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Unit 8 Patterns and Algebra: Variables, Expressions, and Equations

Introduction
This unit focuses on numerical expressions, variables, and equations. This unit describes how to:

- verify equations by calculating expressions on both sides of the equal sign;
- translate words into expressions;
- represent variables and algebraic expressions;
- solve easy equations using guess and check and writing the variable by itself; and
- solve word problems involving addition, subtraction, multiplication, and division using equations.

Meeting Your Curriculum

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Mental Math Minutes

The mental math minutes in this unit are dedicated to:

- multiplication and division skills using skip counting
- solving addition and subtraction equations by comparing the sides of an equation

Assessment
The lessons covered by a quiz or test are as follows:

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**Goals**

Students will verify equations by calculating the expressions on both sides of the equal sign and verifying that they are equal.

**PRIOR KNOWLEDGE REQUIRED**

Can add, subtract, multiply, and divide whole numbers

Knows operations inside brackets have priority

**Mental math minute.** Write “115 ÷ 5 = ____” on the board. SAY: Doubling both numbers in a division does not change the answer. If you double both numbers in this division, you get 230 ÷ 10 = 23. Write the second equation underneath the first and write “23” in the blank.

**Exercises:** Double both numbers in the division to find the answer.

a) 75 ÷ 5  b) 225 ÷ 5  c) 430 ÷ 5  d) 540 ÷ 5

**Answers:** a) 15, b) 45, c) 86, d) 108

**Introduce numerical expressions.** Explain that an expression shows or represents something. For example, a facial expression is when a person’s face shows or represents how that person feels. Demonstrate by making one or two facial expressions to show emotions and ask students to guess which emotions you are showing. SAY: A **numerical expression** represents calculations with numbers. For example, 2 + 3 is a numerical expression.

**Exercises:** Calculate the numerical expression.

a) 2 + 5 + 3  b) 5 × 3 × 2  c) 17 + 14 + 20  d) 5 + 9 – 3

**Answers:** a) 10, b) 30, c) 51, d) 11

**Review brackets.** Remind students that brackets in an expression tell you to do everything inside them before you do all the other operations. For example, 8 – (2 × 3) is calculated as 8 – 6 = 2, but (8 – 2) × 3 = 6 × 3 = 18.

**NOTE:** Students will learn the correct order of operations in a later grade. For now, only assign questions where doing the operations from left to right will give the correct order, or include brackets to ensure that the correct order is used.

Invite volunteers to solve these problems on the board:

(2 × 3) + 5  2 × (3 + 5)

(6 + 5 = 11, 2 × 8 = 16)

**Exercises:** Evaluate the expression.

a) (20 – 5) ÷ 3  b) 5 × (3 + 2)  c) 7 – (4 + 2)  d) 18 ÷ (2 + 4)
Answers: a) 5, b) 25, c) 1, d) 3

Introduce equations. Write on the board:

\[ 8 + 2 = 50 \div 5 \]

SAY: Both expressions are equal to 10. When two equal expressions are separated by an equal sign, we call it an equation. Add an equal sign between the expressions on the board:

\[ 8 + 2 = 50 \div 5 \]

SAY: To verify that an equation is true, you can calculate the numerical expressions on both sides of the equal sign and see whether they have the same value. For example, \( 1 + 2 = 7 - 4 \) is a true equation because both sides have the same value; \( 2 \times 3 = 2 + 3 \) is not true because the left side is equal to 6 and the right side is equal to 5.

Exercises

1. Verify that the equation is true.
   a) \( 20 - 5 = 5 \times 3 \)
   b) \( 5 \times (3 + 2) = 20 + 5 \)
   c) \( 9 + (4 \times 2) = 26 \)
   d) \( 1 + 3 + 5 = 3 \times 3 \)

   \textbf{Answers:} a) true, b) true, c) not true, d) true

2. What’s the mistake in Exercise 1, part c)?

   \textbf{Answer:} the multiplication in the brackets has to be done first:
   \[ 9 + (4 \times 2) = 9 + 8 = 17 \]

Bonus

3. a) Verify that each equation is true.
   \[ 1 + 3 + 5 + 7 = 4 \times 4 \]
   \[ 1 + 3 + 5 + 7 + 9 = 5 \times 5 \]
   \[ 1 + 3 + 5 + 7 + 9 + 11 = 6 \times 6 \]

   b) Look for a pattern in part a). Use the pattern to predict and then check:
   \[ 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = \_\_ \times \_\_ \]

   \textbf{Selected answer:} b) \( 8 \times 8 \)

4. a) Verify that each equation is true.
   \[ 7 + 6 + 10 = (7 + 3) + (6 - 2) + (10 - 1) \]
   \[ 8 + 5 + 12 = (8 + 4) + (5 + 3) + (12 - 7) \]

   b) Look for a pattern in part a). Use the pattern to write another similar equation. Evaluate the expressions on both sides of the equal sign to verify that your equation is true.

   \textbf{Selected sample answer:} b) \( 12 + 31 + 14 = (12 - 2) + (31 - 1) + (14 + 3) \)
Extensions

1. Add brackets where necessary to the equation to make it true. Hint: In some equations, you might need to add one set of brackets inside another.

   a) \(3 + 1 \times 7 - 2 = 20\)  
   b) \(3 + 1 \times 7 \times 2 = 56\)  
   c) \(8 - 4 \times 2 + 5 = 28\)  
   d) \(5 \times 4 - 3 + 2 = 7\)

   **Answers:** a) \((3 + 1) \times (7 - 2)\), b) \((3 + 1) \times 7 \times 2 = 56\),  
   c) \((8 - 4) \times (2 + 5) = 28\), d) \((5 \times (4 - 3)) + 2 = 7\)

2. Verify that the equation is true.

   a) \((5 - 2) \times (1 + 7) = 24\)  
   b) \((2 + 5) \times (8 - 3) = (6 \times 6) - 1\)  
   c) \((3 + 2) \times (9 - 1) \div 4 = 10\)  
   d) \((2 \times 5) + (2 \times 2) = (6 \div 3) \times (4 + 3)\)

3. Each blank represents a single digit. Find the missing digits.

   a) \((2 \times 1000) + (6 \times 100) + (\_ \times 10) + 5 = 2645\)  
   b) \(2 \_ \_ 7 = (\_ \times 100) + (3 \times 10) + \_ \_ \_ \)  
   c) \((3 \times 100\ 000) + (9 \times 10\ 000) + (\_ \times 1000) + (\_ \times 100) + \_ \_ \_ \)  
      \(= \_ \_ \_ \_ 90\ 7 \_ \_ 2\)

   **Answers:** a) 4; b) 3, 2, 7; c) 0, 7, 2, 3, 0
PA5-9  Unknown Quantities and Equations
Pages 2–4

Goals
Students will write and solve easy addition and subtraction equations using models.
Students will solve easy multiplication and division equations using the guess-and-check method and by writing the unknown by itself.

PRIOR KNOWLEDGE REQUIRED
Can add, subtract, multiply, and divide
Understands multiplication as repeated addition
Knows multiplication and division up to $10 \times 10$

MATERIALS
masking tape, string, or ruler
counters
paper bag
calculators

Mental math minute—number talk. Present this problem: Is the equation $11 \times 7 = 10 \times 8$ true? (no) The following strategies could arise:

- The left side is 77, and the right side is 80.
- The right side is a multiple of 10, but the left side is not.
- The left side is 7 more than $10 \times 7$, but the right side is 10 more than $10 \times 7$.

Solving algebraic equations. Divide a desk in half using masking tape, string, or a ruler. Put five counters on one side of the dividing line and put two counters and a paper bag containing three more counters on the other side. (Don’t let students see how many counters are in the bag.) Tell students that the number of counters in total is the same on both sides of the line. Have students guess how many counters are in the bag. (3) Repeat the example with different numbers of counters, but don’t use more than one bag.

Draw a representation of the first concrete model of the equation on the board. ASK: How many counters are on the right side? (5) How many counters can you see on the left side? (2) Write the numbers 2 and 5 under the corresponding number of circles (see example in margin).

SAY: The box represents the bag. To find how many circles are in the box, I put a circle in the box. Draw a circle inside the box. ASK: How many circles can you see on the left side in total? (3) Erase the number 2 and write 3 instead, but write 3 a little to the left, between the box and the two counters (see example in margin).
ASK: Is the number of circles on both sides equal? (no) SAY: I’m going to add another circle in the box. Draw another circle inside the box. ASK: How many circles can you see on the left side in total? (4) Erase the number 3 and write 4.

SAY: The number of circles on both sides is still not equal, so I’m going to put another circle in the box. Draw another circle inside the box. Erase the number 4 and write 5 (see example in margin).

ASK: How many circles are on the left side in total? (5) Is the number of circles on both sides equal? (yes) How many circles are in the box? (3) SAY: We can replace the dotted line with an equal sign because the two sides are equal. Erase the dotted line and draw an equal sign in its place, as shown in the margin, and SAY: This shows that the two sides are equal.

Have a student open the bag to check that there are three counters inside. Students should also see that they can find the number of hidden counters either by counting up from 2 to 5 or by subtracting 2 from 5. Have students check that the equation was solved correctly. ASK: Does the number of counters drawn in the box make the equation true? (yes)

Exercises: Draw circles in the box until the number of circles is the same on both sides.

a) \[ \square + 2 = 5 \]
b) \[ 5 + \square = 7 \]

Explain that it is inconvenient to draw counters all the time. SAY: We can use numbers to represent all the quantities instead. Have students practice writing equations that represent pictures, similar to Question 2 on AP Book 5.2 p. 2. For example, the equation for the first example in this lesson is \[ \square + 2 = 5 \].

Challenge students to solve several more examples. Students can create models for equations that involve addition using counters. They can also draw models; in this case, ask students to use a box for the unknown (the “hidden” number or the number we don’t know) and use a set of circles to model the numbers in the equation. For example, a model for the equation \[ \square + 2 = 7 \] is shown in the margin.

Ask students to make a model with boxes and circles to solve the following exercises. They should explain how many circles they would put in each box to make the equation true.

**Exercises**

a) \[ 7 + \square = 11 \]  
b) \[ 6 + \square = 13 \]

c) \[ 4 + \square = 10 \]  
d) \[ 9 + \square = 12 \]

**Answers:** a) 4, b) 7, c) 6, d) 3

**Equations with addition.** Read several word problems to students. Invite volunteers to draw models, write equations using boxes and numbers, and solve the equations. Some examples are shown on the next page.
a) There are 10 trees in the garden. Three of them are apple trees. All the rest are cherry trees. How many cherry trees are in the garden?

b) Jane has 12 T-shirts. Three of them are plain. All the rest have designs. How many of Jane’s T-shirts have designs?

**Exercises:** Write an equation to solve the problem.

a) There are 15 plants in the flowerbed. Six are lilies. All the rest are peonies. How many peonies are in the flowerbed?

**Bonus**

b) There are 150 pirates on two ships, a galleon and a schooner. Forty of the pirates are on the schooner. How many are on the galleon?

c) A dragon has 15 heads. A knight cut off some of the heads. The dragon has seven remaining heads. How many remaining heads did the knight remove?

**Equations with subtraction.** Present this word problem: Simon has a box of apples. He took two apples from the box. Four were left. How many apples were in the box before he removed any?

Draw the box with four apples in it. Draw two more apples in the box and cross them out to show that they have been taken away (see example in margin).

ASK: How many apples were in the box at the beginning? (6) Explain that, when we write a subtraction equation, we draw it a bit differently than the addition equations we drew earlier. We draw a box for the number we don’t know (the original number), show a minus sign, and then draw the apples we took out of the box. We show the four apples that were left in the box on the other side of the equation (see example in margin).

SAY: To solve the equation, we have to put all the apples into the box—the ones that we took out and the ones that we left there. Remind students that they also learned to write equations using numbers. ASK: How could you write this equation using numbers? (\(9 - 2 = 4\))

Draw several models for subtraction equations (like those in Questions 4 and 5 on AP Book 5.2 pp. 2–3) and ask students to write the equations for them. Ask volunteers to present the answers on the board.

**Exercises:** Draw the model to solve the equation.

a) \( - 6 = 9 \)  b) \( - 7 = 12 \)  c) \( - 5 = 3 \)  d) \( - 3 = 10 \)

**Guessing and checking.** Tell students that someone tried to solve the equation \(9 + \square = 28\) by guessing and checking. The first guess was 16. Write “16” in the box and then ask students to add 9 + 16 to see if the equation is true. (no, 9 + 16 = 25) ASK: Should the next guess be more than 16 or less than 16? (more) Students can indicate their answer with thumbs up or thumbs down. Have a volunteer try 15 for the unknown. ASK: Did we get closer to 28? (no) Have another volunteer try 17. ASK: Did we get closer to 28? (yes) Have students make more guesses until they
find the answer. (19) Repeat with the equation □ + 8 = 31, with a first guess of 25.

**Exercises:** Solve the equation by guessing and checking.

a) 5 + □ = 13  
   b) □ + 7 = 22  
   c) □ - 8 = 4  
   **Bonus:** □ - □ = 14

**Answers:** a) 8, b) 15, c) 12, Bonus: 17

**Equations with multiplication.** Tell students that they can also write equations for multiplication problems. Remind students that “2 ×” means that some quantity will be two times as large—in other words, it will be doubled. Examples:

\[ 2 \times □ = \begin{array}{cc} 0 & \vdots & 0 & \vdots & 0 & \vdots & 0 & \vdots & 0 & \vdots & 0 & \vdots & 0 \\
\end{array} \quad \text{and} \quad 2 \times \begin{array}{cc} 0 & \vdots & 0 & \vdots & 0 & \vdots & 0 & \vdots & 0 \\
\end{array} \]

Present the problem shown in the margin and ask students to draw the appropriate number of circles in the box. (6) Students should solve the problem by dividing the circles on the right side into two equal groups.

**ASK:** How would you write an equation with numbers for this problem? (2 × □ = 12)

**Solving multiplication equations (missing second factor) without using a diagram.** **ASK:** What mathematical operation are you performing when you split circles into equal groups? (division) Have students write the division equations for the problem they solved above using a diagram. Is the number of circles they drew in the diagram the same as the number they wrote in the equation? (It should be!)

**NOTE:** Students can use a calculator to check their answers for the Bonus exercises below.

**Exercises:** Solve the multiplication equation by rewriting it using division.

a) 3 × □ = 24  
   b) 5 × □ = 45  
   c) 6 × □ = 42  
   d) 9 × □ = 72  
   e) 3 × □ = 240  
   f) 5 × □ = 4500  
   g) 6 × □ = 42 000  
   h) 9 × □ = 720 000

**Answers:** a) 8, b) 9, c) 7, d) 8, Bonus: e) 80, f) 900, g) 7000, h) 80 000

**Using division to solve multiplication equations of different types.** **SAY:** Multiplication is commutative. Write the two equations below on the board and ask students whether they should have the same answer.

\[ 3 \times □ = 27 \quad \text{□} \times 3 = 27 \]

Ask students to explain their thinking. Remind them that the box replaces a missing number, so they can do with the box all the same things they would do with a number. **ASK:** How would you solve the first equation? (27 ÷ 3 = 9)
Would the same solution work for the second equation? (yes) Why? (because order does not matter in multiplication, we can just switch the numbers we are multiplying together)

**Exercises:** Solve the multiplication equation by rewriting it using division.

a) \( \square \times 3 = 21 \)  
b) \( \square \times 4 = 48 \)  
c) \( \square \times 7 = 63 \)  
d) \( \square \times 8 = 56 \)

**Bonus**

\( e) \square \times 3 = 3000 \)  
\( f) \square \times 4 = 1200 \)

\( g) \square \times 7 = 4200 \)  
\( h) \square \times 8 = 24000 \)

**Answers:** a) 7, b) 12, c) 9, d) 7, Bonus: e) 1000, f) 300, g) 600, h) 3000

**Solving equations with a missing dividend.** Present the equation \( \square \div 5 = 7 \). ASK: What number divided by 5 gives you 7? (35) How did you get 35 from 5 and 7? (by multiplying) Point out that the equation describes this situation: You had some apples that you divided into 5 equal groups. There are 7 apples in each group. Draw 5 circles and 7 dots in one circle, then ASK: What would you draw to find the total number of apples? (7 dots in each of the remaining circles) How would you then find the total number of dots? (multiply 5 \times 7) Emphasize that to find the number that is being divided into groups, you multiply the number of groups by the number in each group.

**Exercises:** Solve the division equation by rewriting it using multiplication.

a) \( \square \div 4 = 10 \)  
b) \( \square \div 6 = 3 \)  
c) \( \square \div 4 = 8 \)  
d) \( \square \div 5 = 12 \)

**Answers:** a) 40, b) 18, c) 32, d) 60

**Solving equations with a missing divisor.** Have students solve several equations with a missing divisor, such as \( 45 \div \square = 5 \). (9) Students who still have trouble with multiplication and division facts and use guessing and checking to find the answer can use skip counting instead.

**Exercises:** Solve the equation.

a) \( 25 = 5 \times \square \)  
b) \( \square = 6 \times 3 \)  
c) \( 35 \div \square = 7 \)

d) \( \square \times 3 = 15 \)  
e) \( \square \div 2 = 8 \)  
f) \( 21 \div 3 = \square \)

**Answers:** a) \( \square = 5 \), b) \( \square = 18 \), c) \( \square = 5 \), d) \( \square = 5 \), e) \( \square = 16 \), f) \( \square = 7 \)

**Rewriting equations with the unknown by itself.** Write on the board:

\( 5 \times \square = 35 \)

ASK: How can we rewrite this multiplication as division? (\( \square = 35 \div 5 \)) Write “\( \square = 35 \div 5 \)” on the board to the right of the first equation. Ask students to find the unknown in each equation. (7, 7) Explain to students that solving the equation on the right is easier because the unknown is by
itself, so to find the unknown, they just need to do an easy division with two known numbers. SAY: Writing the unknown by itself makes the equation easier to solve because you just need to calculate a numerical expression.

**Exercises:** Solve the equation by rewriting it with the unknown by itself.

a) $3 \times \Box = 12$

b) $\Box \div 7 = 6$

c) $\Box - 11 = 4$

**Bonus:** $36 \div \Box = 3$

**Selected answers:** a) $\Box = 12 \div 3$, $\Box = 4$; c) $\Box = 11 + 4$, $\Box = 15$;

**Bonus:** $\Box = 36 \div 3$, $\Box = 12$

**Extensions**

1. The same symbol in the equation means the same number. What does each symbol represent?

   a) $\Box + \Box = 12$

   b) $\Diamond + \Diamond + \Diamond = 9$

   c) $5 + \Box + \Box = 13$

   d) $9 + \Box + \Box + \Box = 15$

   **Answers:** a) 6, b) 3, c) 4, d) 2

2. Two birds each laid the same number of eggs. Seven eggs hatched, three did not. How many eggs did each bird lay?

   **Answer:** 5

3. Sixty baby alligators hatched from three alligator nests of the same size. We know that only half of the total number of eggs hatched. How many eggs were in each nest? Hint: How many eggs were laid in total?

   **Answer:** 40
Goals

Students will write easy addition, subtraction, multiplication, and division equations.

Students will translate between mathematical phrases written in words and numerical expressions.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, multiply, and divide

Mental math minute. SAY: Remember, an equal sign means “the same as.” To check if an equation is true, you can use what you know about multiplication and division without calculating both sides. For example, you know that doubling both numbers in division does not change the quotient. And you know that doubling one factor and halving another factor keeps the product the same.

Present the equations in the following exercises one at a time and have students signal the answers using thumbs up for “yes” and thumbs down for “no.”

Exercises: Is the equation true?

a) 24 ÷ 4 = 48 ÷ 8  b) 12 ÷ 3 = 36 ÷ 9  c) 4 × 3 = 8 × 6

d) 20 × 8 = 10 × 4  e) 34 ÷ 2 = 68 ÷ 4  f) 11 × 4 = 22 × 2

Answers: a) yes, b) yes, c) no, d) no, e) yes, f) yes

Associating words and phrases with operations. SAY: You can use clues to write expressions. The words give clues to the operations you need to use. On the board, make a table with four columns and these headings: Add, Subtract, Multiply, and Divide.

Have students discuss which operation each phrase makes them think of. Based on the class response, create a table on the board like the one below, with each phrase under its correct heading.

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<th>Subtract</th>
<th>Multiply</th>
<th>Divide</th>
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<td>increased by</td>
<td>less than difference</td>
<td>product times</td>
<td>divided by</td>
</tr>
<tr>
<td>sum</td>
<td>more than</td>
<td>twice as many</td>
<td>divided</td>
</tr>
<tr>
<td>more than</td>
<td>total</td>
<td>reduced by</td>
<td>share equally</td>
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<tr>
<td>total</td>
<td></td>
<td>fewer than</td>
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Exercises: Translate the phrase into an expression.

a) 5 more than 7  
   b) 5 less than 7

c) 5 times 7  
   d) the product of 7 and 5

e) 7 reduced by 5  
   f) 7 divided by 5

g) 5 divided by 7  
   h) 7 decreased by 5

i) 7 increased by 5  
   j) the sum of 7 and 5

k) 5 fewer than 7  
   l) the product of 5 and 7

Bonus: 3 multiplied by 7 then increased by 5

Answers: a) $7 + 5$, b) $7 - 5$, c) $5 \times 7$, d) $7 \times 5$, e) $7 - 5$, f) $7 \div 5$, g) $5 \div 7$, h) $7 - 5$, i) $7 + 5$, j) $7 + 5$, k) $7 - 5$, l) $5 \times 7$, Bonus: $(3 \times 7) + 5$

Associating phrases with expressions with brackets. Discuss expressions with more than one operation. Write on the board:

Multiply 2 and 3. Then subtract 1.

Ask a volunteer to write the numerical expression for the phrase. $(2 \times 3) - 1$.

Exercises: Translate the phrase into an expression with more than one operation. Use brackets to indicate which operation has to be done first.

a) Divide 6 by 2. Then add 3.  
   b) Add 4 and 6. Then divide by 5.

c) Multiply 5 and 4. Then add 2.  
   d) Divide 8 by 4. Then multiply by 3.

e) Subtract 2 from 5. Then multiply by 4. Then add 3.

Answers: a) $(6 \div 2) + 3$, b) $(4 + 6) \div 5$, c) $(5 \times 4) + 2$, d) $(8 \div 4) \times 3$, e) $((5 - 2) \times 4) + 3$

Writing mathematical expressions in words. Start with an easy expression. On the board, write “3 + 5” and ask students to read the expression. If some students answer “three plus five,” say that, rather than using the word “plus,” we prefer to use the verb “add.” Write on the board “add 3 and 5.”

Exercises: Write the operation in words.

a) $5 \times 2$  
   b) $7 - 4$  
   c) $4 + 7$  
   d) $15 \div 3$

Answers: a) multiply 5 by 2, b) subtract 4 from 7, c) add 4 and 7, d) divide 15 by 3

Teach students how to write a mathematical expression with two or more operations in words. Write on the board:

$(3 + 2) \times 4$

ASK: Which operation would you do first, addition or multiplication? (addition, because it’s in brackets) Write “Add 3 and 2” on the board. SAY: The first operation is done, so we have to end the sentence. Put a period to show the sentence is ended. ASK: What operation would you do
next? (multiplication) Ask a volunteer to write “Multiply by 4.” on the board, following your first sentence. (Add 3 and 2. Multiply by 4.)

**Exercises:** Write the mathematical expression in words.

a) \((5 + 1) \times 2\)  
b) \((9 - 3) \div 3\)  
c) \((4 \times 2) - 3 + 7\)

**Bonus:** \((6 + (3 \times 4)) \div 9\)

**Answers:** a) Add 5 and 1. Then multiply by 2. b) Subtract 3 from 9. Then divide by 3. c) Multiply 4 and 2. Then subtract 3. Then add 7.  
**Bonus:** Multiply 3 and 4. Then add 6. Then divide by 9.

**Interpreting expressions.** SAY: A parking lot charges $3 per hour, so you need to pay \(2 \times 3\) dollars for two hours. ASK: How much do you need to pay for four hours? \((4 \times 3)\) ASK: If the cost of parking is \(5 \times 3\), how many hours does that pay for? \((5 \text{ hours})\)

**Exercises:** To rent skates, you must pay $6 for each hour. Complete the meaning of the expression.

a) \(3 \times 6\) is the cost of renting skates for ___ hours.

b) \(2 \times 6\) is the cost of renting skates for ___ hours.

C) \(5 \times 6\) is the cost of renting skates for ___ hours.

**Answers:** a) 3, b) 2, c) 5

**Writing word problems as mathematical expressions.** SAY: A movie ticket costs $8 for adults and $5 for students. ASK: How much will it cost for two adults to watch the movie? \((2 \times 8)\) If some students answer “16,” say that we don’t want to calculate the expressions right now; instead, we just want to write a proper numerical expression that describes the problem. ASK: How much will it cost for three students? \((3 \times 5)\) How much will it cost for two adults and three students? \(((2 \times 8) + (3 \times 5))\)

**Exercises**

1. Six people can travel in one van. Sixteen students and two teachers go to the museum. Write an expression to show the number of vans that they need.

   **Answer:** \((16 + 2) \div 6\)

2. Kim wants to buy a video game that costs $45. Kim has already saved $9.

   a) Write an expression to show how much money she needs to save.

   b) Kim decides to save the same amount of money each month for the next four months. Write an expression to show the amount of money that she has to save each month.

   **Bonus:** Kim’s father agrees to give her $8 per month for the next three months. Write an expression to show the amount of money that she has to save each month.
**Answers:** a) 45 – 9, b) (45 – 9) ÷ 4, Bonus: ((45 – 9) – (3 × 8)) ÷ 4

**Extensions**

1. Ferry tickets cost $5 for kids and $8 for adults. Write an expression to represent the cost of tickets for three kids and seven adults.

**Answer:** (3 × 5) + (7 × 8)

2. Fifteen students from each class go on a trip. There are six classes as well as two teachers and three parents for each class. How many buses will be needed if 34 people can ride in each bus?

Encourage students to model the situation using a single equation with brackets and to explain how their equation shows the situation.

**Sample answer:** There are 6 classes and each class has 15 students, 2 teachers, and 3 parents, so the expression for the total number of people is 6 × (15 + 2 + 3). Since 34 people can ride in each bus, I can find the number of buses needed by dividing the total number of people by 34: (6 × (15 + 2 + 3)) ÷ 34 = (6 × 20) ÷ 34 = 120 ÷ 34 = 3 R 18. So 4 buses (3 full buses and 1 bus with 18 people) will be needed.

3. a) Is the equation (35 ÷ 5) + 2 = 35 ÷ (5 + 2) true? Explain.

b) Use your answer to part a) to explain why brackets are important.

**Answers:** a) No. The left side of the equation is 7 + 2 = 9 and the right side is 35 ÷ 7 = 5, so the two expressions are not equal; b) Brackets are important because they show which operations need to be performed first. The answer to part a) shows that changing where the brackets go can change the value of an expression.

4. Jake says that (40 ÷ 4) × 2 = 40 ÷ (4 × 2) since the expressions on both sides of the equal sign have the same numbers and the operations are in the same order.

Mary says that (30 ÷ 3) × 1 = 30 ÷ (3 × 1) since the expressions on both sides of the equal sign have the same numbers and the operations are in the same order.

a) Explain why you agree or disagree with each person’s reasoning.

b) How are Mary’s example and Jake’s example the same? How are they different?

**Answers:** a) both Mary and Jake use incorrect reasoning because they are ignoring the brackets, which tell us that the order of operations in each pair of expressions is not the same; b) The examples are the same because both equations have division followed by multiplication on the left side and multiplication followed by division on the right side, and both Mary and Jake use the same incorrect reasoning. The examples are different because Mary’s equation is true (even though her reasoning is incorrect), while Jake’s equation is false.
Goals

Students will substitute values for the variables in algebraic expressions and translate simple word problems into algebraic expressions.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, and multiply
Can find rules for patterns

Variables and algebraic expressions. Explain to students that today they will learn to write equations the way mathematicians write them. Instead of drawing a square or a diamond for the unknown, mathematicians usually write letters. We call the letters variables. Remind students that they have used letters in formulas before. Write “2 × (3 + 4)” on the board. SAY: This is a numerical expression. If I replace some numbers with variables, then I have an algebraic expression. Erase the number 3 and write n in its place on the board as shown in the margin.

Explain that when letters are used in an expression, the multiplication sign (×) is often omitted to avoid confusion with the letter “x” and to make the notation shorter. SAY: In this case, instead of writing 2 × (n + 4), we can simply write 2(n + 4).

Exercises

1. Rewrite the expression.
   a) 2 × n          b) (2 × n) + 3        c) 2 × (n + 3)
   Answers: a) 2n, b) (2n) + 3 or 2n + 3, c) 2(n + 3)

2. Write the expressions.
   a) A boat travels at a speed of 10 km per hour. What distance will it cover in 2 hours? In 5 hours? In h hours?
   b) A house has 12 windows. How many windows do 3 houses have? 7 houses? n houses?
   Answers: a) 10 × 2, 10 × 5, 10 × h or 10h; b) 12 × 3, 12 × 7, 12 × n or 12n

Equations and tables. Draw the figures in the margin on the board and then make a table:

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>
ASK: How can you get the number of blocks from the figure number? 
(Number of Blocks = Figure Number + 4) Change the headings of the T-table to A and B and ask a volunteer to write the equation for the new table. Explain to students that, even if the names of the columns change, the rule for the T-table will still have the same form. SAY: Now the rule is B = A + 4.

Draw on the board:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

ASK: What is the difference between the two numbers in the first row? (2) In the second row? (4) Can you add the same number to the numbers in the first column to get the numbers in the second column? (no) SAY: So the numbers in columns B and A are not related by addition. Let’s try multiplication. Point to the table and ASK: What number can I multiply by 1 to get 3? (3) By 2 to get 6? (3) By 3 to get 9? (3) SAY: The numbers in the second column are 3 times as much as the numbers in the first column. Write on the board “Second column = 3 × First column,” then “B = 3 × A.”

**Exercises:** Find the rule, then write an equation for the table.

a)  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>

b)  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

c)  

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Answers: a) B = A + 10, b) B = 2 × A, c) B = 4 × A

**Substituting numbers for variables and evaluating expressions.** Write “n + 4” on the board. Tell students that we can replace n with a number and get a numerical expression. For example, if we replace n with 3, then the expression becomes 3 + 4, which is 7. Writing 7 is called “evaluating the expression” because we are saying the value of the expression. Write the words “evaluate” and “value” on the board with underlining as shown to emphasize the connection.

**Exercises:** Replace n with 3 in each expression and evaluate the expression.

a) n + 2  b) n – 1  c) 5 – n  d) 7 + n

Answers: a) 5, b) 2, c) 2, d) 10

Tell students you are going to try to trick them and write “5n” on the board. Have students replace n with 3. Discuss the problem that students run into. SAY: The answer looks like the number 53, but 5n really means 5 × n, so we mean 5 × 3, not 53. To avoid confusion, we write brackets around the
number that replaces the variable. Tell students that “5(3)” is another way to write “5 × 3.”

**Exercises:** Evaluate.

a) 5(4)   b) 7(3)   c) 6(2)  
d) 9(6)  **Bonus:** 9(2000)

**Answers:** a) 20, b) 21, c) 12, d) 54, Bonus: 18 000

SAY: After evaluating an expression, we can add to it or subtract from it. Write “5(4) + 3” on the board. Tell students that this means “Multiply 5 and 4, then add 3.” SAY: We should probably write it like this to show what we mean. Continue writing on the board:

\[(5(4)) + 3\]

SAY: But that’s awkward because there are too many brackets, so we’ll just write it like this. Continue writing on the board:

\[5(4) + 3\]

SAY: We’ll all understand that it means “do 5(4) first.”

**Exercises:** Evaluate the expression.

a) 3(5) + 4  b) 2(3) + 7  
c) 3(4) – 5  d) 2(4) – 7

Have students combine the steps: replace the variable with a number and evaluate the resulting expression.

**Exercises:** Replace \(n\) with 5, then evaluate.

a) 3\(n\)  b) 10\(n\)  c) 10\(n\) + 1  
d) 10\(n\) – 2  e) 10\(n\) + 4  f) 8\(n\) – 7

**Answers:** a) 15, b) 50, c) 51, d) 48, e) 54, f) 33

**Interchangeable expressions.** Write on the board:

\[2n + 3\]  \[2p + 3\]  \[2t + 3\]  \[2w + 3\]

Explain to students that using different variables in the same expression doesn’t change the meaning of the expression. You can ask students to verify that all the expressions have the same value for the same number; for example, \(n = 5\), \(p = 5\), \(t = 5\), and \(w = 5\).
**Extension**

In the magic trick below, the magician can always predict the result of the sequence of operations performed on any chosen number. Try the trick with students, then encourage them to figure out how it works. Students can use blocks to represent the mystery number and counters to represent the ones that are added. Give students lots of hints as they manipulate the concrete materials.

<table>
<thead>
<tr>
<th>The Trick</th>
<th>The Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick any number.</td>
<td>Use a square to represent the mystery number.</td>
</tr>
<tr>
<td>Add 4.</td>
<td>Use 4 circles to represent the 4 ones that were added.</td>
</tr>
<tr>
<td>Multiply by 2.</td>
<td>Create 2 sets of squares and circles to show the doubling.</td>
</tr>
<tr>
<td>Subtract 2.</td>
<td>Take away 2 circles to show the subtraction.</td>
</tr>
<tr>
<td>Divide by 2.</td>
<td>Remove one set of squares and circles to show the division.</td>
</tr>
<tr>
<td>Subtract the mystery number.</td>
<td>Remove the square.</td>
</tr>
</tbody>
</table>

The result is 3. No matter what number you choose, after performing the operations in the magic trick, you will always get the number 3. The model above shows why the trick works.

a) Explain why the trick works using variables.

b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

**Sample solution:** a) Let \( x \) be the mystery number. Adding 4 can be shown as \( x + 4 \). Multiplying by 2 gives \( 2 \times (x + 4) = 2x + 8 \). Subtracting 2 gives \( 2x + 8 - 2 = 2x + 6 \). Dividing by 2 gives \( (2x + 6) \div 2 = x + 3 \). Subtracting the original mystery number, \( x \), gives \( x + 3 - x = 3 \).
Goals

Students will write and solve equations for one-step word problems involving sums or differences of parts.

PRIOR KNOWLEDGE REQUIRED

Can perform the four basic operations
Knows that the variable in an equation represents an unknown
Can solve simple, one-step addition and subtraction equations

Mental math minute. Present the equation \( x + 5 = 5 + 3 \). SAY: Let’s compare the two sides of the equation. Cover the 3 with your hand and SAY: On the left side of the equal sign we have some number and 5 added to it. On the other side, we have something added to 5. The sides are equal, so it looks like the addends are simply switched. Remove your hand and ASK: What is the missing number? (3) Write “3 + 5 = 5 + 3” underneath the first equation and ASK: Is this true? (yes) SAY: The answer is \( x = 3 \).

Exercises:

Solve the equation by comparing the sides.

a) \( 2 + 8 = x + 8 \)  
   b) \( 11 + 32 = 32 + x \)  
   c) \( 2 + 3 + 4 = 3 + 4 + x \)  
   Bonus: \( 18 + 5 + 44 = 5 + 44 + x \)

Answers: a) \( x = 2 \), b) \( x = 11 \), c) \( x = 2 \), Bonus: \( x = 18 \)

Identifying parts, totals, and unknowns. Draw the table below on the board but fill in only the first column. Tell students that they can use the information in the first column to fill in two of the next three columns; the value that goes in the third column is unknown. ASK: What do we use to represent a number we are not given? (a variable) Point to the columns in turn for each row, and have students say which number goes in each column. They can make the letter \( x \) with their fingers when you point to the column that contains the unknown.

<table>
<thead>
<tr>
<th>Green Grapes</th>
<th>Purple Grapes</th>
<th>Total Number of Grapes</th>
<th>Another Way to Write the Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 green grapes 2 purple grapes</td>
<td>3</td>
<td>2</td>
<td>( x )</td>
</tr>
<tr>
<td>7 green grapes 9 grapes altogether</td>
<td>7</td>
<td>( x )</td>
<td>9</td>
</tr>
<tr>
<td>5 green grapes 3 purple grapes</td>
<td>5</td>
<td>3</td>
<td>( x )</td>
</tr>
<tr>
<td>10 grapes altogether 6 green grapes</td>
<td>6</td>
<td>( x )</td>
<td>10</td>
</tr>
<tr>
<td>5 purple grapes 9 grapes altogether</td>
<td>( x )</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>
Writing addition equations for the data. ASK: How can you get the total number of grapes from the number of green grapes and the number of purple grapes? (add them) Students can record the expression for each total in the last column. \((3 + 2, 7 + x, 5 + 3, 6 + x, x + 5)\)

Remind students that every equation has two parts and the equal sign between them shows that the parts are equal. So if you have two expressions showing the same thing (the total), you can write an equation. For example, the first row of the table would produce the equation \(3 + 2 = x\). Have students write an equation for each row of the table.

Identifying more parts and totals. SAY: In all the questions about grapes above, there were parts—the number of green grapes and the number of purple grapes—and a total—the number of all grapes together. Write “parts” and “total” on the board. Explain that a total is always made up of parts, and you can find the total by adding the different parts together.

SAY: If you have many different apples, the parts could be the number of green apples, yellow apples, and red apples, and “all apples” would be the total. If you look at the students in your class, students who have siblings and students who don’t have a sibling can be parts, and the whole class can be the total. The 10-year-olds and the 11-year-olds could also be parts (but the total would be the same: the whole class). Have students name more situations in which there are parts and a total. Other students can identify the parts and the total in each case.

Present the exercises below and have students identify the total in each one. (If you write the sentences on separate lines, students can raise the number of fingers corresponding to the number of the line that shows the total.)

**Exercises:** Identify the total.

a) There are 7 red marbles. There are \(x\) blue marbles. There are 13 marbles in total.

b) There are \(x\) red apples. There are 9 green apples. There are 35 apples in total.

c) Liz has 51 marbles. Ronin has 36 marbles. Liz and Ronin have \(x\) marbles altogether.

d) There are 25 hockey cards. There are \(x\) sports cards altogether. There are 17 cards that are not hockey cards.

e) Rayder had 59 baseball cards. He gave \(x\) cards away. He has 37 cards left.

**Answers:** a) 13, b) 35, c) \(x\), d) \(x\), e) 59

Writing equations to find a total from parts. Have students write the equation that calculates the total from the parts for the exercises above.

(a) \(7 + x = 13\), b) \(x + 9 = 35\), c) \(51 + 36 = x\), d) \(25 + 17 = x\), e) \(59 = x + 37\)
**Exercises:** Write the equation that calculates the total from the parts.

a) Nora has 5 apples. Arsham has 6 apples. Nora and Arsham have \( x \) apples together.

b) There are 15 birds in a tree. \( x \) birds are robins. 7 birds are not robins.

c) Marko had 12 cookies. He gave 9 cookies to his friends. He has \( x \) cookies left.

d) Shelly paid \( x \) dollars for a scarf and a hat. The scarf cost $12. The hat cost $15.

**Answers:** a) \( 5 + 6 = x \), b) \( x + 7 = 15 \), c) \( 12 = x + 9 \), d) \( 12 + 15 = x \)

**Review solving equations.** SAY: You can rewrite an equation with the variable by itself the same way you rewrote equations with the unknown by itself. Invite a volunteer to rewrite one of the equations above so that the variable, \( x \), is by itself, and then solve the equation. (For example, in part b), rewrite \( x + 7 = 15 \) as \( x = 15 - 7 \) and solve it: \( x = 8 \).) Repeat with all the other equations from the two previous exercises.

**Writing and solving equations for stories (simple word problems) that involve parts and totals.** Tell students that they will now solve problems from beginning to end: they will have to determine the parts and the total, write an equation to find the total from the parts, and solve it. For the exercises below, work through the first problem together, then have students solve the rest individually.

**Exercises:** Write an equation to solve the problem.

a) Rani hiked 15 km on Saturday. She hiked \( x \) km on Sunday. She hiked 32 km over the weekend.

b) There were 25 birds in a tree. \( x \) birds flew away. 17 birds were left.

c) John raised $35 for charity. Megan raised $27. They raised \( x \) dollars together.

d) Jun weighs 33 kg. Jun and his cat weigh 38 kg together. The cat weighs \( x \) kilograms.

**Bonus:** There were \( x \) black-footed ferrets at the end of 1988. Seventy-eight black-footed ferret kits were born in captivity in 1989. At the end of 1989, there were 120 black-footed ferrets.

**Answers:** a) \( 15 + x = 32, x = 17 \); b) \( 25 = x + 17, x = 8 \); c) \( 35 + 27 = x, x = 62 \); d) \( 33 + x = 38, x = 5 \); Bonus: \( x + 78 = 120, x = 42 \)

**The difference.** Review the meaning of the word “difference”: how much larger one number is than another. Draw two rows of blocks on the board and have a volunteer show the difference between the two rows, as shown in the margin.

**Identifying the larger number in a situation.** Point out that many problems deal with a situation in which there is a larger number, a smaller number, and the difference between them. Write “There are 3 green apples. There are 2 more red apples than green apples.” on the board. ASK: What objects...
are in this situation? (green apples and red apples) Draw a table with three columns on the board and label the first two columns “red apples” and “green apples.” ASK: Which piece of information is given: the number of green apples or the number of red apples? (green) Have students signal the number of green apples and add it to the table.

ASK: Which sentence tells us which kind of apples we have more of? (There are 2 more red apples than green apples.) Ask students to write down that sentence, then to cover the number 2 with their fingers. ASK: Which kind of apples are there more of? (red apples) Circle “red apples” in the sentence on the board. ASK: How many more red apples are there? (2) Label the third column “difference” and fill it in. Finally, write \( x \) in the empty (red apples) column.

Repeat the above for this situation: There are 3 fewer red apples than green apples. There are 12 red apples.

Provide the exercises below one at a time, and have students signal the number to put in each column. If the data for the column is not known, students can signal the letter \( x \). Then have students signal which part is larger and circle it.

Exercises

a) 5 red apples; 3 more green apples than red apples
b) 6 more green apples than red apples; 2 red apples
c) 9 green apples; 3 fewer green apples than red apples
d) 7 green apples; 3 fewer red apples than green apples
e) 9 red apples; 4 more red apples than green apples

Identifying the larger number and writing an equation. Remind students that to find the difference, they subtract the smaller number from the larger number. Remind students also that when they can express the same amount—the difference—in two different ways, they can write an equation. Add a fourth column to the table on the board, label it “another way to write the difference,” and have volunteers help you fill it in. Finally, have students write the equations for all the rows of the table. (a) \( x - 5 = 3 \), b) \( x - 2 = 6 \), c) \( x - 9 = 3 \), d) \( 7 - x = 3 \), e) \( 9 - x = 4 \)

Tell students that you will now make the task harder: they will need to write an equation without using the table. Present the following exercises one at a time. Have students identify the larger part and then write the equation.
Exercises

a) 7 green apples
   x red apples
   3 more red apples than green apples
b) x spoons
   9 forks
   3 more forks than spoons

c) 3 cars
   6 buses
   x more buses than cars
d) There are 13 pears.
   There are x apples.
   There are 10 more apples than pears.

e) There are x rats.
   There are 9 fewer mice than rats.
   There are 2 mice.
f) There are 14 hats.
   There are 4 fewer scarves than hats.
   There are x scarves.

g) Zack has x hats.
   Zack has 7 scarves.
   He has 2 more scarves than hats.
h) A cat weighs 8 kg.
   A dog weighs x kilograms less than the cat.
   The dog weighs 5 kg.

i) Marla studied math for 30 minutes.
   She read for 10 minutes less than she studied math.
   She read for x minutes.

Answers: a) 3 = x - 7, b) 9 - x = 3, c) x = 6 - 3, d) x - 13 = 10,
         e) x - 2 = 9, f) 14 - x = 4, g) 2 = 7 - x, h) 8 - 5 = x, i) 30 - x = 10

Solving equations. Remind students how to solve equations by writing an equation in which x is by itself. Solve the first two equations students wrote in the exercises above as a class and then have students work individually on the rest. Students who struggle can use guessing and checking to solve the equations. (a) 10, b) 6, c) 3, d) 23, e) 11, f) 10, g) 5, h) 3, i) 20)

What parts are being compared? Present this situation: Ray spent $12 on a book. He spent $4 less than that on a snack. How much money did he spend on the snack? ASK: What objects or quantities appear in this problem? (cost of book, cost of snack) Write on the board:

- cost of book
- cost of snack

Then ask students how much the book costs. ASK: Are we given the cost of the snack? (no) Remind students that we can use a variable, x, for the unknown number. Write these amounts beside the labels.
**Exercises:** Write an equation for the word problem.

a) A cat weighs 9 kg, and a dog weighs 12 kg more than the cat. The dog weighs \( x \) kilograms. How many kilograms does the dog weigh?

b) Jasmin bikes 8 km before lunch and \( x \) km after lunch. She bikes 7 km less before lunch than after lunch. How many kilometres did Jasmin bike in total?

c) On Monday, 27 people came to the book fair. On Tuesday, 34 people came to the book fair. How many more people came on Tuesday than on Monday?

d) Cameron has 41 Canadian stamps. He has 26 more Canadian stamps than stamps from Brazil. How many stamps from Brazil does he have?

e) A flight attendant served 18 vegetarian meals. There were 69 fewer vegetarian meals than meat meals. How many meat meals were there?

As a class, find the solutions to the equations in the exercises above.
(a) 21 kg, b) 15 km, c) 7 people, d) 15 stamps, e) 87 meat meals)

**Writing and solving equations for simple word problems that involve differences.** Tell students that the next task is more challenging. Now they will solve the problems from beginning to end: determine which part is larger, write an equation, and solve the equation.

**Exercises:** Write an equation for the word problem, then solve it.

a) Jennifer read for 35 minutes. She spent 10 minutes less than that on math. She spent \( x \) minutes on math.

b) Main Street is 30 m wide. King Street is \( x \) metres narrower than Main Street. King Street is 25 m wide.

c) Ren has 27 stamps from Canada. He has \( x \) stamps from Mexico. He has 15 fewer stamps from Mexico than from Canada.

d) Ansel has 29 blue marbles and \( x \) red marbles. He has 17 more blue marbles than red marbles.

**Bonus:** Kate weighs 42 kg with her cat in her arms and 36 kg without it. The cat weighs \( x \) kilograms.

**Answers:** a) \( 35 - 10 = x, \ x = 25 \); b) \( 30 - 25 = x, \ x = 5 \); c) \( 27 - x = 15, \ x = 12 \); d) \( 29 - x = 17, \ x = 12 \); Bonus: \( 36 + x = 42, \ x = 6 \)
Extensions

1. Matt asked Grade 4 and 5 students about the sports that they like. He wrote the results in a table, but his baby brother spilled water on some parts of the table. Help Matt find the missing numbers in the table.

<table>
<thead>
<tr>
<th>Hockey Fans</th>
<th>Soccer Fans</th>
<th>Baseball Fans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 4</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Grade 5</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>Total</td>
<td>22</td>
<td>25</td>
</tr>
</tbody>
</table>

**Answers:** Total hockey fans: 29, Grade 5 soccer fans: 15, Grade 4 baseball fans: 11

2. Have students solve the following problem that involves three quantities instead of two.

There are 3 red apples, \(x\) green apples, and 7 more yellow apples than red apples in a large basket of fruit. There are 21 apples in total inside the basket. How many green apples are there?

**Answers:** \(3 + x + (3 + 7) = 21\), so there are 8 green apples
PA5-13 Problems and Equations—Addition and Subtraction

Goals
Students will solve word problems involving differences or totals using addition and subtraction equations.

PRIOR KNOWLEDGE REQUIRED
Can write an equation for finding the total or the difference from the parts given a one-step word problem
Knows that the variable in an equation represents an unknown
Can solve addition and subtraction equations

MATERIALS
BLM Word Problem Cards (p. K-43)

Mental math minute. Have students complete the exercises below, which practise the skills learned in the mental math minute in Lesson PA5-12.

Exercises: Solve the equation by comparing sides.

a) 79 – 53 = x – 53  b) 489 + 5 = 5 + x  c) 62 – 43 = 62 – x

d) 8 + 35 = 35 + x  Bonus: 12 356 – 736 = 12 356 – x

Answers: a) x = 79, b) x = 489, c) x = 43, d) x = 8, Bonus: x = 736

Organizing data. Explain that when a word problem is long, it is convenient to write out the data in point form. Write on the board:

Armand spent 25 minutes doing his math homework. He spent 15 minutes more on his science project than on math homework. How much time did Armand spend on his science project?

25 minutes on math homework
15 minutes more on science project than on math homework
x minutes on science project

ASK: Is it easier to write an equation using the original problem or the data in point form? (the data in point form) Where did I get the last point? (this sentence answers the question in the problem)

Exercises: Write out the data in point form.

a) Iva has 16 marbles. Seven of them are red. How many are not red?
b) There are 6 rats and 3 hamsters in a store. How many rats and hamsters altogether are in the store?
c) There are 18 cars in a parking lot. There are 7 fewer vans than cars in the lot. How many vans are in the lot?

Selected answer: b) 6 rats, 3 hamsters, x rats and hamsters altogether
Review totals and differences. Draw the picture in the margin on the board. Remind students that there are two things they can find given these two numbers: the difference between the two numbers and the total. Ask volunteers to show both in the model and to find what each is equal to. (the difference is 4 and the total is 10) Review writing the equation for the total and the difference. Repeat with this situation: larger number 8, smaller number \(x\), difference 3, total 13.

Difference or total? Create a table with the headings “Parts,” “Total,” and “Difference” on the board. Look at the exercises below as a class and have students identify which piece of data belongs in which column. In parts f) and g), students need to decide which piece of data is the unknown (\(x\)).

Exercises: Which piece of data belongs in which column?

a) \(x\) spoons, 8 forks, 19 forks and spoons altogether
b) 12 cars, \(x\) buses, 18 cars and buses in a parking lot
c) There are 5 bananas. There are \(x\) kiwis. There are 3 fewer bananas than kiwis.
d) A cat weighs 8 kg. A dog weighs 2 kg less than the cat. The dog weighs \(x\) kg.
e) Jayden paid $7 for a hat. He paid $9 for a pair of mitts. He paid \(x\) for the mitts and the hat.
f) Alexa studied math for 30 minutes. Math and reading took 50 minutes altogether. How long did she read for?
g) A salad recipe calls for 2 onions and 3 more tomatoes than onions. How many tomatoes are needed?

ACTIVITY (Essential)

Give students cards from BLM Word Problem Cards. Have them sort the cards according to the problems (the cards belonging to the same problem have the same picture). Then ask them to write an answer sentence, with \(x\) in place of the answer, below the question. For example, “How much does Hero weigh?” should have the sentence “Hero weighs \(x\) kilograms.” written below it. Have students place the cards in the table they created during the previous exercises.

Writing and solving an equation for a word problem. Have students write the equation for each situation or problem above, including the problems from BLM Word Problem Cards. Work together through the first two. Then solve the first two equations together and have students solve the equations for the rest of the problems individually. You can show students how to do the problem using the guess-and-check method or by finding an equation that has the variable by itself. For example, for part a), write “\(x + 8 = 19\)” on the board. ASK: How can we get a related equation that has \(x\) all by itself on one side of the equation? (19 is 8 more than \(x\), so \(x\) is 8 fewer than 19).
Write "x = 19 – 8" on the board. ASK: How can we find the value of x now? (evaluate the expression on the right, 19 – 8) Write "x = 11" underneath the previous equation on the board.

(a) x + 8 = 19, x = 11; b) 12 + x = 18, x = 6; c) x – 5 = 3, x = 8;
d) 8 – x = 2, x = 6; e) 7 + 9 = x, x = 16; f) 30 + x = 50, x = 20;
g) 2 + 3 = x, x = 5; BLM: 5 + 7 = x, x = 12; 16 – 12 = x, x = 4;
12 + 16 = x, x = 28; 55 – 20 = x, x = 35; 16 + 9 = x, x = 25)

Organizing data, writing an equation, and solving it. Work through the first two exercises below as a class, then have students work individually.

Exercises: Write an equation and solve it.

a) Ken bought 9 books and 3 magazines. How many books and magazines did he buy altogether?
b) A book costs $8 and a poster is $3 cheaper than the book. How much does the poster cost?
c) A pet store sells parrots and canaries. There are 16 canaries in the store. There are 6 fewer parrots than canaries. How many parrots are in the store?
d) Avril read 9 pages on Sunday. She read 4 pages more on Sunday than on Monday. How many pages did she read on Monday?
e) A cake recipe calls for 5 cups of berries. Jack has 3 cups of raspberries and some blueberries. How many cups of blueberries will he need for the cake?

Answers: a) 12, b) $5, c) 10, d) 5, e) 2

Extension

Write an equation and solve it.

a) There are 32 students in a class. 14 of them are playing soccer, and the rest are playing baseball. How many more students are playing baseball than are playing soccer?
b) There are 29 students in a class. 7 of them don’t wear eyeglasses. How many more students wear eyeglasses than do not wear eyeglasses?
c) Braden baked 24 oatmeal cookies and 32 chocolate chip cookies. He brought 42 cookies to a bake sale. How many cookies did he leave at home?
d) Jessica got $100 for her birthday. She paid $39 for a construction toy and $12 for beads. How much money does she have left?

Bonus: Jin bought a book for $9 and two magazines for $7 each. He paid with a $20 bill and a $10 bill. How much change did he get?

Answers: a) 4, b) 15, c) 14, d) $49, Bonus: $7
**Goals**

Students will solve word problems with “times as many” using models.

**PRIOR KNOWLEDGE REQUIRED**

- Can solve a one-step word problem requiring addition or subtraction
- Understands the expression “times as many”
- Can identify the parts, total, and/or difference in a problem

---

**Drawing a model for “times as many” situations.** Tell students that two people, Sun and Raj, have some stickers. Write “Sun has four times as many stickers as Raj” on the board. SAY: I want to draw a model to represent this situation. ASK: Who has more stickers, Sun or Raj? (Sun) Draw a small rectangle or bar on the board and explain that it represents Raj’s stickers. Label the bar “Raj’s stickers.” ASK: How can we show that Sun has four times as many stickers as Raj? Accept all reasonable answers. Then explain that you are going to draw a model a specific way. You have drawn a bar to represent all of Raj’s stickers, and now you will draw the same bar four times to represent all of Sun’s stickers. Draw the picture in the margin on the board and keep if for future reference. Explain that this type of model is called a tape diagram. (This model is similar to the model students used for problems such as “Raj has 4 stickers; Sun has 2 more stickers than Raj.”)

Present the following situation: Karen has three times as many nickels as dimes. Draw on the board:

- a) number of dimes
- number of nickels
- b) number of dimes
- number of nickels
- c) number of dimes
- number of nickels
- d) number of dimes
- number of nickels

Ask which would fit the situation and which would not. Have students explain why the models that do not fit the situation do not work. (a) yes; b) yes; c) no, there are more dimes than nickels; d) no, there are 4 times as many nickels as dimes)

ASK: How do you know that the short bar should be the number of dimes? (there are more nickels than dimes)

Present this situation: Lyn is twice as old as Abella. ASK: Whose age will be the smaller bar? (Abella’s) Why? (because Lyn is older, so her age is larger) Ask students to draw a model for this situation and make sure students understand the meaning of “twice.”
Repeat with the exercises below. For each part, ask students to first identify the smaller number and remind them that this should be the shorter bar.

**Exercises:** Draw a tape diagram for the situation.

a) Bill is three times as tall as his baby brother.

b) Tina’s full name is four times as long as Josh’s.

c) There are eight times as many students in the school as in our class.

d) A book is twice as thick as a notebook.

**Answers**

\[
\begin{array}{ll}
\text{a) Bill} & \hspace{1cm} \text{b) Tina} \\
\text{Brother} & \hspace{1cm} \text{Josh} \\
\text{c) school} & \hspace{1cm} \text{d) book}
\end{array}
\]

**Finding the length of the bars when the smaller part is given.** Return to the situation and model with Raj and Sun. Tell students that Raj has 3 stickers. Write “3” in Raj’s block. Remind students that the blocks are the same and write “3” in each of Sun’s blocks. (see margin) **ASK:** Can you tell from the model how many stickers Sun has? (yes, 12) **How do you know?** (there are 4 blocks of 3) Have students write the multiplication statement for the length of the longer bar. \(3 \times 4 = 12\)

Have students draw a model and find the lengths of the bars for this situation: Ella has 3 red marbles. She has twice as many green marbles as red marbles. Invite volunteers to show the answers.

**Exercises:** Draw a model and find the length of each bar.

a) A car holds 5 people. A van holds three times as many people.

b) Don’s apartment building is 3 storeys high. Hanna’s building is five times as high as Don’s.

c) Ethan is 5 years old. David is four times as old as Ethan.

**Bonus:** A sparrow has 4 eggs in its nest. A duck has three times as many eggs in its nest as a sparrow. An ostrich has five times as many eggs in its nest as a sparrow.

**Answers**

\[
\begin{array}{ll}
\text{a) car} & 5 \\
\text{van} & 5 \hspace{1cm} 5 \hspace{1cm} 5 \\
\text{b) Don’s building} & 3 \\
\text{Hanna’s building} & 3 \hspace{1cm} 3 \hspace{1cm} 3 \hspace{1cm} 3 \hspace{1cm} 3 \\
\text{c) Ethan} & 5 \\
\text{David} & 5 \hspace{1cm} 5 \hspace{1cm} 5 \hspace{1cm} 5 \\
\text{Bonus: sparrow} & 4 \\
\text{duck} & 4 \hspace{1cm} 4 \hspace{1cm} 4 \hspace{1cm} 4 \\
\text{ostrich} & 4 \hspace{1cm} 4 \hspace{1cm} 4 \hspace{1cm} 4 \hspace{1cm} 4 \\
\end{array}
\]
Solving problems when the larger part is given. Write on the board:

Tasha has 20 stickers. Tasha has four times as many stickers as Eric.

Invite a student to draw the bars for the situation, without writing the numbers. ASK: How many blocks are in Tasha’s bar? (4) SAY: Tasha has 20 stickers. ASK: How many stickers does each block represent? (5) How do you know? (20 ÷ 4 = 5) How many stickers does Eric have? (5)

Have students draw bars and find the length of each block for the exercises below. Work through the first one as a class and have students work individually on the rest.

Exercises: Draw a block diagram to solve the problem.

a) There are 6 apples on the table. There are twice as many apples as pears. How many pears are there?

b) A mini-bus holds 16 people. The mini-bus holds twice as many people as a van. How many people can the van hold?

c) Jay’s apartment building is 30 storeys high. Jay’s building is five times as high as Vicky’s building. How tall is Vicky’s building?

d) Luc is 14 years old. Luc is 7 times as old as Lily. How old is Lily?

Bonus: A sugar pinecone is 45 cm long. It is three times as long as an eastern pinecone. The sugar pinecone is nine times as long as a jack pinecone. How long are the eastern pinecone and the jack pinecone?

Answers: a) each block is 3, there are 3 pears; b) each block is 8, the van can hold 8 people; c) each block is 6, Vicky’s building is 6 storeys; d) each block is 2, Lily is 2 years old; Bonus: the eastern pinecone is 15 cm long (the block is 15), and the jack pinecone is 5 cm long (the block is 5)

Finding the size of a single block when the difference is given.

Explain that a student you know drew the model in the margin for some word problem. In the problem, the student was given that the difference between the parts was 18. ASK: What does this mean? (the longer bar is 18 more than the shorter bar) Show how to mark this on the diagram by adding a bracket below the difference and marking it as 18, as shown in the margin. ASK: How many blocks is the difference? (3) What is the length of each block? (6) How do you know? (18 ÷ 3 = 6)

Exercises: What is the size of one block?

a) b) 12

c) d) 15 32
Answers: a) 4, b) 6, c) 5, d) 8

Finding the size of a single block when the total is given. Explain that another student you know drew the model in the margin for a different word problem. Again, all the blocks are the same size, and in the problem the student was given, the total was 18. Show how to mark this on the diagram using a vertical bracket. ASK: How many blocks are there in total? (9) What is the length of each block? (2) How do you know? \(18 \div 9 = 2\)

Exercises: What is the size of one block?

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20 12

c) d)  

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21 90

Answers: a) 4, b) 3, c) 3, d) 9

Now combine the two types of problems: problems with the total given and problems with the difference given.

Exercises: What is the size of one block?

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a) b)  

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45 16

c) Bonus:  

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60 260

Answers: a) 9, b) 8, c) 15, Bonus: 20

Solving problems with the difference or total given. Tell students that now they will need to draw the models themselves. Write on the board:

Rick is four times as old as Sara. Sara is 15 years younger than Rick. How old is Sara?

Ask students to draw a model that fits the first sentence. Have a volunteer present the answer. ASK: What does the second sentence give us: the difference, the total, or one of the parts? (the difference) Have students mark that on the diagram. ASK: How large is one block? (5) How do you know? \(15 \div 3 = 5\) How many blocks long is Sara’s bar? (1 block) How old is Sara? (5) How long is Rick’s bar? (4 blocks) How old is Rick? (20 years old)

Work through the first two exercises on the following page as a class, then have students work individually.
Exercises

a) Zara saved three times as much pocket money as Anton. Anton saved $18 less than Zara. How much money do they have together?

b) Rob and Clara used all their pocket money to buy a common present for their grandmother. They had $60 together. Rob had twice as much money as Clara had. How much money did each of them have?

c) The number of students in the school who are not in Grade 5 is eight times as large as the number of students in Grade 5. There are 248 students in the school who are not in Grade 5. How many students are in the school altogether?

d) A number is five times as large as another number. If you add the two numbers together, you get 54. What are the numbers?

Answers: a) Zara saved $27, Anton saved $9; b) Clara had $20, Rob had $40; c) 279 students in the school in total; d) 9 and 45

Extensions

1. Kyle reads the same number of pages every weekday and twice as many pages every weekend day. He finished a book of 108 pages in a week. How many pages did he read on Monday? How many pages did he read on Sunday?

   Answer: 12 pages on Monday, 24 pages on Sunday

2. Choose any model from the lesson and invent a word problem that would fit the model. Have a partner solve the problem.
Goals

Students will use equations to solve multiplication and division word problems.

PRIOR KNOWLEDGE REQUIRED

Knows that a variable can replace a number in an equation
Understands the expression “times as many/much”

Mental math minute. Ask students to solve multiplication questions within the range of $0 \times 1$ to $10 \times 10$. For each number, first go through the questions in order, such as $0 \times 3, 1 \times 3,$ and so on to $10 \times 3$, then in reverse order. After that, go through the same questions out of order. Then progress to a different number.

NOTE: Students who find it difficult to write equations without first drawing a diagram can draw diagrams as needed.

Scale factor. SAY: Sally has 3 marbles. Sam has four times as many marbles as Sally. Have a volunteer draw a diagram for this situation. Explain that the number that tells us how many times as large one part is than the other is called the scale factor. SAY: In this situation, the scale factor is 4.

Exercises:

a) There are 3 green apples and four times as many red apples as green apples.

b) Glen is three times as old as Nina. Nina is 2 years old.

c) cat’s weight

dog’s weight

d) Kathy’s savings

Marcel’s savings

Answers: a) 4, b) 3, c) 2, d) 4

Writing an equation using a scale factor to find parts. Explain that you can use the scale factor to write two equations that show how to use one part to find the other part. Write on the board:

\[
\text{Larger Part} = \text{Smaller Part} \times \text{Scale Factor}
\]

\[
\text{Smaller Part} = \frac{\text{Larger Part}}{\text{Scale Factor}}
\]

Look at the diagrams from the previous exercises with the class to make sure that both equations make sense.
**Exercises:** Write two equations for the situation, one equation telling how to get the larger part and the other telling how to get the smaller part.

a) \(w \) mice, 4 rats, two times as many mice as rats

b) 6 blue marbles, \(w \) green marbles, four times as many green marbles as blue marbles

c) 8 bananas, \(w \) oranges, twice as many bananas as oranges

**Answers:**
a) \(w = 4 \times 2, 4 = w \div 2\); b) \(w = 6 \times 4, 6 = w \div 4\);
c) \(w = 8 \div 2 = 4, 8 = w \times 2\)

Remind students that equations in which the unknown number is by itself are very easy to solve—you only need a calculation. Have students identify the equations in which \(w\) is by itself in the previous exercises.

Tell students that in the next exercises, you want them to write only one equation, the one that has \(w\) by itself.

**Exercises:** Write an equation with \(w\) by itself.

a) Neka earned $36 babysitting. Jen earned \(w\) dollars mowing lawns. Neka earned three times as much as Jen.

b) Amir hiked 16 km on Monday. He hiked \(w\) km on Tuesday. He hiked twice as far on Monday as on Tuesday.

c) A recipe calls for 3 cups of oatmeal and twice as much flour. How much flour is needed?

d) A dog weighs three times as much as a rabbit. The dog weighs 12 kg. How much does the rabbit weigh?

**Answers:**
a) \(w = 36 \div 3\), b) \(w = 16 \div 2\), c) \(w = 3 \times 2\), d) \(w = 12 \div 3\)

If students have trouble with problems that include units (distance, weight, etc.), point out that they can treat units such as kilometres the same way as they treat objects such as marbles. Three times as many as 4 marbles is 12 marbles, and three times as far as 4 km is 12 km. **ASK:** If Lewis is three times as old as Mandy, and Mandy is 4 years old, how old is Lewis? (12 years old) If a table is three times as heavy as a chair, and the chair weighs 4 kg, how heavy is the table? (12 kg)

**Review the connection between sets and multiplication.** Remind students that we use multiplication to find the total number of objects in equal sets. **SAY:** For example, 5 people can sit in each car. There are 3 cars. **ASK:** How many people are in 3 cars? (15) How do you know? (\(3 \times 5 = 15\)) Remind students that to find the total number of objects, they need to multiply the number in each set by the number of sets. To find either of the other two numbers, they need to divide the total by one of those numbers.

**Write an equation for a story with an unknown number.** Present a few situations and have students identify which number in the situation shows the total, which number shows the number of objects in a set, and which number shows the number of sets.
Draw on the board:

<table>
<thead>
<tr>
<th>Number in Each Set</th>
<th>Number of Sets</th>
<th>Total Number of Objects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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Point at the Total Number of Objects column and SAY: If we don’t know the total, we multiply. When we don’t know either of the other two numbers, we divide. Have students use the headings on the board in the following exercises.

**Exercises:** Fill in a row in the table for the situation.

a) There are 7 people in each van. There are \( w \) vans. There are 21 people altogether.

b) There are \( w \) pears in each basket. There are 5 baskets. There are 45 pears altogether.

c) There are 4 juice boxes in each pack. There are 15 packs. There are \( w \) juice boxes altogether.

d) There are 6 pens in each pack. Aputik bought \( w \) packs. Aputik bought 42 pens altogether.

**Bonus:** An octopus has 8 arms. There are \( w \) octopuses. There are 88 arms altogether.

**Answers**

<table>
<thead>
<tr>
<th>Number in Each Set</th>
<th>Number of Sets</th>
<th>Total Number of Objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 7</td>
<td>( w )</td>
<td>21</td>
</tr>
<tr>
<td>b) ( w )</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>c) 4</td>
<td>15</td>
<td>( w )</td>
</tr>
<tr>
<td>d) 6</td>
<td>( w )</td>
<td>42</td>
</tr>
<tr>
<td>Bonus</td>
<td>8</td>
<td>88</td>
</tr>
</tbody>
</table>

Add a column to the table with the heading “Equation”. Go through the problems in the previous exercises one by one and ask students to write a multiplication or a division equation in which the unknown is by itself. Remind students who are struggling that they need to multiply to find the total number of objects and divide in the other two cases.

Have students solve the equations and find the unknown numbers.
(a) \( w = 21 ÷ 7 = 3 \), b) \( w = 45 ÷ 5 = 9 \), c) \( w = 4 \times 15 = 60 \),
d) \( w = 42 ÷ 6 = 7 \), Bonus: \( w = 88 ÷ 8 = 11 \)

Present a few word problems and have students use the whole process together to solve them: writing the information (in a table as needed), writing and solving the equation, and writing the answer statement.
Exercises

a) Alex is 9 years old. Yu is three times as old as Alex. How old is Yu?

b) Ivan is 7 years old. Tess is four times as old as Ivan. How old is Tess?

c) A book costs $16. The book is twice as expensive as a notebook. How much does a notebook cost?

d) A pen costs $7. A book costs three times as much as the pen. How much does the book cost?

e) A pair of eyeglasses costs $75. The pair of eyeglasses is three times as expensive as a pair of pants. How much do the pants cost?

f) A magazine costs $11. A book costs three times as much as the magazine. How much does the book cost?

g) Anna spends three times as much time reading as she spends on math. She spends 45 minutes reading. How much time does she spend on math?

h) Fred spends twice as much time reading as he spends on science. He spends 20 minutes on science. How much time does he spend reading?

Bonus

i) Sumatran tigers are endangered. Only about 500 Sumatran tigers are left in the wild. There are about five times as many Bengal tigers as there are Sumatran tigers left in the wild. How many Bengal tigers are left in the wild?

j) There are about ten times as many Sumatran tigers as Indo-Chinese tigers left in the wild. How many Indo-Chinese tigers are left in the wild?

Sample solution: a) $9 \times 3 = w$, $27 = w$, Yu is 27 years old

Answers: b) 28, c) $8$, d) $21$, e) $25$, f) $33$, g) 15 minutes, h) 40 minutes, Bonus: i) 2500, j) 50

Extensions

1. A subway station has two parking lots. There are 30 rows of 25 parking spots in the east lot and 15 rows of 30 parking spots in the west lot.

   a) How many cars can park in each lot? Which lot has more parking spots?

   b) How many cars can park at the subway station in total?

   Answers: a) 750 in the east lot and 450 in the west lot, b) 1200

2. Someone wants to donate money to 15 different charities for a total of $30 000. If each charity gets the same amount, how much money will each charity get?

   Answer: $2000
3. Seven people can ride in a mini-van. Six times as many people can ride on a regular bus. A double-decker bus can hold ten times as many people as a mini-van. How many people altogether can ride in a mini-van, a bus, and a double-decker bus?

Solution: $7 + 42 + 70 = 119$
Goals
Students will solve word problems with additive and multiplicative comparisons.

PRIOR KNOWLEDGE REQUIRED
Can solve a single-step word problem requiring addition, subtraction, multiplication, or division
Can identify the scale factor and the difference in a single-step word problem
Understands the expression “times as many/much”

Identifying scale factor or difference. Remind students what a difference between two quantities is (for example, 5 is 2 more than 3, so 2 is the difference). Then remind them what a scale factor is (for example, 12 is 4 times as many as 3, so 4 is the scale factor).

Ask students to say which words in a problem let them know what is given in it—a scale factor or a difference. Present the problems in the exercises below one at a time and have students say if they are given the difference or the scale factor. Students can use crossed fingers to signal the scale factor (as in multiplication) and a horizontal finger to signal the difference (as in subtraction). Fill in the table for the first two exercises as a class, then have them work individually on the rest.

Exercises: Fill in the table for each problem. Write $x$ for the unknown number. Cross out the cell that does not apply to the problem.

<table>
<thead>
<tr>
<th>Parts</th>
<th>How Many?</th>
<th>Scale Factor: _____</th>
<th>Difference: _____</th>
</tr>
</thead>
</table>

a) An apple-pear cake recipe calls for 16 apples. The recipe calls for four times as many apples as pears. How many pears are needed for the cake?

b) There are 50 cars in a parking lot. There are 10 fewer vans than cars. How many vans are at the parking lot?

c) A van holds 9 people. A bus holds five times as many people as the van. How many people can the bus hold?

d) It takes Tristan 20 minutes to do math homework. He spends twice as much time reading as he spends on math. How much time does he spend reading?

e) Lela decorates her house with balloons and stars. She uses 70 balloons. She uses 10 more balloons than stars. How many stars are there?
f) A table costs $80. A chair costs $40 less. How much does the chair cost?

g) Amy is 12 years old. Amy is three times as old as Ben. How old is Ben?

**Answers**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) apples</td>
<td>16</td>
<td>Scale Factor: 4</td>
</tr>
<tr>
<td></td>
<td>pears</td>
<td>x</td>
</tr>
<tr>
<td>b) cars</td>
<td>50</td>
<td>Scale Factor:</td>
</tr>
<tr>
<td></td>
<td>vans</td>
<td>x</td>
</tr>
<tr>
<td>c) people in a van</td>
<td>9</td>
<td>Scale Factor: 5</td>
</tr>
<tr>
<td></td>
<td>people in a bus</td>
<td>x</td>
</tr>
<tr>
<td>d) math time</td>
<td>20</td>
<td>Scale Factor: 2</td>
</tr>
<tr>
<td></td>
<td>reading time</td>
<td>x</td>
</tr>
<tr>
<td>e) balloons</td>
<td>70</td>
<td>Scale Factor:</td>
</tr>
<tr>
<td></td>
<td>stars</td>
<td>x</td>
</tr>
<tr>
<td>f) cost of table</td>
<td>80</td>
<td>Scale Factor:</td>
</tr>
<tr>
<td></td>
<td>cost of chair</td>
<td>x</td>
</tr>
<tr>
<td>g) Amy’s age</td>
<td>12</td>
<td>Scale Factor: 3</td>
</tr>
<tr>
<td></td>
<td>Ben’s age</td>
<td>x</td>
</tr>
</tbody>
</table>

**Identifying which part is the larger quantity.** Ask students to decide which quantity is larger in each exercise above. Students can signal the answer. Circle the larger quantity each time and underline the smaller quantity. For each problem, ask students to say whether the quantity they need to find is the larger quantity. Underline or circle the quantity that needs to be found depending on whether it is the larger quantity or the smaller quantity.

Finally, have students find the missing quantity for each of the exercises above. They can either use an equation or a simple computation to solve the problem. Have students find the difference between the quantities in parts a), c), d), and g) and the totals in parts a) to f). Ask them to try to find a word that describes both quantities in the problem. For example, in part a), there are 20 fruits, and in part b), there are 90 vehicles.

**Exercises:** Find the missing quantity in the previous exercises.

**Answers:** a) 4 pears, b) 40 vans, c) 45 people, d) 40 minutes, e) 60 stars, f) $40, g) 4 years old; differences: a) 12 apples, c) 36 cars, d) 20 minutes, g) 8 years; totals: a) 20 fruits, b) 90 vehicles, c) 54 people, d) 60 minutes, e) 130 decorations, f) $120

**Problems using total and difference.** Remind students that in many problems they need to find the total or the difference between amounts, so the problem will have several steps.
Exercises

a) Cody reads 7 books over the holidays. Emma reads three times as many books as Cody. How many more books did Emma read?

b) There are 12 bicycles parked at the entrance. There are twice as many bicycles as scooters parked there. How many bicycles and scooters are parked at the entrance?

c) A movie pass costs $24. A concert ticket costs $10 more than the movie pass. How much do both tickets cost together?

Bonus

d) A jack pine is 11 m tall. A red pine is three times as tall as the jack pine. A red cedar is six times as tall as the jack pine. A giant sequoia (a giant redwood tree) is eight times as tall as the jack pine. How tall is each tree?

e) A truck driver drives 810 km on Monday. On Tuesday, she drives 50 km more than on Monday, and that is twice as much as she drives on Wednesday. How many kilometres did she drive in three days?

f) Another driver needs to travel the same distance as the driver in part e). He decides to drive the same distance each day for three days. How many kilometres did he drive each day?

Answers: a) 14 books, b) 18 bicycles and scooters, c) $58, Bonus: d) red pine: 33 m, red cedar: 66 m, giant sequoia: 88 m; e) 810 + 860 + 430 = 2100 km; f) 700 km

Solving problems involving division with remainders. Remind students that there is sometimes a remainder when they divide. SAY: For example, 22 \(\div\) 5 = 4 R 2. ASK: If 5 children share 22 grapes, how many does each child get? (4 grapes) What do we do with the remainder? (disregard it) If 22 children need to go on a boat trip, and each boat can hold only 5 children, how many boats will they need? (5) Why 5 and not 4? (we need to add another boat for the remainder) Tell students that the next exercises will be the hardest they have done yet: there will be several steps, and when dividing, they will need to decide what to do with the remainder.

Exercises

a) Kyle has 12 marbles, Grace has 16 marbles, and Jax has 10 marbles. Can they share all the marbles equally? How many marbles will each person get? What can they do with the remainder?

b) There are 135 apples in a crate. 29 apples are spoiled. Alice packs the rest into bags of 6 apples each. How many full bags can she make?

c) Four classes go on a field trip. There are 29 children in one class, 31 in the second class, 35 in the third class, and 28 in the fourth class. Each bus has 45 seats. How many buses do they need?
**Bonus:** On another field trip for the classes in part c), three volunteers and one teacher go with each class. Now how many buses do they need?

**Answers:** a) no, each person gets 12 marbles if they share equally and the remainder is disregarded, or one person gets 12 marbles and the other two people get 13 marbles if the remainder is shared; b) 17; c) 3; Bonus: 4

**NOTE:** Tell students that some problems in the AP Book will require them to answer one or more questions before the question in the problem can be answered. Remind them to look for comparisons with “times” that require division or multiplication and comparisons that do not have the words “times” or “twice” that require addition or subtraction. Students can use equations, tape diagrams, or simple computations to solve the problems.

**Extension**

Ed is training Dory on how to put tires on cars at a factory. They have to put all 4 tires on 354 cars. If Ed puts on five times as many tires as Dory, how many tires does Dory put on? Show your work using a tape diagram and labels.

**Answer**

Tires Ed puts on: \[354 \times 4 = 1416 \text{ tires}\]

Tires Dory puts on: \[\text{The total number of tires that Dory and Ed put on is 1416. From the tape diagram, that's six times the number that Dory put on, so I found } 1416 \div 6 = 236. \text{ Dory put on 236 tires.}\]
Fluffy the cat weighs 5 kg. Hero the dog weighs 7 kg more than Fluffy the cat. How much does Hero weigh?

There are 12 girls in Ms. A’s class. There are 16 boys in Ms. A’s class. How many more boys than girls are in Ms. A’s class?

There are 12 girls in Ms. A’s class. There are 16 boys in Ms. A’s class. How many boys and girls are in Ms. A’s class?

The lunch period is 55 minutes long. Sandy spent 20 minutes eating. Sandy went to the library for the rest of the period. How much time did she spend in the library?

Randi earned $16 by tutoring. Evan earned $9 more than Randi by babysitting. How much money did Evan earn?
PS5-5  Guessing, Checking, and Revising

Teach this lesson after:
Unit 8

VOCABULARY
guess-check-revise
perfect square
search systematically
square

Goals
Students will make organized guesses and use the result of the previous guess to improve their next guess.

PRIOR KNOWLEDGE REQUIRED
Can use the strategy of searching systematically
Can multiply up to three-digit numbers by single-digit numbers
Can round two-digit whole numbers to the nearest ten
(for Problem Bank 2)
Can convert measurements in years to measurements in months
(for Problem Bank 9)
Can convert measurements in centimetres to millimetres
(for Problem Bank 10)
Can convert measurements in decades to measurements in years
(for Problem Bank 12)
Can substitute numbers for variables in expressions involving fractions
(for Problem Bank 14)
Can identify equivalent fractions (for Problem Bank 14)

MATERIALS
books that are at least 150 pages long
calculators

Review the guess-check-revise strategy. Hide an object in the room and have a volunteer try to find the object. If the volunteer finds it quickly, play again until finding the object takes a while. When the volunteer finds the object, ASK: What strategy did you use? (guessed and tried again) Play again, but this time tell the volunteer whether they are “hot” or “cold” as they try to find the object. ASK: What strategy did you use this time? (guessed and tried again) When you tried again, was it easier than before? Why? Lead the discussion to the conclusion that, when students have more information about their wrong guess other than just that it’s wrong, they can use that information to improve their next guess. Write on the board:

guess-check-revise

SAY: When you play hide-and-seek, you are using a guess-and-check strategy, but when you are told whether you are hot or cold, you are using a three-step process: guess, check your guess, and then improve your next guess. This three-step strategy—guess-check-revise—is very useful in math.

Make sure students have a book that is at least 150 pages long. Have students try to open the book to page 60 on the first try. Have different volunteers tell you what page number they turned to on their first try.
Point out how all the attempts were fairly close to 60. SAY: No one’s first try was page 5 and no one’s first try was page 145. Everyone was pretty close to 60. Now have students use their first guess to make a second guess. ASK: Which way in the book should you turn? Should you turn a lot of pages or only a few? How close was your first guess?

**Using the guess-check-revise strategy to find a mystery number.** Write on the board:

\[ N \times N \times N \text{ is } 343. \text{ What number is } N? \]

SAY: Let’s start by checking the numbers in order. A table is a good way to do this. Draw on the board:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1 \times 1 \times 1 = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 2 \times 2 = 4 \times 2 = 8 )</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 3 \times 3 = 9 \times 3 = 27 )</td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 4 \times 4 = 16 \times 4 = 64 )</td>
</tr>
<tr>
<td>5</td>
<td>( 5 \times 5 \times 5 = 25 \times 5 = 125 )</td>
</tr>
</tbody>
</table>

Have volunteers continue filling out the last three rows of the table, as shown below:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1 \times 1 \times 1 = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 2 \times 2 = 4 \times 2 = 8 )</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 3 \times 3 = 9 \times 3 = 27 )</td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 4 \times 4 = 16 \times 4 = 64 )</td>
</tr>
<tr>
<td>5</td>
<td>( 5 \times 5 \times 5 = 25 \times 5 = 125 )</td>
</tr>
<tr>
<td>6</td>
<td>( 6 \times 6 \times 6 = 36 \times 6 = 216 )</td>
</tr>
<tr>
<td>7</td>
<td>( 7 \times 7 \times 7 = 49 \times 7 = 343 )</td>
</tr>
</tbody>
</table>

ASK: Are we getting closer to the answer? (yes) SAY: So, we can continue in this way. Leave the table on the board for use in the exercise below.

**Exercise:** Complete the table for \( N \times N \times N \). Stop when you get the answer 343. What is \( N? \)

**Solution**

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1 \times 1 \times 1 = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 2 \times 2 = 4 \times 2 = 8 )</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 3 \times 3 = 9 \times 3 = 27 )</td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 4 \times 4 = 16 \times 4 = 64 )</td>
</tr>
<tr>
<td>5</td>
<td>( 5 \times 5 \times 5 = 25 \times 5 = 125 )</td>
</tr>
<tr>
<td>6</td>
<td>( 6 \times 6 \times 6 = 36 \times 6 = 216 )</td>
</tr>
<tr>
<td>7</td>
<td>( 7 \times 7 \times 7 = 49 \times 7 = 343 )</td>
</tr>
</tbody>
</table>

\( N = 7 \)
Write on the board:

If \( N \times N \times N = 46656 \), what is \( N \)?

Point to the table on the board for \( N \times N \times N \). ASK: Would continuing the chart be a good strategy for this question? (no) SAY: The answers are getting closer to the number, so you are getting warmer, but not much warmer, because you still have a long way to go to find the answer. Maybe we can take bigger steps to find the answer. Instead of trying 1, 2, 3, and so on, maybe we should start with 10, 20, 30, and so on.

**Exercises:** If \( N \times N \times N = 46656 \), what is \( N \)?

a) Complete the chart up to 50.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>20</td>
<td>8000</td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

b) Which two tens is \( N \) between? Explain how you know.

**Answers:** a) 27,000, 64,000, 125,000; b) \( N \) is between 30 and 40 because \( N \times N \times N \) is between 27,000 and 64,000.

SAY: Now we know that \( N \) is between 30 and 40. Write on the board:

\[
\begin{align*}
30 \times 30 \times 30 &= 27000 \\
N \times N \times N &= 46656 \\
40 \times 40 \times 40 &= 64000
\end{align*}
\]

ASK: Do you think \( N \) is a lot closer to 30 or to 40, or do you think \( N \) is in the middle of 30 and 40? (in the middle) Why? (46,656 is in the middle between 27,000 and 64,000) SAY: Let’s try 35. Write on the board:

\[
35 \times 35 \times 35 = ______
\]

Have a volunteer do the calculation on a calculator and write the answer on the board. (42,875) ASK: Is 35 the right answer, too low, or too high? (too low) What should we try next? (36) Write on the board:

\[
36 \times 36 \times 36 = ______
\]

Again, have a volunteer do the calculation on a calculator and write the answer on the board. (46,656) ASK: Is 36 the right answer, too low, or too high? (the right answer) Write on the board:

So \( N = 36 \)

Students may use a calculator for the following exercises.
Exercises: Find $N$ so that $N \times N \times N$ is …

a) 103 823  

b) 28 094 464

Answers: a) 47, b) 304

Review searching systematically when two related quantities are changing. SAY: A farmer has cows and chickens. Marko counts all the legs and Anna counts all the heads. Write on the board:

Marko counts 26 legs. Anna counts 10 heads.

SAY: I want to know how many cows and how many chickens there are. Remember, to solve this type of problem, you can start by choosing one of the two quantities and systematically moving up in order through all the possibilities. Draw on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remind students that cows have four legs and chickens have two legs. SAY: Ten heads means there are 10 animals altogether. Point to the first row and ASK: If there are no cows, how many chickens are there? (10) If there is one cow, how many chickens are there? (9) Continue filling in the first four rows of the table, as shown below:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>26</td>
</tr>
</tbody>
</table>

ASK: Do I need to continue the table? (no) Why not? (with 3 cows and 7 chickens, we now have a total of 10 animals and 26 legs) SAY: So there are three cows and seven chickens altogether. ASK: If you move down from one row to the next in the chart, how much does the total number of legs increase by? (2) SAY: When you start at the top of the table, you have 10 chickens. When you move down a row, you replace a chicken with a cow. But every time you replace a chicken with a cow, you replace two legs with four legs, so you have two more legs than before.
Exercises: How many cows and chickens are there if you have ...

a) 34 legs and 12 animals
b) 38 legs and 12 animals
c) 48 legs and 12 animals

Answers: a) 5 cows and 7 chickens, b) 7 cows and 5 chickens, c) 12 cows and 0 chickens

Searching systematically to solve problems. SAY: We often combine the guess-check-revise and searching systematically strategies. Let’s say pens and pencils both cost a whole number of dollars. Write on the board:

4 pens and 3 pencils cost $43.
3 pens and 4 pencils cost $41.

SAY: Four pens and three pencils cost more than three pens and four pencils. ASK: Which costs more, a pen or a pencil? (pen) How do you know? (adding an extra pen adds more than adding an extra pencil) SAY: I would like to find out how much each pen and each pencil costs. I will start with four pens and three pencils costing $43, and with the cheaper item (pencils) costing $1 each. Draw on the board:

<table>
<thead>
<tr>
<th>1 Pencil</th>
<th>3 Pencils</th>
<th>4 Pens</th>
<th>1 Pen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$3</td>
<td>$43 – $3 = $40</td>
<td>$10</td>
</tr>
<tr>
<td>$2</td>
<td>$6</td>
<td>$43 – $6 = $37</td>
<td>(\times)</td>
</tr>
<tr>
<td>$3</td>
<td>$9</td>
<td>$43 – $9 = $34</td>
<td>(\times)</td>
</tr>
<tr>
<td>$4</td>
<td>$12</td>
<td>$43 – $12 = $31</td>
<td>(\times)</td>
</tr>
<tr>
<td>$5</td>
<td>$15</td>
<td>$43 – $15 = $28</td>
<td>$7</td>
</tr>
</tbody>
</table>

Ask a volunteer to calculate how much three pens and four pencils cost if one pencil costs $5 and one pen costs $7:

\[3 \times 7 + 4 \times 5 = 41\]

SAY: When the pens cost $7 each, both the first and second sentences are true, so this is the answer we are looking for. Write on the board:

Each pen costs $7 and each pencil costs $5.
Exercises: Notebooks and erasers each cost a whole number of dollars. Three notebooks and six erasers cost $30. Four notebooks and five erasers cost $34.

a) Which costs more, a notebook or an eraser?

b) How much does each notebook and each eraser cost?

Selected solution

<table>
<thead>
<tr>
<th>1 Eraser</th>
<th>6 Erasers</th>
<th>3 Notebooks</th>
<th>1 Notebook</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$6</td>
<td>$30 − $6 = $24</td>
<td>$8</td>
</tr>
<tr>
<td>$2</td>
<td>$12</td>
<td>$30 − $12 = $18</td>
<td>$6</td>
</tr>
<tr>
<td>$3</td>
<td>$18</td>
<td>$30 − $18 = $12</td>
<td>$4</td>
</tr>
</tbody>
</table>

You don’t need to continue the table because in the next row the eraser would be more expensive than the notebook. The answer is the second row because, with the eraser costing $2 and the notebook costing $6, the second sentence is true.

Selected answer: a) notebook

Problem Bank

1. When you multiply me by 9, the result is between 730 and 770. What numbers might I be? Hint: Evaluate $10 \times 9, 20 \times 9, 30 \times 9$, and so on until you get close to 700.

   Answers: 82, 83, 84, 85

2. Multiply me by 9, then round to the nearest ten. The result is 370. What number am I? Use the table below, then make a new table that increases the numbers by 1 instead of by 10.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number × 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>90</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

   Answer: 41

3. When you multiply me by 6, the result is less than 500. When you multiply me by 7, the result is more than 550. When you multiply me by 8, the result is more than 660. What number am I?

   Answer: 83
4. What are the two numbers?

a) The bigger number is five times the smaller number. The product of the two numbers is 180. Use the table below.

<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>Bigger Number</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

b) The bigger number is five times as big as the smaller number. The product of the two numbers is 18 000.

c) The bigger number is five times as big as the smaller number. The product of the two numbers is 14 045.

**Answers:** a) 6 and 30, b) 60 and 300, c) 53 and 265

5. If \(N \times N \times N \times N = 187 388 721\), what is \(N\)?

**Answer:** 117

6. A school bake sale sells muffins and pieces of cake. A muffin costs $2 and a piece of cake costs $3. The bake sale sold 47 items and made $111 in total. How many muffins and how many pieces of cake were sold?

**Answers:** 30 muffins and 17 pieces of cake

7. Use a calculator to answer these questions. Remember that two whole numbers are consecutive if there is no whole number between them.

a) Calculate the product.
   i) \(1 \times 2\) ii) \(2 \times 3\) iii) \(3 \times 4\) iv) \(4 \times 5\) v) \(5 \times 6\)

b) Is 14 the product of two consecutive whole numbers? Explain how you know.

c) Can 160 be the product of two consecutive whole numbers? Explain how you know.

d) Can 992 be the product of two consecutive whole numbers? Explain how you know.

e) Write 6972 as a product of two consecutive whole numbers.

**Answers**

a) i) 2, ii) 6, iii) 12, iv) 20, v) 30
b) no, it is between \(3 \times 4\) and \(4 \times 5\)
c) no, it is between \(12 \times 13 = 156\) and \(13 \times 14 = 182\)
d) yes, it is \(31 \times 32\)
e) \(83 \times 84\)
8. A perfect square is the product of a whole number with itself.
   a) Calculate the product.
      i) $1 \times 1$  ii) $2 \times 2$  iii) $3 \times 3$  iv) $4 \times 4$  v) $5 \times 5$
   b) Is 25 the product of two consecutive whole numbers? Explain how you know.
   c) Write 400 as a perfect square.
   d) Can you write 400 as the product of two consecutive whole numbers? Explain how you know.
   e) Explain why a perfect square cannot be the product of two consecutive whole numbers.

   **Answers**
   a) i) 1, ii) 4, iii) 9, iv) 16, v) 25
   b) no, because it is between $4 \times 5 = 20$ and $5 \times 6 = 30$ and there is no product of consecutive whole numbers between those two
   c) $400 = 20 \times 20$
   d) no, because it is between $19 \times 20 = 380$ and $20 \times 21 = 420$
   e) Any perfect square is between two consecutive products of consecutive whole numbers, so it cannot be the product of two consecutive whole numbers. For example, $15 \times 15$ is in between $14 \times 15$ and $15 \times 16$.

9. Today is Ben’s birthday. His age in years is 121 less than his age in months. How old is Ben? Hint: Use a chart:

<table>
<thead>
<tr>
<th>Age in Years</th>
<th>Age in Months</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

   **Answer:** 11 years old

10. John measured his pencil in millimetres and centimetres. The length in centimetres is 63 less than it is in millimetres. How long is John’s pencil?

   **Answer:** 7 cm or 70 mm

11. Lily calculated the number of weeks left until the summer holidays. Marko calculated the number of days (including weekends) left until the summer holidays. Marko’s answer is 42 more than Lily’s answer. How long until the summer holidays?

   **Answer:** 7 weeks or 49 days

12. In 2047, Canada will be 162 more years old than it will be decades old. How old will Canada be in 2047?

   **Answer:** 180 years or 18 decades
13. Three times a number is 20 more than half the number. What is the number?

Answer: 8

14. a) If \( \frac{A - 1}{A + 1} = \frac{4}{5} \), what is A?

b) If \( \frac{A \times A}{A + A} = 4 \), what is A?

c) If \( \frac{A + 2}{(A \times 2) + 1} = \frac{2}{3} \), what is A?

Answers: a) 9, b) 8, c) 4
PS5-6 Using Patterns in Sequences

Goals
Students will look for patterns in sequences, find gaps, find remainders, write formulas, and solve problems with sequences.

PRIOR KNOWLEDGE REQUIRED
Can multiply a multi-digit number by a one-digit number using the standard algorithm
Can perform long division
Can do operations on decimals

Review sequences with constant gap. Remind students that the gap in a sequence is the difference between two consecutive terms of a sequence. Write on the board:

4, 7, 10, 13, 16, ...

ASK: What is the gap? (3) Draw the numbers on a number line, as shown below:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Draw a new number line and ask a volunteer to show the positive multiples of 3 up to 15, as shown below:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

SAY: The two sequences have the same gap, but every term in the top sequence is moved over one unit to the right. ASK: What is the remainder if we divide 3, 6, 9, 12, and 15 by 3? (0) Why? (the terms are multiples of 3) What is the remainder if we divide 4 by 3? (1) If we divide 7 by 3? (1) If we divide 10 by 3? (1) Write on the board:

4 ÷ 3 = 1 R 1
7 ÷ 3 = 2 R 1
10 ÷ 3 = 3 R 1
13 ÷ 3 = 4 R 1

Point to the sequence 4, 7, 10, 13, 16, ... and SAY: The gap is 3, and if you divide each term by 3 you get the same remainder: 1. Write on the board:

20, 23, 26, 29, ...
ASK: What is the gap? (3) Have a volunteer divide each term by 3 and find the remainder, as shown below:

\[
\begin{align*}
20 \div 3 &= 6 \text{ R } 2 \\
23 \div 3 &= 7 \text{ R } 2 \\
26 \div 3 &= 8 \text{ R } 2 \\
29 \div 3 &= 9 \text{ R } 2
\end{align*}
\]

Explain to students that because the gap is the same, when they divide each term by the gap, they get the same remainder.

**Exercises**

1. Find the gap.
   a) 4, 6, 8, 10, ...  
   b) 6, 11, 16, 21, ...  
   c) 7, 11, 15, 19, ...  
   d) 12, 16, 20, 24, ...  
   e) 2, 8, 14, 20, ...  
   f) 11, 18, 25, 32, ...

**Answers:**

a) 2,  b) 5,  c) 4,  d) 4,  e) 6,  f) 7

2. For each part in Exercise 1, divide each term by the gap and find the remainder. Is the remainder the same for the whole sequence?

**Answers:**

a) 0, 0, 0, 0, yes;  
   b) 1, 1, 1, 1, yes;  
   c) 3, 3, 3, 3, yes;  
   d) 0, 0, 0, 0, yes;  
   e) 2, 2, 2, 2, yes;  
   f) 4, 4, 4, 4, yes

**Finding a term using the remainder.** Write on the board:

\[
\begin{align*}
3, 7, 11, 15, ...
\end{align*}
\]

ASK: If I continue the sequence, will I see the number 35? Ask a volunteer to write the next terms of the sequence, as shown below:

\[
3, 7, 11, 15, 19, 23, 27, 31, 35
\]

SAY: Thirty-five is a term of the sequence and we could find it by continuing the terms. ASK: How can we find if 135 belongs to the sequence? Have students share their ideas. Guide students to see how they can use the property they’ve just learned. Point to the sequence and ASK: Is the gap always the same? (yes) What is the gap? (4) Ask a volunteer to divide 3, 7, and 11 by 4 and find the remainder, as shown below:

\[
\begin{align*}
3 \div 4 &= 0 \text{ R } 3 \\
7 \div 4 &= 1 \text{ R } 3 \\
11 \div 4 &= 2 \text{ R } 3
\end{align*}
\]

SAY: We know that 35 is in the sequence because when 35 is divided by 4, the remainder is 3. Write on the board:

\[
35 \div 4 = 8 \text{ R } 3
\]

ASK: So how can we determine if 135 will belong to the sequence or not? (divide 135 by 4 and find the remainder) Ask a volunteer to divide 135 by 4 using long division, as shown on the following page.
Point to the remainder and SAY: When we divide 135 by 4, the remainder is 3, just the way it is for the earlier numbers in the sequence. So we know that if we continue the sequence, we will see the number 135 in the sequence.

Exercises

1. Is 500 in the sequence?
   a) 10, 16, 22, 28, … 
   b) 8, 14, 20, 26, …

Solutions
   a) the gap is 6, and if you divide the first term by the gap, the remainder is 4; 500 ÷ 6 = 83 R 2, so 500 is not in the sequence
   b) the gap is 6, and 8 ÷ 6 = 1 R 2; 500 ÷ 6 = 83 R 2, so 500 is in the sequence

2. Consider the sequences below.
   A. 6, 10, 14, 18, … 
   B. 9, 13, 17, 21, …
   C. 15, 19, 23, 27, … 
   D. 24, 28, 32, 36, …

Jen says 255 is in sequence A and Rani says 255 is in sequence B. Who is correct?

Solution: Both are incorrect. For all the sequences, the gap is 4 and the remainders are as follows: 2 for A, 1 for B, 3 for C, and 0 for D. 255 ÷ 4 = 63 R 3, so 255 is only in sequence C.

Review writing rules for sequences. Write on the board:

9, 16, 23, 30, …

ASK: How could you describe this sequence without saying all of the individual terms? (start at 9 and add 7 each time) SAY: When you say where to start and how to get each term from the one before it, you are saying a rule for the sequence.

Exercises

1. Write a rule for the sequence.
   a) 7, 11, 15, 19, … 
   b) 29, 23, 17, 11, … 
   c) 45, 53, 61, 69, … 
   d) 104, 93, 82, 71, … 
   e) 1.3, 2.2, 3.1, 4, …
Answers: a) start at 7 and add 4 each time, b) start at 29 and subtract 6 each time, c) start at 45 and add 8 each time, d) start at 104 and subtract 11 each time, e) start at 1.3 and add 0.9 each time

2. Consider the sequences below.

A. Start at 6 and add 7.  
B. Start at 9 and add 7.  
C. Start at 3 and add 7.  
D. Start at 27 and add 7.

Jen says 251 is in sequence A and Rani says 251 is in sequence D. Who is correct?

Answer: Both are correct.

Problem Bank

1. Each shape is made from toothpicks.

<table>
<thead>
<tr>
<th>Shape 1</th>
<th>Shape 2</th>
<th>Shape 3</th>
</tr>
</thead>
</table>

a) Make a table to show the number of toothpicks in each shape.
b) Find the gap and write a rule for the sequence.
c) Determine how many toothpicks Shape 5 will have.
d) Will any figure have 500 toothpicks? Explain how you know.

Selected answers: b) the gap is 6 and the rule is start at 16 and add 6; c) Shape 5 has 40 toothpicks; d) no, because 500 has remainder 2 when divided by 6, but all terms in the pattern have remainder 4 when divided by 6

2. Is 1386 in the sequence?

a) 1, 6, 11, 16, ...  
b) 2, 8, 14, 20, ...  
c) 1, 4, 7, 10, ...  
d) 9, 18, 27, 36, ...  
e) 1, 3, 5, 7, ...  
f) 7, 14, 21, 28, ...  

Answers: a) yes, b) no, c) no, d) yes, e) no, f) yes

3. Use \(941 \div 7 = 134 \text{ R } 3\) to evaluate \(1641 \div 7\). Explain your answer.

Answer: 1641 is 700 more than 941, so you need 100 more groups with the same number left over; \(1641 \div 7 = 234 \text{ R } 3\)

4. Which column is 400 in if the numbers continue as shown?

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
</tbody>
</table>

Solution: \(400 \div 6 = 66 \text{ R } 4\), so 400 is in Column 4.
5. a) Which column is 365 in if the numbers continue as shown?

<table>
<thead>
<tr>
<th>Sun</th>
<th>Mon</th>
<th>Tues</th>
<th>Wed</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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</tr>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>16</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

b) The year is not a leap year and January 1 is a Sunday. What day of the week is December 31?

c) Another year is a leap year and starts with January 1 on a Wednesday. What day of the week is January 1 of the following year?

Solutions

a) \(365 \div 7 = 52 \text{ R } 1\), so 365 is in the Sunday column

b) Sunday, because December 31 is the 365th day of the year

c) If January 1 is a Wednesday and the year is a leap year, the year will end on the 366th day—a Thursday—and the next year will start on Friday, January 1.

6. January 1 is a Thursday and this year is not a leap year. Braden plays soccer every Monday. His birthday is December 1. Will he play soccer on his birthday?

