PS4-4  Using Systematic Search and Structure

Teach this lesson after: 4.1 Measurement

Goals:
Students will use structure (place value, properties of operations) to reduce the amount of searching needed to solve a problem.

Prior Knowledge Required:
Can solve simple addition puzzles with different letters representing different digits
Can fluently add and subtract multi-digit numbers using the standard algorithm
Can use the order of numbers to organize their search to find all possible answers to a problem
Can multiply single-digit numbers
Can use systematic search to solve problems (for Problem Bank 2)

Vocabulary: equation, regroup, sum

Materials:
BLM Secret Meeting Place with Addition (pp. 22–24, see Problem Bank 3)

Solving addition puzzles with different letters and repeated letters in the same puzzle.
Write on the board:

A6
+ AB
74

Point to the first puzzle and SAY: Remember that we have used letters to replace missing digits before. There are two A’s and one B in this puzzle. Each letter stands for a different digit. ASK: What does that tell you? (the two A’s are the same digit and the B is different) Point to the second puzzle and SAY: If the puzzle had just boxes for the missing digits, you wouldn’t have as much information. Having the A’s and B’s instead of boxes tells you something that will help you solve the puzzle.

SAY: Now, let’s try to solve this puzzle. ASK: What is 6 + B? (14) How do you know it’s not four? (because 4 is less than 6) Write on the board:

6 + B = 14

ASK: What is B? (8) Erase B and write “8” in its place, as shown below:

A6
+ A8
74
SAY: Six plus eight is 14, so we have to regroup one to the tens. Write “1” above the two A’s to show the regrouping, circle the tens column, and write the equation for the tens as shown below:

\[
\begin{array}{c}
7 \\
A 6 \\
+ A 8 \\
7 4 \\
\end{array}
\]

\[1 + A + A = 7\]

ASK: What is A + A before I add the one? (6) PROMPT: What is one less than seven? Write on the board:

\[A + A = 6\]

ASK: So, what is A? (3) PROMPT: What number do you double to get six? Write on the board:

\[A = 3\]

SAY: Let’s write that into the puzzle and make sure we get the correct answer. Write on the board:

\[
\begin{array}{c}
36 \\
+ 38 \\
\end{array}
\]

Have a volunteer do the addition. (74) SAY: So, we know we solved the puzzle correctly.

NOTE: For part c) in the exercises below, it is not immediately clear whether A is 2 or 7. Suggest to students who are struggling that they try both to see which one works. A similar strategy will be useful for part e).

Exercises: Solve the puzzle.

a) \[A 36\]  b) \[A 7\]  c) \[13A\]  d) \[A 8\]  e) \[A 2B\]  

\[+ 5AB\]  \[+ AB\]  \[+ B5A\]  \[+ AB\]  \[+ 1AB\]

\[978\]  \[69\]  \[694\]  \[94\]  \[576\]  \[+ AB\]  \[70\]

Answers: a) A = 4, B = 2; b) A = 3, B = 2; c) A = 7, B = 5; d) A = 4, B = 6; e) A = 4, B = 8; Bonus: A = 2, B = 4

Solving puzzles in which one letter must represent zero, due to the ones digit not changing. Write on the board:

\[
\begin{array}{c}
A 3 \\
+ 2B \\
83 \\
\end{array}
\]

ASK: What is 3 + B? (3) PROMPTS: Can three plus a one-digit number be 13? (no) Why not? (because the number would have to be 10, which is two digits)
Write on the board:

\[ 3 + B = 3 \]

ASK: So, what is B? (0) SAY: Sometimes, you can tell right away that one of the digits has to be zero. SAY: Once you know that B is zero, you can complete the puzzle. Erase B in the vertical addition and write "0" in its place. Have a volunteer find A. (6)

**Exercises:** Solve the puzzle.

- a) \( A7 \)
- b) \( 36 \)
- c) \( 7A \)
- d) \( AA \)

**Bonus:** \( AB \)

\[
\begin{array}{ccc}
+6B & +AB & +AB \\
97 & 86 & 9A \\
\end{array}
\]

**Answers:**

- a) A = 3, B = 0
- b) A = 5, B = 0
- c) A = 2, B = 0
- d) A = 4, B = 0
- Bonus: A = 9, B = 0

Deciding whether regrouping is done based on the structure of the puzzle. Write on the board:

\[
\begin{array}{c}
AB \\
+ B \\
B4
\end{array}
\]

ASK: What does the addition of the ones digits tell you? (B + B = 4 or B + B = 14) Write on the board:

\[
B + B = 4 \quad \text{or} \quad B + B = 14
\]

SAY: We don't know yet if B + B is 4 or 14. But there is something in the puzzle that tells you the answer. ASK: Can A and B stand for the same letter? (no) Why not? (that's the rule of the puzzle) PROMPT: Remember that there is a rule that different letters stand for different digits and the same letter always stands for the same digit. SAY: Let's try the first option.

