Goals
Students will find equivalent fractions using division and reduce fractions to lowest terms.

PRIOR KNOWLEDGE REQUIRED
Can divide a 2-digit number by a 1-digit number
Knows the relationship between multiplication and division

Drawing equivalent fractions. Draw several of the following diagrams on the board:

Have volunteers name the equivalent fractions shown by the pictures. (2/4, 3/6, 4/8, and 5/10)

Have another volunteer group all shaded parts together by drawing a thick line around the shaded parts. Then have another volunteer group the unshaded parts in the same way.

SAY: Now all of the drawings represent 1/2. ASK: Are 1/2 and 2/4 equivalent? (yes) How do you know? As an answer, write on the board:

\[
\frac{2}{4} = \frac{1}{2} \quad \frac{3}{6} = \frac{1}{2} \quad \frac{4}{8} = \frac{1}{2} \quad \frac{5}{10} = \frac{1}{2}
\]

Draw on the board:

SAY: If I group all shaded parts together and all unshaded parts together, then I’ll have two equal parts, one shaded and the other unshaded. So the fraction of the whole that is shaded is 1/2.

Point to the dotted line and SAY: Now I would like to erase the dotted line:

Have a volunteer erase the dotted line and then name the fraction. (2/4)

Emphasize that from a picture students can make different equivalent fractions, then write on the board:
Number and Operations—Fractions 5-12

Exercises: Imagine erasing the broken line. Write the equivalent fraction.

a) [Diagram of a circle divided into 8 equal parts, with 4 shaded]
   b) [Diagram of a circle divided into 8 equal parts, with 4 shaded]
   c) [Diagram of a circle divided into 8 equal parts, with all shaded]
   d) [Diagram of a circle divided into 6 equal parts, with 4 shaded]

Answer: a) \(\frac{2}{4} = \frac{1}{2}\), b) \(\frac{4}{8} = \frac{2}{4}\), c) \(\frac{4}{8} = \frac{1}{2}\), d) \(\frac{4}{6} = \frac{2}{3}\)

Using division to write an equivalent fraction. Draw the first picture below on the board:

\[
\frac{4}{6} = \frac{2}{3}
\]

ASK: How many parts do I have to group together to get 3 parts for the whole? (2) PROMPT: Six divided by what is three? (2 because \(6 \div 2 = 3\)) Group the parts and then show this relationship as in the second picture above. ASK: How many parts are shaded now? (2 of the total 3 parts are shaded now) Fill in the numerator (2).

Exercises: Divide to find the missing numerator.

a) \(\frac{3}{5} = \frac{?}{5}\)  
   b) \(\frac{12}{16} = \frac{3}{4}\)  
   c) \(\frac{10}{12} = \frac{5}{6}\)  
   d) \(\frac{90}{100} = \frac{9}{10}\)

Bonus: \(\frac{350}{1000} = \frac{35}{100}\)

Answers: a) 1, b) 3, c) 5, d) 9, Bonus: e) 35

Sample solution: a) \(\frac{3}{15} = \frac{3 \div 2}{15 \div 2} = \frac{1}{5}\)

Reducing a fraction to lowest terms. Remind students that they can make an equivalent fraction using bigger numbers by multiplying both the numerator and the denominator by the same number.

Write the fraction \(\frac{21}{35}\) on the board and ASK: If you have a fraction like 21/35 (point to it), how can you make an equivalent fraction with smaller numbers, like 3/5? (divide the numerator and denominator by the same number)

\[
\frac{21 \div 7}{35 \div 7} = \frac{3}{5}
\]

Exercises: Divide the numerator and denominator by the same number to make an equivalent fraction.

a) \(\frac{15}{20}\)  
   b) \(\frac{16}{20}\)  
   c) \(\frac{8}{12}\)  
   d) \(\frac{5}{30}\)

Answers: a) 3/4, b) 4/5, c) 2/3, d) 1/6
Point out that the number students are dividing by (7 in the example of $\frac{21}{35}$) is a factor of both the numerator and the denominator, so it is a common factor. If necessary, students can repeatedly divide the numerator and the denominator by a common factor until there is no common factor except 1. Demonstrate doing so to solve the first exercise below, and have students do the remaining ones.

**Exercises:** Divide the numerator and the denominator by a common factor until there is no common factor except 1.

a) $\frac{84}{126}$  

b) $\frac{300}{350}$  

c) $\frac{220}{330}$  

d) $\frac{84}{108}$

**Sample answers:**

a) $\frac{84}{126} = \frac{42}{63} = \frac{6}{9} = \frac{2}{3}$,  
b) $\frac{300}{350} = \frac{30}{35} = \frac{6}{7}$,  
c) $\frac{2}{3}$,  
d) $\frac{7}{9}$

When solving part a), point out how useful it is to know the times tables. If students can recognize 42 and 63 as both being in the 7 times table, then they can recognize 7 as a common factor.

When students finish, tell them that the process they just did results in the equivalent fraction that uses the smallest possible numbers, that is, the fraction in lowest terms. This process is called reducing the fraction to lowest terms.

**Extension**

You can reduce a fraction to lowest terms using the greatest common factor (GCF).