Solution: There are 365 days in the year, and 30 of them are after Braden’s birthday, so his birthday is the 335th day of the year. Since \(335 \div 7 = 47 \text{ R } 6\), his birthday is the sixth day of the core. The core starts Thursday, and the sixth term is Tuesday. No, Braden won’t play soccer on his birthday.

7. Hanna’s birthday is April 21 and Simon’s birthday is October 13. Are their birthdays on the same day of the week?

Solution: How many days after April 21 is October 13? 9 in April, 31 in May, 30 in June, 31 in July, 31 in August, 30 in September, and 13 in October, for a total of \(9 + 31 + 30 + 31 + 31 + 30 + 13 = 175\) and \(175 \div 7 = 25 \text{ R } 0\), so there are 25 full weeks from April 21 to October 13. Yes, their birthdays are on the same day of the week.

8. Ken used one kind of block to build a structure. He added the same number of blocks to his structure at each stage of its construction. But he made a mistake when he copied the number of blocks at each stage into the table below. Can you find his error and correct it?

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
</tbody>
</table>

Answer: The number of blocks in Stage 2 should be 11, not 12, because the gap is 7.
9. The table shows the number of line segments in a figure, but some information is missing. The same number of line segments was added at each stage. Fill in the missing numbers. Hint: First find the gap.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Line Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

**Solution:** There are four gaps between Figure 1 and Figure 5. The number of line segments increased by $20 - 8 = 12$, so the gap is $12 \div 4 = 3$. The missing numbers are 11, 14, 17.

10. The table shows the number of blocks used to build a structure. The same number of blocks was added at each stage. Fill in the missing numbers.

a) Stage Number of Blocks
   - 1: 9
   - 2: 
   - 3: 
   - 4: 15

b) Stage Number of Blocks
   - 1: 6
   - 2: 
   - 3: 
   - 4: 39

c) Stage Number of Blocks
   - 1: 11
   - 2: 
   - 3: 23
   - 4: 

**Answers:** a) 11, 13; b) 17, 28; c) 17, 29

11. Suppose you want to construct a block wall following the steps shown below. You would like the finished wall to be 11 blocks long. Each block costs 7¢ and you have $1.25 in total. Do you have enough money to buy all the blocks you need? Hint: Make a table with four columns: Step, Number of Blocks Used, Number of Blocks Long, and Cost (¢).
Solution

<table>
<thead>
<tr>
<th>Step</th>
<th>Number of Blocks Used</th>
<th>Number of Blocks Long</th>
<th>Cost (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>77</td>
<td>7</td>
<td>77</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>9</td>
<td>98</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>11</td>
<td>119</td>
</tr>
</tbody>
</table>

In Step 5, the wall will be 11 blocks long and will need 17 blocks. Since 17 blocks costs $1.19, there is enough money to buy all the blocks needed.

12. Make a table and extend it to predict the number of line segments in Figure 10. Hint: First find a pattern for the gap because the gap is not the same.

Solution: The sequence begins: 0, 1, 3, 6, 10, …, with gaps 1, 2, 3, 4, …. The sequence of gaps continues 5, 6, 7, 8, …, so the sequence itself continues: 15, 21, 28, 36, 45. Figure 10 has 45 line segments.
Unit 9 Number Sense: Fractions

Introduction
This unit extends existing concepts in fractions and introduces new concepts using pictures, standard notation, and word problems to explore:

- naming fractions;
- comparing fractions;
- equivalent fractions;
- mixed numbers and improper fractions; and
- multiplicative relationships.

Meeting Your Curriculum

<table>
<thead>
<tr>
<th></th>
<th>ALBERTA</th>
<th>BRITISH COLUMBIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>NS5-34 to 40, 44</td>
<td>Required NS5-34 to 40, 44 including Extension 1 in NS5-37 and Extension 1 in NS5-44</td>
</tr>
<tr>
<td>Optional</td>
<td>NS5-41 to 43, 45</td>
<td>Optional NS5-41 to 43, 45</td>
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<table>
<thead>
<tr>
<th></th>
<th>MANITOBA</th>
<th>MONCTON</th>
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<tbody>
<tr>
<td>Required</td>
<td>NS5-34 to 40, 44</td>
<td>Required NS5-34 to 37, 39, 41, 42, 44, 45</td>
</tr>
<tr>
<td>Optional</td>
<td>NS5-41 to 43, 45</td>
<td>Optional NS5-40</td>
</tr>
</tbody>
</table>

| Required             | NS5-34 to 37, 39, 41, 42, 44, 45           | Required NS5-38, 43                                   |
| Recommended          | NS5-40                                      | We strongly recommend that students learn to compare and order fractions with different denominators. |

Mental Math Minutes

The mental math minutes in this unit:
- multiplication
- division

Generic BLMs

The Generic BLM used in this unit is:
**BLM 1 cm Grid Paper** (p. S-1)

This BLM can be found in Section S.
Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>NS5-34 to 37</td>
<td>NS5-34 to 37</td>
<td>NS5-34 to 37</td>
<td>NS5-34 to 37</td>
</tr>
<tr>
<td>Quiz</td>
<td>NS5-38 to 40</td>
<td>NS5-38 to 40</td>
<td>NS5-38 to 40</td>
<td>NS5-38, 39, 41</td>
</tr>
<tr>
<td>Quiz</td>
<td>NS5-44</td>
<td>NS5-44</td>
<td>NS5-44</td>
<td>NS5-42 to 45</td>
</tr>
<tr>
<td>Test</td>
<td>NS5-34 to 40, 44</td>
<td>NS5-34 to 40, 44</td>
<td>NS5-34 to 40, 44</td>
<td>NS5-34 to 37, 39, 41, 42, 44, 45</td>
</tr>
</tbody>
</table>

Additional Information for This Unit

**Fraction notation**

We show fractions in two ways in our lesson plans:

- Stacked: \( \frac{1}{2} \)
- Not stacked: \( 1/2 \)

If you show your students the non-stacked form, remember to introduce it as new notation.
Goals

Students will understand that fractions can represent equal parts of a whole.
Students will name fractions of a whole.
Students will understand how to find the whole length if given a specific fraction of the whole length.

Prior Knowledge Required

Can name fractions

Materials

straightedge or metre stick
BLM Dark 1 cm Grid Paper (p. L-53)
centimetre rulers
two identical paper rectangles
grid paper or BLM 1 cm Grid Paper (p. S-1)
pattern blocks (see Extension 3)

Importance of equal parts in fractions. Draw on the board:

Say: These circles both have the same amount shaded, but one circle has one out of four parts shaded and the other circle has two out of five parts shaded. Ask: What is the fraction shaded, one fourth or two fifths? (one fourth) How do you know? (all the parts are the same size in the first picture but not in the second picture) Write the fraction 1/4 on the board.
Ask: What is the top number in a fraction called? (the numerator) What is the bottom number in a fraction called? (the denominator)

Draw on the board the shapes in the margin. Pointing to a) to d) in turn, ask: Does the shape have one half shaded? Students can signal their answers. Ask volunteers to articulate their reasoning when they answer “no.” (a) yes, b) no—the parts are not equal, c) no—the parts are not equal, d) yes) Pointing to e) to g) in turn, ask: Does the shape show 1/4? (e) yes, f) no—the parts are not equal, g) no—there are five parts, not four)

Dividing rectangles into equal parts. Draw some rectangles and mark their tops and bottoms at equal intervals, as shown in the margin. Ask volunteers to divide the rectangles into equal parts by joining the marks using a long straightedge, such as a metre stick; demonstrate joining the first pair of marks.

Draw three more rectangles on the board (50 cm long, 60 cm long, 80 cm long) with different amounts shaded (20 cm shaded, 10 cm shaded, and...
50 cm shaded, respectively. Use the 50 cm rectangle and show students how to divide the top and bottom sides into equal parts 10 cm long. Then have volunteers use the marks to divide the rectangle into equal parts. 

ASK: What fraction of the rectangle is shaded? (2/5) Repeat for the other two rectangles but have volunteers divide the top and bottom sides into equal parts (again, 10 cm long).

**ACTIVITY (Optional)**

Students work in pairs. Students will need BLM Dark 1 cm Grid Paper and another sheet of paper. Each student traces a rectangle from the grid paper onto the other sheet. The student then shades a fraction of the rectangle. Partners switch rectangles and use a centimetre ruler to determine what fraction their partner shaded. Partners check their answers with each other. **NOTE:** The shaded part does not have to be at one end of the rectangle.

**Pieces can have equal area but different shape.** Take two identical rectangles cut from paper. Tell students you want to cut them in half to share with four people. Cut one of them in half length-wise and the other in half width-wise, as shown in the margin. Ask for four volunteers. Give one piece to each of the four volunteers. ASK: Is this fair? (yes) Is one of these pieces bigger? (no—they are all the same size; they have equal area) SAY: They are both one of two equal parts of the same thing, so they are both the same size. Draw the following pictures on the board:

Point to the first pair and ASK: What fraction of each picture is shaded? (one fourth) Do they have the same shape shaded? (no) Do they have the same amount shaded? (yes) Repeat with the second pair. Now draw this rectangle (made of two adjacent, identical squares) on the board:

ASK: Are all four parts equal? (yes) Point out that each part is half of the same square, so they are equal parts of the rectangle. That means each part is one fourth of the rectangle.

**Exercises:** What fraction is shaded? How do you know?

a) 

b)

**Answers:** a)1/4, b) 1/8. All parts are equal because they are half of the same square.

**Creating wholes given one fractional part.** Draw on the board the first rectangle shown in the margin. It should be 20 cm long. SAY: This rectangle
is half of a longer rectangle. That means it’s one of two equal parts. Then have a volunteer use a ruler to extend the rectangle to make the whole. Repeat for the second rectangle, but this time it is one of three equal parts, so the volunteer will have to add two more parts.

Have students draw three rectangles on grid paper that are each three squares long and label them with the fractions 1/2, 1/3, and 1/4.

**Exercises**

1. Extend the rectangles to make a whole.

   **Answers:** The whole rectangles are 6 squares long, 9 squares long, and 12 squares long, respectively.

2. Without drawing a picture, predict how long the extended rectangle would be if the original rectangle was three squares long and \( \frac{1}{20} \) of the whole. Explain how you know.

   **Answer:** 60 squares long because you multiply \( 3 \times 20 \).

**Creating wholes from any fractional parts.** Draw a third rectangle on the board, also 10 cm long, but mark it as 2/3. SAY: This rectangle is two out of three equal parts. ASK: How long is each part if two of them are 10 cm long? (5 cm) SAY: This rectangle is two out of three parts—how many more parts do I need to get a whole? (one) Have a volunteer extend the rectangle to make a whole. (they should draw one more piece, 5 cm long) Repeat for a rectangle, also 10 cm long, but marked as 2/5. This time, the volunteer will need to make three more parts, each 5 cm long.

**Extensions**

1. For each part, draw a rectangle 6 squares long and 1 square high on grid paper or BLM 1 cm Grid Paper. Extend the rectangles to make a whole. Hint: For part b), first figure out how long one third is and then extend the drawing.

   a) one half  
   b) two thirds  
   c) three fifths  
   d) two fifths  
   e) three fourths

   **Answers:** The whole rectangles are a) 12 squares long, b) 9 squares long, c) 10 squares long, d) 15 squares long, e) 8 squares long.

2. On grid paper, draw three 4 by 4 grids. Show three different ways to shade half of the grid. Hint: The picture in the margin shows one way.

3. Give students pattern blocks. Each student will need one hexagon, two trapezoids, three rhombuses, and three triangles to determine which pattern block is the whole if …

   a) a triangle is one half.  
   b) a triangle is one third.  
   c) a trapezoid is one half.  
   d) a rhombus is one third.  
   e) a rhombus is two thirds.

   **Answers:** a) rhombus, b) trapezoid, c) hexagon, d) hexagon, e) trapezoid
4. Tasha cuts a rectangle into four parts, as shown below.

Do the four parts have the same area? Explain how you know.

**Answer:** Yes, I did the same thing to a sheet of paper and realized that if I fold each triangle in half (as shown below), they are all the same, so each triangle is twice of the same triangle, so they are all equal.

5. a) Divide a rectangle into thirds in two different ways.
   b) Divide a rectangle into fourths in three different ways.

6. Five eighths of a pizza was eaten. What fraction is left? (3/8) How do you know?
Goals
Students will understand that fractions can represent equal parts of a set.

PRIOR KNOWLEDGE REQUIRED
Understands fractions as equal parts of a whole

MATERIALS
geoboards and elastics
deck of cards (see Extension 1)
Snakes and Ladders board (see Extension 1)
red and blue counters (see Extension 4)

Mental math minute—number string.

String 1: $200 \div 2$, $200 \div 4$, $200 \div 8$ (100, 50, 25)

Dividing into twice as many groups is like dividing each group in the previous answer into two groups. So if $200 \div 2$ gives two groups of 100, then $200 \div 4$ gives 4 groups of $100 \div 2 = 50$. Each answer is obtained by dividing the previous answer by 2.

String 2: $2000 \div 2$, $2000 \div 4$, $2000 \div 8$ (1000, 500, 250)

Equal parts. Tell students that the whole for a fraction might not be a shape like a circle or square. Tell them that the whole can be anything that can be divided into equal parts. Brainstorm with the class other things that the whole might be: a line segment, an angle, a container, apples, oranges, amount of flour for a recipe. Tell them that the whole could even be a group of people. For example, SAY: The Grade 5 students in this class are a whole set, and I can ask questions such as these: What fraction of students in this class have birthdays in July? What fraction of students in this class are 11 years old? What fraction of students wear glasses? ASK: What do I need to know to find the fraction of students with birthdays in July? (the total number of students and the number of students with birthdays in July) Which number do I put on top: the total number of students or the number of students with birthdays in July? (the number of students with birthdays in July) What does the denominator represent? (the total number of students) What fraction of students in this class have birthdays in July?

Ensure that students say the correct name for the fraction—for example, $10/25$ is “ten twenty-fifths.” Tell them that the students from the example above don’t have to be the same size; they are still equal parts of a set. ASK: What fraction of your family is older than 16? Younger than 16? Some of these fractions, for some students, will have the numerator 0, and this should be pointed out. Avoid asking questions that will lead them to fractions with a denominator of 0. For example, the question “What fraction of your siblings are male?” will lead some students to say 0/0.
**Identifying fractions of sets.** Draw on the board:

```
\[
\begin{array}{c}
\triangle \square \bigcirc \bigcirc \\
\end{array}
\]
```

ASK: What fraction of these shapes are shaded? (2/5) What fraction are circles? (3/5) What fraction of the circles are shaded? (1/3)

Draw on the board:

```
\[
\begin{array}{c}
\triangle \bigtriangleup \square \bigtriangleup \bigtriangleup \\
\end{array}
\]
```

ASK: What fraction of these shapes are shaded? (3/5) What fraction are not shaded? (2/5) What fraction are squares? (2/5) Triangles? (3/5) What fraction of the triangles are shaded? (1/3) What fraction of the squares are shaded? (2/2) What fraction of the squares are not shaded? (0/2)

**Exercises:** Answer the questions for each set of shapes.

i)  
```
\[
\begin{array}{c}
\triangle \square \bigcirc \bigcirc \\
\end{array}
\]
```

ii)  
```
\[
\begin{array}{c}
\triangle \bigtriangleup \bigcirc \square \bigtriangleup \bigtriangleup \\
\end{array}
\]
```

iii)  
```
\[
\begin{array}{c}
\triangle \bigtriangleup \bigcirc \bigcirc \bigtriangleup \bigcirc \\
\end{array}
\]
```

a) What fraction are circles? b) What fraction are shaded? c) What fraction are squares? d) What fraction are triangles?

**Selected answers:** for i), a) 3/5 are circles, b) 2/5 are shaded, c) 1/5 are squares, d) 1/5 are triangles

**Bonus**

a) In the exercise above, the sets of shapes had two attributes, shape (triangle, circle, square) and colour (shaded or unshaded). The following set has three attributes: shape, colour, and size. Answer the questions from the previous exercise for the following set of shapes.

```
\[
\begin{array}{c}
\square \bigcirc \triangle \bigtriangleup \bigtriangleup \square \bigcirc \\
\end{array}
\]
```

b) What fraction of the triangles in the set above are shaded? What fraction of the shaded shapes are triangles?

**Answers:** a) 2/9, 4/9, 4/9, 3/9; b) 2/3, 2/4
**Asking fraction questions.** Draw the following diagram on the board. Have students make up questions to ask each other about the diagram.

![Diagram](image)

Have some students volunteer questions to ask and others volunteer answers.

Then have students write fraction statements in their notebooks for similar pictures.

**Exercises:** Solve the word problem.

a) A basketball team played 5 games and won 2 of them. What fraction of the games did the team win?

b) A basketball team won 3 games and lost 1 game. How many games did they play altogether? What fraction of their games did they win?

c) A basketball team won 4 games, lost 1 game, and tied 2 games. How many games did they play? What fraction of their games did they win?

**Answers:** a) 2/5; b) played 4, won 3/4; c) played 7, won 4/7

Also give word problems that use words such as “and,” “or,” and “not”—for example, Sally has 4 red marbles, 2 blue marbles, and 6 green marbles. Ask questions like the ones below, writing them on the board as you go:

a) What fraction of her marbles are red? (4/12)

b) What fraction of her marbles are blue or red? (6/12)

c) What fraction of her marbles are not blue? (10/12)

d) What fraction of her marbles are not green? (6/12)

For an additional challenge, ASK: Which two questions have the same answer? (b and d) Why? (the fraction that is blue or red equals the fraction that is not green) Challenge students to write another question that uses “or” that will have the same answer as: What fraction of Sally’s marbles are not blue?

**Finding fractions using different attributes in a set.** Draw on the board:

![Diagram](image)

**Exercises**

a) What fraction of the shapes are shaded and circles?

b) What fraction of the shapes are shaded or circles?

c) What fraction of the shapes are not circles?

d) Write a question that has the same answer as c):
   What fraction of the shapes are _____ or _____?

**Answers:** a) 1/7; b) 3/7; c) 5/7; d) triangles, squares
Have students create their own questions and challenge a partner to answer them.

**Solving word problems that include fractions.** Ask students to find the number of each item given the fractions.

**Exercises**

a) A team played 5 games. They won \( \frac{2}{5} \) of their games and lost \( \frac{3}{5} \) of their games. How many games did they win? Lose?

b) There are 7 marbles. \( \frac{2}{7} \) are red, \( \frac{4}{7} \) are blue, and \( \frac{1}{7} \) are green. How many blue marbles are there? Red marbles? Green marbles?

**Answers:** a) 2, 3; b) 4, 2, 1

Then tell students that you have five squares and circles. Some are shaded and some are not. Have students draw shapes that fit the puzzles below.

a) \( \frac{2}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the shapes are shaded. 1 circle is shaded. How many squares are not shaded?

b) \( \frac{3}{5} \) of the shapes are squares. \( \frac{2}{5} \) of the shapes are shaded. No circle is shaded. How many squares are not shaded?

c) \( \frac{3}{5} \) of the shapes are squares. \( \frac{3}{5} \) of the shapes are shaded. \( \frac{1}{3} \) of the squares are shaded. How many circles are shaded?

**Selected answer:** a) ◼ ◼ ◼☐☐ 1 square is not shaded.

**ACTIVITY (Optional)**

Have students enclose a given fraction of the pegs with an elastic on a geoboard. For example, they could enclose 10/25, as shown below:

![Geoboard Illustration](image)

**ASK:** What fraction of the pegs are not enclosed? (15/25) Students should see that the two numerators (in this case 10 and 15) always add up to the denominator (25).
Extensions

1. Have students investigate:
   a) In a deck of cards, what fraction of the cards are face cards (Jack, Queen, King)? What fraction of the cards are hearts?
   b) On a Snakes and Ladders board, on what fraction of the board would you be forced to move down a snake?

   Have students make up their own fraction question about a game they like and tell you the answer next class.

   Answers: a) 12/52, 13/52; b) answers will vary

2. There are 7 triangles and squares. \( \frac{2}{7} \) of the shapes are triangles. \( \frac{3}{7} \) are shaded. 2 triangles are shaded. Draw a picture to show these shapes.

   Answer: two shaded triangles, one shaded square, four unshaded squares

3. There are 5 squares and circles. \( \frac{3}{5} \) of the shapes are squares. \( \frac{1}{3} \) of the squares are big. \( \frac{3}{5} \) of the shapes are shaded. \( \frac{2}{3} \) of the squares are shaded. \( \frac{2}{5} \) of the shapes are big. No shaded shape is big. Draw a picture to show the shapes.

   Answer
   
   
   4. Give students red and blue counters or any other pair of colours. Then ask them to solve the following problems by making a model.
   a) Half of the counters are red. There are 10 red counters. How many are blue?
   b) Two fifths of the counters are blue. There are 6 blue counters. How many are red?
   c) \( \frac{3}{4} \) of the counters are red. 9 are red. How many are blue?

   Answers: a) 5, b) 9, c) 3

5. What fraction of the letters of the alphabet are …
   a) in the word “fractions”? b) not in the word “fractions”?

   Answers: a) 9/26, b) 17/26

6. For the month of September in this year, what fraction of all the days are …
   a) Sundays? b) Wednesdays? c) school days?
   d) not school days? e) divisible by 5?
7. By the following time, what fraction of an hour has passed since 9:00?
   a) 9:07   b) 9:15   c) 9:30   d) 9:40
   **Answers:** a) 7/60, b) 15/60, c) 30/60, d) 40/60

8. SAY: There are 5 circles and triangles. ASK: Can you draw a set so that …
   a) \(\frac{4}{5}\) are circles and \(\frac{2}{5}\) are striped?
      Have volunteers show the different possibilities for a) before moving on. Ask questions like: How many are striped circles? How many different answers are there?
   b) \(\frac{4}{5}\) are circles and \(\frac{2}{5}\) are triangles?
      ASK: What is the same about questions a) and b)? (the fractions are the same in both) What is different? (one is possible, the other is not; one uses the same attribute, shape, in both; the other uses two different attributes, shape and shading)
Goals
Students will look at pictures to compare and order fractions.

PRIOR KNOWLEDGE REQUIRED
Can name fractions
Knows that fractions show quantities made of same-sized pieces

MATERIALS
two identical, rectangular strips of paper
BLM Fraction Strips (p. L-54)

Comparing fractions. In advance, prepare two identical rectangular strips of paper, as shown below:

```
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
```

Divide one in half and the other into three equal parts. Then demonstrate to the class, by folding, that in each rectangle, the parts are equal. Tell students to pretend each rectangle is a chocolate bar. ASK: Would you get a bigger piece if you had to share it between two people or between three people? (two people)

SAY: One half of the chocolate bar is more than one third of the chocolate bar. Tell students that one half of a shape is always more than one third of the same shape. Write on the board:

\[
\frac{1}{2} \text{ is more than } \frac{1}{3}
\]

SAY: I want to know which fraction is bigger, 1/2 or 2/3. Draw on the board:

```
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
```

ASK: Which picture makes it easier to tell which is more? (the second one)
Now draw this picture on the board:

```
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
```

ASK: Does this picture make it easy to tell which is bigger, 1/2 or 2/3? (no)
Point out that the strips have to be lined up and the shaded parts have to be side by side at the same end for it to be easy to tell which has more shading. Have students use fractions strips for 2/3, 1/2, 3/4, and 3/5 from BLM Fraction Strips to compare fractions.
Using the signs for less than (<) and greater than (>). Remind students that > means greater than and < means less than.

**Exercises:** Write the correct sign, < or >.

a) \( \frac{2}{3} \) \( \square \) \( \frac{1}{2} \)  

b) \( \frac{1}{2} \) \( \square \) \( \frac{3}{4} \)  

c) \( \frac{2}{3} \) \( \square \) \( \frac{3}{4} \)  

d) \( \frac{3}{4} \) \( \square \) \( \frac{3}{5} \)  

**Bonus**

e) \( \frac{1}{2} \) \( \square \) \( \frac{5}{8} \)  

f) \( \frac{2}{3} \) \( \square \) \( \frac{5}{8} \)  

g) \( \frac{3}{4} \) \( \square \) \( \frac{5}{8} \)  

h) \( \frac{3}{5} \) \( \square \) \( \frac{5}{8} \)  

**Answers:** a) >, b) <, c) <, d) >, Bonus: e) <, f) >, g) >, h) <  

**Ordering fractions using pictures.** Explain that just as you used pictures to compare pairs of fractions you can use pictures to order fractions. Draw four identical rectangles on the board, all 36 cm long. Divide them, as shown below, into three equal parts, twelve equal parts, four equal parts, and six equal parts:

[Diagram of rectangles divided into parts]

Have volunteers write a fraction beside each strip to represent the shaded region. (2/3, 3/12, 3/4, 5/6) Draw on the board:

[Diagram of fractions ordered from greatest to least]

Have volunteers order the fractions from greatest to least. (5/6, 3/4, 2/3, 3/12) ASK: How did you decide on the order? (the more shading a strip has the greater the fraction)

**Introduce equivalent fractions.** Give students the remaining fraction strips from BLM Fraction Strips (4/6, 3/6, 6/8, 6/10).

**Exercises:** Write >, <, or =.

a) \( \frac{3}{5} \) \( \square \) \( \frac{6}{10} \)  

b) \( \frac{6}{10} \) \( \square \) \( \frac{3}{6} \)  

c) \( \frac{3}{4} \) \( \square \) \( \frac{4}{6} \)  

d) \( \frac{3}{4} \) \( \square \) \( \frac{6}{8} \)  

e) \( \frac{1}{2} \) \( \square \) \( \frac{3}{6} \)  

f) \( \frac{1}{2} \) \( \square \) \( \frac{4}{6} \)  

g) \( \frac{2}{3} \) \( \square \) \( \frac{4}{6} \)  

h) \( \frac{1}{2} \) \( \square \) \( \frac{6}{8} \)  

**Answers:** a) =, b) >, c) >, d) =, e) =, f) <, g) =, h) <

Tell students that two fractions can have different numbers but actually show the same amount. Have volunteers name pairs of fractions that they found in the previous exercise that look different but still represent the same amount.
Extensions

1. Find the missing denominator.

\[
\frac{1}{3} = \frac{4}{\phantom{4}}
\]

Answer: 12

2. Complete the fraction using the pictures.

a) \[
\frac{1}{4} = \frac{25}{100}
\]

b) \[
\frac{2}{5} = \frac{4}{10}
\]

Answers: a) 25/100, b) 4/10

3. Two rectangular gardens are the same size. 3/4 of one garden is planted, and 7/8 of the other garden is planted. Start by making two fraction strips of exactly the same size. Fold the strips to decide which garden has more planted.

Answer: the garden with 7/8 planted
Goals
Students will compare fractions by using number lines.

PRIOR KNOWLEDGE REQUIRED
Can name fractions shown with fraction strips
Can compare fractions using fraction strips

MATERIALS
elastic bands

From strips to number lines. Remind students that we can take a fraction of a line the same way we can take a fraction of a strip. Then SAY: Let’s take a number line from 0 to 1 and use it as the object that we want to take a fraction of. You can divide the number line into any number of equal parts the same way you divide a fraction strip into four equal parts. Draw on the board:

\[
\begin{array}{cccccccc}
\text{0} & \text{1} \\
\text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6}
\end{array}
\]

ASK: What fraction of the number line is shaded? (3/4) SAY: It is divided into four parts and three of them are shaded, so three fourths are shaded.

Exercises: What fraction of the number line is shaded?

a) \[
\begin{array}{cccccccc}
\text{0} & \text{1} \\
\text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6}
\end{array}
\]

b) \[
\begin{array}{cccccccc}
\text{0} & \text{1} \\
\text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6}
\end{array}
\]

c) \[
\begin{array}{cccccccc}
\text{0} & \text{1} \\
\text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6} & \text{6}
\end{array}
\]

Answers: 1/4, b) 2/5, c) 3/5

Ordering fractions with the same denominator using number lines.

Draw on the board:

Have a volunteer mark an X on the number line to show 1/6. Continue with the next fraction in the group written below the number line until they have all been marked. ASK: Are the fractions in order now, from least to greatest? (yes) Point out that the number line orders the fractions for you. Write the fractions in order in the boxes. (1/6 < 2/6 < 4/6 < 6/6)
Exercises

1. Use the number line to order the fractions. Draw an X for each fraction.

a) \(\frac{2}{9}, \frac{7}{9}, \frac{5}{9}, \frac{8}{9}, \frac{1}{9}, \frac{4}{9}\)

b) \(\frac{5}{5}, \frac{3}{5}, \frac{0}{5}, \frac{2}{5}\)

Answers: a) 1/9, 2/9, 4/9, 5/9, 7/9, 8/9; b) 0/5, 2/5, 3/5, 5/5

2. Write a fraction that is between the two fractions.

a) \(\frac{3}{9}\) and \(\frac{8}{9}\)  
b) \(\frac{1}{5}\) and \(\frac{5}{5}\)  
c) \(\frac{1}{4}\) and \(\frac{3}{4}\)  
d) \(\frac{5}{10}\) and \(\frac{8}{10}\)

Sample answers: a) 5/9, b) 4/5, c) 2/4, d) 6/10

NOTE: Under the Ontario curriculum, when using standard fraction notation (but not pictorial representations), students only need to compare and order fractions with the same denominator. However, we recommend that they also compare and order fractions with different denominators.

Comparing fractions with different denominators using number lines. Draw on the board two 20 cm fraction strips, one showing \(\frac{4}{5}\) (16 cm) shaded and the other showing \(\frac{2}{4}\) (10 cm) shaded. Leave enough space between them to add a number line later. Ask: Which fraction is greater? (\(\frac{4}{5}\)) How do you know? (more of the strip is shaded). Write on the board:

\[\frac{4}{5} > \frac{2}{4}\]

Then add a number line from 0 to 1 between them and label the two fractions on the number line, as shown in the margin.

Ask: Which fraction is farther to the right, \(\frac{4}{5}\) or \(\frac{2}{4}\)? (\(\frac{4}{5}\)) Point out that because \(\frac{4}{5}\) is farther right on the number line than \(\frac{2}{4}\), the number line also shows that \(\frac{4}{5}\) is greater than \(\frac{2}{4}\). So one fraction is greater than another if it is farther to the right. Point out the connection with comparing whole numbers: one whole number is also greater than another if it is farther right on a number line.
Using number lines to find fractions that represent the same amount.

Draw on the board a 48 cm long number line divided into four equal parts and marked 0 and 1 at either end. ASK: How many parts is the distance from 0 to 1 divided into? (4) Have a volunteer label the fractions. (1/4, 2/4, 3/4) Now draw another number line, also 48 cm long, underneath, this one divided into sixths. ASK: How many parts is the distance from 0 to 1 divided into on this number line? (6) What labels will you use for the scale of this number line? (1/6, 2/6, 3/6, 4/6, 5/6) Have a volunteer label the fractions.

NOTE: The fraction 2/4 on the top number line should be vertically aligned with the fraction 3/6 on the bottom number line.

Draw a single oval encompassing both 2/4 and 3/6 on the number lines. Beside the number line, write:

\[
\frac{2}{4} = \frac{3}{6}
\]

Tell students that even though the numerators and denominators in the two fractions aren’t the same, the fractions are equal because they represent the same place on the number line.

**ACTIVITY (Optional)**

First, show students how they can make fraction strips out of wide elastics. Draw four marks 1 cm apart on the unstretched elastic (see margin) and label the first mark 0 and the last mark 1.

Show a picture of this on the board. Point out that there are three equal parts from 0 to 1, so each part is a third. Next, draw three marks 1 cm apart on a second unstretched elastic to make halves, again labelling the first mark 0 and the last mark 1. Point out that now they can stretch the smaller unit to make it the same length as the bigger unit. Now they can compare the fractions:

Students, working in groups, can order three fractions at a time:

a) \( \frac{2}{3}, \frac{3}{4}, \frac{3}{5} \)  b) \( \frac{1}{3}, \frac{1}{4}, \frac{2}{5} \)  c) \( \frac{2}{3}, \frac{2}{4}, \frac{4}{5} \)  d) \( \frac{1}{3}, \frac{2}{4}, \frac{1}{5} \)

Students can also create elastics divided into sixths, eighths, or tenths and use these to find equivalent fractions.

NOTE: Extension 1 is required in order to cover the British Columbia curriculum.
Extensions

1. Estimate approximately where the fraction goes by writing the letter above the number line.

   a) A. \(\frac{18}{20}\)  B. \(\frac{9}{20}\)  C. \(\frac{1}{20}\)  D. \(\frac{12}{20}\)

   

   

   0 closer to 0  closer to \(\frac{1}{2}\)  closer to \(\frac{1}{2}\)  closer to 1 1

   less than \(\frac{1}{2}\)  greater than \(\frac{1}{2}\)

   

   b) A. \(\frac{7}{9}\)  B. \(\frac{7}{50}\)  C. \(\frac{7}{15}\)  D. \(\frac{7}{12}\)

   

   0 closer to 0  closer to \(\frac{1}{2}\)  closer to \(\frac{1}{2}\)  closer to 1 1

   less than \(\frac{1}{2}\)  greater than \(\frac{1}{2}\)

   

   Answers: a) C B D A  b) B C D A

2. Find the missing denominator by continuing to divide the bottom number line and then labelling the tick marks on both number lines.

   \[\frac{1}{3} = \frac{5}{?}\]
Goals
Students will compare and order fractions based on the size and the number of fractional parts.

PRIOR KNOWLEDGE REQUIRED
Can name fractions
Understands and can use < and > properly
Can compare and order whole numbers

MATERIALS
BLM Fraction Cards (pp. L-55–57)
BLM Ordering with Fraction Strips (p. L-58)
BLM Ordering Fractions (pp. L-59–60, see Extension 1)

Mental math minute. Have groups of four students add three-digit numbers by adding hundreds, tens, and ones separately. Give an addition problem, such as 355 + 467. The first student adds the hundreds: 300 + 400 = 700; the second adds the tens: 50 + 60 = 110; and the third student adds the ones: 5 + 7 = 12; and the fourth student finishes the addition: 700 + 110 + 12 = 822, so 355 + 467 = 822. Start with problems that do not require regrouping, such as 253 + 346, and continue to questions that require regrouping ones and then ones and tens.

Comparing fractions with the same denominator. Draw on the board:

ASK: Are these circles the same size? (yes) Do they have the same number of parts? (yes) Are the parts the same size? (yes) Have volunteers name the fractions. (1/4 and 3/4) Write them on the board. ASK: Which is more, one fourth of the circle or three fourths of the circle? SAY: Three fourths of something is always greater than one fourth of the same thing since the fourths are the same size. Have a volunteer write < or > between the fractions. Draw on the board:

3/6

5/6
ASK: Which strip has a greater amount shaded? (the bottom one) Which is more, five sixths or three sixths of the strip? (5/6 of the strip) Write on the board:
\[
\frac{5}{6} \quad \frac{3}{6}
\]
Have a volunteer write the correct sign between the fractions. (>)

**Exercises:** Write < or >.

a) \(\frac{2}{15} \quad \frac{4}{15}\)  

b) \(\frac{3}{9} \quad \frac{2}{9}\)

c) \(\frac{6}{10} \quad \frac{9}{10}\)  

**Bonus:** \(\frac{37}{7159} \quad \frac{27}{7159}\)

**Answers:** a) <, b) >, c) <, Bonus: >

Write on the board:
\[
\frac{4}{9} \quad \frac{9}{9}
\]

ASK: What numbers could we put in the blank to make the relationship correct? (0, 1, 2, and 3) Have students suggest all possibilities. Repeat with \(\frac{7}{10} < \ldots\). (8, 9, 10)

**Exercises:** Write any number in the blank that makes the relationship correct.

a) \(\frac{6}{10} \quad \ldots\)  

b) \(\frac{2}{5} \quad \ldots\)  

c) \(\frac{1}{2} \quad \ldots\)  

d) \(\frac{7}{7} \quad \ldots\)

**Answers:** a) 5, 4, 3, 2, 1, or 0; b) 3, 4, or 5; c) 2; d) 6, 5, 4, 3, 2, 1, or 0

**ACTIVITY 1 (Essential)**

1. Give each student a card from BLM Fraction Cards. Have students group themselves according to the denominators on their cards. Then ask students to line up from least (at the front) to greatest (at the back).

**Ordering fractions with the same denominator by considering the numerators.** Draw on the board:
\[
\frac{1}{6} \quad \frac{6}{6} \quad \frac{2}{6} \quad \frac{4}{6}
\]

ASK: What does the denominator “6” mean? (6 parts) Since the denominators are all the same, what tells us how to order the fractions? (numerator) If we do that, which fraction is first and which is last? (one sixth, six sixths) Have volunteers write the fractions in the blanks, in order from least to greatest. (1/6 < 2/6 < 4/6 < 6/6)
Exercises: Order the fractions from least to greatest.

a) \(\frac{2}{9}, \frac{7}{9}, \frac{5}{9}, \frac{8}{9}, \frac{1}{9}, \frac{4}{9}\)  
b) \(\frac{5}{5}, \frac{3}{5}, \frac{0}{5}, \frac{2}{5}\)

Answers: a) \(1/9, 2/9, 4/9, 5/9, 7/9, 8/9\); b) \(0/5, 2/5, 3/5, 5/5\)

Comparing fractions with the same numerator. Draw on the board:

ASK: Are the strips the same length? (yes) Do they have the same number of parts? (no) Are the parts the same size? (no) Have volunteers name the fractions. (one fourth, one third, one half) Write the fractions beside the strips using standard fraction notation. ASK: Which is more, one fourth of the strip or one half of the strip? (one half) Why? (one half is bigger) Point out that the more parts something is divided into, the smaller each part is.

Write on the board:

\[
\frac{1}{4}, \frac{1}{9}
\]

ASK: Which fraction is greater? (1/4) Have a volunteer write the correct inequality sign in the box. (>) Repeat for 3/8 and 3/5. (3/8 < 3/5)

Exercises

1. Write < or >.

   a) \(\frac{2}{8}, \frac{2}{3}\)  
   b) \(\frac{1}{2}, \frac{1}{10}\)  
   c) \(\frac{4}{5}, \frac{4}{4}\)  
   d) \(\frac{1}{10}, \frac{1}{100}\)  
   e) \(\frac{12}{15}, \frac{12}{18}\)  
   f) \(\frac{7}{20}, \frac{7}{8}\)  
   g) \(\frac{6}{12}, \frac{6}{16}\)  
   Bonus: \(\frac{235}{1000}, \frac{235}{400}\)

   Answers: a) <, b) >, c) <, d) >, e) >, f) <, g) >, Bonus: <

2. Write any number in the blank that makes the relationship correct.

   a) \(\frac{1}{10} > \frac{12}{5}\)  
   b) \(\frac{3}{7} < \frac{5}{5}\)  
   c) \(\frac{5}{9} > \frac{5}{9}\)  
   Bonus: \(\frac{172}{983} < \frac{172}{983}\)

   Sample answers: a) 1, b) 3, c) 40, Bonus: 175
ACTIVITY 2 (Optional)

2. Repeat Activity 1, but have students with the same numerator group together and then order themselves.

Ordering fractions with the same numerator using fraction strips.

Draw on the board:

$$\begin{align*}
\frac{1}{4} & \quad \frac{1}{10} & \quad \frac{1}{2} & \quad \frac{1}{5} & \quad \frac{1}{3} \\
\end{align*}$$

Explain that you want to order the fractions from least to greatest.

ASK: What do the denominators represent? (the number of parts)

SAY: The more parts the strip is divided into, the smaller the parts are, which is why the largest denominator represents the smallest shaded fraction. Have volunteers match each fraction to a fraction strip, shade it, and then write the fraction in the box below the strip. The finished diagram should look like this:

$$\begin{align*}
\frac{1}{10} & \quad \frac{1}{5} & \quad \frac{1}{4} & \quad \frac{1}{3} & \quad \frac{1}{2} \\
\end{align*}$$

ASK: Are the fractions ordered from least to greatest? (yes)

Exercise: Complete BLM Ordering with Fraction Strips.

Answers:
1. a) $\frac{2}{9}, \frac{2}{6}, \frac{2}{5}, \frac{2}{3}$; b) $\frac{3}{16}, \frac{3}{12}, \frac{3}{10}, \frac{3}{7}$
2. a) $\frac{1}{1}, \frac{1}{3}, \frac{1}{8}, \frac{1}{10}$; b) $\frac{4}{6}, \frac{4}{8}, \frac{4}{15}, \frac{4}{18}$

Ordering fractions with the same numerator by considering the numerators and denominators. Draw on the board:

$$\begin{align*}
\frac{2}{4} & \quad \frac{2}{5} & \quad \frac{2}{10} & \quad \frac{2}{3} \\
\end{align*}$$

ASK: What does the numerator “2” mean here? (the number of shaded parts) If we order the fractions from greatest to least, which fraction will be first? (2/2) Second? (2/3) Last? (2/10) Write all the fractions in the boxes, from greatest to least. (2/2, 2/3, 2/4, 2/5, 2/10)
Exercises: Order the fractions in part a) from least to greatest and the fractions in part b) from greatest to least.

a) \[
\frac{4}{18} \quad \frac{4}{5} \quad \frac{4}{11} \quad \frac{4}{21}
\]

\[
\begin{array}{cccc}
\ & < & < & < \\
\end{array}
\]

b) \[
\frac{12}{15} \quad \frac{12}{12} \quad \frac{12}{31} \quad \frac{12}{38}
\]

\[
\begin{array}{cccc}
> & > & > & > \\
\end{array}
\]

Answers: a) 4/21, 4/18, 4/11, 4/5; b) 12/12, 12/15, 12/31, 12/38

Extensions

1. Complete BLM Ordering Fractions.

   Answers: 1. a) 8/4, 10/4, 9/4; b) 8/4 < 9/4 < 10/4; 2. a) 9/3, 10/3, 8/3; b) 8/3 < 9/3 < 10/3; 3. a) 8/2, 9/2, 7/2; b) 7/2 < 8/2 < 9/2; 4. a) 16/10, 20/10, 18/10; b) 16/10 < 18/10 < 20/10; 5. >, <, >

2. Use two number lines to decide if the fraction is closer to 0, \(\frac{1}{2}\), or 1.

\[
\begin{array}{ccccccc}
0 & \quad \frac{1}{4} & \quad \frac{2}{4} & \quad \frac{3}{4} & \quad \frac{4}{4} \\
\frac{4}{4} & \quad \frac{4}{4} & \quad \frac{4}{4} & \quad \frac{4}{4} & \quad \frac{4}{4} \\
\frac{7}{7} & \quad \frac{7}{7} & \quad \frac{7}{7} & \quad \frac{7}{7} & \quad \frac{7}{7} \\
\end{array}
\]

Write 0, \(\frac{1}{2}\), or 1.

a) \(\frac{4}{7}\) is closer to _____

b) \(\frac{3}{7}\) is closer to _____

c) \(\frac{6}{7}\) is closer to _____

Answers: a) 1/2, b) 1/2, c) 1

3. Compare the fractions \(\frac{3}{4}\) and \(\frac{4}{5}\) by comparing how much of a whole pie is left if the given amounts are eaten. Hint: The smaller fraction is from the pie with a bigger piece left over.

Answer: 1/4 is more than 1/5, so 3/4 is less than 4/5.

4. Write the fractions in order from least to greatest.

\[
\begin{array}{cccc}
\frac{1}{5} & \frac{3}{8} & \frac{7}{8} & \frac{1}{8} \\
\end{array}
\]

Answer: 1/8, 1/5, 3/5, 7/8
**NS5-39 Equivalent Fractions**

**Goals**

Students will find equivalent fractions using multiplication.

**PRIOR KNOWLEDGE REQUIRED**

Can name fractions
Can use the phrase “times as many as” to compare two numbers

**MATERIALS**

BLM Equivalent Fractions Memory (pp. L-61–63)
three identical strips of paper per student, each about 22 cm long
(see Extension 3)

**Mental math minute.** Give students multiplication questions that can be done by skip counting by 20, 30, 40, 50, or 100. Have students skip count out loud to answer multiplication questions.

**Breaking all parts into two equal parts to create equivalent fractions.**

Draw the first picture below on the board. Tell students that a parent and a child were sharing a cake, so the parent divided the cake into two pieces. The child said he wanted two pieces, so the parent cut the cake again and gave the child two pieces. Show this with the second picture, below:

ASK: Did the child get more cake by getting two pieces? (No) Write the first equation below on the board:

\[
\frac{1}{2} = \frac{2}{4} \rightarrow \frac{1 \times 2}{2 \times 2} = \frac{2}{4}
\]

SAY: The fractions are equal, or *equivalent*, because the pictures have the same amount shaded. Then show how the numerators and denominators are related by multiplication, as in the second equation above. SAY: Both people get twice as many pieces but the same amount of cake as before.

**Exercises:** Copy the picture into your notebook. Break each part in half to create equivalent fractions.

a)  

b)  

c)  

d)  

**Answers:** a) \(\frac{1}{3} = \frac{2}{6}\), b) \(\frac{2}{3} = \frac{4}{6}\), c) \(\frac{1}{4} = \frac{2}{8}\), d) \(\frac{3}{4} = \frac{6}{8}\)

**Sample pictures for a):**

---

**VOCABULARY**

denominator

equiv

tal equivalent

t fraction

fraction names (half, third, fourth, and so on)

numerator

**CURRICULUM REQUIREMENT**

AB: required
BC: required
MB: required
ON: required

---

**CURRICULUM REQUIREMENT**

AB: required
BC: required
MB: required
ON: required

---

**VOCABULARY**

**denominator**

**equivalent**

**fraction**

**fraction names (half, third, fourth, and so on)**

**numerator**

---

**CURRICULUM REQUIREMENT**

AB: required
BC: required
MB: required
ON: required

---

**VOCABULARY**

denominator

equivalent

fraction

fraction names (half, third, fourth, and so on)

numerator
Breaking all parts into the same number of equal parts to create equivalent fractions. Draw the pairs of pictures shown below on the board. Have students signal (by holding up the correct number of fingers) how many times as many parts the first picture has compared to the second picture. (a) 4, b) 2, c) 3, d) 4)

![Picture A](image1)

Now shade the same amount of each picture in a pair and have students signal how many times as many shaded parts there are in the first pictures. (a) 4, b) 2, c) 3, d) 4) If this is difficult because there are many parts, show students how to cover all but one part and count how many parts it has been divided into. For example, for a), cover either the left or right half of the circle with more parts and count how many parts they see.

![Picture B](image2)

Point out that because all original parts were divided into the same number of parts, the shaded parts were also divided into that number of parts.

**Exercises:** Write equivalent fractions from the picture on the board.

**Answers:** a) 4/8 = 1/2, b) 2/6 = 1/3, c) 3/6 = 1/2, d) 8/12 = 2/3

Point out how the numerators and denominators of both fractions are related by multiplication. For example:

\[
\frac{1}{2} \times 4 = \frac{4}{8}
\]

**Using a single picture to write two equivalent fractions.** Tell students that you can use the same picture to show two fractions. Draw on the board the picture in the margin. SAY: You can look at the big parts or the small parts. The big parts show the fraction two thirds because two of the three big parts are shaded. The small parts show the fraction eight twelfths because eight of the twelve small parts are shaded. The fractions are shown by the same picture, so they are equivalent. Write on the board:

\[
\frac{2}{3} = \frac{8}{12}
\]

**Exercises:** Write two equivalent fractions for the picture.

a) ![Picture C]

**Answers:** a) 1/3, 2/6; b) 1/3, 3/9; c) 3/5, 9/15
Bonus: Write as many equivalent fractions as you can for the picture without adding more lines.

a) \[
\begin{array}{c}
\hline
\hline
\hline
\hline
\end{array}
\]

b) \[
\begin{array}{c}
\hline
\hline
\hline
\hline
\end{array}
\]

Answers: a) 2/3, 4/6, 8/12; b) 1/2, 2/4, 3/6, 6/12

Using multiplication to write an equivalent fraction. Draw the first picture below:

\[
\frac{2}{3} = \frac{12}{18}
\] \[
\frac{2}{3} \times \frac{4}{4} = \frac{12}{18}
\]

ASK: How many parts do I have to break each piece into to get 12 parts altogether? (4) PROMPT: Three times what is 12? Again, you can focus on just one of the original parts to make it clearer. Divide the parts, then show this relationship as in the second picture above. ASK: How many parts are shaded now? (8) Fill in the numerator. SAY: That’s two groups of four that are shaded.

Exercises: Use multiplication to find the missing numerator.

a) \[
\frac{1}{5} = \frac{4}{20}
\]

b) \[
\frac{3}{4} = \frac{12}{16}
\]

c) \[
\frac{5}{9} = \frac{10}{18}
\]

d) \[
\frac{7}{10} = \frac{70}{100}
\]

Bonus: \[
\frac{15}{1000} = \frac{10}{10000}
\]

Answers: a) 4, b) 12, c) 10, d) 70, Bonus: 150

Skip counting to find lists of equivalent fractions. Write on the board:

\[
\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{3 \times 3}{5 \times 3} = \frac{3 \times 4}{5 \times 4} = \frac{3 \times 5}{5 \times 5}
\]

Remind students that they can skip count to multiply. So they can create equivalent fractions by skip counting. Write on the board:

\[
\frac{3}{5} = \quad = \quad = \quad = \quad
\]

Demonstrate skip counting by 3s to get the numerators, and then by 5s to get the denominators. Fill in the fractions. Point to each fraction in turn, and SAY: Two times as many parts, three times as many parts, four times as many parts, five times as many parts.

Exercise: Write four fractions equivalent to \[
\frac{4}{5}
\]

Bonus: Write more fractions equivalent to \[
\frac{4}{5}
\]

Sample answers: 8/10, 12/15, 16/20, 20/25, 24/30, 28/35
ACTIVITY (Optional)

Play Picking Pairs and then Memory using BLM Equivalent Fractions Memory. The third page uses more difficult multiplication; distribute it only to students who can quickly do the required multiplications (in the 6 to 11 times tables).

Picking Pairs. Students play in pairs or individually. Place cards face up in an array. Students take turns picking pairs of matching cards and placing them in a common discard pile. When there are no more pairs in the array, more cards are added to it. The goal is to place all the cards into the discard pile. If students have any non-matching cards left at the end, then some of their cards must have been matched incorrectly.

Memory. Place cards face down in an array. Students turn over two cards at a time looking for a match. If the cards match, students set them aside; otherwise, they turn them face down and continue playing. Students can play individually or co-operatively in pairs. In either case, the goal is to find all the matches. If playing with a partner, Player 1 leads by choosing and turning over a card, and Player 2 follows by choosing and turning over another card. Players switch roles after each turn.

Extensions

1. Use multiplication to find the missing denominator.
   
   a) \( \frac{3}{4} = \frac{15}{x} \)  
   b) \( \frac{1}{8} = \frac{7}{x} \)  
   c) \( \frac{5}{6} = \frac{30}{x} \)  
   d) \( \frac{4}{10} = \frac{40}{x} \)

   Bonus
   
   e) \( \frac{3}{1000} = \frac{3000}{x} \)  
   f) \( \frac{22}{100} = \frac{2200}{x} \)

   Answers: a) 20, b) 56, c) 36, d) 100, Bonus: e) 1 000 000, f) 10 000

2. Is there a fraction equivalent to \( \frac{3}{8} \) with an odd denominator? Explain.

   Answer: No. The denominator will always be a multiple of 8, so it will always be even.

3. Give each student three identical strips of paper. Ask students to fold one strip into halves, one into quarters, and one into eighths. Use the strips to find a fraction with a different denominator that is between ...

   a) \( \frac{3}{8} \) and \( \frac{5}{8} \)  
   b) \( \frac{1}{4} \) and \( \frac{2}{4} \)  
   c) \( \frac{5}{8} \) and \( \frac{7}{8} \)

   Bonus: Find four fractions equivalent to \( \frac{24}{192} \) with the following numerators: 2, 3, 4, 8. Hint: You can find equivalent fractions using division.

   Answers: a) 1/4 or 1/2; b) 3/8; c) 3/4; Bonus: 2/16, 3/24, 4/32, 8/64
Goals
Students will compare fractions with different numerators and denominators.

PRIOR KNOWLEDGE REQUIRED
Can create equivalent fractions
Can compare fractions using pictures
Can compare fractions having the same denominator
Can use the signs for greater than (>), and less than (<) correctly

Review comparing fractions using a chart. Draw on the board the chart in the margin. Have volunteers fill in the remaining boxes and solve the first three exercises below as a class.

Exercises: Name a fraction that is …

a) less than one third
b) greater than two thirds
c) between one half and two thirds
d) between three fifths and four fifths
e) between one quarter and one half
f) equivalent to one half
g) greater than three fifths
h) equivalent to one whole

Answers: a) 1/4 or 1/5; b) 3/4, 4/4, 4/5, or 5/5; c) 3/5; d) 2/3 or 3/4; e) 1/3 or 2/5; f) 2/4; g) 2/2, 2/3, 3/3, 3/4, 4/4, 4/5, or 5/5; h) 2/2, 3/3, 4/4, or 5/5

Review comparing fractions with the same denominator.
Exercises: Use the chart to decide which fraction is greater.

a) 2/4 or 3/4  b) 2/5 or 4/5  c) 1/3 or 2/3  d) 1/5 or 3/5

Point out that when the fractions have the same denominator, the one with the greater numerator is more. That’s because when the parts are the same size, more pieces are a greater fraction of the whole.

Exercises: Which fraction is greater?

a) 3/8 or 4/8  b) 8/9 or 7/9  c) 15/16 or 12/16  Bonus 38/10000 or 51/10000

Comparing fractions in which one denominator is divisible by the other.
Write on the board:

\[
\frac{2}{3} \quad \frac{5}{6}
\]
Point out that there are more pieces shaded in the 5/6 (5 instead of 2), but they are smaller pieces (sixths instead of thirds) so these fractions are hard to compare. Tell students that you would like to find a way to compare these fractions without drawing a picture. ASK: Is there a way to turn this problem into one we already know how to do? PROMPT: If both fractions had the same denominator, you would know what to do. Can we change one of the fractions to make it have the same denominator as the other fraction? Is there a fraction with denominator 6 that is equal to 2/3? Write on the board:

\[
\frac{2}{3} = \frac{4}{6}
\]

Have a volunteer write the missing numerator. (4) Review the multiplicative relationship between the numerators and denominators: multiplying both parts of a fraction by the same number gets an equivalent fraction. In this case, multiply both parts by 2:

\[
\frac{2 \times 2}{3 \times 2} = \frac{4}{6}
\]

Then write on the board:

\[
\frac{4}{6} < \frac{5}{6}
\]

ASK: Is 4/6 more or less than 5/6? (less) Is 2/3 more or less than 5/6? (less) How do you know? (2/3 = 4/6, which is less than 5/6) Point out that by changing 2/3 to 4/6, we turned the problem into one we already know how to do.

**Exercises:** Rewrite the first fraction so that it has the same denominator as the second fraction. Then compare the fractions.

a) \(\frac{1}{2}\) and \(\frac{4}{10}\)  

b) \(\frac{4}{5}\) and \(\frac{7}{10}\)  

c) \(\frac{1}{2}\) and \(\frac{5}{8}\)

**Answers:** a) 5/10 > 4/10, b) 8/10 > 7/10, c) 4/8 < 5/8

**Bonus:** Use the answers to parts a) and c). What is greater, \(\frac{4}{10}\) or \(\frac{5}{8}\) ?

**Comparing fractions with any denominator.** Write on the board the fractions 1/3 and 2/5. ASK: What makes comparing these fractions harder than the fractions we compared before? (there’s no number we can multiply 3 by to get 5, so we can’t make an equivalent fraction with denominator 5) Write on the board:

\[
\frac{1}{3} = \frac{5}{5}
\]

SAY: 5 is not a multiple of 3, so we’re stuck, aren’t we? What if we tried to change both denominators? We don’t have to make the denominator of 1/3 equal to 5; we just have to make it a multiple of 5—then we can change 2/5 too. ASK: What is a quick way to get a multiple of 3 that is also a multiple of 5?
PROMPT: What two numbers can you multiply? (multiply 3 \times 5) SAY: To change 1/3, multiply the top and bottom by 5; to change 2/5, multiply the top and bottom by 3. Show this on the board:

\[
\begin{align*}
\frac{1}{3} \times 5 & \quad \frac{2}{5} \times 3 \\
\frac{2}{5} & \quad \frac{6}{15}
\end{align*}
\]

SAY: Because 3 \times 5 = 5 \times 3, the denominators are now the same. Have volunteers show what number to multiply the numerators and denominators by to make the fractions have the same denominator.

a) \(\frac{1}{2} \times \frac{1}{5} = \frac{4}{10}\)  
b) \(\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}\)  
c) \(\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}\)  
d) \(\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}\)

Demonstrate solving the first exercise below, then have students do the remaining exercises individually. Students do not need to multiply by the lowest possible number to get a common denominator; instead, students should just be encouraged to multiply each numerator and denominator by the denominator of the other fraction.

**Exercises**

1. Write these fractions as fractions with the same denominator. Then write < or > as appropriate.
   a) \(\frac{1}{3} \times \frac{4}{5}\)  
   b) \(\frac{2}{3} \times \frac{1}{4}\)  
   c) \(\frac{1}{2} \times \frac{3}{4}\)  
   d) \(\frac{3}{4} \times \frac{5}{6}\)

   **Answers:** a) \(\frac{5}{15} < \frac{12}{15}\), so \(\frac{1}{3} < \frac{4}{5}\); b) \(\frac{4}{12} > \frac{3}{12}\), so \(\frac{1}{3} > \frac{1}{4}\); c) \(\frac{4}{8} < \frac{6}{8}\), so \(\frac{1}{2} < \frac{3}{4}\); d) \(\frac{18}{24} < \frac{20}{24}\), so \(\frac{3}{4} < \frac{5}{6}\)

2. Verify your answer to part d) in the previous exercise by drawing a picture.

**Extensions**

1. Write a number that fits. Justify your choice.

   \[\frac{3}{5} < \frac{20}{??}\]

   **Answer:** Any number greater than 12, because \(\frac{3}{5} = \frac{12}{20}\).

2. a) Teach students to compare fractions by finding a common numerator instead of a common denominator. For example, to compare 2/5 and 3/8, multiply 2 and 3:

   \[
   \frac{2 \times 3}{5 \times 3} > \frac{6}{15} > \frac{6}{16}, \text{ so } \frac{2}{5} > \frac{3}{8}
   \]

   b) Write a denominator that fits. Justify your choice.

   \[\frac{2}{3} > \frac{6}{??}\]

   **Answer:** Any number greater than 9, because \(\frac{2}{3} = \frac{6}{9}\).
Mixed Numbers and Improper Fractions (Introduction)

Goals
Students will be introduced to mixed numbers and improper fractions using pictures.

PRIOR KNOWLEDGE REQUIRED
Can name proper fractions
Can draw models to represent proper fractions

MATERIALS
Cuisenaire rods

Mental math minute—number talk. Present this problem: $15 \times 11$. (165)
The following strategies could arise:

$$(10 \times 11) + (5 \times 11) = 110 + 55$$
$$(15 \times 10) + (15 \times 1) = 150 + 15$$
$$(6 \times 11) + (9 \times 11) = 66 + 99 = 66 + 100 - 1$$

Introduce mixed numbers. Ask students if they have ever answered the question “How old are you?” with a fraction. Point out that many younger children say their age using a whole number and a fraction, for example: I am 7 1/2 years old. Draw on the board:

7
top number
1
bottom number
2

Tell students that this kind of number is called a mixed number because it is a mixture of a whole number and a fraction. ASK: Where else have you seen or heard mixed numbers before? (sample answers: in recipes, for example, 3 1/2 cups of flour, or when sharing food, for example, having 1 1/2 apples)

Naming mixed numbers. Draw on the board the picture in the margin. Tell students that some people ordered three pizzas, and the shaded part of the picture shows how much of the pizza they ate. ASK: How many whole pizzas did they eat? (2) What fraction of another pizza did they eat? (1/2) SAY: So they ate 2 1/2 pizzas. Write 2 1/2 on the board.

Exercises: Write a mixed number for the picture.

a)

b)


Answers: a) 1 1/4, b) 2 1/2, c) 3 3/8, Bonus: 1 5/16
Drawing pictures to represent mixed numbers. Draw the following picture on the board. **NOTE:** The last circle is divided into 4 because the denominator of the fraction means we are counting fourths.

![Picture of mixed number](image)

Tell students that you drew more circles than they need. Ask a volunteer to shade the correct number of wholes and the correct number of parts. (3 wholes and 1 part)

**Exercises:** Sketch the pies for the mixed number.

- a) $2\frac{1}{4}$
- b) $2\frac{1}{2}$
- c) $3\frac{3}{4}$
- d) $3\frac{1}{8}$
- e) $1\frac{3}{8}$

**Introduce improper fractions.** Draw on the board:

![Diagram of improper fractions](image)

SAY: I had a party and made five kinds of pizzas. There was one piece from each pizza left over. **ASK:** How much pizza was left over? **PROMPT:** How much is each piece? (one fourth of a pizza) How many fourths are left? Count the fourths as a class: one fourth, two fourths, ..., five fourths. Then have a volunteer write them on the board: $\frac{1}{4}$, $\frac{2}{4}$, ..., $\frac{5}{4}$. Have students extend the sequence to $\frac{11}{4}$ in their notebooks. Repeat for halves and thirds. Write on the board the fraction $\frac{5}{4}$. **ASK:** Can I fit them all on one plate? (no) **SAY:** I can only fit four fourths on a plate, so I need another plate. Show this on the board:

![Diagram of improper fraction](image)

**ASK:** Is $\frac{5}{4}$ greater than 1? (yes) **How can you tell if a fraction is greater than 1?** (if the numerator is greater than the denominator) Tell students that fractions that are greater than or equal to 1 are called *improper fractions*. SAY: When drawing a picture of an improper fraction, we usually fill up as many full circles as we can, like this (point to the picture of two circles above showing 1 full pizza and $\frac{1}{4}$ pizza) instead of like this (point to the picture of five circles showing $\frac{1}{4}$ each above).

Draw on the board:

![Diagram of improper fraction](image)

**ASK:** How many pieces are shaded? (9) **How many pieces are in one whole?** (4) Have a volunteer write the improper fraction that shows the amount shaded and then say the fraction aloud. ($\frac{9}{4}$, nine fourths)
Exercises: Write an improper fraction for the picture.

a)  

b)  

c)  

d)  

e)  

f)  

Answers: a) \(\frac{7}{4}\), b) \(\frac{9}{4}\), c) \(\frac{17}{5}\), d) \(\frac{10}{6}\), e) \(\frac{13}{8}\), f) \(\frac{14}{3}\)

Representing improper fractions with pictures. Write the fraction \(\frac{15}{4}\) on the board and draw a series of circles divided into four parts each (because 4 is the denominator of the fraction).

Tell students that you have drawn more circles than they need, so they have to know when to stop shading. Have a volunteer shade 15 pieces, starting from the left, and the class count out loud for each shaded portion until the class sees the correct number shaded; at this point, the class should tell the volunteer to stop. The volunteer should shade three circles and three parts of the fourth circle. Before doing the following exercises, write each improper fraction on the board and have the class say its name aloud. (three halves, nine halves, eleven fourths, nineteen eighths, ten fourths, twelve fifths, fifteen eighths, ten thirds)

Exercises: Sketch the pies for the fraction.

a) \(\frac{3}{2}\)  

b) \(\frac{9}{2}\)  

c) \(\frac{11}{4}\)  

d) \(\frac{19}{8}\)

e) \(\frac{10}{4}\)  

f) \(\frac{12}{5}\)  

g) \(\frac{15}{8}\)  

h) \(\frac{10}{3}\)

Finding the equivalent improper fraction for a mixed number. Draw on the board:

ASK: How do we represent this picture as an improper fraction? \(7/2\)  

SAY: We can also represent it as a mixed number by counting fully shaded circles rather than counting the number of halves that are shaded.

ASK: How do we represent this picture as a mixed number? \(3 1/2\)  

SAY: \(7/2\) and \(3 1/2\) are equivalent because they both represent the same shaded amount.
Exercises: Write a mixed number and an improper fraction for the picture.

a) 

b) 

c) 

Bonus 

Answers: a) 2 3/4, 11/4; b) 3 5/8, 29/8; c) 1 6/9, 15/9; Bonus: 2 8/15, 38/15

ACTIVITY (Optional)

Use Cuisenaire rods to represent improper fractions. Put the red rod on top of the yellow rod and ask students to name the fraction. (2/5) Flip the rods so that yellow is on top of red and tell students the new arrangement shows 5/2. Ask students to use two other colours to show 5/2. (sample answer: orange on top of purple) Ask students to show each of the following improper fractions using Cuisenaire rods: 3/2, 4/3, 5/1, 5/3, 6/5, 7/2, 7/3, 7/6, 8/5, 8/7, 9/4, 9/5, 9/6, 10/3, and 10/7.

Extensions

1. Ask students to order these mixed numbers from least to greatest: 3 1/5, 1 5/7, 7 1/11. (1 5/7, 3 1/5, 7 1/11) ASK: Did you need to look at the fractional parts at all or just the whole numbers? (just the whole numbers) Why? (a mixed number with a bigger whole part is bigger)

Bonus: Order the numbers from least to greatest:

\[3 \frac{1}{8}, 5 \frac{2}{7}, 2 \frac{1}{11}, 6 \frac{1}{5}, 8 \frac{5}{7}, 4 \frac{3}{10}.\]

Answers: 2 1/11, 3 1/8, 4 3/10, 5 2/7, 6 1/5, 8 5/7

2. If the triangle represents a whole, draw \[1 \frac{1}{2}.\]

3. If \[\frac{2}{3}\] of a structure looks like this, what could the whole look like?

4. If \[\frac{9}{10}\] of a structure looks like this, what could the whole look like?
Goals

Students will convert between improper fractions and mixed numbers using multiplication and division.

PRIOR KNOWLEDGE REQUIRED

Can compare and order proper fractions that have the same denominator.

Can draw pictures to show improper fractions and mixed numbers

Can name the improper fraction or mixed number shown by a picture

Finding how many halves are in a whole number. Draw on the board:

\[
\begin{align*}
1 &= \frac{2}{2} \\
3 &= \frac{6}{2}
\end{align*}
\]

Point out that 1 whole circle is equal to 2 half circles. ASK: How many halves are 3 wholes equal to? Guide students to find the answer by drawing on the board:

\[
\begin{align*}
3 \text{ wholes} \times 2 \text{ halves in each whole} &= 6 \text{ halves altogether}
\end{align*}
\]

Exercises: Find how many halves are in the given number of wholes. Write the whole number as an improper fraction with denominator 2. Draw a picture to justify your answer.

a) 4  b) 2  c) 7  d) 10

Answers: a) 8/2, b) 4/2, c) 14/2, d) 20/2

Finding how many fourths are in a whole number. Draw on the board:

ASK: How many fourths are in 1 pie? (4) How many fourths are in 2 pies? (8) How many are in 3 pies? PROMPT: What operation can we use to tell us the answer? (multiplication) Show this on the board:

\[
\begin{align*}
4 + 4 + 4 &= 3 \times 4 = 12
\end{align*}
\]
**Exercises:** Fill in the blank.

a) 2 pies = ____ thirds  
   *Answers:* a) 6

b) 3 pies = ____ fifths  
   *Answers:* b) 15

c) 5 pies = ____ fourths  
   *Answers:* c) 20

d) 2 pies = ____ eighths  
   *Answers:* d) 16

**Converting mixed numbers to improper fractions using multiplication and addition.** Draw on the board:

ASK: How many halves are in the 4 whole pies? (8) SAY: There is one half left over, so there are:

\[(4 \times 2) + 1\] halves

\[= 9\] halves

\[= \frac{9}{2}\]

**Exercises:** Write the mixed number as an improper fraction.

**Bonus**

a) \(\frac{5}{2}\)  
   *Answers:* a) \(\frac{11}{2}\)

b) \(\frac{2}{2}\)  
   *Answers:* b) \(\frac{5}{2}\)

c) \(\frac{10}{2}\)  
   *Answers:* c) \(\frac{21}{2}\)

d) \(\frac{13}{2}\)  
   *Answers:* d) \(\frac{27}{2}\)

e) \(\frac{432}{2}\)  
   *Answers:* e) \(\frac{865}{2}\)

f) \(\frac{1043}{2}\)  
   *Answers:* f) \(\frac{2087}{2}\)

Have students justify their answer to part a) with both a picture and an equation. Struggling students should be encouraged to draw pictures for parts a) to d) to help them consolidate the pattern.

Write on the board: \(3 \frac{2}{5}\). ASK: How many parts are in each whole? (5) How many fifths are in the 3 wholes? (15) Write on the board: \(3 \times 5 = 15\). ASK: How many fifths are in three and two fifths? (17) Draw a picture to show this, then write on the board:

\[
3 \frac{2}{5} = 3 \text{ pies} + \frac{2}{5} \text{ pies}  
= \text{____ fifths} + \text{____ fifths}  
= \text{____ fifths}
\]

Ask volunteers to fill in the blanks. Summarize what is done as follows: Multiply 3 and 5, then add 2.

\[
15 \text{ pieces} \quad \rightarrow \quad 3 \frac{2}{5} \quad \rightarrow \quad + \text{ 2 extra pieces}
\]

**Exercises:** Write the mixed number as an improper fraction.

**Bonus**

a) \(\frac{3}{5}\)  
   *Answers:* a) \(\frac{16}{5}\)

b) \(\frac{4}{3}\)  
   *Answers:* b) \(\frac{23}{5}\)

c) \(\frac{4}{3}\)  
   *Answers:* c) \(\frac{13}{3}\)

d) \(\frac{9}{8}\)  
   *Answers:* d) \(\frac{73}{8}\)

e) \(\frac{1}{23}\)  
   *Answers:* e) \(\frac{47}{23}\)

f) \(\frac{4}{32}\)  
   *Answers:* f) \(\frac{100}{32}\)
Writing improper fractions as mixed numbers. ASK: If I have the improper fraction 10/2, how can I find the number of whole pies it represents? Write on the board:

\[ \text{whole pies} = \frac{10}{2} \text{ pies} \]

Show students that they can simply divide 10 into groups of 2 to find the answer. (5 whole pies)

SAY: If I have the improper fraction 7/2, how can I know how many whole pies there are and how many pieces are left over? I want to divide 7 into groups of 2 to find out how many whole groups there are and if there are any extra pieces. ASK: What operation should I use? (division) What is the leftover part called? (the remainder) Write on the board:

\[ 7 \div 2 = 3 \text{ Remainder 1}, \text{ so } \frac{7}{2} = 3 \frac{1}{2}. \]

Repeat for 11/4. Draw the following picture on the board with three number statements.

\[ 7 \div 2 = 3 \text{ R 1} \]

\[ 11 \div 4 = 2 \text{ Remainder 3} = 2 \frac{3}{4} \]

Repeat for several examples: 7/3, 12/5, 13/6.

Exercises: Write the improper fraction as a mixed number.

a) \[ \frac{8}{5} \]

b) \[ \frac{10}{6} \]

c) \[ \frac{23}{4} \]

d) \[ \frac{30}{7} \]

Answers: a) 1 3/5, b) 1 4/6, c) 5 3/4, d) 4 2/7

Comparing and ordering mixed numbers. Remind students that any two fractions with the same denominator can be compared by comparing the numerators. Write on the board:

\[ \frac{2}{5} \quad \frac{4}{5} \]

ASK: Which fraction is greater? (4/5) Write on the board:

\[ 1 \frac{2}{5} \quad 1 \frac{4}{5} \]

SAY: The whole-number part of these two mixed numbers is the same, so we compare them by comparing the fractional part. ASK: Which mixed number has the greater fractional part? (1 4/5) So which mixed number is greater? (1 4/5) Write on the board:

\[ 1 \frac{2}{5} \quad 2 \frac{4}{5} \]
ASK: Are the whole-number parts of these mixed numbers the same? (no)
Which mixed number has the greater whole number? (2 4/5) So which mixed number is greater? (2 4/5) Did we even need to look at the fractional parts to determine which is greater? (no)

**Exercises:** Circle the greater mixed number.

<table>
<thead>
<tr>
<th>a)</th>
<th>11 4/17</th>
<th>b)</th>
<th>9 1/21</th>
<th>c)</th>
<th>6 20/21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonus</td>
<td>1034 316</td>
<td>350 354</td>
<td>1034 354</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 11 13/17, b) 9 1/21, Bonus: 1034 350/354

Write on the board:

1 8/9 2 1/9 1 4/9

ASK: Which is the greatest mixed number? (2 1/9) How do you know? (it has the largest whole number) Which is the smallest mixed number? (1 4/9) How do you know it’s smaller than 1 8/9? (they have the same whole number and 1 4/9 has the smaller fractional part) Have a volunteer write the mixed numbers in the boxes in order from greatest to least. (2 1/9 > 1 8/9 > 1 4/9)

**Exercises**

1. Order the mixed numbers from greatest to least using > signs.

2 7/11 8 9/11 2 9/11

**Answers:** 8 9/11 > 2 9/11 > 2 7/11

2. Order the mixed numbers from least to greatest using < signs.

14 1/16 14 2/16 13 15/16

**Answers:** 13 15/16 < 14 1/16 < 14 2/16

**Comparing and ordering improper fractions.** SAY: To compare and order improper fractions that have the same denominator we use the numerators to decide on the order. Write on the board:

9/5 6/5

ASK: Which fraction is less? (6/5) How do you know? (the denominators are the same and 6/5 has the smaller numerator)

**Exercises:** Circle the smaller improper fraction.

<table>
<thead>
<tr>
<th>a)</th>
<th>24/23</th>
<th>28/23</th>
<th>b)</th>
<th>16/14</th>
<th>15/14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonus</td>
<td>10672/423</td>
<td>12067/423</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 24/23, b) 15/14, Bonus: 10 672/423
Write on the board:

\[
\begin{array}{ccc}
10 & 14 & 8 \\
7 & 7 & 7
\end{array}
\]

ASK: Which is the greatest improper fraction? \(\frac{14}{7}\) How do you know? (they all have the same denominator and \(\frac{14}{7}\) has the greatest numerator)

Which is the smallest improper fraction? \(\frac{8}{7}\) Have a volunteer write the improper fractions in the boxes in order from greatest to least.

\(\frac{14}{7} > \frac{10}{7} > \frac{8}{7}\)

Exercise: Order the improper fractions from greatest to least using > signs.

\[
\begin{array}{ccc}
32 & 34 & 23 \\
19 & 19 & 19
\end{array}
\]

Bonus: Order the improper fractions from least to greatest using < signs.

\[
\begin{array}{ccc}
108 & 101 & 104 \\
94 & 94 & 94
\end{array}
\]

Answers: \(\frac{34}{19} > \frac{32}{19} > \frac{23}{19}\), Bonus: \(\frac{101}{94} < \frac{104}{94} < \frac{108}{94}\)

Extensions

1. Use long division to write the equivalent mixed number.

   a) \(\frac{741}{2}\)
   
   b) \(\frac{683}{3}\)
   
   c) \(\frac{1462}{4}\)

2. What picture better represents \(2 \frac{3}{4}\)? How do you know?

   A.  
   
   B.  

   Answer: B, because the wholes are all the same.

3. Which fractions show more than a whole? How do you know?

   \[
   \frac{5}{3} \quad \frac{2}{8} \quad \frac{4}{7}
   \]

   Answer: \(\frac{5}{3}\) because the numerator is greater than the denominator, and \(\frac{2}{3}/8\) because it is more than \(2\)

4. Find a fast way to put the numbers in order from least to greatest.

   \[
   \frac{3}{5} \quad \frac{1}{7} \quad \frac{7}{11}
   \]

   Answer: I just had to look at the whole-number part, so \(\frac{3}{5} < \frac{1}{7} < \frac{7}{11}\).
Goals
Students will find fractions of whole numbers when the answer is a whole number.

PRIOR KNOWLEDGE REQUIRED
Can find the fraction of an area or a set

MATERIALS
20 counters for each student
BLM Circle Fifths (p. L-64)

Mental math minute. Ask students to solve multiplication questions within the range of $6 \times 6$ to $10 \times 10$ and to solve the corresponding division questions. For each number, go through the questions in order, such as $6 \times 6$, $36 \div 6$, $7 \times 6$, $42 \div 6$, and so on, up to $10 \times 6$, $60 \div 6$. Then repeat with a different number. Next, try questions out of order, but keep the corresponding multiplication and division together.

Real-life fractions. Brainstorm the types of things students can find fractions of (examples: circles, squares, pies, pizzas, groups of people, angles, hours, minutes, years, lengths, areas, capacities, apples). Say each of the following sentences aloud and have students signal thumbs up (yes) or thumbs down (no) for whether or not each one makes sense:

a) 3 1/2 people went skiing. (no)
b) I ate 3 1/2 pancakes. (yes)
c) I folded the sheet of paper 3 1/4 times. (no)
d) Half of me was covered in paint. (yes)
e) I walked 2 1/3 kilometres. (yes)
f) I bought 2 1/3 marbles. (no)

Taking a fraction of a number. Tell students that you have 6 hats, and you will keep half for yourself and give half to a friend. Then draw dots for hats on the board:

\[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]

ASK: How many do I keep? (3) Circle the hats you keep and the hats you give away.

\[ \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \]

SAY: I kept these (pointing to the first group) and I gave these away (pointing to the second group). Now tell students that you have 6 apples and half of them are red. ASK: How many are red? (3) Point out that now
the same dots can mean apples instead of hats. ASK: If I have a pie cut into 6 pieces and half the pieces are eaten, how many are eaten? (3) Now what do the dots represent? (pieces of pie) SAY: No matter what you have 6 of, half is always 3. This means that the number 3 is half of the number 6. Point out that students can find half of a number by drawing that number of dots and making two equal groups.

**Exercises:** Draw dots to find half of the number.

a) 10  b) 4  c) 8  d) 16

**Bonus:** Draw tens blocks to find half of the number.

e) 40  f) 80  g) 60  h) 140

**Answers:** a) 5; b) 2; c) 4; d) 8, Bonus: e) 20, f) 40, g) 30, h) 70

**Using pictures to find unit fractions of a number.** Tell students that you have 6 apples and one third of them are red. Draw on the board 6 dots to represent the 6 apples. Remind students that one third of anything is one out of three equal parts and ask a volunteer to make three equal groups (see margin). ASK: How many apples are red? (2) SAY: So one third of 6 is 2. Write this as follows:

$$\frac{1}{3} \text{ of } 6 = 2$$

Draw on the board 12 dots in a row. Tell students you have 12 hats and one fourth of them are cowboy hats. Point out that you need one out of four equal groups. Ask a volunteer to make four equal groups. Then ASK: What is one fourth of 12? (3)

**Exercises:** Use the picture to find the fraction of the number.

a)  

\[ \frac{1}{5} \text{ of } 10 = \underline{\text{___}} \]

b)  

\[ \frac{1}{4} \text{ of } 12 = \underline{\text{___}} \]

c)  

\[ \underline{\text{___}} \text{ of } 8 = \underline{\text{___}} \]

**Bonus:** Draw dots and make equal groups to find \( \frac{1}{3} \) of 15.

**Answers:** a) 2; b) 3; c) \( \frac{1}{4} \) or 2; Bonus: 5
Dividing to find unit fractions of a number. Refer students to the picture showing 1/5 of 10. Point out that 10 objects are divided among 5 equal groups. ASK: How many are in each group? (2) Write on the board:

\[
\frac{10}{5} = 2
\]

ASK: What operation are we doing when we ask how many are in each group? (division) Have a volunteer write the correct sign in the equation. \((10 \div 5 = 2)\) Point out that since fractions are made from equal groups, we are really just dividing the whole amount, which is 10, by 5 to find one fifth. ASK: What do we divide by to find one fourth? (4) What do we divide by to find one third? (3)

Exercises: Divide to find the fraction of the number.

a) \(\frac{1}{2}\) of 14  b) \(\frac{1}{2}\) of 40  c) \(\frac{1}{3}\) of 24  d) \(\frac{1}{5}\) of 40  

Bonus

e) \(\frac{1}{2}\) of 846  f) \(\frac{1}{5}\) of 5000  g) \(\frac{1}{3}\) of 3300  h) \(\frac{1}{3}\) of 3096

Answers: a) 7, b) 20, c) 8, d) 8, Bonus: e) 423, f) 1000, g) 1100, h) 1032

Finding any fraction of a number. Remind students that, once you have divided a set of objects into equal parts, you can take any number of those parts. Draw on the board:

\[\text{SAY: There are 10 dots in five equal groups. ASK: What is one fifth of 10? (2) What is two fifths of 10? (4) (PROMPT: How many are in two groups?) ASK: What is three fifths of 10? (6) Four fifths of 10? (8) Summarize on the board:}
\]

\[
\frac{1}{5} \text{ of } 10 = 2 \quad \frac{2}{5} \text{ of } 10 = 4 \quad \frac{3}{5} \text{ of } 10 = 6
\]

\[
\frac{4}{5} \text{ of } 10 = 8 \quad \frac{5}{5} \text{ of } 10 = 10
\]

Leave the picture on the board.

Exercises: Use the picture to find the fraction of the number.

a) \(\frac{3}{4}\) of 8 = 

b) \(\frac{2}{3}\) of 6 = 

c) _____ of 9 = 

Number Sense 5-43  L-43
d) \[
\begin{array}{cccc}
\circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ \\
\end{array}
\]

\[ \frac{4}{5} \text{ of } 15 \]

**Bonus:** Draw dots and make equal groups to find \( \frac{3}{4} \) of 12.

**Answers:** a) 6; b) 4; c) 2/3, 6; d) 4/5, 12; Bonus: 9

**ACTIVITY (Optional)**

Give students 20 counters and a large circle divided into five equal parts (e.g., use BLM Circle Fifths). Ask students to use the circle and the counters to find 4/5 of 10, 3/5 of 20, and then 2/5 of 15.

Using division and multiplication to find a fraction of a whole number.

Refer students to the picture on the board showing 10 objects divided among 5 groups. Tell students that you want to find a way to find 3/5 of 10 without using the picture. **ASK:** How could you find 1/5 of 10 without using the picture? (10 \( \div \) 5 = 2) **SAY:** So each group has 2 dots. That means that 3/5 of 10 is 3 groups of 2 dots. How can we write 3 groups of 2 dots mathematically? (3 \( \times \) 2) Write on the board:

\[
\frac{1}{5} \text{ of } 10 = 10 \div 5 = 2
\]

So \( \frac{3}{5} \) of 10 = 3 \( \times \) 2 = 6

**ASK:** What is 1/4 of 12? (3) Have a volunteer write the division that shows this. (12 \( \div \) 4 = 3) **ASK:** What is 3/4 of 12? (9) Have a volunteer write the multiplication that shows this. (3 \( \times \) 3 = 9) Repeat for 1/3 of 12 (12 \( \div \) 3 = 4) and 2/3 of 12 (4 \( \times \) 2 = 8).

**Exercises**

1. Use division and then multiplication to find the fraction of the whole number.
   a) \( \frac{1}{3} \) of 21 and then \( \frac{2}{3} \) of 21  
   b) \( \frac{1}{5} \) of 20 and then \( \frac{4}{5} \) of 20  
   c) \( \frac{1}{7} \) of 35 and then \( \frac{3}{7} \) of 35  
   d) \( \frac{1}{6} \) of 48 and then \( \frac{5}{6} \) of 48

   **Answers:** a) 7, 14; b) 4, 16; c) 5, 15; d) 8, 40

2. Find the fraction of the whole number.
   a) \( \frac{5}{8} \) of 24  
   b) \( \frac{5}{6} \) of 30  
   c) \( \frac{3}{4} \) of 80  
   d) \( \frac{6}{8} \) of 40

   **Answers:** a) 15, b) 25, c) 60, d) 30
Word problems.

Exercises

1. Lewis has 10 oranges. He gives away \( \frac{4}{5} \) of the oranges.
   
   a) How many oranges did he give away?
   
   b) How many did he keep?

2. Nina has a collection of 24 shells. One third of the shells are scallop shells and one quarter of the shells are conch shells. The rest of the shells are cone shells. How many of Nina's shells are cone shells?

3. Neka has 20 marbles. Two fifths are blue and one quarter are yellow. The rest are green. How many are green?

4. Sandy put 6 of her 10 shells on a shelf. Explain how you know she put \( \frac{3}{5} \) of her shells on the shelf.

Bonus: Sun had 12 apples. She gave \( \frac{1}{4} \) of her apples to Braden, and she gave 2 apples to Ann. Sun says that she has half left. Is she correct?

Answers: 1. a) 8, b) 2; 2. 10; 3. 7; 4. One fifth of 10 is 2, so three fifths of 10 is 6; Bonus: No, she has 7 left, which is more than half.

Extensions

1. Find \( \frac{2}{3} \) of 12 and \( \frac{3}{4} \) of 12. Use your answers to determine which is greater, \( \frac{2}{3} \) or \( \frac{3}{4} \). Then find \( \frac{2}{3} \) of 15 and \( \frac{3}{5} \) of 15. Which is greater, \( \frac{2}{3} \) or \( \frac{3}{5} \)?

Answers: \( \frac{2}{3} \) of 12 is 8 and \( \frac{3}{4} \) of 12 is 9, so \( \frac{3}{4} \) is greater than \( \frac{2}{3} \). Also, \( \frac{2}{3} \) of 15 is 10 and \( \frac{3}{5} \) of 15 is 9, so \( \frac{2}{3} > \frac{3}{5} \).

2. Teach students another way to find fractions of a number: to find \( \frac{2}{3} \) of 12, draw 12 dots and colour 2 out of every 3 dots. SAY: The number of dots you coloured is \( \frac{2}{3} \) of 12.

3. Find the first three, then predict the fourth:

   a) \( \frac{2}{5} \) of 5   
   b) \( \frac{3}{4} \) of 4   
   c) \( \frac{5}{8} \) of 8   
   d) \( \frac{941}{8112} \) of 8112

Answers: a) 2, b) 3, c) 5, d) 941

4. Dory had 28 stickers. She kept \( \frac{1}{7} \) for herself and divided the rest evenly among 6 friends. How many stickers did each friend get?

Answer: 4
NS5-44 Fractions and Word Problems

Goals
Students will review naming area and set models, comparing fractions, and equivalent fractions.

PRIOR KNOWLEDGE REQUIRED
Can name fractions using area and set models
Can create sets of equivalent fractions
Can interpret models of fractions

Mental math minute—number talk. Present this problem: $220 \div 5$. (44)
The following strategies could arise:

$$(200 \div 5) + (20 \div 5)$$
$$(250 \div 5) - (30 \div 5)$$
doubling both numbers: $440 \div 10$

This lesson reviews concepts in fractions.

Exercises
1. Add lines to make the parts equal. What fraction is shaded?
   a)    b)
   c)    d)

2. Create a set of five shapes (circles and squares) such that:
   • $\frac{3}{5}$ are squares
   • $\frac{3}{5}$ are shaded
   • $\frac{2}{5}$ are big
   • $\frac{1}{3}$ of the squares are big
   • $\frac{2}{3}$ of the squares are shaded
   • No shaded shape is big

3. Shade the fraction strips to show that $\frac{3}{7}$ is greater than $\frac{2}{7}$.
4. Write as many equivalent fractions as you can using the picture.

5. Write a fraction equivalent to \( \frac{2}{3} \) that has an even denominator.

**NOTE:** Extension 1 is required in order to cover the British Columbia curriculum.

**Extensions**

1. Estimate approximately where the improper fraction goes by writing the letter above the number line.

   **a)**
   - A. \( \frac{11}{6} \)
   - B. \( \frac{2}{6} \)
   - C. \( \frac{8}{6} \)
   - D. \( \frac{5}{6} \)

   | 0 | closer to 0 | closer to 1 | closer to 1 | closer to 2 | 2 |
   |---|-------------|-------------|-------------|-------------|
   |   | less than 1 | greater than 1 |

   **b)**
   - A. \( \frac{5}{8} \)
   - B. \( \frac{17}{8} \)
   - C. \( \frac{3}{8} \)
   - D. \( \frac{13}{8} \)

   | 0 | closer to 0 | closer to 1 | closer to 1 | closer to 2 | 2 |
   |---|-------------|-------------|-------------|-------------|
   |   | less than 1 | greater than 1 |

   **Answers:** a) B D C A b) C A D B

2. Order the mixed numbers from greatest to least.

   \( \frac{8}{3} \)
   \( \frac{3}{10} \)
   \( \frac{3}{5} \)
   \( \frac{9}{20} \)
   \( \frac{3}{20} \)

   Answer: \( 9/3/20 > 8/3/5 > 8/3/10 > 8/3/20 \)

3. Order the improper fractions from least to greatest.

   \( \frac{25}{10} \)
   \( \frac{9}{7} \)
   \( \frac{25}{3} \)
   \( \frac{25}{7} \)

   Answer: \( 9/7 < 25/10 < 25/7 < 25/3 \)
Multiplicative Relationships and Times as Many

Goals
Students will demonstrate an understanding of simple multiplicative relationships involving whole-number rates.
Students will describe multiplicative relationships between quantities by using simple fractions.

PRIOR KNOWLEDGE REQUIRED
Recognizes units of currency (dollars and cents)
Recognizes units used to measure distance (km, m, and cm)
Recognizes units used to measure time (weeks, hours, minutes)

Mental math minute. Present this problem: How many nickels make 75¢?
(15) The following strategies could arise:

75 ÷ 5 = (50 ÷ 5) + (25 ÷ 5)
skip count by 5s
75 is 3 quarters, each quarter is 5 nickels, 3 × 5 = 15

Comparing quantities measured with different units. ASK: What is 1 m plus 2 cm? (102 cm or 1 m 2 cm) Why isn’t it 3? (you can’t add things when the units are different) SAY: We cannot add or subtract things that have different units, but sometimes we compare them. For example, we can’t add minutes and kilometres, but we might say that it takes 15 min to walk 1 km.
ASK: If I said that 3 lemons cost $2, I am comparing lemons and what? (dollars) The following exercises can be done aloud, as a class. Leave them on the board for the discussions that follow.

Exercises: What is being compared?

a) 5 pears cost $2.  b) $1 for 3 kiwis

b) 4 tickets cost $7.  d) 1 kiwi costs 35¢.

e) Sara is driving at 50 km/hour.

f) On a map, 1 cm represents 3 m in real life.
g) Jayden earns $6 an hour for babysitting.
h) The recipe calls for 1 cup of flour for every teaspoon of salt.

Answers: a) pears and dollars, b) dollars and kiwis, c) tickets and dollars, d) kiwis and cents, e) km and hours, f) cm on the map to m in real life, g) dollars and hours babysitting, h) cups of flour and teaspoons of salt

Understanding unit rate. Explain that sometimes one of the things being compared is equal to one. Ask students which examples in the previous exercise are like this. (b, d, e, f, g, h) For a few of the examples, ask students what there is one of. (b) dollars, d) kiwis, e) hours, f) cm, g) hours of babysitting, h) cups of flour or teaspoons of salt
Exercises: Which of the things being compared is there one of?

a) Add 8 cups of water per kilogram of rice.

b) 1 apple costs 25¢.

c) Rani makes $20 for 1 hour of work.

d) 1 can of juice costs 50¢.

e) The speed limit is 60 km per hour.

f) She walks 1 km in 15 minutes.

Answers: a) 1 kg of rice, b) 1 apple, c) 1 hour of work, d) 1 can of juice, e) 1 hour, f) 1 km

Point to part g) of the previous exercises that you left on the board. SAY: In part g), we read that Jayden earns $6 each hour he babysits. ASK: How much will he earn if he works for 2 hours? ($12) How did you work that out? (multiplied by 2) SAY: If I know how much money he makes for 1 hour, I can just multiply to find out how much he gets paid for 2 hours or 3 hours, or any number of hours. In part b), we saw that 3 kiwis cost $1. ASK: How much would 6 kiwis cost? ($2) What about 9 kiwis? ($3)

Exercises

a) 1 apple costs 30¢. How much do 3 apples cost?

b) Zack walks 1 km in 12 min. How long does it take him to walk 2 km?

c) 2 cans of juice cost $1. How much juice can I buy with $5?

d) Mary walks 4 km in 1 hour. How far can she walk in 5 hours?

e) To cook rice, you need 2 cups of water for each cup of rice. Amir is cooking 3 cups of rice. How much water does he need?

Bonus: There are 100 cm in 1 m. How many centimetres are in 4 m?

Answers: a) 90¢, b) 24 min, c) 10 cans, d) 20 km, e) 6 cups, Bonus: 400

Half as many. Draw on the board:

```
1
2
3
4
```

SAY: The picture shows there are four of something, for example, 4 plums. To find half of 4, I shade half of the circle. It should look like this:

```
1
2
3
4
```
SAY: Shading half of the circle was the same as shading two parts, so half as many as 4 is 2.

Draw on the board:

1 2 3 4
8 7 6

Ask a volunteer to shade half of 8. ASK: How many parts are shaded? (4) So how many is half as many as 8? (4)

**Exercises:** Shade half of the circle. How many parts did you shade?

a)  

b)  

c)  

**Answers:** a) 3, b) 1, c) 5

**One and a half times as many.** Draw on the board:

1 2 3 4

SAY: To find one and a half times as many as 2, I shade one whole circle and one half of another circle. Have a volunteer do the shading. It should look like this:

1 2 3 4

SAY: Shading one and a half times as many as 2 was the same as shading 3 parts, so one and a half times as many as 2 is 3.

Draw on the board:

1 2 3 4

5 6

8 7

Ask for a volunteer to shade one and a half times as many as 4. It should look like this:

1 2 3 4

5 6

8 7
SAY: Shading one and a half times as many as 4 is the same as shading 6, so one and a half times as many as 4 is 6.

**Exercises:** Find $1 \frac{1}{2}$ times as many by shading a whole circle and half of the next circle.

![Shading circles](image)

**Answers:** a) 12, b) 9

**Bonus**

a) Find $2 \frac{1}{2}$ times as many as 2.

![Shading circles](image)

b) Find $3 \frac{1}{2}$ times as many as 2.

![Shading circles](image)

**Answers:** a) 5, b) 7

**Extensions**

   a) If 4 books cost $24, how much do 3 books cost?
   b) If 7 books cost $35, how much do 5 books cost?
   c) If 11 L of soy milk cost $33, how much does 5 L cost?

   **Answers:** a) 1 book costs $6, 3 books cost $18; b) 1 book costs $5, 5 books cost $25; c) 1 L costs $3, 5 L cost $15.

2. For every rate, there can be 2 different unit rates. For example, I can tell you how many kilometres I can walk in 1 hour or how many hours it takes to walk 1 km. The rate that is most useful depends on the question being asked. For example, assume I am buying cans of juice. If I know that I need to buy 3 cans of juice, then it is most helpful to know how much money it costs for 1 can of juice. If I have $5 to spend entirely on juice, then I want to know how many cans of juice I can buy for $1. Say which rate you need to answer the question.
a) How long does it take to bike 10 km? Use kilometres and minutes.

b) How far can I bike in 3 hours? Use kilometres and hours.

c) I have only 3 eggs but lots and lots of flour. What unit rate do I need to make pancakes?

**Answers:** a) minutes per kilometre, b) kilometres per hour, c) amount of flour per egg

3. A pancake recipe calls for:
   - 300 g flour
   - 10 g baking powder
   - 2 eggs
   - 250 mL milk
   - 10 mL vanilla

   a) What is the unit rate of flour to eggs?

   b) Find three other unit rates from the recipe.

   c) You have 3 eggs and would like to use them all to make pancakes. How much will you need of the other ingredients if you keep all the unit rates in the recipe the same?

   **Answers:** a) 150 g to 1 egg; b) answers will vary; c) 450 g of flour, 15 g baking powder, 375 mL of milk, 15 mL vanilla
Fraction Strips

\[
\begin{array}{c}
\frac{2}{3} \\
\frac{4}{6} \\
\frac{1}{2} \\
\frac{3}{6} \\
\frac{3}{4} \\
\frac{6}{8} \\
\frac{3}{5} \\
\frac{6}{10}
\end{array}
\]
<table>
<thead>
<tr>
<th>Fraction Cards (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{15} )</td>
</tr>
<tr>
<td>( \frac{2}{18} )</td>
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<tr>
<td>( \frac{2}{24} )</td>
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<td>( \frac{2}{25} )</td>
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<tr>
<td>Fraction Cards (2)</td>
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<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>9/15</td>
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<td>9/18</td>
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<td>9/24</td>
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<tr>
<td>9/25</td>
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</tbody>
</table>
# Fraction Cards (3)

<p>| | | |</p>
<table>
<thead>
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</thead>
<tbody>
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<td>3</td>
<td>5</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>31</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>31</td>
<td>31</td>
<td>31</td>
</tr>
</tbody>
</table>
Ordering with Fraction Strips

1. Shade the strips to show the fractions. Order the fractions from least to greatest.
   a) \(\frac{2}{5}, \frac{2}{9}, \frac{2}{3}, \frac{2}{6}\)
   b) \(\frac{3}{12}, \frac{3}{16}, \frac{3}{7}, \frac{3}{10}\)

2. Shade the strips to show the fractions. Order the fractions from greatest to least.
   a) \(\frac{1}{8}, \frac{1}{3}, \frac{1}{10}, \frac{1}{1}\)
   b) \(\frac{4}{15}, \frac{4}{6}, \frac{4}{18}, \frac{4}{8}\)
Ordering Fractions (1)

1. a) Count how many fourths are shaded.
   
   ![Fraction Diagrams]

   b) Write the fractions in order from least to greatest.
   
   < < <

2. a) Count how many thirds are shaded.
   
   ![Fraction Diagrams]

   b) Write the fractions in order from least to greatest.
   
   < < <
Ordering Fractions (2)

3. a) Count how many halves are shaded.

   ![Fraction Diagram]

   b) Write the fractions in order from least to greatest.

   \[
   \frac{1}{2} < \frac{1}{2} < \frac{1}{2} \]

4. a) Count how many tenths are shaded. Write the fractions in order from greatest to least.

   ![Fraction Diagram]

   b) Write the fractions in order from least to greatest.

   \[
   \frac{1}{10} < \frac{1}{10} < \frac{1}{10} \]

5. Write < or >.

   If \( \frac{8}{8} = 1 \) then \( \frac{9}{8} \) \( \text{\underline{>}} \) 1.

   If \( \frac{5}{5} = 1 \) then \( \frac{4}{5} \) \( \text{\underline{<}} \) 1.

   Since \( \frac{9}{8} > 1 \) and \( \frac{4}{5} < 1 \), then \( \frac{9}{8} \) \( \text{\underline{>}} \) \( \frac{4}{5} \).
## Equivalent Fractions Memory (1)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>3/5</td>
<td>1/3</td>
</tr>
<tr>
<td>2/4</td>
<td>6/10</td>
<td>3/9</td>
</tr>
<tr>
<td>1/4</td>
<td>3/7</td>
<td>2/5</td>
</tr>
<tr>
<td>3/12</td>
<td>6/14</td>
<td>6/15</td>
</tr>
</tbody>
</table>
Equivalent Fractions Memory (2)

\[
\frac{2}{3} \quad \frac{3}{4} \quad \frac{3}{8} \\
\frac{8}{12} \quad \frac{9}{12} \quad \frac{6}{16} \\
\frac{5}{8} \quad \frac{7}{8} \quad \frac{4}{5} \\
\frac{10}{16} \quad \frac{21}{24} \quad \frac{16}{20}
\]
### Equivalent Fractions Memory (3)

<table>
<thead>
<tr>
<th>7/9</th>
<th>5/6</th>
<th>6/7</th>
</tr>
</thead>
<tbody>
<tr>
<td>56/72</td>
<td>45/54</td>
<td>36/42</td>
</tr>
<tr>
<td>4/9</td>
<td>5/11</td>
<td>7/12</td>
</tr>
<tr>
<td>28/63</td>
<td>55/121</td>
<td>63/108</td>
</tr>
</tbody>
</table>
Circle Fifths
PS5-7 Using Structure to Add Sequences

Teach this lesson after:
Unit 9

VOCABULARY
denominator
equivalent fractions
exponents
guess-check-revise
numerator
powers of 10
sequence
term

Goals
Students will use structure to understand the relationship between patterns.

