F8-18  Finding the y-intercept from Ordered Pairs

Pages 15–16

Standards: 8.F.A.3, 8.F.B.4

Goals:
Students will find the y-intercept of a line from a set of ordered pairs.

Prior Knowledge Required:
Can add, subtract, and divide integers
Can find the slope and the y-intercept of a line from its graph, its equation, or its table

Vocabulary: ordered pair, quadrant, rise, run, slope, x-coordinate, y-coordinate, y-intercept

Review plotting ordered pairs. On a grid on the board, mark the point (1, −2) and ASK: What is the x-coordinate of this point? (1) Write on the board:

\[ x = 1 \]

Repeat with the y-coordinate. ASK: How do we write this as an ordered pair? (as (1, −2))
Repeat with a point at (−1, −2), emphasizing the importance of the order of the coordinates.
Mark a point in the second quadrant, such as (−3, 4), and show students how to identify the signs of its coordinates. SAY: The point is on the negative side of the x-axis and the positive side of the y-axis, so the signs for the coordinates of the ordered pair will be (−, +). Repeat with a point in the first quadrant.

Exercises: Plot the points on a grid.

\[ A (5, 1) \quad B (-1, -4) \quad C (-3, -5) \quad D (-4, -2) \]
\[ E (2, 2) \quad F (-3, 2) \quad G (4, -3) \quad H (-2, 4) \]

Answers:

\[ y \]
\[ \quad \]
\[ \quad \]
\[ \quad \]
\[ \quad \]
\[ x \]

L-34  Teacher Resource for Grade 8 — Unit 1 Functions
(MP.7) Finding the \( y \)-intercept from ordered pairs by graphing. Explain to students that, when they plot two ordered pairs on a grid, they can join the points to make a line and then find the \( y \)-intercept by extending the line. As an example, plot two points, \((1, 3)\) and \((2, 5)\), on a grid on the board. Ask a volunteer to join them to make a line and then extend the line to find the \( y \)-intercept. (\( y \)-intercept: 1; see completed graph below)

![Graph showing the \( y \)-intercept](image)

**Exercises:** Graph the two ordered pairs and join them to make a line. Extend the line to find the \( y \)-intercept.

- a) \((-1, 3), (2, 6)\)
- b) \((2, 4), (1, 1)\)
- c) \((-2, 1), (1, -2)\)

**Answers:**

- a) \( y \)-intercept: 4
- b) \( y \)-intercept: -2
- c) \( y \)-intercept: -1

**Finding the \( y \)-intercept from ordered pairs without graphing.** SAY: You just learned how to find the \( y \)-intercept from ordered pairs by graphing. But the graphing method only works well when two points are close to the \( y \)-axis. Write on the board:

- \((14, 5)\)
- \((8, 11)\)

SAY: You would need a big grid to plot these ordered pairs, draw the line that goes through the points, and extend it to cross the \( y \)-axis. However, as in the previous lesson, you can use a table to find the \( y \)-intercept. Draw on the board:

<table>
<thead>
<tr>
<th>( x )</th>
<th>slope ( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
ASK: What is the run? PROMPT: The run is the change in $x$. (−6) ASK: What is the rise? (the change in $y$, +6) Write the run and rise in the circles, as shown below:

<table>
<thead>
<tr>
<th>$x$</th>
<th>slope $\times x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−6</td>
<td>+3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

ASK: What is the slope? ($6/(−6)=−1$) Multiply the $x$-coordinate in each row by the slope and complete the table. Draw an arrow underneath the table, from the second column to the third column. The table should look like this:

<table>
<thead>
<tr>
<th>$x$</th>
<th>slope $\times x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−6</td>
<td>−6</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>−8</td>
<td>11</td>
</tr>
</tbody>
</table>

ASK: What must you add to the second column to get $y$? (19) Write “Add 19” underneath the arrow and SAY: The $y$-intercept is 19.

(MP.7) Exercises: For the line that goes through the given points in the previous exercise, find the $y$-intercept without graphing.

a) ((1, 3), (2, 6))

b) (2, 4), (1, 1)

c) (−2, 1), (1, −2)

Solutions:

a) $\begin{array}{c|c|c}
 x & slope \times x & y \\
-1 & -1 & 3 \\
2 & 2 & 6 \\
\end{array}$

Add 4

slope = \frac{rise}{run} = \frac{+3}{+3} = 1

$y$-intercept: 4

b) $\begin{array}{c|c|c}
 x & slope \times x & y \\
2 & 6 & 4 \\
1 & 3 & 1 \\
\end{array}$

Subtract 2

slope = \frac{rise}{run} = \frac{−3}{−1} = 3

$y$-intercept: −2

c) $\begin{array}{c|c|c}
 x & slope \times x & y \\
−2 & 2 & 1 \\
1 & -1 & −2 \\
\end{array}$

Subtract 1

slope = \frac{rise}{run} = \frac{−3}{+3} = −1

$y$-intercept: −1

Summarize finding the $y$-intercept. Remind students that they can find the $y$-intercept using the following methods:

