Goals
Students will use relative frequency tables to construct circle graphs from circles already divided into 100 equal parts.

PRIOR KNOWLEDGE REQUIRED
Can convert between tallies and numerals
Can convert between fractions and percents
Understands when to use the mean, median, or mode

MATERIALS
BLM A Large Circle Graph (p N-40)
BLM Small Circle Graphs (p N-41)

Introduce relative frequency tables with fractions. Tell students that you surveyed 80 students in grades 7 and 8 about their favourite type of movie. You tallied the results as follows:

<table>
<thead>
<tr>
<th>Favourite type of movie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comedy</td>
</tr>
<tr>
<td>Action</td>
</tr>
<tr>
<td>Horror</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>

Have students use the data to complete this relative frequency table:

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Number of People</th>
<th>Fraction of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comedy</td>
<td>12</td>
<td>12/80 = 3/20</td>
</tr>
<tr>
<td>Action</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horror</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Point out that the table shows both the number of times a data value occurs in a set (e.g., comedy is the favourite for 12 people) and the fraction of time each data value occurs (e.g., comedy is the favourite for 12/80 people).

(A frequency table shows only the former.)

Encourage students to add the total numbers and fractions. **ASK**: What should the numbers add to? (80) Why? (because you surveyed 80 people)
What should the fractions add to? (1) Why? (because 80 out of 80 people is all of them, so the fraction is 1)

Indeed, the numbers add to $12 + 16 + 32 + 20 = 80$ and the fractions add to $3/20 + 1/5 + 2/5 + 1/4 = (3 + 4 + 8 + 5)/20 = 20/20 = 1$.

**Review converting fractions to percents.** Tell students that people often write frequency tables using percents instead of fractions. Then remind students how to convert fractions to percents: change the fraction to a fraction with denominator 100 and then write that fraction as a percent (EXAMPLE: $3/20 = 15/100 = 15\%$).

Have students convert these fractions to percents:

a) $3/10$  
b) $4/5$  
c) $6/25$  
d) $3/50$  
e) $9/20$  
f) $3/4$

Remind students that sometimes the denominator of the fraction does not divide evenly into 100. In some such cases, students can reduce the fraction to make it have a denominator that does divide evenly into 100. Have students do this to convert these fractions to percents:

a) $8/40$  
b) $2/8$  
c) $18/75$  
d) $14/70$  
e) $9/15$  
f) $36/48$

**ANSWERS:** a) 20%  b) 25%  c) 24%  d) 20%  e) 60%  f) 75%

**Introduce relative frequency tables with percents.** Have students add another column to the relative frequency table above, with heading Percent of People, and fill it in.

Explain to students what the terms “even strength,” “power play,” and “short handed” mean in hockey, or ask a volunteer to do so. (Even strength means neither team has a penalty, so both teams have the same number of players on the ice. When one team has a penalty, that team has one fewer players than the other team and plays short handed. The team with more players on the ice is said to have a power play.) Then have students complete this relative frequency table:

<table>
<thead>
<tr>
<th>Goals Scored by a Hockey Team</th>
<th>Number</th>
<th>Fraction</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even strength</td>
<td>45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power play</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short handed</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask questions about this data:

1. Do you think it is harder for a team to score on the power play or at even strength? (even strength)

2. Why did this team score so many more goals at even strength than on the power play? (they were probably playing at even strength for a much greater portion of each game)

3. These goals were scored over 20 games. What is the average number of goals scored per game? ($60 \div 20 = 3$)
4. How did you know which average—mean, median, or mode—the question above referred to? (It was referring to the mean, because not enough information is given for the other two—we would have to know the number of goals scored per game to figure out the median and the mode)

**Introduce circle graphs.** Demonstrate how to convert the data for the hockey team into a circle graph. Use a transparency of *BLM A Large Circle Graph* with an overhead projector. Emphasize that the circle is already divided into 100 equal parts, so it is easy to use for a circle graph. Brainstorm a title for the circle graph (e.g., When Goals are Scored), and demonstrate labelling each section “even strength,” “power play,” or “short handed.”

**Using circle graphs to compare data.** Give students two copies of *BLM A Large Circle Graph* and have students transfer the data used at the beginning of the lesson, about favourite type of movie, to a circle graph. Have students label the circle graph appropriately and title it.

