Goals
Students will understand that dividing 10 times as many objects among the same number of groups will result in 10 times as many objects in each group (e.g. 60 ÷ 3 is 10 times as much as 6 ÷ 3).

PRIOR KNOWLEDGE REQUIRED
Knows that division can be used to find the number of objects in each group
Can represent numbers using base ten blocks

MATERIALS
hundreds blocks, tens blocks, and many ones blocks

Review representing numbers with base ten blocks. Give students 9 tens blocks and 9 ones blocks. Have students model these numbers with their blocks.

Answers

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>b)</td>
<td>c)</td>
<td>d)</td>
<td></td>
</tr>
</tbody>
</table>

Demonstrate the convenience of using tens blocks to divide. Take a pile of 60 ones blocks and a pile of 6 tens blocks. Demonstrate dividing 60 ÷ 2 by placing 2 ones blocks at a time, and then dividing 60 ÷ 2 by placing 2 tens blocks at a time. Emphasize that you get the same answer both ways, but dividing up the tens blocks was a lot faster, and so was seeing the answer from the division. SAY: Using tens blocks is very convenient to divide.

Dividing single place values among equal groups and writing the division statement. Give each student or pair of students 8 ones blocks. Have students divide the blocks into two equal groups. Write on the board:

8 tens ÷ 2 = _____ tens
80 ÷ 2 = _____

Have a volunteer fill in the blanks (4, 40). Repeat with dividing 6 hundreds blocks between 2 groups (600 ÷ 2 = 300).

Ensure that all students or groups of students have 9 tens blocks and 9 hundreds blocks.

Exercises: Use blocks to divide.

a) 60 ÷ 3 b) 400 ÷ 2 c) 90 ÷ 3 d) 800 ÷ 4 e) 800 ÷ 2

Answers: a) 20, b) 200, c) 30, d) 200, e) 400
Compare division when 10 times as many objects are divided. Draw on the board:

a) □ □ □ □ □ □ □ □

b) □ □ □ □ □ □ □ □

c) □ □ □ □ □ □ □ □

d) □ □ □ □ □ □ □ □

Have volunteers divide the blocks into 2 equal groups and write what division statement is shown. Prompt them if necessary by asking what number is being divided and what number is in each group.

Then have students look at all the division statements (see margin).

ASK: What do you think 80,000 ÷ 2 is? (40,000) Point out that when there are 10 times as many objects being divided between 2 groups, there are 10 times as many objects in each group.

Give each student or group of students 12 ones blocks. Have students divide the objects among 3 equal groups and then write the division statement. Repeat with tens blocks and hundreds blocks. Point out that 12 objects divided among 3 equal groups always gives 4 objects in each group, and that is true whether the objects are strawberries, or oranges, or ones blocks, or tens blocks, or hundreds blocks.

**Exercises:** Divide.

a) \( 4 \div 2 = \) _______  b) \( 9 \div 3 = \) _______
\( 40 \div 2 = \) _______  \( 90 \div 3 = \) _______
\( 400 \div 2 = \) _______  \( 900 \div 3 = \) _______
\( 4,000 \div 2 = \) _______  \( 9,000 \div 3 = \) _______

**Answers:** a) 2, 20, 200, 2,000, 20,000, 200,000  b) 3, 30, 300, 3,000

Ask students to use \( 35 \div 5 = 7 \) to divide \( 35,000,000 \div 5 \). (7,000,000)

**Exercises:** Divide.

a) \( 6,000,000,000 \div 2 \)  b) \( 48,000,000 \div 8 \)
c) \( 60,000,000 \div 3 \)  d) \( 420,000,000,000 \div 7 \)

Tell students that the next ones are a bit tricky. For part e), instead of starting with \( 4 \div 8 \), students should start with \( 40 \div 8 \).
Bonus

e) $400,000,000 \div 8$

f) $30,000,000 \div 6$

Answers: a) $3,000,000,000$, b) $6,000,000$, c) $20,000,000$, d) $60,000,000,000$

Bonus: e) $50,000,000$, f) $5,000,000$

Extensions

1. Teach students to use numbers that are easy to divide mentally to estimate answers to other problems. For example, $5,472 \div 9$ is close to $5,400 \div 9$. But $5,400$ is easy to divide by $9$ mentally, because $54$ is a multiple of $9$. We know that $54 \div 9 = 6$, so $5,400 \div 9 = 600$. Have students use mental math to estimate the answers to these problems.

   a) $63,441 \div 9$
   b) $32,548 \div 4$
   c) $56,368 \div 8$

   Answers: a) $7,000$, b) $8,000$, c) $7,000$

   Students can then check their estimates using a calculator.

