Grade 4  Problem-Solving Lessons

Introduction

What is a problem-solving lesson? A JUMP Math problem-solving lesson generally follows the format of a regular JUMP Math lesson, with some important differences:

- There are no AP Book pages that accompany the problem-solving lessons.
- Problem-solving lessons focus on one or more problem-solving strategies rather than focusing on meeting the Common Core State Standards (CCSS). These lessons apply the concepts learned through the standards—often crossing several domains, clusters, or standards—but they are not necessary to complete the standards. Regular lessons, on the other hand, focus on completing the standards and sometimes require problem-solving strategies to do so.
- While regular lessons expose students to all of the problem-solving strategies, the problem-solving lessons provide a way to isolate and focus on the strategies.
- Instead of including extensions, each problem-solving lesson includes an extensive Problem Bank. These questions give students a variety of opportunities to practice the problem-solving strategy from the lesson and to learn new math in the process. Students will need to have mastered the material in the problem-solving lesson (which they do by completing the exercises) in order to tackle the Problem Bank.
- Both the lesson plan and the Problem Bank questions apply the CCSS. All of the standards covered in the lesson are mentioned at the beginning of the lesson plan.
- Some problem-solving lessons include an opportunity for students to complete one or more Performance Tasks. Performance Tasks are multi-part problems intended to determine how well students can apply grade-level CCSS in a new context. While most questions in the task can be done independently of the problem-solving lesson, some questions provide an opportunity to specifically apply the problem-solving strategy. These questions might be challenging for students who have not been taught the problem-solving lesson. Performance Tasks can cover several domains, clusters, or standards at once, all from material covered to date, and so can often be used as a cumulative review.
- While regular lessons cover the standards completely, problem-solving lessons cover clusters of major standards according to the CCSS, and some also cover supporting and additional standards. These lessons provide more challenging independent work while still focusing on the standards.

How do I use problem-solving lessons? Ten problem-solving lessons are provided for Grade 4. The problem-solving lessons can be taught at any point in the grade after the unit indicated in the table on the next page. We recommend using as many problem-solving lessons throughout the year as your class time allows, and suggest using them in the order in which they are indicated below. However, if required, you can pick and choose based on a careful review of the prior knowledge required for each problem-solving lesson.

We recommend teaching more problem-solving lessons toward the end of the year rather than toward the beginning, as this allows time for students to consolidate their mathematical knowledge and gain confidence before attempting more challenging problems. For this reason,
some classes may benefit from using the problem-solving lessons later in the grade than when they can first be used. Stronger classes that need fewer bridging lessons for review will have time to finish more of the problem-solving lessons. We recommend that classes needing most of the bridging lessons try at least a few problem-solving lessons.

Some of the Problem Banks include more problems than students can complete in one period. You might wish to use these as extension problems, or have students complete them as problems of the day throughout the year.

**Performance Tasks.** Performance Tasks are included at the end of some problem-solving lessons. Each Performance Task has at least one question that applies the problem-solving strategy covered in the lesson, but most questions can be done independently of the lesson. The Performance Tasks, together with any preparation, require a separate period each. Blackline Masters (BLMs) for each Performance Task are provided at the end of the lesson in which they are cited.

**Problem-solving strategies for Grade 4 and when to use them.** We consider the following problem-solving strategies as most important for this grade level:

- Recognizing and using structure
- Searching systematically
- Using a diagram
- Making a similar, but simpler, problem
- Guessing, checking, and revising
Teach this lesson after: 4.2 Unit 6

Standards: 4.OA.A.3, 4.NBT.A.2, 4.NBT.B.5

Goals: 
Students will make organized guesses and will use the result of the previous guess to revise their next guess.

Prior Knowledge Required:
Can use organized search
Can round multi-digit whole numbers to any place
Can multiply multi-digit numbers by single-digit numbers
Can compare two multi-digit numbers using place value
Can identify fractions of a number in the range 1–100

Vocabulary: divisor, guess-check-revise, product, remainder, round

Materials:
calculators
BLM Hockey Jerseys (pp. Q-36–37, see Performance Task)

Introduce the guess-check-revise strategy. Hide an object in the room and have a volunteer try to find the object. If the volunteer finds it quickly, play again until it takes a while. When they find it, ASK: What strategy did you use? (sample answer: I guessed and tried again) Play again, but this time tell the volunteer whether they are hot or cold as they try to find the object. Use hints such as “freezing cold” for very far away from the object, “lukewarm” for getting close, and “burning hot” for very close. ASK: What strategy did you use? (sample answer: I guessed and tried again) When you tried again with a hint, was it easier than last time? Why? Tell students that when they have more information about their guess, they can use that information to revise their next guess. Write on the board:

guess-check-revise

SAY: When you play hide-and-seek you are using a guess and check strategy, but when you play with hints such as “burning hot,” “lukewarm,” and “freezing cold,” you are guessing, checking, and revising the next guess. This guess-check-revise strategy is very useful in math.

