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Unit 8  Probability and Data Management: Graphs

Introduction
The lessons in this unit develop skills for collecting, displaying, and analyzing data. This unit provides both artificial data for students to work with and a longer term project using student-generated data to teach how to:

- create a survey;
- gather data from a survey or online research;
- present the data in the form of a graph; and
- analyze the graph to draw conclusions.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit:
- review number sense skills developed thus far
- practise finding the middle between two numbers, an essential skill for determining median values

Generic BLMs
The Generic BLM used in this unit is:
BLM 1 cm Grid Paper (p. S-2)
This BLM can be found in Section S.
Materials

In this unit you will need:

- overhead projector
- transparencies of:
  - BLM Colours of Cubes
  - BLM Snack Bar Graphs
  - BLM Bar Graphs for Display
  - BLM Double Bar Graphs
  - BLM Double Bar Graph Template

Assessment

The lessons covered by a quiz or test are as follows:

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Additional Information for This Unit

Unit Project

Throughout this unit are optional activities that reference BLM Project. Taken together, these activities create a project in which students develop a research question, conduct a survey or do research to answer their question, display their data in different ways, and draw conclusions. The steps in the project are presented in BLM Project. To complete the project, students must do Parts 1 through 3, at least one of the available options for Part 4, and Part 6. The unit project can be used throughout this unit or in lessons on other subjects, such as technology, science, or social studies, with appropriate research questions. This project is required for Alberta and Ontario.
Goals

Students will distinguish between primary and secondary data.
Students will formulate good survey questions.
Students will formulate questions requiring secondary data.

PRIOR KNOWLEDGE REQUIRED

Can read and draw tally charts

MATERIALS

BLM Project (p. K-46, optional)

Mental math minute. Arrange students in a line and have them add two-digit numbers by adding tens and adding ones separately. For each addition, such as $35 + 46$, each student needs to say three steps: adding the tens ($30 + 40 = 70$), adding the ones ($5 + 6 = 11$), and finishing the addition ($70 + 11 = 81$, so $35 + 46 = 81$). The next student in line gets a new problem. Start with problems that do not require regrouping, such as $25 + 34$, and continue to questions that require regrouping ones.

Introduce data management. SAY: In this unit we are going to ask questions, collect answers, and display our answers in graphs. The kinds of questions that we are going to ask will have answers that use numbers, or data. You can get answers to some of the questions by asking people around you. This is called a survey. Survey questions are questions such as “What are the favourite books in your class?” or “How many students in our class bike to school?” Some of the questions can be answered through observation, which means watching or looking. A question such as “How many students biked to school today?” can be answered by observing and counting students coming to school by bike. To answer questions such as “How much did it rain this week?” you would have to measure the amount of rain.

Exercises: Would you use a survey, observation, or measurement to answer the question?

a) Who is the tallest student in the class?
b) How many people in our class are wearing a blue shirt today?
c) Who is the most popular superhero?

Answers: a) measurement, b) observation, c) survey

Introduce first-hand and second-hand data. SAY: Data you collect by yourself is called first-hand or primary data. Some ways you can collect first-hand data are by measuring items, conducting an experiment, or conducting a survey. Explain to students that data collected by someone
else is called second-hand data or secondary data. SAY: You can get second-hand data from books, magazines, the internet, or commercials.

**Exercises:** Would you use primary or secondary data to answer the question?

a) What is the most popular movie in your class?
b) What was the most popular movie of all time?
c) How much time do you spend doing homework each night?
d) How much time does the average Grade 4 student spend doing homework each night?
e) How much time does the average student in your class spend doing homework each night?

**Answers:** a) primary, b) secondary, c) primary, d) secondary, e) primary

**Good survey questions.** Ask students if they have ever taken a survey in person, online, or over the phone. Ask them how it worked. Tell students that a survey usually asks a question and gives the person taking the survey a choice of answers. A lot of thought goes into both the question and the answers. Tell students that a very important first step for a survey is to make sure that the people you want to survey can answer the question you are asking. Ask for examples of questions students think their classmates can and cannot answer.

**Exercises:** Do you think most people would know the answer to the question?

a) What colour is your hair?
b) Who was the prime minister in 1930?
c) What is the best treatment for the flu?
d) What is your favourite flavour of ice cream?

**Answers:** a) yes, b) no, c) no, d) yes

Tell students that almost anyone can answer a) and d). ASK: Who do you think could answer c)? (answers may vary, could include parents, doctors) Would this be a good survey question for Grade 4 students? (no) Would it be a good survey question for nurses? (yes)

**Wording the question to receive only one answer.** Emphasize to students that the question has to be worded so that each person can give only one answer.

**Exercises:** Will the question receive one answer or multiple answers from each person you ask?

a) What is your favourite ice cream flavour?
b) What flavours of ice cream do you like?
c) Who will you vote for in the election?
d) Which of the candidates do you like in the election?

e) What is your favourite colour?

f) Which colours do you like?

**Answers:** a) one, b) multiple, c) one, d) multiple, e) one, f) multiple

**Having a good choice of answers.** Ask students what their favourite flavour of ice cream is. Keep a list of answers on the board. ASK: How many tubs of ice cream would you have to buy to accommodate everyone’s choice? How can the question be changed to reduce the number of flavours you would need to buy?

Explain to students that it is helpful to predict the most popular answers to a survey question before a survey is conducted. ASK: Why is it important to predict the most popular answers? Could the three most popular flavours of ice cream have been predicted?

Have volunteers predict the most popular answers for the following survey questions:

- What is your favourite colour?
- What is your favourite animal?

Students may disagree on the choices. Explain to them that a good way to predict the most popular choices for a survey question is to ask a few people the survey question before asking everyone.

**Including the “other” category.** Tell students that sometimes you need to add “other” as a choice to make sure that everyone can answer the survey question. Write on the board:

What is your favourite food group?

- ☐ Vegetables and fruits
- ☐ Meat and alternatives
- ☐ Milk and alternatives
- ☐ Grain products

What is your favourite food?

- ☐ Soup
- ☐ Spaghetti
- ☐ Tacos
- ☐ Salad

ASK: Which of these questions needs “other” as a choice? (the favourite food question) How do you know when an “other” category is needed? (when there are too many possible answers or too few options to choose from) For the following exercises, write each question on the board one by one, and discuss whether the question requires an “other” category and why.

**Exercises:** Does the question require an “other” category? Why?

a) What is your favourite day of the week?

- ☐ Sunday
- ☐ Monday
- ☐ Tuesday
- ☐ Wednesday
- ☐ Thursday
- ☐ Friday
- ☐ Saturday
b) Which is your favourite day?
- Friday
- Saturday
- Sunday

c) What is your favourite animal?
- horse
- cow
- dog
- pig
- cat

d) How many siblings do you have?
- 0
- 1
- 2
- 3
- 4
- 5 or more

e) Who will you vote for in the student council election?
- Student A
- Student B
- Student C

Selected sample answers: a) The category “other” is not necessary because all days are listed. c) The category “other” is necessary because some people like other animals.

Questions requiring secondary data. SAY: The examples so far have been questions about what the class likes best. The best way to answer that kind of question is to do a survey. But there are lots of questions that we can’t answer by doing a survey. Sometimes that’s because we have no way to talk to the people we want answers from—for example, if we wanted to know the favourite flavour of ice cream in the world or how many siblings every person in Canada has. Sometimes a survey won’t work because the question isn’t about individual people. For example, you wouldn’t use a survey to learn what the warmest city in the world is. These kinds of questions need secondary data to be answered. That means you have to look up the answers by using someone else’s work. Many different kinds of questions can be answered using secondary data, but for math class, we want to ask questions that can be answered in class. ASK: Have you ever looked up the answer to a question on a computer or in a book? What kind of questions could be answered this way? Write the suggestions on the board. Examples: What was the biggest dinosaur? What part of Canada gets the most rain? What is Canada’s favourite ice cream?

Answering a research question. Using an example of a research question, lead a discussion about where you might look to find answers to questions requiring secondary data. For example, if you chose to examine the question “What part of Canada gets the most rain?” you could have the following discussion. ASK: Can we find out exactly how much rain there is everywhere in Canada? (no) Why not? (it’s too big; every place is a little bit different) SAY: We have to make the question a little bit smaller. Suggest that students could choose one city from each province and territory and look at the annual rainfall for those cities. Ask for ideas about how to pick the city (Examples: use capitals, look at a map and pick the city closest to the centre). Ask for suggestions about how to find the annual rainfall. Tell students about the kind of secondary data that is available from Environment Canada.
NOTE: You may want to research these questions as a class.

Exercises: Use a computer to answer the question.

a) What is the average rainfall for this day or the year where you live?

b) What is the most rain ever recorded on this day where you live?

c) What are the five fastest animals? How fast do they go?

Answers: a) answers will vary; b) answers will vary; c) sample answer: peregrine falcon at 322 km/h, frigate bird at 153 km/h, sail fish at 109 km/h, cheetah at 98 km/h, and pronghorn antelope at 97 km/h

NOTE: The project described in the following activity is required to meet the curriculum requirements in Alberta and Ontario. It can be done progressively throughout this unit or it can be used in conjunction with lessons on other subject matter, such as science or social science. To meet Alberta technology requirements, students should choose a question in Part 1 that requires secondary data and research their question using electronic sources. In addition, they should display their findings using spreadsheet software, as in Part 4.

ACTIVITY (Optional)

Students can complete Part 1 of BLM Project.

Extensions

1. Discuss a graph that has an “other” category with a bar higher than at least one other bar. Draw on the board:

Favourite Pizza Topping

<table>
<thead>
<tr>
<th></th>
<th>Olives</th>
<th>Cheese</th>
<th>Pepperoni</th>
<th>Other</th>
</tr>
</thead>
<tbody>
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<td>X</td>
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ASK: If we ask, “What is the least popular pizza topping shown on this graph?” what will the answer be? SAY: “Olives” has the fewest votes, but the “other” bar may contain three different answers; in which case, each one is the least popular. We do not know what answers are included in the “other” bar. Also, there might be toppings that had no votes and so are not recorded at all. This means we cannot say what the least favourite topping is.
2. Have students research other questions that will generate data. Examples:
   a) What was the hottest day each year for the past 20 years in [a given city]?
   b) How many people were living in Canada every 10 years since 1900?
   c) What is the average lifespan in Canada versus [other countries]?
   d) What size are the planets in our solar system, and what is their distance from the sun?

3. Have students look through textbooks or media articles to find charts or graphs. ASK: Do the graphs show survey data or measurement data? What were the choices of answers? Who did the research or took the survey?

NOTE: This is a good opportunity to discuss the reliability of media sources.
Questions

Students will demonstrate an understanding of many-to-one correspondence.

Students will compare pictographs in which the same data has been displayed using one-to-one and many-to-one correspondences.

Prior Knowledge Required

Can skip count

Materials

BLM Project (p. K-46, optional)

Mental Math Minute. Ask a division question with a dividend within 100, such as $30 \div 6$. Repeat the question as “$30$ is how many times as many as $6$?” Eventually ask a “times as many” question directly without first asking the division question; for example, “$40$ is how many times as many as $5$?”

Introduce pictographs. Explain that you are going to conduct a survey and your question is “What is your favourite season?” SAY: There are four seasons, so my possible answers are winter, spring, summer, and fall. Write the four seasons on the board in a column. Have students suggest a title for the survey. (sample answer: Favourite Season) Write the title on the board above the seasons, and draw a grid next to the seasons (see below).

Favourite Season

<table>
<thead>
<tr>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
</tr>
</thead>
</table>

Tell students that instead of tally marks, each student will draw a happy face in the row next to their favourite season. Invite them to come up and draw a happy face one at a time, starting from the left side, not skipping any boxes. Explain to students that what they have created is called a pictograph. Write “pictograph” on the board. SAY: A graph is a way to show information from a survey or from research in a picture. A pictograph uses a symbol to show the numbers. The symbol we used was a happy face.

Analyzing a pictograph. ASK: Which season is the most popular? (answers will vary depending on class results) How does our graph make this easy to see? (the row for that season is the longest) Which season is the least popular? How do you know? How many people like summer best? How many people prefer spring? How can we see that? (count the number of smiley faces in the row for that season)
How many more/fewer? Help students compare the results for two seasons with questions of the form “How many more students chose summer over winter?” or “How many fewer students chose fall over spring?” Have students identify the differences on the graph. Explain that the numbers of people who chose each season are easy to compare, because the happy faces all start at the same place and there is one happy face per square.

Introduce scaled pictographs. Tell students that you would like to make a pictograph for the flowers that grow in your garden. You have counted 40 daffodils, 50 buttercups, and 30 daisies. ASK: What would be a good symbol for my flowers? (answers will vary) Suggest using a simple flower picture as shown below. Tell students that the graph might be kind of big with all those flowers, and ask for suggestions of what to do. Tell students that for large numbers, symbols can sometimes stand for more than one. Draw on the board:

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<table>
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<th>Flowers in My Garden</th>
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</table>
c) ≈ 4 flowers. How many flowers are there altogether?

**Answers:** a) 4, b) 12, c) 48

**Features of pictographs.** Draw on the board:

<table>
<thead>
<tr>
<th>Favourite Drink</th>
</tr>
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<tbody>
<tr>
<td>Apple juice</td>
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<tr>
<td>Milk</td>
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<tr>
<td>Orange juice</td>
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<tr>
<td>Water</td>
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<tr>
<td>Apple juice</td>
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<td>Orange juice</td>
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<td>Water</td>
<td>☺☺</td>
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= 4 people

**ASK:** What is the title of this graph? (Favourite Drink) What are the choices? (apple juice, milk, orange juice, water) What is being counted? (people) What is the scale on the graph? (one happy face is four people)

**Using a half symbol.** **ASK:** How many people chose apple juice? (24) Point to the half smiley face in the milk column. **ASK:** In this row, there is a new symbol—what does it look like? (a half face) What do you think it means? (half of 4 people) **SAY:** Each smiley face stands for four people, and so a half smiley face stands for two people, because two is half of four. **ASK:** How many people chose milk? (10) Orange juice? (12) Water? (6) Write the numbers beside each of the rows. Leave the graph on the board.

**Exercises**

1. **What does the half symbol mean?**
   - a) ☺☺ = 8 people ☺ = ?
   - b) ☺☺ = 6 people ☺ = ?
   - c) ☺☺☺ = 12 people ☺ = ?
   **Answers:** a) 4, b) 3, c) 6

2. **What number do the symbols represent?**
   - a) ☺ = 2 people ☺☺☺ = ?
   - b) ☺ = 6 people ☺☺☺ = ?
   - c) ☺☺ = 8 people ☺☺ = ?
   - d) ☺☺☺ = 10 people ☺☺☺ = ?
   **Answers:** a) 7, b) 21, c) 20, d) 45

**Times as many questions.** **Refer back to the “Favourite Drink” graph on the board.** **ASK:** How many more people chose apple juice than orange juice? (12) How many times as many people chose apple juice as orange juice? (2) **SAY:** 24 is 2 times 12, so 2 times as many people chose apple juice.
ACTIVITY (Optional)

Students can complete Parts 1–3 on BLM Project.

Extensions

1. The pictograph shows the results of a survey about favourite colours.

<table>
<thead>
<tr>
<th>Favourite Colour</th>
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<tbody>
<tr>
<td>Red</td>
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<tr>
<td>Blue</td>
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<tr>
<td>Yellow</td>
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</table>

The scale is not given in this pictograph. Answer the question with the given scale.

a) $\star = 1$ person  
b) $\star \star = 2$ people  
c) $\star \star \star = 10$ people

i) How many more people chose blue than yellow?  
ii) How many more people chose blue than red?  
iii) How many times as many people chose yellow as red?  
iv) How many times as many people chose blue as red?

d) What do you notice about your answers?

Answers: a) i) 1, ii) 2, iii) 2, iv) 3; b) i) 2, ii) 4, iii) 2, iv) 3; c) i) 10, ii) 20, iii) 2, iv) 3; d) The answers to the “times as many” questions, parts iii) and iv), don’t change even when the scale does.

2. Explain that sometimes people use the same symbol in all rows of a pictograph and sometimes they use different symbols. Draw the graph below on the board, and explain that it shows how many times during the week students have different after-school classes:

<table>
<thead>
<tr>
<th>After-School Classes</th>
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<tbody>
<tr>
<td>Art</td>
</tr>
<tr>
<td>Music</td>
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<td>Soccer</td>
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</table>

ASK: I think that there are more art classes during the week than music classes or soccer classes—is that correct? (no) Why not? (there are fewer symbols for art than for music and soccer) I think there are more soccer classes than music classes—is that correct? (no) Why not? (there are fewer symbols for soccer than for music) Why might I make mistakes? (the paintbrushes are longer; the soccer balls are not lined up with the other symbols) How could we redraw the pictograph to make it easier to read? (make the symbols the same size and line
them up; use different symbols that are all the same size; use the same symbol in every row) Have students redraw the pictograph using one or two of the suggestions made.

3. Discuss what is wrong with the pictograph:

Favourite Sports of Students in Class A

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<td>Soccer</td>
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<td>Basketball</td>
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SAY: One happy face means one student who picked that sport as their favourite. ASK: Which sport is the most popular? (hockey) Which sport has the longest row of faces? (soccer) Why is it easier to read the pictograph when all the faces are the same size? (you can just look for the longest row without counting) Have students redraw the pictograph correctly.
Goals
Students will create pictographs, including selecting the scale and justifying their choice.

PRIOR KNOWLEDGE REQUIRED
Can read pictographs with one symbol representing multiple items
Can read pictographs with half symbols
Can skip count

MATERIALS
a ball
BLM Graph Template (p. K-47)
30 assorted connecting cubes in blue, green, red, brown, and yellow, with at least one cube of each colour, per pair of students
BLM Project (p. K-46, optional)
large piece of cardboard, string, tape, clothespins (see Extension 2)
BLM 1 cm Grid Paper (p. S-2, see Extension 2)

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $10 \times 10$ and corresponding division questions. You can toss a ball to the student you want to answer the question, and have the student toss the ball back to you after answering.

Review reading pictographs, including using a half symbol. Remind students that a symbol on a pictograph can mean more than one item.

ASK: If a happy face stands for 10 people, how many people does half of a happy face stand for? (5)

Exercises: How many people does the picture show?

a) \[\smiley \smiley \smiley = 10 \text{ people} \]
b) \[\smiley \smiley = 4 \text{ people} \]
c) \[\smiley = 5 \text{ people} \]

i) \[\smiley \smiley \smiley \]  
ii) \[\smiley \smiley \]  
iii) \[\smiley \smiley \smiley \smiley \smiley \]  

Answers: a) i) 30, ii) 20, iii) 15; b) i) 8, ii) 16, iii) 14; c) i) 15, ii) 25, iii) 30

Creating pictographs, including using half symbols. Write on the board:

\[\smiley \smiley \smiley = 10 \text{ people} \]

30 people =  
40 people =  
5 people =  
35 people =
ASK: How can we show 30 on a pictograph with the scale given? (3 smiley faces) How do you know? (3 \times 10 = 30) Have a volunteer fill in the answer on the board. ASK: How can we show 40? (4 smiley faces) How do you know? (4 \times 10 = 40) Have a volunteer fill in the answer on the board. ASK: How can we show 5? (half of a smiley face) How do you know? (10 \div 2 = 5) Have a volunteer fill in the answer on the board. ASK: How can we show 35? (3 full faces and 1 half of a smiley face) Have a volunteer fill in the answer on the board. ASK: How can you show 15? (1 full face and 1 half of a smiley face)

**ACTIVITY 1 (Essential)**

Give each student a copy of BLM Graph Template. Give each pair of students a collection of 30 connecting cubes in any combination of the following colours: blue, green, red, brown, and yellow. Have partners work together to sort the cubes by colour and then work independently to create a pictograph of their collection on BLM Graph Template, using the scale 1 square = 2 cubes. Partners should compare the two graphs for their cube collection.

**Answering questions using a pictograph.** Write on the board:

a) For which colour are there the most cubes?

b) For which colour are there the fewest cubes?

c) How many red and blue cubes do you have altogether?

d) How many cubes that are not green do you have?

e) How many cubes do you have in total?

Have students answer the questions about their own graphs from Activity 1.

**Choosing a scale for data.** Explain that sometimes students will need to decide what scale to use for data. SAY: Suppose you surveyed 200 people about their favourite team sport and gave them three answers to choose from: baseball, soccer, and ice hockey. In the survey, 100 people chose baseball, 45 chose soccer, and 55 chose ice hockey. Write on the board:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball</td>
<td>100</td>
</tr>
<tr>
<td>Soccer</td>
<td>45</td>
</tr>
<tr>
<td>Ice hockey</td>
<td>55</td>
</tr>
</tbody>
</table>

ASK: If I used a scale of 1 symbol representing 2 people, how many symbols would I need to show 100 people? (50) Have students try to count by 2s to reach 50. When they see that they do not have enough fingers on their hands to keep track, ASK: Does it make sense to use 2s, or should we count by a bigger number? Students can repeat with counting by 3s and by 5s. ASK: What number should we count by? (10s) Have students count by 10s to see that they need 10 symbols to represent the number of people who chose baseball.

NOTE: These should be random, unmatched collections, but each student pair should have at least one cube of each colour.
ASK: Can we represent 45 or 55 people using the scale, where one symbol is equal to 10 people? (yes) How would you represent these numbers? (4 full and 1 half symbols for 45, 5 full and 1 half symbols for 55) SAY: So, 10 seems like a reasonable scale in this case. Point out that if you had, say, 42 people choosing soccer and 58 people choosing ice hockey, you would have trouble using 10 because you would not be able to use halves to show 2 people and 8 people.

Change the numbers in the table to 15, 6, and 9. ASK: Is 10 a good scale for these new numbers? (no) Why not? (6 and 9 cannot be shown with a whole symbol or with half a symbol) SAY: Let’s try 1 symbol equals 2 people. Have students say how many symbols they would use for each number. (7 1/2, 3, 4 1/2) SAY: We say 15, 6, and 9 when we count by something other than 2. ASK: What number can we count by? (3) Have students say how many symbols they should draw for each of the numbers with the scale 1 symbol = 3 people. (5, 2, 3)

SAY: I am going to write groups of numbers. Each group of numbers is data. We need to show this data on a pictograph. Have students signal the answers for each part in the following exercises.

Exercises: Which scale should we use—1 symbol equals 2, 3, or 5?

a) 6, 10, 8  
   b) 6, 10, 7  
   c) 15, 20, 10  
   d) 9, 12, 21  

Bonus: 18, 15, 9, 21, 27, 30

Answers: a) 2, b) 2, c) 5, d) 3, Bonus: 3

**ACTIVITY 2 (Optional)**

Students can complete Parts 1–4.a) of BLM Project.

**Extensions**

1. Make a pictograph for the letters in the city name Mississauga. Use each letter as its own symbol in the graph.

Answer:

<table>
<thead>
<tr>
<th>Letters in Mississauga</th>
</tr>
</thead>
<tbody>
<tr>
<td>M M</td>
</tr>
<tr>
<td>I II</td>
</tr>
<tr>
<td>S SSSS</td>
</tr>
<tr>
<td>A AA</td>
</tr>
<tr>
<td>U U</td>
</tr>
<tr>
<td>G G</td>
</tr>
</tbody>
</table>
2. In advance, prepare materials to conduct a survey and record the results. On a large piece of cardboard, write the title and labels shown below and attach strings that hang down from each label. In the survey, have students attach a clothespin to the appropriate string to show their answers. Ensure that students hang their clothespins the same distance apart or touching.

<table>
<thead>
<tr>
<th>Our Birthplaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
</tr>
<tr>
<td>North America</td>
</tr>
<tr>
<td>South America</td>
</tr>
<tr>
<td>Africa</td>
</tr>
<tr>
<td>Asia</td>
</tr>
<tr>
<td>Europe</td>
</tr>
<tr>
<td>Oceania</td>
</tr>
</tbody>
</table>

Tell students you are going to conduct a survey to find out where they were born. Distribute a clothespin to each student. Show a world map, point out each continent, show where Canada is, and show the areas of North America that are not Canada. Point out that Australia is both a country and a continent, and that the continent of Antarctica is missing because nobody lives there permanently. ASK: Where were you born? Have students record their answers by each attaching their clothespin to the appropriate string. Help students who know the country of their birth but are not sure which continent it is on. Have volunteers count the clothespins and record the answers in a table. Have students make a pictograph on grid paper or **BLM 1 cm Grid Paper** to show the results.

ASK: Where were the largest number of people in our class born? Where were the smallest number of people in our class born? How do you know? How many people were born in Canada? Make a comparison based on your class results. For example, ASK: How many people were born in Asia? How many more people were born in Canada than in Asia?

3. Have students conduct a survey of their classmates and create a pictograph of the results on BLM Graph Template. Some ideas for quick surveys:
   - How old are you?
   - What shoe size do you wear?
   - How many siblings do you have?
   - How many people live in your home?
Goals

Students will compare bar graphs in which the same data has been displayed using one-to-one and many-to-one correspondences. Students will interpret bar graphs involving many-to-one correspondence to draw conclusions. Students will answer questions related to information presented in bar graphs.

Prior Knowledge Required

Can read data from a table
Can read pictographs with one symbol representing multiple items
Can read pictographs with half symbols
Can answer times as many questions

Materials

overhead projector
transparency of BLM Colours of Cubes (p. K-48)
erasable markers of different colours (blue, green, red, black)
transparency of BLM Snack Bar Graphs (p. K-49)
transparencies of BLM Bar Graphs for Display (1) to (2) (pp. K-50–51)
BLM Project (p. K-46, optional)

Mental math minute. Give students division problems that they can do by skip counting.

Review pictographs. Remind students of the features of pictographs: they always have titles, symbols, and labels; they often include a grid on which the results are shown and a scale. SAY: Sometimes instead of more complicated pictures, we use squares as symbols to show our data.

Introduce bar graphs. Explain that to make graphs simple, you can join the squares on graphs into columns or rows of squares. These joined-together columns or rows of squares are called bars. Explain that you will use one grid square for one cube. Project BLM Colours of Cubes on the board. The graphs on the BLM show the following data:

<table>
<thead>
<tr>
<th>Colour of Cubes</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>4</td>
</tr>
<tr>
<td>Green</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>5</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
</tr>
</tbody>
</table>
Explain that both graphs show the same data, or the same collection of cubes. SAY: The graph on the left is a pictograph, like the ones you have looked at and created before. The graph on the right is called a bar graph, because it shows the data in bars. The blocks in each bar are usually squares, but sometimes there is not enough space to make the blocks square, so people use rectangles as well. In this bar graph, each square block in each bar means 1 cube. The bar for blue cubes is 4 blocks long, so we know that there are 4 blue cubes in the collection. ASK: How many green cubes are there? (3) How do you know? (the bar is 3 blocks long) How many red cubes are there? (5) How many yellow cubes? (4)

Introduce vocabulary. Ask students what the two graphs have that is the same, besides showing the same data. (the title, the labels, both go sideways) Circle the title in both graphs using a red marker and write “title” in red beside the graphs. Circle the labels that are shared in both graphs using a blue marker and write “labels” in blue. Explain that a bar graph has more labels than a pictograph. All other markings in words (not numbers) on a bar graph are also called labels. Circle the rest of the labels in blue.

Point out the general organization of the bar graph. Trace the axes, and explain that these two lines make an L-shape and the lines are called axes. SAY: When we talk about one of these, we call it an axis, but when there are two of them, we call them axes. Write both words on the board, and trace the axes on the bar graph with a black marker. Have a volunteer underline the part that is different in the two words. (the third letter)

Explain that axes and their labels help us understand what we are looking at on bar graphs. SAY: One of the lines has numbers. Cover the rest of the graph so that only the bottom axis and the numbers are visible. ASK: What do you call a line that has numbers in counting order under it? (number line) What other type of graph has a number line? (line plot) SAY: The numbers on the number line are called a scale. Point out that a pictograph also has a scale, something that tells you how many pieces of data each symbol means. Explain that a scale in a bar graph plays a similar role and that you will talk about that more in the next lesson. Circle the scales on both graphs with a green marker and write the word “scale” in green beside the graphs.

Draw students’ attention to the fact that the bars on a bar graph usually have spaces between them. Explain that this makes a bar graph easier to read. Space is usually added on either side of each bar, including the bar closest to the number line.

Reading a bar graph. Project the “Favourite Snacks” graph from BLM Snack Bar Graphs on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>5</td>
</tr>
<tr>
<td>Bagels</td>
<td>4</td>
</tr>
<tr>
<td>Fruit</td>
<td>7</td>
</tr>
<tr>
<td>Cheese</td>
<td>3</td>
</tr>
</tbody>
</table>
Ask students to find the title, labels, and scale on the graph. ASK: Who was surveyed to make this graph? (students) What question were they asked? (What is your favourite snack?) What were the possible answers? (muffins, bagels, fruit, cheese)

Have students signal the numerical answers for the next questions. ASK: What was the most common answer? (fruit) What was the least common answer? (cheese) How many students chose muffins as their favourite snack? (5) How many students chose fruit? (7) How many students voted for baked snacks? (9) How do you know? (muffins and bagels are both baked snacks, $5 + 4 = 9$)

ASK: How many students were surveyed in total? (19) How do you know? ($5 + 4 + 7 + 3 = 19$) What is the most popular snack of the four, or the snack that was chosen the most number of times? (fruit) What is the least popular snack, or the snack that was chosen the fewest number of times? (cheese)

Project “Snacks Eaten Today” from BLM Snack Bar Graphs for use in the following exercises.

**Exercises**

a) What is the title of the graph?
b) What was counted to make the graph?
c) What was the most common snack?
d) How many times as many people ate fruit as vegetables?
e) How many times as many people ate fruit as muffins?
f) Can you tell from the graph what the most popular snack was? Explain.

**Answers:**
a) Snacks Eaten Today, b) students, c) bagels, d) 2 times, e) 3 times, f) No, you can only tell what was most common because people don’t always eat their favourite snack.

**Introduce scaled bar graphs.** Remind students that, in a pictograph, sometimes the symbol stands for more than one thing. Project “Number of Snow Days in Calgary, AB” from BLM Bar Graphs for Display (1). Explain that you can do the same thing with bars: you can use one block to mean more than one thing. SAY: We show the scale by skip counting on the number line. ASK: What did we skip count by in this graph? (3)

Ask questions to ensure that students can read data on the graph; for example, ASK: How many days of snow are there in December and January? (21) How many more days of snow are there in February and March than in December and January? (6) If there are 6 more days of snow, why is the January-February column only two squares longer? (because each square represents 3 days)

Project “Sam’s Reading on Weekdays” from BLM Bar Graphs for Display (1) for use in the following exercises.
Exercises

a) What is the title of the graph?

b) What was counted to make the graph?

c) On which day did Sam read the fewest pages? How many pages did he read?

d) On which day did Sam read twice as many pages as he read on Monday?

Bonus: On which day did Sam read three times as many pages as he read on Tuesday?

Answers: a) Sam’s Reading on Weekdays; b) pages read; c) Wednesday, 4; d) Friday; Bonus: Friday

Introduce half-squares. Project “Favourite Winter Activities” from BLM Bar Graphs for Display (2). ASK: What does this graph show? (favourite winter activities) What choices were given? (building snow forts, building snowmen, skiing, sledding, snowshoeing) SAY: Notice that the bar for snowshoeing comes to about the middle of a square, between two marks on the number line. ASK: How many people do you think picked snowshoeing? (15) SAY: Just like a half-symbol in a pictograph is half as much as a whole symbol, a half-square on a bar graph is half as much as a whole square.

Exercises

a) How many people chose building snow forts as their favourite activity?

b) How many people were surveyed altogether?

c) How many times as many people chose building snow forts as snowshoeing?

Bonus: You have two buses to take everyone to their favourite activity. You would like everyone going to the same activity to be on the same bus and the number of people on each bus to be as close to the same as possible. Who will go on each bus?

Answers: a) 45, b) 200, c) 3, Bonus: skiing and building snow forts on one bus, sledding, building snowmen, and snowshoeing on the other.

ACTIVITY (Optional)

Students can continue with Parts 1–4.b) of BLM Project.
Extensions

Remind students of the value of each coin in cents.

1. The bar graph shows the coins in Jayden’s pocket.

   **Coins in Jayden’s Pocket**
   
<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickels</td>
<td>6</td>
</tr>
<tr>
<td>Dimes</td>
<td>3</td>
</tr>
<tr>
<td>Quarters</td>
<td>9</td>
</tr>
<tr>
<td>Loonies</td>
<td>3</td>
</tr>
</tbody>
</table>

   a) For each type of coin, how much money does Jayden have?
   b) How much money does Jayden have in total?

   **Answers:**
   a) 75¢ in nickels, 120¢ in dimes, 300¢ in quarters, and 300¢ in loonies;
   b) 795¢

2. Sharon made a bar graph showing the coins in her piggy bank, but she forgot to label the scale. She has 30 nickels.

   **Coins in Sharon’s Piggy Bank**
   
<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickels</td>
<td>30</td>
</tr>
<tr>
<td>Dimes</td>
<td>10</td>
</tr>
<tr>
<td>Quarters</td>
<td>15</td>
</tr>
<tr>
<td>Loonies</td>
<td>5</td>
</tr>
</tbody>
</table>

   a) What number did she skip count by for the scale?
   b) How many coins of each type does she have?
   c) How much money does she have in her piggy bank in total?

   **Answers:**
   a) 5s;
   b) 30 nickels, 10 dimes, 15 quarters, and 5 loonies;
   c) $1.125 = 75¢ + 100¢ + 375¢ + 500¢$
3. Marko drew a bar graph of the coins in his piggy bank, but he forgot to label the scale. He has 40 coins in total.

**Coins in Marko's Piggy Bank**

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickels</td>
<td>10</td>
</tr>
<tr>
<td>Dimes</td>
<td>15</td>
</tr>
<tr>
<td>Quarters</td>
<td>5</td>
</tr>
<tr>
<td>Loonies</td>
<td>10</td>
</tr>
</tbody>
</table>

a) How many coins of each type does he have?
b) What number did he skip count by for the scale? 
c) How much money does he have in his piggy bank in total?

**Answers**
a) 10 nickels, 15 dimes, 5 quarters, 10 loonies; b) 5s; 
c) $0.50 + 1.50 + 1.25 + 10.00 = 13.25$
**Goals**

Students will construct bar graphs, including selecting the scale and justifying their choice.

**PRIOR KNOWLEDGE REQUIRED**

- Can read data from a table
- Can read and create pictographs with one symbol representing multiple items
- Can read bar graphs with number lines that skip count
- Can read bar graphs with bars that end halfway between two markers on the number line

**MATERIALS**

- transparency of BLM Graph Template (p. K-47) and one copy per student
- BLM Project (p. K-46, optional)

**Mental math minute.** Have students add by using 10. Say the addition you want students to do (such as $18 + 6$). Have a student say the in-between addition step, $20 + 4$, and have another student finish the addition. Start with adding one-digit numbers to two-digit numbers, and progress to adding two-digit numbers to two-digit numbers. As a challenge, use three-digit and four-digit numbers, such as $345 + 8$, or vary the order, e.g., $8 + 56$.

**Drawing bar graphs from tally charts.** Draw on the board:

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Tally</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>![Tally]</td>
<td></td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>![Tally]</td>
<td></td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>![Tally]</td>
<td></td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td>![Tally]</td>
<td></td>
</tr>
</tbody>
</table>

Explain that the table shows the tally of a survey of when students in a class go to bed. For example, a student who goes to bed at 8:45 p.m. is counted with the students in the row that says 8:30 p.m.–8:59 p.m. Ask a volunteer to complete the “Count” column. (2, 8, 14, 10, 4)

Point to the “Count” column and SAY: Now we have all the data values, so we can draw a bar graph to show the data.

Distribute copies of BLM Graph Template and project a copy on the board. ASK: What would be a good title for this graph? (answers will vary) Choose
an appropriate title suggested by students and fill it in on your graph. Invite students to fill in the title on their graphs. Next, fill in the scale and label on the horizontal axis together. Repeat for the vertical axis. Tell students that you will label each bar exactly the way it was labelled on the tally chart. Finally, have volunteers fill in the bars as students complete their own graphs. The result is shown below:

![Bar graph of bedtimes for Grade 4 Students]

Choosing scales for bar graphs. Write on the board:

<table>
<thead>
<tr>
<th>Days of Week</th>
<th>Number of Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>12</td>
</tr>
<tr>
<td>Tuesday</td>
<td>10</td>
</tr>
<tr>
<td>Wednesday</td>
<td>4</td>
</tr>
<tr>
<td>Thursday</td>
<td>8</td>
</tr>
<tr>
<td>Friday</td>
<td>24</td>
</tr>
</tbody>
</table>

Tell students that they will be making a bar graph for the data presented. ASK: Do you have to skip count on the number scale or can you count by 1s? (skip count) Why? (the graph won’t fit otherwise) Have students count how many squares there are across the graph template. (14) Point out that the highest number, 24, is greater than the number of available squares. Ask students what they would skip count by and why. Summarize the discussion. SAY: We could skip count by 2s or 4s. All of these numbers are multiples of 2. ASK: Are they all multiples of 4? (no) Which numbers aren’t multiples of 4? (10) Can you still show 10 on the graph if you are counting by 4s? (yes) Draw on the board:

![Number line for choosing scales]

Have a volunteer indicate where 10 is on this number line.
### Exercises

1. Fill in the missing numbers on the scale.
   
   a) \[0 \quad 5 \quad 15 \quad 20 \quad 30\]
   
   b) \[0 \quad 4 \quad 8 \quad 12 \quad 16\]
   
   c) \[0 \quad 40 \quad 80 \quad 120\]

   **Answers:** a) 10, 25; b) 8, 12, 20; c) 20, 60, 100

2. Put an \(\times\) on number line b) where 6 goes.

   **Answer:**
   
   \[0 \quad 4 \quad 8 \quad 10 \quad 12 \quad 16 \quad 20\]

3. Put an \(\times\) on number line c) where 90 goes.

   **Answer:**
   
   \[0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100 \quad 120\]

### ACTIVITY 1 (Essential)

Have students draw bar graphs for the data on the board using skip counting by 2s or 4s.

### Exercises: What scale would you use for the following data?

a) 3, 24, 9, 15  
   
   b) 20, 4, 8, 40  
   
   c) 20, 40, 45, 70  
   
   **Bonus:** 6, 12, 9, 14

**Answers:** a) 3, b) 4, c) 10, Bonus: 2 or 3

Discuss the answers students gave for the bonus exercise above. Draw on the board the number line in the margin.

ASK: Which two numbers on the number line is 14 between? (12 and 15) Is 14 closer to 12 or 15? (15) Have a volunteer place an X approximately where 14 belongs on the number line (see margin). SAY: It is easier to show numbers that are exactly halfway between points, but we can show other numbers too, sometimes. You have to be careful, though, because they can be hard to read.

### ACTIVITY 2 (Optional)

Students can complete Parts 1–3 and 4.a) or b) of BLM Project.

Students who will not be doing any further lessons in this unit should also do Part 6 of the project.
NOTE: Extension 1 is required to cover the Alberta curriculum.

Extensions

1. Have students use the data from their projects, or data from any of the graphs used in this unit so far, to create new graphs using spreadsheet software.

2. Give students a paragraph of text (for example, from a favourite story read in class or from another subject) and ask them to each create a bar graph that shows the number of words on each line.

3. Give students a paragraph of text (for example, from a favourite story read in class or from another subject) and ask them to each create a bar graph that shows the frequency of each letter in the text.

ASK: Which were the three most frequent letters in your text? Have students research the most frequent letters in the English language (E, T, and A). ASK: Were the most frequent letters in your text also E, T, and A?

4. Using a Caesar cypher, messages are encoded by shifting the letters in the alphabet. For example, in the table below, a becomes b, b becomes c, and so on. So “Happy New Year” becomes “Ibqqz Ofx Zfbs.”

\[
\begin{array}{cccccccccccccccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} \\
\text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} & \text{a} \\
\end{array}
\]

a) Encrypt “Have a nice day” using the cypher in the table above.

b) The message below was encrypted using this table. What does it say?

\[
\begin{array}{cccccccccccccccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} \\
\text{d} & \text{e} & \text{f} & \text{g} & \text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} & \text{a} & \text{b} & \text{c} \\
\end{array}
\]

Wkh sdvvzrug lv prqnhb

c) The message below was encrypted using a Caesar cypher. Use the fact that the most common letter in the English language is E to find the cypher and decrypt the message.

\[
\begin{array}{cccccccccccccccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} \\
\end{array}
\]

Bpmzm qa l jqzl qv bpm bzmm

Answers

a) Lbwf b ogdf ebz
b) The password is monkey

\[
\begin{array}{cccccccccccccccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} & \text{h} & \text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} \\
\text{i} & \text{j} & \text{k} & \text{l} & \text{m} & \text{n} & \text{o} & \text{p} & \text{q} & \text{r} & \text{s} & \text{t} & \text{u} & \text{v} & \text{w} & \text{x} & \text{y} & \text{z} & \text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} \\
\end{array}
\]

There is a bird in the tree
Goals

Students will read and create double bar graphs.
Students will answer questions about the data presented in double bar graphs.

PRIOR KNOWLEDGE REQUIRED

Can read and create bar graphs with number lines that skip count and bars that end between markings

MATERIALS

overhead projector
transparency of BLM Double Bar Graphs (p. K-53)
transparency of BLM Double Bar Graph Template (p. K-54) and one copy per student
two erasable markers in different colours
two pencil crayons in different colours, per student
BLM Project (p. K-46, optional)

Mental math minute. Draw a piece of a number line on the board going from 11 to 19. Show students that the number halfway between 12 and 14 is 13, and the number halfway between 15 and 17 is 16. Then ask students the number that is halfway between any two numbers within 100 that are two digits apart.

Introduce double bar graphs. Tell students that sometimes you want to compare data presented in graphs. You can do this by putting two separate bar graphs side by side or by creating one combined graph that has bars for each set of data. Tell students that these combined graphs are called double bar graphs.

Reading double bar graphs. Project the first graph from BLM Double Bar Graphs on the board. ASK: What is the title of the graph? (Monthly Precipitation in Victoria and Edmonton) Explain the meaning of “precipitation.” ASK: What is the scale on the vertical axis? (count by 20s) What are the categories on the horizontal axis? (months) What is different about this graph from a normal bar graph? (there are two bars in every category) Where does the graph say what each bar means? (underneath the graph) SAY: On double bar graphs, the two bars are filled in with different colours or patterns so that you can tell which one is which. Somewhere on the graph, in this case it is at the bottom, there are two little pieces of filled-in squares showing you what each bar represents. On this graph, one of the bars is striped and the other one is coloured in. Double bar graphs often use different colours instead. ASK: Which city is striped? (Victoria) Which one is coloured in? (Edmonton) SAY: Notice that the bars never trade places. The striped bar representing Victoria is always on the left of the coloured-in bar representing Edmonton.
Ask questions about the graph to ensure that students can read it. For example, ASK: How much rain does Victoria get in June? (30 mm) Which city gets more precipitation in January? (Victoria) How many times as much precipitation does Victoria get as Edmonton in January? (7) In which months of the year does Edmonton get more precipitation than Victoria? (May, June, July, August, and September)

Creating double bar graphs. Draw on the board:

**Average Daytime Highs in Vancouver and Toronto**

<table>
<thead>
<tr>
<th></th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vancouver</td>
<td>9°C</td>
<td>17°C</td>
<td>21°C</td>
<td>10°C</td>
</tr>
<tr>
<td>Toronto</td>
<td>1°C</td>
<td>18°C</td>
<td>24°C</td>
<td>8°C</td>
</tr>
</tbody>
</table>

Distribute BLM Double Bar Graph Template to each student. Explain the term “daytime high.” Tell students that they will draw a double bar graph using the data in this table. Project a copy of BLM Double Bar Graph Template. ASK: Where does the title of the graph go? (on the line above the graph) Tell students to fill in the title. They can use the title of the chart or a different title that has the same meaning. ASK: Where does the number line go? (down the left side of the graph) What is the tallest bar that you will need to draw? (summer in Toronto) How tall is it? (24) Will the bar fit if you count by ones on your graph? (no) SAY: Make sure that you pick a scale that will work for all of the numbers. Have students fill in the numbers on the graph. Prompt students to fill in the label on the vertical axis, and the label and four seasons on the horizontal axis.

Tell students that they are almost ready to draw the bars. First, they have to pick a colour for each city. They can then colour in the rectangles with their chosen colours and write the names of the cities next to each one. Demonstrate filling in the two centre columns of the four columns labelled “Winter.” Let students complete the graph on their own.

**ACTIVITY (Optional)**

Students can complete Parts 1–4.c) of BLM Project.

**Extensions**

1. Bar graphs can compare any number of things. On the next page is an example of a triple bar graph. It shows average precipitation in millimetres in three cities. Churchill, Manitoba, is in the subarctic region; Bogota, Colombia, is near the equator; and Miami, Florida, is in between Churchill and Bogota.
a) Which city gets the most precipitation overall?
b) During which months does Bogota not have the most precipitation?
c) Which two months get the most precipitation for each city?
d) In your answer to part c), what seasons are those months in?
e) Which seasons do you think get the most precipitation where you live?

**Answers:**
a) Bogota; b) July and August; c) Churchill: August and September, Miami: June and September, Bogota: April and October; d) Churchill: summer, Miami: summer, Bogota: spring and fall

2. People who study weather measure temperatures hourly and then record the highest, lowest, and average temperatures for each day. The table shows the average daytime highs and lows for each month in Winnipeg. Notice that some of the numbers have minus signs in front. This means the temperature is less than 0°C.

a) Round the numbers to the nearest multiple of 5.
b) Make a double bar graph for the rounded data. If you need help, look at some temperature graphs that have negative temperatures.
Answer: a)

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>High (°C)</td>
<td>−10</td>
<td>−10</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>−10</td>
</tr>
<tr>
<td>Low (°C)</td>
<td>−20</td>
<td>−20</td>
<td>−10</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>0</td>
<td>−10</td>
<td>−20</td>
</tr>
</tbody>
</table>

3. Find the hottest and coldest cities in the world. Create a bar graph of their monthly temperatures using BLM Double Bar Graph Template.
Goals
Students will build stem and leaf plots from data.
Students will use stem and leaf plots to sort data.

MATERIALS
BLM Project (p. K-46)

Mental math minute. Draw a piece of a number line on the board going from 11 to 19. Show students that the number halfway between 12 and 16 is 14, and the number between 13 and 17 is 15. Then ask students the number that is halfway between any two numbers less than 100 that are four digits apart. Also ask students which number is halfway between any two consecutive powers of 10, such as 10 and 20, or 40 and 50.

Introduce stem and leaf plots. Tell students that 20 adults were asked for their height in centimetres. Write the results on the board so you can use them later:

157, 159, 163, 172, 152, 181, 176, 190, 178, 183, 165, 166, 169, 173, 176, 186, 168, 171, 180, 191

Point out that it is a lot of data to work with. Tell students that a stem and leaf plot is a tool to help organize the data.

Defining stems and leaves. SAY: To make stem and leaf plots, we first divide numbers into stems and leaves. The leaves on a tree are at the very end of a stem. The leaf of a number is its rightmost digit, at the very end of the number. Underline the 7 in 157. ASK: What is the leaf of 159? (9) 163? (3) SAY: The rest of the number is the stem. Circle the stem (15) of 157. Have volunteers circle the stems of 159 and 163. (15, 16) Then have volunteers underline the leaves and circle the stems of the rest of the numbers. SAY: This is Step 1 of creating a stem and leaf plot. Write on the board where it can be kept for later: “Step 1: Find the stems and leaves of the data.” Point out that all the data are three-digit numbers, so the stems are always two digits, but that isn’t always true. For one-digit numbers, the leaf is that digit and the stem is just 0.

Exercise: Step 1: Underline the leaf and circle the stem.
29 12 27 7 25 19 107 102 8

Answers: 29, 12, 27, 7, 25, 19, 107, 102, 8
Creating stem and leaf plots. Draw on the board:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

SAY: To make a stem and leaf plot, we start by writing down all the stems in order in the column on the left. ASK: What is the first stem? (15) Write “15” on the stem side of the plot. Have students read out the rest of the stems. Tell them that you write each stem exactly once. When you are done, point out that the stems are written in order:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>7, 9</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

Write on the board below Step 1: “Step 2: Write the stems in order in the stem column.”

Exercise: Step 2: Write the stems from the data in Step 1 in order in a stem and leaf plot.

Answers:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

SAY: The next step is to write all the leaves that go with each stem. Write under Step 2: “Step 3: Write each leaf in the second column next to its stem.” Point to 157 in the data. ASK: What is the stem of 157? (15) What is the leaf? (7) SAY: The 7 goes in the leaf column (indicate the leaf column) next to 15, its stem. Write “7” in the stem and leaf plot. Repeat with 159 and 163:
Have volunteers add the rest of the leaves to the plot:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>7 9 2</td>
</tr>
<tr>
<td>16</td>
<td>3 5 6 9 8</td>
</tr>
<tr>
<td>17</td>
<td>2 6 8 3 6 1</td>
</tr>
<tr>
<td>18</td>
<td>1 3 6 0</td>
</tr>
<tr>
<td>19</td>
<td>0 1</td>
</tr>
</tbody>
</table>

Exercise: Step 3: Add the leaves to your stem and leaf plot.

Answers: Stem | Leaf
---|---
0 | 7 8
1 | 2 9
2 | 9 7 5
10 | 7 2

Tell students that the last step is to sort the leaves in order. Write on the board: “Step 4: Put the leaves in each row in order, from smallest to largest.” Demonstrate on the first row. Do the remaining rows with the help of volunteers:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2 7 9</td>
</tr>
<tr>
<td>16</td>
<td>3 5 6 8 9</td>
</tr>
<tr>
<td>17</td>
<td>1 2 3 6 6 8</td>
</tr>
<tr>
<td>18</td>
<td>0 1 3 6</td>
</tr>
<tr>
<td>19</td>
<td>0 1</td>
</tr>
</tbody>
</table>

Exercise: Step 4: Arrange the leaves in your stem and leaf plot in order.

Answers: Stem | Leaf
---|---
0 | 7 8
1 | 2 9
2 | 5 7 9
10 | 2 7

**ACTIVITY 1 (Optional)**

1. Have students measure the length of their shoes in centimetres and make a stem and leaf plot of the data.

Finding largest, smallest, and repeating numbers in a stem and leaf plot. Show students how to read out the numbers from the stem and leaf plot, one row at a time. Have students read the numbers from the finished plot. As they read the numbers, write them on the board: 152, 157, 159, 163, 165, 166, 168, 169, 171, 172, 173, 176, 176, 178, 180, 181, 183, 186,
ASK: What do you notice about the numbers? (they are in order from smallest to largest) Why are they in order? (because the stems are in order and the leaves are in order) SAY: We ordered the tens and hundreds digits by writing the stems in order, and then we sorted the ones digits, so all the numbers are in order. ASK: Where is the smallest number in the stem and leaf plot? (have a volunteer point to the 2 in the 15 row) Where is the largest number? (1 in the 19 row) Which number appears twice? (176) How can you see that in the stem and leaf plot? (there are two 6s in the 17 row)

**Exercises:** Find the largest and smallest numbers in the stem and leaf plot. Which number appears more than once?

<table>
<thead>
<tr>
<th></th>
<th>Stem</th>
<th>Leaf</th>
<th></th>
<th>Stem</th>
<th>Leaf</th>
<th></th>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>1</td>
<td>2 4 5</td>
<td>23</td>
<td>5 6 6</td>
<td>9</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 9</td>
<td>24</td>
<td>3 7 8</td>
<td>10</td>
<td>1 2 3 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1 1 7 8</td>
<td>25</td>
<td>2 5 9</td>
<td>11</td>
<td>4 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6</td>
<td>27</td>
<td>0 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 46, 12, 31; b) 271, 235, 236; c) 114, 99, 114

**ACTIVITY 2 (Optional)**

2. Students can complete Parts 1–4 of BLM Project.

**Extensions**

1. Stem and leaf plots can include more than numbers. Here is a stem and leaf plot of friends’ birthdays—the stem is the month and the leaf is the date:

<table>
<thead>
<tr>
<th></th>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>2 8 11 14 21</td>
<td></td>
</tr>
<tr>
<td>April</td>
<td>4 4 29 30</td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>13 14 22 27</td>
<td></td>
</tr>
</tbody>
</table>

a) Draw a bar graph from this data, with bars for each month. The height of each bar is the number of birthdays in that month.

b) How many birthdays are in the first half of a month?

c) How many birthdays are in April?

d) Which month has the most birthdays? The fewest?

e) Which month has a birthday closest to the beginning of the month? The end of the month?

f) If you were given only the bar graph of the birthdays, could you have built the stem and leaf plot? Explain.
2. John counts the total number of pages in each of his favourite novels: 148, 520, 589, 550, 224, 562, 494, 469.
   a) Draw a stem and leaf plot for the data.
   b) How many leaves does each stem have?
   c) Make a new stem and leaf plot using the hundreds digit as the stem and the last two digits as the leaves.
   d) What does using hundreds instead of tens tell you about John’s favourite novels that you couldn’t see as easily in the first plot?

   **Answers:**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>8</td>
<td>1</td>
<td>48</td>
</tr>
<tr>
<td>22</td>
<td>4</td>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>46</td>
<td>9</td>
<td>4</td>
<td>69 94</td>
</tr>
<tr>
<td>49</td>
<td>4</td>
<td>5</td>
<td>20 50 62 89</td>
</tr>
<tr>
<td>52</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **Sample answer:** d) Half of John’s favourite novels are over 500 pages.

3. Use a stem and leaf plot to create a set of data in which ...
   a) the smallest number is 100.
   b) the largest number is 100.

   **Sample answers:**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 1 4 5</td>
<td>8</td>
<td>1 4 7</td>
</tr>
<tr>
<td>11</td>
<td>2 6 8</td>
<td>9</td>
<td>3 6 8</td>
</tr>
<tr>
<td>12</td>
<td>4 6 9</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Goals

Students will find the range, median, and mode in a set of data by sorting the data and by creating stem and leaf plots.

PRIOR KNOWLEDGE REQUIRED

Can create a stem and leaf plot from a set of data
Can find the highest and lowest data values in a stem and leaf plot
Can find a whole number that is halfway between two whole numbers

MATERIALS

BLM Project (p. K-46, optional)

Mental math minute. Have pairs of students find the number that is in the middle of two given numbers: the first student adds the two numbers together, and the second student divides by 2. For example, to find the number halfway between 4 and 8, the first student adds \( 4 + 8 = 12 \), and the second student divides 12 by 2 to get 6. Explain that 6 is 2 more than 4 and 2 less than 8. Start with one-digit numbers and advance to two-digit numbers. Always make sure that the given numbers are either both even or both odd. Use consecutive multiples of 10 as examples (e.g., 85 is halfway between 80 and 90).

Introduce analyzing data. SAY: You have learned a lot about collecting and classifying data. However, to understand what the data you have collected means, you have to analyze it. Today we will begin learning how to analyze data.

Finding the range. SAY: Sometimes knowing the highest and lowest values in a set or group of data can be helpful. Suppose you know that the temperatures next week are predicted to be 15, 14, 12, 17, 11, 10, and 18 degrees. Write the temperatures on the board, and ASK: What is the highest temperature? (18) What is the lowest temperature? (10) Do you need to know all the temperatures to decide if you will need snow pants next week? (no) What do you need to know? (the lowest temperature) Do you need all the temperatures to decide if you will need a pair of shorts? (no) What do you need to know? (the highest temperature) Explain that the range of a set of numbers is the difference between the highest and the lowest numbers. Calculate the range of the temperatures in the above example with students. (\( \text{range} = \text{highest} - \text{lowest} = 18 - 10 = 8 \))

Exercises: Find the highest value, lowest value, and the range of the data.

\begin{align*}
\text{a) } & \quad 5 \quad 9 \quad 13 \quad 17 \quad 11 \\
\text{b) } & \quad 26 \quad 93 \quad 15 \quad 88 \quad 11 \quad 32 \\
\text{Bonus:} & \quad 947 \quad 852 \quad 512 \quad 1046 \quad 431 \\
\text{Answers:} & \quad \text{a) } 17, 5, 12; \quad \text{b) } 93, 11, 82; \quad \text{Bonus: } 1046, 431, 615
\end{align*}
Finding the median. SAY: Sometimes you need to find the number in the middle of a set of data. This number is called the median. The median gives you an idea of what is usual. To find the median of the temperatures, first we have to write them out in order. Here is a list of temperatures for a week in spring. Write on the board:

10 11 12 14 15 17 18

SAY: Then we cross numbers out in pairs, one from each end, until there is only one number left. Demonstrate this with the numbers on the board by crossing out the 10 and the 18, then the 11 and the 17, and ASK: Which two numbers do I cross out next? (12 and 15) Circle the 14 and SAY: The number that is left, 14, is the median. We could say that the temperature will be around 14 degrees and we won’t be far wrong.

10 11 12 14 15 17 18

Exercises: Rewrite the numbers in order and find the median.

a) 12 42 35  
   b) 46 98 27 29 15

   c) 91 75 22 46 83 37 19

Answers: a) 12 35 42, 35; b) 15 27 29 46 98, 29; c) 19 22 37 46 75 83 91, 46

Finding the median of an even number of data points. SAY: Here are the expected temperatures for a six-day trip. Write this set of six temperatures on the board:

10 11 12 14 15 21

Have volunteers cross out two pairs of numbers:

10 11 12 14 15 21

SAY: This time, because we have an even number of temperatures, there isn’t a middle number. The median is the number that is halfway between 12 and 14. ASK: What is halfway between 12 and 14? (13)

Exercises: Find the middle numbers. Then find the median.

a) 18 35 37 41  
   b) 23 26 28 30 34 39

   c) 72 80 90 93  

   Bonus: 22 35 35 40

Answers: a) 35 37, 36; b) 28 30, 29; c) 80 90, 85; Bonus: 35 35, 35

Using stem and leaf plots to find the range and median. SAY: Remember that it is easy to find the highest and lowest numbers on a stem and leaf plot, so it is also easy to find the range. Have volunteers help you with the following example on the board:

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 1 4 5</td>
<td>highest: 131</td>
</tr>
<tr>
<td>11</td>
<td>2 6 8</td>
<td>lowest: 100</td>
</tr>
<tr>
<td>12</td>
<td>4 6 9</td>
<td>range: 131 - 100 = 31</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Leave the example and the following exercises on the board.

**Exercises:** Find the highest value, lowest value, and range.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
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<tr>
<td>0</td>
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<td>5</td>
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<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

**Answers:** a) 95, 71, 24; b) 37, 2, 35; c) 309, 291, 18

SAY: We can also use the stem and leaf plot to find the median. We can do this two ways. The first way is to use the plot to help us to write the numbers in order. And then we can use the list to find the median. Have a volunteer write the data in order. Then have another volunteer find the median:

100 101 104 105 112 116 118 124 126 129 131

SAY: We don’t have to write out all the data, since it is already in order in the plot. Demonstrate crossing out numbers in pairs in the stem and leaf plot on the board:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Exercises:** Find the median of the data in the previous exercises.

**Answers:** a) 83, b) 21, c) 300

**Introducing mode.** SAY: Here is another set of expected temperatures for a trip. Write on the board:

<table>
<thead>
<tr>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°C</td>
<td>20°C</td>
<td>22°C</td>
<td>30°C</td>
<td>30°C</td>
<td>30°C</td>
</tr>
</tbody>
</table>

ASK: What is the range of temperatures? (15°C) What is the median temperature? (26°C) SAY: 26 degrees is nice and warm but not too hot. ASK: What kind of clothes should you pack for that kind of weather? (answers will vary) What do you notice about the weather? (possible answers: it gets hotter Monday–Thursday, there are 3 days at 30°C) Do you need more clothes for cooler weather or for hotter weather days? SAY: You probably need more clothes appropriate for the hotter weather than the cooler weather. In a data set, the number that occurs the most often is called the **mode**. The median of these temperatures is 26°C, but the mode is 30°C.

---

**Probability and Data Management 4-8**

K-39
Exercises: Find the mode.

a) 2 2 3 3 4 4 4 5 5 6 7  b) 2 3 4 5 4 6 1  
c) 31 35 36 38 36 39 36 45  

Answers: a) 4, b) 4, c) 36  

Multiple modes. SAY: Data can have more than one mode. Write on the board:

2 3 3 4 5 6 6 7  

SAY: 3 and 6 both appear twice. Everything else appears less often, so that data has two modes: 3 and 6. If all of the numbers appear exactly as often as each other, we say that there are no modes.  

Exercises: Find the modes.

a) 2 2 3 3 4 4 4 5 5 6 6 7  b) 2 4 6 3 4 6 1 4 6 2  

Bonus: 29 30 31 34 34 35 35 36 36 37 38 40  

Answers: a) 3 and 4, b) 4 and 6, Bonus: 34, 35, and 36  

Using stem and leaf plots to find the mode. ASK: Can we see if a number appears more than once on a stem and leaf plot? (yes) How? (the same leaf will be written more than once in the same row) Can we find the mode from a stem and leaf plot? (yes) How? (find the leaf that is repeated the most times in the same row)  

Exercises: Find the mode or modes of the stem and leaf plots on the board.  

Answers: a) 80; b) 15, 21, and 33; c) 292 and 303  

ACTIVITY (Optional)  
Students can complete Parts 1–5 of BLM Project.  

Extensions  
1. Write three different data sets that have median 9 and range 5.  
2. Write a data set with the same mode and median.  
3. Write a data set with different mode and median.  
4. The mode and the median are two kinds of averages, or what is usual. Another kind of average is the mean. The mean is calculated by adding up all the data and then dividing by the number of data points. For example, to find the mean of the data 10 11 15 18 21, we add all five numbers to get 10 + 11 + 15 + 18 + 21 = 75. Then we divide by 5: 75 ÷ 5 = 15.
Find the mean and median of the data.

a) 1 3 5  
   b) 1 3 7 9  
   c) 1 3 7 13  
   d) 1 3 7 17  
   e) 2 4 6 14 29  

**Answers:**

a) mean: 3, median: 3  
   b) mean: 5, median: 5  
   c) mean: 6, median: 5  
   d) mean: 7, median: 5  
   e) mean: 11, median: 6
Goals

Students will describe and compare data presented in bar graphs and double bar graphs using observation and by calculating range, median, and mode.

PRIOR KNOWLEDGE REQUIRED

Can find the range, median, and mode for a set of data
Can read bar graphs and double bar graphs

MATERIALS

transparency of BLM Bar Graphs for Display (3) (p. K-52)
transparency of BLM Double Bar Graphs (p. K-53)
BLM Project (p. K-46, optional)

Mental math minute. Have students subtract 9, 8, 90, 99, or 98 from two-digit and three-digit numbers by subtracting 10 or 100 and then compensating by adding. Give the subtraction problem, such as 135 − 98. The first student says what needs to be done: subtract 100 and then add 2. The second student subtracts 100: 135 − 100 = 35. The third student adds 2: 35 + 2 = 37.

Analyzing bar graphs. Project the graph on BLM Graphs for Display (3). ASK: What does the graph show? (number of pages Sam reads each day) On which day did Sam read the most? (Sunday) How can you tell? (the bar is longest) How many pages did Sam usually read each day? (7) What do we call the number that shows up most often? (the mode) What does the range mean? (difference between the highest number and the lowest number) What is the range in this graph? (15) What is a median? (the middle number if you write the numbers in order) Can you tell the median by looking at the graph? (not easily)

Exercises

a) Write out the numbers from the graph.

b) Write the numbers in order.

c) Find the median.

Answers: a) 15, 7, 6, 7, 8, 0, 12; b) 0, 6, 7, 7, 8, 12, 15; c) 7

Analyzing double bar graphs. Project the “Temperature Highs in Halifax and Cape Town” graph from BLM Double Bar Graphs. Discuss the graph as you did the previous one, making sure that students understand what the graph shows. You may want to discuss why Halifax is warmer in our summer months.
SAY: Double bar graphs make it easy to compare different graphs but can make it a little harder to see things such as mode and range. To make it easier, let’s make a stem and leaf plot from the data. Have students read the Halifax data one value at a time while you add it to a stem and leaf plot on the board. Rearrange any leaves that are out of order once you are done.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 1 2 7 9</td>
</tr>
<tr>
<td>1</td>
<td>3 5 8 9</td>
</tr>
<tr>
<td>2</td>
<td>3 3</td>
</tr>
</tbody>
</table>

Prompt students to find the high and low temperatures and the range. (23, 0, 23). ASK: What is the median? (11) How many modes are there? (2) What are they? (0 and 23)

**Exercises**

a) Make a stem and leaf plot for the Cape Town temperatures.

b) What is the temperature range?

c) What is the median temperature?

**Answers:**

a) Stem | Leaf  
1 | 7 8 8 9  
2 | 0 2 2 3 5 5 6 6

b) 9, c) 22

ASK: Which city has a larger range of temperatures? (Halifax) Which city is warmer? (Cape Town) Is it always warmer? (no) Discuss different ways to compare temperature. Point out that the high temperatures in Cape Town aren’t much higher than in Halifax (26°C vs. 23°C), but the low temperatures in Halifax are much lower than in Cape Town (0°C vs. 17°C).

**ACTIVITY (Optional)**

Students can complete Parts 1–6 of BLM Project.
**Extensions**

1. The graph below is missing labels. Can you fill them in by answering?

![Snowfall Graph](image)

   a) What units do you think were used to measure snowfall? Label the vertical axis.

   b) In which month do you think there was the most snow? Label that month, then label the other months.

**Bonus:** This graph is either for Ottawa or Vancouver. Which city do you think it is and why?

**Answers:**

   a) cm, b) The highest snowfall is in January, so the non-zero months are October through April. December or February could have been reasonable guesses as well. If students assumed that the data was from the southern hemisphere, it could have been June. c) Ottawa, because Vancouver usually gets more rain than snow

2. a) Find the range, mode, and median for the months that get snow.

   b) Find the range, mode, and median for the year.

**Bonus:** Find the mean snowfall for the months that have snow. (See Extension 4 of Lesson PDM4-9 for an explanation of mean.)

**Answers:**

   a) range: 40 cm, mode: 5 cm, median: 30 cm
   b) range: 45 cm, mode: 0 cm, median: 5 cm; Bonus: 25 cm
3. The scales on all three graphs are the same.

a) Which two graphs have the same range but different medians?

b) Which two graphs have the same median but different ranges?

**Answers:** a) A and C, b) A and B
Project

Part 1. Write a question to answer by using a survey or doing research.

________________________________________________________

Part 2. If you chose a survey question, write possible answers. If you chose a research question, where will you look for answers?

________________________________________________________

________________________________________________________

________________________________________________________

Part 3. Conduct your survey or research, and record your results.

Part 4. Choose at least one of the following ways to display your results:
   a) pictograph
   b) bar graph
   c) double bar graph
   d) stem and leaf plot
   e) graphing software

Part 5. (Optional) Find the range, median, and mode of your data.

Part 6. What did you learn from your survey or research?

________________________________________________________

________________________________________________________

________________________________________________________

________________________________________________________
Graph Template
Colours of Cubes

- Blue: 1 cube
- Green: 2 cubes
- Red: 5 cubes
- Yellow: 4 cubes

Number of Cubes: 0 1 2 3 4 5

Control of Cubes: Blue, Green, Red, Yellow
Snack Bar Graphs

Snacks Eaten Today

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>7</td>
</tr>
<tr>
<td>Bagels</td>
<td>6</td>
</tr>
<tr>
<td>Fruit</td>
<td>5</td>
</tr>
<tr>
<td>Vegetables</td>
<td>4</td>
</tr>
<tr>
<td>Cheese</td>
<td>3</td>
</tr>
<tr>
<td>Vegetables</td>
<td>1</td>
</tr>
<tr>
<td>Fruit</td>
<td>0</td>
</tr>
</tbody>
</table>

Favourite Snacks

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
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<tr>
<td>Bagels</td>
<td>6</td>
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<td>5</td>
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<td>Cheese</td>
<td>3</td>
</tr>
<tr>
<td>Vegetables</td>
<td>1</td>
</tr>
<tr>
<td>Fruit</td>
<td>0</td>
</tr>
</tbody>
</table>
Bar Graphs for Display (1)

Sam’s Reading on Weekdays

<table>
<thead>
<tr>
<th>Days of Week</th>
<th>Number of Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon</td>
<td>0</td>
</tr>
<tr>
<td>Tue</td>
<td>8</td>
</tr>
<tr>
<td>Wed</td>
<td>4</td>
</tr>
<tr>
<td>Thu</td>
<td>0</td>
</tr>
<tr>
<td>Fri</td>
<td>24</td>
</tr>
</tbody>
</table>

Number of Snow Days in Calgary, AB

<table>
<thead>
<tr>
<th>Months</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct–Nov</td>
<td>0</td>
</tr>
<tr>
<td>Nov–Dec</td>
<td>3</td>
</tr>
<tr>
<td>Dec–Jan</td>
<td>6</td>
</tr>
<tr>
<td>Jan–Feb</td>
<td>9</td>
</tr>
<tr>
<td>Feb–Mar</td>
<td>12</td>
</tr>
<tr>
<td>Mar–Apr</td>
<td>15</td>
</tr>
<tr>
<td>Apr–May</td>
<td>18</td>
</tr>
<tr>
<td>May–Jun</td>
<td>21</td>
</tr>
<tr>
<td>Jun–Jul</td>
<td>24</td>
</tr>
</tbody>
</table>

K-50 Blackline Master — Probability and Data Management — Teacher Resource for Grade 4
Bar Graphs for Display (2)

Favourite Winter Activities

- Building snow forts
- Building snowmen
- Skiing
- Sledding
- Snowshoeing

Number of People

0 10 20 30 40 50 60
Bar Graphs for Display (3)

Sam's Reading

Day of the Week
Sun Mon Tue Wed Thurs Fri Sat

Number of Pages
0 2 4 6 8 10 12 14 16

Santana Reading
Double Bar Graphs

Monthly Precipitation in Victoria and Edmonton

Temperature Highs in Halifax and Cape Town
Double Bar Graph Template
Introduction
This unit focuses on fractions of wholes and sets. Fractions are used in real-life contexts, and the meaning and value of the numerator and denominator are evaluated in order to:

• name fractions;
• compare fractions, with and without benchmarks; and
• order fractions, with and without benchmarks.

Meeting Your Curriculum

<table>
<thead>
<tr>
<th>ALBERTA</th>
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<tr>
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<td>46, 48,</td>
</tr>
<tr>
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<td>50</td>
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<tr>
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<td>46, 48 to 50</td>
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<tr>
<td>Optional</td>
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<td>51</td>
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<tbody>
<tr>
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<td>NS4-45,</td>
<td>46, 48,</td>
</tr>
<tr>
<td>Optional</td>
<td>NS4-47,</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
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<th>ONTARIO</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>NS4-45 to 49</td>
<td>including Extension 1 in NS4-48</td>
</tr>
<tr>
<td>Recommended</td>
<td>NS4-50</td>
<td>preparation for Unit 15</td>
</tr>
<tr>
<td>Optional</td>
<td>NS4-51</td>
<td></td>
</tr>
</tbody>
</table>

Mental Math Minutes
The mental math minutes in this unit:

• review doubling and halving to support work related to the benchmark 1/2
• practise multiplication and related division more generally

Generic BLMs
The Generic BLM used in this unit is:
BLM Pattern Blocks (p. S-1)
This BLM can be found in Section S.
Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>NS4-45, 46</td>
<td>NS4-45, 46</td>
<td>NS4-45, 46</td>
<td>NS4-45 to 47</td>
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<tr>
<td>Quiz</td>
<td>NS4-48, 49, 51</td>
<td>NS4-48 to 50</td>
<td>NS4-48, 49, 51</td>
<td>NS4-48 to 50</td>
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<td>Test</td>
<td>NS4-45, 46, 48, 49, 51</td>
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<td>NS4-45, 46, 48, 49, 51</td>
<td>NS4-45 to 49</td>
</tr>
</tbody>
</table>

Additional Information for This Unit

Fraction notation

We show fractions in two ways in our lesson plans:

Stacked: \[ \frac{1}{2} \] Not stacked: \( 1/2 \)

Remember to only show students the stacked form when teaching fractions.

Recurring games

1. **Picking Pairs.** Students play in pairs or individually. Place cards from BLM Fraction Memory or BLM Equivalent Fractions Memory face up in an array. Students take turns picking pairs of matching cards and placing them in a common discard pile. When there are no more pairs in the array, more cards are added to it. The goal is to place all the cards into the discard pile. If students have any non-matching cards left at the end, then some of their cards must have been matched incorrectly.

2. **Memory.** This version of the well-known game is played like Picking Pairs, but the cards are face down. Students turn over two cards at a time looking for a match. If the cards match, students set them aside; otherwise, they turn them face down again and continue playing. Students can play individually or co-operatively in pairs. In either case, the goal is to find all the matches. If playing with a partner, Player 1 leads by choosing and turning over a card, and Player 2 follows by choosing and turning over another card. Players switch roles after each turn.
Goals

Students will name fractions shown by pictures divided into equal areas and record fractions using standard fractional notation.

MATERIALS

- a piece of food that is easily broken (e.g., banana, bread stick)
- BLM Fractions Memory (pp. L-38–40)
- BLM Are the Shaded Amounts Equal? (p. L-41)
- pattern blocks or shapes cut out from BLM Pattern Blocks (p. S-1, see Extension 1)
- tangram pieces or shapes cut out from BLM Tangram (p. L-42, see Extension 3)

Mental math minute. Give a student a small number to double, such as 4. Then have successive students double the previous answer: the first student saying 8, the next 16, the next 32, and so on. Occasionally ask students to explain how they got the answer. When students have doubled several three-digit numbers, start with a new one-digit number.

Review “half.” Hold up a banana or another piece of food that is easily broken. Break it into two very unequal pieces. SAY: This is one of two pieces. ASK: Is this half of the banana? (no) Why not? (because the parts are not equal) Emphasize that the parts have to be equal to be called half.

Introduce other unit fractions. SAY: Just as whole numbers count whole objects, fractions are numbers that count part of an object. Draw on the board:

Point to the second circle and SAY: Because there are three equal parts in the circle, each part is called “one third” of the circle. Repeat for four equal parts and one fourth. Point out that people often say “one quarter,” “two quarters,” “three quarters,” and “four quarters” instead of “one fourth,” “two fourths,” “three fourths,” and “four fourths.” Have volunteers say what each part is called when there are five, six, or ten equal parts. Then write on the board:

one third, one fourth, one fifth, one sixth, ______, ______

Have volunteers continue the pattern. (one seventh, one eighth) ASK: What is each part called if there are 9 equal parts? (one ninth) 10 equal parts? (one tenth) 13 equal parts? (one thirteenth)
Exercises: Write the names for these fractions.

a) one of 11 equal parts  
  b) one of 15 equal parts  
  c) one of 32 equal parts  
  d) one of 100 equal parts  

Bonus: one of 10,000 equal parts  

Answers: a) one eleventh, b) one fifteenth, c) one thirty-second, 
  d) one hundredth, Bonus: one ten-thousandth  

Counting how many equal parts are shaded. Draw on the board:

Point to the circle and ASK: How many equal parts is this circle divided into? (6) What is each equal part called? (a sixth) SAY: I’m going to count how many sixths are shaded: one sixth, two sixths, three sixths, four sixths, five sixths. ASK: How many sixths are shaded? (5) Write on the board: 

5 sixths  

Repeat for the next two pictures (3 eighths, 4 ninths), but have volunteers count all the parts, name what the parts are called, and then count the number of shaded parts.  

Exercises: Count all the parts. Write the name of the parts. Write how much is shaded.  

a)  
  b)  
  c)  
  d)  

Bonus: e)  
  f)  

Answers: a) sixths, 5 sixths; b) ninths, 7 ninths; c) thirds, 2 thirds; 
  d) fourths, 3 fourths; Bonus: e) eighths, 3 eighths; f) twelfths, 5 twelfths  

Fraction notation. SAY: When we write a fraction using numbers, the fraction has a top number and a bottom number. Then write the answers to parts a) through c) above in fraction notation. (5/6, 7/9, 2/3), Challenge students to predict the fraction notation for part d). (3/4) Show more pictures with different fractions shaded (see margin for examples) until all students are confidently volunteering and writing answers.  

ASK: What does the bottom number in a fraction count? (how many equal parts are in the shape) What does the top number in a fraction count? (how many of the equal parts are shaded)
Exercises

1. Write the top and bottom parts for the shaded fraction.

   a) \[ \text{ } \]
   b) \[ \text{ } \]
   c) \[ \text{ } \]
   d) \[ \text{ } \]

   **Answers:** a) 3/4, b) 2/5, c) 6/8, d) 7/8

2. Write the fraction as a top and bottom number.

   a) three eighths
   b) one half
   c) four tenths
   d) Bonus: seventeen hundredths

   **Answers:** a) 3/8, b) 1/2, c) 4/10, Bonus: 17/100

3. Write the fraction using words.

   a) \[ \frac{4}{9} \]
   b) \[ \frac{6}{8} \]
   c) \[ \frac{2}{3} \]
   d) \[ \frac{3}{4} \]
   e) Bonus: \[ \frac{19}{23} \]

   **Answers:** a) four ninths, b) six eighths, c) two thirds, d) three fourths,
   Bonus: nineteen twenty-thirds

**ACTIVITY (Essential)**

Play Picking Pairs (see unit introduction) with the cards from BLM Fractions Memory. Students who finish early can use the same cards to play Memory (see unit introduction). Two cards match if they show the same fraction, regardless if it’s a picture, numbers, or words (see margin). For each fraction represented, there are four matching cards. As long as students have two of the four matching cards, they have a match.

**Numerator and denominator.** Tell students that the top and bottom numbers have special names that are used a lot, so students should try to remember them. SAY: The top number is called the numerator, and the bottom number is called the denominator. The denominator tells you what you are counting (halves, thirds, fourths, etc.). The numerator tells you how many halves, thirds, fourths, etc., are in the fraction.

**Same fraction, different amount.** Draw two circles on the board, one about twice the size of the other. SAY: We’re going to pretend that each circle represents a whole cake. ASK: Which would you rather have? (the big one) Why? (there is more of it) Write on the board:

   \[ 1 \text{ small cake} = 1 \text{ large cake} \]

   ASK: Is it true that one small cake equals one large cake? (no) Circle the 1s and ASK: But these are both 1s, so why aren’t the cakes equal? (they’re not the same size) Erase the equal sign. Explain that there are two important pieces of information: the number tells us how many, and the size tells us how big. Repeat, but with two equal-sized small circles and two equal-sized...
large circles. Repeat with one small and one large square and shade the left half of each. ASK: What fraction of each square is shaded? (one half)

Write on the board:

\[
\frac{1}{2} \text{ small cake} = \frac{1}{2} \text{ large cake}
\]

ASK: Is it true that one half of a small cake equals one half of a large cake? (no)

In the following exercise, complete Question a) on the BLM as a class.

**Exercise:** Complete BLM Are the Shaded Amounts Equal?

**Answers:** a) 5/10, not equal; b) 1/2, not equal; c) 1/2, equal; d) 3/4, not equal; e) 1/2, not equal; f) 4/6, not equal; Bonus: 4/8, equal

**Extensions**

1. Give students pattern blocks or shapes cut from BLM Pattern Blocks. Each student will need one hexagon, two trapezoids, three rhombuses, and six triangles to answer these questions.

a) What fraction of the hexagon is the trapezoid?

b) What fraction of the hexagon is the rhombus?

c) What fraction of the hexagon is the triangle?

d) What fraction of the trapezoid is the triangle?

**Bonus:** What fraction of the trapezoid is the rhombus?

**Answers:** a) 1/2, b) 1/3, c) 1/6, d) 1/3, Bonus: 2/3

2. Add lines to make the parts equal. What fraction is shaded?

   a) 
   
   b) 
   
   c) 
   
   d) 

**Answers**

a) \(\frac{1}{8}\), b) \(\frac{3}{11}\), c) \(\frac{1}{8}\), d) \(\frac{3}{8}\)
3. Use tangram pieces (or BLM Tangram) to answer.
   a) What fraction of the large tangram square is the smallest tangram triangle?
   b) Find three pieces that all look different but are all the same size as two small triangles. What fraction of the whole are each of these shapes?
   
   **Answers:** a) 1/16; b) the small square, the parallelogram, and the medium triangle; 2/16
Goals
Students will compare fractions using the common benchmarks 0, 1/2, and 1.

PRIOR KNOWLEDGE REQUIRED
Understands the relationship between half and double

MATERIALS
two identical transparent glasses
enough counters or beans to completely fill either glass
BLM Fractions Memory (pp. L-38–40, see Extension 4)

Mental math minute. Give a student a large even number to halve, such as 144. Then have successive students halve the previous answer, the first student saying 72, the next 36, and so on. Occasionally ask a student to explain how they got the answer. When students reach an odd number, start with a new large, even number.

Ways to write one half using pictures. Draw on the board:

ASK: What fraction is shaded? (1/2) Write the fraction 1/2. Draw a horizontal line across the circle and ASK: Now what fraction is shaded? (2/4) Write the fraction 2/4. The diagram should now show:

\[
\frac{1}{2} = \frac{1}{4}
\]

SAY: The amount of shading is the same, but the number of parts changed. The picture now shows both one shaded part out of two parts and two shaded parts out of four parts. The two fractions show the same amount of shading. Point out the equal sign between the fractions.

Exercises: Write two fractions for the picture. Write an equal sign between the fractions.

a) b) c)

Answers: a) 1/2 = 2/4, b) 1/2 = 4/8, c) 1/2 = 8/16

Ways to write one half by doubling and halving. Draw several pictures of one half on the board:
SAY: In a picture showing one half, there are always twice as many parts in the whole, the denominator, as there are in the shaded part, the numerator. So, you can double the numerator to get the denominator.

**Exercises:** Write the missing denominator.

a) \( \frac{1}{2} = 5 \)  
   b) \( \frac{1}{2} = 7 \)  
   c) \( \frac{1}{2} = 12 \)  
   **Bonus:** \( \frac{1}{2} = 4132 \)

**Answers:** a) 10, b) 14, c) 24, **Bonus:** 8264

SAY: If you know the denominator of a fraction equal to 1/2, you can divide by 2 to get the numerator.

**Exercises:** Write the missing numerator.

a) \( \frac{1}{2} = \frac{10}{2} \)  
   b) \( \frac{1}{2} = \frac{36}{36} \)  
   c) \( \frac{1}{2} = \frac{50}{50} \)  
   d) \( \frac{1}{2} = \frac{120}{120} \)

**Answers:** a) 5, b) 18, c) 25, d) 60

**Comparing a fraction to one half by shading a picture.** Fill one transparent glass completely with something that is easy to pour without spilling (e.g., counters, beans), and leave an identical transparent glass empty. Pour some of the beans into the empty glass, obviously more than half, then ASK: Did I pour more than half or less than half? (more) How can you tell? (if there is more in the second container than in the first, you poured more than half; if there is more in the first container, you poured less than half)

Ask for two volunteers. Tell students that you have eight counters (or beans) and you are going to give some to each volunteer. Give one volunteer three counters and the other five. ASK: Did they each get half? (no) Who got more than half? Who got less than half?

Draw on the board a circle with 2 out of 6 equal parts shaded. ASK: How many parts are shaded? (2) How many parts are not shaded? (4) Are more parts shaded or not shaded? (not shaded) Is 2/6 more than half or less than half? (less) Repeat for a circle with 5 out of 8 equal parts shaded. SAY: This time, there are more parts shaded (5) than not shaded (3), so 5/8 is more than half. Write on the board:

\[
\frac{2}{6} < \frac{1}{2} \quad \text{and} \quad \frac{5}{8} > \frac{1}{2}
\]

**Exercises:** What fraction is shaded? Is it more or less than half?

a)  
   b)  

**Answers:** a) 3/8, less than half; b) 6/10, more than half

**Comparing a fraction to one half without using a picture.** Draw on the board:

Ask: What fraction is shaded? (6/10) Is that more or less than half? (more)
Write on the board:

\[ \frac{6}{10} \]

ASK: How can you tell from the fraction, without even looking at the picture, how many pieces are shaded in the picture? (the numerator is 6, so 6 parts are shaded) How can you tell how many pieces are not shaded, just from the fraction? (10 – 6 = 4, so 4 pieces are not shaded)

**Exercises:** How many pieces are shaded? How many pieces are not shaded?

<table>
<thead>
<tr>
<th>a) [ \frac{3}{7} ]</th>
<th>b) [ \frac{5}{9} ]</th>
<th>c) [ \frac{11}{20} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>___ shaded</td>
<td>___ shaded</td>
<td>___ shaded</td>
</tr>
<tr>
<td>___ not shaded</td>
<td>___ not shaded</td>
<td>___ not shaded</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d) [ \frac{47}{100} ]</th>
<th>e) [ \frac{100}{1000} ]</th>
<th>f) [ \frac{600}{1000} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>___ shaded</td>
<td>___ shaded</td>
<td>___ shaded</td>
</tr>
<tr>
<td>___ not shaded</td>
<td>___ not shaded</td>
<td>___ not shaded</td>
</tr>
</tbody>
</table>

**Answers:** a) 3, 4; b) 5, 4; c) 11, 9; d) 47, 53; e) 100, 900; f) 600, 400

SAY: If more parts are shaded than not shaded, the fraction is more than one half. If fewer parts are shaded, the fraction is less than one half. Pointing to each fraction in the exercises above, ASK: Is the fraction more than half or less than half? Students can signal thumbs up for more than half and thumbs down for less than half. (a) less, b) more, c) more, d) less, e) less, f) more)

**Exercises:** Is the fraction more than half or less than half?

<table>
<thead>
<tr>
<th>a) [ \frac{5}{8} ]</th>
<th>b) [ \frac{11}{20} ]</th>
<th>c) [ \frac{17}{35} ]</th>
<th>d) [ \frac{22}{45} ]</th>
</tr>
</thead>
</table>

**Bonus:** Which fractions are more than half? Write down the letters. What do they spell?

<table>
<thead>
<tr>
<th>O. [ \frac{3}{5} ]</th>
<th>T. [ \frac{3}{7} ]</th>
<th>R. [ \frac{11}{20} ]</th>
<th>P. [ \frac{5}{16} ]</th>
<th>A. [ \frac{3}{4} ]</th>
<th>N. [ \frac{25}{40} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>G. [ \frac{8}{14} ]</td>
<td>E. [ \frac{52}{100} ]</td>
<td>Y. [ \frac{12}{30} ]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) more, b) more, c) less, d) less, Bonus: ORANGE

**Using 1/2 as a benchmark.** Write on the board the fractions 3/8 and 2/3. Point to each fraction and ASK: Is it more or less than half? (3/8 is less, 2/3 is more) Now write on the board:

\[
\boxed{ } < \frac{1}{2} < \boxed{ }
Have a volunteer write the fractions in the correct boxes. SAY: 3/8 is less than 2/3 because 3/8 is less than 1/2, and 1/2 is less than 2/3. If students are having trouble with this, draw on the board a number line with a tick mark at each end and one in the middle. Label the tick marks 0, 1/2, and 1. ASK: Is 3/8 less than 1/2 or greater than 1/2? (less) How do you know? (there are fewer shaded parts than not shaded) Circle the left half of the number line and write “3/8” over it. Repeat for 2/3. Then have a volunteer write the appropriate fractions in the boxes.

**Exercises:** Compare the fractions. Write < or >. Hint: First compare them both to \( \frac{1}{2} \).

a) \( \frac{5}{9} \) \( \frac{6}{15} \)  
b) \( \frac{2}{3} \) \( \frac{4}{10} \)  
c) \( \frac{10}{18} \) \( \frac{12}{30} \)  
d) \( \frac{4}{9} \) \( \frac{5}{8} \)

**Bonus:** \( \frac{6000}{20000} \) \( \frac{760}{900} \)

**Answers:** a) >, b) >, c) >, d) <, Bonus: <

**Word problems practice.**

**Exercises**

a) David walked \( \frac{5}{11} \) of a kilometre. Is that more than or less than half a kilometre?

b) On a soccer team, \( \frac{8}{15} \) of the players are girls. Are there more boys or girls on the team?

c) Luc gave away \( \frac{3}{5} \) of a cake and kept the rest. Did he keep more or less than half?

d) Kate ate \( \frac{4}{9} \) of a chocolate bar and Raj ate \( \frac{6}{11} \) of the chocolate bar. Who had more?