Then tell students that you surveyed a group of 20 students in grades 5 and 6 about their favourite type of movie and found the following results:

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Number of People</th>
<th>Fraction of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comedy</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Action</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Horror</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Have students copy and complete this relative frequency table in their notebooks and then convert the data to a circle graph on the second BLM. Students can title this one Another Survey of Favourite Movies.

Have students look at the two circle graphs to compare the data. **ASK:** More people in grades 7 and 8 chose comedy than in grades 5 and 6. Why doesn’t the circle graph show this? Emphasize that the circle graphs compare not the frequencies, but the relative frequencies of data values.

**ASK:** According to this data, in which grades did a greater percentage of people choose horror as their favourite type of movie? How can you tell from the circle graph?

**The total percents must add to 100%.** Tell students that you asked students in 6 different kindergarten classes to name their favourite colour, but you might have made a mistake recording some of the data. Have students translate the data into circle graphs using *BLM Small Circle Graphs* (they should create six graphs—one graph per class).

| Class A: | Red: 35% | Blue: 25% | Yellow: 10% | Other: 35% |
| Class B: | Red: 20% | Blue: 30% | Yellow: 20% | Other: 30% |
| Class C: | Red: 30% | Blue: 20% | Yellow: 10% | Other: 40% |
| Class D: | Red: 15% | Blue: 40% | Yellow: 15% | Other: 40% |
Class E:  
Red: 10%  
Blue: 15%  
Yellow: 25%  
Other: 50%

Class F:  
Red: 18%  
Blue: 15%  
Yellow: 26%  
Other: 31%

**ASK:** Which data did you have trouble graphing? (data for classes A, D, and F) Why? (because they total either more or less than 100%)

Explain that the percents in a circle graph must always total 100%. This is because when you count all the data, you count 100% of it. Encourage students to use this to check their work for the relative frequency table in Question 4, on Workbook page 65, before completing the circle graph.

**A tip for struggling students.** In Question 3c), Calli surveyed 200 students, so each 1 marking on her graph represents 2 students, whereas Bilal surveyed only 50 students, so 2 markings on his graph represent 1 student. Noticing this can make it easier to fill in the graphs: divide the number of students by 2 to get the number of markings for each section on Calli’s graph, and multiply the number of students by 2 to get the number of markings on Bilal’s graph.

**After students finish Workbook page 65 Question 3.** Have students perform the survey, to determine whose school really is more like theirs, and compare the result to their prediction in part e).

**Circle graphs that look different can still represent the same data.** Draw two circle graphs that show the same data (you can use BLM Large Circle Graph) but rotate one of the graphs 90° before you label it. Discuss how the graphs are different and how they are the same. Emphasize that two circle graphs can represent the same data even though they look different. Then include a circle graph that shows the same data but in a different order, and discuss again how the graphs look different though the data is the same.

**ACTIVITY**

**Circle graph posters.** Encourage students to look for circle graphs in books, in magazines, on the Internet, on TV (e.g., on the weather network) or in brochures (e.g., from financial institutions). Have students record properties that are common to all the circle graphs. How many categories are used in each circle graph? About how many categories do most circle graphs use? Have students cut out various circle graphs and make a poster titled Circle Graphs. Keep these posters for PDM7-14.
Have students draw a circle graph, using BLM Large Circle Graph, with the following data about the percent of people who use each mode of transportation to get to school:

<table>
<thead>
<tr>
<th>Mode of Transportation</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>30%</td>
</tr>
<tr>
<td>Bike</td>
<td>25%</td>
</tr>
<tr>
<td>Walk</td>
<td>20%</td>
</tr>
<tr>
<td>Car</td>
<td>15%</td>
</tr>
<tr>
<td>Other</td>
<td>10%</td>
</tr>
</tbody>
</table>

Then, have students use a protractor to measure the angle of each region (or “pie piece”) to fill in the following chart:

<table>
<thead>
<tr>
<th>Mode of Transportation</th>
<th>Percent</th>
<th>Angle in Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bike</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Walk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The angles in a circle add to 360°. When students are finished the chart, have them add the angles. What do they total? (360°) Why? (because the central angles of a circle add to 360°)