Make sure everyone has a copy of their JUMP Math AP Book. Tell students to open to page 80 on their first try. Have different volunteers tell you what page number they turned to on their first try. Point out how all the attempts are fairly close to 80. SAY: No one’s first try was page 5 and no one’s first try was page 175. Everyone picked a page pretty close to 80. Now have students use their first guess to make a second guess. ASK: From the first page you turned to, which way in the book should you turn? Should you turn a lot of pages or only a few?
SAY: When you use your first guess to help you make your second guess, you are using the guess-check-revise strategy.

**Review systematic search when two related quantities are changing.** SAY: A farmer has cows and chickens. Jayden counts all the legs and Alice counts all the heads. Write on the board:

- Jayden counts 16 legs.
- Alice counts 6 heads.

ASK: Are there more heads or legs? (legs) Why does that make sense? (because each animal has more legs than heads) SAY: I want to know how many cows and how many chickens there are. Remember, to do this type of problem, you can start by guessing one of the two quantities and go up in order through all the possibilities. Draw on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>1</td>
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<td>6</td>
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</table>

ASK: How can you get the number of chickens from the number of cows? (they add to 6) SAY: There are six heads, so the total number of animals is six. Have a volunteer complete the second column. (6, 5, 4, 3, 2, 1, 0)

**Exercise:** Copy and complete the chart. How many cows and how many chickens are there if there are 16 legs in total?

**Answers:** 12, 14, 16, 18, 20, 22, 24; 2 cows and 4 chickens

Have a volunteer complete the third column of the chart on the board. ASK: If you move down a row, does the total number of legs get bigger or smaller? (bigger) How much bigger? (by 2) SAY: When you start at the top of the table, you have six chickens. When you move down a row, you replace a chicken with a cow, so now you have one cow and five chickens. Every time you replace a chicken with a cow, you replace two legs with four legs, so you have two more legs than before.

**Searching from either direction.** SAY: Jayden and Alice went to another farm that has cows and chickens. Write on the board:

- Jayden counts 36 legs.
- Alice counts 10 heads.
ASK: How many animals are there altogether? (10) How do you know? (the number of heads)

Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>20</td>
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<tr>
<td>1</td>
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<td>2</td>
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<td>9</td>
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<td>40</td>
</tr>
<tr>
<td>10</td>
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</tbody>
</table>

Have a volunteer complete the second column. (10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0) ASK: If there are no cows and 10 chickens, how many legs are there? (20) Write “20” in the first row of the third column. ASK: If there are 10 cows and no chickens, how many legs are there? (40) Write “40” in the last row of the third column. ASK: Do you think the number of cows in our answer will be closer to zero or to 10? (10) Why? (the number of legs is closer to 40 than to 20) PROMPT: Is the actual number of legs closer to 20 or 40? (40) So, is it better to start our guess closer to zero or to 10? (10) SAY: We could save ourselves a lot of work by starting at 10 cows and zero chickens and moving up the chart instead of starting at zero cows and 10 chickens. ASK: How many legs do nine cows have? (36) Write on the board:

36 +

ASK: How many legs does one chicken have? (2) Continue writing on the board:

36 + 2 = 38

Write “38” as the total in the row for 9 cows and 1 chicken. Repeat for the row with 8 cows and 2 chickens. (32 + 4 = 36) SAY: So, eight cows and two chickens have a total of 36 legs. Starting from 10 cows and searching is a lot less work than starting from zero cows and going all the way up to eight cows. Leave the chart on the board for later use.

(MP.1, MP.5) Exercises: How many cows and how many chickens are there on the farm?

a) Jayden counts 22 legs. Alice counts 9 heads.
b) Jayden counts 26 legs. Alice counts 7 heads.
c) Jayden counts 32 legs. Alice counts 15 heads.
d) Jayden counts 52 legs. Alice counts 14 heads.