**Answers:** a) less, b) girls, c) less, d) Raj

**Many ways to write 1 as a fraction.** SAY: Like the fraction 1/2, the number 1 can also be written different ways. A whole pie is a whole pie, no matter how many pieces it is divided into. Draw on the board three fully shaded circles as shown in the margin. Point to the first circle and ASK: How many parts are shaded? (2) How many parts are in the whole circle? (2) Write “1 = \( \frac{2}{2} \)” on the board. Repeat for the second and third circles, but have volunteers write the fractions equal to 1:

\[
1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4}
\]

Point out that a fraction is equal to 1 if the numerator is the same as the denominator because that means that all the parts are shaded.
Exercises: Write the missing number in the box to make the fraction equal to 1.

a) \[
\begin{array}{c}
7
\end{array}
\]
b) \[
\begin{array}{c}
10
\end{array}
\]
c) \[
\begin{array}{c}
6
\end{array}
\]
d) \[
\begin{array}{c}
9
\end{array}
\]

Answers: a) 7, b) 10, c) 6, d) 9, Bonus: 182

Pictures of more than one whole. Draw an empty circle on the board and ASK: How do we shade this circle to show that it is one whole circle? (shade all of it) Draw another circle of the same size beside the first, and ASK: How could we use shading in this second circle to show that now there is a little bit more than one whole? (shade a little bit) Shade a bit of the second circle (see margin).

SAY: Now this shows one whole and a little bit more. Erase a little bit more of the shading (see margin). ASK: Now that the shaded piece is even smaller, does the picture still show a bit more than one whole? (yes)

Pictures of less than one whole. Draw a new circle, shade all of it, and ASK: How much does this picture show? (one whole) Erase a little as shown in the margin. ASK: Now does it show one whole? (no) Is it more than one whole or less than one whole? (less) Erase a little bit more of the shading and ASK: Is this one whole, more than one whole, or less than one whole? (less) Write the fraction 9/10 and SAY: This shows approximately 9/10, but it’s hard to tell because there aren’t any lines showing the parts. Ask for other examples of fractions that are less than one whole. ASK: In order for a fraction to be less than a whole, does the numerator have to be less than the denominator, equal to the denominator, or greater than the denominator? (less)

Exercises: Circle the fractions that are not less than one whole.

a) \[
\begin{array}{c}
5
\end{array}
\]
\[
\begin{array}{c}
8
\end{array}
\]
b) \[
\begin{array}{c}
7
\end{array}
\]
\[
\begin{array}{c}
7
\end{array}
\]
c) \[
\begin{array}{c}
11
\end{array}
\]
\[
\begin{array}{c}
429
\end{array}
\]
d) \[
\begin{array}{c}
999
\end{array}
\]
\[
\begin{array}{c}
1000
\end{array}
\]
e) \[
\begin{array}{c}
4306
\end{array}
\]
\[
\begin{array}{c}
4306
\end{array}
\]
f) \[
\begin{array}{c}
1
\end{array}
\]
\[
\begin{array}{c}
2
\end{array}
\]

Answers: b) and e) are not less than one whole.

Extensions

1. Estimate, very approximately, where the fraction goes. Write the letter above the number line.

A. \[
\begin{array}{c}
1
\end{array}
\]
\[
\begin{array}{c}
10
\end{array}
\]
B. \[
\begin{array}{c}
10
\end{array}
\]
\[
\begin{array}{c}
11
\end{array}
\]
C. \[
\begin{array}{c}
4
\end{array}
\]
\[
\begin{array}{c}
10
\end{array}
\]
D. \[
\begin{array}{c}
5
\end{array}
\]
\[
\begin{array}{c}
8
\end{array}
\]

less than \[
\begin{array}{c}
1
\end{array}
\]
\[
\begin{array}{c}
2
\end{array}
\]
greater than \[
\begin{array}{c}
1
\end{array}
\]
\[
\begin{array}{c}
2
\end{array}
\]

A

closer to 0 closer to \[
\begin{array}{c}
1
\end{array}
\]
\[
\begin{array}{c}
2
\end{array}
\]
closer to \[
\begin{array}{c}
1
\end{array}
\]
\[
\begin{array}{c}
2
\end{array}
\]
closer to 1

L-12

Teacher Resource for Grade 4
Answers: A C D B

2. Write any number to make the fraction greater than \( \frac{1}{2} \), but less than 1.
   a) \( \frac{7}{12} \)  b) \( \frac{12}{20} \)  c) \( \frac{6}{10} \)  Bonus: \( \frac{7}{12} \)
   Answers: a) 4, 5, or 6; b) 7, 8, 9, 10, or 11; c) 4 or 5; Bonus: 8–13

3. Write any number to make the fraction greater than 0, but less than \( \frac{1}{2} \).
   a) \( \frac{7}{12} \)  b) \( \frac{4}{12} \)  c) \( \frac{6}{12} \)  d) \( \frac{3}{12} \)  e) \( \frac{12}{12} \)  Bonus: \( \frac{7}{12} \)
   Answers: a) 1–3, b) 1, c) 1 or 2, d) 1, e) 1–5, Bonus: any number > 14

4. Use one set of BLM Fractions Memory cards. Place all the cards face down. Turn over a card and write a fraction for the part that is not named or shaded. For example, if the card shows \( \frac{2}{3} \), “2 thirds,” “two thirds,” or a picture that shows two thirds, you write \( \frac{1}{3} \), “1 third,” “one third,” or draw a picture of \( \frac{1}{3} \).

5. Identify whether the fraction is closest to the benchmark 0, \( \frac{1}{2} \), or 1.
   a) \( \frac{1}{9} \) is closest to 0
   b) \( \frac{5}{11} \) is closest to 
   c) \( \frac{8}{14} \) is closest to 
   d) \( \frac{13}{15} \) is closest to 
   e) \( \frac{3}{20} \) is closest to 
   f) \( \frac{98}{100} \) is closest to
   Answers: b) 1/2, c) 1/2, d) 1, e) 0, f) 1
Goals
Students will find equivalent fractions using multiplication.

PRIOR KNOWLEDGE REQUIRED
Can use the phrase “times as many as” to compare two numbers

MATERIALS
BLM Equivalent Fractions Memory (pp. L-43–45)

Mental math minute. Give students multiplication questions that can be done by skip counting by 2, 3, 4, 5, or 10. Have students skip count out loud to answer multiplication questions.

Breaking all parts into two equal parts to create equivalent fractions.
Draw the first picture below on the board. Tell students that a parent and a child were sharing a cake, so the parent divided the cake into two pieces. The child said he wanted two pieces, so the parent cut the cake again and gave the child two pieces. Show this with a second picture:

ASK: Did the child get more cake by getting two pieces? (no) Write the first equation below on the board:
\[
\frac{1}{2} = \frac{2}{4}
\]
\[
\frac{1}{2} \times 2 = \frac{2}{4}
\]
SAY: The fractions are equal, or equivalent, because the pictures have the same amount shaded. Then show how the numerators and denominators are related by multiplication, as in the second equation above. SAY: Both people get twice as many pieces but the same amount of cake as before.

Exercises: Copy the picture into your notebook. Break each part in half to create equivalent fractions.

a)  b)  c)  d)

Answers: a) \( \frac{1}{3} = \frac{2}{6} \), b) \( \frac{2}{3} = \frac{4}{6} \), c) \( \frac{1}{4} = \frac{2}{8} \), d) \( \frac{3}{4} = \frac{6}{8} \)

Sample pictures for a):
Breaking all parts into the same number of equal parts to create equivalent fractions. Draw the pairs of pictures shown below on the board. Have students signal (by holding up the correct number of fingers) how many times as many parts the first picture has compared to the second picture. (a) 4, b) 2, c) 3, d) 4)

![Picture A]  ![Picture B]

![Picture C]  ![Picture D]

Now shade the same amount of each picture in a pair, and have students signal how many times as many shaded parts there are in the first pictures. (a) 4, b) 2, c) 3, d) 4) If this is difficult because there are many parts, show students how to cover all but one part and count how many parts it has been divided into. For example, for a), cover either the left or right half of the circle with more parts and count how many parts they see.

![Shaded Picture A]  ![Shaded Picture B]

![Shaded Picture C]  ![Shaded Picture D]

Point out that because all original parts were divided into the same number of parts, the shaded parts were also divided into that number of parts.

**Exercises:** Write equivalent fractions from the picture on the board.

**Answers:** a) 4/8 = 1/2, b) 2/6 = 1/3, c) 3/6 = 1/2, d) 8/12 = 2/3

Point out how the numerators and denominators of both fractions are related by multiplication. For example:

\[
\frac{1 \times 4}{2 \times 4} = \frac{4}{8}
\]

**Using a single picture to write two equivalent fractions.** Tell students that you can use the same picture to show two fractions. Draw on the board the picture in the margin. SAY: You can look at the big parts or the small parts. The big parts show the fraction two thirds because two of the three big parts are shaded. The small parts show the fraction eight twelfths because eight of the twelve small parts are shaded. The fractions are shown by the same picture, so they are equivalent. Write on the board:

\[
\frac{2}{3} = \frac{8}{12}
\]
Exercises: Write two equivalent fractions from the picture.

a) 

b) 

c) 

Answers: a) 1/3, 2/6; b) 1/3, 3/9; c) 3/5, 9/15

Bonus: Write as many equivalent fractions as you can from the picture without adding more lines.

a) 

b) 

Answers: a) 2/3, 4/6, 8/12; b) 1/2, 2/4, 3/6, 6/12

Using multiplication to write an equivalent fraction. Draw the first picture below:

\[
\frac{2}{3} = \frac{12}{12}
\]

ASK: How many parts do I have to break each piece into to get 12 parts altogether? (4) PROMPT: Three times what is 12? Again, you can focus on just one of the original parts to make it clearer. Divide the parts, then show this relationship as in the second picture above. ASK: How many parts are shaded now? (8) Fill in the numerator. SAY: That’s two groups of four that are shaded.

Exercises: Use multiplication to find the missing numerator.

a) \( \frac{1}{5} = \frac{\_}{15} \)  
b) \( \frac{3}{4} = \frac{\_}{16} \)  
c) \( \frac{5}{6} = \frac{\_}{12} \)  
d) \( \frac{7}{10} = \frac{\_}{100} \)

Sample solution

a) \( \frac{\_}{5} \times \frac{3}{3} = \frac{3}{15} \)

Answers: a) 3, b) 12, c) 10, d) 70, Bonus: 150

Skip counting to find lists of equivalent fractions. Write on the board:

\[
\begin{align*}
\frac{3}{5} & = \frac{3 \times 2}{5 \times 2} = \frac{3 \times 3}{5 \times 3} = \frac{3 \times 4}{5 \times 4} = \frac{3 \times 5}{5 \times 5} \\
\end{align*}
\]

Remind students that they can skip count to multiply. So they can create equivalent fractions by skip counting. Write on the board:

\[
\frac{3}{5} = \_ = \_ = \_ = \_
\]

Demonstrate skip counting by 3s to get the numerators, and then by 5s to get the denominators. Fill in the fractions. Point to each fraction in turn, and SAY: Two times as many parts, three times as many parts, four times as many parts, five times as many parts.
Exercise: Write four fractions equivalent to $\frac{2}{5}$.

Bonus: Write more fractions equivalent to $\frac{2}{5}$.

Sample answers: 4/10, 6/15, 8/20, 10/25, 12/30, 14/35

**ACTIVITY (Optional)**

Play Picking Pairs and then Memory (see unit introduction) using BLM Equivalent Fractions Memory. The third page uses more difficult multiplication; distribute it only to students who can quickly do the required multiplications (in the 6, 7, 8, 9, 11, and 12 times tables).

Extensions

1. Use multiplication to find the missing denominator.
   
   a) $\frac{3}{4} = \frac{15}{20}$  
   b) $\frac{1}{8} = \frac{7}{56}$  
   c) $\frac{5}{6} = \frac{30}{36}$  
   d) $\frac{4}{10} = \frac{40}{50}$

   **Bonus**
   
   e) $\frac{3}{1000} = \frac{3000}{10000}$  
   f) $\frac{22}{100} = \frac{2200}{10000}$

   **Answers:** a) 20, b) 56, c) 36, d) 100, Bonus: e) 1 000 000, f) 10 000

2. Is there a fraction equivalent to $\frac{3}{8}$ with an odd denominator? Explain.

   **Answer:** No. The denominator will always be a multiple of 8, so it will always be even.
Goals

Students will compare and order fractions based on the size and the number of fractional parts.

Students will compare and order fractions using number lines and fraction strips.

PRIOR KNOWLEDGE REQUIRED

Knows that 1/2 can be written in different ways
Understands and can use < and > properly
Can compare and order whole numbers

MATERIALS

BLM Fraction Cards (pp. L-46–48)
BLM Ordering with Fraction Strips (p. L-49)
BLM Ordering Fractions (pp. L-50–51, see Extension 1)

Mental math minute. Have groups of three students add two-digit numbers by adding tens and ones separately. Give an addition problem, such as 35 + 46. The first student adds the tens: 30 + 40 = 70; the second adds the ones: 5 + 6 = 11; and the third student finishes the addition: 70 + 11 = 81, so 35 + 46 = 81. Start with problems that do not require regrouping, such as 25 + 34, and continue to questions that require regrouping ones.

Comparing fractions with the same denominator. Draw on the board:

ASK: Are these the same size? (yes) Do they have the same number of parts? (yes) Are the parts the same size? (yes) Have volunteers name the fractions. (1/4 and 3/4) Write them on the board. ASK: Which is more, one fourth of the circle or three fourths of the circle? SAY: Three fourths of something is always greater than one fourth of the same thing, since the fourths are the same size. Have a volunteer write “<” or “>” between the fractions. Draw on the board:
ASK: Which strip has a greater amount shaded? (the bottom one) Which is more, five sixths or three sixths of the strip? (5/6 of the strip) Write on the board:

\[
\frac{5}{6} \quad \frac{3}{6}
\]

Have a volunteer write the correct sign between the fractions. (>)

**Exercises:** Write < or >.

a) \[\frac{2}{5} \quad \frac{4}{5}\]  

b) \[\frac{3}{4} \quad \frac{2}{4}\]

c) \[\frac{6}{10} \quad \frac{9}{10}\]  

**Bonus:** \[\frac{37}{59} \quad \frac{27}{59}\]

**Answers:** a) <, b) >, c) <, Bonus: >

Write on the board:

\[
\frac{4}{9} \quad \frac{7}{9}
\]

ASK: What numbers could we put in the blank to make the relationship correct? (0, 1, 2, and 3) Have students suggest all possibilities. Repeat with 7/10 < ____/10. (8, 9, 10)

**Exercises:** Write any number in the blank that makes the relationship correct.

a) \[\frac{6}{10} \quad \frac{7}{10}\]

b) \[\frac{2}{5} \quad \frac{5}{5}\]

c) \[\frac{1}{2} \quad \frac{2}{2}\]

d) \[\frac{7}{7} \quad \frac{7}{7}\]

**Answers:** a) 5, 4, 3, 2, 1, or 0; b) 3, 4, or 5; c) 2; d) 6, 5, 4, 3, 2, 1, or 0

**ACTIVITY 1 (Essential)**

1. Give each student a card from BLM Fraction Cards. Have students group themselves according to the denominators on their cards. Then ask students to line up from least (at the front) to greatest (at the back).

**Ordering fractions with the same denominator using number lines.**

Draw on the board:

\[
\begin{array}{cccccccc}
& & & & & & & \\
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
6 & 6 & 6 & 6 & 6 & 6 & 6 \\
\end{array}
\]

Have a volunteer mark an X on the number line to show 1/6. Continue with the next fraction to the right in the group written below the number line until all these fractions have been marked. ASK: Are the fractions in order.
now, from least to greatest? (yes) Point out that the number line orders the fractions for you. Write the fractions in order in the boxes. \((1/6 < 2/6 < 4/6 < 6/6)\)

**Exercises**

1. Use the number line to order the fractions. Draw an X for each fraction.

   a) \[
   \begin{array}{ccccccc}
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   & & & & & & \\
   \end{array}
   \]

   \[
   \frac{2}{9} \quad \frac{7}{9} \quad \frac{5}{9} \quad \frac{8}{9} \quad \frac{1}{9} \quad \frac{4}{9}
   \]

   b) \[
   \frac{5}{5} \quad \frac{3}{5} \quad \frac{0}{5} \quad \frac{2}{5}
   \]

   **Answers:** a) 1/9, 2/9, 4/9, 5/9, 7/9, 8/9; b) 0/5, 2/5, 3/5, 5/5

2. Write a fraction that is between the two fractions.

   a) \(\frac{3}{9}\) and \(\frac{8}{9}\)  
   b) \(\frac{1}{5}\) and \(\frac{5}{5}\)  
   c) \(\frac{1}{4}\) and \(\frac{3}{4}\)  
   d) \(\frac{5}{10}\) and \(\frac{8}{10}\)

   **Sample answers:** a) 5/9, b) 4/5, c) 2/4, d) 6/10

**Ordering fractions with the same denominator by considering the numerators.** Draw on the board:

\[
\begin{array}{cccc}
1 & 6 & 2 & 4 \\
6 & 6 & 6 & 6 \\
\end{array}
\]

ASK: What does the denominator “6” mean? (6 parts) Since the denominators are all the same, what tells us how to order the fractions? (numerator) If we do that, which fraction is first and which is last? (one sixth, six sixths) Have volunteers write the fractions in the blanks, in order from least to greatest. \((1/6 < 2/6 < 4/6 < 6/6)\)

**Exercises:** Order the fractions from least to greatest.

a) \[
\frac{2}{9} \quad \frac{7}{9} \quad \frac{5}{9} \quad \frac{8}{9} \quad \frac{1}{9} \quad \frac{4}{9}
\]

b) \[
\frac{5}{5} \quad \frac{3}{5} \quad \frac{0}{5} \quad \frac{2}{5}
\]

**Answers:** a) 1/9, 2/9, 4/9, 5/9, 7/9, 8/9; b) 0/5, 2/5, 3/5, 5/5
Comparing fractions with the same numerator using fraction strips.

Draw on the board:

```
\[
\begin{array}{c}
\frac{1}{4} \\
\frac{1}{9}
\end{array}
\]
```

ASK: Are the strips the same length? (yes) Do they have the same number of parts? (no) Are the parts the same size? (no) Have volunteers name the fractions. (one fourth, one third, one half) Write the fractions beside the strips, using standard fraction notation. ASK: Which is more, one fourth of the strip or one half of the strip? (one half) Why? (one half is bigger) Point out that the more parts something is divided into, the smaller each part is.

Write on the board:

```
\frac{1}{4} \quad \frac{1}{9}
```

ASK: Which fraction is greater? (1/4) Have a volunteer write the correct inequality sign in the box. (>) Repeat for 3/8 and 3/5. (3/8 < 3/5)

**Exercises**

1. Write < or >.

   a) $\frac{1}{8} \quad \frac{1}{3}$  
   b) $\frac{1}{2} \quad \frac{1}{10}$  
   c) $\frac{1}{5} \quad \frac{1}{4}$  
   d) $\frac{1}{10} \quad \frac{1}{100}$  
   e) $\frac{2}{5} \quad \frac{2}{8}$  
   f) $\frac{7}{20} \quad \frac{7}{8}$

   g) $\frac{6}{12} \quad \frac{6}{16}$  
   **Bonus:** $\frac{35}{1000} \quad \frac{35}{40}$

   **Answers:** a) <, b) >, c) <, d) >, e) >, f) <, g) >, **Bonus:** <

2. Write any number in the blank that makes the relationship correct.

   a) $\frac{1}{10} \quad \frac{12}{12}$  
   b) $\frac{3}{7} \quad \frac{5}{5}$  
   c) $\frac{5}{9} \quad \frac{5}{5}$  

   **Bonus:** $\frac{172}{983} \quad \frac{172}{172}$

   **Sample answers:** a) 1, b) 3, c) 40, **Bonus:** 175

**ACTIVITY 2 (Essential)**

2. Repeat Activity 1, but have students with the same numerator group together and then order themselves.
Ordering fractions with the same numerator using fraction strips.

Draw on the board:

\[
\begin{array}{cccc}
\frac{1}{2} & \frac{1}{5} & \frac{1}{10} & \frac{1}{4} & \frac{1}{3} \\
\end{array}
\]

Explain that you want to order the fractions from least to greatest.

ASK: What do the denominators represent? (the number of parts) 
SAY: The more parts the strip is divided into, the smaller the parts are, which is why the largest denominator represents the smallest shaded fraction. Have volunteers match each fraction to a fraction strip, shade it, and then write the fraction in the box below the strip. The finished diagram should look like this:

\[
\begin{array}{cccc}
\frac{1}{2} & \frac{1}{5} & \frac{1}{10} & \frac{1}{4} & \frac{1}{3} \\
\end{array}
\]

ASK: Are the fractions ordered from least to greatest? (yes)

Exercise: Complete BLM Ordering with Fraction Strips.

Answers: 1. a) 2/9, 2/6, 2/5, 2/3; b) 3/16, 3/12, 3/10, 3/7;
2. a) 1/1, 1/3, 1/8, 1/10; b) 4/6, 4/8, 4/15, 4/18

Ordering fractions with the same numerator by considering the numerators and denominators.

Draw on the board:

\[
\begin{array}{cccc}
\frac{2}{4} & \frac{2}{5} & \frac{2}{10} & \frac{2}{3} \\
\end{array}
\]

ASK: What does the numerator “2” mean here? (the number of shaded parts) If we order the fractions from greatest to least, which fraction will be first? (2/2) Second? (2/3) Last? (2/10) Write all the fractions in the boxes, from greatest to least. (2/2, 2/3, 2/4, 2/5, 2/10)

Exercises: Order the fractions in part a) from least to greatest, and the fractions in part b) from greatest to least.

a) \[
\begin{array}{cccc}
\frac{3}{18} & \frac{3}{5} & \frac{3}{11} & \frac{3}{21} \\
\end{array}
\]
Comparing fractions with different numerators and denominators using the benchmark 1/2. Draw on the board:

Have students name several fractions, with any denominator, between 0 and 1/2 and then between 1/2 and 1. Write the fractions on the appropriate side of 1/2, but not in order. Then, choose one fraction from each side and write them in a pair, in either order, with a box in between. For example, if 1/3 and 2/10 are on the left of 1/2 and 4/5 and 3/4 are on the right, you might write:

Have students decide if a greater than or less than sign should go in the box. Write several pairs on the board, and have students copy them and write in inequality signs on their own. Take them up together. Erase everything. Repeat, but use a table instead of a number line:

Word problems practice.

Exercises

a) Marla thinks that \(\frac{1}{2}\) of a cake is equal to \(\frac{1}{2}\) of the moon, but Cathy doesn’t think that is true. She explained why to her teacher, who said her explanation was correct. What was Cathy’s explanation?

b) Arsham doesn’t think that \(\frac{1}{2}\) of a cherry pie can weigh the same as a whole apple pie. Is he correct? Explain.

Sample answers

a) One half means one whole divided into two equal parts. The moon is bigger than a cake, so half of the moon cannot equal half of a cake.
b) No. If a cherry pie is much bigger than an apple pie, half the cherry pie could weigh the same as the whole apple pie.
NOTE: Extension 1 is required to cover the Ontario curriculum.

Extensions

1. Complete BLM Ordering Fractions.

   **Answers:** 1. a) 8/4, 10/4, 9/4; b) 8/4 < 9/4 < 10/4; 2. a) 9/3, 10/3, 8/3; b) 8/3 < 9/3 < 10/3; 3. a) 8/2, 9/2, 7/2; b) 7/2 < 8/2 < 9/2; 4. a) 16/10, 20/10, 18/10; b) 16/10 < 18/10 < 20/10; 5. >, <, >

2. Use two number lines to decide if the fraction is closer to 0, \(\frac{1}{2}\), or 1.

   \[
   \begin{array}{cccccc}
   0 & \frac{1}{4} & 1 & \frac{2}{4} & 2 & \frac{3}{4} & 3 & \frac{4}{4} & 4 \\
   \hline
   \frac{0}{7} & \frac{1}{7} & \frac{2}{7} & \frac{3}{7} & \frac{4}{7} & \frac{5}{7} & \frac{6}{7} & \frac{7}{7}
   \end{array}
   \]

   Write 0, \(\frac{1}{2}\), or 1.

   a) \(\frac{5}{7}\) is closer to _______  
   b) \(\frac{1}{7}\) is closer to _______ 
   c) \(\frac{6}{7}\) is closer to _______

   **Answers:** a) 1/2, b) 0, c) 1

3. Compare the fractions \(\frac{3}{4}\) and \(\frac{4}{5}\) by comparing how much of a whole pie is left if the given amounts are eaten. Hint: The smaller fraction is from the pie with a bigger piece left over.

   **Answer:** 1/4 is more than 1/5, so 3/4 is less than 4/5.

4. Write the fractions in order from least to greatest.

   \[
   \frac{1}{5}, \frac{3}{8}, \frac{6}{8}
   \]

   **Answer:** 1/8, 1/5, 3/5, 6/8
NS4-49  Equal Parts of a Set

Goals

Students will name fractions of a set and recognize that equal parts of a set do not have to have the same area.

PRIOR KNOWLEDGE REQUIRED

Can name fractions from area models within one

MATERIALS

10 counters in one colour and 10 counters in another for each student (see Extension 3)

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $10 \times 10$ and the corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, up to $10 \times 3$ and $30 \div 3$. Then repeat with a different number. Next, try questions out of order, but keep the corresponding multiplication and division together.

Review equal parts of a whole. Ask students to brainstorm all the things they can take a fraction of (Examples: a circle, a square, other shapes; a line, a distance, a cup, a liter, a pound; prompt students to think of things that are not shapes, if necessary).

Introduce fractions of a set. Tell students that they can take a fraction of a set of objects. Draw on the board:

ASK: What fraction of the circles are shaded? (3/8) Now draw on the board:

SAY: There are still 8 circles, and 3 of them are shaded. We can say that three eighths of the circles are shaded, even though the circles are not all the same size. ASK: What fraction of the circles are big? (5/8)

Exercises: Find the fraction of the shapes that are …

a) circles  b) triangles  c) shaded  d) not shaded

Bonus

e) circles  f) small  g) shaded

h) not shaded  i) big  j) triangles
Answers: a) 3/5, b) 2/5, c) 2/5, d) 3/5, Bonus: e) 3/5, f) 3/5, g) 4/5, h) 1/5, i) 2/5, j) 2/5

Exercises: What does the fraction describe?

\[ \square \bigtriangleup \bigtriangleup \bigcirc \square \bigtriangleup \bigtriangleup \]

a) \( \frac{3}{8} \) of the shapes are ________________

b) \( \frac{1}{8} \) of the shapes are ________________

Bonus: \( \frac{1}{2} \) of the shapes are ________________

Answers: a) triangles, b) circles, Bonus: squares, shaded, or not shaded

Bonus

a) Describe the picture using the fraction \( \frac{3}{5} \) in two different ways.

\[ \square \bigtriangleup \bigtriangleup \square \bigtriangleup \bigtriangleup \]

b) Describe the picture using the fraction \( \frac{3}{5} \) in three different ways.

\[ \bigcirc \bigcirc \bigtriangleup \bigcirc \bigtriangleup \]

Answers: a) 3/5 are shaded, 3/5 are squares; b) 3/5 are circles, 3/5 are not shaded, 3/5 are small

Using “and,” “or,” and “not.” Draw on the board:

\[ \square \bigtriangleup \bigtriangleup \bigcirc \bigcirc \square \bigtriangleup \bigtriangleup \bigcirc \bigcirc \]

Exercises: What fraction of the shapes are …

a) circles  b) squares  c) circles or squares  
   d) not circles  e) not triangles

Bonus

f) Which two parts above have the same answer?

g) Make up another question that has the same answer as part d).

Answers: a) 2/9, b) 4/9, c) 6/9, d) 7/9, e) 6/9, Bonus: f) parts c) and e), g) What fractions of the shapes are squares or triangles?

Tell students that they can take a fraction of any kind of set, not just shapes. ASK: What fraction of your fingers are thumbs? (1/5 or 2/10) Point out that both answers are correct. Emphasize that your fingers do not have to all be the same size; they are still equal parts of a set.

Then ask students questions about themselves. ASK: What fraction of the students in the class wear glasses? What fraction play a musical instrument?
SAY: A basketball team played five games and won two of them.  
ASK: What fraction of the games did the team win? (2/5)  

**Exercises:** What fraction of their games did the team win?  
a) Team A played 6 games and won 4.  
b) Team B won 5 games and lost 3.  
c) Team C played 9 games and won 7.  
d) Team D won 4 games and lost 5.  

**Answers:** a) 4/6, b) 5/8, c) 7/9, d) 4/9

ASK: Which team won more games, Team C or Team D? (Team C)  
Which team lost more games, Team C or Team D? (Team D)  
How can you tell which team is better? (compare the fractions of games won)

**Extensions**

1. Explain how you can use your hands to show that $\frac{1}{2}$ is equivalent to $\frac{5}{10}$.  

2. What word do you get when you combine ...  
a) the first $\frac{2}{3}$ of sun and the first $\frac{1}{2}$ of person?  
b) the first $\frac{1}{2}$ of grease and the first $\frac{1}{2}$ of ends?  
c) the first $\frac{1}{2}$ of wood and the last $\frac{2}{3}$ of arm?  

**Answers:** a) super, b) green, c) worm  

**Bonus:** Try making up your own such questions.

3. a) Write as many equivalent fractions as you can for the picture.  

   ![Equivalent Fractions](image)

   b) Make a model (using counters) of a fraction that can be described in two ways. Example:  

   ![Fraction Model](image)  

   $\frac{3}{6}$ of the counters are white  

   $\frac{1}{2}$ of the counters are white
c) Using 10 counters of one colour and 10 counters of a different colour, make a model of a fraction that can be described in at least three different ways. Then write the three fractions.

Sample answers:

\[
\begin{align*}
\text{Model 1:} & \quad \frac{1}{2} = \frac{2}{4} = \frac{4}{8} \\
\text{Model 2:} & \quad \underline{\text{Model}} \\
\end{align*}
\]

4. Draw a set of five shapes (circles and squares) such that:

a) \(\frac{2}{5}\) are squares
\(\frac{2}{5}\) are shaded
One circle is shaded

Answer

```
[Square] [Square] [Circle] [Circle] [Circle]
```

b) \(\frac{3}{5}\) are squares
\(\frac{2}{5}\) are shaded
No circle is shaded

Answer

```
[Square] [Square] [Square] [Circle] [Circle]
```

c) \(\frac{3}{5}\) are squares
\(\frac{3}{5}\) are shaded
\(\frac{1}{3}\) of the squares are shaded

Answer

```
[Square] [Square] [Square] [Circle] [Circle]
```
Bonus

\[
\frac{3}{5} \text{ are squares} \\
\frac{3}{5} \text{ are shaded} \\
\frac{2}{5} \text{ are big} \\
\frac{1}{3} \text{ of the squares is big} \\
\frac{2}{3} \text{ of the squares are shaded} \\
\text{No shaded shape is big}
\]

Answer

[Diagram of shapes]
Goals

Students will find fractions of whole numbers when the answer is a whole number.

PRIOR KNOWLEDGE REQUIRED

Can find the fraction of an area or a set

MATERIALS

20 counters for each student
BLM Circle Fifths (p. L-52)

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $10 \times 10$ and to solve the corresponding division questions. For each number, go through the questions in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, up to $10 \times 3$ and $30 \div 3$. Then repeat with a different number. Next, try questions out of order, but keep the corresponding multiplication and division together.

Real-life fractions. Brainstorm the types of things students can find fractions of (Examples: circles, squares, pies, pizzas, groups of people, angles, hours, minutes, years, lengths, areas, capacities, apples).

Say each of the following sentences, and have students signal thumbs up (yes) or thumbs down (no) for whether or not each one makes sense:

a) 3 1/2 people went skiing. (no)
b) I ate 3 1/2 pancakes. (yes)
c) I folded the sheet of paper 3 1/4 times. (no)
d) Half of me was covered in paint. (yes)
e) I walked 2 1/3 miles. (yes)
f) I bought 2 1/3 marbles. (no)

Taking a fraction of a number. Tell students that you have 6 hats, and you will keep half for yourself and give half to a friend. Then draw dots for hats on the board:

```
  ○ ○ ○ ○ ○ ○
```

ASK: How many do I keep? (3) Circle the hats you keep and the hats you give away.

```
  ○ ○ ○          ○ ○ ○
```

SAY: I kept these (pointing to the first group) and I gave these away (pointing to the second group). Now tell students that you have 6 apples and half of them are red. ASK: How many are red? (3) Point out that now
the same dots can mean apples instead of hats. ASK: If I have a pie cut into 6 pieces and half the pieces are eaten, how many are eaten? (3) Now what do the dots represent? (pieces of pie) SAY: No matter what you have 6 of, half is always 3. This means that the number 3 is half of the number 6. Point out that students can find half of a number by drawing that number of dots and making two equal groups.

**Exercises:** Draw dots to find half of the number.

a) 10  b) 4  c) 8  d) 12

**Bonus:** Draw tens blocks to find half of the number.

e) 40  f) 80  g) 60  h) 140

**Answers:** a) 5; b) 2; c) 4; d) 6; Bonus: e) 20, f) 40, g) 30, h) 70

**Using pictures to find unit fractions of a number.** Tell students that you have 6 apples and one third of them are red. Draw on the board 6 dots to represent the 6 apples. Remind students that one third of anything is one out of three equal parts, and ask a volunteer to make three equal groups (see margin). ASK: How many apples are red? (2) SAY: So one third of 6 is 2. Write this as follows:

$$\frac{1}{3} \text{ of } 6 = 2$$

Draw on the board 12 dots in a row. Tell students you have 12 hats and one fourth of them are cowboy hats. Point out that you need one out of four equal groups. Ask a volunteer to make four equal groups. Then ASK: What is one fourth of 12? (3)

**Exercises:** Use the picture to find the fraction of the number.

a)  

$$\frac{1}{5} \text{ of } 10 = \underline{____}$$

b)  

$$\frac{1}{4} \text{ of } 12 = \underline{____}$$

c)  

$$\underline{____} \text{ of } 8 = \underline{____}$$

**Bonus:** Draw dots and make equal groups to find \(\frac{1}{3}\) of 15.

**Answers:** a) 2; b) 3; c) 1/4, 2; Bonus: 5

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**Number Sense 4-50**

L-31
Dividing to find unit fractions of a number. Refer students to the picture showing 1/5 of 10. Point out that 10 objects are divided among 5 equal groups. ASK: How many are in each group? (2) Write on the board:

\[
\frac{10}{5} = 2
\]

ASK: What operation are we doing when we ask how many are in each group? (division) Have a volunteer write the correct sign in the equation. \((10 \div 5 = 2)\) Point out that since fractions are made from equal groups, we are really just dividing the whole amount, which is 10, by 5 to find one fifth. ASK: What do we divide by to find one fourth? (4) What do we divide by to find one third? (3)

Exercises: Divide to find the fraction of the number.

a) \(\frac{1}{2}\) of 10  
b) \(\frac{1}{2}\) of 40  
c) \(\frac{1}{3}\) of 18  
d) \(\frac{1}{5}\) of 40

Bonus

e) \(\frac{1}{2}\) of 486  
f) \(\frac{1}{5}\) of 5000  
g) \(\frac{1}{3}\) of 1800  
h) \(\frac{1}{3}\) of 3096

Answers: a) 5, b) 20, c) 6, d) 8, Bonus: e) 243, f) 1000, g) 600, h) 1032

Finding any fraction of a number. Remind students that, once you have divided a set of objects into equal parts, you can take any number of those parts. Draw on the board:

\[
\begin{array}{cccccc}
\,
& \,
& \,
& \,
& \,
& \,
\end{array}
\]

SAY: There are 10 dots in five equal groups. ASK: What is one fifth of 10? (2) What is two fifths of 10? (4) (PROMPT: How many are in two groups?) ASK: What is three fifths of 10? (6) Four fifths of 10? (8) Summarize on the board:

\[
\frac{1}{5} \text{ of } 10 = 2 \quad \frac{2}{5} \text{ of } 10 = 4 \quad \frac{3}{5} \text{ of } 10 = 6
\]

\[
\frac{4}{5} \text{ of } 10 = 8 \quad \frac{5}{5} \text{ of } 10 = 10
\]

Leave the picture on the board.

Exercises: Use the picture to find the fraction of the number.

a) \[
\begin{array}{cccccc}
\,
& \,
& \,
& \,
& \,
& \,
\end{array}
\]
\(\frac{3}{4}\) of 8 = ______

b) \[
\begin{array}{cccccc}
\,
& \,
& \,
& \,
& \,
& \,
\end{array}
\]
\(\frac{2}{3}\) of 6 = ______

c) \[
\begin{array}{cccccc}
\,
& \,
& \,
& \,
& \,
& \,
\end{array}
\]
_____ of 9 = ______
d)  

\[ \begin{array}{cccc}
\circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ
\end{array} \]

_____ of 15 = ______

**Bonus:** Draw dots and make equal groups to find \( \frac{3}{4} \) of 12.

**Answers:** a) 6; b) 4; c) 2/3, 6; d) 4/5, 12; Bonus: 9

**ACTIVITY (Optional)**

Give students 20 counters and a large circle divided into five equal parts (e.g., use BLM Circle Fifths). Ask students to use the circle and the counters to find 4/5 of 10, 3/5 of 20, and then 2/5 of 15.

**Using division and multiplication to find a fraction of a whole number.**

Refer students to the picture on the board showing 10 objects divided among 5 groups. Tell students that you want to find a way to find 3/5 of 10 without using the picture. ASK: How could you find 1/5 of 10 without using the picture? (10 \( \div \) 5 = 2) SAY: So each group has 2 dots. That means that 3/5 of 10 is 3 groups of 2 dots. How can we write 3 groups of 2 dots mathematically? (3 \( \times \) 2) Write on the board:

\[
\frac{1}{5} \text{ of } 10 = 10 \div 5 = 2
\]

So \( \frac{3}{5} \) of 10 = 3 \( \times \) 2 = 6

ASK: What is 1/4 of 12? (3) Have a volunteer write the division that shows this. (12 \( \div \) 4 = 3) ASK: What is 3/4 of 12? (9) Have a volunteer write the multiplication that shows this. (3 \( \times \) 3 = 9) Repeat for 1/3 of 12 (12 \( \div \) 3 = 4) and 2/3 of 12 (4 \( \times \) 2 = 8).

**Exercises**

1. Use division and then multiplication to find the fraction of the whole number.

   a) \( \frac{1}{3} \) of 15 and then \( \frac{2}{3} \) of 15  
   b) \( \frac{1}{5} \) of 20 and then \( \frac{4}{5} \) of 20

   c) \( \frac{1}{7} \) of 35 and then \( \frac{3}{7} \) of 35  
   d) \( \frac{1}{6} \) of 18 and then \( \frac{5}{6} \) of 18

**Answers:** a) 5, 10; b) 4, 16; c) 5, 15; d) 3, 15
2. Find the fraction of the whole number.
   a) \( \frac{3}{8} \) of 24   
   b) \( \frac{5}{6} \) of 30   
   c) \( \frac{3}{4} \) of 80   
   d) \( \frac{5}{8} \) of 40

   **Answers:** a) 9, b) 25, c) 60, d) 25

**Word problems.**

**Exercises**

1. Lewis has 10 oranges. He gives away \( \frac{3}{5} \) of the oranges.
   a) How many oranges did he give away?
   b) How many did he keep?

   **Answers:** a) 6, b) 4

2. Nina has a collection of 12 shells. One third of the shells are scallop shells and one quarter of the shells are conch shells. The rest of the shells are cone shells. How many of Nina’s shells are cone shells?

   **Answer:** 5

3. Neka has 20 marbles. Two fifths are blue and one quarter are yellow. The rest are green. How many are green?

   **Answer:** 7

4. Sandy put \( \frac{2}{5} \) of her 10 shells on a shelf. Explain how you know she put \( \frac{2}{5} \) of her shells on the shelf.

   **Answer:** One fifth of 10 is 2, so two fifths of 10 is 4

**Bonus**

Sun had 12 apples. She gave \( \frac{1}{4} \) of her apples to Braden, and she gave 2 apples to Ann. Sun says that she has half left. Is she correct?

**Answer:** No, she has 7 left, which is more than half.

**Extensions**

1. Find \( \frac{2}{3} \) of 12 and \( \frac{3}{4} \) of 12. Use your answers to determine which is greater. \( \frac{2}{3} \) or \( \frac{3}{4} \). Then find \( \frac{2}{3} \) of 15 and \( \frac{3}{5} \) of 15. Which is greater, \( \frac{2}{3} \) or \( \frac{3}{5} \)?

   **Answers:** 2/3 of 12 is 8 and 3/4 of 12 is 9, so 3/4 is greater than 2/3. Also, 2/3 of 15 is 10 and 3/5 of 15 is 9, so 2/3 \( > \) 3/5.

2. Teach students another way to find fractions of a number: to find 2/3 of 12, draw 12 dots and colour 2 out of every 3 dots. SAY: The number of dots you coloured is 2/3 of 12.
3. Find the first three, then predict the fourth:
   a) \( \frac{2}{5} \) of 5    b) \( \frac{3}{4} \) of 4    c) \( \frac{5}{8} \) of 8
   d) \( \frac{941}{3562} \) of 3562

   **Answers:** a) 2, b) 3, c) 5, d) 941

4. Dory had 28 stickers. She kept \( \frac{1}{7} \) for herself and divided the rest evenly among 6 friends. How many stickers did each friend get?

   **Answer:** 4
Goals
Students will name fractions.
Students will compare fractions with the same numerator.
Students will describe sets using fractions.

PRIOR KNOWLEDGE REQUIRED
Can name the shaded and not shaded fractions in a shape
Can compare fractions with the same denominators
Can compare fractions with the same numerators
Can use fractions to describe sets

MATERIALS
deck of cards with face cards removed

Mental math minute. Remove the face cards from a deck of cards. Shuffle the deck. Divide students into pairs. Each student in the pair will select a card at random. Students will create two multiplication equations using the cards selected. For example, if Student A selects a 7 and Student B selects an 8, they will create the equations $7 \times 8 = 56$ and $8 \times 7 = 56$. If the card selected is an ace, students treat it as the number 1. Continue until all students have had an opportunity to participate.

Word problems. Work through the word problems in the exercises below as a class.

Exercises
1. \( \frac{1}{3} \) of a park is used for playing basketball. What fraction is not used for playing basketball?
   Answer: \( \frac{2}{3} \)

2. A pitcher of lemonade is a mixture of water and canned lemon juice.
   a) If \( \frac{1}{4} \) of the juice comes from the can, what fraction of the lemonade is water?
   b) How would the taste of the lemonade change if \( \frac{1}{2} \) of it was canned lemon juice instead of \( \frac{1}{4} \)?
   c) If you added some club soda to a glass of lemonade, would the fraction of canned lemon juice in your glass get bigger or smaller? Explain.
Answers: a) $\frac{3}{4}$, b) would taste stronger, c) smaller because there would be more liquid in the glass overall but the same amount of canned lemon juice.

3. Ethan made a design from some shapes.

![Shapes]

a) What fraction of the shapes are pentagons?
b) What other group has the same fraction as a)?
c) What fraction are not circles?

Answers: a) $\frac{4}{9}$, b) the shaded shapes, c) $\frac{7}{9}$

4. The shaded part of the picture represents classrooms that have both desks and tables. The part that is not shaded represents classrooms that only have desks.

![Shaded and Unshaded Areas]

a) In what fraction of the classrooms are there only desks?
b) What fraction of the classrooms have both desks and tables?
c) Which type of classroom are there more of?

Answers: a) $\frac{2}{8}$, b) $\frac{6}{8}$, c) classrooms with both desks and tables

5. Ronin and Grace both played their musical instruments at the year-end concert. Ronin played for $\frac{3}{8}$ of the time, and Grace played for $\frac{3}{5}$ of the time. Who played for longer?

Answer: Grace

Extension

On June 21, 2018, the sun was above the horizon for approximately $\frac{5}{8}$ of the day.

a) Write a fraction with denominator 24 that is equivalent to $\frac{5}{8}$.
b) There are 24 hours in a day. For approximately how many of those 24 hours was the sun above the horizon?
c) For approximately how many of those 24 hours was the sun not above the horizon?

Answers: a) $\frac{15}{24}$, b) 15 h, c) 9 h
Fractions Memory (1)

\[
\begin{array}{ccc}
\frac{1}{2} & \frac{2}{3} & \frac{1}{4} \\
\text{one half} & \text{two thirds} & \text{one fourth} \\
\end{array}
\]
Fractions Memory (2)

$$\frac{3}{5} \quad \frac{5}{6} \quad \frac{2}{7}$$

three fifths  five sixths  two sevenths
Fractions Memory (3)

three eighths  
two ninths  
seven tenths

\[\frac{3}{8} \quad \frac{2}{9} \quad \frac{7}{10}\]
Are the Shaded Amounts Equal?

Write the fraction that is shaded and if the shaded amounts are “equal” or “not equal.”

a) 
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]

b) 
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]

c) 
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]

d) 
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]

e) 
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]

f) 
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]

BONUS
\[
\begin{align*}
\text{fraction:} & \quad \text{shaded amounts:} \\
\end{align*}
\]
Tangram
Equivalent Fractions Memory (1)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
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<td>4</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>
Equivalent Fractions Memory (2)

\[
\begin{array}{ccc}
\frac{2}{3} & \frac{3}{4} & \frac{3}{8} \\
\frac{8}{12} & \frac{9}{12} & \frac{6}{16} \\
\frac{5}{8} & \frac{7}{8} & \frac{4}{5} \\
\frac{10}{16} & \frac{21}{24} & \frac{16}{20}
\end{array}
\]
Equivalent Fractions Memory (3)

\[
\begin{array}{ccc}
\frac{7}{9} & \frac{5}{6} & \frac{6}{7} \\
\frac{56}{72} & \frac{45}{54} & \frac{36}{42} \\
\frac{4}{9} & \frac{5}{11} & \frac{7}{12} \\
\frac{28}{63} & \frac{55}{121} & \frac{63}{108}
\end{array}
\]
Fraction Cards (1)

\[
\begin{array}{ccc}
\frac{2}{15} & \frac{3}{15} & \frac{5}{15} \\
\frac{2}{18} & \frac{3}{18} & \frac{5}{18} \\
\frac{2}{24} & \frac{3}{24} & \frac{5}{24} \\
\frac{2}{25} & \frac{3}{25} & \frac{5}{25}
\end{array}
\]
Fraction Cards (2)

\[
\begin{array}{ccc}
\frac{9}{15} & \frac{11}{15} & \frac{14}{15} \\
\frac{9}{18} & \frac{11}{18} & \frac{14}{18} \\
\frac{9}{24} & \frac{11}{24} & \frac{14}{24} \\
\frac{9}{25} & \frac{11}{25} & \frac{14}{25}
\end{array}
\]
Fraction Cards (3)

\[
\begin{array}{ccc}
\frac{2}{30} & \frac{3}{30} & \frac{5}{30} \\
\frac{9}{30} & \frac{11}{30} & \frac{14}{30} \\
\frac{2}{31} & \frac{3}{31} & \frac{5}{31} \\
\frac{9}{31} & \frac{11}{31} & \frac{14}{31}
\end{array}
\]
Ordering with Fraction Strips

1. Shade the strips to show the fractions. Order the fractions from least to greatest.
   a) $\frac{2}{5}$, $\frac{2}{9}$, $\frac{2}{3}$, $\frac{2}{6}$
   b) $\frac{3}{12}$, $\frac{3}{16}$, $\frac{3}{7}$, $\frac{3}{10}$

2. Shade the strips to show the fractions. Order the fractions from greatest to least.
   a) $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{1}$
   b) $\frac{4}{15}$, $\frac{4}{6}$, $\frac{4}{18}$, $\frac{4}{8}$
Ordering Fractions (1)

1. a) Count how many fourths are shaded.

b) Write the fractions in order from least to greatest.

2. a) Count how many thirds are shaded.

b) Write the fractions in order from least to greatest.
Ordering Fractions (2)

3. a) Count how many halves are shaded.

[Diagram of shaded fractions]

b) Write the fractions in order from least to greatest.

<  <  

4. a) Count how many tenths are shaded. Write the fractions in order from greatest to least.

[Diagram of shaded fractions]

b) Write the fractions in order from least to greatest.

<  <  

5. Write < or >.

If 8/8 = 1 then 9/8  1.

If 5/5 = 1 then 4/5  1.

Since 9/8 > 1 and 4/5 < 1, then 9/8  4/5.
Circle Fifths
Unit 10 Number Sense: Decimals

Introduction
This unit focuses on decimal tenths and hundredths within and beyond 1. It describes how to:

• relate decimals to fractions;
• use the principles of place value learned with whole numbers to estimate, add, subtract, compare, and order decimals; and
• use decimals in the context of money.

Meeting Your Curriculum

<table>
<thead>
<tr>
<th>Curriculum</th>
<th>Required</th>
<th>Optional</th>
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<tbody>
<tr>
<td>ALBERTA</td>
<td>NS4-52 to 54, 56 to 63</td>
<td>NS4-55</td>
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<td>BRITISH COLUMBIA</td>
<td>NS4-52 to 54, 56 to 63</td>
<td>NS4-55</td>
</tr>
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<td>MANITOBA</td>
<td>NS4-52 to 54, 56 to 63</td>
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<td>ONTARIO</td>
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<td></td>
<td>NS4-59 to 61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>supports proper decimal notations to hundredths for later lessons on money</td>
<td></td>
</tr>
</tbody>
</table>

Mental Math Minutes

The mental math minutes in this unit:
• practise addition, subtraction, multiplication, and division strategies for whole numbers
• consider different strategies for adding decimals in a whole class number talk

Generic BLMs

The Generic BLM used in this unit is:
BLM 1 cm Grid Paper (p. S-2)
This BLM can be found in Section S.
Assessment

The following table indicates the lessons covered by a quiz or test for each curriculum.

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
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<tr>
<td>Quiz</td>
<td>NS4-52</td>
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<tr>
<td></td>
<td>56 to 63</td>
<td>56 to 63</td>
<td>56 to 63</td>
<td>62, 63</td>
</tr>
</tbody>
</table>

Additional Information for This Unit

The Penny

In this unit, students will learn the relationships among coins, including the penny. Although the penny is no longer used in Canada, the value of a penny as one cent is still important as the base value of all the other coins. One cent is also important in the calculation of taxes and still appears in some prices.
NS4-52 Decimal Tenths and Place Value

Pages 36–38

Goals
Students will use decimal notation for fractions with denominators of 10.
Students will place decimal tenths on number lines.
Students will identify the place value of digits to tenths.

PRIOR KNOWLEDGE REQUIRED
Knows that, on number lines, greater whole numbers appear to the right of lesser whole numbers
Can name fractions from area models, tens block models, and number lines
Understands the phrase “times as many”
Understands place value for ones, tens, hundreds, and thousands

Mental math minute. Review pairs that add to 10. You can say a number from 0 to 10 and have students raise the correct number of fingers to signal the number that makes 10 together with the number you said. Have students add by using 10. Say the addition you want students to do (such as 8 + 6). Have a student say the in-between addition step, 10 + 4, and have another student finish the addition. Start with addition problems within 20, such as 7 + 5 or 9 + 3, and progress to harder problems, such as 23 + 8 or 76 + 7. As a challenge, use three-digit and four-digit numbers, such as 345 + 8, or vary the order to 8 + 56.

Introduce decimal tenths. Tell students that the fraction 1/10 can be represented in various ways. Show three ways on the board:

\[
\frac{1}{10} = 0.1
\]

Point out that each way means 1 part out of 10 equal parts. Tell students that mathematicians have invented a simpler way to write one tenth, called decimal notation. Write the equation shown in the margin on the board.

SAY: The dot is called a decimal point. People write the number this way because it takes up less space on the page, it shows we are counting in decimals, and it is easier to write. Ask volunteers to show how they would write 2 tenths (0.2), 3 tenths (0.3), and other numbers up to 9 tenths (0.9) using decimal notation. SAY: The 0 before the decimal point tells you that the number is less than 1. Explain that people say decimals in different ways. Write “0.1” on the board and SAY: This is zero point one, but sometimes people also call it one tenth.

Representing decimal tenths on a number line. Draw a number line from 0 to 1 and ask students to place various decimal tenths on the number line (e.g., 0.8, 0.5, 0.2, 0.7). Write 0.0 under 0 and 1.0 under 1. Explain that people write 0 and 1 both ways. Writing them with “.”0” shows you are using decimal notation for all the numbers on the number line, including 0 and 1.

Number Sense 4-52

M-3
Exercises: Write the decimal and fraction for each marked point.

```
0.0  | 1.0
```

**Answers:** 0.3, 3/10, 0.4, 4/10, 0.9, 9/10

**Representing decimal tenths using pictures.** Draw various shapes on the board, such as circles, squares, or rectangles, and have volunteers represent various numbers given in decimal notation. Count the sections in each shape to demonstrate that they all are divided into 10 equal parts.

a) 0.2  
```
\[ \text{Circle divided into 10 equal parts, 2 parts shaded.} \]
```

b) 0.3  
```
\[ \text{Line divided into 10 equal parts, 3 parts shaded.} \]
```

c) 0.5  
```
\[ \text{Rectangle divided into 10 equal parts, 5 parts shaded.} \]
```

d) 0.6  
```
\[ \text{Square divided into 10 equal parts, 6 parts shaded.} \]
```

**Writing decimal notation for pictures.** Now ask students to do the reverse.

**Exercises:** Write the decimal for the picture.

```
a)  
```
```
b)  
```
```
c)  
```
```
d)  
```
```

**Answers:** a) 0.8, b) 0.7, c) 0.5, d) 0.8

**The value of a digit.** Write “3982.6” on the board. Underline one digit at a time, in no particular order, and ask students to identify the place value of the underlined digit. Repeat with other numbers.

Write “1057.4” on the board and ask students to state the place value and value of a particular digit without underlining it. **Exercises:** Find the place value and value of the digit 4 in these numbers: 2401.7, 4230.9, 46.5, 24.7, 9.4. **Answers:** hundreds, 400; thousands, 4000; tens, 40; ones, 4; tenths, 4 tenths. Continue until students can identify place value and value correctly and confidently. Include examples where you ask for the place value of the digit 0. SAY: Notice that, although the digit 0 always has a value of 0, its place value changes with position the same as any other digit.

**Exercises:** Find the value of the underlined digit.

```
a) 7133.8  
```
```
b) 259.6  
```
```
c) 8340.1  
```
```
d) 126.4  
```
```
e) 3025.7  
```
```

**Answers:** a) 100, b) 9, c) 8000, d) 4 tenths, e) 20
**Introduce the place value chart.** Have students write the digits from the number 231.7 in the correct column:

<table>
<thead>
<tr>
<th>Number</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>231.7</td>
<td></td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

Add more numbers to the place value chart together. Have volunteers come to the board to write the digits in the correct columns.

**Expanded form.** Write "2836.4" on the board. Explain that 2836.4 is just a short way of writing $2000 + 800 + 30 + 6 + 0.4$. Write several other numbers on the board and have students write them in expanded form. Include numbers that contain the digit zero.

**Comparing the value of digits.** Write the following numbers on the board:

- 2350.6
- 5031.4
- 143.5
- 437.5

Ask students to identify which digit, the 5 or the 3, is worth more in each number. Students should be using the phrases introduced in the lesson—stands for, has a value of. (Example: In 2350.6, the 5 stands for 50 and the 3 stands for 300, so the digit 3 is worth more.)

**Exercises:** Copy and complete the table with rows added for parts b) through f).

<table>
<thead>
<tr>
<th>Number</th>
<th>How much is the 5 worth?</th>
<th>How much is the 3 worth?</th>
<th>Which is worth more, the 3 or 5?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3500.7</td>
<td>500</td>
<td>3000</td>
<td>3</td>
</tr>
<tr>
<td>b) 405.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 5243.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 735.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) 3000.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) 3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: How can you tell when the 3 is worth more than the 5 without even completing the chart? (When the 3 is to the left of the 5, it is worth more.)

**Extensions**

1. Write the missing number.
   a) $\frac{7}{10} = 0.7$
   b) $\frac{9}{10} = 0.9$
   c) $\frac{1}{10} = 0.8$
   d) $\frac{4}{10} = 0.4$

   **Answers:** a) 10, b) 10, c) 8, d) 4

2. a) For the picture in the margin, write a fraction to represent the shaded region of the picture on the left or right.
   b) Add lines to both pictures, in two different ways, to make the pictures show tenths.
   c) Write a fraction for each picture, different from the fraction in part a), to represent the shaded regions with the lines you added.

   **Answers:** a) 1/5 for both, b) $\frac{3}{10}$, $\frac{3}{10}$, c) 2/10, 2/10
3.  

![Number Line](image)

a) What fraction of the number line is between the two dots? Explain how you found your answer.

b) What fraction of the number line is not between the two dots? Explain how you found your answer.

**Answers**

a) 4/10, Sample explanation: I counted the spaces from 2 to 6; I subtracted 6 − 2.

b) 6/10, Sample explanation: I counted the spaces from 0 to 2 and from 6 to 10; I know that the whole line is 10, and I subtracted the number of spaces between the dots (4) from the number of spaces in the whole line, 10 − 4 = 6.

4.  

![Shaded Region](image)

a) What fraction of the picture is shaded?

b) What is the area of the shaded region of the picture in mm²?

c) What is the area of the whole picture in mm²?

d) Express your answers to parts b) and c) as a fraction:

<table>
<thead>
<tr>
<th>Shaded Area (mm²)</th>
<th>Total Area (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 mm × 10 mm = 200 mm²</td>
<td>100 mm × 10 mm = 1000 mm²</td>
</tr>
</tbody>
</table>

e) What do you notice about the relationship between your answer to parts a) and d)?

**Answers:**

a) 2/10, b) 20 mm × 10 mm = 200 mm², c) 100 mm × 10 mm = 1000 mm², d) 2/10 = 200/1000, e) You have to multiply the number by 5 to get the denominator for both.

5. Find the value of each bold digit. How many times as much as the second digit is the first digit worth?

<table>
<thead>
<tr>
<th>Number</th>
<th>Value of first bold digit</th>
<th>Value of second bold digit</th>
<th>How many times as much?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 414.2</td>
<td>400</td>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>b) 7115.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 582.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 7347.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** b) 100, 10, 10; c) 80, 8/10, 100; d) 7000, 7, 1000
6. Tell students that there is a way to determine how many times as much as the second digit the first digit is worth without determining the value of each digit. Draw the following table on the board:

<table>
<thead>
<tr>
<th>Number</th>
<th>How many times as much?</th>
<th>How many places apart are the two digits?</th>
</tr>
</thead>
<tbody>
<tr>
<td>323.5</td>
<td>100</td>
<td>2</td>
</tr>
</tbody>
</table>

Show that the 3s are 2 places apart as shown in the margin.

Add four rows and the numbers in parts b) to e) to the table and then have volunteers complete the rows.

b) 4332.6  c) 343.1  d) 3243.7  e) 7321.3

Answers: b) 10, 1, c) 100, 2, d) 1000, 3, e) 3

7. a) Which is worth more in the following numbers, the 3 or the 6? How many times more?
   i) 63.4  ii) 623.7  iii) 634.2  iv) 36.1
   v) 376.9  vi) 3006.4  vii) 6731.8

   If students need help, ask them how they could turn each problem into one that they already know how to solve. For example, if part ii) was 323.7 instead of 623.7, you would know how to solve it (the first 3 is worth 100 times more than the second 3). ASK: How is 623.7 different from 323.7? (6 is twice as much as 3, so the 6 is worth 200 times more than the 3)

b) How many times more is the 2 worth than the 5 in the number?
   i) 25.6  ii) 250.1  iii) 2500.8  Bonus: 2.5

   If students need help, ask them how they could turn each problem into one that they already know how to solve. For example, if part ii) was 323.7 instead of 623.7, you would know how to solve it (the first 3 is worth 100 times more than the second 3). ASK: How is 623.7 different from 323.7? (6 is twice as much as 3, so the 6 is worth 200 times more than the 3)

   c) How many times more is the 2 worth than the 5 in:
      i) 25  ii) 235  iii) 2465
      Bonus: 27 465

   Students will see that the answer multiplies by 10 each time, so they can pretend the numbers are right next to each other and then add a zero for each place they have to move over.

   Answers: a) i) the 6, 20 times more; ii) the 6, 200 times more; iii) the 6, 20 times more; iv) the 3, 5 times more; v) the 3, 50 times more; vi) the 3, 500 times more; vii) the 6, 200 times more; b) i) 4, ii) 4, iii) 4, Bonus: 4; c) i) 4, ii) 40, iii) 400, Bonus: 4000
**Goals**

Students will relate fractions to decimals, within 1, on number lines.

**PRIOR KNOWLEDGE REQUIRED**

Can count by tenths from 0 tenths to 10 tenths and by fifths from 0 fifths to 5 fifths

**MATERIALS**

metre stick

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**Mental math minute.** Have students stand in a line. Give the first student a problem that does not need regrouping, such as $88 - 13$. Students in line repeatedly subtract a number, in this case 13, by each student saying one subtraction aloud. When a student says the subtraction that involves regrouping, emphasize that this answer was a bonus. Example: Student 1 says, “$88 - 13 = 75$.” Student 2 says, “$75 - 13 = 62$.” Bonus: Student 3 says, “$62 - 13 = 49$.” Continue without regrouping until Student 6 says, “$23 - 13 = 10$,” then start a new chain.

**Relating fractional tenths to decimal tenths.** On the board, use a metre stick to draw a number line divided into 10 equal sections:

```
|   |   |   |   |   |   |   |   |   |   |
```

Explain that you want to write a fractional scale above the number line and a decimal scale below it. SAY: The number line is divided into 10 equal parts, so each part is one tenth of the distance from one end of the number line to the other. Write 0/10 and 1/10 above the first and second tick marks, respectively, and 10/10 above the last tick mark. Explain that 1/10 means one tenth of the distance from 0/10 to 10/10. SAY: 0/10 means none of the distance on the number line. ASK: Since the scale goes up by tenths, what will the next fraction be? (2/10) What does 2/10 mean? (two tenths of the distance from 0/10 to 10/10) And what will the next fraction be? (3/10)

Have volunteers fill in the remaining fractions on the board. Explain that 10/10 means the entire distance from 0/10 to 10/10. Write “0.0” and “1.0” below the leftmost and rightmost tick marks, respectively, and have students fill in the rest of the decimals. Point to a decimal or fraction on the number line and ask students for the fraction or decimal it is equal to. Repeat several times.
Exercises: Write the fraction or decimal that is equal to …

a) 0.8  
b) \( \frac{7}{10} \)  
c) \( \frac{3}{10} \)  
d) 0.2  
e) \( \frac{10}{10} \)  
f) 0.1  
g) \( \frac{4}{10} \)  
h) 0.6  
i) 0.9  
j) \( \frac{5}{10} \)  
k) 0.0  
l) \( \frac{8}{10} \)

Answers: a) 8/10, b) 0.7, c) 0.3, d) 2/10, e) 1.0, f) 1/10, g) 0.4, h) 6/10, i) 9/10, j) 0.5, k) 0/0, l) 0.8

Draw the number lines again, but with the decimal scale in order from 0.0 to 1.0 and the fractional scale with some fractions out of order. Point to a fraction and ask if it equals the decimal below it. If the answer is “no,” cross out the wrong fraction and then ask for and write in the correct fraction. Repeat for all remaining fractions. Repeat, but have the fractions in order and some decimals out of order.

Relating fractional halves to decimal tenths using number lines. Draw on the board:

\[
\begin{array}{c|c|c}
0 & \frac{1}{2} & \frac{2}{2} \\
\hline
\frac{2}{2} & & \\
0.0 & & \\
\end{array}
\]

SAY: The top number line is divided into two halves, so the scale is written using halves. The bottom number line is divided into tenths because I want to write its scale using decimals. Have a volunteer complete the decimal scale. Explain that you want to relate the fractions above the top number line to decimals below the bottom number line. Point to \( \frac{0}{2} \) and ASK: What decimal does this equal? (0.0) Write \( \frac{0}{2} = 0.0 \) on the board. Repeat for the other two fractions.

Relating fractional fifths to decimal tenths using number lines. Draw on the board:

\[
\begin{array}{c|c|c}
0 & \frac{3}{5} & \\
\hline
\frac{3}{5} & & \\
0.0 & & \\
\end{array}
\]

Have volunteers complete the scales. Point to \( \frac{3}{5} \) and ASK: What decimal does this equal? (0.6) Write \( \frac{3}{5} = 0.6 \) on the board. Repeat for the other fractions on the number line.
Extensions

1. a)  
   i) Write a fraction and a decimal to represent the darkened section of the number line.
   ii) Draw two different shapes to represent the fraction you just wrote.

   b) Write a fraction and a decimal to represent the darkened section of the number line.

   c) What do you notice about the fractions you wrote in part a) and part b)?

   d) How else could you show that same fraction on a number line?

Answers
a) i) 3/10, 0.3  
b) 3/10, 0.3  
c) they are the same  

Sample Answers
a) ii)  
d)  

2. Write >, <, or = between the two numbers.

   a) 0.6 $\frac{7}{10}$  
b) $\frac{4}{10}$ 0.3  
c) 0.8 $\frac{9}{10}$  
   d) 1.0 $\frac{1}{10}$  
e) $\frac{5}{10}$ 0.5  
f) 0.1 $\frac{10}{10}$  
   g) $\frac{2}{10}$ 1.0  
h) 0.4 $\frac{4}{10}$  
i) $\frac{3}{10}$ 0.5  
   j) 0.0 $\frac{1}{10}$  
k) 0.7 $\frac{5}{10}$  

Answers: a) <, b) >, c) <, d) >, e) =, f) <, g) <, h) =, i) <, j) <, k) >
Goals

Students will use number lines and area models to represent decimals greater than 1.
Students will write decimals greater than 1 in words.
Students will count by decimal tenths from any number.

PRIOR KNOWLEDGE REQUIRED

Can write tenths as decimals up to 1.0

MATERIALS

grid paper or **BLM 1 cm Grid Paper** (p. S-2)

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**Mental math minute.** Ask students to solve multiplication questions within the range of $0 \times 1$ to $10 \times 10$. For each number, first go through the questions in order, such as $0 \times 3$, $1 \times 3$, and so on to $10 \times 3$, and then in reverse order. After that, go through the same questions out of order, and then progress to a different number. You can pass a ball to the student you want to answer the question and have students pass the ball back to you as they answer.

**Counting in decimal tenths on number lines.** Draw on the board:

```
0 1 10 20
```

Have a volunteer number the unlabelled tick marks between 1 and 20. Circle only the ones digits from 0 to 20. SAY: The ones digits follow a pattern, from 0 to 9 and then from 0 to 9 again. Explain that the arrow shows that the number line continues. Draw on the board:

```
0.0 0.1
```

Have students say the first two numbers as a class. (zero point zero, zero point one) Have students continue to 0.9 and write in the decimals on the board as they do so. SAY: The next tick mark shows ten tenths, and we call it one point zero. Write 1.0 above it. Point to the next tick mark and ASK: How many tenths is this? (11) How do we say that as a decimal? (one point one) Point to the next tick mark and ASK: How many tenths is this? (12) How do we say that as a decimal? (one point two) Continue this way up to twenty tenths/two point zero. Circle all the tenths digits (not the decimal points). SAY: The tenths digits follow the same pattern as the ones digits did, from 0 to 9 and then from 0 to 9 again. Repeat, counting and labelling on the number line from 2.0 to 4.0, and then from 3.7 to 5.7. Explain that it
doesn’t matter what the first number is as the pattern is always the same. Leave this on the board as a reference for the following exercises.

**Exercises:** Draw and label a number line on grid paper for the range of numbers.

a) 2.0 to 3.0  
b) 15.2 to 16.2  
c) 234.5 to 235.5

**Answers**
a) 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0  
b) 15.2, 15.3, 15.4, 15.5, 15.6, 15.7, 15.8, 15.9, 16.0, 16.1, 16.2  
c) 234.5, 234.6, 234.7, 234.8, 234.9, 235.0, 235.1, 235.2, 235.3, 235.4, 235.5

**Decimal tenths greater than 1 using area models.** Draw the picture in the margin on the board. Have a volunteer shade the picture to show four tenths. Repeat with a new blank picture for 1.0. Write “1.0” on the board and **ASK:** How many tenths is 1.0? (10) Write “1.1” on the board and **ASK:** How many tenths is 1.1? (11) Draw just the left block shown in the margin on the board and **ASK:** How many sections of this do we shade to show 10 tenths or 1.0? (all) Shade them. Draw the second block and **ASK:** How many sections of this block do we have to shade so that the two blocks show 11 tenths or 1.1 altogether? (one) Shade one section of the second block.

**SAY:** To show 11 tenths or 1.1, we had to shade all 10 sections of one block and one section of the other. Let’s use a picture to show 2.0. **ASK:** How many blocks should we draw? (two) **ASK:** How many sections of each block should we shade? (all of them) Draw a third block and **ASK:** How many sections of this block do we have to shade for the whole picture to show 2.7? (seven) Shade seven sections of the third block. Repeat for 3.3 and 4.5.

**Exercises:** Draw and shade blocks to show the amount.

a) 1.6  
b) 2.3  
c) 4.9

**Answers:**

a)

b)

c)

**Writing decimals in words.** Tell students that just as they have already written number words to show numbers, they can write decimals in words too. **SAY:** We use the word “and” to represent the decimal point. Write on the board:

three and seven tenths

3 . 7

Keep this on the board for later reference.
Exercises: Fill in any missing words.

a) \[4.8 = \underline{\phantom{0}}\phantom{.} \text{and eight tenths}\]

b) \[17.6 = \underline{\phantom{0}}\phantom{.} \text{and six tenths}\]

c) \[16.5 = \text{sixteen and } \underline{\phantom{0}}\phantom{.} \text{tenths}\]

d) \[38.4 = \text{thirty-eight and } \underline{\phantom{0}}\phantom{.} \text{tenths}\]

e) \[30.8 = \text{thirty and eight } \underline{\phantom{0}}\phantom{.} \text{tenths}\]

f) \[4.1 = \text{four } \underline{\phantom{0}}\phantom{.} \text{one } \underline{\phantom{0}}\phantom{.} \text{tenths}\]

Answers: a) four, b) seventeen, c) five, d) seven, e) four, f) tenths, g) tenths, h) and, tenth

Exercises: Write the decimal in words.

a) \[3.8\] b) \[26.9\] c) \[30.4\] d) \[41.5\]

Bonus: \[3007.5\]

Answers: a) three and eight tenths, b) twenty-six and nine tenths, c) thirty and four ten, d) forty-one and five tenths

Bonus: three thousand seven and five tenths

Writing the decimal for the word. Refer to the number words and decimal that you previously wrote on the board and SAY: The “and” tells you where to put the decimal point, and the fraction word tells you what to put after the decimal point.

Exercises: Write the decimal.

a) twelve and three tenths b) fifty and three tenths
c) two and five tenths d) two hundred and five tenths
e) two hundred five and six tenths

Answers: a) 12.3, b) 50.3, c) 2.5, d) 200.5, e) 205.6

Extensions

1. Point out the symmetry in the place values on either side of the ones position. For example, for \[743.61\], the place values are as follows:

<table>
<thead>
<tr>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
<th>tenths</th>
<th>hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
<td>3</td>
<td>.</td>
<td>6</td>
</tr>
</tbody>
</table>

Challenge students to name the place values in the number \[3.612\] by using this symmetry. (ones, tenths, hundredths, thousandths)

NOTE: Some students might look for a “oneths” position, from thinking of the decimal point as the centre of symmetry. This might seem natural because the decimal point is the only part of the number that looks different. However, the ones are the basic units and in fact are the basis for the symmetry.

2. Have students look for decimals greater than 1 in the media and write them both as numbers and in words.
Goals

Students will compare numbers with decimal tenths with and without using base ten blocks.
Students will order numbers with decimal tenths.

PRIOR KNOWLEDGE REQUIRED

Can name whole numbers from base ten models
Can find the greater/lesser of two whole numbers
Can arrange whole numbers in ascending or descending order

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $10 \times 10$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3, 3 \div 3, 2 \times 3, 6 \div 3$, and so on to $10 \times 3$ and $30 \div 3$. Then progress to a different number. Next, try questions out of order, but keep multiplication and corresponding division together.

Comparing numbers using base ten models. Remind students that they have compared numbers before, just not numbers with decimals. Draw on the board:

![Base ten models](image)

SAY: The model on the left has four large squares that represent four ones blocks. ASK: How many tenths does it have? (two) Write “4” in the ones column and “2” in the tenths column in the table above the left model.
ASK: What number does this model show? (4.2) ASK: if we want to compare 4.2 with the model on the right, do we need to count the blocks? (no) Explain that we can already see that the model on the right has more ones blocks, so it has to be the greater number. ASK: What number does the model on the right show? (6.1) Write “6” in the ones column and “1” in the tenths column in the table above the right model.
Exercises: Indicate which model shows the greater number.

a)  

b)  

c)  

Answers: a) A, b) B, c) B

Comparing numbers by finding the pair of digits that are different. Write the following on the board, with decimals aligned as shown below:

\[3914.2\]
\[3814.2\]

SAY: We’re going to compare these numbers. The first thing we always do is to make sure the decimals are perfectly lined up. ASK: Are they? (no) Rewrite either number so the decimals are aligned. ASK: Which digits are different? (9 and 8) Circle the “9” and “8.” ASK: Which number is greater? (3914.2)

NOTE: Align the decimals for all the numbers in the following exercise except parts c), e), and f).

Exercises: Circle the digits that are different and then identify the greater number:

a) 178.6  
179.6
b) 7460.2  
9460.2
c) 54.3  
44.3
d) 611.9  
611.7
e) 2003.1  
2013.1
f) 74.8  
74.7

Answers: a) ones, 179.6; b) thousands, 9460.2; c) tens, 54.3; d) tenths, 611.9; e) tens, 2013.1; f) tenths, 74.8

Comparing numbers by finding the leftmost pair of digits that are different. Write on the board:

\[372.5\]
\[378.5\]

SAY: We can find the greater number by reading the digits from left to right and identifying the first pair that is different. ASK: Reading from the left, which is the first pair of digits that is different? (ones, 2 and 8) Which number is greater? (378.5)
Exercises: Circle the first pair of digits, starting from the left, that are different and then identify the greater number.

a) 293.7  
   292.6  
   b) 1406.8  
   1604.8  
   c) 84.5  
   84.2  
   d) 703.9  
   603.2  
   e) 6500.4  
   6510.4  
   f) 1133.8  
   2138.3

Answers: a) ones should be circled, 293.7; b) hundreds should be circled, 1604.8; c) tenths should be circled, 84.5; d) hundreds should be circled, 703.9; e) tens should be circled, 6510.4; f) thousands should be circled, 2138.3

Comparing numbers with different numbers of digits. Write on the board:

935.2  1352.2

Point to each number and ASK: How many digits does this number have? (4, 5) Erase the “1352.2.” Then write “1352.2” under the “935.2,” aligning the 1 and the 9 rather than the decimals. ASK: Are we ready to compare these numbers? (no) Why? (the decimals aren’t on top of each other) Erase the “1352.2” and then rewrite it with the decimals aligned. ASK: Are we ready to compare these numbers now? (yes) Which number is greater? (1352.2)

Exercises: Write the second number below the first number with the decimals lined up. Then circle the greater number.

a) 2166.8  816.2  
   b) 583.3  1305.8  
   c) 897.2  1241.1  
   d) 1000.8  999.8

Answers: Teacher to check alignment. a) 2166.8, b) 1305.8, c) 1241.1, d) 1000.8

Finding the greatest number. Write on the board:

13.2  14.8  13.9

ASK: Which of the first two numbers is greater? (14.8) Underline “14.8” on the board. ASK: Which number is greater, 14.8 or the third number? (14.8) Circle “14.8” on the board. ASK: Which is the greatest of the three numbers? (14.8) Repeat for 364.5, 374.5, 474.5.

Exercises: Find the greatest number.

a) 2019.4  2018.4  2018.9  
   b) 772.6  782.6  776.2  
   c) 56.1  55.9  56.6  
   d) 3003.1  3003.0  3029.9

Answers: a) 2019.4, b) 782.6, c) 56.6, d) 3029.9
Arranging numbers in descending order. Write on the board:

6208.4   6218.4   6214.8

ASK: Which is the greatest number? (6218.4) Write “6218.4” in the first blank. ASK: Which of the other two numbers is greater? (6214.8) Write “6214.8” in the second blank. Which number is the smallest? (6208.4) Write “6208.4” in the third blank.

Finding a number that fits. Write on the board:

a)     140.1  141.0
b) 140.1     141.0
c) 140.1  140.3

SAY: For each question, we need to find a number that fits the order. ASK: Which question is the most challenging? (b) Why? (the answer has to be greater than the first number and less than the third number) SAY: The best number to choose for question b) is the smallest number that is greater than 140.1. ASK: What number is that? (140.2) Why is that the best number? (no other number could be between 140.1 and 140.2) Have students suggest answers for parts a) and c). Write on the board:

75.8   ______    76.8

ASK: What is the smallest number that is greater than 75.8? (75.9) Is 75.9 less than 76.8? (yes) SAY: That means we’ve found a number that works. Write “75.9” in the blank.

Exercises: Find a number that fits.

a) 461.8, ______, 464.8
b) 73.5, 75.3, ________, 73.5, ______, 75.3
   461.8, 464.8, ______
   461.8, 464.8, ______, 75.3

c) ______, 2810.2, 2820.1
   2810.2; 2820.1, ______
   2810.2, ______, 2820.1

Sample answers: a) 461.9, 461.7, 464.9; b) 75.4, 73.6, 73.4; c) 2810.1, 2820.2, 2810.3

Using a number line to order numbers. Draw on the board:

| 6.0 | 6.1 | 6.5 | 6.3 | 6.7 |
| 7.0 | 7.2 | 7.9 |
| 8.0 |

7.2, 6.1, 7.9, 6.5, 6.3, 7.7

Point to the group of numbers and ASK: Are these numbers in order? (no) Have a volunteer mark an X for 7.2 on the number line and write “7.2” under the X. Repeat for the remaining numbers. ASK: Are the numbers in order now? (yes) SAY: We can use a number line to order a group of numbers no matter how they are arranged at the start.
Extensions

1. Write on the board:

\[ \square \cdot 2 < \square \cdot 4 \]

ASK: What numbers can we put in the box in front of .2 to make this statement true? (1, 2, or 3) Take more than one answer. ASK: What digits can we write in the box after 4 to make this statement true? (any) Remind students that we do not write “0” at the beginning of a number unless it is immediately followed by a decimal point.

Now write on the board:

\[ \square \cdot 2 < \square \cdot 9 \]

ASK: Are there any numbers that will make this statement true?
PROMPTS: How many tens are in the second number? (none) Can a number with no tens be greater than a number that has tens? (no)

Example:

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ ] [ ]</td>
<td>[ ] [ ]</td>
</tr>
</tbody>
</table>

2. Create base ten models of a pair of two-digit numbers. Ask students to say how they know which number is greater. You might make one of the numbers in non-standard form, as shown for the first number at the left. To compare the numbers, students could remodel the first number in standard form by regrouping ones blocks as tens blocks.

3. Ask students to create base ten models of two numbers, in which one of the numbers …
   a) is 30 more than the other.
   b) is 50 less than the other.
   c) has hundreds digit equal to 6 and is 310 more than the other.

4. Ask students where they tend to see many numbers in increasing order (houses, mailboxes, lineups when people need to take a number tag, apartment numbers).

5. Arrange the numbers in ascending order.
   a) 2.1, 8.4, 2.2, 8.7
   b) 12.9, 11.5, 18.0, 11.6
   c) 714.2, 710.4, 784.2, 721.4
   d) 87.2, 89.8, 28.7, 23.1, 87.5, 23.6
   e) 514.2, 451.2, 541.2, 452.2, 4520.1, 515.2

Answers
   a) 2.1, 2.2, 8.4, 8.7
   b) 11.5, 11.6, 12.9, 18.0
   c) 710.4, 714.2, 721.4, 784.2
d) 23.1, 23.6, 87.2, 87.5, 89.8  
e) 451.2, 452.2, 514.2, 515.2, 541.2, 4520.1

6. Find a number that satisfies all the criteria.

a) the tens digit is the largest digit and the tenths digit is the smallest digit
b) the ones digit is greater than the tens digit and greater than the tenths digit
c) the ones digit is greater than the tens digit and less than the tenths digit
d) the tenths digit is the greatest digit and is 2 greater than the tens digit
e) the tens digit is 2 times the tenths digit and less than the ones digit
f) the tenths digit is 3 times the ones digit and greater than the tens digit
g) the ones digit is 4 times the tenths digit and less than the tens digit
h) the ones digit is 7 less than the tenths digit and 1 greater than the tens digit
i) the tenths digit is 2 times the tens digit and 4 more than the ones digit
j) the tenths digit is 3 times the tens digit and 7 greater than the ones digit

Answer: j) 32.9

Sample answers: a) 54.3, b) 56.4, c) 12.3, d) 21.4, e) 89.4, f) 52.6, g) 98.2, h) 12.9, i) 44.8
Mental math minute. Arrange students in a line and have them add three-digit numbers by adding hundreds, tens, and ones separately in groups of four. Give the addition problem, such as $135 + 243$. The first student in the line adds hundreds: $100 + 200 = 300$; the second adds the tens: $30 + 40 = 70$; the third adds the ones: $5 + 3 = 8$; and the fourth student finishes the addition: $300 + 70 + 8 = 378$, so $135 + 243 = 378$. The next student in the line gets a new problem. Start with problems that do not require regrouping, such as $325 + 634$, and continue to questions that require regrouping ones or regrouping tens, but not both.

Regrouping with decimals. Explain that students already have a lot of experience with regrouping. Regrouping with decimals works exactly the same way as all the regrouping they have already done. SAY: Earlier, we regrouped 10 ones to make 1 ten, 10 tens to make 1 hundred, and 10 hundreds to make 1 thousand. Now we are going to regroup 10 tenths blocks to make 1 ones block. Work through the first question in the exercise below as a class.

**Exercises:** Use base ten blocks to regroup so that each place value has a single digit.

a) 3 ones + 12 tenths  
   b) 7 ones + 18 tenths  
   c) 8 ones + 15 tenths

**Answers:** a) 4 ones + 2 tenths, b) 8 ones + 8 tenths, c) 9 ones + 5 tenths

SAY: You can regroup tens, ones, and tenths in the same way.

**Exercises:** Regroup so that each place value has a single digit. You may need to regroup twice.

a) 7 tens + 5 ones + 16 tenths  
   b) 3 tens + 11 ones + 18 tenths  
   c) 5 tens + 10 ones + 14 tenths  
   d) 4 tens + 12 ones + 13 tenths
e) 7 tens + 21 ones + 16 tenths

**Answers:** a) 7 tens + 6 ones + 6 tenths, b) 4 tens + 2 ones + 8 tenths, c) 6 tens + 1 ones + 4 tenths, d) 5 tens + 3 ones + 3 tenths, e) 9 tens + 2 ones + 6 tenths

**Review aligning decimal places.** Explain that you can add decimals the same way you have always done addition—but now you’re lining up the decimal points and adding tenths. Draw on the board:

```
3.6 + 5.1

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>
```

Point to the decimal points and SAY: I made sure to line up the decimal points. ASK: What is 3.6 + 5.1? (8.7) Write the answer in the table. ASK: Was there any regrouping? (no) Repeat for 6.2 + 1.7, but this time misalign the numbers so the 7 is under the 6:

```
6.2 + 1.7

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>+</td>
<td></td>
</tr>
<tr>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>
```

Invite a volunteer to do the addition. If no one says anything before the student begins, ASK: Are the numbers lined up correctly? (no) Ask where the numbers should be, rewrite them, and then have the volunteer do the addition.

**Exercises:** Add by lining up the decimals on grid paper.

- a) 3.4 + 1.5
- b) 4.6 + 2.1
- c) 8.5 + 1.2
  - **Bonus:** 134.3 + 245.5

**Answers:** a) 4.9, b) 6.7, c) 9.7, Bonus: 379.8

**Adding decimals with regrouping, with the same number of digits to the right of the decimal point.** Work through the first two questions together. Then have students work individually to add the numbers. Remind students to align the place values.
Exercises: Add.

a) 23.5  
   + 1.4  
   24.9

b) 2.7  
   + 3.5  
   6.2

c) 192.8  
   + 15.4  
   208.2

d) 4.1  
   + 1.2  
   5.3

e) 15.4  
   + 1.6  
   17.0

Answers: a) 24.9, b) 6.2, c) 208.2, d) 5.3, e) 17.0

Adding whole numbers and decimals. Write on the board:

32 + 4.7

ASK: How can you line up the decimal points when 32 has no decimal point? PROMPT: Where should the decimal point go in 32? (after the 2) SAY: You can look at 32 as 32.0, or 32 and 0 tenths. A whole number is always understood to have a decimal point immediately to the right of the ones digit even if it is not shown. Now you can line up the decimal points and add. Have a volunteer do so on the board:

32.0 + 4.7  
36.7

Exercises: Add.

a) 4 + 13.7  
   b) 16 + 2.3  
   c) 38 + 14.7

Answers: a) 17.7, b) 18.3, c) 52.7

Word problems practice.

Exercises

1. Josh placed a table that is 1.2 m long along a wall that is 3 m long. If his bed is 2.1 m long, will it fit along the same wall? Explain.

   Answer: no; 1.2 m + 2.1 m = 3.3 m

2. Stephen solves one physics problem in 36 seconds and another in 17.9 seconds. How long does it take Stephen altogether to solve the problems?

   Answer: 53.9 seconds

Subtracting decimals. Explain that subtraction with decimals works in a similar way to addition with decimals. SAY: you need to line up the decimal points, and then perform the operation as you did earlier. Show an example without regrouping, such as 73.4 – 1.3 = 72.1.

Exercises: Subtract using grid paper.

a) 7.4 – 2.1  
   b) 6.9 – 4.5  
   c) 8.5 – 3.4  
   d) 6.5 – 3.2

Answers: a) 5.3, b) 2.4, c) 5.1, d) 3.3
Subtracting decimals with regrouping. Subtract with an example that requires regrouping, such as 53.2 – 6.7 = 46.5. Then have students practise individually.

Exercises: Subtract.

a) 7.1 – 4.4   b) 3.7 – 2.9   c) 34.8 – 5.6

d) 6.432 – 2.941  e) 1.0 – 0.5   f) 10.1 – 2.4

g) 15.7 – 2.8   h) 2.8 – 0.9

Answers: a) 2.7, b) 0.8, c) 29.2, d) 3.5, e) 0.5, f) 7.7, g) 12.9, h) 1.9

Subtracting decimals with different numbers of digits to the right of the decimal point. Present the following example: 3 – 1.4. Invite a volunteer to write it in a place value chart. ASK: Do both numbers have a tenths digit? (no) How do I subtract the tenths when one of the numbers appears not to have a tenths digit? If no one answers, ask how they dealt with this in addition. (add .0 after the 3) Repeat with 14.6 – 8.

Now have students write the subtraction 7 – 2.3 in the place value chart. ASK: Should I start by subtracting the ones or the tenths? (tenths) How do I subtract the tenths when one of the numbers doesn’t have a tenths digit? (write .0 after that number) Write the subtraction in vertical form with 7.0 as the top number and 2.3 as the bottom number. ASK: Will we need to regroup? (yes) Why? (you need a number other than zero to subtract) SAY: You can write “.0” after the ones digit for addition and for subtraction.

Exercises: Subtract.

a) 34 – 0.6   b) 6.4 – 2   c) 21.7 – 9

d) 20 – 5.9   e) 60 – 0.5   f) 100 – 92.4

Answers: a) 33.4, b) 4.4, c) 12.7, d) 14.1, e) 59.5, f) 7.6

Students can check their answers using addition.

Word problems with decimals. Solve the first exercise below as a class, and then have students work on the other one individually.

Exercises

a) Jennifer made a 0.8 L milkshake by adding ice cream to 0.6 L of milk. How much ice cream did she add?

b) Lynn cut a piece of wood board to make a shelf that is 0.7 m long. The leftover piece of board is 1.2 m long. How long was the board before she cut the wood for the shelf?

Answers: a) 0.2 L milk added (0.8 L – 0.6 L = 0.2 L), b) 1.9 m
Extensions

1. a) Add mentally.
   
   i) $2.6 + 3.4$  
   ii) $0.8 + 19.2$  
   iii) $5.7 + 5.3$

   b) Add the two numbers that are easiest to add first. Then find the total.
   
