Grade 6 Problem-Solving Lessons

Introduction

What is a problem-solving lesson? A JUMP Math problem-solving lesson generally follows the format of a regular JUMP Math lesson, with some important differences:

• There are no AP Book pages that accompany the problem-solving lessons.
• Problem-solving lessons focus on one or more problem-solving strategies rather than focusing on meeting the Common Core State Standards (CCSS). These lessons apply the concepts learned through the standards—often crossing several domains, clusters, or standards—but they are not necessary to complete the standards. Regular lessons, on the other hand, focus on completing the standards and sometimes require problem-solving strategies to do so.
• While regular lessons expose students to all of the problem-solving strategies, the problem-solving lessons provide a way to isolate and focus on the strategies.
• Instead of including extensions, each problem-solving lesson includes an extensive Problem Bank. These questions give students a variety of opportunities to practice the problem-solving strategy from the lesson and to learn new math in the process. Students will need to have mastered the material in the problem-solving lesson (which they do by completing the exercises) in order to tackle the Problem Bank.
• Both the lesson plan and the Problem Bank questions apply the CCSS. All of the standards covered in the lesson are mentioned at the beginning of the lesson plan.
• Some problem-solving lessons include an opportunity for students to complete one or more Performance Tasks. Performance Tasks are multi-part problems intended to determine how well students can apply grade-level CCSS in a new context. While most questions in the task can be done independently of the problem-solving lesson, some questions provide an opportunity to specifically apply the problem-solving strategy. These questions might be challenging for students who have not been taught the problem-solving lesson. Performance Tasks can cover several domains, clusters, or standards at once, all from material covered to date, and so can often be used as a cumulative review.
• While regular lessons cover the standards completely, problem-solving lessons cover clusters of major standards according to the CCSS, and some also cover supporting and additional standards. These lessons provide more challenging independent work while still focusing on the standards.

How do I use problem-solving lessons? Nine problem-solving lessons are provided for Grade 6. The problem-solving lessons can be taught at any point in the grade after the unit indicated in the table that follows. We recommend using as many problem-solving lessons throughout the year as your class time allows, and suggest using them in the order in which they are indicated in the table. However, if required, you can pick and choose based on a careful review of the prior knowledge required for each problem-solving lesson.

We recommend teaching more problem-solving lessons toward the end of the year rather than toward the beginning, as this allows time for students to consolidate their mathematical knowledge and gain confidence before attempting more challenging problems. For this reason,
we recommend using only two of the nine problem-solving lessons during Part 1 of Grade 6. Stronger classes that need fewer bridging lessons for review will have time to finish more of the problem-solving lessons. We recommend that classes needing most of the bridging lessons try at least a few problem-solving lessons.

Some of the Problem Banks include more problems than students can complete in one period. You might wish to use these as extension problems, or have students complete them as problems of the day throughout the year.

Performance Tasks. Performance Tasks are included at the end of some problem-solving lessons. Each Performance Task has at least one question that applies the problem-solving strategy covered in the lesson, but most questions can be done independently of the lesson. The Performance Tasks, together with any preparation, require a separate period each. Blackline Masters (BLMs) for each Performance Task are provided at the end of the lesson in which they are cited.

Problem-solving strategies for Grade 6 and when to use them. We consider the following problem-solving strategies as most important for this grade level:
• Recognizing and using structure
• Searching systematically
• Guessing, checking, and revising
• Using a diagram
• Using logical reasoning
• Finding patterns
• Making a similar, but simpler, problem
PS6-4  **Multiplication and Division Puzzles**

**Teach this lesson after:** 6.2 Unit 1

**Standards:** 6.EE.A.1, 6.NS.B.2

**Goals:**
Students solve long multiplication and long division puzzles with different letters standing for different unknown digits, using remainders.

