PS6-2 Using Structure I: Multiplication Puzzles

Teach this lesson after:
Unit 4

VOCABULARY
area
product
thousands

Goals
Students will mentally compute the ones digit of a product of multi-digit numbers.
Students will solve multi-digit multiplication puzzles involving missing digits, where different letters stand for different digits and identical letters stand for identical digits.

PRIOR KNOWLEDGE REQUIRED
Can use the guess-check-revise strategy
Can use systematic search
Can multiply 2 two-digit numbers
Can multiply a multi-digit number by a one-digit number
Can apply the distributive property

Mentally determining the ones digits of products. Start with the following exercises.

Exercises
a) Multiply the number pairs.

\[
\begin{align*}
4 \times 3 & \quad 14 \times 3 & \quad 34 \times 3 & \quad 24 \times 3 & \quad 74 \times 3 \\
\end{align*}
\]

b) Circle the ones digit in the answers from part a). What do you notice?

Answers: a) 12, 42, 102, 72, 222; b) the ones digit is always 2

ASK: Why do you think the ones digit is always the same? (the ones digits being multiplied are always the same; you are always multiplying 4 \times 3 to get the ones digit) Write on the board:

\[
24 \times 3 = 20 \times 3 + 4 \times 3 \\
= 60 + 12 \\
= 72
\]

SAY: Adding 60 doesn’t change the ones digit, so the ones digit of 24 \times 3 is the same as the ones digit of 4 \times 3. You can do that with any number. You can break up the tens and ones. Multiplying the tens by the ones doesn’t contribute to the ones digit; multiplying the ones by the ones does.

Exercises: Mentally determine the ones digit of the product.

a) 76 \times 3 \\
b) 87 \times 4 \\
c) 62 \times 9 \\
d) 54 \times 6

Answers: a) 8, b) 8, c) 8, d) 4

SAY: You can do the same thing with multiplying multi-digit by one-digit numbers. You can break down a three-digit number into hundreds, tens, and ones. You can break down a four-digit number into thousands,
hundreds, tens, and ones. The only part that contributes to the ones digit is when you multiply the ones.

**Exercises:** Mentally determine the ones digit of the product.

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<tr>
<td>a)</td>
<td>243 \times 7</td>
<td>b) 182 \times 6</td>
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<tr>
<td>c)</td>
<td>1435 \times 2</td>
<td>d) 807 431 613 \times 3</td>
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**Answers:** a) 1, b) 2, c) 0, d) 9

Write on the board:

\[
\begin{array}{c}
34 \\
\times 53
\end{array}
\]

ASK: Without doing the full multiplication, how can you find the ones digit of the answer? (multiply 4 \times 3; the ones digit of 12 is 2, so the ones digit of the answer is 2) Draw on the board:

\[
\begin{array}{c|c}
50 & 3 \\
30 & \\
4 &
\end{array}
\]

ASK: How does this rectangle show the product 34 \times 53? (the area of the rectangle is 34 \times 53 because one side is 34 units long and the other side is 53 units long) Have volunteers write the area of each smaller rectangle in the diagram, as shown below:

\[
\begin{array}{c|c|c}
50 & 3 & \\
30 & 1500 & 90 \\
4 & 200 & 12 \\
\end{array}
\]

SAY: The whole area is 1500 + 90 + 200 + 12. ASK: What is the only part that contributes to the ones digit? (12) SAY: The ones digit of 34 \times 53 is the same as the ones digit of 4 \times 3, which is easy to calculate.
**Exercises:** Mentally determine the ones digit of the product.

