Unit 1  Number Sense

In this unit, students review place value, properties of operations, and long multiplication. Students also learn why division by 0 does not make sense.

Meeting your curriculum
Most of this unit (except for new material in NS7-5) is review, but it is essential. Students who can use the correct order of operations will have a much easier time later on, when applying the order of operations to new situations. Students need to understand how to check for equality of expressions with numbers before they can do so with variables. Understanding properties of operations will be essential for understanding how to manipulate variables when they come to do algebra. Lessons NS7-6 to NS7-8 accomplish three things at once: they are a good application of the concepts learned so far, they teach problem-solving strategies such as using a model and splitting into simpler problems, and they allow students to review the rules for long multiplication in a way that is neither tedious nor boring. Understanding the procedure for long multiplication in a deep conceptual way will be essential when students learn how to operate on polynomials in higher grades. The knowledge gained in this unit will be applied in most of the subsequent units in this course.

Grid paper
Students can use grid paper to line up place values when performing long multiplication.
**Goals**
Students will use the area model of multiplication to develop mental math strategies for multiplication, such as multiplying one term and dividing the other by the same number, students will also develop mental math strategies for finding quotients.

**VOCABULARY**
the place value terms:
- ones, tens, hundreds, thousands,
- ten thousands, hundred thousands, millions, and so on

**PROCESS EXPECTATION**
Looking for a pattern

**CURRICULUM EXPECTATIONS**
Ontario: 6m8, 6m13, 7m1, 7m6, review
WNCP: 5N1, 6N1, review, [R, CN]

**Identify place values.** Write a 14-digit whole number on the board, without spaces between the digits, and discuss why it is difficult to read.

Write 4 538 902 761. Have students identify and write the digits for the following place values, in order: ten thousands, ones, thousands, ten millions, billions, hundred millions. Answers will be 0, 1, 2, 3, 4, and 5.

Ask students to continue the pattern: What should the next digit be? (6) What is the place value of that digit? (tens). Repeat until you have listed all the digits and place values in the number.

**Bonus**
Tell students the names of the groups of three digits in any number, from right to left, are: ones, thousands, millions, billions, trillions, quadrillions, quintillions. Then have them identify the place value of various digits in a 20-digit number.

**Add spaces to numbers.** Have students write the following numbers with the correct spacing:
- 3412      34125      864312      9004999      60548902      432157809

Tell students that another student once wrote:
- 78939045672 as 789 390 456 72.

**ASK:** Is this correct? Why not? What was the student thinking? Why did they make this mistake? Tell students that a good way to avoid this mistake is to start at the right and put dividing lines in front of every third digit:

**EXAMPLE:** 78939045672 → 78|939|045|672 → 78 939 045 672

**Bonus**
Rewrite this number with the correct spacing, then write the place value of the digit 5: 80924310156200387

**ANSWER:** 80 924 310 156 200 387; 5 has place value ten millions

**EXTRA PRACTICE for Question 5:** Write each number in expanded form.

**ANSWERS:**
- i) 63 801
  - 60 000 + 3 000 + 800 + 1
- ii) 750 041
  - 700 000 + 50 000 + 40 + 1
- iii) 30 200
  - 30 000 + 200
- iv) 907 008
  - 900 000 + 7 000 + 8
**EXTRA PRACTICE for Question 6:** Write the number for each expanded form.

**ANSWERS:**

i) \( 80 000 + 50 + 4 \quad 80\,054 \\
ii) \( 300 000 + 4 000 + 500 + 20 \quad 304\,520 \\
iii) \( 700 000 + 50 \quad 700\,050 \\

**Bonus**

i) \( 500 000 + 30 + 700 \quad 500\,730 \\
ii) \( 2000 + 9 + 400 000 \quad 402\,009 \\

**PROCESS EXPECTATION**

Reflecting on what made the problem easy or hard

**Other ways to write numbers.** Write 45 3890 2761. **ASK:** What is the place value of the 5? Was it harder to read the place value in the number the way it is written? How is the spacing in this number different from what we usually write? The names of the place values partially repeat every three digits (e.g. "hundreds" is in the name of the 3rd digit from the right, the 6th, the 9th, and so on), so grouping by four is awkward.

Then explain that in Japan, the names of place values are:

- **one**
- **ten**
- **hundred**
- **thousand**
- **man**
- **ten man**
- **hundred man**
- **thousand man**
- **oku**
- **ten oku**
- **hundred oku**
- **thousand oku**
- **chou**
- **ten chou**
- **hundred chou**
- **thousand chou**

Tell students that man means 10 000. Have volunteers write what oku and chou mean (oku means 100 000 000 and chou means 1 000 000 000 000).

Discuss whether it would be easier for a Japanese person to read the number 45 3890 2761 or 4 538 902 761, and why. Have students read the number the Japanese way: forty-five oku, three thousand ninety man, two thousand seven hundred sixty-one. Explain that, just like we would find it hard to read numbers with Japanese spacing, Japanese people would find it hard to read numbers with our spacing.

**Review comparing and ordering numbers.** See Questions 8, 9, and 10.

**PROCESS EXPECTATION**

Looking for a pattern

**Compare the values of digits.** Review what it means for a number to be so many times another number, e.g., 30 is 10 times 3, so 30 is 10 times as big as 3.

1. How many times more is the first 3 worth than the second 3?

**ANSWERS:**

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<td>i)</td>
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<td>k)</td>
<td>342 173</td>
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<td>l)</td>
<td>2 356 783 542</td>
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Notice that the number of digits between the two 3s determines the answer. The digits before the first 3 and after the second 3 do not matter. Students could make a T-table with headings “Number of digits between the 3s” and “How many times more.” Notice that there is always one more 0 in the answer than the number of digits between the 3s.

2. How many times more is the 6 worth than the 3?

**ANSWERS:**

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<td>l)</td>
<td>2 642 173 542</td>
<td>200 000</td>
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3. How many times more is the 3 worth than the 6?