1. From a graph: Students can plot two points on a graph, join the points, and extend the line to cross the $y$-axis. The intersection point is the $y$-intercept.
2. From an equation: By substituting 0 for $x$, students can find the $y$-intercept. Point out this trick: in an equation with the form $y = mx + b$, $b$ is the constant term and is the $y$-intercept.
3. From a table: Students can write the coordinates in a table and find the run, rise, and slope. For each $x$ value, multiply by the slope in the second column. The number that students must add (or subtract) to the second column to get $y$, in the third column, is the $y$-intercept.
4. From ordered pairs: Students can write the ordered pairs in a table and find the $y$-intercept from the table. Where $x = 0$, $y$ is the $y$-intercept.
Note to students that the graphing method is the general method for finding the y-intercept. Even if students have the equation—or table or ordered pairs—they might choose to graph to find the y-intercept. The other methods are options for students who do not want to graph.

(MP.1, MP.3, MP.5) Exercises:
a) Four linear functions are represented in different ways below. Find the y-intercept for each.

<table>
<thead>
<tr>
<th>A.</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-7</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

b) Which has the highest y-intercept?
c) Which has a negative y-intercept?
d) Which goes through the origin?

Answers: a) A: -3, B: 0, C: +2.5, D: +1; b) C; c) A; d) B

Extensions
(MP.3, MP.7) 1. Without graphing, find the x-intercept of the line that goes through the points (-2, 1) and (1, -2). Justify your answer.

Sample answers:
• As y decreases from 1 to -2, x increases from -2 to 1, so as y decreases by 3, x increases by 3. But in order to increase x to 0 from -2, x needs to increase by 2, so y needs to decrease from 1 by 2, which means y needs to decrease to -1. The x-intercept is -1.
• I noticed that x = 0 is 2/3 of the way from -2 to 1, so the y-value when x = 0 must be 2/3 of the way from 1 to -2, so the y-value when x = 0 must be -1. The x-intercept is -1.

Whole-class follow-up: Present both solutions above, or have volunteers do so. Then compare the solutions. ASK: Are both solutions correct? Do they get the same answer? Which way is faster? (the second way) Which way always works? (the first way) Would the second way work for the line that goes through (2, 4) and (1, 1)? (no, because 0 is not partway from 2 to 1; yes, but you would have to use that 1 is halfway between 2 and 0 for the x-coordinate, so 1 is also halfway between 4 and the y-coordinate when x = 0, so that is -2)

(MP.6, MP.7) 2. Could the statement be true? Explain.
a) line A is y = 3x and line B is y = 4x
b) line A is y = 3x and line B is y = -4x
c) line A is y = 0.8x and line B is y = 0.6x
d) line A is y = -0.8x and line B is y = -0.6x

Answers: a) no, because line A is steeper than line B, so its slope should be greater; b) no, because both lines have positive slope; c) yes, because both lines have positive slope and line A is steeper, so it should have the greater slope; d) no, because both lines have positive slope
F8-19 Writing an Equation of a Line Using the Slope and \( y \)-intercept

Pages 17–19

Standards: 8.F.A.3, 8.F.B.4

Goals:
Students will write the equation of a line in slope-intercept form.

Prior Knowledge Required:
Can add, subtract, and divide integers
Can find the slope and the \( y \)-intercept from a graph, table of values, or ordered pairs

Vocabulary: coefficient, constant term, rise, run, slope, slope-intercept form, \( y \)-intercept

Review slopes between any two points on a straight line are equal. Draw on the board:

\[
\begin{array}{c|c|c}
\hline
\text{x} & \text{y} \\
\hline
-4 & -5 \\
-3 & -3 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c}
\hline
\text{x} & \text{y} \\
\hline
-3 & -3 \\
1 & 5 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c}
\hline
\text{x} & \text{y} \\
\hline
-4 & -5 \\
1 & 5 \\
\hline
\end{array}
\]

ASK: What is the run from \( A \) to \( B \)? (+1) What is the rise from \( A \) to \( B \)? (+2) What is the slope of \( AB \)? (2/1 or 2) Have a volunteer find the slope of \( BC \). (run = +4, rise = +8, slope = 8/4 or 2) Then have another volunteer find the slope of \( AC \). (run = +5, rise = +10, slope = 10/5 or 2) ASK: Is the slope the same between any two points on a straight line? (yes) Point to the line and SAY: The slope is equal between every set of two points on a straight line, and that means that, if we found the slope using a table of values for that linear function, we would get the same answer.

Draw on the board:
Ask a volunteer to find the slope for each linear function. (a) run = +1, rise = +2, slope = 2/1 = 2; 
b) run = +4, rise = +8, slope = 8/4 = 2; c) run = +5, rise = +10, slope = 10/5 = 2)

ASK: For the graph we did earlier, what is the y-intercept? (3) SAY: The coordinates of the 
y-intercept are (0, 3). Ask a volunteer to find the slope from a table with two points, the 
y-intercept (0, 3) and C (1, 5). (see solution below)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

run = +1, rise = +2, slope = \frac{rise}{run} = \frac{2}{1} = 2

ASK: Among all the points we have used to find the slope on the graph, which two x-coordinates 
were the easiest to use to find the slope? (x equals 0 and x equals 1) SAY: When you use x = 0 
and x = 1, the run is 1 and the rise is the slope of the line.