Use the angles to verify percentages of 360°. ASK: The percentage of people biking to school was 25% and the angle you found was 90°.
ASK: Does that make sense? Why? (yes, because 90° is 25% of 360°)

Demonstrate this to students. First, remind students how to find the percentage of a number. For example, to find 25% of 360, write 25% as a fraction (25/100 or 1/4) and then replace "of" with "×." So 25% of 360° = 25/100 × 360° = 360° × 25 ÷ 100 = 90°. Or, write 25% as 1/4 to obtain 1/4 × 360° = 360° ÷ 4 = 90°. A quick explanation of why this works: To find 1% of 360°, divide 360° by 100. But 25% of 360° is 25 times more than 1% of 360°, so 25% of 360° is 25 × 360° ÷ 100 = 90°.

Have students use this method to verify the angles they measured: Does the percentage of 360° you calculate for each mode of transportation match the measurement?

Drawing circle graphs using a protractor. First mark the centre of a circle you will draw, then draw a circle with radius about 3 cm around that centre point using a compass. Draw a line from the edge of the circle to the centre of the circle (a radius). Demonstrate drawing a circle graph from the data above. Emphasize that this circle is not already divided into 100 equal parts, so students now have to use the angle in the circle to draw the regions.

Have students do questions similar to Workbook page 67 Question 2, but have them draw the circle themselves using a compass. Students should be sure to mark the centre point first, so that they can draw a line from the centre to any point on the circle; this will help them create the first region.

EXAMPLE: Survey results: Favourite kind of snack

<table>
<thead>
<tr>
<th>Percent</th>
<th>Angle in Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetables 30%</td>
<td></td>
</tr>
<tr>
<td>Crackers 10%</td>
<td></td>
</tr>
<tr>
<td>Chips 25%</td>
<td></td>
</tr>
<tr>
<td>Fruit 15%</td>
<td></td>
</tr>
<tr>
<td>Other 20%</td>
<td></td>
</tr>
</tbody>
</table>

Review writing fractions as percents. See Workbook page 68 Question 3.

Writing fractions with denominator 360. See Workbook page 68 Question 4. Not all fractions can be written this way, but if they can, the corresponding angle in a circle is particularly easy to find—it is just the numerator of the fraction with denominator 360. All fractions that students will find in Workbook page 68 Question 5 can be written this way.

Find the percentage of 360° that a given angle represents. See Workbook page 69 Questions 1–3. For Questions 2 and 3, students will need to measure the angle first and then determine which percentage of 360° the angle represents.

Sometimes the angle in a circle does not correspond to a whole number percent of 360°. Provide the example shown in the box at the top of Workbook page 70.
Review changing fractions to decimals. Use long division or estimation, then check your answer on a calculator. See Workbook page 70 Question 4.

Find the decimal percentage of $360^\circ$ that a given angle represents.
Have students round to one decimal place. See Workbook page 70 Question 5.

Show students the steps required to draw a circle graph, when the circle is not already divided into 100 parts.

**EXAMPLE:**

In a grade 7 class, 10 students walk to school, 5 travel by bus, 5 bicycle, and 5 skateboard.

**Step 1:** Find the total number of students. (25)

**Step 2:** Express each piece of data as a fraction of the total (reduce to lowest terms).

\[
\begin{align*}
\frac{10}{25} &= \frac{2}{5} \text{ walk} \\
\frac{5}{25} &= \frac{1}{5} \text{ bus} \\
\frac{5}{25} &= \frac{1}{5} \text{ bicycle} \\
\frac{5}{25} &= \frac{1}{5} \text{ skateboard}
\end{align*}
\]

**Step 3:** Change each fraction to an equivalent fraction out of 360.

\[
\begin{align*}
\frac{2}{5} &= \frac{?}{360} \\
\frac{2}{5} &\times \frac{72}{360} = 144
\end{align*}
\]

The angle for the part of the circle graph that represents the students who walked to school should be $144^\circ$.

\[
\begin{align*}
\frac{1}{5} &= \frac{?}{360} \\
\frac{1}{5} &\times \frac{72}{360} = 72
\end{align*}
\]

The angle for the parts of the graph that represent the students who bicycled, rode the bus, or skateboarded to school should each be $72^\circ$.