**Step 1:** Find the GCF of the numerator and the denominator. (See Extension 4 in NF5-11.)

**Step 2:** Divide the numerator and the denominator by the GCF.

Example: To reduce $\frac{24}{40}$ to lowest terms, first find the GCF of 24 and 40.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

So the GCF of 24 and 40 is 8. Divide the numerator and the denominator by 8.

\[
\frac{24}{40} \div 8 = \frac{3}{5}
\]

Reduce to lowest terms:

a) $\frac{6}{9}$  
b) $\frac{15}{20}$  
c) $\frac{12}{30}$  
d) $\frac{24}{60}$

**Answers:** a) $\frac{2}{3}$,  
b) $\frac{3}{4}$,  
c) $\frac{2}{5}$,  
d) $\frac{2}{5}$
Review adding and subtracting fractions. Write on the board:

\[
\begin{align*}
\frac{1}{5} + \frac{2}{5} &= \frac{3}{5} \\
\frac{3}{4} - \frac{1}{4} &= \frac{2}{4} \\
\frac{4}{7} + \frac{2}{7} &= \frac{6}{7} \\
\frac{5}{6} - \frac{1}{6} &= \frac{4}{6}
\end{align*}
\]

Have volunteers write the answers, reducing them to lowest terms. (3/5, 2/4 = 1/2, 6/7, 4/6 = 2/3) Point out that the denominators say what objects are being counted; pointing to each sum or difference in turn, SAY: fifths, fourths, sevenths, or sixths, so they are in the answer also. Point out that the numerators do the counting. SAY: 1 fifth plus 2 fifths is 3 fifths. So the numerators get added or subtracted, and the denominators stay the same.

**Exercises:** Add or subtract.

a) \(\frac{9}{14} + \frac{3}{14}\)  
b) \(\frac{3}{7} + \frac{3}{7}\)  
c) \(\frac{8}{9} - \frac{7}{9}\)  
d) \(\frac{17}{25} - \frac{11}{25}\)

**Answers:** a) 12/14 = 6/7, b) 6/7, c) 1/9, d) 6/25

Using what you know to add fractions with unlike denominators. After adding fractions with the same denominator, challenge students to find a way to add fractions with different denominators. Discuss the strategy of students changing the problem into one they already know how to solve. Emphasize that, if students know how to add fractions with the same denominator, and they can change fractions into equivalent ones with the same denominator, then they can add any pair of fractions. Remind them that comparing fractions with the same denominator was easier than comparing fractions with different denominators, but they still managed to find a way to compare fractions with different denominators.

First challenge students to find two fractions with the same denominator when one denominator is a multiple of the other. In this case students need to find an equivalent fraction for just one fraction.

**Exercises:** Multiply to find an equivalent fraction.

a) \(\frac{1}{3} = \frac{\_}{6}\)  
b) \(\frac{1}{4} = \frac{\_}{12}\)  
c) \(\frac{3}{5} = \frac{\_}{10}\)  
d) \(\frac{3}{4} = \frac{\_}{16}\)

**Answers:** a) 2/6, b) 3/12, c) 6/10, d) 12/16
After reviewing equivalent fractions, tell students that they can add or subtract fractions with different denominators by first changing one fraction to an equivalent fraction with the same denominator. Demonstrate with the example below.

Example: \( \frac{2}{5} \times \frac{1}{2} + \frac{7}{10} = \frac{2}{10} + \frac{7}{10} = \frac{9}{10} \)

**Exercises:** Add or subtract fractions by changing to equivalent fractions with the same denominator. Reduce to lowest terms.

a) \( \frac{1}{3} - \frac{1}{6} \)  

b) \( \frac{1}{4} + \frac{5}{12} \)  

c) \( \frac{3}{5} + \frac{1}{10} \)  

d) \( \frac{3}{4} - \frac{7}{16} \)

**Answers:** a) 1/6, b) 8/12 = 2/3, c) 7/10, d) 5/16

**Extensions**

1. Add or subtract and then reduce to lowest terms.

   a) \( \frac{1}{3} + \frac{1}{6} \)  

   b) \( \frac{3}{4} - \frac{1}{12} \)  

   c) \( \frac{3}{10} + \frac{1}{5} \)

   **Answers:** a) 3/6 = 1/2, b) 8/12 = 2/3, c) 5/10 = 1/2

2. Add by changing to equivalent fractions with the same denominator. Then reduce to lowest terms.

   a) \( \frac{1}{2} + \frac{1}{6} + \frac{2}{3} \)  

   b) \( \frac{1}{3} + \frac{5}{6} + \frac{1}{2} \)

   **Answers:** a) 8/6 = 4/3 = 1 1/3, b) 4/6 = 2/3

3. You can use fraction strips to show subtraction. For example, for the subtraction \( \frac{2}{5} - \frac{1}{10} = \frac{4}{10} - \frac{1}{10} = \frac{3}{10} \), you can use the following fraction strip model:

   Ask students to draw a model for each subtraction.

   a) \( \frac{1}{3} - \frac{1}{6} \)  

   b) \( \frac{3}{4} - \frac{1}{2} \)  

   c) \( \frac{3}{5} - \frac{5}{10} \)

   **Answers:** a) 1/6, b) 1/4, c) 1/10

4. John ran \( \frac{1}{2} \) mile on Tuesday, \( \frac{1}{3} \) mile on Wednesday, and \( \frac{5}{6} \) mile on Friday. How many miles did John run on these three days?