ASK: If B + B = 4, what is B? (2) SAY: Let's put that into the puzzle. Write on the board:

\[
\begin{array}{c}
A2 \\
+ 2 \\
24
\end{array}
\]

ASK: If B is 2, what is A? (2) But, can A and B be the same number? (no) Refer students back to the original puzzle. Circle the A in AB and the B in B4 and SAY: You can tell by the tens digits being different that the ones digits require regrouping. Even though you don't know what A and B are, this type of puzzle tells you that they are different and the only way adding 0 tens to A can get B is if there was regrouping from the ones. Leave the puzzle on the board.

**Exercises:** Do the ones digits require regrouping? Explain how you decided.

- a) \( BB \)
- b) \( AB \)
- c) \( AA \)
- d) \( AA \)
- e) \( AA \)

\[
\begin{array}{ccc}
+ B & + B & + A \\
A4 & B8 & A6 \\
\end{array}
\]

\[
\begin{array}{ccc}
+ A & + A & + A \\
B6 & B6 & B0
\end{array}
\]
Answers: a) yes, because the tens digit of BB is different from the tens digit of A4
b) yes, because the tens digit of AB is different from the tens digit of B8
c) no, because the tens digits of AA and A6 are the same
d) yes, because the tens digits of AA and B6 are different
e) yes, because the tens digits of AA and B0 are different

SAY: Once you know whether the ones digits require regrouping, you can figure out what the ones digits are. Refer back to the example on the board and SAY: You know that B + B is 14, not 4, because the ones digits need regrouping, so you know right away that B is 7, not 2. Write on the board:

\[
\begin{align*}
1 \\
A7 \\
+ & 7 \\
\hline
74
\end{align*}
\]

Have a volunteer write the equation for the ones (7 + 7 = 14) and another volunteer write the equation for the tens (1 + A = 7) SAY: 1 + A is 7. ASK: What is A? (6) Have a volunteer add 67 and 7 to show that the answer is indeed 74.

Exercises: Solve the puzzles from the previous exercises.

Answers: a) A = 8, B = 7; b) A = 8, B = 9; c) A = 3; d) A = 8, B = 9; e) A = 5, B = 6

Solving puzzles in which a digit must be 1 based on the regrouping structure of the puzzle. Write on the board:

\[
\begin{align*}
3B \\
+ & 81 \\
\hline
B1A
\end{align*}
\]

SAY: Another rule of these puzzles is that no number can start with zero. So, if you see a letter at the beginning of a number, it cannot be zero. Point to “B1A” on the board and SAY: In this puzzle, A can be zero, but B cannot.

SAY: Here, 2 two-digit numbers add to a three-digit number. ASK: What do you know about the three-digit number—how big can it be? (it must be less than 200) PROMPT: Can it be more than 200? (no) Why not? (because each of the two numbers being added is less than 100, so the sum cannot be 200 or greater) ASK: So, what does B have to be? (1) Can B be zero? (no) Why not? (B is at the beginning of a number) SAY: This tells us a lot about the puzzle already. Ask a volunteer to erase the B’s and write “1” in their places, as shown below:

\[
\begin{align*}
31 \\
+ & 81 \\
\hline
11A
\end{align*}
\]

Have another volunteer complete the puzzle. (A = 2)
Exercises: Decide which letter must be 1. Then solve the puzzle.

a) 6B  
b) 29  
c) B8  
d) 7A  
+ 78  
+ 7A  
+ A5  
+ 8A  
B3A  
ABB  
BB3  
A5B  

Answers: a) A = 9, B = 1; b) A = 1, B = 0; c) A = 9, B = 1; d) A = 1, B = 2

Deciding if regrouping is required using even and odd numbers. Write on the board:

