Grade 3
End of Year Teacher Pack

Includes:

- Lesson Plans
- Blackline Masters
- Assessment & Practice Book Answers
- Quizzes and Unit Tests
- Rubrics for Scoring Quizzes and Tests
- Common Core State Standards Correlations
Unit 4 Number and Operations in Base Ten: Rounding and Estimating

This Unit in Context

Students learned in Grades 1 and 2 how to apply properties of place value to add and subtract—within 100 beginning in Grade 1 (1.NBT.C.4) and within 1,000 beginning in Grade 2 (2.NBT.B.7). In 3.1 Unit 2, students continued to practice adding and subtracting within 1,000 in order to reach fluency (3.NBT.A.2). In this unit, students will use their knowledge of adding and subtracting within 1,000 to estimate sums and differences (3.OA.D.8). Students will also add three-digit numbers where the result exceeds 1,000.

Students first learned that 10 is a special number in our number system in Kindergarten when they learned how to represent numerals between 11 and 19 as 10 ones and some more ones, as well as that the number of extra ones beyond ten is seen in how the number is written (K.NBT.A.1). In Grades 1 and 2, students used the fact that adding 10 with a one-digit number is a way to make addition easier, noticing, for example, that $4 + 9$ is 3 more than $4 + 6$, and so it is $10 + 3 = 13$. Students also learned in Grade 2 that 10 is an easy number to skip count by (2.NBT.A.2) and multiples of 10 are easy to add (30 + 40 is 70 because 3 tens + 4 tens is 7 tens). In this unit, students will use rounding to estimate sums and differences, making use of the fact that numbers that are multiples of 10 or 100 are particularly easy to add and subtract (3.OA.D.8). From here, students in Grade 4 will round whole numbers to any place value (4.NBT.A.3), and in Grade 5, students will round decimals to any place value (5.NBT.A.4) as well as recognize that multiplying and dividing decimals by powers of 10 is easy (5.NBT.A.2). Students will learn to use their estimates to assess the reasonableness of their answers (3.OA.D.8) and continue to make estimates for this purpose in higher grades (4.OA.A.3, 5.NF.A.2, and 7.EE.B.3).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1, MP3, MP5, MP7, and MP8. Here are some examples of how students can show that they have met a standard.

**MP3:** In NBT3-19 Extension 2, students construct an argument when they explain why a sum will be more or less than the estimated answer.

**MP5:** In NBT3-18 Extension 3, students choose tools strategically to construct an argument about which garden has more flowers. T-tables make it easier to see the pattern, but arrays make it easier to explain the pattern.

**MP7:** In NBT3-22 Extension 8, students look for and make use of structure when they find the perimeter of a rectangle by using the fact that opposite sides of a rectangle are equal.
**MP.8:** In the exercises in NBT3-16, students multiply one-digit numbers by multiples of ten and, on p. P-7, they notice regularity in that the same argument that works for one such multiplication can be used for any such multiplication.
Unit 4 Number and Operations in Base Ten: Rounding and Estimating

Introduction

In this unit students will use understanding of place value and the properties of operations to perform multi-digit arithmetic. They will round whole numbers to the nearest ten or hundred, and they will use rounding to estimate sums and differences within 1,000 and to evaluate the reasonableness of the answers to word problems. They will also use the standard algorithm to add 3-digit numbers where the result exceeds 1,000.
NBT3-16  Multiplying Tens

Page 90–91

STANDARDS
3.NBT.A.3

VOCABULARY
1-digit number
2-digit number
digit
hundreds
multiply
ones
product
regrouping
tens

Goals
Students will multiply 1-digit numbers by 2-digit multiples of 10.

PRIOR KNOWLEDGE REQUIRED
Can multiply up to 10 × 10
Is familiar with place values up to hundreds
Can represent 2- and 3-digit numbers with base ten materials
Can represent a product as an array

MATERIALS
20 tens blocks and 1 hundreds block for each student

NOTE: Students have not yet learned to use a comma to separate the thousands from the hundreds. They will learn to do that later in this unit, in Lesson NBT3-20.

Review multiplying 1-digit numbers by 10. Have students find 5 × 10, 2 × 10, 7 × 10, 6 × 10. (50, 20, 70, 60) Ask them to explain how they know the answer, or how they know the answer is correct. Make sure students mention that they can add 10 three times to get 3 × 10. ASK: How can you represent 3 × 10 with base ten blocks? (3 tens blocks) PROMPT: You need 3 groups of 10. What is a group of 10 in base ten blocks? (a tens block)

Multiplying 2-digit numbers by 10. ASK: How much is 10 × 10? (100) Ask a volunteer to represent 10 × 10 with base ten blocks. (10 tens blocks) ASK: What block is that equal to? (hundreds block) Draw students’ attention to the fact that a hundreds block has 10 rows of 10 blocks each, so it shows an array of 10 × 10 blocks.

Give each student 20 tens blocks and 1 hundreds block. Ask them to use these blocks to make 12 × 10. (12 tens blocks) ASK: Is there a part that we could replace with a hundreds block? (yes, 10 tens blocks) Have students do the regrouping. ASK: What number did you get? (120) Write on the board:

12 × 10 = 120

Have students use their blocks to find 15 × 10, 16 × 10, and 19 × 10. (150, 160, 190)

(MP8) Ask students to predict 17 × 10 without using the blocks. (170) ASK: How do you know? Students might have noticed a pattern: the answer is the number being multiplied by 10 with a zero written at the end.

Exercises: Use the pattern to multiply by 10.

a) 25 × 10   b) 33 × 10   c) 40 × 10   d) 18 × 10
Bonus

e) 50 \times 10  
f) 100 \times 10  
g) 123 \times 10

Answers: a) 250, b) 330, c) 400, d) 180, Bonus: e) 500, f) 1,000, g) 1,230

The number of tens in a 3-digit number. Ask students to represent the number 140. SAY: In the other exercises you replaced 10 tens blocks with a hundreds block. Can you do the opposite? Have students replace the hundreds block with 10 tens blocks. Point out that the blocks still show 140. Remind students that the process they have just performed is called regrouping.

SAY: 14 tens blocks show 140. This means that the number of tens in 140 is 14. How many tens are in 160? (16) PROMPT: Make 160 using only tens blocks. Point out that when you use only tens blocks and no hundreds block the number of tens is different from the tens digit: the tens digit in 140 is 4, but the number of tens is 14. Repeat with 190. (19) ASK: How many tens are in 240? (24) How do you know? (if you make 240 with only tens blocks, you need 24 tens blocks) How can you tell from the number how many tens are in it without actually making it with tens blocks? (Cover up the ones digit, and look at the rest. The remaining two digits are the number of tens.)

Exercises: How many tens are in the number?
a) 250  
b) 890  
c) 560  
d) 470

Answers: a) 25, b) 89, c) 56, d) 47

Reverse the task. ASK: What number is 17 tens? (170) How do you know? (If I have 17 tens blocks, I can regroup 10 tens as 100. Then I have 100 and 7 tens, or 70, and together they make 170.)

Exercises: Regroup tens as hundreds to write the number.
a) 24 tens  
b) 35 tens  
c) 36 tens  
d) 81 tens

Bonus: 100 tens

Answers: a) 240, b) 350, c) 360, d) 810, Bonus: 1,000

Using base ten blocks to represent products. Draw on the board:

Remind students that we often call both multiplication and its result a product. So both $2 \times 4$ and 8 are products of 2 and 4. SAY: The first picture shows $3 \times 2$, because there are 3 groups of 2. Write "$3 \times 2$" under the first picture. Now look at the second picture. ASK: What number is inside each
group? (20) What product does the picture show? (3 × 20) Write that under the second picture.

Have a volunteer draw pictures for 4 × 3 and 4 × 30. Remind students that they can draw lines for tens blocks and dots for ones blocks. (see answers in the margin)

**Exercises:** Draw a picture for the product.

a) 4 × 20  

b) 5 × 30  

c) 2 × 50  

d) 3 × 60

**Sample answer**

a) 

**Multiplying using a model.** Return to the picture showing 3 × 20. Write on the board:

\[3 \times 20 = 3 \times \_ \text{ tens} = \_ \text{ tens} = \_\]

ASK: How many tens are in 20? (2) Fill in the first blank. Point to the second blank and ASK: How many tens are in the product? (6) Fill in the second blank. ASK: What number is 6 tens? (60) Fill in the last blank.

**Exercises:** Multiply using the pictures you drew in the last exercise.

a) 4 × 20 = 4 × \_ tens = \_ tens = \_  

b) 5 × 30 = 5 × \_ tens = \_ tens = \_  

c) 2 × 50 = 2 × \_ tens = \_ tens = \_  

d) 3 × 60 = 3 × \_ tens = \_ tens = \_

**Answers:** a) 2, 8, 80; b) 3, 15, 150; c) 5, 10, 100; d) 6, 18, 180

**Multiplying without a model.** Explain that you want students to imagine the picture they would draw to fill in the blanks for the next exercise.

**Exercises:** Multiply by using tens.

a) 4 × 70 = 4 × \_ tens = \_ tens = \_  

b) 5 × 50 = 5 × \_ tens = \_ tens = \_  

c) 8 × 50 = 8 × \_ tens = \_ tens = \_  

d) 9 × 60 = 9 × \_ tens = \_ tens = \_

**Answers:** a) 7, 28, 280; b) 5, 25, 250; c) 5, 40, 400; d) 6, 54, 540

Write on the board:

\[
5 \times 5 = \_ \quad 4 \times 2 = \_ \quad 5 \times 3 = \_ \quad 2 \times 5 = \_ \\
5 \times 50 = \_ \quad 4 \times 20 = \_ \quad 5 \times 30 = \_ \quad 2 \times 50 = \_ 
\]

Invite volunteers to fill in the numbers using the calculations they have done earlier.
ASK: Can you see a pattern in the multiplication? (the answer is found by multiplying the 1-digit numbers to get the number of tens and then writing a zero for the number of ones)

**Exercises:** Multiply.

- **a)** $6 \times 2 = \_\_\_\_\_\_\_\_\_\_\_\_\_$
- **b)** $8 \times 3 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **c)** $9 \times 5 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **d)** $7 \times 6 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **e)** $6 \times 20 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **f)** $8 \times 30 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **g)** $9 \times 50 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **h)** $7 \times 60 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **e)** $6 \times 60 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **f)** $7 \times 80 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **g)** $5 \times 70 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **h)** $3 \times 90 = \_\_\_\_\_\_\_\_\_\_\_\_$

**Bonus:** $3 \times 100 = \_\_\_\_\_\_\_\_\_\_\_\_$

**Answers:**

- **a)** $12, 120$
- **b)** $24, 240$
- **c)** $45, 450$
- **d)** $42, 420$
- **e)** $360$
- **f)** $560$
- **g)** $350$
- **h)** $270$
- **Bonus:** $300$

Remind students that order does not matter in multiplication, so they can change the order of the numbers they multiply. For example, $30 \times 5 = 5 \times 30 = 150$.

**Exercises:** Multiply.

- **a)** $60 \times 5 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **b)** $80 \times 4 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **c)** $90 \times 7 = \_\_\_\_\_\_\_\_\_\_\_\_$
- **d)** $70 \times 3 = \_\_\_\_\_\_\_\_\_\_\_\_$

**Answers:** $300, b) 320, c) 630, d) 210$

**Multiplying tens to solve word problems.** Solve the first two word problems in the exercise below as a class, and then have students work independently on the last four. Students attempting the bonus might need a reminder that a nickel is worth 5 cents.

**Exercises**

- **a)** Ms. K. bought 30 boxes with 6 markers in each. How many markers did she buy?
- **b)** A parking lot has 8 rows of cars. Twenty cars can park in each row. How many cars can park in the lot?
- **c)** A store ordered 5 boxes of books. There are 20 books in each box. How many books did the store order?
- **d)** Ron earns $30 per hour by washing windows. How much money does he get after 8 hours of work?
- **e)** A bus can hold 50 people. How many people can ride in 9 buses?

**Bonus:** How much money is in a roll of 50 nickels?

**Answers:**

- **a)** 180 markers, b) 160 cars, c) 100 books, d) $240,
- **e)** 450 people, Bonus: 250 cents, or $2.50
Extensions

1. Solve the problem. Remember: You can find the area of a rectangle by multiplying length and width.

   a) A playground is a rectangle 20 m long and 8 m wide. What is the area of the playground?

   b) A soccer field is 9 m wide and 30 m long. What is its area?

   c) A wall is 9 feet tall and 60 feet long. What is the area of the wall?

   **Answers:** a) 160 m², b) 270 m², c) 540 ft²

2. Another way to show how $3 \times 50 = 150$ uses the associative property of multiplication. Remind students that when they have a product of three numbers, such as $3 \times 2 \times 5$, they can find the answer two ways:

   \[(3 \times 2) \times 5 = 6 \times 5 = 30 \quad \text{and} \quad 3 \times (2 \times 5) = 3 \times 10 = 30\]

   We know that $50 = 5 \times 10$, so we can write $3 \times 50$ as $3 \times (5 \times 10)$. Then we can change the order of multiplication:

   \[3 \times 50 = 3 \times (5 \times 10)\]
   \[= (3 \times 5) \times 10\]
   \[= 15 \times 10\]
   \[= 150\]

   Have students multiply using this method in the following exercises. Then have them check that they get the same answers as with the previous method.

   a) \[4 \times 20 = 4 \times (\_ \times 10)\]
   \[= (4 \times \_ ) \times 10\]
   \[= \_ \times 10\]
   \[= \_ \]

   b) \[6 \times 70 = 6 \times (\_ \times 10)\]
   \[= (6 \times \_ ) \times 10\]
   \[= \_ \times 10\]
   \[= \_ \]

   c) \[8 \times 80 = 8 \times (\_ \times 10)\]
   \[= (8 \times \_ ) \times 10\]
   \[= \_ \times 10\]
   \[= \_ \]

   **Answers:** a) 2, 2, 8, 80; b) 7, 6 × 7, 42, 420; c) 8, 8 × 8, 64, 640

3. Sixty students each found 7 jokes to tell in class. If all the jokes are different, how many jokes did they find altogether?

   **Answer:** 420 jokes
4. There are two hotels beside each other. One has 4 floors with 30 rooms on each floor and the other has 30 floors with 4 rooms on each floor.

a) How many rooms does each hotel have?

b) If you know the answer for one hotel do you need to multiply to find the answer for the other hotel? Explain.

c) Do the hotels look the same or different?

**Answers:**
a) each hotel has 120 rooms; b) no, because both answers are obtained by writing zero after the product of one-digit numbers, and the products $3 \times 4$ and $4 \times 3$ are equal; c) different
Goals
Students will round 2- and 3-digit numbers to the nearest ten.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to hundreds
Can count by 10s to 1,000

MATERIALS
number line from 0 to 100, e.g., made from an enlarged copy of
BLM Hundreds Chart (p. P-42)
a ball
cards from BLM Number Cards (pp. P-43-44)

NOTE: In this lesson students will round numbers to the nearest ten
in two steps:

Step 1: Find the next and the previous multiple of 10.
Step 2: Decide which of them the number is nearest to.

Next, students will be introduced to the rule for rounding based on the ones
digit. We go through these steps first for 2-digit numbers, then for 3-digit
numbers. The rule for rounding is the same for both.

Review relevant vocabulary. Write on the board:

345

ASK: What digit shows the hundreds? (3) What digit shows the ones? (5)
What digit shows the tens? (4) Students can signal their answers by raising
the correct number of fingers. Remind students that a common expression
that describes hundreds, tens, and ones is “place value.”

Remind students that when we skip count by a number starting from
zero, the numbers we say are multiples of that number. Skip count by 10
as a class, recording the numbers on the board. (0, 10, 20, 30, and so
on) SAY: These are the multiples of 10. Write “multiples of 10” under the
numbers on the board.

Finding the multiples of 10 right before and after a number. Display
a number line from 0 to 100. You can create one by cutting an enlarged
copy of BLM Hundreds Chart into strips and taping the strips together.
Have students identify the multiples of 10 on the number line. Then ask a
volunteer to find the number 63 on the number line. ASK: Which multiple
of 10 comes right before 63? (60) Repeat with 47, 82, 35, (40, 80, 30)
ASK: How can you tell what the multiple of 10 right before the number is
without looking at the number line? (pretend the ones digit is zero)
Exercises: Find the multiple of 10 right before the number.

a) 51  b) 64  c) 29  d) 75  e) 12  Bonus: 104

Answers: a) 50, b) 60, c) 20, d) 70, e) 10, Bonus: 100

**ACTIVITY 1**

Pass a ball to a student while saying a number. The student who catches the ball says the multiple of 10 before the number while passing the ball back to you. Repeat with different numbers until all students have had at least one turn.

Have volunteers find the multiple of 10 that is right after the numbers 63, 75, 82, 11, 9, and 27. (70, 80, 90, 20, 10, 30) Then discuss how students can find the multiple of 10 right after a number without looking at the number line. One way is to count up by 1s from the number and see what number you say when the tens digit changes. Another way is to find the multiple of 10 immediately before the number and add 10. Have students try both methods while solving the exercises below, and then discuss which method they prefer.

Exercises: Find the multiple of 10 right after the number.

a) 51  b) 64  c) 29  d) 75  e) 12  Bonus: 94

Answers: a) 60, b) 70, c) 30, d) 80, e) 20, Bonus: 100

Repeat Activity 1 above asking for the multiple of 10 after the number.

Write on the board:

previous  next

Have students read the words aloud. Ask if anyone knows what the words mean. If necessary, explain that *previous* means the one right before, and *next* means the one right after. Now ask students to find the two multiples of 10, previous and next, for 53. (50, 60)

Repeat Activity 1 above, this time asking for both the previous and the next multiples of 10 for each number.

Exercises: Find the previous and the next multiples of 10.

a) 34  b) 49  c) 38  d) 81  e) 92

**Bonus**

f) 104  g) 7

Answers: a) 30, 40; b) 40, 50; c) 30, 40; d) 80, 90; e) 90, 100; Bonus: f) 100, 110; g) 0, 10
ACTIVITIES 2–3

2. Give each pair of students a set of cards from BLM Number Cards and have them sort the cards into multiples of 10 and not multiples of 10. Have students lay the multiples of 10 face up on the table in random order and place the remaining cards (not multiples of 10) face down in a pile.

Player 1 picks a card from the face-down pile and places it face up on the table. Player 2 identifies the previous and the next multiples of 10 from the face-up pile and places them on either side of the card, in the correct order. Players return the multiples of 10 to the table and discard the middle card. Then players switch roles. Play continues until all of the cards in the face-down pile have been used.

After players have played several rounds, they can try the more complicated game below.

3. **Multiples of 10 Train**

   **Materials:** Cards from BLM Number Cards

   **Object of the Game:** To arrange all cards in a “train” from smallest to largest.

   **Preparation:** Have players sort the deck into multiples of 10 and not multiples of 10. They should spread out the multiples of 10 in random order and place the other cards face down in a pile.

   Have students play in pairs. Player 1 picks a card from the face-down pile. Player 2 must find the two multiples of 10 the number is between and place them on either side of the card in the correct order, creating a train. If one of the multiples is part of an existing train, the player adds the card and the second multiple to the train. If both multiples are already in trains, Player 2 should either fit the card inside an existing train, or combine two trains into one.

   **Variation:** To shorten the game, limit the number of cards that are not multiples of 10 to 4 cards.

**Deciding which multiple of 10 is closest.** Draw a number line from 0 to 10 on the board. Circle the 2 and ask if the 2 is closer to the 0 or to the 10. (0) Students can signal the answer by making a circle with one hand or raising 10 fingers. Draw an arrow from the 2 to the zero to show that 2 is closer to zero than to 10. Repeat with 7, 9, 4, and 1. (10, 10, 0, 0) ASK: Which numbers from 1 to 9 are closer to 0? (1, 2, 3, 4) Which numbers are closer to 10? (6, 7, 8, 9) What about 5? (it is right in the middle, the same distance from 0 as from 10)

Draw on the board:

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10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
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Circle various numbers (avoiding 10, 15, 20, 25, or 30) and ask volunteers to draw an arrow to the closest, or nearest, multiple of 10. SAY: The arrow points to the nearest multiple of 10. ASK: Which numbers have an arrow pointing left, to the previous multiple of 10? Which numbers have an arrow pointing right, to the next multiple of 10? Make lists on the board, as shown in the margin.

ASK: What are the ones digits of the numbers that are closer to the previous multiple of 10? (1, 2, 3, 4) What are the ones digits of the numbers that are closer to the next multiple of 10? (6, 7, 8, 9) Which two numbers that don’t have ones digit 0 are exactly the same distance from the previous and the next multiples of 10? (15 and 25) What do you notice about their ones digit? (it is 5)

SAY: I want you to try to predict which multiple of 10 is nearest without the number line. Try to use the pattern we found for numbers from 0 to 30. Repeat Activity 1 above asking for the nearest multiple of 10.

Introduce rounding. Remind students that when measuring, they do not always need to find exact length but only a measurement that is close to the actual length. This is called an estimate of length. Explain that estimates are useful in other situations too. For example, imagine that you are in a store. You buy an eraser that costs 39 cents, and you pay for it with a $1 bill. You get some change, and you want to be sure you are getting the right amount of change. You can calculate the exact amount of change, or you can estimate and see if the change you receive is about the right amount.

One of the simplest ways to estimate is to use a multiple of 10 that is nearest to the number. This is called rounding the number. When you look for a multiple of 10 that is nearest to the number, you round to the nearest multiple of 10.

SAY: I want to round 24 to the nearest multiple of 10. ASK: What is the nearest multiple of 10? (20) SAY: 24 rounded to the nearest multiple of 10 is 20.

The rule for rounding. SAY: When we round to the nearest multiple of 10, and the answer is smaller than the number, we say that we round down. When we round to the the nearest multiple of 10, and the answer is larger than the number, we say that we round up.

Write on the board:

Step 1: Write the previous and the next multiples of 10.

Step 2: If the ones digit is
   1, 2, 3, or 4, round down (previous)
   6, 7, 8, or 9, round up (next)

SAY: I want to round 32 to the nearest multiple of 10. What two multiples of 10 is 32 between? (30 and 40) What is the ones digit? (2) Which list is it in? (the first one) Will we round up or down? (down) What is 32 rounded to the nearest multiple of 10? (30) Repeat the questions for 58. (60)
Repeat Activity 1 above, this time asking students to round 2-digit numbers to the nearest multiple of 10. Do not include numbers with ones digit 0 or 5.