**ANSWERS:**

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<td>364 507</td>
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<td>2 342 176 542</td>
<td>50 000</td>
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**Numbers on cheques.** Draw a copy of a cheque and explain why it’s important to write the amount using both words and numerals. Show students how easy it is to change an amount such as “348.00” to “1348.00” or “3148.00” by adding the digit 1. On the other hand, it would be an obvious forgery if someone tried to add the words “one thousand” before “three hundred forty-eight.” How could you change the words “three hundred forty-eight” to match the number 3148? (add “thousand one” between “three” and “hundred”) Would this be an obvious forgery? (yes)

Preventing forgery is also why people draw a line after the number words on their cheques:

Three hundred forty-eight ____________________________ 00 cents
The line means no one can change the amount by adding a word at the end. No one would accept a cheque that looks like this:

Three hundred forty-eight thousand ___________________ 00 cents

**EXTRA PRACTICE for Workbook Question 12:** Write each number in words.

**ANSWERS:**

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<tr>
<td>i)</td>
<td>41 003 540</td>
<td>forty-one million three thousand five hundred forty</td>
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<td>ii)</td>
<td>8 503 060</td>
<td>eight million five hundred three thousand sixty</td>
</tr>
<tr>
<td>iii)</td>
<td>400 700 200</td>
<td>four hundred million seven hundred thousand two hundred</td>
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<td>iv)</td>
<td>30 040 050</td>
<td>thirty million forty thousand fifty</td>
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<tr>
<td>9 999 999 999</td>
<td>nine trillion nine hundred ninety-nine million nine hundred ninety-nine thousand nine hundred ninety-nine</td>
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Extensions

1. What whole number between 1 and 1,000,000 has the most syllables? 
   **Answer:** 777 777. To see this, note that 7 is the only 1-digit number that has two syllables.

2. How many number words from 1 to 1,000,000 have the word “thirty” in them at least once? 
   **Solution:** 
   The word is in 30–39 (10 times), 130–139 (10 times), 230–239 (10 times), ..., 930–939 (10 times). That’s \(10 \times 10 = 100\), so from 1 to 1000, there are 100 such numbers. 

   From 1001 to 2000, there are again 100 such numbers, because you are just adding the words “one thousand” before the number. So from 1 to 29 999, there are \(100 \times 30 = 3000\) such numbers. 

   From 30 000 to 39 999, all numbers (10 000 of them!) have the word “thirty” in them at least once (because every number starts with “thirty thousand” and \(10 000 + 3000 = 13 000\). So from 1 to 39 999, there are 13 000 such numbers. 

   From 40 000 to 99 999, there are again 100 numbers out of every 1000 with the word “thirty” in them (40 030 to 40 039, 40 130 to 40 139, ..., 40 930 to 40 939), so that makes \(60 \times 100 = 6 000\) such numbers. So from 1 to 100 000, there are \(13 000 + 6 000 = 19 000\) such numbers.

Activities

1. a) Use ones, tens, hundreds, and thousands blocks. Model five different numbers that use exactly 7 blocks. In each case, what is the sum of the digits? Why? (Sample answers: 232, 16, 7000, 4003. Note that the number of blocks is equal to the sum of the digits.) 
   b) A palindrome is a number which looks the same written backwards or forwards. Find as many palindromes as you can whose digits add to 10. (Sample answers: 2332 1441, 2120212, 343, 55, 262.) 

2. Show 1123 using 16 blocks, in two different ways. (Possible answers: 11 hundreds, 2 tens, and 3 ones; 1 thousand, 12 tens, and 3 ones; 1 thousand, 1 hundred, 1 ten, and 13 ones) 

3. I have 4 digits. My digits are all the same. Use 12 blocks to make me. (**Answer:** 3333) 

4. List all the possible numbers that satisfy: My digits are all the same. They multiply to 16. (**Answer:** 2222 or 44) 

5. I have 3 digits. The sum of my ones and tens digits equals my hundreds digit. Make me with 11 blocks. **Answer:** Make 101 using ten tens blocks and 1 ones block.
In each interval of 100 000, there are 19 000 numbers with the word “thirty” in them. Since there are 10 such intervals in 1 000 000, there are 190 000 such numbers in the number words from 1 to 1 000 000!

3. a) How many numbers from 1 to 100 have the digit 3 in them?
   \textbf{ANSWER:} 19 numbers: 3, 13, 23, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 53, 63, 73, 83, 93.

b) How many numbers from 1 to 1000 have the digit 3 in them?
   \textbf{ANSWER:} 100 (all the 3-digit numbers starting with 3)
   \[+19 \times 9\text{ (all the numbers from part a) repeated 9 times, for example, 3, 103, 203, 403, 503, 603, 703, 803 and 903)\]
   \[=271\]

c) How many numbers from 1 to 10 000 have the digit 3 in them?
   \textbf{ANSWER:} \[1000 + (271 \times 9) = 3439\]

d) How many numbers from 1 to 100 000 have the digit 3 in them?
   \textbf{ANSWER:} \[10 000 + (3439 \times 9) = 40 951\]

e) How many numbers from 1 to 1 000 000 have the digit 3 in them?
   \textbf{ANSWER:} \[100 000 + (40 951 \times 9) = 468 559\]

4. Compare your answers to Extension 2 and Extension 3 e). Which is more? Why is this the case?
   \textbf{ANSWER:} There are more numbers with digit 3 because all numbers with “thirty” in them have a digit 3, but there are numbers with a digit 3 that don’t include the word “thirty” (e.g., 103, 403, 10 003).
**NS7-2 Order of Operations**

**Goals**

Students will understand the need for brackets in expressions and for assigning an order to the operations.

**Prior Knowledge Required**

Adding, subtracting, multiplying, and dividing 1-digit and small 2-digit numbers

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**The Need for an Order of Operations.** Have students solve the following problem: \(8 - 5 + 2\). Discuss how to get the answer 5 (subtract 5 from 8 then add 2) and how to get the answer 1 (add 5 and 2 first, then subtract from 8). **Ask:** What could we do to make it clear which operation to do first? (Students may suggest ideas other than brackets if they are not familiar with brackets yet; accept all answers.)

**Introduce Brackets.** The brackets tell you to do the operations in brackets first. Writing \((8 - 5) + 2\) means \(3 + 2 = 5\); writing \(8 - (5 + 2)\) means \(8 - 7 = 1\). If there are no brackets, we do the addition and subtraction from left to right, so \(8 - 5 + 2\) means the same thing as \((8 - 5) + 2\).

Have students evaluate:

i) \((8 - 3) + 3\) and \(8 - (3 + 3)\)

ii) \((10 - 4) + 2\) and \(10 - (4 + 2)\)

iii) \((3 + 7) - 4\) and \(3 + (7 - 4)\)

iv) \((6 + 3) - 2\) and \(6 + (3 - 2)\)

Predict and then check whether these expressions have the same answer:

a) \((7 + 5) - 2\) and \(7 + (5 - 2)\)

b) \((7 - 4) + 2\) and \(7 - (4 + 2)\)

**When Order Doesn’t Matter.** Have students investigate whether changing the order of the numbers affects the answer if only addition and subtraction are in the expression.