2. Teach students to pick the closest 2-digit multiple of the number being divided by to use for estimating. For example, to divide $31,752 \div 4$, circle the first two digits of the dividend (circle 31) and find the closest multiple of 4 to that number. (32) So use $32,000 \div 4 = 8,000$ as an estimate for $31,752 \div 4$. Have students use mental math to estimate the answers to these problems.

   a) $49,314 \div 6$
   b) $3,458 \div 7$
   c) $59,865 \div 5$

   Answers: a) $8,000$, b) $500$, c) $12,000$

3. Teach students to predict whether the estimate will be lower or higher than the actual answer. For example, since $32,000$ is greater than $31,752$, the estimated quotient, $8,000$, will be greater than the actual answer. Have students predict whether the estimated answers above are greater or less than the actual answer, and verify their prediction using a calculator.

   (MP1, MP3, MP4)

4. Lela reads 5 pages of a book per day. It takes her 37 days to finish the book. What are the possible numbers of pages in the book? Explain how you know.

   Answer: $36 \times 5 = 180$ and $37 \times 5 = 185$, so the possible number of pages in the book is $181, 182, 183, 184, or 185$. If there were 180 or fewer pages, she could read the book in 36 days at most. If there were 186 or more pages, she would need at least 38 days.

   Redirecting students: ASK: What is the possible number of pages if it only took her 1 day to finish the book? What about 2 days? 3 days? 7 days?
**Long Division—2-Digit by 1-Digit**

**Goals**
Students will divide 2-digit numbers by 1-digit numbers.

**Prior Knowledge Required**
- Understands division as equal sharing
- Understands place value

**Materials**
tens and ones blocks

**Dividing using tens and ones blocks.** If you have an advanced class, start with the activity below to emphasize the importance of using place value when dividing. If not, you can skip the activity and go right into discovering what the long division notation means.

**Activity**
Have students divide 3 into 63 using tens and ones blocks. Emphasize how much more convenient it is to use 6 tens blocks and 3 ones blocks, rather than 63 ones blocks. Repeat for dividing 2 into 85 (now there will be a remainder) and then dividing 2 into 75 (now they will have to exchange a tens block for 10 ones blocks).

**Introduce the “” symbol.** Write on the board:

\[
egin{align*}
3 & \longdiv{6} \\
4 & \longdiv{12} \\
5 & \longdiv{20} \\
6 & \longdiv{18} \\
9 & \longdiv{18}
\end{align*}
\]

Tell students that the numbers on top were written based on a rule. Students who know the rule should not say the rule, but can volunteer answers. Ask volunteers to fill in the answers on top until everyone is confidently volunteering (4, 3, 2). Point out that the first example on the board is another way of writing \(6 \div 3 = 2\). If you divide 6 objects among 3 groups, you get 2 in each group.

**Dividing using dimes and pennies.** NOTE: This can also be done using tens and ones blocks, but dimes and pennies are more meaningful to students.

Write on the board:

\[
3 \longdiv{72}
\]

SAY: This can be interpreted to mean: 3 friends wish to share 7 dimes and 2 pennies, 72 cents, as equally as possible.
Exercises: How many friends, dimes, and pennies are there in each case?

a) $4 \div 9 \qquad b) \ 2 \div 7 \qquad c) \ 3 \div 7 \qquad d) \ 5 \div 8$

Answers: a) 4, 9, 6; b) 2, 7, 4; c) 3, 7, 5; d) 5, 8, 0

(MP2) Dividing the dimes. Have students draw a picture to show how, for the division $3 \div 72$, they would divide the dimes among the friends. You could have them use a circle for each friend and a D for each dime.

![Picture of dimes divided among friends]

ASK: What does the picture mean? (each friend gets two dimes and there is one dime left over)

Exercises: Draw a picture to show how you would divide the dimes in the exercises above: a) $4 \div 96$, b) $2 \div 74$, c) $3 \div 75$, d) $5 \div 80$

Answers: a) 2 dimes in each of 4 boxes, plus 1 extra dime; b) 3 dimes in each of 2 boxes, plus 1 extra dime; c) 2 dimes in each of 3 boxes, plus 1 extra dime; d) 1 dime in each of 5 boxes, plus 3 extra dimes

Beginning the long division algorithm. Tell students that if they happened to see someone carrying out the first two steps of the long division algorithm, this is what they would see:

![Long division example]

Challenge students to figure out what the steps in the algorithm mean by identifying where they see each number in their picture. ASK: Where in the picture does the 2 on top come from? (each friend got 2 dimes) Where in the picture does the 6 come from? (6 dimes were shared so far) Point out that the 2, 7, and 6 are all counting dimes, so they are written one above the other. Encourage students to use grid paper to line up the digits. Point out the similarity with addition, subtraction, and multiplication, where all the same place values are lined up, tens with tens and ones with ones.

