Goals

Students will identify and describe various patterns in addition tables.

PRIOR KNOWLEDGE REQUIRED

Can add two numbers within 20
Can find the difference, or gap, between two numbers
Can identify and write the rule for a number pattern made by adding a constant gap

Introduce addition sentences, addends, and sums. Write on the board:

\[ 3 + 1 = 4 \]

Read the sentence above as you point to the symbols: “3 plus 1 equals 4.” SAY: “3 + 1 = 4” is called an addition sentence. The sentence tells you that 3 plus 1 (point to “3 + 1”) is equal (point to “=” to 4 (point to the “4”). NOTE: You could tell your students that the sentence 3 + 1 = 4 is also called an equation, because it has an equal sign. However, the word equation will be formally introduced later.

SAY: The different parts of an addition sentence have special names. “3” and “1” are called addends, because they are added together. The answer, “4,” is called the sum. Write on the board:

\[
\begin{array}{ccc}
3 & + & 1 \\
\downarrow & & \downarrow \\
\text{addend} & \text{addend} & \text{sum}
\end{array}
\]

Write a few more addition sentences on the board and have volunteers identify which numbers are the addends and which are the sums.

Examples: 2 + 3 = 5 (addends: 2, 3; sum: 5), 4 + 6 = 10 (addends: 4, 6; sum: 10), 5 + 7 = 12 (addends: 5, 7; sum: 12).

Exercises: Copy the addition sentences and answer the questions.

1. \[ 3 + 6 = 9 \]
   a) Which numbers are the addends? Circle them.
   b) Which number is the sum? Draw a box around it.
   
   Answers: a) 3 and 6, b) 9

2. \[ 7 + 7 = 14 \]
   a) What are the two addends? Circle them.
   b) What is the sum? Draw a box around it.
   
   Answers: a) 7 and 7, b) 14
3. $17 = 8 + 9$

   a) What are the addends? Circle them.

   b) What is the sum? Draw a box around it.

   **Answers:** a) 8 and 9, b) 17

**Bonus:**

$2 + 1 + 3 = 6$

   a) What are the addends? Circle them.

   b) What is the sum? Draw a box around it.

   **Answers:** a) 2, 1, and 3; b) 6

**Introduce addition tables.** Draw on the board:

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**SAY:** This is an *addition table*. An addition table shows the sum of two addends. Circle the “2” from the shaded column and the “3” from the shaded row. **ASK:** Which addends did I circle? (2 and 3) **What is the sum of 2 and 3?** (5) Write “2 + 3 = 5” beside the table. **SAY:** You can find the box where the sum 5 belongs by looking at the row for addend “2” and the column for addend “3.” Trace one hand along the row for addend 2, and trace your other hand down the column for addend 3, until your hands meet. Pointing to that cell, **SAY:** This box is where the row and column meet. This is where we write 5, the sum of 2 and 3. Write 5 in the cell. Draw a box around 5. **ASK:** What is the addition sentence for these addends and this sum? (2 + 3 = 5) Explain to students that we write 2 before 3 since we write the addend from the shaded column on the left before the addend from the shaded row on top in an addition table (by convention).

Repeat this process for more addition sentences: $1 + 2 = 3$, $2 + 4 = 6$, $1 + 1 = 2$. Then proceed to fill in the row for addend 0 from left to right, one cell at a time, asking students each time what the addends are and what the sum is. You might use signaling. Repeat for the other two rows, filling in the missing cells so that the whole table is filled in.

**Exercises:** Copy and complete the addition tables.

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### Answers:
- a) row for addend 5: 10, 11, 12, 13; row for addend 6: 11, 12, 13, 14; row for addend 7: 12, 13, 14, 15; row for addend 8: 15, 16, 17, 18; row for addend 9: 16, 17, 18, 19; row for addend 10: 17, 18, 19, 20

### Exercises:
Copy the addition table to answer the questions. Use grid paper.

1. Look at the addition sentence $1 + 2 = 3$.

   \[
   \begin{array}{cccc}
   + & 0 & 1 & 2 & 3 \\
   0 & 0 & 1 & 2 & 3 \\
   1 & 1 & 2 & 3 & 4 \\
   2 & 2 & 3 & 4 & 5 \\
   \end{array}
   \]

   a) Circle the addends in the table.
   b) Draw a box around the sum in the table.

   **Answers:**
   - a) circle 1 in the left shaded column and 2 in the top shaded row;
   - b) draw a box around the 3 in the row for addend 1

2. Look at the addition sentence $5 + 4 = 9$.

   \[
   \begin{array}{ccccccc}
   + & 3 & 4 & 5 & 6 & 7 \\
   3 & 6 & 7 & 8 & 9 & 10 \\
   4 & 7 & 8 & 9 & 10 & 11 \\
   5 & 8 & 9 & 10 & 11 & 12 \\
   6 & 9 & 10 & 11 & 12 & 13 \\
   \end{array}
   \]

   a) Circle the addends in the table.
   b) Draw a box around the sum in the table.

   **Answers:**
   - a) circle 5 in the left shaded column and 4 in the top shaded row;
   - b) draw a box around the 9 in the row for addend 5

### Patterns in addition tables.
Draw on the board:

\[
\begin{array}{cccc}
+ & 1 & 2 & 3 & 4 \\
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 4 & 5 & 6 \\
3 & 4 & 5 & 6 & 7 \\
4 & 5 & 6 & 7 & 8 \\
\end{array}
\]

Ask a volunteer to circle the row for addend 2. (Ensure the volunteer does not circle the 2 from the shaded column; just the numbers 3, 4, 5, 6 should be circled.) Write the numbers on the board: 3, 4, 5, 6. **ASK:** Is this a number pattern made by adding the same number each time? (yes) **PROMPT:** What is the gap between 3 and 4? (1) Between 4 and 5? (1) Between 5 and 6? (1)
So what number do we add each time? (1) Which number does this number pattern start with? (3) Write on the board, underneath “3, 4, 5, 6”:

Start at _____ and add ______.

Have a volunteer fill in the blanks. (3, 1) SAY: The rule “Start at 3 and add 1” describes the pattern in the row for addend 2. After erasing the circle (so that the row for addend 2 is no longer circled), circle the column for addend 4 and repeat this process with those numbers. (number pattern: 5, 6, 7, 8; gap: 1; rule: “Start at 5 and add 1”) Then circle the numbers along the diagonal, “2, 4, 6, 8,” and repeat the process. (number pattern: 2, 4, 6, 8; gap: 2; rule: “start at 2 and add 2”)

**Exercises:** Use the addition table on the board to answer the questions.

1. Look at the numbers from the column of addend 3.
   a) Write down the numbers, in order: __, __, __, __
   b) Write a rule for this number pattern: Start at ____ and add ____.
   **Answers:** a) 4, 5, 6, 7; b) 4, 1

2. Look at the numbers from the row of addend 1.
   a) Write down the numbers, in order: __, __, __, __
   b) Write a rule for this number pattern: Start at ____ and add ____.
   **Answers:** a) 2, 3, 4, 5; b) 2, 1

3. Look at the numbers from the row of addend 4.
   a) Write down the numbers, in order.
   b) Write a rule for this number pattern.
   **Answers:** a) 5, 6, 7, 8; b) Start at 5 and add 1

**Commutativity in addition tables.** Draw on the board:

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Tell students you want to find the sum of 2 and 3 using the addition table. Write on the board:

2 + 3 = ___

**ASK:** What is the first addend we wrote? (2) Where do we find the first addend, 2, on the addition table? Have a volunteer come and circle the shaded column at the left of the addition table (not the addend 2 in the top shaded row of the table). Remind students that the first
addend comes from the shaded row at the left of the table. Then, pointing to the addition sentence on the board, ASK: What is the second addend we wrote? (3) Have a volunteer come and circle the 3 from the top shaded row of the table.