   $4.7 + 7.9 + 5.3$

   c) Would you use pencil and paper to add the numbers or would you add mentally?
   
   i) $3.5 + 4.5$  
   ii) $43.6 + 82.7$  
   iii) $7.6 + 2.4$

   **Answers:** a) i) 6, ii) 20, iii) 11; b) $4.7 + 5.3 = 10$ and $10 + 7.9 = 17.9$; c) i) mentally, ii) paper and pencil, iii) mentally

2. Make up two decimals that add to 4.5. Check your answer by adding them.

3. Continue the pattern:
   
   a) 0.1, 0.4, 0.7, _____, _____
   
   b) 3.2, 3.5, 3.8, _____, _____
   
   c) 50.7, 51.2, 51.7, _____, _____

   **Answers:** a) 1.0, 1.3, b) 4.1, 4.4, c) 52.2, 52.7

4. Continue the pattern:
   
   a) 17.3, 16.8, 16.3, _____, _____
   
   b) 3.9, 3.5, 3.1, _____, _____
   
   c) 60.9, 60.6, 60.3, _____, _____

   **Answers:** a) 15.8, 15.3; b) 2.7, 2.3; c) 60.0, 59.7

5. Kim’s house and Jasmin’s house are 11 km apart. Kim started walking toward Jasmin’s house. Kim walked 4.9 km in the first hour and 4.4 km in the second hour. How many kilometres did Kim walk in 2 hours? What distance does she still have to go to get to Jasmin’s house?

   **Answers:** 9.3 km and 1.7 km
Goals
Students will round decimals to the nearest one.
Students will estimate sums and differences by rounding the numbers being added or subtracted to the nearest one.

PRIOR KNOWLEDGE REQUIRED
Can round whole numbers to any place value, including regrouping
Can add and subtract with regrouping to tenths

Mental math minute. Arrange students in a line and have them add decimal tenths by adding ones and adding tenths. For each addition problem, such as 3.5 + 4.4, students need to say three steps: adding the ones: 3.0 + 4.0 = 7.0; adding the tenths: 0.5 + 0.4 = 0.9; and finishing the addition: 7.0 + 0.9 = 7.9, so 3.5 + 4.4 = 7.9. The next student in line gets a new problem. Start with problems that do not require regrouping, such as 2.5 + 3.4, and continue to problems that require regrouping of tenths, such as 4.2 + 3.9.

Review rounding to the nearest 10 or 100. On the board, draw a number line that goes from 10 to 20 by ones. Put an X on 13. SAY: We’re going to round to the nearest ten. ASK: Which ten is 13 closer to, 10 or 20? (10) Repeat, including rounding 15. Draw a number line that goes from 100 to 200 by tens. Explain that, this time, you are going to round to the nearest hundred. Mark points on various tens on the number line and ask students which hundred they would round to, 100 or 200.

Introduce rounding decimals. Draw a number line on the board from 1.0 to 3.0, with 1.0, 2.0, and 3.0 a different colour than the other numbers, as shown below:

```
1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 | 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 | 3.0
```

SAY: In the past, we have rounded numbers to the nearest ten or hundred. Now, we’re going to round to the nearest one. Circle the numbers 1.3, 1.8, 2.1, and 2.6, one at a time, and ask volunteers to draw an arrow showing which number (1.0, 2.0, or 3.0) the circled number is closest to. Tell students that sometimes people think of a number in terms of the closest whole number because they don’t need to be precise at that time and whole numbers are easier to work with. SAY: This is called rounding to the nearest one. Leave the number line on the board for reference.

Exercises: Round to the nearest one.

a) 1.4  
**Answers:** a) 1.0, b) 2.0, c) 3.0, d) 2.0
Write "3.7" on the board. SAY: 3.7 is between 3.0 and 4.0. Is 3.7 closer to 3.0 or to 4.0? (4.0) Repeat with 9.4. (9.4 is between 9.0 and 10.0; it is closer to 9.0)

**Exercises:** Round to the nearest one.

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</thead>
<tbody>
<tr>
<td>a)</td>
<td>9.7</td>
<td>b)</td>
<td>3.4</td>
<td>c)</td>
</tr>
<tr>
<td>e)</td>
<td>8.1</td>
<td>f)</td>
<td>6.9</td>
<td>g)</td>
</tr>
</tbody>
</table>

**Answers:** a) 10.0, b) 3.0, c) 5.0, d) 7.0, e) 8.0, f) 7.0, g) 0, h) 1.0

**Rounding tenths to the nearest one.** Make a table with two headings: “Closer to 3.0” and “Closer to 4.0.” Name several decimals (3.4, 3.6, 3.1, 3.8, 3.9, 3.3, 3.2, 3.5, 3.7) and ask students to signal whether the decimals are closer to 3 (thumbs down) or to 4 (thumbs up). Place the decimals in their correct table column as students answer.

ASK: What digit are you looking at to decide? (the tenths digit) SAY: When the tenths digit is 0, 1, 2, 3, or 4, round down. When the tenths digit is 5, 6, 7, 8, or 9, round up.

**Exercises:** What is the nearest one?

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<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>4.5</td>
<td>b)</td>
<td>6.6</td>
<td>c)</td>
</tr>
</tbody>
</table>

**Answers:** a) 5, b) 7, c) 10, d) 48, e) 90

**Estimations in calculations.** Show students how to estimate 5.2 + 3.4 by rounding each number to the nearest one: 5 + 3 = 8. SAY: Since 5.2 is close to 5 and 3.4 is close to 3, 5.2 + 3.4 will be close to, or approximately, 5 + 3. Mathematicians have invented a sign to mean "approximately equal to." It's a squiggly equal sign: "≈." So, we can write 5.2 + 3.4 ≈ 8. It would not be right to put 5.2 + 3.4 = 8 because they are not actually equal; they are just close to, or approximately, equal.

Tell students that when they round up or down before adding, they aren’t finding the exact answer, they are just estimating. They are finding an answer that is close to the exact answer. ASK: When do you think it might be useful to estimate answers? (Sample answer: in a grocery store, estimating total price or change expected)

Have students estimate the sums in the exercises below by rounding each to the nearest one. Remind them to use the approximately equal to sign.

**Exercises:**

<p>| | | | |</p>
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<thead>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>4.1 + 3.8</td>
<td>b)</td>
<td>5.2 + 1.1</td>
</tr>
<tr>
<td>d)</td>
<td>8.4 + 1.3</td>
<td>e)</td>
<td>9.2 + 3.7</td>
</tr>
</tbody>
</table>

9.3 − 2.1 ≈ 9 − 2

9.3 − 2.1 ≈ 9 − 2

Then ASK: How would you estimate 9.3 − 2.1? Write the estimated difference on the board (see margin).

Have students estimate the differences by again rounding each number to the nearest one.
Exercises

a) \(5.3 - 2.1\)  
b) \(7.2 - 2.9\)  
c) \(6.8 - 5.3\)  
d) \(4.8 - 1.7\)  
e) \(6.3 - 1.2\)  
f) \(7.4 - 3.7\)

Answers: a) 3, b) 4, c) 2, d) 3, e) 5, f) 3

Exercises: Calculate both the actual sums and the rounded sums. Circle the larger sum.

a) \(3.2 \ + \ 4.1\)  
b) \(2.3 \ + \ 4\)  
c) \(4.2 \ + \ 7.3\)  

Answers: a) 7.3, 7, b) 8.7, 8, c) 11.5, 11. The actual sum should be circled in all cases.

ASK: Which sum is larger, the actual sum or the rounded sum? (always the actual sum) Why was the actual sum always larger? (because the rounded numbers were smaller than the actual numbers; we always rounded down)

Exercises: Calculate both the actual sums and the rounded sums. Circle the larger sum.

a) \(3.6 \ + \ 4.8\)  
b) \(2.9 \ + \ 8.6\)  
c) \(3.7 \ + \ 5.6\)  

Answers: a) 8.4, 9; b) 11.5, 12; c) 9.3, 10. The rounded sum should be circled in all cases.

Extensions

1. Round 628.3 to the nearest one. Then round it to the nearest ten. Finally, round it to the nearest hundred.

Answers: 628.0, 630, 600

2. Decide what place value it makes sense to round each of the following numbers to. Round to the place value you selected. Justify your decisions. NOTE: students should remember that they made similar decisions in Lesson NS4-44: Division Word Problems, when they interpreted remainders.

   Height of person: 1.5 m  
   Height of tree: 13.1 m  
   Length of bug: 1.2 cm  
   Distance between Iqaluit and Hong Kong: 10 445.7 km  
   Floor area of an apartment: 27.9 square metres  
   Area of Prince Edward Island: 405 212.9 km²  
   Time it takes to blink: 0.3 seconds  
   Speed of a car: 96.5 km/h
**Answers**: Answers will vary. The larger the number, the less important the smaller place values become. The use to which the measurement will be put is also a factor. For example, the time it takes to ski a downhill course would be noted with greater precision to determine a world record than to keep training records.

3. Estimate $42.7 + 51.6$ by rounding both numbers to the nearest ten. Is your estimate higher or lower than the actual answer?

**Answer**: estimate is lower

4. Have students investigate when rounding one number up and one number down is better than rounding each to the nearest ten by completing the following chart and circling the estimate that is closest to the actual answer:

<table>
<thead>
<tr>
<th></th>
<th>a) 76.3</th>
<th>b) 79.6</th>
<th>c) 64.8</th>
<th>d) 60.2</th>
<th>e) 32.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>75.1</td>
<td>38.9</td>
<td>63.9</td>
<td>31.2</td>
<td>73.6</td>
</tr>
<tr>
<td>Actual Answer</td>
<td>151.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round to the Nearest Ten</td>
<td>80 + 80 = 160</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round One Up and Round One Down</td>
<td>70 + 80 = 150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers**: b) 118.5, 120, 110; c) 128.7, 120, 130; d) 91.4, 90, 100; e) 106.5, 100, 110
Goals
Students will express a fraction with the denominator 10 as an equivalent fraction with the denominator 100.

PRIOR KNOWLEDGE REQUIRED
Knows how many pennies/cents or dimes it takes to make one dollar
Knows that 10 pennies/cents are worth one dime
Knows that different fractions can be equivalent
Can name fractions from area models and number lines

MATERIALS
a penny
overhead projector
transparency of grid paper or BLM 1 cm Grid Paper (p. S-2)
BLM Squares Divided into Hundredths (p. M-55)

Mental math minute—number talk. Present this problem: 2.5 + 3.7.
The following strategies could arise:

\[
\begin{align*}
(2.0 + 3.0) + (0.5 + 0.7) &= 5.0 + 1.2 = 6.2 \\
2 + 0.5 + 3 + 0.5 + 0.2 &= 3 + 2 + 0.5 + 0.5 + 0.2 \\
2.5 + 3.7 + 0.3 - 0.3 &= 2.5 + 4.0 - 0.3 \\
2.5 + 0.5 - 0.5 + 3.7 &= 3.0 + 3.7 - 0.5
\end{align*}
\]

Pennies and cents. Hold up a penny and ask if anyone knows what it is. (some students may know) ASK: Do we use these? (no) SAY: This is a penny coin. A penny has a value of one cent, so we can also call it a cent. Even though we don’t use penny coins anymore, the prices of things we buy still have the number of cents, so we need to know how to do the calculations for cents.

Introduce dimes and cent as fractions of a dollar. ASK: How many dimes is a dollar worth? (10) What fraction of a dollar is a dime? (one tenth) How many cents is a dollar worth? (100) What fraction of a dollar is a cent? (one hundredth) Write on the board:

\[
\begin{align*}
\text{A dime is } \frac{1}{10} \text{ of a dollar.}
\end{align*}
\]

\[
\begin{align*}
\text{A cent is } \frac{1}{100} \text{ of a dollar.}
\end{align*}
\]

ASK: What fraction of a dollar is 2 dimes? (2/10) What fraction of a dollar is 3 cents? (3/100)
Exercises: What fraction of a dollar is the amount?

a) 5 dimes  

b) 5 cents  

c) 8 dimes  

d) 17 cents

Answers: a) 5/10, b) 5/100, c) 8/10, d) 17/100

Using dimes and cents to make equivalent fractions. ASK: How many cents are 3 dimes worth? (30) Write on the board:

3 dimes = 30 cents

Ask a volunteer to write the fraction of a dollar each amount shows.

3 dimes = 30 cents

\[
\frac{3}{10} = \frac{30}{100}
\]

Tell students that 3/10 and 30/100 are equivalent fractions because 3 dimes and 30 cents are the same amount.

Exercises: Write the equivalent amount, then write a fraction equation.

a) 4 dimes = _____ cents  

b) 7 dimes = _____ cents

Answers: a) 40, 4/10 = 40/100; b) 70, 7/10 = 70/100

Using pictures to make equivalent fractions. Project a sheet of grid paper or BLM 1 cm Grid Paper onto the board and draw a hundreds square or use BLM Squares Divided into Hundredths. SAY: I want to shade 1/10 of the square. ASK: What is an easy way to do that? (shade a row or a column) ASK: How would I show 4 tenths shaded? (shade 4 columns) Shade 4 columns, then ASK: How many hundredths are shaded? (40) Demonstrate counting by tens to count the hundredths: ten, twenty, thirty, forty hundredths. Write on the board:

\[
\frac{4}{10} = \frac{40}{100}
\]

Exercises: Write two equivalent fractions for the picture.

a) 

b) 

c) 

d) 

Answers: a) 3/10 = 30/100, b) 7/10 = 70/100, c) 6/10 = 60/100, d) 2/10 = 20/100

Using pictures to compare tenths to hundredths. Draw on the board:
Have students name the fraction shaded in both squares. (33/100 and 7/10 or 70/100) Demonstrate how they can count by tens, then by ones, to count the number of hundredths that are shaded in the first square.

ASK: Which is more, 33 hundredths or 7 tenths? (7 tenths) How can you tell by the picture? (more is shaded) Which is worth more, 33 cents or 7 dimes? (7 dimes) SAY: 7 tenths of a dollar is worth more than 33 hundredths of a dollar because 7 tenths is a greater fraction of anything than 33 hundredths is.

Provide students with BLM Squares Divided into Hundredths.

**Exercises:** Shade and label the fractions. Then compare them.

a) \(\frac{24}{100}\), \(\frac{5}{10}\)  
b) \(\frac{6}{100}\), \(\frac{9}{10}\)  
c) \(\frac{54}{100}\), \(\frac{3}{10}\)

**Answers:** a) \(\frac{24}{100} < \frac{5}{10}\), b) \(\frac{6}{100} < \frac{9}{10}\), c) \(\frac{54}{100} > \frac{3}{10}\)

**Extensions**

1. Use squares from BLM Squares Divided into Hundredths to show that

\[
\frac{1}{4} \text{ is equal to } \frac{25}{100}, \quad \frac{2}{4} \text{ is equal to } \frac{50}{100}, \quad \text{and } \frac{3}{4} \text{ is equal to } \frac{75}{100}.
\]

What fraction of a dollar do you have if you have 3 quarters?

2 quarters? 4 quarters? 1 quarter?

**Answers:** \(\frac{75}{100}\), \(\frac{50}{100}\), \(\frac{100}{100}\), \(\frac{25}{100}\)

2. Use squares from BLM Squares Divided into Hundredths to show that

\[
\frac{25}{100} \text{ does not have an equivalent fraction with denominator 10.}
\]

Hint: You will need to divide 1 hundredth square into 10 tenths.
Goals

Students will use decimal notation for fractions with denominators 10 and 100.

Students will place decimal hundredths on number lines.

Students will order decimal hundredths using a number line.

Students will compare and order decimal tenths and hundredths.

PRIOR KNOWLEDGE REQUIRED

Knows that, on number lines, greater whole numbers appear to the right of lesser whole numbers

Can name fractions from area models and number lines

Can use number lines to order whole numbers

MATERIALS

overhead projector

transparency of grid paper or BLM 1 cm Grid Paper (p. S-2)

BLM Squares Divided into Hundredths (p. M-55)

Mental math minute. Ask students to solve multiplication questions within the range of $3 \times 3$ to $10 \times 10$ and corresponding division questions. For each number, go through the questions in order, such as $3 \times 3$, $9 \div 3$, $4 \times 3$, $12 \div 3$, and so on to $10 \times 3$ and $30 \div 3$. Then progress to a different number. Next try questions out of order, but keep corresponding multiplication and division together.

Introduce decimal hundredths. Tell students that the fraction $\frac{1}{100}$ can also be represented in various ways. Show four ways on the board:

$$\frac{1}{100} \quad \text{one hundredth} \quad 0.01$$

Point out how one hundredth is written differently from one tenth—there are two digits after the decimal point instead of only one. Ask a volunteer to show how she would write two hundredths as a decimal (0.02), then read it as “zero point zero two.” ASK: How would you write three hundredths as a decimal? (0.03)

Exercises: Write the fraction as a decimal.

a) $\frac{9}{100}$

b) $\frac{4}{100}$

c) $\frac{8}{100}$

d) $\frac{7}{100}$

e) $\frac{5}{100}$

Answers: a) 0.09, b) 0.04, c) 0.08, d) 0.07, e) 0.05
Writing fractions with denominator 100 as decimal hundredths. Write on the board:

\[
\frac{83}{100} = 0.83 \quad \frac{49}{100} = 0.49 \quad \frac{60}{100} = 0.60
\]

SAY: To write hundredths as decimals, you have to use two places after the decimal point. If there are more than 9 hundredths, you can write the number of hundredths right after the decimal point. Ask volunteers to show how to write various hundredths (28 hundredths, 4 hundredths, 70 hundredths) as decimals (0.28, 0.04, 0.70). Remind volunteers to put in any missing 0s.

**Exercises:** Write the hundredths as a decimal.

a) 81 hundredths  
b) 30 hundredths  
c) 6 hundredths  
d) \( \frac{9}{100} \)  
e) \( \frac{74}{100} \)  
f) \( \frac{50}{100} \)

**Answers:** a) 0.81, b) 0.30, c) 0.06, d) 0.09, e) 0.74, f) 0.50

Introduce equivalent tenths and hundredths as fractions and decimals. Draw on the board:

\[
\frac{3}{10} = \frac{30}{100}
\]

ASK: How many tenths are shaded? (3) Fill in the first numerator. SAY Each column is one tenth, and three of them are shaded. ASK: How many tenths are shaded? (3) How many hundredths are shaded? (30) PROMPT: How many hundredths are in each column? (10) So there are 10, 20, 30 hundredths shaded. Fill in the second numerator. ASK: How would you write 3 tenths as a decimal? (0.3) How would you write 30 hundredths as a decimal? (0.30) Write on the board:

\[0.3 = 0.30\]

SAY: These are equivalent decimals.

**Exercises:** Write two equivalent fractions and two equivalent decimals for the amount shaded.

a)  

\[
\frac{5}{10} = \frac{50}{100} = 0.5 = 0.50 \]

b)  

\[
\frac{2}{10} = \frac{20}{100} = 0.2 = 0.20\]

c)  

\[
\frac{7}{10} = \frac{70}{100} = 0.7 = 0.70\]

d)  

\[
\frac{4}{10} = \frac{40}{100} = 0.4 = 0.40\]

**Answers:** a) 5/10 = 50/100 = 0.5 = 0.50, b) 2/10 = 20/100 = 0.2 = 0.20, c) 7/10 = 70/100 = 0.7 = 0.70, d) 4/10 = 40/100 = 0.4 = 0.40
Reading decimals. Although it is correct to read 0.7 as “zero point seven,” it is not correct to read 0.70 as “zero point seventy.” Each digit after the decimal point should be read separately, so 0.70 becomes “zero point seven zero.” Always be sure to correct students who read 0.70 as “zero point seventy.” Students who are allowed to do so are more likely to incorrectly believe that 0.70 is greater than 0.8, since 70 > 8. Ask volunteers to use this way to read various decimals: 0.9, 0.09, 0.90, 0.13, 0.31, 0.03.

Ordering decimals using hundreds squares. Write on the board:

0.8  0.12

Have volunteers read the numbers aloud. Then project grid paper or BLM 1 cm Grid Paper onto the board (or BLM Squares Divided into Hundredths) and draw two blank hundreds squares on the board. Ask volunteers to shade each decimal above. ASK: Which decimal is larger? (0.8) PROMPT: Which square has more shaded?

Provide students with BLM Squares Divided into Hundredths.

Exercises: Shade and label the decimals. Then compare them.

a) 0.4, 0.30  b) 0.08, 0.7  c) 0.36, 0.4

Answers: a) 0.4 > 0.30, b) 0.08 < 0.7, c) 0.36 < 0.4

Practising word problems. Students might use BLM Squares Divided into Hundredths to help them solve the following word problems.

Exercises

a) Arsham lives 0.93 km from the park and 0.72 km from the community centre. Does he live closer to the park or to the community centre?

b) Zara walked 0.65 km before lunch and 0.85 km after lunch. When did she walk farther?

c) Jen lives 0.41 km from her best friend’s house and 0.83 km from her aunt’s house. Who does she live closer to?

Answers: a) community centre, b) after lunch, c) best friend

Extensions

1. Write the fraction as tenths, then as a decimal.

a) \( \frac{1}{2} \)  b) \( \frac{1}{5} \)  c) \( \frac{2}{5} \)  d) \( \frac{3}{5} \)  e) \( \frac{4}{5} \)

Answers: a) 1/2 = 5/10 = 0.5, b) 1/5 = 2/10 = 0.2, c) 2/5 = 4/10 = 0.4, d) 3/5 = 6/10 = 0.6, e) 4/5 = 8/10 = 0.8

2. Write the missing number.

a) \( \frac{7}{10} \) = 0.7  b) \( \frac{9}{100} \) = 0.09  c) \( \frac{8}{10} \)  d) \( \frac{71}{100} \) = 0.71

Answers: a) 10, b) 100, c) 8, d) 71
3. a) Shade $\frac{1}{4}$ of the squares.

![Diagram showing shading of squares]

b) Write the fraction as hundredths and then a decimal for

i) $\frac{1}{4} = \frac{25}{100} = 0.25$

ii) $\frac{3}{4} = \frac{75}{100} = 0.75$

**Answers:** a) Teacher to check shading, b) $\frac{1}{4} = \frac{25}{100}$, $\frac{3}{4} = \frac{75}{100}$

4. Predict how to write $\frac{1}{1000}$, $\frac{1}{10000}$, and $\frac{1}{1\,000\,000\,000}$ as a decimal.

**Answers:** $0.001$, $0.0001$, $0.000000001$

(Students might put commas after every 3 place values in accordance with what is done before the decimal point. This is not the convention, but it is not incorrect thinking.)

5. Make up a word problem that requires comparing $0.4$ to $0.26$. Ask a partner to solve your problem.

6. Represent $\frac{6}{10}$ in as many ways as you can.

**Sample answers:** $0.6$, $\frac{60}{100}$, $0.60$, six tenths, three fifths, sixty hundredths. Students may also write “zero point six” and “zero point six zero.” These answers are correct as ways of orally representing these numbers but should not be encouraged as a way of writing the numbers. Students can also draw pictures that show the amount shaded.

7. Explain how you know that $0.7 = \frac{7}{10} = \frac{70}{100} = 0.70$

**Answer:** $0.7 = \frac{7}{10} = \frac{70}{100} = 0.70$
Goals
Students will express hundredths in terms of tenths and hundredths.

PRIOR KNOWLEDGE REQUIRED
Can write equivalent tenths and hundredths as fractions
Can write equivalent tenths and hundredths as decimals
Can write the value of dimes and cents as fractions of a dollar
Can name fractions from area models and number lines

MATERIALS
BLM Number Lines Divided into Hundredths (p. M-56)
grid paper or BLM 1 cm Grid Paper (p. S-2)

Mental math minute. Arrange students in a line and have them add three-digit numbers by adding hundreds, tens, and ones separately in groups of four. Give students an addition problem, such as 254 + 643. The first student in the line adds hundreds: 200 + 600 = 800; the second adds the tens: 50 + 40 = 90; the third adds the ones: 4 + 3 = 7; and the fourth student finishes the addition: 800 + 90 + 7 = 897, so 254 + 643 = 897. The next student in the line gets a new problem. Start with problems that do not require regrouping, such as 325 + 634, and continue to questions that require regrouping ones or regrouping tens, but not both.

Using a picture to show a combination of tenths and hundredths. Draw the first picture below on the board:

ASK: How many hundredths are shaded? (30) How many tenths are shaded? (3) Then shade two more hundredths. ASK: Now how many hundredths are shaded? (32) Summarize by saying that 32 hundredths = 3 tenths and 2 more hundredths. Write on the board:

32 hundredths = 3 tenths 2 hundredths

Exercises: Describe the fraction shaded as hundredths and as tenths and hundredths.

a) b) c) d)
Answers: a) 64 hundredths = 6 tenths 4 hundredths, b) 47 hundredths = 4 tenths 7 hundredths, Bonus: c) 85 hundredths = 8 tenths 5 hundredths, d) 86 hundredths = 8 tenths 6 hundredths

Relating tenths and hundredths to money. Remind students that a dime is one tenth of a dollar and a cent is one hundredth of a dollar. SAY: “Zero point seven three dollars” can be represented in different ways:

7 dimes 3 cents
7 tenths 3 hundredths
73 cents
73 hundredths

Exercises: Write the amount in three more ways.
a) 8 dimes 5 cents b) 0 dimes 6 cents c) 9 dimes 0 cents
Answers: a) 8 tenths 5 hundredths, 85 cents, 85 hundredths; b) 0 tenths 6 hundredths, 6 cents, 6 hundredths; c) 9 tenths 0 hundredths, 90 cents, 90 hundredths

Relating tenths and hundredths to place value. Tell students that just like there is a ones place and a tens place in whole numbers, there is a tenths place and a hundredths place in decimals. Draw on the board:

68 hundredths = 6 tenths 8 hundredths

\[
\frac{68}{100} = 0.68
\]

SAY: The first digit to the right of the decimal point is the number of tenths, and the second digit is the number of hundredths. Write on the board:

\[
\frac{100}{100} \ \ \text{tenths} \ \ \ \text{hundredths} \ \ \ \ 0. \ \ \ \ \text{______}
\]

Exercises: Describe the hundredths using the three ways shown above (written on the board).
a) 54 hundredths b) 8 hundredths c) 37 hundredths
Answers: a) \(\frac{54}{100}, 5\ \text{tenths} 4\ \text{hundredths}\), 0.54; b) \(\frac{8}{100}, 0\ \text{tenths} 8\ \text{hundredths}\), 0.08; c) \(\frac{37}{100}, 3\ \text{tenths} 7\ \text{hundredths}\), 0.37

Expanded form. Explain that 23.85 is just a short way of writing \(20 + 3 + 0.8 + 0.05\). SAY: The 2 has a value of 20, the 3 has a value of 3, the 8 has a value of 8/10, and the 5 has a value of 5/100. ASK: What is the expanded form of 4.92? 38.06? 206.57? 9601.82? Write out the expanded form for each number on the board.

Exercises: Write the number in expanded form.
a) 4.03 b) 16.92 c) 354.81 d) 9052.63
Answers: a) \(4 + 0.0 + 0.03\), b) \(10 + 6 + 0.9 + 0.02\), c) \(300 + 50 + 4 + 0.8 + 0.01\), d) \(9000 + 50 + 2 + 0.6 + 0.03\)
Relating tenths and hundredths to number lines. Project onto the board BLM Number Lines Divided into Hundredths. Label the tenths as shown below:

<table>
<thead>
<tr>
<th>0.00</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
<th>1.00</th>
</tr>
</thead>
</table>

Demonstrate counting 4 tenths, then 3 more hundredths (demonstrate doing so), then demonstrate counting 43 hundredths (count by ten hundredths until 40, then one hundredths until 43). Mark 0.43 on the number line.

Exercises: Write the fraction of the distance from 0.00 to 1.00 as hundredths and as tenths and hundredths.

<table>
<thead>
<tr>
<th>0.00</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 0.09 = 0 tenths 9 hundredths</td>
<td>b) 0.28 = 2 tenths 8 hundredths</td>
<td>c) 0.52 = 5 tenths 2 hundredths</td>
<td>d) 0.70 = 7 tenths 0 hundredths</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercises: Write the positions on the number line marked with an “X” as decimal hundredths and as numbers of ones, tenths, and hundredths.

<table>
<thead>
<tr>
<th>1.00</th>
<th>1.10</th>
<th>1.20</th>
<th>1.30</th>
<th>1.40</th>
<th>1.50</th>
<th>1.60</th>
<th>1.70</th>
<th>1.80</th>
<th>1.90</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 1.07 = 1 one 0 tenths 7 hundredths</td>
<td>b) 1.29 = 1 one 2 tenths 9 hundredths</td>
<td>c) 1.52 = 1 one 5 tenths 2 hundredths</td>
<td>d) 1.84 = 1 one 8 tenths 4 hundredths</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimating decimals on number lines. Draw on the board:

<table>
<thead>
<tr>
<th>0.00</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
<th>0.70</th>
<th>0.80</th>
<th>0.90</th>
<th>1.00</th>
</tr>
</thead>
</table>

Tell students you want to know where to place 0.61. ASK: How many tenths are in 61 hundredths? (6) Is 0.61 closer to 6 tenths or 7 tenths? (6 tenths) SAY: The decimal 0.61 is only one more hundredth than 0.6, but 0.7 is ten more hundredths than 0.6. Ask a volunteer to mark where he estimates 0.61 will be on the number line. Repeat for 0.48 (closer to 0.5 than to 0.4) and 0.95 (equally close to 0.9 and 1).

Students can use grid paper or BLM 1 cm Grid Paper to draw the number line.

Exercises: Estimate the location of the decimal on a number line divided into tenths from 0.00 to 1.00.

<table>
<thead>
<tr>
<th>a) 0.75</th>
<th>b) 0.37</th>
<th>c) 0.29</th>
<th>d) 0.94</th>
</tr>
</thead>
</table>
A centimetre is one hundredth of a metre. ASK: How many centimetres are in 1 m? (100) What fraction of a metre is a centimetre? (one hundredth)

Write on the board:

\[ \begin{align*}
1 \text{ cm} & = 0.01 \text{ m} \\
5 \text{ cm} & = \_\_\_ \_ \text{ m} \\
17 \text{ cm} & = \_\_\_ \_ \text{ m}
\end{align*} \]

SAY: 1 cm is one hundredth of 1 m, so 5 cm is 5 hundredths of 1 m. Have a volunteer write the decimal to show this. (0.05) ASK: What fraction of 1 m is 17 cm? (17/100) Ask a volunteer to write the decimal. (0.17)

**Exercises:** Write the decimal.

a) 37 cm = \_\_\_ m  

b) 4 cm = \_\_\_ m  

c) 90 cm = \_\_\_ m  

d) 16 cm = \_\_\_ m  

**Answers:** a) 0.37, b) 0.04, c) 0.90, d) 0.16

**Extensions**

1. What is the largest numerator you can use?

\[ \frac{34}{100} \leq \frac{34}{10} \]

**Answer:** 3

2. How are these questions related? Use number lines to explain your answer.

Is 3 closer to 0 or to 10?

Is 3 tenths closer to 0 or to 1?

Is 3 hundredths closer to 0 or to 0.1?

**Answer:** The number lines that are used to answer these questions will be identical, except for the scale, and the points will be marked at exactly the same place on each number line:

The scale will be 0 to 10, 0 to 1, or 0.0 to 0.1, but that is the only difference. In all cases, the number is closer to 0.

3. Plot the numbers on a number line. Find the number that is halfway between …

a) 0 and 1  

b) 0.3 and 0.4  

c) 0.5 and 0.54  

d) 0.1 and 0.4

**Answers:** a) 0.5, b) 0.35, c) 0.52, d) 0.25

4. Plot the fractions on a number line divided into hundredths to find a pattern. What are the next two fractions in the pattern?

\[ \frac{5}{100} \quad \frac{2}{10} \quad \frac{35}{100} \quad \frac{5}{10} \quad \frac{65}{100} \]

**Answer:** They increase by 15 hundredths; the next two fractions are 8/10 and 95/100.
**Goals**

Students will add and subtract to hundredths.

**PRIOR KNOWLEDGE REQUIRED**

Can add using the standard algorithm
Can subtract using the standard algorithm

**MATERIALS**

grid paper or BLM 1 cm Grid Paper (p. S-2)
base ten blocks

**Mental math minute.** Arrange students in a line and have them add three-digit numbers by adding hundreds, tens, and ones separately in groups of four. Give the addition problem, such as 135 + 246. The first student in the line adds hundreds: 100 + 200 = 300; the second adds the tens: 30 + 40 = 70; the third adds the ones: 5 + 6 = 11; and the fourth student finishes the addition: 300 + 70 + 11 = 381, so 135 + 246 = 381. The next student in the line gets a new problem. Start with problems that do not require regrouping, such as 325 + 634, and continue to questions that require regrouping ones or regrouping tens, but not both.

**Regrouping hundredths.** Ask: When we regrouped tenths, how many tenths were needed to make 1 one? (10) Ask: How many hundredths do you think are needed to make 1 tenth? (10) Work through the first question in the following exercise as a class.

**Exercises:** Use base ten blocks to regroup so that each place value has a single digit.

a) 5 tenths + 13 hundredths

b) 6 ones + 17 tenths

c) 5 ones + 13 tenths + 15 hundredths

d) 6 tens + 11 ones + 16 tenths + 12 hundredths

**Answers**

a) 6 tenths + 3 hundredths

b) 7 ones + 7 tenths

c) 6 ones + 4 tenths + 5 hundredths

d) 7 tens + 2 ones + 7 tenths + 2 hundredths

**Adding by lining up the decimals without regrouping.** Write on the board:

\[
\begin{align*}
5.34 + 1.25 &= 6.59 \\
5.34 &= 5.34 \\
+ 1.25 &= +1.25 \\
\end{align*}
\]
SAY: It is easier to add numbers when they are in vertical form, but it only works when we align the decimal points. Have a volunteer complete the addition on the board.

**Exercises:** Write the addition in vertical form. Add.

a) $2.63 + 7.16$  
   b) $27.54 + 31.04$  
   c) $20.13 + 59.32$

**Answers:** Teacher to check vertical alignment. a) 9.79, b) 58.58, c) 79.45

Adding by lining up the decimals with regrouping. Write on the board:

$$
6.94 + 2.37 \\
6.94 \\
+ __\_ \_ $$

ASK: To get the decimals lined up, what digit do we write under the 6? (2) Write “2.37” on the board lined up correctly under 6.94. Work through the addition as a class.

**Exercises:** Write the additions in vertical form on grid paper and then add. You may need to regroup more than once.

a) $6.72 + 1.46$  
   b) $15.89 + 71.43$  
   c) $63.47 + 24.86$

**Answers:** Teacher to check vertical alignment. a) 8.18, b) 87.32, c) 88.33

Adding decimals with regrouping, with the same number of digits to the right of the decimal point. Work through the first question of the exercises below as a class. Then have students work individually to add the numbers. Remind students to align the place values.

**Exercises:** Write the additions in vertical form on grid paper or BLM 1 cm Grid Paper and then add. You may have to regroup more than once.

a) $1.18 + 6.35$  
   b) $4.57 + 2.56$  
   c) $23.67 + 15.64$  
   d) $55.96 + 37.25$

**Answers:** a) 7.53, b) 7.13, c) 39.31, d) 93.21

Regrouping tenths to hundredths to subtract. Write on the board:

$$
1 \text{ tenth} + 4 \text{ hundredths} $$

ASK: What can I do with these if I need more hundredths? (trade the tenth for ten hundredths)

**Exercises:** Use base ten blocks to help you fill in the blanks.

a) $5 \text{ tenths} + 0 \text{ hundredths} = _____ \text{ tenths} + _____ \text{ hundredths}$

b) $4 \text{ tenths} + 6 \text{ hundredths} = _____ \text{ tenths} + _____ \text{ hundredths}$

c) $3 \text{ tenths} + 8 \text{ hundredths} = _____ \text{ tenths} + _____ \text{ hundredths}$

d) $1 \text{ tenth} + 7 \text{ hundredths} = _____ \text{ tenths} + _____ \text{ hundredths}$

**Answers:** a) 4, 10, b) 3, 16, c) 2, 18, d) 0, 17
Subtracting by lining up the decimals, without regrouping. Write on the board:

\[
\begin{array}{r}
5.36 \\
-1.25
\end{array}
\]

SAY: It is easier to subtract numbers when they are in vertical form, but it only works when we align the decimal points. Have a volunteer complete the subtraction on the board.

Exercises: Write the subtraction in vertical form. Subtract.

a) 7.16 \(-\) 2.14  

b) 38.94 \(-\) 27.53  

c) 69.88 \(-\) 48.32

Answers: a) 5.02, b) 11.41, c) 58.58

Subtracting by lining up the decimals, with regrouping. Write on the board:

\[
\begin{array}{r}
9.64 \\
-2.58
\end{array}
\]

ASK: To get the decimals lined up, what number do we write under the 9? (2) Write 2.58 under 9.64. Work through the subtraction as a class.

Exercises: Write the subtractions in vertical form on grid paper and then subtract. You may need to regroup more than once.

a) 7.82 \(-\) 1.73  

b) 16.15 \(-\) 13.57  

c) 35.21 \(-\) 24.86

Answers: a) 6.09, b) 2.58, c) 10.35

Subtracting numbers with a different number of digits to the right of the decimal point. Write “4.6 \(-\) 2.85” on the board: SAY: So far we have subtracted numbers with the same number of digits to the right of the decimal point, but all numbers are not like that. Just as with addition, we treat missing numbers as a zero. Write on the board:

\[
\begin{array}{r}
4.60 \\
-2.85
\end{array}
\]

SAY: We know 4.6 is equivalent to 4.60, so we put a zero in the empty hundredth place. Work through the subtraction on the board together as a class.

Exercises: Subtract.

a) 3.4 \(-\) 0.78  

d) 21.1 \(-\) 15.68  

g) 78.24 \(-\) 0.6

b) 5.1 \(-\) 4.83  

e) 18.05 \(-\) 9.6  

f) 26.3 \(-\) 6.31

Answers: a) 2.62, b) 0.27, c) 23.81, d) 5.42, e) 8.45, f) 19.99, g) 77.64, h) 18.19
Word problems.

Exercises: Jane has three pieces of string. The strings are 7.32 cm, 34.05 cm, and 16.92 cm long.

a) What is the total length of all the strings?

b) Jane cuts 1.42 cm from the string that is 16.92 cm long. What is the total length of all the strings now, not including the part Jane just cut off?

c) Did you need to know which string Jane cut to answer part b)? Explain.

Answers

a) 7.32 + 34.05 + 16.92 = 58.29 cm

b) 7.32 + 34.05 + 16.92 − 1.42 is the same as 58.29 − 1.42 = 56.87 cm

c) no, because the total length of all the strings will always be 1.42 cm less no matter which one got cut

Extensions

1. Extend the pattern.

   a) 2, 4, 6, 8, _____, _____

   b) 0.2, 0.4, 0.6, 0.8, _____, _____

   c) 0.02, 0.04, 0.06, 0.08, _____, _____

   Bonus: Use what you learned in parts a), b), and c) to extend the pattern.

   d) 0.002, 0.004, 0.006, 0.008, _____, _____

   e) 0.0002, 0.0004, 0.0006, 0.0008, _____, _____

   Answers: a) 10, 12; b) 1.0, 1.2; c) 0.10, 0.12; Bonus: d) 0.010, 0.012; e) 0.0010, 0.0012

2. Fill in the missing numbers.

\[
\begin{array}{cccc}
7 & 4 & \cdot & 2 \\
+ & 3 & 5 & 7 \\
\hline & 9 & 0 & 0 & 2 \\
\end{array}
\]

Answer: 764.29 + 135.73 = 900.02

3. Find five pairs of numbers that add to 37.05.

Sample answer: 36.05 + 1.00, 30.05 + 7.00, 37.00 + 0.05, 27.05 + 10.00, 17.05 + 20.00
4. Find two groups of three numbers that add to 90.67.
   **Sample answer:** 80.00 + 10.00 + 0.67, 40.00 + 50.00 + 0.67

5. Find five pairs of numbers with a difference of 18.26.
   **Sample answer:** 19.26 – 1.00, 20.26 – 2.00, 19.00 – 0.74, 28.26 – 10.00, 38.26 – 20.00

6. A squirrel runs 15.58 m straight up a tree. Then it runs 9.47 m more straight up. Then it runs another 16.25 m straight up the tree.
   a) How far up the tree is the squirrel now?
   b) If the squirrel now runs 16.25 m straight down, how far up the tree will it be?

   **Answers**
   a) 15.58 + 9.47 + 16.25 = 41.30 m
   b) 15.58 + 9.47 + 16.25 = 41.30 – 16.25 = 25.05 m

7. Show students how to subtract decimals from 1 by first subtracting from 0.99:
   \[ 1 – 0.74 = 0.01 + 0.99 – 0.74 = 0.01 + 0.25 = 0.26 \]
   Then ask students to subtract ...
   a) 1 – 0.37  b) 1 – 0.19  c) 1 – 0.08  d) 1 – 0.01

   **Answers:** a) 0.63, b) 0.81, c) 0.92, d) 0.99
**NS4-62  Dollar and Cent Notation**

Pages 62–63

**Goals**

Students will express monetary values in dollar and cent notation. Students will convert from cent to dollar notation and from dollar to cent notation.

**PRIOR KNOWLEDGE REQUIRED**

Can use the cent symbol correctly
Can use dollar notation correctly for amounts given in whole dollars
Is familiar with Canadian coins
Knows the relative values of Canadian coins

**MATERIALS**

BLM Money-Matching Memory Game (p. M-57)

**Mental math minute.** Have students stand in a line. Give the first student a problem that does not need regrouping, such as $3.7 - 0.3$. Students in line repeatedly subtract a number, in this case 0.3, with each student saying one subtraction aloud. When a student says the subtraction that involves regrouping, emphasize that this answer was a bonus. Example: Student 1 says, “$3.7 - 0.3 = 3.4$.” Student 2 says, “$3.4 - 0.3 = 3.1$.” Bonus: Student 3 says, “$3.1 - 0.3 = 2.8$.” Continue without regrouping until Student 12 says, “$0.4 - 0.3 = 0.1$.” Then start a new chain.

**Writing dimes and cents in cent notation.** Tell students to pretend that there is a vending machine that takes only dimes and cents. ASK: What would you use to buy a toy that costs 38¢? (3 dimes and 8 cents) SAY: The tens digit tells you how many dimes you need, and the ones digit tells you how many cents you need.

**Exercises:** How much money is there?

a) 2 dimes and 6 cents  
  b) 8 dimes and 9 cents  
  c) 5 dimes and 5 cents  
  d) 4 dimes and 2 cents

**Answers:** a) 26¢, b) 89¢, c) 55¢, d) 42¢

**Writing dimes and cents in dollar notation.** Ask a volunteer to write 26¢ in dollar notation. If no one knows, write the answer ($0.26) on the board. SAY: In dollar notation, the number of dimes goes right after the decimal point and the number of cents goes right after that.

**Exercises:** Write the amount in dollar notation.

a) 3 dimes and 8 cents  
  b) 4 dimes and 2 cents  
  c) 41¢  
  d) 29¢  
  e) 35¢  
  f) 81¢

**Answers:** a) $0.38, b) $0.42, c) $0.41, d) $0.29, e) $0.35, f) $0.81
Now write "8¢" on the board. ASK: How many dimes are in 8 cents? (0)
Show this on the board by writing 8¢ in dollar notation (see margin).
SAY: The 0 tells us there are no dimes, and the 8 tells us there are 8 cents.

**Exercises:** Write the amount in dollar notation.

a) 0 dimes and 6 cents  
   b) 3¢  
   c) 7¢  
   d) 5 dimes and 0 cents  
   e) 40¢  
   f) 90¢  

**Answers:** a) $0.06,  
                b) $0.03,  
                c) $0.07,  
                d) $0.50,  
                e) $0.40,  
                f) $0.90  

**Writing multiples of a hundred cents in dollar notation.** Remind students that one hundred cents can be written as one dollar. Write on the board:

\[ 100¢ = $1 \quad 200¢ = 1300¢ = $13 \]

Have volunteers fill in the blanks. ($2 and $13) SAY: Two hundred cents equals two dollars and thirteen hundred cents equals thirteen dollars.

**Exercises:** Write the amount in dollar notation.

a) 300¢  
   b) 800¢  
   c) 1000¢  
   d) 1200¢  
   e) 38 400¢

**Answers:** a) $3,  
               b) $8,  
               c) $10,  
               d) $12,  
               e) $384

**Converting cent notation to dollar notation.** Write on the board:

\[ 348¢ = 300¢ + 48¢ \quad 1746¢ = 1700¢ + 46¢ \]

Ask volunteers to write 300¢, 48¢, 1700¢, and 46¢ in dollar notation:

\[ = $3 + $0.48 \quad = $17 + $0.46 \]

Show students how to combine them into a single dollar notation:

\[ = $3.48 \quad = $17.46 \]

Point out that the decimal point is always in front of the last two digits.

**Exercises:** Write the amount in dollar notation.

a) 156¢  
   b) 704¢  
   c) 1804¢  
   **Bonus:** 9604¢

**Answers:** a) $1.56,  
               b) $7.04,  
               c) $18.04,  
               **Bonus:** $96.04

**Changing dollar notation to cent notation.** Write on the board:

\[ $13.85 = 13 \text{ dollars 85 cents} \]

ASK: How many cents are in 13 dollars? (1300) Write on the board:

\[ = 1300¢ + 85¢ \]

\[ = 1385¢ \]

**Exercises:** Write the dollar amount in cent notation.

a) $8.00  
   b) $17.00  
   c) $0.28  
   d) $0.04  
   e) $3.54  
   f) $9.03  
   g) $30.42  
   h) $10.05  

**Answers:** a) 800¢,  
               b) 1700¢,  
               c) 28¢,  
               d) 4¢,  
               e) 354¢,  
               f) 903¢,  
               g) 3042¢,  
               h) 1005¢
Making a given amount of money using the fewest possible coins and bills. **ASK:** What coins could you use to make 80¢? (8 dimes, 3 quarters and 1 nickel, 3 quarters and 5 cents, 2 quarters and 3 dimes, and so on) Allow different volunteers to give answers, including repeated answers. Now tell students that you want to make 80¢ using the fewest possible coins. **ASK:** How can I do that? (3 quarters and 1 nickel) Point out that you need to use as many quarters as you can because quarters have the greatest value of any coin that is less than 80¢, so you'll need fewer of them to make 80¢.

**Exercises:** Make the amount using the fewest coins possible.

| a) 71¢ | b) 85¢ | c) 54¢ | d) 26¢ | e) 13¢ |

**Answers:** a) 2 quarters, 2 dimes, 1 cent; b) 3 quarters, 1 dime; c) 2 quarters, 4 cents; d) 1 quarter, 1 cent; e) 1 dime, 3 cents

**ASK:** What dollar amounts do loonies and toonies come in? ($1 and $2) What dollar amounts do bills come in? (Sample answers: $5, $10, $20)

**Exercises:** Make the amount using the fewest coins and bills possible.

| a) $6 | b) $16 | c) $3.25 | d) $8.30 | e) $10.05 |

**Answers:** a) $5, $1; b) $10, $5, $1; c) $1, $1, $1, 25¢; d) $5, $2, $1, 25¢, 5¢; e) $10, 5¢

Making a given amount of money using a given number of coins. Now tell students that you want to make 80¢ using exactly 5 coins. **ASK:** How can I do that? (2 quarters and 3 dimes) Write the following problem on the board: Make $6.30 using exactly 5 coins. Tell students that a good strategy is to first find one way—any way—of making $6.30, then decide whether they have too many or too few coins. **ASK:** What one way do we already know? (find the one with the fewest coins) Have students do so: $2, $2, $2, 25¢, 5¢. Then challenge students to increase the number of coins by 1, but keep the value the same. (replace the quarter and nickel with three dimes)

**Exercises**

a) Make $2.50 using 6 coins. b) Make $1.75 using 7 coins.

**Answers:** a) $1, $1, 25¢, 10¢, 10¢, 5¢; b) $1, 25¢, 10¢, 10¢, 10¢, 10¢, 10¢

**ACTIVITY (Optional)**

Use BLM Money-Matching Memory Game to have students individually play the two following games.

**Picking Pairs.** Place 12 cards face up in a 3 × 4 array. Students take turns picking pairs of matching cards and placing them into a common discard pile. When there are no more pairs in the array, more cards are added to it. The goal is to place all the cards into the discard pile. If students have any non-matching cards left at the end, then some of their cards must have been matched incorrectly.
Memory. This version of the well-known game is played like Picking Pairs but the cards are face down. Students turn over two cards at a time looking for a match (what constitutes a match will depend on the lesson). If the cards match, students set them aside; otherwise, they turn them face down again and continue playing. Students can play individually or cooperatively in pairs. In either case, the goal is to find all the matches. If playing with a partner, Player 1 leads by choosing and turning over a card and Player 2 follows by choosing and turning over another card. Players switch roles after each turn.

Extensions

1. What coin is being used for skip counting?
   a) $1.00, _____, _____, _____, $1.20
   b) $2.00, _____, _____, _____, $3.00
   **Answers:** a) 5¢, b) 25¢

2. a) Convert $8 to cent notation.
   b) Convert 8 m to centimetres.
   c) How are parts a) and b) the same?
   **Answers:** a) 800¢, b) 800 cm, c) You do the questions the same way, by multiplying both numbers by 100.

3. a) I am less than 40¢. You need 4 coins, nickels and dimes only, to make me. What am I?
   b) I am less than 50¢. I am a multiple of 5 but not a multiple of 10. You can make me with 2 coins. What could I be?
   **Answers:** a) 35¢, b) 15¢ or 35¢

4. Without changing to cent notation, compare the money amounts. Did you need to compare the cent amounts or just the whole-dollar amounts?
   a) $3.84, $2.95
   b) $14.71, $14.36
   **Answers:** a) $3.84 is more than $2.95. Three dollars and anything is more than two dollars and anything, so you do not need to compare the cent amounts.
   b) The dollar amounts are the same, so you need to compare the cent amounts. Since 71¢ is more than 36¢, $14.71 is more than $14.36.
Goals

Students will find the change owed for up to 100 dollars.

PRIOR KNOWLEDGE REQUIRED

Can count up by 10s to subtract multiples of 10
Can use dollar and cent notation

Mental math minute. Ask students to solve multiplication questions within the range of 4 \times 4 to 10 \times 10 and corresponding division questions. For each number, go through the questions in order, such as 4 \times 4, 16 \div 4, 4 \times 5, 20 \div 4, and so on to 10 \times 4 and 40 \div 4. Then progress to a different number. Next, try questions out of order, but keep multiplication and corresponding division together. You can pass a ball to the student you want to answer the question and have students pass the ball back to you as they answer.

Finding the difference owed for up to 10 dollars when prices are in whole dollars. Write on the board:

\[
\text{Price} = \$7 \quad \text{Amount paid} = \$10
\]

SAY: An item costs $7, but I paid $10. SAY: I want to know how much change I will get back. Write "$10 - $7 = " on the board. ASK: What is $10 - $7? ($3) SAY: $3 is the difference owed to me.

Exercises: Find the difference owed.

a) Price = $2 Amount paid = $10
b) Price = $4 Amount paid = $10

Answers: a) $10 - $2 = $8, b) $10 - $4 = $6

Making change for up to 1 dollar when prices are in multiples of 10. Write on the board:

\[
\text{Price} = 40¢ \quad \text{Amount paid} = 100¢
\]

SAY: An item costs 40 cents, but I paid 1 dollar or 100 cents. ASK: Did I pay too much or not enough? (too much) How do you know? (100¢ > 40¢) How can I calculate the difference? (subtract the price from the amount paid) What is 100¢ - 40¢? (60¢) SAY: So the difference owed, or change due, is 60 cents.

Exercises: Find the difference owed.

a) Price = 60¢ Amount paid = 100¢
b) Price = 30¢ Amount paid = $1

Answers: a) 100¢ - 60¢ = 40¢, b) $1 = 100¢; 100¢ - 30¢ = 70¢
Finding the difference to the next highest dollar. Leaving the box and blank empty, write on the board:

$5.20 \quad 80¢ \quad \rightarrow \quad $6.00

ASK: What is the next whole dollar after $5.20? ($6.00) What is the difference between $5.20 and $6.00? (80¢). Write “80¢” in the box and “$6.00” in the blank.

Exercises: Find the next whole dollar and the difference.

a) $11.40

b) $38.80

c) $67.10

Answers: a) $12.00 and 60¢, b) $39.00 and 20¢, c) $68.00 and 90¢

Finding the difference to $10.00. SAY: People might pay for items with a 10-dollar bill. We’re going to find the difference owed when paying with a 10-dollar bill. Write on the board:

Price = $6.30  Amount paid = $10.00

ASK: What is the next whole dollar after $6.30? ($7.00)

Draw on the board:

$6.30 \quad \rightarrow \quad $7.00 \quad \rightarrow \quad $10.00

ASK: What is the difference between $6.30 and $7.00? (70¢) Write “70¢” in the first box on the board. ASK: What is the difference between $7.00 and $10.00? ($3.00) Write “$3.00” in the second box. ASK: How do we find the difference between $6.30 and $10.00? (add) What is $70¢ + $3.00? ($3.70) Repeat with “Price = $2.20” and “Amount paid = $10.00.” (difference owed: $7.80)
Exercises: Find the difference owed from $10.00 for the given price.

a) $3.30  

\[
\begin{array}{c}
\text{70¢} \\
$3.30 \\
\end{array} 
\begin{array}{c}
\text{4.00} \\
$4.00 \\
\end{array} 
\begin{array}{c}
\text{10.00} \\
$10.00 \\
\end{array} 
\]

Difference owed = \(70¢ + 6.00 = 6.70\)

b) $1.40  

\[
\begin{array}{c}
\text{60¢} \\
$1.40 \\
\end{array} 
\begin{array}{c}
\text{2.00} \\
$2.00 \\
\end{array} 
\begin{array}{c}
\text{10.00} \\
$10.00 \\
\end{array} 
\]

Difference owed = \(60¢ + 8.00 = 8.60\)

c) $8.60  

\[
\begin{array}{c}
\text{40¢} \\
$8.60 \\
\end{array} 
\begin{array}{c}
\text{9.00} \\
$9.00 \\
\end{array} 
\begin{array}{c}
\text{10.00} \\
$10.00 \\
\end{array} 
\]

Difference owed = \(40¢ + 1 = 1.40\)

Finding the difference to the next highest dime. SAY: Sometimes, the price of the item is not a multiple of 10¢. It is easier to count up to the next dollar if we first count to the next highest multiple of 10¢ or dime. Write on the board:

Price = $2.65  
Amount paid = $10.00

ASK: What is the next multiple of 10¢ after $2.65? (2.70¢) What is the difference between $2.65 and $2.70? (5¢) What is the next dollar after $2.65? ($3.00) What is the difference between $2.70 and $3.00? (30¢) What is the difference between $3.00 and $10.00? ($7.00) Draw on the board:

\[
\begin{array}{c}
\text{5¢} \\
$2.65 \\
\end{array} 
\begin{array}{c}
\text{30¢} \\
$2.70 \\
\end{array} 
\begin{array}{c}
\text{7.00} \\
$3.00 \\
\end{array} 
\begin{array}{c}
\text{10.00} \\
$10.00 \\
\end{array} 
\]

ASK: How can we use the three differences to find the total difference? (add) What is 5¢ + 30¢ + $7.00? ($7.35) Write on the board:

Difference owed = 5¢ + 30¢ + $7.00 = $7.35

Repeat with “Price = $4.35.” and “Amount paid = $10.00” (difference owed: $5.65)
Exercises: Find the difference owed from $10.00 for the given price.

a) $5.15   b) $7.35

Solutions

a) $5.15  $5.20  $6.00  $10.00

Difference owed = 5¢ + 80¢ + $4.00 = $4.85

b) $7.35  $7.40  $8.00  $10.00

Difference owed = 5¢ + 60¢ + $2.00 = $2.65

Rounding money to the nearest nickel and then finding the difference owed. SAY: Because the prices of things we buy include the number of cents, we need to know how to find the difference for amounts with both dollars and cents. We round to the nearest nickel before we find the difference owed. Write on the board:

Price = $4.63  Amount paid = $10.00

ASK: What is the nearest multiple of 5¢ before $4.63? ($4.60) What is the nearest multiple of 5¢ after $4.63? ($4.65) Is $4.63 closer to $4.60 or $4.65? ($4.65) SAY: We can pretend the price of the product is $4.65 instead of $4.63. Then we find the difference owed, using $4.65. Draw on the board:

$4.65  ___  ___  ___  $10.00

ASK: What is the next multiple of 10¢ after $4.65? ($4.70) Write "$4.70" in the first blank. ASK: What is the difference from $4.65 to $4.70? (5¢) Write "5¢" in the first box. ASK: What is the next highest dollar after $4.70? ($5.00) Write "$5.00" in the second blank. ASK: What is the difference between $4.70 and $5.00? (30¢) Write "30¢" in the next box. ASK: What is the difference between $5.00 and $10.00? ($5.00) Write "$5.00" in the last box. ASK: How do you get the difference owed between $4.65 and $10.00? (add) What is 5¢ + 30¢ + $5.00? ($5.35) Write on the board:

Difference owed = 5¢ + 30¢ + $5.00 = $5.35
**Exercises:** The amount shown is the price. You have a 10-dollar bill. Round the amount to the nearest nickel and then find the difference owed.

a) $4.26  

**Solutions**

\[
\begin{align*}
\text{Rounded price} &= 4.25 \\
\text{Difference owed} &= 75¢ + 5.00 = 5.75
\end{align*}
\]

b) $7.84  

\[
\begin{align*}
\text{Rounded price} &= 7.85 \\
\text{Difference owed} &= 5¢ + 10¢ + 2.00 = 2.15
\end{align*}
\]

**NOTE:** Extensions 1 to 5 are required in order to cover the British Columbia curriculum.

**Extensions**

1. Mandy earns $10 every weekend by babysitting.

   a) She wants to buy concert tickets for herself and some of her friends to see her favourite musician. Tickets cost $30 each. How long will it take her to save enough money to buy the number of tickets?

      i) 1 ticket  
      ii) 2 tickets  
      iii) 3 tickets  
      iv) 4 tickets

   b) Mandy now wants to buy a new bicycle that costs $200. How long will it take her to save enough money to buy the bicycle?

   c) Will Mandy be able to buy 2 concert tickets and the bicycle in 30 weeks?

**Solutions**

a) i) 1 ticket costs $30, so it would take Mandy 3 weeks to buy 1 ticket  
ii) 2 tickets cost $60, so it would take Mandy 6 weeks to buy 2 tickets  
iii) 3 tickets cost $90, so it would take Mandy 9 weeks to buy 3 tickets  
iv) 4 tickets cost $120, so it would take Mandy 12 weeks to buy 4 tickets

b) $200 \div $10/week = 20 weeks

c) It will take 20 weeks to buy the bicycle and 6 more weeks to buy 2 tickets. That is a total of 26 weeks, which is less than 30 weeks, so, yes, she will be able to buy 2 tickets and the bicycle in that time.
2. Glen needs to travel 4 km by taxi and wants to know how much two different taxi companies would charge. Quick Cabs charges $4 no matter how far you go plus $1 for every kilometre travelled. Speedy Cabs charges $1 no matter how far you go plus $2 for every kilometre travelled. Which taxi company should Glen use to travel 4 km if he wants to pay the lesser amount?

Answer: Quick Cabs will cost $4 + $1 + $1 + $1 + $1, which is $8. Speedy Cabs will cost $1 + $2 + $2 + $2 + $2, which is $9. Glen should choose Quick Cabs because it will be less expensive.

3. Use the internet or other library resources to explain what the term means.

a) barter
b) credit card

Answers
a) The exchange of goods or services for other goods or services without using money.
b) A small plastic card from a bank that allows the holder to purchase goods or services and pay for them later.

4. Why are decimals important for writing amounts of money?

Sample answer: The decimals in money make it possible to show that something may cost part of a dollar. Without decimals, we could only talk about whole dollars such as $1, $2, and $3.

5. Use the internet or library resources to research equitable trade rules.
Squares Divided into Hundredths
Number Lines Divided into Hundredths

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
## Money-Matching Memory Game

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<td>$0.20</td>
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<td>$1</td>
</tr>
<tr>
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<td>100¢</td>
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<tr>
<td>$2.02</td>
<td>$2.20</td>
<td>22¢</td>
</tr>
<tr>
<td>202¢</td>
<td>220¢</td>
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Unit 11  Patterns and Algebra: Equations

Introduction
This unit focuses on:
• equations (addition, subtraction, multiplication, and division); and
• the use of symbols to represent unknown values.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit
• focus on increasing fluency in all four operations

Assessment
The lessons covered by a quiz or test are as follows:

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Mental math minute. Have students add by using 10. Say the addition you want students to do (such as 48 + 6), then have one student say the in-between addition step (50 + 4) and another student finish the addition (54). As additional challenges, use three- and four-digit numbers, such as 345 + 8, or vary the order, such as 8 + 56.

NOTE: Demonstrations throughout this lesson feature apples (to match the pictures in the AP Book). In place of real apples, you could use paper cutouts of apples, counters, connecting cubes, or any other roughly identical objects.

Introduce equality in diagrams. Draw a line across your desk. Place 5 apples on one side of the line and 3 apples on the other side. ASK: Are the sides equal? (no) Repeat with 4 apples on one side and 3 apples on the other, then with 3 apples in different arrangements on both sides. When students identify that the sides are equal, place a card with an equal sign drawn on it between the apples. Repeat with the situation in the margin, using a card with the plus sign.

Repeat with another similar situation, this time placing the plus sign on the other side of the line.

Making diagrams for addition equations. Put 5 apples in a cardboard box or opaque bag. Show students the box and explain that sometimes we don’t know how many apples there are in a situation. Place the box on one side of the line on your desk. On the other side of the line, place a group of 2 apples and a group of 3 apples with the plus sign between them to represent the situation 3 + 2. ASK: Can you tell how many apples are in the box? (no) Place a card with the equal sign on the line and SAY: I have given
you some more information. I have told you that the number of apples on each side of the equal sign is the same. ASK: Now can you tell how many apples are in the box? (yes, 5) Verify the answer by revealing the contents of the box. Repeat with the situation in the margin.

Present the situation in the margin. ASK: Can you tell how many apples are in the box? (4) How did you figure this out? (counting up from 3 to 7, subtracting 7 − 3, or using the fact family of 3 + 4 = 7) Repeat with a few more examples, placing the box on different sides of the line.

**Representing problems in pictures.** Tell students that it is easy to use a picture to represent the problem they just solved. Draw the pictures in the margin on the board.

ASK: How are the pictures the same? (both pictures represent the same number of objects—they show a box and 3 objects on the left side of the equal sign and 7 objects on the right side of the equal sign) How are they different? (one of the pictures has apples, the other has circles) Which picture is easier to draw? (the picture with circles) Do both pictures allow us to easily figure out that there are 4 apples in the box? (yes)

Explain that people draw different pictures for different purposes. SAY: In art, we might try to draw apples as realistically as possible. We would pay attention to colour, shading, shape, and other details. ASK: Does colour help us to solve the mathematical problem of how many apples are in the box? (no) Does including leaves on the apples help us to solve the mathematical problem? (no) SAY: These details don’t help us to solve the mathematical problem, so we don’t need to include them. In mathematics, we want simple pictures that help us to solve problems but don’t take too much time to draw. ASK: What should we pay attention to in the pictures we draw to solve mathematical problems? (the number of objects, creating a picture that is not messy, drawing circles that are not too large and not bigger than the box so that we are not distracted)

Have students copy the picture with the circles and solve the problem by drawing the necessary number of circles in the box. (4)

**Exercises:** Draw the circles in the box to solve the equation.

a) \[
\begin{array}{c}
\text{\bigcirc} \\
+ \text{\bigcirc} \\
\end{array}
\]

b) \[
\begin{array}{c}
\text{\bigcirc} \\
\text{\bigcirc} \\
\text{\bigcirc} \\
\text{\bigcirc} \\
\text{\bigcirc} \\
\text{\bigcirc} \\
\end{array}
\]

**Writing addition equations from diagrams.** Point out that it is somewhat inconvenient to draw apples or circles all the time. ASK: What if you have a box and 79 apples on one side of the line and 125 apples on the other side? What would be more convenient to use than a picture? (numbers) SAY: The picture then becomes \[\text{\bigcirc} + 79 = 125.\] Explain that this number sentence is called an *equation*. Write the word “equation” on the board and underline the first four letters. ASK: Do you know any other word that starts with the same letters? (equal) Ask students to describe what an equation is. (a number sentence with an equal sign, showing that two parts—on different sides of the equal sign—are equal)
Have students complete Questions 1–2 on AP Book 4.2 p. 67.

**Drawing diagrams to solve addition equations.** Show students how to draw a diagram for an equation. Write “□ + 2 = 5” on the board and circle “□ + 2” on the left side of the equal sign and “5” on the right side. Emphasize that the equation means that the two circled parts are equal. Remind students that at the beginning of the lesson, you started by drawing a line to separate the two equal sides and then later added an equal sign across the line to show that the parts are equal. Tell students that they do not have to draw the line now—they should use the equal sign alone.

Draw an equal sign on the board. Then draw a large box for the unknown number on the left side of the equal sign. Draw a plus sign and 2 circles beside the box since 2 is the number added to the unknown number. Then draw 5 circles on the right side of the equal sign. Remind students that they are drawing a picture in mathematics, so the quantity of circles is important, but they should not spend time making the picture pretty. ASK: How many circles should be in the box? (3) Have a volunteer draw them.

**Exercises:** Draw a diagram. Add the number of circles in the box, then write the missing number in each equation.

a) 5 + □ = 8  
b) 4 + □ = 9  
c) 3 + □ = 10  
d) 5 + □ = 11  

**Bonus:** 1 + 3 + □ = 8

**Answers:** a) 3, b) 5, c) 7, d) 6, Bonus: 4

Encourage students who are struggling to solve the equation by counting up from the number of circles outside the box to the number of circles on the other side of the equal sign.

**Guessing and checking.** Write “9 + □ = 28” on the board. Tell students that someone tried to solve the equation by guessing and checking. The first guess was 16. Write “16” in the box, then ask students to add 9 + 16 to see if the numbers on both sides of the equal sign are equal. (no, 9 + 16 is 25, not 28) ASK: Should the next guess be more than 16 or less than 16? (more) Students can indicate their answer with thumbs up or thumbs down. Have a volunteer try 17. ASK: Did we get closer to 28? (yes) Have students make more guesses until they get the answer. (19) Repeat with the equation □ + 8 = 31, using 25 as the first guess.

**ACTIVITY (Optional)**

**Equation Dominoes**

*Object of the game:* To create a single chain incorporating all the game cards.

*Materials:* cards from BLM Equation Dominoes (Addition)
Instructions: Students play in pairs or groups of three. Player 1 deals out six cards to every player and places one card face up in the middle (this card is the first card in the first chain). The remaining cards stay face down in a pile. Players take turns placing a card and drawing a new card from the pile.

On each turn, a player can either start a new chain or add a card to an existing chain. This is possible when the number on a card is the missing number in the equation at the end of an existing chain or when the equation on a card is solved by the number at the end of a chain. If the end of a chain then matches the end of another chain, the player combines the chains.

Variation: Students create their own domino cards with equations on both ends of the card. In this case, cards match when the missing number in both equations is the same.

**Subtraction with unknowns.** Repeat the progression used above to teach subtraction with unknowns. Start with real objects and a box and have students write equations to represent them. Examples:

- 7 apples $-$ □ = 2 apples
- □ $-$ 6 apples = 8 apples
- 5 = □ $-$ 6 apples

Emphasize that the equal sign between the two sides makes it possible to solve the equations. Then provide subtraction equations and have students draw diagrams to solve them.

**Exercises:** Draw a picture for the equation. Use your picture to solve the equation.

a) 12 = □ $-$ 9  

Selected answer: a) \[ \begin{array}{c}
\text{\includegraphics{apples.png}} \\
\text{\includegraphics{2_apples.png}} \\
\text{\includegraphics{9_apples.png}} \\
\end{array} \]

b) □ $-$ 4 = 9

c) 15 $-$ □ = 8

Finally, have students solve equations by guessing and checking before they complete the exercises below.

**Exercises:** Solve the equation by guessing and checking.

a) 28 $-$ □ = 19  

b) □ = 81 $-$ 6

c) 7 = □ $-$ 52  

**Bonus:** 1350 = □ $-$ 50

**Answers:** a) 9, b) 75, c) 59, Bonus: 1400
Extensions

1. Place the same number in both boxes to make the equation true.
   a) $\square + \square + 3 = 7$
   b) $\square + \square + 6 = 12$
   c) $\square + \square + \square + 5 = 23$

   **Answers:** a) 2, b) 3, c) 6

2. a) Make the equation true by placing one number in the small box and the next counting number in the big box.
   
   $12 - \square = 13 - \square$

   b) Find two more ways to solve the equation in part a). Are there any other possible solutions? How do you know?

   c) Make up an equation similar to the one in part a) and have someone solve your equation.

   d) Make up an equation that is similar to the one in part a) but that you make true by writing a number in the big box that is 2 less than the number you write in the little box. Have someone solve your equation.

   e) Would the solutions in part a) also solve the equation
   
   $13 - \square = 12 - \square$? Explain.

   **Sample answers**
   
   a) $12 - 5 = 13 - 6$
   
   b) 1 and 2, 6 and 7, 12 and 13
   
   c) $15 - \square = 16 - \square$
   
   d) $30 - \square = 32 - \square$

   **Answers:** b) yes; e) no, the number on the right side has to be 1 more than the number on the left side
GOALS

Students will use diagrams to write and solve equations of the form 
\( a \times \square = b \) (where \( \square \) is an unknown number).

Students will use the relationship between multiplication and division
to solve equations of the form \( a \times \square = b \) (where \( \square \) is an unknown number).

PRIOR KNOWLEDGE REQUIRED

- Can use skip counting to multiply and divide
- Understands that multiplication is repeated addition
- Knows that a box can replace a missing number in an equation
- Understands division as making equal groups
- Understands the relationship between multiplication and division

MATERIALS

- BLM Equation Dominoes (Multiplication) (p. N-41)

Mental math minute. Ask students to solve multiplication questions within the range of \( 1 \times 1 \) to \( 10 \times 10 \) and corresponding division questions. For each number, go through the questions in order, such as \( 1 \times 3, 3 \div 3, 2 \times 3, 6 \div 3, \) and so on to \( 10 \times 3 \) and \( 30 \div 3 \). Then progress to a different number. Next, try questions out of order, but keep the corresponding multiplication and division together.

Review the term “operations.” Remind students that just as red, white, and blue can all be described by a common word—colour—there is a word that describes addition, subtraction, multiplication, and division. These are all mathematical operations.

Using division to solve an equation with repeated addition. Draw the picture below on the board and tell students that there should be the same number of apples in each box.

Invite a volunteer to draw the apples (as circles) in each box. Remind students how they can use division to find the number of objects in each of the equal groups. ASK: How did you know how many apples to put in each box? What strategy did you use to find out? (divided by 3 because there are three equal groups, or shared apples one at a time) If students shared the apples one at a time, ask them what mathematical operation...
they performed when they did this. (division) Have students complete a picture with 4 boxes and 12 apples, then with 2 boxes and 8 apples.

Ask students to write an equation for each picture drawn above. For example, the first picture would give the equation \( \square + \square + \square = 6 \).

**ASK:** What number goes in each box? (2) Have a volunteer fill in the boxes. (2 + 2 + 2 = 6)

**Exercises:** Draw the same number of circles in each box. Write the equation for the picture.

a) \[
\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array} +
\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array} =
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array}
\]
\( \square + \square + \square + \square = \bigcirc \bigcirc \bigcirc \bigcirc \)

b) \[
\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array} +
\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
\end{array} =
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array}
\]
\( \square + \square + \square = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)

**Answers**

a) \[
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} +
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} +
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} +
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} =
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array}
\]
\( 3 + 3 + 3 + 3 = 12 \)

b) \[
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} +
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} +
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array} =
\begin{array}{c}
\bigcirc \\
\bigcirc \\
\bigcirc \\
\bigcirc \\
\end{array}
\]
\( 5 + 5 + 5 = 15 \)

**Review multiplication as repeated addition.** Remind students that multiplication is a short form for repeated addition.

**Exercises:** Rewrite the sums as products.

a) \(4 + 4 + 4 = \)

b) \(5 + 5 + 5 = \)

c) \(10 + 10 + 10 = \)

d) \(\square + \square + \square = \)

**Answers:** a) \(3 \times 4\), b) \(3 \times 5\), c) \(3 \times 10\), d) \(3 \times \square\)

Have students rewrite the addition equations they wrote for the pictures with apples as multiplication equations. (\(3 \times \square = 6\), \(4 \times \square = 12\), \(2 \times \square = 8\))
Solving multiplication equations using a diagram. Draw on the board:

4 \times \square = 20

5 \times \square = 15

Have students indicate how many apples should be in each box. Have volunteers draw the correct number of apples. Point out that since there is now only one box to fill, students can’t share the apples equally as they did before by drawing them one at a time in different boxes. Have students share their methods for solving the problems. (split the apples on the right side of the equation into equal groups; count up by the number the box is multiplied by while raising a finger for each group of that number you count, then draw the same number of apples as there are fingers raised)

Then, referring to the equations beside the pictures, ASK: What number goes in the box in the equation? (4, 3) Have volunteers complete the equations. (4 \times 4 = 16, 5 \times 3 = 15) Write “4 \times 4” and “3 \times 5” on the board.

ASK: Would the answers be the same? (yes)

Solving multiplication equations (missing second factor) without using a diagram. ASK: What mathematical operation are you performing when you split apples into equal groups? (division) What mathematical operation are you performing when you skip count and write the number of fingers you raised when you reached the target number? (division) Have students write the division equations for each problem they solved using a diagram.

ASK: Is the number of apples you drew in the diagram the same as the number you wrote in the equation? (yes) Provide a few more equations without the corresponding diagrams and have students use division to solve them.

Exercises: Rewrite the multiplication as division, then solve the equation.

a) 3 \times \square = 24  
b) 5 \times \square = 45  
c) 6 \times \square = 42  
d) 9 \times \square = 72

Bonus

e) 3 \times \square = 240  
f) 5 \times \square = 4500

Answers: a) 24 \div 3 = \square, 8; b) 45 \div 5 = \square, 9; c) 42 \div 6 = \square, 7;  
d) 72 \div 9 = \square, 8; Bonus: e) 240 \div 3 = \square, 80; f) 4500 \div 5 = \square, 900
Using division to solve multiplication equations of different types.

SAY: Remember, multiplication is commutative. Write on the board:

\[ 3 \times \square = 27 \quad \square \times 3 = 27 \]

Ask students whether the two equations should have the same answer and to explain their thinking. Remind them that the box replaces a missing number, so they can do the same things with the box that they would do with a number. ASK: How would you solve the first equation? (27 ÷ 3 = 9)

Would the same solution work for the second equation? (yes) Why? (order does not matter in multiplication; we can just switch the numbers we are multiplying together) Repeat for 4 × \square = 32 and \square × 4 = 32 and for 20 × \square = 100 and \square × 20 = 100. (32 ÷ 4 = 8, 100 ÷ 20 = 5)

Exercises: Rewrite the multiplication as division, then solve the equation.

a) 3 × \square = 21  b) \square × 5 = 55  c) 7 × \square = 70  d) \square × 5 = 65

Bonus

e) 4 × \square = 84  f) \square × 5 = 95
g) \square × 3 = 3000  h) 5 × \square = 5000

Answers: a) 21 ÷ 3 = \square, 7; b) 55 ÷ 5 = \square, 11; c) 70 ÷ 7 = \square, 10; d) 65 ÷ 5 = \square, 13; Bonus: e) 84 ÷ 4 = \square, 21; f) 95 ÷ 5 = \square, 19; g) 300 ÷ 3 = \square, 100; h) 5000 ÷ 5 = \square, 1000

For additional practice, play the Equation Dominoes game (see the activity in Lesson PA4-12) with cards from BLM Equation Dominoes (Multiplication).

Solving equations with a missing dividend. Present the equation \square ÷ 5 = 7. ASK: What number divided by 5 gives you 7? (35) How did you get 35 from 5 and 7? (by multiplying) Point out that the equation describes this situation: You had some apples that you divided into 5 equal groups. There are 7 apples in each group. Draw 5 circles and 7 dots in one circle, then ASK: What would you draw to find the total number of apples? (7 dots in each of the remaining circles) How would you then find the total number of dots? (multiply 5 × 7) Emphasize that to find the number that is being divided into groups, you multiply the number of groups by the number in each group.

Exercises: Rewrite the division as multiplication, then solve the equation.

a) \square ÷ 5 = 20  b) \square ÷ 6 = 100  c) \square ÷ 10 = 17  d) \square ÷ 40 = 80

Answers: a) 5 × 20 = \square, 100; b) 6 × 100 = \square, 600; c) 10 × 17 = \square, 170; d) 40 ÷ 80 = \square, 3200

Solving equations with a missing divisor. Have students solve several equations with a missing divisor, such as 45 ÷ \square = 5. (9) Students who still have trouble with multiplication and division facts and using guessing and checking to find the answer can use skip counting instead.
PA4-14 Totals and Equations

Pages 71–72

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
equation
part
solve
symbol
total
unknown

Goals
Students will write and solve equations for single-step word problems requiring finding the total or part of the total.
Students will use letters to represent unknowns in equations.

PRIOR KNOWLEDGE REQUIRED
Can perform the four basic operations
Can use a box to represent a missing number in an equation
Can identify sides of an equation
Can add and subtract within 100

Mental math minute. Have students add three-digit numbers by adding hundreds, tens, and ones separately in groups of four. Give an addition problem, such as $135 + 246$. The first student adds the hundreds, $100 + 200$; the second adds the tens, $30 + 40 = 70$; the third adds the ones, $5 + 6 = 11$; and the fourth student finishes the addition, $300 + 70 + 11 = 381$, so $135 + 246 = 381$. Start with problems that do not require regrouping in any place, such as $325 + 634$, and progress to questions that require regrouping ones or regrouping tens, but not both.

Using symbols in equations. SAY: In the last two lessons, when we didn’t know the value of a number in an equation, we represented it with a box. We call the numbers we do not know unknown numbers or unknowns. Explain that instead of boxes, you can use different shapes, smiley faces, question marks, or even letters. Give students a number of equations in which a box represents a missing number and ask them to rewrite the equation, replacing the box with a different symbol.

Exercises: Rewrite the equation using $w$ instead of the box to represent the unknown number.

1. a) $\square = 4 \times 10$ b) $\square = 4 + 9$ c) $48 \div 8 = \square$
d) $\square = 27 - 23$ e) $55 = \square \times 5$ f) $\square + 2 = 14$

Answers: a) $w = 4 \times 10$, b) $w = 4 + 9$, c) $48 \div 8 = w$, d) $w = 27 - 23$, e) $55 = w \times 5$, f) $w + 2 = 14$

2. Solve the equation.
   a) $25 = 5 \times w$ b) $a = 6 \times 3$ c) $45 \div p = 9$
d) $m \times 3 = 15$ e) $b \div 2 = 8$ f) $21 \div 3 = w$

Answers: a) $w = 5$, b) $a = 18$, c) $p = 5$, d) $m = 5$, e) $b = 16$, f) $w = 7$

Have students look at the exercises above and indicate which equations have the unknown number on a side by itself without any other number. Point at the equations one at a time and have students show thumbs up if the unknown is by itself and thumbs down if it is not. (1. a, b, c, d; 2. b, f)
Parts and totals. Write “There are 5 green apples and 7 red apples. How many apples are there in total?” on the board. Explain that in this problem we call the number of green apples and the number of red apples “parts” and the total number of apples the “total” or “whole.” ASK: What do you do with two parts to get the total? (add them) Write on the board:

\[ \text{Total} = \text{Part 1} + \text{Part 2} \]

Write “There are 5 green apples and \( w \) red apples. There are 12 apples in total. How many red apples are there?” on the board. ASK: What are the parts? (green apples and red apples) How many of each part? (5 green, \( w \) red) What is the total? (the total number of apples, 12) What does the unknown number represent, one of the parts or the total? (one of the parts) If you know the total and one of the parts, how do you get the other part? (subtract) Continue writing on the board:

\[ \text{Part 1} = \text{Total} - \text{Part 2} \]
\[ \text{Part 2} = \text{Total} - \text{Part 1} \]

Draw on the board:

<table>
<thead>
<tr>
<th>Total</th>
<th>Part 1</th>
<th>Part 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

SAY: This table shows the Total on the top and Part 1 and Part 2 on the bottom. I’m going to put in some numbers so it’s easier to understand. Draw the table below beside the first table:

<table>
<thead>
<tr>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

Generate the three pairs of equations below by first saying and then writing a word equation from the table on the left and then saying and writing the corresponding number equation from the table on the right:

\[ \text{Total} = \text{Part 1} + \text{Part 2} \quad 10 = 4 + 6 \]
\[ \text{Part 1} = \text{Total} - \text{Part 2} \quad 4 = 10 - 6 \]
\[ \text{Part 2} = \text{Total} - \text{Part 1} \quad 6 = 10 - 4 \]

Draw on the board:

<table>
<thead>
<tr>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

SAY: In this example, the value of the total is unknown, and we are representing it with the letter \( m \). As we did for the previous example, we’re going to write a number equation for the Total, for Part 1, and for Part 2. Then we’ll decide which is easiest to solve. Write the three word equations as above on the board and then, one at a time, ask for the corresponding number equations. \( (m = 5 + 4, 5 = m - 4, 4 = m - 5) \) Point to the number equation for the Total and SAY: This one is the easiest to solve because the
unknown is by itself on one side of the equal sign. ASK: What is the value of \( m \)? (9) Underline the number equation for the Total and write “\( m = 9 \)” beside it.

Draw on the board:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
</tr>
</tbody>
</table>

ASK: Is the unknown in this example the Total, Part 1, or Part 2? (Part 1)
What are we using to represent the unknown? (\( k \)) Have a volunteer write the word equation and the number equation for the Total. (Total = Part 1 + Part 2, 11 = \( k + 9 \)). Repeat with a second and a third volunteer for the equations for Part 1 and Part 2, respectively. (Part 1 = Total − Part 2, \( k = 11 − 9 \); Part 2 = Total − Part 1, \( 9 = 11 − k \)) ASK: Which equation is easiest to solve? (\( k = 11 − 9 \)) Why? (\( k \) is alone on one side of the equal sign) What is the value of \( k \)? (2) Underline the equation for Part 1 and write “\( k = 2 \)” beside it.

Repeat for the table shown below:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

\( 20 = 13 + t, \ 13 = 20 − t, \ t = \frac{20}{13}; \ t = 7 \)

**Exercises**

1. Write three equations for the table. Underline the one that is easiest to solve and solve it.

   a) |   |   |
    | 16 | 7 |
    | w | w |

   b) |   |   |
    | 17 | w |
    | 10 | w |

   c) |   |   |
    | w | 7 |
    | 4 | w |

   d) |   |   |
    | 19 | w |
    | 12 | w |

**Answers**

a) \( 16 = 7 + w, \ 7 = 16 − w, \ w = 16 − 7, \ w = 11 \)
b) \( 17 = w + 10, \ w = 17 − 10, \ 10 = 17 − w, \ w = 7 \)
c) \( w = 7 + 4, \ 7 = w − 4, \ 4 = w − 7, \ w = 11 \)
d) \( 19 = 12 + w, \ 12 = 19 − w, \ w = 19 − 12, \ w = 7 \)

2. Write the equation where the unknown is by itself and solve it.

   a) |   |   |
    | w | 8 |
    | 2 | w |

   b) |   |   |
    | 16 | w |
    | 11 | w |

   c) |   |   |
    | 37 | w |
    | 16 | w |

   **Bonus:** |   |   |
    | 100 | w |
    | 75 | w |

**Answers:** a) \( w = 8 + 2, \ w = 10 \); b) \( w = 16 − 11, \ w = 5 \);
c) \( w = 37 − 16, \ w = 21 \); **Bonus:** \( w = 100 − 75, \ w = 25 \)
Writing an equation for parts and total with one of the components represented by a letter. On the board, write the column headings and the given information in the first column in the table below, leaving room for another column on the right. Tell students that each situation only gives the information to fill in two of the next three columns. ASK: What do we use for a number we are not given? (a letter) Remind students that you can use any letter or symbol. Explain that they will put the letter \( x \) in the column where the information not given should be. Starting with part a), point to each column in turn and have students signal which number from the situation goes in each column. (They can make the letter \( x \) with their fingers for the column that contains the unknown.) Repeat for parts b) to f), one row at a time. (see completed table below)

<table>
<thead>
<tr>
<th></th>
<th>Green Grapes</th>
<th>Purple Grapes</th>
<th>Total Number of Grapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>3 green grapes, 2 purple grapes</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>b)</td>
<td>7 green grapes, 9 grapes altogether</td>
<td>7</td>
<td>( x )</td>
</tr>
<tr>
<td>c)</td>
<td>5 green grapes, 3 purple grapes</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>d)</td>
<td>10 grapes in total, 6 green grapes</td>
<td>6</td>
<td>( x )</td>
</tr>
<tr>
<td>e)</td>
<td>5 purple grapes, 9 grapes altogether</td>
<td>( x )</td>
<td>5</td>
</tr>
<tr>
<td>f)</td>
<td>35 grapes in total, 12 purple grapes</td>
<td>( x )</td>
<td>12</td>
</tr>
</tbody>
</table>

Writing equations for the data. ASK: In each of the situations in the table, what are the parts? (the number of green or purple grapes) What is the whole or the total? (the number of grapes altogether) Add a column and label it “Equation.” SAY: Now we’re going to write an equation for each situation. The equation we write has to have the unknown by itself on one side of the equal sign. Have students tell you what the equation should be for the first two rows. (a) \( x = 3 + 2 \), b) \( x = 9 - 7 \) Then have students write equations for each row of the table. (c) \( x = 5 + 3 \), d) \( x = 10 - 6 \), e) \( x = 9 - 5 \), f) \( x = 35 - 12 \) Have students solve each equation. (a) 5, b) 2, c) 8, d) 4, e) 4, f) 23)

Identifying parts and total. Explain that many of the problems students deal with have parts and total, not just the questions about grapes and apples. If you look at your class, 10-year-olds can be a part, 9-year-olds can be another part, and the whole class is the total. Have students describe a few situations in which there are parts and a total. For each situation given, the class should say what the parts are and what the total is.

In Exercise 1 below, have students identify which piece of information gives the total. You can point to each piece of data and have students signal
thumbs down for parts and thumbs up for total. Or you can write the pieces of information in each question on separate lines and have students raise the correct number of fingers to indicate the line that shows the total.

**Exercises:**

1. Identify the total in the situation.
   a) There are 6 red marbles. There are $x$ blue marbles. There are 11 marbles in total.
   b) There are $x$ red apples. There are 17 green apples. There are 21 apples in total.
   c) Emma has 5 marbles. Ren has 6 marbles. Emma and Ren have $x$ marbles altogether.
   d) There are 5 hockey cards. There are $x$ sport cards altogether. There are 7 cards that are not hockey cards.
   e) Vicky has 5 cousins. 3 of them are girls. $x$ cousins are boys.
   f) Josh had 52 hockey cards. He gave $x$ cards away. He has 27 cards left.

   **Answers:** a) 11, b) 21, c) $x$, d) $x$, e) 5, f) 52

2. For each situation in Exercise 1, write an equation with the unknown by itself and solve it.

   **Answers:** a) $x = 11 - 6, x = 5$; b) $x = 21 - 17, x = 4$; c) $x = 5 + 6, x = 11$; d) $x = 7 + 5, x = 12$; e) $x = 5 - 3, x = 2$; f) $x = 52 - 27, x = 25$

3. Write the equation for the situation. Then solve the equation.
   a) Cam has 5 stamps. Eric has 6 stamps. They have $x$ stamps altogether.
   b) 15 birds are in a tree. $x$ birds are robins. 7 birds are not robins.
   c) Edmond has 12 cookies. He gives 9 cookies to his friends. He has $x$ cookies left.
   d) Jane pays $x$ dollars for a scarf and a hat. The scarf costs $12. The hat costs $15.

   **Bonus:** Black-footed ferrets are critically endangered animals living in North America. In 1987, the last known ferrets were captured in Wyoming.

   Write an equation for the situation and solve it.
   a) 11 male and 7 female black-footed ferrets were captured in Wyoming. $x$ ferrets in total were captured.
   b) 18 ferrets were captured. $x$ kits were born in captivity the same year. There were 25 ferrets in total in 1987.
**Answers:** a) \( x = 5 + 6, x = 11 \); b) \( x = 15 - 7, x = 8 \); c) \( x = 12 - 9, x = 3 \); d) \( x = 12 + 15, x = 27 \); Bonus: e) \( x = 11 + 7, x = 18 \); f) \( x = 25 - 18, x = 7 \)

**Extensions**

1. a) Kate tried to solve the equations to find what number the letter represents. She did some correctly and some incorrectly. Use Kate’s answer to determine if she solved the equation correctly. Which equations did she write incorrectly?