**Step 4:** Draw a circle and then draw a radius.

**Step 5:** Use a protractor to construct a radius for each of the angles you found in step 3.

**Step 6:** Title and label the circle graph. Include the fraction or percent of the total that each region represents.
How we get to school

- Walk $\frac{2}{5} = 40\%$
- Bus $\frac{1}{5} = 20\%$
- Skateboard $\frac{1}{5} = 20\%$
- Bike $\frac{1}{5} = 20\%$

Have students do Workbook Question 6. Notice that the total is found to be 100% for part c), but not for part d). Emphasize that this is because rounding makes the results less accurate and hence can make the total appear to be different from 100%.

**Importance of drawing the angles accurately.** Ask students how the following circle graph is misleading. (Students should be able to estimate what the angles should look like. For instance $\frac{3}{5}$ is greater than $\frac{1}{2}$, but the part marked H covers less than $\frac{1}{2}$ the circle. Also $\frac{1}{5}$ is double $\frac{1}{10}$ so the parts marked B and S should each cover twice as much area as the part marked O.)

**Favourite sport**

- H: Hockey $\frac{3}{5}$
- S: Soccer $\frac{1}{5}$
- B: Baseball $\frac{1}{5}$
- O: Other $\frac{1}{10}$

Ask students why it is important to measure each angle accurately when drawing a circle graph.

**Extensions**

1. The following question is based on an actual reasoning mistake seen on a web page. An opinion poll asks people to strongly agree, agree, disagree, or strongly disagree with an opinion. Here’s the graph representing the answers of a group of adults who were surveyed:

What fraction of people surveyed:

- Agree? _____
- Disagree? _____
- Strongly agree? _____
- Strongly disagree? _____

What fraction of people surveyed either agree or strongly agree? _____

The survey concludes: “Not counting the lunatic $\frac{1}{10}$ of people who strongly disagree, only $\frac{4}{10}$ of people disagree with us.” Is this correct? Explain.
**ANSWER:** No, this conclusion is incorrect. Out of 10 people, you would expect 3 to agree, 2 to strongly agree, 4 to disagree, and 1 to strongly disagree. If you aren’t going to count the 1 who strongly disagrees, you have to remove 1 from the total: the fraction of people who disagree is 4/9. This is slightly more than 4/10. As well, the data shows that 50% of people disagree with the opinion. *(NOTE: the number of people who disagree is the number of people who disagree to any extent, so the number who “disagree” plus the number who “strongly disagree.”)*

**ASK:** Does the circle graph show this? (yes, but it’s not obvious) Have students redraw the circle graph so that this fact—that 50% of people disagree—is prominent (put the “disagree” and “strongly disagree” sections next to each other).

2. The following data shows the number of deaths due to each recreational activity in one year.

   - Boating 80
   - Swimming 40
   - Biking 36
   - Jet skiing 4

   a) Make a frequency table showing the fraction of deaths due to each recreational activity.

   b) Draw a circle graph showing the data.

   c) Can you conclude that biking is more dangerous than jet skiing? Why or why not? What extra information would be relevant? (For example, the number of hours people spend biking is likely significantly larger than the number of hours people spend jet skiing, so even if jet skiing is more dangerous per hour, there could still be a lot more deaths from biking than deaths from jet skiing.)
**Goals**

Students will find the mean, median, and mode using the relative frequency of data values instead of the frequency of data values.

**PRIOR KNOWLEDGE REQUIRED**

- Can make relative frequency tables from frequency tables
- Can draw and read circle graphs
- Can find the mean, median, and mode from given frequency tables
- Understands that \( a \div b = (a \times c) \div (b \times c) \)

**Using circle graphs to find the mean, median, and mode.** Tell students that you surveyed 80 people about the number of cars their family has. Show the data in the first two columns of this relative frequency table and have students fill in the last column.