PRIOR KNOWLEDGE REQUIRED
Can multiply a multi-digit number by a one-digit number using the standard algorithm
Can add and subtract decimal tenths and hundredths
(for Problem Banks 4, 5)
Can evaluate a fraction of a whole number (for Problem Bank 6)
Can divide whole numbers by 10 and get decimal tenths
(for Problem Bank 8)

MATERIALS
overhead projector
transparency of grid paper or BLM 1 cm Grid Paper (p. L-72)
BLM 1 cm Grid Paper (p. L-72)

Using area models to discover patterns. Write on the board:

\[
\begin{align*}
1 &= \quad \\
1 + 2 &= \quad \\
1 + 2 + 3 &= \quad \\
1 + 2 + 3 + 4 &= \quad \\
1 + 2 + 3 + 4 + 5 &= \\
\end{align*}
\]

Fill in the blanks as volunteers tell you the sums. (1, 3, 6, 10, 15) SAY: The gaps increase because that’s how we made the sequence, but I want to know if there is a way to get an expression that will help me find any term. One way to think of the sums is as an area. Project a transparency of grid paper or BLM 1 cm Grid Paper onto the board and draw the following shape:

SAY: Let’s count the squares inside the shape to find the area. ASK: How many squares are in the first row? (1) In the second row? (2) Third row? (3) Fourth row? (4) Fifth row? (5) SAY: So, we can add all these together to find the total. Write on the board:

\[
\text{Area} = 1 + 2 + 3 + 4 + 5 = 15 \text{ square units}
\]
Exercises

1. Write the area as an addition by adding the number of squares in each row.

   a)
   b)
   c)

   **Answers:** a) $3 + 4 + 5 = 12$ square units, b) $1 + 2 + 3 + 4 = 10$ square units, c) $3 + 5 + 7 = 15$ square units

   **NOTE:** Provide students with grid paper or BLM 1 cm Grid Paper for the following exercises.

2. Draw an area model for the expression.

   a) $2 + 3 + 4$  
   b) $4 + 5 + 6 + 7$  
   c) $2 + 5 + 8$

   **Answers**

   a)
   b)
   c)

   On grid paper, have students draw two identical shapes like the one on the board for $1 + 2 + 3 + 4 + 5$ and then cut them out. Challenge students to arrange the shapes to make a rectangle, then ask them to find the area of each shape. When students are finished, draw on the board:

   $$(1 + 2 + 3 + 4 + 5) \times 2 = 5 \times 6 = 30$$

   So $1 + 2 + 3 + 4 + 5 = 15$

   **Exercises:** Use two copies of a shape to make a rectangle. Then write the multiplication.

   a) $(1 + 2 + 3) \times 2 = \_\_ \times \_\_\_$  
   b) $(3 + 5 + 7) \times 2 = \_\_ \times \_\_\_$  
   c) $(1 + 2 + 3 + 4) \times 2 = \_\_ \times \_\_\_$  
   d) $(2 + 5 + 8) \times 2 = \_\_ \times \_\_\_$

   **Bonus:** Which two questions have the same answer? Why does that make sense?

   **Answers:** a) $3 \times 4$; b) $3 \times 10$; c) $4 \times 5$; d) $3 \times 10$; Bonus: parts b) and d), $3 + 5 + 7 = 2 + 5 + 8$, so multiplying either by 2 gets the same answer

   SAY: Once you know the area of the rectangle, you can divide by 2 to find the area of the original shape.
Draw on the board:

<table>
<thead>
<tr>
<th>Sum</th>
<th>Equivalent Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + 2 + 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 + 2 + 3 + 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 + 2 + 3 + 4 + 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: From the exercises you just did, \((1 + 2 + 3) \times 2 = 3 \times 4\), so that means you can divide \(3 \times 4\) by 2 to get an expression that equals \(1 + 2 + 3\). Write that in the table, and have volunteers write the equivalent expression for the next two rows. Then have volunteers write the value of each expression in the third column.

<table>
<thead>
<tr>
<th>Sum</th>
<th>Equivalent Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + 2 + 3</td>
<td>((3 \times 4) ÷ 2)</td>
<td>6</td>
</tr>
<tr>
<td>1 + 2 + 3 + 4</td>
<td>((4 \times 5) ÷ 2)</td>
<td>10</td>
</tr>
<tr>
<td>1 + 2 + 3 + 4 + 5</td>
<td>((5 \times 6) ÷ 2)</td>
<td>15</td>
</tr>
</tbody>
</table>

SAY: There looks like a pattern here in the first two columns. Have volunteers complete the next two rows.

<table>
<thead>
<tr>
<th>Sum</th>
<th>Equivalent Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 + 2 + 3 + 4 + 5 + 6</td>
<td>((6 \times 7) ÷ 2)</td>
<td>21</td>
</tr>
<tr>
<td>1 + 2 + 3 + 4 + 5 + 6 + 7</td>
<td>((7 \times 8) ÷ 2)</td>
<td>28</td>
</tr>
</tbody>
</table>

ASK: How can you get the equivalent expression from the sum? (start with the last number being added, multiply it by the next number, and then divide by 2) SAY: You can do that for larger numbers too. Write on the board:

\[1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9\]

ASK: What would be the equivalent expression? \(\left((9 \times 10) ÷ 2\right)\) SAY: If you want to add all nine numbers, you can use either expression because they will both get you the same answer. Have students do it both ways, then ASK: Which way was faster? (answers may vary) SAY: When there are a lot of numbers to add, using the multiplication method will be faster.

**Exercises:** Evaluate.

a) \(1 + 2 + 3 + 4 + \ldots + 20\) \hspace{1cm} b) \(1 + 2 + 3 + 4 + \ldots + 30\)

**Bonus:** \(1 + 2 + 3 + 4 + \ldots + 100\)

**Answers:**

a) \(20 \times 21 = 210\), b) \(30 \times 31 ÷ 2 = 465\),

Bonus: \((100 \times 101) ÷ 2 = 5050\)
When students have completed the exercises, point out how much time they saved by not having to do all that addition. SAY: Even with a calculator, adding 30 numbers takes a long time.

Write on the board:

\[
21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30
\]

SAY: I can add the numbers 1 to 20 and I can add the numbers 1 to 30. ASK: How can I add the numbers 21 to 30? \(465 - 210 = 255\)

Exercises:

Evaluate the sum.

a) \(31 + 32 + 33 + 34 + \ldots + 40\)

b) \(41 + 42 + 43 + 44 + \ldots + 50\)

c) \(51 + 52 + 53 + 54 + \ldots + 60\)

Answers

a) \((40 \times 41 \div 2) - (30 \times 31 \div 2) = 820 - 465 = 355\)

b) \((50 \times 51 \div 2) - (40 \times 41 \div 2) = 1275 - 820 = 455\)

c) \((60 \times 61 \div 2) - (50 \times 51 \div 2) = 1830 - 1275 = 555\)

When students finish, challenge them to look for a pattern in their answers. (each answer is 100 more than the previous one) ASK: Why does this happen? (each term is 10 more than in the previous sum, so you are adding ten 10s) Write on the board:

\[
21 + 22 + 23 + \ldots + 30 + 10 + 10 + 10 + \ldots + 10 + 31 + 32 + 33 + \ldots + 40
\]

SAY: You are adding 10 to each term and there are ten 10s being added, so you are adding 100 in total. That's why the answer is always 100 more.

Exercises: Add.

a) \(1 + 2 + 3 + 4 + 5 + \ldots + 50 = \quad \)

b) \(1 + 2 + 3 + 4 + 5 + \ldots + 50 = \quad 2 + 4 + 6 + 8 + 10 + \ldots + 100 = \quad \)

SAY: You are adding 10 to each term and there are ten 10s being added, so you are adding 100 in total. That's why the answer is always 100 more.
Answers: a) 1275, b) 1275 + 1275 = 2550, c) 2550 − 50 = 2500, d) 1275 + 1225 = 2500, e) 1275 + 1275 + 1275 = 3825

When students finish, ASK: In which two parts did you add the same number in different ways? (parts c) and d) Did you get the same answer both times? (yes) SAY: Look at parts b) and e). Instead of repeatedly adding the same sum, you can multiply instead. Write on the board:

\[ 2 + 4 + 6 + 8 + 10 + \ldots + 100 = 2 \times (1 + 2 + 3 + 4 + 5 + \ldots + 50) \]
\[ 3 + 6 + 9 + 12 + 15 + \ldots + 150 = 3 \times (1 + 2 + 3 + 4 + 5 + \ldots + 50) \]

SAY: Multiplying each term by the same number gets the same answer as multiplying the whole sum by that number.

Exercises: Fill in the blanks.

a) \[ 3 + 6 + 9 + 12 + 15 = \_ \times (1 + 2 + 3 + 4 + 5) \]
b) \[ 31 + 62 + 93 = \_ \times (1 + 2 + 3) \]
c) \[ 9 + 12 + 15 = 3 \times (\_ + \_ + \_) \]

Answers: a) 3; b) 31; c) 3, 4, 5

SAY: You can use this strategy to evaluate sums that look hard to do. Write on the board:

\[ 11 + 22 + 33 + 44 + 55 + 66 + 77 + 88 + 99 = \_ \times (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \]

Have a volunteer fill in the blank. (11) ASK: If you know that the sum of the first nine numbers is 45, what is the sum on the left? (11 \times 45) How could you calculate 11 \times 45 mentally? (use 10 \times 45 = 450 and then add 45 to get 495)

Exercises

a) Using \[ 3 \times (12924 + 208) = 39396 \], determine what 12924 + 208 is without adding.
b) Using \[ 3 \times (394 + 367 + 442) = 3609 \], determine what 394 + 367 + 442 is without adding.
c) Using \[ 2 + 5 + 8 = 15 \], what is \[ 6 + 15 + 24 \]?
d) Using \[ 1 + 4 + 7 + 10 = 22 \], what is \[ 4 + 16 + 28 + 40 \]?
e) Using \[ 5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 = 180 \], what is \[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \]?

**Bonus:** If \[ 7 + 6 - 1 = 12 \], what is \[ 21 + 18 - 3 \]?

Answers: a) 13 132, b) 1203, c) 45, d) 88, e) 36, Bonus: 36
SAY: Sometimes, you just need to subtract some terms. Write on the board:

\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28 \]
\[ -1 + 2 + 3 = 6 \]
So  \[ 4 + 5 + 6 + 7 = 22 \]

**Exercises:** Use \[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55 \] to evaluate the addition.

a) \[ 4 + 5 + 6 + 7 + 8 + 9 + 10 \]

b) \[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 \]

c) \[ 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 \]

d) \[ 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 \]

**Bonus:** Evaluate \[ 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 \] using the following:

\[ 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 \]
\[ -2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 \]
\[ 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 \]

**Answers:** a) 49, b) 45, c) 165, d) 65, Bonus: 100

**Problem Bank**

1. Use the first sum to evaluate the second sum. Explain your strategy.
   a) Using \[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36, \]
      what is \[ 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11? \]
   b) Using \[ 2 + 3 + 4 + 5 + 6 = 20, \]
      what is \[ 34 + 51 + 68 + 85 + 102? \]
   c) Using \[ 1 + 2 + 3 + 4 + 5 + ... + 15 = 120, \]
      what is \[ 1 + 3 + 5 + 7 + 9 + ... + 29? \]

**Sample answers**
   a) \[ 36 + 8 \times 3 = 36 + 24 = 60, \]
      because I added 3 to each of the eight terms.
   b) \[ 20 \times 17 = 340, \]
      because I multiplied each term by 17.
   c) \[ 120 + (120 - 15) = 125, \]
      because I added term by term the sums \[ (1 + 2 + 3 + ... + 15) \]
      and \[ (0 + 1 + 2 + ... + 14) \]
      and the second sum is 15 less than the first sum.

2. Evaluate \[ 46 + 47 + 48 + 49 + ... + 60 \] in two ways. Make sure you get the same answer both ways.
   a) \[ 1 + 2 + 3 + 4 + ... + 45 + 46 + 47 + ... + 60 = \]
      \[ - (1 + 2 + 3 + 4 + ... + 45) = \]
      \[ 46 + 47 + ... + 60 = \]
   b) \[ 1 + 2 + 3 + ... + 15 = \]
      \[ 45 + 45 + 45 + ... + 45 = \]
      \[ 46 + 47 + 48 + ... + 60 = \]

**Answers:** a) 1830 − 1035 = 795, b) 120 + 675 = 795
3. Evaluate \(24 + 26 + 28 + 30 + 32 + 34\) in two ways. Make sure you get the same answer both ways.

   a) Use \(12 + 13 + 14 + 15 + 16 + 17\).

   b) Use \(1 + 3 + 5 + 7 + 9 + 11\).

   **Answers:** a) \(24 + 26 + 28 + 30 + 32 + 34 = 2 \times (12 + 13 + 14 + 15 + 16 + 17) = 2 \times 87 = 174\), b) \(24 + 26 + 28 + 30 + 32 + 34 = (1 + 3 + 5 + 7 + 9 + 11) + (23 + 23 + 23 + 23 + 23) = 36 + 6 \times 23 = 36 + 138 = 174\).

4. Using \(1 \frac{1}{2} + 1 \frac{1}{3} + 1 \frac{1}{4} + 1 \frac{1}{5} + 1 \frac{1}{6} = 2.45\), what is \(1 \frac{1}{2} + 1 \frac{1}{3} + 1 \frac{1}{4} + 1 \frac{1}{5} + 1 \frac{1}{6}\) ? Explain.

   **Answer:** \(2.45 - 1 = 1.45\)

5. a) Using \(1 \frac{1}{2} + 1 \frac{1}{3} + 1 \frac{1}{4} + 1 \frac{1}{5} + 1 \frac{1}{6} = 1.45\), what is \(1 \frac{1}{3} + 1 \frac{1}{4} + 1 \frac{1}{5} + 1 \frac{1}{6}\) ?

   b) If 2.83 is a good estimate for \(1 \frac{1}{2} + 1 \frac{1}{3} + \ldots + 1 \frac{1}{9}\), what is a good estimate for \(1 \frac{1}{2} + 1 \frac{1}{3} + \ldots + 1 \frac{1}{10}\)?

   c) Using \(1 \frac{1}{2} + 1 \frac{1}{3} + 1 \frac{1}{4} + 1 \frac{1}{5} + 1 \frac{1}{6} = 2.45\), what is \(2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6}\) ?

   **Answers:** a) 0.95, b) 2.93, c) 8.45

6. Using \(\frac{1}{3}\) of 24 is 8, what is \(\frac{1}{2}\) of \(\frac{1}{3}\) of 24?

   **Answer:** 4

7. If \(4 \times (A + 5) = 32\), what is \(A + 5\)? What is \(A\)?

   **Answer:** \(A + 5 = 8\), \(A = 3\)

8. a) Calculate.

   \[
   \begin{align*}
   1 + 2 + 3 + 4 + 5 \\
   + 10 + 9 + 8 + 7 + 6
   \end{align*}
   \]

   b) Use your answer from part a) to calculate.

   \[
   8 + 16 + 24 + 32 + 40 + 48 + 56 + 64 + 72 + 80
   \]

   c) Use your answer from part b) to calculate.

   \[
   0.8 + 1.6 + 2.4 + 3.2 + 4.0 + 4.8 + 5.6 + 6.4 + 7.2 + 8.0
   \]

   **Answers:** a) 55, b) 440, c) 44
1 cm Grid Paper

...
PS5-8 Using Tape Diagrams

**Goals**
Students will use tape diagrams to solve multistep word problems involving all four operations and fractions.

**Prior Knowledge Required**
- Can interpret statements involving “times as many”
- Can represent fractions using fraction bars
- Can divide to find a fraction (with numerator 1) of a whole number amount
- Can determine the volume of a right rectangular prism given its dimensions (for Extended Problem)
- Knows that 1 litre converts to 1000 millilitres (for Extended Problem)
- Can recognize “for each” as representing a multiplicative relationship (for Extended Problem)

**Materials**
BLM Swimming Pool (pp. 84–85, see Extended Problem)

**Drawing a diagram for a “times as many” situation.** Tell students that Kim and Rob have some stickers. Write on the board: “Kim has three times as many stickers as Rob.” Say: I want to draw a diagram to represent this situation. Ask: Who has more stickers, Kim or Rob? (Kim) Draw a small rectangle on the board and explain that this rectangle represents Rob’s stickers. Label the bar as shown below:

<table>
<thead>
<tr>
<th>Rob’s stickers</th>
</tr>
</thead>
</table>

Ask: How can we show that Kim has three times as many stickers as Rob? Accept all reasonable answers. Then explain that you are going to use a specific way to draw a diagram. You will make a bar that contains three of Rob’s bar of stickers. Finish the picture on the board as shown below and keep it for future reference:

<table>
<thead>
<tr>
<th>Rob’s stickers</th>
<th>Kim’s stickers</th>
</tr>
</thead>
</table>

Explain that this type of diagram is called a tape diagram.

Say: Tristan has four times as many nickels as dimes. Draw on the board:

<table>
<thead>
<tr>
<th>A. dimes</th>
<th>nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. dimes</td>
<td>nickels</td>
</tr>
<tr>
<td>C. nickels</td>
<td>dimes</td>
</tr>
<tr>
<td>D. nickels</td>
<td>dimes</td>
</tr>
</tbody>
</table>
ASK: Which of these tape diagrams fit Tristan’s situation? (A and C both work, but B and D do not) Have volunteers explain why B and D don’t work. (B shows more dimes than nickels; D shows five times as many nickels as dimes) ASK: How do you know that the short bar should be the number of dimes? (there are more nickels than dimes) If you want to show that David is twice as old as Karen, whose age would be the shorter bar? (Karen’s) Why? (David is older, so his bar will be longer)

**Exercises:** Draw a tape diagram for the situation.

a) Rani is three times as tall as her baby brother.
b) Mary’s full name (including her last name) is four times as long as Josh’s full name.
c) There are eight times as many students in the school as in our class.
d) The library is three times as far from my home as the school is.
e) A book is twice as thick as a notebook.

**Answers**
a) Rani’s height | brother’s height  
b) Mary’s name | Josh’s name  
c) students in class | students in school  
d) distance to library | distance to school  
e) thickness of book | thickness of notebook  

**Finding the length of the bars when the smaller part is given.** Write on the board:

Ivan has five times as many crayons as pencils. Ivan has four pencils.

- crayons
- pencils

ASK: What information do we have that is not shown on the tape diagram? (Ivan has 4 pencils)

SAY: Let’s show that on the tape diagram. Write “4” in Ivan’s “pencils” bar, as shown below:

- crayons
- pencils

SAY: Each of the crayon blocks is also four, so let’s write that in the diagram. The final picture should look like this:

- crayons
- pencils
ASK: How many crayons does Ivan have? (20) SAY: Ivan has five blocks of four crayons. Write on the board:

\[ 5 \times 4 = 20 \]

**Exercises:** Draw a tape diagram and find the length of the bars.

a) There are six apples on the table. There are twice as many bananas as apples.

b) A car holds five people. A van holds three times as many people.

c) Don’s apartment building is three storeys high. Ella’s building is five times as high as Don’s.

d) Rob is five years old. Sally is four times as old as Rob.

**Bonus:** A sparrow has four eggs in its nest. A duck has three times as many eggs in its nest as the sparrow. An ostrich has five times as many eggs in its nest as the sparrow.

**Answers**

| a) apples | 6 | 6 |
| b) car | 5 | 5 |
| bananas | 6 | 6 | 12 | van | 5 | 5 | 5 | 15 |
| c) Don’s building | 3 | 3 |
| Ella’s building | 3 | 3 | 3 | 3 | 15 |
| d) Rob | 5 | 5 |

**Bonus**

| sparrow’s nest | 4 | 4 |
| duck’s nest | 4 | 4 | 4 | 12 |
| ostrich’s nest | 4 | 4 | 4 | 4 | 4 | 20 |

**Using tape diagrams to solve problems when the larger part is given.**

Write on the board:

Marla has four times as many stickers as Amir.

Have a volunteer draw the bars for the situation, as shown below:

| Amir |
| Marla |

ASK: How many blocks are in Marla’s bar? (4) Write “Marla has 20 stickers.” on the board. SAY: Marla’s bar has four blocks and together the blocks represent 20 stickers. Show this on the picture:

Marla has four times as many stickers as Amir. Marla has 20 stickers.

| Amir |
| Marla |

20
ASK: How many stickers does each bar represent? (5) SAY: Four bars represent 20 stickers, so one bar represents five stickers. ASK: How many stickers does Amir have? (5) How do you know? (Amir has one bar) Write “5” in each bar in the diagram to emphasize this.

**Exercises:** Draw bars and find the length of each block for the situation.

a) There are six apples on the table. There are twice as many apples as pears. How many pears are there?

b) A mini-bus holds 16 people. The mini-bus holds twice as many people as a van. How many people can the van hold?

c) Jun’s apartment building is 30 storeys high. Jun’s building is five times as high as Cathy’s building. How tall is Cathy’s building?

d) Edmond is 14 years old. Edmond is seven times as old as Nina. How old is Nina?

**Bonus:** A sugar pine cone is 45 cm long. It is three times as long as an eastern white pine cone. The sugar pine cone is nine times as long as a jack pine cone. How long is the eastern white pine cone? How long is the jack pine cone?

**Answers:** a) 3 pears; b) 8 people; c) 6 storeys; d) 2 years old; Bonus: eastern white pine cone = 15 cm long, jack pine cone = 5 cm long

**Finding the size of a single block when the difference is given.** Write on the board:

Rick has four times as many white shirts as black shirts.

Have a volunteer show the situation using a tape diagram, then continue writing on the board:

He has 18 more white shirts than black shirts.

ASK: What does this mean on the tape diagram? (the longer bar represents 18 more than the shorter bar) Point to the extra part and SAY: This extra part here represents 18. Show this on the tape diagram:

| white shirts |  |  |  |
| black shirts |  |  | 18 |

ASK: How many extra blocks are there in the longer bar? (3) How many shirts do those three bars represent altogether? (18) So how many shirts does each block represent? (6) What division sentence can you write to show that? (18 ÷ 3 = 6) Write “6” in each block. SAY: All the blocks represent six shirts. ASK: How many black shirts does Rick have? (6) How many white shirts does he have? (24) How did you get that? (4 × 6 = 24)
Exercises: What is the size of each block?

a)  

b)  

c)  

d)  

Answers: a) 5, b) 8, c) 7, d) 11

Finding the size of the block when the total is given. SAY: Sometimes you are given the total instead. For example, Lewis might have 20 shirts altogether. Draw on the board:

white shirts  
black shirts  

ASK: How many blocks represent 20 shirts? (5) So how many shirts does one block represent if five blocks represent 20 shirts? (4) Write “4” in each block. SAY: When each block represents four shirts, five blocks represent 20 shirts.

Exercises

1. What is the size of one block?

a)  

b)  

c)  

d)  

Answers: a) 3, b) 4, c) 4, d) 12

2. Either the total is given or the difference is given. What is the size of one block?

a)  

b)  

c)  

d)  

Answers: a) 11, b) 14, c) 12, d) 10
Solving problems with the difference, the total, or one part given.

SAY: Sometimes you are given the difference, or the total, or one part.

Write on the board:

Raj has five times as many T-shirts as sweaters.

A. Raj has 60 T-shirts.
B. Raj has 60 T-shirts and sweaters altogether.
C. Raj has 60 more T-shirts than sweaters.

\[
\begin{array}{|c|c|c|}
\hline
\text{T-shirts} & \text{A.} & \text{B.} & \text{C.} \\
\hline
\text{sweaters} & & & \\
\hline
\end{array}
\]

Point out that each diagram shows five times as many T-shirts as sweaters. Have volunteers show what the 60 is representing in each situation, as shown below:

\[
\begin{array}{|c|c|c|}
\hline
\text{T-shirts} & \text{A.} & \text{B.} & \text{C.} \\
\hline
\text{sweaters} & & & 60 \\
\hline
\end{array}
\]

ASK: Which picture shows being given one part? (A) Which picture shows being given the difference? (C) Which picture shows being given the total? (B) SAY: Now that you know what 60 represents, you can decide what each bar represents. Pointing to each situation (A, B, and C) in turn, ASK: What does each bar represent? (12 in A, 10 in B, and 15 in C) Work through the first two exercises below as a class, then have students work individually.

Exercises: Solve.

a) Mary saved three times as much money as Sun. Sun saved $18 less than Mary. How much money did they save together?

b) Cam and Amy put all their money together to buy a gift for their grandmother. They have $60 together. Cam has twice as much money as Amy has. How much money does each of them have?

c) Abella is three times as tall as her baby brother Jax. She is 80 cm taller than Jax. How tall is Jax? How tall is Abella?

d) Zara’s full name is four times as long as Ronin’s. Ronin’s full name is 54 letters shorter than Zara’s. How long is each full name? Hint: Take the full name into account; Ronin’s name is not five letters long.

e) The number of students in the school who are not in Grade 5 is eight times as large as the number of students in Grade 5. There are 248 students in the school who are not in Grade 5. How many students altogether are in the school?

Answers: a) $36; b) Amy had $20, Cam had $40; c) Jax is 40 cm tall, Abella is 120 cm tall; d) Ronin’s full name is 18 letters long, Zara’s full name is 72 letters long; e) 279 students in the school in total.
Problem Bank

1. The library is four times as close to Emma’s home as the school. Emma walks from school to her home, then goes to the library. This makes a walk of 15 blocks. How far from Emma’s home are the school and the library?

   **Answers:** The library is 3 blocks away from Emma’s home, and the school is 12 blocks away.

2. A number is five times as large as a smaller number. If you add the two numbers together, you get 54. What are the numbers?

   **Answers:** 9 and 45

3. Armand reads the same number of pages every school day and twice as many pages every weekend day. He finished a book of 108 pages in a week. How many pages does he read on Monday? How many pages does he read on Sunday?

   **Answers:** 12 pages on Monday, 24 pages on Sunday

4. Kim has \(\frac{3}{5}\) as many blue shirts as black shirts. Which diagram shows the situation correctly? What is wrong with the other two?

   **Answers:** C shows the situation correctly. A shows 3/4 as many blue shirts as black shirts, and B shows 3/5 as many black shirts as blue shirts.

5. Draw a diagram to solve the problem.

   a) Jane had $36. She spent \(\frac{3}{4}\) of her money on a pair of shoes. How much money does she have left?

   b) Kyle spent \(\frac{2}{5}\) of his money on a toy. He has $15 left. How much did the toy cost?

   c) Alice spent \(\frac{2}{5}\) of her money on a poster that cost $8. How much money did she have before she bought the poster?

   d) Kate spent \(\frac{2}{5}\) of her money on a shirt and a hat. The shirt cost $18, and the hat cost $6. How much money did Kate have at first?
Solutions
a) $36

\[
\begin{align*}
\text{shoes leftover} &= \$9 \\
\text{toy} &= \$10 \\
\text{poster} &= \$4
\end{align*}
\]

b) $15

\[
\begin{align*}
\text{toy} &= \$5
\end{align*}
\]

c) total before = $20

\[
\begin{align*}
\text{poster} &= \$4
\end{align*}
\]

d) The shirt and hat together cost $24, so each block is $24 \div 2 = $12. Five blocks together is $5 \times $12 = $60.

\[
\begin{align*}
\text{shirt + hat} &= \$24
\end{align*}
\]

6. Eric has some eggs. He uses \( \frac{3}{7} \) of them to make pancakes and \( \frac{1}{2} \) of the remainder to make sandwiches. Now Eric has six eggs left. How many eggs did he use to make pancakes? How many eggs did he have at first?

Answers: Eric used 9 eggs to make pancakes, and he had 21 eggs at first.

7. Ray had 30 stickers. He gave \( \frac{2}{5} \) of his stickers to his brother and \( \frac{1}{2} \) of the rest to his friend. How many stickers did Ray’s brother get? How many stickers are left?

Answers: Ray’s brother got 12 stickers. At the end, there are 9 stickers left.

8. The next time Ray has stickers, he decides to give \( \frac{2}{5} \) of his 30 stickers to his brother and \( \frac{5}{6} \) of the remainder to his friend.

a) How many stickers did Ray’s brother get?

b) How many stickers did Ray’s friend get?

c) How many stickers are left?

Solution

\[
\begin{align*}
\text{total stickers} &= 30 \\
\text{brother} &= 12 \\
\text{remainder} &= 18 \\
\text{friend} &= 15 \\
\text{left} &= 3
\end{align*}
\]
9. Nora has some stickers. She colours \( \frac{1}{4} \) of them red and \( \frac{2}{5} \) of the remainder green. If Nora doesn’t colour 9 stickers, how many stickers does she have in total?

Answer: 20

10. Evan spent \( \frac{3}{5} \) of his money on a book and \( \frac{3}{4} \) of the remainder on some music. Evan has $4 after he paid for the book and the music. How much money did Evan have initially?

Solution

\[
\begin{align*}
\text{initial money} &= \$40 \\
\text{left after book} &= \$16 \\
\text{book} &= \$24 \\
\text{left after book} &= \$16 \\
\text{music} &= \$12 \\
\text{left after music} &= \$4
\end{align*}
\]

11. Shelly received some money for her birthday. She donated \( \frac{1}{5} \) to charity, and she saved \( \frac{2}{3} \) of the remainder in her savings. Of what was left, \( \frac{1}{4} \) was used to buy a gift card for an ice cream store. She used the rest of the money to buy 3 books for $7 each, a T-shirt for $10, and a basketball for $8. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each other part (donation, savings, gift card)?

Answers: the books are $7 each, the T-shirt is $10, the basketball is $8, so she spent $39 for purchases, plus $13 for the ice cream store gift card; $104 went to savings; $39 was given to charity; total birthday gift was $195

12. Randi reads 10 pages of a book on Saturday and she reads \( \frac{3}{4} \) of the rest of the book on Sunday. She still has 17 pages to read. How many pages are in the book?

Answer: 78 pages

13. A convenience store has some ice cream treats. It sells \( \frac{2}{5} \) of them on Friday, \( \frac{1}{4} \) of the remainder of Saturday, and \( \frac{2}{3} \) of the rest on Sunday. The store has 30 ice cream treats left by the end of the day on Sunday.

a) How many ice cream treats did the store have initially?

b) On which day did the store sell the most ice cream treats?

Answers: a) 200; b) Friday, 80
Extended Problem: Swimming Pool

MATERIALS

BLM Swimming Pool (pp. 84–85)

Extended Problem: Swimming Pool. Give students BLM Swimming Pool. Tell students that they will be doing an extended problem involving a swimming pool and volume.

Answers: 1. 96 m³; 2. 480 mL; 3. about $4, because 480 mL is about half of 1 L; 4. a) $12, b) $40, c) $16, d) 2 boxes, e) 4 times
Swimming Pool (1)

Jay’s pool is in the shape of a rectangular prism. It is 8 m long, 6 m wide, and 2 m deep.

1. Find the capacity of the pool in cubic metres.

2. Jay disinfects the water in the pool every day by adding 5 mL of chlorine for each cubic metre of water in the pool. How many millilitres of chlorine does Jay need to put in the pool when the pool is full?

3. Each 1 L bottle of chlorine costs $8.00. About how much does it cost each time Jay adds chlorine to a full pool? Explain your estimate.
Swimming Pool (2)

4. Jay spent \(\frac{2}{5}\) of his money at a pool store to buy some chlorine and half of the remainder on a pool toy. Jay has $12.00 after he paid for the chlorine and the pool toy.
   a) How much did Jay pay for the pool toy?

   b) How much money did Jay have when he went to the store?

   c) How much did Jay pay for the chlorine?


   e) How many times can Jay disinfect the water in the pool with the amount of chlorine he bought?
# Unit 10  Number Sense: Decimals

## Introduction

This unit extends existing concepts in decimals and the relationship between decimals and fractions to explore:

- using, comparing, and ordering decimal tenths and hundredths and their equivalent fractions;
- understanding decimals as an extension of the place value system;
- adding decimal tenths and hundredths; and
- subtracting decimal tenths and hundredths.

## Meeting Your Curriculum

### ALBERTA

<table>
<thead>
<tr>
<th>Required</th>
<th>NS5-46 to 48, 50 to 55</th>
<th>including Extension 3 in NS5-51, Extension 5 in NS5-52, Extension 6 in NS5-53, Extension 2 in NS5-54, and Extension 2 in NS5-55</th>
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### BRITISH COLUMBIA

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<th>including Extension 5 in NS5-52, Extensions 1 and 6 in NS5-53, Extension 2 in NS5-54, and Extension 2 in NS5-55</th>
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### MANITOBA

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### ONTARIO

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<tr>
<th>Required</th>
<th>NS5-46 to 50, 52 to 55</th>
<th>including Extensions 1 to 3 in NS5-52, Extensions 2 to 5 in NS5-53, Extension 1 in NS5-54, and Extension 1 in NS5-55</th>
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<tbody>
<tr>
<td>Optional</td>
<td>NS5-51</td>
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</tbody>
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## Mental Math Minutes

The mental math minutes in this unit:

- review strategies for making equivalent fractions
- practise adding and subtracting decimals to hundredths

## Generic BLMs

The Generic BLM used in this unit is:

**BLM 1 cm Grid Paper** (p. S-1)

This BLM can be found in Section S.
Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tbody>
<tr>
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NS5-46  Decimal Tenths and Hundredths

Pages 45–46

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
decimal
decimal hundredth
decimal notation
decimal point
decimal tenth
fraction
hundredth
tenth

Goals
Students will use decimal notation for fractions with denominators 10 and 100, place decimal hundredths on number lines, and order decimal hundredths using a number line.

PRIOR KNOWLEDGE REQUIRED
Knows that, on number lines, greater whole numbers appear to the right of lesser whole numbers
Can name fractions from area models and number lines

MATERIALS
BLM Representing Hundredths with Pictures (p. M-43)
transparency of BLM Number Lines Divided into Tenths and Hundredths (p. M-44)
overhead projector
base ten blocks (optional, see Extension 3)

Introduce decimal tenths. Tell students that the fraction 1/10 can be represented in various ways. Show three ways on the board:

\[
\frac{1}{10} \quad \frac{1}{10} \quad \frac{1}{10}
\]

Point out that each way means 1 part out of 10 equal parts. Tell students that mathematicians have invented a simpler way to write one tenth, called decimal notation. Show this on the board:

\[
\frac{1}{10} = 0.1
\]

SAY: The dot is called a decimal point. People write the number this way because it takes up less space on the page and is easier to write. Ask volunteers to show how they would write 2 tenths (0.2), 3 tenths (0.3), and other numbers up to 9 tenths (0.9) as a decimal. SAY: The 0 before the decimal point tells you that the number is less than 1.

Representing decimal tenths on a number line. Draw a number line from 0 to 1 and ask students to place various decimal tenths on the number line (e.g., 0.8, 0.5, 0.2, 0.7).

Exercises: Write the decimal for each marked point.

\[
\begin{array}{c}
\ast \ast \ast \ast \ast \\
0 \quad 1
\end{array}
\]

Answers: 0.3, 0.4, 0.9
Representing decimal tenths using pictures. Draw various shapes on the board, such as circles, squares, or rectangles, and have volunteers represent various numbers given in decimal notation.

a) 0.2  b) 0.3  c) 0.5  d) 0.6

Writing decimal notation for pictures. Now have students do the reverse.

Exercises: Write the decimal for the picture.

a)  

b)  

c)  

d)  

Answers: a) 0.8, b) 0.7, c) 0.5, d) 0.8

Introduce decimal hundredths. Tell students that the fraction 1/100 can also be represented in various ways. Show four ways on the board:

\[
\frac{1}{100} \quad \text{one hundredth} \quad 0.01
\]

Point out how one hundredth is written differently from one tenth—there are two digits after the decimal point instead of only one. Ask a volunteer to show how she would write two hundredths as a decimal (0.02), then read it as “zero point zero two.” ASK: How would you write three hundredths as a decimal? (0.03)

Exercises: Write the fraction as a decimal.

a) \(\frac{9}{100}\)  b) \(\frac{4}{100}\)  c) \(\frac{8}{100}\)  d) \(\frac{7}{100}\)  e) \(\frac{5}{100}\)

Answers: a) 0.09, b) 0.04, c) 0.08, d) 0.07, e) 0.05

Writing two-digit hundredths. Write on the board:

\[
\frac{83}{100} = 0.83 \quad \frac{49}{100} = 0.49 \quad \frac{60}{100} = 0.60
\]

SAY: To write decimal hundredths, you have to use the first two places after the decimal point. If there are more than 9 hundredths, you can write the number of hundredths right after the decimal point. Ask volunteers to show how to write various hundredths (28 hundredths, 4 hundredths, 70 hundredths) as decimals (0.28, 0.04, 0.70). Remind volunteers to put in any missing zeros.
Exercises: Complete BLM Representing Hundredths with Pictures.

Answers: a) 0.18, b) 0.30, c) 0.06, d) 0.09, e) 0.74, f) 0.70

Showing decimal hundredths on a number line. Project BLM Number Lines Divided into Tenths and Hundredths on the board. Point out how the number line is counting in tens of hundredths. Point to each appropriate mark in turn and read the marks: 10 hundredths, 20 hundredths, 30 hundredths, and so on to 90 hundredths. SAY: One hundred hundredths is 1 whole. Ask a volunteer to show where 47 hundredths would be. Demonstrate on the number line how to check the volunteer’s answer by counting by tens to 40 and then by ones to 47. Write on the board:

A. 0.34  B. 0.87  C. 0.06  D. 0.50

Have volunteers read out loud the number of hundredths (34, 87, 6, 50), then show the points on the number line.

Exercises: Write the decimals that are marked.

Use BLM Number Lines Divided into Tenths and Hundredths to display these exercises.

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1

Answers: 0.08, 0.30, 0.60, 0.75

Comparing hundredths on number lines. Remind students that when two numbers are placed on a number line, the number to the right is always greater. Demonstrate this with whole numbers. Draw a number line from 0 to 10 on the board and SAY: 6 is to the right of 5 because 6 is greater than 5. Write on the board:

0.60 0.75

ASK: Which number is greater? (0.75) SAY: 75 hundredths is more than 60 hundredths because 75 of anything is more than 60 of the same thing. ASK: Which symbol goes between the numbers, < or >? Remind students that the bigger (wider) side points toward the bigger (greater) number. Have a volunteer write the correct symbol (<).

Exercises: Write the marked decimals in order from least to greatest.

Use BLM Number Lines Divided into Tenths and Hundredths to display these exercises.

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1

Answers: 0.03 < 0.34 < 0.50 < 0.81

Connect decimal notation to dollar notation. ASK: Where have you seen this kind of notation before, with a dot between numbers? (dollar notation) If some students answer less precisely by saying “money notation,” write 34¢ on the board and point out that there is no dot between numbers. Point out that the dot in dollar notation makes sense because each cent is one hundredth, or 0.01, of a dollar, so 34 cents is 34 hundredths of a dollar, or 0.34 of a dollar.
Practising word problems.

Exercises

a) Amir lives 0.85 km from the library and 0.67 km from school. Does he live closer to the library or to school?

b) Kate jogged 0.96 km on Monday and 0.48 km on Tuesday. On which day did she jog farther?

c) Nora lives 0.76 km from school and Jake lives 0.83 km from school. Who lives closer to school?

Answers: a) school, b) Monday, c) Nora

Extensions

1. Write the fraction as hundredths, then as a decimal.

   a) \(\frac{3}{20}\)  
   b) \(\frac{7}{50}\)  
   c) \(\frac{12}{50}\)  
   d) \(\frac{8}{25}\)  
   e) \(\frac{9}{20}\) 

   Answers: a) \(\frac{15}{100} = 0.15\), b) \(\frac{14}{100} = 0.14\), c) \(\frac{24}{100} = 0.24\),
   d) \(\frac{32}{100} = 0.32\), e) \(\frac{45}{100} = 0.45\)

2. Have students look for decimals (less than 1) in the media and write the decimals as fractions.

3. Is 8 a factor of 1672? Use one or more of the following tools to check: base ten blocks, paper and pencil, or mental math.

   Sample solutions
   • I used paper and pencil. \(1672 \div 8 = 209\ R\ 0\), so 8 is a factor of 1672.
   • I used mental math. I did \(1600 \div 8 = 200\) and \(72 \div 8 = 9\), so \(1672 \div 8 = 209\); so 8 is a factor.
   • I used base ten blocks. I made 8 rows, and there were 2 hundreds blocks and 9 ones blocks in each row. There was nothing left over, so 8 is a factor.
Comparing and Ordering Decimal Tenths and Hundredths

Goals
Students will compare and order decimal tenths and hundredths.

Prior Knowledge Required
- Can write tenths and hundredths as decimals
- Can use number lines to order whole numbers
- Can name fractions from area models and number lines

Materials
- Transparency of grid paper, BLM 1 cm Grid Paper (p. S-1) or BLM Squares Divided into Hundredths (p. M-45)
- Overhead projector
- BLM Squares Divided into Hundredths (p. M-45)
- Transparency of BLM Number Lines Divided into Tenths and Hundredths (p. M-44)
- BLM Number Lines Divided into Tenths and Hundredths (p. M-44)

Introduce equivalent tenths and hundredths as fractions and decimals.

Draw on the board:

\[
\frac{3}{10} = \frac{30}{100}
\]

ASK: How many tenths are shaded? (3) Fill in the first numerator. SAY: Each column is one tenth, and three of them are shaded. ASK: How many hundredths are shaded? (30) PROMPT: How many hundredths are in each column? (10) So there are 10, 20, 30 hundredths shaded. Fill in the second numerator. ASK: How would you write 3 tenths as a decimal? (0.3) How would you write 30 hundredths as a decimal? (0.30) Write on the board:

\[
0.3 = 0.30
\]

SAY: These are equivalent decimals because they represent the same amount.

Exercises: Write two equivalent fractions and two equivalent decimals for the amount shaded.

a)

\[
\frac{5}{10} = \frac{50}{100} = 0.5 = 0.50
\]

b)

\[
\frac{2}{10} = \frac{20}{100} = 0.2 = 0.20
\]

c)

\[
\frac{7}{10} = \frac{70}{100} = 0.7 = 0.70
\]

d)

\[
\frac{4}{10} = \frac{40}{100} = 0.4 = 0.40
\]

Answers: a) 5/10 = 50/100 = 0.5 = 0.50, b) 2/10 = 20/100 = 0.2 = 0.20, c) 7/10 = 70/100 = 0.7 = 0.70, d) 4/10 = 40/100 = 0.4 = 0.40
**Reading decimals.** Although it is correct to read 0.7 as “zero point seven,” it is not correct to read 0.70 as “zero point seventy.” Each digit after the decimal point should be read separately, so 0.70 becomes “zero point seven zero.” Always be sure to correct students who read 0.70 as “zero point seventy.” Students who are allowed to do so are more likely to incorrectly believe that 0.70 is greater than 0.8 since 70 > 8. Ask volunteers to use this way to read various decimals: 0.9, 0.09, 0.90, 0.13, 0.31, 0.03, 0.00003, 0.0000700.

**Comparing decimals using hundreds squares.** Write on the board:

0.8   0.12

Have volunteers read the numbers aloud. Then project grid paper, BLM 1 cm Grid Paper, or BLM Squares Divided into Hundredths) and draw two blank hundreds squares on the board. Ask volunteers to shade each decimal above. ASK: Which decimal is larger? (0.8) PROMPT: Which square has more shaded?

Provide students with BLM Squares Divided into Hundredths.

**Exercises:** Shade and label the decimals. Then compare them.

a) 0.4, 0.30   b) 0.08, 0.7   c) 0.36, 0.4

**Answers:** a) 0.4 > 0.30, b) 0.08 < 0.7, c) 0.36 < 0.4

**Ordering decimals using number lines.** Write on the board:

0.60   0.08   0.34

Ask volunteers to read the numbers as decimals (zero point six zero, zero point zero eight, zero point three four), then as hundredths (60 hundredths, 8 hundredths, 34 hundredths). Point out how reading the numbers as hundredths makes it easy to place the numbers on a number line divided into hundredths since they can just count the hundredths. Project BLM Number Lines Divided into Tenths and Hundredths and demonstrate counting out 60 hundredths by tens. Then have volunteers show where they would place 8 hundredths and 34 hundredths. ASK: How does the number line show you which number is greater? (the greater number is on the right)

Ask a volunteer to write the decimals in order from least to greatest. Point out that the order makes sense because 8 of anything is less than 34 of the same thing, so 8 hundredths is less than 34 hundredths.

Provide students with BLM Number Lines Divided into Tenths and Hundredths.

**Exercises:** Mark the decimals on one number line, then write the decimals in order from least to greatest.

a) 0.18, 0.65, 0.39   b) 0.84, 0.08, 0.40

**Answers:** a) 0.18 < 0.39 < 0.65, b) 0.08 < 0.40 < 0.84
Writing decimals as fractions. Remind students that when written as a decimal, fractions with denominator 10 have one digit after the decimal point and fractions with denominator 100 have two digits after the decimal point. For example, 3/10 is 0.3, but 3/100 is 0.03.

Exercises: Write the fraction that the decimal represents.

a) 0.4  
b) 0.04  
c) 0.24  
d) 0.5  
e) 0.87

Answers: a) 4/10, b) 4/100, c) 24/100, d) 5/10, e) 87/100

Exercises: Write the letters for the incorrect equations. Then write the correct fraction to go with the decimal.

A. \(0.03 = \frac{3}{10}\)  
B. \(0.4 = \frac{4}{10}\)  
C. \(0.05 = \frac{5}{100}\)

D. \(0.6 = \frac{6}{100}\)  
E. \(0.24 = \frac{24}{100}\)  
F. \(0.81 = \frac{81}{10}\)

Answers: A, D, F; The correct fractions for the decimals are 3/100 (A), 6/10 (D), and 81/100 (F)

Comparing tenths and hundredths by writing both as hundredths.
Write “0.4” and “0.38” on the board. SAY: It is hard to compare 4 tenths to 38 hundredths because tenths are bigger than hundredths. ASK: How many hundredths is 4 tenths equal to? (40) Is it easier to compare 40 hundredths to 38 hundredths? (yes) Why? (because now we are comparing hundredths to hundredths, and 40 of them is more than 38 of them) Show this on the board:

\[0.4 > 0.38\] because \(0.40 > 0.38\)

Exercises: Write both decimals as hundredths, then compare them.

a) 0.3, 0.25  
b) 0.05, 0.4  
c) 0.76, 0.6

Answers: a) 0.30 > 0.25, b) 0.05 < 0.40, c) 0.76 > 0.60

Word problems practice.

Exercises

a) John ran 0.3 km on Monday and 0.24 km on Tuesday. On which day did he run farther?

b) In Tina’s collection, 0.34 of her stamps are US stamps and 0.2 of her stamps are Canadian stamps. Does she have more US or Canadian stamps?

c) The school is 0.5 km from the library and 0.48 km from the skating rink. Which is closer to the school, the library or the skating rink?

Answers: a) Monday, b) US, c) the skating rink
Extensions

1. Predict how to write \( \frac{1}{1000} \), \( \frac{1}{10,000} \), and \( \frac{1}{1,000,000,000} \) as a decimal.

   **Answers:** 0.001, 0.0001, 0.000000001

2. Make up a word problem that requires comparing 0.4 to 0.26. Ask a partner to solve your problem.

3. Represent \( \frac{6}{10} \) in as many ways as you can.

   **Sample answers:** 0.6, \( \frac{3}{5} \), \( \frac{60}{100} \), 0.60, six tenths, three fifths, sixty hundredths. Students may also write “zero point six” and “zero point six zero.” These answers are correct as ways of orally representing these numbers but should not be encouraged as a way of writing the numbers. Students can also draw pictures that show the amount shaded.

4. Explain how you know that \( 0.7 = 0.70 \).

   **Answer:** \( 0.7 = \frac{7}{10} = \frac{70}{100} = 0.70; \) 7/10 is equivalent to 70/100; therefore, 0.7 is equivalent to 0.70.

5. Find a fast way to answer the question below.

   Orange juice comes in cases of 24 cans. The school went through 15 cases in December and 18 cases in January. How many more cans of juice did the school go through in January than in December?

   **Solution:** The school went through 3 more cases in January compared to December (because 18 is 3 more than 15), and each case has 24 cans, so I did \( 24 \times 3 = 72 \). The school went through 72 more cans of juice in January than in December.
**Goals**

Students will express hundredths in terms of mixed units (tenths and hundredths).

**PRIOR KNOWLEDGE REQUIRED**

- Can write equivalent tenths and hundredths as fractions
- Can write equivalent tenths and hundredths as decimals
- Can name fractions from area models and number lines

**MATERIALS**

- transparency of BLM Number Lines Divided into Tenths and Hundredths (p. M-44)
- overhead projector
- grid paper or BLM 1 cm Grid Paper (p. S-1)

**Using a picture to show a combination of tenths and hundredths.** Draw the first picture below on the board:

![First picture](image1)

ASK: How many hundredths are shaded? (30) How many tenths are shaded? (3)

Then shade two more hundredths. ASK: Now how many hundredths are shaded? (32)

Summarize by saying that 32 hundredths = 3 tenths and 2 more hundredths. Write on the board:

32 hundredths = 3 tenths 2 hundredths

**Exercises:** Describe the fraction shaded as hundredths and as tenths and hundredths.

- a)
- b)
- c)
- d)

**Bonus**

- a) 64 hundredths = 6 tenths 4 hundredths
- b) 47 hundredths = 4 tenths 7 hundredths
- c) 85 hundredths = 8 tenths 5 hundredths
- d) 86 hundredths = 8 tenths 6 hundredths

**Answers:** a) 64 hundredths = 6 tenths 4 hundredths, b) 47 hundredths = 4 tenths 7 hundredths, Bonus: c) 85 hundredths = 8 tenths 5 hundredths, d) 86 hundredths = 8 tenths 6 hundredths

**Relate tenths and hundredths to money.** Remind students that ten cents, or a dime, is one tenth of a dollar and one cent is one hundredth of a dollar. SAY: “Zero point seven three dollars” can be represented in different ways.
Write on the board:

7 tenths 3 hundredths
73 cents
73 hundredths

Relate tenths and hundredths to place value. Tell students that just like there is a ones place and a tens place in whole numbers, there is a tenths place and a hundredths place in decimals. Show this on the board:

\[
\frac{68}{100} = 0.68
\]

SAY: The first digit to the right of the decimal point is the number of tenths, and the second digit is the number of hundredths. Write on the board:

\[
\frac{\text{____}}{100} \quad \text{____ tenths} \quad \text{____ hundredths} \quad 0.\text{____}
\]

Exercises: Describe the hundredths using the three ways shown.

a) 54 hundredths   b) 8 hundredths   c) 37 hundredths

Answers: a) \(\frac{54}{100}, 5 \text{ tenths} 4 \text{ hundredths}, 0.54\); b) \(\frac{8}{100}, 0 \text{ tenths} 8 \text{ hundredths}, 0.08\); c) \(\frac{37}{100}, 3 \text{ tenths} 7 \text{ hundredths}, 0.37\)

Relate tenths and hundredths to number lines. Project BLM Number Lines Divided into Tenths and Hundredths on the board. Label the tenths as shown.

Demonstrate counting 4 tenths, then 3 more hundredths, then demonstrate counting 43 hundredths (count by ten hundredths until 40, then one hundredths until 43). Mark 0.43 on the number line.

Exercises: Write the fraction of the distance from 0 to 1 as hundredths and as tenths and hundredths.

Answers: a) 9 hundredths = 0 tenths 9 hundredths, b) 28 hundredths = 2 tenths 8 hundredths, c) 52 hundredths = 5 tenths 2 hundredths, d) 70 hundredths = 7 tenths 0 hundredths

Estimating decimals on number lines. Draw on the board:

Use BLM Number Lines Divided into Tenths and Hundredths to display these exercises.
Tell students you want to know where to place 0.61. ASK: How many tenths are in 61 hundredths? (6) Is 0.61 closer to 6 tenths or 7 tenths? (6 tenths) SAY: The decimal 0.61 is only one more hundredth than 0.6, but 0.7 is ten more hundredths than 0.6. Ask a volunteer to mark where he estimates 0.61 will be on the number line. Repeat for 0.48 (closer to 0.5 than to 0.4) and 0.95 (equally close to 0.9 and 1).

**Exercises:** Estimate the location of the decimal on a number line divided into tenths from 0 to 1.

a) 0.75  

b) 0.37  

c) 0.29  

d) 0.94  

### A centimetre is one hundredth of a metre.

ASK: How many centimetres are in 1 m? (100) What fraction of a metre is a centimetre? (one hundredth) Write on the board:

\[
1 \text{ cm} = 0.01 \text{ m} \quad 5 \text{ cm} = \_\_\_\_\_ \text{ m} \quad 17 \text{ cm} = \_\_\_\_\_ \text{ m}
\]

SAY: 1 cm is one hundredth of 1 m, so 5 cm is 5 hundredths of 1 m. Have a volunteer write the decimal to show this. (0.05) ASK: What fraction of 1 m is 17 cm? (17/100) Ask a volunteer to write the decimal. (0.17)

**Exercises:** Write the decimal.

a) 37 cm = \_\_\_\_\_\_\_\_ m  

b) 4 cm = \_\_\_\_\_\_\_\_ m  

c) 90 cm = \_\_\_\_\_\_\_\_ m  

d) 16 cm = \_\_\_\_\_\_\_\_ m  

**Answers:** a) 0.37, b) 0.04, c) 0.90, d) 0.16

### Extensions

1. What is the largest numerator you can use?

a) \[\frac{34}{100} < \frac{901}{1000}\] 

b) \[\frac{901}{1000} > \frac{1}{10}\] 

c) \[\frac{1}{10} > \frac{1000}{10}\] 

**Answers:** a) 3, b) 9, c) 99

2. How are these questions related? Use number lines to explain your answer.

Is 3 closer to 0 or to 10?  
Is 3 tenths closer to 0 or to 1?  
Is 3 hundredths closer to 0 or to 0.1?  

**Answer:** The number lines that are used to answer these questions will be identical, except for the scale, and the points will be marked at exactly the same place on each number line:

```
\begin{center}
\begin{tikzpicture}
\draw[very thick, -latex] (0,0) -- (10,0);
\foreach \i in {0,1,2,3,4,5,6,7,8,9,10}
{\draw[thin, gray,latex-latex] (\i,0.1) -- ++(0,0.2) node [midway, fill=white] {\i};}
\end{tikzpicture}
\end{center}
```

The scale will be 0 to 10, 0 to 1, or 0 to 0.1, but that is the only difference. In all cases, the number is closer to 0.
3. Lily enters a contest to run as many kilometres as she can in two weeks. At the end of the contest, she earns two tickets for each kilometre she runs. She runs \( \frac{3}{4} \) of a kilometre each day for 5 days in the first week and 3 days in the second week. Lily can pick one prize for every 5 tickets she earns. What is the greatest number of prizes she can pick?

Show your work using a picture. Say what each part of your picture means.

**Sample solution:** I used number lines. The numbers on the number line show the number of kilometres. The arrows show how far Lily runs each day.

```
0  1  2  3  4  5  6
```

She ran a total of 6 kilometres, so she gets 12 tickets \((2 \times 6 = 12)\). She can pick 1 prize for every 5 tickets, so she can get 2 prizes \((12 \div 5 = 2 \text{ R } 2, \text{ and she can't use the leftover } 2 \text{ tickets})\).

**NOTE:** There are different ways students might determine that Lily ran a total of six kilometres. For example:

- I drew fraction circles. Since 4 fourths make a whole, I grouped them into groups of 4, and there were 6 groups. Each circle is a day, and the shaded part of the circle shows the fraction of a kilometre Lily runs each day. So Lily ran 6 whole kilometres.

```
\[
\begin{array}{cccc}
1 & 1 & 2 & 2 \\
1 & 2 & 3 & 3 \\
\end{array}
\]
```

- I drew fraction circles too, but I used different numbers for different days. Each circle is a whole kilometre, so she ran 6 kilometres in the 8 days.

```
\[
\begin{array}{cccc}
3 & 4 & 5 & 6 \\
4 & 4 & 5 & 6 \\
5 & 6 & 7 & 7 \\
6 & 6 & 7 & 8 \\
7 & 8 & 8 & 8 \\
\end{array}
\]
```
**Goals**

Students will convert between mixed numbers, improper fractions, and decimals greater than 1.

**PRIOR KNOWLEDGE REQUIRED**

Can write tenths and hundredths as decimals
Can write an improper fraction as a mixed number

**VOCABULARY**

decimal
decimal hundredths
decimal point
decimal tenths
denominator
digit
equivalent
fraction
improper fraction
mixed number
numerator

**NS5-49 Decimals Greater Than 1**

Pages 51–52

**CURRICULUM REQUIREMENT**

AB: optional
BC: optional
MB: optional
ON: required

**Review writing fractions as decimals.** Remind students that, as decimals, tenths are written with one digit to the right of the decimal point and hundredths are written with two digits to the right of the decimal point.

**Exercises:** Write the fraction as a decimal.

<table>
<thead>
<tr>
<th>a) $\frac{6}{10}$</th>
<th>b) $\frac{34}{100}$</th>
<th>c) $\frac{8}{100}$</th>
<th>d) $\frac{7}{10}$</th>
<th>e) $\frac{7}{100}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.34</td>
<td>0.08</td>
<td>0.7</td>
<td>0.07</td>
</tr>
</tbody>
</table>

**Writing mixed numbers as decimals.** Tell students that mixed numbers can be written as decimals too. Write on the board:

$$2\frac{3}{10} = 2.3$$

**Exercises:** Write the mixed number as a decimal.

<table>
<thead>
<tr>
<th>a) $13\frac{74}{100}$</th>
<th>b) $8\frac{6}{10}$</th>
<th>c) $9\frac{6}{100}$</th>
<th>d) $83\frac{5}{100}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.74</td>
<td>8.6</td>
<td>9.06</td>
<td>83.05</td>
</tr>
</tbody>
</table>

**Writing decimals as mixed numbers.** Write on the board:

3.14

**Exercises:** Write the decimal as a mixed number.

<table>
<thead>
<tr>
<th>a) 2.7</th>
<th>b) 3.07</th>
<th>c) 4.80</th>
<th>d) 235.6</th>
<th>e) 17.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 7/10</td>
<td>3 7/100</td>
<td>4 80/100</td>
<td>235 6/10</td>
<td>17 9/10</td>
</tr>
<tr>
<td>Bonus: 3.801</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 2.7, b) 3.07, c) 4.80, d) 235.6, e) 17.9

**Bonus:** 3.801

**Answers:** a) 2 7/10, b) 3 7/100, c) 4 80/100, d) 235 6/10, e) 17 9/10, Bonus: 3 801/1000
**Writing decimals in words.** Tell students that if they can write whole numbers and fractions in words, then they can write decimals in words too. SAY: Just like a decimal point separates the whole-number part and the fractional part, use the word “and” to separate the whole number from the fractional part. Write on the board:

three and seventeen hundredths
3.17

**Exercises**

1. Write the missing words.
   a) 4.08 = _______ and eight hundredths
   b) 17.6 = _______ and six tenths
   c) 16.5 = sixteen and _______ tenths
   d) 3.07 = three and _______ hundredths
   e) 38.14 = thirty-eight and _______ hundredths
   f) 30.8 = thirty and eight _______
   g) 3.08 = three and eight _______
   h) 4.17 = four _______ seventeen _______

   **Answers:** a) four; b) seventeen; c) five; d) seven; e) fourteen; f) tenths; g) hundredths; h) and, hundredths

2. Write the decimal in words.
   a) 3.8  b) 26.09  c) 30.40  d) 41.5

   **Bonus:** 3 000 000.45

   **Answers:** a) three and eight tenths, b) twenty-six and nine hundredths, c) thirty and forty hundredths, d) forty-one and five tenths,

   **Bonus:** three million and forty-five hundredths

**Writing the decimal for the words.** Refer to what you wrote on the board above:

three and seventeen hundredths
3.17

SAY: The “and” tells you where to put the decimal point, and the fraction word tells you how many digits to put after the decimal point.

**Exercises:** Write the decimal.
   a) twelve and thirteen hundredths
   b) fifty and three tenths
   c) two and fifty-three hundredths
   d) two hundred and five hundredths
   e) two hundred five and six tenths

Some students might find it helpful to circle or underline the word “and.”
Review writing improper fractions as mixed numbers. Remind students that they can change improper fractions to mixed numbers by dividing.

Write on the board:
\[
\frac{7}{3} = 2 \frac{1}{3} \text{ because } 7 \div 3 = 2 \text{ R } 1. \quad \frac{24}{10} = 2 \frac{4}{10} \text{ because } 24 \div 10 = 2 \text{ R } 4.
\]

Exercises: Write the improper fraction as a mixed or whole number.

a) \(\frac{31}{10}\)  
   b) \(\frac{14}{10}\)  
   c) \(\frac{852}{100}\)  
   d) \(\frac{73}{10}\)  
   e) \(\frac{500}{100}\)

Exercises: Write the improper fraction as a decimal.

a) \(\frac{28}{10}\)  
   b) \(\frac{154}{100}\)  
   c) \(\frac{769}{100}\)  
   d) \(\frac{61}{10}\)  
   e) \(\frac{32}{10}\)

Shortcut for converting improper fractions to decimals. Write the answers to the exercises above on the board:

\[
\frac{28}{10} = 2.8 \quad \frac{154}{100} = 1.54 \quad \frac{769}{100} = 7.69 \quad \frac{61}{10} = 6.1 \quad \frac{32}{10} = 3.2
\]

SAY: The numerator tells you what number to write, and the denominator tells you how many digits go after the decimal point. Write on the board:

\[
\frac{384}{100} = 3.84 \quad \frac{384}{10} = 38.4 \quad \frac{43}{10} = 4.3
\]

Have volunteers show where to put the decimal point. (3.84, 38.4, 4.3)  
SAY: The denominator 10 means 1 digit after the decimal point, and the denominator 100 means 2 digits after the decimal point.

Exercises: Write the decimal.

a) \(\frac{497}{100}\)  
   b) \(\frac{84}{10}\)  
   c) \(\frac{604}{100}\)  
   d) \(\frac{307}{10}\)

Bonus: \(\frac{785 \ 234}{10}\)

Answers: a) 4.97, b) 8.4, c) 6.04, d) 30.7, Bonus: 78 523.4
Writing decimals as improper fractions. SAY: You can go the other way too. Write on the board:

\[ 314.08 = \frac{31408}{?} \]

SAY: Write the number without the decimal point as the numerator. Then write the denominator. ASK: Will the denominator be 10 or 100? (100) How do you know? (there are 2 digits after the decimal point)

Exercises: Write the decimal as an improper fraction.

a) 5.4    b) 6.07    c) 80.3    d) 54.76

Answers: a) 54/10, b) 607/100, c) 803/10, d) 5476/100

Introduce equivalent tenths and hundredths. Write on the board:

\[ \frac{7}{10} = \frac{70}{100} \quad \text{so} \quad \frac{37}{10} = \frac{370}{100} \]

SAY: Seven tenths equals seventy hundredths, so three and seven tenths equals three and seventy hundredths. Ask volunteers to write both fractions and both mixed numbers as decimals:

0.7 = 0.70    so    3.7 = 3.70

Exercises

1. Write the equivalent decimal hundredths.

a) 4.8    b) 3\frac{5}{10}    c) 4\frac{50}{100}    d) 8.7

Answers: a) 4.80, b) 3.50, c) 4.50, d) 8.70

2. Write the equivalent decimal tenths.

a) 5.80    b) 17\frac{4}{10}    c) 6\frac{90}{100}    d) 174.30

Answers: a) 5.8, b) 17.4, c) 6.9, d) 174.3

Extensions

1. Point out the symmetry in the place values on either side of the ones position. For example, in the number 743.61, the place values are as follows:

\[
\begin{array}{cccccc}
\text{hundreds} & \text{tens} & \text{ones} & \text{tenths} & \text{hundredths} \\
7 & 4 & . & 3 & 6 & 1
\end{array}
\]

Challenge students to name the place values in the number 3.612349 by using this symmetry. (ones, tenths, hundredths, thousandths, ten thousandths, hundred thousandths, millionths)

NOTE: Some students might look for a “oneths” position, from thinking of the decimal point as the centre of symmetry. This might seem natural.
because the decimal point is the only part of the number that looks different. However, the ones are the basic units and in fact are the centre of symmetry.

2. Add whole numbers, tenths, and hundredths.
   Example: \( 3 + \frac{8}{10} + \frac{5}{100} = 3.85 \)

3. Have students look for decimals greater than 1 in the media and write the decimals as mixed numbers.
Goals
Students will understand decimal place value as a natural extension of whole-number place value as each place is worth ten times as much as the place immediately on its right.

PRIOR KNOWLEDGE REQUIRED
Understands place value for whole numbers and the use of zero as a placeholder
Understands the phrase “times as many”
Understands place value for ones, tens, hundreds, and thousands
Can write equivalent fractions

Mental math minute. Write on the board:
\[
\frac{4}{6} = \frac{?}{12}
\]
SAY: We can make equivalent fractions by doubling the numerator and the denominator. Since we are doing the same thing to the numerator and the denominator, the new fraction is equivalent to the old fraction. Double 6 is 12. ASK: What is the double of 4? (8) Replace the question mark with 8.

Exercises: Find the number that makes the fractions equivalent.

a) \( \frac{3}{7} = \frac{6}{?} \)  
b) \( \frac{9}{13} = \frac{?}{26} \)  
c) \( \frac{8}{25} = \frac{16}{?} \)  
d) \( \frac{3}{4} = \frac{?}{100} \)

Bonus: \( \frac{2}{3} = \frac{?}{6} = \frac{8}{24} = \frac{?}{?} \)

Answers: a) 14; b) 18; c) 50; d) 75; Bonus: 4, 12, 16

Review place value system with whole numbers. Write “4183” on the board. SAY: We’re going to consider the place value of the digits in 4183. Cover up all but the 4 and tell students that the 4 is the thousands digit. Uncover the 1 and ASK: What is the place value of the 1? (hundreds) How do you know? (hundreds are always just to the right of thousands) Repeat for 8 and 3.

Write on the board:

\[
\text{thousands} \quad \text{hundreds} \quad \text{tens} \quad \text{ones}
\]

ASK: How many hundreds are in a thousand? (10) How many tens are in a hundred? (10) How many ones are in a ten? (10) SAY: A thousand is ten times as many as a hundred, a hundred is ten times as many as ten, and ten is ten times as many as one. Draw “× 10” three times as shown below:
SAY: In this system, every place value, such as thousands, hundreds, tens, and ones, in a number is ten times the value of the digit on its right. A digit’s position in a number tells us what its value is. Leave this diagram on the board for later use.

**Introduce decimal fractions.** SAY: Fractions with denominator 10, 100, or 1000, and so on are called decimal fractions. Mathematicians call them decimal fractions because they convert to decimal numbers easily. Write on the board:

\[
\begin{array}{cccccccc}
3 & 42 & 7 & 10 & 1 & 117 & 3 & 645 & 1 \\
10 & 100 & 9 & 50 & 2 & 200 & 4 & 1000 & 100000000
\end{array}
\]

As a class, determine which of the fractions shown are decimal fractions. (3/10, 42/100, 645/1000, 1/100000000) Point out that although 10/50 has a ten in the numerator, it is not a decimal fraction because the denominator does not fit the pattern just described and that although the denominators 50 and 200 are multiples of ten, they also do not fit the pattern.

**Exercises:** Is the fraction a decimal fraction?

a) \( \frac{2}{3} \)  
b) \( \frac{11}{100} \)  
c) \( \frac{999}{1000} \)  
d) \( \frac{400}{500} \)  
e) \( \frac{6}{10} \)  
f) \( \frac{1}{40} \)

**Bonus:** \( \frac{100}{1000000001} \)

**Answers:** a) no, b) yes, c) yes, d) no, e) yes, f) no, Bonus: no

**Extending the place value system to decimals.** Refer to the place value diagram on the board from earlier. ASK: What is the next place value to the right of the ones? (tenths) How many tenths are in a one? (10) What is the next place value to the right of tenths? (hundredths) How many hundredths are in a tenth? (10) What do you think the next place value to the right of hundredths is? (thousandths) How many thousandths do you think are in a hundredth? (10) SAY: One is ten times as many as a tenth, a tenth is ten times as many as a hundredth, and a hundredth is ten times as many as a thousandth. Extend the diagram as shown below:

\[
\begin{array}{cccccccc}
\times 10 & \times 10 & \times 10 & \times 10 & \times 10 & \times 10 \\
thousands & hundreds & tens & ones & tenths & hundredths & thousandths
\end{array}
\]

ASK: Where does the decimal point go in this diagram? (between ones and tenths) Draw it in.

**Identifying the place value and value of a digit.** Write on the board:

\[
119.08 \quad 0.15 \quad 62.3 \quad 3402.70
\]

Have students identify the place value and value of selected digits. For example, ASK: What is the place value of 8 in 119.08? (hundredths) What is its value? (8 hundredths) Write “\( \frac{8}{100} \), 8 hundredths” on the board. ASK: What is the place value of 0 in 0.15? (ones) What is its value? (zero)
Students should identify the place value and value of both zero and non-zero digits.

**Exercises:** Identify the place value and value of the underlined digit. Write the value as a number and in words.

a) 10.01  

b) 4.00  

c) 367.4  

d) 9990.9  

e) 4808.65  

**Bonus:** 17 390 045.02

**Answers:**  
a) hundredths, 1/100, one hundredth;  
b) tenths, 0, zero;  
c) tens, 60, sixty;  
d) thousands, 9000, nine thousand;  
e) ones, 8, eight;  

**Bonus:** hundreds, 0, zero

**Ordering numbers based on place value.** Draw on the board:

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.6, 6.02, 10.58, 6.2</td>
</tr>
</tbody>
</table>

Have volunteers write the numbers in the place value chart. Then have students order the numbers from least to greatest in their notebooks. (6.02, 6.2, 10.58, 10.6)

**Placing the decimal to create the given value of the digit.** Write “735” on the board. ASK: Where should I place the decimal point so the 5 has a value of 5 hundredths? (between 7 and 3) Add in the decimal point. Repeat for 1020 to make the 2 have a value of 2 tenths, for 908 to make the 9 have a value of ninety, and for 64 071 to make the 4 have a value of four hundred. (10.20, 90.8, 6407.1)

**Extensions**

1. Identify the place value and value of the underlined digit. Write the value as a number and in words.
   a) 6.019  
   b) 138.247  
   c) 80.039

   **Answers:** a) thousandths, 9/1000, nine thousandths;  
   b) hundredths, 4/100, four hundredths;  
   c) tenths, 0, zero

2. Write 1 as a decimal fraction.

   **Sample answers:** 10/10, 100/100
3. There is symmetry in the place value system with the “ones” at the centre. Continue the place values in both directions.

\[ 100 \quad 10 \quad 1 \quad \frac{1}{10} \quad \frac{1}{100} \]

**Answers:** 10 000, 1000, 1/1000, 1/10 000

4. A decimal number has a 2 and a 5 in it. The 2 is worth more than the 5. What do you know about the positions of the 2 and 5?

**Answer:** The 2 is to the left of the 5.

5. Write the missing number.

   a) \( \frac{70}{100} = 0.7 \)  
   b) \( \frac{9}{100} = 0.09 \)  
   c) \( \frac{80}{100} = 0.8 \)  
   d) \( \frac{4}{10} = 0.4 \)

**Answers:** a) 100, b) 100, c) 80, d) 4

6. 

<table>
<thead>
<tr>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
</table>

a) What fraction of the number line is between the two dots? Explain how you found your answer.

b) What fraction of the number line is not between the two dots? Explain how you found your answer.

**Answers**

a) 4/10, Sample explanation: I counted the spaces from 20 to 60. There are 4 spaces.

b) 6/10, Sample explanation: I counted the spaces from 0 to 20 and from 60 to 100; I know that the whole line has 10 spaces, and I subtracted the number of spaces between the dots (4) from the number of spaces in the whole line, 10 – 4 = 6.
Goals
Students will use decimal notation for fractions with denominator 10, 100, and 1000.
Students will write decimals and their equivalent fractions for tenths, hundredths, and thousandths.

PRIOR KNOWLEDGE REQUIRED
Understands decimal numbers with up to two decimal places and their equivalent fractions

MATERIALS
transparency of BLM One Thousandth (p. M-46)
overhead projector

Introduce decimal thousandths. Tell students that just as the fractions 1/10 and 1/100 can be represented in various ways, 1/1000 can also be represented in various ways. Project BLM One Thousandth and write on the board:

\[
\frac{1}{1000} \quad \text{one thousandth} \quad 0.001
\]

Point out how one thousandth is written differently from one tenth and one hundredth—there are three digits after the decimal point instead of only one or two. Point to 0.001 and SAY: The first zero shows there are no ones, and the second zero shows there are no tenths. ASK: What do you think the third zero shows? (there are no hundredths) Ask a volunteer to show how to write six thousandths as a decimal (0.006), then read it as “zero point zero zero six.” ASK: How would you write three thousandths as a decimal? (0.003)

Exercises: Write the fraction as a decimal.

a) \(\frac{4}{1000}\)  
b) \(\frac{9}{1000}\)  
c) \(\frac{2}{1000}\)  
d) \(\frac{7}{1000}\)  
e) \(\frac{5}{1000}\)

Answers: a) 0.004, b) 0.009, c) 0.002, d) 0.007, e) 0.005

Writing two-digit thousandths as decimals. Write on the board:

\[
\frac{14}{1000} = 0.014 \quad \frac{27}{1000} = 0.027 \quad \frac{60}{1000} = 0.060
\]

SAY: 0.014 has three digits after the decimal point. ASK: What does that tell us about the number? (it is thousandths) What does the zero after the decimal point tell us? (there are no tenths) How do we write 27/1000 as a decimal? (0.027) How do we write 60/1000 as a decimal? (0.060)
Exercises: Write the fraction as a decimal.

\[
\begin{align*}
&\text{a)} \quad \frac{11}{1000} \\
&\text{b)} \quad \frac{70}{1000} \\
&\text{c)} \quad \frac{32}{1000} \\
&\text{d)} \quad \frac{50}{1000} \\
&\text{e)} \quad \frac{99}{1000} \\
&\text{f)} \quad \frac{10}{1000}
\end{align*}
\]

Answers: a) 0.011, b) 0.070, c) 0.032, d) 0.050, e) 0.099, f) 0.010

Writing three-digit thousandths as decimals. Write on the board:

\[
\begin{align*}
&\frac{128}{1000} \quad \frac{400}{1000} \quad \frac{901}{1000} \quad \frac{530}{1000}
\end{align*}
\]

Ask for volunteers to write the decimal form of the fractions. (0.128, 0.400, 0.901, 0.530)

Exercises: Write the fraction as a decimal.

\[
\begin{align*}
&\text{a)} \quad \frac{101}{1000} \\
&\text{b)} \quad \frac{12}{1000} \\
&\text{c)} \quad \frac{3}{1000} \\
&\text{d)} \quad \frac{470}{1000} \\
&\text{e)} \quad \frac{602}{1000} \\
&\text{f)} \quad \frac{100}{1000}
\end{align*}
\]

Answers: a) 0.101, b) 0.012, c) 0.003, d) 0.470, e) 0.602, f) 0.100

Writing decimal thousandths as fractions with denominator 1000. SAY: Now we’re going to convert from decimal thousandths to fractions. Write on the board:

\[
\begin{align*}
0.012 & \quad 0.120 & \quad 0.102 & \quad 0.002
\end{align*}
\]

Ask for volunteers to write the fraction form of the decimals. (12/1000, 120/1000, 102/1000, 2/1000)

Exercises: Write the decimal as a fraction.

\[
\begin{align*}
&\text{a)} \quad 0.090 \\
&\text{b)} \quad 0.900 \\
&\text{c)} \quad 0.009 \\
&\text{d)} \quad 0.110 \\
&\text{e)} \quad 0.011
\end{align*}
\]

Answers: a) 90/1000, b) 900/1000, c) 9/1000, d) 110/1000, e) 11/1000

Equivalent fractions and decimals. Draw on the board:

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Tenths</th>
<th>Fraction</th>
<th>Hundredths</th>
<th>Fraction</th>
<th>Thousandths</th>
<th>Decimal</th>
<th>Tenths</th>
<th>Decimal</th>
<th>Hundredths</th>
<th>Decimal</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2}{10})</td>
<td></td>
<td>(\frac{0}{10})</td>
<td></td>
<td>(\frac{0}{10})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have volunteers complete the table by writing the equivalent fractions and decimals. (20/100, 200/1000, 0.2, 0.20, 0.200)

Exercises: Write all equivalent tenths, hundredths, and thousandths in fraction and decimal form.

\[
\begin{align*}
&\text{a)} \quad 0.80 \\
&\text{b)} \quad \frac{300}{1000} \\
&\text{c)} \quad 0.5 \\
&\text{d)} \quad \frac{60}{100} \\
&\text{e)} \quad 0.400
\end{align*}
\]

Answers: a) 8/10, 80/100, 800/1000, 0.8, 0.800; b) 3/10, 30/100, 0.3, 0.30, 0.300; c) 5/10, 50/100, 500/1000, 0.50, 0.500; d) 6/10, 600/1000, 0.6, 0.60, 0.600; e) 4/10, 40/100, 400/1000, 0.4, 0.40

NOTE: Extension 3 is required in order to cover the Alberta and Manitoba curricula.
Extensions

1. Multiply the top and bottom by the same number, either 10 or 100, to create an equivalent fraction with denominator 1000.

   a) \( \frac{3}{10} \times \frac{100}{100} = \frac{300}{1000} \)
   b) \( \frac{7}{100} \times \frac{100}{100} = \frac{700}{1000} \)
   c) \( \frac{20}{100} \times \frac{100}{100} = \frac{2000}{1000} \)
   d) \( \frac{8}{10} \times \frac{100}{100} = \frac{800}{1000} \)
   e) \( \frac{81}{100} \times \frac{100}{100} = \frac{8100}{1000} \)
   Bonus: \( \frac{100}{100} \times \frac{100}{100} = \frac{100000}{10000} \)

   Answers: a) 300, b) \( \times 10 = 70 \), c) \( \times 10 = 200 \), d) \( \times 100 = 800 \), e) \( \times 10 = 810 \), Bonus: \( \times 10 = 1000 \)

2. How many more zeros does the denominator on the right have? Write the numerator on the right with this many more zeros.

   a) \( \frac{6}{10} = \frac{600}{1000} \)
   b) \( \frac{12}{100} = \frac{120}{1000} \)
   Bonus: \( \frac{20}{100} = \frac{2000000000000}{1000000000000} \)

   Answers: a) 600, b) 120, Bonus: 20 000 000 000

3. Order the numbers from least to greatest.

   a) \( \frac{99}{100}, \frac{9}{10}, \frac{999}{1000} \)
   b) \( \frac{1}{10}, \frac{8}{100}, \frac{75}{1000} \)

   Answers: a) 9/10, 99/100, 999/1000; b) 75/1000, 8/100, 1/10

4. Estimate, very approximately, where the fraction goes. Write the letter above the number line.

   A. \( \frac{1}{10} \)
   B. \( \frac{10}{11} \)
   C. \( \frac{4}{10} \)
   D. \( \frac{5}{8} \)

   A closer to 0  B closer to \( \frac{1}{2} \)  C closer to \( \frac{1}{2} \)  D closer to 1

   less than \( \frac{1}{2} \)  greater than \( \frac{1}{2} \)
Comparing and Ordering Decimal Fractions and Decimals

**Goals**

Students will use number lines to compare decimal fractions and decimals.

**Prior Knowledge Required**

Understands decimals and fractions on number lines
Understands decimal numbers with up to two decimal places and their equivalent fractions
Can use number lines

**Materials**

BLM Number Lines Divided into Tenths and Hundredths (p. M-44)
transparency of BLM Number Lines Divided into Tenths and Hundredths (p. M-44)
overhead projector

**Mental Math Minute.** Write on the board:

\[
\frac{6}{9} = \frac{2}{?}
\]

SAY: We can make equivalent fractions by dividing the numerator and the denominator by the same number. Since we are dividing the top and bottom by the same number, the new fraction is equivalent to the old fraction. I see that both the numerator and the denominator divide by the same number.

ASK: What number is it? (3) What is \(6 \div 3\)? (2) What is \(9 \div 3\)? (3) Replace the question mark with "3."

**Exercises:** Find the number that makes the fractions equivalent.

a) \(\frac{14}{21} = \frac{?}{3}\)  
   b) \(\frac{36}{45} = \frac{?}{5}\)  
   c) \(\frac{44}{77} = \frac{4}{?}\)  
   d) \(\frac{45}{100} = \frac{?}{20}\)

**Answers:** a) 2, b) 4, c) 7, d) 9

**Review Number Lines with Fractional Tenths and Hundredths.** Draw on the board:

```
0 1 2 3 4 5 6 7 8 9 1
10 10 10 10 10 10 10 10 10 10
```

Have students count out loud with you from 0 to 1 by tenths: zero, one tenth, two tenths, ..., nine tenths, one. Then have a volunteer write the equivalent decimal for \(1/10\) on top of the number line:

```
0.1
0 1 2 3 4 5 6 7 8 9 1
10 10 10 10 10 10 10 10 10 10
```
Continue in random order until all the equivalent decimals have been added to the number line. SAY: Now let’s modify the scales so they show hundredths instead of tenths. ASK: How do we write the decimal equivalent of 0.1 in hundredths? (0.10) How do we write the decimal equivalent of 0.2 in hundredths? (0.20) Have volunteers convert the decimals to hundredths. (0.10, 0.20, ..., 0.90) ASK: How do we write the fraction equivalent of 1/10 in hundredths? (10/100) How do we write the fraction equivalent of 2/10 in hundredths? (20/100) Have volunteers convert the fractions to hundredths. (10/100, 20/100, ..., 90/100)

**Identifying decimal and fractional tenths on number lines.** Draw a metre-long number line like the one below on the board:

```
A B C
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
```

Have students write a fraction for each point in their notebooks. (A. 2/10, 0.2; B. 7/10; C. 9/10) Leave the number line on the board.

**Identifying decimal and fractional tenths and hundredths on number lines.** Give students BLM Number Lines Divided into Tenths and Hundredths. Write on the board:

```
A. 0.04  B. \frac{18}{100}  C. \frac{2}{10}  D. 0.3
```

Have students mark the points on the top number line. Suggest that they first convert all points into hundredths in both fraction and decimal form. Take up the answers as a class. (A. 4/100, B. 0.18, C. 20/100, 0.20, D. 30/100, 0.30)

Project BLM Number Lines Divided into Tenths and Hundredths onto the board. Mark a variety of points, each with its own upper case letter, onto the top number line. Have students write each as a fraction and decimal hundredth and when possible as the equivalent fraction and decimal tenth. For example, a point at 0.15 can be represented only in hundredths (0.15, 15/100), whereas a point at 0.80 can be represented as hundredths and tenths (0.80, 80/100, 0.8, 8/10). Take up the answers as a class.

**Estimating decimal and fractional tenths and hundredths on number lines.** Again, write a variety of points on the board using decimal and fraction form for tenths and hundredths. Have students mark them onto the middle number line on BLM Number Lines Divided into Tenths and Hundredths. Repeat once more using different fractions and decimals, this time having students mark points onto the bottom number line.

**Ordering decimal and fractional tenths and hundredths.** Write on the board:

```
\frac{6}{10}  0.09  \frac{57}{100}
```

Have students write all the numbers as equivalent fractions with denominator 100 in their notebooks. (60/100, 9/100, 57/100) Have students
order them from least to greatest. (9/100, 57/100, 60/100) Repeat with 0.08, 11/100, 2/10 but this time have students write all the numbers as decimals with 2 digits after the decimal point. (0.08, 0.11, 0.20) Ask students if they found it easier to order the numbers when rewritten as fractions or as decimals.

**NOTE:** Extensions 1 to 3 are required to cover the Ontario curriculum. Extension 5 is required in order to cover the Alberta, British Columbia, and Manitoba curricula. If you choose to do Extension 4, do Extension 3 first.

**Extensions**

1. Use all the digits 4, 7, and 8 once to write a number between the given numbers.
   a) $4.78 < _____ < 8.74$
   b) $7.48 < _____ < 8.74$
   **Bonus:** $0.478 < _____ < 7.48$
   **Sample answers:** a) 7.48, b) 7.84, Bonus: 4.78

2. a) Mark the decimal or fraction on the number line with a dot and a letter.

   ![Number Line Diagram]

   A. 1.3
   B. 2.9
   C. 0.3
   D. $\frac{5}{10}$
   E. $\frac{7}{10}$
   F. $\frac{1}{10}$
   G. 1.3
   **Bonus:** $\frac{21}{10}$

   b) Order B, D, and F from least to greatest.

   c) What whole number is each decimal or fraction in part a) closest to? Explain why there can be no answer for point D.
   **Selected answers:** b) $\frac{5}{10} < 1 \frac{1}{10} < 2.9$; c) A is closest to 1, B is closest to 3, C is closest to 0, E is closest to 3, F is closest to 1, G is closest to 1; Bonus is closest to 2; D is the same distance from 0 as it is from 1

3. Write a decimal for the fraction by first changing the fraction to an equivalent fraction with denominator 100.
   a) $\frac{1}{2}$
   b) $\frac{1}{4}$
   c) $\frac{1}{5}$
   d) $\frac{1}{20}$
   e) $\frac{1}{25}$
   f) $\frac{1}{50}$
   **Answers:** a) $\frac{50}{100} = 0.50$, b) $\frac{25}{100} = 0.25$, c) $\frac{20}{100} = 0.20$, d) $\frac{5}{100} = 0.05$, e) $\frac{4}{100} = 0.04$, f) $\frac{2}{100} = 0.02$

4. If you can run 6 km in 60 minutes, how many kilometres can you run in one minute? Write your answer as a decimal number. Hint: Write a fraction equivalent to 6/60 that has a denominator of 10.
   **Answer:** 0.1 km
5. Order the numbers from least to greatest.

\[
\begin{array}{ccc}
60 & 6 & 0.006 \\
1000 & 10 & \\
\end{array}
\]

Answer: 0.006, 60/1000, 6/10
Goals
Students will use fractions (halves, quarters, and fifths) as benchmarks for decimals.

Prior Knowledge Required
Can place decimals and fractions on number lines
Understands decimal numbers with up to two decimal places and their equivalent fractions

Materials
transparency of BLM Hundredths Block (p. M-47)
overhead projector
BLM Relationships Between Fractions and Their Equivalent Decimals (p. M-48, see Extension 2)

Comparing fractional halves to decimal tenths using number lines.
Draw on the board:

```
<table>
<thead>
<tr>
<th>0</th>
<th>1/2</th>
<th>1</th>
</tr>
</thead>
</table>
```

SAY: The top number line is divided into two halves. The bottom number line is divided into tenths. Have a volunteer complete the decimal scale. Explain that you want to relate the fractions above the top number line to decimals below the bottom number line. Point to $\frac{1}{2}$ and ASK: What decimal does this equal? (0.5) Write $\frac{1}{2} = 0.5$ on the board. ASK: Which decimals between 0 and 1 are less than one half? (0.1, 0.2, 0.3, 0.4) Which decimals are greater than one half? (0.6, 0.7, 0.8, 0.9) Have a volunteer convert the decimal tenths to decimal hundredths and repeat the questions. ($1/2 = 0.50$, 0.10 to 0.40 are less than one half, 0.60 to 0.90 are greater than one half)

Comparing fractional quarters and halves to decimal tenths using number lines. Draw on the board:

```
<table>
<thead>
<tr>
<th>0</th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1</th>
</tr>
</thead>
</table>
```

1.0 tenths

1 halves

NS5-53 Comparing and Ordering Fractions and Decimals

Pages 57–58

Curriculum Requirement
AB: required
BC: required
MB: required
ON: required

Vocabulary
equivalent
Have a volunteer label the bottom number line by decimal tenths. (0.1, 0.2, ..., 0.9) Write on the board:

\[
\begin{align*}
\frac{3}{4} & \quad 0.6 \\
0.4 & \quad \frac{1}{4} \\
0.8 & \quad \frac{3}{4}
\end{align*}
\]

For each pair, have students refer to the number lines to decide which number is greater and then signal whether to use the greater than or less than symbol to make the relationship true. Fill in the symbols on the board. (>, >, >)

**Exercises: Write > or < to make the relationship true.**

\[
\begin{align*}
a) \quad \frac{1}{4} & \quad 0.1 \\
b) \quad 0.7 & \quad \frac{3}{4} \\
c) \quad \frac{3}{4} & \quad 0.4
\end{align*}
\]

**Answers: a) >, b) <, c) >**

Comparing fractional quarters and halves to decimal hundredths using number lines. Have a volunteer convert the scale on the bottom number line from tenths to hundredths. (0.10, 0.20, ..., 0.90) Modify the bottom number line by adding a zero to 1.0, and changing “tenths” to “hundredths” as shown below:

```
0 0.1 0.2 0.3 0.4 1
\hline
\frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4}
\hline
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
```

Point out that you added tick marks between 0.20 and 0.30 and between 0.70 and 0.80 just to show that 1/4 equals 0.25 and 3/4 equals 0.75. Explain that you could have added tick marks to the whole number line, but they wouldn’t have shown anything more than what the students already know, that is, that 1/2 equals 0.50.

**Exercises**

1. Write > or < to make the relationship true

\[
\begin{align*}
a) \quad 0.2 & \quad \frac{1}{4} \\
b) \quad 0.81 & \quad \frac{3}{4} \\
c) \quad \frac{1}{4} & \quad 0.27
\end{align*}
\]

**Answers: a) <, b) >, c) <**

2. Have students use their answers to Exercise 1 to order 0.27, 0.2, and \(\frac{1}{4}\) from least to greatest.

**Answer: 0.2, 1/4, 0.27**
Comparing fractional fifths to decimal tenths and hundredths using number lines. Draw on the board:

\[
\begin{array}{cccccc}
0 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 \\
0 & 0.10 & 0.20 & 0.30 & 0.40 & 0.50 & 0.60 & 0.70 & 0.80 & 0.90 & 1.0
\end{array}
\]

Point to \(\frac{3}{5}\) and ASK: What decimal does this equal? (0.60) Write “\(\frac{3}{5} = 0.60\)” on the board. Repeat for the other fractions on the number line.

Write on the board:

\[
\begin{array}{ccc}
\frac{2}{5} & 0.2 & 0.45 \\
\frac{4}{5} & 0.60 \\
\frac{3}{5} &
\end{array}
\]

For each pair, ASK: Are they equal? If the answer is no, have the class signal whether to use the greater than or less than symbol to make the relationship true. Fill in the symbols on the board. (> , < , =)

**Exercises:** Write > , < , or = to make the relationship true.

a) 0.25 \(\frac{1}{5}\) b) 0.5 \(\frac{2}{5}\) c) \(\frac{3}{5}\) 0.35 d) \(\frac{1}{5}\) 0.2

**Answers:** a) > , b) > , c) > , d) =

Comparing fractional and decimal fourths using hundredths blocks.

Project BLM Hundredths Block on the board. SAY: We’re going to compare fractions and decimals using a hundredths block. Divide it into 4 quarters as shown below:

![Hundredths block diagram]

Shade the top left quadrant and write on the board:

\[
\frac{1}{4} = \frac{25}{100}
\]

ASK: How many hundredths is 1/4 equivalent to? (25) Fill in the numerator.

ASK: How do we write that as a decimal? (0.25) Can we represent 25/100 or 0.25 as an equivalent number of tenths? (no) Why not? (there are 5 hundredths, and there’s no way to represent them as a number of tenths)

Shade the top right quadrant. ASK: How many quarters are shaded? (2) How can we represent that as a fraction with denominator 100? (50/100) Can we represent 50/100 as an equivalent number of tenths? (yes) How many tenths is 50/100? (5) Write on the board:

\[
\frac{1}{4} \quad \frac{3}{10} \quad 0.20
\]
ASK: How can we easily compare and order these numbers? (write them all as hundredths) Have students write the numbers as hundredths and then write them in order from least to greatest. They can choose whether to use decimals or fractions. (0.20, 0.25, 0.30, or 20/100, 25/100, 30/100)

**Comparing fractional and decimal fifths using hundredths blocks.**

Project BLM Hundredths Block on the board. SAY: Let’s do the same thing we just did but with fifths instead of fourths. Divide the block into 5 fifths as shown below:

```
\[
\begin{array}{cccc}
\hline
& & & & \\
\hline
& & & & \\
& & & & \\
& & & & \\
& & & & \\
\hline
\end{array}
\]
```

Shade the top fifth and write on the board:

\[
\frac{1}{5} = \frac{100}{100}
\]

ASK: How many hundredths is 1/5 equal to? (20) Fill in the numerator.

ASK: How do we write that as a decimal? (0.20) Can we represent 20/100 or 0.20 as an equivalent number of tenths? (yes) How many? (2)

(Shade the next two fifths and ASK: How many fifths is that? (3) How can we represent that as a fraction with denominator 100? (60/100) Can we represent 60/100 as an equivalent number of tenths? (yes) How many tenths is 60/100? (6)

Write on the board:

\[
\frac{4}{5} \quad \frac{73}{100}
\]

Have students write the numbers as hundredths and then write them in order from greatest to least. (0.85, 0.80, 0.73, or 85/100, 80/100, 73/100).

**NOTE:** Extension 1 is required in order to cover the British Columbia curriculum. Extensions 2 to 5 are required to cover the Ontario curriculum. Extension 6 is required in order to cover the Alberta, British Columbia, and Manitoba curricula.

**Extensions**

1. a) Estimate the position of the fraction by writing the letter and a dot.

```
0 \quad \frac{1}{2} \quad 1
```

A. \(\frac{3}{8}\)  B. \(\frac{7}{8}\)  C. \(\frac{7}{12}\)  D. \(\frac{11}{12}\)

b) What benchmark is each fraction in part a) closest to: 0, \(\frac{1}{2}\), or 1?

Sample answers: a) A. C, B. D, C. 1/2, D. 1

After reviewing the answer, you may proceed to the next page.
2. Have students complete **BLM Relationships Between Fractions and Their Equivalent Decimals**.

**Answers:**

a) iii) \(\frac{25}{100}, 0.25\); iv) \(\frac{50}{100}, 0.50; \frac{5}{10}, 0.5\)

b) i) 0.04; ii) \(\frac{20}{10}, 0.20, \frac{2}{10}, 0.2\); iii) \(\frac{40}{100}, 0.40, \frac{4}{10}, 0.4\);

iv) \(\frac{80}{100}, 0.80, \frac{8}{10}, 0.8\)

c) i) 0.02; ii) \(\frac{10}{100}, 0.10, \frac{1}{10}, 0.1\); iii) \(\frac{20}{100}, 0.20, \frac{2}{10}, 0.2\);

iv) \(\frac{50}{100}, 0.50, \frac{5}{10}, 0.5\)

3. Write the numbers in the place value chart. Order the numbers from least to greatest. Hint: Write all numbers with two decimal places.

a) 1.4, 1.35, 1.09

b) 20.6, 2.06, 2.6

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Selected answers:** a) 1.09, 1.35, 1.4; b) 2.06, 2.6, 20.60

4. Have students count forward by hundredths. They should count ten hundredths from any decimal number expressed to two decimal places. For example, start from 2.96 and count forward by hundredths to 3.06: “Two and ninety-six hundredths, two and ninety-seven hundredths, two and ninety-eight hundredths, two and ninety-nine hundredths, three, three and one hundredth, three and two hundredths, three and three hundredths, three and four hundredths, three and five hundredths, three and six hundredths.”

5. Draw on the board:

```
1 2
3 4
```

SAY: To find 1.5 times as many as 2, I shade one whole circle and one half of another circle. Have a volunteer do the shading. It should look like this:

```
1 2
3 4
```
Draw on the board:

ASK: How do we find 1.5 times as many as 10? (shade a whole circle and half of the next circle) Have a volunteer do the shading. Write on the board:

We shaded ____ parts.
So 1.5 times as many as 10 is ____.
We shaded ____ times as many as 10.

Have students copy what you wrote in their notebooks and fill in the missing information.

**Answers:** 15, 15, 1.5

6. Write the numbers in the place value chart. Order the numbers from least to greatest. Hint: Write all numbers with three decimal places.

a) 0.404, 0.4, 0.04

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

b) 0.099, 0.1, 0.09

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Selected answers:** a) 0.04, 0.4, 0.404; b) 0.09, 0.099, 0.1

7. Write one digit in each blank that will make the statement true.

____.8 < ____.1

**Answer:** Use any numbers as long as the number in the first box is smaller than the number in the second box. Example: 2 and 3
NS5-54 Adding Decimals
Pages 59–61

Goals
Students will add decimals.

PRIOR KNOWLEDGE REQUIRED
Can represent decimals using base ten materials
Understands decimal place values
Can translate between fractions with denominator 10, 100, or 1000
and decimals
Can add whole numbers with regrouping using the standard algorithm

MATERIALS
base ten blocks
grid paper or BLM 1 cm Grid Paper (p. S-1)

Mental math minute. Have groups of three students add decimals with up to two digits after the decimal point by adding tenths and adding hundredths. Give an addition problem, such as 0.35 + 0.46. The first student adds the tenths, 0.30 + 0.40 = 0.70; the second student adds the hundredths, 0.05 + 0.06 = 0.11; and the third student finishes the addition, 0.70 + 0.11 = 0.81, so 0.35 + 0.46 = 0.81. Start with problems that do not require regrouping. Include problems such as 0.64 + 0.07 and 0.38 + 0.5. In the latter type, students first change 0.5 to 0.50.

Using base ten blocks to add. Draw on the board:

| 1 tenth | 1 hundredth |

Remind students that tenths and hundredths work just like other place values—each value is 10 times the value to the right. So you can regroup them the same way. Work through the first part of the exercises below as a class.

Exercises: Use base ten blocks to regroup so that each place value has a single digit.

a) 3 tenths + 12 hundredths  
b) 7 tenths + 18 hundredths  
c) 5 tenths + 14 hundredths

Answers: a) 4 tenths + 2 hundredths, b) 8 tenths + 8 hundredths, c) 6 tenths + 4 hundredths

Review aligning place values in vertical addition. Write the incorrectly aligned vertical addition question below on the board:

```
    3 5 1
+ 1 2
```
ASK: Have I written the addition correctly? (no) Have students explain what you should change. Review how to align the digits and how to perform the sum. SAY: We align the place values and then add each place value separately.

**Adding decimals.** Write on the board:

\[
\frac{21}{100} + \frac{14}{100} = \frac{35}{100}
\]

Write the equation in vertical format as decimals:

\[
\begin{array}{c}
0.21 \\
+ 0.14 \\
\hline
0.35
\end{array}
\]

Explain that you can add decimals the same way you add whole numbers—line up the place values—but instead of adding whole numbers, you’re adding tenths, hundredths, and so on.

**Exercises:** Add by lining up the place values on grid paper or BLM 1 cm Grid Paper.

a) \(0.34 + 0.15\)  
b) \(0.46 + 0.21\)  
c) \(0.85 + 0.12\)

**Answers:** a) 0.49, b) 0.67, c) 0.97

**Adding decimals with regrouping, with the same number of digits to the right of the decimal point.** Work through the first part of the exercises below together. Then have students work individually to add the numbers. Remind students to align the place values.

**Exercises:** Add.

a) \[0.35 + 0.16\]  
b) \[0.74 + 0.18\]  
c) \[0.67 + 0.23\]  

**Answers:** a) 0.51, b) 0.92, c) 0.90

**Adding decimals with a different number of digits to the right of the decimal point.** Write on the board:

\[
\begin{array}{c}
0.7 \\
+ 0.15 \\
\hline
0.85
\end{array}
\]

Tell students that when they are using the standard algorithm, they should use the same structure as in the place value chart, aligning the place values. And as in the place value chart, when a digit is missing, we regard it as zero. Show the addition in a grid on the board:

\[
\begin{array}{c|c|c}
| & 0 & 7 \ \\
+ & 0 & 15 \\
\hline
0 & 8 & 5
\end{array}
\]

Remind students that 0.7 is equivalent to 0.70, so regarding the empty spot after the 7 as a zero makes perfect sense. Have students check the addition. ASK: Does adding \(0.70 + 0.15\) give the same result as you’ve got? (yes)
Exercises: Add.

a) \(0.38 + 0.4\)  
b) \(0.37 + 0.49\)  
c) \(0.85 + 0.1\)

Answers: a) 0.78, b) 0.86, c) 0.95

**NOTE:** Extension 1 is required in order to cover the Ontario curriculum. Extension 2 is required in order to cover the Alberta, British Columbia, and Manitoba curricula.

Extensions

1. Add by lining up the place values on grid paper.
   
a) \(1.38 + 2.21\)  
b) \(6.47 + 1.45\)  
c) \(2.96 + 5.8\)
   
d) \(7.91 + 1.09\)  
**Bonus:** \(364.88 + 259.26\)

   Answers: a) 3.59, b) 7.92, c) 8.76, d) 9.00, Bonus: 624.14

2. Add by lining up the place values on grid paper.
   
a) \(0.138 + 0.221\)  
b) \(0.647 + 0.145\)
   
c) \(0.296 + 0.58\)  
d) \(0.791 + 0.109\)

   Answers: a) 0.359, b) 0.792, c) 0.876, d) 0.900

3. a) Add mentally.
   
i) \(2.5 + 3.5\)  
ii) \(0.8 + 2.2\)  
iii) \(1.7 + 1.3\)
   
b) Add the two numbers that are easiest to add first. Find the total by adding the third number. \(2.3 + 1.9 + 4.7\)

   Answers: a) i) 6.0, ii) 3.0, iii) 3.0; b) 2.3 + 4.7 = 7.0, 7.0 + 1.9 = 8.9
Goals
Students will subtract decimals.

PRIOR KNOWLEDGE REQUIRED
Can represent decimals using base ten materials
Understands decimal place values
Can translate between fractions with denominator 10 or 100 and decimals
Can subtract whole numbers with regrouping using the standard algorithm

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. S-1)

Mental math minute. Give the first student a problem that does not need regrouping, such as 0.97 − 0.12. Students repeatedly subtract the same number, in this case 0.12, by each student formulating and answering one subtraction aloud. When a student says the subtraction that involves regrouping, emphasize that this answer was a bonus. Example: Student 1 says, “0.97 − 0.12 = 0.85.” Student 2 says, “0.85 − 0.12 = 0.73.” Student 3 says, “0.73 − 0.12 = 0.61.” Bonus: Student 4 says, “0.61 − 0.12 = 0.49.” Continue without regrouping until Student 8 says, “0.13 − 0.12 = 0.01,” then start a new chain.

Subtracting decimals. Explain that subtraction with decimals works in a similar way to addition with decimals: you need to line up the place values and then perform the operation as if you had whole numbers. Show an example without regrouping, such as 0.45 − 0.31 = 0.14.

