)  6.09  615\%

15. Determine the amount mentally.
    a) 300\% of 20 = _____  b) 250\% of 50 = _____  c) 110\% of 6 = _____  d) 330\% of 2 = _____

16. If 30\% = 150, what is 10\%? _____  What is 100\%? _____

17. Determine 100\% mentally.
    a) If 40\% = 200, then 100\% = _____.  b) If 5\% = 20, then 100\% = _____.
    c) If 150\% = 12, then 100\% = _____.  d) If 300\% = 18, then 100\% = _____.

18. Estimate the solution. Use a calculator to check your estimate. Was your estimate close?
    a) What percent of 20 is 30?  b) What percent of 45 is 87?  c) What percent of 2 is 17?
    d) What percent of 7 is 13?  e) What percent of 1.5 is 4.4?  f) What percent of 1.1 is 59.3?
Percent Problems

1. Calculate.
   a) $80\% - 65\% + 22\% = \underline{\hspace{1cm}}$
   b) $41\% + \underline{\hspace{1cm}} = 100\%$
   c) $96\% - \underline{\hspace{1cm}} = 25\%$

2. What is the sales tax where you live? ________________
   Calculate the amount of tax you would pay on each price.
   a) $15 \underline{\hspace{1cm}}$
   b) $40 \underline{\hspace{1cm}}$
   c) $67.25 \underline{\hspace{1cm}}$
   d) $82.52 \underline{\hspace{1cm}}$

3. In the school elections, $\frac{2}{5}$ of the students voted for Anne and 17\% voted for Ravi. The rest voted for Yen. What percent voted for Yen?

4. A builder spent $400.00 on equipment. Complete the chart.

<table>
<thead>
<tr>
<th>Item</th>
<th>Money spent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction</td>
</tr>
<tr>
<td>Drywall</td>
<td></td>
</tr>
<tr>
<td>Paint</td>
<td>$\frac{3}{20}$</td>
</tr>
<tr>
<td>Wallpaper</td>
<td></td>
</tr>
</tbody>
</table>

5. A student hopes to raise $500 for his favourite charity. He has already raised $100 by having a garage sale. What percent of the $500 does he still need to raise?

6. Complete the chart.

<table>
<thead>
<tr>
<th>Item</th>
<th>Regular Price</th>
<th>Discount (percent)</th>
<th>Discount ($ amount)</th>
<th>Sale Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gloves</td>
<td>$36.00</td>
<td>10%</td>
<td>$3.60</td>
<td>$36.00 - $3.60 = $32.40</td>
</tr>
<tr>
<td>Shoes</td>
<td>$49.92</td>
<td>25%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>$14.90</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Clare bought a computer at a 40\% discount. She paid $800. How many dollars did she save by buying the computer at a discount?

8. John spent $720 on furniture. He spent 25\% on a chair, $327.60 on a table, and the rest on a sofa. What fraction and what percent of the $720 did he spend on each item?

9. Erik had 1 400 stamps. 20\% of the stamps were Canadian. Recently he bought 300 new Canadian stamps. How many Canadian stamps does he have now? What percent and what fraction of his stamps are Canadian?
NS8-96 Relating Fractions, Ratios, and Percents

1. Write the number of boys (b), girls (g), and children (c) in each class.
   a) There are 7 boys and 6 girls in a class. b _____ g _____ c _____
   b) There are 5 boys and 9 girls in a class. b _____ g _____ c _____
   c) There are 18 boys and 22 girls in a class. b _____ g _____ c _____
   d) There are 15 girls in a class of 27 children. b _____ g _____ c _____

2. Write the number of boys, girls, and children in each class. Then write the fraction of children who are boys and the fraction who are girls in the boxes provided.
   a) There are 6 boys and 9 girls in a class. b __________ g __________ c ___
   b) There are 17 children in the class and 9 are boys. b __________ g __________ c ___

3. Fill in the missing numbers for each classroom.

<table>
<thead>
<tr>
<th>Ratio of boys to girls</th>
<th>Fraction of boys</th>
<th>Fraction of girls</th>
<th>Percentage of boys</th>
<th>Percentage of girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3 : 2</td>
<td>(\frac{3}{5})</td>
<td>(\frac{2}{5})</td>
<td>(\frac{3}{5} = \frac{60}{100} = 60%)</td>
<td>40%</td>
</tr>
<tr>
<td>b) 1 : 5</td>
<td></td>
<td>(\frac{11}{20})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td>(\frac{12}{25})</td>
<td></td>
<td>30%</td>
</tr>
<tr>
<td>d)</td>
<td></td>
<td>(\frac{27}{50})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) 32 : 18</td>
<td></td>
<td>45%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f)</td>
<td></td>
<td>19%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number Sense 8-96
4. Fill in the missing numbers for each classroom.

<table>
<thead>
<tr>
<th></th>
<th>Number of students</th>
<th>Fraction of boys</th>
<th>Fraction of girls</th>
<th>Number of boys</th>
<th>Number of girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>20</td>
<td>$\frac{4}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{4}{5} \times 20 = 16$</td>
<td>4</td>
</tr>
<tr>
<td>b)</td>
<td>40</td>
<td>$\frac{1}{5}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>24</td>
<td></td>
<td>$\frac{1}{4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>38</td>
<td>$\frac{5}{19}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Determine the number of girls and boys in each class.
   a) There are 20 children and $\frac{2}{5}$ are boys.
   b) There are 42 children and $\frac{3}{7}$ are girls.
   c) There are 15 children. The ratio of girls to boys is 3 : 2.
   d) There are 24 children. The ratio of girls to boys is 3 : 5.
   e) There are 25 children and 60% are girls.
   f) There are 28 children and 25% are boys.

6. For each question, say which classroom has more girls.
   a) In classroom A, there are 40 children and 60% are girls. In classroom B, there are 36 children. The ratio of boys to girls is 5 : 4.
   b) In classroom A, there are 28 children. The ratio of boys to girls is 5 : 2. In classroom B, there are 30 children and $\frac{3}{5}$ of the children are boys.

7. Ron and Ella shared $35 in the ratio 4 : 3. What fraction of the money did each person receive? What amount of money did each person receive?

8. Students in a class each chose one sport to participate in for a sports day. Complete the chart. How did you find the number of students who chose swimming?

