PS7-1  Guessing, Checking, and Revising

Teach this lesson after: 7.1 Unit 4

Goals:  
Students will use binary search to play guessing games.  
Students will understand why binary search is more efficient than linear search.

Prior Knowledge Required:  
Can name special quadrilaterals  
Can classify shapes using geometric properties (for Problem Bank 5)

Vocabulary: acute angle, binary search, equilateral, linear search

Materials:  
overhead projector  
transparency of a large hundreds chart  
BLM Geometry Words (p. E-70, see Problem Bank 5)  
BLM Guess My Shape (pp. E-71–72, see Problem Bank 6)

Guesses that get a lot of information are efficient. Draw on the board:

A.  B.  C.  D.  E.  F.  G.  H.

Tell students that you are thinking of one of these shapes and you want students to determine which shape you are thinking about by asking yes/no questions. SAY: For example, you could ask, “Is it shape F?” or you could ask, “Is it a circle?” ASK: What other questions could you ask? (sample answers: Is it shape B? Is it a triangle? Is it small? Is it shaded? Is it a big triangle?) Have volunteers choose a shape (from A to H on the board) and demonstrate the very poor strategy of guessing the letters in order: Is it A? Is it B? and so on, until they are correct. Repeat a few times and then have a volunteer pick the shape that will take you the longest time to guess (they should pick shape H). Demonstrate guessing shapes A to G in order and getting the answer “no” each time. SAY: Because I know it isn’t A to G, I know your shape is H. But it took me seven guesses for eight shapes. You knew how to make it hard for me because you knew the strategy I was using.

Tell students to try to guess the shape by asking only three questions. After several volunteers have had a chance to play, tell students that there are three questions that will always work. Write on the board:

Is the shape a circle? Yes  
Is the shape shaded? No  
Is the shape big? No

ASK: Based on the questions and answers, which shape am I thinking of? (G) Erase the answers only, and repeat with various other combinations of answers, such as: Yes, No, Yes (E); or No, No, Yes (A); or No, Yes, Yes (B). Keep the questions on the board for the exercises on the next page.
Exercises: Determine the shape based on the answers to the three questions.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Answers:</th>
<th>E, G, A, F, C, H, B, D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
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<tr>
<td>B</td>
<td></td>
<td></td>
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<td>C</td>
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<td>D</td>
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<td>E</td>
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<td>F</td>
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<td>G</td>
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<td>H</td>
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</tbody>
</table>

Is the shape big? Yes Yes Yes No Yes No No No

Is the shape a circle? Yes Yes No Yes No Yes No No

Is the shape black? Yes No Yes Yes No No Yes No

When students finish the exercises, refer back to the example on the board. SAY: After one question, only half the shapes were left, either all the circles or all the triangles, and that's true no matter what the answer is—you don't need to be lucky at all. Write on the board:

Start with 8 shapes.
After 1 question, only 4 shapes are left.

ASK: After two questions, how many shapes are left? (2) SAY: Again, half of the four shapes that are left will have answer “yes,” and the other half will have answer “no.” Write on the board:

After 2 questions, only 2 shapes are left.

ASK: After three questions, how many shapes are left? (1) SAY: Now, one shape will have answer “yes” and one shape will have answer “no.” Write on the board:

After 3 questions, only 1 shape is left.

SAY: With this strategy, there is nothing you can do to make it hard for me to know what your shape is.

Exercises: A friend is thinking of one of the eight objects, and you need to ask yes/no questions to determine which of the eight objects your friend is thinking of. Write three yes/no questions that you can ask to determine the answer.

a) Is the shape a circle? Is it shaded? Is it small?
b) Is the hundreds digit a 1? Is the tens digit a 3? Is the ones digit a 5?
c) Is the picture of a happy face? Does the picture show hair? Does the picture show a nose?

Sample answers: a) Is the shape a circle? Is it shaded? Is it small?; b) Is the hundreds digit a 1? Is the tens digit a 3? Is the ones digit a 5?; c) Is the picture of a happy face? Does the picture show hair? Does the picture show a nose?
Play the guessing number game. Tell students that this time, instead of shapes, they are going to guess numbers, and instead of only eight shapes, they are going to choose from 100 numbers. Tell students that you are thinking of a number from 1 to 100, and you are going to tell them whether the answer is correct, too high, or too low. Project a hundreds chart on the board. Think of a number, say 72, and have volunteers guess your number. If a guess is too high, say so and cross out their guess and all the numbers greater than their guess. For example, if they guess 80, SAY: 80 is too high. Cross it out. ASK: Can my number be 81 now that you know that 80 is too high? (no) Cross that out too. ASK: Can it be 82? (no) Cross out 82 and all the numbers greater than 82. Do the same if their guess is too low. ASK: How many numbers did I cross out after your guess? Continue in this way, always bringing their attention to how many answers they could eliminate after each guess.