Compare the answers.

i) \(9 + 4 - 3\) and \(9 - 3 + 4\) \(13 - 3 = 10\) and \(6 + 4 = 10\)

ii) \(4 + 5 - 3\) and \(4 - 3 + 5\) \(9 - 3 = 6\) and \(1 + 5 = 6\)

For example, adding 4 and then subtracting 3 gives the same answer as subtracting 3 then adding 4.

**Bonus**

i) \(8 - 3 + 4 - 5\) and \(8 + 4 - 3 - 5\) and \(8 - 3 - 5 + 4\)

ii) \(7 + 3 - 2 - 2 + 5 - 6 + 4\) and \(7 + 3 + 5 + 4 - 2 - 2 + 6\) and \(3 + 4 + 5 - 6 + 7 - 2 - 2\)

Emphasize that changing the order of the numbers doesn’t change the answer when there are no brackets, as long as the same numbers appear with the same operation (+ or −) in front of them.
The order of operations for expressions involving only addition, subtraction, and brackets. Explain that when there are brackets, we evaluate the expressions in brackets first, then write the expression without brackets, and then solve from left to right.

EXTRA PRACTICE:

ANSWERS:

i) $13 - (4 + 5) + 3 = 13 - 9 + 3 = 4 + 3 = 7$

ii) $13 - (4 + 5) - 3 = 1$

iii) $13 - (4 + 5 - 3) = 7$

iv) $13 - (4 + 5 + 3) = 1$

Which pairs of expressions above have the same answer? (ANSWERS: i and iii; ii and iv)

Bonus

i) $15 - (4 + 2 + 3) + 6$

ii) $12 - 4 + 3 - (5 + 4) + 12 - (1 + 2 + 3)$

Students can use Question 4 as a model for placing the brackets in Question 5. Some students may think of using nested brackets, such as $15 - (7 - (3 - 1))$. Allow them to investigate this case. Note, however, that there will be no new answer, since, for example: $15 - (7 - (3 - 1)) = 15 - 7 + 3 - 1 = 15 - (7 - 3) - 1$.

Bonus

Add brackets in different ways to get as many different answers as you can: $15 + 7 - 3 - 1$. Note that this will result in only one answer, like part i) of 5, even though both addition and subtraction are involved. The reason is that the subtraction is at the end.

Expressions involving only multiplication, division, and brackets.

Have students solve $15 ÷ 5 × 3$. Discuss how to get the answer 9 and how to get the answer 1. Using the same rule as for addition and subtraction (i.e., moving from left to right) what’s the right answer—9 or 1? ($15 ÷ 5 × 3 = 3 × 3 = 9$). How would students add brackets if they want to ensure someone will get the answer 1? $15 ÷ (5 × 3)$

Does changing the order of the numbers in expressions involving multiplication and division change the answer? To find out, have students solve these three expressions: $60 ÷ 3 × 8 ÷ 4 × 7 ÷ 5$ and $8 × 7 ÷ 4 × 60 ÷ 5 ÷ 3$ and $8 × 7 × 60 ÷ 5 ÷ 4 ÷ 3$. **ANSWERS:** 56 and 56 and 56.

When there is just multiplication and division, changing the order in which the numbers are listed doesn’t change the answer, as long as the same numbers are multiplied and divided from left to right. The answer **will** change if you don’t do the operations from left to right. **EXAMPLE:**

$80 ÷ (5 × 2) = 80 ÷ 10 = 8$ has a different answer from $80 ÷ 5 × 2 = 16 × 2 = 32$.

**Question 7:** Add brackets to $3 × 5 × 40 ÷ 10$ in different ways to get as many different answers as you can. (There will be only one answer.)

Discuss when changing the order the operations are done in affects the answer. (Problems with only multiplication will have only one answer, no
matter where you add brackets, and problems where the only division is at the end will have only one answer. Changing the order for all other combinations of multiplication and division changes the answer.

Students can now complete Questions 8 and 9. (Notice that they do not need to know that multiplication and division are done before addition and subtraction to complete the questions; they only need to know to do what’s in brackets first.)

**Why multiplication is done before addition.** Write $3 + 4 \times 5$. Explain that since $4 \times 5$ is really a short form for adding four 5s, this expression can be written as $3 + 5 + 5 + 5 + 5 + 5$. Look back at the original expression. **ASK:** To get the same answer as the new expression, would you do $3 + 4$ first and then multiply by 5, or $4 \times 5$ first, and then add 3? ($4 \times 5$ first)

Have students write these expressions out using only addition:

**ANSWERS:**

a) $4 \times 3 + 5 \times 5$ = $3 + 3 + 3 + 3 + 5 + 5 + 5 + 5 + 5$

b) $2 \times 7 + 3 \times 7$ = $7 + 7 + 7 + 7 + 7$

c) $3 + 4 \times 2 + 5$ = $3 + 2 + 2 + 2 + 2 + 5$

d) $5 + 2 \times 3 + 4 \times 4$ = $5 + 3 + 3 + 4 + 4 + 4 + 4$

Notice that $4 \times 3 + 5 \times 5 = 3 + 3 + 3 + 3 + 5 + 5 + 5 + 5 + 5 = 12 + 25 = 37$

This is the same as doing the multiplication first: $(4 \times 3) + (5 \times 5) = 12 + 25 = 37$.

Explain that when we see multiplication in an expression with addition, we always calculate the multiplication first because multiplication is just a short form for repeated addition.

Explain to students that they assumed this order of operations—that multiplication is done before addition—in the last lesson, when they wrote numbers in expanded form.

**EXAMPLE:** $345 = 3 \times 100 + 4 \times 10 + 5$

For the example above, **ASK:** What answer would you get if you did all the operations from left to right? (**ANSWER:** $300 + 4 \times 10 + 5 = 3040 + 5 = 3045$)

**ASK:** What answer would you get if you did all the addition first, and then multiplication? (**ANSWER:** $3 \times 104 \times 15 = 312 \times 15 = 4680$)

These answers are both wrong! The only correct way to evaluate the expression is to do all the multiplication first: $300 + 40 + 5 = 345$. That’s what we mean when we write numbers in expanded form. So in a way, this order of operations is nothing new.

**The order of operations for all operations and brackets.** Now tell students the order hierarchy:

1. Evaluate all expressions in brackets.
2. Do multiplication and division from left to right.
3. Do addition and subtraction from left to right.

