Unit 1 Patterns and Algebra: Patterns

Introduction
This unit focuses on patterns. It describes how to:

- review multiplication and division;
- find given terms in repeating patterns;
- extend and create growing and shrinking patterns;
- identify, apply, and distinguish between two types of pattern rules; and
- use variables to represent changing quantities.

Meeting Your Curriculum

**ALBERTA**

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<thead>
<tr>
<th>Required</th>
<th>PA6-4 to 8</th>
<th>including Extension 1 in PA6-7</th>
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<tr>
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<td>PA6-1, 3</td>
<td>support material in later lessons</td>
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<td>Optional</td>
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**BRITISH COLUMBIA**

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**MANITOBA**

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<th>PA6-4 to 8</th>
<th>including Extension 1 in PA6-7</th>
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**ONTARIO**

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<td>Recommended</td>
<td>PA6-1</td>
<td>supports material in later lessons</td>
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Mental Math Minutes
The mental math minutes in this unit:

- review basic addition, subtraction, and multiplication

This unit introduces number talks. See p. A-23 for more information.

Generic BLMs
The Generic BLMs used in this unit are:

- BLM Hundreds Charts (p. J-1)
- BLM Pattern Blocks (p. J-2)
- BLM Filling a Blank Multiplication Chart (p. J-3)

These BLMs can be found in Section J.
Assessment

The lessons covered by a quiz or test are as follows:

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<tr>
<td>Quiz</td>
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<td>Test</td>
<td>PA6-4 to 8</td>
<td>PA6-3 to 8</td>
<td>PA6-4 to 8</td>
<td>PA6-2 to 8</td>
</tr>
</tbody>
</table>

Additional Information for This Unit

Notation
When an expression involves more than one operation, we recommend using brackets to show which operation is done first, as students have not yet learned the order of operations.

Student readiness
Students who have not memorized the times tables can use multiplication charts. Students who struggle with finding the difference between two numbers can count up on their fingers. Dedicate some time to help these students memorize the times tables, and addition and subtraction facts to 20 (see Mental Math, p. A-27).
Goals

Students will describe pattern rules in words that generate patterns by adding, subtracting, multiplying, or dividing by a constant to get the next term.

PRIOR KNOWLEDGE REQUIRED

Can extend patterns made by adding, subtracting, multiplying, or dividing to get the next term.

Mental math minute—number string.

String 1: $2 \times 7, 4 \times 7, 8 \times 7, 16 \times 7$ (14, 28, 56, 112)

Present why doubling one factor doubles the product using number lines and jumps of size 7:

```
0 7 14
0 7 14 21 28
0 7 14 21 28 35 42 49 56
```

Four jumps of 7 go twice as far as two jumps of 7, and eight jumps go twice as far as four jumps.

String 2: $4 \times 4, 8 \times 4, 16 \times 4, 16 \times 8$ (16, 32, 64, 128)

**Identifying how to get the next term in a pattern.** Tell students you are going to write a pattern on the board and you are making the pattern by doing the same operation over and over again. Tell students to signal thumbs up when they know what operation you are doing. Write on the board the first few terms, pausing for students to signal thumbs up after the third term and every term after that:

```
2, 6, 10, 14, ...
```

Sometimes, allow a volunteer who knows the operation to continue the pattern, but only allow a volunteer to say the operation once everyone has signalled they know the operation that is being repeated, in this case adding 4. Check that the operation works for every term in the pattern.
Then repeat with more sequences, as shown below:

5, 10, 20, 40, ...
38, 35, 32, 29, ...
1024, 512, 256, 128, 64, ...

**Using pattern rules to create patterns.** Have students make a pattern in which they get the next term by adding 5. Have various volunteers say what pattern they made, and encourage them to come up with different patterns. Discuss why many of the patterns are different (they start at different numbers). Write several of the patterns on the board.

**Exercises:** Use the pattern rule to make the pattern. Write the first five terms.

a) start at 3 and add 4 each time
b) start at 4 and add 3 each time
c) start at 1 and multiply by 3 each time
d) start at 3 and multiply by 1 each time

**Answers:** a) 3, 7, 11, 15, 19; b) 4, 7, 10, 13, 16; c) 1, 3, 9, 27, 81; d) 3, 3, 3, 3

**Finding a given term from a pattern rule given in words.** SAY: If you know the pattern rule, you can write the sequence, and if you write the sequence, you can find any term you want. Refer students to the exercises they just did. ASK: What is the fourth term in each sequence? (15, 13, 27, 3) What is the third term? The fifth term?

**Identifying pattern rules.** SAY: Once you see how the pattern starts and how it continues, you can write the pattern rule. Write on the board:

2, 9, 16, ...

SAY: This pattern was made by repeating one operation. ASK: What is the pattern rule for this sequence? (start at 2 and add 7 each time)

**NOTE:** If students struggle with the following exercises, encourage them to try addition and multiplication if the numbers are getting bigger, and subtraction and division if the numbers are getting smaller.
**Exercises:** The pattern was made by repeating one operation. Use the first three terms in the pattern to find the rule. Then write the next term.

a) $33, 34, 35, \ldots$ The rule is: start at 33 and ____________

b) $2, 8, 14, \ldots$ The rule is: _______________________

c) $3, 30, 300, \ldots$ The rule is: _______________________

d) $32, 16, 8, \ldots$ The rule is: _______________________

**Bonus:** Write the first three terms for two different patterns where you get the next term by subtracting 2 each time. Then write the pattern rules for the patterns you made.

**Answers:** a) start at 33 and add 1, 36; b) start at 2 and add 6, 20; c) start at 3 and multiply by 10, 3000; d) start at 32 and divide by 2, 4

**Solving word problems using sequences.** Write on the board:

A baby is born weighing 2500 g and gains 200 g per week.

SAY: This sentence involves a sequence of numbers. ASK: What is the pattern rule? (start at 2500 and add 200 each time) PROMPTS: What do you start at? (2500) What do you add each time? (200) Ask a volunteer to write the first five terms of the sequence on the board. (2500, 2700, 2900, 3100, 3300) ASK: After how many weeks does the baby weigh 3300 g? (4 weeks) If some students say 5 weeks, point out that 2500 is the weight at birth, or zero weeks, and 2700 is the weight after the first week.

**NOTE:** You can use Exercise 5 to diagnose students’ ability to solve multistep problems.

**Exercises**

1. Write the pattern rule in words.

   a) A plant that is 14 cm tall grows 3 cm each day.

   b) A baby that is 50 cm tall grows 1 cm each week.

   c) A pencil shrinks 2 cm each month from being sharpened. A new pencil is 18 cm long.

