**Goals:** Students will develop and use rules for congruence of triangles.

**Prior Knowledge Required:**
- Can measure angles and sides of polygons
- Is familiar with notation for equal sides and equal angles
- Can name angles and polygons
- Is familiar with the symbols for angle, triangle, and congruence
- Can identify congruent triangles
- Can write a congruence statement for two triangles
- Knows that the sum of the angles in a triangle is $180^\circ$
- Can classify triangles

**Vocabulary:** angle-side-angle (ASA), congruence rule, congruence statement, congruent, conjecture, corresponding angles, corresponding sides, corresponding vertices, counterexample, isosceles, side-angle-side (SAS), side-side-side (SSS)

**Materials:**
- BLM Investigating Congruence (pp. D-126–127, optional)
- Straws of different lengths and 2 pipe cleaners for each student (optional)
- Scissors (optional)
- The Geometer's Sketchpad® (optional)
- BLM Congruence Rules on The Geometer's Sketchpad® (pp. D-128–130, optional)

**Corresponding sides and angles.** Remind students that, when we want to check whether shapes are congruent, we ask ourselves if we could place the shapes one on top of the other so that they match exactly. SAY: When we place the shapes one on top of the other, the sides of the different shapes that will sit on top of one another are called corresponding sides. The vertices that will sit one on top of the other are called corresponding vertices. And the angles of the different shapes that will sit one on top of the other are called corresponding angles. If the shapes are congruent, corresponding sides and angles will be equal. Draw on the board:

![Diagram of corresponding sides and angles]

SAY: These two triangles are congruent. Some of the equal sides are marked, and a pair of equal angles is also marked. ASK: If we imagine placing one triangle on top of the other, what
do we need to do so they will match? (turn the top triangle clockwise a little) Are the sides marked with thick lines corresponding sides? (no) Point to different sides of the triangle on the bottom and ASK: Does this side correspond to the thick side of the triangle on the top? Have students signal yes or no. Students can also signal the answers to the exercises below. After each question, have a volunteer explain the answer.

**Exercises:** Are the two thick sides corresponding sides?

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**Answers:** a) yes, b) no, c) yes)

**Introduce the idea of congruence rules.** SAY: Congruent triangles have three equal corresponding sides and three equal corresponding angles. However, we do not always need to check all six pairs of elements to decide that two triangles are congruent. Today we will be looking for shortcuts—ways to check fewer pairs of angles and sides. Draw on the board:

![Triangles](image4.png)

ASK: If we check two pairs of angles and find that these two pairs of angles are equal, do we need to check that the third pair of angles is equal? (no) How do you know? (the three angles of a triangle always add to 180°) What is the measure of the third angle in both triangles? (113°) SAY: This means we do not have to check all six elements of a pair of triangles; checking three sides and two angles will be enough. Now let's see if we can check even fewer.

**SSS, SAS, ASA rules.** Have students investigate one of the congruence rules (side-side-side, side-angle-side, or angle-side-angle) in Activity 1 using The Geometer’s Sketchpad®. Alternatively, have students investigate all three rules using BLM Investigating Congruence.

**Activity**

Use The Geometer’s Sketchpad® for this activity.

Divide students into groups of three. Students will work on the construction individually, each using one page of BLM Congruence Rules on The Geometer’s Sketchpad®, sharing the results with the group. Students should tell which elements of the construction could be modified. (For example: I could modify the first triangle any way I want by moving any of the vertices. I could only move around the second triangle by moving vertex E, and I could only turn the triangle by moving vertex D. When I tried to move vertex D, it would only go along a circle, because it was constructed so that ED has a fixed length.)

Students might need to reflect the triangles they created during the activity to place them on top of each other. In this case, have students place the triangles so that they share a side and look like mirror images of each other. Then have students select the common side and use the
Transform menu option to declare the side a mirror line; they need to choose the option “Mark mirror” in the Transform menu options. Now if they select one of the triangles and reflect it using the Transform menu options, they will be able to see that the triangles match exactly.

In their groups of three, have students match each BLM with the congruence rule it seems to be showing—side-side-side (SSS), side-angle-side (SAS), or angle-side-angle (ASA). Have them label the BLM with the full name of the congruence rule. For example, page 2 shows the side-angle-side (SAS) rule by keeping two side lengths and the angle between them constant, which forces the triangle to be fixed.

(end of activity)

Summarize the congruence rules on the board (as in the “Congruence Rules for Triangles” box on AP Book 8.1 p. 99). Emphasize that the order of elements in the congruence rules is important: in the side-angle-side (SAS) rule, the equal angles have to be between the corresponding equal sides; in the angle-side-angle (ASA) rule, the equal sides have to be between the corresponding equal angles.

**Exercises:** Identify the congruence rule that tells you that the triangles are congruent.

a) ![Triangle ABC](image1.png)

b) ![Triangle DEF](image2.png)

c) ![Triangle GHI](image3.png)

d) ![Triangle VOP](image4.png)

e) ![Triangle MRT](image5.png)

f) ![Triangle ABD](image6.png)

**Answers:** a) ASA, b) SAS, c) ASA, d) SAS, e) SSS, f) ASA

Remind students that, in a congruence statement, the corresponding vertices match. So for example, in part a) above, the congruence statement is \( \triangle ABC \cong \triangle FDE \), because if you try to turn triangle DEF and place it on top of triangle ABC, vertex A corresponds to vertex F (and \( \angle A = \angle F \)), vertex B corresponds to vertex D (and \( \angle B = \angle D \)), and vertex C corresponds to vertex E (and \( \angle C = \angle E \)).

**Exercises**

(MP.6) Write the congruence statements for the pairs of triangles in parts b)–f) of the previous exercises.

**Answers:** b) \( \triangle GHL \cong \triangle JKI \), c) \( \triangle BAT \cong \triangle DGO \), d) \( \triangle UVW \cong \triangle POQ \) or \( \triangle UVW \cong \triangle PQO \),
e) \( \triangle LMN \cong \triangle TSR \), f) \( \triangle USA \cong \triangle YZX \)
Bonus: In part d) above, write two different congruence statements with the letters in the first triangle in the same order. Can you do this for another pair of triangles in this exercise? Draw another pair of triangles for which you can do this and write two congruence statements.

Sample answer:
\[ \triangle UVW \cong \triangle POQ \quad \text{and} \quad \triangle UVW \cong \triangle PQU \]

No, you cannot write a statement like for another pair of triangles in the exercise.

\[ \triangle ABC \cong \triangle DEF \quad \text{and} \quad \triangle ABC \cong \triangle FED \]

Using congruence rules to show congruence.

Ask students to sketch the triangles in their notebooks and label the equal sides and angles in the triangles. Have a volunteer label the equal angles and sides on the board, as shown below:

\[ \triangle ABC \quad \text{and} \quad \triangle DEF \]

\[ \angle A = \angle D = 45^\circ \]
\[ \angle C = \angle F = 70^\circ \]
\[ BC = EF = 30 \text{ cm} \]

Ask (MP.6) students if they think these triangles could be congruent? (yes) Is there a congruence rule that tells us these triangles are congruent based on what we know now? (no) Why not? (because the equal sides in the triangles are not between the corresponding equal angles)

PROMPT: What elements are marked as equal in each of these triangles? (two angles and a side) Is the side between the corresponding equal angles? (no) SAY: So this situation does not fit the angle-side-angle (ASA) congruence rule.

ASK: Is the order of equal, or matched, elements the same in both triangles? (yes, the equal sides are \( BC = EF = 30 \text{ cm} \), the angles opposite those sides match: \( \angle A = \angle D = 45^\circ \), and there is another match: \( \angle C = \angle F = 70^\circ \)) If you try to place triangle \( DEF \) on top of triangle \( ABC \) to make the triangles match, would the equal sides fall one on top of the other? (yes) What about the angles? (yes)

SAY: So these triangles have two pairs of corresponding equal angles and one pair of corresponding equal sides. I would like to use a congruence rule to show the triangles are
congruent, but we can’t use the angle-side-angle (ASA) rule yet because the equal sides are not between the corresponding equal angles. Maybe we can deduce some more information to see if the rule applies. ASK: What pair of sides would we need to know are equal to use the ASA congruence rule? (AC, DF) Do we know that these sides are equal? (no) What pair of angles would we need to know are equal to use the ASA congruence rule? (∠B = ∠E)

ASK: How can you find the measure of ∠B from the rest of the angles of the triangle? Have students write down the expression for the measure of the angle. (∠B = 180° − (45° + 70°) = 65°)

Repeat for ∠E. (∠E = 180° − (45° + 70°) = 65°) ASK: Are ∠B and ∠E equal? (yes) Write on the board:

∠B = ∠E = 65°

Mark angles ∠B and ∠E as equal on the picture and ASK: Can we now use a congruence rule? (yes) Which rule? (angle-side-angle, ASA) Invite a volunteer to circle the equalities between sides and angles that allow us to use the ASA rule. ASK: So, based on this additional information, can we say that the triangles are congruent? (yes)

(MP.3, MP.6) Exercise: Explain why the triangles ABC and KML are congruent.

Answer: ∠B = 180° − (35° + 120°) = 25°, ∠M = 180° − (35° + 120°) = 25°, so ∠B = ∠M. ∠B = ∠M, ∠A = ∠K, AB = KM, so with the ASA rule, the triangles are congruent.

Two pairs of equal angles and one pair of equal sides do not always mean that the triangles are congruent. Draw on the board:

(MP.3, MP.6) ASK: Do these triangles have two pairs of equal angles? (yes) Do they have a pair of equal sides? (yes) Can we apply one of the congruence rules? (no) Why not? (answers will vary, but students will likely point out that the triangles do not look congruent) Are the third angles equal in these triangles? (yes) How do you know? (in both triangles the size of the third angle is 180° − (90° + 56°) = 34°) Invite a volunteer to label the equal angles and sides in the triangles. ASK: Why can we still not use congruence rules here? (in triangle PQR, the side PQ
is opposite \( \angle R = 34^\circ \), but in triangle \( \triangle DEF \) the side \( DF \) is opposite \( \angle E = 56^\circ \).