**Exercises:** Make a table with x = 0 and x = 1 for the given equation. Find the slope and y-intercept. 
a) \(y = 2x + 3\) 
b) \(y = 1.5x - 2\) 
c) \(y = -x + 1.5\)

**Answers:**
a) 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

y-intercept: 3 
run = +1, rise = +2 
slope = 2/1 = 2

b) 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

y-intercept: -2 
run = +1, rise = +1.5 
slope = 1.5/1 = 1.5

c) 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

y-intercept: 1.5 
run = +1, rise = -1 
slope = -1/1 = -1

Remind students that, in an equation such as \(y = 2x + 3\), the coefficient of the x (in this case, 2) 
is the slope of the line and the constant term (in this case, +3) is the y-intercept of the line. Write 
on the board:

\[y + 1 = 4x + 3\]

SAY: I had a student who said that the y-intercept of this equation is +3. ASK: How can we 
check if that is right or wrong? (by replacing x with 0 and finding y) Write on the board:

\[y + 1 = 4(0) + 3\]
\[y + 1 = 3\]
\[y = 3 - 1\]
\[y = 2\]
ASK: What is the y-intercept? (+2) Explain to students that, to find the y-intercept in an equation, first they need to rearrange the equation to isolate $y$. The constant term in the rearranged equation is the y-intercept. Write on the board:

$$y + 1 = 4x + 3$$
$$y = 4x + 3 - 1$$
$$y = 4x + 2$$  
So the y-intercept is +2.

**Exercises:** Find the slope and the y-intercept of the line from the equation.

a) $y = -3x + 0.5$    

b) $y = -x - 2$

c) $y = 2 - x$

d) $y - 4 = 5 + 2x$

**Sample solution:** d) $y - 4 = 5 + 2x$, so $y = 2x + 9$, so the slope is +2 and the y-intercept is +9

**Answers:** a) slope: -3, y-intercept: +0.5; b) slope: -1, y-intercept: -2; c) slope: -1, y-intercept: 2

(MP.1, MP.3, MP.4) Writing an equation in slope-intercept form. SAY: To write an equation for a line, multiply $x$ by the slope, add the y-intercept, and write the result equal to $y$. Write on the board:

$$
\text{slope} = 2, \ y - \text{intercept} = 5, \ so \ the \ equation \ is \ y = 2x + 5
$$

Write on the board:

$$
y = mx + b
$$

SAY: If $m$ is the slope of a line and $b$ is the y-intercept, then $y = mx + b$. Mathematicians call this way of writing the equation the **slope-intercept form** of the line.

**Exercises:** For a line with the given slope and y-intercept, write the equation of the line in slope-intercept form.

a) slope = -2, y-intercept = 3    
b) slope = 0.5, y-intercept = -1

c) slope = -1, y-intercept = 1.5    
d) slope = $\frac{1}{2}$, y-intercept = 1

e) slope = -1, y-intercept = $-\frac{2}{3}$    
f) slope = $-\frac{3}{4}$, y-intercept = $-\frac{1}{5}$

**Answers:** a) $y = -2x + 3$, b) $y = 0.5x - 1$, c) $y = -x + 1.5$, d) $y = \frac{1}{2}x + 1$, e) $y = -x - \frac{2}{3}$, f) $y = -\frac{3}{4}x - \frac{1}{5}$

Writing the equation of a line using the y-intercept and a point. Explain to students that they can write the equation of a line using the y-intercept and another point. SAY: Whenever there are two points, you can find the slope of the line that goes through the points. Write on the board:

$y$-intercept = 2, $P$ (1, 3)
SAY: The y-intercept is 2, which means the line crosses the y-axis at (0, 2). Let’s name the y-intercept point A. Write on the board:

\[ A(0, 2), P(1, 3) \]

SAY: With two points \( A \) and \( P \), you can find the slope. Then you can use the slope and the y-intercept to write the equation of the line. Ask a volunteer to find the slope and write the equation of the line, as shown below.

\[
\text{run} = +1, \text{rise} = +1, \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1
\]

equation: \( y = x + 2 \)

**Exercises:**
1. Find the slope and the y-intercept. Write the equation of the line.
   a) \( A(0, -1), B(2, 3) \)  
   b) \( A(3, 1), B(0, -2) \)  
   c) \( A(-2, -3), B(0, 0) \)

**Answers:**
   a) run = +2, rise = +4, slope = \( \frac{4}{2} = 2 \), y-intercept = -1, equation: \( y = 2x - 1 \)
   b) run = -3, rise = -3, slope = \( \frac{-3}{-3} = 1 \), y-intercept = -2, equation: \( y = x - 2 \)
   c) run = +2, rise = +3, slope = \( \frac{3}{2} = 1.5 \), y-intercept = 0, equation: \( y = 1.5x \)

2. Check your answers to Exercise 1 by substituting one point in each equation.
   **Sample answer:** a) \( y = 2x - 1 \), \(-1 = 2(0) - 1, -1 = -1 \checkmark\)