ASK: How do we find the sum of these addends on the addition table? (find the spot where the row for addend 2 and the column for addend 3 meet) Trace one hand along the row for addend 2 and your other hand down the column for addend 3 until they meet. Draw a box around the number 5 in this cell. Pointing to this cell, ASK: What is the sum of 2 and 3? (5) Write “5” in the blank of the addition sentence on the board. ASK: Did the addition table give us the right answer? (yes) Have students check that 2 + 3 really does equal 5 by counting up from 2 using their fingers.

Repeat this process for 3 + 2 =. SAY: First we did 2 + 3, then we did 3 + 2. Did we get the same sum both times? (yes) Why? (the order of the addends doesn’t matter in addition) SAY: When you’re adding, the order of the addends doesn’t matter. No matter which addend you write first, you get the same sum. This is an important rule about addition. This rule is called a property of addition.

Repeat this process for 2 + 1 = (3) and 1 + 2 = (3)

Exercises: Copy the addition table to answer the questions. Use grid paper.

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1. Look at the addition sentences 9 + 7 = and 7 + 9 =.
   a) Draw a box around the sum for 9 + 7 in the table.
   b) Draw a box around the sum for 7 + 9 in the table.
   c) Are the sums in parts a) and b) the same?
   d) What property of addition does this show?
   **Answers:** a) circle 16 from the row for addend 9; b) circle 16 from the row for addend 7; c) yes, they’re both 16; d) when you’re adding, the order of the addends doesn’t matter

2. Look at the addition sentences 8 + 6 = and 6 + 8 =.
   a) Shade the sum for 8 + 6.
   b) Shade the sum for 6 + 8.
   c) What do you notice about parts a) and b)?
   d) What property of addition does this show?
Answers: a) shade 14 from the row for addend 8; b) shade 14 from the row for addend 6; c) you get the same sum: \(8 + 6 = 14\) and \(6 + 8 = 14\); d) when adding, the order of the addends doesn’t matter

Addition sentences with the same sum. Using the addition table from 0 to 3 on the board, shade all sums of 3, and write the corresponding addition sentences beside the table (using blanks) as shown below:

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\[\_ + \_ = 3\]
\[\_ + \_ = 3\]
\[\_ + \_ = 3\]

SAY: I have shaded all the squares for the sum 3. Pointing to the shaded cell in the row for addend 0, ASK: What is the addition sentence for this box? (0 + 3 = 3) Fill in the blanks with “0” and “3” in the first addition sentence.

Repeat for the shaded 3 in the row for addend 1 (1 + 2 = 3), then the row for addend 2 (2 + 1 = 3), and finally, the row for addend 3 (3 + 0 = 3).

Exercise: Use the addition table on the board to answer the questions.

a) Write three addition sentences for the sum 2 from the table.

b) Write three addition sentences for the sum 4 from the table.

c) Write an addition sentence for the sum 4 not shown in the table.

Answers: a) \(0 + 2 = 2, 1 + 1 = 2, 2 + 0 = 2\); b) \(1 + 3 = 4, 2 + 2 = 4, 3 + 1 = 4\); c) \(0 + 4 = 4\) or \(4 + 0 = 4\)

Adding zero to a number. Using the addition table on the board (from 0 to 3), circle the column for addend 0. Write the addition sentences on the board:

\(0 + 0 = \_\) \(1 + 0 = \_\) \(2 + 0 = \_\) \(3 + 0 = \_\)

ASK: What is \(0 + 0\)? (0) PROMPT: Look at the circled column in the addition table. Write 0 in the blank. Repeat with \(1 + 0\) (1), \(2 + 0\) (2), and \(3 + 0\) (3).

ASK: When you add 0 to a number, what is the result? (you get that number as the answer)

Circle the row for addend 0 and repeat the process for those addition sentences (\(0 + 0 = 0, 0 + 1 = 1, 0 + 2 = 2, 0 + 3 = 3\)). Ensure students understand that if one of the addends in an addition sentence is 0, the sum will be the other addend. Explain that this is another important rule, or property, of addition.
Exercises: Use the addition table to answer the question.

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a) Write the addition sentence for the sums in the column for addend 0.
b) When you add 0 to a number, what is the result?

Answers: a) $6 + 0 = 6, 7 + 0 = 7, 8 + 0 = 8, 9 + 0 = 9$; b) you get that number as the answer

Extensions
1. Look at boxes in the table.

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a) How many boxes are there for the sum 0?
b) How many boxes are there for the sum 1?
c) How many boxes are there for the sum 2?
d) How many boxes are there for the sum 3?
e) How many boxes are there for the sum 4?
f) How many boxes are there for the sum 5?
g) How many boxes are there for the sum 6?

Answers: a) 1, b) 2, c) 3, d) 4, e) 3, f) 2, g) 1

2. Use the table to answer the questions.

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a) List all the addition sentences from the table for the sum 4.
b) List the number pattern in the row for addend 3.
c) List the number pattern in the row for addend 4.
d) What is the same about the number patterns in parts b) and c)?
e) What is different about the number patterns in parts b) and c)?
f) List the number pattern in the column for addend 3.
g) What do you notice about the number patterns in parts b) and f)?

**Answers:**
a) \(0 + 4 = 4\), \(1 + 3 = 4\), \(2 + 2 = 4\), \(3 + 1 = 4\), \(4 + 0 = 4\);
b) 3, 4, 5, 6, 7; c) 4, 5, 6, 7, 8; d) the gap is 1 for both number patterns; e) the number pattern in part b) starts at 3, while the number pattern in part d) starts at 4; f) 3, 4, 5, 6, 7; g) the number patterns are the same.

3. If you taught your students about diagonals, have them find a diagonal in the addition table of Extension 2 with the number pattern 0, 2, 4, 6, 8. Then have them find two diagonals with the number pattern 1, 3, 5, 7.

a) What is the gap in these diagonal number patterns?
b) What is the gap in any number pattern along a row?
c) What is the gap in any number pattern along a column?

**Answers:** a) 2, b) 1, c) 1

4. Use the addition table to answer the questions.

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a) List the number pattern in the row for addend 1. Write a rule for this number pattern.
b) List the number pattern in the row for addend 2. Write a rule for this number pattern.
c) What is the gap in the number pattern of any row in this table?
d) List the number pattern in the column for addend 4. Write a rule for this number pattern.
e) List the number pattern in the column for addend 6. Write a rule for this number pattern.
f) What is the gap in the number pattern of any column in this table?
**Answers:** a) 1, 3, 5, 7; start at 1 and add 2; b) 2, 4, 6, 8; start at 2 and add 2; c) 2; d) 4, 5, 6, 7; start at 4 and add 1; e) 6, 7, 8, 9; start at 6 and add 1; f) 1

5. Alex bought a computer for $718 and virus protection for $65. Ethan bought a television for $586 and a DVD for $39. Who spent more money? How much more?

**Answer:** Alex spent $158 more than Ethan.

(MP3) 6. Fred makes a pattern by starting at 345 and adding 267 each time. Lily makes a pattern by starting at 267 and adding 345 each time. Whose pattern will have the larger third term? Explain how you know without finding the third term.

**Answer:** Lily’s pattern will have the larger third term. The second term in both patterns are equal, because they are $345 + 267$ or $267 + 345$. You can find Lily’s third term by adding 345 to that number, but you only add 267 to that same number to get Fred’s third term. Since 345 is larger than 267, Lily’s pattern has a larger third term.
Goals

Students will add numbers mentally by noticing pairs that add to 10 (or multiples of 10).
Students will split an addend into two parts to make mental addition easier.