a) $27 \times 54$  

b) $16 \times 32$  

c) $85 \times 37$  

d) $14 \times 29$  

e) $243 \times 117$  

f) $614 \times 516$  

g) $235 \times 912$  

h) $817 \times 4367$  

**Bonus**

i) $11 \times 22 \times 33 \times 44 \times 55$  

j) $31 \times 71 \times 81 \times 21 \times 51$  

**Answers:** a) 8, b) 2, c) 5, d) 6, e) 1, f) 4, g) 0, h) 9, Bonus: i) 0, j) 1, k) 2

**Introduce missing digit puzzles.** Write on the board:

$7 \times A = B2$  
$6 \times C = 4C$

SAY: The rule is that the same letters stand for the same digit and different letters stand for different digits. In the first puzzle, $A$ and $B$ stand for different digits. In the second puzzle, both Cs stand for the same digit. Pointing to the first puzzle, ASK: What number in the seven times table has ones digit 2? (42) SAY: If you don’t have the times table memorized, you can skip count until you get ones digit 2: 7, 14, 21, 28, 35, 42. So, $B = 4$ in the first puzzle. ASK: What is $A$ in the first puzzle? (6) How do you know? ($7 \times 6 = 42$) Pointing to the second puzzle, ASK: What numbers in the 6 times table are in the forties? (42 and 48) What is $C$? (8) PROMPTS: Does $6 \times 2$ equal 42? (no) Does $6 \times 8 = 48$? (yes) SAY: Remember that both Cs have to be the same, so both Cs stand for 8 in this case.

**Exercises:** Solve the puzzle.

a) $9 \times A = 4A$  

b) $7 \times A = 5B$  

c) $A \times A = 2A$  

d) $A \times A = 4B$

**Answers:** a) $A = 5$; b) $A = 8$, $B = 6$; c) $A = 5$; d) $A = 7$, $B = 9$

**Solving puzzles multiplying two digits by one digit.** Write on the board:

\[
\begin{array}{c}
1A \\
\times 4 \\
\hline
48
\end{array}
\quad \begin{array}{c}
1A \\
\times 4 \\
\hline
68
\end{array}
\]

Point to the first multiplication and ASK: When the ones were multiplied, was anything regrouped to the tens? (no) How do you know? ($4 \times 1$ is 4; if something was regrouped, you would have had to add it to get the number of tens) Point to the second multiplication and ASK: When the ones were multiplied, was anything regrouped to the tens? (yes) How do you know? (you need to add 2 to $4 \times 1$ to get 6 tens) Keep these examples on the board.
Exercises: Was anything regrouped to the tens? If so, how many tens?

a) \[
\begin{array}{c}
3A \\
\times 4
\end{array}
\]
\[128\]
b) \[
\begin{array}{c}
3A \\
\times 4
\end{array}
\]
\[148\]
c) \[
\begin{array}{c}
7A \\
\times 3
\end{array}
\]
\[234\]
d) \[
\begin{array}{c}
7A \\
\times 3
\end{array}
\]
\[219\]

Answers: a) no; b) yes, 2 tens; c) yes, 2 tens; d) no

Refer students back to the examples on the board. SAY: In the first puzzle, there was no regrouping, but in the second puzzle, 2 tens were regrouped when you multiplied the ones. Write on the board:

\[
A \times 4 = 8 \quad A \times 4 = 2 \text{ tens} + 8 \text{ ones} = 28
\]

SAY: In the first puzzle, \(A \times 4\) is just 8 because there is no regrouping.

ASK: So, what is \(A\)? (2)

SAY: In the second puzzle, \(A \times 4\) is 28 because 2 tens were regrouped. ASK: So, what is \(A\)? (7)

Exercises: Solve the puzzle. Hint: Write the puzzle vertically.

a) \[
\begin{array}{c}
2A \\
\times 6
\end{array}
\]
\[126\]
b) \[
\begin{array}{c}
2A \\
\times 6
\end{array}
\]
\[156\]
c) \[
\begin{array}{c}
5A \\
\times 3
\end{array}
\]
\[171\]
d) \[
\begin{array}{c}
5A \\
\times 3
\end{array}
\]
\[156\]

Answers: a) \(A = 1\), b) \(A = 6\), c) \(A = 7\), d) \(A = 2\)

SAY: You can also solve this type of puzzle by using long division because there is only one unknown digit. But solving this type of puzzle by writing it vertically will help you solve harder problems with more unknown digits.