ASK: When you round down, what happens to the tens digit? (it stays the same) What happens to the tens digit when you round up? (it goes up) Write on the board (leave space between "ones digit" and the list in each row):

When you round a number with:
ones digit  1, 2, 3, 4  the tens digit stays the same
ones digit  6, 7, 8, 9  the tens digit goes up

SAY: I want to round 20 to the nearest multiple of 10. Oh wait: it is already a multiple of 10! I do not need to go up or down. ASK: What happens with the tens digit? (it stays the same) Add 0 to the list of ones digits in the first row on the board, before the 1. ASK: What should we do with 5? Have students suggest options and explain why they prefer the option suggested. Remind students that mathematicians have agreed that a measurement that is exactly halfway between two numbers, say 5 cm and 6 cm, is always rounded up, in this case to 6 cm. Explain that just as we round a measurement up to the next unit, we round numbers that have ones digit 5 up. So 25 rounded to the nearest ten is 30 and 15 rounded to the nearest ten is 20. ASK: Which list should we add 5 to? (the second) Add 5 to the second list on the board, before the 6. Point out that this gives both lists five digits.

Repeat Activity 1 above, this time asking students to round any 2-digit numbers to the nearest multiple of 10.

**Multiples of 10 for 3-digit numbers.** Write on the board:

70, 80, 90, 100

Ask students to extend the list. (110, 120, 130, etc.) ASK: How can we know that a number is a multiple of 10? (it has ones digit 0) ASK: Give me an example of a multiple of 10 that is bigger than 342. (350, 360, etc.) Give me an example of a multiple of 10 that is bigger than 200 but smaller than 300. (210, 220, ..., 290)

**ACTIVITY 4**

Name a 3-digit multiple of 10. Students pass a ball from one to the other, each saying the next multiple of 10. You can play several rounds of the game. Start with numbers such as 280, 390, or 790, so that students have to count across hundreds (390, 400, 410, ...) several times in each round. Each student should try to pass the ball to a student who has not yet had it, to ensure that everyone gets a turn.

ASK: What is the previous multiple of 10 for 238? (230) How do you know? (you leave the digits in front of the ones digit the same, and change the ones digit to 0) Repeat Activity 4 above, asking for the previous multiple of 10 for 3-digit numbers.
ASK: What is the next multiple of 10 for 238? (240) Have students explain how they know. Answers might include finding the previous multiple of 10 and taking the next after it, imagining counting up by ones and saying the next number that has ones digit 0, or other methods. Repeat Activity 4 above, asking for the next multiple of 10 for 3-digit numbers. Then ask students to say the previous and the next multiples of 10 for several 3-digit numbers.

**Exercises:** Which two multiples of 10 is the number between?

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| a) 53 | b) 153 | c) 671 | d) 809 | e) 978 **Bonus:** 998

**Answers:**

- a) 50, 60; b) 150, 160; c) 670, 680; d) 800, 810; e) 970, 980; Bonus: 990, 1,000

**Rounding 3-digit numbers to the nearest multiple of 10.** Draw a number line from 150 to 160 on the board. Have volunteers decide which multiple of 10 is the nearest one for 154, 158, 157, and 152, and draw an arrow to that multiple of 10. (150, 160, 160, 150) Then discuss the rounding rule the same way you discussed it for 2-digit numbers. Make sure students understand that the rounding rule is exactly the same: you round down if the ones digit is 1, 2, 3, or 4 and you round up for 6, 7, 8, or 9. Explain that it would not make sense to make the rule for rounding numbers with ones digit 5 different for 2-digit numbers and 3-digit numbers, so 3-digit numbers with ones digit 5 are still rounded up.

Explain that sometimes people simply say “round to the nearest ten” instead of “round to the nearest multiple of 10.” It means the same thing; it is just a short form.

**Exercises:** Round to the nearest ten.

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| a) 73 | b) 183 | c) 651 | d) 309 | e) 477 **Bonus:** 997

**Answers:**

- a) 80, b) 180, c) 650, d) 310, e) 480, Bonus: 1,000

**Extensions**

1. Explain that rounding can help you estimate the answers to multiplication problems. You will not get the same answer as if you did not round, but you can get a good idea of how large the product is. For example, if a banana costs 49 cents, and you want to buy 6 bananas, you can calculate \( 6 \times 50 = 300 \) cents and know that you need about $3 to buy the bananas. So if you have a $5 bill, you know you have enough money. On the other hand, if you have only a $1 bill, you know you do not have enough.

Have students round the 2-digit number in each product to the nearest ten before multiplying the numbers to say about how large the product is.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3 \times 49</td>
<td>b) 6 \times 38</td>
<td>c) 9 \times 21</td>
<td>d) 8 \times 32</td>
<td></td>
</tr>
</tbody>
</table>
Answers: a) $3 \times 50 = 150$, b) $6 \times 40 = 240$, c) $9 \times 20 = 180$, d) $8 \times 30 = 240$

Now, have students use a calculator to find the exact products and then round to the nearest ten. Did they get the same answer? How close were they?

Answers: a) 147, round to 150, no difference; b) 228, round to 230, difference of 10; c) 189, round to 190, difference of 10; d) 256, round to 260, difference of 20

2. Introduce another method of finding the next multiple of ten.

Step 1: Change the ones digit to 0.

Step 2: Add 1 to the tens digit.

Step 3: Keep the hundreds digit the same.

Have students use this method to find the next multiple of 10.

a) 238  b) 769  c) 451  d) 103

Answers: a) 240, b) 770, c) 460, d) 110

Then ask students to try to use this method on 792. What problem do they encounter? (the tens digit is 9, so when they add 1 it becomes 10) Explain that in this case we need to regroup. ASK: What are 10 tens equal to? (1 hundred) Explain that we need to add 1 to the hundreds digit, so the hundreds digit becomes 8. ASK: Are there any tens left over? (no) This means we need to make the tens digit 0, so the next multiple of 10 after 792 is 800. Have students find the next multiple of 10 for 395, 97, and 299. (400, 100, 300) Then have them think about what happens for 999. Students should see that after regrouping 10 tens as 1 hundred, they also need to regroup the hundreds, so the answer is 1,000. Finally, have students check that their answers make sense by counting up.

3. Megan has many pennies in her piggy bank. If she has about $5.60 worth of pennies, how many pennies can she have exactly?

Answer: 555 to 564 pennies

(MP5)  

4. Calculate $18 \div 3$. Use one of the following tools:

- a related multiplication equation
- a picture of equal groups
- an array
- a number line
- an easier division

Answer: 6

Redirecting students: If students struggle to select a tool, have them choose from only two options, such as drawing equal groups and drawing a number line.
Whole-class follow-up: Compare solutions. For example, guide students toward similarities between drawing groups with 3 dots each, drawing an array with 3 in each row, drawing a number line with jumps of size 3, and using the multiplication equation $6 \times 3 = 18$. All these solutions express the number of groups of size 3 that make 18.

Some students might have used an easier division. For example, $15 \div 3 = 5$ (because 18 is one more three than 15) or $9 \div 3 = 3$ (because 18 is double 9). In these cases, ASK: How are these strategies the same? (they both use an easier division) How are they different? (they use different easier divisions)

**NOTE:** Students who use a multiplication or an easier division are noticing and making use of structure (MP7).
Goals

Students will round 3-digit numbers to the nearest hundred.

PRIOR KNOWLEDGE REQUIRED

Is familiar with place values up to hundreds
  Can count by 10s to 1,000
  Can count by 100s to 1,000
  Can find the multiples of 10 before and after a number
  Can round 2- and 3-digit numbers to the nearest ten

MATERIALS

a ball
15 blank cards per student pair (see Extension 2)

NOTE: In this lesson students will round numbers to the nearest hundred using steps similar to the ones in the previous lesson:

Step 1: Find the next and the previous multiple of 100.

Step 2: Decide which of them the number is nearest to.

After that students will be introduced to a rule for rounding based on the tens digit.

Multiples of 100. Remind students that in the previous lesson they talked about multiples of 10. Ask them to explain what a multiple of 10 is and how they know whether a number is a multiple of 10. ASK: If multiples of 10 are the numbers you say when skip counting by 10s starting from 0, what are multiples of 100? (numbers you say when skip counting by 100s starting from 0) Count by 100s as a class starting at 0, and record the multiples of 100 up to 1,000 on the board.

Finding the multiples of 100 before and after a number. Remind students that in the previous lesson they identified the previous and the next multiples of 10 for a number. ASK: What is the previous multiple of 10 for 56? (50) How do you know? (changed ones digit to 0) What is the next multiple of 10 for 56? (60) How do you know? (go to the next multiple of 10 after 50; imagine counting by 1s from 56—the tens digit changes at 60, so this is then next multiple of 10)

Ask students to think of a method to get the previous and the next multiples of 100 for 256. Give students a few minutes to think alone, then have them work in pairs to consolidate their ideas. Debrief as a class. Students can keep the hundreds digit of the number and write zeros for the tens and ones digits to get 200, and then count up by 100 to get 300. Have students use this method to find the previous and the next multiples of 100 for 319 and for 890.
ACTIVITY

As in the previous lesson, use a ball to give students practice in identifying multiples. Pass the ball to a student while asking for the next or the previous multiple of 100 for a 3-digit number. The student passes the ball back to you as they say the answer. Repeat with different numbers and different students.

Exercises: What two multiples of 100 is the number between?

a) 126  
b) 397  
c) 860  
d) 549  
e) 951  

Bonus: 78

Answers:  
a) 100, 200; b) 300, 400; c) 800, 900; d) 500, 600; e) 900, 1,000;  
Bonus: 0, 100

Deciding whether a multiple of 10 is closer to 0 or to 100. Display a number line from 0 to 100 on the board that has only the multiples of 10 marked on it. ASK: Is 30 closer to 0 or to 100? (0) Draw an arrow from 30 to 0. ASK: Is 60 closer to 0 or to 100? (100) Draw an arrow from 60 to 100.

Repeat with 80 and 20. (100, 0) ASK: Is 50 closer to 0 or to 100? (neither, it is right in the middle)

Locating numbers that are not multiples of 10 on a number line.

ASK: Where on this number line is 33? Have a volunteer show the position, and ask the class to show thumbs up or thumbs down to signal whether or not they agree with the answer. Then have another volunteer explain why 33 should be placed between 30 and 40, and slightly closer to 30 than to 40. (30 is the previous multiple of 10, and 40 is the next multiple of 10, but 33 is closer to 30 than to 40) Repeat with 59, 92, 47, and 3. The number line should look like this:

```
   0  10  20  30  40  50  60  70  80  90  100

  3  33  47  59  92
```

Determining the closest multiple of 100 (0, 100, or 200) for numbers that are not multiples of 10. ASK: Is 33 closer to 0 or to 100? (0) Have a volunteer draw an arrow from 33 to 0. Repeat with the rest of the numbers on the number line. ASK: For all these numbers, what is the previous multiple of 100? (0) What is the next multiple of 100? (100) Label the 0 on the number line “previous multiple of 100” and label the 100 “next multiple of 100.”

Without erasing the first number line, draw a number line from 100 to 200 directly beneath it and have students first identify the location and then whether the number is closer to 100 or to 200 for 140, 190, 125, 179, 148, and 151. (100, 200, 100, 200, 100, 200) As before, add labels to the number line for the previous multiple of 100 (100) and next multiple of 100. (200) Keep the second number line on the board.
Ask students to say how they can decide whether a 2-digit number is closer to 0 or to 100 without a number line. (numbers from 1 to 49 are closer to 0, numbers from 51 to 99 are closer to 100) ASK: What number is right in the middle, the same distance from both? (50) Repeat with the numbers from 101 to 199: Which are closer to 100 and which are closer to 200? (numbers from 101 to 149 are closer to 100; numbers from 151 to 199 are closer to 200; 150 is the same distance from 100 and 200)

Draw on the board:

<table>
<thead>
<tr>
<th>Numbers from</th>
<th>1 to 49</th>
<th>101 to 149</th>
<th>51 to 99</th>
<th>151 to 199</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closer to</td>
<td>previous multiple of 100</td>
<td>next multiple of 100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using a pattern to determine the closest multiple of 100 for any 3-digit number. Ask students to predict using the pattern they see in the table whether 320 is closer to the previous or to the next multiple of 100. (previous) Repeat with 680, 444, 555. (next, previous, next) Have students explain their answers using the pattern. (numbers with the last two digits making a number smaller than 50 are closer to the previous multiple of 100, and numbers with the last two digits making a number greater than 50 are closer to the next multiple of 100) ASK: What are the last two digits of the numbers that are the same distance from both multiples of 100, that is, right in the middle? (50)

ASK: What multiples of 100 are these numbers between: 234, 235, 237, 230, 239? (200 and 300) Are these numbers closer to 200 or to 300? (200) What do these numbers have in common? (the first two digits) What is different in each number? (the ones digit) SAY: These numbers all have different ones digits, but they all are closer to the same multiple of 100. I think this means that the ones digit does not matter when you are deciding which multiple of 100 is closer. Let’s check if this is true for other numbers. Ask students to write four different 3-digit numbers with the same hundreds and tens digit in their notebooks. Then ask them to swap notebooks with a partner and to see which multiple of 100 the numbers are closest to.

ASK: Did anyone find numbers that are closer to different multiples of 100? (no) Note that if anyone had a number that ends with 50, this number is a special case, and will be dealt with later.

ASK: Who had numbers with tens digit 0, 1, 2, 3, or 4? Was the nearest multiple of 100 the one before or the one after the number? (before) Who had numbers with tens digit 6, 7, 8, or 9? Was the nearest multiple of 100 the one before or after the number? (after) ASK: How can you tell from the tens digit which multiple of a hundred a number is nearest to? Emphasize that the number is closer to the higher multiple of 100 if its tens digit is 6, 7, 8, or 9 and it’s closer to the lower multiple of 100 if its tens digit is 1, 2, 3, or 4.
ASK: Who had numbers with tens digit 5? If the tens digit is 5, then any ones digit except 0 will make it closer to the higher multiple. Only when the tens digit is 5 and the ones digit is 0 do we have a special case where the number is not closer to either.

**Introduce rounding to 100.** Remind students that in the previous lesson they rounded numbers to the nearest multiple of 10. Ask them to explain what that means. (finding the multiple of 10 that is nearest to the number) Explain that today they will round numbers to the nearest multiple of 100, or to the nearest hundred. Remind them that when they rounded to the previous multiple of 10, they replaced the ones digit with a zero, and that was called rounding down. When they rounded to the next multiple of 10, that was called rounding up. Add a row to the table above, as shown below:

<table>
<thead>
<tr>
<th>Numbers from</th>
<th>1 to 49</th>
<th>101 to 149</th>
<th>51 to 99</th>
<th>151 to 199</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closer to</td>
<td>previous multiple of 100</td>
<td>next multiple of 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round</td>
<td>down</td>
<td>up</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: What should we do with numbers, like 150, that are halfway between two multiples of 100? Ask for suggestions and have students explain their reasoning. Explain that when rounding to the nearest hundred, numbers that end with 50 are rounded up, the same way numbers with tens digit 5 are rounded up.

Write on the board:

To round to the nearest hundred:

**Step 1:** Write the previous and the next multiples of 100.

**Step 2:** If the tens digit is

- 0, 1, 2, 3, 4, round down
- 5, 6, 7, 8, 9, round up

SAY: I want to round 326 to the nearest hundred. What two multiples of 100 is it between? (300 and 400) What is the tens digit? (2) Which list is it in? (the first one) Will we round up or down? (down) What is 326 rounded to the nearest hundred? (300) Repeat the questions for 581. (500 and 600, 8, the second list, round up, 600)

Give students the numbers in the exercises below and have them signal with thumbs up or thumbs down whether they will round the number up or down to the nearest hundred. Then have them round the number.

**Exercises:** Round to the nearest hundred.

a) 124  b) 396  c) 860  d) 749  e) 451  
  f) 73   g) 205  h) 555  i) 650  
**Answers:** a) 100, b) 400, c) 900, d) 700, e) 500, f) 100, g) 200, h) 600, i) 700
Explain that it is enough to imagine the two multiples of 100 without writing them down. Give students 3-digit numbers and have them try to find the nearest multiple of 100 mentally. For more practice, repeat Activity 1, asking for the nearest multiple of 100.

**Rounding to the nearest ten and to the nearest hundred.** Tell students that you want them to round the numbers in the exercises above to the nearest ten instead of the nearest hundred. Ask: Which numbers do not need rounding? (c) 860, i) 650) Why do they not need rounding? (they are already multiples of 10) Have students round the rest of the numbers. (a) 120, b) 400, d) 750, e) 450, f) 80, g) 210, h) 560)

Ask: Which number produced the same answer when rounded to the nearest ten and to the nearest hundred? (396) Can you think of other numbers that will be rounded to the same number when rounding to the nearest ten and to the nearest hundred? (sample answers: 395, 397, 398, 399, 495, 496, 497, 498, 499) To prompt students to think about numbers that are a little over a multiple of 100, ask them to try the numbers that are 1 less than the numbers in the exercises above (123, 395, 859, 748, 450, 72, 204, 554, 649) Ask: Which of these numbers rounds to the same number when rounded to the nearest hundred and to the nearest ten? Students will see that 204 rounds to 200 when rounded both to the nearest ten and to the nearest hundred.

**Extensions**

(MP.7) 

1. Guess the number from the clues.

   a) This number is the largest number that rounds down to 300 when rounded to the nearest hundred.

   b) This number is the smallest number that rounds up to 300 when rounded to the nearest hundred.

   c) This number is the smallest 2-digit number that rounds up to a 3-digit number.

   d) When you round this number to the nearest hundred it becomes 11 more than it was. Its hundreds digit is 7.

   e) This number has hundreds digit 4 and ones digit 2. When you round it to the nearest hundred, you round up. When you round it to the nearest ten, the answer is 10 less than when you round to the nearest hundred.

   f) This number has hundreds digit 3 and ones digit 3. When you round it to the nearest hundred or to the nearest ten, you get the same number.

Answers: a) 349, b) 250, c) 50, d) 789, e) 492, f) 303
2. If students enjoyed the game in Activity 2 of the previous lesson, they can play it again using 3-digit numbers and multiples of 100. First, they will have to make a new set of playing cards. Give pairs of students 15 blank cards. Have them write all the multiples of 100, from 0 to 1,000, on separate cards. Then have them write random 3-digit numbers, each with a different hundreds digit, on each of the remaining 4 cards. Finally, have pairs swap the cards they created with another pair and play the game as before.

(MP3, MP5, MP8) 3. Jun plants flowers in an array. His garden has the same number of rows and columns. Marla plants flowers in another array. Her garden has one more row than Jun’s garden and one fewer column. Who planted more flowers? Do you think the answer will always be the same? Explain why or why not. Use one or more of these tools:
- arrays
- number lines
- a T-table

Sample answers
- I made a T-table. I started with Jun’s square array and then found how many plants are in Marla’s array.

<table>
<thead>
<tr>
<th>Number of Plants in Jun’s Garden</th>
<th>Number of Plants in Marla’s Garden</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 1 = 1</td>
<td>2 × 0 = 0</td>
</tr>
<tr>
<td>2 × 2 = 4</td>
<td>3 × 1 = 3</td>
</tr>
<tr>
<td>3 × 3 = 9</td>
<td>4 × 2 = 8</td>
</tr>
<tr>
<td>4 × 4 = 16</td>
<td>5 × 3 = 15</td>
</tr>
</tbody>
</table>

I noticed a pattern that the number in Marla’s garden is one less than the number in Jun’s garden, so I think it will always be true.
- I drew arrays for different examples. For example, I compared 4 × 4 to 5 × 3 (see margin). I noticed the extra column has 4 and the extra row only has 3. This was true for other examples, too. The extra column always has one more than the extra row, so I think it will always be true.

NOTE: Students can also use number lines, but in order to notice which product is always one greater than the other, students will always have to put the same product first, which will be more difficult.

Whole-class follow-up: Show both solutions provided above and ASK: How does the T-table make it easy to see the pattern? (the number of plants in each garden are in each row, so they are easy to compare) How does the array make it easy to see why the answers are always one apart? (the extra column always has one more than the extra row)
Goals
Students will estimate sums and differences by rounding numbers to the nearest ten or hundred.

PRIOR KNOWLEDGE REQUIRED
- Is familiar with place values up to hundreds
- Can round to the nearest ten or hundred
- Can add and subtract using the standard algorithm

Review rounding to the nearest ten and hundred. Remind students of the rule for rounding to the nearest ten. Explain that even though 5 is the same distance from 0 and 10, and any number with ones digit 5 is the same distance from the two nearest multiples of 10, we still round up.

Exercises: Round to the nearest ten.

a) 34  b) 59  c) 45  d) 128
e) 202  f) 391  g) 497  h) 655

Answers: a) 30, b) 60, c) 50, d) 130, e) 200, f) 390, g) 500, h) 660

Remind students of the rule for rounding to the nearest hundred. ASK: When rounding to the nearest hundred, what digit do we look at? (tens digit) Remind them that if the tens digit is 0, 1, 2, 3, or 4, they round down, and if it is 5, 6, 7, 8, or 9, they round up. Remind students that the ones digit does not matter when they round to the nearest hundred: for example, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249 are all rounded down to 200, because they are all closer to 200 than to 300 and they all have the same tens digit, 4.

Exercises: Round to the nearest hundred.

a) 230  b) 640  c) 790  d) 60  e) 36
f) 459  g) 871  h) 548  i) 235  j) 547
k) 650  l) 850  m) 495

Answers: a) 200, b) 600, c) 800, d) 100, e) 0, f) 500, g) 900, h) 500, i) 200, j) 500, k) 700, l) 900, m) 500

Write on the board:

to the nearest ten  to the nearest hundred
547  547

SAY: In this lesson, we will sometimes round to the nearest ten and sometimes to the nearest hundred. To keep track of what we are rounding to, we can underline the place value to which we are rounding. So when...
rounding to the nearest ten, underline the tens digit. Invite a volunteer to underline the tens in the first number on the board. SAY: When rounding to the nearest hundred, underline the hundreds digit. Have a volunteer underline the hundreds digit in the second number on the board. SAY: Now we need to look at the digit to the right of the underlined one. Draw an arrow pointing to the next digit in both cases, as shown below:

![Diagram](image)

SAY: If the digit after the underlined one is 0, 1, 2, 3, or 4, we round down. If the digit is 5, 6, 7, 8, or 9, we round up. Point to the first number and ASK: Should we round up or down? (up) What will the number be? (550) Repeat with the second number. (500)

**Exercises:** Round.

a) 673 to the nearest ten  
b) 406 to the nearest ten  
c) 809 to the nearest hundred  
d) 330 to the nearest hundred  
e) 685 to the nearest ten  
f) 258 to the nearest hundred

**Answers:** a) 670, b) 410, c) 800, d) 300, e) 690, f) 300

Explain that most 3-digit numbers, when rounded to the nearest hundred, will have 3 digits. Which numbers are exceptions? (any number from 950 to 999, they are rounded to 1,000)

**Introduce estimating in calculations.** Remind students that when measuring they sometimes need an exact measurement, while other times they need only a sense of how large something is. For example, if I want to buy curtains for a window, it matters whether the window is 1 m wide or 4 m wide. But if the window is a little over 1 m, it does not really matter by how much, as long as the measurement is close. Explain that a similar situation can happen with calculations.

