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Unit 9  Number Sense: Adding and Subtracting Decimals

Introduction
This unit focuses on place value and adding and subtracting decimals. It describes how to:

• extend the place value system to decimals;
• add and subtract decimals up to the thousandths place using multi-digit addition and subtraction;
• round decimals to the nearest one (nearest whole number), tenth, hundredth, or thousandth; and
• round to the nearest whole number, tenth, or hundredth to estimate sums and differences.

Meeting Your Curriculum

ALBERTA
Required  NS6-40, 45 to 47  including Extensions 1 and 2 in NS6-40
Recommended NS6-38 to 39, 44  supports material in later lessons
Optional  NS6-41 to 43

BRITISH COLUMBIA
Required  NS6-40 to 47
Recommended  NS6-38 to 39  supports material in later lessons

MANITOBA
Required  NS6-40  including Extensions 1 and 2 in NS6-40
Recommended  NS6-38 to 39, 44 to 47  supports material in later lessons
Optional  NS6-41 to 43

ONTARIO
Required  NS6-38, 40 to 47  including Extension 1 in NS6-43
Recommended  NS6-39  supports material in later lessons

Mental Math Minutes
The mental math minutes in this unit:
• practise making equivalent fractions to order decimals and strategies for adding decimals

Generic BLMs
The Generic BLM used in this unit is:
BLM 1 cm Grid Paper (p. T-1)
This BLM can be found in Section T.
## Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
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<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>NS6-38 to 40</td>
<td>NS6-38 to 43</td>
<td>NS6-38 to 40</td>
<td>NS6-38 to 43</td>
</tr>
<tr>
<td>Quiz</td>
<td>NS6-44 to 47</td>
<td>NS6-44 to 47</td>
<td>NS6-44 to 47</td>
<td>NS6-44 to 47</td>
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<tr>
<td>Test</td>
<td>NS6-40, 45 to 47</td>
<td>NS6-40 to 47</td>
<td>NS6-40</td>
<td>NS6-38, 40 to 47</td>
</tr>
</tbody>
</table>
Goals
Students will use decimal notation for fractions with denominators 10 and 100, place decimal hundredths on number lines, and order decimal hundredths using a number line.

PRIOR KNOWLEDGE REQUIRED
Knows that, on number lines, greater whole numbers appear to the right of lesser whole numbers
Can name fractions from area models and number lines

MATERIALS
overhead projector
transparency of BLM Hundredths Number Lines (p. L-45)
BLM Hundredths Number Lines (p. L-45)

Mental math minute. Write on the board:
\[
\frac{2}{6} = \frac{1}{?}
\]
SAY: We can make equivalent fractions by halving the numerator and denominator. This is similar to halving both numbers in a division statement; the quotient does not change. ASK: Half of 2 is 1, so what is half of 6? (3) Replace the question mark with 3 in the equation.

Exercises: Find the number that makes the fractions equivalent.

a) \(\frac{4}{10} = \frac{?}{5}\)  
b) \(\frac{14}{8} = \frac{?}{4}\)  
c) \(\frac{44}{26} = \frac{?}{13}\)  
d) \(\frac{124}{436} = \frac{62}{?}\)

Answers: a) 2, b) 7, c) 22, d) 218

Review decimal tenths. Remind students that the fraction 1/10 can be represented in various ways. Show three ways on the board:

\[
\frac{1}{10}
\]

Point out that each way means 1 part out of 10 equal parts. Remind students that mathematicians have invented an even easier way to write one tenth, called decimal notation. Show this on the board:

\[
\frac{1}{10} = 0.1
\]
SAY: The dot is called a *decimal point*. Mathematicians use decimal notation because it takes up less space on the page and is easier to write. Ask volunteers to show how they would write the following numbers: 2 tenths (0.2), 3 tenths (0.3), and other numbers up to 9 tenths (0.9).

Representing decimal tenths on a number line. Draw a number line from 0 to 1 and ask students to place various decimal tenths on the number line (0.8, 0.5, 0.2, 0.7).

Exercises: Write the decimal for each marked point.

```
  0   *   *   *   *   |   1
```

Answers: 0.3, 0.4, 0.9

Representing decimal tenths using pictures. Draw various shapes, such as circles, squares, or rectangles, and have volunteers represent various numbers given in decimal notation by shading the pictures.

a) 0.2 b) 0.3 c) 0.5 d) 0.6

Writing decimal notation for pictures. Now ask students to do the reverse.

Exercises: Write the decimal for the shaded part of the picture.

a) b) c) d)

Answers: a) 0.8, b) 0.7, c) 0.5, d) 0.8

Introduce decimal hundredths. Tell students that the fraction 1/100 can also be represented in various ways. Show four ways on the board:

```
\[
\frac{1}{100} \quad \text{one hundredth} \quad 0.01
\]
```

Point out how a hundredth is written differently from a tenth: there are two digits after the decimal point instead of only one. Ask a volunteer to show how they would write two hundredths as a decimal (0.02) and then read it as “zero point zero two.” ASK: How would you write three hundredths as a decimal? (0.03).
Exercises: Write the fraction as a decimal.

a) \( \frac{9}{100} \)  

b) \( \frac{4}{100} \)  

c) \( \frac{8}{100} \)  

d) \( \frac{7}{100} \)  

e) \( \frac{5}{100} \)

Answers: a) 0.09, b) 0.04, c) 0.08, d) 0.07, e) 0.05

Writing equivalent tenths and hundredths as fractions and decimals.

Draw on the board:

\[
\begin{array}{ccc}
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\end{array}
\]

\[
\frac{10}{100}
\]

ASK: How many tenths are shaded? (3) Fill in the first numerator.

SAY: Each column is one tenth, and three of them are shaded.

ASK: How many hundredths are shaded? (30) PROMPT: How many hundredths are in each column? (10) So there are 10, 20, 30 hundredths shaded. Fill in the second numerator.

ASK: How would you write 3 tenths as a decimal? (0.3) How would you write 30 hundredths as a decimal? (0.30)

Write on the board:

\[0.3 = 0.30\]

SAY: These decimals are called equivalent decimals because one equals the other, just as equivalent fractions equal each other.

Exercises: Write two equivalent fractions and two equivalent decimals for the amount shaded.

a) 

b) 

c) 

d) 

Answers: a) \( \frac{5}{10} = \frac{50}{100} = 0.5 = 0.50 \), b) \( \frac{2}{10} = \frac{20}{100} = 0.2 = 0.20 \), c) \( \frac{7}{10} = \frac{70}{100} = 0.7 = 0.70 \), d) \( \frac{4}{10} = \frac{40}{100} = 0.4 = 0.40 \)

Using a picture to show a combination of tenths and hundredths.

Draw the first picture below on the board.

\[
\begin{array}{ccc}
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\end{array}
\]

ASK: How many hundredths are shaded? (30) How many tenths are shaded? (3) Then shade two more hundredths. ASK: Now how many hundredths are shaded? (32) Summarize by saying that 32 hundredths equals 3 tenths and 2 more hundredths. Write on the board:

\[32 \text{ hundredths} = 3 \text{ tenths} + 2 \text{ hundredths}\]
**Exercises:** Describe the fraction shaded as hundredths and as tenths and hundredths.

- **a)**
  - Diagram:
  - **Answers:** 64 hundredths = 6 tenths + 4 hundredths

- **b)**
  - Diagram:
  - **Answers:** 47 hundredths = 4 tenths + 7 hundredths

- **c)**
  - Diagram:
  - **Answers:** 85 hundredths = 8 tenths + 5 hundredths

- **d)**
  - Diagram:
  - **Answers:** 86 hundredths = 8 tenths + 6 hundredths

**Bonus**

**Exercises:** Describe the fraction shaded as hundredths and as tenths and hundredths.

- **a)**
  - Diagram:
  - **Answers:** 64 hundredths = 6 tenths + 4 hundredths

- **b)**
  - Diagram:
  - **Answers:** 47 hundredths = 4 tenths + 7 hundredths

**Relating tenths and hundredths to place value.** Tell students that just like there is a ones place and a tens place in whole numbers, there is a tenths place and a hundredths place in decimals. Show this on the board:

\[
\frac{68}{100} = 6 \text{ tenths } 8 \text{ hundredths}
\]

**Exercises:** Describe the hundredths using the three ways shown above.

- **a)** 54 hundredths
- **b)** 8 hundredths
- **c)** 37 hundredths

**Answers:**

- **a)** 54/100, 5 tenths 4 hundredths, 0.54
- **b)** 8/100, 0 tenths 8 hundredths, 0.08
- **c)** 37/100, 3 tenths 7 hundredths, 0.37

Tell students that they can change a decimal written in tenths to one written in hundredths by writing a zero to the right of the decimal:

**Examples:**

- 0.4 = 0.40 Four tenths is the same as forty hundredths.
- 0.9 = 0.90 Nine tenths is the same as ninety hundredths.

**Exercises:** Write both decimals as hundredths. Which one is greater?

- **a)** 0.5 and 0.42
- **b)** 0.6 and 0.78
- **c)** 0.3 and 0.05

**Answers:**

- **a)** 0.50 > 0.42
- **b)** 0.60 < 0.78
- **c)** 0.30 > 0.05

**Relating tenths and hundredths to number lines.** Project onto the board BLM Hundredths Number Lines. Label the tenths as shown below:

```
0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0
```

```
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
```

Demonstrate counting 4 tenths and then 3 more hundredths. Then demonstrate counting 43 hundredths (count by 10 hundredths until 40, then 1 hundredths until 43). Mark 0.43 on the number line.
Use BLM Hundredths Number Lines to display the following exercises.

**Exercises:** Write the fraction of the distance from 0 to 1 as hundredths and as tenths and hundredths.

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
<td>0.70</td>
</tr>
<tr>
<td>0.80</td>
<td>0.90</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 9 hundredths = 0 tenths 9 hundredths, b) 28 hundredths = 2 tenths 8 hundredths, c) 52 hundredths = 5 tenths 2 hundredths, d) 70 hundredths = 7 tenths 0 hundredths

**Introduce decimal thousandths.** Tell students that the fraction 1/1000 can be represented as 0.001. Write on the board:

\[
\frac{1}{1000} = 0.001
\]

Tell students that 1/1000 or 0.001 means 1 part out of 1000 equal parts. Ask volunteers to show how they would write 4 thousandths as a decimal. (0.004) ASK: What about 7 thousandths? (0.007)

**Exercises:** Describe the decimal using tenths, hundredths, and thousandths. Then describe it using only thousandths.

a) 0.471  b) 0.962  c) 0.508

**Answers**

a) 4 tenths 7 hundredths 1 thousandth, 471 thousandths
b) 9 tenths 6 hundredths 2 thousandths, 962 thousandths
c) 5 tenths 8 thousandths, 508 thousandths

**Extensions**

1. Write the fraction as tenths and then as a decimal.

   a) \(\frac{1}{2}\)  b) \(\frac{1}{5}\)  c) \(\frac{2}{5}\)  d) \(\frac{3}{5}\)  e) \(\frac{4}{5}\)

2. Write the fraction as hundredths and then as a decimal.

   a) \(\frac{3}{20}\)  b) \(\frac{7}{50}\)  c) \(\frac{12}{50}\)  d) \(\frac{8}{25}\)  e) \(\frac{9}{20}\)

3. Explain how you know that 0.7 = 0.70.

**Answers:** 1. a) 5/10 = 0.5, b) 2/10 = 0.2, c) 4/10 = 0.4, d) 6/10 = 0.6, e) 8/10, 0.8; 2. a) 15/100 = 0.15, b) 14/100 = 0.14, c) 24/100 = 0.24, d) 32/100 = 0.32, e) 45/100 = 0.45; 3. 0.7 = 7/10 = 70/100 = 0.70
**NS6-39 Decimal Fractions**

**Goals**
Students will represent decimal fractions in expanded form.

**PRIOR KNOWLEDGE REQUIRED**
- Recognizes increasing and decreasing patterns
- Can use grids to represent tenths and hundredths
- Can write equivalent fractions
- Can add fractions with like denominators

**NOTE:** This lesson is not required by the provinces’ curricula. However, it is highly recommended because it relates decimals and fractions on which students can perform operations. In addition, adding and subtracting decimals is shown later in this unit as an application of adding and subtracting decimal fractions.

**Mental math minute.** Write on the board:

\[
\frac{6}{9} = \frac{2}{?}
\]

SAY: We can make equivalent fractions by dividing the numerator and denominator by the same number. This is similar to dividing both numbers in a division statement by the same number. Just as the quotient does not change, the new fraction is equivalent to the old fraction. ASK: What number do you divide 6 by to get 2? (3) So what number should you divide 9 by? (3) What is \(9 \div 3\)? (3) Replace the question mark with a 3 in the equation.

**Exercises:** Find the number that makes the fractions equivalent.

a) \(\frac{21}{14} = \frac{3}{?}\)

b) \(\frac{36}{45} = \frac{?}{5}\)

c) \(\frac{44}{77} = \frac{4}{?}\)

d) \(\frac{100}{45} = \frac{20}{?}\)

**Answers:** a) 2, b) 4, c) 7, d) 9

**Review powers of 10.** Write this pattern on the board:

\[
10 = 10 \\
10 \times 10 = \underline{______} \\
10 \times 10 \times 10 = \underline{______} \\
10 \times 10 \times 10 \times 10 = \underline{______}
\]

Have volunteers fill in the blanks. Tell students that these numbers are called powers of 10.

**Review multiplying by powers of 10.** SAY: Multiplying by powers of 10 is easy because you just write zeros at the end of the number.

**Exercises:** Multiply.

a) \(10 \times 100\)  

b) \(10 \times 10\)  

c) \(1000 \times 10\)  

d) \(100 \times 100\)
Answers: a) 1000, b) 100, c) 10 000, d) 10 000

For the exercise below, students who are struggling can write the number of zeros under each power of 10. For example, in part a), write “1” under 10 and “3” under 1000.

Exercises: What do you multiply by?

a) $10 \times \square = 1000$  
b) $100 \times \square = 1000$  
c) $10 \times \square = 100$

Bonus: d) $100 \times 
\square = 10000$  
e) $1000 \times \square = 1000000000$

Answers: a) 100, b) 10, c) 10, Bonus: d) 100, e) 10 000 000

Introduce decimal fractions. Display a table with the headings “decimal fractions” (with examples such as $\frac{5}{10}$, $\frac{4}{10}$, $\frac{3}{100}$, $\frac{425}{1000}$) and “not decimal fractions” (with examples such as $\frac{1}{2}$, $\frac{2}{5}$, $\frac{4}{17}$, $\frac{9}{20}$, $\frac{289}{3000}$). Have volunteers suggest additional fractions for the table and have the rest of the class signal in which group each fraction should be placed. Have students guess the rule for putting the fractions in each group.

NOTE: Do not use decimal notation in this lesson. Focus on the concept of a decimal fraction.

Explain that a decimal fraction is a fraction whose denominator is a power of 10. Decimal fractions are important because powers of 10 are easy to work with. Point out that while some of the denominators in the “not decimal fractions” group are multiples of 10, they are not powers of 10. Also, some fractions (such as $\frac{1}{2}$ and $\frac{2}{5}$) are equivalent to decimal fractions but are not decimal fractions.

Review equivalent tenths and hundredths. Draw the two squares on the board. SAY: The picture shows why $\frac{3}{10}$ equals $\frac{3}{100}$. The second square has 10 times as many shaded parts and 10 times as many parts altogether.

Write on the board:

$$\frac{3}{10} \times \frac{10}{10} = \frac{30}{100}$$

Exercises: Write an equivalent fraction with denominator 100. Show your work.

a) $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100}$  
b) $\frac{4}{10} = \frac{40}{100}$  
c) $\frac{9}{10} = \frac{90}{100}$

Answers: The numerators are: a) 70, b) 40, c) 90

Equivalent tenths, hundredths, and thousandths. Write on the board:

$$\frac{3}{10} \times \frac{10}{10} = \frac{30}{100} \quad \frac{7}{10} \times \frac{10}{10} = \frac{70}{100} \quad \frac{5}{10} \times \frac{10}{10} = \frac{50}{100}$$

SAY: Now you have to decide what to multiply the numerator by to get an equivalent fraction. You have to decide what the denominator was multiplied by and then multiply the numerator by the same thing. Have
volunteers tell you what to multiply by, then have other volunteers fill in the numerators (30, 700, 50). SAY: To make an equivalent decimal fraction, you just have to add the same number of zeros to the numerator and the denominator.

**Exercises:** Write the missing numerator in the equivalent fraction.

<p>| | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| a) | \[
\frac{8}{100} = \frac{1000}{1000}
\] |
| b) | \[
\frac{3}{10} = \frac{1000}{1000}
\] |
| c) | \[
\frac{9}{10} = \frac{1000}{1000}
\] |
| Bonus | \[
\frac{3}{10} = \frac{100000}{100000}
\] |

**Answers:** a) 80, b) 300, c) 900, d) 30, Bonus: 30 000

**Adding tenths and hundredths.** Draw the picture below on the board:

\[
\frac{3}{10} + \frac{6}{100} = \frac{36}{100}
\]

SAY: If you can add hundredths, and if you can change tenths to hundredths, then you can add tenths and hundredths. For example, 3 tenths is 30 hundredths, and 6 more hundredths is 36 hundredths. Remind students that they can change the tenths to hundredths without using a picture:

\[
\frac{3}{10} \times \frac{10}{10} + \frac{6}{100} = \frac{30}{100} + \frac{6}{100} = \frac{36}{100}
\]

**Exercises:** Add. Show your work.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| a) | \[
\frac{4}{10} + \frac{3}{100} = \frac{43}{100}
\] |
| b) | \[
\frac{7}{10} + \frac{4}{100} = \frac{74}{100}
\] |
| c) | \[
\frac{5}{10} + \frac{8}{100} = \frac{58}{100}
\] |
| Bonus | \[
\frac{3}{10} + \frac{9}{10} = \frac{93}{100}
\] |

**Adding tenths, hundredths, and thousandths.** Write on the board:

\[
\frac{3}{10} + \frac{9}{100} + \frac{6}{1000}
\]

SAY: To make fractions easier to add, change all denominators to 1000. Continue writing on the board:

\[
\frac{300}{1000} + \frac{90}{1000} + \frac{6}{1000} = \frac{396}{1000}
\]

Have volunteers complete the equation: 300/1000 + 90/1000 + 6/1000 = 396/1000. Point out how adding fractions with denominators 10, 100, and 1000 is easy because it’s just using expanded form.

**NOTE:** For the bonus in the following exercise, students will need to carefully look at the denominators.
Exercises: Add.

a) \[ \frac{4}{10} + \frac{3}{100} + \frac{9}{1000} \]
b) \[ \frac{5}{10} + \frac{2}{100} + \frac{1}{1000} \]
c) \[ \frac{9}{10} + \frac{7}{100} + \frac{8}{1000} \]

Bonus: \[ \frac{3}{100} + \frac{8}{1000} + \frac{4}{10} \]

Answers: a) 439/1000, b) 521/1000, c) 978/1000, Bonus: 438/1000

Adding decimal fractions with missing tenths or hundredths. Write the equation on the board but without the answer shown in italics.

\[ \frac{4}{10} + \frac{9}{1000} = \frac{409}{1000} \]

ASK: How many thousandths are in 4/10? (400) So, how many thousandths are there altogether? (400 + 9 = 409) Write the answer, then SAY: 4 tenths, 0 hundredths, and 9 thousandths add to 409 thousandths. Have students add more tenths and thousandths.

Exercises: Add.

a) \[ \frac{3}{10} + \frac{7}{1000} \]
b) \[ \frac{9}{10} + \frac{1}{1000} \]
c) \[ \frac{2}{10} + \frac{6}{1000} \]

Answers: a) 307/1000, b) 901/1000, c) 206/1000

Repeat the process with 4/100 + 9/1000. Then write on the board:

0 tenths + 4 hundredths + 9 thousandths = 49 thousandths

SAY: We might be tempted to write this as 049/1000, but we do not write the zero at the beginning of a number.

Exercises: Predict the answer, then check by adding.

a) \[ \frac{3}{100} + \frac{7}{1000} \]
b) \[ \frac{8}{100} + \frac{2}{1000} \]
c) \[ \frac{5}{100} + \frac{6}{1000} \]

SAY: Be careful to watch for which decimal fraction is missing.

d) \[ \frac{8}{10} + \frac{2}{1000} \]
e) \[ \frac{3}{10} + \frac{7}{1000} \]
f) \[ \frac{5}{10} + \frac{8}{1000} \]

Bonus: g) \[ \frac{8}{1000} + \frac{2}{10} \]
h) \[ \frac{8}{100} + \frac{2}{10000} \]

Answers: a) 37/1000, b) 82/1000, c) 56/1000, d) 802/1000, e) 37/1000, f) 508/1000, Bonus: g) 208/1000, h) 802/10 000

SAY: You might need to add thousandths or just hundredths.

Exercises: Add the decimal fractions. Write the answer as a decimal fraction.

a) \[ \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} \]
b) \[ \frac{8}{10} + \frac{6}{1000} \]
c) \[ \frac{9}{10} + \frac{6}{100} \]

d) \[ \frac{3}{100} + \frac{5}{1000} \]
e) \[ \frac{8}{10} + \frac{9}{100} + \frac{2}{1000} \]
f) \[ \frac{5}{10} + \frac{1}{100} \]
Bonus: g) $\frac{6}{100} + \frac{8}{1000} + \frac{2}{10} \quad h) \frac{5}{10} + \frac{5}{100000}$

Answers: a) $\frac{333}{1000}$, b) $\frac{806}{1000}$, c) $\frac{96}{100}$, d) $\frac{35}{1000}$, e) $\frac{892}{1000}$, f) $\frac{51}{100}$, Bonus: g) $\frac{268}{1000}$, h) $\frac{50005}{100000}$

Extensions

1. Write 1 as a decimal fraction.

   Sample answers: $\frac{10}{10}$, $\frac{100}{100}$

2. a) Is there a largest power of 10? (no, because you can multiply any power of 10 by 10 to get an even larger one)

   b) Is there a smallest decimal fraction? How do you know? (no, because you can make the fraction smaller by making the denominator a larger power of 10)

3. Find the value of $x$. Explain how you can find your answer in parts b) and f).

   a) $\frac{4}{10} + \frac{x}{10} = \frac{9}{10}$
   b) $\frac{3}{10} + \frac{x}{100} = \frac{38}{100}$
   c) $\frac{x}{10} + \frac{5}{100} = \frac{45}{100}$

   Bonus

   d) $\frac{6}{10} + x = \frac{67}{100}$
   e) $\frac{4}{10} + x = \frac{67}{100}$
   f) $x + \frac{7}{100} = \frac{4}{10}$

   Answers: a) 5, b) 8, c) 4, Bonus: d) 7/100, e) 27/100, f) 33/100
Mental math minute. Write on the board
\[
\frac{4}{6} = \ ?
\]
SAY: Six is doubled to get 12, so you have to double 4 to make the fractions equivalent. This is similar to doubling both numbers in a division statement; the quotient does not change. ASK: The double of 6 is 12; what is the double of 4? (8) Replace the question mark with 8 in the equation.

Exercises: Find the number that makes the fractions equivalent.

a) \[\frac{3}{7} = \ ?\]
   b) \[\frac{13}{9} = \ ?\]
   c) \[\frac{25}{8} = \ ?\]
   d) \[\frac{3}{4} = \ ?\]

Bonus:
\[
\frac{2}{3} = \ ? = \frac{8}{?} = \frac{?}{24}
\]

Answers: a) 14, b) 18, c) 50, d) 75, Bonus: 4, 12, 16

Review the place value system. Write on the board:

\[5834 = 5000 + 800 + 30 + 4\]

SAY: We use place value to write numbers. That means that where a digit is placed in the number tells you its value. Because the digit 5 is in the thousands place, it is worth 5000.

Exercises: What does the digit 7 represent?

a) 6742   b) 9017   c) 6572   d) 7904

Answers: a) 700, b) 7, c) 70, d) 7000

Point out that the place value system extends to include tenths. Write on the board:

\[\text{thousands} \quad \text{hundreds} \quad \text{tens} \quad \text{ones}\]

SAY: The place values get 10 times as big. For example, tens are 10 times as big as ones, hundreds are 10 times as big as tens, and so on.
Tell students that you want to continue using the place value system so that you can use place value for fractions too. ASK: What is 1 ten times as big as? PROMPT: Ten of what make one whole? (a tenth) To guide students, draw pictures of 10 equal parts fitting into 1 whole on the board:

Tell students that there is a way to show 1/10 that uses place value. Write on the board:

\[
\begin{align*}
\frac{1}{10} &= 0.1 \\
\frac{3}{10} &= 0.3 \\
\frac{8}{10} &= 0.8 \\
\frac{27}{10} &= 2.7
\end{align*}
\]

SAY: Decimals are like mixed numbers. Point to \(\frac{3}{10} = 0.3\) on the board and SAY: There’s a whole-number part to the left of the decimal point and a fractional part to the right. However, when the number is less than one whole, we write 0 as the whole-number part.

**Exercises:** Write the decimal for the number.

\begin{itemize}
  \item a) \(\frac{5}{10}\)
  \item b) \(\frac{38}{10}\)
  \item c) \(\frac{746}{10}\)
  \item Bonus: \(\frac{8003}{10}\)
\end{itemize}

**Answers:** a) 0.5, b) 3.8, c) 74.6, Bonus: 800.3

**Extending the place value system beyond tenths.** Write the following sequence on the board:

\[
\text{hundreds} \quad \text{tens} \quad \text{ones} \quad \text{tenths}
\]

ASK: What should the next place value be? (hundredths) PROMPT: Ten of what fit into a tenth? Point out that there is symmetry in the place value names: to some extent, they mirror each other on either side of the “ones”:

\[
\text{hundreds} \quad \text{tens} \quad \text{ones} \quad \text{tenths} \quad \text{hundredths}
\]

SAY: There is also symmetry in the values; in other words, what each place is worth is mirrored on either side of the “ones.” Draw on the board the picture below. Ask volunteers to continue the place values in both directions.

\[
\begin{array}{cccc}
100 & 10 & 1 & \frac{1}{10} \\
& & \frac{1}{100} & \\
\end{array}
\]

Show students how to write decimals for one-digit hundredths and one-digit thousandths:

\[
\begin{align*}
\frac{3}{100} &= 0.03 \\
\frac{8}{1000} &= 0.008
\end{align*}
\]

SAY: The next place value after tenths is for hundredths. The one after that is for thousandths.
**Exercises:** Write the decimal for the fraction.

a) \( \frac{7}{100} \)  

b) \( \frac{4}{1000} \)  

c) \( \frac{5}{100} \)  

d) \( \frac{8}{1000} \)  

e) \( \frac{6}{1000} \)

**Answers:** a) 0.07, b) 0.004, c) 0.05, d) 0.008, e) 0.006

**Expressing decimal fractions in different ways.** Tell students that there are two ways to say 0.03 out loud: “zero point zero three” and “three hundredths.” We use the second way to spell the number out in words, on paper.

**Exercises:** Write the decimal in words.

a) 0.04  

b) 0.8  

c) 0.009  

d) 0.07  

e) 0.003

**Answers:** a) four hundredths, b) eight tenths, c) nine thousandths, d) seven hundredths, e) three thousandths

**Writing a decimal with more than one non-zero digit.** Write on the board:

\[
9 + \frac{6}{10} + \frac{7}{100} = 9.67
\]

Read the place values in the decimal to show how they correspond to the expanded form: 9 ones, 6 tenths, and 7 hundredths. Tell students that they can say 9.67 out loud as “nine point six seven.”

**NOTE:** Reading and saying 9.67 as “nine point sixty-seven” is incorrect and should be discouraged because it can create the misconception that 9.67 is greater than 9.8 (since 67 > 8).

**Exercises:** Write the decimal.

a) \( 3 + \frac{4}{10} + \frac{9}{100} \)  

b) \( 8 + \frac{5}{10} + \frac{3}{100} \)  

c) \( 6 + \frac{2}{10} + \frac{1}{100} \)  

d) \( 2 + \frac{7}{10} + \frac{5}{100} + \frac{9}{1000} \)  

e) \( 4 + \frac{3}{10} + \frac{8}{100} + \frac{1}{1000} \)  

f) \( 6 + \frac{5}{10} + \frac{8}{100} + \frac{2}{1000} \)

**Answers:** a) 3.49, b) 8.53, c) 6.21, d) 2.759, e) 4.381, f) 6.582

**Using 0 as a placeholder.** Write on the board:

\[
\frac{3}{10} = 0.3 \quad \frac{5 + \frac{3}{10}}{100} = 5.3 \quad \frac{3}{100} = 0.03 \quad \frac{5 + \frac{3}{10}}{1000} = 0.003
\]

Ask a volunteer to write the last decimal. (5.03) Point out that because there are no tenths, the tenths place has a zero. Write on the board:

\[
\frac{5}{10} + \frac{3}{100} \quad \frac{5}{10} + \frac{3}{1000}
\]

Ask volunteers to write the decimals. (0.53, 0.503) Point out how the denominator tells you how many places after the decimal point the digit goes: tenths go one place after the decimal, hundredths go two places,
and thousandths go three places. SAY: You have to be careful because some place values might be missing. You’ll have to write zeros in those positions as placeholders.

**Exercises:** Write the decimal.

a) \(2 + \frac{3}{100}\)  
b) \(3 + \frac{5}{10} + \frac{2}{1000}\)  
c) \(5 + \frac{7}{1000}\)  
d) \(8 + \frac{3}{100} + \frac{9}{1000}\)  
e) \(\frac{5}{100}\)  
f) \(8 + \frac{8}{1000}\)

**Answers:** a) 2.03, b) 3.502, c) 5.007, d) 8.039, e) 0.05, f) 8.008

**Exercises:** Write the value of the digit 6 as a fraction or a whole number.

a) 0.642  
b) 0.063  
c) 0.603  
d) 26.453  
e) 13.456

**Answers:** a) 6/10, b) 6/100, c) 6/10, d) 6, e) 6/1000

**NOTE:** Extensions 1 and 2 are required in order to cover the Alberta and Manitoba curricula.

**Extensions**

1. The place value on the right of the thousandths is the ten-thousandths and on the right of the ten-thousandths is the hundred-thousandths, and so on. What is the place value of 4 in the decimal?

   a) \(0.00941\)  
b) \(0.532041\)  
   **Bonus:** 12.60893742

   **Answers:** a) ten-thousandths, b) hundred-thousandths,  
   **Bonus:** ten-millionths

2. What place are the zeros holding in the number?

   a) \(0.3402\)  
b) \(0.34206\)

   **Answers:** a) ones place and thousandths place, b) ones place and ten-thousandths place

3. How much more is the 2 worth than the 5 in the decimal 0.324067568?

   **Solution:** Because each place value is 10 times the next value to the right, the relative values between the 2 and the 5 are the same as for 240 675. In 240 675, the 2 is worth 200 000 and the 5 is worth 5.

   ASK: How many times as much as 5 is 200 000 worth? (200 000 ÷ 5 = 40 000)

   So the 2 is worth 40 000 times as much as the 5.

4. Write the correct decimal:

   $700 + $80 + 9¢ = $____ ____ . ____

   **Answer:** $780.09
Goals
Students will place decimal numbers and mixed fractions on number lines and compare them.

Prior Knowledge Required
Understands decimals and fractions on number lines
Understands decimal numbers with up to two decimal places and their equivalent fractions (proper or mixed)
Can use number lines

Review number lines with fractions. Draw on the board:

\[
\begin{array}{cccccccccc}
0 & \frac{1}{10} & \frac{2}{10} & \frac{3}{10} & \frac{4}{10} & \frac{5}{10} & \frac{6}{10} & \frac{7}{10} & \frac{8}{10} & \frac{9}{10} & 1 \\
\end{array}
\]

Have students count out loud with you from 0 to 1 by tenths: zero, one tenth, two tenths, ..., nine tenths, one. Then have a volunteer write the equivalent decimal for \(\frac{1}{10}\) on top of the number line:

\[
\begin{array}{cccccccccc}
0 & \frac{1}{10} & \frac{2}{10} & \frac{3}{10} & \frac{4}{10} & \frac{5}{10} & \frac{6}{10} & \frac{7}{10} & \frac{8}{10} & \frac{9}{10} & 1 \\
0 & 0.1 & & & & & & & & & \\
\end{array}
\]

Continue in random order until all the equivalent decimals have been added to the number line. Then have students write in their notebooks the equivalent decimals and fractions for the spots marked on these number lines.

Answers: a) 0.7, b) 0.2, c) 0.5, d) 0.6, 1.2, 1.9, 2.6

Draw the number line below on the board and ask students to draw it in their notebooks. Have volunteers mark the location of the following numbers on the number line with an \(\text{X}\) and the corresponding letter.

A. 0.7  B. \(\frac{7}{10}\)  C. 1.40  D. \(\frac{8}{10}\)  E. \(\frac{9}{10}\)

Invite any students who don’t volunteer to participate. Help them with prompts and questions such as: Is the number more than 1 or less than 1? How do you know? Is the number between 1 and 2 or between 2 and 3? How do you know?

Review writing improper fractions as mixed numbers. Then ask students to write the following improper fractions as mixed numbers and then place them on a number line from 0 to 3:

\[
\begin{array}{cccc}
a) \frac{17}{10} & b) \frac{23}{10} & c) \frac{14}{10} & d) \frac{28}{10} & e) \frac{11}{10} \\
\end{array}
\]
Answers: a) 1 7/10, b) 2 3/10, c) 1 4/10, d) 2 8/10, e) 1 1/10

When students are done, ASK: When the denominator is 10, what is an easy way to tell whether the improper fraction is between 1 and 2 or between 2 and 3? (look at the number of tens in the numerator; it tells you how many ones are in the number)

ASK: How many tens are in 34? (3) 78? (7) 123? (12) 345? (34) How many ones are in 34/10? (3) 78/10? (7) 123/10? (12) 345/10? (34)

Exercises: Which two whole numbers is each fraction between?

Answers:

a) 2 and 3, b) 2 and 3, c) 12 and 13, d) 8 and 9, e) 14 and 15, f) 31 and 32

Invite volunteers to answer a) and b) on the board, and then have students do the rest in their notebooks.

Tell students that there are two different ways of saying the number 12.4. We can say “twelve decimal point four” or “twelve and four tenths.” Both are correct. Note that “twelve point four” is also commonly used. Point out the word “and” between the ones and the tenths and tell students that we always include it when a number has both a whole-number part and a fraction part.

Have students place the following fractions on a number line from 0 to 3:

- a) three tenths
- b) two and five tenths
- c) one and seven tenths
- d) one decimal point two
- e) two decimal point eight

Draw a number line from 0 to 3 on the board. Mark the following points with an X—no numbers—and have students write the number words for the points you marked:

1.3 0.7 2.4 0.1 2.8 2.1

Draw a line on the board with end points 0 and 1 marked:

0 1

Ask volunteers to mark the approximate position of each number with an X:

a) 0.4 b) 6/10 c) 0.9

Then draw a number line from 0 to 3 with whole numbers marked:
Ask volunteers to mark the approximate positions of these numbers with an X:

a) 2.1  
b) \( \frac{13}{10} \)  
c) \( \frac{29}{10} \)  
d) 0.4  
e) \( \frac{22}{10} \)

Continue with more numbers and number lines until all students have had a chance to participate. (Example: Draw a number line from 0 to 2 with whole numbers marked. Ask students to mark the approximate positions of 0.5, 1.25, and others.) **Bonus:** Use larger whole numbers and/or longer number lines.

**Extensions**

1. Fill in the missing numbers.

```
0    0.4    1
```

2. Show 1.5 on the number line.

```
0    0.5
```
NS6-42  Comparing Fractions and Decimals  

Goals  
Students will use fractions (one half, one quarter, and three quarters) as benchmarks to compare decimals.

PRIOR KNOWLEDGE REQUIRED  
Can place decimals and fractions on number lines  
Understands decimal numbers with up to two decimal places and their equivalent fractions (proper or mixed)

MATERIALS  
grid paper or **BLM 1 cm Grid Paper** (p. T-1)  
scissors  
string  
coloured thread  
clothespins

Mental math minute—number string.  
String 1: 8 ÷ 4, 1/4 of 8 (2, 2)  
Present the strategy using groups of dots, as shown below. One fourth is one of 4 equal parts, so divide 8 into 4.

String 2: 12 ÷ 3, 1/3 of 12, 1/4 of 20, 1/7 of 21, 1/9 of 45 (4, 4, 5, 3, 5)  
String 3: 40 ÷ 10, 1/10 of 40, 1/10 of 100, 1/10 of 400, 1/100 of 400, 1/1000 of 567 000 (4, 4, 10, 40, 4, 567)

Review number lines with decimals. Draw on the board:

| 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |

**NOTE:** Invite students, in small groups if necessary, to gather near the board as you go through the first part of the lesson.

Have a volunteer show where 1/2 is on the number line. Draw another number line the same length and divided into two equal parts and superimpose that second number line over the first number line so that students see that 1/2 is exactly at the 0.5 mark. ASK: Which decimal is equal to 1/2? Is 0.2 between 0 and 1/2 or between 1/2 and 1? Is 0.7 between 0 and 1/2 or between 1/2 and 1? What about 0.6? 0.4? 0.3? 0.9?

Go back to the decimal 0.2 and ASK: We know that 0.2 is between 0 and 1/2, but is it closer to 0 or to 1/2? Draw the number line shown on the following page on the board.
ASK: Is 0.6 between 0 and 1/2 or between 1/2 and 1? (1/2 and 1) Which number is it closer to, 1/2 or 1? (1/2) Have a volunteer show the distance to each number with arrows. Which arrow is shorter? Which number is 0.4 closest to: 0, 1/2, or 1? (1/2) Which number is 0.8 closest to? (1) Repeat the questioning for all of the remaining decimal tenths between 0 and 1.

Creating number lines. On grid paper or BLM 1 cm Grid Paper, have students draw a line 10 squares long. Then have them cut out the line—leaving space above and below for writing—and fold it in half so that the two end points meet. They should mark the points 0, 1/2, and 1 on their line. Now have students re-fold the line and then fold it in half a second time. Have them unfold the line and look at the folds. ASK: What fraction is exactly halfway between 0 and 1/2? (1/4) How do you know? (the sheet is folded into 4 equal parts, so the first fold must be 1/4 the distance from 0 to 1) SAY: So every half is two quarters. ASK: What fraction is halfway between 1/2 and 1? How do you know? (3/4, because the sheet is folded into 4 equal parts, so the third fold must be 3/4 of the distance from 0 to 1)

Have students mark the fractions 1/4 and 3/4 on their number line. Then have students write the decimal numbers from 0.1 to 0.9 in the correct places on their number line using the squares on the grid paper to help them.

Comparing numbers on number lines. Tell students to look at the number line they’ve created and to fill in the blanks in the following questions by writing “less than” or “greater than” in their notebooks.

**Exercises:** Write “less than” or “greater than.”

a) 0.4 is ________ 1/4.  b) 0.4 is ________ 1/2.  c) 0.8 is ________ 3/4.

d) 0.2 is ________ 1/4.  e) 0.3 is ________ 1/2.  f) 0.7 is ________ 3/4.

**Answers:** a) greater than, b) less than, c) greater than, d) less than, e) less than, f) less than.

Have students rewrite each statement using the “greater than” and “less than” symbols: > and <.

**Exercises:** Which whole number is each decimal, mixed fraction, or improper fraction closest to?

a) 0.7  b) 1 4/10  c) 2.3  d) 18/10  e) 2 6/10  f) 1.1
15 16 17 18

g) 17.2  h) 16.8  i) 16 $\frac{3}{10}$  j) 17 $\frac{4}{10}$  k) 15.9  l) 15.3

**Answers:** a) 1, b) 1, c) 2, d) 2, e) 3, f) 1, g) 17, h) 17, i) 16, j) 17, k) 16, l) 15

**Exercises:** Which decimal is halfway between …

a) 1 and 2?  b) 17 and 18?  c) 31 and 32?  d) 0 and 3?

e) 15 and 18?  f) 30 and 33?  g) 25 and 28?

**Bonus:** Which whole number is each decimal closest to?

h) 23.4  i) 39.8  j) 314.1  k) 235.6  l) 981.1  m) 999.9

**Answers:** a) 1.5, b) 17.5, c) 31.5, d) 1.5, e) 16.5, f) 31.5, g) 26.5, 

**Bonus:** h) 23, i) 40, j) 314, k) 236, l) 981, m) 1000

**Comparing numbers.** Draw on the board:

Ask volunteers to write two different fractions for the amount shaded in the pictures. Have other volunteers change the fractions to decimals.

**ASK:** Do these four numbers all have the same value? How do you know? What symbol do we use to show that different numbers have the same value? (the equal sign) Write on the board: $0.9 = 0.90 = \frac{9}{10} = \frac{90}{100}$.

Have students change more decimals to fractions with denominator 100.

**Exercises:** Write as a fraction with denominator 100.

a) 0.6  b) 0.1  c) 0.4  d) 0.8  e) 0.35  f) 0.42

**Answers:** a) 60/100, b) 10/100, c) 40/100, d) 80/100, e) 35/100, f) 42/100

**Exercises:** Put the group of numbers in order from smallest to largest by first changing all numbers to fractions with denominator 100.

a) 0.3, 0.7, 0.48  b) $\frac{38}{100}$, $\frac{4}{10}$, 0.39  c) $2\frac{17}{100}$, $2\frac{3}{10}$, 2.2

**Answers:** a) 30/100, 48/100, 70/100; b) 38/100, 39/100, 40/100;  c) 217/100, 220/100, 230/100

Now show a hundreds block with half the squares shaded:
SAY: This hundreds block has 100 equal squares. How many of the squares are shaded? (50) So what fraction of the block is shaded? (50/100)

Challenge students to give equivalent answers with different denominators, namely 10 and 2. PROMPTS: If we want a fraction with denominator 10, how many equal parts do we have to divide the block into? (10) What are the equal parts in this case, and how many of them are shaded? (the rows; 5) What fraction of the block is shaded? (5/10) What are the equal parts if we divide the block up into 2? (blocks of 50) What fraction of the block is shaded now? (1/2)

Ask students to identify which fraction of the following blocks is shaded:

Challange students to find a suitable denominator by asking themselves: How many equal parts, that are the size of the shaded area, will make up the whole? For example, is the shaded part equal to 1/5 of the whole? 1/2? Have students convert their fractions into equivalent fractions with denominator 100. (1/5 = 20/100, 1/4 = 25/100, 1/20 = 5/100)

Finding equivalent fractions. Write on the board:

\[
\frac{2}{4} = \frac{3}{6} = \frac{100}{100}, \quad \frac{3}{4} = \frac{75}{100}
\]

and have volunteers fill in the blanks. Then have students copy the following questions into their notebooks and fill in the blanks.

a) \(\frac{2}{5} = \frac{3}{100} \quad \frac{4}{5} = \frac{80}{100} \quad \frac{5}{5} = \frac{100}{100}\)

b) \(\frac{2}{20} = \frac{1}{100} \quad \frac{3}{20} = \frac{15}{100} \quad \frac{4}{20} = \frac{20}{100}\)

Bonus: \(\frac{17}{20} = \frac{85}{100}\)

Answers: a) 40, 60, 80, 100; b) 10, 15, 20, 25; Bonus: 85

Exercises: Circle the larger number. First change the numbers to fractions with denominator 100.

a) \(\frac{1}{2} \text{ or } 0.43 \quad \frac{3}{2} \text{ or } 1.6 \quad 3.7 \text{ or } 3\frac{1}{2}\)

d) \(\frac{1}{2} \text{ or } 0.57 \quad \frac{1}{4} \text{ or } 0.23 \quad \frac{3}{5} \text{ or } 0.7\)

Answers: a) 1/2, b) 1.6, c) 3.7, d) 0.57, e) 1/4, f) 0.7

Give students groups of fractions and decimals to order from least to greatest by first changing all numbers to fractions with denominator 100. Include mixed, proper, and improper fractions. Start with groups of only three numbers and then move to groups of more numbers.
ACTIVITY (Optional)

Make a number line divided into five equal parts from string that is any length. Students can mark the whole numbers with thread. Have students use clothespins to mark the following numbers in the most appropriate location:

\[\frac{13}{3} \quad 4.57 \quad 4.05 \quad \frac{14}{3} \quad 4.17\]

Extensions

1. Have students rearrange the following words to create different numbers. What are the smallest and largest numbers that can be made using all the words?

   hundredths hundred thousand two five nine thirty-seven and

   **Answers:** Greatest number: Thirty-seven thousand nine hundred five and two hundredths, Least number: Two thousand five hundred nine and thirty-seven hundredths

2. Write a decimal for each fraction by first changing the fraction to an equivalent fraction with denominator 100.

   a) \(\frac{2}{5}\)  
   b) \(\frac{1}{2}\)  
   c) \(\frac{1}{4}\)  
   d) \(\frac{3}{5}\)  
   e) \(\frac{11}{25}\)  
   f) \(\frac{47}{50}\)

   **Answers:** a) 0.4, b) 0.5, c) 0.25, d) 0.6, e) 0.44, f) 0.94

3. If you can run 12 km in an hour, how many kilometres can you run in a minute? Write your answer as a decimal number. (Hint: Reduce the fraction and then change to a fraction with denominator 100.)

   **Answer:** 0.2 km

4. Compare without using pictures and determine which number is larger in the pair:

   a) \(2\frac{3}{4}\) and 17 sevenths  
   b) 1.7 and 17 elevenths

   c) 1.5 and 15 ninths  
   d) 2.9 and 26 ninths

   **NOTE:** Students will have to convert all of the numbers into fractions in order to compare them. Is it best to use improper or mixed fractions? You can invite students to try both and see which types of fractions are easier to work with in this case (Mixed numbers work better for parts a) and d); improper fractions work better for parts b) and c).

   **Answers:** a) 2 3/4, b) 1.7, c) 15 ninths, d) 2.9

5. Write digits in the boxes that will make the statement true.

   \[\_ \_ .5 < \_ \_ .3\]

   **Answer:** any numbers \(a\) and \(b\) so that \(a < b\) (Example: 2 and 3)
NS6-43 Ordering Decimals

Pages 13–14

Goals
Students will order decimals using place value.

PRIOR KNOWLEDGE REQUIRED
Can order whole numbers
Can write equivalent fractions and decimals
Can order proper and improper fractions with the same denominator
Understands decimal place values
Can translate between fractions with denominator 10, 100, or 1000 and decimals

MATERIALS
play money (dimes and pennies)
grid paper or BLM 1 cm Grid Paper (p. T-1)

Mental math minute—number string.
String 1: 10 ÷ 5, 1/5 of 10, 3/5 of 10 (2, 2, 6)

Present the strategy of using groups of dots, as shown in the margin.
One fifth is one of five equal parts, so divide 10 into 5 equal groups.
Three fifths is three such groups, so multiply the answer by 3.

String 2: 12 ÷ 4, 1/4 of 12, 3/4 of 12, 1/5 of 20, 4/5 of 20 (3, 3, 9, 4, 16)
String 3: 1/10 of 30, 7/10 of 30, 1/10 of 600, 9/10 of 600 (3, 21, 60, 540)

Comparing decimals by comparing their equivalent fractions. Write on the board:

\[
\begin{array}{ll}
\frac{2}{10} & \frac{7}{10} \\
0.2 & 0.7
\end{array}
\]

ASK: Which fraction is greater? (7/10) So which decimal is greater? (0.7)
SAY: You can compare decimals by comparing the fractions they are equivalent to.

Exercises: Write the decimals as fractions. In the pair, which decimal is greater?

a) 0.4 or 0.3 b) 0.35 or 0.27 c) 0.8 or 0.9 d) 0.76 or 0.84

Answers: a) 0.4 b) 0.35, c) 0.9, d) 0.84
Comparing different place values. Write on the board:

\[
\begin{array}{ccc}
0.5 & 0.36 \\
5 & 36 \\
\hline
10 & 100 \\
50 & 36 \\
\hline
100 & 100
\end{array}
\]

Have students signal which number is the larger in each pair, starting from the bottom pair. SAY: You can compare decimals by writing them as fractions with the same denominator.

NOTE: Students who are struggling with comparing decimals with one and two decimal places (i.e., students who are saying that \(0.17 > 0.2\)) can use play money (dimes and pennies).

Exercises: Write the decimals as fractions with the same denominator. Then decide which decimal is greater.

a) 0.3 and 0.24  
   b) 0.57 and 0.614  
   **Bonus:** 0.009 and 0.0045

Answers: a) 0.3, b) 0.614, Bonus: 0.009

Ordering decimals by rewriting them to the smallest place value. Write on the board:

\[0.7 = 0.70 \quad \text{and} \quad 0.64\]

SAY: I want to compare 0.7 to 0.64. Writing them both as hundredths makes comparing them easy—70 hundredths is more than 64 hundredths. Write the “\(>\)” sign between the decimals: \(0.7 > 0.64\).

Exercises: Write both decimals in the pair as hundredths. Then compare them with the greater than or less than symbol.

a) 0.4 0.51  
   b) 0.5 0.47  
   c) 0.3 0.28

Answers: a) 0.40 < 0.51, b) 0.50 > 0.47, c) 0.30 > 0.28

Write on the board:

\[0.5 \quad 0.487\]

Ask a volunteer to write 0.5 as thousandths, in decimal form. (0.500)

ASK: What’s greater, 500 thousandths or 487 thousandths? (500 thousandths) Write the inequality \(0.5 > 0.487\) on the board.

Exercises: Make both decimals have the same number of digits after the decimal point. Then compare them.

a) 0.35 and 0.4  
   b) 0.006 and 0.03  
   c) 0.786 and 0.31  
   **Bonus:** 0.024 and 0.01904

Answers: a) 0.35 < 0.40, b) 0.006 < 0.030, c) 0.786 > 0.310,  
   Bonus: 0.02400 > 0.01904
Comparing decimals with the same whole-number part. SAY: When the whole-number parts of a number are the same, you just have to compare the decimal parts. When the whole-number parts are different, you only need to compare the whole numbers. Write on the board:

\[
0.5 > 0.36, \text{ so } 4.5 > 4.36, \text{ so } 5 > 3, \text{ so } 5.16 > 3.247
\]

Exercises: Compare the pair of numbers. Which is larger?

a) 3.4 and 3.067, b) 8.56 and 17.001, c) 2.012 and 20.05

d) 0.54 and 0.346, e) 0.3 and 0.295, f) 61.3 and 62.104

Bonus: 8.4444444444 and 88.4

Answers: a) 3.4, b) 17.001, c) 20.05, d) 0.54, e) 0.3, f) 62.104, Bonus: 88.4

Contrasting the importance of digits after the decimal point with that of digits before the decimal point. Many students have difficulty comparing and ordering decimals. A common mistake is regarding decimals with more digits after the decimal point as larger.

Write on the board:

\[
37.9999999999 < 364.3
\]

ASK: Which number is greater? (364.3) How do you know? (because 364 is greater than 37) Point out that more digits on the whole-number side of a decimal mean a larger number, but more digits on the fraction side of a decimal do not necessarily mean a larger number.

Using place value to compare decimals. Write on the board:

\[
0.48 \quad 0.473
\]

SAY: You can compare 0.48 to 0.473 by comparing 480 to 473 because 0.48 is 480 thousandths and 0.473 is 473 thousandths. Then you can compare 480 to 473 by using place value. Write on the board:

\[
\begin{array}{l}
480 \\
473
\end{array} 
\begin{array}{l}
0.48 \\
0.473
\end{array}
\]

SAY: When we compare 480 to 473, they both have 4 hundreds, but 8 tens is more than 7 tens. We can compare the decimals in the same way. They both have 4 tenths, but 8 hundredths is more than 7 hundredths, so 0.48 is greater than 0.473.

Point out how lining up the place values made it easy to find the first place value where the numbers are different. Then SAY: You can line up the place values by lining up the decimal points because the decimal point is always between the ones and the tenths.

Write on the board:

\[
0.703 \quad 0.6194 \quad 13.41
\]
\[
0.71 \quad 0.6185 \quad 3.42
\]
For each pair, ASK: What is the largest place value where the decimals are different? (hundredths, thousandths, tens) Which decimal is greater? (0.71, 0.6194, 13.41)

For the exercises below, make sure students align the place values, not only the decimal points. Encourage them to write each digit in its own cell on grid paper.

**Exercises:** Order the decimals from least to greatest by lining up the decimal points.

a) 0.6, 0.78, 0.254  
b) 0.25, 0.234, 0.219  
c) 2.3, 2.04, 20.1, 2.195  

**Answers:**  
a) 0.254, 0.6, 0.78;  
b) 0.219, 0.234, 0.25;  
c) 2.04, 2.195, 2.3, 20.1  

**Identifying a decimal between two other decimals.** Write 0.4 and 0.9 on the board. Have students name a decimal between these two numbers. Write 4.3 and 4.4 on the board. ASK: Are there any numbers between 4.3 and 4.4? (yes) See what students say. Some may say “no” because they are only thinking about the tenths.

Write the two decimals as hundredths: 4.30 and 4.40. ASK: Are there any numbers between 4.30 and 4.40? (yes) Have students identify a few decimals between 4.30 and 4.40 (for example, 4.35). SAY: 4.35 is between 4.30 and 4.40. Is it also between 4.3 and 4.4? (yes) SAY: If there are no decimal tenths between the numbers, you can always change them to hundredths. If there are no hundredths between them, you can always change them to thousandths.

**Exercises:** Find a decimal between the two numbers.

a) 7.8 and 7.9  
b) 0.25 and 0.26  
c) 0.2 and 0.24  
d) 6.3 and 6.37  

**Bonus**  
e) 3.789 and 3.79  
f) 21.9000099 and 21.90001

**Sample answers:**  
a) 7.83,  
b) 0.251,  
c) 0.22,  
d) 6.34,  
**Bonus:**  
e) 3.7896,  
f) 21.90000995

**NOTE:** Extension 1 is required in order to cover the Ontario curriculum.

**Extensions**

1. With a calculator, find the bigger fraction using division.  
   a) $\frac{3}{8}$ and $\frac{2}{5}$  
   b) $\frac{3}{7}$ and $\frac{4}{9}$  
   c) $\frac{2}{9}$ and $\frac{1}{5}$  

   **Answers:**  
a) $\frac{2}{5}$,  
b) $\frac{4}{9}$,  
c) $\frac{2}{9}$  

2. Use each of the digits 1 to 5 only once to make the statement true. Create as many different answers as you can.  
   a) 0.__ __ > 0.__ __  
   b) 0.__ __ __ < 0.__

   **Sample answers:**  
a) 0.31 > 0.245,  
b) 0.4321 < 0.5
3. Use the digits 0, 1, and 2 once each to create as many different decimals as you can that are …
   a) larger than 1.2   b) between 0.1 and 0.2
   c) between 1.0 and 2.0

   Hint: 0.201 can be also written as .201; 2.1 can be written as 2.10.

   **Answers:** a) 2.01, 2.10, 10.2, 12.0, 20.1, 21.0; b) 0.12 or .102; c) 1.20 or 1.02

4. a) Draw two squares so that 0.2 shaded in one is more than 0.3 shaded in the other.
   
   b) Research to find what is worth more when comparing currency: 0.2 Canadian dollars or 0.3 Brazilian reals.

   **Answer:** a) [Diagram of two squares, one with 0.2 shaded in one and the other with 0.3 shaded]

5. Find the number halfway between …
   a) 0.4 and 0.7   b) 0.2 and 0.3   c) 0.2 and 0.4
   d) 0.56 and 0.57   e) 1.35 and 1.36   f) 0.2 and 0.36

   **Answers:** a) 0.55, b) 0.25, c) 0.3, d) 0.565, e) 1.355, f) 0.28

6. Have students use each of the digits from 0 to 9 once to fill in the 10 spaces and make these statements true.

   __ · __ < __ · __
   __ __ · __ > __ · __

   **Sample answers:** 1.2 < 3.4; 95.8 > 7.06
NS6-44  Adding Decimals
Pages 15–17

CURRICULUM REQUIREMENT
AB: recommended
BC: required
MB: recommended
ON: required

VOCABULARY
algorithm
hundredth
regrouping
tenth
thousandth
decimal
equivalent fractions

Goals
Students will add decimals.

PRIOR KNOWLEDGE REQUIRED
Can represent decimals using base ten materials
Can add fractions with the same denominator
Understands decimal place values
Can translate between fractions with denominator 10, 100, or 1000 and decimals
Can add multi-digit whole numbers

MATERIALS
base ten blocks
grid paper or BLM 1 cm Grid Paper (p. T-1)

Mental math minute. Write the equation in the margin on the board.
SAY: We can make equivalent fractions by multiplying the numerator and the denominator by the same number. This is similar to multiplying both numbers in a division statement by the same number; the quotient does not change. ASK: What number do we multiply 6 by to get 30? (5) What is \( \frac{4}{6} \times 5? \) (20) Replace the question mark with 20 in the equation.

For the following exercises, write each equation and the four possible answers on the board. Present them one at a time and have students signal which answer they think is correct by raising the corresponding number of fingers.

Exercises: What number makes the fractions equivalent?

a) \( \frac{8}{7} = \frac{24}{?} \)
   A. 27  B. 21  C. 14  D. 56

b) \( \frac{3}{14} = \frac{9}{?} \)
   A. 28  B. 27  C. 42  D. 100

c) \( \frac{25}{4} = \frac{?}{16} \)
   A. 100  B. 4  C. 5  D. 50

d) \( \frac{23}{25} = \frac{?}{100} \)
   A. 26  B. 50  C. 92  D. 98

Bonus: \( \frac{2}{3} = \frac{?}{9} \)
A. 4, 18, 48  B. 6, 27, 72  C. 8, 25, 71  D. 6, 36, 48

Answers: a) B, b) C, c) A, d) C, Bonus: D
Using a hundreds block as a whole. Tell students that they can use a hundreds block as one whole. Draw on the board:

![Hundreds block diagram]

1 whole 1 tenth 1 hundredth

SAY: Tenths and hundredths work just like other place values—each value is 10 times the next when moving to the left and 1/10 times the next when moving to the right. So you can regroup them the same way.

Exercises: Use base ten blocks to regroup so that each place value has a single digit. Work through one question with students.

a) 3 tenths + 12 hundredths  
   b) 7 ones + 18 tenths  
   c) 7 ones + 15 tenths + 14 hundredths

Answers: a) 4 tenths + 2 hundredths, b) 8 ones + 8 tenths,  
   c) 8 ones + 6 tenths + 4 hundredths

SAY: You can regroup thousandths in the same way.

Exercises: In your head, regroup without using base ten blocks.

a) 7 hundredths + 13 thousandths  
   b) 1 thousandth + 35 thousandths  
   c) 3 tenths + 25 thousandths

You may need to regroup twice.

d) 6 tenths + 14 hundredths + 13 thousandths  
   e) 5 tenths + 34 hundredths + 26 thousandths

Answers: a) 8 hundredths + 3 thousandths, b) 4 hundredths + 5 thousandths,  
   c) 3 tenths + 2 hundredths + 5 thousandths,  
   d) 7 tenths + 5 hundredths + 3 thousandths, e) 8 tenths + 6 hundredths + 6 thousandths

Review aligning place values in vertical addition. Write the incorrectly aligned vertical addition question below on the board:

```
   3 5 1
+   1 2
```

ASK: Have I written the addition correctly? (no) Have students explain what you should change. Review how to align the digits and how to perform the sum. SAY: We align the place values and then add each place value separately.

Adding decimals. Write on the board:

\[
\begin{align*}
21 + 14 &= 35 \\
\frac{21}{10} + \frac{14}{10} &= \frac{35}{10} \\
\frac{21}{100} + \frac{14}{100} &= \frac{35}{100}
\end{align*}
\]
Write the first equation in vertical format and then ask volunteers to write the other two equations in vertical format, as decimals:

\[
\begin{array}{ccc}
21 & 2.1 & 0.21 \\
+14 & +1.4 & +0.14 \\
35 & 3.5 & 0.35 \\
\end{array}
\]

Explain that you can add decimals the same way you add whole numbers—line up the place values—but instead of adding whole numbers, you’re adding tenths, hundredths, thousandths, and so on.

**Exercises:** Add by lining up the place values on grid paper.

**Bonus**

a) \(3.4 + 1.5\)  

b) \(4.6 + 2.1\)  

c) \(8.53 + 1.26\)  

\[134.3 + 245.5\]

**Answers:** a) 4.9, b) 6.7, c) 9.79, Bonus: 379.8

**Adding decimals with regrouping, with the same number of digits to the right of the decimal point.** Work through the first two questions together. Then have students work individually to add the numbers. Remind students to align the place values.

**Exercises**

a) \(23.5\)  

b) \(2.74\)  

c) \(192.8\)  

d) \(4.18\)  

e) \(15.47\)

\[+1.4 \quad +3.58 \quad +15.4 \quad +1.23 \quad +1.63\]

**Answers:** a) 24.9, b) 6.32, c) 208.2, d) 5.41, e) 17.10

**NOTE:** Point out that in part e) the answer can be written as 17.1.

**Adding decimals with a different number of digits to the right of the decimal point.** Write the equation shown in the margin on the board. Tell students that when they are using the standard algorithm, they should use the same structure as in the place value chart, aligning the place values. And as in the place value chart, when a digit is missing, we regard it as zero. For example, to add 3.7 + 2.15, you can line up the place values.

\[
\begin{array}{c}
3.7 \\
+2.15 \\
\hline
5.85 \\
\end{array}
\]

Remind students that 3.7 is equivalent to 3.70, so regarding the empty spot after the 7 as a zero makes perfect sense. Have students check the addition. **ASK:** Does adding 3.70 + 2.15 give the same result as you’ve got? (yes) Do the first few exercises below together before students work individually.

**Exercises:** Add.

a) \(0.78 + 0.4\)  

b) \(0.37 + 0.495\)  

c) \(34.85 + 65.1\)  

d) \(1.43 + 2.904\)

**Answers:** a) 1.18, b) 0.865, c) 99.95, d) 4.334
Adding whole numbers and decimals. Write “32 + 4.7” on the board. ASK: How can you line up the decimal points when 32 has no decimal point? PROMPT: Where should the decimal point go in 32? (after the 2) SAY: You can look at 32 as 32.0, or 32 and 0 tenths. A whole number is always understood to have a decimal point immediately to the right of the ones digit even if it is not shown. Now you can line up the decimal points and add. Have a volunteer do so:

\[
\begin{align*}
32.0 \\
+ 4.7 \\
\hline
36.7
\end{align*}
\]

Exercises: Add.

a) \[4 + 13.7\]  
b) \[16 + 2.3\]  
c) \[38 + 14.71\]

Answers: a) 17.7, b) 18.3, c) 52.71

Word problems practice. Marko placed a 1.23-m-long table along a wall 3 m long. If his bed is 1.89 m long, will it fit along the same wall? Explain. (no; 1.23 m + 1.89 m = 3.12 m and 3.12 m > 3 m)

Extensions

1. a) Add mentally.
   i) \[2.6 + 3.4\]  
   ii) \[0.8 + 19.2\]  
   iii) \[5.7 + 5.3\]

   b) Add the two numbers that are easiest to add first. Then find the total: \[4.7 + 7.9 + 5.3\].

   c) Would you use pencil and paper to add or would you add mentally?
   i) \[3.5 + 4.5\]  
   ii) \[3.69 + 2.74\]  
   iii) \[7.63 + 2.37\]

   Answers: a) i) 6, ii) 20, iii) 11; b) \[4.7 + 5.3 = 10\] and \[10 + 7.9 = 17.9\]; c) i) mentally, ii) paper and pencil, iii) mentally

2. Make up two decimals that add to 4.53. Check your answer by adding them.

3. Write the numbers as decimals and add: \[3 + \frac{7}{10} + \frac{3}{100}\].

   Answer: 3.73

4. Continue the pattern:
   a) \[0.1, 0.4, 0.7, \ldots, \ldots\]  
   b) \[3.2, 3.5, 3.8, \ldots, \ldots\]  
   c) \[5.07, 5.12, 5.17, \ldots, \ldots\]

   Answers: a) 1.0, 1.3; b) 4.1, 4.4; c) 5.22, 5.27
NS6-45 Subtracting and Adding Decimals
Pages 18–19

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: recommended
ON: required

VOCABULARY
regrouping
thousandth

Goals
Students will add and subtract decimals.

PRIOR KNOWLEDGE REQUIRED
Can represent decimals using base ten materials
Understands decimal place values
Can translate between decimal fractions with and decimals
Can add multi-digit whole numbers

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. T-1)

Subtracting decimals. Explain that subtraction with decimals works in a similar way to addition with decimals: you need to line up the place values and then perform the operation as if you had whole numbers. Show an example without regrouping, such as \( 73.45 - 1.31 = 72.14 \).

Exercises: Subtract using grid paper.

a) \( 7.4 - 2.1 \)  
b) \( 6.93 - 4.52 \)  
c) \( 8.56 - 3.44 \)  
d) \( 6.5 - 3 \)  

Answers: a) 5.3, b) 2.41, c) 5.12, d) 3.5

Subtracting decimals with regrouping. Subtract with an example that requires regrouping, such as \( 53.21 - 6.72 = 46.49 \). Then have students practise individually. Tell students that sometimes they will be able to rewrite the answer with fewer digits to the right of the decimal point—for example, \( 0.62 - 0.42 = 0.20 \), so you can rewrite the answer as 0.2.

Exercises: Subtract.

a) \( 0.71 - 0.44 \)  
b) \( 0.37 - 0.29 \)  
c) \( 34.85 - 0.68 \)  
d) \( 6.432 - 2.341 \)  
e) \( 1.00 - 0.52 \)  
f) \( 10.01 - 2.41 \)  
g) \( 15.76 - 2.67 \)  
h) \( 2.894 - 0.987 \)  

Answers: a) 0.27, b) 0.08, c) 34.17, d) 4.091, e) 0.48, f) 7.6, g) 13.09, h) 1.907

Subtracting decimals with different numbers of digits to the right of the decimal point. Present the following example: \( 1.987 - 0.45 \). Invite a volunteer to write it in a place value chart. ASK: How do I subtract the thousandths? What is the thousandths digit in 1.987? (7) What is the thousandths digit in 0.45? (there is no thousandths digit, so it is zero) Have students tell you what to do in each step as you perform the subtraction. \( (1.987 - 0.450 = 1.537) \)

Now have students write the subtraction 7.8 - 2.345 in the place value chart. ASK: Should I start by subtracting the ones or the thousandths? (thousandths) ASK: How do I subtract the thousandths? What is the thousandths digit in 7.8? (there is no thousandths digit, so it is zero)
Remind students that you can put zeros after the last place value to the right of the decimal point to make subtraction easier. Put zeros where they will make the subtraction easier and have students tell you what to do at each next step as you perform the subtraction. Then demonstrate how you would perform the same subtraction without the place value chart simply by aligning the place values on grid paper.

Remind students that they can add the decimal point after a whole number, so $1 = 1.000$. Demonstrate how to subtract $1.00 - 0.87$. Then have students practise subtraction individually.

**Exercises:** Subtract.

a) $34.85 - 0.6$  

b) $6.432 - 2.34$  

c) $1.7 - 0.42$

d) $20.37 - 5.294$  

e) $2 - 0.52$  

f) $10 - 2.413$  

**Answers:** a) 34.25, b) 4.092, c) 1.28, d) 15.076, e) 1.48, f) 7.587

Students can check their answers using addition.

**Word problems with decimals.** Solve the first problem as a class, then have students work on the other one individually.

**Exercises**

a) Jennifer made a 0.8 L milkshake by adding ice cream to 0.67 L of milk. How much ice cream did she add?

b) Shelly cut a piece of wood board to make a shelf that is 0.79 m long. The leftover piece of board is 1.27 m long. How long was the board before she cut the wood for the shelf?

**Answers:** a) 0.13 L milk added (0.8 L - 0.67 L = 0.13 L), b) 2.06 m

**Extensions**

1. Continue the pattern:

   a) 3.9, 3.5, 3.1, ___, ___, b) 6.09, 6.06, 6.03, ___, ___,

   c) 17.32, 17.27, 17.22, ___, ___

**Answers:** a) 2.7, 2.3; b) 6.00, 5.97; c) 17.17, 17.12

2. Kyle’s house and Jake’s house are 11 km apart. Kyle started walking toward Jake’s house. Kyle walked 4.9 km in the first hour and 4.4 km in the second hour. How many kilometres did Kyle walk in 2 hours? What distance does he still have to go to get to Jake’s house?

**Answers:** 9.3 km and 1.7 km

3. Show students how to subtract decimals from 1 by first subtracting from 0.99: $1 - 0.74 = 0.01 + 0.99 - 0.74 = 0.01 + 0.25 = 0.26$. Then ask students to subtract.

   a) $1 - 0.37$  

   b) $1 - 0.19$  

   c) $1 - 0.08$  

   d) $1 - 0.01$

**Answers:** a) 0.63, b) 0.81, c) 0.92, d) 0.99
NS6-46  Money and Decimals
Pages 20–21

CURRICULUM REQUIREMENT
AB: required
BC: required
MB: recommended
ON: required

VOCABULARY
cent
dime
dollar
loonie
nickel
penny
quarter
regrouping
toonie

Goals
Students will add or subtract money amounts and solve word problems involving money.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract decimals
Can add and subtract with regrouping
Is familiar with money and Canadian currency

MATERIALS
play money
grid paper or BLM 1 cm Grid Paper (p. T-1)
flyers from local businesses

Mental math minute—number string.
String 1: $76 + 24, 7.6 + 2.4, 0.76 + 0.24, 0.076 + 0.024, 0.760 + 0.240, 0.761 + 0.239 (100, 10, 1.00 or 1, 0.1, 1, 1)

To explain why the strategy works, make sure students understand they are doing the same addition, $76 + 24$, but the units they are adding change. In $76 + 24$, they add ones; in $7.6 + 2.4$, they add 76 tenths + 24 tenths = 100 tenths = 10 ones = 10; in $0.76 + 0.24$, they add 76 hundredths + 24 hundredths and so on.

String 2: $17 + 83, 1.7 + 8.3, 0.17 + 0.83, 0.017 + 0.083, 0.170 + 0.830, 0.172 + 0.828 (100, 10, 1.00 or 1, 0.1, 1, 1)

Go over the steps for adding money. Tell students that you will now add money. Compared with other addition, the big difference is in lining up the numbers using the decimal point. ASK: Are the ones still lined up over the ones? (yes) The tens over the tens? (yes) The dimes over the dimes? (yes) SAY: If the decimal point is lined up, all the other digits must be lined up correctly too because the decimal point is between the ones and the dimes. Students can model regrouping of terms using play money: for instance, in $2.33 + 2.74$, they will have to group 10 dimes as a dollar (see completed addition in the margin).

Ask students to add these numbers on grid paper or BLM 1 cm Grid Paper.

Exercises: Add.

\[
\begin{align*}
a) & \quad \$5.08 + \$1.51 = & \quad \$6.59 \\ b) & \quad \$3.13 + \$2.98 = & \quad \$6.11 \\ c) & \quad \$1.07 + \$1.52 = & \quad \$25.96 \\ d) & \quad \$2.39 + \$4.22 = & \quad \$66.23
\end{align*}
\]

Answers: a) $6.59, b) $6.11, c) $25.96, d) $66.23
Adding money in word problems. Use volunteers to solve several word problems, such as: Yu spent $14.98 for a T-shirt and $5.78 for a sandwich. How much did she spend in total? ($20.76)

Students should also practice adding coins and writing the amount in dollar notation.

Examples:
- There are 19 pennies, 23 nickels, and 7 quarters in Jane’s piggy bank. How much money does she have? ($3.09)
- Hanna has a five-dollar bill, 4 toonies, 4 loonies, and 9 quarters in her pocket. How much money does she have? ($19.25)
- Ray paid 2 twenty-dollar bills, 5 toonies, 8 loonies, 5 quarters, and 7 dimes for a parrot. How much did his parrot cost? ($59.95)
- A mango fruit costs 69¢. I have a toonie. How many mangos can I buy? If I add a dime, will it be enough for another one? (2, yes)

Exercises: Add.

a) $18.25
+ $71.46

b) $23.89
+ $67.23

c) $45.08
+ $8.87

d) $78.37
+ $4.79

Answers: a) $89.71, b) $91.12, c) $53.95, d) $83.16

Word problems practice. Sharon saved 6 loonies, 7 dimes, and 3 nickels from babysitting. Her brother Ronin saved a five-dollar bill, a loonie, and 3 quarters from mowing a lawn.

a) Who has saved more money?

b) They want to share their money to buy a present for their mother. How much money do they have altogether?

c) They’ve chosen a teapot for $13.99. Do they have enough money?

Answers: a) Sharon (Sharon saved $6.85 and Ronin saved $6.75), b) $13.60, c) no

Subtracting money. Start with a review of two-digit and three-digit subtraction questions. Model a few examples on the board and involve volunteers if possible. As you demonstrate, show some examples on the board of numbers lined up correctly or incorrectly and have students decide which ones are aligned correctly.

Start with some examples that do not require regrouping. Demonstrate the steps: line up the numbers correctly and subtract the digits in each column in turn, starting from the right.

Move on to questions that require regrouping.

Examples: $86 - 27$, $567 - 38$, $782 - 127$, $673 - 185$, $467 - 369$

Have students subtract money.

**Exercises:** Subtract.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$98.89$</td>
<td>$89.00$</td>
<td>$45.00$</td>
<td>$78.37$</td>
</tr>
<tr>
<td>$71.64$</td>
<td>$67.23$</td>
<td>$38.87$</td>
<td>$9.79$</td>
</tr>
</tbody>
</table>

**Answers:** a) $27.25$, b) $21.77$, c) $6.13$, d) $68.58$

Next, teach students the following fast way of subtracting from powers of 10 (such as 10, 100, 1000, and so on) to help them avoid regrouping: For example, you can subtract any money amount from a dollar by taking the amount away from 99¢ and then adding 1¢ back to the result.

$$
egin{array}{c}
1.00 \\
0.99 + 1\text{¢} \\
-0.57 \\
= -0.57 \\
0.42 + 0.01 = 0.43 = 43\text{¢}
\end{array}
$$

Another example is that you can subtract any money amount from $10.00 by taking the amount away from $9.99 and adding one cent to the result.

$$
egin{array}{c}
10.00 \\
9.99 + 0.01 \\
-8.63 \\
= -8.63 \\
1.36 + 0.01 = 1.37
\end{array}
$$

**NOTE:** If students know how to subtract any one-digit number from 9, then they can easily perform the subtractions shown above mentally.

**Exercises**

1. Subtract.

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<td>$96.85$</td>
<td>$75.00$</td>
<td>$55.00$</td>
<td>$78.37$</td>
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<td>$71.64$</td>
<td>$61.23$</td>
<td>$39.88$</td>
<td>$9.88$</td>
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2. Amir went to a grocery store with $15.00. He would like to buy buns for $1.69, ice cream for $6.99, and tomatoes for $2.50. Does he have enough money? If yes, how much change will he get?

3. Use skip counting to find the answer to the question mentally.

   a) How much do 4 pencils cost at $1.25 each?

   b) Oranges cost 60¢ each. How many could you buy with $3.00?

   c) Stickers cost $1.20 each. How many could you buy if you had $10.00?

**Answers:** 1. a) $25.21$, b) $13.77$, c) $15.12$, d) $68.49$; 2. yes; $3.82$; 3. a) $5$, b) $5$, c) $8$
Sample solution: c) $1.20, $2.40, $3.60, $4.80, $6.00, $7.20, $8.40, $9.60

ACTIVITY (Optional)

Bring in flyers from local businesses. Ask students to select items to buy for a friend or relative as a birthday gift. They must choose at least two items. They have a $20.00 budget. What is the total cost of their gifts?

Extensions

1. Eric is paid $7 for every lawn he mows. He wants to buy a bicycle for $199.99. This week he mowed 2 lawns every weekday and 7 lawns on both Saturday and Sunday together. How much more money does he need to earn to have enough money to buy the bicycle?

   Answer: $80.99

2. A skateboard costs $69.75, plus tax of $9.07. How much does the skateboard cost with tax?

   Answer: 78.82
Goals
Students will round decimals to the nearest one (nearest whole number), tenth, hundredth, or thousandth.

Students will round to the nearest whole number, tenth, or hundredth to estimate sums and differences.

PRIOR KNOWLEDGE REQUIRED
Can round whole numbers to any place value, including regrouping
Can add and subtract decimals with regrouping

MATERIALS
- coloured chalk or markers
- grid paper or BLM 1 cm Grid Paper (p. T-1)

Mental math minute—number string.

String 1: 64 + 36, 64 + 37, 6.4 + 3.7, 0.64 + 0.37, 2.64 + 1.37, 0.064 + 0.037, 4.064 + 4.037 (100, 101, 10.1, 1.01, 4.01, 0.101, 8.101)

Use the strategy from the previous lesson of adding tenths, hundredths, or thousandths using familiar compatible pairs of ones and then compensating.

String 2: 534 + 468, 53.4 + 46.8, 5.34 + 4.68, 0.534 + 0.468, 0.000534 + 0.000468 (1002, 100.2, 10.02, 1.002, 0.001002)

Introduce rounding decimals. Draw a number line on the board from 1.0 to 3.0, with 1.0, 2.0, and 3.0 a different colour than the other numbers.

Circle the numbers 1.3, 1.8, 2.1, and 2.6, one at a time, and ask volunteers to draw an arrow showing which whole number is closest to the number you just circled. Tell students that sometimes people think of a number in terms of the closest whole number because they don’t need to be precise at that time and whole numbers are easier to work with. This is called rounding.

Exercises: Round to the nearest whole number.

a) 1.4    b) 1.9    c) 2.7    d) 2.2

Answers: a) 1.0, b) 2.0, c) 3.0, d) 2.0

Write on the board:
3.7
SAY: 3.7 is between 3.0 and 4.0. Is 3.7 closer to 3.0 or to 4.0? (4.0) Repeat with 9.4. (9.4 is between 9.0 and 10.0; it is closer to 9.0)

**Exercises:** Round to the nearest whole number.

\[
\begin{align*}
\text{a)} & \quad 9.7 & \text{b)} & \quad 3.4 & \text{c)} & \quad 4.6 & \text{d)} & \quad 7.2 & \text{e)} & \quad 8.1 & \text{f)} & \quad 6.9 & \text{g)} & \quad 0.3 & \text{h)} & \quad 0.7 \\
\end{align*}
\]

**Answers:** a) 10.0, b) 3.0, c) 5.0, d) 7.0, e) 8.0, f) 7.0, g) 0, h) 1.0

**Rounding decimals to the nearest whole number.** Make a table with two headings: “Closer to 3.00” and “Closer to 4.00.” Name several decimals (3.42, 3.56, 3.12, 3.85, 3.52, 3.31, 3.27, 3.90, 3.09, 3.51) and ask students to signal whether the decimals are closer to 3 (thumbs down) or to 4 (thumbs up). Place the decimals in their correct table column as students answer.

**ASK:** What digit are you looking at to decide? (the tenths digit) **SAY:** When the tenths digit is 0, 1, 2, 3, or 4, round down. When the tenths digit is 5, 6, 7, 8, or 9, round up.

**Exercises:** What is the nearest whole number?

\[
\begin{align*}
\text{a)} & \quad 4.57 & \text{b)} & \quad 6.12 & \text{c)} & \quad 9.08 & \text{d)} & \quad 7.92 & \text{e)} & \quad 7.29 \\
\end{align*}
\]

**Answers:** a) 5, b) 6, c) 9, d) 8, e) 7

Tell students that numbers less than 3.50 are rounded down to 3 and numbers more than 3.50 are rounded up to 4. **ASK:** Is 3.50 closer to 3 or to 4? (neither; it is the same distance from both) **SAY:** I want to pick 3 or 4 anyway, and I only want to have to look at the tenths digit to decide. **ASK:** Where are all the other decimals with the tenths digit 5? (in the “Closer to 4” column) **SAY:** When a decimal is equally close to both whole numbers, round up.

**Exercises:** Round to the nearest whole number.

\[
\begin{align*}
\text{a)} & \quad 2.50 & \text{b)} & \quad 0.50 & \text{c)} & \quad 8.50 & \text{d)} & \quad 9.50 & \text{e)} & \quad 6.50 \\
\end{align*}
\]

**Answers:** a) 3, b) 1, c) 9, d) 10, e) 7

**SAY:** Do the same thing when rounding decimals with one digit after the decimal point.

**Exercises:** Round to the nearest whole number by looking at the tenths digit.

\[
\begin{align*}
\text{a)} & \quad 2.5 & \text{b)} & \quad 4.5 & \text{c)} & \quad 9.5 & \text{d)} & \quad 0.5 & \text{e)} & \quad 5.5 & \text{f)} & \quad 7.5 & \text{g)} & \quad 3.5 \\
\end{align*}
\]

**Answers:** a) 3.0, b) 5.0, c) 10.0, d) 1.0, e) 6.0, f) 8.0, g) 4.0

**Rounding decimals.** Tell students that you use the same rule to round to decimals as you do to round to whole numbers.

Example: Round 2.365 to the nearest tenth.
Step 1: Underline the digit you are rounding to and put your pencil on the digit to the right of the one you are rounding to.

\[
\begin{array}{cccc}
2 & 3 & 6 & 5 \\
\end{array}
\]

Step 2: Beside the grid, write “round up” if the digit under your pencil is 5, 6, 7, 8, or 9 and “round down” if the digit is 0, 1, 2, 3, or 4.

\[
\begin{array}{cccc}
2 & 3 & 6 & 5 \\
\end{array}
\]

Step 3: Round the underlined digit up or down according to the instruction you have written. Write your answer in the grid and copy all digits to the left of the rounded digit as they were.

\[
\begin{array}{cccc}
2 & 3 & 6 & 5 \\
2 & 4 \\
\end{array}
\]

SAY: So 2.365 rounded to the nearest tenth is 2.4. That makes sense because the number is between 2.3 and 2.4, but it is closer to 2.4 than to 2.3.

Exercises: Round to the underlined place value.

a) 13.451  
\textbf{b) 38.479}  
\textbf{c) 612.389}  
\textbf{d) 804.749} 

\textbf{Answers:} a) 13.5, b) 38, c) 612.39, d) 804.7

Rounding with regrouping. Write on the board: 2.965.

Demonstrate rounding 2.965 to the nearest tenth.

\[
\begin{array}{cccc}
2 & 9 & 6 & 5 \\
\end{array}
\]

SAY: 2.965 rounded to the nearest tenth is 3.0.

Exercises: Round to the stated place value. Use grid paper.

a) 43.698, hundredths  
b) 74.953, tenths  
c) 59.517, ones  
d) 84.09971, thousandths

\textbf{Answers:} a) 43.70, b) 75.0, c) 60, d) 84.100
Estimating sums and differences by rounding to the nearest whole number to check for reasonableness. Tell students that they can check whether the answers to sums and differences are reasonable by rounding to the nearest whole number. Write on the board:

\[ 162.34 + 16.234 \approx 162 + 16 = 178 \]

**Exercises:** Somebody punched these numbers into a calculator and got these answers. Are they reasonable?

a) \[ 23.52 + 11.18 = 34.70 \]

\[ 387.52 - 53.31 = 5.21 \]

**Answers:** a) yes, b) no

Estimating sums and differences in word problems. Teach students that, for estimating sums and differences of decimal numbers, they can round to various place values—the nearest ten, one, tenth, hundredth, and so on. Also teach students to reflect on whether their estimate will be higher or lower than the actual answer. For example, consider the problem: Neka had \$15.87. He paid \$11.02 for a book. Approximately how much does he have? How much does he have exactly? \$15.87 – \$11.02 is about \$16 – \$11 = \$5, but he has exactly \$4.85.

**Exercises:** Estimate the answer.

a) Avril wants to buy three items that cost \$14.79, \$12.25, and \$37.50. If she has \$65 with her, does she have enough money to buy all three items?

b) Mars is about 227.94 million kilometres from the Sun. Earth is about 149.60 million kilometres from the Sun. How much farther from the Sun is Mars?

c) A weather station reports an average high temperature in Yellowknife, NWT, of 10.3°C during September. It is 9.5°C warmer in Ottawa, ON, during September. What is the average high temperature in Ottawa during September?

d) There are approximately 20.69 million people living in Beijing, China. Approximately 67.52 million people live in Hunan province in China. How many more people live in Hunan?

**Answers:** a) yes, b) around 80 million kilometres, c) around 20°C, d) around 47 million

**Extensions**

1. Round 628.327 to the nearest one. Then round it to the nearest tenth. Finally, round it to the nearest hundredth.

**Answers:** 628, 628.3, 628.33

2. Decide what place value it makes sense to round each of the following to. Round to the place value you selected. Justify your decisions.

**NOTE:** Students should remember that they made similar decisions.
in 6.1 Unit 5: Measurement: Length, Perimeter, and Mass when they interpreted remainders.

- Height of person: 1.524 m
- Height of tree: 13.1064 m
- Length of bug: 1.267 cm
- Distance between Edmonton, AB, and Hong Kong: 10 409.24 km
- Distance today between Earth and the Moon: 384 403 km
- Population of Kolkata, India, in 2011: 4 486 679
- Floor area of an apartment: 27.91 m²
- Area of Alberta: 661 848.15 km²
- Angle between two streets: 82.469°
- Time it takes to blink: 0.33 seconds
- Speed of a car: 96.560639 km/h

**Answers:** Answers will vary. The larger the number, the less important the smaller place values become. The use to which the measurement will be put is also a factor. For example, the time it takes to ski a downhill course would be noted with greater precision to determine a world record than to keep training records.

3. To insert the decimal point, estimate the answer (rather than carrying out the operation).
   
   a) \(16.32 + 743.5 = 759.82\)  
   b) \(49.17 - 3.5 = 45.67\)

   **Answers:** a) 759.82, b) 45.67

4. Without calculating the sum, how can you tell whether the sum is greater than or less than 375?

   \(12.5 + 0.46 + 358.63\)

   **Answer:** The sum is less than \(13 + 1 + 359 = 373\), so it is less than 375.

5. Estimate the value of \(14.502 - 13.921\) by rounding both numbers to the nearest …

   a) ten  
   b) one  
   c) tenth  
   d) hundredth

   What place value did you round to that made estimating the difference the fastest? What place value made estimating the difference the most accurate?

   **Answers:** a) \(10 - 10 = 0\), b) \(15 - 14 = 1\), c) \(14.5 - 13.9 = 0.6\),  
   d) \(14.50 - 13.92 = 0.58\); rounding to the nearest ten or one makes estimating the fastest, but rounding to the nearest hundredth makes it most accurate. **NOTE:** There is always a trade-off between speed and accuracy.
Unit 10 Number Sense: Multiplying and Dividing Decimals

Introduction
This unit is dedicated to multiplying and dividing decimals by whole numbers. It describes how to:

- multiply and divide decimals by 10, 100, and 1000 (powers of 10);
- multiply and divide decimals up to the hundredths place (for money applications) by whole numbers using base ten materials, place value, and multiplication and division algorithms;
- round the divisor to estimate the quotient when dividing by two-digit numbers; and
- solve word problems involving decimals, including problems that require conversions between metric units.

Meeting Your Curriculum

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Mental Math Minutes
The mental math minutes in this unit:
- practise operations (multiplication and division) on decimals and whole numbers

Generic BLMs
The Generic BLM used in this unit is:
BLM Filling a Blank Multiplication Chart (p. T-2)
This BLM can be found in Section T.
Assessment

The lessons covered by a quiz or test are as follows:

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Goals
Students will multiply decimals by 10, 100, and 1000.

PRIOR KNOWLEDGE REQUIRED
Knows the factors in a multiplication statement are interchangeable
Can multiply whole numbers by 10, 100, and 1000
Understands decimal place value
Can regroup
Can read decimals in terms of smallest place value

MATERIALS
small cards with a large dot

Mental math minute. SAY: Remember, an equal sign means “is the same as.” To check if an equation is true, use the addition and subtraction strategies you know, without actually calculating both sides. For example, you know that moving 1 from one addend to the other addend on the same side does not change the answer. And you know that adding 1 to both numbers in a subtraction does not change the answer. Present the equations in the exercises below one at a time and have students signal the answer using thumbs up for “yes” and thumbs down for “no.”

Exercises: Is the equation true?
a) $61 + 34 = 60 + 35$
b) $61 - 34 = 60 - 35$
c) $87 - 19 = 88 - 20$

Answers: a) yes, b) no, c) yes

Review multiplying whole numbers by 10. Ask students to describe how they can multiply a whole number by 10. Students might say “add a zero.” In this case, ask them to be more specific. Point out that it is not “adding”; it is writing a zero to the right of a number. Write an incorrect statement, such as $34 \times 10 = 304$, and ASK: Is this correct? (no) In other words, require that students clearly state that the zero has to be written at the end, as in $34 \times 10 = 340$, so that the ones digit becomes the tens digit and the zero becomes the ones digit.

Discuss how this pattern makes sense because each place value gets replaced by the place value that is 10 times as great. Write on the board:

$$34 = 3 \text{ tens} + 4 \text{ ones}$$
$$\text{So } 34 \times 10 = 3 \text{ hundreds} + 4 \text{ tens} = 340$$
Using place value to multiply decimals by 10. Write on the board and underline as shown:

\[ 0.4 \times 10 = 4 \text{ tenths} \times 10 \]

ASK: Which place value is 10 times the tenths? (ones) Write on the board:

\[ = 4 \text{ ones} = 4 \]

Draw the picture below to remind students of the connection between place values:

\[ \times 10 \quad \times 10 \quad \times 10 \quad \times 10 \]

\[ \text{tens} \quad \text{ones} \quad \text{tenths} \quad \text{hundredths} \quad \text{thousandths} \]

Exercises

1. Multiply the place value by 10.
   a) hundredths \( \times 10 \)  
   b) ones \( \times 10 \)  
   c) tenths \( \times 10 \)  
   d) thousandths \( \times 10 \)

   **Bonus:** tens \( \times 10 \)

   **Answers:** a) tenths, b) tens, c) ones, d) hundredths, Bonus: hundreds

2. Use place value to multiply the number by 10.
   a) 3 hundredths \( \times 10 \)  
   b) 4 tenths \( \times 10 \)  
   c) 5 ones \( \times 10 \)  
   d) 7 thousandths \( \times 10 \)

   **Answers:** a) 3 tenths or 0.3, b) 4 ones or 4, c) 5 tens or 50,  
   d) 7 hundredths or 0.07

Write on the board:

\[ 0.005 \times 10 = 0.05 \]

SAY: If 5 is in the thousandths position, then after multiplying by 10 it will be in the hundredths position.

**Exercises:** Multiply using place value.

a) 0.5 \( \times 10 \)  
   b) 0.02 \( \times 10 \)  
   c) 0.006 \( \times 10 \)  
   d) 0.09 \( \times 10 \)

**Answers:** a) 5, b) 0.2, c) 0.06, d) 0.9

Moving the decimal point to multiply decimals by 10. Ahead of time, draw a large decimal point on several small cards. Write the numbers below on the board, and tape the cards to the board so that they act as a decimal point:

\[ 5_{4} 6_{0} 3_{4} 3_{0} 0_{4} \]

Ask volunteers to move the decimal point to show multiplying by 10:

\[ 5_{4} 6_{0} 3_{4} 3_{0} 0_{4} \]
Ask the rest of the class to look for a pattern in how the decimal point is being moved. (Multiplying a number by 10 always means moving the decimal point one place to the right.)

**Exercises:** Move the decimal point one place to the right to multiply by 10.

a) $3.2 \times 10$  
b) $0.58 \times 10$  
c) $10 \times 0.216$  
d) $10 \times 7.46$

**Bonus:** $98763.60789 \times 10$

**Answers:** a) 32, b) 5.8, c) 2.16, d) 74.6, Bonus: 987 636.0789

**Moving the decimal point to multiply decimals by 100 and 1000.**

SAY: One hundred is 10 times 10, so to multiply a number by 100, you can multiply by 10 and then multiply the result by 10 again. Write on the board:

$$3.462 \times 100 = 3.462 \times 10 \times 10$$

SAY: Move the decimal point once to multiply by 10 and then once more to multiply by 10 again. Show this on the board:

$$3 . 4 \ 6 \ 2$$  
So $3.462 \times 100 = 346.2$

SAY: To multiply by 100, move the decimal point two places to the right.

**Exercises:** Move the decimal point two places to the right to multiply by 100.

a) $3.62 \times 100$  
b) $0.725 \times 100$  
c) $1.673 \times 100$  
d) $0.085 \times 100$

**Answers:** a) 362, b) 72.5, c) 167.3, d) 8.5

SAY: We moved the decimal point once to multiply by 10 and twice to multiply by 100. ASK: How many times do we move the decimal point to multiply by 1000? (three times) Show this on the board:

$$2 . 4 \ 6 \ 7$$  
So $2.467 \times 1000 = 2467$

**Exercises:** Move the decimal point to multiply by 1000.

a) $0.462 \times 1000$  
b) $11.241 \times 1000$  
**Bonus:** $9.32416 \times 1000$

**Answers:** a) 462, b) 11 241, Bonus: 9324.16

**Using a zero as a placeholder when multiplying decimals.** Write $3.42 \times 1000$ in a grid on the board, using the card with a large dot for the decimal point so that it can be moved, as shown:

```
3 4 2
```

ASK: How many places do I have to move the decimal point when I multiply by 1000? (three) Move the decimal point three times, as shown:

```
3 4 2
```

ASK: Are we finished writing the number? (no) Why not—what’s missing? (the zero) SAY: Each digit is worth 1000 times as much as it was before multiplying. Pointing to each digit in the first grid, SAY: The number was
3 ones, 4 tenths, and 2 hundredths. Pointing to each digit in the second grid, SAY: Now it is 3 thousands, 4 hundreds, and 2 tens. So the number is 3420.

If students struggle with the exercises below, encourage them to write each place value in its own cell on a grid. Suggest that students draw arrows to show how they moved the decimal point. An example is shown below for Exercise 1 part b):

5 2 4

Exercises

1. Multiply.
   a) 0.4 \times 1000  
   b) 5.24 \times 1000  
   c) 23.6 \times 1000  
   d) 0.01 \times 1000
   
   **Answers:** a) 400, b) 5240, c) 23 600, d) 10

2. Multiply.
   a) 0.6 \times 100  
   b) 7.28 \times 10  
   c) 25.6 \times 1000  
   d) 1.8 \times 100  
   e) 21.9 \times 1000  
   f) 326.3 \times 1000  
   g) 0.002 \times 10  
   **Bonus:** 2.3 \times 10 000
   
   **Answers:** a) 60, b) 72.8, c) 25 600, d) 180, e) 21 900, f) 326 300,  
   g) 0.02, **Bonus:** 23 000

Connect multiplying whole numbers by 10 to multiplying decimals by 10. Write the number 34 on the board, leaving enough room to place the card with the decimal point between the digits. ASK: What is 34 \times 10? (340) SAY: We can also multiply 34 \times 10 by moving the decimal point. Write on the board “34.0,” but place the card in the position of the decimal point. Move the card one place right, and point out that this is the same answer you get the other way.

\[
\begin{array}{c}
3 & 4 & 0 \\
\end{array} \quad \Rightarrow \quad \begin{array}{c}
3 & 4 & 0 & 0 \\
\end{array}
\]

SAY: Multiplying whole numbers uses the same method we use to multiply decimals.

Word problems practice.

a) Sara makes $12.50 an hour mowing lawns. How much does she make in 10 hours?

b) A clothing-store owner wants to buy 100 coats for $32.69 each. How much will the coats cost?

c) A dime is 0.122 cm thick. How tall would a stack of 100 dimes be?

d) A necklace has 100 beads. Each bead has a diameter of 1.32 mm. How long is the necklace?

**Answers:** a) $125, b) $3269.00, c) 12.2 cm, d) 132 mm
Extensions

1. Fill in the blank.
   a) ______ × 10 = 38.2  
   b) ______ × 100 = 67.4
   c) 42.3 × ______ = 4230  
   d) 0.08 × ______ = 0.8
   
   **Answers:** a) 3.82, b) 0.674, c) 100, d) 10

2. Complete the pattern.
   a) 0.0007, 0.007, 0.07, ______, ______
   b) 3.895, 38.95, 389.5, ______, ______
   
   **Answers:** a) 0.7, 7; b) 3895, 38 950

3. Find the answer mentally by multiplying the numbers in the easiest order.
   a) (3.2 × 5) × 20
   b) (6.73 × 2) × 50
   c) (7.836 × 5) × (25 × 8)
   
   **Answers:** a) 320, b) 673, c) 7836

4. Create a word problem with decimals that requires multiplying by 1000. Have a partner solve the problem.

5. One marble weighs 3.5 g. A marble bag weighs 10.6 g. How much does the bag weigh with 100 marbles in it?
   
   **Answer:** 360.6 g
Goals
Students will multiply and divide decimals by 10, 100, and 1000 by shifting the decimal point.

PRIOR KNOWLEDGE REQUIRED
Can multiply whole numbers and decimals by 10, 100, and 1000
Understands decimal place value
Knows that multiplication and division are opposite operations
Can read decimals in terms of smallest place value

MATERIALS
small cards with a large dot
scales and beans (see Extension 4)

Dividing by 10 using base ten materials. Draw on the board:

1.0 0.1 0.01

Tell students that you will represent one whole by a big square, so one tenth is a column or row, and one hundredth is a little square. Write several picture equations on the board, and have volunteers write the decimal equations shown below the pictures:

\[
\begin{align*}
2.0 & \div 10 = 0.2 \\
0.5 & \div 10 = 0.05 \\
3.1 & \div 10 = 0.31
\end{align*}
\]
Exercises: Draw pictures on grid paper, or use base ten blocks, to divide.

a) 3.0 ÷ 10  

b) 0.4 ÷ 10  

c) 3.4 ÷ 10  

d) 2.7 ÷ 10  

Answers: a) 0.3, b) 0.04, c) 0.34, d) 0.27  

Dividing by 10 by inverting the rule for multiplying by 10. Write the following digits on the board and use a card with a large dot to show the decimal point as follows:

\[ 3 \quad 4 \quad \underline{2} \quad 5 \]

Invite a volunteer to move the decimal point to multiply by 10. (34.25) Write on the board:

\[ 34.25 \times 10 = 342.5, \text{ so } 342.5 \div 10 = \underline{\ \ \ \ \ \ \ \ \ \ \ \ \ \} \]

ASK: What number goes in the blank? (34.25) How do you know? (division undoes multiplication) Now write on the board:

\[ 3 \quad 4 \quad 2\underline{5} \]

Have a volunteer move the card with the decimal point in 342.5 to get the answer for 342.5 ÷ 10. (In other words, move the decimal point one place to the left.) SAY: Division is the opposite of multiplication. Division “undoes” the effects of multiplication. When you multiply by 10, you move the decimal point one place to the right. When you divide by 10, you move the decimal point one place to the left.

Exercises: Divide by 10.

a) 14.5 ÷ 10  

b) 64.8 ÷ 10  

c) 9.22 ÷ 10  

d) 0.16 ÷ 10  

Answers: a) 1.45, b) 6.48, c) 0.922, d) 0.016  

Dividing by 100. Write on the board:

\[ 5.831 \times 100 = 583.1, \text{ so } 583.1 \div 100 = \underline{\ \ \ \ \ \ \ \ \ \ \ \ \ \} \]

Ask a volunteer to fill in the blank. (5.831) Point out that the equations are in the same fact family, so knowing how to multiply by 100 also tells us how to divide by 100. ASK: How do we move the decimal point to divide by 100? (we move it two places left) Point out that you had to move it two places right to multiply 5.831 by 100 and then, to get 5.831 back, you needed to move it left two places.

Exercises: Divide by 100.

a) 14.5 ÷ 100  

b) 464.8 ÷ 100  

c) 9.22 ÷ 100  

d) 0.6 ÷ 100  

Answers: a) 0.145, b) 4.648, c) 0.0922, d) 0.006  

Dividing whole numbers by 10 and 100. Write the number 67 on the board, again leaving room between the digits for the decimal point card. Tell students you want to know the answer to 67 ÷ 10. Then SAY: I would
do the division by moving the decimal point, but I don’t see any decimal point here. ASK: What should I do? (write the decimal point to the right of the ones because $67 = 67.0$) Do so, using the decimal point card, then invite a volunteer to move the decimal point one place to the left to get $67 \div 10 = 6.7$. Repeat the process with $18 \div 100$ and $1987 \div 100$.

Exercises: Divide by 10 or 100.

a) $236 \div 10$ b) $573 \div 100$ c) $1230 \div 100$ d) $14889 \div 10$

Answers: a) 23.6, b) 5.73, c) 12.3, d) 1488.9

Dividing by 1000. ASK: How would you shift the decimal point to divide by 1000? (move it three places to the left) Show an example done on a grid:

```
  4 5
  0 4 5
```

So $45 \div 1000 = 0.045$.

Exercises: Divide by 1000.

a) $2934 \div 1000$ b) $423 \div 1000$ c) $18.9 \div 1000$ d) $1.31 \div 1000$

Bonus: $423 \div 100\,000$

Answers: a) 2.934, b) 0.423, c) 0.0189, d) 0.00131, Bonus: 0.000423

Using strategies for remembering which way to move the decimal point. SAY: Remember, multiplying by 10, 100, or 1000 makes the number bigger, so the decimal point moves right. Dividing makes the number smaller, so the decimal point moves left.

If students have trouble deciding which direction to move the decimal point when multiplying and dividing by 10, 100, or 1000, one hint that some students might find helpful is to use the case of whole numbers as an example. ASK: Which way is the decimal point moving when multiplying $34 \times 10 = 340$? (right)

Exercises: Multiply or divide.

a) $78678 \div 1000$ b) $2.423 \times 100$ c) $18.9 \div 10$

d) $1.31 \times 1000$ e) $6 \div 100$ f) $0.082 \times 10$

g) $0.2 \div 100$ h) $5.1 \times 100$ i) $0.31 \times 1000$

Bonus

j) $31498.76532 \div 1000000$ k) $31498.76532 \times 1000000$

Answers: a) 78.678, b) 242.3, c) 1.89, d) 1310, e) 0.06, f) 0.82, g) 0.002, h) 510, i) 310, Bonus: j) 0.03149876532, k) 31498765320

Remind students who are struggling to write each place value in its own cell of grid paper when multiplying or dividing decimals by powers of 10.
Word problems practice.

a) In 10 months, a charity raises $26,575.80 through fundraising. How much does the charity raise each month on average?

b) A stack of 100 cardboard sheets is 13 cm high. How thick is a sheet of the cardboard?

c) One thousand people attended a “pay what you can” event. The total money paid was $5,750. Ray paid $0.60. Did he pay more or less than average?

d) One hundred walruses weigh 121.5 tonnes (1 tonne = 1000 kg). How much does one walrus weigh on average, in kilograms?

e) A box of 1000 nails costs $12.95.

i) How much did each nail cost, to the nearest cent?

Bonus: One hundred of the nails have been used. What is the cost for the nails that are left, to the nearest cent? Hint: Use the actual cost of a nail in your calculations, not the rounded cost from part i).

Answers: a) $2657.58; b) 0.13 cm; c) the average was $5.75, so he paid less than average; d) 1215 kg; e) i) 1¢, Bonus: $11.66

NOTE: Extension 1 is required to cover the Ontario curriculum.

Extensions

1. A dime has a width of 18.03 mm. How long would a line of 10,000 dimes laid end-to-end be in millimetres?

Answer: 180,300 mm

2. a) Ten of an object laid end-to-end have a length of 48 cm. How long is the object?

b) One hundred of an object laid end-to-end have a length of 2.38 m. How long is the object, in centimetres?

c) One thousand of an object laid end-to-end have a length of 274 m. How long is the object, in centimetres?

Answers: a) 4.8 cm, b) 2.38 cm, c) 27.4 cm

3. Create your own word problems that require multiplying and/or dividing decimals by powers of 10. Then trade with a partner and solve each other’s problems.

4. Find the mass of one bean by weighing 100 or 1000 beans. Use a calculator to determine how many beans are in a 2 pound (908 g) package.

5. How would you shift the decimal point to divide by 100,000?

Answer: Move it five places to the left.
Goals
Students will multiply decimals up to the tenths place by a whole number.

PRIOR KNOWLEDGE REQUIRED
Can multiply a multi-digit number by a single-digit number using the standard algorithm
Can use base ten materials to model decimal operations involving regrouping
Can multiply a multi-digit decimal number by powers of 10

MATERIALS
base ten blocks
grid paper

Mental math minute. Review multiplying decimals by 10, 100, and 1000. Remind students that they can multiply a decimal by 10 by shifting the place values: ones become tens, tens become hundreds, and so on. In particular, emphasize that tenths become ones and hundredths become tenths.

For the following exercises, write each multiplication and the four possible answers on the board. Present the questions one at a time and have students signal the answer they think is correct by raising the corresponding number of fingers.

Exercises: Which answer is correct?

a) 3.5 × 10
   1. 350    2. 35    3. 30.5    4. 0.35
b) 63.2 × 10
   1. 6320    2. 6320    3. 632    4. 6.32
c) 3.26 × 100
   1. 3260    2. 326    3. 3.26    4. 0.326
d) 0.678 × 10
   1. 678    2. 67.8    3. 6.78    4. 0.678
e) 0.008 × 10
   1. 80    2. 8    3. 0.8    4. 0.08
f) 0.008 × 100
   1. 80    2. 8    3. 0.8    4. 0.08
g) \(3.079 \times 100\)

1. 307.9  
2. 30.79  
3. 0.3079  
4. 0.03079

h) \(0.0065 \times 1000\)

1. 65  
2. 6.5  
3. 0.65  
4. 0.065

Answers: a) 2, b) 3, c) 2, d) 3, e) 4, f) 3, g) 1, h) 2

**Review base ten materials.** Review the use of base ten materials when using decimals. Draw on the board:

- = 1
- = 0.1
- = 1 one
- = 1 tenth

**NOTE:** In the context of decimals, we are now using the hundreds block as a ones block. One column or row of the ones block is now a tenths block.

*ASK:* How many hundredths are in 1? (100) How many tenths are in 1? (10) How many hundredths are in 1 tenth? (10) Have students model the decimal 3.2 on their desks with base ten materials:

**Multiplying whole numbers by tenths.** *ASK:* How many tenths are in a one? (10) Draw on the board:

- \(10 \times 0.1 = 1\)
- \(8 \times 0.1 = 0.8\)

*Say:* If you add up 10 tenths you get 1, so 10 tenths equals a one. Write on the board: \(10 \times 0.1 = 1\). *ASK:* What do 8 tenths add up to? (0.8) Write on the board: \(8 \times 0.1 = 0.8\). Draw on the board:

- \(13 \times 0.1 = 1.3\)

*Say:* There are 13 tenths. You can regroup 10 tenths as a one, and 3 tenths remain. *ASK:* What do 13 tenths add up to? (1.3) Write on the board:
The connection between multiplying by one tenth and dividing by 10.

SAY: I would like to find $13 \div 10$ using two different methods. In the first method, I’m dividing a whole number by 10. ASK: What is $13 \div 10$? (1.3) SAY: In the second method, I’m multiplying whole numbers by decimals. Draw on the board:

\[ \begin{array}{|c|c|c|c|c|c|c|} 
\hline 
\times & 1 & 3 \\
\hline 
\hline 
\hline 
\end{array} \]

\[ \div 10 = \begin{array}{c} \hline 
\hline 
\hline 
\end{array} \]

ASK: What is $1 \div 10$? (0.1) SAY: There are 13 ones in 13, so when you divide 13 by 10, you get 13 tenths. ASK: What is $13 \times 0.1$? (1.3) Explain to students that both methods give the same answer, and write on the board:

\[ 13 \div 10 = 13 \times 0.1 = 1.3 \]

ASK: How many places do you shift the decimal point to the left when you divide by 10? (one place) SAY: So to multiply by 0.1, you can shift the decimal point the same way, one place to the left.

Exercises: Multiply.

a) $29 \times 0.1$  

b) $37 \times 0.1$  

c) $681 \times 0.1$  

d) $3974 \times 0.1$  

e) $0.1 \times 32$  

Bonus

f) $123456789 \times 0.1$  

g) $0.1 \times 6$

Answers: a) 2.9, b) 3.7, c) 68.1, d) 397.4, e) 3.2, Bonus: f) 12345678.9, g) 0.6

Multiplying whole numbers by 0.01 and 0.001. Explain to students that the same way they divide numbers by 100, they can multiply numbers by 0.01 because there are 100 hundredths in a whole. SAY: To multiply by 0.01, you can shift the decimal point two places to the left. To multiply by 0.001, you can shift the decimal point three places to the left.

Exercises: Multiply.

a) $317 \times 0.01$  

b) $452 \times 0.01$  

c) $36 \times 0.01$  

d) $2768 \times 0.01$  

e) $2768 \times 0.001$  

f) $3 \times 0.01$  

g) $29 \times 0.001$  

h) $4 \times 0.001$  

Bonus: $4375109 \times 0.01$

Answers: a) 3.17, b) 4.52, c) 0.36, d) 27.68, e) 2.768, f) 0.03, g) 0.029, h) 0.004, Bonus: 43751.09

Multiplying decimals up to tenths by whole numbers with base ten materials. Write on the board: $3 \times 5$. ASK: What addition question can be used to find the product? Ask for a volunteer to write the answer on the board. $(5 + 5 + 5)$ Write on the board: $3 \times 2.1$. ASK: What addition question can be used to find the product? Ask for a volunteer to write the
answer on the board. \((2.1 + 2.1 + 2.1)\) Draw the following on the board to show students how to use base ten materials to add \(2.1 + 2.1 + 2.1\):

\[
\begin{array}{ccc}
\text{1} & \text{2} & \text{3} \\
\times & \text{2} \\
\hline
\text{2} & \text{6} & \text{4}
\end{array}
\]

ASK: How many ones altogether? (6) How many tenths altogether? (3) What is \(2.1 \times 3\)? (6.3) What could we have done to the digits in 2.1 to get the answer 6.3? (multiply each digit separately by 3)

Exercises: Find the product mentally by multiplying each digit separately.

\begin{align*}
a) & \quad 3.2 \times 2 \\
b) & \quad 2.3 \times 3 \\
c) & \quad 1.4 \times 2 \\
d) & \quad 1.1 \times 4 \\
e) & \quad 4.3 \times 2 \\
f) & \quad 2.4 \times 2 \\
g) & \quad 7.2 \times 4 \\
h) & \quad 13.2 \times 2
\end{align*}

Answers: a) 6.4, b) 6.9, c) 2.8, d) 4.4, e) 8.6, f) 4.8, g) 28.8, h) 26.4

Multiplying a decimal up to the tenths by a whole number using a grid.
Ask students to multiply \(132 \times 2\) using a grid and compare the answer to part h) of the previous exercises. ASK: What is the only difference in the appearance of the answers? (the answer to part h) has a decimal point)

Write on the board:

The decimal points line up on the grid.

\[
\begin{array}{ccc}
\text{1} & \text{3} & \text{2} \\
\times & \text{2} \\
\hline
\text{2} & \text{6} & \text{4}
\end{array}
\]

SAY: When multiplying a decimal number by a whole number, place the decimal point in the answer underneath the decimal point in the number with the decimal above.

Exercises: Find the product using grid paper. You may have to regroup more than once.

\begin{align*}
a) & \quad 36.4 \times 2 \\
b) & \quad 52.8 \times 3 \\
c) & \quad 62.7 \times 5 \\
d) & \quad 49.3 \times 8 \\
e) & \quad 70.4 \times 9
\end{align*}

Bonus: 91 345.7 \times 8

Answers: a) 72.8, b) 158.4, c) 313.5, d) 394.4, e) 633.6, Bonus: 730 765.6

Word problems practice.

Exercises

a) Each person runs 1.3 km in a four-person relay. How far is the relay run in total?

b) Each lion cub weighs 1.5 kg. How many kilograms do three lion cubs weigh?

c) The side length of a hexagon is 5.4 m. Find the perimeter of the hexagon.

Answers: a) 5.2 km, b) 4.5 kg, c) 32.4 m
Extensions

1. a) The dimensions of the bottom of a rectangular prism are 2.5 m and 1.3 m. What is the combined area of the top face and the bottom face of the prism in square metres?

b) Find the answer for part a) by converting the measurements to centimetres.

**Solutions:** a) Students have to find $2.5 \times 1.3 \times 2$. Since students cannot multiply decimals by decimals at this point, they can determine $2.5 \times 2$ and then multiply by 1.3: $(2.5 \times 2) \times 1.3 = 5 \times 1.3 = 6.5 \text{ m}^2$.

b) $250 \times 130 \times 2 = 65000 \text{ cm}^2$; dividing by 10000 the answer is 6.5 m$^2$.

2. The average salary for a professional hockey player is $1.5$ million. The team has 20 players. What is the approximate total salary of the team?

**Answer:** $30$ million or $30000000$
Goals
Students will multiply decimals up to the hundredths place by a whole number.

PRIOR KNOWLEDGE REQUIRED
Knows how to multiply a multi-digit number by a single-digit number using the standard algorithm
Can use base ten materials to model decimal arithmetic and multiplication involving regrouping
Can multiply a multi-digit decimal number by multiples of 10

MATERIALS
base ten blocks
play money
grid paper

Mental math minute—number string.

String 1: \(15 \div 5\), \(\frac{1}{5}\) of 15 (3, 3)

Present the strategy of using groups of dots, as shown below. One fifth is one of five equal parts, so divide 15 into 5.

String 2: \(21 \div 3\), \(\frac{1}{3}\) of 21, \(\frac{1}{4}\) of 100, \(\frac{1}{6}\) of 54, \(\frac{1}{10}\) of 450 (7, 7, 25, 9, 45)

String 3: \(4 \div 10\), \(\frac{1}{10}\) of 4, \(\frac{1}{100}\) of 10, \(\frac{1}{100}\) of 40, \(\frac{1}{1000}\) of 400, \(\frac{1}{1000}\) of 40 (0.4, 0.4, 0.1, 0.4, 0.4, 0.04)

Review base ten materials. Ask students to model the decimal 2.13 on their desks with base ten materials (see diagram below).

Model multiplying a decimal by a whole number with base ten materials and without regrouping. Write on the board:

\(3 \times 5\)

ASK: What addition question can you use to find the product? Ask for a volunteer to write the answer on the board. (5 + 5 + 5)
Write on the board:

\[ 2.13 \times 3 \]

ASK: What addition question can you use to find the product? Ask for a volunteer to write the answer on the board. \((2.13 + 2.13 + 2.13)\) Ask students to use base ten materials to add \(2.13 + 2.13 + 2.13\). \((6.39; \text{ see diagram below})\)

What could we have done to the digits in 2.13 to get the answer 6.39? (multiply each digit separately by 3)

**Exercises:** Find the product mentally by multiplying each digit separately.

a) \(3.24 \times 2\)  
   b) \(2.31 \times 3\)  
   c) \(1.43 \times 2\)  
   d) \(1.12 \times 4\)  
   e) \(4.31 \times 2\)  
   f) \(2.43 \times 2\)  
   g) \(2.21 \times 4\)  
   h) \(2.31 \times 2\)  

**Answers:** a) 6.48, b) 6.93, c) 2.86, d) 4.48, e) 8.62, f) 4.86, g) 8.84, h) 4.62

**Multiplying a decimal by a whole number using place values.** ASK: How can we write 2.13 using place values? (2 ones + 1 tenth + 3 hundredths)

What is \(3 \times 2\) ones? (6 ones) What is \(3 \times 1\) tenth? (3 tenths) What is \(3 \times 3\) hundredths? (9 hundredths)

How can we write the answers in decimal notation? (6.39)

Write on the board:

\[
\begin{align*}
2.13 & = 2 \text{ ones} + 1 \text{ tenth} + 3 \text{ hundredths} \\
\times 3 & = \frac{2 \text{ ones} + 1 \text{ tenth} + 3 \text{ hundredths}}{\times 3} \\
6.39 & = 6 \text{ ones} + 3 \text{ tenths} + 9 \text{ hundredths}
\end{align*}
\]

**Exercises:** Multiply using place values.

a) \(3.12 \times 3\)  
   b) \(4.12 \times 2\)  
   c) \(1.33 \times 3\)  

**Answers**

a) 9 ones + 3 tenths + 6 hundredths = 9.36  
   b) 8 ones + 2 tenths + 4 hundredths = 8.24  
   c) 3 ones + 9 tenths + 9 hundredths = 3.99
Using base ten materials, model multiplying a decimal by a whole number with regrouping. Point out to students that none of the questions so far have involved regrouping.

Write on the board:

\[ 2.16 \times 3 \]

Ask students to use base ten materials at their desks to calculate the product using addition (see diagram below).

ASK: How many hundredths do we have? (18) What can we use to replace 10 hundredths? (1 tenth) How many hundredths remain? (8) Ask students to replace the 10 hundredths with a tenth block, and read the answer. (6.48)

Using money, model multiplying a decimal by a whole number with regrouping hundredths for tenths (pennies for dimes). Some students will benefit from a demonstration using play money. Consider the decimal 2.16 as \$2.16. Write \$2.16 on the board. ASK: How many dollars are there? (2) How many dimes are there? (1) How many pennies are there? (6) If your class has play money, ask students to represent \$2.16 \times 3 using addition. If not, draw the following on the board and tell students that D and P represent dimes and pennies.

ASK: What can we replace 10 pennies with? (1 dime) How many pennies are left? (8) How many dimes are there now? (4) How much money is there? (\$6.48)
Multiplying a decimal by a whole number and regrouping tenths for ones (dimes for dollars). Write on the board:

\[ 1.63 \times 2 \]

Ask students to use base ten materials or money models to find the product using addition. ASK: What can we replace 10 tenths with? (a one) How many ones are there now? (3) How many tenths? (2) How many hundredths? (6) Ask a student to read the final answer. (3.26 or $3.26; see diagrams below)

Replace 10 tenths with a one.

or

Replace 10 dimes with 1 dollar.

Exercises: Use base ten materials or play money to find the product.

a) \( 2.37 \times 2 \)  
b) \( 2.71 \times 3 \)  
c) \( 3.17 \times 3 \)

Answers: a) 4.74, b) 8.13, c) 9.51

Multiplying a decimal number involving regrouping using place values. Write on the board:

\[ 2.63 \times 2 \]

ASK: How do we write 2.63 using place values? (2 ones + 6 tenths + 3 hundredths) What is 2 ones \( \times 2 \)? (4 ones) What is 6 tenths \( \times 2 \)? (12 tenths) What is 3 hundredths \( \times 2 \)? (6 hundredths)

Write on the board:

\[
\begin{align*}
2.63 & = 2 \text{ ones} + 6 \text{ tenths} + 3 \text{ hundredths} \\
\times 2 & \quad \times 2 \\
4 \text{ ones} + 12 \text{ tenths} + 6 \text{ hundredths} & \\
\end{align*}
\]

ASK: We have 12 tenths—what can we use to replace 10 tenths? (a ones block) How many tenths are left? (2 tenths) How many ones are there altogether? (5) What is the answer in decimal form? (5.26) Write on the board:

\[
\begin{align*}
& = 5 \text{ ones} + 2 \text{ tenths} + 6 \text{ hundredths} \\
& = 5.26
\end{align*}
\]
Write on the board:
\[
\begin{align*}
2.48 & = 2 \text{ ones} + 4 \text{ tenths} + 8 \text{ hundredths} \\
\times 2 & \quad \times 2 \\
4 \text{ ones} + 8 \text{ tenths} + 16 \text{ hundredths}
\end{align*}
\]

ASK: As we have 16 hundredths, what can we use to replace 10 hundredths? (a tenth) How many hundredths are left? (6) How many tenths are there altogether? (9) What is the answer in decimal form? (4.96)

Write on the board:
\[
\begin{align*}
= 4 \text{ ones} + 9 \text{ tenths} + 6 \text{ hundredths} \\
= 4.96
\end{align*}
\]

**Exercises:** Multiply using place values.

a) \(2.61 \times 3\)  

b) \(1.52 \times 3\)  

c) \(1.28 \times 3\)  

d) \(5.29 \times 2\)  

**Bonus**

e) \(2.76 \times 3\)  

f) \(3.48 \times 5\)

**Answers:** a) 7.83, b) 4.56, c) 3.84, d) 10.58, Bonus: e) 8.28, f) 17.40

**Selected solution:** d) 10 ones + 4 tenths + 18 hundredths  

\[= 1 \text{ ten} + 5 \text{ tenths} + 8 \text{ hundredths} = 10.58\]

**Multiplying a decimal by a whole number using a grid.** Ask students to multiply 237 \(\times 2\) using a grid and compare the answer to 2.37 \(\times 2 = 4.74\).

ASK: What is the only difference in the answers? (the decimal point) Write on the board:

\[
\begin{align*}
The \text{ decimal points line up on the grid.}
\begin{array}{c}
2 \ 3 \ 7 \\
\times \ 2 \\
4 \ 7 \ 4
\end{array}
\end{align*}
\]

SAY: When you multiply a decimal number by a whole number, place the decimal point in the answer _underneath_ the decimal point in the decimal number.

**Exercises:** Find the product using grid paper. You may have to regroup more than once.

a) \(3.64 \times 2\)  

b) \(5.28 \times 3\)  

c) \(6.27 \times 5\)  

d) \(4.93 \times 8\)  

e) \(7.04 \times 9\)  

**Bonus:** 9134.57 \(\times 8\)

**Answers:** a) 7.28, b) 15.84, c) 31.35, d) 39.44, e) 63.36, Bonus: 73 076.56

**Multiplying a decimal by multiples of 10.** Write on the board:

\[
23 \times 10
\]
ASK: What is the fastest way to multiply a number by 10? (move the decimal point one place to the right) Write on the board:

\[ 23 \times 10 = 230. \]

SAY: The same rule applies to multiplying a decimal by 10. Write on the board:

\[ 2.3 \times 10 = 23. \]

SAY: We can use the associative property to help us multiply decimals by multiples of 10. Write on the board:

\[ 20 \times 2.3 \]

ASK: How do we write 20 as a multiple of 10? \((2 \times 10)\) Continue writing on the board:

\[ = (2 \times 10) \times 2.3 \]

SAY: The associative property lets us move the brackets.

\[ = 2 \times (10 \times 2.3) \]

ASK: What is \(10 \times 2.3\)? \((23)\) SAY: Now we can multiply whole numbers.