Missing tens digit with regrouping of ones. Write on the board:

\[
\begin{array}{c}
A3 \\
\times 4
\end{array}
\]
\[252\]

SAY: I want to find \(A\). If I do the multiplication, I start with the ones digits.

ASK: What is \(3 \times 4\)? (12)

Show the regrouping on the board:

\[
1
\begin{array}{c}
A3 \\
\times 4
\end{array}
\]
\[252\]

SAY: By adding an extra ten, we get 25 tens. ASK: How many tens would we get without regrouping? (24) Write on the board:

\[
A \times 4 = 24
\]

ASK: What is \(A\)? (6) Write on the board:

\[
\begin{array}{c}
63 \\
\times 4
\end{array}
\]

Have a volunteer complete the multiplication to verify that the answer is 252.
Exercises: What is $A \times 7$?

\[
\begin{array}{cccc}
\text{a)} & 3 A5 & \times 7 & = 315 \\
\text{b)} & 1 A1 & \times 7 & = 427 \\
\text{c)} & 1 A2 & \times 7 & = 644 \\
\text{d)} & 5 A8 & \times 7 & = 266 \\
\end{array}
\]

Answers: a) 28, b) 42, c) 63, d) 21

SAY: You were able to determine what $A \times 7$ is because you knew how much was regrouped when multiplying the ones. Now you will have to multiply the ones first to see how much was regrouped.

Exercises

1. $A7 \times 6$

   \[
   \begin{array}{c}
   \underline{342} \\
   \end{array}
   \]
   a) Multiply the ones.
   b) What is $A \times 6$? Explain how you know.
   c) What is $A$? Explain how you know.
   d) Check your answer by doing the multiplication.

   Answers: a) 42; b) 30, because $30 + 4 = 34$; c) 5, because $5 \times 6 = 30$; d) check: $57 \times 6 = 342$

2. Find A. Check your answer by doing the multiplication.

   \[
   \begin{array}{cccc}
   \text{a)} & 3 A8 & \times 3 & = 294 \\
   \text{b)} & A5 & \times 4 & = 260 \\
   \text{c)} & A3 & \times 9 & = 387 \\
   \text{d)} & A6 & \times 4 & = 224 \\
   \text{e)} & A8 & \times 3 & = 234 \\
   \end{array}
   \]

   Answers: a) 9, b) 6, c) 4, d) 5, e) 7

3. Find B and then A. Check your answer by doing the multiplication.

   \[
   \begin{array}{cc}
   \text{a)} & A2 \times 7 & = 8B \\
   \text{b)} & A5 \times 7 & = 45B \\
   \text{c)} & A6 \times 9 & = 32B \\
   \end{array}
   \]

   Answers: a) $B = 4, A = 1$; b) $B = 5, A = 6$; c) $B = 4, A = 3$

Solving multi-digit multiplication puzzles. Write on the board:

\[
\begin{array}{c}
4A \\
\times B3 \\
\underline{2491} \\
\end{array}
\]

ASK: What is the ones digit of the product? (1) Which digits in the puzzle multiply to give you ones digit 1? ($A \times 3$) What does $A$ have to be? (7) Tell students that if they don’t have the three times table memorized, they can skip count through it to check for a number with 1 as the ones digit. Write on the board:

$7 \times 3 = 21$, so $A = 7$
Erase “A” in the first equation and write “7” on the board, as shown below:

\[
\begin{array}{c}
47 \\
\times \quad B3 \\
\hline
2491
\end{array}
\]

SAY: Now we have to find B. We can try 1, 2, 3, and so on as B, but, instead of doing all the multiplying, let’s estimate to see which products are most likely to be close to 2491. ASK: 47 times what multiple of 10 is close to 2491? (50) PROMPT: 47 is close to 50, so 50 times what multiple of 10 is close to 2491? (50) SAY: By rounding and multiplying only multiples of 10, you are getting a good estimate. B = 5 is a good first guess and, even if it’s not right, you’ll know by multiplying whether to make the next guess higher or lower. Have a volunteer solve 47 \times 53 on the board, as shown below:

\[
\begin{array}{c}
47 \\
\times \quad 53 \\
\hline
2491
\end{array}
\]

SAY: So, in the puzzle, A is 7 and B is 5.