<table>
<thead>
<tr>
<th>Number of Cars in Family</th>
<th>Number of People</th>
<th>Fraction of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>1/10</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>1/4</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>2/5</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>1/5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1/20</td>
</tr>
</tbody>
</table>

**ASK:** Out of every 20 people who answered the survey, how many families have no cars? (2) Repeat for 1 car (5), 2 cars (8), 3 cars (4), and 4 cars (1). Write down the data for 20 people:

0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 3 3 3 3 4

Have students find the mean, median, and mode number of cars for these 20 data values. (ANSWERS: mean: 1.85, mode: 2, median: 2)

Then remind students that there were not actually 20 people surveyed, but 80 people. Since these are the data for every 20 people, the actual data set is:

0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 4
0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 4
0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 4
0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 4

Discuss how to find the new mean, mode, and median. The new sum of data values is

\[
(0 + 0 + 1 + 1 + 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 4) \times 4 = \text{old sum} \times 4.
\]
The new number of data values is just the old number of data values multiplied by 4, so the new mean is:

\[
\frac{(\text{old sum of data values}) \times 4}{(\text{old number of data values}) \times 4} = \frac{\text{old sum of data values}}{\text{old number of data values}} = \text{old mean}
\]

The new median is also the same: the middle number (2) is still in the middle. Why? Each value now occurs four times as often. Since half the data values were 2 or less and half were 2 or more to start, there are now four times as many that are 2 or less and four times as many that are 2 or more, so the same number of data values are still 2 or less as are 2 or more.

Finally, the new mode is also the same: whichever data value was most common before each one was repeated the same number of times is still the most common!

Tell students that they have just shown that, instead of using the frequencies of each data value to find the mean, median, and mode, they can use the relative frequencies. Even if you survey a million families, you can find the percentage of families with each number of cars, and then find the mean, median, and mode, as though you only surveyed 100 people. Display this circle graph:

**Number of cars per family in Country A**

Measure together the angle for the “0” region. **ASK:** What percentage of families have 0 cars? (Write the fraction 54/360 as a fraction over 100: 54/360 = 15/100, so 15%.) Repeat for 1 car (126°, 35%), 2 cars (90°, 25%), 3 cars (54°, 15%), 4 cars (36°, 10%). Then **ASK:** Out of every 100 families, how many have 0 cars? (15) Repeat for 1 car (35), 2 cars (25), 3 cars (15), 4 cars (10). Tell students to pretend that there are only 100 families in the country. **ASK:** What are the mean, median, and mode for every 100 families? **ANSWERS:**

mean: \[
\frac{15 \times 0 + 35 \times 1 + 25 \times 2 + 15 \times 3 + 10 \times 4}{100} = 1.7
\]

median: Since the circle graph has the data values in order around the circle, the half way point is found by drawing a diameter from before the region for 0 cars—the diameter is exactly between the regions for 1 and 2 cars, so the median is 1.5

mode: the largest region on the circle graph is for 1 car so 1 is the mode

**ASK:** How does this tell you the mean, median, and mode for the whole country? (they are the same, because you can think of the country as divided into many groups of 100 families, all repeating the same data values, i.e., 15 with no cars, 35 with 1 car, and so on)
Explain purchasing power. The incomes given in Question 2 on Workbook page 71 are in Canadian dollars (CAD) and represent the equivalent of what you can buy in a year with that amount. For example, if someone who lives in Prague, in the Czech Republic, could buy as much in Prague as someone in Canada making $30 000 could buy in Canada, we would record their income as $30 000 CAD. Most of the people living in this developing country are making enough money to buy in one year what a Canadian living in Canada could buy for $50. (In fact, this is how the figures are given when the United Nations states the poverty line as $1.25 USD per day. If you can buy in your country as much as someone making $1.25 a day in the US can buy in the US, then you are on the borderline of living in absolute poverty.)

Extensions

1. After students do Workbook page 71 Question 1c), **ASK:** If Tina accidentally divides by 10 instead of by 5, how can she tell that she is wrong?

   **ANSWER:** The average she would get is 6/10 or 0.6. This is closer to 0 than to 2, but there are more families with 2 cars than with 0 cars, so the actual average should be closer to 2 than to 0.

2. Discuss conditions that could affect a country’s average number of cars per family. For example, could a country’s hilliness affect whether people choose to bike or drive? What about how large the country is? Or how densely populated the country is? For example, Canada is likely to have a greater number of cars per household than Denmark or Holland. Students might like to research this.
A Large Circle Graph
Small Circle Graphs