Grade 3  Problem-Solving Lessons

Introduction

What is a problem-solving lesson? A JUMP Math problem-solving lesson generally follows the format of a regular JUMP Math lesson, with some important differences:

• There are no AP Book pages that accompany the problem-solving lessons.
• Problem-solving lessons focus on one or more problem-solving strategies rather than focusing on meeting the Common Core State Standards (CCSS). These lessons apply the concepts learned through the standards—often crossing several domains, clusters, or standards—but they are not necessary to complete the standards. Regular lessons, on the other hand, focus on completing the standards and sometimes require problem-solving strategies to do so.
• While regular lessons expose students to all of the problem-solving strategies, the problem-solving lessons provide a way to isolate and focus on the strategies.
• Instead of including extensions, each problem-solving lesson includes an extensive Problem Bank. These questions give students a variety of opportunities to practice the problem-solving strategy from the lesson and to learn new math in the process. Students will need to have mastered the material in the problem-solving lesson (which they do by completing the exercises) in order to tackle the Problem Bank.
• Both the lesson plan and the Problem Bank questions apply the CCSS. All of the standards covered in the lesson are mentioned at the beginning of the lesson plan.
• Some problem-solving lessons include an opportunity for students to complete one or more Performance Tasks. Performance Tasks are multi-part problems intended to determine how well students can apply grade-level CCSS in a new context. While most questions in the task can be done independently of the problem-solving lesson, some questions provide an opportunity to specifically apply the problem-solving strategy. These questions might be challenging for students who have not been taught the problem-solving lesson. Performance Tasks can cover several domains, clusters, or standards at once, all from material covered to date, and so can often be used as a cumulative review.
• While regular lessons cover the standards completely, problem-solving lessons cover clusters of major standards according to the CCSS, and some also cover supporting and additional standards. These lessons provide more challenging independent work while still focusing on the standards.

How do I use problem-solving lessons? Ten problem-solving lessons are provided for Grade 3. The problem-solving lessons can be taught at any point in the grade after the unit indicated in the table on the next page. We recommend using as many problem-solving lessons throughout the year as your class time allows, and suggest using them in the order in which they are indicated below. However, if required, you can pick and choose based on a careful review of the prior knowledge required for each problem-solving lesson.

We recommend teaching more problem-solving lessons toward the end of the year rather than toward the beginning, as this allows time for students to consolidate their mathematical knowledge and gain confidence before attempting more challenging problems. For this reason, we recommend using only three of the ten problem-solving lessons during Part 1 of Grade 3.
Stronger classes that need fewer bridging lessons for review will have time to finish more of the problem-solving lessons. We recommend that classes needing most of the bridging lessons try at least a few problem-solving lessons.

Some of the Problem Banks include more problems than students can complete in one period. You might wish to use these as extension problems, or have students complete them as problems of the day throughout the year.

Performance Tasks. Performance Tasks are included at the end of some problem-solving lessons. Each Performance Task has at least one question that applies the problem-solving strategy covered in the lesson, but most questions can be done independently of the lesson. The Performance Tasks, together with any preparation, require a separate period each. Blackline Masters (BLMs) for each Performance Task are provided at the end of the lesson in which they are cited.

Problem-solving strategies for Grade 3 and when to use them. We consider the following problem-solving strategies as most important for this grade level:
• Recognizing and using structure
• Searching systematically
• Using a diagram
• Making a similar, but simpler, problem
PS3-9  Making a Simpler Problem

Use this lesson after: 3.2 Unit 7

Standards: 3.OA.B.6, 3.OA.D.8, 3.NBT.A.2, 3.NBT.A.3, 3.NF.A.1

Goal:
Students will, given a problem, make a simpler problem and use the solution to the simpler problem to solve the harder problem.

Prior Knowledge Required:
Can add and subtract two-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can represent fractions by using lengths and areas

Vocabulary: add, perimeter, subtract

Materials:
two pre-made sticks of different lengths and different colors
BLM Fraction Strips and Circles (p. S-82, see Problem Bank 12)
scissors (see Problem Bank 13)
BLM Planting a Flower Garden (pp. S-84–85, see Performance Task)

Using a given simpler problem to help solve a harder problem. Write on the board:

There are 300 people in line. How many people are behind the 7th person?