**Exercises:** Solve the puzzle. Hint: Write the puzzle vertically.

a) \(6A \times B7 = 6111\)  
b) \(A4 \times 6B = 4884\)  
c) \(A57 \times 3B = 17366\)

**Answers:** a) \(A = 3, B = 9\); b) \(A = 7, B = 6\); c) \(A = 4, B = 8\)

**Puzzles with more digits missing.** Write on the board:

\[
AB \times 7 = 13C
\]

ASK: How many digits are missing in this puzzle? (3) SAY: That might seem like a lot of missing digits, but if we can just find one of them, then we are down to only two missing digits. Let’s take this one step at a time. By telling us that the product is in the one hundred thirties, the puzzle clues are already telling us quite a bit. ASK: What happens if you multiply a number in the twenties by 7—what would you get? (at least 140) Write on the board:

\[
20 \times 7 = 140
\]

ASK: Is that too high or too low? (too high) SAY: 2 is too high for A, and another rule for this type of puzzle is that no number can start with zero. ASK: What does that tell you about A? (it must be 1) SAY: You just reduced the problem to an easier one with only two unknown digits. Write on the board:

\[
\begin{array}{c}
1B \\
\times \quad 7 \\
\hline
13C
\end{array}
\]

SAY: 7 \times 1 is 7, but the answer says to write 13 tens. ASK: How many tens must have been regrouped from 7 \times B? (6) Continue writing on the board:

\[
\begin{array}{c}
6 \\
1B \\
\times \quad 7 \\
\hline
13C \\
B \times 7 = 6C
\end{array}
\]
SAY: B times 7 is sixty-C. ASK: What number times 7 is in the sixties? (9) So, what is C? (3) PROMPT: 9 × 7 is sixty-what? SAY: So, A = 1, B = 9, and C = 3.

Exercises

1. Multiply 19 × 7. Do you get 133?

   **Answer:** yes

2. Solve the puzzle. Hint: Write the puzzle vertically.

   a) \( AB \times 7 = 26C \)
   b) \( AB \times 6 = 34C \)
   c) \( AB \times 8 = 62C \)
   d) \( AB \times 9 = 32C \)

   **Answers:** a) A = 3, B = 8, C = 6; b) A = 5, B = 7, C = 2; c) A = 7, B = 8, C = 4; d) A = 3, B = 6, C = 4

Using structure to reduce the search required to solve a puzzle. Write on the board:

\[ 4 \times AB = BBC \]

SAY: Remember the rules: the three Bs stand for the same digit, and A, B, and C all stand for different digits. Another rule for this kind of puzzle is that no number can start with zero. So AB is a two-digit number and BBC is a three-digit number. ASK: Can B equal zero? (no) Why not? (it starts the number BBC) SAY: AB is a two-digit number, so it's less than 100. ASK: What does that tell you about 4 times AB? (it is less than 400) What does that tell you about B? (it is 1, 2, or 3) SAY: Let's try B = 1, 2, and 3 in order. Write on the board:

B = 1

ASK: If B is 1, what is C? (4) To guide students, write on the board:

\[ \begin{array}{c}
   \times 4 \\
   A1 \\
   \hline
   114
   \end{array} \]

Point to the two 1s in 114 and ASK: How did I know these were 1s? (all the Bs are 1s) What does that tell us that A × 4 is? (11) SAY: To complete the multiplication, you start by multiplying the 1 and you get 1 × 4 = 4, then you multiply the tens and you get A × 4 = 11. ASK: Is there a whole number A that works here? (no) SAY: So, B = 1 doesn't work.