   **Answer:** \( \frac{10}{6} \) mi = \( \frac{5}{3} \) mi = \( \frac{12}{3} \) mi
**NF5-14 Adding and Subtracting Fractions II**

**Goals**
Students will add and subtract fractions with different denominators.

**PRIOR KNOWLEDGE REQUIRED**
- Can find equivalent fractions
- Can add and subtract fractions with the same denominator
- Can add and subtract fractions when one denominator is the multiple of the other denominator

**Using what you know to add fractions with unlike denominators.** Write on the board: \( \frac{1}{3} + \frac{2}{5} \)

**ASK:** How is this different from other fractions we’ve added? (the fractions have different denominators and we can’t multiply one denominator to become the same as the other) Tell students that you want to find a way to add fractions with different denominators. **ASK:** How have we done that already? (we have changed fractions to equivalent fractions to get fractions with the same denominator) Point out that by doing this, they changed a problem that they didn’t know how to do into one that they already knew how to do. Emphasize that if students know how to add fractions with the same denominator, and can change fractions into ones with the same denominator, then they can add any pair of fractions.

**Finding a common denominator.** Have students find two fractions with the same denominator, one equivalent to 1/3 and the other equivalent to 2/5. They can copy and complete the lists below until they find two fractions with the same denominator.

\[
\begin{align*}
\frac{1}{3} &= \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} \\
\frac{2}{5} &= \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25}
\end{align*}
\]

When students finish, fill in the lists and circle the two that have denominator 15. Explain that since

\[
\frac{1}{3} = \frac{5}{15} \quad \text{and} \quad \frac{2}{5} = \frac{6}{15}
\]

then

\[
\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}
\]

**Exercises:** Use this method to add the fractions.

a) \( \frac{2}{3} + \frac{1}{5} \)  

b) \( \frac{2}{3} + \frac{1}{4} \)  

c) \( \frac{3}{4} + \frac{1}{5} \)
Answers: a) $\frac{13}{15}$, b) $\frac{11}{12}$, c) $\frac{19}{20}$

Students should create lists of equivalent fractions for each fraction being added. They can then use their lists to subtract the same fractions as above.

Exercises: Subtract.

a) $\frac{2}{3} - \frac{1}{5}$  

b) $\frac{2}{3} - \frac{1}{4}$  

c) $\frac{3}{4} - \frac{1}{5}$

Answers: a) $\frac{7}{15}$, b) $\frac{5}{12}$, c) $\frac{11}{20}$

Reviewing the common denominator. Explain that by finding the first fraction in each list with the same denominator, students are finding the number that is a multiple of both denominators. ASK: When else have you used the common denominators of fractions? (when comparing fractions, we found two fractions with the same denominator) Review comparing fractions with different denominators. For example, remind students that to compare $\frac{1}{3}$ and $\frac{2}{5}$, first they need to find equivalent fractions with common denominators. SAY: We can multiply $3$ by $5$ to change $\frac{1}{3}$, and we can multiply $5$ by $3$ to change $\frac{2}{5}$:

\[
\frac{1 \times 5}{3 \times 5} = \frac{2 \times 3}{5 \times 3}
\]

SAY: Because $3 \times 5 = 5 \times 3$, the denominators are now the same, and $\frac{1}{3} < \frac{2}{5}$ because $\frac{5}{15} < \frac{6}{15}$. Emphasize that to find the common denominator, students can multiply the numerator and denominator of the first fraction by the denominator of the second and multiply the numerator and denominator of the second fraction by the denominator of the first, so to find a common denominator students can multiply both denominators together.

Adding and subtracting fractions by finding a common denominator. Write on the board:

\[
\frac{2}{5} + \frac{3}{4}
\]

ASK: What is a common denominator of the fractions? (20) How do you know? (it is $5 \times 4$) Point out that we have to multiply 5 and 2 by 4 and 4 and 3 by 5 to change both fractions into fractions with the denominator 20.

\[
\frac{4 \times 2}{4 \times 5} + \frac{3 \times 5}{4 \times 5} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20} = 1 \frac{3}{20}
\]

Demonstrate again by adding the fractions $\frac{1}{6} + \frac{3}{8}$. Have a volunteer find a common denominator of the fractions. Remind students that a common denominator of the fractions can be found by multiplying the two denominators.
Extensions

1. a) Subtract.
   
   i) \( \frac{2}{3} - \frac{1}{2} \)  
   ii) \( \frac{3}{4} - \frac{2}{3} \)  
   iii) \( \frac{4}{5} - \frac{3}{4} \)  
   iv) \( \frac{5}{6} - \frac{4}{5} \)

   b) Find the next two subtractions in the pattern in part a). Then predict and check your answers.

   c) Predict: \( \frac{99}{100} - \frac{98}{99} \). Explain using math words.

   d) Make up a subtraction problem that has the answer \( \frac{1}{999,000} \).

   **Answers:**
   a) i) \( \frac{1}{6} \), ii) \( \frac{1}{12} \), iii) \( \frac{1}{20} \), iv) \( \frac{1}{30} \), b) \( \frac{6}{7} - \frac{5}{6} = \frac{36}{42} - \frac{35}{42} = \frac{1}{42} \);  
   c) \( \frac{7}{8} - \frac{6}{7} = \frac{49}{56} - \frac{48}{56} = \frac{1}{56} \), c) The pattern in the answers is to have 1 as the numerator and the common denominator equal to the product of the two denominators. So the answer is \( \frac{1}{9,900} \). d) Use the pattern from part a). The question \( \frac{999}{1000} - \frac{998}{999} \) has the answer \( \frac{1}{999,000} \).