Calculate \(2 \times 23\) mentally. \((46)\) Write on the board:

\[ = 2 \times 23 \]
\[ = 46 \]

Exercise: Calculate using this method.

a) \(30 \times 1.2\)  
b) \(40 \times 2.1\)  
c) \(60 \times 1.1\)

Answers: a) 36, b) 84, c) 66

Selected solution: b) \((4 \times 10) \times 2.1 = 4 \times (10 \times 2.1) = 4 \times 21 = 84\)

NOTE: Extension 4 is required to cover the British Columbia curriculum.

Extensions

1. Matt shopped at the local grocery store. This is what he bought:

<table>
<thead>
<tr>
<th>Product</th>
<th>Unit Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk</td>
<td>$4.95</td>
<td>3</td>
</tr>
<tr>
<td>Bread</td>
<td>$2.93</td>
<td>4</td>
</tr>
<tr>
<td>Cereal</td>
<td>$5.99</td>
<td>2</td>
</tr>
</tbody>
</table>

How much did Matt spend altogether?

Answer: $38.55
2. Marla has relatives in Laos, Moldova, and Samoa. She calls them each month and keeps track of how many minutes each call lasts. Here are the calls Marla made last month:

<table>
<thead>
<tr>
<th>Country Called</th>
<th>Length of Call (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laos</td>
<td>2</td>
</tr>
<tr>
<td>Moldova</td>
<td>4</td>
</tr>
<tr>
<td>Samoa</td>
<td>3</td>
</tr>
<tr>
<td>Moldova</td>
<td>3</td>
</tr>
<tr>
<td>Moldova</td>
<td>1</td>
</tr>
<tr>
<td>Laos</td>
<td>3</td>
</tr>
<tr>
<td>Samoa</td>
<td>4</td>
</tr>
<tr>
<td>Laos</td>
<td>3</td>
</tr>
</tbody>
</table>

Marla’s telephone service charges for long distance calls per minute are the following:

<table>
<thead>
<tr>
<th>Country</th>
<th>Laos</th>
<th>Moldova</th>
<th>Samoa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per Minute</td>
<td>$1.49</td>
<td>$1.26</td>
<td>$1.29</td>
</tr>
</tbody>
</table>

Find the total cost of Marla’s long distance calls last month.

**Answer:** $31.03

3. The price per litre of gas in Toronto, Ontario is $1.19. Simon’s motorcycle has a gas tank that holds 19 litres. While on vacation, Simon filled his tank 5 times. Suppose he paid the same price per litre on his trip as he did in Toronto. How much did Simon spend on gas?

**Answer:** $113.05


a) \(2.321 \times 3\)  
b) \(5.122 \times 4\)  
c) \(0.421 \times 4\)  
**Bonus:** \(0.534 \times 2\)

**Answers:** a) 6.963, b) 20.488, c) 1.684, Bonus: 1.068

5. Multiply using grid paper. You may have to regroup more than once.

a) \(3.425 \times 3\)  
b) \(7.243 \times 6\)  
c) \(0.662 \times 5\)  
**Bonus:** \(2.5341 \times 3\)

**Answers:** a) 10.275, b) 43.458, c) 3.31, Bonus: 7.6023
**Goals**

Students will divide decimals by whole numbers using base ten materials and place values.

**PRIOR KNOWLEDGE REQUIRED**

Knows how to multiply whole numbers using base ten materials and place values

Can divide using long division

Knows place value relationships for decimals to tenths

**MATERIALS**

base ten materials

**Mental math minute—number string.**

String 1: $15 \div 5$, $\frac{1}{5}$ of $15$, $\frac{4}{5}$ of $15$ ($3, 3, 12$)

Present the strategy of using groups of dots, as shown below. One fifth is one of five equal parts, so divide $15$ into $5$ equal groups. Four fifths is $4$ such groups, so multiply the answer by $4$.

![Diagram showing division of 15 into 5 equal groups and multiplication by 4]

String 2: $20 \div 4$, $\frac{1}{4}$ of $20$, $\frac{3}{4}$ of $20$, $\frac{1}{5}$ of $100$, $\frac{3}{5}$ of $100$ ($5, 5, 15, 20, 60$)

String 3: $\frac{1}{10}$ of $90$, $\frac{7}{10}$ of $90$, $\frac{1}{10}$ of $6000$, $\frac{11}{10}$ of $6000$ ($9, 63, 600, 6600$)

**Review base ten materials.**

![Diagram showing base ten materials]

Have students review base ten materials by having them represent the following numbers at their desks (see diagram below for sample answer to part a)).

a) 1.4  

b) 1.7  

c) 3.1
Use base ten materials to model division of decimals by whole numbers without regrouping. Write on the board:

\[ 3.6 ÷ 3 \]

Ask students to represent 3.6 using base ten materials (see diagram below).

Ask students to divide the materials into three equal groups (see diagram below).

ASK: What is the division statement? \( 3.6 ÷ 3 = 1.2 \)

**Exercises:** Use base ten materials to perform the division.

a) \( 4.2 ÷ 2 \)  
b) \( 8.4 ÷ 4 \)  
c) \( 9.3 ÷ 3 \)

**Answers:** a) 2.1, b) 2.1, c) 3.1

Use place values to model division of decimals by whole numbers without regrouping. Write on the board:

\[ 6.8 ÷ 2 \]

ASK: How do we write 6.8 using place values? (6 ones + 8 tenths) What is 6 ones ÷ 2? (3 ones) What is 8 tenths ÷ 2? (4 tenths)

Write on the board and SAY:

\[ 6.8 ÷ 2 = (6 \text{ ones} + 8 \text{ tenths}) ÷ 2 \]
\[ = 3 \text{ ones} + 4 \text{ tenths} \]

ASK: How do we write this in decimal notation? \( 3.4 \) Write the answer on the board.

**Exercises:** Use place values to divide.

a) \( 4.2 ÷ 2 \)  
b) \( 6.9 ÷ 3 \)  
c) \( 48.4 ÷ 4 \)

**Answers**

a) 2 ones + 1 tenth = 2.1  
b) 2 ones + 3 tenths = 2.3  
c) 1 ten + 2 ones + 1 tenth = 12.1
Recognizing that dividing decimals by whole numbers can be done by dividing without the decimal and later placing the decimal point.

Write on the board:

\[
\begin{array}{c}
 2 \\
\end{array}
\]

\[
\begin{array}{c}
 6 \\
- 4 \\
\end{array}
\]

\[
\begin{array}{c}
 8 \\
\end{array}
\]

\[
\begin{array}{c}
 64.8 \\
\end{array}
\]

\[
\begin{array}{c}
 64 \\
\end{array}
\]

\[
\begin{array}{c}
 8 \\
\end{array}
\]

\[
\begin{array}{c}
 64.8 \\
\end{array}
\]

\[
\begin{array}{c}
 6 \\
\end{array}
\]

\[
\begin{array}{c}
 4 \\
\end{array}
\]

\[
\begin{array}{c}
 0 \\
\end{array}
\]

\[
\begin{array}{c}
 8 \\
\end{array}
\]

\[
\begin{array}{c}
 8 \\
\end{array}
\]

\[
\begin{array}{c}
 0 \\
\end{array}
\]

\[
\begin{array}{c}
 8 \\
\end{array}
\]

\[
\begin{array}{c}
 0 \\
\end{array}
\]

\[
\begin{array}{c}
 0 \\
\end{array}
\]

\[
\begin{array}{c}
 64.8 \div 2 \\
\end{array}
\]

\[
\begin{array}{c}
 (6 \text{ tens } + 4 \text{ ones } + 8 \text{ tenths}) \div 2 \\
\end{array}
\]

Ask two students to come to the board and perform the divisions: the first using the division algorithm (see below) and the second using place values.

(3 tens + 2 ones + 4 tenths = 32.4)

Ask: What is the same about the quotients? (same digits) What is different? (when the dividend has a decimal point, the quotient has a decimal point)

What do you notice about the position of the decimal points in the quotient and the dividend in the second question? (they are in the same place)

Say: To divide a decimal by a whole number, perform the division using the algorithm as if there were no decimal point and then place the decimal point in the correct place in the quotient.

Write on the board:

\[
82.4 \div 2
\]

Ask students to perform the division in their notebooks. When they have had enough time, ask a student to perform the division on the board (see answer in margin).

Write on the board:

If 824 \div 2 = 412
then 82.4 \div 2 = ???

Ask a student to come to the board to complete the division equation.

(82.4 \div 2 = 41.2)
Exercises

1. Divide by using the division algorithm. First ignore the decimal point, and then place the decimal point in the quotient.
   a) $63.9 \div 3$  b) $42.8 \div 2$  c) $52.6 \div 2$  d) $42.3 \div 3$

   **Answers:** a) 21.3, b) 21.4, c) 26.3, d) 14.1

2. a) Alex earns $77.50 for 5 hours mowing lawns. How much does Alex earn in 1 hour?
   b) Alice bikes 4.8 km in 8 minutes. How far does Alice bike in 1 minute?

   **Answers:** a) $15.50, b) 0.6 km or 600 m

Bonus

a) Use the fact that $3173255 \div 5 = 634651$ to divide $317325.5 \div 5$

b) A stack of 6 toonies has a height of 10.5 mm. What is the thickness of a toonie?

   **Answers:** a) 63 465.1, b) 1.75 mm

Extensions

1. Ava earns $97.20 working for 6 hours at a part-time job.
   a) What is her pay per hour? Hint: When writing numbers in dollar notation, two decimal digits are required.
   b) When Ava works on a holiday, she gets paid extra. She is paid 2 times as much per hour. What is her pay per hour on a holiday?
   c) How much does Ava earn for 8 hours of work on a holiday?

   **Answers:** a) $16.20, b) $32.40, c) $259.20

2. A website about fuel economy says that a particular car model will drive 14.7 km per litre of gas.
   a) The Benitez family drove that same model of car for 114.4 km using 8 litres of gas. Did they do better or worse than predicted by the website?
   b) How much farther would the Benitez family travel on 8 litres if their car drove as far as the website said it would?

   **Answers:** a) no, they traveled 14.3 km per litre, so they did worse; b) $(8 \times 14.7) - 114.4 = 3.2$ km
Goals
Students will divide decimals by whole numbers using base ten materials, money, and the division algorithm.

PRIOR KNOWLEDGE REQUIRED
Knows how to multiply whole numbers using base ten materials and place values
Can divide whole numbers using long division
Understands decimal place value to hundredths

MATERIALS
BLM Filling a Blank Multiplication Chart (p. T-2)
base ten materials
play money

Mental math minute. Give students BLM Filling a Blank Multiplication Chart. Have them fill in the chart as much as they can in three minutes, using the strategies on the BLM as needed.

Use money to model division of decimals by whole numbers without regrouping. Write on the board:

$6.39 \div 3$

ASK: How can we represent $6.39$ using $1$ coins, dimes, and pennies? (6 dollars, 3 dimes, and 9 pennies)

Use play money to model or draw the following on the board:

$\begin{array}{cccc}
$1 & $1 & $1 \\
$1 & $1 & $1 \\
\end{array}$

$\begin{array}{cccc}
D & D & D \\
P & P & P & P \\
P & P & P & P \\
\end{array}$

ASK: If we divide the $6$ among three friends, how many $1$ coins will each friend get? (2) If we divide 3 dimes among three friends, how many dimes does each get? (1) If we divide 9 pennies among three friends, how many pennies does each get? (3) So how much money does each friend get? ($2.13$)

Exercises: Divide the money.

a) $8.46 \div 2$

b) $6.99 \div 3$

c) $4.84 \div 4$

Answers: a) $4.23$, b) $2.33$, c) $1.21$
Use base ten materials to model division of decimals by whole numbers, using the division algorithm where regrouping is required.

Write on the board:

\[ 2 \longdiv{\begin{array}{c} 7.34 \end{array}} \]

Ask students to model the steps of the division algorithm at their desks using base ten materials.

SAY: Use base ten materials to represent 7.34 (see diagram below).

Ask students to follow the steps at their desks using base ten materials.

**Step 1:** Divide the ones blocks into two equal groups.

Continue writing on the board as you ask the following questions.

ASK: How many ones are in each group? (3) How many were placed in groups? (6) How many ones remain? (1)

\[
\begin{array}{c}
3 \\
2 \longdiv{\begin{array}{c} 7.34 \\
- 6 \\
1 \\
\end{array}}
\end{array}
\]
Step 2: SAY: Exchange the ones block for 10 tenths.

ASK: How many tenths are there now? (13) Continue writing on the board:

\[
\begin{array}{c}
\phantom{0}3 \\
2) 7.34 \\
\underline{- 6} \\
\phantom{0}1 \phantom{3}
\end{array}
\]

Exercises: Carry out the first two steps of the division.

a) 6.45 ÷ 5  

b) 9.52 ÷ 7

Bonus: 3.76 ÷ 8

SAY: For the bonus, you have to place the first digit in the second space, since the divisor is bigger than the left digit of the dividend. Emphasize that if they don’t start from the second place, then they might put the decimal point in the wrong place.

Step 3: SAY: Divide the tenths into two equal groups.

Continue writing on the board as you ask the following questions.

ASK: How many tenths are in each group? (6) How many tenths were placed in groups? (12) How many tenths remain? (1)
Step 4: SAY: Exchange the tenths block for 10 hundredths blocks.

ASK: How many hundredths blocks are there now? (14) Continue writing on the board:

\[
\begin{array}{c}
3 & 6 \\
2) 7.34 \\
- 6 \\
\hline
1 & 3 \\
- 1 & 2 \\
\hline
1 & 4
\end{array}
\]

Step 5: Divide the 14 hundredths blocks into two equal groups.

SAY: Place the decimal point in the quotient directly above the decimal point in the dividend so that you line up tenths with tenths and hundredths with hundredths. Add the decimal point between the 3 and the 6 in the quotient.

Continue writing on the board, and ASK: How many hundredths are in each group? (7) How many hundredths were placed altogether? (14) How many hundredths are remaining? (0)
ASK: What decimal is represented in each group of base ten materials? (3.67)

Use money to model division of decimals by whole numbers, using the division algorithm where regrouping is required. Some students will benefit from using a money model.

**ACTIVITY (Optional)**

Model \(2 \div 7.34\) using play money.

ASK: How can we represent \$7.34 using play money? Draw on the board:

\[
\begin{array}{ccccccc}
\$1 & \$1 & \$1 & \$1 & \$1 \\
\$1 & \$1 & D & D & D & P & P & P
\end{array}
\]

Ask students to follow these steps on their own to model the division.

**Step 1:** Divide the loonies into two equal groups.
**Step 2:** Exchange a loonie for 10 dimes.
**Step 3:** Divide the resulting dimes into two groups.
**Step 4:** Exchange the remaining dime for 10 pennies.
**Step 5:** Divide the pennies into two groups.

The final model should look like this:

\[
\begin{array}{cccccccc}
\$1 & \$1 & \$1 & D & D & D & D & D \\
 & & D & D & D & P & P & P & P
\end{array}
\]

ASK: How much money is in each group? (\$3.67)

**NOTE:** In the following exercises, students should notice that the division is exactly like dividing using whole numbers and then putting the decimal point in the correct place. Encourage students to estimate mentally to check their answers; for example, in part a), \(7 \div 3\) is 2 and a remainder.

**Exercises:** Divide.

1. a) \(7.17 \div 3\) \hspace{1cm} b) \(49.44 \div 4\) \hspace{1cm} c) \(1.17 \div 9\)

**Bonus:** 11 111.04 \(\div 9\)

**Answers:** a) 2.39, b) 12.36, c) 0.13, Bonus: 1234.56

2. a) \(821.43 \div 3\) \hspace{1cm} b) \(126.14 \div 2\) \hspace{1cm} c) \(11 246.48 \div 4\)

**Answers:** a) 273.81, b) 63.07, c) 2811.62
NOTE: Extension 4 is required to cover the Manitoba curriculum.

Extensions

1. To turn a fraction into a decimal, write the numerator using at least three decimal digits and then divide this decimal by the denominator. For example, \( \frac{1}{4} = 1.000 \div 4 \). Find decimal representations for the fraction.
   a) \( \frac{1}{2} \)  
   b) \( \frac{1}{4} \)  
   c) \( \frac{1}{5} \)  
   d) \( \frac{1}{8} \)

   **Answers:** a) 0.500, b) 0.250, c) 0.200, d) 0.125

2. In baseball, a batter’s average is a decimal with three decimal digits. To find the decimal, divide the number of hits by the number of times at bat. Rewrite the number of hits using three decimal places to make the division easier. (Example: 3 = 3.000). Fill in the table. Which batter has the highest average?

<table>
<thead>
<tr>
<th>Batter</th>
<th>Number of Hits</th>
<th>Number of Times at Bat</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>David</td>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Mandy</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Josh</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

   **Answers:** David 0.375, Mandy 0.250, Josh 0.400. Josh has the highest average.

3. A pack of three pens costs $5.85.
   a) How much does each pen cost? Estimate and then find the exact answer.
   b) Lily estimates that 20 pens will cost $42. Is her estimate reasonable? Explain.

   **Answers:** a) under $2 each, $1.95; b) No, because the cost of each pen is less than $2, so the cost of 20 pens should be less than $40.

4. To divide by 20, you can divide by 10 and then divide by 2. For example, to find 48.6 \( \div 20 \), divide 48.6 \( \div 10 = 4.86 \) and then 4.86 \( \div 2 = 2.43 \). Divide by multiples of 10. Hint: First divide by 10.
   a) 39.6 \( \div 30 \)
   b) 28.4 \( \div 40 \)
   c) 142.8 \( \div 20 \)

   **Bonus:** 301.5 \( \div 50 \)

   **Answers:** a) 1.32, b) 0.71, c) 7.14, **Bonus:** 6.03
Goals
Students will use rounding to estimate the quotient when dividing by two-digit numbers for cases in which doing so gets the correct quotient.

PRIOR KNOWLEDGE REQUIRED
Can divide using a number line
Can divide multi-digit numbers by one-digit numbers

MATERIALS
base ten materials
play money

Mental math minute. Review multiplying and dividing by skip counting using a number line.

Exercises: Show the multiplication or division on a number line.

a) $8 \times 3$  
b) $28 \div 7$  
Bonus: $23 \div 5$

Answers

\[\begin{align*}
a) & \quad 0 \quad 3 \quad 6 \quad 9 \quad 12 \quad 15 \quad 18 \quad 21 \quad 24 \\
b) & \quad 0 \quad 7 \quad 14 \quad 21 \quad 28 \\
\text{Bonus} & \quad 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 21 \quad 22 \quad 23 \\
\end{align*}\]

So $23 \div 5$ is equal to $4 \text{ R } 3$.

Introduce rounding to estimate the quotient, using multiples of 10.

Draw on the board:

\[\begin{align*}
& \quad 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \\
& \quad 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \\
23 \div 5 & \quad 237 \div 50
\end{align*}\]

Ask volunteers to place 23 approximately where it would go on the first number line and 237 on the second number line. ASK: What is the whole-number quotient before finding the remainders? (they are both 4)
Point out to students that if they know which two multiples of 5 that 23 is between (20 and 25), then they know which two multiples of 50 that 237 is between (200 and 250). Write on the board:

\[
\frac{29}{8} \quad \frac{296}{80}
\]

ASK: Which two multiples of 8 is 29 between? (24 and 32) So which two multiples of 80 is 296 between? (240 and 320) Write on the board:

\[
\frac{253}{30}
\]

ASK: What is an easier division that has the same quotient? (25 ÷ 3) Which two multiples of 3 is 25 between? (24 and 27) So which two multiples of 30 is 253 between? (240 and 270) Show both long divisions on the board:

\[
\begin{align*}
\frac{8}{3} & \frac{25}{-24} \\
& \frac{25}{-240} \\
& \frac{253}{-13}
\end{align*}
\]

SAY: The remainders are different, but the quotients are the same, so you can use the easier division to help you do the harder division. We know the quotient is 8 because 3 goes into 25 eight times. Once you know the quotient, you can finish the long division.

**Exercises:** Divide.

a) \(20 \div 75\)  
   b) \(30 \div 43\)  
   c) \(50 \div 84\)  
   d) \(40 \div 52\)

**Bonus**

e) \(80 \div 715\)  
   f) \(60 \div 473\)  
   g) \(90 \div 608\)  
   h) \(70 \div 571\)

**Answers:** a) 8 R 15, b) 4 R 23, c) 7 R 34, d) 8 R 32, Bonus: e) 8 R 75, f) 7 R 53, g) 6 R 68, h) 8 R 11

**Rounding the divisor to estimate the quotient.** Write on the board:

\[
\frac{49}{167}
\]

ASK: What makes this problem harder than the other ones we’ve done already? (49 is not a multiple of 10) Is it close to a multiple of 10? (yes) Which one? (50) SAY: 49 is close to 50, so 49 will go into 167 about the same number of times as 50 does. ASK: How many times does 50 go into 167? (3) How did you get that? (because 5 goes into 16 three times) Write “3” as the quotient, then SAY: We don’t know for sure that 3 is the right quotient; we’re just guessing because 49 is so close to 50. Keep the following exercises on the board for the rest of this lesson.

**Exercises:** Estimate the quotient by rounding the divisor.

a) \(19 \div 64\)  
   b) \(38 \div 251\)  
   c) \(41 \div 251\)  
   d) \(81 \div 342\)

**Bonus**

e) \(79 \div 581\)  
   f) \(62 \div 502\)  
   g) \(91 \div 557\)  
   h) \(78 \div 724\)

**Answers:** a) 8, b) 6, c) 6, d) 4, Bonus: e) 7, f) 8, g) 6, h) 9
Multiplying the divisor by the estimated quotient. Refer to the division on the board:

\[
\begin{array}{c}
\frac{3}{49} \overline{)167}
\end{array}
\]

ASK: Now that we have an estimate for what the quotient is, what’s the next step? (multiply the quotient by the divisor) Emphasize that it is not the rounded divisor you multiply by but the actual divisor. SAY: We just used the 50 to find the 3; now that we have the 3, we can use it to finish the division. Ask a volunteer to multiply 49 \times 3 on the board. Have another volunteer show where to put the answer in the division:

\[
\begin{array}{c}
2 \\
4 9
\end{array}
\times
\begin{array}{c}
3
\end{array}
\quad \begin{array}{c}
1 4 7
\end{array}
\]

Exercises

1. Multiply the estimated quotient by the divisor (not the rounded divisor) for the division you estimated on the board.
   a) 19 \frac{164}{167} \\
   b) 38 \frac{251}{285} \\
   c) 41 \frac{251}{285} \\
   d) 81 \frac{342}{385}
   
   **Bonus**
   e) 79 \frac{581}{585} \\
   f) 62 \frac{502}{505} \\
   g) 91 \frac{557}{561} \\
   h) 78 \frac{724}{724}

   **Answers:** a) 152, b) 228, c) 246, d) 324, Bonus: e) 553, f) 496, g) 546, h) 702

2. Round the divisor to estimate the quotient, then multiply the divisor by your estimate.
   a) 18 \frac{143}{143} \\
   b) 52 \frac{274}{274} \\
   c) 48 \frac{361}{361} \\
   d) 31 \frac{194}{194}
   
   **Bonus**
   e) 78 \frac{650}{650} \\
   f) 71 \frac{444}{444}

   **Answers:** a) 7, 126; b) 5, 260; c) 7, 336; d) 6, 186; Bonus: e) 8, 624; f) 6, 426

Finishing the long division. Refer to the example on the board:

\[
\begin{array}{c}
\frac{3}{49} \overline{)167}
\end{array}
\]

\[
\begin{array}{c}
147
\end{array}
\]

\[
\begin{array}{c}
20
\end{array}
\]

SAY: We are sharing 167 objects among 49 groups. ASK: How many are in each group? (3) How many objects have been divided so far? (147) How many have not been divided? (20) Have a volunteer show where to put the answer. Add in the subtraction sign to emphasize that they got the answer by subtracting. ASK: Are we done? (yes) How do you know? (the ones have been divided)
**Exercises:** Subtract to finish the long division you started before.

a) \(18 \div 43\)  
   b) \(52 \div 74\)  
   c) \(48 \div 61\)  
   d) \(31 \div 94\)

**Bonus**

e) \(78 \div 50\)  
   f) \(71 \div 44\)

**Answers:** a) 7 R 17, b) 5 R 14, c) 7 R 25, d) 6 R 8, Bonus: e) 8 R 26, f) 6 R 18

SAY: Now put all the steps together.

**Exercises:** Divide.

a) \(327 \div 51\)  
   b) \(184 \div 28\)  
   c) \(148 \div 31\)  
   d) \(211 \div 47\)

**Bonus**

e) \(583 \div 62\)  
   f) \(642 \div 91\)

**Answers:** a) 6 R 21, b) 6 R 16, c) 4 R 24, d) 4 R 23, Bonus: e) 9 R 25, f) 7 R 5

**Extensions**

1. Without solving, predict the answer to \(14 \div 1.4\). Explain your prediction. Then check your prediction using long division.

   **Answers:** 14 is 10 times 1.4, so \(14 \div 1.4 = 10\). Using long division, \(140 \div 14 = 10\).

2. a) Divide \(99 \div 0.9\) and \(99 \div 1.1\). Which answer is greater than 99? Why does that make sense?

   b) Predict which answer will be greater than 84, then check by long division: \(84 \div 2.1\) or \(84 \div 0.3\).

   **Answers**
   a) \(99 \div 0.9 = 110\) and \(99 \div 1.1 = 90\). This makes sense since dividing by a smaller number gets a larger answer.
   b) \(84 \div 0.3\) should be greater than 84, \(84 \div 0.3 = 280\) and \(84 \div 2.1 = 40\)

3. Billy collects baseball cards and stores them in cardboard storage boxes. Each box holds 72 cards. He can sell each box for $9.50. How much will he get if he sells his 8856 cards?

   **Solution:** \(8856 \div 72 = 123\) boxes, \(123 \times 9.50 = $1168.50\)
2-Digit Division—Guess and Check

Goals
Students will use rounding to estimate the quotient when dividing by two-digit numbers, including for cases in which doing so requires adjusting the estimate.

PRIOR KNOWLEDGE REQUIRED
Can use long division to divide by two-digit numbers
Can divide decimals by whole numbers

MATERIALS
BLM Filling a Blank Multiplication Chart (p. T-2)
BLM Hundreds Charts (M-46)

Mental math minute. Give students BLM Filling a Blank Multiplication Chart. Have them fill in the chart as much as they can in three minutes, using the strategies on the BLM as needed.

Review the guess, check, and revise strategy. Give each student a copy of BLM Hundreds Charts. Ask a volunteer to pick a number from 1 to 100 and circle it on the first hundreds chart. Have other students try to guess the answer. The volunteer is only allowed to answer “yes” or “no.” Students can use a pencil to cross out on their chart any number that got “no” so they know not to use it again. If students guess the correct number quickly, play again until it becomes clear that the strategy is not very effective.

Then change the rules. Tell students that now the volunteer is allowed to answer “too high” or “too low.” Have a different volunteer choose a number. This time, students can cross out all the numbers that are ruled out by the volunteer’s answer. For example, if 29 is too low, then so are 1 to 28.

Tell students that when mathematicians talk about the guess, check, and revise strategy, they don’t mean to check only if the guess is wrong, but how it’s wrong. That way, they can use the information to make a better guess.

Applying the guess, check, and revise strategy to division. Tell students that different people guessed the quotient for different divisions. Challenge students to decide whether the quotient guessed is too high or too low.

Write on the board:

\[
\begin{align*}
5 & \div 9 \quad 9 & \div 25 \quad 9 & \div 18 \quad 6 & \div 15 \\
3 & ) 19 & 3 & ) 25 & 2 & ) 17 \quad 2 & ) 15 \\
-15 & -25 & -18 & -12 \\
\quad 4 & \quad 27 > 25 \quad 18 > 17 \quad \frac{6}{3} \\
\end{align*}
\]

Point to the first one and ASK: We have 4 left over—can we put one more in each group? (yes) Is 5 too low or too high? (too low) Point to the second one and SAY: With 9 objects in each group, we would place 27 objects,
but we only have 25 objects. ASK: Is 9 too low or too high? (too high) Point to the third one and ASK: Is 9 too low or too high? (too high) How do you know? (because 18 objects is too many—we only have 17) Repeat for the fourth one. (6 is too low because we have 3 left over, so we can put one more in each group) Write on the board:

a) \[ \begin{array}{c}
7 \\
-161
\end{array} \]

b) \[ \begin{array}{c}
6 \\
-102
\end{array} \]

c) \[ \begin{array}{c}
8 \\
-152
\end{array} \]

d) \[ \begin{array}{c}
9 \\
-279
\end{array} \]

e) \[ \begin{array}{c}
8 \\
-264
\end{array} \]

Have students signal thumbs up for too high, thumbs down for too low, or flat hand for just right. (a) too high, b) too low, c) too low, d) just right, e) too high)

SAY: Now do the subtraction yourself to decide whether the estimate is too low, too high, or just right.

a) \[ \begin{array}{c}
5 \\
-240
\end{array} \]

b) \[ \begin{array}{c}
9 \\
-288
\end{array} \]

c) \[ \begin{array}{c}
7 \\
-406
\end{array} \]

d) \[ \begin{array}{c}
6 \\
-168
\end{array} \]

e) \[ \begin{array}{c}
4 \\
-188
\end{array} \]

Allow students time to subtract, then have all students signal their answers. (a) 51, too low; b) too high; c) 59, too low; d) 25, just right; e) 47, too low)

Exercises: Multiply the estimated quotient by the divisor. Is the estimate too high, too low, or just right?

a) \[ \begin{array}{c}
7 \\
-36298
\end{array} \]

b) \[ \begin{array}{c}
6 \\
-27183
\end{array} \]

c) \[ \begin{array}{c}
6 \\
-73435
\end{array} \]

d) \[ \begin{array}{c}
9 \\
-24198
\end{array} \]

Bonus

e) \[ \begin{array}{c}
7 \\
-82573
\end{array} \]

f) \[ \begin{array}{c}
6 \\
-68455
\end{array} \]

g) \[ \begin{array}{c}
7 \\
-86702
\end{array} \]

h) \[ \begin{array}{c}
6 \\
-71422
\end{array} \]

Answers: a) 252, too low; b) 162, just right; c) 438, too high; d) 216, too high; Bonus: e) 574, too high; f) 408, just right; g) 602, too low; h) 426, too high

Revising the estimate. Write on the board:

\[ \begin{array}{c}
7 \\
-23156
\end{array} \]

too high

\[ \begin{array}{c}
6 \\
-23156
\end{array} \]

too high

\[ \begin{array}{c}
8 \\
-23156
\end{array} \]

too high

ASK: What is a better guess to try next, 6 or 8? (6) How do you know? (even 7 was too much) Erase the “8” guess. Have a volunteer multiply 23 \times 6 on the board. (138) Have another volunteer show where to put the answer and how to finish the division.

Exercises

1. Use the first estimate to make a better estimate. Then divide.

a) \[ \begin{array}{c}
3 \\
-1978
\end{array} \]

b) \[ \begin{array}{c}
4 \\
-42164
\end{array} \]

c) \[ \begin{array}{c}
4 \\
-72430
\end{array} \]

d) \[ \begin{array}{c}
7 \\
-75638
\end{array} \]

21 > 19, too low! 21 > 19, too low! 21 > 19, too low! 113 > 75, too low!
Bonus

e) \[ 35 \longdiv{295}{245} \]
\[ -245 \]
\[ 50 \]
\[ 59 \]
f) \[ 53 \longdiv{262}{265} \]
\[ -265 \]
\[ 0 \]
g) \[ 66 \longdiv{544}{462} \]
\[ -462 \]
\[ 82 \]
\[ 59 \]
h) \[ 58 \longdiv{291}{232} \]
\[ -232 \]
\[ 59 \]

Answers: a) 4 R 2; b) 3 R 38; c) 5 R 70; d) 8 R 38, Bonus: e) low, 8 R 15; f) high, 4 R 50; g) low, 8 R 16; h) low, 5 R 1

2. Divide.

a) \[ 850 \div 32 \]  
\[ 26 \text{ R } 18 \]
b) \[ 231 \div 17 \]  
\[ 13 \text{ R } 10 \]
c) \[ 654 \div 86 \]  
\[ 7 \text{ R } 52 \]
d) \[ 632 \div 57 \]  
\[ 11 \text{ R } 5 \]
e) \[ 984 \div 31 \]  
\[ 31 \text{ R } 23 \]

Dividing four-digit numbers by two-digit numbers. Write on the board:

\[ 46 \longdiv{9631}{3887} \]

Have volunteers circle the first part of the number that is at least as big as 46. (96, 388) Then have volunteers start the long division by dividing the circled number by 46. For the following exercises, remind students to always place the first digit of the quotient over the right-most digit of the circled number.

Exercises

1. Start the long division by dividing the circled number by 46. Then finish dividing.

a) \[ 46 \longdiv{1542}{1894} \]  
\[ 155 \text{ R } 12 \]
b) \[ 46 \longdiv{304}{2961} \]  
\[ 41 \text{ R } 8 \]
c) \[ 46 \longdiv{202}{1761} \]  
\[ 202 \text{ R } 12 \]
d) \[ 46 \longdiv{64}{5781} \]  
\[ 64 \text{ R } 17 \]

Answers: a) 155 R 12, b) 41 R 8, c) 202 R 12, d) 64 R 17

2. Divide using long division.

a) \[ 49 \longdiv{6532}{2807} \]  
\[ 133 \text{ R } 15 \]
b) \[ 38 \longdiv{4781}{5738} \]  
\[ 73 \text{ R } 33 \]
c) \[ 47 \longdiv{101}{5738} \]  
\[ 101 \text{ R } 34 \]
d) \[ 55 \longdiv{104}{5738} \]  
\[ 104 \text{ R } 18 \]

Answers: a) 133 R 15, b) 73 R 33, c) 101 R 34, d) 104 R 18

Review dividing decimals. Remind students that as long as they can divide whole numbers by whole numbers, they can divide decimals by whole numbers too. Write on the board:

\[ 34 \longdiv{176528}{7652.8} \]

Point to the second division and ASK: Once you do the long division, how do you know where to put the decimal point? (in the same place it is in the dividend) Do the long division and have a volunteer place the decimal point. SAY: The answer for the second division will be one tenth (0.1) of the first division. (5192, 519.2)

Exercises: Use the guess, check, and revise strategy to divide.

a) \[ 27 \longdiv{37.8}{763.8} \]  
\[ 1.4 \]  
Bonus: \[ 840 \longdiv{9811.2}{9811.2} \]  
\[ 11.68 \]

Answers: a) 1.4, b) 13.4, Bonus: 11.68
Word problems practice.

Exercises

a) Lela has 4.2 kg of cheese. She needs 50 g of cheese for each sandwich. How many sandwiches can she make?

b) Tristan has $12.50. How many 50¢ candies can he buy?

c) A shelf is 41.4 cm long. How many 23 mm thick books can the shelf hold?

Answers: a) 84, b) 25, c) 18

Extensions

1. Investigate: are the estimates more likely to be correct when the divisor is closer to the rounded number you used to make your estimate? For example, when the divisor is 31 rounded to 30, is your estimate more likely to be correct than when the divisor is 34 rounded to 30? Try these examples:

   \[
   \begin{array}{lllll}
   31 & 243 \div 31 & 249 & 31 & 257 & 31 & 265 & 31 & 274 \\
   34 & 243 & 34 & 249 & 34 & 257 & 34 & 265 & 34 & 274 \\
   \end{array}
   \]

   Answers: Rounding 31 to 30 gives the right answer in all cases, except the first and last cases. Rounding 34 to 30 doesn’t give the right answer in any case. So rounding 31 to 30 gives the correct quotient more often than rounding 34 to 30.

2. Dividing by three-digit numbers. Circle the first part of the dividend that is at least as big as the divisor.

   a) \(2136 \div 512\) \quad b) \(842\ 605 \div 512\)
   
   c) \(51\ 284\ 030 \div 512\) \quad d) \(256\ 813 \div 357\)
   
   e) \(3\ 941\ 078 \div 357\) \quad f) \(35\ 725\ 321\ 098 \div 357\)

   Answers: a) 2136, b) 842, c) 512, d) 2568, e) 394, f) 357

3. Do the long divisions for the problems in Extension 2.

   Answers: a) 4 R 88, b) 1645 R 365, c) 100 164 R 62, d) 719 R 130, e) 11 039 R 155, f) 100 070 927 R 159

   Encourage students to check their answers by multiplication. Emphasize that doing so is very important when dividing multi-digit numbers because there is so much opportunity for making mistakes.
4. Fill the boxes using the digits 3 to 9 once each.

\[\square \square \times \square \square \square \square \]

Multiply the numbers using a calculator or long multiplication. Then give your partner the following problem:

\[
\text{(the product you found)} \div \text{(the two-digit number)}
\]

Check that your partner gets the same five-digit number you started with.

5. The total team salary for a professional soccer team is $34.5 million. The team has 23 players. If each player earns an equal share of the money, how much is each player paid?

**Answer:** $1.5 million or $1,500,000

6. Decide how the first triangle was made. Then finish the second triangle using the same rule.

```
a) 20
   13 7
   9 4 3

   6
   4.2
   2.52

b) 48
   6 8
   3 2 4

   76.5
   15
   5
```

**Answers:**

```
a) 6
   4.2 1.8
   2.52 1.68 0.12

b) 76.5
   15 5.1
   5 3 1.7
```
Goals
Students will solve word problems involving dividing decimals by two-digit numbers.

PRIOR KNOWLEDGE REQUIRED
Knows how to divide a multi-digit number by a two-digit number using the standard algorithm
Knows how to divide decimals by whole numbers
Knows how to multiply decimals by whole numbers

Word problems. This lesson provides practice with division of two-digit decimal numbers using word problems.

Extensions
1. A class of 18 students buys supplies for a party. Three students spend $5.31 each. Seven students spend $4.65 each. Eight students spend $2.31 each.
   a) How much do the students spend altogether?
   b) The students want to share the cost of the party equally. How much should each student pay?

   Answers: a) $66.96, b) $3.72

2. A parent council is helping to make pancakes for a breakfast party at the school. The recipe they are using will make six pancakes and calls for the following ingredients:
   - 2.5 cups of pancake mix
   - 2 tablespoons of sugar
   - 0.5 teaspoons of cinnamon
   - 0.25 teaspoons of nutmeg
   - 0.25 teaspoons of ground ginger
   - 2 eggs
   - 1.5 cups of milk

   The parents have plenty of the other ingredients but only 45 cups of pancake mix and 25.5 cups of milk. Without having to buy more ingredients, how many pancakes can they make?

   Solution: 25.5 cups of milk ÷ 1.5 = 17, 17 x 6 = 102 pancakes
Goals

Students will see connections between numbers and real-life situations, review concepts learned to date, and use them to solve word problems using decimals.

PRIOR KNOWLEDGE REQUIRED

Understands decimal numbers
Understands times as many
Can solve word problems with whole numbers
Can add and subtract decimals with regrouping
Can multiply and divide decimals by powers of 10

MATERIALS

index cards
BLM Always, Sometimes, or Never True (Decimals) (p. M-47)

NOTE: This lesson reviews both Units 9 and 10, so adding and subtracting decimals is reviewed as well as the material from this unit.

Reviewing decimals. This lesson is mostly a cumulative review. The following exercises provide some additional problems that you can use for cumulative review.

Exercises

a) Two friends ate \( \frac{6}{10} \) of a pizza. Write the fraction of the pizza they ate as a decimal.

b) A carpenter used \( \frac{4}{10} \) of a box of 100 nails on Monday and \( \frac{3}{100} \) of the box on Tuesday. Write the total fraction of the nails used as a decimal.

c) A carpenter used 0.5 of the nails in a box of 1000 nails. How many nails did the carpenter use?

d) Ken ran 2.51 km, Jessica ran 2.405 km, and Kate ran 2.6 km. Who ran the farthest?

e) Three plants are 0.6 m, 0.548 m, and 0.56 m tall. Order the heights of the plants from least to greatest.

f) Write a decimal between the two given decimals: 45.79 and 45.8. There are many correct answers.

Answers: a) 0.6; b) 0.43; c) 500; d) Kate; e) 0.548 m, 0.56 m, 0.6 m;

Sample answer: f) 45.791
ACTIVITY (Optional)

Give each student an index card and a card from BLM Always, Sometimes, or Never True (Decimals). Have students decide whether the statement on the card is always true, sometimes true, or never true.

They should write reasons for their answers, such as an explanation for “always true” or “never true” statements, and two examples (one true, one false) for the statements that are “sometimes true” on the index card and glue the card with the statement to the other side of the index card.

Have students pair up. Partners exchange cards and verify each other’s answers. Then players exchange cards and seek a partner with a card they have not seen yet.

Extensions

1. Doctors study the body. Here are some facts a doctor might know:
   a) FACT: “The heart pumps about 0.06 L of blood with each beat.” How much blood would the heart pump in 3 beats?
   b) FACT: “The heart beats about 80 times a minute.” How long would it take the heart to beat 240 times?
   c) FACT: “Each minute, your heart pumps all of your blood.” How many times in one day would your heart pump all your blood?
   d) FACT: “About $\frac{55}{100}$ of human blood is a pale yellow liquid called plasma.” How much plasma would there be in 2 L of blood?
   
   **Answers:** a) 0.18 L, b) 3 minutes, c) 1440, d) 1.1 L

2. Which is a better deal: 3 pencils for $2.34 or 5 pencils for $3.95?

   **Solution:** $2.34 \div 3 = 0.78$ and $3.95 \div 5 = 0.79$, so 3 pencils for $2.34 is the better deal
# Hundreds Charts

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<td>If you multiply a three-digit whole number by a one-digit whole number, the answer will be a three-digit whole number.</td>
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<tr>
<td>An improper fraction is larger than a proper fraction.</td>
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<td>The product of two even numbers is an even number.</td>
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<td>The sum of two odd numbers is an odd number.</td>
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<td>A prime number is a whole number.</td>
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<td>A mixed number is larger than 1.</td>
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<td>When you divide whole numbers, the remainder is less than the number you are dividing by.</td>
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<td>A multiple of 5 is also a multiple of 2.</td>
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<td>In two fractions, the greater fraction has the smaller denominator.</td>
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<td>An improper fraction is greater than 1.</td>
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<td>A whole number is a composite number.</td>
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<td>A sum of two decimals smaller than 1 is smaller than 1.</td>
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<tr>
<td>A difference between two decimals smaller than 1 is smaller than 1.</td>
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<tr>
<td>A whole number made from the digits 1, 2, and 3 used once each is smaller than a whole number made from the digits 4, 5, and 6 used once each.</td>
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PS6-7 Combining Systematic Search with Guess, Check, and Revise

Teach this lesson after:
Unit 10

Goals
Students will search systematically and efficiently by skipping numbers and using the order of the numbers to inform when they have gone too far.
Students will learn to choose a starting point in order to find the answer quicker.

PRIOR KNOWLEDGE REQUIRED
Can order and compare multi-digit whole numbers
Can substitute whole numbers for variables in expressions
Can multiply multi-digit whole numbers by one-digit whole numbers
Knows to evaluate expressions in brackets first
Can search systematically to find mystery numbers
Understands how guessing a middle number can make guessing efficient
Can multiply decimal hundredths by whole numbers (for Problem Bank 16)

MATERIALS
calculator

Review searching systematically to find mystery numbers. Write on the board:

If \( N \) is a whole number so that \( N \times N \times N \times N = 2401 \), what is \( N \)?

SAY: Remember that we can solve equations that look hard if we know that the answer is a whole number. Let’s try the whole numbers in order to see if we can find the answer quickly. Draw on the board:

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1 \times 1 \times 1 \times 1 = 1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 2 \times 2 \times 2 = 16 )</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 3 \times 3 \times 3 = 81 )</td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 4 \times 4 \times 4 = 256 )</td>
</tr>
</tbody>
</table>

ASK: Are we getting closer to the answer? (yes) Are the numbers getting big quickly? (yes) SAY: Let’s keep going because we might find the answer fairly quickly.

Allow students to use a calculator for the following exercises.
Exercises

1. Continue the table until you get \( N \times N \times N \times N = 2401 \). What is \( N \)?

   \[ \text{Answer: } N = 7 \]

2. \( N \) is a whole number so that \( N \times N \times N = 512 \). What is \( N \)?

   \[ \text{Answer: } N = 8 \]

**Searching faster by skipping numbers.** Write on the board:

If \( N \times N \times N \times N = 1\,048\,576 \), what is \( N \)?

SAY: We just solved a problem like this. ASK: How did we do it? (we made a table and started at 1 and moved up the numbers in order) ASK: Would continuing the table be a good strategy for this question? (no) Why not? (it will take too long to get to the answer) SAY: The answers are getting closer to the answer but not much closer; you still have a long way to go to find the answer. Maybe you need to take bigger steps to find the answer. Instead of trying 1, 2, 3, and so on, maybe we should start with 10, 20, 30, and so on.

**Exercises**

a) Complete the table up to 50.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N \times N \times N \times N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( 10 \times 10 \times 10 \times 10 = 10,000 )</td>
</tr>
<tr>
<td>20</td>
<td>( 20 \times 20 \times 20 \times 20 = 160,000 )</td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

b) Using \( N \times N \times N \times N = 1\,048\,576 \), what two 10s is \( N \) between? Explain how you know.

**Answers**

a) 810 000, 2 560 000, 6 250 000;

b) \( N \) is between 30 and 40 because \( N \times N \times N \times N \) is between 810 000 and 2 560 000.

SAY: Now we know that \( N \) is between 30 and 40. Write on the board:

\[
\begin{align*}
30 \times 30 \times 30 \times 30 &= 810\,000 \\
N \times N \times N \times N &= 1\,048\,576 \\
40 \times 40 \times 40 \times 40 &= 2\,560\,000
\end{align*}
\]
SAY: Now we can move up by ones until we get the answer because we know that we are pretty close. Write on the board:

<table>
<thead>
<tr>
<th>N</th>
<th>N × N × N × N</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30 × 30 × 30 × 30 = 810 000</td>
</tr>
<tr>
<td>31</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>

Ask a volunteer to use a calculator to complete each row of the table until you get the correct answer. (31 × 31 × 31 × 31 = 923 521 and 32 × 32 × 32 × 32 = 1 048 576, so N is 32)

**Exercises**

1. Find N so that N × N × N × N is ...
   a) 10 556 001  
   b) 45 212 176  
   **Bonus:** 1 698 181 681. Hint: Move up by hundreds, then by tens, then by ones.
   **Answers:** a) 57, b) 82, Bonus: 203

2. Find N so that N × (N + 1) × (N + 2) is ...
   a) 24 360  
   b) 157 410  
   **Bonus:** 70 444 584. Hint: Start moving up by hundreds, then by tens, and then by ones.
   **Answers:** a) 28, b) 53, Bonus: 412

**Searching from either direction.** SAY: Cameron and Avril went to a farm that has cows and chickens. Write on the board:

Cameron counts 36 legs.  
Avril counts 10 heads.

**ASK:** How many animals are there altogether? (10) How do you know? (the number of heads is the same as the number of animals) Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**ASK:** If there are zero cows and 10 chickens, how many legs are there? (20)
Write “20” in the first row of the third column. **ASK:** If there are 10 cows and zero chickens, how many legs are there? (40) Write “40” in the last row of the third column. **ASK:** Do you think the number of cows in our answer
will be closer to zero or 10? (10) Why? (the number of legs is closer to 40 than to 20) PROMPT: Is the actual number of legs closer to 20 or 40? (40)

ASK: So, is it better to start our search closer to zero or to 10? (10) SAY: We save ourselves a lot of work by starting at 10 cows and zero chickens instead of starting at zero cows and 10 chickens. Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

ASK: How many legs do nine cows have? (36) Write on the board:

36 +

ASK: How many legs does one chicken have? (2) Continue writing on the board:

36 + 2 = 38

Write “38” as the total in the row for 9 cows and 1 chicken. Repeat for the row with 8 cows and 2 chickens. (32 + 4 = 36) SAY: So, 8 cows and 2 chickens have a total of 36 legs. Starting from 10 cows and searching is a lot less work than starting from zero cows and moving all the way up to 8 cows.

**Exercises:** If all the heads Avril counts belong to cows, how many legs are there? If all the heads Avril counts belong to chickens, how many legs are there?

a) Avril counts 30 heads.

b) Avril counts 37 heads.

c) Avril counts 28 heads. **Bonus:** Avril counts 1000 heads.

**Answers:** a) 120, 60; b) 148, 74; c) 112, 56; Bonus: 4000, 2000

SAY: Once you know how many legs there are if all the animals are cows and if all the animals are chickens, you can compare those numbers with the total number of legs given. Then you can decide which option to use to start your search.

**Exercises:** How many cows and how many chickens are there?

a) Cameron counts 22 legs. Avril counts 9 heads.

b) Cameron counts 52 legs. Avril counts 14 heads.

c) Cameron counts 114 legs. Avril counts 30 heads.

d) Cameron counts 140 legs. Avril counts 37 heads.

e) Cameron counts 60 legs. Avril counts 28 heads.

**Bonus:** Cameron counts 3996 legs. Avril counts 1000 heads.
**Answers:** a) 2 cows, 7 chickens; b) 12 cows, 2 chickens; c) 27 cows, 3 chickens; d) 33 cows, 4 chickens; e) 2 cows, 26 chickens; Bonus: 998 cows, 2 chickens

SAY: Once you decide where to start, you might want to skip count by tens first and then by ones to get to the answer quickly.

**Exercises:** There are 100 cows and chickens altogether. How many cows and how many chickens are there?

a) Cameron counts 240 legs.

b) Cameron counts 372 legs.

c) Cameron counts 300 legs.

**Bonus:** There are 1000 cows and chickens altogether. Cameron counts 2366 legs. How many cows and how many chickens are there? Hint: Count by hundreds, then by tens, then by ones.

**Answers:** a) 20 cows, 80 chickens; b) 86 cows, 14 chickens; c) 50 cows, 50 chickens; Bonus: 183 cows, 817 chickens

**Using the guess-check-revise strategy when two quantities are changing.** SAY: Because the total number of legs increases as the number of cows increases and the number of chickens decreases, you can guess an answer and know right away whether your answer is too high or too low. This allows you to play a game like “too high” or “too low” when guessing numbers. Write on the board:

- Cameron counts 344 legs.
- Avril counts 100 heads.

**How many cows and how many chickens are there?**

SAY: There are 100 heads. That means that there are 100 animals altogether, some of them are cows and the rest are chickens, but I don’t know how many of each there are. There might be more cows or there might be more chickens. For my first guess, I’m going to assume a middle situation—that there is the same number of each type of animal.

ASK: How many cows am I assuming there are? (50) SAY: 50 is right in the middle, between zero and 100. ASK: Why is that a good starting guess? (it eliminates half the answers no matter what, whether 50 is too low or too high) And how will I know whether 50 cows is too high or too low? (If the number of legs with 50 cows is less than 344, then I need to add more cows and subtract chickens to get the number of legs up to 344. If the number of legs is more than 344, I need to add more chickens and subtract cows.)

SAY: Let’s see how many legs there are if there are 50 cows and 50 chickens. Write on the board:

- 50 cows have _____ legs altogether.
- 50 chickens have _____ legs altogether.
Have volunteers tell you what to put in the blanks. (200, 100) ASK: How many legs is that altogether? (300) Is that too many legs or too few? (too few) SAY: We need more legs. ASK: Does that mean we need more cows or more chickens? (more cows) Why do you say that? (cows have more legs than chickens) SAY: The number of cows we guessed was too low, so we need more cows. ASK: How many cows and chickens should we guess next? (take various answers) Would 51 cows be a good guess? (no) Why not? (that would only eliminate one more number if it doesn't work; there are quite a few more legs than 300; adding one more cow won't add that many more legs) Encourage students to pick a number that is somewhere in the middle of the numbers left to check, such as 75. Suppose students guess 75 cows. ASK: So, how many chickens will there be? (25) How do you know? (there are 100 animals altogether) Write on the board:

75 cows have ______ legs altogether.
25 chickens have ______ legs altogether.

Have volunteers tell you what to put in the blanks. (300, 50) ASK: How many legs is that altogether? (350) Is that too many legs or too few? (too many) SAY: So, we need fewer legs. ASK: Does that mean we need more cows or fewer cows? (fewer cows) SAY: Our first guess of 50 cows got us 300 legs and our next guess of 75 cows got us 350 legs in total. ASK: How many legs in total are we aiming for altogether? (344) Is that closer to 300 legs or to 350 legs? (350 legs) SAY: We actually have more information than just that the number of cows is too high. We also have a sense that our guess is not too far off. That means we can make our next guess closer to 75 than to 50; we don’t have to guess right in the middle. Continue in this way until the correct number of cows is guessed. (72 cows and, hence, 28 chickens)

**Exercises:** Cameron counts 272 legs. Avril counts 100 heads.

a) How many cows and how many chickens are there? Keep track of your guesses.

b) How many guesses did you use to answer part a)?

**Bonus:** Cameron counts 3166 legs. Avril counts 1000 heads. How many cows and how many chickens are there?

**Selected answers:**

a) 36 cows and 64 chickens, Bonus: 583 cows and 417 chickens

**Problem Bank**

1. Use systematic search to find a whole number so that \(3 \times N + 5 = 29\).

**Answer:** 8
2. Answer all the following questions without using long division.
   a) Is there a whole number $N$ so that $3 \times N = 414$?
   b) Is there a whole number $N$ so that $3 \times N = 415$? Explain how you know.
   c) Is there a whole number $N$ so that $3 \times N = 718$? Explain how you know.
   d) Is there a whole number $N$ so that $3 \times N + 5 = 2162$? Explain how you know.

   **Answers:** a) 138; b) no, sample explanation: because it is one more than 414; c) no, sample explanation: because it is 2 less than 720, which is $3 \times 240$; d) yes, $N = 719$

3. Use a calculator to find $N$ if $N \times N = 1849$.

   **Answer:** 43

4. Is there a whole number $N$ so that $N \times N = 7541$? How do you know?

   **Answer:** no, because $86 \times 86 = 7396$, so 86 is too low, but $87 \times 87 = 7569$, so 87 is too high

5. a) Jessica wants to find a whole number $N$ so that $N \times N \times N = 46656$. She starts by guessing $N = 100$. Is that too high or too low? How do you know?
   b) Find a whole number $N$ so that $N \times N \times N = 46656$.
   c) Find a whole number $N$ so that $N \times N \times N \times N = 187388721$.

   **Answers:** a) 100 $\times$ 100 $\times$ 100 = 1 000 000, so 100 is too high; b) 36; c) 117

6. Is there a whole number $N$ so that $N \times N = 85$. How do you know?

   **Answer:** no, because $9 \times 9 = 81$, so 9 is too low, but $10 \times 10 = 100$, so 10 is too high

7. Find $N$ so that …
   a) $(2 \times N) + 1 = 177$
   b) $(N \times 3) + N = 228$
   c) $(N \times 5) + 5 = 320$

   **Answers:** a) 88, b) 57, c) 63

8. Megan’s mom was 32 when she had Megan. Ten years from today, the sum of Megan’s age and her mother’s age will be 80. How old is Megan now?

   **Answer:** 14
9. In 2037, Canada will be 153 more years old than it will be decades old. How old will Canada be in 2037?
   Answer: 170 years, or 17 decades

10. Find the whole number $A$.
   a) If $\frac{A - 1}{A + 1} = \frac{4}{5}$, what is $A$?
   b) If $\frac{A \times A}{A + A} = 4$, what is $A$?
   c) If $\frac{A + 2}{(A + 2) + 1} = \frac{2}{3}$, what is $A$?
   d) If $\frac{A + 4}{A \times A} = \frac{1}{2}$, what is $A$?
   Answers: a) 9, b) 8, c) 4, d) 4

11. There are five-headed dragons and nine-headed dragons. Altogether, 100 dragons have 608 heads. How many of each kind of dragon are there?
   Answer: 27 nine-headed dragons and 73 five-headed dragons

12. Matt built some bicycles and tricycles. Altogether, he made 100 vehicles. If he used 233 wheels altogether, how many bicycles and how many tricycles did he make?
   Answer: 67 bicycles and 33 tricycles

13. A vending machine has quarters and dimes. Altogether, 100 coins have a value of $17.50 (that's 1750 cents). How many of the coins are quarters?
   Answer: 50

14. What are the two numbers?
   a) The bigger number is seven times the smaller number. Their product is 252.
   b) The bigger number is seven times the smaller number. Their product is 11 200.
   c) The bigger number is seven times the smaller number. Their product is 47 068.
   Answers: a) 6 and 42, b) 40 and 280, c) 82 and 574

15. A school fundraiser has a bake sale that sells muffins and cake. A muffin costs $2 and a piece of cake costs $3. The bake sale sold 30 items altogether and made $71. How many muffins and how many pieces of cake were sold?
   Answer: 19 muffins and 11 pieces of cake
16. A school bake sale sells muffins and pieces of cake. A muffin costs $2.50 and a piece of cake costs $3.50. The bake sale sold 47 items and made $134.50 in total. How many muffins and how many pieces of cake were sold?

**Answer:** 30 muffins and 17 pieces of cake

17. Use a calculator to answer the question. Remember that two whole numbers are consecutive if there is no whole number between them.

   a) Calculate the product.
      i) $1 \times 2$
      ii) $2 \times 3$
      iii) $3 \times 4$
      iv) $4 \times 5$
      v) $5 \times 6$

   b) Is 14 the product of two consecutive whole numbers? Explain how you know.

   c) Can 160 be the product of two consecutive whole numbers? Explain how you know.

   d) Can 992 be the product of two consecutive whole numbers? Explain how you know.

   e) Write 6972 as a product of two consecutive whole numbers.

   **Answers:** a) i) 2, ii) 6, iii) 12, iv) 20, v) 30; b) no, it is between 3 $\times$ 4 and 4 $\times$ 5; c) no, it is between 12 $\times$ 13 = 156 and 13 $\times$ 14 = 182; d) yes, it is 31 $\times$ 32; e) 83 $\times$ 84

18. A perfect square is the product of a whole number with itself.

   a) Calculate the product.
      i) $1 \times 1$
      ii) $2 \times 2$
      iii) $3 \times 3$
      iv) $4 \times 4$
      v) $5 \times 5$

   b) Is 25 the product of two consecutive whole numbers? Explain how you know.

   c) Write 400 as a perfect square.

   d) Can you write 400 as the product of two consecutive whole numbers? Explain how you know.

   e) Explain why a perfect square cannot be the product of two consecutive whole numbers.

   **Answers:** a) i) 1, ii) 4, iii) 9, iv) 16, v) 25; b) no, because it is between 4 $\times$ 5 = 20 and 5 $\times$ 6 = 30 and there is no product of consecutive whole numbers between those two; c) 400 = 20 $\times$ 20; d) no, because it is between 19 $\times$ 20 = 380 and 20 $\times$ 21 = 420; e) Any perfect square is between two consecutive products of consecutive whole numbers, so it cannot be the product of two consecutive whole numbers. For example, 15 $\times$ 15 is in between 14 $\times$ 15 and 15 $\times$ 16.
PS6-8 Using Logical Reasoning

Teach this lesson after:
Unit 10

VOCABULARY
- counter-example
- divisible
- false
- good
- true

Goals

Students will identify false statements of the form “all [of these] are [like this]” by using counter-examples.
Students will identify true statements of the same form by checking all examples or by using reasoning.

PRIOR KNOWLEDGE REQUIRED

- Can identify numbers divisible by 2, 5, and 10
- Can identify even and odd numbers
- Can order and compare multi-digit numbers
- Can use long division to divide by one- and two-digit numbers
- Can recognize multiples of 10

Introduce the term “counter-example.” Draw on the board:

All the circles are shaded.

Have a volunteer identify which circle shows that the statement is false. Repeat for the two different statements and picture below:

All the squares have a horizontal side.
All the squares are shaded.

Tell students that an example that proves a statement false is called a counter-example to the statement.

NOTE: Draw the triangles so that A and D are isosceles, B is right scalene, and C is equilateral but rotated.

Exercises: Which shape is the counter-example to the statement?

A.    B.    C.    D.

a) All triangles are striped.
b) All triangles have a horizontal side.

Bonus: All triangles have at least two equal sides.

Answers: a) D, b) C, Bonus: B
Recognizing when a statement does not apply to all examples. Draw on the board:

All the circles are shaded.

A. \begin{tikzpicture}
\fill[gray!50] (0,0) circle (0.5);
\end{tikzpicture}
B. \begin{tikzpicture}
\fill[gray!50] (1,0) rectangle (2,1);
\end{tikzpicture}
C. \begin{tikzpicture}
\fill[gray!50] (3,0) circle (0.5);
\end{tikzpicture}
D. \begin{tikzpicture}
\fill[gray!50] (4,0) rectangle (5,1);
\end{tikzpicture}
E. \begin{tikzpicture}
\fill[gray!50] (6,0) circle (0.5);
\end{tikzpicture}
F. \begin{tikzpicture}
\fill[gray!50] (7,0) rectangle (8,1);
\end{tikzpicture}

ASK: What is this statement about? (circles) Underline all the circles. Emphasize that the statement refers only to the circles; it doesn’t matter whether any of the other shapes are shaded or not. ASK: Are all circles shaded? (no) Have a volunteer circle the counter-example. (E) Erase the underlining and the circling and repeat with new statements (see below), underlining the relevant shapes first. Emphasize in each case that the sentence is only about the shapes you underline; the shapes that are not underlined don’t matter.

- All the squares are big. (D)
- All the squares are shaded. (D and F)
- All the big squares are shaded. (F)
- All the small circles are shaded. (E)

Exercises: Name the counter-example for the statement, using the same picture as above.

a) All the shaded shapes are circles.
b) All the white shapes are small.
c) All the shaded shapes are big.
d) All the white shapes are squares.
e) All the big shapes are squares.
f) All the big shapes are shaded.
g) All the small shapes are white.
h) All the small white shapes are squares.


As students complete the exercises above, encourage them to first write down the shapes that the statement is talking about. (For example, the statement in part a) is about the shaded shapes: A, B, and C.) These are where the students should look for a counter-example. For students who need extra help, you can draw the all shapes in their notebook for them, and they can underline the shapes each question is referring to (and erase the underlining before starting each new question). Write on the board:

All words start with the letter b.
Ask if each of the examples below is a counter-example to the statement and have students explain why or why not:

- bat (no, because it does start with b)
- cat (yes, it is a word that does not start with b)
- boat (no, because it does start with b)
- bxcv (no, because it does start with b; or no, it is not a word)
- xcvb (no, because it is not a word, and the statement only talks about words, so something that is not a word cannot be a counter-example)

**Exercises:** Find the counter-example among the listed examples.

a) All nouns have the letter e.
   - red
   - brown
   - truck
   - bike

b) All even numbers have a digit 2.
   - 23
   - 32
   - 34
   - 43

c) All numbers divisible by 5 have a ones digit 5.
   - 35
   - 40
   - 52
   - 55

**Answers:** a) truck, b) 34, c) 40

**Proving a statement is true by checking all examples.** Draw on the board:

All the squares are black.

A. B. C. D.
E. F. G. H.
I. J. K.

Demonstrate checking all the squares to see whether they are black. They are, so the statement is true. Repeat with the statement “All triangles have a horizontal side” and have volunteers check all the triangles. (again, the statement is true) Repeat with “All squares have a horizontal side.” (this statement is false; I is a counter-example)

Point out that in order to show that a statement is true, students need to check all examples. To show that a statement is false, students just need to identify any one counter-example.
**Exercises:** Decide whether the statement is true or false. If it is false, provide the counter-example.

a) All striped shapes are big.
b) All triangles are big.
c) All big circles are black.
d) All small squares have a horizontal side.
e) All small shapes have a horizontal side.

**Bonus:** All large black triangles are equilateral.

**Answers:** a) true; b) false, D; c) false, H or K; d) true; e) false, J;

**Bonus:** false, E

**Using reasoning to prove a statement true.** Write on the board:

Whenever it is raining, there are clouds.

ASK: Is this statement true or false? (true) Do you have to check for clouds every time it rains to know that the statement is true? (no) How do you know without checking that it is true? (rain can only come from clouds) Tell students that there is often a reason why a statement is true. When there is, students don’t have to check all examples to prove it. Review the words “even” and “odd” as they apply to numbers. (even numbers are multiples of 2, odd numbers are not even) Write on the board:

All even numbers have an even digit.
All even numbers have an odd digit.

Tell students that one of the statements is true and the other is false.

ASK: Which statement is true? (all even numbers have an even digit) How do you know it’s true? (the ones digit is always even for any even number) Explain that if you had to check all even numbers, one by one, you would be checking forever! SAY: Because we know the reason this statement is true, we don’t have to check every example. Have a volunteer name a counter-example to the second statement. Again, point out that students don’t need to check every even number—one counter-example is enough to prove it’s false.

**Exercises:** Either explain why the statement is true or find a counter-example.

a) All three-digit numbers less than 200 have a digit 1.
b) All three-digit numbers more than 200 have a digit 1.
c) All three-digit numbers less than 900 have a digit 9.
d) All three-digit numbers more than 900 have a digit 9.

**Answers:** a) true, because all three-digit numbers less than 200 are in the hundreds, so their hundreds digit is 1; b) false, sample counter-example: 202; c) false, sample counter-example: 100; d) true, because all three-digit numbers more than 900 are in the 900s and so have hundreds digit 9
Using systematic search to investigate conjectures. Write on the board:

A two-digit number is called **good** if it is divisible by the sum of its digits.

SAY: We’ll call a two-digit number **good** if it is divisible by the sum of its digits. Ask volunteers to come to the board and divide various two-digit numbers by the sum of their digits: 36, 42, 43, 12, 19, 84, 55, 70, 90. Ask: Which of these numbers are good? (36, 42, 12, 84, 70, 90)

**Exercises:** Investigate if the statement is true by moving up in order through all possibilities. Write “true” if it is true; if it is false, write the first counter-example that you found.

a) All two-digit numbers that are multiples of 10 are good.

b) All two-digit numbers that are multiples of 3 are good.

c) All two-digit numbers that are multiples of 6 are good.

d) All two-digit numbers that are multiples of 9 are good.

e) All two-digit numbers whose digits add to 6 are good.

f) All two-digit numbers whose digits add to 3 are good.

g) All two-digit numbers whose digits add to 9 are good.

**Answers:** a) true; b) false, 15; c) false, 66; d) false, 99; e) false, 15; f) true; g) true

**Problem Bank**

1. What is the smallest number that will make the statement true?

   "All three-digit numbers more than ___ have a digit 9."

   **Answer:** 888, because 889 has ones digit 9, and any number in the 890s has tens digit 9, and any number in the 900s has hundreds digit 9

2. Remember that the letters a, e, i, o, u, and sometimes y are vowels.

   a) For which of these statements is “Bob” a counter-example?

      A. All names have two vowels.

      B. All names have three letters.

      C. All names have four letters.

      D. All boys’ names start with D.

      E. All names are boys’ names.

      F. All names read the same backwards as they do forward.

   b) Marcel wants to find a counter-example to each of the three statements for which “Bob” is not a counter-example (i.e., statements B, E, and F). Find one example that works as a counter-example to all three statements at the same time.
c) Explain why there cannot be a counter-example to all six statements at the same time. Hint: Look at statements D and E.

**Answers:** a) A, C, and D; b) sample answer: Sara; c) To be a counter-example to D, the name would have to be a boy’s name. On the other hand, to be a counter-example to E, the name would have to not be a boy’s name. So, there cannot be a counter-example to both D and E at the same time. Therefore, there cannot be a counter-example to all six statements at the same time.

3. Make up a statement so that …
   a) the word “run” is a counter-example.
   b) the number 8 is a counter-example.

4. How many numbers do you have to check to show that the following statement is true?

   “When written out in words, no numbers less than one thousand have a letter A.”

   **Solution:** Number words to check: zero to twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred. That’s it! Every other number less than one thousand is written as a combination of these words, and so also will not have a letter A. Examples: three hundred forty-two, one hundred seventeen. **NOTE:** The word “and” is reserved for mixed numbers and decimals, such as writing 3.2 as “three and two tenths,” so 342 is not written as “three hundred and forty-two” as is commonly believed.

5. A three-digit number is called good if it is divisible by the sum of its digits. Are the three-digit numbers described always good?
   a) numbers that are multiples of 10
   b) numbers that are multiples of 100
   c) numbers whose sum of digits is 3
   d) numbers whose sum of digits is 6
   e) numbers whose sum of digits is 9

   **Answers:** a) no, b) yes, c) yes, d) no, e) yes

6. Make at least two statements about which four-digit numbers are always good. Verify your statement.
   a) Numbers that are multiples of ___ are always good.
   b) Numbers whose sum of digits is ___ are always good.

   **Answers:** a) 1000, b) 3 or 9
7. Show that the numbers in this sequence are all good: 42, 402, 4002, 40 002, ....

Hint: Find a pattern in the quotients.

8. Provide students with scientific statements that can be proven true using logic or proven false using a counter-example. Examples:

a) All solids expand when they melt.

b) All ice cubes are colder than 10°C.

Answers: a) ice is a counter-example; b) true, because all ice cubes have temperature at most 0°C, the freezing point of water

9. Have students decide whether statements of the form “all [of these] are not [like this]” are true or false. Example: For the shapes below, determine whether each statement is true or false.

![Shapes A, B, C, D, E, F]

a) All triangles are not equilateral.

b) All squares are not black.

c) All circles are not big.

d) All circles are not white.

e) All black shapes are not squares.

f) All black shapes are not small.

Answers: a) false, B; b) true; c) true; d) false, F; e) true; f) false, D

10. Remember, a number is even if that many objects can be paired up without a remainder. For each statement, either explain why the statement is true or find a counter-example.

a) The product of any two numbers is greater than their sum.

b) The sum of any two even numbers is even.

c) The sum of any two odd numbers is odd.

d) The sum of any three even numbers is even.

e) The sum of any three odd numbers is odd.

NOTE: A counter-example would consist of two numbers in parts a) to c) and three numbers in parts d) and e). Some students might need this pointed out to them.
Answers: a) sample counter-examples: 1 and 5 have product 5 and sum 6; 0 and 3 have product 0 and sum 3; b) true, because if you can pair up, for example, 8 objects without a remainder and you can pair up 10 objects without a remainder, then you can do the same to $10 + 8 = 18$ objects by just combining your pairs; c) any pair of odd numbers is a counter-example, (e.g., 3 and 5 add to 8); d) true, because combining three sets of paired-up objects still leaves everything paired up; e) true, because combining three sets of paired-up objects, where one object from each set is not paired up, leaves three unpaired objects, two of which make a pair with one left over.

11. a) What are the ones digits of the multiples of 2?
   
b) What are the ones digits of the multiples of 5?
   
c) The numbers that are multiples of both 2 and 5 are the numbers that have ones digit ___.
   
d) Explain why this statement is true: All numbers that are multiples of both 2 and 5 are multiples of 10.
   
e) Find a counter-example for this statement: All numbers that are multiples of both 4 and 6 are multiples of 24.
   
Answers: a) 0, 2, 4, 6, or 8; b) 0 or 5; c) 0; d) The numbers that are multiples of both 2 and 5 must have ones digit 0 because that is the only number in both lists. But the numbers with ones digit 0 are exactly the multiples of 10; e) 12 is a multiple of both 4 and 6 but not a multiple of 24.
PS6-9 Making a Simpler Problem

Teach this lesson after:
Unit 10

Goals
Students will learn a variety of strategies to make a problem easier or clearer.

PRIOR KNOWLEDGE REQUIRED
Can add decimal tenths
Can multiply decimal tenths by a whole number

MATERIALS
grid paper (e.g., from BLM 1 cm Grid Paper, p. M-76)
BLM Fraction Strips and Circles (p. M-77, see Problem Bank 7)

Using smaller numbers to make a simpler problem. Write on the board:

A teacher tells her students to read pages 287 to 354 for homework. How many pages is that?

ASK: What makes this problem hard? (sample answer: 287 and 354 are big numbers) Would it be easier to know how many pages the students have to read if the teacher tells them to read pages 353 to 355 for homework? (yes, you could just count the pages: 353, 354, and 355 are three pages) SAY: So, it’s not exactly how big the numbers are that makes this problem hard. ASK: Can you find a more precise way to say what makes this problem hard? (the numbers are far apart) Have volunteers give you similar, simpler problems that you could solve first. (for example, make the numbers smaller and closer together) Write all the suggestions on the board.

Exercise: Solve all the simpler problems on the board. Do you see a pattern in your answers?

Answer: In all cases, you can find the number of pages by subtracting the smaller number from the bigger number and then adding 1.

Have a volunteer tell you the pattern in the exercise. (subtract the numbers and add 1) SAY: Now that you know the pattern, you can solve any problem of the same type.

Exercises: A teacher tells her students to read pages in a textbook for homework. How many pages do the students need to read?

a) from 352 to 386
b) from 298 to 314
c) from 408 to 451

Answers: a) 35, b) 17, c) 44
Listing similar problems in an organized way. Tell students that it can be helpful to examine the simpler problems in an organized way. Refer students to the problem about reading from pages 287 to 354. Write on the board:

<table>
<thead>
<tr>
<th>Pages Read</th>
<th>How Many Pages?</th>
</tr>
</thead>
<tbody>
<tr>
<td>287 to 288</td>
<td>2</td>
</tr>
<tr>
<td>287 to 289</td>
<td>3</td>
</tr>
<tr>
<td>287 to 290</td>
<td>4</td>
</tr>
<tr>
<td>287 to 291</td>
<td>5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>287 to 354</td>
<td>?</td>
</tr>
</tbody>
</table>

SAY: By being organized, you might find the pattern quicker. Patterns can be easier to see when you have something organized to look at, like a table.

**Exercises:** Make several simpler problems until you see the pattern to complete the harder problem. Organize the simpler problems.

a) A fence is made using 42 posts, each 1 m apart. How long is the fence?  
b) A fence is made using 34 posts, each 2 m apart. How long is the fence?

**Answers:** a) 41 m, b) 66 m

The importance of seeing given information visually. Tell students that you want them to think of a word that has the letters l, t, and r. (sample answers: letter, later, trail, rattle, teller, retail, trailer, relent, relate) After some students tell you an answer, ASK: Did anyone write the letters down so that you didn’t have to remember them? SAY: There are many ways to make a problem easier. One of them is to write down details so you don’t have to keep everything in your head.

**Drawing a diagram to solve a problem.** Write on the board:

Kyle decided to go for a walk in his neighbourhood.  
He started by going 1 block east.  
Then he turned left and went 2 more blocks.  
Then he turned left again and went 3 more blocks.  
He kept turning left and going 1 more block than the previous turn.  
His school is 5 blocks east of his home.  
How many blocks did he walk when he passed his school?

Give students time to read the problem, then ASK: What makes this problem hard? (there are a lot of words, it’s hard to picture what is happening) Tell students you are going to read the problem aloud again, but this time you want students to close their eyes and imagine the diagram as you read it. Remind students that when facing north, east is on the right. Draw on the board the picture in the margin.
Read the problem aloud, and then have a volunteer draw a map on the board of the first few turns of Kyle’s walk. (see example)

If “Home” is not already labelled, have a volunteer mark where it is. SAY: When you have a diagram drawn, you don’t have to keep everything in your head. That means you can focus on solving the problem. ASK: Where is the school? (5 blocks east of home) Have different volunteers estimate where it is on the map. (see example below)

Tell students that it is hard to estimate because it is hard to see exactly how far from home each vertical line is. SAY: One tool we can use to make this easier is grid paper. Draw a grid on the board or project BLM 1 cm Grid Paper, and redraw the diagram, as shown below. Point out how much easier it is to say for sure how far each point is from home in each direction.

Point to various corners and have volunteers tell how far north or south and east or west of home the point is. ASK: Does the diagram show walking all the way to the school yet, or do we still have to draw more? (there will likely be more to draw; if so, have a volunteer do so)

Then label all the number of blocks on the diagram as one greater, except for the last block, as shown below:
Pointing to the last turn, ASK: How many blocks north did he walk on his last turn? (4) How does the grid make it easy to see this? (I can just count the squares) SAY: Without the grid, you would have to calculate how many blocks south he went altogether, so you would have to look at all the times he went north and south and see how they cancel each other out. You can still do it, but it would take more work.

SAY: If you are ever taking a test, and you don’t have grid paper, you can draw the grid yourself. Show students a rough drawing of the grid on the board. SAY: Now that you know all the distances, you can find the total distance. Write on the board:

\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 4 = \]

Ask volunteers to find and explain a quick way to add this long list of numbers. If necessary, remind students that there is an easy way to add many numbers together: look for pairs that make 10 and add those first: 

\[(1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 5 + 4 = 49.\] Write “49” in the blank.

**Exercises:**

Draw a diagram to solve the problem.

a) Yu walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, then turns right again and walks 4 blocks. She then turns right again and walks 4 blocks, turns right again and walks 3 blocks, then turns right again and walks 2 blocks, and then turns right again and walks 1 block. Where does she end up, relative to home?

b) Yu follows the same pattern as in part a) but goes 10 blocks before starting to count down. Where does she end up, relative to her home?

c) Yu walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, then turns right again and walks 4 blocks. She then turns left and walks 4 blocks, turns left again and walks 3 blocks, then turns left again and walks 2 blocks, and then turns left again and walks 1 block. How far does she end up from home, and in which direction?

**Bonus:** Yu follows the same pattern as in part c) but changes the direction of turns from right to left after 4n blocks instead of after 4 blocks. How far does she end up from home and in what direction?

**Answers:**

a) at home, b) 1 block west and 1 block south, c) 4 blocks west, Bonus: 4n blocks west

**Focusing only on relevant information to make a problem simpler.**

Draw on the board the pictures in the margin. Point to the first diagram and ASK: What is this problem asking you to do? (find the length of the thicker stick) What are the other two problems asking you to do? (find the length of the thicker stick) What makes the first problem look easier to do than the other two? (the numbers are whole numbers; the third problem looks harder because the sticks are not right next to each other) SAY: There’s a lot of extra information in this third problem, so it looks harder, but it actually has exactly the same answer as the other one, so you might as
well complete the easier one. The total length of the two sticks at the bottom is still 17.6, they are just not side by side anymore.

**Exercises:** All measurements are in centimetres. Find what the question mark stands for by making the problem into a simpler problem.

a) ![Diagram](image1)

b) ![Diagram](image2)

c) ![Diagram](image3)

d) ![Diagram](image4)

**Answers:** a) 7 cm, b) 9.2 cm, c) 4.73 cm, d) 1.05 cm

SAY: If you need to find a vertical edge—straight up and down—then colour over all the vertical lines. If you need to find a horizontal edge, colour over all the horizontal lines.

**Exercises:** Find what the question mark stands for by making the problem into an easier problem.

a) ![Diagram](image5)

b) ![Diagram](image6)

c) ![Diagram](image7)

d) ![Diagram](image8)

**Answers:** a) coloured vertical, ? = 5 cm; b) coloured horizontal, ? = 8.65 cm; c) coloured horizontal, ? = 2.567 cm; d) coloured vertical, ? = 1.28 cm
Point out to students that by colouring over the horizontal or vertical lines, they changed the problem into an easier problem.

**Finding perimeter without knowing all the side lengths.** Remind students that to find the perimeter of a shape, they add up the lengths of all the sides. Draw on the board:

```
  5
  |
  |
  3
```

SAY: I want to find the perimeter of this shape. It looks like a hard problem because there are a lot of missing side lengths. Ask a volunteer to mark three sides that you do not know the length of. (the two bottom horizontal sides and the right side) SAY: There are two kinds of sides in this shape: horizontal sides and vertical sides.

ASK: How long is the top side? (20) How long are the two bottom sides put together? (20) How do you know? (put together, they are the same length as the top side) How long are the two sides on the left of the shape? (5 and 3) How long is the side on the right? (8) How do you know? (It's the same as the two left sides put together) Write on the board:

<table>
<thead>
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<th>Horizontal edges add to</th>
<th>Vertical edges add to</th>
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<tbody>
<tr>
<td>Perimeter is ___ + ___ = ___</td>
<td></td>
</tr>
</tbody>
</table>

Have volunteers fill in the blanks. (40, 16, 40 + 16 = 56)

**Exercises:** Find the perimeter of the shape.

a)  
```
  13
  3
  8
```

b)  
```
  4.6
  5.7
  4
  5.8
  8
```

c)  
```
  3
  8
  1
```

**Answers:** a) 48, b) 56.2, c) 34
Problem Bank

1. When everyone in Tom’s class stands in line, Tom is 14th in line and 11th from the end of the line. How many people are in the class?
   
   Answer: 24

2. There are 126 people in line. How many people are behind the 94th person?
   
   Answer: 32

3. Make several simpler problems until you see how to complete the harder problem.
   
   a) A fence is made using 53 posts, each 3 m apart. How long is the fence?
   
   b) A fence is made using 61 posts, each 2.5 m apart. How long is the fence?
   
   Answers: a) 156 m, b) 150 m

4. How many posts are needed to make the fence?
   
   a) A fence is 47 m long with posts at 1 m intervals.
   
   b) A fence is 100 m long with posts at 2.5 m intervals.
   
   c) A fence is 84 m long with posts at 3.5 m intervals.
   
   Answers: a) 48, b) 41, c) 25

5. A fence for a square garden is made with posts 1.5 m apart, including a post at each corner. How many posts are needed for the garden? Hint: Start with a garden that is 1.5 m by 1.5 m and then move on to 3 m by 3 m, 4.5 m by 4.5 m, and so on.
   
   a) The garden is 12 m by 12 m.
   
   b) The garden is 21 m by 21 m.
   
   Answers: a) 32, b) 56

6. Predict each answer before checking. A field is a square 30 m by 30 m. How many posts are needed if the posts are …
   
   a) 1 m apart?  
   
   b) 2 m apart?  
   
   c) 1.5 m apart?  
   
   d) 2.5 m apart?  
   
   Bonus: 60 cm apart?
   
   Answers: a) 120, b) 60, c) 80, d) 48, Bonus: 200
7. Cut out the strips and circles from BLM Fraction Strips and Circles (you may cut the line down to the centre of the circles). Estimate to colour the given amount. Use folding to check your estimate.

   a) one fifth of a strip of paper, starting from the left
   b) two fifths of a strip of paper, starting from the left

   Hint: Use your answers to parts a) and b) to help you determine a strategy for parts c) and d). Hold the circle so that the cut line is at the top.

   c) one fifth of a circle, starting from the top
   d) two fifths of a circle, starting from the top

8. a) Find the perimeter.

     \[ \text{Perimeter} = 3.6 + 8.2 + 1.1 + 5.4 = 18.3 \]

   b) Is there enough information to find the area of this shape? Explain how you know.

   Answers: a) 36.6; b) no, we don't have the side length for the small rectangles

9. What is the length of the thick-line path from A to B?

   Solution: \[ 7 + 7 + 15 = 29, \text{ so } 29 \text{ m} \]

10. Each shape was made by placing a small square on top of a large square. All measurements are in centimetres.

    a) Find the perimeter of each shape.

    i) \[ \text{Perimeter} = 1 + 1 + 11 = 13 \]
    ii) \[ \text{Perimeter} = 2 + 11 + 11 = 24 \]
    iii) \[ \text{Perimeter} = 3 + 11 + 11 = 25 \]
    iv) \[ \text{Perimeter} = 4 + 11 + 11 = 26 \]
b) Make a table with headings “Size of Smaller Square” and “Total Perimeter.” Use the pattern from part a) to solve the problem.

i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?

ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?

Answers: a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm
1 cm Grid Paper
Fraction Strips and Circles
Unit 11  Geometry: Transformations

Introduction
This unit will focus on:

• performing, describing, and identifying translations, reflections, and rotations;
• using combinations of transformations to create designs and patterns;
• identifying and plotting points in the first quadrant of a Cartesian coordinate plane; and
• performing and describing transformations of shapes in a Cartesian coordinate plane.

Meeting Your Curriculum

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<th>ALBERTA</th>
<th>Required</th>
<th>G6-13 to 20</th>
<th>including Extension 3 in G6-14 and Extension 2 in G6-15</th>
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<td>Required</td>
<td>G6-13 to 20</td>
<td>including Extensions 1 and 2 in G6-17</td>
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Mental Math Minutes

The mental math minutes in this unit:

• review properties of division with remainders
• use multiplication and division patterns to solve equations by guessing and checking

Generic BLMs

The Generic BLM used in this unit is:

BLM 1 cm Grid Paper (p. T-1)
This BLM can be found in Section T.

Assessment

The lessons covered by a quiz or test are as follows:

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<th>BC</th>
<th>MB</th>
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<td>G6-13 to 20</td>
<td>G6-13 to 20</td>
<td>G6-13 to 20</td>
</tr>
</tbody>
</table>
Additional Information for This Unit

**Technology: dynamic geometry software**
The Alberta curriculum requires performing transformations using technology. Some of the extensions in this unit use a program called The Geometer's Sketchpad®. If you are not familiar with The Geometer's Sketchpad®, the built-in Help Centre provides explicit instructions for many constructions. Use phrases such as "How to reflect polygons" or "How to construct a line segment of given length" when searching the Index.
**Goals**

Students will perform translations on a grid.

**PRIOR KNOWLEDGE REQUIRED**

Can perform and identify translations
Can measure sides and angles of polygons
Can identify congruent shapes

**MATERIALS**

2 identical L-shaped pieces of paper
round counter (e.g., integer tile, round game counter)
rectangular block or a matching paper rectangle
rulers and protractors
paper square (see Extension 1)

**Mental math minute—number string.**

String 1: Divide. Write your answer with remainder. $24 \div 4, 25 \div 4,
26 \div 4, 27 \div 4, 28 \div 4, 29 \div 4, 30 \div 4 (6 \ R \ 0, 6 \ R \ 1, 6 \ R \ 2, 6 \ R \ 3,
7 \ R \ 0, 7 \ R \ 1, 7 \ R \ 2)$

Present the pattern using an array with four dots in a row, adding one
dot to the last row, until the row is full. The row that is not full represents
the remainder.

String 2: $369 \div 3, 370 \div 3, 394 \div 3 (123, 123 \ R \ 1, 131 \ R \ 1)$
String 3: $400 \div 4, 404 \div 4, 406 \div 4 (100 \ R \ 0, 101 \ R \ 0, 101 \ R \ 2)$

**Introduce transformations.** Show students two copies of an L-shape made
of paper that are oriented in different directions, beside each other, as
shown in the margin.

Explain that the shapes are identical. Ask students if they remember the
correct mathematical term for identical shapes. (congruent shapes)

Tell students that you want to move the shapes so that they line up
exactly, with one on top of the other, facing the same direction so that one
congruent shape completely covers the other. Show moving the shapes,
as shown in the margin. Return the shapes to their original position.

Tell students to pretend that the shapes are actually very heavy, very hot
sheets of metal, so you need to program a robot to move them. To write the
computer program, you have to divide the process of lining up the shapes
into very simple steps.
It is always possible to move a figure into any position in space by using some combination of the following three movements:

- Sliding the shape along a straight line without allowing it to turn. This is called translation.
- Flipping the shape over. This is called reflection.
- Turning the shape around some fixed point. This is called rotation.

A rotation can be a turn around a fixed point that is inside the shape, on the edge of the shape, on its corner, or outside the shape.

Have students tell you, the robot, what steps to perform to position the hot L-shaped sheets of metal one on top of the other. Explain that there are different ways to bring one shape on top of the other, so there are no right or wrong answers. However, some instructions are more efficient than others; in other words, some ways will require fewer steps. When students direct you to rotate or to reflect the shapes, point out that there are very many different ways to reflect or to rotate the shape, so you will need more detail. You need to know the direction of rotation and how much you need to rotate it; for example, is it a quarter turn, or half a turn, or maybe some other turn? Demonstrate reflections using the paper shapes as shown below as you SAY: For a reflection, you also need to know if you flip the shape horizontally or vertically.

SAY: In this unit you will learn to describe these movements precisely.

**Introduce terminology.** SAY: These three changes to a figure—translation, reflection, and rotation—are all examples of transformations. When a point or a shape is changed by a transformation, the resulting point or shape is called the image of the original point or shape. We often add a star (*) or a prime symbol (') to the name of the original point to label the image. For example, the image of point A can be labeled as A'. We can also use an arrow to show the change from the original to the image. Write on the board:

\[ A \rightarrow A' \text{ or } B \rightarrow B' \]

SAY: We read these as “A is transformed into A-prime” and “B is transformed into B-star.” We also say that A' is the image of A under transformation and that B* is the image of B under a transformation.

**Translating points on a grid.** Explain to students that in this lesson they will only perform translations. Use a grid on the board and a round counter to demonstrate sliding a point on a grid; place the counter on a grid intersection and physically slide the point, represented by the counter, on
the grid. The diagram below shows a translation of 3 units right. You might want to mark the starting point on the grid.

Have students first signal the direction in which the point is translated (right or left, up or down) and then ask them to hold up the number of fingers equal to the number of units the dot is translated. You may wish to draw a large letter L on the left side of the board and a letter R on the right side of the board to help students who have trouble distinguishing between left and right. Demonstrate several new translations with the counter on the grid and ask students to signal the direction and number for each translation.

Invite volunteers to translate a point and have other volunteers describe the translations. Then reverse the task: have volunteers describe the translation and have other volunteers perform them with a counter. Remind students that the result of a transformation is called an image under that transformation. SAY: For example, the result of a translation of a point or shape is called the \textit{image under translation}.

\textbf{Exercises:} Draw a point on a grid and label it A. Draw a point A' that is the image of A under the given translation.

a) 2 units up 

b) 3 units down 

c) 5 units right 

d) 1 unit left 

\textbf{Sample answers}

\begin{itemize}
  \item a) \[\begin{array}{c}
    \text{A} \\
    \text{A'}
  \end{array}\]
  \item b) \[\begin{array}{c}
    \text{A} \\
    \text{A'}
  \end{array}\]
  \item c) \[\begin{array}{c}
    \text{A} \\
    \text{A'}
  \end{array}\]
  \item d) \[\begin{array}{c}
    \text{A} \\
    \text{A'}
  \end{array}\]
\end{itemize}

SAY: You can also combine translations. For example, you can move 3 units right and 2 units down. Demonstrate with a counter and draw arrows to show the translations, as shown below:

\textbf{Exercises:} Draw a point on a grid and label it A. Draw a point A' that is the image of A under the given translation.

a) 2 units left, 1 unit down 

b) 4 units right, 3 units up 

c) 6 units right, 2 units down 

d) 3 units left, 4 units up 

e) 3 units right, 1 unit up 

f) 5 units left, 3 units down
Sample answers

a)  

b)  

c)  

d)  

e)  

f)  

NOTE: Students who are struggling can draw the arrows showing each part of the slide.

**Describing translations.** SAY: To describe a translation, you need to say how much the point moved and in which direction. Draw the picture in the margin on the board. SAY: You can imagine the arrow from A to A’ as a combination of two arrows, horizontal and vertical. Trace the dashed arrows with a finger. ASK: How much did point A move in the horizontal direction? (4 units) Did it move right or left? (right) Repeat with the vertical arrow. (2 units up) SAY: So the point A moved 4 units right and 2 units up.

**ACTIVITY (Essential)**

Students work in pairs. Partner 1 draws a pair of points on a grid and an arrow from one point to the other. Partner 2 describes the translation. Partner 1 verifies the answer. Partners switch roles.

**How much did the shape slide?** Draw on the board the picture in the margin. ASK: How far did the rectangle slide to the right from Position 1 to Position 2? Accept all answers and record them on the board. Call for a vote if you wish. Students might say the rectangle moved anywhere between 2 and 6 units right. Take a rectangular block or a matching paper rectangle and perform the actual slide, one square at a time, counting the units as a class. The correct answer is 4 units.

**Corresponding points.** Draw a point at the top right vertex of the rectangle in Position 1. ASK: Can this make it easier to see that the translation was 4 points to the right? (yes) Why? (we know how to translate points) SAY: The vertex I marked and its image are corresponding points under a translation. When we talk about transformations, we want to know where each point went to. Invite a volunteer to mark the image of the marked point on the second rectangle. Keep the picture on the board for later use.

Draw the pictures in the exercises below, one pair of figures at a time, and have students signal the answer by raising the correct number of fingers.
Exercises: Which vertex, 1, 2, 3, or 4, is the image of the vertex marked with a dot under the translation?

a) 

b) 

c) 

Answers: a) 3, b) 2, c) 4

Under translations, all points on a shape move the same amount in the same direction. Label the vertices of the rectangle in Position 1 from earlier as A, B, C, and D. Add the same labels to the vertices of the paper rectangle. Translate the paper rectangle again, from the initial position to the position 4 units to the right and 2 units down. Invite volunteers to label the vertices of the image as A'B'C'D' to show the correspondence. Draw arrows from each vertex to its image. SAY: These arrows are called translation arrows. ASK: What do you notice about the translation arrows? (they are all parallel and they are all the same length) Explain that this means that all points on a shape move the same amount in the same direction, so it is enough to draw only one translation arrow to describe a translation. SAY: However, you need to be careful to draw the arrow between a vertex and its image, not any other vertex. Also, the fact that all arrows are the same gives you a way to translate polygons: you can translate each vertex separately and then join the images of the vertices to form the image of the polygon.

Translations preserve length of line segments and size of angles. Give students rulers and protractors. Ask students each to draw a scalene triangle on a grid and measure its sides and angles. Then ask them each to write a translation of their choice. Have students each translate the triangle they drew by using the translation they described and then measure the sides and the angles of the image.

Discuss findings from the translation students just performed. Students should notice that the side lengths of the triangle under translation stayed the same and so did the angle measures. Point out that the result is the same for everyone even though students all drew different triangles and performed different translations.

Translations take polygons to congruent polygons. Ask students to remind you what they know about the sides and angles of congruent polygons. (Congruent polygons have corresponding equal sides and corresponding equal angles; the equal sides and angles come in the same
order in both polygons.) ASK: Do translations change the order of vertices? (no) Do they preserve lengths of sides? (yes) Do translations preserve angle sizes? (yes) Do translations take polygons to congruent polygons? (yes) Write on the board:

If polygon A is an image of polygon B under a translation, then polygons A and B are congruent.

ASK: Do you think it works the other way around? Write on the board:

If polygons A and B are congruent, then polygon A is the image of polygon B under a translation.

Explain that if you want to prove a statement is false, you can find just one example that shows that the statement is false. ASK: What would such an example look like for this second statement? (a pair of polygons that are congruent but are not a translation of each other) Ask students to try to draw a pair of polygons like that. (see example in the margin)

Ask students to explain how they know that one polygon is not the translation of the other polygon. To prompt students to see the answer, ask them to say from which vertex they would draw a line to another vertex in the shape and show that the line segments joining the vertices are not parallel and not equal in length, so the line segments are not translation arrows. Students should conclude that the line segments that form each shape’s sides are equal and the angles are equal, so the two shapes are congruent, but the shape on the left has not been translated to become the shape on the right.

Combining translations. Have students do the following exercises in pairs.

Exercises
a) Draw a quadrilateral that is not symmetrical in any way and label it P.
b) Write a description of a translation of your choice.
c) Translate the polygon P using the description from part b). Label the image P'.
d) Translate the polygon P' using the description your partner wrote in part b). Label the image P**.
e) Describe the transformation that takes P to P**.
f) Compare your answer in part e) to the answer of your partner. What do you notice?

Selected answer: f) the answers are the same

Discuss the results of the exercises. All students should see that the resulting transformation is a translation. Discuss how the translations combine:

• If both translations have components that move in the same direction, these add. For example, if one translation is 3 units right and another is 2 units right, the total translation is 5 units to the right.
If one translation has an “up” component and another has a “down” component, they partially cancel out each other. For example, 2 units up followed by 3 units down results in 1 unit down. Have multiple pairs present their answers. Students should also see that the order in which translations are made does not change the overall translation.

**Exercises:** Emma translated polygon Q 3 units right and 2 units down and then translated the image 4 units left and 5 units down. She labelled the final image Q*. Which translation takes Q to Q*?

**Bonus:** Which translation takes Q* to Q?

**Answers:** 1 unit left and 7 units down, Bonus: 1 unit right and 7 units up

**Extensions**

1. Draw the picture in the margin on the board, first without the arrows. Use a paper square to demonstrate as you SAY: I flipped the square over a horizontal line (demonstrate) and translated it a little to the right. Here is how the vertices changed. Draw the arrows and name each arrow as you draw it: AE, BF, CG, DH.

   SAY: The arrows are not all the same length and not all parallel. I think there is no translation that would take the first square, ABCD, to the second square, EFGH. ASK: Is this correct? (no) Why not? (if you draw the arrows in a different way, you can show a translation with parallel arrows of equal length) How should we draw the arrows to show a translation? (AH, BG, CF, DE) Have a volunteer draw the new arrows. Point out that if there is more than one way to draw the correspondence between the vertices, you would need to check all the possible ways to label the vertices of the image. However, if the shapes are not symmetrical, you do not need to worry about that. Have students look at the shapes that they drew before the recent exercises to show shapes that are not translations of each other. Students can change their examples to make the shapes not symmetrical.

2. **Using properties of parallelograms to explain why translations preserve the length of line segments.** Explain that properties of parallelograms give us a way to explain why translations preserve the length of line segments. Draw a line segment and label it AB. SAY: If we take a line segment AB and translate it, we translate the point A and the point B in the same way. Draw two identical translation arrows, AA' and BB'. ASK: What do you know about the translation arrows? (they are parallel and equal in length) Label the arrows and join the points A' and B', as shown in the margin.

   ASK: What type of quadrilateral is A'B'BA? (a parallelogram) How do you know? (the opposite sides AA' and BB' are parallel and equal) What does this say about AB and A'B'? (they are parallel and equal) SAY: So, we can see if a line segment has been translated by checking for the same properties as in parallelograms: if line segments AA' and
$BB'$ are equal and parallel, they show translation. Have students decide if the line segments $AB$ and $CD$ are translations of each other.

**Answers:** a) yes, b) yes, c) no

3. a) Draw a rectangle $ABCD$ on grid paper, such that $AB = 4$ units and is the horizontal top side and $BC = 5$ units and is the vertical right side.

b) For the points $A$ and $B$, draw a broken line (a collection of line segments) that starts at $A$ and ends at $B$ but does not follow the straight line segment $AB$ and does not intersect it. The broken line will look like a “scenic route” or wandering path. Translate the broken line 5 units down so that it starts at $D$ and ends at $C$. Erase the old lines $AB$ and $CD$.

c) Draw another broken line that starts at $A$ and ends at $D$, so that it does not intersect any of the lines you drew in part b) or the sides $AB$, $BC$, or $CD$. It can go along parts of $AD$ or intersect it. Translate this broken line 4 units to the right. It should start at $B$ and end at $C$. Erase the old lines $BC$ and $AD$.

d) You have created a polygon with vertices $ABCD$. Draw a copy of it away from $ABCD$.

e) Translate the polygon you drew in part d) 4 units to the left. Translate the image 4 units to the left again. Repeat several times.

f) Translate the polygon you drew in part d) 5 units down. Translate the image 4 units to the left. Translate the image 4 units to the left again. Repeat several times.

g) A pattern made of congruent shapes that cover the grid without gaps or overlaps is called a tessellation. Does the shape you created produce a tessellation?

**Selected sample answers**

a) 

b) 

c)
f)

g) yes
Goals

Students will reflect points and shapes and describe reflections. Students will verify that reflections take polygons to congruent polygons.

PRIOR KNOWLEDGE REQUIRED

- Can identify and draw perpendicular lines
- Can measure sides and angles of polygons
- Can identify congruent shapes
- Knows the definition of congruent polygons in terms of sides, angles, and order of elements

MATERIALS

- 2 identical L-shaped pieces of paper, such as those used in Lesson G6-13
- rulers and protractors
- coloured chalk or markers
- The Geometer’s Sketchpad® (see Extension 3)

Mental math minute. Present the following set of problems.

\[
\begin{align*}
30 ÷ 4 & & 34 ÷ 4 & & 38 ÷ 4 & & 42 ÷ 4 & & 46 ÷ 4 \\
\end{align*}
\]

Have students write division with remainder for the first problem. (7 R 2)
ASK: What pattern do you see in this set of problems? (the dividends increase by 4 from problem to problem) Draw arrays with four dots in a row to represent the first and the second problems. ASK: How do arrays help us to see the answer to each next problem? (the dividends increase by 4, so each next array just has 1 more full row, and the same row that is not full, so the quotients will grow by 1 and the remainders will stay the same) Record the answers for the rest of the problems. (8 R 2, 9 R 2, 10 R 2, 11 R 2) Have students signal the answer in the exercises below by showing a 0 or 2 with their fingers. NOTE: Students will solve harder division problems with a variety of remainders and divisors in the next lesson.

Exercises: Is the remainder 2 or 0?

a) \(40 ÷ 4\)    b) \(440 ÷ 4\)    c) \(450 ÷ 4\)    d) \(452 ÷ 4\)

e) \(800 ÷ 4\)    f) \(888 ÷ 4\)    g) \(890 ÷ 4\)    h) \(8082 ÷ 4\)

Answers: a) 0, b) 0, c) 2, d) 0, e) 0, f) 0, g) 2, h) 2

Introduce reflections. Show students two L-shaped pieces of paper. Affix them to the board, as shown in the margin. SAY: I would like to place the shape on the left on top of the shape on the right. ASK: What should I do to the shape on the left? (flip it) Point out that this requires taking the shape
off the board. SAY: Another way to make the shape on the left look like the
shape on the right is to look at it in a mirror. For that reason, we can say
that these two shapes are mirror images of each other. Mathematically,
we say that the shape on the right is a reflection of the shape on the left.
A reflection is another type of transformation.

Remind students that when you “flip” or reflect a shape, there are different
ways to do it. Demonstrate several different ways to reflect the L shape on
the left, as shown below:

SAY: Suppose there is a mirror between each pair of shapes, so that the
original shape is a real one and the other one is the shape in the mirror.
ASK: In each case, where would the mirror be? Have a volunteer hold up a
sheet of paper as if it were a mirror between the shapes, as shown below
(the dashed line represents the mirror):

SAY: If we look at what happens in a plane, the mirror becomes a line. This
line is called a mirror line. Students might recall that a line of symmetry
is also called a mirror line. Point out that the mirror line is indeed a line of
symmetry when you consider the original shape and the image as parts
of the same picture. Explain that in this unit students will mostly work with
horizontal and vertical mirror lines and will work on a grid.

Reflecting points in a line. Draw a vertical line $m$ on the board and a
point $A$ away from the line. SAY: The vertical line is a mirror line, and I am
going to reflect point $A$ in this line. Demonstrate the steps below and write
each step on the board:

Step 1: Draw a line perpendicular to $m$ through $A$. Extend the
line beyond $m$.

Step 2: Measure the distance from $A$ to $m$ along the perpendicular line.

Step 3: Mark point $A'$ on the other side of $m$ so that $A$ and $A'$ are the
same distance from the mirror line $m$.

SAY: The point $A'$ is the mirror image of point $A$. Mathematicians say that
point $A'$ is the image of $A$ under reflection.
Exercises

a) Draw a horizontal line $m$ and mark a point $A$ away from the line.

b) Reflect point $A$ in the mirror line $m$.

c) Draw a vertical line $n$ and mark a point $B$ away from the line.

d) Reflect point $B$ in the mirror line $n$.

**Reflections preserve length of line segments and size of angles.**

Provide students with rulers and protractors. Ask students each to draw a scalene triangle on a grid, label the vertices, and measure its sides and angles. Then ask them to draw a horizontal or a vertical line of their choice. Encourage some students to draw a horizontal line, some to draw a vertical line, and some to draw a line away from the triangle. Ask at least one student to draw a line that passes through one of the vertices of the triangle and ask another student to draw a line that intersects two of the sides of the triangle. Explain that the line that each student drew will serve as the mirror line for that student’s triangle.

Explain that we can reflect the vertices of a triangle and then join the images to reflect the triangle. Have students each reflect the triangle they drew in the mirror line and then measure the sides and the angles of the image. Have them use the prime symbol to label the images of the vertices.

Discuss the findings. Remind students that they showed that translations “preserved” the lengths of the sides and the size of angles. **ASK:** Does the same happen with reflections? Students should notice that, from their triangle to its image under reflection, the side lengths stayed the same and so did the angle measures. Point out that the result is the same for everyone, although they all had different triangles and different mirror lines, so reflections also preserve lengths of sides and angle sizes.

Draw students’ attention to the order of vertices in the original triangle and its image. For example, if the original triangle was $ABC$ and you needed to go clockwise to get from $A$ to $B$ to $C$, as shown in the margin, the order in the image triangle is counter-clockwise. Have all students verify that on their triangles.

**Reflection and congruence.** Ask students what they know about the sides and angles of congruent polygons. (congruent shapes have the same size and shape, so congruent polygons have corresponding sides of equal length, corresponding angles that are equal, and the same order of equal sides and equal angles) Point out that although reflections “flip” the shape, which reverses the order of vertices from clockwise to counter-clockwise, they do not mix up the order of vertices and sides: if two vertices are adjacent to a side of the same length, their images are adjacent to the side of the same length. **ASK:** Do reflections preserve angle sizes? (yes) Do they preserve lengths of sides? (yes) Do reflections take polygons to congruent polygons? (yes) Write on the board:

If polygon $A$ is a mirror image of polygon $B$,
then the polygons $A$ and $B$ are congruent.
ASK: Do you think it works the other way around? Write on the board:

If polygons A and B are congruent, then polygon A is the mirror image of polygon B.

Point out that another way to say the second sentence is: “If two polygons are congruent, then they are mirror images of each other.” Give students a few minutes to think about if the statement is correct and then have them find an example showing that this is not true. (see sample example in margin)

**Distinguishing between reflections and translations visually.** Have students draw a horizontal and a vertical line to act as mirror lines, dividing a sheet of grid paper into four approximately equal parts. Have them pick one part and draw in it a right trapezoid. Have them label the trapezoid $DEFG$ so that they read the name clockwise around the trapezoid and shade the shape to identify it clearly as original.

Have students reflect the trapezoid they drew in the horizontal line, labelling the image using ‘$. Then have them translate the original shape so that it ends in the free region of the page, diagonally from the original shape. Students should use * for labelling the translated polygon.

Have students compare the results of reflection and the translation.

ASK: How are the images different? Students are likely to say that the image under reflection “points” in a different direction from the original shape, when the image under translation points in the same direction. Draw students’ attention to the order of the letters around the polygon: if you want to start with D and read the letters alphabetically, you need to continue clockwise when reading the letters from the original shape and from the image under translation, but counter-clockwise if you are reading the letters from the mirror images. SAY: When the order of vertices changes to the opposite, say, from clockwise to counter-clockwise, we say that the orientation of the shape changes.

Have students reflect the original polygon in the vertical line and use the ‚ symbol to label the image. Repeat the discussion. Students will notice that the orientation (in the sense of the order of letters) of both reflected polygons is the same, but they still “point” in different directions, and both polygons “point” in a different direction from the original and from the translation.
Using line segments joining corresponding vertices to distinguish between reflections and translations. Remind students that when they perform a translation, they translate the vertices the same way, so translation arrows are the same length and parallel to each other. Have students draw the translation arrows and verify that.

Ask students to draw the line segments joining the vertices of $DEFG$ and the corresponding vertices of one of its mirror images. Have some students use $D'E'F'G'$ and others use $D''E''F''G''$ for this purpose. ASK: Are the line segments parallel? (yes) Are they the same length? (no) Ask multiple students. Point out that students have different trapezoids and reflect in different lines but get the same result.

Midpoints of line segments between original and image are on the mirror line. SAY: The point that is exactly halfway between the end points of a line segment is called the midpoint of the line segment. ASK: If a line segment is 6 units long, how far from each end point is the midpoint? (3 units) Draw a line segment on the board and invite a volunteer to find the midpoint. Then ask students to find the midpoints of the line segments joining the vertices of $DEFG$ and one of its mirror images. ASK: What do you notice about the midpoints? (they are all on the mirror line) What angle does the line segment make with the mirror line? (right angle) Why does this make sense? (To construct the mirror image, you draw a line segment that is perpendicular to the mirror line and mark the image so that the distance from the mirror line is the same to the image and to the original point. This means that the mirror line intersects the line segment at the midpoint.)

Draw the picture in the margin on the board. Use a different colour for each shape. SAY: These two polygons are reflections of each other. I would like to find the mirror line. How can I use the midpoints of the line segments joining the corresponding vertices to find the mirror line? (Draw line segments between corresponding vertices. Find their midpoints. Join the midpoints to find the mirror line.) Invite a volunteer to demonstrate.

**ACTIVITY (Essential)**

To allow students to practise finding a mirror line, have them draw a non-symmetrical polygon on a grid. Explain that you want them to reflect it in a line of their choice, but instead of drawing the line, they can place a ruler or a pencil to mark the position of the line so that their partners can find the mirror line afterwards. Then have partners exchange notebooks and find the mirror line.

Draw the picture in the margin on the board. ASK: Are these mirror images of each other? Answers may vary; point out that the shapes “point” in opposite directions, so they might be mirror images.
To check, invite volunteers to draw the line segments between the corresponding vertices and find their midpoints. Students will see that the midpoints all fall on the same line (see second image in the margin), but the line segments are not parallel, and there is no line that is perpendicular to all the line segments at the same time. Explain that this means that the shapes are not mirror images of each other. In this case, the shape was first reflected and then translated 2 units down. Invite a volunteer to draw the reflected shape as an intermediate step.

**Combining a reflection and a translation.** Draw the picture in the margin on the board. Ask students to copy the picture. Then divide students into three groups and have students in each group work in pairs. In each pair, one student will reflect the shape first, then translate the image, and label the final image $T'$. The other student will reverse the order—translate first and then reflect—and label the final image $T^*$. Each group has a different translation, as shown below. The following shows the original shape, the mirror line, and the results of the transformations.

**Group 1:** 2 units up  

$T$  

$T'$  

$T^*$

**Group 2:** 4 units right  

$T$  

$T'$  

$T^*$

**Group 3:** 3 units right and 1 unit down  

$T$  

$T'$  

$T^*$

Have students in each pair compare the results (Did you get the same answers? How are the shapes $T'$ and $T^*$ the same, and how do they differ from the original shape $T$?). Have students discuss the results in their groups and then have groups report on the findings to the class. Students should notice that only Group 2 got the same result for both combinations.
Students should notice that all shapes are congruent, and congruent to the original shape, and that all original shapes “point” the same way, with the lowest vertex on the right, and all final images are oriented the opposite way, with the highest vertex again on the right. Students might also notice that the orientation changed from the original polygon for all the images.

ASK: Could there be a single translation that takes \( T \) to \( T^* \) or to \( T' \)? (no) 
How do you know? (the images “point” in a different direction from \( T \), the orientation changed, and translations do not change orientation)
What about a single reflection? (maybe) How can we check? (join the corresponding vertices of the original to the vertices of the image and find the midpoints. If all midpoints are on the same line that is perpendicular to the line segments, that line is the new mirror line) Have students join the vertices, find the midpoints, and report their findings. Only Group 1 will be able to identify a reflection between \( T \) and \( T' \) and between \( T \) and \( T^* \). See the pictures below.

Emphasize that sometimes there is no single transformation that takes one shape onto the other, and two transformations are needed. Moreover, there are multiple ways to take one shape onto the other.

NOTE: Extension 3 is required to cover the Alberta curriculum.

**Extensions**

1. Triangle \( ABC \) is reflected in a vertical line \( m \) to get triangle \( A'B'C' \). Which transformation will take triangle \( A'B'C' \) to triangle \( ABC \)?

   **Answer:** reflection in the same mirror line \( m \)

2. a) Draw a quadrilateral \( Q \) without a line of symmetry on grid paper. Draw a horizontal line \( m \) away from the quadrilateral.
   b) Reflect the quadrilateral in the line and then translate the image 3 units right and 2 units up. Label the final image \( Q^* \).
   c) Describe a way to get from \( Q^* \) to \( Q \). Use translation and then reflection.
   d) Describe a different way to get from \( Q^* \) to \( Q \). Use reflection and then translation.
e) Did you use the same translation in parts c) and d)?

Sample answers

a–b)

\[ Q \quad Q^* \]

\[ m \]

c) Translate \( Q^* \) 3 units left and 2 units down. Reflect the image in the line \( m \).
d) Reflect \( Q^* \) in the line \( m \). Translate the image 3 units left and 2 units up.

Answer: e) no

3. **Investigating reflections using The Geometer’s Sketchpad®.**

Teach students to reflect polygons in The Geometer’s Sketchpad®. Demonstrate how to draw a polygon using the Polygon tool. Tell students that when they want to reflect a shape, they need to create a mirror line and select it using the “Mark Mirror” option in the Transform menu. Then they can select the shape and “Reflect” in the Transform menu. Demonstrate the process. Then have students follow the instructions below.

a) Draw a quadrilateral and label it \( ABCD \).
b) Draw a line away from \( ABCD \). Label it \( m \).
c) Select line \( m \) and use the “Mark Mirror” option in the Transform menu to label it as the mirror line.
d) Select the quadrilateral \( ABCD \). Use the Transform menu to reflect it in mirror line \( m \). Label the image quadrilateral \( A'B'C'D' \). Does it look congruent to \( ABCD \)? How is it different from \( ABCD \)?
e) Move the vertices of \( ABCD \) to change the shape. Are the quadrilaterals still congruent? Is the image quadrilateral still different from the original in the same way?
f) Move line \( m \) without turning it. How does the quadrilateral \( A'B'C'D' \) change? Does your answer to part e) change?
g) Select one of the points used to create line \( m \) and move it to turn line \( m \). How does the quadrilateral \( A'B'C'D' \) change? Does your answer to part e) change?

**Selected sample answers:**
d) the quadrilaterals are congruent, but \( A'B'C'D' \) is facing the opposite way from \( ABCD \); e) yes, yes;
f) the quadrilateral \( A'B'C'D' \) is reflected farther away or closer to the quadrilateral \( ABCD \), but the quadrilaterals are still congruent, and the image is still facing the opposite way from \( ABCD \); g) the quadrilateral \( A'B'C'D' \) turns in its position and moves away or closer to the quadrilateral \( ABCD \), but the quadrilaterals are still congruent.
Goals

Students will rotate points and shapes 90° around a given centre.

Students will verify that rotations take polygons to congruent polygons.

PRIOR KNOWLEDGE REQUIRED

- Can identify and draw perpendicular lines
- Knows the terms clockwise and counter-clockwise
- Knows that the size of an angle is a measure of rotation
- Can identify congruent shapes
- Knows the definition of congruent shapes in terms of sides, angles, and order of elements

MATERIALS

- set squares
- rulers
- protractors
- BLM Rotating a Triangle (p. N-55)
- scissors
- BLM Rotations Without a Grid (p. N-56) (see Extension 1)
- The Geometer’s Sketchpad® (see Extension 2)
- BLM Find a Flip (pp. N-57–58, see Extension 3)

Mental math minute. Write “450 ÷ 4” on the board. SAY: I can see an easy number that divides by 4 and is smaller than 450. I am separating 450 into 440 and 10. There are other ways to separate 450, such as 400 + 40 + 10, but I am using 440 and 10. Start recording the solution on the board as shown below. Remind students that when dividing a sum they divide each addend separately. Point out that this is like dividing parts of a very large array. The first array has 110 rows of 4 dots, and the second has 2 rows of 4 dots and 2 dots leftover. ASK: What is the total number of full rows? (110 + 2 = 112) What is the total leftover in the division? (2, just the leftover in the second part)

\[
\begin{align*}
450 & \div 4 \\
& = (440 + 10) \div 4 \\
& = (440 \div 4) + (10 \div 4) \\
& = 110 + 2 \text{ R } 2 \\
& = 112 \text{ R } 2
\end{align*}
\]

Exercises: Use the method on the board to divide.

a) 58 ÷ 4  b) 430 ÷ 4  c) 889 ÷ 4  d) 1237 ÷ 4

Answers: a) 14 R 2, b) 107 R 2, c) 222 R 1, d) 309 R 1
Review clockwise and counter-clockwise and describe turns. Review the meanings of “clockwise” and “counter-clockwise.” Draw several arcs pointing clockwise and counter-clockwise on the board and have students signal thumbs up if the arc points clockwise and thumbs down if it points counter-clockwise.

Draw the picture in the margin on the board. ASK: If this arrow turns 90° clockwise, where will it point? Have students show the direction with their arms and then have a volunteer draw the arrow. (pointing left) Repeat with other starting arrows and other directions; include turns of 180°. Explain that it takes too much time and space to write “clockwise” or “counter-clockwise,” so you will be using short forms: “CW” for clockwise and “CCW” for counter-clockwise.

Exercises: From the dark arrow, draw an arc showing the given turn. Draw the arrow after turning.

\[
\begin{align*}
\text{a) 90° CCW} & \quad \text{b) 180° CW} & \quad \text{c) 90° CW} & \quad \text{d) 90° CCW}
\end{align*}
\]

Rotating points. Draw the picture in the margin on the board. SAY: I want to rotate point \( P \) around point \( O \) 90° clockwise. Demonstrate and start making a list of the steps on the board:

**Step 1:** Draw the line segment \( OP \). Measure its length.

**Step 2:** Draw an arc clockwise to show the direction of rotation.

**Step 3:** Place a set square so that:
- the arc points at the diagonal side,
- the right angle is at point \( O \), and
- one arm of the right angle aligns with \( OP \).

At this point, explain that the line segment \( OP \) is like a hand on the clock, attached at point \( O \). If we turn in the direction of the arc, is it passing through the set square? Trace the turn in the direction of the arc with a finger to check. Turn the set square upside down if needed. Continue demonstrating with the following steps:

**Step 4:** Draw a ray from point \( O \) along the side of the square corner. Remove the set square.
Step 5: On the new ray, measure and mark the image point \( P' \), so that \( OP' = OP \).

For the exercises below, have students always start with points on grid intersections.

**Exercises**

a) Draw two points on the same horizontal line and label them \( S \) and \( T \). Rotate point \( T \) around point \( S \) 90° clockwise.

b) Draw point \( C \) and draw point \( D \) underneath it, on the same vertical line. Rotate \( D \) around \( C \) 90° counter-clockwise.

c) Draw two points \( U \) and \( V \) that are not on the same horizontal or vertical line. Rotate point \( V \) around \( U \) 90° clockwise.

d) Rotate point \( U \) around \( V \) 90° counter-clockwise.

SAY: The point around which you rotate other points is called the **centre of rotation**. You used points that were grid line intersections as centres of rotation. The points you rotated also were grid line intersections. ASK: Were the image points also grid line intersections? (yes)

**The centre of rotation is a fixed point.** SAY: When you perform a transformation, such as reflection, rotation, or translation, some points move, and some points do not. For example, when you make a rotation, the centre of rotation does not move. All other points do. We call points that do not move **fixed points**. A rotation has only one fixed point, the centre. ASK: When you reflect points in a line, are there some points that do not move? (yes) Which points? (the points on the mirror line itself) When you translate shapes or points, are there points that do not move? (no, all points move) SAY: Translations have no fixed points: all points move under translation. Rotations have only one fixed point, the centre of rotation. In any reflection, points on the mirror line never move, so there are infinitely many fixed points in any reflection; there are so many points that you cannot even count them.

**Rotating polygons.** Draw the picture in the margin on the board. SAY: I want to rotate the triangle 90° clockwise around the centre of rotation \( O \). To do that, I need to rotate each vertex and then join the images to form the image of the triangle. Have students draw a similar picture and perform the rotation using a set square, with a volunteer doing the same on the board. Students can use slightly different triangles and different centres of rotation, but for practical purposes, have them use a point on one of the sides of the triangle.

Have students measure the sides and the angles of the original triangle and the image using rulers and protractors. Discuss the results. Students should notice that the side lengths and the angle sizes are preserved in rotation. ASK: Does rotation take polygons to congruent polygons? (yes) If two triangles are congruent, does this mean that one is a rotation of the other? (no) What other transformations can take triangles to congruent triangles? (translations, reflections)
Using the grid to perform rotations of $90^\circ$. Draw a right triangle and demonstrate how to rotate the triangle $90^\circ$ clockwise around the vertex $O$, as shown below. First draw the arc showing the direction of rotation, draw the image of the side adjacent to $O$ that aligns with the grid, draw the side perpendicular to the first image side, and then finish the triangle with the third side. (see images below)

Emphasize that you are using the lengths of the short sides of the right triangle and the ones that can be measured by counting squares: the vertical side is 1 unit long, and it rotates into a horizontal side 1 unit long; the horizontal side is 4 units long, and it rotates into a vertical side 4 units long.

Exercises

a) Draw four right triangles on a grid. Make sure the triangles are not congruent and they are oriented differently (point in different directions) on a page.

b) On each triangle, label one of the acute angles as $O$. The vertex $O$ will be the centre of rotation.

c) Rotate the first two triangles $90^\circ$ clockwise and the other two triangles $90^\circ$ counter-clockwise around $O$.

Have students exchange notebooks with partners to check their answers.

Rotating slanted line segments $90^\circ$ around an end point. Draw a slanted line segment between two points on a grid and label one of the end points $O$.

ASK: How could we use right triangles to rotate this line segment $90^\circ$ clockwise around the point $O$? Invite volunteers to draw the right triangles that might help with the rotation. The example in the margin shows two possible $1 \times 4$ triangles, one above and the other below the line segment.

Invite volunteers to perform the rotation. ASK: Does the answer depend on the triangle used? (no)

Exercises: Rotate the line segment $90^\circ$ clockwise around $O$.

a)  

b)  

c)  

d)  

Answers

Rotating slanted lines by imagining triangles. Tell students that now you want them to rotate line segments by only imagining the triangles, not drawing them. Emphasize the rule of changing the lengths of the horizontal and the vertical sides: if the original line segment goes 2 units up or down, the line segment rotated 90° will go 2 units right or left depending on the direction of rotation. Have partners draw slanted line segments for each other and label one of the vertices as the centre of rotation. Then have students rotate the line segments 90° clockwise around the marked centre of rotation.

Rotating polygons. Point out that to rotate a point, you can simply draw the line segment joining the point to the centre of rotation and then rotate it following the method students have just used. The grid gives students a shortcut to performing 90° rotations.

Students who are struggling imagining the triangles can draw only the sides of the triangles that follow the grid lines. For example, in question b) above, they can draw the picture in the margin to rotate point $A$.

