Grade 5  Problem-Solving Lessons

Introduction

What is a problem-solving lesson? A JUMP Math problem-solving lesson generally follows the format of a regular JUMP Math lesson, with some important differences:

• There are no AP Book pages that accompany the problem-solving lessons.
• Problem-solving lessons focus on one or more problem-solving strategies rather than focusing on meeting the Common Core State Standards (CCSS). These lessons apply the concepts learned through the standards—often crossing several domains, clusters, or standards—but they are not necessary to complete the standards. Regular lessons, on the other hand, focus on completing the standards and sometimes require problem-solving strategies to do so.
• While regular lessons expose students to all of the problem-solving strategies, the problem-solving lessons provide a way to isolate and focus on the strategies.
• Instead of including extensions, each problem-solving lesson includes an extensive Problem Bank. These questions give students a variety of opportunities to practice the problem-solving strategy from the lesson and to learn new math in the process. Students will need to have mastered the material in the problem-solving lesson (which they do by completing the exercises) in order to tackle the Problem Bank.
• Both the lesson plan and the Problem Bank questions apply the CCSS. All of the standards covered in the lesson are mentioned at the beginning of the lesson plan.
• Some problem-solving lessons include an opportunity for students to complete one or more Performance Tasks. Performance Tasks are multi-part problems intended to determine how well students can apply grade-level CCSS in a new context. While most questions in the task can be done independently of the problem-solving lesson, some questions provide an opportunity to specifically apply the problem-solving strategy. These questions might be challenging for students who have not been taught the problem-solving lesson. Performance Tasks can cover several domains, clusters, or standards at once, all from material covered to date, and so can often be used as a cumulative review.
• While regular lessons cover the standards completely, problem-solving lessons cover clusters of major standards according to the CCSS, and some also cover supporting and additional standards. These lessons provide more challenging independent work while still focusing on the standards.

How do I use problem-solving lessons? Ten problem-solving lessons are provided for Grade 5. The problem-solving lessons can be taught at any point in the grade after the unit indicated in the table on the next page. We recommend using as many problem-solving lessons throughout the year as your class time allows, and suggest using them in the order in which they are indicated below. However, if required, you can pick and choose based on a careful review of the prior knowledge required for each problem-solving lesson.

We recommend teaching more problem-solving lessons toward the end of the year rather than toward the beginning, as this allows time for students to consolidate their mathematical knowledge and gain confidence before attempting more challenging problems. For this reason,
we recommend using only four of the ten problem-solving lessons during Part 1 of Grade 5. Stronger classes that need fewer bridging lessons for review will have time to finish more of the problem-solving lessons. We recommend that classes needing most of the bridging lessons try at least a few problem-solving lessons.

Some of the Problem Banks include more problems than students can complete in one period. You might wish to use these as extension problems, or have students complete them as problems of the day throughout the year.

**Performance Tasks.** Performance Tasks are included at the end of some problem-solving lessons. Each Performance Task has at least one question that applies the problem-solving strategy covered in the lesson, but most questions can be done independently of the lesson. The Performance Tasks, together with any preparation, require a separate period each. Blackline Masters (BLMs) for each Performance Task are provided at the end of the lesson in which they are cited.

**Problem-solving strategies for Grade 5 and when to use them.** We consider the following problem-solving strategies as most important for this grade level:

- Recognizing and using structure
- Searching systematically
- Using a diagram
- Making a similar, but simpler, problem
- Using patterns
- Guessing, checking, and revising
PS5-9 Using Tape Diagrams

Teach this lesson after: 5.2 Unit 6

Standards: 5.NF.A.1, 5.NF.A.2, 5.NF.B.4a

Goals:
Students will use tape diagrams to solve multistep word problems involving all four operations and fractions.

Prior Knowledge Required:
Is familiar with tape diagrams
Can add and subtract fractions with like and unlike denominators

Materials:
BLM Swimming Pool (pp. Q-96–97, see Performance Task)

Vocabulary: tape diagram

(MP.1) Identifying parts of a diagram and solving a problem given the diagram. SAY: Marta had 35 dollars and spent 3/5 of her money on a shirt. We are going to represent this problem with a diagram. Draw a long rectangle divided into five equal parts. Ask a volunteer to shade the part of the diagram that represents the part that was spent on the shirt. (3 blocks) ASK: How much money does each block represent? ($7) How do you know? (there are 5 blocks and the total is $35, so one block is 35 ÷ 5 = $7) Finish the diagram as shown below:

\[ \text{\$35} \]

\[ \text{\$7} \]

shirt

ASK: How much money did Marta spend on the shirt? ($21) How do you know? (3 × 7 = $21) How much money does she have left? ($14) How do you know? (35 − 21 = $14; or 2 unshaded blocks, 2 × 7 = $14) Point out that, even though the first solution is correct, the second solution shows that you can find the leftover in the diagram without calculating how much money was spent on the shirt.