Explain to students that they can estimate the answer to a calculation by simplifying the numbers first. Their answer when they do this will be close to the exact answer of the original calculation. Estimating an answer before calculating can help them to catch mistakes because when they see the answer, they will already have an idea of what it should be. For example, when you use a calculator, you can sometimes press a wrong number by mistake. If I add 25 and 38 on a calculator, would 173 be a reasonable answer? Have students add the numbers. (25 + 38 = 63) Point out that 173 is much larger than the correct answer, so it is unreasonable. Explain that students could have used an estimate instead of the exact answer to check if 173 was reasonable. The most common way to estimate a calculation is to round the numbers before calculating.
Estimating a sum or a difference. Write on the board:

\[
25 \quad + \quad 38
\]

Say: When working with 2-digit numbers, round to the closest ten.
AsK: What is 25 rounded to the nearest ten? (30) What is 38 rounded to the nearest ten? (40) Have a volunteer write the rounded numbers in the boxes, then add them. (70) Point out that the estimate, 70, is very close to the actual answer, 63. If students had estimated the answer first, they would have known right away that 173 was unreasonable, because it is much larger than 70.

Exercises

1. Round the numbers to the nearest ten. Then add.
   a) \(41 + 38\)  
   b) \(52 + 11\)  
   c) \(73 + 19\)  
   d) \(84 + 13\)  
   e) \(92 + 37\)  
   f) \(83 + 24\)  
   Answers: a) \(40 + 40 = 80\), b) \(50 + 10 = 60\), c) \(70 + 20 = 90\), d) \(80 + 10 = 90\), e) \(90 + 40 = 130\), f) \(80 + 20 = 100\)

2. Add the numbers in Exercise 1 without rounding them first, and compare your answers.
   Answers: a) 79, b) 63, c) 92, d) 97, e) 129, f) 107

Ask: How close are the answers? Is the difference between the estimate and the actual calculation ever larger than 10? (no) Explain that when we round to the nearest ten, getting an answer that is less than 10 away from the actual answer means that the estimate is good.

Ask: How would you estimate \(93 - 21\)? Write on the board:

\[
93 \quad - \quad 21
\]

Ask students to round the numbers, then to subtract. (90 – 20 = 70) Then ask them to subtract the numbers without rounding. (93 – 21 = 72) Again, compare the answers. Are they close? (yes) Which calculation is easier? (90 – 20 = 70)

Exercises

1. Round the numbers to the nearest ten, then subtract.
   a) \(53 - 21\)  
   b) \(72 - 29\)  
   c) \(68 - 53\)  
   d) \(48 - 17\)  
   e) \(63 - 12\)  
   f) \(74 - 37\)  
   Answers: a) \(50 - 20 = 30\), b) \(70 - 30 = 40\), c) \(70 - 50 = 20\), d) \(50 - 20 = 30\), e) \(60 - 10 = 50\), f) \(70 - 40 = 30\)
2. Subtract the numbers in Exercise 1 without rounding them first, and compare your answers.

**Answers:** a) 32, b) 43, c) 15, d) 31, e) 51, f) 37

**ASK:** How close are the answers? Is the difference between the estimate and the actual calculation ever larger than 10? (no) Were our estimates good? (yes)

Explain that for 3-digit numbers, sometimes you can use an even rougher estimate. In these cases, you can round to the nearest hundred before calculating. Do the first two exercises below as a class, then have students do the rest individually. Have students check their estimates by doing each calculation without rounding and then comparing the answers. Point out that since students are now rounding to the nearest hundred, their estimates are good if they are within 100 of the actual answer, and the closer, the better.

**Exercises:** Round the numbers to the nearest hundred, then add or subtract. Check your estimate by adding or subtracting without rounding.

a) \(421 + 159\)  
b) \(904 - 219\)  
c) \(498 + 123\)  
d) \(450 - 151\)  
e) \(378 + 449\)  
f) \(473 - 188\)

**Answers:** a) \(400 + 200 = 600, \) actual 580; b) \(900 - 200 = 700, \) actual 685; c) \(500 + 100 = 600, \) actual 621; d) \(500 - 200 = 300, \) actual 299; e) \(400 + 400 = 800, \) actual 827; f) \(500 - 200 = 300, \) actual 285

**Bonus:** Round the numbers to the nearest ten, then add or subtract. Do you get a better estimate? Is the calculation harder?

**Answers:** You get a better estimate most of the time, but the calculation is harder.

a) \(420 + 160 = 580, \) actual 580; b) \(900 - 220 = 680, \) actual 685; c) \(500 + 120 = 620, \) actual 621; d) \(450 - 150 = 300, \) actual 299; e) \(380 + 450 = 830, \) actual 827; f) \(470 - 190 = 280, \) actual 285

**Using rounding in word problems.** SAY: A student I know added 273 and 385, and got the answer 958. Does this answer seem reasonable? Have students round each number to the nearest hundred to check the answer. (300 + 400 = 700) **ASK:** Is the answer reasonable? (no)

Write on the board:

A store sold 58 red apples and 21 green apples. How many apples did the store sell altogether?

SAY: It would be easier to add these numbers if they were multiples of 10. **ASK:** Are these numbers close to multiples of 10? (yes) What is the nearest multiple of 10 to 58? (60) To 21? (20) What is 60 + 20? (80) Do you think that 80 is a good estimate for 58 + 21? (yes) Point out that 58 is only 2 less than 60, and 21 is only 1 more than 20, so we changed the numbers very little and can expect a good estimate. What is the actual answer to 58 + 21? (79) Was 80 a good estimate? (yes)
**Exercises:** Estimate the total number of apples the store sold. Then find the actual answer. Was your estimate close?

a) 27 red apples and 42 green apples  

b) 46 red apples and 78 yellow apples  

c) 52 Granny Smith apples and 31 Golden Delicious apples  

d) 42 Red Delicious apples and 29 Gala apples  

**Answers:** a) $30 + 40 = 70$, actual 69; b) $50 + 80 = 130$, actual 124;  

c) $50 + 30 = 80$, actual 83; d) $40 + 30 = 70$, actual 71  

Leave the exercises above on the board. Continue writing on the board:

About how many more green apples than red apples were sold in part a)?

**ASK:** What word in this question tells you I only want an estimate? (about) Does the question ask for the sum of green apples and red apples or the difference between them? (the difference) How do you know? (the question asks for “how many more”) What operation should I use to find the difference—addition or subtraction? (subtraction) Have students estimate the differences in all four exercises above. (a) about 10 more green than red, b) about 30 more yellow than red, c) about 20 more Granny Smith than Golden Delicious, d) about 10 more Red Delicious than Gala)

Write on the board:

Greg collected 247 stamps, Ron collected 172 stamps, and Sara collected 349 stamps.

Have students use the information in the following exercise.

**Exercises:** Round to the nearest hundred to estimate.

a) About how many more stamps did Sara collect than Greg?  

b) Do the boys have more stamps altogether than Sara?  

c) About how many stamps do all three of them have altogether?

**Answers:** a) $300 - 200 = 100$, b) the boys have about $200 + 200 = 400$ stamps. Sara has about 300 stamps. Yes, the boys have more stamps altogether. c) $200 + 200 + 300 = 700$. They have about 700 stamps altogether.

**ASK:** About how many more stamps does Greg have than Ron? Have students estimate to the nearest hundred. **ASK:** What is the answer? (0) Does the answer make sense? (no, because the boys don’t have identical numbers of stamps—there is a difference, so the answer must be more than 0) What happened? (both numbers are 200 when rounded to the nearest hundred, so the estimated difference is 0) Have students estimate by rounding to the nearest ten instead. ($250 - 180 = 70$, so Greg has about 70 more stamps than Ron) Explain that estimation using rounding to the
nearest hundred gives only a rough sense of the numbers, and sometimes we need a better estimate (so we round to the nearest ten) or even an exact answer.

**Sometimes estimation does not make sense.** SAY: Jay is 4 years old. ASK: If you round 4 to the nearest ten, what do you get? (0) Does it make any sense to say that someone is about 0 years old? (no) Explain that estimation is a good thing in general, but there are some situations where it makes no sense.

**Extensions**

1. a) Jayden and Tasha are trying to estimate $46 + 25$.
   Jayden says: I round both numbers to the nearest ten, so the answer is about $50 + 30 = 80$. Tasha says: 25 is as close to 20 as it is to 30. If I round 46 up, I will get a bigger number than the actual answer. If I round 25 up too, the answer will be even bigger. I will round 25 down instead, so the answer should be close to $50 + 20 = 70$.
   Who made a better estimate? Check against the actual answer.
   
b) Use both methods, Jayden’s and Tasha’s, to estimate.
   i) $29 + 55$ ii) $87 + 35$ iii) $15 + 98$ 
   Which method gives you a better estimate?
   
c) Pedro thinks it could be better to round 25 down when estimating $22 + 25$. Which method gives you a better estimate—Jayden’s or Pedro’s?

**Answers**

a) $46 + 25 = 71$, so Tasha’s method is better
b) i) Jayden’s method: $30 + 60 = 90$, Tasha’s method: $30 + 50 = 80$, actual answer 84, Tasha’s method is better; ii) Jayden’s method: $90 + 40 = 130$, Tasha’s method: $90 + 30 = 120$, actual answer 122, Tasha’s method is better; iii) Jayden’s method: $20 + 100 = 120$, Tasha’s method $10 + 100 = 110$, actual answer 113, Tasha’s method is better
   c) Jayden’s method: $20 + 30 = 50$, Pedro’s method: $20 + 20 = 40$, actual answer 47, Jayden’s method is better

**2.** $390 + 425$ is about $400 + 400 = 800$. Without adding the numbers, say if the actual answer is more than 800 or less than 800. Explain your thinking.

**Answer:** 390 is 10 less than 400, and 425 is 25 more than 400. Since $25 > 10$, the error in rounding down is more than the error in rounding up. This means the actual sum is more than 800.
3. You have a $1 bill. Do you have enough money to buy all of the items in this list?

- A pencil for 12¢
- An eraser for 25¢
- A notebook for 29¢
- A pen for 19¢

Explain your strategy.

**Answer:** Round the numbers to the nearest ten. Since $10 + 30 + 30 + 20 = 90¢$, a dollar bill should be enough. However, a better strategy would be to round all the numbers up. If you get at most 100¢, you know you have enough money. $20 + 30 + 30 + 20 = 100$, so you know for sure you have enough money.

4. Use estimation to find incorrect equations.

   a) $453 + 278 = 631$
   b) $483 + 287 = 770$
   c) $529 - 146 = 283$
   d) $785 - 295 = 490$

**Answers:** a) $500 + 300 = 800$, so 631 is too low—the equation cannot be correct; b) $500 + 300 = 800$, about the same; c) $500 - 100 = 400$, 283 is too low—the equation cannot be correct; d) $800 - 300 = 500$, about the same.
Goals
Students will learn the place values in 4-digit numbers.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to hundreds
Can read and write numbers to 1,000

MATERIALS
cards from BLM Place Value Cards, (p. P-45), 4 for each student and 4 for demonstration

Review place value to hundreds. Give each student four cards from BLM Place Value Cards (each student should get a ones, a tens, a hundreds, and a thousands card). Remind students that the words on the cards are place values. Write the number 37 on the board and ASK: What is the place value of the 7? Students should raise the card with the correct place value (ones) simultaneously, so that you can assess everyone at the same time. Repeat with the 3. (tens) Then have students identify the place value of each digit in 237, and then in 409.

Introduce thousands. Write on the board:

4,156

ASK: How is this number different from most of the numbers we have worked with this year? (it has 4 digits, it has a comma) Underline the 4 and tell students that its place value is thousands. Explain that there are 10 hundreds in a thousand.

Identifying place values. Write on the board:

4,856

Affix a place value card under each digit. (thousands, hundreds, tens, ones) Repeat with 4,237, asking students to tell you the correct place value as you point to each digit. Students can raise the card with the correct place value as you name the digit. Example: What is the place value of 3 in 4,237? (tens)

Write the numbers in the exercises below one at a time on the board and point to different digits, out of order. Students should raise the card with the correct place value as you point to the digit.

Exercises: Identify the place value of each digit.

a) 6,198  b) 8,739  c) 9,065  d) 8,402  e) 3,760
Selected answer: a) 6: thousands, 1: hundreds, 9: tens, 8: ones
Continue until students can identify place value correctly and confidently. Include examples where you ask for the place value of the digit 0. Point out that, although the digit 0 always has a value of 0, its place value changes with position the same as any other digit.

Explain that to read a 4-digit number you say the number of thousands first, and then read the 3-digit number that is left. For example, 4,856 is “four thousand, eight hundred fifty-six.” Point out that just as we say “eight hundred” and not “eight hundreds,” we also say “four thousand” and not “four thousands.” Have students read the numbers in the exercises above.

Introduce the place value chart. Draw a place value chart on the board. Have students write the digits from the number 231 in the correct columns, as shown below:

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>231</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Add more numbers to the first column of the place value chart. Include numbers with 1, 2, 3, and 4 digits, and have volunteers come to the board to write the digits in the correct columns. Include zeros in the numbers in all places except the thousands.

Writing 4-digit numbers. Explain that when numbers have more than 3 digits, it is hard to read them and hard to keep track of the place values. To help keep track of the place values, people write a comma (point to it) between the thousands and the hundreds. Read the numbers in the exercise below out loud and have students write them down as numerals.

Exercises: Write the number as a numeral.

a) seven thousand, one hundred sixty-eight
b) four thousand, two hundred thirty-nine
c) five thousand, four hundred sixty-five
d) one thousand, seven hundred thirty-two

Answers: a) 7,168; b) 4,239; c) 5,465; d) 1,732

ASK: How would you write the number four thousand? Have students think, then explain that since there are only thousands, and no additional hundreds, tens, or ones, the number is written with a 4 in the thousands place and zeros in all three other places. Have a volunteer write 4,000 on the board. Then have different volunteers write the numbers eight thousand (8,000) and one thousand (1,000).

Repeat with the numbers eight thousand four hundred thirty (8,430), five thousand one (5,001), three thousand five hundred (3,500), and four thousand eighty (4,080).
**The value of a digit.** Remind students that there are 10 ones in a ten and 10 tens in a hundred. This means that each place value to the left is 10 times larger than the one to the right (and each place value to the right is 10 times smaller than the one to the left). For example, in the number 333 the first 3 stands for 300, the second 3 stands for 30, and the third 3 is just 3. Ask students how much each digit in 456 is worth. (4 stands for 400, 5 stands for 50, 6 stands for 6)

Write 2,836 on the board. **ASK:** What is the place value of the digit 2? (thousands) **SAY:** The 2 is in the thousands place, so it stands for 2,000. We also say that the value of 2 in 2,863 is 2,000. What does the digit 8 stand for? (800) What is the value of 3? (30) And the 6? (6) Repeat with 8,902, 7,431, and 9,006.

**Exercises:** What does the digit 2 stand for in the number?

- a) 3,297
- b) 2,985
- c) 7,892
- d) 2,095
- e) 8,132
- f) 9,002
- g) 3,020
- h) 8,200

**Answers:** a) 200, b) 2,000, c) 2, d) 2,000, e) 2, f) 2, g) 20, h) 200

**Extensions**

1. Round 4-digit numbers to all possible places.

**Example:** 1,382
- To the nearest thousand: 1,000
- To the nearest hundred: 1,400
- To the nearest ten: 1,380

<table>
<thead>
<tr>
<th>Number</th>
<th>Nearest 1,000</th>
<th>Nearest 100</th>
<th>Nearest 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 4,562</td>
<td>5,000</td>
<td>4,600</td>
<td>4,560</td>
</tr>
<tr>
<td>b) 6,081</td>
<td>6,000</td>
<td>6,100</td>
<td>6,080</td>
</tr>
<tr>
<td>c) 2,345</td>
<td>2,000</td>
<td>2,300</td>
<td>2,350</td>
</tr>
</tbody>
</table>

**Answers**

2. Teach students the Egyptian system for writing numerals, to help them appreciate the utility of place value. Explain that Egyptians used the following symbols in their system:

- 1 = (stroke)
- 10 = (arch)
- 100 = (coiled rope)
- 1,000 = (lotus leaf)
Write the following numbers on the board using both our system and the Egyptian system:

- 234
- 848
- 423

Invite students to study the numbers for a moment, then ASK: What is different about the Egyptian system for writing numbers? (You have to show the number of ones, tens, and so on individually—if you have 7 ones, you have to draw 7 strokes. In our system, a single digit, 7, tells you how many ones there are.)

Ask students to write a few numbers the Egyptian way and to translate those Egyptian numbers into our numerals. Have students write a number that is really long the Egyptian way (Example: 798). ASK: How is our system more convenient? Why is it helpful to have a place value system, with the ones, tens, and so on, always in the same place? (the number is much shorter) Tell students that the Babylonians, who lived at the same time as the ancient Egyptians, were the first people to use place value in their number system. Students might want to invent their own number system using the Egyptian system as a model.

3. Have students identify and write numbers given specific criteria and constraints. Examples:
   a) Write a number between 30 and 40.
   b) Write an even number with a 6 in the tens place.
   c) Write a number that ends with a zero.
   d) Write a 2-digit number.
   e) Write an odd number greater than 70.
   f) Write a number with a tens digit one more than its ones digit.

   **Sample answers:** a) 39, b) 36, c) 80, d) 56, e) 75, f) 65

4. a) Which number has both digits the same: 34, 47, 88, 90?
   b) Write a number between 50 and 60 with both digits the same.

   **Answers:** a) 88, b) 55

5. a) Find the sum of the digits in each of these numbers: 37, 48, 531, 225, 444, 372.
   b) Write a 3-digit number where the digits are the same and the sum of the digits is 15.

   **Answers:** a) 10, 12, 9, 9, 12, 12; b) 555
6. a) Which of these numbers has a tens digit one less than its ones digit: 34, 47, 88, 90?

b) Write a 2-digit number with a tens digit eight less than its ones digit.
c) Write a 3-digit number where all three digits are odd.
d) Write a 3-digit number where the ones digit is equal to the sum of the hundreds digit and the tens digit.

**Answers:** a) 34; b) 19; c) 111, 333, 555, 777, or 999; d) sample answers: 123, 347

7. Explain how you can use $3 \times 7$ to find $3 \times 70$ and why this works. Use one or more of these tools:

- an array
- a number line
- an addition equation
- a multiplication equation
- base ten blocks

**Sample answers**

- I used base ten blocks. If you make 3 groups of 7 ones blocks, you get 21 ones blocks. Then if you replace all the ones blocks with tens blocks, you get $3 \times 70$ and 21 tens blocks. That number is 210, so you could add a 0 after 21 to get the answer.
- I used two number lines and noticed that skip counting by 70 is always ten times as much as skip counting by 7. It works because counting by 10s is like counting by 1s but with a 0 at the end of the answer. To count by 70s, say every seventh number when counting by 10s and to count by 7s you say every seventh number when counting by 1s.
- I wrote $3 \times 70$ as $3 \times (7 \times 10)$, which is the same as $(3 \times 7) \times 10$, so you can multiply the answer to $3 \times 7$ by 10 to get the answer to $3 \times 70$.

Redirecting students: If students struggle to select a tool (MP5), have them choose from only two options, base ten blocks and an addition equation.

When students explain their reasoning (MP3), you may need to prompt them for more details. For example, if students say “skip counting by 70 is like skip counting by 7, but with a 0 at the end,” ask them how they know.
Goals

Students will add 3-digit numbers that require regrouping hundreds as a thousand.

PRIOR KNOWLEDGE REQUIRED

Is familiar with place values up to thousands
Can add 3-digit numbers that require regrouping ones and tens

Review adding 3-digit numbers using the standard algorithm. Remind students that when they add large numbers it is convenient to write the numbers one above the other, aligning place values. Have a volunteer show how to write the addition vertically for 235 + 341 on the board. Remind students that they can add ones, tens, and hundreds separately. Have a volunteer perform the addition. (see answer below)

\[
\begin{array}{c}
2 & 3 & 5 \\
+ & 3 & 4 & 1 \\
\hline
5 & 7 & 6 \\
\end{array}
\]

Exercises: Write the numbers one above the other. Add.

a) 249 + 450  
   b) 502 + 374  
   c) 803 + 25

Answers: a) 699, b) 876, c) 828

Review regrouping ones as tens and tens as hundreds. Write on the board:

\[
\begin{array}{c}
2 & 4 & 7 \\
+ & 2 & 2 & 5 \\
\hline
\end{array}
\]

Start adding and have students tell you what to do. Remind them that when they add 5 and 7, they get 12, which is not a digit because it is greater than 9. So we replace the 10 ones in 12 with a ten, and we write the remaining 2 in the sum and the additional ten above the tens column in the addition, as shown in the margin. Remind students that this is called regrouping.

Finish the addition. (472) Repeat with 241 + 392, regrouping 10 tens as 1 hundred.

Exercises: Write the numbers one above the other. Add. You will need to regroup.

a) 249 + 435  
   b) 504 + 376  
   c) 673 + 152  
   d) 328 + 590

Answers: a) 684, b) 880, c) 825, d) 918
Remind students that sometimes they need to regroup twice. Have students tell you what to do to add $345 + 386$. (731)

**Exercises:** Add. You will need to regroup twice.

a) $275 + 455$  
 b) $583 + 178$  
 c) $898 + 25$

**Answers:** a) 730, b) 761, c) 923

**Regrouping hundreds as thousands.** Remind students that 10 hundreds make a thousand. Write on the board:

\[
\begin{array}{c}
832 \\
+ 451 \\
\hline
\end{array}
\]

**ASK:** What does the digit 8 stand for in 832, or what is the value of 8? (800) Fill in the first blank in the first line. Repeat with the other digits in both numbers. Students can signal the number that should be written in each blank by holding up the appropriate number of fingers.

**SAY:** We add ones, tens, and hundreds separately. Add another row of "____ hundreds + ____ tens + ____ ones" and have students help you fill it in. The completed picture will look like this:

\[
\begin{array}{c}
832 \\
+ 451 \\
\hline
1283 \\
\end{array}
\]

**SAY:** 10 hundreds is 1 thousand, so 12 hundreds is 1 thousand and 2 hundreds. This means that the sum has 1 thousand, 2 hundreds, 8 tens, and 3 ones. Have a volunteer write the number using only numerals in the vertical addition. (1,283)

Write another vertical addition, such as $743 + 825$, and have students tell you what to do to finish the calculation. Then have students complete the exercises below individually. Explain that they will need to do more regrouping starting from part e).