Answers: a) 2 cows, 7 chickens; b) 6 cows, 1 chicken; c) 1 cow, 14 chickens; d) 12 cows, 2 chickens
Refer students to the chart on the board. SAY: You don’t have to start the chart with zero cows and then move up the chart from the end. You can start with 10 cows and move down the chart instead. Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>36</td>
</tr>
</tbody>
</table>

**Exercises:** If all the animals are cows, how many legs are there? If all the animals are chickens, how many legs are there?

a) Alice counts 30 heads.  
b) Alice counts 37 heads.  
c) Alice counts 28 heads.  
   **Bonus:** Alice counts 1,000 heads.

**Answers:** a) 120, 60; b) 148, 74; c) 112, 56; Bonus: 4,000, 2,000

SAY: Once you know how many legs there are if all the animals are cows and if all the animals are chickens, you can compare those numbers with the total number of legs given. Then you can decide which option to start your search with.

**(MP.1, MP.5) Exercises:** How many cows and how many chickens are there?

a) Jayden counts 114 legs. Alice counts 30 heads.  
b) Jayden counts 140 legs. Alice counts 37 heads.  
c) Jayden counts 60 legs. Alice counts 28 heads.  
   **Bonus:** Jayden counts 3,996 legs. Alice counts 1,000 heads.

**Answers:** a) 27 cows, 3 chickens; b) 33 cows, 4 chickens; c) 2 cows, 26 chickens;  
Bonus: 998 cows, 2 chickens

**Making guesses up or down by 10.** SAY: Now there is a bigger farm with cows and chickens. Write on the board:

Jayden counts 344 legs  
Alice counts 100 heads.  
How many cows and chickens are there?

ASK: If all the animals are cows, how many legs are there? (400) If all the animals are chickens, how many legs are there? (200) Is 344 closer to 400 or to 200? (400) Do you think there are more cows or chickens? (cows) SAY: Let’s start the search for the answer with 100 cows and zero chickens. Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>97</td>
<td>3</td>
<td></td>
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</tbody>
</table>
Have volunteers fill in the chart. (400, 398, 396, 394) SAY: We’re getting closer to 344 legs, but it’s going to take a while. ASK: How could I make the search go faster? Take students’ suggestions, then SAY: I am going to count by 10s instead of by 1s so that I can find the answer faster. Erase the chart on the board. Draw on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
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<tr>
<td>90</td>
<td>10</td>
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<td>80</td>
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<td>70</td>
<td>30</td>
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<tr>
<td>60</td>
<td>40</td>
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</tbody>
</table>

Exercises: a) Complete the chart. (MP.3, MP.7) b) What 2 tens is the number of cows between? Explain how you know.

Answers: a) 400, 380, 360, 340, 320; b) The number of cows is between 70 and 80, because 70 cows and 30 chickens have a total of 340 legs and 80 cows and 20 chickens have a total of 360 legs.

Further narrowing the search. Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>30</td>
<td>340</td>
</tr>
<tr>
<td>80</td>
<td>20</td>
<td>360</td>
</tr>
</tbody>
</table>

ASK: Is the actual number of cows closer to 70 or to 80? (70) A lot closer or a little closer? (a lot closer) Why? (because 344 is a lot closer to 340 than to 360) What number should we try next? (71 or 72)

Exercises:
1. Complete the chart until the total number of legs is 344.

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>30</td>
<td>340</td>
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<tr>
<td>71</td>
<td>29</td>
<td></td>
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<tr>
<td>72</td>
<td>28</td>
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<tr>
<td>73</td>
<td>27</td>
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</tbody>
</table>

Answer: 72 cows and 28 chickens
Problem Bank

(MP.1, MP.7) 1. What numbers might I be?
   a) When you multiply me by 7, the result is between 500 and 550.
   b) When you multiply me by 5, the result is less than 400. When you multiply me by 6, the result is more than 400.
   **Answers:** a) 72, 73, 74, 75, 76, 77, or 78; b) 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, or 79

(MP.1, MP.7) 2. What number am I?
   a) Multiply me by 7. Then round to the nearest 10. The result is 260.
   b) Multiply me by 88. Then round to the nearest hundred. The result is 5,000.
   c) When you multiply me by 800 and then round to the nearest 1,000, the result is 5,000.
   **Answers:** a) 37, b) 57, c) 6

3. What are the numbers?
   a) The quotient of two numbers is 8 and their product is 200.
   b) The quotient of two numbers is 8 and their product is 20,000.
   c) The quotient of two numbers is 8 and their product is 10,952.
   **Answers:** a) 5 and 40, b) 50 and 400, c) 37 and 296

(MP.1, MP.4) 4. A school fundraiser has a bake sale that sells muffins and cake. A muffin costs $2 and a piece of cake costs $3. The bake sale sold 30 items altogether and made $71. How many muffins and how many pieces of cake were sold?
   **Answer:** 19 muffins and 11 pieces of cake