Exercises: Subtract using grid paper or BLM 1 cm Grid Paper.

a) 0.74 − 0.21
b) 0.93 − 0.52
c) 0.56 − 0.44
d) 0.65 − 0.12

Answers: a) 0.53, b) 0.41, c) 0.12, d) 0.53

Subtracting decimals with regrouping. Subtract with an example that requires regrouping, such as 0.21 − 0.02 = 0.19. Then have students practise individually. Tell students that sometimes they will be able to rewrite the answer with fewer digits to the right of the decimal point—for example, 0.62 − 0.42 = 0.20, so you can rewrite the answer as 0.2.

Exercises: Subtract.

a) 0.71 − 0.44
b) 0.37 − 0.29
c) 0.85 − 0.68
d) 0.43 − 0.34

Answers: a) 0.27, b) 0.08, c) 0.17, d) 0.09
Subtracting decimals with different numbers of digits to the right of the decimal point. Present the following example: 0.98 – 0.4. Invite a volunteer to write it in a place value chart. ASK: How do I subtract the hundredths? (there is no hundredths digit, so write a zero and then subtract) Have students tell you what to do in each step as you perform the subtraction. 

\((0.98 - 0.40 = 0.58)\)

**Exercises:** Subtract.

a) 0.85 – 0.6  

b) 0.49 – 0.3  

c) 0.67 – 0.4  

d) 20.37 – 5.29  

e) 2 – 0.52  

f) 10 – 2.41  

**Answers:** a) 0.25, b) 0.19, c) 0.27, d) 15.08, e) 1.48, f) 7.59

Students can check their answers using addition.

**Word problems with decimals.** Solve the first part of the exercises below as a class, then have students work on the other one individually.

**Exercises**

a) Jennifer made a 0.85 L milkshake by adding ice cream to 0.6 L of milk. How much ice cream did she add?  

b) Josh cut a piece of wood board that is 0.79 m long to make a shelf. The leftover piece of board is 0.24 m long. How long is the shelf?

**Answers:** a) 0.85 L – 0.6 L = 0.25 L, b) 0.55 m

**NOTE:** Extension 1 is required in order to cover the Ontario curriculum. Extension 2 is required in order to cover the Alberta, British Columbia, and Manitoba curricula.

**Extensions**

1. Subtract by lining up the place values on grid paper.

   a) 2.39 – 1.25  

   b) 6.41 – 3.02  

   c) 8.64 – 4.5  

   d) 11.12 – 1.09  

   **Bonus:** 201.37 – 59.38

   **Answers:** a) 1.14, b) 3.39, c) 4.14, d) 10.03, Bonus: 141.99

2. Subtract by lining up the place values on grid paper.

   a) 0.239 – 0.125  

   b) 0.641 – 0.302  

   c) 0.864 – 0.45  

   d) 0.112 – 0.089

   **Answers:** a) 0.114, b) 0.339, c) 0.414, d) 0.023

3. Continue the pattern:

   a) 6.4, 5.9, 5.4, ____ , ____  

   b) 10.30, 10.20, 10.10, ____ , ____  

   c) 4.09, 4.06, 4.03, ____ , ____

   **Answers:** a) 4.9, 4.4; b) 10.00, 9.90; c) 4.00, 3.97
4. Solve the following: $6.34 + 5.77 - 10.12$. Hint: Use grid paper and do the addition first.

**Solution**

```
  6.34
+ 5.77
  12.11
```

- $6.34$
- $5.77$
- $12.11$

5. Continue the pattern:

a) $0.37, 0.32, 0.27, \ldots$

b) $0.49, 0.45, 0.41, \ldots$

c) $0.69, 0.66, 0.63, \ldots$

**Answers:**

a) $0.22, 0.17$

b) $0.37, 0.33$

c) $0.60, 0.57$

6. Kim’s house and Raj’s house are 9.8 km apart. Kim starts walking toward Raj’s house. Kim walks 4.9 km in the first hour and 4.4 km in the second hour. What distance does she still have to walk to get to Raj’s house?

**Solution:**

$4.9 + 4.4 = 9.3$. $9.8 - 9.3 = 0.5$. Kim still needs to walk 0.5 km.
Representing Hundredths with Pictures

1. Write the hundredths as a decimal. Then shade squares to show the number.
   
a) 18 hundredths =

```
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
```

b) 30 hundredths =

```
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
```

c) 6 hundredths =

```
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
```

d) \( \frac{9}{100} \) =

```
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
```

e) \( \frac{74}{100} \) =

```
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
```
f) \( \frac{70}{100} \) =

```
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
|     |     |     |     |
```
Number Lines Divided into Tenths and Hundredths

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

M-44

Blackline Master — Number Sense — Teacher Resource for Grade 5
Squares Divided into Hundredths

a)  

b)  

c)  

d)  

e)  

f)  

Blackline Master — Number Sense — Teacher Resource for Grade 5  
M-45
One Thousandth
Hundredths Block
Relationships Between Fractions and Their Equivalent Decimals

1. Use shading to determine the equivalent hundredths, and tenths when possible.

a) 

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c) 

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Unit 11  Number Sense: Using Decimals

Introduction
This unit applies concepts of decimals to real-world contexts that involve:

- money; and
- distance/length.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit:

- focus on decimal addition and subtraction.

Generic BLMs
The Generic BLM used in this unit is:

BLM 1 cm Grid Paper (p. S-1)
This BLM can be found in Section S.

Assessment
The lessons covered by a quiz or test are as follows:

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<td>NS5-56 to 58</td>
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<td>NS5-56 to 59, 62</td>
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Goals

Students will express monetary values in dollar and cent notation.

Students will convert from cent to dollar notation and from dollar to cent notation.

Students will compare monetary values expressed in dollar notation, in cent notation, and in words.

PRIOR KNOWLEDGE REQUIRED

Can use the cent symbol correctly

Can use dollar notation correctly for amounts given in whole dollars

Is familiar with Canadian coins

Knows the relative value of Canadian coins

Can compare multi-digit numbers using place value

Can read number words

Mental math minute. Have students stand in a line. Give the first student a problem that does not need regrouping, such as 3.7 − 0.3. Students in line repeatedly subtract a number, in this case 0.3, with each student saying one subtraction aloud. When a student says a subtraction that involves regrouping, emphasize that this answer was a bonus. Example: Student 1 says, “3.7 − 0.3 = 3.4.” Student 2 says, “3.4 − 0.3 = 3.1.” Bonus: Student 3 says, “3.1 − 0.3 = 2.8.” Continue until Student 12 says, “0.4 − 0.3 = 0.1.” Then start a new chain.

Writing dimes and cents in cent notation. Tell students to pretend that there is a vending machine that takes only dimes and cents. You may wish to remind students that pennies were each worth one cent, but they are no longer in circulation. ASK: What would you use to buy a toy that costs 46¢? (4 dimes and 6 cents) SAY: The tens digit tells you how many dimes you need, and the ones digit tells you how many cents you need.

Exercises: How much money is there?

a) 7 dimes and 6 cents      b) 8 dimes and 9 cents
   c) 5 dimes and 5 cents     d) 4 dimes and 2 cents

Answers: a) 76¢, b) 89¢, c) 55¢, d) 42¢

Writing dimes and cents in dollar notation. Ask a volunteer to write 26¢ in dollar notation. If no one knows, write the answer ($0.26) on the board. SAY: In dollar notation, the number of dimes or tenths of a dollar goes right after the decimal point and the number of cents or hundredths of a dollar goes right after that.

Exercises: Write the amount in dollar notation.

a) 3 dimes and 8 cents      b) 4 dimes and 2 cents
   c) 57¢                   d) 29¢      e) 35¢                   f) 81¢

Answers: a) $0.38, b) $0.42, c) $0.57, d) $0.29, e) $0.35, f) $0.81

VOCABULARY

bill
cent (¢)
cent notation
coin
decimal point
digit
dime
dollar ($)\ndollar notation
fewer
fewest
greater
less
loonie ($1)
nickel
ones
penny
quarter
tens
toonyie ($2)
whole number

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

NS5-56 Dollar and Cent Notation
Pages 64–66

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

bill
cent (¢)
cent notation
coin
decimal point
digit
dime
dollar ($)\ndollar notation
fewer
fewest
greater
less
loonie ($1)
nickel
ones
penny
quarter
tens
toonyie ($2)
whole number

NS5-56 Dollar and Cent Notation
Pages 64–66

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

bill
cent (¢)
cent notation
coin
decimal point
digit
dime
dollar ($)\ndollar notation
fewer
fewest
greater
less
loonie ($1)
nickel
ones
penny
quarter
tens
toonyie ($2)
whole number

NS5-56 Dollar and Cent Notation
Pages 64–66

CURRICULUM REQUIREMENT

AB: required
BC: required
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ON: required

VOCABULARY

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digit
dime
dollar ($)\ndollar notation
fewer
fewest
greater
less
loonie ($1)
nickel
ones
penny
quarter
tens
toonyie ($2)
whole number
Write “8¢” on the board. ASK: How many dimes are in 8 cents? (0) Show this on the board by writing 8¢ in dollar notation (see margin). SAY: The 0 in the tenths place tells us there are no dimes, and the 8 tells us there are 8 cents.

**Exercises:** Write the amount in dollar notation.

a) 0 dimes and 6 cents  
   b) 3¢  
   c) 7¢  
   d) 5 dimes and 0 cents  
   e) 40¢  
   f) 92¢  

**Answers:** a) $0.06, b) $0.03, c) $0.07, d) $0.50, e) $0.40, f) $0.92

**Writing multiples of a hundred cents in dollar notation.** Remind students that one hundred cents can be written as one dollar. Write on the board:

\[
100¢ = 1 \\
200¢ = __________ \\
1300¢ = ______
\]

Have volunteers fill in the blanks. ($2, $13) SAY: Two hundred cents equals two dollars and thirteen hundred cents equals thirteen dollars.

**Exercises:** Write the amount in dollar notation.

a) 300¢  
   b) 800¢  
   c) 1000¢  
   d) 1200¢  
   e) 48 400¢  

**Answers:** a) $3, b) $8, c) $10, d) $12, e) $484

**Converting cent notation to dollar notation.** Write on the board:

\[
348¢ = 300¢ + 48¢ \\
1746¢ = 1700¢ + 46¢
\]

Ask volunteers to write 300¢, 48¢, 1700¢, and 46¢ in dollar notation:

\[
348¢ = $3 + $0.48 \\
1746¢ = $17 + $0.46
\]

Show students how to combine them into a single dollar notation:

\[
348¢ = $3.48 \\
1746¢ = $17.46
\]

Point out that the decimal point is always in front of the last two digits.

**Exercises:** Write the amount in dollar notation.

a) 156¢  
   b) 704¢  
   c) 1804¢  
   **Bonus:** 29 604¢  

**Answers:** a) $1.56, b) $7.04, c) $18.04, Bonus: $296.04

**Changing dollar notation to cent notation.** Write on the board:

\[
$13.85 = 13 \text{ dollars} \ 85 \text{ cents}
\]

ASK: How many cents are in 13 dollars? (1300) Write on the board:

\[
$13.85 = 1300¢ + 85¢ \\
\quad = 1385¢
\]

**Exercises:** Write the dollar amount in cent notation.

a) $8.00  
   b) $17.00  
   c) $0.28  
   d) $0.04  
   e) $3.54  
   f) $9.03  
   g) $30.42  
   **Bonus:** $999.99  

**Answers:** a) 800¢, b) 1700¢, c) 28¢, d) 4¢, e) 354¢, f) 903¢, g) 3042¢, Bonus: 99999¢
Comparing money amounts in cents. SAY: All numbers more than 100 are greater than all numbers less than 100, so numbers with 3 digits are greater than numbers with 2 digits. Write on the board:

$$2564\,\text{¢} \quad 917\,\text{¢}$$

ASK: Which is more money? (2564¢) How do you know? (it has 4 digits, so it is more than 1000; 917 has 3 digits, so it is less than 1000)

Exercises: Which is more money?

a) 80¢ or 205¢  
b) 3579¢ or 543¢  
c) 344¢ or 7102¢

Answers: a) 205¢, b) 3579¢, c) 7102¢

SAY: When two numbers with the same number of decimal places have different numbers of digits, the one with more digits is greater. When they have the same number of digits, look for the highest place value where they are different. Demonstrate this on the board:

$$\begin{array}{l}
3\,\underline{4} \,1 \,7 \,\text{¢} \\
3\,\underline{5} \,1 \,8 \,3 \,\text{¢}
\end{array}$$

SAY: So 35 183¢ is more money.

Exercises: Which is more money?

a) 274 108¢  
b) 31 763¢  
c) 2 400 381¢

Answers: a) 274 108¢, b) 31 763¢, c) 2 400 381¢

Comparing money amounts given in different units ($ and ¢). Write “234¢ and $2.15” on the board. Tell students that you want to know which amount represents more money. ASK: What is different about this problem? (one amount is in cents and the other is in dollars) Ask a volunteer to change the amount in dollar notation to cent notation (215¢), then have another volunteer say which is greater. (234¢) ASK: How did changing to cents make it easier to compare them? (if they’re both in cents, just compare the whole numbers)

Exercises: Which is greater?

a) 1234¢ or $7.56  
b) $9875 or 98 756¢  
c) 9¢ or $0.08  
d) 40¢ or $0.04

Answers: a) 1234¢, b) $9875, c) 9¢, d) 40¢

Writing money amounts in different ways. Tell students that once they know how to write number words, they can write money amounts in words too. Write “$17.04” on the board. ASK: How many dollars are there? (17) How many cents are there? (4) SAY: To write the amount in words, you just need to write how many dollars and how many cents. Write on the board:

seventeen dollars and four cents
Point out that they write the words “dollars” and “cents” just like they say them, and the word “and” goes in between.

Exercises

1. Write the money amount in words.
   
   a) $3.50  
   b) $5.03  
   c) $30.18  
   d) $80.04  

   **Bonus:** $10 000 000.35

   **Answers:** a) three dollars and fifty cents, b) five dollars and three cents, c) thirty dollars and eighteen cents, d) eighty dollars and four cents, Bonus: ten million dollars and thirty-five cents

2. Write the money amount in dollar notation.
   
   a) four dollars and forty cents  
   b) eighteen dollars and five cents  
   c) seventy-three dollars and eighty-four cents  
   d) fifty dollars and sixty cents  

   **Bonus:** seven hundred thousand dollars and eight cents

   **Answers:** a) $4.40, b) $18.05, c) $73.84, d) $50.60, Bonus: $700 000.08

3. Write the amounts in cent notation. Then circle the greater amount.
   
   a) ten dollars and thirty-five cents, 1125¢  
   b) three dollars and fifty cents, 305¢  
   c) $15, 746¢  
   d) sixty-three cents, $6.30

   **Answers:** a) 1035¢ and 1125¢, circle 1125¢; b) 350¢ and 305¢, circle 350¢; c) 1500¢ and 746¢, circle 1500¢; d) 63¢ and 630¢, circle 630¢

Present a problem: Jane emptied her piggy bank. She asked her older brother Josh to help her count the money. He suggested she sort the coins and bills by value—a stack of pennies, a stack of nickels, and so on—and count each stack separately. Write on the board:

19 pennies  8 nickels  21 dimes  
5 quarters  6 loonies  3 five-dollar bills

**ASK:** How much money is in each stack? Have students write the amounts less than a dollar in cent notation and the amounts more than a dollar in dollar notation. (19¢ in pennies, 40¢ in nickels, $2.10 in dimes, $1.25 in quarters, $6.00 in loonies, $15.00 in five-dollar bills)

**Making a given amount of money using the fewest possible coins and bills.** **ASK:** What coins could you use to make 80¢? (8 dimes, 3 quarters and 1 nickel, 3 quarters and 5 cents, 2 quarters and 3 dimes, and so on) Allow different volunteers to give answers, including repeated answers.
Now tell students that you want to make 80¢ using the fewest possible coins. ASK: How can I do that? (3 quarters and 1 nickel) Point out that you need to use as many quarters as you can because quarters have the greatest value of any coin that is less than 80¢, so you’ll need fewer of them to make 80¢.

**Exercises:** Make the amount using the fewest coins possible.

a) 71¢  
   b) 85¢  
   c) 54¢  
   d) 26¢

**Answers:**
a) 2 quarters, 2 dimes, 1 cent  
b) 3 quarters, 1 dime  
c) 2 quarters, 4 cents  
d) 1 quarter, 1 cent

ASK: What dollar amounts do loonies and toonies come in? ($1 and $2) What dollar amounts do bills come in? ($5, $10, $20, and so on)

**Exercises:** Make the amount using the fewest coins and bills possible.

a) $6  
b) $16  
c) $3.25  
d) $8.30  
e) $10.05

**Answers:**
a) $5, $1  
b) $10, $5, $1  
c) $1, $1, $1, 25¢  
d) $5, $2, $1, 25¢, 5¢  
e) $10, 5¢

**Making a given amount of money using a given number of coins.** Tell students that you want to make 80¢ using exactly 5 coins. ASK: How can I do that? (2 quarters and 3 dimes) Write “Make $6.30 using exactly 5 coins.” on the board. Tell students that a good strategy is to first find any way of making $6.30, then decide whether they have too many or too few coins. ASK: What one way do we already know? (find the one with the fewest coins) Have students solve the problem on the board. ($2, $2, $2, 25¢, 5¢) Then challenge students to increase the number of coins by 1 but keep the value the same. (replace a toonie with two loonies, replace the quarter and nickel with three dimes)

**Exercises**
a) Make $2.50 using 6 coins.  
b) Make $1.75 using 7 coins.

**Sample answers:**
a) $1, $1, 25¢, 10¢, 10¢, 5¢  
b) $1, 25¢, 10¢, 10¢, 10¢, 10¢, 10¢

**Extensions**

1. What coin is being used for skip counting?
   a) $1.00, ___, ___, ___, ___, $1.20  
b) $2.00, ___, ___, ___, ___, $6.00

   **Answers:** a) 5¢, b) $1

2. a) Convert $8 to cent notation.  
b) Convert 8 m to centimetres.  
c) How are parts a) and b) the same?

   **Answers:** a) 800¢, b) 800 cm, c) you do the questions the same way, by multiplying both numbers by 100
3. a) I am an amount less than 40¢. You need 4 coins, nickels and dimes only, to make me. What amount am I?

b) I am an amount less than 50¢. I am a multiple of 5 but not a multiple of 10. You can make me with 2 coins. What amount could I be?

**Answers:** a) 25¢ or 30¢ or 35¢, b) 15¢ or 35¢

4. Without changing to cent notation, compare the money amounts. Did you need to compare the cent amounts or just the whole-dollar amounts?

a) $3.84, $2.95  
b) $14.71, $14.36

**Answers**
a) $3.84 is more than $2.95. Three dollars and anything is more than two dollars and anything, so you do not need to compare the cent amounts.

b) The dollar amounts are the same, so you need to compare the cent amounts. Since 71¢ is more than 36¢, $14.71 is more than $14.36.
Goals
Students will add or subtract money amounts and solve word problems involving money.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract decimals
Can add and subtract with regrouping
Is familiar with money and Canadian currency

MATERIALS
play money
grid paper or BLM 1 cm Grid Paper (p. S-1)

Mental math minute. Give the first student an addition problem, such as $0.12 + 0.5$. Have students add 0.5 repeatedly: the next student adds $0.17 + 0.5$, the next student adds $1.2 + 0.5$, and so on.

Go over the steps for adding money. Tell students that you will now add money. SAY: Compared with other addition, the big difference is in lining up the numbers using the decimal point. ASK: Will the ones still be lined up over the ones? (yes) The tens over the tens? (yes) The dimes over the dimes? (yes) SAY: If the decimal point is lined up, all the other digits must be lined up correctly too because the decimal point is between the ones and the dimes. Students can model regrouping of tenths or hundredths using play money: for instance, in $2.33 + 2.74$, they will have to group 10 dimes as a dollar (see completed addition in the margin).

Have students complete the following exercises on grid paper or BLM 1 cm Grid Paper.

Exercises: Add.

\[
\begin{array}{c}
\$2.33 + \$2.74 = \$5.07 \\
\$5.08 + \$1.51 = \$6.59 \\
\$3.13 + \$2.98 = \$6.11 \\
\$1.07 + \$1.52 = \$2.59 \\
\$4.22 + \$2.88 = \$7.10 \\
\end{array}
\]

Answers: a) $6.59, b) $6.11, c) $25.96, d) $66.23

Adding money in word problems. Use volunteers to solve several word problems, such as: Yu spent $14.98 for a T-shirt and $5.78 for a sandwich. How much did she spend in total? ($20.76)

Students should also practise adding coins and writing the amount in dollar notation. Examples:

- Hanna has a five-dollar bill, 4 toonies, 4 loonies, and 9 quarters in her pocket. How much money does she have? ($19.25)
• Ray paid 2 twenty-dollar bills, 5 toonies, 8 loonies, 5 quarters, and 7 dimes for a parrot. How much did his parrot cost? ($59.95)

• A mango fruit costs $2.20. I have a ten-dollar bill. How many mangoes can I buy? If I add a loonie, will it be enough for another one? (4, yes)

**Exercises:** Add.

a) $18.25  
   + $71.46  
   = $89.71

b) $23.89  
   + $67.23  
   = $91.12

c) $45.08  
   + $8.87  
   = $53.95

d) $78.37  
   + $4.79  
   = $83.16

**Answers:** a) $89.71, b) $91.12, c) $53.95, d) $83.16

**Word problems practice.**

**Exercises:** Sharon saved 5 loonies, 5 dimes, and 3 nickels from babysitting. Her brother Ronin saved a five-dollar bill, a loonie, and 3 quarters from shovelling snow.

a) Who has saved more money?

b) They want to share their money to buy a present for their mother. How much money do they have altogether?

b) They’ve chosen a teapot for $12.50. Do they have enough money?

**Answers:** a) Ronin (Sharon saved $5.65 and Ronin saved $6.75), b) $12.40, c) no

**Subtracting money.** Start with a review of two-digit and three-digit subtraction questions. Model a few examples on the board and involve volunteers if possible. As you demonstrate, show some examples on the board of numbers lined up correctly or incorrectly and have students decide which ones are aligned correctly.

Start with some examples that do not require regrouping. Demonstrate the steps: line up the numbers correctly and subtract the digits in each column in turn, starting from the right. Examples: 45 − 23, 78 − 67, 234 − 123, 678 − 354.

Move on to questions that require regrouping. Examples: 86 − 27, 567 − 38, 782 − 127, 673 − 185, 467 − 369.

**Exercises**

1. Subtract.

   a) $98.89  
      − $71.64  
      = $27.25

   b) $89.00  
      − $67.23  
      = $21.77

   c) $45.00  
      − $38.87  
      = $6.13

   d) $78.37  
      − $9.79  
      = $68.58

**Answers:** a) $27.25, b) $21.77, c) $6.13, d) $68.58

2. Amir went to a grocery store with $15.00. He would like to buy buns for $1.45, ice cream for $6.50, and tomatoes for $2.50. Does he have enough money? If yes, how much change will he get?

**Answers:** yes, $4.55
3. Use skip counting to find the answer to the question mentally.
   a) How much do 4 pencils cost at $1.10 each?
   b) Oranges cost 60¢ each. How many could you buy with $3.00?
   c) Stickers cost $1.20 each. How many could you buy if you had $10.00?

   **Sample solution:** c) $1.20, $2.40, $3.60, $4.80, $6.00, $7.20, $8.40, 
   $9.60, so you could buy 8 stickers

   **Answers:** a) $4.40, b) 5

   **NOTE:** Extensions 1 and 2 are required in order to cover the British Columbia curriculum.

**Extensions**

1. Teach students the following decomposition strategy to calculate how much is owed when an item is paid for with a whole-dollar amount, such as $1, $2, $3, and so on.

   SAY: It is easier to subtract from $0.99 than from $1.00, easier to subtract from $1.99 than from $2.00, and so on. The strategy is to subtract from, for example, $0.99 and then add $0.01 back to the answer. Write on the board:

   \[
   \begin{align*}
   1.00 & - 0.99 + 0.01 \\
   - 0.57 & - 0.57 \uparrow \\
   & 0.42 + 0.01 = 0.43 = $0.43
   \end{align*}
   \]

   Repeat with 2.00 – 1.46. The final answer is shown below:

   \[
   0.53 + 0.01 = 0.54 = $0.54
   \]

   Use the method you just learned to subtract on grid paper or BLM 1 cm Grid Paper. Make sure you start by lining up the decimal places.

   a) 1.00 – 0.29       b) 1.00 – 0.08       c) 2.00 – 0.63

   d) 2.00 – 1.17        **Bonus:** 3.00 – 1.68

   **Answers:** a) 0.70 + 0.01 = 0.71, b) 0.91 + 0.01 = 0.92, c) 1.36 + 0.01 = 1.37, d) 0.82 + 0.01 = 0.83, **Bonus:** 1.31 + 0.01 = 1.32

2. Teach students the following counting-up strategy to calculate how much is owed when an item is paid for with a whole-dollar amount, such as $1, $2, $3, and so on.

   Write on the board:

   Randi bought a book for $7.00. She paid with a ten-dollar bill. How much is she owed?

   \[
   \begin{array}{c}
   \$7.00 \\
   \rightarrow \\
   \$10.00
   \end{array}
   \]
ASK: How much do I have to add to $7.00 to get $10.00? ($3.00)
Write “$3.00” in the box. SAY: Randi is owed $3.00.

SAY: Billy bought a book too. It cost $6.40, and he also paid with a ten-dollar bill. Write on the board:

\[ \boxed{\$6.40} \rightarrow \boxed{\$10.00} \]

ASK: What is the next whole-dollar amount after $6.40? ($7.00)
Write “$7.00” in the blank and explain that it’s easy to count up from any amount to the next whole dollar, and that’s why we use $7.00.

ASK: How much do I have to add to $6.40 to get $7.00? (60¢)
Write “60¢” in the first box. SAY: We already know how to get from $7.00 to $10.00. Write “$3.00” in the second box. SAY: We counted up from $6.40 to $7.00 and then from $7.00 to $10.00. ASK: How much is that altogether? ($3.60)
SAY: Billy is owed $3.60. The final diagram should look like this:

\[ \boxed{\$6.40} \rightarrow 60¢ \rightarrow \boxed{\$7.00} \rightarrow \boxed{\$10.00} \]

Repeat with the following examples.

a) price: $3.15, bill used: $5.00
b) price: $4.65, bill used: $10.00
c) price: $2.10, bill used: $20.00
d) price: $1.25, bill used: $50.00

NOTE: Some students won’t need to use all the steps shown in the solution below. For example, in part a), some students might go from $3.15 to $4.00 directly, and in part c), some students might go from $3.00 to $20.00 directly.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
<th>Total Owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $3.20 (5¢)</td>
<td>$4.00 (80¢)</td>
<td>$5.00 ($1)</td>
<td>n/a</td>
<td>$1.85</td>
</tr>
<tr>
<td>b) $4.70 (5¢)</td>
<td>$5.00 (30¢)</td>
<td>$10.00 ($5)</td>
<td>n/a</td>
<td>$5.35</td>
</tr>
<tr>
<td>c) $3.00 (90¢)</td>
<td>$10.00 ($7)</td>
<td>$20.00 ($10)</td>
<td>n/a</td>
<td>$17.90</td>
</tr>
<tr>
<td>d) $1.30 (5¢)</td>
<td>$2.00 (70¢)</td>
<td>$10.00 ($8)</td>
<td>$50.00 ($40)</td>
<td>$48.75</td>
</tr>
</tbody>
</table>

Find the difference owed.

a) price: $5.65, bill used: $10.00
b) price: $11.35, bill used: $20.00

Bonus: price: $3.05, bill used: $100.00

Answers: a) $4.35, b) $8.65, Bonus: $96.95
3. Eric is paid $5 for every lawn he mows. He wants to buy a bicycle for $150.00. This week he mowed 2 lawns every day of the week, including the weekend. How much did he earn this week? How much more money does he need to earn to have enough money to buy the bicycle?

**Answers:** $70.00, $80.00

4. A skateboard costs $69.75, plus tax of $9.07. How much does the skateboard cost with tax?

**Answer:** $78.82

5. Tina needs $164.25 to buy a snowboard. She has one $50 bill, five $20 bills, one $10 bill, three $5 bills, two toonies, one quarter, and five nickels.

a) Does she have enough money to buy the snowboard?

b) Determine two different combinations of bills and coins she could use that add up to exactly $164.25.

**Bonus:** If she pays with all of her fifty-, twenty-, and ten-dollar bills and with one five-dollar bill, how much change will she get?

**Sample answer:** b) one $50 bill, five $20 bills, one $10 bill, 2 toonies, and 1 quarter; one $50 bill, five $20 bills, two $5 bills, two toonies, and five nickels

**Answers:** a) yes, Bonus: $0.75
NS5-58  Rounding Decimals
Pages 69–70

Goals

Students will round decimals to the nearest one (nearest whole number) or tenth

PRIOR KNOWLEDGE REQUIRED

Can round whole numbers to any place value, including regrouping
Can add and subtract with regrouping

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1)

Introduce rounding decimals. Draw the number line below on the board with 1.0, 2.0, and 3.0 a different colour than the other numbers:

1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0

Circle the numbers 1.4, 1.8, 2.6, and 2.7, one at a time, and ask volunteers to draw an arrow showing which whole number is closest to the number you just circled. Tell students that sometimes people think of a number in terms of the closest whole number because they don’t need to be precise at that time and whole numbers are easier to work with. This is called **rounding**.

Exercises: Round to the nearest whole number.

a) 1.4  b) 2.8  c) 5.7  Bonus: 2110.3

Answers: a) 1, b) 3, c) 6, Bonus: 2110

Write on the board:

3.7

SAY: 3.7 is between 3.0 and 4.0. ASK: Is 3.7 closer to 3.0 or to 4.0? (4.0)

Repeat with 9.4. (9.4 is between 9.0 and 10.0; it is closer to 9.0)

Exercises: Round to the nearest whole number.

a) 7.9  b) 5.3  c) 8.1

d) 0.2  e) 0.9  Bonus: 6459.6

Answers: a) 8, b) 5, c) 8, d) 0, e) 1, Bonus: 6460

Rounding decimals to the nearest whole number. Make a table with two headings: “Closer to 3.00” and “Closer to 4.00.” Name several decimals (for example, 3.42, 3.56, 3.12, 3.85, 3.52, 3.31, 3.27, 3.90, 3.09, 3.51) and ask students to signal whether the decimals are closer to 3 (thumbs down) or to 4 (thumbs up). Place the decimals in their correct column as students answer.
ASK: What digit are you looking at to decide? (the tenths digit) SAY: When the tenths digit is 0, 1, 2, 3, or 4, round down. When the tenths digit is 5, 6, 7, 8, or 9, round up.

**Exercises:** What is the nearest whole number?

<table>
<thead>
<tr>
<th>Value</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.57</td>
<td>5</td>
</tr>
<tr>
<td>6.12</td>
<td>6</td>
</tr>
<tr>
<td>9.08</td>
<td>9</td>
</tr>
<tr>
<td>37.92</td>
<td>38</td>
</tr>
<tr>
<td>Bonus: 537.29</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 5, b) 6, c) 9, d) 38, Bonus: 537

Tell students that numbers less than 3.50 are rounded down to 3 and numbers more than 3.50 are rounded up to 4. ASK: Is 3.50 closer to 3 or to 4? (neither, it is the same distance from both) SAY: I want to pick 3 or 4 anyway, and I only want to have to look at the tenths digit to decide. ASK: Where are all the other decimals with the tenths digit 5? (in the closer to 4 column) SAY: When a decimal is equally close to both whole numbers, round up.

**Exercises:** Round to the nearest whole number.

<table>
<thead>
<tr>
<th>Value</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td>3</td>
</tr>
<tr>
<td>0.50</td>
<td>1</td>
</tr>
<tr>
<td>18.50</td>
<td>19</td>
</tr>
<tr>
<td>29.50</td>
<td>30</td>
</tr>
<tr>
<td>Bonus: 438.50</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 3, b) 1, c) 19, d) 30, Bonus: 439

SAY: Do the same thing when rounding decimals with one digit after the decimal point.

**Exercises:** Round to the nearest whole number by looking at the tenths digit.

<table>
<thead>
<tr>
<th>Value</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>14.5</td>
<td>15</td>
</tr>
<tr>
<td>39.5</td>
<td>40</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>55.5</td>
<td>56</td>
</tr>
<tr>
<td>87.5</td>
<td>88</td>
</tr>
<tr>
<td>Bonus: 10 000.5</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 3, b) 15, c) 40, d) 1, e) 56, f) 88, Bonus: 10 001

**Rounding decimals.** Tell students that you use the same rule to round to decimals as you do to round to whole numbers. Example: Round 2.36 to the nearest tenth.

**Step 1:** Underline the digit you are rounding to.

```
2 3 6
```

**Step 2:** Put your pencil on the digit to the right of the one you are rounding to.

```
2 3 6
```

**Step 3:** Beside the grid write “round up” if the digit under your pencil is 5, 6, 7, 8, or 9 and “round down” if the digit is 0, 1, 2, 3, or 4.

```
2 3 6 round up
```
Step 4: Round the underlined digit up or down according to the instruction you have written. Write your answer in the grid.

\[
\begin{array}{ccc}
2 & 3 & 6 \\
4 & & \\
\end{array}
\]

Step 5: Copy all digits to the left of the rounded digit as they were.

\[
\begin{array}{ccc}
2 & 3 & 6 \\
2 & 4 & \\
\end{array}
\]

SAY: So 2.36 rounded to the nearest tenth is 2.4. That makes sense because the number is between 2.3 and 2.4, but it is closer to 2.4 than to 2.3.

Exercises: Round to the underlined place value.

a) 13.45  
b) 38.47  
c) 612.38  
d) 804.74

Answers: a) 13.5, b) 38, c) 612.4, d) 804.7

Rounding with regrouping. Write “2.96” on the board. Demonstrate rounding 2.96 to the nearest tenth:

\[
\begin{array}{ccc}
2 & 9 & 6 \\
& & \\
\end{array}
\]

SAY: 2.96 rounded to the nearest tenth is 3.0.

Exercises: Round to the stated place value. Use grid paper.

a) 43.69, ones  
b) 74.95, tenths  
c) 59.51, ones  
d) 84.09, tens

Answers: a) 44, b) 75.0, c) 60, d) 80

NOTE: Extension 1 is required to cover the Alberta, British Columbia, and Manitoba curricula.

Extensions

1. Round 628.327 to the nearest whole number. Then round it to the nearest tenth. Finally, round it to the nearest hundredth.

   Answers: 628, 628.3, 628.33

2. Decide what place value it makes sense to round each of the following to. Round to the place value you selected. Justify your decisions.
Remember: You made similar decisions in Unit 4, when you interpreted remainders.

Height of person: 1.524 m
Height of tree: 13.1064 m
Length of bug: 1.267 cm
Distance between Quebec City and Hong Kong: 12 301 km
Distance today between Earth and the moon: 384 403 km
Population of Kolkata, India, in 2018: 5 202 405
Floor area of an apartment: 27.91 square metres
Area of Newfoundland and Labrador: 405 212 km²
Angle between two streets: 82.469°
Time it takes to blink: 0.33 seconds
Speed of a car: 96.560639 km/h

**Answers:** Answers will vary. The larger the number, the less important the smaller place values become.

3. Kate uses a different way to round numbers to the nearest tenth. She adds 0.05 to the number and then removes all digits smaller than tenths. For example:
   - To round 34.58, add 0.05 to get 34.63, then remove digits to get 34.6.
   - To round 49.74, add 0.05 to get 49.79, then remove digits to get 49.7.

   a) Does Kate’s method always work to round numbers to the nearest tenth? Explain why or why not.

   b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

**Selected sample answer:** a) Kate’s method works. If a number’s hundredths digit is less than 5, adding 0.05 will not result in regrouping, so the tenths digit will stay the same. If a number’s hundredths digit is 5 or more, adding 0.05 will result in regrouping, and the tenths digit will go up.
NS5-59 Estimating Sums and Differences for Decimals

Pages 71–72

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
approximately
approximately equal to (=)
estimating
exactly
reasonable
regrouping
rounding

Goals
Students will round to the nearest whole number and to the nearest tenth to estimate sums and differences.

PRIOR KNOWLEDGE REQUIRED
Can round numbers with up to two decimal places to any place value
Can add and subtract decimals with regrouping

Estimating sums and differences by rounding to the nearest whole number to check for reasonableness. Tell students that they can check whether the answers to sums and differences are reasonable by rounding to the nearest whole number. Write on the board:

\[ 162.34 + 16.23 \approx 162 + 16 = 178 \]

Exercises: Somebody punched these numbers into a calculator and got these answers. Are they reasonable?

a) \[ 21.52 + 8.18 = 32.70 \]
b) \[ 87.52 - 53.31 = 5.21 \]

Answers: a) yes, b) no

Estimating sums and differences in word problems. Teach students that, for estimating sums and differences of decimal numbers, they can round to various place values—the nearest ten, one, tenth, hundredth, and so on. Have students reflect on whether their estimate will be higher or lower than the actual answer. For example, consider the problem: Ivan had $17.45. He paid $10.95 for a new dictionary. Approximately how much does he have left? How much does he have left exactly? ($17.45 - $10.95 is about $17 - $11 = $6, but he has exactly $6.50 left)

Exercises: Estimate the answer.

a) Avril wants to buy three items that cost $4.79, $2.25, and $7.50. If she has $15 with her, does she have enough money to buy all three items?

b) The distance from Vancouver to Victoria, BC is 93.1 km by plane and 109.9 km by car. How many kilometres farther is the car ride than the flight? If 78.2 km of the route by car is on the ferry, what is the remaining driving distance by land?

c) A weather station reports an average high temperature in Resolute, NU, of 7.3°C. It is 12.2°C warmer in Inuvik, NT. What is the average high temperature in Inuvik, NT?

d) The population of Beaumont, AB, in 2018 was about 18.83 thousand people. In 2011, the population was about 13.28 thousand people. How many more people lived in Beaumont in 2018 than in 2011?
**Answers:** a) yes; b) the car ride is about 20 km longer than the plane route, and the driving distance by land is about 30 km; c) around 19°C; d) about 6 thousand more people

**Extensions**

1. To insert the decimal point, estimate the answer (rather than carrying out the operation).
   
   a) \(16.32 + 743.5 = 759.82\)  
   
   b) \(49.17 - 3.5 = 45.67\)

   **Answers:** a) 759.82, b) 45.67

2. Without calculating the sum, how can you tell whether the sum is greater than or less than 275?

   \[11.9 + 0.46 + 258.63\]

   **Answer:** The sum is less than \(12 + 0 + 259 = 271\), so it is less than 275

3. Estimate the value of \(14.50 - 13.92\) by rounding both numbers to the nearest …

   a) ten.   
   
   b) one.   
   
   c) tenth.

   **Answers:** a) \(10 - 10 = 0\), b) \(15 - 14 = 1\), c) \(14.5 - 13.9 = 0.6\); rounding to the nearest ten or one makes estimating the fastest, but rounding to the nearest tenth makes it most accurate

4. Luc says that 0.40 and 0.04 are equal because both of these decimal numbers have one 4 and two zeros. Do you agree with Luc’s reasoning? Explain why or why not.

   **Sample answer:** I disagree with Luc’s reasoning because the different position of the 4s and the zeros in these numbers makes them different. The numbers can also be written as \(40/100\) and \(4/100\), and these are clearly not equal.

5. Ava says that \(0.8 = 0.80\). Do you agree with Ava? Explain why or why not.

   **Sample answer:** I agree with Ava. \(0.8 = 8/10 = 80/100\) and \(80/100\) can also be written as 0.80.
NS5-60 Multiplying Decimals by Powers of 10
Pages 73–75

Goals
Students will multiply decimals by 10, 100, 1000, and 10 000

PRIOR KNOWLEDGE REQUIRED
Knows the factors in a multiplication statement are interchangeable
Can multiply whole numbers by 10, 100, 1000, and 10 000
Understands decimal place value
Can regroup

MATERIALS
small cards with a large dot
tape

Mental math minute. SAY: Remember, an equal sign means “is the same as.” To check if an equation is true, use the addition and subtraction strategies you know, without actually calculating both sides. For example, you know that moving 1 from one addend to the other addend on the same side does not change the answer. And you know that adding 1 to both numbers in a subtraction does not change the answer. Present the equations in the exercises below one at a time and have students signal the answer using thumbs up for “yes” and thumbs down for “no.”

Exercises: Is the equation true?

a) \(21 + 34 = 20 + 35\)
b) \(21 - 34 = 20 - 35\)
c) \(27 - 16 = 28 - 17\)

Answers: a) yes, b) no, c) yes

Review multiplying whole numbers by 10. Ask students to describe how they can multiply a whole number by 10. Students might say “add a zero.” In this case, ask them to be more specific. Point out that it is not “adding”; it is writing a zero to the right of a number. Write an incorrect statement, such as \(26 \times 10 = 206\), and ASK: Is this correct? (no) Make sure students clearly state that the zero has to be written at the end, as in \(26 \times 10 = 260\), so that the ones digit becomes the tens digit and the zero becomes the ones digit.

Discuss how this pattern makes sense because each place value gets replaced by the place value that is 10 times as great. Write on the board:

\[26 = 2 \text{ tens} + 6 \text{ ones}\]
\[26 \times 10 = 2 \text{ hundreds} + 6 \text{ tens} = 260\]
Using place value to multiply decimals by 10. Write on the board:

\[ 0.4 \times 10 = 4 \text{ tenths} \times 10 \]

ASK: How many tenths does that make? (40) Which place value is 10 times the tenths? (ones) Write on the board:

\[ = 40 \text{ tenths} = 4 \text{ ones} = 4 \]

Repeat with \(0.06 \times 10 = 6 \text{ hundredths} \times 10 = 60 \text{ hundredths} = 6 \text{ tenths} = 0.6\).

Draw the picture below to remind students of the connection between place values:

\[ \times 10 \quad \times 10 \quad \times 10 \quad \times 10 \]

tens \quad ones \quad tenths \quad hundredths \quad thousandths

Exercises

1. Multiply the place value by 10.
   a) hundredths \(\times 10\)  
   b) ones \(\times 10\)  
   c) tenths \(\times 10\)  
   d) thousandths \(\times 10\)

   **Bonus:** tens \(\times 10\)

   **Answers:** a) tenths, b) tens, c) ones, d) hundredths, **Bonus:** hundreds

2. Use place value to multiply the number by 10.
   a) 5 ones \(\times 10\)  
   b) 2 tenths \(\times 10\)  
   c) 3 hundredths \(\times 10\)  
   d) 7 tens \(\times 10\)

   **Answers:** a) 50 ones = 5 tens or 50, b) 20 tenths = 2 ones or 2, 
   c) 30 hundredths = 3 tenths or 0.3, d) 70 tens = 7 hundreds or 700

Write on the board:

\[ 0.05 \times 10 = 0.5 \]

SAY: If 5 is in the hundredths position, then after multiplying by 10, it will be in the tenths position.

**Exercises:** Multiply using place value.

a) 0.5 \(\times 10\)  
   b) 0.02 \(\times 10\)  
   c) 0.09 \(\times 10\)

   **Answers:** a) 5, b) 0.2, c) 0.9

Moving the decimal point to multiply decimals by 10. Ahead of time, draw a large decimal point on several small cards. Write the numbers below on the board and tape the cards to the board so that they act as a decimal point:

\[ 5.4 \quad 6.03 \quad 300.4 \]
Ask volunteers to move the decimal point to show multiplying by 10:

5.4  603  3004

Ask the rest of the class to look for a pattern in how the decimal point is being moved. (multiplying a number by 10 always results in moving the decimal point one place to the right)

**Exercises:** Move the decimal point one place to the right to multiply by 10.

a) 3.2 \times 10  

b) 0.58 \times 10  

c) 10 \times 0.20

d) 10 \times 7.46

**Bonus:** 98 763.60789 \times 10

**Answers:** a) 32, b) 5.8, c) 2.0, d) 74.6, Bonus: 987 636.0789

**Moving the decimal point to multiply decimals by 100 and 1000.**

SAY: One hundred is 10 times 10, so to multiply a number by 100, you can multiply by 10 and then multiply the result by 10 again. Write on the board:

3.462 \times 100 = 3.462 \times 10 \times 10

SAY: Move the decimal point once to multiply by 10 and then once more to multiply by 10 again. Continue writing on the board:

3 . 4 6 2  So 3.462 \times 100 = 346.2

SAY: To multiply by 100, move the decimal point two places to the right.

**Exercises:** Move the decimal point two places to the right to multiply by 100.

a) 3.62 \times 100  

b) 0.72 \times 100  

c) 1.673 \times 100  

d) 0.08 \times 100

**Answers:** a) 362, b) 72, c) 167.3, d) 8

SAY: We moved the decimal point once to multiply by 10 and twice to multiply by 100. ASK: How many times do we move the decimal point to multiply by 1000? (three times) Write on the board:

2 . 4 6 7  So 2.467 \times 1000 = 2467

**Using a zero as a placeholder when multiplying decimals.** Write 3.42 \times 1000 in a grid on the board, using the card with a large dot for the decimal point so that it can be moved, as shown below:

| 3 | 4 | 2 |

ASK: How many places do I have to move the decimal point when I multiply by 1000? (three) Move the decimal point three times, as shown below:

| 3 | 4 | 2 |

ASK: Are we finished writing the number? (no) Why not—what’s missing? (the zero) SAY: Each digit is worth 1000 times as much as it was before multiplying. The number was 3 ones, 4 tenths, and 2 hundredths. Now it is 3 thousands, 4 hundreds, and 2 tens. So the number is 3420.
If students struggle with the exercises below, encourage them to write each place value in its own cell on a grid. Suggest that students draw arrows to show how they moved the decimal point. An example is shown below for Exercise 1 part b):

```
  5 4 2 4
```

**Exercises**

1. Multiply.
   a) $0.4 \times 1000$
   b) $5.24 \times 1000$
   c) $23.6 \times 1000$
   d) $0.01 \times 1000$

   **Answers:** a) 400, b) 5240, c) 23600, d) 10

2. Multiply.
   a) $0.6 \times 100$
   b) $7.28 \times 10$
   c) $25.6 \times 1000$
   d) $1.8 \times 100$
   e) $21.9 \times 1000$
   f) $326.3 \times 1000$
   g) $0.002 \times 10$

   **Bonus:** $2.3 \times 10000$

   **Answers:** a) 60, b) 72.8, c) 25600, d) 180, e) 21900, f) 326300, g) 0.02, Bonus: 23000

**Connect multiplying whole numbers by 10 to multiplying decimals by 10.**

**ASK:** What is $34 \times 10$? (340)

**SAY:** We can also multiply $34 \times 10$ by moving the decimal point. Use the card with the large dot to write “34.0” on the board. Move the card one place right as shown below, and point out that this is the same answer you get the other way.

```
  3 4 0
  3 4 0 0
```

**SAY:** Multiplying whole numbers uses the same method we use to multiply decimals.

**Word problems practice.**

**Exercises**

a) Sara makes $12.50 an hour shovelling snow. How much does she make in 10 hours?

b) A clothing-store owner wants to buy 100 coats for $32.69 each. How much will the coats cost?

c) A dime is 0.12 cm thick. How tall would a stack of 100 dimes be?

d) A string of Tibetan Buddhist prayer flags has 100 flags on it. Each flag is 16.51 cm wide. How long is the entire string of flags?

**Answers:** a) $125$, b) $3269.00$, c) 12 cm, d) 1651 cm
Extensions

1. Fill in the blank.
   a) _____ \times 10 = 38.2  
   b) _____ \times 100 = 67.4  
   c) 42.3 \times _____ = 4230  
   d) 0.08 \times _____ = 0.8  
   **Answers:** a) 3.82, b) 0.674, c) 100, d) 10

2. Complete the pattern.
   a) 0.0003, 0.003, 0.03, _____, ____  
   b) 3.895, 38.95, 389.5, _____. ____  
   **Answers:** a) 0.3, 3; b) 3895, 38 950

3. Find the answer mentally by multiplying the numbers in the easiest order.
   a) (3.2 \times 5) \times 20  
   b) (6.73 \times 2) \times 50  
   **Answers:** a) 320, b) 673

4. One marble weighs 3.5 g. A marble bag weighs 10.6 g. How much does the bag weigh with 10 marbles in it?
   **Answer:** 45.6 g
Goals

Students will multiply decimals by 10, 100, 1000, and 10 000 and divide decimals by 10 and 100 by shifting the decimal point.

PRIOR KNOWLEDGE REQUIRED

Can multiply whole numbers by 10, 100, 1000 and 10 000
Can divide whole numbers by 10 and 100
Understands decimal place value
Knows that multiplication and division are opposite operations

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1) or base ten blocks
small cards with a large dot

Dividing by 10 using base ten materials. Draw on the board:

Tell students that you will represent one whole by a big square, so one tenth is a column or row, and one hundredth is a little square. Write the picture equations shown below on the board and have volunteers write the decimal equations underneath the pictures. For the first example, shade the first column of each hundredths block to show a tenth of each. For the second example, shade one tenth of each tenths block.

\[
\begin{align*}
1.0 & \div 10 = 0.1 \\
0.1 & \div 10 = 0.01 \\
2.0 & \div 10 = 0.2 \\
0.5 & \div 10 = 0.05 \\
3.1 & \div 10 = 0.31
\end{align*}
\]
Exercises: Draw pictures on grid paper, or use base ten blocks, to divide.

a) 3.0 ÷ 10  

b) 0.4 ÷ 10  

c) 3.4 ÷ 10  

d) 2.7 ÷ 10

Answers: a) 0.3, b) 0.04, c) 0.34, d) 0.27

Dividing by 10 by inverting the rule for multiplying by 10. Write the following on the board using a card with a large dot to show the decimal point:

3.42 5

Invite a volunteer to move the decimal point to multiply by 10. (342.5) Write on the board:

34.25 × 10 = 342.5, so 342.5 ÷ 10 =

ASK: What number goes in the blank? (34.25) How do you know? (division undoes multiplication) Have a volunteer move the card to get the answer for 342.5 ÷ 10. (move the decimal point one place to the left) SAY: Division is the opposite of multiplication. Division “undoes” the effects of multiplication. When you multiply by 10, you move the decimal point one place to the right. When you divide by 10, you move the decimal point one place to the left.

Exercises: Divide by 10.

a) 14.5 ÷ 10  

b) 64.8 ÷ 10

c) 9.22 ÷ 10  

d) 0.16 ÷ 10

Answers: a) 1.45, b) 6.48, c) 0.922, d) 0.016

Dividing by 100. Write on the board:

0.58 × 100 = 58, so 58 ÷ 100 =

Ask a volunteer to fill in the blank. (0.58) Point out that the equations are in the same fact family, so knowing how to multiply by 100 also tells us how to divide by 100. ASK: How do we move the decimal point to divide by 100? (move it two places left) Point out that you had to move it two places right to multiply by 100 and then two places to the left to divide by 100.

Exercises: Divide by 100.

a) 14.5 ÷ 100  

b) 464.8 ÷ 100

c) 9.22 ÷ 100  

d) 0.6 ÷ 100

Answers: a) 0.145, b) 4.648, c) 0.0922, d) 0.006

Dividing whole numbers by 10 and 100. Write “67” on the board, leaving room between the digits for the decimal point card. Tell students you want to know the answer to 67 ÷ 10. SAY: I would do the division by moving the decimal point, but I don’t see any decimal point here. ASK: What should I do? (write the decimal point to the right of the ones because 67 = 67.0) Add the decimal point card to the right of the digit 7, then invite a volunteer to move the decimal point one place to the left to get 67 ÷ 10 = 6.7. Repeat with 18 ÷ 100 and 1987 ÷ 100. (0.18, 19.87)
Exercises: Divide by 10 or 100.

a) \(236 \div 10\)  
   b) \(573 \div 100\)  
   c) \(1230 \div 100\)  
   d) \(14889 \div 10\)

Answers: a) 23.6, b) 5.73, c) 12.3, d) 1488.9

Using strategies for remembering which way to move the decimal point.

SAY: Remember, multiplying by 10, 100, 1000, or 10 000 makes the number bigger, so the decimal point moves right. Dividing by 10 or 100 makes the number smaller, so the decimal point moves left.

If students have trouble deciding which direction to move the decimal point when multiplying and dividing by powers of 10, one hint that some students might find helpful is to use the case of whole numbers as an example.

ASK: Which way is the decimal point moving when multiplying \(34 \times 10 = 340\)? (right)

Exercises: Multiply or divide.

a) \(78678 \div 100\)  
   b) \(2.42 \times 100\)  
   c) \(18.9 \div 10\)  
   d) \(1.31 \times 1000\)  
   e) \(6 \div 100\)  
   f) \(0.082 \times 1000\)  
   g) \(0.2 \div 100\)  
   h) \(15.1 \times 100\)  
   i) \(0.31 \times 1000\)  
   Bonus: \(31498.76 \div 100\)

Answers: a) 786.78, b) 242, c) 1.89, d) 1310, e) 0.06, f) 820, g) 0.002, h) 1510, i) 310, Bonus: 314.9876

Remind students who are struggling to write each place value in its own cell of grid paper when multiplying or dividing decimals by powers of 10.

Word problems practice.

Exercises

a) In 10 months, a charity raises $2650.00 through fundraising. How much does the charity raise each month on average?

b) A stack of 100 cardboard sheets is 13 cm high. How thick is a sheet of the cardboard?

c) One hundred people attended a “pay what you can” event. The total money paid was $600. Jax paid $0.60. Did he pay more or less than average?

d) A box of 1000 nails costs $12.95. How much did each nail cost, to the nearest cent? Hint: Start by converting from dollars to cents.

Answers: a) $265; b) 0.13 cm; c) the average was $6.00, so he paid less than average; d) 1¢
Extensions

1. A dime is 18.03 mm wide. How long would a straight line of 10 000 dimes with no gaps be in millimetres?
   
   **Answer:** 180 300 mm

2. a) Ten of an object laid end to end have a length of 48 cm. How long is the object?
   
   b) One hundred of an object laid end to end have a length of 2.38 m. How long is the object, in centimetres?
   
   c) One thousand of an object laid end to end have a length of 274 m. How long is the object, in centimetres?
   
   **Answers:** a) 4.8 cm, b) 2.38 cm, c) 27.4 cm

3. How would you shift the decimal point to divide by 100 000?
   
   **Answer:** Move it five places to the left.
Goals

Students will apply their knowledge of decimals to real-world situations involving money, distance, and mass.

PRIOR KNOWLEDGE REQUIRED

Understands decimal numbers
Can solve word problems with whole numbers
Can add and subtract decimals with regrouping

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1)
BLM Budget (p. N-32, see Extension 2)

Mental math minute—number talk. Present this problem: $3.5 + 4.7$. (8.2)
The following strategies could arise:

\[
(3.0 + 4.0) + (0.5 + 0.7) = 7.0 + 1.2
\]
\[
(3.0 + 4.0) + (0.5 + 0.5 + 0.2) = 7.0 + 1.2
\]
\[
3 + 0.5 + 4 + 0.5 + 0.2 = 4 + 3 + 0.5 + 0.5 + 0.2
\]
\[
3.5 + 0.5 + 4.7 = 4.0 + 4.7 - 0.5
\]
\[
3.5 + 4.7 + 0.3 - 0.3 = 3.5 + 5.0 - 0.3
\]

Review adding and subtracting decimals using the standard algorithm.

Write the following questions on the board and have students answer them on grid paper or BLM 1 cm Grid Paper:

\[
a) \ 17.54 + 12.46 \quad b) \ 35.11 - 9.22 \quad \text{Bonus: } 4102.65 + 8.56
\]

(a) 30.00, b) 25.89, Bonus: 4111.21

Word problems practice.

Exercises

1. Tom ran 2.41 km, Cathy ran 2.15 km, and Sun ran 5.1 km.

a) Who ran the shortest distance?

b) Could you use rounding to the nearest whole number to answer the question? Explain.

\textbf{Answers:} a) Cathy; b) no, because Tom’s and Cathy’s distances would both be 2 km

2. Ethan is trying to decide whether to buy a shirt for $28.99 and a pair of pants for $52.49 or a new backpack for $73.49.

a) Which purchase would be less expensive, a shirt and pants or a backpack?
b) How much less would the less expensive purchase be?

**Answers:** a) a shirt and pants cost $81.48 altogether, so the backpack would be less expensive; b) $7.99

3. An airline permits every passenger to take no more than 25.00 kg of luggage. Two friends want to get on a plane. One suitcase weighs 15.91 kg, and the other weighs 31.85 kg. Should they try to move enough weight into the lighter suitcase for both to weigh less than 25.00 kg? Explain.

**Answers:** Yes. Two people can carry a total of 50.00 kg of luggage, and the total weight of the luggage the friends are carrying is $31.85 + 15.91 = 47.76$ kg. So they should be able to move enough weight for both suitcases to weigh less than 25.00 kg.

**NOTE:** Extensions 1 and 3 are required in order to cover the British Columbia curriculum.

**Extensions**

1. An ecologist is studying how fast kernels of corn dry out after being removed from the cob. She removed a kernel of corn from a cob and weighed it three times. Her results are shown in the table below:

<table>
<thead>
<tr>
<th>Weighing Number</th>
<th>Time of Weighing</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Right after the kernel is removed from the cob</td>
<td>0.489 grams</td>
</tr>
<tr>
<td>2</td>
<td>1 week after the kernel is removed from the cob</td>
<td>0.364 grams</td>
</tr>
<tr>
<td>3</td>
<td>2 weeks after the kernel is removed from the cob</td>
<td>0.327 grams</td>
</tr>
</tbody>
</table>

a) Determine how much the weight changed between Weighing Number 1 and Weighing Number 2.

\[
0.489 - 0.364 = \text{weight change in first week}
\]

b) Determine how much the weight changed between Weighing Number 2 and Weighing Number 3.

\[
0.364 - 0.327 = \text{weight change in second week}
\]

c) Determine the total change in weight.

\[
\text{answer from part a)} + \text{answer from part b)} = \text{total change in weight}
\]

**Answers:** a) 0.125 grams, b) 0.037 grams, c) 0.162 grams
2. Complete the multiplication equation. Then use the solution to write a related division equation.

a) \[0.03 \times 10 = \frac{0.3}{0.3 \div 10 = 0.03}\]

b) \[0.05 \times 100 = \]

c) \[0.08 \times 1000 = \]

d) \[0.09 \times 10000 = \]

e) \[0.37 \times 1000 = \]

f) \[0.62 \times 100 = \]

g) \[1.09 \times 10 = \]

h) \[4.87 \times 100 = \]

**Bonus:** \[572.31 \times 10000 = \]

**Answers:** b) 5, c) 80, d) 900, e) 370, f) 62, g) 10.9, h) 487, Bonus: 5 723 100

3. Give each student **BLM Budget**. Have students brainstorm about the ways they might earn and spend money in the course of a month. Then have them create a simple budget to determine if their income would cover their costs. Encourage students to fill in at least two rows of income activities and expenses. Example:

<table>
<thead>
<tr>
<th>Income Activity</th>
<th>Income ($)</th>
<th>Expense</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babysit one night a week</td>
<td>$15</td>
<td>Buy 3 ice cream cones</td>
<td>$4 + $4 + $4</td>
</tr>
<tr>
<td>Cut the grass on Saturday and Sunday</td>
<td>$10 + $10</td>
<td>Download 4 albums</td>
<td>$10 + $10 + $8 + $8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pay back $10 borrowed from a friend</td>
<td>$10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$35</strong></td>
<td></td>
<td><strong>$58</strong></td>
</tr>
</tbody>
</table>

**Conclusion:** Earnings do not cover cost.

4. a) Multiplying by 100 gives the same answer as multiplying by 10 _two_ times.

b) Multiplying by 1000 gives the same answer as multiplying by 10 _three_ times.

c) Multiplying by 10 000 gives the same answer as multiplying by 10 _four_ times.

d) Dividing by 100 gives the same answer as dividing by 10 _two_ times.
e) Dividing by 10,000 gives the same answer as dividing by 10 _____ times.

f) Multiplying by 10 and then 100 gives the same answer as multiplying by ____.

g) Dividing by 10 _____ times gives the same answer as dividing by 100.

**Answers:** b) three, c) four, d) two, e) four, f) 1000, g) two

5. Complete the table.

<table>
<thead>
<tr>
<th>Number</th>
<th>How many places to the right of the first 9 is the second 9?</th>
<th>What fraction of the value of the first 9 is the second 9 worth?</th>
<th>The second 9 is how many times the value of the first 9?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 349.092</td>
<td>2</td>
<td>1/100</td>
<td>100</td>
</tr>
<tr>
<td>b) 95.089</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 95,091.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 799.186</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) 0.93409</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** b) 4, 1/10,000, 10,000; c) 3, 1/1000, 1000; d) 1, 1/10, 10; e) 4, 1/10,000, 10,000

6. Divide.

a) 34 ÷ 1000  
b) 962 ÷ 1000  
c) 5907 ÷ 1000  
d) 2.75 ÷ 1000  
e) 0.681 ÷ 1000  

**Bonus:** 0.034 ÷ 10,000

**Answers:** a) 0.034, b) 0.962, c) 5.907, d) 0.00275, e) 0.000681, 
**Bonus:** 0.0000034
## Budget

<table>
<thead>
<tr>
<th>Activity</th>
<th>Income ($)</th>
<th>Expense ($)</th>
<th>Cost ($)</th>
<th>Total</th>
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<tbody>
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<td></td>
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### Conclusion:

[Blank space for conclusion]
Unit 12  Geometry: Coordinates and Transformations

Introduction
This unit focuses on:
• coordinate grid systems; and
• translations, reflections, and rotations of 2-D shapes.

Meeting Your Curriculum

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<tr>
<td>Optional</td>
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Mental Math Minutes
The mental math minutes in this unit:
• explore and review strategies for addition, subtraction, multiplication, and division.

Generic BLMs
The Generic BLMs used in this unit are:
BLM 1 cm Grid Paper (p. S-1)
BLM Pentominoes (p. S-2)
These BLMs can be found in Section S.
Assessment
The lessons covered by a quiz or test are as follows:

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<th>BC</th>
<th>MB</th>
<th>ON</th>
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<td>G5-15, 17 to 20</td>
<td>G5-15, 17 to 20</td>
<td>G5-12, 13, 15 to 18</td>
</tr>
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</table>
Goals
Students will identify coordinates of points and draw points according to their coordinates in an array of dots.

PRIOR KNOWLEDGE REQUIRED
Knows that arrays consist of columns of equal numbers of objects and rows of equal numbers of objects

MATERIALS
deck of cards (NOTE: if available, use magnetic cards with a cookie sheet or other metal board for demonstrations)
grid paper or BLM 1 cm Grid Paper (p. S-1)
dividers, such as binders

Review the terms “column” and “row.” Draw an array of dots on the board. Remind students that columns are vertical and rows are horizontal. Use the picture in the margin as a reminder for students. Point out that the shape looks like a lower case “t.” It wouldn’t look like a “t” if we wrote “column” across instead of down. (Briefly show the second picture in the margin on the board but erase it quickly.) Point out that if students ever forget which word goes across, they can write the words both ways and choose the one that looks like a “t.” Have students draw the correct version in their notebooks.

Using coordinate systems to make it easy to find data. To illustrate the idea of a coordinate system, start with a card trick:

1. Deal out 9 cards—face up—in an array of 3 cards by 3 cards.

2. Ask a student to select a card in the array and tell you what column that card is in but not identify the card in any other way (e.g., by number or by suit).

3. Gather up the cards column by column—first one column that was not selected, then the other that was not selected, and finally the column that was selected on the top of the deck. Show clearly how to make sure the 3 cards of the selected column are on the top of the deck.

4. Deal the cards face up in another 3 by 3 array so that the top 3 cards of the deck are in the top row of the array.

5. Ask the volunteer to tell you what column the chosen card is in now. The chosen card will always be the top card in that column.

Repeat the trick several times and ask students to try to figure out how it works. Ask students to watch how you place the cards. You might also repeat the trick with a 2 by 2 array.
When students understand how the trick works, ASK: Would there be any point to the trick if the volunteer told me both the column number and the row number of the selected card? (no) Trace a column and a row to demonstrate that there is only one card that is in both the chosen column and the chosen row. Explain that two pieces of information are enough to clearly identify a position in an array or on a grid. Point out that labelling the columns and rows creates a structure that allows us to describe a location in a very effective way.

ASK: Will the trick work with a larger array? Have students try the trick with a 4 by 4 array. They should see that the trick works for any square array of cards that has the same number of columns and rows. (It can be modified to fit a rectangular array—see Extension 1.)

ASK: Would the trick work if you switched the words “columns” and “rows”? Have volunteers show how to perform the new trick.

**Labelling columns and rows in arrays.** Draw an array of 3 columns and 3 rows on the board and number the columns and rows as shown in the margin. Stress that we label the columns from left to right and the rows from bottom to top. Have students copy the picture.

**Identifying dots in an array.** Using the array on the board, ask students to trace specific columns or rows (e.g., the second column, the third row) and then to circle specific dots (e.g., the dot in the third row and the first column). Encourage students who have trouble locating a particular dot to join the dots in the given row and column prior to circling the dot itself. (The dot they are looking for will be at the intersection of the lines.)

**Identifying the column and the row for a location.** Draw an array of dots on the board and circle a dot. Ask students to write the coordinates of the dot: Column __, Row __. Have students practise identifying the column and the row using the following Activity.

**ACTIVITY (Essential)**

Students work in pairs and use grid paper or BLM 1 cm Grid Paper. Partners use a divider, such as a binder, to conceal their grids from each other. Each partner draws a square array of dots on the grid starting with a 3 by 3 array and increasing the challenge by increasing the number of columns and rows. Partner 1 circles a dot in the array and tells Partner 2 the column and the row it is in. Partner 2 identifies the dot on his own grid and circles it. Partners switch roles a number of times before checking their answers.

**Introduce ordered pairs.** SAY: Imagine you have to write the coordinates of 100 points. Would you like to write the words “column” and “row” 100 times? What could you do to shorten the information? Students might suggest making a T-table or even writing a pair of numbers. ASK: How do you know which is first, the column number or the row number? What if you have to ask a partner to find a dot with a pair of numbers but do not
tell your partner which number identifies the column and which number identifies the row? SAY: Here are two numbers: 2 and 3. I won’t tell you which one is the column number and which one is the row number. ASK: How many points can you find that could go with these two numbers? (2; see margin)

Explain that mathematicians around the world have agreed to give the location of a point by two numbers in parentheses. The column number is always on the left, and the row number is always on the right: (column, row). SAY: This means that the numbers in the pair have a specific order, so we call them an ordered pair. Give students several ordered pairs and ask them to identify the corresponding points in an array of dots.

Exercises

a) Draw a 4 by 4 array of dots. Label the columns with letters A to D and the rows with numbers 1 to 4, starting from the bottom left corner.

b) Mark the points (A, 4), (B, 2), (C, 3).

c) Mark two other points on the array and ask a partner to name them.

Extensions

1. Modify the card trick used to introduce the coordinate system for non-square arrays as follows. Deal out an array of 3 columns and 5 rows of cards. Have a student select a card and tell you what column it’s in. Gather up the cards so that the 5 cards in the selected column are on the top. Deal out the cards again as an array of 5 columns and 3 rows and with the cards of the selected column now in the top row of the new array. Ask the volunteer to tell you what column the chosen card is in now. The top card in that column is the chosen card.

2. You will need 27 cards. Lay the cards out as shown in the margin—in groups of threes in an array of 3 by 3. Ask a student to select a card and tell you only what column it’s in. Collect the cards so that the 3 triples in the chosen column are the 9 top cards on the deck. (You do not need to keep the triples together.) Deal the cards again so that the top 9 cards form 3 triples in the top row of a new array. Ask the volunteer in which column the card is now. The chosen card is now in the top triple of that column. Gather the cards and deal again so that the 3 cards from that triple become the bottom cards laid down for 3 new triples of the top row. Finally, ask the volunteer which column the card is in now. The first (bottom) card in the triple in the top row of that column will be the chosen card.

This version of the trick illustrates a powerful principle in science and mathematics: when you are looking for a solution to a problem, it is often possible to eliminate many possibilities by asking a well-formulated question. In the card trick, the card dealer can single out 1 of 27 possibilities by asking only 3 questions. Repeat the trick, asking students how many possibilities were eliminated by the first question (there were only 9 possible cards left, so $27 - 9 = 18$ possibilities were eliminated), by the second question ($9 - 3 = 6$ eliminated), and by the third question ($3 - 1 = 2$ eliminated).
Goals

Students will identify and plot points in the first quadrant of a coordinate grid.

PRIOR KNOWLEDGE REQUIRED

Can identify points on a number line made by skip counting

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1)
dividers, such as binders
rulers
BLM Grid with Tens (p. O-50)
two dice of different colours (see Extension 3)

Mental math minute. Students work in groups of 4. Have students add three-digit numbers by adding hundreds, tens, and ones separately. First, give each group an addition problem, such as $135 + 246$. The first student adds the hundreds ($100 + 200 = 300$), the second student adds the tens ($30 + 40 = 70$), the third student adds the ones ($5 + 6 = 11$), and the fourth student finishes the addition ($300 + 70 + 11 = 381$), so $135 + 246 = 381$. Start with problems that do not require regrouping, such as $325 + 634$, and continue to questions that require regrouping ones or regrouping tens but not both.

Introduce the axes and origin. Draw the coordinate grid shown in the margin but without the labels. SAY: In geometry, we use grids with number lines that always meet at 0. The grid is called the coordinate grid. The number lines are called axes. Point out the axes and mention that "axes" is the plural of axis. SAY: The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. The point at which the two axes intersect is called the origin. Label the axes, the origin, and numbers 0 to 4 on the axes.

Point out that both axes start at 0. Explain that since the origin is the intersection of two number lines, which meet at 0, its ordered pair is $(0, 0)$. It makes sense to draw only one 0 at the place where both number lines meet.

Introduce the x-coordinate and y-coordinate. On a grid on the board, mark the point $(4, 3)$, as shown in the margin, and have students identify the ordered pair. You may need to remind students that the column number is always listed first and then the row number. SAY: In the ordered pair $(4, 3)$, the numbers 4 and 3 are called the coordinates of the point. The number 4 is the x-coordinate of this point. Trace your finger down from the point and show that it is directly above the number 4 on the x-axis. Then trace with your finger left from the point $(4, 3)$ to the y-axis to look at the
number on the y-axis. SAY: The number 3 is the y-coordinate of this point. We can write the x-coordinate and the y-coordinate as \( x = 4 \) and \( y = 3 \).

Write on the board:

\[
x = 4 \quad y = 3
\]

Mark several points on the grid drawn on the board and have students identify the \( x \) and the \( y \) for these points. SAY: The \( x \)-coordinate is often called the first coordinate, because it is written first, and the \( y \)-coordinate is called the second coordinate.

**Exercises:** Rewrite the coordinates of the point as \( x = \underline{\hspace{1cm}} \), \( y = \underline{\hspace{1cm}} \).

a) (3, 1) b) (1, 4) c) (5, 2) d) (0, 3)

**Answers:** a) \( x = 3, y = 1 \); b) \( x = 1, y = 4 \); c) \( x = 5, y = 2 \); d) \( x = 0, y = 3 \)

**ACTIVITY (Essential)**

Students work in pairs and use grid paper. Partners use a divider, such as a binder, to conceal their grids from each other. Each partner draws a coordinate grid and labels it from 0 to 4. Partner 1 marks a point on the grid and tells Partner 2 its ordered pair. Partner 2 marks the point on her own grid. Partners switch roles a number of times before checking that their grids match.

**Coordinates of points on the axes.** Remind students that the axes intersect at 0. Mark the point (3, 0) on the coordinate grid and have students identify the coordinates. Point out that the \( y \)-coordinate is in fact the height above the horizontal axis, and since the height is 0, the point should be on the horizontal axis itself. Repeat with the point (0, 2), explaining that the \( x \)-coordinate shows the distance from the point to the vertical axis, measured along the grid line on which the point is situated.

Have students use grid paper for the exercises below. Point out that they can refer to the coordinate grid drawn on the board to help them draw their own.

**Exercises:** Use a ruler to draw a coordinate grid and label it with numbers from 0 to 5. Mark and label the points on the grid.

\( A (0, 3), B (2, 0), C (1, 0), D (0, 5) \)

**Answers:**

\[
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\]
Coordinate grids with scales made by skip counting by 2s. On a grid on the board, draw axes and mark them with intervals of 2. Explain that just as number lines on graphs can be marked with skip counting, number lines on coordinate grids can be marked by skip counting too. Mark several points and ask students to find the coordinates of these points. Start with points that are on the grid lines, such as (2, 4), continue to points that are on only one grid line, such as (2, 3) and (5, 4), and progress to points that are not on grid lines, such as (5, 3).

Exercises

a) Draw a coordinate grid with axes that skip count by 2s from 0 to 10.

b) Mark and label the points on the grid.
   - Z (4, 8), Y (3, 6), X (7, 5), W (0, 7), V (9, 0), U (3, 5), T (7, 9)

Answers:

Marking numbers on a number line that only shows tens. Draw a number line from 0 to 40 but label only the tens. Point to a few locations on the line and have students identify the number for each location. Write several numbers and point at locations on the number line. Have students signal with thumbs up and thumbs down to indicate whether the location you are pointing at is the number you wrote.

Coordinate grid with scales made by skip counting by 10s and 5s. Display BLM Grid with Tens. Use the point A (18, 24) to show students how to determine the coordinates of the point and then invite volunteers to draw lines from other points to the axes. Point to the locations on the grid for each coordinate pair and have students signal thumbs up or thumbs down to indicate whether this point is the one given by the coordinates.

Have students draw a coordinate grid whose axes are marked with intervals of 5 from 0 to 25. Points to plot: A (15, 4), B (3, 20), C (0, 24), D (16, 0), E (22, 6), F (23, 5), G (9, 20)

Extensions

1. Draw the points on a coordinate grid. (2, 6), (4, 4), (5, 7), (7, 8), (5, 2), (3, 4), (2, 1), (0, 0) Join the points in the order you drew them. Join the first point to the last point. What letter did you make?

Answer: N
2. Players will need a divider to conceal the coordinate grids they are working on from partners. Player 1 draws a square or a rectangle on the coordinate grid and tells Player 2 the coordinates of the vertices. Player 2 tries to visualize the shape and guess what kind it is before plotting the vertices. Player 2 plots the vertices and checks the answer.

Advanced: Use other shapes, such as rhombuses, parallelograms, and trapezoids.

3. Students will need a pair of dice of different colours, say red and blue. A student rolls the dice and records the results as a pair of coordinates: (the number on the red die, the number on the blue die). She draws axes on grid paper and plots a point that has this pair of coordinates. She rolls the dice a second time and obtains a second point in the same way. The student joins the points to form a line segment. She then has to draw a rectangle so that the line she drew is a diagonal of the rectangle. There could be several rectangles drawn this way. If the line is neither vertical nor horizontal, the simplest solution is to make the sides of the rectangle horizontal and vertical. In this case, ask your students if they see a pattern in the coordinates of the vertices.
Goals

Students will identify and produce congruent and non-congruent shapes.
Students will identify lines of symmetry in shapes.
Students will draw shapes with a line of symmetry.

PRIOR KNOWLEDGE REQUIRED

Can count sides and vertices of polygons

MATERIALS

2 identical paper L's
1 tall paper L
1 small paper L
12 pattern block squares per student
grid paper or BLM 1 cm Grid Paper (p. S-1)
paper squares, rectangles, and isosceles triangles
shapes from BLM Shapes for Folding (pp. O-51–54)
Miras
shapes from BLM Polygons (p. O-55)
4 pattern blocks or BLM Pattern Blocks (p. O-56)
rulers
paper shapes (see Extension 1)
tangram pieces or BLM Tangrams (p. O-57, see Extension 2)
pictures from magazines, scissors, and glue (see Extension 4)
geoboard (see Extension 5)

Mental math minute. Present a subtraction problem that does not need regrouping, such as 97 – 12. Have each student subtract the same number, in this case 12, from the answer the previous student gave. When a student says a subtraction that involves regrouping, emphasize that this answer was a bonus. Stop when students can no longer subtract (that is, don't use negative numbers). Example: Student 1 says, “97 – 12 = 85.” Student 2 says, “85 – 12 = 73.” Student 3 says, “73 – 12 = 61.” Student 4 says, “61 – 12 = 49” (this is a bonus). Continue without regrouping until Student 8 says “13 – 12 = 1,” then start a new chain.

Congruent shapes. Affix two identical paper L’s to the board as shown below:

SAY: I would like to know if these two shapes are the same. ASK: How could I check? (sample answer: place one on top of the other) Invite a volunteer to remove one of the shapes and place it on top of the other. ASK: Are the shapes the same? (yes) Do they match exactly, or does one of the shapes
“stick out”? (they match exactly) SAY: These two shapes are exactly the same in size and shape. If I place one on top of the other, I can turn it so that the shapes match exactly. Such shapes are called **congruent**.

Remove one L and affix the tall paper L to the board, as shown below:

![Shape](image)

ASK: Are these two shapes congruent? Take votes, then have a volunteer try to put one of the shapes on top of the other. Turn the shapes over so that students can see that the horizontal “wings” are different lengths and so the shapes are not the same. SAY: These two shapes do not match exactly. They are not the same shape. These shapes are not congruent. Repeat with the L and the small L (as shown below), discussing how these two shapes are not the same size.

![Shape](image)

Draw the shapes in the exercise below, one pair at a time, and have students signal thumbs up if the shapes are congruent and thumbs down if the shapes are not congruent.

**Exercises:** Are the shapes congruent?

![Shapes](image)

**Answers:** a) yes, b) yes, c) no, d) no, e) yes, Bonus: no

Ask students why some pairs of shapes in the previous exercises might seem at first glance to not to be congruent, even though they are congruent. (for parts b) and e), the shapes are pointing in different directions) Then ask students why some pairs of shapes in the exercises might seem at first glance to be congruent, even though they are not congruent. (for part c), both shapes are trapezoids, but one is wider; for part d), both are squares, but one is larger; for the bonus, both are triangles, but the shapes of the triangles are different)

**Identifying congruent shapes regardless of non-geometric attributes.**

Remind students that shapes can have many different attributes: size, shape, design, colour, or direction. Explain that only size and shape need to be the same for shapes to be congruent. Colour, design, and direction do not matter. Have students signal their answers to the following exercises. If students’ answers vary, say for parts b) and the Bonus, invite suggestions for how students can determine whether the shapes are congruent.
(sample answers: measure the lengths of the sides of the shapes and compare them; trace one of the shapes and compare it to the other shape.)

**Exercises:** Are the shapes congruent?

- a)  
- b)  
- c)  
- d)  
- e)  

**Bonus:**

**Answers:** a) no, b) yes, c) yes, d) yes, e) no, Bonus: yes

Have volunteers explain why the pairs in parts a) and e) are not congruent. (the shapes are different)

**ACTIVITY 1 (Optional)**

1. Give each student 12 pattern block squares. If pattern block squares are unavailable, have students do the exercise on grid paper or **BLM 1 cm Grid Paper**. Draw the shapes below one at a time. Have students make the shapes from pattern block squares and then make shapes congruent to those shapes but pointing in different directions.

   ![Pattern Block Shapes](image)

**Folding shapes so that parts match exactly.** Using a paper square, show students how to fold it along the horizontal line of symmetry so the parts match perfectly. Give each student a paper square and have them repeat what you did. ASK: Do the parts match perfectly? (yes) Flip the folded square over and ask again if the parts match perfectly. (yes) Unfold your square—what parts do you see? (2 rectangles) Are the shapes of the same kind? (yes) Are the shapes of the same size? (yes) What do you call two shapes of the same shape and size? (congruent) How did you see that the parts are congruent when the square was folded? (everything was covered from both sides) Repeat, but this time fold along the vertical line of symmetry.

   ![Folding Square](image)

Repeat with a rectangle, but first have students make predictions about whether both folds will result in parts that match perfectly. Repeat again with an isosceles triangle folding only once, as shown in the margin.

**Introduce lines of symmetry.** Have students unfold their paper shapes and explain that the line that the fold makes is called a **line of symmetry**. This line indicates that the parts of the shape match exactly when folded over. For each paper shape, ASK: How many lines of symmetry do you see? (square, 2; rectangle, 2; triangle, 1)
ACTIVITY 2 (Essential)

2. Give students several shapes from BLM Shapes for Folding. Students fold the shapes along the dotted lines. ASK: Do the parts match exactly? Is the fold a line of symmetry?

Are all fold lines also lines of symmetry? Demonstrate folding a paper square along one of its diagonals, from corner to corner, then have students do the same. ASK: Was that a line of symmetry? (yes) How do you know? (the parts match perfectly) Ask students if they think folding the square between the other two corners (the other diagonal) will have the same result and then have them test their predictions. Explain that since all four ways of folding produced parts that match perfectly, a square has four lines of symmetry.

Show a paper rectangle and ask students if they think that folding a rectangle from corner to corner will produce parts that match perfectly. (answers may vary) Have students fold their paper rectangles along the diagonal to test their predictions. ASK: Did the parts match perfectly? (no) Have students try folding along the other diagonal. ASK: Of the four ways we folded the rectangle, how many produced parts that match perfectly after folding? (two) Explain that this means a rectangle only has two lines of symmetry, unlike a square, which has four. Have students predict whether the triangle will have another line of symmetry and then test their predictions.

Using Miras. Ask students what object shows an exact match of anything. (a mirror) Give students Miras. If students have not used Miras before, ASK: What does this kind of mirror do? Let students experiment with personal objects. When students understand that this mirror is transparent, show them how to check whether parts of objects match exactly. For example, you could draw on the board an asymmetrical E that is about 20 centimetres tall and ask whether the top part is exactly the same as the bottom part. (see example in margin)

ASK: If you could fold the board, would the top of this E fall exactly on the bottom? (no) How could we check? Take guesses, then show students how to check this using the Mira. Students should clearly see that the parts do not match.

ACTIVITY 3 (Optional)

3. Give students shapes from BLM Polygons and have them use Miras to try to find and draw lines of symmetry on the shapes. Tell students that there are shapes that have no lines of symmetry, one line of symmetry, two lines of symmetry, and more than two lines of symmetry. NOTE: Students do not need to find all the lines of symmetry.
Creating shapes with lines of symmetry.

**ACTIVITY 4 (Essential)**

4. Using exactly four pattern blocks (for example, from BLM Pattern Blocks), students build as many designs as they can that have at least one line of symmetry. Students draw the designs in their notebooks.

**Exercises:** Copy the picture. Sketch the missing part so that the dashed line is a line of symmetry. Use grid paper or a ruler.

![Diagram](image)

**Selected answers**

![Answer Diagram](image)

**Congruence and symmetry.** In advance, cut out the square and parallelogram from BLM Shapes for Folding (3) and (4). Fold the square along one line of symmetry at a time and after every fold, ASK: Do the sides match? (yes) Is this a line of symmetry? (yes) Are the parts congruent? (yes) Fold the parallelogram along any line. ASK: Do the sides match? (no) Is this a line of symmetry? (no) Are the parts congruent? (answers may vary) Cut the parallelogram along the fold line and flip one of the resulting shapes so the two parts can be placed directly on top of each other. ASK: Are the parts congruent? (yes) Explain that you had to cut the shape to show the congruent parts. SAY: If you have to cut along a fold line to show congruent parts, then it is not a line of symmetry. It is only a line of symmetry if you do not have to cut to show that the parts match.
Extensions

1. a) Give students two congruent paper shapes each made from four or five squares, with one corner on one of the shapes marked with a dot. Ask students to determine whether the shapes are congruent. If the shapes are congruent, have students identify the matching corners by drawing a dot that matches the dot on the other shape. Repeat with two congruent non-isosceles trapezoids, then with two congruent quadrilaterals with all sides of different lengths so that all angles are different and there is just one correct answer.

b) Using the shapes they marked from part a), have students identify and mark matching sides.

2. Give each student a set of tangram pieces (for example, from BLM Tangrams). One at a time, have each student make a shape using three tangram pieces (with the other students not watching). The other students then use their own tangram pieces to make a congruent shape. Repeat with four and then five tangram pieces. NOTE: Students do not have to use the same pieces as one another. For instance, one student could use a small square while another student could combine two small triangles to make a square of the same size.

3. Which provinces and territories have flags with a line of symmetry?
   Answers: BC, NL, QC

4. Find a photo of an animal or human face from a magazine that is looking straight at the camera. Cut out half of the face and glue it on a piece of paper. Draw the missing half to make a complete face.

5. Make a shape on a geoboard or grid paper (or BLM 1 cm Grid Paper) with the given number of lines of symmetry.
   a) a quadrilateral with 1 line of symmetry
   b) a quadrilateral with 4 lines of symmetry
   c) a triangle with 1 line of symmetry
   d) a pentagon with 1 line of symmetry

   Sample answers

   a) ![Sample Shape A](image1)
   b) ![Sample Shape B](image2)
   c) ![Sample Shape C](image3)
   d) ![Sample Shape D](image4)

6. Discuss lines of symmetry in designs. Have students look for lines of symmetry in designs on their clothes.
G5-15 Translations
Pages 87–89

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
congruent
horizontal
image
transformation
translation
translation arrow
vertex
vertical
vertices

Goals
Students will perform translations on a grid.

PRIOR KNOWLEDGE REQUIRED
Can measure sides of polygons
Can identify congruent shapes

MATERIALS
2 identical L-shaped pieces of paper
round magnet or counter (e.g., integer tile, round game counter)
rectangular block or a matching paper rectangle
2 identical paper arrows
BLM 1 cm Grid Paper (p. S-1)
BLM Pentominoes (p. S-2)
scissors (optional)
magnetic grid and round magnet (see Extension 1)
chalk or masking tape and a ball (see Extension 2)
geoboards and rubber bands of two colours (see Extension 3)
2 colours of pencil crayons and BLM Dot Paper (p. O-58, see Extension 4)
The Geometer’s Sketchpad® (see Extension 5)

Mental math minute—number string.

String 1: Divide. Write your answer with remainder. 24 ÷ 4, 25 ÷ 4, 26 ÷ 4, 27 ÷ 4, 28 ÷ 4, 29 ÷ 4, 30 ÷ 4 (6 R 0, 6 R 1, 6 R 2, 6 R 3, 7 R 0, 7 R 1, 7 R 2)

Present the pattern using an array with four dots in a row, adding one dot to the last row, until the row is full. The row that is not full represents the remainder.

String 2: 369 ÷ 3, 370 ÷ 3, 394 ÷ 3 (123 R 0, 123 R 1, 131 R 1)

String 3: 400 ÷ 4, 404 ÷ 4, 406 ÷ 4 (100 R 0, 101 R 0, 101 R 2)

Introduce transformations. Show students two copies of an L-shape made of paper that are oriented in different directions, beside each other, as shown in the margin.

Explain that the shapes are identical. Ask students if they remember the correct mathematical term for identical shapes. (congruent shapes)

Tell students that you want to move the shapes so that they line up exactly, with one on top of the other, facing the same direction so that one congruent shape completely covers the other. Show moving the shapes, as shown in the margin. Return the shapes to their original position.
Tell students to pretend that the shapes are actually very heavy, very hot sheets of metal, so you need to program a robot to move them. To write the computer program, you have to divide the process of lining up the shapes into very simple steps.

It is always possible to move a figure into any position in space by using some combination of the following three movements:

• Sliding the shape along a straight line without allowing it to turn.
• Flipping the shape over.
• Turning the shape around some fixed point.

Explain that these three types of movements are called **transformations**.

Have students tell you, the robot, what steps to perform to position the hot L-shaped sheets of metal one on top of the other. Explain that there are different ways to bring one shape on top of the other, so there are no right or wrong answers. However, some instructions are more efficient than others; in other words, some ways will require fewer steps. When students direct you, they need not be so precise at this point. But tell students that they will learn to describe these movements or transformations more precisely throughout the unit.

**Translating points on a grid.** Explain to students that in this lesson they will only perform slides, which are also called **translations**. Write the word “translations” on the board. Use a grid on the board and a round magnet or counter to demonstrate sliding a point on a grid; place the counter on a grid intersection and physically slide the point, represented by the counter, on the grid. The diagram below shows a translation of 3 units right. You might want to mark the starting point on the grid.

![Translation Diagram]

Have students first signal the direction in which the point is translated (right or left, up or down) and then ask them to hold up the number of fingers equal to the number of units the dot is translated. You may wish to draw a large letter L on the left side of the board and a letter R on the right side of the board to help students who have trouble distinguishing between left and right. Demonstrate several new translations with the counter on the grid and ask students to signal the direction and number for each translation.

Invite volunteers to translate a point and have other volunteers describe the translations. Then reverse the task: have volunteers describe the translation and have other volunteers perform them with a counter. Explain that the point (or shape) in the new position after a translation is called the **image**.
**Exercises:** Draw a point on a grid and label it $A$. Draw a point $B$ that is the image after the given translation.

a) 2 units down  
   b) 3 units up  
   c) 4 units left  
   d) 1 unit right

**Sample answers**

a) ![Sample answer a]  
   b) ![Sample answer b]  
   c) ![Sample answer c]  
   d) ![Sample answer d]

SAY: You can also combine translations. For example, you can move 3 units right and 2 units down. Demonstrate with a counter and draw arrows to show the translations, as shown below:

**Exercises:** Draw a point on a grid and label it $A$. Draw a point $B$ that is the image after the given translation.

a) 2 units right, 1 unit up  
   b) 5 units right, 2 units up  
   c) 6 units right, 2 units down  
   d) 3 units left, 4 units up  
   e) 3 units right, 1 unit down  
   f) 5 units left, 4 units down

**Sample answers**

a) ![Sample answer a]  
   b) ![Sample answer b]  
   c) ![Sample answer c]  
   d) ![Sample answer d]  
   e) ![Sample answer e]  
   f) ![Sample answer f]

**NOTE:** Students who are struggling can draw the arrows showing each part of the slide.

**Describing translations.** SAY: To describe a translation, you need to say how much the point moved and in which direction. Draw the picture in the margin on the board. Explain to students that the arrow going from $A$ to $B$ is called a **translation arrow**. SAY: You can imagine the arrow from $A$ to $B$ as a combination of two arrows, horizontal and vertical. Trace the dashed arrows with a finger. ASK: How much did point $A$ move in the horizontal direction?
(4 units) Did it move right or left? (right) Repeat with the vertical arrow.
(2 units up) SAY: So the point A moved 4 units right and 2 units up.

**ACTIVITY 1 (Essential)**

1. Students work in pairs. Partner 1 draws a pair of points on a grid and an arrow from one point to the other. Partner 2 describes the translation. Partner 1 verifies the answer. Partners switch roles.

How much did the shape slide? Draw on the board the picture in the margin. ASK: How far did the rectangle slide to the right from Position 1 to Position 2? Accept all answers and record them on the board. Call for a vote if you wish. Students might say the rectangle moved anywhere between 2 and 6 units right. Take a rectangular block or a matching paper rectangle and perform the actual slide, one square at a time, counting the units as a class. The correct answer is 4 units.

**Corresponding points.** Draw a point at the top right vertex of the rectangle in Position 1. ASK: Can this make it easier to see that the translation was 4 points to the right? (yes) Why? (we know how to translate points) SAY: The vertex I marked and its image are corresponding points under a translation. When we talk about transformations, we want to know where each point went to. Invite a volunteer to mark the image of the marked point on the second rectangle. Keep the picture on the board for later use.

Draw the pictures in the exercises below, one pair of figures at a time, and have students signal the answer by raising the correct number of fingers.

**Exercises:** Which vertex, 1, 2, 3, or 4, is the image of the vertex marked with a dot after the translation?

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<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 3, b) 2, c) 4

Under translations, all points on a shape move the same amount in the same direction. Label the vertices of the rectangle in Position 1 from earlier as A, B, C, and D. Add the same labels to the vertices of the paper rectangle. Translate the paper rectangle again, from the initial position to the position 4 units to the right and 2 units down. Draw arrows from each vertex to its image. SAY: These arrows are called translation arrows.
ASK: What do you notice about the translation arrows? (they are all parallel, and they are all the same length) Explain that this means that all points on a shape move the same amount in the same direction, so it is enough to draw only one translation arrow to describe a translation. SAY: However, you need to be careful to draw the arrow between a vertex and its image, not any other vertex. Also, the fact that all arrows are the same gives you a way to translate polygons: you can translate each vertex separately and then join the images of the vertices to form the image of the polygon.

A shape and its image after a translation face the same direction. In advance, cut out two identical arrows using two sheets of paper. Post one arrow on the board, pointing right, as shown below:

ASK: In which direction is the arrow pointing? (right) Now affix the other arrow to the board pointing left and position it as shown below:

ASK: In which direction is the second arrow pointing? (left) Challenge students to find a translation that takes the first arrow to the second arrow. After tracing the position of the first arrow, slide the arrow according to the translations students suggest. After each translation, ASK: In which direction is the arrow pointing? (right) After several tries, ASK: Is there any translation that will change the direction of the first arrow? (no, the arrow will always point right) ASK: What could we do to make the first arrow match the second arrow? Students might suggest turning (rotating) or flipping (reflecting) the arrow. Explain that a shape and its image after any translation always point in the same direction or face the same way.

**ACTIVITIES 2–3 (Essential)**

2. Provide each pair of students with grid paper or BLM 1 cm Grid Paper and the L-shape from BLM Pentominoes. Or you might provide students with scissors and have them cut out the L-shape themselves.

Partner 1 positions the shape anywhere on the grid, in any position, but so that the edges of the shape line up with the grid lines. Partner 1 traces the shape in this starting position and then writes a translation for Partner 2 to perform. (For example, translate this shape 3 units right and 4 units down.)
Partner 2 predicts where the image will be without using the paper shape and sketches the predicted image. Partner 2 then slides the shape to perform the translation and draws the image. Both partners compare the prediction to the actual image, and Partner 2 corrects any mistakes. Partners switch roles and repeat several times, drawing the starting shape on an unused part of the grid or erasing as needed. Students can repeat the activity with different pentominoes.

3. Repeat Activity 2, except Partner 1 performs the translation and sketches both the starting position and the image. Partner 2 must describe the translation. Partners switch roles and repeat.

Extensions

1. Draw a hockey rink on a magnetic grid and place a small circular magnet on the grid to act as the puck. Invite volunteers to move the magnet, as if they were passing a puck, according to your instructions. You can start with instructions that require moving either only horizontally or only vertically (e.g., pass the puck three units right; five units left; seven units down; two units up) and then proceed to instructions that involve both (e.g., pass the puck two units left and five units up). You might position several small figures or drawings of hockey players on grid intersections on the rink and ask students questions that involve the players (e.g., Player 3 passes the puck five units right and two units up. Who receives the pass? Player 5 wants to pass the puck to Player 7. How many units left and how many units down should the puck go?)

2. In the school yard, draw a grid on the ground using chalk. Alternatively, make a grid using masking tape on the floor of the classroom. Have students stand on points on the grid, facing the same direction, and tell them they are facing forward. Give one student a ball. Give translations such as: “The ball slides three units to the right and two units backward.” All students squat down except the student who is the image after the translation. The student with the ball then throws it to the correct student on the grid.

3. Provide each pair of students with geoboards and rubber bands of two different colours, for example, blue and red. Partner 1 creates any polygon with the blue rubber band and then creates the image of the shape after a translation using the red rubber band. Partner 2 describes the translation. Partners switch roles and repeat several times.

4. Provide each pair of students with BLM Dot Paper, two pencil crayons of different colours, and BLM Pentominoes. Students repeat a version of Extension 3, using dot paper and drawn shapes. The starting shape can be any polygon students wish to draw as long as all the vertices lie on dots. Students may refer to BLM Pentominoes for examples of shapes they can draw.
NOTE: The following extension uses a computer program called The Geometer’s Sketchpad®. If students have not used this program before, you will need to teach them basics, such as plotting points and drawing triangles.

5. Translating shapes in The Geometer’s Sketchpad® by distance. Explain that the default grid in The Geometer’s Sketchpad® shows increments of 1 cm, so a point at (2, 0) is 2 cm to the right of the origin. Have students follow the steps below to translate a triangle and then answer the questions.

Step 1: On the Graph menu, select Show Grid. Use the Graph menu again and select Snap Points.

Step 2: Draw a triangle and label it $ABC$.

Step 3: Use the Arrow tool to select all elements of the triangle.

Step 4: On the Transform menu, select Translate. Choose Rectangular in the dialog box. To translate the triangle 1 cm right and 2 cm down, set a fixed horizontal distance of 1 cm and a fixed vertical distance of $-2$ cm.

a) Do the triangles seem to be congruent?

b) Move different vertices around. Do both triangles seem to stay congruent?

c) As you move the vertices, does the position of one triangle change relative to the other triangle?

d) Use Steps 3 and 4 to translate triangle $ABC$ as given.

   i) 1 unit right, 2 units up
   ii) 3 units left, 5 units down
   iii) 6 units left, 8 units up

Selected answers: a) yes, b) yes, c) no
Goals

Students compare grid systems commonly used on maps (i.e., the use of numbers and letters to identify an area) with coordinate grids used in previous lessons.

PRIOR KNOWLEDGE REQUIRED

Can locate points on a coordinate grid

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1)
map with grid
BLM Map of Saskatchewan (p. O-59)
overhead projector (optional)
transparency of BLM Map of Saskatchewan (p. O-59, optional)
BLM Coordinate Grid Map of Saskatchewan (p. O-60)
transparency of BLM Coordinate Grid Map of Saskatchewan (p. O-60, optional)
maps (e.g., atlas or device with internet access) (see Extensions 1 and 2)

Mental math minute—number string.

String 1: $45 \div 5, 450 \div 5, 4500 \div 5, 45 000 \div 5, 45 000 \div 5$, (9, 90, 900, 9000). Present the reason why the strategy works:

- $45 \div 5 = 9$ objects
- $450 = 45$ tens $\div 5 = 9$ tens $= 90$
- $4500 = 45$ hundreds $\div 5 = 9$ hundreds $= 900$
- $45 000 = 45$ thousands $\div 5 = 9$ thousands $= 9000$

String 2: $36 \div 6, 360 \div 6, 3600 \div 6, 36 000 \div 6, (6, 60, 600, 6000)$

String 3: $42 \div 7, 420 \div 7, 4200 \div 7, 42 000 \div 7, (6, 60, 600, 6000)$

Reading positions on a grid. Draw a grid on the board like the one shown in the following activity. Explain that just as coordinates can be used to specify the location of a point or dot on a grid, they can also be used to identify squares on a grid. Point to various squares on the board in random order and have students identify the coordinates of the square. Remind students that they must say the column first before the row. Then write coordinates for various points on the grid (i.e., (A, 5), (D, 3), (B, 1), (F, 4), (A, 2), (E, 6)) and point to a few incorrect squares before pointing to the correct one, having students signal thumbs up or down to indicate whether you’ve pointed to the correct square on the grid.
**ACTIVITY (Essential)**

Pairs of students will play a short version of the game sometimes known as "Battleship." Distribute grid paper or **BLM 1 cm Grid Paper** to each student. Have them mark off two 6 by 6 squares, leaving one row above and one row to the left of each. Students label the columns of the squares with the letters A to F and number the rows 1 to 6. Without letting their partners see, students then draw four non-overlapping rectangles anywhere on the first grid: two 1 by 2 rectangles, one 1 by 3, and one 1 by 4 (see example below).