**Exercises:** Add. You will need to regroup.

a) $849 + 430$  
 b) $506 + 572$  
 c) $973 + 112$  
 d) $828 + 540$

e) $938 + 437$  
 f) $457 + 581$  
 g) $873 + 869$  
 h) $728 + 672$

**Answers:** a) 1,279; b) 1,078; c) 1,085; d) 1,368; e) 1,375; f) 1,038; g) 1,742; h) 1,400

Have students estimate the answers to the previous exercises by rounding the numbers to the nearest hundred and then adding. Point out that if an estimate differs a lot (more than 100) from the actual answer, they need to look for a mistake.
Exercises: Estimate the answers to the previous exercises by rounding to the nearest hundred.

**Bonus:** Find two numbers (not multiples of 100) so that the estimate of their sum by rounding to the nearest hundred is exactly equal to the sum itself, as in part h).

**Answers:**
- a) 800 + 400 = 1,200; 
- b) 500 + 600 = 1,100; 
- c) 1,000 + 100 = 1,100; 
- d) 800 + 500 = 1,300; 
- e) 900 + 400 = 1,300; 
- f) 500 + 600 = 1,100; 
- g) 900 + 900 = 1,800; 
- h) 700 + 700 = 1,400;

**Bonus:** answers will vary

**ACTIVITY**

Player 1 writes a 3-digit number. Player 2 writes another 3-digit number so that the addition of the two will require regrouping in all three digits. Both players estimate the sum and then find the actual sum. They compare answers to check each other's work before switching roles.

**Extensions**

1. If you taught students Egyptian writing (see Extension 2 in Lesson NBT3-20) you could ask them to show adding and regrouping using Egyptian writing. Examples:

   **Regrouping:**

   ![Egyptian regrouping image]

   **Adding:**

   ![Egyptian addition image]

2. Have students add more than 2 numbers at a time.

   Example: 427 + 382 + 975 + 211. (1,995)

   (MP1)

3. Fill in the missing numbers to make each addition correct.

   a)  
   ![Missing numbers in addition a)](image)

   **Answers:** a) 361, b) 295 + 531 = 826
Goals
Students will use estimation while solving word problems.

PRIOR KNOWLEDGE REQUIRED
Is familiar with place values up to hundreds
Can read and write numbers to 1,000

MATERIALS
map of the Western United States (optional)

Using estimation to check if an answer is reasonable. Remind students that they can use estimation to check if an answer is reasonable. For example, if you add $678 + 234$ and you get $712$, you can estimate by rounding to the nearest hundred to see whether the answer makes sense. Have students estimate the answer. ($700 + 200 = 900$) Remind students that a good estimate is within 100 of the correct answer. ASK: If 900 is a good estimate, is 712 a reasonable answer? (no, it is too small) Repeat with $629 - 384 = 345$. ($600 - 400 = 200$, the answer is too large)

Exercises: Estimate by rounding to the nearest hundred to check the equation. Is it correct?

a) $376 - 192 = 174$  
b) $279 + 451 = 630$

c) $288 + 441 = 739$  
d) $974 - 588 = 586$

Answers: a) $400 - 200 = 200$, about the same; b) $300 + 500 = 800$, so $630$ is too small, equation is incorrect; c) $300 + 400 = 700$, about the same; d) $1,000 - 600 = 400$, so $586$ is too large, equation is incorrect

SAY: With a simple calculation that you did in your head, you were able to find two incorrect equations, parts b) and d). However, rounding to hundreds does not always catch a problem. Let’s round to tens, maybe we can see more incorrect equations. Have students round to tens to estimate the answers in parts a) and c). (a) $380 - 190 = 190$, so $174$ is too small, equation is incorrect; c) $290 + 440 = 730$, about the same)

SAY: This calculation was harder, but you still managed to catch a mistake. Now have students perform the actual calculation in c). The answer is 729, so the equation is in fact incorrect. Point out that estimation is a good strategy, but it does not always work to catch mistakes. When an estimate by rounding to the nearest hundred differs from the actual answer by more than 100, you know there is a problem: the actual answer is wrong, or the estimation was done incorrectly, or this method of estimation makes no sense.
Choosing what to round to. Write the following list of distances from San Francisco, CA, on the board:

<table>
<thead>
<tr>
<th>Place</th>
<th>Distance from San Francisco, CA (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Las Vegas, NV</td>
<td>567</td>
</tr>
<tr>
<td>Los Angeles, CA</td>
<td>382</td>
</tr>
<tr>
<td>Portland, OR</td>
<td>636</td>
</tr>
<tr>
<td>Salt Lake City, UT</td>
<td>736</td>
</tr>
<tr>
<td>San Diego, CA</td>
<td>523</td>
</tr>
</tbody>
</table>

If available, show a map of the western United States and have students locate the places listed in the table on the map. SAY: I want to estimate how long the drive from San Diego to Portland is if I go through San Francisco. Have students estimate the distance. (523 + 636 is about 500 + 600 = 1,100 miles) Have students calculate the actual distance. (1,159 miles) ASK: How much larger is the actual distance than the estimate? (59 miles) SAY: I would like to get a closer estimate, by rounding to the nearest ten. Have students round the numbers to the nearest ten and estimate again. (520 + 640 = 1,160 miles) Point out that this answer is much closer to the actual distance; it is almost exactly the same.

Have students estimate the distance from Portland to Las Vegas if driving through San Francisco by rounding to the nearest ten. Again, have them compare the estimate to the actual distance. (estimate: 640 + 570 = 1,210 miles, actual distance: 636 + 567 = 1,203 miles)

SAY: Let’s try to use rounding to the nearest hundred to estimate how much closer Las Vegas is to San Francisco than Portland. Have students round both distances to 100. ASK: What do you notice? (both round to 600) ASK: Can we use rounding to the nearest hundred to estimate the difference? (no) Why not? (the estimate would be zero, which is not what we need, because the numbers are not the same) What should we do? (round to the nearest ten) Have students estimate by rounding to the nearest ten and then have them calculate the actual difference. (estimate: 640 − 570 = 70 miles, actual: 636 − 567 = 69 miles, so 70 is a good estimate)

Repeat with the distances to Las Vegas and San Diego. This time point out that though the distances round to different numbers, 600 and 500, the difference is then estimated as 100, which makes no sense—both numbers are between 500 and 600, so the difference should be much smaller. The actual distance is 567 − 523 = 44 miles.

SAY: If I want to go from Salt Lake City to Los Angeles, does it make sense to drive through San Francisco? If they have a map, students can identify the shorter route between the two cities, figuring out that the drive through Las Vegas is shorter. SAY: It is 689 miles to go directly from Salt Lake City to Los Angeles. How much longer is it to drive from Salt Lake City to
Los Angeles through San Francisco than to go there directly? Remind students to estimate first. Have students figure out the answer, then discuss the solution as a class. (see answer below)

Distance from Salt Lake City to Los Angeles through San Francisco:

Estimate: $740 + 380 = 1,120$ miles
Actual distance: $736 + 382 = 1,118$ miles

Difference:
Estimate: $1,120 - 690 = 430$ miles
Actual: $1,118 - 689 = 429$ miles

**Review multiplying 1-digit numbers by tens.** SAY: To plan a trip, I estimate that I can drive 50 miles in 1 hour. To find how far I can drive in 6 hours, I need to multiply $6 \times 50$ miles. Remind students that to multiply $6 \times 50$ they actually take $6 \times 5$ tens, so the answer is 30 tens, or 300. Write on the board:

\[
6 \times 50 = 6 \times 5 \text{ tens} = 30 \text{ tens} = 300
\]

ASK: How far can I drive in 9 hours? ($9 \times 50 = 450$ miles) SAY: I cannot drive more than 8 hours in one day. ASK: How far can I drive in 1 day? (400 miles) Which of the distances in the table can I complete in 1 day? (the drive from San Francisco to Los Angeles)

**Extensions**

1. When estimating sums or differences of 2-digit numbers, it sometimes makes sense to use numbers with ones digit 5 as well as multiples of 10. This is rounding to the next multiple of 5. Example: To estimate $27 + 44$, use $25 + 45 = 70$.

Have students round to the nearest multiple of 5 to estimate the sum or difference. Students should then calculate the actual sum or difference.

a) $34 + 89$  b) $89 - 34$  c) $78 - 33$  d) $92 + 47$

**Answers**

a) estimate: $35 + 90 = 125$, actual: 123
b) estimate: $90 - 35 = 55$, actual: 55
c) estimate: $80 - 35 = 45$, actual: 45
d) estimate: $90 + 45 = 135$, actual: 139

2. Find five different pairs of numbers with different ones digits so that the numbers add to 1,000.

**Sample answers:** $123 + 877$, $234 + 766$, $345 + 655$, $456 + 544$, $678 + 322$
3. Remind students that in this lesson they used estimation to find potential errors in addition or subtraction equations. Explain that another way to check for potential errors in addition or subtraction is to look at the ones digits. Remind students that earlier in the year (in Lesson OA3-15) they learned that the sum of two even numbers is always an even number, and the sum of two odd numbers is also an even number. Only when you add an even number to an odd number do you get an odd number. The same rule applies to subtraction. Have students answer the following questions.

a) Is the answer odd or even? Predict, then add or subtract to check.
\[ 234 + 786 \quad 678 + 237 \quad 567 - 294 \quad 789 - 233 \quad 355 + 221 \]
b) Is the answer to \[ 215 + 352 + 427 \] even or odd? Predict and explain your prediction. Then add to check.

c) Is the equation \[ 215 + 127 + 428 = 861 \] correct? Predict without adding. How do you know?

**Answers**

a) even, 1,020; odd, 915; odd, 273; even, 566; even, 576
b) even, because \[ 215 + 352 \] is odd, so when we add another odd number the result is even, actual sum: 994
c) No. The sum of the first two numbers is even, then we add another even number, so the sum should be even. 861 is odd, so the equation is incorrect.

4. Which two numbers have a sum of 12 and a difference of 4? Suggest that students search systematically for all numbers with a sum of 12.

**Answers:** 8 and 4

**NOTE:** Extensions 5–8 must be done in order so that students can discover shortcuts to determine the perimeter of a square. Students will get another chance to do this in MD3-37.

5. Find the other side lengths of the square. Then find the perimeter.

\[ \begin{array}{c}
2 \text{ cm} \\
3 \text{ cm} \\
4 \text{ m}
\end{array} \]

**Answers:** a) 2 cm, 2 cm, 2 cm, perimeter 8 cm; b) 3 cm, 3 cm, 3 cm, perimeter 12 cm; c) 4 m, 4 m, 4 m, perimeter 16 m

6. Write a general rule about how you can find the perimeter of a square using only one side length. What property of squares did you use?

**Answer:** Perimeter = length + length + length + length or \( 4 \times \text{(length)} \).

I used the property that each side length of a square is the same.

Redirecting students: Ask students to look at their answers to Extension 5. Are they doing the same thing in each part of Extension 5? Why is that?
7. Find the other side lengths of the rectangle. Then find the perimeter.

   a) 3 cm  
   b) 4 cm  
   c) 5 cm  
   
   2 cm  
   2 cm  
   3 cm  

   Answers: a) 2 cm, 3 cm, perimeter 10 cm; b) 2 cm, 4 cm, perimeter 12 cm; c) 3 cm, 5 cm, perimeter 16 cm

(MP7, MP8)

8. Write a general rule about how you can find the perimeter of a rectangle using only the length and the width. What property of rectangles are you using?

Answer: Perimeter = length + width + length + width or $2 \times (\text{length} + \text{width})$. I used the property that opposite sides of a rectangle are equal.

Look for students to recognize the structure (MP7) inherent in squares (all sides are equal) and rectangles (opposite sides are equal) and to use it to find all the side lengths and the perimeter of each shape. Look for students to recognize that they are using the same reasoning for each square or for each rectangle (MP8).
# Hundreds Chart

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### Number Cards (2)

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- Blackline Master — Number and Operations in Base Ten — Teacher Resource for Grade 3
Place Value Cards

Ones  Ones
Tens  Tens
Hundreds  Hundreds
Thousands  Thousands
This Unit in Context

The concept of measuring time is more difficult than other measurement attributes students have seen before, partly because of the cyclical nature of the clock (it starts over after 12) and partly because of how time is recorded using both hours and minutes. When measuring length, for example, you start with measuring feet and inches separately, and only combine the units after students have had some exposure to each. Starting in Grade 1, students learned to tell time using analog clocks, digital clocks, and words like “o’clock” and “half past.” By Grade 2, students had learned to tell time to the nearest five minutes (2.MD.C.7) and assign times of day to events by placing events on a timeline (which looks like a number line), distinguishing between events that occur in the a.m. and p.m. (2.MD.C.7). In this unit, students will tell time to the nearest minute using both digital and analog clocks (3.MD.A.1).

In earlier grades, students learned to measure time by associating events with the time of day that event occurs (for example, recess starts at 10:30 a.m.). This is similar to how students approach other measurement attributes—for instance, they associate a given length with a particular object. However, the process of measuring time in this way is fundamentally different from other measurement attributes—there is no starting and ending point between which to measure, only an instantaneous time that we see by looking at the clock.

In this unit, students will use their knowledge of associating events with specific times in order to measure time intervals—how long an event takes by subtracting the time the event begins from the time the event ends (3.MD.A.1). As a result, more similarities between time and other measurement attributes will become apparent: just as students learned in Grade 2 that they can place a ruler along an object to see how many centimeters or inches are taken up by the object, regardless of whether the student measures from 0 (2.MD.B.5), students will learn that they can look at what the clock says when an event starts and when it ends to find how much time the event takes.

Using the three-step progression learned in Grades 1 and 2 for measuring length and other measurement attributes, students will directly compare elapsed time lengths, indirectly compare time lengths, and then finally be guided to the idea of measuring time. Exposing students to this similarity deepens their understanding of elapsed time as a measurement attribute. Students will go through a similar progression with other measurement attributes in this and higher grades, such as areas (3.MD.C.5, 6), liquid volumes (3.MD.A.2), and angles (4.MD.C.5, 6). By repeating this progression several times, students make important connections and increase their flexibility in thinking about measurement attributes. Students will continue to learn about different measurement attributes in later grades. They will be introduced to volume in Grade 5, when they study volumes of right rectangular prisms with whole number side lengths (5.MD.C.3).
In Grade 6, they will continue this study of volume for right rectangular prisms with fractional side lengths (6.G.A.2).

In this unit, students will solve word problems involving the addition and subtraction of time intervals in minutes (3.MD.A.1). Students will use addition, subtraction, multiplication, or division to solve problems involving masses and volumes in 3.2 Unit 8 (3.MD.A.2). This extends in Grade 4 to using the four operations to solve word problems involving distances, time intervals, liquid volumes, masses, and money—including problems involving simple fractions or decimals (4.MD.A.2).

**Mathematical Practices in This Unit**

In this unit, you will have the opportunity to assess MP.1 to MP.7. Here are some examples of how students can show that they have met a standard.

**MP.2:** In MD3-19 Extension 4, students reason abstractly and quantitatively when they use the real-world context of minutes and hours to explain why $60 \div 4 = 15$ and $60 \div 5 = 12$. In MD3-16 Extension 2, students reason quantitatively when they recognize that the hour hand moves 5 tick marks in 60 minutes. They reason abstractly when they use that to determine how long it takes the hour hand to move 3 tick marks.

**MP.4:** In MD3-17 Extension 2, students model a real-world problem when they find the elapsed time and then divide the number of minutes by the number of pages read in each minute.

**MP.5:** In MD3-20 Extension 3, students choose tools strategically when they choose between rounding to the nearest ten and to the nearest hundred. Students recognize that rounding to the nearest hundred is faster but rounding to the nearest ten is more accurate.

**MP.7:** In MD3-13 Extension 3, students look for and make use of structure to find $270 \div 6$. Students may write 270 as $3 \times 90$, as $300 - 30$, or as $240 + 30$ to help them divide.
Introduction

In this unit, students will tell and write time to the nearest minute using both digital and analog clocks. Students will learn the concept of elapsed time and will measure time intervals. Students will also solve word problems involving addition and subtraction of time intervals.

Materials. BLM Time Memory Cards contains many cards showing different ways to tell time, including analog and digital clock faces, as well as text descriptions of time. To help students combine these different ways to show time, have them play Picking Pairs or Memory using the cards from the BLM. Cut out and laminate the cards to allow students to use them multiple times, whenever they have a few spare minutes. The complete descriptions of the games Picking Pairs and Memory appear on p. F-2.

If students do not use grid paper notebooks, provide them with BLM 1 cm Grid Paper (p. V-1) for adding and subtracting time in Lessons MD3-21 and MD3-22.
**Goals**

Students will tell time to the nearest 1 minute from a digital clock.

**PRIOR KNOWLEDGE REQUIRED**

Can read and write two-digit numbers

**MATERIALS**

digital clock for demonstration
BLM Reading Digital Times (p. Q-58)
BLM Time Memory Cards (1) to (2) (pp. Q-59–60)

**NOTE:** This lesson assumes you use a digital clock that shows the hours with two digits and the minutes with two digits (e.g., the time 5 minutes past 9 is shown as 09:05). If the demonstration clock you have does not show the first zero, draw students' attention to the difference between the clock you use and the clocks in the AP Book.

**Discuss the need for clocks.** Show students a digital clock. SAY: This is a digital clock. It shows the time in digits only—that is, only numbers, no hands. ASK: Why do we need clocks? Record students' ideas. (e.g., to know the time of the day, to measure time) ASK: Why do we need to know the time of the day? Again, record students' ideas. (e.g., you need a way to tell the time so you can get to school before lessons start) Have students give examples of events they know of that happen at a precise time. (e.g., school starts at 8:30, lunch is at 12:25, karate class starts at 5:15)

Point out that sometimes you need to know the time precisely. For example, you need to know the time a train leaves precisely so that you will not be late; if a train leaves at 7:33, you will miss it if you arrive at the station at 7:35.

**Introduce hours and minutes.** Remind students that a day is divided into hours, and hours are divided into minutes. There are 24 hours in a day, and 60 minutes in an hour. A clock shows 12:00 at noon and at midnight. **NOTE:** The concepts of "a.m." and "p.m." will be introduced later in the unit. If students mention them, explain that you will use these labels later.

**Hours and minutes on digital clocks.** Point out the parts on the clock that show the hours and the minutes. Set the time on the demonstration clock to 07:05. Explain that digital clocks show hours using two-digit numbers and minutes using two-digit numbers. Even when the number of hours or minutes is a one-digit number, such as 5, the clock still uses two digits. ASK: How many tens are in 5? (0) SAY: There are 0 tens in 5, so the clock shows 5 minutes as 05. ASK: What hour does the clock show? (7 hours) What does the minute part of the clock show? (5 minutes)
Exercises: What hour does the clock show? What does the minute part of the clock show?

a) 12:15  b) 4:23  c) 10:01  d) 9:02

Answers: a) 12 hours, 15 minutes; b) 4 hours, 23 minutes; c) 10 hours, 1 minute; d) 9 hours, 2 minutes

Writing the time from a digital clock. Draw students’ attention to the symbol dividing the hours and the minutes on a digital clock. Say: This symbol is called a colon. When we write time, we use a colon to separate the numbers that show the hours and those that show the minutes, just like on the digital clock. We do not need to write the hours as a two-digit number, but we do write the minutes as a two-digit number. Using the example of 07:05 again, say: The hour (write 7 on the board) is separated from the minutes (write 05 to the right of the 7), by a colon (insert the colon between the 7 and 05). Have students write the times from the exercises above. (a) 12:15, b) 4:23, c) 10:01, d) 9:02)

Reading the time from a digital clock. Remind students that when you ask somebody what the time is, they usually do not answer “9 hours 5 minutes,” but they say something like “5 minutes after 9.” Have students think of other ways to say the time. You might wish to record the answers. Explain that in this lesson you will read the time in the form “5 minutes past 9.” Emphasize that we say the minutes first. Have students read the times in the exercises above using this format. (a) 15 minutes after 12, b) 23 minutes after 4, c) 1 minute after 10, d) 2 minutes after 9)

Exercises: Write the time in numbers and words.

a) 1:27  b) 8:46  c) 10:08

d) 4:07  e) 2:04  f) 1:03

Bonus: Ron thinks that 02:10 is 2 minutes past 10. Explain his mistake.

Answers: a) 27 minutes past 11, b) 46 minutes past 8, c) 8 minutes past 10, d) 7 minutes past 4, e) 4 minutes past 2, f) 3 minutes past 1, Bonus: Ron read the hours first instead of reading the minutes first; the time is 10 minutes past 2

Students might need more practice with times where the number of minutes is 12 or less, because there is a greater chance of mixing up minutes and hours. Remind students to start at the second number, the number of minutes, when saying the time. Students who repeatedly read 9:05 as 9 minutes past 5 can benefit from Activity 1, below.
**ACTIVITY 1**

Have students cut out the cards from BLM Reading Digital Times. Students pick two of the cards at random and place them face up in the top row of the remaining BLM, creating a time shown on a digital clock. To get the time in words, they switch the order of the cards, placing them in the second row, and then read the time. After going through all six cards in random order, students remix the cards and try to say the time in words first, and check the answer by creating the time as shown on a digital clock.

**Showing the given time on a digital clock.** Remind students that when we say the time in words and numbers, we say the minutes first. When we write the time in numbers, or as it is shown on a digital clock, we write the hours first and then the minutes, so 12 minutes past 3 is 03:12, not 12:03. Write on the board:

12 minutes past 3 is 03:12.

Remind students to write a zero in front of one-digit numbers, both hours and minutes. Volunteers can show the time in the exercises below on the demonstration clock.

**Exercises:** How would the time look on a digital clock?

- a) 15 minutes past 12
- b) 9 minutes past 11
- c) 7 minutes past 10
- d) 12 minutes past 6
- e) 10 minutes past 7
- f) 8 minutes past 9

**Answers:**

- a) 12:15
- b) 11:09
- c) 10:07
- d) 06:12
- e) 07:10
- f) 09:08

Students who forget to use two digits for both hours and minutes might benefit from a template with spots for two digits, as shown below:

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**ACTIVITY 2**

Play Picking Pairs or Memory (see unit introduction) using the cards from BLM Time Memory Cards (1) to (2). Cards that match show the same time.