Explain why multiplication and division are treated as a group, while addition and subtraction are treated as a separate group, as follows:

Addition and subtraction are naturally grouped together because
a) they undo each other, and
b) strings of addition and subtraction can be done in any order and give the same answer. For example, \( 8 + 4 - 5 = 12 - 5 \) and \( 8 - 5 + 4 = 3 + 4 \) have the same answer. (Be careful not to confuse this with saying that we can add the 5 and 4 before subtracting from the 8.)

Multiplication and division are grouped together for the same reason. For example: \( 30 \div 2 \div 3 \times 4 \) has the same answer as \( 30 \times 4 \div 3 \div 2 \).

On the other hand, \( 8 + 4 \times 5 = 8 + 20 \) and \( 8 \times 5 + 4 = 40 + 4 \) do not have the same answer, even though we just wrote the same numbers, with the same operation symbols in front of them, in different orders. Even if we did the operations from left to right, \( 8 + 4 \times 5 \) (would be \( 12 \times 5 \)) and \( 8 \times 5 + 4 \) would have different answers. So multiplication and addition cannot be grouped together.

We already said that multiplication is done before addition. Since multiplication is grouped with division and addition is grouped with subtraction, we do all instances of multiplication and division before all instances of addition and subtraction, unless there are brackets (in which case we do whatever is in the brackets first).

**Bonus**

**Question 10:** Evaluate each expression using the correct order of operations.

i) \( (3 \times 5 - 7) \times 5 \div (16 - 6) \)
   **ANSWER:** \( (15 - 7) \times 5 \div 10 = 8 \times 5 \div 10 \)
   \( = 40 \div 10 \)
   \( = 4 \)

ii) \( 90 \div (13 - 2 \times 5) - (4 + 3 \times 2) \times 2 + 5 \)
   **ANSWER:** \( 90 \div (13 - 10) - (4 + 6) \times 2 + 5 = 90 \div 3 - 10 \times 2 + 5 \)
   \( = 30 - 20 + 5 \)
   \( = 10 + 5 \)
   \( = 15 \)

iii) \( (80 \div (1 + 2 + 3 + 4) \times 5 - (1 + 2 + 3 + 4)) \div 6 \)
   **ANSWER:** \( (80 \div 10 \times 5 - 10) \div 6 = (8 \times 5 - 10) \div 6 \)
   \( = (40 - 10) \div 6 \)
   \( = 30 \div 6 \)
   \( = 5 \)

**PROCESS EXPECTATION**  
Looking for a pattern

Explain that sometimes, we don’t need to write any brackets if we change some of the operation symbols. Have students calculate these three expressions:
10 – (3 + 2) 10 – 3 + 2 10 – 3 – 2

ASK: Which expression without brackets has the same answer as the expression with brackets?

**Extension**

Using exactly four 4s each time, make expressions equal to each number from 0 through 10. You may use brackets and any of the four operations.

**EXAMPLE:** \((4 \times 4) ÷ (4 + 4) = 16 ÷ 8 = 2\)

**HINT:** You may need to use the 2-digit number 44.

Possible answers:
- \(0 = (4 - 4) \times (4 + 4)\)
- \(1 = 4 ÷ 4 \times 4 ÷ 4\)
- \(2 = 4 \times 4 ÷ (4 + 4)\)
- \(3 = (4 + 4 + 4) ÷ 4\)
- \(4 = (4 - 4) \times 4 + 4\)
- \(5 = (4 \times 4 + 4) ÷ 4\)
- \(6 = (4 + 4) ÷ 4 + 4\)
- \(7 = 44 ÷ 4 - 4\)
- \(8 = 4 \times 4 - (4 + 4)\)
- \(9 = 4 ÷ 4 + 4 + 4\)
- \(10 = (44 - 4) ÷ 4\)

How many different expressions can your students come up with?
NS7-3  Equations
Pages 6–7

CURRICULUM EXPECTATIONS
Ontario: 6m9, 6m18, 7m1, 7m2, 7m6, 7m7, review
WNCP: 5N3, 5N4, review, [R, ME, V, T, C]

VOCABULARY
area

Goals
Students will use the area model of multiplication to develop mental math strategies for multiplication, such as multiplying one term and dividing the other by the same number. Students will also develop mental math strategies for finding quotients.

PRIOR KNOWLEDGE REQUIRED
Understands the concept of area
Can add, subtract, multiply, and divide

Have students work through Workbook p. 6–7. Use the questions below as bonus problems for each question as stated.

**Bonus**

**Question 3:** Verify that each equation is true.
1 + 3 + 5 = 3 × 3
1 + 3 + 5 + 7 = 4 × 4
1 + 3 + 5 + 7 + 9 = 5 × 5

Predict and then check:
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = ___ × ___

**Question 4:** Verify that each equation is true.
i) 7 + 6 + 10 = (7 + 3) + (6 - 2) + (10 - 1)
ii) 8 + 5 + 12 = (8 + 4) + (5 + 3) + (12 - 7)

**Question 5:** Rewrite each pair of equations as a single equation. Leave out the number on the right.
i) (7 + 3) + (10 + 2) + (8 - 5) = 25 and 7 + 10 + 8 = 25
ii) (5 + 2) + (6 - 3) + (4 + 1) = 15 and 5 + 6 + 4 = 15

**Question 6:** Write the correct number to make the equation true.
i) (8 + 7) + (5 - 4) + (9 - ___) = 8 + 5 + 9
ii) (8 - 6) + (5 - 2) + (9 + ___) = 8 + 5 + 9

You want students to notice that the numbers added or subtracted to the numbers in the brackets on the left side of the equation should add to 0. (EXAMPLE: In i), if I add 7 and then take away 4, I have to take away 3 more, so the correct number is 3.) Students who don’t see this right away can calculate each expression in brackets separately first, to see what goes in the blank.

**EXAMPLE:**
(8 + 7) + (5 - 4) + (9 - ___) = 8 + 5 + 9
15 + 1 + (9 - ___) = 22
16 + (9 - ___) = 22
9 - ___ = 6 so the correct number is 3
Encourage students who solve the problem this way to reflect on the answer and to see if they could have predicted it.