<table>
<thead>
<tr>
<th>Chosen sport</th>
<th>Fraction of the class that chose the sport</th>
<th>Percent</th>
<th>Decimal</th>
<th>Number of students who chose the sport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer</td>
<td>$\frac{1}{5}$</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Swimming</td>
<td></td>
<td>40%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseball</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gymnastics</td>
<td></td>
<td>.15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Finding the Whole from the Part

\[ \frac{2}{3} \text{ of a number is 100. What is the number?} \]

\[ \frac{2}{3} \times 100 = \frac{2 \times 100}{3} = \frac{200}{3} \]

The number is 150.

1. Find the number.
   a) \( \frac{2}{5} \) of a number is 4.
   b) \( \frac{3}{7} \) of a number is 9.
   c) \( \frac{5}{11} \) of a number is 25.

2. A box holds red and blue beads. Find the total number of beads in the box.
   a) \( \frac{3}{4} \) of the beads are red. Six beads are red.
   b) \( \frac{3}{5} \) of the beads are blue. Twelve beads are blue.
   c) 60% of the beads are red. Fifteen beads are red.
   d) The ratio of red to blue beads is 4 : 5. There are 20 red beads.

3. Ron and Lisa share a sum of money. Ron receives \( \frac{2}{5} \) of the money. Lisa receives $24.
   a) What fraction of the sum does Lisa receive?  
   b) How much money do Ron and Lisa share?

4. At Franklin Middle School, \( \frac{3}{8} \) of the students take a bus to school, \( \frac{3}{5} \) walk, and the rest bike. There are 20 students who bike to school. How many students are in the school?

5. In a fish tank, \( \frac{2}{3} \) of the fish are red, \( \frac{1}{4} \) are yellow, and the rest are green. There are 42 more red fish than green fish.
   a) What fraction of the fish are green?
   b) What fraction of the total number of fish does 42 represent? Hint: 42 is the difference between the number of red and green fish.
   c) How many fish are in the tank?

6. In Tina’s stamp collection, 70% of the stamps are Canadian and the rest are international. Tina has 500 more Canadian stamps than international stamps. How many stamps does she have?

7. On a neon sign, \( \frac{1}{5} \) of the lights are yellow and the rest are blue and red. There are twice as many blue lights as yellow lights, and there are 200 red lights on the sign. How many lights of all colours are on the sign?
**Further Percent Problems**

1. A pair of jeans costs $80. Now the jeans are 20% off. Find the new cost of the jeans in two ways:
   a) Find 20% of 80. Then subtract this amount from 80.
   b) \(100\% - 20\% = 80\%\). Find 80% of 80.

2. A daily planner cost $12.50 last year. The price has increased 20%. What is the new price?

3. A game sells for $25 plus 14% tax. Is $28 enough to buy the game?

4. a) \(115\% = 46\). What is 100%?  
    b) \(120\% = 80\). What is 100%?  
    c) \(150\% = 45\). What is 100%?
    
    \[
    \begin{align*}
    115\% \div 115 &= 46 \div 115 \\
    1\% &= 46 \div 115 = 0.4 \\
    100\% &= 1\% \times 100 \\
    &= 0.4 \times 100 \\
    &= 40
    \end{align*}
    \]

5. The total cost of a T-shirt, including 14% tax, is $23.00. The total cost is 114% of the price before taxes. What is the price of the T-shirt before taxes?

6. A grocery buys organic apples at 80 cents each and sells them for $1 each. What percent does the store mark up the price of each apple?

7. This year, 20 more students joined the band than last year. That is a 10% increase.  
   a) How many students were in the band last year?  
   b) How many students are in the band this year?

8. The tax on a purchase of $20 is $2.80. How much tax will there be on a purchase of $45.50?

9. Suppose you bought something that was priced at $6.95, and the total bill was $7.61. What is the sales tax rate in this city? (Round your answer to one decimal place.)

10. Calculate the percent. Round to one decimal place if necessary.  
    a) \(25\%\) of 50%  
    b) \(10\%\) of 60%  
    c) \(80\%\) of 30%  
    
    \[
    \begin{align*}
    25\% \text{ of } 50 &= 0.25 \times 50 \\
    10\% \text{ of } 60 &= 0.1 \times 60 \\
    80\% \text{ of } 30 &= 0.8 \times 30
    \end{align*}
    \]

11. Ravi gave 60% of his stamp collection to his brother. He sold 20% of the remaining amount. What percent of his collection did he sell?
1. An 8-slice pizza is shared among 3 people. Mayah eats 2 pieces, Tegan eats 3 pieces, and Matias eats 3 pieces. The pizza costs $12.99 plus 14% tax. How much should each person pay?

2. Two hockey goalies, Dillon and Melissa, are comparing their records. Dillon saved 53 out of 60 shots in 3 games. Melissa saved 65 out of 70 shots in 2 games. Find...
   a) the percentage of shots each person saved (to one decimal place).
   b) the average number of goals allowed per game by each person (to one decimal place).
   c) Who do you think is the better goalie? Why?

3. Philip gave away 45% of his hockey cards.
   a) What fraction of his cards did Philip keep?
   b) Philip put his remaining cards in a scrapbook. Each page held 18 cards and he filled $23\frac{5}{6}$ pages. How many cards did he put in the book?
   c) How many cards did Philip have before he gave part of his collection away?

4. Pure gold is 24 karat, so 12-karat gold is 50% pure and 18-karat gold is 75% pure.
   a) What percentage of pure gold is in 15-karat gold?
   b) Rita has a gold bracelet weighing 50 g. It is 15-karat gold. If pure gold costs $23.64/g, what is a fair price for the bracelet?

5. Kevin gave $\frac{2}{7}$ of his savings to charity and spent $\frac{3}{5}$ of the remainder on holiday gifts.
   a) What fraction of Kevin’s money was left?
   b) If Kevin had $300 left, how much money did he have to start with?

6. A book costs $17.50. The salesperson tells you that the total price, including taxes, is $21.43. How can you tell if the total price is reasonable without using a calculator?

7. You invest $4 000 in a fund that earns 10% interest each year. If you leave the money in the fund and do not touch it, how much money will you have after two years?

8. Two years ago, a calculator cost $120. The price increased by 10% last year. This year, last year’s price increased by 12%. What is this year’s price? What percent did the price increase over the two years?