**Writing the equation of a line using the graph of the line.** Remind students that, to find the slope of a line from a graph, it is easier to use two points with integer coordinates. Then, remind students that they can also extend the line to cross the y-axis to find the y-intercept. Draw on the board:

[Diagram of a graph with points A and B]

ASK: What is the run from \( A \) to \( B \)? (+1) What is the rise from \( A \) to \( B \)? (+2) What is the slope of \( AB \)? (2/1 or 2) Extend the line to find the y-intercept (3), then write the equation of the line on the board in slope-intercept form, as shown below:

\[ y = 2x + 3 \]
Exercises:

(MP.1, MP.3) 1. Extend the line to find the \( y \)-intercept. Mark two points with integer coordinates and find the slope of the line.

\begin{align*}
\text{a) } & \quad y\text{-intercept } = 2, \text{ run } = +1, \text{ rise } = +1, \text{ slope } = 1/1 = 1; \\
\text{b) } & \quad y\text{-intercept } = -3, \text{ run } = +1, \text{ rise } = +2, \text{ slope } = 2/1 = 2; \\
\text{c) } & \quad y\text{-intercept } = 0.5, \text{ run } = +2, \text{ rise } = -1, \text{ slope } = -1/2 = -0.5
\end{align*}

Answers:

2. Write the equation for each line in Exercise 1 in slope-intercept form.

\begin{align*}
\text{a) } & \quad y = x + 2, \quad \text{b) } y = 2x - 3, \quad \text{c) } y = -0.5x + 0.5
\end{align*}

3. Write the equation in the form \( y = mx + b \) and state the slope and \( y \)-intercept.

\begin{align*}
\text{a) } & \quad x = 2y + 3 \quad \text{b) } -7 = -x - y \quad \text{c) } 0 = -3x + 5y + 7 \\
\text{d) } & \quad -2x - 3y = 6y + 8x \quad \text{e) } -3(-2 + 3x) = 3y \quad \text{f) } 0.5y - 1.3x = 8 \\
\text{g) } & \quad \frac{2}{3}x + \frac{4}{5}y - 3\frac{1}{2} = 0 \quad \text{h) } 0.8 = 0.3(5x - 2y) \quad \text{i) } 5x - \frac{2}{3}y = \frac{3}{5}x + 13
\end{align*}

Selected answers:

\begin{align*}
\text{a) } & \quad y = \frac{1}{2}x - \frac{3}{2}, \text{ slope is } \frac{1}{2}, \text{ y-intercept is } -\frac{3}{2}; \\
\text{b) } & \quad y = -x + 7; \quad \text{c) } y = \frac{3}{5}x - \frac{7}{5}; \\
\text{d) } & \quad y = -\frac{10}{9}x; \quad \text{e) } y = -3x + 2; \quad \text{f) } y = 2.6x + 16; \quad \text{g) } y = -\frac{5}{6}x + 4\frac{3}{8}; \quad \text{h) } y = 2.5x - 1.3
\end{align*}

Extensions

(MP.8) 1. a) Find the \( x \)-intercept of the line. Show your work.

\begin{align*}
\text{i) } & \quad y = 2x + 6 \quad \text{ii) } y = 3x - 6 \quad \text{iii) } y = 2x - 5 \quad \text{iv) } y = -3x - 6
\end{align*}

b) Describe what you are always doing the same in part a).

c) Find a formula for finding the \( x \)-intercept of the line \( y = mx + b \).

Answers:

\begin{align*}
\text{a) } & \quad i) 0 = 2x + 6, \text{ so } -6 = 2x, \text{ so } x = -3; \quad ii) 0 = 3x - 6, \text{ so } 3x = 6, \text{ so } x = 2; \quad iii) 0 = 2x - 5, \text{ so } 2x = 5, \text{ so } x = 5/2; \quad iv) 0 = -3x - 6, \text{ so } -3x = 6, \text{ so } x = -2
\end{align*}

b) I am always setting \( y = 0 \) and then solving the equation for \( x \).

c) I put \( mx + b = 0 \), so \( mx = -b \), so \( x = -b/m \).

2. Determine \( m \) and \( b \), then use the formula that you found in Extension 1.c) to find the \( x \)-intercept of the line.

\begin{align*}
\text{a) } & \quad y = x + 1 \quad \text{b) } y = 1.5x \quad \text{c) } y = -2x + 2 \quad \text{d) } y = 2x - 1
\end{align*}

Answers:

\begin{align*}
\text{a) } & \quad m = 1, \text{ } b = 1, \text{ x-intercept } = -1/1 = -1; \quad \text{b) } m = 1.5, \text{ } b = 0, \text{ x-intercept } = 0/1.5 = 0; \\
\text{c) } & \quad m = -2, \text{ } b = 2, \text{ x-intercept } = -2/-2 = 1; \quad \text{d) } m = 2, \text{ } b = -1, \text{ x-intercept } = 1/2 = 0.5
\end{align*}
F8-20 Comparing Linear Functions

Pages 20–21

Standards: 8.F.A.2

Goals:
Students will compare linear functions represented in different ways by graphing.