**Extensions**

1. Explain that minutes are divided into seconds. Some digital clocks also show seconds, after minutes—for example, 09:08:07. The first two digits show the hour, the next two show the minutes, and the final two show the seconds. Explain that when you say the time on a clock with seconds, you say minutes first, seconds after, and only then say “past the hour.” For example, 09:08:07 is “8 minutes, 7 seconds past 9” (or “8 minutes and 7 seconds past 9”).
Have students write the time on each digital clock as they would say it, using words and numbers.

a) 10:27:32  b) 17:46:15  c) 12:08:21

d) 04:07:21  e) 2:04:09  f) 1:03:02

Answers: a) 27 minutes 32 seconds past 10, b) 46 minutes 15 seconds past 7, c) 8 minutes 21 seconds past 12, d) 7 minutes 21 seconds past 4, e) 4 minutes 9 seconds past 2, f) 3 minutes 2 seconds past 1

2. These clocks are broken. Explain what is wrong.

a) 25:04  b) 1:48  c) 5:68

Answers: a) the hour number is too high, the hours cannot go beyond 12; b) the minutes are shown with only one digit, but should be shown with two digits; c) the number of minutes is too high, the minutes cannot go beyond 59

(MP.2, MP.4, MP.7)

3. If you use five $50 bills to pay for a $219 tennis racket, how much money should you get back? Use equations to explain how you know.

Write what each equation means in the situation.

Solution: I used five $50 bills, so I found 5 \times 50 = 250. I paid a total of $250. But the racket cost only $219, so I found 250 − 219 = 31. I should get $31 back.

Look for students to recognize how to model (MP.4) with equations the amount paid, the amount they get back, and then to interpret the results of each calculation (MP.2). Look for students to recognize and make use of structure (MP.7) in multiplying tens.

Individual or small-group follow-up: If some students still don’t understand, ask them to start by making sure they have enough money to pay for the racket. This should lead them to determine how much money five $50 bills is. If students struggle with that, ASK: How much would two $50 bills be? (some students may need to count up by tens) Three $50 bills? Four $50 bills? Five $50 bills? Record the skip counting by 50 on the board and ask if skip counting reminds them of another operation? (multiplying) Have students record the skip counting as multiplication (5 \times 50 = 250), then ASK: Is there a shortcut to do 5 \times 50 without skip counting? PROMPT: Can you use 5 \times 5? (5 \times 5 is 25, so 5 \times 50 is ten times as much) Then have students read the question again to find the next step in the problem. Then have them write the equation to get the answer to that step. (250 − 219 = 31, so I should get $31 back)
MD3-11 Analog Clock Faces and Hands
Pages 108–109

STANDARDS
3.MD.A.1

VOCABULARY
analog clock
face
hour
hour hand
minute
minute hand
number line
time unit

Goals
Students will identify elements of an analog clock face and will use analog clocks to compare intervals of time directly and indirectly.

PRIOR KNOWLEDGE REQUIRED
Can read and write two-digit numbers
Can read a number line
Has experience with direct (object to object) and indirect (using a third object) comparisons of length

MATERIALS
two pencils of different lengths and different colors for demonstration
analog clock with hour, minute, and second hands
cards with numbers from 1 to 12
masking tape or a Hula-hoop
paper plates
BLM Make Your Own Clock (p. Q-67)
scissors
paper fasteners
pencils
BLM Empty Clock Faces (p. Q-68)
song recording and lyrics (see Extension 1)
metronome (see Extension 2)

NOTE: In Activity 2, students will need to copy the hands on the clock at different times throughout the day. You might want to start the lesson as early in the day as possible, and return to the material later in the day.

Compare times directly. ASK: What will take more time: walking around the room or doing 10 sit-ups? Ask students to vote based on their ideas. Explain that you will check the prediction by asking one person to do one activity and another person to do the other, and seeing who finishes first. Emphasize that this is not a race! The only reason you need two volunteers is because one person can’t do both things at the same time. Volunteers should do each activity at a normal pace (demonstrate a regular pace versus a rushed or slowed pace). Explain that the volunteers have to start moving at the same time for you to compare the time each activity takes. ASK: How can we make sure the two volunteers start at the same time? (for example, say “1, 2, 3, GO!”) Have two volunteers perform the activity and a third volunteer tell them when to start. When the volunteers are finished, ASK: What took longer, walking around the room or doing 10 sit-ups? How do you know?
ACTIVITY 1

Student pairs compare the time it takes to do five jumping jacks and say the alphabet. Each pair decides who will do which activity. Remind students to start at the same time and not to rush. Students answer these questions: Who stopped first? Who stopped second? Which activity took more time? Which activity took less time?

Compare measuring time to measuring length. Point out that comparing time is similar in a way to comparing length. To compare the lengths of two pencils, you can place pencils one beside the other, aligning their endpoints. The pencil that “sticks out” is longer. Demonstrate what you mean with two pencils of different lengths and different colors. SAY: Imagine I want to compare the lengths of two pencils, one here and another in Brazil. How could I do that? Students are likely to suggest measuring the pencils with a ruler to compare the length in units.

Now consider how to measure time with direct and indirect comparisons. SAY: We found that it takes this student (name the student—for example, Kate) longer to do 10 sit-ups than it takes the other student (name the student—for example, Will) to walk around the room. But now I would like to know whether it will take me longer to do 10 sit-ups than it will take me to go around the room. I am not sure whether Will walks as fast as I do. I am not sure whether I do sit-ups at the same pace as Kate does. I don’t think I can use what they do to compare what will take me longer. What could I do to compare the times? (measure the times)

Introduce analog clocks. Have students think of what tools are used to measure time. (stopwatch, clock) Point out that although a clock is usually used to show the time of the day, it can be also used to measure time. Explain that a clock with hands, like the classroom clock, is called an analog clock, and it usually has three hands. It is easy to see one of the hands moving. We will call this hand “the fast-moving hand,” or “the fast hand” for short. NOTE: Since students are not going to learn about seconds in this unit, and since the word “second” can be confused with the ordinal number, we use “fast hand” instead of “second hand.”

Explain that all the hands on a clock move, but two move so slowly that they don’t look like they’re moving. Compare the slower hands to the sun in the sky; you don’t see the position of the sun move in the sky, but it is in a different place after school than it was when school started. You might have students note where the sun is in the sky at the start of the day and then again at the end. To help students develop a sense of the hands moving, you can do Activity 3 below throughout the day.

Compare clocks to number lines. Point out how the clock face has numbers all around, beginning with 1 and ending with 12. Draw a number line on the board and label it from 1 to 15.

Discuss how the clock is like a number line that goes in a circle. ASK: What comes right before 3 on the number line? (2) What comes right before 3
on the clock? (2) Repeat with other numbers between 2 and 11. Ask students what numbers come after some numbers. Then ASK: What comes after 12 on the number line? (13) What comes after 12 on the clock? (1) Emphasize that clocks are like number lines, except the clock has numbers from 1 to 12 and then starts over again at 1.

Add the numbers to a clock face. Ahead of time, create a large circle on the floor using masking tape or a Hula-hoop and prepare cards with numbers 1 to 12. Place the cards face down in the correct positions around the circle to serve as hour markings.

Tell students to look closely at a clock face and to try to remember where all the numbers go. Then gather the class around the circle on the floor, making sure students have their backs to any actual clocks in the room or hide them if possible. Ask a volunteer to choose a number card on the clock to predict and turn over the card. If the volunteer was correct, then leave the number face up. If not, replace it face down. Repeat with different volunteers until all cards are face up. Play again if students wish.

ASK: Which numbers were easier to remember? Why were they easier to remember? (Most students will likely say 12 and 6; some will say 3 and 9.) Explain that the numbers at the top, bottom, left, and right are usually the easiest to remember. Put the cards for 3, 6, 9, and 12 in position, face down, and play with only these cards. Then take away all the cards and ASK: Where does the 12 go? Have a volunteer place it in the correct position. Repeat with 6, 3, and 9. ASK: What numbers come right before and right after the 3? (2 and 4) Have volunteers place those numbers and repeat for 6 (5 and 7) and 9 (8 and 10). End with 12 (11 and 1). Emphasize that the clock face shows only numbers 1 to 12.

SAY: We said that 2 comes right before 3 and that 1 comes right after 12. That means the 1 comes right before 2. ASK: Does that make sense? (yes) PROMPT: Does 1 come right before 2 on a number line? (yes) Repeat with more numbers on the clock. Explain how solving the same problem in two different ways and getting the same answer suggests that students have the right answer. If we know that 5 comes right before 6 and right after 4, and both pieces of information make the number go in the same place, the answer must be right.

ACTIVITY 2

Make your own clock. Students use a paper plate, a copy of BLM Make Your Own Clock, scissors, a paper fastener, and a pencil and follow these steps to make their own clocks:

1. Cut out the circle from the BLM and glue it to the inside of the paper plate.
2. Use a sharpened pencil to poke a hole in the center of the clock face.
3. Write the numbers in the correct positions on the clock face.
4. Cut out the hands from the BLM and attach the hands to the plate with the paper fastener. Students can use the clock they create to show times in later lessons.

**Introduce hour and minute hands.** Point out to students that the clock they made during the activity has only two hands. Explain that many clocks do not have the fast-moving hand, including the clocks in the AP book. Explain that the short and thick hand on the analog clock is called the *hour hand*, and the long, thin hand is called the *minute hand*. Show hands in different positions on an analog clock, so that at least one of the hands points directly at a number, and have students identify which hand points to the number. For example, set the clock to show 6:15 and ASK: Which hand points at 3? (minute hand) Repeat with other times.

**ACTIVITY 3**

Give each student a copy of BLM Empty Clock Faces. Direct students’ attention to the classroom clock. ASK: What number is the hour hand pointing at now? Show how to draw the hour hand on the clock. Make the hand thick and short, and have students do the same. Repeat with the minute hand, but make it thin and going all the way to the number. Explain that the fast hand moves too fast to copy it. Repeat the activity throughout the day several times, so that students copy the hour and the minute hands at various times. Discuss how the hands have moved. Example: From the start of the math lesson (say, 1:15) to the end of the math lesson (say, 1:50), both hands moved but the short hand didn’t move very much—it is still between the 1 and the 2, but closer to the 2 than the 1.

**Introduce time units.** Draw students’ attention to the fast-moving hand on the analog clock and watch it move. Point out how the hand moves at a steady pace, or makes regular jumps to the next mark, all the way around the clock. SAY: When people measure length, they use units that are all the same length. When they measure time, they also need units that are the same. The time it takes the fast hand to move from one number to the next is always the same, so we can use the time it takes the fast-moving hand to get from one number to the next to measure time. We will call these *time units*.

Have a volunteer start doing 10 jumping jacks when the fast hand is at the 12 and have the other students tell you where the hand is pointing when the volunteer is done. Remind students that the length of objects—such as pencils—is not usually an exact number of centimeters. For example, a pencil might be almost 5 cm, so we record the length as “about 5 cm.” We can do the same with time. To make matters simple, when telling time we will always say the last number the clock hand passed. So if the hand starts at 12 and is later close to 5, but has not reached 5, we will say that four time units have passed. This makes sense because only four full time
units have passed, and it might have taken some time to turn your head to look at the clock.

**Compare time using time units.** Have a volunteer clap hands 10 times starting when the fast hand is at the 12, and have the other students tell you where the hand is pointing when the volunteer is finished the 10 claps. Then ASK: Which activity took more time or more time units: clapping 10 times or doing 10 jumping jacks? How do you know? How many time units passed while clapping 10 times or doing 10 jumping jacks? Repeat with other tasks that take less than a minute to complete and note the results on the board. Examples: count to 30, hop on one foot 10 times, count to 20, say the alphabet, walk around the room. ASK: Which task took the longest time? Which took the shortest time? Did any two tasks take about the same amount of time? How can you tell? (the measurements are close)

### ACTIVITY 4

Students work in pairs to time each other doing first one task and then another. Examples: write your first and then your last name; write the first five letters of the alphabet (a to e) and then the last five (v to z); find page 3 of a book and then page 43. Students might predict which task they think will take longer (i.e., which takes more time units) before starting. Partners should choose different activities to avoid competing.

ASK: Who took longer to write their first name than their last name? Who took longer to write their last name? Why do you think some people needed more time to write their first name and some needed more time to write their last name? (depends on the length of the names) Discuss why some activities might take more time than others.

### Extensions

1. **Use music to compare times (indirect comparison).** Play a familiar song while doing various tasks (the example below uses “Twinkle, Twinkle, Little Star”). To make keeping track of the time easier, have students work in pairs and provide them with lyrics for the song. While one partner performs a task, the other keeps track of the words, circling the word in the song that is reached when the task is finished. Each partner performs two different tasks.

   Explain with the example of doing jumping jacks versus walking around the classroom to the tune of “Twinkle, Twinkle, Little Star”: if John finished three jumping jacks when the word “sky” in the last line of the first verse was playing but he finished walking around the classroom when the next line was playing, it took him more time to walk around the room than to do three jumping jacks.

   **NOTE:** If playing music is not an option, have some students sing a familiar song while other students perform a task and then switch roles.
2. **Count the beats from a metronome to measure the time a task takes.** As their partners do various short tasks, students either count each beat of a metronome or make a tick mark for each beat and then count the total number of tick marks at the end.

3. **Estimate and measure elapsed time.** Students work in pairs to estimate, measure, and compare the number of time units it takes to trace one hand and then color the tracing. Students record their estimates and measurements.

4. Ron bought five t-shirts. He paid with four $20 bills and got three $5 bills back. How much did each t-shirt cost? Show your work with equations. Write what each equation means in the situation.

**Sample answer:** Ron paid with four $20 bills, so I found \(4 \times 20 = 80\). He paid $80 altogether, but he got back three $5 bills, so I found \(3 \times 5 = 15\). He got $15 back. To find the cost of all five t-shirts, I found \(80 - 15 = 65\), so they cost $65. But he bought 5 t-shirts, so I found \(65 \div 5 = 13\). Each t-shirt cost $13.

Look for students to write the multiplications and divisions to model the problem (MP.4). Look for students to notice and use structure—for example, when multiplying \(4 \times 20\), do students recognize that they can use \(4 \times 2 = 8\) (MP.7) or do they need to skip count? Look for students to interpret the results of their calculations in the context of the situation at each step (MP.2).

Redirecting students: ASK: What is the problem asking you to find out? (how much each t-shirt costs) What information would make the problem easier if you had it? (how much all the 5 t-shirts cost) Does the problem tell you directly how much Ron pays for all 5 t-shirts? (no) Encourage students to figure out from the problem how much Ron pays for all 5 shirts.
MD3-12  The Hour Hand
Pages 110–111

STANDARDS
preparation for 3.MD.A.1

VOCABULARY
analog clock
digital clock
hour
hour hand
minute
minute hand
o’clock
time unit

Goals
Students will distinguish between the exact hour (o’clock) and not-exact hour and will write time to the hour as both ___ o’clock and ___:00.

PRIOR KNOWLEDGE REQUIRED
Is familiar with the analog clock face
Can distinguish between the hour hand and the minute hand
Can read time on a digital clock

MATERIALS
analog clock for demonstration
clocks made in Activity 2 of Lesson MD3-11
digital clock for demonstration
BLM Time Memory Cards (1) to (4) (pp. Q-59–62)
BLM Empty Clock Faces (p. Q-68, see Extension 1)
clock with only the hour hand, e.g., from BLM Make Your Own Clock (p. Q-67, see Extension 2)

Review hands on a clock. Remind students that the shortest hand on the clock is the hour hand, and the longer, thinner hand is the minute hand. Use a demonstration analog clock to show various times and ask students what number the hour hand is closest to. Students can signal the answer by holding up the corresponding number of fingers.

Introduce hours. Explain that it always takes the same amount of time for the hour hand to move from one number to the next, exactly 1 hour. For this reason, this hand is called the hour hand. Explain that it takes the minute hand 1 hour to do a full circle around the clock, so while the minute hand turns a full circle, the hour hand moves from one number to the next. For example, it moves from 3 to 4.

Introduce “o’clock.” Explain that when the minute hand is at 12, the hour hand points exactly at one of the numbers on the clock face. We call a time like that o’clock. For example, when the minute hand is at 12 and the hour hand is at 9, we say the time is 9 o’clock. In other words, it is an exact hour or the time is on the hour. To check if the time is an exact hour, you need to look at the minute hand. Show different positions of hands on the clock and have students say whether the time is an exact hour or not—for example, 10:00, 3:15, 4:00, 7:30, and so on. Vary your questions: Is it an exact hour? Is the time on the hour? Is it o’clock? Students can signal the answers with thumbs up for yes and thumbs down for no.

Identifying time to the hour. Show 7:00 on the clock. ASK: Is this o’clock? (yes) Explain that when it is the exact hour, you need to look at the hour hand to tell what the time is. ASK: What number is the hour hand pointing...
to? (7) SAY: So, the clock shows 7 o’clock. Show different times (using exact hours only) on a demonstration clock and have students tell you what “o’clock” it is.

Reverse the task. Have students use the clocks they made in Activity 2 of Lesson MD3-11 to show the time. Say specific “o’clock” times and have students show the hour hand in the correct position (say times sequentially at first and then in random order).

**Write ____ o’clock with numbers.** Remind students that digital clocks don’t have hands but show the time using only numbers. Explain that they show 9 o’clock as 09:00. Show students a digital clock to illustrate. Remind students that when they write the time with numbers, they would write this time as 9:00. Explain that the o’clock times always have two zeros for minutes. Show 3 o’clock on an analog clock, and ASK: How do you write this time with numbers? (3:00) Repeat with other o’clock times.

Summarize the four different ways of showing or writing the same time: 9 o’clock, 9:00, 09:00 on a digital clock, and hour hand at 9, minute hand at 12 on an analog clock. Again, show different times on the demonstration analog clock and have students write the time both ways, as ____ :00 and as ____ o’clock.

**ACTIVITY**

Play **Picking Pairs** and then **Memory** (see unit introduction) with the cards on **BLM Time Memory Cards (3) to (4)**. Students who need a further challenge can also use some of the cards from **BLM Time Memory Cards (1) to (2)**.

**A little before or after the o’clock times.** ASK: How do you know the time is not “o’clock?” (the minute hand is not at 12) Remind students that the hour hand moves the same way as the other hands. So when the minute hand is not at the 12, the hour hand is somewhere between two numbers. Remind students that when they measured time using the fast hand in the previous lesson, they said how many time units passed from 12. For example, when a volunteer did 10 jumping jacks, she started when the fast hand was at 12 and finished when the fast hand was between 4 and 5. We said that four time units passed. Similarly, when the hour hand is between the numbers 4 and 5, we say that the hour is 4. Point at the classroom clock and ASK: What hour is it now? Then show students different times on the demonstration analog clock and have them say what hour it is.

To help students who struggle deciding what hour it is, draw a clock with an hour hand pointing a little before the 6. Use a ruler to extend the hour hand by drawing a dotted line to the clock boundary. ASK: What two numbers is the line between? (5 and 6) What hour is it? (5) Have students read various clocks by first drawing a dotted line to help them. Students can also use the same method when doing Questions 3–4 on AP Book 3.2 p. 111.
Extensions

1. Give students BLM Empty Clock Faces. Have them draw the hands to show the time.

   a) 4:00  b) 9:00  c) 12:00  d) 10:00  
   e) 5 o’clock  f) 8 o’clock  g) 1 o’clock  h) 11 o’clock

   **Answers**

   ![Clock Images]

2. A little before or after the o’clock times. At various times of the day, stop and look at the clock when the hour hand is not pointing directly at a number, but is closer to one number than the other. ASK: Which number is the hour hand closest to? Is it pointing a little before or a little after the number?

   Explain that we can say times such as “a little before 2 o’clock” or “a little after 11 o’clock.” Show a clock with only an hour hand (e.g., from BLM Make Your Own Clock) and have students tell you first which number the hour hand is closest to and then read the time as “a little before (or after) o’clock.”

   Say more such times (for example, a little after 3 o’clock; a little before 7 o’clock) and ask students to show you on their clocks where the hour hand will be pointing at that time.
MD3-13 The Minute Hand
Pages 112–114

STANDARDS
3.OA.A.3, 3.OA.C.7

GOALS
Students will use skip counting and multiplication to tell how many minutes passed from the last hour.

PRIOR KNOWLEDGE REQUIRED
Is familiar with analog clock face
Can distinguish between the hour hand and the minute hand
Can write time using numbers
Can skip count by 5s
Can multiply one-digit numbers by 5
Knows that multiplication is repeated addition
Is familiar with multiplication strategies such as adding on

MATERIALS
2 analog clocks for demonstration
clock with only the minute hand, e.g., from BLM Make Your Own Clock (p. Q-67)
clocks made in Activity 2 of Lesson MD3-11
2 dice for each pair of students

How the minute hand moves every minute. Draw students’ attention to the minute markings on the analog clock. Explain that the marks show divisions of an hour into minutes. SAY: When the fast hand moves all the way around the clock, the minute hand (point to the minute hand) moves from one tick mark to the next. Verify this by watching the clock for a minute.

Compare the motion of the minute hand to that of the hour hand. SAY: The hour hand moves past a number every hour; the minute hand moves past a tick mark every minute, and that’s why it (point to the minute hand) is called the minute hand.

How the minute hand moves every hour. Explain that the minute hand goes all the way around the clock in one hour. Demonstrate with two analog clocks, one showing 4:00 and the other showing 5:00.

Explain that it looks like the minute hand didn’t move from 4:00 to 5:00, but the minute hand actually just moved around the circle and back to
where it started. Show the movement of both hands from 4:00 to 5:00 on a demonstration clock.

**Count by 5s to determine the number of minutes after the hour.** Show 9:00 on a demonstration clock, and then move the hands to show 9:15.

SAY: I want to know how many minutes passed when the minute hand moved from the 12 to the 3. Draw on the board:

![Diagram of a clock with minute hand pointing to 3 and 12]

SAY: This picture shows two minute hands. The gray hand shows where the minute hand was at 9 o’clock. The black minute hand shows where the minute hand moved to. Count together as a class the number of minutes that passed, emphasizing every fifth number (when the minute hand passed numbers 1, 2, and 3 on the clock face).

SAY: Like the hour hand and the fast hand, the minute hand takes the same amount of time, five minutes, to pass from any number to the next.

ASK: How many minutes pass when the minute hand moves from the 12 to the 1? (five) Draw an arrow showing a jump and write 5 next to the 1.

Repeat for the 12 to the 2 (10), the 12 to the 3 (15), the 12 to the 4 (20), and so on until 11.

ASK: What are we counting by? (5s) Explain that by noticing the pattern, we have made it easier to count the number of minutes after the hour.

Use a clock with only a minute hand (e.g., from BLM Make Your Own Clock) to show the minute hand pointing at various numbers. Have students tell how many minutes passed from when the minute hand was at the 12. Then add the hour hand and ask students to concentrate on the minute hand only. Show 7:15 on the clock and write on the board:

7: ___

Have students count by 5s to tell you the number of minutes. Repeat with other times, always providing the hour.

**Using multiplication to find the number of minutes after the hour.**

Remind students that when they skip count by 5s, they are actually adding 5 repeatedly. For example, when they skip count 5, 10, 15, it is the same as adding $5 + 5 + 5$. ASK: How can we write this calculation in a shorter way? (using multiplication) Have students write the multiplication sentence. $(3 \times 5 = 15)$ Remind students that we also call this a multiplication equation.