**Exercise:** Try B = 2 and B = 3 in the puzzle on the board. Are there any possible values for A, B, and C?

**Solution:** If B = 2, then A2 × 4 = 22C, so C = 8 and A × 4 = 22, which again doesn’t work. If B = 3, then 4 × A3 = 33C, so C = 2 and 4 × A = 32 and so A = 8, which is the only answer because B cannot be greater than 3. So, the answer is A = 8, B = 3, C = 2.
When students finish the exercise, SAY: By using information about how big the product can be, you were able to reduce your work by a lot and check only three possibilities for $B$. That makes it a lot less overwhelming.

**Exercise:** Solve the puzzle: $5 \times AB = BCC$.

**Answer:** $A = 3$, $B = 1$, $C = 5$

**Problem Bank**

1. Fill in the blank.
   a) $(63 \times 2) + (63 \times 6) = 63 \times ____$
   b) $(63 \times 41) + (41 \times 2) = 41 \times ____$
   c) $3 \times 5 \times 7 \times 9 \times 11 = 5 \times 7 \times 9 \times 11 \times ____$
   d) $(579 \times 853) - (579 \times 852) = ____$
   e) $(2 \times 7) + (2 \times 6) = (2 \times 10) + (2 \times ____)$
   f) $(2 \times 3 \times 8) + (2 \times 3 \times 5) = (2 \times 3 \times 10) + (2 \times 3 \times ____)$
   g) $(82 \times 41) + (82 \times 3) + (3 \times 39) + (3 \times 5) = 85 \times ____$

   **Answers:** a) 8, b) 65, c) 3, d) 579, e) 3, f) 3, g) 44

2. Evaluate.
   a) $(47 \times 8) + (8 \times 27) + (26 \times 8)$
   b) $(35 \times 3) + (17 \times 3) - (3 \times 19)$
   c) $(13 \times 19) + (25 \times 13) - (14 \times 13) - (29 \times 13)$
   d) $(172 \times 27) - (27 \times 135) - 27 \times 26$

   **Solutions:** a) $(47 + 27 + 26) \times 8 = 800$, b) $(35 + 17 - 19) \times 3 = 99$, c) $(19 + 25 - 14 - 29) \times 13 = 13$, d) $(172 - 135 - 26) \times 27 = 297$

3. The key with digit 5 on your calculator isn’t working. What could you press to find …
   a) $315 + 64$    b) $351 + 64$    c) $34 \times 15$    d) $52 \times 8$

   **Sample answers:** a) $310 + 69$, b) $300 + 110 + 1 + 4$, c) $34 \times 14 + 34$, d) $42 \times 8 + 10 \times 8$

4. Fill in the blanks.
   a) $2700 = 27 \times ____$, so $2727 = 27 \times ____$
   b) $272700 = 27 \times ____$, so $272727 = 27 \times ____$
   c) $272727 = 27 \times ____$
   d) $534000 = 534 \times ____$, so $534534 = 534 \times ____$
   e) $277277277 = 277 \times ____$
Answers: a) 100, 101; b) 10 100, 10 101; c) 1 010 101; d) 1000, 1001; e) 1 001 001

5. a) \(805 \times 805 = 1001 \times \) ____
   b) Use \(7 \times 11 \times 13 = 1001\). Which of 7, 11, and 13 are factors of the given number?
      i) 805 812     ii) 805 805     iii) 805 850
      iv) 805 818     v) 805 882
   Answers: a) 805; b) i) 7 only; ii) 7, 11, and 13; iii) none; iv) 13 only; v) 7 and 11

6. Find the ones digit.
   a) \(11 \times 21 \times 31 \times 41 \times 51 \times 61 \times 71 \times 81 \times 91\)
   b) \(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\)
   Answers: a) 1, b) 4