Exercises: Draw a diagram to solve the problem.

a) Jane had $36. She spent \( \frac{3}{4} \) of her money on a pair of shoes. How much money does she have left?

b) John spent \( \frac{2}{5} \) of his money on a toy. He has $15 left. How much did the toy cost?

c) Nancy spent \( \frac{2}{5} \) of her money on a poster that cost $8. How much money did she have before she bought the poster?
Solutions:

a)  $36

\[ \begin{array}{c}
\text{shoes} \\
\text{leftover} = \$9
\end{array} \]

b)  $15

\[ \begin{array}{c}
\text{toy} = \$10
\end{array} \]

c)  total before = $20

\[ \begin{array}{c}
\text{poster}
\end{array} \]

SAY: Now we are going to solve the same kind of problem but with decimals.

**Exercise:** Kate spent \( \frac{2}{5} \) of her money on a shirt and a hat. The shirt cost $18 and the hat cost $5.50. How much money did Kate have at first?

**Solution:** The shirt and hat together cost $23.50, so each block is $23.50 \div 2 = $11.75. Five blocks together is 5 \times $11.75 = $58.75.

\[ \begin{array}{c}
\text{shirt + hat} = \$23.50
\end{array} \]

**Solving problems where an amount is divided and then further divided into unequal parts.** Write on the board:

Cam has some eggs. He uses \( \frac{3}{7} \) of them to make pancakes and \( \frac{1}{2} \) of the remainder to make sandwiches. Now Cam has 6 eggs left. How many eggs did Cam use to make pancakes? How many eggs did Cam have at first?

Point out that the first two sentences are similar to the problem you just did, so to solve this problem we can start with the same type of diagram as earlier. Draw on the board:

\[ \begin{array}{c}
\text{pancakes}
\end{array} \]

ASK: Which part of the diagram shows the leftover? (the unshaded squares) Cover up the shaded part and ASK: Which part is half of the leftover? (2 squares) Mark that on the diagram, as shown below:

\[ \begin{array}{c}
\text{pancakes}
\end{array} \]

\[ \begin{array}{c}
\text{sandwiches}
\end{array} \]

ASK: How many eggs are left after he made the pancakes and sandwiches? (6) So how many eggs does each block represent? (3) How do you know? (6 ÷ 2 = 3) How many eggs did Cam
use for the pancakes? (9) How many eggs did Cam have initially? (21) How do you know?
(7 blocks, \(7 \times 3 = 21\))

Write on the board:

Raj had 30 stickers. He gave \(\frac{2}{5}\) of his stickers to his brother and \(\frac{1}{2}\) of the rest to his friend. How many stickers did Raj’s brother get? How many stickers are left?

\[
\text{total stickers} = 30
\]
\[
\text{brother}
\]

ASK: How many blocks are there in total? (5) How many stickers does each block represent? (6) How do you know? (\(30 \div 5 = 6\)) How many blocks did Raj’s brother get? (2) So how many stickers did Raj’s brother get? (12) SAY: Raj’s friend got half of the rest. Draw a dashed line to show Raj’s friend’s stickers, as shown below:

\[
\text{total stickers} = 30
\]
\[
\text{brother} \quad \text{friend}
\]

ASK: How many blocks did Raj’s friend get? (one and a half) How many stickers does each half block represent? (3) How do you know? (each block represents 6 so half represents 3) How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

Explain to students that it is sometimes easier to take away the first part of the question and then continue with the rest of the problem. SAY: At the beginning of the question, we know Raj has 30 stickers and he gave \(\frac{2}{5}\) of his stickers to his brother. Draw the original tape diagram on the board again:

\[
\text{total stickers} = 30
\]
\[
\text{brother}
\]

ASK: How many stickers does each block represent? (6) How many are left? (18) What fraction of the leftover goes to Raj’s friend? (half) Draw on the board:

\[
\text{stickers left} = 18
\]
\[
\text{friend}
\]
Point to the new tape diagram and ASK: How many stickers does each block represent? (9)
How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

**Exercises:** The next time Raj has stickers, he decides to give \(\frac{2}{5}\) of his 30 stickers to his brother and \(\frac{5}{6}\) of the remainder to his friend.

a) How many stickers did Raj’s friend get?
b) How many stickers are left?

**Solution:**

\[
\begin{align*}
\text{total stickers} & = 30 \\
\text{remainder} & = 18 \\
\text{brother} & = 12 \\
\text{friend} & = 15 \\
\text{left} & = 3
\end{align*}
\]

**Solving problems backward.** Write on the board:

Emma has some stickers. She colors \(\frac{1}{4}\) of them red and \(\frac{2}{5}\) of the remainder green.