Exercises: Start the long division for the exercises you just drew a picture for.

a) $4 \div 96$ \quad b) $2 \div 74$ \quad c) $3 \div 75$ \quad d) $5 \div 80$

Answers: a) $2 \div 96$, b) $3 \div 74$, c) $2 \div 75$, d) $1 \div 80$

Refer students' attention again to the division on the board dividing 3 into 72. ASK: How could you get the 6 knowing that 3 friends each got 2 dimes? (multiply 3 x 2) Point out that in each answer above, the number of dimes given out is the number of friends times how many each friend got. Point out that all the dimes are written in the same column: the dimes in each group, the dimes started with, and the dimes shared so far.
**Exercises**: Multiply to get the number of dimes shared so far.

a) \(4 \div 9\) 2  

b) \(3 \div 7\) 4  

c) \(2 \div 9\) 7  

d) \(3 \div 8\) 7  

**Answers**: a) 8, b) 6, c) 8, d) 6

**Subtracting to find the leftover dimes.** Now show students the next step of the long division algorithm.

\[
\begin{array}{c|cc}
\text{3} & \text{7} & \text{2} \\
\hline
\text{6} & \text{D} & \text{D} \\
\text{D} & \text{D} & \text{D} \\
\text{D} & \text{D} & \text{D} \\
\end{array}
\]

**ASK**: Where do you see the 1 in the picture? (it is the number of dimes left over) Point out that this makes sense. When you subtract the number of dimes shared from the number of dimes available, you get the number of leftover dimes.

**Exercises**: Subtract to get the number of dimes that are left over.

a) \(4 \div 9\) 5  

b) \(3 \div 7\) 7  

c) \(3 \div 5\) 4  

d) \(3 \div 9\) 5  

**Answers**: a) 1, b) 1, c) 2, d) 0

**Exercises**: Multiply to see how many dimes were shared, then subtract to see how many dimes are left.

a) \(5 \div 7\) 5  

b) \(4 \div 9\) 1  

c) \(3 \div 8\) 7  

d) \(2 \div 9\) 5  

**Answers**: a) \(5 \div 7\) 5 , b) \(4 \div 9\) 1 , c) \(3 \div 8\) 7 , d) \(2 \div 9\) 5  

**Exercises**: Divide the dimes.

a) \(5 \div 8\) 4  

b) \(3 \div 8\) 7  

c) \(2 \div 7\) 5  

d) \(2 \div 5\) 6  

**Answers**: a) \(5 \div 8\) 4 , b) \(3 \div 8\) 7 , c) \(2 \div 7\) 5 , d) \(2 \div 5\) 6  

**The importance of exchanging dimes for 10 pennies.** Now refer students to the picture above and point out that the picture so far only shows the dimes that haven’t been placed. Ask a volunteer to write P for each penny that still has to be placed. **ASK**: How much money needs to be placed? (12 cents) Ask for 3 volunteers, and show students the 1 dime and 2 pennies. **SAY**: I want to share this equally among the three volunteers. Give two students a penny each and one student a dime. If nobody protests, **ASK**: Is that fair? (no) **THEN** **ASK**: How would you divide up the money to make it fair? (trade the dime for 10 pennies)
Using the picture for \( \frac{3}{7} \) that you started, cross out the leftover dime and replace it with 10 pennies:

\[
\begin{array}{cccc}
  & & D & D \\
  & & D & D \\
  & & D & D \\
  & & P & P \\
  & & P & P \\
  & & P & P \\
  & & P & P \\
\end{array}
\]

Tell students that this process of regrouping the tens as ones, or dimes as pennies, is actually a step in the long division process. Most adults call this the “bring down” step, but very few understand it.

\[
\begin{array}{c}
  \underline{3 \overline{7_2}} \\
  -6 \\
  \underline{12} \quad 1 \text{ dime and } 2 \text{ pennies } = 12 \text{ pennies}
\end{array}
\]

Point out that this “bringing down” step is a lot easier if they align the tens, or dimes, in the first place.

**Exercises:** Show the “bringing down” step for each exercise:

- a) \( \frac{2}{4} \frac{95}{8} \)
- b) \( \frac{2}{7} \frac{77}{6} \)
- c) \( \frac{3}{5} \frac{4}{3} \)
- d) \( \frac{3}{9} \frac{5}{9} \)

**Answers:**

- a) \( \frac{2}{4} \frac{9}{15} \)
- b) \( \frac{2}{7} \frac{7}{17} \)
- c) \( \frac{3}{5} \frac{4}{24} \)
- d) \( \frac{3}{9} \frac{5}{05} \)

Emphasize that, even though there are no dimes left in part d), there are still pennies left, so we are not done dividing up the money.