7. Find the ones digit of the sum of the numbers.
   a) The numbers from 1 to 10.
   b) The numbers from 1 to 100.
   c) The numbers from 1 to 1000.
   d) The numbers from 1 to 10 000.
   e) The numbers from 1 to 1 000 000.
   Answers: a) 5, b) 0, c) 0, d) 0, e) 0

8. A number is called a perfect square if you can write it as a product of a whole number times itself. The first five perfect squares are:
   \(1 \times 1 = 1 \quad 2 \times 2 = 4 \quad 3 \times 3 = 9 \quad 4 \times 4 = 16 \quad 5 \times 5 = 25\)
   What can be the ones digit of a perfect square?
   Answers: 0, 1, 4, 5, 6, 9

9. Is there a whole number \(N\) with \(N \times N = 12347\)? Decide using two methods. Which method is quicker?
   a) Use systematic search.
   b) Use the possible ones digit of a number times itself.
   Answers: a) 12 347 is between 111 \(\times\) 111 and 112 \(\times\) 112, so there is no such \(N\); b) no, because 7 cannot be the ones digit of a perfect square; using the possible ones digit of a perfect square was quicker

10. Is there a number \(N\) where \(N \times N = 342816517\)? Explain how you know.
   Answer: no, because 7 cannot be the ones digit of a perfect square
11. Draw an area model for $547 \times 613$ and use it to explain why its ones digit is the same as the ones digit for $7 \times 3$.

**Answer:** In the diagram below, the only region where the area is not a multiple of 10 is $7 \times 3$, so $7 \times 3$ provides the only contribution to the ones digit.

![Area model diagram](image)

12. Check that $24 \times 63$ has the property in which reversing both numbers gets the same answer (i.e., $42 \times 36$ equals $24 \times 63$). Draw area models for both multiplications and compare them to explain why this is true. Then find more pairs of numbers that have the same property.

**Solution:** The four products in each case are:

- $24 \times 63$  
- $42 \times 36$
- $20 \times 60$  
- $40 \times 30$
- $20 \times 3$  
- $2 \times 30$
- $4 \times 60$  
- $40 \times 6$
- $4 \times 3$  
- $2 \times 6$

The reason these products are equal is because $2 \times 6 = 4 \times 3$. Thus, this will also work for pairs such as: $64 \times 23$ because $6 \times 2 = 4 \times 3$, $13 \times 93$ because $1 \times 9 = 3 \times 3$, $26 \times 93$ because $2 \times 9 = 6 \times 3$, $23 \times 96$ because $2 \times 9 = 3 \times 6$, $14 \times 82$, $12 \times 84$, $12 \times 42$, $48 \times 63$, $43 \times 68$

13. a) Solve the puzzle.
   i) $9 \times B = AB$
   ii) $9 \times A = BA$

   b) How are the puzzles the same? How are they different?

   **Answers:** a) i) $A = 4$, $B = 5$; ii) $A = 5$, $B = 4$; b) they are the same puzzle but with $A$ and $B$ switched

14. Solve the puzzle. Hint: Write the puzzle vertically.

   a) $A7 \times 2B = 1482$
   b) $A8 \times 4B = 1786$

   **Bonus:** $A34 \times 2B = 12586$

   **Answers:** a) $A = 5$, $B = 6$; b) $A = 3$, $B = 7$; Bonus: $A = 4$, $B = 9$
15. Solve the puzzle $A7 \times A2 = 4154$.

**Answer:** $A = 6$

**NOTE:** Solving some of the problems below will be easier when the problem is written vertically. Allow students to struggle before providing any hints.