**PROMPT:** The equal side is opposite one of the angles in both triangles. Which angle? (in triangle \( \triangle PQR \), the side \( PQ \) is opposite \( \angle R \), in triangle \( \triangle DEF \) the side \( DF \) is opposite \( \angle E \)). What is the size of the opposite angles in each triangle? (\( \angle R = 34^\circ \), \( \angle E = 56^\circ \)). Emphasize that this means the order of equal sides and equal angles is different in these two triangles, so the congruence rule does not apply.

**SAY:** And indeed, the triangles do not look congruent at all! They have the same shape, but they are obviously different sizes.

**(MP.3, MP.6) Exercise:** Explain why the ASA congruence rule cannot be used for these two triangles.

![Diagram of two triangles](image)

**Answer:** In triangle \( \triangle ABC \), the given side is between angles \( A \) and \( B \), which have the measures \( 40^\circ \) and \( 85^\circ \). In triangle \( \triangle XYZ \), the side \( XZ \) that is equal to \( AB \) is between \( \angle X = 40^\circ \) and \( \angle Z = 180^\circ - (40^\circ + 85^\circ) = 55^\circ \). The order of equal pairs of angles and equal sides is different in the two triangles, so the ASA rule cannot be applied. The triangles are not congruent.

**Bonus:** Use a ruler and a protractor to draw \( \triangle ABC \) and \( \triangle XYZ \) with the measurements given.

**AAA is not a congruence rule.** Write on the board:

If two triangles have three pairs of corresponding equal angles, then the triangles are congruent.

True or false?

**(MP.3)** Have students vote on the question above. (the statement is false) **PROMPT:** Think of the triangles you saw in the previous exercise. Have students draw a counterexample to the statement. Students who wish and have time could draw more than one counterexample.

**SSA is not a congruence rule.** **SAY:** We checked what happens with three pairs of angles, three pairs of sides, two pairs of angles and one pair of sides, but we did not check what happens when two triangles have two pairs of corresponding equal sides and one pair of corresponding equal angles. Let’s see if the order matters in the side-angle-side (SAS) congruence rule. Draw on the board:
(MP.3) SAY: I drew a copy of triangle \( ABC \) and called it \( DEF \). Then I drew an isosceles triangle \( EFG \), so that \( EG = EF \). Now look at the triangles \( ABC \) and \( DEG \). ASK: Which angles are equal? (\( \angle A = \angle D \)) Which sides are equal? (\( AB = DE, BC = EG \)) Are the equal angles opposite corresponding equal sides? (yes, \( \angle A \) is opposite \( BC \), \( \angle D \) is opposite \( EG \)) Are the triangles congruent? (no) Which description identifies the equal elements, in order, in your triangles: side-angle-side or side-side-angle? (side-side-angle, SSA) Of the two descriptions, which is a congruence rule and which is not? (side-angle-side, or SAS, is a congruence rule) How do the triangles I drew help you to decide? (they are a counterexample to this statement) Emphasize that the order of equal corresponding elements matters in triangles. The pair of equal angles has to be between the pairs of equal corresponding sides for the triangles to be congruent.

(MP.3) Exercise: On grid paper, draw a counterexample to SSA. Use a ruler.

Sample answer:

Using congruence rules to find sides or angles in congruent triangles. Draw on the board:

SAY: I want to find the size of the angles and the lengths of the sides in both of these triangles. Let’s see if these triangles are congruent. If the triangles are congruent, we will know that the lengths of all the sides in both triangles match even though we haven’t measured them. Have students sketch the triangles and label all the known information. Then ask them to label the angles that are the same with arcs and the equal sides with the same number of hash marks. Point out that we do not know yet if the triangles are congruent, and so we cannot, for example, mark angles \( T \) and \( V \) as equal.

ASK: How many pairs of equal sides have you marked? (1 pair, \( UW = RS \)) How many pairs of equal angles do we see? (2 pairs, \( \angle W = \angle R \), \( \angle U = \angle S \)) Which congruence rule would we like to apply? (angle-side-angle, or ASA) Is the pair of equal sides between the corresponding equal angles? (yes) SAY: Then we can apply the congruence rule ASA. Have students write the congruence statement and the rest of the angle and side equalities. Then ask them to find the
lengths of all the sides and angles. (see answers below) Ask volunteers to explain how they found the answers.

\[ \triangle UVW \cong \triangle STR \]
\[ UV = ST = 20 \text{ cm} \]
\[ VW = TR = 10 \text{ cm} \]
\[ \angle V = \angle T = 180^\circ - (90^\circ + 30^\circ) = 60^\circ \]

Remind students that, when several lines or rays meet at a point and there are overlapping angles, we use three letters to label an angle with the vertex label always in the middle.

Draw the picture in the exercises below on the board, mark different angles with arcs one at a time, and ask students to name them.

**Exercises:** Explain why the triangles are congruent. Then write the congruence statement and find the missing angle measures.

\[ \angle A = 43^\circ \]
\[ \angle E = 83^\circ \]
\[ AB = CD = 3 \text{ cm} \]
\[ AC = CE = 3.7 \text{ cm} \]
\[ BC = DE = 2.5 \text{ cm} \]

**Answers:** The triangles have 3 pairs of corresponding equal sides, so by the side-side-side, or SSS, rule they are congruent, and \( \triangle ABC \cong \triangle CDE \). Therefore, \( \angle A = \angle DCE = 43^\circ \), \( \angle BCA = \angle E = 83^\circ \), and \( \angle B = \angle D = 180^\circ - (43^\circ + 83^\circ) = 54^\circ \).

**Extensions** (MP.1, MP.3, MP.7) 1. In the diagram below, \( AC \perp BD \), \( AO = CO \), and \( BO = DO \).

\[ A \]
\[ B \]
\[ O \]
\[ C \]
\[ D \]

a) Copy the diagram and label the equal sides and right angles.
b) Use triangles \( AOB \) and \( COD \) to explain why \( AB = CD \).
c) Explain why \( AD = BC \).
d) Use triangles \( AOB \) and \( COB \) to explain why \( AB = CB \).
e) What type of quadrilateral is \( ABCD \)?

**Selected solutions:**
b) Since \( AC \perp BD \), \( \angle AOB = \angle COD = 90^\circ \) and we know that \( AO = CO \) and \( BO = DO \). This means we have two pairs of corresponding equal sides and a pair of corresponding equal angles between them, so by the SAS rule \( \triangle AOB \cong \triangle COD \). Therefore \( AB = CD \).
d) Since \( AC \perp BD \), \( \angle AOB = \angle COB = 90^\circ \) and we know that \( AO = CO \). The side \( BO \) is in both triangles, so we have another equal side in both triangles. This means we have two pairs of
corresponding equal sides and a pair of corresponding equal angles between them, so by the SAS rule \( \triangle AOB \cong \triangle COB \). Then \( AB = CB \).
e) \( ABCD \) is a rhombus.

**MP.1, MP.3, MP.7**  2. Maria drew 2 right triangles as shown below, with \( PQ = TR \) and \( QT = RS \). She thinks that the points \( P, T, \) and \( S \) are on the same line. Is she correct? Explain.

\[ \text{Solution:} \quad \text{Maria is correct. Since } PQ = TR \text{ and } QT = RS \text{ and the angles between the corresponding pairs of equal sides are equal, } \angle Q = \angle R = 90^\circ, \text{ the triangles are congruent by the SAS rule. This means } \angle PTQ = \angle S. \text{ From the sum of the angles in a triangle we know that:} \]
\[ \angle S = 180^\circ - (90^\circ + \angle RTS) = 90^\circ - \angle RTS. \]
\[ \angle PTS = \angle PTQ + \angle QTR + \angle RTS \]
\[ = \angle S + 90^\circ + \angle RTS \]
\[ = 90^\circ - \angle RTS + 90^\circ + \angle RTS \]
\[ = 180^\circ \]

The angle \( PTS \) is a straight angle, so the points \( P, T, \) and \( S \) are on the same line.

**MP.1, MP.3**  3. \( \triangle ABC \) and \( \triangle DEF \) are both isosceles triangles. \( \angle A = \angle D \) and \( AB = DE \). Are \( \triangle ABC \) and \( \triangle DEF \) always congruent? Explain.

**Hint:** Make a sketch that includes all the information you have been given. Try making more than one sketch using a different position for the equal angles.

**Answer:** The triangles do not have to be congruent. Sample counterexample:

**MP.3**  4. Sketch a counterexample to show why the statement is false.
\( \triangle ABC \) has \( AB = BC = 7 \text{ cm} \) and \( \triangle DEF \) has \( DE = EF = 7 \text{ cm} \). So \( \triangle ABC \cong \triangle DEF \).

**Sample answer:**
G8-12  Congruence (Advanced)

Pages 102–103


Goals:
Students will use informal arguments involving congruent triangles to solve problems.

Prior Knowledge Required:
Can name angles and polygons
Can measure angles and sides of polygons
Is familiar with notation for equal sides and angles
Can classify triangles
Knows that the sum of the angles in a triangle is 180°
Is familiar with the symbols for angle, triangle, and congruence
Can identify congruent triangles
Knows the SAS, ASA, and SSS congruence rules
Can write a congruence statement for two triangles

Vocabulary: ASA (angle-side-angle), congruence rule, congruence statement, congruent, conjecture, corresponding angles, corresponding sides, corresponding vertices, counterexample, isosceles, midpoint, SAS (side-angle-side), SSS (side-side-side), supplementary angles, vertical angles

Materials:
BLM Two Pentagons (p. D-131)
sissors

Review congruence rules and properties of isosceles triangles. Remind students that congruence rules are shortcuts that allow us to determine whether or not triangles are congruent by checking only three elements (sides or angles). ASK: What three elements could we use? (3 sides, 2 sides and 1 angle, 1 side and 2 angles) Can we use any three elements? (no; for example, we can’t use 3 angles) Remind students that the order of the elements is important. Draw on the board:

Remind students that, in an isosceles triangle, the angles between the equal sides and the third side are equal. SAY: Both these triangles are isosceles right triangles. ASK: What is the size of the angles that are not marked? (45°) How do you know? (angles in a triangle add to 180°, and
one angle is 90°, so the other two angles add to 180° − 90° = 90°; since they are equal, they measure 90° ÷ 2 = 45° each) Mark the angles in the triangles on the board as 45°.