<table>
<thead>
<tr>
<th>Equation</th>
<th>Kate’s Answer</th>
<th>Checking Kate’s Answer</th>
<th>Correct or Incorrect?</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) ( n + 6 = 31 )</td>
<td>( n = 25 )</td>
<td>( 25 + 6 = 31? )</td>
<td>correct</td>
</tr>
<tr>
<td>ii) ( 100 - s = 108 )</td>
<td>( s = 8 )</td>
<td>( 100 - 8 = 108? )</td>
<td>incorrect</td>
</tr>
<tr>
<td>iii) ( 76 + o = 132 )</td>
<td>( o = 67 )</td>
<td>( 76 + ___ = 132? )</td>
<td></td>
</tr>
<tr>
<td>iv) ( 91 - b = 0 )</td>
<td>( b = 91 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v) ( e \times 4 = 48 )</td>
<td>( e = 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vi) ( o \div 2 = 80 )</td>
<td>( o = 40 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vii) ( 1100 \times t = 5500 )</td>
<td>( t = 5 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>viii) ( 8 \times p = 868 )</td>
<td>( p = 181 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ix) ( w \div 2 = 2143 )</td>
<td>( w = 4286 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Write the letters used for the unknowns for Kate’s incorrect answers.

   ______ ______ ______

c) Unscramble the letters to make a word related to this question. _______!

**Answers:** a) ii), iii), vi), viii); b) s, o, p; c) oops

2. Sam looks at the equation \( 12 + m = 13 + n \). To make the equation true, Sam decided to use 5 as the value of \( m \). Then he solved to find the value of \( n \).

\[
12 + 5 = 13 + n \\
17 = 13 + n \\
17 = 13 + 4 \\\nn = 4
\]

a) Choose a new value for \( m \) and find \( n \).

b) Choose another new value for \( m \) and find \( n \).

c) What do you notice about \( m \) and \( n \)?
**Sample answers:** a) if \( m = 3 \), I get \( 12 + 3 = 13 + n \), \( 15 = 13 + n \), \( 15 = 13 + 2 \), \( n = 2 \); b) if \( m = 11 \), I get \( 12 + 11 = 13 + n \), \( 23 = 13 + n \), \( 23 = 13 + 10 \), \( n = 10 \)

**Answer:** c) \( m \) is always 1 more than \( n \)

3. Find the value of the unknown.

<p>| | | |</p>
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<tbody>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \begin{array}{|c|c|c|} \hline w & 8 - 3 & 2 \\ \hline \end{array} \) \( \begin{array}{|c|c|c|} \hline z & 4 & 9 + 2 \\ \hline \end{array} \) \( \begin{array}{|c|c|c|} \hline 12 + 13 & x & 7 \\ \hline \end{array} \)

\( \begin{array}{|c|c|c|} \hline k & 16 & 12 \div 3 \\ \hline \end{array} \) \( \begin{array}{|c|c|c|} \hline 20 & 6 + 7 & y \\ \hline \end{array} \) \( \begin{array}{|c|c|c|} \hline 10 + 14 & s & 9 - 6 \\ \hline \end{array} \)

**Answers:** a) \( w = 7 \), b) \( z = 15 \), c) \( x = 18 \), d) \( k = 20 \), e) \( y = 7 \),

Bonus: \( s = 21 \)

4. Make up two more questions like the ones in Extension 3 and give them to a partner to solve.
Mental math minute. Give students a subtraction problem that does not require regrouping, such as $97 - 12$. Students should then take turns repeatedly subtracting the same number, with each student formulating and answering their subtraction aloud. When a student’s subtraction involves regrouping, emphasize that this answer was a bonus. For example: Student 1 says, "$97 - 12 = 85." Student 2 says, "$85 - 12 = 73." Student 3 says, "$73 - 12 = 61." Student 4 says, "$61 - 12 = 49." SAY: That was a bonus! Continue until Student 8 says, "$13 - 12 = 1," then start a new chain.

Difference. Review the meaning of the word “difference”—how many more one number is than another. For example, 7 is 4 more than 3 (write the phrase on the board), so the difference between 7 and 3 is 4. Repeat with "3 is 2 less than 5." 

SAY: In the last lesson, we looked at how two parts, when added together, make the total. Today we’re going to subtract one part from the other to find the difference between them. Draw on the board:

Point to the two parts and SAY: The top part is larger than the bottom part, so we call them the Larger Part and the Smaller Part. Write “Larger Part” in the top and “Smaller Part” in the bottom. ASK: Which is larger, 10 or 6? (10)

Write the numbers beside the labels as shown below:

```
Larger Part  10
Smaller Part  6
```

ASK: How much larger is 10 than 6? (4) Draw the third box and label it “Difference.” SAY: The difference between 10 and 6 is 4. Write “4” as shown below:

```
Larger Part  10
Smaller Part  6  Difference  4
```
Explain that just as you could write three equations for every situation in which you had two parts and a total, you can write three equations when you have two parts and a difference. Write on the board:

\[ \text{Difference} = \text{Larger Part} - \text{Smaller Part} \]

Point to the Larger Part and the Smaller Part of the diagram you drew and ASK: If you know the Larger Part is 10 and the Smaller Part is 6, how do you find the Difference? (subtract) Point to the Smaller Part and the Difference and ASK: When you know the Smaller Part and the Difference, how do you find the Larger Part? (add) Under the word equation on the board, write “Larger Part = Smaller Part + Difference.” ASK: When you know the Larger Part and the Difference, how do you find the Smaller Part? (subtract) Under the two word equations on the board, write “Smaller Part = Larger Part − Difference.”

Writing three equations for a situation.

**Exercises:** Write three equations for the table using the words “Difference,” “Larger Part,” and “Smaller Part,” and then using the given numbers and letter.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>17</td>
<td>w</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>b)</td>
<td>7</td>
<td>w</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>c)</td>
<td>w</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>d)</td>
<td>28</td>
<td>w</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
</tr>
</tbody>
</table>

**Answers**

a) Difference = Larger Part − Smaller Part, 5 = 17 − w; Larger Part = Smaller Part + Difference, 17 = w + 5; Smaller Part = Larger Part − Difference, w = 17 − 5

b) Difference = Larger Part − Smaller Part, w = 7 − 4; Larger Part = Smaller Part + Difference, 7 = 4 + w; Smaller Part = Larger Part − Difference, 4 = 7 − w

c) Difference = Larger Part − Smaller Part, 12 = w − 23; Larger Part = Smaller Part + Difference, w = 23 + 12; Smaller Part = Larger Part − Difference, 23 = w − 12

d) Difference = Larger Part − Smaller Part, 13 = 28 − w; Larger Part = Smaller Part + Difference, 28 = w + 13; Smaller Part = Larger Part − Difference, w = 28 − 13

For each equation in the exercises above, have students identify which equations have the unknown number by itself. Point at the equations one at a time and have students show thumbs up if the unknown is by itself and thumbs down if it is not.

**Identifying the larger number in a situation.** Point out that many problems deal with a situation in which there is a larger number, a smaller number, and the difference between them. Write on the board:

- There are 3 green apples.
- There are 2 more red apples than green apples.
ASK: What types of objects are in this situation? (green apples and red apples) On the board, start a table with columns labelled “Red Apples” and “Green Apples.” ASK: Which piece of information is given: the number of green apples or the number of red apples? (green) Have students signal the number of green apples and write the number in the table. (3)

ASK: Which sentence tells us which colour of apple we have more of? (There are 2 more red apples than green apples.) Ask students to write down that sentence, then to cover the number 2 in it: “There are □ more red apples than green apples.” SAY: Now I’m going to read that sentence again as if the 2 isn’t there at all. “There are more red apples than green apples.” ASK: Which colour of apple is there more of? (red apples) Circle “red apples” on the board. ASK: How many more red apples are there? (2) Add the column “Difference” to the table on the board and write “2” in it. Write “x” in the Red Apples column. Leave the table on the board.

Write on the board:

There are 3 fewer red apples than green apples.
There are 12 red apples.

Put your finger over the 3 and ask a volunteer to read the sentence as if there is no 3. (There are fewer red apples than green apples.) ASK: Are there fewer red apples or green apples? (red)

Write the situations below on the board one at a time. For each situation, have students identify the number that comes immediately before either “more” or “fewer” and ask a volunteer to read the sentence without the number. Then have students signal the number that goes in each column in the table on the board. If the piece of data for a column is not known, students can signal the letter x by showing crossed fingers. After filling in the row for each situation, have students signal which part is the largest and circle the larger part.

a) 7 red apples, 5 more green apples than red apples
b) 5 fewer red apples, 8 green apples
c) 5 more green apples than red apples, 3 red apples
d) 9 green apples, 3 fewer green apples than red apples
e) 3 fewer red apples than green apples, 7 green apples
f) 7 red apples, 2 more red apples than green apples

Identifying the larger number and writing an equation. Add a column labelled “Equation” to the table on the board. Explain that you want to write only the equation in which the unknown number is by itself. Have volunteers help you fill in the column for part a). Have students write the equations for the rest of the rows in the table. (a) \( x = 7 + 5 \), b) \( x = 8 - 5 \), c) \( x = 3 + 5 \), d) \( x = 9 + 3 \), e) \( x = 7 - 3 \), f) \( x = 7 - 2 \)
Tell students that you will now make the task harder: they need to write the equation without the table. Present the situations in the exercises below one at a time.

**Exercises:** Circle the part that is larger, then write the equation.

a) 6 green apples  
   x red apples  
   2 more red apples than green apples

b) x spoons  
   8 forks  
   5 more forks than spoons

c) 5 cars  
   12 buses  
   x more buses than cars

d) There are 6 pears.  
   There are x apples.  
   There are 10 more apples than pears.

e) There are x rats.  
   There are 5 fewer mice than rats.  
   There are 4 mice.

f) There are 5 hats.  
   There are 4 fewer scarves than hats.  
   There are x scarves.

g) Clara has x hats.  
   Clara has 3 scarves.  
   She has 2 more scarves than hats.

h) A cat weighs 5 kilograms.  
   A dog weighs x kilograms less than the cat.  
   The dog weighs 4 kilograms.

i) Mary studied math for 20 minutes.  
   She read for 5 minutes less than she studied math.  
   She read for x minutes.

**Answers:** a) \( x = 6 + 2 \), b) \( x = 8 - 5 \), c) \( x = 12 - 5 \), d) \( x = 6 + 10 \), e) \( x = 4 + 5 \), f) \( x = 5 - 4 \), g) \( x = 3 - 2 \), h) \( x = 5 - 4 \), i) \( x = 20 - 5 \)

**Solving equations.** Remind students that equations in which the unknown is by itself require only a calculation. Have students solve the equations above.

**Answers:** a) \( x = 8 \), b) \( x = 3 \), c) \( x = 7 \), d) \( x = 16 \), e) \( x = 9 \), f) \( x = 1 \), g) \( x = 1 \), h) \( x = 1 \), i) \( x = 15 \)

**What is compared?** Write on the board:

Raj spent $12 on a book.  
The app he bought cost 4 dollars less than the book.  
How much money did he spend on the app?


- cost of book
- cost of app
ASK: How much does the book cost? ($12) Are we given the cost of the app? (no) Point out that in such situations we write $x$ for the unknown number. Write the amounts beside the labels.

**Exercises:** Solve the equation.

a) A cat weighs 5 kilograms. A dog weighs 14 kilograms more than the cat. The dog weighs $x$ kilograms.

b) Anton bikes 12 km before lunch and $x$ km after lunch. He bikes 7 km fewer before lunch than after lunch.

c) 38 people went to the book fair on Monday. On Tuesday, 45 people went to the book fair. On Tuesday, $x$ more people went to the book fair than went on Monday.

d) A male African elephant is 3 m tall. A female African elephant is 2 m tall. How much taller is the male elephant than the female elephant?

e) Sharon has 35 Canadian stamps. She has 17 more Canadian stamps than Brazilian stamps. How many Brazilian stamps does she have?

f) A flight attendant served 35 vegetarian meals. There were 88 fewer vegetarian meals than regular meals. How many regular meals were there?

**Answers:** a) 19 kg, b) 19 km, c) 7 people, d) 1 m, e) 18 stamps, f) 123 regular meals

**Writing and solving an equation given a situation or a word problem.**
Tell students that the next task is more challenging. They will need to do all the things together: decide which part is the larger number, write an equation, and solve it.

Work through the first two examples below as a class, then have students work individually to complete the rest.

a) Jane read for 25 minutes. She spent 10 minutes less on math. She spent $x$ minutes on math.

b) A large street is 30 metres wide. A smaller street is $x$ metres narrower. The smaller street is 23 metres wide.

c) Ren has 39 Canadian stamps. He has $x$ Mexican stamps. He has 15 fewer Mexican stamps than Canadian stamps.

d) Ronin has 43 blue marbles and $x$ red marbles. He has 17 more blue marbles than red marbles.

e) Rani walks 1250 m to school. Ava walks $x$ m to school. Ava walks 345 m less than Rani walks.

f) The school year is 216 days long in Russia. It is $x$ days shorter in Quebec. The school year is 180 days long in Quebec.

**Bonus:** Kate weighs 33 kilograms with her cat in her arms and 30 kilograms without it. The cat weighs $x$ kilograms.
Answers: a) 15 min on math, b) 7 m narrower, c) 24 Mexican stamps, d) 26 red marbles, e) Ava walks 905 m, f) school year in Quebec is 36 days shorter, Bonus: 3 kg

Extension

The table shows population growth data for some territories from 2011 to 2016. The population increased in all three regions during that period. Fill in the missing numbers.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Northwest Territories</td>
<td>41 462</td>
<td>44 291</td>
<td>2829</td>
</tr>
<tr>
<td>Yukon</td>
<td>33 897</td>
<td>37 193</td>
<td>3296</td>
</tr>
<tr>
<td>Nunavut</td>
<td>31 906</td>
<td>37 174</td>
<td>5268</td>
</tr>
</tbody>
</table>

Answers
Addition and Subtraction Word Problems

Mental math minute. Have students add three-digit numbers by adding hundreds, tens, and ones separately in groups of four. Give an addition problem, such as 135 + 246. The first student adds the hundreds, 100 + 200; the second adds the tens, 30 + 40 = 70; the third adds the ones, 5 + 6 = 11; and the fourth student finishes the addition, 300 + 70 + 11 = 381, so 135 + 246 = 381. Start with problems that do not require regrouping in any place, such as 325 + 634, and progress to questions that require regrouping ones or regrouping tens, but not both.

Organizing data. SAY: Remember, in word problems, we need to decide which piece of data is the unknown. Explain that with long and complex problems, it is convenient to write out the data in short form. Write on the board:

Karen spends 25 minutes doing her math homework.

She spends 15 minutes more on her science project than on math homework.

How much time did Karen spend on her science project?

Tell students that they can write the data in short form. Continue writing on the board:

25 minutes for math homework

15 minutes more on science project than on math homework

x minutes for science project

ASK: Which way of presenting the problem makes it easier to write an equation? (the second way) Where did I get the part “x minutes for science project” from? (it answers the question for the problem)
Exercises: Write out the data in short form. Then solve the problem.

a) Jayden has 25 marbles. 7 of the marbles are green. How many are not green?

b) There are 7 rats and 9 hamsters in a store. How many rats and hamsters altogether are in the store?

c) There are 25 cars in a parking lot. There are 7 fewer vans than cars in the lot. How many vans are in the lot?

d) Nina hiked 7 km before lunch and 10 km after lunch. How far did she hike altogether?

Answers: a) 18 marbles are not green, b) 16 rats and hamsters, c) 18 vans, d) 17 km

Review totals and differences. Draw the picture in the margin on the board. Remind students that there are two things they can find given these two numbers: the difference between the two numbers and the total. Ask volunteers to show the difference and the total in the diagram and to find the answer for each one. (the difference is 4, the total is 10) Review writing an equation for the total and an equation for the difference. Now present students with a situation where the larger number and the total are given and they need to find the smaller number: larger number 8, smaller number \(x\), total 13. (5) Finally, present students with a situation where the smaller number and the difference are given and they need to find the larger number: large number \(x\), small number 10, difference 4. (14)

Difference or total? Draw a table with headings “Parts,” “Total,” and “Difference” on the board. Complete the exercises below together as a class. Have students identify which piece of data belongs in which column.

Exercises

a) 6 green grapes, 8 more black grapes than green grapes

b) 10 forks, 22 forks and spoons altogether

c) 5 cars, 21 cars and buses in a parking lot

d) There are 6 bananas. There are 10 fewer bananas than kiwis.

e) A cat weighs 6 kilograms. A dog weighs 5 kilograms more than the cat.

f) Lily pays $12 for a hat and $15 for a pair of mitts. She pays \(x\) in total for the mitts and the hat.

g) Abella studies math for 20 minutes. Math and reading take 45 minutes altogether. For how long did she read?

h) A salad recipe asks for 3 onions. Five more tomatoes than onions are needed for the salad. How many tomatoes are needed?
ACTIVITY (Essential)

Give students cards from BLM Word Problem Cards. Students sort the cards according to the problems. (The cards belonging to the same problem have the same picture.) Students write an answer sentence with an x below the question. For example, the question “How much does Fido weigh?” should have the answer sentence “Fido weighs x kilograms.” Students place the cards in the table they created during the previous exercises. Have students solve each equation. (4 kg, 4 more minutes, 28 push-ups and sit-ups, 35 minutes, $25)

Writing and solving an equation for a word problem. Have students write and solve the equation for each situation or problem in the previous exercises. Work through the first two problems together as a class. (a) 14 black grapes, b) 12 spoons, c) 16 buses, d) 16 kiwis, e) 11 kilograms, f) $27 total, g) 25 minutes, h) 8 tomatoes

Combining tasks: organizing data, writing, and solving the equation. Work through the first two parts in Exercise 1 below as a class, then have students complete the rest individually.

Exercises
1. Organize the data, write the equation, then solve.
   a) Braden bought 12 books and 7 magazines. How many books and magazines did he buy altogether?
   b) A book costs $10. A poster is $4 cheaper than the book. How much does the poster cost?
   c) A bird store sells parrots and canaries. There are 27 canaries in the store. There are 12 fewer parrots than canaries. How many parrots are in the store?
   d) Iva read 12 pages on Sunday. She read 7 pages more on Sunday than on Monday. How many pages did she read on Monday?
   e) A cake recipe calls for 5 cups of berries. Randi has 3 cups of raspberries and some blueberries. How many cups of blueberries will she need for the cake?

   Answers: a) 19 books and magazines, b) $6, c) 15 parrots, d) 5 pages, e) 2 cups

2. There are 32 students in a class. 13 of them play lacrosse. The rest do not play lacrosse.
   a) How many students do not play lacrosse?
   b) How many more students do not play lacrosse than students who do?

   Answers: a) 19, b) 6 more don’t play as do
3. Rick bought 8 hockey cards and 10 baseball cards. He gave away 3 cards.
   a) How many cards did he buy altogether?
   b) How many cards does he have left?

   **Answers:** a) 18 cards altogether, b) 15 cards left

**Bonus:** The population of Prince Edward Island in 1861 was 80,857. In 2016, the population of Prince Edward Island was 142,907. How many years passed between the two dates? How much did the population grow in that time?

   **Answers:** 155 years, 62,050 people

**Extensions**

1. Liz found data about the population of four provinces in 2017.

<table>
<thead>
<tr>
<th>Province</th>
<th>Population in 2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Brunswick</td>
<td>747,101</td>
</tr>
<tr>
<td>Newfoundland and Labrador</td>
<td>519,716</td>
</tr>
<tr>
<td>Nova Scotia</td>
<td>923,598</td>
</tr>
<tr>
<td>Prince Edward Island</td>
<td>142,907</td>
</tr>
</tbody>
</table>

   Estimate by rounding to the largest place value that produces a meaningful estimate, then calculate the exact answer.

   a) How many more people lived in New Brunswick than in Newfoundland and Labrador?
   b) How many fewer people lived in Prince Edward Island than in Nova Scotia?
   c) How many more people lived in Nova Scotia than in New Brunswick?

   **Bonus:** How many people lived in Nova Scotia and Prince Edward Island altogether?

   **Answers:** a) 227,385, b) 780,691, c) 176,497, Bonus: 1,066,505
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<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Edmonton, AB</td>
<td>730 372</td>
<td>81 829</td>
<td>932 546</td>
</tr>
<tr>
<td>Calgary, AB</td>
<td>988 812</td>
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a) What was the population of Edmonton in 2011?
b) How much did Edmonton grow in population from 2011 to 2016?
c) How much did Calgary grow from 2011 to 2016?
d) Which city, Edmonton or Calgary, grew more from 2011 to 2016?
e) Use estimation to check your answers.

**Answers:**
a) 812 201, b) 120 345, c) 142 387, d) Calgary,
e) Estimations: a) $730 000 + 80 000 = 810 000$; b) $930 000 - 810 000 = 120 000$; c) $1 240 000 - 990 000 - 110 000 = 140 000$; d) 140 000 > 120 000, so Calgary has grown more.
Models and Times as Many

Goals
Students will use tape diagrams to solve word problems with “times as many.”

PRIOR KNOWLEDGE REQUIRED
Can solve a single-step word problem requiring addition or subtraction
Understands the expression “times as many”
Can identify parts, total, and difference in a problem
Can multiply up to $9 \times 9$ and perform the corresponding divisions

Mental math minute. Ask students to solve multiplication questions within the range of $0 \times 1$ to $10 \times 10$. For each number, first go through the questions in order, such as $0 \times 3$, $1 \times 3$, and so on to $10 \times 3$, then in reverse order. After that, go through the same questions out of order. Then progress to a different number.

Drawing a diagram for a “times as many” situation. Tell students that Kim and Rob have some stickers. Write on the board:

Kim has three times as many stickers as Rob.

SAY: I want to draw a diagram to represent this situation. ASK: Who has more stickers, Kim or Rob? (Kim) Draw a small rectangle on the board and explain that this rectangle represents Rob’s stickers. Label the bar as shown in the margin. ASK: How can we show that Kim has three times as many stickers as Rob? Accept all reasonable answers. Then explain that you are going to use a specific way to draw a diagram. You will make a bar that contains three of Rob’s bar of stickers. Finish the picture in the margin on the board and leave it for future reference. Explain that this type of diagram is called a tape diagram because the picture looks like two pieces of tape—one for Rob’s stickers and one for Kim’s stickers.

SAY: Tasha has four times as many pennies as dimes. Draw the tape diagrams below on the board and ask which of them would fit the situation and which would not. Have students explain why the diagrams that do not fit the situation do not work.

A. number of dimes (yes)
   number of pennies

B. number of dimes
   number of pennies (no, all the bars have to be the same size)
C. number of dimes  
number of pennies  
(no, there are more dimes than pennies)

D. number of dimes  
number of pennies  
(no, there are five times as many pennies as dimes)

ASK: How do you know that the short bar should be the number of dimes? (there are more pennies than dimes)

SAY: Kyle is twice as old as Amy. ASK: Whose age will have the shorter bar? (Amy’s) Why? (because Kyle is older, so his age bar will be longer) Have students draw a tape diagram for the situation.

As you present each situation in the exercises below, ask students to first identify and draw a bar for the smaller number and remind them that this should be the shorter bar. Point out that it doesn’t matter which bar is above the other: it could be the shorter or the longer one. For the last exercise, make sure students understand the meaning of "twice."

**Exercises:** Draw a tape diagram for the situation.

a) Cathy is three times as tall as her baby brother.

b) Sandy’s full name is four times as long as Matt’s.

c) There are eight times as many students in the school as in our class.

d) The library is three times as far from my home as the school.

e) A book is twice as thick as a notebook.

**Sample answers**

a) Cathy’s height  
Brother’s height  
b) Sandy’s name  
Matt’s name  
c) Students in class  
Students in school  
d) Distance to library  
Distance to school  
e) Thickness of book  
Thickness of notebook  

Finding the length of the bars when the smaller part is given. Return to the first diagram you created for the situation with Rob and Kim. Tell students that Rob has 4 stickers. Write “4” in Rob’s block. Remind students that the blocks are the same and then write “4” in each of Kim’s blocks.

ASK: Can you tell from the diagram how many stickers Kim has? (yes, 12) How do you know? (there are 3 blocks of 4) Have students write the multiplication statement for the length of the longer bar. ($4 \times 3 = 12$)
Have students draw a tape diagram and find the lengths of the bars for the situation: Ella has 5 red marbles. She has twice as many green marbles as red marbles. Ask volunteers to show the answers on the board. (10 green marbles)

**Exercises:** Draw a model for the story.

a) There are 6 apples on the table. There are twice as many bananas as apples.

b) A car holds 4 people. A van holds three times as many people.

c) Don’s apartment building is 3 stories high. Yu’s building is five times as high as Don’s.

d) Cody is 6 years old. Jin is four times as old as Cody.

**Bonus:** A sparrow has 4 eggs in its nest. A duck has three times as many eggs in its nest as the sparrow. An ostrich has five times as many eggs in its nest as the sparrow.

**Answers**

a) apples  
   bananas

b) car  
   van

   \[ \frac{20}{4} = 5 \]

   Cody

   \[ \frac{20}{4} = 5 \]

   Jin

   \[ \frac{20}{4} = 5 \]

   \[ \frac{20}{4} = 5 \]

   \[ \frac{20}{4} = 5 \]

**Using tape diagrams to solve problems when the larger part is given.**

Write on the board:

Shelly has 20 stickers.
Shelly has four times as many stickers as Luc.

Invite a student to draw the bars for the situation, without writing the numbers.

**ASK:** How many blocks are in Shelly’s bar? (4) **SAY:** Shelly has 20 stickers.

**ASK:** How many stickers does each block represent? (5) **How do you know?** (20 \( \div 4 = 5 \)) How many stickers does Luc have? (5)

Work through the first exercise below as a class and have students work individually on the rest.
Exercises: Draw the model and find the size of each block.

a) There are 6 apples on the table. There are twice as many apples as pears. How many pears are there?

b) A mini-bus holds 16 people. The mini-bus holds twice as many people as a van. How many people can the van hold?

c) Tristan’s apartment building is 30 stories high. Tristan’s building is five times as high as Anne’s building. How tall is Anne’s building?

d) Ethan is 14 years old. Ethan is seven times as old as Tina. How old is Tina?

Bonus: A sugar pine cone is 45 cm long. It is three times as long as an eastern white pine cone. The sugar pine cone is nine times as long as a jack pine cone. How long is the eastern white pine cone? How long is the jack pine cone?

Answers: a) 3 pears, b) 8 people, c) 6 stories, d) 2 years old, 
Bonus: eastern white pine cone = 15 cm long, jack pine cone = 5 cm long

Finding the size of a single block when the difference is given. Explain that a student drew the diagram in the margin for a word problem. The problem stated that the difference between the parts was 18. ASK: What does this mean? (the longer bar is 18 more than the shorter bar) Show how to mark this on the diagram by adding a bracket below the difference and writing “18.” ASK: How many blocks long is the difference? (3) What is the size of each block? (6) How do you know? (18 \( \div \) 3 = 6)

Exercises: What is the size of each block?

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b) \[
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15

16


Answers: a) 5, b) 8, c) 7, d) 11

Finding the size of a single block when the total is given. Explain that a student drew the tape diagram in the margin for a different word problem. Again, all the blocks are the same length. The problem stated that the total was 18. Show how to mark this on the diagram using a vertical bracket. ASK: How many blocks are there in total? (9) What is the size of each block? (2) How do you know? (18 \( \div \) 9 = 2)
Exercises

1. What is the size of each block?
   a) \[15\]  
   b) \[16\]  
   c) \[28\]  
   d) \[120\]  
   **Answers:** a) 3, b) 4, c) 4, d) 12

2. All the blocks are the same size. What is the size of each block?
   a) \[55\]  
   b) \[28\]  
   c) \[48\]  
   **Bonus:** \[130\]  
   **Answers:** a) 11, b) 14, c) 12, Bonus: 10

**Solving problems with difference or total given.** Tell students that now they will need to draw the tape diagrams themselves. Write on the board:

Amir is three times as old as Evan.
Evan is 16 years younger than Amir.
How old is Evan?

Ask students to draw a diagram that fits the first sentence. Have a volunteer draw it on the board. ASK: What does the second sentence give us: the difference, the total, or one of the parts? (the difference) Have students mark that on the diagram. ASK: How large is one block? (8) How do you know? (16 ÷ 2 = 8) How many blocks long is Evan’s bar? (1 block) How old is Evan? (8 years old) How long is Amir’s bar? (3 blocks) How old is Amir? (24 years old)

Work through the first two exercises below as a class, then have students work individually on the rest.

**Exercises**

a) Eddy saved three times as much money as Tina. Tina saved $18 less than Eddy. How much money do they have altogether?

b) Jin and Yu put all their money together to buy a gift for their grandmother. They have $60 together. Jin has twice as much money as Yu has. How much money does each of them have?
c) Lela is three times as tall as her baby brother Jax. She is 42 cm taller than Jax. How tall is Jax? How tall is Lela?

d) Alex’s full name is four times as long as Rayder’s. Rayder’s full name is 54 letters shorter. How long is each full name?

e) The number of students in the school who are not in Grade 4 is eight times as large as the number of students in Grade 4. There are 248 students in the school who are not in Grade 4. How many students altogether are in the school?

f) The library is four times as close to Arsham’s home as the school. Arsham walks home from school, then goes to the library. This makes a walk of 15 blocks. How far from Arsham’s home are the school and the library?

g) A number is five times as large as a smaller number. If you add the two numbers together, you get 54. What are the numbers?

**Answers:**
a) Eddy saved $27, Tina saved $9, $36 altogether; b) Yu had $20, Jin had $40; c) Jax is 21 cm tall and Lela is 63 cm tall; d) Rayder’s full name is 18 letters long, Alex’s full name is 72 letters long; e) 279 students in the school in total; f) the library is 3 blocks away from Arsham’s home, the school is 12 blocks away; g) 9 and 45

**Bonus:** Jack reads the same number of pages every school day and twice as many pages every weekend day. He finished a book of 108 pages in a week. How many pages does he read on Monday? How many pages does he read on Sunday?

**Answer:** 12 pages on Monday, 24 pages on Sunday

**Extension**

Choose one of the tape diagrams from the last exercises in the lesson and invent a new word problem that would fit it. Have a partner solve the problem.
PA4-18 Problems and Equations—Multiplication and Division

Goals
Students will use equations to solve one-step multiplication word problems.

PRIOR KNOWLEDGE REQUIRED
Knows that a variable can replace a number in an equation
Understands the expression “times as many/much”
Can multiply up to 9 × 9 and perform the corresponding divisions

Mental math minute. Ask students to solve multiplication questions within the range of 0 × 1 to 10 × 10. For each number, first go through the questions in order, such as 0 × 3, 1 × 3, and so on to 10 × 3, then in reverse order. After that, go through the same questions out of order. Then progress to a different number.

NOTE: Students who find it difficult to write equations without first drawing a diagram can draw diagrams as needed.

Introduce scale factor. Present a situation: Rani has 3 marbles. Sam has four times as many marbles as Rani. Have a volunteer draw a diagram for this situation. Explain that the number that tells us how many times larger one part is than the other is called the scale factor. SAY: In this situation, the scale factor is 4.

Exercises: Identify the scale factor.

a) There are 2 green apples and three times as many red apples as green apples.

b) Glen is four times as old as Nina. Nina is 2 years old.

c) cat’s weight

d) Kathy’s savings

dog’s weight

Answers: a) 3, b) 4, c) 2, d) 4

Writing an equation using a scale factor to find parts. Explain that you can use the scale factor to write two equations that show how to use one part to find the other part. Write on the board:

Larger Part = Smaller Part × Scale Factor

Smaller Part = Larger Part ÷ Scale Factor

Look at the diagrams from the previous exercises with the class to make sure that both equations make sense.
Exercises: Write two equations for the situation, one equation telling how to get the larger part, the other telling how to get the smaller part.

a) \( w \) mice, 6 rats, three times as many mice as rats
b) 3 blue marbles, \( w \) green marbles, five times as many green marbles as blue marbles
c) 8 boys, \( w \) girls, twice as many boys as girls

Answers: a) \( w = 6 \times 3, 6 = w \div 3; \) b) \( w = 3 \times 5, 3 = w \div 5; \) c) \( w = 8 \div 2 = 4, 8 = w \times 2 \)

Remind students that equations in which the unknown number is by itself are very easy to solve—you only need a calculation. Have students identify the equations in which \( w \) is by itself in the previous exercises.

Tell students that in the next exercises, you want them to write only one equation, the one that has \( w \) by itself.

Exercises: Write an equation with \( w \) by itself.

a) Neka earned $42 babysitting. Jen earned \( w \) dollars mowing lawns. Neka earned three times as much as Jen.
b) Amir hiked 9 km on Monday. He hiked \( w \) km on Tuesday. He hiked three times as far on Monday as on Tuesday.
c) A recipe calls for 2 cups of oatmeal and three times as much flour. How much flour is needed?
d) A dog weighs twice as much as a rabbit. The dog weighs 6 kg. How much does the rabbit weigh?

Answers: a) \( w = 42 \div 3, \) b) \( w = 9 \div 3, \) c) \( w = 2 \times 3, \) d) \( w = 6 \div 2 \)

If students have trouble with problems that include units (distance, weight, etc.), point out that they can treat units such as kilometres the same way as they treat objects such as marbles. Three times as many as 4 marbles is 12 marbles, and three times as far as 4 km is 12 km. ASK: If Lewis is three times as old as Mandy, and Mandy is 4 years old, how old is Lewis? (12 years old) If a table is three times as heavy as a chair, and the chair weighs 4 kg, how heavy is the table? (12 kg)

Review the connection between sets and multiplication. Remind students that we use multiplication to find the total number of objects in equal sets. SAY: For example, 5 people can sit in each car. There are 3 cars. ASK: How many people are in 3 cars? (15) How do you know? (\( 3 \times 5 = 15 \)) Remind students that to find the total number of objects, they need to multiply the number in each set by the number of sets. To find either of the other two numbers, they need to divide the total by one of those numbers.

Write an equation for a story with an unknown number. Present a few situations and have students identify which number in the situation shows the total, which number shows the number of objects in a set, and which number shows the number of sets.
Draw on the board:

<table>
<thead>
<tr>
<th>Number in Each Set</th>
<th>Number of Sets</th>
<th>Total Number of Objects</th>
</tr>
</thead>
</table>
| Point at the Total Number of Objects column and SAY: If we don’t know the Total, we multiply. When we don’t know either of the other two numbers, we divide. Have students use the headings on the board in the following exercises.

**Exercises:** Fill in a row in the table for the situation.

a) There are 8 people in each van. There are \( w \) vans. There are 24 people altogether.

b) There are \( w \) cookies on each plate. There are 12 plates. There are 48 cookies altogether.

c) There are 4 markers in each pack. There are 11 packs. There are \( w \) markers altogether.

d) There are 5 erasers in each pack. Aputik bought \( w \) packs. Aputik bought 35 erasers altogether.

**Bonus:** An octopus has 8 arms. There are \( w \) octopuses. There are 240 arms altogether.

**Answers**

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<tr>
<th>Number in Each Set</th>
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<td>12</td>
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Add a column to the table with the heading “Equation”. Go through the problems in the previous exercises one by one and ask students to write a multiplication or a division equation in which the unknown is by itself. Remind students who are struggling that they need to multiply to find the total number of objects and divide in the other two cases.

Have students solve the equations and find the unknown numbers.

(a) \( w = 24 ÷ 8 = 3 \), b) \( w = 48 ÷ 12 = 4 \), c) \( w = 4 \times 11 = 44 \),

d) \( w = 35 ÷ 7 = 5 \), Bonus: \( w = 240 ÷ 8 = 30 \)

Present a few word problems and have students use the whole process together to solve them: writing the information (in a table as needed), writing and solving the equation, and writing the answer statement.
Exercises

a) The Bead Club makes and sells bracelets for charity. They sold 12 bracelets. There are five times as many bracelets left as they sold. How many bracelets are left?

b) Jayden read 18 books for Battle of the Books. Jayden read three times as many books as Sara. How many books did Sara read for Battle of the Books?

c) Alex is 8 years old. Yu is three times as old as Alex. How old is Yu?

d) Ivan is 8 years old. Ivan is four times as old as Tess. How old is Tess?

e) A book costs $12. The book is twice as expensive as a magazine. How much does a magazine cost?

f) An app costs $9. A book costs three times as much as the app. How much does the book cost?

g) A pair of gloves costs $12. The pair of gloves is three times as expensive as a pair of socks. How much do the socks cost?

h) A puzzle costs $11. A construction toy costs three times as much as the puzzle. How much does the construction toy cost?

i) Anna spends twice as much time reading as she spends on math. She spends 20 minutes reading. How much time does she spend on math?

j) David spends three times as much time reading as he spends on French. He spends 10 minutes on French. How much time does he spend reading?

k) Kate lives twice as far from school as Ronin. Ronin lives 3 blocks from school. How far from school does Kate live?

l) Matt walks 8 blocks to school. The walk to school is twice as long as the walk to the library. How far is the library?

m) A spaniel weighs 15 kg. A Rottweiler is three times heavier. How heavy is the Rottweiler?

Bonus

n) Sumatran tigers are endangered. Only about 500 Sumatran tigers are left in the wild. There are about five times as many Bengal tigers as there are Sumatran tigers left in the wild. How many Bengal tigers are left in the wild?

o) There are about ten times as many Sumatran tigers as Indo-Chinese tigers left in the wild. How many Indo-Chinese tigers are left in the wild?

Sample solution: a) \[12 \times 5 = w, 60 = w, 60\] bracelets are left

Answers: b) 6, c) 24, d) 2, e) $6, f) $27, g) $4, h) $33, i) 10 minutes, j) 30 minutes, k) 6 blocks, l) 4 blocks, m) 45 kg, Bonus: n) 2500, o) 50
Extensions

1. A subway station has two parking lots. There are 20 rows of 35 parking spots at the East lot and 25 rows of 30 parking spots at the West lot.
   a) How many cars can park at each lot? Which lot has more parking spots?
   b) How many cars can park at the subway station in total?

   **Answers:** a) 700 in the East lot and 750 in the West lot, b) 1450

2. Someone wants to donate money to 20 different charities for a total of $20 000. If each charity gets the same amount, how much money will each charity get?

   **Answer:** $1000

3. Eight people can ride in a van. Six times as many people can ride on a regular bus. A double-decker bus can hold ten times as many people as a van. How many people altogether can ride in a van, a bus, and a double-decker bus?

   **Solution:** \(8 + 48 + 480 = 536\)
### Equation Dominoes (Addition)

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<td>+ 6 = 19</td>
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<td>+ 3 = 18</td>
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<td>+ 3 = 18</td>
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<td>+ 3 = 19</td>
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<tbody>
<tr>
<td>4</td>
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<td>3</td>
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<td>20</td>
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<td>1</td>
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**Equation Dominoes (Multiplication)**

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<tr>
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<tbody>
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<td>$2 \times \square = 20$</td>
<td>$3 \times \square = 27$</td>
<td>$5 \times \square = 40$</td>
<td>$5 \times \square = 35$</td>
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<td>14</td>
<td>8</td>
<td>12</td>
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<tr>
<td>$4 \times \square = 24$</td>
<td>$9 \times \square = 45$</td>
<td>$7 \times \square = 28$</td>
<td>$2 \times \square = 6$</td>
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<td>3</td>
<td>7</td>
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<tr>
<td>$22 \times \square = 44$</td>
<td>$79 \times \square = 79$</td>
<td>$\square \times 8 = 24$</td>
<td>$\square \times 3 = 21$</td>
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<td>11</td>
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<td>$\square \times 7 = 49$</td>
<td>$\square \times 6 = 48$</td>
<td>$\square \times 7 = 63$</td>
<td>$\square \times 3 = 15$</td>
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<tr>
<td>$\square \times 2 = 28$</td>
<td>$\square \times 8 = 96$</td>
<td>$\square \times 4 = 44$</td>
<td>$\square \times 2 = 24$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>
Word Problem Cards

Sara's cat Fluffy weighs 5 kilograms. Sara's dog Fido weighs 1 kilogram more than Fluffy. How much does Fido weigh?

Lela read for 12 minutes. Lela biked for 16 minutes. How many more minutes did Lela bike than read?

Amir did 12 push-ups. Amir did 16 sit-ups. How many push-ups and sit-ups did Amir do altogether?

The lunch recess is 55 minutes long. Sandy spent 20 minutes eating. Sandy went to the library for the rest of recess. How much time did she spend in the library?

Rani earned $16 by tutoring. Simon earned $9 more than Rani by babysitting. How much money did Simon earn?
PS4-6 Searching Systematically II

Teach this lesson after:
Unit 11

VOCABULARY
clockwise
domino
length
operation
pentomino
perimeter
rectangle
square
tetromino
triangle
triomino

Goals
Students will organize their search to find all possible answers to a given problem by using order, such as the order in numbers or clockwise order.
Students will learn the strategy of working backward.

PRIOR KNOWLEDGE REQUIRED
Can find the perimeter of a rectangle given its side lengths
Can multiply one-digit numbers
Can convert between months and years (for Problem Bank 7)
Can convert between millimetres and centimetres (for Problem Bank 8)
Can convert between days and weeks (for Problem Bank 9)
Can convert between years and decades (for Problem Bank 10)
Can determine the missing factor, up to 11, when the missing factor is multiplied by a one-digit number (for Problem Bank 11)

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. N-53)

Finding one unknown side length when the other side length and the perimeter are given. Draw on the board:

```
5 cm

? Perimeter = 18 cm
```

SAY: The length of the given side of the rectangle is five centimetres, and I want to find the length of the other side. Let’s start at one and then go up to try possible lengths in order. That way, we know we won’t miss any. Draw on the board:

<table>
<thead>
<tr>
<th>Given Side (cm)</th>
<th>Other Side (cm)</th>
<th>Rectangle</th>
<th>Perimeter (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td><img src="5+5+1+5=12" alt="Rectangle 1" /></td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td><img src="5+5+1+5=12" alt="Rectangle 2" /></td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>
Have volunteers complete each row of the chart. After each row, ASK: Is the perimeter 18? (no, no, no, yes) SAY: So the other side is four centimetres.

**Exercises**

a) The perimeter of a rectangle is 20 cm. One side is 3 cm. What is the length of the other side?

b) The perimeter of a rectangle is 14 cm. One side is 6 cm. What is the length of other side?

c) The perimeter of a rectangle is 16 cm. One side is 1 cm. What is the length of other side?

**Bonus:** The perimeter of a rectangle is 30 cm. One side is 3 cm. What is the length of other side?

**Answers:** a) 7 cm, b) 1 cm, c) 7 cm, Bonus: 12 cm

Using a chart to find the pattern when trying numbers in order. Draw on the board:

\[
\begin{align*}
A & \times 5 \rightarrow \square + 3 \rightarrow \square \times 2 \rightarrow \square - 2 \rightarrow 74 \\
1 & \times 5 \rightarrow \square + 3 \rightarrow \square \times 2 \rightarrow \square - 2 \rightarrow \_ \\
2 & \times 5 \rightarrow \square + 3 \rightarrow \square \times 2 \rightarrow \square - 2 \rightarrow \_ \\
3 & \times 5 \rightarrow \square + 3 \rightarrow \square \times 2 \rightarrow \square - 2 \rightarrow \_
\end{align*}
\]

**NOTE:** In the following exercises, students will find the ending numbers of the operations on the board.

**Exercises:** Find the ending number for the starting number.

a) 1     b) 2     c) 3

**Answers:** a) 14, b) 24, c) 34

After students finish the exercises, have volunteers show the steps for each one on the board. SAY: Let’s put the answers into a table. Draw the table on the next page on the board, but have volunteers put in the ending numbers.
ASK: Can you predict the next ending number? (44) How did you get that? (added 10) Have a volunteer complete the ending number row until they see 74, as shown below:

<table>
<thead>
<tr>
<th>Starting Number</th>
<th>Ending Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td></td>
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<td>5</td>
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<td>6</td>
<td></td>
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<td>7</td>
<td></td>
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<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

ASK: What starting number has the ending number 74? (7) How did the chart help you have less work? (I saw a pattern, so I didn’t have to try each number) SAY: When you see a pattern in the ending numbers, you don’t have to repeat all the calculations each time, but instead you can just use the pattern. It’s easier to see a pattern if you try the numbers in order, starting at 1.

NOTE: Some students might notice that the ending number can be obtained from the starting number by just writing the ones digit 4 after the starting number. This works as an even faster method for this example, but a similar trick will not work for most of the following exercises.

Exercises

1. Do the operations that are on the board (× 5, + 3, × 2, − 2) in order starting with 7. Make sure you get 74.

Answers: 7, 35, 38, 76, 74
2. Eddy does the same operations to the starting numbers 1, 2, and 3, and records the ending numbers in a table.

<table>
<thead>
<tr>
<th>Starting Number</th>
<th>Ending Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>49</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
</tr>
</tbody>
</table>

Predict what starting number has an ending number of 73. Explain how you found your prediction.

Sample answer: I extended the table: 4, 61; 5, 67; 6, 73; so I predict 6.

3. Find A so that you get the ending number after doing all the operations.

a) \[ A \times 2 + 4 \times 2 - 1 \rightarrow 39 \]

b) \[ A + 3 \times 5 - 4 \times 2 \rightarrow 82 \]

c) \[ A \times 2 - 1 \times 2 + 3 \rightarrow 41 \]

d) \[ A \times 25 + 3 \times 4 - 2 \rightarrow 910 \]

e) \[ A + 4 \times 5 + 316 \times 4 \rightarrow 1504 \]

Answers: a) 8, b) 6, c) 10, d) 9, e) 8

NOTE: In Problem Bank 11, students will learn how to do this type of problem by working backward instead of by looking for a pattern.

Organizing possible answers in order of size. Draw on the board:

SAY: There are many rectangles here. Draw on the board:

SAY: There are math problems that ask you to figure out how many ways you can draw a rectangle in a big rectangle, using the grid lines shown. We want to look for an organized way to draw all possible rectangles. ASK: What is different about the three rectangles I outlined? (they are different sizes—1, 4, and 3 squares; they are in different places) What are all the possible sizes of the
rectangles? (1, 2, 3, 4, 5, or 6 squares) Erase the previous rectangles and draw on the board:

Now, draw a blank grid on the board:

Point to the rectangle of one square and ask a volunteer for another way to draw a rectangle that size in the big rectangle. Continue until all six possibilities are found. Write on the board:

There are 6 possible rectangles (actually squares) of size 1 square.

Now point to the rectangle of two squares outlined and ask a volunteer for another way to draw a rectangle that size. Point out that the two squares can be anywhere in the big rectangle, but they have to be next to each other. There are five possibilities altogether, so volunteers should find four more. The possibilities are shown below:

Write on the board:

There are 5 possible rectangles of size 2 squares.

SAY: When you try possible answers in order up through the different sizes, starting at one, then two, and so on, then you know that you won’t miss any. Also, when you shift the rectangle of two squares by one square each time, you find all possible locations. That’s why it’s important to be organized.

NOTE: Provide students with grid paper or BLM 1 cm Grid Paper for the following exercises.

Exercises: How many rectangles are there …

a) of size 3? b) of size 4? c) of size 5?

d) of size 6? e) of sizes 1, 2, 3, 4, 5, or 6 altogether?

Answers: a) 4, b) 3, c) 2, d) 1, e) 21

ASK: How did organizing the search help to make sure you didn’t miss any? (because when you go through the sizes in order, you know you won’t miss any sizes; because when you shift the rectangle by 1 space, you know you won’t miss any locations)
Write on the board:

How many squares of any size are there in a 3 by 4 rectangle?

SAY: Now we are looking for just squares, not rectangles. ASK: How many different sizes of squares are there? (three: 1 by 1, 2 by 2, and 3 by 3) Have volunteers show examples on the board. (see sample answers below)

Remind students that the squares can go anywhere in the rectangle as long as they go along the grid lines, and that you want to know how many ways there are of doing that. Point to each size of square and ASK: How many of this size are there? (12 of size 1 by 1, 6 of size 2 by 2, 2 of size 3 by 3) Draw the first three squares of size 2 by 2 and point out how you found the first three possibilities in an organized way by starting at the top and noting them from left to right. Have a volunteer show the next three by starting at the bottom and again going from left to right. The six squares are shown below:

Have another volunteer show the two squares of size 3 by 3, as shown below:

ASK: So, how many squares are there altogether, of any size? (20)

Exercises: How many squares are there in the rectangle altogether? Hint: Find how many there are of each size first.

a) 

b) 

c) 

d) 

Selected solution: c) There are 16 squares of size 1 by 1, 9 squares of size 2 by 2, 4 squares of size 3 by 3, and 1 square of size 4 by 4, for a total of 30 squares.

Answers: a) 8, b) 14, d) 40

Using clockwise order to organize. SAY: Sometimes in a problem, the tricky part is figuring out how to organize possibilities. Draw on the board:
SAY: Here is a domino. It has two squares. A *triomino* is like a domino, but it has three squares. You can make a triomino from a domino by adding a square. You have to add a square so that it shares an edge with a square in the domino. Draw on the board:

![Diagram of dominoes: one correct, one incorrect, one with an edge not shared.]

SAY: I want to know all the different shapes that I can get by adding a square in this way. To do that, I need an organized way of adding squares. ASK: How can I add squares in an organized way? (go in order around the shape) SAY: You can pick any edge to start with, but it makes sense to start at the top left corner and go around the shape in clockwise order. Draw on the board:

![Diagram of triominos: all the possible ways to add a square, showing six shapes.]

SAY: There are six ways that a new square can share an edge with one of the squares in the domino. You can attach a square to any of those six places around the shape. Add a square to each rectangle as shown below:

![Diagram of all possible triominos, with an example of two different shapes crossed out.]

SAY: It looks like there are six different shapes, but some of them are really the same. Let’s cross out all the shapes that are the same as one that comes before it. Point to the first shape and SAY: Here’s one shape. Point to the second shape and ASK: Is this the same or different than the one before it? (same) SAY: You can turn it or flip it to fit onto the other shape, so it’s really the same shape. Cross out the second shape. Pointing to the third shape, ASK: Is this the same or different from the first one? (different) Continue in this way to see that, in fact, there are only two different shapes, the first and the third. SAY: There is only one domino, and there are only two triominos.

SAY: A *tetromino* is made of four squares in the same way.

**NOTE:** If students are struggling with the exercises below, have them count the number of places a new square can touch the shape (this will be equal to the perimeter of the shape). Students may need to put a dot on the top left corner to remember where they started so that they don’t count past that point. Students should then copy the shape that many times before they add the edges.

**Exercise:** How many different tetrominoes can you make by adding one square to the triomino?

![Diagram of tetrominoes: three possible shapes shown.]  

**Solution:** 3

![Diagram of all possible tetrominoes, with three different shapes shown.]
Problem Bank

1. How many different shapes can you make by adding one square to the figure?
   a) [Diagram]  b) [Diagram]  c) [Diagram]

   **Sample solution:** c) 5

   ![Additional diagrams showing different shapes](image)

   **Answers:** a) 4, b) 1

2. How many different tetrominoes are there? Remember: A tetromino is made of four squares.

   **Solution:** 5

   ![Tetromino examples](image)

3. a) How many rectangles (including squares) of any size can you make in the figure?
   i) [Diagram]  ii) [Diagram]  iii) [Diagram]  iv) [Diagram]  v) [Diagram]

   b) Use the pattern to predict how many rectangles of any size you can make in this figure:

   ![Pattern example](image)

   **Answers:** a) i) 1, ii) 3, iii) 6, iv) 10, v) 15; b) 55

4. How many upright triangles (like △, but not ▽) are there, of any size, in the figure?

   a) [Diagram]  b) [Diagram]  c) [Diagram]  d) [Diagram]

   **Answers:** a) 4, b) 10, c) 20, d) 22
5. A *pentomino* is made of 5 squares in the same way as dominoes, triominoes, and tetrominoes are made. How many different pentominoes are there?

**Solution:** 12

![Pentominoes](image)

6. For these puzzles, the rule is that the path must go up or right.

a) How many different possible paths are there from point A to point B?

i) ![Path A](image)

ii) ![Path B](image)

iii) ![Path C](image)

iv) ![Path D](image)

v) ![Path E](image)

b) Use the pattern to predict how many paths from point A to point B there are in this figure:

![Path F](image)

**Answers:** a) i) 2, ii) 3, iii) 4, iv) 5, v) 6; b) 11

7. Today is Sara’s birthday. Her age in years is 77 less than her age in months. How old is she? Hint: Use a chart.

<table>
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<tr>
<th>Age in years</th>
<th>Age in months</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

**Answer:** 7 years old

8. John measures his pencil in millimetres and centimetres. His answer in centimetres is 54 less than his answer in millimetres. How long is John’s pencil?

**Answer:** 6 cm or 60 mm
9. Karen calculates how many weeks are left until the summer holidays. Alex calculates how many days (including weekends) are left until the summer holidays. Alex’s answer is 48 more than Karen’s answer. How long is it until the summer holidays?

**Answer:** 8 weeks or 56 days

10. In 2037, Canada will be 153 more years old than it will be decades old. How old will Canada be in 2037?

**Answer:** 170 years or 17 decades

11. a) What number goes in the box?

i) $30 \times 5 \rightarrow 30$

ii) $\square \rightarrow 7$

iii) $\square \times 7 \rightarrow 63$

iv) $\square \times 4 \rightarrow 28$

v) $\square \rightarrow 60$

vi) $\square + 123 \rightarrow 143$

b) Find the starting number by working from the ending number.

i) $\square \times 5 \rightarrow \square \rightarrow 33$ Hint: What number plus 3 is 33?

ii) $\square \times 3 \rightarrow \square \rightarrow 20$

iii) $\square \times 8 \rightarrow \square \rightarrow 45$

iv) $\square \rightarrow \square \times 3 \rightarrow 33$

v) $\square \rightarrow \square \times 3 \rightarrow -2 \rightarrow \square \times 2 \rightarrow 44$

**Answers:** a) i) 6, ii) 4, iii) 9, iv) 7, v) 6, vi) 20; b) numbers in the order found: i) 30, ii) 18, 6, iii) 40, 5, iv) 11, 7, v) 22, 24, 8, 5

**NOTE:** You might point out to students that the strategy they just used is called working backward, because they started with the ending number to find the starting number.
1 cm Grid Paper
Unit 12 Measurement: 2-D Shapes

Introduction
This unit explores the measurement and manipulation of two-dimensional shapes, and provides applications of equations using variables. Topics include:

- perimeter;
- reflections;
- area;
- scale drawings; and
- grids.

Meeting Your Curriculum

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<tr>
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<tbody>
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<td>Optional</td>
<td>ME4-9 to 12, 17 to 20</td>
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</table>

| ONTARIO         | Required   | ME4-9 to 20  |

Mental Math Minutes
The mental math minutes in this unit
- practise arithmetic skills with an emphasis on multiplication and division.

Generic BLMs
The Generic BLMs used in this unit are:
BLM 1 cm Grid Paper (p. S-2)
BLM Pattern Blocks (p. S-1)
These BLMs can be found in Section S.
Materials

Lesson ME4-12 requires students to analyze patterns. Have students bring in examples of patterns. For example, pieces of wrapping paper; pages from colouring books; or pictures of sweaters, blankets, or architectural friezes.

Lesson ME4-20 requires one map for every three or four students. You can provide all students with the same map, or you can use different maps to illustrate differences. Maps are often available for free at local tourist offices.

Assessment

The lessons covered by a quiz or test are as follows:

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<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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<td>ME4-13 to 16</td>
<td>ME4-9, 10</td>
<td>ME4-13 to 16</td>
<td>ME4-9 to 20</td>
</tr>
</tbody>
</table>
Goals
Students will calculate the perimeter of polygons with given side lengths.
Students will select the most appropriate unit for measuring the perimeter of a given polygon and justify their answers.
Students will estimate and measure perimeter in centimetres and millimetres, using a variety of tools.

PRIOR KNOWLEDGE REQUIRED
Can measure length in centimetres and millimetres

MATERIALS
markers or chalk in two different colours
geoboards and elastics, BLM 1 cm Grid Paper (p. S-2), or BLM Dot Paper (p. O-43)
metre stick
rulers
ones blocks (see Extension 1)

Mental math minute. Have students skip count by 4s within 40, starting from multiples of 4.

Review perimeter. Draw a rectangle on the board. ASK: What do you call the measurement around the outside of a two-dimensional shape? (perimeter) What units can you use to measure perimeter? (units of length: cm, m, mm, etc.) Remind students that perimeter is a length, so you measure it using units of length.

Calculating perimeter by counting. Draw on the board:

Tell students that each edge measures exactly 1 cm. ASK: What is the perimeter of the shape? (6 cm) Why is the perimeter 6 cm when I drew seven edges? (the edge in the middle doesn’t count; only the edges on the outside of the shape are part of the perimeter) Trace around the outside of the shape in another colour to show the perimeter.

ASK: What would the perimeter be if each edge measured exactly 5 mm instead? (30 mm) Demonstrate counting by 5s around the edge of the shape to find the perimeter.
**ACTIVITY 1 (Essential)**

1. In pairs, have students take turns using elastics to make a shape on a geoboard and finding the shape’s perimeter. Shapes should have only vertical and horizontal edges—no diagonals. Alternatively, students can draw shapes on BLM 1 cm Grid Paper or BLM Dot Paper.

**Calculating perimeter by adding.** Draw on the board:

```
2 m
3 m
3 m
4 m
```

**ASK:** How can we calculate the perimeter of this trapezoid? (add the side lengths) Three metres is written here twice—do I need to add it twice? (yes) Write the addition sentence for the perimeter of the trapezoid, pointing to each side before adding it to the equation \((2 + 3 + 4 + 3)\). Ask students for the final answer. (12 m)

Repeat with the rectangle shown below. Ask students why the numbers appear twice in the addition. (opposite edges have the same length and both need to be added)

```
5 km
3 km
```

**Perimeter** = \(5 + 3 + 5 + 3 = 16\) km

**Exercises:** Add to find the perimeter of the shape.

a) \(2 + 4 + 2 + 4 = 12\) dm, b) \(5 + 12 + 13 = 30\) mm, c) \(1 + 3 + 1 + 4 + 5 + 4 = 18\) km

**Choosing units.** **ASK:** Which unit would you use to measure the length of the room? (m) Which unit would you use to measure its perimeter? (m) **SAY:** Since you can calculate perimeter by adding lengths, you usually use the same units for perimeter that you would use for length. Ask students which units they would use to measure the perimeter of objects of various sizes, such as boards (m), parks (km), desks (cm or dm), provinces (km), erasers (mm), or rings that you wear on your fingers (mm).
Calculating perimeter by measuring (m, cm, mm). Tell students that you want to measure the perimeter of the classroom door. You can use a window or the board instead if those are more convenient. Ask students what you need to do before you can start measuring. (choose a unit) Have the class agree on a unit of measure. Tell students that since you will need to make more than one measurement, it is helpful to sketch the object first. Draw on the board a rectangle with dimensions of roughly the same ratio as the door. Have volunteers use a metre stick to measure the height and width of the door. After making each measurement, record it on the sketch on the board. Finally, have a volunteer calculate the perimeter.

Calculating perimeter by estimating, then measuring. Remind students that they can estimate lengths in metres with large steps and in centimetres with the width of a finger. Tell them that they can use the same techniques to estimate perimeter before measuring it. Demonstrate how five fingers spread out a bit can be used as an estimate of one decimetre. Draw on the board a rectangle of random size and use five slightly-spread fingers to estimate the lengths of the sides in decimetres. Have a volunteer use your estimates to approximate the perimeter. Then have one or two more volunteers first measure the lengths of the sides in decimetres using a ruler and then add to find the perimeter. You may wish to have another volunteer measure and calculate in centimetres.

**ACTIVITY 2 (Essential)**

1. Give each pair of students a list of objects in the classroom or schoolyard. Tell them that they will be estimating and then calculating perimeters. For each object, students need to first choose an appropriate unit. Then they sketch the object on BLM 1 cm Grid Paper or BLM Dot Paper, write down their length estimates, and use them to estimate the perimeter. Finally, students measure and calculate the actual perimeter of the object.

**Extensions**

1. Make the figure shown using ones blocks.

   ![Figure](image)

   a) Calculate the perimeter.
   b) Add a block to the shape so that the perimeter increases by 2.
   c) Add a block to the shape so that the perimeter stays the same.
   d) Add a block to the shape so that the perimeter decreases by 2.

   **Answer:** a) 14
Sample answers

b)  

c)  

d)  

2. Can 500 toothpicks line the entire perimeter of your school? Calculate an estimate.

3. Sally wants to arrange eight square posters into a rectangle.
   a) How many different rectangles can she create?
   b) She plans to border the posters with a trim. For which arrangement would she need the least trim? Explain how you know.

   Answers: a) 4; b) the $2 \times 4$ or $4 \times 2$ arrangement, because the perimeter is 12 compared to 18 for the $1 \times 8$ and $8 \times 1$ arrangements

4. Find the perimeter.

   a)  
   b)  
   c)  

   Answers: a) 130 mm, b) 234 cm, c) 3800 m
Goals
Students will develop a formula for finding the perimeter of rectangles. Students will use the relationship between side lengths and perimeter to find perimeter or missing side lengths.

PRIOR KNOWLEDGE REQUIRED
Can represent unknown quantities as letters
Can solve additive equations containing unknowns

MATERIALS
BLM 1 cm Grid Paper (p. S-2)
geoboard and elastics (optional, see Extension 1)

Mental math minute. Practise doubling and halving by giving the first student a small number to double. Each successive student doubles the previous answer. When students have doubled several three-digit numbers, give the next student an even three-digit number to halve. Each successive student then halves the previous answer. When students reach an odd number, start doubling again with a new small number.

Developing a formula for the perimeter of a rectangle. Draw on the board the rectangles shown below. Label one length and width in each, and ask students what the lengths of the opposite sides should be. Then ask students to write the expressions that allow them to find a perimeter. Have volunteers find the perimeter of each rectangle. The board should look like this:

- Length = 3 cm
  Width = 2 cm
  \(3 \text{ cm} + 3 \text{ cm} + 2 \text{ cm} + 2 \text{ cm} = 10 \text{ cm}\)

- Length = 5 cm
  Width = 1 cm
  \(5 \text{ cm} + 5 \text{ cm} + 1 \text{ cm} + 1 \text{ cm} = 12 \text{ cm}\)

- Length = 4 cm
  Width = 2 cm
  \(4 \text{ cm} + 4 \text{ cm} + 2 \text{ cm} + 2 \text{ cm} = 12 \text{ cm}\)

Have volunteers circle the lengths in the equations and have others draw squares around the widths. ASK: How are the equations the same? (they all have two circles and two squares) Ask students to draw a template for an equation to find the perimeter of another rectangle. Explain that you need...
a general rule for finding the perimeter of any rectangle, so you want to replace the actual length with the unknown $\ell$ to stand for length. Write “$\ell$” in the circles. Repeat with $w$ for width. Rewrite the formula without circles and squares: $\ell + \ell + w + w$. SAY: Now we have a formula for the perimeter. This formula is a rule that allows you to find perimeter by replacing the unknowns with different numbers. For example, if a rectangle has length 10 cm and width 7 cm, you can just replace the letters with actual numbers and find the perimeter. Write “10 cm” below each $\ell$. Repeat for $w$. Insert the addition and equal signs in the equation. Have students find the perimeter using the rule. (34) ASK: Is there another way to write $\ell + \ell$? (2 $\times \ell$) Write “(2 $\times \ell$)” below $\ell + \ell$ and then repeat for $w + w$. (2 $\times w$) Write the plus sign in between. ASK: Will the two formulas give you the same answer: $\ell + \ell + w + w$ and $(2 \times \ell) + (2 \times w)$? (yes)

**Exercises:** Calculate the perimeter two ways.

a) ![Rectangle 1 cm x 4 cm]
b) ![Rectangle 3 m x 8 m]
c) ![Rectangle 8 km x 11 km]

**Answers:** a) 10 cm, b) 22 m, c) 38 km

**Developing a formula for the perimeter of a square.** Draw a square on the board and label its side length as 3 cm. Have a volunteer write the addition statement to find its perimeter. (3 + 3 + 3 + 3) Ask if there is any way to simplify this expression. (4 $\times 3$) Have a volunteer write the final answer with units. (12 cm) Draw another square and label its side length with the variable $s$. Have a volunteer write the addition statement to find the perimeter of the square. ($s + s + s + s$) Then have another volunteer write the multiplication to find the perimeter of the square. (4 $\times s$)

**Exercises:** Find the perimeter of the square from side length $s$.

a) $s = 5$ mm  
 b) $s = 7$ km  
 c) $s = 12$ dm  
 d) $s = 35$ m

**Answers:** a) 20 mm, b) 28 km, c) 48 dm, d) 140 m

**Finding missing sides.** Draw a rectangle on the board. SAY: The perimeter of this rectangle is 14 cm. The length of one side is 3 cm. Ask students to find the lengths of the other sides. Encourage students to present more than one solution to the problem. For example, they could say that $14 - 3 - 3 = 8$, so the sum of the other two sides is 8 and each side is 4 cm. Another solution would be to use the fact that the sum of two adjacent sides is exactly half of the perimeter: half of 14 is 7, so 3 plus something equals 7, which means that each of the other sides is 4 cm.

To lead students to the second solution, draw a big dot on one corner of the rectangle and ask students to imagine walking around the rectangle, starting from the dot. Trace your finger along the entire perimeter to illustrate walking. SAY: You’ve gone exactly halfway around the perimeter. ASK: Where are you now? (at the opposite corner) How can you find the...
distance you’ve passed? (add length to width) How can you find the whole perimeter from this number? (multiply by 2)

In the following exercise, if students find only one (or zero) rectangle, show them a systematic method of finding the answer. On BLM 1 cm Grid Paper, draw a line 1 cm long and ask students to finish the rectangle so that the perimeter is 14 units. Repeat with a side 2 cm long. ASK: What side length would you try next? (3 cm) If students produce more than four rectangles, ask them to compare rectangles to find which are congruent to each other.

Exercise: Use grid paper or a geoboard to find all rectangles with perimeter 16 cm.

Answers: 1 cm by 7 cm, 2 cm by 6 cm, 3 cm by 5 cm, 4 cm by 4 cm

Extensions

1. Have students use BLM 1 cm Grid Paper or a geoboard and elastics to determine if the following rectangles are possible. NOTE: Sides must measure an exact number of units (e.g., 4, not 4.5).

   a) A rectangle with perimeter 7 units.
   b) A rectangle with a perimeter that is an odd number of units (e.g., 9, 11, 17).

   Answers: Both are impossible. Invite students to explain why.

2. Repeat Extension 1, but allow half-units.

   Sample answers: a) 1.5 by 2, b) 2.5 by 2

3. The sides of a pentagon are all 5 cm long. What is the pentagon’s perimeter?

   Answer: 25 cm

4. The perimeter of a hexagon with all equal sides is 42 cm. Find the length of its sides.

   Answer: 7 cm

5. Find the perimeter.

   a) 45 mm
   b) 98 dm
   c) 450 m

   Answers: a) 110 mm, b) 596 dm, c) 2900 m
**Goals**

Students will perform reflections on a grid.
Students will create and extend symmetrical designs and patterns resulting from reflections.

**PRIOR KNOWLEDGE REQUIRED**

Can identify and describe locations on a grid
Can place points on a grid

**MATERIALS**

overhead projector
blank transparency
transparency of BLM 1 cm Grid Paper (p. S-2)
Miras
pentominoes or BLM Pentominoes (p. O-44, optional)
BLM 1 cm Grid Paper (p. S-2)
red and blue chalk or markers
rulers (see Extension 3)

**Mental math minute.** Have students add 9, 8, 90, 99, or 98 to two-digit and three-digit numbers by adding 10 or 100 and then compensating by subtracting as follows: give an addition problem, such as 135 + 98. The first student says what needs to be done (add 100 and then subtract 2). The second student adds 100 (135 + 100 = 235). The third student subtracts 2 (235 – 2 = 233).

**Review lines of symmetry.** Draw on the board:

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<-- ---- -->
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**ASK:** What is the line called? (line of symmetry) Remind students that if you fold a shape along a line of symmetry, the halves will overlap exactly. Ask what happens if you place a mirror along a line of symmetry. (The visible half of the shape and the mirror image make up the whole shape.)

**Exercises:** Draw all the lines of symmetry.

a) ![Reflection](image)

b) ![Reflection](image)

c) ![Reflection](image)
Reflecting shapes in a mirror. Tell students that they are going to find mirror images of whole shapes. Using a blank transparency projected on the board, draw the triangle and reflecting line shown below:

Invite a volunteer to trace the triangle and the line on the board. Then flip the transparency horizontally so that the reflecting lines on the transparency and the board coincide and the projected triangle is the reflection of the traced triangle. Have a volunteer trace the projected triangle.

ASK: Is this triangle a mirror image of the first triangle? (yes) Tell students you created the second triangle by flipping the first one around the mirror line or reflecting line. Demonstrate this by placing the transparency on top of the original triangle and then flipping it over so that it coincides with the mirror image. Remind students that this is also called reflecting and that shapes made this way are called reflections or mirror images.

NOTE: Project BLM 1 cm Grid Paper on the board for the remainder of the lesson. Leave the following exercises on the board for later reference. If you do not have Miras for Exercise 1, give students pentominoes or shapes from BLM Pentominoes instead of the shapes provided, have them trace their pentomino on BLM 1 cm Grid Paper, and reflect the pentomino by flipping it over a line of reflection. It is not important that students copy the shapes accurately. It is important to verify that they have properly reflected the shapes as they have drawn them.

Exercises

1. Copy the picture onto grid paper. Place a Mira along the dashed line. Draw the reflection of the shape.

   a)  
   b)  
   Bonus:  

Answers

   a)  
   b)  
   Bonus:  

2. The dashed line is the mirror line. Copy the picture onto grid paper, then draw the reflection of the shape. Check your answer using a Mira.

![Image of shapes](image1.png)

**Answers**

![Image of answers](image2.png)

**Reflecting points.** Draw on the board the following image in blue:

![Image of blue triangle](image3.png)

Have a volunteer draw the mirror image in red, including the dot on the vertex. Point out that the original triangle is above the reflecting line and the reflection is below it. Draw students’ attention to the blue vertex. ASK: How far away from the mirror line is the blue dot? (one square) How far away is its red image? (one square) Did the image move left or right? (no)

Draw an X on another vertex. ASK: Where did this vertex go when the image was reflected? Have a volunteer draw an X on the corresponding vertex in the reflected image. Repeat the questions above about distance from the mirror line and left/right movement. ASK: What happens to points that are already on the reflecting line? (they stay where they are) Draw students’ attention to the edges in the previous exercises that lie along the mirror line (Exercise 1, part a) and the bonus). Those edges and vertices are the same in the original and reflected images.

For Exercise 1 below, add points to the shapes in the previous Exercise 2. Students can add these same points to their answers and find the images.

**Exercises**

1. Draw the mirror images of the points. Check your answers using a Mira.

![Image of points](image4.png)

**Answers**

![Image of answers](image5.png)
2. Draw the reflection of the points. Check your answers using a Mira.

\[ \text{Answers} \]

\[ \begin{array}{c}
\text{a)} \\
\text{b)}
\end{array} \]

**Reflecting vertices to reflect shapes.** Tell students that one way to find the reflection of a shape is to first reflect all of its vertices, then connect the vertices to make the reflection. To demonstrate, draw on the board the shape on the left and the line of reflection shown below. Have volunteers find the reflections of the vertices and reconnect them to form the image.

\[ \begin{array}{c}
\text{Exercises: The dashed line is the mirror line. Copy the picture onto grid paper. Reflect each vertex, then connect the vertices to make the reflection of the shape. Use a Mira to check your answer.} \\
\text{a)} \\
\text{b)}
\end{array} \]

\[ \text{Answers} \]

\[ \begin{array}{c}
\text{a)} \\
\text{b)}
\end{array} \]

**Creating patterns using reflection.** Ask students for examples of attributes of shapes. (size, colour, direction, etc.) Remind students that they can create patterns by repeatedly changing an attribute of a shape. ASK: Which attribute changes when you reflect a shape? (direction or position) Can you use this to make a pattern? (yes) Draw on the board:
Invite a volunteer to draw the reflection of the shape in the dashed line. Then draw two more dashed lines, each four squares apart, and have other volunteers draw the mirror images. (see below)

![Reflection Diagram]

**ACTIVITY 1 (Essential), ACTIVITY 2 (Optional)**

1. Give each student a pentomino from a commercial set or BLM Pentominoes. Students place the piece on BLM 1 cm Grid Paper and trace it. They draw a reflecting line one square to the right of the rightmost edge of the shape or one square below the bottom edge of the shape and then reflect the shape in the line. They can check the results by flipping the pentomino. Students produce a pattern with at least five terms. They then exchange patterns with a partner and produce the next three shapes in their partner’s pattern.

2. Students repeat Activity 1, except that they reflect their shape both vertically and horizontally to produce a two-dimensional pattern.

**Extensions**

1. Fold a piece of grid paper in half twice, once vertically and once horizontally. Draw a picture in the upper left-hand part of the folded grid paper. Draw the mirror images of your drawing by reflecting through the fold lines.

2. Draw the reflection of the shape by first reflecting the vertices.

![Reflection Diagrams]

**Answers**

![Answer Diagrams]
3. a) Starting near the top-right corner of a piece of grid paper, draw a dot on an intersection of two grid lines. Then draw another dot that is one square to the left and one square down. Continue drawing dots one square over and one square down from the previous dot. Connect all the dots with a ruler to make a line. It should look like this:

b) Draw a pentomino to the left of your line using the squares of the grid.

c) Find the mirror image of the pentomino by drawing lines from the vertices that go one square right and one square down. Make sure the reflected vertices are the same distance from the reflecting line as the original vertex.

d) What angle do the lines from the vertices make with the connecting lines?

e) Draw other shapes and their reflections.

Sample answers: a) to c)

Answer: d) 45°

4. Draw a shape in the top-left corner of a piece of grid paper. Reflect your shape horizontally from left to right. Then reflect the new shape vertically from up to down. Then reflect that shape horizontally from right to left. Finally, reflect that shape vertically from down to up. What happens?

Answer: the shape rotates clockwise and then returns to its original location
**Goals**

Students will identify and describe reflections.
Students will identify patterns made by reflection.

**PRIOR KNOWLEDGE REQUIRED**

Can reflect a shape through a given line
Can reflect points through a given line

**MATERIALS**

overhead projector
transparency of BLM 1 cm Grid Paper (p. S-2)
pentominoes or BLM Pentominoes (p. O-44, optional)
images of patterns

**Mental math minute.** Give the first student a subtraction problem involving two-digit numbers that does not need regrouping, such as $97 - 12$. Students repeatedly subtract the same number, in this case 12, with each student formulating and answering one subtraction aloud. When a student has a subtraction that involves regrouping, emphasize that this answer was a bonus. Example: Student 1 says, "$97 - 12 = 85".$ Student 2 says, "$85 - 12 = 73."$ Student 3 says, "$73 - 12 = 61."$ Student 4 says, "$61 - 12 = 49$" (point out that this subtraction was a bonus question). Continue without regrouping until Student 8 says, "$13 - 12 = 1,"$ then start a new chain.

**NOTE:** Project BLM 1 cm Grid Paper on the board for the remainder of the lesson.

**Finding vertical reflecting lines.** Draw on the board:

![Diagram of two triangles reflected vertically]

ASK: Are these triangles congruent? (yes) Are they mirror images? (yes)
Have a volunteer draw the mirror line. Redraw the same triangles two squares apart and repeat the process. (There is no need to distinguish the triangles by colouring.) The board should look like this:

![Diagram with same triangles and mirror line]

Repeat again with the same triangles, but four squares apart. Then repeat with other shapes at a variety of distances apart.
**Exercises:** Copy the shapes onto grid paper. Find the line of reflection.

a) ![Shape A]

b) ![Shape B]

c) ![Shape C]

**Bonus:**

![Bonus Shape]

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**Answers**

a) ![Answer A]

b) ![Answer B]

c) ![Answer C]

**Bonus:**

![Bonus Answer]

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**Finding horizontal reflecting lines.** Draw on the board:

![Horizontal Line]

ASK: Are these triangles congruent? (yes) Are they mirror images? (yes) In which direction is the reflecting line? (horizontal) Have volunteers draw lines connecting corresponding vertices. Then have another volunteer draw the line of reflection.

**Exercises:** Copy the shapes onto grid paper. Find the reflecting line.

a) ![Shape A]

b) ![Shape B]

c) ![Shape C]

**Bonus:**

![Bonus Shape]
Identifying reflected images without a grid. Tell students that all reflected images are congruent, but not every congruent image is a reflection. Draw on the board:

A. \hspace{1cm} B. \hspace{1cm} C.

Ask students which pair of triangles was created using a reflection. (A) Have volunteers draw lines connecting corresponding vertices. Ask students to compare these lines. (in A and B, the lines are parallel. In C, the lines cross. In B, the lines are all the same length.) ASK: How can you tell which are reflections by looking at the lines? (sample answer: the lines are parallel, but they are different lengths) As an additional challenge, ask students if they can guess what kind of transformations were used in B and C. (translation, i.e., sliding, and rotation)

ACTIVITIES 1–2 (Essential), ACTIVITY 3 (Optional)

1. Place a sheet of grid paper so that the long edge is horizontal. Copy a pentomino (from a standard set or from BLM Pentominoes) into the top-left corner. Create a pattern by alternating reflection with one other transformation. What would the 10th, 20th, and 33rd figures in your pattern look like? Exchange patterns with a partner. What shape was reflected? What other transformation was used?

2. Analyze images of patterns to find congruent shapes and examples of reflections.

3. Create patterns on the rest of the page from Activity 1 by following these steps.
   a) Create a pattern of two or three other pentominoes down the left side of the page.
   b) Fill in the rest of the page by repeating the transformations you used in the first row in each of the other rows.

Extensions

1. Copy the shapes onto grid paper. Find the line of reflection. Hint: It will be diagonal.
   a) b) Bonus:
2. Copy the shapes onto grid paper. Find the line of reflection. Hint: It will be inside the shape and not a line of symmetry.

Answers

a)  

b)  

Bonus:

Answers

a)  

b)  

Bonus:
ME4-13  Area in Square Centimetres

Page 92

Goals

Students will use square centimetres to measure the areas of regular and irregular shapes.

PRIOR KNOWLEDGE REQUIRED

Can use a ruler to draw line segments
Can measure in centimetres

MATERIALS

two grid-paper rectangles for every student
scissors
glue or tape
metre stick
grid paper or BLM 1 cm Grid Paper (p. S-2)
geoboards (optional)
pattern block squares from BLM Pattern Blocks (p. S-1) or a commercial set, or connecting cubes (optional)

Mental math minute. Have students skip count forwards by 3s, starting at multiples of 3 within 30.

Introduce area. Give students two identical grid-paper rectangles. Ask students to fold one of their rectangles in half, then cut along the fold to make two smaller rectangles (vertically or horizontally). Ask students to rearrange the pieces to make a different rectangle than the one they started with and glue or tape the new rectangle onto scrap paper. Invite volunteers to tape the original rectangle and the two possible rearranged rectangles onto the board so that students can see all three shapes together.

ASK: Which shape is larger? How is it larger? Did one shape use more paper than the other? Did they use the same amount of paper? Explain that the amount of paper you need to make the shape is the shape’s area.

ASK: When you cut the rectangle in two and rearranged the pieces, did you change the amount of paper you were using? (no) Does your shape have the same shape as the rectangle I gave you? (no) Does it have the same area? (yes) How do you know? (the same amount of paper was used to make it)

Introduce squares as units of area. Draw two rectangles as shown in the margin. ASK: Which rectangle has a larger area? PROMPT: Imagine these are two pieces of cake. Which one has more cake? Explain that squares that cover the shape are used as units to measure area. Point out that units should cover the shape without gaps or overlaps. Draw several shapes made of squares (see examples in the margin). Ask students to count the number of squares in each shape and write it as “[6] square units.”
Determining the area of shapes on grids. Draw several rectangles and mark their sides at regular intervals, as shown in the margin. Ask volunteers to divide the rectangles into squares by using a metre stick to join the marks. Ask students to find the area of these rectangles.

**ACTIVITY (Essential)**

Students work in pairs. One student draws a shape on grid paper or BLM 1 cm Grid Paper, or creates one using whole squares on a geoboard, and the other student calculates the area. ASK: How many shapes can you draw with an area of 8 squares? Have students work in their pairs to find the answer.

Introduce square centimetres. Explain that exercises in the AP Books use squares that have all sides equal to 1 cm. They are called **square centimetres**. Write “square centimetre” and “cm²” on the board and explain that these are two ways to write square centimetres. Show students a ones block and tell them that it covers about 1 square centimetre. Another example is the top of the pinky finger.

Draw a rectangle that is 3 squares wide and 2 squares tall. Tell students that these squares are each 1 cm by 1 cm. ASK: What is the length of each side? Have students copy the rectangle and write the lengths on the sides. Then ask students to find the area of the rectangle. ASK: What unit is length measured in? SAY: Length is measured in centimetres, not square centimetres. Emphasize that these are different (though connected) units.

Students who have trouble with Question 4 on AP Book 4.2 p. 92 can use pattern block squares (from BLM Pattern Blocks or a commercial set) or connecting cubes to solve the problems.

**Extensions**

1. On grid paper or a geoboard, make as many shapes as possible with an area of 6 squares. For a challenge, try making shapes that have at least one line of symmetry. For instance, the shapes in the margin have an area of 6 square units and one line of symmetry.

2. Mario says that shape A at left has an area of 4 squares and shape B has an area of 3 squares, so shape A has a larger area than shape B. Explain his mistake.

**Answer:** Each square in shape B is larger than each square in shape A, and three larger squares can have a larger total area than four smaller squares.
goals

Students will use square metres to measure the areas of regular and irregular shapes.
Students will determine appropriate units for measuring area.

Prior Knowledge Required

Understands area as the space taken up by a two-dimensional shape or flat surface
Can estimate the areas of regular and irregular shapes in square centimetres
Can choose appropriate units for measuring length

MATERIALS

metre stick
hundreds block
wrapping paper or old newspapers
ones blocks (optional)

Mental Math Minute. Ask students to solve multiplication problems within the range of 0 × 1 to 10 × 10. For each number, first go through the problems in order, such as 0 × 3, 1 × 3, and so on to 10 × 3, then in reverse order, and after that, go through the same problems out of order. Then progress to a different number.

Introduce Different Units. Explain that just as length is measured in different units, so is area. Square centimetres are squares with length and width 1 cm. Ask: What will square metres be? (squares with length and width 1 m) Use a metre stick to draw a square metre on the board. Ask students for examples of objects around the room that are about the same size. Examples might include a desktop, a small bookcase, or a window.

Comparing Unit Squares. Ask students to guess about how many square centimetres fit in 1 square metre. Ask if it is bigger or smaller than 100 square centimetres. Hold up a hundreds block. Ask: What is the area of a hundreds block in square centimetres? (100 cm²) Prompt: How many ones are in a hundreds block? Hold the hundreds block up in front of the square metre to demonstrate how much bigger the square metre is.

Note: Keep the squares made in Activity 1 for use in Activity 2.
ACTIVITY 1 (Essential)

1. Making and comparing unit squares. Assign different sizes of squares (1 cm², 10 cm × 10 cm, 1 m²) to students and have them use wrapping paper or old newspapers to make and label squares of the given size. Have students who were given the two smaller units make several squares. Students compare and order the units in groups. ASK: Will a 30 cm by 20 cm rectangle fit inside 1 square metre? How many 10 cm by 10 cm squares do you think will fit in a metre square?

Choosing appropriate units. Point to some small objects in the classroom and ask students if they would measure the area of these objects in square centimetres or square metres. Have students explain their reasoning. Point out that those objects are too small to measure in square metres. Then indicate some larger objects and ask the same questions. Ask students for examples of objects that could reasonably be measured using either. These will tend to be things that measure around 1 m².

Estimating and measuring area. In advance, mark out a rectangle on the floor that is a whole number of metres on each edge, or choose a rectangular surface to be measured, such as the board or a window. Hold up a metre square from Activity 1 next to the surface. Ask students to estimate, by comparing the sizes, how many square metres will fit on the surface. Record the answers on the board. Demonstrate measuring the surface with the help of volunteers. Place the metre square in the lower-left corner. If the surface is vertical, keep the metre square in place by holding or taping the corners. Place another metre square right next to the first. Point out that the squares leave no gaps and do not overlap. Then move the first square to the right of the second square. Tell students that this is like pacing out distances. Have students count how many metre squares fit in your surface as you go (to the nearest whole number). When you can no longer go to the right, place the next square above the previous one. Then begin counting back to the left. Compare the measured area to the estimated areas on the board.

ACTIVITY 2 (Essential)

2. Write on the board a list of large and small objects to be measured (for example, the area of the door, the surface of a book, the area of floor tiles or the carpet). If possible, include shapes that are not rectangular. Working in small groups, students choose the appropriate unit for measuring the area of each item. They first estimate and then measure the area using the squares made in Activity 1 or ones blocks.
Extensions

1. Find the area of the rectangle.
   a) 4 km
   b) 27 dm
   c) 7 mm

   Answers: a) 12 km², b) 216 dm², c) 49 mm²

2. Find the area of the rectangle.
   a) 3 cm
   b) 3 dm
   c) 7 cm

   Answers: a) 750 mm², b) 270 cm², c) 4830 mm²

3. a) What is the area of the rectangle?

   b) The rectangle is divided into two triangles. Are they congruent?
   c) Do congruent shapes have the same area? Explain. (Hint: What
does congruent mean? What is area?)
   d) What is the area of the grey triangle? How do you know?

   Answers: a) 20 cm²; b) yes; c) congruent shapes have the same area
   because they can be placed on top of each other to cover exactly
   the same space; d) 10 cm², because each triangle is exactly half
   the rectangle

4. Find the area of the shaded part.

   Answer: 18 − 4 = 14 cm²
Goals

Students will derive the formula for the area of a rectangle.
Students will express the formula for the area of a rectangle using variables.
Students will use a formula to calculate the areas of rectangles.

PRIOR KNOWLEDGE REQUIRED

Understands the concept of area
Understands that symbols can be used in equations to represent numbers
Can use a ruler to draw and measure line segments
Can use multiplication to find the number of objects in an array
Can multiply and divide up to 10 × 10
Can multiply up to two-digit numbers by one-digit numbers
Can multiply two-digit numbers by powers of ten

MATERIALS

overhead projector
transparency of BLM 1 cm Grid Paper (p. S-2)
BLM 1 cm Grid Paper (p. S-2)
rulers

Mental math minute. Give students multiplication questions that can be done by skip counting by 2s, 3s, 4s, 5s, or 10s. For example, ask a student to find 8 × 4 by skip counting by 4s. Have students skip count aloud to answer the questions.

Introduce rectangles as arrays of squares. Draw a 3 by 4 array of dots on the board. ASK: How many dots are in the array? How did you count the dots? Have students write the corresponding multiplication statement on the board.

Project BLM 1 cm Grid Paper on the board. Draw a 3 by 4 rectangle on the grid. Have students copy it on grid paper or BLM 1 cm Grid Paper and write the length and width of the rectangle. ASK: How are the rectangle and the array the same? What multiplication statement gives us the area of the rectangle? To prompt students to see the answer, draw a dot in each square of the array. Ask students to write the multiplication statement for the area of the rectangle.

Draw several rectangles on the grid. Ask students to write the length and the width and calculate the area.

Determining the formula for area of rectangles. On the board, draw a rectangle (not on a grid) and tell students that it is 50 squares long and 30 squares wide. ASK: What is the area of this rectangle? (1500) How did
you find the answer? (50 × 30 = 1500) SAY: Because area is measured in squares, we can think about any rectangle as being made of squares.

Draw a 25 cm by 20 cm rectangle on the board. Use this opportunity to review using a ruler to draw line segments of a given length by having students tell you what to do next. Write the length and the width beside the sides of the rectangle. SAY: The rectangle is 25 cm long. ASK: How many 1 cm squares long is it? (25) How many 1 cm squares wide is it? (20) How many 1 cm squares are needed to cover the rectangle? (500) Have students write the multiplication statement. (25 × 20 = 500) ASK: What is the area of the rectangle? (500 cm²) Did we divide the rectangle into squares to find the answer? (no) How did we find the answer? (multiplied length by width) Do you think this method will work to find the area of any rectangle? (yes) Summarize on the board:

Area of rectangle = length × width

**ACTIVITY (Essential)**

Finding areas of rectangles. Students work in pairs to draw various rectangles with length and width in whole centimetres, then exchange notebooks. Partners then measure the sides of the rectangles, record their length and width, and find the area. Partners check each other’s work. Have students work in their pairs to find as many rectangles as they can with area 12 cm².