2. A team won \( \frac{2}{3} \) of its games and lost \( \frac{1}{5} \) of its games. What fraction of the games did the team tie? Explain how you know.

   **Solution:** The fraction of games the team either won or lost is \( \frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15} \). The fraction of games that were won, lost, or tied is 1, because this represents all the games. We can write 1 as \( \frac{15}{15} \), and \( \frac{15}{15} - \frac{13}{15} = \frac{2}{15} \). So the team tied \( \frac{2}{15} \) games.

3. Find the missing number.

   a) \( \frac{1}{3} + \frac{x}{5} = \frac{11}{15} \)  
   b) \( \frac{y}{4} + \frac{3}{5} = \frac{17}{20} \)  

   **Bonus:** \( \frac{3}{4} + \frac{6}{6} = \frac{11}{12} \)

   **Selected sample solution:** a) \( \frac{1}{3} = \frac{5}{15} \). \( \frac{5}{15} + \frac{6}{15} = \frac{11}{15} \).  
   b) \( \frac{6}{15} = \frac{2}{5} \), so the missing number is 2.

   **Answers:** b) 1, Bonus: 1

Individual or small-group follow-up: If students struggle, ASK: What makes this problem hard? (the denominators are all different) What could I do first to make the problem easier? (find a common denominator) For part a), once students have found that the known fraction is \( \frac{5}{15} \) and the missing fraction is \( \frac{6}{15} \), ASK: Is 6 the answer to the question? (no) What else do you need to do? (I need to find the numerator when the denominator is 5).
**Goals**

Students will identify multiples of a number, common multiples of two numbers, and the lowest common multiple (LCM) of two numbers (up to 12).

**PRIOR KNOWLEDGE REQUIRED**

- Can skip count by 2, 3, 4, or 5
- Can multiply 1-digit numbers
- Can find equivalent fractions
- Knows the times table
- Can reduce fractions to lowest terms

**Review whole numbers.** Remind students that whole numbers are the numbers 0, 1, 2, 3, and so on. Write “whole number” on the board, and contrast whole numbers with fractions, which represent parts of a whole.

**Introduce multiples.** The *multiples* of a whole number are the numbers you get by multiplying the number by another whole number. For example, 6 is a multiple of 2 because $2 \times 3 = 6$.

Write “multiple” and “multiply” on the board. Point out how similar the words are—only the last letter of each is different, “e” versus “y.” Emphasize that this makes it easy to remember what a multiple is: You get the *multiples* of a number by *multiplying* that number by whole numbers.

**Skip counting to find multiples.** List the first five multiples of 4, including 0, on the board as shown. Have students continue the list by writing the next five multiples of 4. ($4 \times 5 = 20$, $4 \times 6 = 24$, $4 \times 7 = 28$, $4 \times 8 = 32$, $4 \times 9 = 36$)

**ASK:** Is 26 a multiple of 4? (no) How can you tell? (it is not on the list; it is between 24 and 28, which are right next to each other) Point out that we can list all the multiples of 4 by skip counting: 0, 4, 8, ...

**Exercises:** Skip count to decide if each number is a multiple of 3.

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a) 8</td>
<td>b) 15</td>
</tr>
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</table>

**Answers:** a) no, b) yes, c) yes, d) no

**Zero is a multiple of any number.** Have students write a multiplication equation that proves that 0 is a multiple of 3. ($3 \times 0 = 0$) **ASK:** Which whole numbers is 0 a multiple of? (all of them, because any whole number can be multiplied by 0 to make 0)

**The multiples of 0 and 1.** **ASK:** What are the multiples of 0? (only 0 is a multiple of 0) Demonstrate this by skip counting by 0s from 0: 0, 0, 0, .... **ASK:** What are the multiples of 1? (every whole number is a multiple of 1) **PROMPT:** What numbers do we get when we skip count by 1’s? (all whole numbers)
Common multiples. Draw two number lines to 12 on the board, one for the multiples of 2 and the other for the multiples of 3. Mark the multiples of 2 on the first number line with Xs. Point out that you start at 0 and mark every second number. Ask a volunteer to mark the multiples of 3, starting at 0 and marking every third number. The number lines on the board should now look like this:

2: \[ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \]
3: \[ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \]

ASK: Which numbers are multiples of both 2 and 3? (0, 6, and 12)

Exercises: Draw two number lines on grid paper …
a) to 12 to find numbers that are multiples of 2 and 5
b) to 12 to find numbers that are multiples of 3 and 6
c) to 16 to find numbers that are multiples of 3 and 5

Tell students that a number that is a multiple of two numbers is called a common multiple of the two numbers. Ask volunteers to say some common multiples of 2 and 3. (Examples: 0, 6, 12) Challenge students to name more common multiples of 2 and 3. (18, 24, and so on) ASK: Does anyone know the phrase “in common”? What does it mean if I say two people have something in common? (the two people share a quality, characteristic, like, or dislike) Have students pair up and find something they have in common with each other. (example: we both like reading) In their pairs, students should take turns saying something about themselves until they find something in common. Relate this to the term “common multiple”: the numbers 2 and 3 both have 12 as a multiple, so that is something the numbers 2 and 3 have in common. That is why 12 is called a common multiple of 2 and 3.