**Finishing the long division.** Have students show you how they would divide up the pennies on their picture. Now show students the completed long division algorithm, and challenge students to tell you where the remaining numbers come from:

\[
\begin{array}{c}
  \underline{3 \overline{7_2}} \\
  -6 \\
  \underline{12} \\
  -12 \\
  \underline{0}
\end{array}
\]

\[
\begin{array}{cccc}
  & & D & D \\
  & & D & D \\
  & & D & D \\
  & & P & P \\
  & & P & P \\
  & & P & P \\
  & & P & P \\
\end{array}
\]

(MP.2) **ASK:** Where do you see the 4 in your picture? (There are 4 pennies in each group.) Pointing to the second 12, **ASK:** Where do you see the 12? (12 pennies were shared.) What does the 0 tell you? (There are no pennies left over.)

Tell students that the long division is done when all the tens and all the ones are divided, so there will be two steps. Any leftover ones are the remainder. Have students complete the problems in the exercises above, and then do more exercises from start to finish.
Exercises: Divide.

a) \[ \frac{5}{80} \]  

b) \[ \frac{4}{65} \]  

c) \[ \frac{2}{85} \]  

d) \[ \frac{3}{97} \]  

e) \[ \frac{3}{72} \]  

f) \[ \frac{4}{86} \]  

g) \[ \frac{2}{75} \]  

h) \[ \frac{3}{95} \]  

Answers: a) 16 R 0, b) 16 R 1, c) 42 R 1, d) 32 R 1, e) 24 R 0, f) 21 R 2, g) 37 R 1, h) 31 R 2

Some students might want to stop when they see a remainder of 0 partway through the long division, as in parts c), d), f), and h). Point out that the 0 they see only tells them how many dimes they need to exchange for pennies—they still have to divide the pennies even if there aren't any dimes. Encourage students to verify this by using dimes and pennies to divide.

Checking the division. Have students check their answers with multiplication. For example, if long division gives \[ \frac{69}{4} = 17 \text{ R } 1 \], then students can multiply \[ 17 \times 4 = 68 \], and add 1 to get 69.

Word problems practice.

a) How many weeks are in 91 days?
b) How many 3-person canoes will 54 children need?
c) How many 5-person cars will 85 people need?
d) How many $6 T-shirts can you buy with $84?

Answers: a) 13, b) 18, c) 17, d) 14

Extensions

1. Teach students short division notation. For example, to divide 3 into 74, start by noticing that 3 goes into 7 twice with remainder 1. Write the 2 as usual, but write the 1 between the 7 and the 4 in the dividend. Now repeat the process on dividing 3 into 14, which is 4 remainder 2. Notice that students have to keep the products at each stage in their heads.

\[
\begin{array}{c}
3 \overline{)74} \\
\underline{6} \\
14 \\
\underline{12} \\
2
\end{array}
\Rightarrow
\begin{array}{c}
3 \overline{)74} \\
\underline{6} \\
14 \\
\underline{12} \\
2
\end{array}
\]

(MP7)

2. Divide.

a) \[ 720,000 ÷ 3 \]  

b) \[ 6,800 ÷ 2 \]  

c) \[ 8,100,000 ÷ 3 \]  

Answers: a) 240,000, b) 3,400, c) 2,700,000
3. Find and correct the mistake in the long divisions.

a) \[
\begin{array}{c}
3 \overline{)85} \\
\underline{-9} \\
5 \\
\underline{-3} \\
2
\end{array}
\]
So 85 \(\div\) 3 = 21 R 2.

b) \[
\begin{array}{c}
3 \overline{)67} \\
\underline{-6} \\
0
\end{array}
\]
So 67 \(\div\) 3 = 2 R 0.

**Answers:** a) 8 – 6 = 2 needs to be subtracted, and 85 \(\div\) 3 = 28 R 1.
b) The 7 needs to be brought down and the division continued, so 67 \(\div\) 3 = 22 R 1.

(MP3) 4. Without dividing, decide if the answers to the following problems will have one or two digits. Explain your thinking.

\[
\begin{array}{c}
3 \overline{)72} \\
4 \overline{)38} \\
9 \overline{)74} \\
6 \overline{)82} \\
6 \overline{)34}
\end{array}
\]

**Selected solution:** 3 \(\times\) 10 = 30 is less than 72, so 72 \(\div\) 3 is greater than 10 and so has two digits.

**Answers:** two, one, one, two, one

(MP2) 5. Write a word problem to express 96 \(\div\) 4.