9. A population increased by 10% one year and then increased by 15% the next year. Explain why there was not a 25% increase in population over the two years.
A rate is a ratio of two quantities measured in different units. Rates are written with a slash instead of a colon or as a fraction. Example: $2 / 3 \text{ min}$ (we read this as “$2 \text{ per } 3 \text{ minutes}””)

1. Find the equivalent rate.
   a) $\frac{10 \text{ km}}{2 \text{ h}} = \frac{5 \text{ km}}{1 \text{ h}}$
   b) $\frac{18 \text{ km}}{3 \text{ h}} = \frac{6 \text{ km}}{1 \text{ h}}$
   c) $\frac{20 \text{ km}}{8 \text{ s}} = \frac{2.5 \text{ km}}{1 \text{ s}}$
   d) $\frac{42 \text{ km}}{3 \text{ L}} = \frac{14 \text{ km}}{1 \text{ L}}$
   e) $\frac{\$35}{7 \text{ kg}} = \frac{\$5}{1 \text{ kg}}$
   f) $\frac{\$96}{6 \text{ h}} = \frac{\$16}{1 \text{ h}}$
   g) $\frac{\$1.05}{10 \text{ min}} = \frac{\$0.105}{1 \text{ min}}$
   h) $\frac{8 \text{ m}^2}{0.5 \text{ L}} = \frac{16 \text{ m}^2}{1 \text{ L}}$

In a unit rate, the second term is equal to 1. The 1 is often left out. Example: $60 \text{ km} / 1 \text{ h} = 60 \text{ km/h}$

2. Find the unit rate for each rate (include the units).
   a) $\frac{20 \text{ km}}{5 \text{ h}} = \frac{4 \text{ km}}{1 \text{ h}}$
   b) $\frac{\$12}{2 \text{ boxes}} = \frac{\$6}{1 \text{ box}}$
   c) $\frac{\$70}{2 \text{ h}} = \frac{\$35}{1 \text{ h}}$
   d) $\frac{96 \text{ m}}{12 \text{ s}} = \frac{8 \text{ m}}{1 \text{ s}}$
   e) $\frac{\$45}{9 \text{ jars}} = \frac{\$5}{1 \text{ jar}}$
   f) $\frac{\$32}{4 \text{ kg}} = \frac{\$8}{1 \text{ kg}}$

3. Change both prices to a unit rate to find out which offer is a better buy.
   a) 6 golf balls for $10 or 12 golf balls for $24?
   b) $112 for 7 CDs or $68 for 4 CDs?
   c) $36.52 for 2 cans of paint or $46.20 for 3 cans?

4. Density is the ratio of mass to volume measured in grams per cubic centimetre (g/cm$^3$).
   a) 500 cm$^3$ of human blood weigh 612 g. What is the density of human blood?
   b) One litre of milk weighs 1.003 kg. Is milk denser than human blood?
   c) The density of gasoline is 0.737 g/cm$^3$. Which is heavier: 500 mL of gasoline or 400 mL of milk?

5. Anne donates blood at a rate of 200 mL in 3 minutes. How long will it take Anne to donate 500 mL of blood?

6. A space shuttle flies at a speed of 11 km/s. The Moon is 380 000 km from Earth. How long will it take for the space shuttle to get from Earth to the Moon?

7. Jade is sick. She needs to take 0.5 mL of antibiotic per kilogram of her body weight each day. Jade weighs 42 kg.
   a) How much antibiotic does she need each day?
   b) Jade takes the antibiotic 3 times per day. How much antibiotic does she need to take each time?
   c) Jade will take the antibiotic for 10 days. How much antibiotic will she get in total?

8. Estimate to the nearest half hour how long it would take to drive each distance at 100 km/h.
   a) 254 km
   b) 723 km
   c) 1 425 km
**NS8-102 Using Unit Rates**

**REMINDER** ► Often a unit rate is written with the 1 left out. Example: 100 km / 1 h is written 100 km/h.

1. Use the unit rates in the chart to convert the measurements.

   Example: Convert 25 mm to centimetres.

   \[
   \frac{25 \text{ mm}}{1 \text{ cm}} \times 2.5 = 2.5 \text{ cm}
   \]

   So, 25 mm = 2.5 cm.

   a) 50 mm to cm  
   b) 25 cm to m  
   c) 3 200 m to km  
   d) 4 500 mL to L

   e) 6 900 mg to g  
   f) 240 s to min  
   g) 4 200 min to h  
   h) 120 h to days

2. The scale on this map is 120 km/1 cm. Measure the distance between Edmonton and each other place on the map in centimetres. Use the scale to determine the distances in real life.

<table>
<thead>
<tr>
<th>Distance on map</th>
<th>Distance in real life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edmonton and Calgary</td>
<td></td>
</tr>
<tr>
<td>Edmonton and Jasper</td>
<td></td>
</tr>
<tr>
<td>Edmonton and Fort McMurray</td>
<td></td>
</tr>
</tbody>
</table>

3. On a map, 2.5 cm represents 40 km. How many kilometres do 17.5 cm on the map represent?

4. Cars are rated according to their fuel efficiency. David's car gets 11.45 km/L, Felicity's gets 12.76 km/L, and Jack's gets 38 km/L. How many kilometres can they each travel on 50 L of gas?

5. A spacecraft accelerates from the speed of 7 793 m/s to the speed of 10 822 m/s. It adds 9.52 m/s to its speed every second. How long will the acceleration take?

6. Connor is using paint that covers 12 square metres per can of paint. He needs to cover 160 m². How many cans of paint will he need? Explain your answer.

7. Grapes cost $2.80/kg, watermelon costs $1.50/kg, and peaches cost $1.80/kg. Julie made a fruit salad for a party with grapes, watermelon, and peaches in a ratio of 1 : 2 : 3. She made 18 kg of fruit salad. How much did she spend on fruit?
**Problem:** A granola recipe uses \( \frac{1}{2} \) cup of raisins for every 3 cups of oats. How many cups of oats are needed for 2 cups of raisins?

**Solution:** Write the names of the quantities being compared.

<table>
<thead>
<tr>
<th>cups of raisins</th>
<th>cups of oats</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
</tr>
</tbody>
</table>

Find an equivalent ratio so that both terms are whole numbers.

Write the given quantities under their names, as a ratio.

Solve the two ratios that consist of whole numbers only. \( 1 : 6 = 2 : ? \)

In this case, \( ? = 12 \), so 12 cups of oats are needed.