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>6</td>
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<td></td>
</tr>
</tbody>
</table>
```

To begin the game, Player 1 guesses a position on the grid by using its column letter and row number. If this position is part of a rectangle on Player 2’s grid, Player 2 replies “hit”; otherwise, it is a “miss.” Player 1 records the guess on the second grid, marking a hit with an X and a miss with an O. If it is a hit, Player 2 also marks it by crossing out that square of the rectangle. Player 2 then has a turn guessing. When all squares of a player’s rectangle are crossed out, that player says “one down.” Play continues until all rectangles have been found.

**Grids commonly used on maps.** Tell students that the grid they just used to play the game, using letters for columns and numbers for rows, is the type of grid often used on maps. SAY: We can call this a grid square system. Show students an example of a map with a grid. Point out other features of the map, such as the scale of the map and how it is indicated, how icons and colour are used to highlight attractions such as parks and schools, and the alphabetical list of streets with grid positions. Note that streets usually cross more than one square in the grid, so their positions are indicated by a range (for example, B7 to F7). Point out that there can be several items on a map that fall into the same grid square.

Distribute a copy of **BLM Map of Saskatchewan** to each student for the following exercises. You might wish to project a transparency of BLM Map of Saskatchewan as well.

**Exercises:** Use BLM Map of Saskatchewan to answer the question.

a) What are the coordinates of Saskatoon?

b) What are the coordinates of Regina?

c) What are the coordinates of Uranium City?
d) Which grid square is located 2 squares east and 3 squares south of Uranium City? What is in that square?

e) Which grid square is 3 squares north and 1 square west of Swift Current? What is in that square?

f) What can you find in the squares D5 and D4?

g) What can you find 3 squares to the east of Maple Creek?

**Bonus:** Which square on this map has more than one city? Name the cities.

**Answers:** a) B2, b) C1, c) A5, d) C2, Prince Albert, e) A4, Clearwater River Provincial Park, f) Wollaston Lake, g) Weyburn, Bonus: C1, Moose Jaw and Regina

**Maps that use a coordinate grid.** Explain that another way to make a map is to use a coordinate grid. SAY: Instead of labelling the grid squares, we can label the grid lines, as we did with coordinate grids. We can call this a coordinate grid system. Distribute BLM Coordinate Grid Map of Saskatchewan and ask students to compare this version of the map with the one they used for the exercises. You might wish to also project a transparency of BLM Coordinate Grid Map of Saskatchewan to help in demonstrating exercises. **ASK:** What is different between the two maps?

If the following differences do not come out of the discussion, bring them up yourself: the first map uses numbers and letters, whereas the second map uses only numbers; in the first map, the coordinates refer to large grid squares, whereas in the second map, the coordinates refer to grid lines and points where grid lines intersect. You might mention to students that in each map, 1 cm represents about 47.5 km in real life.

Complete the first couple of exercises below as a class before having students complete the rest individually.

**Exercises:** Use BLM Coordinate Grid Map of Saskatchewan.

a) What are the coordinates of Swift Current?

b) What are the coordinates of Weyburn?

c) Which city has coordinates (8, 40)?

d) Which city has coordinates (16, 16)?

e) Which city is 36 units south of Uranium City?

f) Which city is 4 units west and 7 units north of Moose Jaw?

g) Describe the location of Regina from Maple Creek.

**Bonus:** Describe the location of Maple Creek from the southern tip of Wollaston Lake.

**Answers:** a) (8, 4), b) (23, 2), c) Uranium City, d) Prince Albert, e) Swift Current, f) Saskatoon, g) 17 units east and 3 units north, Bonus: 19 units west and 29 units south
Comparing the two systems for maps. Discuss the differences and pros and cons of the two systems for maps. ASK: Which system made it easier to find a place on the map quickly, the grid square system, or the coordinate grid system? Students’ answers may vary. Some students might choose the grid square system since the grid squares are larger and you can find the place you’re looking for once you know its grid square. Other students might choose the coordinate grid system since the coordinates can tell you the exact position of the place you’re looking for. Emphasize that with the grid square system, a single grid square can contain many different places. For example, if cities are represented by small dots on a map, then there can be more than one city in a large grid square, whereas in the coordinate grid system, the coordinates can give you a more precise location of the city.

Extensions

1. Have students find a map that uses the grid square system. Students can look in an atlas or search online. Have them come up with 3 to 5 questions about how to locate one landmark on the map from another. Students exchange maps with a partner and answer their partner’s questions.

2. Repeat Extension 1 using a map that uses a coordinate grid system (where the coordinate grid lines are numbered rather than the grid squares).

3. Students make a map of a location of their choosing (such as the classroom, their home, the playground, or a made-up fictional place). They make the map in two ways: first using grid squares, then with a coordinate grid. There should be about 8 to 10 landmarks on the map represented by dots. Students create 5 questions about directions from one landmark to another and exchange them with a partner. Partners must answer the questions using both types of maps. Discuss as a class which map made the task easier and why.
Goals

Students will perform reflections on a grid.
Students will create and extend symmetrical designs and patterns resulting from reflections.

PRIOR KNOWLEDGE REQUIRED

Can identify and describe relative locations on a grid
Can place points on a grid

MATERIALS

overhead projector
blank transparency
transparency of BLM 1 cm Grid Paper (p. S-1)
Miras
pentominoes or BLM Pentominoes (p. S-2)
BLM 1 cm Grid Paper (p. S-1)
scissors (optional)
red and blue chalk or markers
geoboards and rubber bands of two colours (see Extension 2)
BLM Dot Paper (p. O-58) and two colours of pencil crayons (see Extension 3)

Mental math minute. Have students add 9, 8, 90, 99, or 98 to two-digit and three-digit numbers by adding 10 or 100 and then compensating by subtracting as follows: give an addition problem, such as 135 + 98. The first student says what needs to be done (add 100 and then subtract 2). The second student adds 100 (135 + 100 = 235). The third student subtracts 2 (235 − 2 = 233).

Review lines of symmetry. Draw the picture shown in the margin on the board.

ASK: What is the line called? (line of symmetry) Remind students that if you fold a shape along a line of symmetry, the halves will overlap exactly. Ask what happens if you place a mirror along a line of symmetry. (The visible half of the shape and the mirror image make up the whole shape.)

Exercises: Draw all the lines of symmetry.

a)  

b)  

c)  

Reflecting shapes in a mirror. Tell students that they are going to find mirror images of whole shapes. Using a blank transparency projected on the board, draw the triangle and reflecting line shown below:

Invite a volunteer to trace the triangle and the line on the board. Then flip the transparency horizontally so that the reflecting lines on the transparency and the board coincide and the projected triangle is the reflection of the traced triangle. Have a volunteer trace the projected triangle.

ASK: Is this triangle a mirror image of the first triangle? (yes) Tell students you created the second triangle by flipping the first one over the mirror line or reflecting line. Demonstrate this by placing the transparency on top of the original triangle and then flipping it over so that it coincides with the mirror image. Explain that this is also called reflecting and that shapes made this way are called reflections or mirror images.

NOTE: Project BLM 1 cm Grid Paper on the board for the remainder of the lesson. Leave the following exercises on the board for later reference. If you do not have Miras for Exercise 1, give students pentominoes or shapes from BLM Pentominoes instead of the shapes provided, have them trace their pentomino on BLM 1 cm Grid Paper, and reflect the pentomino by flipping it over a line of reflection. It is not important that students copy the shapes accurately. It is important to verify that they have properly reflected the shapes as they have drawn them.

Exercises

1. Copy the picture onto grid paper. Place a Mira along the dashed line. Draw the reflection of the shape.

   a) b) Bonus:

Answers

a) b) Bonus:
2. The dashed line is the mirror line. Copy the picture onto grid paper, then draw the reflection of the shape. Check your answer using a Mira.

![Diagram](image)

**Answers**

![Solution](image)

**Reflecting points.** Draw the image in the margin on the board, with the triangle and dot drawn in blue. Have a volunteer draw the mirror image in red, including the dot on the vertex. Point out that the original triangle is above the reflecting line and the reflection is below it. Draw students’ attention to the blue vertex. ASK: How far away from the mirror line is the blue dot? (one square) How far away is its red image? (one square) Did the image move left or right? (no)

Draw an X on the top vertex. ASK: Where did this vertex go when the triangle was reflected? Have a volunteer draw an X on the corresponding vertex in the reflected image. Repeat the questions above about distance from the mirror line and left/right movement. ASK: What happens to points that are already on the reflecting line? (they stay where they are) Draw students’ attention to the edges in the previous exercises that lie along the mirror line (Exercise 1, part a) and the bonus). Those edges and vertices are the same in the original and reflected images.

For Exercise 1 below, add points to the shapes in the previous Exercise 2. Students can add these same points to their answers and find the images.

**Exercises**

1. Draw the mirror images of the points. Check your answers using a Mira.

![Diagram](image)

**Answers**

![Solution](image)
2. Draw the reflection of the points. Check your answers using a Mira.

a) ![Reflection](image1)

b) ![Reflection](image2)

Answers

a) ![Answer](image3)

b) ![Answer](image4)

**ACTIVITIES 1–2 (Essential)**

1. Provide each pair of students with BLM 1 cm Grid Paper and the L-shape from BLM Pentominoes. Or you might provide students with scissors and have them cut out the L-shape themselves.

   Partner 1 positions the shape anywhere on the grid, in any position, but so that the edges of the shape line up with the grid lines. Partner 1 traces the shape in this starting position and then draws a line of reflection (vertical or horizontal). Without using the paper shape, Partner 2 predicts where the image will be after reflection in the line and sketches the image. After sketching the prediction, Partner 2 takes the shape from Partner 1 and flips the shape to perform the reflection, drawing the image. Both partners compare the prediction to the actual image, and Partner 2 corrects any mistakes. Partners switch roles and repeat several times. Students can repeat the activity with different pentominoes.

2. Repeat Activity 1, except this time Partner 1 performs the reflection and sketches both the starting position and the image but not the line of reflection. Partner 2 must describe the reflection by indicating the line of reflection. Partners switch roles.

**Reflecting vertices to reflect shapes.** Tell students that one way to find the reflection of a shape is to first reflect all of its vertices, then connect the vertices to make the reflection. To demonstrate, draw on the board the shape on the left and the line of reflection shown below. Have volunteers find the reflections of the vertices and reconnect them to form the image. Remind students that a vertex and its image are the same distance away from the reflecting line, on opposite sides. The final image on the board should look like this:

![Reflection Diagram](image5)
Exercises: The dashed line is the mirror line. Copy the picture onto grid paper. Reflect each vertex, then connect the vertices to make the reflection of the shape. Use a Mira to check your answer.

![Exercises Example](image)

Answers

![Answers Example](image)

Creating patterns using reflection. Ask students for examples of attributes of shapes. (size, colour, direction, etc.) Remind students that they can create patterns by repeatedly changing an attribute of a shape. ASK: Which attribute changes when you reflect a shape? (direction) Can you use this to make a pattern? (yes) Draw on the board:

![Drawing Example](image)

Invite a volunteer to draw the reflection of the shape in the dashed line. Then draw two more dashed lines, each four squares apart, and have other volunteers draw the mirror images. (see below)

![Mirror Images Example](image)

ASK: Are all the shapes in this pattern congruent? (yes) How do you know? (it is the same shape that is just getting reflected or flipped over and over to make all the figures in the pattern) What is different about the first shape and the second shape in the pattern? (the direction) Do the 1st shape and the 3rd shape face the same direction? (yes) Do the 2nd shape and the 4th shape face the same direction? (yes) Which other shapes in the pattern will face the same direction as the 1st and 3rd shapes? (the 5th, 7th, 9th, and so on; all the odd-numbered shapes) Which other shapes in the pattern will face the same direction as the 2nd and 4th shapes? (the 6th, 8th, 10th, and so on; all the even-numbered shapes) Ask students to predict what the 55th shape in the pattern will look like and to explain how they know. (it will look like the 1st shape and all other odd-numbered shapes since 55 is an odd number) Repeat with an even position, such as the 82nd shape in the pattern.
ACTIVITY 3 (Essential), ACTIVITY 4 (Optional)

3. Give each student a pentomino from a commercial set or BLM Pentominoes. Students place the piece on BLM 1 cm Grid Paper and trace it. They draw a reflecting line one square to the right of the rightmost edge of the shape or one square below the bottom edge of the shape and then reflect the shape in the line. They can check the results by flipping the pentomino. Students produce a pattern with at least five terms. They then exchange patterns with a partner and produce the next three shapes in their partner’s pattern. Students must also sketch what the 43rd and 50th shapes in their partner’s pattern will look like.

4. Students repeat Activity 3, except that they reflect their shape both vertically and horizontally to produce a two-dimensional pattern.

Extensions

1. Fold a piece of grid paper in half twice, once vertically and once horizontally, and then lay it out flat again. Draw a picture in the upper left-hand part of the folded grid paper. Reflect your shape horizontally from left to right in the folded line. Draw the image and then reflect the new shape vertically from up to down. Then reflect that shape horizontally from right to left. Finally, reflect that shape vertically from down to up. What happens?

   **Answer:** The final image matches the original shape.

2. Provide each pair of students with geoboards and rubber bands of two different colours, for example, blue and red. Partner 1 creates any polygon with the blue rubber band and then creates the image of the shape after a reflection using the red rubber band. Partner 2 describes the reflection, indicating the line of reflection by tracing the line with a finger. Partners switch roles and repeat several times.

3. Provide each pair of students with **BLM Dot Paper**, two pencil crayons of different colours, and BLM Pentominoes. Students repeat a version of Extension 2, using dot paper and drawn shapes. The starting shape can be any polygon students wish to draw as long as all the vertices lie on dots. Students may refer to BLM Pentominoes for examples of shapes they can draw.
Goals

Students will identify and describe reflections.
Students will distinguish between reflections and translations.
Students will identify, extend, and create patterns made by reflection and translation.

PRIOR KNOWLEDGE REQUIRED

Can reflect a shape through a given line
Can reflect points through a given line
Can translate points and shapes

MATERIALS

overhead projector
transparency of BLM 1 cm Grid Paper (p. S-1)
grid paper or BLM 1 cm Grid Paper (p. S-1)
a large paper pentomino
pentominoes or BLM Pentominoes (p. S-2)
images of patterns and designs that involve reflections and translations
geobords and rubber bands of two colours (see Extension 3)
BLM Dot Paper (p. O-58, see Extension 4)
pattern blocks or BLM Pattern Blocks (p. O-56, see Extension 5)

Mental math minute—number string.


Use rounding and compensating to explain one possible strategy: for 95 – 48, since 50 is 2 more than 48, subtracting 50 is subtracting 2 too many; 95 – 48 is 2 more than 95 – 50. Similarly, 91 – 62 is 2 less than 91 – 60 because subtracting 60 is 2 less than needed.


NOTE: You can project BLM 1 cm Grid Paper on the board for the parts of this lesson that require a grid on the board.

Finding vertical reflecting lines. Draw on the board:

ASK: Are these triangles congruent? (yes) Are they mirror images? (yes)
Have a volunteer draw the mirror line. Redraw the same triangles two squares apart and repeat the process. (There is no need to distinguish the triangles by colouring.)
The board should look like this:

Repeat again with the same triangles, but four squares apart. Then repeat with other shapes at a variety of distances apart. ASK: How do you know where to draw the line of reflection? (the line of reflection is halfway between the shapes; the original shape and the image must be the same distance from the line of reflection).

**Exercises:** Copy the shapes onto grid paper. Find the line of reflection.

a) ![Shape](image1)

b) ![Shape](image2)

c) ![Shape](image3)

**Bonus:**

Finding horizontal reflecting lines. Draw the picture shown in the margin on the board. ASK: Are these triangles congruent? (yes) Are they mirror images? (yes) In which direction is the reflecting line? (horizontal) Have volunteers draw lines connecting corresponding vertices. Then have another volunteer draw the line of reflection.

**Exercises:** Copy the shapes onto grid paper. Find the reflecting line.

a) ![Shape](image4)

b) ![Shape](image5)

c) ![Shape](image6)

**Bonus:**

Answers

a) ![Shape](answer1)

b) ![Shape](answer2)

c) ![Shape](answer3)

**Bonus:**

![Shape](answer4)
Identifying reflected images and translated images. Tell students that all reflected images are congruent, but not every congruent image is a reflection. Draw on the board:

A.  
B.  
C.  

Ask students which pair of triangles was created using a reflection. (A) Have volunteers draw lines connecting corresponding vertices. Ask students to compare these lines. (In A and B, the lines are parallel. In C, the lines cross. In B, the lines are all the same length.) ASK: How can you tell which are reflections by looking at the lines? (sample answer: the lines are parallel, but they are of different lengths) ASK: Which pair of triangles was created using a translation? (B) How do you know? (sample answer: because the triangles are congruent and facing the same direction) How could you use the lines we drew to tell which one is a translation? (sample answer: in a translation, the lines are parallel and all the same length) Emphasize that in a reflection, the original shape and the image face in opposite directions, whereas in a translation, they face in the same direction.

NOTE: Extensions 1 and 2 explore in more depth the direction and orientation of images after reflection and translation.
2. For the shapes you drew in Exercise 1, draw a line of reflection for the shapes that were created by a reflection. Draw a translation arrow for the shapes that were created by a translation.

**Answers**

a)   b)   c)   

![Reflection](image1)

![Translation](image2)

**Bonus:**

![Reflection](image3)

3. Use check marks to fill in the table.

<table>
<thead>
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<th>The Original Shape and the Image are Congruent.</th>
<th>The Original Shape and the Image Face the Same Direction.</th>
<th>The Original Shape and the Image Face Opposite Directions.</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td>After a Translation</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Answers**

<table>
<thead>
<tr>
<th>The Original Shape and the Image are Congruent.</th>
<th>The Original Shape and the Image Face the Same Direction.</th>
<th>The Original Shape and the Image Face Opposite Directions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>After a Reflection</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>After a Translation</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Analyzing patterns made by reflection and translation.** Choose a pentomino piece that does not have a line of symmetry and make a large one from paper using **BLM Pentominoes** as a guide. Show students how to make a pattern by alternating between reflections and translations by tracing the pentomino on the board. For example:
Have volunteers extend the pattern by alternating reflection and translation. Then ASK: What is the core of the pattern? Students can answer by saying how many shapes are in the core of the pattern. (4) Have a volunteer circle the core of the pattern. Have another volunteer circle the repetitions of the core. ASK: How could we find out what the 8th shape in the pattern will look like by using the core? (there are 4 shapes in the core, so the 8th shape will look just like the 4th shape) How could we figure out the 40th shape in the pattern? (40 is a multiple of 4, so it will look just like the 4th shape) How about the 41st shape? (41 is one more than 40, which is a multiple of 4, so the 41st shape will look just like the 1st shape) How about the 83rd shape? (83 is 3 more than 80, which is a multiple of 4. So the 83rd shape will look just like the 3rd shape.) Explain to students that they can also use the remainder after dividing by 4 (the size of the core for this pattern) to predict the shape. For example: 83 ÷ 4 = 20 R 3. Since the remainder is 3, the 83rd shape will look just like the 3rd shape.

ACTIVITIES 1–2 (Essential), ACTIVITY 3 (Optional)

1. Place a sheet of grid paper (or BLM 1 cm Grid Paper) so that the long edge is horizontal. Copy a pentomino (from a commercial set or from BLM Pentominoes) into the top-left corner. Create a pattern by alternating reflection with translation. What would the 10th, 20th, and 33rd figures in your pattern look like? Exchange patterns with a partner. What shape was reflected? What translation was used? Repeat the activity but have students create patterns using only translations. Students must circle the core of their partners’ pattern and extend the pattern by adding at least two repetitions of the core.

2. Analyze images of patterns to find congruent shapes and examples of reflections and translations.

3. Create patterns on the rest of the page from Activity 1 by following these steps.
   a) Create a pattern of two or three other pentominoes down the left side of the page.
   b) Fill in the rest of the page by repeating the transformations you used in the first row in each of the other rows.
NOTE: Extensions 1 and 2 should be done in order.

Extensions

1. Draw on the board:

\[ \begin{array}{c|c}
1 & 2 \\
\hline
\end{array} \]

ASK: Can triangle 2 be obtained from triangle 1 by reflection? (yes; reflect in the dashed line) Do the triangles face the same direction or opposite directions? (Students’ answers may vary. Some may say that both directions “face up” since the triangles seem to point up. Others may say they face opposite directions since they are reflections of each other.) ASK: Can triangle 2 be obtained from triangle 1 by translation? (yes; translate to the right) Ask again if students think the triangles face the same direction. SAY: We normally say that shapes face in opposite directions after reflection, but it may not look like this when shapes have a line of symmetry parallel to the line of reflection. ASK: Does triangle 1 have a line of symmetry? (yes, a vertical line through the top vertex) Sketch the line of symmetry and ASK: Is this line of symmetry parallel to the line of reflection? (yes) Ask students to draw a few different pairs of shapes on grid paper that can be made using either a reflection or a translation. (Emphasize that the shapes they draw will need to have a line of symmetry parallel to the line of reflection.) Have them sketch the line of reflection and a translation arrow.

2. Explain that one way to tell the difference between a reflection and a translation when either will produce the image is to label the vertices. Ask students to choose how each vertex should be labelled in the first triangle below and write the labels. For example:

\[ \begin{array}{c|c|c}
A & B & C \\
\hline
\end{array} \]

ASK: If you go around the shape in alphabetical order, which direction are you moving in, clockwise or counter-clockwise? (clockwise) Tell students that one way to label corresponding vertices after a translation or reflection is to use prime notation. Write A' on the board somewhere far away from the picture and tell students that this is how you write “A prime.” Repeat with B' and C'. Tell students that we will perform a translation to obtain the second triangle and label corresponding vertices using prime notation. ASK: Where is the image of A after translation? (the top vertex) Write A' on the top vertex of the image. Draw a translation arrow from A to A'. Repeat with B' and C'. ASK: If we go around the image triangle in alphabetical order, which direction are
you moving in, clockwise or counter-clockwise? (clockwise) Explain that because the original and the image were both clockwise, we can say that the shapes have the same orientation. Explain that if the corresponding vertices of the original and the image in alphabetical order are both clockwise or both counter-clockwise, we say the orientation has not changed, whereas if one is clockwise and the other counter-clockwise, we say the orientation has changed. Redraw and repeat the above exercise, but this time have students tell you how to label the vertices for a reflection. The picture on the board should look like this:

![Diagram of translation and reflection]

ASK: After reflection, if we go around the image triangle in alphabetical order, which direction are we moving in, clockwise or counter-clockwise? (counter-clockwise) Did the orientation change? (yes) So what is the difference between a translation and a reflection, in terms of orientation? (translations do not change the orientation, but reflections do change the orientation) Have students choose one or two pairs of shapes they drew in Extension 1 and illustrate the difference between a translation and a reflection by labelling the vertices before and after using prime notation.

3. Provide each pair of students with geoboards and rubber bands of two different colours. Partner 1 creates a shape on the geoboard with one rubber band. She then creates another shape with a different colour of rubber band that is the image of the first shape after a translation or reflection. Partner 2 must identify whether the two shapes are related by a reflection or by a translation and must describe the reflection or translation (e.g., a reflection in this horizontal line or a translation 2 units left and 1 unit down). Partners switch roles and repeat.

4. Provide each pair of students with BLM Dot Paper and BLM Pentominoes. Students repeat a version of the previous extension using dot paper. The starting shape can be any polygon students wish to draw as long as all the vertices lie on dots. Students may refer to BLM Pentominoes for examples of shapes they can draw.

5. Provide each student with pattern blocks or shapes cut from BLM Pattern Blocks. Students choose one pattern block and use it to create a repeating pattern that results from translations. Students trace the pattern block into their notebooks to create the pattern. They should draw at least two repetitions of the core of the pattern. Students exchange patterns with a partner, circle the core of their partner’s pattern, and extend the pattern by adding at least one more repetition of the core.
Goals
Students will describe and perform rotations of 2-D shapes.

PRIOR KNOWLEDGE REQUIRED
Can identify the fractions $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of a circle

MATERIALS
large clock with a minute hand that can be turned in both directions for display
cardboard or bristol board, paper arrow, and pin to affix paper arrow to cardboard (optional)
grid paper or BLM 1 cm Grid Paper (p. S-1)
blue and red pencil crayons
a large paper L-shape
BLM Pentominoes (p. S-2)
BLM Polygons (p. O-55, see Extension 1)
scissors and BLM Isometric Grid Paper (p. O-61, see Extension 5)

Review fractions of a circle. Draw the circle in the margin on the board.
ASK: How many equal parts are shown on this circle? (4) How many parts are shaded? (1) So what fraction of the circle is shaded? (one quarter)

How do I write that as a fraction? (1 over 4; $\frac{1}{4}$) Write $\frac{1}{4}$ beside the circle.

Repeat, but this time shade both the top and bottom quarters of the circle. (2; one half; $\frac{1}{2}$) Remind students that it is more common to write this fraction as $\frac{1}{2}$ than as $\frac{2}{4}$. Repeat again with all but the top left quarter shaded (3; three quarters; $\frac{3}{4}$), and finally with all 4 quarters shaded.

For the last example, remind students that the amount shaded can be described as “one whole” or “1.”

Exercises: Write the amount shaded as one whole (1) or as a fraction.

a) b) c) d)

Answers: a) $\frac{1}{2}$, b) $\frac{1}{4}$, c) 1, d) $\frac{3}{4}$

Explain clockwise and counter-clockwise movement. Display a large clock at the front of the class. Alternatively, draw a clock face on cardboard or bristol board and pin a paper arrow to the centre so that the arrow can rotate like a clock hand. ASK: In which directions do the hands move on a clock? Have students signal their answers by motioning. Then have a volunteer come and show the direction on the clock at the front. SAY: This direction is called clockwise because it’s the direction that clock hands move. If we move the hands in the opposite direction (show the motion),
it’s called \textit{counter-clockwise}. Write the words “clockwise” and “counter-clockwise” on the board with a curved arrow beside each word to show the direction.

\textbf{Describing the rotation of clock hands.} Position the minute hand on the display clock so that it points straight up (at the 12). SAY: I’m going to turn the minute hand. Watch closely so that you can describe how I turned it. Sketch lightly in pencil the original position of the arrow and then turn the minute hand until it points to the 3. ASK: What direction did the minute hand turn, clockwise or counter-clockwise? (clockwise) Did it turn all the way around the clock or just a fraction of a full turn? (a fraction) What fraction did it turn? (1/4) Explain that students can look at the part of the circle the clock hand moved across to help them figure out the fraction of the turn. Write “\(\frac{1}{4}\) turn clockwise” on the board. Repeat with several more examples using only multiples of a quarter turn (1/4, 1/2, 3/4, or 1 whole turns). At first, do examples where the minute hand always begins pointing at 12 and the motion is clockwise, then progress to include examples of both directions and then examples of different starting points (where the arrow starts pointing at 3, 6, or 9 instead of 12).

\textbf{NOTE:} For the exercises in this lesson, you might wish to let students write the abbreviation CW for clockwise and CCW for counter-clockwise. Write “CW” and “CCW” beside the respective words on the board and underline the letters in the words that form the abbreviations.

\textbf{Exercises:} Write the fraction of the turn and the direction.

\begin{enumerate}
  \item [a)] \hspace{1cm} \hspace{1cm}
  \item [b)] \hspace{1cm} \hspace{1cm}
  \item [c)] \hspace{1cm} \hspace{1cm}
  \item [d)] \hspace{1cm} \hspace{1cm}
\end{enumerate}

\textbf{Bonus:}

\begin{center}
\begin{tikzpicture}
  \draw (0,0) circle (1cm);
  \fill (0,0) circle (2pt);
  \draw[->] (0,0) -- (180:1cm);
\end{tikzpicture}
\end{center}

\textbf{Answers:} a) 1/2 turn CCW, b) 3/4 turn CW, c) 1/2 turn CW, d) 1/4 turn CW, Bonus: 3/4 turn CCW

\textbf{ACTIVITY 1 (Optional)}

\begin{center}
1. Students stand up facing the front of the class with one arm stretched out in front. Tell them to imagine they are standing at the centre of a clock on the floor and that their outstretched arm is the minute hand. Explain that you will call out a turn by stating the fraction of the turn and the direction.
\end{center}
Students must turn their bodies and their outstretched arm according to the turn you describe. Repeat with several examples, such as 1/4 turn CW, 3/4 turn CCW, 1/2 turn CW, 1 whole turn CCW.

Show the steps for rotating an arrow. Draw and write the following on the board, making sure to draw the arrow the same size as the paper arrow from the display clock:

\[
\frac{1}{4} \text{ turn clockwise}
\]

SAY: Now we will turn an arrow using pictures only. To turn this arrow 1/4 turn clockwise, the first step is to determine the direction of the turn.
ASK: Which way should my curved arrow point? (clockwise) Have students show the direction of the curved arrow by motioning with their hands.
SAY: The next step is to determine the size of the curved arrow. ASK: How will I know when to stop drawing the curved arrow? (when you are 1/4 of the way around the circle) Add the curved arrow to the drawing on the board, as shown below:

\[
\frac{1}{4} \text{ turn clockwise}
\]

SAY: Now we can draw the final position of the straight arrow after a quarter turn clockwise. Finish the drawing on the board:

\[
\frac{1}{4} \text{ turn clockwise}
\]

Show students how to verify that this is the correct turn by placing the paper arrow on the starting position and turning it. ASK: Which is the only point of the arrow that did not move when we did this turn? (the end opposite to the arrowhead; the end that you hold down when you’re turning) Explain to students that the turns they have been exploring in this lesson are called rotations. The end point of the arrows that is at the centre of the pictures is called the centre of rotation, the point you turn around. It is the one point that does not move during a rotation. Repeat with several examples. Explain to students that the arrow at the finish position is also called the image after the rotation.

Students should use grid paper or BLM 1 cm Grid Paper for the following exercises.
Exercises: Draw the given rotation. Draw the arrow in the starting position in blue and draw the image after the rotation in red. Label the centre of rotation as point $P$.

a) $\frac{1}{4}$ turn clockwise  

b) $\frac{1}{2}$ turn counter-clockwise  

c) $\frac{3}{4}$ turn counter-clockwise  

d) 1 whole turn clockwise  

Bonus  

e) three $\frac{1}{4}$ turns clockwise  

f) two $\frac{1}{2}$ turns counter-clockwise  

Rotating 2-D shapes. Draw the picture in the margin on the board, using an L-shaped piece of paper to trace the shape. Show students how to rotate the shape $1/4$ turn clockwise by first rotating the dark line (the vertical edge in this case) just as they rotated arrows and then drawing the rest of the shape. Verify the drawing of the image by rotating the actual L-shaped piece of paper. Emphasize the importance of using a finger or the point of a pencil firmly pressed on the central point (the centre of rotation) to keep that point from moving. Repeat with several examples of different rotations of the L-shape, asking students to predict the position of the image before performing the rotation.

ACTIVITIES 2–3 (Essential)

2. Provide each pair of students with BLM 1 cm Grid Paper and the L-shape from BLM Pentominoes. Or you might provide students with scissors and have them cut out the L-shape themselves.

On the grid paper, Partner 1 traces a central horizontal grid line and a central vertical grid line and labels the point in the middle as point $P$. Point $P$ will be the centre of rotation. Partner 2 positions the shape so that it lies in one of the four regions and so that an edge lines up with one of the darkened grid lines and a vertex from that edge is on point $P$.

Partner 2 traces the shape in this starting position and then describes a rotation for Partner 1 to perform (for example, $3/4$ turn counter-clockwise around point $P$). Partner 1 predicts where the image will be after the rotation. After sketching the prediction, Partner 1 takes the shape from Partner 2 and performs the rotation, drawing the image.

Both partners compare the prediction to the actual image. Partners switch roles and repeat several times. Students can repeat the activity with different pentominoes.

3. Repeat Activity 2, except this time Partner 2 performs the rotation and sketches both the starting position and the image. Partner 1 must describe the rotation by indicating the fraction of the turn, the direction, and the centre of rotation (which will always be point $P$).
Extensions

1. Have students experiment doing 1/4 and 1/2 rotations with shapes from BLM Polygons. Ask students to find shapes that look like they are pointing in the same direction after a half turn around any vertex. (shapes B. (square), E. (rectangle), and K. (rhombus)) ASK: How about after a quarter turn? (only the square) Challenge students to think of a shape (not from the BLM) that would look like it is pointing in the same direction after a rotation of any amount. (a circle) Ask students if they can think of a way to show that a circle has been rotated. (sample answer: mark one point on the circle; after a rotation, the marked point moves)

2. As in Extension 1, students explore rotating polygons with shapes from BLM Polygons, but this time students focus on polygons that have an obvious centre point. Students mark the central point of the shape with a pencil and rotate the shape around the centre by pressing the point of the pencil firmly on the central point as they turn the shape. Challenge students to explain how they could use this kind of rotation to figure out which shapes will look like they are pointing in the same direction after various rotations. (sample answer: trace the shape in the starting position; shapes will look like they are pointing in the same direction after a certain rotation if they overlap perfectly with the starting position)

3. Have students perform rotations using grid paper and a pentomino from BLM Pentominoes. Students should use a pentomino that has no lines of symmetry. Ask students to perform the given counter-clockwise rotations and see if they can find a single clockwise rotation that gives the same result (in other words, the same ending position).

   a) \(\frac{3}{4}\) turn CCW
   b) \(\frac{1}{4}\) turn CCW
   c) 1 turn CCW
   d) \(\frac{1}{2}\) turn CCW
   e) \(\frac{1}{4}\) turn CCW
   f) \(\frac{3}{2}\) turn CCW
   g) 2 turns CCW
   h) \(\frac{3}{4}\) turn CCW

   Bonus

   e) \(\frac{1}{4}\) turn CCW
   f) \(\frac{3}{2}\) turn CCW

   Answers: a) 1/4 turn CW, b) 3/4 turn CW, c) 1 turn CW (or 0 turns CW), d) 1/2 turn CW, Bonus: e) 3/4 turn CW, f) 1/2 turn CW, g) 1 turn CW (or 2 turns CW, or 0 turns CW), h) 1/4 turn CW

4. Have students experiment with sequences of rotations using grid paper and a pentomino from BLM Pentominoes. Students should use a pentomino that has no lines of symmetry. Ask students to perform two rotations in a row (for example, a 1/4 turn CW followed by a 1/4 turn CW). ASK: How could that rotation be described as a single rotation? (1/2 turn CW or CCW) Students verify by trying the single rotation from the starting position. For the questions below, students determine and
write down a single rotation that gives the same result as the given sequence of rotations. Challenge students to find the smallest single rotation that gives the same ending position.

a) $\frac{1}{4}$ turn CW, $\frac{3}{4}$ turn CCW  
   b) $\frac{1}{2}$ turn CCW, $\frac{3}{4}$ turn CW  
   c) $\frac{3}{4}$ turn CW, $\frac{1}{2}$ turn CCW  
   d) $\frac{3}{4}$ turn CCW, $\frac{3}{4}$ turn CCW

**Bonus**

e) $\frac{1}{4}$ turn CCW, $\frac{3}{4}$ turn CW, $\frac{1}{2}$ turn CW  
   f) $\frac{3}{4}$ turn CCW, $\frac{3}{4}$ turn CCW, $\frac{3}{4}$ turn CW

**Answers:** a) $\frac{1}{2}$ turn CCW (or CW), b) $\frac{1}{4}$ turn CW, c) $\frac{1}{4}$ turn CW, d) $\frac{1}{2}$ turn CCW (or CW), Bonus: e) 0 turns CW (or CCW), f) $\frac{1}{4}$ turn CW

5. Have students cut the triangle out from shape card A of BLM Polygons. Provide students with BLM Isometric Grid Paper. Students position the triangle close to the centre of the isometric grid, with the three edges of the triangle lining up with grid lines. Students mark a dot on the grid at the vertex of the triangle closest to the centre of the grid. They trace the starting outline of the triangle and then rotate the paper triangle slowly around that vertex until all the edges of the triangle again line up with grid lines. They trace the triangle in that position and continue in this way until they return to the starting position. ASK: How many positions of the triangle did you trace altogether? (6) What 2-D shape do all the triangles together make? (a hexagon) What fraction of the hexagon is each triangle? (1/6) So if you rotate the triangle to go from one position to the next, what fraction of a full turn is that? (1/6)

Working in pairs, students take turns asking their partners to show any of the following rotations in either direction of the triangle around the marked vertex: $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, or 1 whole turn. Remind students that $1/3 = 2/6$, $1/2 = 3/6$, and $2/3 = 4/6$. 

Geometry 5-19

O-45
Goals

Students will compare and distinguish between translations, reflections, and rotations.

PRIOR KNOWLEDGE REQUIRED

Can identify a translation, reflection, and rotation of a 2-D shape
Can describe a translation, reflection, and rotation of a 2-D shape
Can perform a translation, reflection, and rotation of a 2-D shape

MATERIALS

overhead projector
transparency of BLM 1 cm Grid Paper (p. S-1)
2 identical L-shaped pieces of paper
flag made from a straw and paper
BLM Triangle Transformations (p. O-62), tracing paper, and scissors
BLM 1 cm Grid Paper (p. S-1)
BLM Pentominoes (p. S-2)
images of designs made by transformations (see Extension 1)
materials gathered outside (see Extension 5)

Mental math minute. Remind students that to multiply whole numbers mentally by 10, 100, or 1000 they need to write zeros to the right of the last digit. ASK: How many zeros do you write when you multiply by 10? (1) by 100? (2) by 1000? (3) Invite a volunteer to explain why $3 \times 10 = 30$, $4 \times 100 = 400$, and $6 \times 1000 = 6000$, referring to base ten blocks. Give students 1-digit numbers and have them multiply each number by 10, 100, or 1000. Progress to 4-digit numbers. Use 10, 100, and 1000 in random order and use as many volunteers as possible to check answers.

NOTE: You can project BLM 1 cm Grid Paper on the board for the parts of this lesson that require a grid on the board.

Review translations, reflections, and rotations. Tell students that they will play a game again from a previous lesson (G5-15) about describing a series of transformations to move one L-shape on top of another. Affix one paper L-shape to the board with tape and hold another identical L-shape farther away and oriented differently. Remind students that translations, reflections, and rotations are all called transformations. Students must describe one transformation at a time, which you must perform on the L-shape you are holding so that it lands perfectly on the affixed L-shape. Emphasize that the instructions must be very precise since they are supposed to be performed by a robot. To guide students to be precise about describing the transformations, ask questions such as: How many units and in which direction should I translate the shape? Which line of reflection should
I reflect in? What fraction of a turn should I rotate the shape, in which direction, and around which point?

**Direction of shapes after transformations.** Draw the cardinal directions somewhere on the board and position a flag made from a straw and paper somewhere on the grid, pointing east:

```
\[ \text{N} \quad \text{E} \quad \text{S} \quad \text{W} \]
```

SAY: We will use the directions north, east, south, and west to describe the direction the flag is pointing. ASK: In which direction is the flag pointing? (east) Ask volunteers to perform several translations of the flag one at a time (for example, 2 units east and 1 unit south) After each translation, ask students which direction the flag points. (east) ASK: After a translation, does the image flag point in the same direction as the original flag? (yes) Repeat with several examples where the flag begins pointing in different directions. Emphasize that the direction of the flag does not change after a translation.

Repeat the above for reflections, making sure to use at first only lines of reflection that are parallel to the flag pole: if the flag is pointing east or west, use a vertical line of reflection; if the flag is pointing north or south, use a horizontal line of reflection. ASK: After a reflection, do the original shape and the image face the same direction or opposite directions? (opposite)

Tell students you will show them a tricky example now. With the flag upright and pointing east, draw a horizontal line of reflection underneath the flag. Trace the original and then ask a volunteer to perform a reflection in the horizontal line and draw the image. ASK: Which way was the flag pointing at first? (east) In which direction is the flag pointing in the image? (east) Does this mean the shape and its image face in the same direction? Explain that in order to understand this example, students need to look at the entire shape, which includes the flag piece and the flag pole. Even though the flag is pointing east before and after the reflection, the flag is upright at first and then upside-down in the reflection. SAY: So we can still say the shape and its image are facing in opposite directions, even though the flags are both pointing east.

Perform examples of rotations with the flag, always rotating around the “bottom” end point of the flag pole, and ask students to say in which direction the original and the image face. ASK: Is there any rotation you can do so that the original flag and the image face the same direction? (yes, a whole turn in either direction)
ACTIVITIES 1–3 (Essential)

1. Provide students with **BLM Triangle Transformations**. Have student use tracing paper to trace one of the triangles on the page and cut it out. Students use the paper triangle to help them answer all the questions on the BLM.

2. Provide each pair of students with grid paper or **BLM 1 cm Grid Paper** and the L-shape from **BLM Pentominoes**. Or you might provide students with scissors and have them cut out the L-shape themselves.

   On grid paper, Partner 1 positions the L-shape so that it lines up with the grid lines. Partner 1 traces the shape in this starting position and then writes a transformation (either a translation, reflection, or rotation) for Partner 2 to perform. Partner 1 must specify the details of the transformation: the number of units and directions for translations; the line of reflection for reflections; the centre of rotation, the direction of a turn, and the fraction of a turn for rotations. Partner 2 predicts where the image will be after the transformation without using the paper shape. After sketching the prediction, Partner 2 takes the shape from Partner 1 and performs the transformation, drawing the image. Both partners compare the prediction to the actual image. Partners switch roles and repeat several times. Students can repeat the activity with different pentominoes.

3. Repeat Activity 2, except this time Partner 1 performs the transformation and sketches both the starting position and the image. Partner 2 must identify and describe the transformation, including all the details. Partners switch roles and repeat several times. Encourage students to consider the direction of the original shape and the image when trying to figure out what transformation was performed.

**NOTES:** For Questions 5 – 7 on AP Book 5.2 pp. 23 – 24, students who are struggling can use the paper triangle they cut out in Activity 1 for assistance. Extension 1 is required in order to cover the British Columbia curriculum.

**Extensions**

1. Show students images of designs (such as designs made by weaving, or in traditional clothing, tiles, or cedar baskets) made by translations, reflections, and/or rotations of shapes. Students choose one or more designs and describe how the design was generated by starting from a shape and then reflecting, rotating, and/or translating the shape. Encourage students to be as precise as possible when describing the transformations. Ask guiding questions such as: Can you trace the shape that was reflected with your finger? Where is the line of
reflection? What is the centre of rotation? What is the direction of the rotation? Can you trace a translation arrow?

2. Students work in pairs. Partner 1 chooses a transformation (translation, reflection, or rotation). Partner 2 explains what the transformation means and provides an example (by drawing or using concrete materials). Partners alternate until each partner has had a chance to explain each type of transformation. Partners cannot use an example that has already been used. (For example, if Partner 2 has already provided an example of a reflection, when Partner 1 provides an example of a reflection, she must use a different shape or a different line of reflection.)

3. a) Nora says that reflecting in a horizontal line and then a vertical line is the same as reflecting in a diagonal line. Do you agree with Nora? Explain why or why not.

b) Is there a single transformation that is the same as reflecting first in a horizontal line and then in a vertical line? If so, describe the transformation. If not, explain why not.

Answers: a) no, the images will not necessarily be the same; b) yes, a half-turn rotation will produce the same image

4. Provide each student with grid paper or BLM 1 cm Grid Paper. Students play the following game in pairs. Player 1 draws a shape that has no lines of symmetry on his grid. Player 2 copies the shape onto her grid in the same location. Player 1 performs a single transformation: he rotates the shape 1/4, 1/2, or 3/4 of a turn in the direction of his choice around a vertex of the shape, reflects the shape through a vertical or a horizontal line, or translates the shape. He draws the image, but he does not let Player 2 see the image. Then he describes the image based on its position relative to the original shape and its direction/orientation. Player 2 tries to draw the image on her grid and identifies whether the transformation performed by her partner was a translation, reflection, or rotation. Players switch roles and repeat.

5. Have students use local materials gathered outside to represent examples of patterns and designs that could be created by transformations. Students explain to a partner or to the whole class where they see translations, reflections, and rotations in the materials they have gathered.
Grid with Tens

$B (10, 35) \quad C (4, 20) \quad D (27, 6) \quad E (41, 33) \quad F (0, 23)$
Shapes for Folding (1)
Shapes for Folding (2)
Shapes for Folding (3)
Shapes for Folding (4)
Polygons

A. 

B. 

C. 

D. 

E. 

F. 

G. 

H. 

I. 

J. 

K. 

L. 

Blackline Master — Geometry — Teacher Resource for Grade 5
Pattern Blocks

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Triangle

Square

Parallelogram

Trapezoid

Hexagon
Tangrams
Dot Paper
Map of Saskatchewan

- Uranium City
- Wollaston Lake
- Cleanwater River Provincial Park
- Prince Albert
- Saskatoon
- Moose Jaw
- Regina
- Weyburn
- Swift Current
- Maple Creek
Coordinate Grid Map of Saskatchewan

- Uranium City
- Wollaston Lake
- Clearwater River Provincial Park
- Prince Albert
- Saskatoon
- Moose Jaw
- Regina
- Swift Current
- Maple Creek
- Weyburn
Triangle Transformations

1. Describe a transformation used to move the triangle from position 1 to position 2.
The side of each small square is one unit.

   a)     b)
   \[
   \begin{array}{c}
   \text{Line 2} \\
   \hline
   2 \\
   P \\
   1 \\
   \text{Line 1}
   \end{array}
   \quad
   \begin{array}{c}
   \text{Line 2} \\
   \hline
   2 \\
   P \\
   1 \\
   \text{Line 1}
   \end{array}
   \]

   c)     d)
   \[
   \begin{array}{c}
   \text{Line 2} \\
   \hline
   1 \\
   P \\
   2 \\
   \text{Line 1}
   \end{array}
   \quad
   \begin{array}{c}
   \text{Line 2} \\
   \hline
   1 \\
   P \\
   2 \\
   \text{Line 1}
   \end{array}
   \]

2. Trace your triangle in a starting position. Perform either a translation, rotation, or reflection and draw the image. Describe the transformation you used.

   a)     b)
   \[
   \begin{array}{c}
   \text{Line 2} \\
   \hline
   P' \\
   \text{Line 1}
   \end{array}
   \quad
   \begin{array}{c}
   \text{Line 2} \\
   \hline
   P' \\
   \text{Line 1}
   \end{array}
   \]
Unit 13  Geometry: 3-D Shapes

Introduction
This unit explores the geometry of 3-D shapes, including:
• describing and categorizing prisms and pyramids;
• constructing skeletons of three-dimensional figures;
• identifying parallel, intersecting, perpendicular, vertical, and horizontal edges and faces in 3-D shapes;
• identifying prisms and pyramids from their nets; and
• constructing nets of prisms and pyramids.

Meeting Your Curriculum

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<td>Optional</td>
<td>G5-25, 26</td>
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<td>ONTARIO</td>
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<tr>
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<td>G5-24</td>
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Mental Math Minutes
The mental math minutes in this unit:
• provide practice in addition, subtraction, multiplication, and division

Generic BLMs
The Generic BLM used in this unit is:
BLM 1 cm Grid Paper (p. S-1)
This BLM can be found in Section S.
Materials

The following materials are used in several lessons and should be collected in advance:

- rectangular and triangular prisms, such as cereal boxes and confectionary boxes
- prisms and pyramids with various bases (if none are available, they can be constructed using BLM Nets)
- modelling clay
- toothpicks or other sticks of various lengths
- plastic knives

Assessment

The lessons covered by a quiz or test are as follows:

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<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tr>
<td>Quiz</td>
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<td>G5-21 to 23</td>
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<tr>
<td>Quiz</td>
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<td>G5-25, 26</td>
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<tr>
<td>Test</td>
<td>G5-21 to 24</td>
<td>G5-21 to 24</td>
<td>G5-21 to 24</td>
<td>G5-21 to 23, 25, 26</td>
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G5-21 3-D Shapes

Pages 104–106

Goals

Students will identify faces, vertices, and edges of 3-D shapes.
Students will count vertices and edges of 3-D shapes using actual shapes and pictures.
Students will identify intersecting faces and edges in 3-D shapes.

PRIOR KNOWLEDGE REQUIRED

Can identify and count sides and vertices of 2-D shapes
Can identify and name polygons
Can identify intersecting sides in 2-D shapes

MATERIALS

cube, prism, and pyramid per student
large paper square
various 3-D shapes, including a cube, a square-based prism, a square-based pyramid, and some everyday objects (e.g., boxes, a ball, a can, a hockey puck)
BLM Matching 3-D Shapes (p. P-31, see Extension 1)
flashlight or overhead projector (see Extension 3)
cone, cylinder, and sphere (see Extension 4)

Mental math minute. Have groups of three students add two-digit numbers by adding tens and adding ones. First give an addition problem, such as 35 + 46. The first student adds the tens (30 + 40 = 70). The second student adds the ones (5 + 6 = 11). The third student finishes the addition (70 + 11 = 81), so 35 + 46 = 81.

Introduce faces. Give each student a cube, a prism, and a pyramid—use a variety of different shapes. Hold up a large cube and ask if anyone remembers what it is called. Students should be familiar with cubes from earlier grades. Have students identify the cube in their collections and hold it up.

Explain that the flat sides of a 3-D shape are called faces. Point to the faces on the large cube. ASK: What polygons are the faces of a cube? (squares) SAY: Hold up a shape that has some faces that are not squares. Ask volunteers to show the shape to the class, point out the face that is not a square, and identify the shape of that face. Point out that some 3-D shapes have faces that are triangles and some 3-D shapes have faces that are rectangles. ASK: Does anyone have a shape that has only triangles as faces? Have a volunteer show this shape to the class if they have one, turning it so that everyone can see that it has only triangular faces. Repeat with a shape that has only rectangles for faces. Remind students that squares are a special case of rectangle, so a cube qualifies. Ask students if anyone has a shape that has only rectangles as faces but is not a cube. Again, have a volunteer show this shape if they have one.
Counting faces. Ask students to count the faces on their cubes. Discuss strategies to keep track of faces counted. If the following strategy does not arise, show it to students. Count the top and the bottom first, then look at the shape from the top. From the top, the cube looks like a square. But each side of that square is the side of another “hidden” square that you can see when you look at the cube from the side. Because we know that a square has four sides, there are four “hidden” squares. Four squares (on the sides) and two squares (top and bottom) equals six squares altogether, which means that a cube has six faces.

Have students count faces on the other two shapes in their collections. Invite volunteers to show their shapes and explain how they kept track of the faces. Students who finish early can exchange shapes with a partner and count the faces on their partner’s shapes.

Introduce edges of 3-D shapes. Hold up a cube. Run your finger along one of the edges and explain that the place where two faces meet is called an edge. Ask students to show an edge on their cubes.

Pointing to two adjacent faces on the cube, SAY: These two faces meet, or intersect, at this edge. Trace the edge again with your finger. SAY: If two faces on a 3-D shape meet at an edge, they are called intersecting faces. Show various pairs of faces and ask students to signal thumbs up or down to show whether the faces intersect. If they intersect, have a volunteer trace the edge where they meet.

NOTE: The faces of a 3-D shape are sometimes referred to informally as the sides of the shape, but the edges of a 2-D shape are also often called the sides of the shape. In the case of 3-D shapes, encourage students to use the precise words “edges” and “faces” rather than “sides.”

Counting edges. Hold up a square-based prism and explain that you want to count the edges of this shape. Place the prism on the desk, square face down. SAY: I see three groups of edges. There are edges on the bottom face, the edges that run along the desk. They are the sides of the bottom face. Lift up the prism and trace the edges of the bottom face. ASK: How many edges like this do we have? (4) Write “4” on the board. Place the prism on the desk again. SAY: There are edges that only touch the desk at one end, the longer, vertical edges. Trace these edges with a finger and have students show the same edges on their square-based prisms. ASK: How many edges like this do we have? (4) Write “+ 4” on the board. SAY: There are edges that do not touch the desk at all. They are the sides of the top face. Trace them with a finger. ASK: How many edges like this does the shape have? (4) Write “+ 4” on the board. ASK: Did we miss any edges? (no) How many edges are there in total? (12) Write “= 12” on the board. SAY: The shape has 12 edges.

Have students count the edges of their other two shapes. Have them discuss strategies in pairs. Students who finish early can exchange shapes with a partner and count the edges of their partner’s shapes.
Introduce vertices of 3-D shapes. Show students a large paper square. Invite a volunteer to identify the vertices. ASK: How do you know this is a vertex? (it is a corner where sides meet) Do vertices feel different? (they are pointy, they are sharp turns, they feel the sharpest) Ask students to hold a cube. Ask them to feel their cubes and show the places on the cubes that feel pointy. Explain that these are also called vertices. Ask a volunteer to show the vertices of their cube. Have students show the vertices of their other 3-D shapes. Point out that edges of 3-D shapes meet at vertices. SAY: Just as sides of flat shapes meet at vertices, edges in 3-D shapes also meet at vertices.

Counting vertices. Discuss strategies for counting vertices of a cube. Guide students through the following strategy if no one suggests it. Set the cube on a desk. Point at two vertices, one on the bottom face and the other on the top face. ASK: How are these two vertices different? (one is on the top and the other is on the bottom, or one touches the table and the other does not). Can we first count all of the vertices on the bottom, then all of the vertices on the top? (yes) How many vertices are on the bottom? (4) How many vertices are on the top? (4) Are there any vertices in the cube that we did not count? (no) Write on the board:

\[ 4 + 4 = 8 \]

A cube has 8 vertices.

SAY: By counting the bottom vertices separately from the top vertices, we solved two simpler problems and used them to solve the harder problem. ASK: Why was it so easy to find the number of vertices on the bottom? (the bottom is a square) How is the top face similar to the bottom? (it is also a square) How many vertices do two squares have? (8) How many vertices does a cube have? (8)

Ask students to count the vertices of their other two shapes. Students can problem-solve in pairs how to track the number of vertices. Have students share their strategies with the class. Students who finish early can exchange shapes and count the vertices on their partner’s shapes.

Vertices and edges on pictures of 3-D shapes. Draw on the board:

ASK: What shape is this? (a cube) Invite a volunteer to place a dot on each vertex she can see in the picture. Count the vertices together and write the numbers beside the dots as you do so. ASK: How many vertices are in the picture? (7) How many vertices does a cube have? (8) Why did we get seven instead of eight? (there is a corner on the back that we do not see)

Repeat with edges. Students will see that there are nine edges visible and three edges on the back.

Introduce hidden edges. Explain that in mathematics people often draw the parts of shapes that we cannot see (because they are hidden behind other parts) with dashed lines. The dashed lines are behind the solid lines.
and would only be seen if the shape were made of a clear, see-through material, such as glass. Use your hands or two pencils to show the relative positions of a visible edge and a hidden edge in the cube that look like they intersect. SAY: We call the edges we cannot see in a picture hidden edges. Add the dashed lines to the cube on the board and invite a volunteer to mark the hidden vertex. (see margin)

Draw on the board:

SAY: I drew dots on the vertices of this box. ASK: Did I draw everything correctly? (no) Point to each dot one at a time. ASK: Is this a vertex? Have students signal the answer with thumbs up for yes and thumbs down for no. Erase (or cross out) the two incorrect vertices and have volunteers count the vertices (8) and the edges (12).

**Faces that intersect at a vertex.** Show students a square-based pyramid. Point out two opposite triangular faces. ASK: Do these faces share an edge? (no) Do these faces share a vertex? (yes) Point to several vertices before pointing to the apex and each time have students signal thumbs up or down to show whether the vertex lies on both faces. SAY: These two faces are called intersecting faces because they meet, or touch, or intersect each other. They don’t meet at an edge, but they meet at a vertex. Point out that all four triangular faces on the pyramid meet at the same vertex. Write “vertex” on the right side of the board and “edge” on the left. Point to various pairs of faces on the pyramid and ask students to point to the correct side of the board to signal whether the faces intersect at an edge or a vertex. After students have answered, point to the edge or vertex where the faces meet or have a volunteer do so.

**ACTIVITY (Essential)**

Students work in groups of three or four. Provide each group with at least three 3-D shapes, including a pyramid and a rectangular prism. Player 1 chooses a 3-D shape, points to two edges on the shape, and asks Player 2 to say whether the edges are intersecting edges and, if so, to point to the vertex where they meet. Player 2 then chooses two different edges and asks the same questions to Player 3. Play continues until all students have had a turn to choose two edges. Students repeat the process for faces, identifying whether two faces intersect and, if so, pointing to the edge or vertex where they meet. Have groups repeat the activity for edges and faces using at least three different 3-D shapes.
Extensions

1. Have students play the games below using the cards from BLM Matching 3-D Shapes. The cards match if they show the same shape. For example, the long, thin rectangular prism on card 13 would not match the short, thick rectangular prism on card 11.

   **Picking Pairs.** Students play in pairs or individually. Place cards face up in an array. Students take turns picking pairs of matching cards and placing them in a common discard pile. When there are no more pairs in the array, more cards are added to it. The goal is to place all the cards into the discard pile. If students have any non-matching cards left at the end, then some of their cards must have been matched incorrectly.

   **Memory.** This version of the well-known game is played like Picking Pairs, but the cards are face down. Students turn over two cards at a time looking for a match. If the cards match, students set them aside; otherwise, they turn them face down again and continue playing. Students can play individually or cooperatively in pairs. In either case, the goal is to find all the matches. If playing with a partner, Player 1 leads by choosing and turning over a card and Player 2 follows by choosing and turning over another card. Players switch roles after each turn.

2. Ask students to hold cubes in various positions (for example: face-on, looking at a vertex, and so on) so that they see it from different angles. Ask students to describe what the faces look like when seen from different angles. The faces will look like squares, rectangles, parallelograms, or rhombuses depending on the position.

3. Have students use a cube and a flashlight or overhead projector. Tell students that by holding a cube in different positions, they can produce shadows of different shapes. Ask students what polygons they can produce as a shadow.

   **Answers:** square, rectangle, hexagon, trapezoid, rhombus

4. Show students a cone. **ASK:** How many flat faces are on the cone? (1) What is the shape of the flat face? (circle) Is a circle a polygon? (no) Ask students if they think there are any other faces on the cone. Explain that the curved surface of the cone can be called a curved face. **ASK:** Do the curved face and the flat circular face intersect? (yes) Do the faces meet at an edge or a vertex? Explain that the circular edge of the flat circular face is the edge where the two faces meet. The edge is not a straight edge but a curved edge. Have students examine a cone, a cylinder, and a sphere and write down the number of flat faces, curved faces, edges, and vertices of each.

   **Answers:** cone: 1 flat face, 1 curved face, 1 edge, 1 vertex; cylinder: 2 flat faces, 1 curved face, 2 edges, 0 vertices; sphere: 0 flat faces, 1 curved face, 0 edges, 0 vertices

**NOTE:** Some definitions of “vertex” do not include the apex of a cone since there are no edges that meet at the apex. For the purpose of this grade, it will be considered a vertex.
**Goals**

Students will construct skeletons of triangular and rectangular prisms. Students will compare skeletons with actual shapes. Students will name and distinguish between prisms by the shapes of their bases. Students will explore intersecting and non-intersecting faces on prisms.

**Prior Knowledge Required**

- Can recognize and name polygons and cubes
- Can identify faces, vertices, and edges on 3-D shapes

**Materials**

- paper triangle that is an enlarged copy of a triangular pattern block
- triangular and square pattern blocks
- toothpicks or sticks of various lengths
- modelling clay
- triangular and rectangular prisms
- various blocks, including prisms from a commercial set or built from BLM Nets (1) to (4) (pp. P-32–35, see Extension 1)
- plastic knives (see Extension 2)
- connecting cubes (see Extension 3)

**Mental Math Minute.** Have students subtract three-digit numbers by subtracting hundreds, tens, and ones separately in groups of four. Give a subtraction problem, such as 542 − 231. The first student subtracts the ones (2 − 1 = 1). The second student subtracts the tens (40 − 30 = 10). The third student subtracts the hundreds (500 − 200 = 300). The fourth student adds the results to complete the subtraction (300 + 10 + 1 = 311, so 542 − 231 = 311). Start with problems that do not require regrouping in any place, such as 635 − 324, and continue to problems that require regrouping tens or hundreds, but not both. If tens have been regrouped, for example, the student subtracting tens must adjust.

**Introduce Prisms.** Show students a paper triangle that is an enlarged copy of a triangular pattern block. Ask them to identify the shape. Show students the pattern block and ask them how it is different from the paper triangle. (the paper triangle is larger and flat, or 2-D; the pattern block has thickness and is a 3-D shape) Have several volunteers stack different numbers of triangular pattern blocks, one on top of the other. Discuss what the result looks like. (it has a triangle on top and the same triangle on the bottom, the other faces are rectangles) Explain that this shape is called a **triangular or triangle-based prism**. Tell students that a **prism** is a 3-D shape with two congruent faces called bases and faces that are rectangles or parallelograms connecting the two bases. **SAY:** You can make prisms with
any polygon as the base. Then have students stack square pattern blocks and introduce the terms square prism and rectangular prism.

Ask students for examples of prisms in the real world. Examples of square or rectangular prisms include tall buildings and food boxes. Examples of triangular prisms include some chocolate bar boxes, closed binders, wedges, ramps, and pointed rooftops. ASK: What is the difference between rectangular prisms and cubes? (rectangular prisms can have faces shaped like any rectangle, the faces of cubes are all squares) SAY: Cubes are a special example of rectangular prisms. Square prisms have some square faces but not necessarily all square faces, so they are special cases of rectangular prisms too.

Building skeletons of prisms. Tell students that a skeleton of a prism is the frame of the prism, the vertices and edges without the faces. Demonstrate making a skeleton of a triangular prism using toothpicks and modelling clay. Write the following steps on the board as you demonstrate them. When demonstrating Step 1, explain that the polygon you start with is the base of the prism and that each prism has two bases, which are on opposite sides of the figure.

Step 1: Make two copies of the same polygon using clay balls for vertices and toothpicks for edges.

Step 2: Add one toothpick to each vertex of one of the polygons.

Step 3: Attach the other polygon on top of the loose ends of the toothpicks.

ACTIVITY 1 (Essential)

1. Have students use toothpicks and modelling clay to construct the skeletons of two types of prisms, triangular and rectangular. Discuss how the skeletons that students created are the same as prisms and how they are different. (skeletons have vertices and edges only, no faces)

Tell students that skeletons make it easier to count edges because there are no hidden edges.

Counting vertices and edges of prisms. Draw on the board:

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<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Have students help you fill in the table. Point to each cell and have students count the vertices or edges in the skeletons they made in the activity and signal their answers. The completed table is shown below.

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

**ACTIVITIES 2–3 (Essential)**

2. Provide groups of students with triangular and rectangular prisms. Each student must record the number of triangular and rectangular faces for a triangular prism (2 triangular faces, 3 rectangular faces) and for a rectangular prism (0 triangular faces, 6 rectangular faces). Group members compare results with each other and then with the class. **ASK:** Do all triangular prisms have the same number of triangular and rectangular faces? (yes) Do all rectangular prisms have six rectangular faces and no other faces? (yes)

3. Using the same 3-D objects as in Activity 2, have students explore which faces intersect and which do not intersect in triangular and rectangular prisms. Students take turns pointing to various pairs of faces, and another student in the group says whether the faces intersect. After repeating this several times, have students consider these questions: Is there a pair of faces in a triangular prism that never intersect? (yes, the bases) Are there any pairs of faces in a rectangular prism that never intersect? (yes, any pair of opposite faces)

**Bonus:** Are there any faces in a triangular prism that intersect every other face? (yes, any of the rectangular faces)

**Extensions**

1. Give each student or pair of students blocks, including several prisms from a commercial set or built from **BLM Nets (1) to (4)**, and have them create structures. Ask students to identify the prisms in their structures and then in structures built by other students.

2. Have students make prisms out of modelling clay. They can use plastic knives to cut the clay and create flat surfaces.

3. Have students create rectangular prisms using connecting cubes.

4. Give each student a rectangular prism. Have students find all of the congruent faces.
5. Hanna says the shape below is a square-based prism because there are two square faces on opposite ends of the shape and four matching side faces. Do you agree with Hanna? Explain.

Sample answer: I disagree with Hanna. A square-based prism must have two square faces on opposite ends that are congruent. This shape has two square faces on opposite ends, but they are not congruent since one square is smaller than the other. Also, the faces of a square-based prism that are not the bases are supposed to be rectangles or parallelograms, but the other faces on this shape are trapezoids.
Goals
Students will name prisms and pyramids by the shapes of their bases.
Students will construct skeletons of prisms and pyramids with various bases.
Students will use various properties to distinguish between prisms and pyramids.

PRIOR KNOWLEDGE REQUIRED
Knows that a prism has two identical bases on opposite ends
Can recognize and name polygons
Can construct skeletons of triangular and rectangular prisms

MATERIALS
trapezoidal, pentagonal, and hexagonal pattern blocks
toothpicks or sticks of various lengths
modelling clay
skew prism or shape built from BLM Nets (5) (p. P-36)
several pyramids with different bases
pictures of real-life pyramids
pyramid-shaped teabags (optional)
plastic knives (see Extension 1)
BLM Faces (p. P-47, see Extension 6)

Mental math minute. Ask students to solve multiplication problems within 1 × 1 to 10 × 10 and the corresponding division problems. For each number, go through the problems in order, for example, from 1 × 3, 3 ÷ 3, 2 × 3, 6 ÷ 3, and so on, to 10 × 3 and 30 ÷ 3. Then progress to a different number. Finally, try problems out of order but keep corresponding multiplications and divisions together.

Introduce other bases. Tell students that prisms don’t need to have triangular or rectangular bases. Distribute trapezoidal, rhombic, and hexagonal pattern blocks. Have students choose one of these pattern block shapes and create a prism by stacking more of them. Ask them to show and name their prism. (trapezoid-based prism, rhombus-based prism, hexagon-based prism) Explain that the faces of a prism that are not the bases are often called the side faces.

Making skeletons. Remind students how they constructed skeletons of prisms by making two copies of the base and then attaching them. Ask: What was the advantage of having a skeleton? (it was easy to count vertices and edges) Why isn’t a skeleton a real prism? (it doesn’t have faces)
ACTIVITY 1 (Essential)

1. In groups of three, have students use toothpicks and modelling clay to construct skeletons of three types of prisms: trapezoidal, pentagonal, and hexagonal.

Counting edges, vertices, and faces. Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>trapezoid</th>
<th>pentagon</th>
<th>hexagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Edges in the Base</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Vertices in the Prism</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Number of Edges in the Prism</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Have students help you fill in the table. Point to each cell and have students use the skeletons they made in the activity to count the vertices or edges and then signal their answers. The completed table is shown below.

Ask students to consider the columns in the table. ASK: Do you see any patterns? (the number of vertices in the prism is double the number of edges in the base; the number of edges in the prism is three times the number of edges in the base)

Sketching faces. Write the names of the prisms from the activity on the board. For each, have a volunteer draw a complete set of faces. Then have the volunteer or the class count the total number of faces and write the number below the drawings. (see sample drawings below)

ASK: How many edges does a trapezoid have? (4) How many side faces does a trapezoid-based prism have? (4) Why are these numbers the same? (you have one side face for each edge) How many faces does the prism have altogether? (6) SAY: It has a side face for every edge in the base plus two more for the top and bottom faces. ASK: How many faces would a prism have if its base had eight edges? (10)

Right prisms and skew prisms. In advance, use toothpicks and modelling clay to build a skeleton of a pentagonal prism. Place it on a flat horizontal surface where all students can see it. Tell students to imagine the skeleton
is covered with paper, so that the shape has faces and is an actual prism rather than just a skeleton of a prism. ASK: Where are the two bases on this pentagon-based prism? (on the top and bottom) Is the top base directly above the bottom base? (yes) What shape are the side faces? (rectangles)

Tell students you will change the shape slightly. Push two adjacent vertical sticks forward so that the top base is no longer directly above the bottom base. ASK: In this new shape, is the top base directly above the bottom base? (no) Are all the side faces rectangles? (no) What shape are the side faces? (parallelograms) Explain that this shape is called a skew prism. In a skew prism, the side faces are parallelograms (not necessarily rectangles) and the top base is not directly above the bottom base. Explain to students that most of the prisms they have seen so far are called right prisms. SAY: In a right prism, the top base is directly above the bottom base, and the side faces are all rectangles. Place a right prism beside a skew prism, each with one base flat on a table. ASK: When I place a right prism on a flat table, does it have vertical edges? (yes, the edges of all the side faces are vertical) Remind students that edges are vertical if they run straight up and down. ASK: When I place a skew prism on a flat table, does it have vertical edges? (no) Show students at least one skew prism, such as the rhombus-based skew prism built from the net in BLM Nets (5).

NOTE: Skew prisms are also called oblique prisms, but you need not teach students this term.

ACTIVITY 2 (Optional)

2. Have students use the prism skeletons they created in Activity 1 to create skeletons for skew prisms.

Introduce pyramids. Hold up several pyramids and place them base-down in front of students. Point out the similarities among the shapes: they all have a polygon at the bottom, and they all have one vertex (point to it) opposite that polygon. Hold up the pyramids one at a time and show students that the base changes in each of the shapes. Explain that all of these shapes are pyramids. Ask students if they have heard this word before. Write "pyramid" on the board and read the word aloud together. Show pictures of pyramids in real life, such as the Great Pyramid of Giza (Egypt), the Louvre Pyramid (Paris, France), the Muttart Conservatory (Edmonton, AB), and some tents. If possible, show pyramid-shaped teabags.

Building skeletons of pyramids. Demonstrate how to make a rectangular (rectangle-based) pyramid with toothpicks and modelling clay. You will need six longer and two shorter toothpicks. Write the following steps on the board as you demonstrate them. When demonstrating the first step, explain that as with prisms, the polygon you start with is called the base of the pyramid.

Step 1: Make a polygon using clay balls for vertices and toothpicks for sides.

Step 2: Add a toothpick to each vertex of the polygon.

Step 3: Join the loose ends of the toothpicks to form one vertex at the top.
ACTIVITY 3 (Essential)

3. Have groups of three students use toothpicks and modelling clay to construct the skeletons of three types of pyramids: triangular, rectangular (or square), and pentagonal.

Discuss the similarities and differences between prisms and pyramids; for example, both have bases and are named by the shapes of their bases, but pyramids have only one base, whereas prisms have two.

Counting edges, vertices, and faces. Draw on the board:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
<th>pentagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Vertices in the Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Edges in the Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students help you fill in the table. Point to each cell and have students count the vertices or edges in the skeletons they built in the activity and signal their answers. The completed table is shown below:

<table>
<thead>
<tr>
<th>Shape of the Base</th>
<th>triangle</th>
<th>rectangle</th>
<th>pentagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Vertices in the Base</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Number of Vertices in the Pyramid</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Number of Edges in the Pyramid</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Ask students to look at the second and third rows in the table. ASK: What pattern do you see? (there is one more vertex in the pyramid than in the base) Could you tell when you were making it that there would always be one more vertex in the finished pyramid than in the base? (yes) How? (you make the base first and then add one more vertex) Explain that the vertex opposite the base is called the apex of the pyramid.

Have students look at the second and fourth rows in the table. ASK: What pattern do you see? (the fourth row is double the second row) Will that be true with different bases too? (yes) Why? (You start with the base that has as many edges as vertices. Then you add one extra edge at each vertex, so you have twice as many edges as vertices in the base.)

Drawing faces. Tell students that, as with prisms, the faces that aren’t bases are called side faces. ASK: What shape are the side faces in pyramids? (triangles) Write “Triangular Pyramid,” “Square-Based Pyramid,” and “Pentagonal Pyramid” on the board. For each pyramid, have a volunteer draw a complete set of faces. Then count the total number of faces and write the number below each drawing. (see sample drawings on the next page)
Triangular Pyramid

Square-Based Pyramid

Pentagonal Pyramid

ASK: How do the faces tell you that these are pyramids and not prisms? (there are a lot of triangular faces) SAY: The side faces on a prism are rectangles or parallelograms. So a prism can never have more than two triangular faces, which would be the bases. ASK: How many rectangular faces can a pyramid have? (1) Why? (the side faces on pyramids are triangles, so the only face that can be a rectangle is the base) Which pyramid can have all congruent faces? (the triangular pyramid) SAY: Not every triangular pyramid has all congruent faces. Invite students to guess the property a triangular pyramid must have for all of its faces to be congruent. (its faces must be equilateral triangles)

ASK: If you know how many edges are in the base of a pyramid, can you tell how many faces it has? (yes) What is the pattern? (there is one more face in the pyramid than edges in its base) If necessary, prompt by looking at the examples one at a time and giving students time to think between each example.

**Review faces that intersect at a vertex.** Show students a square-based pyramid. Ask a volunteer to point to two faces that intersect only at a vertex. (two opposite triangular faces) Ask another volunteer to point to two faces that intersect at an edge. (the base and any side face or any two adjacent side faces) ASK: Is there one point that is common to all the side faces? (yes, the apex) Repeat with several pyramids with different bases. ASK: In a pyramid, do all the side faces meet at the apex? (yes)

Encourage students to look at actual 3-D shapes to help them with the exercises below.

**Exercises**

a) Grace says that in a pyramid, any two faces intersect. Is Grace correct? Explain why or why not.

b) David says that in a prism, any two faces intersect. Is David correct? Explain why or why not.

**Answers:** a) yes, b) no

**Sample explanations:** a) the base of a pyramid and any other face will meet at an edge, and any two side faces will meet at the apex; b) the two bases of a prism are always on opposite ends, so they are not intersecting faces.
NOTE: Extension 5 is required in order to cover the British Columbia curriculum.

Extensions

1. Have students make prisms and pyramids out of modelling clay. They can cut the clay with plastic knives to create flat surfaces.

2. Build skeletons of pyramids by building a base and then adding an apex that is not centred over the base. How does the shape change as you move the apex relative to the base? What are the shapes of the side faces? Can you place the apex so that some of the side faces are right-angle triangles?

   Answer: The side faces are still triangles but are no longer congruent. If you place the apex directly above one of the vertices of the base, you will produce two side faces that are right-angle triangles.

3. Compare a cone to a pyramid. How is it similar, and how is it different?

   Answer: Like a pyramid, a cone has a flat face and an apex opposite the flat face. Unlike a pyramid, this flat face is a circle, not a polygon, and a cone does not have flat side faces, only one curved side face.

4. a) How many faces are in a triangular prism?

   b) Can you make a prism with fewer faces than a triangular prism?

   c) In pairs, explain your answers to part b). Do you agree with each other? Discuss why or why not.

   Answers: a) 5; b) no; c) A prism must have two matching polygons as bases and one side face for each edge of the base. There are no polygons with fewer edges than a triangle.

5. Have students find at least five different prisms or objects that are almost prisms in their environment (for example, at school, at home, or outside). Students name the object that is shaped like a prism, and if only part of the object is a prism, they describe which part is a prism. Students sketch the objects, explain whether the object is a perfect prism or slightly different, and name the prisms found in the objects.

6. Have students play in pairs. The goal is to assemble the faces needed for a prism or pyramid. Give each pair of students four copies of BLM Faces and have them cut out the cards. The shapes on the cards represent the faces of pyramids or prisms. To begin play, students deal five cards each and place the rest face down in a pile. The first player takes a card off the top of the deck. If the player’s cards contain all the faces needed to make a prism or pyramid, he puts down those cards and wins the game. If not, he places one card face up on the discard pile. On each subsequent turn, a player can take either a new card from the deck or the top card from the discard pile. The first player with cards for all the faces needed to create a pyramid or prism wins; players can use up to six cards and do not need to use all cards in their hand.
Goals

Students will identify edges and faces of 3-D objects that are horizontal, vertical, parallel, and perpendicular. Students will use the terms “horizontal,” “vertical,” “parallel,” and “perpendicular” when describing prisms and pyramids.

PRIOR KNOWLEDGE REQUIRED

Can identify vertical, horizontal, parallel, and perpendicular edges on 2-D shapes
Can identify edges, faces, and vertices on 3-D shapes
Can identify bases and side faces on prisms and pyramids

MATERIALS

triangular prism
rectangular prism
various other 3-D shapes (prisms and pyramids with different bases)
string with an eraser (or other weight) tied to it
empty shoeboxes
skew prisms
newspapers, magazines, or access to the internet (see Extension 3)

Review horizontal and vertical edges in 2-D shapes. Draw on the board:

ASK: What is a vertical edge on a 2-D shape? (an edge that runs straight up and down) What is a horizontal edge? (an edge that runs perfectly left and right) Remind students that they can use vertical edges on the board (the left and right edges) or horizontal edges on the board (the top and bottom edges) to check if lines on the board are vertical or horizontal. Point to various edges on the octagon on the board and ask if each is horizontal, vertical, or neither. Students can signal with an arm straight up for vertical, an arm sideways for horizontal, and thumbs down for neither.

Horizontal and vertical depend on the position of a shape. Hold a blank sheet of paper against the board so that two edges are horizontal. ASK: What shape is this? (a rectangle) Point to the bottom edge and ASK: Is this edge horizontal? (yes) Now turn the paper so that none of the edges are horizontal or vertical. Point to the same edge and ASK: Is this edge horizontal now? (no) Is it vertical? (no) Keep turning the paper so that the edge is vertical. ASK: Is the edge horizontal now? (no) Is it vertical? (yes) Remind students that moving a shape can change whether an edge is horizontal, vertical, or neither.
**Horizontal faces in 3-D shapes.** Tell students that they will learn to describe horizontal and vertical edges in 3-D shapes, as well as horizontal and vertical faces. Show students a triangular prism. SAY: Just like with 2-D shapes, whether an edge or face is vertical or horizontal depends on how we position the shape. For this lesson, we will always position a shape on a flat surface, such as the top of a table, a desk, or the floor. Place a base of the triangular prism flat on a table. SAY: For 3-D shapes, any face that runs flat left and right in the same direction as the top of the table and the floor is called a **horizontal face**. ASK: Is the floor a horizontal face? (yes) Is the top of the table a horizontal face? (yes) What shape is the face that is on the bottom of the prism now? (a triangle) Is that triangular face horizontal? (yes) What is the shape of the top face of the prism right now? (a triangle) Is the top triangular face also horizontal? (yes) Point to one of the rectangular side faces and ASK: Does this face run in the same direction as the table? (no) Is this a horizontal face? (no)

Point to various faces of the triangular prism and ASK: Is the face horizontal? Have students signal thumbs up for “yes” and thumbs down for “no.” Repeat with the prism on a side face and then with a rectangular prism and a pyramid.

**Horizontal edges in 3-D shapes.** Explain that for 3-D shapes, any edge that runs flat along a horizontal face is a **horizontal edge**. Place a base of the triangular prism flat on the table and ASK: How many edges are on the bottom face of this prism? (3) Are those edges all horizontal? (yes) Why? (they run in the same direction as the bottom face, which is horizontal) How many edges are on the top face? (3) Are they all horizontal edges? (yes) Why? (they run in the same direction as the top and bottom faces, which are horizontal) Point to one of the vertical edges on one of the rectangular side faces. ASK: Is this edge horizontal? (no) Why not? (it does not run in the same direction as a horizontal face)

Point to various edges of the triangular prism ASK: Is the edge horizontal? Have students signal their answers. Repeat with the prism on a side face and then with a different prism and a pyramid.

**Vertical edges in 3-D shapes.** Remind students that in 3-D shapes, just as in 2-D shapes, any edge that runs straight up and down is called a **vertical edge**. Show students a string with an eraser (or other weight) tied to it and hold the string so that the weight hangs straight down without swinging. SAY: When you hold a string like this one with a weight tied to it, the string will be vertical. Tell students that they can usually tell whether an edge of a 3-D shape is vertical just by looking at it, but if they are unsure, they can check by holding a weighted string beside the edge.

Point to various edges of the triangular prism and ASK: Is the edge horizontal? Have students signal their answers. Repeat with the prism on a different face and then with a different prism and a pyramid.

**Vertical faces in 3-D shapes.** Explain to students that **vertical faces** run straight up and down, like walls. Place a rectangular prism on a table and point to a side face. ASK: Is this face vertical? (yes) Repeat with the other side faces. Point to the top face and ASK: Is this face vertical? (no) Is the top face horizontal? (yes) Is the bottom face vertical or horizontal? (horizontal)
Place a triangular prism side face down on the table so that the other side faces are slanted. Point to a side face and ASK: Is this face vertical? (no) Then place the triangular prism base down. Point to the same side face as before and ASK: Is the face vertical now? (yes) Explain that one way to check if a face is vertical is to place it against a flat face that they already know is vertical, such as a wall or the side of a shoebox. As long as students keep the same face down, they can move the shape because it will not change which edges and faces are vertical or horizontal.

With the triangular prism base down, tell students you will check if the side faces are vertical. Move the prism against a wall. SAY: I need to check whether the side face can be placed flat against the wall while keeping the base down. Place one of the side faces flat against the wall and ASK: Is this face vertical? (yes) Repeat with the other two side faces. Then place the prism side face down so that one side face is not vertical and show students how to check if that face is vertical. ASK: If I place the bottom edge of that face against the wall, is the rest of the face flat against the wall? (no) Is the face vertical? (no)

**Vertical faces with no vertical edges.** Place the triangular prism side face down on a table so that the other two side faces are not vertical. Have a volunteer use the wall to check if the triangular faces are vertical. ASK: Are the edges of the triangular face vertical? (no) Point out that sometimes a face is vertical even if none of its edges are. Repeat using skew prisms with a base down.

### ACTIVITY 1 (Essential)

1. **Horizontal and vertical in 3-D shapes.** Provide pairs of students with 3-D shapes, including prisms and pyramids with different polygons as bases. Also provide empty shoeboxes or have students use a classroom wall. Partner 1 chooses a 3-D shape and places one face down on the table, floor, or other flat horizontal surface. Partner 2 points to various edges and faces on the shape, and Partner 1 says if they are horizontal, vertical, or neither, using the empty box or wall to help if necessary. Partners switch roles and repeat several times.

   **Variation:** Include skew prisms.

**Review parallel and perpendicular in 2-D shapes.** ASK: What does it mean for two edges to be parallel in 2-D shapes? (the edges run in the same direction, and they are always the same distance apart) Draw on the board:

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   □
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Point to various pairs of edges and ask each time if the pair is parallel or not.
ASK: What does perpendicular mean in 2-D shapes? (two lines or edges meet at a right angle) Remind students that they can check whether an angle is a right angle using a square corner. Demonstrate this using a piece of paper and the shape on the board. Point to various pairs of edges and have students signal each time whether the edges are perpendicular or not.

Parallel and perpendicular edges in 3-D shapes. Tell students that checking whether edges are parallel or perpendicular in 3-D shapes is similar to checking them in 2-D shapes. SAY: In this lesson, we will only consider two edges from a 3-D shape that lie on the same face. The first thing you need to do is find the face that the two edges are on. Point to two edges on a rectangular prism that lie on one face and ASK: Which face are both of these edges on? Point to several faces and have students signal thumbs up or down until they have identified the correct face. ASK: Are these two edges parallel, perpendicular, or neither? Repeat with several pairs of edges. Then repeat with different 3-D shapes.

**ACTIVITY 2 (Essential)**

2. Provide pairs of students with 3-D shapes, including prisms and pyramids with different polygons as bases. Also provide blank paper to check for square corners. Partner 1 chooses a 3-D shape. Partner 2 points to various pairs of edges on the shape that are on the same face. Partner 1 identifies the face that includes both edges and then says if the edges are parallel, perpendicular, or neither.

Parallel and perpendicular faces in 3-D shapes. Explain that two faces on 3-D shapes are **parallel faces** if they run in the same direction and are always the same distance apart. SAY: To check whether two faces are parallel, place one face flat on a horizontal surface and check if the other face is also horizontal. Point to pairs of faces on various 3-D shapes (some parallel and some not) and have students signal whether they think the faces are parallel or not. Then have a volunteer check by placing one face flat on the table.

Explain to students that two faces on 3-D shapes are **perpendicular faces** if they meet at right angles. Tell students that to check whether two faces are perpendicular, place one face flat on a horizontal surface and check if the other face is vertical. Point to pairs of faces on various 3-D shapes and have students signal whether they think the faces are perpendicular or not. Then have a volunteer check by placing one face flat on the table.

**ACTIVITY 3 (Essential)**

3. Provide pairs of students with 3-D shapes, including prisms and pyramids with different polygons as bases. Also provide empty shoeboxes or have students use a classroom wall. Partner 1 chooses a 3-D shape. Partner 2 points to various pairs of faces on the shape, and Partner 1 says if they are parallel, perpendicular, or neither, using a horizontal surface and a vertical “wall” to help.
Exercises: Use 3-D shapes to answer the questions.

a) Ray places a right prism on a table with one base down. Will all side faces be vertical?

b) Anne places a skew prism on a table with one base down. Will all side faces be vertical?

c) Are the bases in a right prism always parallel?

d) Are the bases in a skew prism always parallel?

Answers: a) yes, b) no, c) yes, d) yes

NOTE: Encourage students to use actual 3-D shapes to help them answer the questions in the AP Book or to check their answers.

Extensions

1. Have students sketch prisms or pyramids as though they are lying flat on a table. Students can use actual 3-D shapes on their desks to help them make their sketches. Students should draw at least three different shapes and then label at least one of each of the following:

   • a vertical edge and a horizontal edge
   • a vertical face and a horizontal face
   • a pair of intersecting edges and a pair of intersecting faces
   • a pair of parallel edges and a pair of parallel faces
   • a pair of perpendicular edges and a pair of perpendicular faces

2. Students make a list of at least five objects from their environment (for example, at school, at home, or in their communities) that show parallel, intersecting, perpendicular, vertical, and horizontal line segments and faces. Students sketch the objects. In pairs, students share their lists and sketches and describe the edges and faces using the words “parallel,” “intersecting,” “perpendicular,” “vertical,” and “horizontal.”

3. Provide students with newspapers, magazines, or access to the internet. Students find at least five images that show parallel, intersecting, perpendicular, vertical, and horizontal line segments and faces. In pairs, students describe the edges and faces they found using the words “parallel,” “intersecting,” “perpendicular,” “vertical,” and “horizontal.”
Goals

Students will construct prisms from nets.
Students will match prisms with their nets.
Students will sketch the faces of prisms.

PRIOR KNOWLEDGE REQUIRED

Can recognize and name polygons and prisms
Can identify faces, edges, and vertices on 3-D shapes

MATERIALS

triangular prism
cut-out nets from \textit{BLM Nets (2) to (4) and (6)} (pp. P-33–35, P-37)
glue or tape
set of prisms made from \textit{BLM Nets (1) to (4) and (6)} (pp. P-32–35, P-37)
\textit{BLM Nets (1) to (4) and (6)} (pp. P-32–35, P-37) per pair of students
\textit{BLM Nets (7)} (p. P-38, see Extension 2)
connecting cubes (see Extension 3)

Mental math minute. Ask students to solve multiplication problems within 1 \times 1 to 10 \times 10 and the related division problems.

Introduce nets. Hold up a triangular prism. ASK: What is this 3-D shape called? (a triangular prism or triangle-based prism) What are the shapes of the faces? (triangles and rectangles) How many triangular faces does it have? (2) How many rectangular faces does it have? (3)

In advance, cut out the nets from \textit{BLM Nets (2) to (4) and (6)}. Show students the cut-out net of a triangular prism. ASK: How many triangles does this picture have? (2) How many rectangles does it have? (3) Explain that you can fold this picture (demonstrate as you do so) and glue or tape it together to make a 3-D shape. ASK: What shape does this picture make? (a triangular prism) SAY: A picture or pattern that we can fold to make a 3-D shape is called a net of the 3-D shape. This was the net of a triangular prism.

Hold up a cut-out net of a square-based prism and ask students to describe the shapes they see. (2 squares and 4 rectangles) ASK: If this is a prism, what shape is its base? (square or rectangle) Can it be a prism? (yes) Repeat for the cut-out net of a rectangular prism and then a pentagonal prism.
ACTIVITY (Essential)

Display in random order a set of completed prisms from BLM Nets (1) to (4) and (6) and label them A to E. Give each pair of students BLM Nets (1) to (4) and (6). Have the students cut out the nets. Then have students predict which shape the net will make. Finally, have students fold the nets and check that they have identified the shapes correctly.

Sketching faces of prisms. Have each student select one prism from the shapes they made in the activity. Ask: How many bases does a prism have? (2) Write “Triangular Prism,” “Square-Based Prism,” “Rectangular Prism,” “Cube,” and “Pentagon-Based Prism” on the board. Have volunteers sketch the bases of their prisms under the appropriate heading. Remind students that the faces that aren’t bases are called side faces.

Ask: What shape are the side faces on your prism? (rectangle, square for cubes) Have students count and sketch the side faces. Students who are having trouble counting can try counting from the top or marking the first face (or all faces) with a dot or sticker. Have volunteers sketch the faces for each prism named on the board. (see sample sketches below)

Extensions

1. Arsham tries to draw a net for an octagon-based prism. He draws 8 matching rectangular faces and 2 matching octagons sharing an edge.

a) Will the net make a prism?

b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

Answers: a) no; b) In a prism, the bases are opposite faces, so they do not intersect. If the octagon faces share an edge in the net, they will share an edge in the 3-D shape.
2. Construct a shape using **BLM Nets (7)**. What shape is it? Why is it not a prism? How is it similar to a prism, and how is it different?

**Answers:** A cylinder. It is not a prism because the base is not a polygon. It is similar to a prism because it has two congruent bases on opposite sides of the shape. It is different because the bases are not polygons and it has no flat side faces, only one curved side face.

3. One way to describe a 3-D shape is to draw a mat plan. A mat plan is a drawing of the base of the shape with a number in each section showing the height at that point. For example, below is the mat plan for a 2 by 2 by 3 prism. The base, a 2 by 2 square, has been drawn. At each place on the square, there is a stack of cubes that is three cubes high.