   **Bonus:** A tree that is 2 m tall grows 15 cm each week.

   **Answers:** a) start at 14 and add 3 each time, b) start at 50 and add 1 each time, c) start at 18 and subtract 2 each time, Bonus: start at 200 and add 15 each time

2. A plant that is 11 cm tall grows 8 cm each week. How tall is the plant after 4 weeks?

   **Answer:** 43 cm
3. A pencil that is 13 cm long gets sharpened and shrinks 2 cm each month. How long is the pencil after 3 months?

Answer: 7 cm

4. Jax is on page 45 of a book when the winter vacation starts. He reads 12 pages a day.
   a) Write the pattern rule in words.
   b) What page is he on after 6 days?
   c) The book is 165 pages long. How many days of winter vacation would he need to finish the book?
   d) Winter vacation is 9 days long. Will he finish the book before the vacation ends?

Answers: a) start at 45 and add 12 each time, b) page 117, c) 10 days, d) no

5. a) A bike Zara wants is on sale for $360. The sale will end after five weeks. Zara gets $10 a week in allowance and she already has $40 saved. She can make $9 an hour mowing lawns. How many hours a week does she need to work for the next five weeks to be able to buy the bike on the last day of the sale?
   b) Compare your answer to part a) with a partner and try to agree on the correct answer. Make sure you understand each other’s reasoning before agreeing on the answer.

Answer: a) 6 hours a week

Extensions
1. The sequence was made by repeatedly adding or subtracting a number. Find the missing number.
   a) 2, 4, ___, 8
   b) 3, ___, 11, 15

Answers: a) 6, b) 7

2. The sequence was made by repeatedly adding a number. Write the pattern rule.
   a) 5, ___, 65
   b) 5, ___, ___, 65
   c) 5, ___, ___, ___, 65
   d) 5, ___, ___, ___, ___, 65
   e) 5, ___, ___, ___, ___, ___, 65

Answers: start at 5 and: a) add 30 each time, b) add 20 each time, c) add 15 each time, d) add 12 each time, e) add 10 each time
3. The scientist Edmond Halley noticed that the comets that appeared in 1531, 1607, and 1682 were all the same object, now called Halley’s Comet.
   a) About how many years apart are the appearances?
   b) The last appearance was in 1986. How many times did the comet appear between 1682 and 1986?
   c) Predict the next three years it will return to Earth.
   d) Use the Internet to check your prediction from part c). When do scientists actually predict the next appearance of Halley’s Comet? Was your prediction close?

   **Answers:**
   a) about 75 or 76; b) 3 times; c) using the 76 years gap from your answer to part b), predict 2062, 2138, and 2214; d) 2061

4. A baby blue whale weighs 2700 kg. It gains 4 kg each hour. How much does the baby whale weigh after 6 days?

   **Solution:** In 6 days, there are $24 \times 6 = 144$ hours. So, the baby whale gained $144 \times 4 = 576$ kg, which brings its weight to 3276 kg after 6 days.

5. Create a word problem that goes with the sequence.
   a) 7, 9, 11, 13, ____, ____
   b) 5, 9, 13, 17, ____, ____

6. Ben is playing a game at a fair. He rolls two dice and tries to make the largest number he can using the following rules. He makes a sequence by starting at 1 and using one dice number as the gap in the sequence and the other dice number as the number of terms he can make. He wants the last number in the sequence to be as high as possible. Does it matter which dice he uses for each role? Explain.

   **Answer:** No, it doesn’t matter, because the number that he gets to will always be the product of the two dice numbers, plus the 1 he started with.

7. Find the next three terms of the sequence.
   Rule: add the two previous terms
   1, 3, 4, ____, ____, ____

   **Answer:** 7, 11, 18

8. Extend the sequence until you see a pattern repeating.
   Rule: subtract the smaller of the two previous terms from the larger term
   3, 7, 4, 3, 1, ...

   **Answer:** 2, 1, 1, 0, 1, 1, 0, 1, 1, 0, (the 1, 1, 0 continues to repeat)
Goals
Students will create a table of values for visual sequences and for pattern rules given in words.
Students will use tables of values for patterns to write pattern rules in words.

Prior Knowledge Required
Can write pattern rules for patterns made by adding, subtracting, multiplying, or dividing to get the next term

Materials
pattern blocks or BLM Pattern Blocks (p. J-2)

Mental Math Minute—Number Talk. Present this problem: double 169.
(338) The following strategies could arise:

169 = 100 + 60 + 9, so the double is 200 + 120 + 18
169 = 160 + 9, so the double is 320 + 18
169 = 170 − 1, so the double is 340 − 2

Making a Table of Values. Tell students that you are planning to organize a community picnic. Four people can sit at each square picnic table, but you want to put the picnic tables together to make one long table, because you will have benches, not chairs. Show how the long table changes as you add small tables to it, and how people sit around each resulting table. Use squares (for the picnic tables) and dots (for the people).

Tell students that you want to keep track of the number of people that can sit at the long table, to make sure that you set out enough plates and cutlery. Ask a volunteer to write the number of people at each table.

Number of people: 4 6 8

SAY: The number of people forms a sequence of numbers, 4, 6, 8, .... Is this an increasing or decreasing sequence? (increasing) Remind students that when the numbers get bigger, the sequence is increasing. When the numbers get smaller, the sequence is decreasing. SAY: I want to keep track of the number of people sitting at the table, but I also want to know how
many of the small picnic tables I need for different numbers of people. Write on the board:

<table>
<thead>
<tr>
<th>Number of people</th>
<th>Number of picnic tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

A simple way to keep track of how many people can sit at each table is to make a **table of values**. Draw the table at left on the board and ask students to help you fill in the number of picnic tables and the number of people.

**Tables of values and patterns.** Ask students to describe how the number of people changes as picnic tables are added. They should notice that the number of people in each successive picture on the board increases by 2, or that the difference between successive terms in the right-hand column is 2. Write the number 2 (the gap between terms) in a circle between each pair of terms, as shown in the margin.

Ask students if they can state a rule for the pattern in the table (start at 4 and add 2) and predict how many people can sit around the fourth set of tables. Students should see that they can continue the pattern in the table (by adding the gap to each new term) to find the answer. Point out that the table of values allows them to calculate the number of people who can sit before they put the picnic tables together.

**Exercises**

1. Look at the sequences of tables and chairs.

   ![Tables](image)

   a) Determine the gap.

   ![Tables](image)

   b) How many people can sit around the fourth set of tables?

   ![Tables](image)

   c) How many tables do you need to put together to seat 26 people?

   ![Tables](image)

   d) How many people can sit around the first four sets of tables altogether?