If Emma doesn’t color 9 stickers, how many stickers does Emma have in total?

Explain to students that, like in the previous problem, they can draw tape diagrams, one for each step of the problem. Draw on the board:

\[
\begin{align*}
\text{total stickers} & = ? \\
\text{stickers left} & = ? \\
\text{red} & = ? \\
\text{stickers left} & = ? \\
\text{green} & = ? \\
\text{not colored} & = 9
\end{align*}
\]

SAY: In the diagram on the left, all parts are unknown. ASK: Can I start with the diagram on the left? (no) SAY: Look at the diagram on the right. ASK: How many stickers are not colored? (9) How many stickers does each block represent? (3) How do you know? (9 \(\div 3 = 3\)) How many blocks are green? (2 \(\times 3 = 6\)) Erase the question mark beside “green” and write “6” in its place, as shown below:

\[
\begin{align*}
\text{total stickers} & = ? \\
\text{stickers left} & = ? \\
\text{red} & = ? \\
\text{stickers left} & = ? \\
\text{green} & = 6 \\
\text{not colored} & = 9
\end{align*}
\]

Point to the diagram on the right and ASK: How many stickers are shown here in total? (9 \(\div 6 = 15\)}
So how many are left after Emma colors some red? (15) Erase the question mark beside “stickers left” in both diagrams and write “15” in its place, as shown below:

![Diagram](image)

Ask a volunteer to solve the diagram on the left and find the total numbers of stickers. (20)

**Exercises:**

1. Mike spent $\frac{3}{5}$ of his money on a book and $\frac{3}{4}$ of the remainder on some music. Mike has $4 after he paid for the book and the music. How much money did Mike have initially?

**Solution:**

<table>
<thead>
<tr>
<th>Initial money</th>
<th>Left after book</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40$</td>
<td>$16$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Book</th>
<th>Left after book</th>
<th>Music</th>
<th>Left after music</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24$</td>
<td>$16$</td>
<td>$12$</td>
<td>$4$</td>
</tr>
</tbody>
</table>

**Problem Bank**

1. Zara received some money for her birthday. She donated $\frac{1}{5}$ to charity, and she saved $\frac{2}{3}$ of the remainder in her savings. Of what was left, $\frac{1}{4}$ was a gift card to an ice-cream store. She used the rest of the money to buy 3 books for $6.99 each, a T-shirt for $5.99, and a basketball for $7.99. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each other part (donation, savings, gift card)?

**Answers:** the books are $6.99 each, the T-shirt is $5.99, the basketball is $7.99, so $34.95 for purchases plus $11.65 for the ice cream parlor gift card; $93.20 went to savings; $34.95 was given to charity; total birthday gift was $174.75; Zara had $139.80 after making the donation and $46.60 after putting money in her savings.

2. Kate reads 10 pages of a book on Saturday and she reads $\frac{3}{4}$ of the rest of the book on Sunday. She still has 17 pages to read. How many pages are in the book?

**Answer:** 78 pages
3. A convenience store has some ice cream treats. It sells \( \frac{2}{5} \) of them on Friday, \( \frac{1}{4} \) of the remainder of Saturday, and \( \frac{2}{3} \) of the rest on Sunday. The store has 30 ice cream treats left by the end of Sunday.

a) How many ice cream treats did the store have initially?

b) On which day did the store sell the most ice cream treats?

**Answers:**
a) 200; b) Friday, 80
Performance Task: Swimming Pool

Materials:
BLM Swimming Pool (pp. Q-96–97)

(MP.1) Performance Task: Swimming Pool. Give students BLM Swimming Pool. Tell students that they will be doing a performance task involving a swimming pool and volume. Answers: 1. 1,440 ft³; 2. 10,800 gallons; 3. 6.48 oz; 4. $0.50; 5. a) $12, b) $40, c) $16, d) 2 boxes
Swimming Pool (1)

Jay’s pool is in the shape of a rectangular prism. It is 24 feet long, 12 feet wide, and 5 feet deep.

1. Find the capacity of the pool in cubic feet.

2. Each cubic foot is about 7.5 gallons. About how many gallons of water are in the pool when it is full?

3. Jay disinfects the water in the pool every day by adding 0.0006 ounces of chlorine for each gallon of water in the pool. How many ounces of chlorine does Jay need to put in the pool when the pool is full?
Swimming Pool (2)

4. Each box of the chlorine weighs 1 pound and costs $8.00. How much does each ounce of the chlorine cost?

5. Jay spent \( \frac{2}{5} \) of his money at a pool store to buy some chlorine and half of the remainder on a pool toy. Jay has $12.00 after he paid for the chlorine and the pool toy.
   a) How much did Jay pay for the pool toy?

   b) How much money did Jay have when he went to the store?

   c) How much did Jay pay for the chlorine?