Play at least one full game, and continue until students get a sense that some guesses give you a lot of information and some give you only a bit of information (this may take more than one game if students quickly guess correctly). To summarize, SAY: Some guesses allowed you to cross out a lot of information and other guesses allowed you to cross out only a bit of information.

Discover the strategy of choosing a middle number. ASK: What number could you start with so that no matter what my answer is, too high or too low, you can cross out at least half the numbers? (start with 50 or 51) If I guess 50 and it’s too low, what numbers can you cross out? (1 to 50) If I guess 50 and it’s too high, what numbers can you cross out? (50 to 100) SAY: If you guess 50 and it happens to be correct, great! But even if it’s not correct, you can still cross out at least half the numbers left, either 1 to 50 or 50 to 100. It is a really good question when you can eliminate half the choices because then you’ve reduced your work by a lot right away. Now instead of choosing from 100 numbers, you can choose from only 50 numbers.

Investigate the strategy of choosing a middle number. SAY: I want to investigate the strategy of choosing a middle number of all the remaining numbers. Suppose that 31 to 50 are left. ASK: How many numbers are left? (20) How did you get that? (50 – 30) SAY: To decide how many numbers there are from 31 to 50, you can count the numbers from 1 to 50—there are 50 of them—and then take away the numbers from 1 to 30—there are 30 of them. So, there are 20 numbers left. ASK: What is a good middle number? (students will likely say 40, but 41 also works) SAY: Sometimes there are two middle numbers and you can pick either one. In this case, the tenth number, 40, or the eleventh number, 41, is a good middle number.

Exercises: What is a good middle number to choose? Explain your choice.

a) 66 to 80 are left
b) 35 to 74 are left
c) 49 to 61 are left

Answers: a) 73, because it is the 8th number out of 15; b) 54, because it is the 20th number out of 40, or 55, because it is the 21st number out of 40; c) 55, because it is the 7th number out of 13

Now have students guess your number, and as you say “too high” or “too low” have volunteers cross out numbers on the hundreds chart projected on the board. Challenge students to always choose the middle number, or one of the two middle numbers, based on which numbers are left.
Activity
Play the guessing number game with a partner, guessing numbers from 1 to 100. Try to guess the number within seven guesses.
(end activity)

Changing the next question based on previous guesses for guessing shapes. SAY: When we played guessing shapes, you looked at the shapes and decided all the questions you were going to ask based on the attributes that were changing. But when you played guessing numbers, you changed your next question based on the answer to the previous question. ASK: If I told you that 50 is too high, would your next guess be 25 or 75? (25) If I said that 50 is too low, what would your next guess be? (75) SAY: This can happen with shapes too. Draw on the board:

A. B. C. D.

SAY: We’re going to play a guessing game for shapes again. Just like guessing numbers, you sometimes have to change your next question based on the answer to the previous question. ASK: What are some questions you could ask that will eliminate exactly two of the shapes? (Is the shape a parallelogram? Is the shape a trapezoid?) SAY: Suppose we ask, “Is the shape a parallelogram?” There are two possible answers, and we would ask different questions for each answer, so a flow chart would be convenient here. Draw on the board:

Is the shape a parallelogram?
Yes           No

SAY: Suppose the shape is a parallelogram, so we know it's one of these two. (circle the two left-most shapes in the diagram) ASK: What question can you ask next so you know which shape it is? (sample answers: Is the shape a rectangle? Are there any right angles?) Continue to fill in the flow chart, as shown below:

Is the shape a parallelogram?
Yes           No
Is the shape a rectangle?

ASK: Would that question be useful if I said the shape was a trapezoid? (no) If the shape was a trapezoid, what question could you ask? (Is there a pair of equal sides?) Would that question be useful if I said the shape was a parallelogram? (no) Continue to fill in the flow chart, as shown below:

Is the shape a parallelogram?
Yes           No
Is the shape a rectangle?
Is there a pair of equal sides?
SAY: Sometimes you have to adjust your question based on the answer to your first question. At each stage, you are sorting the shapes into two groups, in which one category is the “Yes” group and the other category is the “No” group. SAY: Now let’s continue the flow chart to get the answer. Continue to fill in the flow chart, as shown below:

```
<table>
<thead>
<tr>
<th>Is the shape a parallelogram?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
</tr>
<tr>
<td>Is the shape a rectangle?</td>
</tr>
<tr>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
</tr>
<tr>
<td>Is there a pair of equal sides?</td>
</tr>
<tr>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
</tr>
</tbody>
</table>
```

Have volunteers fill in the letter names of the shape that fits the answers to the question. (in order from left to right: B, A, C, D)

**Exercises:** Decide how to ask yes/no questions so that half the possible answers are eliminated with each question.

a) A. B. C. D.

b) A. B. C. D. E. F. G. H.