Show 10:15 on the demonstration analog clock. ASK: How many minutes past the hour is it? (15) How many numbers do you say when you skip count by 5s to get to the answer? (3) What number is the minute hand pointing at? (3) Repeat with 10:20, then with 10:30. Have students guess
how they can use the number the minute hand is pointing at to write a multiplication sentence for the number of minutes that passed after the last o’clock or exact hour. (multiply the number the minute hand points at by 5 to get the number of minutes after the hour)

**Exercises:** Write the multiplication equation for the minutes after 8 o’clock. Then write the time.

a) b) c) d)

**Answers:** a) $3 \times 5 = 15$, 8:15; b) $6 \times 5 = 30$, 8:30; c) $2 \times 5 = 10$, 8:10; d) $7 \times 5 = 35$, 8:35

Show a clock showing 8:50 and SAY: A student I know thinks that the time is 8:10. ASK: Is he correct? (no) SAY: The hour hand is very close to 9, but it is not at 9 yet. So the hour is 8. How many minutes passed after 8 o’clock? (50) What multiplication sentence gives 50 as the product? ($10 \times 5 = 50$) What mistake did the student make? (he did not use multiplication—he just wrote the number the minute hand is pointing at; the student might have also counted the minutes in the wrong direction, from 12 to 11 to 10)

Move the minute hand to 11. Ask students what multiplication they should use for the number of minutes in this case. ($11 \times 5$) Write on the board:

$11 \times 5 = ___$

Have them think how they can find the answer. Students might suggest counting by 5s, or adding 5 to 50. Have students find the answer both ways, and remind them that the answers should agree. Have a volunteer fill in the answer. (55)

ASK: What if five more minutes passed? What multiplication will that give us? ($12 \times 5$) Write on the board:

$12 \times 5 = ___$

Again, have students think of how to find the answer. Have a volunteer fill in the answer. (60) Then point out that the minute hand has moved around the whole circle. This means that one hour has passed. ASK: How many minutes are in an hour? (60)

**ACTIVITY**

Each pair of students will need two dice and a clock made in Activity 2 of Lesson MD3-11. Player 1 rolls the dice, adds the results, and points the minute hand at the sum. (For example, if Player 1 rolls 3 and 4, she adds $3 + 4 = 7$, and sets the minute hand pointing at 7.) Player 1 writes down the minutes past the hour given by the minute hand. ($7 \times 5 = 35$ minutes) Player 2 checks Player 1’s answers, and then players switch roles.
Extensions

1. Have students use a clock and count by 5s to the number of minutes (given below) to determine where the minute hand points. Students can keep track on their fingers. For example, if 20 minutes passed, students should count 5, 10, 15, 20. They have four fingers up, so the minute hand points at 4.
   a) 25 minutes    b) 40 minutes    c) 30 minutes    d) 45 minutes
   **Answers:** a) 5, b) 8, c) 6, d) 9

2. Have students use division by 5 to determine where the minute hand points for the minutes given below. For example, if 20 minutes passed from the last hour, students can divide \(20 \times 5 = 4\) to conclude that the minute hand points at 4.
   a) 15 minutes    b) 10 minutes    c) 35 minutes    d) 50 minutes
   **Answers:** a) 3, b) 2, c) 7, d) 10

(MP3, MP7)

3. Find \(270 \div 6\). Explain how you know your answer is correct.

**Sample solutions**

- I used \(270 = 300 - 30\), so \(270 \div 6 = (300 \div 6) - (30 \div 6) = 50 - 5 = 45\).
- I used \(270 = 240 + 30\), so \(270 \div 6 = (240 \div 6) + (30 \div 6) = 40 + 5 = 45\).
- I imagined sharing $270 with 6 people. I started by pretending there were only 3 people. This is an easier division because I know 3 \(\times\) 90 is 270. But if I have to share between 6 people instead of 3, everyone gets half as much—instead of $90, everyone gets $45. So \(270 \div 6 = 45\).

Look for students to notice and use structure (MP7)—such as using easier divisions or using multiplication to divide—and to articulate their reasoning (MP3). Note that the third solution above uses the real-world context of sharing (MP2) to help think about the abstract division problem.

Whole-class follow-up: Ask volunteers to explain their strategies and articulate why they chose them (MP5). For example, students might explain their choice for the solutions above as follows:

- I know that 300 (or 240) is close to 270 and easy to divide by 6, so I started there.

- I think it’s easier to think of sharing money than to think of division.
MD3-14 Time to the Five Minutes

Goals

Students will tell time from an analog clock when the minute hand shows a multiple of 5 minutes.

Prior Knowledge Required

- Is familiar with analog clock face
- Can distinguish between the hour hand and the minute hand
- Can write time using numbers
- Can skip count by 5s
- Can multiply one-digit numbers by 5
- Knows that multiplication is repeated addition
- Is familiar with multiplication strategies such as adding on

Materials

- Analog clock for demonstration
- 2 dice for each pair of students
- Clocks made in Activity 2 of Lesson MD3-11
- Cards from BLM Time Memory Cards (3) to (5) (pp. Q-61–63)

Review analog clocks. Ask students to explain how to distinguish the hands on a clock. (the minute hand is short and thick, the hour hand is longer) Review telling what hour it is using an analog clock. (look at the number the hour hand is pointing to) Show 9:00 on the demonstration analog clock and ASK: Is it o’clock? (yes) How do you know? (the minute hand points at the 12) What time is it? (9 o’clock) How do you know the hour? (the hour hand is pointing at the 9) Then show 9:15. Point out that the hour hand is now between 9 and 10. ASK: What hour is it? (9) Move the clock to 9:45. Remind students that, even when the hour hand is closer to 10 than to 9, the hour is still 9. Review telling how many minutes passed after the last hour. Remind students that they can either count by 5s around the clock starting at 12, or multiply the number the minute hand points at by 5 to get the number of minutes. Demonstrate both methods for 9:15, then have students tell how many minutes past the hour it is for 9:45.

Write digital times using the number of minutes after the hour. Show 7:10 on the demonstration analog clock. Explain that you want students to write the time in numbers. Remind them that they should write the hour first. ASK: Which hand is close to 7? (hour hand) Which hand points at the 2? (minute hand) What is the hour? 7 (SAY: So when we write the time as numbers, we start by writing the hour.

Write on the board:

7:___
ASK: How many minutes after 7 o’clock is it? (10) How do you know? (the minute hand points at 2, and $2 \times 5 = 10$) Finish writing the time: 7:10. Repeat for 7:50. Finally, show 7 o’clock and ask how many minutes after 7 o’clock it is. Explain that because it is 0 minutes after 7 o’clock, we just write 7:00.

**Exercises:** Write the time on the clock in numbers.

a) [Image of clock showing 1:30]

b) [Image of clock showing 4:35]

c) [Image of clock showing 10:10]

d) [Image of clock showing 6:15]

**Answers:** a) 1:30, b) 4:35, c) 10:10, d) 6:15

**ACTIVITY**

Each pair of students will need two dice and a clock made in Activity 2 of Lesson MD3-11. Player 1 rolls the dice, adds the results, and points the hour hand at the sum. Player 2 rolls the dice, adds the results, and points the minute hand at the sum. Both players write the time in numbers. Partners check each other’s answers and switch roles.

Review writing the time in the format of a digital clock. Remind students that digital clocks show both hours and minutes as two-digit numbers, so if the number of hours or minutes is a one-digit number, we write 0 first. For example, 5 minutes after 7:00 is shown as 07:05. Show several times on an analog clock and have students write the times in digital format. For more practice, you can either repeat the activity above, having partners write the time in digital format, or have them play Picking Pairs and then Memory (see introduction to Unit 5, p. F-2) using the cards from BLM Time Memory Cards (5). Include a few pairs from BLM Time Memory Cards (3) to (4) to make the game more challenging and increase the number of pairs.

**Review saying time in words.** Remind students that when the time is, say, 7:10, we say it as 10 minutes after 7, which is short for “10 minutes past (or after) 7 o’clock.” This means we say the minutes first and the hours after.

**Exercises:** Say the time in words.

a) 8:20  
b) 7:45  
c) 9:10  
d) 12:40

**Answers:** a) 20 minutes past 8, b) 45 minutes past 7, c) 10 minutes past 9, d) 40 minutes past 12

Show 3:05 on an analog clock and have students first write the time in numbers, then say it in words. Repeat with other times, such as 6:15, 10:30, 1:25, and 12:35.
Extension

1. Each group of three students will need two dice and the clock made in Activity 2 of Lesson MD3-11. Player 1 rolls the two dice. The player adds the results and writes them down as the hour (for example, $5 + 6 = 11$, so 11:__). Player 2 then rolls a single die, multiplies the result of the roll by 10, and chooses whether or not to add a bonus 5 for the minutes (for example, roll 4, $4 \times 10 = 40$, add 5 if wanted, so the minutes could be :40 or :45). If the roll is 6, the player should write :00 or :05 instead of :60 or :65. Player 3 sets the clock to this time. Players rotate roles after each turn.

(MP1, MP2, MP4) 2. Emma draws a picture of a house. The house has two windows. She uses yarn to make a trim around the outside of the windows. If she uses 16 cm of yarn for the smaller window, how much will she need for the larger window?

Use number sentences to explain your answer. Write what each number sentence means in the situation.

Solution: I started by finding the side length of the small squares: eight side lengths make 16 cm, so I found $16 \div 8 = 2$, so each side is 2 cm long. There are 12 of these around the outside of the large window, so I found $12 \times 2 = 24$. Emma will need 24 cm of yarn for the larger window.

Look for students to reread the problem (MP1) to ensure that each step of their work makes sense or when they realize an approach isn’t working so they can change course. Look for students to model the situation (MP4) using number sentences. Look for students to do the calculations and interpret the results of each calculation in the context (MP2).

Redirecting students: ASK: What do you have to know in order to find the perimeter of the shape? (how long each side of the small squares is) PROMPTS: Why can’t you just count around the outside of the shape and say the perimeter is the number of small square edges? (we need to know how long those small square edges are) Do you know how long each one is? (no) Encourage students to figure out how long they are from the information in the problem.

Whole-class follow-up: Take up different approaches to solving $12 \times 2$. (sample answers: it is the same as $2 \times 12$ which is $12 + 12$; it is $(10 \times 2) + (2 \times 2)$ which is $20 + 4$)
STANDARDS
3.OA.A.3, 3.OA.C.7

VOCABULARY
analog clock
digital clock
half
half past
hour
hour hand
minute
minute hand
multiply
o’clock
quarter
quarter past
skip count
whole

Goals
Students will tell time in the format of “half past” and “quarter past” the hour.

PRIOR KNOWLEDGE REQUIRED
Can tell time to the 5 minutes on an analog clock
Can tell time to the minute on a digital clock
Can write time using numbers
Can skip count by 5s
Can multiply one-digit numbers by 5
Can identify halves and quarters

MATERIALS
analog clock for demonstration
digital clock for demonstration
BLM Time Memory Cards (6) to (8) (pp. Q-64–66)

Review “whole” and “half.” Draw some basic shapes on the board, such as a square, a rectangle, and a circle, and ask volunteers to show wholes and halves by shading or re-drawing the shapes. Then draw the face of an analog clock and ask students to show half of the clock face. If students do not draw the division line vertically from 12 to 6, ask them specifically to start the line at the 12.

Introduce “half past” an hour. Show 2:00 on an analog clock and ask students to say or write the time in different ways. (2 o’clock, 2:00, and 02:00) Remind students that when the hour hand is at 12, the time is “o’clock,” as in 2 o’clock. SAY: In one hour, the hour hand will move from 2 to 3. How does the minute hand move? (it turns a full circle)

Trace your finger around half the picture of the clock. SAY: In half an hour, the minute hand turns half a circle. What number will the minute hand point at when half an hour has passed? (6) Change the clock so that it shows 2:30 and explain that we can read a time like that as half past 2, because half an hour has passed after 2 o’clock.

Draw on the board:

The time is half past 2.
Show 4:30 on the analog clock. ASK: What hour is it? (4) SAY: The minute hand is at 6, so we know half an hour passed after 4. ASK: What is the time? (half past 4) Repeat with 7:30, 9:30, and 12:30.

Writing “half past” in two ways. Remind students that they have two methods to find how many minutes past the hour it is. One is to count minutes by 5s. Count by 5s as a class while tracing the five-minute skips from 12 through 6 on a clock showing 12:30. ASK: How many minutes are in half an hour? (30) Remind students that a faster way to say how many minutes passed from the last hour is to multiply by 5. ASK: What number do we multiply by 5 to get the number of minutes? (the number the minute hand points at) What number does the minute hand point at? (6) Have students write the multiplication equation for the number of minutes. (6 \times 5 = 30) ASK: How would you write this time in numbers? (12:30) Have students say the time as “half past” and write it in numbers for the following times shown on an analog clock: 8:30, 3:30, 5:30, 10:30.

Exercises: Write the time in numbers.

a) half past 3 b) half past 6 c) half past 11 d) half past 9

Answers: a) 3:30, b) 6:30, c) 11:30, d) 9:30

Reading time in the “half past” form from a digital clock. Show a digital clock. ASK: How can you tell on a digital clock if the time is half past an hour? (the number of minutes is 30) Show several different times (Examples: 04:30, 07:30, 02:30) and have students tell the time in the form “half past” the hour.

Explain that some digital clocks show the times without the first zero. For example, the clock might show 2:30 instead of 02:30.

Exercises: Write the time in words and numbers.

a) 9:30 b) 3:30 c) 11:30 d) 5:30

Answers: a) half past 9, b) half past 3, c) half past 11, d) half past 5

Distinguishing between exact hours and half past times. ASK: How can you tell if the clock is showing “half past” or “o’clock”? (the minute hand points to the 12 for o’clock and to the 6 for half past) Where does the hour hand point for o’clock? (directly at the number for the hour) And for half past? (halfway between two numbers) Show times on the hour and on the half hour on an analog clock face and have students identify each time. Examples: 7:00, 4:30, 12:30, 6:00, 3:30. Then have students write the time in two ways—in numbers and in words and numbers. Emphasize that for o’clock—in other words, for the exact hour—it is 0 minutes after the hour, since it is exactly on the hour, but for half past, it is 30 minutes after the hour, so o’clock is written :00 and half past is written :30. Finally, show several times on a digital clock and have students identify the time. Examples: 8:00, 05:30, 11:00, 01:00, 02:30.
ACTIVITY 1

Play **Picking Pairs** and then **Memory** (see introduction to Unit 5, p. F-2) with cards from BLM Memory Cards (6) to (7).

**Quarters on a clock.** Draw some basic shapes on the board, such as a square, a rectangle, and a circle, and ask volunteers to show quarters by shading or drawing. Draw a clock face on the board, divide it in half (draw a line from the 12 to the 6), and then have a volunteer divide it into quarters by drawing a line from 9 to 3. Shade the quarter between 12 and 3. Explain that when the minute hand passes the shaded part of the clock face, one quarter of an hour has passed. When it is a quarter of an hour after 7 o’clock, the hour hand is pointing a little after 7 and the minute hand is a quarter of the way around the clock and pointing at the 3.

**Quarter past.** Explain that when a quarter of an hour has passed after 7:00, we say it is “quarter past 7.” ASK: What time is a quarter of an hour after 9 o’clock? (quarter past 9) Repeat with various “quarter past” times.

**Writing “quarter past” times.** ASK: When it is quarter past 4, where does the minute hand point? (at the 3) How many minutes after 4 o’clock is that? (15) How do you know? (count the five-minute skips starting at the 12 to the 3: 5, 10, 15; multiply $3 \times 5 = 15$) Have students write the multiplication equation. ASK: How do we write 15 minutes after 4 o’clock? (4:15) How do we read the time? (15 minutes past 4) How would we write quarter past 6 on a digital clock? (6:15 or 06:15) Quarter past 9? (9:15 or 09:15) Repeat with various “quarter past” times, having students both write and say the time (e.g., 8:15, 15 minutes past 8).

Show times on a clock face and have students identify the time. Use various times that are all a quarter past the hour. Give times sequentially at first, from 12:15 to 11:15, and then in random order. Then show the same times, sequentially and then in random order, but have students say or write the digital times. Finally, include o’clock and half past times as well, such as 7:15, 3:00, 5:30, 9:15, 4:30, and 11:00.

ACTIVITY 2

Students play **Picking Pairs** and then **Memory** (see introduction to Unit 5, p. F-2) with selected matching cards from BLM Time Memory Cards (6) to (8). Playing with all the Time Memory Cards might be too much, so use only those from the specified pages.

**Saying times in different formats.** Ask students for what times they know special ways to say the time. (exact hour, such as 1 o’clock; 30 minutes past the hour, such as half past 2; and 15 minutes past the hour, such as quarter past 3) Remind students that in other cases they simply say how many minutes past the hour it is. Remind them to multiply the number the minute hand points at on the analog clock by 5 to get the number of minutes past the hour. Students who struggle with multiplication can use...
skip counting by 5s to say the number of minutes past the hour. Show
different times on the clock and have students say the time in all formats
that they can think of. Use as many volunteers as possible. Repeat with
a digital clock, including times that are not multiples of 5 minutes
(examples: 5:15, 2:37, 9:00, 6:30, 12:01, 3:00, 4:14, 4:15)

Extensions

1. Write the times in order.
   a) 4:30, 2:30, 5:30
   b) 3:15, 11:15, 8:15
   c) 6:15, 3:00, 5:30
   d) 4:30, quarter past 5,
      10 minutes after 5
   e) 6 o’clock, 5:50,
      14 minutes past 6

   **Bonus:** 1:30, 5:00, 4:30, 2:15

   **Answers:** a) 2:30, 4:30, 5:30; b) 3:15, 8:15, 11:15; c) 3:00, 5:30, 6:15;
d) 4:30, 10 minutes after 5, quarter past 5; e) 5:50, 6 o’clock,
   14 minutes past 6; Bonus: 1:30, 2:15, 4:30, 5:00

2. Record the start and finish times of a favorite television show. Express
   the times using o’clock, half past, or quarter past if you can.

3. Rani plays soccer for 80 minutes a week for 5 weeks. Cody plays
   soccer for 60 minutes a week for 6 weeks. What is the difference in
   the total amount of time that Rani played soccer and the total amount
   of time that Cody played soccer? Explain your answer using number
   sentences. Say what each number sentence means in the situation.

   **Answer:** Rani played soccer for 400 minutes (because $80 \times 5 = 400$)
   and Cody played soccer for 360 minutes (because $60 \times 6 = 360$), so
   the difference is 40 minutes (because $400 - 360 = 40$).

   Look for students to write the correct number sentences (MP4), to
   notice the structure that makes multiplying tens easier than skip
   counting (MP7), and to say what each number sentence means in
   the situation (MP2).
Goals
Students will tell time from an analog clock to the nearest 1 minute in the format “33 minutes past 3.”

PRIOR KNOWLEDGE REQUIRED
Can tell time to the 5 minutes on an analog clock
Can tell time to the minute on a digital clock
Can write time using numbers
Can skip count by 5s
Can multiply one-digit numbers by 5

MATERIALS
analog clock for demonstration
clock with only the minute hand, e.g., from BLM Make Your Own Clock (p. Q-67)
clocks made in Activity 2 of Lesson MD3-11
2 dice for each pair of students
BLM Telling Time (The Second Hand) (p. Q-69, see Extension 1)

Review telling time to the five minutes. Set the demonstration analog clock to 3:35 and have students tell you what the time is. Ask them to explain how they found the answer. (the hour hand is between 3 and 4, so the hour is 3; the minute hand points at 7, so either skip count or multiply $7 \times 5 = 35$, so there are 35 minutes after the hour) Repeat with 8:10 and 11:25.

Telling time to the minute. Remind students that, on an analog clock face, the space between two numbers is divided into five parts, for 5 minutes. Set the demonstration clock to 10:07. If the minute hand does not reach the tick marks for minutes, draw a line on the clock face, as shown below:

As a class, count how many minutes after the hour the clock shows. (7) SAY: The time is 7 minutes past 10. Have a volunteer write the time in digital form. (10:07)

Set the demonstration clock to 10:22. SAY: I want to know how many minutes past the hour it is, I can count the minutes one by one to tell the time, but I think it will take a very long time. Have students suggest ideas for faster counting. Make sure the following two methods arise.
**Method 1:** Skip count by 5s from the 12 until you get to the number just before the minute hand (in this case, count 5, 10, 15, 20), and then count on by 1s for each part after (21, 22).

**Method 2:** The minute hand has just passed 4. When the minute hand was at 4, these minutes had passed after the hour: \(4 \times 5 = 20\). After that the hand moved 2 more minutes. The total number of minutes is \(20 + 2 = 22\).

To prompt students to see both methods, remind them of a similar problem.

SAY: Imagine you have 4 nickels and 2 pennies. How many cents do you have in total? (22) How did you find the answer? (count first by 5s, then by 1s, or multiply by \(4 \times 5 = 20\), then add 2) Have students write the multiplication and addition equation to show how to find the number of minutes in the second method. \((4 \times 5) + 2 = 22\)

Draw several analog clocks with the minute hand only, or show a clock with only the minute hand (e.g., from BLM Make Your Own Clock). Point the minute hand at 3, 7, and 9. Ask students to tell how many minutes past the hour it is. (15, 35, 45) After that, add the hour hand and ask students to write the exact time using numbers only, and then using both numbers and words. (for example, 3:05, 5 minutes past 3)

**Exercises:** Write the time in numbers. Then write the time in numbers and words.

a) \[\begin{array}{c}
9 \\
8 \\
7 \\
6 \\
5 \\
4 \\
3 \\
2 \\
1 \\
0
\end{array}\]

b) \[\begin{array}{c}
9 \\
8 \\
7 \\
6 \\
5 \\
4 \\
3 \\
2 \\
1 \\
0
\end{array}\]

c) \[\begin{array}{c}
9 \\
8 \\
7 \\
6 \\
5 \\
4 \\
3 \\
2 \\
1 \\
0
\end{array}\]

**Bonus:** Write the multiplication and addition equation for the number of minutes on each clock.

**Answers:** a) 5:19, 19 minutes past 5; b) 4:39, 39 minutes past 4; c) 9:07, 7 minutes past 9; Bonus: a) \(3 \times 5) + 4 = 19\), b) \(7 \times 5) + 4 = 39\), c) \(1 \times 5) + 2 = 7\)

**ACTIVITY**

Each pair of students will need 2 dice and a clock made in Activity 2 of Lesson MD3-11. Player 1 rolls the dice, adds the numbers, and points the hour hand to the sum. Then Player 1 rolls the dice again, multiplies the numbers, and points the minute hand to the minute given by the product. Player 2 writes the time in two forms: numbers and words and numbers. Player 1 checks the work and players switch roles. **NOTE:** Students who struggle can use addition instead of multiplication to determine the position of both hands. This way, instead of dealing with three concepts (addition, multiplication, and telling time), they are only dealing with two (addition and telling time).
Extensions

1. You can find the number of seconds after the minute the same way you find the number of minutes after the hour. Example: The clock shows 9:07:23.