PRIOR KNOWLEDGE REQUIRED

Can add two numbers within 20

MATERIALS

BLM Decomposing Addends for Mental Addition (p. D-48)

Modeling addition sentences using circles and lines. SAY: There are different ways to show how to add two numbers. One way is to draw a row of circles, and draw a line separating the circles into two groups. Draw on the board:

\[ \bullet \bullet | \bullet \bullet \bullet \]

ASK: How many circles are to the left of the line? (2) How many circles are to the right of the line? (3) SAY: This picture shows \(2 + 3 = 5\). Write “2 + 3 = 5” on the board beside the row of circles. SAY: The number of circles to the left of the line is the first addend, and the number of circles to the right of the line is the second addend. The total number of circles is the sum. Draw on the board:

\[ \bullet \bullet \bullet \bullet | \bullet \bullet \bullet \bullet \]

ASK: What addition sentence does this picture show? (3 + 4 = 7)

Exercises: Write the addition sentence for the picture.

a) \[ \bullet \bullet \bullet | \bullet \]

b) \[ \bullet \bullet \bullet \bullet \]

c) \[ \bullet \bullet \bullet \bullet \bullet \]

Answers: a) \(4 + 1 = 5\), b) \(1 + 3 = 4\), c) \(5 + 2 = 7\)

Addition sentences for a given sum. Draw four rows of five circles, with a dividing line in each row, as follows:

\[ \bullet \bullet \bullet \bullet | \bullet \]

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Write the corresponding addition sentences for each row, having students signal the addends and the sums, one by one. (1 + 4 = 5, 2 + 3 = 5, 3 + 2 = 5, 4 + 1 = 5) ASK: How would we show the addition sentence “0 + 5 = 5”? Have a volunteer draw it on the board. (vertical line followed by five circles) ASK: How many circles are to the left of the line? (0) How many circles are to the right? (5) Repeat with the addition sentence “5 + 0 = 5.” (five circles followed by a vertical line)

Write on the board:

\[
\begin{align*}
8 &= 1 + 7 \\
8 &= 2 + 6 \\
8 &= 3 + 5 \\
8 &= 4 + 4 \\
8 &= 5 + 3 \\
8 &= 6 + 2 \\
8 &= 7 + 1 \\
\end{align*}
\]

ASK: How would you fill in the missing numbers? Illustrate the following possible answers:

- Draw eight circles and a vertical line between them at various points and then count the circles to the right of the vertical line. (Alternatively, use a vertical line that can be moved, such as a magnet.) Start with the vertical line to the right of one circle, and then keep moving the vertical line one circle to the right each time.

- Start at the bottom and write the numbers 1, 2, 3, 4, 5, 6, and 7 in order from the bottom up.

- Start at the top, and write the numbers in reverse order, starting at 7.

- Draw two boxes and eight dots for apples and say you want to put eight apples into two boxes. Draw the various possibilities, starting with one apple in the first box.

Emphasize that from one addition sentence to the next, you are adding one to the first addend and subtracting one from the second addend, so you are not changing the total (or sum). Point out that moving from one addition sentence to the next is just like moving the vertical line one more circle to the right.

**Exercises:** Fill in the missing numbers.

\[
\begin{align*}
a)\ 4 &= 0 + \_ \\
b)\ 9 &= 1 + \_ \\
c)\ 10 &= 1 + \_
\end{align*}
\]

\[
\begin{align*}
a)\ 4 &= 1 + 3 \\
b)\ 9 &= 2 + 7 \\
c)\ 10 &= 2 + 8
\end{align*}
\]

\[
\begin{align*}
a)\ 4 &= 2 + 2 \\
b)\ 9 &= 3 + 6 \\
c)\ 10 &= 3 + 7
\end{align*}
\]

\[
\begin{align*}
a)\ 4 &= 3 + 1 \\
b)\ 9 &= 4 + 5 \\
c)\ 10 &= 4 + 6
\end{align*}
\]

\[
\begin{align*}
a)\ 4 &= 4 + 0 \\
b)\ 9 &= 5 + 4 \\
c)\ 10 &= 5 + 5
\end{align*}
\]

**Answers:** a) 4, 3, 2, 1, 0; b) 8, 7, 6, 5, 4; c) 9, 8, 7, 6, 5
Pairs that add to 10. Teach students an easy way to find pairs that make 10. Hold up all your fingers on both hands. ASK: How many fingers do I have up? (10) Then hold up seven fingers and ASK: How many fingers do I have up? (7) How many fingers are not up? (3) What is 7 + 3? (10) How do you know? (because you have 10 fingers in total) Repeat with several examples: 6 + 4, 8 + 2, 2 + 8, and 9 + 1. Demonstrate clearly how to use your fingers from the right hand (up or down) to count fingers from the left hand (up or down), and vice versa. Then write on the board:

4 + ___ = 10

ASK: How could I use my fingers to find the missing number? How many fingers should I hold up? (4) How many fingers are not up? (6) So what number do we need to add to 4 to get 10? (6) Write “6” in the blank.

Write three numbers on the board:

4  5  6

ASK: Does 4 add to 10 with either of the next two numbers? (yes, 4 + 6 = 10) Circle the 4 and 6. Then write the three numbers “2 3 7” and ASK: Does 2 add to 10 with either of the other numbers? (no) What number added to 2 gives 10? (8) PROMPT: Hold up two fingers and count the number of fingers you are not holding up. SAY: Look at the last two numbers, 3 and 7. Do they add to 10? (yes) Circle the 3 and 7.

Exercises: Circle the pair that adds to 10.

a) 2  4  8  b) 6  1  9

C) 7  3  5  d) 4  8  6

e) 5  4  5  f) 6  3  7

Bonus

g) 3  4  5  6  h) 2  3  4  7  9

Answers: a) 2 and 8, b) 1 and 9, c) 7 and 3, d) 4 and 6, e) 5 and 5, f) 3 and 7; Bonus: g) 4 and 6, h) 3 and 7

If students have difficulty remembering which pairs of numbers add to 10, have them practice by playing Modified Go Fish, as described on p. A-45.

Demonstrate the first exercise below before having students complete the rest.

Exercises: Circle the pair that adds to 10. Write the number left over in the blank.

a) 3 + 4 + 7 = 10 + ___  b) 2 + 9 + 1 = 10 + ___

c) 4 + 7 + 6 = 10 + ___  d) 5 + 2 + 8 = 10 + ___

e) 7 + 3 + 8 = 10 + ___  f) 6 + 3 + 4 = 10 + ___
Answers: a) circle 3 and 7, blank: 4; b) circle 9 and 1, blank: 2; c) circle 4 and 6, blank: 7; d) circle 2 and 8, blank: 5; e) circle 7 and 3, blank: 8; f) circle 6 and 4; blank: 3

Adding 1-digit numbers to multiples of 10. Write on the board:

\[
\begin{align*}
10 + 4 &= \underline{\quad} \\
10 + 6 &= \underline{\quad} \\
10 + 8 &= \underline{\quad}
\end{align*}
\]

ASK: What is \(10 + 4\)? (14) What is \(10 + 6\)? (16) What is \(10 + 8\)? (18) Fill in the blanks. ASK: Why is it easy to add a one-digit number to 10? (because 10 has a “0” in the ones place) SAY: Adding a one-digit number to a larger number that has a “0” in the ones place is easy: you just write the larger number, but replace the 0 in the ones place with the one-digit number. Write on the board:

\[
\begin{align*}
30 + 2 &= 32 \\
90 + 7 &= 97 \\
160 + 3 &= 163
\end{align*}
\]

ASK: What is \(30 + 2\)? (32) PROMPT: Replace the 0 in 30 with 2. Repeat with \(90 + 7\) (97) and \(160 + 3\) (163). Then repeat with examples written horizontally: \(50 + 6\) (56), \(340 + 8\) (348), \(820 + 9\) (829).