16. Solve the puzzle.
   
   a) $AAA \times 7 = 6216$
   
   b) $BAA \times 7 = 6916$
   
   c) $AAB \times 7 = 4655$
   
   d) $BAB \times 7 = 5159$
   
   e) $BAB \times 9 = 5814$
   
   f) $AAA \times 6 = 4662$
   
   g) $BAA \times 3 = 2631$
   
   h) $AAB \times 5 = 2245$

**Answers:**
   
   a) $A = 8$; b) $A = 8$, $B = 9$; c) $A = 6$, $B = 5$; d) $A = 3$, $B = 7$; e) $A = 4$, $B = 6$; f) $A = 7$; g) $A = 7$, $B = 8$; h) $A = 4$, $B = 9$

17. a) When Tasha multiplies 2 one-digit numbers, the answer has the ones digit 3. What might the two numbers be? List all possible answers.

b) Solve the puzzle $6A \times 5B = 3933$

c) A two-digit number $AB$ is multiplied by its reverse $BA$, with $A < B$. The product is a four-digit number with ones digit 3. What are $A$ and $B$?

**Answers:**
   
   a) 1 and 3, 7 and 9; b) $A = 9$ and $B = 7$; c) $A = 7$ and $B = 9$

18. Solve the puzzle $AB \times 5B = 4399$.

**Solution:** Looking at the ones digit (9), $B$ is either 3 or 7 because $B \times B$ gives an answer with the ones digit 9. Check the two cases $A3 \times 53 = 4399$ and $A7 \times 57 = 4399$. Now, $A7 \times 57 = 4399$ doesn’t have an answer because 87 x 57 is too high (4959) and 77 x 57 is too low (4389). If we check $A3 \times 53 = 4399$ with $A = 8$, we get $83 \times 53 = 4399$, which is correct.

19. $AB$ and $BA$ are both two-digit numbers, so that neither $A$ nor $B$ is 0, and $5 \times AB = 6 \times BA$.

   a) Explain how you know that $A$ must be 5.
   
   b) Explain how you know that $B$ must be even.
   
   c) Use the information from parts a) and b) to solve the puzzle.

**Solutions**
   
   a) Because $5 \times AB = 6 \times BA$, then $6 \times BA$ is a multiple of 5. It is even, so it is a multiple of 10. So, $6 \times BA$ has ones digit 0 and $6 \times A$ has ones digit 0. But $A$ isn’t 0, so $A$ is 5.
b) \(5 \times AB = 6 \times BA\), but \(6 \times BA\) is even, so \(AB\) has to be even for it to be multiplied by 5 and come to an even number, so its ones digit \(B\) is even.

c) We know \(A = 5\) and \(B\) is even and not 0, so \(AB\) is 52, 54, 56, or 58. Trying each in turn, we find that \(5 \times 54 = 6 \times 45\) works, so \(A = 5\) and \(B = 4\).

20. Solve the puzzle. Hint: You need to solve an addition puzzle before you solve the multiplication puzzle.

\[
\begin{array}{c}
ABC \\
\times D3 \\
\hline
654 \\
CEA0 \\
\hline
9374
\end{array}
\]

Answers: \(A = 2, B = 1, C = 8, D = 4, E = 7\)

21. A four-digit number \(ABCD\) has all different digits. When it is multiplied by 9, the answer is the reverse, also a four-digit number: \(DCBA\). What is the original four-digit number? Hint: Write the multiplication vertically and determine one digit at a time. Use the fact that the answer to multiplying a number by 9 is not a five-digit number.

Solution: \(A\) must be 1 because a number in the two thousands multiplied by 9 would be at least 18,000, which has five digits. But then \(D \times 9\) has ones digit 1, so \(D\) must be 9. So far, we have:

\[
\begin{array}{c}
\phantom{1}BC9 \\
\times \phantom{6}9 \\
\hline
\phantom{9}CB1
\end{array}
\]

\(B\) must be 0 or 1 because 2 or greater would carry over to the thousands, but there is no regrouping, and \(B\) isn’t 1 because \(A\) is 1, so \(B\) is 0. Then, since \(9 \times C + 8\) has ones digit 0, then \(9 \times C\) has ones digit 2, and that makes \(C = 8\). Check: 1089 \(\times 9 = 9801\).