(MP.3, MP.6) ASK: What makes this problem hard? (students might say because 300 is a lot of people) Would it be easier if I asked how many people are behind the 299th person in line? (yes, there is only 1 person) SAY: So, it is not how big 300 is that makes this problem hard. ASK: Can you say what exactly makes it hard? (7 and 300 are far apart) Write on the board:

There are 8 people in line. How many people are behind the 7th person?

SAY: I’m going to draw the 8 people. Draw on the board:

Front of line: ☃ ☃ ☃ ☃ ☃ ☃ ☃

Have a volunteer circle the 7th person in line. ASK: How many people are behind the 7th person? (1) SAY: You don’t have to draw happy faces. You could just draw dots. Draw on the board:

● ● ● ● ● ● ● ●

7th
**Exercises:** Draw a picture to show your answer.

a) There are 6 people in line. How many people are behind the 5th person?
b) There are 5 people in line. How many people are behind the 3rd person?
c) There are 9 people in line. How many people are behind the 4th person?

**Answers:** a) 1, b) 2, c) 5

**(MP.8)** **ASK:** For exercises like the ones you just did, how can you get the answer from the two numbers given? (subtract) **PROMPT:** What operation can you do? (subtract) Draw on the board:

```
● ● ● ● ● | ● ● ● ● ●
```

**SAY:** This picture shows there are 9 people in line, and I want to know how many people are after the 5th person. **ASK:** Where are the dots that represent the people who are after the 5th person? (to the right of the line) **SAY:** So I have to subtract all the dots that are before the line. **ASK:** How many people are before the line? (5) **SAY:** So you subtract 9 − 5 to get how many people are after the 5th person. **ASK:** What is 9 − 5? (4) **SAY:** So there are four people after the 5th person. Now you know that you can do any problem like this with subtraction.

**Exercises:**

a) There are 300 people in line. How many people are behind the 12th person?
b) There are 487 people in line. How many people are behind the 30th person?

**Bonus:** There are 3,459 people in line. How many people are behind the 1,459th person?

**Answers:** a) 300 − 12 = 288; b) 487 − 30 = 457; Bonus: 3,459 − 1,459 = 2,000

**ASK:** How did solving the easier problems make it easier to solve the harder problems? (doing so told me that the right approach is to subtract: number of people in line − the position of the person in line)

**Off-by-one errors.** Tell students that you are waiting in line to get on a rollercoaster ride. You are 37th in line and you see your friend who is 7th in line. **ASK:** How many people are between my friend and me? Note various guesses. Most students will likely guess 37 − 7 = 30. If they do, **SAY:** That answer is close, but not quite right. Let’s draw a simple picture using smaller numbers to see what is going on. Write on the board:

```
Front of line: ● ● ● ● ● ● ● ● ● ●
```

**SAY:** Each dot represents a person. **ASK:** How many dots did I draw? (9) **SAY:** For this simpler problem, suppose I am 9th in line. Have a volunteer circle the last dot. **SAY:** My friend is 5th in line. Have another volunteer circle the 5th dot. Label the dots, as shown below:

```
Front of line: ● ● ● ● ● ● ● ● ● ●
               5th  9th
```

**ASK:** How many people are between the 5th and 9th person? (3) Is that equal to 9 − 5? (no) **SAY:** It is close, but not quite equal.
**Exercises:** Draw a picture to decide how many people are between the given positions.

- a) the 7th and 8th person
- b) the 7th and 9th person
- c) the 7th and 10th person
- d) the 7th and 11th person
- e) the 7th and 12th person
- f) the 7th and 37th person

**Answers:** a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

**ASK:** Did subtracting give exactly the right answer? (no) Did it give close to the right answer? (yes) How can you get the number of people between two people given their positions in line? (subtract the smaller position from the other and then subtract 1 from the difference)

**SAY:** Sometimes, it is easier to start by using smaller numbers than what is given in the problem. Then you will see patterns and how to solve the harder problem. Now that you know the pattern for finding the number of people between any two positions, you can use that method with any numbers.