Exercises

a) Draw a quadrilateral that is not symmetrical in any way. Choose a point $O$ away from the quadrilateral. Rotate the quadrilateral 90° counter-clockwise around $O$.

b) Draw a pentagon that is not symmetrical in any way and has a right angle. Rotate the pentagon 90° clockwise around the vertex with the right angle.

ACTIVITY (Essential)

Give students BLM Rotating a Triangle and have them cut out the bottom triangle. Have them place the cut-out triangle on top of the triangle at the top of the page to see if the triangles are congruent and if the black dots are in the same places on both triangles. Explain that students will use the black dots as the centres of rotation for the triangle.

When students have placed the cut-out triangle on top of the other triangle and lined up the black dots, have them press the tip of a pencil to the black dot labelled $O$ and rotate the top triangle around it clockwise so that the horizontal side becomes the vertical side, coinciding with one of the vertical lines on the grid. Have them hold the cut-out triangle in place, trace it onto the BLM, and label the
corresponding vertices using *. Remove the cut-out triangle and compare the results. ASK: How is the rotated image different from the original? (the image turned; it is pointing in a different direction)

Repeat with the second black dot, labelled Q. Label the image triangle G'H'I'. ASK: Which transformation takes triangle G'H*I* to triangle G'H'I'? (translation 7 units left, 5 units down) If I were to rotate the original triangle 90° clockwise around O and then translate it 7 units left, 5 units down, which triangle would I get? (G'H'I') Have students verify.

Explain that when you want to take a polygon onto a congruent polygon, you can rotate the polygon on a grid to the position of the congruent polygon and then find a translation to bring it to the location you need it to be.

Discuss what students can notice from the activity. Draw students’ attention to the order of the letters in the original and the images. In the original triangle, if you want to read the letters in alphabetical order, you go clockwise around the triangle. ASK: In which direction do you go in the image? (also clockwise) Is it true for both images? (yes) Which transformation changes the order? (reflection reverses the order)

Discuss how the position of the centre of rotation influences the image under rotation. When students rotated shapes on a grid, they usually rotated shapes around a point outside the shape, a vertex, or a point on a side of the shape. The images were usually drawn away from the original shape or beside it. ASK: Does this happen when the centre of rotation is inside the shape? (no, the image overlaps the original shape)

**Exercises:** Fill in the table to summarize. What happens to a polygon that is reflected? Translated? Rotated?

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Lengths of Sides</th>
<th>Sizes of Angles</th>
<th>Orientation</th>
<th>Position on Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Translation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers**

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Lengths of Sides</th>
<th>Sizes of Angles</th>
<th>Orientation</th>
<th>Position on Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td>same</td>
<td>same</td>
<td>opposite</td>
<td>changed</td>
</tr>
<tr>
<td>Translation</td>
<td>same</td>
<td>same</td>
<td>same</td>
<td>moved only</td>
</tr>
<tr>
<td>Rotation</td>
<td>same</td>
<td>same</td>
<td>same</td>
<td>changed</td>
</tr>
</tbody>
</table>

**NOTE:** Students will need protractors for Question 4 on AP Book 6.2 p. 55.
NOTE: Extension 2 is required to cover the Alberta curriculum.

Extensions

1. Have students complete BLM Rotations Without a Grid.

   Selected answers: 2. a) 300° counter-clockwise, b) 340° clockwise, c) 210° clockwise, d) 180° counter-clockwise; 3. parallelogram, because the quadrilateral is made from two congruent triangles, with opposite sides the same length

2. Investigating rotations using The Geometer's Sketchpad®. Teach students how to perform rotations using The Geometer's Sketchpad®. Explain that before performing a rotation, one needs to mark a centre of rotation and select an angle of rotation. The software always uses the counter-clockwise direction, so that does not need to be specified. One way to select an angle of rotation is to create a new parameter using the Number menu. Remind students to select the "Angle" option for the new parameter. Then have them follow the instructions below to investigate the effects of changing the centre of rotation on the image of the shape.

   a) Use the Polygon tool to draw a triangle.
   b) Use the Number menu to create a new parameter equal to 90°. Use the Transform menu and the "Mark Angle" option to mark the parameter as the angle of rotation.
   c) Draw a point away from the triangle. Label it A.
   d) Use the Transform menu and the "Mark Center" option to mark point A as the centre of rotation.
   e) Select the triangle, including the interior, the vertices, and the edges. Use the Transform menu to rotate the triangle around point A by the angle of 90°.
   f) Move the vertices of the triangle around. Does the image seem to be congruent to the original triangle?
   g) Move the centre of rotation A around, including moving it to the sides, the vertices, and the interior of the triangle. How does the image triangle change? Does it change its shape? Do the angles or the lengths of the sides change?
   h) Moving the centre of rotation A around is the same as rotating the triangle around the new centre. What type of transformation—reflection, rotation, or translation—moves one image triangle to another image triangle?

   Selected answers: f) yes; g) the image triangle moves around, but its shape, angles, and side lengths do not change; h) translation

   Emphasize that moving the centre of rotation around is equivalent to rotating the shape around a different centre. So two shapes rotated the same way around different centres are translations of each other.
3. **Find a Flip.** Students can play this game in groups of 2 to 4. To make sure that students identify the transformations correctly, ensure that at least one student can reliably check the answers of the other players in the group.

**Objective:** Students create 2 by 2 squares of cards with each card’s shape the result of one transformation from the adjacent cards. Students can play cooperatively, working together toward creating eight squares of four cards each (in other words, 2 by 2 squares).

**Materials:** BLM Find a Flip. There are 32 cards in total: each row on the BLM has four identical shapes, and the next row shows their reflections; thus, there are four sets (or suits) of eight congruent shapes each. The cards do not have a clear top or bottom, so their orientation does not matter.

**Preparation:** Provide BLM Find a Flip to each group of students. Students cut out the cards on the BLM. The players shuffle the cards and deal out six cards to each player. Players sort the cards they are given by suit and identify the cards that are reflections of each other.

**Instructions:** Students play cooperatively in groups of three to four players so they can see each other’s cards, or competitively, in which case, the cards should remain hidden from other players. Players play in turn. Player 1 starts by placing a card at the centre. If Player 2 has a card with a shape that can be obtained from the card already on the table by a single transformation, Player 2 places the card adjacent to the card already in the centre, with the objective of creating a 2 by 2 square of cards. For example, see the pair of cards in the margin: The card on the right is a reflection of the card on the left, and the common vertical side is the mirror line: a student who demonstrates this reflection can flip one card onto the other and look at the cards together against a light source.

Player 2 says what transformation takes one card to the other, demonstrates the transformation, and picks a new card from the deck. Then Player 3 tries to place another card in the 2 by 2 square of cards. The players continue to take turns and work toward the objective of creating a 2 by 2 square of cards. If any player in turn does not have a card that is a single transformation of any of the cards on the table, that player must pick up all the cards already put down for the square (whether that means one, two, or three cards), and the next player starts a new square but does not need to take another card from the deck. When there are three cards in a square, the fourth card must be placed so that it can be obtained by a transformation from each of the adjacent cards.

Example: The second picture in the margin shows a completed 2 by 2 square of cards, with all cards showing a reflection of the adjacent cards in the common side. When a 2 by 2 square of cards is completed, all four cards are placed in the common score pile.
Goals
Students will rotate points and shapes around a given centre.
Students will perform combinations of rotations and combine rotations with reflections and translations.
Students will identify and describe rotations.

PRIOR KNOWLEDGE REQUIRED
Knows the terms clockwise and counter-clockwise
Knows that the size of an angle is a measure of rotation
Knows that angle sizes are additive
Can rotate a shape on a grid 90° clockwise or counter-clockwise
Can identify congruent shapes
Knows the definition of congruent shapes in terms of sides, angles, and order of elements

MATERIALS
BLM Rotating a Triangle (p. N-55)
scissors
rulers
BLM 1 cm Grid Paper (p. T-1)
BLM Find a Flip (pp. N-57–58, see Extension 1)

Mental math minute. Remind students that when dividing a sum, they divide each addend separately. Point out that this is like dividing parts of a very large array.

For the following exercises, write the division and the four possible answers on the board. Present one question at a time and have students signal which answer they think is correct by raising the correct number of fingers.

Ask volunteers to explain why some of the answers are not correct. For example, in part a), the remainder of #1 is larger than the divisor, so this cannot be a correct answer.

Exercises: Which answer is correct?

a) 58 ÷ 5
   1. 10 R 8  
   2. 11 R 3  
   3. 11 R 2  
   4. 10 R 3

b) 508 ÷ 5
   1. 100 R 1  
   2. 110 R 3  
   3. 101 R 3  
   4. 11 R 3

c) 508 ÷ 4
   1. 120 R 0  
   2. 127 R 2  
   3. 127 R 0  
   4. 102 R 0
d) \(3692 \div 3\)
   
1. 123 R 2  
2. 1230 R 2  
3. 1232 R 0  
4. 1232 R 2

\[\]

\[\]

e) \(7149 \div 7\)

\[\]

\[\]

\[\]

1. 121 R 2  
2. 1020 R 9  
3. 1021 R 2  
4. 121 R 0

**Answers:** a) 2, b) 3, c) 3, d) 2, e) 3

**Rotations in opposite directions can produce the same result.** Ask students to watch how much you are rotating and in which direction. Rotate your entire body, with one arm outstretched, one full turn and ASK: How many degrees did I turn? (360°) In which direction? Repeat, turning in the other direction. Point out that the result of your rotation is the same, regardless of the direction you turned in. Repeat with rotating 90° clockwise and 270° counter-clockwise. SAY: The amount of rotation you perform or the angle that you turn in is called the angle of rotation.

Remind students that they actually know that rotations around the same point can be added. Draw the picture in the margin on the board. SAY: If we rotate the top ray clockwise 90°, we get the ray in the middle, and the result is an angle of 90°. We know angle measures can be added. If we rotate the ray in the middle, the image, a further 90° clockwise, we get another angle of 90°, and the total rotation is 180°. When two rotations in the same direction have the same centre of rotation, we can simply add the angles of rotation to get the final angle of rotation.

Ask students again to watch how much you rotate and where you are facing at the beginning and at the end. You can have students count every 90° out loud to keep track. Rotate 180° clockwise and then another 270° (3/4 turn) clockwise. ASK: How much did I turn during the first rotation? (180°) the second rotation? (270°) How many degrees was that in total? (180° + 270° = 450°) What rotation is that equivalent to? (90°) How do you know? (you turned one full turn and another 90°) PROMPT: How many degrees are in a full turn? (360°) Repeat with two consecutive rotations of 270° counter-clockwise to produce a 180° counter-clockwise turn.

On the diagram from earlier, mark O on the centre of rotation, point P on the upward-pointing ray, and point P' as its image on the ray pointing downward to the right. SAY: Now let’s look at rotations in opposite directions. Point P' is an image of point P under a 90° clockwise rotation around O. ASK: What rotation in the opposite direction—counter-clockwise—gets you from the point P to the point P''? (270°) How do you know? (360° – 90° = 270°) Draw an arc to show this rotation and write the subtraction to label it.

**Exercises:** What turn in the opposite direction would produce the same result?

\[a) \text{90° CCW} \quad b) \text{270° CCW} \quad c) \text{180° CW}\]

**Answers:** a) 270° CW, b) 90° CW, c) 180° CCW
ASK: When we perform a rotation of 180°, does it matter if the rotation is clockwise or counter-clockwise? (no) If we want to rotate a point in a coordinate plane 270° clockwise, what simpler rotation could we do instead and get the same result? (90° counter-clockwise) Have students plot a pair of points on a grid, but not on the same horizontal or vertical grid line, label them P and O, and rotate P 270° clockwise around O. Use this opportunity to review rotating points 90° by drawing or imagining a right triangle with the longest, slanted side being OP, where O is the centre of rotation, as in the previous lesson.

**ACTIVITY (Essential)**

Give students BLM Rotating a Triangle and have them cut out the triangle from the bottom of the page. Have them place the cut-out triangle on top of the triangle at the top of the page and line up the triangles. Have students press the tip of a pencil to the black dot labelled O and rotate the top triangle around it 90° clockwise, as they did in the previous lesson, and then rotate the triangle again, around the same point, an additional 90°. ASK: How much did you rotate the triangle in total? (180°) Have them hold the cut-out triangle in place, trace it onto the BLM, and label the vertices using *.

Remove the cut-out triangle and compare the results. ASK: How is the rotated image different from the original? (the image turned upside down) Point out that the side that was horizontal in the original triangle is horizontal in the image triangle as well. Contrast the result with rotation of 90°: when rotating 90° clockwise or counter-clockwise, horizontal sides become vertical and vertical sides become horizontal. Have students also verify that rotating the triangle 180° clockwise and counter-clockwise produces the same result.

Finally, draw students’ attention to the order of the letters in the original triangle and the image. Students should see that the triangle is still labelled clockwise after rotation. Emphasize that reflection reverses the order; rotation and translation do not.

**Rotating points 180°.** Remind students that when rotating a point 90° around another point, they start with drawing a line segment between the points, then use a set square to create the right angle, and then mark the image point the same distance from the centre as the original point. Draw two points, O and P, on the board and invite a volunteer to draw the line segment between them and measure its length. SAY: I want to rotate point P around point O clockwise by 180°. How do I draw an angle of 180° with vertex O and side OP? (extend the line segment OP beyond O to create a straight angle) Demonstrate the construction on the board following the steps below.

**Step 1:** Draw line segment OP. Measure its length.

**Step 2:** Extend OP beyond point O.

**Step 3:** Mark the point P′ so that OP′ = OP.
**Exercises:**

a) Draw two points, $A$ and $B$, on grid line intersections but not on the same horizontal or vertical line.

b) Rotate $B$ around $A$ $180^\circ$ clockwise.

c) Rotate $A$ around $B$ $180^\circ$ counter-clockwise.

**Using a grid to rotate points $180^\circ$.** Remind students that they used a grid to rotate points $90^\circ$ clockwise or counter-clockwise. Remind them that they drew or imagined a right triangle and rotated the triangle around one of its vertices. Draw the picture in the margin on the board and explain that it shows rotating point $A$ around point $O$. ASK: Is this a $90^\circ$ rotation around $O$? (no) How much was the triangle turned? (half a turn, $180^\circ$, clockwise or counter-clockwise) Discuss how the triangle $AOB$ and its image are the same and how they are different. (The triangles are congruent, the horizontal sides are the same length, and the vertical sides are the same length, but if you move from $O$ to $A$, you move $2$ units right, $1$ unit down. When you move from $O$ to $A'$, you move $2$ units left and $1$ unit up, so the directions are opposite.)

Point out that the picture with triangles $AOB$ and $A'O'B'$ gives us a quick way to rotate points on a grid by $180^\circ$ similar to what students did with $90^\circ$ clockwise or counter-clockwise. SAY: All you need to do is to mentally extend the slanted line beyond point $O$ as if you were drawing a congruent triangle with horizontal and vertical sides of the same length but in the opposite directions from the centre of rotation. If you go $2$ units left and $1$ unit up from $A$ to $O$, continue another $2$ units left and $1$ unit up from $O$ to $A'$. Demonstrate using the first exercise below: move $1$ unit right and $3$ units down from $A$ to $O$ and then move $1$ unit right and $3$ units down from $O$ to $A'$.

**Exercises**

1. Rotate the point $A$ $180^\circ$ clockwise around point $O$. Draw the line segment $AA'$ to check.

![Diagram of grid with points and triangles]
2. a) Draw a right trapezoid.

b) Rotate the right trapezoid 180° CCW around the vertex with the acute angle.

c) Draw a pentagon that has no lines of symmetry.

d) Choose a point away from the pentagon. Rotate the pentagon 180° CW around the chosen point.

Distinguishing rotations of 90° from rotations of 180°. Draw the picture in the margin on the board. SAY: The grey shape was rotated 90° counter-clockwise to get one of the shapes and 180° (clockwise or counter-clockwise) to get the other shape. ASK: Which shape is produced by a rotation of 90° counter-clockwise and which is produced by a rotation of 180°? (the shape on the top is the image of a 90° CCW rotation; the shape on the bottom is the image of 180° CW or CCW rotation) Have students explain how they know. Answers will vary; make sure students understand that horizontal sides remain horizontal after a rotation of 180°, clockwise or counter-clockwise, and become vertical after a rotation of 90°, clockwise or counter-clockwise.

Finding the centre of rotation. Have students copy the grey shape on grid paper and cut it out. (Students can use BLM 1 cm Grid Paper.) Have students copy the picture on the board, place the cut-out shape on top of it, and perform both rotations to check. Have them rotate the shape using various points by pressing the tip of a pencil to different points on the outline of the grey shape. After students tried several different points, present the following exercises. Students can signal the answer by raising the number of fingers to show the number of the answer that they think is correct.

Exercises

1. The grey shape was rotated 90° CCW to get the white shape. Which point is the centre of rotation?

a)    b)    Bonus
2. The grey shape was rotated 180° CW to get the white shape. Which point is the centre of rotation?

Answers: a) 1, b) 3, Bonus: 4

Discuss strategies to find the correct centre of rotation. Point out that when the shape and the image touch each other, it makes sense to look for points that are on the common edge. Moreover, it makes sense to check the vertices first: are there any corresponding vertices that match? The centre of rotation is the only fixed point in rotation, so it should be on the same spot on both the original shape and the image.

In the case of 180° rotation, there is another way to look for the centre of rotation. If students are familiar with rotational symmetry, have them think of the point that they would rotate the whole picture, original and image, around to make it fall back onto itself, or the visual centre of the whole picture.

NOTE: Students who are struggling with the exercises below can use the cut-out shape to try to figure out the answers.

Exercises: The white shape is the image of the grey shape under rotation. Find the centre of rotation and describe the rotation.

Answers: a) 90° CW (or 270° CCW) rotation around point 1, b) 180° CW or CCW rotation around point 2, c) 90° CCW (or 270° CW) rotation around point 3, d) 180° CW or CCW rotation around point 3, Bonus: 180° CW or CCW rotation around point 3
Two reflections in perpendicular mirror lines produce a 180° rotation. Remind students that they can model reflection by flipping a shape over. Flipping it over a horizontal side is like reflection in the mirror line containing this horizontal side. Have students draw a horizontal and a vertical line on grid paper and place the L shape they used as shown in the margin.

Have students trace the shape, then reflect the shape in the vertical mirror line, and then reflect the image in the horizontal mirror line and trace the image. Repeat with reflecting in the opposite order, in the horizontal line followed by the reflection in the vertical line. ASK: What do you notice? (the result is the same) Is there one transformation that would take the original shape to the image? (yes) Which transformation? (180° rotation around the intersection of the mirror lines)

Repeat by placing the shape differently. Compare answers with the whole class. Did everyone place the shape the same way? (no) Did everyone get the same result: the order of reflections does not matter, and the resulting image is also a 180° rotation around the intersection of the mirror lines? (yes) Point out that the fact that everyone got the same result means that the answer is likely not dependent on the shape and is true for all shapes.

Extensions

1. Have students play Find a Flip (see Extension 3 in Lesson G6-15) with cards from BLM Find a Flip. First, have students place the cards so that each card they place is a reflection of the adjacent cards. Then ask the students to use rotations instead so that each card they place is a 90° rotation of the adjacent card. The fourth card will be a 90° rotation of both adjacent cards. Players can even place cards out of order—for example, both sequences are valid: first, second, third, fourth and first, second, fourth, third. Players must say around which vertex of the card the rotation was made and in which direction. In the example in the margin, the card on the right appears to be a 90° counter-clockwise rotation of the card on the left around the common vertex on the top.

After students have played the game both ways (reflections and rotations) several times, discuss similarities and differences between the two games. ASK: How many 2 by 2 squares of cards can you complete with cards from the same suit? (2 squares) How many different types of cards are in each suit? (each suit has eight cards of congruent shapes: four identical shapes and four that are their reflections) How are the squares completed in each game different—in other words, how many cards of each type does each square contain? (for rotations, a square has four of the same; for reflections, there are two of one type and two of another type) In which game was it harder to complete the first square? Why? (It is harder to complete the first square in the game with rotations. With rotations, when you place the first card of a suit, there are only three cards you can add to it. With reflections, when you place the first card, there are more card options, so the reflection game is easier.)
Is there a difference in constructing the second 2 by 2 square of cards with the same suit? (no, you have four cards from the same suit left in both cases, and they can always make a square)

2. Describe the transformation (including the translation arrow, the mirror line, or the amount of rotation around point O) that takes the rectangle $ABCD$ onto the other rectangle so that …

a) $A \rightarrow P$  

b) $B \rightarrow P$  

c) $D \rightarrow P$

Answers: a) reflection in the vertical line through $O$, b) translation 4 units right, c) 180° rotation (clockwise or counter-clockwise) around $O$

3. a) Copy the picture.

b) Rotate the point around point $O$ as given.

i) $P \rightarrow P'$: 90° clockwise

ii) $P' \rightarrow P''$: 180° clockwise

iii) $P'' \rightarrow P^*$: 270° clockwise

c) Point $P^*$ can be obtained by rotating point $P$ around point $O$

$90° + 180° + 270° - 360° =$ _______° clockwise.

Explain where each number in the equation comes from.

**Selected sample answer:** c) 180°; all the rotations in part b) are performed clockwise around point $O$, so the measures are added to get the combined amount of rotation: $90° + 180° + 270° = 540°$. This is more than a full rotation. A full rotation of 360° brings the point $P$ to the initial position, so we can subtract 360° to get the rotation after the full turn. $540° - 360° = 180°$, so point $P^*$ is directly down from point $O$. 

Geometry 6-16

N-35
Goals
Students will identify transformations used to create patterns and designs.
Students will use transformations and combinations of transformations to create patterns and designs.

PRIOR KNOWLEDGE REQUIRED
Can identify and perform translations, reflections, and rotations
Can identify congruent shapes
Can extend a repeating pattern

MATERIALS
pictures of designs created from transformations
BLM Find a Flip (pp. N-57–58)
scissors
glue
blank square about 4 cm by 4 cm (see Extension 3)
pattern blocks or BLM Pattern Blocks (p. N-59, see Extension 4)

Identifying the smallest part that repeats in the design. Show students several pictures of patterns and designs created by repeatedly using a transformation, such as a frieze pattern (see example in the margin). Explain that by repeatedly using one or several transformations on a simple shape, you can create a beautiful pattern or design. Invite a volunteer to show the part that repeats in the design. For each part that students identify, ask them to explain which transformation is used to repeat this part and create the design.

If students identify the part that repeats, but it is not the smallest, explain that there can be a smaller region that repeats, using more transformations. For example, in the second picture in the margin, students might identify either of the two white rectangles as a repeating part. The largest rectangle is translated to the right to create the repeating pattern. However, the smaller white rectangle can be reflected in the horizontal line to create the larger white rectangle, so the whole pattern can be created using a reflection and translation. Moreover, the grey rectangle can be rotated 180° around the midpoint of one of the vertical sides, as well as reflected in the horizontal line to create the same pattern, so the grey rectangle is the smallest part that was used to create the repeating pattern.

Exercises: What is the smallest part that creates the pattern?

```
a) b)
```
Sample answers: a) b) 

NOTE: Some students might identify half the square shown in the margin as the smallest part creating the pattern. Ask these students to identify all the required mirror lines. This is the correct answer, but it involves reflections in a slant line, which students are not expected to focus on.

Creating designs and patterns by repeated rotation and identifying the rotation.

ACTIVITY 1 (Essential)

1. Give each student BLM Find a Flip and have them cut out eight cards of the same suit (showing the same shape). Assign different suits to different students. Have them create a design that meets the rules below and glue each design to a separate strip of paper. Have students describe the direction, angle, and centre of rotation on the back of the strip of paper.

   a) a 4 by 1 rectangle so that each card is the same 90° CW or CCW rotation of the adjacent cards around a common vertex of the cards

   b) a 4 by 1 rectangle so that each card is a 180° rotation of the adjacent cards around the midpoint of the common side

Have students swap strips with a partner who used a different suit. Students need to identify the transformations used in the design. They can verify the answer by checking the back of the strip of paper.

Exercises: Copy the picture on grid paper.

a) Create a design by repeatedly rotating the shape 90° clockwise around point O.

b) Create a pattern by repeatedly rotating the shape 90° counter-clockwise around the top-right corner.

c) Create a pattern by repeatedly rotating the shape 180° clockwise around the middle of the right side.
Answers

a) 

b) 

c) 

NOTE: Students who are struggling with imagining the image after each rotation can cut out the shape, rotate it by pressing the tip of the pencil to the correct point, and then copy the shape.

Creating designs by repeated reflection and identifying the transformation.

**ACTIVITY 2 (Essential)**

2. Repeat Activity 1 for the rules below. Have students use cards from a different suit.
   
a) a 2 by 2 square so that each card in the square is the same 90° CW or CCW rotation of the adjacent cards around the centre of the 2 by 2 square
   
b) a 2 by 2 square so that each card in the square is a reflection of the adjacent cards in the common side
   
c) a 4 by 1 rectangle so that each card is a reflection of the adjacent cards in the common side

To make the guessing part harder, have students add one of the strips from Activity 1 as the fourth strip. This will force students to distinguish between reflections and rotations. Have students work with other partners than the ones they worked with in Activity 1.

**Exercises:** Copy the pictures on grid paper.

A. 

B. 
a) Create designs by repeatedly reflecting the shapes in the given mirror lines.

b) Create patterns by repeatedly reflecting the shapes in a vertical line through the right side.

**Bonus:** Create a design by repeatedly reflecting the polygon in the mirror lines.

![Diagram](image)

**Answers**

a) A. ![Pattern A](image)  

  B. ![Pattern B](image)

b) A. ![Pattern A](image)  

  B. ![Pattern B](image)

**Bonus:** ![Pattern](image)

**Identifying transformations in patterns.** Display the patterns in the following exercises one at a time. Have a volunteer identify the part that repeats. Discuss which transformation or combination of transformations
takes the repeated shape to the adjacent shapes. Have students explain how they know which transformation to use. (the shapes are exactly the same and point the same way, so it is a translation; the shapes point in opposite ways, and if I join corresponding vertices, I get parallel line segments of different lengths, so this is a reflection; the shape points in a different direction, and horizontal lines became vertical, so there is a 90° rotation) Have students also identify the mirror lines and the centres of rotation. Encourage multiple answers.

**Exercises:** Identify the part that repeats.

a)

b)

c)

d)

e)

f)

g)

### Answers

a) reflection in a horizontal line to get the shapes in the bottom row from the shapes in the top row, translation or rotation of 180° around the middle of the common side to get the shapes in the same horizontal row

b) reflection in a horizontal line to get the shapes in the bottom row from the shapes in the top row, reflection in the vertical line to get the shapes in the same horizontal row; translation down and right or rotation of 180° around the point in the centre of the space between each 4 shapes to get the shape situated diagonally

c) reflection in a horizontal line and translation left or right to get from 1 to 2, translation right or reflection in a vertical line to get from 1 to 3; rotation of 180° CW or CCW around the point marked (see margin) to get from 1 to 2 or from 2 to 3; students might also notice that half the leaf can be used to generate the full leaf using a reflection in a vertical line
d) reflection in a vertical line or rotation of 180° around the point midway between the tips of the leaves  
e) reflection in a horizontal line and translation to the right  
f) reflection in a vertical line  
g) reflection in a vertical line and reflection in a horizontal line  

**NOTE:** Students might also replace any rotation of 180° by a combination of reflections, in a horizontal and a vertical line that intersect at the centre of rotation.  

Discuss why some patterns allow more descriptions than other patterns. Students should notice that shapes that have some symmetry, such as lines of symmetry or rotational symmetry (students in Ontario should be familiar with the concept from Unit 6), produce more options because reflecting or rotating them results in the same shape as translating.

### ACTIVITY 3 (Optional)

3. Find a real-life example of a pattern or design that is made from repeating shapes. Identify the part that is used to create the pattern and describe the transformations used to create the pattern. Students can make a class display of the patterns and designs they found.

**NOTE:** Extensions 1 and 2 are required in order to cover the British Columbia curriculum.

### Extensions

1. Explain that a frieze pattern is a pattern that is created from repeated tiles placed in a row. Several shapes can appear on the same tile, and often the same shape is reflected, rotated, or translated to create the tile. There are seven different types of frieze patterns. Have students follow the instructions below to create a tile for each of the seven types of frieze patterns, starting from the same shape that has no lines of symmetry and no rotational symmetry.

   A. Draw a shape that has no lines of symmetry on grid paper. Your shape is the tile. To create the frieze pattern, translate the shape several times to the right.

   B. Reflect the shape in the horizontal line. Your tile consists of the original and the image.

   C. Reflect the shape in the vertical line. Your tile consists of the original and the image.

   D. Imagine that your shape is drawn on a rectangle. Rotate the shape 180° CW or CCW around the bottom-right corner of the rectangle. The tile consists of the original and the image.
E. Reflect the shape in the horizontal line and then translate it a little to the right. Your tile consists of two shapes, the original and the reflected and translated image.

F. Reflect the shape in the horizontal line. Reflect both shapes in the vertical line. Your tile consists of four shapes.

G. Reflect the shape in the vertical line. Reflect both shapes in the horizontal line and translate them to the right. The tile consists of four shapes.

2. Match the frieze pattern to the description in Extension 1.

\[ \text{Answers: } a) \text{ C, b) D, c) F, d) G} \]

3. Draw a shape that has no line of symmetry on a square. Use the square with the shape to create a tile pattern.

a) Rotate the square 90° clockwise around the bottom-right corner repeatedly. Translate the whole row you created 1 unit left and 1 unit down. Repeat, translating the row.

b) Rotate the square 90° clockwise around the bottom-left corner. Translate the whole column 1 unit right and 1 unit up. Repeat, translating the column.

c) Did you get the same tiling pattern both ways?

Selected answer: c) yes

4. Use pattern blocks or cut them out from BLM Pattern Blocks to create a shape that has no lines of symmetry. Use rotations to create a design based on the shape you created. Describe the rotations you used.
**Goals**

Students will identify and plot points in the first quadrant of a coordinate grid.

**PRIOR KNOWLEDGE REQUIRED**

Can read line graphs and broken line graphs
Can identify points on a number line made by skip counting

**MATERIALS**

binders or other dividers
rulers
BLM Grid with Tens (p. N-60)

**Review using grids in line graphs.** Draw the picture in the margin on the board. Remind students that in line plots they used points on a grid to show the relationship between two quantities. For example, this graph shows the relationship between the number of songs downloaded and the cost in cents. Each dot refers to two numbers. Point at the dots in random order and have students identify what the number of songs and the cost are for each dot. Then draw a dot at (1, 150) and repeat the question. Point out that you could describe any point on a grid using the numbers on the number lines.

SAY: Mathematicians around the world have agreed to give the location of a point on a grid by two numbers in parentheses, or brackets. The first number is always the number above which the point is on the horizontal number line, and the second number is the number to the left, on the vertical number line. This means that the numbers in the pair have a specific order, so we call them an ordered pair. Go through all the marked points in the picture on the board and have students identify the ordered pairs. ((1, 50), (2, 100), (3, 150), (4, 200), and the extra point (1, 150))

**Introduce the axes and origin.** Draw the coordinate grid shown in the margin, but without the labels. SAY: In geometry, we use grids with number lines that always meet at 0. The grid is called the coordinate grid. The number lines are called axes. Point out the axes and mention that “axes” is the plural of axis. SAY: The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. The point at which the two axes intersect is called the origin. Label the axes, the origin, and numbers 0 to 4 on the axes.

Point out that both axes start at 0. Explain that since the origin is the intersection of two number lines, which meet at 0, its ordered pair is (0, 0). It makes sense to draw only one 0 at the place where both number lines meet because in geometry the grids have no squiggly lines or breaks; they are just regular number lines always meeting at 0.
ACTIVITY (Essential)

Students work in pairs and use grid paper. Partners use a divider, such as a binder, to conceal their grids from each other. Each partner draws a coordinate grid and labels it from 0 to 4. Partner 1 marks a point on the grid and tells Partner 2 its ordered pair. Partner 2 marks the point on her own grid. Partners switch roles a number of times before checking that their grids match.

Introduce the x-coordinate and y-coordinate. On a grid on the board, mark the point (4, 3) and have students identify the ordered pair. SAY: The ordered pair (4, 3) is called the coordinates of the point. Mathematicians say that the point (4, 3) has \( x = 4 \) or the x-coordinate is 4. Trace your finger down from the point and show that it is directly above the number 4 on the x-axis.

Then trace with your finger left from the point (4, 3) to the y-axis to look at the number on the y-axis. SAY: Mathematicians say that this point has \( y = 3 \) or the y-coordinate is 3.

Exercises: Rewrite the coordinates of the point as \( x = \_), \( y = \_ \).

a) (2, 1)    b) (1, 3)    c) (4, 2)    d) (3, 4)

Answers: a) \( x = 2, y = 1 \); b) \( x = 1, y = 3 \); c) \( x = 4, y = 2 \); d) \( x = 3, y = 4 \)

Mark several points on the grid and have students identify the x and the y for these points. SAY: The x-coordinate is often called the first coordinate, because it is written first, and the y-coordinate is called the second coordinate.

Coordinates of points on the axes. Remind students that the axes intersect at 0. Mark the point (3, 0) on the coordinate grid and have students identify the coordinates. Point out that the y-coordinate is in fact the height above the horizontal axis, and since the height is 0, the point should be on the horizontal axis itself. Repeat with the point (0, 4), explaining that the x-coordinate shows the distance from the point to the vertical axis, measured along the grid line on which the point is situated.

Exercises: Draw a coordinate grid and label it with numbers from 0 to 5. Mark and label the points on the grid.

\( A (0, 4) \), \( B (2, 0) \), \( C (5, 0) \), \( D (0, 1) \)

Answers:
Coordinate grids with scales made by skip counting by 2s. On a grid on the board, draw axes and mark them with intervals of 2. Explain that just as number lines on graphs can be marked with skip counting, number lines on coordinate grids can be marked by skip counting too. Mark several points and ask students to find the coordinates of these points. Start with points that are on the grid lines, such as (2, 4), continue to points that are on only one grid line, such as (2, 3) and (5, 4), and progress to points that are not on grid lines, such as (5, 7).

**Exercises**

a) Draw a coordinate grid with axes that skip count by 2s from 0 to 10.

b) Mark and label the points on the grid.

\[ Z (4, 6), Y (3, 8), X (7, 9), W (0, 7), V (5, 0), U (3, 5), T (1, 9) \]

**Answers:**

Review marking numbers on a number line that only shows tens. Draw a number line from 0 to 40 but label only the tens. Point to a few locations on the line and have students identify the number for each location. Write several numbers and point at locations on the number line. Have students signal with thumbs up and thumbs down to indicate whether the location you are pointing at is the number you wrote.

Coordinate grid with scales made by skip counting by 10s and 5s. Display BLM Grid with Tens. Use the point \( A (18, 24) \) to show students how to determine the coordinates of the point and then invite volunteers to draw lines from other points to the axes. Point to the locations on the grid for each coordinate pair and have students signal thumbs up or thumbs down to indicate whether this point is the one given by the coordinates.

Have students draw a coordinate grid whose axes are marked with intervals of 5 from 0 to 25. Points to plot: \( A (15, 6), B (3, 10), C (0, 24), D (6, 0), E (22, 6), F (23, 5), G (1, 20) \)

**Extensions**

1. Draw the points on a coordinate grid.
   \( (2, 6), (4, 4), (5, 7), (7, 8), (5, 2), (3, 4), (2, 1), (0, 0) \)
   Join the points in the order you drew them. Join the first point to the last point. What letter did you make?

   **Bonus:** If you rotate the shape 270° counter-clockwise around any point, what letter will you make?
Answer: N, Bonus: Z

2. Players will need a divider to conceal the coordinate grids they are working on from partners. Player 1 draws a square or a rectangle on the coordinate grid and tells Player 2 the coordinates of the vertices. Player 2 tries to visualize the shape and guess what kind it is before plotting the vertices. Player 2 plots the vertices and checks the answer.

Advanced: Use other shapes, such as rhombuses, parallelograms, and trapezoids.

3. a) Draw a coordinate grid with axes from 0 to 10.
   b) Plot the pair of points.
      i) (3, 0) and (7, 0)   ii) (4, 1) and (9, 1)   iii) (5, 2) and (1, 2)
   c) Find the distance between the points in the pair.
   d) Which coordinates are the same in each pair, the first or the second? What is the same about the location of the points?
   e) Which coordinate is not the same? How can you find the distance between the points using the coordinate that is not the same?

   Selected answers: c) i) 4, ii) 5, iii) 4; d) second, the points are on the same horizontal line; e) x-coordinates, subtract the smaller coordinate from the larger

4. a) Draw a coordinate grid with axes from 0 to 10.
   b) Plot the pair of points.
      i) (3, 0) and (3, 6)   ii) (7, 1) and (7, 9)   iii) (0, 2) and (0, 5)
   c) Find the distance between the points in the pair.
   d) Which coordinates are the same in each pair, the first or the second? What is the same about the location of the points?
   e) Which coordinate is not the same? How can you find the distance between the points using the coordinate that is not the same?

   Selected answers: c) i) 6, ii) 8, iii) 3; d) first, the points are on the same vertical line; e) y-coordinates, subtract the smaller coordinate from the larger
Goals

Students will perform and describe translations and reflections of points and polygons in the first quadrant of a coordinate grid.

PRIOR KNOWLEDGE REQUIRED

Can identify and perform translations and reflections
Can identify and plot points on a coordinate grid

MATERIALS

BLM Maps (pp. N-61–62, see Extension 3)

Mental math minute. Remind students that they can use division with remainders to convert improper fractions to mixed numbers. For example, \(\frac{11}{4} = 2 \frac{3}{4}\), so \(\frac{11}{4}\) can be represented as \(2\frac{3}{4}\). Demonstrate using circles divided into fourths, as shown below. Each circle shows a group of four fourths, and the remainder is three fourths, or the fractional part.

\[
\begin{align*}
11/4 &= 2 \text{ wholes} + 3/4 = 2\frac{3}{4} \\
\end{align*}
\]

Exercises: Convert the improper fraction to a mixed number.

a) \(\frac{13}{4}\)  
b) \(\frac{11}{3}\)  
c) \(\frac{17}{5}\)  
d) \(\frac{51}{8}\)  
e) \(\frac{77}{8}\)  
f) \(\frac{77}{9}\)

Answers: a) \(3\frac{1}{4}\), b) \(3\frac{2}{3}\), c) \(3\frac{2}{5}\), d) \(6\frac{3}{8}\), e) \(9\frac{5}{8}\), f) \(8\frac{5}{9}\)

Investigate the change in the coordinates under a horizontal or vertical translation. Mark the points \((4, 5)\), \((0, 6)\), and \((1, 7)\) on a grid on the board and ask students to identify the coordinates. Have students draw a coordinate grid (with axes counting by 1s from 0 to 10) and plot the points, joining them into a triangle. Have students translate the vertices of the triangle 3 units to the right and write the original and image coordinates in a table as shown below.

<table>
<thead>
<tr>
<th>Original</th>
<th>(4, 5)</th>
<th>(0, 6)</th>
<th>(1, 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>(7, 5)</td>
<td>(3, 6)</td>
<td>(4, 7)</td>
</tr>
</tbody>
</table>

ASK: Which coordinate changed during the translation? (the \(x\)-coordinate)
What about the other coordinate? (it stayed the same) How did the coordinate change—did it increase or decrease? (increase) By how much? (by 3) How do you express it as an operation? (add 3 to the \(x\)-coordinate)
Repeat the investigation by translating the points 4 units down. Students will see that the y-coordinate changes, decreasing by 4, and the x-coordinate stays the same.

**Generalizing the rule for combined directions.** On a new grid, draw the point (4, 5). SAY: When we move this point 3 units right, the x-coordinate increases by 3. ASK: What happens if we move the point 3 units left? (the x-coordinate will decrease by 3) PROMPT: Which coordinate changes? (the y-coordinate) Moving left is the opposite of moving right. Does the x-coordinate increase or decrease? (decrease) Demonstrate both movements with arrows. Write on the board:

- 3 units right: add 3 to the x-coordinate
- 3 units left: subtract 3 from the x-coordinate

ASK: So if I wanted to translate a point, say, (7, 7) 3 units left, what point would I get? (4, 7) Invite a volunteer to check. ASK: What if I wanted to translate point (7, 7) 2 units right? (9, 7) Again, invite a volunteer to check.

Repeat the questioning with moving the point (4, 1) 3 units up and 3 units down. Point out that, this time, the y-coordinate changes. Write on the board:

- 3 units up: add 3 to the y-coordinate
- 3 units down: subtract 3 from the y-coordinate

Remind students that they can combine translations in different directions. For example, students can translate a shape 3 units right and 4 units down. Ask students to predict the change in the coordinates in the point (5, 5) and check.

**Exercises**

a) Draw a coordinate grid. Plot the points A (4, 5), B (3, 6), C (5, 6), and D (6, 5).

b) How will the coordinates change in each translation?

   i) 2 units right, 1 unit up 
   ii) 3 units left, 2 units down

c) Predict the coordinates of the images for each translation.

d) Perform the translations and check your prediction.

**Selected answers**

b) i) x-coordinate increases by 2, y-coordinate increases by 1; 
   ii) x-coordinate decreases by 3, y-coordinate decreases by 2

   c) i) A' (6, 6), B' (5, 7), C' (7, 7), D' (8, 6); ii) A* (1, 3), B* (0, 4), C* (2, 4), D* (3, 3)

**Investigate the change in the coordinates under a reflection in a vertical line.** Mark the points (3, 5), (0, 6), and (1, 2) on a grid on the board and ask students to identify the coordinates. Have students draw a coordinate grid (with axes counting by 1s from 0 to 10) and plot the points, joining them into a triangle. Draw the vertical line through point (4, 0) and have students copy it as well. Have students reflect the vertices of the triangle in the line (see the completed table on the following page).
ASK: Which coordinate changed during the reflection? (the x-coordinate)
What about the other coordinate? (it stayed the same) Repeat with a reflection in another vertical line through the point (2, 0). (the image points are (1, 5), (4, 6), and (3, 2)) Discuss why the y-coordinate did not change. To prompt students to see the answer, have them draw a horizontal line through one of the vertices of the original triangle and look at the y-coordinates of different points on the same line. Students will notice that points on the same horizontal line have the same second coordinate. Have them think how they perform a reflection in a vertical line: they start by drawing a line perpendicular to the mirror line, so a horizontal line, and mark a point on the same horizontal line. Naturally, the second coordinate of the image will be the same as the second coordinate of the original—they are on the same horizontal line by construction.

ASK: Are there points on the triangle that had no change of coordinates after the second reflection? (yes) Have students give examples. SAY: The coordinates of the points on the mirror line did not change under reflection because they are fixed points—points on the mirror line stay the same under reflection.

Repeat the investigation by using two horizontal mirror lines, for example, a horizontal line through (0, 3) and another line through (0, 6). Students will see that the y-coordinate changes and the x-coordinate stays the same because both the original and the image under reflection are on the same vertical line.

**Extensions**

1. A shape has vertices $A(2, 2)$, $B(3, 5)$, $C(5, 5)$, and $D(4, 2)$. Under a translation, vertex $A$ moved to $A'(7, 8)$. Find the coordinates of the other vertices under the translation. Check by plotting $ABCD$ and performing the translation.
   
   **Answer:** $B'(8, 11)$, $C'(10, 11)$, $D'(9, 8)$

2. Marcel translated triangle $ABC$ 5 units left and 3 units up. The image triangle has vertices $A^* (2, 4)$, $B^* (1, 5)$, and $C^* (2, 3)$. What were the coordinates of the vertices of triangle $ABC$? Explain.
   
   **Sample answer:** Triangle $ABC$ was translated 5 units left and 3 units up to get $A^*B^*C^*$. This means that translating triangle $A^*B^*C^*$ 5 units right and 3 units down will get us back to triangle $ABC$. Therefore, the coordinates of the vertices are $A(7, 1)$, $B(6, 2)$, and $C(7, 0)$.
3. Have students complete **BLM Maps**.

**Selected answers**

1. a) Dubhe; b) (60, 10); c) Alkaid; Bonus: Alioth, (10, 16)
2. b) 10; c) 10; south, 5; d) 5, east, 10, north; e) 10 paces west, 10 paces north
3. b) Red Rock, Tall Fir; c) 10 paces west, 10 paces south
4. a) Bear Cave, Treasure, The Fort; b) (14, 8), (12, 2), (12, 10); c) Clear Spring, Swamp, Bonus: Ear Wood; d) 4 km west; 6 km south; 6 km east, 2 km south; 2 km north, 2 km west; 4 km west, 6 km south; 2 km north, 6 km west, 6 km north
Goals

Students will perform and describe transformations of points and polygons in the first quadrant of a coordinate grid.

PRIOR KNOWLEDGE REQUIRED

Can identify, describe, and perform translations, reflections, and rotations
Can identify and plot points on a coordinate grid

Mental math minute—number talk. Present this division with remainder: $650 \div 8$. (81 R 2) The following strategies could arise:

- use $648 \div 8 = (600 \div 8) + (48 \div 8)$
- use $648 \div 8 = (640 \div 8) + (8 \div 8)$

\[(640 \div 8) + (10 \div 8) = 80 + 1 R 2 = 81 R 2\]

Investigate the change in the coordinates under a 90° rotation.

Draw a coordinate grid with axes marked from 0 to 10 on the board and have students draw a similar grid in their notebooks. On the grid, draw a trapezoid with vertices (3, 3), (3, 5), (8, 5), and (7, 3). Ask students to identify the coordinates of the vertices and fill in the first row of the table below for the original points. Have students plot the points on the coordinate grid they drew and join them into the quadrilateral. Have students rotate the quadrilateral 90° CCW around the three points given in the table and fill in the table. (see completed table and grid below)

<table>
<thead>
<tr>
<th>Original</th>
<th>(3, 3)</th>
<th>(3, 5)</th>
<th>(8, 5)</th>
<th>(7, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image under 90° CCW rotation around</td>
<td>R (2, 3)</td>
<td>(2, 4)</td>
<td>(0, 9)</td>
<td>(2, 8)</td>
</tr>
<tr>
<td>P (5, 4)</td>
<td>(6, 2)</td>
<td>(4, 2)</td>
<td>(4, 7)</td>
<td>(6, 6)</td>
</tr>
<tr>
<td>Q (8, 5)</td>
<td>(10, 0)</td>
<td>(8, 0)</td>
<td>(8, 5)</td>
<td>(10, 4)</td>
</tr>
</tbody>
</table>
Have students look at each rotation separately. ASK: In the first rotation, is there any vertex that had even one coordinate, $x$ or $y$, left unchanged? (no) Repeat with the second and third rotations. In the third rotation, students will notice that the vertex $(8, 5)$ stayed the same. ASK: How is this point special? (it is the centre of rotation) What do we know about fixed points of rotations? (the centre of rotation is the only fixed point of a rotation)

**Investigate the change in the coordinates under a $180^\circ$ rotation.** Repeat the previous investigation with quadrilateral $ABCD$ with vertices $A (3, 6)$, $B (3, 8)$, $C (5, 6)$, $D (5, 4)$, rotating it $180^\circ$ clockwise around the point $(3, 4)$. Have students use $'$ to label the images of the vertices. The resulting vertices are shown below.

<table>
<thead>
<tr>
<th>Original</th>
<th>$A (3, 6)$</th>
<th>$A (3, 8)$</th>
<th>$A (5, 6)$</th>
<th>$A (5, 4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>$A' (3, 2)$</td>
<td>$B' (3, 0)$</td>
<td>$C' (1, 2)$</td>
<td>$D' (1, 4)$</td>
</tr>
</tbody>
</table>

Students will notice that some vertices have the same first or second coordinate before and after the rotation. Discuss why this happens. To prompt students to see the answer, draw their attention to the construction they use to rotate points $180^\circ$: the original point, the image, and the centre of rotation are always on the same line, so if the original and the centre of rotation are on the same horizontal line, so is the image, and the $y$-coordinate of all three points is the same. Similarly, the $x$-coordinate of the points on the same vertical line as the centre of rotation does not change when rotating it $180^\circ$. Keep the picture on the board for later use.

**Rotating polygons with rotational symmetry.** Have students rotate the quadrilateral $ABCD$ on the board $180^\circ$ around the point $(4, 6)$. Have them use $'$ to label the image vertices. ASK: What do you notice? (the image coincides with the original; $A' = C$, $B' = D$, $C' = A$, $D' = B$) Discuss why this happened. ($ABCD$ is a parallelogram, which is made of two congruent triangles that are a $180^\circ$ rotation of each other) If students are familiar with rotational symmetry (see Unit 6, Lesson G6-11), they should recognize that a parallelogram has rotational symmetry of order 2, and students are rotating it around the centre, so it should coincide with itself.
Exercises

a) Draw a coordinate grid.

b) Plot points $E(1, 2)$ and $F(1, 6)$.

c) Find two points $G$ and $H$, so that $EFGH$ is a square.

d) Find the centre of the square $O$. What are the coordinates of $O$?

e) What is the smallest rotation around $O$ that would bring $EFGH$ to a square that occupies the same place?

Bonus: Explain how you know.

Selected answers: c) $G(5, 6), H(5, 2)$; d) $O(3, 4)$; e) $90^\circ$ clockwise or counter-clockwise; Bonus: A square has rotational symmetry of order 4, so a rotation of $90^\circ$ should bring it to the same place. OR: If you turn a square $90^\circ$ around the centre, it does not change.

NOTE: Parts c) and d) have another possible answer, but it involves points in the second quadrant.

Using different transformations to produce the same image. Return to the quadrilaterals $ABCD$ and $A'B'C'D'$ on the board. Have students translate $ABCD$ 2 units left and 4 units down, using " to label the images. Discuss what students notice. ($A'B'C'D'$ and $A''B''C''D''$ occupy the same space, $A'' = C', B'' = D', C'' = A', D'' = B'$) Point out that since a parallelogram does not change when rotated $180^\circ$ around the centre, rotation and translation produce results that occupy the same space, although they have different image vertices.

Exercises:

a) Draw a coordinate grid.

b) Draw a rectangle on the grid.

c) Reflect or translate the rectangle using a transformation of your choice. Describe the transformation you used.

d) Describe two transformations of a different type that would take the rectangle to the image you produced.

Bonus: Use two different transformations in a row on the rectangle you drew. Then find two different transformations that take the original rectangle to the image rectangle.

NOTE: Encourage students who are familiar with rotational symmetry to use order of rotational symmetry in their explanation in Question 5.d) on AP Book 6.2 p. 69.
Extensions

1. Find a sequence of transformations that takes $MNSP$ to $M*N*S*P*$, keeping the vertices in the same order ($M$ to $M*$, and so on). Describe the transformations and the change of vertices.

   a) 
   
<table>
<thead>
<tr>
<th>Original</th>
<th>$M$ (0, 5)</th>
<th>$N$ (1, 7)</th>
<th>$S$ (4, 7)</th>
<th>$P$ (3, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>$M^*$ (10, 3)</td>
<td>$N^*$ (9, 5)</td>
<td>$S^*$ (6, 5)</td>
<td>$P^*$ (7, 3)</td>
</tr>
</tbody>
</table>

   b) 
   
<table>
<thead>
<tr>
<th>Original</th>
<th>$M$ (2, 3)</th>
<th>$N$ (1, 5)</th>
<th>$S$ (2, 7)</th>
<th>$P$ (3, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>$M^*$ (4, 3)</td>
<td>$N^*$ (6, 4)</td>
<td>$S^*$ (8, 3)</td>
<td>$P^*$ (6, 2)</td>
</tr>
</tbody>
</table>

   Sample answers
   
   a) 
   
<table>
<thead>
<tr>
<th>Original</th>
<th>$M$ (0, 5)</th>
<th>$N$ (1, 7)</th>
<th>$S$ (4, 7)</th>
<th>$P$ (3, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection in a vertical mirror line through (5, 0)</td>
<td>(10, 5)</td>
<td>(9, 7)</td>
<td>(6, 7)</td>
<td>(7, 5)</td>
</tr>
<tr>
<td>Translation 2 units down</td>
<td>$M^*$ (10, 3)</td>
<td>$N^*$ (9, 5)</td>
<td>$S^*$ (6, 5)</td>
<td>$P^*$ (7, 3)</td>
</tr>
</tbody>
</table>

   b) 
   
<table>
<thead>
<tr>
<th>Original</th>
<th>$M$ (2, 3)</th>
<th>$N$ (1, 5)</th>
<th>$S$ (2, 7)</th>
<th>$P$ (3, 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ$ CCW rotation around (2, 3)</td>
<td>(2, 3)</td>
<td>(4, 4)</td>
<td>(6, 3)</td>
<td>(4, 2)</td>
</tr>
<tr>
<td>Translation 2 units right</td>
<td>$M^*$ (4, 3)</td>
<td>$N^*$ (6, 4)</td>
<td>$S^*$ (8, 3)</td>
<td>$P^*$ (6, 2)</td>
</tr>
</tbody>
</table>

2. 

   a) Rotate the polygon $PQRS$ $90^\circ$ clockwise around point $U$. What are the coordinates of the vertices of the image?

   b) Describe another transformation or sequence of transformations that would take $PQRS$ to occupy the same place, but with $P^*$ being (10, 1).

   ![Diagram](image)

   Answer: a) $P'$ (10, 3), $Q'$ (11, 2), $R'$ (10, 1), $S'$ (7, 2)

   Sample answer: b) rotate $PQRS$ $90^\circ$ clockwise around point $U$, then reflect the image in the horizontal line through the point $U$
Rotating a Triangle
Rotations Without a Grid

To rotate point $P$ around point $O$ $60^\circ$ clockwise:

**Step 1:** Draw line segment $OP$. Measure its length.

**Step 2:** Draw an arc clockwise to show the direction of rotation.

**Step 3:** Place the protractor so that the origin is at point $O$ and the base line aligns with $OP$.

**Step 4:** Does the scale that counts clockwise have a $0$ on the line segment? If not, turn the protractor upside-down.

**Step 5:** Make a mark at $60^\circ$ on the scale that counts clockwise. Remove the protractor and draw a ray through the mark, starting at $O$.

**Step 6:** On the new ray, measure and mark the image point $P'$ so that $OP' = OP$.

1. Rotate point $P$ around point $O$ by the given angle and direction.
   a) $60^\circ$ CW
   b) $20^\circ$ CCW
   c) $150^\circ$ CCW
   d) $180^\circ$ CW

2. For points $O$ and $P$ in Question 1, what rotation in the opposite direction around point $O$ will take point $P$ to the same image?
   a) ________________
   b) ________________
   c) ________________
   d) ________________

**BONUS** Use a ruler to draw a triangle $ABC$. Find the midpoint of side $AC$ and label it $M$. Rotate $ABC$ $180^\circ$ clockwise around point $M$.

What type of quadrilateral do $ABC$ and its image make together? Explain.
Find a Flip (1)
Find a Flip (2)
Pattern Blocks
Grid with Tens

(y-axis)

(x-axis)

B (10, 35)  C (4, 20)  D (27, 6)  E (41, 33)  F (0, 23)
Maps (1)

1. This is the star map of the Big Dipper, part of the Great Bear constellation.
   a) The star at point (60, 20) is the official star of Utah.
      What is it called? ______________________
   b) What are the coordinates of Merak?
      ( _____, _____ )
   c) Which star is 50 units west of Merak?
      ______________________
   d) Galaxy M81 is located 10 units north and 10 units east from Dubhe. Mark it on the map.
      BONUS ▶ What star is at point (30, 16)? ______________________
      BONUS ▶ The Pinwheel Galaxy is located 20 units west of Alioth.
      Mark it on the map and write its coordinates. ( _____, _____ )

2. This map shows part of Feral Cat Island, where pirates have buried gold, silver, and weapons. Fill in the directions.
   a) From the Tall Fir, walk __10__ paces (steps) __west____ to the Red Rock.
   b) From the Red Rock, walk _____ paces north to the Large Birch.
   c) From the Red Rock, walk _____ paces _____________ and _____ paces east to the Rose Bush.
   d) From the Rose Bush, walk _____ paces _____________ and _____ paces _____________ to the Tall Fir.
   e) From the Tall Fir, walk ________________ and ________________ to the Large Birch.

3. Mark on the map in Question 2 the point where some treasure is buried.
   a) Gold (G): From the Tall Fir, walk 5 paces east and 10 paces north.
      Weapons (W): From the Rose Bush, walk 10 paces west and 5 paces south.
      Silver (S): From the Large Birch, walk 10 paces south and 5 paces east.
   b) What two landmarks is the silver buried between? ______________________
   c) Write directions for walking from Gold to Silver.
      ______________________
Maps (2)

4. This map shows all of Feral Cat Island. Each square on the map has sides 2 km long.
   a) Round Lake is at point (4, 8). What is located at each point?
      (6, 4) ____________________
      (6, 10) ____________________
      (10, 10) ____________________
      (13, 7) ____________________
   b) Give the coordinates for the landmark.
      Old Lighthouse _____________
      Lookout Hill _____________
      Clear Spring _____________
   c) Name the landmark located at the point described.
      2 km east of the Fort _____________
      4 km south of Round Lake _____________
      BONUS ► 2 km north and 3 km west of the Treasure _____________
   d) Fill in the blanks.
      From Round Lake, the Old Lighthouse is ___10___ km _____east_____.
      From the Fort, walk _____ km _____________ to the Treasure.
      From the Treasure, the Bear Cave is _____ km _____________.
      To walk from the Bear Cave to Lookout Hill, walk _____ km _____________ and
      _____ km south.
      From the Old Lighthouse, walk _____ km _____________ and _____ km
      _____________ to the Clear Spring.
      From the Fort, walk _________________ to the Bear Cave.
      From Lookout Hill to the Treasure, walk _________________.
   e) Write your own question that asks for directions and uses the map. Ask your partner to answer it.

__________________________________________________________________________________________

__________________________________________________________________________________________
Unit 12 Patterns and Algebra: Equations and Graphs

Introduction
This unit is about equations, graphs, and tables, including:

- solving one-step equations using different methods, including balances, preserving equality, guessing and checking, and logic;
- solving word problems using equations; and
- finding connections between sequences, graphs, tables, and equations (formulas).

Meeting Your Curriculum

**ALBERTA**
Required PA6-9 to 20

**BRITISH COLUMBIA**
Required PA6-9 to 12, 14 to 20 including Extensions 1 and 2 in PA6-11
Recommended PA6-13 supports solving equations with larger numbers

**MANITOBA**
Required PA6-10 to 20
Recommended PA6-9 supports material in later lessons

**ONTARIO**
Required PA6-9 to 18 including Extension 3 in PA6-11
Optional PA6-19, 20

Mental Math Minutes
The mental math minutes in this unit:
- use shortcuts to solve equations by comparing sides and realizing what changes from side to side.

Generic BLMs
The Generic BLMs used in this unit are:
- BLM 1 cm Grid Paper (p. T-1)
- BLM Filling a Blank Multiplication Chart (p. T-2)
These BLMs can be found in Section T.
Materials

In this unit you will need:

- several identical objects that are heavier than a paper bag, such as small fruit of equal size (apples will match the pictures on the AP Book), metal spoons, rolls of masking tape, tennis balls, or cereal bars
- a pan balance or, if one is not available, a concrete model and/or pictures, such as a seesaw, to explain how a pan balance works
- grid paper or BLM 1 cm Grid Paper for students to use as a basis for drawing coordinate grids. You will also need to draw coordinate grids on the board—if your board does not have a grid, you can project BLM 1 cm Grid Paper instead.

Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>PA6-9 to 11</td>
<td>PA6-9 to 11</td>
<td>PA6-9 to 11</td>
<td>PA6-9 to 11</td>
</tr>
<tr>
<td>Quiz</td>
<td>PA6-12 to 15</td>
<td>PA6-12 to 15</td>
<td>PA6-12 to 15</td>
<td>PA6-12 to 15</td>
</tr>
<tr>
<td>Quiz</td>
<td>PA6-16 to 20</td>
<td>PA6-16 to 20</td>
<td>PA6-16 to 20</td>
<td>PA6-16 to 18</td>
</tr>
<tr>
<td>Test</td>
<td>PA6-9 to 20</td>
<td>PA6-9 to 12, 14 to 20</td>
<td>PA6-10 to 20</td>
<td>PA6-9 to 18</td>
</tr>
</tbody>
</table>
Goals

Students will write and solve one-step equations.
Students will solve easy two-step equations by converting to one-step equations.

PRIOR KNOWLEDGE REQUIRED

Can add, subtract, multiply, and divide
Knows that the equal sign means expressions on both sides of it are equal
Can write an equation to solve a one-step addition problem

MATERIALS

paper bag
counters

Mental math minute. SAY: Remember, an equal sign means “the same as.” To check if an equation is true, you can use what you know about multiplication and division without actually calculating both sides. For example, you know that doubling both numbers in division does not change the quotient. And you know that doubling one factor and halving another factor keeps the product the same. Present the equations in the following exercises one at a time and have students signal the answers using thumbs up for “yes” and thumbs down for “no.”

Exercises: Is the equation true?

a) $\frac{84}{11} = \frac{42}{22}$

b) $\frac{42}{13} = \frac{84}{26}$

c) $43 \times 20 = 430 \times 2$

d) $5 \times 13 = 10 \times 26$

e) $\frac{34}{0.5} = \frac{68}{1}$

f) $11 \times 2.5 = 22 \times 5$

g) $11 \div 2.5 = 44 \div 10$

h) $22 \div 2.5 = 11 \times 5$

Answers: a) no, b) yes, c) yes, d) no, e) yes, f) no, g) yes, h) yes

Equations with addition. Have students practise writing equations that represent pictures, as in Question 2 on AP Book 6.2 p. 70. For instance, the equation for the picture in the margin is

$\square + \bigcirc = \square \square \square \square \square$

ASK: Are the numbers of circles on both sides equal? (yes) How many circles are on the left side in total? (5) How many circles are in the box? (3) Challenge students to solve several more examples. Students can create models for equations that involve addition using counters. They can also draw models: A square could stand for the unknown (the “hidden” number,
the number we don’t know) and a set of circles could be used to model the numbers in the equation. Write on the board:

\[ \square + 2 = 7 \]
\[ \square + \square = \square \square \square \square \square \square \square \square \]

SAY: This equation has this model. Ask students to make a model with squares and circles to solve the following problems. Then they should explain how many circles they would put in each square to make the equation true.

a) \( 7 + \square = 11 \)  
   b) \( 6 + \square = 13 \)  
   c) \( 4 + \square = 10 \)  
   d) \( 9 + \square = 12 \)

**Writing equations to solve word problems.** Read the word problems below. Invite volunteers to draw models, write equations using squares and numbers, and solve the equations.

**Exercises:** Write an equation to solve the problem.

a) There are 10 trees in the garden. Three of them are apple trees. All the rest are cherry trees. How many cherry trees are in the garden?

b) Jane has 12 T-shirts. Three of them are plain. All the rest have designs. How many of Jane’s T-shirts have designs?

c) There are 15 flowers in the flowerbed. Six are lilies. All the rest are peonies. How many peonies grow in the flowerbed?

**Bonus**

d) There are 150 bloodthirsty pirates on two ships, a galleon and a schooner. Forty of the pirates are on the schooner. How many are on the galleon?

e) A multi-headed dragon has 15 heads. Some of them were cut off by a mighty and courageous knight. The dragon ran away from the knight with seven remaining heads. How many heads were removed by the knight?

**Sample answer:** c) \( \square + 6 = 15 \), so there are 9 peonies in the flowerbed.

**Equations with subtraction.** Present this word problem: Sindi has a box of apples. She took 2 apples from the box and 4 were left. How many apples were in the box before she removed any? Draw the box (shown in the margin) with 4 apples left.

Draw two more apples and cross them out to show that they are taken away.

ASK: How many apples were there in the beginning? (6) SAY: When we write a subtraction equation, we draw it a bit differently. We use a square to show the initial situation (the number we don’t know) and we draw the apples that we took out outside the box preceded by a minus sign, to show that they were taken away. Demonstrate this on the board as in the picture in the margin.
SAY: We show the four apples that were left in the box on the other side of the equation. Finish the picture as shown in the margin.

To solve the equation, we have to find how many apples were in the box to make the equation true. Remind students that they also learned to write equations using numbers. Can they figure out how to write this equation using numbers? \((\bigcirc - 2 = 4)\)

Draw several models for subtraction equations (like those in Questions 4 to 5 on AP Book 6.2 p. 71), and ask students to write the equations for them. Ask volunteers to present the answers on the board. Then give students the following equations and ask them to draw the corresponding models and solve the equations:

\[ \bigcirc - 6 = 9 \quad \bigcirc - 7 = 12 \quad \bigcirc - 5 = 3 \quad \bigcirc - 3 = 10 \]

**Equations with multiplication.** Tell students that they can also write equations for multiplication problems. Remind students that “2 \(\times\)” means that some quantity is taken two times.

\[ 2 \times \bigcirc \bigcirc \bigcirc = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \quad \text{and} \quad 2 \times \bigcirc = \bigcirc \bigcirc \]

Present the problem shown in the margin and ask students to draw the appropriate number of circles in the box.

Students should solve the problem by dividing the circles on the right side into two equal groups. Present more such problems and ask students to write and solve the corresponding numerical equations. Then show students how to write an equation for a word problem involving multiplication. Use this word problem:

Tony has four boxes of pears. Each box holds the same number of pears. He has 12 pears in total. How many pears are in each box?

Students might reason in the following way: We can represent the thing that we do not know (the unknown) by a square. Four times the unknown makes 12, and we have the equation \(4 \times \bigcirc = 12\).

**Exercises**

1. Write and solve the equation for the problem.
   
   a) Jenny used three eggs to bake muffins. Seven eggs remained in the carton. How many eggs were in the carton?
   
   b) Bob has nine pets. Three of them are snakes. All the rest are iguanas. How many iguanas does Bob have?

   **Answers:** a) \(\bigcirc - 3 = 7\), \(\bigcirc = 10\); b) \(3 + \bigcirc = 9\), \(\bigcirc = 6\)

2. Solve the equation.
   
   a) \(3 + \bigcirc = 8\)  
   
   b) \(3 \times \bigcirc = 15\)
   
   c) \(\bigcirc - 4 = 11\)  
   
   d) \(2 \times \bigcirc = 14\)

   **Answers:** a) 5, b) 5, c) 15, d) 7
3. Draw models to solve the problem.
   a) Aputik has 12 stamps. Four of them are Canadian. How many foreign stamps does she have?
   b) Lewis has 15 stamps. Five of them are French and the rest are German. How many German stamps does he have?
   
   **Answers:** a) 8, b) 10

**Bonus:** Solve the equations.
   a) \(243 + \square = 248\)  
   b) \(8 \times \square = 56\)  
   c) \(\square - 4 = 461\)  
   d) \(60 \times \square = 240\)
   
   **Answers:** a) 5, b) 7, c) 465, d) 4

**Recognizing true and false equations.** Write on the board:
   \(3 \times 5 = 6 + 7\)
   
   Point to the equation and **ASK:** What is the value of the left side? (15) The right side? (13) **SAY:** This equation is read as "three times five is the same number as six plus seven" which is false because 15 is not equal to 13.
   
   Write on the board:
   \[5 + 3 \quad 9 - 3 \quad 2 \times 4\]
   
   **ASK:** Which two numerical expressions have the same value? (the first and the third) **SAY:** Since they are equal, I can write an equation to show that. Write on the board:
   \[5 + 3 = 2 \times 4\]

**Exercises:** Find the missing number that makes all three expressions equal.
   \[\square + 6 \quad 4 \times \square \quad 10 - \square\]
   
   **Answer:** \(\square = 2\)

**Equations with variables.** Explain to students that a variable is a letter or symbol that represents a number. Write on the board:
   \[\square + 5 = 8\]
   
   Point to the box and **SAY:** This represents a number, so it is a variable, but I would like to show this variable with a letter instead. Erase the box and replace it with variable \(x\), as shown below:
   \[x + 5 = 8\]
   
   **ASK:** What number do you add to 5 to get 8? (3) **SAY:** So \(x = 3\) makes the equation true. Remind students that solving an equation means finding a number that makes the equation true. Below \(x + 5 = 8\), continue writing on the board:
   \[3 + 5 = 8\]
Exercises: Solve the equation.

a) \( x + 3 = 8 \)  

b) \( 9 + x = 11 \)

c) \( 13 = x + 3 \)  

d) \( x + 2 = 5 + 2 \)

e) \( x - 4 = 2 \)  

f) \( 7 - x = 2 \)

g) \( 21 = x \times 7 \)  

h) \( x \div 5 = 4 \)

i) \( 12 \div x = 2 \)  

j) \( 17 - x = 17 - 11 \)

Bonus

k) \( 6 \times 4 = 8 \times w \)  

l) \( (t \div 3) + 1 = 5 + 1 \)

Answers: a) \( x = 5 \), b) \( x = 2 \), c) \( x = 10 \), d) \( x = 5 \), e) \( x = 6 \), f) \( x = 5 \), g) \( x = 3 \), h) \( x = 20 \), i) \( x = 6 \), j) \( x = 11 \), Bonus: k) \( w = 3 \), l) \( t = 15 \)

Using equations with variables to solve word problems. Remind students that they can use a letter as a variable to write an equation the same way they used a box symbol as a variable. Write on the board:

Dory has 21 red and green balloons. 13 of them are red. How many are green?

Ask a volunteer to use a box and numbers to write the equation on the board \( [x] + 13 = 21 \). Erase the box and replace it with the variable \( x \), as shown below:

\[ x + 13 = 21 \]

Ask a volunteer to solve the equation. \( (x = 8) \)

Exercises: Write an equation with a variable for the situation. Solve the equation.

a) There are 50 green and yellow papers in a box. 17 of them are yellow. How many are green?

b) Ansel saved $23 in March and $21 in April. How much did he save in March and April?

c) Shelly saved $31 in November and $67 in November and December together. How much did she save in December?

d) Raj has some stickers. Avril has twice as many stickers as Raj. Avril has 18 stickers. How many stickers does Raj have?

Answers: a) \( g + 17 = 50 \), \( g = 33 \); b) \( 23 + 21 = t \), \( t = 44 \); c) \( 31 + d = 67 \), \( d = 36 \); d) \( r \times 2 = 18 \), \( r = 9 \)

Solving two-step equations. Write on the board:

\[ x + 3 = 2 \times 5 \]
SAY: Sometimes you need to do one step to make the equation ready to solve. Point to the equation on the board and ASK: Which operation can we do first, addition or multiplication? (multiplication) What is $2 \times 5$? (10) Write on the board:

$$x + 3 = 10$$

SAY: Now this equation is easy to solve. ASK: What number do I add to 3 to get 10? (7) Write “$x = 7$” on the board below the equation.

**Exercises:** Solve the equation.

a) $x + 5 = 3 \times 4$   
b) $4 + x = 2 \times 5$   
c) $4 \times 6 = x + 5$

d) $x - 4 = 5 \times 3$   
e) $7 \times 4 = x - 2$   
f) $5 \times 6 = x - 9$

**Bonus**

g) $7 - w = 3 \times 2$   
h) $3 \times 5 = 17 - t$

**Answers:** a) $x = 7$, b) $x = 6$, c) $x = 19$, d) $x = 19$, e) $x = 30$, f) $x = 39$,

**Bonus:** g) $w = 1$, h) $t = 2$

**Extensions**

1. The same symbol in the equation means the same number. 
   Solve the equation.

   a) $\square + \square = 12$   
   b) $\lozenge + \lozenge + \lozenge = 9$

   c) $5 + \bigcirc + \bigcirc = 13$   
   d) $9 + \blacksquare + \blacksquare + \blacksquare = 15$

   **Answers:** a) 6, b) 3, c) 4, d) 2

2. Two birds laid the same number of eggs. Seven eggs hatched, and three did not. How many eggs did each bird lay?

   **Answer:** 5

3. Sixty baby alligators hatched in total from three alligator nests of the same size. We know that only half of the total number of eggs hatched. How many eggs were in each nest? (Hint: How many eggs were laid in total?)

   **Answer:** 40

4. Solve the equation.

   a) $19 = x + x + 5$   
   b) $x = 10 - x$

   c) $t \times t + 1 = 10$   
   d) $32 \div w = 10 - 2$

   **Bonus:** $x + 4 = 12 + 6 - 3$

   **Answers:** a) $x = 7$, b) $x = 5$, c) $t = 3$, d) $w = 4$, Bonus: $x = 11$
PA6-10 Modelling Equations

Pages 73–76

Goals

Students will use pictures and balances to model and solve equations.

PRIOR KNOWLEDGE REQUIRED

Can use variables to represent an unknown value
Can solve simple equations to find an unknown value
Is familiar with balances
Can substitute numbers for unknowns in an expression
Can check whether a number solves an equation

MATERIALS

paper bags
counters
apples
pan balance
ruler, masking tape, or string
connecting cubes

Mental math minute—number talk. Present this problem: Is the equation $61 \times 34 = 60 \times 35$ true? (no) The following strategies could arise:

- The right side is a multiple of 10, but the left side is not.
- The right side is a multiple of 5, but the left side is not.
- The left side is 34 more than $60 \times 34$, but the right side is 60 more than the same number.

Finding the unknown in a concrete model. Divide students into pairs and have them play the following game:

Step 1: Player 1 takes a small number of paper bags (at most 7) and puts an equal number of counters (at most 7) in each bag.

Step 2: Player 1 shows Player 2 the model and tells Player 2 the total number of counters placed in the bags. Example: If Player 1 places 2 counters in each of 5 bags, Player 1 tells Player 2 “I’ve placed 10 counters altogether.”

Step 3: Player 2 has to figure out (without looking in the bags) how many counters are in each bag. Players then switch roles.

After playing the game several times, discuss strategies for finding the number of counters in each bag. Player 2 can use division or skip counting to figure out the number of counters in each bag. After two or three turns, students can play a more advanced variation of the game: In Step 1, Player 1 puts some counters outside the bags too. Example: Player 1 places 2 counters in each of 5 bags and 3 counters outside the bags.

CURRICULUM REQUIREMENT

AB: required
BC: required
MB: required
ON: required

VOCABULARY

balance
equation
expression
sides (of an equation)
unknown
variable

VOCABULARY

balance
equation
expression
sides (of an equation)
unknown
variable
In Step 2, Player 1 tells Player 2 the total number of counters placed both in the bags and outside the bags. For the previous example, Player 1 would say “I’ve placed 13 counters altogether.” Step 3 is the same as above.

Students might discover the strategy of reducing problems. Reducing a problem means changing it into a problem you already know how to do. In this case, students know how to do problems in which no counters are left outside the bag. They can solve problems in which some counters are left outside the bag by removing those counters and subtracting that number from the total, thus “reducing” the problem to one they already know how to do. Example: “If there are 13 counters altogether and I see 3 counters outside the bags, there must be 10 counters in the bags. There are 5 bags, so there must be 2 in each bag.”

**Relating the model to algebra.** ASK: What is the unknown you were looking for in the game? (the number of counters in each bag) Use the variable $x$ to represent the unknown. ASK: How can you get the number of counters in all the bags from $x$? (multiply $x$ by the number of bags) How can you get the total number of counters from $x$? ($x$ × the number of bags + the number of counters outside the bags)

Draw on the board a model like the one shown in the margin and build the corresponding algebraic expression step by step:

- There are $x$ counters in each bag.
- There are $4x$ (or $4 \times x$) counters in all the bags because there are 4 bags.
- There are $4x + 3$ counters altogether.

Then tell students that there are 35 counters altogether. ASK: What equation can we write? ($4x + 3 = 35$) Challenge students to determine the number of counters in each bag by solving the equation using the strategy of reducing problems. ($x = 8$ because $35 - 3$ is equal to 32 and there are 4 bags, which means there must be 8 in each bag) Explain to students that they can use this type of model only for addition equations like $4x + 3 = 35$. They can’t use the same model for subtraction equations like $4x - 3 = 25$.

**Drawing a model to verify an answer.** Explain to students that after writing an equation they need to solve the equation and find the unknown. SAY: We used the reducing strategy above, but there is another strategy we can use to solve equations: the organized approach. Write on the board the equation $2x + 5 = 11$. SAY: For the equation $2x + 5 = 11$, start by drawing 2 containers and 5 counters, then add 1 counter to each container until you have 11 counters altogether. Show this on the board as follows:

Students can solve equations using this organized approach. The model they draw in the process will verify their answer—they can see the answer in the last picture. Students should draw their models so that the items inside the bags (e.g., counters, apples) are clearly visible.
Review pan balances. Show students a pan balance. Place the same number of identical (or nearly identical) apples on both pans, and show that the pans balance. Remind students that when the pans, or scales, are balanced, it means there is the same number of apples on each pan.

Removing the same number of apples from each pan keeps them balanced. Place some apples in a paper bag and place it on one pan, then add some apples beside the bag. Place the same total number of apples on the other pan. ASK: Are the pans balanced? (yes) What does this mean? (the same number of apples is on each pan) Take one apple off each pan. ASK: Are the pans still balanced? Repeat with two apples. Continue removing the same number of apples from each pan until one pan has only the bag with apples on it. ASK: Are the pans balanced? Can you tell how many apples are in the bag? Show students the contents of the bag to check their answer. Repeat the exercise with a different number of apples in the bag.

Writing equations from a balance model. Divide a desk in half (use a ruler, masking tape, or string) and explain that the parts on either side of the line will be the pans. Ask students to imagine that the pans are balanced. Remind them that they can use variables to represent numbers they do not know. Place a paper bag with 5 apples and 2 more apples on one side of the line, and place 7 apples on the other side of the line. Ask students to write an expression for the number of apples on the side with the paper bag. Explain that an equation is like a pair of balanced pans (or scales), and the equal sign shows that the number of apples on each pan is the same. Remind students that the parts of the equation on either side of the equal sign are called the sides of the equation. Each pan of the balance becomes a side in the equation and the "balance" on the desk becomes \( x + 2 = 7 \).

Create more such models and have students write the equation for each one. After you have done a few models that follow this pattern, start placing the bag on different sides of the line, so that students have to write the expressions with unknown numbers on different sides of the equation.

Solving addition equations using the balance model. Return to the model that corresponds to the equation \( x + 2 = 7 \). ASK: What do you need to do to find out how many apples are in the bag? (remove two apples from each side) Invite a volunteer to remove the apples, then have students write both the old equation and the new one \( x = 5 \), one below the other. Repeat with a few different examples.

ASK: What mathematical operation describes taking the apples away? (subtraction) Write the subtraction for each of the equations above vertically. Example:

\[
\begin{align*}
x + 2 &= 7 \\
-2 &= -2
\end{align*}
\]

ASK: How many apples are left on the right side of the equation? (5) What letter did we use to represent the number of apples in the bag? (x) Remind students that we write this as “\( x = 5 \)” Repeat with the other equations used earlier.
Solving addition equations without using the balance model. Present a few equations without a corresponding model. Have students signal how many apples need to be subtracted from both sides of the equation, and then write the vertical subtraction for both sides.

**Exercises**

a) \( x + 5 = 9 \)  
   b) \( n + 17 = 23 \)  
   c) \( 14 + n = 17 \)  
   d) \( p + 15 = 21 \)

Students who have trouble deciding how many apples to subtract without drawing a model can complete the following problems.

**Exercises:** Write the missing number.

a) \( x + 15 \)  
   b) \( x + 55 \)  
   c) \( x + 91 \)  
   d) \( x + 38 \)

**Answers:** a) 15, b) 55, c) 91, d) \( x \)

Finally, give students a few equations and have them work through the whole process of subtracting the same number from both sides to find the unknown number.

**Exercises**

a) \( x + 5 = 14 \)  
   b) \( x + 9 = 21 \)  
   c) \( 2 + x = 35 \)  
   d) \( x + 28 = 54 \)

**Selected solution**

a) \( x + 5 = 14 \)

\[
\begin{array}{c}
\quad \downarrow 5 \\
\quad \downarrow 9 \\
\quad \downarrow x = \quad 9
\end{array}
\]

**Bonus:** The scales in the margin are balanced. Each bag has the same number of apples in it. How many apples are in the bag? Hint: You can cross out whole bags too!

**Answer:** 2 apples in each bag

Solving multiplication equations given by a model. Divide a desk into two parts and place 3 bags (with 4 cubes in each) on one side of the line and 12 separate cubes on the other side. Tell students that the "pans" are balanced. ASK: What does this say about the number of cubes on both pans? (they are equal) How many cubes are on the pan without the bags? (12) How many cubes are in the bags in total? (12) How many cubes are in each bag? (4) How do you know? (divide 12 into 3 equal groups, \( 12 \div 3 = 4 \)) Invite a volunteer to group the 12 cubes into 3 equal groups to check the answer. Show students the contents of the bags to confirm the answer.

Repeat the exercise with 4 bags and 20 cubes, 5 bags and 10 cubes, 2 bags and 6 cubes. Students can signal the number of cubes in one bag each time.

Writing and solving equations from models. Remind students that the pans of the balance become the sides of an equation, and that the equal sign in the equation shows that the pans are balanced. If students have,
say, 3 bags with the same number of cubes in each, they write the total number of cubes in the bags as \(3 \times b\). Present a few equations in the form of a model, and have students write the corresponding equations using the letter \(b\) for the unknown number.

**Exercises**

a) Write the equation. Use \(b\) for the variable.

i) ![Model image]

b) Solve the equations in part a). Write the solution below the equation.

**Answers:** a) i) \(2 \times b = 12\), ii) \(12 = 4 \times b\)

**Selected answer:** b) i) \(2 \times b = 12\)

\[b = 6\]

**Using division to find the missing factor.** ASK: Which mathematical operation did you use to write an equation for the model? (multiplication) Which mathematical operation did you use to find the number of cubes in each bag? (division) Have students show the division in the models in Exercise a), part i) above by circling equal groups of dots (they should circle two equal groups of dots). ASK: What number do you divide by? (the number of bags)

**Multiplying and dividing by the same number does not change the starting number.** Have students calculate the following expressions:

a) \((5 \times 2) ÷ 2\) b) \((3 \times 2) ÷ 2\) c) \((8 \times 2) ÷ 2\)

d) \((5 \times 4) ÷ 4\) e) \((9 \times 3) ÷ 3\) f) \((10 \times 6) ÷ 6\)

SAY: Look at the expressions you calculated. ASK: How are they all the same? (you start with a number, then multiply and divide by the same number) Did you get back to the same number you started with? (yes) Does it matter what number you started with? Does it matter what number you multiplied and divided by as long as it was the same number? (no)

Write on the board:

\[(\text{[ ]} \times 3) ÷ 3 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\]

ASK: What will we get when we perform the multiplication and the division? (the box) Repeat with expressions that include letters as variables, as in the following exercises.

**Exercises:** Evaluate the expression.

a) \((b \times 3) ÷ 3\) b) \((b \times 5) ÷ 5\)

c) \((b \times 6) ÷ 6\) d) \((b \times 10) ÷ 10\)

**Answers:** a) \(b\), b), b, c) \(b\), d) \(b\)
Solving equations by dividing both sides by the same number. Write on the board the questions below and have students signal the number they would divide the product by to get back to \( b \).

\[
\begin{align*}
(b \times 7) \div \underline{} &= b \\
(b \times 2) \div \underline{} &= b \\
(b \times 4) \div \underline{} &= b \\
(b \times 8) \div \underline{} &= b \\
(b \times 12) \div \underline{} &= b \\
(b \times 9) \div \underline{} &= b
\end{align*}
\]

If available, show students a pan balance with 3 bags of 5 apples on one pan and 15 apples on the other pan. Invite a volunteer to write the equation for the balance on the board \((3 \times 5 = 15)\). ASK: How many apples are in one bag? (5) Have a volunteer make three groups of 5 apples on the side without the bags. Point out that there are three equal groups of apples on both sides of the balance. Remove two of the bags from one side, and two of the groups from the other side. SAY: I have replaced three equal groups on each side with only one of these groups. ASK: What operation have I performed? (division by 3) Are the scales still balanced? (yes) Point out that when you perform the same operation on both sides of the balance, the scales remain balanced. ASK: What does that mean in terms of the equation? Write the equation that shows the division below the original equation:

\[
b \times 3 \div 3 = 15 \div 3
\]

Have students calculate the result on both sides \((b \text{ on the left side, 5 on the right side})\). Write on the board “\( b = 5 \)” (align the equal signs vertically). Demonstrate that the bags indeed contain 5 apples. Repeat with a few more examples.

**Exercises:** Solve the equation by dividing both sides by the same number.

- a) \( b \times 7 = 21 \)
- b) \( b \times 2 = 12 \)
- c) \( b \times 4 = 20 \)
- d) \( b \times 6 = 42 \)
- e) \( b \times 3 = 27 \)
- f) \( b \times 9 = 72 \)

**Bonus**

- g) \( 3 \times b = 270 \)
- h) \( 8 \times b = 4000 \)
- i) \( 7 \times b = 42000 \)
- j) \( 6 \times b = 720000 \)

**Selected solution**

- a) \( b \times 7 = 21 \)
  
  \[
b \times 7 \div 7 = 21 \div 7
  \]
  
  \[
b = 3
  \]

**Answers:** b) 6, c) 5, d) 7, e) 9, f) 8, Bonus: g) 90, h) 500, i) 6000, j) 120 000

**Solving two-step multiplication and addition equations using a balance model.** Write on the board:

\[
2 \times b + 3 = 11
\]

SAY: I would like to draw a balance model using bags and apples for this equation. ASK: How can I model the right side of the equation? (draw
11 apples) How can I model the left side of the equation? (draw two bags and three apples) Draw on the board:

SAY: To solve the equation, we need to find how many apples balance one bag. Then we will know what the variable equals. Let’s start by removing apples. ASK: How many apples can I take from each pan and keep both pans in balance? (3) Cross out three circles from each side, as shown below:

Ask a volunteer to write the equation for the new situation and solve it on the board. (see sample solution below)

\[ b \times 2 = 8 \]
\[ (b \times 2) \div 2 = 8 \div 2 \]
\[ b = 4 \]

Invite a different volunteer to substitute \( b = 4 \) into the original equation to check the answer. SAY: There are 4 apples in each bag.

**Exercises:** Solve the two-step equation by modelling with a balance.

a) \( 3 \times b + 1 = 7 \)  
   b) \( 12 = 2 \times b + 4 \)  
   c) \( 4 \times b + 1 = 13 \)

**Answers:** a) \( b = 2 \), b) \( b = 4 \), c) \( b = 3 \)

**Extensions**

1. Clara threw 3 darts and scored 12 points (see margin). The dart in the centre is worth twice as much as each dart in the outer ring. How much is each dart worth?

   **Answer:** Let \( n \) be the value of a dart in the outer ring. The dart in the centre is worth twice as much as the dart in the ring, so it’s worth \( 2 \times n \) or \( 2n \). The total value of the darts is \( n + n + 2n = 12 \), or \( 4n = 12 \), so \( n \) is 3. A dart in the outer ring is worth 3; a dart in the centre is worth 6.

2. Find as many solutions as you can for the following equation where \( x \) and \( y \) are whole numbers: \( 10x + 4y = 74 \). Hint: Use the organized approach. The value of \( x \) must be between 0 and 7, so try each in turn.

   **Answer:** \( (x, y) = (1, 16), (3, 11), (5, 6), (7, 1) \). See the table in the margin.

3. All three angles in a triangle are equal. What are the measures of the angles?

   **Solution:** \( a + a + a = 180^\circ \), \( 3 \times a = 180^\circ \), so \( a = 60^\circ \).
Goals
Students will “undo” more than one operation to get back to where they started when starting with a number, and will “undo” one operation when starting with a variable.

PRIOR KNOWLEDGE REQUIRED
Knows that addition and subtraction undo each other
Knows that multiplication and division undo each other
Can substitute numbers for variables
Knows that the equal sign means expressions on both sides of it are equal

Mental math minute. Present the equation: \( x + 5 = 5 + 29 \). SAY: Let’s compare the sides of the equation. Cover the 29 with your hand and SAY: On the left side of the equal sign we have some number and 5 added to it. On the other side, we have something added to 5. The sides are equal, so it looks like the addends are simply switched. Remove your hand and ASK: What is the missing number? (29) Write “29 + 5 = 5 + 29” underneath the first equation and ASK: Is this true? (yes) SAY: The answer is \( x = 29 \).

Exercises: Solve the equation by comparing the sides.

a) \( 7 + 8 = x + 8 \)  
b) \( 13 + 24 = 24 + x \)

c) \( 2 + 3 + 4 + 5 = 3 + 4 + 5 + x \)  
d) \( 18 + 5 + 44 = 5 + 44 + x \)

Answers: a) \( x = 7 \), b) \( x = 13 \), c) \( x = 2 \), d) \( x = 18 \)

Undoing one operation.

ACTIVITY (Essential)
Have students pair up. Player 1 chooses a secret number. Player 2 gives Player 1 an operation—either multiplication or addition—to do to the secret number (for example: multiply by 3, add 7). Player 1 carries out the operation and tells Player 2 the answer. Player 2 has to find the secret number using a different operation. Players switch roles and repeat.

Discuss how students “undid” operations in the activity to find—or get back to—the numbers their partners started with. For example, ASK: How did you get back to the original number if your partner multiplied the number by 3? (divided the answer by 3) How did you get back to the original number if you told your partner to add 7? (subtracted 7 from the answer)
**Exercises:** How could you undo the operation to get back to the number you started with?

a) add 5  

b) subtract 13  

c) divide by 7  

d) multiply by 11  

**Answers:** a) subtract 5, b) add 13, c) multiply by 7, d) divide by 11

**Undoing operations done to variables.** SAY: You undo operations done to variables in the same way you undo operations done to numbers.

**Exercises:** Write the operation and the number that makes the equation true.

a) \( x - 7 = x \)  

b) \( t \div 3 = t \)  

c) \( n \times 4 = n \)  

d) \( x + 5 = x \)  

**Answers:** a) \( +7 \), b) \( \times 3 \), c) \( \div 4 \), d) \( -5 \)

Tell students that to add 3 to \( x \), you would write \( x + 3 \). ASK: What would you write to add 3 to \( x \)? \( (x + 3) \) Continue with other operations, as in Question 4 on AP Book 6.2 p. 77.

**Exercises:** Show the result of the operation.

a) Multiply \( x \) by 4  

b) Subtract 2 from \( x \)  

c) Divide \( x \) by 8  

d) Add \( x \) to 9  

**Answers:** a) \( 4x \), b) \( x - 2 \), c) \( x \div 8 \), d) \( 9 + x \)

**Preserving equality.** Give students some equations as in the next exercises that involve one operation and one variable.

**Exercises:** Describe what was done to the variable \( x \). How do you undo that operation to find \( x \)?

a) \( 3x = 12 \)  

b) \( x + 3 = 12 \)  

c) \( x \div 3 = 5 \)  

d) \( x - 3 = 5 \)  

**Answers**  

a) \( x \) was multiplied by 3, so divide 12 by 3 to get \( x: x = 12 \div 3 = 4 \)  
b) \( 3 \) was added to \( x \), so subtract 3 from 12 to get \( x: x = 12 - 3 = 9 \)  
c) \( x \) was divided by 3 to get 5, so multiply 5 by 3 to get \( x: x = 5 \times 3 = 15 \)  
d) \( 3 \) was subtracted from \( x \) to get 5, so add 3 to 5 to get \( x: x = 5 + 3 = 8 \)

When students are comfortable undoing operations to find \( x \), show them how to relate this to the algebra. Write on the board:

\[
\begin{align*}
3x &= 12 \\
3x \div 3 &= 12 \div 3 \\
x &= 4
\end{align*}
\]

Point to the first two lines and say that if \( 3x \) and 12 are the same number, then dividing them both by 3 will still result in a true equation. Explain that you divided both sides by 3 because you wanted to undo the multiplication by 3 to find the value of \( x \) and solve the equation. Demonstrate how to check the answer by substituting 4 for \( x \) in \( 3x = 12 \) and evaluating. SAY: Since both sides are equal, \( x = 4 \) is the solution.
Treating variables as numbers. Tell students that to subtract 3 from $x$, you would write $x - 3$. ASK: How can you get just $x$? (add 3) What would you write to subtract $x$ from 3? ($3 - x$) SAY: To get the original number, 3, you can add $x$ to $3 - x$ because $x$ is really a number, and you can treat it the same way you treat a number.

Exercises: Write the operation and variable that make the equation true.

a) $5 - x = 5$  
   b) $4 = 4 - t$
   c) $9 - z = 9$  
   d) $8 = 8 - x$

Answers: a) $+ x$, b) $+ t$, c) $+ z$, d) $+ x$

NOTE: Extensions 1 and 2 are required in order to cover the British Columbia curriculum. Extension 3 is required in order to cover the Ontario curriculum.

Extensions

1. Preserving equality in equations with decimals. Explain to students that they can use the same method to solve equations that involve decimals.

   Example:
   
   $x + 3.5 = 5.1$
   $x + 3.5 - 3.5 = 5.1 - 3.5$
   $x = 1.6$

   Solve the equation.

   a) $x - 2.3 = 1.2$  
   b) $4.1 + x = 5.6$  
   c) $13x = 3.9$

   Bonus: $0.6x = 2.7$

   Answers: a) 3.5, b) 1.5, c) 0.3, Bonus: 4.5


   Example:
   
   $2x + 5 = 11$
   $2x + 5 - 5 = 11 - 5$
   $2x = 6$
   $2x ÷ 2 = 6 ÷ 2$
   $x = 3$

   Solve the equation.

   a) $3x - 1 = 11$  
   b) $4 + 2x = 14$
   c) $29 = 4x + 5$  
   d) $51 = 6x - 3$

   Answers: a) 4, b) 5, c) 6, d) 9
3. Explain to students that sometimes they can solve equations with more than one variable. Write on the board:

\[ x + y = 8 \]

Point to the equation and SAY: This equation has two variables, \( x \) and \( y \). ASK: What would \( y \) be if you know that \( x \) is 5? (3) Write on the board:

\[ x + 3 + y = 9 \]

ASK: What would \( x \) be if you know that \( y \) is 4? Replace \( y \) with 4 and ask a volunteer to solve the equation. (see solution below)

\[ x + 3 + 4 = 9, \text{ so } x + 7 = 9, \text{ so } x = 2 \]

Solve the equation with the given value.

a) \( p + 1 + q = 14 \) and \( q \) is 4  
   b) \( n + 1 = 15 \) and \( n + 1 + s = 19 \)

\textbf{Answers:} a) \( p \) is 9, b) \( s \) is 4

4. One angle of a parallelogram is 50°. What are the measures of the other angles?

\textbf{Answer:} 130°, 50°, and 130°
Goals

Students will solve one-step equations using logic and the concept of operations.

PRIOR KNOWLEDGE REQUIRED

Knows that addition and subtraction undo each other
Knows that multiplication and division undo each other

MATERIALS

BLM Filling a Blank Multiplication Chart (p. T-2)

Mental math minute. Give students BLM Filling a Blank Multiplication Chart. Have them fill in as much of the chart as they can in three minutes, using the strategies on the BLM as needed.

Review adding and subtracting on a number line. Remind students how they can use a number line to add (by moving to the right) and subtract (by moving to the left). For example, to show $4 + 3 = 7$ and $5 - 3 = 2$ they can draw the pictures shown below:

ASK: What number added to 6 gives 9? (3) Have a volunteer write the equation and draw the number line on the board:

SAY: You can get 9 from 6 by counting up 3 units, so by counting down 3 units from 9 you can get 6. Draw the number line shown below on the board as you make this point:

ASK: Should I use this method to solve $237 + x = 314$? (no) Why not? (the number line and also the number of steps would be huge) How can I find the answer, the value of $x$, without counting and without using a number line? (by subtracting the smaller number from the larger number) SAY: When we solve equations by using the concepts of addition and subtraction instead of a number line, we are using logic.
Solving equations using logic.

**Exercises:** Use logic to solve the equation.

a) \(38 = 8 + x\)  
   b) \(21 = x + 12\)  
   c) \(x + 22 = 33\)  
   d) \(7x = 56\)

e) \(48 = 2x\)  
   f) \(14 = 2x\)  
   g) \(x \div 3 = 6\)  
   **Bonus:** \(5 = \frac{x}{2}\)

**Solutions:** a) \(x = 38 - 8 = 30\), b) \(x = 21 - 12 = 9\), c) \(x = 33 - 22 = 11\),  
   d) \(x = 56 \div 7 = 8\), e) \(x = 48 \div 2 = 24\), f) \(x = 14 \div 2 = 7\), g) \(x = 6 \times 3 = 18\),  
   **Bonus:** \(x = 5 \times 2 = 10\)

**Extensions**

1. Share the problem:

   Rani has 10 balloons and would like to share some of them with Cam. She wants to keep 7 balloons for herself. How many balloons should Rani give Cam?

   Rani needs to solve the equation \(10 - x = 7\). Tell students that they can solve this type of equation using logic too! Tell students that Rani thinks she can take her 7 balloons first and then give the rest to Cam, in which case she needs to solve an easier equation, \(10 - 7 = x\). The answer to that equation is \(x = 3\), so Rani can give 3 balloons to Cam. Emphasize that the two equations, \(10 - x = 7\) and \(10 - 3 = x\), have the same answer.

   Solve the equation.

   a) \(12 - x = 4\)  
   b) \(9 = 23 - x\)

   c) \(33 = 149 - x\)  
   d) \(1310 - x = 459\)

   **Solutions:** a) \(12 - 4 = x, x = 8\); b) \(x = 23 - 9, x = 14\); c) \(x = 149 - 33, x = 116\); d) \(1310 - 459 = x, x = 851\)

2. Solve the two-step equation using logic.

   a) \(\frac{x}{3} + 1 = 2\)  
   b) \(4 + \frac{x}{5} = 7\)

   c) \(5 + \frac{x}{2} = 17\)  
   **Bonus:** \(5 - \frac{x}{2} = 2\)

   **Answers:** a) \(x = 3\), b) \(x = 15\), c) \(x = 24\), **Bonus:** \(x = 6\)
Goals
Students will solve equations by guessing values for $x$, checking by substitution, and then revising their answers.

PRIOR KNOWLEDGE REQUIRED
Can read tables
Can substitute numbers for variables in equations
Can check whether a number solves an equation

MATERIALS
paper bags
counters
calculator

Introduce using a table to solve equations. Write on the board:

There are $x$ counters in each bag.
There are $4x$ counters in all the bags because there are 4 bags.
There are $4x + 3$ counters altogether.

Make a model using 4 bags and 31 counters. Put 3 counters outside the bags and put 1 counter at a time in each bag until all 31 are used. ASK: What equation can we write for this model? ($4x + 3 = 31$)

Show students how to solve $4x + 3 = 31$ by using a table and substituting different values of $x$ in sequence ($x = 1, x = 2, x = 3$, and so on) into the expression on the left side of the equation (see margin).

Point out the connection between the table and the method using counters: In the table, each time we increase the value of $x$ by 1, it is as though we are adding a counter to each bag (4 counters for 4 bags) and checking how many counters are used in total. We stop when we see that all 31 counters are used.

Introduce the guess-and-check method to solve equations. Show the equation $7h + 2 = 44$. Tell students that you are going to solve this equation by guessing and checking. Start by guessing $h = 5$. ASK: If $h = 5$, what is $7h + 2$? (37) Should $h$ be higher or lower to make $7h + 2 = 44$? (higher)

What would your next guess be? (6) If $h = 6$, what is $7h + 2$? (44) Is the equation true? (yes) Draw the table at left on the board and SAY: $h = 6$ makes the equation $7h + 2 = 44$ true, so $h = 6$ is the answer.

Compare the two methods of solving equations. ASK: Which method requires less work? Which method is quicker? (the guess-and-check method is quicker) Which method is more like looking up a word in the dictionary using alphabetical order? (guess and check) Which method is more like looking up a word in the dictionary without knowing or using alphabetical order?
order? (using the table) Have students explain the connection. (In a dictionary, the words at the top of each page you turn to tell you whether to look to the right or to the left; they tell you if you have gone too far or not far enough.)

**Exercises**

a) Replace \( x \) with 5 and say whether 5 is too high or too low.

\[
\begin{array}{ccc}
   & 4x + 1 & \text{Answer} \\
 5 & 21 & \text{too low} \\
   & 5x + 3 & \text{Answer} \\
 5 & 28 & \text{too high} \\
   & 2x + 4 & \text{Answer} \\
 5 & 14 & \text{too low}
\end{array}
\]

b) Use the answers in part a) to try a higher or lower number and solve each equation.

**Answers:** a) i) 21, too low; ii) 28, too high; iii) 14, too low; b) i) \( x = 6 \) works; ii) \( x = 4 \) works; iii) \( x = 6 \) works

**Using the guess-and-check strategy to find a mystery number.** Write on the board:

\( N \times N \times N \) is 343. What number is \( N \)?

SAY: Let’s start by checking the numbers in order. A table is a good way to do this. Draw on the board:

\[
\begin{array}{c|c|c}
   N   & N \times N \times N \\
   1   & 1 \times 1 \times 1 = 1 \\
   2   & 2 \times 2 \times 2 = 4 \times 2 = 8 \\
   3   & \\
   4   & \\
   5   & 
\end{array}
\]

Have volunteers continue the chart. (3 \( \times 3 \times 3 = 9 \times 3 = 27 \), 4 \( \times 4 \times 4 = 16 \times 4 = 64 \), 5 \( \times 5 \times 5 = 25 \times 5 = 125 \)) ASK: Are we getting closer to the answer? (yes) SAY: So we can continue as we’re doing.

**Exercises:** Continue the chart on the board. Stop when you get the answer 343. What is \( N \)?

**Answer:** \( N \) is 7

**Using a calculator to guess and check.** Write on the board:

\[
\text{If } N \times N \times N = 46\,656, \text{ what is } N?
\]

ASK: Would continuing the chart be a good strategy for this question? (no) SAY: The answers are getting closer to the answer, but not much closer. Instead of trying 1, 2, 3, and so on, maybe we should start with 10, 20, 30, and so on.
Exercises

a) Complete the chart up to 50.

<table>
<thead>
<tr>
<th>N</th>
<th>(N \times N \times N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>20</td>
<td>8000</td>
</tr>
<tr>
<td>30</td>
<td>(_)</td>
</tr>
<tr>
<td>40</td>
<td>(_)</td>
</tr>
<tr>
<td>50</td>
<td>(_)</td>
</tr>
</tbody>
</table>

b) What two tens is \(N\) between? How do you know?

Answers: a) 1000, 8000, 27 000, 64 000, 125 000; b) \(N\) is between 30 and 40 because \(N \times N \times N\) is between 27 000 and 64 000

SAY: Now we know that \(N\) is between 30 and 40. Write on the board:

\[
30 \times 30 \times 30 = 27 000 \\
N \times N \times N = 46 656 \\
40 \times 40 \times 40 = 64 000
\]

ASK: Do you think that \(N\) is a lot closer to 30 or to 40, or do you think \(N\) is about in the middle? (about in the middle) Why? (46 656 is about in the middle between 27 000 and 64 000) SAY: Let’s try 35. Write on the board:

\[
35 \times 35 \times 35 = __
\]

Have a volunteer do the calculation on a calculator and fill in the answer. (42 875) ASK: Is 35 the answer, too low, or too high? (too low) What should we try next? (36) Write on the board:

\[
36 \times 36 \times 36 = __
\]

Again, have a volunteer do the calculation on a calculator and fill in the answer (46 656) ASK: Is 36 the answer, too low, or too high? (36 is the answer) Write on the board:

So \(N = 36\)

Exercises: Find \(N\) so that \(N \times N \times N\) is the given number.

a) 103 823 b) 28 094 464

Answers: a) 47, b) 304

Extensions

1. How many digits does the solution to \(3x + 5 = 8000\) have? Explain.
   Hint: One-digit numbers are between 1 and 9, two-digit numbers are between 10 and 99, and so on.

Solution: To determine the number of digits in the solution, we need to determine the first power of 10 (10, 100, 1000, etc.) that is greater than
the solution. We can substitute increasing powers of 10 for the variable until the answer is larger than 8000:

\[
\begin{align*}
3(10) + 5 &= 35 \\
3(100) + 5 &= 305 \\
3(1000) + 5 &= 3005 \\
3(10000) + 5 &= 30005
\end{align*}
\]

So \(x\) is between 1000 and 10000, which means that it has four digits. (Indeed, \(x = 2665\).)

2. How many solutions can you find to \(2x + 1 = 4y - 1\) if \(x\) and \(y\) are whole numbers?

**Sample solution:** Find \(2x + 1\) for various values of \(x\):

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x + 1)</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

Now find \(4y - 1\) for various values of \(y\):

<table>
<thead>
<tr>
<th>(y)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4y - 1)</td>
<td>3</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
</tr>
</tbody>
</table>

Look for numbers that are the same in the second rows:

\[
\begin{align*}
2x + 1 &= 3 = 4y - 1 \text{ when } x = 1 \text{ and } y = 1 \\
2x + 1 &= 7 = 4y - 1 \text{ when } x = 3 \text{ and } y = 2 \\
2x + 1 &= 11 = 4y - 1 \text{ when } x = 5 \text{ and } y = 3
\end{align*}
\]

Students might continue the pattern to find more solutions \((x = 7 \text{ and } y = 4 \text{ is the next one})\).

3. I am a number. Multiply me by 7. Then round to the nearest ten. The result is 330. What number am I?

**Answer:** 47
PA6-14   Word Problems—Addition and Subtraction Equations
Pages 82–83

Goals
Students will solve one- and two-step word problems involving differences or totals using addition and subtraction equations.

PRIOR KNOWLEDGE REQUIRED
Can write an equation for finding the total or the difference from the parts given a one-step word problem
Knows that the variable in an equation represents an unknown
Can solve addition and subtraction equations

MATERIALS
BLM Word Problem Cards (p. O-55)

Mental math minute. Present the equation \( x + 30 = 5 + 29 \). SAY: Let’s compare the sides of the equation. Cover the 5 with your hand and SAY: On the left side of the equal sign we have some number and 30 added to it. On the other side we have something else with 29 added to it. The sides are equal, so it looks like we moved 1 from one addend to the other. ASK: One addend went down from 30 to 29, so did the other addend go up or down? (up) If the addend increased by 1 and became 5 (remove your hand from the 5), what was it? (4) Write “4 + 30 = 5 + 29” underneath the first equation and ASK: Is this true? (yes) SAY: The answer is \( x = 4 \).

Exercises: Solve the equation by comparing sides.

a) \( 9 + 5 = x + 6 \)  
b) \( x + 33 = 9 + 32 \)
c) \( 19 + 5 = x + 6 \)  
d) \( 29 + 5 = x + 6 \)

Answers: a) \( x = 8 \), b) \( x = 8 \), c) \( x = 18 \), d) \( x = 28 \)

Organizing data. Explain that when a word problem is long, it is convenient to write out the data in point form. Write the following problem on the board, then write the data in point form:

Nazim spent 25 minutes doing his math homework. He spent 15 minutes more on his science project than on math homework. How much time did Nazim spend on his science project?

25 minutes on math homework
15 minutes more on science project than on math homework
\( x \) minutes on science project

ASK: Is it easier to write an equation using the original problem or the data in point form? (the data in point form) Where did I get the last part (\( x \) minutes on science project)? (this sentence answers the question in the problem)
Exercise: Write out the data in point form.

a) Rona has 25 marbles. Seven of them are red. How many are not red?

b) There are 7 rats and 9 hamsters in a store. How many rats and hamsters altogether are in the store?

c) There are 25 vehicles in a parking lot. There are 7 fewer vans than cars in the lot. How many vans are in the lot?

Selected sample answer: b) 7 rats, 9 hamsters, how many altogether

Review totals and differences. Draw the picture in the margin on the board. Remind students that there are two things they can find given these two numbers: the difference between the two numbers and the total. Ask volunteers to show both in the model and to find what each is equal to (the difference is 4, and the total is 10). Review writing the equation for the total and the difference. Repeat with this situation: larger number 8, smaller number $x$, difference 3, total 13.

Difference or total? Create a table with headings “Parts,” “Total,” and “Difference” on the board. Look at the situations below as a class and have students identify which piece of data belongs in which column. In parts f) and g), students need to decide which piece of data is the unknown ($x$).

Exercises: Which piece of data belongs in which column?

a) $x$ spoons, 10 forks, 22 forks and spoons altogether

b) 5 cars, $x$ buses, 21 cars and buses in a parking lot

c) There are 6 bananas. There are $x$ kiwis. There are 10 fewer bananas than kiwis.

d) A cat weighs 7 kg. A dog weighs 3 kg less than the cat. The dog weighs $x$ kg.

e) Jayden paid $12 for a hat. He paid $15 for a pair of mitts. He paid $x$ for the mitts and the hat.

f) Mary studied math for 20 minutes. Math and reading took 45 minutes altogether. How long did she read for?

g) A salad recipe calls for 3 onions and 5 more tomatoes than onions. How many tomatoes are needed?

ACTIVITY (Essential)

Give students cards from BLM Word Problem Cards. Have them sort the cards according to the problems (the cards belonging to the same problem have the same picture). Then ask them to write an answer sentence, with $x$ in place of the answer, below the question. For example, “How much does Hero weigh?” should have the sentence “Hero weighs $x$ kilograms.” written below it. Have students place the cards in the table they created during the previous exercise.
Writing and solving an equation for a word problem. Have students write the equation for each situation or problem above, including the problems from BLM Word Problem Cards. Work together through the first two. Then solve the first two equations together and have students solve the equations for the rest of the problems individually.

(a) $x + 10 = 22$, $x = 12$; b) $5 + x = 21$, $x = 16$; c) $x - 10 = 6$, $x = 16$; d) $12 - 3 = x$, $x = 9$; e) $12 + 15 = x$, $x = 27$; f) $20 + x = 45$, $x = 25$; g) $3 + 5 = x$, $x = 8$; BLM: $5 + 7 = x$, $x = 12$; $16 - 12 = x$, $x = 4$; $12 + 16 = x$, $x = 28$; $55 - 20 = x$, $x = 35$; $16 + 9 = x$, $x = 25$)

Organizing data, writing an equation, and solving it. Work through the first two exercises below as a class, then have students work individually.

Exercises: Write an equation and solve it.

a) Ken bought 12 books and 7 magazines. How many books and magazines did he buy altogether?

b) A book costs $10 and a poster is $4 cheaper than the book. How much does the poster cost?

c) A pet store sells parrots and canaries. There are 27 canaries in the store. There are 12 fewer parrots than canaries. How many parrots are in the store?

d) Monique read 12 pages on Sunday. She read 7 pages more on Sunday than on Monday. How many pages did she read on Monday?

e) A cake recipe calls for 5 cups of berries. Jose has 3 cups of raspberries and some blueberries. How many cups of blueberries will he need for the cake?

Bonus: The population of Canada in 1867 was 3 463 000. In 2016, the population of Canada was 35 151 728.

i) How many years passed between the two dates?

ii) How much did the population grow in that time?

Answers: a) 19, b) $6$, c) 15, d) 5, e) 2, Bonus: i) 149, ii) 31 688 728

Two-step problems: identifying the data needed to answer the question in the second step. Present the problem below.

Sheila bought 8 hockey cards and 10 baseball cards. She gave away 3 cards. How many cards does she have left?

Explain that many word problems are like this one. You cannot answer the main question (How many cards does she have left?) without first answering another question (How many cards did she buy altogether?). Part of solving this type of problem (two-step) is finding which question to ask and answer before approaching the main question.
Present some two-step problems and have students tell which question they need to ask and answer before they can answer the question of the problem itself. In each case, ask students to explain how they know which question to ask.

**Exercises:** Write an equation and solve it.

- **a)** There are 29 students in the class. Fifteen of them have siblings. How many more students have siblings than have no siblings?
- **b)** Mr. B paid $22 for a book and a CD. The CD cost $8. How much more expensive was the book than the CD?
- **c)** Brandi bought a book for $12 and a magazine for $6. She paid $20. How much change did she get?

**Answers:**
- **a)** How many students have no siblings?,
- **b)** How much did the book cost?,
- **c)** How much did Brandi pay for the book and the magazine together? OR How much money is left after she paid for the book?

**Solving two-step problems.** Point out that it is sometimes easier to solve one part of the problem with an equation, and the other without an equation. For example, in part a) above, it is easy to answer the question “How many students have no siblings?” using an equation:

\[
\begin{align*}
29 \text{ students} \\
15 \text{ have siblings} \\
x \text{ have no siblings} \\
\text{Equation: } 15 + x = 29 \\
\text{so } x = 29 - 15, x = 14
\end{align*}
\]

However, to tell how many more students have siblings than have no siblings, you do not have to use an equation. It is easy to see that there is 1 more student with siblings in the class: you already know 15 students have siblings and 14 students don’t have siblings.

Solve the problems in the previous exercises above together as a class.
- **a)** $1, **b)** $6, **c)** $2) Students can use a method of their choice to solve each of the parts. Encourage multiple solutions (using equations for one or more steps or not using equations at all). Then present a few more two-step word problems and have students work on them independently.

**Exercises:** Write an equation and solve it.

- **a)** There are 32 students in a class. 14 of them are playing soccer and the rest are playing baseball. How many more students are playing baseball than are playing soccer?
- **b)** There are 29 students in a class. 7 of them don’t wear eyeglasses. How many more students wear eyeglasses than do not wear eyeglasses?
c) Chen baked 24 oatmeal cookies and 32 chocolate chip cookies. He brought 42 cookies to a bake sale. How many cookies did he leave at home?

d) Marie got $100 for her birthday. She paid $39 for a construction toy and $12 for beads. How much money does she have left?

**Bonus:** Esteban bought a book for $9 and two magazines for $7 each. He paid with $20 and $10. How much change did he get?

**Answers:** a) 4, b) 15, c) 14, d) $49, Bonus: $7

**Extensions**

1. Jada found data about the population of some provinces in 2016. Estimate the answer to each question by rounding, then calculate the exact answer.

<table>
<thead>
<tr>
<th>Province</th>
<th>Population in 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Brunswick</td>
<td>747 101</td>
</tr>
<tr>
<td>Nova Scotia</td>
<td>923 598</td>
</tr>
<tr>
<td>Saskatchewan</td>
<td>1 098 352</td>
</tr>
<tr>
<td>Prince Edward Island</td>
<td>142 907</td>
</tr>
</tbody>
</table>

a) How many more people lived in Nova Scotia than in New Brunswick?

b) How many fewer people lived in Prince Edward Island than in Nova Scotia?

c) How many more people lived in New Brunswick than in Prince Edward Island?

**Bonus:** Did more people live in Nova Scotia and Prince Edward Island altogether than in Saskatchewan?

**Answers:** a) 176 497, b) 780 691, c) 604 194, Bonus: no

2. Orbital speed is how fast a planet travels on its orbit around the sun. The table shows the orbital speed of some planets in our solar system.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Orbital Speed (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>107 208</td>
</tr>
<tr>
<td>Mars</td>
<td>86 677</td>
</tr>
<tr>
<td>Mercury</td>
<td>172 332</td>
</tr>
<tr>
<td>Saturn</td>
<td>34 884</td>
</tr>
<tr>
<td>Venus</td>
<td>126 072</td>
</tr>
</tbody>
</table>

a) How much faster than Mars does Earth travel around the sun?

b) How much slower than Mercury does Saturn travel around the sun?
Bonus: The table shows the distance of some planets from the sun.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Distance from the Sun (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>149 600 000</td>
</tr>
<tr>
<td>Mars</td>
<td>227 900 000</td>
</tr>
<tr>
<td>Mercury</td>
<td>57 900 000</td>
</tr>
<tr>
<td>Saturn</td>
<td>1 427 000 000</td>
</tr>
<tr>
<td>Venus</td>
<td>108 200 000</td>
</tr>
</tbody>
</table>

Do you see a relationship between orbital speed and distance from the sun?

Answers: a) 20 531 km/h; b) 137 448 km/h; Bonus: yes, planets that are closer to the sun have greater orbital speed
Goals
Students will solve one-step multiplication word problems using equations.
Students will solve word problems with “times as many”.

PRIOR KNOWLEDGE REQUIRED
Can solve a one-step word problem requiring addition or subtraction
Understands the expression “times as many”
Can identify the parts and total in a problem

Mental math minute. This mental math minute combines the skills of the previous two mental math minutes.

Exercises: Solve the equation by comparing sides.

\[
\begin{align*}
a) \quad 79 - 53 &= x - 53 \\
b) \quad 489 + 5 &= 5 + x \\
c) \quad 62 + 43 &= 63 + x \\
d) \quad 8 + 15 &= 15 + x \\
e) \quad 16 + 7 &= 17 + x
\end{align*}
\]

Answers: a) \(x = 79\), b) \(x = 489\), c) \(x = 42\), d) \(x = 8\), e) \(x = 6\)

Review solving multiplication equations. Write on the board a few multiplication equations, such as \(3 \times w = 27\) and \(w \times 5 = 35\). Remind students that they can rewrite the equations as division equations so that \(w\) is by itself. (see margin) Remind students how to write the solution. Students can also use multiplication facts to solve some equations.

Scale factor. Present a situation: Shelly is three times as old as Edmond.
ASK: Who is older, Shelly or Edmond? (Shelly) How many times? (3 times)
Write the equation \(S = 3 \times E\) on the board. Remind students that the number that tells us how many times larger or smaller one part is than the other is called the scale factor.

Review the connection between sets and multiplication. SAY: Remember, we can use multiplication to find the total number of objects in equal sets. For example, 5 people can sit in each car. There are 3 cars. ASK: How many people can sit in the 3 cars altogether? (15) How do you know? (3 \(\times 5 = 15\) Emphasize that the total is always the product of the number of objects in a set and the number of sets.

Writing an equation for a story with an unknown number. Present a few situations and have students identify which number in the situation is the total, which number is the number of objects in a set, and which is the number of sets.

Exercises: Identify the total, the number of objects in a set, and the number of sets in the situation.

\[
\begin{align*}
a) \quad & \text{There are 8 people in each van. There are } w \text{ vans.} \\
& \text{There are 24 people altogether.}
\end{align*}
\]
b) There are \( w \) cookies on each plate. There are 12 plates.
There are 48 cookies altogether.

c) There are 4 markers in each pack. There are 11 packs.
There are \( w \) markers altogether.

d) There are 5 erasers in each pack. Ms. A bought \( w \) packs.
Ms. A bought 35 erasers altogether.

**Bonus:** An octopus has 8 arms. There are \( x \) octopuses.
There are 240 arms altogether.

**Answers:**
a) total: 24, objects in each set: 8, sets: \( w \);
b) total: 48, objects in each set: \( w \), sets: 12;
c) total: \( w \), objects in each set: 4, sets: 11;
d) total: 35, objects in each set: 5, sets: \( w \), Bonus: total: 240, objects in each set: 8, sets: \( x \)

**ASK:** Which number is the largest: the number of sets, the number of objects in each set, or the total? (total) Remind students that since the total is the largest number, they can use it to write the equation for the problem: they need to multiply the other two numbers (the number of sets and the number of objects in one set) to get the total. Write an equation for part a) from the previous exercises as a class. The number of objects in one set is 8, the number of sets is \( w \), and the total is 24, so the equation is \( 8 \times w = 24 \).

**ASK:** What is the scale factor in this equation? (8)

Ask students to write an equation for the rest of the previous exercises. Then have them solve the equations to find the unknown numbers.

(b) \( 12 \times w = 48 \), \( w = 4 \);
c) \( 4 \times 11 = w \), \( w = 44 \);
d) \( 5 \times w = 35 \), \( w = 7 \);
Bonus: \( 8 \times x = 240 \), \( x = 30 \)

**Writing an equation for a problem with sets.** Remind students that the first step in solving a word problem is identifying the information, or data, they are given, including the unknown. In the problems students have been solving so far, there are always three pieces of information. For example, present this problem:

There are 5 cookies on each plate. There are 45 cookies altogether.
How many plates are there?

Have students identify the three pieces of information in this word problem. (5 cookies on each plate, \( w \) plates, 45 cookies altogether) Then ask students to write and solve an equation for the total number of cookies. \( 5 \times w = 45 \)
Keep the data and the solution on the board for future reference.

**Exercises:** Write the data, then write and solve a multiplication equation.

a) There are 12 cookies divided equally among 2 plates. How many cookies are on each plate?

b) There are 5 cages with mice in a pet store. There are 50 mice altogether. How many mice are in each cage?

c) Rani packed 42 books into boxes. She packed 6 books into each box. How many boxes did she use?
Bonus: A train has 12 cars. It can carry 2,400 people. How many people
fit into each train car?

Answers: a) \(2 \times w = 12, w = 6\); b) \(5 \times w = 50, w = 10\);
c) \(6 \times w = 42, w = 7\); Bonus: \(12 \times w = 2,400, w = 200\)

Solving word problems with “times as many” using equations.
Explain that problems with “times as many” can also be solved using
a multiplication equation. Present the situation below and ask students
to think of the equation it produces.

21 cats
\(w\) dogs
3 times as many cats as dogs

PROMPT: What is the larger number, the number of cats or the number
of dogs? (the number of cats) The larger number is three times the smaller
number, so \(21 = 3 \times w\). Repeat with other situations, gradually increasing
the difficulty. Factors that make a situation or problem more difficult include
bigger numbers, and a longer or more complicated story (more words).
Explain to students that they have to determine what is given, the larger
or the smaller part. SAY: If the smaller part is given, then you multiply the
smaller part by the scale factor to find the larger part, but if the larger part
is given, you need to make it equal to the smaller part multiplied by the
scale factor. Then you divide the larger part by the scale factor to solve
the equation.

Exercises: Write a multiplication equation for the problem and then solve it.

a) \(w\) mice, 6 rats, three times as many mice as rats
b) 3 blue marbles, \(w\) green marbles, five times as many green marbles
   as blue marbles
c) 8 boys, \(w\) girls, twice as many boys as girls
d) Mario earned $42 babysitting. Jenna earned \(w\) dollars mowing lawns.
   Mario earned three times as much as Jenna.
e) Jeremy hiked 9 km on Monday. He hiked \(w\) km on Tuesday.
   He hiked three times as far on Monday as on Tuesday.
f) A recipe calls for 2 cups of oatmeal and three times as much flour.
   How much flour is needed?
g) Anna’s dog weighs twice as much as her cat. The dog weighs 12 kg.
   How much does the cat weigh?