   **Answers:**
   a) 4
   b) 18
   c) 6
   d) $6 + 10 + 14 + 18 = 48$

2. In the following pattern, each term is made up of squares. Terms 2, 3, and 4 are shown.

   ![Squares](image)

   a) Draw Terms 1 and 5.

   ![Squares](image)

   b) How many squares are needed to make the first five terms altogether?
c) Compare your answer to part b) with a partner and try to agree on the correct answer. Make sure you understand each other’s reasoning before agreeing on the answer.

**Selected answer:** b) $3 + 5 + 7 + 9 + 11 = 35$

**Creating the table of values given a pattern rule in words.** SAY: Just like you can create a sequence from a pattern rule in words, you can create a table of values from a pattern rule in words. In a table of values, one column shows the *term numbers* and the other column shows the *term values*. Write on the board:

Start at 3 and add 5 each time.

SAY: Write the term numbers in one column and the term values in the other column. Draw on the board:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Pointing to the first row, SAY: The first term, or term number 1, is 3. Pointing to the second row, ASK: What is the second term? (8) How did you get that? (I added 5) SAY: So the second term, or term number 2, is 8. Ask a volunteer to complete the table of values.

**Exercises**

1. Make a table of values for the pattern rule to find the term value or term number asked for.
   a) Start at 2 and add 6 each time. Find the fifth term.
   b) Start at 18 and subtract 1 each time. Which term is equal to 10?
   c) Start at 5 and multiply by 2 each time. Which term is equal to 320?
   d) Start at 1000 and divide by 2 each time. Find the fourth term.

**Selected solution**

\[
\begin{array}{|c|c|}
\hline
\text{Term Number} & \text{Term Value} \\
\hline
1 & 1000 \\
2 & 500 \\
3 & 250 \\
4 & 125 \\
\hline
\end{array}
\]

The fourth term is 125.

**Answers:** a) 26, b) 9th, c) 7th
2. Kyle makes a table of values for the rule. Is his table correct? If not, describe his mistake.

a) Start at 8 and add 5 each time.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

b) Start at 3 and multiply by 2 each time.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

Answers: a) no, he started at 5 instead of 8; b) no, he didn’t double 12 to get the fourth term

SAY: You can do the same thing when the pattern rule has two operations. As long as you can make the sequence from the pattern rule, you can make the table of values too.

Exercises

1. Complete a table of values for the pattern rule. Show the first four terms.

a) Start at 3. Multiply by 2 and subtract 1 each time.

b) Start at 4. Subtract 3 and multiply by 5 each time.

c) Start at 2. Add 5 and multiply by 2 each time.

Selected answer

c) | Term Number | Term Value |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>86</td>
</tr>
</tbody>
</table>

2. Compare your answers to Exercise 1 with a partner and try to agree on the correct answer. Make sure you understand each other’s reasoning before agreeing on the answer.
Bonus: Tessa made a table of values for the pattern rule. Is her table correct? If not, explain her mistake.

a) Start at 5. Subtract 2 and multiply by 3 each time.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>93</td>
</tr>
</tbody>
</table>

b) Start at 0. Multiply by 3 and add 1 each time.

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

Answers: a) no, to get the third term, she added 2 instead of subtracted 2 and then multiplied by 3; b) yes

Writing pattern rules from tables of values. Write on the board:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

ASK: Where do you see the sequence from the table of values? (in the second column) What number does the pattern start at? (5) Write on the board:

Start at 5.

ASK: What operation is being repeated? (add 2) Continue the pattern rule on the board:

Start at 5. Add 2 each time.

Exercises: Write the pattern rule from the table of values.

a) | Term Number | Term Value |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>

b) | Term Number | Term Value |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17 000</td>
</tr>
<tr>
<td>2</td>
<td>1700</td>
</tr>
<tr>
<td>3</td>
<td>170</td>
</tr>
</tbody>
</table>

Answers: a) start at 8 and add 3 each time, b) start at 17 000 and divide by 10 each time
ACTIVITY (Essential)

Ask students to construct a sequence of shapes (for instance, castles or letters of the alphabet) that grow in a fixed way. Students could use pattern blocks (for example, from BLM Patterns Blocks) for this activity. Examples:

```
    o   o   o
    o   o   o
    o   o   o
    o   o   o
    o   o   o
    o   o   o
```

Ask students to describe how their pattern grows and to predict how many blocks they would need to make the 6th figure. (If students have trouble finding the answer in a systematic way, suggest that they use a table of values to organize their calculation.)

Extensions

1. The following tables show the number of blocks used in building different structures. Each figure corresponds to a stage in the construction process. The same number of blocks was added to each structure at each stage. Fill in the missing numbers.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

Answers: 9, 11; 7, 10; 15, 20

2. Cody used one kind of block to build a structure. He added the same number of blocks to his structure at each stage of its construction. He made a mistake when he copied the number of blocks at each stage into the table. Can you find his error and correct it?

Answer: The number of blocks in Figure 2 should be 8, not 7.

3. You want to construct a block castle following the steps shown below. You would like each tower in the finished castle to be 5 blocks high. Each block costs 5¢ and you have 80¢ altogether. Do you have enough money to buy all the blocks you need? (Hint: Make a table with three columns: Step, Number of Blocks, and Cost.)

```
Step 1
  o   o   o
  o   o   o
  o   o   o

Step 2
  o   o   o   o   o
  o   o   o   o   o
  o   o   o   o   o

Step 3
  o   o   o   o   o   o   o   o   o   o
  o   o   o   o   o   o   o   o   o   o
  o   o   o   o   o   o   o   o   o   o
```

Step 1
Step 2
Step 3
Answer

<table>
<thead>
<tr>
<th>Step</th>
<th>Number of Blocks</th>
<th>Cost (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>85</td>
</tr>
</tbody>
</table>

You need 85¢, so you don’t have enough money to buy all the blocks you need.

4. Edith’s maple sapling grows 5 cm in July. It grows 4 cm each month after that. Ron’s sapling grows 7 cm in July. It grows 3 cm each month after that. Whose sapling is taller by the end of October?

Answer

<table>
<thead>
<tr>
<th>Month</th>
<th>Height of Edith’s Sapling (cm)</th>
<th>Height of Ron’s Sapling (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>August</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>September</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>October</td>
<td>17</td>
<td>16</td>
</tr>
</tbody>
</table>

Edith’s sapling is taller by the end of October.

5. a) Make and complete the table below for this sequence of shapes.