**Exercises:** How many people are in line between the given positions?

- a) the 8th and 78th person
- b) the 314th and 1,000th person
- c) the 492nd and 613th person

**Answers:** a) 69, b) 685, c) 120

**Making the problem easier by finding what is relevant.** SAY: Sometimes making the problem easier has nothing to do with using smaller numbers and finding a pattern. Sometimes all you need to do is eliminate information that’s not relevant, and moving objects around can help with that.

Affix pre-made sticks of different colors and different lengths to the board, end to end. Label one length and their combined length. The following is an example for 8 cm and 12 cm but your sticks can be other lengths:

![Diagram](image)

Tell students that all the measurements are in centimeters. **ASK:** How long is the second stick? (12 cm) **SAY:** It is easy to see with sticks, but now I’m going to move these sticks around. Slide the gray stick down and draw the lines around it, as shown below:

![Diagram](image)
ASK: How did I move the sticks? (you slid one of them down) SAY: This now looks like a problem to do with shapes and the lengths of missing sides. There's a lot of extra information in this second problem compared with the first problem, so it looks harder, but it actually has exactly the same answer as the other one. The total length of the two sticks is still 20 cm—I just slid one of the sticks down so that they are not side by side anymore.

**Exercises:** Find what the ? stands for by pretending the sticks are side by side.

a) ![Diagram](a.png)
b) ![Diagram](b.png)
c) ![Diagram](c.png)
d) ![Diagram](d.png)

**Bonus:**
e) ![Diagram](e.png)
f) ![Diagram](f.png)

**Answers:** a) 7, b) 9, c) 40, d) 368, Bonus: e) 17, f) 13

SAY: By pretending that the sticks were side by side, you turned the problem into an easier problem.
**Making the problem easier by emphasizing what is relevant.** SAY: We can look at a problem and focus on what matters most. For example, if you need to find a vertical edge—straight up and down—then bold all the vertical lines. If you need to find a horizontal edge, bold all the horizontal lines.

**Exercises:** Find what the ? stands for by making the problem into an easier problem.

a) ![Diagram A](image1)

b) ![Diagram B](image2)

**Answers:** a) bold vertical, ? = 5; b) bold horizontal, ? = 12

Point out to students that by bolding the horizontal or vertical lines, they changed the problem into an easier problem.

**Finding perimeter by finding missing side lengths.** Remind students that, to find the perimeter of a shape, they have to add up the lengths of all the outside edges.

**Exercises:** Find the perimeter by finding missing sides, then adding all the sides.

a) ![Diagram C](image3)

b) ![Diagram D](image4)

**Answers:** a) 48, b) 30

**Problem Bank**

(MP.1) 1. How many posts are needed to build the fence?
   a) A fence 38 m long made with posts 1 m apart.
   b) A fence 50 m long made with posts 5 m apart.
   c) A fence 60 m long made with posts 3 m apart.

**Answers:** a) 39, b) 11, c) 21

2. Ken wants to cut a rope into 20 equal parts. How many cuts does he need to make?

**Answer:** 19
(MP.7, MP.8) 3. A teacher tells the class to read pages for homework.
   a) How many pages do students read if they are assigned pages 87 and 88?
   b) How many pages do students read if they are assigned pages 87, 88, 89, and 90?
   c) How many pages do students read if they are assigned pages 87 to 91?
   d) How many pages do students read if they are assigned pages 87 to 92?
   e) How can you get the number of pages from the two page numbers given? Hint: Compare your answers to the result of subtracting the two numbers.
   f) How many pages do students read if they are assigned pages 87 to 104?
   **Answers:** a) 2, b) 4, c) 5, d) 6, e) subtract and add 1, f) 18

(MP.4, MP.7) 4. Ben reads every night at home. How many pages does he read?
   a) from pages 352 to 386   b) from pages 298 to 314   c) from pages 408 to 451
   **Answers:** a) 35, b) 17, c) 44

(MP.1) 5. Ava reads pages 354 to 412 except for pages 363 to 389, which have illustrations only. How many pages does Ava read?
   **Answer:** 32

(MP.1) 6. How many whole numbers are greater than 11 and less than 45?
   **Answer:** 33