Finding the area of rectangles using the formula. On the board, draw the picture in the margin. Ask students to copy the dimensions of the rectangle. Demonstrate how to use a formula to record the process of finding the area, as shown below:

Length = 10 cm
Width = 5 cm
Area = length × width
  = 10 cm × 5 cm
  = 50 cm²

Remind students to write the unit for each measurement. In the first three exercises below, provide a picture of a rectangle labelled with the length and the width. For the rest of the exercises, just write the length and width. Review multiplying multi-digit numbers before assigning part g) onwards.

**Exercises:** Use the formula to find the area.

a) length 5 m, width 4 m
b) length 6 m, width 7 m
c) length 20 cm, width 15 cm
d) length 5 cm, width 8 cm
e) length 6 m, width 11 m
f) length 12 m, width 4 m
g) length 56 cm, width 20 cm
h) length 42 m, width 42 m
i) length 16 m, width 11 m
j) length 32 m, width 4 m
k) length 510 cm, width 9 cm
l) length 470 cm, width 8 cm

**Bonus:** length 2 121 000 cm, width 400 cm
Answers: a) 20 m², b) 42 m², c) 300 cm², d) 40 cm², e) 66 m², f) 48 m², g) 1120 cm², h) 1764 m², i) 176 m², j) 128 m², k) 4590 cm², l) 3760 cm², Bonus: 848 400 000 cm²

Extension

a) Draw a rectangle so that the number representing the area (in square centimetres) is the same as the number representing the perimeter (in centimetres).

b) Convert the measurements of the sides to millimetres. Find the area and perimeter of the rectangle with the new measurements. Is the area in square millimetres still equal to the perimeter in millimetres?

Sample answers: a) 18 = 3 × 6 (area) = 2 × 3 + 2 × 6 (perimeter), or 16 = 4 × 4 (area and perimeter); b) 30 mm by 60 mm, so area = 1800 mm² and perimeter = 180 mm; or 40 mm by 40 mm, area = 1600 mm² and perimeter = 160 mm; the area is not equal to the perimeter

NOTE: This exercise shows that the numbers representing area and perimeter can be identical in one set of units and different in another set of units, even though the rectangles do not change. This means that area and perimeter cannot be compared because the units they are measured in are incomparable.
Goals
Students will find areas of irregular polygons by decomposing them into rectangles.

PRIOR KNOWLEDGE REQUIRED
Knows that area is additive
Can find the area of a rectangle
Can multiply and divide up to two-digit numbers by one-digit numbers
Can solve an equation of type \( a \times x = b \)

MATERIALS
overhead projector
transparency of BLM 1 cm Grid Paper (p. S-2)
pencil crayons

Mental math minute. Give students division questions that can be done by skip counting by 2s, 3s, 4s, 5s, or 10s. For example, ask a student to find \( 21 \div 3 \) by skip counting by 3s. Have students skip count aloud to answer the questions.

Finding the area of shapes composed of two rectangles (on a grid).
Project BLM 1 cm Grid Paper on the board. On the grid, draw several shapes composed of two rectangles. Have students draw the line that divides the composite shape into the two simple ones. Then ask students to find the area of each shape.

Exercises: Find the area.

a) ![Rectangle](image)

b) ![Rectangle](image)

c) ![Rectangle](image)

Answers: a) 6 square units, b) 10 square units, c) 11 square units

Point out that part a) has two different solutions. Encourage students to find both and have volunteers show both solutions (\( 1 \times 4 + 1 \times 2 \) and \( 1 \times 2 + 2 \times 2 \)).

Finding the area of shapes composed of two rectangles (not on a grid).
Draw several shapes composed of two rectangles (not on a grid), and mark the dimensions on four of the sides, as shown in the following exercises. Ask students to shade each rectangle with its own colour and then circle the dimensions that belong to the rectangle in the same colour. Have students find the area of each rectangle and add the areas of the rectangles in each shape to find the total area.
Exercises

a) 

\[
\begin{array}{c}
3 \text{ m} \\
2 \text{ m} \\
1 \text{ m}
\end{array}
\]

b) 

\[
\begin{array}{c}
8 \text{ m} \\
7 \text{ m}
\end{array}
\]

\[
\begin{array}{c}
5 \text{ m}
\end{array}
\]

Answers: a) 8 m², b) 68 m², c) 130 cm², Bonus: 193 cm²

Show students the shape in the margin. ASK: Is 7 cm the length of the short side of the shaded rectangle (trace it with your finger)? (no) Is it the length of the short side of the unshaded rectangle? (no) Point out that 7 cm is the length of the combined side, so 7 cm should be the sum of the lengths of two other sides. ASK: What is the length of the short side of the unshaded rectangle? (3 cm) How do you know? (7 cm − 4 cm = 3 cm) Finally, have students find the areas of both rectangles and add them to find the total area of the shape. (5 cm × 3 cm + 11 cm × 4 cm = 15 cm² + 44 cm² = 59 cm²)

Present the second shape at left and ask students to find the length of the longer side of the shaded rectangle. (21 cm) Have students find the area of the shape. (7 cm × 10 cm + 9 cm × 21 cm = 70 cm² + 189 cm² = 259 cm²) Then have students work individually to practise finding the area of composite shapes.

Exercises

a) 

\[
\begin{array}{c}
2 \text{ m} \\
2 \text{ m} \\
3 \text{ m} \\
1 \text{ m}
\end{array}
\]

b) 

\[
\begin{array}{c}
2 \text{ m}
\end{array}
\]

\[
\begin{array}{c}
4 \text{ m}
\end{array}
\]

\[
\begin{array}{c}
3 \text{ m}
\end{array}
\]

Answers: a) 9 m², b) 10 m², c) 360 m²
Present the shape in the margin and shade one of the rectangles as shown in the first picture. Ask students to identify the side lengths of the shaded rectangle. Point to each label in order and have students signal thumbs up if the label shows the length of a side of the shaded rectangle or thumbs down if it does not. Circle the labels for the shaded rectangle, then repeat with the side lengths of the unshaded rectangle. Then present the same shape with the same labels, but broken into two rectangles in a different way (as in the second picture), and repeat the exercise. Point out that, in each case, students need to use four of the six labels to find the area of the shape. The remaining two labels are unused, and the unused labels differ in each case. Have students use both ways to find the area of the shape. (89 cm²) ASK: Did you get the same answer?

Have students find the area of the shapes below. In the first two shapes, all side lengths are given. In the rest of the shapes, only four side lengths are given.

Exercises: Find the area.

a) 8 m 7 m 4 m 7 m
   14 m 12 m

b) 6 cm 5 cm 16 cm 5 cm
   10 cm 22 cm

c) 12 m 15 m 12 m
   23 m

d) 4 m 3 m
   5 m 5 m

Bonus: Find a different way to split these shapes into rectangles and use them to find the area. Did you get the same answers as before?

Selected answers: a) 140 m², b) 140 cm², c) 456 m², d) 57 m²

NOTE: The rest of the lesson deals with perimeter, which is optional for Alberta and Manitoba.

Finding the perimeter of a rectangle given the area and side. Review finding the length or the width of a rectangle from the area and the other dimension. For example, have students find the width of a rectangle with an area of 14 m² and a length of 7 m. (2 m) Remind students that the perimeter of a shape is the distance around it and have them find the perimeter of the rectangle. (18 m)

Exercises: Find the missing length and the perimeter of the rectangle.

a) area = 20 cm², width = 4 cm   b) area = 20 cm², width = 2 cm

Answers: a) length 5 cm, perimeter 18 cm; b) length 10 cm, perimeter 24 cm
Finding the area of a rectangle given the perimeter and side. Discuss ways of finding the perimeter of a rectangle. (add all four sides; add two sides and double; double length and width, then add them) Draw a rectangle on the board and mark one side as 5 m. SAY: The perimeter of this rectangle is 22 m. ASK: What are the other sides of the rectangle? Have students present multiple solutions. To prompt students, have them first find the length of the opposite side. SAY: These two sides together add to 10 m. ASK: What do the other two sides add to? (12 m) How do you know? (22 m – 10 m = 12 m) How long is each side? (6 m) How do you know? (they are equal and add to 12 m, so each is 12 m ÷ 2 = 6 m) SAY: Another way of thinking about the perimeter is: Two touching sides should be half of 22, so two touching sides add to 11 m. One of the sides is 5 m. ASK: How long is the other side? (6 m)

Then have students find the area of the rectangle. (5 m × 6 m = 30 m²)

Exercises: Find the missing length, then find the area of the rectangle.

a) perimeter 20 cm, width 4 cm  
   b) perimeter 20 cm, width 2 cm

Answers: a) length 6 cm, area 24 cm²; b) length 8 cm, area 16 cm²

Finally, draw a square and tell students that its perimeter is 24 cm. ASK: How long is each side? (6 cm) How do you know? (the sides are equal, so perimeter is 24 cm ÷ 4 = 6 cm) What is the area of the square? (36 cm²)

Extension

a) If 1 m = 100 cm and 1 m² = 1 m × 1 m, then 1 m² = ______ cm².

b) A book is about 33 cm long and about 25 cm wide. How many of these books will you need to lay in a row for a total length of about 1 m?
   How many will you need to lay in a row for a total width of about 1 m?
   How many books do you need to cover an area of about 1 m²?

c) Find the area of the book in part b). Round the answer to the nearest hundred.

d) Divide your answer in part a) (the total number of square centimetres in 1 square metre) by the area of the book. How many books will fit in 1 square metre? What did you do with the leftover? Does your answer match the answer to part b)?

Answers: a) 10 000; b) length 3 books, width 4 books, 3 × 4 = 12 books will cover about 1 m²; c) about 800 cm²; d) 12 books, ignore the leftover
**ME4-17  Problems with Area and Perimeter**

Pages 98–99

**CURRICULUM REQUIREMENT**

AB: optional  
BC: optional  
MB: optional  
ON: required

**VOCABULARY**

area  
centimetre (cm)  
equation  
kilometre (km)  
metre (m)  
perimeter  
square centimetre (cm²)  
square metre (m²)  
unknown

---

**Goals**

Students will solve problems that require the ability to distinguish area and perimeter.

**PRIOR KNOWLEDGE REQUIRED**

Knows the relative size of units of length measurements within the metric and Imperial systems  
Can find the perimeter and area of a rectangle  
Can multiply and divide up to two-digit numbers by one-digit numbers  
Can solve an equation of type \( a \times x = b \)

**MATERIALS**

deck of cards  
overhead projector  
transparency of **BLM Rectangles** (p. O-45)  
**BLM Rectangles** (p. O-45)  
**BLM 1 cm Grid Paper** (p. S-2, see Extension 1)

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**Mental math minute.** Remove the face cards from a deck of cards, then shuffle the deck. Divide students into pairs. Each partner will select a card at random, and the pair will then create two division equations using the selected cards. For example, if Partner 1 selects a 7 and Partner 2 selects an 8, the pair will create the equations \( 56 \div 8 = 7 \) and \( 56 \div 7 = 8 \). When an ace is selected, have students treat it as the number 1. Continue until all students have had an opportunity to participate.

**Finding the missing dimension in a rectangle.** Tell students that you want to find the length of a rectangle with area 24 cm² and width 4 cm. Explain that you want to organize what you know and what you need to find. Show how to do it, as shown in the margin. ASK: What could I write instead of the length? SAY: Remember, in word problems, we often use a letter for the unknown number. I will use the letter \( \ell \) as the short form of "length." Write \( \ell \text{ cm} \) in the blank. ASK: What is the area of a rectangle with length \( \ell \) cm and width 4 cm? \( \ell \times 4 \text{ What equation does this make? } \) \( \ell \times 4 = 24 \text{ What equation does this make? } \) \( \ell \times 4 = 24 \text{ What equation does this make? } \)

Remind students that to solve this multiplication equation, they can, for example, think what number multiplied by 4 produces 24 or use an equation from the same fact family, so that the letter \( \ell \) is by itself, \( 24 \div 4 = \ell \). ASK: What is \( \ell \) equal to? (6)

Repeat with a rectangle with area 24 cm² and length 8 cm. Use \( w \) for the unknown number. Then have students practise.
Exercises

1. Find the missing length or width.
   a) length 7 m, width \( w \) m, area 42 m\(^2\)
   b) length 5 cm, width \( w \) cm, area 45 cm\(^2\)
   c) length \( \ell \) m, width 9 m, area 72 m\(^2\)
   d) length \( \ell \) cm, width 11 cm, area 99 cm\(^2\)
   e) length 4 m, width \( w \) m, area 72 m\(^2\)
   f) length 5 cm, area 20 cm\(^2\), find the width
   g) width 2 m, area 24 m\(^2\), find the length
   h) width 5 m, area 75 m\(^2\), find the length
   i) width 7 m, area 84 m\(^2\), find the length

   **Bonus:** A square has area 16 m\(^2\). What is its width? Hint: What can you say about the length and the width of a square? What number multiplied by itself equals 16?

   **Answers:** a) 6 m, b) 9 cm, c) 8 m, d) 9 cm, e) 18 m, f) 4 cm, g) 12 m, h) 15 m, i) 12 m, Bonus: 4 m

2. Find the perimeters of the rectangles in the previous exercise.

   **Answers:** a) 26 m, b) 28 cm, c) 34 m, d) 40 cm, e) 44 m, f) 18 cm, g) 28 m, h) 40 m, i) 38 m, Bonus: 16 m

Comparing area and perimeter. Project BLM Rectangles on the board. Then draw on the board the blank table below and have students fill it in. Ask students to list the rectangles from least to greatest by area. (D, C, A, E, B or D, C, E, A, B) Then ask them to list the rectangles from least to greatest by perimeter. (C, A, B, E, D or C, B, A, E, D)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>6</td>
<td>24</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>5</td>
<td>25</td>
<td>20</td>
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<tr>
<td>C</td>
<td>6</td>
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<td>D</td>
<td>11</td>
<td>1</td>
<td>11</td>
<td>24</td>
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<tr>
<td>E</td>
<td>3</td>
<td>8</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>

ASK: Are your lists the same? (no) Does the rectangle with the greatest area also have the greatest perimeter? (no) Does the rectangle with the smallest perimeter also have the smallest area? (no) Which rectangles have the same area? (A, E) Do they have the same perimeter? (no) Which rectangles have the same perimeter? (A, B) Do they also have the same area? (no) Emphasize that when one shape has an area larger than the other, you cannot tell which shape will have greater perimeter until you calculate it. The same is true the other way around: When one shape has greater perimeter than another, you still need to calculate which shape will have greater area. Area and perimeter do not depend on each other and are measured in different units.
Using area and perimeter. Discuss situations in which you would need to find the area or perimeter of a shape. For example, when installing a hardwood floor, you would need to know how much flooring to buy.

ASK: Would you need to find the area or the perimeter of the room you are renovating? (area) If you need to install quarter round around a room (explain what quarter round is), will you need to know the area or the perimeter of the room? (perimeter) If you are planning a race around a city park, do you need to know the area or the perimeter of the park? (perimeter) If you need to decide how much fertilizer to put in your garden, do you need to know the area or the perimeter of the garden? (area) If you need to put a fence around the garden, do you need to know the area or the perimeter of the garden? (perimeter) Encourage students to think of examples of situations in which they would need to find area or perimeter.

Extensions

1. On grid paper or BLM 1 cm Grid Paper, draw a square with sides 7 units long.
   a) Find the area and the perimeter of the square.
   b) Inside the square, draw a shape that has an area smaller than the area of the square and a perimeter larger than the perimeter of the square. There are many possible answers.

   **Answers:** a) area = 49 square units, perimeter = 28 units; b) for the shape in the margin: area = 17 square units, perimeter = 36 units

2. Draw two rectangles that have an area of 20 cm² but different perimeters. Calculate their perimeters.
   **Sample answer:** 4 cm × 5 cm, perimeter 18 cm; and 2 cm × 10 cm, perimeter 24 cm

3. a) Draw a square with an area of 9 cm². Calculate the perimeter.
   b) Draw a rectangle with the same perimeter as the square in part a). Find its area.
   c) Which of your shapes has a greater area?
   d) Try to find a different answer in part b). Did your answer to part c) change?

   **Answers:** a) 3 cm by 3 cm, perimeter 12 cm; b) sample answers: 2 cm by 4 cm, area 8 cm², 1 cm by 5 cm, area 5 cm²; c) the square has a greater area; d) no, the square will always have the greatest area
Goals
Students will solve problems connected to the area and perimeter of rectangles.

PRIOR KNOWLEDGE REQUIRED
Knows that area is additive
Knows the relative size of units of length measurements within the metric and Imperial systems
Can find the area and perimeter of a rectangle
Can multiply and divide up to two-digit numbers by one-digit numbers
Can convert between dollar and cent notations

Mental math minute. Remind students that they can double twice to multiply by 4 and double three times to multiply by 8. For example, to multiply $4 \times 6$, do $2 \times 6 = 12$ and then do $2 \times 12 = 24$. Then, to get $8 \times 6 = 48$, you can double 24. Also remind students that order does not matter in multiplication, so they can find the answer to $9 \times 4$, for example, by doubling 9 twice. Ask students multiplication problems in which one of the factors is 4 or 8 (for example, $7 \times 4$).

This lesson is a review of area. Work through the problems below as a class before assigning AP Book 4.2 p. 100. Review dollar and cent notation for money along the way.

Exercises
1. A room has the shape as shown below.

   a) Split the shape into two rectangles. Then find its area.
   b) Carpet costs $15.00 per square metre. How much will carpet cost for the room?
   c) Find the perimeter of the room.
   d) Carpet tack strips need to be attached along all the walls to hold the carpet in place. Tack strips cost 18¢ for 1 m. How much will the tack strips cost for the room?

   Answers: a) 24 m², b) $360, c) 22 m, d) $3.96
2. Mandy’s backyard is a square 30 m long. The house is along one side of the square, as shown in the margin.
   a) How many square metres of grass does Mandy need to cover the backyard?
   b) Grass seed with fertilizer costs 98¢ for 100 m². How much will the seed for the backyard cost?
   c) Mandy wants to fence the backyard. How much fencing does she need?
   d) Fencing costs $27.00 for 1 m. How much will the fence cost?
   **Answers:** a) 900 m², b) $8.82, c) 90 m, d) $2430

3. Use the table below to find all possible rectangles with sides in whole centimetres and an area of 24 cm². Then find the perimeter of the rectangles. Which rectangle has the smallest perimeter?

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Width (cm)</th>
<th>Area (cm²)</th>
<th>Perimeter (cm)</th>
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</tbody>
</table>

   **Answers:** 1 × 24, perimeter 50 cm; 2 × 12, perimeter 28 cm; 3 × 8, perimeter 22 cm; 4 × 6, perimeter 20 cm; rectangle with smallest perimeter is 4 cm × 6 cm, perimeter 20 cm

**Extension**

Find a counterexample to the statement. Explain why this is a counterexample.

a) All rectangles with the same area have the same perimeter.

b) All rectangles with the same perimeter have the same area.

**Sample answers:** a) rectangles with area of 24 cm²: 2 cm × 12 cm, perimeter 28 cm; 4 cm × 6 cm, perimeter 20 cm; 3 cm × 8 cm, perimeter 22 cm; b) rectangles with perimeter of 20 cm: 2 cm × 8 cm, area 16 cm²; 4 cm × 6 cm, area 24 cm²
Goals
Students will determine actual distance from scale drawings.

Prior Knowledge Required
Understands unit rates
Can measure in centimetres
Can calculate perimeter

Materials
overhead projector
transparency of BLM 1 cm Grid Paper (p. S-2)
BLM 1 cm Grid Paper (p. S-2)
centimetre rulers

Mental math minute. Ask students to solve multiplication problems within the range $1 \times 1$ to $10 \times 10$ and the corresponding division problems. For each number, go through the problems in order, for example, $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, to $10 \times 3$ and $30 \div 3$. Then progress to a different number. Next, try problems out of order, but keep corresponding multiplications and divisions together.

Introduce scale drawing. SAY: Sometimes it’s important to show sizes when you are drawing real objects, even if the object is too big to draw full size. For example, if you draw a building, it’s important to get the ratio of the height to width right so that it looks like it does in real life. One way to do this is to make a scale drawing. A scale drawing is a picture with all measurements scaled up or down by the same amount. Let’s say I need a scale drawing of a park that measures 20 km by 30 km. ASK: Can I draw a 20 km park? (no) Why not? (it’s too big) SAY: A scale that I could use is 1 cm for every kilometre.

Carefully measure and draw a 20 cm by 30 cm rectangle on the board. Write “1 cm : 1 km” above it. Tell students that this indicates the scale. The first number or measure is on the drawing, and the second number is what it would be in real life. Have volunteers measure the sides in centimetres and label the sides in kilometres. (20 km and 30 km) Draw the diagonal and have a volunteer measure it approximately in centimetres (36 cm) and then label it in kilometres. (36 km) ASK: What would be the area of this rectangle in real life? (600 km$^2$) What would be its perimeter? (100 km)
Repeat with a rectangle measuring 20 cm by 25 cm and a scale of 1 cm : 3 m. The finished drawing should look like this:

\[
\begin{array}{c}
\text{1 cm : 3 m} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{75 m} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{96 m} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{60 m} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\text{Area = 4500 m}^2 \\
\text{Perimeter = 270 m}
\end{array}
\]

For the following exercises, project BLM 1 cm Grid Paper on the board and distribute the BLM to students as well.

**Exercises:** Copy the rectangle onto BLM 1 cm Grid Paper. Use the scale to label the side lengths. Then find the area and the perimeter.

a) 1 cm : 2 m 

b) 1 cm : 5 km

**Bonus:** Approximately how long are the diagonals?

**Answers**

a) 
\[
\begin{array}{c}
\text{12 m} \\
\text{6 m}
\end{array}
\]

Area = 72 m$^2$ 

Perimeter = 36 m

b) 
\[
\begin{array}{c}
\text{20 km} \\
\text{25 km}
\end{array}
\]

Area = 500 km$^2$ 

Perimeter = 90 km

**Bonus:** a) 13 m, b) 32 km

**Scales on maps.** Tell students that if a scale drawing is made accurately, they can use it to estimate distances that they didn’t measure beforehand, like the diagonal of the rectangle.

**ACTIVITY (Optional)**

Design your own park. Use a scale of 1 cm : 2 m. The area of the playground should be at least 1600 m$^2$. You can add structures to your playground. Each structure requires at least the amount of space listed below:

- swings: 6 m$^2$
- wading pool: 10 m$^2$
- slide: 3 m $\times$ 1 m
- basketball court: 10 m$^2$
- sandbox: 25 m$^2$
Extensions

1. Copy the rectangle onto BLM 1 cm Grid Paper. Use the scale to label the side lengths. Then find the perimeter.
   a) 1 cm : 5 km  
   b) 1 cm : 7 dm  
   c) 1 cm : 8 mm
   
   ![Grid Paper Images]

   Answers: a) 90 km, b) 126 dm, c) 192 mm

2. Draw and measure the diagonals of the rectangles in Extension 1. Approximately what would the diagonal measure in real life?

   Answers: a) 35 km, b) 42 dm, c) 64 mm

3. Draw a scale drawing of your classroom on grid paper. What scale did you use?
**Goals**

Students will identify and describe the location of points and objects on a grid.
Students will place points and objects on a grid at a given location.

**PRIOR KNOWLEDGE REQUIRED**

Can determine actual distance from a scale drawing

**MATERIALS**

BLM 1 cm Grid Paper (p. S-2)
maps with grids, one for every 3 or 4 students

**Mental math minute.** Remind students that they can multiply by 4 by doubling twice and multiply by 8 by doubling three times. Also remind them that halving is the opposite of doubling. For example, they can divide by 4 by halving twice and divide by 8 by halving 3 times. Have students work in groups of three to divide numbers using repeated halving. Give a three-digit number that is a multiple of 8, such as 224. The first student then halves the number and says “224 ÷ 2 = 112,” the second students halves that answer and says “224 ÷ 4 = 56,” and the third student halves that answer and says “224 ÷ 8 = 28.”

**Reading positions on a grid.** Activity 1 uses grids that are identical to those used on maps.

**ACTIVITY 1 (Essential)**

1. Pairs of students will play a short version of the game sometimes known as "Battleship." Distribute BLM 1 cm Grid Paper to each student. Have them mark off two 6 by 6 squares, leaving one row above and one row to the left of each. Students label the columns of the squares with the letters A to F and number the rows 1 to 6. Without letting their partners see, students then draw four non-overlapping rectangles anywhere on the first grid: two 1 by 2 rectangles, one 1 by 3, and one 1 by 4 (see example below).
To begin the game, Player 1 guesses a position on the grid by using its column header and row number. If the position called is part of a rectangle on Player 2’s grid, Player 2 replies “hit”; otherwise, it is a “miss.” Player 1 records the guess on the second grid, marking a hit with an X and a miss with an O. If it is a hit, Player 2 also marks it by crossing out that square of the rectangle. Player 2 then has a turn guessing. When all squares of a player’s rectangles are crossed out, that player says “one down.” Play continues until all rectangles have been found.

Grids on maps. Tell students that the grid they just used to play the game, using letters for columns and numbers for rows, is the type of grid often used on maps. Show students an example of a map with a grid. Point out other features of the map, such as the scale of the map and how it is indicated, how icons and colour are used to highlight attractions such as parks and schools, and the alphabetical list of streets with grid positions. Note that streets usually cross more than one square in the grid, so their positions are indicated by a range (for example, B7 to F7).

**ACTIVITY 2 (Optional)**

2. Distribute maps with grids to groups of three or four students. Have students find features on their maps, such as the grid, the scale of the map, and the list of streets.

   In addition, have students locate several points of interest on their maps and record their locations using grid numbers. If all students are using the same map, prepare questions in advance, such as “Where is the park on the map?” or “What museum is at D9?” Students can also find their street in the list of streets and locate it on the map.

   **Bonus:** Have students measure the distance between two locations on the map and use the scale to estimate the distance in real life.

**Extensions**

1. Draw a map of your classroom on grid paper. Make sure to show where doors and windows are as well as the board and your desk.

2. Chess players often keep track of their moves using a grid system. The columns across the 8 by 8 grid are labelled “a” to “h,” and the rows are numbered 1 to 8, with the closest row labelled number 1. They record the moves by writing a letter for the type of piece and the end position. For example, Qe2 means that the queen moved to position e2.
If the pieces are in the positions shown, what will the board look like after the given moves?

Moves: Qb8, Ra6, Nf5, pd6

Answer

```
 a b c d e f g h
8 Q 8
7 7
6 R p 6
5 N 5
4 4
3 3
2 2
1 K 1
```

```
 a b c d e f g h
```
Dot Paper
Pentominoes
Rectangles
PS4-7 Using Structure II

Teach this lesson after:
Unit 12

VOCABULARY
area
area model
increase
length
product
row
sequence
sum
term
width

Goals
Students will use pictures to understand why two expressions are equal.

PRIOR KNOWLEDGE REQUIRED
Can fluently add and subtract multi-digit whole numbers using the standard algorithm
Can multiply a whole number of up to two digits by a one-digit whole number
Can divide two-digit numbers by 2
Can apply the distributive property of multiplication over addition
Knows that expressions within brackets are evaluated first
Can find the area of a rectilinear shape drawn on grid paper
Can compute the area of a rectangle from its side lengths

MATERIALS
15 counters per student
transparency of grid paper or BLM 1 cm Grid Paper (p. O-55)
grid paper or BLM 1 cm Grid Paper (p. O-55)
scissors

Discovering patterns in sums. Start by having students do the exercises below.

Exercises: Add and then find a pattern. What will the next addition equation be?

\[ 1 + 2 + 3 = \_
\]
\[ 2 + 3 + 4 = \_
\]
\[ 3 + 4 + 5 = \_
\]
\[ 4 + 5 + 6 = \_
\]

Answers: 6, 9, 12, 15; 5 + 6 + 7 = 18

ASK: How did the answers in the exercises change? (they increased by 3)
How did the additions change? (they started with a number 1 greater each time; each number increased by 1)
Give students 12 counters each. Tell them to arrange the counters to show \( 3 + 4 + 5 \). Students should make three groups—a group of three, a group of four, and a group of five.

SAY: Now I want you to show \( 4 + 5 + 6 \) instead of \( 3 + 4 + 5 \). ASK: How many more counters do I need to give you? (3) How do you know? (because the sum increases by 3 each time; because each of the three numbers increases by 1; because 4 + 5 is in both additions and 6 is 3 more than 3)

Give students three more counters each. Challenge them to add the
three counters, without moving the original counters, to show $4 + 5 + 6$.
(add 1 more to each group)

Draw on the board:

```
  O O O
  O O
  O O O
```

SAY: The white dots show $3 + 4 + 5$. The black dots show adding one more dot to each group. The whole picture shows $4 + 5 + 6$.

Exercises

a) If you have dots showing $8 + 9 + 10 + 11$, how many more dots would you need to show $9 + 10 + 11 + 12$?
b) If you have dots showing $5 + 7 + 9$, how many more dots would you need to show $6 + 8 + 10$?

Answers: a) 4, b) 3

Patterns using the times tables. Write on the board:

```
1 + 2 + 3 = 6 = 3 \times \_
2 + 3 + 4 = 9 = 3 \times \_
3 + 4 + 5 = 12 = 3 \times \_
4 + 5 + 6 = 15 = 3 \times \_
```

Have volunteers fill in the blanks. (2, 3, 4, 5) SAY: All these additions are three times a number. ASK: How can you know what number to multiply three by from just looking at the sum or addition? (It’s the middle number)
PROMPT: Where can you see the underlined number in the addition? Have volunteers circle where the underlined number is in each addition. ASK: Is it always in the same place? (yes) Can you predict where the number will be for the next sum? (yes, the middle number) SAY: When you can write the numbers in the sequence as a product, you can predict any term without having to find all the terms in between. Draw on the board:

```
<table>
<thead>
<tr>
<th>1 + 2 + 3</th>
<th>2 + 3 + 4</th>
<th>3 + 4 + 5</th>
<th>4 + 5 + 6</th>
<th>9 + 10 + 11</th>
<th>99 + 100 + 101</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 \times 2 = 6</td>
<td>3 \times 3 = 9</td>
<td>3 \times 4 = 12</td>
<td>3 \times 5 = 15</td>
<td>3 \times _ = _</td>
<td>3 \times _ = _</td>
</tr>
</tbody>
</table>
```

Have a volunteer fill in the blanks. (10, 30; 100, 300) Have students check the answers by doing the addition and the multiplication.

Exercises: Add the numbers and then multiply the middle number by 3. Did you get the same answer?

a) $16 + 17 + 18$  

b) $29 + 30 + 31$  

Bonus: $999 + 1000 + 1001$

Answers: a) 51, 51, yes; b) 90, 90, yes; c) 129, 129, yes; Bonus: 3000, 3000, yes
Using models to understand the pattern. Have students use 12 counters to again show $3 + 4 + 5$. Challenge them to rearrange the counters to show $3 \times 4$ by moving only one counter. (move 1 counter from the group of 5 to the group of 3; this makes 3 groups of 4) When all students have found the answer, demonstrate it on the board as shown below:

Repeat with $2 + 3 + 4$ and have students change it to $3 \times 3$ by moving only one counter. (move 1 counter from the group of 4 to the group of 2)

**Exercises:** Draw a picture to show why the sum equals the product.

a) $5 + 6 + 7 = 3 \times 6$

b) $7 + 8 + 9 = 3 \times 8$

**Answers:** a) moving a dot from the group of 7 to the group of 5 makes three groups of 6, b) moving a dot from the group of 9 to the group of 7 makes three groups of 8

SAY: When you can see a pattern and understand the reasons for it, you can sometimes see many other patterns.

**Exercises:** Draw a picture or use counters to show the addition. Then ...

a) move two dots to show that $2 + 4 + 6 = 3 \times 4$.

b) move three dots to show that $2 + 5 + 8 = 3 \times 5$.

c) move three dots to show that $4 + 5 + 6 + 7 + 8 = 5 \times 6$.

**Bonus:** move six dots to show that $1 + 2 + 3 + 4 + 5 + 6 + 7 = 7 \times 4$.

**Answers**

a) moving two dots from the group of 6 to the group of 2 makes three groups of 4

b) moving three dots from the group of 8 to the group of 2 makes three groups of 5

c) moving one dot from the group of 7 to the group of 5 and moving two dots from the group of 8 to the group of 4 makes five groups of 6

**Bonus:** moving one dot from the group of 5 to the group of 3, and two dots from the group of 6 to the group of 2, and three dots from the group of 7 to the group of 1 makes 7 groups of 4

**Using area models to discover patterns.** Write on the board:

\[
\begin{align*}
1 &= \\
1 + 2 &= \\
1 + 2 + 3 &= \\
1 + 2 + 3 + 4 &= \\
1 + 2 + 3 + 4 + 5 &= 
\end{align*}
\]

Fill in the blanks as volunteers tell you the sums. $(1, 3, 6, 10, 15)$ SAY: The gaps increase because that’s how we made the sequence, but I want to
know if there is a way to get an expression that will help me find any term. One way to think of the sums is as an area.

Project a transparency of grid paper or BLM 1 cm Grid Paper onto the board and draw the following shape on the board:

```
  +---+
  |   |
  |   |
  +---+
```

SAY: Let's count the squares inside the shape to find the area. ASK: How many squares are in the first row? (1) In the second row? (2) Third row? (3) Fourth row? (4) Fifth row? (5) SAY: So, we can add all these together to find the total. Write on the board:

\[
\text{Area} = 1 + 2 + 3 + 4 + 5 = 15 \text{ square units}
\]

**Exercises**

1. Write the area as a sum by adding the number of squares in each row.

   a) \[3 + 4 + 5\]
   b) \[1 + 2 + 3 + 4\]
   c) \[3 + 5 + 7\]

   **Answers:** a) \[3 + 4 + 5\], b) \[1 + 2 + 3 + 4\], c) \[3 + 5 + 7\]

   **NOTE:** Provide students with grid paper or BLM 1 cm Grid Paper for the following exercises.

2. Draw an area model for the expression.

   a) \[2 + 3 + 4\]
   b) \[4 + 5 + 6 + 7\]
   c) \[2 + 5 + 8\]

   **Answers**

   a) 
   b) 
   c) 

Have students draw two identical shapes like the one on the board on grid paper for \(1 + 2 + 3 + 4 + 5\) and then cut them out. Challenge students to arrange the shapes to make a rectangle. Then ask them to determine the area of each shape.
When students are finished, draw on the board:

(1 + 2 + 3 + 4 + 5) × 2 = 5 × 6 = 30
So, 1 + 2 + 3 + 4 + 5 = 15

SAY: Remember that you do the expression inside the brackets first, so you add to find the area of one shape, and then you multiply by 2 to get the area. ASK: How did I know to multiply 5 × 6 to get the area? (the area of a rectangle is length times width)

**Exercises:** Use two copies of a shape to make a rectangle. Then write the multiplication.

a) (1 + 2 + 3) × 2 = ___ × ___  
b) (3 + 5 + 7) × 2 = ___ × ___

c) (1 + 2 + 3 + 4) × 2 = ___ × ___  
d) (2 + 5 + 8) × 2 = ___ × ___

**Bonus:** Which two questions have the same answer? Why does that make sense?

**Answers:** a) 3 × 4; b) 3 × 10; c) 4 × 5; d) 3 × 10; Bonus: parts b) and d), 3 + 5 + 7 = 2 + 5 + 8, so multiplying both by 2 gets the same answer

**Using layers instead of rows in an area model.** Draw on the board:

```
1 + 3 + 5 + 7 + 9
```

ASK: How does the picture show 1 + 3 + 5 + 7 + 9? (the layers have 1, 3, 5, 7, and 9 squares) PROMPT: Where do you see the numbers 1, 3, 5, 7, and 9 in the picture? ASK: How does the picture show 5 × 5? (there are 5 rows of 5 squares) SAY: When you have two ways to show the number represented, you can write an equation. Continue writing on the board:

1 + 3 + 5 + 7 + 9 = 5 × 5

**Exercises**

1. Draw a picture using layers to write the sum as a product.

   a) 1 + 3
   b) 1 + 3 + 5
   c) 1 + 3 + 5 + 7
   d) 1 + 3 + 5 + 7 + 9 + 11

   **Bonus:** Draw a picture to show that 2 + 4 + 6 + 8 + 10 = 5 × 6.
Selected solution: d) $6 \times 6$

Answers: a) $2 \times 2$, b) $3 \times 3$, c) $4 \times 4$

2. Draw a picture using rows instead of layers to show that $(1 + 3 + 5 + 7) \times 2 = 4 \times 8$. Does this match the answer you got in Exercise 1.c)? How do you know?

Selected answer: Yes, because $4 \times 8 = 4 \times 4 \times 2$.

Problem Bank

1. a) Use grid paper to make a rectangle from two copies of a shape with area $8 + 9 + 10 + 11 + 12$.
   
   b) What is the area of the rectangle?
   
   c) What is the area of the original shape?
   
   d) What rectangle has the same area as two shapes with area $1 + 2 + 3 + \ldots + 7$?
   
   e) What rectangle has the same area as two shapes with area $1 + 2 + 3 + \ldots + 12$?
   
   f) How can you get your answer to part e) from your answers to parts b) and d)? Write an equation that shows this equality.
   
   g) Use your equation in part f) to mentally calculate $12 \times 13$.

Answers

a) 

b) 100 square units

b) 100 square units

c) 50 square units

d) $7 \times 8$

e) $12 \times 13$

f) $(1 + 2 + 3 + 4 + 5 + 6 + 7) + (8 + 9 + 10 + 11 + 12) = 1 + 2 + 3 + \ldots + 12$, so $12 \times 13 = (7 \times 8) + (5 \times 20)$

g) 156

2. a) Fill in the blanks.
   
   i) $1 + 2 + 3 = (\_ \times \_) ÷ 2$
   
   ii) $1 + 2 + 3 + 4 = (\_ \times \_) ÷ 2$
iii) \(1 + 2 + 3 + 4 + 5 = \left( \_ \times \_ \right) \div 2\)
iv) \(1 + 2 + 3 + 4 + 5 + 6 = \left( \_ \times \_ \right) \div 2\)
v) \(1 + 2 + 3 + 4 + 5 + 6 + 7 = \left( \_ \times \_ \right) \div 2\)

b) Look for a pattern in your answers to part a). Then predict:
\[1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9\]

**Answers:** a) i) 3, 4; ii) 4, 5; iii) 5, 6; iv) 6, 7; v) 7, 8; b) \((9 \times 10) \div 2 = 45\)

3. a) Draw a picture with layers to show that \(2 + 4 + 6 + 8 + 10 = 5 \times 6\).

b) Draw a picture with rows to show that \(2 + 4 + 6 + 8 + 10 = 5 \times 6\).

c) Draw a picture with rows to show that \(2 \times (1 + 2 + 3 + 4 + 5) = 5 \times 6\).

d) Explain how you could have predicted that \(2 + 4 + 6 + 8 + 10 = 2 \times (1 + 2 + 3 + 4 + 5)\).

e) Calculate \(1 + 2 + 3 + 4 + 5\) using \(5 \times 6 = 30\).

**Answers**

a)

![Diagram a]

b)

![Diagram b]

c)

![Diagram c]

d) When you double each term, you double the total

**e) 30 ÷ 2 = 15**

4. Draw a picture to show that \(2 + 4 + 6 + 8 + 10 + 12 = 6 \times 7\).

**Answer**

![Diagram d]
5. a) Draw a picture for the sum and move one dot to show that the sum is equal to the product.
   i) \(3 + 5 = 2 \times 4\)   ii) \(5 + 7 = 2 \times 6\)   iii) \(6 + 8 = 2 \times 7\)

b) Draw a picture for the sum and move two dots to show that the sum is equal to the product.
   i) \(1 + 5 = 2 \times 3\)   ii) \(2 + 6 = 2 \times 4\)   iii) \(3 + 7 = 2 \times 5\)

c) Use a number line to show that the number being doubled in part b) is halfway between the two numbers being added.

d) Use a number line to find the number halfway between the two numbers. Then check that double that number is the sum of the two numbers.
   i) 1 and 7   ii) 2 and 7   iii) 11 and 16

Selected answers

a) i) 
   ![Picture for 3 + 5 = 2 \times 4]

b) i) 
   ![Picture for 1 + 5 = 2 \times 3]

d) iii) 
   ![
   ![Number line for 11 to 16 with 13.5 marked]
   13.5 + 13.5 = 27 and 11 + 16 = 27

6. Draw a picture to show that \(10 + 11 + 12 + 13\) is 4 more than \(9 + 10 + 11 + 12\).

Answer

![Picture for 10 + 11 + 12 + 13]

7. If \(33 + 158\) is 191, what is \(34 + 159\)?

Answer: 193

8. If \(375 + 406 = 781\), what is \(378 + 409\)?

Answer: 787

9. If \(74 + 75 + 76 + 77\) is 302, what is \(75 + 76 + 77 + 78\)? How do you know?

Answer: 306, because if you draw rows of 74, 75, 76, and 77, and then add 1 to each of the four rows, you get rows of 75, 76, 77, and 78
1 cm Grid Paper
PS4-8 Using a Diagram

Goals
Students will create number lines to solve problems involving multiplication and division of numbers up to 10 000 by one-digit numbers.

PRIOR KNOWLEDGE REQUIRED
Can apply the distributive property of multiplication
Can multiply up to 10 × 10
Can multiply whole numbers by 10, 100, and 1000
Can interpret products in terms of repeated addition
Can multiply one-digit whole numbers by multiples of 10 up to 90
Can add and subtract multi-digit numbers
Understands division as a missing factor problem
Knows that expressions in brackets are done first
Can find the perimeter of a rectangle given its side lengths (for Extended Problem)
Can multiply two-digit numbers by multiples of 10 up to 90 (for Extended Problem)
Can convert metres to centimetres (for Extended Problem)

MATERIALS
calculators (optional, see Problem Bank 3)
BLM Posters (pp. O-68–69, see Extended Problem)

Review the distributive property. Write on the board:

\[
\begin{align*}
5 + 2 &= 7 \\
3 + 3 + 3 + 3 + 3 + 3 + 3 &= (5 \times 3) + (2 \times 3) = 7 \times 3
\end{align*}
\]

SAY: Just like five plus two is seven, five 3s plus two 3s is seven 3s.

Exercises: Write 12 × 8 as a sum of smaller products.

a) \[12 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8} \]
   \[= \underline{8} + \underline{8} \]

b) \[12 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8 + 8} \]
   \[= \underline{8} + \underline{8} \]

c) \[12 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8 + 8 + 8} \]
   \[= \underline{8} + \underline{8} \]
d) \[12 \times 8 = (8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8)\]

\[= \underline{80} + \underline{16}\]

e) \[12 \times 8 = (8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8)\]

\[= \underline{80} + \underline{16}\]

f) \[12 \times 8 = (8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8)\]

\[= \underline{80} + \underline{16}\]

Answers: a) \((11 \times 8) + (1 \times 8)\), b) \((10 \times 8) + (2 \times 8)\), c) \((9 \times 8) + (3 \times 8)\), d) \((8 \times 8) + (4 \times 8)\), e) \((7 \times 8) + (5 \times 8)\), f) \((6 \times 8) + (6 \times 8)\)

ASK: Which way of separating 12 eights into two smaller numbers of eights would be easiest to use to calculate \(12 \times 8\) mentally? (split 12 into 10 and 2) SAY: Multiplying by 10 is easy to do, and the result is easy to add. Write on the board:

\[10 \times 8 = 80\]
\[2 \times 8 = 16\]

so

\[12 \times 8 = 80 + 16 = 96\]

SAY: You would get the same answer using any other way, but using the 10 times table is easiest.

Showing the distributive property on a number line. Draw on the board:

\[\begin{array}{c}
0 \\
10 \times 8 \\
2 \times 8 \\
\end{array} \quad \begin{array}{c}
80 \\
+ \\
16 \\
\end{array} = 96\]

\[10 \times 8 + 2 \times 8 = 12 \times 8\]

SAY: You can keep track of the multiplication in parts by sketching a number line. This isn’t a precise number line because I didn’t try to make the numbers the correct distance apart. But the sketch is good enough to help us keep track of the numbers that we are adding. You can use this method to help you multiply.

Draw on the board:

\[\begin{array}{c}
0 \\
10 \times 7 \\
\_ \times 7 \\
\_ \times 7 = 14 \times 7 \\
\end{array} \quad \begin{array}{c}
70 \\
+ \\
= \underline{98}\end{array}\]

SAY: For the multiplication \(14 \times 7\), I started with 10 sevens, because that’s easy to multiply. ASK: How many more sevens do I need? (4) PROMPT: I need 14 sevens altogether. Write “4” in the bottom blank. ASK: What is \(4 \times 7\)? (28) Write “28” in the blank above “\(4 \times 7\).” ASK: So, what is \(14 \times 7\)? (98) How do you know? (70 + 28 = 98) Write “98” in the final blank.
Exercises
1. Use the diagram to multiply.
   a) 13 × 6

   \[
   \begin{array}{c}
   60 \\
   + \\
   =
   \end{array}
   \]

   0 10 6 + \_ \_ \times 6 = 13 \times 6

   b) 14 × 8

   \[
   \begin{array}{c}
   80 \\
   + \\
   =
   \end{array}
   \]

   0 10 8 + \_ \_ \times 8 = 14 \times 8

   c) 17 × 8

   \[
   \begin{array}{c}
   80 \\
   + \\
   =
   \end{array}
   \]

   0 10 8 + \_ \_ \times 8 = 17 \times 8

   d) 16 × 7

   \[
   \begin{array}{c}
   70 \\
   + \\
   =
   \end{array}
   \]

   0 10 7 + \_ \_ \times 7 = 16 \times 7

   Answers: a) 3 × 6 = 18, so 13 × 6 = 78; b) 4 × 8 = 32, so 14 × 8 = 112; c) 7 × 8 = 56, so 17 × 8 = 136; d) 6 × 7 = 42, so 16 × 7 = 112

2. Fill in the blanks to multiply.
   a) 13 × 7 = (10 × 7) + (\_ \_ \times 7)

   \[
   = \_ \_ + \_ \_ \\
   = \_
   \]

   b) 18 × 4 = (10 × 4) + (\_ \_ \times 4)

   \[
   = \_ \_ + \_ \_ \\
   = \_
   \]

   c) 16 × 9 = (10 × 9) + (\_ \_ \times 9)

   \[
   = \_ \_ + \_ \_ \\
   = \_
   \]

   Answers: a) 3, 70 + 21, 91; b) 8, 40 + 32, 72; c) 6, 90 + 54, 144
Review multiplying tens. SAY: Remember, if you can multiply $7 \times 3$, then you can multiply $70 \times 3$; the answer is just 10 times as much. Write on the board:

$7 \times 3 = 21$, so $70 \times 3 = 210$

Exercises: Multiply.

a) $60 \times 4$  
   b) $60 \times 8$  
   c) $90 \times 5$  
   d) $50 \times 4$

Bonus: Multiply in order: $8 \times 4$, $80 \times 4$, $800 \times 4$, $800 \times 40$

Answers: a) 240; b) 480; c) 450; d) 200; Bonus: 32, 320, 3200, 32 000

Multiplying two-digit numbers by one-digit numbers using a number line. Write on the board:

$78 \times 3$

SAY: You can calculate $78 \times 3$ by multiplying parts of $78 \times 3$ separately. Let’s split 78 into 70 + 8. Draw on the board:

ASK: What is $70 \times 3$? (210) Write “210” above “$70 \times 3$” on the number line.  
ASK: What is $8 \times 3$? (24) Write “+ 24 =” above “$8 \times 3$” on the number line, as shown below:

ASK: So, what is $78 \times 3$? (234) Write “234” above “$78 \times 3$” on the number line. SAY: Keeping track of the numbers you’re adding on a number line picture means you don’t have to remember them mentally, and that makes adding the numbers easier.

Exercises: Sketch a number line to help you multiply.

a) $64 \times 3$  
   b) $36 \times 5$  
   c) $87 \times 2$  
   d) $48 \times 7$

Selected solution

a) $180 + 12 = 192$

Answers: b) $(30 \times 5) + (6 \times 5) = 150 + 30 = 180$  
   c) $7 \times 2 = (80 \times 2) + (7 \times 2) = 160 + 14 = 174$  
   d) $(40 \times 7) + (8 \times 7) = 280 + 56 = 336$
Introduce the distributive property for division. SAY: Because multiplication and division are related, you can use the same idea to make dividing easier too. Draw on the board:

\[
26 \div 2
\]

\[
\begin{array}{c}
0 \\
10 \times 2 \\
+ \_ \times 2 = \_ \times 2
\end{array}
\]

SAY: I want to divide 26 by 2, so I need to find out how many twos I need to make 26. Point to the \( \times 2 \) under the 26 and SAY: I need to find out what times two is 26. I already have 20 from \( 10 \times 2 \). ASK: How much more is 26 than 20? (6) Write “6” in the top blank. ASK: How many twos are in six? (3) Write “3” in the blank below “6.” SAY: We needed 10 twos and 3 more twos to make 26. ASK: How many twos did we need altogether? (13) Write “13” in the last blank. The final picture should look like this:

\[
26 \div 2
\]

\[
\begin{array}{c}
0 \\
10 \times 2 \\
+ \_ \times 2 = 13 \times 2
\end{array}
\]

Then, underneath the picture, write on the board:

\[
13 \times 2 = 26, \text{ so } 26 \div 2 = 13
\]

**Exercises:** Fill in the blanks and then use the number line to divide.

a) \( 24 \div 2 \)

\[
\begin{array}{c}
0 \\
10 \times 2 \\
+ \_ \times 2 = \_ \times 2
\end{array}
\]

b) \( 28 \div 2 \)

\[
\begin{array}{c}
0 \\
10 \times 2 \\
+ \_ \times 2 = \_ \times 2
\end{array}
\]

c) \( 48 \div 4 \)

\[
\begin{array}{c}
0 \\
10 \times 4 \\
+ \_ \times 4 = \_ \times 4
\end{array}
\]
d) $39 \div 3$

\[
\begin{array}{c}
0 \\
10 \times 3 \\
\_ \times 3 = ___ \times 3
\end{array}
\]

\[
\begin{array}{c}
30 \\
\_ \times 3
\end{array}
\]

= 39

Answers: a) 4, 2, 12, so $24 \div 2 = 12$; b) 8, 4, 14, so $28 \div 2 = 14$; c) 8, 2, 12, so $48 \div 4 = 12$; d) 9, 3, 13, so $39 \div 3 = 13$

SAY: You can divide bigger numbers this way, too. You will just have more to keep track of on the number line. Draw on the board:

\[
\begin{array}{c}
76 \div 2
\end{array}
\]

SAY: When you’re dividing by 2, you can count by 20s until you get close to the number you’re dividing, in this case 76. You count by 20s because that’s ten times two. Show the skip counting on the board:

\[
\begin{array}{c}
76 \div 2
\end{array}
\]

SAY: Adding another 20 would be more than 76, so we can stop skip counting by 20s at 60. ASK: From 60, how much more do we need to get 76? (16) 16 is what times 2? (8) Continue writing on the board:

\[
\begin{array}{c}
76 \div 2
\end{array}
\]

\[
\begin{array}{c}
10 \times 2 \\
10 \times 2 \\
10 \times 2 \\
8 \times 2 = 16
\end{array}
\]

So \_ \times 2 = 76 and $76 \div 2 = ___$

SAY: So, now it’s just a matter of totalling what we multiplied by. Circle the three 10s and the 8 above the number line. SAY: We added 10 twos, then 10 more, then 10 more, then 8 more. ASK: So, how many twos did we add altogether? (38) Write “38” in the first blank. ASK: So, what is $76 \div 2$? (38) Write “38” in the second blank.

Exercises: Show counting by 20s, then by 2s, on a number line to divide.

a) $52 \div 2$  b) $74 \div 2$  c) $68 \div 2$  d) $90 \div 2$

Answers: a) 26, b) 37, c) 34, d) 45

Write on the board:

$285 \div 5$
ASK: What can you start skip counting by to divide by 5? (50s) Why? (50 is 10 × 5) Show the skip counting by 50s on a number line, as shown below:

| 0 | 50 | 100 | 150 | 200 | 250 |

SAY: Skip counting one more time would pass 285, so we can stop skip counting at 250. ASK: From 250, how much more do we need to get to 285? (35) Continue writing on the board:

| 10 × 5 | 10 × 5 | 10 × 5 | 10 × 5 | 10 × 5 | 7 × 5 = 35 |

So ___ × 5 = 285 and 285 ÷ 5 = ___

Circle the five 10s and the 7 above the number line. SAY: There are five 10s and a 7. ASK: What number did we multiply by five altogether? (57) So, what is 285 ÷ 5? (57) Write “57” in both blanks. SAY: Counting by 50s allows you to count by 10s when you find the answer. When you count by 10s to find the answer, the last product will only require multiplying by one-digit numbers. ASK: If you are dividing by 4, what would you count by on the number line? (40) If you are dividing by 7, what would you count by? (70) If you are dividing by 8, what would you count by? (80)

Exercises: Sketch a number line to divide. Check your answer using long division.

a) 190 ÷ 5  
b) 136 ÷ 4  
c) 364 ÷ 7

Answers: a) 38, b) 34, c) 52

Problem Bank

1. Find 678 ÷ 2 by counting by 200s and then by 20s. Sketch a number line to show your skip counting.

Solution

| 0 | 200 | 400 | 600 | 620 | 640 | 660 | 678 |

339 × 2 = 678, so 678 ÷ 2 = 339

2. a) How does 99 × 2 compare to 100 × 2? Explain.
   b) Use 100 × 2 to calculate 99 × 2.
   c) Use 100 × 3 to calculate 99 × 3.
   d) Use 100 × 17 to calculate 99 × 17.
   e) Use 1000 × 2 to calculate 999 × 2.
f) Use $1000 \times 3$ to calculate $999 \times 3$.
g) Use $1000 \times 49$ to calculate $999 \times 49$.

**Answers:** a) $99 \times 2$ is one fewer $2$ than $100 \times 2$, so it is $2$ less than $100 \times 2$; b) $200 - 2 = 198$; c) $300 - 3 = 297$; d) $1700 - 17 = 1683$; e) $2000 - 2 = 1998$; f) $3000 - 3 = 2997$; g) $49000 - 49 = 48951$

3. a) Calculate each product.
   - $9 \times 7 = ___$
   - $99 \times 7 = ___$
   - $999 \times 7 = ___$
   - $9999 \times 7 = ___$
   b) Use the pattern in part a) to calculate $999999999 \times 7$.
   c) Check your answer to part b) by doing the multiplication. You may use a calculator.

**Answers:** a) $63, 693, 6993, 69993$; b) $699999993$

4. Multiply in order.
   a) $7 \times 10, 7 \times 13, 7 \times 130, 7 \times 131$
   b) $8 \times 10, 8 \times 20, 8 \times 30, 8 \times 32, 8 \times 320, 8 \times 321$

**Answers:** a) $70, 91, 910, 917$; b) $80, 160, 240, 256, 2560, 2568$

5. a) Multiply $4126 \times 3$ using a number line.

b) Sketch a number line to multiply.
   i) $852 \times 7$
   ii) $613 \times 9$
   iii) $4444 \times 4$

**Bonus:** $312403 \times 2$

**Selected solution**

b) i) $5600 + 350 + 14 = 5964$

**Answers:** a) $12378$; b) ii) $5517$, iii) $17776$; Bonus: $624806$
6. Find the missing number two ways. Make sure you get the same answer both ways.

a) \( ? \times 6 = 192 \)

\[
\begin{array}{cccc}
10 \times 6 & 10 \times 6 & 10 \times 6 & 2 \times 6 \\
0 & 60 & 120 & 180 & 192 \\
\end{array}
\]

\[\text{?} = + + + = \]

\[40 \times 6 = 240\]

\[
\begin{array}{cccc}
3 \times 6 & 5 \times 6 \\
0 & 192 & 210 & 240 \\
\end{array}
\]

\[? = - - - = \]

b) \( ? \times 8 = 312 \)

\[
\begin{array}{cccc}
30 \times 8 & 9 \times 8 \\
0 & 240 & 312 \\
\end{array}
\]

\[? = + = \]

\[40 \times 8\]

\[
\begin{array}{cccc}
\text{?} = - - = \\
0 & 312 & 320 & 1 \times 8 \\
\end{array}
\]

c) Use your answers to parts a) and b) to divide.

i) \( 192 \div 6 \)

ii) \( 312 \div 8 \)

d) Sketch two different number lines to divide \( 294 \div 6 \). Make sure you get the same answer both ways.

**Answers:** a) \( 10 + 10 + 10 + 2 = 32 \) or \( 40 - 5 - 3 = 32 \); b) \( 30 + 9 = 39 \) or \( 40 - 1 = 39 \); c) i) 32, ii) 39; d) 49
**Extended Problem: Posters**

**MATERIALS**

BLM Posters (pp. O-68–69)

**Preparation for the extended problem.** Tell students that the extended problem involves the following situation: students are making posters for parents to come and see in a poster show. Each student makes a poster on a large sheet of paper. Draw on the board:

```
20 cm
```

```
27 cm
```

Have students show with their hands about how wide and how tall the posters are. Attach some regular sheets of paper to the board, with some really close together and others really far apart. ASK: Is this a good way to display the posters? (no) Why not? (for example, it doesn’t look very good) Tell students that the extended problem is partly about decorating posters to make them look good and partly about how to display them to make them look good. Tell students that one of the things they will need to remember how to do is to convert metres to centimetres. ASK: How many centimetres are in 1 metre? (100) In 2 metres? (200) 3 metres? (300) SAY: You can always multiply by 100 to get the number of centimetres if you know the number of metres.

**Extended Problem: Posters.** Give students BLM Posters. Question 7 is an opportunity to apply the strategy of using a diagram in an unfamiliar context.

**Answers:** 1. 94 cm; 2. 2910 cm; 3. yes, because $100 \times 30 = 3000$ cm; 4. 14 cm; 5. from one edge of one poster to the next, 8 cm; 6. 7; 7. 9 cm (the poster itself is 6 cm from the door, so the pinhole is 9 cm from the door)
Posters (1)

Your class is holding a poster show for parents to come and see some of your class's artwork. The posters will be hung in the school all along the hallway. Each poster is 20 cm wide by 27 cm tall. Each student can decorate the edges of their poster with yarn.

1. How much yarn do you need to decorate the edges of one poster?

2. Each student takes an extra 3 cm of yarn, to make sure they have enough. How much yarn, in centimetres, will a class of 30 students need?

3. A ball of yarn has 30 m of yarn. Is that enough for the whole class?
4. You put two pins near the top of the poster, 3 cm from each edge. How far apart are the pins?

5. You decide to place all the pinholes the same distance apart. How far apart will the posters be?

6. Doors along the hallway are 2 m apart. How many posters can you put between each pair of doors?

7. You decide to centre the posters so that the posters closest to the doors are the same distance from the door. How far from the door should you make the first pinhole?
Unit 13  Measurement: Time

Introduction
This unit explores time. Topics include:

• reading time to the minute on digital and analog clocks;
• time and date notation; and
• estimating and measuring time intervals and total elapsed time.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit:

• practise arithmetic skills, with an emphasis on multiplication and division.

Materials
Cut out and laminate the cards from BLM Time Memory Cards so students can use them multiple times.
Assessment

The lessons covered by a quiz or test are as follows:

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Additional Information for This Unit

**Recurring Games**

1. **Picking Pairs.** Place cards face up in an array (the deck of cards used and the dimensions of the array will depend on the lesson). Students play individually or in teams and take turns picking pairs of matching cards and placing them into a common discard pile. When there are no more pairs in the array, students can add more cards to it. The goal is to match all the cards.

2. **Memory.** Set up an array of cards face down on the table. Students turn over two cards at a time. If the cards match by time, students set these cards aside; otherwise, they turn them face down again and continue playing. Students can play individually or co-operatively in pairs. In either case, the goal is to find all the matching pairs.

**NOTE:** It is a good idea for students to play Picking Pairs first—to practise making and recognizing matches—before they play Memory.
Goals
Students will tell time to the nearest minute from a digital clock.

PRIOR KNOWLEDGE REQUIRED
Can read and write two-digit numbers

MATERIALS
large digital clock

Mental math minute. Arrange students in a line and have groups of three students add two-digit numbers by adding tens and adding ones. Give an addition problem, such as 35 + 46. The first student in line adds the tens (30 + 40 = 70), the second student adds the ones (5 + 6 = 11), and the third student finishes the addition (70 + 11 = 81, so 35 + 46 = 81). Then, give the next three students in the line a new problem. Start with problems that do not require regrouping, such as 25 + 34, then continue on to problems that require regrouping ones.

Discuss the need for clocks. Show students a large digital clock. SAY: This is a digital clock. It shows the time in digits only—that is, only numbers, no hands. ASK: Why do we need clocks? (sample answers: to know the time of the day, to measure time) Record students’ ideas. ASK: Why do we need to know the time of the day? (sample answer: you need a way to tell the time so you can get to school before lessons start) Again, record students’ ideas. Have students give examples of events that happen at a precise time. (sample answers: school starts at 8:30, lunch is at 12:25, karate class starts at 5:15)

Point out that sometimes you need to know the time precisely. For example, you need to know precisely the time a train leaves so that you will not be late; if a train leaves at 7:33, you will miss it if you arrive at the station at 7:35.

Introduce hours and minutes. SAY: We use hours and minutes to show time and to measure time. A day is divided into hours, and hours are divided into minutes. There are 24 hours in a day and 60 minutes in an hour. A clock shows 12:00 at noon and at midnight. NOTE: The concepts of “a.m.” and “p.m.” will be introduced later in the unit. If students mention them, explain that you will use these labels later.

Hours and minutes on digital clocks. Point out the parts on the digital clock that show the hours and the minutes. Set the time on the clock to 7:05. Explain that digital clocks show hours and the minutes separated by a colon. The number of minutes is always shown as a two-digit number even when it is less than ten. On some clocks, the number of hours is also shown as a two-digit number.
Exercises: How many hours does the clock show? How many minutes does the clock show?

a) 12:15  
b) 04:23  
c) 10:01  
d) 09:02

Answers: a) 12 hours, 15 minutes; b) 4 hours, 23 minutes; c) 10 hours, 1 minute; d) 9 hours, 2 minutes

Reading the time from a digital clock. Remind students that when you ask somebody what the time is, they usually do not answer “9 hours 5 minutes.” They say something like “5 minutes after 9.” Have students think of other ways to say the time. You might want to record the answers. Explain that you will read the time in the form “5 minutes past 9.” Emphasize that we say the minutes first. Have students read the times in the previous exercises using this format. (a) 15 minutes past 12, b) 23 minutes past 4, c) 1 minute past 10, d) 2 minutes past 9) Tell students that you can also read the times as minutes after the hour and that we don’t always say the word “minutes.” Point to part a) and SAY: 15 past (or after) 12. Have students read the other times using this format. (b) 23 past 4, c) 1 past 10, d) 2 past 9)

Exercises: Write the time in numbers and words.

a) 11:27  
b) 8:46  
c) 10:08

d) 04:07  
e) 02:04  
f) 1:03

Bonus: Ren thinks that 02:10 is 2 minutes past 10. Explain his mistake.

Answers: a) 27 minutes past (or after) 11; b) 46 minutes past 8; c) 8 minutes past 10; d) 7 minutes past 4; e) 4 minutes past 2; f) 3 minutes past 1;
Bonus: Ren read the hours first instead of reading the minutes first, so the time is 10 minutes past 2.

Introduce “o’clock.” Ask students if they know how to read the time when there are zero minutes after the hour. SAY: For these times, we say o’clock. Write “7:00” on the board, and SAY: For example, this would be 7 o’clock. Write a few more examples, and have volunteers say the time.

Showing times on a digital clock. Remind students that when they say the time in words and numbers, they sometimes say the minutes first. SAY: When we write the time in numbers (or as it is shown on a digital clock), we write the hours first and then the minutes, so 12 minutes past 3 is 03:12, not 12:03. Write on the board:

12 minutes past 3 is 03:12 or 3:12.

Remind students to write a zero in front of any one-digit numbers of minutes. Volunteers can show the times in the following exercises on the digital clock.

Exercises: How would the time look on a digital clock?

a) 15 minutes past 12  
b) 9 minutes past 11  
c) 7 minutes past 10

d) 12 minutes past 6  
e) 10 minutes past 7  
f) 8 o’clock

Answers: a) 12:15, b) 11:09, c) 10:07, d) 06:12, e) 07:10, f) 08:00
Finding minutes to the hour. Tell students that when the time is close to changing hours, we sometimes say how many minutes before the next hour.

ASK: How many minutes in an hour? (60) Write “9:61” on the board. ASK: Is this a time you will see on a clock? (no) Why not? (the most minutes you can have is 59) What happens after 59? (the hour changes) If the time is 58 minutes after 9, is it closer to 9 o’clock or 10 o’clock? (10 o’clock) Write “9:58” on the board. ASK: How many minutes to 60? (2) SAY: In 2 more minutes, it will be 10 o’clock.

Exercises: How many minutes until the hour changes?

a) 9:51  b) 3:42  c) 1:45  d) 11:37

Answers: a) 9, b) 18, c) 15, d) 23

Telling time in minutes to the hour. Write “5:40” on the board. ASK: How many minutes until the hour changes? (20) What time will it be when the hour changes? (6 o’clock)

Exercises: What time will it be when the hour changes in the previous exercises?

Answers: a) 10 o’clock, b) 4 o’clock, c) 2 o’clock, d) 12 o’clock

SAY: When the number of hours is close to changing, we sometimes read the time as minutes to the next hour. Instead of saying 9:58, we read the time as 2 minutes before 10 o’clock or 2 minutes to 10. Write a few examples on the board and have volunteers read the times both ways. Make sure that the number of minutes are always between 31 and 59. Again point out the variety of ways that people will say the time (e.g., 18 minutes before 4 o’clock, 18 minutes to 4). Students can do the following exercises to practise writing the times.

Exercises: Write the times in the previous exercises as minutes to the hour.

Answers: a) 9 minutes to 10, b) 18 minutes to 4, c) 15 minutes to 2, d) 23 minutes to 12

Extensions

1. Explain that minutes are divided into seconds. Some digital clocks also show the seconds after the minutes—for example, 09:08:07. The first two digits show the hour, the next two show the minutes, and the final two show the seconds. Explain that when you say the time on a clock with seconds, you say minutes first, seconds after, and only then say “past the hour.” For example, 09:08:07 is “8 minutes, 7 seconds past 9” (or “8 minutes and 7 seconds past 9”).

Have students write the time on each digital clock as they would say it, using words and numbers.

a) 10:27:32  b) 07:46:15  c) 12:08:21
d) 04:07:21  e) 02:04:09  f) 01:03:02
Answers: a) 27 minutes, 32 seconds past 10; b) 46 minutes, 15 seconds past 7; c) 8 minutes, 21 seconds past 12; d) 7 minutes, 21 seconds past 4; e) 4 minutes, 9 seconds past 2; f) 3 minutes, 2 seconds past 1

2. The clock is broken. Explain what is wrong.
   a) 25:04   b) 11:4   c) 5:68
   Answers: a) the hour number is too high, the hours cannot be greater than 12; b) the minutes are shown with only one digit, but should be shown with two digits; c) the number of minutes is too high, the minutes cannot be greater than 59

3. Jane is going to swimming class in 124 minutes. It is 3:00 now. When will Jane go?
   Answer: 5:04
Goals

Students will identify times of the day using a.m. and p.m.
Students will convert between a.m./p.m. notation and the 24-hour clock.

PRIOR KNOWLEDGE REQUIRED

Can read times on a digital clock
Knows times on the hour are read as o’clock

MATERIALS

analog clock with hour and minute hands
BLM Make Your Own Clock (p. P-40)
scissors
glue
paper plates
pencils
paper fasteners
BLM Numbers on a Clock Face (p. P-41, optional)

Mental math minute. Give students division questions that can be done by skip counting by 2, 3, 4, 5, or 10. Have students skip count aloud to answer the division questions.

Introduce analog clocks. Show students an analog clock. SAY: There are two types of clocks people often use. One type shows the time with digits only; it is called a digital clock. A clock with hands is called an analog clock. Point out that the clock face has numbers all around it going from 1 to 12. SAY: These numbers show the hour.

ACTIVITY (Essential)

Make your own clock. Give each student BLM Make Your Own Clock, scissors, glue, a paper plate, a pencil, and a paper fastener. Have them follow these steps to make their own clocks:

Step 1: Cut out the circle and the hands from the BLM.
Step 2: Glue the circle to the inside of the paper plate.
Step 3: Use a sharpened pencil to poke a hole in the centre of the clock face.
Step 4: Write the numbers in the correct positions on the clock face.
Step 5: Attach the hands to the plate with the paper fastener.

Have students keep the clocks they make for use in later lessons.
NOTE: Students who are struggling with filling in the numbers on a clock face will benefit from doing BLM Numbers on a Clock Face.
Introduce a.m. and p.m. ASK: How many hours are in the day? (24) SAY: If there are 24 hours, why do we use a clock that has only 12 hours on its face? The 12-hour clock comes from ancient Egypt, where people divided the night and the day into 12 hours each. Later, we started to use midday as the division, and that created the a.m./p.m. clock that we use. Write “12:00” on the board. ASK: What time is it? (12 o’clock) Is it nighttime or daytime? (it could be either) Ask students if anyone knows what we can write next to the time to tell whether it is 12:00 at night or 12:00 during the day. Write “a.m.” and “p.m.” on the board. SAY: We add a.m. to times between midnight and just before noon, and p.m. to times from noon to just before midnight. You might want to mention that these names or labels are short forms for expressions that originated in ancient Rome: a.m. stands for “ante meridiem” (before noon) and p.m. stands for “post meridiem” (after noon). Draw on the board:

Writing times using a.m. and p.m. List several events with times of day on the board, and ask students to say whether each time should have a.m. or p.m. after the numbers. (see examples below—the answers are in parentheses) Students can signal the answers by pointing to the correct side of the picture on the board.

Breakfast at 8:00 (a.m.)
Plane departure at 3:15 in the afternoon (p.m.)
Train arrival at 11:45 in the morning (a.m.)
Library visit at 5:30 (p.m.)
Dentist appointment at 1:15 (p.m.)
3 hours after midnight is 3:00 (a.m.)
7 o’clock in the morning is 7:00 (a.m.)
I ate ice cream at 4:30 (p.m.)
Half past 9 in the morning (a.m.)

Exercises: Is the time a.m. or p.m.?

a) Lunch time: 11:55  b) School ends: 3:15
  c) Swimming pool closes: 9:30  d) School bus leaves: 8:05

Answers: a) a.m., b) p.m., c) p.m., d) a.m.

Introduce the 24-hour clock. SAY: Although we have adopted a 12-hour clock, it is often more convenient to use a 24-hour clock to record and measure time because you can tell by looking at the time if it is a.m. and p.m. For example, schedules for buses or trains often use 24-hour clocks. SAY: The 24-hour clock starts at midnight. Midnight is 00:00 or 24:00.
Converting a.m. hours to 24-hour time. Draw a long timeline on the board:

SAY: This is a *timeline*—it is like a number line, except that it is marked with times instead of numbers. Tell students that on a 24-hour clock, the hour is always written as two digits. The hours until noon go as in the regular clock, except that you don’t have to say a.m. So 1:00 a.m. is 01:00, 2:00 a.m. is 02:00, and so on. Mark 2:00 a.m. and 02:00 on the timeline. Mark 6:00 a.m. on the board and have a volunteer mark 06:00. Repeat for 7:30. The timeline should look like this:

Write a few more a.m. times on the board or on the timeline and have volunteers convert them to 24-hour notation. Leave the timeline on the board.

**Exercises:** Write the time as it would appear on a 24-hour clock.

a) 4:00 a.m.  b) 11:00 a.m.  c) 3:25 a.m.  d) 9:46 a.m.

**Answers:** a) 04:00, b) 11:00, c) 03:25, d) 09:46

ASK: What did you do to the time to convert to 24-hour time? (made the hour two digits, didn’t write a.m.)

Converting p.m. hours to 24-hour time. Tell students that from noon back to midnight, the hours continue to count up on a 24-hour clock. Add 1:00 p.m., 2:00 p.m., and 3:00 p.m. to the top of the timeline. SAY: On a 12-hour clock, after 12:59 p.m. we go back to counting from 1:00. On a 24-hour clock, the hours keep counting up. So we write 1:00 p.m. as 13:00. Ask how to write 2:00 p.m. and 3:00 p.m. (14:00, 15:00) Add 4:30 p.m. to the timeline and have a volunteer write the time as it would appear on a 24-hour clock. (16:30)

**Exercises:** Write the time as it would appear on a 24-hour clock.

a) 4:00 p.m.  b) 11:00 p.m.  c) 3:25 p.m.  d) 9:46 p.m.

**Answers:** a) 16:00, b) 23:00, c) 15:25, d) 21:46

ASK: What change did you make to convert the p.m. times? (added 12 to the hour) What happened to the minutes? (they stayed the same) Do you need to write p.m.? (no)

Converting 12:00 to 1:00 a.m./p.m. to 24-hour time. SAY: Here is a trick question. Write “12:45 a.m.” on the board. Ask how to write this time as it would look on a 24-hour clock. Take answers without confirming or correcting them. Refer back to the timeline on the board. Have a volunteer
mark 12:45 a.m. on the timeline. (see the first × in the picture below) Have another volunteer mark on the timeline where 12:45 on a 24-hour clock would be. (see the second × in the picture below)

ASK: Are these times the same? (no) What are the hours on a 24-hour clock? (0 to 24) Ask again how to write 12:45 a.m. on a 24-hour clock. (00:45) How do you write 12:45 p.m. on a 24-hour clock? (12:45)

Exercises: Write the time as it would appear on a 24-hour clock.

a) 12:30 a.m.  b) 12:30 p.m.  c) 12:25 a.m.
d) 12:25 p.m.  e) 12:17 a.m.  f) 12:58 p.m.

Answers: a) 00:30, b) 12:30, c) 00:25, d) 12:25, e) 00:17, f) 12:58

Extensions

1. Which countries use the 12-hour clock and which countries use the 24-hour clock? What are the advantages and disadvantages of each system?

2. Explain that the ancient Romans also divided the day into 12 equal parts, starting at around what we call 6 a.m. and ending at around 7 p.m. SAY: Here are more examples of times in ancient Rome and their modern-day equivalents. Write on the board:

<table>
<thead>
<tr>
<th>Time in Ancient Rome</th>
<th>Time Today</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st hour</td>
<td>about 7 a.m.</td>
</tr>
<tr>
<td>6th hour</td>
<td>about 12 p.m.</td>
</tr>
<tr>
<td>7th hour</td>
<td>about 1 p.m.</td>
</tr>
<tr>
<td>12th hour</td>
<td>about 6 p.m.</td>
</tr>
</tbody>
</table>

Have students extend the table to show all the hours before noon, or have them create a timeline that shows all daylight times (from 7 a.m. to 6 p.m.) as they would have been written in ancient Rome and now. Have students look for patterns. ASK: What number do you need to subtract from modern a.m. times to get the Roman day time? (6 hours) What number do you need to add to modern p.m. times to get the Roman day time? (6 hours) What Roman hour is an exception to this rule? (6th hour, because you still need to subtract 12 – 6 to get the Roman time)

3. Imagine a free day when you can do anything you want. Write six things you would do. Write the start time and the end time for each, using a.m. or p.m. for each time. Make a timeline and find the time each activity takes. Try to include an activity that begins in the a.m. and ends in the p.m.
**Goals**

Students will tell time in 15-minute increments as o’clock, half past, quarter past, or quarter to.

**PRIOR KNOWLEDGE REQUIRED**

Can identify times of the day using a.m. and p.m.

Can convert between a.m./p.m. notation and the 24-hour clock

Can read times as minutes past or minutes to the hour

**MATERIALS**

- analog clock with hour and minute hands
- clocks made in the activity from Lesson ME4-22
- BLM Time Memory Cards (6) to (8) (pp. P-42–44)
- BLM Half and Quarter Hours (pp. P-50–51, optional)

**Mental math minute.** Ask students to solve multiplication problems within the range of $0 \times 1$ to $10 \times 10$. For each number, first go through the problems in order, such as $0 \times 3, 1 \times 3$, and so on, to $10 \times 3$; then in reverse order; and after that, go through the same problems out of order. Then, progress to a different number.

**Introduce the minute and hour hands.** Display an analog clock with hour and minute hands. SAY: Analog clocks use two hands to show the time. A shorter, thicker hand, called the hour hand, shows the hour. The longer, thinner hand is the minute hand. If your clock has a second hand, tell students that the very thin hand moving quickly is the second hand, but they won’t be using it today.

**Reading “o’clock” times.** Turn your clock so that it reads 9:00. SAY: When the minute hand is pointing straight up to the 12, the hour hand points directly to a number. This clock shows 9 o’clock. Turn your clock to a few other “o’clock” times and have volunteers read the times. Finish with 1 o’clock. SAY: Every hour, the minute hand makes one full circle, and the hour hand moves ahead one number. Demonstrate this by moving the clock ahead, drawing attention first to the minute hand and then to the hour hand. Point out that when the minute hand is halfway around the clock, the hour hand is halfway between two numbers. ASK: Which direction do the hands move in? SAY: This is called the clockwise direction. ASK: Why do you think clock faces are round? Set your clock to read 1:00 and PROMPT: What time comes one hour after 1 o’clock? (2:00) ASK: How does the minute hand move between 1 o’clock and 2 o’clock? (it moves one full circle around the clock) What does the hour hand do? (it moves from 1 to 2) Move the hour hand clockwise forward to 2 o’clock. Move the hour hand forward (clockwise) once or twice more, and ask the same question. Finally, advance
the clock to 12 o’clock and ask what time comes next. Again, ASK: Why do you think the clock face is round? (the hours count to 12, then start again at 1; the times go round in a circle)

For the following exercises, students will need the clocks they made in the activity from Lesson ME4-22.

**Exercises:** Show the time on your clock.

a) 7 o’clock  b) 11:00  c) 02:00  d) 4 o’clock

**Introduce “half past” an hour.** Show 2:00 on an analog clock, and ask students to say or write the time in different ways. (2 o’clock, 2:00, 02:00) SAY: In a full hour, the minute hand moves a full circle around the clock. ASK: How far around the circle will the minute hand move in half an hour? (halfway around) What number will the minute hand point at when half an hour has passed? (6) Change the clock so that it shows 2:30 and explain that we can read a time like this as half past 2, because half an hour has passed after 2 o’clock. Point out that the hour hand has moved halfway towards the 3.

Show 4:30 on the analog clock. ASK: What hour is it? (4) SAY: The minute hand is at 6, so we know half an hour passed after 4. ASK: What is the time? (half past 4) Repeat with 7:30, 9:30, and 12:30.

**Writing “half past” in two ways.** ASK: How many minutes are in an hour? (60) How many minutes are in half an hour? (30) SAY: So, half past 12 is also 30 minutes after 12. ASK: How would you write this time in numbers? (12:30) Show the following times on an analog clock and have students say the time as “half past” and write it in numbers: 8:30, 3:30, 5:30, 10:30. Point out that students can add a.m. or p.m. if they like.

**Exercises:** Write the time in numbers.

a) half past 3  b) half past 6  c) half past 11  d) half past 9

**Answers:** a) 3:30, b) 6:30, c) 11:30, d) 9:30

**Reading time in the “half past” form from written times.** ASK: How can you tell on a digital clock if the time is half past an hour? (the number of minutes is 30) Write different times on the board (for example, 04:30, 07:30, 02:30) and have students tell the time in the form “half past” the hour.

**Exercises:** Write the time in words with numbers.

a) 9:30  b) 3:30  c) 11:30  d) 5:30

**Answers:** a) half past 9, b) half past 3, c) half past 11, d) half past 5

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**ACTIVITY 1 (Optional)**

1. Play **Picking Pairs** and then **Memory** (see unit introduction) with cards from BLM Time Memory Cards (6) to (7).
Quarters on a clock. Draw a clock face on the board, divide it in half (draw a line from the 12 to the 6), and then have a volunteer divide it into quarters by drawing a line from 9 to 3. Shade the quarter between 12 and 3. Explain that when the minute hand passes the shaded part of the clock face, one fourth or one quarter of an hour has passed. When it is a quarter of an hour after 7 o’clock, the hour hand is pointing a little after 7 and the minute hand is a quarter of the way around the clock and pointing at the 3. Leave the clock face on the board for later use.

Introduce “quarter past.” SAY: When a quarter of an hour has passed after 7:00, we say it is quarter past 7. ASK: What time is a quarter of an hour after 9 o’clock? (quarter past 9) Repeat with various “quarter past” times.

Writing “quarter past” times. ASK: How many minutes are in an hour? (60) How many minutes are in a quarter of an hour? (15) How do we write 15 minutes after 4 o’clock as a time? (4:15) How do we read the time? (15 minutes past 4, quarter past 4) How would we write quarter past 6 on a digital clock? (6:15 or 06:15) Quarter past 9? (9:15 or 09:15) Repeat with various “quarter past” times, having students both write and say the time (e.g., 8:15, 15 minutes past 8).

Show various “quarter past” times on a clock face and have students identify the times. Give times sequentially at first, from 12:15 to 11:15, and then in random order. Tell students several times in these formats and have them show the time on their own clocks.

Introduce “quarter to.” Refer to the clock face on the board showing a “quarter past” time. Shade the quarter between 9 and 12. Remind students that when it is 5 minutes before 7 o’clock, we say 5 minutes to 7. SAY: When it is a quarter of an hour before 7:00, we say it is a quarter to 7. ASK: What time is a quarter of an hour before 9 o’clock? (quarter to 9) Repeat with various “quarter to” times.

Writing “quarter to” times. ASK: How do we write 15 minutes to 4 o’clock as a time? (3:45) How do we read the time? (45 minutes past 3, 15 minutes to 4, quarter to 4) How would we write quarter to 6 on a digital clock? (5:45 or 05:45) Quarter to 9? (8:45 or 08:45) Repeat with various “quarter to” times, having students both write and say the times (e.g., 7:45, 15 minutes to 8, quarter to 8).

Show times on a clock face and have students identify the time. Use various times that are all a quarter to the hour. Give times sequentially at first, from 12:45 to 11:45, and then in random order. Then, show the same times, first sequentially and then in random order, but have students say or write the digital times. Finally, include “o’clock,” “half past,” and “quarter past” times as well (for example, 7:15, 3:00, 5:30, 9:15, 4:30, and 11:00). Tell students several times in these formats and have them show the time on their own clocks.
ACTIVITY 2 (Essential)

2. Students play **Picking Pairs** and then **Memory** (see unit introduction) with cards from BLM Time Memory Cards (6) to (8).

For additional practice, students can use **BLM Half and Quarter Hours**.

**Extensions**

1. Write the times in order.
   a) 4:30 a.m., 2:30 p.m., 5:30 a.m.
   b) 03:15, 21:45, 08:15
   c) 6:15 a.m., 3:45 p.m., 5:30 p.m.
   d) 4:30 a.m., quarter past 5 p.m., 10 minutes to 5 a.m.
   e) 6 o’clock, 5:50, 14 minutes past 6
   f) 1:45, 5:00, 4:30, 2:15

   **Answers:** a) 4:30 a.m., 5:30 a.m., 2:30 p.m.; b) 03:15, 08:15, 21:45; c) 6:15 a.m., 3:45 p.m., 5:30 p.m.; d) 4:30 a.m., 10 minutes to 5 a.m., quarter past 5 p.m.; e) 5:50, 6 o’clock, 14 minutes past 6; f) 1:45, 2:15, 4:30, 5:00

2. How much time passes between quarter to 3 and quarter past 3? Draw a clock face to show your answer.

   **Answer:** 30 minutes, the minute hand moves through the top half of the clock

3. a) What time is a quarter to noon? Write it as many different ways as you can.
   b) What time is a quarter past midnight? Write it as many different ways as you can.

   **Answers:** a) 11:45 a.m., 11:45, 15 minutes to 12, 45 minutes after 12; b) 12:15 a.m., 00:15, 15 minutes after 12

4. There are 15 minutes in a quarter of an hour. How many minutes are in one fifth, three fifths, one sixth, five sixths, and one tenth of an hour?

   **Answers:** 12, 36, 10, 50, 6

5. Show the hour hand at various positions between two hours, and have students predict where the minute hand will be. Will it be closer to 12, 3, 6, or 9? Examples: The hour hand is about halfway between 7 and 8, so the minute hand will be near the 6; the hour hand is just a little after the 4, so the minute hand will be near the 3.
Goals

Students will tell time on an analog clock when the minute hand shows a multiple of 5 minutes.

PRIOR KNOWLEDGE REQUIRED

- Is familiar with analog clock faces
- Can distinguish between the hour hand and the minute hand
- Can write time using numbers
- Can skip count by 5s
- Can multiply one-digit numbers by 5
- Can divide by 5 up to 50 ÷ 5
- Knows that multiplication is repeated addition
- Is familiar with multiplication strategies such as adding on

MATERIALS

- ball (optional)
- analog clock with hour and minute hands
- dice, 2 per pair of students
- clocks made in the activity from Lesson ME4-22
- BLM Telling Time Two Ways (p. P-52, optional)
- BLM The Second Hand (1) (p. P-53, see Extension 3)

Mental math minute. Ask students to solve questions that require multiplying and dividing by 5. First, go through the questions in order, such as 1 × 5, 5 ÷ 5, 2 × 5, 10 ÷ 5, and so on, to 12 ÷ 5 and 60 ÷ 5. Then, ask the same questions out of order, but keep each multiplication and its corresponding division together. Finally, ask both types of questions separately. You can pass a ball to the student you want to answer the question, who then passes the ball back to you after answering.

Review analog clocks. Ask students to explain how to distinguish between the hands on a clock. (the hour hand is shorter and thicker, the minute hand is longer and thinner) Review how to tell the hour on an analog clock. Show 9:00 on the analog clock and ASK: Is it o’clock? (yes) How do you know? (the minute hand points at the 12) What time is it? (9 o’clock) How do you know the hour? (the hour hand is pointing at the 9) Show 9:15 on the clock. Point out that the hour hand is now between 9 and 10. ASK: What hour is it? (9) Show 9:45 on the clock. Remind students that even when the hour hand is closer to 10 than to 9, the hour is still 9.

Counting minutes by 5. ASK: How many minutes are in an hour? (60) Tell students that they can get the number of minutes after the hour by counting by 5s around the clock starting at 12 or multiplying the number the minute hand points to by 5. Demonstrate counting around one full hour.
to verify that you count 60 minutes in total. Then point out that $12 \times 5$ is 60. Demonstrate both methods for 9:15 and 9:45.

**Writing times using the number of minutes after the hour.** Show 7:10 on the analog clock. Explain that you want students to write the time in numbers. Remind them that they should write the hour first. ASK: Which hand is close to 7? (the hour hand) Which hand points at the 2? (the minute hand) What is the hour? (7) SAY: So, when we write the time as numbers, we start by writing the hour. Write on the board:

7:____

ASK: How many minutes after 7 o’clock is it? (10) How do you know? (the minute hand points at 2, and $2 \times 5 = 10$) Write “10” in the blank. Repeat with 7:50. Finally, show 7 o’clock and ask how many minutes after 7 o’clock it is. Explain that because it is 0 minutes after 7 o’clock, we just write 7:00.

**Exercises:** Write the time on the clock in numbers.

a)  

b)  

c)  

d)  

**Answers:** a) 1:30, b) 4:35, c) 10:10, d) 6:25

**ACTIVITY (Essential)**

Each pair of students will need two dice and a clock made in the activity from Lesson ME4-22. Player 1 rolls the dice, adds the results, and points the hour hand at the sum. Player 2 rolls the dice, adds the results, and points the minute hand at the sum. Both players write the time in numbers. Partners compare answers and switch roles.

**Review saying the time in words.** Remind students that when the time is, say, 7:10, they say it is 10 minutes after 7, which is short for “10 minutes past (or after) 7 o’clock.” SAY: This means we say the minutes first and the hours after.

**Exercises:** Say the time in words.

a) 8:20  

b) 7:45  

c) 9:10  

d) 12:40

**Sample answers:** a) 20 minutes past 8, b) 15 minutes to 8, c) 10 minutes past 9, d) 20 minutes to 1

Show 3:05 on an analog clock and have students first write the time in numbers, then say it in words. Repeat with other times, such as 6:15, 10:30, 1:25, and 12:35. For additional practice, students can use BLM Telling Time Two Ways.
Extensions

1. Each group of three students will need two dice and the clocks made in the activity from Lesson ME4-22. Player 1 rolls the two dice, adds the results, and writes them down as the hour (for example, \(5 + 6 = 11\), so \(11:\__\)). Player 2 then rolls a single die, multiplies the result of the roll by 10, and chooses whether or not to add a bonus 5 for the minutes (for example, roll 4, \(4 \times 10 = 40\), add 5 if wanted, so the minutes could be :40 or :45). If the roll is 6, the player should write :00 or :05 instead of :60 or :65. Player 3 sets the clock to this time. Players rotate roles after each turn.