<table>
<thead>
<tr>
<th>Number of Unshaded Squares</th>
<th>Number of Shaded Squares</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Extend the table as required to answer these questions:

b) How many shaded squares will be needed for a figure with 7 unshaded squares?

c) How many squares will be needed for a figure with 15 shaded squares?
Answers

a)  

<table>
<thead>
<tr>
<th>Number of Unshaded Squares</th>
<th>Number of Shaded Squares</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>

b) 17 shaded squares, c) 21 squares

6. This table shows the amount of fuel left in an airplane as it travels toward an airport. Extend the table as required to answer the questions below.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Litres of Fuel</th>
<th>Distance from Airport (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1200</td>
<td>525</td>
</tr>
<tr>
<td>5</td>
<td>1150</td>
<td>450</td>
</tr>
<tr>
<td>10</td>
<td>1100</td>
<td>375</td>
</tr>
<tr>
<td>15</td>
<td>1050</td>
<td>300</td>
</tr>
<tr>
<td>20</td>
<td>1000</td>
<td>225</td>
</tr>
<tr>
<td>25</td>
<td>950</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>900</td>
<td>75</td>
</tr>
<tr>
<td>35</td>
<td>850</td>
<td>0</td>
</tr>
</tbody>
</table>

a) How much fuel will be left in the airplane after 25 minutes?
b) How far from the airport will the plane be after 30 minutes?
c) How much fuel will be left in the airplane when it reaches the airport?

Answers

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Litres of Fuel</th>
<th>Distance from Airport (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1200</td>
<td>525</td>
</tr>
<tr>
<td>5</td>
<td>1150</td>
<td>450</td>
</tr>
<tr>
<td>10</td>
<td>1100</td>
<td>375</td>
</tr>
<tr>
<td>15</td>
<td>1050</td>
<td>300</td>
</tr>
<tr>
<td>20</td>
<td>1000</td>
<td>225</td>
</tr>
<tr>
<td>25</td>
<td>950</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>900</td>
<td>75</td>
</tr>
<tr>
<td>35</td>
<td>850</td>
<td>0</td>
</tr>
</tbody>
</table>

a) 950 L, b) 75 km, c) 850 L
Goals

Students will use variables for changing quantities.
Students will substitute values for variables in algebraic expressions involving one operation.

PRIOR KNOWLEDGE REQUIRED

none

MATERIALS

blocks and counters (see Extension 4)

Mental math minute—number string.

String 1: \(10 \times 7, 11 \times 7, 12 \times 7\) (70, 77, 84)

To present the strategy of adding on 7, write each multiplication as repeated addition, as shown below:

\[
11 \times 7 = 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = (10 \times 7) + 7
\]

Emphasize that 11 sevens is 1 more seven than 10 sevens.

String 2: \(10 \times 8, 11 \times 8, 12 \times 8\) (80, 88, 96)

Using variables to represent a changing value. Write on the board:

\[
1 + 5 \\
2 + 5 \\
3 + 5 \\
4 + 5
\]

ASK: What is staying the same in all of these expressions? (the 5 being added) What is changing? (the first number) SAY: Let’s use the letter \(n\) for the number that is changing. Write on the board:

\[n + 5\]

SAY: The letter \(n\) is called a variable because it represents a number that is changing. “Vary” is a verb that means the same as “change.” When something can change, it can vary. The number 5 does not change, so it is not a variable.
Exercises: Replace the changing number with the variable \( n \).

a) \( 5 \times 3 \)  
\[ 5 \times 4 \]  
\[ 5 \times 5 \]  
\[ 5 \times 6 \]  

b) \( 8 - 1 \)  
\[ 8 - 2 \]  
\[ 8 - 3 \]  
\[ 8 - 4 \]  

\[ \text{Bonus: } (3 + 1) \times 2 \]

\[ (3 + 2) \times 2 \]  
\[ (3 + 3) \times 2 \]  
\[ (3 + 4) \times 2 \]  

Answers: a) \( 5 \times n \), b) \( 8 - n \), c) \( n + 4 \), Bonus: \( (3 + n) \times 2 \)

Write on the board:

Ethan wants to work 5 hours a week. He has a choice of 3 different jobs.

Cutting lawns: $8 an hour
Delivering newspapers: $7 an hour
Tutoring math: $9 an hour

How much money can he make in a week?

ASK: What value can change, the number of hours or the amount of money per hour? (the amount of money per hour) Write on the board:

\[ \text{(number of hours worked) } \times \text{(dollars per hour) } = \text{amount Ethan makes} \]

SAY: The total amount of money Ethan makes is the number of hours he works times the amount he gets paid each hour. ASK: How many hours does he work in a week? (5) Write on the board:

\[ 5 \times \text{(dollars per hour)} \]

SAY: The number of dollars per hour is the number that changes. Let’s use a variable. Let’s use \( d \) for dollars. Write on the board:

He makes \( 5 \times d \) dollars in a week, where \( d \) is the number of dollars per hour.

SAY: When you write the number that changes as a variable, you have to say what the variable represents. In this case, it is the number of dollars per hour.

Exercises: Write an expression for the amount of money, in dollars, that Eric earns in a week. Write the number that changes as a variable.

a) He has a job that pays $10 an hour. He sometimes works 3 hours, sometimes 4 hours, and sometimes 5 hours.

b) He works 6 hours a week tutoring. During the school year, he is paid $10 an hour, and in the summer he is paid $8 an hour.

c) Compare your answer to parts a) and b) with a partner and try to agree on the correct answers. Make sure you understand each other’s reasoning before agreeing on the answers.

Answers: a) \( 10 \times h \), where \( h \) is the number of hours a week that Eric works; 
b) \( 6 \times d \), where \( d \) is the number of dollars per hour that he is paid
Variables and algebraic expressions. On the board, write “$2 \times (3 + 4)$.”

SAY: This is a numerical expression. If I replace some numbers with variables, then I have an algebraic expression. Erase the number 3 on the board and write $n$ in its place:

$$2 \times (n + 4)$$

Explain to students that when letters are used in an expression, the multiplication sign ($\times$) is often omitted to avoid confusion with the letter “x” and to make the notation shorter. For example, instead of writing $2 \times (n + 4)$, we can simply write $2(n + 4)$.

Exercises: Rewrite the expression.

a) $2 \times n$  
   b) $(2 \times n) + 3$  
   c) $2 \times (n + 3)$

Answers: a) $2n$, b) $(2n) + 3$, c) $2(n + 3)$

Substituting numbers for variables and evaluating expressions.

On the board, write the expression $n + 4$. Tell students that we can replace $n$ with a number and get a numerical expression. For example, if we replace $n$ with 3, then the expression becomes $3 + 4$, which is 7. Writing 7 is called “evaluating the expression,” because we are saying the value of the expression. Write the words on the board with underlining, as shown in the margin, to emphasize the connection.