Prior Knowledge Required:
Can graph a line using its equation
Can find the absolute value of a number

Vocabulary: absolute value, greater slope, slope-intercept form, steeper, steepness

(MP.1, MP.3) Comparing lines by graphing. Draw on the board:

A. \( y = x + 2 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

B. \( y = 2x + 1 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

Ask a volunteer to find the slope of each line. (1, 2) ASK: Which line has the greater slope? (B)

Draw on the board to the right of graphs A and B:

C. \( y = -3x + 7 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

ASK: What is the slope of this line? (-3/1 = -3) Point to the three graphs on the board and SAY: I would like to compare these three graphs. Ask a volunteer to order all the lines from smallest to largest slope. (C, B, A) ASK: Which has the greatest slope? (B) SAY: Suppose that you want to compare steepness, or how steep they are. Point out that comparing the steepness of different lines is like comparing the slopes of different mountains; students will need to look at two points on each graph and think about which line would be the most difficult to walk up.
ASK: Which graph has the steepest slope? (C) Does a greater slope always mean a steeper slope? (no) SAY: The sign (positive or negative) for a slope indicates which direction the line goes in, but the absolute value indicates whether a slope will be more or less steep than another one. Remind students that the absolute value of a number is its distance from zero, regardless of whether it is to the left or right of zero; for example, |−3| = 3 and |+2| = 2. Ask a volunteer to order the three slopes graphed earlier by absolute value, from smallest to largest. (A, B, C) SAY: A steeper slope means a higher absolute value for the slope.

**Exercises:** Graph the two functions on the same grid. Determine which function has the greater slope and which is steeper.

a) \(y = x + 1.5\)  
\(y = 3x - 1\)

**Answers:**

a) 

![Graph of y = x + 1.5 and y = 3x - 1]

greater slope: \(y = 3x - 1\)  
steeper: \(y = 3x - 1\)

You can ask some higher-level questions if all students answered the previous exercises. ASK: If two lines are parallel, what do you know about their slopes? (their slopes are equal) What do you know about how steep they are? (they are equally steep) If two lines intersect, what do you know about their slopes? (they have different slopes) If two lines are equally steep, do they have the same slope? (it depends—for example, two lines each with slope 3 have equal slopes and are equally steep, but a line with slope 3 and another line with slope −3 are equally steep, but have different slopes.)

**Exercises:**

a) The information needed to find a slope is given in four different ways. Find the slopes of each of the given linear functions.

A. \[
\begin{array}{c|c}
\text{x} & \text{y} \\
\hline
-3 & 1 \\
-1 & 5 \\
0 & 7 \\
2 & 11 \\
\end{array}
\]  
B. \[
\begin{array}{c|c}
\text{x} & \text{y} \\
\hline
-2 & -1 \\
0 & 1 \\
2 & 3 \\
\end{array}
\]

b) Which two have the same slope?
c) Which has the greatest slope?
d) Which has the steepest slope?
e) Write the equation in slope-intercept form for A, B, and D.

**Answers:**
a) slopes: A: 2, B: \(-2.5\), C: \(-2\), D: \(-2\); b) C and D; c) A; d) B; e) A: \(y = 2x + 7\), B: \(y = -2.5x + 1\), D: \(y = -2x + 2\)

**Extensions**
1. You can find the intersection of two lines by graphing. Graph both functions on the same grid and find the coordinates of the intersection point.

   a) \(y = x + 1\)  
   b) \(y = -2x + 1\)  
   c) \(y = 2x\)  

   **Answers:**
   a) intersection point: (1, 2)  
   b) intersection point: (\(-1, 3\))  
   c) intersection point: (1, 2)

2. A line passes through the points A (2, \(y\)) and B (\(-3, 9\)) and has slope 2. Find the value of \(y\).

   **Solution:** run = \(-3 - 2 = -5\), rise = \(9 - y\), so slope = \(\frac{9 - y}{-5} = 2\). After cross multiplying, \(9 - y = -10\) and \(y = 9 + 10 = 19\)

   **(MP.3)** 3. A line passes through A (\(-3, 4\)) and has slope 2. Will it pass through B (7, 23)? Explain.

   **Sample answers:**
   • No, the line will not pass through point B. As the slope is 2, the run from point A to point B would be \(7 - (-3) = 10\) and the rise would be 20 (because \(20/10 = 2\)), which means the line would pass through \((7, 4 + 20)\) or \((7, 24)\) rather than B (7, 23).
   • No, the slope of the line through points A and B is \((23 - 4)/(7 - (-3)) = 19/10 = 1.9\), not 2.

   **NOTE:** The following extension is for advanced students only.

4. A line passes through A (2, \(-3\)) and B (2\(k\), \(-3k\)).

   a) Can you determine a line when \(k = 1\)?
   b) What will the slope be for all other values of \(k\)?