Exercises: Add mentally.

\[
\begin{align*}
a) \quad 10 + 3 &= \underline{\quad} \\
b) \quad 10 + 9 &= \underline{\quad} \\
c) \quad 20 + 7 &= \underline{\quad} \\
d) \quad 70 + 8 &= \underline{\quad} \\
e) \quad 210 + 4 &= \underline{\quad} \\
f) \quad 350 + 8 &= \underline{\quad} \\
g) \quad 460 + 5 &= \underline{\quad} \\
h) \quad 710 + 2 &= \underline{\quad} \\
i) \quad 910 + 6 &= \underline{\quad} \\
j) \quad 180 + 1 &= \underline{\quad} \\
k) \quad 270 + 4 &= \underline{\quad} \\
l) \quad 970 + 3 &= \underline{\quad}
\end{align*}
\]

Answers: a) 13, b) 19, c) 27, d) 78, e) 214, f) 358, g) 465, h) 712, i) 916, j) 181, k) 274, l) 973

Decomposing addends to make pairs that add to 10. SAY: We can add two 1-digit numbers by splitting the second addend into two parts. Write on the board:

\[
9 + 6 = 9 + \underline{\quad} + \underline{\quad}
\]

SAY: We can split 6 into two smaller numbers. We saw in the last set of exercises that adding a one-digit number to 10 is easy. So we can split 6 into two parts so that one part adds to 9 to make 10. ASK: What number do we need to add to 9 to get 10? (1) Write 1 in the first blank. SAY: Since we used 1 from 6, what number is left over? (5) PROMPT: What number do we need to add to 1 to get 6? (5) Write 5 in the second blank. Write the following labels on the board:

\[
9 + 6 = 9 + 1 + 5
\]

Repeat with \(7 + 5 = 7 + \underline{\quad} + \underline{\quad}\) (3 and 2), and \(8 + 7 = 8 + \underline{\quad} + \underline{\quad}\) (2 and 5). Provide each student with a copy of BLM Decomposing Addends for Mental Addition as a reference sheet for the different ways to
decompose one-digit addends. This will help struggling students with the following exercises.

**Exercises:** Fill in the blanks.

a) \[9 + 5 = 9 + \_ + \_\] these make 10 \hspace{1cm} \text{left over}

b) \[8 + 4 = 8 + \_ + \_\] these make 10 \hspace{1cm} \text{left over}

c) \[7 + 6 = 7 + \_ + \_\]
d) \[9 + 4 = 9 + \_ + \_\]
e) \[6 + 5 = 6 + \_ + \_\]
f) \[8 + 6 = 8 + \_ + \_\]

**Answers:** a) 1, 4; b) 2, 2; c) 3, 3; d) 1, 3; e) 4, 1; f) 2, 4

**Adding 1-digit numbers by decomposition.** Write on the board:

\[9 + 6 = 9 + 1 + 5 = 10 + 5 = \_\] these make 10 \hspace{1cm} \text{left over}

**Add up 1-digit numbers by decomposition.** Write on the board:

\[a) \ 9 + 7 = 9 + \_ + \_ = 10 + 6 = \_\] these make 10 \hspace{1cm} \text{left over}

\[b) \ 8 + 5 = 8 + \_ + \_ = \_\] these make 10 \hspace{1cm} \text{left over}

\[c) \ 7 + 6 = 7 + \_ + \_ = \_\] these make 10 \hspace{1cm} \text{left over}

\[d) \ 6 + 5 = 6 + \_ + \_ = \_\] these make 10 \hspace{1cm} \text{left over}

**Answers:** a) 9 + 1 + 6 = 10 + 6 = 16; b) 8 + 2 + 3 = 10 + 3 = 13; c) 7 + 3 + 3 = 10 + 3 = 13; d) 6 + 4 + 1 = 10 + 1 = 14

**Adding to larger numbers using decomposition.** SAY: You can follow these kinds of steps to add even with larger numbers. Write on the board:

\[28 + 6 = 28 + \_ + \_ = \_ + \_ = \_\] these make 30 \hspace{1cm} \text{left over}
SAY: We want to add $28 + 6$. What is the ones digit in 28? (8) What number do you add to 8 to get 10? (2) PROMPT: Use your fingers. Write “2” in the first blank. ASK: We used 2 from 6, so what number is left over? (4) PROMPT: Count up from 2 until you reach 6. Write “4” in the second blank. ASK: If you add $28 + 2$, what number will you get? (30) Write “30” in the third blank. ASK: What number is left over? (4) Write “4” in the 4th blank. ASK: What is $30 + 4$? (34) PROMPT: Adding $30 + 4$ is easy because “30” has ones digit “0.” Just replace the “0” with “4.” Repeat with the first exercise below, and then have students complete the rest.

**Exercises:** Add by following the steps.

a) $39 + 7 = 39 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} + 6 = \underline{\hspace{1cm}}$
   - These make 40
   - Left over

b) $48 + 8 = 48 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   - These make 50
   - Left over

c) $67 + 6 = 67 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   - These make 70
   - Left over

d) $86 + 5 = 86 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   - These make 90
   - Left over

**Answers:**
a) $39 + 7 = 39 + 1 + 6 = 40 + 6 = 46$

b) $48 + 8 = 48 + 2 + 6 = 50 + 6 = 56$

c) $67 + 6 = 67 + 3 + 3 = 70 + 3 = 73$

d) $86 + 5 = 86 + 4 + 1 = 90 + 1 = 91$

Tell students that with enough practice, they'll be able to add numbers by using these steps mentally, without having to write anything down.

**Word problems for adding by decomposition.** Write on the board:

58 students went on a field trip. 6 teachers went with them.

How many people went on the field trip?

Have a volunteer read the problem. ASK: How do we find the number of people on the field trip? (add 58 and 6) Write “58 + 6 = ” on the board. SAY: Let's add these numbers by following the steps we've been using. ASK: What is the ones digit of 58? (8) What number do we need to add to 8 to get 10? (2) If we use 2 from 6, what number is left over? (4) PROMPT: What number added to 2 gives 6? Continue writing on the board: “58 + 6 = 58 + 2 + 4 = ”. ASK: What is 58 + 2? (60) Continue writing: “58 + 6 = 58 + 2 + 4 = 60 + 4 = ”. What is 60 + 4? (64) So, how many people went on the field trip? (64) Tell students to follow these steps to answer the next questions. Encourage them to try to find the answers mentally, but tell them they can write the steps if they need to.
**Exercises:** Find the answers by adding the numbers mentally.

a) Ava had 49 dollars. Her parents gave her 8 dollars. How much money does Ava have now?

b) Sal has 18 markers. Zara has 7 markers. How many do they have altogether?

**Answers:** a) $49 + 8 = 57$, b) $18 + 7 = 25$

**Extensions**

1. Add the first pair of numbers mentally. Then use your answer to add the next pair mentally.

   a) $38 + 5 = \underline{\hspace{2cm}}$  
      $380 + 50 = \underline{\hspace{2cm}}$

   b) $47 + 6 = \underline{\hspace{2cm}}$  
      $470 + 60 = \underline{\hspace{2cm}}$

   c) $59 + 8 = \underline{\hspace{2cm}}$  
      $590 + 80 = \underline{\hspace{2cm}}$

   **Answers:** a) 43, 430; b) 53, 530; c) 67, 670

2. Add the first pair of numbers mentally. Then use your answer to add the next pair mentally.

   a) $38 + 7 = \underline{\hspace{2cm}}$  
      $138 + 7 = \underline{\hspace{2cm}}$

   b) $47 + 5 = \underline{\hspace{2cm}}$  
      $647 + 5 = \underline{\hspace{2cm}}$

   c) $59 + 6 = \underline{\hspace{2cm}}$  
      $859 + 6 = \underline{\hspace{2cm}}$

   **Answers:** a) 45, 145; b) 52, 652; c) 65, 865

3. Explain how you would add $48 + 7$ mentally.

   **Answer:** Add 2 to 48 to get 50. Since $7 = 2 + 5$, add 5 to 50 to get 55.

**NOTE:** Students must complete Extensions 1 and 2 before doing Extension 4.

4. a) Look at your answers to Extension 1. In pairs, explain why you can use the answer to $38 + 5$ to get the answer to $380 + 50$. Do you agree with each other? Discuss why or why not.

   b) Look at your answers to Extension 2. In pairs, explain why you can use the answer to $47 + 5$ to get the answer to $647 + 5$. Do you agree with each other? Discuss why or why not.

   **Sample explanations:** a) There are 38 tens in 380 and 5 tens in 50, so there are 38 + 5 tens in 380 + 50. You can just write the answer to 38 + 5 with a 0 after it to get 380 + 50; b) I split 647 into 600 + 47, so 647 + 5 is the same as 600 + 47 + 5. Since it doesn’t matter which two numbers I add first, I can add 47 + 5 and then add 600.