```
3 3
3 3
```

Use connecting cubes to build the shape shown in the mat plan. Is it a prism?

a) b) c) d) Can you tell from the mat plan if the shape will be a prism without building it? Explain.

**Answers:** a) yes; b) no; c) yes; d) yes, because in a prism, all the numbers in the mat plan will be the same.
Goals
Students will build pyramids and prisms from nets.
Students will identify correct and incorrect nets.
Students will add missing faces to nets.
Students will draw nets for prisms and pyramids.

PRIOR KNOWLEDGE REQUIRED
Knows that a right prism has two identical bases and rectangular side faces
Knows that a pyramid has one base and triangular side faces
Can recognize and name polygons, prisms, and pyramids
Can construct skeletons of prisms and pyramids

MATERIALS
completed shapes from BLM Nets (5), (6), (8) to (15)
(p. P-36, P-37, P-39–46)
BLM Nets (5), (6), (8) to (15) per small group (pp. P-36, P-37, P-39–46)
grid paper or BLM 1 cm Grid Paper (p. S-1)
square-based prism that is not a cube
various prisms and pyramids with bases that are regular polygons
rulers
Polydrons (see Extension 4)
BLM Isometric Dot Paper (p. P-48, see Extension 6)

Mental math minute. Have groups of four students add three-digit numbers by adding hundreds, tens, and ones. Give an addition problem, such as 235 + 546. The first student adds the ones (5 + 6 = 11). The second student adds the tens (30 + 40 = 70). The third student adds the hundreds (200 + 500 = 700). The fourth student finishes the addition (700 + 70 + 11 = 781), so 235 + 546 = 781.

ACTIVITY 1 (Essential)
1. Display in random order a completed set of shapes from BLM Nets (5), (6), (8) to (15), labelled A to J. Give each small group of students several pages from BLM Nets (5), (6), (8) to (15). Have students cut out the nets and then predict the shape each net will make. Then have students fold the nets to check whether they correctly identified the shapes. Have students name the shapes they made from the nets.

Correct nets. ASK: How many squares are there in a net for a cube? (6)
Will any arrangement of six squares make a net for a cube? (answers may vary) Draw on the board the shape on the next page.
ASK: Does this picture have the correct number of faces to make a cube? Yes Count the faces to verify that there are six. ASK: Can you fold this to make a cube? No Why not? (the faces are all lined up; you don’t have a top or bottom) How many squares do I have to move to make a net? 2 Erase two squares and ASK: Where should I move them? (answers will vary)

SAY: Let’s add a square to make the top of the cube. ASK: Where should it go? (sticking out on top) Does it matter which square it attaches to? No SAY: As long as it attaches to the top, it doesn’t matter where it goes. Draw a square above the second square in the chain, as shown below:

ASK: Where should I put the last square? (on the bottom) Draw the sixth square sticking out anywhere on the bottom, as shown below:

Exercise: Draw two nets for cubes, one that works and one that doesn’t. Trade with a partner and correct the net that does not work.

Missing faces on nets. Draw on the board:

SAY: This is part of a net. It is missing a face. ASK: Is this a prism or a pyramid? (a prism) How do you know? (it has 2 rectangular faces, it doesn’t have enough triangles to be a pyramid) What kind of prism will this net make? (triangular) How do you know? (it has 2 triangular faces) What shape is the missing face? (a rectangle) SAY: In a lot of the nets we have seen, the side faces were all the same. The side faces on this net are different. ASK: Why? (the sides or edges of the triangle are different lengths) Label the sides of the triangle a, b, and c, as shown in the margin. Point out that the side faces are all the same height but have different widths. SAY: The side face on the left is as wide as side a. The middle side face is as wide as side b. ASK: How wide should the last side face be? (as wide as side c) Have a volunteer sketch the missing face. The completed net is shown in the margin.
Exercises: Copy the net onto grid paper or BLM 1 cm Grid Paper. The net is missing one face. Draw the missing face.

a) b) c) 

d) e) 

Bonus:

Sample answers

a) b) c) 

d) e) 

Bonus:

Drawing nets of prisms. Hold up a square-based prism that is not a cube and demonstrate how to draw a net for it by following the steps below. Point out before you start that because the base is a regular polygon, all of the side faces are the same shape and size (congruent). So if you can draw one, you can draw all of them. Write each step on the board and leave them there for the activity that follows. (see sample net in margin)

Step 1: Put the prism in front of you standing on its base.
Step 2: Measure and draw the side face that is facing you.
Step 3: Draw the remaining side faces in a row.
Step 4: Measure and draw the base on the top.
Step 5: Draw the base on the bottom.
ACTIVITY 2 (Essential)

2. Have students choose a prism with a base that is a regular polygon and draw a net for it on grid paper. Encourage students to use a ruler to help them draw or check their nets.

Drawing nets of pyramids. Demonstrate how to draw a net for an equilateral triangle-based pyramid by following these steps. Write the steps on the board and leave them there for the activity that follows. (see sample net in margin)

Step 1: Measure and draw the base.
Step 2: Measure a side face.
Step 3: Draw a side face attached to each edge of the base.

ACTIVITY 3 (Essential)

3. Have students choose a pyramid with a base that is a regular polygon and draw a net for it on grid paper. Encourage students to use a ruler to help them draw or check their nets.

Extensions

1. There are three different ways to add the missing face to the net. Find all three.

Answer

2. Draw a net for a rectangular prism that is not square based. Repeat for a rectangular pyramid that is not square based.
3. How many different nets can you draw for the same cube?

Answer: There are six different nets for the cube. Other nets can be made by flipping or rotating one of these.

![Net Diagrams]

4. Show students how to construct a net for a cube using Polydrons. Demonstrate how to fold the net into the cube and how to unfold the cube back into the net. Have students use the Polydrons themselves to construct as many nets of pyramids and prisms as they can. Students should sketch the nets they were able to form. If students are not sure which shapes to try, suggest some examples, such as a cube, a square-based prism, a triangular prism, a triangle-based pyramid, and a square-based pyramid.

5. Have students search online for a tool to construct nets—they can use the search term “geometric solids application.” Students can construct a net of a familiar 3-D shape, print it, cut it out, and fold it to check whether it creates a 3-D shape or not.

6. Use BLM Isometric Dot Paper to construct a net for the given shape. Try to make sure all vertices lie on dots.

   a) a hexagonal prism with all edges in the base 2 cm long and all side edges 17 mm long
   b) a triangular prism with all edges in the base 2 cm long and all side edges 35 mm long
   c) a triangular pyramid with all edges 3 cm long
   d) a triangular pyramid with faces that are not all congruent
   e) a hexagonal pyramid

Selected sample answers:

![Sample Answers Diagrams]
Matching 3-D Shapes

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

12. 

13. 

14. 

15. 

16.
Nets (1)
Nets (2)
Nets (3)
Nets (4)
Nets (5)
Nets (6)
Nets (7)
Nets (8)
Nets (9)
Nets (11)
Nets (12)
Nets (13)
Nets (14)
Nets (15)
Faces

---

Blackline Master — Geometry — Teacher Resource for Grade 5
Isometric Dot Paper
Goals

Students will solve applied problems across the Grade 5 curriculum using a variety of problem-solving strategies, such as using structure, making diagrams, and making a simpler problem.

PRIOR KNOWLEDGE REQUIRED

- Can multiply multi-digit numbers by multiples of 10 (see Problem Bank 3)
- Can evaluate a fraction of a whole number (for Problem Bank 4, 5)
- Can use decimal notation for tenths and hundredths (for Problem Bank 8)
- Can identify and extend patterns (for Problem Bank 9)
- Can add and subtract decimals to hundredths (for Problem Bank 10)
- Can write a rule to get the value of each term in a pattern with constant gaps from the term number (for Problem Bank 11)
- Can evaluate expressions at a given value for the variable (for Problem Bank 11)
- Can determine the coordinates of points in the first quadrant (for Problem Bank 11)

MATERIALS

counters
BLM Banquet Hall (pp. 56–58, see Extended Problem)
BLM Electric! (pp. 61–63, see Extended Problem)

Introduce the buckets problem. Write on the board:

You have a 5 L bucket, a 2 L bucket, and you are beside a river.

ASK: If you have a 5 L bucket and a 2 L bucket, how can you fill the 5 L bucket with 3 L of water? (fill the 5 L bucket and pour as much as possible into the 2 L bucket; when the 2 L bucket is full, the 5 L bucket has 3 L in it) Model this on the board with counters representing litres, as shown below:

SAY: Start with the two empty buckets again. ASK: How can you fill the 5 L bucket with 4 L? (fill the 2 L bucket and pour the water into the 5 L bucket and then repeat) Have volunteers show this process on the board with counters representing litres.
ASK: When the 5 L bucket has 4 L in it, how can you fill the 2 L bucket with 1 L? (fill the 2 L bucket with water and pour all that you can into the 5 L bucket until the 5 L bucket is full; the amount left in the 2 L bucket is 1 L) Again, have volunteers show this process on the board using counters.

Write on the board:

You have a 4 L bucket, a 9 L bucket, and you are beside a river.

SAY: Suppose you want to fill the 9 L bucket with 6 L. ASK: Why would it help to start with 1 L in the 4 L bucket? (you can fill the 9 L bucket and then pour into the 4 L bucket until the 4 L bucket is full; that means you poured 3 L from the 9 L bucket into the 4 L bucket to leave 6 L in the 9 L bucket)

Exercises

a) How would you fill the 4 L bucket with 1 L so that you can still follow the steps to get 6 L in the 9 L bucket?

b) You have a 3 L bucket, a 5 L bucket, and you are beside a river. How can you get exactly 4 L of water from the river?

Solutions

a) Fill the 9 L bucket and then pour 4 L into the 4 L bucket. Empty the 4 L bucket into the river and pour another 4 L from the 9 L bucket into the 4 L bucket. The 9 L bucket now has 1 L. Empty the 4 L bucket and pour the 1 L from the 9 L bucket into the 4 L bucket. The 4 L bucket now has 1 L. Now you can follow the steps to get 6 L in the 9 L bucket.

b) Fill the 5 L bucket and use it to fill the 3 L bucket. Empty the 3 L bucket and pour the remaining 2 L from the 5 L bucket into the 3 L bucket. There is now 2 L in the 3 L bucket and the 5 L bucket is empty. Fill the 5 L bucket and pour 1 L into the 3 L bucket so that the 3 L bucket is full. The 5 L bucket now has 4 L in it. Visual representation of solution:

```
\[ 5 \text{ L} \rightarrow 3 \text{ L} \rightarrow 5 \text{ L} \rightarrow 3 \text{ L} \rightarrow 5 \text{ L} \rightarrow 3 \text{ L} \rightarrow 5 \text{ L} \rightarrow 3 \text{ L} \rightarrow 5 \text{ L} \rightarrow 3 \text{ L} \]
```

NOTE: The following Problem Bank questions reflect a selection of the problem-solving strategies used in the problem-solving lessons for Grade 5. Students will need to choose among all the strategies they have learned this year to solve the problems.

Problem Bank

1. You have an 8 L bucket, a 5 L bucket, and a 3 L bucket. There is no river nearby. The 8 L bucket is full and the other two buckets are empty. How can you pour half of the water from the 8 L bucket into the 5 L bucket, so that 4 L is in the 5 L bucket and 4 L is in the 8 L bucket?
Solution: Fill the 5 L bucket with water from the 8 L bucket, leaving 3 L in the 8 L bucket.

![Diagram](8 L 0 L 0 L → 3 L 5 L 0 L)

Then fill the 3 L bucket with water from the 5 L bucket, leaving 2 L in the 5 L bucket.

![Diagram](3 L 2 L 3 L)

Fill the 8 L bucket with water from the 3 L bucket, and then pour the 2 L from the 5 L bucket into the 3 L bucket.

![Diagram](6 L 2 L 0 L → 6 L 0 L 2 L)

Fill the 5 L bucket with water from the 8 L bucket, and then fill the 3 L bucket with water from the 5 L bucket.

![Diagram](1 L 5 L 2 L → 1 L 4 L 3 L)

Pour the water from the 3 L bucket into the 8 L bucket.

![Diagram](4 L 4 L 0 L)

2. It is 7:34 a.m. on Wednesday. What day and what hour was it 9 hours and 45 minutes ago?

Solution: Nine hours ago it was 10:34 p.m. on Tuesday. Forty-five minutes before that it was 9:49 p.m. on Tuesday.

3. The heartbeat of an adult is around 70 beats per minute. How many times does the heart beat in a solar year? Hint: A solar year to the nearest minute is 365 days, 5 hours, and 49 minutes.

Solution: The number of minutes in a solar year is 

\[ (365 \times 24 \times 60) + (5 \times 60) + 49 = 525\,949 \]

so the number of beats per year is about 

\[ 70 \times 525\,949 = 36\,816\,430. \]
4. Colour the shape to show the fraction of the whole number and then evaluate the fraction of the whole number.

a) \[ \frac{1}{2} \] of 8

b) \[ \frac{2}{3} \] of 12

**Answers:** a) 4, b) 8

5. Aputik reads \( \frac{3}{4} \) of a book and Megan reads \( \frac{1}{2} \) of another book. Megan says she read more pages than Aputik. Is that possible? Explain with an example.

**Sample answer:** Yes, it is possible. For example, if Megan’s book was 200 pages long and Aputik’s book was only 100 pages long, then Megan read 100 pages and Aputik only read 75 pages.

6. Billy wants to add 345 + 687 on his calculator, but he accidentally presses “345 ×” instead of “345 +.” What could he press next so that he doesn’t have to re-enter 345?

**Answer:** 1 (345 × 1 + 687)

7. Put the decimal point in the appropriate position to make the data realistic.

a) height of a room: 27 m
b) gas in a car tank: 245 L
c) length of a pencil: 145 cm
d) patient’s body temperature: 385°C

**Answers:** a) 2.7 m, b) 24.5 L, c) 14.5 cm, d) 38.5°C

8. What number am I?

a) I am a two-digit number. My digits add to 13 and my ones digit is 3 less than my tens digit.

b) I am a two-digit number that is greater than 60. I am a multiple of 26.

c) I am an odd three-digit number. The sum of my digits is 18 and I am a multiple of 5, 7, and 9.

d) I am a two-digit decimal between 0.20 and 0.30. My hundredths digit is 4 times my tenths digit.

e) I am a three-digit decimal between 5 and 6. My tenths digit and hundredths digit are equal. My digits add to 13.

f) I am a three-digit decimal between 4 and 5. My digits add to 9 and my hundredths digit is 3 more than my tenths digit.

**Answers:** a) 85, b) 78, c) 945, d) 0.28, e) 5.44, f) 4.14
9. How many shaded and unshaded triangles would be in Figure 10?
   Hint: First look at the number of shaded triangles only.

   \[
   \text{Solution: The sequence for shaded triangles is: } 1, 3, 6, 10, \ldots \\
   \text{The sequence does not have a common gap, but the tenth term is} \\
   1 + 2 + 3 + 4 + \ldots + 10. \text{ Find the answer by adding:} \\
   \begin{align*}
   &1 + 2 + 3 + 4 + 5 \\
   &+ 10 + 9 + 8 + 7 + 6 \\
   &= 11 + 11 + 11 + 11 + 11
   \end{align*}
   \]

   The total for shaded triangles in Figure 10 is 55. The sequence for the unshaded triangles is 0, 1, 3, 6, 10, \ldots. Figure 10 has the same number of unshaded triangles as Figure 9 has shaded triangles, which is 10 less than 55, so the total for unshaded triangles in Figure 10 is 45.

10. The number on top is equal to the sum of two numbers on the bottom. Find the missing number.

   a) b) c) d)
   \[
   \begin{align*}
   &5 \quad 3 \\
   &0.37 \quad 0.14 \\
   &9 \quad 2 \\
   &5.2 \quad 3.4
   \end{align*}
   \]

   Answers: a) 8, b) 7, c) 0.51, d) 1.8

11. A number is equal to the sum of the two numbers below it. Find the top number.

   a) b)
   \[
   \begin{align*}
   &4 \quad 2 \quad 2 \quad 4 \\
   &2.2 \quad 1.1 \quad 1.1 \quad 1.1
   \end{align*}
   \]

   Answers: a) 20, b) 8.8
12. What will be the coordinates of the centre of the 100th rectangle in the pattern? Each grid mark represents one unit on the coordinate plane.

Solution: The coordinates of the terms are: (2, 2), (5, 5), (8, 8), and so on. Each coordinate is equal to $3 \times \text{term number} - 1$, so the coordinates of the centre of the 100th rectangle are (299, 299).
Extended Problem: Banquet Hall

**MATERIALS**

BLM Banquet Hall (pp. 56–58)

**Extended Problem: Banquet Hall.** Give students BLM Banquet Hall. Tell students that this extended problem involves hosting an event at a banquet hall.

**Selected answers**

1. a) 6, 10, 14; b) 22; c) 7
2. a) i) 13.5 m and 28 people, ii) 16.5 m and 32 people
   b) i) the arrangement requires exactly 22 m, including the 1 m space at the ends of the banquet hall, and it seats 38 people; ii) the arrangement would require 24 m, which is too long; iii) the arrangement would require 25 m, which is too long
   Bonus: a) 40; b) the arrangement would require 23.5 m, which is too long; c) make any two groups—such as a group of 4 and a group of 5, a group of 3 and a group of 6, or a group of 2 and a group of 7—or 1 single table and a group of 8 (1 long table will fit but will only seat 38 people, while splitting the tables into 3 groups will not fit because it will need 23 m)
Banquet Hall (1)

1. Sara is preparing for a banquet. She puts tables together in three different ways:

   - 
   - 
   - 

   a) Use the picture to fill in the table below and find how many people can fit around one, two, or three joined tables.

<table>
<thead>
<tr>
<th>Number of Joined Tables</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

   b) If Sara puts five tables in a row, how many people will fit around the table?

   c) Sara wants to make one long table for 30 people. How many tables will she need to join together to fit that many people?
Banquet Hall (2)

2. Each table is 2 m long and there needs to be 1.5 m of space between the table groupings.
   a) How much space is required for the arrangement of six tables? How many people can sit in this arrangement?
      i) 
      
      
      ii) 
      
      b) The banquet hall is 22 m long and only fits one row of tables. There needs to be 1 m of space from the ends of the table to the walls of the banquet hall.
      i) Draw a picture to show how four separate tables and three joined tables (seven tables in total) can fit in the banquet hall. How many people can sit in this arrangement?

      ii) Draw a picture to show how one long table made from 11 joined tables cannot fit in the banquet hall.

      iii) Draw a picture to show how seven separate tables cannot fit in the banquet hall.
Banquet Hall (3)

BONUS ▶ Sara has a party for 40 people. She wants to have her party in the 22 m long banquet hall that only fits one row of tables. Remember: Each table is 2 m long, there needs to be 1.5 m of space between the table groupings, and there needs to be 1 m of space from the ends of the table to the walls of the banquet hall.

a) How many people can sit around five separate tables and two joined tables (seven tables in total)?

b) Draw a picture to show that the arrangement in part a) cannot fit in the banquet hall.

c) Draw a picture to show how to arrange nine tables in the banquet hall and seat 40 people.
Extended Problem: Electric!

Preparation for the extended problem. Write on the board:

\[ 3 \times 4 = \_ \quad 30 \times 4 = \_ \]

Have volunteers fill in the blanks. (12, 120) ASK: How can you use \( 3 \times 4 \) to get \( 30 \times 4 \)? (multiply the answer by 10)

SAY: You can do the same thing with two-digit numbers. Write on the board:

\[
\begin{array}{c}
\times \\
31 \\
14 \\
\end{array}
\]

so \( 31 \times 140 = \_ \)

Have one volunteer complete the first multiplication and another volunteer complete the second. (434, 4340) ASK: How can you use \( 31 \times 14 \) to get \( 31 \times 140 \)? (multiply the answer by 10)

Tell students they are going to do an extended problem about the amount of power that we use at home and they will need to use this multiplication strategy. SAY: Different appliances use different amounts of power.

ASK: Which do you think uses more power, a ceiling fan or an air conditioner? (air conditioner)

SAY: Just like length can be measured using centimetres and metres, electricity can be measured using watts or amps. Write on the board:

\[
\begin{align*}
1 \text{ metre} &= 100 \text{ centimetres} \\
\text{A 1 amp appliance in a home uses 120 watts.}
\end{align*}
\]

ASK: How can you convert five metres to centimetres? (multiply 5 by 100) Write on the board:

\[ 5 \times 100 = 500, \text{ so } 5 \text{ m} = 500 \text{ cm} \]

ASK: How can you convert five amps to watts? (multiply 5 by 120) Write on the board:

\[ 5 \times 120 = 600, \text{ so a 5 amp appliance uses 600 watts} \]

SAY: \( 5 \times 12 \) is 60, so \( 5 \times 120 \) is 600. Draw on the board:

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blender</td>
<td>360</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1200</td>
</tr>
<tr>
<td>Electric can opener</td>
<td>150</td>
</tr>
<tr>
<td>Kettle</td>
<td>1800</td>
</tr>
<tr>
<td>Microwave</td>
<td>1080</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>720</td>
</tr>
<tr>
<td>Coffee maker</td>
<td>960</td>
</tr>
</tbody>
</table>
SAY: These are some typical appliances in a home along with how much power they might use. ASK: Which appliance uses the most power? (the kettle) SAY: You don’t use a kettle for a long time, but when you do use it, it uses a lot of power. A kettle will use less power in a year than a microwave, because you use it for less time. ASK: Which appliance uses the least amount of power? (the can opener) Which appliance do you think will use the most power in a year? (the refrigerator) PROMPTS: Which one gets used the most? Which one is almost always running?

Tell students that the way power works in a house is that each outlet has a maximum number of watts you can use at one time. SAY: For example, in some houses, if you are using a kettle, you can’t use a microwave plugged into the same outlet at the same time, because you will blow the fuse, and the power will go out. Extension cords allow you to plug in more than two appliances to the same outlet, but you have to be careful not to go over the number of watts allowed. The extended problem that follows investigates that kind of situation.

NOTE: The word “outlet” is being used imprecisely here; the precise word that should be used is “circuit,” but we use outlet here since students will be more familiar with the word and also because the concept of a circuit is not necessary for this task.

Extended Problem: Electric! Provide students with BLM Electric! On the BLM, Question 4 and the Bonus question are good opportunities for students to apply the guess-check-revise and using structure problem-solving strategies learned this year. If students have not done these problem-solving lessons, they will likely find these questions difficult.

Answers
1. a) kettle; b) i) yes, ii) no, iii) yes
2. a) 1560 watts; b) 600 watts; c) television only, lamp only, stereo only, the television and lamp
3. 120 more watts because 1 amp is the same as 120 watts
4. 9 amps
Bonus: 1001 amps
Electric! (1)

Electricity is needed to power an appliance. The amount of power an appliance uses is given in watts.

These are some typical appliances in a kitchen and how much power they might use.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric can opener</td>
<td>150</td>
</tr>
<tr>
<td>Blender</td>
<td>360</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>720</td>
</tr>
<tr>
<td>Coffee maker</td>
<td>960</td>
</tr>
<tr>
<td>Microwave</td>
<td>1080</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1200</td>
</tr>
<tr>
<td>Kettle</td>
<td>1800</td>
</tr>
</tbody>
</table>

1. The number of watts used by an outlet should not pass the outlet’s capacity. One outlet in the kitchen has a capacity of 1620 watts.
   a) Which appliance can never be used in that outlet?

   b) Can the given appliances be used in the outlet at the same time?
      i) the blender and dishwasher
      
      ii) the microwave and refrigerator
      
      iii) can opener, blender, and coffee maker
Electric! (2)

2. Sometimes the capacity of an outlet is given in amps. In a typical home, to get the number of watts from the number of amps, multiply by 120.

These are some typical appliances in a living room and how much power they might use.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp</td>
<td>72</td>
</tr>
<tr>
<td>Television</td>
<td>240</td>
</tr>
<tr>
<td>Stereo</td>
<td>360</td>
</tr>
<tr>
<td>Window air conditioner</td>
<td>960</td>
</tr>
<tr>
<td>Heater</td>
<td>1200</td>
</tr>
</tbody>
</table>

a) An outlet in the living room has a capacity of 13 amps. How many watts can be used at the outlet?

b) The outlet from part a) is already being used by a window air conditioner. How many additional watts can be used at the outlet?

c) In the winter, a heater is plugged into the living room outlet instead of the air conditioner. Which of these items can be used in that outlet at the same time as the heater? List all possible combinations.
   A. Television   B. Lamp   C. Stereo
Electric! (3)

3. How many more watts could an outlet with a capacity of 10 amps use than an outlet with a capacity of 9 amps? Explain how you know.

4. An outlet has a capacity that is a whole number of amps. The number of watts it can use, to the nearest 100, is 1100. What is the outlet’s capacity in amps?

BONUS ▶ In a building, 120 120 watts are being used at the same time. How many amps is that?
Unit 14 Measurement: Perimeter, Area, Volume, and Mass

Introduction
This unit focuses on:

- estimating, measuring, and recording perimeter, area, volume, capacity, and mass;
- selecting appropriate metric units and justifying the selection;
- developing formulas for the perimeter and area of rectangles;
- calculating and estimating the volume of rectangular prisms;
- solving problems requiring conversion of units of volume and capacity; and
- converting between measurement units.

Meeting Your Curriculum

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<td>ME5-12</td>
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<tr>
<td>Required</td>
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<tr>
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<tr>
<td>Optional</td>
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<tr>
<td>Recommended</td>
<td>ME5-20, 21, 23</td>
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</tbody>
</table>

Mental Math Minutes
The mental math minutes in this unit:
- practise multiplication
- review addition and subtraction skills

Generic BLMs
The Generic BLMs used in this unit are:
- BLM Pentominoes (p. S-2)
- BLM 1 cm Grid Paper (p. S-1)
These BLMs can be found in Section S.
Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
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<th>BC</th>
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<th>ON</th>
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<td>ME5-20 to 22</td>
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<td>ME5-12 to 16</td>
<td>ME5-13 to 18, 20 to 22</td>
<td>ME5-12 to 19, 22, 24</td>
</tr>
</tbody>
</table>
Goals

Students will estimate and measure the perimeter of shapes using appropriate units.

Students will explore how the perimeter of a figure changes when shapes are added to the figure in different places.

PRIOR KNOWLEDGE REQUIRED

Knows different units of measurement: mm, cm, m, km

Can estimate using m, cm, and mm and measure using a ruler

Can identify patterns

Can name geometric shapes

MATERIALS

stapler

a variety of objects (a book, a coin, etc.)

chart paper

geoboards and string

rulers and metre sticks

BLM Pentominoes (p. S-2, see Extension 4)

Mental math minute. Remind students that to multiply whole numbers mentally by 10, 100, or 1000 they need to write zeros to the right of the last digit. ASK: How many zeros do you write when you multiply by 10? (1) By 100? (2) By 1000? (3) Invite a volunteer to explain why \(2 \times 10 = 20\), \(5 \times 100 = 500\), and \(3 \times 1000 = 3000\) using base ten blocks. Give students one-digit numbers and have them multiply each number by 10, 100, or 1000. Progress to two-digit numbers; use 10, 100, and 1000 in random order and include as many volunteers as possible.

Review the best unit to measure in. Demonstrate that a stapler is, for example, 12 cm in length. If we measure it to the closest millimetre, it is, for example, 123 mm long. If we measure it to the closest metre, it is 0 m long. ASK: How many kilometres long is the stapler? (0 km) SAY: 12 cm is a smaller number than 123 mm, and it gives us more information about the stapler than 0 m and 0 km.

Review the term “most appropriate.” Explain that the unit that gives the best way to write a measurement is called the most appropriate unit. It makes the number simple, but it gives the most information about the object measured.

Choosing the appropriate unit to measure in. Display a variety of objects (a book, a stapler, a coin, etc.) and ask students to tell you which unit of measurement will best express the length of the object. ASK: Is the length of a pencil case expressed best in centimetres or in metres? Is the width of a coin expressed best in millimetres or centimetres?
Have students select five items in the classroom and guess which unit of measurement will best express each item’s height, width, or length. Have students measure their five objects. ASK: Which units of measurement did you use? Do these units of measurement result in simple numbers? Would other units of measurement offer simpler measurements?

Ask students to list in their notebooks five things that could be measured and are not in the room. (a bicycle, cell phone, a rocket ship, anything) Ask students to arrange the five things in order from smallest to largest (by height, length, or width). Then ask them to indicate which unit of measurement will give the simplest measurement for each item.

**ACTIVITY 1 (Optional)**

1. Divide the class into four groups and assign one unit of measurement (mm, cm, m, or km) to each group. Give each group a sheet of chart paper and ask students to write the full name of their unit of measurement and the abbreviation at the top of the page. Have students list as many things as they can that could be measured with that unit of measurement. Set a target quantity (maybe 20) and ask students to try to list more than that quantity. Have each group share their ideas.

**Review perimeter.** Write the word perimeter on the board and remind students that perimeter is the measurement around the outside of a shape. Illustrate the perimeters of some classroom items by running your hand along the outside of a desk, the blackboard, or a blackboard eraser. Write the phrase “the measurement around the outside of a shape.”

Draw on the board:

```
1 cm

1 cm
```

Explain that each edge of the squares represents 1 cm and that perimeter is calculated by totaling the outside edges. Demonstrate a method for calculating the perimeter by marking or crossing out each edge as it is counted. Demonstrate this several times.

Have students practise finding the perimeter of figures by counting the edges.

**Exercises:** Each edge is 1 cm long. Find the perimeter of the figure.

a) ![Figure 1](image1.png)

b) ![Figure 2](image2.png)

c) ![Figure 3](image3.png)

**Answers:** a) 8 cm, b) 8 cm, c) 16 cm

**Perimeter is not additive.** Display the following sequence:

```
△ △ △ △
```
As a class, find the perimeter of each shape in the sequence. Examine the quantities and then ask students how the perimeters change with the addition of each triangle. (The perimeter increases by one.)

ASK: Why does the perimeter only change by one even though each added shape has three sides? Reiterate that perimeter is the measurement around the outside of a figure or shape only. SAY: Every time a new triangle is added to the sequence it covers one of the edges that had previously been on the outside.

Extend the sequence and have students predict the perimeter of the three subsequent figures. Draw each figure to check the predictions.

**Adding squares in different places produces different perimeters.** Draw on the board:

![Diagram of squares]

Ask a volunteer to find the perimeter. Then ask additional volunteers to add a square to all possible perimeter positions and identify how the perimeter measurement changes. Summarize the results in a table. (It is also good to identify identical shapes. Do they have the same perimeter?) ASK: Why does the perimeter change the way it does? How many edges that had previously been on the outside are now inside?

**Exercises:** Add a square to the shape at left so the perimeter’s measurement …

![Shape with a square added]

a) increases by two.  

b) remains the same.  

c) decreases by two.

**Sample answers**

a) ![Shape with a square added]

b) ![Shape with a square added]

c) ![Shape with a square added]

**ACTIVITY 2 (Essential)**

2. Distribute one piece of string, about 30 cm long, and a geoboard to each student. Have students tie the ends of the string together to form a loop and then create a variety of shapes on the geoboard with the string. Explain that the shapes will all have the same perimeter because the length of the string, which forms the outside edges, is fixed. How many different shapes can all have the same perimeter?
Draw an irregular figure with some edges that are neither horizontal nor vertical.

Have a volunteer use a ruler or metre stick to measure each side and then count the sides to confirm that they’ve all been measured.

Remind students that they can estimate the lengths of the edges of a shape with their fingers.

**ACTIVITY 3 (Optional)**

3. Using metre sticks and rulers, challenge students to measure the perimeters of various objects in the classroom and to specify the best unit of measurement for each.

**Extensions**

1. Introduce students to the principle of metric prefixes. Explain to them that the metric system is composed of many different units of measurement, all dependent on the dimensions being measured—distance is measured in metres, volume is measured in litres, weight is measured in kilograms.

    Then explain that each unit of measurement has a base unit (for these lessons, metres) that shares prefixes with the other base units. Write the word “metre” and ask students to provide you with the prefixes.

    Write the prefixes and explain a bit about the etymology of each. For example, “centi” means one hundredth. That’s why there are 100 centimetres in a metre. Also, there are 100 cents in a dollar. There are 100 years in a century.

    “Milli” means one thousandth. That’s why there are 1000 millimetres in a metre. Also, there are 1000 years in a millennium.

    “Deci” means one tenth. That’s why there are 10 decimetres in a metre.

    “Kilo” means 1000. That’s why there are 1000 m in a kilometre.

2. Have students research (and prepare reports or descriptive posters on) alternative systems of measurement, such as Ancient Egyptian, Chinese, or Old English systems. How long were their respective units of measurement, and what were they called? Are they still in use?

3. For a challenge, try to guide students toward developing a formula for the figure they worked with before the exercises. (new perimeter = old perimeter + 4 − twice the number of sides that become inside edges) To do so, draw the original figure and a new square beside it, as shown below:
ASK: How many edges does a square have? (4) SAY: This means we add 4 new edges. Write on the board:

\[ \text{new perimeter} = \text{old perimeter} + ____ \]

Have a volunteer fill in the blank. SAY: Some edges of the original figure are not counted anymore. ASK: What do we need to do with their number? (subtract) Why are they not counted? (they become inside edges, when the square is attached to them) Write on the board:

\[ \text{new perimeter} = \text{old perimeter} + 4 - \text{edges that became inside edges} \]

Draw the new figure with the square attached. ASK: Are the edges we subtracted also edges of the square? (yes) If we add 4 when we add the sides of the square, do we add these edges too? (yes) SAY: We do not need these edges. ASK: What should we do? (subtract them) Add another subtraction to the formula on the board, so it looks like this:

\[ \text{new perimeter} = \text{old perimeter} + 4 - \text{edges that became inside edges} - \text{edges that became inside edges} \]

SAY: We subtract the same number twice. ASK: Can we multiply by 2 first and subtract later? Check with students that this process works with numbers: is \(10 - 3 - 3\) the same as \(10 - (2 \times 3)\)? Is \(15 - 1 - 1\) the same as \(15 - (2 \times 1)\)? Is \(14 - 5 - 5\) the same as \(14 - (2 \times 5)\)? When students are convinced that this operation is valid, rewrite the formula.

4. Distribute Pentomino pieces (a set of 12 figures each made of 5 squares; see BLM Pentominoes, p. S-2) and have students calculate the perimeter of each figure and fill in the table below.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter</th>
<th>Number of Inside Edges</th>
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<tbody>
<tr>
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a) Which shape is different from all other shapes?

b) Double the number of inside edges and add to the perimeter for all shapes. What do you notice? (they all add to 20)

**Explanation:** Each inside edge belongs to 2 squares. Each edge in the perimeter belongs to 1 square. The answer in part b) is the total number of edges on all squares. There are 5 squares, so the total number of edges is \(5 \times 4 = 20\).
Goals

Students will find the perimeter of given and self-created figures using addition equations, find missing sides of rectangles given the perimeter, and develop a formula for the perimeter of a rectangle.

PRIOR KNOWLEDGE REQUIRED

Can calculate perimeter
Knows that rectangles have equal opposite sides

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1)
geoboards (optional, see Extension 1)

Mental math minute—number string.

String 1: 10 × 3, 11 × 3, 12 × 3 (30, 33, 36)

To present the strategy of adding on 3, write each multiplication as repeated addition, as shown below:

\[ 11 \times 3 = (10 \times 3) + 3 \]

Emphasize that 11 threes is 1 more three than 10 threes.

String 2: 10 × 4, 11 × 4, 12 × 4 (40, 44, 48)

Review perimeter. Review perimeter, its definition, and how it is calculated by totalling the outside edges of a figure.

Demonstrate the method for calculating perimeter by counting the entire length of each side and creating an addition equation. Write the length of each side on the picture.

Draw several figures on a grid and ask students to find the perimeter of each figure, supposing that each side of the square is 1 cm.

Exercises

a) b) c)

\[
\begin{align*}
&\text{a)} \\
&\text{b)} \\
&\text{c)} \\
&\text{d)} \\
&\text{e)} \\
&\text{f)}
\end{align*}
\]

Answers: a) 10 cm, b) 12 cm, c) 12 cm, d) 12 cm, e) 14 cm, f) 16 cm
Ask students to draw several figures of their own design on grid paper or BLM 1 cm Grid Paper and exchange them with a partner. Have students find the perimeter of the shapes created by their partners.

Finally, suggest that students draw a letter from the alphabet on grid paper and find the perimeter of the letter.

**Bonus:** In the figure below, write the length beside each side and calculate the perimeter. Do not miss any sides—there are 10!

![Figure](image)

**Answer:** 20 cm

**Finding missing sides.** Draw a rectangle on the board. Write a length on one of the longer sides, for example, 6 cm. Ask students what the length of the opposite side should be. (6 cm) Add the length of one of the shorter sides, for example, 4 cm, and ask students what the length of the last side should be. (4 cm) ASK: What is the perimeter of the rectangle? (20 cm) Ask a volunteer to write the addition equation for the perimeter.

Draw another rectangle and say that its perimeter is 14 m. Say that the length of one side is 3 m. Ask students to find the lengths of the other sides. Encourage students to present more than one solution to the problem. For example, they could say that $14 - 3 - 3 = 8$, so the sum of the other two sides is 8, so each side is 4 m. Another solution would be to use the fact that the sum of two adjacent sides is exactly half of the perimeter: half of 14 is 7, so $3 +$ something $= 7$, which means each of the other sides is 4 m.

To lead students to the second solution, draw a big dot on one corner of the rectangle and ask students to imagine walking around the rectangle, starting from the dot. Trace your finger along the perimeter to illustrate walking. SAY: You’ve gone exactly halfway around the perimeter. ASK: Where are you now? (at the opposite corner) How can you find the distance you’ve passed? (add length to width) How can you find the whole perimeter from this number? (multiply by 2)

Ask students to find all the rectangles with a perimeter of 14 units and sides with lengths in whole units (e.g., 3 units, not 3.5). If students find only one (or zero) rectangle, show them a systematic method of finding the answer. On grid paper, draw a line 1 unit long, and ask students to finish the rectangle so that the perimeter is 14 units. Repeat with a side 2 units long. ASK: What side length would you try next? (3 units) When students draw a rectangle starting with 4 units, ask them if this rectangle is different from the previous one. It is not, so explain that we can stop because we have stopped producing new rectangles. Students can try to produce a rectangle starting with sides 5, 6, and 7 to see that they are not producing anything new. (Students will see that a rectangle with perimeter 14 and sides 7 is impossible. Prompt students to compare rectangles with sides 5 or 6 with rectangles drawn earlier. They should see that they have already drawn these rectangles, but in a different orientation.)
Exercises: Find all rectangles with the given perimeter. (Lengths and widths are whole numbers.)

a) 6 units   b) 10 units   c) 16 units

Answers

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<table>
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<td>3</td>
<td>5</td>
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</table>

Developing a formula for the perimeter of a rectangle. Draw three rectangles, one above the other on the board, label one length and width in each, and ask students to write the expressions that allow them to find a perimeter. If there are several solutions, choose one (e.g., addition statement with 4 terms; make sure all terms are written in the same order for all three rectangles) and leave the other methods for later. The board will look like this:

Length = 3 cm
Width = 2 cm
3 cm + 3 cm + 2 cm + 2 cm = 10 cm

Length = 5 cm
Width = 1 cm
5 cm + 5 cm + 1 cm + 1 cm = 12 cm

Length = 4 cm
Width = 2 cm
4 cm + 4 cm + 2 cm + 2 cm = 12 cm

Invite volunteers to circle the lengths in the equations. Invite more volunteers to draw a square around the widths. ASK: How are all the equations the same? (all have two circles first and two squares after) Explain that two \( \bigcirc \) can be written as \( 2 \times \bigcirc \) and two \( \bigboxdot \) can be written as \( 2 \times \bigboxdot \) by writing this on the board. Ask students to draw a template for an equation for finding the perimeter for another rectangle. Explain that you need a general rule for finding the perimeter, so you want to replace the actual length with the word “length.” Write “length” in the circles. Repeat with width. Rewrite the formula without circles and squares. Write on the board:

\[
\text{Perimeter} = (2 \times \text{length}) + (2 \times \text{width})
\]

Explain that you’ve now got a formula for the perimeter. SAY: The formula is a rule that allows you to find perimeter by just replacing the words with different numbers. For example, if a rectangle has length 10 cm and width 7 cm, you can just replace the words with actual numbers and find the perimeter. Erase the word “length” and write 10 cm instead. Repeat with “width.” Have students find the perimeter using the formula.
Extensions

1. a) Is it possible to draw a rectangle on grid paper with whole number side lengths and a perimeter of 7 units? Investigate using grid paper or a geoboard.

   b) Is it possible to draw a rectangle on grid paper with whole number, side lengths and a perimeter that is an odd number of units? Explain how you know.

   **Answers:** a) no, b) no, because going halfway around is a whole number, so going the whole way around the rectangle is double that, and so is even.

2. The perimeter of a hexagon with all equal sides is 42 cm. How can you find the length of its sides?

3. Can 500 toothpicks line the entire perimeter of your school? Calculate an estimate. Use any tool you think will help.

   **Sample answer:** I measured a toothpick with a ruler and found that it is about 6 cm long, so 500 of them would be about 3000 cm. I estimated the perimeter of the school by counting large steps, and I used a metre stick to make sure that my steps were about a metre. I counted about 80 m around the school, and that is 8000 cm, so, no, 500 toothpicks cannot line the entire perimeter.
Goals

Students will use square centimetres or square metres to measure the areas of regular and irregular shapes. Students will determine appropriate units for measuring area.

PRIOR KNOWLEDGE REQUIRED

Can use a ruler to draw line segments
Can measure in centimetres
Can choose appropriate units for measuring length

MATERIALS

metre stick
grid paper or BLM 1 cm Grid Paper (p. S-1)
geoboards (optional)
ones block
hundreds block
wrapping paper or old newspapers

Mental math minute. Ask students to solve multiplication problems within the range of $0 \times 1$ to $10 \times 10$. For each number, first go through the problems in order, such as $0 \times 3$, $1 \times 3$, and so on to $10 \times 3$, then in reverse order, and after that, go through the same problems out of order. Then progress to a different number.

Review squares as units of area. Remind students that the area of a flat shape is the amount of space it takes up. Draw two rectangles as shown in the margin.

ASK: Which rectangle has a larger area? PROMPT: Imagine these are two pieces of cake. Which one has more cake? Explain that squares that cover the shape are used as units to measure area. Point out that units should cover the shape without gaps or overlaps. Draw several shapes made of squares (see examples in the margin).

Ask students to count the number of squares in each shape and write it as “[6] square units.”

Determining the area of shapes on grids. Draw several rectangles and mark their sides at regular intervals, as shown below:

Ask volunteers to divide the rectangles into squares by using a metre stick to join the marks. Ask students to find the area of these rectangles.
ACTIVITY 1 (Optional)

1. Students work in pairs. One student draws a shape on grid paper or BLM 1 cm Grid Paper or creates one using whole squares on a geoboard, and the other student calculates the area. ASK: How many shapes can you draw with an area of 8 squares? Have students work in their pairs to find the answer.

Review square centimetres. Remind students that squares that have all sides equal to 1 cm are called square centimetres. Write “square centimetre” and “cm²” on the board and explain that these are two ways to write square centimetres. Show students an ones block and tell them that it covers about 1 square centimetre. Another example is the top of the pinky finger.

Draw a rectangle that is 3 squares wide and 2 squares tall. Tell students that these squares are each 1 cm by 1 cm. ASK: What is the length of each side? Have students copy the rectangle and write the lengths on the sides. (3 cm, 2 cm, 3 cm, 2 cm) Then ask students to find the area of the rectangle. (6 cm²) ASK: What unit is length measured in? SAY: Length is measured in centimetres, not square centimetres. Emphasize that these are different (though connected) units.

Exercise: On grid paper or a geoboard, make as many shapes as possible with an area of 6 squares. For a challenge, try making shapes that have at least one line of symmetry. For instance, the shapes below have an area of 6 square units and one line of symmetry.

Review different units. Remind students that just as length is measured in different units, so is area. Square centimetres are squares with length and width 1 cm. ASK: What will square metres be? (squares with length and width 1 m) Use a metre stick to draw a square metre on the board. Ask students for examples of objects around the room that are about the same size. Examples might include a desktop, a small bookcase, or a window.

Exercise: Marko says that shape A at left has an area of 4 squares and shape B has an area of 3 squares, so shape A has a larger area than shape B. Explain his mistake.

Answer: Each square in shape B is larger than each square in shape A, and three larger squares can have a larger total area than four smaller squares.

Comparing unit squares. Ask students to guess about how many square centimetres fit in 1 square metre. Ask if it is bigger or smaller than 100 square centimetres. Hold up a hundreds block. ASK: What is the area of a hundreds block in square centimetres? (100 cm²) PROMPT: How many ones are in a hundreds block? Hold the hundreds block up in front of the square metre to demonstrate how much bigger the square metre is.
NOTE: Keep the squares made in Activity 2 for use in Activity 3.

ACTIVITY 2 (Optional)

2. **Making and comparing unit squares.** Assign different sizes of squares (1 cm², 10 cm × 10 cm, 1 m²) to students and have them use wrapping paper or old newspapers to make and label squares of the given size. Have students who were given the two smaller units make several squares. Students compare and order the units in groups. **ASK:** Will a 30 cm by 20 cm rectangle fit inside 1 square metre? How many 10 cm by 10 cm squares do you think will fit in a metre square?

**Choosing appropriate units.** Point to some small objects in the classroom and ask students if they would measure the area of these objects in square centimetres or square metres. Have students explain their reasoning. Point out that those objects are too small to measure in square metres. Then indicate some larger objects and ask the same questions. Ask students for examples of objects that could reasonably be measured using either. These will tend to be things that measure around 1 m².

**Estimating and measuring area.** In advance, mark out a rectangle on the floor that is a whole number of metres on each edge or choose a rectangular surface to be measured, such as the board or a window. Hold up a metre square from Activity 2 next to the surface. Ask students to estimate, by comparing the sizes, how many square metres will fit on the surface. Record the answers on the board. Demonstrate measuring the surface with the help of volunteers. Place the metre square in the lower-left corner. If the surface is vertical, keep the metre square in place by holding or taping the corners. Place another metre square right next to the first. Point out that the squares leave no gaps and do not overlap. Then move the first square to the right of the second square. Tell students that this is like pacing out distances. Have students count how many metre squares fit in your surface as you go (to the nearest whole number). When you can no longer go to the right, place the next square above the previous one. Then begin counting back to the left. Compare the measured area to the estimated areas on the board.

ACTIVITY 3 (Optional)

3. Write on the board a list of large and small objects to be measured (for example, the area of the door, the surface of a book, the area of floor tiles or the carpet). If possible, include shapes that are not rectangular. Working in small groups, students choose the appropriate unit for measuring the area of each item. They first estimate and then measure the area using the squares made in Activity 2 or ones blocks.
Exercises: Find the area of the rectangle. Don’t forget to write the appropriate unit.

a) 4 km  

b) 7 cm  

c) 7 mm

Answers: a) 12 km², b) 14 cm², c) 49 mm²

Extensions

1. Find the area of the rectangle.

a) 3 cm  

b) 3 dm  

c) 4 cm

Answers: a) 6 cm², b) 3 dm², c) 12 cm²

2. a) What is the area of the rectangle? (See margin.)

b) The rectangle in part a) is divided into two triangles. Are they congruent?

c) Do congruent shapes have the same area? Explain. (Hint: What does congruent mean? What is area?)

d) What is the area of the grey triangle? How do you know?

Answers: a) 20 cm²; b) yes; c) congruent shapes have the same area because they can be placed on top of each other to cover exactly the same space; d) 10 cm², because each triangle is exactly half

3. a) What is the area of the shaded part?

b) Copy the figure onto grid paper. Draw a straight line to divide the shape into 2 parts of equal area. What is the area of each part?

Answers: a) 1 cm²; b) Since the total area is 14 square units, each part has an area of 7 square units.
ME5-15  Area and Perimeter of Rectangles  
Pages 126–128

CURRICULUM REQUIREMENT  
AB: required  
BC: required  
MB: required  
ON: required

VOCABULARY  
area  
centimetre (cm)  
metre (m)  
square centimetre (cm²)  
square metre (m²)

Goals  
Students will express the formula for the area of a rectangle using variables.  
Students will use a formula to calculate the areas of rectangles.  
Students will investigate how perimeter and area are related to but not dependent on each other.

PRIOR KNOWLEDGE REQUIRED  
Understands the concept of area  
Can use a ruler to draw and measure line segments  
Can multiply and divide up to 10 \times 10  
Can multiply up to two-digit numbers by one-digit numbers  
Can multiply two-digit numbers by powers of ten

MATERIALS  
overhead projector  
transparency of BLM 1 cm Grid Paper (p. S-1)  
grid paper or BLM 1 cm Grid Paper (p. S-1)  
rulers

Mental math minute. Give students multiplication questions that can be done by skip counting by 2s, 3s, 4s, 5s, or 10s. For example, ask a student to find 8 \times 4 by skip counting by 4s. Have students skip count aloud to answer the questions.

Review rectangles as arrays of squares. Draw a 3 by 4 array of dots on the board. ASK: How many dots are in the array? How did you count the dots? Have students write the corresponding multiplication statement on the board.

Project BLM 1 cm Grid Paper on the board. Draw a 3 by 4 rectangle on the grid. Have students copy it on grid paper or BLM 1 cm Grid Paper and write the length and width of the rectangle. ASK: How are the rectangle and the array the same? What multiplication statement gives us the area of the rectangle? To prompt students to see the answer, draw a dot in each square of the rectangle. Ask students to write the multiplication statement for the area of the rectangle. Remind students that the area of the rectangle is the number of square units that cover the rectangle.

Draw several rectangles on the grid. Ask students to write the length and the width and calculate the area.

Determining the formula for area of rectangles. On the board, draw a rectangle (not on a grid) and tell students that it is 50 squares long and 30 squares wide. ASK: What is the area of this rectangle? (1500) How did you find the answer? (50 \times 30 = 1500) SAY: Because area is measured in squares, we can think about any rectangle as being made of squares.
Draw a 25 cm by 20 cm rectangle on the board. Use this opportunity to review using a ruler to draw line segments of a given length by having students tell you what to do next. Write the length and the width beside the sides of the rectangle. SAY: The rectangle is 25 cm long. ASK: How many 1 cm squares long is it? (25) How many 1 cm squares wide is it? (20) How many 1 cm squares are needed to cover the rectangle? (500) Have students write the multiplication statement. (25 \times 20 = 500) ASK: What is the area of the rectangle? (500 cm²) Did we divide the rectangle into squares to find the answer? (no) How did we find the answer? (multiplied length by width) Do you think this method will work to find the area of any rectangle? (yes) Summarize on the board:

Area of rectangle = length \times width

**ACTIVITY (Optional)**

**Finding areas of rectangles.** Students work in pairs to draw various rectangles with length and width in whole centimetres, then exchange notebooks. Partners then measure the sides of the rectangles, record their length and width, and find the area. Partners check each other’s work. Have students work in their pairs to find as many rectangles as they can with area 12 cm².

**Finding the area of rectangles using the formula.** Draw on the board:

```
10 cm
\[\text{\textmd{5 cm}}\]
```

Ask students to copy the dimensions of the rectangle. Demonstrate how to use a formula to record the process of finding the area, as shown below:

\[
\text{Length} = 10 \text{ cm} \\
\text{Width} = 5 \text{ cm} \\
\text{Area} = \text{length} \times \text{width} \\
\hspace{1cm} = 10 \text{ cm} \times 5 \text{ cm} \\
\hspace{1cm} = 50 \text{ cm}^2
\]

Remind students to write the unit for each measurement. In the first three exercises below, provide a picture of a rectangle labelled with the length and the width. For the rest of the exercises, just write the length and width. Review multiplying multi-digit numbers before assigning part g) onwards.

**Exercises:** Use the formula to find the area.

a) length 5 m, width 4 m  
b) length 6 m, width 7 m  
c) length 20 cm, width 15 cm  
d) length 5 cm, width 8 cm  
e) length 6 m, width 11 m  
f) length 12 m, width 4 m  
g) length 56 cm, width 20 cm  
h) length 42 m, width 42 m
Comparing area and perimeter of rectangles. Draw several rectangles on a grid: 4 × 6, 5 × 5, 6 × 3, 9 × 1, and 3 × 8. Label them A to E and ask volunteers to find the area and the perimeter of each one. Tell students that each side of a grid square represents 1 m. Ask students to list the rectangles from least to greatest by area. (D: 9 m², C: 18 m², A: 24 m², E: 24 m², B: 25 m²)

Then ask them to list the rectangles from least to greatest by perimeter. (C: 18 m, A: 20 m, B: 20 m, D: 20 m, E: 22 m) ASK: Are your lists the same? (no) Does the rectangle with the greatest area also have the greatest perimeter? (no) Does the rectangle with the smallest perimeter also have the smallest area? (no) Are there rectangles with the same area? (yes, A and E) Do they have the same perimeter? (no) Are there rectangles that have the same perimeter? (yes, A, B, and D) Do they also have the same area? (no)

What do you do with the length and width of a rectangle to calculate the area? (multiply) What do you do with the length and width of a rectangle to calculate perimeter? (add, then multiply by 2) SAY: Perimeter and area are related to each other but not dependent on each other.

Exercises: Draw 2 rectangles that have the same area—10 cm²—but different perimeters. Calculate the perimeters.

Bonus: Bill drew 3 shapes with the same perimeter but different areas. The sides of Bill’s shapes are measured whole centimetres. One shape is a square with area 9 cm². The other two shapes are rectangles. Draw the rectangles.

Answers: 1 × 10 and 2 × 5, perimeters are 22 cm and 14 cm, Bonus: 1 × 5 and 2 × 4 rectangles

NOTE: Extension 1 is required in order to cover the British Columbia curriculum.

Extensions

1. a) A pit house is a structure that is partly dug into the ground and covered by a roof. These structures provide shelter from extremes of weather. They may also be used to store food and for cultural activities such as the telling of stories, dancing, singing, and celebrations.

Aputik wants to make a rectangular base pit house with a perimeter of 24 m. Which dimensions of the base of the pit house will provide the greatest area?
b) Invite a local Elder or knowledge keeper to talk about traditional measuring and estimating techniques for hunting, fishing, and building.

**Answer:** a) a 6 m × 6 m square

2. David wants to fence a rectangular flower bed with an area of 36 m². Which dimensions will give the least perimeter of the flower bed?

**Answer:** a 6 m × 6 m square

3. Find the area of the shaded part.

![Diagram](image)

**Answer:** $18 - 4 = 14 \text{ cm}^2$
**Goals**

Students will solve problems connected to the area and perimeter of rectangles.

**PRIOR KNOWLEDGE REQUIRED**

- Can multiply and divide, including multi-digit whole numbers
- Can find the area of a rectangle using the formula for the area of a rectangle
- Can solve an equation $a \times x = b$
- Knows that a variable can replace a number
- Can find the perimeter of a rectangle

**Mental math minute—number string.**

String 1: $10 \times 7, 9 \times 7, 8 \times 7$ (70, 63, 56)

Present the strategy of subtracting from 10 times the number by writing each multiplication as repeated addition. Emphasize that 9 sevens is 1 less than 10 sevens.

String 2: $10 \times 9, 9 \times 9, 8 \times 9$ (90, 81, 72)

**Review using a formula for area of a rectangle.** Remind students that they can find the area of a rectangle by multiplying length by width. Since order does not matter in multiplication, they can write the factors in any order. Remind them also that we often use letters instead of whole words in formulas, so instead of writing "Area = length \times width," we can write "Area = ℓ \times w." Write both versions of the formula on the board.

**Exercises:** Find the area of the rectangle. Show your calculations.

a) length 8 m, width 6 m  
   b) length 2 cm, width 4 cm  
   c) length 15 km, width 8 km  
   **Bonus:** length 24 mm, width 15 mm

**Answers:** a) 48 m², b) 112 cm², c) 120 km², Bonus: 360 mm²

**Using equations for the area of a rectangle to find the missing dimension.** Remind students that they can use a letter for an unknown number in a problem. For example, a rectangle has a length of 7 cm and a width of $w$ cm. The rectangle’s area is 35 cm². Have students record this information. SAY: In this case, we can write an equation for the area: $w \times 7 = 35$. Write the equation on the board and have students copy it. ASK: What number will make the equation true? (5) How do you know? (5 \times 7 = 35) Remind students that multiplication and division are related, so if $5 \times 7 = 35$, there is an equation from the same fact family that has 5 as the answer. ASK: What equation is that? (35 ÷ 7 = 5) Remind students that they’ve learnt to write an equation where the unknown number is by itself, and that’s what they could do to solve this equation.
Write the solution on the board:

\[ w \times 7 = 35 \]
\[ w = 35 \div 7 = 5 \]

ASK: What units will the answer be in? (cm) How do you know? (the area is in cm² and the length is in centimetres, so the width has to be in centimetres)

Add the units to the answer: \( w = 5 \text{ cm} \).

**Exercises:** Write an equation for the area of the rectangle. Then find the unknown length.

a) Width = 5 cm  
\begin{align*}
\text{Length} & = \ell \text{ cm} \\
\text{Area} & = 55 \text{ cm}^2 \\
\end{align*}

b) Width = 6 km  
\begin{align*}
\text{Length} & = \ell \text{ km} \\
\text{Area} & = 96 \text{ km}^2 \\
\end{align*}

Bonus: Width = 15 m  
\begin{align*}
\text{Length} & = \ell \text{ m} \\
\text{Area} & = 675 \text{ m}^2 \\
\end{align*}

**Answers:** a) 11 cm, b) 16 km, Bonus: 45 m

**Exercises:** Write an equation for the area of the rectangle. Then find the unknown length or width.

a) Width = 5 m  
\begin{align*}
\text{Length} & = \ell \text{ m} \\
\text{Area} & = 45 \text{ m}^2 \\
\end{align*}

b) Width = \( w \) cm  
\begin{align*}
\text{Length} & = 7 \text{ cm} \\
\text{Area} & = 42 \text{ cm}^2 \\
\end{align*}

c) Width = 3 m  
\begin{align*}
\text{Length} & = \ell \text{ m} \\
\text{Area} & = 210 \text{ m}^2 \\
\end{align*}

**Answers:** a) 9 m, b) 6 cm, c) 70 m

**Shapes composed of two rectangles.** Draw the shapes composed of two rectangles from the exercises below on the board and mark the dimensions on four of the sides. Ask students to copy the shapes and dimensions, shade each rectangle with its own colour, and then circle the dimensions that belong to the rectangle in the same colour.

**Exercises:** Find the area of each rectangle. Add the areas of the rectangles in each shape to find the total area of the shape.

a) 
\begin{align*}
2 \text{ m} & \quad 2 \text{ m} \\
3 \text{ m} & \quad 1 \text{ m} \\
\end{align*}

b) 
\begin{align*}
7 \text{ dm} & \quad 3 \text{ dm} \\
4 \text{ dm} & \quad 6 \text{ dm} \\
\end{align*}

 Bonus: 
\begin{align*}
6 \text{ cm} & \quad 7 \text{ cm} \\
5 \text{ cm} & \quad 8 \text{ cm} \\
23 \text{ cm} & \\
\end{align*}

**Answers:** a) 8 m, b) 46 dm, c) 130 m², Bonus: 201 cm²
Determining the missing side lengths to find area. Draw on the board:

![Rectangle](image)

ASK: Is 7 cm the length of the short side of the shaded rectangle? Trace it with your finger. (no) ASK: Is it the length of the short side of the unshaded rectangle? (no) Point out that 7 cm is the length of one side of the unshaded rectangle and one side of the shaded rectangle combined—in other words, 7 cm is the sum of the two side lengths. ASK: What is the length of the short side of the unshaded rectangle? (3 cm) How do you know? (7 cm − 4 cm = 3 cm) Finally, have students find the areas of both rectangles and add them to find the total area of the shape. (5 cm × 3 cm = 15 cm², 11 cm × 4 cm = 44 cm², total area = 59 cm²)

Draw on the board:

![Rectangle](image)

Ask students to find the length of the longer side of the shaded rectangle. (10 m + 11 m = 21 m) Have students find the area of the shape. (7 m × 10 m = 70 m², 9 m × 21 m = 189 m², total area = 259 m²)

Exercises: Find the area.

a) ![Rectangle](image)  

b) ![Rectangle](image)

Answers: a) 9 m², b) 10 cm²

Finding the area two ways by dividing the shape differently. Draw on the board:

![Rectangle](image)
Shade one of the rectangles as shown. Ask students to identify the side lengths of the shaded rectangle. Point to each label in order and have students signal thumbs up if the label shows the length of a side of the shaded rectangle and thumbs down if it does not. Circle the labels for the shaded rectangle and then repeat with the side lengths of the unshaded rectangle. Then present the same shape with the same labels but broken into two rectangles in a different way (as shown in the margin) and repeat the exercise. Point out that, in each case, students need to use four of the six labels to find the area of the shape. The remaining two labels are unused, and the unused labels differ in each case. Have students find the area of the shape both ways. (89 cm²) Did they get the same answer? If not, ask them to find their mistakes.

**Exercises:** Divide the shape into two rectangles. Then find the area of the shape.

a)

```
12 m

23 m

15 m

12 m
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b)

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8 cm

13 cm

7 cm

4 cm

6 cm

12 cm
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**Bonus:** Find a different way to split these shapes into rectangles and find the area the new way. Did you get the same answer as before?

**Selected answers:** a) 456 m², b) 128 cm²

**Review perimeter.** Remind students that the perimeter of a shape is the distance around the shape. To find it, you add the measurements of all sides. As well, remind students that, since a rectangle has equal opposite sides, students can find the perimeter of a rectangle by adding length and width and multiplying by 2.

**Exercises:** Find the perimeter of the rectangle with the given length and width.

a) 21 cm by 11 cm  b) 26 m by 23 m  c) 7 km by 10 km

**Answers:** a) 64 cm, b) 98 m, c) 34 km

Draw a rectangle on the board and mark one side as 5 m. SAY: The perimeter of this rectangle is 22 m. What are the other sides of the rectangle? Have students present multiple solutions. To prompt students, have them first find the length of the opposite side. SAY: These two sides together add to 10 m. (5 m + 5 m = 10 m) What do the other two sides add to? How do you know? (22 m – 10 m = 12 m) How long is each side? (6 m) How do you know? (they are equal and add to 12 m, so each is 12 m ÷ 2 = 6 m) Another way of thinking about the perimeter is that two adjacent sides should be half of 22, so two adjacent sides add to 11 m. One of the sides is 5 m. How long is the other side? (6 m) What is the area of the rectangle? (5 m × 6 m = 30 m²)
**Exercise:** Find the missing length of the rectangle. Then find the area.

a) perimeter 20 cm, width 4 cm  

b) perimeter 20 cm, width 2 cm

**Answers:** a) length 6 cm, area 24 cm²; b) length 8 cm, area 16 cm²

**Extensions**

1. a) Draw a rectangle so that the number representing the area (in cm²) is the same as the number representing the perimeter (in cm).

b) Convert the measurements of the sides to millimetres. Find the area and perimeter of the rectangle with the new measurements. Is the number representing the area in mm² still equal to the number representing the perimeter in mm?

**Sample answer:** a) 18 = 3 × 6 (area) = 2 × 3 + 2 × 6 (perimeter) or 16 = 4 × 4 (area and perimeter); b) 30 mm by 60 mm, so area = 1800 mm² and perimeter = 180 mm. Or 40 mm by 40 mm, area = 1600 mm², perimeter = 160 mm. The numbers are different.

**NOTE:** This exercise shows that the numbers representing area and perimeter can be identical in one set of units and different in another set of units, even though the rectangles do not change. This means that area and perimeter simply cannot be compared; the units they are measured in are incomparable.

2. On a grid, draw a square with sides 7 units long.

a) Find the area and the perimeter of the square.

b) Inside the square, draw a shape that has an area smaller than the area of the square and perimeter larger than the perimeter of the square. There are many possible answers.

**Sample answer**  
b) The shape below is one possibility, with area = 17 square units and perimeter = 36 units

![Sample shape diagram]

**Answer:** a) area = 49 square units, perimeter = 28 units
Goals
Students will find the number of cubes in a rectangular stack.
Students will develop the formula length $\times$ width $\times$ height for the number of cubes in a stack.

PRIOR KNOWLEDGE REQUIRED
Can find the area of a rectangle using the formula for the area of a rectangle

MATERIALS
connecting cubes

Mental math minute. Write on the board:

$37 \times 10 = \_
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Exercises

a) Write a multiplication equation for the number of blocks in the top layer.

i) $2 \times 3 = 6$, ii) $2 \times 3 = 6$, iii) $4 \times 3 = 12$;

b) Write a multiplication equation for the total number of blocks for each structure in part a).

Answers: a) i) $2 \times 3 = 6$, ii) $2 \times 3 = 6$, iii) $4 \times 3 = 12$;
b) i) $2 \times 3 \times 3 = 6 \times 3 = 18$, ii) $2 \times 3 \times 4 = 6 \times 4 = 24$,
iii) $4 \times 3 \times 3 = 12 \times 3 = 36$

Counting cubes in a stack using a vertical layer. Invite students to look at this last stack and calculate the number of cubes by adding vertical layers instead of horizontal layers as shown below:

ASK: How many cubes are at the end of the stack? ($3 \times 2 = 6$) Separate the stack into four vertical layers. ASK: How many cubes are in each vertical layer? (6) How many vertical layers are in the stack? (4) Invite volunteers to write the addition and multiplication statements for the total number of cubes using the number of cubes in the vertical layer. ASK: Does this method produce a different result than the previous method? (no) Why should we expect the same answer? (it is the same stack of cubes, only counted differently)

Determining the number of cubes in a stack as a product of length, width, and height. Remind students that, to find the number of squares in a rectangle, they can multiply length by width. Now they have a three-dimensional stack, but it is very similar to a rectangle. Remind students that the vertical dimension in 3-D objects is called the height. Display the picture in the margin and identify the length, width, and height of the stack. Then write the following equation on the board and use the terms “length,” “width,” and “height” to label the multiplication statement that gives the number of cubes in the stack:

$$4 \times 2 \times 3 = 24$$

length width height

Draw several stacks on the board and ask students to find the number of cubes in each stack by multiplying length, width, and height. Remind students that order does not matter in multiplication, so the number of cubes can be found in other ways, such as height $\times$ length $\times$ width or width $\times$ length $\times$ height.
Exercises: Use the stacks in the previous exercises.

a) Write a multiplication equation for the number of blocks in a vertical layer.

b) Write a multiplication equation for the total number of blocks.

Answers: a) i) $3 \times 3 = 9$, ii) $3 \times 4 = 12$, iii) $3 \times 3 = 9$

b) i) $3 \times 3 \times 2 = 9 \times 2 = 18$, ii) $3 \times 4 \times 2 = 12 \times 2 = 24$

ACTIVITY (Optional)

Students work in pairs. Each student creates several rectangular stacks of connecting cubes. Students exchanges stacks with his or her partner. Have students find the number of cubes in the stacks created by their partner by multiplying length, width, and height.

Discuss how the dimensions in the formula length $\times$ width $\times$ height are related to the layers in a stack. ASK: What part of the formula gives you the number of cubes in one horizontal layer? (length $\times$ width) SAY: Then you multiply by the number of horizontal layers, which is height. Repeat with a vertical layer. (width $\times$ height)

Extension

Use connecting cubes.

a) How many ways can you build a rectangular stack of 36 cubes with a height of 3 cubes?

b) How many ways can you build a rectangular stack with 48 cubes?

Answers

a) 3 ways; dimensions in order of length $\times$ width $\times$ height: $12 \times 1 \times 3$, $6 \times 2 \times 3$, $4 \times 3 \times 3$

b) 9 combinations of dimensions (with different arrangements of the same dimensions possible): $48 \times 1 \times 1$, $24 \times 2 \times 1$, $16 \times 3 \times 1$, $12 \times 4 \times 1$, $12 \times 2 \times 2$, $8 \times 3 \times 2$, $8 \times 6 \times 1$, $6 \times 4 \times 2$, $4 \times 4 \times 3$

Redirecting students: If students struggle to find all the combinations, ASK: How can you be organized to make sure you check all combinations without repeating any? (for example, list possibilities in increasing order) How can you simplify your work? (start by finding the possible heights, then find the possible lengths and widths given the height) Which numbers don’t need to be checked for the dimensions? (numbers that are not factors of the total number of cubes)
Goals

Students will develop and use the formula Volume = length \times width \times height for the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED

Can find the area of a rectangle using the formula
Can find the number of blocks in a rectangular stack
Can multiply two-digit numbers
Is familiar with units for measuring length and can convert between the units within the same system

MATERIALS

connecting cubes
old newspapers or wrapping paper tubes
tape
BLM Cube Skeleton (p. Q-59)
centimetre cubes for demonstration
prism sample, such as a facial tissue box

Mental math minute—number talk. Present this problem: 7 \times 9. (63)
The following strategies could arise:

\[
\begin{align*}
(7 \times 10) & - 7 \\
(10 \times 9) & - (3 \times 9) \\
(5 \times 9) & + (2 \times 9) \\
(7 \times 5) & + (7 \times 4)
\end{align*}
\]

Introduce volume. Remind students that area is the amount of two-dimensional (flat) space a shape takes up. We measure area in square units—squares that have all sides 1 length unit long.

Hold up two shapes made from connecting cubes, one made from 3 cubes and another made from 4 cubes. ASK: Which shape takes up more three-dimensional space? (the shape made from 4 cubes) How do you know? (it is made from more cubes)

Explain that the space taken up by a box or a cupboard or any other three-dimensional object—in other words, an object that has length, width, and height—is called volume. We measure volume in cubic units: units that are cubes.

Introduce standard cubic units. Draw several cubes on the board and mark the sides as 1 cm for one cube, 1 m for another cube, and so on. Explain that these represent different cubic units: cubic centimetre, cubic metre, etc.
Show the abbreviated form for each unit:

cubic centimetre (cm³)
cubic decimetre (dm³)
cubic metre (m³)

**ACTIVITY (Optional)**

Divide students into two groups and assign a unit to each group: dm³ or m³. Have students roll old newspapers into tubes slightly longer than the unit they were assigned (to allow for binding at the ends); alternatively, students can use wrapping paper tubes. Mark the unit on each tube as shown below at left. Students might need to combine several newspapers to produce a longer tube. Have each group produce 12 tubes. Give each group tape and BLM Cube Skeleton and have them create skeletons of their assigned units to scale (see sample cube skeleton below at right).

Comparing cubic units. Display and compare the relative sizes of 1 m³ and 1 cm³ represented by a centimetre cube. Point out that cubic metres are very large. For example, to fill an aquarium with a volume of one cubic metre, you would need about 100 large pails of water!

Finding the volume of rectangular prisms. Explain that a three dimensional object, such as a cupboard or a box, has length, width, and height. Show a 3 × 4 × 2 stack of centimetre cubes and review how to find the number of cubes in a stack. (multiply length, width, and height) ASK: How many cubes are in the stack? (3 × 4 × 2 = 24) Explain that the volume of this stack is 24 cubic centimetres.

Draw several stacks of cubes on the board and explain that the cubes in each stack are unit cubes. Mark the size of each cube drawing, using different units for different pictures. Have students find the volume of each stack.

On the board, draw the rectangular prism shown in the margin.

Explain that mathematicians call rectangular boxes **rectangular prisms**. ASK: What is the length of this prism? (5 m) What is the width of this prism? (3 m) What is the height of this prism? (4 m) Record the information on the board:

\[
\text{Length} = 5 \text{ m} \\
\text{Width} = 3 \text{ m} \\
\text{Height} = 4 \text{ m}
\]
SAY: Imagine this is a container, and I want to pack it with cubes that each measure 1 m by 1 m by 1 m. How many cubes would fit along the length of this container? (5) Width? (3) Height? (4) How many cubes in total would fit in the container? (60) How do you know? (5 \times 3 \times 4 = 60; \text{volume} = \text{length} \times \text{width} \times \text{height}) Have students write the multiplication statement. What is the volume of the prism? (60 \text{ m}^3)

Point out that students found the volume of the prism even though you did not draw the cubes that filled it. ASK: What did you do to find the volume? (multiplied the dimensions: length \times width \times height) Write the formula \(V = \ell \times w \times h\) on the board and explain that it is also convenient to use the short form \(V = \ell \times w \times h\).

Draw the prism below on the board:

![Prism diagram]

Have students write the multiplication statement for its volume. Explain that, when finding volume, it helps to write the units for each measurement in the multiplication statement to prevent mistakes. For example, for this prism, the multiplication statement should be:

\[
\text{Volume} = 6 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm} \\
= 36 \text{ cm}^3
\]

Explain that writing the units in the equation helps students to see that there are three dimensions. Explain also that a few lessons later, students will see prisms where the dimensions for one prism are given in different units, so writing the units in the equations will be even more important.

Point out that the raised 3 that appears in the cubic units is the number of length measurements that were multiplied together to make the cubic unit: you multiplied three measurements—length, width, and height—to get the volume. All three measurements were in centimetres, so the result is in cubic centimetres. Point to the raised 3 in "cm³."

Point out that, in area, we also use a raised number in the units. In units of area, we multiply two dimensions—length and width—and the result is square units, for example, square centimetres. Write "cm²" on the board and point to the raised 2.

**Exercises**

1. A rectangular prism has dimensions 2 dm by 3 dm by 4 dm. What is its volume?

   **Answer:** 24 dm³
2. Find the volume of the rectangular prism. Include the units in your calculation and answer.

a) \(8 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm}\)

b) \(7 \text{ m} \times 8 \text{ m} \times 2 \text{ m}\)

c) \(5 \text{ km} \times 2 \text{ km} \times 3 \text{ km}\)

d) \(8 \text{ m} \times 2 \text{ m} \times 10 \text{ m}\)

e) length 10 cm, width 10 cm, height 10 cm

f) length 100 cm, width 100 cm, height 100 cm

Answers: a) 80 cm³, b) 112 m³, c) 30 km³, d) 60 m³, e) 1000 cm³, f) 1 000 000 cm³

Show that volume does not depend on orientation. Show students a prism that has different measurements for length, width, and height, such as a tissue box. Write its dimensions to the nearest centimetre on the board. Have students find the volume of the prism. Then turn the prism.

ASK: Did the height of the prism change? (yes) The length? (yes) The width? (yes) Do you expect the volume to change? (no) Why not? (rotating the box does not change the volume, the space that the prism takes up; the dimensions are the same, but we multiply them in a different order)

Constructing rectangular prisms for a given volume. Draw two different rectangular prisms on the board with the volume of 4 cubic units:

\[\ell = 4, \ w = 1, \ h = 1\]

\[\ell = 2, \ w = 1, \ h = 2\]

ASK: Can you make one rectangular prism from the other by turning or rotating it? (no) Explain to students that the left rectangular prism has a dimension equal to 4, but the right prism doesn’t have such a dimension.

SAY: These are two different rectangular prisms with the same volume.

Exercises: Construct two different rectangular prisms for the given volume.

a) 6 unit³

b) 10 unit³

Bonus: Construct four different rectangular prisms with the volume of 12 unit³.

Selected sample answer: Bonus
NOTE: Extension 4 is required in order to cover the Alberta and Manitoba curricula.

Extensions

1. Find the volume of the shape.

\[
\text{Answer: } (3 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm}) + (2 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm}) = 42 \text{ cm}^3
\]

2. a) Estimate the length, width, and height of the classroom in metres and then estimate the volume of the classroom.

b) Measure the length, width, and height of the classroom in metres. Then calculate the volume of the classroom. How close was your guess?

3. During an excavation of an old military fort from the War of 1812, an underground ammunition storage room was discovered. The room is 10 m long, 6 m wide, and 4 m deep. Several ammunition crates measuring 2 m by 1 m by 1 m each were found in the room. What is the greatest number of crates that would fit in the room?

Answer: The 2 m edge can be placed along any side of the storage room. One way to do so would be to place 5 crates along the length of the storage room, 6 along the width, and 4 along the height. This means 120 crates \((5 \times 6 \times 4 = 120)\) could fit in the room.

4. Construct four different rectangular prisms with the volume of 18 cm\(^3\).

Answer: Dimensions: \(1 \times 1 \times 18\), \(1 \times 2 \times 9\), \(1 \times 3 \times 6\), \(2 \times 3 \times 3\)
Volume and Area of One Face

Goals

Students will develop and use the formula Volume = area of horizontal face × height for the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED

Can use the formula \( V = l \times w \times h \) to find the volume of a rectangular prism

Can multiply and divide multi-digit whole numbers and decimals

MATERIALS

cube for demonstration
rectangular box for demonstration and 1 per student

Mental math minute—number string.

String 1: 10 × 6, 9 × 6, 8 × 6 (60, 54, 48)

Present the strategy of subtracting from 10 times the number by writing each multiplication as repeated addition. Emphasize that 9 sixes is 1 less six than 10 sixes.

String 2: 10 × 9, 9 × 9, 8 × 9 (90, 81, 72)

Review terminology. Hold up a cube. Remind students that different parts of a 3-D shape are called faces, edges, and vertices. Show the faces on a cube, run your finger along edges, and point out the vertices. Count the faces, edges, and vertices of a cube together and write on the board:

A cube has 6 faces, 12 edges, 8 vertices.

Draw a cube on the board as shown in the margin. Count the faces, edges, and vertices on the picture. (3 faces, 9 edges, 7 vertices) Explain that the edges and vertices that are behind the faces you see are called hidden edges and can be shown with dashed lines. Add the dashed lines to the picture, as in the second picture.

Label the faces of the cube as shown below:

Shade the front face and explain that the face opposite to the front face is called the back face.
Review the fact that opposite faces of a rectangular prism match. Show students a rectangular prism. **ASK:** In a rectangular box, or in a rectangular prism, can the top face be larger than the bottom face? (no) Can the bottom face be larger than the top face? (no) Can they have different shapes? (no) **SAY:** The top face and the bottom face are always the same rectangles. If some students have difficulty seeing that the top face and the bottom face match exactly, give them a box, have them trace the bottom face of the box, and check that the top face matches the bottom face exactly. Repeat with other pairs of opposite faces.

**Review the formula** \( V = l \times w \times h \). Ask students how they can find the volume of their boxes. **ASK:** What do you need to measure? (length, width, height) Remind students that a formula is a short way to write the instructions for how to calculate something. **ASK:** What is the formula for the volume of a rectangular prism? (Volume = length \( \times \) width \( \times \) height) How do we write it the short way? \( (V = l \times w \times h) \) Write both formulas on the board.

Remind students that they used a formula to find areas of rectangles and how they recorded the solution. For example, write the solution for a problem “Find the area of a rectangle with the length 3 cm and the width 2 cm” on the board, as shown in the margin. Keep the formula on the board for future use.

Give each student a rectangular box. Have students measure and record the dimensions of the boxes to the nearest centimetre. Then have them find the volume of their boxes in cubic centimetres.

**Developing the formula Volume = area of horizontal face \( \times \) height.**
Write on the board the formula for the volume of a rectangular prism:

\[
\text{Volume} = l \times w \times h
\]

Ask students whether they see the formula for the area of a rectangle hidden in that formula. (length \( \times \) width) Draw on the board:

```
+----+
|    |
|    |
+----+
```

**ASK:** Which face or faces of the prism have the same length and width as the prism itself? (top face and bottom face) Explain that we can describe both the top and the bottom faces as horizontal faces.

Write on the board:

\[
\text{Volume} = \text{area of horizontal face} \times h
\]

Explain that we now have a new formula: \( V = (\text{area of horizontal face}) \times h \). Write that formula on the board as well.
Using the formula to find the volume. Solve the first exercise as a class and then have students work individually.

**Exercises:** Find the volume.

a) ![Image of a prism with a 16 m² base and a height of 2 m]

b) ![Image of a prism with an 18 cm² base and a height of 3 cm]

c) height 5 m, area of top face 19 m²

d) height 12 mm, area of bottom face 2250 mm²

**Answers:** a) 32 m³, b) 54 cm³, c) 95 m³, d) 27000 mm³

**Units in the formula.** Point out that area is given in square units. Square units mean that there were two length units multiplied to get the square unit, which we can see in the raised 2 in square units—for example, m². In volume, we need to multiply three length units, so we need to multiply area by another quantity in length units—height—to get the volume:

\[
\text{Volume} = \text{area of horizontal face} \times \text{height}
\]

\[
\text{m}^3 = \text{m}^2 \times \text{m}
\]

3 lengths multiplied \quad 2 lengths multiplied \quad 1 length multiplied

Draw on the board:

SAY: This prism is made from centimetre cubes. ASK: How many cubes are in this prism? (6 cubes) What is the volume of the prism? (6 cm³) What is the area of the shaded face? (6 cm²) Write on the board:

Volume: 6 cm³

Area of the horizontal face: 6 cm²

ASK: What is the difference between these two measurements? (the units) SAY: The number might be the same, but the units are different, because volume measures the amount of space taken by the 3-D shape, while the area of the face belongs to the flat object, the face.

**NOTE:** Question 6 on AP Book 5.2 p. 137 shows a different way of developing the same formula for the volume of a rectangular prism, emphasizing the equality between the height and the number of horizontal layers in the prism. You might want to point out this connection to the students when they have finished working on the AP Book questions for this lesson.
Extensions

1. Find the volume of the prism.
   a) length 8 cm
      area of right-side face 12 cm²
   b) width 22 m
      area of front face 33 m²
   c) area of right-side face 9.4 cm², length 5 cm
   d) area of front face 23.1 m², width 6 m

   **Answers:** a) 96 cm³, b) 726 m³, c) 47 cm³, d) 138.6 m³

2. Find the volume of the prism or explain why you cannot.
   a) length 4 cm
      area of bottom face 12 cm²
   b) width 13 m
      area of top face 39 m²
   c) area of top face 4 cm²
   d) area of front face 225 m²
      length 2 cm
      width 15 m

   **Sample solution:** a) The measurements given are the bottom face (so length and width) and the length again, but not a third dimension, so we cannot calculate volume.

   **Answers:** b) cannot calculate, c) cannot calculate, d) 3375 m³

3. A cedar basket is made from strips 2.5 cm wide. The bottom is a square made from 6 strips overlapping 6 other strips at a right angle. The height of the basket is 10 strips. Estimate the volume of the basket. Explain how you know.

   **Answer:** The bottom of the basket is a square 8 strips wide = 6 × 2.5 cm = 15 cm, so the area of the bottom is 225 cm². The height of the basket is 10 strips = 10 × 2.5 = 25 cm, so the volume of the basket is about 225 cm² × 25 cm = 5625 cm³.
Goals

Students will use referents for litres and millilitres and convert measurements in whole number litres to millilitres.

PRIOR KNOWLEDGE REQUIRED

Can multiply numbers by 1000
Is familiar with metric prefixes
Can convert from larger to smaller metric units

MATERIALS

cup or mug
containers for 1 L
funnel
large pan, bowl, or tub to contain any spills
water or other pourable substance
other containers, including empty medicine bottles
200 mL and 250 mL glasses
measuring cups of different sizes

Mental math minute—number string.

String 1: $10 \times 8, 9 \times 8, 8 \times 8$ (80, 72, 64)

Present the strategy of subtracting from 10 times the number by writing each multiplication as repeated addition. Emphasize that 9 eights is 1 less eight than 10 eights.

String 2: $10 \times 12, 9 \times 12, 8 \times 12$ (120, 108, 96)

**Introduce liquid volume.** Explain that, when students first looked at the volume of a box, they imagined the box packed with cubes. Show students a cup or a mug and tell them that you would like to find its volume. **ASK:** Should I fill the cup with cubes? (no) Why not? (there will be many gaps between the cubes) Should I measure the length, width, and height of the cup and multiply those measurements? (no) Why not? (there is no clear length and width, and the formula only works for rectangular prisms, not for round objects such as cups) Explain that the way to measure the volume of the cup is to fill it with something that fills the cup without gaps, such as rice, sand, sugar, or water, and then pour the contents of the cup into a container with a volume that can be measured.

**Introduce litres.** Explain that the volume of liquids can be measured in different units. One of the common units is a litre. Write the word “litre” and its abbreviation “L” on the board and explain that these are both ways to write litre. Show students a container holding 1 L of water, and explain that this is a litre.
Hold up several containers, one at a time, and have students signal thumbs up if they think the container can hold more than 1 L, thumbs down for less than 1 L, and thumbs to side if they think it is about 1 L. If possible, have volunteers pour the water from your 1 L container into the containers to check their estimates. You will need a funnel and a large pan, bowl, or tub to contain spills.

Discuss with students whether 1 L is a large quantity of liquid. Is 1 L enough to water the plants in your home? (probably, yes) In your garden? (no) Is 1 L enough to take a bath or a shower? To wash the dishes? To fill an aquarium?

**Introduce millilitres.** Explain that for quantities of liquid smaller than 1 L, there is another unit: millilitres. Millilitres are very small—for example, there are about 5 millilitres in one teaspoon. Introduce the short form of the unit (mL) as well.

**ACTIVITY (Essential)**

**Developing of the sense of the size of containers.** Give each student one empty medicine bottle. Have students find the labels that show the volume of their bottle and round the amount to the closest 10 mL. Have students work in pairs and guess the volume of their partner’s bottle rounded to the nearest 10 mL. Students can provide hints (too high or too low) and then, when both partners have correctly guessed the capacity, change partners.

**Most appropriate unit.** Review the term “appropriate”—specifically, an appropriate unit is the unit that gives the most convenient number for the measurement. For example, it is more convenient to measure the volume of a bathtub in litres than in millilitres because the number of millilitres would be really large. PROMPT: A teaspoon contains 5 mL of water. How many teaspoons would we need to fill a bathtub? (It’s hard to tell!) As well, most people do not need to know the volume of a bathtub with great precision. In contrast, if you need to give medication to your pet, you need to know the volume very precisely to avoid over dosing or under dosing your pet so you will measure the medication in millilitres.

Take some of the containers used during the lesson and ask students to decide which unit—litre or millilitre—is more appropriate for measuring how much the container can hold. Write both units on the board and have students point to the correct unit to signal their answers.

**1 L = 1000 mL.** Write on the board:

\[
1 \text{ metre} = 1000 \text{ millimetres} \quad 1 \text{ m} = 1000 \text{ mm}
\]

Invite volunteers to circle the common parts in the words “metre” and “millimetre.” Ask students to guess what the part “milli” means. Explain that “milli” means 1000 in Latin. Explain to students that the prefix “kilo” is used to create large units, and “milli” is used to create small units. So when they see a “milli” in a measurement unit, they know right away that there are 1000 smaller units (“milli”-units) in the large unit.
Write on the board:

\[
1 \text{ litre} = \underline{\text{____} \text{ millilitres}} \quad 1 \text{ L} = \underline{\text{____} \text{ mL}}
\]

ASK: What number goes in the blanks? (1000)

**Converting litres to millilitres.** Draw a conversion table on the board as shown in the margin and have students fill in the millilitres column. Ask students to look for regularity: How can we get the number of millilitres from the number of litres? What number would we multiply the number of litres by to get the number of millilitres? (1000) Why? (because there are 1000 mL in 1 L) Remind students that this is very similar to getting grams from kilograms, or metres from kilometres, or millimetres from metres.

**Exercises:** Multiply by 1000 to convert the measurement to millilitres.

- a) 9 L
- b) 18 L
- c) 42 L
- d) 100 L
- e) 394 L
- f) 1000 L

**Answers:**

- a) 9000 mL
- b) 18 000 mL
- c) 42 000 mL
- d) 100 000 mL
- e) 394 000 mL
- f) 1 000 000 mL

**Compare measurements in different units.** Remind students that when they compare two measurements in different units, they need to convert one of the measurements so that the units become the same. Ask students to give an example of how they did it with other units. Have students convert the measurement in litres to millilitres before they compare the measurements. If you present the problems one by one, students can point to the larger measurement to signal the answer.

**Exercises:** Which measurement is larger?

- a) 473 mL or 4 L
- b) 73 L or 7300 mL
- c) 25 678 mL or 25 L

**Bonus**

- d) 456 654 mL or 400 L
- e) 1 000 000 mL or 10 000 L

**Answers:**

- a) 4 L
- b) 73 L
- c) 25 678 mL
- d) 456 654 mL
- e) 10 000 L

**Filling containers.** Show students a 1 L carton of milk or juice and a small (200 mL) glass. ASK: How can we determine how many millilitres of liquid this glass can hold? You might ask a volunteer to check how many glasses can be filled from the container. ASK: How many times can we fill the glass? (5 times) How many millilitres of liquid can this glass hold? (200 mL) PROMPT: How many millilitres of liquid are in the carton? (1000 mL) How can you find how many millilitres are in this glass? (1000 \(\div\) 5) Now bring out a larger glass (250 mL). ASK: How many glasses of this size can be filled from the carton? (4 glasses) Ask a volunteer to check. **NOTE:** You will need a large bowl or container in which to empty out the glasses as they are filled.

**Exercises:** A bottle can hold the given volume of liquid. How many bottles do you need to make 2 L?

- a) 100 mL
- b) 200 mL
- c) 500 mL
- d) 250 mL

**Answers:**

- a) 20
- b) 10
- c) 4
- d) 8
Extensions

1. a) How many teaspoons are needed to fill a 1 L bottle of water? (Remember: 1 teaspoon = 5 mL)
   
   b) A bathtub can hold 220 L of water. How many teaspoons would you need to fill the bathtub?
   
   c) A cup holds 250 mL of water. How many cups will you need to fill the bathtub from part b)?
   
   d) It takes 45 seconds to fill a cup with water in a bathroom sink, empty it into the bathtub, and return to the sink. If you want to fill the bathtub in part b) with cups, how much time will it take? Express your answer in seconds, minutes, and hours.
   
   **Answers:** a) 200, b) 44 000, c) 880, d) \(880 \times 45\) sec \(= 39 600\) sec \(= 660\) min \(= 11\) h

2. **Measuring volume.** Show students measuring cups of different sizes and draw their attention to the marks. Point out that 1 mL is a small unit, so when a liquid needs to be measured to the closest millilitre, you need to use a small measuring cup or a graduated cylinder that can hold a very small amount of liquid. Demonstrate using a measuring cup to measure the volume of a container by filling a container with water, then pouring the water into a measuring cup. Choose a container that holds an amount of liquid that is not a round number so that the water level is between adjacent marks on the measuring cup. Remind students that they need to look at the mark that is closest to the water level. Point out that the measurement produced in this way is approximate.

   Give students a variety of containers and have them estimate and then measure their volume in millilitres. Students will need funnels and a large bowl or tub to work over to contain any spills.
Goals

Students will convert the measurement in litres (including decimal litres) to millilitres and solve problems involving converting units of liquid volume.

PRIOR KNOWLEDGE REQUIRED

Knows the concept of multiplying decimals by 1000
Knows that liquid volume is measured in litres and millilitres
Knows that $1 \text{ L} = 1000 \text{ mL}$
Can convert from metres to millimetres or kilometres to metres

MATERIALS

grid paper or BLM 1 cm Grid Paper (p. S-1)
containers of specific capacities (see Extension 1)

NOTE: Students in Ontario need to be familiar with thousandths decimals to do converting measurements in this lesson.

Mental math minute—number talk. Present this problem: $9 \times 12$. (108)
The following strategies could arise:

- $(9 \times 10) + (9 \times 2)$
- $(10 \times 12) - 12$
- $2 \times (9 \times 6)$
- double $9 \times 3$ twice

Review multiplying decimals and whole numbers by 1000. Remind students that they shift the decimal point three places to the right to multiply by 1000. Remind them that they can write zeros after the decimal point, and the number will not change. For example, $6.7 = 6.70 = 6.700$. Ask: Why are these numbers equal? (the decimal part is $7/10 = 70/100 = 700/1000$)
As well, remind students that, even though we do not write a whole number with a decimal point, we can still add the decimal point without changing the number. For example, $12 = 12.000$. Use the questions below as a test to make sure all students can perform the multiplication required.

Exercises: Calculate.

- a) $0.004 \times 1000$
- b) $4.356 \times 1000$
- c) $1.79 \times 1000$
- d) $0.07 \times 1000$
- e) $0.3 \times 1000$

Answers: a) 4, b) 4356, c) 1790, d) 70, e) 300

Use multiplication to convert from litres to millilitres (decimals). Remind students that, to convert from kilometres to metres, they multiplied by 1000. Ask: What other conversions require multiplying by 1000? (kilograms to grams, metres to millimetres) Why do you multiply by 1000? (there are
1000 metres in a kilometre, etc.) How will you convert a measurement in litres to millilitres then? (multiply by 1000)

For example, convert 0.12 L to mL:

\[ 0.12 \text{ L} = 0.12 \times 1000 \text{ mL} = 120 \text{ mL} \]

Move the decimal point three places to the right.

**Exercises:** Convert to millilitres.

a) 7.239 L  

b) 10.825 L  

c) 0.002 L  

d) 0.04 L  

e) 0.063 L  

f) 0.41 L  

g) 10.89 L  

h) 2.3 L  

**Answers:** a) 7239 mL, b) 10 825 mL, c) 2 mL, d) 40 mL, e) 63 mL,  
f) 410 mL, g) 10 890 mL, h) 2300 mL

**Mixed measurements.** Remind students that they wrote length as a mixed measurement: for example, 3 km 456 m. Remind students how they converted mixed measurements to metres, as shown below:

\[
3 \text{ km} = 3000 \text{ m}, \text{ so } 3 \text{ km 456 m} \\
= 3 \underline{0} \underline{0} \underline{0} \underline{0} \text{ m} \\
+ 4 \underline{5} \underline{6} \text{ m} \\
= 3 \underline{4} \underline{5} \underline{6} \text{ m}
\]

Explain to students that every thousand metres is one kilometre and every thousand millilitres is one litre. ASK: Would the same method work for litres and millilitres? (yes) Why does it make sense? (we used a similar method for other units, such as millimetres and metres)

Convert mixed measurements to millilitres. Do the first three exercises below as a class, and then have students do the remaining exercises individually, working on grid paper or **BLM 1 cm Grid Paper**.

**Exercises:** Convert to millilitres.

a) 2 L 345 mL  

b) 17 L 67 mL  

c) 4 L 8 mL  

d) 2 L 371 mL  

e) 45 L 604 mL  

f) 658 L 400 mL  

g) 8 L 75 mL  

h) 30 L 5 mL

**Bonus**

i) 100 L 100 mL  

j) 1000 L 10 mL

**Answers:** a) 2345 mL, b) 17 067 mL, c) 4008 mL, d) 2371 mL, e) 45 604 mL,  
f) 658 400 mL, g) 8075 mL, h) 30 005 mL, Bonus: i) 100 100 mL,  
j) 1 000 010 mL

**Converting measurements in litres to mixed measurements.** Explain that in a measurement in litres, such as 6.527 L, the whole part shows the litres, and the decimal part shows the number of millilitres, so 6.527 L = 6 L 527 mL.
Exercises: Convert to a mixed measurement.

a) 6.998 L   b) 4.708 L   c) 2.039 L   d) 3.007 L

Answers: a) 6 L 998 mL, b) 4 L 708 mL, c) 2 L 39 mL, d) 3 L 7 mL

SAY: A student I know thinks that 2.5 L is 2 L 5 mL. Is he correct? (no)

Explain the mistake: 1 mL is one thousandth of a litre, so 2 L and 5 mL is 2.005 mL, not 2.5 mL. ASK: What do you need to do to convert 2.5 L to mL? (write 2.5 with digits in the thousandths place, 2.5 = 2.500, note the 2 as litres or 2 L, and then move the decimal point right three places for the millilitres) Have students rewrite the measurement and then perform the conversion. (2.5 L = 2 L 500 mL)

Exercises: Expand the measurement to the thousandths place. Convert to a mixed measurement.

a) 6.9 L   b) 4.7 L   c) 2.19 L   d) 3.87 L

Answers: a) 6 L 900 mL, b) 4 L 700 mL, c) 2 L 190 mL, d) 3 L 870 mL

Converting measurements in millilitres to mixed measurements. Ask students to think about how to convert 3046 mL to a mixed measurement. One possible way is to write the whole thousand part as litres and the rest as millilitres. (3 L 46 mL)

Point out that we could take the hundreds, tens, and ones to be the number of millilitres and the number of thousands to be the number of litres. Demonstrate with 34 678 mL = 34 L 678 mL.

Exercises: Convert to a mixed measurement.

a) 81 032 mL   b) 7643 mL   c) 35 650 mL   d) 200 002 mL

Bonus: 87 654 321 mL

Answers: a) 81 L 32 mL, b) 7 L 643 mL, c) 35 L 650 mL, d) 200 L 2 mL, Bonus: 87 654 L 321 mL

Word problems with conversions. Solve the following problems as a class by converting all the measurements in litres to millilitres and working with millilitres. Convert the final answer to a mixed measurement.

1. a) To make juice from concentrate, you need to mix a 355 mL can of concentrate with three cans of water. How much juice do you get? (355 mL × 4 = 1420 mL = 1 L 420 mL)

b) Raj makes a fruit drink by mixing juice from concentrate (one 355 mL can and 3 cans of water) with a 946 mL bottle of cranberry juice and 1.5 L bottle of ginger ale. How much fruit drink does Raj mix? (3866 mL = 3 L 866 mL)

c) Raj plans that each of 12 people at a party needs at least one 300 mL glass of fruit drink. Did he make enough? (12 × 300 mL = 3600 mL = 3 L 600 mL; yes, Raj made enough fruit drink)
2. A pack of six bottles of apple juice, each 300 mL, costs $4 per pack. A pack of eight apple juice boxes, each 125 mL, costs $3 per pack.

a) How much juice is in each pack? Write your answer in two ways: in millilitres and in mixed measurements. (6 bottles contain 1800 mL = 1 L 800 mL juice, 8 boxes contain 1000 mL = 1 L juice)

b) Which contains more juice, five packs of six juice bottles or nine packs of eight juice boxes? (both are 9 L)

c) What costs more, five packs of six bottles or nine packs of eight juice boxes? (five packs of six bottles cost $20, nine packs of eight juice boxes cost $27)

d) Which way of buying the juice is cheaper by volume? Explain. (buying the juice in bottles is cheaper by volume)

Extensions

1. a) Show students two containers, one that can hold 500 mL and another that can hold 300 mL. ASK: How could you use only these containers to measure 200 mL of water? Have students develop a solution mentally or on paper. They can use the containers to check their solution afterward. (You could make the requisite containers by filling two 1-quart juice containers with the different amounts of water, marking the water level, and cutting the containers at that height.)

b) Use the containers from part a) to measure 400 mL of water.

c) Use containers that can hold 1 L, 250 mL, and 100 mL to measure out 750 mL in three different ways.

Solutions

a) fill the 500 mL container with water. Fill the 300 mL container with water from the 500 mL container. There will be 200 mL of water left in the larger container.

b) use the method from part a) to measure 200 mL of water. Pour the 200 mL of water into the 300 mL container so that you have only 100 mL of room in the small container. Fill the 500 mL container. Pour water from the 500 mL container into the 300 mL container until the 300 mL container is full. There will be 400 mL of water left in the 500 mL container.

c) 1 L = 250 mL = 750 mL, 3 × 250 mL = 750 mL, 5 × 100 mL + 250 mL = 750 mL

2. Find the volume of the prism.

a) 1 m × 1 km × 1 m  

b) 5 cm × 3 dm × 2 m

Bonus: 1 mm × 1 m × 1 km

Answers: a) 1000 m³, b) 30 000 cm³, Bonus: 1 000 000 000 mm³
Goals

Students will recognize the connection between millilitres, litres, and cubic units.
Students will solve problems involving capacity and volume.

PRIOR KNOWLEDGE REQUIRED

- Can multiply numbers by 1000
- Knows that liquid volume is measured in litres and millilitres
- Knows that 1 L = 1000 mL
- Knows that 1 dm = 10 cm
- Can convert metric units of length and units of liquid volume
- Can find the volume of a rectangular prism

MATERIALS

- rectangular aquarium
- cube with 10 cm sides (e.g., a thousands block)
- large graduated pitcher
- water
- ruler
- an object with a complicated shape, such as a toy (see Extension 5)

Mental math minute—number talk. Present this problem: 12 \times 12. (144)
The following strategies could arise:

\[(12 \times 10) + (12 \times 2)\]
\[\text{double } 3 \times 12 \text{ twice}\]

Introduce capacity. Explain that the amount of liquid (or sand, rice, or anything else that can be poured) a container can hold is called the *capacity* of the container. Capacity is often measured in litres and millilitres. Remind students that, when they found the volume of boxes, they found that volume in cubic units, such as cubic centimetres. If available, show students a rectangular aquarium. Explain that we can find the volume of the inside of the aquarium in cubic units. But we also can pour water into it and find its capacity—or the volume of the liquid—in millilitres or litres. Explain that there is a connection between these two types of units.

Review decimetres and cubic decimetres. Remind students that 1 decimetre is a unit of length equal to 10 centimetres.

A cube that has sides of 1 decimetre (1 dm) has a volume of 1 dm³. Show students a cube with sides 1 dm and explain that this is a *cubic decimetre*. Write on the board:

\[1 \text{ dm} = 10 \text{ cm}\]
\[1 \text{ dm} \times 1 \text{ dm} \times 1 \text{ dm} = 1 \text{ dm}^3\]
Introduce the connection between dm³ and litres. If available, show students a large graduated pitcher with water in it so that at least 1 L can be added to it. Have a volunteer say how much water is in the pitcher. (say, 2 L) Place the 1 dm cube into the pitcher and have students observe that the water level rises. Explain that we say that the cube displaces some water in the pitcher—in other words, it takes the place of the water. The volume of water the cube displaces equals the volume of the cube. What is the total volume of the liquid and the cube in the pitcher? Have a volunteer check. (3 L) Ask: How much water did the cube displace? (1 L) What is the volume of the cube in litres? (1 L) Write on the board:

\[ 1 \text{ dm}^3 = 1 \text{ L} \]

Point out that both cubic decimetres and litres are measurements of volume; however, cubic decimetres are metric units, and litres are equal to the metric units. The volume of liquids and capacity are usually measured in litres, and geometric volume is usually measured in cubic units.

Introduce the connection between cm³ and mL. Remind students that a millilitre is one thousandth of a litre, so there are 1000 mL in a litre. Write on the board:

\[ 1 \text{ L} = 1000 \text{ mL} \]

Draw a cube on the board and mark its length, width, and height as 1 dm = 10 cm. Ask students to find the volume of the cube in cubic centimetres. Write the calculation on the board:

\[ 1 \text{ dm}^3 = 10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} = 1000 \text{ cm}^3 \]

Write on the board:

\[ 1 \text{ dm}^3 = 1000 \text{ cm}^3 \]

\[ 1 \text{ L} = 1000 \text{ mL} \]

Ask: How many millilitres are in 1 cubic centimetre? (1) How do you know? (1000 mL = 1000 cm³, so 1 mL and 1 cm³ should be the same thing)

Finding the volume of an aquarium. Draw a prism on the board and mark the sides as 5 dm, 6 dm, and 4 dm. Ask students to find the volume of the prism in cubic decimetres. (5 dm \times 6 \text{ dm} \times 4 \text{ dm} = 120 \text{ dm}^3) Then tell them that this prism is an aquarium and ask them how much water will fit into the aquarium. (120 L)

Ask students to convert the dimensions of the prism into centimetres and find the volume in cubic centimetres and in millilitres. (50 cm \times 60 cm \times 40 cm = 120 000 \text{ cm}^3 = 120 000 \text{ mL}) Do the answers match? (yes, 120 L = 120 000 mL)

Invite a volunteer to measure the inner dimensions of the rectangular aquarium you showed students at the beginning of the lesson. Ask them to find the volume of the aquarium in cubic centimetres. Then ask: How many millilitres of water will fit into the aquarium? (the number of millilitres will be the same number as the volume in cm³) Have them convert the answer to litres.
Pour several litres of water into the aquarium using a pitcher. Have a volunteer use a ruler to measure the height in centimetres the water reaches and have students find the volume of water in the aquarium in cubic centimetres. Again, have students convert the answer to millilitres and then have students write the volume as a mixed measurement (using L and mL). Is the answer close to the amount of water you poured in? The answer is likely not to be exactly the same, so discuss why there might be a difference. (The measurements are only approximations. The exact height might have been, say, 12.3 cm, which we rounded to 12 cm. The same applies to length and width.)

**Finding the height of water.** Show students how to find the missing dimension when given the volume of a prism. For example, if a prism has a volume of 120 cm³, and it is 10 cm long and 6 cm wide, what is its height? Remind students of the formula Volume = length × width × height and that length and width give the area of the bottom or the top face. So, in the example, 10 cm × 6 cm = 60 cm², and the height is 120 cm³ ÷ 60 cm² = 2 cm.

Draw another prism on the board and mark the length and the width as 75 cm and 40 cm. Tell students that this is an aquarium, and you want to pour, say, three pails equal to 30 litres of water into it. How high do you think the water will be? Have students think how they can find the height and discuss the potential solution in pairs. Go through the solution as a class:

- Volume of water = 30 L = 30 000 mL = 30 000 cm³
- Area of bottom face = 75 cm × 40 cm = 3000 cm²
- Height = 30 000 cm³ ÷ 3000 cm² = 10 cm

Point out that this is very little water for such an aquarium—it is only 10 cm of water, and aquariums usually have water levels much higher than that!

Repeat the exercise with an aquarium of the size you might see, say, in a doctor’s office. For example: an aquarium measures 1.5 m long and 60 cm wide and contains 819 L of water (including sand, decorations, and fish). What is the height of the water? This time students will need to convert 1.5 m to centimetres. (height = 91 cm)

**Exercise:** An aquarium that is 1.4 m long and 50 cm wide contains 770 L of water including sand, decorations, and fish. How high is the water in the aquarium?

**Answer:** 110 cm = 1.1 m

**Extensions**

1. How can we “count” water? Read students the chapter on capacity from *Anno’s Math Games II* by Mitsumasa Anno.

2. Tom measured the dimensions of a 2 L juice carton and calculated the volume to be 200 cm³. Is his answer correct? Explain how you know.

**Answer:** No; 2 L = 2000 mL = 2000 cm³, so Tom’s answer is incorrect.
3. The volume of very large amounts of liquid is measured in cubic metres.
   a) How many cubic centimetres are in 1 m³?
   b) How many litres are in 1 m³?
   c) A community swimming pool has a volume of 1800 m³. What is the volume in litres?
   d) Why do we need a unit between 1 m³ and 1 cm³?

   **Answers:** a) 1 000 000 cm³, b) 1000 L, c) 1 800 000 L, d) because each cubic metre is 1 000 000 cm³ and there is a big gap between these two units

4. Rob wants an aquarium that will hold 24 L of water. Write a set of whole number dimensions (length, width, and height) in decimetres and centimetres that give the required capacity.

   **Sample answer:** 4 dm, 3 dm, and 2 dm or 40 cm, 30 cm, and 20 cm

5. **Using displacement of water to find volume.** Explain that displacement allows you to find the volume of objects that are not rectangular prisms. Pour some water into a graduated pitcher and have a volunteer check the level of the water. Place an object with a complicated shape, such as a toy, into the water and have another volunteer check the new level of the water. SAY: The water level now shows the volume of the water plus the volume of the toy. You can find the volume of the toy using subtraction.

   An aquarium is 25 cm wide and 40 cm long.
   a) Lily pours 8 L of water into the aquarium. How high is the water?
      **Hint:** Find the volume of the water in cubic centimetres first.
   
   b) Lily places a stone in the aquarium. The water level rises to 9 cm.
      What is the volume of the stone and the water together?
   
   c) What is the volume of the stone?

   **Answers:** a) 8 cm, b) 9 L, c) 1 L

   a) How many cubic metres are in 1 km³?
   b) The volume of water in Lake Michigan is 4920 km³. Find the volume of Lake Michigan in cubic metres and in litres. How many zeros are in each of these numbers?

   **Answers**
   a) 1 000 000 000 m³
   b) 4 920 000 000 000 m³ = 4 920 000 000 000 000 L;
      10 zeros for m³; 13 zeros for L
**Goals**

Students will review metric units of mass.

**PRIOR KNOWLEDGE REQUIRED**

- Can multiply decimals and whole numbers by 1000
- Can compare and sort objects by weight (heavier/lighter)
- Understands the concept of measuring weight
- Can add and subtract decimals
- Can multiply 2-digit numbers by 2-digit numbers
- Can multiply and divide numbers up to 4 digits by 1-digit numbers
- Can solve word problems using the four operations
- Can convert between metric units of length measurement

**MATERIALS**

- a collection of items such as dry food with weights marked in grams and kilograms
- grid paper or BLM 1 cm Grid Paper (p. S-1)

**Mental math minute—number talk.** Present this problem: 8 \times 9. (72)

The following strategies could arise:

\[
(8 \times 10) - 8 \\
2 \times (4 \times 9) \\
(10 \times 9) - (2 \times 9) \\
(5 \times 9) + (3 \times 9) \\
(8 \times 5) + (8 \times 4)
\]

**Review mass.** Ask students how people can check how heavy an object is. (weigh it, put it on a scale) Tell students that mass is the amount of matter in an object. Explain that in science we usually use the word mass to describe how heavy objects are, and we say that the heavier object has the greater mass.

Name some pairs of objects that have similar mass: a pen and a pencil, a full water bottle and a book, a truck and a bus. Then name several everyday objects and have students name an object with a similar mass: What has a mass similar to an eraser? (e.g., a glue stick)

**Review grams.** Ask: In what units is weight measured? Students are likely to be familiar with pounds but may remember other units (ounces, kilograms, grams) from previous grades. Explain that in the metric system, the mass of small objects is measured in grams. Write “gram” on the board, circle the letter “g,” and write the abbreviation “g.” Explain that these are two ways to write grams. Explain that a large paper clip and a large chocolate chip weigh 1 gram each. A nickel weighs 4 grams.
Point out that since 1 gram is such a small weight, there are very few everyday items that weigh less than that. For example, grains of rice, pills, and most insects, such as ants, weigh less than 1 gram. Only very large insects weigh more than 1 gram.

**ACTIVITY (Essential)**

Give students a collection of everyday items that have their weight listed in metric units (canned food, plastic spice jars, and so on). Have them sort the objects into three groups by mass: less than 100 g, between 100 g and 500 g, and more than 500 g. Then show more objects, without telling students the mass, and have them signal which group these objects should be sorted into. Students can show hands at desk level for the group with mass less than 100 g, hands at shoulder level for the medium-sized group, and hands up for the objects heavier than 500 g. This will help students to develop a feel for the size of grams.

**Review kilograms.** Point out a few larger objects in the “heavy” group. ASK: Would 1 gram be a convenient unit to measure the weight of a laptop, a pile of books, or a human? (no) Why not? (the number of grams will be very large) Explain that the unit used to measure the mass of objects of that size is the **kilogram**. Write “kilogram” on the board, circle the letters “k” and “g,” and write the abbreviation “kg.” Explain that these are two ways to write kilogram.

Write on the board:

\[
1 \text{ kilometre} = \underline{\hspace{2cm}} \text{metres} \quad 1 \text{ km} = \underline{\hspace{2cm}} \text{m}
\]

ASK: What number goes in the blanks? (1000) Invite a volunteer to circle the common parts in the words “kilometre” and “metre.” Ask students to guess what the part “kilo” means. Explain that kilo means 1000 in Greek. When you see kilo in a measurement unit, you know right away that there are 1000 smaller units in the large unit. Write on the board:

\[
1 \text{ kilogram} = \underline{\hspace{2cm}} \text{grams} \quad 1 \text{ kg} = \underline{\hspace{2cm}} \text{g}
\]

ASK: What number goes in the blanks? (1000)

Have students look through the objects in the group “more than 500 g” to find any objects that weigh more than 1000 g. If there are no such objects, ask why that could be. Show an object such as a large package of flour. Point out that it is more convenient to write the weight of such a heavy object in kilograms, using decimals, than to write it in grams.

**Review converting kilograms to grams.** Remind students that they multiplied by 1000 to convert the measurement in kilometres to metres. ASK: What number would we multiply the number of kilograms by to get the number of grams? (1000) Why? (there are 1000 g in 1 kg) As well, remind students that they can shift the decimal point 3 places to the right to multiply decimals by 1000.
Exercises: Convert to grams.

a) 7 kg  
   b) 12 kg  
   c) 3.545 kg

d) 2.27 kg  
   e) 34.5 kg  
   f) 0.8 kg

Answers: a) 7000 g, b) 12 000 g, c) 3545 g, d) 2270 g, e) 34 500 g, f) 800 g

Comparing measurements in different units. Remind students that when they compare two measurements in different units, they need to convert one of the measurements so the units become the same. For example, 7 km is larger than 300 m, even though 7 is smaller than 300, because 7 km = 7000 m, and 7000 m > 300 m.

Exercises: Convert the measurement in kilograms to grams, then compare the measurements. To signal the answers, students can show a sideways “V” with their fingers open in the direction of the larger measurement to resemble a “<” or “>” sign.

a) 354 g or 3 kg  
   b) 78 kg or 7800 g  
   c) 24 897 g or 20 kg

d) 37 890 g or 3.789 kg  
   e) 2.78 kg or 278 g

Bonus

f) 456 789 kg or 30 000 g  
   g) 1 000 000 g or 10 000 kg

Answers: a) 354 g < 3 kg, b) 78 kg > 7800 g, c) 24 897 g > 20 kg,  
d) 37 890 g > 3.789 kg, e) 2.78 kg > 278 g,  
Bonus: f) 456 789 kg > 30 000 g, g) 1 000 000 g < 10 000 kg

Review mixed measurements. Remind students that they can write length as a mixed measurement; for example, a shelf is 5 m 23 cm long. Explain that mass can also be written as a mixed measurement, such as 3 kg 456 g. Point out that since grams are thousandths of kilograms, the conversion from kilograms to mixed measurements is easy: all you need to do is to write the measurement in kilograms up to thousandths. The whole part then becomes the number of kilograms, and the number of thousandths after the decimal point is the number of grams. Show an example:

2.35 kg = 2.350 kg = 2 kg 350 g

Exercises: Write as a mixed measurement.

a) 3.789 kg  
   b) 78.089 kg  
   c) 5.78 kg  
   d) 2.4 kg

Answers: a) 3 kg 789 g, b) 78 kg 89 g, c) 5 kg 780 g, d) 2 kg 400 g

Remind students that to convert a mixed measurement such as 5 m 23 cm to centimetres, they need to convert metres to centimetres and add the remaining centimetres:

5 m = 500 cm, so 5 m 23 cm = 500 cm + 23 cm = 523 cm

ASK: How can you convert 3 kg 456 g to grams? Take up different ideas, then lead students to the idea of converting the part in kilograms and
adding the remaining grams. Students should perform the addition on grid paper or BLM 1 cm Grid Paper as shown below:

\[
\begin{array}{c}
3 \text{ kg} = 3000 \text{ g}, \text{ so } 3 \text{ kg } 456 \text{ g} \\
= \begin{array}{c}
3 \, 0 \, 0 \, 0 \\
\hline
4 \, 5 \, 6 \\
\end{array} \\
\hline
3 \, 4 \, 5 \, 6 \, \text{ g}
\end{array}
\]

Complete the first two exercises below as a class, and then have students individually convert the measurements to grams.