(MP.3) Point to the triangles and SAY: These triangles have all angles the same size, and they have some sides that are equal. ASK: Are they congruent? (no) Why not? (they are different sizes) Why can you not apply any of the congruence rules here? (the sides that are equal in both triangles are in different places in relation to the equal angles: the equal side is between the 45° angles in the smaller triangle but between the 90° angle and one of the 45° angles in the other) Remind students that the equal sides need to be adjacent to the equal angles for the triangles to be congruent using the side-angle-side (SAS) rule.

ASK: When you have two pairs of equal sides and a pair of equal angles, where does the equal angle have to be for the triangles to be congruent? (in both triangles, the equal angle has to be between the 2 pairs of equal sides) Draw on the board:

Point out that the two triangles have two pairs of equal sides and a pair of equal angles, but the triangles are not congruent. SAY: Even though the order of the equal elements is the same in both triangles—angle-side-side—the triangles are still not congruent because the congruence rule requires the order side-angle-side.

Remind students that the third congruence rule is the side-side-side rule: if two triangles have all sides of the same length, they are congruent. Also remind students that a congruence statement lists the vertices of the triangles so that you can say which angle is equal to which angle and which side equals which side. For example, if \( \triangle ABC \cong \triangle PQR \), then we know that \( \angle A = \angle P \), \( \angle B = \angle Q \), \( AB = PQ \), and so on.

**Exercises:** Which congruence rule tells that the two triangles are congruent? Write the congruence statement.

a)

b)

c)
Identifying congruent triangles and explaining why they are congruent when triangles have a common side or vertex. Explain that sometimes you see triangles that share a side or a vertex. Draw on the board:

Have students copy the picture. SAY: To prove that triangles $ABD$ and $BCD$ are congruent, you need to use the fact that the common side $BD$ belongs to both triangles and therefore makes a pair of equal sides. ASK: What sides are equal in these triangles? ($AD = CD, BD = BD$) Point out that we know that $AD$ and $DC$ are equal, so we can describe $D$ as at the midpoint in the line segment $AC$—in other words, $D$ divides the line segment in half. Write the equalities between the sides on the board and have students write them in their notebooks. ASK: What angles are equal in these triangles? ($\angle ADB = \angle CDB$) PROMPT: What is the size of angle $ADB$? ($90^\circ$) Have students write the equality. ASK: What congruence rule can you apply? (SAS) What do you need to check to be sure the rule applies? (that the matching angle is between the sides listed in the equalities) Is this the case? (yes, angles $\angle ADB$ and $\angle CDB$ are between the sides $AD$ and $DB$, and $CD$ and $DB$) Have a volunteer write the congruence statement on the board and have students write it in their notebooks. ($\triangle ABD \cong \triangle CBD$) Leave this picture on the board for later use.

**Exercises:** The pairs of equal sides and angles are marked in the diagram. Which congruence rule can you use to prove that the triangles are congruent?

a) 

b) 

c) 

d) 

**Answers:** a) ASA, b) SAS, c) SSS, d) SSS

Draw two intersecting lines on the board and remind students which angles are vertical angles, and that vertical angles are equal.
(MP.6) Exercises:
a) Write the equalities between the sides and the angles in the triangles.

i) \[ AD = CD, \quad AB = CB, \quad BD = BD \]

ii) \[ AO = CO, \quad BO = DO, \quad \angle COD = \angle AOB \]

iii) \[ KO = MO, \quad \angle KON = \angle MOL, \quad \angle OKN = \angle OML \]; Bonus: \[ ZY = XW, \quad \angle ZYW = \angle XWY, \quad YW = WY \]

b) Which congruence rule can you use to show the triangles are congruent?

c) Write the congruence statement.

Answers:
a) i) SSS; ii) SAS; iii) ASA; Bonus: SAS

b) i) \( \triangle ABD \cong \triangle CBD \); ii) \( \triangle ABO \cong \triangle CDO \); iii) \( \triangle KNO \cong \triangle MLO \); Bonus: \( \triangle YZW \cong \triangle WXY \)

ASK: In part ii), how do you know the angles \( \angle COD \) and \( \angle AOB \) are equal? (they are vertical angles, and vertical angles are equal) What other pair of vertical angles did you see in this exercise? (\( \angle KNO \) and \( \angle MLO \))

Using congruence. Return to the picture below, from earlier in the lesson.

\[ AD = CD \]
\[ BD = BD \]
\[ \angle ADB = \angle CDB \]
\[ \triangle ABD \cong \triangle CBD \]

Remind students that they showed that the triangles are congruent using the side-angle-side congruence rule. Ask students to write the rest of the equalities between the sides and the angles of the triangles. (\( AB = BC, \quad \angle A = \angle C, \quad \angle ABD = \angle CBD \)) ASK: What have we just written about the larger triangle, \( ABC \)? (the triangle is isosceles) To prompt students to see the answer, trace the larger triangle with a finger and ask students to classify it.
Add the information in the exercises below to the triangles from Exercise 2, above. Students might need a prompt to identify $\angle ADB$ as a right angle in part a). If so, ask them what the measure of angle $ADC$ is, and what they know about the two angles with vertex $D$. When students have finished their work, discuss solutions as a class.

**Exercises:** Find all the missing sides and angles that you can.

a) \[ \triangle ABC \] with $BC = 5$ cm, $\angle C = 64^\circ$, $\angle ADB = \angle CDB = 90^\circ$.

b) \[ \triangle BCD \] with $CD = 7$ in, $\angle COD = 60^\circ$, $\angle OCD = 70^\circ$, $\angle OBA = \angle ODC = 50^\circ$.

c) \[ \triangle LMN \] with $LM = KN = 6$ m, $\angle KON = \angle MOL = 51^\circ$, $\angle MLO = \angle KNO = 39^\circ$.

**Answers:** a) $\angle C = 64^\circ$, $\angle ADB = \angle CDB = 90^\circ$, $BC = 5$ cm; b) $CD = 7$ in, $\angle COD = 60^\circ$, $\angle OCD = 70^\circ$, $\angle OBA = \angle ODC = 50^\circ$; c) $LM = KN = 6$ m, $\angle KON = \angle MOL = 51^\circ$, $\angle MLO = \angle KNO = 39^\circ$.

**Bonus:** Use the picture to prove that, in an isosceles triangle, the angles between the equal sides and the third side are equal (or $\angle A = \angle C$).

**Answer:** $AB = CB$, $BD = BD$, and $\angle ABD = \angle CBD$, so by the SAS congruence rule, $\triangle ABD \cong \triangle CBD$. From the congruence statement we know that $\angle C = \angle A$, so in $\triangle ABC$, $AB = BC$.

**Using notation shortcuts in diagrams.** Draw the picture below on the board and have students copy it:

\[ FE = GH \]
\[ \angle EFH = \angle GHF \]

SAY: We have a quadrilateral $EFGH$ that is made from two triangles with a common side, another pair of equal sides, and equal angles. I would like to see what properties this quadrilateral has. ASK: What can you tell about triangles $EFH$ and $FGH$? (they are congruent) How do you know? (they have two pairs of equal sides and a pair of equal angles between them, so by the SAS congruence rule the triangles are congruent) Ask students to write the congruence statement for the triangles. ($\triangle EFH \cong \triangle GHF$) Then ask them to write the equalities.
for the sides and the angles that follow from the congruence statement. \((EH = GF, \angle FEH = \angle HGF, \\
\angle EHF = \angle GFH)\)

Point out that listing all the angle names takes a long time, and it is often hard to see which angle name is equal to which angle on the diagram. In addition, if you are talking about several angles, it becomes difficult to use arcs. SAY: In such cases, we often mark the size of the angles and sides with letters, using the same letter for angles of the same size. Label \(\angle EFH = a, \angle FEH = b, \text{ and } \angle EHF = c\), and have students to label all the equal angles on the diagram with these letters. Have a volunteer do the same on the board. The picture will look like this:

SAY: Now it is clear from the picture that the quadrilateral also has equal opposite angles. Angles \(E\) and \(G\) are equal, but so are angles \(EFG\) and \(GHE\). ASK: Using letters, what are angles \(EFG\) and \(GHE\) equal to? \((a + c)\)

ASK: What type of a quadrilateral does \(EFGH\) seem to be? (a parallelogram) Point out that parallelograms have equal opposite sides and equal opposite angles, but students have not proven that the quadrilateral has parallel opposite sides. The fact that \(EFGH\) is a parallelogram remains a conjecture—something we think is true but have not proved using logic. Explain that students will be able to prove this conjecture later in this unit.

\((\text{MP.3, MP.7})\) \textbf{Exercise:} Prove that quadrilateral \(PQRS\) has equal opposite angles. Use the shortcut notations for the angles.

\textbf{Answer:} \(PQ = RS, QR = SP, \text{ and } PR = PR\) (common side), so by the SSS congruence rule, \(\triangle PQR \cong \triangle RSP\). From the congruence statement, the corresponding angles in the triangles are equal, as labeled below.

This means \(\angle Q = \angle S\) and \(\angle QPS = a + b = \angle SRQ\), so the opposite angles in the quadrilateral \(PQRS\) are equal.
Bonus: XYZW is a rhombus with line segment XZ.

a) Sketch the situation.

b) Prove that \( \angle YXZ = \angle WXZ = \angle YZX = \angle WZX \).