   **Solutions:**
   a) No, \(k = 1\) gives point B (2, \(-3\)), which has the same coordinates as point A, and you cannot use a single point to determine a line.
   b) rise = \(-3k - (-3) = -3k + 3 = -3(k - 1)\), run = \(2k - 2 = 2(k - 1)\), so the slope for all other values of \(k\) will be \(\frac{-3(k - 1)}{2(k - 1)} = \frac{-3}{2} = -1.5\)
F8-21 Using the Equation of a Line to Solve Word Problems

Pages 22–23

Standards: 8.F.B.4

Goals:
Students will write the equation of a function to model a linear relationship between two quantities.
Students will understand that the y-intercept is the initial value or flat fee.

Prior Knowledge Required:
Can add, subtract, multiply, and divide integers
Can graph a linear function on grid paper
Can find the slope and the y-intercept of a line from a graph, equation, or table of values

Vocabulary: flat fee, ordered pairs, rise, run, slope, y-intercept

(MP.4) Solving word problems using linear functions. Explain to students that many real-world problems involve linear relationships. Draw on the board:

SAY: The graph shows the cost of renting an e-bike (electric bike) for a $10 flat fee plus $5 per hour. ASK: How much do you have to pay to rent the e-bike for 1 hour? ($15) For 5 hours? ($35) Ask a volunteer to find the slope of the line. (5) ASK: How are the slope and the cost related? (the slope is the cost per hour) What is the y-intercept of the line? (10) How are the y-intercept and the cost related? (the y-intercept is the flat fee or initial cost) If Beth paid $50 for renting an e-bike, how many hours did she rent the e-bike for? (8) Explain to students that they can use the graph to find the answer, or they can use the equation of the line. SAY: The slope is 5 and the y-intercept is 10. Write the equation of the line on the board:

\[ y = 5x + 10 \]
SAY: This is the equation for the straight line. Ask a volunteer to find the value for x when y = 50. ASK: In other words, how many hours can be paid for with $50? (see solution below)

\[ 50 = 5 \times x + 10 \]
\[ 50 - 10 = 5x \]
\[ 40 = 5x, \text{ so } x = 8 \]

Explain to students that using algebra seems to take longer for small numbers, but for large numbers, it might be the best or only solution.

Solve the word problems in the following exercises with the class.

**Exercises:**

**(MP.4)** 1. A phone company charges a $5 flat fee, plus $0.10 for each international text message.
   a) Create a table of values for the cost of sending 1, 2, and 3 international text messages.
   b) Plot the ordered pairs from the table of values in part a) on a grid paper.
   c) Extend the line to find the y-intercept.
   d) Find the run, rise, and slope of the line.
   e) Write an equation to represent the cost of sending x international text messages.
   f) Substitute x = 3 in the equation to find the cost of sending 3 international text messages.
   g) Ben has $10. What is the maximum number of international text messages he can send?

**Solutions:**

a) | Number of Messages | Cost ($) |
---|-------------------|---------|
| 1    | 5.10             |
| 2    | 5.20             |
| 3    | 5.30             |

b) ![Graph showing the relationship between the cost and number of messages](image)

c) y-intercept = 5.00

d) run = 1, rise = 0.10, slope = 0.1

e) \( y = 0.10x + 5.00 \)

f) \( y = 0.10(3) + 5.00 = 5.30 \)

\[ 10 = 0.10 \times x + 5.00, \text{ so } 10 - 5 = 0.10x, \text{ and } x = 50. \]

Ben can send 50 international text messages with $10.
2. A balloon is released from the top of a tower. The graph shows the height of the balloon over time.

a) What is the y-intercept of the line?
b) What is the height of the tower?
c) What is the slope of the line?
d) What is the speed of the balloon?
e) Write the equation of the line.
f) What is the height of the balloon after 1 minute?
g) How long does it take for the balloon to reach a height of 2000 m?

Answers:
a) 100
b) 100 m
c) 20
d) 20 m/s
e) \( y = 20x + 100 \), \( x \) stands for seconds and \( y \) shows the height of the balloon after \( x \) seconds
f) 1,300 m
g) 95 seconds

See Questions 1–4 on AP Book 8.2, pp. 22–23 for more practice.

Extensions
(MP.4) 1. A sports arena will host a new event soon. The arena manager estimates that if tickets are priced at $7.50 each, then 500 tickets will be sold per day, and if tickets are priced at $10.00 each, then 350 tickets will be sold per day.

a) Fill in the table, then plot the ordered pairs from the table of values on a grid.

<table>
<thead>
<tr>
<th>Ticket Price ($)</th>
<th>Estimated Ticket Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) What is the rate of change relating estimated ticket sales to ticket price?
c) Write an equation for the line.
d) Use the equation to estimate how many tickets will sell in a day if each ticket costs $9.00.
e) Use the equation to estimate the ticket price needed to sell 440 tickets per day.
Answers:

a) 

<table>
<thead>
<tr>
<th>Ticket Price ($)</th>
<th>Estimated Ticket Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>500</td>
</tr>
<tr>
<td>10</td>
<td>350</td>
</tr>
</tbody>
</table>

b) run = 2.5, rise = −150, slope = −150/2.5 = −60

c) \( y = −60x + 950 \)
d) \( y = −60(9) + 950 = 410 \)
e) \( 440 = −60x + 950 \), so \( x = 8.5 \)

(MP.4, MP.5) 2. Grace gets to school by biking or by taking public transportation. She records the amount of time (in minutes) she takes to get to school each day for 20 days, and how she got to school that day.