1. Make both terms a whole number without changing the ratio.
   a) 3 km walked / \( \frac{1}{2} \) h =
   b) 5 km rowed / \( \frac{1}{3} \) h =
   c) \( \frac{1}{4} \) cup of flour : 5 cups of flour =

   In parts d) and e), multiply each term by 10.
   d) 0.7 km / 3 L of gas =
   e) 1.7 mL of ginger ale : 0.3 mL of orange juice =

2. Solve the proportions.
   a) \( \frac{2.4}{4} = \frac{24}{40} = \frac{20}{20} \)
   b) \( \frac{5}{0.9} = \frac{50}{9} = \frac{45}{45} \)
   c) \( \frac{3.9}{0.2} = \frac{39}{2} = \frac{18}{1} \)
   d) \( \frac{6}{2.1} = \frac{20}{10} \)
   e) \( \frac{7.2}{3.2} = \frac{9}{4} \)
   f) \( \frac{5.4}{0.6} = \frac{54}{6} = \frac{18}{2} \)

3. Solve the problem by changing the ratio into a more convenient form.
   a) Rhonda can ride her bike 6 km in \( \frac{1}{4} \) of an hour. How far can she ride in 3 hours?
   b) A plant grows 0.5 cm in 4 days. How many days will it take to grow 9 cm?
   c) On a map, 0.3 cm equals 50 m. How many metres does 7 cm on the map represent?
   d) A recipe uses \( \frac{1}{3} \) cup milk to 2 cups flour. How much milk do you need if you use 3 cups of flour?
1. Write the correct operation and number to get back where you started.
   a) \( n + (-3) - (-3) = n \)  
   b) \( n \times (-3) = n \)  
   c) \( (-5)m = m \)  
   d) \( x - (+5) = x \)  
   e) \( x + (-7) = x \)  
   f) \( x \div (-9) = x \)  
   g) \( \frac{z}{-5} = z \)  
   h) \(-7y = y \)  
   i) \( r \times 8 = r \)

2. Circle the expressions that always equal \( m \), for any number \( m \). Check your answers for \( m = -5 \).
   \[ 6m - 6 \quad -6m \div (-6) \quad m \div 6 \times 6 \quad 6 \div m \times 6 \quad 6 + m + (-6) \quad 6 - m + 6 \]

3. Solve for \( x \) by doing the same thing to both sides of the equation. Check your answer.
   a) \( (-3)x = 12 \)  
   b) \( x - 4 = 11 \)  
   c) \( -4x = 20 \)  
   d) \( 3 + x = -9 \)  
   e) \( -2x = -6 \)  
   f) \( 5x = -15 \)  
   g) \( -10 + x = 90 \)  
   h) \( x + (-4) = -12 \)  
   i) \( \frac{x}{-3} = 7 \)  
   j) \( \frac{x}{5} = -7 \)

4. Solve for the variable. Your answer will be a fraction or decimal. Check your answer by substituting it back into the equation.
   a) \( 3s = 29 \)  
   b) \( 2t = 11 \)  
   c) \( 7y = 8 \)  
   d) \( 5x = 21 \)  
   e) \( 6a = 9 \)

5. Start with \(-3\). Multiply by \(-2\). Then add 4.
   Which sequence of operations will get you back to the number you started with \((-3)\)?
   i) Divide your answer by \(-2\), then subtract 4.
   ii) Subtract 4 from your answer, then divide by \(-2\).

6. Solve for the variable by undoing each operation, working backwards. Then check your answer by substitution.
   a) \( (-8)x + 4 = 28 \)  
   b) \( 4h - 3 = -39 \)  
   Check: \( (-8)(-3) + 4 = 24 + 4 = 28 \)
   \[ (-8)x + 4 - 4 = 28 - 4 \]  
   \[ (-8)x = 24 \]  
   \[ (-8)x \div (-8) = 24 \div (-8) \]  
   \[ x = -3 \]  
   \[ 4h - 3 + 3 = -39 + 3 \]  
   \[ 4h = -36 \]  
   \[ 4h \div 4 = -36 \div 4 \]  
   \[ h = -9 \]

   c) \( (-3)s - 4 = 29 \)  
   d) \( 2t + 3 = -11 \)  
   e) \( \frac{x}{3} + (-5) = 7 \)  
   f) \( \frac{x}{-2} - 4 = 7 \)
   g) \( 3s - 5 = 27 \)  
   h) \( 3t + 3 = 14 \)  
   i) \( (-4) + 5y = 42 \)  
   j) \( 6z + 14 = 16 \)
PA8-19 Concepts in Equations

1. The scales are balanced. Find the mass of one triangle in circles. Write your answer as a decimal or as a fraction.

   a) \[
   \Delta = \frac{\text{4.5 circles}}{} \]

   b) \[
   \Delta = \frac{}{} \]

   c) \[
   \Delta = \frac{}{} \]

   \[
   \Delta = \text{4.5 circles} \]

2. Here are two ways to solve \((-2)x + 6 = -4\).

   **Method 1**
   
   \[\begin{align*}
   (-2)x + 6 &= -4 \\
   (-2)x + 6 - 6 &= -4 - 6 \\
   (-2)x &= -10 \\
   x &= 5
   \end{align*}\]

   **Method 2**
   
   \[\begin{align*}
   (-2)x + 6 &= -4 \\
   (-2)(x + 3) &= -4 \\
   (-2) \div (-2) &= 4 \div (-2) \\
   x + 3 &= 2 \\
   x &= -1
   \end{align*}\]

   a) Do both methods give the same answer? __________

   b) Substitute \(x = 5\) into the expression \((-2)x + 6\). Do you get \(-4\)? __________

   Substitute \(x = -1\) into the expression \((-2)x + 6\). Do you get \(-4\)? __________

   Which method gives the correct answer? __________

   c) Substitute \(x = 5\) into the left side of each step of Method 2. Where was the mistake made?

   d) Correct the mistake in Method 2 and solve the equation using the corrected method.

3. Decide whether each solution is correct by substituting the answer into the original expression.

   a) \(-3x + 6 = -21\)

   b) \(-3x + 6 = -21\)

   c) \(-3x + 6 = -21\)

   d) \(-3x + 6 = -21\)

   \[\begin{align*}
   (-3)(x - 6) &= -21(-3) \\
   (x + 2) &= -21 \\
   (-3)(x - 2) &= -21 \\
   -3x + 6 - 6 &= -21 - 6 \\
   x - 6 &= 7 \\
   x &= 2 + 7 \\
   x - 2 &= 7 \\
   -3x &= -27 \\
   x - 6 + 6 &= 7 + 6 \\
   x &= 2 + 7 + 2 \\
   x &= 3 \\
   x &= 9 \\
   x &= 9 \\
   x &= 9
   \end{align*}\]

   x = 13

   x = 5

   x = 9

   x = 9

   x = 9

   x = -9

4. For each incorrect solution above, describe the mistake. If the solution is correct, write “Correct.”

5. To find the average temperature, we add the temperatures and divide the sum by the number of days. Use equations to find the missing temperatures.