**Prior Knowledge Required:**
Can multiply multi-digit numbers using the standard algorithm
Can divide multi-digit numbers using long division
Can find the percentage of a number

**Vocabulary:** division, long division, multiple, multiplication, percent, percentage, quotient, reciprocal

**Materials:**
calculators

**Review the rules for solving puzzles with different or identical letters.** Write on the board:

\[ 7 \times A = B2 \quad 6 \times A = 4A \]

SAY: A and B stand for different digits in the first puzzle. In the second puzzle, both As stand for the same one-digit number. Pointing to the first puzzle, ASK: What number in the 7 times table has the ones digit 2? (42) PROMPT: Let's say the multiples of 7 together: 7, 14, 21, 28, 35, 42. SAY: So B = 4 in the first puzzle. ASK: What is A? (6) How do you know? (because \( 7 \times 6 = 42 \))

Pointing to the second puzzle, ASK: What numbers in the 6 times table are in the forties? (42 and 48) Again, prompt students by reciting the multiples of 6 until you finish the forties: 6, 12, 18, 24, 30, 36, 42, 48. ASK: What is A? (8) SAY: That makes sense because \( 6 \times 8 = 48 \), so both As stand for 8.

**Exercises:** Solve the puzzle.

a) \( 9 \times A = 4A \)  
 b) \( 7 \times A = 5B \)  
 c) \( A \times A = 2A \)  
 d) \( A \times A = 4B \)

**Answers:** a) A = 5; b) A = 8, B = 6; c) A = 5; d) A = 7, B = 9

**Reducing the amount of search required to solve a puzzle.** Write “\( 4 \times AB = BBC \)” on the board. ASK: What are the possibilities for B? (1, 2, or 3) How do you know B cannot be 4 or greater? (because AB would have to be at least 100, but we know that AB is a two-digit number)

PROMPT: If B is 4, BBC is 400 or greater. What would AB have to be for \( 4 \times AB \) to equal 400 or greater?
SAY: Let’s try B = 1. Write on the board:

\[
\begin{array}{c}
A1 \\
\times 4 \\
\end{array}
\]
\[11C\]

SAY: To do this multiplication, start by multiplying the ones digits (in this case, \(1 \times 4 = 4\)). Write on the board:

\[
\begin{array}{c}
A1 \\
\times 4 \\
\end{array}
\]
\[114\]

SAY: If B is 1, then C is 4. ASK: What does that tell you about A? (\(A \times 4 = 11\)) PROMPT: What is \(A \times 4\)? (11) Write on the board:

\[A \times 4 = 11\]

ASK: Can you find what A equals? (no) SAY: So B = 1 is not correct. Let’s try B = 2. Write on the board:

\[
\begin{array}{c}
A2 \\
\times 4 \\
\end{array}
\]
\[22C\]

ASK: What would C be? (8) What is \(A \times 4\)? (22) Can you find what A equals? (no)

**Exercise:** Try B = 3 in \(4 \times A = BBC\). Are there any possible values for A and C?

**Answer:** B = 3 works because if B = 3, then C has to be 2.\(4 \times A3 = 332\) gives A = 8, which is the only answer because B cannot be greater than 3. The answers are: A = 8, B = 3, C = 2.

**Solving multi-digit multiplication puzzles.** Write on the board:

\[
\begin{array}{c}
4A \\
\times B3 \\
\end{array}
\]
\[2,491\]

Explain to students that the ones digit of the product is equal to the ones digit of \(A \times 3\). Write on the board:

\[A \times 3 = 11 \quad A \times 3 = 21 \quad A \times 3 = 31\]

ASK: Can \(A \times 3\) be 11 or 31? (no) Why? (because 11 and 31 are not multiples of 3) Can \(A \times 3\) be 41, or 51, or more? (no) Why? (because A is a one-digit number, so \(A \times 3\) is less than 30) ASK: What can \(A \times 3\) be? (21) So, what is A? (7)
Erase A and write “7” on the board in its place, as shown below:

\[
\begin{array}{c}
47 \\
\times \quad B3 \\
\hline
2,491 \\
\end{array}
\]

SAY: Now we have to find B. Write on the board:

\[
\begin{array}{cccccccc}
47 & 47 & 47 & 47 & 47 & 47 & 47 & 47 \\
\times 13 & \times 23 & \times 33 & \times 43 & \times 53 & \times 63 & \times 73 & \times 83 & \times 93 \\
\end{array}
\]

SAY: We can try 1, 2, 3, and so on for B, but instead of doing all the multiplying, let’s estimate to see which products are most likely to be close to 2,491. ASK: How can we estimate 47 \times 13 without doing a lot of work? (multiply 50 \times 10 instead) SAY: By rounding both numbers to the nearest ten, you can get a good estimate. Pointing to each product in turn, ASK: What is the estimated product? Is that close to 2,491? (50 \times 10 = 500; 50 \times 20 = 1,000; 50 \times 30 = 1,500; 50 \times 40 = 2,000; 50 \times 50 = 2,500; 50 \times 60 = 3,000; 50 \times 70 = 3,500; 50 \times 80 = 4,000; 50 \times 90 = 4,500) SAY: Only 47 \times 43 and 47 \times 53 are close. ASK: Which one is closer to 2,491? (47 \times 53) Write on the board:

\[
\begin{array}{c}
47 \\
\times \quad 53 \\
\hline
2,491 \\
\end{array}
\]

Ask a volunteer to check the answer. (47 \times 53 = 2,491) SAY: So in the puzzle, A is 7 and B is 5.