AB
+ AB
56

ASK: What can you say about B + B? (B + B = 6 or B + B = 16) So, what can B equal? (3 or 8) SAY: Let’s try B = 3. Write on the board:

A3
+ A3
56

ASK: If B is 3, what is A + A? (5) What digit, doubled, is 5? (there is no such digit) Why not? (because 5 is odd and doubling results in an even number) SAY: So, that’s another way to tell if regrouping is required. Write on the board:

1
A8
+ A8
56

ASK: What is A + A? (4) How do you know? (because adding 1 gets 5) Write on the board:

A + A = 4

ASK: So, what is A? (2) Have a volunteer add 28 + 28 to check the solution. (56)

Exercises: Solve the puzzle. Then check your answer by doing the addition.

a) AB  
b) AB  
c) AB  
d) AB  
+ AB  
+ AB  
+ AB  
+ AB  
78  
42  
90  
84

Bonus:

e) AAB  
f) AAB  
g) AAB  
h) AAB  
+ AB  
+ AB  
+ AB  
+ AB  
A78  
B78  
A46  
C46

Answers: a) A = 3, B = 9; b) A = 2, B = 1; c) A = 4, B = 5; d) A = 4, B = 2; Bonus: e) A = 3, B = 9; f) A = 8, B = 9; g) A = 2, B = 3; h) A = 7, B = 3, C = 8
Problem Bank

1. Solve the puzzle. Hint: In part a), how big does A need to be for AA + 11 to be 11C?

   a)  
   \[ \begin{array}{c}
   \text{AA} \\
   + \text{BB}
   \end{array} \]

   b)  
   \[ \begin{array}{c}
   \text{AB} \\
   + \text{AB}
   \end{array} \]

   c)  
   \[ \begin{array}{c}
   \text{ABC} \\
   + \text{ABC}
   \end{array} \]

   \[ \begin{array}{c}
   \text{BBC} \\
   \text{BCC}
   \end{array} \]

   \[ \begin{array}{c}
   \text{BCDC}
   \end{array} \]

   **Solutions:**

   a) We know that A and B cannot be 0, but C could be 0. Because BBC must be less than 200, we know B = 1, so AA + 11 = 11C. But A can’t be 8 or less because 88 + 11 is only 99, not a three-digit number, so A is 9 and 99 + 11 = 110, so C = 0.

   b) For the sum to be BCC, B must be 1, so C must be 2. That makes A + A = 12, so A = 6.

   c) For the sum to be BCDC, B must be 1 and C must be 0, as that is the only one-digit number which, doubled, has itself as a ones digit. So, B + B has ones digit D and, since B is 1, D must be 2. We know so far that A10 + A10 = 1020, so A = 5.

2. Kim wants to solve this puzzle:

   \[ \begin{array}{c}
   \text{AB} \\
   + \text{A}
   \end{array} \]

   \[ \begin{array}{c}
   \text{B5}
   \end{array} \]

   a) Will B + A require regrouping? How do you know?

   b) What is B + A?

   c) Which is greater, A or B? How much greater?

   d) Find A and B that satisfy the conditions from parts b) and c). Hint: Start at A = 1 and go up in order trying the different possibilities for A.

   **Selected solution:**

   d) Try the numbers in order that satisfy B is one more than A, until you get B + A = 15:

   \[
   \begin{array}{c|c}
   A & B \\
   \hline
   0 & 1 \\
   1 & 2 \\
   2 & 3 \\
   3 & 4 \\
   4 & 5 \\
   5 & 6 \\
   6 & 7 \\
   7 & 8 \rightarrow 7 + 8 is 15 \\
   \end{array}
   \]

   So, A = 7, B = 8, and 78 + 7 does equal 85.

   **Answers:** a) yes, because A and B in the tens digits are different; b) 15; c) B is one greater than A because when you regroup the 1 ten and add it to A, you get B

3. Provide students with BLM Secret Meeting Place with Addition. It is an opportunity to do multi-part problems with a context.