Answers: a) \(w = 6 \times 3, w = 18\); b) \(w = 3 \times 5, w = 15\);
c) \(8 = 2 \times w, w = 4\); d) \(42 = 3 \times w, w = 14\); e) \(9 = w \times 3, w = 3\);
f) \(2 \times 3 = w, w = 6\); g) \(w \times 2 = 12, w = 6\)
If students have trouble with problems that involve units of measurements (distance, weight, etc.), point out that they can treat units, such as kilometres, the same way they treat objects, such as marbles. Three times as many as 4 marbles is 12 marbles, and three times as far as 4 km is 12 km. ASK: If Jennifer is three times as old as Kyle, and Kyle is 4, how old is Jennifer? (12) If a table is three times as heavy as a chair, and the chair weighs 4 kg, how heavy is the table? (12 kg)

Present the word problems and have students go through the whole process to solve them: write the information (data), write and solve the equation, and write the answer as a sentence or statement.

**Exercises:** Solve the problem.

a) The Bead Club makes and sells bracelets for charity. They sold 12 bracelets. There are five times as many bracelets left over as were sold. How many bracelets are left?

b) Mahmoud read 18 books for Battle of the Books. Mahmoud read three times as many books as Joanna. How many books did Joanna read for Battle of the Books?

c) Alex is 8 years old. Angela is three times as old as Alex. How old is Angela?

d) Ali is 8 years old. Ali is four times as old as Ben. How old is Ben?

e) A book costs $12. The book is twice as expensive as a magazine. How much does the magazine cost?

f) A binder costs $9. A book costs three times as much as the binder. How much does the book cost?

g) A pair of gloves costs $12. The pair of gloves is three times as expensive as a pair of socks. How much do the socks cost?

h) A puzzle costs $12. A construction toy costs three times as much as the puzzle. How much does the construction toy cost?

i) Anne spent twice as much time reading as she spent on math. She spent 20 minutes reading. How much time did she spend on math?

j) Eric spent three times as much time reading as he spent on French. He spent 10 minutes on French. How much time did he spend reading?

k) Brenda lives twice as far from school as Jeremy. Jeremy lives 3 blocks from school. How far from school does Brenda live?

l) Matt walks 8 blocks to school. The walk to school is twice as long as the walk to the library. How far is the library?

m) A spaniel weighs 20 kg. A Rottweiler is three times as heavy. How heavy is the Rottweiler?
**Bonus:** There are about 400 endangered Sumatran tigers left in the wild.

i) There are about six times as many Bengal tigers left in the wild. How many Bengal tigers are left in the wild?

ii) There are about eight times as many Sumatran tigers as Malayan tigers left in the wild. How many Malayan tigers are left in the wild?

**Answers:**
a) 60 bracelets are left over, b) Joanna read 6 books, c) Angela is 24 years old, d) Ben is 2 years old, e) The magazine costs $6, f) The book costs $27, g) The socks cost $4, h) The construction toy costs $36, i) Anne spent 10 minutes on math, j) Eric spent 30 minutes reading, k) Brenda lives 6 blocks from school, l) The library is 4 blocks away, m) The Rottweiler weighs 60 kg, Bonus: i) About 2400 Bengal tigers are left in the wild, ii) There are about 50 Malayan tigers left in the wild.

**Extensions**

1. A mall has two parking lots. There are 20 equal rows of 35 parking spots in the eastern lot and 25 equal rows of 30 parking spots in the western lot.
   
a) How many cars can park in each lot? Which lot has more parking spots?
   
b) How many cars can park at the mall in total?

   **Answers:**
a) eastern lot: 700, western lot: 750, the western lot has more parking spots; b) 1450

2. A person wants to donate $200 000 to 50 charities. If each charity gets the same amount, how much money will each charity get?

   **Answer:** $4000

3. Eight people can ride in a van. Six times as many people can ride on a regular bus. A double-decker bus can hold ten times as many people as a van. How many people can ride in a van, a bus, and a double-decker bus altogether?

   **Answer:** 136
PA6-16 Graphing Sequences
Pages 86–88

Goals
Students will graph sequences and determine if the graph shows an increasing or decreasing sequence.

PRIOR KNOWLEDGE REQUIRED
Can write a sequence as a set of ordered pairs
Can plot points in the first quadrant of a coordinate grid

Mental math minute—number string.
String 1: Solve for $x$ by comparing sides: $x + 17 = 17 + 3$, $x + x = x + 3$, $2x = x + 3$, $2x + x = x + x + 3$, $3x = 2x + 3$, $100x = 99x + 3$.
$(3, 3, 3, 3, 3, 3)$

Present the strategy by identifying the parts that are the same and the parts that are different, then analyzing the parts that are different, but should still be equal. Note that the first equation in String 2 is the same as the last equation in String 1.

String 2: Solve for $x$ by comparing sides: $100x = 99x + 3$, $500x = 499x + 10$, $500x = 498x + 10$, $500x = 495x + 10$.
$(3, 10, 5, 2)$

Review coordinate grids. Draw a coordinate grid on the board. Invite volunteers to locate points by their coordinates and to give the coordinates of various points on the grid. Remind them that the first number in the coordinate pair is the $x$-coordinate, or the distance along the $x$-axis, and the second number is the $y$-coordinate, or the distance along the $y$-axis.

Writing a sequence as a set of ordered pairs. SAY: You can think of the sequence as a set of ordered pairs in the form (Term Number, Term). For example, consider the sequence 1, 3, 5, 7, 9. Write on the board:

First term

$1, 3, 5, 7, 9$

Second term

SAY: You can write the term numbers and the term values in a sequence table. Draw on the board:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
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<tr>
<td>2</td>
<td>3</td>
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<td>3</td>
<td>5</td>
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<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>
SAY: The first term is 1, so the ordered pair for the first term is (1, 1).
The second term is 3, so the ordered pair for the second term is (2, 3).
Continue writing on the board:

1, 3, 5, 7, 9 → (1, 1), (2, 3), (3, 5), (4, 7), (5, 9)

Leave the sequence and the ordered pairs on the board for later reference.

**Exercises:** Write the sequence as a set of ordered pairs.

a) 4, 6, 8, 10, 12  b) 13, 10, 7, 4, 1

Answers: a) (1, 4), (2, 6), (3, 8), (4, 10), (5, 12); b) (1, 13), (2, 10), (3, 7), (4, 4), (5, 1); c) (1, 0), (2, 1), (3, 4), (4, 9), (5, 16); d) (1, 4), (2, 2), (3, 4), (4, 2), (5, 4)

**Graphing sequences.** Explain to students that to graph a sequence, they need to change the sequence to a set of ordered pairs—in other words, give the term number (x-coordinate) and then the term value (y-coordinate)—and then plot the ordered pairs on a graph. Draw on the board the graph for the sequence 1, 3, 5, 7, 9 using the ordered pairs you created earlier, as shown in the margin.

Remind students that sequences in which numbers get larger are called “increasing,” and sequences in which numbers get smaller are called “decreasing.” SAY: The graph shows an increasing sequence because as the term number increases, the term value increases as well. Increasing graphs start from bottom-left and extend to top-right.

**Exercise:** Plot the sequences from the previous exercises on a graph and determine if the graph shows an increasing or decreasing sequence.

Answers: a) increasing, b) decreasing, c) increasing, d) neither

**Selected graph:** b)
Graphing sequences using pattern rules. Explain to students that to graph a sequence using a pattern rule, they can find the terms using the rule to make a list of ordered pairs and graph the ordered pairs. Write on the board:

Add 3 to the term number.

Ask a volunteer to find the first five terms for the rule and write them on the board. (4, 5, 6, 7, 8) Ask students to change the sequence into a set of ordered pairs and then graph the sequence in their notebooks as shown below:

(1, 4), (2, 5), (3, 6), (4, 7), (5, 8)

ASK: Is the graph increasing or decreasing? (increasing)

Exercises: Plot the sequence from the pattern rule using the first five terms. Determine if the graph is increasing or decreasing.

a) Double the term number.

b) Add 2 to the term number.

c) Triple the term number and subtract 3.

d) Subtract the term number from 10.

Selected answers: a) increasing, b) increasing, c) increasing, d) decreasing

Selected graph: d)
Extensions

1. The sequence of numbers gives a set of ordered pairs for points on a graph that can be joined by a straight line. The first and the third terms are given. Find the fourth term by drawing the graph and extending the line that joins the first and third points.

   a) 5, __, 11, __  
   b) 2, __, 10, __  
   c) 10, __, 4, __

   **Selected answers:** a) 14, b) 14, c) 1

   **Selected graph:**

```
  10 8 6 4 2 0
  1 2 3 4 5
```

2. Is 2020 in the sequence? If so, which term is it?

   a) 1, 6, 11, 16, ...  
   b) 4, 10, 16, 22, ...  
   c) 1, 4, 7, 10, ...  
   d) 9, 18, 27, 36, ...  
   e) 1, 3, 5, 7, ...  
   f) 11, 18, 25, 32, ...

   **Answers:** a) no; b) yes, 337th term; c) yes, 674th term; d) no; e) no;  
   f) yes, 288th term
Making Sequences From Graphs

**Goals**

Students will create a table of values and a sequence from a graph.

Students will compare graphs represented in different ways.

**PRIOR KNOWLEDGE REQUIRED**

- Can write a sequence as a set of ordered pairs
- Can identify and plot points on coordinate grids
- Can make a table to record ordered pairs

**MATERIALS**

- dice in different colours

**Plotting and identifying ordered pairs on a grid.** Remind students that they can consider ordered pairs as (Term Number, Term) and use them to show a sequence. Remind them that the first number in the coordinate pair is the number of the column and the second number is the number of the row. Review coordinate grids by having students do the following activity.

**ACTIVITY (Essential)**

Students play in pairs. Each pair will need two dice in different colours (for example, blue and red) and a coordinate grid. Player 1 rolls the dice so that Player 2 does not see the result. Player 1 marks a point on the coordinate grid, where the coordinates of the point are given by the dice: the blue die gives the first number and the red die gives the second number. Player 2 writes the coordinates of the point.

**Exercises**

a) Write a list of ordered pairs for the graph.

b) What is the value of the second term?

c) What is the value of the third term?

d) What term number has term value equal to 8?

e) Does the graph show an increasing or decreasing sequence?
Answers: a) (1, 2), (2, 4), (3, 6), (4, 8), (5, 10); b) 4; c) 6; d) 4; e) increasing

Leave the exercise on the board for later use.

Writing a table using ordered pairs. SAY: You can write a set of ordered pairs in a table. To do this you can make a table with two columns, one for the term numbers and the other for the term values. Draw on the board:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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</table>

Ask a volunteer to fill the table using the ordered pairs from the previous exercise. (see completed table in the margin)

Point to the table and ASK: What is the gap between the numbers in the first column? (1) What is the gap between the numbers in the second column? (2) Point to the graph from the previous exercise and explain that the gap between the term numbers shows the horizontal distance between two consecutive points and the gap between the term values shows the vertical distance between two consecutive points. SAY: To get the next point on a graph from the previous point you go one unit to the right and 2 units upwards because the horizontal gap is 1 and the vertical gap is 2.

Exercises
1. Three patterns are shown.

   a) List the ordered pairs for each graph.
   b) Draw a table for each graph.
   c) Find the gap between the term values in each table.
   d) Continue the patterns the same way. Which pattern reaches a term with a value of 12 first?
Answers
a) A: (1, 5), (2, 6), (3, 7); B: (1, 0), (2, 3), (3, 6); C: (1, 2), (2, 4), (3, 6)
b) A:

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
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B:

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<td>3</td>
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C:

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<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

c) A: 1, B: 3, C: 2
d) B

2. Two sequences are given.

Sequence A: Start with 4 and add 5 to get the next term.

Sequence B: 21, 23, 25, ...

a) Which sequence will reach a term with a value of 39 first? How do you know?

b) Draw both sequences on one grid to show your answer in part a) is correct.

Selected answer: a) Sequence A reaches 39 first. Sequence A: 4, 9, 14, 19, 24, 29, 34, 39, so 39 is the 8th term. Sequence B: 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, so 39 is the 10th term.

Extensions

1. The sequence is made by repeatedly multiplying by 2 and adding a number. Find the added number, then complete the sequence. Hint: Use guess and check or write an equation with x as the added number.

   a) 3, __, 45

   Bonus: 3, __, __, 45

   Answers: a) added number: 11, sequence: 3, 17, 45; Bonus: added number: 3, sequence: 3, 9, 21, 45

2. The sequence was made by repeatedly multiplying the previous term by a number and then adding 2. Find the multiplied number, then complete the sequence. Hint: Use guess and check.

   a) 2, __, 26

   b) 3, __, 58

   Answers: a) multiplied number: 3, sequence: 2, 8, 26; b) multiplied number: 4, sequence: 3, 14, 58
Goals

Students will learn about dependent and independent variables and interpret linear graphs.

Prior Knowledge Required

Can identify and plot points on coordinate grids
Can read and create graphs
Can use the input to find the output

Interpreting graphs. Draw on the board:

Explain to students that this graph represents the cost of parking a car in a lot. ASK: How much will you pay for entering the parking lot, even before you park the car there? ($3.00) How much does it cost to park for 1 hour? ($5) For 2 hours? ($7) For 3 hours? ($9) Does the hourly charge for the first 3 hours vary? (no) How do you know? (it's $2 for each hour) As a challenge, ask students to give a rule that allows you to calculate the cost of parking. (For example, “$3 for parking plus $2 for each hour up to 3 hours, up to a maximum of 3 hours (or $9).” Or, in a more mathematical way, “If you park for less than 3 hours, multiply the time by $2 and add $3. If you park for 3 hours or more, the price is $9.”)

Explain that a nearby parking lot charges $2 per hour. ASK: What is the mathematical rule for the cost of parking there? (multiply the number of hours by 2) Ask students to write ordered pairs for time and cost for the second parking lot, and ask them to plot the graph for the second rule. ASK: Which parking lot will be cheaper to stay in for 2 hours? (the second one because it’s $4) For 5 hours? (the first one because the cost doesn’t increase after 3 hours) The table in the margin summarizes the answers students should get from their graphs.

For a challenge, students might plot the cost of a third parking lot that charges $1 for any time less than 1 hour and $4 per hour after that.
Exercise: Which of the three parking lots is best for parking times of 1, 2, and 5 hours?

Bonus: Joe and Zoe left their cars at parking lots 2 and 3, respectively. They parked for the same amount of time but Zoe paid $5 more than Joe. How long did they park for?

Answers: Lot 3 is best for 1 hour, Lot 2 is best for 2 hours, Lot 1 is best for 5 hours; Bonus: 4 hours

Dependent and independent variables. Ask a volunteer to write the equation for the second parking lot ($2 per hour). Tell the volunteer to use the variables $c$ for cost and $t$ for time. ($c = 2t$) ASK: What variable in this equation is the input? ($t$) SAY: You can park in the parking lot for any number of hours you want, but the cost depends on the duration of park-time, so $c$ changes depending on $t$ and because of this, in the equation $c = 2t$, we call $t$ the independent variable and $c$ the dependent variable or output.

Let students analyze another graph that shows the motion of two objects over time. Use the situation below, or create your own.

Exercises: A boat leaves port at 9:00 a.m. and travels at a steady speed. Some time later, a man jumps into the water and starts swimming in the same direction as the boat. The graph in the margin shows the motion of the boat and the man over time. Use the graph to answer the questions.

a) How many minutes passed between the time the boat left port and the time the man jumped into the water? When did the man jump into the water?

b) How far from the port was the man at 9:15 a.m.?

c) When did the boat overtake the man?

d) How far can the boat travel in 1 hour?

e) How long does it take the man to swim 1 kilometre?

f) i) How far did the man swim before he was taken aboard?  
ii) How can you see from the graph when the man was picked up by the boat?

g) Did the boat continue travelling when it met the man or did it stay at the same place for some time? How do you know?

h) The boat docked at another port 9 km from the starting point. At what time did this happen?

Answers: a) 15 minutes, 9:15 a.m.; b) 3 km; c) 10 a.m.; d) 6 km; e) 15 minutes; f) i) 3 km, ii) The two lines intersect where the boat and the man meet. There is only one line on the graph after that time, which means the boat and the man are travelling together; g) The graph is horizontal, which means the distance didn’t change over time; h) 9 a.m. + 105 minutes = 10:45 a.m.
Extension

A movie theatre is showing a new movie. The manager estimates that if the ticket price is $8, then 400 tickets will be sold per day, and if the ticket price is $10, then 300 tickets will be sold per day.

a) Determine the independent and the dependent variables.
b) Fill in the table, then plot the ordered pairs from the table of values on a graph.

<table>
<thead>
<tr>
<th>Ticket Price ($)</th>
<th>Ticket Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>400</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
</tr>
</tbody>
</table>

c) How many tickets per day will the theatre sell if the ticket price is $9?
d) Use the graph to find the price the theatre should charge to sell 500 tickets per day.

Answers

a) ticket price is the independent variable and number of tickets sold is the dependent variable
b) |

<table>
<thead>
<tr>
<th>Ticket Price ($)</th>
<th>Ticket Sold</th>
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<tbody>
<tr>
<td>8</td>
<td>400</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
</tr>
</tbody>
</table>

c) 350 tickets
d) $6
Goals
Students will produce formulas for linear relations given in numerical and geometric forms.

PRIOR KNOWLEDGE REQUIRED
Can create and extend a table for a pattern
Can identify a sequence that varies directly with the term number
Can produce a formula for a sequence that varies directly with the term number
Is familiar with variables
Can identify increasing and decreasing sequences
Can find the gaps in a sequence

Mental math minute. Present the equations and answers in the exercises below one at a time. Have students compare the sides of the equations and signal which solution is correct by raising the corresponding number of fingers.

Exercises: Which answer is correct?

a) $100x = 99x + 5$
   1. $x = 100$
   2. $x = 99$
   3. $x = 5$
   4. $x = 1$

b) $13 + 4 = 4 + x$
   1. $x = 13$
   2. $x = 4$
   3. $x = 17$
   4. $x = 9$

c) $13 + 4 = x + 13$
   1. $x = 13$
   2. $x = 4$
   3. $x = 17$
   4. $x = 9$

d) $x + 13 = 3 + 12$
   1. $x = 13$
   2. $x = 12$
   3. $x = 3$
   4. $x = 2$

e) $1 + 3 + 4 = x + 3$
   1. $x = 8$
   2. $x = 3$
   3. $x = 5$
   4. $x = 4$

Answers: a) 3, b) 1, c) 2, d) 4, e) 3

Introduce formulas and step-by-step rules. Write on the board:

$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \leftarrow \text{Term number}$

$3, 5, 7, 9, 11 \quad \leftarrow \text{Term value}$

Ask students to write the rule for this sequence in two different ways. ("Start at 3 and add 2" or "Multiply the term number by 2 and add 1") Write the sequence as a table of values with space to write gaps (see next page) and then write both versions of the rule on the board. Below the second version, write "Term number $\times 2 + 1 = 2n + 1."
Point to the table of values and the two rules, and ASK: How are these rules different? (one uses the term number and the other gives directions on how to get the next number from the previous one; one is based on a variable and the other is not)

Explain that the rule that uses a variable is called a formula; but the other rule tells you how to get the sequence starting from the first term, step by step. ASK: Which of the rules is easier to use to find the 100th term? (the formula) Why? (it is based on the term number so you can substitute 100 into the formula)

**Producing pattern rules from sequences.** Point to the sequence on the board. Demonstrate how to find the pattern rule for the sequence from the gaps by writing the gaps in the circles between the numbers in the sequence. (the gaps are always +2, so the rule is “Start at 3 and add 2 each time”)

**Exercises:** Find the pattern rule for the sequence.

a) 0, 4, 8, 12, 16  
   b) 13, 11, 9, 7, 5  
   c) 8, 11, 14, 17, 20  
   d) 11, 10, 9, 8, 7  
   e) 4, 7, 10, 13, 16  
   f) 0, 2, 4, 6, 8  
   g) 5, 3, 1, –1, –3  
   h) 8, 4, 0, –4, –8

**Bonus**

g) 5, 3, 1, –1, –3  
   h) 8, 4, 0, –4, –8

**Answers:** a) start at 0, add 4 each time; b) start at 13, subtract 2 each time; c) start at 8, add 3 each time; d) start at 11, subtract 1 each time; e) start at 4, add 3 each time; f) start at 0, add 2 each time; Bonus: g) start at 5, subtract 2 each time; h) start at 8, subtract 4 each time

**Formulas for sequences that do not vary directly with the term number.**
Write on the board:

3, 6, 9, 12

Ask a volunteer to make a table for the sequence, as shown below:
ASK: What is the gap in the term values? (3) How you can find the term value from the term number? (by multiplying the term number by 3)

SAY: You can consider the term number the input and the term value or term the output.

Draw on the board:

| a) 7, 10, 13, 16 |
| b) 5, 9, 13, 17 |
| c) 12, 19, 26, 33 |

Have students fill in the first and the third columns in the chart for each of the three sequences. Then have students find the gap between the terms, multiply the gap by the input, and write the product in the middle column of the table. Have students compare the numbers in the second and the third columns. ASK: How can you obtain the numbers in the third column from the numbers in the second column? (add the same number, for example add 4 for part a) The completed table for part a) is shown below:

<table>
<thead>
<tr>
<th>Input (n)</th>
<th>Input × Gap</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

Have students write both a general rule for the sequence (multiply the input by , then add ), and the formula ( ). (a) rule: multiply by 3 then add 4, formula: ; b) rule: multiply by 4, then add 1, formula: ; c) rule: multiply by 7, then add 5, formula: 

Next, present several patterns where the adjustment factor should be subtracted from the product of the term number and the gap.

**Exercises:** Find the general rule for the sequence. Then write the formula.

| a) 1, 4, 7, 10 | b) 3, 7, 11, 15 | c) 4, 11, 18, 25 |

**Answers:** a) rule: multiply by 3, then subtract 2, formula: ; b) rule: multiply by 4 then subtract 1, formula: ; c) rule: multiply by 7, then subtract 3, formula: 

Emphasize that this method only works when the gap between terms is a constant.

**Exercises:** Find the general rule and the formula for the sequence by making a table and finding the gap.

| a) 2, 7, 12, 17 | b) 21, 33, 45, 57 | c) 2, 23, 44, 65 |

**Answers:** a) term number × 5 – 3, 5n – 3; b) term number × 12 + 9, 12n + 9; c) term number × 21 – 19, 21n – 19
Draw on the board:

![Figure 1](image1)

![Figure 2](image2)

![Figure 3](image3)

Ask students to make a table for the number of blocks in each figure and then find a formula for the pattern. (see table below, formula: figure number × 3 – 2)

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

NOTE: The extensions provide applications of finding formulas for sequences.

Extensions

1. **Applications of pattern rules.** Simon builds towers by placing cubes that have a side length of 7 cm one on top of the other.

   a) Find the formula for the surface area of the tower if he uses ...

      i) 1 cube
      ii) 2 cubes
      iii) 3 cubes
      iv) n cubes

   b) What is the height and the surface area of a tower that is 30 cubes tall?

   c) Simon keeps adding cubes until the surface area for the tower in part b) is greater than 1 m². How many cubes are in the tower now?

   d) How tall is the tower from part c) in metres?

Answers

a) Each cube face has an area of 49 cm². So the surface areas are:

   i) 6 × 49 = 294 cm², ii) 10 × 49 = 490 cm², iii) 14 × 49 = 686 cm²

<table>
<thead>
<tr>
<th>Number of Cubes (n)</th>
<th>n × Gap</th>
<th>Surface Area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>196</td>
<td>294</td>
</tr>
<tr>
<td>2</td>
<td>392</td>
<td>490</td>
</tr>
<tr>
<td>3</td>
<td>588</td>
<td>686</td>
</tr>
</tbody>
</table>

Formula: 196n + 98

b) A tower that is 30 cubes tall has a height of 7 cm × 30 = 210 cm.

   Its surface area is 196 × 30 + 98 = 5978 cm².

   c) 1 m² = 100 cm × 100 cm, so the tower must have a surface area greater than 10 000 cm². 196n + 98 = 10 000 gives n = 50.51/98.
   So the tower will be 51 cubes high.

   d) 51 × 7 cm = 357 cm = 3.57 m
2. a) For the following figures, find the formulas for the perimeter and the area. Use your formulas to predict the perimeter and the area of Figure 15 in the sequence.

![Figures 1, 2, 3]

b) How many inner line segments does the 20th figure in this pattern have?

**Answers**

a) perimeter sequence: 12, 16, 20, …; perimeter formula: \(4n + 8\); perimeter of Figure 15 is 68; area sequence: 7, 11, 15, …; area formula: \(4n + 3\); area of Figure 15 is 63

b) inner line sequence: 8, 14, 20, …; inner line formula: \(6n + 2\); number of inner lines in Figure 20 is 122

3. Find the rule for the first table and then use it to find a rule for the second table.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
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<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
</tr>
</tbody>
</table>

**Answer:** The rule for the first table is “Multiply the first number by 3, then add 2.” The rule for the second table is “Subtract 2 from the first number, then divide by 3.” In the second rule and second table, we are just undoing the operations in the first rule and table, so we get back to where we started. In other words, we have to do the inverse operations in backwards order.

4. The table shows the area of rectangles based on their length.

<table>
<thead>
<tr>
<th>Length ((L))</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Write a formula for the perimeter of the rectangles.

**Solution:** Based on the formula for the area \((3 \times L)\) the width of the rectangle is 3, so the perimeter is \(6 + 2 \times L\) or \(2 \times (3 + L)\)
Goals
Students will find equations (formulas) for graphs.

PRIOR KNOWLEDGE REQUIRED
Can identify and plot points on coordinate grids
Can make a table to record ordered pairs
Can find rules for tables

Review coordinate grids. Draw a coordinate grid on the board, and invite volunteers to locate points by their coordinates and to give the coordinates of various points on the grid. Remind them that the first number in the coordinate pair is the number of the column and the second number is the number of the row.

Finding rules for linear graphs. Draw a line on the grid so that it passes through several grid vertices. (For example, draw a line that passes through the points (1, 4) and (5, 8).) Invite volunteers to mark points on the line and to write their coordinates on the board. ASK: What is the second coordinate of a point with first coordinate 11? (14) How can you find the answer to this question? (you can extend the line on the grid)

ASK: How would you find the second coordinate of a point with first coordinate 100? Suggest to students that they try to derive an equation or formula using a table, as they did earlier (see AP Book 6.2 pp. 93–95). Ask students how they could make a T-table from the graph. Students might suggest making a T-table with headings “Input” and “Output.” SAY: You can also use “first number” and “second number” instead of “Input” and “Output.” Mathematicians usually use \( x \) for Input and \( y \) for Output, so they can make tables with short headings \( x \) and \( y \). Invite a volunteer to make such a table. Let students practise making T-tables from several different graphs. They should mark points on the line, list their coordinates, and make a T-table with headings as above.

Exercises
1. On grid paper, make a T-table and graph the rule “Multiply by 2 and add 3.”

   Answer:

   ![Graph with T-table]

   \[
   \begin{array}{c|c}
   \text{Input} & \text{Output} \\
   \hline
   0 & 3 \\
   1 & 5 \\
   2 & 7 \\
   3 & 9 \\
   4 & 11 \\
   \end{array}
   \]
2. Mark four points on the line segment. Write a list of ordered pairs, complete the T-table, and write a rule that tells you how to calculate the output \((y)\) from the input \((x)\).

\[
\begin{array}{|c|c|}
\hline
\text{Input (x)} & \text{Output (y)} \\
\hline
0 & 0 \\
1 & 3 \\
2 & 6 \\
3 & 9 \\
\hline
\end{array}
\]

Answer: \begin{align*}
\text{Rule: } 3 \times \text{Input} &= \text{Output}
\end{align*}

**Extensions**

1. Wilson has $30. For every book he reads, his mother gives him $5.
   a) Create a T-table that shows the number of books as input and the amount of money Wilson has as output.
   b) Write a rule for the amount of money Wilson has after reading \(n\) books.
   c) How many books does Wilson need to read to have $50? $100?
   
   **Selected answers:** b) \(5n + 30\); c) 4, 14

2. Tell students that you want to add \(1 + 2 + 3 + 4\) and so on, all the way up to adding 100. ASK: How would you approach the problem? Would you just start adding and hope to finish quickly? Would you look for a pattern? Let students try the method of their choice. Then explain that the mathematician C.F. Gauss came up with an answer that avoided both methods. He noticed that you could write the problem in two rows of numbers to add, as shown below:

\[
\begin{align*}
1 & + 2 & + 3 & + \ldots & + 48 & + 49 & + 50 \\
+ 100 & + 99 & + 98 & + \ldots & + 53 & + 52 & + 51 \\
\hline
101 & + 101 & + 101 & + \ldots & + 101 & + 101 & + 101
\end{align*}
\]
Point out the pattern of numbers 1 to 50 from left to right and 51 to 100 from right to left, and then the additions lined up. ASK: How many 101s are in the sum? (50) How do you know? (there are 50 numbers from 1 to 50) What is the sum? (101 \times 50 = 5050)

Have students find the sum using the method they just learned.

a) \[1 + 2 + 3 + 4 + \ldots + 1000\]  

b) \[1 + 3 + 5 + \ldots + 99\]

c) \[2 + 4 + 6 + \ldots + 100\]

**Answers:** a) 500 500, b) 2500, c) 2550
Word Problem Cards

Fluffy the cat weighs 5 kg.  Hero the dog weighs 7 kg more than Fluffy the cat.  How much does Hero weigh?

There are 12 girls in Ms. A’s class.  There are 16 boys in Ms. A’s class.  How many more boys than girls are in Ms. A’s class?

There are 12 girls in Ms. A’s class.  There are 16 boys in Ms. A’s class.  How many boys and girls are in Ms. A’s class?

The lunch period is 55 minutes long.  Sandy spent 20 minutes eating.  Sandy went to the library for the rest of the period.  How much time did she spend in the library?

Rina earned $16 by tutoring.  Sean earned $9 more than Rina by babysitting.  How much money did Sean earn?
Unit 13  Measurement: Area

Introduction
This unit focuses on:

• developing formulas for areas of rectangles, parallelograms, triangles, and trapezoids;
• estimating and calculating area of polygons;
• investigating relationships between units of area; and
• solving problems related to area.

Meeting Your Curriculum

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<tr>
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Mental Math Minutes

The mental math minutes in this unit:

• concentrate on strategies to make multiplying and dividing decimals easier.

Proficiency in multiplying and dividing decimals will be very useful for students when they solve problems related to area.

Generic BLMs

The Generic BLM used in this unit is:

BLM 1 cm Grid Paper (p. T-1)

This BLM can be found in Section T.
Materials

In most lessons of this unit you will need a grid on the board. If you do not have a grid available, photocopy BLM 1 cm Grid Paper to a transparency and project it on the board. This allows you to erase shapes without erasing the grid.

Assessment

The lessons covered by a quiz or test are as follows:

<table>
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<th></th>
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<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tr>
<td>Quiz</td>
<td>ME6-8 to 10</td>
<td>ME6-8 to 10</td>
<td>ME6-8 to 10</td>
<td>ME6-8 to 10</td>
</tr>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>ME6-11 to 14</td>
<td>n/a</td>
<td>ME6-11 to 13</td>
</tr>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>ME6-15, 16</td>
<td>n/a</td>
<td>ME6-15, 16</td>
</tr>
<tr>
<td>Test</td>
<td>ME6-8 to 10</td>
<td>ME6-8 to 15</td>
<td>ME6-8 to 10</td>
<td>ME6-9, 11 to 13, 15, 16</td>
</tr>
</tbody>
</table>
Goals

Students will find the area of rectangles with whole-number sides and review units of area learned in previous grades.

PRIOR KNOWLEDGE REQUIRED

- Knows the relative size of units of length measurement within the metric system
- Can use multiplication to find the number of objects in an array
- Can multiply and divide mentally up to $10 \times 10$
- Can multiply two 2-digit numbers and decimals

MATERIALS

- metre stick
- 1 cm cube
- paper square with sides 1 m
- ruler
- grid paper or BLM 1 cm Grid Paper (p. T-1) or a geoboard (see Extension 1)

Mental math minute—number string.

String 1: $8 \times 0.5$, $4 \times 1$ ($4, 4$)

Present the strategy of halving one factor and doubling the other using number lines:

When jumps are twice as long, you need half as many jumps to get to the same spot.

String 2: $2 \times 3$, $4 \times 1.5$, $8 \times 0.75$ ($6, 6, 6$)
String 3: $6 \times 4.5$, $16 \times 2.25$, $7.5 \times 12$ ($27, 36, 90$)

Review counting squares in a rectangular array. Remind students that, when they have an array of rows and columns of the same length, they can use skip counting, repeated addition, or, more efficiently, multiplication to find the total number of squares. To find the number of squares in an array, such as the one in the margin, students usually multiply the number of squares along the length of the array (5) by the number of squares along the width of the array (4) for the total (20).
Introduce square centimetres as units of area. SAY: Squares that have sides of 1 cm each are called square centimetres. We write 1 cm\(^2\) for 1 square centimetre. When we cover a flat shape with squares of the same size so that there are no gaps or overlaps, the number of squares needed to cover the shape is called its area. A square centimetre is then a unit for measuring area.

Draw a rectangle and a square as shown in the margin. ASK: Which shape has a larger area? How do you know? (the square is covered by 4 squares and the rectangle is covered by 3 squares, so the square has a larger area)

Determining area of rectangles covered by squares. On the board, draw several rectangles and mark their sides at regular intervals, as shown in the margin, so that students will be able to extend the marks into squares. Ask volunteers to divide the rectangles into squares by using a metre stick to join the marks. Ask students to find the area of the rectangles.

Draw a rectangle on the board as shown in the margin. Tell students that the squares represent square centimetres since actual square centimetres are too small to draw on the board. Explain that you want to find the area of the rectangle, which means counting the total number of squares needed to cover the rectangle without gaps or overlaps. How can you do it without drawing all the squares? (multiply the number of rows by the number of columns, or multiply the number of squares along the length by the number of squares along the width) ASK: How many squares fit along the length, the longer side, of the rectangle? (6) How many squares fit along the width of the rectangle, the shorter side? (4) How many columns will there be? (6) How many squares will there be in each column? (4) What is the total number of squares? (6 \(\times\) 4 = 24 squares) What is the area of the rectangle? (24 cm\(^2\))

Emphasize that finding area is counting squares; the trick is to do it efficiently. SAY: In a rectangle, the squares are arranged in an array, so we multiply the number of columns by the number of rows.

Determining the formula for area of rectangles. Draw another rectangle and mark its sides as 5 cm and 3 cm. ASK: How many 1 cm squares will fit along the length of the rectangle? (5) How do you know? (the rectangle is 5 cm long) How many squares will fit along the width of the rectangle? (3) How many square centimetres are needed to cover the entire rectangle? (15) How do you know? (3 rows of 5 squares, or 5 columns of 3 squares, make an array of 15 squares) What is the area of the rectangle? (15 cm\(^2\))

Explain that, because area is measured in squares, we can think about any rectangle as being made of squares. ASK: Did we divide the rectangle into squares to find the answer? (no) How did we find the answer? (we multiplied length by width) Do you think this method will work to find the area of any rectangle? (yes) Summarize on the board:

\[
\text{Area of rectangle} = \text{length} \times \text{width}
\]
Finding areas of rectangles. Have students work in pairs. Ask each student to draw various rectangles, label the length and width of the rectangles in whole centimetres (not necessarily to scale), and then exchange notebooks. Then ask partners to find the area of the rectangles and check each other’s work.

Introduce different units. Explain that, just as length is measured in different units, so is area. Square centimetres are squares with length and width each of 1 cm. ASK: What other square units can you think of? Write the students’ suggestions on the board. Add units as necessary until you have at least the units shown below. Introduce the short forms of the units.

- square millimetre (mm²)
- square centimetre (cm²)
- square metre (m²)
- square kilometre (km²)

Show a centimetre cube and explain that each square face is exactly 1 cm². Show a paper square that is 1 m long, or draw a 1 m by 1 m square on the board to show the size of a square metre. Point out that 1 mm² is so tiny that it is hard to show it, and have students try to draw a square that is 1 mm long and wide using rulers. Explain that square kilometres are squares with sides of 1 km. SAY: Since a square has 4 sides, the distance around the square is 4 km, and since the average person takes 15 min to walk 1 km, it would take an average person a whole hour to walk around 1 km².

Ask students to think what areas would be measured in each of the four units. Record multiple suggestions on the board. (mm²: area of one side of a coin, pill or small button; cm²: area of a desk, floor tile, side of a box; m²: area of a room, backyard, soccer field; km²: area of a large plot of land, a country, surface of a large lake)

Finding area of rectangles in different units. Work through the following problem as a class. Ensure students pay attention to the numbers as well as the units in part b).

a) Calculate the area of each rectangle. Include the units.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>M</th>
<th>E</th>
<th>L</th>
<th>P</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3 km</td>
<td>8 mm</td>
<td>15 cm</td>
<td>13 cm</td>
<td>21 cm</td>
</tr>
<tr>
<td>Width</td>
<td>2 km</td>
<td>7 mm</td>
<td>6 cm</td>
<td>7 cm</td>
<td>15 cm</td>
</tr>
</tbody>
</table>

(area: 6 km², 56 mm², 90 cm², 91 cm², 315 m²)

b) Jake thinks that Rectangle A fits inside Rectangle M. Is he correct? Explain. (yes, because 21 m and 15 m are both smaller than 1 km; 1 km = 1000 m, so this rectangle will fit inside a rectangle that is 3 km long and 2 km wide)

c) List the rectangles from greatest area to least area. What word does your answer spell? (maple)
Estimating and measuring area of rectangles. Draw a rectangle on the board, or show a rectangular object, such as a book. SAY: I would like to estimate the area of this rectangle. Have students suggest ways to do that. If the idea of estimating length and width and multiplying the estimates does not arise, suggest that students think of how they found the area of rectangles, and how they estimated other things this year. Have students estimate the length and the width of the rectangle. Record several estimates on the board and have students explain their estimation process. Then have students multiply the estimates for the dimensions to get the estimates for the area and record these as well. Have a volunteer measure the sides of the rectangle and calculate the actual area.

Exercises

a) Estimate the area of the front cover of a JUMP Math AP Book in square centimetres.

b) Measure the length and the width of the book to the closest centimetre.

c) Calculate the actual area to check your estimate.

Sample answer: a) 22 cm × 30 cm = 660 cm²

Answers: b) 28 cm, 21 cm; c) 588 cm²

Show a book to the class that has length or width that is not a whole number of centimetres. Invite a volunteer to measure the length as precisely as possible. Discuss different ways to record the length. Point out that the book is, say, 17 cm wide to the closest centimetre, but in fact its width is between 16 and 17 cm. ASK: How can you record the width in a more precise way? (use millimetres only, use a mixed measurement) Ask students to record the measurement in these ways. (165 mm, 16 cm 5 mm)

ASK: There are 10 mm in 1 cm, so what fraction of 1 cm is each millimetre? (one tenth) If the book is 16 cm 5 mm wide, how can you record this width in centimetres as a decimal? (16.5 cm) SAY: When you measured length in centimetres and millimetres, you can record the measurement to the closest tenth of a centimetre. This means you record the answer as a decimal, with tenths.

Exercise: Measure the length and the width of the front cover of the JUMP Math AP Book to the closest tenth of a centimetre.

Answer: 27.6 cm, 21.3 cm

Have students decide which of the measurements they took is closer to a whole number of centimetres and round it to the nearest whole number, then recalculate the area of the front cover of the book using a decimal for one of the measurements. (587.88 cm²)

NOTE: Extension 3 is required in order to cover the British Columbia curriculum.
Extensions

1. On grid paper, or a geoboard, make as many shapes as possible with an area of 6 squares. For a challenge, try making shapes that have at least one line of symmetry. For instance, the shapes in the margin each have an area of 6 square units and one line of symmetry, shown as a thin line.

2. Marcel says that Shape A in the margin has an area of 4 squares and Shape B has an area of 3 squares, so Shape A has a larger area than Shape B. Explain his mistake.

**Answer:** Each square in Shape B is larger than each square in Shape A, and three larger squares can have a larger total area than four smaller squares.

3. An area of pavement is covered in ants. There are about 4 ants on each square centimetre of the pavement.

   a) If the area of pavement covered in ants is a rectangle about 80 cm wide and 120 cm long, about how many ants are there?

   **Solutions:** Area of rectangle: \(80 \text{ cm} \times 120 \text{ cm} = 9600 \text{ cm}^2\), so there are about \(9600 \times 4 = 38400\) ants; b) \(1 \text{ 600 000} \div 4 = 400 000\), so the area covered with ants is about \(400 000 \text{ cm}^2\). If this is a rectangle with width 80 cm, the length of the rectangle is \(400 000 \text{ cm}^2 \div 80 \text{ cm} = 5000 \text{ cm} = 50 \text{ m}\)
ME6-9  Area and Perimeter
Pages 100–101

Goals
Students will investigate the connection between the perimeter and area of rectangles and see that these are independent.

PRIOR KNOWLEDGE REQUIRED
Can find factors of a number
Knows that perimeter is the distance around a shape
Can find the area and perimeter of a rectangle
Can multiply and divide decimals to tenths by a whole number

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. T-1)
centimetre rulers

Mental math minute. SAY: You can double both numbers in a division and the result does not change. This is the same as doubling an array by doubling the size of each column. The array doubles, but the number of columns (the answer) does not change. Demonstrate this on the board as shown in the margin. SAY: This works with decimals too.

Exercises: Double both numbers in the division once or twice to get an easier division. Divide.

a) $0.9 \div 5$  b) $4.5 \div 25$  c) $0.4 \div 5$  d) $24.1 \div 5$  e) $22.5 \div 25$

Answers: a) $1.8 \div 10 = 0.18$, b) $9 \div 50 = 18 \div 100 = 0.18$, c) $0.8 \div 10 = 0.08$, d) $48.2 \div 10 = 4.82$, e) $45 \div 50 = 90 \div 100 = 0.9$

Review perimeter of rectangles. Draw a rectangle on the board and mark its sides as 2 m and 5 m. ASK: What do we call the distance around the rectangle? (perimeter of the rectangle) How do you find the perimeter? (add the side lengths) How can you use the fact that a rectangle has some equal sides to find the perimeter more efficiently? (double the length and the width, then add them; add the length and the width and double the result) Have students find the perimeter for the rectangle you drew and have volunteers record the equations for the perimeter on the board, as shown below:

$\frac{(2 \times 2 \text{ m}) + (2 \times 5 \text{ m})}{2(2 \text{ m} + 5 \text{ m})} = 4 \text{ m} + 10 \text{ m} = 14 \text{ m}$

To help students see that they can find the perimeter by adding the length and the width, then doubling the result, mark a dot on one vertex of the rectangle and draw the way along two adjacent sides to the opposite vertex. ASK: How far did I travel from the dot? (2 m + 5 m = 7 m) What fraction of the perimeter have I covered? (half) What do I need to get the whole perimeter from 7 m? (double the result) How do you know?
(two halves equal one whole) Keep the rectangle and the equations on the board for further reference. Label the rectangle “D.”

**Exercises:** Find the perimeter of the rectangle.

![Rectangle A](A 3 m × 4 m)

![Rectangle B](B 2.5 m × 4 m)

![Rectangle C](C 6 m × 2 m)

**Answers:** a) 14 m, b) 13 m, c) 16 m

**Comparing perimeter and area.** ASK: How do you find the area of a rectangle? (multiply length by width) What is the area of the rectangle on the board? (2 m × 5 m = 10 m²) Invite a volunteer to write the calculation on the board.

**Exercises:** Find the area of the rectangles in the previous exercises.

**Answers:** a) 12 m², b) 10 m², c) 12 m²

Referring students to the rectangles from the previous exercises, ASK: Which rectangles have the same perimeter? (A and D) Have students order the rectangles by perimeter, from least to greatest. (B, D, A, C or B, A, D, C)

ASK: Which rectangles have the same area? (A and C, B and D) Can two rectangles have the same area but different perimeters? (yes) Which rectangles have the same area but different perimeters? (A and C, B and D) Can two rectangles have the same perimeter but different area? (yes, A and D)

**Drawing rectangles with a given area.** Display a grid on the board. SAY: I want to draw several different rectangles with area 16 square units. ASK: Is it possible to do? (yes) Have students try to draw different rectangles with this area, and then have students present different answers. SAY: I want to try to find all rectangles of this sort. I want to limit myself to rectangles with whole number sides. ASK: How can we be sure that we have found all the rectangles with area of 16 square units? PROMPT: To find the area of a rectangle, you multiply two numbers, length and width. What do we call two numbers that multiply to 16? (factors of 16) How do you find all the factors of 16? Students can recall using a table to find all the factors. Have students use a table to find all the factors in their notebooks and have a volunteer complete the resulting table on the board, as shown in the margin.

Have students check the rectangles they drew and add any missing rectangles. Point out that when a rectangle is rotated, it is not changed, so a rectangle that measures 2 units across and 8 units up is the same as a rectangle that measures 8 units across and 2 units up. You can also remind students that the longest side is usually called the length of the rectangle. They can also call the distance across “width,” and the length of the vertical side “height.” The formula for the area of a rectangle then becomes “width × height.”
Add a column to the table on the board and have students find the perimeter of the rectangles they found. (34 units, 20 units, 16 units)

**Exercises:** On grid paper, use a ruler to draw all the rectangles with area 10 square units and whole unit sides. Find the perimeter of each rectangle.

**Bonus:** Draw a rectangle with area 10 square units with some sides that are not whole numbers.

**Answers:** 1 by 10, 2 by 5, perimeters 22 units and 14 units

**Sample answer:** Bonus: 2.5 units by 4 units, perimeter 13 units

SAY: A student I know thinks that a rectangle with a larger perimeter than another rectangle always has larger area too. ASK: Is that student correct? (no) Remind students that to show that a claim is not correct students can find just one example, showing that the claim is not true. ASK: What pair of rectangles are we looking for to show this claim is not correct? (two rectangles with different perimeters, so that a rectangle with larger perimeter has area that is not larger than the area of the rectangle with the smaller perimeter) SAY: Let’s use some of the rectangles in the last exercise. They all have the same area. Let’s try to draw a big rectangle that has even smaller area, say 9 square units.

Have students try to draw a rectangle that has an area of 9 square units, but has perimeter that is larger than the perimeters of some of the rectangles they found in the last exercise. PROMPT: Try to make rectangles with area 9 square units. They all have area that is less than 10 square units. ASK: Is there at least one of them that has perimeter larger than 14 units? (yes, a rectangle with width 1 unit and length 9 units has perimeter of 20 units, so has larger perimeter but smaller area)

**Finding rectangles with the same perimeter.** Remind students that they can add the length and the width and then double them to find the perimeter of the shape. SAY: I would like to find all the rectangles with perimeter of 12 units and sides that are whole numbers. Again, have students try to draw different rectangles, present answers, and then discuss using an organized search. To prompt students using organized search, suggest that they make a table similar to the table they used to find all rectangles with the given area. ASK: If the rectangle has perimeter of 12 units, what do the length and the width add to? (6 units) If the width is 1 unit, what is the length? (5 units) Repeat until all rectangles are found, and students realize that they are starting to draw some rectangles twice. (1 by 5 units, 2 by 4 units, 3 by 3 units; 4 by 2 units is a repetition) Finally, have students find the areas of the rectangles they drew. (5 square units, 8 square units, 9 square units)

**Exercises:** Draw and label all possible rectangles with perimeter 16 units and sides that are whole numbers. Find the area of all rectangles you drew.

**Bonus:** Draw a rectangle with area 18 square units and perimeter 18 units.
Answers: 1 by 7 units, 2 by 6 units, 3 by 5 units, 4 by 4 units. Areas: 7 sq. units, 12 sq. units, 15 sq. units, 16 sq. units; Bonus: 3 by 6 units

Uses of area and perimeter. Discuss as a class situations in which people need to find the area and perimeter of a shape. For example, when fencing a yard, building a path around a pool or a flower bed, or doing an art project that involves making a trim, you need to know the perimeter of the shape. When installing flooring or wallpaper, you need to know the area of the surface (floor, wall, and so on).

Extensions

1. Mr. Green wants to make a rectangular flower bed with perimeter 24 m and sides that are whole numbers of metres. Which dimensions of the flower bed will provide the greatest area? (6 m × 6 m)

2. Ms. Brown wants to make a rectangular flower bed with sides that are whole numbers of metres and area 36 m². Which dimensions will give her the least perimeter? (6 m × 6 m)

3. a) On grid paper, draw three figures (made of whole squares sharing whole sides, see examples in margin) with the same area, 6 cm², but different perimeters: 14 cm, 12 cm, 10 cm. Hint: Try attaching one square to different figures made with 5 squares in different positions.

Sample answers

b) Is it possible to draw a figure made of squares with area 9 cm² and perimeter more than 20 cm? Investigate using the table below.

<table>
<thead>
<tr>
<th>Number of 1 cm Squares</th>
<th>Possible Shapes</th>
<th>Area (cm²)</th>
<th>Perimeter (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the largest perimeter for each area? Predict the largest perimeter for a shape made of 5 squares. (12 cm)

Predict the answer for 9 squares. Is it more than 20 cm?
Answer: The pattern of the perimeters is 4, 6, 8, 10, so the largest perimeter for shapes of 5 tiles is 12. The largest perimeter that a shape made of squares sharing whole sides can have is achieved in a 1 by 5 rectangle (and other shapes).

For 9 squares, the largest perimeter possible is the perimeter of a 9 cm × 1 cm rectangle, equal to 20 cm.

4. Can two squares have the same perimeter but different areas? Explain.

Answer: No. If two squares have the same perimeter (say, 20 cm), they have the same side length (5 cm). This means they have the same area too. This argument works for any side length, so it is true for all squares.
ME6-10  Area of Shapes Made from Rectangles

Pages 102–103

Goals

Students will find the area of shapes composed of rectangles.

PRIOR KNOWLEDGE REQUIRED

Knows that area is additive
Can find the area of a rectangle by multiplying length by width
Can multiply decimals to tenths by a whole number

MATERIALS

coloured pencils

Mental math minute. Remind students that if they multiply the dividend (the first number in a division) by 10, 100, or 1000, the result of the division is also multiplied by 10, 100, or 1000. Write “0.18 ÷ 3” on the board. SAY: I can multiply the dividend (the first number) by 100 to get 18 ÷ 3 = 6 (write that division underneath the first question) and then I know that the answer, 6, is 100 times the answer to the first division. I can divide the answer by 100 and get 0.06. For the following exercises, write each division and the four possible answers on the board. Present them one at a time and have students signal which answer they think is correct by raising the correct number of fingers.

Exercises:

Divide to find the right answer.

a) 0.20 ÷ 4
   A. 5    B. 0.5    C. 0.05    D. 0.005

b) 0.208 ÷ 8
   A. 0.026    B. 0.26    C. 0.021    D. 0.251

c) 3.6 ÷ 6
   A. 6    B. 0.6    C. 0.06    D. 0.006

d) 4.5 ÷ 9
   A. 5    B. 0.5    C. 0.05    D. 0.005

e) 0.2 ÷ 5
   A. 40    B. 4    C. 0.4    D. 0.04

Answers: a) C, b) A, c) B, d) B, e) D

Shapes composed of two rectangles (on a grid). On a grid, draw several shapes composed of two rectangles, and have students draw the line that divides the composite shape into rectangles.
Exercises: Find the area of the shape.

a)   

b)   

c)   

Answers: a) 6 square units, b) 10 square units, c) 11 square units

Point out that part a) has two different solutions. Encourage students to find both, and have volunteers show both solutions ($1 \times 4 + 1 \times 2$ and $1 \times 2 + 2 \times 2$).

Shapes composed of two rectangles (not on a grid). Draw several shapes composed of two rectangles not on a grid and mark the dimensions on four of the sides, as shown in the Exercises below. Ask students to use coloured pencils to shade each rectangle with its own colour and then circle the dimensions that belong to the rectangle in the same colour.

Exercises: Find the area of the shape by adding the areas of the rectangles.

a)   

b)   

c)   

Bonus:   

Answers: a) $8 \text{ m}^2$, b) $68 \text{ cm}^2$, c) $130 \text{ cm}^2$, Bonus: $199.5 \text{ cm}^2$

Identifying dimensions of shapes. Show students the shape in the margin. ASK: Is 7 cm the length of the short side of the shaded rectangle (trace it with your finger)? (no) Is it the length of the short side of the unshaded rectangle? (no) Point out that 7 cm is the length of the combined side, so 7 cm should be the sum of the lengths of two other sides. ASK: What is the length of the short side of the unshaded rectangle? How do you know? ($7 \text{ cm} - 4 \text{ cm} = 3 \text{ cm}$) Finally, have students find the areas of both rectangles and add them to find the total area of the shape. ($5 \text{ cm} \times 3 \text{ cm} + 11 \text{ cm} \times 4 \text{ cm} = 15 \text{ cm}^2 + 44 \text{ cm}^2 = 59 \text{ cm}^2$)

Present the shape in the margin, and ask students to find the length of the longer side of the shaded rectangle. (21 m) Have students find the area of the shape. ($7 \text{ m} \times 10 \text{ m} + 9 \text{ m} \times 21 \text{ m} = 70 \text{ m}^2 + 189 \text{ m}^2 = 259 \text{ m}^2$) Then have them practise finding areas individually.
Exercises: Find the area of the shape.

a) \[ 2 \text{ m} \times 1 \text{ m} = 2 \text{ m}^2 \]

b) \[ 2 \text{ cm} 	imes 4 \text{ cm} = 8 \text{ cm}^2 \]

c) \[ 8 \text{ m} \times 3 \text{ cm} = 24 \text{ cm}^2 \]

Answers: a) 2 m², b) 8 cm², c) 24 cm²

Present the shape shown in the margin, and shade one of the rectangles as shown in the first picture. Ask students to identify the side lengths of the shaded rectangle. (13 cm and 5 cm) Repeat with the side lengths of the unshaded rectangle. (4 cm and 6 cm) Then present the same shape with the same labels but broken into two rectangles in a different way (as in the second picture) and repeat the exercise. (shaded: 5 cm by 7 cm, unshaded: 6 cm by 9 cm) Have students find the area of the shape both ways. (89 cm²) Did they get the same answer?

Exercises: Find the area of the shape.

a) \[ 12 \text{ m} \times 7 \text{ m} = 84 \text{ m}^2 \]

b) \[ 14 \text{ m} \times 6 \text{ cm} = 84 \text{ cm}^2 \]

c) \[ 23 \text{ m} \times 12 \text{ m} = 276 \text{ m}^2 \]

d) \[ 4 \text{ m} \times 3 \text{ m} = 12 \text{ m}^2 \]

e) \[ 2 \text{ cm} \times 3 \text{ cm} = 6 \text{ cm}^2 \]

Answers: a) 140 m², b) 140 cm², c) 456 m², d) 57 m², e) 4.4 cm²

Bonus: Find a different way to split these shapes into rectangles and find the area the new way. Did you get the same answer as before?

Area and perimeter of composite shapes. Remind students how to find the perimeter of a shape. Point out that to find the perimeter of a composite shape there is no need to break it into rectangles—students just need to “walk” around the shape and write an addition equation using all the side lengths. ASK: How could we check that we did not forget any of the sides? (one strategy is to mark the sides as you go and check that all sides are marked; another strategy is to count the number of sides first, and then check that the number of side lengths in the addition equation matches the total number of sides in the shape)

Exercises: Find the perimeter of the shapes in the previous exercises.

Answers: a) 52 m, b) 64 cm, c) 100 m, d) 34 m, e) 10.4 cm
Bonus: The perimeter of the shape in the margin is 30 cm. Find $x$.

Answer: 8 cm

NOTE: For composite shapes made of rectangles with side lengths that are whole numbers, such as shapes drawn on grids, the perimeter is always an even number. Students do not need to know this at this point, but it will help you to quickly identify some types of errors.

Present a composite shape on a grid (such as the shape in the margin) and tell students that each square on the grid is 5 cm by 5 cm. Have students write the side lengths for all of the sides of the shape and then find the perimeter. (110 cm) Next, ask students to split the shape into rectangles and find the area. (300 cm$^2$)

NOTE: When students are solving Question 6 on p. 111 of AP Book 6.2, make sure they break the shapes into rectangles that have at least one side that has a whole-number length to avoid multiplying a decimal by another decimal.

Extensions

1. Teach students to check their calculations of the area of composite shapes on a grid, like those in Questions 5 and 6 on AP Book 6.2 p. 111. Look at the last shape you worked with during the lesson (in the margin above). ASK: What is the area of the shape in square units? (12 sq. units) What is the area of one square? ($5 \text{ cm} \times 5 \text{ cm} = 25 \text{ cm}^2$) If one square has area 25 cm$^2$, what is the area of 12 such squares? ($12 \times 25 = 300 \text{ cm}^2$) What is the area of the whole shape? Did we get the same answer as when we used the first method?

2. Each square on the grid is 0.5 cm by 0.5 cm.
   a) How many squares make 1 cm$^2$? What is the area of each square on the grid?
   b) Find the area and the perimeter of the rectangle.
   c) Jax thinks the perimeter of this rectangle is larger than its area. How would you explain to Jax his mistake?

   Answers: a) 4 squares, 0.25 cm$^2$; b) perimeter 10 cm, area 5.25 cm$^2$; c) Perimeter and area are measured in different type of units. Centimetres cannot be converted into square centimetres, or vice versa, and so the measurements cannot be compared.

3. a) Draw a rectangle so that the number representing the area (in cm$^2$) is the same as the number representing the perimeter (in cm).
   b) Convert the measurements of the sides to millimetres. Find the area and perimeter of the rectangle with the new measurements. Is the number representing the area in mm$^2$ still equal to the number representing the perimeter in mm?
c) How does this exercise help you explain that perimeter and area can never be the same?

**Sample answers:**

a) Area: $18 \text{ cm}^2 = 3 \text{ cm} \times 6 \text{ cm}$, Perimeter: $18 = 2 \times 3 \text{ cm} + 2 \times 6 \text{ cm}$;  
b) $30 \text{ mm} \times 60 \text{ mm}$, so area $= 1800 \text{ mm}^2$ and perimeter $= 180 \text{ mm}$, the numbers are different;  
c) This exercise shows that the numbers representing area and perimeter can be identical in one set of units and different in another set of units for the same shape. Area and perimeter can never be the same because the units they are measured in cannot be compared.

4. On a grid, draw a square with sides that are 7 units long.

a) Find the area and the perimeter of the square.

b) Inside the square, draw a shape that has an area smaller than the area of the square and perimeter larger than the perimeter of the square. There are many possible answers.

**Answer:**

a) area $= 49$ square units, perimeter $= 28$ units

**Sample answer**

b) area $= 17$ square units, perimeter $= 36$ units
Goals

Students will develop the formula for the area of a parallelogram.

PRIOR KNOWLEDGE REQUIRED

Is familiar with decimals to tenths
Can multiply or divide decimals by multiples of 10
Can convert units of length
Knows the formula for the area of a rectangle
Can make a simple geometric sketch
Can draw a perpendicular to a line through a point
Can draw parallel lines

MATERIALS

parallelograms drawn on grid paper for every student
scissors and tape
grid paper or BLM 1 cm Grid Paper (p. T-1)
a large, flexible rectangle made from cardboard strips and paper fasteners (see picture on p. P-20)
set squares and rulers
rectangular sheets of paper
BLM Area of Parallelograms (Advanced) (p. P-49, see Extension 2)

Turning parallelograms into rectangles of the same area. Give each student a parallelogram drawn on grid paper. Ask students to use scissors to cut a piece off the parallelogram and to use tape to reattach it so as to turn the parallelogram into a rectangle. Students may need to experiment to find the answer; provide extra copies of parallelograms as needed. If students need a hint, you can suggest cutting off a triangle as shown in the margin. Give everyone a chance to find the answer, and, if necessary, invite one or more students to explain it to the class. ASK: Are the sides of the rectangles the same length as the sides of the parallelogram? How many sides preserve their length after being rearranged? (2)

Introduce base and height. Explain that the sides of the parallelogram that have the same length as the sides of the rectangle students created are called the bases of the parallelogram. SAY: A base is something that an object can stand on. If we think of a parallelogram as standing on its base (hold one of the parallelograms used earlier vertically, as if it was standing on a desk on its base), we can ask how tall the parallelogram is. Draw a height on the parallelogram you are using, and explain that the height of a parallelogram is always measured at a right angle to the base. Point out that “base” is also short for “the length of the base” and “height” refers to both the distance between the bases and the perpendicular it is measured along. The parallelogram students were using is drawn on a grid, and the
base runs along the grid lines, so it is really easy to draw the height of the parallelogram. Give students another copy of the parallelogram and ask them to find the base and the height.

**Exercises:** Find and record the length of the base and the height of the parallelogram.

a) ![Parallelogram](image)

b) ![Parallelogram](image)

c) ![Parallelogram](image)

**Answers:** a) length of base: 4 units, height: 4 units; b) length of base: 4 units, height: 3 units; c) length of base: 5 units, height: 3 units

Ask students to copy the parallelograms above onto grid paper and draw where they would cut them so as to convert them to rectangles. Ask students to draw each piece they cut off in the position they would reattach it, creating pictures like the one in the margin.

**The area of a rectangle can be expressed as width \times height.** Remind students that the area of a rectangle has a formula, “length \times width.” Point out that it is sometimes hard to say which side is the “length” and which side is the “width.” Usually, the longer side is the length and the shorter side is the width. However, in real life we often think of width as distance across. For a rectangle such as the one in the margin, the distance across is 5, so is the width 4—the length of the shorter side—or is it 5—the distance across? Luckily, we can write the formula for the area of a rectangle as “width \times height” as well as “length \times width.” When we use the first version, there is no confusion about which side shows what. The width in this formula is 5, and the height is 4. Point out that in both cases you multiply the lengths of the adjacent sides (sides that share a vertex) to get the area of a rectangle.

**Developing the formula for the area of a parallelogram.** ASK: Did the area of the parallelogram change when you cut and rearranged the pieces? (no) Remind students that the base of the parallelogram is the side that does not change when the parallelogram is converted into a rectangle. Have students write the height for each rectangle and compare it to the height and the base of the parallelogram. ASK: What is the height of each rectangle you produced equal to? (the height of the original parallelogram) SAY: To find the area of a rectangle, we multiply width by height. ASK: What should we multiply to find the area of a parallelogram? (length of base and height) Ask students to try to write a formula for the area of a parallelogram using the base and height. (Area = length of base \times height or base \times height) Remind students that sometimes we use a shortcut for formulas. SAY: For example, we could use \( A \) for area, \( b \) for the length of the base, and \( h \) for height. In this case we would write the formula as \( A = b \times h \).

**Finding the area of parallelograms using a formula.** Remind students to write the units when multiplying base and height. This habit becomes very useful when students work with different units, as it helps to avoid multiplying lengths in incompatible units (such as metres and centimetres).
Demonstrate finding the area of a parallelogram using the formula. Example:

Area of parallelogram = base × height
Base = 5 cm
Height = 4 cm
Area of parallelogram = 5 cm × 4 cm
= 20 cm²

**Exercises:** Find the area of the parallelogram.

a) base 3 cm, height 4 cm  
b) base 3 m, height 4.5 m  
c) base 3.5 km, height 3 km  
d) base 4 mm, height 2.5 mm

**Answers:** a) 12 cm², b) 13.5 m², c) 10.5 km², d) 10 mm²

The area of a parallelogram is not defined by side lengths. Draw two identical rectangles using different colours. Mark the lengths of the sides and have students find the area of one of the rectangles. ASK: What is the area of the second rectangle? How do you know? (the rectangles have the same sides, so they have the same area)

Have on hand a large, flexible rectangle made from cardboard strips and paper fasteners. ASK: Do you think two parallelograms with the same side lengths have the same area? How could we find out? Take various answers, then hold up the rectangle and begin to deform it into a shape that has area close to zero.

Point out that you aren’t changing the length of the sides, you’re just moving them and creating different parallelograms. ASK: Have we answered the question? How? (Students should see, qualitatively if not quantitatively, that parallelograms with the same side lengths can have different areas.) Emphasize that the length of the sides does not define the area of a parallelogram the way it does the area of a rectangle. SAY: All the parallelograms created this way have the same perimeter, but different areas.

Any side of a parallelogram could be a base. Draw a parallelogram on the board so that it does not have any vertical or horizontal sides. Explain that since this parallelogram is not on a grid, it does not matter which pair of opposite sides you chose as bases. SAY: Actually, you can choose any pair of opposite sides to be the bases. Pick one pair of sides and highlight it, saying that you want to use these sides as bases. ASK: How could you now find the height of the parallelogram?

Explain that if your parallelogram was made of paper, you could turn it so that your chosen base was horizontal and then measure the height vertically. SAY: However, you can’t turn the drawing on the board. ASK: What could you do instead? Take suggestions, then remind students that the height is perpendicular to the base, so you need to draw a line that is perpendicular to the base.
Constructing a perpendicular to a given line. Remind students how to use a set square to construct a perpendicular.

The distance between parallel lines. Ask students to look at the parallelograms on grid paper that they used at the beginning of the lesson. Ask them to draw the height in these parallelograms in more than one place. Ask: Is the height the same no matter where you draw it in a parallelogram? (yes) Say: When you work on a grid, the height does not depend on where you measure it. Ask: Do you think the same will happen when you are working on blank paper?

Ask students to draw a pair of slant parallel lines by drawing lines along both sides of a ruler. Have students draw perpendiculars to one of the lines (so that they intersect with the other line) in three different places. Remind students that a line perpendicular to one of the parallel lines is also perpendicular to the other parallel line. Then ask students to measure the distance between the lines along the three perpendiculars. Do students get the same answer? If not, have students check the angles between the lines (they should be right angles) and the measurements. Students can also exchange their pictures with a partner and check each other’s work. Point out that they have discovered an important fact—the distance between parallel lines (measured along a perpendicular) does not depend on where the perpendicular is drawn.

The height of a parallelogram can be measured anywhere. Return to the parallelogram already on the board (the one you used to show that any side can be a base). Ask: Does the height of a parallelogram depend on where it is measured? (No) Why or why not? (Because the opposite sides are parallel lines and the height is the distance between the parallel lines, so it is the same everywhere)

Estimating and measuring areas of parallelograms. Have pairs of students divide a rectangular sheet of paper into two unequal rectangles by folding and cutting along the fold. Partners can each use one of the two created rectangles. Then have them fold one of the rectangles, as shown in the left image below, and cut off a right triangle. Have them then place the right triangle over the other side of the rectangle and cut off an identical triangle, as shown in the right image below, creating a parallelogram.

Cut off

Cut off

Show a paper parallelogram to the class and say: I want to estimate the area of this parallelogram. Ask: How can I do this? Prompt: How did you estimate area of rectangles? (Estimated length and width, then multiplied the estimates) Have students choose the base, estimate its length, and then estimate the distance between the bases. Discuss ways to estimate the height. Point out that the distance along a perpendicular line is always shorter than the distance along a slanted line, so the height is smaller than the length of the sides that are not bases. Students can then estimate the length of the other side and then reduce the estimate to get the height.
Have students estimate the area of the parallelogram they cut out earlier and record their estimates both for the lengths and for the area. Have partners exchange parallelograms and estimate the area again. Then have them draw the height on the parallelogram using a set square, measure the height and the base, and calculate the area. Show students how to find the height without a set square by folding the parallelogram so that the bases fold onto themselves. They can round the measurements to the closest centimetre. Have students exchange parallelograms again and calculate the area of the second parallelogram. Partners should compare answers to the estimates, compare answers between partners, and look for reasons for discrepancies (such as measurement mistakes).

Parallelograms with height falling outside the parallelogram. Have students copy the parallelograms in the exercises below on grid paper. Explain that you want to find the area of these parallelograms. Is it possible to convert them to rectangles using the method from the beginning of the lesson? Why not? (the triangle that you cut off does not contain all of the slanted side of the parallelogram)

Ask students to extend the bases to create two longer parallel lines. Remind students that the height is the distance between the bases, measured along a perpendicular line.

Exercises: Find the distance between the bases. Then find the area of the parallelogram.

Finding the area of parallelograms in different ways. Have students use the paper parallelograms they used earlier in the lesson to find the area using the second pair of sides as bases. Ask: Should the two methods give the same answer? Why? (Yes, because the area is the same no matter how you measure it.) Are the two answers the same? If not, what could cause the answers to be different? (imprecise measurements, a mistake in drawing the heights)

Exercise: On grid paper, draw at least 4 different parallelograms with base 3 units and height 2 units. What do you know about the area of these parallelograms?

Sample answer: The area of all the parallelograms is base $\times$ height $= 3 \times 2 = 6$ square units.
NOTE: In this lesson, students learned that the area of a parallelogram can be written using variables:

\[ A = b \times h \]

where \( b \) is the base and \( h \) is the height. They also learned the formula using words:

Area = base \( \times \) height

In the following lessons, students will learn formulas for the area of various shapes. We recommend you give students practice writing and applying these formulas using both words and variables.

Extensions

1. Cutting and rearranging parallelograms with height outside the parallelogram to find the area. ASK: Will cutting and rearranging a parallelogram change its area? (no) Use the two parallelograms in the exercises from the end of the lesson for this Extension. Students can’t cut and rearrange these parallelograms the same way they did those at the beginning of the lesson, but invite them to try cutting and rearranging them a different way. The goal is to create a new parallelogram with at least one pair of parallel sides that align with the grid lines. Students should start by cutting each parallelogram out and tracing it (to preserve the original shape). They may want to cut out several copies (or you may want to provide multiple copies), so that they can test various solutions. They should find the heights and bases of the old and new parallelograms to calculate and compare their areas.

Sample solutions for a)

2. On BLM Area of Parallelograms (Advanced) (p. P-49) students will find the area of parallelograms using non-horizontal sides as bases.

3. A parallelogram has height 80 cm and base 1.5 m.

   a) Tom says: The area is \( 80 \text{ cm} \times 1.5 \text{ m} = 120 \text{ cm}^2 \). Is he correct? Explain.

   b) Convert both measurements to centimetres and find the area.

   Base: _____ cm Height: _____ cm Area: _____ cm²

**Answers:** a) Tom is not correct, the length of base and the height should be in the same unit. b) Base: 150 cm, Height: 80 cm, Area: 12 000 cm²
Goals
Students will develop the formula for the area of a triangle, using the area of a rectangle, and then apply it.

PRIOR KNOWLEDGE REQUIRED
- Is familiar with decimals to tenths
- Can multiply and divide decimals by a whole number
- Can find the area of a rectangle using a formula
- Knows that two identical right triangles make a rectangle
- Understands that area is additive
- Is proficient in measuring with a ruler
- Can identify and draw right angles using a set square

MATERIALS
- paper rectangles
- BLM Triangles on Grid Paper (p. P-50)
- tape
- scissors
- grid paper or BLM 1 cm Grid Paper (p. T-1)
- BLM Obtuse Triangles on a Grid (p. P-51, see Extension 2)

Relating the areas of a right triangle and a rectangle. Give each student a paper rectangle. Ask them to fold the rectangle along a line joining the opposite corners. Ask: If you cut along the line, what shapes will you get? (triangles) What kind of triangles are they? (right triangles) Are these triangles different? (no) Have students cut the rectangle along the diagonals and make sure the triangles are congruent.

Review how to find the area of a rectangle. Tell students that the area of the rectangle was, say, 300 cm². Ask: What is the area of each triangle? (150 cm²) How do you know? (a right triangle is half of a rectangle) Emphasize that the rectangle has the same length and width (or width and height) as the sides of the triangle that meet at the right angle. Display the picture in the margin and keep it on the board as a reminder during this lesson.

Splitting a triangle into two right triangles to find its area. Draw several acute and obtuse triangles on a grid so that the longest side is horizontal (see margin).

Ask students to copy the triangles and to split them into two right triangles. Then have students draw a rectangle around each triangle and use the rectangle to find the area of both the original triangle and the two right triangles. Ask: What fraction of the area of the big rectangle is the area of the big triangle? (half) How do you know? (the triangle is made of two right
triangles, each right triangle is half of a rectangle, and the two rectangles together make the big rectangle, as shown in the margin) Point out that this is a general property of numbers: half of $20 + \text{half of } 8 = \text{half of } (20 + 8) = \text{half of } 28$.

**Introduce base and height in triangles.** Point out that when we split a triangle into two right triangles, we draw a perpendicular from one of the vertices to the opposite side. The perpendicular (and its length) is called the height of the triangle and the side we draw the perpendicular to is called the base. Ask students to identify the base and the height in each triangle they split into right triangles earlier. As a class, draw and fill in a table for the triangles you used earlier, using these headings:

<table>
<thead>
<tr>
<th>Base of Triangle</th>
<th>Height of Triangle</th>
<th>Width of Rectangle</th>
<th>Height of Rectangle</th>
<th>Area of Rectangle</th>
<th>Area of Triangle</th>
</tr>
</thead>
</table>

**The formula for the area of a triangle.** Write on the board:

\[
\text{Area of rectangle} = \text{width of rectangle} \times \text{height of rectangle}
\]

\[
\text{Area of triangle} = \frac{\text{area of rectangle}}{2}
\]

Point out that since we know how to find the area of a rectangle, we can replace the words “area of rectangle” by the formula. Erase the words “area of rectangle” in the second formula and write:

\[
\text{Area of triangle} = \text{width of rectangle} \times \text{height of rectangle} ÷ 2
\]

ASK: Is the width of the rectangle the same as the height or the base of the triangle? (base) Erase “width of rectangle” and replace it with “base of triangle.” ASK: Is the height of the rectangle the same as the height or the base of the triangle? (height) Erase “height of rectangle” and replace it with “height of triangle.” Point out that when everything is written in terms of the triangle only, we do not need to say “of triangle,” so we can revise the formula further. Write the new formula below the “extended” one:

\[
\text{Area of triangle} = \text{base} \times \text{height} ÷ 2
\]

Again, point out that you can use a short form of the formula: \( A = b \times h ÷ 2 \).

**ACTIVITY (Essential)**

Give students half of BLM Triangles on Grid Paper, so that each student gets two copies of the right triangle and two copies of the acute triangle. Provide additional copies if needed. Have students use scissors to cut out the triangles. Ask them to try to cut and rearrange the right triangle to create a rectangle with the same area. Students can use tape to hold the pieces of the rectangles together. Students should work individually first, then they can discuss possible solutions in pairs (provide a time limit for each partner to talk, and have partners provide positive and constructive feedback before they present their own ideas). Point out that there are two solutions possible, and encourage students to find both (see answer in margin). Debrief as a class.
Repeat the activity with the acute triangles from the same BLM. As a hint, point out that to change a right triangle to a rectangle, students cut off and moved one piece of the triangle. With the acute triangle, they can either cut off one piece and split it in two, or cut off two separate pieces. If the solutions shown in the margin do not arise, show them. Ask students to compare the dimensions of the resulting rectangles (they will be different) and to check that the areas of the rectangles are the same by multiplying.

**Finding the area of a triangle using the formula.** Have students find the area of triangles using the formula they learned earlier.

**Exercises:** Find the area of the triangle.

a) base 10 cm, height 3 cm  

b) base 2 m, height 7 m  

c) base 4 m, height 3.2 m

**Answers:** a) 15 cm², b) 7 m², c) 6.4 m²

**Constructing triangles with the given perimeter and area.** SAY: I would like to construct a triangle with an area of 10 grid squares. Have students think of different ways to do so and discuss the options. PROMPTS: The area of a triangle is half the area of a rectangle with length and width equal to the base and the height of the triangle. ASK: Ten is half of which number? (20) Can you construct a rectangle with area of 20 square units? What can dimensions of such a rectangle be? (1 by 20, 2 by 10, 4 by 5 units) Have students construct a rectangle with area of 20 square units and divide it into two triangles. Have students present different solutions to the class.

SAY: Another way to construct a triangle with area of 10 square units would be to reverse the steps of constructing a rectangle from a triangle: construct a rectangle with area of 10 squares, then cut off a piece, as shown in the margin, and turn it to construct a triangle. Have a volunteer show the method, and then ASK: Is the resulting triangle the same as one of the triangles constructed using the other method? (yes)

**Exercises:** On grid paper, construct a triangle with the given area.

a) 12 square units  
b) 16 square units  
c) 18 square units

**Bonus:** 2 different triangles with area 15 square units

**Sample answers**

a)  

b)  

c)
Bonuss

Have volunteers show multiple answers to the previous exercises. Point out that any two triangles with the same base and the same height have the same area. Draw the picture below on the board and have students find the area of the triangles. (all triangles have an area of 8 square units)

Students can copy the triangles to grid paper, cut them out, and try to cut and rearrange pieces so as to verify that all the triangles have the same area. Students can work in pairs to save time checking different triangles.

Extensions

1. Look at the solutions for the acute triangle in the Activity for this lesson. Discuss with students how each rearrangement is reflected in the formula. PROMPTS: What is the height of the rectangle? How is it related to the height of the triangle? What is the width of the rectangle? How is it related to the base of the triangle?

For Solution 1, the width of the rectangle is the same as the base of the triangle, and the height of the rectangle is half the height of the triangle. For Solution 2, the height of the rectangle is the same as the height of the triangle, and the width of the rectangle is half the base of the triangle.

Point out that to get the area of a triangle, you multiply base by height and divide by 2. This means you take half of the product of the base and height. In the rectangles mentioned above, you take half of either the base or the height, so the whole product is also halved.

2. Find the area of triangle T (from Question 4 on AP Book 6.2 p. 106). Use the BLM Obtuse Triangles on a Grid (p. P-51).

3. Find the areas of triangles A and B (see margin) using the formula for the area of a triangle. What fraction of the area of the rectangle is the area of A? The area of B?

**Answer**

Area of A = 6 × 5 ÷ 2 = 15
Area of B = 10 × 3 ÷ 2 = 15

Area of A = Area of B, so the rectangle in the picture is divided into four parts of equal area. The area of A and the area of B are each 1/4 of the area of the rectangle.
**Goals**

Students will develop the formula for the area of a triangle using parallelograms, and then apply it.

**PRIOR KNOWLEDGE REQUIRED**

- Is familiar with decimals to tenths
- Can multiply and divide decimals by a whole number
- Can find the area of a parallelogram using a formula
- Understands that area is additive
- Can copy triangles onto a grid in orientations different from the original
- Can plot points in the 1st quadrant of a coordinate grid

**MATERIALS**

- BLM Filling a Blank Multiplication Chart (p. T-2)
- Various paper triangles or BLM Triangles (p. P-52)
- Grid paper or BLM 1 cm Grid Paper (p. T-1)
- Ruler

**Mental math minute.** Give students BLM Filling a Blank Multiplication Chart. Have them fill in the chart as much as they can in three minutes, using the strategies on the BLM as needed.

**Review area of a parallelogram.** Remind students that any pair of parallel sides can be chosen to be the base; the height is the distance between the bases measured along a line perpendicular to the bases; and the formula for the area of a parallelogram is base \(\times\) height.

**Comparing area of a parallelogram to area of a triangle.** Start with the activity below.

**ACTIVITY (Essential)**

Divide students into pairs. Give one student in each pair two copies of a scalene acute triangle and the other student two copies of a scalene obtuse triangle. (Sample shapes are provided on BLM Triangles.)

Each student should try to create as many shapes as possible using their two copies of the same triangle by joining the triangles along a pair of sides of the same length. Ask students to trace the shapes they make and to identify them if possible. Pairs should discuss the shapes they produced. How many shapes does each triangle make? Are all the shapes quadrilaterals? What types of quadrilaterals can be created? What special features do they have?

**Answers:** Each type of triangle produces 6 shapes: 3 parallelograms and 3 shapes with two pairs of equal adjacent sides; 2 of the shapes produced by the obtuse triangles will have indentations.
Ask students to think about the area of the shapes produced from the same pair of triangles. ASK: What can you say about the area of the shapes you’ve obtained? (the areas are the same) For each pair of triangles, which quadrilateral can you use to find the area? (parallellogram) What fraction of the area of the parallelogram is the area of the triangle? (half) Have students keep the triangles to use in Extension 1.

Comparing the base and height in triangles and parallelograms.
Remind students that the height in a triangle is the distance between the vertex opposite the base and the base itself, and it is measured along the line perpendicular to the base. So if your triangle has one side running along a gridline, it is very convenient to use it as a base.

Exercises: Copy the triangle on grid paper. Draw a copy of the triangle to create a parallelogram. Find the area of the parallelogram and the area of the triangle.

a)    b)    Bonus

Sample Answers

a) , b) , Bonus

Answers: a) parallelogram: 24 square units, triangle: 12 square units; b) parallelogram: 15 square units, triangle: 7.5 square units; Bonus: parallelogram: 12 square units, triangle: 6 square units

ASK: For each triangle and parallelogram in the exercises, what do you notice about the base of the parallelogram and the base of the triangle? (they are the same) What do you notice about the height of the parallelogram and the height of the triangle? (they are the same)

The formula for the area of a triangle. Write on the board:

\[
\text{Area of parallelogram} = \text{base of parallelogram} \times \text{height of parallelogram}
\]
\[
\text{Area of triangle} = \frac{\text{area of parallelogram}}{2}
\]

Point out that since we know how to find the area of a parallelogram, we can replace the words “area of parallelogram” by the formula. Erase the words “area of parallelogram” in the second formula and write:

\[
\text{Area of triangle} = \text{base of parallelogram} \times \text{height of parallelogram} \div 2
\]

ASK: Is the base of the parallelogram the same as the height or the base of the triangle? (base) Erase “base of parallelogram” and replace it with “base of triangle.” ASK: Is the height of the parallelogram the same as the height or the base of the triangle? (height) Erase “height of parallelogram” and replace it with “height of triangle.” Point out that when all the data is...
in terms of a triangle only, we do not need to say “of triangle,” so we can revise the formula further. Write the new formula below the “extended” one:

\[
\text{Area of triangle} = \text{base} \times \text{height} \div 2 \quad \text{or} \quad A = b \times h \div 2
\]

Finding the area of a triangle using the formula. Ask students to give their partners from the previous Activity one of the triangles they used in the Activity, so that each student has two triangles of different kinds. Ask students to use the longest side as the base. Have them draw the height on one triangle. Have students use rulers to measure the base and the height and find the area of the triangle using the formula. Repeat with the other triangle. Have students compare the answers. ASK: Did everyone get the same answers? Point out that the answers might be slightly different, because students might be a little off when drawing the perpendicular line and measuring. Point out that any measurement with a ruler produces an approximation—we can only measure to the closest millimetre.

The area of a right triangle when the base is one of the legs. Draw a right triangle and choose one of the legs (legs are sides adjacent to the right angle) as a base. Ask students to find the height. Since the triangle is a right triangle, the height is the other leg. ASK: Does the formula “base \times height \div 2” produce the same answer as “length \times width \div 2”? (yes) Summarize by pointing out that in the case of a right triangle we can talk about length and width—as the length and width of the rectangle that was cut in half to make the triangle—and these are the same as base and height if we regard one of the short sides as the base.

Finding the area of a triangle when the base is not horizontal. Remind students that any side in a triangle can be chosen as a base. Then draw and label several triangles with non-vertical heights already drawn for the students and have them identify the base in each case. For example, the triangle in the margin has a base of \( AB \).

Exercises: The height is given in the triangle. Name the base.

\[
\begin{align*}
\text{a)} & \quad \triangle ABC \\
\text{b)} & \quad \triangle ABC \\
\text{c)} & \quad \triangle ABC \\
\text{d)} & \quad \triangle ABC
\end{align*}
\]

Answers: a) \( BC \), b) \( AC \), c) \( AB \), d) \( AB \)
Next, provide triangles with dimensions and non-horizontal bases, and have students find their areas.

**Exercises:** Find the area of the triangle.

a) ![Triangle A](image)
   - Base: 5 cm
   - Height: 11 cm
   - Area: 27.5 cm²

b) ![Triangle B](image)
   - Base: 10 m
   - Height: 20 m
   - Area: 100 m²

c) ![Triangle C](image)
   - Base: 15 mm
   - Height: 27 mm
   - Area: 202.5 mm²

d) ![Triangle D](image)
   - Base: 2 km
   - Height: 6.4 km
   - Area: 6.4 km²

**Answers:** a) 27.5 cm², b) 100 m², c) 202.5 mm², d) 6.4 km²

Finally, have students solve several simple word problems that require finding the area of a triangle.

**Exercises:** Solve the problem.

a) A triangular banner has base 15 cm and height 12 cm. What is its area?

b) A flower bed is a triangle with base 330 cm and height 180 cm. What is its area?

c) A plot of land is a triangle with base 151 m and height 51 m. What is its area?

**Bonus:** The triangles in the picture (in the margin) are congruent. Find the area of the hexagon.

**Answers:** a) 90 cm², b) 29 700 cm², c) 3850.5 m², Bonus: 42 cm²

**Area of shapes on coordinate grids.** Review plotting points on coordinate grids. Display a coordinate grid; remind students that the first number in an ordered pair tells you how far you need to go from the origin along the x-axis (the horizontal axis), and the second number tells you how far to go parallel to the vertical axis, or y-axis. Have students plot the points below on a 10 by 10 coordinate grid and join the points to form a quadrilateral or a triangle.

**Exercises:** Plot the points. Identify the shape.

a) A (0, 1), B (0, 4), C (2, 4), D (2, 1)

b) E (0, 6), F (2, 10), G (5, 10), H (3, 6)

c) I (3, 0), J (5, 4), K (8, 0)

d) L (9, 0), M (9, 3), N (10, 2)

e) O (5, 8), P (9, 9), Q (9, 7), R (5, 6)
Selected answers: a) rectangle, b) parallelogram, c) triangle, d) triangle, e) parallelogram.

Exercises: Use the formulas you’ve learned to find the area of the shape in the previous exercises.

Answers: a) 6 sq. units, b) 12 sq. units, c) 10 sq. units, d) 1.5 sq. units, e) 8 sq. units

Estimating areas of triangles. Draw a triangle on the board without using a grid. SAY: I would like to estimate the area of this triangle. Let’s use what we did for estimating area of parallelograms to estimate area of triangles. ASK: How did we estimate area of parallelograms? (chose a base, drew the perpendicular to it, estimated the length of base and the height, then multiplied the estimates) Can we use the same method here? (yes) Invite a volunteer to choose a base and draw the height, and then invite another volunteer to estimate the distances. Have students calculate an estimate for the area of the triangle. Then invite volunteers to measure the base and the height and have students calculate the resulting area.

Exercises
a) Draw a triangle not on a grid. Choose a base and draw the height.
b) Estimate the area of your triangle.
c) Choose a different side as the base and re-estimate the area.
d) Calculate the area of the triangle to check your estimates.

Extensions
1. Another method to find the height of a triangle is to fold the triangle through the vertex opposite the base so that the base folds onto itself. Students can check that this method produces a right angle. Have students use this method to estimate and measure the area of the triangles they used in the Activity during the lesson.

2. Draw a right triangle with the same area as the triangle. Can you draw a right triangle with the right same area and the same base? (You do not need to cut and shift pieces.)

   a)

   b)

3. a) Drawing the height of an obtuse triangle when one of the shorter sides is chosen as the base. Draw the picture in the margin on the board. Ask students to suggest how to draw the height in this triangle if the bottom side is chosen to be the base. If the solution does not arise, explain that when we draw perpendiculars, they are perpendicular to the whole line containing the base,
not just the line segment itself. We can draw a perpendicular to the line that contains the base by first extending the base.

Draw several triangles on a grid (so that each has a short horizontal side) and mark the horizontal sides as bases. Ask students to copy the triangles and to draw heights.

\[ \text{i) } \quad \text{ii) } \quad \text{iii) } \]

b) Do the triangles $T_1$, $T_2$, $T_3$, and $T_4$ all have the same area? Explain.

**Answer:** Yes, because they all have equal bases and the same height.
ME6-14 Area of Trapezoids and Parallelograms
Pages 110–111

CURRICULUM REQUIREMENT
AB: optional
BC: required
MB: optional
ON: optional

VOCABULARY
area
base
height

Goals
Students will develop the formula for the area of a trapezoid using parallelograms, and then apply it.

PRIOR KNOWLEDGE REQUIRED
Is familiar with correct order of operations
Can multiply and divide decimals by a whole number
Can find the area of a parallelogram, rectangle, and triangle on a grid using a formula
Understands that area is additive
Can perform rotations on a grid
Can draw parallel lines

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. T-1)
sissors
folk art items that use geometric shapes, stripes of beading, birch bark biting, or embroidery (see Extension 1)

Finding the area of trapezoids drawn on grids. Display several right trapezoids on a grid, as in the exercises below.

Exercises: Copy the trapezoid on grid paper. Draw a line to split the trapezoid into a triangle and a rectangle.

a)  

b)  

c)  

Review how to find the area of a rectangle and right triangle. Ask students to find the area of their trapezoids by finding the areas of the rectangles and triangles and adding them.

Have a volunteer show how to divide the trapezoid in the margin into a rectangle and two right triangles, then find the area of the trapezoid.

(8 sq. units + 2 sq. units + 6 sq. units = 16 sq. units)

Exercises: Divide the trapezoid into a rectangle and two right triangles. Find the area of the trapezoid.

a)  

b)  

c)  

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**Bonus**

d)\[\begin{array}{c}
\hline
\midrule
\text{Grid 1} \\
\hline
\end{array}\]
e) Find the area of the surrounding rectangle, then use subtraction to find the area of the trapezoid.

**Selected answers:** a) 16 sq. units, b) 15 sq. units, c) 15 sq. units.

**Answers:** Bonus: d) area of rectangle is 8, areas of triangles are 2 and 6, so area of trapezoid is 16 sq. units; e) area of rectangle is 24, areas of triangles are 6 and 10, so area of trapezoid is $24 - 6 - 10 = 8$ sq. units.

**Finding the area of trapezoids not drawn on a grid.** Draw several right trapezoids without an underlying grid and mark the bases and the sides perpendicular to them. Invite volunteers to once again divide the trapezoids into triangles and rectangles. Ask students to label all four sides of each rectangle with their lengths and then figure out the bases of the right triangles from the length of the bases of the trapezoids. After that, students should find the areas of the trapezoids.

**Exercises:** Find the area of the trapezoid.

*a*)\[\begin{array}{c}
\hline
\midrule
\text{8 m} \\
\hline
\end{array}\]

*b*)\[\begin{array}{c}
\hline
\midrule
\text{7 cm} \\
\hline
\end{array}\]

*c*)\[\begin{array}{c}
\hline
\midrule
\text{9 cm} \\
\hline
\end{array}\]

**Answers:** short side of triangle: a) $10 - 8 = 2$ m, b) 2 cm, c) 3 cm; area of trapezoid: a) $8 \times 5 = 40$ m², $2 \times 5 \div 2 = 5$ m², so area of trapezoid $= 40 + 5 = 45$ m², b) 15 cm², c) 115.5 cm²

**Review area of parallelogram.** Remind students that any pair of parallel sides can be chosen to be the base, and that the height is the distance between the bases measured along a line perpendicular to the bases, and that area of parallelogram $= \text{base} \times \text{height}$.

**Making parallelograms with two copies of a trapezoid.** Ask students to fold a sheet of grid paper in two along a grid line, draw a trapezoid that is not symmetrical and has no right angles (see example in margin), and cut the trapezoid through both halves of the sheet to obtain two copies of the same trapezoid. Ask students to draw the height on both trapezoids. Ask them if they can arrange these two trapezoids to create a rectangle without cutting. They should quickly realize that they can’t.
ASK: Can you rearrange the trapezoids to make another kind of quadrilateral? What type of quadrilateral is it? Students should find that they can make parallelograms.

Developing a formula for the area of a trapezoid. ASK: Can you make different parallelograms with your trapezoids? (yes) Do these parallelograms have the same area? (yes) How could you find the area of the parallelogram? How can you get the length of the base of the parallelogram from the lengths of the bases of the trapezoid? (add the bases) Write on the board:

\[
\text{base of parallelogram} = \text{base 1 of trapezoid} + \text{base 2 of trapezoid}
\]

ASK: What is the height of the parallelogram? (the same as the height of the trapezoid) Write on the board: “height of parallelogram = height of trapezoid.” ASK: Which part, or fraction, of the parallelogram is the trapezoid? (half) How could you find the area of the trapezoid from the area of the parallelogram? (divide by 2)

Write on the board:

\[
\text{Area of parallelogram} = \text{base of parallelogram} \times \text{height of parallelogram}
\]
\[
\text{Area of trapezoid} = \frac{\text{area of parallelogram}}{2}
\]

Point out that students have faced a similar problem before, when they developed a formula for the area of a triangle using the formula for the area of a parallelogram. Remind students that it is common in mathematics to look at how you solved a similar problem and to try to apply the same method in a new setting. SAY: When developing the formula for the area of a triangle, we replaced parts of the formula that used the elements of a parallelogram with corresponding parts that used the elements of a triangle. Have volunteers help you to replace the parts of the formula that use elements of a parallelogram with parts that use elements of a trapezoid. Replace:

“area of a parallelogram” with “base of parallelogram \times height of parallelogram,”

“base of parallelogram” with “base 1 of trapezoid + base 2 of trapezoid,”

and “height of parallelogram” with “height of trapezoid.”

After the second replacement, point out that we need to ensure that the addition is done first, before the multiplication. ASK: What can we use to ensure that addition is done first? (brackets) Have students add brackets around the sum before the next replacement. Once all the replacements have been made, point out that since everything in the formula now relates to the trapezoid, we do not need to say “of trapezoid” every time, so we can rewrite the formula. Write the revised formula below the “extended” one on the board:

\[
\text{Area of trapezoid} = (\text{base 1} + \text{base 2}) \times \text{height} \div 2
\]
**Exercises:** Find the area of the trapezoid.

a) ![Trapezoid 1](image1)

\[ b_1 = 4 \text{ cm}, \quad b_2 = 3 \text{ cm}, \quad h = 6 \text{ cm} \]

b) ![Trapezoid 2](image2)

\[ b_1 = 4 \text{ m}, \quad b_2 = 2 \text{ m}, \quad h = 7 \text{ m} \]

c) ![Trapezoid 3](image3)

\[ b_1 = 3 \text{ mm}, \quad b_2 = 6 \text{ mm}, \quad h = 9 \text{ mm} \]

d) ![Trapezoid 4](image4)

\[ b_1 = 6.2 \text{ m}, \quad b_2 = 2 \text{ m}, \quad h = 4.5 \text{ m} \]

**Answers:** a) 15 cm², b) 11 m², c) 22.5 mm², d) 10.7 m²

Explain that when you want to write a short form for the formula of the area, you need a way to distinguish between the bases. SAY: The common way to do so is to write the numbers 1 and 2 beside the letters, but make the numbers visibly smaller than the letter, similar to the 2 in m². However, to avoid mixing with the notation for the square units (which is used in other cases in mathematics), mathematicians agreed to write those numbers towards the bottom of a letter, like this: \( b_1 \) and \( b_2 \). So we write \( b_1 \) instead of base 1 and \( b_2 \) instead of base 2. Rewrite the formula in the short form: \[ A = \left( b_1 + b_2 \right) \times h \div 2. \]

**NOTE:** Extension 1 is required in order to cover the British Columbia curriculum.

**Extensions**

1. Bring examples of household items decorated with any forms of folk art that uses geometric shapes, stripes of beading, birch bark biting, or embroidery. Invite students to bring from home such items that are related to their heritage. Have students identify geometric shapes as parts of decorations, or the shape of the stripes. Have them measure the shapes and find the area of the decorated parts or polygons.

2. **Magic trick.** Draw an 8 \( \times \) 8 square on grid paper (the smaller the grid, the better the trick works). Cut and rearrange the parts as shown:

![Magic trick](image5)

**ASK:** What is the area of the square? (64 square units) What is the length of the rectangle? (5 + 8 = 13 units) What is its width? (5 units) What is the area of the rectangle? (65 square units) How can that be?

To understand where the additional square comes from, draw a 13 \( \times \) 5 rectangle on 2 cm grid paper. Carefully (using a ruler!) draw parts identical to the parts of the square. What do you notice?
The pieces do not fit together perfectly, leaving a gap in the shape of a parallelogram in the middle. To see the parallelogram properly, you need a fairly large grid.
Measurment 6-15  Variables and Area

Goals
Students will use equations to solve problems involving the area of parallelograms and rectangles.

Prior Knowledge Required
- Is familiar with decimals to tenths
- Can multiply and divide decimals by a whole number
- Knows the formulas for the area of a parallelogram, a rectangle, and a triangle
- Is familiar with variables
- Can solve an equation of the type \( ax = b \)

Materials
- BLM Organizing Data (pp. P-53–54)
- BLM Area Spinners (p. P-55)
- BLM Area of Rhombuses (p. P-56, see Extension 3)

Mental Math Minute.
Write “50 \times 6.8 \times 2” on the board. SAY: Order does not matter in multiplication. I can switch between the last two numbers. Record the rest of the solution on the board, as shown in the margin, and prompt students to tell you what to write at each step.

Exercises: Use numbers that multiply to 10, 100, or 1000 to multiply in an easier way.

\[
\begin{align*}
a) & \quad 500 \times 37 \times 2 \\
b) & \quad 500 \times 91 \times 0.002 \\
c) & \quad 40 \times 2.5 \times 0.74 \\
d) & \quad 2.5 \times 4 \times 0.85
\end{align*}
\]

Answers: a) 37 000, b) 91, c) 74, d) 8.5

Review the formulas for area learned to date. Remind students that the formula for a rectangle can be written as “length \times width” as well as “width \times height,” but in both cases we find the area by multiplying the lengths of the adjacent sides.

Exercises: Find the areas.

\[
\begin{align*}
a) & \quad \text{width } 3 \text{ cm, height } 4 \text{ cm, area of rectangle } = \\
b) & \quad \text{width } 6 \text{ mm, length } 7 \text{ mm, area of rectangle } = \\
c) & \quad \text{base } 3 \text{ m, height } 5 \text{ m, area of parallelogram } = \\
d) & \quad \text{base } 6 \text{ cm, height } 4 \text{ cm, area of triangle } = \\
\end{align*}
\]

Answers: a) 12 cm², b) 42 mm², c) 15 m², d) 12 cm²
**Using variables to organize data.** Tell students that problems often give the area of an item and ask you to figure out one of the components of the formula. For example: “A parallelogram has area 35 cm² and height 5 cm. What is the base of the parallelogram?” Start a table on the board, as in Question 1 on AP Book 6.2 p. 112, and fill in the data that is given in this problem. Leave the “Base or Width” column empty. ASK: What do we use when we have to deal with unknown numbers? (variables) Remind students that we usually use letters to represent unknown numbers, and in this case you will write $b$ so that you do not have to write the whole word “base.” Write the variable $b$ in the column “Base or Width.”

Add more problems to the table, such as those in the exercises below, and work together as a class to fill it in. Do not include problems that require finding the base or height of triangles.

**Exercises:** Record the given information and the unknown variable.

a) Base of parallelogram 4 m, area 24 m²
b) Base of triangle 6 cm, height 4 cm
c) Width of rectangle 7 mm, area 63 mm²
d) ![Parallelogram with area 120 m²](image)
e) ![Rectangle with area 12 km²](image)
f) ![Parallelogram with area 25 cm²](image)
g) A rectangle with area 72 cm² has width 12 cm. What is its height?  
h) A triangle with base 12 cm has height 2.5 cm. What is its area?  
i) A parallelogram with base 7 mm has area 77 mm². What is its height?

**Review solving equations.** Show how to solve the equation $5x = 15$ on the board. Then have students solve the equation $8x = 144$. ($x = 18$)

**Writing and solving equations for area problems.** Return to the first problem above (A parallelogram has area 35 cm² and height 5 cm. What is the length of the base of the parallelogram?). Write the formula for the area of a parallelogram and ask volunteers to help you replace parts of the formula with the known data. Replace the word “base” with the variable $b$. Then solve the equation. At each step, prompt students to help you by asking, What do I do next?

Work through the rest of the table as a class, writing and solving equations for each problem.

**Organizing data in a word problem.** Explain to the students that when solving a word problem, especially a measurement problem that requires using a formula, it really helps to organize the data. **SAY:** When the data is organized, it is easy to see which formula to use and what numbers to substitute into it.
Display the following problem on the board:

A plot of land is a parallelogram with two sides 150 m long.
The distance between these sides is 50 m. What is the area of the
plot of land?

Draw a parallelogram on the board. Point at two adjacent sides and
ASK: Are these the sides the problem is talking about? (no) Why not?
(The problem is talking about the distance between the sides that are
150 m long, so these sides have to be opposite each other.) Add the
information from the problem to your parallelogram, creating the picture
in margin. Then draw on the board:

Given: ________________

____________________

Find: ________________

Formula: ________________

Point out that when you write the area, you want to use words that are as
close to the words in a formula as possible. SAY: Instead of writing “the
area of a plot of land,” you should use the name of the plot’s geometrical
shape. Instead of “the distance between the sides,” use the word “height,”
because the height is the distance between the bases. Ask volunteers to
help you fill in the information on the board.

Repeat with this problem:

A wafer wrapper is a rectangle 9 cm wide. Its area is 140.4 cm².
What is the length of the wrapper?

Keep both problems and data on the board for further use.

**Exercises:** Write out the data given and unknown, and the formula needed
for the problem.

a) A rectangular desk is 75 cm long and 50 cm wide. What is the area
   of the desk?

b) A triangular banner has base 5 cm and height 12 cm. How much cloth
   is needed for the banner?

c) Paper tubes are made from parallelograms 10 cm tall with area
   190 cm². What is the length of the base of each parallelogram?

d) To make a pillowcase, Yu uses 8800 cm² of cloth. It is a rectangle
   55 cm wide. How long is the rectangle Yu uses?

e) A window is a parallelogram with horizontal sides 70 cm and height
   40 cm. How much glass is in the window?

**Bonus**

A window has 4 straight sides, each 45 cm long. The window is 30 cm
high. What is the shape of the window? How much glass is in the window?
Selected answers: a) Given: length: 75 cm, width: 50 cm, Find: area of rectangle, Formula: area = length \times width; Bonus: rhombus; Given: base: 45 cm, height: 30 cm, Find: area of rhombus, Formula: area = base \times height

NOTE: Students who have trouble organizing data can benefit from BLM Organizing Data, which concentrates on problems that require finding area without going into the complication of finding missing information.

Writing an equation for a word problem. Point out that it is inconvenient to write the whole word for each piece of information each time. Remind students that they can use letters instead of words in a formula. Use the two examples on the board to show how to substitute the given information and the variable. You will get the equations $150 \times 50 = A$ for the first problem and $l \times 9 = 140.4$ for the second problem. Have students solve the equations, and check answers on the board.

Exercises: Organize the data. Write and solve the equation to solve the problem.

a) What is the base of a parallelogram with height 3 m and area 24 m²?

b) A garden path is a rectangle 90 cm wide. Glen used 22 500 cm² of tiles to cover the path. How long is the path?

c) A triangular traffic island is covered in grass. The island has two perpendicular sides, 8.3 m and 2 m. How much grass is needed to cover the traffic island?

d) A parking spot is a parallelogram 4 m wide. It covers an area of 26 m². How long is the parking spot?

e) Hanna’s bedroom has area 10.5 m², and it is 3 m long. How wide is the room?

Bonus
A lawn is a triangle with all sides 20 m. The distance from one side to the opposite vertex is 17.1 m.

f) How much space does the lawn take up?

g) Clara needs 20 grams of fertilizer for each square meter of lawn. How much fertilizer does she need in total?

Selected answers: a) 8 m, b) 250 cm, c) 8.3 m², d) 6 m, e) 3.5 m, Bonus: f) 171 m², g) 3420 g

ACTIVITY (Essential)

Students can work in pairs to create problems for each other to solve. They can use the spinners on BLM Area Spinners to generate a variety of problems. Each player spins both spinners and secretly picks two numbers to be the dimensions of the shape. Each player finds the area of the shape chosen by the spin, then writes a problem with one of the data values missing, according to what the spinner shows. Players solve each other’s problems and check the solutions.
Example:
Spinners show: metres,

Player 1 picks height 2.5 m and base 3 m, and finds the area, 7.5 m².

Problem for Player 2: A parallelogram has base 3 m and area 7.5 m². What is its height?

Variations: To modify the level of difficulty, you can ask students to use only whole numbers, or to use decimals for the numbers they pick. More advanced students can also identify a real-world context for their problems; the shapes could be flower beds, pins with logos of a favourite team, and so on.

Extensions
1. Jasmin writes out the data for a problem:

   Given: height = 5 km²
   area = 20 km

   How do you know she has made a mistake?

2. What is the height of the trapezoid in the margin? Write and solve an equation.

   Answer
   Base 1 = 3 cm
   Base 2 = 5 cm
   Area of trapezoid = 24 cm²
   Formula: \( A = \frac{(b_1 + b_2) \times h}{2} \)
   \( 24 \text{ cm}^2 = (3 \text{ cm} + 5 \text{ cm}) \times h \div 2 \)
   \( 24 \text{ cm}^2 = 8 \text{ cm} \times h \div 2 \)
   \( 24 = 4 \times h \)
   \( h = 24 \div 4 \)
   \( = 6 \text{ cm} \)

3. A special formula for the area of a rhombus. Introduce the term diagonal—a line segment that joins two vertices that do not have a common edge. Each quadrilateral has two diagonals. Draw several quadrilaterals on a grid, and have students copy them and draw diagonals in them. Then have students work through BLM Area of Rhombuses (p. P-56). They will discover that the area of a rhombus equals half the product of the diagonals: \( \text{diagonal 1} \times \text{diagonal 2} \div 2 \). Note that this formula is not a special case of the formula for the area of a parallelogram, because this formula does not use the base or the height of the rhombus, but uses the lengths of the diagonals.

   Selected answers: 1. b) 4, c) 1/2, d) 1/2; 2. b) 2; 3. a) 20 cm², b) 6 m², c) 36 m², d) 37.5 m²
Goals

Students will convert measurements expressed as a decimal to a smaller unit.

Students will investigate connections between different units of area.

Students will solve problems related to area, including problems that require converting units.

PRIOR KNOWLEDGE REQUIRED

Is familiar with decimals to thousandths

Can perform operations with decimals

Can find areas of rectangles, parallelograms, and triangles

Is familiar with metric units of length and area

MATERIALS

metre stick

Mental math minute—number talk. Present this problem: 2.5 ÷ 5. (0.5)

The following strategies could arise:

(double of 2.5) ÷ (double 5) = 5 ÷ 10

(10 × 2.5) ÷ (10 × 5) = 25 ÷ 50 = 1/2 = 0.5

the divisor is double the dividend, so the fraction is equivalent to 1/2

(10 × 2.5) ÷ 5 = 10 × answer, so find 25 ÷ 5 and divide by 10

Converting decimal measurements in metres to centimetres.

ASK: How many centimetres are in 1 m? (100) How many centimetres are in 5 m? (500) How do you convert measurements in metres to centimetres? (multiply by 100) What fraction of 1 m is 1 cm? (1/100) What other unit (that is not a unit of length) do you know that has 100 small units in one large unit? (100 cents in 1 dollar)

Remind students that they measured objects, such as a blackboard, in metres and in centimetres. They found that the blackboard is 4 m long to the closest metre, or 3 m 75 cm long. Remind students that when they have an amount such as 3 dollars and 75 cents, they write it as $3.75. SAY: The decimal point separates the dollars from the cents. We can use decimal notation to write a measurement in metres: for example, 3 metres 75 centimetres can be written as 3.75 m.

SAY: I want to convert 3.75 m to centimetres. You said we multiply by 100 to convert measurements in metres to centimetres. ASK: What is 3.75 × 100? (375) How do you multiply a decimal by 100? (move the decimal point 2 places to the right) SAY: Let’s check that we got the correct answer.
Write on the board:

\[
\begin{align*}
3.75 \text{ m} &= 3.75 \times 100 \text{ cm} = 375 \text{ cm} \\
3 \text{ m } 75 \text{ cm} &= \underline{300} \text{ cm} + \underline{75} \text{ cm} = \underline{375} \text{ cm}
\end{align*}
\]

ASK: How many centimetres are in 3 m? (300 cm) Fill in the first blank.
ASK: What do we do now? (add the leftover centimetres) Fill in the second blank, and then have a volunteer finish the calculation.

Remind students that when they do not have enough digits after the decimal point to move the decimal point two places to the right, they can write zeros to the right of the last non-zero digit. For example, to convert 3.4 m to centimetres, they can write 3.4 m as 3.40 m.

**Exercises:** Convert the measurement from metres to centimetres.

a) 5.92 m  

b) 34.02 m  

c) 0.45 m  

d) 6.6 m  

**Bonus:** 233.458 m

**Answers:** a) 592 cm, b) 3402 cm, c) 45 cm, d) 660 cm, Bonus: 23 345.8 cm

Remind students that there are 10 mm in 1 cm, and 1000 m in 1 km.

Have volunteers convert 3.9 cm to millimetres and 2.45 km to metres.

(39 mm, 2450 m)

**Exercises:** Convert the measurement.

a) 4.7 cm to mm  

b) 1.609 km to m  

c) 2.6 km to m  

d) 0.7 cm to mm  

e) 0.789 km to m  

**Bonus:** 0.7 m to mm

**Answers:** a) 47 mm, b) 1609 m, c) 2600 m, d) 7 mm, e) 789 m, f) 30 m, Bonus: 700 mm

**Converting units of area.** Draw a square on the board using a metre stick and mark two sides as 1 m. Ask students to convert the side lengths to centimetres. Then ask them to find the area in square centimetres. Repeat with several other squares with lengths in metres, and summarize the data in a table, as shown below. ASK: What number do you multiply area in square metres by to get area in square centimetres? (10 000)

<table>
<thead>
<tr>
<th>Area (m²)</th>
<th>Area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 000</td>
</tr>
<tr>
<td>4</td>
<td>40 000</td>
</tr>
<tr>
<td>9</td>
<td>90 000</td>
</tr>
</tbody>
</table>

**Exercises:** Convert the measurement to square centimetres.

a) 5 m²  

b) 1.2 m²  

c) 100 m²  

d) 0.86 m²  

**Bonus:** 0.005 m²

**Answers:** a) 50 000 cm², b) 12 000 cm², c) 1 000 000 cm², d) 8600 cm², Bonus: 50 cm²
Data needs to be in units of the same type. Draw on the board:

\[
\text{Area of rectangle} = 2 \text{ m} \times \underline{\quad} = 12000 \text{ cm}^2
\]

ASK: What should we put in the blank? Will 6000 m work? Have students check: \(2 \text{ m} \times 6000 \text{ m} = 12000 \text{ m}^2\), which is not the area given. SAY: 6000 m does not work. ASK: Will 6000 cm work? Have students check again: \(2 \text{ m} \times 6000 \text{ cm}\). Remind students that when we multiply measurements, they must be in the same units. ASK: What should we do to find the missing length in this problem? Point out that mathematicians often try to change a problem into another problem that they already know how to solve. SAY: We know how to multiply metres by metres or centimetres by centimetres. ASK: Could we change the problem with the rectangle into one of these two? How? (by changing 2 metres to centimetres) Have students perform the conversion: \(2 \text{ m} = 200 \text{ cm}\). Write the new equation on the board:

\[
\text{Area of rectangle} = 200 \text{ cm} \times \underline{\quad} = 12000 \text{ cm}^2
\]

ASK: Which measurement fits in the blank? (60 cm) Point out that at this point there is no question about which units the missing measurement should be in; both lengths are in centimetres, and the area is in square centimetres.

**Exercises:** Fill in the missing unit.

a) \(5 \text{ cm} \times 7 \text{ cm} = 35 \underline{\quad}\)  
b) \(5 \text{ mm} \times 3 \underline{\quad} = 15 \text{ mm}^2\)

c) \(4 \text{ m} \times 3 \underline{\quad} = 12 \text{ m}^2\)

**Answers:** a) cm², b) mm, c) m

**Solving problems that require converting measurements.** Present the following problem:

Jayden uses a parallelogram with area 1950 mm² to make a logo for a soccer team's badge. The badge is 6 cm long at the base. What is the height of the badge?

Use the problem to review organizing data. Point out that the measurements are in different units. ASK: What do you have to do? (convert centimetres to millimetres) Have students change the data, then write an equation and solve the problem. Have volunteers show the solution on the board. Point out that writing the units in the equation can help them to make sure all units match. \((60 \text{ mm} \times h = 1950 \text{ mm}^2, h = 32.5 \text{ mm})\) Point out that in the exercises below students will sometimes need to convert the units of length, and sometimes convert the units of area.
**Exercises:** Organize the data. Then solve the problem.

a) The area of a parallelogram is 24 m\(^2\). Its height is 60 cm. What is the length of the base of the parallelogram?

b) A triangular traffic island has base 5 m and height 380 cm. The traffic island needs to be covered in grass. How much grass is needed?

c) A backyard is a rectangle 850 cm wide. Its area is 255 m\(^2\). What is the length of the yard?

**Bonus:** Write your answers to the questions above in both metres and centimetres.

**Answers:** a) 4000 cm = 40 m, b) 95 000 cm\(^2\) = 9.5 m\(^2\), c) 3000 cm = 30 m

**Finding the area of shapes made from other shapes.** Draw the picture below on the board and work as a class through finding the area. (change the lengths to the same unit, find the area of each part, then add the areas)

The area of the shape is \(240 \text{ cm} \times 85 \text{ cm} \div 2 + (240 \text{ cm} \times 120 \text{ cm}) = 39 000 \text{ cm}^2\).

![Diagram of a shape with dimensions 240 cm, 120 cm, and 0.85 m, with calculations for finding the area.]

**Exercises:** Find the area of the shape.

**Answers:** a) 125 000 cm\(^2\), b) 40 000 cm\(^2\)

**Extensions**

1. a) Write the length and width of each square in millimetres. The squares are not drawn to scale.

   i) \(1 \text{ cm} = \_\_\_ \text{ mm}\)

   ii) \(2 \text{ cm} = \_\_\_ \text{ mm}\)

   iii) \(3 \text{ cm} = \_\_\_ \text{ mm}\)
b) Find the area of each square in square centimetres and in square millimetres.

c) To convert square centimetres to square millimetres, what number do you multiply by?

d) \(1 \text{ m} = 100 \text{ cm}\), so \(1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10 000 \text{ cm}^2\)

\(1 \text{ cm} = 10 \text{ mm}\), so \(1 \text{ cm}^2 = \text{ mm} \times \text{ mm} = \text{ mm}^2\)

\(1 \text{ km} = 1000 \text{ m}\), so \(1 \text{ km}^2 = \text{ m} \times \text{ m} = \text{ m}^2\)

**Selected answers:**
bi) \(1 \text{ cm}^2 = 100 \text{ mm}^2\), ii) \(4 \text{ cm}^2 = 400 \text{ mm}^2\),
iii) \(9 \text{ cm}^2 = 900 \text{ mm}^2\),
c) 100,
d) 10, 10, 100, 1000, 1 000 000

2. Find the area of the shaded trapezoid in two ways.

![Trapezoid Diagram]

a) Use the formula for the area of a trapezoid.

b) Find the area of the rectangle and subtract the areas of the two triangles.

**Answers:**
a) \((7 \text{ cm} + 5 \text{ cm}) \times 2 \text{ cm} \div 2 = 12 \text{ cm} \times 2 \text{ cm} \div 2 = 12 \text{ cm}^2\),
b) area of rectangle: \(2 \text{ cm} \times 8 \text{ cm} = 16 \text{ cm}^2\), area of larger triangle:
\(2 \text{ cm} \times 3 \text{ cm} \div 2 = 3 \text{ cm}^2\), area of smaller triangle: \(1 \text{ cm} \times 2 \text{ cm} \div 2 = 1 \text{ cm}^2\), so shaded area = \(16 \text{ cm}^2 - 3 \text{ cm}^2 - 1 \text{ cm}^2 = 12 \text{ cm}^2\).

3. Find the area of the triangle.

![Triangle Diagram]

**Answer:** 2.5 square units
Area of Parallelograms (Advanced)

1. The grid was rotated. Can you still find the area of the parallelogram?
   a) 
   b)
   
   Base: __________
   Height: __________
   Area: __________

2. Trace the dashed line that represents the height.
   a)  
   b)  
   c)  
   d)  
   base
   base
   base
   base

3. The base is drawn as a thick line. Draw a height to the base.
   a)  
   b)  
   c)  
   d)  

4. Draw the perpendicular to the thick side. Use the thick side as the base and find the area.
   Did you get the same answer both ways? If not, find your mistake.
Triangles on Grid Paper
Obtuse Triangles on a Grid

1. a) Shade triangle $ADC$ in the first picture.
   
   What fraction of rectangle $ABCD$ is triangle $ADC$? _____
   
   What is the area of triangle $ADC$? _____ $\times$ _____ $\div$ 2 = _____

b) Shade triangle $ECD$ in the second picture.
   
   What is the area of triangle $ECD$? _____

c) Shade $AEC$ in the third picture.
   
   Since area of $ACD$ = area of $AEC$ + area of $ECD$
   
   then area of $AEC$ = area of $ACD$ – ____________

d) What is the area of triangle $AEC$? _____

e) Use the same method to find the area of the triangles below.

   i)  
   
   Area of triangle $ACD$ _____
   
   Area of triangle $ECD$ _____
   
   Area of triangle $AEC$ _____

   ii)  
   
   Area of triangle $ACD$ _____
   
   Area of triangle $ECD$ _____
   
   Area of triangle $AEC$ _____

   iii)  
   
   Area of triangle $ACD$ _____
   
   Area of triangle $ECD$ _____
   
   Area of triangle $AEC$ _____

   iv)  
   
   Area of triangle $ACD$ _____
   
   Area of triangle $ECD$ _____
   
   Area of triangle $AEC$ _____
Triangles
Organizing Data (1)

1. Match each area formula to a shape: rectangle, triangle, or parallelogram.
   - Area of ________________ = base × height
   - Area of ________________ = length × width = width × height
   - Area of ________________ = base × height ÷ 2

2. Which formula from Question 1 would you use to find the area of the shape?
   a) square _______ Area of rectangle = length × width _______
   b) rhombus ________________________________

3. Write the shape whose area you need to find and the formula for that area.
   a) A square has sides 5 cm long. What is its area?
      Given: width = 5 cm   Find: area of _______ rectangle _______
      length = 5 cm   Formula: Area = length × width

   b) A book cover is 30 cm long and 20 cm wide. What is the area of the cover?
      Given: length = 30 cm   Find: area of
      width = 20 cm   Formula:

   c) Find the area of a rhombus with base 3 cm and height 2.5 cm.
      Given: base = 3 cm   Find: area of
      height = 2.5 cm   Formula:

   d) A parking lot is a parallelogram with two sides 200 m long.
      It measures 75 m between these sides. What is the area of the parking lot?
      Given: base = 200 m   Find: area of
      width = 75 m   Formula:

   e) A traffic island is a triangle with perpendicular sides 1.5 m
      and 4 m. How many square metres of grass are needed to
      cover the traffic island?
      Given: base = 4 m   Find: area of
      height = 1.5 m   Formula:

4. Solve each problem in Question 3.
Organizing Data (2)

5. Organize the data. Write the formula for the area you need to find. Substitute the data into the formula to find the area.

a) A park has the shape of a parallelogram. It has two sides 200 m long that run parallel to each other, and the sides are 135 m apart. How much space does the park take up?

Given: base = 200 m  height = 135 m

Formula: Area = base \times height

Find: area of parallelogram

\[
\text{Area} = 200 \, \text{m} \times 135 \, \text{m} = 27,000 \, \text{m}^2
\]

b) Find the area of a triangle with base 2 cm and height 3 cm.

Given: base = 2 cm  height = 3 cm

Formula: Area = \frac{1}{2} \times \text{base} \times \text{height}

Find: area of triangle

\[
\text{Area} = \frac{1}{2} \times 2 \, \text{cm} \times 3 \, \text{cm} = 3 \, \text{cm}^2
\]

c) A rectangular curtain is 3 m wide and 1.8 m long. How many square metres of cloth is the curtain made from?

Given: base = 3 m  height = 1.8 m

Formula: Area = \text{base} \times \text{height}

Find: area of rectangle

\[
\text{Area} = 3 \, \text{m} \times 1.8 \, \text{m} = 5.4 \, \text{m}^2
\]

d) A flower bed is a parallelogram with base 4 m and height 1.7 m. How much space does the flower bed take up?

Given: base = 4 m  height = 1.7 m

Formula: Area = base \times height

Find: area of parallelogram

\[
\text{Area} = 4 \, \text{m} \times 1.7 \, \text{m} = 6.8 \, \text{m}^2
\]

e) A triangular pin has a horizontal top side 27 mm long. Its height is 30 mm. How much space does the pin take up?

Given: base = 27 mm  height = 30 mm

Formula: Area = \frac{1}{2} \times \text{base} \times \text{height}

Find: area of triangle

\[
\text{Area} = \frac{1}{2} \times 27 \, \text{mm} \times 30 \, \text{mm} = 405 \, \text{mm}^2
\]

f) A square window has sides 75 cm. How much glass is in the window?

Given: side = 75 cm

Formula: Area = \text{side} \times \text{side}

Find: area of square

\[
\text{Area} = 75 \, \text{cm} \times 75 \, \text{cm} = 5,625 \, \text{cm}^2
\]
Area Spinners

\[
\text{Area} = ?
\]

\[
\text{?}
\]

\[
\text{Area} = ?
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\text{?}
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\text{Area} = ?
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\text{Area} = ?
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\text{?}
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\[
\text{Area} = ?
\]

\[
\text{?}
\]
Area of Rhombuses

1. a) Draw diagonals in each rhombus.

   ![Diagrams of rhombuses with diagonals drawn]

   b) How many right triangles are in each rhombus? _____

   c) The diagonals of the rhombus break the rectangle around each rhombus into smaller rectangles. What fraction of each smaller rectangle is each triangle? _______________

   d) What fraction of the large rectangle is the rhombus? _____

2. a) Find the diagonals of the rhombus. Then fill in the table.

   ![Table with columns for vertical diagonal, horizontal diagonal, width of rectangle, height of rectangle, area of rectangle, and area of rhombus]

<table>
<thead>
<tr>
<th>Vertical Diagonal</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Diagonal</td>
<td>6</td>
</tr>
<tr>
<td>Width of Rectangle</td>
<td>6</td>
</tr>
<tr>
<td>Height of Rectangle</td>
<td>4</td>
</tr>
<tr>
<td>Area of Rectangle</td>
<td>24</td>
</tr>
<tr>
<td>Area of Rhombus</td>
<td>12</td>
</tr>
</tbody>
</table>

   b) Look at the table. Finish writing the formula for the area of a rhombus.