(MP.1, MP.4) 7. When everyone in Tom’s class stood in line, Tom was 12th in line and 15th from the end of the line. How many people are in the class?
   **Answer:** 26

(MP.1, MP.4) 8. Make several easier problems until you see the pattern to help you do the harder problem.
   a) A fence is made using 42 posts, each 1 meter apart. How long is the fence?
   b) A fence is made using 34 posts, each 2 meters apart. How long is the fence?
   **Answers:** a) 41 meters, b) 66 meters

(MP.1, MP.4) 9. Jen builds a fence for a square garden with posts 1 m apart, including a post at each corner. How many posts does she need for a garden of the given size?
   a) 10 m by 10 m
   Hint: Start with a garden that is 1 m by 1 m and then move on to 2 m by 2 m, 3 m by 3 m, and so on.
   b) 20 m by 20 m
   **Answers:** a) 40, b) 80

(MP.1, MP.4) 10. Ravi builds a fence around a square field that is 20 m by 20 m. He uses a post at each corner.
   a) How many posts are needed if the posts are 1 m apart?
   b) How many posts are needed if the posts are 2 m apart?
   c) How many posts are needed if the posts are 4 m apart?
   d) How many posts are needed if the posts are 5 m apart?
   **Answers:** a) 80, b) 40, c) 20, d) 16
11. Each line segment of the path below has a length of 1 meter. What is the path’s total length?

Solution: 18 vertical meters plus 17 horizontal meters = 35 meters altogether

12. Cut out the strips and circles from BLM Fraction Strips and Circles (you may cut the line down to the center of the circles).

a) Use folding to check that one fifth of strip A is shaded.

b) Use folding to check that two fifths of strip B is shaded.

Hint: Use your answers to parts a) and b) to help you determine a strategy for parts c) and d).

c) Use folding to check that one fifth of circle C is shaded.

d) Use folding to check that two fifths of circle D is shaded.
Fraction Strips and Circles

A

B

C

D
Performance Task: Planting a Flower Garden

Materials:
BLM Planting a Flower Garden (pp. S-84–85)

Preparation for the performance task. Write on the board:

There are 7 trees in 4 rows. How many trees are there altogether?

ASK: How can you solve this problem? (multiply 4 × 7) Erase the 7 and write “18” in its place.
ASK: How can you solve this problem? (multiply 4 × 18) SAY: But I don’t have the 18 times table memorized. What else can I do instead of multiply? (18 + 18 + 18 + 18) PROMPT: What is 4 × 18 short for? Write the addition vertically on the board and have a volunteer do the addition, as show below:

```
  3
+18
+18
+18
+18
  72
```

SAY: Another way to make a problem easier is to turn it into one that you already know how to do. If you find one of the questions difficult, but you need the answer to do the next question, guess an answer and use it in the next question. You can still get the next question right based on your answers to the other question, even if your answer to the other question is wrong.

Performance Task: Planting a Flower Garden. Provide students with BLM Planting a Flower Garden. Question 6 provides an opportunity to apply the problem-solving strategy of making an easier problem. All students might find an answer to Question 6, but students who notice that the strategy can be used will find the problem easier.

Answers: 1. 50 feet; 2. $150; 3. $100; 4. $251; 5. 4; 6. 19, I subtracted 20 − 1; 7. 76
Planting a Flower Garden (I)

Ivan has a flower bed. His flower bed is 5 ft wide by 20 ft long.

1. Ivan wants to put a fence around his flowerbed to keep his pet dog out. How much fencing does he need?

2. If fencing costs $3 for each foot, how much will the fence cost?

3. Ivan needs to add more soil to his garden. The added soil will cost $1 for each square foot. How much will the soil cost?

4. Ivan will grow the flowers from seeds. He buys the seeds for $1. How much will Ivan’s total cost for the flower bed be?
Planting a Flower Garden (2)

5. To plant his flower bed, Ivan will:
   - plant each flower 1 foot apart,
   - start planting 1 foot from each edge of his flower bed.

   His flower bed is 5 feet wide and 20 feet long.

   How many rows of flowers can Ivan plant?

   20 ft
   5 ft

6. How many flowers can Ivan plant in each row? Explain how you got your answer.