![Clock Image]

Write the time in numbers.

a)  

b)  

c)  

Answers: a) 1:00:07, b) 8:38:20, c) 1:45:38

Have students complete the BLM Telling Time (The Second Hand).

(1.a) 5:01:37, b) 8:24:55, c) 2:17:46, d) 1:30:50, e) 7:15:18, f) 10:40:08)

(MP2)  2. If the hour hand is in the position shown, what number will the minute hand point at?

![Clock Image]

Answer: The hour hand moved three tick marks after the 3, so $3 \times 12 = 36$ minutes have passed since the last hour. This means the time is 3:36, so the minute hand points to the first minute after the 7.

![Clock Image]

Whole-class follow-up: Take up the solution provided. ASK: When the hour hand moves from the 3 to the 4, how many minutes pass? (60 minutes) How many parts is the space between the 3 and the 4 divided into? (5 parts) If 60 minutes are divided into five equal parts, how many minutes are in each part? (12 minutes, because $60 ÷ 5 = 12$) When the hour hand moves from one tick mark to the next, how many minutes pass? (12 minutes)
**Goals**

Students will find elapsed time within 1 hour in increments of 5 minutes, using analog clocks.
Students will solve problems requiring finding elapsed time or finishing time.

**PRIOR KNOWLEDGE REQUIRED**

Can tell time to the 5 minutes
Can read and write time using numbers
Can skip count by 5s

**MATERIALS**

analog clock for demonstration
clocks made in Activity 2 of Lesson MD3-11

**Review the idea of time passage.** Remind students that when they want to tell the time from an analog clock, they determine the hour and the number of minutes that passed from the last hour. Show 7:15 on the demonstration clock. ASK: How much time passed since 7 o’clock? (15 minutes) SAY: So if I wake up at 7 o’clock, and start eating breakfast at 7:15, I know that 15 minutes passed between these two events.

**Exercises:** How much time passed from 7:00 to the time shown?

![Clocks](image)

**Answers:** a) 25 minutes, b) 40 minutes, c) 50 minutes

**Skip counting by 5s to find the elapsed time.** SAY: Emma wakes up at 7:15 in the morning. She eats breakfast starting at 7:40. ASK: How much time passes between the time that Emma wakes up and the time that she starts eating breakfast? Turn the hand on the demonstration clock slowly and ask a volunteer to count the minutes that pass by 5s. (5, 10, 15, 20, 25) SAY: 25 minutes passed from 7:15 to 7:40.

**Draw on the board:**
SAY: This clock shows two minute hands to show how the minute hand moved. The white minute hand shows where the minute hand was when recess started, and the black minute hand shows the time when it ended. We will call the time the white minute hand shows the start time, and the time the black minute hand shows the end time. Have a volunteer label the hands “start time” and “end time.”

Trace a finger around the clock from the start time to the end time and SAY: This is how much the minute hand moved. We can count by 5s to say how many minutes passed. Draw arrows to show the skips as shown below, and have the class count by 5s as you go.

ASK: How many minutes passed between the start and the end time? (15 minutes)

Repeat with the clocks below:

(10 minutes, 35 minutes)

SAY: I cannot draw arrows on a real clock, but I think I can track the time that passed without drawing arrows. One way is to count by 5s, and just imagine the arrows. Do the first exercise below as a class and then have students work individually.

Exercises: Count by 5s to find out how much time passed. The white minute hand shows the start time.

a)  

b)  

c)  

Bonus:  

Answers: a) 25 minutes, b) 35 minutes, c) 45 minutes, Bonus: 15 minutes

Solving problems with elapsed time. Present the next exercises one at a time. Have a volunteer show the start time on a demonstration clock, and have students show the same time on the clocks made in Activity 2
of Lesson MD3-11. Then have them turn the minute hand to show how much time passed to find the elapsed time. For these exercises there is no need to set the hour hand precisely; a rough position within the correct hour interval is enough.

**Exercises:** How much time has passed?

a) Jenny woke up at 7:00 and left for school at 7:55. How much time passed?

b) Tom got to school at 8:15 and played until school started. School started at 8:55. How long did he play for?

c) Tessa had a piano lesson from 3:10 to 3:55. How long was the lesson?

d) Jayden read from 6:30 to 7:00. How long did he read for?

**Bonus:** Sun got home from school at 3:55 and had a snack at 4:05. How much time passed from the time she got home from school to the time she had a snack?

**Answers:** a) 55 minutes, b) 40 minutes, c) 45 minutes, d) 30 minutes, Bonus: 10 minutes

**Finding the end time using an analog clock.** SAY: I am baking muffins. I know that muffins need to bake in the oven for 25 minutes. I need to know when to take them out. Imagine I put the muffins in the oven at 5:25. Invite a volunteer to show the time on the demonstration clock, and have students show the time on their clocks. ASK: How can I know when to take the muffins out of the oven? Have students try to figure out when to take the muffins out.

Draw a clock showing 5:25 on the board and label the minute hand as the “start time.” SAY: I know the start time, and I know that 25 minutes passed. How can I show the 25 minutes on the picture? (draw arrows) Have a volunteer draw the arrows. The final picture will look like this:

```
  15
  20
  25
  30
  35
  40
  45
  50
  55
      1
      2
      3
      4
      5
      6
      7
      8
      9
     10
     11
     12
     13
     14
     15
     16
     17
     18
     19
     20
     21
     22
     23
     24
     25
      Start time
```

ASK: What number will the minute hand point to after 25 minutes? (10) How many minutes past the hour is that? (50) How do you know? (10 \times 5 = 50) Will the minute hand pass 12? (no) SAY: This means the hour will not change, the hour will still be 5. What will the time be after 25 minutes? (6:50, or 50 minutes past 5) SAY: This means I need to take the muffins out at 50 minutes past 5.

SAY: What if I put the muffins in at 4:10? Repeat the exercise with the new start time. (end time will be 4:35) Repeat again starting at 6:35. Note that
the minute hand reaches 12 this time, so the hour changes and the end time is 7:00. For an extra challenge, ask students to figure out the end time if the muffins are put in the oven at 8:55. (9:20)

**Solving problems requiring finding the end time.** Have students solve the problems below by showing the start time on an analog clock (e.g., using the clocks they made in Activity 2 of Lesson MD3-11) and skip counting along the clock face without drawing the actual arrows.

**Exercises:** Use a clock to solve the problem.

a) Don went out to play soccer at 7:30. He played for 20 minutes. When did he finish?

b) A TV show starts at 8:15. It lasts 30 minutes. When does it end?

c) Vicky left home at 6:10. It takes her 35 minutes to walk to the library. When does she arrive at the library?

**Bonus:** It takes Carlos 2 minutes to read 1 page. He read 15 pages. How long did he read for? Carlos started reading at 6:50. When did he stop reading?

**Answers:** a) 7:50, b) 8:45, c) 6:45, Bonus: 30 minutes, 7:20

**Extensions**

1. Count back by fives on a clock to find the start time.

   a) Ron finished reading at 7:20. He read for 15 minutes. When did he start reading?

   b) It takes Tasha 25 minutes to get to her friend’s house. She needs to be there at 6:40. When should she leave?

   c) A pizza takes 35 minutes to cook in the oven. Kyle wants to serve pizza at 2:50. When should he put the pizza in the oven?

   **Answers:** a) 7:05, b) 6:15, c) 2:15

   (MP.4)

2. Use a clock to solve the problems.

   a) Sam reads 1 page in 3 minutes. He started reading at 8:20 and finished at 8:50. How many pages did he read?

   b) Alice reads 1 page in 2 minutes. She started reading at 8:00 and finished at 8:14. How many pages did she read?

   **Answers:** a) 10 pages, b) 7 pages

3. In a story, a witch is cooking a potion. She adds 3 dead mice to the potion at 11:10.

   a) The witch needs to stew the potion for 20 minutes after adding the dead mice and then add 7 snakes. When should she add the snakes?
b) After adding the snakes, she needs to let the potion cool for 15 minutes and then stir it quickly 12 times. When should she stir the potion?

c) After stirring, she needs to heat the potion for 10 more minutes, then add 1 cup of chopped toenails, and heat the potion for 5 more minutes. Then the potion is ready. When will the potion be ready?

**Answers:** a) 11:30, b) 11:45, c) 12:00
MD3-18 Elapsed Time

Goals
Students will find elapsed time within 1 hour.

PRIOR KNOWLEDGE REQUIRED
Can tell time to 1 minute
Can skip count by 5s
Can multiply by 5
Understands the connection between skip counting and multiplication

MATERIALS
analog clock for demonstration
ball

Review finding elapsed time to the 5 minutes on an analog clock. Draw on the board:

Have students say how much time passed if the minute hand moves from the start time, at the white position, to the end time, at the black position.
(a) 25 minutes, b) 40 minutes, c) 35 minutes)

Finding elapsed time by counting by 5s, then counting by 1s. SAY: The amount of time that passed between two events or times is also called elapsed time.

Show 2:23 on a demonstration clock. Review by asking students to explain how they can find what the time is. (The hour hand is between 2 and 3, so the hour is 2. The minute hand points between 4 and 5, so count by 5s to 20, then count by 1s to 23, or multiply \(4 \times 5 = 20\), and then add 3 to get 23.) Point out that by saying the number of minutes, students have found elapsed time from the last hour, 2 o’clock. Explain that students can use a very similar method to find elapsed time between any two times.

Draw on the board:

Start time

End time

STANDARDS
3.MD.A.1, 3.OA.A.3

VOCABULARY
analog clock
elapsed time
end time
hour
hour hand
minute
minute hand
multiply
o’clock
skip count
start time
Have students identify the start time and the end time (8:05, 8:24). SAY: I want to know how much time passed or elapsed from 8:05 to 8:24. Let’s count by 5s first. Have students count by 5s and draw skipping arrows around the clock as shown below:

```
1 2 3 6 7 8 9 10 12 11 4 5
```

SAY: We have 15 minutes and then some more. The minute hand for the end time is between 4 and 5, so I cannot count another 5. Let’s count on by 1s. Count each minute as a class. (16, 17, 18, 19) SAY: The elapsed time from 8:05 to 8:24 is 19 minutes.

**Exercises:** Find the elapsed time. Count by 5s, and then count by 1s.

a) from 2:40 to 2:51  
b) from 11:15 to 11:37

c) from 7:25 to 7:43  
**Bonus:** from 10:50 to 11:24

**Answers:** a) 11 minutes, b) 22 minutes, c) 18 minutes, **Bonus**: 34 minutes

**Practice saying times in order, counting by 5s.** Explain that for the next task students will need to say times in order: 2:00, 2:05, 2:10, 2:15, and so on. Have students finish the sequence of times from 2:15 to 3:00. Remind students that there are 60 minutes in 1 hour, so when they are counting minutes by 5s and reach 55, the next time is the next hour: for example, 1:55 is followed by 2:00.

**ACTIVITY**

Start by saying a time, such as 3:00, and passing a ball to a student. The student who receives the ball says the next time, counting minutes by 5s. (3:05) Students continue passing the ball to students who have not yet had it, each saying the next time. (3:10, 3:15, and so on)
Using multiplication to find elapsed time. SAY: I want to know how much time passed from 2:30 to 2:45, but I do not have a clock. Try to imagine a clock to check how much time passed. Explain that you can keep track on your fingers. Demonstrate how to do so by saying the times and raising fingers one at a time, as shown below:

2:30  2:35  2:40  2:45

SAY: I have three fingers up. This means I counted three groups of 5 minutes. So $3 \times 5$ minutes have passed. ASK: How many minutes is that? (15) Write on the board:

\[ 3 \times 5 = 15 \text{ minutes} \]

15 minutes passed from 2:30 to 2:45

Emphasize that we say the start time with a closed fist. ASK: How many minutes passed from 3:15 to 3:50? Count as a class by 5s, keeping track on fingers. (3:15, 3:20, 3:25, 3:30, 3:35, 3:40, 3:45, 3:50) ASK: How many fingers did you raise? (seven) Have students write a multiplication equation for the number of minutes ($7 \times 5 = 35$ minutes) and record the answer. (35 minutes passed from 3:15 to 3:50)

**Exercises:** Count by 5s from the start time to the end time. Write the multiplication equation and the elapsed time.

a) from 7:20 to 7:35  
   b) from 9:25 to 9:50  
   c) from 12:05 to 12:45  
   d) from 1:25 to 1:55  

**Bonus**  
e) from 10:45 to 11:00  
f) from 12:45 to 1:15

**Answers:** a) $3 \times 5 = 15$ minutes, b) $5 \times 5 = 25$ minutes,  
c) $8 \times 5 = 40$ minutes, d) $6 \times 5 = 30$ minutes,  
Bonus: e) $3 \times 5 = 15$ minutes, f) $6 \times 5 = 30$ minutes

**Extensions**

1. Example: It takes 10 minutes to boil an egg. Helen puts an egg to boil at 3:45. When is the egg ready?  
   
   10 minutes $= 2 \times 5$ minutes, so we need to raise two fingers. Count up until two fingers are up: 3:45, 3:50, 3:55. The egg is ready at 3:55.  
   
   Count up by 5 minutes to find the end time.  
   
   a) Ray starts baking a cake at 2:15. It takes him 25 minutes to make the batter. When is the batter ready?
b) Ray puts the cake in the oven right away. The cake bakes for 20 minutes. When does Ray take the cake out?

c) Ray decorates the cake right after baking. It takes him 15 minutes to decorate the cake. When is the cake ready?

**Answers:** a) 2:40, b) 3:00, c) 3:15

2. a) Liz wakes up at 7:05. It takes Liz 5 minutes to wash her face and brush her teeth, 10 minutes to dress, 10 minutes to eat breakfast, and 5 minutes to get to the bus stop. The bus leaves at 7:50. If Liz also spends 20 minutes finding her homework before leaving home, is she late for the bus? Explain.

b) Tom wakes up at 7:20. It takes him 5 minutes to brush his teeth, 3 minutes to shower, 4 minutes to dress, 12 minutes to eat, and 4 minutes to get to the bus stop. Then he realizes that he has forgotten his phone and he runs home and back. It takes him 2 minutes to run home and 2 minutes to run back. The bus leaves at 7:53. Is he late for the bus? Explain.

**Answers**
a) Liz spends 30 minutes in total to wash her face, brush teeth, dress, eat, and get to the bus stop. If she spends 20 more minutes finding her homework, the total is 50 minutes. She arrives at the bus stop at 7:55, too late for the bus.
b) Tom spends 28 minutes on his morning routine, plus 4 more minutes to get his phone. This is 32 minutes in total. He finally arrives at the bus stop with his phone at 7:52, in time for the bus.

3. As a class, count by minutes by 5s to and past the exact hour. Example: 2:50, 2:55, 3:00, 3:05, 3:10.

a) Count minutes by 5s to find the elapsed time.

From 2:50 to 3:15
From 5:45 to 6:25
From 8:55 to 9:30

b) Find the end time when the elapsed time passes the exact hour.

20 minutes from 7:45
35 minutes from 9:40
55 minutes from 7:30

**Answers:** a) 25 minutes, 40 minutes, 35 minutes; b) 8:05, 10:15, 8:25

4. Tom wants to make thank you cards for friends who visited him in the hospital. Tom draws 4 dogs, 3 cats, and 5 bunnies on each card. If Tom draws a total of 20 bunnies, how many dogs and how many cats does he draw? Show your work using number sentences. Say what each number sentence means in the situation.

**Answers:** 20 ÷ 5 = 4, so Tom makes 4 cards, and 4 × 4 = 16, so Tom draws 16 dogs. Also, 4 × 3 = 12, so Tom draws 12 cats.
**Goals**

Students will draw diagrams to find elapsed time within 1 hour.

**PRIOR KNOWLEDGE REQUIRED**

- Can tell time to 1 minute
- Can read and write time using numbers
- Can find elapsed time in increments of 5 minutes

**MATERIALS**

- analog clock for demonstration

**Finding elapsed time when two times are very close.** Show 2:35 on a demonstration analog clock. Have students identify the time. Write “2:35” on the board. Slowly move the minute hand on the demonstration clock to 2:37. **ASK:** What is the time now? (2:37) Record this end time on the board under the start time, being sure to align the place values, as shown below:

| 2:35 | 2:37 |

**ASK:** How many minutes passed from 2:35? (2 minutes) **How do you know?** (the minute hand moved two marks further along the clock face) Record that on the board underneath the two times. Repeat with 3:30 to 3:34, 4:55 to 4:58, and 1:40 to 1:46.

**SAY:** The times in each case are very close. Is there a way to tell how many minutes passed without counting every minute? (yes) **ASK:** Did the hour change in each case? (no) **SAY:** Only the minutes changed. Is there a mathematical operation—addition, subtraction, division, or multiplication—that you could do to find how much time passed? (yes, subtraction) **SAY:** You can subtract the minutes to find how much time passed. Invite volunteers to write the subtraction sentences for the minutes for the examples above. (37 – 35 = 2 minutes, 34 – 30 = 4 minutes, 58 – 55 = 3 minutes, 46 – 40 = 6 minutes) **ASK:** Did we get the same answer with both methods: counting every minute on the clock face and subtracting? (yes)

**Exercises:** How much time has passed?

- a) from 5:10 to 5:14
- b) from 8:25 to 8:27
- c) from 11:41 to 11:44

**Bonus:** from 9:32 to 9:40

**Answers:** a) 4 minutes, b) 2 minutes, c) 3 minutes, Bonus: 8 minutes

**Drawing a picture to find elapsed time.** Review finding elapsed time by counting by 5s for times such as 3:25 to 3:40, 6:15 to 6:50, and 9:10 to 9:35.
Have students count by 5s as a class to find the elapsed time. (15 minutes, 35 minutes, 25 minutes)

SAY: I want to know how much time passed from 3:25 to 3:41. I can draw a picture to find out. Draw on the board:

3:25 3:40 3:41

ASK: How much time passed from 3:25 to 3:40? (15 minutes) Write “15” in the first box. ASK: How much time passed from 3:40 to 3:41? (1 minute) Write “1” in the second box. ASK: How much time passed in total from 3:25 to 3:41? (16 minutes) How do you know? (15 + 1 = 16) Write “+” between the boxes and write “= 16 minutes” to finish the addition sentence. The final picture should look like this:

3:25 3:40 3:41

15 1

Repeat the exercise with the time interval from 4:15 to 4:52. This time draw blanks under the tick marks in the picture and have students tell you what to write in the blank. The final picture should look like this:

4:15 4:50 4:52

35 + 2 = 37 minutes

**Exercises:** Draw a picture and find the elapsed time.

a) from 3:05 to 3:32  b) from 8:15 to 8:43  c) from 12:10 to 12:54

**Answers:** a) 27 minutes, b) 28 minutes, c) 44 minutes

**Finding the finish time when times are close.** SAY: My dentist says I should brush my teeth for 2 minutes. If I start brushing my teeth at 7:15, when should I finish brushing them? (7:17) How do you know? Have students come up with different ways to find the answer. Suggestions might include using an analog clock and counting 2 minutes up from 7:15, counting up by ones (7:15, 7:16, 7:17), or adding 2 to the number of minutes (15 + 2 = 17).

**Exercises:** Eddie brushes his teeth for 2 minutes. Find the time he needs to stop brushing.

a) Eddie starts at 7:20  b) Eddie starts at 9:55  c) Eddie starts at 6:57

**Answers:** a) 7:22, b) 9:57, c) 6:59
Now have students find the end time in different situations. Solve the first two exercises as a class and then have students work individually. Encourage students to write addition sentences for minutes.

**Exercises**

a) Sandy talked on the phone for 7 minutes. She started at 10:15. When did she finish?

b) Will started eating lunch at 12:25. He ate for 14 minutes. When did he finish?

c) The school bus left at 8:30. The ride to school is 9 minutes long. When did the bus arrive at school?

d) It takes Kathy 7 minutes to bike to school. She left home at 8:35. When did she arrive at school?

**Bonus:** It takes Miss K. 23 minutes to drive to work. She leaves home at 9:20. When does she get to work?

**Answers:** a) 10:22, b) 12:39, c) 8:39, d) 8:42, Bonus: 9:43

**Extensions**

1. The table shows some flight times for flights from Seattle, WA, to Portland, OR.

   a) Fill in the missing times and lengths of flight.

<table>
<thead>
<tr>
<th>Day</th>
<th>Departure</th>
<th>Arrival</th>
<th>Length of Flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>11:04</td>
<td>11:32</td>
<td></td>
</tr>
<tr>
<td>Tuesday</td>
<td>11:07</td>
<td>11:32</td>
<td></td>
</tr>
<tr>
<td>Wednesday</td>
<td>11:06</td>
<td></td>
<td>28 minutes</td>
</tr>
<tr>
<td>Thursday</td>
<td></td>
<td>11:46</td>
<td>30 minutes</td>
</tr>
<tr>
<td>Friday</td>
<td>11:03</td>
<td></td>
<td>27 minutes</td>
</tr>
</tbody>
</table>

   b) On which two days did the flights take the same time?

c) On which day did the flight take the longest time?

d) On which day did the flight take the shortest time?

e) How much longer was the flight on Thursday than on Monday?