**Exercises:** Convert to grams.

a) 2 kg 4 g  
 b) 13 kg 67 g  
 c) 5 kg 159 g  
 d) 2 kg 391 g  
 e) 52 kg 604 g  
 f) 643 kg 600 g  
 g) 8 kg 15 g  
 h) 30 kg 1 g  

**Bonus**

i) 100 kg 100 g  
 j) 1000 kg 10 g

**Answers:** a) 2004 g, b) 13 067 g, c) 5159 g, d) 2391 g, e) 52 604 g, f) 643 600 g, g) 8015 g, h) 30 001 g, **Bonus:** i) 100 100 g, j) 1 000 010 g

Now reverse the task. Write on the board:

\[
23 \, 765 \, \text{ g} = \_ \_ \_ \_ \_ \_ \, \text{kg} \_ \_ \_ \_ \_ \_ \, \text{g}
\]

**ASK:** How many whole kilograms are in 23 765 g? (23) Have a volunteer fill in the first blank. **SAY:** This is the number of thousands in 23 765 because there are 1000 g in 1 kg. Have a volunteer fill in the second blank. (765)

**Exercises:** Convert to mixed measurements.

a) 7893 g  
 b) 82 081 g  
 c) 35 950 g  
 d) 100 009 g  

**Bonus:** 7 123 456 g

**Answers:** a) 7 kg 893 g, b) 82 kg 81 g, c) 35 kg 950 g, d) 100 kg 9 g, **Bonus:** 7123 kg 456 g

**Solving word problems with masses.** Work through the following problems as a class. Invite and encourage as many students as possible to contribute answers, explanations, and suggestions.

**Exercises**

a) A newborn Siberian tiger cub weighs about 1 kg. It gains 700 grams per week. How much weight does it gain in 4 weeks? How much does the tiger weigh after 4 weeks? Express your answer in grams and as a mixed measurement.

b) A raccoon weighs 7.45 kg. A beaver is 3800 g heavier than the raccoon. How much does the beaver weigh?
c) A paper coffee cup with a lid weighs about 8 grams. Express your answer in grams and as a mixed measurement.

i) Ken buys one cup of coffee every day. What mass of garbage does he generate from paper cups in a week? In a year?

ii) Zara buys coffee 3 times a day. What mass of garbage from paper cups does she generate in a year?

d) A small bag of rice weighs 1816 g, and a big bag of rice weighs 3 kg. Three big bags cost as much as 5 small bags. Which combination is a better buy? Hint: Find the weight of 3 big bags and of 5 small bags first. Which gives more rice for the same price?

**Answers:**
a) The tiger cub gains 2800 g in 4 weeks, so its weight is 3800 g = 3.8 kg = 3 kg 800 g; b) 11 250 g = 11.25 kg = 11 kg 250 g; c) i) Ken generates 56 g of garbage from paper cups in a week, 2912 g = 2.912 kg = 2 kg 912 g in a year; ii) Zara generates 8736 g = 8.736 kg = 8 kg 736 g of garbage from paper cups in a year; d) 3 big bags weigh 9 kg, and 5 small bags weigh 9080 g = 9.08 kg, so 5 small bags are a better buy.

**Extensions**

1. A small box of rice weighs 1362 g and costs $5.60. A large bag of rice weighs 2.27 kg and costs $9. How much do 5 small boxes weigh and cost? How much do 3 large bags weigh and cost? Which combination is a better buy?

   **Answer:** 5 small boxes weigh 6.81 kg and cost $28. 3 large bags weigh 6.81 kg and cost $27. Three large bags are a better buy.

2. **Milligrams.** We measure very small weights in milligrams (mg).

   1 g = 1000 mg

   a) Convert to milligrams.

      i) 3 g = ______  ii) 7 g = ______  iii) 12 g = ______

   b) Which is heavier: 3 g or 456 mg? 5 g or 5890 mg?

   c) How many milligrams are in 1 kilogram?

   **Answers:** a) i) 3000 g, ii) 7000 g, iii) 12 000 g; b) 3 g > 456 mg, 5 g < 5890 mg; c) 1 000 000 mg

3. **Metric Tonnes.** We measure very large weights in metric tonnes (t).

   1 t = 1000 kg

   a) Convert to kilograms.

      i) 4 t = ______  ii) 15 t = ______  iii) 100 t = ______

   b) A male elephant weighs 5 metric tonnes. A female elephant weighs 3897 kg. How much heavier is the male than the female?

   **Answers:** a) i) 4000 kg, ii) 15 000 kg, iii) 100 000 kg; b) 1103 kg
Goals

Students will compare measurements in different units (tonnes, kilograms, grams, milligrams).

Students will choose the most appropriate unit to measure mass.

Prior Knowledge Required

Understands the concept of measurement
Understands the concept of mass
Can compare masses of objects

Materials

two different-shaped containers (see Extension 1)
water (see Extensions 1 and 3)
scales (see Extensions 1 and 3)
funnels and a tray to prevent spills (see Extensions 1 and 3)
two small identical containers, one full of oil (see Extension 3)
a larger container that can contain the oil and water together (see Extension 3)

Mental math minute. Have students add by using 10. Say the addition you want students to do (such as $8 + 6$). Have a student say the in-between addition step, $10 + 4$, and have another student finish the addition. Start with addition questions within 20, such as $7 + 5$ or $9 + 3$, and progress to harder questions, such as $23 + 8$ or $76 + 7$. As a challenge, use three-digit and four-digit numbers, such as $345 + 8$, or vary the order (e.g., to $8 + 56$).

Comparing masses in grams and in kilograms. ASK: Which is heavier, 150 g or 1000 g? (1000 g) How do you know? (1000 is greater than 150) Which is heavier, 1 kg or 150 g? (1 kg) How do you know? (1 kg is 1000 g, and 1000 is greater than 150) SAY: A student I know thinks that 359 g is heavier than 1 kg because 359 is a greater number than 1. ASK: Is that student correct? (no) Have students explain the mistake. (Sample answers: 1 kg equals 1000 g, and 1000 is greater than 359; you need to compare measurements that are in the same units.) ASK: What is the largest three-digit number? (999) Is 999 g heavier than 1 kg? (no) Have students explain again.

Exercises

1. a) Write a measurement in grams that is between 4 kg and 5 kg.
   b) Write a measurement in kilograms that is between 2837 g and 3591 g.

Bonus: Write a measurement in grams that is more than 7 kg.

Sample answers: a) 4003 g, b) 3 kg, Bonus: 7001 g
2. How many whole kilograms are in the measurement?
   a) 5472 g  b) 2091 g  c) 7305 g  **Bonus:** 902 g

   **Answers:** a) 5 kg, b) 2 kg, c) 7 kg, Bonus: 0 kg

Write on the board:

   2314 g  3 kg

   **ASK:** Which is heavier? (3 kg) **How do you know?** (Sample answers: 3 kg = 3000 g, 3000 is more than 2314; 2314 g is 2 kg and some more, but less than another kilogram, so it is lighter than the full 3 kg.) Encourage multiple explanations. Then write the pairs of measurements found in Exercise 1 on the next page on the board, and have students point their thumbs in the direction of the lighter measurement. Invite a volunteer to underline the number of whole kilograms in the measurement in grams and explain why the selected measurement is lighter.

**Exercises**

1. Which measurement is lighter?
   a) 5390 g  6 kg  b) 4003 g  4 kg  c) 2 kg  2807 g
   **Bonus:** 6 kg  696 g

   **Answers:** a) 5390 g, b) 4 kg, c) 2 kg, Bonus: 696 g

2. Order the measurements in Exercise 1 from heaviest to lightest.

   **Answers:** 6 kg, 5390 g, 4003 g, 4 kg, 2807 g, 2 kg, 696 g

   **Introduce milligrams.** **ASK:** What is lighter than 1 g? (sample answers: a grain of sand, a grain of rice, an insect, a single sheet of paper) **Explain that to measure the mass of very light objects, we use milligrams (mg). Write on the board:**

   milligram   mg

   **Explain that these are two ways to write milligram, and have a volunteer circle the first “m” and the “g” in the full form. ASK:** **Which other unit does milligram remind you of?** (millimetre) **How many millimetres are in 1 m? (1000)** **PROMPT:** **How many centimetres are in 1 m? (100)** Each centimetre is 10 millimetres, so how many millimetres are in 100 cm? (1000) **Write on the board:**

   \[1 \text{ metre} = 1000 \text{ millimetres} \quad 1 \text{ m} = 1000 \text{ mm}\]

   \[1 \text{ gram} = \_\_\_\_\_\_ \text{ milligrams} \quad 1 \text{ g} = \_\_\_\_\_\_ \text{ mg}\]

   **ASK:** **What does “milli” stand for?** (1000) **Remind students that “milli” means 1000 and comes from Latin. You might want to point out that scientists agreed to use Greek prefixes for units larger than the base unit, such as gram or metre, and Latin prefixes for units that are smaller than the base unit.**
Explain that milligrams are a very small unit of mass. For example, a piece of tissue paper weighs about 100 mg. A grain of sand weighs about 10 mg. A very small ant weighs about 1 mg. A money bill weighs 930 mg, which is almost a full gram. Write these examples on the board. Explain that people also use milligrams when they need to be very precise. For example, when doctors prescribe medications, they use milligrams.

Write the words “gram” and “milligram” on different sides of the board and leave them up. For the following exercises, have students signal the answer by pointing their thumb toward the unit they would use to measure the mass of the given object.

**Exercises:** Will you use grams or milligrams to measure the mass?

a) lentil  
b) grain of salt  
c) cookie  
d) coin  
e) medication in a pill

**Answers:** a) mg, b) mg, c) g, d) g, e) mg

**Introduce tonnes.** Write on the board:

\[
1000 \text{ mg} = 1 \text{ g} \quad 1000 \text{ g} = 1 \text{ kg} \quad 1000 \text{ kg} = 1 \text{ t}
\]

Ask if anyone knows what we call the unit that is equal to 1000 kg. Explain that 1000 kg is called a **tonne**, and tonnes are used to measure objects with very large masses. SAY: For example, a small car weighs about 1 tonne. Ask students to give examples of an object that has a very large mass that can be measured in tonnes. Examples include very large animals (such as elephants and whales), trucks, train cars, planets, and so on. Explain that the short form for tonne is just the letter "t" and fill in the blank on the board. Write the word “tonne” on the board as well.

**Converting between units.** Ask: What do you do to a measurement in kilograms to convert to grams? (multiply by 1000) Why? (there are 1000 g in every kilogram) What do you do to convert a measurement in grams to milligrams? (also multiply by 1000) How would you convert a measurement in tonnes to kilograms? (multiply by 1000) Have volunteers convert 3 g to milligrams, 15 kg to grams, and 7 tonnes to kilograms. (3000 mg, 15 000 g, 7000 kg)

**Exercises**

1. Convert the measurement.

   a) 17 kg to grams  
   b) 20 t to kilograms  
   c) 13 g to milligrams  
   d) 419 t to kilograms  
   e) 600 g to milligrams  
   f) 280 kg to grams

**Answers:** a) 17 000 g, b) 20 000 kg, c) 13 000 mg, d) 419 000 kg, e) 600 000 mg, f) 280 000 g
2. Choose the most appropriate unit (mg, g, kg, or t) to measure the object.
   
   a) bear  b) ant  c) boulder  d) bag of rice  
   e) notebook  f) lion  g) grain of sugar  

   Answers: a) kg, b) mg, c) t, d) kg, e) g, f) kg, g) mg

Solving problems connected to mass. SAY: A large paper clip weighs about 1 g. ASK: How much do 10 paper clips weigh? (10 g) 100 paper clips? (100 g) 1000 paper clips? (1000 g or 1 kg) SAY: A pill weighs 500 mg. ASK: How much do 2 pills weigh? (1000 mg = 1 g) How do you know? (500 mg + 500 mg = 1000 mg = 1 g) How much do 20 pills weigh? (10 g) How do you know? (20 is 10 groups of 2, each group of two pills weighs 1 gram, so 10 groups weigh 1 gram) How much do 200 pills weigh? (100 g) How do you know? (200 is 100 pairs) How many pills do I need to make 1 kg? (2000) How do you know? (1 kg = 1000 g, each gram is 2 pills, so we need 2000 pills)

Exercises: A grain of sand weighs 5 mg.
   
   a) How much do 2 grains of sand weigh?
   
   b) Do 2000 grains of sand weigh more than 1 g? Explain.

   Answers: a) 10 mg; b) 2000 grains of sand is 1000 pairs of grains of sand. Each pair weighs 10 mg, so 1000 pairs weigh 10 000 mg. 10 000 mg = 10 g, so more than 1 gram.

Extensions

1. Weigh two empty containers of different shapes, then pour some water into the smaller container and weigh it again. You can use a funnel and a tray to help prevent spills. To find the mass of the water, subtract the mass of the container before adding the water from the mass of the container and the water. Pour the water into the empty container and weigh it again. Subtract the masses again. Did you get the same answer in both cases?

   Answer: The answer should be the same in both cases, unless some water is spilled or left in the containers.

2. Draw a number line from 0 kg to 10 kg. Mark the masses of the animals in the table below on the number line you drew.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>raccoon</td>
<td>7 kg</td>
</tr>
<tr>
<td>fox</td>
<td>8 kg 500 g</td>
</tr>
<tr>
<td>squirrel</td>
<td>520 g</td>
</tr>
<tr>
<td>armadillo</td>
<td>4 kg 700 g</td>
</tr>
<tr>
<td>ferret</td>
<td>1100 g</td>
</tr>
</tbody>
</table>
3. **Mass of oil and mass of water.** Students will need two small identical containers, one of which is full of oil; access to water; a larger container that can contain the oil and water together; funnels; and a tray to prevent spills.

   a) Have students weigh the empty small container and the container with oil and subtract the masses to find the mass of oil.

   b) Fill the empty small container with water to the same level as the other container is filled with oil. Weigh the container and subtract the mass of the empty container to find the mass of water.

   c) Compare the masses of oil and water. Which is heavier?

   d) Mix the water and oil together in the larger container, pouring the oil into the container first. Let it stand for a minute or two. What happens?

   **Selected answers:** c) water is heavier than oil; d) oil and water separate, with oil staying atop the water because oil is lighter

4. A blue whale is the largest mammal on Earth. It weighs about 140 tonnes.

   a) An African savannah elephant weighs about 6050 kg. About how many elephants weigh as much as a blue whale?

   b) A narwhal weighs about 940 kg. About how many narwhals weigh as much as a blue whale?

   c) A beluga whale weighs about 1400 kg. About how many belugas weigh as much as a blue whale?

   **Sample answers:** a) about 23 elephants, b) about 150 narwhals, c) about 100 belugas
Cube Skeleton

You will need 12 tubes and tape to make a skeleton of a cube.

1. Use tape to bind tubes together as shown.

2. Make two squares with the tubes. Make sure the squares are 1 unit long and 1 unit wide on the inside.

3. Bind the four leftover tubes to one of the squares as shown.

4. Add the other square to the top.
Problem-Solving Lesson 5-10

Goals
Students will, when given a problem, make a simpler problem and use the solution to the simpler problem to help solve the original problem.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract decimals up to hundredths
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can write an expression for a given term in a pattern
Can convert measurements expressed in metres to centimetres (for Extended Problem)

MATERIALS
BLM Fraction Strips and Circles (p. Q-71, see Problem Bank 7)
BLM Class Art Show (pp. Q-74–75, see Extended Problem)

Using a given simpler problem to help solve a harder problem.
Write on the board:

There are 300 people in line. How many people are after the 12th person?

ASK: What makes this problem hard? (students will likely say that 300 is a big number) Would it be easier if I asked how many people are after the 299th person in line? (yes) SAY: So, it's not exactly how big 300 is that makes this problem hard. ASK: Can you find a more precise way to say what makes it hard? (12 and 300 are far apart)

Exercises: Answer the question.

a) There are 13 people in line. How many people are after the 12th person?
b) There are 5 people in line. How many people are after the 3rd person?
c) There are 300 people in line. How many people are after the 12th person?

Bonus: There are 3459 people in line. How many people are after the 1459th person?

Answers: a) 1, b) 2, c) 288, Bonus: 2000

ASK: How did solving the easier problems make it easier to solve the harder problems? (it told me that the right approach was to subtract: number of people in line − position of the person in line)

Using off-by-one patterns to solve problems. Tell students that you are waiting in line to get on a roller coaster. SAY: You are 37th in line and you see your friend, who is 7th in line. ASK: How many people are between us? (students will likely say 30)
Draw on the board:

| 0 | 7 | 37 |

SAY: Thirty is a good guess because, on a number line, the length of the part of the line between 7 and 37 is 30. Add 8 to the number line. ASK: If you are 7th in line and I'm 8th in line, how many people are between us? (none) SAY: But 8–7 is 1, not 0, so although subtracting gets us close to the right answer, it's not exactly right. Let's try to figure out what is going on.

**Exercises:** How many people are in between? Hint: Use a number line to solve.

a) the 7th and 8th person
b) the 7th and 9th person
c) the 7th and 10th person
d) the 7th and 11th person
e) the 7th and 12th person
f) the 7th and 37th person

**Answers:** a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the right answer? (no) Did it give close to the right answer? (yes) How can you get the number of people between two people, given their positions in line? (the number is off by 1, so I can find the difference between the positions and then subtract 1 more) How many people are between the 37th person and the 7th person? (29) How did starting with smaller numbers help? (it made the problem clearer) SAY: Sometimes, it's easier to start by using smaller numbers rather than using what is given in the problem. Then you will see things that help you solve the harder problem. Now that you know the pattern, you can find the number of people between any two positions.

**Exercises:** How many people are in between?

a) the 8th and 78th person
b) the 314th and 1000th person
c) the 492nd and 613th person

**Answers:** a) 69, b) 685, c) 120

Write on the board:

A teacher tells her class to read pages 287 to 304 for homework. How many pages is that?

Have volunteers give you similar, simpler problems that you can solve first (for example, make the numbers smaller). Write down all the volunteers' suggestions on the board.
Exercises: Solve all the simpler problems on the board. Do you see a pattern in your answers?

Answers: In all cases, you can find the number of pages by subtracting the smaller number from the bigger number and then adding 1.

Have a volunteer tell you the pattern. (subtract the numbers and add 1)

SAY: Now that you know the pattern, you can solve any problem of the same type.

Exercises: A teacher tells her class to read pages in a textbook for homework. How many pages of reading do the students need to do?

a) from 352 to 386
b) from 298 to 314
c) from 408 to 451

Answers: a) 35, b) 17, c) 44

Tell students that it can be helpful to examine the simpler problems in an organized way. Refer students to the problem about reading from pages 287 to 304. Write on the board:

<table>
<thead>
<tr>
<th>Pages Read</th>
<th>How Many Pages?</th>
</tr>
</thead>
<tbody>
<tr>
<td>287 to 288</td>
<td>2</td>
</tr>
<tr>
<td>287 to 289</td>
<td>3</td>
</tr>
<tr>
<td>287 to 290</td>
<td>4</td>
</tr>
<tr>
<td>287 to 291</td>
<td>5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>287 to 354</td>
<td>?</td>
</tr>
</tbody>
</table>

SAY: By being organized, you might find the pattern sooner. Patterns can be easier to see when you have something organized to look at, like a table or a diagram.

Exercises: Make several simpler problems until you see the pattern to do the harder problem. Organize the simpler problems.

a) A fence is made using 42 posts, each 1 m apart. How long is the fence?
b) A fence is made using 34 posts, each 2 m apart. How long is the fence?

Answers: a) 41 m, b) 66 m

SAY: You can extend this type of problem to fences that go all the way around a field.

Exercises: A fence for a square field is made with posts 1 m apart, including a post at each corner. How many posts are needed for a field that is …

a) 10 m by 10 m? Hint: Start with a field that is 1 m by 1 m and then move on to 2 m by 2 m, 3 m by 3 m, and so on.
b) 20 m by 20 m?
Answers: a) 40, b) 80

NOTE: For the following exercises, encourage students to predict the answer before checking.

Exercises: A square field is 20 m by 20 m. How many posts are needed if the posts are ...

a) 1 m apart?  b) 2 m apart?  c) 4 m apart?  d) 5 m apart?

Bonus: 40 cm apart?

Answers: a) 80, b) 40, c) 20, d) 16, Bonus: 200

Using whole numbers instead of decimals to make a problem easier.

Draw on the board:

\[
\begin{align*}
6.7 & \quad 17.6 \\
\text{?} & \\
\end{align*}
\]

Tell students that you have two sticks. You know one stick’s length and the total length, but you want to know the length of the second stick.

ASK: What makes this problem hard? (there are decimals) SAY: Let’s solve the problem approximately with whole numbers first. Erase the decimal part of the numbers on the board, as shown below:

\[
\begin{align*}
6 & \quad 17 \\
\text{?} & \\
\end{align*}
\]

ASK: Approximately how long is the second stick? (11) How did you get that? (17 \( - \) 6 = 11) SAY: So you can do the problem the same way when the lengths include decimals. Instead of subtracting 6 from 17, you subtract 6.7 from 17.6. Write on the board:

\[
\begin{align*}
17.6 \\
& \quad - 6.7 \\
& = 10.9
\end{align*}
\]

Have a volunteer complete the subtraction. (10.9) SAY: When we estimated with whole numbers, the answer was 11. ASK: Is the actual answer close to 11? (yes)

Exercises: Find the missing length.

a) \[
\begin{align*}
? & \quad 3.2 \quad 5.68 \\
\end{align*}
\]

b) \[
\begin{align*}
15.34 & \quad \? \quad 12.76 \\
\end{align*}
\]

Bonus: \[
\begin{align*}
\begin{cases}
4.2 \\
6.5 \\
\text{?} \\
8
\end{cases}
\end{align*}
\]

Answers: a) 8.88, b) 2.58, Bonus: 2.7
Focusing only on relevant information to make a problem simpler.
Remind students of the example of $6.7 + ? = 17.6$ from earlier. SAY: I’m going to move these sticks around. Draw on the board:

```
| 6.7 | 17.6 |
```

ASK: How did I move the sticks? (slid one of them down) SAY: This second problem has a lot of extra information so it looks harder, but it actually has exactly the same answer as the other one, so you might as well do the easier one. The total length of the two sticks at the bottom is still 17.6, they are just not side by side anymore.

**Exercises:** Find what the ? stands for by making the problem into a simpler problem.

a)    b)    c)    d)

```
11  ?  18
```

```
15.7  6.5  ?
```

```
3.46  1.27  ?
```

```
5.75  4.4  ?
```

```
3.81  ?
```

**Answers:** a) 7, b) 9.2, c) 4.73, d) 1.05

SAY: By pretending that the sticks are side by side, you turn the problem into an easier problem.

Making a problem easier by emphasizing what is relevant. SAY: We can look at a problem and focus on what matters most. For example, if you need to find a vertical side—straight up and down—then colour over all the vertical lines. If you need to find a horizontal side, colour over all the horizontal lines.

**Exercises:** Find what the ? stands for by making the problem into an easier problem.

a)    b)    c)    d)

```
15  8  ?
```

```
5.12  3.81  ?
```

```
3.53  4.73  8.54
```

```
?    ?
```
Point out to students that by colouring over all the horizontal or vertical lines, they changed the problem into an easier problem.

**Finding perimeter without knowing all the side lengths.** Remind students that to find the perimeter of a shape, we add up the lengths of all the sides. Draw on the board:

```
  20
  5
  3
```

SAY: I want to find the perimeter of this shape. It looks like a hard problem, because there are a lot of missing side lengths. Ask a volunteer to mark three sides that are missing lengths. (the two bottom horizontal sides and the right side) SAY: There are two kinds of sides in this shape: horizontal sides and vertical sides.

ASK: How long is the top side? (20) How long are the two bottom sides put together? (20) How do you know? (put together they are the same length as the top side) How long are the two sides on the left of the shape? (5 and 3) How long is the side on the right? (8) How do you know? (it’s the same as the two left sides put together) Write on the board:

```
Horizontal edges add to     Vertical edges add to     
```

Perimeter is ___ + ___ = ___

Have volunteers fill in the blanks. (40, 16, 40 + 16 = 56)

**Exercises:** Find the perimeter of the shape. All measurements are in centimetres.

a) ![Image](image1.png)

Answers: a) 48 cm, b) 560 cm
SAY: You can find the area of these shapes by taking away rectangles, finding the area of each rectangle you took away, and subtracting the results from the area of the big rectangle.

**Exercises**

1. Find the area of each shape from the previous exercises.
   **Answers:** a) 128 cm², b) 13 262 cm²

2. Find the perimeter.

```
\begin{array}{c}
<table>
<thead>
<tr>
<th>3</th>
<th>8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\end{array}
```

**Answer:** 34

**Bonus:** Is there enough information to find the area of the shape in Exercise 2? Explain.

**Answer:** no, because to find the area you need the length of each part

**Problem Bank**

1. When everyone in Tom’s class stands in line, Tom is 14th in line and 11th from the end of the line. How many people are in the class?
   **Answer:** 24

2. There are 126 people in line. How many people are behind the 94th person?
   **Answer:** 32

3. Make several simpler problems until you see how to do the harder problem.
   a) A fence is made using 53 posts, each 3 m apart. How long is the fence?
   **Answers:** 156 m
   b) A fence is made using 61 posts, each 2.5 m apart. How long is the fence?
   **Answers:** 150 m

4. How many posts are needed to make the fence?
   a) A fence is 47 m long, with posts at 1 m intervals.
   b) A fence is 100 m long, with posts at 2.5 m intervals.
   c) A fence is 84 m long, with posts at 3.5 m intervals.
   **Answers:** a) 48, b) 41, c) 25
5. A fence for a square garden is made with posts 1.5 m apart, including a post at each corner. How many posts are needed for the garden? Hint: Start with a garden that is 1.5 m by 1.5 m and then move on to 3 m by 3 m, 4.5 m by 4.5 m, and so on.
   a) The garden is 12 m by 12 m.
   b) The garden is 21 m by 21 m.
   **Answers:** a) 32, b) 56

6. Predict each answer before checking. A square field is 30 m by 30 m. How many posts are needed if the posts are...
   a) 1 m apart?  
   b) 2 m apart?  
   c) 1.5 m apart?  
   d) 2.5 m apart?  
   **Bonus:** 60 cm apart?
   **Answers:** a) 120, b) 60, c) 80, d) 48, Bonus: 200

7. Cut out the strips and circles from **BLM Fraction Strips and Circles** (you may cut the line down to the centre of the circles). Estimate to colour the given amount. Use folding to check your estimate.
   a) one fifth of a strip of paper, starting from the left
   b) two fifths of a strip of paper, starting from the left
   Hint: Use your answers to parts a) and b) to help you determine a strategy for parts c) and d). Hold the circle so that the cut line is at the top.
   c) one fifth of a circle, going clockwise starting from the top
   d) two fifths of a circle, going clockwise starting from the top

8. Each line segment is 1 m long. What is the total length of this path? Look for a quick way to answer.

   **Answer:** 18 vertical metres plus 17 horizontal metres = 35 metres altogether

9. a) Find the perimeter.

   **Answer:**

   b) Is there enough information to find the area of this shape? Explain.
Answers: a) 36.6; b) no, we don’t have the side length for the small rectangles

10. Each shape was made by placing a small square on top of a large square. All measurements are in centimetres.

a) Find the perimeter of the shape.

i)  

ii)  

iii)  

iv)  

b) Make a table with the headings “Size of Smaller Square” and “Total Perimeter.” Use the pattern from part a) to solve the problems.

i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?

ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?

Answers: a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm

11. a) Convert the measurements in metres to centimetres. Hint: 1 m = 100 cm.

i) 2 m = ___ cm  

ii) 3 m = ___ cm  

Bonus: 183 m = ___ cm

b) Find the perimeter in centimetres.

i)
250 cm
1 m
4 m
75 cm

Answers: a) i) 200, ii) 300, Bonus: 18 300; b) i) 1040 cm, ii) 2050 cm

12. Find the missing length.

Answer: 8 m

13. In the figures below, each square has a side length of 1 m.

a) Complete the table for the figures.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Perimeter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
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<td>2</td>
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<td>3</td>
<td>4</td>
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</table>

b) What is the perimeter of the 10th figure?
c) Which figure has perimeter 48 m?

Answers: a) 8, 12, 16; b) 40 m; c) 12th figure
Fraction Strips and Circles
Extended Problem: Class Art Show

MATERIALS

BLM Class Art Show (pp. Q-74–75)

Extended Problem: Class Art Show. Give students BLM Class Art Show. Tell students the context for the extended problem: You are holding an event for parents to view posters created by your class. The posters will be hung in the school hallway. Tell students to use diagrams in their answers.

Answers: 1. a) 10 cm, b) 155 cm; 2. a) 104 cm, b) 12 posters
Class Art Show (1)

1. Each student makes a poster that is 22 cm tall and 17 cm wide. You put two pins in the poster 1 cm below the top of the poster and 1 cm from each edge.

The centre of the posters should be at eye level so that they can be seen easily. For adults, eye level is about 145 cm above the floor.

a) How high is each pinhole above the level of the centre of the poster?

b) How high above the floor is each pinhole?
Class Art Show (2)

2. Two doors along the school hallway are 3 m apart. Posters on the wall between the doors should be 8 cm apart from each other. The posters should also be centred so that the posters nearest to the doors are each the same distance from a door.

   a) If you put 4 posters between the two doors, how far will the posters nearest to the doors be from the door’s edge?

   b) What is the maximum number of posters you can hang between the two doors?
Unit 15 Probability and Data Management: Likelihood and Probability

Introduction
This unit focuses on:
• naming and quantifying outcomes;
• measuring the likelihood of events;
• predicting the frequencies of events;
• comparing predictions with data from experiments; and
• determining the probability of winning in games.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit:
• practise multiplication properties and strategies
• review addition and subtraction skills
• practise converting fractions to decimals

Assessment
The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>PDM5-9 to 11</td>
<td>PDM5-9, 12 to 15</td>
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<td>PDM5-9, 12 to 15</td>
</tr>
<tr>
<td>Test</td>
<td>PDM5-9 to 11</td>
<td>PDM5-9, 12 to 15</td>
<td>PDM5-9 to 11</td>
<td>PDM5-9, 12 to 15</td>
</tr>
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</table>
Goals
Students will identify the outcomes of events involving chance.

PRIOR KNOWLEDGE REQUIRED
Knows that a half can be represented in different ways using standard fractional notation
Knows that half of a circle can be represented using different pictures
Can recognize when a circle is divided into equal or unequal parts
Can draw circles divided into equal or unequal parts

MATERIALS
die
coin
letter-size piece of craft paper
sharpened pencil and paper clips
blue and red markers or chalk
paper cups

Mental math minute. Give a student a large, even number to halve, such as 288. Successive students halve the previous answer; for example, the first student says 144, the next says 72, and so on. Occasionally ask students to explain how they got the answer. When students reach an odd number, start with a new large, even number.

Introduce results and outcomes. Hold up a die. Have students make predictions about what number you will roll and then test the predictions to demonstrate that predictions and actual results can differ. SAY: The result of rolling a die once is the number facing up when it lands. For each roll, there is one result. The possible outcomes of rolling a die are all of the possible results. ASK: What are the possible outcomes? (1, 2, 3, 4, 5, 6) How many outcomes are there in total? (6) Repeat for flipping a coin. (heads, tails, 2 possible outcomes) SAY: Rolling a die, tossing a coin, spinning a spinner, or picking a marble out of a bag without looking are all examples of experiments. Any time you do something where different results are possible, you are doing an experiment. Repeat for a soccer game. (one team wins, the other team wins, neither team wins, 3 possible outcomes)

Outcomes of spinning a spinner. On a letter-size piece of craft paper, draw a 10 cm diameter circle, marked as shown in the margin. Demonstrate how to use the tip of a sharpened pencil, a paper clip, and the circle as a spinner. Then label the regions 1 to 4 and list the outcomes on the board:

1. The pointer lands in region 1.
2. The pointer lands in region 2.
3. The pointer lands in region 3.
4. The pointer lands in region 4.
ASK: How many outcomes does this spinner have? (4) SAY: When the spinner lands in each region, it is a different outcome.

Exercises: How many outcomes does the spinner have?

a)  

b)  

c)  

Answers: a) 2, b) 4, c) 6

Outcomes of a spinner with several regions of the same colour. Using the spinner you made on craft paper, colour regions 1, 2, and 3 blue and region 4 red. Then label regions 1, 2, and 3 with a “B” and region 4 with an “R.” SAY: When I spin the spinner now, the pointer will still land in one of the four regions. The number of regions on the spinner did not change. This means that the number of outcomes did not change either. The spinner still has four outcomes.

Introduce events. SAY: Imagine I am going to play a game with you using this spinner. If the spinner lands on blue, you win. If the spinner lands on red, I win. ASK: How many different ways can the spinner land on blue? (3) How many different ways can the spinner land on red? (1) SAY: This game can have two different results, spinning blue or spinning red. We call each result, spinning blue or spinning red, an event. ASK: How many outcomes make the event “spinning blue”? (3) Which outcomes are these? (pointer lands in any of the regions labelled “B”) How many outcomes make the event “spinning red”? (1) Which possible outcome? (pointer lands in the region labelled “R”)

Exercises

1. How many outcomes does the spinner have? How many outcomes make the event “spinning white”?

a)  

b)  

c)  

d)  

Answers: a) 4, 2; b) 8, 4; c) 6, 3; d) 4, 4

2. You pick a ball from the box. How many outcomes are there? How many outcomes make the event “picking red”?

a)  

b)  

c)  

Answers: a) 5, 1; b) 4, 2; c) 6, 3
3. You pick a marble from the box without looking. How many outcomes are there in total? How many outcomes make the event “not picking a red marble”?

![Marbles Image]

**Answers:** a) 6, 4; b) 7, 4; c) 5, 5

**Unequal outcomes.** SAY: You have to make a spinner with four possible outcomes. ASK: How would you do this? Invite volunteers to draw possible spinners on the board. Draw the spinner shown in the margin on the board.

Shade each region with a different colour and ASK: Are all the outcomes equally likely to occur? (no) Is there one that will likely occur more often than the others? (yes) Which one? (the big one) Draw another spinner on the board with a short pointer, as shown in the margin.

ASK: How many outcomes does this spinner have? (4) Will the pointer ever be in the grey regions? (no) SAY: Because the spinner will never land in the grey regions, we do not consider the grey regions to be outcomes.

**Exercises:** Draw four circles using a paper cup. Using a ruler and a pencil, divide one of the circles into regions according to the description.

a) three equal outcomes  
c) four equal outcomes  
b) three unequal outcomes  
d) four unequal outcomes

**Answers**

![Spinners Image]

**Sample answers**

**ACTIVITY (Optional)**

Students will each need a sharpened pencil, a paper clip, and a spinner like the one shown below, with all regions coloured differently.

![Spinner Image]

Students work in pairs with one partner spinning the spinner 20 times and the other partner tallying the results. Partners switch roles. Which colour occurs the greatest number of times? (Y, yellow)
 Extensions

1. Explain the rules of the game of Lahal or watch a video explaining the rules. You might wish to invite a local First Peoples Elder to talk about the significance of Lahal and how it is played. Describe the probability set-up of a variation of the game: There are four bones. Two of them have a stripe and two do not. Two people hide one of each type of bone in each of their hands. You choose two bones to uncover. If both of them are unstriped, you win. For each striped bone you uncover, you lose one point (one scoring stick).

Let’s number the hands: Hands 1 and 2 belong to one person, and hands 3 and 4 belong to the other person.

   a) How many ways to uncover the bones are there?

   b) How many favourable outcomes (outcomes that uncover both unstriped bones) are there? Hint: Suppose that the unstriped bones are on the outside (Hands 1 and 4).

   c) How many outcomes will make you lose one stick?

   d) How many outcomes will make you lose two sticks?

   **Answers:** a) 4 (hands 1 and 3, 1 and 4, 2 and 3, 2 and 4); b) 1; c) 2; d) 1

2. Design a spinner that has six equal outcomes, two of which make the event “spinning white.”

   **Sample answer:**

3. Design a spinner that has five outcomes, all unequal, two of which make the event “spinning black.”

   **Sample answer:**

4. Mandy and Jun are playing a game. They put 5 red marbles in a bag. To start playing, they each choose a marble out of the bag without looking. How could they set up the game so that not all outcomes are equal?

   **Sample answer:** Use different sizes of marbles.

5. Jack and Marla are playing “Rock, Paper, Scissors.” Describe all the possible outcomes of the game. **NOTE:** “Jack wins” is not an outcome; it is an event. “Jack shows paper, Marla shows rock” is an outcome.
**Goals**

Students will identify situations in which the chances of an event are even.

**PRIOR KNOWLEDGE REQUIRED**

- Can identify outcomes of an experiment
- Can find half of a number
- Can identify visual representations of fractions

**MATERIALS**

- coin
- marbles or counters of different colours (optional)
- pencil crayons
- paper clips

**Mental math minute—number talk.** Present this problem: $46 - 23$. (23)

The following strategies could arise

- 46 is double 23
- 46 is $23 + 3 + 20$
- $46 - 20 - 3$
- 46 is $23 + 7 + 10 + 6$

**Finding half of a number.** Remind students that finding half is the same as dividing by 2. Review how to find half of a number, half of a pie, or half of a set of objects. Draw six squares on the board. Invite a volunteer to circle half of the squares, as shown below:

![Image of six squares with half circled]

Have the volunteer explain their answer. ASK: What is half of 8? (4) How do you know? (double 4 is 8) I need a number that is less than half of eight—which numbers fit this description? (0, 1, 2, or 3) Repeat these questions for different even numbers. Then draw even numbers of shapes and ask students to find several ways to shade more than half of the shapes.

Draw a different set of six squares and ask students to colour less than half of the squares red. (they can colour 0, 1, or 2 squares) Draw another set of six squares and ask students if they can shade a different number of squares blue so that the number of blue squares will still be less than half of six.
Exercises

1. I have 12 marbles. Half of them are blue. How many marbles are blue?
   Answer: 6

2. I have 14 marbles and 7 of them are red. How many of my marbles are red: half, less than half, or more than half?
   Answer: half are red

3. I had 18 marbles. I lost 8 of them. Did I lose more than half, less than half, or exactly half?
   Answer: less than half

4. Complete the statement by writing “more than half,” “half,” or “less than half.”
   a) 4 is ______ of 8.  
   b) 5 is ______ of 10.
   c) 9 is ______ of 16.  
   d) 8 is ______ of 14.
   e) 3 is ______ of 8.  
   f) 7 is ______ of 16.
   Answers: a) half, b) half, c) more than half, d) more than half, e) less than half, f) less than half

5. Shade half of the pieces.
   a)  
   b)  
   c)  
   Sample answers
   a)  
   b)  
   c)  

Even chances in events. Hold up a coin. ASK: How many outcomes are there for flipping this coin? (2) What happens more often, flipping heads or tails? (neither) SAY: The chances are the same. In mathematics, we say that you have an even chance of flipping heads or tails. Write “even chance” on the board and explain that an event has an even chance of occurring when it happens in exactly half of the outcomes. SAY: Flipping a tail is one out of two possible outcomes, and 1 is half of 2. We say flipping heads and flipping tails are equally likely because both have the same chance to happen. ASK: How many outcomes are there when you roll a die? (6) How many outcomes are numbers that are odd? (3) What are those numbers? (1, 3, and 5) SAY: Since half of the numbers are odd, you have an even chance of rolling an odd number. You also have an even chance of rolling an even number. ASK: How many outcomes are even numbers? (3) What are those numbers? (2, 4, and 6)
SAY: There are 12 marbles in a box. I take out one marble without looking.
ASK: How many outcomes are possible? (12, regardless of the colour of
the marbles) What is half of 12? (6) If I would like to have an even chance
of taking out a red marble, how many marbles should be red? (exactly 6)
Does it matter what colour the other marbles are? (no, as long as they are
not red) Invate a volunteer to sketch 12 marbles (or to create a collection
using actual marbles, if available) that give an even chance of picking a
red marble. If the collection uses only two colours, ask another volunteer
to make a collection that uses at least three colours but still gives an even
chance of picking a red marble.

**Exercises**

a) Draw 8 marbles that give an even chance of picking a yellow marble.
b) Draw 10 marbles of 6 colours that give an even chance of picking
a red marble.

**Sample answers:**
a) 

```
Y Y Y Y
R B W G
```
b) 

```
R R R R R
W B Y G P
```

**Determining whether an event has even chances.** Draw the following
spinner on the board and shade half blue and half red:

```
B R
```

ASK: Which part of the spinner is shaded red? (the right half) What are the
possible outcomes for this spinner? (the spinner lands in the blue region,
the spinner lands in the red region) How many outcomes are there? (2)
Which part, or fraction, of the outcomes is the pointer landing in a red
region? (half) Is there an even chance of spinning red? (yes)

**Exercises:** Is there an even chance of spinning blue? Why?

```
R B B G
```

**Answer:** yes, this spinner has four possible equal outcomes because it has
four equal regions, and two of the four regions (that is, half of 4) are blue

Draw on the board:
Ask students to identify the outcomes for each spinner. ASK: In which spinners do you have an even chance of spinning green? (the one on the left and the one on the right)

**Exercises:** Is there an even chance of spinning green?

a) ![Spinner A](image1)  

b) ![Spinner B](image2)  

c) ![Spinner C](image3)  

d) ![Spinner D](image4)  

**Bonus:** ![Spinner E](image5)

**Answers:** a) no, b) yes, c) yes, d) no, Bonus: yes

**Real-world events with even chances.** Ask students to describe several events with even chances for rolling a die. (sample answers: roll a number that is 3 or less; roll an even number; roll 2, 3, or 5; roll an odd number) Encourage students to think of examples that do not involve rolling dice, spinning spinners, or drawing marbles. For example: several pairs of boots are in a closet. I pick a boot in the dark. It is either a left boot or a right boot. So “I pick a left boot” has even chances.

**ACTIVITY (Optional)**

Divide students into three groups. Have each group make one of the five spinners below, spin it 48 times, and record the results. Then have groups make a bar graph of the results.

ASK: Did your group spin green in more than half, less than half, or exactly half of the spins? Is this what you expected? Discuss as a class.

**Extensions**

1. Complete the sentence by writing “more than half” or “less than half.”

   a) 3 is ________ of 5.  
   b) 3 is ________ of 7.  
   c) 6 is ________ of 11.  
   d) 5 is ________ of 11.  
   e) 9 is ________ of 16.  
   f) 8 is ________ of 15.

**Answers:** a) more than half, b) less than half, c) more than half, d) less than half, e) more than half, f) more than half
2. A spinner has five regions and three of them are green. If you spin the spinner 20 times, do you expect to spin green more than 10 times or less than 10 times? Explain.

**Sample answer:** more than 10 times because 3 is more than half of 5, and since half of 20 spins is 10, we expect more than 10 spins to be green
Goals
Students will describe the likelihood of events occurring using the words “impossible,” “possible,” and “certain.”
Students will compare the likelihood of two events using the words “less likely,” “more likely,” or “equally likely.”

PRIOR KNOWLEDGE REQUIRED
Can identify the outcomes of an experiment
Can find half of a number
Can identify visual representations of fractions

MATERIALS
hundreds chart or BLM Hundreds Charts (p. R-33)
pencil crayons
paper clips
marbles or counters of different colours (red, blue, green, and yellow)

Mental math minute—number string.
String 1: 9, 9 + 30, 9 + 31, 9 + 29 (9, 39, 40, 38)

Give each student a hundreds chart (for example, from BLM Hundreds Charts). Have each student place a finger on a target number on the chart.

Present the strategies: adding tens by moving down rows, adding one more by making the answer one greater, subtracting tens by moving up rows, subtracting one by making the answer one greater.

String 2: 52, 52 − 10, 52 − 9 (52, 42, 43)

Likely and unlikely events. SAY: When events happen often, but not always, we say that such events are likely. For example, you are likely to go to school on a Wednesday in November. You might get sick and not go to school, but most Wednesdays during November you go to school, so the event is likely. Ask students to give you examples of likely events.

SAY: When an event happens not so often, but still happens sometimes, or at least can happen in theory, we call such events unlikely. For the following exercises, have students signal thumbs up if the event is likely and thumbs down if it is unlikely.

Exercises: Is the event likely or unlikely?

a) You will be in Grade 6 next year.
b) You will be taller than your teacher next year.
c) An elephant will walk into class in the next minute.
d) You will eat some food within the next 24 hours.

e) You will travel to the moon next year.

f) It will snow in August.

**Answers:** a) likely, b) unlikely, c) unlikely, d) likely, e) unlikely, f) unlikely

Invite students to name some events and have other students say if the events are likely or unlikely.

**Likely and unlikely events in probability experiments.** Draw on the board the spinner below and colour the parts marked “R” red, “Y” yellow, and “G” green.

```
G G G G
Y R
G G
```

ASK: How many outcomes are there? (6) How many parts are green? (4) Is that more than half? (yes) Point to the spinner and SAY: Spinning green is **likely** to happen because more than half of the spinner is green.

ASK: How many parts are red? (1) Is that more than half or less than half? (less) SAY: Less than half of the spinner is red, so spinning red is **unlikely** to happen.

Conduct an experiment with 24 spins. Use a tally chart to record the results. SAY: If the same experiment is done many times, we expect the total result to match the likelihood of the event. Examples:

• An event with an even chance is expected to happen exactly half of the time.
• A likely event is expected to happen more than half of the time.
• An unlikely event is expected to happen less than half of the time.

Write “likely,” “unlikely,” and “even chance” on the board. Using the tally chart, describe the chances of spinning green, red, and yellow in these terms. (green is likely, red and yellow are both unlikely)

**NOTE:** The definitions used here for “likely” and “unlikely” will be easy for students to understand and apply. Note that some resources do not define the terms precisely, and some sources consider events that have a close to even chance of occurring as being neither likely nor unlikely.

**Exercises**

1. There are three pairs of black boots and three pairs of brown boots in a dark closet.
   a) Count the total number of outcomes if you take out a boot from the closet.
b) Count the number of outcomes for the given event. Write whether the event is likely, is unlikely, or has an even chance of happening.

i) taking out a right boot

ii) taking out a black boot

iii) taking out a brown left boot

iv) taking out either a black boot or a brown right boot

v) taking out a boot that is not black

**Bonus:** taking out a boot that is not black left

**Answers:** a) 12; b) i) 6, even chance; ii) 6, even chance; iii) 3, unlikely; iv) 9, likely; v) 6, even; Bonus: 9, likely

2. Describe each event as having an even chance, likely, or unlikely.

a) ![Diagram](image1)

b) ![Diagram](image2)

c) ![Diagram](image3)

**Events:**
- spinning green
- spinning red
- spinning green
- spinning red

**Answers:** a) green: likely, red: unlikely; b) green: even chance, red: unlikely; c) blue: unlikely, red: likely

3. Are the chances likely, unlikely, or even?

a) Pulling a green sock from a box with 7 green socks and 5 black socks.

b) Pulling a nickel from a pocket with 4 nickels and 5 quarters.

c) Pulling a red balloon from a bag with 4 red balloons, 2 blue balloons, 1 green balloon, and 1 yellow balloon.

**Answers:** a) likely, b) unlikely, c) even chance

4. Give an example of a likely event and an unlikely event for rolling a die. Both events should be possible.

**Sample answer:** rolling 2 or more is likely, and rolling 6 is unlikely

**Likely and unlikely events in unequal outcomes.** Draw on the board:
Ask students to describe the possibility of spinning each of the colours.

**ASK:** How many outcomes are there if you spin the spinner? (4) Is it equally likely that you will spin green or spin red? (yes) Is it equally likely that you spin green or blue? (no) **PROMPT:** Look at the angle at the centre. **SAY:** It is more likely to spin blue than to spin green. **ASK:** Is it likely to spin yellow? (yes) **WHY?** (more than half of the circle is yellow) Is it more likely to spin blue or to spin red or green? (spin red or green) **WHY?** (red and green together are equal to a quarter of the spinner, but blue is less than a quarter)

**Certain and impossible events.** Write “unlikely,” “even chance,” and “likely” on the board in that order from left to right. Ask students which word people would use to describe the event of drawing a pink marble out of a bag with 2 blue and 3 green marbles. **ASK:** Can that happen at all? (no) Write the word “impossible” at the left of the list. **SAY:** Impossible describes an event that cannot happen. Ask students what words describe an event that will definitely happen, such as rolling a number less than 7 on a die. (certain, definite, sure, absolute) **SAY:** In probability, an event that will definitely happen is described as **certain.** Add the word “certain” at the right of the list, as shown below.

impossible unlikely even chance likely certain

**Exercises**

1. Describe the event as impossible, unlikely, even chance, likely, or certain.
   a) Pulling a green sock from a drawer with 4 green socks and 2 black socks.
   b) Pulling a $50 bill from a wallet with three $10 bills and one $5 bill.
   c) Growing taller than your mother on your next birthday.
   d) Rolling a number less than 7 on a die.
   e) Drawing a red balloon out of a bag of 20 balloons with 4 red balloons.
   f) Rolling an odd number on a die.

**Answers:** a) likely, b) impossible, c) unlikely, d) certain, e) unlikely, f) even chance

2. Use “impossible,” “unlikely,” “even chance,” “likely,” or “certain” to describe the given event.
   a) If you flip a coin once, you will get a head and a tail.
   b) If you flip a coin once, you will get a head or a tail.
   c) If you roll a die once, you will get a number less than six.

**Answers:** a) impossible, b) certain, c) likely
Ask students to give examples of various events and explain whether the chances of it occurring are likely, unlikely, certain, impossible, or even. Encourage students to think of events that use marbles, dice, money, and other objects, as well as events from daily life, such as meeting a tiger or an astronaut on the way to school.

**Comparing and describing the likelihood of events.** Show students the following collection of marbles or coloured counters:

```
R Y R R B G Y R
```

ASK: How would you describe the event of picking blue? (unlikely) What about picking red? (even chance) Which colour are you most likely to pick? (red) Which colour is less likely to be picked, yellow or green? (green)

**Exercises**

a) For each spinner, describe each event as certain, likely, even chance, unlikely, or impossible.

<table>
<thead>
<tr>
<th>Event</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>spinning green</td>
<td>unlikely</td>
<td>unlikely</td>
<td>unlikely</td>
<td>impossible</td>
<td>unlikely</td>
</tr>
<tr>
<td>spinning red</td>
<td>likely</td>
<td>impossible</td>
<td>likely</td>
<td>impossible</td>
<td>even chance</td>
</tr>
<tr>
<td>spinning blue</td>
<td>unlikely</td>
<td>unlikely</td>
<td>impossible</td>
<td>certain</td>
<td>unlikely</td>
</tr>
<tr>
<td>spinning yellow</td>
<td>impossible</td>
<td>even chance</td>
<td>impossible</td>
<td>impossible</td>
<td>impossible</td>
</tr>
</tbody>
</table>

b) Which colour are you more likely to spin in spinner B, blue or green?

c) Which colour are you more likely to spin in spinner A, blue or green?

**Answers:**

b) blue, c) neither—there is an even chance of spinning blue or green

Ask students to compare the likelihood of events. For example, it is unlikely to snow in August, but it is even more unlikely that an alien will walk into the class.
Exercises: Compare the likelihood of the events for each spinner in the previous exercises and write them in order from most to least likely.

Selected answer: spinner B: yellow, blue, green

ACTIVITY (Optional)
Give students marbles or counters of various colours. One player secretly chooses 12 marbles or counters. The player describes the set to the partner using probability terms (certain, likely, unlikely, and so on). The second player has to reconstruct the set based on the first player’s description.

Extensions
1. a) If you roll a die, are your chances of rolling a number greater than 3 unlikely, even, or likely? Explain your answer.
   b) If you roll a die, are your chances of rolling a number less than 3 unlikely, even, or likely? Explain your answer.
   c) Edmond says that rolling a number less than 3 or greater than 3 should both be events with an even chance since 3 is half of 6. Do you agree with Edmond? Explain.

   Answer: a) Chances are even because 4, 5, and 6 are greater than 3, so there are 3 out of 6 outcomes for the event; b) Chances are unlikely because 1 and 2 are less than 3, so there are only 2 out of 6 outcomes for the event; c) I disagree with Edmond. Although 3 is half of 6, there are only 2 out of 6 outcomes for rolling a number less than 3, so that event does not have an even chance.

2. Draw a spinner to match the description.
   a) You are likely to spin yellow.
   b) You are unlikely to spin green.
   c) It is impossible to spin blue.

   Sample answer: a–c)

3. Draw a collection of marbles of at least three colours to match the description.
   a) You are likely to draw a green marble.
   b) You are unlikely to draw a yellow marble.
   c) It is impossible to draw a purple marble.

   Sample answer: a–c)
4. Draw a spinner that matches the following descriptions:

- It is most likely to spin green.
- It is unlikely to spin red.
- It is most unlikely to spin yellow.
- It is equally likely to spin red or blue.
- It is impossible to spin pink.

**Sample answer**

![Spinner Diagram]

5. Match the spinner with the correct statement.

A. R B Y G
B. R B W G
C. B B B W
D. R B B W

___ Spinning blue is three times as likely as spinning white.
___ Spinning white is twice as likely as spinning red.
___ Spinning any colour is equally likely.
___ Spinning one colour is twice as likely as spinning any other colour.

**Answer:** C, D, A, B

6. Invent or describe a game in which one player’s chance of winning is very close to certain. What are the chances of the other player(s) winning?

**Sample answer:** Two players each roll a die three times. Player 1 wins if she rolls any number other than 6 at least once. Player 2 wins if he rolls a 6 all three times. Player 2’s chance of winning is very close to impossible (it is 1 out of 216 outcomes).
Goals
Students will find the theoretical probability of simple events, including certain and impossible events.
Students will express probability as a fraction.

PRIOR KNOWLEDGE REQUIRED
Has experience with spinning spinners, rolling dice, and tossing coins
Can read, write, and compare fractions
Can produce fractions equivalent to a given fraction
Can identify equally likely outcomes
Can identify certain and impossible events

MATERIALS
two pencils of different lengths

Mental math minute. Remind students that they can convert fractions to decimals by first finding an equivalent decimal fraction. Demonstrate using $\frac{3}{25} = \frac{12}{100} = 0.12$.

Exercises: Convert the fraction to a decimal.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $\frac{1}{4}$</td>
<td>0.25</td>
</tr>
<tr>
<td>b) $\frac{2}{5}$</td>
<td>0.4</td>
</tr>
<tr>
<td>c) $\frac{7}{25}$</td>
<td>0.28</td>
</tr>
<tr>
<td>d) $\frac{9}{20}$</td>
<td>0.45</td>
</tr>
<tr>
<td>e) $\frac{43}{50}$</td>
<td>0.86</td>
</tr>
<tr>
<td>f) $\frac{13}{25}$</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Answers: a) 0.25, b) 0.4, c) 0.28, d) 0.45, e) 0.86, f) 0.52

Measuring likelihood. Show students two pencils of different lengths. Ask students how they could determine which pencil is longer. (place the pencils beside each other; measure each with a ruler and compare the lengths) Present two measurements that cannot be compared directly, such as the length of a ruler and the circumference of a cup. (Students might suggest using a measuring tape to compare them indirectly.) Ask students how they could compare the weight of two objects, such as a book and a cup, or the temperature in two different places. Point out that in all cases, students tried to attach a number to the characteristic or quantity and to compare the numbers. They suggested using different tools—a measuring tape, a scale, or a thermometer—to get a number, or a measurement. Explain that probability is the branch of mathematics that studies how likely events are and expresses this likelihood in numbers. The measure of the likelihood of an event is called probability.

Finding probability. Draw on the board:
ASK: How many regions does the spinner have? (3) How many ways can you spin red? (only one) How many ways can you spin blue? (two ways)
SAY: Since the regions of the spinner are all the same size, it is equally likely that the spinner will land in any of the three regions. Since two regions are blue and only one is red, it is twice as likely that you will spin blue as it is that you will spin red.

SAY: In mathematics we describe probability as a fraction. Point to the spinner and SAY: For this spinner, there are three possible outcomes. (Even though there are only two colours there are three regions.) So the probability of spinning red is 1/3 because one region out of three regions is red. Write on the board:

\[
\text{Probability of spinning red} = \frac{1}{3}
\]

ASK: What fraction of the spinner is red? (1/3) SAY: The probability of spinning blue is 2/3 because two regions out of three regions are blue. ASK: Which fraction of the spinner is blue? (2/3)

Exercises
1. What is the probability of spinning red?
   a) \[ \frac{G}{R} \]  b) \[ \frac{G}{B} \]  c) \[ \frac{G}{Y} \]  d) \[ \frac{R}{R} \]  e) \[ \frac{R}{G} \]  f) \[ \frac{R}{R} \]  
   Answers: a) 2/3, b) 1/3, c) 0/3 = 0, d) 3/3 = 1, e) 2/4 = 1/2, f) 6/8 = 3/4
2. What is the probability of pulling out a marble of the given colour from the collection without looking?
   \[ R \ R \ R \ R \ G \ G \ G \ B \ B \ B \ Y \] 
   a) yellow  b) blue  c) red  d) green
   Answers: a) 1/10, b) 3/10, c) 4/10 = 2/5, d) 2/10 = 1/5

In probability, all outcomes must be equally likely. Draw on the board:

ASK: Which colour are you more likely to spin: red or white? (white) Why? (the white region is bigger than the red region) How many regions does
this spinner have? (5) How many regions are red? (1) How many are white? (1) What do you get if you write a fraction in which the numerator is the number of white regions and the denominator is the total number of all regions? (1/5) Repeat with the red region. (1/5) ASK: Did you get the same fraction? (yes) SAY: However, we want these fractions to measure the likelihood of spinning red and spinning white. If one of them is more likely to happen than the other, that fraction should be greater. This means we made a mistake. Ask students if they can tell what was wrong.

SAY: For a probability fraction, all events must have outcomes that are equal in size. That is, the regions on the spinner need to be the same size. In later grades, you’ll find a way to find the probability of these types of spinners.

**Finding the probability of events made from different types of outcomes.**

Draw the spinner shown in the margin on the board. SAY: Let’s find the probability of spinning either green or blue. ASK: How many ways can you spin green? (2) How many ways can you spin blue? (1) SAY: The outcomes that make up an event are called the *favourable outcomes* for that event. ASK: How many favourable outcomes are there for spinning blue or green? (3) Write on the board:

\[
\text{Ways to spin blue or green: 3}
\]

ASK: How many outcomes are there in total? (8) Write “Total number of outcomes: 8” on the board. ASK: What is the probability of spinning blue or green? (3/8) Write “Probability of spinning blue or green: 3/8” on the board.

SAY: Now let’s find the probability of spinning a colour that is not red.

ASK: How many outcomes that are not spinning red are there? (5) How do you know? (count the outcomes that are not red; subtract 8 – 3 = 5 because there are 3 ways to spin red) Repeat with the probability of “not spinning a colour that is on the Canadian flag.” (4/8 = 1/2)

**Exercises:** Use the spinner on the board to find the probability.

- a) spinning either yellow or green
- b) spinning a colour on the flag of Canada
- c) not spinning blue
- d) spinning a colour that is neither green nor white

**Answers:** a) 3/8, b) 4/8 = 1/2, c) 7/8, d) 5/8

**Bonus:** Draw 8 marbles so that the probability of pulling a red marble is \(\frac{3}{8}\), pulling a green marble is \(\frac{1}{8}\), and pulling a white marble is \(\frac{1}{4}\).

**Answers:** 3 red marbles, 1 green marble, 2 white marbles, 2 marbles of other colours.
Extensions

1. Sam randomly picks a marble from a bag. The probability of picking a red marble is \( \frac{3}{7} \). What is the probability of not picking red? Explain.

   **Answer:** \( \frac{4}{7} \), because if 3 out of every 7 marbles are red, then 4 out of every 7 marbles are not red.

2. A bag contains 26 cards, one with each letter of the alphabet.
   a) Tess randomly picks a letter of the alphabet from the bag. What is the probability that she picks a letter from A to D?
   
   **Answers:** a) \( \frac{4}{26} \); b) \( \frac{21}{26} \)

3. The probability of picking a white balloon from a bag is \( \frac{1}{2} \). Can you say for sure there are 2 balloons in the bag and one of them is white?

   **Answer:** no, there might be 4 balloons in the bag and 2 of them are white, or 3 white balloons out of 6 balloons, and so on.
Goals
Students will determine expected frequencies for simple events.

Prior Knowledge Required
Understands the meaning of the numerator and the denominator of a fraction
Can represent pictures of fractions using standard fractional notation
Can calculate fractions of whole numbers within 1000

Materials
BLM Rearranging Spinner Regions (p. R-34)
scissors
BLM Expected Outcomes (pp. R-35–37)
transparency of BLM Events with Unequal Chances (pp. R-38–39)
overhead projector

Mental Math Minute. Remind students that they can double twice to multiply by 4 and double three times to multiply by 8. For example, to multiply $4 \times 6$, multiply $2 \times 6 = 12$ and then multiply $2 \times 12 = 24$. Then you can double 24 to get $8 \times 6 = 48$. Also, remind students that order does not matter in multiplication, so they can find the answer to $9 \times 4$ by doubling 9 twice. Ask students multiplication questions in which one of the factors is 4 or 8; for example, $7 \times 4$.

Note: This lesson deals with expectations for simple events, which are events made up of one experiment, such as flipping a coin once or rolling a die once. Students do not need to be familiar with the term “simple events” or the distinction between simple and compound events.

Expectations for Simple Events with Equal Chances. Draw on the board:

<table>
<thead>
<tr>
<th>Flipping a Coin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Total Number of Outcomes</td>
</tr>
<tr>
<td>Number of Heads Outcomes</td>
</tr>
<tr>
<td>Number of Tails Outcomes</td>
</tr>
<tr>
<td>Fraction of Outcomes Expected to Be Heads</td>
</tr>
<tr>
<td>Fraction of Outcomes Expected to Be Tails</td>
</tr>
</tbody>
</table>

Say: Since there is an equal chance of flipping heads or tails on a coin toss, we expect half of all flips to be heads and half to be tails. Have students help fill in the chart. (2, 1, 1, 1/2, 1/2)
ASK: If we flip a coin 10 times, how many heads do we expect? (5) How many tails do we expect? (5) What if we flip 100 times? (50 heads, 50 tails)

Draw on the board:

<table>
<thead>
<tr>
<th>Spinning a Spinner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spinner</td>
</tr>
<tr>
<td>Total Number of Outcomes</td>
</tr>
<tr>
<td>Number of Grey Outcomes</td>
</tr>
<tr>
<td>Number of White Outcomes</td>
</tr>
<tr>
<td>Fraction of Outcomes Expected to Be Grey</td>
</tr>
<tr>
<td>Fraction of Outcomes Expected to Be White</td>
</tr>
</tbody>
</table>

Have students help fill in the second column of the chart as they did for the Flipping a Coin chart. Then ASK: If we spin 40 times, how many times do we expect to spin grey? (20) How many times do we expect to spin white? (20) What if we spin 500 times? (250 grey, 250 white)

At the top of the third column of the chart on the board, draw the same spinner again. Then divide it in half, as shown in the margin. Have students help fill in the third column of the chart, as they did for the first spinner. Then ASK: If we spin 40 times, how many times do we expect to spin grey? (20) How many times do we expect to spin white? (20) What if we spin 500 times? (250 grey, 250 white) Did dividing the sections affect the expected outcomes? (no) SAY: As long as the spinner is still half grey and half white, the actual number of outcomes doesn’t affect what is expected to happen.

At the top of the fourth column of the chart on the board, draw the spinner shown in the margin. Have students help fill in the fourth column of the chart, as they did for the first and second spinners. ASK: Did rearranging the sections affect the expected outcomes? (no) SAY: As long as all outcomes are equally likely and half of the regions are grey and half are white, the actual arrangement of the regions doesn’t affect what is expected to happen.

**ACTIVITY (Optional)**

Give each student one section from BLM Rearranging Spinner Regions. Have students cut out the spinners. ASK: Is there an equal chance of grey or white occurring? (yes) Have students cut along the dotted line on the diameter to make two half-circles, and then fold and cut each half-circle in half to make four equal sections in total.
Students then reassemble the spinner in a different way. ASK: How many outcomes are there now? (4) Do they have an equal chance of occurring? (yes) Students can continue to cut sections in half and rearrange them to convince themselves that having more equal sections will not change the outcomes.

More examples of expectation with equal chances. Draw on the board:

![Spinner diagram]

ASK: Is there an equal chance of spinning red, spinning green, and spinning blue? (yes) Write on the board:

- Number of possible outcomes: 3 (1 green, 1 red, 1 blue)
- Chances of spinning blue: 1 out of 3
- Probability of spinning blue: \( \frac{1}{3} \)

SAY: I am going to spin the spinner 12 times. To find how many times we expect to spin blue, we need to find \( \frac{1}{3} \) of 12. Draw 12 dots on the board and ask a volunteer to regroup the dots into 3 equal groups, as shown below:

![Dots regrouped into 3 equal groups]

ASK: How many dots are in each group? (4) SAY: So \( \frac{1}{3} \) of 12 is 4. Explain to students that they can find \( \frac{1}{3} \) of 12 using division (without using a model). Write on the board:

\[ 12 \div 3 = 4, \text{ so } \frac{1}{3} \text{ of } 12 = 4 \]

SAY: If you spin the spinner 12 times, you expect to spin blue 4 times. Remind students that actual outcomes usually differ from expected outcomes. You might not get blue 4 times every time you make 12 spins, but 4 times is the most likely number of times you will spin blue. Review with students how to find a fraction of a set and a fraction of a number using division.

**Exercises**

1. Use division to find the answer.
   
   a) \( \frac{1}{2} \) of 10  
   b) \( \frac{1}{2} \) of 16  
   c) \( \frac{1}{3} \) of 18  
   d) \( \frac{1}{4} \) of 20

   **Answers:** a) 10 \( \div 2 = 5 \), b) 16 \( \div 2 = 8 \), c) 18 \( \div 3 = 6 \), d) 20 \( \div 4 = 5 \)

2. Complete BLM Expected Outcomes (1).

   **Answers:** 1. a) 2, b) 1, c) 1, d) 1/2, e) 1/2, f) 25, g) 25; 2. a) 8, b) 4, c) 4, d) 1/2, e) 1/2, f) 15, g) 15
**Expectations for simple events with unequal chances.** Project a transparency of BLM Events with Unequal Chances (1) on the board. Have students help you fill in the first three rows of the table. SAY: Since the chance of getting the event “spinning grey” is one in four, we expect 1/4 of all spins to be grey. Since the chance of getting the event “spinning white” is three in four, we expect 3/4 of all spins to be white. Have students help fill in the remainder of the chart. (4, 1, 3, 1/4, 3/4, 1, 3, 2, 6) ASK: If we spin four times, how many spins do we expect to be grey? (1) How many do we expect to be white? (3) If we spin eight times, how many spins do we expect to be grey? (2) How many do we expect to be white? (6) If we spin 40 times, how many spins do we expect to be grey? (10) How many do we expect to be white? (30)

Project a transparency of BLM Events with Unequal Chances (2) on the board. Read aloud each row heading on the BLM and fill in the chart as students answer.

**Exercises:** Complete BLM Expected Outcomes (2) and (3).

**Answers**
BLM Expected Outcomes (2): a) 4, b) 3, c) 1, d) 3/4, e) 1/4, f) 3, g) 1, h) 30, i) 10
BLM Expected Outcomes (3): a) 6; b) 2; c) 1, 2; d) 3, 4, 5, 6; e) 2/6; f) 4/6; g) 2; h) 4; i) 4; j) 8; k) 6; l) 12; m) 8; n) 16

**Extensions**
1. A spinner has 25 regions and 12 of them are red. If you spin the spinner 100 times, do you expect to spin red more than 50 times or less than 50 times? Explain.
   **Answer:** We expect to spin red less than 50 times. Since 12 is less than half of 25, we expect the number of spins resulting in red to be less than half of 100 spins.

2. The spinner in the margin has three outcomes: A, B, and C. A soccer game also has three outcomes: one team wins, the other team wins, or no team wins.
   Given that both have three different outcomes, how are the spinner and the soccer game different?
   **Answer:** You can’t alter the chance of winning with the spinner, but you can with soccer. The likelihood of the three outcomes is equal for the spinner but not equal in soccer. One of the two soccer teams playing might be a more experienced team and therefore have a higher chance of winning than the other team.
Goals

Students will compare predictions for experiments with actual results. Students will see that the more data there is for an experiment, the more it looks like the expected outcomes.

PRIOR KNOWLEDGE REQUIRED

Can determine what the outcomes are for simple experiments
Can determine the number of outcomes for simple experiments
Can plot data on a bar graph

MATERIALS

two prepared spinners per pair of students from BLM Experiment Spinners (p. R-40, see details below)
overhead projector
transparency of BLM Experiment Spinners (p. R-40)
BLM Tally Charts for Spinner Experiments (p. R-41)
BLM Bar Graph Templates for Spinner Experiments (p. R-42)
transparency of BLM Bar Graph Templates for Spinner Experiments (p. R-42, optional)
coin
BLM Expected Outcomes for Rolling a Die Twice (p. R-43)
dice (see Extension 3)
computer (see Extension 4)

Mental math minute. Ask students to solve multiplication questions within the range of 1 × 1 to 10 × 10 and corresponding division questions. For each number, go through the questions in order, such as 1 × 3, 3 ÷ 3, 2 × 3, 6 ÷ 3, and so on, to 10 × 3 and 30 ÷ 3. Then progress to a different number. Next, try questions out of order, but keep multiplication and the corresponding division together.

Expected outcomes versus results. In advance, prepare two different spinners for each pair of students, one with the letters A/B/C/D and the other without letters using BLM Experiment Spinners.

Testing predictions for the A/B/C/D spinner. SAY: Now that we know how to determine the outcomes of an experiment and how many of each outcome to expect, we can do some experiments and compare the results with the expected outcomes. Project the A/B/C/D spinner from BLM Experiments Spinners onto the board. Then draw the table on the following page on the board and have students help to fill it in. (4, 1, 1, 1, 1)
Total Number of Outcomes

<table>
<thead>
<tr>
<th>Number of A Outcomes</th>
<th>Number of B Outcomes</th>
<th>Number of C Outcomes</th>
<th>Number of D Outcomes</th>
</tr>
</thead>
</table>

Give every pair of students the A/B/C/D spinner from BLM Experiment Spinners and the corresponding tally chart from BLM Tally Charts for Spinner Experiments. SAY: You are going to spin the spinner 20 times, but before you begin, you need to make predictions about what you expect will happen. Then you will test your predictions by spinning the spinner. For each spin, you will record on the tally chart whether the spinner stopped in region A, B, C, or D. If the spinner stops exactly on the line between two regions, then don’t count that result, and redo that spin. Ask students where to put tally marks for a couple of examples to make sure they know how to mark their results. When all spins have been completed, have students fill in the count column of the chart.

Give every pair of students two copies of the bar graph template for the A/B/C/D spinner experiment from BLM Bar Graph Templates for Spinner Experiments. Have each student plot the data from their tally charts. After all individual data is plotted, ASK: Did everybody get five of each of the letters A, B, C and D? (no) Did anyone get five of each outcome?

Draw or project the bar graph template for the A/B/C/D spinner experiment on the board. You will need to renumber the scale on the vertical axis to accommodate all the data. For example, if 15 pairs did the experiment, the vertical scale will go up to 20 times 15 (20 spins per pair times 15 pairs, i.e., 300 spins). Combine the results for the whole class by adding all the bars from A together, all the bars from B together, all the bars from C together, and all the bars from D together. Plot the data on the board. Compare the class results with those of the pairs. Discuss the difference between the class’ results and the pairs’ results. ASK: Which results are closer to the expected outcomes? (class result) Why is that? (the more times you spin, the more the data will look like what you expect) SAY: The fraction of each letter spun by the class will be closer to a quarter of the total number of spins than will be the fraction for the pairs’ results.

Testing predictions for the grey/white spinner. Repeat the above experiment using the grey/white spinner, the grey/white tally chart, and the grey/white bar graph template. SAY: Now we’re going to do an experiment to see how many times the spinner lands on the grey part and how many times it lands on the white part. ASK: If we spin four times, how many times do we expect to land on the grey part? (1) How many times do we expect to land on a white part? (3) ASK: If we spin 20 times, how many times do you predict it will land on the grey part? (5) How many times do you predict it will land on a white part? (15) Why? (in 4 spins, we expect white 3 times; in 8 spins, 6 times; in 12 spins, 9 times; in 15 spins, 12 times; and in 20 spins, 15 times)
Again, combine the pairs’ results to make a whole class bar graph on the board. Ask which results are closer to the expected outcome—the pairs’ or the class’—and discuss what would happen if they collected a lot more data. (the experimental results would look more like the predictions)

### ACTIVITIES 1–2 (Optional)

1. Have students predict the results of tossing a coin 20 times. Then have each student toss a coin 20 times and record the results in a tally chart. Students should then compare their results to their prediction.

2. Have students do BLM Expected Outcomes for Rolling a Die Twice.

### Extensions

1. Design an experiment with three possible outcomes in which one of the outcomes has a probability near \(
\frac{1}{2}
\).

   **Sample answer:** a spinner with 3 regions, one of them being almost half the circle

2. Choose a novel. Open it to any page and note whether or not the first letter is a “t.” Check 20 pages in this manner. Describe the probability that “t” is the first letter on a page of the book, based on your experiment.

3. Have students do 36 repetitions of rolling a die twice and adding the results. Students record the results of the experiment in a tally chart like the one shown below.

   ![Tally Chart](image)

4. Simulate experiments on a computer (e.g., search for coin toss, rolling die, or spinning spinner simulators online).
Goals
Students will determine the probability of winning in games, decide if a game is fair, and compare the theoretical probability of winning with the experimental probability.

PRIOR KNOWLEDGE REQUIRED
- Has experience with spinning spinners, rolling dice, and tossing coins
- Can identify equally likely outcomes
- Can find the theoretical and experimental probability of a given simple event
- Can determine the expected number of outcomes based on probability
- Can find a fraction of a number
- Can compare two fractions
- Can plot data on a bar graph

MATERIALS
- 3 green, 3 red, and 2 blue marbles per pair of students
- die and shoebox per pair of students

Mental math minute. Present this problem: 8 × 13. (104) The following strategies could arise:

\((8 \times 10) + (8 \times 3)\)
\((10 \times 13) - (2 \times 13)\)
\(2 \times (4 \times 13)\)
\((5 \times 13) + (3 \times 13)\)

Introduce fair games. SAY: I would like to play a game with you. The rules of the game are simple. I spin a spinner. If I spin white, I win; if I spin grey, the class wins. Ask students if they agree to play by these rules. Draw the spinner shown in the margin on the board. ASK: Do you still want to play? (no) Why not? (because you are more likely to spin white than grey) How do you know? (the white part is more than half the spinner, and the part that is coloured grey is less than half the spinner; there is more white than grey) Write “fair game” on the board. SAY: In mathematics, a fair game means that all players have an equal chance of winning, or are equally likely to win.

Present the following game of chance: There are 8 marbles in a box: 3 green, 3 red, and 2 blue. If a red marble is taken out, Player 1 wins, and if a green marble is taken out, Player 2 wins. Taking out a blue marble results in a draw. Ask students if the game is fair and why. (yes, both players have the same probability of winning: 3/8) Tell students that Emma and Glen played this game 24 times, replacing the marble after each game. ASK: How many times do you expect Glen to win? (9 times) How do you know? (in 8 times, we expect 3 green; in 16 times, 6 green; and in 24 times, 9 green) Glen and
Emma tallied the results—which one of the following tables is most likely to be theirs?

<table>
<thead>
<tr>
<th>Glen</th>
<th>Emma</th>
<th>Draw</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Glen</th>
<th>Emma</th>
<th>Draw</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Glen</th>
<th>Emma</th>
<th>Draw</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Let students explain their choice. Then give students marbles and let them play the same game 24 times in pairs. Have them tally their results in tables and compare them to Glen’s and Emma’s. **ASK:** Which tables were nearest to the expected results? Which tables were furthest from the expected results? Add the results for each player from each table. **ASK:** Is the group experimental result for each player nearer to 3/8? Discuss with students the differences between the expected and experimental results and between the individual and group results.

**Exercises:** What is the probability for each player to win when spinning the spinner shown? Is the game fair? If not, which player has a better chance of winning?

![Spinner]

a) Player 1 spins red to win. Player 2 spins blue to win.

b) Player 1 spins white or yellow to win. Player 2 spins blue to win.

c) Player 1 spins white or red to score a point. Player 2 spins blue to score a point.

d) Player 1 spins a colour that is not red to score a point. Player 2 spins a colour that is not white to score a point.

e) Player 1 spins blue to win. Player 2 spins red to win. Player 3 spins white or yellow to win.

f) Player 1 spins blue or white to score a point. Player 2 spins blue or yellow to score a point. Player 3 spins red or white to score a point.

g) Player 1 spins blue or white to score a point. Player 2 spins blue or yellow to score a point. Player 3 spins red or blue to score a point.

**Answers**

a) $\frac{2}{6} = \frac{1}{3}$ for both players, the game is fair

b) $\frac{2}{6} = \frac{1}{3}$ for both players, the game is fair

c) $\frac{1}{2}$ for Player 1, $\frac{1}{3}$ for Player 2, the game is not fair, Player 1 has a better chance of scoring a point

d) $\frac{2}{3}$ for Player 1, $\frac{5}{6}$ for Player 2, the game is not fair, Player 2 has a better chance of scoring a point

e) $\frac{1}{3}$ for all players, the game is fair

f) $\frac{1}{2}$ for all players, the game is fair

g) $\frac{1}{2}$ for Players 1 and 2, $\frac{2}{3}$ for Player 3, the game is not fair, Player 3 has a better chance to score a point
Comparing expected probability in games with experimental results.

SAY: You are now going to play another game. Here are the rules: You play in pairs. One player rolls a die. The other player adds a mark to the tally chart of the results. Players switch roles. If the player rolls 1 or 6, Player 1 scores a point. If the player rolls 3 or 4, Player 2 scores a point. If the player rolls 2 or 5, nobody scores. Write the scoring guide on the board. Emphasize that the game is based entirely on chance and says absolutely nothing about the winner or the loser.

ASK: What are the chances of Player 1 scoring a point? (2/6 = 1/3) What are the chances of Player 2 scoring a point? (2/6 = 1/3) Is the game fair? (yes)

SAY: You will roll the die 30 times. ASK: How many times do you expect Player 1 to score a point? (1/3 of the times, so 10 times) How many points do you expect Player 2 to score? (10 points)

ACTIVITY (Essential)

Give each pair of students a die and a shoebox to prevent the die from rolling away. Have them roll the die 30 times and tally the results using the table shown below. Students can also present the results using a double bar graph.

<table>
<thead>
<tr>
<th></th>
<th>30 rolls</th>
<th>60 rolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1 scores</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 2 scores</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nobody scores</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discuss the results of the activity. ASK: Did some pairs have a tie? Did Player 1 win in some pairs? Did Player 2 win in other pairs? Did both players score 10 points? Record multiple results on the board and discuss how the actual results are different from the expectation. Then have students combine the results of Player 1 and Player 2 with another pair, to get 60 rolls in total.

ASK: How many points do you expect each player to get? (20) How many points did each player score in total? Have students calculate the results for each player to score, in 30 rolls and in 60 rolls. ASK: Did the experiment results get closer to the expected results of scoring in 60 rolls? (yes)

NOTE: Students will need a sharpened pencil and a paper clip to complete Question 4 on AP Book 5.2 p. 161.
Extensions

1. Two juice companies offer prizes.
   Company A: 1 out of every 4 boxes wins $3.
   Company B: 1 out of every 5 boxes wins $4.
   a) If you buy a pack of 20 boxes of each type of juice, how many boxes do you expect will have a prize?
   b) If you buy a pack of 20 boxes of each type of juice, how much money do you expect to win from each company?
   c) Which company would you buy the 20 boxes from if you want better prizes?
   **Answers:** a) 5 from Company A, 4 from Company B; b) $15 from Company A, $16 from Company B; c) Company B

2. Ed and Braden are playing "Rock, Paper, Scissors."
   a) What is the probability that Ed wins? Explain. Hint: “Ed wins” is not an outcome; it is an event. “Ed has rock, Braden has rock” is an outcome.
   b) Ed and Braden can play 15 games in 1 minute. If they play the game for 5 minutes, how many times do you expect Ed to win?
   **Answers**
   a) There are 9 outcomes to the game. Ed wins if: Ed has rock, Braden has scissors; Ed has paper, Braden has rock; Ed has scissors, Braden has paper. The probability that Ed wins is 1/3.
   b) 25 times

3. Compare the following two games:
   Game 1: Players roll a die 30 times. Player 1 gets a point if a 1 or a 6 is rolled. Player 2 gets a point otherwise. The player with more points wins.
   Game 2: Players take turns rolling the die. The player rolling a die gets a point if a 1 or a 6 is rolled. The partner gets a point otherwise. The die is rolled 30 times. The player with more points wins.
   a) How many points do you expect each player to get in each game?
   b) Is each of these games fair? Why or why not?
   **Answers:** a) Game 1: Player 1 gets 10 points and Player 2 gets 20 points, Game 2: both players expect 15 points; b) Game 1 is not fair because the probability of Player 1 winning is 1/3 and for Player 2 it is 2/3, Game 2 is fair because both players have the same probability of winning
Rearranging Spinner Regions
Expected Outcomes (1)

1. Kim has a coin that she plans to flip.
   a) How many outcomes are there in total? _______
   b) How many outcomes are heads? _______
   c) How many outcomes are tails? _______
   d) What fraction of outcomes are expected to be heads? _______
   e) What fraction of outcomes are expected to be tails? _______
   f) Out of 50 flips, how many are expected to be heads? _______
   g) Out of 50 flips, how many are expected to be tails? _______

2. A spinner is divided into eight equal parts: four black and four white.
   a) How many outcomes are there in total? _______
   b) How many outcomes are black? _______
   c) How many outcomes are white? _______
   d) What fraction of outcomes are expected to be black? _______
   e) What fraction of outcomes are expected to be white? _______
   f) Out of 30 spins, how many are expected to be black? _______
   g) Out of 30 spins, how many are expected to be white? _______
Expected Outcomes (2)

3. A spinner is divided into four equal parts. Three of the parts are blue and one of the parts is orange.
   a) How many outcomes are there in total? ______
   b) How many outcomes are blue? ______
   c) How many outcomes are orange? ______
   d) What fraction of outcomes are expected to be blue? ______
   e) What fraction of outcomes are expected to be orange? ______
   f) Out of 4 spins, how many outcomes are expected to be blue? ______
   g) Out of 4 spins, how many outcomes are expected to be orange? ______
   h) Out of 40 spins, how many outcomes are expected to be blue? ______
   i) Out of 40 spins, how many outcomes are expected to be orange? ______
Expected Outcomes (3)

4. A die has six faces. Two of the faces are numbers less than 3 and four of the faces are numbers greater than 2.
   
a) How many outcomes are there in total? _______
   b) How many outcomes are there for the event “less than 3”? _______
   c) What outcomes are there for the event “less than 3”? _____________
   d) What outcomes are there for the event “greater than 2”? _____________
   e) What is the fraction of outcomes for the event “less than 3”? \(\boxed{\text{ _______ }}\)
   f) What is the fraction of outcomes for the event “greater than 2”? \(\boxed{\text{ _______ }}\)
   g) Out of 6 rolls, how many outcomes are expected for the event “less than 3”? ______
   h) Out of 6 rolls, how many outcomes are expected for the event “greater than 2”? ______
   i) Out of 12 rolls, how many outcomes are expected for the event “less than 3”? ______
   j) Out of 12 rolls, how many outcomes are expected for the event “greater than 2”? ______
   k) Out of 18 rolls, how many outcomes are expected for the event “less than 3”? ______
   l) Out of 18 rolls, how many outcomes are expected for the event “greater than 2”? ______
   m) Out of 24 rolls, how many outcomes are expected for the event “less than 3”? ______
   n) Out of 24 rolls, how many outcomes are expected for the event “greater than 2”? ______
## Events with Unequal Chances (1)

<table>
<thead>
<tr>
<th>Total Number of Outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Grey Outcomes</td>
<td></td>
</tr>
<tr>
<td>Number of White Outcomes</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fraction of Outcomes Expected to Be Grey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of Outcomes Expected to Be White</td>
</tr>
</tbody>
</table>

Out of 4 Spins, the Number Expected to Be Grey

Out of 8 Spins, the Number Expected to Be Grey

Out of 4 Spins, the Number Expected to Be White

Out of 8 Spins, the Number Expected to Be White
# Events with Unequal Chances (2)

![Dice](image)

<table>
<thead>
<tr>
<th>Total Number of Outcomes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Number of Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Fraction of Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Fraction of Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 6 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Out of 6 Rolls, the Number of Expected Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 12 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
</tr>
<tr>
<td>Out of 12 Rolls, the Number of Expected Outcomes for the Event “Greater Than 2”</td>
<td></td>
</tr>
<tr>
<td>Out of 18 Rolls, the Number of Expected Outcomes for the Event “Less Than 3”</td>
<td></td>
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<tr>
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Experiment Spinners

- Spinners labeled A, B, C, D
- One spinner shaded half to illustrate probability

R-40
Blackline Master — Probability and Data Management — Teacher Resource for Grade 5
# Tally Charts for Spinner Experiments

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Bar Graph Templates for Spinner Experiments

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Outcomes

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### Expected Outcomes for Rolling a Die Twice

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1 cm Grid Paper
Pentominoes
Patterns and Algebra: Variables, Expressions, and Equations – AP Book 5.2: Unit 8

**AP Book PA5-8**

**page 1**

1. a) 8  
b) 10  
c) 24  
d) 12  
e) 16  
f) 6  
g) 9  
h) 3  
i) 8  

2. b) 5  
c) 6  
d) 1  
e) 15  
f) 6  

3. a) Circle parts a) and i) in Question 1.  
b) 2 + 5 + 1  
   = 10 − (4 + 2)  

4. a)  
   b)  
   c)  
   d)  
   e)  
   f)  
   g)  
   h)  
   i)  

5. a)  
   b)  
   c)  

6. b) 5  
c) 4  
d) 3  
e) 2  
f) 11  
g) 4  
h) 3  
i) 1  

**AP Book PA5-9**

**page 2**

1. b)  
   c)  
   d)  

2. a)  
   b)  
   c)  
   d)  
   e)  
   f)  
   g)  
   h)  
   i)  

3. a)  
   b) 2  
   c) 7 = 4 + 3  
   d) 3 + 1 = 4  
   e) 4 + 4 = 8  

4. a)  
   b)  
   c)  
   d)  
   e)  
   f)  
   g)  
   h)  

5. a)  
   b)  
   c)  

6. b)  
   c)  
   d)  
   e)  
   f)  
   g)  
   h)  
   i)  

7. a) 5, 5  
   b) Teacher to check picture.  
   4 + 4 + 4 = 12  

8. b)  
   c)  
   d)  
   e)  
   f)  
   g)  
   h)  

9. b)  
   c)  
   d)  
   e)  
   f)  
   g)  
   h)  

10. Teacher to check pictures.  
   a) 12  
   b) 6  

11. a) 5  
    b) 9  
    c) 12  
    d) 6  

**AP Book PA5-10**

**page 5**

1. a) 6 divided by 3  
   = 6 ÷ 3  
   2 less than 6  
   = 6 − 2  
   product of 6 and 4  
   = 4 × 6  
   6 decreased by 3  
   = 6 − 3  
   b) 2 divided into 11  
   = 11 ÷ 2  
   11 reduced by 4  
   = 11 − 4  
   11 times 3  
   = 11 × 3  
   twice as many as 11  
   = 2 × 11  
   11 increased by 3  
   = 11 + 3  

2. b) 15 − 8  
   c) 24 + 8  
   e) 67 + 29  
   f) 4 + 35  
   g) 5 × 2  
   h) 15 + 5  
   i) 7 × 4  
   j) 5 × 8  

BONUS  
60  

3. a) 12  
    b) 6 + 2  
    c) 34 + 9  
    d) 7 − 5  
    e) 42 × 2  
    f) 3 − 2  
    g) (8 + 4) + 3  
    h) (8 + 4) + 5  
    i) (4 + 2) + 10 − 4  

4. Answers may vary.  
   Sample answers:  
   b) Add 6 and 1. Then multiply by 2.  
   c) Subtract 5 from 12. Then multiply by 2.  
   d) Subtract 2 from 3. Then multiply by 4.  

BONUS  
Subtract 1 from 3. Then add 5. Then multiply by 4.

5. b) 80 × 4  
    c) 70 × 5  

6. a) ii) 5 × 2  
    iii) 5 × 4  
    b) ii) 2  
    iii) 5  

7. a) ii) 3 × 2  
    iii) 3 × 4  
    b) ii) 2  
    iii) 5  

8. a) 11 + 2  
    b) (11 + 2) × 3  

BONUS  
Answers will vary.  
Sample answer:  
19 people buy lunch. A sandwich costs $11 and a drink costs $2. How much would 19 sandwiches and 19 drinks cost?  

9. 5 × (2 + 2)  

BONUS  
(20 × 5) + 13 + 16  
((20 × 5) + 13 + 16) + 30  
= (100 + 13 + 16) + 30  
= 129 + 30  
= 4 R 9  
5 buses will be needed.

**AP Book PA5-11**

**page 7**

1. b) 3 × 5  
   c) 3 × 6  
   d) 3 × 8
<table>
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<th>Answers</th>
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<td>5 × t, 5t</td>
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<td>4. c)</td>
<td>A + 2 = B</td>
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<td>5. b)</td>
<td>3 × 2, 6</td>
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### Patterns and Algebra: Variables, Expressions, and Equations – AP Book 5.2: Unit 8

#### (continued)

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<td>4. b)</td>
<td>12, 9, x − 12</td>
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<td>6. a)</td>
<td>7 − x = 5</td>
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<td>8. b)</td>
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#### BONUS

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<td>2(2) + 3 = 20</td>
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<td>3(2) − 2 = 6</td>
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### AP Book PA5-12

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### AP Book PA5-13

#### page 12

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<td>4. a)</td>
<td>8 + 10 = x</td>
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<td>5. x = 9</td>
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<td>6. a)</td>
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#### BONUS

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### AP Book PA5-13

#### page 12

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<tr>
<td>3. b)</td>
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<td>4. b)</td>
<td>12, 9, x − 12</td>
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<td>7 − x = 5</td>
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<td>7 − x = 5</td>
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<td>Circle 13 oranges</td>
</tr>
<tr>
<td>6. a)</td>
<td>7 − x = 5</td>
</tr>
<tr>
<td>7. b)</td>
<td>red balloons, green balloons</td>
</tr>
</tbody>
</table>

#### BONUS

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>x + 12 = 21</td>
<td>x = 9</td>
</tr>
<tr>
<td>x − 11 = 8</td>
<td>x = 19</td>
</tr>
<tr>
<td>2(2) + 3 = 20</td>
<td>2(6) + 3 = 15</td>
</tr>
<tr>
<td>3(2) − 2 = 6</td>
<td>3(4) − 2 = 10</td>
</tr>
</tbody>
</table>
7. 60 − x = 20
   60 − 20 = x
   40 = x
   Nina spent 40 minutes on homework.

8. 23 − x = 8
   23 − 8 = x
   15 = x
   The movie pass costs $15.

9. 553 − 442 = x
   111 = x
   The CN Tower is 111 m taller than the Willis Tower.

1.  b) Blue
   c) Red
   d) Yu

2.  b) Rats
    c) Chocolate chip
    d) Math

3.  Teacher to check models.
    b) Seniors: 30
    c) Shoes: 24
    d) Math: 30

4.  b) 16
    c) 9
    d) 8

5.  Teacher to check models.
    b) Vicky is 20 years old and Ella is 5 years old.
    c) There are 36 party balloons and 6 streamers.

BONUS
   Anna needs 48 tablespoons of butter and 144 tablespoons of sugar.

6.  Shoes  
    Wallet  
    The pair of shoes costs $34 and the wallet costs $17.

BONUS
   (2 × 34) + (3 × 17)  
   = 68 + 51  
   = 119  
   Glen would pay $119 for two pairs of shoes and three wallets.

6. a) 60 − x = 20
    60 − 20 = x
    40 = x
    Nina spent 40 minutes on homework.

8. a) 23 − x = 8
    23 − 8 = x
    15 = x
    The movie pass costs $15.

9. a) 553 − 442 = x
    111 = x
    The CN Tower is 111 m taller than the Willis Tower.

AP Book PA5-14  
page 14  
1.  b) Blue
    Red
    c) Red
    Green
    d) Yu
    Nora

2.  b) Rats
    c) Chocolate chip
    Oatmeal

3.  Teacher to check models.
    b) Seniors: 30
    c) Shoes: 24
    d) Math: 30

4.  b) 16
    c) 9
    d) 8

5.  Teacher to check models.
    b) Blue
    Red
    c) Red
    Green
    d) Yu
    Nora

2.  b) Rats
    Hamsters

7 + 14 = 21,  
so Randi has 21 animals altogether.

c) Chocolate chip
    Oatmeal

12 + 72 = 84, so there are 84 cookies in the box altogether.

d) Math
    Science

17 + 68 = 85, so there are 85 books in the library altogether.

3.  Teacher to check models.
    b) Seniors: 30
    c) Shoes: 24
    d) Math: 30

4.  b) 16
    c) 9
    d) 8

5.  Teacher to check models.
    b) Blue
    Red
    c) Red
    Green
    d) Yu
    Nora

2.  b) Circle mangoes, underline kiwis.
    mangoes, 16
    kiwis, n
    n + 2 = n

c) Circle dogs, underline cats.
    dogs, n
    cats, 6
    n = 6 × 2

3.  b) 16 + 7 = p
    d) 2 × 11 = p

4.  b) x, 5, 8, 5 × 8 = x
    40 balloons
    c) 35, 7, x, 35 + 7 = x
    5 students on each team

5.  a) 6 × 2 = x
    12 = x
    The store sold 12 hamsters.

b) 6 + 12 = x
    18 = x
    18 rats and hamsters were sold altogether.

c) 12 − 6 = x
    6 = x
    6 more hamsters than rats were sold.

6.  a) 35 + 5 = x
    7 = x
    Eddy is 7 years old.

b) 35 − 7 = x
    28 = x
    Emma is 28 years older than Eddy.

7.  a) 100 + 5 = x
    20 = x
    The male angler fish is 20 cm long.

b) 100 − 20 = x
    80 = x
    The female angler fish is 80 cm longer than the male.

8. a) Yellow = 3
    Blue = 3 + 12 = 15
    Red = 15 + 3 = 5

b) 3 + 15 + 5 = 23

8. a) 18 + 20 = 38
    The geese travelled for 38 hours.

b) 3200 − 3040 = 160
    They need to fly another 160 km.

38 + 2 = 40
    The geese need 40 hours to fly from BC to Texas.

9.  Adult narwhal: 5 + 3 = 8 m
    Baby narwhal: 8 + 4 = 2 m
    A baby narwhal is 2 m long.

10. a) The pencil is three times as long as the eraser.
    b) The pencil is 10 cm longer than the eraser.

11. Yes.
    4000 ÷ 4 = 1000, so the scale factor is 1000.

12. 8 × 2 = 16 slices
    16 + 5 = 3 R 1
    Each person can have 3 slices with one slice left over.

13. 52 − 13 = 39 avocados
    39 + 5 = 7 R 4
    Zack can make 7 full bags.

14. 24 + 23 = 47 students
    47 + 4 = 11 R 3
    12 cars are needed to transport all the students.
Number Sense: Fractions – AP Book 5.2: Unit 9

AP Book NS5-34

Page 20

1. b) \(\frac{3}{4}\)
   c) \(\frac{1}{4}\)
   d) \(\frac{4}{6}\)
   e) \(\frac{5}{9}\)
   f) \(\frac{9}{12}\)
   g) \(\frac{1}{8}\)
   h) \(\frac{6}{10}\)

2. Answers will vary. Teacher to check shading.

3. b) sixths
   c) fifths
   d) thirds
   e) ninths
   f) halves

4. a) [Diagram of three equal parts]
   b) [Diagram of three equal parts with one shaded]
   c) [Diagram of three equal parts with one shaded]

5. Teacher to check.

6. Teacher to check.

7. Teacher to check.

8. a) \(\frac{1}{3}\)
   b) \(\frac{2}{5}\)

9. Teacher to check.

10. a) The number of parts in the whole pie.
    b) The number of parts of the pie I have.
    c) The parts are not all the same size, so the picture does not show \(\frac{1}{4}\).

AP Book NS5-35

Page 22

1. a) triangles
   b) squares
   c) circles
   d) shaded

2. a) \(\frac{2}{5}\)
   b) \(\frac{1}{5}\)
   c) \(\frac{3}{5}\)

3. \(\frac{3}{5}\) of the shapes are shaded.
   \(\frac{3}{5}\) of the shapes are triangles.

4. a) 9
   b) 6
   c) \(\frac{3}{9}\)
   d) yes

5. a) \(\frac{2}{5}\)
   b) \(\frac{1}{3}\)

6. a) \(\frac{4}{8}\)
   b) \(\frac{4}{8}\)

7. \(\frac{6}{7}\)

8. a) \(\frac{3}{8}\)
   b) \(\frac{2}{8}\)
   c) \(\frac{4}{8}\)
   d) \(\frac{3}{8}\)

9. Sample answer:
   \(\frac{3}{8}\) squares
   \(\frac{1}{8}\) shaded

10. a) Sample picture:
    b) 2 squares are shaded.

BONUS
   Even though the parts are different shapes, they are each half of a half of the whole shape, so they are the same size. One part is shaded, so the picture shows \(\frac{1}{4}\).

AP Book NS5-36

Page 24

1. Circle the following:
   b) \(\frac{5}{8}\)
   c) \(\frac{4}{5}\)
   d) shaded
   e) \(\frac{3}{4}\)
   f) \(\frac{16}{20}\)

2. a) \(\frac{3}{8}\)
   b) \(\frac{4}{5}\)
   c) \(\frac{2}{3}\)
   d) circle \(\frac{3}{4}\)
   e) circle \(\frac{2}{3}\)
   f) circle \(\frac{1}{2}\)

BONUS
   Teacher to check shading.
   \(\frac{9}{12} > \frac{2}{3} > \frac{14}{24}\)
   Simon ate the largest amount.

AP Book NS5-37

Page 26

1. Circle the following:
   a) \(\frac{2}{5}\)
   b) \(\frac{1}{8}\)
   c) \(\frac{2}{3}\)
   d) \(\frac{3}{8}\)
   e) \(\frac{7}{12}\)
   f) \(\frac{7}{8}\)

2. Teacher to check shading.
   Circle the following:
   a) \(\frac{2}{3}\)
   b) \(\frac{4}{5}\)
   c) \(\frac{2}{5}\)
   d) \(\frac{4}{5}\)
   e) \(\frac{2}{5}\)
   f) \(\frac{2}{5}\)
3. Teacher to check number line.

\[
\begin{array}{ccccccc}
1 & 3 & 4 & 6 & 7 & 8 & 9
\
10 & 10 & 10 & 10 & 10 & 10 & 10
\end{array}
\]

4. \(\frac{3}{4}\)

5. a) i) < ii) > iii) >

b) Teacher to check circles.

\[
\begin{array}{cccc}
\frac{5}{6} & \frac{1}{2} & \frac{1}{3}
\end{array}
\]

c) The fraction with the largest denominator has been divided into the greatest number of parts, so each part is smaller. The fraction with the smallest denominator has been divided into the fewest number of parts, so each part is larger.

6. Teacher to check number lines.

b) 4
c) 2
d) 6

AP Book NS5-38

page 28

1. a) 1, 2, 3

b) i) increases ii) stays the same iii) increases

2. Circle the following:

a) \(\frac{4}{5}\)
b) \(\frac{3}{4}\)
c) \(\frac{9}{12}\)
d) \(\frac{3}{3}\)

3. Sample answers:

b) 15
c) 50

BONUS

1

4. The fraction with the greater numerator is greater.

5. a) \[
\frac{0}{5} \cdot \frac{1}{5} \cdot \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{5}{5}
\]
b) \[
\frac{1}{10} \cdot \frac{2}{10} \cdot \frac{4}{10} \cdot \frac{6}{10}
\]
c) Yes. As the denominator increases, the size of each part in each strip grows, so the order is the same in a) and b).

10. No, \(\frac{1}{4}\) of a pie is greater than \(\frac{1}{6}\) because the denominator is smaller.

11. a) Ray thinks it is unfair because none of the pies are the same size, so none of the ninths will be the same size.
b) Lynn thinks it is fair because everyone gets one ninth of a pie.