2. You can find the number of seconds after the minute the same way you find the number of minutes after the hour. Example: The clock shows 1:30:20.

```
10 11 12
9 8 7 6 5
4
3
2
1
```

Write the time in numbers.

a)  

b)  

c)  

d)  

Answers: a) 1:30:45, b) 4:35:05, c) 10:10:30, d) 6:15:40

3. Have students complete BLM The Second Hand (1).

Answers

1. a) 5:00:35, b) 8:25:55, c) 2:15:45, d) 1:30:50, e) 7:15:20, f) 10:40:10

2. b)

c)
**Goals**

Students will tell time to the minute on an analog clock.
Students will read the time as minutes past and minutes to the hour.

**PRIOR KNOWLEDGE REQUIRED**

Can skip count by 5s
Can tell time in 5-minute intervals

**MATERIALS**

- analog clock with hour and minute hands
- **BLM Telling Time (Review)** (pp. P-55–56, optional)
- dice, 4 per pair of students (in two different colours)
- **BLM Empty Clock Faces** (p. P-57)
- **BLM Time Memory Cards** (pp. P-42–49)
- **BLM The Second Hand (2)** (p. P-54, see Extension 2)

**Mental math minute.** Ask students to solve multiplication problems within the range $1 \times 1$ to $10 \times 10$ and the corresponding division problems. For each number, go through the problems in order, such as $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, to $10 \times 3$ and $30 \div 3$. Then, progress to a different number. Next, try problems out of order, but keep corresponding multiplications and divisions together.

**Review telling time to 5 minutes.** Show various times on an analog clock with the minute hand showing a multiple of 5 minutes. Have students tell the time in minutes past and minutes to the hour. Then write the following times on the board, and ask volunteers to show the times on an analog clock and to say the times verbally.

08:30  12:45  10:40  4:15  5:20  2:25  11:05

For additional review, use **BLM Telling Time (Review)**.

**Estimating time to 5 minutes.** Point out that, so far, you have been telling time on analog clocks only when the minute hand has been pointing to a number. Tell students that knowing the time to the nearest 5 minutes is usually close enough. Discuss examples where time to the nearest 5 minutes is close enough (e.g., when dinner time starts, when you arrive at a friend’s house) and when it isn’t (e.g., on a train schedule, when arriving at school on time). Tell students that they will be learning to tell the time to the minute on an analog clock in this lesson but that they will start by estimating times to five minutes.

Set your analog clock to show 12:21. **ASK:** Where is the minute hand pointing? (near the 4; between the 4 and the 5) **Is it on the 4?** (no) **Is it closer to the 4 or the 5?** (4) **What time would it be if the minute hand pointed to the 4?** (12:20) **SAY:** So the time is around 12:20. Repeat with...
other times, starting with examples that are closer to the lower number (e.g., 3:26, 9:47), then moving to examples that require rounding up (e.g., 6:58, 2:14).

Exercises: Approximately what time is it?

a)  

b)  

c)  

d)  

Answers: a) 1:30, b) 3:45, c) 8:20, d) 5:40

Telling time to the minute. Set the analog clock to 10:02. Point out that the arc between the numbers 12 and 1 is divided into five parts of one minute each. So if the minute hand points at the second division after 12, it means that the time is two minutes after 10.

Set the clock to 10:18. Point out that you could count each division to tell the time, but there is a faster way to find the number of minutes past the hour. Skip count by 5s till you get to the number just before the minute hand (in this case, count 5, 10, 15), and then count on by 1s for each division (16, 17, 18).

Draw several analog clocks on the board. Draw only the minute hand on each clock for the following number of minutes: 12, 23, 47, 38, 26, 41. Ask students to tell how many minutes past the hour it is. After that, add the hour hand and ask students to write the exact times two ways (example: 8:46 can be written as “46 minutes past 8” or “14 minutes to 9”).

When completing exercises on paper, students can draw a line from the minute hand to the edge of the clock to identify exactly which division the minute hand is pointing to. If students need help identifying the hour (between which numbers is the hour hand pointing?), they can use their fingers or the end of their pencil to trace a line from the hour hand to the edge of the clock.

Exercises

1. Write the time in words. Example: 7:56 is four minutes to eight or seven fifty-six.
   a) 5:52  
   b) 3:48  
   c) 6:41  
   d) 9:34  

   Answers: a) 8 minutes to 6, b) 12 minutes to 4, c) 19 minutes to 7, d) 34 minutes after 9

2. What time is it?
   a)  
   b)  
   c)  
   d)  

   Answers: a) 5:19, b) 4:39, c) 9:07, d) 11:48
ACTIVITY 1 (Essential), ACTIVITY 2 (Optional)

1. Students will each need four dice, two red and two blue, and BLM Empty Clock Faces. The student rolls the dice. The product of the red numbers gives the position of the minute hand, and the sum of the blue numbers gives the position of the hour hand. The student shows the time on an analog clock (by drawing the hands in the correct positions) and writes the time two ways. Struggling students can use addition instead of multiplication to determine the position of both hands. This way, instead of dealing with three concepts (addition, multiplication, and telling time) they are only dealing with two (addition and telling time).

2. Play Picking Pairs and then Memory (see unit introduction) with cards from BLM Time Memory Cards.

NOTE: Extension 3 is required in order to cover the British Columbia curriculum.

Extensions

1. Students will each need two dice and BLM Empty Clock Faces. The sum of the numbers on the first, second, and third rolls gives the hours, minutes, and seconds, respectively. Students draw the hands in the correct positions (as given by the numbers rolled) on a clock face on the BLM and write the time two ways.

2. Have students complete BLM The Second Hand (2).

3. Investigate how First Peoples use the position of the sun, the moon, and stars to determine times for traditional activities.

4. Draw the picture in the margin on the board. Ask students what number the minute hand would point at if the hour hand was in the position shown. PROMPTS: When the hour hand moves from the 3 to the 4, how many minutes pass? (60 minutes) How many divisions are between the 3 and the 4? (5 divisions) How many minutes does each division represent for the hour hand? (60 ÷ 5 = 12 minutes)

Solution: The position of the hour hand shows that it is $3 \times 12 = 36$ minutes past 3 (see margin), so the minute hand should point to the first interval after the 7.
**Goals**

Students will skip count by 5s and 1s to determine time intervals.

Students will estimate time intervals on an analog clock.

Students will add time intervals to start times to find end times.

**PRIOR KNOWLEDGE REQUIRED**

Can tell time to the minute

Can skip count by 5s

**MATERIALS**

analog clock with hour and minute hands

clocks made in the activity in Lesson ME4-22

BLM Empty Clock Faces *(p. P-57)*

**Mental math minute.** Have students subtract by using 10. Say the subtraction you want students to do (such as 27 – 9). Have a student say the in-between addition step, 20 – 2, and have another student finish the subtraction. As a challenge, use three-digit and four-digit numbers, such as 345 – 8.

**Counting by 5s to find time intervals.** Draw an analog clock on the board showing 7:15. SAY: This is the time at which Ella starts eating breakfast. She finishes breakfast at 7:40. Draw a hollow minute hand showing 40 minutes past the hour.

SAY: We want to find how much time Ella spends eating breakfast. Show 7:15 on an actual analog clock. SAY: The minute hand is pointing to the three. ASK: How many minutes pass when the minute hand moves from 3 to 4? (5) From 4 to 5? (5) Where will the minute hand be pointing when Ella finishes breakfast? (8) SAY: Let’s count the time together. Move the time slowly from 7:15 to 7:40 counting by 5s. SAY: The amount of time between two times is called a time *interval*. Here, the time interval is 25 minutes.

Students can use the clocks they made in the activity in Lesson ME4-22 in the following exercises.

**Exercises**

a) Ella brushed her teeth at 7:30 and left for school at 7:45. How long is the time interval?

b) Ella leaves for school at 8:15. She arrives at school at 8:55. How long is the time interval?

**Answers:** a) 15 minutes, b) 40 minutes.
SAY: Let’s do another example without using our clocks. Ella starts dinner at 6:45 and finishes at 7:05. Draw the start time on a clock and then draw a hollow minute hand showing the end time. Demonstrate skip counting around the clock from the start time to the end time to find the time interval, as shown in the margin. (20 minutes)

Have a volunteer find the time interval from 4:25 to 4:55. Distribute BLM Empty Clock Faces, which will be used for the many of the exercises in this lesson.

**Exercises:** Draw the minute hand for the start and end times. Count by 5s to find the time interval.

a) 8:20 to 8:45  
 b) 1:45 to 2:20  
 c) 12:50 to 1:35  

**Bonus:** 14:15 to 14:55

**Answers:** a) 25 minutes, b) 35 minutes, c) 45 minutes, Bonus: 40 minutes

**ACTIVITY (Essential)**

Throughout the day, have students estimate how long tasks will take to the nearest 5 minutes. Then have them record start and end times to calculate time intervals. Suggested activities to time: How long does it take to complete a worksheet? How long does it take to brush your teeth? Eat dinner? Read 5 pages of a book?

**Counting by 5s and 1s to find time intervals.** Point out to students that in the examples so far, the minutes were multiples of five. Show students how to count the minutes for the example below by first counting by 5s and then by 1s:

8:10 to 8:27

Have a volunteer find the time interval from 9:30 to 9:53. (23 minutes)

**Exercises:** Draw the minute hand for the start and end times. Count by 5s then by 1s to find the time interval.

a) 5:20 to 5:46  
 b) 2:40 to 3:19  
 c) 12:55 to 1:26  

**Bonus:** 20:45 to 21:07

**Answers:** a) 26 minutes, b) 39 minutes, c) 31 minutes, Bonus: 22 minutes
Counting by 5s to find time intervals starting at any time. Tell students that so far, all of the start times have been multiples of 5. Now you are going to start at other places. Write “8:12 to 8:27” on the board. Then, draw a clock face with a minute hand pointing to 12 minutes after the hour. Tell students that this is where the minute hand starts, then demonstrate counting 12 minutes after the hour. Have a volunteer draw the minute hand for the end time. Demonstrate skipping ahead by 5 from the start time. Point out that the minute hand is pointing to the second little mark, so when you skip ahead 5 minutes, you go to the second mark. Skip ahead 5 minutes at a time, drawing jumps as you go, until you reach the end time. ASK: How long is the interval? (15 minutes) SAY: We are still counting by 5s except we aren’t starting from a multiple of 5. The clock should look like this:

Repeat with 3:46 to 4:06. (20 minutes)

Exercises: Draw the minute hand for the start and end times. Count by 5s to find the time interval.

a) 5:21 to 5:46  
   b) 2:48 to 3:18  
   c) 12:59 to 1:09

Answers: a) 25 minutes, b) 30 minutes, c) 10 minutes

Finding any time interval. Demonstrate counting by 5s then by 1s to find the time interval from 7:18 to 7:31. (13 minutes, see clock below)

Exercises: Draw the minute hand for the start and end times. Count by 5s then by 1s to find the time interval.

a) 1:39 to 2:07  
   b) 4:06 to 4:58  
   c) 6:49 to 7:18

Answers: a) 28 minutes, b) 52 minutes, c) 29 minutes

Estimating time intervals. Tell students that it is often enough to know approximately how long a time interval is. They can do that by counting by fives and ignoring the ones. Demonstrate with the time interval from 8:06 to 8:27. (20 minutes) Then repeat with 3:41 to 3:54. (15 minutes) Explain that 10 minutes would also be a reasonable estimate.
Exercises: Draw the minute hand for the start and end times. Estimate the time interval.

a) 2:49 to 3:05  b) 9:10 to 9:42  c) 5:18 to 5:57

Answers: a) 15 minutes, b) 30 minutes, c) 40 minutes

Finding the end time. When you know the start time and the duration of an activity, you can find the end time. SAY: Kim ran 3 km. She started at 5:15. She ran for 20 minutes. Draw an analog clock on the board showing the start time. Draw jumps while skip counting by 5s to count out 20 minutes from the start time. Ask students what the end time is. (5:35)

Exercises

a) Luc leaves with his dog sled at 1:30. It takes him 35 minutes to travel to an area where he can fish. At what time does he arrive?

b) Eddy took 27 minutes to decorate a cake for his father’s birthday. He started at 2:20. At what time did he finish?

c) Aputik started drawing an airplane at 8:17. It took her 42 minutes to complete the airplane. At what time did she finish?

Bonus: A train leaves the station at 16:27 and takes 42 minutes to arrive at its first stop. At what time does it arrive?

Answers: a) 2:05, b) 2:47, c) 8:59, Bonus: 17:09

Finding the end time using addition (no regrouping). Point out to students that they are counting up when they are finding the end time, so they are actually adding time. ASK: If a show starts at 3:25 and lasts 20 minutes, at what time does it end? What will you get if you simply add 20 minutes to 3:25? Write on the board:

\[
\begin{align*}
3:25 \\
+ & \ 0:20 \\
\end{align*}
\]

ASK: Why did I write 20 minutes as 0:20? (there are 0 hours and 20 minutes)

Invite a volunteer to do the addition. (3:45) Point out that the answer is the same as the one students got by counting up. Repeat with a starting time of 2:15 and a show lasting 35 minutes. (end time 2:50)

Exercises: Add to find the end time.

a) \(12:15 + 0:23\)  b) \(8:09 + 3:25\)  Bonus: \(9:24 + 1:12\)

Answers: a) 12:38, b) 11:34, Bonus: 10:36

Finding the end time using addition (with regrouping). Present the following problem: Sam starts reading at 5:45. He reads for 30 minutes. When does he finish reading? Suggest the following solution:

\[
\begin{align*}
5:45 \\
+ & \ 0:30 \\
\end{align*}
\]

5:75
SAY: Sam finished reading at 75 minutes after 5. ASK: Does this sound correct? (no) Why not? (an hour is 60 minutes, so the number of minutes after the hour should be less than 60) Ask for suggestions on how to fix the problem. PROMPT: What do we do when we are adding numbers and the answer has more than ten ones? (regroup ten ones as one ten) SAY: We could do something similar with 75 minutes. ASK: What unit of time is larger than a minute? (an hour) How many minutes are in an hour? (60 minutes) SAY: To regroup 75 minutes as hours and minutes, you would subtract 60 minutes and replace them with 1 hour. Write on the board:

\[ 75 - 60 = \underline{1}, \text{ so } 75 \text{ minutes} = 1 \text{ hour} + \underline{15} \text{ minutes} \]

Have students help you to fill in the missing minutes. Finally, write the regrouped time, 6:15, under 5:75. Ask students to check the answer by counting up by 5s from 5:45.

As a class, solve this problem: Rani started reading at 2:55. She read for 1 hour and 20 minutes. When did she finish reading? (at 4:15)

**Exercises:** Add to find the end time.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3:46</td>
<td>b) 4:51</td>
<td>c) 8:15</td>
</tr>
<tr>
<td>+ 0:34</td>
<td>+ 2:17</td>
<td>+ 3:59</td>
</tr>
</tbody>
</table>

**Bonus:** 12:34 + 3:45

**Answers:** a) 4:20, b) 7:08, c) 12:14, Bonus: 4:19 or 16:19

**Extensions**

1. At various times during the day, ask students to record the time from the classroom analog clock in 12-hour notation. Ask them to calculate the amount of time that passed between each reading.

2. A witch is cooking a potion. Her potion turned purple at 3:45. Twenty minutes after it turns purple, she should add snake heads.
   a) At what time should the witch add snake heads?
   b) Snake heads should stay in the potion for 35 minutes. Then the witch should stir the potion quickly 7 times clockwise and remove the heads. After that, the potion should boil for 35 minutes and then it will be ready. At what time will the witch’s potion be ready?

   **Answers:** a) 4:05, b) 5:15

3. Add the times with seconds.
   a) 12:15:23 + 00:01:12
   b) 11:27:05 + 1:52:17
   c) 09:48:45 + 02:26:31

   **Answers:** a) 12:16:35, b) 13:19:22 or 01:19:22, c) 12:15:16
4. Find the time interval by subtracting the start time from the end time. Check your answers by counting forward on a clock face.
   a) 6:17 to 6:58  
b) 9:07 to 9:33  
c) 10:47 to 11:21
   **Answers:** a) 41 minutes, b) 26 minutes, c) 34 minutes

5. Find the time interval by subtracting the start time from the end time.
   a) 6:17 to 8:58  
b) 9:21 to 11:40  
c) 7:56 to 12:03
   **Answers:** a) 2 hours and 41 minutes, b) 2 hours and 19 minutes, c) 4 hours and 7 minutes

6. Ben studied for $\frac{2}{3}$ of an hour.
   a) How many minutes did he study for?
   b) Ben started studying at 7:10 p.m. At what time did he stop studying?
   c) Ben's father came home at 8:00 p.m. Did Ben finish studying before his father arrived?
   **Answers:** a) 40 minutes, b) 7:50, c) yes

7. Find the missing numbers.
   a) $8 : 3 \quad - \quad 8 : 9 \quad = \quad 2 : 3$
   b) $6 : 5 \quad - \quad 4 : 7 \quad = \quad 2 : 3$
   c) $5 : 1 \quad - \quad 5 : 4 \quad = \quad 7 : 4$
   **Answers:** a) $8 : 32 - 8 : 09 = 0 : 23$, b) $6 : 50 - 4 : 27 = 2 : 23$, c) $12 : 51 - 5 : 47 = 7 : 04$

8. Zara started studying at 7:15. She studied for $\frac{3}{5}$ of an hour. At what time did she stop studying?
   **Answer:** 7:51
**Goals**

Students will find time intervals on a timeline.
Students will determine elapsed times given by consecutive time intervals, with and without a timeline.

**PRIOR KNOWLEDGE REQUIRED**

Can tell time to the minute
Can skip count by 5s and 10s

**Mental math minute.** Remove the face cards from a deck of cards. Shuffle the deck. Divide students into pairs. Each partner will select a card at random, and the pair will then create two division equations using the selected cards. For example, if Partner 1 selects a 7 and Partner 2 selects an 8, the pair will create the equations $56 ÷ 8 = 7$ and $56 ÷ 7 = 8$. Have students treat an ace as the number 1.

**Finding time intervals on a timeline.** Tell students that they can find time intervals using timelines the same way they use number lines. SAY: Suppose it’s 3:40 and Glen needs to be home by 5:10. ASK: How much time does he have? Draw on the board:

![Timeline](image)

SAY: Let’s draw the hours between the start and end time. ASK: What is the next hour after 3:40? (4 o’clock) Add 4:00 and 5:00 to the timeline. ASK: How long is the interval from 3:40 to 4:00? (20 minutes) Draw an arrow from 3:40 to 4:00 and label it “20 minutes.” Draw an arrow from 4:00 to 5:00 and ASK: How long is this time interval? (1 hour) Label the arrow. Repeat for 5:00 to 5:10. (10 minutes) SAY: Now we can add up the times. Indicate the appropriate arrows, and ASK: What is 20 minutes plus 10 minutes? (30 minutes) SAY: The time interval is 1 hour (indicating the 1 hour arrow) and 30 minutes.

Repeat with the following: Ronin works from 11:40 to 3:50. How much time does he work for? Solve the problem as a class, using as many volunteers as possible. Point out that the minutes must be regrouped to get the final answer. (4 hours and 10 minutes)

Give students rulers to use in the following exercises.

**Exercises:** Draw a timeline to find the time interval.

a) 2:30 to 7:15  
b) 12:45 to 3:35  
c) 3:20 to 5:55  
d) 4:35 to 7:50

**Answers:** a) 4 hours and 45 minutes, b) 2 hours and 50 minutes, c) 2 hours and 35 minutes, d) 3 hours and 15 minutes

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**VOCABULARY**

elapsed time
hour
interval
minute
o’clock

**CURRICULUM REQUIREMENT**

AB: optional
BC: optional
MB: optional
ON: required
Finding end times on a timeline. Tell students that you can find end times using timelines, too. Draw on the board:

Sara starts her homework at 6:45. She works on math for 25 minutes.

Mark 6:45 at the beginning of the timeline. Tell students that you are going to mark the timeline in five-minute intervals. Draw small marks on the timeline, counting by 5s until you reach 25. Do not label the marks. Draw an arrow from the start time to the last mark. Tell students that you now have to find the end time by counting up by 5s from the start time. Count from 6:45 to 7 o’clock. Make the mark at 7 o’clock longer and label it. Continue counting until you reach the end time (7:10). Label the end time, as shown:

Repeat with the help of volunteers to find 12:50 plus 2 hours and 40 minutes. Have the volunteers add the minutes first and then add the hours in a single leap as shown (3:30 or 15:30).

Finding elapsed times on a timeline. Tell students that timelines are really useful when they are adding several time intervals. Tell them that this is sometimes called the elapsed time. Write on the board:

Jane starts her homework at 6:45. She works on math for 15 minutes then reads for 20 minutes. At what time does she finish her homework?

Draw a timeline on the board and mark 6:45 at the beginning. Have a volunteer add 15 minutes to the timeline as before. Then have a second volunteer add a further 20 minutes. ASK: How much time did Jane spend doing homework altogether? (35 minutes) At what time did she finish? (7:20)

Repeat with the following: Don starts his homework at 6:50. He works on math for 20 minutes, then reads for 10 minutes. At what time does he finish his homework? (7:20)
Exercises: Use a timeline to find the elapsed time and the end time.

a) Jasmin starts her homework at 5:35. She spends 20 minutes on math, 15 minutes on science, and 25 minutes on history, and then she reads for 1 hour.

b) Marko wakes up at 7:10. He takes 20 minutes to eat breakfast, brushes his teeth for 5 minutes, takes 15 minutes to get dressed, and then walks 25 minutes to school.

Answers: a) elapsed time: 2 hours, end time: 6:35; b) elapsed time: 65 minutes, end time: 8:15

Finding elapsed times by adding. Tell students that they can also find elapsed times by adding. Write on the board:

Eat breakfast: 15 min
Brush teeth: 5 min
Get dressed: 10 min
Pack bag: 5 min

SAY: This is how much time Rob spends getting ready in the morning.
ASK: How much time does Rob spend getting ready altogether? (35 minutes) If he wakes up at 7:00, what time is he ready to leave? (7:35) What if he wakes up at 7:45? (8:20) Verify this last answer by writing the addition on the board.

Exercises

1. Add the times to find the elapsed time. Write your answer in hours and minutes.

   a) Eat breakfast: 30 min  
      Brush teeth: 5 min  
      Get dressed: 25 min  
      Pack bag: 10 min

   Answers: a) 1:10, b) 0:50

2. If the student in Exercise 1 gets up 7:15, what time are they ready to go to school?

   Answers: a) 8:25, b) 8:05

Estimating elapsed times. Write on the board:

Lynn gets home from school at 4:30 and goes to bed at 8:30. In that time, she:

Eats dinner: 35 minutes
Does homework: 45 minutes
Gets ready for bed: 30 minutes
Practises piano: 30 minutes

About how much free time does Lynn have?
SAY: In this kind of question, we don’t need to know exactly how much time Lynn has, so we can estimate. ASK: How much time does Lynn have at home in the evening before bed? (4 hr) How many minutes are in an hour? (60) SAY: We can quickly add the times. Point to the two 30-minute times and tell students that the sum of these is 1 hour. Point to the 35 min and 45 min times and SAY: The sum of these is more than 1 hour. ASK: Is their sum as much as 2 hours? (no) SAY: I estimate that this sum is around one and a half hours, so the total amount of time is about two and a half hours altogether. ASK: How much time is left? (one and a half hours)

Present each part of the following exercises one at a time. Show students how to group the times to make hours (e.g., 45 minutes and 15 minutes make 1 hour).

**Exercises:** Will the student be on time? Calculate the available time, then estimate the elapsed time to find out.


**Answers:** a) yes, b) no

**Extensions**

1. In a triathlon, competitors run, bike, and swim in a race. Here are the times for the top three competitors in hours, minutes, and seconds.

<table>
<thead>
<tr>
<th>Competitor</th>
<th>Swim</th>
<th>Bike</th>
<th>Run</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>00:39:37</td>
<td>01:12:17</td>
<td>57:57</td>
</tr>
<tr>
<td>B</td>
<td>00:45:15</td>
<td>01:01:08</td>
<td>42:53</td>
</tr>
<tr>
<td>C</td>
<td>00:28:19</td>
<td>01:15:12</td>
<td>49:38</td>
</tr>
</tbody>
</table>

a) Who won the triathlon?
b) By how much did the winner beat the other two competitors?

**Answers:** a) B won, b) B beat C by 3 seconds and A by 16 minutes and 45 seconds

2. The Browns take a car trip to visit relatives. They decide to take turns driving and sleeping so that they can get there as quickly as possible. The family leaves on May 5 at 8:00 a.m. They drive for 4 hours and stop for a 1-hour lunch. They continue driving for 6 hours before stopping for a 2-hour dinner. They drive for three 4-hour stretches with half-hour breaks. On what day and at what time do they arrive at their relatives’ house?

**Answer:** May 6 at 10:00 a.m.
3. Clara started doing her homework at 6:00 p.m. She spent $\frac{2}{5}$ of an hour on English, $\frac{1}{6}$ of an hour on math, and $\frac{3}{10}$ of an hour on science. At what time did she finish working?

   Answer: 6:52 p.m.

4. How long is your school day? How much time do you spend in class and how much time do you spend at recess?
**Goals**

Students will write dates in a variety of formats.

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**PRIOR KNOWLEDGE REQUIRED**

Knows the months of the year

**MATERIALS**

- items with printed dates (optional)
- overhead projector
- transparency of *BLM Calendar* (p. P-58)
- *BLM Calendar* (p. P-58) or any other calendar per pair of students
- tokens or markers

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**Mental math minute.** Give the first student an even three-digit number to halve, such as 144. Each successive student then halves the previous answer. In the given example, the first student would say 72, the next 36, and so on. Occasionally ask students to explain how they got the answer. When students reach an odd number, start with another three-digit even number.

**Numbering the months.** Ask students to write the months of the year in order and to number them so that January is the first month. Ask them to add zeros before any numbers having only one digit. **ASK:** Which month is 04? (April) 08? (August) 10? (October) 06? (June) What is the number for May? (05) February? (02) November? (11)

**Writing the date in ascending order.** Write today’s date on the board in the usual written form (e.g., February 12, 2020), and have several volunteers write their birthdays in the same format. These examples will be used throughout the lesson. Make sure there is space under each date to rewrite it at least four times. If available, show the class where to find printed dates on items, such as on receipts or the best-before dates on food. **SAY:** There are many ways to write the date. We usually write the date as month then day, followed by the year separated by a comma. Another way to write the date is from smallest to largest: day, then month, then year. Demonstrate by rewriting the date in this order (e.g., 12 February 2020) under the original date. Have volunteers rewrite the birthdays in this format, underneath the original dates.

**Exercises:** Rewrite the date as day month year.

- a) July 5, 1968   b) December 19, 2000
- c) March 31, 2017 d) September 9, 1684

**Answers:** a) 5 July 1968, b) 19 December 2000, c) 31 March 2017, d) 9 September 1684
Writing the date using numbers in ascending order. Point out that dates are often written using numbers alone, with dashes or slashes separating the numbers. Write today’s date to illustrate (e.g., 12/02/2020). Tell students that when you are filling out a date on a form, it will often show you how they want the date written by writing “d” for days, “m” for months and “y” for years. SAY: So, this format would appear like this. Write “dd/mm/yyyy” on the board. Indicating the appropriate part of the format, SAY: Two d’s means two digits for the day. Then two m’s means two digits for the month. Then four y’s means four digits for the year. Have volunteers convert the birthdays on the board to this format.

Exercises: Rewrite the dates in the previous exercises in the format dd/mm/yyyy.

Answers: a) 05/07/1968, b) 19/12/2000, c) 31/03/2017, d) 09/09/1684

Tell students that another very common format is to write the year with two digits as well. Ask students to look at the dates on the board and ASK: Which two digits would you use? (the last two) Why? (the first two digits of the birthdays are all the same) Ask if students can think of times when using only two digits to show the year would be a bad idea. (sample answer: in history class)

Exercises: Write the date in dd/mm/yy format.

a) January 12, 2015  b) October 18, 1997  c) June 5, 2012

Answers: a) 12/01/15, b) 18/10/97, c) 05/06/12

Introduce international standard dates. Write on the board:

02/05/08      mm/dd/yy

ASK: What date is this if it is written as month, then day, then year? (February 5 2008) Help students work with one part at a time to arrive at this date. Erase “mm/dd/yy,” replace it with “dd/mm/yy,” and repeat the exercise. (May 2, 2008) Tell students that another common way to write dates is from the largest to the smallest time period: year, then month, then day. Erase “mm/dd/yy,” write it with “yy/mm/dd,” and repeat the exercise. (2002 May 8) Tell students that because dates can be written and read so many ways, there is an international standard for writing dates. Write on the board:

yyyy-mm-dd or yyyy/mm/dd

Rewrite today’s date using this standard, e.g., 2020-02-12. Have volunteers rewrite the birthdays on the board using the international standard.

Exercises: Write the date using yyyy/mm/dd.


Answers: a) 1962/04/21, b) 2012/05/31, c) 1998/11/05
**Reading dates.** Write “2015-05-17” on the board and tell students that it is written in the international format. Point out that it is the only format you have seen that starts with a four-digit number. ASK: When we write the date, what do we usually start with? (the month) Which numbers represent the month in this date? (05) What is the month? (May) Write “May” on the board. ASK: What follows the month? (the day) What is the day here? (17) Write “17,” followed by a comma. Prompt for the year and complete the date: May 17, 2015.

Repeat with 1999-12-11 (December 11, 1999).

**Exercises:** Write the date in words.

a) 2022-04-30  b) 1946/02/05  c) 2121-12-21

**Answers:** a) April 30, 2022; b) February 5, 1946; c) December 21, 2121

**Relating dates to days on calendars.** Project BLM Calendar on the board. Show students where the year and month are indicated on the calendar. Write “2021-06-15” on the board. Ask students to read the date. (June 15, 2021) Have a volunteer point to the date on the calendar. ASK: On which day of the week does June 15 fall in 2021? (Tuesday) Point to June 22 and ask a volunteer to write the date in standard form. (2021-06-22)

**ACTIVITY (Essential)**

Give each pair of students BLM Calendar (or one month from any other calendar) and tokens or markers. On scrap paper, Player 1 writes a date that appears on the calendar. Player 2 places a token or mark over that date on the calendar. Players change roles, always choosing a date that has not yet been marked.

**Extensions**

1. Find out on what day your birthday falls for five back-to-back years.
   a) How does the day change most years?
   b) What is special about the year that is different? If you don’t see anything special, check the year before.

   **Answers:** a) it moves ahead 1 day each year, b) it is a leap year (or the year before was a leap year if you were born in January or February)

2. Avril was born on April 7, 2008. John is 6 weeks older than Avril. When is John’s birthday? Hint: 2008 was a leap year, so February had 29 days in 2008.

   **Solution:** 6 weeks $= 6 \times 7 \text{ days} = 42 \text{ days}$. March has 31 days and February had 29 days in 2008. So, April 7 $= \text{March 38} = \text{February 67. 67} - 42 = 25$, so John was born on February 25, 2008.
3. Have students create a calendar that includes days of the week, dates, and personal events.

4. Find the date in a leap year.
   a) 15 days after December 23
   b) 16 days before January 5
   c) 7 days after February 24
   d) 15 days before March 13
   **Answers:** a) January 7, b) December 20, c) March 2, d) February 27

5. Have students use a calendar to estimate how many months are between two events. For example, National Aboriginal Day is on June 21 and school starts in September, so there are about 3 months between them.
### Goals
Students will convert between hours, days, weeks, months, years, decades, and centuries.
Students will find elapsed times in hours, days, weeks, months, and years.

### PRIOR KNOWLEDGE REQUIRED
- Knows there are 24 hours in a day
- Knows there are 7 days in a week
- Knows there are approximately 30 days in a month
- Knows there are 12 months in a year

### MATERIALS
- calendars (see Extension 1)

### Mental math minute.
Ask students to solve multiplication problems and their corresponding division problems. Concentrate on multiples of 5, 7, and 12. Go through the problems in order, such as $1 \times 7$ and $7 \div 7$, $2 \times 7$ and $14 \div 7$, and so on, to $10 \times 7$ and $70 \div 7$. Then, progress to a different number. Next, try problems out of order, but keep corresponding multiplications and divisions together. Practise multiples of 5 from $1 \times 5$ to $12 \times 5$, and multiples of 12 from $1 \times 12$ to $5 \times 12$.

### Converting between hours and days using a table.
ASK: How many hours are in a day? (24) How many hours are in two days? (48) Draw a table on the board, and fill it in with the help of volunteers.

<table>
<thead>
<tr>
<th>Days</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>96</td>
</tr>
</tbody>
</table>

Write “30 hours” on the board. ASK: Is 30 hours more or less than a day? (more) More or less than two days? (less) SAY: There is one whole day in 24 hours. Repeat with 50 hours. (2 whole days)

### Exercises: How many whole days?
- a) 76 hours
- b) 49 hours
- c) 100 hours

**Answers:** a) 3, b) 2, c) 4

SAY: Thirty hours is more than one whole day. ASK: How many more hours than 1 day? (6) Write “$30 - 24 = 6$” on the board. SAY: So 30 hours is the same as 1 day and 6 hours.
Repeat with 73 hours. (2 days and 1 hours)

**Exercises:** Write the time in days and hours.

a) 52 hours  
   b) 81 hours  
   c) 110 hours

**Answers:** a) 2 days and 4 hours, b) 3 days and 9 hours, c) 4 days and 14 hours

**Finding the elapsed time in hours and days.** Tell students that Neka keeps a journal of how he spends his time. Here is a summary. Draw on the board the table in the margin.

ASK: For how many days did Neka record his activities? Ask students to guess how many days Neka kept a journal. Then ask them how to work out exactly how many days he kept a journal. (add up the total number of hours, then convert to days and hours) Have students add the hours individually. (72 hours) ASK: How many days is 72 hours? (3) How many of those three days do you think were school days? (2) How do you know? (we spend about 6 or 7 hours a day in school)

**Converting between weeks and days.** ASK: How many days are in a week? (7) If I tell you how long something is in weeks, can you tell me how many days it is? (yes) PROMPT: Winter break is two weeks long—how many days is winter break? (14) Summer break is 10 weeks long—how many days long is summer break? (70) How do we convert from weeks to days? (multiply by 7)

**Converting between partial weeks and days.** Ask students to imagine that a relative is coming to visit in two weeks and two days. ASK: How many days is this? Students might reason as follows: A week is seven days long, so two weeks are $2 \times 7 = 14$ days long. There are two more days, so $14 + 2 = 16$ days left.

**Exercises:** Write the time in days.

a) 3 weeks and 4 days  
   b) 4 weeks and 1 day  
   c) 6 weeks and 5 days

**Answers:** a) 25, b) 29, c) 47

Tell students that the relative will be staying for 20 days. Ask how many weeks and days that is. Students might reason as follows: 20 days is more than two weeks but less than three weeks, so it is two weeks plus some extra days. To find the extra days, subtract two weeks: $20 - 14 = 6$.

SAY: We multiply by 7 to find how many days in a number of weeks, so we can divide by 7 to find how many weeks in a number of days. ASK: How many weeks in 25 days? Ask students to divide 25 by 7. (3 remainder 4) ASK: How many full weeks are in 25 days? (3) How many extra days are there? (4) Give students a chance to work this out any way they like. ASK: Can you tell from the division that there are four extra days? (yes) How? (it is the remainder)
Exercises: Divide to write the times in weeks and days.

a) 17 days   b) 26 days   c) 52 days

Answers: a) 2 weeks and 3 days, b) 3 weeks and 5 days, c) 7 weeks and 3 days

Converting between months and years. ASK: How many months in a year? (12) As a class, fill in the following table:

<table>
<thead>
<tr>
<th>Years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Months</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>

Exercises

1. Write the time in months.
   a) 1 year and 3 months   b) 5 years and 6 months

   Answers: a) 15 months, b) 66 months

2. Write the time in years and months.
   a) 50 months   b) 31 months

   Answers: a) 4 years and 2 months, b) 2 years and 7 months

3. Alexa’s family moves often. So far she has lived in each of the 10 provinces for 4 months and in each of the 3 territories for 3 months. How old is she?

   Answer: 4 years and 1 month

Deciding between units of time. Write on the board:

Winter break
Summer break
The age of an adult
A period in school
The age of an infant
The age of a country
The age of the pyramids in Egypt

Ask students what units of time are used to measure each entry. Record their answers. (sample answers: weeks, months, years, minutes, weeks, years, years)

Explain to students that longer periods of time can be measured not only in years but also in decades and centuries. Write “decade” and “century” on the board. Explain that a decade is 10 years long and a century is 100 years long. Ask students to give an example of a period of time that is measured in decades or in centuries. (sample answers: age of a country, age of a tree, time since the beginning of the Common Era)
Exercises

a) 50 years = ____ decades
b) 110 years = ____ decades
c) 1 century = ____ decades
d) 90 decades = ____ years = ____ centuries
e) 300 years = ____ centuries = ____ decades
f) 210 years = ____ centuries + ____ decades

Answers: a) 5; b) 11; c) 10; d) 900, 9; e) 3, 30; f) 2, 1

Extensions

1. Have students use calendars to estimate how many months are between two events. For example, National Aboriginal Day is on June 21 and school starts in September, so there are about 3 months between them.

2. As in some other cultures, many First Nations celebrate a Lunar New Year. The Lunar New Year begins on the day of the first full moon of the calendar year. In 2015, the Lunar New Year began on February 19. In 2016, the Lunar New Year began on February 8. How many weeks were there from the Lunar New Year in 2015 to the Lunar New Year in 2016?

Answer: about 51 weeks

3. If a ones block (in base ten materials) represents 1 year, which block would represent ...

a) a decade?  
b) a century?

Answers: a) tens block, b) hundreds block

4. How many weeks and days are there from March 2 to April 4, including the start and end days?

Sample solution: March 2 to March 31 is 30 days. April 1 to April 4 is 4 days. \(30 + 4 = 34\) days = 4 weeks and 6 days.

5. a) How many weeks and days are there in a year?

b) How many weeks are there in 10 years? Find an answer two different ways.

c) Are your two answers to part b) the same? If not, why not?

Answers: a) 52 weeks, 1 day; b) 521 weeks and 2 days, sample solution: 520 weeks + 10 days + 2 leap days = 521 weeks, 5 days or \(365 \times 10 + 2 = 3652, 3652 \div 7 = 521\) remainder 2; c) sample answer: no, my answers are a little different because I rounded.
Make Your Own Clock
Numbers on a Clock Face

Analog clock faces show numbers from 1 to 12 in a circle.
To label a clock face, write in the numbers 12, 6, 3, and 9 first.
Then fill in the rest of the numbers.

1. Fill in the missing numbers on the clock face.
   a) 
   b) 
   c) 
   d) 
   e) 
   f)
Time Memory Cards (1)

quarter to 12

quarter to 3

quarter to 7

quarter to 8

quarter to 11

quarter to 6
Time Memory Cards (2)

6:05
8:15
11:20
12:25
2:35
3:45
Time Memory Cards (3)

- 1:37
- 4:16
- 10:42
- 1:00
- 9:21
- 14:58
- 20:03
Time Memory Cards (4)

12:00
1:00
6:00

7:00
2:00
3:00

8:00
9:00
10:00
Time Memory Cards (5)

- 3:00
- 9:00
- 4:00
- 10:00
- 5:00
- 11:00
- 12:00

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Time Memory Cards (6)

1 o’clock

9 o’clock

2 o’clock

9 o’clock

4 o’clock

8 o’clock

5 o’clock
<table>
<thead>
<tr>
<th>Time Memory Cards (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:30 half past 12</td>
</tr>
<tr>
<td>5:30 half past 5</td>
</tr>
<tr>
<td>1:30 half past 1</td>
</tr>
<tr>
<td>1:30 half past 5</td>
</tr>
<tr>
<td>8:30 half past 8</td>
</tr>
<tr>
<td>8:30 half past 9</td>
</tr>
<tr>
<td>9:30 half past 4</td>
</tr>
</tbody>
</table>
Time Memory Cards (8)

- Quarter past 12
- Quarter past 1
- Quarter past 5
- Quarter past 8
- Quarter past 9
- Quarter past 4
Half and Quarter Hours (1)

When the minute hand (the long hand) travels from the 12 around the clock until it hits the 12 again, 1 hour has passed.

1. Shade the space from the 12 to the minute hand. Then write whether the minute hand is “half past,” “quarter past,” or “quarter to” the hour.

   a)  b)  c)  d)  
   ![Clock A]  ![Clock B]  ![Clock C]  ![Clock D]  
   half past

   e)  f)  g)  h)  
   ![Clock E]  ![Clock F]  ![Clock G]  ![Clock H]  

2. What time is it? Circle the correct answer.

   a)  b)  c)  d)  
   ![Clock I]  ![Clock J]  ![Clock K]  ![Clock L]  
   quarter to 8 or 7  quarter to 10 or 9  quarter to 10 or 11  quarter to 1 or 2

   e)  f)  g)  h)  
   ![Clock M]  ![Clock N]  ![Clock O]  ![Clock P]  
   quarter past 2 or 3  quarter past 4 or 5  quarter past 5 or 6  quarter past 10 or 9
Half and Quarter Hours (2)

3. Write where the minute hand is. Draw the hour hand and write the time.

   a) The minute hand is three quarters of the way from 1:00 to 2:00.
      The time is 1:45.

   b) The minute hand is __________________ from 1:00 to 2:00.
      The time is ________.

   c) The minute hand is __________________ from 4:00 to 5:00.
      The time is ________.

   d) The minute hand is __________________ from 2:00 to 3:00.
      The time is ________.

   e) The minute hand is __________________ from 8:00 to 9:00.
      The time is ________.

   f) The minute hand is __________________ from 7:00 to 8:00.
      The time is ________.

   g) The minute hand is __________________ from 10:00 to 11:00.
      The time is ________.

   h) The minute hand is __________________ from 12:00 to 1:00.
      The time is ________.
Telling Time in Two Ways

1. Shade in the space from the 12 to the minute hand so that the shaded part is less than half of the circle.
   a) [Clock image]
   b) [Clock image]
   c) [Clock image]
   d) [Clock image]
   e) [Clock image]
   f) [Clock image]
   g) [Clock image]
   h) [Clock image]

2. Which hour is the hour hand closer to? Circle the answer.
   a) 11 or 12
   b) 10 or 11
   c) 6 or 7
   d) 8 or 9
   e) 4 or 5
   f) 12 or 1
   g) 3 or 4
   h) 2 or 3

When it is less than half an hour to the next hour, we can tell time two ways:
40 minutes past 5
20 minutes to 6

3. Tell the time in two ways.
   a) [Clock image] ____ minutes past ____ ____ minutes to ____
   b) [Clock image] ____ minutes past ____ ____ minutes to ____
The Second Hand (1)

The second hand is longer and thinner than both the minute and hour hands.
We read seconds the same way we read minutes.
The exact time with seconds is:

1. Write the time in numbers under the clock.
   a)  
   ____ : ____ : ____  
   b)  
   ____ : ____ : ____  
   c)  
   ____ : ____ : ____  
   d)  
   ____ : ____ : ____  
   e)  
   ____ : ____ : ____  
   f)  
   ____ : ____ : ____  

2. Draw the hands on the analog clock to show the time.
   a) 3:50:35  
   b) 7:05:25  
   c) 9:15:05
The Second Hand (2)

Like the minute hand, we count by fives then by ones to read the second hand.

1. Write the time in numbers under the clock.
   a) [ Clock Image ] b) [ Clock Image ] c) [ Clock Image ]
   d) [ Clock Image ] e) [ Clock Image ] f) [ Clock Image ]

2. Draw the hands on the analog clock to show the time.
   a) 2:45:18   b) 4:30:07   c) 6:23:48
Telling Time (Review) (1)

1. How many minutes is it past the hour? Count by 5s around the clock, filling in the boxes as you go.

   a) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   b) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   c) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   d) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   e) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   f) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   The hour hand (the short hand) moves in a clockwise direction. When the hand is between 7 and 8 the hour is still 7.

2. What is the hour?

   a) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   b) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   c) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   d) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   e) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   f) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   g) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]

   h) \[
   \begin{array}{ccc}
   \text{10} & \text{11} & \text{12} \\
   \text{9} & \text{8} & \text{7} \\
   \text{6} & \text{5} & \text{4} \\
   \text{3} & \text{2} & \text{1} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{5} & \text{10} & \text{15} \\
   \text{4} & \text{3} & \text{2} \\
   \text{1} & \text{0} & \text{9} \\
   \end{array}
   \]
Telling Time (Review) (2)

3. What time does the analog clock show? Write the answer in numbers and in words.
   a)  b) 
   12  : 30  
   Thirty minutes after twelve  
   c)  d) 
   : 

4. What time does the digital clock show? Write the answer in words and draw the hands on the analog clock.
   a)  b) 
   11:10  8:20  

5. Draw the hands on the clock to show the time.
   a) 4:45  b) 8:15  c) 2:10
Empty Clock Faces

[Diagrams of empty clock faces with numbers from 1 to 12 and minutes marks]

Blackline Master — Measurement — Teacher Resource for Grade 4
## Calendar

### June 2021

<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
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### June 2021

<table>
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<th>Sunday</th>
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PS4-9 Making a Simpler Problem

Teach this lesson after:
Unit 13

VOCABULARY
- centimetre
- horizontal
- metre
- perimeter
- vertical

Goals
Students will, when given a problem, make a simpler problem and use the solution to the simpler problem to help solve the original problem.

PRIOR KNOWLEDGE REQUIRED
- Can add and subtract numbers within 10 000
- Can use long division to divide two-digit numbers by one-digit numbers
- Can find the perimeter of a shape by adding the side lengths
- Can identify patterns in sequences that increase by the same amount
- Can add and subtract decimal tenths
- Can multiply one-digit numbers by two-digit numbers (for Extended Problem)
- Can find the area of a rectangle given its side lengths (for Extended Problem)
- Can interpret the remainder in division (for Extended Problem)
- Can use long division to divide three-digit numbers by one-digit numbers (for Extended Problem)

MATERIALS
- 2 sticks of different colours and lengths
- chalk of two different colours
- scissors and BLM Fraction Strips and Circles (p. P-70, see Problem Bank 16)
- BLM Flower Garden (pp. P-72–73, see Extended Problem)

Using off-by-one patterns to solve problems. Tell students that you are waiting in line to get on a rollercoaster ride. You are 37th in line and you see your friend who is 7th in line. ASK: How many people are between my friend and me? Note various guesses. Most students will likely guess 37 − 7 = 30. If they do, SAY: That answer is close, but not exactly correct. Let’s draw a simple picture using smaller numbers to see what is going on. Write on the board:

Front of line: ● ● ● ● ● ● ● ● ●

SAY: Each dot represents a person. ASK: How many dots did I draw? (9) SAY: For this simpler problem, suppose I am 9th in line. Circle the last dot. SAY: My friend is 5th in line. Circle the 5th dot. Label the dots, as shown below:

Front of line: ● ● ● ● ● ● ● ● ●

5th

9th
ASK: How many people are between the 5th and 9th person? (3) Is that equal to 9 – 5? (no) SAY: It is close, but not quite equal.

**Exercises:** Draw a picture to decide how many people are between the given positions.

a) the 7th and 8th person  
b) the 7th and 9th person  
c) the 7th and 10th person  
d) the 7th and 11th person  
e) the 7th and 12th person  
f) the 7th and 37th person

**Answers:** a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the correct answer? (no) Did it give close to the correct answer? (yes) How can you get the number of people between two people given their positions in line? (subtract the smaller position from the other and then subtract 1 from the difference) How many people are between the 37th person and the 7th person in the rollercoaster line? (29, because 37 – 7 = 30, 30 – 1 = 29) How did starting with smaller numbers help? (answers will vary) SAY: Sometimes, it is easier to start by using smaller numbers than the numbers given in the problem. Then you will see patterns and learn how to solve the harder problem. Now that you know the pattern for finding the number of people between any two positions, you can use that method with any numbers.

**Exercises:** How many people are in line between the given positions?

a) the 8th and 78th person  
b) the 314th and 1000th person  
c) the 492nd and 613th person

**Answers:** a) 69, b) 685, c) 120

**Making the problem easier by finding what is relevant.** SAY: Sometimes making the problem easier has nothing to do with using smaller numbers and finding a pattern. Sometimes all you need to do is eliminate information that’s not relevant, and moving objects around can help with that.

Affix two pre-made sticks of different colours and different lengths to the board, end to end. Label one length and their combined length. The following is an example for 8 cm and 12 cm, but your sticks can be other lengths:

![Image of sticks](image.png)

Tell students that all the measurements are in centimetres. ASK: How long is the second stick? (12 cm) SAY: It is easy to see with sticks, but now I’m going to move these sticks around. Slide the grey stick down and draw the lines around it, as shown on the following page.
ASK: How did I move the sticks? (you slid one of them down) SAY: This now looks like a problem to do with shapes and the lengths of missing sides. There’s a lot of extra information in this second problem compared with the first problem, so it looks harder, but it actually has exactly the same answer as the other one. The total length of the two sticks is still 20 centimetres—I just slid one of the sticks down so that they are not side by side anymore.

**Exercises:** Find what the ? stands for by pretending the sticks are side by side.

- a) ![Diagram a]
- b) ![Diagram b]
- c) ![Diagram c]
- d) ![Diagram d]
- e) ![Diagram e]
- f) ![Diagram f]

**Answers:** a) 7, b) 9, c) 1240, d) 3368, e) 17, f) 13

SAY: By pretending that the sticks were side by side, you turned the problem into an easier problem.

**Making the problem easier by emphasizing what is relevant.** SAY: We can look at a problem and focus on what matters most. For example, if you need to find a vertical side—straight up and down—then colour along all the vertical lines. If you need to find a horizontal side—side to side—then colour along all the horizontal lines.
Exercises: Find what the ? stands for by making the problem into an easier problem.

a) 
\[
\begin{array}{c}
13 \\
12 \\
\end{array}
\begin{array}{c}
15 \\
3 \\
\end{array}
\begin{array}{c}
8 \\
? \\
\end{array}
\]

b) 
\[
\begin{array}{c}
3819 \\
3514 \\
\end{array}
\begin{array}{c}
4736 \\
3534 \\
\end{array}
\begin{array}{c}
5125 \\
8555 \\
\end{array}
\]

c) 
\[
\begin{array}{c}
62 \\
152 \\
\end{array}
\begin{array}{c}
43 \\
152 \\
\end{array}
\begin{array}{c}
28 \\
? \\
\end{array}
\]

d) 
\[
\begin{array}{c}
1515 \\
3417 \\
\end{array}
\begin{array}{c}
2881 \\
1834 \\
\end{array}
\begin{array}{c}
1830 \\
2531 \\
\end{array}
\]

Answers: a) colour vertical, ? = 5; b) colour horizontal, ? = 8659; c) colour horizontal, ? = 217; d) colour vertical, ? = 948

Point out to students that by colouring along the horizontal or vertical lines, they changed the problem into an easier problem.

Finding perimeter by finding missing side lengths. Remind students that to find the perimeter of a shape, they have to add up the lengths of all the sides.

Exercises: Find the perimeter by finding missing sides, then adding all the sides.

a) 
\[
\begin{array}{c}
343 \\
895 \\
\end{array}
\begin{array}{c}
431 \\
677 \\
\end{array}
\begin{array}{c}
431 \\
677 \\
\end{array}
\]

b) 
\[
\begin{array}{c}
8.2 \\
5.2 \\
\end{array}
\begin{array}{c}
4.5 \\
3.2 \\
\end{array}
\begin{array}{c}
7.8 \\
5.2 \\
\end{array}
\]

Answers: a) 3830, b) 44.4

Finding perimeter without knowing all the side lengths. Draw on the board:

\[
\begin{array}{c}
2 \\
1 \\
\end{array}
\begin{array}{c}
6 \\
\end{array}
\]
SAY: All measurements are in centimetres. Point to the side on the right and ASK: What is this length? (3 cm) Point to the bottom two horizontal sides and ASK: What might these be? (1 and 5, 2 and 4, or 3 and 3) SAY: We don’t know for sure what the missing horizontal sides are, but let’s find the perimeter using different possibilities.

**Exercises:** All lengths are in centimetres. Find the perimeter.

a) ![Image](a.png)

b) ![Image](b.png)

c) ![Image](c.png)

**Answers:** a) 18 cm, b) 18 cm, c) 18 cm

ASK: Did using different possibilities change the answer to the perimeter? (no) Why not? (because the two bottom sides always add to 6, so it didn’t change the total) SAY: The bottom numbers always add to 6 because they have to add to the same as the top side. So, the total perimeter didn’t change.

Using different colours for the horizontal and vertical sides, draw on the board:

```
6
2 3
1 5
```

SAY: Let’s go back to the original picture. There are two kinds of sides in this shape—horizontal and vertical. ASK: How long is the top side? (6 cm) How long are the two bottom sides put together? (6 cm) How do you know? (because if you lined up the two bottom sides together, they would match the top one) How long are the two vertical sides on the left side of the shape? (2 cm and 1 cm) How long is the side on the right? (3 cm) How do you know? (because it’s the same as the two other vertical sides put together) Write on the board:

```
Horizontal sides add to ___ Vertical sides add to ___
Perimeter is ___ + ___ = ___
```

Have volunteers fill in the blanks. (12, 6, 12 + 6 = 18)
**Exercises:** All measurements are in centimetres. Find the perimeter.

a) ![Diagram](a)

b) ![Diagram](b)

**Answers:** a) 8100 cm, b) 12 040 cm

**Problem Bank**

1. A teacher asks students to read some pages of a book for homework. Write the page numbers down, then count them to find out how many pages the students have to read in total.

   a) Read pages 3 to 6.  
   b) Read pages 5 to 10.  
   c) Read pages 2 to 7.  
   d) Read pages 1 to 8.  
   e) Read pages 6 to 11.

**Answers**

a) 3, 4, 5, 6; 4 pages  
   b) 5, 6, 7, 8, 9, 10; 6 pages  
   c) 2, 3, 4, 5, 6, 7; 6 pages  
   d) 1, 2, 3, 4, 5, 6, 7, 8; 8 pages  
   e) 6, 7, 8, 9, 10, 11; 6 pages

2. a) Complete the chart.

<table>
<thead>
<tr>
<th>Pages to Read</th>
<th>How Many Pages</th>
<th>(Largest Page Number) − (Smallest Page Number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) 1, 2, 3, 4, 5</td>
<td>5</td>
<td>5 − 1 = 4</td>
</tr>
<tr>
<td>ii) 2, 3, 4, 5, 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii) 3, 4, 5, 6, 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) 4, 5, 6, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v) 5, 6, 7</td>
<td></td>
<td></td>
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<tr>
<td>vi) 5, 6, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vii) 5, 6, 7, 8, 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>viii) 5, 6, 7, 8, 9, 10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) How can you get the number of pages to read from the result of the subtraction?

**Answers:** a) i) 5, 6 − 2 = 4; ii) 5, 7 − 3 = 4; iv) 5, 8 − 4 = 4;  
v) 3, 7 − 5 = 2; vi) 4, 8 − 5 = 3; vii) 5, 9 − 5 = 4; viii) 6, 10 − 5 = 5;  
b) The number of pages is 1 more than the result of the subtraction.
3. A teacher tells his class to read the given pages in a textbook for homework. How many pages does the class have to read?
   a) from 320 to 387   b) from 352 to 386
   c) from 298 to 314   d) from 408 to 451
   **Answers:** a) 68, b) 35, c) 17, d) 44

4. What was the last page that Ray read?
   a) Ray read 5 pages, starting at page 263.
   b) Ray read 156 pages, starting at page 24.
   **Answers:** a) 267, b) 179

5. A teacher tells her class to read from page 354 to 412 but skip pages 363 to 389. How many pages does the class have to read?
   **Answer:** 32

6. How many whole numbers are greater than 11 and less than 45?
   **Answer:** 33

7. When everyone in Liz's class stands in line, Liz is 12th in line and 15th from the end of the line. How many people are in Liz's class?
   **Answer:** 26

8. There are 800 people in line. How many people are behind the 12th person?
   **Answer:** 788

9. There is a long line-up at a rollercoaster. Edmond is 8th in line and Ava is 78th in line.
   How many people are between Edmond and Ava in the line-up?
   **Answer:** 69

10. How long is the fence?
    a) A fence is made using 42 posts, each 1 metre apart.
    b) A fence is made using 34 posts, each 2 metres apart.
    **Answers:** a) 41 m, b) 66 m

11. How many posts are needed to make the fence?
    a) A fence is 38 metres long with posts 1 metre apart.
    b) A fence is 50 metres long with posts 2 metres apart.
    c) A fence is 63 metres long with posts 3 metres apart.
    **Answers:** a) 39, b) 26, c) 22
12. A fence for a square field is made with posts 1 metre apart, including a post at each corner. How many posts are needed for a field that is …

a) 10 m by 10 m?
   Hint: Start with 1 m by 1 m, then move on to 2 m by 2 m, 3 m by 3 m, etc.

b) 20 m by 20 m?
   
   **Answers:** a) 40, b) 80

**NOTE:** For the following problems, encourage students to predict each answer before checking.

13. A field is a square 20 m by 20 m. How many posts are needed if the posts are …

a) 1 m apart?

b) 2 m apart?

c) 4 m apart?

d) 5 m apart?

**Bonus:** 40 cm apart?

**Answers:** a) 80, b) 40, c) 20, d) 16, Bonus: 200

14. In 1993, an artist named Manfred Laber started a piece of public art called The Time Pyramid. Every 10 years, a cube is added to the pyramid. This will continue until 120 cubes are placed. In what year will the artwork be finished?

**Answer:** 3183

15. The sides of a square are made of 76 dots (like in the sketch below, but with more dots). Each side has the same number of dots. How many dots are on each side?

![Sketch of a square with dots]

**Answer:** 20

16. Cut out the strips and circles from **BLM Fraction Strips and Circles** (you may cut the line down to the centre of the circles).

a) Look at the strip of paper that is partly shaded. Sandy thinks that the amount shaded is one fifth. Is she correct? Use folding to check your answer.
b) Estimate two fifths of the blank strip of paper. Colour to show your estimate. Use folding to check your estimate.

c) Look at the circle that is partly shaded. Lewis thinks that the amount shaded is one fifth. Is he correct? Use folding to check your answer.

d) Estimate two fifths of the blank circle. Colour to show your estimate. Use folding to check your estimate.

17. On this crooked path, each line segment is 1 metre long. What is the total length of the path? Look for a quick way to find the answer.

Solution: 18 vertical metres plus 17 horizontal metres = 35 metres altogether

18. All measurements are in centimetres.

a) Add the horizontal edges and the vertical edges together to find the perimeter.

b) Is there enough information to find the area of this shape? Explain.

Answers: a) 34 cm; b) no, because to find the area you need the measurements of each part

19. Each shape was made by placing a small square on top of a large square. All measurements are in centimetres.

a) Find the perimeter of each shape.

i) 

ii)
b) Make a table with headings “Size of Smaller Square,” and “Total Perimeter.” Use the pattern from part a) to solve the problems.

i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?

ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?

**Answers:** a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm

20. a) Convert the measurements in metres to centimetres. 
   Hint: 1 m = 100 cm.
   i) 2 m = ____ cm   ii) 3 m = ____ cm

   **Bonus:** 183 m = ____ cm

b) Find the perimeter, in centimetres.

   i) 

   ![Diagram](image1)

   ii) 

   ![Diagram](image2)

   **Answers:** a) i) 200, ii) 300, Bonus: 18300; b) i) 1040 cm, ii) 2050 cm
21. Find the missing length.

a) \[\begin{array}{c}
4 \quad 7 \\
? \quad 10
\end{array}\]

b) \[\begin{array}{c}
38 \\
? \\
20
\end{array}\]

Answers: a) 1 cm, b) 8 cm
Fraction Strips and Circles
Extended Problem: Flower Garden

MATERIALS

BLM Flower Garden (pp. P-72–73)

Extended Problem: Flower Garden. Give students BLM Flower Garden. Question 6 is a good opportunity to apply the problem-solving strategy learned in this lesson. Students who have not had the opportunity to do this lesson may find that question difficult.

Answers: 1. 20 m; 2. $60; 3. $147; 4. 16 bags; 5. $223; 6. 6 rows of 14 flowers each, or 84 flowers
Flower Garden (1)

Kyle has a flower garden. His flower garden is 3 metres wide by 7 metres long.

1. He wants to put a fence around his garden to keep animals away from his plants. How many metres of fencing does Kyle need?

2. If fencing costs $3 per metre, how much will the fence cost?

3. Kyle needs to add more soil to his garden. The soil will cost $7 for each square metre. How much will it cost to cover his entire garden?
Flower Garden (2)

4. Kyle decides to plant 144 flowers in his garden. Each bag holds 9 flower seeds. How many bags does Kyle need to buy?

5. Flower seeds cost $1 per bag. What is Kyle’s total cost for his flower garden, including fencing, soil, and seeds?

6. After planting his garden this year, Kyle finds new instructions for planting a flower garden:
   - Plant each flower 50 cm apart.
   - Start planting 25 cm from each edge of the garden.
   If he follows these instructions next year and has the same size of garden, what is the greatest number of flowers he can plant?
Unit 14  Geometry: 3-D Shapes

Introduction
This unit explores the geometry and measurement of 3-D shapes, including:
- describing and categorizing prisms and pyramids;
- constructing skeletons of three-dimensional figures;
- constructing shapes from nets and with connecting cubes;
- understanding and measuring volume; and
- understanding and measuring capacity in litres and millilitres.

Meeting Your Curriculum

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<th>ALBERTA</th>
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Mental Math Minutes
The mental math minutes in this unit:
- review multiplications and divisions used in the lessons.
- provide opportunities for students to further develop fluency in calculation.

Generic BLMs
The Generic BLM used in this unit is:
BLM 1 cm Grid Paper (p. S-2)
This BLM can be found in Section S.

Materials
The following materials are used in several lessons and should be collected in advance:
- examples of rectangular and triangular prisms, such as cereal boxes and confectionary boxes
- modelling clay
- toothpicks or other sticks of various lengths
- plastic knives
Assessment

The lessons covered by a quiz or test are as follows:

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<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tr>
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<td>G4-10 to 12</td>
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<td>G4-10 to 12</td>
<td>G4-10 to 12</td>
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<tr>
<td>Quiz</td>
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<td>n/a</td>
<td>n/a</td>
<td>G4-13 to 15</td>
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<td>n/a</td>
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<td>G4-10 to 12</td>
<td>n/a</td>
<td>G4-10 to 12</td>
<td>G4-11 to 18</td>
</tr>
</tbody>
</table>
Goals

Students will identify faces, vertices, and edges of 3-D shapes.
Students will count vertices and edges of 3-D shapes using actual shapes and pictures.

PRIOR KNOWLEDGE REQUIRED

Can identify and count sides and vertices of 2-D shapes
Can identify and name polygons

MATERIALS

cube, prism, and pyramid per student
large paper square
various 3-D shapes, including a cube, a square-based prism, and some everyday objects (e.g., boxes, a ball, a can, a hockey puck)
BLM Matching 3-D Shapes (p. Q-35, see Extension 1)
flashlight or overhead projector (see Extension 3)

Mental math minute. Have groups of three students add two-digit numbers by adding tens and adding ones. First give an addition problem, such as 35 + 46. The first student adds the tens (30 + 40 = 70). The second student adds the ones (5 + 6 = 11). The third student finishes the addition (70 + 11 = 81), so 35 + 46 = 81.

Introduce faces. Give each student a cube, a prism, and a pyramid—use a variety of different prisms and pyramids. Hold up a large cube and ask if anyone remembers what it is called. Students should be familiar with cubes from earlier grades. Have students identify the cube in their collections and hold it up.

Explain that the flat sides of a 3-D shape are called faces. Point to the faces on the large cube. ASK: What polygons are the faces of a cube? (squares) SAY: Hold up a shape that has some faces that are not squares. Ask volunteers to show the shape to the class, point out the face that is not a square, and identify the shape of that face. Point out that some 3-D shapes have faces that are triangles and some 3-D shapes have faces that are rectangles. ASK: Does anyone have a shape that has only triangles as faces? Have a volunteer show this shape to the class if they have one, turning it so everyone can see that it has only triangular faces. Repeat with a shape that has only rectangles for faces. Remind students that squares are a special case of rectangle, so a cube qualifies. Ask students if anyone has a shape that has only rectangles as faces but is not a cube. Again, have a volunteer show this shape if they have one.

Counting faces. Ask students to count the faces on their cubes. Discuss strategies to keep track of faces counted. If the following strategy does not
arise, show it to students. Count the top and the bottom first, then look at
the shape from the top. From the top, the cube looks like a square. But
each side of that square is the side of another “hidden” square that you
can see when you look at the cube from the side. Because we know that
a square has four sides, there are four “hidden” squares. Four squares (on
the sides) and two squares (top and bottom) equals six squares altogether,
which means a cube has six faces.

Have students count faces on the other two shapes in their collections.
Invite volunteers to show their shapes and explain how they kept track of
the faces. Students who finish early can exchange shapes with a partner
and count the faces on their partner’s shapes.

**Introduce edges of 3-D shapes.** Hold up a cube. Run your finger along
one of the edges and explain that the place where two faces meet is called
an *edge*. Ask students to show an edge on their cubes.

**Counting edges.** Hold up a square-based prism and explain that you want
to count the edges of this shape. Place the prism on the desk, square face
down. SAY: I see three groups of edges. There are edges on the bottom
face, the edges that run along the desk. They are the sides of the bottom
face. Lift up the prism and trace the edges of the bottom face. ASK: How
many edges like this do we have? (4) Write “4” on the board. Place
the prism on the desk again. SAY: There are edges that only touch the desk
at one end, the longer, vertical edges. Trace these edges with a finger
and have students show the same edges on their square-based prisms.
ASK: How many edges like this do we have? (4) Write “+ 4” on the board.
SAY: There are edges that do not touch the desk at all. They are the sides
of the top face. Trace them with a finger. ASK: How many edges like this
does the shape have? (4) Write “+ 4” on the board. ASK: Did we miss any
edges? (no) How many edges are there in total? (12) Write “= 12” on the
board. SAY: The shape has 12 edges.

Have students count the edges of their other two shapes. Have them
discuss strategies in pairs. Students who finish early can exchange shapes
with a partner and count the edges of their partner’s shapes.

**Introduce vertices of 3-D shapes.** Show students a large paper square.
Invite a volunteer to identify the vertices. ASK: How do you know this is a
vertex? (it is a corner where sides meet) Do vertices feel different? (they are
pointy, they are sharp turns, they feel the sharpest) Ask students to hold
up a cube. Ask them to feel their cubes and show the places on the cubes
that feel pointy. Explain that these are also called *vertices*. Ask a volunteer
to show the vertices of their cube. Have students show the vertices of their
other 3-D shapes. Point out that edges of 3-D shapes meet at vertices.
SAY: Just as sides of flat shapes meet at vertices, edges also meet
at vertices.

**Counting vertices.** Discuss strategies for counting vertices of a cube.
Guide students through the following strategy if no one suggests it. Set the
cube on a desk. Point at two vertices, one on the bottom face and the other
on the top face. ASK: How are these two vertices different? (one is on the
top and the other is on the bottom, or one touches the table and the other does not). Can we first count all of the vertices on the bottom, then all of the vertices on the top? (yes) How many vertices are on the bottom? (4) How many vertices are on the top? (4) Are there any vertices in the cube that we did not count? (no) Write on the board:

\[ 4 + 4 = 8 \]

A cube has 8 vertices.

SAY: By counting the bottom vertices separately from the top vertices, we solved two simpler problems and used them to solve the harder problem.

ASK: Why was it so easy to find the number of vertices on the bottom? (the bottom is a square) How is the top face similar to the bottom? (it is also a square) How many vertices do two squares have? (8) How many vertices does a cube have? (8)

Ask students to count the vertices of their other two shapes. Students can problem-solve in pairs how to track the number of vertices. Have students share their strategies with the class. Students who finish early can exchange shapes and count the vertices on their partner’s shapes.

**Vertices and edges on pictures of 3-D shapes.** Draw on the board:

ASK: What shape is this? (a cube) Invite a volunteer to place a dot on each vertex she can see in the picture. Count the vertices together and write the numbers beside the dots as you do so. ASK: How many vertices are in the picture? (7) How many vertices does a cube have? (8) Why did we get seven instead of eight? (there is a corner on the back that we do not see)

Repeat with edges. Students will see that there are nine edges visible and three edges on the back.

**Introduce hidden edges.** Explain that in mathematics people often draw the parts of shapes that we cannot see (because they are hidden behind other parts) with dashed lines. The dashed lines are behind the solid lines and would only be seen if the shape were made of a clear, see-through material, such as glass. Use your hands or two pencils to show the relative positions of a visible edge and a hidden edge in the cube that look like they intersect. SAY: We call the edges we cannot see in a picture hidden edges. Add the dashed lines to the cube on the board and invite a volunteer to mark the hidden vertex. (see margin)

Draw on the board:
SAY: I drew dots on the vertices of this box. ASK: Did I draw everything correctly? (no) Point to each dot one at a time. ASK: Is this a vertex? Have students signal the answer with thumbs up for yes and thumbs down for no. Erase (or cross out) the two incorrect vertices and have volunteers count the vertices (8) and the edges (12).

Extensions

1. Have students play the games below using the cards from BLM Matching 3-D Shapes. The cards match if they show the same shape. For example, the long, thin rectangular prism on card 13 would not match the short, thick rectangular prism on card 11. Players can help each other by asking questions or making suggestions, but they should not tell each other where specific cards are. Example: "I think I know where both rectangular prisms are. Should I turn one of them over?"

Picking Pairs. Students play in pairs or individually. Place cards face up in an array. Students take turns picking pairs of matching cards and placing them in a common discard pile. When there are no more pairs in the array, more cards are added to it. The goal is to place all the cards into the discard pile. If students have any non-matching cards left at the end, then some of their cards must have been matched incorrectly.

Memory. This version of the well-known game is played like Picking Pairs but the cards are face down. Students turn over two cards at a time looking for a match. If the cards match, students set them aside; otherwise, they turn them face down again and continue playing. Students can play individually or cooperatively in pairs. In either case, the goal is to find all the matches. If playing with a partner, Player 1 leads by choosing and turning over a card and Player 2 follows by choosing and turning over another card. Players switch roles after each turn.

2. Ask students to hold cubes in various positions (for example: face-on, looking at a vertex, and so on) so that they see it from different angles. Ask students to describe what the faces look like when seen from different angles. The faces will look like squares, rectangles, parallelograms, or rhombuses depending on the position.

3. Have students use a cube and a flashlight or overhead projector. Tell students that by holding a cube in different positions, they can produce shadows of different shapes. Ask students what polygons they can produce as a shadow.

Answers: square, rectangle, hexagon, trapezoid, rhombus
G4-11 Triangular and Rectangular Prisms
Pages 130–131

CURRICULUM REQUIREMENT
AB: required
BC: optional
MB: required
ON: required

VOCABULARY
3-D shape
base
edge
face
hidden edge
polygon
prism
rectangle
rectangular prism
skeleton
square
square prism
triangle
triangle-based prism
triangular prism
vertex
vertices

Goals
Students will construct skeletons of triangular and rectangular prisms.
Students will compare skeletons with actual shapes.
Students will name prisms by the shapes of their bases.

PRIOR KNOWLEDGE REQUIRED
Can recognize and name polygons and cubes

MATERIALS
paper triangle that is an enlarged copy of a triangular pattern block
triangular and square pattern blocks
toothpicks or sticks of various lengths
modelling clay
various blocks, including prisms from a commercial set or
BLM Nets (1) to (4) (pp. Q-36–39, see Extension 1)
plastic knives (see Extension 2)
connecting cubes (see Extension 3)

Mental math minute. Have students subtract three-digit numbers by subtracting hundreds, tens, and ones separately in groups of four. Give a subtraction problem, such as 542 − 231. The first student subtracts the ones (2 − 1 = 1). The second student subtracts the tens (40 − 30 = 10). The third student subtracts the hundreds (500 − 200 = 300). The fourth student finishes the addition (300 + 10 + 1 = 311), so 542 − 231 = 311. Start with problems that do not require regrouping in any place, such as 635 − 324, and continue to problems that require regrouping tens or hundreds, but not both. If tens have been regrouped, for example, the student subtracting tens must adjust.

Introduce prisms. Show students a paper triangle that is an enlarged copy of a triangular pattern block. Ask them to identify the shape. Show students the pattern block and ask them how it is different from the paper triangle. (the paper triangle is larger and flat, or 2-D; the pattern block has thickness and is a 3-D shape) Have several volunteers stack different numbers of triangular pattern blocks, one on top of the other. Discuss what the result looks like. (it has a triangle on top and the same triangle on the bottom, the sides are rectangles) Explain that this shape is called a triangular or triangle-based prism. Tell students that a prism is a 3-D shape with two congruent bases and sides that are rectangles or parallelograms. SAY: You can make prisms with any polygon as the base. Then have students stack square pattern blocks and introduce the terms square prism and rectangular prism.

Ask students for examples of prisms in the real world. Examples of square or rectangular prisms could include tall buildings and food boxes.
Examples of triangular prisms include some chocolate bar boxes, closed binders, wedges, ramps, and pointed rooftops. ASK: What is the difference between rectangular prisms and cubes? (rectangular prisms can have faces shaped like any rectangle, the faces of cubes are all squares)

SAY: Cubes are a special example of rectangular prisms. Square prisms have some square faces but not all square faces, so they are special cases of rectangular prisms too.

**Building skeletons of prisms.** Tell students that a skeleton of a prism is the frame of the prism, the vertices and edges without the faces. Demonstrate making a skeleton of a triangular prism using toothpicks and modelling clay. Write the following steps on the board as you demonstrate them. When demonstrating Step 1, explain that the polygon you start with is called the base of the prism and that each prism has two bases, which are on opposite sides of the figure.

- **Step 1:** Make two copies of the same polygon using clay balls for vertices and toothpicks for sides.
- **Step 2:** Add one toothpick to each vertex of one of the polygons.
- **Step 3:** Attach the other polygon on top of the loose ends of the toothpicks.

**ACTIVITY (Essential)**

Have students use toothpicks and modelling clay to construct the skeletons of two types of prisms, triangular and rectangular. Discuss how the skeletons that students created are the same as prisms and how they are different. (skeletons have vertices and edges only, no faces)

Tell students that skeletons make it easier to count edges because there are no hidden edges.

**Counting vertices and edges of prisms.** Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students help you to fill in the table. Point to each cell and have students count the vertices or edges in the skeletons they made in the activity and signal their answers. The completed table is shown below.

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>
Extensions

1. Give each student or pair of students blocks, including several prisms from a commercial set or BLM Nets (1) to (4), and have them create structures. Ask students to identify the prisms in their structures and then in structures built by other students.

2. Have students make prisms out of modelling clay. They can use plastic knives to cut the clay and create flat surfaces.

3. Have students create rectangular prisms using connecting cubes.

4. Give each student a rectangular prism. Have students find all of the congruent faces.

**NOTE:** Students will work with hexagon- and pentagon-based prisms in lesson G4-13.

5. Have students construct skeletons for prisms with other shapes as the base, such as hexagons and pentagons. Then have students count the vertices on the base and the edges and vertices on the final shape. **ASK:** Do you see any patterns?

   **Answers:** There are twice as many vertices in the 3-D object as there are in the base and three times as many edges.

6. Can you make a prism with fewer faces than a triangular prism? Explain.
Goals

Students will construct triangular and rectangular prisms from nets.
Students will sort prisms according to their bases.
Students will sketch the faces of prisms.

PRIOR KNOWLEDGE REQUIRED

Can recognize and name polygons and cubes

MATERIALS

triangular prism
cut-out nets from \textit{BLM Nets (2) to (4)} (p. Q-37–39)
glue or tape
one set of prisms made from \textit{BLM Nets (1) to (4)} (pp. Q-36–39)
\textit{BLM Nets (1) to (4)} (pp. Q-36–39) per pair of students
toothpicks or sticks of various lengths (see Extension 1)
modelling clay (see Extension 1)
\textit{BLM Nets (5)} (p. Q-40, see Extension 2)
connecting cubes (see Extension 3)

Mental math minute. Ask students to solve multiplication problems within $1 \times 1$ to $10 \times 10$.

\textbf{Introduce nets}. Hold up a triangular prism. ASK: What is this 3-D shape called? (a triangular prism or triangle-based prism) What are the shapes of the faces? (triangles and rectangles) How many triangular faces does it have? (2) How many rectangular faces does it have? (3)

In advance, cut out the nets from \textit{BLM Nets (2) to (4)}. Show students the cut-out net of a triangular prism. ASK: How many triangles does this picture have? (2) How many rectangles does it have? (3) Explain that you can fold this picture (demonstrate as you do so) and glue or tape it together to make a 3-D shape. ASK: What shape does this picture make? (a triangular prism) SAY: A picture that we can fold to make a 3-D shape is called a \textit{net} of the 3-D shape. This was the net of a triangular prism.

Hold up a cut-out net of a square-based prism and ask students to describe the shapes they see. (2 squares and 4 rectangles) ASK: If this is a prism, what shape is its base? (square or rectangle) Can it be a prism? (yes) Repeat for the cut-out net of a rectangular prism.

\textbf{NOTE}: Activity 2 is required in order to cover the Alberta and Manitoba curricula.
ACTIVITY 1 (Essential), ACTIVITY 2 (Optional)

1. Display in random order a set of completed prisms from BLM Nets (1) to (4) and label them A to D. Give each pair of students BLM Nets (1) to (4). Have the students cut out the nets. Then have students predict which shape the net will make. Finally, have students fold the nets and check that they have identified the shapes correctly.

2. Have students sort the prisms they constructed in Activity 1 into triangular prisms and rectangular prisms.

Sketching faces of prisms. Have each student select one prism from the shapes they made in Activity 1. ASK: How many bases does a prism have?

(2) Write “Triangular Prism,” “Square-Based Prism,” “Rectangular Prism,” and “Cube” on the board. Have volunteers sketch the bases of their prisms under the appropriate heading. SAY: The faces of a prism that aren’t bases are called side faces. ASK: What shape are the side faces on your prism? (rectangle, square for cubes) Is that true for the triangular prisms too? (yes) Have students count and sketch the side faces. Students who are having trouble counting side faces can try counting from the top or marking the first side (or all sides) with a dot or sticker. Have volunteers sketch the faces for each prism named on the board. (see sample sketches below)

<table>
<thead>
<tr>
<th>Triangular Prism</th>
<th>Square-Based Prism</th>
<th>Rectangular Prism</th>
<th>Cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ △</td>
<td>□ □</td>
<td>□ □</td>
<td>□ □</td>
</tr>
<tr>
<td>□ □</td>
<td>□ □</td>
<td>□ □</td>
<td>□ □</td>
</tr>
<tr>
<td>□ □</td>
<td>□ □</td>
<td>□ □</td>
<td>□ □</td>
</tr>
</tbody>
</table>

ASK: What is different about the number of side faces in the triangular prism? (there are only 3) Why is that? (triangles have 3 sides, but squares and rectangles have 4 sides)

Extensions

1. Build a prism skeleton using toothpicks and modelling clay, but instead of placing the top base directly above the bottom, place it a little to the side. This prism is called a skew prism. What shape are the side faces of skew prisms?

Answer: parallelograms

2. Construct a shape using BLM Nets (5). What shape is it? Why is it not a prism?

Answers: A cylinder. It is not a prism because the base is not a polygon.
3. One way to describe a 3-D shape is to draw a mat plan. A mat plan is a drawing of the base of the shape with a number in each section showing the height at that point. For example, below is the mat plan for a 2 by 2 by 3 prism. The base, a 2 by 2 square, has been drawn. At each place on the square, there is a stack of cubes that is three cubes high.

```
 3 3
 3 3
```

Use connecting cubes to build the shape shown in the mat plan. Is it a prism?

a) b) c)  
```
2 2
2 2
1 2
3 4
4 4 4
```

d) Can you tell from the mat plan if the shape will be a prism without building it? Explain.

**Answers:** a) yes; b) no; c) yes; d) yes, because in a prism, all the numbers in the mat plan will be the same.
Goals

Students will name prisms by the shapes of their bases.
Students will construct skeletons of prisms with trapezoidal, pentagonal, and hexagonal bases.

PRIOR KNOWLEDGE REQUIRED

Knows that a prism has two identical bases and rectangular side faces
Can recognize and name polygons
Can construct skeletons of triangular and rectangular prisms

MATERIALS

trapezoidal, pentagonal, and hexagonal pattern blocks
toothpicks or sticks of various lengths
modelling clay
rectangular prisms (see Extension 1)
flashlight or overhead projector (see Extension 1)
heavy duty aluminium foil, tape, paper towel rolls, black construction paper, plastic wrap, sequins, and elastic bands (see Extension 2)
different kinds of prisms (see Extension 3)
plastic knives (see Extension 4)

Mental math minute. Ask students to solve multiplication problems within $1 \times 1$ to $10 \times 10$ and the corresponding division problems. For each number, go through the problems in order, for example, from $1 \times 3$, $3 \div 3$, $2 \times 3$, $6 \div 3$, and so on, to $10 \times 3$ and $30 \div 3$. Then progress to a different number. Finally, try problems out of order but keep corresponding multiplications and divisions together.

Introduce other bases. Tell students that prisms don’t need to have triangular or rectangular bases. Distribute trapezoidal, rhombic, and hexagonal pattern blocks. Have students choose one of these pattern block shapes and create a prism by stacking more of them. Ask them to show and name their prism. (trapezoid-based prism, rhombus-based prism, hexagon-based prism)

Making skeletons. Remind students how they constructed skeletons of prisms by making two copies of the base and then attaching them. ASK: What was the advantage of having a skeleton? (it was easy to count vertices and edges) Why isn’t a skeleton a real prism? (it doesn’t have faces)
**ACTIVITY (Essential)**

In groups of three, have students use toothpicks and modelling clay to construct skeletons of three types of prisms: trapezoidal, pentagonal, and hexagonal.

**Counting edges, vertices, and faces.** Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>trapezoid</th>
<th>pentagon</th>
<th>hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Edges in the Base</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Have students help you to fill in the table. Point to each cell and have students use the skeletons they made in the activity to count the vertices or edges, and then signal their answers. The completed table is shown below.

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>trapezoid</th>
<th>pentagon</th>
<th>hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Edges in the Base</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Ask students to consider the columns in the table. **ASK:** Do you see any patterns? (the number of vertices in the prism is double the number of edges in the base; the number of edges in the prism is three times the number of edges in the base)

**Sketching faces.** Write the names of the prisms from the activity on the board. For each, have a volunteer draw a complete set of faces. Then have the volunteer or the class count the total number of faces and write the number below the drawings. (see sample drawings below)

**Trapezoid-Based Prism**

6

**Pentagon-Based Prism**

7

**Hexagon-Based Prism**

8

**ASK:** How many edges does a trapezoid have? (4) How many side faces does a trapezoid-based prism have? (4) Why are these numbers the same? (you have one side face for each edge) How many faces does the prism have altogether? (6) **SAY:** It has a side face for every edge plus two more for the top and bottom faces. **ASK:** How many faces would a prism have if its base had eight edges? (10)
Extensions

1. Have students use a rectangular prism and a flashlight or an overhead projector to make shadow shapes. (A cube works especially well.) Tell students they can produce shadows of different shapes by holding a rectangular prism in different positions. Ask students what polygons they were able to make as a shadow.

**Answers:** square, rectangle, hexagon, trapezoid, rhombus

2. Follow these steps to make a kaleidoscope with a triangular prism.

**Step 1:** Draw a 10 cm by 20 cm rectangle on heavy duty aluminium foil and cut it out. Draw lines on it to make three 3 cm by 20 cm rectangles and one 1 cm by 20 cm rectangle, as shown in the margin.

**Step 2:** Fold along the lines to make a triangular prism—keep the shiny side in. Tape the thinner rectangle to the other side so that the prism holds its shape.

**Step 3:** Cut a paper towel roll to the same length as your prism. Slide the prism inside the roll.

**Step 4:** Place one end of the paper towel roll on black construction paper and trace a circle around it. Cut out a circle a bit bigger than the one you traced. Tape the circle over one end of the paper towel roll. Poke a small hole in the centre of the circle to make a peephole.

**Step 5:** Tape a 5 cm by 5 cm square of plastic wrap on the other end of the paper towel roll so that it pokes in a little. Fill the plastic wrap with small shiny objects such as sequins.

**Step 6:** Cover the end of the paper towel roll with another piece of plastic wrap and secure it in place with an elastic band. Your kaleidoscope is finished! You can adjust the colourful end of the kaleidoscope as you’d like.

3. Give students several different kinds of prisms and have them find out which can be stacked without any space being left between them.

4. Make a cube out of modelling clay. Can you cut your cube to create two triangular prisms? Use a plastic knife to show how.

**Answer:** yes
Goals

Students will name pyramids by the shapes of their bases.
Students will construct skeletons of pyramids with different shapes of bases.
Students will count the vertices and edges of pyramids.
Students will draw the faces of pyramids.

PRIOR KNOWLEDGE REQUIRED

Knows that a prism has two identical bases and rectangular side faces
Can recognize and name polygons
Can construct skeletons of prisms

MATERIALS

several pyramids with different bases
pictures of real-life pyramids
pyramid-shaped teabags (optional)
toothpicks or sticks of various lengths
modelling clay
plastic knives (see Extension 1)
BLM Nets (6) (p. Q-41, see Extension 3)
BLM Faces (p. Q-50, see Extension 4)

Mental math minute. Have students divide a two-digit number by a one-digit number from memory or by skip counting forward by the divisor and counting the number of jumps. For example, to calculate 56 ÷ 8, students skip count by 8s, keeping track on their fingers, until they reach 56. They should have 7 fingers raised.

Introduce pyramids. Hold up several pyramids and place them base-down in front of students. Point out the similarities among the shapes: they all have a polygon at the bottom, and they all have one vertex (point to it) opposite that polygon. Hold up the pyramids one at a time and show students that the base changes in each of the shapes. Explain that all of these shapes are pyramids. Ask students if they have heard this word before. Write “pyramid” on the board and read the word aloud together. Show pictures of pyramids in real life, such as the Great Pyramid of Giza (Egypt), the Louvre Pyramid (Paris, France), the Muttart Conservatory (Edmonton, AB), and some tents. If possible, show pyramid-shaped teabags.

Building skeletons of pyramids. Demonstrate how to make a rectangular (rectangle-based) pyramid with toothpicks and modelling clay. You will need six longer and two shorter toothpicks. Write the following steps on the board as you demonstrate them. When demonstrating the first step,
explain that as with prisms, the polygon you start with is called the base of the pyramid.

**Step 1:** Make a polygon using clay balls for vertices and toothpicks for sides.

**Step 2:** Add a toothpick to each vertex of the polygon.

**Step 3:** Join the loose ends of the toothpicks to form one vertex at the top.

**ACTIVITY (Essential)**

Have groups of three students use toothpicks and modelling clay to construct the skeletons of three types of pyramids: triangular, rectangular (or square), and pentagonal.

Discuss the similarities and differences between prisms and pyramids; for example, both have bases and are named by the shapes of their bases, but pyramids have only one base, whereas prisms have two.

**Counting edges, vertices, and faces.** Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
<th>pentagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Vertices in the Pyramid</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Edges in the Pyramid</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Have students help you to fill in the table. Point to each cell and have students count the vertices or edges in the skeletons they built in the activity and signal their answers. The completed table is shown below:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
<th>pentagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Number of Vertices in the Pyramid</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Edges in the Pyramid</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Ask students to look at the second and third rows in the table. **ASK:** What pattern do you see? (there is one more vertex in the pyramid than in the base) Could you tell when you were making it that there would always be one more vertex in the finished pyramid than in the base? (yes) **How?** (you make the base first and then add one more vertex)

Have students look at the second and fourth rows in the table. **ASK:** What pattern do you see? (the fourth row is double the second row) Will that be true with different bases too? (yes) **Why?** (You start with the base that has as many edges as vertices. Then you add one extra edge at each vertex so you have twice as many edges as vertices in the base.)
Drawing faces. Tell students that, as with prisms, the faces that aren't bases are called side faces. ASK: What shape are the side faces in pyramids? (triangles) Write “Triangular Pyramid,” “Square-based Pyramid,” and “Pentagonal Pyramid” on the board. For each pyramid, have a volunteer draw a complete set of faces. Then count the total number of faces and write the number below each drawing. (see sample drawings below)

<table>
<thead>
<tr>
<th>Triangular Pyramid</th>
<th>Square-based Pyramid</th>
<th>Pentagonal Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ △</td>
<td>□ △ △</td>
<td>△ △ △ △ △</td>
</tr>
</tbody>
</table>

4 5 6

ASK: How do the faces tell you that these are pyramids and not prisms? (there are a lot of triangular sides) SAY: The side faces on a prism are rectangles. So a prism can never have more than two triangular faces, which are the bases. ASK: How many rectangular sides can a pyramid have? (1) Why? (the side faces on pyramids are triangles, so the only face that can be a rectangle is the base) Which pyramid can have all congruent faces? (the triangular pyramid) SAY: Not every triangular pyramid has all congruent faces. Invite students to guess the property a triangular pyramid must have for all of its faces to be congruent. (its faces must be equilateral triangles)

ASK: If you know how many edges are in the base of a pyramid, can you tell how many faces it has? (yes) What is the pattern? (there is one more face in the pyramid than edges in its base) If necessary, prompt by looking at the examples one at a time and giving students time to think between each example.