   Area of a rhombus = diagonal 1 × diagonal 2 ÷ _____

3. Find the area of the rhombus. Do not forget the units.

   a) diagonal 1 = 5 cm  b) diagonal 1 = 4 m  c) diagonal 2 = 8 cm  diagonal 2 = 3 m

   ![Diagram of rhombus with dimensions]

   Area = __________  Area = __________  Area = __________  Area = __________
Unit 14  Number Sense: Percentages and Ratios

Introduction
This unit is about ratios, fractions, and percentages, including:

- finding equivalent ratios using multiplication, division, and ratio tables;
- finding and comparing unit rates;
- evaluating percentages;
- comparing fractions, decimals, and percentages; and
- using fractions, ratios, and percentages to solve real-world problems.

Meeting Your Curriculum

<table>
<thead>
<tr>
<th>ALBERTA</th>
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<tr>
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<td>NS6-63</td>
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<td>NS6-61, 62, 68</td>
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<td>Optional</td>
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Mental Math Minutes
The mental math minutes in this unit are dedicated to:

- finding fractions of a number, the percentage of a number, and equivalent fractions
- multiplying decimals by multiples of 10
Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tr>
<td>Quiz</td>
<td>NS6-58 to 60, 63</td>
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<td>NS6-64 to 67</td>
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<td>NS6-68 to 70</td>
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<td>NS6-58 to 70</td>
<td>NS6-58 to 60, 64 to 67, 69, 70</td>
<td>NS6-58 to 62, 64 to 67, 69</td>
</tr>
</tbody>
</table>
Goals
Students will understand ratios as a way to compare one part of a whole to a different part of a whole.

PRIOR KNOWLEDGE REQUIRED
Can interpret fractions

Mental math minute. Review the method shown below for finding equivalent fractions.

\[ \frac{6}{9} \times 5 = ? \]

Exercises: Find the number that makes the fractions equivalent.

a) \[ \frac{1}{2} = \frac{6}{?} \]
b) \[ \frac{3}{5} = \frac{?}{45} \]
c) \[ \frac{10}{7} = \frac{40}{?} \]
d) \[ \frac{9}{20} = \frac{?}{100} \]

Answers: a) 12, b) 27, c) 28, d) 45

Fractions and ratios. Start by reminding students what the numerator and denominator of a fraction are. Draw on the board:

\[ \frac{1}{3} \]

SAY: The numerator (1) tells you the number of shaded parts, and the denominator (3) tells you how many equal parts there are in total. This fraction compares a part to a whole.

Introduce the concept of part-to-part comparison. SAY: A team won 2 games, lost 1 game, and tied 4 games. ASK: How many games did the team play? (7) What is the fraction of games won? (2/7) Write on the board:

\[ \frac{2}{7} \]

SAY: Let’s see another team’s results. The second team won 2 games, lost 3 games, and tied 2 games. ASK: How many games did this team play? (7) What fraction of games did this team win? (2/7) Ask students if this is the same fraction of games won as the other team or a different fraction. (the same)

ASK: Can we use these fractions to determine which team had better results? (no) Why not? (because both teams won the same fraction of games) SAY: So we need another way to compare these two teams. Ask students to look at the other information about the games played and to signal which team had better results, the first or the second. (the first team)
SAY: Both teams won 2 games, but the first team lost 1 game and the second team lost 3 games. The first team had fewer losses, which is better.

Tell students that you’d like to compare games won to games lost, but you can’t use fractions to do that because games won is not a part of games lost. You want to compare a part of the games played (games won) to a different part of the games played (games lost) instead of a part to a whole. SAY: Mathematicians use ratios to compare different parts of a whole to each other. Ratios are written as two numbers with a colon between them.

SAY: The ratio of games won to games lost for the first team is 2 : 1, and for the second team it is 2 : 3. Write these ratios on the board. Point to the ratio 2 : 1 and SAY: We can read or say this ratio as “2 won to 1 lost” or “2 won for every 1 lost” or simply “2 to 1” when it is clear that we are talking about the ratio of games won to the games lost. According to these ratios, the first team had better results than the second team.

Look at some ratios in areas other than sports. Examples:

• Count the number of boys and girls in the class. If you have 13 boys and 15 girls in your class, the ratio of boys to girls is 13 : 15 and the ratio of girls to boys is 15 : 13.
• The ratio of the number of vowels to the number of consonants in the word “ratio” is 3 : 2. Students can compare the number of vowels to the number of consonants in various words and then compare the number of consonants to the number of vowels. They can also compare nouns to verbs, adjectives to nouns, or adverbs to verbs in sentences.
• Draw several rectangles on the board and review the words “length” and “width.” Have students find the ratio of length to width of the rectangles in their notebooks.

**NOTE:** Extension 1 is required in order to cover the Alberta, British Columbia, and Manitoba curricula.

**Extensions**

1. Determine if the ratio is part-to-whole or part-to-part. How do you know?
   a) vowels in “band” : letters in “band”
   b) vowels in “blog” : consonants in “blog”
   c) buses : trucks
   d) school buses to buses
   e) school days to days of the week
   f) days in January to days in September

   **Answers:** a) part-to-whole, vowels are part of letters; b) part-to-part, vowels and consonants are both parts of letters; c) part-to-part, buses and trucks are not parts of each other; d) part-to-whole, a school bus is
a type of bus; e) part-to-whole, school days are part of the whole week; 
f) part-to-part, January and September are both parts of a year

2. Tell students that you want to compare three things at a time: squares to circles to triangles.

Have students find all six possible ratios: squares to circles to triangles, squares to triangles to circles, and so on. ASK: How are these ratios the same as ratios that compare two things, and how are they different? (like other ratios, these ratios compare parts to parts, but they compare three different parts instead of two) Can we compare three numbers at a time using fractions? (no) How are ratios different from fractions? (fractions can compare just one part with the whole, but ratios can compare two or more parts to each other)

3. Remind students that area of rectangle = length × width. Investigate the area-to-perimeter ratio of various rectangles. ASK: What does the ratio look like for short and thin rectangles? What does the ratio look like for almost-square rectangles? Note that students will likely not be able to say at this point which ratio is “larger” as this has not been discussed yet.

4. Tell students that sports teams from Ottawa and Vancouver played each other in both hockey and basketball. The final scores were 5–1 in the hockey game and 99–93 in the basketball game. ASK: Which game was closer? Some students will likely use the difference in the scores (5 – 1 = 4 and 99 – 93 = 6) to decide that the hockey game was closer. Some students may consider the relative difference in the scores instead—the difference versus the total number of goals/points scored by the winning team (4 out of 5 vs. 6 out of 99). Students will learn to make the latter comparison using ratios later in this unit.

5. a) Find the ratio of length to width for the bold rectangles.

b) Are all the diagonals of the bold rectangles part of the same line? Draw the diagonals to check.
**Goals**

Students will understand that equivalent ratios are obtained through multiplication, and they will find examples of equivalent ratios.

**PRIOR KNOWLEDGE REQUIRED**

| Is familiar with equivalent fractions |
| Is familiar with ratios |

**Mental math minute.** Review the method shown below for finding equivalent fractions.

\[
\begin{align*}
\frac{6}{9} \div 3 &= \frac{2}{?} \\
\end{align*}
\]

**Exercises:** Find the number that makes the fractions equivalent.

\[
\begin{align*}
a) \quad \frac{14}{20} &= \frac{7}{?} & b) \quad \frac{56}{35} &= \frac{8}{?} & c) \quad \frac{44}{100} &= \frac{?}{25} & d) \quad \frac{27}{45} &= \frac{5}{?} = \frac{?}{10}
\end{align*}
\]

**Answers:** a) 10; b) 5; c) 11; d) 3, 6

**Introduce equivalent ratios.** Tell students that you have a pancake recipe that calls for 6 cups of flour and 2 bananas. Draw the picture shown in the margin on the board.

**ASK:** How many cups of flour do we need for only 1 banana? (3 cups) How can you use that information to find the number of cups of flour you would need for 5 bananas? (multiply 5 \( \times \) 3) Emphasize that the number of cups of flour is always three times the number of bananas, so if students know how many bananas they have, they can deduce the number of cups of flour they need. Have students perform this calculation for 7, 6, and 3 bananas. (21 cups of flour, 18 cups of flour, 9 cups of flour) Tell students that they have just found many equivalent ratios. Write on the board:

\[
\begin{align*}
\end{align*}
\]

**ASK:** Why are these ratios called equivalent? (because they show the same ratio) Tell students that to say the ratio of cups of flour to bananas is 3 to 1 is to say that for every 3 cups of flour, we need 1 banana. Draw the picture shown in the margin on the board. **ASK:** If you have 9 cups of flour, how many bananas do you need? (3) Draw 3 cups of flour and 1 banana and repeat until you have 9 cups of flour, as shown in the margin. How many bananas did you draw? (3)

**ASK:** How many cups of flour would you need for 10 bananas? (30) So the ratio 30 : 10 is equivalent to the ratio 3 : 1. Emphasize that 30 is 3 \( \times \) 10, so students can just multiply the number of bananas by 3 to get the number of cups of flour.
Emphasize that in the previous example, students compared numbers through multiplication rather than through addition. SAY: If we had said, “The recipe calls for 3 cups of flour and 1 banana, so it calls for 2 more cups of flour than the number of bananas,” this would be comparing through addition rather than through multiplication. ASK: If I want to make the recipe with 5 bananas, should I use 7 cups of flour since 7 is 2 more than 5? Would the recipe work if I used 2 cups of flour with no bananas since 2 is two more than zero? Will my pancakes turn out right? (no) Explain to students that in this situation (a recipe), 3 (cups of flour) is 3 times 1 (bananas), and it is this 3 : 1 ratio we want to preserve—the number of cups of flour should always be 3 times the number of bananas.

Show students how they can make a series of equivalent ratios by repeatedly drawing 3 cups of flour and 1 banana. Draw on the board:

<table>
<thead>
<tr>
<th>Cups of Flour</th>
<th>Number of Bananas</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

SAY: I would like to use a table to show these ratios. ASK: What will the headings be in my table? (cups of flour and number of bananas) PROMPT: What are the different units? Draw the table shown in the margin on the board.

**Exercises**

a) Fill in the table. Use the number in the first row in each column to skip count.

<table>
<thead>
<tr>
<th>Cups of Flour</th>
<th>Number of Bananas</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

b) Write the next three equivalent ratios in the sequence.

i) 3 : 5 = 6 : 10 =
ii) 2 : 7 =
iii) 4 : 5 =
iv) 8 : 3 =

**Answers**

a) i) 3 5
   6 10
   9 15
   12 20

b) i) 9 : 15, 12 : 20; ii) 4 : 14, 6 : 21, 8 : 28; iii) 8 : 10, 12 : 15, 16 : 20;
   iv) 16 : 6, 24 : 9, 32 : 12
Finding the missing number in an equation between ratios. Explain to students how to find the missing part of a ratio. Write on the board:

\[ 1 : 4 = \_ : 12 \]

Tell students that they should continue the sequence of equivalent ratios until 12 is the second number, so we have \[ 1 : 4 = 2 : 8 = 3 : 12 \]. SAY: To find the ratios 2 : 8 and 3 : 12 from 1 : 4, you can use skip counting or multiplication.

Ask a volunteer how to find the missing part of the ratio in \[ 5 : 6 = 15 : \_ \]. (continue the sequence of equivalent fractions until 15 is the first number)

**Exercises:** Find the missing term in the pair of equivalent ratios.

a) \[ 3 : 5 = \_ : 20 \]  
b) \[ 3 : 4 = \_ : 12 \]  
c) \[ 3 : 4 = 12 : \_ \]  
d) \[ 3 : 5 = 15 : \_ \]  
e) \[ 3 : 5 = \_ : 15 \]

**Answers:**  a) 12, b) 9, c) 16, d) 25, e) 9

**Writing ratios for word problems.** SAY: You can find many real-life examples of ratios. ASK: How many seasons are in a year? (4) SAY: So the ratio of years to seasons is 1 to 4. Write “1 : 4” on the board.

**Exercises:** Fill in the blank to find the ratio.

a) For every \_ months, there is 1 year, so the ratio of months to years is \_ : 1.

b) For every \_ days, there is \_ week, so the ratio of days to weeks is \_ : \_.

c) For every \_ dozen, there are \_ items, so the ratio of dozens to items is \_ : \_.

d) For every \_ mm, there is \_ cm, so the ratio of mm to cm is \_ : \_.

**Solving word problems using equivalent ratios.** SAY: To solve a word problem using equivalent ratios you need to write the ratio and then write a sequence of equivalent ratios to find the answer. Write on the board:

There are 2 red balloons for every 3 pink balloons at a party. If there are 8 red balloons at the party, how many pink balloons are there?

ASK: What is the ratio of red balloons to pink balloons? (2 : 3) Write on the board:

\[
\text{Red} : \text{Pink} \\
2 : 3 \\
4 : 6 \\
6 : 9 \\
8 : 12
\]
Explain to students that you stopped because the question asked how many pink balloons for 8 red balloons. Circle “12” in the pink column and SAY: There are 12 pink balloons for 8 red balloons.

Exercises

a) A recipe calls for 5 cups of flour for every 2 cups of milk. How many cups of milk are needed for 15 cups of flour?

b) Each year has 52 weeks. How many weeks are in 3 years?

c) To make orange paint, you need 3 cups of yellow paint for every 2 cups of red paint. How many cups of yellow paint do you need if you have 12 cups of red paint?

Answers: a) 6, b) 156, c) 18

Extensions

1. Use your fingers and hands to show that 1 : 2 and 5 : 10 are equivalent ratios.

2. The mass density or density of a material is its mass per unit volume. In other words, the ratio of mass to volume is density. For example, the density of water at 4°C is 1 gram per mL. That means that 1 gram of 4°C water has a volume of 1 mL; the ratio of mass to volume is 1 : 1.

   a) Lily has a full 500 mL bottle of water in the fridge (almost 4°C). What is the mass of the water in the bottle?

   b) Lily’s mother has a small scale in the kitchen, and she can weigh up to 5 kg. She has a star-shaped cake pan and wants to know what volume of liquid can fit into the pan. How can she use 4°C water and the scale to find the volume?

   Answers: a) 500 grams; b) She can fill the pan with 4°C water and then weigh the water with the scale. The ratio of mass (in grams) to volume (in mL) is 1 : 1, so if the mass of the water is, for example, 200 grams, then the volume is 200 mL.

3. A salad recipe calls for 3 bell peppers for every 2 tomatoes.

   a) How many tomatoes do you need if you have 12 bell peppers?

   b) How many bell peppers do you need if you have 12 tomatoes?

   c) Did you get the same answer in parts a) and b)? Explain.

   Answers: a) 8; b) 18; c) No, because the ratios are not the same. In part a) you solve $3 : 2 = 12 : ?$, and in part b) you solve $3 : 2 = ? : 12$. 
Goals

Students will create ratio tables for growing patterns and will use ratio tables to identify rules for number patterns.

Prior Knowledge Required

Can make T-tables
Understands what a ratio is
Can skip count
Can find multiples of a whole number

Mental Math Minute. Write on the board:

\[
\frac{3}{5} = \frac{24}{35}
\]

Ask: Is this equation true? (no) How do you know? (you need to multiply by 8 to get 24 from 3, but you need to multiply by 7 to get from 35 from 5)

Have students signal the answers in the exercises below. For the ones that are not true, have volunteers occasionally explain how they know.

Exercises: Is the equation true?

a) \(\frac{3}{5} = \frac{21}{35}\)

b) \(\frac{8}{12} = \frac{24}{36}\)

c) \(\frac{33}{55} = \frac{3}{11}\)

d) \(\frac{7}{5} = \frac{14}{15}\)

e) \(\frac{8}{5} = \frac{24}{15}\)

f) \(\frac{6}{25} = \frac{24}{100}\)

g) \(\frac{5}{25} = \frac{25}{100}\)

h) \(\frac{5}{7} = \frac{35}{25}\)

Answers: a) yes, b) yes, c) no, d) no, e) yes, f) yes, g) no, h) no

Developing ratio tables. Draw the "castle" shown in the margin on the board. Ask: How many blocks do I need to construct this castle? (4) What is the ratio of the number of triangular blocks, or triangles, to the total number of blocks? (1 : 4) Say: I want to construct another castle. Ask: How many blocks do I need to make two castles? (8) Draw on the board:

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Total Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Point to the first number in the second column. Say: You can find the total number of blocks in two castles by adding 4 or by multiplying by 2. Ask a volunteer to complete the third row of the table. (3, 12) Then point to the second row of the table and ask: What is the ratio of the number of triangles to the total number of blocks when I have two castles? (2 : 8) Emphasize that the total number of the blocks is always 4 times the number of triangular blocks, so the ratios 1 : 4 and 2 : 8 show the same comparison.
Ask students to describe how the numbers in the table change. They should notice that the numbers of blocks in each row are multiples of the numbers in the first row. Draw arrows and write the multiples on the arrows, as shown below:

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Total Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>×2</td>
<td>×2</td>
</tr>
<tr>
<td>×3</td>
<td>×3</td>
</tr>
<tr>
<td>×7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

SAY: Imagine seven of these castles. ASK: How many triangles do I need to construct seven castles? (7) How many blocks do I need in total? (28) How do you know? (because $7 \times 4 = 28$) Extend the table as shown below:

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Total Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>×2</td>
<td>×2</td>
</tr>
<tr>
<td>×3</td>
<td>×3</td>
</tr>
<tr>
<td>×7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
</tr>
</tbody>
</table>

SAY: Mathematicians call this special kind of T-table a ratio table. The ratios in a ratio table show the same comparison. Point to the table and tell students that the ratio of the number of triangles to the total number of blocks is 1 : 4. Emphasize that to make a ratio table they must multiply the numbers in the first row by the same number to get the numbers in another row—otherwise, it is not a ratio table!

Exercises: Make a ratio table with three rows for other blocks in the same castle.

a) Number of triangles to number of cylinders

b) Number of cylinders to total number of blocks

Answers

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Cylinders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cylinders</th>
<th>Total Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

T-tables and ratio tables.

Exercises: Determine which tables are ratio tables and which are not. How do you know?

a) 3 1  
   6 2  
   9 3  
   12 4  

b) 1 2  
   2 3  
   3 4  
   4 5  

<table>
<thead>
<tr>
<th>Cylinders</th>
<th>Total Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>50</td>
</tr>
</tbody>
</table>

| Bonus: | |
|--------| |
| 2      | 5  |
| 4      | 10 |
| 10     | 25 |
| 20     | 50 |
**Answers:**
a) ratio table because every row is a multiple of the first row;
b) not a ratio table because if you multiply the first row by 2 you get 2 and 4, but these are not the numbers in the second row; c) not a ratio table because the second row is not a multiple of the first row; Bonus: ratio table because the second row is the first row times 2, the third row is the first row times 5, and the last row is the first row times 10

**Using ratio tables to solve word problems.** SAY: Each student has to pay $8 for a field trip. ASK: How much do two students have to pay? ($16) How much do three students have to pay? ($24) Show the results in a ratio table, as shown below:

<table>
<thead>
<tr>
<th># of Students</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

SAY: We can use ratio tables to help us solve word problems. If I want to know how much seven students pay for the trip, I can extend the ratio table to seven rows and find out. Have a volunteer do so on the board. (56)

**Exercise:** To make a punch you need 3 cups of ginger ale for every 2 cups of cranberry juice. Use a ratio table to find out how many cups of ginger ale are needed for 10 cups of cranberry juice.

**Answer:** 15 cups

**Using multiplication to find equivalent ratios.** Write on the board:

There are 3 oranges for every 2 bananas.

SAY: Every time you add 2 bananas, you have to add 3 oranges. Draw on the board:

```
      O O O O
      O O O O
      O O O O
      O O O O
      O O O O
```

SAY: If you add 2 bananas four times, then you have to add 3 oranges four times. Write on the board:

```
bananas: 2 + 2 + 2 + 2  oranges: 3 + 3 + 3 + 3
```

ASK: How can you say the same thing in terms of multiplication? (if you multiply 2 times 4, then you have to multiply 3 times 4 also) Write on the board:

```
2 : 3
4 × 2 : 3 × 4
```
Exercises

1. Multiply both terms by 4 to make an equivalent ratio.
   a) \( 3 : 5 \)
   b) \( 1 : 2 \)
   c) \( 6 : 7 \)
   d) \( 10 : 9 \)  
   **Bonus:** \( 1200 : 13 \)
   **Answers:** a) \( 12 : 20 \), b) \( 4 : 8 \), c) \( 24 : 28 \), d) \( 40 : 36 \), Bonus: \( 4800 : 52 \)

2. What number is each term being multiplied by to make the second ratio?
   a) \( \times \) \( \frac{3}{5} \frac{30}{50} \times \) 
   b) \( \times \) \( \frac{2}{3} \frac{10}{15} \times \) 
   **Answers:** a) 10, b) 5

3. Multiply both terms by the same number to make an equivalent ratio.
   a) \( \times 2 \) \( \frac{3}{5} \frac{6}{10} \times 2 \) 
   b) \( \times 3 \) \( \frac{2}{5} \frac{6}{15} \times 3 \) 
   **Answers:** a) \( 6 : 10 \), b) \( 6 : 15 \)

4. What number is the first term multiplied by? Multiply the second term by the same number to make an equivalent ratio.
   a) \( 3 : 4 = 9 : \) 
   b) \( 2 : 7 = 8 : \) 
   c) \( 3 : 8 = 30 : \)  
   **Bonus:** \( 13 : 7 = 65 : \) 
   **Answers:** a) \( 3 \times 3, 12 \); b) \( 4 \times 4, 28 \); c) \( 10 \times 10, 80 \); Bonus: \( 5 \times 5, 35 \)

5. Multiply the first term by the same number the second term was multiplied by.
   a) \( 5 : 8 = \) \( \) \( 24 \) 
   b) \( 3 : 10 = \) \( 60 \) 
   c) \( 2 : 9 = \) \( 36 \)  
   **Bonus:** \( 7 : 100 = \) \( 1000000 \) 
   **Answers:** a) \( 15 \), b) \( 18 \), c) \( 8 \), Bonus: \( 700000 \)

Extension

SAY: You can use a shortened form of a ratio table to solve word problems. For example, in the field trip problem from before, if 11 students want to go on the trip, you can make a ratio table like the one in the margin to find the total cost.

SAY: 11 is \( 1 + 10 \). ASK: Can I use addition in the second column and say the missing number is \( 8 + 10 \)? (no) Why can’t I use addition? (because in a ratio table, every row is a multiple of the first row) SAY: I have to find the number being multiplied by in the first column. Then I multiply by that number in the second column to find the missing number. ASK: What number do I have to multiply by in the first column? (11) So what is the missing number in the second column? \( 11 \times 8 = 88 \)
ASK: How much would 20 students have to pay in total to go on the trip? (20 × 8 = $160) To find the 20th row, do I need to fill in all the rows from 1 to 20? (no) How do you know the numbers in the 20th row? (in a ratio table, every row is a multiple of the first row, and since 20 students is 20 × 1, the cost is 20 × 8)

Have students complete the ratio tables.

### a)  
\[
\begin{array}{cc}
1 & 4 \\
2 & 8 \\
3 & 12 \\
4 & 16 \\
\end{array}
\]

### b)  
\[
\begin{array}{cc}
4 & 3 \\
8 & 6 \\
12 & 9 \\
16 & 12 \\
\end{array}
\]

### c)  
\[
\begin{array}{cc}
3 & 1 \\
9 & 3 \\
12 & 4 \\
21 & 7 \\
\end{array}
\]

### Bonus: 
\[
\begin{array}{cc}
1 & 5 \\
2 & 10 \\
3 & 15 \\
8 & 40 \\
\end{array}
\]

**Answers**

### a) 
\[
\begin{array}{cc}
1 & 4 \\
2 & 8 \\
3 & 12 \\
4 & 16 \\
\end{array}
\]

### b) 
\[
\begin{array}{cc}
4 & 3 \\
8 & 6 \\
12 & 9 \\
16 & 12 \\
\end{array}
\]

### c) 
\[
\begin{array}{cc}
3 & 1 \\
9 & 3 \\
12 & 4 \\
21 & 7 \\
\end{array}
\]

### Bonus: 
\[
\begin{array}{cc}
1 & 5 \\
2 & 10 \\
3 & 15 \\
8 & 40 \\
\end{array}
\]
Goals
Students will understand simple multiplicative relationships involving unit rates.

PRIOR KNOWLEDGE REQUIRED
Knows units for money (dollars and cents)
Knows units for distance (kilometres)
Knows units for time (hours and minutes)

Rates and unit rates. Explain that a rate is the comparison of two quantities in different units. For example, "3 apples cost 50¢" is a rate. The units being compared are apples and cents. Have students identify the units being compared in the following rates:

- 5 pears cost $2.
- $1 for 3 kiwis.
- 4 tickets cost $7.
- 1 kiwi costs 35¢.
- Ronin is driving at 50 km per hour.
- On a map, 1 cm represents 3 m.
- A student earns $6 an hour for babysitting.
- The recipe calls for 1 cup of flour for every teaspoon of salt.

(NOTE: In this example, the units are not cups and teaspoons, they are cups of flour and teaspoons of salt.)

Explain that in a unit rate, one of the quantities is always equal to one. Give several examples of unit rates and have students identify the quantity that makes it a unit rate:

- 1 kg of rice per 8 cups of water. (1 kg)
- 1 apple costs 30¢. (1 apple)
- $1 for 2 cans of juice. ($1)
- 1 can of juice costs 50¢. (1 can of juice)
- The speed limit is 40 km per hour. (1 hour)
- She runs 1 km in 15 minutes. (1 km)

Unit rates and ratio tables. SAY: Knowing a unit rate can help to determine other rates. ASK: If one book costs $3, how much do two books cost?

Three books? Four books? Draw the ratio table shown in the margin on the board. Explain to students that they can use this table to find the cost of different numbers of books. ($6, $9, $12)
Exercises: Complete the table for the unit rate.

<table>
<thead>
<tr>
<th># of Books</th>
<th>Cost ($)</th>
<th># of Kids</th>
<th># of Balls</th>
<th># of Days</th>
<th>Allowance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>4</td>
<td>9</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

Answers

<table>
<thead>
<tr>
<th># of Books</th>
<th>Cost ($)</th>
<th># of Kids</th>
<th># of Balls</th>
<th># of Days</th>
<th>Allowance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4</td>
<td>12</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>9</td>
<td>27</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

Write on the board:

Rani can walk 1 km every 20 minutes and she can bike 1 km every 5 minutes. Her school is 2 km away from her home. How much time can she save by riding her bike to school and back instead of walking?

Guide students through the solution. ASK: How far does Rani travel altogether? (2 km + 2 km = 4 km) Have students complete the ratio tables below:

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>Time (minutes)</th>
<th>Distance (km)</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

ASK: How long does it take to walk to school and back? (80 minutes) How long to bike? (20 minutes) How much time is saved if Rani bikes instead of walks? (80 − 20 = 60 minutes or 1 hour)

NOTE: Students will need rulers to complete Question 4 on AP Book 6.2 p. 122.

Extension

To solve questions like the one about biking time versus walking time at the end of the lesson, tell students that they can use one ratio table with three columns as shown below:

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>Walking Time (minutes)</th>
<th>Biking Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>20</td>
</tr>
</tbody>
</table>

So Rani saves 80 min − 20 min = 60 min.
Goals
Students will use division to find unit rates and compare unit rates.

PRIOR KNOWLEDGE REQUIRED
Can make T-tables
Can skip count
Can divide and multiply decimals by whole numbers

MATERIALS
flyers

Mental math minute. Review finding equivalent fractions using multiplication and division, as shown below:

\[ \frac{25}{45} = \frac{\Box}{9} = \frac{99}{\Box} \]

\[ \frac{\Box}{5} \times 11 \]

Exercises: Find the number that makes the fractions equivalent.

a) \[ \frac{16}{20} = \frac{4}{\Box} = \frac{?}{25} \]

b) \[ \frac{21}{35} = \frac{?}{5} = \frac{?}{10} \]

c) \[ \frac{64}{100} = \frac{?}{25} = \frac{48}{?} \]

d) \[ \frac{42}{120} = \frac{?}{20} = \frac{?}{100} \]

Answers: a) 5, 20; b) 3, 6; c) 16, 75; d) 7, 35

Finding unit rates using ratio tables. ASK: If you know that two books cost $6, how can you determine the cost of one book? What makes this problem different from the problems in the previous lesson? (instead of starting with the cost of 1 book, we are now starting with the cost of 2 books; we are not given a unit rate) Draw the ratio table shown in the margin on the board.

SAY: To find the missing number, first we need to find the number being divided by in the first column. ASK: What is that number? (2) Draw an arrowhead from 2 to 1 with “÷ 2” beside it. SAY: Now we can divide by that number in the second column to find the missing number. Write “6 ÷ 2 = 3” on the board. SAY: So one book costs $3.

Explain that rates higher than the given rates can be determined through multiplication of that rate, but the unit rate can be determined through division. Write on the board:

If 1 peach costs 25¢, then 3 peaches cost 75¢ (3 × 25¢).
If 3 peaches cost 75¢, then 1 peach costs 25¢ (75¢ ÷ 3).
Exercises: Find the unit rate to solve the problem.

a) Four pears cost 80¢. How much does 1 pear cost?

b) Twenty-four cans of juice cost $24. How much does 1 can of juice cost?

c) Two books cost $14. How much does 1 book cost?

d) Three teachers supervise 90 students on a field trip. How many students does each teacher supervise?

Bonus: 5 miles equal about 8 km = 8000 m. About how long is 1 mile in metres?

Answers: a) 20¢, b) $1, c) $7, d) 30 students, Bonus: 1600 m

ACTIVITY (Optional)
Bring in some flyers from a grocery store and ask students to determine unit prices and to calculate the cost of quantities greater than one. For instance, if the unit price is $2.75 per item, how much will three items cost? If students do not know how to multiply a decimal number by a single-digit number, challenge them to select and use an alternate unit to dollars: cents. ASK: How many cents are in $2.75? (275) If each item costs 275¢, how many cents will three items cost? (825) What does that equal in dollars? ($8.25)

Finding unit rates by division. Write on the board:

\[
\begin{align*}
1 \text{ kg} & = 1000 \text{ g} \\
1 \text{ km} & = 1000 \text{ m} \\
1 \text{ m} & = 100 \text{ cm} \\
1 \text{ cm} & = 10 \text{ mm}
\end{align*}
\]

ASK: How many metres are in 3 kilometres? (3000) How did you find 3000? (by multiplying 3 by 1000) SAY: To find a measurement in smaller units from a measurement in larger units you multiply, but if you want to find a measurement in larger units from a measurement in smaller units you divide. For example, 5000 grams is 5 kilograms, and you get 5 kilograms by dividing 5000 grams by 1000. ASK: How many centimetres are in 1.5 metres? (150) How many centimetres are in 3.45 metres? (345) Write on the board:

3 bags of rice weigh 7.5 kg. How many grams does each bag weigh?

SAY: Since the question asks how many grams, you may convert kilograms to grams in the first step. ASK: How many grams are in 7.5 kilograms? (7500) Did you multiply or divide? (multiply) Why? (we are converting from a larger unit to a smaller unit) How much did you multiply by? (1000) SAY: Three bags weigh 7500 grams, so to find the mass of one bag you have to divide 7500 by 3 either by long division or mentally. Ask a volunteer to find the answer. (2500 grams) Explain to students that they could instead find the unit rate in kilograms and then convert it to grams. ASK: If three bags weigh 7.5 kg, how much does one bag weigh? (2.5 kg) How many grams are in 2.5 kg? (2500 grams)
Exercises
a) 5 bags of potatoes weigh 7.5 kg. How many grams does each bag weigh?
b) 4 pears cost $2.80. How many cents does 1 pear cost?
c) 3 pomegranates cost $4.50. How much does 1 pomegranate cost?
d) 8 cans of juice cost $2. How much does 1 can of juice cost?
e) A stack of 10 dimes is 1.2 cm tall. How thick is 1 dime?

Answers: a) 1500 grams, b) 70¢, c) $1.50 or 150¢, d) $0.25 or 25¢, e) 0.12 cm or 1.2 mm

Extensions
1. Determine the unit rate and then solve the problem.
   a) If 4 books cost $20, how much do 3 books cost?
   b) If 7 books cost $28, how much do 5 books cost?
   c) If 4 L of soy milk costs $8, how much does 5 L cost?

Answers
   a) Books | Cost ($)  
            4  | 20  
            1  | 5   
            3  | 15  
   b) Books | Cost ($)  
            7  | 28  
            1  | 4   
            5  | 20  
   c) Soy Milk (L) | Cost ($)  
                     4  | 8   
                     1  | 2   
                     5  | 10  

2. a) A 1.5 L bottle of water costs $0.99. A 0.5 L bottle of water costs 69¢. Find the unit rate to see which is cheaper by volume.
   b) A 2 L bottle of juice costs $2.98. A 0.5 L bottle of juice is on sale for $0.85. Find the unit rate to see which is cheaper by volume.
   c) How can you solve the problems in parts a) and b) by using multiplication only, without finding the unit rate?

Sample solutions
   a) For the 1.5 L bottle, 1 L of water costs $0.99 ÷ 1.5, which is equal to $1.98 ÷ 3 = $0.66 = 66¢ for 0.5 L, which is less than the cost of the 0.5 L bottle, so the 1.5 L bottle is cheaper by volume.
   b) 1 L of juice costs $1.49 in the 2 L bottle and costs $1.70 in the 0.5 L bottle, so the 2 L bottle is cheaper by volume.
   c) For part a), 0.5 L × 3 = 1.5 L, so if I multiply the cost of the 0.5 L bottle by 3, I get the cost of the same amount of drink as in the 1.5 L bottle. 3 × 69¢ = 207¢, which is more than $0.99. For part b), 2 L of juice is 4 of the 0.5 L bottles, so 4 × $0.85 = $3.40, which is more than $2.98.

NOTE: Part c) is an example where the unit rate does not have to be 1; in this case, it is 0.5 L since 1.5 L and 2 L are both multiples of 0.5.

Number Sense 6-62 Q-19
Goals
Students will find equivalent ratios through multiplication rather than through repeated addition.
Students will solve word problems using ratios.

PRIOR KNOWLEDGE REQUIRED
Can identify equivalent ratios

Part-to-part problems where a part is given. Write on the board:

There are 3 boys for every 2 girls in a class. There are 12 girls in the class. How many boys are in the class?

3 : 2
boys girls

Have a volunteer write the first few terms of the sequence of equivalent ratios. ASK: Which ratio in the sequence are we looking for? Which number needs to be 12? (the second number) Continue the sequence: 3 : 2 = 6 : 4 = 9 : 6 = 12 : 8 = 15 : 10 = 18 : 12. So there are 18 boys if there are 12 girls.

Exercise: There are 4 boys for every 5 girls in a class. There are 20 boys in the class. How many girls are in the class?

Answer: 25 girls

Part-to-part problems where the total is given. Write on the board:

An aquarium has just guppy and platy fish. There are 3 guppies for every 2 platies in the aquarium. There are 25 fish in the aquarium. How many guppies are in the aquarium?

Now students are looking for the term in the sequence where the two numbers add to 25. Write the sum of the two numbers under each ratio on the board:

5 10 15 20 25

SAY: So there are 15 guppies in the aquarium. (We know the first number is the number of guppies because we are given the ratio of guppies to platies, not platies to guppies.)
**Exercises:** Solve the problem by writing a sequence of equivalent ratios.

a) A pet shop has 4 dogs for every 7 cats. There are 33 cats and dogs altogether. How many cats are in the pet shop?

b) A pet shop has 6 dogs for every 5 cats. There are 22 cats and dogs altogether. How many dogs are in the pet shop?

c) There are 3 red marbles for every 4 blue marbles in a jar. If there are 28 red and blue marbles altogether, how many of them are red?

**Answers:** a) 21, b) 12, c) 12

Take up the answers to the exercises above with the whole class. For example, for part a), write on the board:

\[
\frac{4}{7} = \frac{8}{14} = \frac{12}{21}
\]

Have a volunteer circle the number in each ratio that represents the number of cats, and have another volunteer write the total number of cats and dogs below each ratio:

\[
\begin{array}{ccc}
4:7 & = & 8:14 \\
11 \text{ cats and dogs} & = & 22 \text{ cats and dogs}
\end{array}
\]

Say: So in a pet shop of 33 cats and dogs, there are 21 cats.

Say: We can use a ratio table to find the number of cats in the pet shop. We have to start with the ratio of cats to the total number of cats and dogs, which is 7 : 11. Draw on the board:

<table>
<thead>
<tr>
<th>Number of Cats</th>
<th>Total Number of Cats and Dogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>21</td>
<td>33</td>
</tr>
</tbody>
</table>

Emphasize that the other rows can be obtained using skip counting or multiplication. Say: Multiplication can be used as a shortcut so that we don’t even need to find the whole table. We can find the missing number in 7 : 11 = ____ : 33 by finding the number we need to multiply 11 by to get 33 using a ratio table. Draw on the board:

<table>
<thead>
<tr>
<th>Number of Cats</th>
<th>Total Number of Cats and Dogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>33</td>
</tr>
</tbody>
</table>

Say: Since \(3 \times 11 = 33\), we multiply \(3 \times 7 = 21\). Draw the arrows and write "21" in the empty cell, as shown below:

<table>
<thead>
<tr>
<th>Number of Cats</th>
<th>Total Number of Cats and Dogs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\times 3)</td>
<td>(\times 3)</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>21</td>
<td>33</td>
</tr>
</tbody>
</table>
Exercises: Find the missing number in the ratio table.

a) \[ \begin{array}{cc}
4 & 7 \\
12 & \end{array} \]

b) \[ \begin{array}{cc}
3 & 5 \\
15 & \end{array} \]

c) \[ \begin{array}{cc}
2 & 9 \\
27 & \end{array} \]

d) \[ \begin{array}{cc}
12 & 14 \\
36 & \end{array} \]

Answers: a) 21, b) 25, c) 6, d) 42

Explain to students that in a ratio table arrows can also point from bottom to top. Emphasize that both arrows must point in the same direction; if you multiply from top to bottom in one column, you have to multiply from top to bottom in the other column.

Exercises: Find the missing number in the ratio table.

a) \[ \begin{array}{cc}
20 & \\
5 & \end{array} \]

b) \[ \begin{array}{cc}
10 & \\
3 & \end{array} \]

c) \[ \begin{array}{cc}
12 & \\
4 & \end{array} \]

d) \[ \begin{array}{cc}
48 & \\
8 & \end{array} \]

Answers: a) 24, b) 15, c) 21, d) 32

Solving problems using equivalent ratios. Show students how to solve word problems using equivalent ratios. Use this problem: If 5 bus tickets cost $9, how much would 20 tickets cost?

Step 1: Make the ratio table. Draw a table with two columns and the headings shown in the margin. Write the ratio of bus tickets to dollars (5 : 9) in the first row. In the second row, write 20 in the “Number of Tickets” column and a question mark for the missing number in the “Cost” column.

Step 2: Find the missing number. Find the number being multiplied by in the first column. Then multiply by that number in the second column to find the missing number. (see completed table in margin)

SAY: Since 5 : 9 = 20 : 36, 20 tickets cost $36.

Have volunteers complete the first few exercises below at the board. Then have students complete the rest individually.

Exercises: Draw a ratio table to solve the problem.

a) If 5 bus tickets cost $4, how much will 15 bus tickets cost?

b) Five bus tickets cost $6. How many can you buy with $30?

c) On a map, 3 cm represents 10 km. How many kilometres does 15 cm represent?

d) Tristan gets paid $45 for 3 hours of work. How much would he get paid for working 6 hours?

e) Three centimetres on a map represents 20 km in real life. If a lake is 6 cm long on the map, what is its actual length?

f) There are 2 apples in a bowl for every 3 oranges. If there are 12 oranges, how many apples are there?

Bonus: A goalie stopped 18 out of every 19 shots. There were 38 shots. How many goals were scored (i.e., how many did she not stop)?

Answers: a) $12, b) 25, c) 50 km, d) $90, e) 40 km, f) 8, Bonus: 2
SAY: The ratio of guppies to platies in an aquarium is 4 : 7. ASK: What is the ratio of guppies to fish? (4 : 11) SAY: The ratio of platies to guppies in an aquarium is 5 : 3. ASK: Are there more platies or guppies in the aquarium? (platies)

**Extensions**

1. There are 6 boys for every 10 girls on a school trip. If there are 35 girls, how many boys are there? (NOTE: To solve this question, you need to reduce the ratio given to smaller numbers.)

2. a) Sun is reading on her way to work. She reads 3 pages on the 1 km bus ride. What is the ratio of pages read to kilometres travelled on the bus?

   b) Sun gets off the bus and gets on the train. She reads 6 pages on the 6 km train ride. What is the ratio of pages read to kilometres travelled on the train?

   c) For each kilometre Sun travels, what is the ratio of pages read on the bus to pages read on the train? Hint: If Sun travelled 6 km on the bus, how many pages would she read?

   d) Sun reads at the same rate on the bus as on the train. Which mode of transportation is faster, the bus or the train? How many times as fast?

   e) Anna is knitting on her way to work. She knits 120 stitches on the 2 km bus ride, switches to the train, and then knits 450 stitches on the 15 km train ride. How much faster is the train than the bus? What assumption did you need to make?

**Bonus:** Whose bus travels faster, Anna’s or Sun’s?

**Answers:** a) 3 : 1; b) 6 : 6 = 1 : 1; c) 3 : 1; d) the train is three times as fast as the bus; e) the bus ratio is 60 stitches : 1 km and the train ratio is 30 stitches : 1 km, so she gets twice as much done per kilometre on the bus as on the train, which means the train is twice as fast as the bus, assuming she knits at the same rate on the bus as on the train; Bonus: Anna’s bus is three times as fast as Sun’s bus, assuming their trains travel at the same speed
Goals

Students will write given fractions as percentages, where the given fractions have a denominator that divides evenly into 100.

PRIOR KNOWLEDGE REQUIRED

Can find equivalent fractions
Can reduce fractions to lowest terms
Can convert decimals to fractions
Can find a decimal equivalent to a fraction

Mental math minute—number string.

String 1: $15 \div 5$, $\frac{1}{5}$ of 15, $\frac{2}{5}$ of 15 (3, 3, 6)

Present the strategy using groups of dots, as shown below. One fifth is one of 5 equal parts, so divide 15 into 5 equal groups. Two fifths is 2 groups, so multiply the answer by 2.

String 2: $100 \div 4$, $\frac{1}{4}$ of 100, $\frac{3}{4}$ of 100, $\frac{1}{5}$ of 100, $\frac{4}{5}$ of 100 (25, 25, 75, 20, 80)

String 3: $\frac{1}{100}$ of 900, $\frac{3}{100}$ of 900, $\frac{1}{100}$ of 6000, $\frac{11}{100}$ of 6000 (9, 27, 60, 660)

Percentages as ratios. Ask students what the word “per” means in these sentences:

- Randi can type 60 words per minute.
- Ivan scores 3 goals per game.
- John makes $15 per hour.
- The car travels at a speed of up to 140 kilometres per hour.

Emphasize that “per” means “for each” or “for every.” Ask volunteers to read the sentences with “for every” replacing “per.” Then write “percent” on the board. ASK: What is a “cent” (an amount of money; 100 cents is a dollar) SAY: In French, cent means 100. “Percent” means “for every 100” or “out of every 100.” For example, a score of 84% on a test means that you got 84 out of every 100 marks or points. Another example: If a survey reports that 72% of people read the newspaper every day, that means 72 out of every 100 people read the newspaper daily.

SAY: Fred got 84% on a test where there were 200 possible points. ASK: How many points did he get? SAY: I can also rephrase the question. A test has 200 possible points. Fred got 84 points for every 100 possible points. ASK: How many points did he get? Write on the board:

$$\frac{84}{100} = \frac{200}{\text{possible points}}$$
Explain to students that a *percentage* is a ratio that compares a number to 100.

**Exercises:** Rephrase the percentage in the statement using the phrases “for every 100 ___” or “out of 100 ___.”

a) 52% of students in the school are girls.

b) 40% of tickets sold were on sale.

c) Ren scored 95% on the test.

d) About 60% of your body weight is water.

**Answers:** a) For every 100 students, 52 are girls, or 52 out of every 100 students in the school are girls; b) For every 100 tickets sold, 40 were on sale, or 40 out of every 100 tickets were on sale; c) For every 100 possible points, Ren scored 95 points on the test, or Ren got 95 out of every 100 points on the test; d) For every 100 kg of body weight, about 60 kg is water, or 60 kg out of every 100 kg of body weight is made up of water

**Percentages as fractions.** Explain to students that a percentage is just a short way of writing a fraction with denominator 100. For example, you can write the fraction 84/100 as 84%.

**Exercises**

1. Write the fraction as a percentage.

   a) \( \frac{28}{100} \)   b) \( \frac{9}{100} \)   c) \( \frac{34}{100} \)

   d) \( \frac{67}{100} \)   e) \( \frac{81}{100} \)   f) \( \frac{3}{100} \)

2. Write the percentage as a fraction.

   a) 6%   b) 19%   c) 8%   d) 54%   e) 79%   f) 97%

**Writing hundredths as percentages.** In the following exercises, have students write each decimal as a percentage by first changing the decimal into a fraction with denominator 100. For example, 0.84 is 84/100, which is 84%.

**Exercises**

a) 0.74   b) 0.03   c) 0.12   d) 0.83   e) 0.91   f) 0.09

**Changing fractions to percentages when the denominator divides evenly into 100.** Write the fraction 3/5 on the board and have a volunteer find an equivalent fraction with denominator 100. (60/100) ASK: If 3 out of every 5 students at a school are girls, how many out of every 100 students are girls? (60) What percentage of the students are girls? (60%) Write on the board:

\[
\frac{3}{5} = \frac{60}{100} = 60\%
\]
Exercises

1. Find the equivalent fraction with denominator 100 and then the equivalent percentage.
   a) \( \frac{2}{5} \)  
   b) \( \frac{4}{5} \)  
   c) \( \frac{1}{5} \)

2. Find the equivalent fractions with denominator 100 and then the equivalent percentage.
   a) \( \frac{4}{10} \)  
   b) \( \frac{9}{20} \)  
   c) \( \frac{3}{4} \)  
   d) \( \frac{1}{2} \)  
   e) \( \frac{29}{50} \)  
   f) \( \frac{21}{25} \)  
   g) \( \frac{17}{25} \)

Selected answers: a) 40/100, 40%; b) 40/100, 40%

Using percentages to order fractions. Point out that percentages are easily ordered because they are all fractions with the same denominator, 100. Use the equivalent percentages to put the above fractions in order from least to greatest.

Writing decimal tenths as percentages. Have students write various decimal tenths as percentages by first changing the decimal to a fraction with denominator 100. Examples: 0.2 (= 2/10 = 20/100 = 20%), 0.3, 0.9, 0.7, 0.5.

Equivalent percentage of a fraction. Explain to students that they can find a percentage of a figure just as they can find a fraction of a figure. Ask students to decide first what fraction and then what percentage of each figure in the margin is shaded. (a) 4/10, 40%; b) 1/4, 25%; c) 7/20, 35%)

Fractions that need to be reduced before changing the denominator to 100. Write the fraction 9/15 on the board. Tell students that you want to find an equivalent fraction with denominator 100 so that you can turn it into a percentage. Ask: How is this fraction different from previous fractions you have changed to percentages? (the denominator does not divide evenly into 100) Is there any way to find an equivalent fraction whose denominator does divide evenly into 100? (reduce the fraction by dividing both the numerator and the denominator by 3) Write on the board:
   \[
   \frac{9}{15} = \frac{3}{5} = \frac{60}{100} = 60\%
   \]


Summarize the steps for finding the equivalent percentage of a fraction.

Step 1: Reduce the fraction so that the denominator is a factor of 100.

Step 2: Find an equivalent fraction with denominator 100.

Step 3: Write the fraction with denominator 100 as a percentage.
Example: Change $\frac{14}{35}$ to a percentage by first reducing it to smaller numbers.

$$\frac{14}{35} \div 7 = \frac{2}{5} \times 20 = \frac{40}{100} = 40\%$$

**Exercises:** Write the fraction as a percentage.

a) $\frac{3}{12}$  
b) $\frac{6}{30}$  
c) $\frac{24}{30}$  
d) $\frac{3}{75}$  
e) $\frac{6}{15}$  
f) $\frac{36}{48}$  
g) $\frac{60}{75}$

**Answers:** a) 25%, b) 20%, c) 80%, d) 4%, e) 40%, f) 75%, g) 80%

**Extensions**

1. Draw on the board:
   
   ASK: How many degrees are in a circle? (360) If I rotate an object 90° counterclockwise, what fraction and what percentage of a complete 360-degree turn has the object made? (1/4, 25%) PROMPT: 90 out of 360 is what out of 100? (90/360 = 1/4 = 25/100 = 25%)
   
   Repeat for 180°, 18°, 126°, 270°, 72°, 216°.

2. Express $\frac{5040}{100\,800}$ as a percentage.

   **Answer:** 0.05 or 5%

3. **Introduce promille or per mille (‰).** SAY: A promille or per mille is $\frac{1}{1000}$ of something.

   Convert $\frac{3}{125}$ to promille.

   **Answer:** 24 per mille (3/125 = 24‰)
Goals
Students will visualize various percentages of different shapes, including rectangles, squares, triangles, and line segments.

PRIOR KNOWLEDGE REQUIRED
Can write equivalent fractions
Understands the relationship between decimal tenths and hundredths and fractions with denominator 100

MATERIALS
metre stick
rulers

Mental math minute—number string.
String 1: 10 × 2.8, 5 × 2.8, 15 × 2.8, 20 × 2.8, 25 × 2.8 (28, 14, 42, 56, 70)
Present the strategies: use halving, doubling, or the distributive law.
For example:
5 is half of 10, so halve the answer for 10 × 2.8 to find 5 × 2.8
Have students complete the last problem in the string in two ways to check the answer:
use 25 = 20 + 5
double 25 twice and halve 2.8 twice
(25 × 2.8 = 50 × 1.4 = 100 × 0.7)

String 2: 100 × 0.76, 50 × 0.76, 25 × 0.76, 75 × 0.76 (76, 38, 19, 57)

String 3: 100 × 31.4, 10 × 31.4, 110 × 31.4, 90 × 31.4 (3140, 314, 3454, 2826)

Percentage of a shape. Draw the hundreds block in the margin on the board and have students write what part of the block is shaded in three different ways: as a fraction, a decimal, and a percentage. (39/100, 0.39, 39%) Have students find 25% of each shape below in various ways (for example, 4 horizontal strips, 4 vertical strips, use diagonals).

Draw the shapes in the following exercises on the board.
**Exercises**: What fraction and what percentage of each shape are shaded?

Hint: Change each fraction to an equivalent fraction with denominator 100 and then to a decimal and a percentage.

- a) ![Fraction](image1)
- b) ![Fraction](image2)
- c) ![Fraction](image3)
- d) ![Fraction](image4)
- e) ![Fraction](image5)

**Answers**: a) $\frac{7}{10}$, 70%; b) $\frac{1}{5}$, 20%; c) $\frac{9}{25}$, 36%; d) $\frac{14}{20}$, 70%; e) $\frac{11}{20}$, 55%

**Percentage of a line.** Draw the double number line below on the board for students to use as reference in the following exercises:

<table>
<thead>
<tr>
<th>0</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

**Exercises**: Add fractions and percentages to each number line in the margin.

Hint: If there are 5 parts in a whole, each part is $\frac{1}{5}$.

**Bonus**: If there are 5 parts in $\frac{1}{2}$, how many parts are in a whole? What fraction is each part?

**Selected answer**: Bonus: 10 parts, each part is $\frac{1}{10}$ (see margin)

**Estimating multiples of 25%**. Teach students the strategy of estimating 25% and 75% using 50%. Once they’ve estimated and marked 50%, they can halve the left part of the line to estimate 25% and halve the right part of the line to estimate 75%.

**Exercises**: Estimate visually and mark the percentage on the line segment.

- a) 25%
- b) 75%
- c) 50%
- d) 75%

Draw on the board a line one metre long and have students estimate the percentage of various marks on the number line (to the nearest 10%). Then, using a metre stick, draw another line of the same length divided into 10 equal parts above or below the first line so that students can check their estimates.

**Extending a line to 100%**. Draw a line segment 2 cm long on the board. Tell students this line segment is 1/3 of a longer line segment; it is part of a whole. Have a volunteer extend the line to make the whole. Point out that we need 3 equal parts and we already have 1, so we need to add 2 more (see margin).
Exercises

a) Draw a 3 cm line segment. It is $\frac{1}{4}$ of a line segment. Extend to make the whole.

b) Draw a 5 cm line segment. It is $\frac{1}{2}$ of a line segment. Extend to make the whole.

c) Draw a 4 cm line segment. It is $\frac{1}{3}$ of a line segment. Extend to make the whole.

Draw a 6 cm line segment on the board. Tell students it is $\frac{2}{3}$ of a line segment. SAY: This line segment is 2 out of 3 equal parts. ASK: How can we find what 1 of the 3 equal parts looks like? (divide the line segment in two equal parts) Do so and then SAY: Now we know what 1 part looks like. ASK: How many more of those parts do we need to draw to get the whole? (1) Have a volunteer draw the 1 extra part.

Repeat for a 6 cm line segment that is $\frac{3}{5}$ of a line segment. SAY: Now we divide the line segment into 3 equal parts, and we need to draw 2 more of those equal parts to get the whole.

Exercises

a) Draw a 6 cm line segment. It is $\frac{2}{5}$ of a line segment. Draw the whole line segment.

b) Draw an 8 cm line segment. It is $\frac{4}{7}$ of a line segment. Draw the whole line segment.

SAY: Notice that 50% = $\frac{1}{2}$, so a given line segment that is 50% of the whole is 1 of 2 equal parts, and you can simply draw another equal part to make the whole. Also, 40% = $\frac{4}{10}$ = $\frac{2}{5}$, so a given line segment that is 40% of the whole is 2 of 5 equal parts. You can divide this line segment into two equal parts and draw three more identical parts.

Estimating how full a container is. Draw on the board:

```
100%
75%
50%
25%
```

SAY: This is a container with the marks 25%, 50%, 75%, and 100%. Imagine that I am pouring some water in this container. Add shading to the container as shown below:

```
100%
75%
50%
25%
```

SAY: The water level is closer to 50% than 75%, so we say the container is about 50% full.
Exercises: Estimate whether the container is closer to 0%, 25%, 50%, 75%, or 100% full.

Answers: a) about 25%, b) about 100%, c) about 0%, d) about 75%

Extensions

1. Have students find 25% of this triangle:

   Answer: \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

2. Extend the line segment to show 100%.

   a) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

   b) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

   c) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

   d) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

   e) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

   f) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

   g) \[
   \begin{array}{c}
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \text{\textbackslash 1}\text{\textbackslash 1} \\
   \end{array}
   \]

3. Change \( \frac{7}{5} \) to a percentage.

   Answer: \( \frac{7}{5} = \frac{140}{100} = 140\% \)

4. a) What percentage of an hour is 6 minutes?

   b) 4.8 hours is what percentage of a day?

   Answers: a) 10%, b) 20%
Goals
Students will compare and order fractions, percentages, and decimals.

PRIOR KNOWLEDGE REQUIRED
Understands percentages as fractions with denominator 100
Can order fractions
Can find equivalent fractions
Knows the signs for less than (<) and greater than (>

Review comparing and ordering:
- fractions with the same denominator (7/10 is greater than 4/10)
- percentages (30% is greater than 24% because 30/100 is greater than 24/100)
- fractions with different denominators (5/10 is greater than 6/20 = 3/10)
- fractions and decimals (3/5 is greater than 0.52 because 60/100 is greater than 52/100)

Comparing fractions and percentages. Remind students of the signs for less than (<) and greater than (>). Teach students how to compare fractions and percentages by changing both to an equivalent fraction with denominator 100.

Exercises
1. Which is larger?
   a) \frac{1}{2} or 38%  
   b) \frac{3}{5} or 70%
   c) \frac{9}{10} or 84%
   d) \frac{7}{25} or 30%
   e) \frac{9}{20} or 46%

   Answers: a) 1/2, b) 70%, c) 9/10, d) 30%, e) 46%

2. Which is closer to 50%? Hint: Change the fractions to percentages first.
   a) \frac{1}{4} or \frac{2}{5} 
   b) \frac{3}{10} or \frac{4}{5} 
   c) \frac{3}{5} or \frac{1}{4} 
   Bonus: \frac{2}{5} or \frac{3}{5}

   Answers: a) 2/5, b) 3/10, c) 3/5, Bonus: they are the same

Comparing decimals and percentages. Compare decimals and percentages by changing both to an equivalent fraction with denominator 100.

Exercises
1. Which is larger?
   a) 0.9 or 10% 
   b) 0.09 or 10% 
   c) 28% or 0.34 
   d) 4% or 0.3

   Answers: a) 0.9, b) 10%, c) 0.34, d) 0.3
2. Write the set of numbers in order from least to greatest by first changing each number to a fraction with denominator 100.

a) 0.28, 42% \(\frac{3}{10}\)  

b) \(\frac{14}{50}\), 23% 0.3  

c) \(\frac{19}{25}\), 0.72, 7%  

d) \(\frac{1}{4}\), 4% 0.4  

**Bonus:** \(\frac{13}{20}\), 0.6, 66% 0.7 7% \(\frac{16}{25}\), 3 0.5  

**Answers:** a) 0.28, 3/10, 42%; b) 23%, 14/50, 0.3; c) 7%, 0.72, 19/25; d) 4%, 1/4, 0.4; Bonus: 3/50, 7%, 0.6, 16/25, 13/20, 66%, 0.7

**Comparing fractions and percentages when a denominator does not divide evenly into 100.**

ASK: How can we compare 35% to 1/3? If we change 35% to a fraction, what would it be? (35/100) Do we have a way to compare 1/3 to 35/100? SAY: We have two fractions with different denominators, but 3 doesn’t divide evenly into 100. ASK: How can we give both fractions the same denominator? (use denominator 300) Have volunteers change both fractions to equivalent fractions with denominator 300 and ask the class to identify which is greater, 35% or 1/3, and to explain how they know. (35% is 105/300 and 1/3 is 100/300, so 35% is greater)

Repeat with various reduced fractions whose denominator does not divide evenly into 100, such as comparing 45% and 3/7 by making both denominators equal to 700. (45% = 45/100 = 315/700 and 3/7 = 300/700, so 45% is greater)

**Exercises:** Compare.

a) \(\frac{5}{6}\) and 85%  

b) \(\frac{3}{7}\) and 42%  

c) \(\frac{2}{9}\) and 21%  

**Bonus:** Compare \(\frac{1}{8}\) and 12.5%. Hint: use half of \(\frac{1}{4}\) and half of 25%.

**Answers:** a) 85% is greater, b) 3/7 is greater, c) 2/9 is greater, Bonus: they are equal because 1/8 is half of 1/4 and 12.5% is half of 25%

Have students order lists of numbers (fractions, percentages, and decimals) in which the fractions do not have denominators that divide evenly into 100.

**Exercises:** Order the numbers from least to greatest.

a) \(\frac{1}{6}\), 0.17, 13%  

b) 0.37, \(\frac{1}{3}\), 28%  

c) \(\frac{5}{7}\), 71%, 0.68  

**Bonus:** \(\frac{7}{9}\), 0.8, \(\frac{4}{7}\), 51%, 0.78, 62%  

**Answers:** a) 13%, 1/6, 0.17; b) 28%, 1/3, 0.37; c) 0.68, 71%, 5/7;  
Bonus: 51%, 4/7, 62%, 7/9, 0.78, 0.8
Extensions

1. Ask students to name percentages that indicate:
   - almost all of something
   - very little of something
   - a little less than half of something
   Ask students to explain their thinking.

2. Ask students to look for percentages in newspapers, flyers, magazines, and other printed materials, such as food packaging, trading cards, and order forms. What kind of information is expressed as a percentage? Ask students to clip examples and to make a collage for a class display.

CONNECTION
Real World
NS6-67 Finding Percentages

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CURRICULUM REQUIREMENT
AB: required
BC: required
MB: required
ON: required

VOCABULARY
percentage

Goals
Students will find multiples of 10% of a number.

PRIOR KNOWLEDGE REQUIRED
Can convert fractions to decimals and vice versa
Understands the relationship between percentages and fractions

MATERIALS
base ten materials

Mental math minute—number string.

String 1: 100 × 3.5, 10 × 3.5, 90 × 3.5 (350, 35, 315)

Present the strategy using place value and then compensating by using the distributive law: 90 × 3.5 = (100 × 3.5) − (10 × 3.5)

String 2: 100 × 1.3, 90 × 1.3, 80 × 1.3, 70 × 1.3 (130, 117, 104, 91)

String 3: 100 × 0.45, 50 × 0.45, 5 × 0.45, 55 × 0.45 (45, 22.5, 2.25, 24.75)

Percentages and base ten representations. Tell students that you will use one thousands block to represent one whole. Given this information, ask students to identify the decimal each model below represents:

(a) 1 or 1.00, b) 0.3 or 0.30, c) 0.33, d) 0.05

Tell students that you want to make a model of the number 1.6, again using one thousands block as one whole. ASK: What do I need to make the model? (1 thousands block and 6 hundreds blocks, see margin)

ASK: How can I show 1/10 of 1.6? (one tenth of a thousands block is a hundreds block and one tenth of a hundreds block is a tens block, so I need 1 hundreds block and 6 tens blocks to make 1/10 of 1.6, see margin) What number is 1/10 of 1.6? (0.16) PROMPT: What do the base ten materials show? Do the following examples together as a class:

a) 1/10 of 1  b) 1/10 of 0.1  c) 1/10 of 0.01
d) 1/10 of 2.3  e) 1/10 of 0.41  f) 1/10 of 5.01
ASK: How can you find 1/10 of any number? (move the decimal point one place to the left) PROMPT: What do you do to the decimal point in the answers above? Remind students that when they move the decimal point one place to the left, each digit becomes worth 1/10 as much, so the entire number becomes 1/10 of what it was before they moved the decimal point. Examples:

4 is \(\frac{1}{10}\) of 40

0.1 is \(\frac{1}{10}\) of 1

4.1 is \(\frac{1}{10}\) of 41

ASK: How else can I find 4.1 from 41—what is this like dividing by? (10) Emphasize that to find 1/10 of anything, you divide it into 10 equal parts; to find 1/10 of a number, you divide the number by 10. ASK: What decimal is the same as 1/10? (0.1) What percentage is the same as 1/10? (10%) Write on the board:

\[
\frac{1}{10} \quad \frac{10}{100} = 10\%
\]

SAY: So to find 10% of a number you can divide the number by 10 or simply move the decimal point one place to the left. Emphasize the fact that 1/10 is 10% not because of the denominator but because of reduction. SAY: A student thinks that 20% is 1/20, so 20% of 20 is 1. ASK: Is she correct? (no) What is her mistake? (20% is 20/100 or 1/5, or 1 out of every 5. In 20, there are 4 groups of 5, so 1/5 of 20 is 4.)

**Exercises:** Find 10% of the number by moving the decimal point.

a) 40  
   b) 4  
   c) 7.3  
   d) 500  
   e) 408  
   f) 3.07  

**Answers:** a) 4, b) 0.4, c) 0.73, d) 50, e) 40.8, f) 0.307, Bonus: 43.25609

**Finding percentages with a number line.** Draw the number line below but omit the numbers on top from 3 to 27. Have a volunteer fill in the missing numbers on the number line.

Ask volunteers to look at the completed number line and identify 10% of 30, 40% of 30, 90% of 30, and 70% of 30 (3, 12, 27, 21)

**Exercise:** Draw a number line from 0 to 21 that shows 0% to 100% of 21. Hint: Start at 0 and add 2.1 each time.

**Answer**

0 2.1 4.2 6.3 8.4 10.5 12.6 14.7 16.8 18.9 21

0 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
ASK: If you know 10% of a number, how can you find 30% of that number? (multiply 10% of the number by 3) Tell students that you would like to find 70% of 12. ASK: What is 10% of 12? (1.2) If I know that 10% of 12 is 1.2, how can I find 70% of 12? (multiply 1.2 \times 7 = 8.4) Review multiplying a decimal by a whole number with an example before students complete the exercises below.

**Exercises:** Find the percentage of the number using the method above.

a) 60% of 15  
b) 40% of 40  
c) 60% of 4  
d) 20% of 1.5  
e) 90% of 8.2  
f) 70% of 4.3  
g) 80% of 5.5  
h) 30% of 3.1

**Answers:** a) $6 \times 1.5 = 9$, b) $4 \times 4 = 16$, c) $6 \times 0.4 = 2.4$, d) $2 \times 0.15 = 0.30$, e) $9 \times 0.82 = 7.38$, f) $7 \times 0.43 = 3.01$, g) $8 \times 0.55 = 4.4$, h) $3 \times 0.31 = 0.93$

**SAY:** To find 1/100 of a number, you divide the number by 100. Explain to students that taking 1% of a number is the same as dividing the number by 100. (The decimal point shifts two places to the left.)

**Exercises:** Find 1% of the number by shifting the decimal point two places to the left.

a) 27  
b) 3.2  
c) 773  
d) 12.3  
e) 68.2

**Answers:** a) 0.27, b) 0.032, c) 7.73, d) 0.123, e) 0.682

**Estimating percentages.** Draw on the board:

```
0 4 8 12 16 20 24 28 32 36 40

0 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
```

Ask a volunteer to complete the number line, as shown below.

```
0 4 8 12 16 20 24 28 32 36 40

0 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
```

ASK: What is 50% of 40? (20) What is 60% of 40? (24) If you want to estimate what percentage of 40 is 21, would your estimate be 50% or 60%? (50%) Mark 21 on the number line as shown below to show that 21 is closer to 50%.

```
0 4 8 12 16 20 24 28 32 36 40

0 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
```

**Exercises**

a) 31 out of 40 marbles in a jar are red. About what percentage of the marbles are red?

b) 26 out of 30 students in a class wear running shoes. About what percentage of the students wear running shoes?

**Answers:** a) about 80%, b) about 90%
Extensions

1. Use 10% to find 5% of the number.
   
   a) 20  b) 50  c) 66  d) 34  
   
   **Bonus:** 26.5
   
   **Answers:** a) 1, b) 2.5, c) 3.3, d) 1.7, **Bonus:** 1.325

2. Use 30% and 40% of 80 to find 35% of 80.
   
   **Solution:** 30% of 80 is 24 and 40% of 80 is 32. 35% is exactly in the middle of 30% and 40%, and the middle of 24 and 32 is 28, so 35% of 80 is 28.

3. Compare.
   
   a) 20% of 60 and 60% of 20  
   b) 30% of 50 and 50% of 30
   
   c) 40% of 20 and 20% of 40  
   d) 70% of 90 and 90% of 70
   
   e) 80% of 60 and 60% of 80  
   f) 50% of 40 and 40% of 50

   What pattern do you see?
   
   **Answers:** Both percentages in each part are the same.
Goals
Students will solve problems involving percentages.

PRIOR KNOWLEDGE REQUIRED
Can reduce fractions
Can multiply decimals
Knows the standard algorithm for multiplying

MATERIALS
BLM Percentage Strips (p. Q-50)

Mental math minute. Remind students that they can represent a percentage as a decimal fraction and then reduce the fraction by dividing both the numerator and denominator by the same number, a common factor of both. Demonstrate using an example: \( \frac{65}{100} = \frac{13}{20} \).

Exercises: Convert the percentage to a fraction. Use the smallest numbers possible.

a) 25% b) 75% c) 50% d) 20% e) 24%

Answers: a) \( \frac{1}{4} \), b) \( \frac{3}{4} \), c) \( \frac{1}{2} \), d) \( \frac{1}{5} \), e) \( \frac{6}{25} \)

Finding percentages using multiplication. ASK: How can we calculate 53% of 12 using what we know about multiplying decimals by whole numbers? Students should notice that 53% is equal to 0.53, so they can find the percentage by first changing the percentage to a decimal, as shown below:

\[
53\% \text{ of } 12 = 0.53 \times 12 = \frac{(53 \times 12)}{100}
\]

Remind students that they can find products like \( 53 \times 12 \) by using long multiplication (or mentally if it is an easy product). Also remind them that dividing by 100 shifts the decimal point two places to the left. Continue writing on the board:

\[
\frac{(53 \times 12)}{100} = 636 \div 100 = 6.36
\]

Students can use estimation to check whether their answers to percentage problems are reasonable. For instance, they can round a given percentage to the nearest multiple of 10 and use the rounded percentage to estimate the answer. ASK: How can we tell if 6.36 is a reasonable answer to 53% of 12? Is there a percentage of 12 that is close to 53% and easy to calculate? (yes, 50%) Will the estimate be lower or higher than the actual answer? (50% of 12 is 6, which is lower than the actual answer because 50% is less than 53%. But 6 is close to 6.36, so the answer seems reasonable.) Students might also round the number as well as the percentage: 12 is
close to 10 and 53 is close to 50. So to estimate 53% of 12, find 50% of 10, which is easy to calculate. \((50 \times 10) \div 100 = 5\)

**Exercises:** Find the percentage.

a) 68% of 33   b) 5% of 42   c) 76% of 85   d) 55% of 21

**Answers:** a) 22.44, b) 2.1, c) 64.6, d) 11.55

Students can use **BLM Percentage Strips** to check their answers for the exercises above. Each of the four numbers in the exercises has been placed on a number line (all of the same length but with a different scale for each). A fifth number line, divided into a hundred parts to represent percentages, is at the bottom of the BLM. To estimate 68% of 33, students can find the number 68 on the percentage number line and then locate the number that is in the same position on the number line for 33.

After students have finished their calculations for the exercises, ask them what method they would use to estimate 68% of 33 to see if their answer is reasonable. Encourage them to give a variety of answers. For example, 68% is close to 70% and 33 is close to 30. I found 70 \times 30 mentally (2100) and then shifted the decimal point two places to the left. So my estimate is 21. For another example, 68% is close to 75%, which is the same as 3/4 (75/100 = 3/4). And 33 is close to 32. I know 1/4 of 32 is 8, so 3/4 of 32 is 24. So my estimate is 24.

**Finding 25% and 75% of a number mentally.** Write on the board:

What is 25% of 60?

SAY: One method to find 25% of 60 is multiplication. Ask a volunteer to find \(25 \times 60\) and then divide by 100. \((1500, 15)\) SAY: 25% as a fraction is 25/100. Write “25/100” on the board and find an equivalent fraction by dividing both the numerator and denominator by 25, as shown in the margin.

SAY: To find 25% of a number you need to find 1/4 of the number. ASK: How do you find half of a number? (divide by 2) How do you find a quarter of a number? (divide by 4) So what is a quarter of 60? \((60 \div 4 = 15)\) SAY: So 25% of 60 is 15.

**Exercises:** Find 25% of the number.

a) 25% of 40   b) 25% of 120   c) 25% of 160   d) 25% of 140

**Answers:** a) 10, b) 30, c) 40, d) 35

SAY: Since 75 is 3 times 25, you can find 75% of a number by multiplying 25% of that number by 3. Write on the board:

\[75\% \text{ of } 60 = 3 \times (25\% \text{ of } 60) = 3 \times 15 = 45\]

**Exercises:** Find 75% of the number.

a) 75% of 40   b) 75% of 120   c) 75% of 160   d) 75% of 140

**Answers:** a) 30, b) 90, c) 120, d) 105
**Finding the number when 10% of the number is given.**

ASK: What is 10% of 34? (3.4) How do you know? (divide 34 by 10 or shift the decimal point one place to the left)

SAY: In this section, we are going to find a number when only 10% of it is given. Explain to students that to find a whole number when 10% of it is given, they can easily multiply by 10 or shift the decimal point one place to the right. ASK: If 10% of a number is 1.2, what is the number? (12) Write on the board:

\[ 1.2 \times 10 = 12 \]

**Exercises:** 10% of a number is given. What is the number?

a) 7  

b) 2.9  

c) 0.71  

d) 4.782

**Answers:** a) 70, b) 29, c) 7.1, d) 47.82

Write on the board:

\[ \frac{30\%}{\text{of } \_\_\_} = 6.3 \]

SAY: Since 30% of the number is given, we can divide that by 3 to find 10% of the number. ASK: What is 6.3 divided by 3? (2.1) SAY: Now you have 10% of a number and it is similar to the previous exercises. ASK: If 10% of a number is 2.1, what is the number? (21)

**Exercises:** The percentage of a number is given. Find the number.

a) 20% of \_\_\_ is 2.6  

b) 80% of \_\_\_ is 0.32  

c) 60% of \_\_\_ is 42

**Bonus:** 40% of \_\_\_ is 10 000

**Answers:** a) 13, b) 0.4, c) 70, Bonus: 25 000

**Finding what percentage of another number is a given number.**

Write on the board:

\[ \_\_\% \text{ of } 31 \text{ is } 6.2 \]

ASK: What is 10% of 31? (3.1) Do you see any relationship between 3.1 and 6.2? (yes, 6.2 is double 3.1) What is double 10%? (20%) SAY: So 6.2 is 20% of 31. Write “20” in the blank on the board.

**Exercises:** Find 10% of the number. Then find what percentage of the original number is the given number.

a) 10% of 30 is \_\_\_, so 9 is \_\_\% of 30.  

b) 10% of 15 is \_\_\_, so 3 is \_\_\% of 15.  

c) 10% of 22 is \_\_\_, so 8.8 is \_\_\% of 22.  

d) 10% of 140 is \_\_\_, so 42 is \_\_\% of 140.

**Answers:** a) 3, 30; b) 1.5, 20; c) 2.2, 40; d) 14, 30

Remind students that to find a specific percentage of a number they have to multiply the percentage by the number and then divide the result by 100.

Write on the board:

What percentage of 23 is 6.9?
SAY: In this question, the initial number is known and the amount of percentage is given; the unknown is the percentage. Since you multiply the percentage by the initial number to find the given amount, you need to divide the given amount by the initial number to find what the percentage of the initial number is. Write on the board:

\[ 6.9 \div 23 = \]

SAY: This is the mathematical way to find what percentage of 23 is 6.9, but you need long division to find the answer, and it makes the solution longer. To avoid long division, we can use the 10% benchmark. ASK: What is 10% of 23? (2.3) How do you get 6.9 from 2.3? (multiply by 3) SAY: So 6.9 is \( 3 \times 10\% \) or 30% of 23.

**Exercises:** Find the missing number.

a) ____% of 12 is 4.8  
b) ____% of 42 is 12.6  
c) ____% of 11 is 8.8  
d) ____% of 15 is 9  
e) ____% of 21 is 14.7  

**Bonus:**  
% of 111,000 is 99,900  

**Answers:** a) 40, b) 30, c) 80, d) 60, e) 70, Bonus: 90

**Extensions**

1. Sara says that to find 10% of a number, she can divide the number by 10. So to find 5% of a number, she can divide the number by 5. Is she right? Explain.

**Answer:** No. 5% of a number is \( \frac{5}{100} \) or \( \frac{1}{20} \) of the number, so to find 5% or \( \frac{1}{20} \) of the number, she should divide it by 20.

**NOTE:** Extension 2 continues from Extension 3 in Lesson NS6-67.

2. Compare.

a) 36% of 24 and 24% of 36  
b) 17% of 35 and 35% of 17  
c) 29% of 78 and 78% of 29  
d) 48% of 52 and 52% of 48

Have students predict a rule and make up another example to check that the rule works. Challenge them to figure out why this pattern holds.

**Sample answer:** a) because 36 \times 24 is equal to 24 \times 36

3. ASK: Does it make sense to talk about 140% of a number? What does 140% of a number mean? Lead the discussion by referring to fractions greater than 1. Discuss what 100% and 40% of a number mean separately. ASK: Could 50% of a number be obtained by adding 20% and 30% of that number? Could 140% be obtained by adding 100% and 40% of the number?

**Answer:** Yes. Operations such as addition, subtraction, multiplication, and division can be used with percentages because percentages are numbers.
Goals
Students will solve word problems involving percentages.

PRIOR KNOWLEDGE REQUIRED
Can convert fractions to decimals
Can calculate the percentage of a number
Can compare decimals, fractions, and percentages

Using percentages to compare fractions. SAY: Marlo got 17 out of 25 on her math test and 14 out of 20 on her science test. ASK: What percentage of the points, or marks, did she get on each test? (68% in math, 70% in science) On which test did she do better? (science) SAY: Even though Marlo got more marks on her math test than on her science test (17 instead of 14), she got a higher percentage of marks on the science test than on the math test (70% instead of 68%). So she did better on the science test. ASK: Is it easier to compare test scores when they are given as fractions or when they are given as percentages? (percentages) PROMPT: Is it easier to compare 17/25 to 14/20 or 70% to 68%? ASK: Why doesn’t the test have to have 100 marks in order for the result to be expressed as a percentage? (we can convert any fraction to a percentage by changing the denominator to 100) Tell students that this is one application of percentages: we can compare two test scores easily, even when the total number of marks in each test is different.

Exercises: Convert Luc’s test scores to percentages and decide which was his best test and which was his worst. Hint: One of the scores will need to be reduced before it can be expressed as a fraction with denominator 100.

Math: $\frac{17}{20}$
Science: $\frac{22}{25}$
Language: $\frac{42}{50}$
Social Studies: $\frac{36}{40}$

Answers: Math 85%, Science 88%, Social Studies 90%, Language 84%. Luc’s best test was Social Studies and his worst test was Language.

Using percentages of numbers to solve real-world problems. Tell students that Aputik has collected 50 stamps from various countries: 31 from Canada, 14 from the United States, and 5 from elsewhere. Ask students to calculate what percentage of Aputik’s stamp collection is from Canada, what percentage is from the United States, and what percentage is from elsewhere. (62% from Canada, 28% from the United States, and 10% from elsewhere)

SAY: Matt, on the other hand, has collected 3000 stamps: 1020 from Canada, 840 from the United States, and 1140 from elsewhere. Ask students to calculate what percentage of Matt’s stamp collection is from
Canada, what percentage is from the United States, and what percentage is from elsewhere. (34% from Canada, 28% from the United States, and 38% from elsewhere)

ASK: Who has more stamps from Canada? (Matt) Who has a greater percentage of stamps from Canada? (Aputik—62% of her stamps are from Canada but only 34% of Matt’s are from Canada) How do percentages help us to compare stamp collections even when one has many more stamps than the other? (percentages compare for every 100 stamps)

The whole is always 100%. Tell students that Arsham’s stamp collection has this distribution: 41% from Canada, 26% from the United States, and an unknown percentage from elsewhere. ASK: What percentage of Arsham’s collection is from somewhere other than Canada or the United States? (33%) Emphasize that percentages must add to 100 because the whole amount of anything is 100%.

Word problems involving percentages, fractions, and decimals.
SAY: Jennifer has stamps from all over the world. In her collection, 2/5 of the stamps are from the United States and 36% are from Canada.
ASK: What percentage of Jennifer’s stamps are from neither the United States nor Canada? (change 2/5 to 40%, then add 40% + 36% = 76%, so the stamps from neither place make up 24% of Jennifer’s collection)

Exercises: Find the missing percentages of other stamps in each collection.

<table>
<thead>
<tr>
<th></th>
<th>Fraction of Trip</th>
<th>Percentage of Trip</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Africa: 3/10, 30%, 150; Europe: 6/10 or 3/5, 60%, 300; Other: 1/10, 10%, 50)
Extensions

1. Five people—2 adults and 3 children—attend a hockey game. What percentage of the group do the children represent? Describe a group of a different size with the same percentage of children.

2. Mr. Bates buys:
   • 5 single-scoop ice cream cones for $1.45 each
   • 3 double-scoop ice cream cones for $2.65 each

A tax of 10% is added to the cost of the cones. Mr. Bates pays with a 20-dollar bill. How much change does he receive? Show your work.

**Answer:** $3.28

3. The chart shows the fraction or percentage of stamps that people have collected from various countries.

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>England</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Braden’s Collection</td>
<td>23%</td>
<td>3/5</td>
<td></td>
</tr>
<tr>
<td>Grace’s Collection</td>
<td>3/4</td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td>Ansel’s Collection</td>
<td>1/2</td>
<td>30%</td>
<td></td>
</tr>
</tbody>
</table>

Which person has the greatest percentage of stamps from other countries?

**Answer:** Ansel

4. a) Discuss: Sally got 171/200 on a national math test. Can this mark be written as a percentage? The answer (yes, it can be written as a decimal percentage) is not as important as the discussion that should arise. Leading questions you might use include: Is this mark better or worse than 80%? How do you know? Is it better or worse than 90%? Than 85%? Than 86%? Is it closer to 85% or to 86%? (It is halfway between them) Is there a number halfway between 85 and 86? (yes, 85.5) Tell students that even though we said that percentages are just fractions with denominator 100, percentages are actually even better than fractions with denominator 100—you can’t write 85.5/100 as a fraction, but you can write 85.5%. (You could tell students that they won’t learn about decimal percentages until Grade 8, but this class is smart enough to know about them in Grade 6.)

b) Teach students a strategy for estimating 12.5%. First, find 50%, then halve one part to estimate 25%, and then halve that part again to estimate 12.5%. (See number line below.)

Have students estimate 37.5%, 62.5%, and 87.5%.
5. Investigate what percentage of car passengers wear seat belts in Canada? In the United States? In other countries? (Emphasize that even though the United States has many more people than Canada, a meaningful comparison can still be made in terms of percentages.)

6. Which has a greater percentage of water by volume, your body or planet Earth? (Emphasize that although Earth has much more water than your body, your body has a greater percentage of water than does Earth.)

7. Which has a greater percentage of water by surface area, Canada or the United States? Canada or Finland? Canada or Russia?
Goals
Students will solve word problems involving fractions, ratios, and percentages.

PRIOR KNOWLEDGE REQUIRED
Can compare fractions, ratios, and percentages
Can convert among fractions, ratios, and percentages

Mental math minute—number talk. Present this problem: Find 75% of 18.
(13.5) The following strategies could arise:

75% = 75/100 = 3/4, 18 ÷ 4 = 4.5, 4.5 × 3
75% = 0.75, 0.75 × 18 = (0.75 × 10) + (0.75 × 8)
75% = 100% − 25%, 25% of 18 = 1/4 of 18 = 4.5, so 75% of 18 is 18 − 4.5
75% = 0.75, 0.75 × 18 = 75 × 18 ÷ 100
= 150 × 9 ÷ 100
= 15 × 9 ÷ 10
= ((10 × 9) + (5 × 9)) ÷ 10
= (90 + 45) ÷ 10

75% = 0.75, 0.75 × 18 = 1.5 × 9 = 9 + 4.5

Recognize part-to-part and part-to-whole ratios. SAY: There are 4 green crayons and 3 blue crayons in a bag with only green and blue crayons. ASK: What is the total number of crayons in the bag? (7) Write on the board:

g: 4  b: 3  c: 7

ASK: What fraction of crayons are green? (4/7) How did you find that? (by dividing the number of green crayons by the total number of crayons) Explain to students that if they want to find the fraction, first they have to find the total number and then write the fraction. SAY: The number of green and blue crayons are called “parts” and the total number of crayons is called “the whole.”

Tell students this lesson involves bags that only contain blue and green crayons.

Ask students to fill in the numbers of blue crayons (b), green crayons (g), and total crayons (c) given various pieces of information:

a) 7 green and 8 blue  b: ___  g: ___  c: ___
b) 6 green in a bag of 20  b: ___  g: ___  c: ___
c) 12 blue in a bag of 30  b: ___  g: ___  c: ___
d) 17 green in a bag of 28  b: ___  g: ___  c: ___
Have students determine the numbers of blue, green, and total crayons, the fraction of green, and the fraction of blue in these bags:

a) There are 6 blue and 5 green crayons.
b) There are 14 blue crayons in a bag of 23.
c) There are 15 green crayons in a bag of 26.

Have students write the fraction of green and blue crayons in these bags:

a) There are 3 blue and 4 green crayons in a bag.
b) There are 7 blue and 13 crayons in total in a bag.
c) There are 8 green and 19 crayons in total in a bag.
d) The ratio of blue to green crayons is 1 : 2.
e) The ratio of green to blue crayons is 2 : 3.
f) The ratio of blue to green crayons is 12 : 11.
g) The ratio of blue to green crayons is 11 : 12.
h) The ratio of green to blue crayons is 11 : 12.

ASK: Which two of the last three questions have the same answer? (parts f) and h)

Can you re-write part d) in a different way that still has the same answer? (the ratio of green to blue crayons is 2 : 1)

Have students determine the number of green and blue crayons in these bags:

a) There are 30 crayons in a bag and \(\frac{3}{5}\) are green.
b) There are 36 crayons in a bag and \(\frac{4}{9}\) are green.
c) There are 21 crayons in a bag and \(\frac{4}{7}\) are blue.
d) There are 18 crayons in a bag. The ratio of blue to green is 7 : 2.
e) There are 18 crayons in a bag. The ratio of green to blue is 2 : 7.
f) There are 18 crayons in a bag. The ratio of blue to green is 2 : 7.
g) There are 30 crayons in a bag and 60% are green.
h) There are 45 crayons in a bag and 40% are green.

Discuss which two of parts d), e), and f) have the same answer (parts d) and e)) and re-write part f) in a different way that still has the same answer.

**Exercises**

1. A baseball player got a hit 2 out of every 3 times at bat. She was at bat 9 times. How many hits did she have?

   **Answer:** 6
2. Answer the question in your notebook.
   a) The ratio of voters who voted for Candidate A to voters who voted for Candidate B is 12 : 13. If 200 students participated in the vote, how many students voted for Candidate A?
   b) The ratio of dogs to cats in a pet shop is 4 : 11. If there are 60 dogs and cats altogether in the pet shop, how many cats are there?
   c) There are 2 apples in a bowl for every 3 oranges. If there are 15 fruits in the bowl, how many apples are there?

   Answers: a) 96, b) 44, c) 6

Extension

To estimate a fraction or ratio, you can change one or both parts slightly.

Example A: 5 out of 11 is close to 5 out of 10, which is close to \( \frac{1}{2} \) or 50%.

Example B: 9 out of 23 is close to 10 out of 25, which is \( \frac{2}{5} \) or 40%.

The chart below shows the lengths of calves and adult whales (in metres). Approximately what fraction and what percentage of each adult's length is the calf's length? Do you need to know how long a metre is to answer the question?

<table>
<thead>
<tr>
<th>Type</th>
<th>Killer</th>
<th>Humpback</th>
<th>Narwhal</th>
<th>Fin-backed</th>
<th>Sei</th>
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<tr>
<td>Calf Length (m)</td>
<td>2</td>
<td>5</td>
<td>1.5</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Adult Length (m)</td>
<td>4</td>
<td>16</td>
<td>4.5</td>
<td>20</td>
<td>19</td>
</tr>
</tbody>
</table>

Answers: Killer: 1/2 and 50%, Humpback: 1/3 and 30%, Narwhal: 1/3 and 30%, Fin-backed: 1/3 or 30%, Sei: 1/4 and 25%; no
Percentage Strips
Goals
Students will use diagrams to solve multistep word problems.

PRIOR KNOWLEDGE REQUIRED
Can represent fractions with a model
Can determine half of a whole number
Can multiply and divide decimal tenths by whole numbers
  (for Problem Banks 1–4)
Can add decimals, up to hundredths (for Problem Bank 7)
Can multiply and divide decimal hundredths by whole numbers
  (for Problem Bank 7 and Extended Problem)
Can compare fractions (for Extended Problem)
Can solve proportions (for Extended Problem)
Can calculate a fraction of a whole number (for Extended Problem)
Can divide by a two-digit whole number (for Extended Problem)

MATERIALS
BLM Making Punch (pp. Q-60–62, see Extended Problem)

Identifying parts of a diagram and solving a problem given the diagram.
SAY: Marla had $35 and spent three fifths of her money on a shirt. We are going to represent this problem with a diagram. Draw on the board:

SAY: I want to draw a diagram to show three fifths, so I drew a rectangle divided into five parts. Ask a volunteer to shade the part of the diagram that represents three fifths. (3 blocks) SAY: This diagram shows the total amount of money that Marla had and the shaded part shows the money spent on a shirt. Label the diagram, as shown below:

ASK: If all five blocks represent $35, how much money does each block represent? ($7) How do you know? ($35 ÷ 5 = 7)

Finish the diagram, as shown below:
ASK: How much money did Marla spend on the shirt? ($21) How do you know? (3 \times 7 = \$21) How much money does she have left? ($14) How do you know? (35 – 21 = \$14; or 2 unshaded blocks, 2 \times 7 = \$14) Point out that, even though the first solution is correct, the second solution shows that you can find the leftover in the diagram without calculating how much money was spent on the shirt.

Exercises: Draw a diagram to solve the problem.

a) Jane had $36. She spent \(\frac{3}{4}\) of her money on a pair of shoes. How much money does she have left?

b) Tristan spent \(\frac{2}{5}\) of his money on a toy. He has $15 left. How much did the toy cost?

c) Nora spent \(\frac{2}{5}\) of her money on a poster that cost $8. How much money did she have before she bought the poster?

Solutions

\[
\begin{align*}
a) & \quad \text{shoes leftover} = \$9 \\
b) & \quad \text{toy} = \$10 \\
c) & \quad \text{total before} = \$20
\end{align*}
\]

Solving problems where an amount is divided and then further divided into unequal parts. Write on the board:

Eric has some eggs. He uses \(\frac{3}{7}\) of them to make pancakes and \(\frac{1}{2}\) of the remainder to make sandwiches. Now Eric has 6 eggs left.

How many eggs did Eric use to make pancakes? How many eggs did Eric have at first?

Point out that the first two sentences are similar to the problem you just did, so to solve this problem we can start with the same type of diagram as earlier. Draw on the board:

pancakes

ASK: Which part of the diagram shows the leftover? (the unshaded squares)

Cover up the shaded part and ASK: Which part is half of the leftover? (2 squares) Mark that on the diagram, as shown below:

pancakes sandwiches
Problem-Solving Lesson 6-10

ASK: How many eggs are left after Eric made the pancakes and sandwiches? (6) So how many eggs does each block represent? (3) How do you know? (6 ÷ 2 = 3) How many eggs did he use for the pancakes? (9) How many eggs did Eric have initially? (21) How do you know? (7 blocks, 7 × 3 = 21)

Exercises

a) Tristan spent \( \frac{2}{5} \) of his money on a toy and \( \frac{2}{3} \) of the remainder on a gift for his sister. He has $8 left. How much did he spend altogether?

b) Marla spent \( \frac{2}{7} \) of her money on a shirt. She spent \( \frac{3}{5} \) of the remainder on a book. She has $10 left. How much did the book cost?

Answers: a) $32, b) $15

Write on the board:

Raj had 30 stickers. He gave \( \frac{2}{5} \) of his stickers to his brother and \( \frac{1}{2} \) of the rest to his friend. How many stickers did Raj’s brother get?

How many stickers are left?

Write on the board:

\[
\begin{align*}
\text{total stickers} & = 30 \\
\text{brother} & \\
\end{align*}
\]

ASK: How many blocks are there in total? (5) How many stickers does each block represent? (6) Explain how you know. (30 ÷ 5 = 6) How many blocks did Raj’s brother get? (2) So how many stickers did Raj’s brother get? (12) SAY: Raj’s friend got half of the rest. Draw a dashed line to show Raj’s friend’s stickers, as shown below:

\[
\begin{align*}
\text{total stickers} & = 30 \\
\text{brother} & \\
\text{friend} & \\
\end{align*}
\]

ASK: How many blocks did Raj’s friend get? (one and a half) How many stickers does each half block represent? (3) How do you know? (each block represents 6 so half represents 3) How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

Explain to students that it is sometimes easier to take away the first part of the question and then continue with the rest of the problem. SAY: At the beginning of the question, we know Raj has 30 stickers and he gave two fifths of his stickers to his brother.
Draw the original diagram on the board again:

- total stickers = 30
- brother

ASK: How many stickers does each block represent? (6) How many are left? (18) What fraction of the leftover goes to Raj’s friend? (half) Draw on the board:

- stickers left = 18
- friend

Point to the new diagram and ASK: How many stickers does each block represent? (9) How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

**Exercises:** The next time Raj has stickers, he decides to give \( \frac{2}{5} \) of his 30 stickers to his brother and \( \frac{5}{6} \) of the remainder to his friend.

a) How many stickers did Raj’s friend get?
b) How many stickers are left?

**Solution**

- total stickers = 30
- remainder = 18
- brother = 12
- friend = 15
- left = 3

**Solving problems backwards.** Write on the board:

Emma has some stickers. She colours \( \frac{1}{4} \) of them red and \( \frac{2}{5} \) of the remainder green.

If Emma doesn’t colour 9 stickers, how many stickers does Emma have in total?

Explain to students that, like in the previous problem, they can draw diagrams, one for each step of the problem. Draw on the board:

- total stickers = ?
- stickers left = ?
- red = ?
- stickers left = ?
- green = ?
- not coloured = 9
SAY: In the diagram on the left, all parts are unknown. ASK: Can I start with the diagram on the left? (no) SAY: Look at the diagram on the right. ASK: How many stickers are not coloured? (9) How many stickers does each block represent? (3) How do you know? ($9 \div 3 = 3$) How many blocks are green? ($2 \times 3 = 6$)

Erase the question mark beside “green” and write “6” in its place, as shown below:

```
|   |   |   |   |   |   |   |   |   |   |
```
```
|   |   |   |   |   |   |   |   |   |   |
```

Point to the diagram on the right and ASK: How many stickers are shown here in total? ($9 + 6 = 15$) SAY: So, how many are left after Emma colours some red? ($15$) Erase the question mark beside “stickers left” in both diagrams and write “15” in its place, as shown below:

```
|   |   |   |   |   |   |   |   |   |   |
```
```
|   |   |   |   |   |   |   |   |   |   |
```

Ask a volunteer to solve the diagram on the left and find the total numbers of stickers. (20)

**Exercise:** Marko spent $\frac{3}{5}$ of his money on a book and $\frac{3}{4}$ of the remainder on some music. Marko has $4$ after he paid for the book and the music. How much money did he have initially?

**Solution**

```
|   |   |   |   |   |   |   |   |   |   |
```
```
|   |   |   |   |   |   |   |   |   |   |
```

```
|   |   |   |   |   |   |   |   |   |   |
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|   |   |   |   |   |   |   |   |   |   |
```

Problem Bank

1. How many minutes are in each decimal number of hours?
   Remember, 60 minutes = 1 hour.
   a) 0.5 hours  b) 0.1 hours  c) 0.3 hours  d) 0.7 hours  
   e) 1.2 hours  f) 1.5 hours  g) 1.6 hours  h) 3.4 hours

   **Answers:** a) 30, b) 6, c) 18, d) 42, e) 72, f) 90, g) 96, h) 204
2. Since 0.2 hours is 12 minutes, you can write 1.2 hours as “1 hour 12 minutes.” Write the decimal number of hours as hours and minutes.
   a) 2.1 hours  b) 1.4 hours  c) 3.5 hours  d) 8.9 hours
   **Answers:** a) 2 hours 6 minutes, b) 1 hour 24 minutes, c) 3 hours 30 minutes, d) 8 hours 54 minutes

3. The number of hours is often given in decimal numbers. Convert the times shown in these situations to hours and minutes.
   a) Listen to 1.6 hours of music.
   b) In a flight course, you can use a flight training device for 1.2 hours.
   c) A group of people surveyed said they use social media networks an average of 2.7 hours per day.
   d) The average person surveyed watches 4.8 hours of television a week.
   **Answers:** a) 1 hour 36 minutes, b) 1 hour 12 minutes, c) 2 hours 42 minutes, d) 4 hours 48 minutes

4. a) This week, Glen spent \( \frac{3}{5} \) of his time doing homework on math and science. He spent 2 hours on math and 4 hours on science. How much time did he spend doing homework altogether?

   b) This week, Kate spent \( \frac{2}{5} \) of her time doing homework on math and science. She spent 1.6 hours on math and 2 hours on science. How many hours of homework did she do altogether?

   c) Did you do part b) by converting the number of hours to minutes and dividing the whole number of minutes by 2, or by dividing the decimal number of hours by 2? Do the question again the other way, and make sure you get the same answer.

   **Selected solutions**
   a) He spent 6 hours on math and science altogether, so each block represents 2 hours. Five blocks together is 10 hours:

   \[
   \text{total} = 10
   \]

   \[
   \text{math + science} = 6
   \]
b) She spent 3.6 hours on math and science altogether, so each block is 3.6 hours ÷ 2 = 1.8 hours. Five blocks together is $5 \times 1.8$ hours = 9 hours.

\[
\text{total} = 9
\]

\[\text{math + science} = 3.6\]

5. Rani reads 10 pages of a book on Saturday and she reads $\frac{3}{4}$ of the rest of the book on Sunday. She still has 17 pages to read. How many pages are in the book?

**Answer:** 78 pages

6. A convenience store has some ice cream treats. It sells $\frac{2}{5}$ of them on Friday, $\frac{1}{4}$ of the remainder on Saturday, and $\frac{2}{3}$ of the rest on Sunday. The store has 30 ice cream treats left by the end of Sunday.

a) How many ice cream treats did the store have initially?

b) On which day did the store sell the most ice cream treats?

**Answers:** a) 200; b) Friday, 80

7. Zara received some money for her birthday. She donated $\frac{1}{5}$ to charity, and she saved $\frac{2}{3}$ of the remainder in her savings. Of what was left, $\frac{1}{4}$ was a gift card to an ice cream store. She used the rest of the money to buy three books for $6.99 each, a T-shirt for $5.99, and a basketball for $7.99. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each part (donation, savings, gift card)?

**Answers:** the books were $6.99 each, the T-shirt was $5.99, and the basketball was $7.99, so she spent $34.95 on purchases; she was given $174.75 altogether; she gave $34.95 to charity; she put $93.20 in savings; and she had $11.65 on the ice cream store gift card
Extended Problem: Making Punch

MATERIALS

BLM Making Punch (pp. Q-60–62)

Extended Problem: Making Punch. Provide students with BLM Making Punch. Explain to students who are not familiar with punch that it is a drink made from fruit juices and soda.

NOTE: The bonus question provides students with an opportunity to use diagrams to solve a word problem. As students apply this problem-solving strategy, they will need to work backwards using the information given in the problem.

Answers
1. a) Recipe A: 3/5, Recipe B: 5/8, Recipe C: 7/12; b) 5/8 is the largest fraction, so Recipe B has the strongest ginger ale taste; c) 7/12 is the smallest fraction, so Recipe C has the strongest cranberry juice taste
2. a) 360 cups; b) 120 cups; c) 72 cups ginger ale and 48 cups cranberry juice; d) Recipe B: 75 cups ginger ale and 45 cups cranberry juice, Recipe C: 70 cups ginger ale and 50 cups cranberry juice, in total: 217 cups ginger ale and 143 cups cranberry juice
Bonus: a) the initial budget was $146.25, b) the committee paid $48.75 for ginger ale
Making Punch (1)

A graduation committee has three recipes to make cranberry juice/ginger ale punch for a graduation party.

*Recipe A:* 3 cups of ginger ale for every 2 cups of cranberry juice

*Recipe B:* 5 cups of ginger ale for every 3 cups of cranberry juice

*Recipe C:* 7 cups of ginger ale for every 5 cups of cranberry juice

1. a) What fraction of each recipe is ginger ale?

   Recipe A: _______  
   Recipe B: _______  
   Recipe C: _______

   b) Find the recipe that has the strongest ginger ale taste. Explain how you found your answer.

   c) Find the recipe that has the strongest cranberry juice taste. Explain how you found your answer.
Making Punch (2)

2. There will be 194 students and 256 parents at the graduation party. The graduation committee decides to make enough punch so that everybody can have one glass. Each plastic glass holds 200 mL of punch.

a) How many cups of punch in total are needed for the party? (Use 1 cup = 250 mL.)

b) The committee decides to make an equal amount of each type of punch. How many cups of each recipe are needed?

c) How many cups of ginger ale are needed for each recipe? How many cups of cranberry juice are needed for each recipe?

d) How many cups of ginger ale and how many cups of cranberry juice are needed for the party in total?
Making Punch (3)

BONUS ▶ The committee spends $\frac{1}{3}$ of its budget buying ginger ale and $\frac{2}{5}$ of the leftover money buying cranberry juice.

a) After buying cranberry juice, the committee has $58.50 left to buy plastic glasses. How much was the initial budget?

b) How much did the committee pay for ginger ale?
Unit 15  Probability and Data Management: Probability, Collecting and Analyzing Data

Introduction

This unit will focus on:

• finding theoretical and experimental probability of simple events;
• performing experiments and comparing theoretical and experimental probability;
• comparing measures of central tendency;
• using measures of central tendency to compare sets of data;
• identifying potential sources of bias in surveys, including biased samples;
• selecting, justifying, and using different methods for collecting data; and
• graphing collected data and analyzing the results.

Meeting Your Curriculum

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Mental Math Minutes

The mental math minutes in this unit review:

• fraction concepts, which are crucial for working with probability
• division skills necessary to work with mean and median
• equations that will be useful in the next unit
Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
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<tbody>
<tr>
<td>Quiz</td>
<td>PDM6-7 to 11</td>
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Additional Information for This Unit

Science connections
Both Alberta and Manitoba science curricula require that students perform experiments and record, present, and discuss the collected data. If students have already performed experiments and the results were kept, you can use the data collected during these experiments in Lessons PDM6-16 and PDM6-17. If not, these two lessons provide an excellent opportunity for combining mathematics with science. Students can conduct the experiment part of these lessons during science class to combine discussing the scientific and the mathematical components.
Goals

Students will identify outcomes of probability experiments, favourable outcomes for events, and equally likely outcomes.

Prior knowledge required

Has experience with spinning spinners, rolling dice, and tossing coins

Materials

A bottle cap, such as from a medicine bottle

Mental math minute. Write on the board:

\[
\frac{1}{3} \quad \frac{1}{2}
\]

Ask: Is one third more than one half or less than one half? (less) How do you know? (same numerator and 3 > 2, so 1/3 < 1/2) Remind students that if a fraction is less than half and you double the number in the numerator, you get less than a whole. If double of the numerator is less than the denominator, then the fraction is less than a half. Demonstrate using a circle divided into 3 parts; shade 1/3 and then shade 2/3 to show the doubling. Since less than a full circle is shaded, one third is less than one half. Students can signal the answer in the exercises below.

Exercises: Is the fraction more than half?

a) \(\frac{5}{9}\)
   b) \(\frac{3}{8}\)
   c) \(\frac{5}{11}\)
   d) \(\frac{8}{15}\)
   e) \(\frac{12}{23}\)
   f) \(\frac{13}{27}\)
   g) \(\frac{24}{47}\)
   h) \(\frac{42}{83}\)

Answers: a) yes, b) no, c) no, d) yes, e) yes, f) no, g) yes, h) yes

Introduce experiments and outcomes. Ask students if they have ever done a science experiment. Have volunteers give examples. Tell students that any time they do something that has results that depend on chance, and they know what all the possible results are, they are doing a probability experiment. Say: Rolling a die is an experiment because there are six possible results: rolling a 1, 2, 3, 4, 5, or 6. Ask: Is tossing a coin an experiment? (yes) What are the possible results? (coin lands heads up or tails up) Say: The different results of an experiment are called outcomes. Ask: What are the possible outcomes when two teams play soccer? (win, lose, or tie)

Hold up a bottle cap and ask: What are the possible outcomes when I toss this lid in the air? (landing with the flat side down, flat side up, or on its side) Then toss it several times until you see all three possibilities come up.
Exercises: What are the possible outcomes when you spin the spinner?

![Spinner Images]

a) 1, 2, 3,

b) 3, 5, 7

c) 6, 8, 10

d) 7

Answers: a) the spinner lands on 1, 2, 3, or 4; b) the spinner lands on 1, 3,
or 5; c) the spinner lands on 6, 7, or 8; d) the spinner lands on 7

Introduce events. SAY: An event is any set of outcomes. For example, when
rolling a die, the event "rolling an odd number" consists of outcomes 1, 3,
and 5. We call outcomes that make a specific event favourable outcomes.
So, the favourable outcomes for rolling an odd number on a regular die
are 1, 3, and 5. ASK: What are the favourable outcomes of rolling an even
number on a regular die? (2, 4, 6)

Exercises

1. List the favourable outcomes for the event when spinning the
spinner shown.

![Spinner Image]

a) spinning a multiple of 3

b) spinning an even number

c) spinning a number greater than 14

d) spinning a factor of 6

e) spinning a prime number

f) spinning a multiple of 7

g) spinning a number greater than 6

Answers: a) 12, 15; b) 10, 12, 14, 16; c) 15, 16, 17; d) none;
e) 11, 13, 17; f) 14; g) 10, 11, 12, 13, 14, 15, 16, 17

2. How many favourable outcomes are there for the event?

a) spinning green (G)

b) spinning an odd number

![Spinner Images]

c) spinning a 5

d) spinning a multiple of 4

Answers: a) 2, b) 6, c) 3, d) 0
Refer students to the spinner in Exercise 2, part a), above. SAY: Spinning green might seem like it is a single outcome because there is one colour, but in fact two regions have green. There are six outcomes because there are six regions on the spinner, and you can define events by combinations of outcomes. The event “spinning green” is a combination of two outcomes. ASK: How many regions are blue? (3) How many outcomes make the event “spinning blue”? (3) How many outcomes are a primary colour—red, yellow, or blue? (4) SAY: You can describe an event using this spinner in multiple ways, so there can be many events. But for this spinner, there are exactly six outcomes.

**Introduce impossible and certain events.** SAY: We call an event *impossible* if there are no possible outcomes that produce it. Refer students to Exercise 2 above and ASK: Which event is impossible? (spinning a multiple of 4 in part d) SAY: An event is *certain* if all the possible outcomes produce it. ASK: Which event is certain? (spinning an odd number in part b) SAY: Anything that is not impossible and not certain is in between. We call such events “possible, but not certain.”

**Exercises:** Jin rolls a regular die. Is the event certain, impossible, or in between?

a) rolling a 5    b) rolling an even number

c) rolling a number less than 7    d) rolling a 0

**Answers:** a) in between, b) in between, c) certain, d) impossible

**Introduce equally likely outcomes.** Draw the spinner shown in the margin on the board. SAY: Suppose we play a game. We spin this spinner. If the spinner lands on white, I win. If the spinner lands on grey, you win. ASK: Who will win more often? (teacher) How do you know? (larger part of the spinner is white) SAY: This spinner has two outcomes, but one of these outcomes will happen more often than the other. We say that these outcomes are not equally likely. If the outcomes should happen the same number of times, for example, the fraction of the circle is the same for each colour on the spinner, we say that the outcomes are *equally likely*. ASK: If you have a regular, fair coin, are heads and tails equally likely? (yes) Are all numbers on a regular die equally likely? (yes) If I have a collection of marbles, all exactly the same size and shape, but different colours, and I put them in a bag and take one without looking, are all marbles equally likely to come out? (yes)

For the exercises below, show the spinners one at a time and have students signal the answers to the questions.

**Exercises:** How many outcomes does the spinner have? Are the outcomes on the spinner equally likely?

![Spinner](image)

**Answers:** a) 8, yes; b) 6, no; c) 6, yes
NOTE: Extension 1 is required to cover the British Columbia curriculum.

Extensions

1. Explain the rules of the game Lahal sticks (or bones), or watch a video explaining the rules. Describe the probability set-up of the first round of the game: There are four bones in four hands, one bone in each hand. Two of the bones are white and two have a stripe. Each player holds one white bone and one bone with a stripe. Two bones are uncovered. If both are white, you win. If you uncover a marked bone, you lose a point (tally stick).

Let’s number the hands 1 to 4: Hands 1 and 2 belong to one person; Hands 3 and 4 belong to the other person.

a) How many ways to uncover the bones are there?

b) How many favourable outcomes (outcomes that uncover both white bones) are there? Hint: Suppose the white bones are on the outside (Hands 1 and 4).

c) How many outcomes will make you lose one stick?

d) How many outcomes will make you lose two sticks?

Answers: a) 4 (1 and 3, 1 and 4, 2 and 3, 2 and 4); b) 1; c) 2; d) 1

2. a) Draw a spinner that has four equally likely outcomes.

b) Draw a spinner that has four outcomes that are not equally likely.

Sample answers: a) A B

3. An experiment consists of tossing two coins: a dime and a nickel.

a) What are the possible outcomes of the experiment?

b) How many outcomes are in the event “the coins land the same way”?

Answers: a) dime tails and nickel tails, dime tails and nickel heads, dime heads and nickel tails, dime heads and nickel heads; b) 2
Goals

Students will find the theoretical probability of simple events, including certain and impossible events.

Students will express probability as a proper non-negative fraction.

PRIOR KNOWLEDGE REQUIRED

- Has experience with spinning spinners, rolling dice, and tossing coins
- Can read, write, and compare fractions
- Can produce fractions equivalent to a given fraction
- Can identify equally likely outcomes
- Can identify certain and impossible events

MATERIALS

- two pencils of different lengths

Mental math minute. Remind students that they can convert fractions to decimals by first finding an equivalent decimal fraction. Demonstrate using \( \frac{3}{25} = \frac{12}{100} = 0.12 \).

Exercises: Convert the fraction to a decimal.

\[
\begin{align*}
a) & \quad \frac{1}{4} \\ b) & \quad \frac{2}{5} \\ c) & \quad \frac{7}{25} \\ d) & \quad \frac{9}{20} \\ e) & \quad \frac{43}{50} \\ f) & \quad \frac{13}{25}
\end{align*}
\]

Answers: a) 0.25, b) 0.4, c) 0.28, d) 0.45, e) 0.86, f) 0.52

Measuring likelihood. Show students two pencils of different lengths. Ask students how they could determine which pencil is longer. Present two measurements that cannot be compared directly, such as the length of a ruler and the circumference of a cup. (Students might suggest using a measuring tape to compare them indirectly.) Ask students how they could compare the weight of two objects, such as a book and a cup, or the temperature in two different places. Point out that in all cases, students tried to attach a number to the characteristic or quantity and to compare the numbers. They suggested using different tools—a measuring tape, a scale, or a thermometer—to get a number, or a measurement. Explain that probability is the branch of mathematics that studies how likely events are and expresses this likelihood in numbers. The measure of the likelihood of an event is called probability.

A formula for finding probability. SAY: In the last lesson, we called outcomes that have the same chance of happening, like pulling same-size marbles out of a sack without looking, or a spinner landing on regions of the same size, equally likely. When all outcomes are equally likely, we define the probability of an event as a fraction. Write on the board:

\[
\text{Probability of Event } A = \frac{\text{number of favourable outcomes for } A}{\text{total number of outcomes}}
\]
SAY: Let’s look at rolling a die. ASK: Are the outcomes equally likely? (yes)
How many outcomes in total are there? (6) What is the probability of rolling a 3? (1/6)
Write on the board:

Event: rolling a 3
Number of favourable outcomes: 1
Total number of outcomes: 6
Probability of rolling a 3 = \( \frac{1}{6} \)

Repeat with the probability of rolling an even number. The probability is 3/6 = 1/2. Use the opportunity to remind students how to reduce fractions to make the numbers as small as possible.

**Exercises:** What is the probability of spinning red?

- a) [Diagram: R R G]
- b) [Diagram: R B G]
- c) [Diagram: Y B G]
- d) [Diagram: R R R]
- e) [Diagram: R B G R]
- f) [Diagram: R G R]

**Answers:** a) 2/3, b) 1/3, c) 0/3 = 0, d) 3/3 = 1, e) 2/4 = 1/2, f) 6/8 = 3/4

**Probability assigns numbers between 0 and 1 to likelihoods of events.** ASK: What is larger in the probability fraction, the numerator or the denominator? (denominator) How do you know? (denominator is the total number of outcomes in the experiment, numerator is just some of these outcomes) What does this mean about the size of this fraction? (it is not more than 1)

Remind students that impossible events are events that cannot happen, and certain events are events that will always happen in some experiments. ASK: In the exercises, in which part was spinning red impossible? (part c) What was the probability? (0) SAY: Look at the definition of probability. ASK: Why does it make sense that impossible events have probability 0? (there are no favourable outcomes for impossible events, so the number of ways an event can happen is 0, so the whole fraction is 0 too)

ASK: In the exercises, in which part was spinning red certain? (part d) What was the probability? (1) SAY: Look at the definition of probability. ASK: Why does it make sense that certain events have probability 1? (all outcomes are favourable, so the number of ways an event can happen is the same as the total number of outcomes, so the fraction is 1)
Exercises

1. What is the probability of spinning red?

   a) \[ \text{G B} \]
   b) \[ \text{R B} \]
   c) \[ \text{R R} \]
   d) \[ \text{R G} \]
   e) \[ \text{R G B Y} \]
   f) \[ \text{R Y R R} \]

   Answers: a) 0, b) \( \frac{1}{2} \), c) 1, d) \( \frac{1}{2} \), e) \( \frac{1}{4} \), f) \( \frac{5}{6} \)

2. What is the probability of pulling out a marble of the given colour from the collection without looking? Write your answer as a fraction with the smallest possible numbers.

   \[ \text{R R R R R G G B B B Y} \]

   a) yellow b) blue c) red d) green

   Answers: a) \( \frac{1}{10} \), b) \( \frac{3}{10} \), c) \( \frac{4}{10} = \frac{2}{5} \), d) \( \frac{2}{10} = \frac{1}{5} \)

Finding the probability of events made from different types of outcomes.

Draw the spinner shown in the margin on the board. SAY: Let’s find the probability of spinning either green or blue. ASK: How many ways can you spin green? (2) How many ways can you spin blue? (1) How many favourable outcomes are there for spinning blue or green? (3) Write on the board:

\[ \text{Ways to spin blue or green: 3} \]

ASK: How many outcomes are there in total? (8) Write “Total number of outcomes: 8” on the board. ASK: What is the probability of spinning blue or green? (3/8) Write “Probability of spinning blue or green: 3/8” on the board.

SAY: Now let’s find the probability of finding a colour that is not red. ASK: How many outcomes that are not spinning red are there? (5) How do you know? (count the outcomes that are not red; subtract 8 − 3 = 5, because there are 3 ways to spin red) Record finding the probability on the board (5/8) and have students tell you how to do it at each step. Repeat with the probability of “not spinning a colour that is on the Canadian flag.” (4/8 = 1/2)

Exercises: Use the spinner on the board to find the probability.

   a) spinning either yellow or green
   b) spinning a colour on the flag of Canada
   c) not spinning blue
   d) spinning a colour that is neither green nor white

   Bonus: spinning a colour that is not on the flag of your province or territory
Answers: a) $\frac{3}{8}$, b) $\frac{4}{8} = \frac{1}{2}$, c) $\frac{7}{8}$, d) $\frac{5}{8}$, Bonus: answers will vary

Finding the probability by subdividing spinner regions into equal parts.

Draw the spinner in the margin on the board. Say: I would like to find the probability of spinning each of the colours on this spinner. But I think there is a bit of a problem with this spinner. Ask: Do you spot it? Prompt: How many outcomes are there for spinning green? (1) How many outcomes are there for spinning yellow? (1) Is the spinner going to land more often on green or on yellow? (on yellow) Why is that? (yellow covers one quarter of the circle, green covers only one eighth)

Ask: When we wrote the formula for the probability of an event, what did we say about the outcomes? (they need to be equally likely, they need to happen equally often) How can we turn this spinner into a spinner with equally likely outcomes? (divide the larger regions into smaller parts so that all parts are equal) Invite a volunteer to divide the spinner and relabel the regions. Then have students calculate the probability of spinning each colour. Have volunteers show the answers on the board. (red: $\frac{3}{8}$, green: $\frac{1}{8}$, blue: $\frac{1}{8}$, yellow: $\frac{2}{8} = \frac{1}{4}$, white: $\frac{1}{8}$)

Exercises: Divide the spinner into equal parts. Then find the probability of the given event.

a) spinning blue  

b) spinning E

c) spinning a letter from “Canada”

Answer: a) $\frac{3}{8}$, b) $\frac{2}{6} = \frac{1}{3}$, c) $\frac{4}{6} = \frac{2}{3}$

Creating experiments with a given probability. Draw a circle. Say: I want to make this into a spinner that would have the probability of spinning red equal to one eighth. How can I do this? Accept all possible ideas and make sure students understand that the circle needs to be divided into eight equal parts with only one of the parts coloured red; the rest can be any other colour or colours, just not red. Mark one eighth as red or colour it. Record the probability of spinning red beside the spinner.

Ask: How can I use the same spinner and get the probability of spinning blue $\frac{1}{4}$? (colour two of the remaining seven parts blue and use any other colour or colours on the other 5 parts) How do you know we need two parts? (the spinner has 8 parts, so we need the probability with denominator 8, and $\frac{1}{4} = \frac{2}{8}$; 2 out of 8 parts need to be coloured blue) Colour the parts and record the probability. Then have students decide how they can make the spinner also have the probability of spinning green $\frac{1}{2}$. (colour 4 parts green) Have students draw spinners with this arrangement and have volunteers present different answers. Make sure students understand that the last part of the spinner can be any other colour, just not red, blue, or green.
Exercises: Draw 10 marbles so that the probability of pulling a red marble is 3/10, pulling a green marble is 1/5, and pulling a white marble is 2/5.

Bonus: Can you make the collection with probabilities as above using five colours? Explain.

Answers: 3 red marbles, 2 green marbles, 4 white marbles, 1 marble of another colour; Bonus: no, because only 1 marble is left to be used, so the collection has to have four colours.

NOTE: Before assigning the AP Book pages, remind students that they can also design a cube with different numbers on its sides by using a net of a cube. The net consists of six squares, on which students can write different numbers or letters, or students can colour them in different colours.

NOTE: Extension 1 is required to cover the British Columbia curriculum, and relies on Extension 1 in Lesson PDM6-7.

Extensions

1. Find the probability of each event in the game Lahal sticks.
   a) What is the probability of winning a guess in Lahal?
   b) What is the probability of losing a guess?
   c) What is the probability of losing one stick?
   d) What is the probability of losing two sticks?

   Answers: a) 1/4, b) 3/4, c) 2/4 = 1/2, d) 1/4

2. Find the probability of the event.
   a) A factor of 24 is chosen at random from the numbers 1 to 24.
   b) A multiple of 3 is chosen at random from all the composite numbers (numbers with more than two factors) from 1 to 20.
   c) A multiple of 3 is chosen at random from the factors of 48.
   d) A word is formed from writing the letters in ART in random order.
   e) A word is formed from writing the letters from OPST in random order.

   Answers: a) 1/3; b) 5/11; c) 1/2; d) 1/2, the three words are ART, RAT, and TAR; e) 1/4, the six words are OPTS, POST, POTS, SPOT, STOP, TOPS

3. A bag contains 26 cards, one with each letter of the alphabet.
   a) Tess randomly picks a letter of the alphabet from the bag. What is the probability that she picks a letter from A to M?
   b) A teacher randomly picks a student from the class. Is the probability that the student’s name starts with a letter from A to M equal to 1/2? Hint: Is each letter equally likely?
Answers: a) 1/2; b) no, because it is unlikely that each student has a name that starts with a different letter from A to Z.

4. Sam randomly picks a marble from a bag. The probability of picking a red marble is 2/5. What is the probability of not picking red? Explain.
Answer: 3/5, because if 2 out of every 5 marbles are red, then 3 out of every 5 marbles are not red.
Goals

Students will compare events by probability.
Students will express probability as fractions, decimals, and percentages.

PRIOR KNOWLEDGE REQUIRED

Has experience with spinning spinners, rolling dice, and tossing coins
Can identify equally likely outcomes
Can find probability of a given simple event
Can compare fractions, decimals, percentages, with or without a number line
Can convert among fractions, decimals, and percentages

Mental math minute. Remind students that they can represent a decimal or a percentage as a decimal fraction and then reduce the fraction by dividing the numerator and the denominator by the same number, a common factor of both. Demonstrate using an example: 48% = 0.48 = 48/100 = 12/25.

Exercises: Convert the percentage or the decimal to a fraction. Use the smallest numbers possible.

a) 75%  
b) 0.36  
c) 0.5  
d) 85%  
e) 0.2

Answers: a) 3/4, b) 9/25, c) 1/2, d) 17/20, e) 1/5

Review the definition of probability as a fraction. Remind students that when all outcomes are equally likely, such as when rolling a fair die or tossing a fair coin, or spinning a spinner divided into equal parts, the probability of an event is a fraction, or a ratio, of the number of favourable outcomes to the total number of outcomes. Write the definition on the board and keep it visible throughout the lesson:

Probability of Event A = \frac{\text{number of favourable outcomes for A}}{\text{total number of outcomes}}

Introduce the probability line. Remind students that in this fraction the numerator cannot be larger than the denominator because the number of outcomes for an event is always included in the total number of outcomes. Therefore, the probability of any event is a number between 0 and 1. Draw a number line from 0 to 1 and divide it into sixths. Label only the 0 and 1 tick marks. SAY: A number line from 0 to 1 that shows probabilities of different events is called a probability line.

ASK: If I roll a regular, fair die, how many possible outcomes do I have? (6) What is the probability of rolling a 5? (1/6) How do you know? (only 1 outcome is 5, and there are 6 outcomes in total) Write on the board:

Probability of rolling 5 = \frac{1}{6}
Invite a volunteer to mark the probability “roll 5” on the probability line. Repeat with the following events: roll an even number, roll a factor of 6, roll a multiple of 3. (3/6 = 1/2, 4/6 = 2/3, 2/6 = 1/3) The number line will look like this:

```
0 roll 5 roll a roll an even roll a
multiple of 3 number factor of 6
```

ASK: Is the probability of rolling an even number greater or smaller than the probability of rolling a factor of 6? (smaller) How does the probability line show that? (the probability of roll an even number is to the left of the probability of roll a factor of 6; 1/2 is smaller than 2/3)

SAY: When the probability of Event 1 is larger than the probability of Event 2, we say that Event 1 is more likely to happen than Event 2. We can use fractions to compare the likelihood of completely unrelated events. For example, imagine that two people play a weird game. Player 1 rolls a die. If he rolls an even number, he scores a point. Player 2 tosses a coin. If she tosses heads, she also scores a point. If we calculate the probability of both players winning, we can say whether one of them has a better chance of winning than the other. ASK: What is the probability of rolling an even number on a die? (3/6 = 1/2) What is the probability of tossing heads? (also 1/2) What can you say about the chances of the players to score a point? (they are the same) What if Player 1 scores a point only if he rolls 5 or 6? What is the probability of him winning then? (2/6 = 1/3) Now who is more likely to score a point, Player 1 or Player 2? (Player 2) How do you know? (the probability of Player 2 scoring a point is 1/2, which is greater than 1/3 for Player 1) Keep the probability line on the board for further discussion.