**Exercises:** Solve the puzzle. Hint: Write the puzzle vertically.

a) 6A \times B7 = 6,111  
   b) A4 \times 6B = 4,884  
   c) A57 \times 3B = 17,366

**Answers:** a) A = 3, B = 9; b) A = 7, B = 6; c) A = 4, B = 8

**Introduce division puzzles.** Write on the board:

\[
9) \quad 2A
\]

Tell students that you want the remainder to be 0, so the number being divided is a multiple of 9 and starts with a 2. ASK: What number must the unknown digit be? (7) SAY: If you have the times tables memorized, you can answer this type of question very quickly. If you don’t, you will need to look at the times table charts to find the answers.

**Exercises:** Find the unknown digit so that there is no remainder.

a) 9) \quad 4A  
   b) 8) \quad 5B  
   c) 7) \quad 6C  
   d) 6) \quad 2D  
   e) 9) \quad 7E  
   f) 8) \quad 6F

**Answers:** a) 5, b) 6, c) 3, d) 4, e) 2, f) 4
Write on the board:

\[
7 \div 13A \text{ has remainder 0. What is } A? \]

Have students brainstorm and express their ideas. If some students suggest that you can use multiplication to solve the division statement, SAY: To write the multiplication, we need to find the number of digits in the quotient. Because \(7 \times 10\) (10 is the lowest two-digit number) is 70 and \(7 \times 100\) (100 is the lowest three-digit number) is 700, which is greater than 13A, the quotient must be a two-digit number. Let’s write BC for the digits of the quotient. Fill in “BC” as the quotient in the division, then write on the board:

\[
\begin{array}{c}
BC \\
\times 7 \\
\hline 13A
\end{array}
\]

ASK: How many unknown digits are in the multiplication puzzle? (3) SAY: We know that B is 1 because \(7 \times 20\) is 140, which is too high. There are still two other unknown digits to find. Write the multiples of 7 on the board:

\[
7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119, 126, 133, 140
\]

SAY: The only multiple of 7 between 130 and 140 is 133. ASK: So, what is A? (3) Explain to students that this method worked because 133 is not a very big multiple of 7. Prior to writing out the multiples of 7, students might notice that they could start with 140, which is very close to 13A, and subtract 7s to find multiples of 7 in the 130s.

Write on the board:

\[
7 \div 66D \text{ has remainder 0. Find } D. \]

SAY: It would take a while to write all multiples of 7 up to six hundred sixty-something to find D. Point to \(7 \div 13A\) and SAY: I would like to find another method to solve this puzzle that works for other puzzles like \(7 \div 66D\) as well. SAY: In the puzzle \(7 \div 13A\), A can be any digit from 0 to 9. Let A be 0 and do the division. Write on the board:

\[
\begin{array}{c}
18 \\
7 \div 130 \\
-7 \\
\hline 60 \\
-56 \\
\hline 4
\end{array}
\]

ASK: What is the remainder? (4) Explain to students that if the dividend is 131 instead of 130, then the remainder will be 5 because after the first step subtraction in long division, the result would be 61 instead of 60. ASK: What will the remainder be when you divide 132 by 7? (6)
When you divide 133 by 7? (0) If some students say 7, correct them and say the remainder cannot be 7 when you divide a number by 7. Write on the board:

\[ 7 \div 133 \text{ has remainder 0, so A is 3.} \]

To solve the puzzle \( 7 \div 660 \), ask a volunteer to find \( 7 \div 660 \) using long division, as shown below:

\[
\begin{array}{c}
94 \\
7 \underline{\div 660} \\
-63 \\
30 \\
-28 \\
2
\end{array}
\]

SAY: With a remainder of 2, you need to add 5 to make the next 7, so the quotient will increase and the remainder will be zero. 660 + 5 = 665, so D is 5.