   **Selected solution:** 2. b) U cannot be 2 because, as the tens digit of the answer, it must be greater than 3, so U is 7 and A is 3.
Answers:
1. a) 6, 12, 24, 30, 36, 42, 48, 54, 60; b) 8
2. a) 2 or 7; b) A = 3, U = 4
3. a) B = 8; b) A = 3, U = 7; c) D = 5, R = 6; d) N = 4, Y = 9; e) B = 8, M = 2, L = 1, E = 0
4. a) ELMANDRUBY; b) no; c) Elm Street, Ruby Street

4. Solve the puzzle. Hints: In part a), how big does A need to be for AA + 1C to be 111? In part b), which digit must be 1? In the bonus, you will need to use almost all the tools you learned in this lesson.

a)     AA  b)    ABA  Bonus:  A BCD  
+  BC  −  CA  +  A BCD  
BBB  AB  DA FGF  

Solutions:

a) B = 1, so the puzzle becomes AA + 1C = 111; looking at the tens digits, A can’t be 8 or less and so A = 9. Then the puzzle becomes 99 + 1C = 111, so C = 2.
b) This can be done either directly or by changing to an addition puzzle. A = 1 and B = 0, so 101 − C1 = 10, so C = 9.

Bonus: We know from the ten thousands digit that D = 1, so in the ones digits, 1 + 1 = F, so F = 2. Now we know that the hundreds digit is F = 2, an even number; so, in the tens digits, C + C can’t require regrouping (B + B + 1 would have the ones digit 2, which would make B + B odd, but the double of B cannot be odd). So B + B is either 2 or 12, so B = 1 or 6; but we can rule out 1, because D = 1, so B = 6. So far, we have:

\[ 1 \]
\[ A 6C1 \]
\[ + A 6C1 \]
\[ 1A 2G2 \]

In the thousands column, 1 + A + A has ones digit A and tens digit 1. Adding 1 + A to A is the same as adding 10 to A. That means 1 + A = 10 and so A = 9. At this point, it looks like there are a lot of possibilities for C and G (1 + 1 = 2, 2 + 2 = 4, 3 + 3 = 6, 4 + 4 = 8). Since 1, 2, and 6 are already taken, C = 4 and G = 8.

5. a) In the shape below, use the numbers 1 to 5 once each so that both lines of three numbers add to 10. Hint: Do a systematic search to see which number can be the corner number.

b) Find the corner number a different way.
   i) What do all the numbers in the circles add to?
   ii) What is the total of the three numbers in one line plus the three numbers in the other line?
   iii) The corner number is counted twice in part ii). Which number must that be? How do you know?

Sample answer: a)
**Answers:** b) i) 15; ii) 20; iii) 5, because 20 is 5 more than 15, so 5 must have been the number that was counted twice

6. In the shape below, three numbers can make a line up the left side, along the bottom, and up the right side.

```
   
  
 
 
```

Use the numbers 1 to 6 once each in the shape so that …

a) each line of three numbers adds to 10. Hints: Find all sums of numbers from 1 to 6 that add to 10. Which numbers are in two sums? Which numbers are in only one sum? How does that tell you which numbers are corner or side numbers?

b) each line of three numbers adds to 11.

c) each line of three numbers adds to 12.

d) each corner number is the sum of the two adjacent numbers. Hint: Should 6 be a corner number or a side number, and why?

**Sample answers:**

a)  

```
2 4
3 6 1
```

b)  

```
4
5 1
```

c)  

```
6
1 2
```

d)  

```
5
3 4
```

7. a) Using three different numbers from 1 to 9, write all the sums that equal 15. Did you find 8 sums? If not, find your mistake.

b) Sally wants to use the numbers 1 to 9 so that each row, column, and diagonal line of three numbers in the shape below adds to 15.

```
     5
  
  
     1
```

i) How many different sums will she need to make?

ii) How many sums is the middle number in?

iii) Look at your answer to part a). Which number is in four different sums?

iv) What number should Sally put in the middle? Explain.

c) Complete the puzzle for Sally. Hint: Which numbers are in three different sums? Which numbers are in only two different sums? What does that tell you about where the numbers must go in the puzzle?

**Sample solution:** c) The numbers 2, 4, 6, and 8 are each in three different sums, whereas 1, 3, 7, and 9 are in only two different sums, so 2, 4, 6, and 8 are corner numbers and 1, 3, 7, and 9 are side numbers.

```
8 3 4
1 5 9
6 7 2
```
**Answers:** a) $9 + 5 + 1, 9 + 4 + 2, 8 + 6 + 1, 8 + 5 + 2, 8 + 4 + 3, 7 + 6 + 2, 7 + 5 + 3, 6 + 5 + 4;$
b) i) 8, ii) 4, iii) 5, iv) 5, because it is the only number that is in four different sums

8. In the puzzles below, the same letters stand for the same numbers in both puzzles and different letters stand for different numbers, so both additions have the same answer.