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{City} & \text{Day 1} & \text{Day 2} & \text{Day 3} & \text{Average temperature over 3 days} \\
   \hline
   \text{Yellowknife, NWT} & -21°C & -34°C & x°C & -27°C \\
   \text{Vancouver, BC} & -3°C & y°C & +5°C & +1°C \\
   \text{Calgary, AB} & z°C & -5°C & +1°C & -9°C \\
   \hline
   \end{array}
   \]
Ariel makes a garden path using square and triangular tiles. He uses 4 triangular tiles for every 1 square tile.

He writes a formula — an equation that shows how to calculate the number of triangles from the number of squares:

\[ 4 \times \text{squares} = \text{triangles} \]

or (for short): \[ 4 \times s = t \]

<table>
<thead>
<tr>
<th>Squares (s)</th>
<th>4 \times s = t</th>
<th>Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 \times 1 = 4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4 \times 2 = 8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>4 \times 3 = 12</td>
<td>12</td>
</tr>
</tbody>
</table>

1. Each chart represents a different design for a path. Complete the charts.

<table>
<thead>
<tr>
<th>Squares (s)</th>
<th>5 \times s = t</th>
<th>Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 \times 1 = 5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5 \times 2 = 10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5 \times 3 = 15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares (s)</th>
<th>7 \times s = t</th>
<th>Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 \times 1 = 7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7 \times 2 = 14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7 \times 3 = 21</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares (s)</th>
<th>6 \times s = t</th>
<th>Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6 \times 2 = 12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6 \times 3 = 18</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6 \times 4 = 24</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares (s)</th>
<th>9 \times s = t</th>
<th>Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9 \times 5 = 45</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>9 \times 6 = 54</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>9 \times 7 = 63</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares (s)</th>
<th>7 \times s = t</th>
<th>Triangles (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write a formula to show how to calculate the number of triangles (t) from the number of squares (s).

<table>
<thead>
<tr>
<th>s</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
</tr>
</tbody>
</table>
Sandra makes a border using rectangular and triangular tiles. The number of triangles is always 4 more than the number of rectangles.

Sandra writes this formula:

\[ \text{rectangles} + 4 = \text{triangles} \]

or (for short): \( r + 4 = t \)

### 3. Each chart represents a different design for a border. Complete the charts.

<table>
<thead>
<tr>
<th>Rectangles ((r))</th>
<th>(r + 4 = t)</th>
<th>Triangles ((t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 + 4 = 5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2 + 4 = 6</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3 + 4 = 7</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares ((s))</th>
<th>(s + 7 = t)</th>
<th>Triangles ((t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 + 7 = 8</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2 + ___ = ___</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3 + ___ = ___</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares ((s))</th>
<th>(s + 3 = r)</th>
<th>Rectangles ((r))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 + 3 = 5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>___ + 3 = ___</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>___ + 3 = ___</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares ((s))</th>
<th>(s + 1 = t)</th>
<th>Triangles ((t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>___ + 1 = ___</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>___ + 1 = ___</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>___ + 1 = ___</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares ((s))</th>
<th>(s + 2 = k)</th>
<th>Kites ((k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kites ((k))</th>
<th>(k - 1 = t)</th>
<th>Triangles ((t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9 - 1 = ___</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4. Write a formula to show how to calculate the number of triangles \((t)\) from the number of squares \((s)\).

<table>
<thead>
<tr>
<th>(s)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(s)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(s)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(s)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>
5. Apply the given rule or formula to the numbers in the first column. Write your answer in the second column.

a) Rule: Add 4 to each input number.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

b) Rule: Subtract 4 from each input number.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

c) Rule: Multiply each input number by 5.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

d) Rule: Divide each input number by 3.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

e) Rule: Add 8 to each input number.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

f) Rule: Multiply each input number by 7.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

g) Formula: Input + 4 = Output

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>15</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

h) Formula: Input − 3 = Output

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

i) Formula: Input × 3 = Output

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

6. For each chart, give a rule or a formula (as in Question 5) that tells how to make the numbers in the second column from the numbers in the first column.

a) Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Rule:

b) Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>32</td>
<td>4</td>
</tr>
<tr>
<td>48</td>
<td>6</td>
</tr>
</tbody>
</table>

Rule:

c) Input | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>21</td>
<td>18</td>
</tr>
</tbody>
</table>

Rule:

d) \(x\) | \(y\)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

Formula:

f) Term Number | Term |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
</tbody>
</table>

Formula:
PA8-25 Graphs

A point on a graph or on a coordinate plane can be represented as an ordered pair — a pair of numbers where order matters.

1. For each set of points, write a list of ordered pairs and complete the T-table.

   a)
   
   b)
   
   c)

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 1)</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

2. Mark 3 grid points on each line segment. Then write a list of ordered pairs and complete the T-table.

   a)
   
   b)
   
   c)

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(1, 3)</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ordered Pairs</th>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( , )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>
3. a) Make a T-table for each set of points on the coordinate grid.

<table>
<thead>
<tr>
<th>Line A</th>
<th>Line B</th>
<th>Line C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>Output</td>
<td>Input</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

b) Write a formula for each T-table.

To graph the sequence 2, 4, 6, 8, 10:

Step 1: Make a list of ordered pairs.
(1, 2) (2, 4) (3, 6) (4, 8) (5, 10)

Step 2: Plot the ordered pairs on a graph.

4. a) Draw a line through the points on the grid.
b) Mark 3 more grid points that lie on the line you drew. Then make a T-table for your set of points.
c) Write a formula for your T-table.
d) A point on your line has coordinates (x, 15). Mark the point. x = ________.
e) Why is there not a 0 marked on the horizontal axis?

5. The graph shows the cost of making a telephone call to India.
a) Fill in the T-table.
b) Write a formula for the T-table.
c) How much would you pay to talk for 10 minutes?
d) If you paid $1, how long would you be able to talk for?

BONUS ▶ If your call is 30 seconds long, how much will you pay?
6. Write a list of ordered pairs based on the T-table provided. Plot the ordered pairs.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

7. Draw a graph for each T-table (as in Question 6). Make sure you look carefully at the scales in parts c) and d).

a)

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

b)

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

c)

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

d)

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

8. a) For each formula, fill in the table, then make a list of ordered pairs: (Term Number, Term).

i) \( \text{Term Number} \times 3 + 2 \)

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>(1, 5)</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>(2, 8)</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>(3, 11)</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>(4, 14)</td>
</tr>
</tbody>
</table>

ii) \( \text{Term Number} \times 4 - 3 \)

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>(2, 5)</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>(3, 9)</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>(4, 13)</td>
</tr>
</tbody>
</table>

b) Draw a coordinate grid on grid paper. Plot the ordered pairs on the grid.

c) What is the ordered pair for Term Number 40?
Sequences and Ordered Pairs

You can think of a sequence as a set of ordered pairs with the form (Term Number, Term).