Introduce lowest common multiples. Remind students that 0 is a multiple of every number. Because of that, it is useless as a multiple and cannot give us extra information. When we want to list multiples, we are only interested in the multiples that are not 0. Tell students that the smallest common multiple of two numbers not including 0 is called the lowest common multiple of the numbers. SAY: Some people call the lowest common multiple the “least common multiple.”

Exercise: Find the lowest common multiple. Write your answer in sentence form. Example: The lowest common multiple of 6 and 10 is 30.

a) 2 and 3  
   b) 2 and 5  
   c) 3 and 6  
   d) 3 and 5  
   e) 4 and 6  
   f) 6 and 8

Answers: a) 6, b) 10, c) 6, d) 15, e) 12, f) 24

Tell students that the term “lowest common multiple” is often written as \( LCM \). Write on the board: lowest common multiple.
Introduce a shortcut for finding lowest common multiples. Have students list the multiples of 3 up to $3 \times 10$ (not including 0):

$3, 6, 9, 12, 15, 18, 21, 24, 27, 30$

Point out that you can use this list to find the lowest common multiple of 3 and 4 by finding the first multiple of 4 that is on the list. (12)

**Exercises:** Use the list of multiples of 3 to find the LCM of …

a) 2 and 3  

b) 3 and 5  

c) 3 and 8  

d) 3 and 6  

e) 3 and 7

**Answers:** a) 6, b) 15, c) 24, d) 6, e) 21

Listing multiples to use the shortcut. Students will learn that is easier to list the multiples of the larger number to use the shortcut. Tell students that you want to find the common multiples of 3 and 8. List the multiples of 3, and have a volunteer name the first multiple of 8 they see in the list. (24)

Then list the multiples of 8 and have a volunteer name the first multiple of 3 they see in the list. (24) **ASK:** Did you get the same answer both ways? (yes) **SAY:** It doesn’t matter which list you make—the multiples of 3 or the multiples of 8—because you will get the same answer either way. So you might as well make the list that requires less work. **ASK:** Which requires less work: listing the multiples of 3 until you find a multiple of 8, or listing the multiples of 8 until you find a multiple of 3? (listing the multiples of 8)

Discuss why that is: 8 is more than 3, so it takes fewer multiples of 8 than multiples of 3 to get to the same number. (24)

**Exercises:** List multiples of the larger number (not including 0) until you find a multiple of the smaller number, to find the LCM of the two numbers.

a) 3 and 7  

b) 4 and 8  

c) 4 and 10  

d) 5 and 9  

e) 3 and 10

**Answers:** a) 7, 14, 21; b) 8; c) 10, 20; d) 9, 18, 27, 36, 45; e) 10, 20, 30

Using the LCM to add fractions with unlike denominators. Write the two problems on the board:

$$\frac{1}{3} + \frac{2}{5} = \quad \frac{1}{5} + \frac{2}{5} =$$

Discuss how they are different (the first has unlike denominators; the second has the same denominators). Then **SAY:** One fifth plus two fifths is three fifths, but one third plus two fifths isn’t three of anything, because thirds are not the same part of the whole as fifths.

Remind students that comparing fractions with the same denominator was easier than comparing fractions with different denominators, but they still managed to do so by changing them to equivalent fractions with the same denominator. Emphasize that if students know how to add fractions with the same denominator and can change fractions into ones with the same denominator, then they can add any pair of fractions.

Remind students that they can use the lowest common multiple (LCM) of the denominators to make fractions with the same denominator. **ASK:** What
is the lowest common multiple of 3 and 5? (15) Demonstrate how to use the LCM to add fractions.

\[
\frac{1}{3} = \frac{5}{15} \quad \text{and} \quad \frac{2}{5} = \frac{6}{15}.
\]

So

\[
\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}.
\]

**Introduce the lowest common denominator (LCD).** 
ASK: When else have you used the lowest common multiple of the denominators of fractions? (when comparing fractions, we found two fractions with the same denominator). Tell students that, because the lowest common multiple of the denominators is so useful, we have a special name for it: lowest common denominator, or LCD.

**Exercises:** Find the LCD to add or subtract.

a) \(\frac{1}{3} + \frac{3}{4}\)  
b) \(\frac{2}{3} - \frac{1}{4}\)  
c) \(\frac{2}{5} + \frac{1}{2}\)  
d) \(\frac{3}{5} - \frac{1}{2}\)

In the following problems, students cannot just multiply the denominators to find the lowest common denominator. Do the first one together.

**Exercises:** Find the LCD to add or subtract.

e) \(\frac{3}{8} - \frac{1}{6}\)  
f) \(\frac{1}{6} + \frac{3}{4}\)  
g) \(\frac{3}{10} - \frac{4}{15}\)  
h) \(\frac{2}{30} + \frac{1}{40}\)

Now, in these problems, students will need to change only one denominator. Again, do the first one together.