Exercises: Replace $n$ with 3 in the expression and evaluate the expression.

a) $n + 2$  
   b) $n - 1$  
   c) $5 - n$  
   d) $7 + n$

Answers: a) 5, b) 2, c) 2, d) 10

Now tell students that you are going to try to trick them. Write $5n$ on the board. Have students replace $n$ with 3. Discuss the problem that students run into. The answer looks like the number 53, but $5n$ really means $5 \times n$, so we mean $5 \times 3$, not 53. To avoid this problem, we include brackets if replacing a variable with a number could cause confusion. This confusion could happen whenever a variable is being multiplied by a number. Tell students that $5(3)$ is another way to write $5 \times 3$.

Exercises: Evaluate.

a) $5(4)$  
   b) $7(3)$  
   c) $6(2)$  
   d) $9(6)$

Bonus: $9(2000)$

Answers: a) 20, b) 21, c) 12, d) 54, Bonus: 18 000

Tell students that since two numbers can be multiplied in any order, a number and a variable can be multiplied in any order too, since a variable stands for a changing number. The convention is to always write the number first. Write on the board:

$$2 \times t$$

and $t \times 2$ are both written as $2t$, not $t2$. 

Patterns and Algebra 6-6  

B-33
NOTE: Extensions 1 and 2 are required in order to cover the Ontario curriculum. Extensions 3 and 4 allow students the opportunity to solve a challenging problem that is unrelated to the lesson they were taught.

Extensions

1. A dragon has 44 teeth and 4 poisonous spikes on its tail. A dragon breeder uses two formulas: \( t = 44d \) and \( s = 4d \).
   a) What does each formula describe?
   b) Write a formula that relates the quantities \( t \) and \( s \), and does not involve \( d \).

   Answers: a) \( t = 44d \) describes the number of teeth on \( d \) dragons and \( s = 4d \) describes the number of spikes on \( d \) dragons, b) \( t = 11s \)

2. Mary babysits once a week. Sometimes she works for 3 hours, sometimes 4 hours, and sometimes 5 hours. Sometimes she gets paid $12 an hour, sometimes $13 an hour, and sometimes $14 an hour.
   a) Write two variables for the changing quantities.
   b) Write an expression for the total amount that Mary makes in a week.
   c) How much money does she make if she works for 5 hours and makes $12 an hour?

   Sample answers: a) let \( h \) be the number of hours Mary babysits in a week and \( d \) be the number of dollars per hour she makes that week, b) Mary makes \( h \times d \) or \( hd \) dollars in a week, c) $60

3. Evaluate the expression using the given values of the variables.
   a) \((2x) + (3y), x = 4, y = 5\)
   b) \((3m) − n, m = 4, n = 10\)
   c) \(x \times (y + 1), x = 2, y = 0\)

   Answers: a) 23, b) 2, c) 2

4. In the following magic trick, the magician can always predict the result of the sequence of operations performed on any chosen number. Try the trick with students and then encourage them to figure out how it works. Students can use blocks to represent the mystery number and counters to represent the numbers that are added. Give students lots of hints as they manipulate the concrete materials.
The Trick | The Algebra
---|---
Pick any number. | Use a square block to represent the mystery number.
Add 4. | Use 4 circles to represent the 4 ones that were added.
Multiply by 2. | Create 2 sets of blocks to show the doubling.
Subtract 2. | Take away 2 circles to show the subtraction.
Divide by 2. | Remove one set of blocks and circles to show the division.
Subtract the mystery number. | Remove the square.

The result is 3. No matter what number you choose, after performing the operations in the magic trick, you will always get the number 3. The model above shows why the trick works.

Encourage students to make up their own tricks of the same type.
Goals

Students will create pattern rules to obtain the output from the input using any of the four operations.

Students will use letters as short form for words in mathematical formulas, and understand that the letters represent changing quantities.

PRIOR KNOWLEDGE REQUIRED

Can write a variable to represent a changing quantity

Mental math minute—number string.

String 1: $10 \times 8, 9 \times 8, 8 \times 8$ (80, 72, 64)

Present the strategy of subtracting from 10 times the number by writing each multiplication as repeated addition. Emphasize that 9 eights is 1 less eight than 10 eights.

String 2: $10 \times 12, 9 \times 12, 8 \times 12$ (120, 108, 96)

Writing equations for patterns made with addition. Write on the board:

Amy invites some friends to a party. She needs one chair for each friend and one for herself.

<table>
<thead>
<tr>
<th>Number of Friends</th>
<th>Number of Chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

ASK: If Amy invites 1 friend, how many chairs will she need? (2) Write this in the first row. Repeat for 2 friends (3 chairs), 3 friends (4 chairs), and 4 friends (5 chairs). SAY: Let’s extend the table so that there are 10 friends. Continue drawing on the board:
ASK: How does the number of chairs change as I add one more friend? (you need 1 more chair each time) Have a volunteer complete the table.

SAY: Suppose I want to know how many chairs Amy needs when she invites 50 friends. ASK: Should I continue the table or is there an easier way? PROMPT: Is there a way to get the number of chairs from the number of friends directly? (add 1; the number of chairs is one more than the number of friends) SAY: The number of chairs is always one more than the number of friends because Amy needs one chair for herself. ASK: How many chairs would she need for 50 friends? (51) For 100 friends? (101) For 1000 friends? (1001) Would you want to continue the table for 1000 rows to find that out? (no) Write on the board:

\[
\text{number of friends} + 1 = \text{number of chairs}
\]

SAY: Let’s rewrite the equation using letters for the variables. ASK: What letter could we use for the number of friends? (sample answer: \( F \)) What letter could we use for the number of chairs? (sample answer: \( C \)) Write on the board:

\[
F + 1 = C
\]

**Exercises:** Write an equation to get the number of chairs (\( C \)) from the number of friends (\( F \)).

a) Liz and Nora invite some friends to a party. How many chairs do they need?

b) Ken, Lewis, and Sam invite some friends to a party. How many chairs do they need?

c) Nina, Rayder, and Cathy invite 20 friends to a party. How many chairs do they need?

**Answers:** a) \( F + 2 = C \), b) \( F + 3 = C \), c) \( 20 + 3 = 23 \)

SAY: A formula tells you how to find the output number directly from the input number. When you calculate the number of chairs you need from the number of friends you invite, the number of friends you invite is the input and the number of chairs you need is the output. Write on the board:

\[
C = F + 1
\]

SAY: When writing a formula, you write the output number first. This formula tells you how to get the number of chairs (point to \( C \)) from the number of friends (point to \( F \)).