**Exercises:** Find the unknown digit so there is no remainder.

a) \( 8 \div 2 \text{ A} \)  

b) \( 6 \div 5 \text{ M} \)  

c) \( 7 \div 5 \text{ X} \)

**Answers:** a) A = 4, b) M = 4, c) X = 6

**Solving division puzzles with an unknown middle digit.** Write on the board:

\[ 9 \div 8 \text{ A2} \]

Tell students that you want to find the middle digit so that there is no remainder. ASK: What makes starting the long division harder than before? (we do not know what we are dividing 9 into) SAY: We do not know what we are dividing 9 into, but we do know that we are dividing it into eighty-something. ASK: What are the possible answers to eighty-something divided by 9? (8 or 9) Prompt students by writing on the board:

\[
\begin{array}{cccccc}
9 & \underline{\div 80} & 9 & \underline{\div 81} & 9 & \underline{\div 82} & \ldots & 9 & \underline{\div 89}
\end{array}
\]

Have volunteers fill in the quotients, as shown below:

\[
\begin{array}{cccccc}
8 & 9 & 9 & 9 & \ldots & 9
\end{array}
\]

SAY: There are two possible quotients. We will need to check them both.

Write on the board:

\[
\begin{array}{ccc}
8 & 9 \\
9 & \underline{\div 82} & 9 & \underline{\div 82} \\
-72 & -81 \\
? 2 & ? 2
\end{array}
\]
Let's work backward. If 9 divides into something-two, what must the something be? (70) Erase both question marks and write “7” in their places, as shown below:

\[
\begin{array}{c}
8 \\
9)82 \\
\hline
72 \\
\hline
10 \\
\end{array}
\]

Point to the first long division and ask: If something minus 72 is 7, what is the something? (79) Say: But our something starts with 8, so that's not possible. Draw a big “X” through the first long division. Ask: If something minus 81 is 7, what is the something? (88) Write “8” as the middle digit. Say: So the middle digit is 8.

**Exercises:**

1. The square represents a single digit. What are the possible quotients?
   - a) \(8\overline{4}\)
   - b) \(7\overline{2}\)
   - c) \(6\overline{5}\)
   - d) \(7\overline{9}\)
   - e) \(3\overline{7}\)
   
   **Answers:** a) 5, 6; b) 2, 3, 4; c) 8, 9; d) 12, 13, 14; e) 23, 24, 25, 26

2. Find the missing digit so there is no remainder.
   - a) \(7\overline{5}\overline{6}\)
   - b) \(9\overline{3}\overline{4}\)
   - c) \(7\overline{8}\overline{8}\)
   
   **Bonus:** Find all possible missing digits so there is no remainder.
   - d) \(8\overline{3}\overline{8}\)
   - e) \(7\overline{4}\overline{6}\)
   - f) \(8\overline{7}\overline{2}\)
   
   **Answers:** a) 4; b) 2; c) 6; Bonus: d) 2, 6; e) 0, 7; f) 1, 5, 9

**Solving division puzzles with an unknown left digit.** Write on the board:

\[
7\overline{\square}\overline{16} \quad \text{The quotient is a two-digit number.}
\]

Say: This looks a bit trickier because now there are two digits after the one we are looking for. But we can still go through the long division as though we know what the missing digit is, and work backward.

Write on the board:

\[
\begin{array}{c}
?? \\
7\overline{\square}\overline{16} \\
\hline
?? \\
\hline
\overline{76} \\
\end{array}
\]

Point to the circled number and say: This number has to be divisible by 7 in order to have no remainder. Ask: What can the question mark be? (5) Say: We can work backward to fill in the other question marks, too. Write on the board:

\[
\begin{array}{c}
\square \overline{1} \\
\hline
?? \\
\hline
\overline{5} \\
\end{array}
\]
ASK: Can 1 minus something be 5? (no) So how can this be? (it must be $11 - ? = 5$)

SAY: Whatever the square stands for, you will need to regroup from it to get 11 ones minus question mark ones equals 5 ones. ASK: So what is the question mark in the ones place? (6)

Erase the question mark in the ones place and write “6” in its place. Circle the number, as shown below:

\[
\begin{array}{c}
\square 1 \\
- \square 6 \\
\hline
5
\end{array}
\]