**Answers:**

a)

![Diagram of a rhombus]

b) \( XY = XW, YZ = WZ, XZ = XZ \), so by the SSS congruence rule, \( \triangle XYZ \cong \triangle XWZ \). From the congruence statement, \( \angle YXZ = \angle WXZ \) and \( \angle YZX = \angle WZX \). Since \( \triangle XYZ \) is isosceles, \( \angle YXZ = \angle YZX \), so all angles are equal.

**Congruence in other polygons.** Remind students that they can talk about congruence of any pair of polygons. SAY: If you can place two polygons one on top of the other and they match exactly, they are congruent. Explain that, in polygons with more than three sides, the order of the equal sides and angles is even more important than in triangles. It is not enough to say that the shapes have all sides the same lengths and that all angles are the same size; the equal sides and angles have to match in order.

(MP.3) Activity

Give students BLM Two Pentagons. Have students work individually to cut out the pentagons and compare the sides and the angles to answer the questions on the BLM. When students are finished, ask the class Question a) and have them signal the number of right angles on each polygon to check their answer. (3 each) Read Question b) to the class, and have them hold up the folded shapes so that they can show that the remaining four angles are all equal. Ask the class Questions c) to h) and have students to signal their answers to each one so that you can check the whole class at the same time. Students can signal thumbs up if their answer is “yes” and thumbs down if their answer is “no.” (c) yes, d) yes, e) no, f) no, g) yes, h) no)

Have students answer Question i). Pair students who are struggling with students who show greater understanding of the material to compare their answers (the former can coach the latter as they come up with a common answer). Repeat with groups of four and groups of eight, and then have the groups share their answers with the whole class. (The order of sides and angles matters. For example, in Pentagon A the obtuse angles are adjacent, but in Pentagon B the obtuse angles are separated by a right angle. The order of the equal sides is not the same on these pentagons, so they are not congruent.)

(end of activity)

Explain that, when we place Pentagons A and B from the BLM one on top of the other, we define the order in which we will check the angles and sides. If we place the pentagons so that at least one pair of sides or angles matches, we cannot make all the remaining corresponding sides and angles match. For example, you can match the shapes so that two angles and the
side between them correspond, but the other sides adjacent to the angles are not equal because the order of the sides in each shape is different.

SAY: When you check for congruence, you either need to try all the possible combinations or show why congruence is impossible. In this case, congruence is impossible because all the right angles are adjacent in Pentagon A but not in Pentagon B.

**Extensions**

1. Look again at the pentagons on BLM Two Pentagons. Which one has greater area? Take the necessary measurements to check.

**Answer:** Pentagon A

(MP.1, MP.3) 2. In the quadrilateral $ABCD$, $AB = CD$ and $BC = AD$. Copy the picture.

![Diagram of quadrilateral ABCD with points O]  

Answer the questions to explain why the point $O$ divides the line segments $BD$ and $AC$ in half. (In other words, explain why $O$ is the midpoint of both $AC$ and $BD$.)

a) $\triangle ABD \cong \triangle CDB$ by the _____ congruence rule, so $\angle ABD = \angle ________$

Label the equal angles with letter $a$.

b) $\triangle ABC \cong \triangle CDA$ by the _____ congruence rule, so $\angle BAC = \angle ________$

Label the equal angles with letter $b$.

c) Shade $\triangle AOB$ and $\triangle COD$.

d) $\triangle AOB \cong \triangle ________$ by the ________ congruence rule, so $BO = ______$ and $AO = ______$, so the point $O$ divides the line segments ______ and ______ in half.

**Answers:**

a) SSS, $CDB$; b) SSS, $DAC$; d) $COD$, ASA, $DO$, $CO$, $AC$, $BD$
3. Tina wants to measure the distance from point $X$ to point $Y$, but a pond stops her from walking directly between the points. She finds a point, $Z$, from which she can walk to both $X$ and $Y$ in a straight line.

Copy the sketch and follow the steps below to see how Tina solves the problem.

a) Tina walks from $X$ to $Z$ and counts her steps. She continues in a straight line from $Z$, walking the same number of steps. She labels the point she stops at $W$. Draw the point $W$ on the sketch. Mark the equal distances.

b) Tina repeats the task in part a), walking from $Y$ through $Z$, to find point $U$ so that $YZ = ZU$. Draw the point $U$ on the sketch. Mark the equal distances.

c) Tina measures the distance $UW$. Explain why this distance is the same as the distance between $X$ and $Y$.

Answers:

c) $XZ = ZW$ and $YZ = ZU$ as constructed. Also, $\angle XZY = \angle WZU$ because they are vertical angles. So $\triangle XZY \cong \triangle WZU$ by the side-angle-side (SAS) congruence rule. From the congruence statement, $XY = WU$.

4. a) Find a value for $x$ that makes the equation true: $81^6 = 27^x$. Explain how you know your answer is correct.

b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

Answer: a) I wrote 81 as $3^4$ and 27 as $3^3$ so that both sides were powers of 3. So, $(3^4)^6 = (3^3)^x$, $3^{24} = 3^{3x}$, so $24 = 3x$. Then $x = 8$.

MP.3 Assessment Opportunity: Question 3 on AP Book 8.1 p. 102
G8-13 Exterior Angles of a Triangle

Pages 104–105

Standards: 8.G.A.5

Goals:
Students will discover, prove, and use the fact that the exterior angle in a triangle equals the sum of the non-adjacent interior angles.

Prior Knowledge Required:
Knows that the sum of the angles in a triangle is 180°
Can identify supplementary and vertical angles
Knows that vertical angles are equal
Can draw and measure with a ruler and a protractor
Can identify and construct parallel lines using a protractor

Vocabulary: conjecture, exterior angle, interior angle, parallel, supplementary angles, vertical angles

Materials:
protractors
The Geometer’s Sketchpad®

Introduce exterior angles. Draw on the board:

\[ \angle a \quad \angle b \quad \angle c \quad \angle x \]

ASK: What do you know about the measures of angles \( a, b, \) and \( c \)? (they add to 180°) What do you know about angles \( c \) and \( x \)? (they add to 180°) What are angles like \( c \) and \( x \) called? (supplementary angles)

Ask students if anyone knows what “exterior” means. (outer, on the outside) Explain that an angle such as angle \( x \), created by extending one of the sides outside the triangle, is called an exterior angle of the triangle because it is outside the triangle. The angles inside the triangle are called interior angles.

Looking for a pattern in the measures of exterior and interior angles. Mark the measures of \( \angle a \) and \( \angle b \) in the triangle on the board as 50° and 57°, respectively. Ask students to find the measure of \( \angle c \). (73°) ASK: How do you know? (180° − (50° + 57°) = 73°) What do you know about \( \angle c \) and the exterior angle, \( \angle x \)? (they are supplementary angles; they add to 180°) Have
students find the measure of $\angle x$. (107°) Draw the table below on the board and fill in the first column with the information from this triangle:

| $\angle a$ | 50° |
| $\angle b$ | 57° |
| $\angle x$ | 107° |

Ask students to each draw a triangle in their notebooks and label the angles $a$, $b$, and $c$. Then ask them to extend one of the sides of the triangle that make angle $c$ beyond the vertex so that the exterior angle is $x$. Have students measure the angles $a$, $b$, and $x$, and write the measures in the table. Then have them exchange notebooks and repeat the exercise with the triangles drawn by their peers.

(MP.7) Ask students to look for a pattern in their tables and have them formulate a conjecture about the sizes of the angles. Have students pair up and to improve the conjecture they have written, using the words “exterior” and “opposite.” Students can improve their conjecture again in groups of four. Have all groups share their conjectures with the class. (2 interior angles add to the measure of the exterior angle that is the supplementary angle of the third interior angle) You might point out that the angles that add to the exterior angle are opposite the third angle in the triangle.

The activity below allows students to check their conjecture using The Geometer’s Sketchpad®.

Activity
Use The Geometer’s Sketchpad® for this activity.

Checking that the exterior angle equals the sum of the interior angles opposite to it.

a) Construct a triangle $ABC$ and measure its angles.
b) Draw a ray $BC$ and mark a point $D$ on the ray, outside the triangle.
c) Measure $\angle ACD$.
d) Using the Number menu option, calculate the sum of the measures of $\angle ABC$ and $\angle BAC$. You can click on the angle measures to make them appear in the calculation windows.
e) What do you notice about the answers in parts c) and d)?
f) Modify the triangle. Did your answer to part e) change?
Answers: e) the answers are the same, f) no
(end of activity)

(MP.3) Proving the conjecture for the size of exterior angle. Return to the triangle above and erase the angle measures. Ask: How can you find the measure of angle $c$ from the measure of angle $x$? ($\angle c = 180° - \angle x$) Write the equation on the board. Ask: How can you find the measure of angle $c$ from the measure of angles $a$ and $b$? ($\angle c = 180° - (\angle a + \angle b)$) Write the second equation underneath the first, as shown below:

$$\angle c = 180° - \angle x$$
$$\angle c = 180° - (\angle a + \angle b)$$
SAY: The expressions on the left side of these equations are the same. This means the expressions on the right side are the same too. ASK: How are the expressions on the right side of the equal sign the same in both equations? (something is subtracted from 180°) Have a volunteer circle the parts that are subtracted. ASK: What can you say about the subtracted parts? (they are the same) Have students write the equation showing this. ($\angle x = \angle a + \angle b$)

Point out that students have now proved their conjecture about the external angle using logic. Summarize on the board:

An exterior angle of a triangle equals the sum of the two angles opposite to it in the triangle.