Find $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, and $I$, so that:

\[
\begin{array}{ccc}
A & A & A \\
A & A & A \\
B & B & B \\
B & B & B \\
+ & C & C & C \\
+ & E & E & E \\
F & G & H & I \\
F & G & H & I \\
\end{array}
\]

**Solution:** In the first equation, the only way $A + B + C$ can have three different answers is if 1 ten is regrouped first and then 2 tens are regrouped after:

\[
\begin{array}{ccc}
2 & 1 \\
A & A & A \\
A & A & A \\
B & B & B \\
B & B & B \\
+ & C & C & C \\
+ & E & E & E \\
F & G & H & I \\
F & G & H & I \\
\end{array}
\]

That means, for the first equation:

$A + B + C = 19$

$A + B + C + 1 = 20$

$A + B + C + 2 = 21$

So $I = 9$, $H = 0$, $G = 1$, $F = 2$, and $FGHI = 2109$. For the same reasons that $A + B + C$ is 19, $A + D + E$ is 19 in the second equation. The digits $A$, $B$, $C$, $D$, and $E$ must be chosen from 3, 4, 5, 6, 7, and 8 (since 0, 1, 2, and 9 are taken). One possibility that works is $8 + 7 + 4 = 8 + 6 + 5$, so one answer is $A = 8$, $B = 7$, $C = 4$, $D = 6$ and $E = 5$. 
Secret Meeting Place with Addition (1)

1. a) Complete the 6 times table.
   
   \[
   \begin{array}{c c}
   1 \times 6 &= 6 \\
   2 \times 6 &= 12 \\
   3 \times 6 &= 18 \\
   4 \times 6 &= 24 \\
   5 \times 6 &= 30 \\
   \end{array}
   \]

   b) In the puzzle below, which digit does B stand for? _____
   
   \[B \times 6 = 4B\]

2. John wants to solve this puzzle:
   
   \[
   \begin{array}{c c c}
   A U \\
   + 3 U \\
   \hline
   U 4 \\
   \end{array}
   \]

   a) Which digits can U be so that U + U has ones digit 4? _____ or _____

   b) Solve the puzzle.
Secret Meeting Place with Addition (2)

3. Solve the puzzle. The rules are:

• Two different letters stand for two different digits, even in different puzzles.
• A letter must stand for the same digit every time it occurs, even in different puzzles.
• No two-digit or three-digit number can begin with 0.

Hint: Use your answers from Questions 1 and 2 to solve parts a) and b).

a) \( B \times 6 = 4B \)  \( B = \) ____

b) \[
\begin{array}{c}
A \\
\underline{+ \quad 3 \\
U 4}
\end{array}
\]  \( A = \) ____  \( U = \) ____

c) \[
\begin{array}{c}
D \\
\underline{+ \quad 7 \\
1 2 9}
\end{array}
\]  \( D = \) ____  \( R = \) ____

d) \[
\begin{array}{c}
N \\
\underline{+ \quad N 5 \\
9 N}
\end{array}
\]  \( N = \) ____  \( Y = \) ____

e) \[
\begin{array}{c}
M \\
\underline{+ \quad B M \\
L L E}
\end{array}
\]  \( B = \) ____  \( M = \) ____  \( L = \) ____  \( E = \) ____

Hint: Which letter do you already know from a previous puzzle?
Secret Meeting Place with Addition (3)

4. All the letters from Question 3 stand for a different digit from 0 to 9. When you put them in order, you can find the name of a secret meeting place.

   a) Write the letters in order.

   0 1 2 3 4 5 6 7 8 9

   _______ _______ _______ _______ _______ _______ _______ _______ _______

   b) Is any number missing a letter? If so, find your mistake.

   c) Decode the message. You will need to:
      • Read the name of the two street names you are to meet at.
      • Find the streets on the map to know where to meet.

   Meet at __________________________ and __________________________.

   Mark the location on the map below.

   ELY STREET

   BURY STREET

   MEL STREET

   BURM STREET

   RUBY STREET

   BRUM STREET

   ELM STREET