**Bonus:**

```
A  B  C  D  E  F  G  H
```

**Sample answers:**

a) Is it a circle?
   - Yes
     - Is it shaded?
       - Yes
         - B
       - No
       - Is it big?
         - Yes
         - D
         - No
         - C
b) Is it a square?
   - Yes
   - No
   - Is it dark?
     - Yes
     - No
     - Is it big?
       - Yes
       - No
       - F
       - H
     - G
   - D
   - C
   - B
   - A

Bonus: Question 1: Does it have four vertices? Regardless of if the answer to Question 1 is yes or no, Question 2: Does it have any curved sides? If the answer to Question 2 is yes, Question 3: Does it have two curved sides? If the answer to Question 2 is no, Question 3: Is it equilateral?

**Linear search and binary search.** Tell students that *linear search* is when you go through all the possibilities in order and *binary search* is when you eliminate half the possibilities each time. SAY: Most cell phones have a four-digit security code that can be from 0000 to 9999. ASK: If you forget your code, would you be able to use binary search or do you have to use linear search? (I would have to use linear search) SAY: Knowing that one combination doesn't work doesn't tell you whether or not any other combination will work, so you have to check every single one. Then tell students they can use binary search when they can divide the possible answers into approximately two equal parts, such as knowing whether the number is too high or too low or whether or not the shape is a circle. SAY: If you're guessing my number and I tell you that 50 is too high, then you know that 51 is too high, 52 is too high, and every number greater than 50 is too high. Once you have two halves, you decide which half of the answer you are looking for is in. Write on the board:

```
binary               bicycle
```

SAY: “Binary” means something has two parts, just like a bicycle has two wheels.

**Exercises:** Decide whether to do linear or binary search.

a) There are 10 boxes and one of them has a prize. You want to find the box with the prize.
b) There are 10 boxes in a row and one of them has a prize. The ones to the left have a brick and the ones to the right have nothing. You want to find the box with the prize.
c) You are looking for a T-shirt that you like in a store that has a rack of 20 T-shirts.
d) You are looking for your name in an alphabetical list of 20 names.

**Answers:** a) linear, b) binary, c) linear, d) binary
Problem Bank

1. Students can play in pairs and use the “guess a middle number” strategy for guessing numbers from 1 to 1000. Students should try to guess within 10 guesses.

2. Students can play in pairs and use the “guess a middle number” strategy for guessing numbers from 1 to 1,000,000. Students should try to guess within 20 guesses.

3. You are shown a set of 16 shapes and your friend is thinking of one of them. Write four yes/no questions that you can ask to determine the shape.

![Sample Image]

**Sample answer:** Is it black? Is it big? Is it a triangle? If yes, is the horizontal side on the top or on the bottom? If no, is the arrow facing right or left?

4. a) Create a set of 4 shapes where you can guess the shape using two yes/no questions.
b) Create a set of 8 shapes where you can guess the shape using three yes/no questions.
c) Create a set of 16 shapes where you can guess the shape using four yes/no questions.
d) Create a set of 32 shapes where you can guess the shape using five yes/no questions.
e) Ethan created a set of 256 shapes using the same strategy. How many yes/no questions would you need to guess the shape?
f) Alexa created another set of shapes using the same strategy. It took 10 yes/no questions to determine the shape. How many shapes are in her set?

**Selected answers:** e) 8, f) 1024

5. Use the smallest number of questions that you can to determine the shape. Sorting attributes might include:
   • the number of sides
   • whether the shape has any line of symmetry
   • the number of lines of symmetry
   • whether the shape is equilateral
   • whether the shape has any equal sides
   • whether the shape has any acute/right/obtuse/reflex angles
   • the number of acute/right/obtuse/reflex angles
   • whether there any pairs of parallel sides
   • the number of pairs of parallel sides
   • whether any adjacent sides are equal
NOTE: If students need to be reminded of the vocabulary, provide BLM Geometry Words.

a)

b)

c)

d)

e)

f)

g)
h) 

i) 

j) 

k) 

Selected sample solution: f) Three questions will be sufficient.

Are there two pairs of parallel sides?  
Yes  
No  

Does the shape have four right angles?  
Yes  
No  

Is the shape a trapezoid?  
Yes  
No  

Does the shape have all sides equal?  
Yes  
No  

Does the shape have six sides?  
Yes  
No  

Is there a right angle?  
Yes  
No  

Are all sides straight?  
Yes  
No

Answers: a) 3 questions, b) 3 questions, c) 2 questions, d) 2 questions, e) 3 questions, g) 3 questions, h) 2 questions, i) 3 questions, j) 2 questions, k) 3 questions
6. On BLM Guess My Shape, show how to ask at most four questions to guess the shape correctly. All questions must have the answer “Yes” or “No” and must ask about specific geometric properties. You cannot ask, “Is it shape L?” But you can ask, “Are there two equal sides?”