**Question 7:** Verify that each equation is true.

i) \((8 + 7) - (2 + 5) - (4 + 2) = 8 - 2 - 4\)

ii) \((8 + 3) - (2 - 1) - (4 + 4) = 8 - 2 - 4\)

iii) \((8 + 5) + (9 - 2) - (4 + 3) = 8 + 9 - 4\)

iv) \((8 + 2) + (9 + 3) - (4 + 5) = 8 - 9 - 4\)

**Question 8:** Write the correct number to make the equation true.

i) \((15 - 7) + (6 - 2) + (8 + \_\_\_) = 15 + 6 + 8\)

ii) \((15 - 7) + (6 + 2) + (8 + \_\_\_) = 15 + 6 + 8\)

iii) \((15 + 5) - (7 + 2) - (4 + \_\_\_) = 15 - 7 - 4\)

iv) \((15 + 5) - (7 - 2) - (4 + \_\_\_) = 15 - 7 - 4\)

v) \((9 + 4) + (7 - 3) - (5 + \_\_\_) = 9 + 7 - 5\)

Students can find the answer using the same method as Question 6, but should again reflect on why the answer makes sense and how they could have predicted it.

**Question 9:** Write the correct operation. Verify your answers.

i) \((12 - 5) + (8 + 3) + (9 \bigcirc 2) = 12 + 8 + 9\)

ii) \((12 - 5) + (8 + 7) + (9 \bigcirc 2) = 12 + 8 + 9\)

iii) \((12 - 5) + (8 + 7) - (9 \bigcirc 2) = 12 + 8 - 9\)

iv) \((12 - 5) + (8 + 3) - (9 \bigcirc 2) = 12 + 8 - 9\)

**Question 10:** Write the correct operation and number.

i) \((12 + 5) - (8 - 3) + (9 \_\_\_) = 12 - 8 + 9\)

ii) \((12 + 5) + (8 - 3) - (9 \_\_\_) = 12 + 8 - 9\)
## Goals

Students will verify equations by calculating the expressions on either side of the equal sign and comparing them. Students will also decide what to add, subtract, multiply, or divide by to make one expression equal to another.

### PRIOR KNOWLEDGE REQUIRED

- Understands the concept of area
- Can add, subtract, multiply, and divide

### MATERIALS

- BLM 2-cm grid paper (p 1-2)
- Large sheets of thick paper (2 colours) with rectangles drawn on square grids as follows:
  - $8 \times 10$ rectangle on red paper
  - $8 \times 10$ rectangle on blue paper
  - $18 \times 15$ rectangle on red paper
  - $18 \times 15$ rectangle on blue paper
- Scissors
- Tape
- 5 index cards, each with 3 rows of 4 dots

### Finding products with the same answer by rearranging rectangles.

Have students cut out a $6 \times 10$ rectangle from 2 cm grid paper (or, to save time, cut the rectangles out for them). Have students fold the rectangle in half one way and ask what the new dimensions are. Have them fold it in half the other way and ask what the new dimensions are.

Draw pictures like the ones on the worksheet (Question 2) on the board, so that students understand how such pictures represent the folding they just did. Repeat with more examples (keep all side lengths even). Discuss the pattern: How do you get the side lengths of the folded sheet from the original? Then provide examples of rectangles that have been rearranged without folding: students should find the lengths from the dimensions of the original rectangle, as shown on the worksheet.

Have students fold, cut, and rearrange many examples of rectangles. Students can then look for a pattern to determine the resulting sides without folding or cutting (see Question 3).

Take two large sheets of thick coloured paper (different colours), and draw the same rectangle on each: an $8 \times 10$ rectangle with $8 \times 10 = 80$ large squares (ensure the grid lines are clearly visible). Cut the rectangles out. Verify by a direct comparison that the two rectangles are identical. Ask students what the area of the rectangle is and display the equation.
8 \times 10 = 80. Cut one of the rectangles in half to form two 4 \times 10 rectangles, and rearrange it to form a new rectangle, as on the worksheet. Ask if you changed the area by rearranging the rectangle. What are the new side lengths? (4 and 20) Display the equation 8 \times 10 = 4 \times 20. Repeat with the other rectangle, this time creating two 8 \times 5 rectangles and rearranging to form a 16 \times 5 rectangle. Write 8 \times 10 = ____ \times ____ and have students fill in the blanks.

Provide more grid paper, and have students make several rectangles with even side lengths (EXAMPLES: 14 \times 16 or 10 \times 18). Then have students cut out and rearrange the rectangles to form other rectangles. Have students write at least two equivalent products for their original product. EXAMPLE: 14 \times 16 = 7 \times 32 or 28 \times 8.

Finding equal products without using rectangles. Discuss the pattern: If you double one factor, what do you have to do to the other to make the products equal? (halve the other factor)

Have students predict what you have to do to the other factor if you multiply one factor by 3. Use this EXAMPLE: 18 \times 15 = 54 \times ___. Check the students’ predictions by cutting and rearranging two large 18 \times 15 rectangle. This time, cut the rectangles into thirds instead of halves. Using side lengths that are multiples of 3 (EXAMPLES: 9 \times 12, 15 \times 24, 30 \times 12, and so on), have students cut out and rearrange the rectangles to form other rectangles. Students should write two equivalent products for their original product.

Have students predict what you have to do to the other factor if you multiply one factor by 4. (divide by 4)

EXAMPLE: 8 \times 20 = 32 \times ____ and 8 \times 20 = ____ \times 80

What would you do to the other factor if you multiplied one factor by 5? (divide by 5)

EXAMPLE: 15 \times 20 = 75 \times ____ and 15 \times 20 = ____ \times 100

Have students draw and cut out rectangles on grid paper to show these equations. Rewrite the equations in the previous examples using the notation in Question 8. EXAMPLE: 8 \times 20 = 32 \times 5 becomes 8 \times 20 = (8 \times 4) \times (20 \div 4). Explain that when you multiply one term by 4, you need to divide the other term by 4 to keep the products the same. Students can verify their answers to Workbook Questions 8, 9, and 10 using a calculator. Here are some bonus questions you can assign, as students finish each question.

**Bonus**

**Question 8:** 5 \times 10 \times 15 = (5 \times 6) \times (10 \div 2) \times (15 \div ____)
4 \times 5 \times 6 = (4 \div 2) \times (5 \times 4) \times (6 \div ____)
12 \times 15 \times 18 = (12 \times 30) \times (15 \div 5) \times (18 \div ____)

**Question 9:** 6 \times 4 \times 10 = (6 \div 3) \times (4 \times 2) \times (10 \times 6)
6 \times 4 \times 10 = (6 \div 3) \times (4 \times 15) \times (10 \times 5)
Question 10: $6 \times 21 \times 14 = (6 \times 14) \times (21 \div 7) \times (14 \div \underline{2})$

$6 \times 21 \times 14 = (6 \underline{\times 14}) \times (21 \div 3) \times (14 \div 2)$

$10 \times 15 \times 42 = (10 \times 2) \times (15 \times 3) \times (42 \underline{\times 2})$

Finding an easier problem with the same answer. See Question 11.