$\angle x = \angle a + \angle b$

**Use the measure of the exterior angle to find missing angles.** Work through the examples below as a class. Then have students work individually on the following exercises.

$$\angle x = 68^\circ + 38^\circ = 106^\circ$$

$$\angle x = 115^\circ - 61^\circ = 54^\circ$$

**Exercises:** Find the measure of angle $a$.

a) $\angle a$

b) $\angle b$

**Bonus:**

**Answers:** a) 90°, b) 82°, Bonus: 22.5°

SAY: Now you will use what you know about exterior angles, vertical angles, and supplementary angles to find the missing angle measures. Draw on the board:

ASK: Which angles are the exterior angles for this triangle? (c and the angle labeled 58°) What is the measure of angle $a$? (43°) How do you know? ($58^\circ - 15^\circ = 43^\circ$) Have students find the rest of the angles in the picture, and then have volunteers explain the solutions. ($\angle b = 122^\circ$, supplementary to 58° or using sum of the angles in a triangle; $\angle c = 58^\circ$, supplementary to $\angle b$ or vertical to 58°; $\angle d = 122^\circ$, supplementary to 58° or vertical to $\angle b$)
Exercises: Find the measures of angles \( x \) and \( y \).

a) \[
\begin{align*}
\angle x &= 54.5^\circ, \\
\angle y &= 109^\circ
\end{align*}
\]

b) \[
\begin{align*}
\angle x &= 36^\circ, \\
\angle y &= 144^\circ
\end{align*}
\]

c) \[
\begin{align*}
\angle x &= 128^\circ, \\
\angle y &= 93^\circ
\end{align*}
\]

Extensions

(MP.3) 1. Josh thinks that the exterior angle in a triangle is larger than each of the interior angles not supplementary to it.

a) Make a sketch of a triangle with an exterior angle.
b) Is Josh’s statement true for your triangle?
c) Is Josh’s statement true for any triangle? Explain.
d) Ted thinks the exterior angle is larger than any angle in a triangle. Is Ted’s statement true? Explain.

Answers: c) Yes, Josh’s statement is true for any triangle. If \( x \) is the exterior angle, 
\( \angle x = \angle a + \angle b \), all the measures are positive numbers, and the measure of \( \angle x \), being the sum, is larger than any of the addends—in other words, larger than any two interior angles that can add to the exterior.
d) Ted’s statement is not true. Counterexample:

The angle adjacent to the exterior angle is an obtuse angle, so it is larger than \( 70^\circ \); in this case it is \( 110^\circ \) and it is the largest of the three interior angles and larger than the exterior angle.

(MP.3) 2. The angles \( QPR \) and \( PQR \) in triangle \( PQR \) are acute. Point \( S \) is on the line \( PQ \) so that \( RS \perp PQ \). Jenny thinks that point \( P \) can be between the points \( Q \) and \( S \). Is she correct? Explain.

Hint: Make a sketch placing point \( P \) between \( Q \) and \( S \). Look at triangle \( PRS \). What can you say about the size of \( \angle QPS \)?

Answer: Jenny is not correct.
However, we are given that angle $PQR$ is acute. An angle cannot be both acute and obtuse, so point $P$ cannot be between points $Q$ and $S$.

**NOTE:** In part b) of Extension 3 and part c) of Extension 4, encourage partners to ask questions to understand and challenge each other’s thinking (MP.3)—see p. A-49 for sample sentence and question stems.

**(MP.3, MP.7)** 3. a) Grace thinks $\angle ABC$ is a right angle. Is she correct? Explain without measuring.

![Diagram of triangle ABC with $\angle A = 110^\circ$](image)

b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

**Sample solution:** a) Angle $C$ is equal to angle $x$, so $2x = 110^\circ$ because $\angle ADB$ is the exterior angle for triangle $BDC$. So $\angle x = 55^\circ$. Triangle $ABD$ is an isosceles triangle, so $\angle A = \angle y$, and from the sum of the angles in triangle $ABD$, $2y = 180^\circ - 110^\circ = 70^\circ$, so $\angle y = 35^\circ$. $\angle ABC = \angle x + \angle y = 55^\circ + 35^\circ = 90^\circ$. Grace is correct: $\angle ABC$ is a right angle.

**(MP.3)** 4. a) Write the numbers in order from least to greatest: $25$, $28$, $2^{-2}$, $2^{-1}$.

b) Marco says that for integers $a$ and $b$, $2^a > 2^b$ when $a > b$. Do you agree with Marco? Why or why not? Be sure to discuss both positive and negative exponents.

c) In pairs, explain your answers to part b). Do you agree with each other? Discuss why or why not.

**Answer:** a) The numbers are $32$, $256$, $1/4$, and $1/2$. In order, they are: $1/4$, $1/2$, $32$, $256$, or $2^{-2}$, $2^{-1}$, $2^5$, $2^8$.

**Sample answer:** b) I agree with Marco. When both exponents are positive, you are multiplying more 2s when the exponent is greater. For example, multiplying eight 2s will be greater than multiplying five 2s. When both exponents are negative, say $-5$ and $-8$, $2^{-5} = \frac{1}{2^5} > \frac{1}{2^8} = 2^{-8}$ because $5 < 8$, but $-5 > -8$, so even when both exponents are negative, the greater exponent gives the greater number. So, Marco’s statement is true for comparing negative numbers. Also, $2^1 > 1 = 2^0 > \frac{1}{2} = 2^{-1}$, so Marco’s statement is true when zero is involved or when one exponent is positive and the other is negative.
G8-14  Corresponding Angles and Parallel Lines

Pages 106–108

Standards: 8.G.A.5

Goals:
Students will use informal arguments to establish facts about corresponding and co-interior angles at parallel lines.

Prior Knowledge Required:
Can identify parallel lines using a protractor
Can draw and measure with a ruler and a protractor
Can identify supplementary and vertical angles
Knows that vertical angles are equal
Knows what counterexamples are

Vocabulary: co-interior angles, conjecture, corresponding angles, counterexample, parallel, supplementary angles, vertical angles

Materials:
transparencies
overhead projector
rulers
protractors
The Geometer’s Sketchpad®

Introduce corresponding angles. Draw on the board:

\[
\begin{array}{c}
1 \quad 2 \\
3 \quad 4 \\
5 \quad 6 \\
7 \quad 8
\end{array}
\]

SAY: We are going to look at different situations when two lines intersect a third line. One situation is when angles create a pattern like in the letter F. We call these corresponding angles. **NOTE:** The term “corresponding angles” here is not the same as the corresponding angles in congruent triangles—in other words, not the angles that match up in congruent shapes.

SAY: In the case of corresponding angles among intersecting lines, the letter F pattern can be flipped from side to side or rotated in a circle. For example, angles 4 and 8 in the picture are corresponding angles. Trace the letter F in the picture with your finger. Then copy the picture
three more times, trace the letter F in each picture, and have students find all four pairs of corresponding angles, as shown below:

If students do not see that the upside-down or reflected pattern resembles an F, copy the picture to a transparency, highlight the letter F, and turn it over or rotate it so that students see the pattern.

Exercises: List the corresponding angles.

a) 

b) 

Answers: a) \(\angle 1\) and \(\angle 3\), \(\angle 2\) and \(\angle 4\), \(\angle 5\) and \(\angle 7\), \(\angle 6\) and \(\angle 8\); b) \(\angle a\) and \(\angle k\), \(\angle b\) and \(\angle m\), \(\angle c\) and \(\angle n\), \(\angle d\) and \(\angle u\)

Corresponding angles for parallel lines are equal. Have students draw a pair of parallel lines by using the opposite sides of a ruler. Ask them to place the ruler across the two lines they drew and draw a third line intersecting both lines. Have them measure all eight angles created this way with protractors and identify which angles are corresponding angles. ASK: What do you notice about the measures of corresponding angles? (they are equal) Did everyone create angles of the same size? (no) Did everyone get the same result: corresponding angles in a pair of parallel lines are equal? (yes)

The activity below allows students to check this result with The Geometer’s Sketchpad®.

Activity 1
Use The Geometer's Sketchpad® for this activity.

Checking that corresponding angles for parallel lines are equal.

a) Draw a line. Label it \(AB\). Mark a point \(C\) not on the line and construct a new line parallel to \(AB\) through \(C\).

b) Draw a line through points \(B\) and \(C\).
c) On the line parallel to $AB$, mark point $D$ on the same side of $BC$ as point $A$.

d) On the line $BC$, mark a point $E$ outside the line segment $BC$, as shown below:

```
here  B  C  or here
```

e) Name two corresponding angles in the diagram.
f) Measure the corresponding angles you named. What do you notice? Make a conjecture.
g) Move the points $A$, $B$, $C$, or $D$ around. Are the corresponding angles always equal?

(MP.5) h) Move point $E$ so that the angles you measured stop being equal (e.g., move point $E$ to be between $B$ and $C$). Look at the pattern the angles create. Are they corresponding angles? Does this create a counterexample to your conjecture from c)? Explain.

Answers: f) corresponding angles at parallel lines are equal; g) yes; h) When angles become not equal, they are not corresponding angles anymore. This does not create a counterexample, because the statement talks about corresponding angles and the unequal angles are not corresponding.

(end of activity)

Practice finding measures of corresponding angles at parallel lines. Draw on the board:

```
137°  x
```

SAY: The lines are parallel. ASK: What do you know about angle $x$ and the angle that measures $137°$? (they are corresponding angles, so they are equal) What is the measure of angle $x$? ($137°$)

Exercises: Find the measure of the corresponding angles.

a)  

```
  70°
  x
```

b)  

```
  23°  a
```

```
  60°  65°
  y  x
```

```
  54°  y
```

c)  

```
  101°  x
```

Bonus:

Answers: a) $x = 70°$, b) $a = 23°$, c) $x = 101°$, $y = 54°$, Bonus: $x = 60°$, $y = 65°$
Find measures of angles between intersecting lines using corresponding, vertical, and supplementary angles. Review what vertical angles are and the fact that they are equal. Remind students what supplementary angles are. Then return to part c) in the previous exercises and work as a class to fill in all the missing angle measures. (see answers below)

Exercises: Find all the angles in the picture.

a) 

b) 

c) 

Bonus: 

Answers:

a) 

b) 

Bonus: 

Teacher Resource for Grade 8 — Unit 3 Geometry
(MP.3) **When corresponding angles are equal, lines are parallel.** Write on the board:

When lines are parallel, corresponding angles are equal.