Extensions

1. Have students make pyramids out of modelling clay. They can cut the clay with plastic knives to create flat surfaces.

2. Build skeletons of pyramids by building a base and then adding a vertex that is not centred over the base. How does the shape change as you move the vertex relative to the base? What are the shapes of the side faces? Can you place the vertex so that some of the side faces are right-angle triangles?

   Answer: The side faces are still triangles but are no longer congruent. If you place the vertex directly above one of the vertices of the base, you will produce two side faces that are right-angle triangles.

3. Build four triangular pyramids using BLM Nets (6). Can you assemble them to make a cube?
4. Have students play in pairs. The object of this game is to assemble the faces needed for a prism or pyramid. Give each pair of students four copies of **BLM Faces** and have them cut out the cards. The shapes on the cards represent the faces of pyramids or prisms. To begin play, students deal five cards each and place the rest face down in a pile. The first player takes a card off the top of the deck. If the player’s cards contain all the faces needed to make a prism or pyramid, he puts down those cards and wins the game. If not, he places one card face up on the discard pile. On each subsequent turn, a player can take either a new card from the deck or the top card from the discard pile. The first player holding cards that contain all the faces needed to create a pyramid or prism wins; players can use up to six cards and do not need to use all cards in their hand.
G4-15  Nets
Pages 138–139

**Goals**

Students will build pyramids and prisms from nets.
Students will identify correct and incorrect nets.
Students will add missing faces to nets.
Students will draw nets for square or triangular prisms or pyramids.

**PRIOR KNOWLEDGE REQUIRED**

Knows that a prism has two identical bases and rectangular side faces
Knows that a pyramid has one base and triangular side faces
Can recognize and name polygons
Can construct skeletons of prisms and pyramids

**MATERIALS**

completed shapes from BLM Nets (6) to (14) (pp. Q-41–49)
BLM Nets (6) to (14) per small group (pp. Q-41–49)
square-based prism that is not a cube
various prisms with bases that are regular polygons
square or triangular prisms and pyramids
pattern blocks (see Extension 4)

**Mental math minute.** Have groups of four students add three-digit numbers by adding hundreds, tens, and ones. Give an addition problem, such as 235 + 546. The first student adds the ones (5 + 6 = 11). The second student adds the tens (30 + 40 = 70). The third student adds the hundreds (200 + 500 = 700). The fourth student finishes the addition (700 + 70 + 11 = 781), so 235 + 546 = 781.

**ACTIVITY 1 (Essential)**

1. Display in random order a completed set of shapes from BLM Nets (6) to (14), labelled A to I. Give each small group of students several pages from BLM Nets (6) to (14). Have students cut out the nets and then predict the shape each net will make. Then have students fold the nets to check whether they correctly identified the shapes. Have students name the shapes they made from the nets.

**Correct nets.** ASK: How many squares are there in a net for a cube? (6)
Will any arrangement of six squares make a net for a cube? (answers may vary) Draw on the board:
ASK: Does this picture have the correct number of faces to make a cube? (yes) Count the faces to verify that there are six. ASK: Can you fold this to make a cube? (no) Why not? (the faces are all lined up; you don’t have a top or bottom) How many squares do I have to move to make a net? (2) Erase two squares and ASK: Where should I move them? (answers will vary) SAY: Let’s add a square to make the top of the cube. ASK: Where should it go? (sticking out on top) Does it matter which square it attaches to? (no) SAY: As long as it attaches to the top, it doesn’t matter where it goes. Draw a square above the second square in the chain, as shown below:

![Diagram](https://via.placeholder.com/150)

ASK: Where should I put the last square? (on the bottom) Draw the sixth square sticking out anywhere on the bottom, as shown below:

![Diagram](https://via.placeholder.com/150)

**Exercise:** Draw two nets for cubes, one that works and one that doesn’t. Trade with a partner and correct the net that does not work.

**Missing faces on nets.** Draw on the board:

![Diagram](https://via.placeholder.com/150)

SAY: This is part of a net. It is missing a face. ASK: Is this a prism or a pyramid? (a prism) How do you know? (it has 2 rectangular faces, it doesn’t have enough triangles to be a pyramid) What kind of prism will this net make? (triangular) How do you know? (it has 2 triangular faces) What shape is the missing face? (a rectangle) SAY: In a lot of the nets we have seen, the side faces were all the same. The side faces on this net are different. ASK: Why? (the sides of the triangle are different) Label the sides of the triangle $a$, $b$, and $c$, as shown in the margin. Point out that the side faces are all the same height but have different widths. The side face on the left is as wide as side $a$. The middle side face is as wide as side $b$. ASK: How wide should the last side face be? (as wide as side $c$) Have a volunteer sketch the last side. The completed net is shown in the margin.
Exercises: Copy the net onto grid paper. The net is missing one face. Draw the missing face.

a) 

b) 

c) 

Answers: The missing face can go in any one of the dashed-outline positions.

a) 

b) 

c) 

Drawing nets of prisms. Hold up a square-based prism that is not a cube and demonstrate how to draw a net for it by following the steps below. Point out before you start that because the base is a regular polygon, all of the side faces are the same shape and size (congruent). So if you can draw one, you can draw all of them. Write each step on the board and leave them there for the activity that follows. (see sample net in margin)

Step 1: Put the prism in front of you standing on its base.
Step 2: Measure and draw the side face that is facing you.
Step 3: Draw the remaining side faces in a row.
Step 4: Measure and draw the base on the top.
Step 5: Draw the base on the bottom.

ACTIVITY 2 (Essential)

2. Have students choose a prism with a base that is a regular polygon and draw a net for it on grid paper.

Drawing nets of pyramids. Demonstrate how to draw a net for an equilateral triangle-based pyramid by following these steps. Write the steps on the board and leave them there for the activity that follows. (see sample net in margin)

Step 1: Measure and draw the base.
Step 2: Measure a side face.
Step 3: Draw a side face attached to each edge of the base.
ACTIVITY 3 (Essential)
3. Have students choose either a square or triangular prism or pyramid and draw a net for it on grid paper.

Extensions
1. There are three different ways to add the missing face to the net. Find all three.

Answer

2. Draw a net for a rectangular prism that is not square-based.

3. How many different nets can you draw for the same cube?

Answer: There are six different nets for the cube. Other nets can be made by flipping or rotating one of these.

4. What shapes can you construct that have all faces congruent? What shapes can be used for the faces? Use pattern blocks to help you.

Answer: cubes, triangular pyramids with faces that are equilateral triangles
Goals

Students will understand volume as the amount of space an object occupies.

Students will determine volume as the number of cubes used to make a 3-D object.

Students will calculate the volume of prisms.

Students will build 3-D objects from pictures and determine their volumes.

Students will build prisms with specified volumes.

PRIOR KNOWLEDGE REQUIRED

Knows that a rectangular prism has two identical rectangular bases and rectangular side faces

Can describe bases and side faces of prisms and pyramids

MATERIALS

connecting cubes

BLM Block Models (p. Q-51)

BLM 1 cm Grid Paper (p. S-2)

Mental math minute. Ask students to multiply three numbers within the range 1 × 1 × 1 to 10 × 10 × 10. For example, 2 × 3 × 4.

Define volume. Draw a 2 by 3 rectangle on the board (use any non-standard units). ASK: What is the word we use to describe how much space a rectangle takes up? (area) SAY: One way to describe the size of a flat object is to calculate its area. For a 3-D object, one way to describe its size is to measure its volume. Volume is a measure of how much space a 3-D object takes up.

Measuring volume. Draw dividing lines on the rectangle on the board to show that it is 2 units by 3 units. SAY: The dimensions of a rectangle are the lengths of its two different sides. The dimensions of this rectangle are 2 units by 3 units. I measured the length with non-standard units. Ask the class what standard units they have used for measuring area. (cm²) SAY: Sometimes we use non-standard units for area, too. I divided this rectangle into square units. ASK: What is its area? (6 square units) How did you get that answer? (by counting or multiplying) SAY: Today we are going to measure volume in cubes by counting how many connecting cubes you need to make an object.

Finding the volume of rectangular prisms. Give students connecting cubes and have them build a rectangular prism that is 2 cubes wide, 3 cubes long, and 1 cube tall. SAY: The dimensions of a rectangular prism are the lengths of its three different edges. ASK: What are the dimensions
of your rectangular prism? (2 units by 3 units by 1 unit) How many cubes did you use altogether? (6) What is the volume? (6 cubes) Have students make two more 2 by 3 by 1 rectangular prisms and then stack them to make a single prism. ASK: What are the dimensions of this prism? (2 units by 3 units by 3 units) How many cubes did you use altogether? (18) What is the volume? (18 cubes) How did you count? (answers will vary) If no one suggests it, ask if anyone noticed that each rectangle used 6 cubes, so 3 rectangles must use 18 cubes.

**Exercises**

1. Make a prism that is 3 by 4 by 5. What is its volume?

   **Answer:** 60

2. Make a prism with a volume of 24 cubes. What are the dimensions of your prism?

   **Bonus:** Make a different prism with a volume of 24 cubes.

   **Sample answers:** 1 by 1 by 24, 1 by 2 by 12, 1 by 3 by 8, 1 by 4 by 6, 2 by 2 by 6, 2 by 3 by 4

SAY: Prisms are easy to describe because we already know their shape. It can sometimes be hard to tell exactly what other 3-D shapes are like from pictures. (You can show an Escher painting or an image of a Mobius strip, if available.)

**ACTIVITY 1 (Essential), ACTIVITY 2 (Optional)**

1. Working individually, students build each shape on BLM Block Models using connecting cubes. They then have a partner verify their work. Students should pay special attention to left and right. Students then find the volume of the object. Students can use BLM 1 cm Grid Paper to make sketches for Question 3.

2. Students use all of the blocks, skeletons, and prisms and pyramids from nets available to build a village.

**Extensions**

1. Draw a mat plan for the figure. What is its volume? (See Lesson G4-12 Extension 3 for an explanation of mat plans.)

   ![Diagram](image)
Answers

a) $\begin{array}{ccc}
2 & 2 & 3 \\
1 & 1 & 1 \\
\end{array}$  
Volume = 10 cubes

b) $\begin{array}{cccc}
2 & 1 & 1 & 3 \\
1 & 1 & 1 & 1 \\
\end{array}$  
Volume = 11 cubes

c) $\begin{array}{cc}
2 \\
1 & 3 & 2 \\
\end{array}$  
Volume = 8 cubes

d) $\begin{array}{ccc}
2 & 1 & 3 \\
1 & 1 & 1 \\
\end{array}$  
Volume = 8 cubes

2. a) Build an object using connecting cubes. Draw a mat plan for your object.

b) Draw a mat plan for a 3-D object. Construct the object from your mat plan.
Goals

Students will understand capacity as volume that an object can hold. Students will develop a sense of the size of 1 L and 1 mL.

PRIOR KNOWLEDGE REQUIRED

Understands the concept of volume

MATERIALS

different waterproof containers to measure and compare capacities
containers with capacity 1 L (or bottles with 1 L mark)
funnels, large pans or tubs, water
empty medicine bottles with labels (or a scale) for capacity, in mL
measuring cups of different sizes, including a graduated cylinder

Mental math minute. Have students add or subtract two-digit numbers by adding (or subtracting) tens and adding (or subtracting) ones in groups of three. Give an addition or subtraction problem, such as 35 + 46. In this case, the first student adds the ones (5 + 6 = 11). The second student adds the tens (30 + 40 = 70). The third student finishes the addition (70 + 11 = 81), so 35 + 46 = 81.

Introduce capacity. Review the story of “Goldilocks and the Three Bears.” Remind students that there were three bowls in the story, and Goldilocks tried the largest bowl first. ASK: What does it mean that the bowl was the largest? Was it the tallest? The widest? The heaviest? (no, it contained the largest amount of porridge) Explain that capacity generally means the volume of something (porridge, water, beans, etc.) a container can hold.

Remind students that to determine which container holds more, they could fill one of the containers with beans or water and then pour the beans or water into the second container. ASK: If the second container is full, and there is some water left in the first container, which one can hold more? (the first container) In this case, we say that the first container has greater capacity. ASK: If the first container is empty, but there is room left in the second container, which has greater capacity? (the second container)

Explain that just as students compared the masses of two objects by comparing two measurements, they can compare the capacities of two containers using measurements. To do so, they need to measure the capacity of both containers.

Introduce litres. Explain that capacity is measured in many different units. ASK: Which units of capacity do you know? Explain that in science, people usually use metric units because they are very convenient to work with. The
main metric unit is called a litre. Show students a container holding 1 litre of water and explain that this is a litre. Explain that the water has a volume of 1 litre. When the container is completely full, it holds 1 litre of water, so it has capacity 1 litre. Stress that when the container is only half full, there is only half a litre of water, but the container still has a 1-litre capacity.

Developing a sense of the size of a litre.

### ACTIVITY 1 (Essential), ACTIVITY 2 (Optional)

1. Give each student a 1 L container, a cup, and another container. Have students fill the 1 L container and pour the water carefully into the other container (students might need a funnel and should work over a large bowl or tub to prevent spills). Which container has larger capacity? How much larger is its capacity? If the difference is less than a whole container (or a whole litre), students can use the cup to find the difference. Students should record how the container they had compares to 1 litre.

2. Have students pair up. Player 1 asks Player 2 to guess (without looking at the recorded difference from Activity 1) whether his container holds more than 1 L, about 1 L, or less than 1 L, and how large the difference is. After Player 2 guesses, Player 1 tells her the right answer. Players switch roles. Then they trade containers (students’ records help them to remember the information) and seek a new partner with a container they have not seen yet.

After students have compared the capacities of all the containers in the activities, collect the containers students compared to 1 L and make three groups: more than 1 L, about 1 L, and less than 1 L. Have students signal which group each container should be sorted into.

Discuss with students whether 1 L is a large volume of liquid. Is 1 L enough to water the plants in your home? (probably, yes) In your garden? (no) Is 1 L enough to take a bath or a shower? (no) To wash the dishes? (no) To fill an aquarium? (no)

**Introduce millilitres.** Explain that for quantities of liquid smaller than 1 L, there is another unit of capacity: millilitres. Millilitres are very small (there are about 5 millilitres in 1 teaspoon).

### ACTIVITY 3 (Optional)

3. Give students empty medicine bottles that have their capacity marked on them. Have students find the capacity of their bottles where it is written on the bottle and round the capacity to the closest 10 mL. Students can pair up and have their partner guess the capacity of the bottle (rounded to the nearest 10 mL). Players can provide a hint (too high or too low) and then, when both partners have correctly guessed the capacity, change partners.
Most appropriate unit. Review the term “appropriate.” Take some of the containers used during both activities and ask students to decide which unit—litre or millilitre—is more appropriate for measuring their capacity. To see all answers simultaneously, write both units on the board and have students point to the answer.

Measuring capacity. Show students measuring cups of different sizes and draw their attention to the marks. Point out that 1 mL is a small unit, so when the liquid needs to be measured to the closest millilitre, you need to use a small measuring cup, or a graduated cylinder, that can hold a very small amount of liquid.

Demonstrate using a measuring cup to measure the capacity of a container by filling a container with water and then pouring the water into a measuring cup. Choose a container with a capacity that is not a round number, so that the water level is between adjacent marks on the measuring cup, and remind students that they need to look at the mark that is closest to the water level. Point out that the measurement produced that way is approximate.

ACTIVITY 4 (Essential)

4. Estimate and measure in millilitres. Give students a variety of containers, and have them estimate and then measure the capacity of the containers in millilitres. Students will need funnels and a large bowl or tub to work over in case of spillage.

Extensions

1. A recipe tells you to mix all the ingredients in a bowl. To make sure your bowl is large enough, you need to estimate the total volume of the ingredients and compare it with the capacity of your bowls. You have one bowl with a capacity of 1000 mL and another bowl with a capacity of 2500 mL. Which bowl will hold all the ingredients in the recipe?

   a) 280 mL pumpkin puree
   120 mL milk
   120 mL cream
   65 mL sugar
   5 mL spices
   65 mL flour
   50 mL butter
   1 beaten egg

   b) 500 mL ground beef
   250 mL uncooked rice
   240 mL water
   500 mL bell peppers
   450 mL tomato juice
   5 mL hot sauce
   20 mL onions

   Answers: a) either bowl, b) the 2500 mL bowl
2. You have two containers, one with capacity 500 mL and another with capacity 300 mL.
   a) How could you use only these containers to measure 200 mL of water?
   b) How could you use only these containers to measure 400 mL of water?

**Answers**

a) Fill the 500 mL container with water. Fill the 300 mL container with water from the 500 mL container. There is 200 mL of water left in the larger container.

b) Use the method from part a) to measure 200 mL of water. Pour the 200 mL of water into the 300 mL container, so there is only 100 mL of capacity left in the small container. Fill the 500 mL container. Pour out the water from the 500 mL container into the 300 mL container until the 300 mL container is full. There is 400 mL of water left in the 500 mL container.
Goals

Students will determine, through investigation, how many millilitres are in a litre.
Students will compare capacities in litres and millilitres.
Students will solve problems involving rates and capacity.

PRIOR KNOWLEDGE REQUIRED

Understands capacity as the amount a container holds
Is familiar with the size of a 1 L container
Is familiar with millilitres
Knows that litres and millilitres are measures of capacity and liquid volume

MATERIALS

collection of containers, at least one of which holds 1 L
1 L container per small group
100 mL plastic cup per small group
beans or water and tub for overflow (if using water)
scale, 1 L container, large transparent plastic bottle, funnel, marker, and a large pan (see Extension 1)

Mental math minute. Ask students to multiply or divide numbers up to 1000 by 10. Give problems with whole number answers. Progress to multiplication and division by 100 and then by 1000.

Review litres and millilitres. Show students a few containers and have them point out any containers that hold approximately 1 L. ASK: How much does a teaspoon hold in millilitres? (5)

Millilitres in a litre. SAY: Litres are pretty large compared with millilitres. ASK: How many millilitres do you think there are in a litre? (answers will vary) Do you think there are more than 100 millilitres in a litre? (yes) How do you think we can check? (answers will vary) Tell students that they are going to make a 100 mL measure to find the answer.

ACTIVITY (Essential)

Working in small groups, students determine how many millilitres are in 1 L by filling a 1 L container with water or dried beans using 100 mL plastic cups.
$1 \text{ L} = 1000 \text{ mL}$. Write on the board:

$$1 \text{ metre} = 1000 \text{ millimetres} \quad 1 \text{ m} = 1000 \text{ mm}$$

Invite volunteers to circle the common parts in the words “metre” and “millimetre.” Ask students to guess what “milli” means. Recall that “milli” is used to create small units. When they see “milli” in a measurement unit, they know there are 1000 of these smaller units (milli-units) in the larger unit. Write on the board:

$$1 \text{ litre} = \underline{\phantom{000}} \text{ millilitres} \quad 1 \text{ L} = \underline{\phantom{000}} \text{ mL}$$

ASK: What goes in the blanks? (1000)

**Whole litres in millilitre measurements.** Write on the board:

$$750 \text{ mL}$$

ASK: Is 750 mL greater than or less than 1 L? (less than) How do you know? (750 is less than 1000 mL) Can you give me an example of a number of millilitres that would be greater than 1 L? (any number greater than 1000)

Write “1750 mL” on the board. ASK: Is 1750 mL greater than or less than 1 L? (greater than) How do you know? (it is greater than 1000) Do you think it is greater than or less than 2 L? (less than) If there are 1000 mL in 1 L, how many millilitres are in 2 L? (2000) SAY: 1750 is greater than 1000 but less than 2000, so 1750 mL is between 1 L and 2 L.

As a class, fill in the table below. Keep the table on the board.

<table>
<thead>
<tr>
<th>L</th>
<th>mL</th>
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<tbody>
<tr>
<td>1</td>
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Have students look at the numbers in the second row of the table. ASK: What digit in each number of millilitres shows the number of litres? (thousands digit) Why does this make sense? (there are 1000 mL in each litre) Write “5679 mL” on the board. ASK: Which two numbers in the second row of the table is this number between? (5000 and 6000) How many whole litres are in 5679 mL? (5) How can you tell that without looking at the table? (the thousands digit in 5679 is 5) Invite a volunteer to circle the thousands digit. SAY: 5679 mL is 5 L and some extra litres.

**Exercises:** How many whole litres are in the measurement?

a) 4326 mL  
b) 2982 mL  
c) 8458 mL  
d) 99 990 mL  
**Bonus:** 995 mL

**Answers:** a) 4, b) 2, c) 8, d) 99, Bonus: 0
Comparing measurements in litres and millilitres. Remind students that when they compare two measurements in different units, they usually convert one of the measurements, so that the units become the same. Have students determine the number of whole litres in the millilitre measurement and then compare the measurements. Students can point to the larger measurement to signal the answers to Exercise 1 below.

Exercises

1. Which measurement is the greater capacity?
   a) 2458 mL or 3 L  
   b) 7002 mL or 7 L
   c) 469 mL or 4 L  
   d) 78 L or 7800 mL
   
   **Bonus:** 25 867 mL or 25 L
   
   **Answers:** a) 3 L, b) 7002 mL, c) 4 L, d) 78 L, Bonus: 25 867 mL

2. Put the measurements in order from smallest to largest.
   4 L    357 mL    2 L    1563 mL    3570 mL
   
   **Answers:** 357 mL, 1563 mL, 2 L, 3570 mL, 4 L

Extensions

1. Tell students that 1 L of water weighs exactly 1 kg. (Students could use a scale and a 1 L container to verify this fact for themselves.)
   
   SAY: 1 L = 1000 mL and 1 kg = 1000 g. ASK: What is the mass of 1 mL of water? (1 g)

   Students can use this fact to make their own measurement bottles. They will need a scale that measures weight in grams, a large transparent plastic bottle, a funnel, and a marker. Students should set the scale in a large pan in case of spillage. Have students weigh their bottles and make a table as shown in the margin (the sample answers in italics are for a bottle that weighs 35 g).

<table>
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<tr>
<th>Water (mL)</th>
<th>Weight of bottle and water (g)</th>
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<tbody>
<tr>
<td>0</td>
<td>35</td>
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<tr>
<td>50</td>
<td>85</td>
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   Pour water into the bottle and let it settle for a few seconds, until the weight reaches the next number in the table, 85 g. Mark the level the water reached—this is the 50 mL mark. Have students repeat the process for other numbers in the table and mark the number of millilitres poured into the bottle beside each mark, as on a ruler.

   Point out that this measurement bottle allows students to measure capacity to the nearest 50 mL. The water will occasionally reach a level between the marks, in which case, students would need to decide which mark the level is closest to. Students can measure capacity using their measurement bottle.
2. Use containers with a capacity of 1 L, 150 mL, and 100 mL to measure the given amount of water in more than one way if possible.
   a) 300 mL   b) 350 mL   c) 900 mL   d) 750 mL   e) 500 mL

Sample solutions
a) Fill the 150 mL container twice, pouring the contents into the 1 L container each time.
b) Fill the 100 mL container twice and the 150 mL container once, pouring the contents into the 1 L container each time.
c) Fill the 1 L container completely. Then remove 100 mL by filling the 100 mL container from the 1 L container.
d) Fill the 1 L container completely. Then remove 250 mL by filling the 100 mL and the 150 mL containers from the 1 L container.
e) Fill the 100 mL and the 150 mL containers twice, pouring the contents into the 1 L container each time.

3. Some of the non-liquid ingredients in a recipe are often given in volume because it is more convenient. Provide recipes from print or online sources. Ask students to choose a recipe and change any volumes of sugar, cocoa, flour, rice, raisins, or nuts to masses, using these equivalencies:

   1 tsp = 5 mL = 4 g sugar = 3 g flour or cocoa
   1 cup = 240 mL = 200 g sugar = 125 g flour or cocoa = 165 g raisins = 100 g nuts = 175 g rice
   1 egg = 60 g = 50 mL
Matching 3-D Shapes

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15.  
16.  

Blackline Master — Geometry — Teacher Resource for Grade 4  Q-35
Nets (1)
Nets (2)
Nets (4)
Nets (5)
Nets (8)
Nets (12)
Nets (13)
## Faces

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Q-50
Block Models

1. Use connecting cubes to build the shape. What is its volume?
   a)
   
   Volume = _____ × _____
   = _____ cubes
   b)
   
   Volume = _____ × _____
   = _____ cubes
   c)
   
   Volume = _____ cubes
   d)
   
   Volume = _____ cubes
   e)
   
   Volume = _____ cubes
   f)
   
   Volume = _____ cubes

2. Build a prism using 20 cubes.

3. Make as many different prisms as you can using 24 cubes. Sketch two of them.
Goals
Students will solve problems and puzzles using any of the problem-solving strategies studied so far in the Grade 4 problem-solving lessons.

PRIOR KNOWLEDGE REQUIRED
Can compare numbers using place value up to decimal tenths (for Problem Bank 1)
Can fluently add and subtract within 1000 (for Problem Banks 2–7, 19)
Can multiply two-digit numbers by one-digit numbers (for Problem Banks 3–5, 9, 20, 21)
Can round four-digit whole numbers to the nearest ten, hundred, or thousand (for Problem Bank 8)
Can divide two-digit numbers by one-digit numbers (for Problem Banks 20, 21)
Can find the area of rectilinear shapes on grid paper (for Problem Bank 22)
Can evaluate a simple fraction of a whole number (for Problem Bank 23)

MATERIALS
scissors (see Problem Bank 22)
grid paper or BLM 1 cm Grid Paper (p. Q-61, see Problem Bank 22)

NOTE: The following Problem Bank questions reflect a selection of the problem-solving strategies used in the problem-solving lessons for Grade 4. Students will need to choose among all the strategies they have learned this year to solve the problems.

Problem Bank
1. I am a decimal with two digits after the decimal point. What number am I?
   a) I am less than 0.1. My hundredths digit is 7.
   b) I am between 0.4 and 0.5. My digits add to 8.
   c) I am less than 1. My tenths digit and hundredths digit are equal. My digits add to 12.
   d) I am between 1 and 10. All my digits are equal. My digits add to 9.
   Answers: a) 0.07, b) 0.44, c) 0.66, d) 3.33
2. Add mentally: $1 + 11 + 111 + 1111$.

**Solution:** In the ones place, there are four 1s, in the tens place, there are three 1s, in the hundreds, there are two 1s, and in the thousands, there is one 1, so the sum is 1234.

3. Kathy has exactly 7 loonies and an unknown number of 5 dollar bills, 10 dollar bills, and 20 dollar bills. Which of these can be the total value of the money: $90, $91, $92, or $93?

**Answer:** $92

4. On your calculator, the key with the digit 4 isn't working. What could you press instead to find …

   a) $214 + 63$  
   b) $241 + 63$  
   c) $841 + 34$

   d) $34 \times 15$  
   e) $42 \times 8$

**Sample answers:** a) $210 + 67$, b) $200 + 100 + 3 + 1$, c) $800 + 70 + 5$, d) $33 \times 15 + 15$, e) $32 \times 8 + 10 \times 8$

5. Fill in the blank.

   a) $(83 \times 2) + (83 \times 4) = 83 \times ___$
   
   b) $(83 \times 41) + (2 \times 41) = ___ \times 41$
   
   c) $(72 \times 41) + (72 \times 3) + (3 \times 39) + (3 \times 5) = 75 \times ___$

**Answers:** a) 6, b) 85, c) 44

6. Add: $98 + 98 + 98 + 98 + 98$.

**Answer:** 490

7. Add: $(35 + 35 + 35 + 35 + 35) + (65 + 65 + 65 + 65 + 65)$.

**Answer:** 600

8. A number rounds to 500 when rounded to the nearest hundred and 450 when rounded to the nearest ten. The digits add to 12. What number is it?

**Answer:** 453

9. Fill in the blank: $3131 = 31 \times ___$.

**Answer:** 101

10. Fill in the blank: $2 \times 3 \times 4 \times 5 \times 6 = 3 \times 4 \times 5 \times 6 \times ___$.

**Answer:** 2

11. a) Jin and Vicky’s ages add to 32. What will they add to 3 years from now?

   b) Jin, Vicky, and Tessa’s ages add to 32. What will they add to 3 years from now?

**Answers:** a) 38, b) 41
12. Jasmin has 85 marbles and Don has 92 marbles. Can Don give some marbles to Jasmin so that they have the same number of marbles? Explain.

**Answers:** No. Don has 7 more marbles than Jasmin, and since 7 is not a multiple of 2, Don can’t give Jasmin half of the extra marbles.

13. Ethan has 5 apples. Hanna has 8 apples. Sally has 11 apples.
   a) How many apples do they have altogether?
   b) They decide to share the apples equally. How many apples should each person get?
   c) Who doesn’t need to give or receive any apples?
   d) How many apples do the other two people need to give or receive?

**Answers:** a) 24, b) 8, c) Hanna, d) Sally needs to give away 3 apples and Ethan needs to receive 3 apples

14. There are three sets of numbers.
   Set A: 2, 3, 5   Set B: 4, 5, 6   Set C: 1, 8, 11
   a) What is the sum of the numbers in each set?
   b) Jane traded exactly two numbers between sets. When she was done, all sets had the same sum.
      i) Which set did she leave alone? How do you know?
      ii) What two numbers did she trade?

**Answers**
   a) A: 10, B: 15, C: 20
   b) i) Jane must have left Set B alone because it has the number in the middle; if one sum increased and another decreased, it must be the sum in the middle that stayed the same; ii) 3 and 8

**NOTE:** Problem Banks 15 to 18 should be done in order.

15. Clara and Tom play a game. The rules are that Player 1 rolls three dice and then Player 2 rolls three dice. They both win when they get the same total. To help them get the same total, players are allowed to trade exactly one die for one die. For example: Player 1 rolls 2, 3, 5 and Player 2 rolls 1, 5, 6. Player 1’s total is 10 and Player 2’s total is 12. Player 1 can trade the 5 for Player 2’s 6 so that they each get the same total of 11.

Clara rolls 2, 3, 6. Tom rolls 4, 5, 6.
   a) What do Clara’s dice add to and what do Tom’s dice add to?
   b) How far apart are their totals?
   c) When they trade, who should give away a bigger number? Why?
d) Clara and Tom make the following trades. Do they win?
   i) They trade Clara’s 3 for Tom’s 4.
   ii) They trade Clara’s 3 for Tom’s 5.
   iii) They trade Clara’s 2 for Tom’s 4.
   iv) They trade Clara’s 2 for Tom’s 6.

e) When they won, how far apart were the numbers they traded?

   **Answers:**
   a) Clara’s dice add to 11 and Tom’s dice add to 15; b) 4;
   c) Tom, because his total is greater; d) i) Clara has 12 and Tom has 14,
   ii) Clara has 13 and Tom has 13, iii) Clara has 13 and Tom has 13,
   iv) Clara has 15 and Tom has 11; e) 2

16. Clara and Tom roll the given numbers. Help them win.
   a) Clara: 4, 6, 6       Tom: 1, 2, 5
   b) Clara: 3, 3, 5       Tom: 1, 2, 4
   c) Clara: 2, 3, 6       Tom: 4, 4, 5

   **Answers:**
   a) trade Clara’s 6 for Tom’s 2, b) trade Clara’s 3 for Tom’s 1,
   c) trade Clara’s 3 for Tom’s 4

17. Clara rolls 1, 4, 5. Tom rolls 2, 3, 6.
   a) What do Clara’s dice add to and what do Tom’s dice add to?
   b) How far apart are their totals?
   c) When they trade, who should give away a bigger number? Why?
   d) Clara and Tom make the following trades. Do they win?
      i) They trade Clara’s 1 for Tom’s 2.
      ii) They trade Clara’s 5 for Tom’s 6.
      iii) They trade Clara’s 1 for Tom’s 3.
   e) Can Clara and Tom win? Explain.

   **Answers:**
   a) Clara’s dice add to 10 and Tom’s dice add to 11; b) 1;
   c) Tom, because his dice add to more; d) i) no, ii) no, iii) no;
   e) no, because no matter what they trade, if Tom gives away a bigger
      number than Clara, Clara’s total will be bigger than Tom’s because
      they were only 1 apart to begin with

   a) Clara rolls 1, 2, 6. Tom rolls 1, 6, 6.
   b) Clara rolls 1, 1, 3. Tom rolls 1, 6, 6.
Answers: a) no, because their totals are 4 apart, so they would have to trade numbers that are 2 apart to win, but there are no such numbers; b) no, because their totals are 8 apart, so they would have to trade numbers that are 4 apart to win, but there are no such numbers.

19. You can find the reflection of a number by placing a mirror to the right of the number.

a) The dashed line is a mirror. Draw the reflection of each digital clock digit.

\[
\begin{array}{cccc}
\text{0} & \text{1} & \text{2} & \text{3} \\
\text{5} & \text{6} & \text{7} & \text{8} \\
\text{4} & \text{9} & \text{9} & \text{8} & \text{7} & \text{6} & \text{5} & \text{4} & \text{3} & \text{2} & \text{1} & \text{0}
\end{array}
\]

b) Which digits, when written like on a digital clock, have a reflection that is also a digit?

c) What is the reflection of the number?

i) 202  ii) 11  iii) 55  iv) 218

d) What can the two numbers be if a number and its reflection ...

i) add to 7  ii) add to 2  iii) multiply to 10  iv) add to 99  v) add to 909  vi) add to 9009  vii) add to 9999  viii) add to 50

**Bonus**

ix) add to 5000  x) are the same

Answers

a) 0, 1, 2, 5, and 8

b) 0, 1, 2, 5, and 8

c) i) 505, ii) 11, iii) 22, iv) 815

d) i) 2 and 5, ii) 1 and 1, iii) 2 and 5, iv) 18 and 81, v) 108 and 801, vi) 1008 and 8001, vii) 1818 and 8181 or 1188 and 8811, viii) 25 and 25, Bonus: ix) 2185 and 2815, x) 0, 1, 8, 25, 52, 205, 502, 215, 512, 285, 582, 2255, 5522, and many more
20. Pens and markers each cost a whole number of dollars. Three pens and two markers cost $34. Two pens and three markers cost $31.

a) If three pens and two markers cost more than two pens and three markers, what costs more, a pen or a marker? Explain.

b) How much does each pen and each marker cost? Use the information from part a) to make sure your answer makes sense.

Sample solution: b) Start the cost of markers at $1 each in the equation 3 pens and 2 markers cost $34, and continue raising the cost of each marker until the cost of a pen becomes less than the cost of a marker.

<table>
<thead>
<tr>
<th>Cost of 1 marker</th>
<th>Cost of 2 markers</th>
<th>Cost of 3 pens</th>
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<tr>
<td>$1</td>
<td>$2</td>
<td>$34 - $2 = $32</td>
<td>X</td>
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<tr>
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<td>$4</td>
<td>$34 - $4 = $30</td>
<td>$10</td>
</tr>
<tr>
<td>$3</td>
<td>$6</td>
<td>$34 - $6 = $28</td>
<td>X</td>
</tr>
<tr>
<td>$4</td>
<td>$8</td>
<td>$34 - $8 = $26</td>
<td>X</td>
</tr>
<tr>
<td>$5</td>
<td>$10</td>
<td>$34 - $10 = $24</td>
<td>$8</td>
</tr>
<tr>
<td>$6</td>
<td>$12</td>
<td>$34 - $12 = $22</td>
<td>X</td>
</tr>
<tr>
<td>$7</td>
<td>$14</td>
<td>$34 - $14 = $20</td>
<td>X</td>
</tr>
<tr>
<td>$8</td>
<td>$16</td>
<td>$34 - $16 = $18</td>
<td>$6</td>
</tr>
</tbody>
</table>

We can stop here because we are at the point where markers cost $8 each and pens cost $6 each, but we need a marker to cost less than a pen. So, there are only two possibilities to check with the second given sentence (2 pens and 3 markers cost $31). Looking at the first possibility, if 1 marker costs $2 and 1 pen costs $10, then 2 pens and 3 markers cost $26; in the second possibility, if 1 marker costs $5 and 1 pen costs $8, then 2 pens and 3 markers cost $31. So 1 marker costs $5 and 1 pen costs $8.

Answer: a) a pen costs more because, compared to buying two pens and three markers, an extra pen adds a greater cost than an extra marker.

21. Shirts and crayons each cost a whole number of dollars.

Anton pays $30 for 3 shirts and 2 crayons.
Lily pays $23 for 1 shirt and 5 crayons.
How much does each shirt and each crayon cost?

Answer: each crayon costs $3 and each shirt costs $8
22. A pentomino is made of 5 squares in the same way a domino is made of 2 squares. The picture below shows all 12 pentominoes. Using grid paper or BLM 1 cm Grid Paper, create and cut out the pentominoes.

a) What is the total area of all 12 pentominoes?
b) For your answer to part a), what pairs of numbers multiply to the number?
c) For which pairs of numbers from part b) can you make the 12 pentominoes into a rectangle with those dimensions? You will need to use the pentominoes you created and cut out.

Answers
a) 60 units$^2$
b) $1 \times 60, 2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$
c) $3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$
23. Solve this problem by working backwards:

\[
\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{4}{5} \text{ of } 30 \text{ is } \underline{\phantom{999}}.
\]

a) \(\frac{4}{5}\) of 30 is \(\underline{\phantom{999}}\). 

b) \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is \(\underline{\phantom{999}}\).

c) \(\frac{2}{3}\) of \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is \(\underline{\phantom{999}}\). 

d) \(\frac{1}{2}\) of \(\frac{2}{3}\) of \(\frac{3}{4}\) of \(\frac{4}{5}\) of 30 is \(\underline{\phantom{999}}\).

**Solutions:**
a) 24; b) 18, because \(\frac{3}{4}\) of 24 is 18; c) 12, because \(\frac{2}{3}\) of 18 is 12; d) 6, because \(\frac{1}{2}\) of 12 is 6
Unit 15  Probability and Data Management: Probability

Introduction
This unit focuses on experiments involving chance. The characteristics of experiments are considered in order to:

- name and quantify outcomes;
- predict the frequencies of events; and
- compare predictions with real data.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit:
- focus on halving and doubling

Assessment
The lessons covered by a quiz or test are as follows:

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<th>MB</th>
<th>ON</th>
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<tbody>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>PDM4-10, 11, 13</td>
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<tr>
<td>Test</td>
<td>n/a</td>
<td>PDM4-10, 11, 13</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Additional Information for This Unit

Fraction notation. We show fractions in two ways in our lesson plans:

Stacked: \(\frac{1}{2}\)  
Not stacked: 1/2

If you show your students the non-stacked form, remember to introduce it as new notation.
Goals

Students will identify the outcomes of events involving chance.

PRIOR KNOWLEDGE REQUIRED

Knows that a half can be represented in different ways using standard fractional notation
Knows that half of a circle can be represented using different pictures
Can recognize when a circle is divided into equal or unequal parts
Can draw circles divided into equal or unequal parts

MATERIALS

die
coin
letter-size piece of craft paper, paper clip, and a sharpened pencil
blue and red markers or chalk
paper cups, pencils, and rulers

Mental math minute. Give a student a large, even number to halve, such as 144. Successive students halve the previous answer; for example, the first student says 72, the next says 36, and so on. Occasionally ask students to explain how they got the answer. When students reach an odd number, start with a new large, even number.

Introduce results and outcomes. Hold up a die and ask students what will happen when you roll it. (it lands with one number facing up) Have students make predictions about what number you will roll and then test the predictions to demonstrate that predictions and actual results can differ. SAY: The result of rolling a die once is the number facing up when it lands. For each roll there is one result. The possible outcomes of rolling a die are all of the possible results. ASK: What are the possible outcomes? (1, 2, 3, 4, 5, 6) How many outcomes are there in total? (6) Repeat for flipping a coin. (heads, tails, 2 possible outcomes) SAY: Rolling a die, tossing a coin, spinning a spinner, or picking a marble out of a bag without looking are all examples of experiments. Each type of experiment has more than one possible outcome. Repeat for a soccer game. (one team wins, the other team wins, neither team wins, 3 possible outcomes)

Outcomes of spinning a spinner. On a letter-size piece of craft paper, draw a 10 cm diameter circle, marked as shown below:
Demonstrate how to use the tip of a sharpened pencil, a paper clip, and the circle as a spinner. Then label the regions 1 to 4 and list the outcomes on the board:

1. The pointer lands in region 1.
2. The pointer lands in region 2.
3. The pointer lands in region 3.
4. The pointer lands in region 4.

ASK: How many outcomes does this spinner have? (4) SAY: When the spinner lands in each region, it is a different outcome.

**Exercises:** You spin a spinner. How many outcomes does the spinner have?

a) ![Image](image1.png)

b) ![Image](image2.png)

c) ![Image](image3.png)

**Answers:** a) 2, b) 4, c) 6

**Outcomes of a spinner with several regions of the same colour.** Using the spinner you made on craft paper, colour regions 1, 2, and 3 blue and region 4 red. Then label regions 1, 2, and 3 with a “B” and region 4 with a “R.” SAY: When I spin the spinner now, the pointer will still land in one of the four regions. The number of regions on the spinner did not change. This means that the number of outcomes did not change either. The spinner still has four outcomes.

**Introduce events.** SAY: Imagine I am going to play a game with you using this spinner. If the spinner lands on blue, you win. If the spinner lands on red, I win. ASK: How many different ways can the spinner land on blue? (3) How many different ways can the spinner land on red? (1) SAY: This game can have two different results, spinning blue or spinning red. We call each result, spinning blue or spinning red, an event. ASK: How many outcomes make the event “spinning blue”? (3) Which outcomes are these? (pointer lands in any of the regions labelled “B”) How many outcomes make the event “spinning red”? (1) Which possible outcome? (pointer lands in the region labelled “R”)

**Exercises**

1. How many outcomes does the spinner have? How many outcomes make the event “spinning white”?

a) ![Image](image4.png)

b) ![Image](image5.png)

c) ![Image](image6.png)

d) ![Image](image7.png)

**Answers:** a) 4, 2; b) 8, 4; c) 6, 3; d) 4, 4
2. You pick a ball from the box. How many outcomes are there? How many outcomes make the event “picking red”?

a)  b)  c) 

Answers: a) 4, 1; b) 5, 2; c) 6, 4

3. You pick a marble out of a box without looking. How many outcomes are there in total? How many outcomes make the event “not picking a red marble”?

a)  b)  c) 

Answers: a) 5, 3; b) 8, 5; c) 4, 4

Unequal outcomes. SAY: You have to make a spinner with four possible outcomes. ASK: How would you do this? Invite volunteers to draw possible spinners on the board. Draw the spinner shown in the margin on the board.

Shade each region with a different colour and ASK: Are all the outcomes equally likely to occur? (no) Is there one that will likely occur more often than the others? (yes) Which one? (the big one) Draw another spinner on the board with a short pointer, as shown in the margin.

ASK: How many outcomes does this spinner have? (4) Will the pointer ever be in the grey regions? (no) SAY: Because the spinner will never land in the grey regions, we do not consider the grey regions to be outcomes.

Exercises: Draw four circles using a paper cup. Using a ruler and a pencil, divide one of the circles into regions according to the description.

a) Three equal outcomes b) Three unequal outcomes

c) Four equal outcomes d) Four unequal outcomes

Answers Sample answers

a) , c)  b) , d)
Extensions

1. Design a spinner that has six equal outcomes, two of which make the event “spinning white.”

   Sample answer
   
   ![Diagram of spinner with six equal outcomes, two of which are white.]

2. Design a spinner that has five outcomes, all unequal, two of which make the event “spinning black.”

   Sample answer
   
   ![Diagram of spinner with five unequal outcomes, two of which are black.]

3. Mandy and Jun are playing a game. They put 5 red marbles in a bag. To start playing, they each choose a marble out of the bag without looking. How could they set up the game so that not all outcomes are equal?

   Sample answer: Use different sizes of marbles.
Goals

Students will determine expected frequencies for simple events.

PRIOR KNOWLEDGE REQUIRED

Understands the meaning of the numerator and denominator of a fraction
Can represent pictures of fractions using standard fractional notation
Can calculate fractions of whole numbers within 1000

MATERIALS

BLM Rearranging Spinner Regions (p. R-18)
scissors
BLM Expected Outcomes (pp. R-19–21)
transparency of BLM Events with Unequal Chances (pp. R-22–23)
overhead projector

Mental math minute. Remind students that they can double twice to multiply by 4 and double three times to multiply by 8. For example, to multiply $4 \times 6$, multiply $2 \times 6 = 12$ and then multiply $2 \times 12 = 24$. Then you can double 24 to get $8 \times 6 = 48$. Also, remind students that order does not matter in multiplication, so they can find the answer to $9 \times 4$ by doubling 9 twice. Ask students multiplication questions in which one of the factors is 4 or 8; for example, $7 \times 4$.

NOTE: This lesson deals with expectations for simple events, which are events made up of one experiment, such as flipping a coin once or rolling a die once. Students do not need to be familiar with the term “simple events” or the distinction between simple and compound events.

Equal chances for events. ASK: When flipping a coin, what are the possible outcomes? (heads, tails) How many possible outcomes are there? (2) SAY: Flipping heads is one out of two possible outcomes, and flipping tails is one out of two possible outcomes. These outcomes have an equal chance of occurring, so we expect to flip heads and tails an equal number of times.

ASK: How many outcomes are there when you roll a die? (6) How many of the outcomes are even numbers? (3) How many are odd numbers? (3) SAY: Since they have an equal chance of occurring, we expect to roll an even number and an odd number an equal number of times.

Expectations for simple events with equal chances. Draw the chart shown on the following page on the board.
Flipping a Coin

<table>
<thead>
<tr>
<th>Total Number of Outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Heads Outcomes</td>
<td></td>
</tr>
<tr>
<td>Number of Tails Outcomes</td>
<td></td>
</tr>
<tr>
<td>Fraction of Outcomes</td>
<td></td>
</tr>
<tr>
<td>Expected to Be Heads</td>
<td></td>
</tr>
<tr>
<td>Fraction of Outcomes</td>
<td></td>
</tr>
<tr>
<td>Expected to Be Tails</td>
<td></td>
</tr>
</tbody>
</table>

SAY: Since there is an equal chance of flipping heads or tails on a coin toss, we expect half of all flips to be heads and half to be tails. Have students help fill in the chart. (2, 1, 1/2, 1/2)

ASK: If we flip a coin 10 times, how many heads do we expect? (5) How many tails do we expect? (5) What about if we flip 100 times? (50 heads, 50 tails) Draw the following chart on the board:

Spinning a Spinner

<table>
<thead>
<tr>
<th>Spinner</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Outcomes</td>
<td></td>
</tr>
<tr>
<td>Number of Grey Outcomes</td>
<td></td>
</tr>
<tr>
<td>Number of White Outcomes</td>
<td></td>
</tr>
<tr>
<td>Fraction of Outcomes</td>
<td></td>
</tr>
<tr>
<td>Expected to Be Grey</td>
<td></td>
</tr>
<tr>
<td>Fraction of Outcomes</td>
<td></td>
</tr>
<tr>
<td>Expected to Be White</td>
<td></td>
</tr>
</tbody>
</table>

Have students help fill in the second column of the chart as they did for the Flipping a Coin chart. Then ASK: If we spin 40 times, how many times do we expect to spin grey? (20) How many times do we expect to spin white? (20) What about if we spin 500 times? (250 grey, 250 white)

At the top of the third column of the chart on the board, draw the same spinner again. Then divide it in half, as shown in the margin. Have students help fill in the third column of the chart, as they did for the first spinner. Then ASK: If we spin 40 times, how many times do we expect to spin grey? (20) How many times do we expect to spin white? (20) What about if we spin 500 times? (250 grey, 250 white) Did dividing the sections affect the expected outcomes? (no) SAY: As long as the spinner is still half grey and half white, the actual number of outcomes doesn’t affect what is expected to happen.

At the top of the fourth column of the chart on the board, draw the spinner shown in the margin. Have students help fill in the fourth column of the chart, as they did for the first and second spinners. ASK: Did rearranging
the sections affect the expected outcomes? (no) SAY: As long as all outcomes are equally likely and half of the regions are grey and half are white, the actual arrangement of the regions doesn’t affect what is expected to happen.

**ACTIVITY (Essential)**

Give each student one section from BLM Rearranging Spinner Regions. Have students cut out the spinners. ASK: Is there an equal chance of grey or white occurring? (yes) Have students cut along the dotted line on the diameter to make two half-circles, and then fold and cut each half-circle in half to make four equal sections in total. Students then reassemble the spinner in a different way. ASK: How many outcomes are there now? (4) Do they have an equal chance of occurring? (yes) Students can continue to cut sections in half and rearrange them to convince themselves that having more equal sections will not change the outcomes.

**Exercises:** Complete BLM Expected Outcomes (1).

**Answers:** 1. a) 2, b) 1, c) 1, d) 1/2, e) 1/2, f) 25, g) 25; 2. a) 8, b) 4, c) 4, d) 1/2, e) 1/2, f) 15, g) 15

Draw a spinner divided into three equal parts on the board, labelled “1,” “2,” and “3.” Next to the spinner, draw the table below and have students help fill it in. (3, 1, 1, 1/3, 1/3, 1/3)

<table>
<thead>
<tr>
<th>A Spinner with Three Equal Parts</th>
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<tbody>
<tr>
<td>Total Number of Outcomes</td>
</tr>
<tr>
<td>Number of “1” Outcomes</td>
</tr>
<tr>
<td>Number of “2” Outcomes</td>
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<tr>
<td>Number of “3” Outcomes</td>
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<tr>
<td>Fraction of “1” Outcomes</td>
</tr>
<tr>
<td>Fraction of “2” Outcomes</td>
</tr>
<tr>
<td>Fraction of “3” Outcomes</td>
</tr>
</tbody>
</table>

ASK: If we spin three times, how many times do we expect to spin each of the numbers 1, 2, and 3? (1, 1, 1) What about if we spin six times? (2, 2, 2) What about if we spin 30 times? (10, 10, 10) 90 times? (30, 30, 30)

**Expectations for simple events with unequal chances.** Project a transparency of BLM Events with Unequal Chances (1) on the board. Have students help you fill in the first three rows of the table. SAY: Since the chance of getting the event “spinning grey” is one in four, we expect 1/4 of all spins to be grey. Since the chance of getting the event “spinning white” is three in four, we expect 3/4 of all spins to be white. Have students help fill in the remainder of the chart. (4, 1, 3, 1/4, 3/4, 1, 3, 2, 6) ASK: If we spin four times, how many spins do we expect to be grey? (1) How many
do we expect to be white? (3) If we spin eight times, how many spins do we expect to be grey? (2) How many do we expect to be white? (6) If we spin 40 times, how many spins do we expect to be grey? (10) How many do we expect to be white? (30)

Project a transparency of **BLM Events with Unequal Chances (2)** on the board. Read aloud each row heading on the BLM and fill in the chart as students answer.

**Exercises:** Complete **BLM Expected Outcomes (2) and (3)**.

**Answers**
BLM Expected Outcomes (2): a) 4, b) 3, c) 1, d) 3/4, e) 1/4, f) 3, g) 1, h) 30, i) 10
BLM Expected Outcomes (3): a) 6; b) 2; c) 1, 2; d) 3, 4, 5, 6; e) 2/6; f) 4/6; g) 2; h) 4; i) 4; j) 8; k) 6; l) 12; m) 8; n) 16

**Extensions**

1. A spinner has 15 regions and 7 of them are blue. If you spin the spinner 60 times, do you expect to spin blue more than 30 times or less than 30 times? Explain.

   **Answer:** We expect to spin blue fewer than 30 times. Since 7 is less than half of 15, we expect the number of spins resulting in blue to be less than half of 60 spins.

2. The spinner in the margin has three outcomes: A, B, and C. A soccer game also has three outcomes: one team wins, the other team wins, or no team wins.

   Given that both have three different outcomes, how are the spinner and soccer game different?

   **Answer:** You can’t alter the chance of winning with the spinner, but you can with soccer. The likelihood of the three outcomes are equal for the spinner but not equal in soccer.
Goals
Students will determine expected frequencies for compound events.

Prior Knowledge Required
Understands the meaning of the numerator and denominator of a fraction
Can represent pictures of fractions using standard fractional notation
Can calculate fractions of whole numbers

Materials
overhead projector
transparency of BLM Expected Outcomes for Rolling a Die Twice (p. R-24)

Mental math minute. Remind students that they can multiply by 4 by doubling twice and multiply by 8 by doubling three times. Also, remind them that halving is the opposite of doubling, so they can divide by 4 by halving twice and divide by 8 by halving 3 times. Have groups of three students divide numbers using repeated halving. You give a number, for example 224. The first student halves the number and says 224 ÷ 2 = 112; the second students halves the answer and says 224 ÷ 4 = 56; and the third student halves again, saying 224 ÷ 8 = 28. Repeat with a new problem.

Note: This lesson deals with expectations for compound events.
Compound events are those made up of more than one experiment. This includes doing the same experiment more than once, such as flipping a coin or rolling a die multiple times, and also includes doing combinations of different experiments, such as rolling a die and flipping a coin. Students do not need to be familiar with the term “compound events,” or the distinction between simple and compound events.

Expectations for events made up of one experiment done repeatedly.
SAY: I want to find out all the possible outcomes of flipping a coin twice.
ASK: What are the outcomes of the first experiment, flipping a coin? (heads, tails) Draw on the board:

First Flip
H
T

ASK: What are the outcomes of the second experiment, flipping a coin again? (heads, tails) Extend the diagram on the board as shown on the following page.
First Flip  Second Flip
H        H
T        T

Add a third column heading called “Final Outcomes.” For each path, have volunteers draw a final arrow and a final outcome as shown below.

First Flip  Second Flip  Final Outcomes
H        H        HH
T        T        HT
H        H        TH
T        T        TT

ASK: How many final outcomes are there? (4) How many final outcomes are two heads? (1) How many are a head and a tail? (2) Why are there more final outcomes that are a head and a tail? (you can get a head first, then a tail, or you can get a tail first, then a head) How many outcomes are there in total? (4) How many outcomes are flipping two heads? (1) What fraction of outcomes are flipping heads twice? (1/4) Write “$\frac{1}{4}$” beside “HH.” Repeat for tails. ASK: How many outcomes are a head and a tail? (2) What fraction of outcomes are flipping a head and a tail? (1/2) Circle the two outcomes that are a head and a tail and write “$\frac{1}{2}$” beside it. ASK: Since we can expect a quarter of flips to be two heads, how many heads-heads should we expect if we flip 40 times? (10) How many tails-tails? (10) How many with one head and one tail? (20) Repeat for 100 flips. (25 HH, 25 TT, 50 HT)

Expectations for compound events made up of two different experiments.

SAY: Edmond is going on a field trip. He has to bring one pair of shorts and one T-shirt. He has two pairs of shorts and two T-shirts in mind, but isn’t sure which to choose. He wants to consider all possible combinations. One T-shirt is plain (P) and the other is striped (S). One pair of shorts is blue (B) and the other is green (G). He is going to use a tree diagram to figure out all the possible outcomes of T-shirt and shorts combinations. He first considers his T-shirt options. Draw on the board:

**T-shirts**

P
S

SAY: He could choose either the plain T-shirt or the striped T-shirt. The arrows indicate the possible outcomes. Then he considers his shorts options. Extend the diagram on the board as shown below:

**T-shirts**  **Shorts**
P  B
S  G

P  G
S  B

G
Point to the path for plain, blue and SAY: He could combine the plain T-shirt with the blue shorts. Point to the path for plain, green and SAY: Or he could combine the plain T-shirt with the green shorts. Point to the path for striped, blue and SAY: Or he could combine the striped T-shirt with the blue shorts. Point to the path for striped, green and SAY: Or he could combine the striped T-shirt with the green shorts. Extend the diagram on the board, as shown below:

<table>
<thead>
<tr>
<th>T-shirts</th>
<th>Shorts</th>
<th>Final Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>B</td>
<td>PB</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>PG</td>
</tr>
<tr>
<td>S</td>
<td>B</td>
<td>SB</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>SG</td>
</tr>
</tbody>
</table>

SAY: By using a tree diagram, Edmond has determined that there are four possible outcomes: plain T-shirt and blue shorts, plain T-shirt and green shorts, striped T-shirt and blue shorts, or striped T-shirt and green shorts. ASK: What fraction of the outcomes are plain T-shirt and blue shorts? (1/4) Repeat for the other three outcomes. (1/4 PG, 1/4 SB, 1/4 SG)

Exercise: A restaurant offers two kinds of pizza: pepperoni (P) and vegetarian (V). It also offers two kinds of ice-cream: chocolate (C) and strawberry (S). Find all the possible outcomes using a tree diagram. How many outcomes are there in total?

Answer: There are four outcomes in total.

<table>
<thead>
<tr>
<th>T-shirts</th>
<th>Shorts</th>
<th>Final Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>C</td>
<td>PC</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>PS</td>
</tr>
<tr>
<td>V</td>
<td>C</td>
<td>VC</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>VS</td>
</tr>
</tbody>
</table>

Expected outcomes for rolling a die twice and adding. Project a transparency of BLM Expected Outcomes for Rolling a Die Twice on the board. SAY: We are going to find all the outcomes of rolling a die twice and adding the results. Do not draw any more arrows and ASK: What are all possible outcomes of the first roll? (1, 2, 3, 4, 5, 6) Explain that the first column shows the outcomes of the first roll. ASK: What does the second column show? (the outcomes of the second roll) What does the third column show? (adding the outcomes of the two rolls) What does the fourth column show? (answers to the additions) Explain the first path of arrows. SAY: When the outcome of the first roll is 1 and the outcome of the second roll is 1 and we add those results, the answer is 2. Have volunteers explain the second and third paths, and then have volunteers complete the second column. Select several outcomes at random, such as “4” in the first column and “2” in the second column, and have students explain what they mean. (the first roll is 4 and the second roll is 2) To fill in the third column, ask students for the answer to each addition one a time from top to bottom. To fill in the fourth column, ask students for each sum one at a time from top to bottom.
Explain that you are going to make a chart to show all the final outcomes and how many of each final outcome there is. Starting from the top of the fourth column, have students identify the final outcomes. (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12) Record them in a chart on the board, as shown below:

<table>
<thead>
<tr>
<th>Final Outcome</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Fraction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/36</td>
<td>2/36</td>
<td>3/36</td>
</tr>
</tbody>
</table>

Again starting from the top of the fourth column, count how many final outcomes of “2” there are, then of “3,” then “4,” and so on, to “12.” Write the counts in the chart, as shown below:

<table>
<thead>
<tr>
<th>Final Outcome</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Fraction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/36</td>
<td>2/36</td>
<td>3/36</td>
</tr>
</tbody>
</table>

ASK: Which outcomes are expected to occur least often? (2, 12) SAY: 2 and 12 occur least often because there is only one way to get each sum. We can only get a sum of 2 if both die rolls are 1, and we can only get a sum of 12 if both rolls are 6. ASK: What sum do we expect to get most often? (7) Why does 7 occur most often? (there are many ways to get a sum of 7) Ask for all the ways to get a sum of 7 and write them down as students say them. Be sure students understand that getting 7 by rolling 1 first and then 6 is different from getting 7 by rolling 6 first and then 1.

SAY: Just as we determined the fractions of expected outcomes previously, we can do the same for rolling a die twice and adding the results. First, we need to find out the total number of outcomes. There is one way to get a final outcome of 2, two ways to get 3, three ways to get 4, and so on. If we add up all the ways of getting all the final outcomes, we have the total number of final outcomes. Have students add the numbers in the second row. (1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36) SAY: Since there is one way to get a final outcome of 2, one out of the 36 final outcomes is getting 2. Write “1/36” in the table. ASK: Since there are two ways to get 3 and there are 36 final outcomes, what fraction of final outcomes are getting 3? (2/36) Continue to complete the table on the board. The final result should look like the table below:

<table>
<thead>
<tr>
<th>Final Outcome</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Extensions

1. When you roll a die twice and add the results, the sum will be either an even number or an odd number. Explain why there is always an odd number of ways to get an even sum and an even number of ways to get an odd sum.

   **Answer:** Every sum that results in an odd number can be written two ways. For example, you can get the sum 5 by adding 2 + 3, but you can also reverse the order of the numbers being added to 3 + 2. 2 + 3 and 3 + 2 are different ways of getting the sum of 5. However, one of the sums that results in any even number can only be written one way. For example, you can get the sum 6 by adding 3 + 3, but reversing these numbers gives you that same way again.

2. Read all the sentences below from top to bottom. For each, consider how likely the outcome is. Explain the trend.

   If you roll a die, you will get a number greater than 0.
   If you roll a die, you will get a number greater than 1.
   If you roll a die, you will get a number greater than 2.
   If you roll a die, you will get a number greater than 3.
   If you roll a die, you will get a number greater than 4.
   If you roll a die, you will get a number greater than 5.
   If you roll a die, you will get a number greater than 6.

   **Answer:** A number greater than 0 is certain to occur because any number you roll makes that event. A number greater than 1 is very likely to occur but not certain, since you might roll 1. A number greater than 2 is a little less likely to occur because now rolling 1 or 2 doesn’t make the event. The event becomes less likely from top to bottom until, for a number greater than 6, the event is impossible because a die has numbers from 1 to 6 only.

3. Roll a die and flip a coin.

   a) List all possible outcomes.
   b) How many of each outcome are there?
   c) What fraction of the time will you get each outcome?
   d) If you roll 60 times, how many times would you expect to get each outcome?

   **Answers:**
   a) 1 H, 2 H, 3 H, 4 H, 5 H, 6 H, 1 T, 2 T, 3 T, 4 T, 5 T, 6 T;
   b) 1, c) 1/12, d) 5
Goals
Students will compare predictions for experiments with actual results. Students will see that the more data there is for an experiment the more it looks like the expected outcomes.

PRIOR KNOWLEDGE REQUIRED
Can determine what the outcomes are for simple experiments
Can determine the number of outcomes for simple experiments
Can plot data on a bar graph

MATERIALS
two prepared spinners per pair of students from BLM Experiment Spinners (p. R-25, see details below)
overhead projector
transparency of BLM Experiment Spinners (p. R-25)
BLM Tally Charts for Spinner Experiments (p. R-27)
BLM Bar Graph Templates for Spinner Experiments (p. R-26)
transparency of BLM Bar Graph Templates for Spinner Experiments (p. R-26, optional)
coin
dice (see Extension 2)

Mental math minute. Ask students to solve multiplication questions within the range of $1 \times 1$ to $10 \times 10$ and corresponding division questions. For each number, go through the questions in order, such as $1 \times 3, 3 \div 3, 2 \times 3, 6 \div 3$, and so on, to $10 \times 3$ and $30 \div 3$. Then progress to a different number. Next, try questions out of order, but keep multiplication and the corresponding division together.

Expected outcomes versus results. In advance, prepare two different spinners for each pair of students, one with the letters A/B/C/D and the other without letters using BLM Experiment Spinners.

Testing predictions for the A/B/C/D spinner. SAY: Now that we know how to determine the outcomes of an experiment and how many of each outcome to expect, we can do some experiments and compare the results with the expected outcomes. Project the A/B/C/D spinner from BLM Experiments Spinners onto the board. Then, draw the table on the following page on the board and have students help to fill it in. (4, 1, 1, 1, 1)
Total Number of Outcomes

<table>
<thead>
<tr>
<th>The Number of A Outcomes</th>
<th>The Number of B Outcomes</th>
<th>The Number of C Outcomes</th>
<th>The Number of D Outcomes</th>
</tr>
</thead>
</table>

Give every pair of students the A/B/C/D spinner from BLM Experiment Spinners and the corresponding tally chart from BLM Tally Charts for Spinner Experiments. SAY: You are going to spin the spinner 20 times, but before you begin, you need to make predictions about what you expect will happen. Then you will test your predictions by spinning the spinner. For each spin, you will record on the tally chart whether the spinner stopped in region A, B, C, or D. Ask students where to put tally marks for a couple of examples to make sure they know how to mark their results. When all spins have been completed, have students fill in the count column of the chart.

Give every pair of students two copies of the bar graph template for the A/B/C/D spinner experiment from BLM Bar Graph Templates for Spinner Experiments. Have each student plot the data from their tally charts. After all individual data is plotted, ASK: Did everybody get five of each of the letters A, B, C and D? (no) Did anyone get five of each outcome?

Draw or project the bar graph template for the A/B/C/D spinner experiment on the board. You will need to renumber the scale on the vertical axis to accommodate all the data. For example, if 15 pairs did the experiment, the vertical scale will go up to 20 times 15 (20 spins per pair times 15 pairs, i.e., 300 spins). Combine the results for the whole class by adding all the bars from A together, all the bars from B together, all the bars from C together, and all the bars from D together. Plot the data on the board. Compare the class results with those of the pairs. Discuss the difference between the class results and the pairs’ results. ASK: Which results are closer to the expected outcomes? (class result) Why is that? (the more times you spin, the more the data will look like what you expect) SAY: The fraction of each letter spun by the class will be closer to a quarter of the total number of spins than will be the fraction for the pairs’ results.

Testing predictions for the grey/white spinner. Repeat the above experiment using the grey/white spinner, the grey/white tally chart, and the grey/white bar graph template. SAY: Now we’re going to do an experiment to see how many times the spinner lands on the grey part and how many times it lands on the white part. ASK: If we spin four times, how many times do we expect to land on the grey part? (1) How many times do we expect to land on a white part? (3) ASK: If we spin 20 times, how many times do you predict it will land on the grey part? (5) How many times do you predict it will land on a white part? (15)

Again, combine the pairs’ results to make a whole class bar graph on the board. Ask which results are closer to the expected outcome—the pairs’
or the class’—and discuss what would happen if they collected a lot more data. (the experimental results would look more like the predictions)

**ACTIVITY (Essential)**

Have students predict the results of tossing a coin 20 times. Then have each student toss a coin 20 times and record the results in a tally chart. Students should then compare their results to their prediction.

**NOTE:** Extension 1 is required in order to cover the British Columbia curriculum. Extension 2 is required in order to cover the Ontario curriculum.

**Extensions**

1. Explore the role of chance in Dene/Kaska hand games or Lahal stick games.

2. Have students do 36 repetitions of rolling a die twice and adding the results. Students record the results in a tally chart like the one shown below and compare the results with the expected outcomes they determined in Lesson PDM4-12: Expectations for Compound Events.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tally</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

3. Simulate experiments on a computer (e.g., search for coin toss, rolling die, or spinning spinner simulators online).
Rearranging Spinner Regions
Expected Outcomes (1)

1. Kim has a coin that she plans to flip.
   a) How many outcomes are there in total? _______
   b) How many outcomes are heads? _______
   c) How many outcomes are tails? _______
   d) What fraction of outcomes are expected to be heads? _______
   e) What fraction of outcomes are expected to be tails? _______
   f) Out of 50 flips, how many are expected to be heads? _______
   g) Out of 50 flips, how many are expected to be tails? _______

2. A spinner is divided into eight equal parts: four black and four white.
   a) How many outcomes are there in total? _______
   b) How many outcomes are black? _______
   c) How many outcomes are white? _______
   d) What fraction of outcomes are expected to be black? _______
   e) What fraction of outcomes are expected to be white? _______
   f) Out of 30 spins, how many are expected to be black? _______
   g) Out of 30 spins, how many are expected to be white? _______
Expected Outcomes (2)

3. A spinner is divided into four equal parts. Three of the parts are blue and one of the parts is orange.
   a) How many outcomes are there in total? _______
   b) How many outcomes are blue? _______
   c) How many outcomes are orange? _______
   d) What fraction of outcomes are expected to be blue? _______
   e) What fraction of outcomes are expected to be orange? _______
   f) Out of 4 spins, how many outcomes are expected to be blue? _______
   g) Out of 4 spins, how many outcomes are expected to be orange? _______
   h) Out of 40 spins, how many outcomes are expected to be blue? _______
   i) Out of 40 spins, how many outcomes are expected to be orange? _______
Expected Outcomes (3)

4. A die has six faces. Two of the faces are numbers less than 3 and four of the faces are numbers greater than 2.

   a) How many outcomes are there in total? ______
   
   b) How many outcomes are there for the event “less than 3”? ______
   
   c) What outcomes are there for the event “less than 3”? ________________
   
   d) What outcomes are there for the event “greater than 2”? ________________
   
   e) What is the fraction of outcomes for the event “less than 3”? __________
   
   f) What is the fraction of outcomes for the event “greater than 2”? __________
   
   g) Out of 6 rolls, how many outcomes are expected for the event “less than 3”? ______
   
   h) Out of 6 rolls, how many outcomes are expected for the event “greater than 2”? ______
   
   i) Out of 12 rolls, how many outcomes are expected for the event “less than 3”? ______
   
   j) Out of 12 rolls, how many outcomes are expected for the event “greater than 2”? ______
   
   k) Out of 18 rolls, how many outcomes are expected for the event “less than 3”? ______
   
   l) Out of 18 rolls, how many outcomes are expected for the event “greater than 2”? ______
   
   m) Out of 24 rolls, how many outcomes are expected for the event “less than 3”? ______
   
   n) Out of 24 rolls, how many outcomes are expected for the event “greater than 2”? ______
Events with Unequal Chances (1)

<table>
<thead>
<tr>
<th>The Total Number of Outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The Number of Grey Outcomes</td>
<td></td>
</tr>
<tr>
<td>The Number of White Outcomes</td>
<td></td>
</tr>
<tr>
<td>The Fraction of Outcomes Expected to Be Grey</td>
<td></td>
</tr>
<tr>
<td>The Fraction of Outcomes Expected to Be White</td>
<td></td>
</tr>
<tr>
<td>Out of 4 Spins, the Number Expected to Be Grey</td>
<td></td>
</tr>
<tr>
<td>Out of 4 Spins, the Number Expected to Be White</td>
<td></td>
</tr>
<tr>
<td>Out of 8 Spins, the Number Expected to Be Grey</td>
<td></td>
</tr>
<tr>
<td>Out of 8 Spins, the Number Expected to Be White</td>
<td></td>
</tr>
<tr>
<td>The Total Number of Outcomes</td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---</td>
</tr>
<tr>
<td>The Number of Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>The Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>The Number of Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>The Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>The Fraction of Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>The Fraction of Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 6 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Out of 6 Rolls, the Number of Expected Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 12 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Out of 12 Rolls, the Number of Expected Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 18 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Out of 18 Rolls, the Number of Expected Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 24 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Out of 24 Rolls, the Number of Expected Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
</tbody>
</table>
### Expected Outcomes for Rolling a Die Twice

<table>
<thead>
<tr>
<th>Outcomes of First Roll</th>
<th>Outcomes of Second Roll</th>
<th>Additions</th>
<th>Final Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1 + 1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1 + 2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1 + 3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
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</table>
Experiment Spinners

A  B
D  C

A  B
D  C

D  C

A  B
D  C
Bar Graph Templates for Spinner Experiments

Number of Spins vs. Outcomes

Outcomes: A, B, C, D

Number of Spins: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Outcomes: grey, white
## Tally Charts for Spinner Experiments

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Prediction</th>
<th>Tally</th>
<th>Count</th>
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<tbody>
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Pattern Blocks
1. a) C
   b) A
   c) C
   d) B
   e) B
2. a) secondary
   b) primary
   c) secondary
   d) secondary
   e) primary
   f) secondary
3. a) C
   b) A
   c) C
   d) C
   e) A
   f) A
4. a) Add “other” category.
   b) All possible responses given.
   c) Add “other” category.
   d) All possible responses given.
5. a) yes
   b) no
   c) yes
   d) no
6. a) other
   b) fall
   c) other
   d) over 1.5 m
7. Answers will vary. Teacher to check.
8. Answers will vary. Teacher to check.

BONUS
Sample answer: helicopter

AP Book PDM4-3
1. a) 40
   b) 25
   c) 10
   d) 8 circles
   e) 5 circles
   f) 2 circles
   g) 4 circles
   h) 2 circles
   i) 1 circle
   j) 4
   k) Answers will vary. Sample answer: Flowers in my Garden

AP Book PDM4-4
1. a) Africa
   b) North America
   c) 2
   d) Europe and Oceania
   e) 9
2. a) 3
   b) 9
   c) 0
   d) 36
   e) No, half of a person is not possible.
3. a) 4
   b) 8
   c) 3
   d) 2
   e) 2
   f) 20
   g) 5
   h) 2
   v) Boating
   vi) Windsurfing
   vii) 29

BONUS
No. Each block represents 2 students, so 2 blocks represent 4 students.
Answers will vary. Sample answer: Flowers in my Garden

AP Book PDM4-5
1. a) 8
   b) 28
   c) 20
   d) 22
   e) 6
   f) 4
   g) yes
   h) 7
   i) Teacher to check.
   j) no
   k) 2
   l) 3
   m) 2
   n) 126 kg
   o) 30
   p) 35
   q) 15
   r) 45
   s) 125
   t) Answers will vary. Teacher to check.
   u) Sample answer: Flowers in my Garden
   v) Teacher to check.
   w) Vegetarian,
   x) Pepperoni,
   y) Plain Cheese,
   z) Hawaiian
BONUS
a) 25
b) 6, 7, 3, 9
c) 4, 5, 2, 6

AP Book PDM4-6
page 11
1. a) May and November
b) Ottawa
c) 65 cm
d) February and March
e) Winnipeg. It has snow for one month longer than Ottawa.
2. a) Yes. Sample explanation: The two sets of data are related to each other so it would make sense to compare them.
b) No. Sample explanation: The favourite movies are likely to be very different, so a double bar graph wouldn’t work.
3. a) Teacher to check.
b) Teacher to check.
c) They collected the same amount.
d) Ms. Ali’s class
e) Monday
f) Friday

AP Book PDM4-7
page 13
1. Underline the following:
   b) 1
c) 2
d) 4
e) 8
f) 0
g) 1
h) 4
i) 5
j) 1
2. Circle the following:
   b) 3
   c) 12
d) 3
e) 5987
f) 1
g) 0
h) 1
i) 432
j) 900
3. a) underline 8 circle 0
   b) underline 3 circle 8
c) underline 1 circle 83
d) underline 0 circle 831
e) underline 1 circle 407
f) underline 9 circle 68
g) underline 7 circle 90
h) underline 9 circle 89
i) underline 3 circle 0
j) underline 9 circle 6245
4. Answers will vary. Sample answers: 90, 09
5. a) 78, 74
   b) 90, 91
c) 77, 76
d) 371, 379
e) 263, 265
f) 390, 394
g) 578, 574
h) 341, 340
i) 28, 29
6. a) 0, 1, 2, 6
   b) 0, 2, 4, 5
c) 9, 10, 12, 13
BONUS
a) Yes. The stem is all the digits except the right-most digit.

AP Book PDM4-8
page 15
1. a) 12, 4, 8
   b) 11, 4, 7
   c) 42, 36, 6
2. Circle the following:
   a) 12
   b) 8
   c) 2
   d) 144
3. a) 7
   b) 14
   c) 42
   d) 15
   e) 40
   f) 68
4. a) circle 6
   b) circle two 3s
   c) circle 13
   d) circle 6 and 10
   e) circle 7 and 13
   f) circle 38 and 42
   g) circle 248
5. a) 47
   b) 24
   c) 106
   d) 112
   e) 112, 91
6. a) 34
   b) 16
   c) 91
   d) 103
7. a) 34
   b) 13
   c) 92
   d) 102, 105
BONUS
a) Answers will vary.
Sample answer:
0, 0, 1, 1, 2, 3, 9, 9, 9
b) Answers will vary.
Sample answer:
0, 0, 1, 2, 3, 4, 4

AP Book PDM4-9
page 17
1. a) 4, 3, 3, 3, 6, 9, 7,
    5, 3, 3, 4
b) 10 – 3 = 7
c) 3
d) 4
e) No, there are more months of 3 days of rain or snow than months with 10 days of rain or snow.
f) summer
BONUS
The mode is 3 and the median is 4.

2. a) i) | Stem | Leaf |
    0   | 5 7 8 |
    1   | 0 1 3 5 |
    7 8
    2   | 0 1 |
ii) | Stem | Leaf |
    0   | 9 9 |
    1   | 0 0 1 2 |
    2 3 4 4
    5 5
b) 21, 5, 16
c) 15, 9, 6
d) 12
e) 12

3. a) Answers will vary.
Sample answer:
Iqaluit and Windsor experienced peak hours of daylight in June and July. Iqaluit had a much greater range in daylight hours than Windsor.
b) Answers will vary.
Teacher to check.
1. a) \( \frac{1}{5} \)  
b) \( \frac{1}{4} \)  
c) \( \frac{1}{6} \)  
d) \( \frac{1}{9} \)  
e) \( \frac{1}{8} \)  
f) \( \frac{1}{10} \)  

2. Teacher to check that one part is shaded.

3. a) one sixth  
b) one fifth  
c) one ninth

4. a) \( \frac{1}{9} \)  
b) \( \frac{4}{9} \)  
c) \( \frac{9}{9} \)  
d) \( \frac{1}{10} \)  
e) \( \frac{5}{10} \)  
f) \( \frac{1}{10} \)  

5. Teacher to check that the following number of parts is shaded:  
a) one  
b) three  
c) six  
d) one  
e) five  
f) seven

6. a) B  
b) D  
c) A  
d) C  

7. Teacher to check shading. Checkmark should be under the following shapes:  
a) right  
b) left

8. Answers may vary. Teacher to check.

Sample answer:  
\( \frac{4}{6} \), \( \frac{8}{10} \), \( \frac{12}{15} \), \( \frac{18}{18} \)
iii) \(\frac{1}{6}\)

c) smaller

8. Circle the following:
   a) \(\frac{2}{3}\)
   b) \(\frac{3}{4}\)
   c) \(\frac{4}{5}\)
   d) \(\frac{3}{3}\)

9. Answers will vary.
   Teacher to check.

10. If two fractions have the same numerator, the fraction with the smaller denominator is larger.

11. Teacher to check shading.
   a) i) \(\frac{1}{10}, \frac{1}{5}\)
      ii) \(\frac{2}{10}, \frac{2}{5}\)
   b) i) \(\frac{1}{10}, \frac{1}{5}\)
      ii) \(\frac{2}{10}, \frac{2}{5}\)
   c) Yes, they are the same. The order should not change whether they are being compared by shaded parts or by numerical values of numerator and denominator.

12. Randi is incorrect. \(\frac{1}{2} - \frac{5}{10}\)
    which is larger than \(\frac{1}{10}\).

13. a) Ray thinks this is unfair because \(\frac{1}{8}\) of a small cake will be a smaller piece than \(\frac{1}{8}\) of a larger cake.
    b) Lynn thinks it is fair because all of the cakes will be split into the same number of pieces.

14. a) | 0 to \(\frac{1}{2}\) | \(\frac{1}{2}\) to 1 |
    | --- | --- |
    | 1 2 3 | 4 7 5 |
    | \(\frac{3}{5}\), \(\frac{5}{10}\) | \(\frac{6}{8}\), \(\frac{8}{9}\) |
    | 1 3 4 | 6 2 |
    | \(\frac{6}{7}\), \(\frac{7}{9}\) | \(\frac{10}{3}\) |

b) i) >
   ii) <
   iii) >
   iv) <
   v) >
   vi) <
   vii) >
   viii) <

15. \(\frac{15}{30} = \frac{1}{2}\), which is larger than \(\frac{1}{10}\).

16. a) Sample answers:
   b) shaded stickers
      and triangles
   c) \(\frac{1}{5}\)
   d) \(\frac{1}{3}\)

5. Answers will vary.
   Sample answers:
   a) \(\frac{2}{5}\) of the shapes are circles.
   b) \(\frac{2}{5}\) of the shapes are squares.
   c) \(\frac{1}{5}\) of the shapes are triangles.
   d) \(\frac{3}{5}\) of the shapes are shaded.

6. b) Shade 4, circle 8
   c) Shade 1, circle 2
   d) Shade 5, circle 10

7. a) Shade 6 boxes
    b) Shade 12 boxes

10. a) 3
    b) 6

11. 3

12. $6

13. 3

BONUS

4. 6

5. Answers will vary.
   Sample answers:
   a) \(\frac{2}{5}\) of the shapes are circles.
   b) \(\frac{2}{5}\) of the shapes are squares.
   c) \(\frac{1}{5}\) of the shapes are triangles.
   d) \(\frac{3}{5}\) of the shapes are shaded.

6. Circle the following:
   a) 3 lines
   b) 5 lines
   c) 6 lines
   d) 7 lines

7. a) Shade 4, circle 8
    b) Shade 1, circle 2
    c) Shade 5, circle 10

8. Shade 4, circle 12

9. a) Shade 6 boxes
    b) Shade 12 boxes

10. a) 3
    b) 6

11. 3

12. $6

13. 3

BONUS

30

AP Book NS4-50

1. b) 4
   c) \(\frac{1}{4}\)
   d) \(\frac{1}{2}\)

2. Circle the following:
   b) three groups of two
   c) four groups of two
   d) three groups of three

3. a)
   b)

4. a)
   b)

5. b) \(\frac{5}{10}\) ÷ 2
   c) \(\frac{8}{16}\) ÷ 2
   d) \(\frac{10}{20}\) ÷ 2
   e) \(\frac{3}{9}\) ÷ 3
   f) \(\frac{5}{15}\) ÷ 3

BONUS

4

4000 ÷ 1000

6. Circle the following:
   a) 3 lines
   b) 5 lines
   c) 6 lines
   d) 7 lines

7. a) Shade 4, circle 8
    b) Shade 1, circle 2
    c) Shade 5, circle 10

8. Shade 4, circle 12

9. a) Shade 6 boxes
    b) Shade 12 boxes

10. a) 3
    b) 6

11. 3

12. $6

13. 3

BONUS

30

AP Book NS4-51

1. \(\frac{4}{9}\)

2. a) \(\frac{3}{4}\)
   b) There would be more orange flavour.
   c) It would get smaller, as the amount of liquid increases the fraction of canned juice decreases.

3. a) shaded stickers
    and triangles
   b) \(\frac{5}{9}\)
   c) \(\frac{2}{5}\)
   d) the number of unshaded stickers that are squares

4. a) \(\frac{7}{12}\)
   b) \(\frac{5}{12}\)
   c) with water

5. a) \(\frac{4}{6}\)
b) $\frac{4}{5}$
c) Ray

6. a) no
b) yes
c) $\frac{4}{10}$
d) Answers will vary.
   Teacher to check that more than 6 pizzas are shaded.

7. a) $\frac{6}{9}$
b) $\frac{4}{9}$
c) $\frac{2}{5}$
d) B
e) C
1. b) \(\frac{2}{10}, 0.2\)
   c) \(\frac{9}{10}, 0.9\)

2. b) 0.7
c) 0.6
d) 0.9
e) 0.2
f) 0.8
g) 0.3

BONUS
0 or 0.0

3. b) shade 8 pieces
c) shade 5 pieces
d) shade 6 pieces

4. Teacher to check.

5. b) tenths
c) tenths
d) hundreds
e) thousands
f) tens
g) tens
h) hundreds
i) ones

6. a) hundreds
b) tenths
c) ones
d) tens
e) thousands
f) tenths
g) hundreds
h) tens
i) thousands

7. | Th | H | T | O | T |
---|---|---|---|---|
b) 8 | 0 | 5 | 3 | 4
| c) | 4 | 8 | 9 | 2
| d) | 2 | 7 | 8
| e) | 9 | 1 | 0 | 4 | 5
| f) | 8 | 7
| g) | 7 | 0 | 6 | 0
| h) | 6 | 1

8. a) 4
   50
   600

b) \(\frac{1}{10}\)
   c) \(\frac{5}{10}\)
   e) \(\frac{1}{10}\)
   f) \(\frac{7}{10}\)
   g) \(\frac{1}{10}\)

9. b) 7
c) 70
d) 7000
e) 7
f) 7

3. a) \(\frac{2}{10}\), \(\frac{3}{10}\), \(\frac{4}{10}\), \(\frac{5}{10}\), \(\frac{6}{10}\)
    b) replace \(\frac{7}{10}\) with \(\frac{1}{10}\), replace \(\frac{5}{10}\) with \(\frac{3}{10}\), replace \(\frac{1}{10}\) with \(\frac{2}{10}\)
    c) \(\frac{7}{10}\), \(\frac{8}{10}\), \(\frac{9}{10}\), \(\frac{10}{10}\), \(\frac{10}{10}\)

4. a) 1, 2
    b) 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0
    b) ii) 0.5
    iii) 1.0
    BONUS
    0.3, 0.4, 0.6, 0.7, 0.8, 0.9

5. a) 24, 2.4
    c) 18, 1.8
    d) 13, 1.3
    e) 36, 3.6
    BONUS
    21, 2.1

6. a) 3.4
circle 3.6
b) 7.6, 5.5
circle 7.6

AP Book NS4-54
page 41

1. a) 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9
b) 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7

3. a) \(\frac{7}{10}\)
   c) 9
   9.7, 8.7

AP Book NS4-55
page 43

1. a) nine and seven tenths
b) twelve and eight tenths

2. Teacher to check drawings.
Circle the following:

3. a) 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9

BONUS
0.5, 0.6, 0.7, 0.9

AP Book NS4-56
page 44

1. a) 1, 2
b) 3

c) 4

d) 5

2. a) 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9

3. a) The scales have different starting values.
b) The scales are all divided into tenths.

3. b) seven
c) six
d) twenty
4. a) seven and four tenths
b) four and nine tenths
c) 19.1
d) 62.4

5. b) 24, 2.4
c) 18, 1.8
d) 13, 1.3
e) 36, 3.6
BONUS
21, 2.1
Number Sense: Decimals – AP Book 4.2: Unit 10

10. a) 5490.1, 954.1, 549.1
   b) 10 002.4, 1300.4, 989.7
   c) 826.7, 800.0, 762.8
   d) 1000.4, 410.0, 400.1

11. Answers will vary. Teacher to check.

12. a) Teacher to check number line.
      519.3, 519.7, 520.0
      520.0, 519.7, 519.3

   b) Answers will vary.

   Sample answer: You would need a really long number line to show numbers in the tens, hundreds, and thousands with decimal tenth accuracy.

AP Book NS4-56

date 46

1. a) 2
   
   [ | | | ]
   3

   b) 1, 6

   c) 2, 3

   d) 4, 9

   e) 5, 2

2. b) 8, 4

   c) 8, 9, 6

   d) 6, 5, 9, 4

   BONUS

   9, 3, 7, 4, 8

   BONUS

   3. b) [H T O T]

   1 4 6 1
   2 2 8
   1 6 8 9

   4. b) [T O T]

   2 5 8
   1 2 6
   3 7 14
   3 8 4

5. b) 1 1 3

   3 2 5
   4 3 8
   6 5 6

   c) 2 3

   d) 3 7 2

   4 2 6

   7 9 8

   1

   2 4 7

   4 3

   2 9 0

   5 7 2

   3 1 9

   8 9 1

   6 b) 1

   2 4 7

   4 3

   2 9 0

   5 7 2

   3 1 9

   8 9 1

   6. b) 1

   2 4 7

   4 3

   2 9 0

   5 7 2

   3 1 9

   8 9 1

   11. Answers will vary. Teacher to check.

   12. a) Teacher to check number line.

   b) 519.3, 519.7, 520.0

   c) 520.0, 519.7, 519.3

   BONUS

   circle the hundreds digits

   732.5

   BONUS

   circle the tens digits

   776.8

4. b) circle the tens digits

   136.0

   c) circle the ones digits

   4858.5

   d) circle the hundreds digits

   732.5

5. a) 4332.3

   b) circle the hundreds digits

   5900.7

   c) circle the tens digits

   776.8

   BONUS

   circle the hundreds digits

   12 146.6

6. Circle the following:

   a) 9147.6

   b) 352.1

   c) 5098.1

7. a) <

   b) >

   c) >

8. Circle the following:

   b) 416.2

   c) 5371.2

   d) 7358.2

9. Circle the following:

   a) 86.1

   b) 319.4

   c) 3670.1

   d) 5228.2

10. a) 5490.1, 954.1, 549.1

    b) 10 002.4, 1300.4, 989.7

    c) 826.7, 800.0, 762.8

    d) 1000.4, 410.0, 400.1

11. Answers will vary. Teacher to check.

12. a) Teacher to check number line.

      519.3, 519.7, 520.0

      520.0, 519.7, 519.3

   b) Answers will vary.

   Sample answer: You would need a really long number line to show numbers in the tens, hundreds, and thousands with decimal tenth accuracy.