Point to the circled number and SAY: This number has to be a multiple of 7. ASK: How do I know that? (you get that number by multiplying the tens digit of the quotient by 7)

Pointing to the long division algorithm, PROMPT: Look at where it is in the long division algorithm. ASK: If something-six is divisible by 7, what must the something be? (5)

Erase the bottom row of question marks in the long division and fill in the missing numbers, as shown below:

\[
\begin{array}{c}
?\square 7 \\
\hline
\square 16 \\
56 \\
\hline
56
\end{array}
\]

ASK: What is $56 + 5$? (61) SAY: So the missing digit is 6. Have students complete the long division $616 ÷ 7$. (88 R 0)

**Exercises:** Find the missing digit.

- a) □15 is a multiple of 9.
- b) □86 is a multiple of 9.
- c) □64 is a multiple of 7.
- d) □55 is a multiple of 7.
- e) 3, □12 is a multiple of 7.

**Answers:** a) 3, b) 4, c) 3, d) 4, e) 6

**Problem Bank**

1. a) Solve the puzzles.
   - i) $9 \times B = AB$
   - ii) $9 \times A = BA$
   b) How are the puzzles the same? How are they different?

   **Answers:** a) i) A = 4, B = 5, ii) A = 5, B = 4; b) They are the same puzzle, but with A and B switched.

2. Solve the puzzle $A7 \times A2 = 4,154$.

   **Answer:** A = 6

3. Solve the puzzle $AB \times 5B = 4,399$.

   **Solution:** Looking at the ones digit (9), B is either 3 or 7 because $B \times B$ gives an answer with the ones digit 9. Check the two cases: $A3 \times 53 = 4,399$ and $A7 \times 57 = 4,399$. $A7 \times 57 = 4,399$ is not correct because 87 × 57 is too high (4,959) and 77 × 57 is too low (4,389). If we check $A3 \times 53 = 4,399$ with A = 8, we get $83 \times 53 = 4,399$, which is correct.
4. Find the missing digit so there is no remainder.
   a) \( \overline{84 \ 2A} \)  
   b) \( \overline{9B \ 2} \)  
   c) \( \overline{98 \ C \ 2} \)

**Answers:** a) \( A = 4 \), b) \( B = 7 \), c) \( C = 8 \)

5. Find \( M \) if …
   a) \( \overline{13 \ M \ 8} \) has remainder 0.   
   b) \( \overline{27 \ M \ 37} \) has remainder 0.

**Answers:** a) \( M = 3 \), b) \( M = 8 \)

6. What is 500% of 50% of 5% of 5?

**Solution:** Work backward. 5% of 5 is 0.25, 50% of 0.25 is 0.125, 500% of 0.125 is 0.625.

7. A number is multiplied by itself and then increased by 3. When the result is doubled, the answer is 104. What is the number?

**Solution:** Work backward. The result, when doubled, is 104, so the result is 52. Before being increased by 3, it must have been 49. What number multiplied by itself is 49? The answer is 7, which can be found by either knowing the times table or checking the numbers in order.

8. The reciprocal of a number is multiplied by 3, then increased by 2. When the result is multiplied by 5, the answer is 40. What is the number?

**Solution:** Work backward. The result that was multiplied by 5 must have been 8. Reduce the question to “The reciprocal of a number is multiplied by 3, then increased by 2. The result is 8.” Before the number was increased by 2 (to give 8), the number was 6. The reciprocal of the number was multiplied by 3 to give 6. So, the reciprocal must have been 2. The number is 1/2.

9. Mark gives Sue the same number of nickels as Sue already has. Sue gives John the same number of nickels as John already has. John gives Mark the same number of nickels as Mark already has. Now, they each have 40 nickels. How many nickels did each person start with?

**Solution:** Work backward. After John gives Mark as many nickels as Mark has, they both have 40, so Mark must have had 20 before that, and John must have had 60: \( J = 60, M = 20, S = 40 \). After Sue gives John as many nickels as John has, Sue has 40 and John has 60. So Sue must have given John 30 nickels, which means she started with 70 and John started with 30: \( J = 30, M = 20, S = 70 \). After Mark gives Sue as many nickels as Sue has, Mark has 20 and Sue has 70. So, Mark must have given Sue 35 nickels. Which means Mark started with 55 and Sue started with 35: \( J = 30, M = 55, S = 35 \).