SAY: We have seen that this statement is true. Let’s change the order in this sentence. Continue writing on the board:

When corresponding angles are equal, lines are parallel.

Point out that when we change an order in a sentence, the meaning changes. For example, the sentence “All boys are people” is true, but the sentence “All people are boys” is not true. ASK: Do you think the statement on the board is true? You might want to have students vote.

Ask students to draw a pair of lines that are not parallel and then a third line that intersects them. Have them pick a pair of corresponding angles in their drawing and measure them. ASK: Did anyone get a pair of equal corresponding angles? (no) Explain that this shows that the statement “When corresponding angles are equal, lines are parallel” is likely to be true, but does not prove it. SAY: However, mathematicians have proven that the conjecture is true. Write “true” beside the statement on the board. **NOTE:** If your class is ready for the challenge, you can have them do Extension 1 to prove the statement.

**Using corresponding angles to identify parallel lines.** Explain that the statement “When corresponding angles are equal, lines are parallel” allows you to tell which lines are parallel and which are not. Return to the picture in part c) of the previous exercises and ask students to circle a pair of corresponding angles for each pair of lines, using different colors. Point out that, to make identifying lines easier, you used small letters to name the lines in this picture. Then ask students to identify which lines are parallel. (b and c)

**Activity 2**

Have students work in pairs to complete this activity.

a) Draw two pairs of lines, one parallel and the other not but looking like it might be. (For example, draw a line and then place a ruler along the line as if to draw another line along the parallel side of the ruler. Then rotate the ruler very slightly.) Do not indicate which pair is which.

b) For each pair of lines, draw a third line intersecting the first and second lines.

c) Exchange your paper with a partner. Measure and compare the corresponding angles to identify the pair of parallel lines.

**(end of activity)**

**Introduce co-interior angles.** Explain that, when two lines intersect a third line, there can be other patterns of angles, which also have special names. Draw on the board:

![Diagram of co-interior angles]

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Explain that angles that create a pattern like in the letter C are called *co-interior angles* or same-side interior angles. For example, angles 4 and 6 in the picture are co-interior angles. Trace the letter C in the picture with your finger. Then copy the picture and have students find the second pair of co-interior angles. (∠3 and ∠5) Trace the letter C in the second picture, as shown below:

In parallel lines, co-interior angles are supplementary. Draw on the board:

Point to different angles in the picture and ASK: Is this angle co-interior with the angle given? Have students signal the answer with thumbs up or thumbs down. When students have identified the correct angle as co-interior, ASK: How can we find its measure? To prompt students to see the answer, label the angles as shown below and ask them to find angle x before finding angle y. (the lines are parallel, so the corresponding angles are equal, so ∠x = 138°; angles x and y are supplementary, so ∠y = 180° – 138° = 42°)

ASK: Do you think this will work for any pair of parallel lines that have a third line intersecting them? (yes) What do co-interior angles add to when lines are parallel? (180°) In other words, what can we call co-interior angles at parallel lines? (supplementary angles) Write on the board:

When lines are parallel, co-interior angles are supplementary so they add to 180°.

**Exercises:** Find the missing angle measures.

a)  

b)  

**Bonus:**

**Answers:** a) x = 128°, y = 52°; b) a = 121°, b = 59°, Bonus: x = y = 130°, z = 70°
Extensions

(MP.3) 1. SAY: Let's see what happens if we have two lines that intersect. Can we still have equal corresponding angles? Draw on the board:

```
  x
  y
  x
```

SAY: It looks like these two lines have equal corresponding angles. Maybe I did not draw them perfectly, but I've marked the angles I think are equal with an x. Circle the x at the bottom and SAY: I see a triangle in this picture. ASK: What do we call the angle I circled for that triangle? (external angle) What do we know about the measure of the external angle in a triangle? (the measure of the external angle equals the sum of the measures of the interior angles opposite it) How can we write it down using the letters in the diagram? (x = x + y) What does this say about the measure of angle y? ( y = 0°) Can that happen? (no) SAY: If the lines have equal corresponding angles they cannot intersect, because if they do we get a 0° angle between them. In mathematics, when you suppose something happens and you logically arrive at nonsense, you conclude that your original idea was incorrect. So in this case, we proved using logic that, if two lines meet a third line making equal corresponding angles, the lines are parallel.

(MP.3) 2. Use co-interior angles to explain why opposite angles in a parallelogram are equal.

Answer:

```
A
  B
  C
  D
```

AB || CD and angles B and C are co-interior angles, so \( \angle C = 180° - \angle B \).
AD || BC and angles B and A are co-interior angles, so \( \angle A = 180° - \angle B \).
This means \( \angle C = \angle A \).
Similarly, AB || CD and angles B and C are co-interior angles, so \( \angle B = 180° - \angle C \).
AD || BC and angles C and D are co-interior angles, so \( \angle D = 180° - \angle C \).
This means \( \angle B = \angle D \).

3. Demonstrate the equality between corresponding angles using translation—a slide. Make two copies of the picture below on transparencies. Place the transparencies one on top of the other and show students that they are identical. Then slide one transparency down the other so that the line KQ slides along the line KL to the position of LR, so that \( \angle MKQ \) slides to match \( \angle MLR \).

This means \( \angle MKQ = \angle MLR \).
Have students complete the statements below.

a) $KQ$ is parallel to ____ and $\angle RLK$ and $\angle QKM$ are _____________ angles, so $\angle QKM = \angle RLK = 70^\circ$.

b) $\angle QKM$ and $\angle TQS$ are corresponding angles and $\angle QKM = 70^\circ = \angle TQS$, so ____ is parallel to ____.

c) The quadrilateral $QRLK$ is a ________________.

**Answers:** a) $LR$, corresponding; b) $KL$, $RS$; c) parallelogram

**MP.1, MP.3, MP.7** 4. Draw a parallelogram that is not a rhombus. Measure each angle of the parallelogram and draw a ray that divides each angle into two equal angles. Extend each ray so that it intersects two other rays. What geometric shape can you see in the middle of the parallelogram? Use the sum of the angles in the shaded triangle to explain why this is so.

**Answer:** The shape in the middle is a rectangle. The acute angles of the triangle are both half of the angles of a parallelogram. The adjacent angles of a parallelogram add to $180^\circ$, so their halves add to $90^\circ$. Because the sum of the angles in a triangle is $180^\circ$, the third angle in each triangle is a right angle. Since there are three other triangles like the shaded triangle in the parallelogram, the shape in the middle has four right angles and must be a rectangle.

**MP.1, MP.3** 5. A restaurant has many windows of unusual shapes. One of them is the trapezoid shown below.

The restaurant owner, Bill, calls a window company to order a replacement for this window. He tells the company representative that the window is a trapezoid with $AB$ parallel to $CD$, and he gives some more information.

For each combination of information below, say whether Bill is giving enough information to create a unique trapezoid of the exact shape and size wanted. If so, provide directions for constructing the trapezoid if they are easy. If there is no unique trapezoid, draw two trapezoids with the given specifications. Hint: Draw a line parallel to $AD$ through point $C$. It separates the trapezoid into a quadrilateral and a triangle. Label the new line $EC$ and label the triangle $EBC$. What type of quadrilateral have you created? Can you construct this quadrilateral and the leftover triangle given the additional information?
a) window is a trapezoid with $AB$ parallel to $CD$; angles $A$ and $B$ and sides $AB$ and $DA$

b) window is a trapezoid with $AB$ parallel to $CD$; angles $A$ and $B$ and sides $AB$ and $CD$

c) window is a trapezoid with $AB$ parallel to $CD$; angles $A$ and $B$ and sides $BC$ and $DA$

d) window is a trapezoid with $AB$ parallel to $CD$; sides $AB$, $BC$, $CD$ and angle $B$

e) window is a trapezoid with $AB$ parallel to $CD$; sides $AB$, $BC$, $CD$, $DA$ and angle $A$
f) window is a trapezoid with $AB$ parallel to $CD$; the four sides, $AB$, $BC$, $CD$, $DA$

Answers:

a) Yes, the information Bill gives creates a unique trapezoid. Construct line $AB$ and two rays to form angles $A$ and $B$, and then draw the side $DA = 7$ cm. Draw a line parallel to $AB$ through $D$ and label it $DC$. The ray creating angle $B$ intersects with line $DC$ at $C$ and finishes the unique trapezoid.

b) Yes. To create the combined shape, create the triangle before the quadrilateral (a parallelogram), but determine the triangle sides and angles from the parallelogram. The parallelogram $AECD$ will have equal opposite sides, so $CD = AE = 5$ cm and $AD = EC$. Since $AD || EC$, $\angle A$ and $\angle CEB$ are corresponding angles at parallel lines and they are equal ($60^\circ$ each). Thus, in triangle $CEB$, $\angle CEB = 60^\circ$, $\angle CBE = \angle B = 66^\circ$, and side $EB = 5$ cm (because $AB = 10$ cm and $AE = DC = 5$ cm, $AB - AE = EB$, thus $10 - 5 = 5$ cm). Start by constructing the triangle. Then extend side $BE$ beyond point $E$ 5 cm to point $A$. Complete the parallelogram by drawing angle $\angle A = 60^\circ$ for line $AD$ and drawing a line $CD$ parallel to $AB$ through point $C$.

c) No. Without knowing the length of side $AB$, we don’t know where to construct angle $B$, so this leaves both sides $AB$ and $CD$ of unknown lengths.

d) Yes. Construct the side $AB$ and angle $B$. Construct the side $BC$. Since $CD || AB$, $\angle C = 180^\circ - \angle B = 114^\circ$, so construct angle $C$ so that $CD$ is 5 cm. Join points $A$ and $D$ to create the fourth side.

e) Yes. Construct line $AB$, angle $A$, and line $AD$. Since $ABCD$ is a trapezoid, $\angle D$ is a co-interior angle with $\angle A$, so $\angle D = 180^\circ - \angle A = 120^\circ$. Construct $\angle D$ and side $CD$ (5 cm). Join points $B$ and $C$.

f) Yes. To create the combined shape, create the triangle before the parallelogram, but determine the triangle sides from the parallelogram. The parallelogram $AECD$ will have equal opposite sides, so $AE = DC$ and $AD = EC$. Start the triangle with the sides $EB = 5$ cm ($AB - EB = AE = DC = 5$ cm), $BC = 6.2$ cm, and $EC = 7$ cm ($AD = 7$ cm = $EC$). Extend side $BE$ beyond point $E$ 5 cm to point $A$. From point $C$, extend line $CD$ 5 cm parallel to line $AB$ and join $AD$ (7 cm) to complete the parallelogram.
G8-15  Alternate Angles and Parallel Lines

Pages 109–111

Standards: 8.G.A.5

Goals:
Students will use informal arguments to establish facts about alternate angles at parallel lines. They will use these facts to informally prove that angles in a triangle add to 180°.