AP Book NS4-56

date 46

1. a) 2

   b) 1, 6

   c) 2, 3

   d) 4, 9

   e) 5, 2

2. b) 8, 4

   c) 8, 9, 6

   d) 6, 5, 9, 4

    BONUS

    9, 3, 7, 4, 8

    BONUS

3. b) [H T O T]

   1 4 6 1
   2 2 8
   1 6 8 9

4. b) [T O T]

   2 5 8
   1 2 6
   3 7 14
   3 8 4

5. b) 1 1 3

   3 2 5

   4 3 8

   6 5 6

   2 3

   3 7 2

   4 2 6

   7 9 8

   1

   2 4 7

   4 3

   2 9 0

   5 7 2

   3 1 9

   8 9 1

   6 b) 1

   2 4 7

   4 3

   2 9 0

   5 7 2

   3 1 9

   8 9 1

   12. b) 2 0 5

      1 0 2

      1 0 3

   c) 1 3 4

      2 2

      1 1 2

   d) 1 6 4

      0 3

      1 6 1

   e) 5 2 5

      1 1 5

      4 1 0

   f) 6 3 7

      2 6

      6 1 1

   g) 7 8 8

      7 1

      7 1 7

8. 6.7 kg

9. 49.6 kg
Number Sense: Decimals – AP Book 4.2: Unit 10

Answer Keys for AP Book 4.2
c) \( \frac{80}{10} = \frac{800}{100} \)

d) \( \frac{50}{100} = 0.5 \)

e) \( \frac{47}{100} = 0.47 \)

**BONUS**

\[ \frac{96}{100} = 0.96 \]

2. a) 0.18

b) 0.09

c) 0.90

d) 0.10

e) 0.52

f) 0.99

3. c) 80

\[ \frac{88}{10} \text{ or } 8.8 \]

\[ \frac{10}{100} \text{ or } 0.1 \]

3. b) 50

\[ \frac{55}{10} \text{ or } 5.5 \]

\[ \frac{0}{100} \text{ or } 0 \]

4. Yu is incorrect.

5 dimes = 50 cents and

37 pennies = 37 cents.

So, 5 dimes is worth more than 37 pennies.

5. a) 30

b) shade 5 columns

6. b) 8

\[ \frac{80}{10} \text{ or } 8 \]

\[ \frac{100}{10} \text{ or } 1 \]

7. a) 36

\[ \frac{36}{100} \]

b) 72

\[ \frac{72}{100} \]

8. Teacher to check.

9. Teacher to check shading.

Circle the following:

a) \( \frac{6}{10} \)

b) \( \frac{7}{10} \)

10. Marko is incorrect. To compare fractions, they need to have the same denominator. \( \frac{8}{10} = \frac{80}{100} \)

which is greater than \( \frac{17}{100} \).

AP Book NS4-60

**page 57**

1. b) 2.05, 2, 0.5

c) 1.30, 1, 0.3

d) 2.20, 2, 2.0

2. b) 2, 8

28, 2, 8

c) 4, 1

41, 4, 1

d) 6, 0

60, 6, 0

e) 5, 3

53, 5, 3

f) 1, 2

12, 1.2

g) 3, 6

36, 3, 6

h) 9, 2

92, 9, 2

3. c) 5, 2

52 hundredths

d) 6, 0, 2

6 and 2 hundredths

e) 8, 3

83 hundredths

f) 5, 5, 5

5 and 55 hundredths

g) 1, 0, 6

1 and 6 hundredths

h) 8, 9, 0

8 and 90 hundredths

4. a) 2, 0.9, 0.05

b) 5000, 400, 0, 8, 0, 0.4, 0.01

c) 200, 30, 7, 0, 0.06

d) 60, 7, 0.2, 0.03

5. A. 1, 4

14

B. 3, 7

37

C. 6, 0

60

D. 8, 4

84

6. b) 0.58, \( \frac{58}{100} \)

c) 0.13, \( \frac{13}{100} \)

d) 0.91, \( \frac{91}{100} \)

e) 0.06, \( \frac{6}{100} \)

f) 0.3, \( \frac{30}{100} \)

AP Book NS4-61

**page 60**

1. b) 3 ones + 4 tenths

+ 3 hundredths

c) 2 ones + 6 tenths

+ 1 hundredth

f) 5 ones + 9 tenths

+ 5 hundredths

2. b) 1, 8, 2

3. c) 8, 5, 0

d) 8, 7, 6

AP Book NS4-60

**page 57**

1. b) 3 ones + 4 tenths

+ 3 hundredths

c) 2 ones + 6 tenths

+ 1 hundredth

f) 5 ones + 9 tenths

+ 5 hundredths

2. b) 1, 8, 2

c) 8, 5, 0

d) 8, 7, 6

**BONUS**

\[ 1, 0, 8, 2 \]

\[ \begin{array}{c|c|c|c|c}
3 & 4 & 6 & 4 \\
2 & 1 & 2 & 7 \\
5 & 5 & 9 & 1 \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
6 & 3 & 8 & 9 \\
2 & 2 & 3 & 1 \\
9 & 1 & 2 & 0 \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
3 & 4 & 9 & 0 \\
5 & 7 & 7 & 4 \\
0 & 6 & 7 & 0 \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
6 & 2 & 9 & 5 \\
2 & 7 & 1 & 0 \\
9 & 0 & 0 & 5 \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
5 & 3 & 8 & 0 \\
8 & 0 & 3 & 6 \\
6 & 1 & 8 & 3 \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
1 & 7 & 8 & 6 \\
1 & 9 & 2 & 6 \\
\end{array} \]

\[ \begin{array}{c|c|c|c|c}
0 & 4 & 1 & 3 \\
3 & 8 & 0 & 4 \\
4 & 2 & 1 & 5 \\
\end{array} \]
Number Sense: Decimals – AP Book 4.2: Unit 10

(continued)

5. b) 8, 14
c) 0, 16
d) 7, 18

6. b) 0, 12
c) 1, 13
d) 0, 14
e) 12
f) 14, 12
g) 2, 10
h) 13
i) $6.00
j) $0.99
k) $12.00
l) $16.04

5. b) $20, 56¢, $20.56
c) $30, 7¢, $30.07
d) $26, 11¢, $26.11

6. quarters
7. Draw 3 loonies and 2 quarters.
8. 5 loonies and 1 quarter and 2 toonies, 1 loonie, 2 dimes and 1 nickel

AP Book NS4-62
page 62

1. b) 5, 0.05
c) 43, 0.43
d) 87, 0.87
e) 54, 0.54
f) 9, 0.09
g) 2, 0.02
h) 75, 0.75
i) 1, 0.01

2. b) 0, 4, 7, $0.47
c) 3, 2, 5, $3.25
d) 0, 0, 3, $0.03
e) 28, 1, 6, $28.16

3. a) 300¢
b) 60¢
c) 9¢
d) 100¢
e) 700¢
f) 1200¢
g) 1500¢
h) 199¢
i) 151¢
j) 98¢
k) 3¢
l) 8¢

4. b) $1.03
c) $2.16
d) $3.75
e) $3.00
f) $0.04
g) $0.07
h) $0.90

AP Book NS4-63
page 64

1. b) $10 − $4, $6
c) $10 − $7, $3
d) $10 − $3, $7
e) $10 − $6, $4

2. b) 100¢ − 40¢, 60¢
c) 100¢ − 60¢, 40¢
d) 100¢ − 90¢, 10¢
e) 100¢ − 50¢, 50¢
f) 100¢ − 10¢, 90¢

3. b) 80¢
c) 60¢, $15.00
d) 30¢, $12.00
e) 40¢, $22.00
f) 90¢, $36.00
g) 60¢, $60.00
BONUS
20¢, $88.00

4. a) 40¢, 6, 6.40
b) 20¢, $13, $7, $7.20
c) 90¢, $18, $2, $20, $2.90
BONUS
d) 80¢, $49, $1, $50, $1.80
e) 90¢, $44, $6, $50, $6.90
f) 30¢, $5, $45, $50, $45.30
g) 10¢, $20, $30, $50, $30.10

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### AP Book PA4-12

1. a) $\square \square \square \square$
   b) 6
   c) $\square \square \square$
   d) $\square \square \square \square \square \square \square$
2. b) $\square \square \square \square \square \square \square$
   7
   c) $\square \square \square \square \square$
   d) $\square \square \square \square \square \square \square \square \square \square$
3. Teacher to check pictures.
   a) 4
   b) 6
4. a) 17
   b) 27
   c) 35
   d) 65
5. a) $\square \square \square \square \square$
   b) $\square \square \square \square \square$
   c) $\square \square \square \square \square$
   d) $\square \square \square \square \square \square \square \square \square \square$

### AP Book PA4-13

1. b) Draw 4 apples in each box.
   $\square + \square + \square = 12$
2. Teacher to check pictures.
   b) 6
   c) 5
   d) 3
   e) 5
   f) 4
3. b) 2
   c) 12
   d) 2
   e) 3
   f) 18
   g) 4
   h) 28

### BONUS

36

### AP Book PA4-14

1. Circle the first, third, and fifth equations.
2. b) $24 = 21 + k$
   $21 = 24 - k$
   $k = 24 - 21$
   c) $17 = k + 3$
   $k = 17 - 3$
   $3 = 17 - k$
   d) $k = 215 + 65$
   $215 = k - 65$
   $65 = k - 215$
   e) $97 = k + 18$
   $k = 97 - 18$
   $18 = 97 - k$
   f) $312 = 78 + k$
   $78 = 312 - k$
   $k = 312 - 78$

3. b) $m = 8 - 5$
   c) $m = 11 + 2$
   d) $m = 9 - 3$
4. b) $5, 3, m, \text{and } x$
   $5 + 3$
   c) $9, m, 11, \text{and } x$
   $11 - 9$
   d) $m, 7, 16, \text{and } x$
   $16 - 3$
   e) $x, 17, 13, \text{and } x$
   $17 + 13$

### BONUS

$100, x, 20,$
$x = 100 - 20$

3. a) underline "4 more hats"
   b) circle "7 scarves"
   underline "5 fewer hats"
   c) circle "6 scarves"
   underline "x fewer hats"

4. b) red marbles, 12
   green marbles, 8
   Difference: $x$
   $x = 12 - 8$
   $x = 4$
   c) dog, 13
   cats, $x$
   Difference: 7
   $x = 13 - 7$
   $x = 20$
   d) bulldog, $x$
   boxer, 35
   Difference: 7
   $x = 35 - 7$
   $x = 28$

5. a) $x = 8 + 3$
   $x = 11$
   b) $x = 17 - 8$
   $x = 9$
   c) $x = 29 - 8$
   $x = 21$
   d) $x = 42 + 12$
   $x = 54$
   e) $x = 38 - 29$
   $x = 9$
   f) $x = 72 - 24$
   $x = 48$

### T-12

**Answer Keys for AP Book 4.2**
1. b) 47 km distance on Tuesday, 54 km
   Total: x
   x = 47 + 54
   x = 101
   c) money raised by Alice, $32
   money raised by Ben, $x
   Difference: 9
   Total: x
   x = 32 − 9
   x = 23
   d) apple juice, 3000 mL
   plum juice, x mL
   Difference: 2000
   Total: x
   x = 3000 + 2000
   x = 5000
   e) white milk, 198 mL
   chocolate milk, x mL
   Difference: 35
   Total: x
   x = 198 + 35
   x = 233
2. 65 m
3. a) i) 9 + 6 = 15
   ii) 15 − 5 = 10
   b) i) 24 − 15 = 9
   ii) 15 − 9 = 6
4. a) Sara bought 8 + 5 = 13 jelly beans in total. So she has 13 − 4 = 9 jelly beans left.
1. a) 12
   b) 14
   c) 14
2. a) 50
   b) 40
   c) 50
   d) 60
3. Answers will vary. Teacher to check.
4. a) cm
   b) km
   c) m
   d) m
   e) km
   f) cm
5. Answers will vary. Sample answer: You would use millimetres if you were measuring a small object and you needed to be exact, like if you were making frames for eyeglasses.
6. a) 24 m
   b) 28 cm
   c) 6 km
   d) 8 dm
7. a) 100 mm
   b) 86 mm
   c) 100 mm
8. Teacher to check estimates.
   a) 16 cm
   b) 16 cm
9. a) 2 + 4 + 2 + 4 = 12 cm
    b) 3
    c) 3 + 3 + 3 + 3 = 12 cm
3. Perimeter = ℓ + ℓ + w + w
   or
   Perimeter = (2 × ℓ) + (2 × w)
4. 4
   4, 8
   8, 4
5. 6
   12 m - 6 m = 6 m
   6 m + 2 = 3
6. a) 1, 1
   b) 4, 4
   c) 3, 3
   d) 5, 5
7. Perimeter = 6 units
   Width | Length
   1 | 2
   Perimeter = 10 units
   Width | Length
   1 | 4
   2 | 3
   Perimeter = 16 units
   Width | Length
   1 | 7
   2 | 6
   3 | 5
   4 | 4
   Perimeter = 18 units
   Width | Length
   1 | 8
   2 | 7
   3 | 6
   4 | 5
8. Perimeter = s + s + s + s
   or
   Perimeter = 4 × s
   a)
   b)
   c)
   d)
   e)
   f)
Measurement: 2-D Shapes – AP Book 4.2: Unit 12

1. a) \( \frac{\text{length}}{\text{width}} \)
   b) \( \frac{\text{base}}{\text{height}} \)
   c) \( \frac{\text{side}}{\text{side}} \)
   d) \( \frac{\text{diameter}}{\text{radius}} \)

2. a) \( \frac{\text{height}}{\text{base}} \)
   b) \( \frac{\text{side}}{\text{side}} \)
   c) \( \frac{\text{circumference}}{\text{diameter}} \)
   d) \( \frac{\text{radius}}{\text{radius}} \)

3. a) \( \frac{\text{length}}{\text{length}} \)
   b) \( \frac{\text{base}}{\text{base}} \)
   c) \( \frac{\text{side}}{\text{side}} \)
   d) \( \frac{\text{diameter}}{\text{diameter}} \)

4. \( \frac{\text{length}}{\text{length}} \)

5. a) \( \frac{\text{side}}{\text{side}} \)
   b) \( \frac{\text{base}}{\text{base}} \)
   c) \( \frac{\text{side}}{\text{side}} \)
   d) \( \frac{\text{diameter}}{\text{diameter}} \)

6. a) \( \frac{\text{length}}{\text{length}} \)
   b) \( \frac{\text{base}}{\text{base}} \)
   c) \( \frac{\text{side}}{\text{side}} \)
   d) \( \frac{\text{diameter}}{\text{diameter}} \)

7. a) \( \frac{\text{length}}{\text{length}} \)
   b) \( \frac{\text{base}}{\text{base}} \)
   c) \( \frac{\text{side}}{\text{side}} \)
   d) \( \frac{\text{diameter}}{\text{diameter}} \)

8. a) \( \frac{\text{length}}{\text{length}} \)
   b) \( \frac{\text{base}}{\text{base}} \)
   c) \( \frac{\text{side}}{\text{side}} \)
   d) \( \frac{\text{diameter}}{\text{diameter}} \)

BONUS

Sample answer: Someone might measure a large area in cm\(^2\) if they needed to be exact. For example, a carpenter putting in wood floors would measure in cm\(^2\) so they wouldn’t waste wood.
3. a) \[2 \times 7 = 14\]
    b) \[3 \times 3 = 9\]
    c) \[5 \times 3 = 15\]

4. Teacher to check drawings.
    a) \[3 \times 3 = 9\]
    b) \[3 \times 2 = 6\]
    c) \[4 \times 3 = 12\]

5. To find the area of a rectangle, multiply its length by its width.

6. a) \[3 \times 3 = 9\] cm²
    b) \[3 \times 2 = 6\] cm²
    c) \[4 \times 3 = 12\] cm²

7. To find the area of a rectangle, multiply its length by its width.

8. a) \[5 \times 3 = 15\] cm²
    b) \[1 \times 2 = 2\] cm²
    c) \[5 \times 2 = 10\] cm²

9. Teacher to check drawings.

10. a) \[3 \times 3 = 9\] cm²
    b) \[3 \times 2 = 6\] cm²
    c) \[4 \times 3 = 12\] cm²

11. Teacher to check drawings.

12. a) \[8 \times 7 = 56\]
    b) \[7 \times 3 = 21\]
    c) \[3 \times 3 = 9\]

13. a) \[4 \times 3 = 12\]
    b) \[7 \times 2 = 14\]
    c) \[7 \times 2 = 14\]

14. Teacher to check drawings.

15. a) \[7 \times 3 = 21\]
    b) \[7 \times 2 = 14\]
    c) \[7 \times 2 = 14\]
3. a) Length: 30 m  
   Width: 20 m
   b) Perimeter: 100 m  
      Area: 600 m²
   c) about 14 m
   d) \(6 \text{ m} \times 8 \text{ m} = 48 \text{ m}^²\)
   e) Teacher to check drawing is 3 cm by 2 cm.

AP Book ME4-20

1. treasure hunt
2. b) A1, D2, B2  
   c) A2, B2, B2, D2, A3, D2
   d) A3, C3, B3, D1, E3, A2, E1
   e) A3, A1, C3, B2, B1, A1, A2, E1, E3, B1
3. Answers will vary. Teacher to check.
4. Answers will vary. Teacher to check.

BONUS

X won the game.

5. a) A2  
    b) E3
    c) D4
    d) A1
6. b) D1, D4
    c) F2, F4
    d) A1, E1
7. about 700 m

BONUS

Teacher to check drawing.

about 1 km
### Measurement: Time – AP Book 4.2: Unit 13

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<td>c) 12:05</td>
<td>b) a.m.</td>
<td>b) 10 o'clock</td>
<td>c) 7:25</td>
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<td>d) 14:23</td>
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<td>c) 2 o'clock</td>
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<td>d) 12 o'clock</td>
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<td>2. b) 03:00</td>
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<td>j) 16:16</td>
<td>2. a) a.m.</td>
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<td>d) 8:35</td>
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<td>3. 2:20, 14:20</td>
<td>b) p.m.</td>
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<td>e) 10:10</td>
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<td>4. a) 10</td>
<td>c) a.m.</td>
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<tr>
<td>b) 20</td>
<td>d) p.m.</td>
<td></td>
<td>3. b) 3:25</td>
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<tr>
<td>c) 25</td>
<td>e) p.m.</td>
<td></td>
<td>c) 7:55</td>
</tr>
<tr>
<td>d) 4</td>
<td>f) p.m.</td>
<td></td>
<td>d) 11:20</td>
</tr>
<tr>
<td>5. b) 18 minutes to 4</td>
<td>g) a.m.</td>
<td></td>
<td>e) 8:40</td>
</tr>
<tr>
<td>c) 5 minutes to 11</td>
<td>h) p.m.</td>
<td></td>
<td>f) 12:50</td>
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<tr>
<td>d) 27 minutes to 8</td>
<td></td>
<td></td>
<td>10 minutes to 1</td>
</tr>
<tr>
<td>e) 2 minutes to 10</td>
<td></td>
<td></td>
<td>4. Teacher to check.</td>
</tr>
<tr>
<td>f) 20 minutes to 2</td>
<td></td>
<td></td>
<td>5. 10:35 p.m.</td>
</tr>
<tr>
<td>6. a) 4:53</td>
<td>b) 19:00</td>
<td>6. Teacher to check.</td>
<td></td>
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<tr>
<td>b) 3:35</td>
<td>c) 09:30</td>
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<td>7. a) quarter past 4</td>
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<tr>
<td>c) 10:50</td>
<td>d) 21:30</td>
<td></td>
<td>b) quarter past 9</td>
</tr>
<tr>
<td>d) 5:58</td>
<td>e) 06:15</td>
<td></td>
<td>c) quarter past 3</td>
</tr>
<tr>
<td>e) 1:20</td>
<td>f) 15:45</td>
<td></td>
<td>8. a) quarter to 6</td>
</tr>
<tr>
<td>f) 12:55</td>
<td>g) 14:20</td>
<td></td>
<td>b) quarter to 12</td>
</tr>
<tr>
<td>7. a) 45 past 11</td>
<td>h) 11:54</td>
<td></td>
<td>c) quarter to 9</td>
</tr>
<tr>
<td>b) 47 past 6</td>
<td>i) 16:23</td>
<td></td>
<td>9. a) 7:15</td>
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<tr>
<td>BONUS</td>
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<td>52 past 12</td>
<td>k) 08:48</td>
<td></td>
<td>c) 11:15</td>
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<tr>
<td>8 minutes to 1</td>
<td>l) 23:19</td>
<td></td>
<td>d) 10:45</td>
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<td></td>
<td></td>
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<td>e) 1:15</td>
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<td></td>
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<td>f) 3:45</td>
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<td>g) 8:45</td>
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<td></td>
<td></td>
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<td>h) 5:15</td>
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<td></td>
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<td>i) 9:30</td>
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<td></td>
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<td>j) 11:14</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>k) 2:30</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>l) 3:15</td>
</tr>
<tr>
<td>10. Teacher to check.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>11. a) 4</td>
<td>b) 5</td>
<td>BONUS</td>
<td>exactly halfway</td>
</tr>
</tbody>
</table>

**Answer Keys for AP Book 4.2**
c) 7:27

1. b) 25 minutes
c) 35 minutes
d) 25 minutes
e) 25 minutes
f) 55 minutes

2. a) 22 minutes
b) 34 minutes
c) 57 minutes
d) 37 minutes
e) 16 minutes
f) 39 minutes

3. a) 40 minutes
b) 25 minutes
c) 20 minutes

4. a) 33 minutes
b) 18 minutes
c) 24 minutes

5. Estimates will vary.

6. He has been playing for 52 minutes.

7. Teacher to check arrows.

a) 5:30
b) 3:52
c) 12:09

8. a) 3:43
b) 8:33
c) 6:48
d) 8:52
e) 11:51

9. b) 8:11
c) 9:20
d) 3:32
e) 2:45

10. b) 8:73
    c) 10:78
    d) 8:68
    e) 4:92

11. a) 8:15 p.m.

b) 6:32 p.m.

12. 54 minutes

13. 5:38 p.m.
b) 2, 2  
c) 4, 4  
d) 3, 3  
3. a) 7, 14, 21, 28, 35, 42, 49  
b) 7  
4. a) 14, 3  
   17  
b) 14, 5  
   19  
c) 21, 2  
   23  
5. There are about 4 weeks in 1 month.  
6. 24, 36, 48, 60  
7. a) 24, 3  
   27  
b) 12, 5  
   17  
c) 36, 11  
   47  
8. a) 6  
  b) 8  
  c) 120  
  d) 4  
9. a) Circle 85 years.  
   7 decades is  
   70 years, so  
   85 years is longer.  
b) Circle 8 centuries.  
   8 centuries is  
   800 years, so it is  
   longer than  
   670 years.  
10. Answers will vary depending on the year.  
   Samples answers for 2018:  
   a) About one and a half centuries.  
   b) About 15 decades.  
11. 2 weeks + 2 days  
   = 14 days + 2 days  
   = 16 days  
   16 ÷ 4 = 4  
   They can spend 4 days in each city.  
12. a) 9 × 6 = 54  
   Most students will spend 54 months in school from Grades 1 to 6.  
b) Most students will spend 4.5 years in school from Grades 1 to 6.  
13. 4 × 8 = 32 months  
    = 2 years and 8 months  
    Students study for 2 years and 8 months in total.  
14. 5 + 16 = 21  
   The family arrives home on July 21.  
   **BONUS**  
   There are 42 days left of summer vacation.  
   There are 6 weeks left of summer vacation.
1. a) rectangle  
b) triangle  
c) hexagon  
d) circle  
e) rectangle

**BONUS**  
square

2. Teacher to check.

3. Teacher to check.

4. Teacher to check.

5. Teacher to check tracing.

6. Teacher to check dots.

a) 12  
b) 6  
c) 8  
d) 9

6. Teacher to check dots.

a) 8  
b) 4  
c) 5  
d) 6

7. Teacher to check.

AP Book G4-11

**page 130**

1. Teacher to check shading.  
b) triangular prism  
c) rectangular prism  
d) triangular prism

2. Cross out the fourth shape in the top row and the first and third shapes in the bottom row.

Shade the bases of the second and last shape in the top row and the second shape in the bottom row.

Circle the first and third shapes in the top row.

3. 6, 8, 8  
9, 12, 12

4. Teacher to check.

5. Teacher to check.

AP Book G4-13

**page 133**

1. Teacher to check shading.  
a) pentagon  
b) trapezoid  
c) square  
d) parallelogram  
e) hexagon  
f) square

2. Teacher to check.

3. Teacher to check last column.  
8, 12, 6  
10, 15, 7  
12, 18, 8

4. a) 8  
b) 12  
c) 6  
d) Answers will vary.  
Teacher to check.

**BONUS**  
8  
16  
24

AP Book G4-14

**page 135**

1. Teacher to check shading and dots.  
b) rectangular pyramid  
c) rectangular pyramid  
d) triangular pyramid

2. Teacher to check.  
a) triangular pyramid  
b) rectangular prism  
c) rectangular pyramid  
d) triangular prism  
e) pentagon-based pyramid  
f) hexagonal pyramid  
g) rectangular pyramid  
h) hexagon-based pyramid

3. Teacher to check.

4. Teacher to check last column.

triangular pyramid; 4, 6, 4  
rectangular pyramid; 5, 8, 5  
pentagonal pyramid; 6, 10, 6  
hexagonal pyramid; 7, 12, 7

b) Teacher to check.  
c) triangles

5. The object is a pyramid.  
The shape of the base has 7 vertices and a pyramid always has one more vertex than the base.

6. triangular pyramid

**BONUS**  
a) It is a pyramid with a base that has 8 sides.  
b) It is a prism with a base that has 7 sides.

AP Book G4-15

**page 138**

1. a) C  
b) A  
c) B

2. a) yes  
b) no  
c) yes  
d) no  
e) no  
f) yes

3. a) yes  
b) yes  
c) no  
d) yes

4. Teacher to check.

5. Teacher to check.

AP Book G4-16

**page 140**

1. a) 3  
b) 6  
c) 6  
d) 10  
e) 8  
f) 14  
g) 5  
h) 9  
i) 10

2. a) 3  
b) 3  
c) 3

3. a) 6  
b) 12  
c) 16

4. a) 2  
b) 3  
c) 2

5. a) 6, 2  
12  
12, 3  
36  
16, 2  
32  
20, 2  
40  
16, 3  
48  
f) 12, 4  
48

6. a) 7  
b) 15  
c) 11
1. Circle the following:
   a) milk carton
   b) bucket
   c) bottle

2. Circle the cup, jam, and spoon.
   Cross out the bucket, sink, and swimming pool.

3. Answers will vary.
   Sample answers:
   a) 1
   b) 3
   c) 10

4. a) 3
   b) 5
   c) 4
   d) 2

4. **BONUS**
   \[ 250 \text{ mL} \times 4 \]
   \[ = 1000 \text{ mL} = 1 \text{ L} \]

4. 15 g
   **BONUS**
   60 months (5 years)

5. a) No, 1 L is a lot to drink all at once.
   b) No, 1 L is not enough to take a bath in.

6. Circle the following:
   a) L
   b) mL
   c) L
   d) mL
   e) L
   f) mL

7. 10 mL

8. Answers will vary.
   Sample answers:
   a) 100
   b) 400
   c) 200
   **BONUS**
   a) 60
   b) 8

---

**AP Book G4-18**

**page 144**

1. 2000, 3000, 4000, 5000, 6000, 7000, 8000
2. B, E, C, D, A, F
3. b) 5
c) 4

---

**Answer Keys for AP Book 4.2**
1. b) A, B, C
   c) A, B, C, D
   d) A

2. 1
3. a) heads, tails; 2
   b) 1, 2, 3, 4, 5, 6; 6
4. b) Y, B, G, R
   c) Y
   d) R, G, Y

5. a) 2
   b) 4
   c) 4
   d) 6

6. a) 3
   b) 12
   c) 7
   d) no

7. Answers will vary.
   Sample answer:
   Y Y Y R
   G B P W

1. a) ii) 4
   iii) 6
   BONUS
   a) i) 2
   ii) 4
   iii) 6

2. a) Teacher to check arrow.
   HT
   TH
   TT
   BONUS
   i) 10, 10
   ii) 10, 10
   iii) 10, 10

3. Sample answer: The coin is likely unevenly weighted, causing it to land with heads facing up every time.

4. a) ii) 2
   iii) 3
   iv) 4
   v) 5
   vi) 6
   BONUS
   a) i) 2
   ii) 6
   iii) 12

5. a) i) 2
   ii) 4
   iii) 6
   b) i) 4
   ii) 8
   iii) 12
   BONUS
   a) i) 2
   ii) 4
   iii) 6

6. a) 3
   b) 1
   c) 7
   d) no

7. Answers will vary.
   Sample answer:
   Y Y Y R
   G B P W

1. a) ii) 4
   b) 4
   c) 6
   d) 12
   BONUS
   i) 10, 10
   ii) 10, 10
   iii) 10, 10

2. a) ii) 10, 10
   iii) 10, 10
   BONUS
   a) ii) 10, 10
   iii) 10, 10

3. a) Teacher to check arrow.
   HT
   TH
   TT
   BONUS
   i) 10, 10
   ii) 10, 10
   iii) 10, 10

4. a) i) 1
   ii) 2
   iii) 3
   iv) 4
   v) 5
   vi) 6
   BONUS
   a) i) 1
   ii) 2
   iii) 3
   iv) 4
   v) 5
   vi) 6

5. a) i) 2
   ii) 4
   iii) 6
   b) i) 4
   ii) 8
   iii) 12

6. a) 3
   b) 1
   c) 7
   d) no

7. Answers will vary.
   Sample answer:
   Y Y Y R
   G B P W
1. a) i) \(\frac{1}{2}\)  
    ii) 1  
    \(\frac{1}{2}\)  

   b) i) 15 
    ii) 15 

c–d) Answers will vary.  
Teacher to check.

2. C is most likely because it is closest to the expected result of 50 heads and 50 tails.

3. a) i) 5  
    ii) 15 

4. a) Teacher to check tree diagrams.  
    i) 1  
    ii) 2  
    iii) 3  
    iv) 2  
    v) 1 

   b) 2 and 6 are equally likely. 
   3 and 5 are equally likely. 

c–e) Answers will vary.  
Teacher to check.

5. Sample answer: I disagree. 
Actual results will vary from the expected and 12 to 8 is close to the expected results.
Grade 4 JUMP Math Correlation to the Alberta Curriculum

NOTES:
JUMP Math strands are represented by:
- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

<table>
<thead>
<tr>
<th>Number</th>
<th>General Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Develop number sense.</td>
</tr>
</tbody>
</table>

| Specific Outcomes | JUMP Math Lessons | |
|-------------------|-------------------|
| 1. Represent and describe whole numbers to 10 000, pictorially and symbolically. [C, CN, V] | Part 1 Unit 1 Lessons NS4-3 to 6 |
| 2. Compare and order numbers to 10 000. [C, CN, V] | Part 1 Unit 1 Lessons NS4-7, 8 |
| 3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by: • using personal strategies for adding and subtracting • estimating sums and differences • solving problems involving addition and subtraction [C, CN, ME, PS, R, V] | Part 1 Unit 1 Lessons NS4-1, 2, 9, 10 |
| | Part 1 Unit 2 Lessons NS4-11 to 20 |
| | Part 1 Unit 7 Lessons NS4-32, 33 |

Note: Students investigate a variety of strategies, including standard/traditional algorithms, to become proficient in at least one appropriate and efficient strategy that they understand.

Note: Through this outcome, students have the opportunity to maintain and refine previously learned addition and subtraction number facts: Apply mental mathematics strategies and number properties in order to understand and recall basic addition facts and related subtraction facts to 18 (Grade 3).
### Number

<table>
<thead>
<tr>
<th></th>
<th>Apply the properties of 0 and 1 for multiplication and the property of 1 for division.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>[C, CN, R]</td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>

| 5. | Describe and apply mental mathematics strategies to determine basic multiplication facts to $9 \times 9$ and related division facts. |
|    | [C, CN, ME, R] |                                                                 |
|    | Understanding and apply strategies for multiplication and related division facts to $9 \times 9$. Recall multiplication and related division facts to $7 \times 7$. |

<table>
<thead>
<tr>
<th></th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>5</td>
<td>PA4-1, 3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td>NS4-24</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>NS4-34, 35</td>
</tr>
</tbody>
</table>

| 6. | Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: |
|    | • using personal strategies for multiplication with and without concrete materials |
|    | • using arrays to represent multiplication |
|    | • connecting concrete representations to symbolic representations |
|    | • estimating products |
|    | • applying the distributive property. |
|    | [C, CN, ME, PS, R, V] |                                                                 |

- **Note:** Students investigate a variety of strategies, including standard/traditional algorithms, to become proficient in at least one appropriate and efficient strategy that they understand.

- **Note:** Through this outcome, students have the opportunity to maintain and refine previously learned addition and subtraction number facts: Apply mental mathematics strategies and number properties in order to understand and recall basic addition facts and related subtraction facts to 18 (Grade 3).
### Number

<table>
<thead>
<tr>
<th>7.</th>
<th>Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• using personal strategies for dividing with and without concrete materials</td>
<td>1</td>
<td>7</td>
<td>NS4-31 to 44</td>
</tr>
<tr>
<td></td>
<td>• estimating quotients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• relating division to multiplication.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[C, CN, ME, PS, R, V]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Note: Students investigate a variety of strategies, including standard/traditional algorithms, to become proficient in at least one appropriate and efficient strategy that they understand.</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Note: Through this outcome, students have the opportunity to maintain and refine previously learned addition and subtraction number facts: Apply mental mathematics strategies and number properties in order to understand and recall basic addition facts and related subtraction facts to 18 (Grade 3).</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial and symbolic representations to:</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td>• name and record fractions for the parts of a whole or a set</td>
<td>2</td>
<td>9</td>
<td>NS4-45, 46, 48, 49, 51</td>
</tr>
<tr>
<td></td>
<td>• compare and order fractions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• model and explain that for different wholes, two identical fractions may not represent the same quantity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• provide examples of where fractions are used.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[C, CN, PS, R, V]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Represent and describe decimals (tenths and hundredths), concretely, pictorially and symbolically.</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td>[C, CN, R, V]</td>
<td>2</td>
<td>9</td>
<td>NS4-52, 54</td>
</tr>
<tr>
<td>10.</td>
<td>Relate decimals to fractions and fractions to decimals (to hundredths).</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td>[C, CN, R, V]</td>
<td>1</td>
<td>7</td>
<td>NS4-52 to 54, 59, 60</td>
</tr>
</tbody>
</table>
### Number

11. Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:
   - using personal strategies to determine sums and differences
   - estimating sums and differences
   - using mental mathematics strategies to solve problems.

[C, CN, ME, PS, R, V]

Note: Through this outcome, students have the opportunity to maintain and refine previously learned addition and subtraction number facts: Apply mental mathematics strategies and number properties in order to understand and recall basic addition facts and related subtraction facts to 18 (Grade 3).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>NS4-56, 57, 61</td>
</tr>
</tbody>
</table>
### Patterns & Relations — Patterns

**General Outcome**

Use patterns to describe the world and to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Identify and describe patterns found in tables and charts.</td>
<td>Part 1, Unit 5, Lessons PA4-4, 9, 10</td>
</tr>
<tr>
<td>[C, CN, PS, V] [ICT: C6-2.3]</td>
<td></td>
</tr>
<tr>
<td>2. Translate among different representations of a pattern, such as a table, a chart or concrete materials. [C, CN, V]</td>
<td>Part 1, Unit 5, Lessons PA4-9, 10</td>
</tr>
<tr>
<td>3. Represent, describe and extend patterns and relationships, using charts and tables, to solve problems. [C, CN, PS, R, V] [ICT: C6-2.3]</td>
<td>Part 1, Unit 5, Lessons PA4-11</td>
</tr>
<tr>
<td>4. Identify and explain mathematical relationships, using charts and diagrams, to solve problems. [CN, PS, R, V] [ICT: C6-2.3]</td>
<td>Part 1, Unit 4, Lessons G4-1, 2</td>
</tr>
</tbody>
</table>

### Patterns & Relations — Variables & Equations

**General Outcome**

Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Express a given problem as an equation in which a symbol is used to represent an unknown number. [CN, PS, R]</td>
<td>Part 2, Unit 11, Lessons PA4-12 to 18</td>
</tr>
<tr>
<td>6. Solve one-step equations involving a symbol to represent an unknown number. [C, CN, ME, PS, R, V]</td>
<td>Part 2, Unit 12, Lessons ME4-15</td>
</tr>
<tr>
<td>Note: Through this outcome, students have the opportunity to maintain and refine previously learned addition and subtraction number facts: Apply mental mathematics strategies and number properties in order to understand and recall basic addition facts and related subtraction facts to 18 (Grade 3).</td>
<td></td>
</tr>
</tbody>
</table>
### Shape & Space — Measurement

**General Outcome**

Use direct and indirect measurement to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> Read and record time, using digital and analog clocks, including 24-hour clocks. [C, CN, V]</td>
<td>Part 2  Unit 13  Lessons ME4-21 to 25</td>
</tr>
<tr>
<td><strong>2.</strong> Read and record calendar dates in a variety of formats. [C, V]</td>
<td>Part 2  Unit Lessons ME4-28</td>
</tr>
</tbody>
</table>
| **3.** Demonstrate an understanding of area of regular and irregular 2-D shapes by:  
• recognizing that area is measured in square units  
• selecting and justifying referents for the units cm² or m²  
• estimating area, using referents for cm² or m²  
• determining and recording area (cm² or m²)  
• constructing different rectangles for a given area (cm² or m²) in order to demonstrate that many different rectangles may have the same area. [C, CN, ME, PS, R, V] | Part 2  Unit 12  Lessons ME4-13 to 16 |

### Shape & Space — 3-D Objects & 2-D Shapes

**General Outcome**

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.</strong> Describe and construct right rectangular and right triangular prisms. [C, CN, R, V]</td>
<td>Part 2  Unit 14  Lessons G4-10 to 12</td>
</tr>
</tbody>
</table>

### Shape & Space — Transformations

**General Outcome**

Describe and analyze position and motion of objects and shapes.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
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</thead>
<tbody>
<tr>
<td><strong>5.</strong> Demonstrate an understanding of congruency, concretely and pictorially. [CN, R, V]</td>
<td>Part 1  Unit 4  Lessons G4-8</td>
</tr>
</tbody>
</table>
| **6.** Demonstrate an understanding of line symmetry by:  
• identifying symmetrical 2-D shapes  
• creating symmetrical 2-D shapes  
• drawing one or more lines of symmetry in a 2-D shape. [C, CN, V] | Part 1  Unit 4  Lessons G4-9 |
### Statistics & Probability — Data Analysis

#### General Outcome
Collect, display and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
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</thead>
<tbody>
<tr>
<td>1. Demonstrate an understanding of many-to-one correspondence.</td>
<td>Part 2 8 PDM4-1</td>
</tr>
<tr>
<td>[CN, R, T, V]</td>
<td></td>
</tr>
<tr>
<td>[ICT: C6-2.2, C6-2.3]</td>
<td></td>
</tr>
<tr>
<td>2. Construct and interpret pictographs and bar graphs involving many-to-one</td>
<td>Part 2 8 PDM4-2 to 5</td>
</tr>
<tr>
<td>correspondence to draw conclusions.</td>
<td></td>
</tr>
<tr>
<td>[C, PS, R, V]</td>
<td></td>
</tr>
</tbody>
</table>
Grade 4 JUMP Math Correlation to the New BC Curriculum

NOTES:

*Italicized* JUMP Math lessons contain prerequisite material required to meet the learning standard.

JUMP Math strands are represented by:

- **NS** Number Sense
- **ME** Measurement
- **G** Geometry
- **PA** Patterns and Algebra
- **PDM** Probability and Data Management

### Big Ideas

- Fractions and decimals are types of **numbers** that can represent quantities.
- Development of computational **fluency** and multiplicative thinking requires analysis of patterns and relations in multiplication and division.
- Regular changes in **patterns** can be identified and represented using tools and tables.
- Polygons are closed shapes with similar **attributes** that can be described, measured, and compared.
- Analyzing and interpreting experiments in **data** probability develops an understanding of chance.

### Content JUMP Math Lessons

<table>
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<th>JUMP Math Lessons</th>
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<td><strong>number concepts</strong> to 1000</td>
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<tr>
<td>• counting:</td>
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</tr>
<tr>
<td>° multiples</td>
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</tr>
<tr>
<td>° flexible counting strategies</td>
<td></td>
</tr>
<tr>
<td>° whole number benchmarks</td>
<td></td>
</tr>
<tr>
<td>• Numbers to 10 000 can be arranged and recognized:</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>° comparing and ordering numbers</td>
<td>Part 1  Unit 1    Lessons NS4-7 to 10</td>
</tr>
<tr>
<td>° estimating large quantities</td>
<td>Part 1  Unit 1    Lessons NS4-2</td>
</tr>
<tr>
<td>• place value:</td>
<td>Part 1  Unit 1    Lessons NS4-3 to 6</td>
</tr>
<tr>
<td>• 1000s, 100s, 10s, and 1s</td>
<td>Part 1  Unit 1    Lessons NS4-3 to 6</td>
</tr>
<tr>
<td>° understanding the relationship between digit places and their value, to 10 000</td>
<td>Part 1  Unit 1    Lessons NS4-3 to 6</td>
</tr>
</tbody>
</table>

| decimals to hundredths                                                 | Part 2  Unit 9    Lessons NS4-45, 49, 50 |
| • Fractions and decimals are numbers that represent an amount or quantity. | Part 2  Unit 10   Lessons NS4-45, 46, 52, 53, 59, 60, 62 |
| • Fractions and decimals can represent parts of a region, set, or linear model. | Part 2  Unit 9    Lessons NS4-45, 49 |
| • Fractional parts and decimals are equal shares or equal-sized portions of a whole or unit. | Part 2  Unit 10   Lessons NS4-52, 53, 59, 60, 62 |
| ° understanding the relationship between fractions and decimals         | Part 2  Unit 10   Lessons NS4-45, 46 |

<p>| ordering and comparing fractions                                        | Part 2  Unit 9    Lessons NS4-45, 46, 49 |
| • comparing and ordering of fractions with common denominators          | Part 2  Unit 9    Lessons NS4-46 |
| • estimating fractions with benchmarks (e.g., zero, half, whole)         | Part 2  Unit 9    Lessons NS4-46 |</p>
<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
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</thead>
<tbody>
<tr>
<td>using concrete and visual models</td>
<td>Part 2  Unit 9  Lessons NS4-45, 46, 49</td>
</tr>
<tr>
<td>equal partitioning</td>
<td>Part 2  Unit 9  Lessons NS4-45, 46</td>
</tr>
<tr>
<td><strong>addition and subtraction</strong> to 10 000</td>
<td>Part 1  Unit 2  Lessons NS4-11 to 20</td>
</tr>
<tr>
<td>using flexible computation strategies, involving taking apart</td>
<td>Part 1  Unit 2  Lessons NS4-11 to 19</td>
</tr>
<tr>
<td>(e.g., decomposing using friendly numbers and compensating) and</td>
<td></td>
</tr>
<tr>
<td>combining numbers in a variety of ways, regrouping</td>
<td></td>
</tr>
<tr>
<td>estimating sums and differences to 10 000</td>
<td>Part 1  Unit 2  Lessons NS4-20</td>
</tr>
<tr>
<td>using addition and subtraction in real-life contexts and</td>
<td>Part 1  Unit 2  Lessons NS4-13, 20</td>
</tr>
<tr>
<td>problem-based situations</td>
<td></td>
</tr>
<tr>
<td>whole-class number talks</td>
<td>Part 1  Unit 2  Lessons NS4-20</td>
</tr>
<tr>
<td>Part 1  Unit 4  Lessons G4-4, 7</td>
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<tr>
<td><strong>multiplication and division</strong> of two- or three-digit numbers by</td>
<td>Part 1  Unit 5  Lessons PA4-1 to 3</td>
</tr>
<tr>
<td>one-digit numbers</td>
<td>Part 1  Unit 6  Lessons NS4-22 to 30</td>
</tr>
<tr>
<td>understanding the relationships between multiplication and</td>
<td>Part 1  Unit 7  Lessons NS4-31 to 37, 39 to 41, 42, 44</td>
</tr>
<tr>
<td>division, multiplication and addition, division and subtraction</td>
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</tr>
<tr>
<td>using flexible computation strategies (e.g., decomposing,</td>
<td>Part 1  Unit 5  Lessons PA4-1, 2</td>
</tr>
<tr>
<td>distributive principle, commutative principle, repeated addition</td>
<td>Part 1  Unit 6  Lessons NS4-22, 33</td>
</tr>
<tr>
<td>and repeated subtraction)</td>
<td>Part 1  Unit 7  Lessons NS4-31, 32, 34 to 36, 41</td>
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<tr>
<td></td>
<td>Part 1  Unit 5  Lessons PA4-1, 3</td>
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<td></td>
<td>Part 1  Unit 6  Lessons NS4-23 to 27, 29</td>
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<tr>
<td></td>
<td>Part 1  Unit 7  Lessons NS4-33, 34, 39 to 41</td>
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<td>JUMP Math Lessons</td>
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</tr>
<tr>
<td>• using multiplication and division in real-life contexts and</td>
<td>Part</td>
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<tr>
<td>problem-based situations</td>
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<td>• whole-class number talks</td>
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<tr>
<td>addition and subtraction of <strong>decimals</strong> to hundredths</td>
<td>Part</td>
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<tr>
<td>• estimating decimal sums and differences</td>
<td>Part</td>
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<tr>
<td>• using visual models, such as base 10 blocks, place-value mats,</td>
<td>Part</td>
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<tr>
<td>grid paper, and number lines</td>
<td>2</td>
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<tr>
<td>• using addition and subtraction in real-life contexts and problem-</td>
<td>Part</td>
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<td>based situations</td>
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<tr>
<td>• whole-class number talks</td>
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<tr>
<td>addition and subtraction facts to 20 (developing **computational</td>
<td>Part</td>
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<tr>
<td>fluency)</td>
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<td></td>
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<tr>
<td>• Provide opportunities for authentic practice, building on previous</td>
<td>Part</td>
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<tr>
<td>grade-level addition and subtraction facts.</td>
<td>1</td>
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<tr>
<td>• flexible use of mental math strategies</td>
<td>Part</td>
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<tr>
<td>multiplication and division <strong>facts</strong> to 100 (introductory computational</td>
<td>Part</td>
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<tr>
<td>strategies)</td>
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<td>Content</td>
<td>JUMP Math Lessons</td>
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<tr>
<td>• Provide opportunities for concrete and pictorial representations of multiplication.</td>
<td>Part</td>
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<td>• building computational fluency</td>
<td>Part</td>
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<tr>
<td>• Use games to provide opportunities for authentic practice of multiplication computations.</td>
<td>Part</td>
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<tr>
<td>• looking for patterns in numbers, such as in a hundred chart, to further develop understanding of multiplication computation</td>
<td>Part</td>
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<tr>
<td>• Connect multiplication to skip-counting.</td>
<td>Part</td>
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<tr>
<td>• Connecting multiplication to division and repeated addition.</td>
<td>Part</td>
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<tr>
<td>• Memorization of facts is not intended for this level.</td>
<td>Part</td>
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<tr>
<td>• Students will become more fluent with these facts.</td>
<td>Part</td>
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<tr>
<td>• using mental math strategies, such as doubling or halving</td>
<td>Part</td>
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<td></td>
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<tr>
<td>• Students should be able to recall the following multiplication facts by the end of Grade 4 (2s, 5s, 10s).</td>
<td>Part</td>
</tr>
<tr>
<td>increasing and decreasing patterns, using tables and charts</td>
<td>1</td>
</tr>
<tr>
<td>• Change in patterns can be represented in charts, graphs, and tables.</td>
<td>Part</td>
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<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
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<td>------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>• using words and numbers to describe increasing and decreasing patterns</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1      5    PA4-5 to 7, 11</td>
</tr>
<tr>
<td>• fish stocks in lakes, life expectancies</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1      5    PA4-11</td>
</tr>
<tr>
<td><strong>algebraic relationships</strong> among quantities</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1      5    PA4-8 to 11</td>
</tr>
<tr>
<td></td>
<td>1      7    NS4-37</td>
</tr>
<tr>
<td></td>
<td>2      11   PA4-12 to 18</td>
</tr>
<tr>
<td>• representing and explaining one-step equations with an unknown number</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2      11   PA4-12 to 18</td>
</tr>
<tr>
<td>• describing pattern rules, using words and numbers from concrete and</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td>pictorial representations</td>
<td>2      5    PA4-8 to 10</td>
</tr>
<tr>
<td>• planning a camping or hiking trip; planning for quantities and</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td>materials needed per individual and group over time</td>
<td>1      5    PA4-11</td>
</tr>
<tr>
<td></td>
<td>1      7    NS4-37</td>
</tr>
<tr>
<td><strong>one-step equations</strong> with an unknown number, using all operations</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2      11   PA4-12 to 18</td>
</tr>
<tr>
<td>• one-step equations for all operations involving an unknown number</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td>(e.g., ____ + 4 = 15, 15 - □ = 11)</td>
<td>2      11   PA4-12 to 17</td>
</tr>
<tr>
<td>• start unknown (e.g., n + 15 = 20; 20 - 15 = □)</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2      11   PA4-12, 14 to 16</td>
</tr>
<tr>
<td>• change unknown (e.g., 12 + n = 20)</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2      11   PA4-12, 14 to 16</td>
</tr>
<tr>
<td>• result unknown (e.g., 6 + 13 = ____ )</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2      11   PA4-12, 14 to 16</td>
</tr>
<tr>
<td><strong>how to tell time</strong> with analog and digital clocks, using 12- and 24-</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td>hour clocks</td>
<td>2      13   ME4-21 to 25</td>
</tr>
<tr>
<td>• understanding how to tell time with analog and digital clocks,</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td>using 12- and 24-hour clocks</td>
<td>2      13   ME4-21, 23</td>
</tr>
<tr>
<td>• understanding the concept of a.m. and p.m.</td>
<td>Part   Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2      13   ME4-22</td>
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<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
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<tr>
<td>-----------------------------------------------------------------------</td>
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<tr>
<td>• understanding the number of minutes in an hour</td>
<td>Part 2  Unit 13</td>
</tr>
<tr>
<td>• understanding the concepts of using a circle and of using fractions in telling time (e.g., half past, quarter to)</td>
<td>Part 2  Unit 13</td>
</tr>
<tr>
<td>• telling time in five-minute intervals</td>
<td>Part 2  Unit 13</td>
</tr>
<tr>
<td>• telling time to the nearest minute</td>
<td>Part 2  Unit 13</td>
</tr>
<tr>
<td>• First Peoples use of numbers in time and seasons, represented by seasonal cycles and moon cycles (e.g., how position of sun, moon, and stars is used to determine times for traditional activities, navigation)</td>
<td>Part 2  Unit 13</td>
</tr>
<tr>
<td>regular and irregular polygons</td>
<td>Part 1  Unit 4</td>
</tr>
<tr>
<td>• describing and sorting regular and irregular polygons based on multiple attributes</td>
<td>Part 1  Unit 4</td>
</tr>
<tr>
<td>• investigating polygons (polygons are closed shapes with similar attributes)</td>
<td>Part 1  Unit 4</td>
</tr>
<tr>
<td>• Yup’ik border patterns</td>
<td>Part 1  Unit 4</td>
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<tr>
<td>perimeter of regular and irregular shapes</td>
<td>Part 2  Unit 12</td>
</tr>
<tr>
<td>• using geoboards and grids to create, represent, measure, and calculate perimeter</td>
<td>Part 2  Unit 12</td>
</tr>
<tr>
<td>line symmetry</td>
<td>Part 1  Unit 4</td>
</tr>
<tr>
<td>• using concrete materials such as pattern blocks to create designs that have a mirror image within them</td>
<td>Part 1  Unit 4</td>
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<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
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<td>------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>• First Peoples art, borders, birchbark biting, canoe building</td>
<td>Part 1 Unit 4 G4-9</td>
</tr>
<tr>
<td>• Visit a structure designed by First Peoples in the local community</td>
<td>Part 1 Unit Lessons</td>
</tr>
<tr>
<td>and have the students examine the symmetry, balance, and patterns</td>
<td></td>
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<tr>
<td>within the structure, then replicate simple models of the architecture</td>
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<tr>
<td>focusing on the patterns they noted in the original.</td>
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</tbody>
</table>

| one-to-one correspondence and many-to-one correspondence,              | Part 2 Unit Lessons|
| using bar graphs and pictographs                                       |                   |
| • many-to-one correspondence: one symbol represents a group or value  | Part 2 Unit Lessons|
| (e.g., on a bar graph, one square may represent five cookies)         |                   |

| probability experiments                                               | Part 2 Unit Lessons|
| • predicting single outcomes (e.g., when you spin using one spinner   | Part 2 Unit Lessons|
| and it lands on a single colour)                                      |                   |
| • using spinners, rolling dice, pulling objects out of a bag          | Part 2 Unit Lessons|
| • recording results using tallies                                     | Part 2 Unit Lessons|
| • Dene/Kaska hand games, Lahal stick games                           | Part 2 Unit Lessons|

| financial literacy — monetary calculations, including making change   | Part 2 Unit Lessons|
| with amounts to 100 dollars and making simple financial decisions     |                   |
| • making monetary calculations, including decimal notation in real-   | Part 2 Unit Lessons|
| life contexts and problem-based situations                            |                   |
| • applying a variety of strategies, such as counting up, counting    | Part 2 Unit Lessons|
| back, and decomposing, to calculate totals and make change            |                   |
| • making simple financial decisions involving earning, spending,      | Part 2 Unit Lessons|
| saving, and giving                                                   |                   |
| • equitable trade rules                                              | Part 2 Unit Lessons|

V-8
The Curricular Competencies in the new BC Mathematics curriculum are addressed throughout JUMP Math's Grade 4 resource. The following table lists a selection of JUMP Math lessons that provide effective illustrations of how each Curricular Competency is addressed.

<table>
<thead>
<tr>
<th>Curricular Competencies</th>
<th>JUMP Math Lessons</th>
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<tbody>
<tr>
<td><strong>Reasoning and analyzing</strong></td>
<td></td>
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<tr>
<td>• Use reasoning to explore and make connections</td>
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<tr>
<td>Part Unit Lessons</td>
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<tr>
<td>1 5 PA4-11</td>
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<tr>
<td>2 10 NS4-53</td>
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<tr>
<td>• Estimate reasonably</td>
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<td>Part Unit Lessons</td>
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<tr>
<td>1 1 NS4-2</td>
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<tr>
<td>2 10 NS4-57</td>
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<tr>
<td>• Develop <em>mental math strategies</em> and abilities to make sense of quantities</td>
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<td>Part Unit Lessons</td>
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<tr>
<td>1 1 NS4-10</td>
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<td>2 9 NS4-46</td>
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<tr>
<td>• Use <em>technology</em> to explore mathematics</td>
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<td>Part Unit Lessons</td>
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<tr>
<td>1 5 PA4-5</td>
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<td>2 8 PDM4-5</td>
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<tr>
<td>• Model mathematics in contextualized experiences</td>
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<td>Part Unit Lessons</td>
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<td>1 7 NS4-32</td>
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<tr>
<td>2 13 G4-22</td>
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<tr>
<td><strong>Understanding and solving</strong></td>
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<tr>
<td>• Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving</td>
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<tr>
<td>Part Unit Lessons</td>
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<tr>
<td>1 5 PA4-11</td>
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<td>2 12 ME-9</td>
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<tr>
<td>• Visualize to explore mathematical concepts</td>
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<td>Part Unit Lessons</td>
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<td>1 5 PA4-2</td>
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<tr>
<td>2 11 PA4-17</td>
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<tr>
<td>• Develop and use <em>multiple strategies</em> to engage in problem solving</td>
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<td>Part Unit Lessons</td>
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<tr>
<td>1 7 NS4-37</td>
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<td>2 10 NS4-60</td>
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</table>
### Curricular Competencies

- Engage in problem-solving experiences that are connected to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>G4-9</td>
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<tr>
<td>2</td>
<td>8</td>
<td>PDM4-2</td>
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</table>

### Communicating and representing

- **Communicate** mathematical thinking in many ways

<table>
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<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>G4-7</td>
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<td>2</td>
<td>12</td>
<td>ME4-10</td>
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</table>

- Use mathematical vocabulary and language to contribute to mathematical discussions

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>NS4-30</td>
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<tr>
<td>2</td>
<td>11</td>
<td>PA4-12</td>
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</table>

- **Explain and justify** mathematical ideas and decisions

<table>
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<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>NS4-42</td>
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<tr>
<td>2</td>
<td>8</td>
<td>PDM4-3</td>
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</tbody>
</table>

- Represent mathematical ideas in **concrete, pictorial, and symbolic forms**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>NS4-7</td>
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<tr>
<td>2</td>
<td>11</td>
<td>PA4-15</td>
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</table>

### Connecting and reflecting

- **Reflect** on mathematical thinking

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>NS4-42</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>NS4-58</td>
</tr>
</tbody>
</table>

- Connect mathematical concepts to each other and to **other areas and personal interests**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>PA4-11</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>PDM4-4</td>
</tr>
</tbody>
</table>

- **Incorporate** First Peoples worldviews and perspectives to make connections to mathematical concepts

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>G4-9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>ME-29</td>
</tr>
</tbody>
</table>
Grade 4 JUMP Math Correlation to the Manitoba Curriculum

NOTES:

JUMP Math strands are represented by:

- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

### Number

#### General Learning Outcome

Develop number sense.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.N.1</strong> Represent and describe whole numbers to 10 000, pictorially and symbolically.</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>[C, CN, V]</td>
<td>1 1 NS4-3 to 6</td>
</tr>
<tr>
<td><strong>4.N.2</strong> Compare and order numbers to 10 000.</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>[C, CN]</td>
<td>1 1 NS4-7, 8</td>
</tr>
<tr>
<td></td>
<td>2 10 NS4-52</td>
</tr>
<tr>
<td><strong>4.N.3</strong> Demonstrate an understanding of addition of numbers with answers to 10 000 and</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>their corresponding subtractions (limited to 3- and 4-digit numerals), concretely,</td>
<td></td>
</tr>
<tr>
<td>pictorially, and symbolically, by</td>
<td></td>
</tr>
<tr>
<td>• using personal strategies</td>
<td></td>
</tr>
<tr>
<td>• using the standard algorithms</td>
<td></td>
</tr>
<tr>
<td>• estimating sums and differences</td>
<td></td>
</tr>
<tr>
<td>• solving problems</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME, PS, R]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1 NS4-1, 2, 9, 10</td>
</tr>
<tr>
<td></td>
<td>1 2 NS4-11 to 20</td>
</tr>
<tr>
<td></td>
<td>1 7 NS4-32, 33</td>
</tr>
<tr>
<td><strong>4.N.4</strong> Explain the properties of 0 and 1 for multiplication, and the property of 1</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>for division.</td>
<td></td>
</tr>
<tr>
<td>[C, CN, R]</td>
<td>1 5 PA4-2</td>
</tr>
<tr>
<td></td>
<td>1 7 NS4-34, 35, 40</td>
</tr>
<tr>
<td>Number</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4.N.5</td>
<td>Describe and apply mental mathematics strategies, such as • skip-counting from a known fact • using doubling, halving • using doubling and adding one more group • using patterns in the 9s facts • using repeated doubling to develop an understanding of basic multiplication facts to $9 \times 9$ and related division facts. [C, CN, ME, PS, R]</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4.N.6</td>
<td>Demonstrate an understanding of multiplication (2- or 3-digit numerals by 1-digit numerals) to solve problems by • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products [C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4.N.7</td>
<td>Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication [C, CN, ME, PS, R, V]</td>
</tr>
<tr>
<td>4.N.8</td>
<td>Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used [C, CN, PS, R, V]</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4.N.9</td>
<td>Describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically. [C, CN, R, V]</td>
</tr>
<tr>
<td>Number</td>
<td>Relate decimals to fractions (to hundredths). [CN, R, V]</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------------</td>
</tr>
<tr>
<td>4.N.10</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number</th>
<th>Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by • using compatible numbers • estimating sums and differences • using mental math strategies to solve problems. [C, ME, PS, R, V]</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.N.11</td>
<td></td>
<td>2</td>
<td>10</td>
<td>NS4-56, 57, 61</td>
</tr>
</tbody>
</table>
Patterns and Relations (Patterns)

General Learning Outcome
Use patterns to describe the world and solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.PR.1</strong> Identify and describe patterns found in tables and charts, including a multiplication chart. [C, CN, PS, V]</td>
<td>Part 1 Unit 5 Lessons PA4-4, 9, 10</td>
</tr>
<tr>
<td><strong>4.PR.2</strong> Reproduce a pattern shown in a table or chart using concrete materials. [C, CN, V]</td>
<td>Part 1 Unit 5 Lessons PA4-9, 10</td>
</tr>
<tr>
<td><strong>4.PR.3</strong> Represent and describe patterns and relationships using charts and tables to solve problems. [C, CN, PS, R, V]</td>
<td>Part 1 Unit 5 Lessons PA4-11</td>
</tr>
<tr>
<td><strong>4.PR.4</strong> Identify and explain mathematical relationships using charts and diagrams to solve problems. [CN, PS, R, V]</td>
<td>Part 1 Unit 4 Lessons G4-1, 2</td>
</tr>
</tbody>
</table>

Patterns and Relations (Variables and Equations)

General Learning Outcome
Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.PR.5</strong> Express a problem as an equation in which a symbol is used to represent an unknown number. [CN, PS, R]</td>
<td>Part 2 Unit 11 Lessons PA4-12 to 18</td>
</tr>
<tr>
<td><strong>4.PR.6</strong> Solve one-step equations involving a symbol to represent an unknown number. [C, CN, PS, R, V]</td>
<td>Part 2 Unit 11 Lessons PA4-12 to 18</td>
</tr>
<tr>
<td><strong>4.PR.6</strong></td>
<td>Part 2 Unit 12 Lessons ME4-15</td>
</tr>
</tbody>
</table>
# Shape and Space (Measurement)

**General Learning Outcome**

Use direct or indirect measurement to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.SS.1</strong> Read and record time using digital and analog clocks, including 24-hour clocks. [C, CN, V]</td>
<td>Part 2 Unit 13 Lessons ME4-21 to 25</td>
</tr>
<tr>
<td><strong>4.SS.2</strong> Read and record calendar dates in a variety of formats. [C, V]</td>
<td>Part 2 Unit 13 Lessons ME4-28</td>
</tr>
<tr>
<td><strong>4.SS.3</strong> Demonstrate an understanding of area of regular and irregular 2-D shapes by • recognizing that area is measured in square units • selecting and justifying referents for the units cm² or m² • estimating area using referents for cm² or m² • determining and recording area (cm² or m²) • constructing different rectangles for a given area (cm² or m²) in order to demonstrate that many different rectangles may have the same area [C, CN, ME, PS, R, V]</td>
<td>Part 2 Unit 12 Lessons ME4-13 to 16</td>
</tr>
</tbody>
</table>

# Shape and Space (3-D Objects and 2-D Shapes)

**General Learning Outcome**

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.SS.4</strong> Solve problems involving 2-D shapes and 3-D objects. [CN, PS, V]</td>
<td>Part 1 Unit 4 Lessons G4-3, 8</td>
</tr>
<tr>
<td><strong>4.SS.5</strong> Describe and construct rectangular and triangular prisms. [C, CN, R, V]</td>
<td>Part 2 Unit 14 Lessons G4-10 to 12</td>
</tr>
</tbody>
</table>

# Shape and Space (Transformations)

**General Learning Outcome**

Describe and analyze position and motion of objects and shapes.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.SS.6</strong> Demonstrate an understanding of line symmetry by • identifying symmetrical 2-D shapes • creating symmetrical 2-D shapes • drawing one or more lines of symmetry in a 2-D shape [C, CN, V]</td>
<td>Part 1 Unit 4 Lessons G4-9</td>
</tr>
</tbody>
</table>
## Statistics and Probability (Data Analysis)

### General Learning Outcome
Collect, display, and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.SP.1</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>Demonstrate an understanding of many-to-one correspondence. [C, R, T, V]</td>
<td>2</td>
</tr>
<tr>
<td><strong>4.SP.2</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]</td>
<td>2</td>
</tr>
</tbody>
</table>
Grade 4 JUMP Math Correlation to the Ontario Curriculum

NOTES:

*Italicized* JUMP Math lessons contain prerequisite material required to meet the learning standard.

Expectation codes source: Ontario Curriculum Unit Planner

JUMP Math strands are represented by:

- **NS** Number Sense
- **PA** Patterns and Algebra
- **ME** Measurement
- **G** Geometry
- **PDM** Probability and Data Management

<table>
<thead>
<tr>
<th>Number Sense and Numeration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Expectations</td>
</tr>
<tr>
<td>4m8</td>
</tr>
<tr>
<td>4m9</td>
</tr>
<tr>
<td>4m10</td>
</tr>
<tr>
<td>4m11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specific Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity Relationships</td>
</tr>
<tr>
<td>4m12</td>
</tr>
<tr>
<td>4m13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m12</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4m13</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
### Number Sense and Numeration

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m14</td>
<td>read and print in words whole numbers to one thousand, using meaningful contexts (e.g., books, highway distance signs);</td>
<td>1</td>
<td>1</td>
<td>NS4-4</td>
</tr>
<tr>
<td>4m15</td>
<td>round four-digit whole numbers to the nearest ten, hundred, and thousand, in problems arising from real-life situations;</td>
<td>1</td>
<td>1</td>
<td>NS4-9, 10</td>
</tr>
<tr>
<td>4m16</td>
<td>represent, compare, and order decimal numbers to tenths, using a variety of tools (e.g., concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings) and using standard decimal notation <em>(Sample problem: Draw a partial number line that extends from 4.2 to 6.7, and mark the location of 5.6.)</em>;</td>
<td>2</td>
<td>10</td>
<td>NS4-52, 54, 55</td>
</tr>
<tr>
<td>4m17</td>
<td>represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered;</td>
<td>2</td>
<td>9</td>
<td>NS4-45, 49, 50</td>
</tr>
<tr>
<td>4m18</td>
<td>compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional parts (e.g., (\frac{4}{5}) is greater than (\frac{3}{5}) because there are more parts in (\frac{4}{5}); (\frac{1}{4}) is greater than (\frac{1}{5}) because the size of the part is larger in (\frac{1}{4});)</td>
<td>2</td>
<td>9</td>
<td>NS4-48</td>
</tr>
<tr>
<td>4m19</td>
<td>compare fractions to the benchmarks of 0, (\frac{1}{2}), and 1 (e.g., (\frac{1}{8}) is closer to 0 than to (\frac{1}{2}); (\frac{3}{5}) is more than (\frac{1}{2}));</td>
<td>2</td>
<td>9</td>
<td>NS4-46</td>
</tr>
<tr>
<td>4m19</td>
<td>demonstrate and explain the relationship between equivalent fractions, using concrete materials (e.g., fraction circles, fraction strips, pattern blocks) and drawings (e.g., “I can say that (\frac{3}{6}) of my cubes are white, or half the cubes are white. This means that (\frac{3}{6}) and (\frac{1}{2}) are equal.”);</td>
<td>2</td>
<td>9</td>
<td>NS4-46</td>
</tr>
<tr>
<td>4m21</td>
<td>read and represent money amounts to $100 (e.g., five dollars, two quarters, one nickel, and four cents is $5.59);</td>
<td>2</td>
<td>10</td>
<td>NS4-59 to 61 (\text{NS4-62})</td>
</tr>
</tbody>
</table>

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**X-2** JUMP Math Correlation to the Ontario Curriculum — Grade 4
### Number Sense and Numeration

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m22</td>
<td>solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 10 000 <em>(Sample problem: How high would a stack of 10 000 pennies be? Justify your answer.)</em></td>
<td>1</td>
<td>1</td>
<td>NS4-1, 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>ME4-5, 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>11</td>
<td>PA4-16</td>
</tr>
</tbody>
</table>

#### Counting

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m23</td>
<td>count forward by halves, thirds, fourths, and tenths to beyond one whole, using concrete materials and number lines (e.g., use fraction circles to count fourths: “One fourth, two fourths, three fourths, four fourths, five fourths, six fourths, …”);</td>
<td>2</td>
<td>9</td>
<td>NS4-45, 48</td>
</tr>
<tr>
<td>4m24</td>
<td>count forward by tenths from any decimal number expressed to one decimal place, using concrete materials and number lines (e.g., use base ten materials to represent 3.7 and count forward: 3.8, 3.9, 4.0, 4.1, …; “Three and seven tenths, three and eight tenths, three and nine tenths, four, four and one tenth, …” <em>(Sample problem: What connections can you make between counting by tenths and measuring lengths in millimetres and in centimetres?)</em>)</td>
<td>2</td>
<td>10</td>
<td>NS4-52, 54</td>
</tr>
</tbody>
</table>

#### Operational Sense

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m25</td>
<td>add and subtract two-digit numbers, using a variety of mental strategies (e.g., one way to calculate 73 − 39 is to subtract 40 from 73 to get 33, and then add 1 back to get 34);</td>
<td>1</td>
<td>2</td>
<td>NS4-11 to 14, 16 to 18, 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>7</td>
<td>NS4-32, 33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>11</td>
<td>PA4-14, 15</td>
</tr>
<tr>
<td>4m26</td>
<td>solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard algorithms (e.g., “I added 4217 + 1914 using 5000 + 1100 + 20 + 11.”);</td>
<td>1</td>
<td>2</td>
<td>NS4-11, 13, 15, 19, 20</td>
</tr>
<tr>
<td>4m27</td>
<td>add and subtract decimal numbers to tenths, using concrete materials (e.g., paper strips divided into tenths, base ten materials) and student-generated algorithms (e.g., “When I added 6.5 and 5.6, I took five tenths in fraction circles and added six tenths in fraction circles to give me one whole and one tenth. Then I added 6 + 5 + 1.1, which equals 12.1.”);</td>
<td>2</td>
<td>10</td>
<td>NS4-56, 57</td>
</tr>
<tr>
<td>4m28</td>
<td>add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools (e.g., currency manipulatives, drawings);</td>
<td>2</td>
<td>10</td>
<td>NS4-59 to 61</td>
</tr>
</tbody>
</table>

JUMP Math Correlation to the Ontario Curriculum — Grade 4

X-3
### Number Sense and Numeration

<p>| 4m29 | multiply to $9 \times 9$ and divide to $81 \div 9$, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting); |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>PA4-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PA4-2, 3</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NS4-21, 24</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>NS4-31, 33 to 35, 37, 40</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>PA4-13, 17</td>
</tr>
</tbody>
</table>

<p>| 4m30 | solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies (e.g., $6 \times 8$ can be thought of as $5 \times 8 + 1 \times 8$); |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>PA4-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PA4-2, 3</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NS4-30</td>
</tr>
</tbody>
</table>

<p>| 4m31 | multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule); |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>NS4-22</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>NS4-39, 40</td>
</tr>
</tbody>
</table>

<p>| 4m32 | multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., base ten materials or drawings of them, arrays), student-generated algorithms, and standard algorithms; |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>PA4-2, 3</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NS4-23, 25 to 27</td>
</tr>
</tbody>
</table>

<p>| 4m33 | divide two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings) and student-generated algorithms; |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>NS4-32, 34, 36, 38 to 41, 44</td>
</tr>
</tbody>
</table>

<p>| 4m34 | use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution (<strong>Sample problem:</strong> A school is ordering pencils that come in boxes of 100. If there are 9 classes and each class needs about 110 pencils, estimate how many boxes the school should buy.). |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS4-20</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NS4-29</td>
</tr>
</tbody>
</table>

### Proportional Relationships

<p>| 4m35 | describe relationships that involve simple whole-number multiplication (e.g., “If you have 2 marbles and I have 6 marbles, I can say that I have three times the number of marbles you have.”); |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>NS4-37</td>
</tr>
</tbody>
</table>
Number Sense and Numeration

<table>
<thead>
<tr>
<th>4m36</th>
<th>determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., decompose $\frac{2}{5}$ into $\frac{4}{10}$ by dividing each fifth into two equal parts showing that $\frac{2}{5}$ can be represented as 0.4);</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>2 10 NS4-52, 53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4m37</th>
<th>demonstrate an understanding of simple multiplicative relationships involving unit rates, through investigation using concrete materials and drawings (e.g., scale drawings in which 1 cm represents 2 m) (Sample problem: If 1 book costs $4, how do you determine the cost of 2 books? … 3 books? … 4 books?).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 7 NS4-37</td>
</tr>
<tr>
<td></td>
<td>2 12 ME-9, 19</td>
</tr>
</tbody>
</table>
## Measurement

### Overall Expectations

<table>
<thead>
<tr>
<th>Code</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m38</td>
<td>estimate, measure, and record length, perimeter, area, mass, capacity, volume, and elapsed time, using a variety of strategies;</td>
</tr>
<tr>
<td>4m39</td>
<td>determine the relationships among units and measurable attributes, including the area and perimeter of rectangles.</td>
</tr>
</tbody>
</table>

### Specific Expectations

#### Attributes, Units, and Measurement Sense

<table>
<thead>
<tr>
<th>Code</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m40</td>
<td>estimate, measure, and record length, height, and distance, using standard units (i.e., millimetre, centimetre, metre, kilometre) (e.g., a pencil that is 75 mm long);</td>
</tr>
<tr>
<td>4m41</td>
<td>draw items using a ruler, given specific lengths in millimetres or centimetres (Sample problem: Use estimation to draw a line that is 115 mm long. Beside it, use a ruler to draw a line that is 115 mm long. Compare the lengths of the lines.);</td>
</tr>
<tr>
<td>4m42</td>
<td>estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest minute;</td>
</tr>
<tr>
<td>4m43</td>
<td>estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in five-minute intervals, hours, days, weeks, months, or years (Sample problem: If you wake up at 7:30 a.m., and it takes you 10 minutes to eat your breakfast, 5 minutes to brush your teeth, 25 minutes to wash and get dressed, 5 minutes to get your backpack ready, and 20 minutes to get to school, will you be at school by 9:00 a.m.?);</td>
</tr>
<tr>
<td>4m44</td>
<td>estimate, measure using a variety of tools (e.g., centimetre grid paper, geoboard) and strategies, and record the perimeter and area of polygons;</td>
</tr>
<tr>
<td>4m45</td>
<td>estimate, measure, and record the mass of objects (e.g., apple, baseball, book), using the standard units of the kilogram and the gram;</td>
</tr>
<tr>
<td>4m46</td>
<td>estimate, measure, and record the capacity of containers (e.g., a drinking glass, a juice box), using the standard units of the litre and the millilitre;</td>
</tr>
</tbody>
</table>

#### JUMP Math Lessons

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>ME4-1 to 6</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>ME4-1 to 4, 6</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>ME4-24, ME4-25, 26</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>ME4-27, 29</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>ME4-9, 10, 13, 14, 16 to 18</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>ME4-7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>G4-17</td>
</tr>
</tbody>
</table>
## Measurement

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m47</td>
<td>estimate, measure using concrete materials, and record volume, and relate volume to the space taken up by an object (e.g., use centimetre cubes to demonstrate how much space a rectangular prism takes up) <em>(Sample problem:)</em> Build a rectangular prism using connecting cubes. Describe the volume of the prism using the number of connecting cubes.).</td>
</tr>
</tbody>
</table>

### Measurement Relationships

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m48</td>
<td>describe, through investigation, the relationship between various units of length (i.e., millimetre, centimetre, decimetre, metre, kilometre);</td>
</tr>
<tr>
<td>4m49</td>
<td>select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure the side lengths and perimeters of various polygons;</td>
</tr>
<tr>
<td>4m50</td>
<td>determine, through investigation, the relationship between the side lengths of a rectangle and its perimeter and area <em>(Sample problem:)</em> Create a variety of rectangles on a geoboard. Record the length, width, area, and perimeter of each rectangle on a chart. Identify relationships.;</td>
</tr>
<tr>
<td>4m51</td>
<td>pose and solve meaningful problems that require the ability to distinguish perimeter and area (e.g., “I need to know about area when I cover a bulletin board with construction paper. I need to know about perimeter when I make the border.”);</td>
</tr>
<tr>
<td>4m52</td>
<td>compare and order a collection of objects, using standard units of mass (i.e., gram, kilogram) and/or capacity (i.e., millilitre, litre);</td>
</tr>
<tr>
<td>4m53</td>
<td>determine, through investigation, the relationship between grams and kilograms <em>(Sample problem:)</em> Use centimetre cubes with a mass of one gram, or other objects of known mass, to balance a one-kilogram mass.;</td>
</tr>
<tr>
<td>4m54</td>
<td>determine, through investigation, the relationship between millilitres and litres <em>(Sample problem:)</em> Use small containers of different known capacities to fill a one-litre container.;</td>
</tr>
<tr>
<td>4m55</td>
<td>select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram) and the most appropriate standard unit to measure the capacity of a container (i.e., millilitre, litre);</td>
</tr>
<tr>
<td>Measurement</td>
<td>Part</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4m56 solve problems involving the relationship</td>
<td>2</td>
</tr>
<tr>
<td>between years and decades, and between decades</td>
<td></td>
</tr>
<tr>
<td>and centuries</td>
<td></td>
</tr>
<tr>
<td>*(Sample problem: How many decades old is</td>
<td></td>
</tr>
<tr>
<td>Canada?*)</td>
<td></td>
</tr>
<tr>
<td>4m57 compare, using a variety of tools (e.g.,</td>
<td>2</td>
</tr>
<tr>
<td>geoboard, patterns blocks, dot paper), two-</td>
<td></td>
</tr>
<tr>
<td>dimensional shapes that have the same perimeter</td>
<td></td>
</tr>
<tr>
<td>or the same area *(Sample problem: Draw, using</td>
<td></td>
</tr>
<tr>
<td>grid paper, as many different rectangles with</td>
<td></td>
</tr>
<tr>
<td>a perimeter of 10 units as you can make on a</td>
<td></td>
</tr>
<tr>
<td>geoboard.*)</td>
<td></td>
</tr>
</tbody>
</table>
### Geometry and Spatial Sense

#### Overall Expectations

<table>
<thead>
<tr>
<th>Code</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m58</td>
<td>identify quadrilaterals and three-dimensional figures and classify them by their geometric properties, and compare various angles to benchmarks;</td>
</tr>
<tr>
<td>4m59</td>
<td>construct three-dimensional figures, using two-dimensional shapes;</td>
</tr>
<tr>
<td>4m60</td>
<td>identify and describe the location of an object, using a grid map, and reflect two-dimensional shapes.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometric Properties</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>4m61</td>
<td>draw the lines of symmetry of two-dimensional shapes, through investigation using a variety of tools (e.g., Mira, grid paper) and strategies (e.g., paper folding) (Sample problem: Use paper folding to compare the symmetry of a rectangle with the symmetry of a square.);</td>
</tr>
<tr>
<td>4m62</td>
<td>identify and compare different types of quadrilaterals (i.e., rectangle, square, trapezoid, parallelogram, rhombus) and sort and classify them by their geometric properties (e.g., sides of equal length; parallel sides; symmetry; number of right angles);</td>
</tr>
<tr>
<td>4m63</td>
<td>identify benchmark angles (i.e., straight angle, right angle, half a right angle), using a reference tool (e.g., paper and fasteners, pattern blocks, straws), and compare other angles to these benchmarks (e.g., “The angle the door makes with the wall is smaller than a right angle but greater than half a right angle.”) (Sample problem: Use paper folding to create benchmarks for a straight angle, a right angle, and half a right angle, and use these benchmarks to describe angles found in pattern blocks.);</td>
</tr>
<tr>
<td>4m64</td>
<td>relate the names of the benchmark angles to their measures in degrees (e.g., a right angle is 90°);</td>
</tr>
<tr>
<td>4m65</td>
<td>identify and describe prisms and pyramids, and classify them by their geometric properties (i.e., shape of faces, number of edges, number of vertices), using concrete materials.</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
</tbody>
</table>

#### Geometric Relationships

<table>
<thead>
<tr>
<th>Geometric Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m66</td>
<td>construct a three-dimensional figure from a picture or model of the figure, using connecting cubes (e.g., use connecting cubes to construct a rectangular prism);</td>
</tr>
</tbody>
</table>
### Geometry and Spatial Sense

<table>
<thead>
<tr>
<th>4m67</th>
<th>construct skeletons of three-dimensional figures, using a variety of tools (e.g., straws and modelling clay, toothpicks and marshmallows, Polydrons), and sketch the skeletons;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>14</td>
<td>G4-11, 13, 14</td>
</tr>
<tr>
<td>4m68</td>
<td>draw and describe nets of rectangular and triangular prisms (<em>Sample problem:</em> Create as many different nets for a cube as you can, and share your results with a partner.);</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>14</td>
<td>G4-15</td>
</tr>
<tr>
<td>4m69</td>
<td>construct prisms and pyramids from given nets;</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>14</td>
<td>G4-12, 15</td>
</tr>
<tr>
<td>4m70</td>
<td>construct three-dimensional figures (e.g., cube, tetrahedron), using only congruent shapes.</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>14</td>
<td>G4-11, 12, 14, 15</td>
</tr>
</tbody>
</table>

### Location and Movement

<table>
<thead>
<tr>
<th>4m71</th>
<th>identify and describe the general location of an object using a grid system (e.g., “The library is located at A3 on the map.”);</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>12</td>
<td>ME4-20</td>
</tr>
<tr>
<td>4m72</td>
<td>identify, perform, and describe reflections using a variety of tools (e.g., Mira, dot paper, technology);</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>12</td>
<td>ME4-11, 12</td>
</tr>
<tr>
<td>4m73</td>
<td>create and analyse symmetrical designs by reflecting a shape, or shapes, using a variety of tools (e.g., pattern blocks, Mira, geoboard, drawings), and identify the congruent shapes in the designs.</td>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>4</td>
<td>G4-9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>12</td>
<td>ME4-11, 12</td>
</tr>
</tbody>
</table>
## Patterning and Algebra

### Overall Expectations

4m74  describe, extend, and create a variety of numeric and geometric patterns, make predictions related to the patterns, and investigate repeating patterns involving reflections;

4m75  demonstrate an understanding of equality between pairs of expressions, using addition, subtraction, and multiplication.

### Specific Expectations

#### Patterns and Relationships

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Description</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m76</td>
<td>extend, describe, and create repeating, growing, and shrinking number patterns (e.g., &quot;I created the pattern 1, 3, 4, 6, 7, 9, …. I started at 1, then added 2, then added 1, then added 2, then added 1, and I kept repeating this.&quot;);</td>
<td>Part 1 Unit 5 Lessons PA4-5 to 11</td>
</tr>
<tr>
<td>4m77</td>
<td>connect each term in a growing or shrinking pattern with its term number (e.g., in the sequence 1, 4, 7, 10, …, the first term is 1, the second term is 4, the third term is 7, and so on), and record the patterns in a table of values that shows the term number and the term;</td>
<td>Part 1 Unit 5 Lessons PA4-9 to 11</td>
</tr>
<tr>
<td>4m78</td>
<td>create a number pattern involving addition, subtraction, or multiplication, given a pattern rule expressed in words (e.g., the pattern rule “start at 1 and multiply each term by 2 to get the next term” generates the sequence 1, 2, 4, 8, 16, 32, 64,…);</td>
<td>Part 1 Unit 5 Lessons PA4-8, 11</td>
</tr>
<tr>
<td>4m79</td>
<td>make predictions related to repeating geometric and numeric patterns (Sample problem: Create a pattern block train by alternating one green triangle with one red trapezoid. Predict which block will be in the 30th place.);</td>
<td>Part 2 Unit 12 Lessons ME4-11, 12</td>
</tr>
<tr>
<td>4m80</td>
<td>extend and create repeating patterns that result from reflections, through investigation using a variety of tools (e.g., pattern blocks, dynamic geometry software, dot paper).</td>
<td>Part 2 Unit 12 Lessons ME4-11, 12</td>
</tr>
</tbody>
</table>

#### Expressions and Equality

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m81</td>
<td>determine, through investigation, the inverse relationship between multiplication and division (e.g., since 4 × 5 = 20, then 20 ÷ 5 = 4; since 35 ÷ 5 = 7, then 7 × 5 = 35);</td>
<td>Part 1 Unit 7 Lessons NS4-33 to 36, 11 Lessons PA4-13</td>
</tr>
<tr>
<td>4m82</td>
<td>determine the missing number in equations involving multiplication of one- and two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (Sample problem: What is the missing number in the equation □ × 4 = 24?);</td>
<td>Part 2 Unit 11 Lessons PA4-13, 17, 18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Part 2 Unit 12 Lessons ME4-15</td>
</tr>
</tbody>
</table>
**Patterning and Algebra**

<table>
<thead>
<tr>
<th>4m83</th>
<th>identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the commutative property of multiplication to facilitate computation with whole numbers (e.g., “I know that 15 \times 7 \times 2 equals 15 \times 2 \times 7. This is easier to multiply in my head because I get 30 \times 7 = 210.”);</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>5</td>
<td>PA4-2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4m84</th>
<th>identify, through investigation (e.g., by using sets of objects in arrays, by drawing area models), and use the distributive property of multiplication over addition to facilitate computation with whole numbers (e.g., “I know that 9 \times 52 equals 9 \times 50 + 9 \times 2. This is easier to calculate in my head because I get 450 + 18 = 468.”).</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>5</td>
<td>PA4-1</td>
</tr>
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<td></td>
<td></td>
<td>1</td>
<td>6</td>
<td>NS4-23, 25</td>
</tr>
</tbody>
</table>
## Data Management and Probability

### Overall Expectations

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m85</td>
<td>collect and organize discrete primary data and display the data using charts and graphs, including stem-and-leaf plots and double bar graphs;</td>
</tr>
<tr>
<td>4m86</td>
<td>read, describe, and interpret primary data and secondary data presented in charts and graphs, including stem-and-leaf plots and double bar graphs;</td>
</tr>
<tr>
<td>4m87</td>
<td>predict the results of a simple probability experiment, then conduct the experiment and compare the prediction to the results.</td>
</tr>
</tbody>
</table>

### Specific Expectations

#### Collection and Organization of Data

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m88</td>
<td>collect data by conducting a survey (e.g., “Choose your favourite meal from the following list: breakfast, lunch, dinner, other.”) or an experiment to do with themselves, their environment, issues in their school or the community, or content from another subject, and record observations or measurements;</td>
</tr>
<tr>
<td>4m89</td>
<td>collect and organize discrete primary data and display the data in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, simple spreadsheets, dynamic statistical software).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>PDM4-1, 6, 7</td>
</tr>
</tbody>
</table>

#### Data Relationships

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m90</td>
<td>read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., temperature data in the newspaper, data from the Internet about endangered species), presented in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs);</td>
</tr>
<tr>
<td>4m91</td>
<td>demonstrate, through investigation, an understanding of median (e.g., “The median is the value in the middle of the data. If there are two middle values, you have to calculate the middle of those two values.”), and determine the median of a set of data (e.g., “I used a stem-and-leaf plot to help me find the median.”);</td>
</tr>
<tr>
<td>4m92</td>
<td>describe the shape of a set of data across its range of values, using charts, tables, and graphs (e.g., “The data values are spread out evenly.”; “The set of data bunches up around the median.”);</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>PDM4-6, 7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>PDM4-8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
<td>PDM4-6, 9</td>
</tr>
</tbody>
</table>
### Data Management and Probability

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4m93</td>
<td>compare similarities and differences between two related sets of data, using a variety of strategies (e.g., by representing the data using tally charts, stem-and-leaf plots, or double bar graphs; by determining the mode or the median; by describing the shape of a data set across its range of values).</td>
<td>2</td>
<td>8</td>
<td>PDM4-6, 9</td>
</tr>
</tbody>
</table>

### Probability

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>4m94</td>
<td>predict the frequency of an outcome in a simple probability experiment, explaining their reasoning; conduct the experiment; and compare the result with the prediction <em>(Sample problem: If you toss a pair of number cubes 20 times and calculate the sum for each toss, how many times would you expect to get 12? 7? 1? Explain your thinking. Then conduct the experiment and compare the results with your predictions.)</em></td>
<td>2</td>
<td>9</td>
<td>NS4-50</td>
</tr>
<tr>
<td>4m95</td>
<td>determine, through investigation, how the number of repetitions of a probability experiment can affect the conclusions drawn <em>(Sample problem: Each student in the class tosses a coin 10 times and records how many times tails comes up. Combine the individual student results to determine a class result, and then compare the individual student results and the class result.)</em></td>
<td>2</td>
<td>15</td>
<td>PDM4-11 to 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>15</td>
<td>PDM4-13</td>
</tr>
</tbody>
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