Example: In the sequence $3, 5, 7, 9, 11, \ldots$
11 has term number 5, because 11 is the 5th term.
The ordered pairs are $(1,3), (2,5), (3,7), (4,9), (5,11), \ldots$

1. Complete the T-table for each sequence.

   a) $5, 8, 11, 14, 17$
   b) $4, 6, 8, 10, 12$
   c) $4, 8, 12, 16, 20$

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>(1,5)</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Ordered Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

2. Change each sequence to a set of ordered pairs.

   a) $1, 7, 13, 19, 25$
   $\quad (1,1), (2,7), (3,13), (4,19), (5,25)$

   b) $3, 5, 7, 9, 11$
   $\quad (1,\quad ), (2, \quad ), (3, \quad ), (4, \quad ), (5, \quad )$

   c) $41, 38, 35, 32, 29$
   d) $10, 20, 40, 80, 160$

3. a) Change each set of ordered pairs to a sequence of numbers.

   A $(1,0), (2,6), (3,12), (4,18), (5,24)$
   B $(1,64), (2,32), (3,16), (4,8), (5,4)$

   C $(1,64), (2,32), (3,16), (4,32), (5,64)$
   D $(1,4), (2,9), (3,4), (4,9), (5,4), (6,9), (7,4), (8,9)$

   E $(1,55), (2,46), (3,37), (4,28), (5,19)$
   F $(1,3), (2,6), (3,11), (4,18), (5,27)$

   b) Which sequence from part a) matches each description?

   i) decreases then increases ______
   ii) increases by the same amount ______
   iii) repeats ______
   iv) decreases by different amounts ______
   v) increases by different amounts ______
   vi) decreases by the same amount ______

   c) Plot each set of ordered pairs on a separate coordinate grid and join the points in order.

   d) Describe the graphs. How are the graphs of increasing and decreasing sequences different? How can you tell from the graph that a sequence repeats?
PA8-27  Graphing Sequences

1. a) Graph each sequence of numbers by first making a list of ordered pairs.

   i) 0, 1, 3, 6, 10, 15
       \((1, 0), (2, 1), (3, 3), (4, 6), (5, 10), (6, 15)\)

   ii) 1, 3, 5, 7, 9, 11
        \((1, 1), (2, 3), (3, 5), (4, 7), (5, 9), (6, 11)\)

   iii) 0, 3, 6, 9, 12, 15
        \((1, 0), (2, 3), (3, 6), (4, 9), (5, 12), (6, 15)\)

   iv) 1, 5, 9, 12, 14, 15
        \((1, 1), (2, 5), (3, 9), (4, 12), (5, 14), (6, 15)\)

   v) 4, 2, 0, –2, –4, –6
        \((1, 4), (2, 2), (3, 0), (4, –2), (5, –4), (6, –6)\)

   vi) 14, 10, 8, 4, 3, 0
        \((1, 14), (2, 10), (3, 8), (4, 4), (5, 3), (6, 0)\)

b) Join the points on each graph by a line. Is the line straight?

   i) \(\)  ii) \(\)  iii) \(\)  iv) \(\)  v) \(\)  vi) \(\)

A sequence is called **linear** if the points on its graph can be joined by a straight line.

2. Which of the sequences in Question 1 are linear? ________________________________

3. Graph each sequence on grid paper by first making a list of ordered pairs. Then decide whether the sequence is linear or not.

   a) 3, 2, 1, 0, –1
   b) 1, –2, 3, –4, 5, –6
   c) –8, –5, –2, 1, 4
INVESTIGATION 1 ▶ How can you tell from the gaps whether or not a sequence is linear?

A. Which sequences from Question 1 are linear? Find their gaps.

   || 1 || 3 || 5 || 7 || 9 || 11
   --- || --- || --- || --- || --- || --- || ---

   --- || --- || --- || --- || --- || --- || ---

   --- || --- || --- || --- || --- || --- || ---

B. Which sequences from Question 1 are not linear? Find their gaps.

   --- || --- || --- || --- || --- || --- || ---

   --- || --- || --- || --- || --- || --- || ---

   --- || --- || --- || --- || --- || --- || ---

C. How can you tell by looking at the gaps of a sequence whether or not the sequence is linear?

4. a) Decide which of these decreasing sequences is linear by finding the gaps between terms.
   A. 12, 10, 7, 5, 2
   B. 14, 11, 8, 5, 2

   ___ is linear because ___

b) Check your answer by graphing both sequences on grid paper and joining the points.

INVESTIGATION 2 ▶ How can you tell from the formula whether or not a sequence is linear?

A. For each formula, write the first 4 terms of the sequence. Then find the gaps.

   i) Term Number × 2 + 1
      ___  ___  ___  ___

   ii) Term Number + 5
       ___  ___  ___  ___

   iii) Term Number × Term Number
        ___  ___  ___  ___

   iv) Term Number × 3 − 2
      ___  ___  ___  ___

   v) Term Number ÷ 4
      0.25  ___  ___  ___

   vi) Term Number × (Term Number − 1)
       ___  ___  ___  ___

B. Looking at the gaps, predict which sequences are linear.

C. Choose one sequence that you think is linear and another that you think is not linear.
   Check your predictions by graphing the sequences on grid paper and joining the points.

D. How are the formulas for non-linear sequences different from the formulas for linear sequences?
ME8-12 Volume of Polygonal Prisms

INVESTIGATION 1 ▶ Does the volume of any prism made from centimetre cubes = area of base × height?

i)  

ii)  

iii)  

A. Complete the chart for each prism shown.

<table>
<thead>
<tr>
<th>Figure</th>
<th>i)</th>
<th>ii)</th>
<th>iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of one layer (cm³)</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of structure (cm³)</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area of base (cm²)</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Height of structure (cm)</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Is the volume of any prism made from blocks equal to (area of base) × (height)?

INVESTIGATION 2 ▶ Does the volume of a triangular prism = area of base × height?

A. What fraction of the rectangle is the triangle? Explain.

a)  

b)  

c)  

B. What fraction of the rectangular prism is the triangular prism? Explain.

a)  

b)  

c)  

C. How can you use the volume of the rectangular prisms to find the volume of the triangular prisms?

\[ \text{volume of triangular prism} = \frac{\text{volume of rectangular prism}}{} \]

D. The volume of the rectangular prism at right is

(a) (area of rectangle) \times (\text{____________} \text{ of prism})

= 2 \times (\text{area of } \text{____________}) \times (\text{____________} \text{ of prism})

So the volume of a triangular prism is

(area of ________) \times (\text{____________} \text{ of prism})
1. Imagine rotating the prism so that the shaded base is on the bottom. Trace an edge that shows the height.