Prior Knowledge Required:
Can identify parallel lines
Can identify supplementary, vertical, corresponding, and co-interior angles
Knows the properties of supplementary, vertical, corresponding, and co-interior angles
Can draw and measure with a ruler and a protractor
Knows what counterexamples are

Vocabulary: alternate angles, co-interior angles, conjecture, corresponding angles, counterexample, parallel, straight angle, supplementary angles, vertical angles

Materials:
rulers
transparencies
overhead projector
protractors
The Geometer’s Sketchpad®

Introduce alternate angles. Draw on the board:

Explain that alternate angles are angles that create a pattern like in the letter Z. For example, angles 3 and 6 in the picture are alternate angles. Trace the letter Z on the picture with your finger. Copy the initial picture, trace the letter Z, and have students find the other pair of corresponding angles, as shown at right below:
If students do not see that the reflected pattern resembles a letter Z, copy the picture onto a transparency, highlight the letter Z, and turn it over so that students see the pattern.

**Exercises:** List the alternate angles.

a) List the alternate angles.

\[
\begin{align*}
\angle 6 \text{ and } \angle 3, \\
\angle 2 \text{ and } \angle 7
\end{align*}
\]

b) List the alternate angles.

\[
\begin{align*}
\angle d \text{ and } \angle k, \\
\angle b \text{ and } \angle n
\end{align*}
\]

**Answers:**

a) \(\angle 6 \text{ and } \angle 3, \angle 2 \text{ and } \angle 7\); b) \(\angle d \text{ and } \angle k, \angle b \text{ and } \angle n\)

**Alternate angles for parallel lines are equal.** Have students draw a pair of parallel lines by using the opposite sides of a ruler. Ask them to place the ruler across the lines they drew and draw a third line intersecting both the first and second lines. Have them identify both pairs of alternate angles and measure them. **ASK:** What do you notice about the measures of alternate angles? (they are equal) Did everyone create angles of the same size? (no) Did everyone get the same result: alternate angles in a pair of parallel lines are equal? (yes)

**ASK:** Are alternate angles always equal? Have students repeat the exercise above, this time starting with a pair of lines that are not parallel.

Students can also do Activity 1 below to discover that parallel lines create equal alternate angles. Part e) encourages students to pay close attention to what they see and to what changes on the screen.

**Activity 1**

Use The Geometer's Sketchpad® for this activity.

**Discovering that parallel lines create equal alternate angles.**

a) Draw a line. Label it \(AB\). Mark a point \(C\) not on the line, and construct a line parallel to \(AB\) through \(C\).

b) Draw a line through points \(B\) and \(C\).

c) On the line parallel to \(AB\), mark point \(D\) on the other side of \(BC\) from point \(A\).

d) Name two alternate angles in the diagram.

e) Measure the alternate angles you named. What do you notice? Make a conjecture.

f) Move the lines or the points \(A\), \(B\), or \(C\) around. Are the alternate angles always equal? **(MP.5)**

g) Pull point \(D\) to the other side of the line \(BC\) to stop the angles you measured being equal. Look at the pattern the angles create. Are they alternate angles? Does this create a counterexample to your conjecture from e)? Explain.

**Answers:**

e) alternate angles at parallel lines are equal; f) yes; g) When angles stop being equal, they are not alternate angles anymore. This does not create a counterexample because the statement talks about alternate angles and the unequal angles are not alternate.

*(end of activity)*
Proving that alternate angles at parallel lines are equal. To review supplementary, vertical, corresponding, and co-interior angles, draw the picture below on the board and have students find all the missing angle measures. Ask them to explain how they found each angle measure. Encourage multiple explanations. (sample answers: \( \angle b \) is co-interior with the given angle, so its measure is \( 180° - 40° = 140° \); \( \angle a \) and the given angle are corresponding angles, so they are equal; angles \( a \) and \( b \) are supplementary angles, so \( \angle b = 180° - 40° = 140° \))

ASK: Which angles in this picture are alternate angles? (\( \angle d \) and the given angle, \( \angle b \) and \( \angle n \)) Are they equal? (yes) Are alternate angles at parallel lines always equal? (yes) How can we explain using logic that these angles are equal? Replace the angle measure with the letter \( k \) and have students explain why angles \( \angle k \) and \( \angle d \) are equal. PROMPT: Which angle is corresponding with \( \angle k \)? (\( \angle a \)) What do we know about corresponding angles? (they are equal) What do we know about angles \( a \) and \( d \)? (they are equal) Why? (they are vertical angles)

Find measures of angles between intersecting lines using alternate, corresponding, vertical, and supplementary angles.

(MP.7) Exercises: Find all the angles in the picture.

(a)

(b)

(c)

Bonus:
Using alternate angles to identify parallel lines. Remind students that when they learned about corresponding angles, they looked at two statements: When lines are parallel, corresponding angles are equal, and when corresponding angles are equal, lines are parallel. Write on the board:

When lines are parallel, alternate angles are equal.

ASK: Is this true or false? (true; we have proved it using logic) Write “true” beside the statement. ASK: If we change the order in this statement, what statement will we get? (when alternate angles are equal, the lines are parallel) Write that statement on the board and explain that you want to investigate whether this statement is true. ASK: What do you need to do? (draw a pair of lines with equal alternate angles and check whether they are parallel) How can we check if lines are parallel? (For example, if the corresponding angles are equal, then the lines are parallel)

Draw on the board:

(Say) These lines have equal alternate angles. Which other angles do we know are equal to $x$? (vertical angles to the given ones) Have a volunteer mark the angles and explain why they are equal to $x$. ASK: What other types of angles can we see in this picture? (corresponding angles) Have another volunteer circle a pair of corresponding angles. ASK: Are the corresponding angles equal? (yes) What do we know about lines that have equal corresponding angles? (they are parallel) SAY: We have just proved that these two lines are parallel, so we have just proved that the statement is true. Write “true” beside the second statement on the board.
Practice identifying parallel lines using the equality between alternate angles. Return to the picture in part c) in the previous exercises and ask students to circle a pair of alternate angles for each pair of lines, using different colors. Ask students to identify which rays are parallel. (rays b and c)

Activity 2
Have students work in pairs to complete this activity.
a) Draw two pairs of lines, one pair that is parallel and the other not but looking like it might be. (For example, draw a line and then place a ruler along the line as if to draw another line along the parallel side of the ruler, but then rotate the ruler very slightly.) Do not indicate which pair is which.
b) For each pair of lines, draw a third line intersecting the first and second line.
c) Exchange your paper with a partner. Measure and compare the alternate angles to identify the pair of parallel lines.
(end of activity)

(MP.3) Proving that sum of the angles in a triangle is 180°. Explain that the properties of alternate angles allow us to prove using logic that the sum of the angles in a triangle equals 180°. Point out that, even though we discovered this fact and used it many times, we have not proven it using logic. Draw a triangle on the board and label the angles a, b, and c as shown below. Explain that you want to use alternate angles, so you are drawing a line parallel to one of the sides, through the opposite vertex. Draw the line, as shown below:

ASK: Are there any equal alternate angles in this picture? (yes) Is there an angle alternate to angle a? (yes) Point to different angles and have students signal thumbs up or thumbs down to show if this is the angle alternate with \(\angle a\). Repeat with \(\angle c\). Point to the three angles at the top of the picture and ASK: What type of angle do the three angles make? (straight angle) What is the measure of a straight angle? (180°) Ask students to write an equation that shows that the three angles add to 180°. \(a + b + c = 180°\) ASK: What did we need to prove to show that angles in a triangle add to 180°? \(a + b + c = 180°\) SAY: We have just proven precisely that fact.
Extensions
1. Demonstrate the equality between corresponding angles using transformations. Make two copies of the picture below on transparencies. Place the transparencies one on top of the other, and show students that they are identical. Have students identify a pair of alternate angles (e.g., \( \angle QKL \) and \( \angle NLK \)). Mark the angles with arcs.

(a) Slide the line \( KQ \) down along the ray \( KL \) to the position of \( LR \) so that \( \angle MKQ \) slides to \( \angle MLR \). This means \( \angle MKQ = \angle MLR \). The angles \( \angle MLR \) and \( \angle NLK \) are vertical angles, so they are equal. But \( \angle MLR = \angle MKQ \), so \( \angle MKQ = \angle NLK \).

(b) Press a pencil to act as a pivot to the point \( O \). Rotate the top transparency 180° around point \( O \) to show how angle \( \angle QKL \) becomes \( \angle NLK \).

(MP.3) 2. Sketch parallelogram \( ABCD \).

Fill in the blanks to prove using logic that opposite sides of a parallelogram are equal.

(a) Parallel lines \( AB \) and _____ intersect with the third line \( DB \). \( \angle ABD \) and \( \angle ____ \) are alternate angles at parallel lines, so \( \angle ABD = \angle ____ \). Label these angles with the same number of arcs.