**Exercises:** Find the LCD to add or subtract.

i) \(\frac{3}{8} + \frac{1}{4}\)  
j) \(\frac{2}{5} + \frac{7}{15}\)  
k) \(\frac{2}{3} - \frac{4}{9}\)  
l) \(\frac{3}{5} - \frac{1}{20}\)

**Answers:**

a) \(\frac{13}{12}\),  
b) \(\frac{5}{12}\),  
c) \(\frac{9}{10}\),  
d) \(\frac{1}{10}\),  
e) \(\frac{5}{24}\),  
f) \(\frac{11}{12}\),  
g) \(\frac{1}{30}\),  
h) \(\frac{11}{120}\),

i) \(\frac{5}{8}\),  
j) \(\frac{13}{15}\),  
k) \(\frac{2}{9}\),  
l) \(\frac{11}{20}\)

**Sample solutions:** e) LCD is 24; \(\frac{3}{8} = \frac{9}{24}\), \(\frac{1}{6} = \frac{4}{24}\); \(\frac{9}{24} - \frac{4}{24} = \frac{5}{24}\)

i) LCD is 8; \(\frac{1}{4} = \frac{2}{8}\); \(\frac{3}{8} + \frac{2}{8} = \frac{5}{8}\)

**ACTIVITY**

Each student will need two dice of one color and two dice of another color. (Alternatively, students can roll the same pair of dice twice.) The student rolls all four dice, adds the results of the dice of the same color, and finds the lowest common multiple of the resulting numbers. For example, if a student rolls 3 and 6 with red dice and 2 and 4 with blue dice, the student has to find the lowest common multiple of 9 \((= 3 + 6)\) and 6 \((= 2 + 4)\).
Extensions

1. Explore the patterns in the ones digits of the multiples of …
   a) 2 and 8  
   b) 3 and 7  
   c) 4 and 6

What do you notice? In particular, if you know the pattern in the ones
digits for multiples of 2, how can you get the pattern in the ones
digits for multiples of 8?

**Answers:**
   a) The pattern for 2 is 0, 2, 4, 6, 8, repeat. The pattern for
      8 is 0, 8, 6, 4, 2, repeat.  
   b) The pattern for 3 is 0, 3, 6, 9, 2, 5, 8, 1, 4, 7, repeat. The pattern for 7 is 0, 7, 4, 1, 8, 5, 2, 9, 6, 3, repeat.  
   c) The pattern for 4 is 0, 4, 8, 2, 6, repeat. The pattern for 6 is 0, 6, 2, 8, 4, repeat.

In each case, after 0, the patterns are the reverse of each other. For
example, if you read 2, 4, 6, 8 (the pattern for 2) in reverse order, you
get 8, 6, 4, 2 (the pattern for 8).

To understand the reason for the overall pattern, first note that each pair
of numbers adds to 10. Then look at the case of 2 and 8. We get
the pattern for 2 by adding 2, so to reverse it, we can subtract 2. But adding
8 to a number gives the same ones digit as subtracting 2 from the same
number, because the results are 10 apart. For example, $15 + 8 = 23$
has ones digit 3, and $15 - 2 = 13$ also has ones digit 3.

2. Would it be helpful to reduce the fractions to lowest terms before solving
   the problem in the margin? Explain.

**Answer:** no; it’s easier to add fractions with the same denominator,
so leave them as is

3. a) What fraction is shaded? Add the fractions from parts i) and ii) to
   solve part iii).

   i)  

   ii)  

   iii)

   b) Find a way to answer part iii) without adding the fractions.

   **Sample answer**
   b) I split the rectangle into two parts so that both parts had 1/4 shaded.

   If each part has 1/4 shaded, the distributive property tells us that 1/4 of
one part and 1/4 of another part is equal to 1/4 of the whole.

   **Answers:** a) i) 1/12, ii) 1/6, iii) $1/12 + 1/6 = 1/4$
Goals
Students will add and subtract mixed numbers using the LCD of the denominators, and reduce answers to lowest terms using flexibility with equivalent fractions.

PRIOR KNOWLEDGE REQUIRED
Can name fractions, including mixed numbers and improper fractions
Can find equivalent fractions
Can find the lowest common denominator
Can reduce a fraction to lowest terms

Review adding whole numbers and mixed numbers. Write on the board:

\[
\begin{array}{c}
2 \quad + \\
& \quad 3 \ 
\end{array}
\]

Have a volunteer tell you the answer. (5 3/4) Point out that when you add a whole number to a mixed number, you add the whole numbers, then write the fraction part of the mixed number.

Exercises: Add:

a) \(3 + 4 \frac{1}{2}\)  
\(b) 3 + 5 \frac{1}{3}\)  
\(c) 2 + 1 \frac{3}{4}\)  
\(d) 4 + 2 \frac{5}{8}\)

Answers: a) \(7 \frac{1}{2}\), b) \(8 \frac{1}{3}\), c) \(3 \frac{3}{4}\), d) \(6 \frac{5}{8}\)

Adding mixed numbers with the same denominator, but no regrouping. Draw on the board:

\[
\begin{array}{c}
\frac{1}{6} \quad + \\
& \quad \frac{2}{6}
\end{array}
\]

Have a volunteer tell you the answer. (3 2/6) Point out that when you add two mixed numbers, you can add the wholes and the parts separately. Write on the board:

\[
\begin{array}{c}
1 + 2 = 3 \\
1 \frac{1}{6} + 2 \frac{1}{6} = \frac{3}{6} \\
\frac{1}{6} + \frac{1}{6} = \frac{2}{6}
\end{array}
\]
Comparing with adding tens and ones. Point out that adding the wholes and the parts separately is similar to adding they’ve done many times before. Write on the board:

\[
\begin{align*}
32 & \quad 30 + 2 \\
+ 46 & \quad 40 + 6 \\
\hline
78 & = 70 + 8
\end{align*}
\]