(MP2, MP4) 6. In September, the students in a Grade 4 class read 84 chapter books and 37 picture books. They read 3 times as many books in October. How many books did they read in September and October altogether?

Show your work using equations. Say what each equation means in the situation.

**Answer:** 84 \(\div\) 37 = 121, so they read 121 books in September.

3 \(\times\) 121 = 363, so they read 363 books in October. 363 \(\div\) 121 = 484, so they read 484 books altogether.

**NOTE:** Some students may notice that the total number of books is 4 times the number of books read in September. If so, these students are looking for and making use of structure (MP7).
Using long division to divide 3-digit numbers by 1-digit numbers. Using base ten materials, explain why the standard algorithm for long division works. Write on the board:

\[ 736 \div 3 \]

**Step 1:** Make a model of 736 units.

```
[Model with 736 units]
```

**Step 2:** Divide the hundreds blocks into 3 equal groups. There will be 2 in each group.

Keep track of the number of units in each of the 3 groups and the number remaining.

\[
\begin{align*}
3 & \longdiv{736} \\
-6 & \\
\hline
1 & \\
\end{align*}
\]

2 hundreds blocks in each group

\[ 2 \times 3 = 6 \text{ hundreds blocks have been divided} \]

1 hundreds block left over

Give each student or group of students 9 hundreds blocks.

**Exercises:** Practice Steps 1 and 2 using hundreds blocks, and record your work using the standard algorithm.

a) \[ 2 \longdiv{512} \]  
   b) \[ 3 \longdiv{822} \]  
   c) \[ 2 \longdiv{726} \]  
   d) \[ 4 \longdiv{912} \]

**Answers:** a) \[ 3 \]  
   b) \[ 2 \]  
   c) \[ 3 \]  
   d) \[ 4 \]

**Step 3:** Divide the remaining hundreds block and the 2 remaining tens blocks among the 3 groups equally.
SAY: There are 136 units left to divide. Write on the board:

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\square & \square & \square & \square \\
\end{array}
\]

SAY: There are not enough hundreds blocks to put one in each group. So it is not convenient to use hundreds blocks any more. Let’s change the last hundreds block to 10 tens blocks. ASK: Now how many tens blocks do we have? (13) How many tens blocks can we put in each group? (4)

Write on the board:

\[
\begin{array}{c}
\begin{array}{c}
2 \quad 4 \\
3 \quad 7 \quad 3 \quad 6 \\
\end{array} \\
\begin{array}{c}
4 \quad 4 \\
6 \quad 1 \quad 3 \\
\end{array} \\
12 \quad 12 \quad 10 \\
16 \quad 16 \\
\end{array}
\quad 12 \text{ tens blocks have been divided}
\quad 16 \text{ ones blocks still need to be divided}
\]

Show what the three groups now look like:

Exercises: Practice Step 3 using tens blocks, and record your work using the standard algorithm for the problems started above.

Answers:

a) \( \begin{array}{c}
2 \quad 5 \\
3 \quad 1 \quad 2 \\
\end{array} \), b) \( \begin{array}{c}
2 \quad 7 \\
3 \quad 8 \quad 2 \quad 2 \\
\end{array} \), c) \( \begin{array}{c}
3 \quad 6 \\
4 \quad 7 \quad 3 \quad 6 \\
\end{array} \), d) \( \begin{array}{c}
2 \quad 2 \\
3 \quad 9 \quad 1 \quad 2 \\
\end{array} \)

Exercises: Practice Steps 1, 2, and 3 using hundreds and tens blocks and record your work using the standard algorithm.

a) \( \begin{array}{c}
2 \quad 5 \quad 5 \\
3 \quad 8 \quad 3 \quad 7 \\
\end{array} \), b) \( \begin{array}{c}
4 \quad 7 \quad 3 \quad 6 \\
5 \quad 9 \quad 5 \quad 2 \\
\end{array} \)

Answers:

a) \( \begin{array}{c}
2 \quad 7 \\
3 \quad 8 \quad 3 \quad 7 \\
\end{array} \), b) \( \begin{array}{c}
4 \quad 7 \quad 3 \quad 6 \\
5 \quad 9 \quad 5 \quad 2 \\
\end{array} \)

Step: Divide the remaining 16 ones blocks among the 3 groups equally. There are now 2 hundreds blocks, 4 tens blocks, and 5 ones blocks in each group.

\[
\begin{array}{c}
2 \quad 4 \quad 5 \\
3 \quad 7 \quad 3 \quad 6 \\
\end{array} \quad 5 \text{ ones blocks have been put in each group}
\quad 1 \text{ ones block is left over}
\]

Number and Operations in Base Ten 4-45, 4-46
Exercises: Finish the problems you started above.