   Encourage partners to ask questions to understand and challenge each other’s thinking—see p. A-49 for sample sentence and question stems to guide students.
Goals
Students will use tape diagrams to record the parts, totals, and differences in problems involving addition and subtraction.

PRIOR KNOWLEDGE REQUIRED
Can add two numbers within 20
Can subtract two numbers within 20
Understands addition as how much/many altogether
Understands subtraction as how much/many more

MATERIALS
BLM Blank Tape Diagrams (p. D-49)

Before the lesson, print out enough copies of BLM Blank Tape Diagrams so that each student can have two.

Using tape diagrams to find the total. Draw on the board:

6 green marbles
2 blue marbles

green
blue

SAY: Let's say we have 6 green marbles and 2 blue marbles. This picture shows the number of green marbles and the number of blue marbles. Point to the top row of squares and ASK: How many squares are on top beside the word green? (6) SAY: Each square stands for one green marble. There are 6 squares because we have 6 green marbles. Pointing to the bottom row of squares, ASK: How many squares beside the word blue? (2) SAY: Each square stands for one blue marble. There are 2 squares because we have 2 blue marbles.

SAY: This picture is called a tape diagram. A tape diagram uses squares or boxes in a row to stand for objects. A tape diagram can help you find the total number of two kinds of objects, such as green marbles and blue marbles. Let's use this tape diagram to find out how many marbles we have in total. Explain that the total is really the sum of the two parts, blue and green. Add a brace, the word total, and a blank line to the diagram on the board, as shown below:

green
blue

SAY: total: ___
ASK: How do we find the total number of marbles? (count the total number of boxes) SAY: There are 6 boxes on top, or 6 green marbles, plus 2 boxes on the bottom, or 2 blue marbles. ASK: What is $6 + 2$? (8) SAY: There are 8 marbles in total. Write “8” in the blank beside “total.” Write “$6 + 2 = 8$” below the tape diagram. Pointing to this addition sentence, explain that the number of green marbles (6) and the number of blue marbles (2) are the addends, and the total number of marbles (8) is the sum.

**Using tape diagrams to find the difference.** ASK: How do we find the difference between the number of blue and green marbles? In other words, how do we find out how many more green marbles there are than blue marbles? ($6 - 2 = 4$, or count the extra squares in the row beside the word “green”) Add to the diagram on the board:

![Tape Diagram](image)

Pointing to the first two boxes in the “green” row, ASK: If we pair up 2 green marbles with 2 blue marbles, how many green marbles are left over? (4) Pointing to the remaining 4 squares in the “green” row, SAY: These squares here show the difference. There are 4 more green marbles than blue marbles, so the difference is 4. Write “4” in the blank beside the word “difference.”

Give each student one copy of BLM Blank Tape Diagrams for the following exercises.

**Exercises:** Write green or blue in the blanks. Shade the boxes for green marbles and blue marbles. Then find the total and the difference.

a) 8 green marbles, 3 blue marbles  
b) 2 green marbles, 5 blue marbles  
c) 5 green marbles, 9 blue marbles  
d) 10 green marbles, 7 blue marbles

**Answers:** a) total: 11, difference: 5; b) total: 7, difference: 3; c) total: 14, difference: 4; d) total: 17, difference: 3

**Problems where only one part is given.** SAY: There are always two parts in these problems: the number of green marbles and the number of blue marbles. Sometimes you know only one of the parts and the difference or the total. For example, you might know the number of green marbles and the difference. You can use a tape diagram to find the number of blue marbles and the total. Or, you might know the number of blue marbles and the total. You can use a tape diagram to find the number of green marbles and the difference.
Write on the board:

6 green marbles
2 more blue marbles than green marbles

<table>
<thead>
<tr>
<th>green</th>
<th>blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: Start by shading the amount you know. ASK: How many green marbles? (6) Shade 6 squares in the row for “green.” ASK: Are there more blue marbles than green marbles or fewer? (more) How many more? (2 more) So how many boxes should we shade in the row for blue? (6 + 2 = 8) PROMPT: Start by shading 6, the number of green marbles, and then shade 2 more. After doing the shading, show students how to indicate the difference and total on the diagram, as before. Have students signal the difference, and then write it in. (2) ASK: What is the total? (14) PROMPT: What is 8 + 6? (14) Write “14” in the blank for total.

Repeat with the following examples, where the larger number is given:

- 5 green marbles, 2 more green marbles than blue marbles (green: 5, difference: 2, blue: 3, total: 8)
- 9 blue marbles, 4 fewer green marbles than blue marbles (blue: 9, difference: 4, green: 5, total: 14)

Then do an example where the total is given: 3 green marbles, 10 marbles altogether. (green: 3, total: 10, blue: 7, difference: 4)

Provide each student with another copy of BLM Blank Tape Diagrams for the following exercises.

**Exercises:** Draw tape diagrams. Shade the boxes for green marbles and blue marbles.

Write the total and the difference.

- a) 5 green marbles, 3 more blue marbles than green marbles.
- b) 2 green marbles, 5 more blue marbles than green marbles
- c) 5 green marbles, 9 marbles altogether
- d) 10 green marbles, 7 fewer blue marbles

**Answers:** a) 8 blue marbles, difference: 3, total: 13; b) 7 blue marbles, difference: 5, total: 9; c) 4 blue marbles, difference: 5, total: 9; d) 3 blue marbles, difference: 7, total: 13
Extensions

1. Have students draw their own tape diagrams using grid paper, or provide them each with two copies of BLM Blank Tape Diagrams.
   a) 5 green fish, 6 blue fish
   b) 4 purple grapes, 5 more green grapes than purple grapes
   c) 8 red cars, 3 more red cars than black cars
   d) 7 green apples, 5 fewer red apples than green apples
   e) 4 green apples, 5 fewer green apples than red apples
   f) 10 Art and Science books in total, 4 Art books

   **Answers:**
   a) total: 11, difference: 1; b) 9 green grapes, difference: 5, total: 13; c) 5 black cars, difference: 3, total: 13; d) 2 red apples, difference: 5, total: 9; e) 9 red apples, difference: 5, total: 13; f) 6 Science books, difference: 2, total: 10

2. For an extra challenge, have students try to find both parts when only the difference and the total are given. At this stage, students should just guess and test their answers. They can use a tape diagram to help. Students should list the pairs of numbers that add to the given total, and test each pair until they find the pair with the given difference.
   a) 5 marbles in total, 1 more green marble than blue marble
   b) 5 marbles in total, 3 more blue marbles than green marbles
   c) 10 blue and green marbles in total, 2 more blue marbles than green marbles
   d) 10 marbles in total, 4 more green marbles than blue marbles
   e) 10 marbles in total, 8 more blue marbles than green marbles
   f) 10 marbles in total, 6 fewer blue marbles than green marbles

   **Answers:**
   a) 3 green marbles, 2 blue marbles; b) 1 green marble, 4 blue marbles; c) 4 green marbles, 6 blue marbles; d) 7 green marbles, 3 blue marbles; e) 1 green marble, 9 blue marbles; f) 8 green marbles, 2 blue marbles

3. Have students use sketches of tape diagrams (as shown below) to find the difference and total in problems that involve larger numbers. The scale is not important in the sketches, as long as the bar for the larger number is longer than the bar for the shorter number. Have students line up the digits to add or subtract by regrouping.
   a) 324 green marbles, 568 blue marbles

   ![Diagram](attachment:image.png)
b) 298 green marbles, 427 blue marbles

difference: __________ total: __________

Answers: a) difference: 244, total: 892; b) difference: 129, total: 725

4. Display a hundreds chart and a calendar for the current month.
Challenge students to find patterns that are the same in both and to explain the patterns using properties of addition.