How does the product change if one factor is multiplied and the other factor doesn’t change? Demonstrate using a $3 \times 4$ array and a $6 \times 4$ array. Show how the $6 \times 4$ array is just two copies of a $3 \times 4$ array. Use two prepared index cards, each with 3 rows of 4 dots. Arrange the cards one above the other to see 6 rows of 4 dots.

ASK: How many copies of a $3 \times 4$ array (3 rows of 4) would you need to make a $15 \times 4$ array? Have ready five index cards, each with a $3 \times 4$ array, and place the cards one above the other to show the 15 rows of 4.

How do you keep the quotient the same if the dividend or divisor is multiplied or divided by a number? See Workbook Questions 15–22.

Here are some extra practice questions you can assign, as students work on each question.

EXTRA PRACTICE:

Question 15: Finish writing the equivalent division statements for each multiplication statement.

$4 \times 5 = 20$ so $20 \div \underline{\_ \_ \_ \_} = 5$

$(4 \times 2) \times 5 = 20 \times 2$ so $20 \times 2 \div (\underline{\_ \_ \_ \_}) = 5$

$(4 \times 7) \times 5 = 20 \times 7$ so $20 \times 7 \div (\underline{\_ \_ \_ \_}) = 5$

Question 16: Write the correct operation and number.

$20 \div 4 = (20 \times 2) \div (4 \underline{\_ \_ \_ \_})$

$20 \div 4 = (20 \times 7) \div (4 \underline{\_ \_ \_ \_})$

Question 17:

$35 \div 7 = (35 \times 4) \div (7 \underline{\_ \_ \_})$

$42 \div 6 = (42 \times 3) \div (6 \underline{\_ \_ \_})$

$84 \div 4 = (84 \times 5) \div (4 \underline{\_ \_ \_})$

**Bonus** $60 \div 3 \div 5 = (60 \times 12) \div (3 \times 6) \div (5 \underline{\_ \_ \_})$

Question 19:

$75 \div 5 = \underline{\_ \_ \_ \_} \div 10$

$80 \div 5 = \underline{\_ \_ \_ \_} \div 10$

$245 \div 5 = \underline{\_ \_ \_ \_} \div 10$

$= \underline{\_ \_ \_ \_}$

$= \underline{\_ \_ \_ \_}$

$= \underline{\_ \_ \_ \_}$

Question 22:

$30 \div 10 = (30 \div 2) \div (10 \underline{\_ \_ \_})$

$40 \div 8 = (40 \div 4) \div (8 \underline{\_ \_ \_})$

$75 \div 15 = (75 \div 3) \div (15 \underline{\_ \_ \_})$

**Bonus** $240 \div 6 \div 8 = (240 \div 12) \div (6 \div 3) \div (8 \underline{\_ \_ \_})$
How 1 is special in multiplication and division. Challenge students to explain, in as many ways as they can, why multiplying by 1 results in the number you started with. What models for multiplication do they know? (EXAMPLES: arrays, areas, repeated addition)

Show students how to get $5 \times 1 = 5$ by using the pattern in the 5 times table:

| $5 \times 6$ | 30 |
| $5 \times 5$ | 25 |
| $5 \times 4$ | 20 |
| $5 \times 3$ | 15 |
| $5 \times 2$ | 10 |
| $5 \times 1$ | ___ |

Since you are subtracting 5 each time, you can get $5 \times 1$ by subtracting 5 from $5 \times 2 = 10$.

“Anything” $\times 0 = 0$. First use repeated addition: adding 0 any number of times is always 0. Then use patterns: continue the pattern above (you can get $5 \times 0$ by subtracting 5 from $5 \times 1 = 5$). Repeat with the 2 times table.

Dividing zero by zero doesn’t make sense. Review again division as related to multiplication: if $2 \times 4 = 8$, then $8 \div 2 = 4$ and $8 \div 4 = 2$. Then write these five equations:

| $1 \times 0 = 0$ | $2 \times 0 = 0$ | $3 \times 0 = 0$ | $4 \times 0 = 0$ | $5 \times 0 = 0$ |

Have students write two division statements for each multiplication statement shown. When they are done, ASK: What is $0 \div 1$? (0) $0 \div 2$? (0) $0 \div 3$? (0) $0 \div 4$? (0) $0 \div 5$? (0) $0 \div 97$? (0)

Then ASK: What is $0 \div 0$? Is it 1? (It seems to be.) Is it 2? (It seems to be that too!) 3? (Well, it appears so.) 4? 5? All of these seem right, but we can’t have more than one answer to a division question! Explain that $0 \div 0$ has too many answers, so it doesn’t make sense to even ask the question.

Dividing a non-zero number by zero doesn’t make sense either. Explain as in Workbook Question 11.
The Area Model for Multiplication

Pages 14–16

CURRICULUM EXPECTATIONS
Ontario: 6m18, 7m1, 7m2, 7m3, 7m6, 7m7, review
WNCP: 5N5, review, [ME, PS, R, V, T, C]

Goals
Students will understand the distributive law of multiplication through the area model.

PRIOR KNOWLEDGE REQUIRED
Can calculate the area of a rectangle
Understands order of operations
Can multiply by single-digit numbers and by 10

VOCABULARY
none (students do not need to use the phrase “distributive law”)

Multiply by doubling. Teach the strategy of doubling. If students know $2 \times 7 = 14$, then $4 \times 7$ is double that, so $4 \times 7$ is $28$. (Doubling one factor and leaving the other the same will double the product.) You could draw two $2 \times 7$ arrays on the board, one above the other, to demonstrate this. If students are familiar with the 2 times table, then they can quickly calculate the 4 times table and the 8 times table.

Have students fill in the blanks:

a) $4 \times 7$ is double $2 \times 7 = 14$, so $4 \times 7 = 28$.
b) $5 \times 8$ is double $2 \times 8 = 16$; so $5 \times 8 = ____$.
c) $6 \times 9$ is double $2 \times 9 = 18$, so $6 \times 9 = ____$.
d) $3 \times 8$ is double $2 \times 8 = 16$, so $3 \times 8 = ____$.
e) $4 \times 8$ is double $2 \times 8 = 16$, so $4 \times 8 = ____$.