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bus, 23</td>
<td>Bike, 25</td>
<td>Bus, 22</td>
<td>Bike, 25</td>
<td>Bus, 31</td>
</tr>
<tr>
<td></td>
<td>Bus, 28</td>
<td>Bike, 24</td>
<td>Bike, 26</td>
<td>Bike, 24</td>
<td>Bus, 22</td>
</tr>
<tr>
<td></td>
<td>Bike, 25</td>
<td>Bus, 33</td>
<td>Bike 25</td>
<td>Bus, 23</td>
<td>Bus, 23</td>
</tr>
<tr>
<td></td>
<td>Bus, 40</td>
<td>Bus, 22</td>
<td>Bike, 22</td>
<td>Bike, 24</td>
<td>Bike, 25</td>
</tr>
</tbody>
</table>

a) On most days, what takes less time for Grace to get to school, taking the bus or riding her bike? Use a box plot, histogram, or scatter plot to decide.

b) It is 8:30 a.m. and Grace needs to give a school presentation at 9 a.m. Should she take the bus or ride her bike? Explain.

c) Explain your choice of graph to a partner.

Sample answers:

a) On most days, taking the bus takes less time. From the box plots, the median time for taking the bus is only 23 minutes, whereas the lowest time for riding her bike is 24 minutes.

b) She should ride her bike because it is much more reliable—the bus times are much more variable and could take up to 40 minutes.

c) I used two box plots and compared them. Using box plots allowed me to compare both the median and the variability at the same time.

MP.2 Assessment Opportunity: Question 2 on AP Book 8.2 p. 22
F8-22 Describing Graphs

Pages 24–26

Standards: 8.F.B.5

Goals:
Students will describe the relationship between two quantities by analyzing a graph.
Students will sketch a graph to represent a verbally described linear function.

Prior Knowledge Required:
Can write an equation of a line from its graph

Vocabulary: decreasing lines, increasing lines, speed, velocity

Materials:
BLM Immigration to the United States, 1900–2000 (p. L-57)
BLM Mountain Climbers (p. L-58)

Review increasing and decreasing lines. Explain to students that the graph of a line represents important and useful information about the relationship between two quantities. Remind students of the terms “increasing line” and “decreasing line” and what they mean.

SAY: For an increasing line, both run and rise are positive, so the slope of an increasing line is positive. When the slope of a line is positive, then the line is increasing and goes from bottom left to top right on a grid.

SAY: For example, Kelly is saving for a new bike; she has $20 already and will save $15 per week; describe the graph that shows the money Kelly saves and time. (sample answer: the line has a positive slope and goes from bottom left to top right) ASK: When will she have enough money for a bike that costs $250? (after 15.33 weeks of saving, or after about 16 weeks)

SAY: In contrast, a decreasing line has a negative slope and goes from top left to bottom right on a grid. Draw on the board:

ASK: Is graph A increasing or decreasing? (increasing) What about graph B? (decreasing)
Can you say graph C is increasing? (no, because it decreases after $x = 1$) Explain to students
that graph C is neither increasing nor decreasing, but can still give useful information about the relationship between two quantities. SAY: Graph C, for example, might show the gallons of water in a bathtub over four minutes. In the first minute after turning on the tap, we could run three gallons of water into the tub, but then we turn off the tap and open the drain, at which point the tub takes three minutes to empty.

**Describing graphs.** Solve the following word problem as an example with the class. Draw on the board:

SAY: Kim starts at her home and exercises for 60 minutes. The graph shows the elapsed time in 10-minute intervals and Kim’s distance from home. Kim starts by running. ASK: How far did Kim run in the first 10 minutes? (1 km) In the next 10 minutes? (2 km) What is the run from O to A? (10 min) What is the rise from O to A? (1 km) Write on the board:

\[ \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{10} = 0.1 \text{ km/min} \]

Point to the unit (km/min) and SAY: **Speed** is the rate of distance traveled in a certain time. Kim runs 1 km in 10 minutes, so her speed is 0.1 km/min. As a class, find Kim’s speed in each 10 minute interval. (OA: 0.1 km/min, AB: 0.2 km/min, BC: 0.2 km/min, CD: 0 km/min, DE: 0.1 km/min) Explain to students the difference between the speed in AB and BC. SAY: The AB line segment shows that Kim runs at a speed of 0.2 km/min away from home. But at point B, she turns toward home and runs at a speed of 0.2 km/min toward home. Both speeds are equal, but they are in different directions. Later, in high school, you will learn that velocity is the speed with the direction considered; in this case, you may say the velocity during AB is 0.2 km/min and the velocity during BC is −0.2 km/min. ASK: Which line segment has a slope equal to 0? (CD) What is Kim’s speed in the CD line segment? (0) Point to the graph and SAY: You can guess Kim had stopped for 20 minutes to rest.