Answers: a) 276, b) 279, c) 184, d) 238

Exercises: Complete all steps to divide.

\[
\begin{align*}
a) & \quad 2 \overline{) 392} \\
b) & \quad 3 \overline{) 528} \\
c) & \quad 2 \overline{) 974} \\
d) & \quad 4 \overline{) 692}
\end{align*}
\]

Answers: a) 196, b) 176, c) 487, d) 173

Explain long division with an intermediate remainder of 0. Write on the board a division where the divisor goes evenly into the leading digit of the dividend, and demonstrate beginning the division:

\[
\begin{align*}
2 & \overline{) 642} \\
-6 & \\
0 &
\end{align*}
\]

Point out that it looks like the remainder is 0, but, in fact, we are really subtracting 642 – 600. There is still 42 left to divide. We show this by bringing down the 4 and continue by dividing the 4 tens into 3 groups. Have a volunteer finish the division.

Exercises: Divide using long division.

\[
\begin{align*}
a) & \quad 4 \overline{) 832} \\
b) & \quad 3 \overline{) 945} \\
c) & \quad 2 \overline{) 634} \\
d) & \quad 3 \overline{) 982}
\end{align*}
\]

Answers: a) 208, b) 315, c) 317, d) 327 R1

SAY: Sometimes the second step has a remainder of 0. Then demonstrate part e).

\[
\begin{align*}
e) & \quad 2 \overline{) 584} \\
f) & \quad 3 \overline{) 846} \\
g) & \quad 4 \overline{) 925} \\
h) & \quad 3 \overline{) 784}
\end{align*}
\]

Answers: e) 292, f) 282, g) 231 R 1, h) 261 R 1

Introduce long division where the leading digit of the dividend is less than the divisor. Write on the board:

\[
\begin{align*}
a) & \quad 4 \overline{) 657} \\
b) & \quad 3 \overline{) 352}
\end{align*}
\]

ASK: How many times does 4 go into 6? (once) Write 1 above the 6. Now move to the second problem. ASK: How many times does 4 go into 3? (none; 3 is less than 4) Write 0 above the 3. Point out that, when doing long division, we always start by asking how many times the divisor goes into the first digit of the dividend, and sometimes that answer is 0. Show how to start the two divisions on the board:

\[
\begin{align*}
1 & \overline{) 657} \\
-4 & \\
25 &
\end{align*}
\]

\[
\begin{align*}
0 & \overline{) 352} \\
-0 & \\
35 &
\end{align*}
\]

Have students finish the two divisions. Remind students that numbers do not start with a "0" digit. So, we can assume that the 3 hundreds have already been traded for 30 tens and start by dividing the 35 tens. Demonstrate this shortcut.
89
4) 357
- 32
  37
- 36
  1

Exercises: Divide.

a) $\overline{5238}$, b) $\overline{8427}$, c) $\overline{4318}$, d) $\overline{2164}$

Answers: a) 47 R 3, b) 53 R 3, c) 79 R 2, d) 82

Introduce long division where the quotient has a 0 digit. Start the division for $936 \div 9$:

$\overline{936}$
- 9
  03

ASK: How many times does 9 go into 3? (none) What is the tens digit of the quotient? (0) Write it in, then ASK: What is $9 \times 0$? (0) Have a volunteer show where to write the answer to $9 \times 0$, then demonstrate finishing the long division:

$\overline{936}$
- 9
  03
- 0
  36
- 36
  0

Exercises: Divide using long division.

a) $\overline{4835}$, b) $\overline{5512}$, c) $\overline{3924}$, d) $\overline{3817}$, e) $\overline{4817}$

Bonus f) $\overline{32185}$, g) $\overline{43629}$, h) $\overline{56947}$, i) $\overline{37418}$, j) $\overline{742163}$

Answers: a) 208 R 3, b) 102 R 2, c) 308, d) 272 R 1, e) 204 R 1, f) 728 R 1, g) 907 R 1, h) 1,389 R 2, i) 2,472 R 2, Bonus: 371,081 R 1

Word problems practice.

a) How many weeks are in 364 days?

b) The distance of a 4-person relay totals 1,100 m. How far does each person run?

c) 3 people share the cost of an $846 vacation. How much does each person pay?

Bonus: A can holds 3 tennis balls. A tennis instructor has 84 students and needs to buy 5 tennis balls for each student. How many cans are needed?