(b) Parallel lines \( AD \) and _____ intersect with the third line \( DB \). \( \angle ADB \) and \( \angle ____ \) are alternate angles at parallel lines, so \( \angle ADB = \angle ____ \). Label these angles with the same number of arcs.

(c) Triangles \( ABD \) and _____ are congruent by the _____ congruence rule because they have two pairs of equal corresponding ___________ and a common ____________.

(d) \( \triangle ABD \cong \triangle ____ \), so \( AB = ____ \) and \( AD = ____ \).

Answers: a) \( CD \), \( CDB \), \( CDB \); b) \( BC \), \( CBD \), \( CBD \); c) \( CDB \), ASA, angles, side; d) \( CDB \), \( CD \), \( CB \)

(MP.1, MP.7) 3. Which is larger, \( 2^{75} \) or \( 3^{50} \)? Do not use a calculator. Explain how you know your answer is correct.

Sample answer: \( 2^3 = 8 < 9 = 3^2 \), so \( (2^3)^{25} < (3^2)^{25} \), so \( 2^{75} < 3^{50} \).

Redirecting students: Encourage students to ask themselves, “What makes this problem hard?” (the exponents are large) Encourage students to explore similar questions with smaller powers of 2 and 3: Which powers of 2 are bigger or smaller than other powers of 3? For example, what does \( 2^2 > 3 \) tell you about \( 2^7 > 3^{50} \)? Encourage students to look for similar patterns.
Investigating Congruence (1)

A conjecture is a statement you think is true, but you have not proved it with logic.

1. Conjecture: If two triangles have 3 pairs of equal sides, then the triangles are congruent.
   a) Test the conjecture:
      Take 3 straws of different lengths and put a pipe cleaner though them. Make a triangle with them and trace it below.

   b) Try to make a different triangle with the same 3 straws.
      Are your triangles congruent? _______

   c) Repeat parts a) and b) with a different set of 3 straws.
      Are your triangles congruent? _______

   d) Do you think this result will be true for all triangles? _______

2. Conjecture: If 2 sides and the angle between them in one triangle are equal to 2 sides and the angle between them in another triangle, the triangles are congruent.
   a) Test the conjecture:
      Draw 3 triangles, each with one side 3 cm long, one side 5 cm long, and a 45° angle between these sides. Try to make the triangles different.

   Are the triangles congruent? _______

   b) Do you think this result will be true for all triangles? _______
Investigating Congruence (2)

3. Continue testing the conjecture from Question 2.
   Pick 3 measurements: $a = \underline{\hspace{2cm}}$ cm, $b = \underline{\hspace{2cm}}$ cm, $\angle C = \underline{\hspace{2cm}}^\circ$
   Draw 3 triangles, each with one side $a$ cm long, one side $b$ cm long, and a $C^\circ$ angle between these sides.

   Are your triangles congruent? ______

4. Conjecture: If 2 angles and the side between them in one triangle are equal to 2 angles and the side between them in another triangle, the triangles are congruent.
   a) Test this conjecture:
      Draw 3 triangles, each with a $60^\circ$ angle, a $45^\circ$ angle, and a side 5 cm long between these angles.

      Are the triangles congruent? ______
   b) Do you think this result will be true for all triangles? ______
   c) Pick 3 new measurements: $a = \underline{\hspace{2cm}}$ cm, $\angle B = \underline{\hspace{2cm}}^\circ$, $\angle C = \underline{\hspace{2cm}}^\circ$
   d) Draw three triangles, each with a $B^\circ$ angle, a $C^\circ$ angle, and a side $a$ cm long between these angles.

      Are your triangles congruent? ______
Congruence Rules on The Geometer’s Sketchpad® (1)

1. Using the Polygon tool, construct a triangle, \( ABC \). Measure the sides of your triangle.

2. a) Construct a point \( D \) outside \( \Delta ABC \). Using a command for constructing circles and the length of \( AB \) as the radius, construct a circle with center \( D \). Construct a line segment \( DE = AB \). Hide the circle.
   
b) Using a command for constructing circles and the length of \( BC \) as the radius, construct a circle with center \( E \).
   
c) Using a command for constructing circles and the length of \( AC \) as the radius, construct a circle with center \( D \).
   
d) Construct a point that is on both circles. Label it \( F \). Use the Polygon tool to construct triangle \( DEF \). Hide the circles.

3. a) Which sides are equal in \( \Delta ABC \) and \( \Delta DEF \)?
   
   
   
   
   
   b) Measure the angles of \( \Delta ABC \) and \( \Delta DEF \). What can you say about \( \Delta ABC \) and \( \Delta DEF \)?

4. Try to move the vertices of \( \Delta DEF \) around.
   
   Can you move \( \Delta DEF \) onto \( \Delta ABC \) to check whether they are congruent? ______

5. Move \( \Delta DEF \) away from \( \Delta ABC \). Try to move the vertices of \( \Delta ABC \) around.
   
   a) What happens to \( \Delta DEF \) when you modify \( \Delta ABC \)?

   
   
   
   
   
   b) What can you say about the triangles \( \Delta ABC \) and \( \Delta DEF \)?

   
   

6. If two triangles have 3 pairs of equal sides, then the triangles are congruent.
   
   True or false? ______
1. Using the Polygon tool, construct a triangle, \( \triangle ABC \). Measure the sides \( AB \) and \( BC \) and the angle \( \angle ABC \) of your triangle.

2.  
   a) Construct a point \( D \) outside \( \triangle ABC \). Using a command for constructing circles and the length of \( AB \) as the radius, construct a circle with center \( D \). Construct the line segment \( DE = AB \). Hide the circle.
   b) Construct a circle with center \( E \) and the radius equal to the length of \( BC \).
   c) Use the Transformation menu options. Select \( E \) as a center of rotation. Select the measure of \( \angle ABC \) as the angle of rotation. Rotate point \( D \) around \( E \) by the angle selected. Construct a ray from \( E \) through the image of \( D \).
   d) Construct a point that is on the ray and the circle. Label it \( F \). Use the Polygon tool to construct triangle \( \triangle DEF \). Hide the circle and the ray \( EF \).
   e) Measure the angle \( \angle DEF \). Did you construct an angle equal to \( \angle ABC \)?

3.  
   a) Which sides and angles are equal in \( \triangle ABC \) and \( \triangle DEF \)?
      
      \[ \quad \]
      \[ \quad \]
      \[ \quad \]
      \[ \quad \]
   b) Measure the rest of the sides and angles of \( \triangle DEF \) and \( \triangle ABC \).
      
      What can you say about \( \triangle ABC \) and \( \triangle DEF \)?

4. Try to move the vertices of \( \triangle DEF \) around.
   a) How does your triangle change?
   b) Can you move \( \triangle DEF \) onto \( \triangle ABC \) to check whether they are congruent?

5. Move \( \triangle DEF \) away from \( \triangle ABC \). Try to move the vertices of \( \triangle ABC \) around.
   a) What happens to \( \triangle DEF \) when you modify \( \triangle ABC \)?
       
       
   b) What can you say about \( \triangle ABC \) and \( \triangle DEF \)?

6. If 2 sides and the angle between them in one triangle are equal to 2 sides and the angle between them in another triangle, the triangles are congruent.
   True or false?
Congruence Rules on The Geometer’s Sketchpad® (3)

1. Using the Polygon tool, construct a triangle, $ABC$. Measure the sides $AB$ and and the angles $ABC$ and $BAC$.

2. a) Construct a point $D$ not on the triangle. Draw a ray starting at point $D$. Label it $l$.
   
   b) Use the Transform menu options to select $D$ as a center and the measure of $\angle BAC$ as the angle of rotation. Rotate the ray $l$ around point $D$ by the angle selected. Label this ray $r$.
   
   c) Using a command for constructing circles and the length of $AB$ as the radius, construct a circle with center $D$. Mark the point where the circle intersects ray $r$ and label it $E$. Construct line segment $DE = AB$. Check that the measure of $DE$ is the same as the measure of $AB$. Hide the circle.
   
   d) Draw the ray $ED$. Remember to start at point $E$. Hide ray $r$.
   
   e) Use the commands in the Transform menu. Select $E$ as a center of rotation. Select the measure of $\angle ABC$ as the angle of rotation. Rotate ray $ED$ around $E$ by the angle equal to the measure of $\angle ABC$. Label the new ray $m$.
   
   f) Construct a point $F$ that is the intersection point of rays $l$ and $m$.
      
      Measure the angle $DEF$. Did you construct $\angle DEF = \angle ABC$? ________
      
      Measure the angle $EDF$. Did you construct $\angle EDF = \angle BAC$? ________
   
   g) Use the Polygon tool to construct triangle $DEF$. Hide the rays.

3. a) Which sides and angles are equal in $\triangle ABC$ and $\triangle DEF$?
      
      ________ = ________, ________ = ________, ________ = ________

   b) Measure the rest of the sides and angles of $\triangle DEF$.
      
      What can you say about $\triangle ABC$ and $\triangle DEF$? __________________________________________

4. Can you move $\triangle DEF$ onto $\triangle ABC$ to check whether they are congruent? ________

5. Move $\triangle DEF$ away from $\triangle ABC$. Try to move the vertices of $\triangle ABC$ around.
   
   a) What happens to $\triangle DEF$ when you modify $\triangle ABC$? __________________________________________

   b) What can you say about $\triangle ABC$ and $\triangle DEF$? __________________________________________

6. If 2 angles and the side between them in one triangle are equal to 2 angles and the side between them in another triangle, the triangles are congruent.

   True or false? ________
Two Pentagons

a) How many right angles does each pentagon have? ______

b) Fold the pentagons so that you can see that the remaining angles are all equal.

c) Does each side on pentagon A have a side of the same length on pentagon B? ______

d) Is there the same number of sides of each length on both pentagons? ______

e) If you place one pentagon on top of the other, do they match? ______

f) Are they the same shape? ______

g) Can we say that these pentagons have the same sides and angles? ______

h) Are the pentagons congruent? ______

i) What makes the answers to g) and h) different?