**(MP.4) Exercise:** Complete BLM Immigration to the United States, 1900–2000.

**Answers:**
1. a) decade ending at 1910 and decade ending at 2000; b) decade ending at 1940 c) decade ending at 1980 d) i) −3, ii) −2, iii) −4, iv) +1, v) +1, vi) +2, vii) 0, viii) +3, ix) +2 e) viii (decade ending at 1990) f) iii (decade ending at 1940) g) vii (decade ending at 1980)
Bonus: possible answers could mention the Great Depression in the United States and economic depressions in other areas around the world

**MP.4 Sketching graphs.** Explain to students that, in the previous section, they described the relationship between two quantities when the graph is given, but, in this section, they have to do the reverse. SAY: Now you will sketch a graph to show the relationship between two quantities. Distribute *BLM Mountain Climbers* and have students complete the BLM as an example. (see answers below)

1. a) 4,500 m
   
   b) | **Time** | 12 p.m. | 2 p.m. | 3 p.m. | 4 p.m. | 4:30 p.m. | 5 p.m. | 6 p.m. |
      |-------|--------|--------|--------|--------|--------|--------|--------|
      | **Height (m)** | 4,000  | 4,500  | 4,500  | 4,700  | 4,600  | 4,600  | 4,700  |

   c) ![Graph](image)

   d) i) slope: 250, speed: 250 m/h, ii) slope: 0, speed: 0 m/h, iii) slope: 200, speed: 200 m/h, iv) slope: −200, speed: 200 m/h downward, v) slope: 0, speed: 0 m/h, vi) slope: 100, speed: 100 m/h

   e) part i) from noon to 2 p.m.

**Extensions**

1. Julie gets on her bike for a ride that includes a hill. First, Julie accelerates for 10 seconds until she reaches a constant speed of 10 m/s. Julie maintains her speed for 30 seconds. She then starts to ride uphill, at which point her speed slows considerably for 20 seconds. When she gets to the top of the hill, she stops for 10 seconds. Julie then starts downhill and reaches a speed of 12 m/s after 5 seconds. Julie slows down over 25 seconds until she stops on the other side of the hill.

Sketch a graph that shows the relationship between Julie’s speed and the time, from the time she gets on her bike until she stops on the other side of the hill. Label each axis with the appropriate variable.
You work for a tennis club that sells clothes to its members. Your boss asks you to decide the best way to stock clothes based on the ages of the members. She gives you a member list and asks you to present the information visually to her. You start by making the chart below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>14</td>
<td>56</td>
<td>48</td>
<td>35</td>
<td>36</td>
<td>20</td>
<td>3</td>
</tr>
</tbody>
</table>

a) Which age groups should the club stock the most clothes for? Show your answer on a box plot, histogram, or scatter plot.
b) Explain your choice of graph to a partner.

**Answers:**
a) They should stock more clothes that will appeal to people in their twenties and thirties.
b) I used a histogram because the information is provided by intervals, not by data points. Also, I’m not looking to see how the frequency changes as age increases; I just want to know what ages have the greatest frequencies; that’s why a histogram is better than a scatter plot. A box plot doesn’t make sense because I know that the halfway mark is somewhere in the 30–40 age group, but I can’t tell exactly where.

Redirecting students: You may need to remind students what a box plot and histogram are.

**MP.2 Assessment Opportunities:** Questions 2–3 on AP Book 8.2 p. 25
Immigration to the United States, 1900–2000

1. The graph shows the number of new immigrant arrivals in the United States in each ten-year interval from 1901–2000 (1901–1910, 1911–1920, and so on). For example, approximately 9 million people immigrated to the US in the decade from 1901–1910. The numbers are rounded to the nearest million.
   a) In what decades did the highest number of people immigrate to the US?
   b) In what decade did the lowest number of people immigrate to the US?
   c) In what decade did the number of immigrants not change from the previous decade?
   d) Label the line segments with Roman numerals. Find the slope of each line segment in millions per decade.
   e) Which line segment has the greatest slope?
   f) Which line segment has the steepest slope?
   g) Which line segment has slope equal to 0?

Bonus ➤ Why do you think there was a low number of immigrants from 1931 to 1940?
Mountain Climbers

1. A group of mountain climbers starts to climb a mountain at 12:00 p.m. (noon). The climbers start from a height of 4,000 meters above sea level. They climb 500 meters in 2 hours and then rest for one hour. Then they climb another 200 meters in an hour, but because of poor weather conditions they then climb downhill 100 meters in half an hour and rest for another half an hour. Finally, the climbers take another path and climb 100 meters in 1 hour and set up their camp at that height.

   a) What is the height of the mountain climbers at 2:00 pm? _______________

   b) Fill the table with the heights of the mountain climbers.

<table>
<thead>
<tr>
<th>Time</th>
<th>12 p.m.</th>
<th>2 p.m.</th>
<th>3 p.m.</th>
<th>4 p.m.</th>
<th>4:30 p.m.</th>
<th>5 p.m.</th>
<th>6 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>4,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c) Sketch a graph using the table in part b).

   ![Graph](image)

   d) Label the line segments with Roman numerals. Find the slope of each line segment to find the climbers’ speed during each part of their climb.

   i) ii) iii) iv) v) vi)

   e) In what period of time does the graph have the steepest slope? _______________