**Exercises**

1. Write the equations in parts a) and b) from the previous exercises as formulas for how to get \( C \) from \( F \).

**Answers:** a) \( C = F + 2 \), b) \( C = F + 3 \)
2. A family invites several guests to a party. The number of chairs (C) they need is \( C = G + 6 \), where G is the number of guests.

   a) How many people are in the family?

   b) If they invited 10 guests, how many chairs will they need?

   c) Compare your answers to parts a) and b) with a partner and try to agree on the correct answers. Make sure you understand each other’s reasoning before agreeing on the answers.

   **Selected answers:** a) 6, b) 16

**Bonus:** Write a problem for the formula \( C = G + 4 \).

   **Sample answer:** Four people are planning a party. They invite several guests. How many chairs do they need?

**Writing formulas from tables that involve any operation.** Draw on the board:

<table>
<thead>
<tr>
<th>Input (A)</th>
<th>Output (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

ASK: What is the difference between the two numbers in the first row? (2) In the second row? (4) Can you add the same number to the input numbers to get the output numbers? (no) SAY: So, you cannot get the numbers in column B from the numbers in column A by addition. Write on the board:

addition    subtraction    multiplication    division

Cross out the word “addition” and SAY: Let’s look for a different operation to use. ASK: Can you get the output numbers by subtracting something from the input numbers? (no) How do you know? (the output numbers are bigger than the input numbers) If some students say that you can subtract a negative number to get a larger number, tell them they are correct and they will learn about that next year, but you want to work with positive numbers for this lesson.

SAY: Let’s try multiplication. Point to the table and ASK: What number can I multiply by 1 to get 3? (3) By 2 to get 6? (3) By 3 to get 9? (3) SAY: The output numbers are 3 times as much as the input numbers in the first column. Write on the board:

\[
\text{output} = 3 \times \text{input} \\
B = 3 \times A \\
B = 3A
\]
SAY: I could have used $O$ for output and $I$ for input. Write on the board:

$$O = 3I$$

ASK: If I did that, what number might $O$ be confused with? (0) SAY: The letter “O” isn’t usually used as a variable, because it can easily be confused with zero. Even the letter “I” can be confused with the number 1. So we should avoid using these letters as variables.

**Exercises**

1. Find the rule, then write a formula for how to get $B$ from $A$.

   a)  
   
   \[
   \begin{array}{c|c}
   A & B \\
   \hline
   1 & 11 \\
   2 & 12 \\
   3 & 13 \\
   \end{array}
   \]

   b)  
   
   \[
   \begin{array}{c|c}
   A & B \\
   \hline
   1 & 2 \\
   2 & 4 \\
   3 & 6 \\
   \end{array}
   \]

   c)  
   
   \[
   \begin{array}{c|c}
   A & B \\
   \hline
   1 & 4 \\
   2 & 8 \\
   3 & 12 \\
   \end{array}
   \]

   **Answers:** a) $B = A + 10$, b) $B = 2A$, c) $B = 4A$

2. Draw a table showing the figure number and the number of squares. Write a formula for the number of squares $S$ from the figure number $N$.

   a)  
   
   \[
   \begin{array}{c}
   \text{Figure 1} \\
   \text{Figure 2} \\
   \text{Figure 3}
   \end{array}
   \]

   b)  
   
   \[
   \begin{array}{c}
   \text{Figure 1} \\
   \text{Figure 2} \\
   \text{Figure 3}
   \end{array}
   \]

   **Answers:** a) $S = 3N$, b) $S = 2N$

When students finish the exercises, refer to Exercise 2 and ASK: What was the input? (the figure number) What was the output? (the number of squares) SAY: In these patterns, you got the output from the input by adding or multiplying, but you can get the output from the input using any operation. As long as there is a rule, you can get the output from the input.
Exercises

1. Copy the table and apply the rule to get the output from the input.
   a) subtract 5 from the input
   b) divide the input by 3
   c) multiply the input by 10

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td>33</td>
<td>11</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>69</td>
<td>23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>12</td>
<td>120</td>
</tr>
<tr>
<td>174</td>
<td>1740</td>
</tr>
</tbody>
</table>

   **Answers:** a) 1, 2, 3, 4; b) 8, 11, 5, 23; c) 30, 80, 120, 1740

2. Draw a table and write a rule that tells you how to get the number of triangles \( t \) from the number of squares \( s \).

   a) \( t = 4s \)
   b) \( t = s + 1 \)
   c) \( t = s - 3 \)
   d) \( t = s \div 3 \)

   **Answers:** a) \( t = 4s \), b) \( t = s + 1 \), c) \( t = s - 3 \), d) \( t = s \div 3 \)

3. Compare your answers to Exercise 2 with a partner and try to agree on the rules.

   **NOTE:** Extension 1 is required in order to cover the Alberta and Manitoba curricula. Extensions 3 to 5 should be done in order.
Extensions

1. Check that all the equations are true by evaluating both sides. Then replace the changing numbers with the two variables \( m \) and \( n \).

   \[
   \begin{align*}
   \text{a)} & \quad 3 + 4 = 4 + 3 \\
   \text{b)} & \quad (3 \times 2) + (4 \times 2) = (3 + 4) \times 2 \\
   & \quad 2 + 6 = 6 + 2 \\
   & \quad (1 \times 2) + (5 \times 2) = (1 + 5) \times 2 \\
   & \quad 8 + 3 = 3 + 8 \\
   & \quad (6 \times 2) + (3 \times 2) = (6 + 3) \times 2 \\
   & \quad 7 + 5 = 5 + 7 \\
   & \quad (7 \times 2) + (4 \times 2) = (7 + 4) \times 2
   \end{align*}
   \]

   **Bonus:** Now use three variables: \( m, n, \) and \( p \).

   \[
   \begin{align*}
   & \quad (8 \div 2) + (6 \div 2) = (8 + 6) \div 2 \\
   & \quad (9 \div 3) + (12 \div 3) = (9 + 12) \div 3 \\
   & \quad (40 \div 10) + (30 \div 10) = (40 + 30) \div 10
   \end{align*}
   \]

   **Answers:**
   \[
   \begin{align*}
   \text{a)} & \quad m + n = n + m \\
   \text{b)} & \quad (m \times 2) + (n \times 2) = (m + n) \times 2 \\
   \text{Bonus:} & \quad (m \div p) + (n \div p) = (m + n) \div p
   \end{align*}
   \]

2. Write a formula to get the number of circles from the number of squares. Start by saying what the variables represent.