Sample answers
• The numbers go up by the number of columns as you go down a row (10 on a hundreds chart and 7 on a calendar). This is because adding 1 seven (or ten) times is the same as adding seven (or ten).
• If you make a 2 by 2 square in either the calendar or hundreds chart, the diagonals add to the same number. For example, in the calendar, you might have the following 2 by 2 square:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

In this square, $4 + 12 = 4 + 11 + 1$ and $5 + 11 = 4 + 1 + 11$, so they are both adding the same three numbers. This is because when you move right on a calendar or on a hundreds chart, the number increases by 1.
• In a calendar and a hundreds chart, the opposite corners of a rectangle add to the same number because each number increases by 1 as you move right.

Whole-class follow-up: Take up two or three sample answers. Start with a conceptually easy-to-understand example, such as the first sample answer provided above.
Goals

Students will use tables to distinguish between the parts, the total, and the difference in problems involving addition and subtraction.
Students will solve word problems involving addition and subtraction by distinguishing between the parts, the total, and the difference.

PRIOR KNOWLEDGE REQUIRED

Can add and subtract 2-digit numbers
Can distinguish between the total (or sum) of and the difference between two numbers

Using tables for parts and totals. Write on the board:

3 red apples, 5 green apples

SAY: Now we will use a table to keep track of the red apples, the green apples, the total, and the difference. Draw on the board:

<table>
<thead>
<tr>
<th>Red Apples</th>
<th>Green Apples</th>
<th>Total</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

ASK: How many red apples do we have? (3) Write “3” in the Red Apples column. ASK: How many green apples? (5) Write “5” in the Green Apples column. ASK: How do we find the total number of apples? (add 3 and 5) What is 3 + 5? (8) Write “8” in the Total column. ASK: How do we find the difference? (subtract 3 from 5) PROMPT: How do we find how many more green apples than red apples? (5 − 3) What is 5 − 3? (2) Write “2 more green apples than red” in the Difference column. SAY: Another way to help find the total and the difference is to draw a tape diagram, as we did in the previous lesson. Draw a tape diagram for this example, as in Lesson OA3-12.

Write on the board:

6 red apples, 8 apples in total

SAY: Let’s write the numbers we know in the table. After adding another row to the table, ASK: How many red apples? (6) Write “6” in the Red Apples column, and ASK: Are we given the number of green apples? (no) SAY: Since we’re not given that number, we leave the Green Apples column blank for now. ASK: How many apples in total? (8) Write “8” in the Total column. ASK: How can we find the number of green apples? (subtract the number of red apples from the total) PROMPT: Draw a tape diagram and fill in the numbers you know: the red apples and the total. SAY: Since 6 apples are red and there are 8 apples in total, 8 − 6 gives us the number of green apples. ASK: What is 8 − 6? (2) Write “2” in the Green Apples column. ASK: Are there more green apples or more red apples? (red) How do we find out how many more? (subtract the number of green apples from the
number of red apples; $6 - 2$) What is $6 - 2$? (4) So, there are 4 more red apples than green apples. Write “4 more red apples than green” in the Difference column.

Repeat this process with another problem: “8 green apples, 1 more red apple than green.” Students should see that the second piece of information given belongs in the Difference column. Ask students if there are more green apples or more red apples. Since there are more red apples, they should see that they need to add the difference to the smaller number; i.e., $8 + 1 = 9$ red apples. Once students have both the number of red and the number of green apples, they should see that they need to add the numbers to find the total, i.e., $8 + 9 = 17$. Repeat this process again “8 green apples, 1 more green apple than red.” (red apples: 7, total: 15)

**Exercises:** Fill in the table.

<table>
<thead>
<tr>
<th>Red Apples</th>
<th>Green Apples</th>
<th>Total</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 5</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 8</td>
<td></td>
<td>3 more red apples than green</td>
<td></td>
</tr>
<tr>
<td>d) 7</td>
<td></td>
<td>3 more red apples than green</td>
<td></td>
</tr>
<tr>
<td>e) 13</td>
<td>21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Bonus**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>f) 10</td>
<td>4 fewer green apples than red</td>
<td></td>
</tr>
<tr>
<td>g) 7</td>
<td>5 fewer green apples than red</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:**

a) total: 12, difference: 6 more green apples than red;
b) red: 2, difference: 3 more green apples than red; c) green: 5, total: 13;
d) red: 10, total: 17; e) green: 8, difference: 5 more red apples than green;
Bonus: f) green: 6, total: 16; g) red: 12, total: 19

**Choosing between addition and subtraction.**

ASK: When you know the number of two kinds of objects, like red apples and green apples, how do you find the total number of objects? (add) How do you find the difference? (subtract the smaller number from the larger number) Write on the board:

\[
\text{Number of yellow beans} + \text{Number of green beans} = \text{Total number of beans}
\]

ASK: What should we write in the circle, “+” (plus) or “−” (minus)? (+

PROMPT: We have two kinds of beans, yellow and green. If we know the number of yellow beans and the number of green beans, how do we find the total number of beans? (add) Write “+” in the circle. ASK: If we subtract instead of add, would we find the total or the difference? (difference)

Repeat the same example, except change the word “beans” to “marbles” so that students see the context doesn’t matter.
Then write:

| Number of red fish | Number of blue fish | = | How many more red fish than blue fish? |

ASK: What should we write in the circle, “+” (plus) or “−” (minus)? (−)

PROMPT: The words “how many more” tell us that we want to find the difference. Write “−” in the circle. ASK: If we add instead of subtract, would we find the total or the difference? (total) Repeat the same example, except change the word “fish” to “marbles” so that students see the context doesn’t matter. Then write:

| Total number of apples | Number of red apples | = | Number of green apples |

ASK: What should we write in the circle, “+” (plus) or “−” (minus)? (−)

SAY: We have two kinds of apples, red and green. If we take away the number of red apples from the total number of apples, what is left over? (the number of green apples)

**Exercises:** Write + or −.

a) Number of red grapes
   Number of green grapes = Total number of grapes

b) Total number of apples
   Number of green apples = Number of red apples

c) Number of red cars
   Number of black cars = How many more red cars than black

d) Number of boys in the class
   Number of girls in the class = Total number of students in the class

**Answers:** a) +, b) −, c) −, d) +

**Tables and word problems.** SAY: We can use tables to solve word problems. Write on the board:

| Red | Green | Total | Difference |

Have a volunteer read the sentence and ASK: Are we given the number of red apples? (yes) How many red apples does Ava have? (4) Write “4” in the Red column. ASK: Are we given the number of green apples? (yes) How many green apples? (5) Write “5” under Green. ASK: Are we given the total
number of apples? (no) How do we find the total number of apples? 
(4 + 5 = 9) Write “9” under Total. Are we given the difference? (no) Are 
there more red or more green apples? (more green) How many more? 
(5 – 4 = 1) Write “1” under Difference. Then complete the word problem 
by adding to part a):

How many apples does Ava have altogether?

After having a volunteer read the sentence, SAY: We already have the 
answer in the table! We just have to find it. ASK: Which number answers 
the question, the total or the difference? (total) PROMPT: The word 
“altogether” tells us we want the number of all apples, both green and red. 
Have a volunteer circle the number in the table that answers the question. 
(the number “9” in the total column) Then erase the sentence “How many 
apples does Ava have altogether?” and replace it with “How many more 
green apples does Ava have than red?” Have a volunteer read the new 
question. ASK: Which number in the table answers the question: the 
number of red apples? (no) the number of green apples? (no) the total? 
(no) the difference? (yes) Have students signal thumbs up or thumbs down 
for each question.

Extend the table on the board by another row, and write under part a):

b) Jake has 15 red apples and 7 more red apples than green apples.

After a volunteer reads the question, ASK: How many red apples? (15) Write 
“15” in the Red column. ASK: Are we told the number of green apples? (no) 
Are we told the total number of apples? (no) What is the difference between 
the number of red apples and the number of green apples? (7) Write “7” 
under Difference. ASK: Are there more red apples or more green apples? 
(more red) PROMPT: What does the question say? (more red apples than 
green) ASK: How many more red apples? (7 more) So, how do we find 
the number of green apples? (15 – 7 = 8) Write “8” under Green. Ensure 
students understand that if the question had said “more green apples than 
red,” then the number of red apples (15) would be the smaller number 
and the difference of 7 would have to be added to 15 to get the number of 
green apples. ASK: Now, how do we find the total number of apples? 
(15 + 8 = 23) Write “23” under Total. Add to the word problem in part b):

How many green apples does Jake have?