AP Book NS5-39

page 31

1. a) 3

b) 4
c) 2
d) 2

2. a) \(\frac{2}{3}\)
b) \(\frac{4}{4}\)
c) \(\frac{8}{8}\)
d) \(\frac{3}{3}\)

3. a) \(\frac{2}{10}\)
b) \(\frac{3}{15}\)
c) \(\frac{2}{25}\)
d) \(\frac{4}{10}\)

4. b) \(\frac{4}{16}\)

c) \(\frac{12}{20}\)

d) \(\frac{5}{10}\)

e) \(\frac{10}{25}\)

BONUS

\[
\begin{array}{c}
70 \\
100 \\
10 \\
\end{array}
\]

5. Teacher to check lines.

a) 2, 3, 4

b) 2, 3, 4

6. Teacher to check lines.

a) 4

b) multiply by 2
c) multiply by 3
d) multiply by 2

e) multiply by 3

f) multiply by 3

g) multiply by 2

h) multiply by 10

i) multiply by 8

8. Sample answers:

\[
\begin{array}{cccc}
4 & 6 & 8 & 10 \\
10 & 15 & 20 & 25 \\
30 & & & 
\end{array}
\]

AP Book NS5-40

page 34

1. Teacher to check lines.

a) 4, 6, 8, 10

b) 6, 9, 12, 15

2. a) \(\frac{10}{15}\)
b) \(\frac{9}{15}\)

c) \(\frac{2}{3}\)

I rewrote \(\frac{2}{3}\) and \(\frac{3}{5}\) as fractions with the same denominator, \(\frac{10}{15}\) and \(\frac{9}{15}\). The order is the same in a), b), and c).
Number Sense: Fractions – AP Book 5.2: Unit 9

(continued)

3. a) 5, 6
circle \( \frac{2}{5} \)
b) 9, 8
circle \( \frac{3}{8} \)
4. a) i) 16
ii) 20
iii) 18
iv) 12
b) \( \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6} \)
5. Teacher to check lines.
a) 2
circle \( \frac{1}{2} \)
b) 4
circle \( \frac{5}{6} \)
6. a) <
b) multiply by 2
6, >
c) multiply by 2
\( \frac{2}{4}, < \)
d) multiply by 3
\( \frac{3}{9}, > \)
e) multiply by 2
\( \frac{6}{10}, < \)
BONUS
multiply by 8
\( \frac{16}{40}, < \)
7. a) 21, 20
> 
b) multiply by 3
\( \frac{3}{6} \)
multiply by 2
\( \frac{4}{6} \)
< 
8. Teacher to check picture.

AP Book NS5-41
page 36
1. b) 1
c) 3
2. b) \( \frac{3}{4} \)
c) \( \frac{1}{2} \)
d) \( \frac{1}{2} \)
e) \( \frac{3}{8} \)
3. Teacher to check.
4. a) 2
b) 1
c) 4, because the denominator is 4.
d) 1
5. a)

b) 

c) 

d) 

6. b) \( \frac{7}{4} \)
c) \( \frac{5}{3} \)
d) \( \frac{19}{8} \)

AP Book NS5-42
page 38
1. a) 2
b) 4
c) 6
d) 4
e) 8
f) 12
h) 6
i) 9
2. c) 6, 1, \( \frac{7}{2} \)
d) 8, 1, \( \frac{9}{2} \)
e) 4, 1, \( \frac{5}{4} \)
f) 4, 2, \( \frac{6}{4} \)
g) 4, 3, \( \frac{7}{4} \)
h) 8, 1, \( \frac{9}{4} \)
i) 3, 1, \( \frac{4}{3} \)
j) 3, 2, \( \frac{5}{3} \)
3. Ella should use scoop A because she will need to measure in thirds.
4. c) \( 2, \frac{3}{4} \)
d) 3, 0, 3
c) 1, 3
 \( \frac{1}{4} \)
d) 8 ÷ 4 = 2 R 0
So \( \frac{8}{4} = 2 \)
e) 10 ÷ 3 = 3 R 1
So \( \frac{10}{3} - \frac{3}{1} \)
f) 11 ÷ 3 = 3 R 2
So \( \frac{11}{3} - \frac{2}{3} \)
6. Circle the following:
a) \( \frac{8}{5} \)
b) \( \frac{18}{7} \)
c) \( \frac{30}{8} \)
BONUS
\( \frac{1582}{36} \)
7. a) \( \frac{8}{3}, \frac{12}{3}, \frac{22}{3}, \frac{34}{3} \)
b) \( \frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5} \)
Number Sense: Fractions – AP Book 5.2: Unit 9

(continued)

3. Sample answer:
   a) 1/7 of the shapes are shaded circles.
   b) 1/7 of the shapes are squares.
   c) 3/7 of the shapes are shaded.
   d) 3/7 of the shapes are triangles.
   e) 4/7 of the shapes are not circles.
   f) 4/7 of the shapes are not shaded.

4. 6 1/2, 3 1/4, 2 1/4

5. Raj's backpack weighs 12/24 kg, so Raj's backpack weighs less.

6. The salmon is longer because it is 3 21/35 m long and the tuna is 3 15/35 m long, and 21/35 > 15/35.

BONUS

7. Teacher to check shading.

BONUS

8. Teacher to check shading.

9. a) 20
   b) 5
   c) 25
Number Sense: Decimals – AP Book 5.2: Unit 10

AP Book NS5-46
page 45

1. b) $\frac{3}{10}$, 0.3
c) $\frac{8}{10}$, 0.8
d) $\frac{2}{10}$, 0.2

2. b) 0.4
c) 0.6
d) 0.9

3. b)
c)
d)

4. b) 0.4
c) 0.6
d) 0.9

5. b) 0.2, 0.40, 0.7
c) 0.8
d) 0.2

2. a) ii) $\frac{7}{10}$, 0.7, $\frac{40}{100}$, 0.4, $\frac{0.4}{0.4}$
   b) 0.2, 0.40, 0.7

3. A. 2
   b) 20
   c) 0.2, 0.20
   B. 6
   d) 60
   e) 0.6, 0.60
   C. 7
   e) 70
   d) 0.7, 0.70

4. Teacher to check number lines.
   a) 0.05, 0.27, 0.40
   b) 0.05, 0.08, 0.80

5. a) 7.70
   b) 48
   c) 9
   d) 30
   e) $\frac{100}{100}$

6. a) 6
   b) 60
   c) 77
   d) 5
   e) 0.9

7. Cross out 0.7 = $\frac{7}{100}$ and 0.02 = $\frac{2}{10}$

8. a) 40, 73
   b) 20, 16
   c) 70, 59

AP Book NS5-47
page 47

1. Teacher to check shading.
   b) 90
   c) 60
   d) 0.60

2. a) $\frac{8}{10}$, 0.8
   b) $\frac{2}{10}$, 0.2

BONUS

94 = 0.94

6. a) 0.18
   b) 0.09
   c) 0.90

7. a) Teacher to check.
   b) 0.06, 0.24, 0.45, 0.70

AP Book NS5-48
page 49

1. b) 47, 4.7
   e) $\frac{13}{10}$

AP Book NS5-49
page 51

1. a) 3.4
   b) 12.5
   c) 8.45
   d) 46.03

2. a) 10
   b) 100
   c) 100

BONUS

3. a) $\frac{3}{10}$$\frac{81}{100}$
   b) $\frac{6}{10}$
   c) $\frac{7.4}{10}$
   d) $\frac{18.15}{10}$
   e) $\frac{13.4}{10}$

9. b) $\frac{38}{10}$, $\frac{380}{100}$, 3.80

BONUS

1007 $\frac{4}{100}$

4. a) hundredths
   b) tenths
   c) hundredths
   d) tenths

5. a) seven and four tenths
   b) four and nine hundredths
   c) 74.11
   d) 20.4

6. a) 7, 4
   b) 6, 25
   c) 625
   d) 100

7. c) $\frac{4.1}{10}$
   d) $\frac{6.42}{100}$
   e) $\frac{64}{100}$
   f) $\frac{8.08}{100}$

8. a) $\frac{38}{10}$
   b) 708
   c) 860
   d) 6004
   e) 708
   f) 175
   g) 3189
   h) 904

9. b) $\frac{380}{10}$, $\frac{38}{100}$, 3.80
c) $\frac{39}{100}, 0.39$

d) $\frac{64}{10}, 0.64$

e) $\frac{594}{10}, 5.94$

f) $7.5, \frac{750}{100}, 7.50$

g) $6.7, \frac{67}{10}, 0.67$

h) $30.8, \frac{3080}{100}, 30.80$

AP Book NS5-50

page 53

1. b) tenths
c) tenths
d) hundredths
e) tenths
f) hundredths

2. a) hundredths
b) hundredths
c) ones
d) ones
e) thousandths
f) tenths

3. b) 0.074
c) 0.009
d) 0.101
e) 0.596
f) 0.110
g) 0.900
h) 0.010
i) 0.100

AP Book NS5-52

page 55

1. b) 0.4
c) 0.7
d) 0.9

AP Book NS5-53

page 57

1. a) $0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$
b) 0.5
c) ii) <
   iii) >
   iv) =
   v) <
   vi) <
e) =
f) >
g) >
h) >

3. a) <
b) <
c) =
d) >
e) <
f) =
g) =
h) <

4. Teacher to check shading.
5, 50

5. Teacher to check shading.
2, 20

6. a) $\frac{37}{100}$
b) $\frac{25}{100}, \frac{52}{100}$
c) $\frac{40}{100}, \frac{42}{100}$
d) $\frac{70}{100}, \frac{60}{100}$
e) $\frac{23}{100}, \frac{20}{100}$
f) $\frac{52}{100}, \frac{50}{100}$

2. a) >
b) <
c) <
d) <

2. Teacher to check.
3. A) $\frac{19}{100}, 0.19$
B) $\frac{8}{100}, 0.08$
C) $\frac{14}{100}, 0.14$
D) $\frac{27}{100}, 0.27$

4. Teacher to check.
5. a) Teacher to check.
b) $\frac{10}{100}, 0.5, \frac{8}{10}$
Number Sense: Decimals – AP Book 5.2: Unit 10

(continued)

9. a) \(0.32 < \frac{1}{2} < 0.7\)
   Sample explanation: I changed all the numbers to fractions with denominator 100, then I compared the numerators.
   b) \(\frac{1}{4} < \frac{3}{5} < 0.63\)
   c) \(0.35 < \frac{2}{5} < \frac{1}{2}\)

AP Book NS5-54
page 59
1. a) Teacher to check drawing.
   2
   13
   3
   3
   b) 1, 4
   c) 2, 3
   d) 4, 9
   e) 6, 7

2. b) 8, 4
   c) 9, 5
   d) 8, 4
   e) 9, 9

3. b) 7, 10
   c) 3, 11
   d) 5, 18
   e) 0, 19

BONUS
0, 90

4. Teacher to check shading.
   b) 0.32, 0.36, 0.68
   c) 0.61, 0.35, 0.96
   d) 0.48, 0.31, 0.79
   e) 0.18, 0.71, 0.89

BONUS
0.45, 0.24, 0.69

5. b) \[\begin{array}{cccc}
3 & 6 & 4 & 8 \\
4 & 2 & 1 & \\
7 & 8 & 5 & 8 \\
\end{array}\]

6. b) \[\begin{array}{cccc}
T & O & T & H \\
3 & 1 & 1 & 4 \\
5 & 7 & 1 & \\
3 & 6 & 11 & 1 \\
3 & 7 & 1 & 1 \\
\end{array}\]

7. Teacher to check grids.
   b) 0.98
   c) 0.88
   d) 0.44

8. Teacher to check grids.
   b) 0.91
   c) 1.47
   d) 1.92

9. Teacher to check grids.
   a) 7.19
   b) 8.90
   c) 8.57
   d) 0.91

10. 10.55 g

11. Bill did not properly align the decimal points in the two numbers being added.

AP Book NS5-55
page 62
1. Teacher to check pictures.
   a) 0.1
   b) 0.26
   c) 0.35
   d) 0.32

2. Teacher to check grids.
   b) 0.41
   c) 1.27
   d) 6.14
   e) 1.00
   f) 1.73
   g) 1.74
   h) 2.12
   i) 0.04
   j) 0.12
   k) 3.11
   l) 7.13

3. Teacher to check grids.
   b) 2.16
   c) 1.95
   d) 2.08
   e) 1.75

   f) 8.8
   g) 0.77
   h) 2.87

4. Teacher to check grids.
   a) 0.61
   b) 2.11
   c) 2.14

5. a) 0.85
   b) 2.11
   c) 3.07
   d) 3.59
   e) 4.11
   f) 7.00
   g) 3.99
   h) 2.23
   i) 2.01

6. a) The quarter is 0.36 mm thicker.
   b) The nickel is 0.18 mm thicker.

7. Sara made 0.90 L of blue-coloured water.

8. The total length of an average house cat is 0.76 m.
Number Sense: Using Decimals – AP Book 5.2: Unit 11

AP Book NS5-56
page 64
1. b) 60¢, $0.60
c) 25¢, $0.25
d) 25¢, $0.25
e) 75¢, $0.75
f) 80¢, $0.80
g) 100¢, $1.00
h) 500¢, $5.00
i) 700¢, $7.00
j) 100¢, $1.00
BONUS
k) 435¢, $4.35
l) 845¢, $8.45
2. b) 4, 7, $0.47
c) 3, 0, 5, $3.05
d) 0, 0, 3, $0.03
BONUS
20, 1, 6, $20.16
3. b) 60¢
c) 9¢
d) 100¢
e) 798¢
f) 1200¢
g) 1000¢
h) 199¢
i) 151¢
j) 98¢
k) 3¢
l) 8¢
m) 2300¢
n) 3106¢
o) 4004¢
4. b) $1.03
c) $2.16
d) $3.75
e) $3.00
f) $0.04
g) $6.07
h) $19.08
i) $6.00
j) $0.99
k) $12.00
BONUS
$90.08
5. b) $8.25¢, $8.25
c) $20, 55¢, $20.55
d) $30, 7¢, $30.07
e) $36, 11¢, $36.11
f) $150, 60¢, $150.60
6. Lela used 3 toonies.
7. 1 five-dollar bill and 5 nickels or 5 loonies and 1 quarter
8. b) $1.00 = 100¢
c) $0.04 = 4¢
d) $5.98 = 598¢
e) $6.05 = 605¢
f) $0.87 = 87¢
circle 100¢
circle 6¢
circle 187¢
9. a) $2.75
c) $22.65
d) $40.51
e) $37.01
f) $45.11
d) $45.08
4. $13.53
+ $ 7.25
$20.78
Jasmin spent $20.78 in total.
5. $389.82
+ $270.25
$660.07
The library spent $660.07 in total.
6. $21.30
+ $21.30
$42.60
Eric paid $42.60 in total.
7. $25.00
− $ 9.50
$15.50
− $10.35
$5.15
Raj will have enough left to buy a second book for $5.10.
8. $69.99
− $10.50
$59.49
Lynn will pay $59.49 today.
BONUS
Two pairs of pants cost $99.90.
Sample explanation: I can find the answer mentally by rounding 49.95 to 50. 50 + 50 = 100, then I can subtract 10¢ from 100 to get $99.90.
9. a) $28.50
b) $28.50
$12.30
The pants cost more than shoes and a soccer ball.
BONUS
Two pairs of pants cost $99.90.
Sample explanation: I can find the answer mentally by rounding 49.95 to 50. 50 + 50 = 100, then I can subtract 10¢ from 100 to get $99.90.
10. a) $8.40
b) 6 apples
c) 3 markers
11. $3.27
+ $4.96
$8.23
Sam spent $8.23 in total.
AP Book NS5-58
page 69
1. Teacher to check that arrows are drawn to the following numbers:
b) 1.0
c) 0
d) 1.0
2. a) i) 0.1, 0.2, 0.3, 0.4
ii) 0.6, 0.7, 0.8, 0.9
b) 0.5 is special because it is the same distance from 0 and from 1.
3. Teacher to check arrows.
a) 2.0, 3.0
b) 3.0, 4.0, 5.0
4. c) ×

Answer Keys for AP Book 5.2
d) ×
e) √
f) ×
g) ×

**BONUS**

5. Teacher to check that arrows are drawn to the following numbers:
   a) 1.00
   b) 0

6. Teacher to check arrow. 0.600 is closer to 1.000.
7. Teacher to check arrows. 1.33 is closer to 0. 1.78 is closer to 2.
8. Teacher to check arrows. 4.26 is closer to 4. 4.72 is closer to 5.

9. c) 7
d) 6
e) 9
f) 9
g) 11
h) 31
i) 20

10. b) 1.8
c) 3.6
d) 3.4
e) 5.6
f) 6.7
g) 6.6
h) 8.5
i) 9.4
j) 7.9
k) 5.0
l) 10.0

11. a) 20
   b) If you rounded to 20, you would fill the tank above the line.

---

**AP Book NS5-60**

**page 73**

1. Teacher to check hundreds blocks diagrams.

2. a) 3, 5
   b) 100, 370
   c) 100, 104
   d) 1000, 9020

3. a) 40
   b) 80
   c) 340

4. 0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.5 + 0.5 = 5

5. Teacher to check.
6. a) 3
   b) 100 × 0.02 = 2

7. b) 350
c) 720
d) 600
e) 34
f) 7

**BONUS**

a) 930
b) 6320
c) 720

8. a) 3, 5
   b) 100
   c) more
d) m, cm
e) as large as
   f) as small as

9. b) 20
   c) 83
d) 490

10. b) 2400
    c) 160
d) 40

11. a) 900
    b) 100, 370
    c) 100, 104
    d) 1000, 9020

12. a) no
   b) Kim needs to convert the measurements into the same units before adding.
      0.15 km = 150 m, so 48 m + 150 m = 198 m.
### Number Sense: Using Decimals – AP Book 5.2: Unit 11

#### Answer Keys for AP Book 5.2

<table>
<thead>
<tr>
<th>Exercise</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Teacher to check pictures.</td>
<td>b) (2.0 \div 10 = 0.2)</td>
<td>c) 0.04</td>
</tr>
<tr>
<td></td>
<td>d) (0.3 \div 10 = 0.03)</td>
<td>e) (0.6 \div 10 = 0.06)</td>
<td>f) 0.11</td>
</tr>
<tr>
<td></td>
<td>g) (2.1 \div 10 = 0.21)</td>
<td>h) (2.3 \div 10 = 0.23)</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>a) 2, right</td>
<td>b) 3, right</td>
<td>c) (0.04)</td>
</tr>
<tr>
<td></td>
<td>d) (0.3 \div 10 = 0.03)</td>
<td>e) (0.6 \div 10 = 0.06)</td>
<td>f) 0.11</td>
</tr>
<tr>
<td></td>
<td>g) (2.1 \div 10 = 0.21)</td>
<td>h) (2.3 \div 10 = 0.23)</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>b) (0.07) or 0.07</td>
<td>c) (0.06) or 0.06</td>
<td>d) (1.50)</td>
</tr>
<tr>
<td></td>
<td>e) (2.54)</td>
<td>f) (2.54)</td>
<td>g) (2.30)</td>
</tr>
<tr>
<td></td>
<td>h) (0.07) or 0.070</td>
<td>i) 0.091 or 0.091</td>
<td>j) 0.91 or 0.910</td>
</tr>
<tr>
<td>4.</td>
<td>b) 3, right</td>
<td>c) 4, right</td>
<td>d) 2, left</td>
</tr>
<tr>
<td></td>
<td>e) 1, left</td>
<td>f) 2, right</td>
<td>g) divide, 1</td>
</tr>
<tr>
<td></td>
<td>h) multiply, 2</td>
<td>i) multiply, 1</td>
<td>j) divide, 2</td>
</tr>
<tr>
<td></td>
<td>k) multiply, 3</td>
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<td></td>
</tr>
</tbody>
</table>

#### BONUS

- a) \(6 + 6 + 13 = 25\)  
  yes
- b) \(6.25 + 6.25 + 13.25 = 25.75\)  
  no
- c) Sample answer: Yes, my answer would have changed. Rounding to the nearest whole number gave an underestimate, and a “yes” answer. Rounding to the nearest tenth gives an overestimate, resulting in a “no” answer.

---

**AP Book NS5-61**

**Page 76**

1. Teacher to check pictures.
   - b) \(2.0 \div 10 = 0.2\)
   - c) 0.04
   - d) \(0.3 \div 10 = 0.03\)
   - e) \(0.6 \div 10 = 0.06\)
   - f) 0.11
   - g) \(2.1 \div 10 = 0.21\)
   - h) \(2.3 \div 10 = 0.23\)

2. a) 2, right
   - 2, left
   - b) 3, right
   - 3, left

3. b) \(0.07\) or 0.07
   - c) \(0.06\) or 0.06
   - d) \(1.50\)
   - e) \(2.54\)
   - f) \(2.30\)
   - g) \(0.07\) or 0.070
   - h) 0.091 or 0.091
   - i) 0.91 or 0.910

4. b) \(0.07\) or 0.07
   - c) \(0.06\) or 0.06
   - d) \(1.50\)
   - e) \(2.54\)
   - f) \(2.30\)
   - g) \(0.07\) or 0.070
   - h) 0.091 or 0.091
   - i) 0.91 or 0.910

<table>
<thead>
<tr>
<th>Exercise</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
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<tbody>
<tr>
<td>1. a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
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<table>
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<tr>
<th>Exercise</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. a) (47.90)</td>
<td>b) (90.80)</td>
<td>c) (78.15)</td>
<td></td>
</tr>
<tr>
<td>3. a) (71.31)</td>
<td>b) (20.11)</td>
<td>c) (24.61)</td>
<td></td>
</tr>
<tr>
<td>4. a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**AP Book NS5-62**

**Page 78**

1. a) \(\square\)
   - b) \(\square\)
   - c) \(\square\)

2. a) \(\$47.90\)
   - b) \(\$90.80\)
   - c) \(\$78.15\)

3. a) \(\$71.31\)
   - b) \(\$20.11\)
   - c) \(\$24.61\)

4. a) \(\$47.90\)
   - b) \(\$90.80\)
   - c) \(\$78.15\)

5. Sample answer:
   I do not agree with Alex because 2.50 and 2.5 are equal. The zero in 2.50 doesn’t make the distance longer. It just shows that there are zero hundredths.

6. a) \(8.08\)
   - b) \(0.06\)
   - c) \(0.92\)

7. Marko: \(\$10.25\)
   Sandy: \(\$10.25 + \$10.25 = \$20.50\)
   Total: \(\$10.25 + \$20.50 = \$30.75\)

8. \(4.99 + 7.20 + 35.15 = \$47.34\)

9. a) Jayden would have to mow 6 lawns.
   - b) Answers will vary.
   Sample answer:
   Mow 1 lawn ($10) and babysit for 3 hours (3 x 15 = $45).
   \(10 + 45 = \$55\)

10. a) 3
    - b) 6

11. a) 19.5
    - b) 7.1
    - c) 0.1

12. a) Estimate: \(1.6 + 18.8 = 20.4\)
   Calculate: \(1.64 + 18.75 = 20.39\)
   b) Estimate: \(23.1 - 17.1 = 6.0\)
   Calculate: \(23.07 - 17.09 = 5.98\)
   c) Estimate: \(104.4 + 0.1 = 104.5\)
   Calculate: \(104.43 + 0.09 = 104.52\)

**BONUS**

- a) \(6 + 6 + 13 = \$25\)  
  yes
- b) \(6.25 + 6.25 + 13.25 = \$25.75\)  
  no
- c) Sample answer: Yes, my answer would have changed. Rounding to the nearest whole number gave an underestimate, and a “yes” answer. Rounding to the nearest tenth gives an overestimate, resulting in a “no” answer.
1. b) 3
   c) 1
   d) 3

2. b) 3
   c) 3
   d) 3

3. Teacher to check circling.
   a) 2
   b) 1
   c) 3
   d) 2

4. a) B (3, 5)  C (6, 4)  D (0, 2)  E (7, 3)  F (2, 0)  G (5, 0)  H (3, 1)  I (4, 2)  J (7, 1)  K (0, 0)  L (2, 2)
   b) F, G, K
   c) D, K
   d) K
   e) K

5. Teacher to check.

6. Teacher to check.

7. a) 2, 3
   b) 3, 2
   c) 1, 2
   d) 3, 1

8. Circle the following shapes:
   a) first and third
   b) first and second
   c) first and third
   d) first and third

9. Teacher to check.

10. To be congruent, shapes need to have the same size and shape. The two squares are not congruent because they are different sizes.

11. b) no
    c) yes
    d) yes
    e) no
    f) yes
    g) no
    h) yes

12. Circle the following shapes:
    a) first and third
    b) first and second
    c) first and third
    d) first and third

13. Teacher to check.

14. To be congruent, shapes need to have the same size and shape. The two squares are not congruent because they are different sizes.

15. b) no
    c) yes
    d) yes
    e) yes
    f) no
    g) no
    h) yes

16. a) a crown

17. Teacher to check circling.
   a) 2
   b) 1
   c) 3
   d) 2

18. Teacher to check.

19. Teacher to check.

20. Teacher to check.

21. Teacher to check.

22. Teacher to check.

23. Teacher to check.

24. Teacher to check.

25. Teacher to check.

26. Teacher to check.

27. Teacher to check.

28. Teacher to check.

29. Teacher to check.

30. Teacher to check.

31. Teacher to check.

32. Teacher to check.

33. Teacher to check.

34. Teacher to check.

35. Teacher to check.

36. Teacher to check.

37. Teacher to check.

38. Teacher to check.

39. Teacher to check.

40. Teacher to check.

41. Teacher to check.

42. Teacher to check.

43. Teacher to check.

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164. Teacher to check.

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166. Teacher to check.

167. Teacher to check.

168. Teacher to check.

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171. Teacher to check.

172. Teacher to check.

173. Teacher to check.

174. Teacher to check.

175. Teacher to check.

176. Teacher to check.

177. Teacher to check.

178. Teacher to check.

179. Teacher to check.

180. Teacher to check.

181. Teacher to check.

182. Teacher to check.

183. Teacher to check.

184. Teacher to check.

185. Teacher to check.

186. Teacher to check.

187. Teacher to check.

188. Teacher to check.

189. Teacher to check.

190. Teacher to check.

191. Teacher to check.

192. Teacher to check.

193. Teacher to check.

194. Teacher to check.

195. Teacher to check.

196. Teacher to check.

197. Teacher to check.

198. Teacher to check.

199. Teacher to check.

200. Teacher to check.
b) 

8. a) 

b) 

c) 

d) 

e) 

f) 

9. a) 

b) 

c) 

BONUS 

Answers will vary. Teacher to check. Sample answer:

AP Book G5-15
page 87

1. a) 4 
   b) 3 units right 
   c) 2 units right 

2. a) 3 
   b) 5 units left 
   c) 2 units left 

3. a) 
   b) 
   c) 

4. a) 4 
   2 
   b) 1 
   3 
   c) 3 
   1 

5. a) 
   b) 
   c) 

6. Teacher to check. 
7. Teacher to check. 
8. a) 

This shape has four lines of symmetry: one vertical, one horizontal, and two diagonal.
12. a) 

b) 

c) 

d) 

13. a) Shape B is the image of Shape A translated 2 units left and 3 units up.

b) The shapes cannot be created by a translation because a flip is also needed.

AP Book G5-16
page 90
1. a) Dubhe
b) Alioth
c) A3
d) E2
Merak
e) 2
f) Mizar

BONUS
Teacher to check map.
A4

2. b) 10
c) 10, south
5
d) 5, east
10, north
e) 10 paces north
10 paces west

3. a) Teacher to check map.
b) Red Rock and Tall Fir

4. a) Bear Cave
Treasure
Mouth Bay
Clear Spring

b) (14, 8)
(12, 2)
(10, 10)

c) Clear Spring
Swamp
Ear Wood

d) 4, west
6, south
6, east
2
3, north, 1
west
4 km west and 6 km south
8 km north and 6 km west
8 km west and 4 km south

e) Answers will vary. Teacher to check.

AP Book G5-17
page 92
1. a) 

b) 

c) 

d) 

2. a) 

b) 

c) 

d) 

3. a) i)
ii)
iii)

b) i) 3 units 3 units
ii) 3 units 3 units
iii) 1 unit 1 unit

c) The distance between the original vertex and the reflecting line is equal to the distance between the image vertex and the reflecting line.

4. a) 

b) 

c) 

d) 

5. a) 

b) 

c) 

d) 

Answer Keys for AP Book 5.2
6. Teacher to check.
7. Teacher to check.

AP Book G5-18

1. a)
   ![Diagram](image1)
   ![Diagram](image2)
   ![Diagram](image3)
   ![Diagram](image4)

2. a)
   ![Diagram](image5)
   ![Diagram](image6)
   ![Diagram](image7)
   ![Diagram](image8)
   ![Diagram](image9)

3. a)
   ![Diagram](image10)
   ![Diagram](image11)
   ![Diagram](image12)
   ![Diagram](image13)
   ![Diagram](image14)

4. Circle parts a), d), f), h), i) and j).
   Check beside parts b), g) and j).

5. a) Teacher to check.
   b) Teacher to check.
   c) part j)
   ![Diagram](image15)

6. a) opposite
   b) same

7. Sample answer: I agree with Tessa. A reflection or a translation does not change the size or shape of a figure.

8. Teacher to check.

9. Teacher to check.

10. Answers will vary.

   Teacher to check.

AP Book G5-19

1. b) 1/2

---

### Table

<table>
<thead>
<tr>
<th>2. c)</th>
<th>3/4</th>
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<tbody>
<tr>
<td>d) 1</td>
<td>e) 1/2</td>
</tr>
<tr>
<td>f) 1/4</td>
<td>g) 1/4</td>
</tr>
<tr>
<td>h) 3/4</td>
<td>i) 1/2</td>
</tr>
<tr>
<td>j) 1/4</td>
<td>k) 1/4</td>
</tr>
</tbody>
</table>

### BONUS

- **3. b)** 3/4
- **c)** 1/2 turn CW
- **d)** 1/4 turn CW
- **e)** 1/4
- **f)** 1/4
- **g)** 3/4 turn CCW
- **h)** 1/2 turn CCW

---

**Answer Keys for AP Book 5.2**
5. a) b) c) d) e) f) g) h)  

BONUS i) j) k) l) 

6. Answers will vary. Teacher to check. 

AP Book G5-20 page 101 
1. b) c) d) e) f) g) h) i) j) k) l) 

3. a) i) ii) iii) iv) 

5. Answers will vary. Teacher to check. 

AP Book G5-20 page 101 
1. b) c) d) e) f) g) h) i) j) k) l) 

3. a) i) ii) iii) iv) 

4. a) i) ii) iii) iv)
iv) S
ii) W
iii) N
iv) E

5. b) rotation: $\frac{1}{4}$ turn CCW around point P

c) reflection in the line M

d) reflection in the line M

e) translation: 1 unit right

f) rotation: $\frac{1}{4}$ turn CW around point P

6. Answers will vary. Teacher to check.

7. a) reflection in Line 1

b) rotation: $\frac{1}{2}$ turn CW or CCW around point P

c) translation: 1 unit right

d) rotation: $\frac{1}{4}$ turn CW around point P or rotation: $\frac{3}{4}$ turn CCW around point P

BONUS

a) Sometimes true. It is true when the shape has a line of symmetry parallel to the line of reflection.

b) Sometimes true. It is true for special shapes and certain rotations, like a $\frac{1}{2}$ turn for a rectangle, any rotation for a circle, and a whole turn for any shape.

8. Answers will vary. Teacher to check.
Geometry: 3-D Shapes – AP Book 5.2: Unit 13

AP Book G5-21

1. a) rectangle
   b) circle
   c) hexagon
   d) triangle
   e) rectangle
   f) square
2. Teacher to check.
3. Teacher to check.
4. Teacher to check.
5. Teacher to check tracing.
   a) 12
   b) 6
   c) 8
   d) 9
6. Teacher to check dots.
   a) 8
   b) 4
   c) 5
   d) 6
7. a) 
   b) 
   c) 
   d) 
   e) 
   f) 
8. Teacher to check pictures.
   b) yes
   c) yes
   d) no
   e) yes
   f) no
   g) yes
   h) no
9. Teacher to check pictures.
   a) yes
   b) no
   c) yes
   d) yes
   e) yes
   f) yes
   g) yes
   h) yes

AP Book G5-22

1. Teacher to check shading.
   b) triangular prism
   c) square-based prism
   d) triangular prism
2. Teacher to check.
3. 
4. a) 
   b) 
   c) 
   d) 
   e) 
   f) 
5. Answers will vary.
   Teacher to check.
6. 
7. The shape does not have two identical opposite bases.
8. Yes, the bases in a prism are opposite from each other so they never intersect.

BONUS
   The faces are triangles because only the faces that do not intersect are the bases.

AP Book G5-23

1. Teacher to check shading.
   a) pentagon
   b) trapezoid
   c) square
   d) heptagon
   e) hexagon
   f) square
2. a) 8
   b) 12
   c) 4
   d) 6
7. Teacher to check dots.
8. a) Teacher to check pictures in last column.

<table>
<thead>
<tr>
<th>Name of Shape</th>
<th>Number of ...</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertices</td>
<td>Edges</td>
</tr>
<tr>
<td>triangular pyramid</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>rectangular pyramid</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>pentagon-based pyramid</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>hexagon-based pyramid</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

b) Teacher to check.

c) triangles

9. The object is a pyramid because the number of vertices in a pyramid is the number of vertices in the base (in this case, 9) plus one (the apex).

10. No, in a pyramid, all the side faces intersect with each other at the apex and with the base, so there are no faces that do not intersect.

BONUS

a) It is a pyramid and its base has 9 sides, because a pyramid always has twice as many edges as there are sides in its base.

b) It is a prism and its base has 8 edges, because a prism always has twice as many vertices and 3 times as many edges in total as there are sides in its base.

AP Book G5-25

page 115

1. Teacher to check pictures in last column.

<table>
<thead>
<tr>
<th>Name of Shape</th>
<th>Number of ...</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertices</td>
<td>Edges</td>
</tr>
<tr>
<td>triangular prism</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>square-based prism</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>rectangular prism</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>hexagon-based prism</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

2. a) C
b) D
c) A
d) E
e) B

3. a) yes
b) no
c) yes
d) no

4. Teacher to check.

Sample answers:

a) [Diagram]
b) [Diagram]
c) [Diagram]
d) [Diagram]

5. Teacher to check.

BONUS

Teacher to check.
1. a) 100 200 300 2500
   b) 10 20 30 370
   c) 1000 2000 3000 75 000
2. a) Teacher to check.
   b) 50
3. a) 128
   b) longer
4. a) 12
   b) 14
   c) 14
5. a) 10 cm
   b) 8 cm
   c) 14 cm
6. a) 24 m, 28 cm, 6 km, 30 cm
   b) PERU
7. Estimates will vary. Teacher to check.
   Actual perimeter:
   a) 18 cm
   b) 16 cm
8. a) 4, 6, 8, 10
    b) Each time a square is added the perimeter increases by 2 units.
    c) 14
9. a) 6, 10, 14, 18
    b) Each time a hexagon is added the perimeter increases by 4 units.
    c) 26
10. a) 10
    b) 12
    Answers will vary. Teacher to check.

8. a) Width Length
    1 3
    2 2
    b) Width Length
    1 5
    2 4
    3 3
    c) Width Length
    1 6
    2 5
    3 4
    4 5

BONUS
Perimeter = (2 × length) + (2 × width)
or
Perimeter = 2ℓ + 2w
9. Answers will vary.
   Teacher to check.
10. a) 6
    b) 12
11. a) m
    b) cm
    c) m
    d) km
    e) cm
    f) km
    g) m
    h) km
12. Answers will vary.
    Teacher to check.
13. a) 20 cm
    b) A square’s length and width are equal, so I can multiply the length of one side by four to get the perimeter.
    c) One side is 3 cm because 12 ÷ 4 = 3.

14. The perimeter of the poster is 4 + 1 + 4 + 1 = 10 m.
The ribbon border will cost 10 m × 15¢ = 150¢ = $1.50
15. Sample answer:
I can wrap the strip of paper around the object, and make a mark where the strip begins to overlap itself. Then I can lay the strip of paper flat, and measure the distance from the end of the strip to the mark with a ruler.
16. Yes.
   Sample explanation:
The two shapes below both have a perimeter of 8 units.

17. Emma is correct.
   Sample explanation:
Using the distributive property of multiplication, 
2 × (length + width) = (2 × length) + (2 × width), which is the formula for the perimeter of a rectangle.
Measurement: Perimeter, Area, Volume, and Mass – AP Book 5.2: Unit 14

5. To find the area of a rectangle, multiply its length by its width.
6. a) 2 × 5 = 10 cm²
   b) 3 × 4 = 12 cm²
   c) 3 × 5 = 15 cm²
7. a) 2 by 4 rectangle with area 8 cm²
    b) 3 by 3 square with area 9 cm²
    c) the 3 by 3 square
8. No.

BONUS
Since a 1 m by 1 m square is also a 100 cm by 100 cm square, the area is 10 000 cm². So, to convert m² to cm² you multiply by 10 000.

AP Book ME5-15

page 126
1. a) 2 × 5 = 10
   b) 4 × 5 = 20
   c) 3 × 3 = 9
   d) 3 × 5 = 15
2. Teacher to check dots.
   b) 4 × 3 = 12
   c) 4 × 4 = 16
   d) 3 × 6 = 18
3. a) 2
    7
    2 × 7 = 14
   b) 3
   c) 3 × 3 = 9
   d) 3 × 5 = 15

4. Teacher to check lines.
   a) 3 × 3 = 9 cm²
   b) 2 × 6 = 12 cm²
   c) 3 × 4 = 12 cm²

5. Sample answers:
   a) 6 m × 9 m = 54 m²
   b) 1 m × 2 m = 2 m²
   54 m² + 2 m² = 27
   Ethan needs 27 pieces of sod.
   c) 27 × 25 = 675
   It will cost Ethan $675 to cover the entire lawn with sod.

AP Book ME5-16

page 129
1. a) 6 × 3
    = 18 m²
   b) 9 × 2
    = 18 km²
   c) 80 × 65
    = 5200 cm²
   d) 66 × 3
    = 198 km²
   e) 93 × 42
    = 3906 mm²
   f) 140 × 25
    = 3500 m²

2. b) w × 2 = 12
   w = 12 ÷ 2
   w = 6 m
   c) w × 6 = 24
   w = 24 ÷ 6
   w = 4 km

3. a) ℓ × 5 = 30
   ℓ = 30 ÷ 5
   ℓ = 6 cm
   b) ℓ × 7 = 63
   ℓ = 63 ÷ 7
   ℓ = 9 km

BONUS
   ℓ × 10 = 1678
   ℓ = 1678 + 10
   ℓ = 167.8 m

4. a) ℓ = 48 ÷ 3
   ℓ = 16 m
   b) w = 5600 + 80
   w = 70 cm
   c) w × w = 16
   w = 4 cm

5. Sample answers:
   a) 4 cm
   b) 6 cm
   c) 16 × 5 = 80
   The fencing will cost $80.
   d) Area = 2 × 6 = 12 m²
   12 × 8 = 96 seeds
   Ivan needs 2 packets of seeds.
   e) 80 + 2 + 2 = 84
   Altogether, the seeds and fencing cost $84.
1. a) 3
   b) 3
   c) 3
2. a) 6
   b) 6
   c) 6
3. a) 6
   b) 6
   c) 6
4. a) 3, 2, 6
   b) i) 3, 2, 2
      12
   ii) 3, 2, 3
      18
   iii) 3, 2, 4
      24
5. a) 5, 4, 2
     40
   b) 4, 4, 3
     48
   c) 2, 6, 4
     48
6. a) 3, 2, 4, 24
   b) 2, 3, 6
   2, 3, 5, 30
   c) 4, 3, 12
   4, 3, 2, 24
   d) 3, 4, 12
   3, 4, 2, 24
7. Sample answer: The three numbers that are multiplied, height, width, and length, are the same regardless of the order of multiplication.
### Measurement: Perimeter, Area, Volume, and Mass – AP Book 5.2: Unit 14

**AP Book ME5-20**

1. Circle the bucket, the pool, and the water tank.

2. Circle the following:
   - a) mL
   - b) L
   - c) L
   - d) mL

3. Circle the following:
   - a) mL
   - b) L
   - c) L
   - d) mL

4. Circle the following:
   - a) 500 mL
   - b) 200 L
   - c) 300 L
   - d) 500 mL

5. a) 2000, 3000, 4000, 5000, 6000, 7000, 8000
   - b) 1000

6. a) 10 000
   - b) 13 000
   - c) 20 000
   - d) 45 000
   - e) 72 000
   - f) 100 000

7. b) 8 L = 8000 mL
c) 23 L = 23 000 mL
d) 6 L = 6000 mL
e) 70 L = 70 000 mL
f) 75 L = 75 000 mL

8. a) Answers will vary.
    - b) 7 L
9. b) 5
c) 2

**AP Book ME5-21**

1. b) 0.590, 590
c) 2.540, 2540
d) 0.020, 20
e) 4
f) 1759
g) 1040
h) 24 700

2. b) 3 L = 3000 mL
c) 17 L = 17 000 mL
d) 2.5 L = 2500 mL
e) 4.6 L = 4600 mL
f) 11.790 L = 11 790 mL
g) 6.95 L = 6950 mL
h) 7.40 L = 7400 mL
i) 3.047 L = 3047 mL

3. a) Answers will vary.
    - b) 4 L

4. b) 3, 247
c) 4, 27
d) 5, 820
e) 5, 8
f) 12, 750
g) 2, 700
h) 58, 100

5. b) 5, 217
c) 4, 367
d) 4, 81

**AP Book ME5-22**

1. a) 10, 10, 10, 1000
   - b) i) 5 g
      - ii) 10 g
      - iii) 25 g
      - iv) 40 g

2. Circle the following:
   - a) 100 g
   - b) 20 g
   - c) 35 kg
   - d) 4 kg

3. 2000, 3000, 4000, 5000, 6000, 7000, 8000

4. a) 1000
   - b) i) 13 000 g
      - ii) 49 000 g
      - iii) 107 000 g

5. b) 91 kg = 91 000 g
   - circle 91 000 g

**BONUS**

- a) 36 cm$^3$
  - 6
- b) 320 dm$^3$
  - 4
- c) 90 cm$^3$
  - 5

**AP Book ME5-23**

1. a) 50 g, 250 g
   - b) i) 5 g
      - ii) 10 g
      - iii) 25 g
      - iv) 40 g

2. Circle the following:
   - a) 100 g
   - b) 20 g
   - c) 35 kg
   - d) 4 kg

3. 2000, 3000, 4000, 5000, 6000, 7000, 8000

4. a) 1000
   - b) i) 13 000 g
      - ii) 49 000 g
      - iii) 107 000 g

5. b) 91 kg = 91 000 g
   - circle 91 000 g
c) 15 kg = 15 000 g
   circle 34 768 g
d) 2 kg = 2000 g
   circle 2222 g
e) 70 kg = 70 000 g
   circle 70 000 g
f) 47 kg = 47 000 g
   circle 47 000 g

6. a) 1600
b) 0.890, 890
c) 2.830, 2830
d) 0.020, 20
e) 4
f) 1789
g) 1030
h) 23600

7. b) 3, 247
c) 4, 27
d) 5, 820
e) 2, 700
f) 58, 100

8. b) 7, 412
c) 6, 274
d) 8, 81
e) 9, 8
f) 57, 400

9. b) 5000
   5000
   + 630
   \hline
   5630
c) 7000
   7000
   + 23
   \hline
   7023
d) 9128
e) 12 237
f) 44 003

10. 300 \times 20 g = 6000 g = 6 kg

11. a) baby Josh is heavier
    because 4 kg = 4000 g, which is greater than 3617 g.
b) 200 \times 4 = 800 g
    3.5 kg = 3500 g
    3500 g + 800 g
    = 4300 g = 4.3 kg

<table>
<thead>
<tr>
<th>AP Book ME5-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>page 146</td>
</tr>
</tbody>
</table>
| 1. Raj is incorrect.
   2 kg = 2000g, which is greater than 15 g. |
| 2. a) Sample answer: 4355 g
   b) 3 kg |
| 3. b) 2, 639
c) 5, 704
d) 3, 410
e) 6, 19
f) 4, 7 |
| 4. Circle the following:
   a) mg
   b) g |
| 5. Circle the following:
   a) 500 mg
   b) 6 g
c) 500 g
d) 3 g |
| 6. 12 \times $4 = $48 |
| 7. a) 5000
   b) 18 000
c) 6000
d) 50 000
e) 1500
f) 7300
g) 4550
h) 260 |
| 8. a) t
   b) g
c) t |
| 9. adults: 3 \times 75 = 225 kg
   children: 21 \times 40 = 840 kg
   total: 225 + 840 = 1065 kg
   1 t = 1000 kg
   1065 kg is more than 1 tonne, so all the people cannot ride in the raft. |

10. a) 5 t
    b) 7 \times 150 kg = 1050 kg
    The elephant eats more than 1 tonne in a week.

11. a) 10 500 kg
    b) 2800 kg
    c) 1000 kg

BONUS

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>bus: 10 500 kg</td>
</tr>
<tr>
<td>people: 42 \times 70 = 2940 kg</td>
</tr>
<tr>
<td>bags: 42 \times 25 = 1050 kg</td>
</tr>
<tr>
<td>total: 10 500 + 2940 + 1050 = 14 490 kg</td>
</tr>
<tr>
<td>The bus can cross the bridge.</td>
</tr>
</tbody>
</table>
1. b) A, B, C
c) A, B, C, D
d) A

2. tails

3. a) black wins; 2
   b) 1, 2, 3, 4, 5, 6; 6
   c) team 1 wins, team 2 wins, both teams tie; 3

4. b) Y, G, B, R
c) G, Y
d) G

5. b) 1, 3, 5, 7; 4
c) 5, 6, 7, 8; 4

6. b) 1, 3, 5; 3
c) 5, 6; 2

7. 1, 3, 5, 7; 4
   BONUS
   no outcomes; 0

8. a) 3
    b) 4
    c) 4
    d) 6
    e) 6
    f) 8

   BONUS
   yes There are 14 – 6 – 4 = 4 blue marbles, which is the same as the number of red marbles.

9. a) no
    b) no

   BONUS
   yes Half of 18 is 9 and 8 < 9, so the team won less than half of its games.

10. a) no
    b) no

   BONUS
   yes Half of 13 is 6.5 and 7 > 6.5, so the team won more than half of its games.

1. b) likely
c) even chance
d) even chance

2. a) likely
    b) unlikely
c) even chance
d) unlikely

3. a) even
    b) likely
c) likely
d) unlikely

   BONUS
   unlikely

4. red
   There are more red marbles than blue marbles, so you are more likely to draw a red marble.

5. a) likely
    b) unlikely
c) certain
d) impossible
e) even chance
    f) likely

   BONUS
   unlikely

7. Answers will vary. Sample answers:
   a) 3, 3
   b) 5, 5
   c) 7, 7

   BONUS
   7, 7

5. There are three sections in the spinner but they are not of equal size and cannot be used to describe probability.

   BONUS
   2/4

6. a) 1/8
    b) 2/8
c) $\frac{2}{8}$

d) $\frac{5}{8}$

e) $\frac{4}{8}$

f) $\frac{3}{8}$

7. a) $\frac{1}{5}$

b) $\frac{2}{5}$

c) $\frac{3}{5}$

d) $\frac{2}{5}$

e) $\frac{3}{5}$

8. Answers will vary. Sample answer:

BONUS
Teacher to check shading.

3. a) i) 10

b) 20

c) Teacher to check.

4. a) Teacher to check table.

b) i) 2

ii) 4

iii) 6

5. I disagree because even though we expect to get each outcome exactly half of the time, in reality there is no guarantee that we will get that exact result.

BONUS

4. a) 8

b) Teacher to check.

c) 10, 8, 8

Circle Jack.

d) 10, 10, 6

e) No, both Jack and Lynn won, but we expected only Jack to win.

6. B

The probability of getting heads or tails is each $\frac{1}{2}$.

So we expect to get heads and tails 10 times each. The results in B are closest to this expectation.

AP Book PDM5-15
page 158

1. a) i) $\frac{1}{2}$

ii) $\frac{1}{2}$

b) yes

c) 50

d) 50

4. a) ii) 2

iii) 3

iv) 4

v) 5

vi) 6

b) ii) 6

iii) 9

iv) 12

v) 15

vi) 18

5. a) ii) 8

iii) 12

b) ii) 4

iii) 6

AP Book PDM5-14
page 158

1. a) i) $\frac{1}{2}$

ii) $\frac{1}{2}$

b) 20

c) Teacher to check.

d) Teacher to check.

2. Circle C.

C is most likely because it is closest to the expected result, which is to get heads 50 times and tails 50 times.

3. Answers will vary. Sample answer:
Grade 5 JUMP Math Correlation to the Alberta Curriculum

NOTES:

Italicized JUMP Math lessons contain prerequisite material required to meet the learning standard.

An asterisk (*) indicates that a JUMP Math lesson covers a curriculum requirement primarily in the lesson plan.

JUMP Math strands are represented by:

- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

### Number

**General Outcome**

Develop number sense.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> Represent and describe whole numbers to 1 000 000. [C, CN, V, T] [ICT: C6-2.2]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>2.</strong> Use estimation strategies in problem-solving contexts. [C, CN, ME, PS, R, V]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>3.</strong> Apply mental mathematics strategies and number properties in order to understand and recall basic multiplication facts (multiplication tables) to 81 and related division facts. [C, CN, ME, R, V]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Underline text: Understand, recall and apply multiplication and related division facts to 9 × 9. [C, CN, ME, R, V]</td>
<td></td>
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<tr>
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<td>1</td>
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<td>1</td>
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<tr>
<td>Number</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>4.</td>
<td>Apply mental mathematics strategies for multiplication. [C, CN, ME, R, V]</td>
</tr>
</tbody>
</table>
| 5.     | Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems. [C, CN, PS, V]  
*Note: Students investigate a variety of strategies, including standard/traditional algorithms, to become proficient in at least one appropriate and efficient strategy that they understand.*  
*Note: Through this outcome, students have the opportunity to maintain and refine previously learned operations of addition and subtraction with whole numbers (Grade 4).* | 1    | 3    | NS5-15, 18, 20, 21 |
| 6.     | Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems. [C, CN, ME, PS, R, V]  
*Note: Students investigate a variety of strategies, including standard/traditional algorithms, to become proficient in at least one appropriate and efficient strategy that they understand.*  
*Note: Through this outcome, students have the opportunity to maintain and refine previously learned operations of addition and subtraction with whole numbers (Grade 4).* | 1    | 4    | NS5-25 to 33 |
| 7.     | Demonstrate an understanding of fractions by using concrete, pictorial and symbolic representations to:  
- create sets of equivalent fractions  
- compare fractions with like and unlike denominators. [C, CN, PS, R, V] | 2    | 9    | NS5-34 to 40, 44 |
<p>| 8.     | Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially and symbolically. [C, CN, R, V] | 2    | 10   | NS5-46 to 48, 51 |
| 9.     | Relate decimals to fractions and fractions to decimals (to thousandths). [CN, R, V] | 2    | 10   | NS5-48, 50 to 53 |</p>
<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
</table>
| **10.** Compare and order decimals (to thousandths) by using:  
  • benchmarks  
  • place value  
  • equivalent decimals.  
  [C, CN, R, V] | Part | Unit | Lessons |
| | 2 | 10 | NS5-50, 53 |

| **11.** Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).  
  [C, CN, PS, R, V] | Part | Unit | Lessons |
| | 1 | 2 | *NS5-5 to 7* |
| | 2 | 10 | NS5-54, 55 |
| | 2 | 11 | NS5-57 to 59, 62 |

*Note: Through this outcome, students have the opportunity to maintain and refine previously learned operations of addition and subtraction with whole numbers (Grade 4).*
Patterns & Relations — Patterns

<table>
<thead>
<tr>
<th>General Outcome</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use patterns to describe the world and to solve problems.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1  1  PA5-1 to 3, 5, 7</td>
</tr>
<tr>
<td></td>
<td>2  8  PA5-10</td>
</tr>
</tbody>
</table>

Patterns & Relations — Variables and Equations

<table>
<thead>
<tr>
<th>General Outcome</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Represent algebraic expressions in multiple ways.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers). [C, CN, PS, R]</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2  8  PA5-8, 10 PA5-11 to 13, 15, 16</td>
</tr>
<tr>
<td>3. Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2  8  PA5-8, 9, 12 to 16</td>
</tr>
</tbody>
</table>

U-4 JUMP Math Correlation to the Alberta Curriculum — Grade 5
Shape & Space — Measurement

General Outcome
Use direct and indirect measurement to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Identify 90° angles. [ME, V]</td>
<td>Part 1 Unit 6 Lessons G5-1, 6</td>
</tr>
<tr>
<td>2. Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations. [C, CN, PS, R, V]</td>
<td>Part 1 Unit 5 Lessons ME5-3, ME5-12 to 16</td>
</tr>
<tr>
<td>3. Demonstrate an understanding of measuring length (mm) by: • selecting and justifying referents for the unit mm • modelling and describing the relationship between mm and cm units, and between mm and m units. [C, CN, ME, PS, R, V]</td>
<td>Part 1 Unit 5 Lessons ME5-1, 2, 4</td>
</tr>
<tr>
<td>4. Demonstrate an understanding of volume by: • selecting and justifying referents for cm³ or m³ units • estimating volume, using referents for cm³ or m³ • measuring and recording volume (cm³ or m³) • constructing right rectangular prisms for a given volume. [C, CN, ME, PS, R, V]</td>
<td>Part 2 Unit 14 Lessons ME5-17, 18</td>
</tr>
<tr>
<td>5. Demonstrate an understanding of capacity by: • describing the relationship between mL and L • selecting and justifying referents for mL or L units • estimating capacity, using referents for mL or L • measuring and recording capacity (mL or L). [C, CN, ME, PS, R, V]</td>
<td>Part 2 Unit 14 Lessons ME5-20 to 22</td>
</tr>
</tbody>
</table>

Shape & Space — 3-D Objects and 2-D Shapes

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are: • parallel • intersecting • perpendicular • vertical • horizontal. [C, CN, R, T, V] [ICT: C6–2.2, P5–2.3]</td>
<td>Part 1 Unit 6 Lessons G5-1, G5-5, 8</td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Part 2 Unit 13 Lessons G5-21 to 24</td>
</tr>
</tbody>
</table>
### Shape & Space — 3-D Objects and 2-D Shapes

7. Identify and sort quadrilaterals, including:
   - rectangles
   - squares
   - trapezoids
   - parallelograms
   - rhombuses
   according to their attributes.

   **[C, R, V]**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>G5-1, 2, 5</td>
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<tr>
<td></td>
<td></td>
<td>G5-6, 9 to 11</td>
</tr>
</tbody>
</table>

### Shape & Space — Transformations

**General Outcome**

Describe and analyze position and motion of objects and shapes.

**Specific Outcomes**  
**JUMP Math Lessons**

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
</table>
| 8. Identify and describe a single transformation, including a translation, rotation and reflection of 2-D shapes.  
   **[C, T, V]**  
   **[ICT: C6–2.1]** | **Part** | **Unit** | **Lessons** |
| 2                | 12               | G5-15, 17 to 20 |
| 9. Perform, concretely, a single transformation (translation, rotation or reflection) of a 2-D shape, and draw the image.  
   **[C, CN, T, V]**  
   **[ICT: C6–2.1]** | **Part** | **Unit** | **Lessons** |
| 2                | 12               | G5-15*, 17*, 18*, 19*, 20* |
### Statistics & Probability — Data Analysis

**General Outcome**
Collect, display and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Differentiate between first-hand and second-hand data. [C, R, T, V] [ICT: C1–2.2, P5–2.3]</td>
<td>Part 1 Unit 7 Lessons PDM5-7</td>
</tr>
<tr>
<td>2. Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V] [ICT: C6–2.2, P5–2.3]</td>
<td>Part 1 Unit 7 Lessons PDM5-1, 2</td>
</tr>
</tbody>
</table>

### Statistics & Probability — Chance and Uncertainty

**General Outcome**
Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Describe the likelihood of a single outcome occurring, using words such as: impossible, possible, certain. [C, CN, PS, R]</td>
<td>Part 2 Unit 15 Lessons PDM5-9, 11</td>
</tr>
<tr>
<td>4. Compare the likelihood of two possible outcomes occurring, using words such as: less likely, equally likely, more likely. [C, CN, PS, R]</td>
<td>Part 2 Unit 15 Lessons PDM5-9 to 11</td>
</tr>
</tbody>
</table>
Grade 5 JUMP Math Correlation to the New BC Curriculum

NOTES:

*Italicized* JUMP Math lessons contain prerequisite material required to meet the learning standard.

An asterisk (*) indicates that a JUMP Math lesson covers a curriculum requirement primarily in the lesson plan.

JUMP Math strands are represented by:

- **NS** Number Sense
- **ME** Measurement
- **G** Geometry
- **PA** Patterns and Algebra
- **PDM** Probability and Data Management

### Big Ideas

**Numbers** describe quantities that can be represented by equivalent fractions.

Computational **fluency** and flexibility with numbers extend to operations with larger (multi-digit) numbers.

Identified regularities in number **patterns** can be expressed in tables.

Closed shapes have **area and perimeter** that can be described, measured, and compared.

**Data** represented in graphs can be used to show many-to-one correspondence.

<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>number concepts</strong> to 1 000 000</td>
<td></td>
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<tr>
<td>• counting:</td>
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<tr>
<td>° multiples</td>
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<tr>
<td>° flexible counting strategies</td>
<td></td>
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<tr>
<td>° whole number benchmarks</td>
<td></td>
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</tbody>
</table>

- **Part**
- **Unit**
- **Lessons**

1 1 PA5-4
1 2 NS5-1 to 3, 10, 11
1 3 NS5-20

1 1 PA5-4
1 1 PA5-4
1 1 PA5-4
1 2 NS5-1, 2, 10
<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Numbers to 1 000 000 can be arranged and recognized:</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
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<td></td>
<td>1</td>
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<tr>
<td>° comparing and ordering numbers</td>
<td>Part</td>
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<td></td>
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<tr>
<td></td>
<td>1</td>
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<tr>
<td>° estimating large quantities</td>
<td>Part</td>
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<tr>
<td></td>
<td>1</td>
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<td></td>
<td>1</td>
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<tr>
<td>• place value:</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>° 100 000s, 10 000s, 1000s, 100s, 10s, and 1s</td>
<td>Part</td>
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<tr>
<td>° understanding the relationship between digit places and their value, to 1 000 000</td>
<td>Part</td>
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<td></td>
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<tr>
<td>• First Peoples use unique counting systems (e.g., Tsimshian use of three counting systems, for animals, people and things; Tlingit counting for the naming of numbers e.g., 10 = two hands, 20 = one person)</td>
<td>Part</td>
</tr>
<tr>
<td></td>
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<tr>
<td>decimals to thousandths</td>
<td>Part</td>
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<tr>
<td></td>
<td>2</td>
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<tr>
<td>equivalent fractions</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>whole-number, fraction, and decimal benchmarks</td>
<td>Part</td>
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<td></td>
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<td></td>
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<tr>
<td>• Two equivalent fractions are two ways to represent the same amount (having the same whole).</td>
<td>Part</td>
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<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>• comparing and ordering of fractions and decimals</td>
<td>Part</td>
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<td>Content</td>
<td>JUMP Math Lessons</td>
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<tr>
<td>------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>• addition and subtraction of decimals to thousandths</td>
<td>Part 2 Unit 10 Lessons NS5-54, 55</td>
</tr>
<tr>
<td></td>
<td>Part 2 Unit 11 Lessons NS5-57</td>
</tr>
<tr>
<td>• estimating decimal sums and differences</td>
<td>Part 2 Unit 11 Lessons NS5-58, 59, 62</td>
</tr>
<tr>
<td>• estimating fractions with benchmarks (e.g., zero, half, whole)</td>
<td>Part 2 Unit 9 Lessons NS5-37, 44</td>
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<tr>
<td></td>
<td>Part 2 Unit 10 Lessons NS5-53</td>
</tr>
<tr>
<td>• equal partitioning</td>
<td>Part 2 Unit 9 Lessons NS5-34</td>
</tr>
<tr>
<td>addition and subtraction of whole numbers to 1 000 000</td>
<td>Part 1 Unit 2 Lessons NS5-4 to 7, 12</td>
</tr>
<tr>
<td>• using flexible computation strategies, involving taking apart (e.g., decomposing using friendly numbers and compensating) and combining numbers in a variety of ways, regrouping</td>
<td>Part 1 Unit 2 Lessons NS5-4 to 6</td>
</tr>
<tr>
<td>• estimating sums and differences to 10 000</td>
<td>Part 1 Unit 2 Lessons NS5-12</td>
</tr>
<tr>
<td>• using addition and subtraction in real-life contexts and problem-based situations</td>
<td>Part 1 Unit 2 Lessons NS5-7</td>
</tr>
<tr>
<td>• whole-class number talks</td>
<td>Part 1 Unit 2 Lessons NS5-12</td>
</tr>
<tr>
<td>multiplication and division to three digits, including division with remainders</td>
<td>Part 1 Unit 3 Lessons NS5-14 to 21</td>
</tr>
<tr>
<td></td>
<td>Part 1 Unit 4 Lessons NS5-24 to 33</td>
</tr>
<tr>
<td>• understanding the relationships between multiplication and division, multiplication and addition, and division and subtraction</td>
<td>Part 1 Unit 3 Lessons NS5-14</td>
</tr>
<tr>
<td></td>
<td>Part 1 Unit 4 Lessons NS5-24, 25</td>
</tr>
<tr>
<td>• using flexible computation strategies (e.g., decomposing, distributive principle, commutative principle, repeated addition, repeated subtraction)</td>
<td>Part 1 Unit 3 Lessons NS5-14 to 18, 21</td>
</tr>
<tr>
<td></td>
<td>Part 1 Unit 4 Lessons NS5-24, 28</td>
</tr>
<tr>
<td>• using multiplication and division in real-life contexts and problem-based situations</td>
<td>Part 1 Unit 3 Lessons NS5-18 to 21</td>
</tr>
<tr>
<td></td>
<td>Part 1 Unit 4 Lessons NS5-25, 32, 33</td>
</tr>
</tbody>
</table>
### Content

- **whole-class number talks**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>NS5-16, 17</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>NS5-33</td>
</tr>
</tbody>
</table>

- **addition and subtraction of decimals to thousandths**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<th>Unit</th>
<th>Lessons</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-54, 55</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-56, 57, 59, 62</td>
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</tbody>
</table>

- **estimating decimal sums and differences**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-59, 62</td>
</tr>
</tbody>
</table>

- **using visual models such as base 10 blocks, place-value mats, grid paper, and number lines**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<tbody>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-54</td>
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<tr>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-56</td>
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- **using addition and subtraction in real-life contexts and problem-based situations**

<table>
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<tr>
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<th>Unit</th>
<th>Lessons</th>
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<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-54, 55</td>
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<td>11</td>
<td>NS5-57, 62</td>
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- **whole-class number talks**

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<tr>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-62</td>
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- **addition and subtraction facts to 20 (extending computational fluency)**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<th>Lessons</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>NS5-8 to 10</td>
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</table>

- **Provide opportunities for authentic practice, building on previous grade-level addition and subtraction facts.**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<th>Lessons</th>
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<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>NS5-8 to 10</td>
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</table>

- **applying strategies and knowledge of addition and subtraction facts in real-life contexts and problem-based situations, as well as when making math-to-math connections (e.g., for 800 + 700, you can annex the zeros and use the knowledge of 8 + 7 to find the total)**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
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<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>NS5-8 to 10</td>
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</table>

- **multiplication and division facts to 100 (emerging computational fluency)**

<table>
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<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>PA5-4, 6</td>
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<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>NS5-14, 16 to 19, 21, 22</td>
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<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>NS5-24, 25, 28</td>
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</tbody>
</table>

- **Provide opportunities for concrete and pictorial representations of multiplication.**

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>NS5-14, 17 to 19, 21, 22</td>
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<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
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<td></td>
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</tr>
<tr>
<td>• Use games to provide opportunities for authentic practice of multiplication computations.</td>
<td>1 3 NS5-14</td>
<td></td>
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</tr>
<tr>
<td>• looking for patterns in numbers, such as in a hundred chart, to further develop understanding of multiplication computation</td>
<td>1 1 PA5-6</td>
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<tr>
<td>• Connect multiplication to skip-counting.</td>
<td>1 1 PA5-4</td>
<td></td>
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</tr>
<tr>
<td>• Connecting multiplication to division and repeated addition.</td>
<td>1 3 NS5-14</td>
<td></td>
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</tr>
<tr>
<td>• Memorization of facts is not intended for this level.</td>
<td>1 4 NS5-24, 25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students will become more fluent with these facts.</td>
<td>1 1 PA5-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• using mental math strategies, such as doubling and halving, annexing, and distributive property</td>
<td>1 3 NS5-14, 16, 17, 19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Students should be able to recall many multiplication facts by the end of Grade 5 (e.g., 2s, 3s, 4s, 5s, 10s).</td>
<td>1 3 NS5-14, 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• developing computational fluency with facts to 100</td>
<td>1 3 NS5-14, 16, 17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rules for increasing and decreasing patterns with words, numbers, symbols, and variables</td>
<td>1 1 PA5-1 to 3, 5, 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>one-step equations with variables</td>
<td>2 8 PS5-8, 10</td>
<td></td>
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<tr>
<td>• solving one-step equations with a variable</td>
<td>2 8 PA5-9, 11 to 16</td>
<td></td>
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<tr>
<td>• expressing a given problem as an equation, using symbols (e.g., (4 + X = 15))</td>
<td>2 8 PA5-10</td>
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<tr>
<td></td>
<td>2 8 PA5-11 to 16</td>
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<td>Content</td>
<td>JUMP Math Lessons</td>
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<tr>
<td>area measurement of squares and rectangles</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 14</td>
<td>ME5-14 to 16</td>
<td></td>
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<tr>
<td>relationships between area and perimeter</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 14</td>
<td>ME5-12 to 16</td>
<td></td>
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<tr>
<td>• measuring area of squares and rectangles, using tiles, geoboards, grid paper</td>
<td>Part Unit Lessons</td>
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<tr>
<td>2 14</td>
<td>ME5-14 to 16</td>
<td></td>
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<tr>
<td>• investigating perimeter and area and how they are related to but not dependent on each other</td>
<td>Part Unit Lessons</td>
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<tr>
<td>2 14</td>
<td>ME5-15, 16</td>
<td></td>
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<tr>
<td>• use traditional dwellings</td>
<td>Part Unit Lessons</td>
<td></td>
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<tr>
<td>2 14</td>
<td>ME5-16</td>
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<tr>
<td>• Invite a local Elder or knowledge keeper to talk about traditional measuring and estimating techniques for hunting, fishing, and building.</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
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<tr>
<td>2 14</td>
<td>ME5-15</td>
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<tr>
<td>duration, using measurement of time</td>
<td>Part Unit Lessons</td>
<td></td>
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<tr>
<td>1 5</td>
<td>ME5-5, ME5-6 to 8, 10</td>
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<tr>
<td>• understanding elapsed time and duration</td>
<td>Part Unit Lessons</td>
<td></td>
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<tr>
<td>1 5</td>
<td>ME5-5, ME5-6 to 8, 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• applying concepts of time in real-life contexts and problem-based situations</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 5</td>
<td>ME5-7, 8, 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• daily and seasonal cycles, moon cycles, tides, journeys, events</td>
<td>Part Unit Lessons</td>
<td></td>
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</tr>
<tr>
<td>1 5</td>
<td>ME5-10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>classification of prisms and pyramids</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 6</td>
<td>G5-1, 5, 6, 8 to 11</td>
<td></td>
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</tr>
<tr>
<td>2 13</td>
<td>G5-21 to 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• investigating 3D objects and 2D shapes, based on multiple attributes</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 6</td>
<td>G5-1, 5, 6, 8 to 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 13</td>
<td>G5-21 to 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• describing and sorting quadrilaterals</td>
<td>Part Unit Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 6</td>
<td>G5-5, G5-6, 9 to 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
<td></td>
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<tr>
<td>---------</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>• describing and constructing rectangular and triangular prisms</td>
<td>Part 2 Unit 13 Lessons G5-22 to 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• identifying prisms in the environment</td>
<td>Part 2 Unit 13 Lessons G5-22*, 23*</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>single transformations</strong></td>
<td>Part 2 Unit 12 Lessons G5-15, 17 to 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• single transformations (slide/translation, flip/reflection, turn/rotation)</td>
<td>Part 2 Unit 12 Lessons G5-15, 17 to 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• using concrete materials with a focus on the motion of transformations</td>
<td>Part 2 Unit 12 Lessons G5-15*, 17*, 18*, 19*, 20*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• weaving, cedar baskets, designs</td>
<td>Part 2 Unit 12 Lessons G5-17, 18, 20*</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>one-to-one correspondence and many-to-one correspondence</strong>, using double bar graphs</td>
<td>Part 1 Unit 7 Lessons PDM5-1, 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• many-to-one correspondence: one symbol represents a group or value (e.g., on a bar graph, one square may represent five cookies)</td>
<td>Part 1 Unit 7 Lessons PDM5-1, 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>probability experiments</strong>, single events or outcomes</td>
<td>Part 2 Unit 15 Lessons PDM5-9, 12 to 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• predicting outcomes of independent events (e.g., when you spin using a spinner and it lands on a single colour)</td>
<td>Part 2 Unit 15 Lessons PDM5-12 to 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• predicting single outcomes (e.g., when you spin using a spinner and it lands on a single colour)</td>
<td>Part 2 Unit 15 Lessons PDM5-9, 12 to 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• using spinners, rolling dice, pulling objects out of a bag</td>
<td>Part 2 Unit 15 Lessons PDM5-12 to 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• representing single outcome probabilities using fractions</td>
<td>Part 2 Unit 15 Lessons PDM5-12 to 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>financial literacy</strong> — monetary calculations, including making change with amounts to 1000 dollars and developing simple financial plans</td>
<td>Part 2 Unit 11 Lessons NS5-56, 57, 59, 62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• making monetary calculations, including making change and decimal notation to $1000 in real-life contexts and problem-based situations</td>
<td>Part 2 Unit 11 Lessons NS5-56, 57, 59, 62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
<td></td>
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<tr>
<td>------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>• applying a variety of strategies, such as counting up, counting</td>
<td>Part  Unit  Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>back, and decomposing, to calculate totals and make change</td>
<td>2  11  NS5-57*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• making simple financial plans to meet a financial goal</td>
<td>Part  Unit  Lessons</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>2  11  NS5-57, 62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• developing a budget that takes into account income and expenses</td>
<td>Part  Unit  Lessons</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2  11  NS5-62*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Grade 5 JUMP Math Exemplar Lessons for Curricular Competencies

The Curricular Competencies in the new BC Mathematics curriculum are addressed throughout JUMP Math’s Grade 5 resource. The following table lists a selection of JUMP Math lessons that provide effective illustrations of how each Curricular Competency is addressed.

<table>
<thead>
<tr>
<th>Curricular Competencies</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reasoning and analyzing</strong></td>
<td></td>
</tr>
<tr>
<td>• Use reasoning to explore and make connections</td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  1 PA5-7</td>
</tr>
<tr>
<td></td>
<td>2  10 NS5-53</td>
</tr>
<tr>
<td>• Estimate reasonably</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  5 ME5-8</td>
</tr>
<tr>
<td></td>
<td>2  11 NS5-59</td>
</tr>
<tr>
<td>• Develop mental math strategies and abilities to make sense of quantities</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  4 NS5-28</td>
</tr>
<tr>
<td></td>
<td>2  11 NS5-61</td>
</tr>
<tr>
<td>• Use technology to explore mathematics</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  6 G5-10</td>
</tr>
<tr>
<td></td>
<td>2  12 G5-15</td>
</tr>
<tr>
<td>• Model mathematics in contextualized experiences</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  5 ME5-10</td>
</tr>
<tr>
<td></td>
<td>2  9 NS5-44</td>
</tr>
<tr>
<td><strong>Understanding and solving</strong></td>
<td></td>
</tr>
<tr>
<td>• Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  2 NS5-7</td>
</tr>
<tr>
<td></td>
<td>2  12 G5-15</td>
</tr>
<tr>
<td>• Visualize to explore mathematical concepts</td>
<td></td>
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<td></td>
<td>Part  Unit Lessons</td>
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<tr>
<td></td>
<td>1  6 G5-8</td>
</tr>
<tr>
<td></td>
<td>2  13 G5-24</td>
</tr>
<tr>
<td>• Develop and use multiple strategies to engage in problem solving</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part  Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1  4 NS5-33</td>
</tr>
<tr>
<td></td>
<td>2  14 ME5-16</td>
</tr>
</tbody>
</table>
### Curricular Competencies

<table>
<thead>
<tr>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engage in problem-solving experiences that are <strong>connected</strong> to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures</td>
<td>1</td>
<td>2</td>
<td>NS5-1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>12</td>
<td>G5-20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Communicating and representing</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Communicate</strong> mathematical thinking in many ways</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Use mathematical vocabulary and language to contribute to mathematical discussions</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
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<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td><strong>Explain and justify</strong> mathematical ideas and decisions</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Represent mathematical ideas in <strong>concrete, pictorial, and symbolic forms</strong></td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connecting and reflecting</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reflect</strong> on mathematical thinking</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Connect mathematical concepts to each other and to <strong>other areas and personal interests</strong></td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td><strong>Incorporate</strong> First Peoples worldviews and perspectives to make <strong>connections</strong> to mathematical concepts</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
Grade 5 JUMP Math Correlation to the Manitoba Curriculum

NOTES:

*Italicized* JUMP Math lessons contain prerequisite material required to meet the learning standard.

JUMP Math strands are represented by:

- **NS** Number Sense
- **ME** Measurement
- **G** Geometry
- **PA** Patterns and Algebra
- **PDM** Probability and Data Management

### Number

**General Learning Outcome**

Develop number sense.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5.N.1</strong> Represent and describe whole numbers to 1 000 000. [C, CN, T, V]</td>
<td>Part 1, Unit 2: NS5-1 to 7; Part 1, Unit 3: NS5-15</td>
</tr>
<tr>
<td><strong>5.N.2</strong> Apply estimation strategies, including • front-end rounding • compensation • compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]</td>
<td>Part 1, Unit 2: NS5-8, 9 NS5-10 to 12; Part 1, Unit 3: NS5-20</td>
</tr>
<tr>
<td><strong>5.N.3</strong> Apply mental math strategies to determine multiplication and related division facts to 81 (9 × 9). [C, CN, ME, R, V]</td>
<td>Part 1, Unit 1: PA5-4, 6; Part 1, Unit 3: NS5-14, 16, 17; Part 1, Unit 4: NS5-24, 28</td>
</tr>
<tr>
<td><strong>5.N.4</strong> Apply mental mathematics strategies for multiplication, such as • annexing then adding zeros • halving and doubling • using the distributive property [C, ME, R]</td>
<td>Part 1, Unit 3: NS5-14 to 17</td>
</tr>
</tbody>
</table>

Recall of multiplication facts to 81 and related division facts is expected by the end of Grade 5.
<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.N.5</td>
<td>Demonstrate an understanding of multiplication (1- and 2-digit multiples and up to 4-digit multiplicands), concretely, pictorially, and symbolically, by • using personal strategies • using the standard algorithm • estimating products to solve problems. [C, CN, ME, PS, V]</td>
<td>1</td>
<td>3</td>
<td>NS5-15, 18 to 22</td>
</tr>
<tr>
<td>5.N.6</td>
<td>Demonstrate an understanding of division (1- and 2-digit divisors and up to 4-digit dividends), concretely, pictorially, and symbolically, and interpret remainders by • using personal strategies • using the standard algorithm • estimating quotients to solve problems. [C, CN, ME, PS]</td>
<td>1</td>
<td>4</td>
<td>NS5-25 to 33</td>
</tr>
<tr>
<td>5.N.7</td>
<td>Demonstrate an understanding of fractions by using concrete and pictorial representations to • create sets of equivalent fractions • compare fractions with like and unlike denominators [C, CN, PS, R, V]</td>
<td>2</td>
<td>9</td>
<td>NS5-34 to 40, 44</td>
</tr>
<tr>
<td>5.N.8</td>
<td>Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically. [C, CN, R, V]</td>
<td>2</td>
<td>10</td>
<td>NS5-47, 48, 51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-56, 62</td>
</tr>
<tr>
<td>5.N.9</td>
<td>Relate decimals to fractions (tenths, hundredths, thousandths). [CN, R, V]</td>
<td>2</td>
<td>10</td>
<td>NS5-48, 50 to 53</td>
</tr>
<tr>
<td>5.N.10</td>
<td>Compare and order decimals (tenths, hundredths, thousandths) by using • benchmarks • place value • equivalent decimals [CN, R, V]</td>
<td>2</td>
<td>10</td>
<td>NS5-50, 53</td>
</tr>
<tr>
<td>5.N.11</td>
<td>Demonstrate an understanding of addition and subtraction of decimals (to thousandths), concretely, pictorially, and symbolically, by • using personal strategies • using the standard algorithms • using estimation • solving problems [C, CN, ME, PS, R, V]</td>
<td>1</td>
<td>2</td>
<td>NS5-5 to 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-54, 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-57 to 59, 62</td>
</tr>
</tbody>
</table>
## Patterns and Relations (Patterns)

### General Learning Outcome

Use patterns to describe the world and solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.PR.1 Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

## Patterns and Relations (Variables and Equations)

### General Learning Outcome

Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.PR.2 Solve problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients, and whole-number solutions. [C, CN, PS, R]</td>
<td>Part</td>
</tr>
</tbody>
</table>
|                             | 2 | 8 | PA5-8  
PA5-9, 12 to 16 |
Shape and Space (Measurement)

**General Learning Outcome**

Use direct or indirect measurement to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5.SS.1</strong> Design and construct different rectangles given either perimeter or area, or both (whole numbers), and draw conclusions. [C, CN, PS, R, V]</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 5  <strong>Lessons</strong> ME5-3  2  14  ME5-12 ME5-13 to 16</td>
</tr>
<tr>
<td><strong>5.SS.2</strong> Demonstrate an understanding of measuring length (mm) by  • selecting and justifying referents for the unit mm  • modelling and describing the relationship between mm and cm units, and between mm and m units [C, CN, ME, PS, R, V]</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 5  <strong>Lessons</strong> ME5-1, 2, 4</td>
</tr>
<tr>
<td><strong>5.SS.3</strong> Demonstrate an understanding of volume by  • selecting and justifying referents for cm³ or m³ units  • estimating volume by using referents for cm³ or m³  • measuring and recording volume (cm³ or m³)  • constructing rectangular prisms for a given volume [C, CN, ME, PS, R, V]</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 14  <strong>Lessons</strong> ME5-17, 18</td>
</tr>
<tr>
<td><strong>5.SS.4</strong> Demonstrate an understanding of capacity by  • describing the relationship between mL and L  • selecting and justifying referents for mL or L units  • estimating capacity by using referents for mL or L  • measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 14  <strong>Lessons</strong> ME5-20 to 22</td>
</tr>
</tbody>
</table>

Shape and Space (3-D Objects and 2-D Shapes)

**General Learning Outcome**

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5.SS.5</strong> Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes, that are  • parallel  • intersecting  • perpendicular  • vertical  • horizontal [C, CN, R, T, V]</td>
<td><strong>Part</strong> 1  <strong>Unit</strong> 6  <strong>Lessons</strong> G5-1 G5-5, 8  2  13  G5-21 to 24</td>
</tr>
</tbody>
</table>
### Shape and Space (3-D Objects and 2-D Shapes)

| 5.SS.6 | Identify and sort quadrilaterals, including  
|        | • rectangles  
|        | • squares  
|        | • trapezoids  
|        | • parallelograms  
|        | • rhombuses according to their attributes.  
|        | [C, R, V] | \[C, R, V\] |
| Part | Unit | Lessons |
| 1   | 6    | G5-1, 2, 5  
|     |      | G5-6, 9 to 11 |

### Shape and Space (Transformations)

#### General Learning Outcome

Describe and analyze position and motion of objects and shapes.

#### Specific Learning Outcomes

| 5.SS.7 | Perform a single transformation (translation, rotation, or reflection) of a 2-D shape, and draw and describe the image.  
|        | [C, CN, T, V] | \[C, CN, T, V\] |
| Part | Unit | Lessons |
| 2   | 12   | G5-15, 17 to 20 |

| 5.SS.8 | Identify a single transformation (translation, rotation, or reflection) of 2-D shapes.  
|        | [C, T, V] | \[C, T, V\] |
| Part | Unit | Lessons |
| 2   | 12   | G5-15, 17 to 20 |
### Statistics and Probability (Data Analysis)

**General Learning Outcome**
Collect, display, and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.SP.1 Differentiate between first-hand and second-hand data.</td>
<td>Part 1, Unit 7, Lessons PDM5-7</td>
</tr>
<tr>
<td>[C, R, T, V]</td>
<td></td>
</tr>
<tr>
<td>5.SP.2 Construct and interpret double bar graphs to draw conclusions.</td>
<td>Part 1, Unit 7, Lessons PDM5-1, 2</td>
</tr>
<tr>
<td>[C, PS, R, T, V]</td>
<td></td>
</tr>
</tbody>
</table>

### Statistics and Probability (Chance and Uncertainty)

**General Learning Outcome**
Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.SP.3 Describe the likelihood of a single outcome occurring, using words such as • impossible • possible • certain</td>
<td>Part 2, Unit 15, Lessons PDM5-9, 11</td>
</tr>
<tr>
<td>[C, CN, PS, R]</td>
<td></td>
</tr>
<tr>
<td>5.SP.4 Compare the likelihood of two possible outcomes occurring, using words such as • less likely • equally likely • more likely</td>
<td>Part 2, Unit 15, Lessons PDM5-9 to 11</td>
</tr>
<tr>
<td>[C, CN, PS, R]</td>
<td></td>
</tr>
</tbody>
</table>
Grade 5 JUMP Math Correlation to the Ontario Curriculum

NOTES:

Underlined JUMP Math lessons are review from a previous grade.

Italicized JUMP Math lessons contain prerequisite material required to meet the learning standard.

An asterisk (*) indicates when a JUMP Math lesson covers a curriculum requirement primarily in the lesson plan.

Expectation codes source: Ontario Curriculum Unit Planner

JUMP Math strands are represented by:

NS  Number Sense
ME  Measurement
G   Geometry
PA  Patterns and Algebra
PDM Probability and Data Management

### Number Sense and Numeration

#### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m8</td>
<td>read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers;</td>
</tr>
<tr>
<td>5m9</td>
<td>demonstrate an understanding of magnitude by counting forward and backwards by 0.01;</td>
</tr>
<tr>
<td>5m10</td>
<td>solve problems involving the multiplication and division of multi-digit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies;</td>
</tr>
<tr>
<td>5m11</td>
<td>demonstrate an understanding of proportional reasoning by investigating whole-number rates.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

<table>
<thead>
<tr>
<th>Quantity Relationships</th>
<th>JUMP Math Lessons</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5m12</td>
<td>represent, compare, and order whole numbers from 0.01 to 100 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>NS5-1 to 3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-46 to 50, 52, 53</td>
</tr>
</tbody>
</table>
### Number Sense and Numeration

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m13</td>
<td>demonstrate an understanding of place value in whole numbers and decimal numbers from 0.01 to 100,000, using a variety of tools and strategies (e.g., use numbers to represent $23,011$ as $20,000 + 3000 + 0 + 10 + 1$; use base ten materials to represent the relationship between 1, 0.1, and 0.01) (<strong>Sample problem</strong>: How many thousands cubes would be needed to make a base ten block for 100,000?)</td>
<td>1</td>
<td>2</td>
<td>NS5-1 to 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-50, 52, 53</td>
</tr>
<tr>
<td>5m14</td>
<td>read and print in words whole numbers to ten thousand, using meaningful contexts (e.g., newspapers, magazines)</td>
<td>1</td>
<td>2</td>
<td>NS5-1, 2</td>
</tr>
<tr>
<td>5m15</td>
<td>round decimal numbers to the nearest tenth, in problems arising from real-life situations</td>
<td>2</td>
<td>11</td>
<td>NS5-58, 59, 62</td>
</tr>
<tr>
<td>5m16</td>
<td>represent, compare, and order fractional amounts with like denominators including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, number lines) and using standard fractional notation</td>
<td>2</td>
<td>9</td>
<td>NS5-34 to 37, 41, 42, 44</td>
</tr>
<tr>
<td>5m17</td>
<td>demonstrate and explain the concept of equivalent fractions, using concrete materials (e.g., use fraction strips to show that $\frac{3}{4}$ is equal to $\frac{9}{12}$)</td>
<td>2</td>
<td>9</td>
<td>NS5-39, 44</td>
</tr>
<tr>
<td>5m18</td>
<td>demonstrate and explain equivalent representations of a decimal number, using concrete materials and drawings (e.g., use base ten materials to show that three tenths [0.3] is equal to thirty hundredths [0.03])</td>
<td>2</td>
<td>10</td>
<td>NS5-50</td>
</tr>
<tr>
<td>5m19</td>
<td>read and write money amounts to $1000$ (e.g., $455.35$ is 455 dollars and 35 cents, or four hundred fifty-five dollars and thirty-five cents)</td>
<td>2</td>
<td>11</td>
<td>NS5-56</td>
</tr>
<tr>
<td>5m20</td>
<td>solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 100,000 (<strong>Sample problem</strong>: How many boxes hold 100,000 sheets of paper, if one box holds 8 packages of paper, and one package of paper contains 500 sheets of paper?)</td>
<td>1</td>
<td>2</td>
<td>NS5-4 to 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>NS5-15, 20</td>
</tr>
</tbody>
</table>
### Number Sense and Numeration

#### Counting

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m21 count forward by hundredths from any decimal number expressed to two decimal places, using concrete materials and number lines (e.g., use base ten materials to represent 2.96 and count forward by hundredths: 2.97, 2.98, 2.99, 3.00, 3.01, …; &quot;Two and ninety-six hundredths, two and ninety-seven hundredths, two and ninety-eight hundredths, two and ninety-nine hundredths, three, three and one hundredth, …&quot;)</td>
<td>2</td>
<td>10</td>
<td>NS5-48</td>
</tr>
</tbody>
</table>

#### Operational Sense

<table>
<thead>
<tr>
<th>JUMP Math Lessons</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m22 solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies (e.g., use the commutative property: $5 \times 18 \times 2 = 5 \times 2 \times 18$, which gives $10 \times 18 = 180$);</td>
<td>1</td>
<td>2</td>
<td>NS5-8, 9, 12</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>NS5-15 to 17</td>
</tr>
<tr>
<td>5m23 add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms (e.g., use $10 \times 10$ grids to add 2.45 and 3.25);</td>
<td>1</td>
<td>2</td>
<td>NS5-4 to 6, 10, 11</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>NS5-54, 55</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11</td>
<td>NS5-56, 57, 59, 62</td>
</tr>
<tr>
<td>5m24 multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms;</td>
<td>1</td>
<td>3</td>
<td>NS5-15, 18, 20, 21</td>
</tr>
<tr>
<td>5m25 divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms;</td>
<td>1</td>
<td>4</td>
<td>NS5-28, 30 to 33</td>
</tr>
<tr>
<td>5m26 multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule);</td>
<td>2</td>
<td>11</td>
<td>NS5-60, 61</td>
</tr>
<tr>
<td>5m27 use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.</td>
<td>1</td>
<td>2</td>
<td>NS5-12</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>NS5-20, 21</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>NS5-29, 32, 33</td>
</tr>
</tbody>
</table>
### Number Sense and Numeration

<table>
<thead>
<tr>
<th>Proportional Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5m28</strong> describe multiplicative relationships between quantities by using simple fractions and decimals (e.g., “If you have 4 plums and I have 6 plums, I can say that I have $1 \frac{1}{2}$ or 1.5 times as many plums as you have.”);</td>
<td>Part</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td><strong>5m29</strong> determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms (e.g., use a 10 × 10 grid to show that $\frac{2}{5} = \frac{40}{100}$, which can also be represented as 0.4);</td>
<td>Part</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td><strong>5m30</strong> demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation using concrete materials and drawings (<em>Sample problem:</em> If 2 books cost $6, how would you calculate the cost of 8 books?);</td>
<td>Part</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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<tr>
<td>2</td>
<td>8</td>
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<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>
# Measurement

## Overall Expectations

5m31 estimate, measure, and record perimeter, area, temperature change, and elapsed time, using a variety of strategies;

5m32 determine the relationships among units and measurable attributes, including the area of a rectangle and the volume of a rectangular prism.

## Specific Expectations

### Attributes, Units, and Measurement Sense

<table>
<thead>
<tr>
<th>Specific Expectation</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m33 estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest second;</td>
<td>Part</td>
</tr>
<tr>
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</tr>
<tr>
<td>5m34 estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in minutes, hours, days, weeks, months, or years (Sample problem: You are travelling from Toronto to Montreal by train. If the train departs Toronto at 11:30 a.m. and arrives in Montreal at 4:56 p.m., how long will you be on the train?);</td>
<td>Part</td>
</tr>
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</tr>
<tr>
<td>5m35 measure and record temperatures to determine and represent temperature changes over time (e.g., record temperature changes in an experiment or over a season) (Sample problem: Investigate the relationship between weather, climate, and temperature changes over time in different locations.);</td>
<td>Part</td>
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</tr>
<tr>
<td>5m36 estimate and measure the perimeter and area of regular and irregular polygons, using a variety of tools (e.g., grid paper, geoboard, dynamic geometry software) and strategies.</td>
<td>Part</td>
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<td>2</td>
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</tbody>
</table>

### Measurement Relationships

<table>
<thead>
<tr>
<th>Specific Expectation</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m37 select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure length, height, width, and distance, and to measure the perimeter of various polygons;</td>
<td>Part</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>5m38 solve problems requiring conversion from metres to centimetres and from kilometres to metres (Sample problem: Describe the multiplicative relationship between the number of centimetres and the number of metres that represent a length. Use this relationship to convert 5.1 m to centimetres.);</td>
<td>Part</td>
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<tr>
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<tr>
<td>5m39 solve problems involving the relationship between a 12-hour clock and a 24-hour clock (e.g., 15:00 is 3 hours after 12 noon, so 15:00 is the same as 3:00 p.m.);</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Measurement</td>
<td></td>
</tr>
<tr>
<td>-------------</td>
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</tr>
<tr>
<td>5m40 create, through investigation using a variety of tools (e.g., pattern blocks, geoboard, grid paper) and strategies, two-dimensional shapes with the same perimeter or the same area (e.g., rectangles and parallelograms with the same base and the same height) (Sample problem: Using dot paper, how many different rectangles can you draw with a perimeter of 12 units? with an area of 12 square units?);</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
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<td>2</td>
<td>14</td>
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<tr>
<td>5m41 determine, through investigation using a variety of tools (e.g., concrete materials, dynamic geometry software, grid paper) and strategies (e.g., building arrays), the relationships between the length and width of a rectangle and its area and perimeter, and generalize to develop the formulas [i.e., Area = length × width; Perimeter = (2 × length) + (2 × width)];</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>5m42 solve problems requiring the estimation and calculation of perimeters and areas of rectangles (Sample problem: You are helping to fold towels, and you want them to stack nicely. By folding across the length and/or the width, you fold each towel a total of three times. You want the shape of each folded towel to be as close to a square as possible. Does it matter how you fold the towels?);</td>
<td></td>
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<tr>
<td>Part</td>
<td>Unit</td>
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<td>14</td>
</tr>
<tr>
<td>5m43 determine, through investigation, the relationship between capacity (i.e., the amount a container can hold) and volume (i.e., the amount of space taken up by an object), by comparing the volume of an object with the amount of liquid it can contain or displace (e.g., a bottle has a volume, the space it takes up, and a capacity, the amount of liquid it can hold) (Sample problem: Compare the volume and capacity of a thin-walled container in the shape of a rectangular prism to determine the relationship between units for measuring capacity [e.g., millilitres] and units for measuring volume [e.g., cubic centimetres].);</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>5m44 determine, through investigation using stacked congruent rectangular layers of concrete materials, the relationship between the height, the area of the base, and the volume of a rectangular prism, and generalize to develop the formula (i.e., Volume = area of base × height) (Sample problem: Create a variety of rectangular prisms using connecting cubes. For each rectangular prism, record the area of the base, the height, and the volume on a chart. Identify relationships.);</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>5m45 select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram, tonne).</td>
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</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
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<td>2</td>
<td>14</td>
</tr>
</tbody>
</table>
### Geometry and Spatial Sense

#### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m46</td>
<td>identify and classify two-dimensional shapes by side and angle properties, and compare and sort three-dimensional figures;</td>
</tr>
<tr>
<td>5m47</td>
<td>identify and construct nets of prisms and pyramids;</td>
</tr>
<tr>
<td>5m48</td>
<td>identify and describe the location of an object, using the cardinal directions, and translate two-dimensional shapes.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometric Properties</strong></td>
<td><strong>JUMP Math Lessons</strong></td>
</tr>
<tr>
<td>5m49 distinguish among polygons, regular polygons, and other two-dimensional shapes;</td>
<td>Part 1 Unit 6 G5-2, 6</td>
</tr>
<tr>
<td>5m50 distinguish among prisms, right prisms, pyramids, and other three-dimensional figures;</td>
<td>Part 2 Unit 13 G5-21 to 23</td>
</tr>
<tr>
<td>5m51 identify and classify acute, right, obtuse, and straight angles;</td>
<td>Part 1 Unit 6 G5-1, 3, 4</td>
</tr>
<tr>
<td>5m52 measure and construct angles up to 90°, using a protractor;</td>
<td>Part 1 Unit 6 G5-3, 4</td>
</tr>
<tr>
<td>5m53 identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral), and classify them according to angle and side properties;</td>
<td>Part 1 Unit 6 G5-5, G5-7</td>
</tr>
<tr>
<td>5m54 construct triangles, using a variety of tools (e.g., protractor, compass, dynamic geometry software), given acute or right angles and side measurements</td>
<td>Part 1 Unit 6 G5-4, 7</td>
</tr>
</tbody>
</table>

#### Geometric Relationships

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m55</td>
<td>identify prisms and pyramids from their nets;</td>
</tr>
<tr>
<td>5m56</td>
<td>construct nets of prisms and pyramids, using a variety of tools (e.g., grid paper, isometric dot paper, Polydrons, computer application).</td>
</tr>
</tbody>
</table>

#### Location and Movement

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m57</td>
<td>locate an object using the cardinal directions (i.e., north, south, east, west) and a coordinate system (e.g., “If I walk 5 steps north and 3 steps east, I will arrive at the apple tree.”);</td>
</tr>
</tbody>
</table>
### Geometry and Spatial Sense

<table>
<thead>
<tr>
<th>5m58</th>
<th>compare grid systems commonly used on maps (i.e., the use of numbers and letters to identify an area; the use of a coordinate system based on the cardinal directions to describe a specific location);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
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<tr>
<td>2</td>
<td>12</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>5m59</th>
<th>identify, perform, and describe translations, using a variety of tools (e.g., geoboard, dot paper, computer program);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
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<tr>
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<td>12</td>
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</tbody>
</table>

| 5m60 | create and analyse designs by translating and/or reflecting a shape, or shapes, using a variety of tools (e.g., geoboard, grid paper, computer program)  
(*Sample problem*: Identify translations and/or reflections that map congruent shapes onto each other in a given design). |
<table>
<thead>
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<tbody>
<tr>
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<td>Unit</td>
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<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>
# Patterning and Algebra

## Overall Expectations

5m61 determine, through investigation using a table of values, relationships in growing and shrinking patterns, and investigate repeating patterns involving translations;

5m62 demonstrate, through investigation, an understanding of the use of variables in equations.

## Specific Expectations

### Patterns and Relationships

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m63</td>
<td>create, identify, and extend numeric and geometric patterns, using a variety of tools (e.g., concrete materials, paper and pencil, calculators, spreadsheets);</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>5m64</td>
<td>build a model to represent a number pattern presented in a table of values that shows the term number and the term;</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>5m65</td>
<td>make a table of values for a pattern that is generated by adding or subtracting a number (i.e., a constant) to get the next term, or by multiplying or dividing by a constant to get the next term, given either the sequence (e.g., 12, 17, 22, 27, 32, ...) or the pattern rule in words (e.g., start with 12 and add 5 to each term to get the next term);</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>5m66</td>
<td>make predictions related to growing and shrinking geometric and numeric patterns (Sample problem: Create growing L's using tiles. The first L has 3 tiles, the second L has 5 tiles, the third L has 7 tiles, and so on. Predict the number of tiles you would need to build the 10th L in the pattern.);</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>5m67</td>
<td>extend and create repeating patterns that result from translations, through investigation using a variety of tools (e.g., pattern blocks, dynamic geometry software, dot paper).</td>
<td>Part Unit Lessons</td>
</tr>
</tbody>
</table>

### Variables, Expressions, and Equations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m68</td>
<td>demonstrate, through investigation, an understanding of variables as changing quantities, given equations with letters or other symbols that describe relationships involving simple rates (e.g., the equations $C = 3 \times n$ and $3 \times n = C$ both represent the relationship between the total cost ($C$), in dollars, and the number of sandwiches purchased ($n$), when each sandwich costs $3$);</td>
<td>Part Unit Lessons</td>
</tr>
</tbody>
</table>
### Patterning and Algebra

<table>
<thead>
<tr>
<th>5m69</th>
<th>demonstrate, through investigation, an understanding of variables as unknown quantities represented by a letter or other symbol (e.g., $12 = 5 + \square$ or $12 = 5 + s$ can be used to represent the following situation: “I have 12 stamps altogether and 5 of them are from Canada. How many are from other countries?”);</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5m70</th>
<th>determine the missing number in equations involving addition, subtraction, multiplication, or division and one- or two-digit numbers, using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (Sample problem: What is the missing number in the equation $8 = 88 ÷ \square$?);</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>
# Data Management and Probability

## Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5m71</td>
<td>Collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including broken-line graphs;</td>
</tr>
<tr>
<td>5m72</td>
<td>Read, describe, and interpret primary data and secondary data presented in charts and graphs, including broken-line graphs;</td>
</tr>
<tr>
<td>5m73</td>
<td>Represent as a fraction the probability that a specific outcome will occur in a simple probability experiment, using systematic lists and area models.</td>
</tr>
</tbody>
</table>

## Specific Expectations

### Collection and Organization of Data

<table>
<thead>
<tr>
<th>Expectation</th>
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</tr>
</thead>
<tbody>
<tr>
<td>5m74</td>
<td>Distinguish between discrete data (i.e., data organized using numbers that have gaps between them, such as whole numbers, and often used to represent a count, such as the number of times a word is used) and continuous data (i.e., data organized using all numbers on a number line that fall within the range of the data, and used to represent measurements such as heights or ages of trees);</td>
</tr>
<tr>
<td>5m75</td>
<td>Collect data by conducting a survey or an experiment (e.g., gather and record air temperature over a two-week period) to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;</td>
</tr>
<tr>
<td>5m76</td>
<td>Collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including broken-line graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales that suit the range and distribution of the data (e.g., to represent precipitation amounts ranging from 0 mm to 50 mm over the school year, use a scale of 5 mm for each unit on the vertical axis and show months on the horizontal axis), using a variety of tools (e.g., graph paper, simple spreadsheets, dynamic statistical software);</td>
</tr>
<tr>
<td>5m77</td>
<td>Demonstrate an understanding that sets of data can be samples of larger populations (e.g., to determine the most common shoe size in your class, you would include every member of the class in the data; to determine the most common shoe size in Ontario for your age group, you might collect a large sample from classes across the province);</td>
</tr>
<tr>
<td>5m78</td>
<td>Describe, through investigation, how a set of data is collected (e.g., by survey, measurement, observation) and explain whether the collection method is appropriate.</td>
</tr>
</tbody>
</table>

### JUMP Math Lessons

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>PDM5-3</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>PDM5-7</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>PDM5-14</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>PDM5-1, 2, 4, 7, 8</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>PDM5-8</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>PDM5-7, 8</td>
</tr>
</tbody>
</table>
### Data Management and Probability

<table>
<thead>
<tr>
<th>Data Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5m79</strong> read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., precipitation or temperature data in the newspaper, data from the Internet about heights of buildings and other structures), presented in charts, tables, and graphs (including broken-line graphs);</td>
<td>Part 1 Unit 7 Lessons PDM5-1, 2, 4, 7, 8</td>
</tr>
<tr>
<td><strong>5m80</strong> calculate the mean for a small set of data and use it to describe the shape of the data set across its range of values, using charts, tables, and graphs (e.g., “The data values fall mainly into two groups on both sides of the mean.”; “The set of data is not spread out evenly around the mean.”);</td>
<td>Part 1 Unit 7 Lessons PDM5-5, 6</td>
</tr>
<tr>
<td><strong>5m81</strong> compare similarities and differences between two related sets of data, using a variety of strategies (e.g., by representing the data using tally charts, stem-and-leaf plots, double bar graphs, or broken-line graphs; by determining measures of central tendency [i.e., mean, median, and mode]; by describing the shape of a data set across its range of values).</td>
<td>Part 1 Unit 7 Lessons PDM5-2, 5, 6</td>
</tr>
</tbody>
</table>

### Probability

<table>
<thead>
<tr>
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<th>JUMP Math Lessons</th>
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</thead>
<tbody>
<tr>
<td><strong>5m82</strong> determine and represent all the possible outcomes in a simple probability experiment (e.g., when tossing a coin, the possible outcomes are heads and tails; when rolling a number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6), using systematic lists and area models (e.g., a rectangle is divided into two equal areas to represent the outcomes of a coin toss experiment);</td>
<td>Part 2 Unit 15 Lessons PDM5-9, 13 to 15</td>
</tr>
<tr>
<td><strong>5m83</strong> represent, using a common fraction, the probability that an event will occur in simple games and probability experiments (e.g., “My spinner has four equal sections and one of those sections is coloured red. The probability that I will land on red is $\frac{1}{4}$.”);</td>
<td>Part 2 Unit 15 Lessons PDM5-12 to 15</td>
</tr>
<tr>
<td><strong>5m84</strong> pose and solve simple probability problems, and solve them by conducting probability experiments and selecting appropriate methods of recording the results (e.g., tally chart, line plot, bar graph).</td>
<td>Part 2 Unit 15 Lessons PDM5-15</td>
</tr>
</tbody>
</table>