   **Answer:** Let \( C \) be the number of circles and \( S \) the number of squares. Then \( C = 2 \times (S - 1) \) or \( C = (S - 1) \times 2 \) or \( C = 2S - 2 \)

3. Apply the rule to get the output \( B \) from the input \( A \). Then write a formula. Use brackets to show the operation you do first.

   \[
   \begin{align*}
   \text{a)} & \quad \text{multiply by 2 and subtract 1} \quad (A \times 2) - 1 \\
   \text{b)} & \quad \text{divide by 2 and add 3} \quad (A \div 2) + 3 \\
   \text{c)} & \quad \text{multiply by 2 and add 6} \quad (A \times 2) + 6 \\
   \text{d)} & \quad \text{add 3 and multiply by 2} \quad (A + 3) \times 2
   \end{align*}
   \]

   **Answers:**
   \[
   \begin{align*}
   \text{a)} & \quad 5, 7, 9, 11 \\
   \text{b)} & \quad 9, 10, 11, 12 \\
   \text{c)} & \quad 8, 10, 12, 14 \\
   \text{d)} & \quad 8, 10, 12, 14
   \end{align*}
   \]
4. Which two formulas from Extension 3 get the same output for each input? Verify this for other input numbers.

**Answer:** \( B = (2 \times A) + 6 \) and \( B = (A + 3) \times 2 \) get the same output for each input. Sample verification: For \( A = 5 \), \( B \) is \( (2 \times 5) + 6 = 10 + 6 = 16 \) and \( B \) is \( (5 + 3) \times 2 = 8 \times 2 = 16 \), so \( B \) is the same for both formulas when \( A = 5 \).

5. Fill in the blank to make the two rules have the same output for each input. Verify your answer for different inputs.

a) Multiply by 4 and then subtract 12.

b) Divide by 3 and then add 12.

Subtract ___ and then multiply by 4.

**Answers:** a) 3; b) 36, sample verification: If \( A = 3 \), then dividing by 3 gets 1 and adding 12 gets 13. Similarly starting with \( A = 3 \), adding 36 gets 39 and dividing by 3 gets 13, as before.

6. What two operations are being done to get the output from the input?

a)  
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<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
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<td>4</td>
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</table>

b)  
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<th>A</th>
<th>B</th>
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<tbody>
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</table>

**Answers:** a) multiply by 2 and add 2, or add 1 and multiply by 2; b) divide by 5 and add 2, or add 10 and divide by 5
Goals

Students will distinguish between pattern rules that require extending patterns to find a given term and pattern rules that require using a formula.

Students will identify visual patterns where the numbers are from the multiplication chart and recognize that this type of pattern is the type that is easy to find a far term.

PRIOR KNOWLEDGE REQUIRED

Can create and follow pattern rules that require continuing a pattern to find a given term
Can create and follow pattern rules that require only knowing the term number to find a given term
Can substitute a number into a formula and evaluate the expression

MATERIALS

BLM Filling a Blank Multiplication Chart (p. J-3)
BLM 9 × 9 Multiplication Chart (p. B-50)

Mental math minute. Give students BLM Filling a Blank Multiplication Chart. Have them fill in as much of the chart as they can in three minutes using the strategies on the BLM as needed.

Using a formula is more efficient than extending the sequence to find terms that are far along in the sequence. Draw on the board:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>6</td>
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<tr>
<td>7</td>
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</tr>
</tbody>
</table>

Have a volunteer fill in the first four rows of the table. Have another volunteer extend the table. Point out that the last volunteer was able to find the number of squares in Figure 7 without drawing it. SAY: Suppose I want to find the number of squares in the 50th figure. ASK: Would it be a good strategy to extend the table? (no) Why not? (it would take too long) If you had time to do it, would it work? (yes) SAY: You could extend the table to 50 rows if you had time, and if you did everything correctly, you would get the right answer, but there is an easier way. ASK: How else could we
determine the number of squares in the 50th figure? (write a formula for how to get the number of squares from the figure number) From the table, how can you get the number of squares from the figure number? (multiply the figure number by 2) Write on the board:

number of squares = 2 × figure number

Ask a volunteer to rewrite the formula using the letter S for the number of squares and the letter F for the figure number. (see below)

\[ S = 2 \times F \]

ASK: How many squares are in the 50th figure? (100) Write on the board:

\[ S = 2 \times 50 \\
= 100 \]

Exercises

1. The table represents a pattern of squares. Extend the table to find the number of squares in the 8th figure. Then write a formula and use it to check your answer.

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<thead>
<tr>
<th>Figure number (F)</th>
<th>Number of Squares (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
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<td>2</td>
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<td>8</td>
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<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

\[ S = F + 4 \]

\[ S = 4F \]

Answers: a) 12, \( S = F + 4 = 8 + 4 = 12 \); b) 32, \( S = 4F = 4(8) = 32 \)

2. For each table in Exercise 1, find the number of squares in the 50th figure. Tell a partner whether you extended the table or used a formula, and why.

Answers: a) 54, b) 200; sample explanation: I used the formula because extending the table to 50 rows would take a long time and I will be more likely to make a mistake when adding 4 so many times.

Translating between the two types of pattern rules. SAY: There are two types of pattern rules. Rules that tell you how to make each term from previous terms and rules that tell you how to make the sequence directly from the term number. Write on the board:

Start at 3 and add 3 each time.

ASK: What kind of pattern rule is this? (it makes the pattern from previous terms) Have a volunteer create the sequence from the rule (3, 6, 9, 12, 15, …) and another volunteer write the rule for making the sequence directly from the term number (multiply the term number by 3).
Exercises

1. Write both types of rules from the sequence.
   a) 4, 5, 6, 7, 8, ...
   b) 8, 16, 24, 32, 40, ...
   **Bonus:** \( n \), \( 2n \), \( 3n \), \( 4n \), \( 5n \), ...
   **Answers:** a) start at 4 and add 1 each time, add 3 to the term number; b) start at 8 and add 8 each time, multiply the term number by 8; Bonus: start at \( n \) and add \( n \) each time, multiply the term number by \( n \)

2. Write the first five terms of the sequence and the other type of rule.
   a) multiply the term number by 5
   b) start at 6 and add 1 each time
   **Bonus:** start at \( n \) and add 1 each time
   **Answers:** a) 5, 10, 15, 20, 25, start at 5 and add 5 each time; b) 6, 7, 8, 9, 10, add 5 to the term number; Bonus: \( n \), \( n+1 \), \( n+2 \), \( n+3 \), \( n+4 \), add \( n-1 \) to the term number

Using patterns in the gaps to continue sequences. Draw on the board:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Squares</th>
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<tbody>
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<td>1</td>
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</tbody>
</table>

Ask a volunteer to draw Figure 4. Then draw on the board:

Have volunteers tell you the number of squares to write for the first four terms in the pattern (1, 4, 9, 16). SAY: Let’s find the gaps between the terms. Draw circles beside the chart and have volunteers tell you the difference between each pair of consecutive terms. Write the answers as they tell them to you. (3, 5, 7) SAY: The gaps are not the same, but there is a pattern in the gaps, so we can still continue the sequence. **ASK:** What is the next gap? (9) **How do you know?** (the gaps are increasing by 2) Write "9" in the next circle. Then **SAY:** So now we know that the fifth term is 9 more than the fourth term. **ASK:** What is the fifth term? (25) **What is the sixth term?** (36) **How did you get that?** (I added 11) Why 11—where did
that come from? (it is 2 more than 9) Write “11” in the next circle. Keep this example on the board for later.