(MP.1, MP.7) 5. Use a calculator to answer these questions. Remember that two whole numbers are consecutive if they differ by 1.
   a) Calculate the products.
      i) $1 \times 2$   ii) $2 \times 3$   iii) $3 \times 4$   iv) $4 \times 5$   v) $5 \times 6$
   b) Is 14 the product of two consecutive whole numbers? Explain how you know.
   c) Can 160 be the product of two consecutive whole numbers? Explain how you know.
   d) Can 992 be the product of two consecutive whole numbers? Explain how you know.
   e) Write 6,972 as a product of two consecutive whole numbers.
   **Answers:** a) i) 2, ii) 6, iii) 12, iv) 20, v) 30; b) no, it is between $3 \times 4$ and $4 \times 5$; c) no, it is between $12 \times 13 = 156$ and $13 \times 14 = 182$; d) yes, it is $31 \times 32$; e) $83 \times 84$

(MP.1, MP.7) 6. Find $N$ so that …
   a) $(2 \times N) + 1 = 177$  b) $(N \times 3) + N = 228$  c) $(N \times 5) + 5 = 320$
   **Bonus:** Use a calculator to find $N$ if $N \times N = 1,849$
   **Answers:** a) 88, b) 57, c) 63, Bonus: 43
(MP.1, MP.7) 7. a) Fill in the blanks with a whole number when you can.

___ × 1 + 6 = 30
___ × 2 + 6 = 30
___ × 3 + 6 = 30
___ × 4 + 6 = 30
___ × 5 + 6 = 30
___ × 6 + 6 = 30

b) Which blanks have a whole number that works? Explain. Hint: Make sure the remainder is less than the divisor.

30 ÷ ____ = 1 R 6
30 ÷ ____ = 2 R 6
30 ÷ ____ = 3 R 6
30 ÷ ____ = 4 R 6
30 ÷ ____ = 5 R 6
30 ÷ ____ = 6 R 6

c) John divides 45 by a number and gets a remainder of 9. What numbers could he have divided by?

Answers:

a) 24, 12, 8, 6, no whole number possible, 4

b) 30 ÷ 24 = 1 R 6, 30 ÷ 12 = 2 R 6, and 30 ÷ 8 = 3 R 6 work here. 6 R 6 and 4 R 6 don’t work because dividing by 6 or 4 can’t leave a remainder of 6. 5 R 6 doesn’t work because 5 is not a factor of 24 (30 ÷ 6).

c) To fill in the blank in 45 ÷ ____ = ? R 9, you need ____ × ? + 9 = 45, so ____ is a factor of 36. ____ also must be bigger than 9—otherwise dividing by it can’t get a remainder of 9. So, the numbers he could have divided by are 12, 18, and 36.
Performance Task: Hockey Jerseys

Materials:
BLM Hockey Jerseys (pp. Q-36–37)

Preparation for the performance task. Tell students that the performance task is about hockey jerseys (the team shirts) and how much it costs overall for a team to play ice hockey. Part of the total cost is for the jerseys and part is for renting the rink. The players themselves supply their skates, hockey sticks, and other equipment. The team buys the jerseys, then sends them out to have numbers printed on them. The numbers start at one and go up in order to the number of players on the team. Numbers with two digits cost more than numbers with one digit, because the printing cost is per digit. Tell students that this will be part of what they investigate in the performance task.

Performance Task: Hockey Jerseys. Provide students with BLM Hockey Jerseys. Question 6 is a good opportunity to apply the problem-solving strategy learned in this lesson. Students who have not had the opportunity to do this lesson might find it difficult.

Answers: 1. $360, 2. 21, 3. $63, 4. $200, 5. $623, 6. 17
Hockey Jerseys (1)

In an ice hockey league, each team buys jerseys for their players before the season begins.

1. Jerseys cost $24 each. The Warriors is a team with 15 players. How much does the team need to pay for jerseys?

2. Each team decides to put numbers on the back of each jersey. Each team starts at 1 and numbers the jerseys in order. How many digits will the Warriors need in total?

3. Each digit costs $3 to put on. How much would a team of 15 players have to pay for the digits?

4. It costs the league $1,800 to rent the ice rink for the year. There are 9 teams in the league. How much does each team pay for the rink rental?
Hockey Jerseys (2)

5. What is the team’s total cost, including rink rental, jerseys, and digits?

6. The Athenas is another team in the league. They paid $75 for the digits on their jerseys. How many players are on that team?