2. The bases of the prism are shaded.
   a) Label the height $h$ along one of the edges.
   b) Explain why the volume of the prism is $15 \times h + 20 \times h$.
   c) Is the expression $15 \times h + 20 \times h$ equal to the expression $(15 + 20) \times h$?
      Check for $h = 1$, $h = 2$, and $h = 3$.
   d) How do your answers to parts b) and c) show that the volume of the prism is equal to area of base $\times$ height?

3. Decompose the base into triangles and rectangles. Then find the area of the base and the volume of the prism.

4. Estimate, then calculate, the volume of a prism with height 10 cm and the base shown.

5. Sketch and label two prisms with different heights that have volume 300 cm$^3$.

6. The volume of a right triangular prism is 600 cm$^3$. Its height is 15 cm. What is the area of the base of this prism? Explain how you found your answer.
ME8-13 Volume of Cylinders

1. Calculate the volume of the prism.
   a) \( V = \phantom{0} \text{cm}^3 \)
   b) \( V = \phantom{0} \text{cm}^3 \)
   c) \( V = \phantom{0} \text{cm}^3 \)
   d) \( V = \phantom{0} \text{cm}^3 \)

A cylinder is like a prism, but with circles for bases.

INVESTIGATION ▶ What is the formula for the volume of a cylinder?

A. Predict the formula for the volume of a cylinder in terms of its base and height. Explain your prediction.

B. The volume of a can of food is given on the label. Bilal estimates the inside radius and inside height of four cans, and creates this table.

<table>
<thead>
<tr>
<th>Can</th>
<th>Inside Radius ((r))</th>
<th>(r^2)</th>
<th>Inside Height ((h))</th>
<th>(h \times r^2)</th>
<th>Volume of Food ((V))</th>
<th>(\frac{V}{hr^2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>pea soup</td>
<td>4.8 cm</td>
<td></td>
<td>11 cm</td>
<td>796 cm³</td>
<td>796 cm³</td>
<td></td>
</tr>
<tr>
<td>pasta sauce</td>
<td>4.1 cm</td>
<td></td>
<td>13 cm</td>
<td>680 cm³</td>
<td>680 cm³</td>
<td></td>
</tr>
<tr>
<td>mixed beans</td>
<td>4 cm</td>
<td></td>
<td>10.8 cm</td>
<td>540 cm³</td>
<td>540 cm³</td>
<td></td>
</tr>
<tr>
<td>tomato paste</td>
<td>2.5 cm</td>
<td></td>
<td>8 cm</td>
<td>156 cm³</td>
<td>156 cm³</td>
<td></td>
</tr>
</tbody>
</table>

Bilal measured the outside height of the pea soup can to be 11.8 cm. Why did he estimate a smaller number for the inside height? ________________

Bilal measured the outside diameter of the pea soup can to be 10 cm. What is the outside radius? _____ Why did he estimate 4.8 cm for the inside radius? ________________

C. Complete Bilal’s table.

D. Which column has values always close to \(\pi \approx 3.14\)? ________________

E. Use your answer in part D to write a formula for the volume of a can. \(\pi \approx V \div (\phantom{0})\) so the formula is: \(V = \pi \times \phantom{0}\)

F. The base of a cylinder is a circle with radius \(r\). The area of the circle is ________________.

G. Use your answer in F to rewrite your formula from E for the volume of a cylinder:
   \(V = (\text{area of base circle}) \times \phantom{0}\)

H. How is the formula for the volume of a cylinder like the formula for the volume of a prism? Why does this make sense? Hint: Think about how cylinders are like prisms.

I. Was your prediction in part A correct? _____
2. The volume of a cylinder is equal to (area of base) \( \times \) height. Find the volume (\( V \)) of the cylinder.

\[
\begin{align*}
\text{a)} & \quad \text{area of base } \approx \quad \text{cm}^2 \\
& \quad \text{height } = \quad \text{cm} \\
& \quad \text{volume } \approx \quad \text{cm}^3 \\
\text{b)} & \quad \text{area of base } \approx \quad \text{cm}^2 \\
& \quad \text{height } = \quad \text{cm} \\
& \quad \text{volume } \approx \quad \text{cm}^3 \\
\text{c)} & \quad \text{area of base } \approx \quad \text{cm}^2 \\
& \quad \text{height } = \quad \text{cm} \\
& \quad \text{volume } \approx \quad \text{cm}^3 
\end{align*}
\]

3. Use the radius or diameter to find the area of the base and the volume of the cylinder.

\[
\begin{align*}
\text{a)} & \quad \text{area of base } \approx \quad \text{cm}^2 \\
& \quad \text{height } = \quad \text{cm} \\
& \quad \text{volume } \approx \quad \text{cm}^3 \\
\text{b)} & \quad \text{area of base } \approx \quad \text{cm}^2 \\
& \quad \text{height } = \quad \text{cm} \\
& \quad \text{volume } \approx \quad \text{cm}^3 
\end{align*}
\]

4. Tina has a jar 20 cm high. She can fit 30 candies flat on the bottom of the jar. Each candy is 1 cm high. How many candies can she fit into the jar if...

a) the jar is a cylinder? 

b) the jar is a right prism?

Are the two answers the same? Why does this happen?

5. Regular polygons with many sides look a lot like circles. Some Canadian pennies are circular, but some are polygonal.

a) Estimate the volume of a penny by pretending all pennies are perfect cylinders.

**Step 1:** Measure the diameter of a penny. Diameter = _____ mm

**Step 2:** Calculate the radius of the penny. Radius = _____ mm

**Step 3:** Measure the height of the penny.

First, measure the height of one coin individually. _____ mm.

Next, measure the height of 10 coins by stacking them.

Then divide your answer by 10.

\[
\text{Height of 1 coin} = \text{Height of 10 coins} \div 10 \\
= \quad \text{cm} \div 10 \\
= \quad \text{mm}
\]

Which answer for the height of one coin is more accurate? Explain.

**Step 4:** Calculate the volume of the penny using the radius and height you found.

b) Sara fills a graduated cylinder to the 30 mL mark. She then adds 10 pennies.

Where should the water level be now?
ME8-16 Surface Area of Prisms

Note: Pictures are not drawn to scale.