Encourage students to find two possible answers for the first blank in part e). (2 $\times$ 8 and 4 $\times$ 4 are both possible)

Ask volunteers to solve in sequence: $2 \times 3, 4 \times 3, 8 \times 3, 16 \times 3$. Then ASK: What is the double of 16? What is $32 \times 3$? What is the double of 32? What is $64 \times 3$? Different volunteers could answer each question.

Repeat with the sequences beginning as follows. Take each sequence as far as your students are willing to go with it.

a) $3 \times 5, 6 \times 5, ...$ b) $2 \times 6, 4 \times 6, ...$ c) $3 \times 4, 6 \times 4, ...

d) $2 \times 7, 4 \times 7, ...$ e) $2 \times 9, 4 \times 9, ...$ f) $3 \times 9, 6 \times 9, ...$

Use arrays to show a product as a sum of two smaller products.
See Questions 1 and 2.

EXTRA PRACTICE for Question 3:
Write $8 \times 17$ as a sum of two smaller products in all the ways possible. Emphasize that the easiest sum to use is $8 \times 10 + 8 \times 7$ because multiplying by 10 and by single digits is easy.

Investigate which operations “distribute.” See Workbook Question 4.

Bonus
Write $=$ (equal) or $\neq$ (not equal)
$4 \times (2 + 3 + 5) \square (4 \times 2) + (4 \times 3) + (4 \times 5)$
$3 \times (1 + 2 + 3 + 4) \square (3 \times 1) + (3 \times 2) + (3 \times 3) + (3 \times 4)$
Use the area model to understand why multiplication distributes over addition and subtraction. See Questions 5–9. Remind students that when they studied the order of operations, they found that \((4 + 6) \times (3 + 2)\) had the same answer as \(4 \times 3 + 4 \times 2 + 6 \times 3 + 6 \times 2\). Have them verify this by drawing rectangles.

Explain that it wasn’t just a coincidence that the two expressions have the same answer; any statement with only multiplication and addition can be written without brackets! (It will just take longer to write if you don’t use brackets.) Challenge students to come up with an expression involving only addition and multiplication that cannot be written without brackets, and then show them how they can indeed write it without brackets. Students can verify that you are correct by calculating both sides.

As students work through Workbook pp. 15–16, you can provide bonus questions as follows.

**Bonus Question 5:**

\[
\begin{align*}
3 + (1 \times 2 \times 3 \times 4) & = (3 + 1) \times (3 + 2) \times (3 + 3) \times (3 + 4) \\
5 \times (4 + 9 - 3) & = (5 \times 4) + (5 \times 9) - (5 \times 3) \\
(8 + 4 + 6) \div 2 & = (8 \div 2) + (4 \div 2) + (6 \div 2)
\end{align*}
\]

**Extension**

Use a cube model to write \((4 + 5) \times (3 + 6) \times (2 + 7)\) without brackets.

This expression represents the volume of the whole cube. Using the diagram at right, challenge students to find the volume of each of the 8 pieces separately, and then the volume of the whole cube. To find the volume of the 8 pieces separately, students will need to write the side
lengths not shown. For example, the edge at the top right of the diagram is divided into parts of length 6 and 3. To find the volume of the entire cube, students add the volumes of the 8 pieces.

ANSWER:
\[
4 \times 3 \times 2 + 4 \times 3 \times 7 + 4 \times 6 \times 2 + 4 \times 6 \times 7 \\
+ 5 \times 3 \times 2 + 5 \times 3 \times 7 + 5 \times 6 \times 2 + 5 \times 6 \times 7
\]
NS7-7 Breaking Multiplication into Simpler Problems

Page 17

CURRICULUM EXPECTATIONS
Ontario: 5m24, 7m1, 7m2, 7m3, review
WNCP: 5N3, 5N4, 5N5, review, [PS, R, ME]

VOCABULARY
none

PROCESS EXPECTATION
Reflecting on what made the problem easy or hard

PROCESS EXPECTATION
Splitting into simpler problems

PROCESS EXPECTATION
Splitting into simpler problems

Goals
Students will understand the usefulness of breaking problems into simpler parts and solving each part.

PRIOR KNOWLEDGE REQUIRED
Can multiply single-digit numbers by 2, 3, 5, and 10
Understands place value

Review multiplying by 10, 100, 1000, and their multiples. ASK: What kinds of numbers are easiest to multiply by? Write a large number on the board, such as: 2 345 711 256 × ______. ASK: What can you put in the blank so that you will know the answer easily? (0, 1, 10, 100, 1000, and so on) Challenge students to prove their assertions—can they in fact write the answer easily?

After students finish Workbook Question 1, provide the following bonus problems.

**Bonus**

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Breaking multiplication into simpler problems by using multiples of 10, 100, or 1000. See Workbook Questions 2–5.

After students finish Workbook Questions 3 and 4, provide these bonus questions.

**Bonus**

**Question 3:** 3 × (1 230 132 113 021)

**Question 4:**
1. Use the 3 times table, the 10 times table, and the 100 times table to write the 113 times table.

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2. Have students find the 6 times table from the 3 times table.

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PROCESS EXPECTATION

Reflecting on the reasonableness of the answer

**EXAMPLE:** $6 \times 7$ is double $3 \times 7$ because

$$6 \times 7 = 7 + 7 + 7 + 7 + 7 + 7$$

$$= (7 + 7 + 7) + (7 + 7 + 7)$$

$$= 2 \times (7 + 7 + 7)$$

Then have students find the 6 times table using the 5 times table.

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**EXAMPLE:** $6 \times 7$ is 7 more than $5 \times 7$ because

$$6 \times 7 = 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$$

$$= (7 + 7 + 7 + 7 + 7) + 7$$

$$= 5 \times 7 + 7$$

Ask students to check if they got the same answers both ways.

**NOTE:** Before doing Question 5, some students may need to determine the 20 times table from the 10 times table by doubling. Students can calculate the 20 times table in a separate chart or add a row to the existing chart. After students finish Workbook Question 5, provide the following bonus question.

**Bonus**

Find the 523 times table from the 3 times table, the 20 times table, and the 500 times table.