**Exercises:** Find the probability of both events. Write “is more likely than,” “is less likely than,” or “is as likely as” in the blank.

a) Rolling 3 on a regular die _______ rolling a number larger than 4.
b) Tossing tails on a coin _______ rolling 6 on a regular die.
c) Spinning green on the spinner shown _______ spinning red.

**Bonus**
d) Spinning green on the spinner shown _______ spinning yellow.
e) Spinning green on the spinner shown _______ spinning a rainbow colour.

**Answers:** a) 1/6, 2/6, is less likely than; b) 1/2, 1/6, is more likely than; c) 3/8, 3/8, is as likely as; Bonus: d) 3/8, 0, is more likely than; e) 3/8, 1, is less likely than

**Introduce “unlikely,” “likely,” and “even chances.”** ASK: Where do you put an event such as rolling 18 on a regular die on the probability line? (at 0) How do you know? (the probability is 0; there are no possible ways to roll 18 on a regular die) What do we call such events? (impossible) Repeat
with rolling a number less than 18. (probability is 1 because all outcomes are favourable, certain event) SAY: Events that have probability less than 1/2, but not 0, such as rolling 4, are called unlikely. Events with probability that is more than 1/2, but not 1, such as rolling a number less than 5, are called likely. Events that have probability that is exactly 1/2, such as rolling an even number or tossing heads, are said to have even chances of occurring. Label the probability line with the five terms.

**Exercises:** Describe the event using words.

a) rolling 3 on a regular die
b) rolling a number larger than 2 on a regular die
c) rolling 6 on a regular die
d) spinning green on the spinner shown

e) spinning red or orange on the spinner
f) spinning yellow on the spinner
g) spinning a rainbow colour on the spinner
h) not spinning blue

**Answers:** a) unlikely, b) likely, c) unlikely, d) unlikely, e) even chances, f) impossible, g) certain, h) likely

**Comparing likelihood without using a number line.** SAY: Imagine that we have a bag of marbles, and we pull out a marble without looking. Let’s say the bag has 2 red marbles, 5 green marbles, and 3 blue marbles. Write on the board:

<table>
<thead>
<tr>
<th>Bag 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 red</td>
</tr>
<tr>
<td>5 green</td>
</tr>
<tr>
<td>3 blue</td>
</tr>
</tbody>
</table>

SAY: Each marble is equally likely to come out. ASK: How many outcomes does this experiment have? (10) How many favourable outcomes does the event “pulling out a blue marble” have? (3) What is the probability of pulling out a blue marble? (3/10) What is the probability of pulling out a green marble? (5/10 = 1/2) Have students find the probability of pulling out a red marble. (2/10 = 1/5) Record the probabilities.

SAY: Imagine that we have a second bag of marbles, with 3 red marbles, 5 green marbles, and 7 blue marbles. Record the contents of the second bag on the board. ASK: If both bags have 5 green marbles, does this mean that the probability of pulling a green marble out of both bags is the same? (no) What is the probability of pulling out a green marble from the second bag? (5/15 = 1/3) Have students find the probability of pulling out of the second bag a red marble (3/15 = 1/5) and pulling out a blue marble (7/15).
Record the probabilities. ASK: Which colour has the same probability to come out of both bags? (red)

**Exercises:** Which event is more likely?

a) pulling out a green marble from bag 1 or from bag 2
b) pulling out a red marble from bag 1 or pulling out a green marble from bag 2
c) pulling out a blue marble from bag 1 or pulling out a red marble from bag 2

**Bonus:** pulling out a green marble from bag 1 or pulling out a blue marble from bag 2

**Answers:** a) pulling out a green marble from bag 1, b) pulling out a green marble from bag 2, c) pulling out a blue marble from bag 1, Bonus: pulling out a green marble from bag 1

**Writing probabilities as decimals and percentages.** SAY: Because a probability is just a number between 0 and 1, it can be written as a fraction, a decimal, or a percentage. Write on the board:

\[
\frac{11}{25}
\]

SAY: The first step to writing the fraction as a decimal is to write it as a fraction with the denominator equal to a power of 10. ASK: What is the smallest power of 10 we can use? (100) Continue writing on the board:

\[
\frac{11}{25} = \frac{100}{25} = 0.44
\]

**Exercises:** Write the probability of spinning grey as a fraction, a decimal, and a percentage.

a) \(\frac{1}{2}\), 0.5, 50%; b) \(\frac{7}{10}\), 0.7, 70%; c) \(\frac{3}{4}\), 0.75, 75%
**Probabilities as decimals and percentages in real life.** SAY: Probabilities are often written as decimals or percentages in real life. Write on the board:

a) the probability that it will rain
b) the probability that a given baseball batter will get a hit
c) the probability of a flood occurring
d) the probability that a given hockey goalie will make a save on the next shot
e) the probability that a given basketball player will sink the next free throw

Have volunteers tell you whether each probability is usually seen as a decimal or as a percentage. (a) percentage, b) decimal, c) percentage, d) percentage, e) percentage)

**Exercises:**Write the probability as a fraction with the smallest numbers.

a) The probability of rain is 60%.
b) The probability of the next free throw being successful is 75%.
c) The probability of a given baseball player getting a hit is .350, meaning the player gets a hit 350 times out of every 1000 chances.

**Answers:** a) 3/5, b) 3/4, c) 7/20

**Review creating experiments with the given probability.** Draw five circles on the board. SAY: I want to label these marbles with colours so that the probability of drawing a red marble is 20% and the probability of drawing a blue marble is 0.4. Record the probabilities on the board. Have students think of how they can do that. Accept multiple answers. Students need to convert the probabilities to fractions out of 5 (1/5 for red, 2/5 for blue), colour one marble red and two marbles blue, and use any other colour or colours for the remaining two marbles.

**Exercise:** Draw a collection of 10 marbles so that the probability of choosing a green marble is 0.3 and the probability of choosing a yellow marble is 40%. Can you use five different colours in your collection?

**Sample answer**

R B W G G Y Y Y Y

**Extensions**

1. Estimate the probability of spinning grey. Write your answer to the nearest 10%.

a) ![Circle A](image)
b) ![Circle B](image)
c) ![Circle C](image)
d) ![Circle D](image)

**Sample answers:** a) 40%, b) 20%, c) 60%, d) 10%
2. Draw a spinner with 10 equal regions, all marked A, B, or C, with the given conditions.

a) Spinning A is as likely as spinning B, and spinning C is the most likely outcome.

b) Spinning A is more likely than spinning B, and spinning B is more likely than spinning C.

c) Spinning A is as likely as spinning B, and spinning C is the least likely outcome, but not impossible.

Sample answers


3. Draw a spinner with 10 equal regions, all marked A, B, C, or D, with the given conditions.

a) Spinning A and B are equally likely, spinning C and D are equally likely, and spinning A is more likely than spinning C.

b) Spinning A, B, and C are equally likely, and spinning D is the most likely, but less than even chances.

c) Spinning A, B, and C are equally likely, and spinning D is the least likely.


Goals

Students will determine the expected number of times an event will happen when an experiment is repeated a given number of times. Students will compare experimental and theoretical probability.

PRIOR KNOWLEDGE REQUIRED

Has experience with spinning spinners, rolling dice, and tossing coins
Can identify equally likely outcomes
Can find the probability of a given simple event
Can find a fraction of a number

MATERIALS

overhead projector
transparency of BLM Spinner (p. R-63)
BLM Spinner (p. R-63), a paper clip, and a pencil for each student

Mental math minute—number strings.

String 1: 60 ÷ $\frac{3}{2}$ of 60, $\frac{2}{3}$ of 60 (20, 20, 40)

String 2: $\frac{1}{6}$ of 60, $\frac{5}{6}$ of 60, $\frac{1}{8}$ of 40, $\frac{3}{8}$ of 40, $\frac{7}{8}$ of 40, $\frac{3}{6}$ of 30

(10, 50, 15, 15, 35, 15)

String 3: $\frac{3}{5}$ of 100, $\frac{3}{10}$ of 50, $\frac{3}{8}$ of 48, $\frac{3}{12}$ of 600 (60, 15, 18, 150)

Expected number of a given event when repeating an experiment.

Project a transparency of BLM Spinner. ASK: Which fraction does the grey part show? (1/4) What is the probability of spinning grey on this spinner? (1/4) If I spin this spinner 20 times, how many times would you expect it to spin grey? (take all guesses) Then do 20 trials and record the number of grey outcomes. To spin the spinner, place the tip of a pencil at the centre of the spinner and a paper clip around the pencil point. Tell students that if the spinner lands between a white region and a grey region, you will not count that result.

NOTE: Students will use the data from this activity and its analysis later in this lesson. Ontario students will also use it in Lesson PDM6-12.

ACTIVITY (Essential)

Provide each student with BLM Spinner, a pencil, and a paper clip. Have students spin the spinner 20 times and keep a tally of how many times they spin white and how many times they spin grey. Students should not count any spins that land exactly between two regions.
Have several students record their results on the board. Then ask the class how many got grey exactly one time, two times, and three times and record the results. Record the results of all students in a bar graph, each bar showing the number of students that spun grey the given number of times. Keep the bar graph for use in Lesson PDM6-12. ASK: What appears to be the expected number spinning of grey now that you have seen everyone's results? (5)

SAY: The expected number of spinning grey when doing 20 spins seems to be 5. This makes sense because 1/4 of the spinner is grey and 1/4 of 20 is 5. Write on the board:

\[
\frac{1}{4} \text{ of } 20 = 20 \div 4 = 5
\]

Exercises: How many times would you expect the spinner to land on grey if you spin it the given number of times?

a) 40 times  b) 80 times  c) 32 times  d) 1000 times

Answers: a) 10, b) 20, c) 8, d) 250

Introduce experimental probability. Refer students to the example above. Point out that 5 isn’t expected in the sense that everyone spun grey exactly 5 times. ASK: Was 5 the most common result? (answers may vary)

SAY: Some of the results were 4 and 6, which is also close to 5.

SAY: When you calculated the probability of spinning grey, you calculated the theoretical probability of spinning grey. This was in theory. In practice, you did not always get grey exactly one quarter of the spins. When you repeat an experiment and then divide the number of times you spin grey by the number of experiments you performed, you calculate experimental probability. Write on the board:

\[
\text{Experimental probability of A} = \frac{\text{number of times A really happened}}{\text{total number of experiments performed}}
\]

Have students calculate the experimental probability of spinning grey in the experiment they have just performed. Then ask them to combine results with another person and calculate the experimental probability again. Repeat with groups of four. ASK: In which part of the previous exercises did you calculate the expected number of spinning grey for 40 spins? (part a) For 80 spins? (part b) Compare the results with the answers in parts a) and b) of the previous exercises. You can also do Extension 1 with students at this point.

The expected number of times when the probability has a numerator greater than 1. ASK: If the spinner is expected to land on grey 5 times out of 20, how many times out of the 20 times is it expected to land on white? (15) How did you get that? (students might subtract 20 − 5, or students might multiply by 3) If both solutions do not come up, ask for other possible
ways of seeing it. Once you have elicited both solutions, SAY: You would expect that 3 out of every 4 spins will land on white. Write on the board:

\[
\frac{3}{4} \text{ of } 20 = 3 \times \frac{1}{4} \text{ of } 20 = 3 \times 5 = 15
\]

SAY: An event that happens \(\frac{3}{4}\) of the time should happen three times as often as an event that happens only \(\frac{1}{4}\) of the time.

**Exercises**

1. Megan spins the spinner 30 times. How many times should she expect the spinner to land on red (R)?

   a) \[\text{R B O Y}\]  
   b) \[\text{R B Y G}\]  
   c) \[\text{R B Y R}\]  
   d) \[\text{R R B R}\]  
   e) \[\text{R R R B}\]  
   f) \[\text{R Y R R}\]

   **Answers:** a) 6, b) 12, c) 18, d) 24, e) 20, f) 25

2. Mentally divide the spinner into equal parts. If you spin the spinner 40 times, how many times do you expect the spinner to land on the given colour?

   a) red  
   b) blue  
   c) white  
   d) grey

   **Answers:** a) 20, b) 15, c) 12, d) 28

**Deciding whether given results are likely or unlikely.** Refer back to BLM Spinner. Write on the board:

White: 6, Grey: 14  
White: 14, Grey: 6

ASK: Which of these results do you think would be more likely? (White: 14, Grey: 6) Why? (because there is more chance to land on white than on grey)
Exercises: Suppose you spin the spinner shown 25 times.

a) How many times would you expect to spin grey?

b) Which of the charts shows a result you would be most likely to get?

A. Grey White
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B. Grey White
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C. Grey White
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c) Which result would surprise you the most? Explain.

Answers: a) 20; b) Chart A; c) Chart C because the spinner is mostly grey, but the results in Chart C show many more white spins than grey

Bonus: Matt spins a spinner 300 times and gets red 96 times. Which spinner did he likely use? Explain.

A. B. C. D.
\[ \begin{array}{ccc}
B & G & B \\
B & G & Y \\
R & B & G \\
R & B & Y \\
\end{array} 

Answer: Spinner C because about 1/3 of the spins were red.

Extensions

1. a) Draw a probability line and mark on it the probabilities below.
   i) theoretical probability of spinning grey in the activity
   ii) experimental probability of spinning grey you obtained after 20 spins in the activity
   iii) experimental probability of spinning grey you obtained after combining results with one partner (40 spins)
   iv) experimental probability of spinning grey you obtained after combining results in a group of four (80 spins)

b) Does your experimental probability get closer to the actual probability as the number of the spins increases?

NOTE: Answers will vary. The majority of students will see the experimental probability getting closer to the theoretical value. Explain that this is what should happen, but experiments are unpredictable, and there is always a chance that your answer deviates from theory. The result should get closer to theory with a really large number of spins.
2. A batting average of .427 means a baseball player had 427 hits in 1000 times at bat.
   
a) Is batting average an example of theoretical probability or experimental probability?
   
b) How many hits, on average, would each player likely get in 60 times at bat? Order the players by the number of the hits you expect, from least to greatest.
   
i) Player A has a .200 batting average
   
ii) Player B has a .250 batting average
   
iii) Player C has a .400 batting average
   
iv) Player D has a .350 batting average
   
c) Check that your answers in part b) make sense. Order the players by the batting average, from lowest to highest. Did you get the same order? Is Player B’s number more than Player A’s number by the same amount that Player C’s number is more than Player D’s? Is the difference between Player D’s and Player B’s numbers twice that amount?
   
**Answers**
   
a) experimental probability
   
b) i) 12, ii) 15, iii) 24, iv) 21; A, B, D, C
   
c) A, B, D, C; yes; 12 < 15 < 21 < 24 with a difference of 3 between Player B and Player A and between Player D and Player C; twice that amount (2 x 3 = 6) is the difference between Player B and Player D
   
3. These nets are folded together to make a die, and each die is rolled 300 times. Match each die net to the correct statement.
   
A.  
   
4 5 3  
5 2
   
B.  
1 4 6 2  
4 5
   
C.  
2 3 1 3  
3 6
   
D.  
3 5 2 4  
5 1
   
   
a) I would expect to roll 4 about 50 times.
   
b) I would expect to roll an even number about 150 times.
   
c) I would expect to roll 3 about 150 times.
   
d) I would expect to roll 1 about the same number of times as rolling 5.
   
**Answers:** a) D, b) A, c) C, d) B
**Goals**

Students will determine the probability of winning in games, decide if a game is fair, and compare the theoretical probability of winning with the experimental probability.

**PRIOR KNOWLEDGE REQUIRED**

- Has experience with spinning spinners, rolling dice, and tossing coins
- Can identify equally likely outcomes
- Can find the theoretical and experimental probability of a given simple event
- Can determine the expected number of outcomes based on probability
- Can find a fraction of a number

**MATERIALS**

die and shoebox per pair of students

cards with numbers 1 to 10 (e.g., playing cards) per student or student pair

**Mental math minute.** Present the problem: Find 30% of 20. SAY: You can find the percentage of a number by multiplying by the percentage and dividing by 100. You can also do that by multiplying by a decimal that represents the percentage. So, to find 30% of 20, find 0.3 × 20. One way to find 0.3 × 20 is 3 × 20 ÷ 10 = 60 ÷ 10 = 6. Record the solution on the board as you explain.

**Exercises:** Find the percentage of the number.

a) 40% of 30  
b) 25% of 40  
c) 15% of 60  
d) 50% of 16  
e) 48% of 200

**Answers:** a) 12, b) 10, c) 9, d) 8, e) 96

**Introduce fair games.** SAY: I would like to play a game with you. The rules of the game are simple. I spin a spinner. If I spin white, I win; if I spin grey, the class wins. Ask students if they agree to play by these rules. Draw the spinner shown in the margin on the board. ASK: Do you still want to play? (no) Why not? (because you are more likely to spin white than grey) How do you know? (the white part is more than half the spinner, and the part that is coloured grey is less than half the spinner; there is more white than grey) Write “fair game” on the board. Ask students to explain what they think this term might mean. Encourage students to use math vocabulary in their explanations. SAY: In mathematics, a fair game means that all players have an equal chance of winning, or are equally likely to win.
ASK: Does this mean that the probability to win should always be 50% or one half for each player for a game to be fair? (no) Draw the second spinner shown in the margin on the board. SAY: Imagine we play with this spinner. If we spin red, I win. If we spin blue, you win. If we spin yellow, nobody wins. ASK: What is the probability that I win? (1/3) What is the probability that you win? (1/3) Is the game fair? (yes) Point out that in some games you do not win right away; you just score points. In this case, you need to see if everyone has the same chance of scoring a point.

**Exercises:** What is the probability for each player to win when spinning the spinner shown? Is the game fair? If not, which player has a better chance of winning?

![Spinner](image)

a) Player 1 spins red to win. Player 2 spins blue to win.

b) Player 1 spins white or yellow to win. Player 2 spins blue to win.

c) Player 1 spins white or red to score a point. Player 2 spins blue to score a point.

d) Player 1 spins a colour that is not red to score a point. Player 2 spins a colour that is not white to score a point.

e) Player 1 spins blue to win. Player 2 spins red to win. Player 3 spins white or yellow to win.

f) Player 1 spins blue or white to score a point. Player 2 spins blue or yellow to score a point. Player 3 spins red or white to score a point.

g) Player 1 spins blue or white to score a point. Player 2 spins blue or yellow to score a point. Player 3 spins red or blue to score a point.

**Answers**

a) $\frac{2}{6} = \frac{1}{3}$ for both players, the game is fair

b) $\frac{2}{6} = \frac{1}{3}$ for both players, the game is fair

c) $\frac{1}{2}$ for Player 1, $\frac{1}{3}$ for Player 2, the game is not fair, Player 1 has a better chance of scoring a point

d) $\frac{2}{3}$ for Player 1, $\frac{5}{6}$ for Player 2, the game is not fair, Player 2 has a better chance of scoring a point

e) $\frac{1}{3}$ for all players, the game is fair

f) $\frac{1}{2}$ for all players, the game is fair

g) $\frac{1}{2}$ for Players 1 and 2, $\frac{2}{3}$ for Player 3, the game is not fair, Player 3 has a better chance to score a point

**Comparing theoretical probability in games with experimental probability.** SAY: You are now going to play a game. Here are the rules: You play in pairs. One player rolls a die. The other player adds a mark to the tally chart of the results. Players switch roles. If the player rolls 1 or 6, Player 1 scores a point. If the player rolls 3 or 4, Player 2 scores a point. If the player rolls 2 or 5, nobody scores. Write the scoring guide on the board.
Emphasize that the game is based entirely on chance and says absolutely nothing about the winner or the loser.

ASK: What are the chances of Player 1 scoring a point? (2/6 = 1/3)
What are the chances of Player 2 scoring a point? (2/6 = 1/3) Is the game fair? (yes)

SAY: You will roll the die 30 times. ASK: How many times do you expect Player 1 to score a point? (1/3 of the times, so 10 times) How many points do you expect Player 2 to score? (10 points)

### ACTIVITY 1 (Essential)

1. Give each pair of students a die and a shoebox to prevent the die from rolling away. Have them roll the die 30 times and tally the results using the table shown below. Students can also present the results using a double bar graph.

<table>
<thead>
<tr>
<th></th>
<th>30 rolls</th>
<th>60 rolls</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Player 1 scores</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Player 2 scores</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Nobody scores</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discuss the results of the activity. ASK: Did some pairs have a tie? Did Player 1 win in some pairs? Did Player 2 win in other pairs? Did both players indeed score 10 points? Record multiple results on the board and discuss how the actual results are different from the expectation. Then have students combine the results of Player 1 and Player 2 with another pair, to get 60 rolls in total. ASK: How many points do you expect each player to get? (20) How many points did each player score in total? Have students calculate the experimental probability for each player to score, in 30 rolls and in 60 rolls. ASK: Did the experimental probability get closer to the theoretical probability of scoring in 60 rolls?

SAY: Suppose you play this game 20 times. ASK: How many times do you expect Player 1 to score a point? Accept all answers. SAY: Since 20 does not divide into 3 without a remainder, you can expect an answer that is close to 20 ÷ 3, so 6 or 7 times. If you write your answer as a mixed number, 20 ÷ 3 = 6 2/3, which is closer to 7 than to 6, seven is a little more likely.

**Exercises:** Three cereal companies offer prizes.
Company A: 1 out of every 3 boxes wins $2.
Company B: 1 out of every 4 boxes wins $3.
Company C: 1 out of every 6 boxes wins $5.

a) If you buy a pack of 24 boxes of each type of cereal, how many boxes will have a prize?

b) If you buy a pack of 24 boxes of each type of cereal, how much money do you expect to win from each company?
c) Which company would you buy the 24 boxes from if you want better prizes?

Answers:  

a) 8 from Company A, 6 from Company B, 4 from Company C;  
b) $16 from Company A, $18 from Company B, $20 from Company C;  
c) Company C

ACTIVITY 2 (Optional)

2. Give each student or student pair 10 cards with numbers 1 to 10. Playing cards with the ace serving as 1 work well. Have students use the cards to play the game in Question 3 on AP Book 6.2 p. 149. Students can tally the results and repeat parts d) and e) in Question 3 using their own data. They can also pool the results from several groups and compare the experimental probability of each player to win to the theoretical probability.

Extensions

1. The chart shows the survival rate, or how many birds will survive, under two different environmental protection programs. If a program could be implemented in only one forest, which one would you choose? Explain.

<table>
<thead>
<tr>
<th>Number of Endangered Birds</th>
<th>Forest A</th>
<th>Forest B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survival Rate</td>
<td>5000</td>
<td>15,000</td>
</tr>
<tr>
<td>80%</td>
<td>2 in 5</td>
<td></td>
</tr>
</tbody>
</table>

Answer: The program in Forest B because it will save 6000 birds, while the program in Forest A will save only 4000 birds.

2. Ed and Braden are playing “Rock, paper, scissors.”

a) What is the probability that Ed wins? Explain. Hint: “Ed wins” is not an outcome; it is an event. “Ed has paper, Braden has rock” is an outcome.

b) Ed and Braden can play 15 games in 1 minute. If they play the game for 5 minutes, how many times do you expect Ed to win?

Answers

a) There are 9 outcomes to the game. Ed wins if: Ed has rock, Braden has scissors; Ed has paper, Braden has rock; Ed has scissors, Braden has paper. The probability that Ed wins is 1/3.

b) 25 times

3. If there are 8 billion people in the world, how many would you expect to be born on February 29th? Explain your reasoning and your assumptions.

Sample answer: February 29th occurs approximately once every four years, so about $1/1461$ of the time. So, you would expect about $8 \ 000 \ 000 \ 000 \div 1461 \approx 5 \ 475 \ 702$ people to have been born on February 29th. This assumes that all birthdays are equally likely.
Goals
Students will find the mean of sets of data, investigate change of mean when the set of data changes, recognize mean as the value that balances the set, and see uses of mean in real-life situations.

PRIOR KNOWLEDGE REQUIRED
Can divide numbers producing a mixed number or a decimal
Can read and analyze bar graphs

MATERIALS
15 blocks
BLM Moving Beads to Find the Mean (p. R-64)
data collected during the activity in Lesson PDM6-10
BLM Investigating Mean with Blocks and Beads (p. R-65, see Extension 2)

Mental math minute. Have students divide $17 \div 5$ with remainder, divide $17 \div 5$ and write the answer as a mixed number, and divide $17 \div 5$ and write the answer as a decimal. (3 R 2, 3 2/5, 3.4)

Draw 17 dots and circle the groups of five. SAY: There are two dots left over. If we write division with remainder, the remainder is 2. ASK: What do these two dots become in the mixed number answer? (numerator of the fractional part) What is the denominator? (the divisor, 5) How do you get a decimal from 3 2/5? (convert 2/5 to 4/10 by multiplying both numerator and denominator by 2) Point out that $17 \div 5$ is also the same as an improper fraction, 17/5, which can be converted to the mixed number 3 2/5 and the decimal 3.4.

Exercises: Divide. Write the answer with remainder, as a mixed number, and as a decimal.

<table>
<thead>
<tr>
<th>Student</th>
<th>Plums Picked</th>
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<tbody>
<tr>
<td>Jane</td>
<td>4</td>
</tr>
<tr>
<td>Amir</td>
<td>2</td>
</tr>
<tr>
<td>Rani</td>
<td>6</td>
</tr>
<tr>
<td>Kyle</td>
<td>1</td>
</tr>
<tr>
<td>Zara</td>
<td>2</td>
</tr>
</tbody>
</table>

a) $38 \div 5$  b) $49 \div 4$  Bonus: $65 \div 25$

Answers: a) 7 R 3, 7 3/5, 7.6; b) 12 R 1, 12 1/4, 12.25; Bonus: 2 R 15, 2 3/5, 2.6

Introduce mean. SAY: Five people want to evenly share the plums they picked. People with a lot could give some to people with fewer until everyone has the same number, or someone could deal them out like cards. Mathematicians call this finding the mean or average number of plums. Draw the table shown in the margin on the board. SAY: Each number is called a data point, and a group or collection of data points is called a data set or set of data. The data points in a set all refer to the same thing; in this case, 4, 2, 6, 1, and 2 all refer to the number of plums each person picked.
Finding the mean by moving objects between groups. SAY: I will use five groups of blocks to represent the plums each person picked. Place 15 blocks in groups of 4, 2, 6, 1, and 2. Have a volunteer move blocks until the five groups are equal. ASK: How many blocks are in each group? (3) SAY: If they share the plums evenly, everyone would get three plums, so the mean is 3.

Give each student one third of BLM Moving Beads to Find the Mean and draw the first picture from it on the board. Demonstrate crossing out one bead on the tallest stack and moving it to the lowest stack and shading the new bead to show students what to do in the exercise below.

Exercise: Move beads to find the mean.

Sharing equally by adding and dividing. SAY: There is another method to find the mean. We put all the plums together and distribute plums to the five friends, one at a time. Put 15 blocks in a pile. Ask a volunteer to distribute the blocks evenly into five groups. ASK: How many blocks are in each group? (3) Is this the mean we found before? (yes)

ASK: When we put all plums in a single pile, which mathematical operation describes this? (addition) Write the corresponding addition on the board:

Finding the mean of 4, 2, 6, 1, 2:

**Step 1:** Add the data values: \(4 + 2 + 6 + 1 + 2 = 15\)

ASK: When we distribute the plums one at a time, what mathematical operation describes this? (division) Write the division on the board:

**Step 2:** Divide the sum by the number of data values: \(15 ÷ 5 = 3\)

ASK: How can we write this in a single calculation? PROMPT: If you want to do addition before division, you need to show that with brackets. \((4 + 2 + 6 + 1 + 2) ÷ 5 = 15 ÷ 5 = 3\) Record this calculation on the board as well.

Exercises: Calculate the mean.

a) 4, 7, 2, 1, 1 
 b) 10, 12, 3, 7

**Bonus:** 0, 8, 15, 16, 19, 24

**Answers:** a) 3, b) 8, Bonus: 14

Using the mean to describe the data. Hold up a pencil and ask students to describe it. Prompt with questions about colour, length, uses, and so on. Explain that we can’t describe everything about it, but a person who couldn’t see it would likely know it was a pencil. SAY: Just as we can describe pencils (or shoes, or animals, or anything), we can describe sets of data, and the mean can be part of that description. It tells us about the centre of the set, the “norm.” Explain that the mean is always useful to know, but it is especially useful for very large sets of data. Let’s say, for example, that we know (from asking 1000 people) that the mean age when a child loses her first tooth is 6. If the mother of a 2-year-old who has just lost his first tooth knows this mean, she may decide to take the child to a doctor to see if everything is okay.
The mean in context: comparing the mean with data points from the set. SAY: Sometimes we need to be able to compare the mean of a set of data with one or more of the data points in the set. For example, let's say the mean weight of newborn babies is 3 kg 300 g. ASK: Is a newborn baby that weighs 1 kg below the mean weight, equal to it, or above it? (below) What about a 2.5 kg baby? (below) Which is farther from the mean? (the 1 kg baby) SAY: Doctors know that babies who weigh a lot less than the mean often need extra care, so knowing the mean weight of newborns helps doctors decide which babies might need a little extra care.

Exercises: A teacher sees that a student has these marks (out of 100) in the subject: 80, 85, 90, 85, 10, 88.

a) Find the mean.
b) Which test should the teacher check again to see if there is a problem with the test or the student’s understanding?

Answers: a) 73, b) the test that got 10

The mean in context: comparing means with each other. Explain that sometimes it is useful to compare means with each other. Let's say that you research how much money it costs to drive a large car and you find the following figures. Write on the board:

Costs of driving 100 km for large cars: $45, $40, $30, $32, $36

Then you do more research and find the costs for medium cars. Write on the board:

Costs of driving 100 km for medium cars: $35, $38, $30, $32, $21

SAY: You can see that no medium car costs $45 for 100 km, but you can see that some numbers for medium cars are higher than some numbers for large cars. You could make a double bar graph to compare the sets, but there is no real order to them; it is not clear which number to compare with which number. Let's find the mean for both sets. Note that the means might not be a whole number of dollars. Have students find the means for the sets. (large: 36.6, medium 31.2) Point out that since these are money amounts, we write the answers to the cent. ($36.60, $31.20) SAY: On average, a medium car costs about 5 dollars less than a large car, for every 100 km of driving. Moreover, you can now use the mean to see, within each set of data, which cars are below the average and so are more efficient than others.

Exercises: Two classes wrote a test that had marks out of 25.

<table>
<thead>
<tr>
<th>Class A’s marks</th>
<th>20, 25, 21, 22, 19, 22, 17, 12, 21, 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class B’s marks</td>
<td>19, 20, 15, 25, 25, 19, 19, 19, 19, 20</td>
</tr>
</tbody>
</table>

a) Which class has a higher mean grade?
b) Which class has a higher lowest grade?
c) In which class did more students get full marks?
d) Which class did better overall? Why do you think so?
Answers
a) Class A mean: 20.2, Class B mean: 20, so Class A has a higher mean
b) Class B's lowest grade (15) is higher than the lowest grade in Class A (12)
c) more students got full marks in class B

Sample answer: d) overall, on average, Class A has higher marks because they have a higher mean; more students got higher grades than the mean

Changing the set of data to produce a related set. SAY: Sometimes you have a set of data and you add some data values. For example, you look at a set of your marks and find the average. Then you write another test, and your average changes. Write on the board:

Test scores: 73, 77, 80, 84, 90, 85, 85

Have students calculate the average mark. (82) SAY: Suppose you miss the next test, and the mark on it is 0. ASK: How do you think this will affect your average? (it will go down) Have students calculate the new average, with the additional 0. (71.75) ASK: Does the mean now accurately represent your grades? (not really) SAY: Suppose you write the test and instead of 0 you now get 80. Add 80 to the list instead of the 0. ASK: What happens with your average? Accept all answers, then have students calculate the new mean. (81.75) ASK: What happened to the original mean? (it decreased) Why did this happen? (the new grade was close to the mean, but a little below it, so it dropped the grade, but only by a small amount) Finally, have students predict what would happen if the new grade, instead of being 80, was 88. (the mean would rise, but more than it dropped in the last case) Again, have students find the new mean. (82.75)

Finding the mean when multiple data values are the same. Display the bar graph you created in Lesson PDM6-10 showing how many times students spun grey when spinning the spinner on BLM Spinner 20 times. SAY: I would like to calculate the mean for this set of data. Multiple students spun grey 4 times, and many students spun grey 5 and 6 times. I could list all the data values for the whole class and add them, but I think it would take a lot of time. ASK: Can you think of a more efficient way of adding the data values? If the idea of using multiplication and addition does not arise, start listing the data values as a sum in order, from smallest to largest, then put brackets around the first group of identical numbers and ask how students could calculate the contents of the brackets more efficiently. Then write the calculation for the data on the board, following this format:

\[ 3 \times \_ + 4 \times \_ + 5 \times \_ + 6 \times \_ + 7 \times \_ \]

ASK: How many students spun grey 3 times? Fill in the first blank. Continue with the rest of the blanks, then have students help you finish the calculation. Have them count the total number of data values and find the mean. The mean is likely to be close to 5, but not exactly 5. Point out that the result is close to 5, which was the expectation for 20 spins.
Exercises: The graph shows grades out of 10 for math and for science.

Grade 6 Math and Science Grades

a) Find the mean of the math grades.

b) Find the mean of the science grades.

c) In which subject did the class do better overall?

Answers: a) 7.4, b) 7.8, c) science

Discuss the exercises. ASK: Is it easier to see in which subject the class did better overall from the means or from the graph? What information do you lose when you only look at the mean? Have students explain using examples. (It is easier to see that the class did better in science from the means because you only compare two numbers, but you do not see how many students got each grade. For example, you do not see that there were two students who got 10 in math, although nobody got 10 in science.) Accept several explanations. Point out that the science grades are closer together, while the math grades are more spread out; you have more bars for math than for science, but the science bars are taller towards the right end. This means that more students got higher grades in science than in math.

ACTIVITY (Essential)

Have students choose two locations in two different provinces in Canada, preferably one in the north and another in the south. Have them use the internet to find the average daily temperatures for both locations during the next two weeks. Students can use computer software to find the mean temperature and then use the mean to compare the two sets of data. Discuss whether the mean shows the whole picture. Are there days when the location with the higher mean was colder than the other location? Does the mean show this? On average, how much colder was one location than the other?
Extensions

1. Evan found some information about the egg sizes and clutch sizes (number of eggs in a nest) of some Alberta birds.

<table>
<thead>
<tr>
<th>Bird</th>
<th>Egg Length (cm)</th>
<th>Clutch Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snowy owl</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>American white pelican</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Herring gull</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Common loon</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Western grebe</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Great blue heron</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>American bittern</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Turkey vulture</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Find the mean of the egg lengths and the clutch sizes.
b) Make a Venn diagram for these categories: 1. Birds with at least average egg length. 2. Birds with at least average clutch size.
c) Evan read that birds with larger eggs produce smaller clutches. Does your Venn diagram support that claim?
d) Evan found data about northern cardinals. They have eggs that are 2.5 cm long, and there are two eggs in a clutch. Without re-calculating the mean, where would you put the northern cardinal in your Venn diagram? Does this data support the claim from part c)?
e) Evan found data about African ostriches. They have eggs that are 18 cm long, and there are nine eggs in a clutch. Without re-calculating the mean, where would you put the African ostrich in your Venn diagram? Does this data support the claim?
f) Evan thinks that the data about northern cardinals and African ostriches does not create a problem for the claim because these birds are very different from the other birds in his table. What might Evan be thinking about?

Sample answer: f) Northern cardinals are much smaller birds than the birds in the table, so they have much smaller eggs. Similarly, African ostriches are large birds, larger than the birds in the table, so their eggs are much larger than other eggs.

Answers: a) egg length: 6 7/8, clutch size 2 7/8; b) see margin, the initials of the birds’ name are used in the diagram; c) yes; d) outside both circles, no; e) central region, no

2. Complete BLM Investigating Mean with Blocks and Beads.

Sample answer: 2. The number of spaces below the mean equals the number of blocks above the mean because the mean balances the data.

Answers: 1. a) 4, 4; b) 5, 5; c) 4, 4; 3. b) 1 + 3, c) 3 + 1 = 2 + 2; 4. b) 2, 2, 3, 6, 7; c) 3, 3, 3, 3, 8
3. Explain that the Gross Domestic Product (GDP) is the value of all goods and services produced by a country in a given period of time. This is a measure of the “income” each country gets. However, when you divide the GDP by the number of people in the country, you get an average income per person, and it can give a very different picture. GDP per person (usually called GDP per capita) is a measure of how much money a country has to spend for every person. “Richer” countries have higher GDP per person, although different countries choose to spend this money in different ways.

Have students find online data to fill in the table below and rank the countries in the list by GDP and by the GDP per person (from highest (1st) to lowest (5th)). Have students explain why the order is different.

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP</th>
<th>Ranking by GDP</th>
<th>Population</th>
<th>GDP per Person</th>
<th>Ranking by GDP per Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iceland</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>India</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Selected sample answer: The ranking is different because the GDP is averaged by population. Iceland has the smallest GDP and the smallest population, so the ratio is higher than, say, in India, which has a high GDP, but the largest population.
Goals

Students will find the median, mode, and range of sets of data.
Students will investigate change of median when the set of data changes.
Students will recognize median as the central value.
Students will compare mean and median, including real-life situations.
Students will compare sets using mean, median, mode, and range.

PRIOR KNOWLEDGE REQUIRED

Can divide numbers producing a mixed number or a decimal
Can read and analyze bar graphs
Can calculate the mean of a set of data
Knows that mean is the value that balances a set of data

VOCABULARY

average
data point
data set
mean
median
mode
range

Mental math minute—number strings.

String 1: \((24 + 31) \div 2, (25 + 30) \div 2, (26 + 29) \div 2, (27 + 28) \div 2\) 
\(27.5, 27.5, 27.5, 27.5\)

ASK: What do you notice about the answers in the string? (they are all the same) To explain why this happens, rewrite the sums by showing the shift of 1 from one addend to the other. We add 1 to the first addend in the brackets and subtract 1 from the second addend in the brackets, so the total in the brackets does not change. Demonstrate using the number line that for all pairs of addends the result is the number midway from one number to the other.

String 2: Find the number halfway between the two numbers. 31 and 32, 30 and 33, 29 and 34, 28 and 35 (31.5, 31.5, 31.5, 31.5)

String 3: Find the number halfway between the two numbers. 131 and 232, 130 and 233, 129 and 234, 128 and 235 (181.5, 181.5, 181.5, 181.5)

Finding the data value in the centre. Invite nine volunteers to be a set of data. Ask them to order themselves from earliest to latest birthday in the year. SAY: Now that these students are in order of birthday, we can say that there is one person in the central position. ASK: Who is in the centre? Repeat, with ordering the students alphabetically by first name. Repeat, with ordering the students by the number of letters in their name.

Introduce median. SAY: When we put sets of data in order, as we have been doing, we call the one in the central position the median. Write on the board:

\[6, 8, 11, 14, 19, 26, 27\]

ASK: Are the numbers in the set in order? (yes) SAY: 14 is in the centre, so it is the median. Underline “14.” SAY: The median is the number such that...
half of the other values in the data set are above it and the other half of the values are below it.

**Finding the median when the number of data values is odd.** Write on the board:

23, 11, 9, 16, 27, 4, 20, 12, 8

SAY: I want to find the number that is in the central position, in the sense that half of the data values are below it and the other half are above it. We’ll order these numbers from least to greatest in an organized way.

ASK: What is the smallest number? (4) Write “4” under 23 and cross out 4 in the original set. SAY: We cross out 4 to show we’ve used it. ASK: What’s next? (8) Write “8” beside the 4 and cross it out in the original set. Continue until the set is in order: 4, 8, 9, 11, 12, 16, 20, 23, 27. Keep the set on the board for later use.

**Exercises:** Order the numbers.

a) 16, 12, 23, 42, 7, 33, 10, 41, 50  
   b) 203, 322, 230, 302, 233, 320, 200

**Answers:**
a) 7, 10, 12, 16, 23, 33, 41, 42, 50  
   b) 200, 203, 230, 233, 302, 320, 322

Return to the example on the board. ASK: What is the value in the central position? (12) SAY: To make sure, we can put fingers on the data values on both ends of the set and move in, one data value at a time. If we have an odd number of data values in the set, we will be left with one number, the median. Demonstrate on the set above. (12) Keep the set on the board.

**Exercises:** Find the median in the data sets in the previous exercises.

**Answers:**
a) 23,  
   b) 233

**Finding the median when the number of data values is even.** SAY: Sets with an odd number of data points always have one number in the central position. Let’s find the median of a set with an even number of data points. Erase the 4 from both the unordered and the ordered version of the set in the example. Demonstrate counting the values from the sides until you have fingers on adjacent data values, 12 and 16. SAY: Sets with an even number of data points have a median that is halfway between the two central numbers. Underline “12” and “16.” Write the numbers from 12 to 16 in order and repeat the process. The number in the middle is 14. 

SAY: Remember, when you calculated the mean, sometimes you got a number that was not in the set. A similar thing can happen to the median. The median number doesn’t need to be in the set. The median for this set is 14.

Write the following pairs of numbers on the board. ASK: What number is halfway between 12 and 14? (13) 5 and 7? (6) 6 and 10? (8) 13 and 19? (16) Remind students that in the mental math minute at the beginning of the lesson, they also found the number halfway between two numbers, and they used a different method: they added the two numbers and divided it by 2. This is another way to find the median.
Exercises: Order the numbers and then find the median.

a) 4, 7, 2, 1, 5, 4  
   b) 9, 12, 3, 7  
   c) 19, 1, 26, 4, 28, 18  
   d) 19, 1, 26, 4, 28, 28

Bonus: 320, 602, 362, 630, 302, 360, 620, 203

Answers: a) 4, b) 8, c) 18.5, d) 22.5, Bonus: 361

Comparing mean and median as centres of data sets. SAY: The mean and median both tell us about the centre of a set, but we find them in different ways: one by balancing the data values, the other using ordering. Write “14, 2, 5” on the board and have students find the mean and the median. (mean 7, median 5)

Exercises: Find the mean and median.

a) 11, 1, 15  
   b) 1, 17, 6, 8  
   Bonus: 108, 106, 110, 108

Answers: a) mean 9, median 11; b) mean 8, median 7; Bonus: mean 108, median 108

Extreme values change the mean, not the median. SAY: When basketball player, Nora, practises her free throws, she usually scores 10 out of 10 baskets. Draw the table below on the board and fill in the data for the first row (usual scores).

<table>
<thead>
<tr>
<th>Data</th>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nora’s Usual Scores</td>
<td>10, 10, 10, 10</td>
<td></td>
</tr>
<tr>
<td>Nora’s Scores Yesterday</td>
<td>10, 10, 10, 0, 10</td>
<td></td>
</tr>
</tbody>
</table>

SAY: These are usual scores. Have students find the median (10) and the mean (10). Fill in the second row. SAY: These are her scores from yesterday’s practice. Have students find the median (10) and the mean (8). ASK: Why is the mean less in the second set? (0 made it go down) SAY: The 0 didn’t change the median because 10 stayed in the central position. Explain that in sets, changing the largest or the smallest number, even drastically, to 0 or to 10 times the rest of the numbers changes the mean but not the median. SAY: The median, which is 10, is much more like Nora's usual score than the mean, which is 8, so we may decide to think of the set’s centre as being its median. The mean and median are both types of centres, but sometimes it makes more sense to use one than the other.

Exercises: A company has the following salaries:

1 person is paid $200 000  
2 people are paid $100 000  
7 people are paid $30 000  

a) What is the mean salary in the company?  

b) What is the median salary in the company?  

c) Does the mean or the median better represent the average salary in the company?
Answers: a) $61 000, b) $30 000, c) median

Discuss the answers in the exercises. ASK: How did you find the mean? If students do not mention the method of multiplying the data values by the number of times they occur, remind them about this shortcut. ASK: How did you find the median? If students have only listed the data values and then counted from both sides, point out that there is a more efficient method. ASK: What is the number of data values? (10) SAY: The median divides the data values into two halves, so half of the data is below the median and the other half is above it. ASK: What is half of the number of data values? (5) If we take the 5 lowest salaries, what are they? (all $30 000) What is the next salary, the lowest in the upper half? (also $30 000) How do you know? (the 7 lowest salaries are the same) SAY: There are 10 data values, so the median will be halfway between the 5th and the 6th data values. ASK: Both of these are $30 000, so what is the median? ($30 000)

Have students explain their thinking in the answer to part c). The mean best represents the data in terms of how much money the company spends, but the median gives a better way to see what you will likely be paid if you were hired by the company because most people there are paid $30 000. You might want to point out that when the mean and the median are so different, it usually means that the data is somehow skewed—there are more data values on one end, towards the median, or there is a data value that is very different from the rest. Both of these are true for the data set in the exercise.

Introduce mode. Remind students that in previous grades they called the most common data value the mode. ASK: What is the mode in the set in the exercises? ($30 000) Is the mode a good representation of the salaries in that company? (yes) Point out that the mode often gives very little information about the set of data and sometimes does not exist at all, but there are cases, like the case in the exercises, in which it is a sensible representation of the data set.

Exercises: Find the mean, median, and mode of the set of data.

a) 0, 0, 0, 1, 1, 2, 10  b) 0, 1, 2, 3, 4

b) 0, 1, 2, 3, 4

c) 5 salaries of $30 000, 20 salaries of $40 000, 3 salaries of $75 000, 1 salary of $100 000, 1 salary of $150 000

Bonus: 100 data values of 0, 150 data values of 2, 199 data values of 10, 1 data value of 5 000 000

Answers: a) mean 2, median 1, mode 0; b) mean 2, median 2, mode none; c) mean $47 500, median $40 000, mode $40 000; Bonus: mean 11 116.2, median 2, mode 10

Introduce range. Draw on the board:

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Highest Mark</th>
<th>Lowest Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>14</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Class 2</td>
<td>14</td>
<td>16</td>
<td>13</td>
</tr>
</tbody>
</table>
SAY: Two teachers gave their classes the same test out of 20. The classes had the same average mark, 14. However, in Class 1, the highest and lowest marks were 20 and 5, whereas in Class 2, the highest and lowest marks were 16 and 13. Discuss why a teacher might want to know this information. (Possible discussion ideas: the smaller range suggests that all students learned roughly the same body of material)

SAY: When we look at how far apart the highest and lowest values are, we are talking about how much the data values are spread out. The difference between the highest and the lowest values is called the range. Write “12, 16, 14, 11” on the board. ASK: What is the highest value? (16) The lowest value? (11) Underline “16” and “11.” Write on the board:

\[
\text{Range} = \text{highest value} - \text{lowest value}
\]

SAY: We find the range by subtracting the lowest value from the highest value. Write “16” and “11” in the first two blanks. ASK: What is the range? (5) Write “5” in the last blank.

Exercises: Underline the highest and lowest values. Find the range.

a) 9, 2, 5 b) 22, 26, 17, 14 c) 410, 140, 104, 401

Bonus: 473, 1347, 374, 743, 1374, 437, 734

Answers: a) 9 – 2 = 7, b) 26 – 14 = 12, c) 410 – 104 = 306, Bonus: 1374 – 374 = 1000

Comparing related sets of data using mean, median, mode, and range.

Write on the board:

<table>
<thead>
<tr>
<th>Science Test Grades</th>
<th>Mean</th>
<th>Median</th>
<th>Mode</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Club</td>
<td>85, 85, 90, 90, 92, 95, 95, 95, 98, 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Students</td>
<td>65, 65, 70, 75, 75, 79, 81, 85, 85, 85, 85, 90, 90, 92, 95, 95, 95, 98, 100, 100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: The two sets of data show the grades of students on the last science test. Some of the students in the grade are members of the science club, and their grades are included in the grades of all students. The grades are already in order.

Have students fill in the first row of the table. (mean 92.5, median 93.5, mode 95, range 15) ASK: Students in the first group are included in the second. Do you expect the mean, median, mode, and range to be the same? (no) What do you think happens to the mean, median, mode, and range when we include the grades for the students who are not in the
science club? Accept all answers and have students present their reasoning. Students are likely to expect the range to grow (they can see that the lowest data value is lower in the second set since the data values are in order) and the mean and the median to decrease because students in the science club are likely to do better in science than all students. Students might also expect the median to grow because of the addition of another 100 in the set.

Have students fill in the second row of the table. (mean 85.25, median 85, mode 85, range 35) Discuss which predictions were reasonable and if they have been verified.

**Extensions**

1. a) Find the range and the median.
   i) 13, 18, 20, 32, 45  
   ii) 6, 23, 27, 68
   b) Add 10 to the highest value in the set. Find the range and median again.
   c) Did the median change? Did the range change? Explain.

**Answers**

a) i) range 32, median 20; ii) range 62, median 25
b) i) range 42, median 20; ii) range 72, median 25

2. a) Find the median and the range.
   i) 2, 3, 5, 7, 9, 10  
   ii) 12, 16, 19, 22, 26, 26, 26
   b) Add 4 to each data point in i) and ii). Find the new median and range.
   c) Why did adding 4 to each data point change the median but not the range?

**Answers**

a) i) median 6, range 8; ii) median 24, range 14
b) i) median 10, range 8; ii) median 28, range 14

3. Write three different sets that have median 9 and range 5.

**Sample answers:** 8, 9, 13; 5, 9, 10; 7, 8, 9, 10, 12; 9, 9, 9, 9, 10, 11, 14; 8, 8, 8, 8, 8, 8, 10, 10, 10, 10, 13
Goals

Students will decide on ways to collect information, including obtaining primary data from a survey, experiment, measurement, or observation and secondary data.

Students will decide if they need to use an entire population or a sample.

Students will use samples to make predictions about the population.

PRIOR KNOWLEDGE REQUIRED

Has experience with conducting surveys
Can find a fraction of a number

MATERIALS

150 g of dry red beans and 100 g of dry white beans mixed together (see Extension 1)

Mental math minute—number strings.

String 1: 80 ÷ 4, 1/4 of 80, 3/4 of 80 (20, 20, 60)

String 2: 1/5 of 60, 4/5 of 60, 1/8 of 120, 5/8 of 120, 7/8 of 120, 3/6 of 120 (12, 48, 15, 75, 105, 60)

String 3: 3/15 of 300, 3/10 of 150, 3/8 of 848, 4/12 of 600 (60, 45, 318, 200)

What are surveys good for? Conduct a survey with students to determine how many of them were born before noon and how many of them were born after noon. After very few (or none) of the students raise their hands for either option, explain that conducting a survey or using another way to obtain data is decided by whether or not the people being surveyed will have answers to the survey questions. Students can signal the answers to the exercises below.

Exercises: Is a survey a good method to determine the answer to the question?

a) What are people’s favourite colours?

b) Are people left-handed or right-handed?

c) Do more people live in Vancouver or in Montreal?

d) How many sit-ups can people do in one minute?

e) What are people’s resting heart rates?

f) Who should be the next person to represent our class in the student council?
g) Does ice melt faster in direct sun or in the shade?

h) How do people get to school?

i) What is the largest insect on earth?

j) Which birds visit the school yard in the spring?

**Answers:** a) yes, b) yes, c) no, d) no, e) no, f) yes, g) no, h) yes, i) no, j) no

**Other options to obtain data.** For questions where students answered “no,” ASK: What other way could you find the answer to this question? For parts c) and i), students can suggest searching the internet. Explain that they are looking for data online, so they should be searching for a reputable source, such as data from Statistics Canada, a large museum, a zoo, or a university, not a site where anybody can change the information, such as Wikipedia, or a website created by a student whose information is hard to verify. NASA, National Geographic, and the BBC are some of the other good sources for scientific data.

For parts d), e), and g), an experiment is the best way to determine the answer. ASK: Which tools do you need to use in your experiment? (timer or stopwatch) For part g), students require additional equipment; for example, they need to make ice, make sure the cubes of ice are the same, and check how much ice has melted. Have students suggest ways to check that. (collect and compare amounts of melted water, weigh the cubes before and during the experiment, etc.) Which tools will be needed to collect water? (measuring tube, graduated cylinder) To weigh the ice cubes? (scale)

For part j), point out that students do not need special tools, but they need to stay in the backyard and record which birds and how many they see. This is called an observation. Observation is similar to an experiment but does not require tools. For the exercises below, write on the board:

<table>
<thead>
<tr>
<th>Observation</th>
<th>Experiment or Measurement</th>
<th>Survey</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Have students signal the answer by pointing their thumbs towards the best method to obtain data.

**Exercises:** What is the best method to determine the answer to the question?

a) How many students in the class wear eyeglasses?

b) How many students in the class wear contact lenses?

c) What are adults’ hair colours?

d) What are adults’ natural hair colours?

e) What are people’s favourite sports?

f) How long are arm spans of people in our class?

g) What colour of shirt is the most common today in our class?
h) What are my classmates’ favourite books?

i) How does the temperature of a glass of ice water change over time?

j) Are more people born in the fall or in the spring?

**Bonus:** Are more people born before noon or after noon?

**Answers:** a) observation, b) survey, c) observation, d) survey, e) survey, f) experiment or measurement, g) observation, h) survey, i) experiment or measurement, j) survey.

**Sample answer:** Bonus: survey—go to a hospital and ask how many babies are born before noon and how many are born after noon; obtain data from a hospital; survey mothers

**Introduce primary and secondary data.** SAY: Sometimes you are able to collect your own data directly and sometimes you need to rely on data that someone else has collected. Data you collect yourself are called *primary* or *first-hand* data. Data somebody else has collected are called *secondary* or *second-hand* data. So, data obtained from Statistics Canada, from the internet, from a book, or data collected in a hospital by somebody else is secondary data. Students can signal the answers in the next exercise by raising one finger for primary data and two fingers for secondary data. Have volunteers explain their answer for each question.

**Exercises:** Will you use primary or secondary data in the situation?

a) You want to find the favourite television show of students in your class.

b) You want to know the amount of rain in your hometown in the past week.

c) You want to know which player won the Stanley Cup the most number of times.

d) You want to know how many push-ups each person in the class can do in a minute.

e) You want to know the world record for the most number of push-ups done in a minute.

f) You want to know the temperature outside.

g) You want to know what the temperature will be during March break.

h) You want to know your classmates’ eyeglasses prescriptions.

i) You want to know how many people in your class wear eyeglasses.

**Answers:** a) primary, b) secondary, c) secondary, d) primary, e) secondary, f) primary or secondary, g) secondary, h) secondary, i) primary

**Choosing between a census and a sample.** SAY: Sometimes you want to know things about very large groups of people, but you may not be able to gather data for everyone. For example, if we want to know how many people in Canada have read *The Wizard of Oz*, it wouldn’t be practical to ask everyone if they’ve read it. In other cases, you have to ask everyone,
or at least try to. For example, when you want to know who should be your next representative in parliament, you conduct elections, which are technically a survey of the entire voting population in your area.

SAY: Collecting information by surveying the entire population is called a **census**. When you are asking only some people, you are using a **sample**. Write the words “census” and “sample” on the board and draw attention to the difference in the first letter, although both words start with the same sound.

SAY: I want to find the average shoe size of everyone in, for example, our province. ASK: Should I check everyone in the province or only some people in the province? (only some people) Why? (it would be too difficult to check everyone in the province; a sample will do) For the exercises below, have students signal thumbs up for a census and thumbs down for a sample and have volunteers explain their reasoning.

**Exercises:** Should I use a census or a sample? Explain.

a) If I want to know what books my classmates read last week, should I survey the whole class or only a sample?

b) If I want to know how many people in Canada watched a particular television show, should I ask everyone in Canada or just a sample?

c) If I am organizing a party and want to know which pizza flavours are preferred by my friends, should I ask everyone or just a sample?

d) If I am organizing a whole-school board game party, should I check the favourite board game of every person or just a few people from each grade?

**Answers:** a) a census, the whole class, it is not too many people; b) a sample, too many people to ask; c) a census, because you want to be fair; d) a sample, because there are too many people to ask

**Using a sample to make predictions about the population.** Explain that you can make predictions about the whole population by using a sample. For example, before elections, newspapers often publish predictions about how the elections will go. The poll companies that produce these predictions use a sample of the population to ask the questions and then make inferences about the whole population.

**Explain that other frequent users of this technique are scientists.** For example, sometimes people release fish into the lake, and this fish is not native to the lake. There might be no natural predators for the fish in the lake, and the fish starts spreading and leaves no room for the native fish in the lake. Suppose that scientists estimate that the lake can support 1000 fish. They catch (and release) 50 fish and check how many of these are not native. Let’s say that they found that 15 out of 50 fish were not native. Write on the board:

- Number of non-native fish in the sample: 15
- Total number of fish in the sample: 50
ASK: What fraction of the sample is non-native? (15 out of 50, $\frac{15}{50} = \frac{3}{10}$)

Write the fractions on the board. SAY: Scientists assume that $\frac{3}{10}$ of the fish in the lake are not native. Write on the board:

Total number of fish in the lake: 1000

Number of non-native fish in the lake: $\frac{3}{10}$ of 1000

ASK: What is three tenths of 1000? (300) How do you know? (1/10 of 1000 is 100, so $\frac{3}{10}$ of 1000 is 300)

SAY: So we estimate that there are 300 non-native fish in the lake.

ASK: Does this method remind you of anything else you calculated recently? (yes, making predictions in a probability experiment)

Point out that you can wonder for each fish in the lake, is it invasive or not? So when you check a sample, this is like finding the experimental probability of the fish being non-native. Then you use that probability to figure out how many fish out of 1000 you expect to be non-native.

**Exercises:** A packaging plant made 2000 packages of tea bags.

a) A quality-control technician opens 20 packages and checks if they contain the correct number of tea bags. One out of 20 packages contains fewer tea bags than required. How many packages would she estimate contain the wrong number of tea bags?

b) A quality-control manager opens 50 packages and checks if they contain the correct number of tea bags. Two out of 50 packages contain 1 fewer tea bag than required. Another one contains one more tea bag than required. One package contains a torn tea bag. How many packages would he estimate are defective?

c) Whose estimate about the number of defective packages is more reliable? Explain.

**Answers:** a) $\frac{1}{20}$ of 2000 = 2000 ÷ 20 = 100; b) $\frac{4}{50}$ of 2000 = 160; c) the manager’s, because his sample is larger, so he can spot more problems

**Extensions**

1. **A large sample produces better results than a small sample.** Ahead of time, thoroughly mix together 150 g of dry red beans and 100 g of dry white beans. Show students the beans in a large bowl or a jar. Have each student close their eyes and choose 10 beans.

   Have students count how many beans of each colour they chose. Record at least eight results on the board. ASK: Do you think we can estimate the fraction of red beans and white beans in the whole bowl based on the fraction in a sample of 10? (no) Why not? (everyone got different answers, so we wouldn’t know whose answer to use)

   Have students pool their results in pairs and then in groups of four (pair up the pairs). Continue pooling results in groups of eight and
then with the whole class. Students can record their results in a table, as shown below.

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>Whole Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Red Beans</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of White Beans</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage of Red Beans</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tell students the exact proportion of beans of each colour in the bowl (60% red, 40% white). **ASK:** Which sample produced the best and most reliable estimate? (the largest one) **SAY:** Some small samples might be closer to the actual proportion, maybe even exact, but we can’t rely on them because the small samples are so different from each other.

2. An adult ticket to an observation Ferris wheel costs $20 and a child ticket costs $12.

a) The table shows the attendance during the first week of July.

<table>
<thead>
<tr>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>70</td>
<td>68</td>
<td>65</td>
<td>60</td>
<td>70</td>
<td>120</td>
</tr>
<tr>
<td>250</td>
<td>150</td>
<td>151</td>
<td>142</td>
<td>137</td>
<td>180</td>
<td>259</td>
</tr>
</tbody>
</table>

What would you expect the sales from the tickets to be during the whole month of July? Round to the nearest dollar.

b) The table shows the attendance during the first week of November.

<table>
<thead>
<tr>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>30</td>
<td>34</td>
<td>30</td>
<td>27</td>
<td>38</td>
<td>60</td>
</tr>
<tr>
<td>130</td>
<td>75</td>
<td>72</td>
<td>60</td>
<td>65</td>
<td>80</td>
<td>144</td>
</tr>
</tbody>
</table>

What would you expect the sales from the tickets to be during the whole month of November? Round to the nearest dollar.

c) What would you expect the sales from the tickets to be during the whole year if the observation Ferris wheel is open every day in the year? Round to the nearest dollar.

**Solutions**

a) A total of 533 adult tickets and 1269 child tickets were sold in a week, so the total revenue was $553 \times 20 + 1269 \times 12 = 26288$. There are 31 days in July, so this revenue is $7/31$ of the expected revenue. Therefore, we can expect $3755.43$ on average daily and $116418$ in the month of July.
b) A total of 274 adult tickets and 626 child tickets were sold in a week, so the total revenue was $274 \times 20 + 626 \times 12 = 12992. So on average, the daily revenue is $1856. There are 30 days in November, so we can expect $55680 in the month of November.

c) The average revenue of 14 days is $39280 \div 14 \approx 2805.71$, so the expected yearly revenue is $1024084$. 
Goals
Students will identify various ways in which the results of a survey can be biased.

PRIOR KNOWLEDGE REQUIRED
Has experience with conducting surveys
Can find a fraction of a number

Mental math minute—number strings.

String 1: Solve for the variable. \( x = 7 \), \( 2x = 2(7) \), \( 3x = 3(7) \) (\( x = 7 \), \( x = 7 \), \( x = 7 \))

To explain why all the solutions are the same, ASK: How can we get the second equation from the first? (multiply both sides by 2) How can we get the third equation from the first? (multiply both sides by 3) Remind students that equations, e.g., \( x = 7 \), can be regarded as pan balances. In this equation, we have a bag on one side and seven apples on the other side. If we double or triple what is on both pans at the same time, the pans remain balanced. Similarly, if we add the same number of apples to both sides, the pans also remain balanced.

String 2: \( x + 3 = 8 \), \( 2x + 6 = 16 \), \( 3x + 9 = 24 \) (\( x = 5 \), \( x = 5 \), \( x = 5 \))

String 3: \( 4x = 8 \), \( 4x = 9 - 1 \), \( 4x + 1 = 9 \) (\( x = 2 \), \( x = 2 \), \( x = 2 \))

Exploring biased samples. Tell students that you want to know if Grade 6 students in Ontario prefer action movies or comedies. ASK: Can I ask only students in Ottawa? Why or why not? (no, because the survey is about the whole province) Can I ask only boys? (no, that sample doesn’t represent the whole population)

SAY: When the results of a survey or an experiment are distorted by the design of the survey or experiment, we say that there is a bias in the results. A biased sample is not similar to the whole population because some part of the population is not represented. In the example, Grade 6 students in Ontario are the whole population. If only boys are surveyed, then other students are not represented. If only Ottawa students are surveyed, then people from other cities and towns in the province are not represented. If only public school students are surveyed, then private school students are not represented. However, the question might focus only on a specific population. For example, if I want to know if boys prefer action movies or comedies, it would make sense to ask only boys.

SAY: A Grade 1 to 8 school is planning a games party and wants to decide what games to buy. Students can signal the answers to the following questions. ASK: Would the sample be biased if the school surveyed all of the Grade 2 students? (yes) All of the members of the school soccer club? (yes) Every 10th student, when listed in alphabetical order? (no) Three people chosen at random from each classroom? (no)
SAY: A sample that is similar to the whole population is called a representative sample. Finding a representative sample is often the most difficult part of conducting a survey. For example, an apartment building manager would like to hold a monthly dance in the games room for tenants. Discuss the bias if, to decide what kind of music should be played at the dance, the building manager:

a) Asks the bridge club. (people in the bridge club are likely to be older than the average building population)
b) Asks the soccer club. (people in the soccer club are likely to be younger)
c) Asks the teen movie club. (only teenagers will be represented)
d) Puts a survey under every 10th door by apartment number. (no bias)
e) Lists the names of people living in the building in alphabetical order and picks every 10th person to ask. (no bias)
f) Asks people at the playground in the morning. (teenagers and working adults are less likely to be represented)

Timing can create bias. Explain that even surveying people at the same location, at different times, can produce different biases. ASK: How would your samples be different if you surveyed people at the mall on a weekday morning or on a weekend morning? Some points to discuss are provided below:

- On weekday mornings, most teenagers and many working people would be excluded.
- People who go to malls on weekday mornings could be unemployed, retired, have a part-time job, work from home, or start working later in the day.
- University students might have class schedules that allow them to go to the mall on a weekday morning.
- Some people go to the mall on weekdays to avoid the crowds on the weekend.
- Any group that was overrepresented (given too much representation) on weekday mornings may be underrepresented (not given enough representation) on the weekend.

Exercise: A mall management team wants to decide whether to rent space to a pet store or a video game store. How and when should the team conduct a survey?

Have multiple students present their answers; prompt students to include additional considerations. Samples:

- Survey customers at different times. PROMPT: If the mall is crowded at certain times, should the management pay more attention to the results during this time? (yes) If more customers visit the mall during the weekend, the results from the weekend should be given more weight.
• Survey people from the surrounding area. PROMPT: Suppose there is a big retirement community near the mall. Should the mall pay more attention to older people? (yes, if they visit the mall more often than other people) The mall is interested in biasing the sample to its customers. If it happens that more elderly people go to that mall, the mall will want the survey to target older people. PROMPT: Suppose there are three apartment buildings next to the mall. The mall decides to survey every 10th apartment unit in these apartment buildings instead of asking mall customers at different times of day. ASK: Is this a representative sample? (no, because only people living in apartments are represented; this will bias the sample against people who live in nearby houses; not all the people who live in the apartments necessarily like visiting malls)

The wording of a question can affect the results of the survey. Discuss the following three cases in which the sample is representative, but the results are biased.

Case 1: A town council is thinking of selling a city park and allowing a department store to be built in its place. Two groups ask different questions:

A: Are you in favour of having a new store that will provide jobs for 50 people in our town?

B: Are you in favour of keeping our neighbourhood quiet and peaceful?

ASK: Which question do you think was proposed by someone in favour of selling the park? (A) Which was proposed by someone against selling the park? (B) Have students write a survey question that is more neutral and does not already suggest an answer. (Sample answers: How do you feel about building a new store on the park site? Would you agree or disagree with building a new store on the park site? Are you in favour of or against building a new store on the park site?)

Case 2: A student council chooses music and food for school events. Most people enjoy the music, but not the food. Two surveys are shown below:

Survey A    Survey B
1. Do you enjoy the music at the events? 1. Do you enjoy the food at the events?
2. Is the student council doing a good job? 2. Is the student council doing a good job?
3. Do you enjoy the food at the events? 3. Do you enjoy the music at the events?

ASK: What is the same about the two surveys? (they have the same three questions) What is different? (the questions are in a different order)

Which survey is more likely to suggest that the student council is doing a good job? Why? (Survey A, because it starts with a question about the music, which is what students are known to like best) How can the order of
the questions affect the results of a survey? (if the question about whether the student council is doing a good job is asked after something students are happy or unhappy with, the results will be biased about how the student council is doing; but if they ask the question about student council performance first, there would be no bias)

Case 3: A service company surveys its customers to see how their service was. The survey is:

**How was your experience with us today?**

Excellent  Very good  Good  Fair

**ASK:** What is the problem with this survey? (the answers do not give enough options) **PROMPT:** What if your experience was bad? If the company posts the results of this “survey” on its website, it creates a false feeling that everybody is quite satisfied with the service the company provides.

**Exercises:** Will the question and/or choices produce a biased result? If yes, explain.

a) TV shows are a great source of information. How many hours per day do you watch TV shows?

- Less than 2 hours
- 2–3 hours
- 4–5 hours
- 6–7 hours

b) How many hours per weekday do you spend reading?

- 0–2 hours
- 3–4 hours
- 5–6 hours
- 7–8 hours
- More than 8 hours

c) How much time in a day do you spend playing sports?

- 0–19 minutes
- 20–39 minutes
- 40–59 minutes
- 1–2 hours
- More than 2 hours

**Answers**

a) yes, the question prompts you to think that you need to spend more time watching TV; also, most people would pick the first option
b) yes, there will be too many people who choose the first two options, with most people choosing option 1; some people will not know whether to include reading for work or study purposes or just reading for pleasure; people will not know if this includes reading newspapers and e-books and reading from websites
c) no
Extensions

1. **Location bias.** Tell students that you stood outside a hockey arena and counted the fans who were wearing the jerseys of each team playing that evening. The results were 60% home team jerseys and 40% away team jerseys. Can you conclude that 40% of the fans at the game supported the away team? Explain. Hint: Is a fan who is wearing a jersey more likely to support the home team or the away team? Why?

   **Sample answer:** Fans of the away team are more likely to be coming from out of town and therefore more likely to make the extra effort to wear their team’s jersey. The fans who wear jerseys are thus a biased sample, more likely to be cheering for the away team.

2. **Sample size bias.** SAY: I want to know if a coin toss is fair. ASK: If I toss a coin once and it comes up heads, can I conclude that it is more likely to come up heads than tails? (no) What if I toss it 3 times and get 2 heads? (no) What if I toss it 300 times and get 280 heads? (yes, the coin toss is not fair) SAY: A friend tells me that most students in Canada are against school uniforms. ASK: Can I make that conclusion after I ask one student if he or she is against school uniforms? (no) What if I ask three students and two of them are against school uniforms? (no) What if I ask 300 students and 280 of them are against school uniforms? (probably yes) When a sample of the population is chosen, is it more likely to be representative of the whole population if it is a large sample or a small sample? Why? (a large sample will include more different people, will have more variety, and so is more likely to be representative)

3. Explain that students will investigate an effect that is called “priming,” which can also create a bias in the results of a survey. Students work in pairs. Partner 1 asks the questions below, in order; Partner 2 answers.

   What colour do you add to yellow to make orange?
   What colour do you add to blue to make purple?
   What colour is blood?
   What colour is a stop sign?
   What colour is a strawberry?
   What colour is a poppy?
   What traffic light colour do you go on?
   Did Partner 2 answer red to the last question? Why could that happen?

   **Answer:** The first six questions had “red” as the answer. As soon as someone hears the beginning of the question “what colour of traffic light,” the person is likely to expect the answer to be “red” again.
Goals
Students will conduct their own surveys and experiments and analyze the data collected.

PRIOR KNOWLEDGE REQUIRED
Has experience with conducting surveys
Can create and analyze various types of graphs
Can compare sets of data using mean, median, and range
Can identify sources of bias in a survey

MATERIALS
tools for conducting an experiment (see Activity 1)
access to pressurized air, 500 mL plastic bottles filled with a small amount of water, paper plates, conical paper cups, cardstock paper, and tape (see Extension 3)

Mental math minute. Present the equation: $3 \times 4 = 4x$. ($x = 3$) Have students try to solve the equation without calculating the left side, observing the similarities between the expressions on both sides of the equal sign instead. There is a number multiplied by 4 on both sides, and the sides are equal, so the numbers multiplied by 4 must be the same.

Exercises: Solve for $x$ by comparing the sides of the equation.

a) $4 \times 5 = 4x$  
   b) $2 \times 3 \times 5 = 5x$
   c) $3 \times 4 \times 8 = 4x$  
   d) $12 \times 6 \times 3 = 6x$

Answers: a) $x = 5$, b) $x = 6$, c) $x = 24$, d) $x = 36$

Potential bias in an experiment. Explain that students will be designing and conducting their own experiment. Tell them that sometimes the trickiest part of doing an experiment is making sure that they are really testing what they want to be testing and that nothing else influences the results. For example, you want to see how the placement of a paper clip affects the distance a paper glider flies. ASK: Apart from the placement of a paper clip, what else can influence the distance the glider flies? (design of the plane, precision of creases, thickness of paper, skill of the launcher, wind, and so on) Discuss ways to counter the suggested differences. (use the same glider; use different gliders made from identical sheets of paper, strengthen the creases to make them precise; have the same person launch the glider the same way; launch them indoors to eliminate wind; if launching the gliders outside, make several identical gliders and launch them at the same time)
Designing an experiment. Present the topic that students will investigate during their experiment. Examples include:

1. How does the design of a glider affect the distance it flies?
2. How does the placement of the paper clip affect the distance the plane flies?

Have students discuss and design the experiment in pairs, then have volunteers present suggestions. Examples:

1. Design and create two different gliders. Adjust both gliders to make sure they fly straight. Fly both designs five times. Measure the distance the glider flies. Compare the sets of data using a double bar graph. If students are familiar with mean and median, find the average distance each design flies and compare the averages. Compare the range of the sets of data.

2. Make a paper glider, adjusting it to make sure it flies straight. Add a paper clip to the bottommost fold of the glider and measure the distance from the nose of the glider to the front of the paper clip. Record the distance and fly the plane three times. Measure and record the distance the plane flies each time. Shift the paper clip 3 cm towards the tail of the plane and repeat. Continue in this manner until the paper clip is close to the end of the plane. Find the mean distance of flight for each position of the paper clip and present the averages using a broken line plot.

NOTE: If your students are not familiar with mean, remind them how they sometimes calculate the average mark in a course by adding all marks together and dividing by the number of marks. Explain that this creates a mark that is “evened out,” to eliminate unusual results. Similarly, it makes sense that to “even out” the distance a glider flies with each position of the paper clip, fly it three times and find the average distance. Alternatively, students can fly the glider with each position of the paper clip only once.

Discuss the measurement tools students need and have them prepare the tools. What do they need to record the data? If students are performing the experiment as part of the science curriculum, discuss the scientific component: Which design creates more lift? What does adding a paper clip do to a plane? (increases the weight but tightens the crease and decreases the drag) Make sure students use the proper terminology for the explanations.

ACTIVITY 1 (Essential)

1. In pairs, students perform the experiment. They record their findings in a graph and as a comparison of mean, median, and range if applicable. Students will present the results of the experiment and discuss them in the next lesson.
Designing a survey. Review what students already know about designing a survey from previous grades. Explain that students are going to conduct a survey on a topic of their choice, with their classmates being the population. They will decide whether a sample is enough and, if so, choose the representative sample. They will also need to make a graph to present the results to the class. Encourage students to choose survey questions that can be related to causes on which they might be willing to write an opinion piece (see Extension 1 in Lesson PDM6-17).

Have several volunteers present an example of the question they are going to ask and the answers. Have the class vote on whether the question is unbiased, and if it is biased, have other volunteers explain what creates the bias. In each case, have the class also vote on whether there are enough answers, if the number of answers is not too large, and whether any additional answers, such as “other,” “more than ____,” or “none,” are needed.

For each question, discuss if a sample is enough and if so, how to choose it. Again, put the suggestions to a vote: is the method creating a bias?

For each suggested question, discuss the best type of graph to display the data.

**ACTIVITY 2 (Essential)**

2. In pairs, students conduct a survey of the class using one of their questions. They display their findings in a graph. Students will present the results of the survey and discuss them in the next lesson.

**NOTE:** Students will need access to the internet for Question 4 on AP Book 6.2 p. 161. They can get the required data by searching for “average high temperatures in ____” or from the Environment Canada website.

**Extensions**

1. Have students make a graph of a different type to display the results of the survey they did in Activity 2. Have them compare the graphs and explain the advantages and shortcomings of each type.

2. Jasmin wants to know how much her classmates read for fun. Is the survey question good? If no, what problems do you see with it?
   a) Do you like to read for fun? YES NO
   b) How many books have you read in the last year for fun? ____
   c) How often do you read for fun?
      Any chance I get  Often  Sometimes
      Not very often  Never
   d) How many pages have you read in the last week for fun?
      0  1–50  51–100  101 or more

**NOTE:** Students can use the results of their survey to answer Question 3 on AP Book 6.2 p. 161. Students will also use these results in Lesson PDM6-17.
Sample answers: a) no, the answers do not give a measure of how much people actually read; b) no, people are unlikely to know how many books they read over last year because that is such a long time; c) yes; d) no, the question is too precise, it is hard to estimate how many pages a person reads in a week.

3. If students have access to pressurized air, they can try the following experiment to investigate the forces of lift, drag, and thrust. Students will use a bottle with a small amount of water so that it has some weight. They fill the bottle with pressurized air and release it, bottom first, measuring the distance it flies. They can fly each of the following designs 3–5 times and find the average distance, then present the results in a bar graph.

1. Bottle with nothing attached to it.
2. Bottle with a paper plate attached to the front (bottom of the bottle). The paper plate will considerably increase the drag and decrease the flight distance.
3. Bottle with a paper cone attached to the front (bottom of the bottle), creating a “nose.” The nose will reduce the drag, increasing the flight distance.
4. Bottle with a paper cone attached to the front and paper wings attached to the sides of the bottle. The wings increase the lift, increasing the flight distance. Students can try different designs of wings or different placements of the wings as well.
5. If the air pressure on the source can be varied, students can fly any of these designs with a different pressure, resulting in a different thrust and changing the flight distance again.
Goals

Students will make inferences from graphs and sets of data.

PRIOR KNOWLEDGE REQUIRED

Can read and interpret various types of graphs
Can find mean, median, mode, and range of data set

MATERIALS

BLM Graphs (pp. R-66–67)
graphs produced in Activities 1 and 2 in Lesson PDM6-16
graphs from the media

NOTE: Students in Alberta and Manitoba are not familiar with mean, median, mode, and range. Parts of the lesson that refer to these concepts, as well as Questions 1 and 3.a) on AP Book 6.2 pp. 162–163, are optional for students in these provinces.

Mental math minute. Present the equation: 2 \times 64 = 4x. (x = 32) Have students try to solve the equation without calculating the left side, observing the similarities between the expressions on both sides of the equal sign instead. ASK: How can you change the left side to look like the right side, 4 times a number? (double 2 and halve 64) Rewrite the equation: 4 \times 32 = 4x.

Exercises: Solve for \( x \) by comparing the sides of the equations and making them the same.

a) \( 2 \times 72 = 4x \)  
   \( b) \ 3 \times 8 = 4x \)  
   \( c) \ 24 \times 3 = 12x \)

Answers: a) \( x = 36 \), b) \( x = 6 \), c) \( x = 6 \)

Presenting a set of data. Write the following set of data on the board:

\[
\begin{array}{cccccccccc}
15 & 15 & 16 & 18 & 19 & 14 & 15 & 18 & 17 & 14 \\
20 & 19 & 15 & 18 & 12 & 16 & 15 & 19 & 16 & 15 \\
\end{array}
\]

Explain that these numbers are grades out of 20 that students got on a test. Point out that it is hard to say whether, say, 16 was a good grade or not in comparison with the rest of the grades. ASK: How can we present the data so that we can easily compare grades? Accept all answers. Explain that one way to show the data is to put it on a number line. ASK: What is the highest grade? (20) The lowest grade? (12) Have students draw a number line from 10 to 20 and do the same on the board. Then show how to mark the grades above the number line, creating a dot plot. Have students create a dot plot in their notebooks. Students might recall creating dot plots in earlier grades.
The finished dot plot will look like this:

ASK: How does this arrangement help us organize the data? (we can see how many of each data value there is; we can see the highest, the lowest, and the most common data values very clearly) What is the mode? (15)

PROMPT: The number of dots above the hash mark on the number line shows how many times the data value appears in the set. What is the most common data value?

**Review finding mean and median.** Have students find the median of the data. (16) Ask a volunteer to explain how she did it. If necessary, remind students that the median is the value that divides the set so that there is the same number of values on both sides of the median. Since there are 20 data values, there should be 10 values on each side of the median. This means the median will be between the 10th and 11th data points, and both the 10th and 11th data values equal 16.

**Review finding the mean using multiplication and addition.** Have students find the mean of the data set. (16.3) Have volunteers mark the median and the mean on the number line.

**Discussing the shape of the data.** ASK: Are there more data values above the mean or below the mean? (below the mean) Is the data spread out more evenly above the mean or below the mean? (above the mean) Are there any data points that are far away from the rest of the data? (no) Point out that 12 is not really far away from the rest of the grades; there is just one missing number between it and the rest of the data.

Discuss if 16 is a good grade. Students can vote on whether they think each claim in the exercises below is correct. Then have students explain their votes.

**Exercises:** Anne got 16 on the test. She is trying to persuade her parents she did well on the test. Is the claim correct? Explain.

a) Most people got a lower mark than I did.
b) Less than half the class did better than me.
c) My grade is average.
d) The most common grade is lower than mine.

**Answers**
a) no, only 9 people got a lower grade; this is not “most”
b) yes, 8 people got a higher grade, which is less than half the class, 10
  c) yes and no; 16 is equal to the average if rounded to the nearest whole number, but this is rounding down; 16 is actually lower than the average
  d) yes, the mode is 15
Review trends in a line plot. Display the first graph on BLM Graphs (1). Explain that the graph shows the number of daylight hours in Windsor, Ontario. Discuss the trends students see in the graph: the number of daylight hours grows from January to June and then decreases from June to December. Display the second graph, covering the place label for Ushuaia, Argentina. Explain that you have added the same data for another city and ask students to describe the pattern in the second graph. (the number of daylight hours decreases from December to June and grows from June to December) Have students try to explain what could create the dramatic difference. Then reveal the place label and explain that Ushuaia is one of the southernmost cities on Earth, so it has more light in our winter months and less light in our summer months.

ASK: Which city gets longer days during the time the cities have long daytime? (Ushuaia) Which city gets longer days during the time the cities have short daytime? (Windsor) When do the cities have about the same amount of daytime? (in March and September)

ACTIVITY 1 (Essential)

1. If students had chosen to use a line graph or broken line graph in one of the surveys or experiments they performed during Lesson PDM6-16, have students present the graphs and have the class discuss what trends they see in the graphs and what other information they can infer from them. Have other students suggest improvements for the argument. For example, if students performed the experiment with a glider and paper clip, they might notice that adding the paper clip in any position increases the distance the glider flies, but placing the paper clip closer (but not too close) to the start of the wings provides the greatest distance. About 6 cm from the nose of the glider is the best position. Encourage precision and numerical comparisons, such as “the glider with the plane at the optimal position flies about twice as far as the plane without the paper clip.”

Comparing data presented in tables. Draw on the board:

<table>
<thead>
<tr>
<th>Month</th>
<th>Windsor, ON</th>
<th>Alert, NU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>9.5</td>
<td>0</td>
</tr>
<tr>
<td>Feb</td>
<td>10.5</td>
<td>0</td>
</tr>
<tr>
<td>Mar</td>
<td>12</td>
<td>10.75</td>
</tr>
<tr>
<td>Apr</td>
<td>13.25</td>
<td>24</td>
</tr>
<tr>
<td>May</td>
<td>14.5</td>
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<tr>
<td>Jun</td>
<td>15.25</td>
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<tr>
<td>Jul</td>
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<td>24</td>
</tr>
<tr>
<td>Aug</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>Sep</td>
<td>12.5</td>
<td>16</td>
</tr>
<tr>
<td>Oct</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Nov</td>
<td>9.75</td>
<td>0</td>
</tr>
<tr>
<td>Dec</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>
Explain that the data in the table shows the same data as on the first line plot, but the last column displays the number of daylight hours in a different location, Alert, Nunavut. The times are rounded to the nearest quarter of an hour, and all are recorded on the 15th of the month.

ASK: What is the largest number of daylight hours in Alert? (24) In which months does this happen? (April, May, June, July, August) Can there be daylight time that is longer than 24 hours in a day? (no) Why not? (there are 24 hours in a day, it is light the entire time) Explain that the time when the sun is in the sky 24 hours in a day is called “polar day.” ASK: Where does polar day happen? (north of the North Polar circle or south of the South Polar circle) Explain that Alert is the northernmost city in Canada, so it is north of the North Polar circle.

ASK: How many hours long is the daylight in January in Alert? (0) Explain that the time when the sun does not rise at all is called “polar night.” ASK: Which other months are also parts of polar night? (February, October, November, December)

Review comparing data using double bar graphs. Display the double bar graph on BLM Graphs (2). Have students try to explain what the graph shows. Students should realize that the double bar graph shows the same data as the table. ASK: How does the bar graph show the polar night? (there are no bars) How does the bar graph show the polar day? (the bars are as high as possible and all the same height)

ASK: When is there more sun in Alert than in Windsor? (from April to September) How do you see that? (the numbers for Alert are larger than the numbers for Windsor) How do you see that from the graph? (the bars are higher) Is it easier to see the answer to this question on a double bar graph or in a table? (double bar graph)

Tell students that you want to calculate the mean for both sets of data. ASK: Is a double bar graph a convenient tool for that? (no) Why not? (the exact numbers are hard to read) Explain that you have calculated the mean for both sets of data on the number of daylight hours. Write on the board:

<table>
<thead>
<tr>
<th>City</th>
<th>Windsor, ON</th>
<th>Alert, NU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean daylight time</td>
<td>12.2</td>
<td>12.2</td>
</tr>
</tbody>
</table>

For the following questions, have students identify the presentation of the data they used and explain what they did. ASK: If the means are the same, does this tell us that Windsor and Alert get the same amount of sunlight on average? (yes) Does it also tell us that the climate is the same in both cities? (no) How can we see that from the data? (The amount of daylight Alert gets is far more in summer than in winter, and in summer Alert gets longer days than Windsor. In winter, Windsor gets some sunlight, when Alert gets nothing at all.) Does this mean that the temperatures will be the same in Alert and in Windsor? (no) Does the fact that Alert gets more sun in summer mean that Alert is warmer than Windsor in summer? (no) Why not? (Alert is much farther to the north than Windsor) You might want to point out that Windsor is the southernmost city in Canada and so one of the warmest.
ACTIVITY 2 (Essential)

2. Repeat Activity 1 with bar graphs or double bar graphs. If there are any other types of graphs left from the previous lesson, include them in this activity as well.

Using a graph from the media to make inferences. Present a graph from the media, related to a topic of your choice. Have students describe what they see on the graph and suggest different things they can infer from it. For example, present the second graph on BLM Graphs (2) showing the amount of time people spend online by age. Students can notice that:

- the amount of time that people spend online decreases as the age increases: the older the person, the less time the person spends online on mobile devices
- young people spend over 3 hours daily online on mobile devices
- the amount of time people spend online on mobile devices drops by approximately 50 minutes from one age group to the next; only the drop between the last two categories is less dramatic

Have students try to make inferences based on other data. For example, ASK: Based on the data on the graph, how much time would you estimate people who are younger than 16 years old spend online on their devices? (about 250 minutes per day) Is this reasonable? How many hours is that? (about 4 hours) How much time do you estimate you spend daily online on your device? Have students discuss.

For another way to make inferences from the graph, suggest that students think which group their parents or caregivers belong to. How much time do people of this age spend online daily? Does this fit with their observations of their parents’ lifestyle? Why would their parents spend less time on mobile devices? Would the time spent on mobile devices decrease with age as people get more responsibilities? Would the time spent on computers increase with age?

NOTE: Students will need a graph from the media or access to the internet for Question 4 on AP Book 6.2 p. 163.

Extensions

1. When students are presenting the results of a survey, ask them to think who could be interested in doing a survey of this kind for a larger population. Ask them to imagine that they are journalists who want to write an article about the cause the survey is connected to. For example, if the survey asks what kind of activities students do on their cell phones, they could role-play a journalist who wants to show that students use their cell phones for learning and social activities far more than they use it for games. How do the data from the survey support this cause? Students can write an opinion piece based on the data from their survey.
2. Grace wrote 10 math tests this year, each worth 100 marks. Her marks were:

<table>
<thead>
<tr>
<th>Test</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>60</td>
<td>50</td>
<td>65</td>
<td>55</td>
<td>70</td>
<td>30</td>
<td>75</td>
<td>45</td>
<td>80</td>
<td>65</td>
</tr>
</tbody>
</table>

a) Draw a line graph showing all her marks.

b) Draw a line graph showing only tests 1, 3, 5, 7, and 9.

c) What trend does the second graph suggest that the first one does not?

d) Grace wants to show her parents how much her math mark is improving. If she shows her parents the second graph, do you think she is being honest? Explain. Use mean, median, and range in your answer.

Sample answers

a) Grace's Math Grades

b) Grace's Math Grades

c) The graph shows steady improvement in the grades.

d) No, Grace's grades do not improve steadily. In fact, she is not very consistent in her marks. The real range of her grades is 50, while the second graph suggests that it is only 20. Grace's mean grade in the course is 59.5, lower than the lowest grade on the second graph, when the second graph suggests a mean grade of 70. Similarly, the real median is 62.5, and the median on the second graph is again 70. If Grace uses the second graph, she is not being honest.
**Spinner**

Place a pencil point at the centre of the circle and a paper clip around the pencil point. Spin the paper clip around the pencil tip.

<table>
<thead>
<tr>
<th>Grey</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td></td>
</tr>
</tbody>
</table>
Moving Beads to Find the Mean

Move beads to find the mean. Cross out the beads you move and draw shaded beads in new positions.

a) \[8 \quad 5 \quad 2\]

b) \[7 \quad 2 \quad 5 \quad 2\]

c) \[1 \quad 3 \quad 9 \quad 3\]

Mean: ____  Mean: ____  Mean: ____
Investigating Mean with Blocks and Beads

1. The horizontal line shows the mean. The $\times$s show spaces. Count the blocks above the mean and the spaces below the mean.

   a) 
   
   
   
   
   
   
   
   
   
   ___ blocks above mean 
   ___ spaces below mean 

   b) 
   
   
   
   
   
   
   
   
   
   ___ blocks above mean 
   ___ spaces below mean 

   c) 
   
   
   
   
   
   
   
   
   
   ___ blocks above mean 
   ___ spaces below mean 

2. Look at your answers to Question 1. What do you notice? Explain.

3. The horizontal line shows the mean. The $\times$s show spaces. Write the number sentence that shows that the number of spaces below the mean equals the number of blocks above the mean.

   a) 
   
   
   
   
   
   
   
   
   
   $3 = 2 + 1$

   b) 
   
   
   
   
   
   
   
   
   
   $2 + 2 + 0 = \square + \square$

   c) 
   
   
   
   
   
   
   
   
   
   \( \square + \square = \square + \square \)

4. Use the number sentence to find a data set with mean 4. Draw circles to help you.

   a) 
   
   
   
   
   
   
   
   
   
   $3 + 2 = 1 + 4$

   Data: 1, 2, 5, 8

   b) 
   
   
   
   
   
   
   
   
   
   $2 + 2 + 1 = 2 + 3$

   Data: ____________

   c) 
   
   
   
   
   
   
   
   
   
   $1 + 1 + 1 + 1 = 4$

   Data: ____________
Graphs (1)

Hours of Daylight in Windsor, ON

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>18</td>
</tr>
<tr>
<td>Feb</td>
<td>16</td>
</tr>
<tr>
<td>Mar</td>
<td>14</td>
</tr>
<tr>
<td>Apr</td>
<td>12</td>
</tr>
<tr>
<td>May</td>
<td>10</td>
</tr>
<tr>
<td>Jun</td>
<td>8</td>
</tr>
<tr>
<td>Jul</td>
<td>6</td>
</tr>
<tr>
<td>Aug</td>
<td>4</td>
</tr>
<tr>
<td>Sep</td>
<td>2</td>
</tr>
<tr>
<td>Oct</td>
<td>0</td>
</tr>
<tr>
<td>Nov</td>
<td>0</td>
</tr>
<tr>
<td>Dec</td>
<td>0</td>
</tr>
</tbody>
</table>

Hours of Daylight in Windsor and Ushuaia

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>18</td>
</tr>
<tr>
<td>Feb</td>
<td>16</td>
</tr>
<tr>
<td>Mar</td>
<td>14</td>
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<tr>
<td>Apr</td>
<td>12</td>
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<tr>
<td>May</td>
<td>10</td>
</tr>
<tr>
<td>Jun</td>
<td>8</td>
</tr>
<tr>
<td>Jul</td>
<td>6</td>
</tr>
<tr>
<td>Aug</td>
<td>4</td>
</tr>
<tr>
<td>Sep</td>
<td>2</td>
</tr>
<tr>
<td>Oct</td>
<td>0</td>
</tr>
<tr>
<td>Nov</td>
<td>0</td>
</tr>
<tr>
<td>Dec</td>
<td>0</td>
</tr>
</tbody>
</table>

- Windsor, ON
- Ushuaia, Argentina
Graphs (2)

### Daylight Hours

<table>
<thead>
<tr>
<th>Month</th>
<th>Windsor, ON</th>
<th>Alert, NU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Feb</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Mar</td>
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<td>Nov</td>
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<tr>
<td>Dec</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Online Device Usage by Age Group

<table>
<thead>
<tr>
<th>Age Group (years)</th>
<th>Daily Usage (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16–24</td>
<td>200</td>
</tr>
<tr>
<td>25–34</td>
<td>150</td>
</tr>
<tr>
<td>35–44</td>
<td>100</td>
</tr>
<tr>
<td>45–54</td>
<td>50</td>
</tr>
<tr>
<td>55–64</td>
<td>20</td>
</tr>
</tbody>
</table>
Unit 16 Measurement: 3-D Shapes, Volume, and Surface Area

Introduction
This unit focuses on:

- estimating, measuring, and recording volume and capacity;
- solving problems requiring conversion of units of volume and capacity;
- calculating and estimating the volume of rectangular and triangular prisms;
- finding the surface area of rectangular and triangular prisms; and
- constructing and drawing 3-D structures made from interlocking cubes.

Meeting Your Curriculum

<table>
<thead>
<tr>
<th></th>
<th>ALBERTA</th>
<th>BRITISH COLUMBIA</th>
<th>MANITOBA</th>
<th>ONTARIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>ME6-17 to 19, 21</td>
<td>ME6-17, 18, 22, 23</td>
<td>ME6-17 to 19, 21</td>
<td>ME6-18, 20 to 23, 25 to 28</td>
</tr>
<tr>
<td>Optional</td>
<td>ME6-20, 22 to 28</td>
<td>Optional</td>
<td>Optional</td>
<td>Recommended</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Optional</td>
<td></td>
<td>ME6-17, 19, 24</td>
</tr>
</tbody>
</table>

Mental Math Minutes
The mental math minutes in this unit:
- practise solving equations by comparing sides;
- practise finding a fraction of a number; and
- practise multiplying and dividing decimals.

Generic BLMs
The Generic BLMs used in this unit are:
- BLM 1 cm Grid Paper (p. T-1)
- BLM Filling a Blank Multiplication Chart (p. T-2)

These BLMs can be found in Section T.
Materials

In preparation for and throughout the unit, collect various sizes of rectangular boxes (such as shoeboxes, small food and medication packaging boxes, facial tissue boxes) so that students can measure and calculate their volumes and surface area. Invite students to add to your collection of boxes by bringing some in from home. In Lesson ME6-23, you might want to show students a rectangular aquarium. In Lessons ME6-17, 27, and 28, students will need connecting cubes.

Assessment

The lessons covered by a quiz or test are as follows:

<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>BC</th>
<th>MB</th>
<th>ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz</td>
<td>ME6-17 to 19, 21</td>
<td>ME6-17, 18</td>
<td>ME6-17 to 19, 21</td>
<td>ME6-17 to 21</td>
</tr>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>ME6-22, 23</td>
<td>n/a</td>
<td>ME6-22 to 26</td>
</tr>
<tr>
<td>Quiz</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>ME6-27, 28</td>
</tr>
<tr>
<td>Test</td>
<td>ME6-17 to 19, 21</td>
<td>ME6-17, 18, 22, 23</td>
<td>ME6-17 to 19, 21</td>
<td>ME6-18, 20 to 23, 25 to 28</td>
</tr>
</tbody>
</table>

Additional Information for This Unit

Volume versus Capacity

Volume and capacity are concepts that are sometimes interchanged. Volume is the amount of space taken up by a three-dimensional object, and capacity is defined as how much a container can hold. While volume is measured in cubic units—such as cubic centimetres (cm³)—capacity is measured in millilitres (mL), litres (L), etc. Units of capacity are also used to measure the volume of any pourable substance, such as sand or water. Litres are not strictly part of the metric system; the SI or International System of Units has basic units and derivatives and avoids multiple units for the same thing. However, litres are compatible with the use of the metric system, specifically for the volume of pourable substances. Litres are also commonly used and appealing because of convenience.
Goals
Students will find the number of cubes in three-dimensional stacks.

PRIOR KNOWLEDGE REQUIRED
Can find the area of a rectangle
Understands that the number of squares in a rectangle can be found by multiplication
Knows that multiplication is commutative

MATERIALS
connecting cubes

Review the area of a rectangle as an array. Draw a rectangle on a grid (or subdivide a rectangle into equal squares). Ask students to write the addition and multiplication equations needed to calculate the area of this rectangle.

Counting cubes in a stack using horizontal layers. Build a $2 \times 4$ “rectangle” using connecting cubes, as shown in the margin. ASK: How many cubes are in this rectangular layer? Ask students to explain how they calculated the number of cubes—did they count the cubes one by one or did they count them another way? You can use addition or multiplication to count the total number of cubes:

- $2 + 2 + 2 + 2 = 8$ (2 cubes in each of 4 columns)
- $2 \times 4 = 8$ (2 rows of 4 cubes OR length $\times$ width)

Add 1 layer to the stack so that it is 2 cubes high, as shown in the margin. ASK: How many cubes are in the new stack? Prompt students to use the fact that the stack has 2 horizontal layers. Remind them that they already know the number of cubes in one layer. Ask students to write the addition and multiplication equations for the number of cubes in the new stack using layers. ($8 + 8 = 16$, $2 \times 8 = 16$) Add a third layer to the stack and repeat. ($8 + 8 + 8 = 24$; $3 \times 8 = 24$)

Exercises

a) Write a multiplication equation for the number of blocks in the top layer.

i) ii) iii)

b) Write a multiplication equation for the total number of blocks for each structure in part a).

Answers: a) i) $4 \times 2 = 8$, ii) $3 \times 5 = 15$, iii) $6 \times 3 = 18$; b) i) $4 \times 2 \times 3 = 8 \times 3 = 24$, ii) $3 \times 5 \times 2 = 15 \times 2 = 30$, iii) $6 \times 3 \times 4 = 18 \times 4 = 72$
Counting cubes in a stack using a vertical layer. Invite students to look at the stack in the margin and calculate the number of cubes by adding vertical layers instead of horizontal layers. ASK: How many cubes are at the end of the stack? (3 \times 2 = 6) How many cubes are in each vertical layer? (6) How many vertical layers are in the stack? (4) Invite volunteers to write the addition and multiplication equations for the total number of cubes using the number of cubes in the vertical layer. ASK: Does this method produce a different result than the previous method? (no, it’s the same answer) Why should we expect the same answer? (it is the same stack, only counted differently)

Exercises: Use the stacks in the previous exercises.

a) Write a multiplication equation for the number of blocks in a vertical layer.

b) Write a multiplication equation for the total number of blocks.

Answers: a) i) 2 \times 3 = 6, ii) 5 \times 2 = 10, iii) 3 \times 4 = 12; b) i) 2 \times 3 \times 4 = 6 \times 4 = 24, ii) 5 \times 2 \times 3 = 10 \times 3 = 30, iii) 3 \times 4 \times 6 = 12 \times 6 = 72

Determining the number of cubes in a stack as a product of length, width, and height. Review the terms “length” and “width.” Remind students that to find the number of squares in a rectangle, they can multiply length by width. Now they have a three-dimensional stack, but it is very similar to a rectangle. Remind students that the vertical dimension in 3-D objects is called the height. Identify the length, width, and height of the same stack in the margin. Then use the terms “length,” “width,” and “height” to label the multiplication equation that gives the number of cubes in the stack:

\[
\frac{4 \times 2 \times 3}{\text{length}} = 24
\]

\[
\text{width} \times \text{height}
\]

Draw several stacks on the board and ask students to find the number of cubes in each stack by multiplying length, width, and height.

Remind students that order does not matter in multiplication, so the number of cubes can be found in other ways, such as height \times length \times width or width \times length \times height.

ACTIVITY (Optional)

Students work in pairs. Each student creates several rectangular stacks of connecting cubes, and then exchanges stacks with his or her partner. The partner finds the number of cubes in the stacks created by his or her partner by multiplying length, width, and height and writes a multiplication equation. Partners then check each other’s work.

Discuss how the dimensions in the formula length \times width \times height are related to the layers in a stack. ASK: What part of the formula gives you the number of cubes in one horizontal layer? (length \times width) SAY: Then you multiply by the number of horizontal layers, which is height. Repeat with a vertical layer. (width \times height)
Extensions

1. Use connecting cubes.
   a) In how many ways can you build a rectangular stack of 36 cubes with a height of 3 cubes?
   b) In how many ways can you build a rectangular stack with 48 cubes?

   **Answers**
   a) 3; 12 \times 1 \times 3, 6 \times 2 \times 3, 4 \times 3 \times 3 (dimensions in order length, width, height)
   b) 9; 48 \times 1 \times 1, 24 \times 2 \times 1, 16 \times 3 \times 1, 12 \times 4 \times 1, 12 \times 2 \times 2, 8 \times 3 \times 2, 8 \times 6 \times 1, 6 \times 4 \times 2, 4 \times 4 \times 3 (different arrangements of the same dimensions omitted)

2. A box is 80 cm long, 60 cm wide, and 40 cm tall. It is filled with thousands blocks, which are cubes with sides of 10 cm.
   a) How many cubes fit into the box?
   b) If ones blocks count as ones, what number is represented by the contents of the box?

   **Answers:** a) 192 cubes, b) 192 000
Goals

Students will estimate, calculate, and record the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED

Can find the number of blocks in a rectangular prism made of blocks using multiplication
Can multiply decimals
Can estimate products using rounding
Can estimate length in metric units
Can measure length in metric units

MATERIALS

piece of thread
old newspapers, tape, and BLM Cube Skeleton (p. S-65)
centimetre cubes
hundreds and thousands blocks
box
ruler
rectangular cedar basket (see Extension 2)

Mental math minute—number talk. Present the problem:

Solve \(2n + 34 = 16 + 34\) for \(n\). \((n = 8)\) The following strategies could arise:

Use \(2n = 16\).
Use \(n + 17 = 8 + 17\).

Guess and check.

Introduce volume. Hold up a piece of thread. Explain that it has only one dimension—length. ASK: What units do we measure length in? \((\text{cm, m, km})\)
Point to the top of a desk or table and explain that it has length and width—that’s two dimensions. SAY: Anything two-dimensional has area. ASK: What units do we measure area in? \((\text{cm}^2, \text{m}^2, \text{km}^2)\)

Emphasize that all these units are squares with sides that measure one length unit.

Explain that a three-dimensional object, such as a cupboard or a box, has length, width, and height. SAY: The space taken up by the box or cupboard or any three-dimensional object is called volume, and we measure it in cubic units: units that are cubes.

Introduce standard cubic units. Draw several cubes on the board and mark the sides as 1 cm for one cube, 1 mm for another cube, and 1 m for the third. Explain that these are different cubic units—cubic centimetre, cubic metre, etc. Show the abbreviated form of each unit.

NOTE: Activity 1 is required in order to cover the British Columbia curriculum.
ACTIVITY 1 (Optional)

1. Divide students into groups and give each group old newspapers, tape, and BLM Cube Skeleton, which shows the instructions for the activity. Have students roll the newspapers into tubes slightly longer than 1 m to allow for binding at the ends. Students might need to combine several newspapers to produce a longer tube; suggest rolling the newspapers diagonally to make tubes thinner at the ends for easier binding. Have students mark 1 m on each tube, as shown in the margin.

Compare the relative sizes of 1 m³ and 1 cm³ represented by a centimetre cube. Point out that cubic metres are very large. For example, to fill an aquarium with a volume of one cubic metre, you would need about 100 large pails of water! Discuss which volumes would be measured in various cubic units. Have students give examples for each unit. (mm³: volume of medication, drop of water; cm³: household objects, such as boxes or jars; m³: volume of a room, a tent, a building; km³: volume of a lake, greenhouse gas emissions)

Finding the volume of rectangular prisms. Show a $3 \times 4 \times 2$ stack of centimetre cubes, and review how to find the number of cubes in a stack (by multiplying length, width, and height). ASK: How many cubes are in the stack? ($3 \times 4 \times 2 = 24$) Explain that the volume of this stack is 24 cubic centimetres.

Draw several stacks of cubes on the board and explain that these cubes are unit cubes. Mark the size of each cube, using different units for different pictures. Have students find the volume of each stack.

Draw the picture in the margin. Remind students that mathematicians call rectangular boxes rectangular prisms. ASK: What is the length of this prism? (5 m) Repeat with width and height, and record the information on the board. ASK: If this were a stack of cubes 1 m by 1 m by 1 m, how many cubes would fit along the length of this prism? (5) Width? (3) Height? (4) How many cubes in total would be in the stack? (60) How do you know? ($3 \times 5 \times 4 = 60$; volume is length × width × height) Have students write down the multiplication equation. ASK: What is the volume of the prism? (60 m³)

Point out that students found the volume of the prism even though you did not draw the cubes that made it. ASK: What did you do to find the volume? (multiplied the dimensions: length × width × height) Write the formula $V = l \times w \times h$ on the board, and explain that it is also convenient to use the short form $V = l \times w \times h$.

On the board, draw the prism in the margin. Have students write the multiplication equation for its volume. Explain that, when finding volume, it helps to write the units for each measurement in the multiplication equation to prevent mistakes. For example, for this prism, the multiplication equation should be:

Volume = $6 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm}$

= 36 cm³
Point out that the raised 3 that appears in the cubic units is the number of length measurements that were multiplied together to make the cubic unit: you multiplied three measurements (length, width, and height) to get the volume. All three measurements were in centimetres, so the result is in cubic centimetres (point to the raised 3 in cm$^3$).

**Exercises:** Find the volume of the rectangular prism. Include the units in your calculation and answer.

a) ![Prism](image1)
   - 10 cm x 2 cm x 6 cm = 120 cm$^3$

b) ![Prism](image2)
   - 7 mm x 8 mm x 2 mm = 112 mm$^3$

c) ![Prism](image3)
   - 7 km x 4 km x 3 km = 84 km$^3$

d) ![Prism](image4)
   - 3 m x 15 m x 4 m = 180 m$^3$

e) Length 11 m, width 6 m, height 11 m
   - 11 m x 6 m x 11 m = 726 m$^3$

f) Length 3.2 cm, width 3 cm, height 3 cm
   - 3.2 cm x 3 cm x 3 cm = 28.8 cm$^3$

g) Length 12 m, width 12 m, height 1.2 m
   - 12 m x 12 m x 1.2 m = 172.8 m$^3$

h) Length 100 cm, width 100 cm, height 100 cm
   - 100 cm x 100 cm x 100 cm = 1 000 000 cm$^3$

**Bonus:** Use one of your answers above to convert 1 m$^3$ to cm$^3$.

**Answers:**
a) 120 cm$^3$, b) 112 mm$^3$, c) 84 km$^3$, d) 180 m$^3$, e) 726 m$^3$, f) 28.8 cm$^3$, g) 172.8 m$^3$, h) 1 000 000 cm$^3$, Bonus: 1 m$^3$ = 1 000 000 cm$^3$

**Showing that volume does not depend on orientation.** Show students a prism that has different measurements for the length, width, and height (e.g., a tissue box). Write its dimensions to the nearest centimetre on the board. Have students find the volume of the prism. Then turn the prism.

**ASK:** Did the height of the prism change? (yes) The length? (yes) The width? (yes) Do you expect the volume to change? (no) Why not? (the order of the dimensions changed, not the dimensions themselves; we multiply the same numbers but in a different order)

**Estimating volume.** Hold up a thousands block. SAY: This cube is 10 cm long, 10 cm wide, and 10 cm tall. **ASK:** What is the volume of this cube? (10 cm x 10 cm x 10 cm = 1000 cm$^3$) Arrange 2 thousands blocks together to show a volume of 2000 cm$^3$, and repeat with different arrangements of 4, 6, and 8 blocks. Then hold up a hundreds block and have students determine the volume. (10 cm x 10 cm x 1 cm = 100 cm$^3$) Have students determine the volume of a stack of 5 hundreds blocks and a stack of of 8 hundreds blocks. Record the volumes on the board or on cards and place the cards beside the arrangements. Leave these arrangements in view and point out that students can visually compare these arrangements to other objects to estimate the volume.
Show students a box and explain that you would like to estimate its volume in cubic centimetres. Discuss strategies. Students might compare the box to objects of known volume, such as a thousands cube. To help students develop a better estimate, ASK: How did you estimate the area of rectangles? (estimate side lengths and multiply length by width) Can you use a similar strategy for volume? (yes, estimate the dimensions and multiply length by width by height) Students can use different strategies to estimate the dimensions, such as comparing to objects of known length or measuring with hands and fingers. Then have them multiply the estimated dimensions and get an estimate for the volume. Have students record their estimates of lengths and volume in their notebooks, and record several answers on the board.

**ACTIVITY 2 (Essential)**

2. Give each student a box and have students estimate the volume in cubic centimetres. Then have students exchange boxes with a partner and repeat the estimating. Partners compare estimates and estimation strategies.

Measuring volume. Remind students that there are 10 mm in 1 cm, so each millimetre is one tenth of a centimetre or 0.1 cm. If an object has a length that is between whole numbers of centimetres, they can record the length not only in millimetres but also in centimetres and millimetres, or in centimetres as a decimal. Draw the picture in the margin on the board and have students record the length of the line segment in three ways. (35 mm, 3 cm 5 mm, 3.5 cm)

Invite a volunteer to measure the dimensions of the box for which students estimated the volume before Activity 2 and record the dimensions in centimetres as decimals. Compare the actual measurements to the estimates made earlier. Have students round some of the dimensions so that only one estimate remains in a decimal, and calculate the volume. Have students compare the answer to the estimates.

**ACTIVITY 3 (Essential)**

3. Have students measure the dimensions of one of the boxes they used in Activity 2. Students share their measurements with the partner they worked with earlier. Have them each calculate the volume of both boxes and compare the answers to all the estimates.

**NOTE:** Extension 2 is required to cover the British Columbia curriculum.

**Extensions**

1. Find the volume of the shape in the margin.

   **Answer:** $3 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm} + 3 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm} = 48 \text{ cm}^3$
2. Have students repeat Activities 2 and 3 with rectangular cedar baskets. Students need to decide how to measure the dimensions, because the corners of the baskets are slightly rounded. Have them use rounding to the nearest whole number of centimetres to estimate the volumes of the baskets they measured.

3. Grace and Jin are packing books into a box. Each book is 28 cm long, 22 cm wide, and 2 cm thick. The box is 50 cm long, 30 cm wide, and 30 cm tall on the inside.
   a) What is the volume of each book?
   b) What is the volume of the box?
   c) Grace divides the volume of the box by the volume of 1 book to find the number of books that will fit in the box. How many books does she think will fit in the box?
   d) Jin thinks: I’ll pack the books in two stacks, as shown in the margin. Then I can fill the leftover space with more books.
      i) How many books will be in each stack?
      ii) How many books will fit in the leftover space? How should Jin place them?
      iii) How many books in total will fit in the box?
   e) Why do Jin and Grace get different numbers of books for their answer? How many books will really fit in the box?

   Answers: a) 1232 cm$^3$, b) 45 000 cm$^3$, c) 36 books, d) i) 15 books, ii) 3 books placed vertically to the right of the stacks. Also, 2 more books can fit into the space between the stacks and the front face of the box, if placed vertically; iii) $2 \times 15 + 3 + 2 = 35$ books, e) Jin’s packing takes the dimensions of the books into account while Grace’s packing does not. In Jin’s packing there will be empty space left, which is equal in volume to about a book and a half, but the dimensions of the empty space do not allow fitting in another book.

4. During an excavation of an old military fort from the War of 1812, an underground ammunition storage room was discovered. The room is 300 cm long, 180 cm wide, and 120 cm deep. Several ammunition crates measuring 60 cm by 30 cm by 30 cm each were found in the room. What is the greatest number of crates that would fit in the room?

   Solution: The 60 cm side can be placed along any side of the storage room. One way to do so would be to place 5 crates along the length of the storage room, 6 along the width, and 4 along the height. This means 120 crates ($5 \times 6 \times 4 = 120$) could fit in the room.
ME6-19  Volume and Area of One Face

Pages 168–170

Goals

Students will develop and use the formula Volume = area of horizontal face × height for the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED

Can use the formula $V = l \times w \times h$ to find the volume of a rectangular prism
Can multiply and divide multi-digit whole numbers and decimals

MATERIALS

cube
rectangular box
box with faces labelled with their names for demonstration
coloured marker

NOTE: In rectangular prisms, any pair of opposite faces can be considered bases. In this lesson, students develop an understanding that they can find the volume of a rectangular prism by multiplying the area of any face by the dimension that is not one of the dimensions of that same face. The term “base” is introduced when students study triangular prisms, in Lesson ME6-20, or in later grades.

Review terminology. Hold up a cube. Remind students that different parts of a 3-D shape are called faces, edges, and vertices. Show the faces on a cube, run your finger along edges, and point out the vertices. Count the faces, edges, and vertices of a cube together and write on the board:

A cube has 6 faces, 12 edges, 8 vertices.

Draw a cube on the board as shown in the margin. Count the faces, edges, and vertices on the picture. (3 faces, 9 edges, 7 vertices) Explain that the edges and vertices that are behind the faces you see are called hidden edges and can be shown with dashed lines. Add the dashed lines to the picture, as in the second picture.

Label the faces of the cube as shown in the margin. Shade the front face and explain that the face opposite to the front face is called the back face.

Review the fact that opposite faces of a rectangular prism match. Show students a rectangular prism. ASK: In a rectangular box, or in a rectangular prism, can the top face be larger than the bottom face? (no) Can the bottom face be larger than the top face? (no) Can they have different shapes? (no) SAY: The top face and the bottom face are always the same rectangles. If some students have difficulty seeing that the top face and the bottom face match exactly, give them a box, have them trace the bottom face of the box, and check that the top face matches the bottom face exactly. Repeat with other pairs of opposite faces.

VOCABULARY

back
down
cubic centimetre (cm³)
cubic kilometre (km³)
cubic metre (m³)
cubic millimetre (mm³)
edge
face
formula
front
height
horizontal face
left side
rectangular prism
right side
top face
vertex, vertices
volume

CURRICULUM REQUIREMENT

AB: required
BC: optional
MB: required
ON: recommended

VOCABULARY

back
down
cubic centimetre (cm³)
cubic kilometre (km³)
cubic metre (m³)
cubic millimetre (mm³)
edge
front
height
horizontal face
left side
rectangular prism
right side
top face
vertex, vertices
volume
Review the formula $V = \ell \times w \times h$. Ask students how they can find the volume of their boxes. Ask: What do you need to measure? (length, width, height) Remind students that a formula is a short way to write the instructions for how to calculate something. Ask: What is the formula for the volume of a rectangular prism? ($Volume = length \times width \times height$) How do we write it the short way? ($V = \ell \times w \times h$) Write both formulas on the board.

Remind students that they used a formula to find areas of rectangles and how they recorded the solution. For example, write the solution for a problem “Find the area of a rectangle with the length 3 cm and the width 2 cm” on the board, as shown in the margin. Keep the formula on the board for future use.

Demonstrate how to record the solution to the problem: “Find the volume of a rectangular prism with length 51 mm, width 33 mm, and height 20 mm.” ($Volume = 33660 \text{ mm}^3$)

Give each student a rectangular box. Have students measure and record the dimensions of the boxes to the nearest millimetre. Then have them find the volume of their boxes in cubic millimetres.

**Estimating to check calculations.** If students use a calculator to solve the problem, they need to estimate the result before punching in the numbers. For example, in the prism above, the volume should be about $50 \text{ mm} \times 30 \text{ mm} \times 20 \text{ mm} = 30000 \text{ mm}^3$, and since we rounded down both the length and the width, the actual number should be larger.

**Developing the formula Volume = area of horizontal face $\times$ height.** Write on the board the formula for the volume of a rectangular prism:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Ask students whether they see the formula for the area of a rectangle hidden in that formula. (length $\times$ width) Draw on the board:

```
    \text{height} \\
    \text{length} \leftrightarrow \text{width} \\
```

Ask: Which face or faces of the prism have the same length and width as the prism itself? (top face and bottom face) Explain that we can describe both the top and the bottom faces as horizontal faces.

Write on the board:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

area of horizontal face

Explain that we now have a new formula: Volume = (area of horizontal face) $\times$ height. Write that formula on the board as well.
Using the formula to find the volume. Solve the first exercise as a class, and then have students work individually.

**Exercises:** Find the volume.

- **a)** \(16 \text{ m}^2 \times 2 \text{ m} = 32 \text{ m}^3\)
- **b)** \(18 \text{ cm}^2 \times 3 \text{ cm} = 54 \text{ cm}^3\)
- **c)** height 5 m, area of top face 19 m²
- **d)** height 12 mm, area of bottom face 2250 mm²

**Answers:** a) 32 m³, b) 54 cm³, c) 95 m³, d) 27 000 mm³

**Units in the formula.** Point out that area is given in square units. Square units mean that there were two length units multiplied to get the square unit, which we can see in the raised 2 in square units—for example, m². In volume, we need to multiply three length units, so we need to multiply area by another quantity in length units—height—to get the volume:

\[
\text{Volume} = \text{area of horizontal face} \times \text{height}
\]

\[
\begin{array}{ccc}
\text{m}^3 & \text{m}^2 & \text{m} \\
3 \text{ lengths} & 2 \text{ lengths} & 1 \text{ length} \\
\text{multiplied} & \text{multiplied} & \text{multiplied}
\end{array}
\]

Draw the picture in the margin on the board. SAY: This prism is made from centimetre cubes. ASK: How many cubes are in this prism? (6 cubes) What is the volume of the prism? (6 cm³) What is the area of the shaded face? (6 cm²) Write on the board:

- **Volume:** 6 cm³
- **Area of the horizontal face:** 6 cm²

ASK: What is the difference between these two measurements? (the units) SAY: The number might be the same, but the units are different, because volume measures the amount of space taken by the 3-D shape, when area of the face belongs to the flat object, the face.

**Finding volume in other ways.** Draw the picture in the margin on the board. Ask students how they can find the volume of the prism. (multiply 25 cm² × 4 cm) Confirm that this is the correct answer. ASK: Do you know the height? (no) Do you know the area of the top face or the bottom face? (no) Then how do you know that this product is the volume? If the following two explanations do not arise, lead students to think about them:

1. If you turn this prism so that the front face becomes a horizontal face, the given length becomes the height, and the formula works. To illustrate this, use a box with the faces labelled on it, trace the width (the edge marked as 4 cm) with a coloured marker, and have a volunteer rotate the prism so that the front face becomes the bottom or the top face. What dimension did the coloured edge become? (height)
2. As an alternative, students can think of the following: in having the height and the area of the front face, they have all the information needed for calculating the volume. The only difference is that they multiply the dimensions in a different order.

\[
\text{Volume} = \text{length} \times \text{width} \times \text{height}
\]

Area of front face = 25 cm\(^2\)

**Exercises:** Find the volume of the prism.

- **a)** length 8 cm  
  area of right-side face 12 cm\(^2\)  
  area of front face 33 cm\(^2\)

- **b)** width 22 m  
  area of front face 33 m\(^2\)

- **c)** area of right-side face 9.4 cm\(^2\), length 5 cm

- **d)** area of front face 23.1 m\(^2\), width 6 m

**Answers:** a) 96 cm\(^3\), b) 726 m\(^3\), c) 47 cm\(^3\), d) 138.6 m\(^3\)

**NOTE:** Question 6 on AP Book 6.2 p. 169 shows a different way of developing the same formula for the volume of a rectangular prism, emphasizing the equality between the height and the number of horizontal layers in the prism. You might want to point out this connection to the students when they have finished working on the AP Book questions for this lesson.

**Extensions**

1. Find the volume of the prism or explain why you cannot.

- **a)** height 9 mm  
  area of front face 18 mm\(^2\)

- **b)** width 13 m  
  area of left-side face 39 m\(^2\)
c) area of top face 4 cm²  
   length 2 cm

\[ \text{area} = \text{length} \times \text{width} \]

\[ 4 \text{ cm}^2 = 2 \text{ cm} \times \text{width} \]

\[ \text{width} = \frac{4 \text{ cm}^2}{2 \text{ cm}} = 2 \text{ cm} \]

\[ \text{Sample solution:} \ a) \text{ The measurements given are the front face} \]
\[ \text{(so length and height) and the height again, but not a third dimension,} \]
\[ \text{so we cannot calculate volume.} \]

\[ \text{Answers:} \ b) \text{ cannot calculate,} \ c) \text{ cannot calculate,} \ d) \text{ 3375 m}^3 \]

2. A cedar basket is made from strips 1.5 cm wide. The bottom is a square
   made from 8 strips overlapping 8 other strips at a right angle. The
   height of the basket is 15 strips. Estimate the volume of the basket.
   Explain how you know.

\[ \text{Answer} \]
\[ \text{The bottom of the basket is a square 8 strips wide} = 8 \times 1.5 \text{ cm} = 12 \text{ cm}, \]
\[ \text{so the area of the bottom is 144 cm}^2. \text{ The height of the basket is} \]
\[ 15 \text{ strips} = 15 \times 1.5 = 22.5 \text{ cm}, \text{ so the volume of the basket is about} \]
\[ 144 \text{ cm}^2 \times 22.5 \text{ cm} = 3240 \text{ cm}^3. \]
**Goals**

Students will develop and use the formula \( \text{Volume} = \text{area of base} \times \text{height} \) for the volume of triangular prisms.