Answers: a) 52 weeks, b) 275 m, c) $282$, Bonus: 140 cans
NOTE: The word problems in NBT4-46 Concepts in Multiplication and Division can be used as cumulative review for concepts in multiplication and division. You may need to remind students that 1 min = 60 s and 1 h = 60 min (see AP Book 4.2 p. 18 Question 7).

Extensions

(MP6, MP7) 1. Write a number sentence to show how 5,638 ÷ 8 and 23,956 ÷ 12 compare. Use one of the symbols <, =, or > in your number sentence. Explain how you know your number sentence is correct, and explain what the symbols you used mean.

Sample answers
• 5,678 ÷ 8 < 8,000 ÷ 8 = 1,000, but 23,456 ÷ 12 > 12,000 ÷ 12 = 1,000, so 5,678 ÷ 8 < 23,456 ÷ 12. I used “÷” to mean “divided by,” “=” to mean “is the same number as,” “<” to mean “is less than,” and “>” to mean “is greater than.”
• I used all the same symbols except I didn’t need to use the less than symbol. I estimated 5,678 ÷ 8 is about 5,600 ÷ 8 = 700, while 23,456 ÷ 12 is about 24,000 ÷ 12 = 2,000, so I estimate that 23,456 ÷ 12 > 5,678 ÷ 8.

Redirecting students: Since students will not have done division by two-digit numbers, encourage them to estimate the quotients instead of directly calculating them.

2. Use short division instead of long division to divide 3-digit numbers by 1-digit numbers. (See Extension 1 from NBT4-44 Long Division—2-Digit by 1-Digit.)

3. The perimeter of a rhombus is 196 cm. How long is each side? (49 cm)

4. Find and correct the mistakes in these long division problems.

\[
\begin{array}{ccc}
\text{18} & \text{15} & \text{8} \\
\text{361} & \text{603} & \text{317} \\
\text{2} & \text{4} & \text{3} \\
\hline
\text{16} & \text{23} & \text{17} \\
\text{16} & \text{20} & \text{16} \\
\hline
\text{0} & \text{3} & \text{1} \\
\end{array}
\]

Answers: a) bring down the 1 and continue dividing; the answer is 180 R 1; b) bring down 0, not 3, beside the 2; the answer is 150 R 3; c) the 8 is the hundreds digit, and 0 should be written as the tens digit when dividing 4 into 1 ten; the answer is 804 R 1
NBT4-47 Mental Math

Page 19

STANDARDS
4.NBT.A.1, 4.NBT.B.6

VOCABULARY
fact family
hundreds
ones
tens

Goals
Students will divide 1-digit multiples of powers of ten (10, 100, 1,000, and so on) by the same multiple of a lesser power of ten, and divide using expanded form when all digits are divisible by the divisor.

PRIOR KNOWLEDGE REQUIRED
Can divide single place values
Understands the relationship between multiplication and division

NOTE: NBT4-48 Mental Math (Advanced) can be used as an extension for students who finish early while you help students who are struggling with this lesson. Do NBT4-48 with your whole class if it is advanced.

Review multiplying by 10, 100, and 1,000. Have a volunteer solve on the board:

\[ 30 \times 100 = \] __________

Point out that the answer has two more zeros than 30 does, because 100 has two zeros.

Exercises: Multiply.

a) \[ 400 \times 10 \]  
b) \[ 50 \times 1,000 \]  
c) \[ 3,000 \times 100 \]  
d) \[ 40 \times 100 \]  

Bonus: \[ 60 \times 100,000 \]

Answers: a) 4,000, b) 50,000, c) 300,000, d) 4,000, Bonus: 6,000,000

Finding the missing factor. Have students go in the other direction. Write on the board:

\[ 400 \times \underline{\hspace{2cm}} = 40,000 \]

ASK: How many zeros did I add to 400 to get 40,000? (2) What number must I be multiplying by if I added two zeros? (100) Write 100 in the blank.

Exercises: Write the missing number.

a) \[ 30 \times \underline{\hspace{2cm}} = 300 \]  
b) \[ 700 \times \underline{\hspace{2cm}} = 700,000 \]  
c) \[ 60 \times \underline{\hspace{2cm}} = 6,000 \]  
d) \[ 2,000 \times \underline{\hspace{2cm}} = 2,000,000 \]  

Bonus: \[ 300 \times \underline{\hspace{2cm}} = 30,000,000 \]

Answers: a) 10, b) 1,000, c) 100, d) 1,000, Bonus: 100,000

Review the relationship between multiplication and division. Write on the board: \[ 3 \times 4 = 12 \]. Then have a volunteer complete the fact family.