ASK: Which number in the table answers the question? (“8” under Green) 
Go through each option and have students signal thumbs up or thumbs 
down. Circle the “8” in the Green column. Demonstrate the first two 
exercises below in the same manner before having students complete the 
rest. Point out that it is harder to make a tape diagram for large numbers, 
so a table is better. NOTE: A tape diagram sketch can be used for larger 
numbers, without showing the individual squares, as in Extension 3 of 
Lesson OA3-12.
**Exercises:** Fill in the table. Circle the number in the table that answers the question.

<table>
<thead>
<tr>
<th>Red</th>
<th>Green</th>
<th>Total</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

a) Ed has 22 apples. 15 are green and the rest are red. How many red apples does he have?

b) Randi has 23 red apples and 11 fewer red apples than green apples. How many apples does she have in total?

c) Lynn has 17 green apples and 25 red apples. How many more red apples does she have than green?

d) Jon has 14 red apples and 3 fewer green apples than red. How many apples does he have?

e) Mona has 21 apples. 11 are red. The rest are green. How many more red apples does she have than green?

f) Raj has 13 green apples and 4 more green apples than red. How many apples does he have altogether?

**Answers:** a) red: 7 (circled), green: 15, total: 22, difference: 8; b) red: 23, green: 34, total: 57 (circled), difference: 11; c) red: 25, green: 17, total: 42, difference: 8 (circled); d) red: 14, green: 11, total: 25 (circled), difference: 3; e) red: 11, green: 10, total: 21, difference: 1 (circled); f) red: 9, green: 13, total: 22 (circled), difference: 4

SAY: You can make your own table to solve word problems. Write on the board:

Nina has 7 pets. 5 are dogs and the rest are cats.
How many cats does she have?

Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

ASK: What word should we write at the top of the first column? (Dogs) At the top of the second column? (Cats) Where does the number “7” belong? (Total column) Where does “5” belong? (under Dogs) How do we find the number of cats? (7 − 5 = 2) Write “2” under Cats. ASK: Which number answers the question? (the number of cats, which is 2) Do we need to find the difference? (no) Demonstrate the first exercise below before having students complete the rest.
Exercises: Make a table to answer the question.

1. Ethan has 13 math books and 7 more art books than math books. How many books does he have altogether?
   Answer: 33 books altogether

2. Grace has 32 toy cars. 18 are red and the rest are blue. How many blue cars does she have?
   Answer: 14 blue cars

3. A class has 28 students. 15 are girls and the rest are boys.
   a) How many students are boys?
   b) How many more girls are there than boys?
   Answers: a) 13 boys, b) 2 more girls than boys

Extensions

1. Use a table to help answer the question. Line up the digits to add and subtract. You might need to regroup.
   a) A school has 342 science books. It has 139 fewer art books than science books. How many art and science books does the school have in total?
   b) Alexa loves to read books. In April she reads 256 pages. In May she reads 108 pages more than in April. How many pages does she read altogether in April and May?
   Answers: a) 545 books in total; b) 620 pages altogether

2. For an extra challenge, have students try to find both parts when only the difference and the total are given, like in Extension 2 of the previous lesson. At this stage, students should just guess and test their answers. Students should list the pairs of numbers that add to the given total, and test each pair until they find the pair with the given difference.

<table>
<thead>
<tr>
<th></th>
<th>Red Apples</th>
<th>Green Apples</th>
<th>Total</th>
<th>Difference</th>
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</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td></td>
<td>9</td>
<td>1 more green apple</td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td>7</td>
<td>3 fewer green apples</td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td></td>
<td>10</td>
<td>2 more red apples</td>
</tr>
<tr>
<td>d)</td>
<td></td>
<td></td>
<td>10</td>
<td>6 fewer red apples</td>
</tr>
<tr>
<td>Bonus</td>
<td>e)</td>
<td></td>
<td>21</td>
<td>1 fewer green apple</td>
</tr>
<tr>
<td>f)</td>
<td></td>
<td></td>
<td>19</td>
<td>5 fewer green apples</td>
</tr>
</tbody>
</table>

Answers: a) red: 4, green: 5; b) red: 5, green: 2; c) red: 6, green: 4; d) red: 2, green: 8; Bonus: e) red: 11, green: 10; f) red: 12, green: 7
NOTE: Before doing Extension 3, explain what a palindrome is: the digits are the same both backwards and forwards. For example, 3,223 and 8,448 are palindromes, but 374 and 6,449 are not.

3. a) What two-digit numbers are palindromes? Share your answer with a partner.
   
   b) Start with a two-digit number that is not a palindrome. Add the number to its reverse. Do you always, sometimes, or never get a palindrome? Explain.
   
   c) John and Sara made statements about what they found in part b).
   - John: Any number where the sum of the digits is 9 or less will get a palindrome.
   - Sara: Any number where the sum of the digits is 10 or more will not get a palindrome.
   
   In pairs, explain why you agree or disagree with each statement. Do you agree with each other? Discuss why or why not.

Answers
a) 11, 22, 33, ..., 99
b) Sometimes. For example, 42 + 24 = 66, which is a palindrome, but 48 + 84 = 132, which is not a palindrome.

Students make sense of the problem (MP.1) by recognizing that they need to categorize numbers by the sum of the digits to decide whether John and Sara’s statements are true. They notice and use structure (MP.7)—for example, the ones and tens digits add to the same total because of the commutative property (NOTE: Students do not need to use this term) to prove John’s conjecture. For part c), encourage partners to ask questions to understand and challenge each other’s thinking (MP.3)—see p. A-49 for sample sentence and question stems to guide students.

Redirecting students: If students struggle with how to begin (MP.1), encourage them to repeat the procedure for many examples to get a feel for when they get a palindrome and when they do not.

For part c), you may need to remind students that to prove a conjecture or statement is false, they only need one example, but to prove a conjecture is true, they need a reason why it is always true or else they need to check every example (MP.3). Encourage students who struggle with one conjecture to see if they can figure out the other, then go back to the first one after they have done one that is easier (MP.1).
Sums and Differences

Goals
Students will solve one-step and two-step word problems involving addition and subtraction. Students might use tables and/or tape diagrams to help them find and distinguish between the parts, totals, and differences in each problem.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract three-digit numbers

Comparing word problems. Demonstrate the first exercise below and have all students try the second one. Students should see that part a) asks for the total, while part b) asks for the difference. If students struggle, have them draw a tape diagram or make a table.

Exercises
1. There are 7 green grapes. There are 5 purple grapes.
   a) How many grapes are there altogether?
   b) How many more green grapes are there than purple grapes?
   
   **Answers:** a) 12, b) 2

2. There are 8 boys and 12 girls on the hockey team.
   a) How many children are on the team?
   b) How many more girls than boys are on the team?

   **Answers:** a) 20, b) 4

Discuss the similarities and differences between Exercises 1 and 2. Students should see that the problems use different situations and different numbers, but the method of solving them is identical.

Deciding when to add or subtract. Ask students what words in a question give them a hint that they need to add? (examples: in total, altogether, in all) Ask students what words in a question give them a hint that they need to subtract? (examples: how many more, how much more, how much taller, how much longer, how much older) Tell students that if they are not sure whether to add or subtract, they can draw a tape diagram or a table to help. Remind students that if the numbers are large, a tape diagram is hard to draw, so a table is better. (NOTE: A tape diagram sketch can be used for larger numbers, without showing the individual squares, as in Extension 3 of Lesson OA3-12.)
Exercises: Add or subtract to answer the question.

a) 22 girls and 18 boys went on a school trip. How many students went on the trip altogether?

b) Vicky is 9 years old. Her father is 37. How much older is her father?

c) A school library has 642 books. 518 books are at the library. The rest are borrowed. How many books are borrowed?

d) Anwar has a math book and an art book. The math book has 184 pages. The art book has 32 more pages than the math book. How many pages does the art book have?