**PRIOR KNOWLEDGE REQUIRED**

- Can find the volume of a rectangular prism
- Can multiply and divide multi-digit whole numbers and decimals

**MATERIALS**

- prisms made from BLM Nets of 3-D Shapes (1) (p. S-66) copied onto blue and green paper

**Mental math minute—number string.**

String 1: \( 6 \times 0.5, 3 \times 1 (3, 3) \)

String 2: \( 2 \times 7, 4 \times 3.5, 8 \times 1.75 (14, 14, 14) \)

String 3: \( 6 \times 9.5, 16 \times 1.25, 17.5 \times 8 (57, 20, 140) \)

Review finding volume of rectangular prisms. Draw a rectangular prism on the board, mark the dimensions as 2 cm, 3 cm, and 4 cm, and have students find the volume of the prism. (24 cm\(^3\)) Encourage multiple explanations; make sure the idea of finding the area of one face and multiplying by the length of the perpendicular side arises.

Volume of triangular prisms with a right triangle in the base. Display one of the blue triangular prisms made from BLM Nets of 3-D Shapes (1). Remind students that this shape is called a triangular prism. Point out that three of its faces are rectangles, and two faces are congruent triangles. SAY: The triangular faces of the prism are called bases. Each prism has two bases, and they are identical.

Ask students to think about how they could calculate the volume of the prism. To prompt students, show the identical prism in green. ASK: Can you make a rectangular prism from these two prisms together? Invite a volunteer to demonstrate. ASK: What fraction of the volume of the rectangular prism does each triangular prism make? (half) How do you find the volume of the triangular prism from the volume of the rectangular prism? (divide by 2)
Exercises: Fill in the table.

<table>
<thead>
<tr>
<th></th>
<th>a) Prism 1</th>
<th>b) Prism 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>[Diagram]</td>
<td></td>
</tr>
<tr>
<td>Volume of</td>
<td>2 m</td>
<td>6 cm</td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td>4 m</td>
<td>25 cm</td>
</tr>
<tr>
<td>8 m</td>
<td>30 cm</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Volume of</th>
<th>Fraction of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Prism</td>
<td>Volume Shaded</td>
<td></td>
</tr>
<tr>
<td>Answers: a) 64 m(^3), 1/2, 32 m(^3); b) 4500 cm(^3), 1/2, 2250 cm(^3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Review how to find the area of a triangle by splitting it into two right triangles. Use the picture in the margin to remind students that you can split a scalene triangle into two right triangles to find its area. Keep the picture you drew to illustrate the process posted for the duration of the lesson.

Volume of triangular prisms with a scalene triangle in the base.
Demonstrate that you can join the congruent faces (numbered 1) of the two green prisms together so that they make a single triangular prism with a scalene obtuse base. ASK: How can I find the volume of this prism? (Divide it into two prisms with right triangular bases, and then find the volume of each of the two prisms by halving the volume of the rectangular prism with the same dimensions.)

Attach the blue prisms to the first combined prism, as shown below, to create a rectangular prism. (Use numbers on the faces of the prism to identify congruent faces.)

ASK: What fraction of the rectangular prism is the combined triangular prism? (one half) How do you know? (the two green triangular prisms are identical to the parts that make the combined prism) SAY: The volume of a triangular prism with any triangle in the base is half the volume of the rectangular prism with the same height and a base that has an area twice as large as the base of the triangular prism.
Exercises: Fill in the table.

<table>
<thead>
<tr>
<th>Volume of Rectangular Prism</th>
<th>Fraction of Volume Shaded</th>
<th>Volume of Triangular Prism</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Prism 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) Prism 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: a) 160 m³, 1/2, 80 m³; b) 4000 cm³, 1/2, 2000 cm³

Introduce height of a prism. Hold up a triangular prism with the base down and SAY: The distance between the bases (trace one of the side edges) is called the height of a prism. If I place a prism base down, the height becomes vertical. Turn the prism on the side and SAY: I can turn the prism, and the edge that shows the distance between the bases is not vertical anymore, but the distance is still called the height of the prism. Have students identify the height of the triangular prisms in the previous exercises. (a) 8 m, b) 16 cm)

For triangular prisms, volume = area of base × height of prism. Combine two triangular prisms into a rectangular prism. Trace the face that is formed from two triangles and SAY: I want to use the area of this face to find the volume of this rectangular prism. I need to multiply the area of this face by the length of one of the edges of the rectangular prism. Trace several edges and have students tell you if this is the correct edge for this purpose. ASK: What did we call the length of this edge in our triangular prisms? (height of a prism) SAY: We can call the face of the prism that is perpendicular to the height the base in a rectangular prism, too. Then the volume of the prism is a product of the area of the base and the height of the prism. Write on the board:

Volume of prism = area of the base × height of prism

SAY: Let’s see if this works for triangular prisms. Draw a table on the board as shown below for the prisms used in the two previous exercises and have students help you fill it in (answers shown in italics).

<table>
<thead>
<tr>
<th>Prism</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of Triangular Prism</td>
<td>32 m³</td>
<td>2250 cm³</td>
<td>80 m³</td>
<td>2000 cm³</td>
</tr>
<tr>
<td>Area of Base</td>
<td>4 m²</td>
<td>75 cm²</td>
<td>10 m²</td>
<td>200 cm²</td>
</tr>
<tr>
<td>Height of Prism</td>
<td>8 m</td>
<td>30 cm</td>
<td>8 m</td>
<td>10 cm</td>
</tr>
<tr>
<td>Area of Base × Height</td>
<td>32 m³</td>
<td>2250 cm³</td>
<td>80 m³</td>
<td>2000 cm³</td>
</tr>
</tbody>
</table>
ASK: Does the formula work for triangular prisms? (yes)

Exercises: Find the volume of the prism.

a)  
\[ \text{Volume} = 2 \text{ cm} \times 3.4 \text{ cm} \times 5 \text{ cm} \div 2 \]

b)  
\[ \text{Volume} = 2 \text{ m} \times 3.4 \text{ m} \times 4 \text{ m} \div 2 \]

c)  
\[ \text{Volume} = 2 \text{ cm} \times 4 \text{ cm} \times 4.5 \text{ cm} \div 2 \]

Answers: a) 17 cm³, b) 13.6 m³, c) 18 cm³

NOTE: There is a lot of room for confusion with terminology for triangular prisms. The words “base” and “height” are used for two different objects each. Each time students talk about base or height, have them say which base they refer to. For example, students can say “area of the base of the prism” and “height of the prism” when talking about the volume, or “height of the base” when talking about the area of the base.

Extensions

1. Discuss how the two ways of finding the volume of triangular prisms are connected. On the board, draw the picture shown in the margin, and have volunteers show which edges of the prism show the following values:
   a) length of the rectangular prism
   b) width of the rectangular prism
   c) height of the triangular prism
   d) height of the rectangular prism
   e) base of the triangle in the base of the prism
   f) height of the triangle in the base of the prism

ASK: Which of these values are the same? (a = e, b = f, c = d)

Then write on the board:

\[ \text{Volume of triangular prism} = \text{length} \times \text{width} \times \text{height of the prism} \div 2 \]

\[ \text{Volume of triangular prism} = \text{area of base} \times \text{height of prism} \]

ASK: How do you find the area of the base of the prism? (multiply the base of the triangle by the height of the base, then divide by 2) Have a volunteer rewrite the formula using this information.

\[ \text{Volume of triangular prism} = (\text{base of triangle} \times \text{height of triangle} \div 2) \times \text{height of prism} \]

Have volunteers underline the parts that are the same in the first and in the third formulas using different underlining for different parts.

ASK: How are the two formulas the same? (they use the same values,
the values are multiplied) How are the two formulas different? (in the first formula you divide at the end, and in the third formula you divide before the last multiplication) Should the volumes be the same or different? (the same)

Point out that this is similar to the strategy students used in multiplication: halving one factor and doubling the other does not change the product. SAY: In the first formula, the last factor is halved. In the third formula, the first part of the equation is halved. This means that the answers are the same.

2. For the picture in the margin, Edmond thinks that the volume of the shaded prism is half the volume of the rectangular prism.


b) Find the volume of the shaded prism. Show your work.

**Answers:**
a) No. The base of the shaded prism is not half of the rectangle that forms the top face of the rectangular prism, because no sides of the rectangle are the same length as the sides of the triangle.

b) The triangle has base 2.4 cm and height 3 cm, so the area of the base is $2.4 \text{ cm} \times 3 \text{ cm} \div 2 = 3.6 \text{ cm}^2$. The volume of the triangular prism is 14.4 cm$^3$. Alternatively, students might realize that the shaded prism is one quarter of the rectangular prism by volume, hence the volume of the triangular prism is $(4.8 \text{ cm} \times 3 \text{ cm} \times 4 \text{ cm}) \div 4 = 14.4 \text{ cm}^3$. 
Goals
Students will solve problems related to the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED
Knows the formulas for the volume of rectangular prisms
Can find volume of rectangular prisms using the formulas
Can find the missing dimension of a given area of a rectangle and the other dimension
Knows the different units for measuring volume

MATERIALS
BLM Pictures of Rectangular Prisms (p. S-70)
rectangular waterproof container, such as a large juice carton, with a ruler attached to the inside
water
small fruit, such as a mandarin orange or a strawberry
several waterproof blocks (not connecting cubes)
calculator
connecting cubes (see Extension 1)
grid paper or BLM 1 cm Grid Paper (p. T-1, see Extension 2)

Review finding the missing length given the area of a rectangle. Remind students that the area of a rectangle is the product of its length and width, or area = length × width. Draw a rectangle and label the width as 5 cm and area as 35 cm². Explain that you want to find the length of the rectangle. Have students explain how to find the length. (the area is 5 cm × length = 35 cm², so 35 cm² ÷ 5 cm = 7 cm)

Remind students that it is important to include the units in their calculations, and even more important to include them in the answers.

Exercises: Find the missing width or length.

a) length 9 m, area 45 m²  b) width 6 mm, area 72 mm²
c) length 8 km, area 48 km²  d) width 8 cm, area 176 cm²

Bonus: length 3500 m, area 70 000 m²

Answers: a) 5 m, b) 12 mm, c) 6 km, d) 22 cm, Bonus: 20 m

Finding length of edges from a sketch. On the board, draw the picture in the margin. Point at the edges one at a time and have students raise the number of fingers equal to the length of the edge. For the first few edges that are not directly labelled, have a volunteer explain how they know the length. Repeat with the two prisms on the following page.
Review finding the volume of rectangular prisms. Remind students of the formulas for volume: Volume = area of the horizontal face × height and \( V = l \times w \times h \). Draw the picture below on the board for reference.

Finding the volume of rectangular prisms when some of the dimensions are not known. Draw the picture below on the board:

Have students identify the missing linear dimension. PROMPT: Which dimensions are you given: length, width, or height? (width and height) ASK: What is the width of the prism? (1 cm) What is the height of the prism? (2 cm) ASK: For which face are you given the area? (front face) Which dimensions do you multiply to find the area of the front face? (length and height) What is the missing length? (4 cm) Have students find the volume of the prism two ways, by multiplying all the dimensions and by multiplying the area of the front face by the width. Remind students that they should get the same answer (8 cm\(^3\)) both ways.

Exercises

a) Find the missing dimension of the prism in the margin.
b) Find the volume of the prism.

Answers: a) width = 35 cm, b) volume = 29 400 cm\(^3\)

ACTIVITY 1 (Essential)

1. Ask students to imagine, sketch, and write the dimensions for a prism. (Provide students who have trouble sketching a prism with BLM Pictures of Rectangular Prisms.) Then ask students to choose one face of the prism to shade. ASK: What is the area of the shaded face? Ask students to mark the area on the face and erase one of the dimensions for that face. Then ask students to form pairs, exchange prisms, and calculate the volume of the prism. Players can then repeat the activity for new prisms.

NOTE: An advanced variation of this activity is to find the missing dimension of the prism.
Finding the area of the base or the height given the volume. Tell students you want to make a box that is 30 cm long and 20 cm wide. You need your box to have a volume of 21 000 cm³. Record the data on the board. ASK: What height should the box be? (35 cm) Let students think about how to solve this problem and encourage multiple solutions.

Emphasize the importance of writing the units in the answer and checking that the answer makes sense: when you are looking for height, you divide the volume (in cubic centimetres) by the area of the face (in square centimetres), so the answer should be in centimetres. Students who have trouble with the units can benefit from the following exercises.

Exercises: Write the missing unit.

a) 5 cm × 6 ___ = 30 cm³
b) 5 mm × 6 mm² = 30 ___
c) 5 ___ × 6 km = 30 km³
d) 5 m² × 6 ___ = 30 m³

Answers: a) cm², b) mm³, c) km², d) m

Then have students practise finding the missing dimension with the exercises below. Students can use a method of their choice to solve the problem. Encourage multiple solutions, and encourage students to check their answers by substitution.

Exercises: Solve the problem.

a) area of top face 3 m², volume 12 m³, height = ?
b) area of top face 30 cm², volume 450 cm³, height = ?
c) height 3 mm, volume 51 mm³, area of top face = ?
d) length 5 m, width 3 m, volume 46.5 m³, height = ?

Bonus: length 8 cm, height 2.5 cm, volume 72 cm³, width = ?

Answers: a) 4 m, b) 15 cm, c) 17 mm², d) 3.1 m, Bonus: 3.6 cm

ACTIVITIES 2–3 (Essential)

For both activities, you will need a waterproof rectangular container, such as a juice carton with the top cut off, with a centimetre ruler attached to the inside of the container. Have a volunteer measure the dimensions of the container.

2. Finding the volume of an object submerged in water. Pour water into the container until it reaches a height of 10 cm. ASK: What is the height of the water in millimetres? (100 mm) How much water is in the container in cubic millimetres? Have students find the volume of the water.

Now show students a small fruit, such as a mandarin orange or a strawberry. Explain that you want to find the volume of the fruit. You will do it the same way the ancient Greek scientist Archimedes found the volume of a crown for a king. (You might tell students the
story of Archimedes and the crown.) Place the fruit in the water, and have a volunteer measure the height of the water in the container.

Have students find the volume of the water with the fruit. SAY: I know the volume of the water and the volume of the water and the fruit together. ASK: What is the volume of the fruit? Have students find the volume of the fruit.

3. **Predicting the height of the water after submerging objects.**

Pour water in the container to a height of 10 cm. Show students several waterproof blocks of the same size and have volunteers measure them (or provide the dimensions). Have students find the volume of the blocks. ASK: If I drop 5 blocks into the water, what will the volume of the water and the blocks together be? How do you know? What will the height of the water be? Have students figure out the answer on a calculator, rounding to whole numbers. Drop the blocks into the water and check the answer.)

### Extensions

1. Use 30 connecting cubes to make as many different rectangular prisms as possible, using all of the cubes.

   **Answer:** There are five possible prisms: $1 \times 1 \times 30$, $1 \times 2 \times 15$, $1 \times 3 \times 10$, $1 \times 5 \times 6$, $2 \times 3 \times 5$

2. Sketching cubes and rectangular prisms. Show students how to sketch a cube, and then have them sketch a cube themselves.

   **Step 1:** On grid paper, draw a square that will become the front face.

   **Step 2:** Draw another square of the same size, so that the centre of the first square is a vertex of the second square.

   **Step 3:** Join the vertices with lines as shown.

   **Step 4:** Turn the lines that represent hidden edges into dashed lines by erasing a few parts of those lines.

   On the board, sketch the two diagrams in the margin. ASK: Which dimension is different in these two shapes? (length) How is Step 2 performed differently in each drawing? (In the second cube, the square that is the back face is farther apart from the square that is the front face. In the drawings, the back face’s vertex is closer to the bottom left vertex of the front face in the shorter shape, and farther from that vertex in the longer shape.)
ASK: What would you do in Step 2 to draw a very long rectangular prism? Invite a volunteer to draw it. If the drawing is difficult to understand (i.e., if edges overlap), suggest to students that the vertex of the back face should not sit on the diagonal (see sample in margin).

Have students sketch rectangular prisms.

3. A wealthy king ordered a new treasure chest. The old chest is 18 cm long, 12 cm wide, and 10 cm high. The king asked four carpenters to each make a chest. Each carpenter changed the dimensions:

1st carpenter: twice as long, but the same width and height
2nd carpenter: twice as high, but the same width and length
3rd carpenter: twice as long and twice as wide, but half the height
4th carpenter: the same height, but a different length and width, which makes the area of the new chest’s bottom twice the area of the old chest’s bottom

a) Which carpenter made the chest with the largest volume? Explain.

b) The fourth carpenter’s chest has length and width in whole centimetres. What might the dimensions of the new treasure chest be? Find as many answers as you can.

Answers

a) All new chests have a volume of 4320 cm$^3$, which is twice the volume of the old chest, 2160 cm$^3$.

b) The area of the base of the old chest is 18 cm $\times$ 12 cm = 216 cm$^2$. The area of the base of the new chest is 432 cm$^2$. If both the length and the width of the fourth carpenter’s chest are different from those of the old chest, the chest might be 16 cm wide and 27 cm long.

Any other pair of whole numbers that multiply to 432 would also work, though some of these dimensions, such as 2 cm by 216 cm, are inconvenient for a treasure chest (2 cm $\times$ 216 cm, 3 cm $\times$ 144 cm, 4 cm $\times$ 108 cm, 6 cm $\times$ 72 cm, 8 cm $\times$ 54 cm, 9 cm $\times$ 48 cm, 16 cm $\times$ 27 cm; note that 12 cm $\times$ 36 cm and 18 cm $\times$ 24 cm are not included because the carpenter changed both the length and the width of the chest).

Bonus: The volume of the combined shape in the margin is 20 cm$^3$. What is the missing height?

Answer: 3 cm
**Goals**

Students will convert between litres and millilitres.
Students will solve problems involving converting units of capacity.

**PRIOR KNOWLEDGE REQUIRED**

Can multiply a decimal by 1000
Can convert kilograms to grams, metres to millimetres, or kilometres to metres

**MATERIALS**

cup or mug
container for 1 L
funnel
large pan, bowl, or tub to contain any spills
water or other pourable substance
other containers, including empty medicine bottles
measuring cups of various sizes (see Extension 4)

**Mental math minute.** Remind students that they shift the decimal point three places to the right to multiply by 1000. Remind them that they can write zeros after the decimal point, and the number will not change. For example, \(6.7 = 6.70 = 6.700\). **ASK:** Why are these numbers equal? (the decimal part is \(7/10 = 70/100 = 700/1000\)) As well, remind students that, even though we do not write a whole number with a decimal point, we can still add the decimal point without changing the number. For example, \(12 = 12.00\).

**Exercises:** Calculate.

- a) \(0.004 \times 1000\)
- b) \(4.356 \times 1000\)
- c) \(1.79 \times 1000\)
- d) \(0.07 \times 1000\)
- e) \(0.3 \times 1000\) **Bonus:** \(1000 \times 2.01\)

**Answers:** a) 4, b) 4356, c) 1790, d) 70, e) 300, Bonus: 2010

**Review liquid volume.** SAY: When you estimate or measure the volume of a box in cubic centimetres, you in fact are looking for the number of centimetre cubes that will fit into the box. Show students a cup or a mug and tell them that you would like to find its volume. **ASK:** Should I fill the cup with cubes? (no) Why not? (there will be many gaps between the cubes) Should I measure the length, width, and height of the cup and multiply those measurements? (no) Why not? (there is no clear length and width, and the formula only works for rectangular prisms, not for round objects such as cups) Explain that the way to measure the volume of the cup is to fill it with something that fills the cup without gaps, such as rice, sand, sugar, or water, and then pour the contents of the cup into a container with a volume that can be measured.
**Review litres.** Explain that the volume of liquids can be measured in different units. One of the common units is a *litre*. Write the word “litre” and its abbreviation “L” on the board and explain that these are both ways to write litre. Show students a container holding 1 L of water, and explain that this is a litre.

Hold up several containers, one at a time, and have students signal thumbs up if they think the container can hold more than 1 L, thumbs down for less than 1 L, and thumbs to side if they think it is about 1 L. If possible, have volunteers pour the water from your 1 L container into the containers to check their estimates. You will need a funnel and a large pan, bowl, or tub to contain spills.

Discuss with students whether one litre is a large quantity of liquid. **ASK:** Is one litre enough to water the plants in your home? (probably, yes) In your garden? (no) Is one litre enough to take a bath or a shower? To wash the dishes? To fill an aquarium?

**Introduce millilitres.** Explain that for quantities of liquid smaller than 1 L, there is another unit: *millilitres*. Millilitres are very small—for example, there are about 5 millilitres in one teaspoon. Introduce the short form of the unit (mL) as well.

**ACTIVITY (Essential)**

**Developing a sense of the size of containers.** Give each student one empty medicine bottle. Have students find the labels that show the volume of their bottle and round the amount to the closest 10 mL. Have students work in pairs and estimate the volume of their partner’s bottle rounded to the nearest 10 mL. Students can provide hints (too high or too low) and then, when both partners have correctly estimated the capacity, change partners.

**Most appropriate unit.** Review the term “appropriate”—specifically, an appropriate unit is the unit that gives the most convenient number for the measurement. For example, it is more convenient to measure the volume of a bathtub in litres than in millilitres because the number of millilitres would be really large. **PROMPT:** A teaspoon contains 5 mL of water. How many teaspoons would we need to fill a bathtub? (It’s hard to tell!) As well, most people do not need to know the volume of a bathtub with great precision. In contrast, if you need to give medication to your pet, you need to know the volume very precisely to avoid over-dosing or under-dosing your pet, so you will measure the medication in millilitres.

Take some of the containers used during the lesson and ask students to decide which unit—litre or millilitre—is more appropriate for measuring how much the container can hold. Write both units on the board and have students point to the correct unit to signal their answers.

**Connecting between litres and millilitres.** Write on the board:

\[
1 \text{ metre} = 1000 \text{ millimetres} \quad \quad 1 \text{ m} = 1000 \text{ mm}
\]
Invite volunteers to circle the common parts in the words “metre” and “millimetre.” Ask students to guess what the part “milli” means. Explain that “milli” means 1000 in Latin. Explain to students that the prefix “kilo” is used to create large units, and “milli” is used to create small units. So when they see a “milli” in a measurement unit, they know right away that there are 1000 smaller units (“milli”-units) in the large unit. Write on the board:

\[
1 \text{ litre} = \underline{1000} \text{ millilitres} \quad 1 \text{ L} = \underline{1000} \text{ mL}
\]

**ASK:** What number goes in the blanks? (1000)

### Converting litres to millilitres.

Draw a conversion table on the board as shown in the margin, and have students fill in the millilitres column. Ask students to look for regularity: How can we get the number of millilitres from the number of litres? What number would we multiply the number of litres by to get the number of millilitres? (1000) Why? (because there are 1000 mL in 1 L) Remind students that this is very similar to getting grams from kilograms, or metres from kilometres, or millimetres from metres.

**Exercises:** Multiply by 1000 to convert the measurement to millilitres.

- a) 9 L  
- b) 18 L  
- c) 42 L  
- d) 100 L  
- e) 394 L  
- f) 1000 L

**Answers:**

- a) 9000 mL  
- b) 18 000 mL  
- c) 42 000 mL  
- d) 100 000 mL  
- e) 394 000 mL  
- f) 1 000 000 mL

### Using multiplication to convert from litres to millilitres (decimals).

As a class, convert 0.12 L to mL:

\[
0.12 \text{ L} = 0.120 \text{ L} = 0.120 \times 1000 \text{ mL} = 120 \text{ mL}
\]

Move the decimal point three places to the right.

**Exercises:** Convert to millilitres.

- a) 7.239 L  
- b) 10.825 L  
- c) 0.002 L  
- d) 0.04 L  
- e) 0.063 L  
- f) 0.41 L  
- g) 10.89 L  
- h) 2.3 L

**Answers:**

- a) 7239 mL  
- b) 10 825 mL  
- c) 2 mL  
- d) 40 mL  
- e) 63 mL  
- f) 410 mL  
- g) 10 890 mL  
- h) 2300 mL

### Comparing measurements in different units.

Remind students that when they compare two measurements in different units, they need to convert one of the measurements so that the units become the same. Ask students to give an example of how they did it with other units. Have students convert the measurement in litres to millilitres before they compare the measurements. If you present the problems one at a time, students can point to the larger measurement to signal the answer.

**Exercises:** Which measurement is larger?

- a) 473 mL or 4 L  
- b) 73 L or 7300 mL  
- c) 25 678 mL or 25 L  
- d) 25 L or 25.089 mL  
- e) 3.6 L or 360 mL  
- f) 10.67 L or 106 700 mL
Word problems with conversions. Solve the following problems as a class by converting all the measurements in litres to millilitres and working with millilitres.

1. a) To make juice from concentrate, you need to mix a 355 mL can of concentrate with three cans of water. How much juice do you get? (355 mL × 4 = 1420 mL)

b) Raj makes a fruit drink by mixing juice from concentrate (one 355 mL can and 3 cans of water) with a 946 mL bottle of cranberry juice and 1.5 L bottle of ginger ale. How much fruit drink does Raj mix? (3866 mL)

c) Raj plans that each of 12 people at a party needs at least one 0.3 L glass of fruit drink. Did he make enough? (12 × 0.3 L = 3.6 L = 3600 mL < 3866 mL; yes, Raj made enough fruit drink)

2. A pack of six bottles of apple juice, each 0.3 L, costs $3.99 per pack. A pack of eight apple juice boxes, each 125 mL, costs $2.29.

a) How much juice is in each pack? (6 bottles contain 1800 mL juice, 8 boxes contain 1000 mL juice)

b) Which contains more juice, five packs of six juice bottles or nine packs of eight juice boxes? (both are 9 L)

c) What costs more, five packs of six bottles or nine packs of eight juice boxes? (five packs of six bottles cost $19.95, nine packs of eight juice boxes cost $20.61)

d) Which pack would you buy? Explain. (Answers will vary, buying the juice in bottles is cheaper by volume, but the preferred pack depends on your needs)

Exercises

1. a) How many teaspoons are needed to fill a 1 L bottle of water? (Remember: 1 teaspoon = 5 mL)

b) A bathtub can hold 220 L of water. How many teaspoons would you need to fill the bathtub?

c) A cup holds 250 mL of water. How many cups will you need to fill the bathtub from part b)?

d) It takes 45 seconds to fill a cup with water in a bathroom sink, empty it into the bathtub, and return to the sink. If you want to fill the bathtub in part b) with cups, how much time will it take? Express your answer in seconds, minutes, and hours.
2. Milk is sold in packs of 3 bags that total 4 L.
   a) How much milk is in each bag? Round your answer to the closest millilitre.
   b) The capacity of a cup is 240 mL. Eric opens a new bag of milk. He uses 2.5 cups of milk for a milkshake and 350 mL of milk for pancakes. How much milk is left in the bag?

   **Answers:** 1. a) 200, b) 44 000, c) 880, d) $880 \times 45 \text{ sec} = 39 600 \text{ sec}$
   $= 660 \text{ min} = 11 \text{ h}; 2. a) 1333 \text{ mL}, b) 2.5 \times 240 + 350 = 600 + 350$
   $= 950$, so $1333 \text{ mL} - 950 \text{ mL} = 383 \text{ mL}$ is left in the bag

**Extensions**

1. Convert litres to millilitres, and then add the leftover millilitres to convert the mixed measurement to a measurement in millilitres only.

   a) 2 L 345 mL  
   b) 17 L 67 mL  
   c) 4 L 8 mL

   d) 2 L 371 mL  
   e) 45 L 604 mL  
   f) 658 L 400 mL

   g) 8 L 75 mL  
   h) 30 L 5 mL

   **Bonus**

   i) 100 L 100 mL  
   j) 1000 L 10 mL

   **Answers:** a) 2345 mL, b) 17 067 mL, c) 4008 mL, d) 2371 mL, 
   e) 45 604 mL, f) 658 400 mL, g) 8075 mL, h) 30 005 mL, 
   Bonus: i) 100 100 mL, j) 1 000 010 mL

2. **Converting measurements in litres to mixed measurements.** Explain that in a measurement in litres, such as 6.527 L, the whole part shows the litres, and the decimal part shows the number of millilitres, so $6.527 \text{ L} = 6 \text{ L} 527 \text{ mL}$.

   Convert to a mixed measurement.

   a) 6.998 L  
   b) 4.708 L  
   c) 2.039 L  
   d) 3.007 L

   **Answers:** a) 6 L 998 mL, b) 4 L 708 mL, c) 2 L 39 mL, d) 3 L 7 mL

3. a) Marco thinks that 2.5 L is 2 L 5 mL. Is he correct? Explain.
   b) Expand the measurement to the thousandths place. Convert to a mixed measurement.

   **Answers**

   a) no, $2.5 \text{ L} = 2.5 \text{ L} \times 1000 = 2500 \text{ mL}$
   b) i) 6 L 900 mL, ii) 4 L 700 mL, iii) 2 L 190 mL, iv) 3 L 870 mL
4. **Measuring volume.** Show students measuring cups of different sizes and draw their attention to the marks. Point out that 1 mL is a small unit, so when a liquid needs to be measured to the closest millilitre, you need to use a small measuring cup or a graduated cylinder that can hold a very small amount of liquid. Demonstrate using a measuring cup to measure the volume of a container by filling a container with water, then pouring the water into a measuring cup. Choose a container that holds an amount of liquid that is not a round number so that the water level is between adjacent marks on the measuring cup. Remind students that they need to look at the mark that is closest to the water level. Point out that the measurement produced in this way is approximate.

Give students a variety of containers, and have them estimate and then measure their volume in millilitres. Students will need funnels and a large pan, bowl, or tub to work over to contain any spills.
**ME6-23  Capacity**

**Pages 177–179**

**Goals**

Students will recognize the connection between millilitres, litres, and cubic units, and will solve problems involving capacity and volume.

**PRIOR KNOWLEDGE REQUIRED**

- Can multiply a decimal by 1000
- Knows that capacity is measured in litres and millilitres
- Knows that 1 L = 1000 mL
- Knows that 1 dm = 10 cm
- Can convert metric units of length and units of capacity
- Can find the volume of a rectangular prism
- Can find the missing dimension given the volume of a prism and other dimensions

**MATERIALS**

- rectangular aquarium
- cube with 10 cm sides (e.g., a thousands block)
- large graduated pitcher
- water
- ruler
- smaller graduated pitcher
- small toy (heavy enough to not float in water)

**Mental math minute.** Remind students that they can double both numbers in a division and the result does not change.

**Exercises:** Double both numbers once or twice to get an easier division. Then divide.

a) \(1.8 \div 5\)  
 b) \(62.5 \div 25\)  
 c) \(2.4 \div 5\)  
 d) \(0.21 \div 5\)

**Bonus:** \(112.5 \div 75\)

**Answer:** a) \(3.6 \div 10 = 0.36\), b) \(125 \div 50 = 250 \div 100 = 2.5\),
  c) \(4.8 \div 10 = 0.48\), d) \(0.42 \div 10 = 0.042\),
  Bonus: \(225 \div 150 = 450 \div 300 = 1.5\)

**Review capacity.** Remind students that the amount of liquid (or sand, rice, or anything else that can be poured) a container can hold is called the capacity of the container. Capacity is often measured in litres and millilitres. Remind students that, when they found the volume of boxes, they found that volume in cubic units, such as cubic centimetres. If available, show students a rectangular aquarium. Explain that we can find the volume of the inside of the aquarium in cubic units. But we also can pour water into it and find its capacity—or the volume of the liquid the aquarium can hold—in millilitres or litres. Explain that there is a connection between these two types of units.
Review decimetres and cubic decimetres. Remind students that 1 decimetre is a unit of length equal to 10 centimetres.

A cube that has sides of 1 decimetre (1 dm) has a volume of 1 dm$^3$. Show students a cube with sides 1 dm (e.g., a thousands block) and explain that this is a cubic decimetre. Write on the board:

\[
1 \text{ dm} = 10 \text{ cm} \\
1 \text{ dm} \times 1 \text{ dm} \times 1 \text{ dm} = 1 \text{ dm}^3
\]

Introduce the connection between dm$^3$ and litres. If available, show students a large graduated pitcher with water in it so that at least 1 L can be added to it. Have a volunteer say how much water is in the pitcher. (say, 2 L) Place the 1 dm cube into the pitcher and have students observe that the water level rises. Explain that we say that the cube displaces some water in the pitcher—in other words, it takes the place of the water. The volume of water the cube displaces equals the volume of the cube. Remind students that they used this method for finding the volume of some fruit in Lesson ME6-21: Using Volume Formulas. ASK: What is the total volume of the liquid and the cube in the pitcher? Have a volunteer check. (3 L) ASK: How much water did the cube displace? (1 L) What is the volume of the cube in litres? (1 L) Write “1 dm$^3$ = 1 L” on the board.

Point out that both cubic decimetres and litres are measurements of volume; however, cubic decimetres are metric units and litres are equal to the metric units. The volume of liquids and capacity are usually measured in litres, and geometric volume is usually measured in cubic units.

Introduce the connection between cm$^3$ and mL. Remind students that a millilitre is one thousandth of a litre, so there are 1000 mL in a litre. Write “1 L = 1000 mL” on the board.

Draw a cube on the board and mark its length, width, and height as 1 dm = 10 cm. Ask students to find the volume of the cube in cubic centimetres. Write “1 dm$^3$ = 10 cm $\times$ 10 cm $\times$ 10 cm = 1000 cm$^3$” on the board.

Write on the board:

\[
\begin{align*}
\text{equal} & \\
1 \text{ dm}^3 & = 1000 \text{ cm}^3 \\
1 \text{ L} & = 1000 \text{ mL}
\end{align*}
\]

ASK: How many millilitres are in 1 cubic centimetre? (1) How do you know? (1000 mL = 1000 cm$^3$, so 1 mL and 1 cm$^3$ should be the same thing)

Finding volume of an aquarium. Draw a prism on the board, and mark the sides as 5 dm, 6 dm, and 4 dm. Ask students to find the volume of the prism in cubic decimetres. (5 dm $\times$ 6 dm $\times$ 4 dm = 120 dm$^3$) Then tell them that this prism is an aquarium, and ask them how much water will fit into the aquarium. (120 L)

Ask students to convert the dimensions of the prism into centimetres and find the volume in cubic centimetres and in millilitres. (50 cm $\times$ 60 cm $\times$ 40 cm = 120 000 cm$^3$ = 120 000 mL) Do the answers match? (yes, 120 L = 120 000 mL)
Invite a volunteer to measure the inner dimensions of the rectangular aquarium you showed students at the beginning of the lesson. Ask them to find the volume of the aquarium in cubic centimetres. Then ASK: How many millilitres of water will fit into the aquarium? (the number of millilitres will be the same number as the volume in cm³)

Pour several litres of water into the aquarium using a pitcher. Have a volunteer use a ruler to measure the height in centimetres the water reaches, and have students find the volume of water in the aquarium in cubic centimetres. Again, have students convert the answer and the initial amount of water poured into the aquarium to millilitres. ASK: Are the answers close? The answer is likely not to be exactly the same, so discuss why there might be a difference. (The measurements are only approximations. The exact height might have been, say, 12.3 cm, which we rounded to 12 cm. The same applies to length and width.)

Finding the height of water. Review finding the missing dimension when given the volume of a prism. For example, if a prism has a volume of 120 cm³, and it is 10 cm long and 6 cm wide, what is its height? Remind students of the formula Volume = length × width × height and that length and width give the area of the bottom or the top face. So, in the example, 10 cm × 6 cm = 60 cm², the height is 120 cm³ ÷ 60 cm² = 2 cm.

Draw another prism on the board and mark the length and the width as 75 cm and 40 cm. Tell students that this is an aquarium, and you want to pour, say, three pails equal to 30 litres of water into it. How high do you think the water will be? Have students think how they can find the height and discuss the potential solution in pairs. Go through the solution as a class:

- Volume of water = 30 L = 30 000 mL = 30 000 cm³
- Area of bottom face = 75 cm × 40 cm = 3000 cm²
- Height = 30 000 cm³ ÷ 3000 cm² = 10 cm

Point out that this is very little water for such an aquarium—it is only 10 cm of water, and aquariums usually have water levels much higher than that!

Repeat the exercise with an aquarium of the size you might see, say, in a doctor’s office. For example: an aquarium measures 1.5 m long, 60 cm wide, and contains 819 L of water (including sand, decorations, and fish). What is the height of the water? This time students will need to convert 1.5 m to centimetres. (height = 91 cm)

Exercise: An aquarium that is 1.4 m long and 50 cm wide contains 770 L of water including sand, decorations, and fish. How high is the water in the aquarium?

Answer: 110 cm

Using displacement of water to find volume. Explain that displacement allows you to find the volume of objects that are not rectangular prisms. Pour some water into a graduated pitcher and have a volunteer check how much water is in the pitcher. Place an object with a complicated shape, such as a toy, into the water and have another volunteer check the level of
the water. SAY: The water level now shows the volume of the water plus the volume of the toy. How can we find the volume of the toy? Have students write the subtraction equation and find the volume of the toy.

**ACTIVITY (Optional)**

Give each student a small toy and a small graduated pitcher. Ask students use displacement to find the volume of the toy. Then have students form pairs, and exchange toys with their partner. Have students find the volume of their partner’s toy, and then compare answers.

**Using estimation in word problems.** SAY: I have a splash pool in my backyard. It is a square, 1.28 m long and wide, and I can fill it up to 40 cm. Write on the board:

Pool length: 1.28 m  width: 1.28 m  height: 40 cm

ASK: How much water does the pool hold? How can we estimate? Accept suggestions. PROMPT: Can you multiply 1.28 m by 40 cm? (no) Why not? (the units are not the same) Is it easy to multiply 128 cm by 40 cm? What can we do to make it easier? (round the numbers) Have students convert the measurements to centimetres and estimate the product. (130 cm × 130 cm × 40 cm = 676 000 cm³) ASK: How many millilitres is that? (676 000 mL) How many litres is that? (676 L) How do you know? (1 L = 1000 mL, so 676 thousands of millilitres equals 676 litres)

SAY: I have an aquarium at home, too. The aquarium is 59 cm long, 32.5 cm wide, and I fill it to a height of 38 cm. Record the dimensions on the board and have students estimate the volume of the water in the aquarium and convert the answer to litres. (60 cm × 30 cm × 40 cm = 72 000 cm³, so the capacity is about 72 000 mL = 72 L)

SAY: I use the same hose to fill the aquarium and the splash pool. It takes me about 4 minutes to fill the aquarium. About how much time should it take me to fill the pool? Have students explain how they can find the answer and calculate the estimate. (sample answer: 700 L ÷ 70 L = 10, so the pool is about 10 times as large as the aquarium, so it will take about 4 minutes × 10 = 40 minutes to fill the pool)

Have students calculate the exact answers to all three questions, rounding the quotient to the nearest whole number before calculating the time.

(Capacity of pool: 655 360 mL, capacity of aquarium: 72 865 mL, the pool is about 9 times as large as the aquarium, so filling the pool will take about 36 minutes)

**Exercise:** A rectangular pail is 33 cm long, 22 cm wide, and 47 cm tall. A rectangular bathtub is 1.2 m long, 0.9 m wide, and can be filled to a height of 48 cm. How many times do you need to empty the pail into the bathtub to fill it? Estimate, then calculate the exact answer to the nearest whole number.

**Answer:** Sample estimate: capacity of pail: 30 L, capacity of bathtub: 540 L, 540 ÷ 30 = 18 times; Exact calculation: capacity of pail 34 122 mL, capacity of bathtub: 518 400 mL, 518 400 ÷ 34 122 ≈ 15 times
Extensions

1. Remind students that when adding, they round one number up and another down to balance the mistakes in estimation. Point out that a similar method might work well in multiplication, especially if you are not changing both numbers by much. For example, when multiplying $128 \times 128 \times 40$, it makes sense to increase one factor to 130 and decrease another factor. However, rounding 128 all the way down to 120 might be too much. If you round the number to just 125, you might get a closer answer. In particular, 125 would work quite well because it is an easy number to multiply by 40: doubling 125 twice and halving 40 twice allows you to multiply by 10. Have students compare the methods of estimation below.

   a) Estimate $128 \times 128 \times 40$ by rounding to the nearest ten.
   b) Estimate $128 \times 128 \times 40$ by rounding one number up and another down to the nearest ten.
   c) Estimate $128 \times 128 \times 40$ by rounding one number up and another down to the nearest five.
   d) Calculate the exact answer. Which estimate was the closest?

   **Answers:** a) 676 000, b) 624 000, c) 650 000, d) 655 360, rounding one number up and another down to the nearest five

2. Tom measured the dimensions of a 2 L juice carton and calculated the volume to be 200 cm$^3$. Is his answer correct? Explain how you know.

   **Answer:** No; 2 L = 2000 mL = 2000 cm$^3$, so Tom’s answer is incorrect.

3. Explore cubic kilometres.

   a) How many cubic metres are in 1 km$^3$?
   b) The volume of water in Lake Erie is 484 km$^3$. Find the volume of Lake Erie in cubic metres and in litres. How many zeroes are in each of these numbers?
   c) The Commerce Court West tower in Toronto, is almost a rectangular prism 239 m tall, 94 m long, and 51 m wide. What is the volume of the tower? Round your answer to the nearest hundred thousand cubic metres.
   d) Estimate the volume of Lake Erie measured in Commerce Court West towers. Use your answer from part c) to estimate.

   **Answers**
   a) 1 000 000 000 m$^3$
   b) $484 \text{ km}^3 = 484 000 000 000 \text{ m}^3 = 484 000 000 000 000 \text{ L}$; 9 zeroes for m$^3$; 12 zeroes for L
   c) tower volume = 1 145 766 m$^3$ ≈ 1 100 000 m$^3$
   d) about 440 000 Commerce Court West
ME6-24 Nets of Prisms

Pages 180–181

Goals
Students will draw nets for rectangular and triangular prisms.

PRIOR KNOWLEDGE REQUIRED
Can draw squares, rectangles, and triangles of given dimensions
Can identify congruent rectangles and triangles

MATERIALS
box labelled with the names of the faces (top face, right side face, etc.)
boxes (e.g., medication boxes)
et cut out from BLM Nets of 3-D Shapes (2) (p. S-67)
ruler
grid paper or BLM 1 cm Grid Paper (p. T-1)
BLM Nets of 3-D Shapes (1), (3), and (4) (pp. S-66, 68, 69)
scissors
 glue

Review names of faces. Display a box with faces labelled “top face,”
“right-side face,” “front face,” “left-side face,” and so on. Review the names
of the faces. Then display a different box, point to different faces, and have
students name them.

Introduce nets. SAY: A net of a 3-D shape is a pattern that can be folded
to make the shape. Show a cut-out from BLM Nets of 3-D Shapes (2) and
show how it folds to produce a prism.

Use the box with the face labels to demonstrate how to make a net of it by
tracing the bottom face onto paper, then rolling the box so that the front
face can be traced onto the paper adjacent to the first face, then the top
face, and then the back face. Then roll the prism to the sides and trace
the side faces, so that each side face attaches to a bottom, front, top, or
back face. Point out that you could attach the side faces at different places,
depending on the face the prism was standing on before you rolled it
sideways. Also, point out that both side faces do not have to be attached to
the same face. For example, all three nets below fold into the same box.
ACTIVITY 1 (Essential), ACTIVITY 2 (Optional)

1. **Producing a net for a rectangular prism.** Give each student a different box and have them produce a net the same way you produced the net for a box. Have them fold the net and check that it creates a copy of the box, and then unfold it.

2. Have students assemble in groups of four, exchange boxes, and mix up their nets. Have students find the net for the box that they now have. Discuss matching strategies. For example, students can try to identify the number of square faces the box has, and look for a net with the same number of square faces.

**Labelling faces on a net.** Return to the box you used earlier. Place the box on the net so that the bottom face touches the net, and label the faces on the net following the order you traced them in, as shown in the margin. Have students label the faces on the nets they produced. Discuss how some faces are congruent—for example, the top face and the bottom face are exactly the same. Which other faces are congruent? (back and front, left side and right side)

On the board, sketch a box as shown in the margin. ASK: What are the dimensions of the top face? (6 m by 4 m) Have students sketch a rectangle that is 6 squares long and 4 squares wide on grid paper and label it “top face.” ASK: What faces touch the top face on the prism? (front, back, and both side faces) What face does not touch the top face? (bottom face) If I now draw an identical rectangle right beside the top face and label it “bottom face,” would that be correct? (no) I want to draw the front face now. Where should I attach it? (to the top or to the bottom edge of the top face) What should the length of the front face rectangle be? (6 squares) The width? (2 squares) How do you know? (The front face dimensions are the length and the height of the prism.) Continue with the other faces.

**Exercises:** On grid paper, draw a net for the prism and label the faces. Use 1 square of the grid for 1 cm².

a) ![Net](image1.png)

b) ![Net](image2.png)

**Sample answers**

a) ![Labelled Net](image3.png)

b) ![Labelled Net](image4.png)
Drawing nets of triangular prisms. Hold up a triangular prism, such as one of the prisms constructed from nets on BLM Nets of 3-D Shapes (1). Ask students to identify the shape. SAY: Rectangular prisms have 6 faces, all of them rectangles. ASK: How many faces do triangular prisms have? (5) What shapes are the faces? (2 triangles, 3 rectangles) Are any of the faces congruent? (yes, the triangles) Are any of the rectangular faces congruent? (no)

Repeat creating the net by placing the prism onto one of the side faces, tracing it, then rolling the prism to a different side face and tracing again. Finally, attach the bases. Point out how the base should be rotated if two bases are attached to different side faces on the net. See pictures below for reference. Point out that if the bases are attached to the same side face, they are reflections of each other.

ACTIVITY 3 (Essential), ACTIVITY 4 (Optional)

3. Give each student a net of a triangular prism from BLM Nets of 3-D Shapes (1), (3), and (4). Have students cut out the nets, and then fold and glue them into prisms. Have students exchange prisms with partners and produce a net for the prism they received. Give each student a different box and have them produce a net the same way you produced the net for a box. Have them fold the net and check that it creates a copy of the box, and then unfold it.

4. Repeat Activity 2 using triangular prisms and nets produced in Activity 3.

Extensions

1. a) Will the net will fold into a cube?
   
   i) ![Net 1](image1)
   
   ii) ![Net 2](image2)

   iii) ![Net 3](image3)

   iv) ![Net 4](image4)

   b) Use six squares to make a different net for a cube.
c) Use six squares to make a different figure that will not be a net for a cube.

**Answers:** a) i) no, ii) yes, iii) yes, iv) no

**Sample answers**

b) ![Net Example](image1)

c) ![Net Example](image2)

2. Explain why the picture is not a net of a triangular prism.

a) ![Triangular Prism Net Example](image3)

b) ![Triangular Prism Net Example](image4)

c) ![Triangular Prism Net Example](image5)

d) ![Triangular Prism Net Example](image6)

**Sample answers:** a) the bases are not congruent; b) a side face is missing; c) the triangles in the bases are not equilateral, but the rectangles are congruent, so the rectangle in the middle needs to be wider; d) the triangle on the bottom points the wrong way, the longer side of the triangle will not be glued to the longest rectangle

3. Copy the net in the margin to grid paper.

a) What shape would you make if you fold the net?

b) Which line segment joins the line segment $AB$ on the folded net?

c) Which line segment joins the line segment $CH$?

d) Which vertex joins vertex $F$?

e) Which vertices join vertex $M$?

**Answers:** a) cube, b) $FG$, c) $GL$, d) $B$, e) $H$, $L
Goals

Students will use nets to find the surface area of rectangular and triangular prisms.

PRIOR KNOWLEDGE REQUIRED

Is familiar with square units of measurement
Can draw nets for rectangular and triangular prisms
Can find the area of rectangles and triangles

MATERIALS

medium-sized cereal box
permanent marker
overhead projector
transparency of BLM 1 cm Grid Paper (p. T-1)
grid paper or BLM 1 cm Grid Paper (p. T-1)
ruler

NOTE: Before class, measure the length, width, and height of a medium-sized cereal box and write the measurements in permanent marker near the edges in large print. Label the faces with the terms “front,” “back,” “right side,” “left side,” “top,” and “bottom.” In the sample answers for this lesson, we use a box with dimensions 24 cm by 7.5 cm by 34 cm.

Mental math minute—number talk. Present the problem: \(3.6 \times 25\). (90)
The following strategies could arise:

\[
\begin{align*}
1.8 \times 50 &= 0.9 \times 100 \\
3.6 \times 20 + 3.6 \times 5 &= 3 \times 25 + 0.6 \times 25 \\
3 \times 25 + 2 \times (3 \times 25) \div 10 &= 
\end{align*}
\]

Introduce surface area. Show students a labelled cereal box. SAY: Suppose a factory that makes cereal wants to redesign its cereal package. They want to know how much cardboard they would need to make the box. To do that, they need to know the area of each face of the box, and then add the numbers. This calculation is called finding the surface area of the box.

Surface area is the total area of all the surfaces, or faces, of a 3-D shape. You might want to know the surface area to figure out how much wrapping paper you would need to cover a box or how much paint you would need to paint a box.

Point out that the term “area” is used for 2-D shapes, so when we talk about 3-D shapes, we say surface area. ASK: What units do you think we use to measure surface area? (square units) PROMPT: What units do we use to measure area?
Labelling the faces on the net of a rectangular prism. Invite a volunteer to draw a net of the cereal box on the board by tracing its faces and rolling the box. Point to different faces of the box, and have students tell you what they are called. (front face, back face, etc.) Then SAY: I want to label these faces on the net. Point to the front face of the box and ASK: Which rectangle on the net could be the front face? Point to different rectangles and have students signal yes or no. Label one of the faces as the front face. Repeat with other faces until all faces are labelled. Keep the net on the board.

Discuss strategies to determine which face is which on the net. For example, the front face of the cereal box is the largest face, so it should correspond to one of the two largest rectangles. The right-side face and the left-side face are both long thin rectangles, with one of the sides being the longest dimension of the prism, and the other side being the shortest dimension. Therefore we need to choose one of the two longest and narrowest rectangles.

Exercise: Draw the net on grid paper and label the faces. Use 1 grid square for 1 m².

![Net of a rectangular prism](image)

Answer: 

Labelling the dimensions on the net of a rectangular prism. Hold up the same cereal box and SAY: I would like to label the length, width, and height of the prism on the net. The longest side of the box is 34 cm long. Which edges on the net are 34 cm long? Point to different edges of the net on the board and have students signal yes or no using thumbs up or thumbs down. Label the correct edges, then repeat with the other two dimensions. The picture should be similar to the picture below. Keep the picture on the board.

![Dimensions of a rectangular prism](image)
Exercises: Copy the net of the rectangular prism on grid paper. On the net, label each face and write the length of each edge.

a) 

3 cm
2 cm
1 cm

b) 

1 cm
1 cm

Answers: a) 

left front right 2 cm
bottom back 2 cm
top 1 cm
3 cm

Using the net to find the surface area of a rectangular prism. SAY: The net of the cereal box shows the shape of every face or surface that makes the cereal box. So we can use the net to find the surface area of the box. Next to the net of the cereal box, write on the board:

Front:
Back:
Right side:
Left side:
Top:
Bottom:

NOTE: If students are using calculators to find the surface area of the cereal box, remind them to estimate the answer first, to help spot any mistakes. For example, when multiplying $34 \times 7.5$, they should expect an answer that is close to 300, not 30 and not 3000.

ASK: What is the area of the front face of the box? ($816 \text{ cm}^2$) Write the calculation on the board:

Front: $34 \text{ cm} \times 24 \text{ cm} = 816 \text{ cm}^2$

Exercises: Write a multiplication equation for the area of each face of the cereal box.

Answers: Front: $34 \text{ cm} \times 24 \text{ cm} = 816 \text{ cm}^2$
Back: $34 \text{ cm} \times 24 \text{ cm} = 816 \text{ cm}^2$
Right side: $7.5 \text{ cm} \times 34 \text{ cm} = 255 \text{ cm}^2$
Left side: $7.5 \text{ cm} \times 34 \text{ cm} = 255 \text{ cm}^2$
Top: $24 \text{ cm} \times 7.5 \text{ cm} = 180 \text{ cm}^2$
Bottom: $24 \text{ cm} \times 7.5 \text{ cm} = 180 \text{ cm}^2$
ASK: If surface area is the total area of all faces, then what is the surface area of this prism? (2502 cm²) Write on the board:

Surface area = 2502 cm²

**Exercises:** Using the rectangular prism nets you labelled in one of the previous exercises, write a multiplication equation for the area of each face of the prism. Then find the surface area.

**Answers**
a) Front: 3 cm × 2 cm = 6 cm²  
Back: 3 cm × 2 cm = 6 cm²  
Right side: 1 cm × 2 cm = 2 cm²  
Left side: 1 cm × 2 cm = 2 cm²  
Top: 3 cm × 1 cm = 3 cm²  
Bottom: 3 cm × 1 cm = 3 cm²  
Surface area = 22 cm²  
b) All faces: 1 cm × 1 cm = 1 cm²  
Surface area = 6 cm²

**Drawing the net of a triangular prism.** Draw the picture shown in the margin on the board. ASK: What type of prism is this? (triangular) How many faces does a triangular prism have? (5) What shapes are the faces? (2 triangles and 3 rectangles) How many different sizes of faces are there? (the 2 triangles are congruent and the 3 rectangles are all different)

On the board, draw the parts of the net one by one. Ask students to tell you what dimensions to make each face, and mark the dimensions on the face. The final picture is shown below:

SAY: The names for the faces of a triangular prism are less specific than for a rectangular prism: we’ll just call them “bases” and “side faces.”

ASK: Which shape are the bases? (triangles) the side faces? (rectangles)

Label the net on the board as shown below:

Leave this net displayed on the board for reference in later exercises.
Exercises: Copy the net of the triangular prism on grid paper. On the net, label each face and write the length of each edge. Use one grid square for 1 m².

![Net of a triangular prism](image)

Using the net to find the surface area of a triangular prism. Refer to the net on the board. List the faces on the board, starting with the bases, in preparation for calculating the surface area. SAY: The base is a triangle. ASK: How do we calculate the area of a triangle? (base × height ÷ 2) What is the area of the base? (6 cm²) Write on the board:

Base 1: 3 cm × 4 cm ÷ 2 = 6 cm².

Exercises: Write a multiplication equation for the area of each face of the prism.

Answers
- Base 1: 3 cm × 4 cm ÷ 2 = 6 cm²
- Base 2: 3 cm × 4 cm ÷ 2 = 6 cm²
- Face 1: 10 cm × 3 cm = 30 cm²
- Face 2: 10 cm × 4 cm = 40 cm²
- Face 3: 10 cm × 5 cm = 50 cm²

ASK: What is the surface area of this prism? (132 cm²) Write on the board:

Surface area of prism = 132 cm²

Exercises: Using the net you drew in the previous exercises, write a multiplication equation for the area of each face of the prism. Find the surface area.

Answers
- Base 1: 8 m × 6 m ÷ 2 = 24 m²
- Base 2: 8 m × 6 m ÷ 2 = 24 m²
- Face 1: 8 m × 3 m = 24 m²
- Face 2: 6 m × 3 m = 18 m²
- Face 3: 10 m × 3 m = 30 m²
- Surface area = 120 m²
ASK: How does drawing the net help you find the surface area of a prism? (it shows you all the faces of a prism so you can find each area, then you add the areas to find the surface area)

Extensions

1. You don’t have to draw a net to find the surface area of a shape. You can just draw the faces separately.
   a) The diagram shows all the faces of a 3-D shape. Name the shape.
      i) [Diagram of rectangular prism]
      ii) [Diagram of triangular prism]
      iii) [Diagram of cube]
   Answers: i) rectangular prism, ii) triangular prism, iii) cube
   b) How many of each face do you need to make the prism? Write the number on the face.
      i) [Diagram with measurements]
      ii) [Diagram with measurements]
   Answers: i) two of each rectangle, ii) two triangles, two 2 by 3 rectangles, one 2.8 by 3 rectangle
   c) Use your answers from part b) to fill in the blanks. Find the surface area of the prism.
      i) \[ SA = \underline{} \times (5 \text{ mm} \times 3 \text{ mm}) + \underline{} \times (2 \text{ mm} \times 3 \text{ mm}) + \underline{} \times (2 \text{ mm} \times 5 \text{ mm}) = \underline{} \]
      ii) \[ SA = \underline{} \times (2 \text{ m} \times 2 \text{ m} \div 2) + \underline{} \times (2 \text{ m} \times 3 \text{ m}) + \underline{} \times (2.8 \text{ m} \times 3 \text{ m}) = \underline{} \]
   Answers: i) 2, 2, 2, 62 mm², ii) 2, 2, 1, 24.4 m²
d) Find the surface area using as few calculations as you can. 
Hint: Draw the faces.

Answers
i) \( SA = 6 \times (2 \text{ cm} \times 2 \text{ cm}) = 24 \text{ cm}^2 \)
ii) \( SA = 2 \times (23 \text{ cm} \times 23 \text{ cm}) + 4 \times (23 \text{ cm} \times 46 \text{ cm}) = 5290 \text{ cm}^2 \)
iii) \( SA = 2 \times (3 \text{ m} \times 3 \text{ m} \div 2) + 2 \times (3 \text{ m} \times 7 \text{ m}) + (4.2 \text{ m} \times 7 \text{ m}) = 80.4 \text{ m}^2 \)

2. A poor king ordered a new treasure chest. The old chest is 15 cm long, 12 cm wide, and 10 cm high. The king asked four carpenters to each make a chest. All chests have a flat lid. Each carpenter changed the dimensions:

1st carpenter: twice as long, but the same width and height
2nd carpenter: twice as high, but the same width and length
3rd carpenter: twice as long and twice as wide, but half the height
4th carpenter: twice as wide, but the same length and height

a) Which carpenter made the chest with the smallest surface area?

b) Can you make a chest with the same volume as the other carpenters, dimensions in whole numbers of centimetres, and smaller surface area than all four carpenters?

Answer
a) 1st carpenter: length 30 cm, width 12 cm, height 10 cm, 
surface area = 1560 \text{ cm}^2; 
2nd carpenter: length 15 cm, width 12 cm, height 20 cm, 
surface area = 1440 \text{ cm}^2; 
3rd carpenter: length 30 cm, width 24 cm, height 5 cm, 
surface area = 1980 \text{ cm}^2; 
4th carpenter: length 15 cm, width 24 cm, height 10 cm, 
surface area = 1500 \text{ cm}^2, so the 2nd carpenter made the chest with the smallest surface area.

Sample answer: b) volume = 3600 \text{ cm}^3, length = width = 15 cm, 
height = 16 cm, surface area = 1410 \text{ cm}^2
Goals
Students will solve problems connected to volume, capacity, and surface area of rectangular and triangular prisms.

PRIOR KNOWLEDGE REQUIRED
- Can find the volume and capacity of a prism
- Can convert larger units to smaller units
- Can convert between units of volume and capacity
- Can draw nets for rectangular and triangular prisms
- Can find surface area of rectangular and triangular prisms

MATERIALS
- box with faces labelled as front, left side, top, etc.
- overhead projector
- transparency of 1 cm Grid Paper (p. T-1)
- grid paper or 1 cm Grid Paper (p. T-1)
- ruler

Mental math minute—number talk. Present the problem: Find 75% of 164.

(123) The following strategies could arise:

- \[ 75\% = 0.75, \text{ so find } 0.75 \times 164 = 1.5 \times 82 = 3 \times 41 \]
- \[ 75\% = \frac{3}{4}, \text{ so find } \frac{3}{4} \text{ of } 164: \ (164 \div 4) \times 3 = 41 \times 3 \]
- \[ 75\% = 100\% - 25\%, \text{ 25\% of } 164 = 164 \div 4 = 41, \text{ so } 164 - 41 \]
- \[ 164 \times 75 \div 100 = 82 \times 150 \div 100 = 41 \times 300 \div 100 = 41 \times 3 \]
- \[ 1\% \text{ of } 164 = 1.64, \text{ find } 1.64 \times 75 \]

Review nets. Show a rectangular prism with labelled faces. Sketch a net for it on the board. Place the prism with the bottom face down, and point to different faces on the net. Have students signal thumbs up if this is one of the horizontal faces and thumbs down if it is not. Label the top and the bottom faces on the net.

Identifying dimensions from the net. ASK: Which dimensions do you multiply to get the area of a horizontal face? (length and width) Point to different edges on the net on the board in turn, and ask students to signal thumbs up if the edge shows the length and thumbs down if it does not. Repeat with width. Finally, label length and width on the sides of the bottom and top faces of the net.

ASK: Which edges on the net show the height of the prism? Point to edges one by one and have students signal thumbs up if they show the height and thumbs down if they do not.

Now show a net on a grid on the board, with the height, length, and width all different measurements. Label one of the faces as the bottom face and have students identify the length, width, and height of the prism. Finally, ask them to find the volume of the prism.
Exercises: Find the length, width, and height of the prism. Then find the volume of the prism. Each square on the grid represents 1 cm².

a) 
\begin{array}{c}
\text{bottom face} \\
\end{array}

\text{length: } 5 \text{ cm, width: } 3 \text{ cm, height: } 2 \text{ cm, volume: } 30 \text{ cm}^3

b) 
\begin{array}{c}
\text{bottom face} \\
\end{array}

\text{length: } 6 \text{ cm, width: } 4 \text{ cm, height: } 1 \text{ cm, volume: } 24 \text{ cm}^3

Finding volume and surface area given a net without one face. Draw a net, as shown in the margin, on a grid on the board, or a projection of BLM 1 cm Grid Paper. Explain that this is a net of a box without a lid, and that each square on the grid is 5 cm long. ASK: Which face is the bottom face? How do you know? (the face in the centre is the only face that has no match, so it is the bottom face)

ASK: When you know which face is the bottom face, how can you tell what the length and the width of the box are? (the dimensions of the bottom face are the length and the width of the whole prism) What are the length and the width of this box? (4 squares by 3 squares, so 20 cm by 15 cm) What is the height of the prism? (10 cm) Have a volunteer indicate an edge of the net that shows the height of the prism. Have students find the volume of the prism. (20 cm \times 15 cm \times 10 cm = 3000 cm³)

Exercises: Use the net to find the dimensions of a box without a lid. Then find the volume. Each square on the grid is 10 cm long.

a) 
\begin{array}{c}
\text{bottom face} \\
\end{array}

\text{length: } 60 \text{ cm, width: } 40 \text{ cm, height: } 20 \text{ cm, volume: } 48000 \text{ cm}^3

b) 
\begin{array}{c}
\text{bottom face} \\
\end{array}

\text{length: } 40 \text{ cm, width: } 20 \text{ cm, height: } 40 \text{ cm, volume: } 32000 \text{ cm}^3

Finding the volume and surface area of prisms with dimensions in different units. On the board, sketch the prism shown in the margin. Discuss with students how to find the volume of the prism. Lead them to the idea of converting all measurements to centimetres before finding...
the volume and the surface area. Emphasize how writing the units when substituting the measurements in the formula helps avoid multiplying different units, such as metres and centimetres.

Length = 125 cm
Width = 100 cm
Height = 70 cm
Volume = \( l \times w \times h = 875 \, 000 \, \text{cm}^3 \)
Surface area = 56 500 cm²

Keep the picture and the solution on the board.

**Exercises:** Find the volume and the surface area of the 3-D shape.

a) 

\[ \begin{array}{ccc}
\text{0.5 cm} & \text{3 mm} & \text{1.2 cm} \\
\end{array} \]

b) 

\[ \begin{array}{ccc}
\text{35 cm} & \text{60 cm} & \text{1.2 m} \\
\end{array} \]

c) 

\[ \begin{array}{ccc}
\text{30 cm} & \text{0.75 m} & \text{16 cm} \\
\end{array} \]

**Answers**

a) volume = 180 mm³, surface area = 222 mm²
b) volume = 252 000 cm³, surface area = 27 000 cm²
c) volume = 18 000 cm³, surface area = 6480 cm²

**Finding capacity of prisms with dimensions in different units.** Return to the prism on the board with dimensions in different units. SAY: Suppose this prism is an aquarium. I want to know the capacity of this aquarium. ASK: What do you know about the connection between millilitres and cubic centimetres? (capacity of 1 cm³ is 1 mL) If the volume of the aquarium is 875 000 cm³, what is the capacity? (875 000 mL) How many litres is that? (875 L) How do you know? (1 L = 1000 mL, so the number of thousands in the number of millilitres is the number of litres)

**Exercises:** Use the prisms in parts b) and c) in the previous exercises. Find the capacity of the prisms in millilitres and in litres.

**Answers:** b) 252 000 mL = 252 L, c) 18 000 mL = 18 L

**Finding the missing height given the volume.** SAY: I’ve used 800 L of water to fill the aquarium. ASK: What height does the water reach in the aquarium? Have students explain how to find the answer. (the area of the bottom face of the aquarium is 125 cm × 100 cm = 12 500 cm², and the volume of the water is 800 000 cm³, so the height of the water is 800 000 cm³ ÷ 12 500 cm² = 64 cm)

**Exercise:** An aquarium is a rectangular prism that is 1.2 m long and 40 cm wide, with the height of 80 cm. The volume of the water, fish, and decorations is 360 L. What height does the water reach?

**Answer:** 75 cm
Refer students to the aquarium on the board again. SAY: I am going to keep freshwater angelfish in the aquarium. I checked in the store and they say that each fish needs at least 25 L of water. ASK: How many fish can I hold in my aquarium, if I fill it to 90% of the height of the aquarium? Again, solve the problem as a class. Discuss rounding: it makes sense to round the volume of the aquarium to the nearest thousand to convert to litres to make the calculation easier, and you need to round down so that each fish gets no less than 25 L. (90% of 70 cm = 63 cm height of water; volume of water in the aquarium = area of bottom face × height of water, so 12 500 cm² × 63 cm = 787 500 cm³ ≈ 787 L, so you can have 787 ÷ 25 ≈ 31 fish)

Exercise: How many freshwater angelfish can you put in the aquarium in the previous exercise, if every fish needs at least 25 L of water? You can ignore the volume of the decorations and the fish itself.

Answer: 14 fish

Extensions

1. A skyscraper is a rectangular prism with all floors exactly the same size. The volume of the skyscraper is 588 000 m³, and it is 280 m tall. The distance between the floors of the skyscraper (from the ceiling of one floor to the ceiling of the floor above it) is 4 m.

   a) How many floors tall is the skyscraper?
   
   b) What is the total area of the floors of the skyscraper?
   
   c) Carpet needs to be installed on half of the total area of the floors of the skyscraper. Installing carpet costs $20 per square metre. How much will installing the carpet in the whole building cost?

   Answers: a) 70 floors; b) Each floor has area 588 000 m³ ÷ 280 m = 2100 m², so the total area is 2100 m² × 70 = 147 000 m²; c) $1 470 000

2. a) Find the volume of a rectangular prism 1.25 m long, 28 cm wide, and 25 cm tall.
   
   b) To make a different rectangular prism with the same volume, but height 50 cm, what could the other two dimensions be?

   Answer: a) 87 500 cm³
   
   Sample answer: b) 1.25 m, 14 cm

3. Students who can multiply decimals by decimals can find the volume and the surface area of the prism shown in the margin. Challenge students who do not know how to multiply decimals by decimals to use strategies for mental multiplication to simplify the product to use only whole numbers.

   NOTE: Another potential strategy students could use is to convert the measurements to millimetres.

   Answer: volume = 1700 cm³, surface area = 884.5 cm²
4. A wealthy queen has a treasure chest as shown in the picture in the margin. The chest is a rectangular prism 55 cm long, 40 cm wide, and 20 cm high, with no top face. The lid of the chest is a triangular prism, with no bottom face. The total height of the chest and the lid is 54.6 cm.

a) The chest is filled with treasure to the base of the lid. What is the volume of the treasure?

b) What is the volume inside the chest, including the lid?

c) What is the surface area of the chest?

Solutions

a) \(55 \text{ cm} \times 40 \text{ cm} \times 20 \text{ cm} = 44000 \text{ cm}^3\)

b) height of lid: \(54.6 \text{ cm} - 20 \text{ cm} = 34.6 \text{ cm}\)

volume of lid: \((40 \text{ cm} \times 34.6 \text{ cm} \div 2) \times 55 \text{ cm} = 692 \text{ cm}^2 \times 55 \text{ cm} = 38060 \text{ cm}^3\), so total volume = \(44000 \text{ cm}^3 + 38060 \text{ cm}^3 = 82060 \text{ cm}^3\)

c) surface area of the chest (only one horizontal face is counted):

\[
(55 \text{ cm} \times 40 \text{ cm}) + 2 \times (55 \text{ cm} \times 20 \text{ cm}) + 2 \times (20 \text{ cm} \times 40 \text{ cm}) = 2200 \text{ cm}^2 + 2200 \text{ cm}^2 + 1600 \text{ cm}^2 = 6000 \text{ cm}^2,
\]

d) surface area of the lid (only two side faces are counted):

\[
2(40 \text{ cm} \times 34.6 \text{ cm} \div 2) + 2(55 \text{ cm} \times 40 \text{ cm}) = 1384 \text{ cm}^2 + 4400 \text{ cm}^2 = 5784 \text{ cm}^2,
\]

so total surface area = \(11784 \text{ cm}^2\)

5. When answering the question, round your answer up to the nearest tenth (Example: 2.322 is rounded to 2.4). Why does it make sense to round the answer up?

a) One litre of paint covers 7 m\(^2\). How much paint would it take to paint a wall that is 6 m by 3 m? What if the wall has a door measuring 2 m high and 80 cm wide that is not being painted? What if the wall also has a window that measures 1 m by 1 m?

b) I bought a standalone closet at a garage sale. The closet is 1 m deep, 2 m wide, and 2.5 m high. I want to paint the outside of the closet—the walls, the door, and the top—but not the bottom. How much paint do I need?

c) Choose a room in your school or at home. Calculate the amount of paint needed to repaint the room. Consider all surfaces and fixtures, such as doors, windows, closets, electrical outlets, built-in shelves, or ledges. What will you paint and what does not need to be painted? Do you want to use more than one colour? Will you need more than one coat of paint? (You will if you are using a dark colour or painting over a dark colour.)

Selected answers: a) whole wall: 2.6 L, without the door: 2.4 L, without the door and window: 2.2 L; It makes sense to round answers up, because if you round down, you might not have enough paint; b) 2.5 L
**Goals**

Students will draw structures on isometric dot paper.
Students will construct structures based on pictures on isometric dot paper.
Students will draw top views and front views of structures.

**PRIOR KNOWLEDGE REQUIRED**

Can identify top, bottom, front, back, left, and right sides of 3-D structures

**MATERIALS**

- isometric dot paper or BLM Isometric Dot Paper (p. S-72)
- rulers
- overhead projector
- transparency of BLM Dot Paper (p. S-71)
- transparency of BLM Isometric Dot Paper (p. S-72)
- connecting cubes
- box with faces labelled “front,” “left side,” “top,” etc.
- grid paper or BLM 1 cm Grid Paper (p. T-1)
- transparency of BLM 1 cm Grid Paper (p. T-1)
- dot paper or BLM Dot Paper (p. S-71, see Extension 1)

**Mental math minute.** Review finding equivalent fractions using multiplication and division using the question in the margin.

**Exercises:** Find the number that makes the fractions equivalent.

- a) \(\frac{12}{20} = \frac{3}{?}\)
- b) \(\frac{84}{100} = \frac{?}{25}\)
- c) \(\frac{126}{15} = \frac{?}{5}\)
- d) \(\frac{114}{120} = \frac{?}{20}\)

**Answers:** a) 5, 15; b) 21, 75; c) 42, 840; d) 19, 95

**Introduce isometric dot paper.** Give each student a sheet of isometric dot paper (e.g., BLM Isometric Dot Paper), and write “isometric” on the board. ASK: Which other words that start with “iso” do you know? (isosceles) What does “iso” mean in that word? (the same) What does “metric” remind you of? (metre) Explain that “metric” means length (or distance). Ask students to join several close dots with line segments. ASK: How is this dot paper different from regular dot paper? (it makes triangles, not squares) Which type of triangles? Have students measure the sides to check. (the triangles are equilateral) Explain that this paper is called isometric dot paper because the distances between adjacent dots are all equal. Project a sheet of regular dot paper on the board, and show how distances between adjacent dots on regular dot paper are not equal (the distance along a diagonal in a square is

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**Measurement 6-27**

**CURRICULUM REQUIREMENT**

- AB: optional
- BC: optional
- MB: optional
- ON: required

**VOCABULARY**

- back
- bottom
- isometric
- face
- front
- left side
- right side
- top

**On isometric dot paper all distances are equal.**

**On regular dot paper the distances are not equal.**
larger than the distance along a side of the same square). SAY: Regular dot paper is not isometric.

Explain that these two kinds of dot paper are used to produce two different types of views of 3-D shapes. Show the images of cubes in the margin and ask students to compare them. ASK: How are the images the same? (both show a cube, both show three faces of a cube) How are they different? (Image A has one face that is not distorted but the other two are distorted so that they look like parallelograms. In Image B, all three faces are distorted the same way—they all look like rhombuses.) Point out that the edges of Image B are all the same length, whereas in Image A the edges perpendicular to the front face look shorter. ASK: Which picture is easier to draw on regular dot paper? (A) on isometric dot paper? (B)

**Drawing cubes on isometric dot paper.** Project a sheet of isometric dot paper onto the board. Show students how to draw a cube using the dots. Start with the top face, then draw the vertical edges (no hidden edges!), and then draw the visible bottom edges. Have students draw a cube on isometric dot paper.

**Drawing 3-D shapes on isometric dot paper.** Explain that to create an isometric drawing, it helps to start from the top. SAY: Look at the topmost layer and draw the top face or faces first. Then draw the vertical edges that are part of the topmost layer as you did with the single cube.

Hold up the shape made with three connecting cubes shown in the margin. Invite a volunteer to draw the top layer, a single cube (see below). SAY: The next layer is made of two cubes. Take two cubes locked together and compare this shape to the original shape made with three cubes. ASK: Which edges of the new shape are hidden by the top cube in the original shape? SAY: We do not need to draw them. ASK: Which visible edges of the new shape are already drawn (because they are the bottom edges of the cube on top)? Ask a volunteer to draw the remaining visible edges of the second layer.

**Exercises:** Copy the shape onto isometric dot paper.

a) ![Image](b) ![Image](c)

**Sample answer:** a) ![Image]
Students may find it easier to copy a shape onto isometric dot paper if they start by shading the top face of the top layer of the shape as the first step, and all the rest of the top faces as the second step.

**Building structures from isometric drawings.** Distribute about 15 connecting cubes to each student. Display the picture in the margin on the board. Point out that pictures on isometric dot paper are often hard to understand. SAY: One way to make the picture easier to build is to shade all faces of the cubes that face a certain way, right or left, for example.

Draw a cube, as shown below, under the structure and shade the two vertical faces using different colours, say blue for the one on the right and green for the one on the left.

Compare the blue, green, and white rhombuses in the picture. SAY: The blue and the green rhombuses (the vertical faces) have vertical sides, while the white rhombus (the top face) does not. In the blue rhombus, the top and bottom sides have the right-side endpoint higher than the left-side endpoint, and in the green rhombus it is the opposite.

Invite a volunteer to identify all the rhombuses that are like the green rhombus in the picture on the board and shade them green. Display a second copy on the board of the same shape and invite another volunteer to shade the rhombuses that are like the blue rhombus on the second shape. See the resulting shapes below. ASK: How many towers does the structure have? (two) How many cubes tall are the towers? (3 cubes tall) Have students construct the structure using connecting cubes.

### ACTIVITY (Essential)

a) Build a structure using at least 10 cubes. The structure must have at least two towers.

b) Draw a picture of your structure using isometric dot paper.

c) Exchange pictures with a partner.

d) Shade the faces that face one side.

e) Construct the new structure using connecting cubes.

f) Have a partner check your structure. Keep the picture you shaded to use later.
Introduce views of a structure. Use the labelled box to remind students what the faces are called (left-side face, front face, and so on). Explain that when they view a structure from one of the sides, they get a side view. When they look at the structure from the front, they get a front view. When they look at the structure from the top, they get a top view. Display the shape in the margin and have students make the same shape from connecting cubes and place it on their desk as shown.

Ask students to look at the shape from the top and draw what they see on grid paper. Have a volunteer show the answer on the grid on the board, as shown in the margin. Repeat, this time holding the shape up at eye level, so that students only see the front face of the shape. Point out that from the front it is hard to see that one of the cubes is closer to the eye than the rest; the shape looks like a rectangle. The answer shown in the margin. Keep the structure and the pictures on the board.

Drawing top and front views using a structure. Have students make a structure that has at least one tower and that does not look like a rectangle from any side and place it on the desk. Then have them use the structure to draw, on grid paper, the top view and the front view of the structure. Have them exchange structures with a partner, draw the top view and the front view of the partner’s structure, and compare pictures.

Drawing top and front views using a picture of a structure. Draw on the board:

SAY: I want to draw the front view of this structure without making it from blocks. One way to do so is to shade the front faces of the shape, starting from the bottom layer. Invite a volunteer to do so. SAY: In this picture I see that there are 3 cubes in the bottom row. One of them closer to me than the rest, but in the front view I cannot see that this cube is closer to me. Have a volunteer draw the bottom row of the front view. Repeat with the other two layers until the picture is finished. Redraw the 3-D structure picture and repeat for the top view. Completed pictures are shown in the margin.

Compare the top view of this structure and the L-shaped structure from the previous example. ASK: What do you notice about the top views? (the top views are the same) How can you make the second structure from the first? (build two towers on the first structure) Point out that towers are not visible in the top view—we are just adding height, not making a bigger base layer of the structure.

To practise, have students use the pictures they and their partners drew in the previous activity and draw the top view and the front view of the structure. Then have them reassemble the shapes from connecting cubes to check their drawings.
Extensions

1. Remind students how to draw a cube on regular dot or grid paper using BLM Dot Paper or BLM 1 cm Grid Paper. (Draw a 2-by-2 square for the front face, then translate it 1 unit up and 1 unit right or left and draw the image. Join the vertices of the squares. Erase the hidden lines.) Have students choose some of the structures drawn on isometric dot paper on AP Book 6.2 pp. 186–187, and have them redraw these structures on regular dot or grid paper.

Example:

2. Use 6 connecting cubes to build as many different structures as you can. Draw them on isometric dot paper.

3. a) Build the two structures shown from connecting cubes. How many cubes did you use?

A.  

B.  

b) How are the structures the same? How are they different?

c) Draw a structure that does not look like a rectangle from any side and that has a hole. Draw your structure on isometric dot paper.

d) Switch pictures with a partner, and build the structure your partner drew.

Selected sample answers

a) A: 9 cubes, B: 8 cubes; b) Both structures are made from one layer of cubes, both look like a thick square but Structure B lacks one cube in the centre, it has a hole, and both structures are three cubes wide and three cubes tall.
Mental math minute. Give students BLM Filling a Blank Multiplication Chart. Have them fill in the chart as much as they can in three minutes, using the strategies on the BLM as needed. Compare the chart from this lesson with the charts done earlier in the year to demonstrate how far students have progressed.

Introduce side views. Review the names of faces of a rectangular prism. Remind students that in the last lesson they studied top views and front views. SAY: When you look at a structure from the right side, you see the right-side view. ASK: What would you call the view you see when you look at a structure from the left side? (left-side view)

Point out that the visible faces may differ from picture to picture. Draw a cube as shown in the margin (A), and explain that the undistorted face is always considered the front face. ASK: Which other two faces are visible on this cube? (top and right side). Repeat with Cube B. Then show Cube C.

Explain that when we draw shapes from a different angle, as we do when we draw them on isometric dot paper, we have to decide which of the two vertical sides to make the front face. Depending on our choice, the other side will be either the left side or the right side.
One 3-D picture might not be enough to build the structure. Distribute connecting cubes, at least 20 for each student. Show students the picture in the margin. ASK: Was it drawn on isometric dot paper? (no) Ask students to build it from connecting cubes. ASK: How many cubes did you use? (7, 8, or 9) What is the volume of your structure? (7 cm³, 8 cm³, or 9 cm³, if students are using 1 cm cubes, and 56 cm³, 64 cm³, or 72 cm³ if students are using 2 cm connecting cubes) Ask if anybody made a structure with a different volume. Have students present solutions with 7, 8, and 9 cubes. ASK: Why are three solutions possible? (because the picture only shows 7 cubes but there might be one or two cubes in the back that are invisible from the front) How could we make clear what the structure looks like? (sample answers: show a second picture from a different angle, tell what the volume of the structure is) Explain that engineers and workers often use pictures of several views of a shape to give all the information about what the shape looks like. Have all students construct the shape pictured with 7 cubes (a $2 \times 2 \times 2$ cube with one cube missing on the back), then ask them to hold the shape so that they see only the front face. Have them draw the front view.

Exercises: Look at the structure below:

![Structure Diagram]

a) Build the structure with connecting cubes. Find the volume of the structure.

b) Draw the top view, right-side view, and front view of the structure on grid paper.

c) Add or remove some cubes from the structure, so that it still looks the same from this angle. Try to change at least one of the side views. Find the volume of the new structure.

d) Draw the top view, right-side view, and front view of the new structure.

e) Compare the views of both structures.

Selected sample answers: a) assuming 1 cm cubes are used: 19 cm³,

- top view
- right-side view
- front view

c) 17 cm³
d) top view
right-side view
front view

e) The right-side view and the front view are the same, but the top view of the second structure misses two cubes: one in the left back corner and the other in the centre.
Point out that some structures are complicated, and you need a lot of information to build the correct structure. **SAY:** Sometimes, you can do that from just one picture from one angle and sometimes you need more than one picture. Sometimes one picture and additional information, such as the number of cubes, will be sufficient. Every case is different.

**Drawing three views together.** Explain to students that when we refer to several views of a shape, we often say “side views,” but we actually mean the top view, the front view, and at least one side view (right or left).

Show students the structure shown in the margin. Have students draw the front view and the right-side view of the structure on regular dot paper. **ASK:** What is the height of the front view? (3) What is the height of the right-side view? (3) Are these the same? (yes) Will this happen for any structure? (yes) **Why?** (the height of both views is the height of the structure) Explain that we emphasize that the views have the same height by drawing the right-side view and the front view side-by-side, aligning the top and the bottom of the views. **SAY:** We also draw the right-side view to the right of the front view, so that the front of the structure is shown on the left of the right-side view, closest to the front view. This provides a self-checking mechanism. If you see that the front view and the side view are not the same height, you know right away that there is a mistake. Repeat with the top view, which is drawn directly above the front view with the front at the bottom, closest to the front view. **SAY:** The top view has the same width as the front view, because the width of both views is the width of the structure. When students finish, draw the three views aligned as shown in the margin. **SAY:** Height and width are two of the dimensions of the structure, and we used them for self-checking. The third dimension of the structure, depth, is perpendicular to the front view. **ASK:** In which two of the three views do we see how deep the structure is? (right-side view and top view) Which dimensions of these views are the same as the depth of the structure? (the “height” of the top view and the “width” of the right-side view) **SAY:** This provides another opportunity for self-checking.

Students can signal the answer to the exercises below by raising the correct number of fingers.

**Exercises:** Which structure has these side views?

a) top view

<table>
<thead>
<tr>
<th>top view</th>
<th>front view</th>
<th>right-side view</th>
</tr>
</thead>
</table>

**Answer:** Structure 2
Building structures from side views. Show the set of side views below and have students use 3 connecting cubes to construct the structure.

**Answer:** Structure 3

**Exercises**

1. Add 1 cube to the structure to make the structure with the given side views.

   a) top view
   
   front view
   
   right-side view

2. a) Use 4 cubes to make a structure with the given side views.

   top view
   
   front view
   
   right-side view
b) Add 1 cube to the structure in part a) to make the structure with the given side views.

   i) top view

   front view right-side view

   ii) top view

   front view right-side view

   Bonus: Turn the structure in part c) ii) around so the front face becomes the back face. Draw the top, front, and right-side views of the structure. Draw it on isometric dot paper or on regular grid paper.

   Answers
   a)  
   b) i)  , ii)  
   c) i)  , ii)  

   Bonus: top view

   front view right-side view

3. Use connecting cubes to make the structure with the given side views.

   top view

   left-side view front view

   Answer
Extensions

1. **Using thick lines to show change of level.** Display a $2 \times 2 \times 2$ cube with one cube missing on the back (one of the shapes used earlier in the lesson). Have students make it with connecting cubes. Remind students that the shape looked like a 2-by-2 square in the top view, front view, and right-side view.

   Ask students to turn the shape so that they see only the right side. ASK: What is its shape? (square) Now ask them to turn the shape so that they only see the left side. ASK: What shape do you see? (square) How is the left side different from the right side? (there is a cube missing)

   Explain that one common way to show the missing cube is to add thick lines to show the change of depth. On the board, draw the picture shown in the margin, and explain that this is the left-side view of the shape. The front face is at the right side, and the back face is at the left side, so the missing cube is in the back.

   Have students draw front, top, right-side, and left-side views of the shape, using thick lines to show change of depth.

   a)  
   b)  
   c)  
   d)  

   **Answers**

   a) top view  
   b) top view  
   c) top view  
   d) top view  

   left-side view  
   front view  
   right-side view  
   left-side view  
   front view  
   right-side view  
   left-side view  
   front view  
   right-side view

2. Use connecting cubes to build the structure. Find the volume of the structures you built.

   a)  

   top view  

   left-side view  
   front view  
   right-side view

Measurement 6-28
b) top view

left-side view  front view  right-side view

Answers

a) front  right side  back  left side

Volume = 19 cm$^2$

b) Volume = 13 cm$^2$
Cube Skeleton

You will need 12 tubes and tape to make a skeleton of a cube.

1. Use tape to bind tubes together as shown.

2. Make two squares with the tubes. Make sure the squares are 1 m long and 1 m wide on the inside.

3. Bind the four leftover tubes to one of the squares as shown.

4. Add the other square to the top.
Nets of 3-D Shapes (1)
Nets of 3-D Shapes (2)
Nets of 3-D Shapes (3)
Nets of 3-D Shapes (4)
Pictures of Rectangular Prisms
Dot Paper
Goals
Students will choose among the strategies learned this year to solve problems.

Prior Knowledge Required
Can solve problems using all the strategies learned
Can create ratios equivalent to a given ratio (for Problem Banks 1, 2, 3)
Can compute the area of a parallelogram (for Problem Bank 1)
Can multiply two-digit numbers by two-digit numbers (for Problem Banks 5, 11, 17)
Can add decimal tenths (for Problem Bank 6)
Can evaluate the mean of a set of numbers (for Problem Bank 8)
Can determine the coordinates of points in the first quadrant (for Problem Bank 13)
Can multiply a decimal by a whole number (for Problem Bank 14)
Can evaluate the LCM and GCF of small numbers (for Problem Bank 20)
Can graph sequences on a coordinate grid with the first coordinate representing the term number and the second coordinate representing the term (for Extended Problem: International Text Cost)
Can write a rule for obtaining the term from the term number in a sequence with constant gaps (for Extended Problem: International Text Cost)
Can calculate the area of a rectangle given its side lengths (for Extended Problem: Volume and Area)
Can calculate the volume of a rectangular prism given its dimensions (for Extended Problem: Volume and Area)
Can apply the additive property of volume (for Extended Problem: Volume and Area)
Can divide decimal tenths by whole numbers (for Extended Problem: Volume and Area)

Materials
BLM Hundreds Chart (p. S-82)
BLM International Text Cost (pp. S-84–86, see Extended Problem: International Text Cost)
BLM Volume and Area (pp. S-88–90, see Extended Problem: Volume and Area)
NOTE: The following Problem Bank questions reflect a selection of the problem-solving strategies used in the problem-solving lessons for Grade 6. Students will need to choose among all the strategies they have learned this year to solve the problems.

Problem Bank

1. The ratio of base to height in a parallelogram is 1 : 2. The area is 50 cm². What is the height of the parallelogram?

   Solution: Make a table for base, height, and area with the ratio 1 : 2 between base and height.

<table>
<thead>
<tr>
<th>Base (cm)</th>
<th>Height (cm)</th>
<th>Area (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>18</td>
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<td>4</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

   The height of the parallelogram is 10 cm.

2. a) The ratio of length to width of a rectangle is 4 : 3. The perimeter is 70 cm. What is the area?
   b) The ratio of length to width of a rectangle is 7 : 4 and its perimeter is 44 mm. What is the area?

   Answers: a) 300 cm², b) 112 mm²

3. The ratio of girls to boys is 5 : 3. There are 8 more girls than boys. How many girls and how many boys are there?

   Answer: 20 girls and 12 boys

4. How many multiples of 9 are there from …
   a) 1 to 900?  b) 1 to 1000?  c) 901 to 1000?

   Answers: a) 100, b) 111, c) 11

5. How many perfect squares \((1 = 1 \times 1, 4 = 2 \times 2, 9 = 3 \times 3, \text{ and so on})\) are there from …
   a) 1 to 900?  b) 1 to 1000?  c) 901 to 1000?

   Answers: a) 30, b) 31, c) 1

6. Add: \(1.1 + 2.2 + 3.3 + 4.4 + \ldots + 9.9\).

   Answer: 49.5

7. A regular hexagon and a regular triangle have the same perimeter. How do their areas compare?
Solution: To have the same perimeter, the triangle must have side lengths twice those of the hexagon because the hexagon has twice as many sides. In the picture in the margin, each of the small equilateral triangles has the same side length and area.

So, the area of the triangle is \(4/6\), or \(2/3\), of the area of the hexagon.

8. The mean of two numbers is 30, and the mean of three other numbers is 40. What is the mean of all five numbers?

Answer: 36

9. The reciprocal of a whole number is the number that you need to multiply it by to get 1. For example, the reciprocal of 2 is \(1/2\) and the reciprocal of 3 is \(1/3\). How many whole numbers have a reciprocal between (and including) ... \(0.3\) and \(0.5\)? \(11/91\) and \(58/91\)?

Answers: a) 2, b) 7

Selected solution: a) reciprocal of 0.3 is \(10/3\) (or 3.33) and reciprocal of 0.5 is 2, so the integers are 2 and 3

10. a) What is the pattern on the left side of the equations? What is the pattern on the right side of the equations?

\[
\begin{align*}
2 &= 1 \times 2 \\
2 + 4 &= 2 \times 3 \\
2 + 4 + 6 &= 3 \times 4 \\
2 + 4 + 6 + 8 &= 4 \times 5
\end{align*}
\]

b) Find the sum of the first 15 even numbers using the 15th row.

c) What is the sum of the first 15 whole numbers? Hint: Divide each even number by 2. What happens?

Answers: a) The left side is the sum of the even numbers starting from 2 and increasing the number of even numbers by 1 each row. The right side is the product of two consecutive numbers and increasing the factors by 1 each row; b) \(15 \times 16 = 240\); c) \(120\) (by dividing each even number by 2, you get a whole number)

11. a) Complete the table.

<table>
<thead>
<tr>
<th></th>
<th>((2 \times 2) - (1 \times 1) = ___)</th>
<th>((3 \times 3) - (2 \times 2) = ___)</th>
<th>((4 \times 4) - (3 \times 3) = ___)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>___</td>
<td>___</td>
<td>___</td>
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<td>2</td>
<td>___</td>
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<td>5</td>
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<td>___</td>
<td>___</td>
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<tr>
<td>6</td>
<td>___</td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>

b) How can you get your answers in each row from the row number?

Problem-Solving Lesson 6-11  S-75
c) Will the answer to \((23 \times 23) - (22 \times 22)\) be in the 22nd row or the 23rd row? Explain how you know.

d) Use your rule in part b) to solve \((23 \times 23) - (22 \times 22)\).

e) Check your answer.

Answers: a) 3, 5, 7, 9, 11, 13; b) multiply the row number by 2 and add 1; c) the 22nd row because the product being subtracted is the one with the row number in it; d) \(2 \times 22 + 1 = 44 + 1 = 45\); e) \(23 \times 23 = 529, 22 \times 22 = 484\), and so \(23 \times 23 - 22 \times 22 = 529 - 484 = 45\)

12. Using \(44 + 45 + 46 + 47 = 182\), what is \(\frac{44}{2} + \frac{45}{2} + \frac{46}{2} + \frac{47}{2}\) ?

Answer: 184

13. What will be the coordinates of the centre of the 100th rectangle in the pattern?

Solution: The coordinates of the terms are (2, 2), (5, 5), (8, 8), and so on. Each coordinate is equal to \(3 \times \text{term number} - 1\), so the coordinates of the centre of the 100th rectangle are (299, 299).

14. In the figures below, each square has a side length of 1.5 m.

a) Complete the table for the figure pattern.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Perimeter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

b) What is the perimeter of the 10th figure?

c) Which figure has perimeter 72 m?

Answers: a) 12, 18, 24; b) 60; c) 12th figure
15. Bowl A has 5 spoonfuls of red paint and 2 spoonfuls of white paint. Bowl B has 1 spoonful of red paint and 1 spoonful of white paint. All the spoons are the same size.

a) Which bowl has paint that is darker red? Explain how you know using fractions.

b) If you pour the contents of Bowl B into the contents of Bowl A, will it make the paint in Bowl A darker or lighter red?

c) What is the new fraction of red paint in the bowl in part b)? Is that fraction greater than or less than \( \frac{5}{7} \)? How can you tell without doing any calculations?

d) Is \( \frac{35}{69} \) more or less than one half?

e) Without doing any calculations, use your answer to part d) to decide if \( \frac{36}{71} \) is more or less than \( \frac{35}{69} \).

Selected solution: e) Suppose I have a mixture of red and white paint with 35 spoonfuls of red paint and 34 spoonfuls of white paint. The mixture is 35/69 red. If I add 1 spoonful of red paint and 1 spoonful of white paint, the mixture becomes 36/71 red. However, doing this will make the paint a lighter colour of red because \( 1/2 < \frac{35}{69} \), so \( \frac{36}{71} \) must be less than \( \frac{35}{69} \).

Answers: a) Bowl A is darker red because the fraction of red paint is 5/7, while Bowl B is 1/2, and 5/7 is greater than half; b) lighter red because you are adding paint that is lighter red to Bowl A; c) 6/9, or 2/3, which is less than 5/7 because the paint is lighter red; d) more

16. How many factors does 1 000 000 000 have?

Solution: Look for a pattern: 10 has 4 factors, 100 has 9 factors, 1000 has 16 factors. These numbers are the perfect squares: add 1 to the number of zeros in the number and multiply the result by itself. There are 9 zeros in the given number, so the number of factors is \( 10 \times 10 = 100 \).

17. A path continues spiralling, as shown below. Each arrow shows one unit along the x- or y-axis. What is the length of the path from (0,0) to (0,10)?

Answer: 120 (one less than 11 \( \times \) 11 = 121)
18. How much greater is \( \frac{2003}{25} + 25 \) than \( \frac{2003}{25} \)?

**Answer:** 24

19. Lewis divides 48 by a number and gets a remainder of 6. What could he have divided by?

**Solution:** If \( 48 \div A = B \ R 6 \), then \( A \times B + 6 = 48 \), so \( A \times B = 42 \). Then, \( A \) is a factor of 42 and \( A \) is at least 7, so \( A \) can be 7, 14, 21, or 42. Indeed, \( 48 \div 7 = 6 \ R 6 \), \( 48 \div 14 = 3 \ R 6 \), \( 48 \div 21 = 2 \ R 6 \), and \( 48 \div 42 = 1 \ R 6 \).

20. How can you get the LCM and GCF of 2000 and 3000 from the LCM and GCF of 2 and 3?

**Solution:** Make a table as follows:

<table>
<thead>
<tr>
<th></th>
<th>LCM</th>
<th>GCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 and 3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>4 and 6</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>6 and 9</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>8 and 12</td>
<td>21</td>
<td>4</td>
</tr>
</tbody>
</table>

The pair 2000 and 3000 would be in the thousandth row, so the LCM and GCF of 2000 and 3000 can be obtained from the LCM of 2 and 3 by multiplying both by 1000. The LCM of 2 and 3 is 6, so the LCM of 2000 and 3000 is 6000. The GCF of 2 and 3 is 1, so the GCF of 2000 and 3000 is 1000.

21. a) Sharon walks 1 block east, then turns right and walks 2 blocks, then turns right and walks 3 blocks, then turns right again and walks 4 blocks. She continues this pattern until she goes 100 blocks. Then she turns right again and goes another 100 blocks, turns right again and goes 99 blocks, turns right again and walks 98 blocks, and so on until walking 1 block. Where does she end up, relative to home?

b) What if she did the same pattern as in part a) but with starting to count down after 97 blocks instead of 100 blocks; now where would she end up relative to home?

**Answers:** a) at home, b) 1 block east and 1 block south

22. A two-digit number is divided by the sum of its digits. What two-digit number will result in the largest remainder? Solve this problem in steps.

a) Start by dividing the two-digit numbers by the sum of their digits, in order:

\[
\begin{align*}
10 \div 1 &= \_R\_ \\
11 \div 2 &= \_R\_ \\
12 \div \_ &= \_R\_ \\
13 \div \_ &= \_R\_ \\
14 \div \_ &= \_R\_
\end{align*}
\]
b) Is the strategy from part a) a good strategy to continue? Why or why not?

c) On a hundreds chart, e.g., BLM Hundreds Chart, calculate the sum of the digits of all the two-digit numbers. Write them on the hundreds chart squares.

d) What is the largest sum of digits a two-digit number can have?

e) The remainder must be smaller than the divisor, which is the sum of the digits. So, make a table starting with the largest sum of the digits.

<table>
<thead>
<tr>
<th>Sum of Digits</th>
<th>Two-Digit Number</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
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<td></td>
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<tr>
<td>16</td>
<td></td>
<td></td>
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<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f) What is the largest remainder you found in part c)?

g) All the two-digit numbers not in the table so far have the sum of the digits at 15 at the most. Can they get a higher remainder than you found in part c)? Explain how you know.

**Answers:** a) 10 ÷ 1 = 10 R 0, 11 ÷ 2 = 5 R 1, 12 ÷ 3 = 4 R 0, 13 ÷ 4 = 3 R 1, 14 ÷ 5 = 2 R 4; b) no, because the remainder can’t be bigger than the sum of the digits, so we should start with numbers that have bigger sums of digits; d) 18,

e) | Sum of Digits | Two-Digit Number | Division |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>99</td>
<td>99 ÷ 18 = 5 R 9</td>
</tr>
<tr>
<td>17</td>
<td>98</td>
<td>98 ÷ 17 = 5 R 13</td>
</tr>
<tr>
<td>17</td>
<td>89</td>
<td>89 ÷ 17 = 5 R 4</td>
</tr>
<tr>
<td>16</td>
<td>97</td>
<td>97 ÷ 16 = 6 R 1</td>
</tr>
<tr>
<td>16</td>
<td>88</td>
<td>88 ÷ 16 = 5 R 8</td>
</tr>
<tr>
<td>16</td>
<td>79</td>
<td>79 ÷ 16 = 4 R 15</td>
</tr>
</tbody>
</table>

f) 15; g) no, the highest remainder you can get when dividing by 15 is 14, so 15 is the largest remainder possible
23. Ivan divides a three-digit number by the sum of its digits. What is the largest possible remainder he can get?

**Solution:** Make a table starting with the largest possible sum of digits.

<table>
<thead>
<tr>
<th>Number</th>
<th>Sum of Digits</th>
<th>Number $\div$ Sum of Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>999</td>
<td>27</td>
<td>37 R 0</td>
</tr>
<tr>
<td>998</td>
<td>26</td>
<td>38 R 10</td>
</tr>
<tr>
<td>989</td>
<td>26</td>
<td>38 R 1</td>
</tr>
<tr>
<td>899</td>
<td>26</td>
<td>34 R 15</td>
</tr>
<tr>
<td>988</td>
<td>25</td>
<td>39 R 13</td>
</tr>
<tr>
<td>898</td>
<td>25</td>
<td>35 R 23</td>
</tr>
<tr>
<td>889</td>
<td>25</td>
<td>35 R 14</td>
</tr>
<tr>
<td>799</td>
<td>25</td>
<td>31 R 24</td>
</tr>
<tr>
<td>979</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>997</td>
<td>25</td>
<td></td>
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</tbody>
</table>

The largest possible remainder is 24, since dividing by 25 or less cannot result in a remainder that is larger than 24, so we can stop here at $799 \div 25 = 31 \text{ R } 24$.

**NOTE:** In Problem Bank 24, parts a) to d) guide students to solve the puzzle. Some students may appreciate the opportunity to solve the puzzle without doing parts a) to d) first.

24. In a "very right" polygon, all angles are either 90° or 270°. Here are some very right polygons.

![A.](image) ![B.](image) ![C.](image) ![D.](image)

Investigate the problem: A very right polygon has 100 edges. How many 90° angles does the polygon have?

a) Count the number of 90° angles, 270° angles, and edges in each shape above.

b) Draw three different very right polygons that have 10 edges. How many 90° angles and how many 270° angles does each of your shapes have?

c) Complete the table.

<table>
<thead>
<tr>
<th>Number of 90° angles</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d) Compare the sequences in part c) to predict the number of 90° angles in a right polygon that has 100 edges.
**Answers:** a) A: 4, 0, 4, B: 5, 1, 6, C: 6, 2, 8, D: 6, 2, 8; b) each shape has seven $90^\circ$ angles and three $270^\circ$ angles; c) 4, 6, 8, 10; d) divide each term in the bottom sequence by 2: 2, 3, 4, 5, ..., then the result is 2 less than the corresponding term in the top row, so when the bottom row is 100, the top row is 2 more than 50 (i.e., 52), so the number of $90^\circ$ angles in a very right polygon with 100 edges is 52.
### Hundreds Chart

<p>| | | | | | | | | | |</p>
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<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
</tr>
</tbody>
</table>
Extended Problem: International Text Cost

MATERIALS

BLM International Text Cost (pp. S-84–86)

Extended Problem: International Text Cost. Provide students with BLM International Text Cost. In this Extended Problem, students plot data from a T-table and find the rule for how to get the output from the input. Students interpret the results in the context of sending international text messages.

Selected answers: 1. a) (2, 0.40), (3, 0.60), (4, 0.80); c) divide by 5 (or multiply by 0.2); d) $4; 2. a) (2, 5.20), (3, 5.30), (4, 5.40); c) divide by 10 (or multiply by 0.1), then add 5; d) $7; e) Company A; f) Company B, because Company A will charge $20 and Company B will only charge $15; Bonus: 50 text messages
**International Text Cost (1)**

1. Phone Company A charges $0.20 for each international text message.
   a) Complete the table for the cost of sending one, two, three, and four international text messages. Write a list of ordered pairs for the table.

<table>
<thead>
<tr>
<th>Text Messages</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   \[ (1, 0.20) \]

   \[ (, ) \]

   \[ (, ) \]

   \[ (, ) \]

   b) Plot the ordered pairs from the table of values in part a) on the grid below.

   ![Graph of Cost vs. Number of Text Messages]

   c) Write a rule that tells you how to calculate the cost ($) from the number of international text messages sent.

   d) Edmond sends 20 international text messages each month. How much will it cost him if he uses Company A?
International Text Cost (2)

2. Phone Company B charges $5 per month, plus $0.10 for each international text message sent.

   a) Create a table for the cost of sending zero, one, two, three, and four international text messages. Write a list of ordered pairs for the table.

<table>
<thead>
<tr>
<th>Text Messages</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.00</td>
</tr>
<tr>
<td>1</td>
<td>5.10</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   \[
   (0, 5.00) \quad (1, 5.10) \quad (\ , \ ) \quad (\ , \ ) \quad (\ , \ )
   \]

   b) Plot the ordered pairs from the table of values in part a) on the grid below.

   c) Write a rule that tells you how to calculate the cost ($) from the number of international text messages sent.
International Text Cost (3)

d) Edmond sends 20 international text messages each month. How much will it cost him if he uses Company B?

e) Which company gives Edmond the lower rate, Company A or Company B?

f) Lily sends 100 international text messages each month. Which company will give her the lower rate?

BONUS ▶ For how many text messages is the cost the same for both companies?
Extended Problem: Volume and Area

MATERIALS

BLM Volume and Area (pp. S-88–90)

Extended Problem: Volume and Area. Give students BLM Volume and Area. In this Extended Problem, students calculate the dimensions of a storage unit, including the height, the area of the rectangular sides, and the volume. Students use a given painter’s rate to determine the cost of having the storage unit painted.

Answers: 1. a) 8 m, b) 41.6 m², c) 291.2 m³; 2. a) 41.6 m³, b) 20.8 m³; 3. 312 m³; 4. 184.8 m²; 5. $974.00; Bonus: Jen is right because a perfect square cannot have ones digit 3.
Volume and Area (1)

A storage unit has a rectangular base and a slant roof with dimensions shown.

1. a) What is the height of the highest part of the storage unit roof from the ground?

b) Find the area of the base of the storage unit.

c) Find the volume of the rectangular prism part of the storage unit.
Volume and Area (2)

2. To get the volume of the top part of the storage unit, you can find half of the volume of the rectangular prism below.

   a) What is the volume of the rectangular prism shown? ____________

   b) What is the volume of the top part of the storage unit? ____________

3. Find the total volume of the storage unit.

4. Find the total area of the four side faces of the rectangular prism part of the storage unit.
Volume and Area (3)

5. A painter charges $5.00 per square metre, plus $50.00 for paint. How much will it cost to have all four side faces of the rectangular prism part painted, not including the slant roof?

BONUS ▶ Rick said he built a storage unit that had a square base with the length of each side a whole number of centimetres. He said the area of the base was 45 293 cm². Jen replied, “That's not possible!” Who is right? Explain how you know.
1 cm Grid Paper
Filling a Blank Multiplication Chart

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Sample strategies

- Fill in all the facts that you have memorized or use numbers that you can skip count by easily, such as 1s, 2s, 3s, 4s, 5s, and 10s.
- Use doubling to fill in the 6s, 8s, and 12s. For example, $6 \times 8$ is double $3 \times 8$ because $3 \times 8 = 8 + 8 + 8$ and $6 \times 8 = (8 + 8 + 8) + (8 + 8 + 8)$.
- Use the 5s and the 2s to fill in the 7s. For example, $7 \times 8 = (8 + 8 + 8 + 8) + (8 + 8)$, which is $(5 \times 8) + (2 \times 8)$.
- Use the 10s and the 1s to fill in the 9s and the 11s.
- Check that the same two numbers always multiply to the same number.
Number Sense: Adding and Subtracting Decimals – AP Book 6.2: Unit 9

AP Book NS6-38

1. b) \( \frac{5}{10} \)
   c) \( \frac{7}{10} \)
   d) \( \frac{1}{10} \)
2. b) \( \frac{3}{10}, 0.3 \)
   c) \( \frac{8}{10}, 0.8 \)
   d) \( \frac{6}{10}, 0.6 \)
3. 0.8, 1.0
4. b) \( \frac{50}{100} = 0.50 \)
   c) \( \frac{50}{100} = 0.50 \)
5. a) 20
   b) \( \frac{50}{100} \)
   c) \( \frac{90}{100} \)
6. b) \( \frac{4}{10}, \frac{40}{100} \)
   c) \( \frac{6}{10}, \frac{60}{100} \)
7. Ben incorrectly thought that 0.9 is 9 hundredths. 0.57 is 57 hundredths, but 0.9 is 90 hundredths. Since 57 is not greater than 90, 0.57 is not greater than 0.9.
8. b) \( \frac{76}{100}, \frac{7}{10} \)
   c) \( \frac{28}{28}, 2, 8 \)
   d) \( \frac{6}{6}, \frac{60}{60}, 6, 0 \)
   e) \( 0.2, 0.2 \)
   f) \( 8, 3 \)
   g) \( 83 \) hundredths
   h) \( 2, 4 \)
   i) \( 24 \) hundredths

AP Book NS6-39

1. Circle \( \frac{4}{10}, \frac{53}{100}, \frac{7}{100} \).
   \( \frac{63}{1000}, \frac{125}{100}, \frac{100}{100} \).
2. a) 30
   b) \( 4, 40 \)
   c) \( 9, 90 \)
3. a) 30
   b) 50
   c) multiply by 10
   d) \( \frac{90}{100} \)
   e) multiply by 10
   f) \( \frac{90}{100} \)
4. a) 50
   b) multiply by 10
   c) multiply by 10
   d) 300
   e) multiply by 100
   f) multiply by 100
   g) multiply by 100
   h) multiply by 10
5. a) \( \frac{1}{1000} \)
   b) \( 0.001 \)

AP Book NS6-40

1. b) \( 5, 3, 7 \)
   c) \( 6, 4, 1 \)
   d) \( 8, 9, 2 \)
   e) \( 4, 2, 4 \)
   f) \( 0, 5, 3 \)
   g) \( 2, 7, 5, 6 \)
   h) \( 3, 4, 1, 0 \)
   i) \( 9, 2, 0, 7 \)
   j) \( 8, 0, 1, 9 \)
2. b) 6, 3, 5
c) \( \frac{3}{100} + \frac{5}{1000} \)
d) \( \frac{0}{10} + \frac{4}{10} + \frac{1}{1000} \)
e) \( \frac{0}{10} + \frac{5}{1000} \)
f) \( \frac{3}{10} + \frac{7}{1000} \)

3. b) tenths
c) 100, hundredths
d) 1000, thousandths
e) \( \frac{9}{1000} \), thousandths
f) \( \frac{9}{10} \), tenths
c) \( \frac{9}{100} \), hundredths

4. b) 6
c) 6
d) 0, 5, 4
e) 1, 8
f) 0, 1, 8
1, 8
g) 0, 2, 4, 3
h) 7, 0, 3, 5
0, 3, 5

5. b) 4, 6, 7, 0
c) 0, 3, 0, 7
d) 2, 7, 2, 7
e) 9, 0, 2, 0

6. Teacher to check underlining.
b) thousandths
c) four hundredths
d) eight thousandths
e) six thousandths
f) nine tenths

7. a) 5.72
b) 1.07
c) 2.8759
d) .71

**AP Book NS6-41**

*page 9*

1. A: \( \frac{0.2}{10} \)
   B: \( \frac{8}{10} \)
   C: \( \frac{1.4}{10} \)
   E: \( \frac{2.7}{10} \)

2. Teacher to check number line.

3. A: \( \frac{0.19}{100} \)
   B: \( \frac{0.08}{100} \)
   C: \( \frac{0.14}{100} \)
   D: \( \frac{0.27}{100} \)

4. Teacher to check number line.

5. change 1.01 to 1.1,
   change 0.2 to 2.0,
   change 1.5 to 2.6

6. a) one
   b) two
   c) two

7. a) \( \frac{80}{100} \)
   \( \frac{35}{100} \)
   \( \frac{20}{100} \)
   \( \frac{80}{100} \)
   \( \frac{35}{100} \)
   \( \frac{20}{100} \)
   \( \frac{90}{100} \)
   \( \frac{27}{100} \)
   \( \frac{25}{100} \)
   \( \frac{130}{100} \)
   \( \frac{122}{100} \)
   \( \frac{139}{100} \)

b) \( \frac{27}{100} \)
   \( \frac{90}{100} \)
   \( \frac{25}{100} \)
   \( \frac{90}{100} \)
   \( \frac{27}{100} \)
   \( \frac{25}{100} \)
   \( \frac{130}{100} \)
   \( \frac{122}{100} \)
   \( \frac{139}{100} \)

8. b) 1000, 100

**BONUS**

10. a) 2.487
   b) 7.284
   c) 8.724

**AP Book NS6-42**

*page 11*

1. a) 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
   b) 0.5
   c) \( \frac{0.1}{10} \)
   \( \frac{0.4}{10} \)
   \( \frac{0.5}{10} \)
   \( \frac{0.6}{10} \)
   \( \frac{0.7}{10} \)
   \( \frac{0.8}{10} \)
   \( \frac{0.9}{10} \)

2. a) >
   b) <
   c) >
   d) <
   e) <
   f) >
   g) <
   h) >

3. a) <
   b) <
   c) =
   d) >
   e) <
   f) =
   g) =
   h) <

4. circle \( \frac{1}{2} \)

8. b) 1000, 100

**BONUS**

9. a) 70, 32, 50
   b) 25, 60
   c) 25, 63
   d) 25, 60
   e) 23, 20
   f) 52, 50
   g) 0.32
   h) \( \frac{1}{2} \)
   i) 0.63
   j) \( \frac{1}{2} \)
Number Sense: Adding and Subtracting Decimals – AP Book 6.2: Unit 9

5. Teacher to check zeros. Circle the following:

   - b) 0.40
   - c) 0.900
   - d) 0.310
   - e) 0.700
   - f) 0.800
   - g) 3.5300
   - h) 12.310
   - i) 3.100

4. b) 32
   c) 32
   d) 68

5. a) 2
   b) 12
   c) 3
   d) 2

6. a) 3
   b) 4
   c) 6
   d) 5

7. a) 2.054
   b) 0.007
   c) 0.011
   d) 0.09

8. a) 0.78
   b) 0.98
   c) 0.88
   d) 0.79

9. a) 0.78
   b) 0.98
   c) 0.88
   d) 0.79

10. a) 2.5
   b) 4.2
   c) 6.7
   d) 8.9
Number Sense: Adding and Subtracting Decimals – AP Book 6.2: Unit 9

(continued)

AP Book NS6-46

1. a) $8.68
   b) $2615
      + $3223
      $5838
   c) $1957
      + $3032
      $4989

2. Teacher to check regrouping.
   a) $40.35
   b) $72.57
   c) $93.93
   d) $60.60
   e) $82.65
   f) $64.77

3. Teacher to check regrouping.
   a) $2.75
   b) $22.65
   c) $38.51
   d) $37.11
   e) $45.11
   f) $45.08

4. Jasmin needs $30.78.

5. The library spent $660.07 in total.


7. The game and book cost $9.50 + $10.35 = $19.85 in total. After buying them, Raj has $25 − $19.85 = $5.15. He has enough money left to buy the second book that costs $5.10.

8. Lela will pay $69.99 − $10.50 = $59.49.

BONUS

The second pair of glasses cost $59.49 − $5.25 = $54.24. Lela will pay $59.49 + $54.24 = $113.73.
9. a) $55.19
   b) The pants and soccer ball ($62.25) cost more than the watch and backpack ($58.53).
   c) Yes, because the items cost $12.30 + $35.47 + $49.95 = $97.72, which is less than $100.
   d) The three most expensive things are the pants, watch, and tennis rackets, which cost $128.31 in total.
   e) Answers will vary. Teacher to check.

10. a) $9.20
    b) 7 apples
    c) 8 markers
    d) No, the book and pen cost $10.09, which is greater than $10.00.
    e) The 3 oranges cost more ($1.35) than the 4 apples ($1.28).

6. Circle the following:
   a) 1.00
   b) 0
   c) 3.00
   d) 6.00
   7. →
   8. Circle the following:
   a) 1.000
   b) 0
   c) 8.000
   d) 5.000
   9. 4.268: ←
      4.723: →
   10. c) 7.0
       d) 11.0
       e) 31.0
       f) 20.0
       11. Teacher to check underlining.
       b) 1.80
       c) 3.60
       d) 3.40
       e) 5.50
       f) 6.70
       12. Teacher to check underlining.
       b) 1.490
       c) 3.550
       d) 4.270
       e) 9.170
       f) 5.320
       13. Teacher to check underlining.
       b) round down
       c) round up
       14. a) rd
           b) rd
           c) ru
           15. a) 2.200
               b) 6.00
               c) 39.900
               16. a) 3, 4, 7
                   b) 7, 5, 2
                   c) 3, 5, 4, 12
   d) 9, 3, 4, 2
   17. b) 0.30
       c) 0.80
       d) 2.60
       e) 0.20
       18. b) 1.350
           c) 0.630
           d) 1.980
           e) 3.140
           f) 2.510
       19. 5.35 to 5.44
         Sample explanation:
         5.34 and under would be rounded down to 5.3, and 5.45 and over would be rounded up to 5.5. Only decimal hundredths from 5.35 to 5.44 are close enough to 5.4 to be rounded to that number.
         20. Estimated total cost
             $25 + $7 + $20
             = $52
             Actual total cost
             $24.99 + $6.50 + $19.99
             = $51.48
             $51.48 > $50, so Mary does not have enough money to buy all three items.
### Number Sense: Multiplying and Dividing Decimals – AP Book 6.2: Unit 10

#### AP Book NS6-48

**Page 26**

1. Teacher to check drawings.
   - b) 2
   - c) 5

2. b) 6
   - c) 14
   - d) 24
   - e) 35
   - f) 145
   - h) 27.5
   - i) 976

3. a) 4
   - b) 8
   - c) 75

4. \(0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 = 4\)

5. Teacher to check.

6. a) 2
   - b) \(100 \times 0.03\)

7. Teacher to check rough work.
   - b) 350
   - c) 720
   - d) 600
   - e) 34
   - f) 7

8. a) 1
   - b) 1

9. 3

**BONUS**

- a) 932
- b) 6325
- c) 720

#### AP Book NS6-49

**Page 28**

1. Teacher to check drawings.
   - b) 3 + 10, 0.3
   - c) 0.04
   - d) 0.3 + 10, 0.03
   - e) 0.6 + 10, 0.06
   - f) 0.11
   - g) 2.1 + 10, 0.21
   - h) 2.3 + 10, 0.23

2. a) 1, right
   - 1, left
   - b) 2, right
   - 2, left

3. Teacher to check rough work.
   - b) 0.07
   - c) 0.06
   - e) 2.6
   - f) 8.14
   - g) 2.54
   - h) 0.032
   - j) 0.07
   - k) 0.091
   - l) 0.91

4. b) 3, right
   - c) 2, left
   - d) 1, left
   - e) 3, left
   - f) 2, right
   - h) divide, 1
   - i) multiply, 2
   - j) multiply, 1
   - k) divide, 2
   - l) multiply, 3

5. Teacher to check rough work.
   - c) 2, left
   - 7.246
   - d) 1, left
   - 90.003

6. Teacher to check rough work.
   - a) 3410
   - b) 500.2
   - c) 7.1
   - d) 124 050
   - e) 0.052
   - f) 8.004
   - g) 2.769
   - h) 4.702
   - i) 31

7. Sample answer: We know \$1 = 100¢\), so \(100¢ + 10 = 10¢\), which is the same as \$0.1\).

8. 0.025 m or 2.5 cm

9. a) 123
   - b) 3412
   - c) 176
   - d) 52 300

**BONUS**

- 402.7

#### AP Book NS6-50

**Page 31**

1. a) 0.2
   - b) 0.2
   - c) 1
   - d) 1
   - e) 1.2
   - f) 1.2

2. a) 2.3
   - b) They have the same answer.
   - c) 1

3. a) 3.1
   - b) 4.9
   - c) 24.2
   - d) 1.7

4. a) 1
   - b) 1
   - c) 1.12
   - d) 1.12
   - e) 2

5. a) 2.31
   - b) 1.95
   - c) 9.31
   - d) 2.8
   - e) 0.17
   - f) 23.56

**BONUS**

- 21 903 309.6

6. b) 2, 1
   - 8, 4
   - 8.4

7. a) 12.6
   - b) 24.8
   - c) 40.5
   - d) 18.6
   - e) 12.4
   - f) 15.9
   - g) 28.4
   - h) 39.6
   - i) 64.8
   - j) 189.6
   - k) 217.7
   - l) 2084.8

**BONUS**

- 21 903 309.6

8. 4.8 cm

9. 90.9 cm

10. 63.6 mm

11. a) 5.8
    - b) 28.8
    - c) 6.8
    - d) 107.0
    - e) 285.9
    - f) 274.2

12. a) 12.8
    - b) 164.5
    - c) 7.5 kg
    - d) 78 m
    - e) 299.7 m

**BONUS**

- 21 903 309.6
Number Sense: Multiplying and Dividing Decimals – AP Book 6.2: Unit 10

Answer Keys for AP Book 6.2
Number Sense: Multiplying and Dividing Decimals – AP Book 6.2: Unit 10

1. a) low
   b) high
   c) high
   d) low

2. a) 26\; 149
     \[ \underline{130} \]
     \[ \underline{19} \]
   b) 17\; 135
     \[ \underline{119} \]
     \[ \underline{16} \]
   c) 17\; 121
     \[ \underline{119} \]
     \[ \underline{2} \]
   d) 23\; 129
     \[ \underline{115} \]
     \[ \underline{14} \]
   e) 34\; 1263
     \[ \underline{238} \]
     \[ \underline{25} \]
   f) 44\; 362
     \[ \underline{252} \]
     \[ \underline{10} \]

3. a) 6 R 33
   b) 9 R 19
   c) 7 R 4
   d) 8 R 66

4. a) circle 512
   b) circle 27

5. a) Yes
   b) Sample answer: Both divisions have the same divisor, so the division with the greater dividend, 2376 ÷ 36, will have the greater answer.

6. Teacher to check Steps 1–5.

7. a) Yes
   b) Sample answer: Both divisions have the same divisor, so the division with the greater dividend, 2376 ÷ 36, will have the greater answer.

8. a) i) 85 R 4
Number Sense: Multiplying and Dividing Decimals – AP Book 6.2: Unit 10

(continued)

10. 27 books

AP Book NS6-56

page 45

1. 36 cases
2. $14
3. 23 buses
4. 39 copies
5. a) 454.5 L
   b) 228 cartons
6. 20,417.4 m

BONUS

3402.9 m, 340.29 m

7. $4250 + $34/hour
   = 125 hours
   = 125 $36 = $4500
   $4500 - $4250 = $250
   She could have earned $250 more.
8. a) $30
    b) 200 $0.20 = $40.
    The $35 unlimited minutes plan is better.
9. a) $160 + ($26.50 $40)
   = $1220
   b) $30.50

AP Book NS6-57

page 46

1. a) i) 2.1 kg a day,
   2.07 kg a day
   ii) 2.07 kg a day
   iii) 2.1 kg a day
   iv) 0.03 kg
   b) about 43 times
   c) 65.1 kg
2. a) 

b)  

c) 

3. a) 3207.02
   b) 10,520.15
   c) 6308.1
   d) 407.02
4. a) >
   b) <
   c) >
   d) <

5. 129
Sample explanation: To undo a division, you multiply the answer by the divisor. 12.9 10
= 129, so 129 was the original number.