**Extensions**

1. Add and subtract the same amounts to rewrite each sum as a product.

   a) $2 + 4 + 6 + 8$
      
      $$+ 3 + 1 - 1 - 3$$
      
      $$= 5 + 5 + 5 + 5 = 4 \times 5$$

   b) $3 + 6 + 8 + 11$

   c) $1 + 2 + 3 + 4 + 5 + 6 + 7$

   d) $5 + 6 + 7 + 9 + 10 + 11$

2. Have students calculate the total number of days in a non-leap year in two ways.

   a) by adding a sequence of 12 numbers (the number of days in each month)

   b) by comparing each number in the sequence to 30 and hence the total sum to $30 \times 12$.

   $$31 + 28 + 31 + \ldots + 30 + 31 =$$

   $$31 + 28 + 31 + \ldots + 30 + 31 = 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 31 + 31 + 31 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 + 31$$

   $$= 30 \times 12 + 5$$

   $$= 360 + 5 = 365$$
**NS7-8 Long Multiplication**

**Goals**
Students will use the standard algorithm to solve 2-digit by 2-digit multiplication problems.

**PRIOR KNOWLEDGE REQUIRED**
- Understands order of operations
- Can apply the distributive property (without using the terminology)

**CURRICULUM EXPECTATIONS**
- Ontario: 5m24, 5m26, 6m19, 7m1, review
- WNCP: 5N5, review, [R]

**VOCABULARY**
one

**The standard algorithm for multiplication.** Teach the steps in the algorithm in sequence, as on the worksheet. After students finish the relevant Workbook question, assign bonus questions as shown below.

**Bonus**

- **Question 1:** 74 231
  \[ \times 2 \]

- **Question 2:** 835 629
  \[ \times 7 \]

- **Questions 4, 5, 6:** 25 674
  \[ \times 9 \]

- **Question 7:**
  a) 342 \times 2000  
  b) 320 \times 6000  
  c) 324 \times 2000  
  d) 623 \times 20 000

**Extensions**

1. a) Which is larger: 28 \times 6 or 26 \times 8? How can you tell without actually multiplying the numbers? Encourage students to do the calculations and then to reflect back on how they could have known which was larger before doing the calculations. For the pairs below, challenge students to first decide which product will be larger and then do the actual calculations to check their predictions.

   - i) 34 \times 5 or 35 \times 4
   - ii) 27 \times 9 or 29 \times 7
   - iii) 42 \times 3 or 43 \times 2
   - iv) 89 \times 7 or 87 \times 9

   Here are three ways to decide which is larger between 34 \times 5 and 35 \times 4:

   - **Compare both products to 34 \times 4.** The product 34 \times 5 is 34 more than 34 \times 4, but the product 35 \times 4 is only 4 more (since 35 \times 4 = (34 + 1) \times 4 = 34 \times 4 + 1 \times 4), so 34 \times 5 is larger than 35 \times 4.

   - **Write each product as a sum.**
     
     - \[ 34 \times 5 = 5 \times 34 = 34 + 34 + 34 + 34 + 34 \]
     
     - \[ 35 \times 4 = 4 \times 35 = 35 + 35 + 35 + 35 \]

     Since 35 is only 1 more than 34, but there is an extra 34 in the top sum, that sum is larger.
• **Look at the digits.** Notice that both products have the same product of their ones digits \((4 \times 5 = 5 \times 4)\), but the product of the tens digits with the single-digit number is different:

- in \(34 \times 5\): 3 tens \(\times\) 5 = 15 tens
- in \(35 \times 4\): 3 tens \(\times\) 4 = 12 tens

Since \(34 \times 5\) has more tens, it must be larger.

b) Which is larger in each pair?

i) \(35 \times 27\) or \(37 \times 25\)
ii) \(36 \times 27\) or \(37 \times 26\)
iii) \(46 \times 25\) or \(45 \times 26\)
iv) \(49 \times 25\) or \(45 \times 29\)

Students should investigate by actually calculating the products and then reflecting back on how they could have solved each problem using the least amount of effort possible. **HINT:** For part a), compare both products to \(35 \times 25\). (The first product adds 70 and the second product adds 50, so the first product is larger.)

2. Explain to students why a 3-digit number multiplied by a 1-digit number must have at most 4 digits: \((3\text{-digit number}) \times (1\text{-digit number})\) is less than \(1000 \times 10 = 10000\), so it must have at most 4 digits.

Have students predict, and then check using their calculators, the maximum number of digits when multiplying

a) 2-digit by 2-digit numbers
d) 3-digit by 4-digit numbers
b) 1-digit by 4-digit numbers
e) 3-digit by 5-digit numbers
c) 2-digit by 3-digit numbers f) 2-digit by 7-digit numbers

Challenge your students to predict the maximum number of digits when multiplying a 37-digit number with an 8-digit number.

**ANSWER:** \(37 + 8 = 45\). The product has to be less than

\[
100000000000000000000000000000 \times 10000000
\]

which is 1 with 45 zeroes. This is the smallest 46-digit number.

3. Explain to students that there are often several different ways of showing the same concept mathematically. Ask students to come up with several ways of showing \(2 \times 5 = 5 \times 2\) (without calculating the products).

**EXAMPLES:**

a) Draw a \(2 \times 5\) array and rotate it to get a \(5 \times 2\) array.

b) Count in a different way. For example, think of 5 pairs of shoes. Each pair consists of 2 shoes, so there are \(2 + 2 + 2 + 2 + 2 = 5 \times 2\) shoes, but there are 5 left shoes and 5 right shoes, so there are \(5 + 5 = 2 \times 5\) shoes. Since counting in different ways doesn’t change the number of shoes we counted, this shows that \(2 \times 5 = 5 \times 2\).

c) Another example of counting in a different way: Pair up corresponding fingers on each hand to get \(2 + 2 + 2 + 2 + 2 = 5 \times 2\) and count by number on each hand to get \(5 + 5 = 2 \times 5\).
Discuss: Can you generalize the first example above to show $3 \times 5 = 5 \times 3$? (Yes, draw a 3 by 5 array and rotate it.)

The second example? (Yes, but only if you have 5 people with 3 feet each!)

The third example? (Yes, but only if you have someone with 3 hands!)

Then show students the following method which they are unlikely to have seen before and is quite elegant. Draw 2 points in one line and 5 points in another parallel line. Then join all the points in one line to all the points in the other line.

Count the lines in two ways. Notice that all the lines go between the top and bottom sets of points; no line goes between the top points and no line goes between the bottom points.

The number of lines connected to the top 2 points is $5 + 5$, and since these are all the lines, there are $5 + 5 = 2 \times 5 = 10$ lines altogether. Counting from the bottom 5 points, the number of lines is $2 + 2 + 2 + 2 + 2 = 5 \times 2 = 10$.

Discuss: Can this method be used to show that $3 \times 5 = 5 \times 3$? Yes! In general, if there are $A$ points in the top and $B$ points in the bottom, there are $A \times B = B \times A$ lines altogether.