Point out that, to add 32 and 46, we add the tens and ones separately. We are doing a similar thing when we add the wholes and the parts separately.

\[
\begin{align*}
\frac{3}{11} + \frac{2}{11} & \quad 3 + \frac{2}{11} \\
+ \frac{4}{11} + \frac{6}{11} & \quad 4 + \frac{6}{11} \\
\hline
\frac{7}{11} & = \frac{7 + 8}{11}
\end{align*}
\]

Exercises: Add:

a) \(3 \frac{2}{5} + 4 \frac{1}{5}\)  

b) \(1 \frac{3}{8} + 4 \frac{2}{8}\) 

c) \(1 \frac{3}{7} + 2 \frac{3}{7}\) 

d) \(5 \frac{3}{10} + 3 \frac{4}{10}\)

Answers: a) \(7 \frac{3}{5}\)  

b) \(5 \frac{5}{8}\) 

c) \(3 \frac{6}{7}\) 

d) \(8 \frac{7}{10}\)

Adding mixed numbers with the same denominator, with regrouping. Write on the board:

\[
\begin{align*}
\frac{3}{8} & \quad 3 + \frac{6}{8} \\
+ 2 \frac{7}{8} & \quad 2 + \frac{7}{8} \\
\hline
\quad & = \quad \frac{5 + 13}{8} = \quad \frac{5 + 5}{8}
\end{align*}
\]

ASK: What is \(3 + 2\)? (5) What is \(6/8 + 7/8\)? (13/8) Write on the board: \(5 \frac{13}{8}\)

ASK: Do we ever write fractions this way? (no; it combines a whole number and an improper fraction) What is different from how we normally write fractions? (the fraction part is greater than 1) What is 13/8 as a mixed number? (1 5/8) Write on the board:

\[
\begin{align*}
\frac{3}{8} & \quad 3 + \frac{6}{8} \\
+ 2 \frac{7}{8} & \quad 2 + \frac{7}{8} \\
\hline
\quad & = \quad \frac{5 + 13}{8} = \quad \frac{5 + 5}{8}
\end{align*}
\]

ASK: What is \(5 + 1\ 5/8\)? (6 5/8) Tell students that this process is called regrouping. We needed to regroup because 6/8 + 7/8 is greater than one whole—in other words, it is more than 8 eighths. Compare this to adding tens and ones:
Point out that 6 and 7 add to more than 10, so we need to regroup. ASK: How do we do that? (We write 50 + 13 as 60 + 3.) SAY: We add 1 to the number of tens and then write the leftover number of ones:

\[
\begin{array}{c}
1\\
36\\
+ 27\\
\hline
63
\end{array}
\]

Have students tell you whether you need to regroup for each addition problem. Students can signal thumbs up for “yes” and thumbs down for “no.” After each question, ASK: How do you know? PROMPT: Do the ones add to 10 or more?

a) 45 + 32  
b) 67 + 21  
c) 46 + 76  
d) 83 + 69  
e) 45 + 74

Answers: a) no, b) no, c) yes, d) yes, e) no

Now write the fraction addition shown in the margin, on the board.

ASK: Do you need to regroup? (yes; 11/11 is 1, so we can add 1 to 5 to make 6) Provide students with pairs of proper fractions to add and have them tell you whether or not they need to regroup. Students can signal their answer with thumbs up for “yes” and thumbs down for “no.” Emphasize that the numerators of the fraction parts have to add to more than the denominator. For example, for fractions with denominator 8, ASK: How many eighths make one whole? How can you tell if the fraction parts add to one whole or more? (the numerators add to 8 or more)

Exercises: Do you need to regroup?

a) \( \frac{5}{7} + \frac{3}{7} \)  
b) \( \frac{3}{5} + \frac{2}{5} \)  
c) \( \frac{3}{5} + \frac{1}{5} \)  
d) \( \frac{3}{8} + \frac{5}{8} \)  
e) \( 2\frac{3}{8} + 3\frac{4}{8} \)

Answers: a) yes, b) yes, c) no, d) yes, e) no, f) yes

Exercises: Now have students add the fractions above, writing their answer as a mixed number when the answer is 1 or more.

Answers: a) 1 1/7, b) 1, c) 4/5, d) 1, e) 5 7/8

Exercises: Add the fractions. Regroup when necessary.

a) \( 2\frac{1}{5} + 3\frac{2}{5} \)  
b) \( 4\frac{5}{7} + 2\frac{3}{7} \)  
c) \( 1\frac{1}{3} + 4\frac{2}{3} \)

Answers: a) 5 3/5, b) 6 8/7 = 7 1/7, c) 5 3/3 = 6

Subtracting mixed numbers without regrouping. Write on the board:

\[
\begin{array}{c}
\frac{4}{5} - \frac{3}{5}
\end{array}
\]
Have a volunteer circle the part of the picture that shows 1 3/5. ASK: How much is left over? (3) Point out that all the parts in the fraction part of 4 3/5 were taken away, so they only need to subtract the whole numbers to find the answer. Exercises: Subtract.

\[
\begin{align*}
\text{a)} & \, \quad \frac{7}{5} - \frac{6}{5} \\
\text{b)} & \, \quad \frac{8}{4} - \frac{5}{4} \\
\text{c)} & \, \quad \frac{5}{2} - \frac{4}{2}
\end{align*}
\]

Answers: a) 1, b) 3, c) 1

Now write on the board:

\[
\frac{4}{5} - \frac{1}{5}
\]

Point out that now we are not taking away all the parts, but just some of them. ASK: How many wholes are we taking away? (1) Cross out one whole circle in the picture. ASK: How many fifths are we taking away? (2) Cross out two parts in the picture.