6. 2 7 2.4 km = 33.6 km
7. 2.81 m
8. a) 75.35 km
   b) 29.7 kg
9. 39.4 mL
10. 0.63 m
11. $2.99

BONUS

a) Luc had $49.75 before buying the shirt. He had $21.40 before starting work on Monday.

b) ($3.20 + $1.25) 2
   = $8.90
1. a) 4
   b) 3 units right
   c) 2 units right
2. a) 3
   b) 4 units left
   c) 2 units left
3. a)
   b)
   c)
4. a) 4
   2
   3
   c) 3
   1
5. a)
   b)
   c)
6. a) i) 29, 90°
   17, 30°
   34, 60°
   ii) 29, 28°
   17, 100°
   36, 52°
   b) Teacher to check.
   c) i) 29, 90°
   17, 30°
   34, 60°
   ii) 29, 28°
   17, 100°
   36, 52°
   d) They are equal.
7. a) True. A triangle and its image under translation have equal angles and side lengths and so are congruent.
   BONUS
   False.
   Teacher to check counterexample.
   Sample answer:
   b)
   c)
8. a) Teacher to check.
   b) Teacher to check arrows.
   The arrows are parallel.
   c) They are equal.
   d) yes
   i) 3, up
   2, right
   ii) 1, left
   1, down
9. a) Quadrilaterals will vary. Teacher to check.
   b) Predictions may vary. Teacher to check.
   Sample answer: 2, right
   7, down
   c) Teacher to check translation.
   Sample answer: yes
10. Jax is correct.
    Sample explanation: Moving up 3, and then down 3 brings the shape back to its original position, as does moving 4 left and then 4 right.

AP Book G6-14

1. a)
c) Teacher to check drawings.
   i) no
   ii) yes
   iii) no

d) i) no
   ii) no
   iii) no

e) Teacher to check mirror line in part a) ii).

f) Triangles T and T* are congruent because reflections and translations preserve lengths of sides and sizes of angles. So a combination of a reflection and a translation will also preserve congruency.

7. a–b) i)

   BONUS

   c) i) Yes, translate R 6 units right.
   ii) Yes, translate R 4 units down.
   iii) no

8. a) 

   BONUS

   The two triangles make a parallelogram.

AP Book G6-16

page 57

1. a) 3
   2
   2
   b) horizontal, vertical

   BONUS

   A full rotation is 360°, so a rotation of 180° CW is the same as the rotation of 360° − 180° = 180° CCW.
3. a) 

b) 

c) \(180^\circ\) CW or \(180^\circ\) CCW

4. a–b) i)

ii)

c) \(180^\circ\) CW or \(180^\circ\) CCW

5. b) \(90^\circ\)

c) \(180^\circ\)

d) \(90^\circ\)

6. b) \(270^\circ\) CW

c) \(90^\circ\) CW

d) \(270^\circ\) CCW

7. a) \(90^\circ\) CW

b) \(180^\circ\) CW or \(180^\circ\) CCW

c) \(90^\circ\) CCW

d) \(90^\circ\) CW

8. b) 

\(180^\circ\) CW or \(180^\circ\) CCW

c) \(90^\circ\) CCW

9. a) S

b) W

c) N

d) W

e) N

t) They are the same.

10. a–d) 

e) They are the same.

f) \(180^\circ\) CW or CCW rotation around the intersection of lines \(\ell\) and \(m\).
Geometry: Transformations – AP Book 6.2: Unit 11

(continued)

Answer Keys for AP Book 6.2

AP Book G6-18
page 63
1. a–b)
2. a) $B(5, 2), B'(6, 6)$
   C(4, 1), C'(8, 2)
   $D(10, 4), D'(5, 1)$
   $E(9, 6), E'(4, 3)$
   $F(6, 5), F'(1, 2)$
   $G(7, 3), G'(2, 0)$
   
   c) rocket ship
   
   2. a) $B(3, 5), B'(4, 3)$
   C(11, 2), C'(7, 4)
   4 increases, 2
   4, left
   2, up
   
   b) $P(1, 4), P'(6, 1)$
   Q(3, 6), Q'(8, 3)
   R(5, 5), R'(10, 2)
   increases, 5
   decreases, 3
   5, right
   3, down

   3. Teacher to check coordinate grids.
   a) $B(2, 6), B'(2, 0)$
   C(7, 1), C'(7, 5)
   $D(9, 6), D'(1, 6)$
   $E(5, 4), E'(5, 4)$
   $F(1, 8), F'(2, 1)$
   
   b) $G(3, 1), G'(3, 7)$
   $H(4, 3), H'(4, 5)$
   $K(6, 4), K'(6, 4)$
   $M(8, 1), M'(8, 7)$
   $N(5, 2), N'(5, 6)$
   
   d) $W(9, 4), W'(3, 4)$
   $X(7, 6), X'(6, 6)$
   $Y(9, 0), Y'(3, 0)$
   $Z(10, 1), Z'(2, 1)$

   4. a) $x$-coordinate
   $y$-coordinate
   
   BONUS
   Teacher to check.

   6. Teacher to check.
   a) Teacher to check.
   b) Teacher to check coordinate grids.
   i) parallelogram
   ii) rectangle

AP Book G6-19
page 65
1. Teacher to check coordinate grids.
   a) $A(3, 2), A'(5, 4)$
   $B(1, 3), B'(6, 6)$
   $C(5, 5), C'(8, 2)$
   ii) $D(8, 1), D'(3, 6)$
   $E(7, 2), E'(2, 5)$
   $Q(3, 1), Q'(3, 1)$
   iii) $F(2, 4), F'(3, 5)$
   $G(7, 4), G'(3, 0)$
   $H(6, 6), H'(5, 1)$
   $l(2, 6), l'(5, 5)$
   iv) $J(4, 5), J'(2, 3)$
   $K(6, 5), K'(2, 5)$
   $L(8, 1), L'(6, 7)$
   $M(3, 1), M'(6, 2)$

   b) $Q$
   It is the centre of rotation.

   c) no
   no

2. a) $i) N(5, 1), N'(5, 5)$
   $P(4, 0), P'(6, 6)$
   $Q(2, 3), Q'(8, 3)$
   $R(4, 3), R'(6, 3)$
   ii) $S(6, 1), S'(6, 5)$
   $T(4, 3), T'(8, 3)$
   $U(7, 6), U'(5, 0)$
   $V(9, 4), V'(3, 2)$

   b) $Q, R, T$
   Hor. Vert.
   i) $Q, R, T$
   $Q', R', T'$
   ii) $T, S$
   $T', S'$
   c) $N, S$
   $Q, R, T$

AP Book G6-20
page 67
1. Teacher to check coordinate grids.
   a) $i) A(3, 2), A'(5, 4)$
   $B(1, 3), B'(6, 6)$
   $C(5, 5), C'(8, 2)$
   ii) $D(8, 1), D'(3, 6)$
   $E(7, 2), E'(2, 5)$
   $Q(3, 1), Q'(3, 1)$
   iii) $F(2, 4), F'(3, 5)$
   $G(7, 4), G'(3, 0)$
   $H(6, 6), H'(5, 1)$
   $l(2, 6), l'(5, 5)$
   iv) $J(4, 5), J'(2, 3)$
   $K(6, 5), K'(2, 5)$
   $L(8, 1), L'(6, 7)$
   $M(3, 1), M'(6, 2)$

   b) $Q$
   It is the centre of rotation.

   c) no
   no

2. a) $i) N(5, 1), N'(5, 5)$
   $P(4, 0), P'(6, 6)$
   $Q(2, 3), Q'(8, 3)$
   $R(4, 3), R'(6, 3)$
   ii) $S(6, 1), S'(6, 5)$
   $T(4, 3), T'(8, 3)$
   $U(7, 6), U'(5, 0)$
   $V(9, 4), V'(3, 2)$

   b) $Q, R, T$
   Hor. Vert.
   i) $Q, R, T$
   $Q', R', T'$
   ii) $T, S$
   $T', S'$
   c) $N, S$
   $Q, R, T$
d) The $x$-coordinate of a point on the same vertical line as the centre of rotation doesn't change in a $180^\circ$ CW or CCW rotation, and the $y$-coordinate of a point on the same horizontal line as the centre of rotation doesn't change in a $180^\circ$ CW or CCW rotation, because any line through the centre of rotation is the same as its image under this rotation.

3. Teacher to check coordinate grid.

$S'T'U'V'$ and $S^*T^*U^*V^*$ are on the same place on the grid, but the images of individual vertices are different. $S' = U^*$, $T' = V^*$, $U' = S^*$, $V' = T^*$

4. a) Simon is correct. The coordinates of the image are $(5, 6)$. Rotation of $180^\circ$ around a point on the same vertical line will result in an image with the same $x$-coordinate.

b) Kathy is correct. The image coordinates are $(2, 3)$. A $90^\circ$ rotation will change the $x$-coordinate.

5. a) Teacher to check.

b) $D^* (6, 1)$

c) $A^* (2, 5)$, $B^* (6, 5)$, $C^* (6, 1)$, $D^* (2, 1)$

d) $ABCD$ and $A^*B^*C^*D^*$ take up the same place on the grid because the centre of rotation was the centre of the shape.

$B = A^*$, $C = B^*$, $D = C^*$, $A = D^*$

BONUS

a) Teacher to check.

b) $H (6, 6)$

c) The smallest clockwise rotation is $180^\circ$ because parallelograms have rotational symmetry of order 2, meaning there are 2 positions that the shape will look exactly the same when rotated around the centre.

6. Answers may vary.

Teacher to check.

Sample answer:

Translate $P$ down 2 units, then reflect $P$ in the line $x = 4$.

7. a) Translate $M$ 6 units right.

Reflect $M$ in the line $x = 5$.

Rotate $M$ $180^\circ$ CCW around $(5, 3)$.

b) Translation:

$(2, 1) \rightarrow (8, 1)$

$(1, 3) \rightarrow (7, 3)$

$(2, 5) \rightarrow (8, 5)$

$(3, 3) \rightarrow (7, 3)$

Reflection:

$(2, 1) \rightarrow (8, 1)$

$(1, 3) \rightarrow (9, 3)$

$(2, 5) \rightarrow (8, 5)$

$(3, 3) \rightarrow (7, 3)$

Rotation:

$(2, 1) \rightarrow (8, 5)$

$(1, 3) \rightarrow (9, 3)$

$(2, 5) \rightarrow (8, 1)$

$(3, 3) \rightarrow (7, 3)$

c) i) top and bottom

ii) none

iii) left and right

8. Answers will vary.

Teacher to check.
1. b) 6 6 6
   c) 6 6 6
   d) 6 6 6

2. a) 6 6
   b) 6 6 6 6
   c) 6 6 6
   d) 6 6 6 6

3. b) 7 3 + □
   c) 8 3 + □
   d) 10 2 + □

4. a) 6 6 6
   b) 6 6 6
   c) 6 6 6
   d) 6 6 6

5. a) 6 6 6
   b) 6 6 6
   c) 6 6 6
   d) 6 6 6

6. a) 3
   b) 3
   c) 4
   d) 3
   e) 4
   f) 2
   g) 3
   h) 5
   i) 3
   j) 8
   k) 15

BONUS
a) 3, 3
b) 4, 4
c) 3, 3

7. Possible answers:
   1 + 1 + 3 = 5
   2 + 2 + 1 = 5
   0 + 0 + 5 = 5

8. b) 2
   c) 3
   d) 4
   e) 5
   f) 6
   g) 7
   h) 8
   i) 9
   j) 10
   k) 15

9. Choice of variable will vary.
   b) 13 + x = 21, so x = 8
   c) 64 + b = 100, so b = 36
   d) 32 + x = 75, so x = 43

10. b) 14
    c) 24 = x + 9
        x = 15
    d) x − 3 = 8
        x = 11
    e) 30 = x − 7
        x = 37
    f) 2 × x = 12
        x = 6
    g) 30 = x × 10
        x = 3
    h) x + 3 = 7
        x = 21

BONUS
a) −63 − 63
   x = −22

AP Book PA6-10
page 73
1. b) x + 3
   c) 3x or 3 × x
   d) 2x + 4

2. b) 2x + 4
   c) 3x
   d) 3x = 15

3. b) 2x + 3 = 13
   c) 5x + 2 = 17
   d) 3x + 2 = 11
   e) 3x + 2 = 14
   f) 3x + 7 = 31

4. No
   We cannot show taking away 4 apples.

5. a) Cross out 2 apples from each side.
    3
   b) Cross out 1 apple from each side.
    5

6. b) 5
   c) 10
   d) 52

7. a) 17
   b) 16
   c) 18
   d) 48

8. a) 5
   b) 14
   c) 15
   d) 8

9. Teacher to check pictures.
   a) 2
   b) 3 × b = 9
   c) 6 = 3 × b
   d) 2b = 6
   e) b = 3

10. a) 2
    b) 4
    c) 5
    d) 6
    e) 7
    f) 8
    g) 9
    h) 10
    i) 11
    j) 12
    k) 13

11. a) 2
    b) 4
    c) 5
    d) 6

12. a) 15
    b) 18
    c) 19

13. a) 3
    b) 8
    c) 5

14. a) ii) b × 5 + 5
    b) b = 4
    iii) b × 2 + 2
        = 16 + 2
        b = 8

15. b) LS: O
    RS: OOOOO
   c) LS: OOOOOOOO
   RS: O

16. a) 3
    b) Cross out 1 apple from each side.
       6 = 3 × b
       2 = b

17. Teacher to check pictures.
   a) 2
   b) 3 × b = 9
   c) 6 = 3 × b
   d) 2b = 6
   e) b = 3
### Patterns and Algebra: Equations and Graphs – AP Book 6.2: Unit 12

#### (continued)

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</tbody>
</table>
| **1.**  
  a) 4  
  b) 3  
  c) 2  
  d) 4  
  e) 6  
  f) 3  
  b) 4  
  c) 3  
  d) 4  
  e) 6  
  f) 3  
  b) 4  
  c) 3  
  d) 4  
  e) 6  
  f) 3  |  
| **2.**  
  a) −  
  b) +  
  c) x  
  d) +  
  e) x  
  f) −  
  b) × 4  
  c) + 2  
  d) + 4  
  e) + 3  
  f) − 2  
  g) + 2  
  h) × 2  
  i) + 2  
  k) + 3  
  l) + 5  
  m) + 5  
  n) − 7  
  o) × 5  |  
| **3.**  
  b) × 4  
  c) + 2  
  d) + 4  
  e) + 3  
  f) − 2  
  g) + 2  
  h) × 2  
  i) + 2  
  k) + 3  
  l) + 5  
  m) + 5  
  n) − 7  
  o) × 5  |  
| **4.**  
  c) x − 5  
  d) 5 − x  
  e) x + 10  
  f) 9 + x  
  g) 8 × x or 8x  
  h) 9 + x  |  
| **BONUS**  
  y + x  |  
| **5.**  
  b) divide by 3  
  c) add 9  
  d) multiply by 2  
  e) subtract 7  
  f) divide by 5  
  g) divide by 2  
  h) multiply by 8  
  i) add x  |  
| **6.**  
  b) x = 18  
  18 + 6 = 3 ✓  |  
|  | **AP Book PA6-12** |  
| **page 79** | **page 79** |
| **1.**  
  b) x = 10 − 3  
  x = 7  
  c) x = 41 − 25  
  x = 16  
  d) x = 34 − 21  
  x = 13  
  e) 28 − 8 = x  
  20 = x  
  f) 41 − 14 = x  
  27 = x  
  g) x = 56 − 17  
  x = 39  
  h) x = 33 − 22  
  x = 11  
  i) x = 34 − 16  
  x = 18  
  j) x = 61 − 35  
  x = 26  
  k) x = 100 − 6  
  x = 94  |  
| **2.**  
  b) x = 5 + 12  
  x = 17  
  c) 26 + 3 = x  
  29 = x  
  d) x = 9 + 19  
  x = 28  
  e) x = 28 + 7  
  x = 35  
  f) x = 22 + 13  
  x = 35  
  g) 14 + 27 = x  
  41 = x  
  h) 29 + 32 = x  
  61 = x  
  i) x = 62 + 15  
  x = 77  
  j) 43 + 19 = x  
  62 = x  
  k) x = 49 + 51  
  x = 100  
  l) 73 + 21 = x  
  94 = x  |  
| **3.**  
  b) 2  
  c) 3  
  d) 3  |  
| **4.**  
  b) 2x + 2 = 10 + 2  
  x = 5  
  c) 6x + 6 = 42 + 6  
  x = 7  
  d) 2x + 2 = 14 + 2  
  x = 7  
  e) 7x + 7 = 28 + 7  
  x = 4  
  f) 6x + 6 = 18 + 6  
  x = 3  
  g) 7x + 7 = 49 + 7  
  x = 7  
  h) 8x + 8 = 48 + 8  
  x = 6  |  
| **5.**  
  b) x = 8 + 2  
  x = 4  
  c) x = 5 × 4  
  x = 20  
  d) x = 8 − 3  
  x = 5  |  
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Patterns and Algebra: Equations and Graphs – AP Book 6.2: Unit 12

(continued)

e) \(x = 6 + 5\)
   
   \(x = 11\)
    
f) \(x = 4 \times 3\)
   
   \(x = 12\)
    
g) \(x = 12 - 5\)
   
   \(x = 7\)
    
h) \(12 + 2 = x\)
   
   \(6 = x\)
    
i) \(15 + 3 = x\)
   
   \(5 = x\)
    
j) \(4 \times 3 = x\)
   
   \(12 = x\)
    
k) \(x = 4 \times 7\)
   
   \(x = 28\)
    
l) \(x = 7 \times 4\)
   
   \(x = 28\)
    
m) \(x = 27 + 3\)
   
   \(x = 9\)
    
n) \(36 + 12 = x\)
   
   \(3 = x\)
    
BONUS
   
   \(x = 5 \times 3\)
   
   \(x = 15\)

AP Book PA6-13

page 81

1. a) \(3(3) + 2 = 11\)
    
   \(3 = 3(4) + 2 = 14\) ✓
    
   \(4\)
    
   b) \(4(2) + 3 = 11\)
    
   \(3 = 4(3) + 3 = 15\) ✓
    
   \(4(4) + 3 = 19\)
   
   \(5 = 4(5) + 3 = 23\) ✓
    
   \(5\)
    
   c) \(5(1) - 2 = 3\)
    
   \(2 = 5(2) - 2 = 8\) ✓
    
   \(3 = 5(3) - 2 = 13\)
    
   \(3\)
    
   2. a) \(3(6) + 2 = 20\)
    
   \(6 = 5(4) + 1 = 21\)
    
   \(5 = 5(5) + 1 = 26\)
    
   \(\text{too high}\)
    
   \(c)\)
    
   \(5 = 2(5) + 3 = 13\)
    
   \(6 = 2(6) + 3 = 15\)
    
   \(\text{too low}\)
    
   d) \(5 = 4(5) + 3 = 23\)
    
   \(6 = 4(6) + 3 = 27\)
    
   \(\text{too low}\)
    
   e) \(4 = 5(4) - 6 = 14\)
    
   \(5 = 5(5) - 6 = 19\)
    
   \(\text{too high}\)
    
   f) \(5 = 3(5) - 3 = 12\)
    
   \(6 = 3(6) - 3 = 15\)
    
   \(\text{too low}\)
    
   3. a) \(n = 4\)
    
   b) \(n = 3\)
   
   c) \(n = 4\)
    
   d) \(n = 6\)
   
   e) \(n = 3\)
    
   f) \(n = 3\)

AP Book PA6-14

page 82

1. b) Saturday, 13 km
   
   Sunday, 14 km
   
   Total: \(x = 13 + 14\)
   
   \(x = 27\)
   
   c) January, $43
   
   February, \(x\)
   
   Difference: $14
   
   \(24 - 14 = x\)
   
   \(x = 10\)
   
   d) Leviathan, 93 m
   
   Kingda Ka, \(x\) m
   
   Difference: 46 m
   
   \(93 + 46 = x\)
   
   \(x = 139\)
   
   e) White, 139
   
   Yellow, \(x\)
   
   Total: 473
   
   \(473 - 139 = x\)
   
   \(x = 334\)
   
   2. a) TV: 45 min
   
   Homework: \(x\) min
   
   Difference: 15 min
   
   \(45 - 15 = x\)
   
   \(x = 30\)
   
   3. a) i) 30 + 45 = 75
   
   Alex read for 75 minutes altogether.
   
   ii) 7:50 − 0:45 − 0:30 = 6:35
   
   Alex started eating dinner at 6:35 p.m.
   
   b) i) 18 − 7 = 11
   
   There are 11 field players on the team.
   
   ii) 11 − 7 = 4
   
   There are 4 more field players than reserve players.
   
   4. a) 16 + 25 = 41
   
   41 − 13 = 28
   
   Mary has 28 stickers left.
   
   b) 28 − 13 = 15
   
   15 − 13 = 2
   
   2 more students wear glasses than don’t wear glasses.
   
   c) 7 + 3 = 10
   
   7 + 10 = 17
   
   Shawn read 17 books altogether.
   
   d) 12 + 32 + 25 = 69
   
   75 − 69 = 6
   
   Ava does not have enough money to buy the pants.
   
   e) White, 139
   
   Yellow, \(x\)
   
   Total: 473
   
   \(x = 473 − 139\)
   
   \(x = 334\)

AP Book PA6-15

page 84

1. b) There are five times as many apples as pears.
   
   c) There are four times as many cats as dogs.
   
   d) Ed’s wallet is one-sixth times as heavy as his suitcase.
   
   e) A kitten is four times as big as a mouse.
   
   f) A bus holds ten times as many people as a car.
   
   2. b) apple: 90 g
   
   cherry: \(x\) g
   
   \(90 = 10 \times x\)
   
   c) computer: \(x\)
   
   tablet: $225
   
   \(x = 3 \times 225\)

23 500 − 12 700
   
   = 10 800 more houses than apartments
   
   23 500 − 750
   
   = 22 750 houses
   
   12 700 + 2400
   
   = 15 100 apartments
   
   22 750 − 15 100
   
   = 7650 more houses than apartments

BONUS

a) 352.24 − 237.57
   
   = $114.67 deposited in July
   
   b) 528.06 − 237.57
   
   = $290.49 deposited in July and August
   
   c) 290.49 − 114.67
   
   = $175.82 deposited in August
   
   175.82 − 114.67
   
   = $61.15 more deposited in August than in July
   
   d) 699.98 − 528.06
   
   = $171.92 more needs to be saved in September.

Answer Keys for AP Book 6.2
Patterns and Algebra: Equations and Graphs – AP Book 6.2: Unit 12

BONUS

Amir: $x$ years old
Lara: 5 years old
$x = 10 \times 5$

3. a) $x = 8 \times 32$
   $x = 256$
   Carl planted 256 tomato plants.

b) $20 = 5x$
   $x = 4$
   The great white shark is 4 m long.

c) $220 = 4x$
   $x = 55$
   The chair weighs 55 kg.

d) $620 = 4x$
   $x = 155$
   The female alligator weighs 155 kg.

4. b) $30, 5, x, 30 = 5x$

c) $24, 6, x, 24 = 6x$

d) $x, 11, 4, x = 11 \times 4$

e) $50, x, 10, 50 = 10x$

5. a) $x = 5$

b) $x = 6$

c) $x = 4$

d) $x = 44$

e) $x = 5$

f) $x = 198$

6. a) $1960 = 10x$
   $x = 196$
   There are 196 seats in each car.

b) $492 = 12x$
   $x = 41$
   41 cars can park in each row.

c) $x = 10 \times 3$
   $x = 30$
   The pine tree is 30 m tall.

d) $19.50 = 3x$
   $x = 6.50$
   The soft toy costs $6.50.

e) $12 = 2x$
   $x = 6$
   Ella is 6 years old.

AP Book PA6-16

page 86

1. a) 7

b) 17

b) 11

c) 13

d) 3

e) 15

2. 1, 1, 2, 3, 5, 8

BONUS

3. 24, 14, 27, 33

4. a) $\begin{bmatrix}
11 \\
14
\end{bmatrix}$

b) $\begin{bmatrix}
50 \\
53 \\
62 \\
79
\end{bmatrix}$

5. b) $\begin{bmatrix}
(1, 2), (2, 4), (3, 6), \\
(4, 8), (5, 10)
\end{bmatrix}$

6. a) $\begin{bmatrix}
(1, 1), (2, 4), (3, 6), \\
(4, 12)
\end{bmatrix}$

b) $\begin{bmatrix}
(1, 3), (2, 6), (3, 9), \\
(4, 12)
\end{bmatrix}$

c) $\begin{bmatrix}
(1, 10), (2, 7), (3, 6), \\
(4, 3)
\end{bmatrix}$

d) $\begin{bmatrix}
(1, 2), (2, 3), (3, 4), \\
(4, 9), (5, 11)
\end{bmatrix}$

e) $\begin{bmatrix}
(1, 11), (2, 9), (3, 7), \\
(4, 5), (5, 3)
\end{bmatrix}$

f) $\begin{bmatrix}
(1, 4), (2, 8), (3, 10), \\
(4, 11), (5, 12)
\end{bmatrix}$

7. Circles parts c) and e).

8. a) $\begin{bmatrix}
4 \\
5 \\
6
\end{bmatrix}$
Patterns and Algebra: Equations and Graphs – AP Book 6.2: Unit 12

(continued)

Answer Keys for AP Book 6.2

b) 3
4
5
c) 1
2
3
2
4
3
6
d) 1
2
5
3
8
2
8
3
11

9. a) 12, 16, 20
b) 5, 6, 7, 8, 9
c) 5, 7, 9, 11
d) 1, 4, 7, 10, 13

10. a) (2, 6), (3, 7), (4, 8)

b) (2, 5), (3, 7), (4, 9)
c) (1, 1), (2, 4), (3, 7), (4, 10)

2. a) (2, 3) 2 3
(3, 4) 3 4
(4, 5) 4 5
b) (1, 9) 1 9
(2, 7) 2 7
(3, 5) 3 5
(4, 3) 4 3
c) (1, 1) 1 1
(2, 4) 2 4
(3, 7) 3 7
(4, 10) 4 10

3. Part b)

4. a) 8, 12, 16
b) 17, 11, 9, 4, 1
c) 2, 3, 4, 3, 2
d) 5, 9, 5, 9, 5, 9, 5, 9

5. Teacher to check graphs.

6. a) A
b) Teacher to check.

7. a) $5
b) $6
c) $7

BONUS

$13

AP Book PA6-17

page 89

1. a) (2, 6), (3, 4), (4, 2), (5, 0)
b) 6
c) 2
d) 2

2. a) 20 km
b) 40 km
c) Yes. Between hours 3 and 4, the graph is horizontal, indicating that the distance travelled did not increase, so Kelly rested during this period of time.

3. a) Dependent: distance
Independent: time
b) i) 40 m
ii) 100 m

e) 5
d) decreasing sequence

b) 6
5
4
3

c) 240 m

4. a) Dependent: cost
Independent: time
b) i) $12
ii) $20
iii) $16
c) $4

5. a) The input increases by 1 each time.
The output increases by 6 each time.
Multiply the input by 6 to get the output.
b) The input increases by 1 each time.
The output increases by 9 each time.
Multiply the input by 9 to get the output.
c) The input increases by 1 each time.
The output increases by 9 each time.
Multiply the input by 9 to get the output.

BONUS

The graphs are the same.

AP Book 6-19

page 93

1. a) 5, 7, 9
b) 6, 8, 10
c) 12, 9, 6
d) 5, 9, 13

2. a) i) 4, 7, 10, 13
gap: +3
ii) 2.5, 3, 3.5, 4
gap: +0.5
iii) 7, 9, 11, 13
gap: +2
iv) 4, 4, 4, 4
gap: 0
b) The number that the rule tells you to multiply by is the same as the gap.

3. a) gap: +4
4, 8, 12
7
4, 7
b) gap: +2
2, 4, 6
3
2, 3
c) gap: +3
3, 6, 9
1
3, 1

6. a) 2 km 900 15
4 km 1800 30
8 km 3600 60

b) Teacher to check graph.
Randi can run 6 km in 45 minutes.

BONUS

2 km
4 km
8 km

The graphs are the same.
Patterns and Algebra: Equations and Graphs – AP Book 6.2: Unit 12

(continued)

4. a) gap: +6
   6, 12, 18
   6, 2

   Subtract 1
   Formula: 2n – 1

b) gap: +11
   11, 22, 33
   10, 20, 30

   Subtract 3
   Formula: 3n – 3

c) gap: +5
   5, 10, 15
   4, 9, 14

   Subtract 2
   Formula: 2n – 2

d) gap: +2
   2, 4, 6
   1, 3, 5

   Subtract 1
   Formula: 2n – 1

5. a) gap: +3
   3, 6, 9
   2, 5, 8

   Subtract 2
   Formula: 3n – 2

b) gap: +4
   4, 8, 12
   3, 7, 11

   Subtract 3
   Formula: 4n – 3

c) gap: +6
   6, 12, 18
   5, 11, 17

   Subtract 4
   Formula: 6n – 4

6. a) gap: +2
   2, 4, 6
   1, 3, 5

   Subtract 1
   Formula: 2n – 1

b) gap: +11
   11, 22, 33
   10, 20, 30

   Subtract 2
   Formula: 11n – 2

c) gap: +5
   5, 10, 15
   4, 9, 14

   Subtract 2
   Formula: 5n – 2

d) gap: +1.5
   1.5, 3, 4.5
   1, 2.5, 4

   Subtract 0.5
   Formula: 1.5n – 0.5

e) gap: +2
   2, 4, 6
   1, 3, 5

   Subtract 1
   Formula: 2n – 1

f) gap: \( \frac{1}{5} \)
   \( \frac{1}{5}, \frac{2}{5}, \frac{3}{5} \)

7. a) gap: +6
   6, 12, 18

   Subtract 1
   Formula: 2n – 1

b) gap: +3
   3, 6, 9

   Subtract 2
   Formula: 3n – 2

3. Teacher to check.
1. b) 5 + 5 = 10
   2 × 5 = 10
   c) 5 + 5 + 5 = 15
   3 × 5 = 15

2. b) 4
   5
   5 × 4 = 20
   c) 3
   7
   7 × 3 = 21

3. Teacher to check lines.
   a) 4 × 1 = 4
   b) 3 × 2 = 6
   c) 4 × 2 = 8

4. b) square millimetre
   c) square kilometre

5. a) ii) 6 cm × 4 cm = 24 cm²
     iii) 8 m × 3 m = 24 m²
   b) length × width

6. a) 8 m × 3 m = 24 m²
   b) 8 cm × 7 cm = 56 cm²
   c) 6 cm × 6.1 cm = 36.6 cm²

7. a) Sample estimates:
     4 cm, 4 cm, 16 cm²
     4 cm, 3.5 cm, 14 cm²
   b) Teacher to check drawing.
     Sample answer:

8. a) 9 m × 7 m = 63 m²
   b) 12 m × 9 m = 108 m²
   c) 16 cm × 8.5 cm = 136 cm²
   d) 27 cm × 11 cm = 297 cm²
   e) 39 mm × 30.2 mm = 1177.8 mm²
   f) 12 km × 3.1 km = 37.2 km²

9. a) 300 m²
   b) $105

10. a) Perimeter:
      B (2 × 3 cm) + (2 × 2 cm) = 10 cm
       C (2 × 3 cm) + (2 × 4 cm) = 14 cm
       D (2 × 5 cm) + (2 × 2 cm) = 14 cm
       E (2 × 2 cm) + (2 × 7 cm) = 18 cm
       F (2 × 2 cm) + (2 × 4 cm) = 12 cm

      Area:
      B 3 cm × 2 cm = 6 cm²
      C 3 cm × 4 cm = 12 cm²
      D 5 cm × 2 cm = 10 cm²
      E 2 cm × 7 cm = 14 cm²
      F 2 cm × 4 cm = 8 cm²

b) no
c) C, D
d) E, A, C and D, F, B
f) no
g) Alice is not correct. For example, shape A has a larger area than shape E, but shape E has a larger perimeter than shape A.

h) Tristan is not correct. For example, shape E has a larger perimeter than shape A, but shape A has a larger area than shape E.

4. a) Teacher to check drawing.
     9 cm²
b) Teacher to check drawing.
     16 cm
c) No, because the only possible side length for a square with perimeter 12 cm is 12 cm ÷ 4 = 3 cm.
d) No, because the only possible side length for a square with area 16 cm² is √16 = 4 cm.

5. a) area
   b) perimeter
   c) area
   d) perimeter

2. Teacher to check drawings.
   a) 28 mm² or 12 mm²
      16 mm² or 32 mm²
      44 mm²
   b) 40 cm² or 24 cm²
      12 cm² or 28 cm²
      52 cm²
   c) 10 m² or 16 m²
      27 m² or 21 m²
      37 m²

3. a) 3
b) 40

4. Teacher to check drawings.
   a) Missing side lengths:
      2 m, 4 m
      4 m² or 12 m²
      24 m² or 16 m²
      28 m²
   b) Missing side lengths:
      3 cm, 7 cm
      18 cm² or 9 cm²
      12 cm² or 21 cm²
      30 cm²

5. a) Area of A
     = 79 200 cm²
     Perimeter of A
     = 1320 cm
     Area of B
     = 82 800 cm²
     Perimeter of B
     = 1200 cm
   b) B
   c) A
6. Area of A = 2.5 km²
Perimeter of A = 9 km
Area of B = 3.5 km²
Perimeter of B = 9 km
7. Answers will vary.
Teacher to check shapes.

AP Book ME6-11
page 104
1. a) Height = 5
   Width = 4
b) Height = 3
   Base = 5
   Height = 3
   Width = 5
c) Height = 5
   Base = 4
   Height = 5
   Width = 5
d) Height = 3
   Base = 5
   Height = 3
   Width = 5
2. a) height
   base
b) base, height
3. a) 35 cm²
b) 12 m²
   12.2 m²
c) 52 mm²
   52 mm²
d) 22.2 cm²
4. First shape:
   Height = 3 cm
   Base = 5.5 cm
   Area = 16.5 cm²
Second shape:
   Height = 5 cm
   Base = 3.3 cm
   Area = 16.5 cm²
5. Teacher to check drawings.
   Estimates will vary.
a) Estimate:
   Height = 3 cm
   Base = 5 cm
   Area = 15 cm²
   Actual:
   Height = 2 cm
   Base = 5 cm
   Area = 10 cm²

b) Estimate:
   Height = 4 cm
   Base = 4 cm
   Area = 16 cm²
   Actual:
   Height = 3 cm
   Base = 4 cm
   Area = 12 cm²
6. The area of one window is 1 m × 1.3 m = 1.3 m².
The area of ten windows is 1.3 m² × 10 = 13 m². So the total cost to replace all ten windows is $23 × 13 m² = $299.

AP Book ME6-12
page 106
1. a) 4
b) 6
   12
   10
   5
   15
d) 3
   6
   9
2. Teacher to check drawings.
b) 6
   6
   12
c) 10
   5
   15
d) 3
   6
   9
3. a) Rectangle A: 10
   Rectangle B: 30
   Rectangle C: 40
   Triangle A: 5
   Triangle B: 15
   Triangle C: 20
b) 1
   2
4. Jun is not correct. Triangle T is not a right triangle and you cannot use two congruent copies of T to make the rectangle shown.

AP Book ME6-13
page 108
1. a) divide in half
   16
   2, 8
b) 6
   6
   12
c) 10
   5
   15
d) 3
   6
   9
3. a) Rectangle A: 10
   Rectangle B: 30
   Rectangle C: 40
   Triangle A: 5
   Triangle B: 15
   Triangle C: 20
b) 1
   2
4. The triangles all have the same base (5 units), height (4 units), and area (10 square units).
   b) Triangles will vary.
   Teacher to check. The areas will be the same.

AP Book ME6-14
page 110
1. b) 12
   6
   18
   4
   10
   14
2. a) 6 cm
   3 cm
   9 cm²
b) 4 cm
   3 cm
   6 cm²
c) 4 cm
   3.5 cm
   7 cm²
3. a) 6 cm²
   b) 12 m²
   c) 10.8 mm²
   d) 12.8 cm²
7. a) 2
   b) 2
   c) A: 9
      B: 4, 1, 2
      5, 2, 5
      C: 4, 3, 3
      7, 3, 10.5
8. height, 2
9. a) 17.1 m²
   b) 13 m²
   c) 20 cm²

AP Book ME6-15

page 112

1. a) ii) 6 m, 3 m, A
   iii) b, 2 m, 5 m²
   iv) 6 cm, h, 18 cm²
   v) w, 3 cm, 24 cm²
   vi) 43 mm, 36 mm, A
   vii) 4 km, h, 20 km²
   vii) 7 m, 6 m, A
   b) ii) 6 m × 3 m = A
      A = 18 m²
      b × 2 m = 5 m²
      b = 5 m² + 2 m
      = 2.5 m
   iv) 6 cm × h = 18 cm²
   h = 18 cm² + 6 cm
   = 3 cm
   v) w × 3 cm = 24 cm²
      w = 24 cm² + 3 cm
      = 8 cm
   vi) A = (43 mm × 36 mm) + 2
      = 1548 mm² + 2
      = 774 mm²
   vii) 4 km × h = 20 km²
   h = 20 km² + 4 km
   = 5 km
   viii) A = 7 m × 6 m
      = 42 m²

2. b) Given:
   base = 3 m
   area = 12 m²
   Find: height of parallelogram
   Formula:
   Area = base × height
   Equation:
   12 m² = 3 m × h
   h = 12 m² + 3 m
   = 4 m
   c) Given:
   width = 10 cm
   area = 300 cm²
   Find: length of rectangle
   Formula:
   Area = length × width
   Equation:
   300 cm² = ℓ × 10 cm
   ℓ = 300 cm² + 10 cm
   = 30 m
   d) Given:
   area = 2.4 m²
   length = 3 m
   Find: width of rectangle
   Formula:
   Area = length × width
   Equation:
   2.4 m² = 3 m × w
   w = 2.4 m² + 3 m
   = 0.8 m
   e) Given:
   height = 80 cm
   area = 6000 cm²
   Find: base of parallelogram
   Formula:
   Area = base × height
   Equation:
   6000 cm² = b × 80 cm
   b = 6000 cm² + 80 cm
   = 75 cm
   f) Given:
   area = 10.2 m²
   length = 3 m
   Find: width of rectangle
   Formula:
   Area = length × width
   Equation:
   10.2 m² = 3 m × w
   w = 10.2 m² + 3 m
   = 3.4 m

BONUS

Given:
hexagon side = 8 cm
hexagon area = 168 cm²
number of rhombuses = 3
Find: height of rhombus
Formula:
Area of rhombus = base × height
Equation:
56 cm² = 8 cm × h
h = 56 cm² + 8 cm
= 7 cm
The area of each rhombus is 56 cm².
The height of each rhombus is 7 cm.

AP Book ME6-16

page 114

1. b) 521
   c) 297
   d) 670
   e) 203
   f) 32

2. a) i) 100
       100
      ii) 200
       200
      iii) 300
       300
   b) i) 1
      100, 100
     10 000
    ii) 4
     200, 200
    40 000
   iii) 9
     300, 300
    90 000
   c) 10 000
3. a) 120 000
b) 9500
c) 24 000
4. a) area = 25 m^2
   length = 250 cm = 2.5 m
   width = 25 m × 2.5 m = 10 m
   The width of the rectangle is 10 m.
   b) area = 0.24 m^2
      height = 40 cm
      area = base × height
      2400 cm^2 = b × 40 cm
      b = 2400 cm^2 ÷ 40 cm = 60 cm
      The base of the parallelogram is 60 cm.
5. a) no
   Lynn did not convert the base to centimetres to match the height.
b) no
   Cam did not convert the height to metres to match the base.
c) 1.6
   d) 200
   16 000
e) Teacher to check.
6. a) The area of the square is 2 m × 2 m = 4 m^2.
The area of each triangle is (4.3 m × 2 m) ÷ 2 = 4.3 m^2.
So the total area is 4 m^2 + 4.3 m^2 + 4.3 m^2 = 12.6 m^2.
b) The area of the rectangle is (3 cm + 3 cm) × 4.4 cm = 6 cm × 4.4 cm = 26.4 cm^2.
The area of the first triangle is (3 cm × 4 cm) ÷ 2 = 6 cm^2.
Convert the measurement in mm to cm: 30 mm = 3 cm.
The area of the second triangle is (3 cm × 4.4 cm) ÷ 2 = 6.6 cm^2.
So the total area is 26.4 cm^2 + 6 cm^2 + 6.6 cm^2 = 39 cm^2.
c) Convert the measurement in m to km: 5990 m = 5.99 km.
The area of each parallelogram is 5.99 km × 3 km = 17.97 km^2.
So the total area is 17.97 km^2 × 2 = 35.94 km^2.
7. a) 5 m
b) 1 m × 5 m = 5 m^2
c) (7 m × 5 m) − 5 m^2 = 35 m^2 − 5 m^2 = 30 m^2
   The area of the flower beds is 30 m^2.
d) The cost of covering the path in tiles is $3 × 5 m^2 = $15.
The cost of planting the flower beds is $5 × 30 m^2 = $150.
   So the total cost of creating the garden is $15 + $150 = $165.
8. The base of the smaller parallelogram is 5 m and the height is 280 cm = 2.8 m. So the area is 5 m × 2.8 m = 14 m^2.
The area of the larger parallelogram is 32.5 m^2 − 14 m^2 = 18.5 m^2.
Area = base × height, so 8.5 m^2 = 5 m × h.
h = 8.5 m^2 ÷ 5 m = 1.7 m
The height of the larger parallelogram is 1.7 m.
BONUS
The base of the rhombus-shaped field is 5 kaans.
Area = base × height, so 20 = 5 × height. The height is 20 ÷ 5 = 4 kaans.
1. b) 1 : 2
c) 3 : 3
d) 3 : 6
e) 3 : 2
f) 3 : 15

2. b) 3 : 3
c) 3 : 3
d) 2 : 2

3. a) 3 : 6
b) 2 : 6
c) 4 : 1
d) 4 : 2
e) 3 : 5

BONUS
5 : 11

4. a) 9
b) 4 : 9

5. a) the ratio of triangles to circles
b) the ratio of squares to shapes

6. Answers will vary.
Sample answer: □□□□□□□□□□

AP Book NS6-59
page 117

1. a) 1, 2
b) 4, 8
c) 5, 15
d) 4, 12

2. □□□□□□□□□□
4 : 6, 9
□□□□□□□□□□
3 : 1, 9 : 3

3. a) 9 : 6
b) 6 : 10, 9 : 15
c) 10 : 16, 15 : 24
d) 6 : 20, 9 : 30
e) 10 : 8, 15 : 12
f) 8 : 18, 12 : 27

4. a) 6
b) 12, 15

AP Book NS6-60
page 119

1. a) 16
b) 15
20
c) 6
12
d) 4
6
e) 14
28
f) 6
18

2. b) 3, 6
4, 8
c) 6, 2
9, 3
12, 4
d) 2, 14
3, 21
4, 28

e) 4, 6
6, 9
8, 12
f) 10, 4
15, 6
20, 8
g) 12, 8
18, 12
24, 16
h) 6, 10
9, 15
12, 20

3. a) 21
b) 12
c) 6

BONUS
5

4. No. You cannot multiply the first row by the same number to get the other rows.

5. Circle the first and third tables.

6. 10, 6
15, 9
She needs 15 cups of ginger ale.

BONUS
10, 6, 16
15, 9, 24
20, 12, 32
25, 15, 40
She needs 25 cups of ginger ale and 15 cups of cranberry juice to make 40 cups of punch.

AP Book NS6-61
page 121

1. b) 9, 3
25, 1
50, 2
75, 3
d) 1, 5
2, 10
3, 15
e) 2, 120
3, 180

f) 6, 1
12, 2
18, 3

2. 2, 40
3, 60
4, 80

3. b) 15
c) 75

4. a) 4
200
b) 3
150
c) 9
450

5. a) $60
b) 300

6. 15
1
30
2
45
3
60
4
75
5
90
6
He will earn $90 in 6 hours.

7. a) Car A
c) Car B
2, 24
2, 30
3, 36
3, 45
4, 48
4, 60
5, 60
b) 5 × $1.10 = $5.50
4 × $1.10 = $4.40
c) Car B. It can travel farther on less gas.

AP Book NS6-62
page 123

1. a) $3
b) $4
c) $4
d) $8
e) $10

BONUS
20

2. a) $11
b) $15

3. a) 8
b) 12
c) 20
5. a) $21
   b) $23
   c) 21

6. Circle C.

7. a) $21
   b) $23
   c) 21

8. Nickels Width (cm)
   10 1.76
   1 0.176

0.176 cm = 1.76 mm
One nickel is 1.76 mm thick.

9. a) 300
    b) 1700
    c) 9240

10. a) 2%
     b) 31%
     c) 52%
     d) 100%
     e) 17%
     f) 88%
     g) 7%
     h) 1%

11. a) 3, 2
     b) 9, 6
     There are 6 apples.

12. a) 3, 5
     b) 9, 15
     You can buy 15 tickets.

13. a) 27
    b) 100 = 27%

14. a) 20
   b) 9
   c) 25
   d) 54
   e) 18
   f) 28
   g) 765
   h) 100
   i) 2
   j) 17
   k) 12
   l) 28
   m) 28

15. a) 15 : 9 = 20 : 12
     b) 2 : 11 = 4 : 22
     c) 6 : 5 = 12 : 10
     d) 3 : 5 = 6 : 10 = 9 : 15
     e) 12 : 20 = 15 : 25
     f) 18 : 30 = 21 : 35
     g) 24 : 40

16. a) 3, 2
     b) 3, 4
     c) 3, 9

17. Circle C.

18. a) 45
    b) 150

19. Stickers Cost ($)
   3 4.95
   12 19.8

20. 12 stickers cost $19.80.
3. a) \[ \frac{60}{100} \cdot \frac{42}{100} \cdot \frac{73}{100} = 42\%, \quad \frac{3}{5} , 0.73 \]

b) \[ \frac{50}{100} \cdot \frac{67}{100} \cdot \frac{80}{100} = 10\% , 0.67 , 80\% \]

c) \[ \frac{25}{100} \cdot \frac{9}{100} \cdot \frac{15}{100} = 0.09 , 15\% , \frac{1}{4} \]

d) \[ \frac{200}{300} \cdot \frac{171}{300} = 57\% , 0.62 , \frac{2}{3} \]

AP Book NS6-67

page 132

1. a) 0.7
b) 3.2
c) 12
d) 0.38
e) 0.25
f) 0.9

2. a) 0.9
b) 0.57
c) 0.405
d) 0.635
e) 0.006
f) 2.11

3. a) i) 1.5
   ii) 1.5 , 6
b) i) 25 , 2.5
   ii) 6 , 2.5 , 15
c) i) 31 , 3.1
   ii) 9 , 3.1 , 27.9

4. a) 60\% of 120 is 72.
   70\% of 120 is 84.
   I would estimate 70\%.

b) 50\% of 32 is 16.
   About 50\% of students walk to school.

AP Book NS6-68

page 133

1. a) \[ 45 \times 32 = 1440 \]
   \[ 1440 \div 100 = 14.4 \]
   So 45\% of 32 is 14.4.

b) \[ 28 \times 63 = 1764 \]
   \[ 1764 \div 100 = 17.64 \]
   So 28\% of 63 is 17.64.

2. a) 1.17
b) 3.64
c) 5.2
d) 7.02
e) 9.66
f) 11.56
g) 29.6
h) 46.5

AP Book NS6-69

page 135

1. b) 80\%, 10\%, 10\%
c) 50\%, 40\%, 10\%
d) 22\%, 60\%, 18\%
e) 75\%, 15\%, 10\%
f) 75\%, 10\%, 15\%

2. \[ \frac{1}{10} = 10\% \]
   \[ 40\% , \$200 \]
   \[ \frac{1}{2} , \$250 \]

3. a) 25\%
   \[ 0.05 , 3 \]
   \[ \frac{1}{2} , 0.5 , 30 \]
   \[ \frac{1}{5} , 20\% , 12 \]

b) Answers will vary.
   Sample answer: 50\% is \[ \frac{50}{100} = \frac{1}{2} \].
   Half of 60 minutes is 30 minutes.

4. 15 \times 8 = 120
   120 \div 100 = 1.20
   He will pay $1.20 in tax.

5. \[ \frac{3}{4} = \frac{75}{100} = 75\% \]
   75\% of 12 is 9, so 9 green balloons have writing on them.
   60\% of 15 is 9, so 9 blue balloons have writing on them.
   9 + 9 = 18 balloons in total have writing on them.

AP Book NS6-70

page 136

1. b) 4 , 7 , 11
   \[ 12 , 15 , 27 \]
   \[ 11 , 9 , 20 \]
   \[ 7 , 3 , 10 \]

2. a) \[ \frac{6}{11} = \frac{11}{15} , \frac{7}{7} , \frac{15}{15} \]
   \[ 8 \]

3. a) \[ \frac{12}{17} \]
b) \[ \frac{3}{5} : \frac{2}{5} \]

c) \[ \frac{11}{20} : \frac{9}{20} \]

d) \[ \frac{5}{14} : \frac{9}{14} \]

e) \[ \frac{8}{15} : \frac{7}{15} \]

f) \[ 10 : 11 \]

g) \[ 11 : \frac{13}{25} \]

h) \[ 12 : \frac{13}{25} \]

4. b) \[ 3 : 7, 4 : 3, 3 : 4, \]

\[ \frac{4}{7}, \frac{3}{7} \]

c) \[ 3 : 4, 1 : 4, 3 : 1, \]

\[ \frac{1}{3}, \frac{3}{4} \]

d) \[ 27 : 50, 27 : 23, \]

\[ \frac{23}{50}, \frac{27}{50} \]

e) \[ 16 : 25, 9 : 25, 16 : 9, \]

\[ \frac{16}{25}, \frac{9}{25} \]

f) \[ 1 : 2, 1 : 2, 1 : 1, \]

\[ \frac{1}{2} \]

g) \[ 7 : 17, 10 : 17, 10 : 7, \]

\[ \frac{7}{10}, \frac{10}{17} \]

h) \[ 12 : 15, 3 : 15, 12 : 3, \]

\[ \frac{3}{15} \]

i) \[ 31 : 56, 25 : 56, \]

\[ \frac{31}{56}, \frac{25}{56} \]

5. b) \[ 65\% : \frac{65}{100}, \frac{35}{100}, \]

\[ \frac{65}{35} \]

c) \[ 25\% , 75\% , \frac{1}{4} : 1 : 3 \]

d) \[ 40\% , 60\% , \frac{10}{25}, \frac{15}{25} \]

e) \[ 50\% , 50\% , \frac{1}{2} : 1 : 1 \]

f) \[ 35\% , \frac{65}{100}, \frac{35}{100}, \]

\[ \frac{65}{35} \]

g) \[ 46\% , 54\% , \frac{23}{50}, \frac{27}{50} \]

6. a) \[ 12 \text{ kayaks, 8 canoes} \]

b) \[ 18 \text{ kayaks, 24 canoes} \]

c) \[ 9 \text{ kayaks, 6 canoes} \]

d) \[ 9 \text{ kayaks, 15 canoes} \]

7. a) Marina A = 24 kayaks

Marina B = 16 kayaks

Marina A has more kayaks.

b) Marina A = 8 kayaks

Marina B = 12 kayaks

Marina B has more kayaks.

8. a) \[ 4 : 6 \]

b) \[ \frac{4}{10} \]

c) \[ 60\% \]

9. Answers will vary.

Sample answer:

\[ \frac{1}{20} = \frac{5}{100} = 5\% \]

\[ 0.2 = \frac{20}{100} = 20\% \]

\[ \frac{1}{20} = 20\%, 0.2 \]

10. 30\% of 360 is 108.

50\% of 360 is 180.

360 − 108 − 180 = 72

So she has 108 Montreal Canadiens cards,

180 Detroit Red Wings cards, and 72 Edmonton Oilers cards.
2. a) part d)  
   \[
   \frac{3}{4} \]
   b) part c)  
   \[
   \frac{3}{3}
   \]
   c)  
   \[
   \frac{2}{6} \]
   d)  
   \[
   \frac{1}{3}
   \]

3. b)  
   \[
   \frac{5}{6}, \frac{1}{3}
   \]
   c)  
   \[
   \frac{2}{3}, \frac{3}{6} = \frac{1}{2}
   \]
   d)  
   \[
   \frac{1}{6}
   \]
   e)  
   \[
   \frac{1, 2, 3, 4, 6; 5}{6}
   \]

4. a)  
   \[
   \frac{2}{10} = \frac{1}{5}
   \]
   b)  
   \[
   \frac{8}{10} = \frac{4}{5}
   \]
   c)  
   \[
   \frac{5, 5}{10} = \frac{1}{2}
   \]
   d)  
   \[
   \frac{8, 8}{10} = \frac{4}{5}
   \]
   e)  
   \[
   \frac{7, 7}{10}
   \]

BONUS

Pulling out a marble that is not red is the same as pulling out a marble that is either blue or white.

5. The spinner is not evenly divided. When properly divided, Y has 4 outcomes, R has 1 outcome, and B has 1 outcome.

6. Teacher to check drawn lines.
   a)  
   \[
   \frac{2}{6} = \frac{1}{3}
   \]
   b)  
   \[
   \frac{1}{4}
   \]
   c)  
   \[
   \frac{3}{4}
   \]
   d)  
   \[
   \frac{3}{8}
   \]

7. a) one R square  
   b) three G squares  
   c) one B square, two R squares  
   d) three W squares, two Y squares
Probability and Data Management: Probability, Collecting and Analyzing Data – AP Book 6.2: Unit 15 (continued)

iv) 16

b) Answers will vary. Teacher to check.

1. Circle the second, third, and fifth spinners.

2. BONUS

   a) \( \frac{1}{3}, \frac{1}{3} \)  
   b) 10, 10
   c) Lily
   d) 90, 30, 10, 30, or 10
   e) Lily
   f) 19, 21, or 7

   Part a), because the zero she got on the missed test decreases her mean.

3. a) 6
   b) 12 + 0 + 4 + 8 + 6 = 30, 5, 30 + 5 = 35
   c) 1 + 16 + 8 + 11 + 9 = 45, 5, 45 + 5 = 50
   d) 21 + 6 + 12 + 1 = 40, 40 + 4 = 10
   e) 100 + 400 + 300 + 200 = 1000, 4, 1000 + 4 = 250
   f) 1000 + 1400 + 600 = 3000, 3, 3000 + 3 = 1000

   No. 35 flips would never amount to an equal number of heads and tails because it is an odd number.

4. a) 7
   b) 10
   c) 17, 18
   d) B, C, A

   BONUS

   No. 35 flips would never amount to an equal number of heads and tails because it is an odd number.

5. a) 4, 6, 6, 6, 7, 7, 36
   b) 1, 3, 2, 4, 18, 14, 36
   c) yes
   d) 36, 6, 6

   Yes, the mean stays the same.

6. a) \( (3 \times 2) + (2 \times 4) + (1 \times 8) + (3 \times 3) + (1 \times 9) + 6 + 8 + 9 + 9 + 40 + 40 + 10 = 4 \)
   b) \( (6 \times 5) + (4 \times 3) = 30 + 12 = 42 \)
   c) \( (2 \times 500) + (2 \times 300) + (1 \times 450) + 1000 + 600 + 450 = 2050 \)
   d) \( 2050 + 5 = 410 \)

   Yes, the mean stays the same.

7. a) 36
   b) 6, 60
   c) yes
   d) Predictions will vary. 164 + 16 = 10.25

   Yes, the mean stays the same.

8. a) English: 86
   b) 44
   c) yes
   d) 13

   Yes, the mean stays the same. 13 + 44 + 643 = 80.

9. a) 1.4
   b) 3
   c) yes

AP Book PDM6-13

1. a) 3
   b) 4
   c) 18

   Yes, the mean stays the same.

2. a) 2, 4, 6, 7, 8
   b) 2, 3, 3, 8
   c) 1, 4, 7, 9, 13, 26

   No, because the mean is not a whole number.

3. a) 2, 5
   b) 2, 6

   The mean increases.

4. Science: 400 + 5 = 80
   Math: 643 + 8 = 80.375

   He did better in math.
5. a) \(60 \div 6 = 10\) 
\[x = 10\]
\[\frac{+ 15}{+ 60}\]
\[x = 13\]
b) \(360 + 4 = 90\) 
\[x = 90\]
\[\frac{+ 80 + 93 + 91}{\cancel{360}}\]
\[x = 96\]
You would need 96 on the next test.

6. a) \(200 000 \times 2 \times 75 000 \times 17 \times 29 500\)
\[= 200 000 + 150 000 + 501 500 = 851 500\]
\[851 500 + 20 = 42 575\]
The mean salary is \$42 575.
b) The median salary is \$29 500.
c) The median better reflects the company's salaries because many more people earn salaries closer to the median.
d) Answers will vary. Sample answer: No, the median salary is actually much lower than \$40 000. The mean does not show how few people make a salary of above \$40 000.

7. a) \(10, 10, 10, 0\)
\[10, 10, 10, 18\]
\[10, 10, 10, 18\]
b) no
c) yes
d) no

8. a) \(2, 2, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6\)
b) \(16\)
\[8\]
\[4\]
c) The mean is 4. The modes are 4 and 5.
d) The mean is 2.75. The median is 2.5. The mode is 1.

AP Book PDM6-14

1. a) primary
b) secondary
c) primary
d) secondary
e) secondary
f) primary
g) secondary

2. Answers will vary. Teacher to check.

3. a) survey
b) measurement
c) observation
d) observation
e) survey
f) measurement

4. Answers will vary. Teacher to check.

5. a) census
b) census

b) biased representative
Sample explanation: Option A is biased because members of a track and field team are probably in better shape than students in general.

4. A: People at a beach may be biased towards a swimming pool.
B: People at a bookstore may be biased towards a library.
C: People at a professional hockey game may be biased towards a hockey arena.
D: A shopping mall is the best site because there is no obvious bias for any of the three options on the survey.

5. a) Kim's is more biased because students in her class probably know her very well.
b) Eddy will probably win because his representative survey shows that more students in the school will vote for him.

6. a) B
b) Answers will vary. Teacher to check.

7. Answers will vary. Sample answer: Josh's survey results will be biased because the question prompts you to think that spending time playing video games is good for your brain.

AP Book PDM6-16

1. a) Kathy
b) Tom: He waters the tomato plant in the sun more than the one in the shade.
Iva: She places the tomato plants in very different places with different climates.
Anton: The cat may eat the tomato plant.

2. Teacher to check.
3. Teacher to check.
4. Teacher to check.

AP Book PDM6-17
page 162

1. a) Teacher to check.
b) 75
   74
   76.75
c) Teacher to check
   number line.
   above the mean
   above the median
d) Answers will vary.
   Sample answers:
i) I agree, his mark is higher than the median.
ii) I disagree, his mark is slightly lower than the mean.
iii) I agree, his mark is the mode.
iv) I agree.
v) I agree, his mark is the mode.

2. a) Candidate A’s support increased slightly at first but then decreased over time.
   Candidate B’s support increased at first but then decreased to the original level of support.
   Candidate C’s support increased over time.
b) Candidate C, because their level of support has increased over time and I would expect it to continue to do so.

3. a) Randi is not correct.
   The mean calculated is for the average precipitation overall, but it does not reflect precipitation for each season.
b) Jake is correct. We can show this by calculating the means for the summer and winter months only and comparing them.
c) Hanna is correct.
   She can improve her argument by finding the total rainfall for Victoria and Edmonton and comparing them.

4. Answers will vary. Teacher to check.
### Measurement: 3-D Shapes, Volume, and Surface Area – AP Book 6.2: Unit 16

**AP Book ME6-17 page 164**

1. **b)** 4, 4  
   2, 4, 8  
   c) 4, 4, 4, 4, 4  
   20  
   5, 4, 20  
2. **a)** 3  
   **b)** 3  
   **c)** 3  
3. **a)** 4, 4, 4, 12  
   **b)** 3, 4, 12  
4. **a)** 6  
   **b)** 6, 6, 6, 6, 24  
   **c)** 6, 24  
5. **a)** 2, 2, 2  
   6  
   2, 6  
   **b)** 10, 10, 10, 10  
   40  
   4, 10, 40  
   **c)** 12, 12, 12  
   36  
   3, 12, 36  
6. **a)** 2, 3, 6  
   **b)** i) 2, 3, 2  
   12  
   ii) 2, 3, 3  
   18  
   iii) 2, 3, 4  
   24  
7. **a)** 4, 3, 2, 24  
   **b)** 5, 4, 3, 60  
   **c)** 6, 4, 2, 48  
8. **a)** 3, 2, 4, 24  
   **b)** 4, 2, 8  
   4, 2, 3, 24  
   **c)** 4, 3, 12  
   4, 3, 2, 24  
9. **a)** length × width × height  
   **b)** i) 12 cm² × 5 cm  
   60 cm²  
   ii) 80 mm² × 5 mm  
   400 mm³  
10. **a)** Yes, Tom is correct. Multiplication is commutative, so the order in which you multiply length, width, and height does not matter.

**BONUS**

- **AP Book ME6-18 page 166**
1. **a)** 3  
   **b)** 11  
   **c)** 12  
2. **b)** 8 cm³  
3. Teacher to check.  
4. Teacher to check.  
5. 12 cm²  
6. Estimates will vary.  
   a) Estimate: 60 × 40 × 250 = 600 000 m³  
   Actual: 616 512 m³  
   b) Estimate: 40 × 50 × 300 = 600 000 m³  
   Actual: 598 828 m³  
   c) 32 dm³  
3. **b)** 3 cm  
   2 cm  
   2 cm  
   2 cm  
4. **a)** 25 × 4 × 6 = 600 m³  
   **b)** 40 × 15 × 35  
   = 21 000 cm³  
   **c)** 4 × 2.5 × 8 = 80 m³  
   **d)** 4.5 × 4 × 3 = 54 cm³  
5. Estimates may vary.  
   a) Estimate: 10 × 10 × 10 = 1000 mm³  
   Actual: 1848 mm³  
   b) Estimate: 2 × 2 × 5  
   = 20 m²  
   Actual: 29.4 m²  
   c) Estimate: 6 × 2 × 10  
   = 120 m³  
   Actual: 117 m³  
   d) 2 × 2 × 0.5 = 2 cm³  
   Actual: 2.4 cm³  
6. **a)** ii) 12 cm²  
   12 cm³  
   3 cm  
   3  
   36 cm³  
   iii) 15 cm²  
   15 cm³  
   2 cm  
   2  
   30 cm³  
   iv) 16 cm²  
   16 cm³  
   4 cm  
   4  
   64 cm³  
7. **length × width × height**  
8. **a)** 64 m², 3 m  
   192 m³  
   **b)** 84 cm², 5 cm  
   420 cm³  
   **c)** 16.8 m³, 2 m  
   33.6 m³  
   **d)** 25 mm³, 10 mm  
   250 mm³  
   **e)** 256 m³, 4.5 m  
   1152 m³  
   **f)** 15.3 cm², 3 cm  
   45.9 cm³  
   a) 12 cm² × 5 cm  
   60 cm³  
   **b)** i) 12 cm² × 5 cm  
   60 cm³  
   ii) 80 mm² × 5 mm  
   400 mm³  
   **BONUS**  
   16 m² × 2.5 m  
   40 m³  
10. **a)** Yes, Tom is correct. Multiplication is commutative, so the order in which you multiply length, width, and height does not matter.
b) Yes, Tom is correct.
   width × length × height
   = 6 cm × 30 cm
   = 180 cm³

   c) i) 120 cm³
      ii) 480 mm³
      iii) 574 m³

AP Book ME6-20
page 171
1. a) one half
   b) 4
   c) 2
2. b) 70 cm³
   one half
   35 cm³
   c) 40 cm³
   one half
   20 cm³
3. a) one half
   b) one half
4. one half
5. a) 27 m³
   one half
   13.5 m³
   b) 160 m³
   one half
   80 m³
   c) 54 m³
   one half
   27 m³
6. Circle the following:
   a) 2 cm on the right
   b) 5 m
   c) 3 cm
7. a) i) 8 cm²
      1 cm
      8 cm³
      ii) 16 cm³
      8 cm²
      2 cm
      16 cm³
      iii) 24 cm³
      8 cm²
      3 cm
      24 cm³
   b) They are the same.
   c) area of base × height
8. a) 18 cm³
   b) 27 mm³
   c) 60 m³
   d) 10.5 cm³
   e) 4.5 cm³
   f) 20 570 m³

AP Book ME6-21
page 173
1. a) 5
   b) 4
   c) 8
2. Teacher to check.
3. Teacher to check lengths of thick edges.
   b) 6
   c) 7
4. a) 120 m³
   b) 72 cm³
   c) 252 mm³
5. a) 25 cm²
   b) 10 m²
   c) 16.5 m²
6. a) 6
   b) 6.3
   c) 4.5
   d) 2
   e) 12
   f) 9
   g) 5.5
   h) 0.6
   i) 5
7. 334 000 cm³
   a) 18 600 cm³
   b) 22 600 cm³
   c) 11.3 cm
8. a) 1925 cm³
   b) 9 cm
9. 9 cm

AP Book ME6-22
page 175
1. a) mL
   b) L
2. Circle the following:
   a) mL
   b) L
   c) mL
   d) L
3. Circle the following:
   a) 300 mL
   b) 200 L
   c) 14 000 L
   d) 330 mL
4. 2000, 3000, 4000, 5000, 6000, 7000, 8000, 1000
5. 9000
   b) 12 000
   c) 40 000
   d) 35 000
   e) 132 000
   f) 200 000
6. a) 0.590, 590
   b) 2.540, 2540
   c) 0.020, 20
   d) 4
   f) 7159
   g) 1040
   h) 24 700
7. a) circle 3 L
   b) 8000 mL
   circle 8 L
   c) 23 000 mL
   circle 23 567 mL
   d) 66 600 mL
   circle 66 666 mL
   e) 70 823 mL
   circle 70.823 L
   f) 65 200 mL
   circle 65 203 mL
8. a) Answers may vary.
   Sample answer: 7895 mL
   b) 6.906 L

AP Book ME6-23
page 177
1. a) 10, 10, 10, 1000
   b) 1000, 1000, 1
2. a) 4 L
   b) 450 mL
   c) 330 cm³
   d) 1.89 dm³
3. b) 37 cm
   20 cm
   20 cm
   37 cm × 20 cm
   × 20 cm
   14 800 cm³
   14 800 mL
   c) 5 dm
   3 dm
   3 dm
   5 dm × 3 dm × 3 dm
   45 dm³
   45 L
   d) 16 dm
   4 dm
   8 dm
   16 dm × 4 dm × 8 dm
   512 dm³
   512 L
4. a) 60 000, 60 000
   b) 5, 3, 4
   c) 60, 60
   d) The answers in parts a) and c) should be equal since they describe the same prism using different units. 60 L × 1000
   = 60 000 mL
5. a) 100, 60, 33
   b) 198 000, 198 000
   c) 198
6. 14 000, 14
7. a) 36 cm³
   b) 0.590, 590
   c) 2.540, 2540
   d) 0.020, 20
   e) 4
   f) 7159
   g) 1040
   h) 24 700
   i) 5
   j) 334 000 cm³
   k) 18 600 cm³
   l) 22 600 cm³
   m) 11.3 cm
   n) 1925 cm³
   o) 9 cm
   p) Answers may vary.
   q) Sample answer: 7895 mL
   r) 6.906 L

   c) Buying the juice in cans is cheaper because you get more volume for almost equal cost.
b) 320 dm³

c) 90 cm³

8. a) 1 L

b) 250 mL or 0.25 L

c) 1.5 L or 1500 mL

9. Circle the following:

a) 20 L

b) 250 mL or 0.25 L

c) 1.5 L or 1500 mL

a) 1 L

b) 250 mL or 0.25 L

c) 1.5 L or 1500 mL

11. a) Estimates will vary.

Sample answer:

Estimate: 100 × 50 × 20 = 10 000 mL

b) Estimates will vary.

Sample answer:

250 × 50 × 100 = 1 250 000 mL

c) Estimated volume of office aquarium + estimated volume of Avril’s aquarium

= 1 250 000 + 10 000

= 125

The water from the dental office aquarium can fill Avril’s aquarium about 125 times.

d) Avril: 12 320 mL

dentist: 1 524 000 mL

about 124 times

12. a) 100, 100, 100, 1 000 000

b) 1 800 000

c) The difference between 1 m³ and 1 cm³ is too large. In many cases neither unit would be appropriate for accuracy.

BONUS

Answers will vary.

Sample answer:

2 dm × 3 dm × 4 dm

6. Answers may vary.

Sample answer:

The net makes it easier to see all the faces and to label them clearly so that no area is missed.

b) 320 dm³

3

4

b) 24 cm²

c) 231 cm²

AP Book ME6-26

page 184

1. a) 1 L

b) 250 mL or 0.25 L

c) 1.5 L or 1500 mL

a) 52 cm²

b) 24 cm²

c) 231 cm²

AP Book ME6-27

page 186

1. Teacher to check.

BONUS

a) (7 × 5) × 2 = 70 m²

70 ÷ 0.3 ≈ 233

About 233 shingles are needed.

Check: (7.2 × 5) × 2 = 72 m²

72 ÷ 0.3 = 240

240 shingles are actually needed.

2. Teacher to check.

3. Teacher to check.

4. b) 3

c) 2

d) 4
BONUS

5. b) [Diagram]
   c) [Diagram]
   d) [Diagram]

6. Teacher to check shading.
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

7. Teacher to check shading.
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

AP Book ME6-28
page 189

1. Teacher to check shading.
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

2. Circle the fourth shape.

3. Teacher to check shading.
   b) [Diagram]
   c) [Diagram]

4. Teacher to check shading.
   b) [Diagram]
   c) [Diagram]
   d) [Diagram]

5. a) front
   b) top
   c) left-side

6. b) top:
   [Diagram]
   front:
   [Diagram]
   right side:
   [Diagram]

5. a) front
   b) top
   c) left-side

6. b) top:
   [Diagram]
   front:
   [Diagram]
   right side:
   [Diagram]

5. a) front
   b) top
   c) left-side

6. b) top:
   [Diagram]
   front:
   [Diagram]
   right side:
   [Diagram]

BONUS

7. Teacher to check.
## Number

### General Outcome

Develop number sense.

### Specific Outcomes

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Demonstrate an understanding of place value, including numbers that are:</td>
<td>Part  Unit   Lessons</td>
</tr>
<tr>
<td>• greater than one million</td>
<td>1  2         NS6-1 to 4</td>
</tr>
<tr>
<td>• less than one thousandth.</td>
<td>2  9         NS6-38, 39 NS6-40</td>
</tr>
<tr>
<td>[C, CN, R, T]</td>
<td></td>
</tr>
<tr>
<td>2. Solve problems involving whole numbers and decimal numbers.</td>
<td></td>
</tr>
<tr>
<td>[ME, PS, T]</td>
<td></td>
</tr>
<tr>
<td>[ICT: C6-2.4]</td>
<td></td>
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<tr>
<td><strong>Note:</strong> Through this outcome, students have the opportunity to maintain and</td>
<td></td>
</tr>
<tr>
<td>refine previously learned multiplication and division number facts (Grade 5) and</td>
<td></td>
</tr>
<tr>
<td>operations with whole numbers (Grades 4 and 5).</td>
<td></td>
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</tbody>
</table>
### Number

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 3.     | Demonstrate an understanding of factors and multiples by:  
        - determining multiples and factors of numbers less than 100  
        - identifying prime and composite numbers  
        - solving problems using multiples and factors.  
        [CN, R, V]  
        Note: Through this outcome, students have the opportunity to maintain and refine previously learned multiplication and division number facts (Grade 5). | 1    | 7    | NS6-20  
         |             |      |      | NS6-18, 19, 21, 23 |
| 4.     | Relate improper fractions to mixed numbers and mixed numbers to improper fractions.  
        [CN, ME, R, V] | 1    | 8    | NS6-26, 28, 32, 34, 35  
         |             |      |      | NS6-29 to 31, 33, 36, 37 |
| 5.     | Demonstrate an understanding of ratio, concretely, pictorially and symbolically.  
        [C, CN, PS, R, V] | 2    | 14   | NS6-63  
         |             |      |      | NS6-58 to 60 |
| 6.     | Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.  
        [C, CN, PS, R, V] | 2    | 14   | NS6-64 to 67, 69, 70 |
| 7.     | Demonstrate an understanding of integers, concretely, pictorially and symbolically.  
        [C, CN, R, V] | 1    | 2    | NS6-7, 8 |
| 8.     | Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).  
        [C, CN, ME, PS, R, V] | 1    | 4    | NS6-12, 13, 16  
         |             |      |      | NS6-9 to 11, 14 |
|         |             | 2    | 9    | NS6-44 |
|         |             | 2    | 10   | NS6-48 to 53, 57 |
| 9.     | Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).  
        [C, CN, ME, PS, V]  
        [ICT: C6-2.4, C6-2.7]  
        Note: Through this outcome, students have the opportunity to maintain and refine previously learned multiplication and division number facts (Grade 5) and operations with whole numbers (Grades 4 and 5). | 1    | 7    | NS6-24 |
### Patterns & Relations — Patterns

**General Outcome**
Use patterns to describe the world and to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Represent and describe patterns and relationships, using graphs and tables. [C, CN, ME, PS, R, V]</td>
<td>Part 2 Unit 12 Lessons PA6-16, 17, 20</td>
</tr>
<tr>
<td>2. Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R]</td>
<td>Part 1 Unit 1 Lessons PA6-3, PA6-4, 5, 7, 8, ME6-4</td>
</tr>
<tr>
<td></td>
<td>Part 1 Unit 5 Lessons ME6-4</td>
</tr>
<tr>
<td></td>
<td>Part 2 Unit 12 Lessons PA6-16, 17, 19, 20</td>
</tr>
</tbody>
</table>

### Patterns & Relations — Variables and Equations

**General Outcome**
Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. Represent generalizations arising from number relationships, using equations with letter variables. [C, CN, PS, R, V]</td>
<td>Part 1 Unit 1 Lessons PA6-6 to 8</td>
</tr>
<tr>
<td></td>
<td>Part 2 Unit 12 Lessons PA6-18 to 20</td>
</tr>
<tr>
<td>4. Express a given problem as an equation in which a letter variable is used to represent an unknown number. [C, CN, PS, R]</td>
<td>Part 2 Unit 12 Lessons PA6-9, 10, 14, 15</td>
</tr>
<tr>
<td>5. Demonstrate and explain the meaning of preservation of equality, concretely and pictorially. [C, CN, PS, R, V]</td>
<td>Part 2 Unit 12 Lessons PA6-10 to 13</td>
</tr>
</tbody>
</table>
### Shape & Space — Measurement

#### General Outcome
Use direct and indirect measurement to solve problems.

#### Specific Outcomes

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
</table>
| 1. Demonstrate an understanding of angles by:  
  • identifying examples of angles in the environment  
  • classifying angles according to their measure  
  • estimating the measure of angles, using 45°, 90° and 180° as reference angles  
  • determining angle measures in degrees  
  • drawing and labelling angles when the measure is specified.  
  [C, CN, ME, V] | Part Unit Lessons  
  1 6 G6-1 to 5 |
| 2. Demonstrate that the sum of interior angles is:  
  • 180° in a triangle  
  • 360° in a quadrilateral.  
  [C, R] | Part Unit Lessons  
  1 6 G6-9 |
| 3. Develop and apply a formula for determining the:  
  • perimeter of polygons  
  • area of rectangles  
  • volume of right rectangular prisms.  
  [C, CN, PS, R, V] | Part Unit Lessons  
  1 5 ME6-1, 2 ME6-4, 5  
  2 13 ME6-8 to 10  
  2 16 ME6-17 to 19, 21 |

### Shape & Space — 3-D Objects and 2-D Shapes

#### General Outcome
Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

#### Specific Outcomes

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
</table>
| 4. Construct and compare triangles, including:  
  • scalene  
  • isosceles  
  • equilateral  
  • right  
  • obtuse  
  • acute  
  in different orientations.  
  [C, PS, R, V] | Part Unit Lessons  
  1 6 G6-7 G6-8 |
| 5. Describe and compare the sides and angles of regular and irregular polygons.  
  [C, PS, R, V] | Part Unit Lessons  
  1 6 G6-6 |
## Shape & Space — Transformations

### General Outcome
Describe and analyze position and motion of objects and shapes.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.</strong> Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image. [C, CN, PS, T, V]</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> G6-13 to 17</td>
</tr>
<tr>
<td><strong>7.</strong> Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V]</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> G6-17</td>
</tr>
<tr>
<td><strong>8.</strong> Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs. [C, CN, V]</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> G6-18</td>
</tr>
<tr>
<td><strong>9.</strong> Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices). [C, CN, PS, T, V]</td>
<td><strong>Part</strong> 2  <strong>Unit</strong> 11  <strong>Lessons</strong> G6-19, 20</td>
</tr>
</tbody>
</table>
### Statistics & Probability — Data Analysis

**General Outcome**

Collect, display and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> Create, label and interpret line graphs to draw conclusions. [C, CN, PS, R, V]</td>
<td>Part 1, Unit 3, Lessons PDM6-1, PDM6-3 to 6</td>
</tr>
<tr>
<td><strong>2.</strong> Select, justify and use appropriate methods of collecting data, including: • questionnaires • experiments • databases • electronic media. [C, CN, PS, R, T]</td>
<td>Part 2, Unit 15, Lessons PMD6-14, 16</td>
</tr>
<tr>
<td><strong>3.</strong> Graph collected data, and analyze the graph to solve problems. [C, CN, PS, R, T]</td>
<td>Part 2, Unit 15, Lessons PDM6-16, 17</td>
</tr>
</tbody>
</table>

### Statistics & Probability — Chance and Uncertainty

**General Outcome**

Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

<table>
<thead>
<tr>
<th>Specific Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.</strong> Demonstrate an understanding of probability by: • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment. [C, ME, PS, T]</td>
<td>Part 2, Unit 15, Lessons PDM6-7 to 11</td>
</tr>
</tbody>
</table>
Grade 6 JUMP Math Correlation to the New BC Curriculum

NOTES:

Underlined JUMP Math lessons are review from a previous grade.

Italicized JUMP Math lessons contain prerequisite material required to meet the learning standard.

An asterisk (*) indicates that a JUMP Math lesson covers a curriculum requirement primarily in the lesson plan.

JUMP Math strands are represented by:

NS  Number Sense
ME  Measurement
G   Geometry
PA  Patterns and Algebra
PDM Probability and Data Management

Big Ideas

Mixed numbers and decimal numbers represent quantities that can be decomposed into parts and wholes.

Computational fluency and flexibility with numbers extend to operations with whole numbers and decimals.

Linear relations can be identified and represented using expressions with variables and line graphs and can be used to form generalizations.

Properties of objects and shapes can be described, measured, and compared using volume, area, perimeter, and angles.

Data from the results of an experiment can be used to predict the theoretical probability of an event and to compare and interpret.

<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>small to large numbers (thousandths to billions)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
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<td>2</td>
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<td>2</td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>• place value from thousandths to billions, operations with thousandths to billions</td>
<td>Part 1 Unit 2 Lessons NS6-1, 2, 4</td>
</tr>
<tr>
<td>• numbers used in science, medicine, technology, and media</td>
<td>Part 1 Unit 2 Lessons NS6-1 to 4</td>
</tr>
<tr>
<td>• compare, order, estimate</td>
<td>Part 1 Unit 2 Lessons NS6-3, 5, 6</td>
</tr>
<tr>
<td>• mental math strategies (e.g., the double-double strategy to multiply $23 \times 4$)</td>
<td>Part 1 Unit 4 Lessons NS6-10, 11, 13, 14</td>
</tr>
<tr>
<td>multiplication and division facts to 100 (developing computational fluency)</td>
<td>Part 1 Unit 4 Lessons NS6-10, 11, 13, 14</td>
</tr>
<tr>
<td>order of operations with whole numbers</td>
<td>Part 1 Unit 7 Lessons NS6-24</td>
</tr>
<tr>
<td>• includes the use of brackets, but excludes exponents</td>
<td>Part 1 Unit 8 Lessons NS6-36</td>
</tr>
<tr>
<td>• quotients can be rational numbers</td>
<td>Part 1 Unit 8 Lessons NS6-36</td>
</tr>
<tr>
<td>factors and multiples — greatest common factor and least common multiple</td>
<td>Part 1 Unit 7 Lessons NS6-18 to 23</td>
</tr>
<tr>
<td>• prime and composite numbers, divisibility rules, factor trees, prime factor phrase (e.g., $300 = 2^2 \times 3 \times 5^3$)</td>
<td>Part 1 Unit 7 Lessons NS6-22, 23</td>
</tr>
<tr>
<td>• using graphic organizers (e.g., Venn diagrams) to compare numbers for common factors and common multiples</td>
<td>Part 1 Unit 7 Lessons NS6-18, 21</td>
</tr>
<tr>
<td>improper fractions and mixed numbers</td>
<td>Part 1 Unit 8 Lessons NS6-26, 28</td>
</tr>
<tr>
<td></td>
<td>Part 1 Unit 6 Lessons NS6-29 to 34, 36, 37</td>
</tr>
<tr>
<td></td>
<td>Part 2 Unit 9 Lessons NS6-42</td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
</tbody>
</table>
| using benchmarks, number line, and common denominators to compare and order, including whole numbers | Part 1 Unit 8 Lessons NS6-29 to 34, 36, 37  
Part 2 Unit 9 Lessons NS6-42 |
| using pattern blocks, Cuisenaire Rods, fraction strips, fraction circles, grids | Part 1 Unit 8 Lessons NS6-26, 28  
NS6-29, 30 |
| birchbark biting                                                      | Part 1 Unit 8 Lessons NS6-30 |
| introduction to ratios                                                | Part 2 Unit 14 Lessons NS6-58 to 63 |
| comparing numbers, comparing quantities, equivalent ratios            | Part 2 Unit 14 Lessons NS6-58 to 63 |
| part-to-part ratios and part-to-whole ratios                          | Part 2 Unit 14 Lessons NS6-58 |
| whole-number percents and percentage discounts                        | Part 2 Unit 14 Lessons NS6-64 to 70 |
| using base 10 blocks, geoboard, 10 × 10 grid to represent whole number percents | Part 2 Unit 14 Lessons NS6-64, 65 |
| finding missing part (whole or percentage)                            | Part 2 Unit 14 Lessons NS6-67, 68 |
| 50% = 1/2 = 0.5 = 50:100                                              | Part 2 Unit 14 Lessons NS6-66, 69, 70 |
| multiplication and division of decimals                                | Part 1 Unit 4 Lessons NS6-12, 15, 16  
Part 1 Unit 8 Lessons NS6-30  
Part 2 Unit 10 Lessons NS6-48 to 53, 57 |
<p>| 0.125 × 3 or 7.2 ÷ 9                                                  | Part 2 Unit 10 Lessons NS6-48 to 53, 57 |
| using base 10 block array                                             | Part 2 Unit 10 Lessons NS6-51 to 53, 57 |</p>
<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• birchbark biting</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1  8  NS6-30</td>
</tr>
<tr>
<td>increasing and decreasing patterns, using expressions, tables, and</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td>graphs as functional relationships</td>
<td>1  1  PA6-3 to 8</td>
</tr>
<tr>
<td></td>
<td>1  3  PDM6-3</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-16 to 20</td>
</tr>
<tr>
<td>• limited to discrete points in the first quadrant</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-16 to 18, 20</td>
</tr>
<tr>
<td>• visual patterning (e.g., colour tiles)</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1  1  PA6-5, 7</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-16</td>
</tr>
<tr>
<td>• Take 3 add 2 each time, (2n + 1), and 1 more than twice a number</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td>all describe the pattern 3, 5, 7, …</td>
<td>1  1  PA6-3 to 8</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-17 to 20</td>
</tr>
<tr>
<td>• graphing data on First Peoples language loss, effects of language</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td>intervention</td>
<td>1  3  PDM6-3</td>
</tr>
<tr>
<td>one-step equations with whole-number coefficients and solutions</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-13</td>
</tr>
<tr>
<td></td>
<td>PA6-9 to 12, 14, 15</td>
</tr>
<tr>
<td>• preservation of equality (e.g., using a balance, algebra tiles)</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-10, 11</td>
</tr>
<tr>
<td>• (3x = 12, x + 5 = 11)</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>2  12  PA6-13</td>
</tr>
<tr>
<td></td>
<td>PA6-9 to 12, 14, 15</td>
</tr>
<tr>
<td>perimeter of complex shapes</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td>1  5  ME6-1, 2</td>
</tr>
<tr>
<td></td>
<td>ME6-4</td>
</tr>
<tr>
<td>• A complex shape is a group of shapes with no holes (e.g., use</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td>colour tiles, pattern blocks, tangrams).</td>
<td>1  5  ME6-4</td>
</tr>
<tr>
<td>area of triangles, parallelograms, and trapezoids</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td>• grid paper explorations</td>
<td>2  13  ME6-10 to 15</td>
</tr>
<tr>
<td></td>
<td>2  13  ME6-10 to 14</td>
</tr>
<tr>
<td>Content</td>
<td>JUMP Math Lessons</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>• deriving formulas</td>
<td>Part 2 Unit 13 Lessons ME6-8 to 12, 14</td>
</tr>
<tr>
<td>• making connections between area of parallelogram and area of rectangle</td>
<td>Part 2 Unit 13 Lessons ME6-11 to 13</td>
</tr>
<tr>
<td>• birchbark biting</td>
<td>Part 2 Unit 13 Lessons ME6-14</td>
</tr>
</tbody>
</table>

**angle** measurement and classification

<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• straight, acute, right, obtuse, reflex</td>
<td>Part 1 Unit 6 Lessons G6-1 to 5</td>
</tr>
<tr>
<td>• constructing and identifying; include examples from local environment</td>
<td>Part 1 Unit 6 Lessons G6-1 to 4</td>
</tr>
<tr>
<td>• estimating using 45°, 90°, and 180° as reference angles</td>
<td>Part 1 Unit 6 Lessons G6-3</td>
</tr>
<tr>
<td>• angles of polygons</td>
<td>Part 1 Unit 6 Lessons G6-5</td>
</tr>
<tr>
<td>• Small Number stories: Small Number and the Skateboard Park (mathcatcher.irmacs.sfu.ca/stories)</td>
<td>Part 1 Unit 6 Lessons G6-2</td>
</tr>
</tbody>
</table>

**volume and capacity**

<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• using cubes to build 3D objects and determine their volume</td>
<td>Part 2 Unit 16 Lessons ME6-17, 18, 22, 23</td>
</tr>
<tr>
<td>• referents and relationships between units (e.g., cm³, m³, mL, L)</td>
<td>Part 2 Unit 16 Lessons ME6-17, 18</td>
</tr>
<tr>
<td>• the number of coffee mugs that hold a litre</td>
<td>Part 2 Unit 16 Lessons ME6-22, 23</td>
</tr>
<tr>
<td>• berry baskets, seaweed drying</td>
<td>Part 2 Unit 16 Lessons ME6-23</td>
</tr>
</tbody>
</table>

JUMP Math Correlation to the New BC Curriculum — Grade 6
<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangles</td>
<td>Part</td>
</tr>
</tbody>
</table>
| • scalene, isosceles, equilateral                                      | 1    | 6    | G6-7
<p>|                                                                          |      |      | G6-8    |
| • right, acute, obtuse                                                 | 1    | 6    | G6-8    |
| • classified regardless of orientation                                | 1    | 6    | G6-8    |
| combinations of transformations                                        | Part | Unit | Lessons |
| • plotting points on Cartesian plane using whole-number ordered pairs   | 2    | 11   | G6-16   |
|                                                                          |      |      | G6-18 to 20 |
| • translation(s), rotation(s), and/or reflection(s) on a single 2D shape | 2    | 11   | G6-16   |
|                                                                          |      |      | G6-18 to 20 |
| • limited to first quadrant                                            | 2    | 11   | G6-16 to 20 |
| • transforming, drawing, and describing image                         | 2    | 11   | G6-16 to 20 |
| • Use shapes in First Peoples art to integrate printmaking (e.g., Inuit, Northwest coastal First Nations, frieze work) (mathcentral.uregina.ca/RR/database/RR.09.01/mcdonald1/) | 2    | 11   | G6-16 |
| line graphs                                                            | Part | Unit | Lessons |
| • table of values, data set; creating and interpreting a line graph from a given set of data | 1    | 3    | PDM6-1 |
|                                                                          |      |      | PDM6-2 to 4, 6 |
| single-outcome probability, both theoretical and experimental          | Part | Unit | Lessons |
| • single-outcome probability events (e.g., spin a spinner, roll a die, toss a coin) | 2    | 15   | PDM6-7 to 11 |</p>
<table>
<thead>
<tr>
<th>Content</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>• listing all possible outcomes to determine theoretical probability</td>
<td>Part 2 Unit 15</td>
</tr>
<tr>
<td></td>
<td>Lessons PDM6-7 to 11</td>
</tr>
<tr>
<td>• comparing experimental results with theoretical expectation</td>
<td>Part 1 Unit 8</td>
</tr>
<tr>
<td></td>
<td>Lessons NS6-35</td>
</tr>
<tr>
<td></td>
<td>Part 2 Unit 15</td>
</tr>
<tr>
<td></td>
<td>Lessons PDM6-10, 11</td>
</tr>
<tr>
<td>• Lahal stick games</td>
<td>Part 2 Unit 15</td>
</tr>
<tr>
<td></td>
<td>Lessons PDM6-7, 8</td>
</tr>
<tr>
<td>financial literacy — simple budgeting and consumer math</td>
<td>Part 2 Unit 14</td>
</tr>
<tr>
<td></td>
<td>Lessons NS6-62, 63</td>
</tr>
<tr>
<td>• informed decision making on saving and purchasing</td>
<td>Part 2 Unit 14</td>
</tr>
<tr>
<td></td>
<td>Lessons NS6-62, 63</td>
</tr>
<tr>
<td></td>
<td>Part 2 Unit 16</td>
</tr>
<tr>
<td></td>
<td>Lessons ME6-22*</td>
</tr>
<tr>
<td>• How many weeks of allowance will it take to buy a bicycle?</td>
<td>Part 1 Unit 1</td>
</tr>
<tr>
<td></td>
<td>Lessons PA6-5</td>
</tr>
</tbody>
</table>
### Curricular Competencies

#### Reasoning and analyzing

- **Use logic and patterns** to solve puzzles and play games
  - Part Unit Lessons
  - 1 1 PA6-4
  - 2 10 NS6-55

- **Use reasoning and logic** to explore, analyze, and apply mathematical ideas
  - Part Unit Lessons
  - 1 6 G6-8
  - 2 11 G6-14

- **Estimate reasonably**
  - Part Unit Lessons
  - 1 2 NS6-6
  - 2 13 ME6-8

- **Demonstrate and apply** mental math strategies
  - Part Unit Lessons
  - 1 1 PA6-2
  - 1 4 NS6-11
  - 2 9 NS6-47

- **Use tools or technology to explore and create patterns and relationships, and test conjectures**
  - Part Unit Lessons
  - 1 4 NS6-17
  - 2 11 G6-14

- **Model** mathematics in contextualized experiences
  - Part Unit Lessons
  - 1 5 ME6-5
  - 2 15 PDM6-11
  - 2 16 ME6-23
## Curricular Competencies

### Understanding and solving

- **Apply multiple strategies** to solve problems in both abstract and contextualized situations

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>PDM6-2</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>NS6-20</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>G6-17</td>
</tr>
</tbody>
</table>

- **Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>G6-8</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>ME6-11</td>
</tr>
</tbody>
</table>

- **Visualize to explore mathematical concepts**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>PDM6-5</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>ME6-9</td>
</tr>
</tbody>
</table>

- **Engage in problem-solving experiences that are connected to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>G6-2</td>
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<tr>
<td>2</td>
<td>15</td>
<td>PDM6-7</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>ME6-18</td>
</tr>
</tbody>
</table>

### Communicating and representing

- **Use mathematical vocabulary and language to contribute to mathematical discussions**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>PDM6-4</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>NS6-21</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>G6-16</td>
</tr>
</tbody>
</table>

- **Explain and justify** mathematical ideas and decisions

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>NS6-12</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>G6-3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>PA6-20</td>
</tr>
</tbody>
</table>

- **Communicate** mathematical thinking in many ways

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>NS6-20</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>NS6-30</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>PA6-12</td>
</tr>
</tbody>
</table>

- **Represent mathematical ideas in concrete, pictorial, and symbolic forms**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>NS6-32</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>NS6-53</td>
</tr>
</tbody>
</table>
### Curricular Competencies

#### Connecting and reflecting

<table>
<thead>
<tr>
<th>Reflect on mathematical thinking</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td></td>
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<td>2</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Connect mathematical concepts to each other and to other areas and personal interests</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
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<td>2</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Use mathematical arguments to support personal choices</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
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<tr>
<td></td>
<td>2</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Incorporate First Peoples worldviews and perspectives to make connections to mathematical concepts</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
Grade 6 JUMP Math Correlation to the Manitoba Curriculum

NOTES:

**Underlined** JUMP Math lessons are review from a previous grade.

**Italicized** JUMP Math lessons contain prerequisite material required to meet the learning standard.

JUMP Math strands are represented by:

- **NS** Number Sense
- **ME** Measurement
- **G** Geometry
- **PA** Patterns and Algebra
- **PDM** Probability and Data Management

### Number

#### General Learning Outcome

Develop number sense.

#### Specific Learning Outcomes

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
</table>
| **6.N.1** Demonstrate an understanding of place value for numbers  
  • greater than one million  
  • less than one-thousandth | Part 1 Unit 2 Lessons NS6-1 to 4  
Part 2 Unit 9 Lessons NS6-38, 39 NS6-40 |
| **6.N.2** Solve problems involving large numbers, using technology. | Part 1 Unit 2 Lessons NS6-5  
NS6-4, 6  
Part 1 Unit 4 Lessons NS6-15, 17 |
| **6.N.3** Demonstrate an understanding of factors and multiples by  
  • determining multiples and factors of numbers less than 100  
  • identifying prime and composite numbers  
  • solving problems involving factors or multiples | Part 1 Unit 7 Lessons NS6-20  
NS6-18, 19, 21, 23 |
| **6.N.4** Relate improper fractions to mixed numbers. | Part 1 Unit 8 Lessons NS6-26, 28, 32, 34  
NS6-29 to 31, 33, 36, 37 |
<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.N.5</td>
<td>Demonstrate an understanding of ratio, concretely, pictorially, and symbolically. [C, CN, PS, R, V]</td>
<td>2</td>
<td>14</td>
<td>NS6-63</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NS6-58 to 60</td>
</tr>
<tr>
<td>6.N.6</td>
<td>Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically. [C, CN, PS, R, V]</td>
<td>2</td>
<td>14</td>
<td>NS6-64 to 67, 69, 70</td>
</tr>
<tr>
<td>6.N.7</td>
<td>Demonstrate an understanding of integers, concretely, pictorially, and symbolically. [C, CN, R, V]</td>
<td>1</td>
<td>2</td>
<td>NS6-7, 8</td>
</tr>
<tr>
<td>6.N.8</td>
<td>Demonstrate an understanding of multiplication and division of decimals (involving 1-digit whole-number multipliers, 1-digit natural number divisors, and multipliers and divisors that are multiples of 10), concretely, pictorially, and symbolically, by • using personal strategies • using the standard algorithms • using estimation • solving problems [C, CN, ME, PS, R, V]</td>
<td>1</td>
<td>4</td>
<td>NS6-9 to 11, 12, 13, 14, 16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NS6-44 to 47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NS6-48 to 54, 57</td>
</tr>
<tr>
<td>6.N.9</td>
<td>Explain and apply the order of operations, excluding exponents (limited to whole numbers). [CN, ME, PS, T]</td>
<td>1</td>
<td>7</td>
<td>NS6-24</td>
</tr>
</tbody>
</table>
## Patterns and Relations (Patterns)

### General Learning Outcome
Use patterns to describe the world and solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.PR.1 Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>1 1 PA6-3, PA6-4, 5, 7, 8</td>
<td></td>
</tr>
<tr>
<td>1 5 ME6-4</td>
<td></td>
</tr>
<tr>
<td>2 12 PA6-16, 17, 19, 20</td>
<td></td>
</tr>
<tr>
<td>6.PR.2 Represent and describe patterns and relationships using graphs and tables. [C, CN, ME, PS, R, V]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>2 12 PA6-16, 17, 20</td>
<td></td>
</tr>
</tbody>
</table>

## Patterns and Relations (Variables and Equations)

### General Learning Outcome
Represent algebraic expressions in multiple ways.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.PR.3 Represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>1 1 PA6-6 to 8</td>
<td></td>
</tr>
<tr>
<td>2 12 PA6-18 to 20</td>
<td></td>
</tr>
<tr>
<td>6.PR.4 Demonstrate and explain the meaning of preservation of equality, concretely, pictorially, and symbolically. [C, CN, PS, R, V]</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td>2 12 PA6-9, PA6-10 to 15</td>
<td></td>
</tr>
</tbody>
</table>
# Shape and Space (Measurement)

## General Learning Outcome

Use direct or indirect measurement to solve problems.

## Specific Learning Outcomes

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.SS.1</strong> Demonstrate an understanding of angles by</td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>• identifying examples of angles in the environment</td>
<td>1</td>
</tr>
<tr>
<td>• classifying angles according to their measure</td>
<td></td>
</tr>
<tr>
<td>• estimating the measure of angles using 45°, 90°,</td>
<td></td>
</tr>
<tr>
<td>and 180° as reference angles</td>
<td></td>
</tr>
<tr>
<td>• determining angle measures in degrees</td>
<td></td>
</tr>
<tr>
<td>• drawing and labelling angles when the measure is specified</td>
<td></td>
</tr>
<tr>
<td>[C, CN, ME, V]</td>
<td></td>
</tr>
<tr>
<td><strong>6.SS.2</strong> Demonstrate that the sum of interior angles is</td>
<td></td>
</tr>
<tr>
<td>• 180° in a triangle</td>
<td>1</td>
</tr>
<tr>
<td>• 360° in a quadrilateral</td>
<td></td>
</tr>
<tr>
<td>[C, R]</td>
<td></td>
</tr>
<tr>
<td><strong>6.SS.3</strong> Develop and apply a formula for determining the</td>
<td></td>
</tr>
<tr>
<td>• perimeter of polygons</td>
<td>1</td>
</tr>
<tr>
<td>• area of rectangles</td>
<td></td>
</tr>
<tr>
<td>• volume of right rectangular prisms</td>
<td>2</td>
</tr>
<tr>
<td>[C, CN, PS, R, V]</td>
<td>2</td>
</tr>
</tbody>
</table>

# Shape and Space (3-D Objects and 2-D Shapes)

## General Learning Outcome

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

## Specific Learning Outcomes

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.SS.4</strong> Construct and compare triangles, including</td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>• scalene</td>
<td>16</td>
</tr>
<tr>
<td>• isosceles</td>
<td></td>
</tr>
<tr>
<td>• equilateral</td>
<td></td>
</tr>
<tr>
<td>• right</td>
<td></td>
</tr>
<tr>
<td>• obtuse</td>
<td></td>
</tr>
<tr>
<td>• acute</td>
<td></td>
</tr>
<tr>
<td>in different orientations.</td>
<td></td>
</tr>
<tr>
<td>[C, PS, R, V]</td>
<td></td>
</tr>
<tr>
<td><strong>6.SS.5</strong> Describe and compare the sides and angles of regular and irregular polygons.</td>
<td></td>
</tr>
<tr>
<td>[C, PS, R, V]</td>
<td>1</td>
</tr>
</tbody>
</table>
# Shape and Space (Transformations)

## General Learning Outcome

Describe and analyze position and motion of objects and shapes.

## Specific Learning Outcomes | JUMP Math Lessons
--- | ---
6.SS.6 | Perform a combination of transformations (translations, rotations, or reflections) on a single 2-D shape, and draw and describe the image. [C, CN, PS, T, V]  
Part 2  Unit 11  Lessons G6-13 to 17

6.SS.7 | Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations. [C, CN, T, V]  
Part 2  Unit 11  Lessons G6-17

6.SS.8 | Identify and plot points in the first quadrant of a Cartesian plane using whole-number ordered pairs. [C, CN, V]  
Part 2  Unit 11  Lessons G6-18

6.SS.9 | Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole-number vertices). [C, CN, PS, T, V]  
Part 2  Unit 11  Lessons G6-19, 20
# Statistics and Probability (Data Analysis)

## General Learning Outcome
Collect, display, and analyze data to solve problems.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.SP.1</strong> Create, label, and interpret line graphs to draw conclusions. [C, CN, PS, R, V]</td>
<td>Part 1 Unit 3 Lessons PDM6-1 PDM6-3 to 6</td>
</tr>
<tr>
<td><strong>6.SP.2</strong> Select, justify, and use appropriate methods of collecting data, including • questionnaires • experiments • databases • electronic media [C, PS, T]</td>
<td>Part 2 Unit 15 Lessons PDM6-14, 16</td>
</tr>
<tr>
<td><strong>6.SP.3</strong> Graph collected data and analyze the graph to solve problems. [C, CN, PS]</td>
<td>Part 2 Unit 15 Lessons PDM6-16, 17</td>
</tr>
</tbody>
</table>

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# Statistics and Probability (Chance and Uncertainty)

## General Learning Outcome
Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

<table>
<thead>
<tr>
<th>Specific Learning Outcomes</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.SP.4</strong> Demonstrate an understanding of probability by • identifying all possible outcomes of a probability experiment • differentiating between experimental and theoretical probability • determining the theoretical probability of outcomes in a probability experiment • determining the experimental probability of outcomes in a probability experiment • comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]</td>
<td>Part 1 Unit 8 Lessons NS6-35</td>
</tr>
<tr>
<td><strong>6.SP.4</strong></td>
<td>Part 2 Unit 15 Lessons PDM6-7 to 11</td>
</tr>
</tbody>
</table>
Grade 6 JUMP Math Correlation to the Ontario Curriculum

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Expectation codes source: Ontario Curriculum Unit Planner

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- NS Number Sense
- ME Measurement
- G Geometry
- PA Patterns and Algebra
- PDM Probability and Data Management

### Number Sense and Numeration

#### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m8</td>
<td>read, represent, compare, and order whole numbers to 1,000,000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;</td>
</tr>
<tr>
<td>6m9</td>
<td>solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;</td>
</tr>
<tr>
<td>6m10</td>
<td>demonstrate an understanding of relationships involving percent, ratio, and unit rate.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

<table>
<thead>
<tr>
<th>Quantity Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6m11</strong></td>
<td></td>
</tr>
<tr>
<td>represent, compare, and order whole numbers and decimal numbers from 0.001 to 1,000,000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);</td>
<td>Part</td>
</tr>
<tr>
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<td></td>
<td>2</td>
</tr>
<tr>
<td><strong>6m12</strong></td>
<td></td>
</tr>
<tr>
<td>demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1,000,000, using a variety of tools and strategies (e.g., use base ten materials to represent the relationship between 1, 0.1, 0.01, and 0.001) (Sample problem: How many thousands cubes would be needed to make a base ten block for 1,000,000?);</td>
<td>Part</td>
</tr>
<tr>
<td></td>
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</tr>
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<td></td>
<td>2</td>
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<tr>
<td><strong>6m13</strong></td>
<td></td>
</tr>
<tr>
<td>read and print in words whole numbers to one hundred thousand, using meaningful contexts (e.g., the Internet, reference books);</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
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</tbody>
</table>
Number Sense and Numeration

6m14 represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, number lines, calculators) and using standard fractional notation. **Sample problem:** Use fraction strips to show that $1\frac{1}{2}$ is greater than $\frac{5}{4}$.

6m15 estimate quantities using benchmarks of 10%, 25%, 50%, 75%, and 100% (e.g., the container is about 75% full; approximately 50% of our students walk to school).

6m16 solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1 000 000. **Sample problem:** How would you determine if a person could live to be 1 000 000 hours old? Show your work.

6m17 identify composite numbers and prime numbers, and explain the relationship between them (i.e., any composite number can be factored into prime factors) (e.g., $42 = 2 \times 3 \times 7$).

Operational Sense

6m18 use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers (e.g., use the commutative property: $4 \times 16 \times 5 = 4 \times 5 \times 16$, which gives $20 \times 16 = 320$; use the distributive property: $(500 + 15) \div 5 = 500 \div 5 + 15 \div 5$, which gives $100 + 3 = 103$).

6m19 solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms).

6m20 add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators;

6m21 multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators (e.g., calculate $4 \times 1.4$ using base ten materials; calculate $5.6 \div 4$ using base ten materials);

6m22 multiply whole numbers by 0.1, 0.01, and 0.001 using mental strategies (e.g., use a calculator to look for patterns and generalize to develop a rule).
## Number Sense and Numeration

### 6m23
multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies (e.g., “To convert 0.6 m² to square centimetres, I calculated in my head 0.6 × 10 000 and got 6000 cm².”) *(Sample problem:)* Use a calculator to help you generalize a rule for multiplying numbers by 10 000;*

<table>
<thead>
<tr>
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<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>NS6-48, 49</td>
</tr>
</tbody>
</table>

### 6m24
calculate the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution *(Sample problem:)* Mori used a calculator to add 7.45 and 2.39. The calculator display showed 31.35. Explain why this result is not reasonable, and suggest where you think Mori made his mistake;*

<table>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS6-6</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>NS6-47</td>
</tr>
</tbody>
</table>

### 6m25
explain the need for a standard order for performing operations, by investigating the impact that changing the order has when performing a series of operations *(Sample problem:)* Calculate and compare the answers to $3 + 2 \times 5$ using a basic four-function calculator and using a scientific calculator;*

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>NS6-24</td>
</tr>
</tbody>
</table>

## Proportional Relationships

### 6m26
represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation *(Sample problem:)* In a classroom of 28 students, 12 are female. What is the ratio of male students to female students?*

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14</td>
<td>NS6-58 to 60</td>
</tr>
</tbody>
</table>

### 6m27
determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents (e.g., use a $10 \times 10$ grid to show that $\frac{1}{4} = 0.25$ or 25%);*

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14</td>
<td>NS6-64 to 67, 69</td>
</tr>
</tbody>
</table>

### 6m28
represent relationships using unit rates *(Sample problem:)* If 5 batteries cost $4.75, what is the cost of 1 battery?*

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14</td>
<td>NS6-63  NS6-61, 62</td>
</tr>
</tbody>
</table>
## Measurement

### Overall Expectations

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m29</td>
<td>estimate, measure, and record quantities, using the metric measurement system;</td>
</tr>
<tr>
<td>6m30</td>
<td>determine the relationships among units and measurable attributes, including the area of a parallelogram, the area of a triangle, and the volume of a triangular prism.</td>
</tr>
</tbody>
</table>

### Specific Expectations

#### Attributes, Units, and Measurement Sense

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m31</td>
<td>demonstrate an understanding of the relationship between estimated and precise measurements, and determine and justify when each kind is appropriate <em>(Sample problem: You are asked how long it takes you to travel a given distance. How is the method you use to determine the time related to the precision of the measurement?)</em>;</td>
</tr>
<tr>
<td>6m32</td>
<td>estimate, measure, and record length, area, mass, capacity, and volume, using the metric measurement system.</td>
</tr>
</tbody>
</table>

#### Measurement Relationships

<table>
<thead>
<tr>
<th>Expectation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m33</td>
<td>select and justify the appropriate metric unit (i.e., millimetre, centimetre, decimetre, metre, decametre, kilometre) to measure length or distance in a given real-life situation <em>(Sample problem: Select and justify the unit that should be used to measure the perimeter of the school.)</em>;</td>
</tr>
<tr>
<td>6m34</td>
<td>solve problems requiring conversion from larger to smaller metric units (e.g., metres to centimetres, kilograms to grams, litres to millilitres) <em>(Sample problem: How many grams are in one serving if 1.5 kg will serve six people?)</em>;</td>
</tr>
<tr>
<td>6m35</td>
<td>construct a rectangle, a square, a triangle, and a parallelogram, using a variety of tools (e.g., concrete materials, geoboard, dynamic geometry software, grid paper), given the area and/or perimeter <em>(Sample problem: Create two different triangles with an area of 12 square units, using a geoboard.)</em>;</td>
</tr>
</tbody>
</table>
## Measurement

<table>
<thead>
<tr>
<th>6m36</th>
<th>determine, through investigation using a variety of tools (e.g., pattern blocks, Power Polygons, dynamic geometry software, grid paper) and strategies (e.g., paper folding, cutting, and rearranging), the relationship between the area of a rectangle and the areas of parallelograms and triangles, by decomposing (e.g., cutting a parallelogram into a rectangle and two congruent triangles) and composing (e.g., combining two congruent triangles to form a parallelogram) <em>(Sample problem:)</em> Decompose a rectangle and rearrange the parts to compose a parallelogram with the same area. Decompose a parallelogram into two congruent triangles, and compare the area of one of the triangles with the area of the parallelogram.;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>13</td>
<td>ME6-10 ME6-11 to 13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6m37</th>
<th>develop the formulas for the area of a parallelogram (i.e., ( \text{Area of parallelogram} = \text{base} \times \text{height} )) and the area of a triangle (i.e., ( \text{Area of triangle} = \frac{(\text{base} \times \text{height})}{2} )), using the area relationships among rectangles, parallelograms, and triangles <em>(Sample problem:)</em> Use dynamic geometry software to show that parallelograms with the same height and the same base all have the same area.;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>13</td>
<td>ME6-11 to 13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6m38</th>
<th>solve problems involving the estimation and calculation of the areas of triangles and the areas of parallelograms <em>(Sample problem:)</em> Calculate the areas of parallelograms that share the same base and the same height, including the special case where the parallelogram is a rectangle.;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>13</td>
<td>ME6-15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6m39</th>
<th>determine, using concrete materials, the relationship between units used to measure area (i.e., square centimetre, square metre), and apply the relationship to solve problems that involve conversions from square metres to square centimetres <em>(Sample problem:)</em> Describe the multiplicative relationship between the number of square centimetres and the number of square metres that represent an area. Use this relationship to determine how many square centimetres fit into half a square metre.;</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>13</td>
<td>ME6-16</td>
</tr>
</tbody>
</table>
### Measurement

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m40</td>
<td>determine, through investigation using a variety of tools and strategies (e.g., decomposing rectangular prisms into triangular prisms; stacking congruent triangular layers of concrete materials to form a triangular prism), the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula ( V = \text{area of base} \times \text{height} ) (Sample problem: Create triangular prisms by splitting rectangular prisms in half. For each prism, record the area of the base, the height, and the volume on a chart. Identify relationships.);</td>
<td>2</td>
<td>16</td>
<td>ME6-20</td>
</tr>
<tr>
<td>6m41</td>
<td>determine, through investigation using a variety of tools (e.g., nets, concrete materials, dynamic geometry software, Polydrons) and strategies, the surface area of rectangular and triangular prisms;</td>
<td>2</td>
<td>16</td>
<td>ME6-25, 26</td>
</tr>
<tr>
<td>6m42</td>
<td>solve problems involving the estimation and calculation of the surface area and volume of triangular and rectangular prisms (Sample problem: How many square centimetres of wrapping paper are required to wrap a box that is 10 cm long, 8 cm wide, and 12 cm high?);</td>
<td>2</td>
<td>16</td>
<td>ME6-21, 26</td>
</tr>
</tbody>
</table>
# Geometry and Spatial Sense

## Overall Expectations

6m43 classify and construct polygons and angles;

6m44 sketch three-dimensional figures, and construct three-dimensional figures from drawings;

6m45 describe location in the first quadrant of a coordinate system, and rotate two-dimensional shapes.

## Specific Expectations

### Geometric Properties

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m46 sort and classify quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools (e.g., geoboard, dynamic geometry software) and strategies (e.g., using charts, using Venn diagrams);</td>
<td>Part Unit Lessons 1 6 G6-6, 7 G6-10, 12</td>
</tr>
<tr>
<td>6m47 sort polygons according to the number of lines of symmetry and the order of rotational symmetry, through investigation using a variety of tools (e.g., tracing paper, dynamic geometry software, Mira);</td>
<td>Part Unit Lessons 1 6 G6-6, 7 G6-11</td>
</tr>
<tr>
<td>6m48 measure and construct angles up to 180° using a protractor, and classify them as acute, right, obtuse, or straight angles;</td>
<td>Part Unit Lessons 1 6 G6-1 to 4</td>
</tr>
<tr>
<td>6m49 construct polygons using a variety of tools, given angle and side measurements (Sample problem: Use dynamic geometry software to construct trapezoids with a 45° angle and a side measuring 11 cm.).</td>
<td>Part Unit Lessons 1 6 G6-5</td>
</tr>
</tbody>
</table>

### Geometric Relationships

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m50 build three-dimensional models using connecting cubes, given isometric sketches or different views (i.e., top, side, front) of the structure (Sample problem: Given the top, side, and front views of a structure, build it using the smallest number of cubes possible.);</td>
<td>Part Unit Lessons 2 16 ME6-27, 28</td>
</tr>
<tr>
<td>6m51 sketch, using a variety of tools (e.g., isometric dot paper, dynamic geometry software), isometric perspectives and different views (i.e., top, side, front) of three-dimensional figures built with interlocking cubes.</td>
<td>Part Unit Lessons 2 16 ME6-27, 28</td>
</tr>
</tbody>
</table>
## Geometry and Spatial Sense

<table>
<thead>
<tr>
<th>Location and Movement</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6m52</strong> explain how a coordinate system represents location, and plot points in the first quadrant of a Cartesian coordinate plane;</td>
<td>Part 2  Unit 11  Lessons G6-18, 19</td>
</tr>
<tr>
<td><strong>6m53</strong> identify, perform, and describe, through investigation using a variety of tools (e.g., grid paper, tissue paper, protractor, computer technology), rotations of 180° and clockwise and counterclockwise rotations of 90°, with the centre of rotation inside or outside the shape;</td>
<td>Part 2  Unit 11  Lessons G6-15, 16, 20</td>
</tr>
<tr>
<td><strong>6m54</strong> create and analyse designs made by reflecting, translating, and/or rotating a shape, or shapes, by 90° or 180° (Sample problem: Identify rotations of 90° or 180° that map congruent shapes, in a given design, onto each other.).</td>
<td>Part 2  Unit 11  Lessons G6-13 to 17</td>
</tr>
</tbody>
</table>
### Patterning and Algebra

#### Overall Expectations

<table>
<thead>
<tr>
<th>Code</th>
<th>Expectation</th>
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</thead>
<tbody>
<tr>
<td>6m55</td>
<td>describe and represent relationships in growing and shrinking patterns (where the terms are whole numbers), and investigate repeating patterns involving rotations;</td>
</tr>
<tr>
<td>6m56</td>
<td>use variables in simple algebraic expressions and equations to describe relationships.</td>
</tr>
</tbody>
</table>

#### Specific Expectations

<table>
<thead>
<tr>
<th>Patterns and Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Patterns and Relationships</strong></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td>6m57</td>
<td>identify geometric patterns, through investigation using concrete materials or drawings, and represent them numerically;</td>
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<tr>
<td>6m58</td>
<td>make tables of values for growing patterns, given pattern rules in words (e.g., start with 3, then double each term and add 1 to get the next term), then list the ordered pairs (with the first coordinate representing the term number and the second coordinate representing the term) and plot the points in the first quadrant, using a variety of tools (e.g., graph paper, calculators, dynamic statistical software);</td>
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<td>6m59</td>
<td>determine the term number of a given term in a growing pattern that is represented by a pattern rule in words, a table of values, or a graph (<em>Sample problem</em>: For the pattern rule &quot;start with 1 and add 3 to each term to get the next term&quot;, use graphing to find the term number when the term is 19.);</td>
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<tr>
<td>6m60</td>
<td>describe pattern rules (in words) that generate patterns by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (e.g., for 1, 3, 5, 7, 9, …, the pattern rule is &quot;start with 1 and add 2 to each term to get the next term&quot;), then distinguish such pattern rules from pattern rules, given in words, that describe the general term by referring to the term number (e.g., for 2, 4, 6, 8, …, the pattern rule for the general term is &quot;double the term number&quot;);</td>
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<tr>
<td>6m61</td>
<td>determine a term, given its term number, by extending growing and shrinking patterns that are generated by adding or subtracting a constant, or multiplying or dividing by a constant, to get the next term (<em>Sample problem</em>: For the pattern 5000, 4750, 4500, 4250, 4000, 3750, …, find the 15th term. Explain your reasoning.);</td>
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<tr>
<td>6m62</td>
<td>extend and create repeating patterns that result from rotations, through investigation using a variety of tools (e.g., pattern blocks, dynamic geometry software, geoboards, dot paper).</td>
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</tbody>
</table>
### Patterning and Algebra

#### Variables, Expressions, and Equations

<table>
<thead>
<tr>
<th>6m63</th>
<th>demonstrate an understanding of different ways in which variables are used (e.g., variable as an unknown quantity; variable as a changing quantity);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6m64</th>
<th>identify, through investigation, the quantities in an equation that vary and those that remain constant (e.g., in the formula for the area of a triangle, ( A = \frac{b \times h}{2} ), the number 2 is a constant, whereas ( b ) and ( h ) can vary and may change the value of ( A ));</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
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<tr>
<td>1</td>
<td>5</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>6m65</th>
<th>solve problems that use two or three symbols or letters as variables to represent different unknown quantities (Sample problem: If ( n + l = 15 ) and ( n + l + s = 19 ), what value does the ( s ) represent?);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>6m66</th>
<th>determine the solution to a simple equation with one variable, through investigation using a variety of tools and strategies (e.g., modelling with concrete materials, using guess and check with and without the aid of a calculator) (Sample problem: Use the method of your choice to determine the value of the variable in the equation ( 2 \times n + 3 = 11 ). Is there more than one possible solution? Explain your reasoning.);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>
## Data Management and Probability

### Overall Expectations

6m67 collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including continuous line graphs;

6m68 read, describe, and interpret data, and explain relationships between sets of data;

6m69 determine the theoretical probability of an outcome in a probability experiment, and use it to predict the frequency of the outcome.

### Specific Expectations

#### Collection and Organization of Data

<table>
<thead>
<tr>
<th>Specific Expectation</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6m70</strong> collect data by conducting a survey (e.g., use an Internet survey tool) or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;</td>
<td>Part 2  Unit 15  Lessons PDM6-16</td>
</tr>
<tr>
<td><strong>6m71</strong> collect and organize discrete or continuous primary data and secondary data (e.g., electronic data from websites such as E-Stat or Census At Schools) and display the data in charts, tables, and graphs (including continuous line graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes), and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, spreadsheets, dynamic statistical software);</td>
<td>Part 1  Unit 3  Lessons PDM6-1 to 3, 5  Part 2  Unit 15  Lessons PDM6-14</td>
</tr>
<tr>
<td><strong>6m72</strong> select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied, such as pictographs, horizontal or vertical bar graphs, stem-and-leaf plots, double bar graphs, broken-line graphs, and continuous line graphs);</td>
<td>Part 1  Unit 3  Lessons PDM6-6</td>
</tr>
</tbody>
</table>

6m73 determine, through investigation, how well a set of data represents a population, on the basis of the method that was used to collect the data (Sample problem: Would the results of a survey of primary students about their favourite television shows represent the favourite shows of students in the entire school? Why or why not?).
# Data Management and Probability

<table>
<thead>
<tr>
<th>Data Relationships</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m74 read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., sports data in the newspaper, data from the Internet about movies), presented in charts, tables, and graphs (including continuous line graphs);</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 3 PDM6-3, 4</td>
</tr>
<tr>
<td></td>
<td>2 15 PDM6-17</td>
</tr>
<tr>
<td>6m75 compare, through investigation, different graphical representations of the same data <em>(Sample problem:)</em> Use technology to help you compare the different types of graphs that can be created to represent a set of data about the number of runs or goals scored against each team in a tournament. Describe the similarities and differences that you observe.;</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 3 PDM6-4</td>
</tr>
<tr>
<td>6m76 explain how different scales used on graphs can influence conclusions drawn from the data;</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>1 3 PDM6-1</td>
</tr>
<tr>
<td>6m77 demonstrate an understanding of mean (e.g., mean differs from median and mode because it is a value that “balances” a set of data – like the centre point or fulcrum in a lever), and use the mean to compare two sets of related data, with and without the use of technology <em>(Sample problem:)</em> Use the mean to compare the masses of backpacks of students from two or more Grade 6 classes.;</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>2 15 PDM6-12, 13</td>
</tr>
<tr>
<td>6m78 demonstrate, through investigation, an understanding of how data from charts, tables, and graphs can be used to make inferences and convincing arguments (e.g., describe examples found in newspapers and magazines).</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>2 15 PDM6-17</td>
</tr>
</tbody>
</table>

## Probability

<table>
<thead>
<tr>
<th>Probability</th>
<th>JUMP Math Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m79 express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely (e.g., the theoretical probability of rolling an odd number on a six-sided number cube is ( \frac{3}{6} ) because, of six likely outcomes, only three are favourable – that is, the odd numbers 1, 3, 5);</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>2 15 PDM6-7, 8</td>
</tr>
<tr>
<td>6m80 represent the probability of an event (i.e., the likelihood that the event will occur), using a value from the range of 0 (never happens or impossible) to 1 (always happens or certain);</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td>2 15 PDM6-9</td>
</tr>
</tbody>
</table>
### Data Management and Probability

<table>
<thead>
<tr>
<th>6m81</th>
<th>predict the frequency of an outcome of a simple probability experiment or game, by calculating and using the theoretical probability of that outcome (e.g., “The theoretical probability of spinning red is $\frac{1}{4}$ since there are four different-coloured areas that are equal. If I spin my spinner 100 times, I predict that red should come up about 25 times.”).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Sample problem: Create a spinner that has rotational symmetry. Predict how often the spinner will land on the same sector after 25 spins. Perform the experiment and compare the prediction to the results.).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
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<td>NS6-35</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>PDM6-10, 11</td>
</tr>
</tbody>
</table>
Grade 6 Essential Lessons for EQAO Test Preparation

EQAO test questions cover the majority of Ontario math curriculum topics. However, if you find that your class has been progressing too slowly and you are unable to cover the complete curriculum before the EQAO test, make sure to cover the most crucial topics.

If, by the beginning of April, you have not started on Unit 12, be sure to teach the lessons in the list below.

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<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
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<tr>
<td>Area</td>
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<td>ME6-11, 12, 13, 15, 16</td>
</tr>
<tr>
<td>Percentages</td>
<td>14</td>
<td>NS6-64 to 66</td>
</tr>
<tr>
<td>Probability</td>
<td>15</td>
<td>PDM6-7, 8, 9, 10</td>
</tr>
<tr>
<td>Volume and Surface Area</td>
<td>16</td>
<td>ME6-18, 20, 21, 25, 26</td>
</tr>
</tbody>
</table>