1. In each prism, shade all the edges that have the same length as the edge marked.

   a) ![Prism A](image1)
   b) ![Prism B](image2)
   c) ![Prism C](image3)
   d) ![Prism D](image4)

2. Find the missing edge length for the prism.

   a) ![Prism A with missing edge](image5)
   b) ![Prism B with missing edge](image6)
   c) ![Prism C with missing edge](image7)

3. Shade the face that has the same area as the shaded face.

   a) ![Face A](image8)
   b) ![Face B](image9)
   c) ![Face C](image10)

4. The area of each visible face is given. What is the area of each hidden face?

   a) ![Visible Face A](image11)
   b) ![Visible Face B](image12)
   c) ![Visible Face C](image13)

   back
   bottom
   left

   back
   bottom
   left

   back

5. Write the area of each visible face directly on the face. Then double each area to find the total area of each pair of congruent faces.

   ![Face with areas](image14)

   \[
   \text{front} + \text{back} = 6 \text{ cm}^2 \times 2 = 12 \text{ cm}^2
   \]

   \[
   \text{top} + \text{bottom} = \quad = \quad
   \]

   \[
   + \quad = \quad = \quad
   \]
The **surface area** of a 3-D shape is the total area of all the faces of the shape.

6. Calculate the surface area of the prism.
   a) ![Diagram of a rectangular prism]
   b) ![Diagram of a rectangular prism]
   c) ![Diagram of a rectangular prism]

7. Miki calculates the surface area of the prism to be 40 cm². What did she do wrong?

8. a) Alexandra says that she needs to find the area of only two faces of this prism to calculate the surface area. Is she correct? Explain.
   b) What is the surface area of the prism?

9. Write the name of each face of the prism on the net, then mark the length of each edge on the net.
   a) ![Diagram of a rectangular prism net]
   b) ![Diagram of a rectangular prism net]
   c) ![Diagram of a rectangular prism net]

10. Find the surface area of each prism from Question 9. Include the units.
    a) ____________________
    b) ____________________
    c) ____________________

How is the surface area of a prism related to the area of its net? Explain.
11. Find the missing length.
   a) \[
   \begin{array}{c}
   3 \text{ m} \\
   \hline
   \text{Area} = 12 \text{ m}^2 \\
   \hline
   \end{array}
   \]
   b) \[
   \begin{array}{c}
   \_ \text{ m} \\
   \hline
   \text{Area} = 15 \text{ m}^2 \\
   \hline
   \end{array}
   \]
   c) \[
   \begin{array}{c}
   2 \text{ m} \\
   \hline
   \text{Area} = 14 \text{ m}^2 \\
   \hline
   \end{array}
   \]

12. Find the missing edge length.
   a) \[
   \begin{array}{c}
   4 \text{ m} \\
   \hline
   6 \text{ m} \\
   \hline
   20 \text{ m}^2 \\
   \hline
   \_ \text{ m} \\
   \end{array}
   \]
   b) \[
   \begin{array}{c}
   3 \text{ m} \\
   \hline
   7 \text{ m} \\
   \hline
   15 \text{ m}^2 \\
   \hline
   \_ \text{ m} \\
   \end{array}
   \]
   c) \[
   \begin{array}{c}
   12 \text{ m}^2 \\
   \hline
   3 \text{ m} \\
   \hline
   2 \text{ m} \\
   \hline
   \_ \text{ m} \\
   \end{array}
   \]

13. Edges \( a \), \( b \), and \( c \) have lengths that are whole numbers. The surface area of each face is written directly on the face. What are some possible lengths for edges \( a \), \( b \), and \( c \)? (Hint: Why can edge \( a \) not be 4 m long?)

14. Write a formula for the surface area of the prism using the length (\( \ell \)), width (\( w \)), and height (\( h \)).

15. Calculate the surface area of the prism. Be careful with the units!

16. It costs $0.40 per square metre to paint a room. How much would it cost to paint the walls of this room (not including the door and windows)?

17. Look at the prism at right. It is not drawn to scale.
   a) Draw a better sketch.
   b) Find the volume and surface area of this prism.

18. a) Write a rule that tells you how to calculate the surface area of the figures from the figure number. (Each cube has length, width, and height 1 cm.)
   b) Use your rule to predict the surface area of the 20th figure.
ME8-17 Surface Area of Cylinders

1. A paper towel tube is a cylinder without the top and bottom circles.
   a) Cut a paper towel tube vertically. What shape do you get when you open it up?
      
   b) Cut a paper towel tube diagonally. What shape do you get when you open it up?
      
   c) If the tube has circumference 15 cm and height 12 cm, what are the base and height of the shapes in parts a) and b)?
      i) 
      ii ) 
      
   d) Do the shapes in parts a) and b) have the same area? 
      Why does this make sense?
      
   e) What is the surface area of the tube? 

2. Find the surface area of the tube by finding the length and width of a rectangle with the same area.
   a) 
   b) 
   c) 

3. Write a formula for the surface area (SA) of a tube in terms of...
   a) the circumference C of the circular base and the height h. 
      
   b) the diameter d of the circular base and the height h. 
      
   c) the radius r of the circular base and the height h. 
      
4. Which of these shapes can be the net for a tube? Explain.
   A   B   C   D   E   F
5. A can of food is a cylinder. It has both top and bottom circles.
   a) Find the surface area of the can.
      \[
      \begin{align*}
      \text{Area of rectangle} & = \\
      \text{Area of top circle} & = \\
      \text{Area of bottom circle} & = \\
      \hline
      \text{Surface area of can} & = 
      \end{align*}
      \]

5 cm \[\text{length} = 2\pi r \approx 18.8 \text{ cm}\]

b) Which net matches the can in part a)? What is wrong with the other net?

6. a) Which cylinder matches each net?

A \hspace{1cm} B \hspace{1cm} C \hspace{1cm} D \hspace{1cm} E

b) Sketch nets for the two cylinders that were not matched.

c) How does the surface area of a cylinder compare to the area of its net? Explain.

7. Find the surface area of each can by first finding the area of a rectangle and the area of the top and bottom circles.

a) \[
\begin{align*}
\text{5 cm} & \hspace{1cm} \text{5 cm} \\
\end{align*}
\]

b) \[
\begin{align*}
\text{2 cm} & \hspace{1cm} \text{9 cm} \\
\end{align*}
\]

c) \[
\begin{align*}
\text{9 cm} & \hspace{1cm} \text{2 cm} \\
\end{align*}
\]

8. A cylindrical can has a circular base of radius \(r\) and height \(h\). Write a formula for its surface area. Do not forget to include the top and bottom.