**Exercises**

a) Draw Figure 4 for the pattern.

i) [Figure 1] [Figure 2] [Figure 3]

ii) [Figure 1] [Figure 2] [Figure 3]

**Bonus**

[Figure 1] [Figure 2] [Figure 3]

b) How many squares are in the sixth figure? Hint: Make a table of values and continue the table to term number 6.

**Answers**

a) i) [Figure] [Figure] Bonus: [Figure]

ii) [Figure] [Figure] Bonus: [Figure]

b) i) 42, ii) 48, Bonus: 54

**Using the multiplication chart to find far terms.** Refer to the pattern on the board (1, 4, 9, 16, 25, 36). SAY: I want to know the number of squares in the 50th figure, but I don’t want to have to continue the sequence to 50 terms. I want to find a formula for how to get the number of squares from the figure number. That way, I can just use 50 as the value of the variable in the formula. Give students **BLM 9 × 9 Multiplication Chart**. Ask students to find the numbers from the pattern in the chart. ASK: Is there a pattern for where you can find the numbers on the chart? (yes, they are all on the diagonal) SAY: Let’s write the numbers on the diagonal. Write on the board:

1 × 1
2 × 2
3 × 3
4 × 4
5 × 5
6 × 6
ASK: What would the next number be? (7 × 7 or 49) SAY: The two numbers being multiplied are the same as the figure number. ASK: What would the 50th number in the diagonal be? (50 × 50 or 2500) What is the number of squares in the 50th figure? (2500)

Students can refer to their multiplication charts for the following exercises. Go through part a) together as a class. Emphasize that the correct prediction is 10 × 11, because the figure number is always being multiplied by one more than the figure number. SAY: In these exercises, you need to predict two numbers, the first number being multiplied and the second number being multiplied.

**Exercises:** Look at the patterns you found in the previous exercises. Complete the table for the first four figures. Then predict the number of squares in the tenth figure.

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Figure 2</th>
<th>Figure 3</th>
<th>Figure 4</th>
<th>Prediction for Figure 10</th>
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**Answers:** i) 1 × 2 = 2, 2 × 3 = 6, 3 × 4 = 12, 4 × 5 = 20, 10 × 11 = 110; ii) 1 × 3 = 3, 2 × 4 = 8, 3 × 5 = 15, 4 × 6 = 24, 10 × 12 = 120; Bonus: 1 × 4 = 4, 2 × 5 = 10, 3 × 6 = 18, 4 × 7 = 28, 10 × 13 = 130

**Extensions**

1. A newborn Siberian tiger cub weighs 1300 g. It gains 100 g every day. A newborn baby boy weighs 3300 g. He gains 200 g every week.
   a) The cub and the baby are born on the same day. Who weighs more after …
      i) 2 weeks?         ii) 6 weeks?
   b) After how many weeks would the cub and the baby have the same weight?
   c) Did you use a formula or extend a table to answer parts a) and b)? Why?

**Answers:** a) i) the baby, ii) the cub; b) 4

**Sample answer:** c) I extended the table, because the terms were very early in the sequence, so it was easy to use a table.
2. Look at the pattern below:

![Figure 1](image1) Figure 2 Figure 3 Figure 4

a) Complete the table. How many squares are needed altogether to make the figures?

<table>
<thead>
<tr>
<th></th>
<th>How many squares are needed?</th>
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</thead>
<tbody>
<tr>
<td>The first figure</td>
<td></td>
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<tr>
<td>The first two figures</td>
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<tr>
<td>The first three figures</td>
<td></td>
</tr>
<tr>
<td>The first four figures</td>
<td></td>
</tr>
</tbody>
</table>

b) Find your answers to part a) in the multiplication table. Write each answer to part a) as a multiplication fact.

c) How many squares are needed altogether to make the first 10 figures? Explain how you know.

**Answers:** a) 1, 4, 9, 16; b) $1 \times 1$, $2 \times 2$, $3 \times 3$, $4 \times 4$; c) $10 \times 10$ because you just multiply the number of figures by itself.

3. The picture below shows the second, third, and fourth figures in a pattern. How many squares are needed to make the first five terms altogether?

![Figure 2](image2) Figure 3 Figure 4

**Answer:** $2 + 4 + 6 + 8 + 10 = 30$

4. a) How many 9s would there be in the sequence?
   
   i) 1 3 5 5 5 ...
   
   ii) 1 1 2 2 2 3 3 3 3 3 ...

b) In part a), did you extend the sequences or use a formula? Explain your choice.

**Answers:** a) i) 5, ii) 18

**Sample answers:** b) i) I extended the sequences because I couldn’t find a formula using one operation to get the number of times a number occurs directly from the number itself; also, 9 comes right after 7, so it wasn’t much work; ii) I used a formula, because there is always twice as many of each number as the number itself, so I didn’t have to do all that work extending the sequence.
## 9 × 9 Multiplication Chart

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Pattern Blocks
**Filling a Blank Multiplication Chart**

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**Sample strategies**

- Fill in all the facts that you have memorized or use numbers that you can skip count by easily, such as 1s, 2s, 3s, 4s, 5s, and 10s.

- Use doubling to fill in the 6s, 8s, and 12s. For example, \(6 \times 8\) is double \(3 \times 8\) because \(3 \times 8 = 8 + 8 + 8\) and \(6 \times 8 = (8 + 8 + 8) + (8 + 8 + 8)\).

- Use the 5s and the 2s to fill in the 7s. For example, \(7 \times 8 = (8 + 8 + 8 + 8) + (8 + 8)\), which is \((5 \times 8) + (2 \times 8)\).

- Use the 10s and the 1s to fill in the 9s and the 11s.

- Check that the same two numbers always multiply to the same number.