Answers: a) 40 students, b) 28 years, c) 124 books, d) 216 pages

Problems involving money and change. Write on the board:

Amit paid 75 cents for a pen that cost 53 cents. How much change did he get back?

SAY: The pen only cost 53 cents, but Amit paid 75 cents. Since Amit paid more than 53 cents, he has to get the extra money back. The money he gets back is called “change.” SAY: How do we find the change that Amit gets back? (subtract the price, 53 cents, from the amount paid, 75 cents) PROMPT: If Amit had paid 54 cents, how much change would he get back? (1 cent) How did you find the answer? (54 − 53) So how do we find the change if Amit paid 75 cents? (75 − 53) Have a volunteer line up the digits to subtract 75 − 53. (22)

Exercises: Add or subtract to find the answer.

a) Sharon paid 50 cents for a pencil that costs 39 cents. How much change does she get back?

b) Ben gave 352 dollars to a charity and Zara gave 467 dollars. How much money did they give in total?

Answers: a) 11 cents, b) 819 dollars

Word problems where you need the answer to part a) in order to do part b).
Tell students that in some problems, in order to do part b), they will need to use their answer to part a). Demonstrate with the following problem:

There are 4 more green grapes than purple grapes. There are 6 purple grapes.

a) How many green grapes are there?

b) How many grapes are there altogether?

For part a), ASK: How many purple grapes are there? (6) Are there more green grapes or more purple grapes? (green) How many more green grapes? (4 more) How do we find the number of green grapes? (6 + 4 = 10) Beside part a), write: “There are 10 green grapes.” Then ASK: How do we find the number of grapes altogether for part b)? (add the number of
purple grapes to the number of green grapes) Point out that they can now solve part b), but only because they know the answer to part a). SAY: There are 6 purple grapes and 10 green grapes. How many grapes are there altogether? \(6 + 10 = 16\)

**Exercises:** Find the answer to part a). Use the answer for part a) to do part b).

1. There are 8 green grapes. There are 15 grapes altogether.
   a) How many purple grapes are there?
   b) How many more green grapes are there than purple?
   **Answers:** a) 7 purple grapes, b) 1 more green grape than purple

2. There are 19 green grapes. There are 8 fewer purple grapes than green grapes.
   a) How many purple grapes are there?
   b) How many grapes are there altogether?
   **Answers:** a) 11 purple grapes, b) 30 grapes altogether

**Two-step word problems.** Now tell students that you are only going to give them part b) and they have to decide which question they should answer in part a) to solve part b). Students should think about what else they need to know to solve part b). Demonstrate the first exercise before having all students complete the second exercise.

**Exercises:** Complete the question for part a). Find the answer to part a). Use the answer for part a) to do part b).

1. There are 8 dogs in a shelter. There are 5 more cats than dogs in the shelter.
   a) How many ______ are in the shelter?
   b) How many cats and dogs are there altogether?
   **Answers:** a) cats, 13; b) 21

2. There are 18 people on a soccer team. 7 of them are boys.
   a) How many ______ are on the team?
   b) How many more girls than boys are on the team?
   **Answers:** a) girls, 11; b) 4

Explain to students that the following questions will require two steps. The first step will be to find a piece of information they need to answer the question.

**Exercises:** Answer the question. You will need to find a missing number first.

a) There are 7 red apples. There are 2 fewer green apples than red apples. How many apples are there altogether?
b) There are 12 raspberry bushes in Jenny’s backyard. There are 3 more blueberry bushes than raspberry bushes. How many berry bushes are in the backyard altogether?

Answers: a) 12 apples altogether, b) 27 berry bushes altogether, c) 2 more forks than spoons

Selected solution: c) The number of forks is \(12 - 5 = 7\), so there are \(7 - 5 = 2\) more forks than spoons.

Extensions

1. Challenge students to try two-step problems involving three kinds of objects.

a) Zack has knives, forks, and spoons in his kitchen drawer. He pulled out 16 objects without looking. He counted 8 knives and 3 spoons. How many forks did he pull out?

b) Clara has blue, green, and red marbles. She has 5 green marbles, 11 red marbles, and 23 marbles in total. How many blue marbles does she have?

c) There are red, green, and yellow apples in a fruit basket. Seven of the apples are green and 4 are red. There are 2 more yellow apples than green apples. How many apples are in the fruit basket altogether?

d) Sam had 93 stickers. He gave 25 to his brother and 39 to his sister. How many stickers did he keep?

Answers: a) 5 forks, b) 7 blue marbles, c) 20 apples altogether, d) 29 stickers

Selected solutions: a) \(8 + 3 = 11\) objects that are not forks, \(16 - 11 = 5\) forks; d) Sam gave \(25 + 39 = 64\) stickers to his brother and sister, and he kept \(93 - 64 = 29\)

2. Challenge students to try three-step problems involving two or three kinds of objects.

a) Amy has 21 football cards and 13 baseball cards. She gives away 4 football cards and 5 baseball cards. How many cards does she have left?

b) Peter has red cars, blue cars, and black cars. He has 15 red cars, and 6 more blue cars than red cars. He has 2 more blue cars than black cars. How many red and black cars does Peter have?

c) Tasha has blue, green, and red marbles. She has 15 green marbles, and 8 more red marbles than green marbles. She has 50 marbles in total. How many blue marbles does she have?
**Answers:** a) 25 cards left, b) 34 red and black cars, c) 12 blue marbles

**Solutions:** a) $21 - 4 = 17$ football cards left, $13 - 5 = 8$ baseball cards left, $17 + 8 = 25$ cards left; b) $15 + 6 = 21$ blue cars, $21 - 2 = 19$ black cars, $15 + 19 = 34$ red and black cars; c) $15 + 8 = 23$ red marbles, $15 + 23 = 38$ red and green marbles, $50 - 38 = 12$ blue marbles

3. Challenge students to solve three-step problems involving three-digit numbers.

a) A school library has 243 math books, 139 more art books than math books, and 108 more math books than science books. How many art and science books does the library have?

b) Jane, Sun, and Ravi raised money for a charity. Ravi raised 468 dollars. Sun raised 273 dollars more than Ravi. Ravi raised 129 dollars more than Jane. How much more money did Sun raise than Jane?

c) Blanca has blue, green, and red marbles. She has 250 green marbles, and 128 more green marbles than red marbles. She has 712 marbles in total. How many blue marbles does she have?

**Answers:** a) 517 art and science books, b) 402 dollars, c) 340 blue marbles

**Solutions:** a) $243 + 139 = 382$ art books, $243 - 108 = 135$ science books, $382 + 135 = 517$ art and science books; b) Sun raised $468 + 273 = 741$ dollars, Jane raised $468 - 129 = 339$ dollars, Sun raised $741 - 339 = 402$ dollars more than Jane; c) Blanca has $250 - 128 = 122$ red marbles, so she has $250 + 122 = 372$ green and red marbles, and there are $712 - 372 = 340$ blue marbles.

(MP3) 4. Kim has an addition chart. She notices that when she starts at the bottom left and goes up diagonally, all the squares have the same number:

<table>
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<tr>
<th>+</th>
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<th>2</th>
<th>3</th>
<th>4</th>
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<td>1</td>
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</table>

Is that true for every diagonal? Explain how you know.

**Answer:** Yes, because when you move right, you add 1, but when you move up in the chart, you subtract 1. Adding 1 and then subtracting 1 gets you back where you started.
## Decomposing Addends for Mental Addition

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<tr>
<th></th>
<th>3 = 1 + 2</th>
<th>4 = 1 + 3</th>
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<td>5 = 1 + 4</td>
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<td>8 = 6 + 2</td>
</tr>
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</tbody>
</table>

<table>
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<tr>
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<td>10 = 4 + 6</td>
</tr>
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<td>10 = 5 + 5</td>
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<tr>
<td>p</td>
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10 = 9 + 1
Blank Tape Diagrams

difference: __________
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\begin{array}{c}
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total: __________

difference: __________
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difference: __________
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total: __________

difference: __________
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\begin{array}{c}
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total: __________