**Sample solution:**

![Diagram showing a decision tree for guessing a shape.]

7. Ella and Marko play Guess My Number from 1 to \( N \). Marko picks a number and Ella guesses Marko’s number. Marko tells Ella whether her answer is too high, too low, or just right. Ella wants to use the fewest questions she can. If she uses the best strategy, at most how many guesses will she need?

a) Fill in the table.

<table>
<thead>
<tr>
<th>( N )</th>
<th>Number of Guesses Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
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<td>8</td>
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<tr>
<td>9</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
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</tbody>
</table>

b) Continue the pattern from part a) up to \( N = 20 \).

c) Predict the number of guesses Ella needs if Marko picks a number from 1 to 500.

**Selected solution:** c) starting at one shape, doubling the number of shapes requires one more guess; 256 shapes requires 8 guesses, and 512 shapes requires 9 guesses, and everything in between requires 8 guesses, so 500 shapes requires 8 guesses

**Answers:** a) 1, 1, 2, 2, 2, 2, 3, 3, 3; b) 3, 3, 3, 3, 4, 4, 4, 4
8. Lewis and Nina play the guessing shapes game. Lewis picks one of \( N \) shapes. Nina asks Lewis yes/no questions about his shape. Nina chooses questions so that at least half the answers are eliminated after each question. She wants to know at most how many guesses she will need to ask.

a) Fill in the table.

<table>
<thead>
<tr>
<th>( N )</th>
<th>Number of Guesses Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>9</td>
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<td>10</td>
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</tbody>
</table>

b) Continue the pattern from part a) up to 20.

c) Predict the number of guesses Nina needs if Lewis picks a shape from a set of 500 shapes.

Answers: a) 1, 2, 3, 3, 4, 4; b) 4, 4, 4, 4, 5, 5, 5, 5; c) 9 guesses

9. Explain how the tables in Problem Banks 7 and 8 are the same and how they are different. For which numbers are the number of guesses the same? For which numbers does one game require more guesses than the other?

Bonus: Why does the guessing number game usually need fewer guesses?

Sample answers: In both tables, if you start at 1, the number of guesses increases by 1 as you double the number of shapes. So, the numbers 1, 2, 4, 8, 16, … all require the same number of guesses in each game. But all the numbers in between the doubling require the smaller number of guesses for the number game and they require the larger number of guesses for the shape game.

Bonus: The guessing number game has three possible answers (just right, too high, or too low), whereas the guessing shape game has only two possible answers (yes or no), so you get slightly more information with the guessing number game than you do with the guessing shape game.
Geometry Words

The **sides** of a shape are the lines that form the boundary of the shape.

A **vertex** is a point where two sides of a figure meet.

A **square corner**, or **right angle**, is an angle matching that found at the corner of a square (and is also called a 90-degree angle).

A shape is **equilateral** if all of its sides are of the same length.

A **line of symmetry** is a line that divides a shape into two matching parts. To test whether a line is a line of symmetry, fold the shape along the line. If the two parts of the shape on either side of the line do not match up exactly, the line is not a line of symmetry. The line shown in the diagram below is not a line of symmetry: even though the two parts of the shape are the same shape and size, they do not match up when the shape is folded along the line.

A square has four lines of symmetry, but a rectangle has only two.

Lines are **parallel** if they are straight and if they would never meet when extended.

An angle is called **acute** if it is less than 90 degrees, **obtuse** if it is between 90 and 180 degrees, and **reflex** if it is between 180 and 360 degrees.

A closed shape with three straight sides is a **triangle**, one with four sides is a **quadrilateral**, one with five sides is a **pentagon**, one with six sides is a **hexagon**, and one with eight sides is an **octagon**.

Some triangles have special names. A triangle is called **equilateral** if it has all equal sides, **isosceles** if it has two equal sides, and **right angle** if it has a right angle. Any shape, including a triangle, is called **scalene** if all sides are of different lengths.

Some quadrilaterals have special names. A **square** has four equal sides and four right angles. A **rectangle** has opposite sides that are equal and four right angles. A **rhombus** has four equal sides but not necessarily any right angles. A square is both a rhombus and a rectangle.

In a **parallelogram**, opposite sides are parallel and of equal length. A square, rectangle, and rhombus are all parallelograms. A **trapezoid** has exactly one pair of parallel sides.
Guess My Shape (1)

Here are some shapes. The arrows mark parallel sides, the lines mark equal sides, and the small squares mark right angles.

Using the flow chart on the next page, show how to ask at most four questions to guess the shape correctly. All questions must have the answer “yes” or “no,” and must ask about specific geometric properties.

For example, you cannot ask, “Is it Shape L?” but you can ask, “Are there two equal sides?”

When you know what the answer is, put the letter of the shape in the grey box.
Guess My Shape (2)