ASK: How many whole circles are left? (3) How many parts are left? (1) Write the answer on the board:

\[
\frac{4}{5} - \frac{1}{5} = \frac{3}{5}
\]

Point out that you are subtracting the wholes separately: 4 - 1 = 3, and the parts separately: 3/5 – 2/5 = 1/5.

Exercises: Subtract the wholes and the parts separately.

\[
\begin{align*}
\text{a)} & \, \quad \frac{3}{7} - \frac{2}{7} \\
\text{b)} & \, \quad \frac{5}{9} - \frac{2}{9} \\
\text{c)} & \, \quad \frac{5}{8} - \frac{4}{8} & \text{Bonus} & \, \quad \frac{180}{5} - \frac{176}{5} \\
\end{align*}
\]

Answers: a) 2 3/7, b) 3 2/9, c) 1 2/8 = 1 1/4, Bonus: 4

**Subtracting mixed numbers with regrouping.** Tell students that you want to subtract 9 1/4 – 5 3/4. ASK: What makes this problem harder than other problems we’ve solved so far? (we can’t subtract the parts easily because we can’t take 3/4 away from 1/4) SAY: I wonder if there is an easier problem that we can get ideas from. PROMPT: Have we ever run into a similar problem when subtracting tens and ones? Ask a volunteer to write such a problem on the board (Example: 82 – 35). ASK: How did we solve this problem? (we wrote 82 as 7 tens + 12 ones instead of 8 tens + 2 ones; that gave us enough ones to subtract 5 ones) Demonstrate the subtraction on the board.

SAY: I wonder how we can use this reasoning to subtract the fractions. Write on the board: 9 1/4 = 8 + 1 1/4 = 8 + 5/4. ASK: Can we subtract 3/4 from 5/4? (yes) SAY: Now we can subtract the wholes and parts separately. Demonstrate on the board:

\[
9 \frac{1}{4} - 5 \frac{3}{4} = 8 \frac{5}{4} - 5 \frac{3}{4} = 3 \frac{2}{4} = 3 \frac{1}{2}
\]
SAY: Just as we can add 10 to the ones in a number to make sure we have enough ones to subtract from, we can add 1 to the fraction part of a mixed number to make sure we have enough in the fraction part to subtract from. Point to the 3/4 in 5 3/4 and SAY: This number is less than 1 because it's the fraction part of a mixed number. Point to the 1/4 in 9 1/4 and SAY: If we regroup to make this more than 1, we can be sure to have enough to subtract from. **Exercises:**

1. Regroup to make the fraction part more than 1.
   a) $6\ 3/5 = 5 + 1\ 3/5 = b) 5\ 4/9 = 4 + 1\ 4/9 =$
   c) $3\ 1/7 = 2 + 1\ 1/7 = d) 7\ 5/8$
   e) $2\ 6/11$ **Bonus:** $1\ 2/5$

2. Regroup the first number and then subtract.
   a) $3\ 2/5 - 1\ 3/5$ b) $5\ 1/3 - 2\ 2/3$ c) $6\ 1/8 - 2\ 5/8$

**Answers:** 1. a) 5 8/5, b) 4 13/9, c) 2 8/7, d) 6 13/8, e) 1 17/11, Bonus: 7/5; 2. a) 2 7/5 – 1 3/5 = 1 4/5, b) 2 2/3, c) 3 4/8

**Using the LCD to add and subtract.** For some mixed numbers, the fraction parts must be changed so that they have a denominator equal to the LCD of the fractions. Some problems will require regrouping. Do the first one with students. **Exercises:** Subtract.

a) $5\ 3/4 - 2\ 1/3$ b) $5\ 3/7 + 3\ 2/21$ c) $3\ 1/4 - 1\ 5/8$

d) $4\ 3/5 - 2\ 2/3$ e) $3\ 4/9 + 1\ 5/7$ **Bonus:** $7\ 4/9 - 2\ 5/7$

**Answers:** a) 5 3/4 – 2 1/3 = 5 9/12 – 2 4/12 = 3 5/12, b) 8 11/21, c) 1 5/8, d) 1 14/15, e) 5 10/63, Bonus: 4 46/63

**Extensions**

1. Subtract mixed numbers by adding parts and whole portions of the difference on a number line. Example: $9\ 1/4 - 5\ 3/4$.

![Number line diagram](image)

**Sample solution:** Ron sold 4 3/8 pies ($5 - 1\ 5/8 = 4\ 3/8$). Multiply to find the number of pieces in the 4 whole pies sold: $8 \times 4 = 32$ pieces. He also sold 3 pieces of the fifth pie, which means he sold 35 pieces in total $(32 + 3 = 35)$. Since Ron sold each piece for $3, he made $35 \times 3 = $105.