PROMPTS: What is another multiplication fact you can get from this one? (4 \times 3 = 12) What are two more division equations you can get? (12 \div 3 = 4, 12 \div 4 = 3)
Dividing mentally when the answer is 10, 100, or 1,000. Write on the board:

\[ 4,000 \div 40 = \] 100

SAY: This is asking 40 times what is 4,000? Then have a volunteer fill in the blank (100). Point out that you can subtract the number of zeros in the two numbers to see how many zeros were added to 40 to get 4,000. Since 3 − 1 = 2, two zeros were added.

**Exercises:** Divide mentally.

a) 300,000 ÷ 3,000  
   b) 80,000 ÷ 80  
   c) 700,000 ÷ 70,000

**Answers:** a) 100, b) 1,000, c) 10

Review using tens and ones blocks to divide. Remind students that they can use tens and ones blocks to divide. Demonstrate 69 ÷ 3:

So 69 ÷ 3 = 23.

**Exercises**

a) 84 ÷ 2  
   b) 64 ÷ 2  
   c) 48 ÷ 4

**Answers:** a) 42, b) 32, c) 12

Using expanded form to divide. Point out that when students divide using tens and ones blocks, they might as well be dividing the tens and ones separately into groups. Write on the board:

\[ 84 \div 2 = (80 \div 2) + (4 \div 2) \]

ASK: Why did I write brackets? (because the brackets tell you what to do first) SAY: We can add the number in each of these groups (point to the right-hand groups) to get the number in each of these groups (point to the left-hand groups). Write on the board: 68 = 60 + 8. ASK: How can I use expanded form to divide 68 ÷ 2? (add the results of 60 ÷ 2 and 8 ÷ 2)

Write on the board:

\[ 68 \div 2 = (60 \div 2) + (8 \div 2) \]

= 30 + 4

= 34

**Exercises:** Divide using expanded form.

a) 96 ÷ 3 = (90 ÷ 3) + (6 ÷ 3)  
   b) 48 ÷ 2 = (40 ÷ 2) + (8 ÷ 2)  
   c) 48 ÷ 4 = (40 ÷ 4) + (8 ÷ 4)  
   d) 63 ÷ 3 = (60 ÷ 3) + (3 ÷ 3)  
   e) 86 ÷ 2 = (___) + (___)  
   f) 64 ÷ 2 = (___) + (___)
Answers: a) 32, b) 24, c) 12, d) 21, e) 43, f) 32

Verifying the answer to division with multiplication. Remind students that they can check their answers using multiplication. For example, to check that $68 \div 2 = 34$, calculate $34 \times 2$; the product should be the first number in the division, in this case 68. Allow time for students to check their answers.

Using long division notation. Point out that by using expanded form, students are really just dividing each digit one at a time. Write on the board:

$$648 \div 2 = \_\_\_\_$$

ASK: What is 6 hundreds divided by 2? (3 hundreds) SAY: So we write 3 in the hundreds position. Demonstrate doing so. Repeat for 4 tens divided by 2 (2 tens) and 8 ones divided by 2 (4 ones). Then show how convenient the long division notation is for this:

$$\begin{array}{c|cc}
\text{324} \\
\hline
2
\end{array}$$

Point out that each digit is written directly above the digit that is being divided. Do exercise a) together, and remind students that $0 \div 4$ is 0.

Exercises: Divide mentally.  

<table>
<thead>
<tr>
<th>Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 480</td>
</tr>
<tr>
<td>b) 468</td>
</tr>
<tr>
<td>c) 390</td>
</tr>
<tr>
<td>d) 400</td>
</tr>
</tbody>
</table>

Answers: a) 20, b) 234, c) 3021, Bonus: 100,201

Extensions

1. Give each student 20 tens blocks and 9 ones blocks. Have students make the 3-digit number with tens and ones blocks and then divide those blocks into the correct number of groups.

   a) $124 \div 2$  b) $186 \div 3$  c) $186 \div 2$  d) $168 \div 4$
   e) $108 \div 2$  f) $120 \div 3$  g) $104 \div 2$

   Answers: a) 62, b) 62, c) 93, d) 42, e) 54, f) 40, g) 52

   Have students check their answers by multiplication.

2. Divide $480,000 \div 600$. Show how you solved the problem.

   Answer: I used $600 \times \_ = 480,000$. I know that $6 \times 8 = 48$, so $600 \times 8 = 4,800$, and $600 \times 800 = 480,000$, so $480,000 \div 600 = 800$.

   Whole-class follow-up: Have volunteers show how they solved the problem.