**Answers:** a) 28 minutes, 25 minutes, 11:34, 11:16, 11:30; b) Monday and Wednesday; c) Thursday; d) Tuesday; e) 2 minutes
2. Finish the picture and find the elapsed time.

a) from 4:45 to 5:12

\[
\begin{array}{c}
4:45 \\
+ \\
5:00 \\
5:12
\end{array}
\]

\[= \text{ ___ minutes} \]

b) from 8:15 to 9:07

\[
\begin{array}{c}
8:15 \\
+ \\
9:00 \\
9:07
\end{array}
\]

\[= \text{ ___ minutes} \]

c) from 12:40 to 1:14

\[
\begin{array}{c}
12:15 \\
+ \\
1:00 \\
1:14
\end{array}
\]

\[= \text{ ___ minutes} \]

Answers: a) \(15 + 12 = 27\) minutes, b) \(45 + 7 = 52\) minutes, c) \(20 + 14 = 34\) minutes

3. Finish the picture and find the elapsed time.

a) from 4:42 to 5:12

\[
\begin{array}{c}
4:42 \\
+ \\
4:45 \\
5:00 \\
5:12
\end{array}
\]

\[= \text{ ___ minutes} \]

b) from 7:12 to 8:07

\[
\begin{array}{c}
7:12 \\
+ \\
7:15 \\
8:00 \\
8:07
\end{array}
\]

\[= \text{ ___ minutes} \]

c) from 2:44 to 3:16

\[
\begin{array}{c}
2:44 \\
+ \\
2:45 \\
3:00 \\
3:16
\end{array}
\]

\[= \text{ ___ minutes} \]

Answers: a) \(3 + 15 + 12 = 30\) minutes, b) \(3 + 45 + 7 = 55\) minutes, c) \(1 + 15 + 16 = 32\) minutes
4. Use minutes and hours to explain in pairs why the division is correct. Use math words in your explanation. Do you agree with each other? Discuss why or why not.

a) \( 60 \div 4 = 15 \)  
b) \( 60 \div 5 = 12 \)

**Sample answers:** a) If you divide 60 minutes into 4 equal parts, you get a quarter of an hour or 15 minutes, so \( 60 \div 4 = 15 \); b) a clock divides 60 minutes into parts that are 5 minutes in size, and there are 12 equal parts, so \( 60 \div 5 = 12 \)

Look for students to use math words (MP6), including “minutes,” “hours,” “equal parts,” “quarter” or “fourth.” Look for students to reason about the equation using the given context (MP2). Encourage partners to ask questions to understand and challenge each other’s thinking (MP3) and use of terminology (MP6)—see p. A-49 for sample sentence and question stems to guide students.)
**Goals**

Students will identify times of the day using a.m. and p.m.

Students will draw and use timelines to find elapsed time.

**PRIOR KNOWLEDGE REQUIRED**

- Can tell time to 1 minute
- Can read and write time using numbers
- Can find elapsed time within 1 hour
- Can read and draw a number line

**MATERIALS**

- analog clock for demonstration

**Introduce a.m. and p.m.** Write “12:00” on the board. ASK: What time is it? (12 o’clock) Is it nighttime or daytime? (it could be either) Ask students if anyone knows what we can write next to the time to tell whether it is 12:00 at night or 12:00 during the day. Write “a.m.” and “p.m.” at different ends of the board. Explain that we add *a.m.* to times between midnight and noon and *p.m.* to times from noon to midnight. These names or labels are short forms for expressions that originated in Ancient Rome: *a.m.* stands for *ante meridiem* (“before noon”) and *p.m.* stands for *post meridiem* (“after noon”).

Draw on the board:

```
<table>
<thead>
<tr>
<th>a.m.</th>
<th>noon</th>
<th>p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>midnight</td>
<td>noon</td>
<td>midnight</td>
</tr>
</tbody>
</table>
```

**Write times using a.m. and p.m.** List several events with times of day on the board and ask students to say whether each time should have a.m. or p.m. after the numbers. (see examples below) Students can signal the answer by pointing to the correct side of the picture above.

- Breakfast at 8:00 (a.m.)
- Plane takes off at 3:15 in the afternoon (p.m.)
- Train arrives at 11:45 in the morning (a.m.)
- Library visit at 5:30 (p.m.)
- Dentist appointment at 1:15 (p.m.)
- 3 hours after midnight is 3:00 (a.m.)
- 7 o’clock in the morning is 7:00 (a.m.)
- I ate ice cream at 4:30 (p.m.)
- Half past 9 in the morning (a.m.)

Show the following times on the demonstration analog clock and have students write the time in numbers and using a.m. or p.m.
Lunch time: 11:55  
School ends: 3:15  
Swimming pool closes: 9:30  
School bus leaves: 8:07  
(a.m., p.m., p.m., a.m.)

Remind students that when the minute hand is at 6, the time is half past the hour, and when the minute hand is at 3, we say the time is quarter past the hour. For example, in the exercises above, school ends at quarter past 3 p.m., and the swimming pool closes at half past 9 p.m. ASK: If I leave home to go to work at half past 7, what is the time in numbers? (7:30 a.m.) If I return home after school at quarter past 6, what is the time in numbers? (6:15 p.m.)

**Exercises:** Write the time in numbers using a.m. or p.m.

a) Math lesson ends at  
   half past 10.

b) Karate class starts at quarter past 7.

**Bonus:** Sal’s little sister has a nap during the day from half past 11 to quarter past 1.

**Answers:** a) 10:30 a.m., b) 7:15 p.m., Bonus: 11:30 a.m. to 1:15 p.m.

Ask students to list six things they do and the time they do each thing, making sure that three times are before noon, and three times are after noon. For example, I wake up at 6:50 a.m., and I go to bed at 9:50 p.m.

**Introduce timelines.** Ask students to tell you some of the things they do in the a.m. or p.m. Write down several of the suggestions but make sure that they are out of chronological order. Explain that you would like to organize the list of the activities students suggested so that they are in the order that they happen. For example, waking up should come before breakfast. Point out that the times students indicated provide a natural order of things. Remind students that when they try to order numbers, they can mark them on a number line, and then read them from left to right. Explain that we can use a similar tool with times, called a *timeline*. Draw on the board:

```
7:00 a.m. ___ ___ ___ 8:00 a.m. ___ ___ 9:00 a.m.
```

SAY: This is a timeline from 7 o’clock in the morning to 9 o’clock in the morning. Ask a volunteer to show which part shows one hour from 7:00 to 8:00 and which part shows one hour from 8:00 to 9:00. ASK: How many parts is each hour divided into? (four parts) What fraction of an hour is each part? (quarter, fourth) How many minutes is one quarter of an hour? (15 minutes) PROMPT: If a quarter hour passed from 7 o’clock, what time is it? (7:15) SAY: This means the timeline shows marks for every 15 minutes. Have students copy the timeline and tell you what times to put in each blank. Repeat with a timeline from 2:00 p.m. to 5:00 p.m. with increments of half an hour.
Using a timeline to solve problems. Draw a timeline for a school morning with 15-minute increments, but label only the increments for hours—for example, from 9:00 a.m. to 11 a.m. ASK: How many parts is each hour divided into? (four) How many minutes is each part? (15 minutes) Point to different marks and ask students to say what time each mark shows.

NOTE: Revise the following as needed based on the times at your school.

SAY: This is the timeline of a school morning. School starts at 9 o’clock. Label that on the timeline. SAY: The first lesson is math. It starts at quarter past 9. Where is this time on the number line? Have a volunteer mark it. Present the events in the exercises below or ones that reflect your school morning, and point to different increments. Have students signal thumbs up if you are showing the right position on the number line and thumbs down if not.

Exercises
a) Math lesson ends at quarter past 10.
b) Science lesson starts at half past 10.
c) Quiz starts at 9:45.

Answers

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 a.m.</td>
<td>School starts</td>
</tr>
<tr>
<td></td>
<td>Math lesson starts</td>
</tr>
<tr>
<td>10:00 a.m.</td>
<td>Pop quiz starts</td>
</tr>
<tr>
<td></td>
<td>Math lesson ends</td>
</tr>
<tr>
<td>11:00 a.m.</td>
<td>Science lesson starts</td>
</tr>
</tbody>
</table>

ASK: How long is the break between the end of the math lesson and the start of the science lesson? (15 minutes) Have students explain how they know. Lead students to both using the number line (one quarter of an hour between the arrows, so the break is 15 minutes) and finding the elapsed time (15 minutes elapsed from 10:15 to 10:30) ASK: How much time elapsed from the start of the math lesson to the start of the quiz? (30 minutes) Again, have students explain how they determined the answer. SAY: The quiz was only 15 minutes long. When did the quiz end? (at 10:00) How do you know? Students can either count on by 5s to add 15 minutes to 9:45, or use the number line: 15 minutes is one quarter of an hour, so the next mark on the timeline after the start of the quiz should be the time the quiz ends. Make sure to discuss both solutions.

Extend the timeline for another hour. SAY: The science lesson was 45 minutes long. When did it end? Encourage students to use the timeline to solve the problem, rather than to count on by 5s. To help students to see that 45 minutes are three increments of 15 minutes each, you can draw a clock face and have a volunteer shade 45 minutes on the clock. ASK: What fraction of the clock face is shaded? (three quarters) SAY: This means 45 minutes equals three quarters of an hour. ASK: How many spaces on the timeline is that? (three) Have students count three spaces on the timeline, starting at 10:30. ASK: When did the science lesson end? (11:15)
ACTIVITY

Students record when they do different things over one day on the weekend. Then they make a timeline of the day, marking the things they did. Students write the elapsed time and invent problems for their peers to solve. Example: I read from 8:15 a.m. to 9:00 a.m. How long did I read?

Extensions

1. **Time in Ancient Egypt.** Tell students that our 12-hour clock comes from Ancient Egypt, where people divided the day into 12 equal parts and the night into 12 equal parts. This meant that the length of each part or hour would change over the seasons because the day would get longer in the summer and shorter in the winter. The Egyptians used sundials to measure time during the day, and water clocks to measure time during the night. Students can research ways to construct sundials and water clocks, as well as their origins. Good online search phrases are “how to make a sundial for kids” and “how to make a water clock for kids.”

The Ancient Romans also divided the day into 12 equal parts, starting at around what we call 6 a.m. and ending at around 7 p.m. Here are more examples of times in Ancient Rome and their modern-day equivalents.

<table>
<thead>
<tr>
<th>Time in Ancient Rome</th>
<th>Time Today</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st hour</td>
<td>about 7 a.m.</td>
</tr>
<tr>
<td>6th hour</td>
<td>about 12 p.m.</td>
</tr>
<tr>
<td>7th hour</td>
<td>about 1 p.m.</td>
</tr>
<tr>
<td>12th hour</td>
<td>about 6 p.m.</td>
</tr>
</tbody>
</table>

Have students extend the table above to show all hours before noon or create a timeline that shows all day times (from 7 a.m. to 6 p.m.) as they would have been written in Ancient Rome and now. Ask students to look for patterns. What number do you need to subtract from modern a.m. times to get the Roman day time? (6 hours) What number do you need to add to modern p.m. times to get the Roman day time? (6 hours) What Roman hour is an exception to this rule? (6th hour: you still need to subtract 12 – 6 to get the Roman time)

2. Imagine a free day when you can do anything you want. Write six things you would do. Write the start time and the end time for each, using a.m. or p.m. for each time. Make a timeline and find the elapsed time for each activity.
3. a) Estimate each sum to check if the answer is reasonable.

A. \(234 + 146 = 380\)

B. \(213 + 96 = 562\)

C. \(146 + 332 = 458\)

b) Did you round to the nearest ten or to the nearest hundred? Explain your choice to a partner.

**Sample answers**

a) A. \(230 + 140 = 380\), so the answer is reasonable; B. \(200 + 100 = 300\), so the answer is not reasonable; C. \(150 + 330 = 480\), so the answer is not reasonable

b) For example, for C, I started by rounding to the nearest hundred because it’s faster. I got \(100 + 300 = 400\), but 458 rounds to 500, so I thought the answer was too high. Then I rounded to the nearest ten and got \(150 + 330 = 480\), so now I think the answer is actually too low.

Look for students to recognize that rounding to the nearest hundred is faster, but that rounding to the nearest ten is more accurate. This demonstrates that they can choose tools strategically (MP.5) and recognize the degree of precision required to solve the problem (MP.6).

**Whole-class follow-up:** ASK: Is the answer to C too high or too low? Students who estimated to the nearest hundred may think the answer is too high, but students who estimated to the nearest ten may think the answer is too low. Ask students to check by doing the addition. (the answer is too low) Have students discuss with a partner why rounding to the nearest hundred made it look like the answer was too high when in fact it was too low (both numbers were rounded down by quite a bit, so the estimate wasn’t very accurate). Summarize by saying that if the answer looks like it might be close, it is better to use rounding to the nearest ten, but if the answer is far off, it is better to use rounding to the nearest hundred.
MD3-21 Adding Time
Pages 131–132

STANDARDS
3.MD.A.1, 3.OA.D.9

VOCABULARY
analog clock
colon
elapsed time
hour
hour hand
minute
minute hand
multiplication
o’clock
operation
skip count

Goals
Students will add times (without regrouping minutes as hours).
Students will solve problems that require adding times.

PRIOR KNOWLEDGE REQUIRED
Can tell time to 1 minute
Can read and write time using numbers
Can find elapsed time
Can add using the standard algorithm

Writing elapsed time using a colon. Ask students to say what time it is now and ask a volunteer to write the time in numbers on the board. Remind students that the two dots (one above the other) that separate the hours and the minutes are a symbol called a colon. Explain that sometimes it is convenient to write elapsed time the same way we write time, with a colon.
For example, if the time is 1:05 p.m., this means that 5 minutes have passed since 1 o’clock. ASK: How much time has passed since noon? (1 hour and 5 minutes) How do you know? (1 hour from noon to 1 o’clock, 5 more minutes to 1:05 p.m.) Explain that it is sometimes convenient to write the elapsed time as 1:05. Remind students that they need to write a zero as the tens digit if the number of minutes is a one-digit number. Point out that 1:05 p.m. is the time of day, for example the time now. On the other hand, if there is no a.m. or p.m., the time is actually elapsed time, so 1:05 means 1 hour and 5 minutes passed.

Exercises: Write the elapsed time using a colon.

a) 5 hours 12 minutes  b) 9 hours 23 minutes  c) 2 hours 2 minutes

Answers: a) 5:12, b) 9:23, c) 2:02

ASK: What if only 45 minutes passed? How many whole hours is that? (0) How can we write that 45 minutes passed using a colon? (0:45) How will you write the elapsed time if exactly 3 hours passed, no extra minutes? (3:00)

Exercises: Write the elapsed time using a colon.

a) 5 hours  b) 23 minutes  c) 2 minutes  Bonus: 27 hours

Answers: a) 5:00, b) 0:23, c) 0:02, Bonus: 27:00

Review finding the end time by adding minutes. SAY: I made a phone call that lasted 3 minutes and started at 2:40 p.m. ASK: When did I finish the call? (2:43) Have students explain how they found the answer. Make sure that the idea of adding the minutes is discussed.
Finding the end time using addition. ASK: How can we write 3 minutes with a colon? (0:03) Write on the board:

\[
\begin{array}{c}
\text{I started talking:} \\
\text{Elapsed time:}
\end{array}
\quad \begin{array}{c|c|c}
& & \\
\hline
2 & : & 4 \\
+ & 0 & 3 \\
\hline
2 & 4 & 3
\end{array}
\]

SAY: We can vertically add times on a grid, exactly the same way we add numbers. Show how to do the addition, starting from the right, writing each digit in its own cell of the grid and aligning the place values. Point out that the answer came out to be exactly the same as with adding mentally. Remind students that the end time is not elapsed time, so they should write a.m. or p.m. when they write the final answer. Record the final answer on the board, as shown below:

\[
\begin{array}{c}
\text{I started talking:} \\
\text{Elapsed time:} \\
\text{I finished talking at 2:43 p.m.}
\end{array}
\quad \begin{array}{c|c|c}
& & \\
\hline
2 & : & 4 \\
+ & 0 & 3 \\
\hline
2 & 4 & 3
\end{array}
\]

Write on the board:

Kim ran for 20 minutes. She started at 5:15 p.m. When did she finish?

ASK: How do we write 20 minutes as elapsed time? (0:20) Demonstrate solving the problem using addition on a grid. (Kim finished running at 5:35 p.m.) Have students prompt you about what to write. Students can signal what digit to write, one at a time, from right to left.

Repeat with the following problem, showing the regrouping in minutes:

Carlos read for 1 hour 15 minutes. He started reading at 6:27 p.m. When did he finish reading?

The final solution should look like this:

\[
\begin{array}{c}
\text{Carlos started reading:} \\
\text{Elapsed time:} \\
\text{Carlos finished reading at 7:42 p.m.}
\end{array}
\quad \begin{array}{c|c|c}
& & \\
\hline
6 & : & 2 \\
+ & 1 & 5 \\
\hline
7 & 4 & 2
\end{array}
\]

Exercises: Add the times to find the end time.

\begin{align*}
a) \quad & 12:15 + 0:23 \\
b) \quad & 8:09 + 3:25 \\
c) \quad & 9:24 + 1:12
\end{align*}

Answers: a) 12:38, b) 11:34, c) 10:36
Solving problems by adding time. Remind students to add a.m. or p.m. to the answer and to align the place values properly when doing addition.

**Exercises:** Add the times to solve.

a) Baseball practice is 45 minutes long. It starts at 5:05 p.m. When does it finish?

b) A movie is 2 hours 23 minutes long. It starts at 4:30 p.m. When does it end?

c) It takes Glen 4 hours 22 minutes to drive from Detroit, MI, to Chicago, IL. Glen leaves Detroit at 3:15 p.m. When does he arrive in Chicago?

**Bonus:** Anna drives from Salt Lake City, UT, to Yellowstone National Park. The drive takes 5 hours 8 minutes. Anna also stops for 20 minutes to rest. If she leaves Salt Lake City at 6:15 a.m., when does she arrive at Yellowstone?

**Answers:** a) 5:50 p.m., b) 6:53 p.m., c) 7:37 p.m., Bonus: 11:43 a.m.

**Extensions**

1. Add times with seconds. Example:

   \[
   \begin{align*}
   &10:15:23 \\
   + &0:01:12 \\
   \hline
   &10:16:35
   \end{align*}
   \]


   **Answers:** a) 8:55:32, b) 10:57:52, c) 6:55:55

2. The table shows when some flights leave New York City. Find the arrival time for each flight.

<table>
<thead>
<tr>
<th>Flight to</th>
<th>Departure</th>
<th>Flight Length</th>
<th>Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Atlanta, GA</td>
<td>2:05 p.m.</td>
<td>2 hours 27 minutes</td>
<td></td>
</tr>
<tr>
<td>b) Chicago, IL</td>
<td>7:24 a.m.</td>
<td>2 hours 31 minutes</td>
<td></td>
</tr>
<tr>
<td>c) Columbus, OH</td>
<td>8:13 p.m.</td>
<td>1 hour 45 minutes</td>
<td></td>
</tr>
<tr>
<td>d) Detroit, MI</td>
<td>6:29 p.m.</td>
<td>2 hours 4 minutes</td>
<td></td>
</tr>
<tr>
<td>e) Indianapolis, IN</td>
<td>3:50 p.m.</td>
<td>2 hours 8 minutes</td>
<td></td>
</tr>
<tr>
<td>f) Miami, FL</td>
<td>2:29 p.m.</td>
<td>3 hours 3 minutes</td>
<td></td>
</tr>
<tr>
<td>g) Milwaukee, WI</td>
<td>3:11 p.m.</td>
<td>2 hours 18 minutes</td>
<td></td>
</tr>
<tr>
<td>h) Minneapolis, MN</td>
<td>8:15 a.m.</td>
<td>3 hours 8 minutes</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 4:32 p.m., b) 9:55 a.m., c) 9:58 p.m., d) 8:33 p.m., e) 5:58 p.m., f) 5:32 p.m., g) 5:29 p.m., h) 11:23 a.m.
3. Calculate $12 \times 5$ in three ways:

- split 12 into a product of two smaller numbers
- split 12 into a sum of two smaller numbers
- use minutes and hours

**Sample strategies**

- $12 \times 5 = (6 \times 2) \times 5 = 6 \times (2 \times 5) = 6 \times 10 = 60$
- $12 \times 5 = (3 \times 4) \times 5 = 3 \times (4 \times 5) = 3 \times 20 = 60$
- $10 \times 5 = 50$ and $2 \times 5 = 10$, so $12 \times 5 = 50 + 10 = 60$
- $12 \times 5 = (2 \times 6) \times 5 = 2 \times (6 \times 5) = 2 \times 30 = 60$
- $12 \times 5 = (4 \times 3) \times 5 = 4 \times (3 \times 5) = 4 \times 15 = 60$, because a quarter of an hour is 15 minutes
- there are $12 \times 5$ minutes in an hour, but there are 60 minutes in an hour, so $12 \times 5 = 60$

Look for students to recognize structure (MP.7)—for example, when they use the associative and distributive properties **(NOTE: Students do not need to use these terms)**. Look for students to recognize how using the context of minutes and hours can help them solve a mathematical problem (MP.2).

Whole-class follow-up: Have partners share strategies and then take up a few of them and compare them as a class. For example, in the sample strategies above, some use the same property of multiplication, one of them uses a different property, and two of them use clocks, but in different ways.
Goals
Students will subtract times (without regrouping hours as minutes).
Students will solve problems that require finding elapsed time.

PRIOR KNOWLEDGE REQUIRED
Can tell time to 1 minute
Can read and write time using numbers
Can find elapsed time
Can subtract using the standard algorithm

MATERIALS
analog clock for demonstration

Review writing elapsed time with a colon. Remind students that they can write elapsed time in the same format as the time of the day, only without a.m. or p.m. after the numbers. Remind them that when the elapsed time is hours only, there are no additional minutes, so we write 00 for the number of minutes. On the other hand, when only a few minutes have passed, we write 0 for the number of hours. Remind students to use two-digit numbers for the number of minutes, with a 0 in the tens place if needed.

Exercise: Write the elapsed time using a colon.

a) 3 hours 42 minutes  b) 2 hours 3 minutes
  c) 4 hours  d) 2 minutes

Answers: a) 3:42, b) 2:03, c) 4:00, d) 0:02

Write on the board:

Elapsed time: 8:15

ASK: If the elapsed time is 8:15, how much time passed? (8 hours 15 minutes) PROMPT: Which part shows the hours? (the part before the colon, 8) Circle the hours and write “hours” underneath using the same color. Which part shows the minutes? (the part after the colon, 15) Circle the minutes using a different color and write “minutes” underneath. Repeat with 3:07, 0:16, and 9:00.

Determining elapsed time using subtraction. SAY: Last lesson we added times to find the end time. ASK: Do you think we could subtract time too? (yes) What problems could we solve by subtracting time? (finding elapsed time; students might also mention finding the start time) Tell students that today they will use subtraction to find elapsed time. SAY: Ken left home at 8:15 a.m. and got to school at 8:35 a.m. ASK: How long did it take him to get to school?
Ask students to find the elapsed time by counting up from the start time to the end time. (20 minutes) Then show how to solve the same problem by subtraction. Draw on the board:

\[
\begin{array}{c}
8 : 35 \\
- 8 : 15 \\
\hline
0 : 20
\end{array}
\]

Explain that the end time is later than the start time, so you subtract the start time from the end time. Point out that when doing rough work, students do not need to write the a.m. or p.m. after the start time or the end time. SAY: If the answer is 0:20, how much time has passed? (20 minutes) PROMPT: How many hours? (0) How many minutes? (20) ASK: Did we get the same answer as with counting up? (yes)

Repeat the subtraction with longer periods of time that require counting up by many 5s, such as from 7:05 a.m. to 7:50 a.m. ASK: Which method is faster: counting up, or subtracting? (subtracting) Finally, remind students that for such periods of time, they need to first count by 5s, then count by 1s, and also draw a picture to keep track of your counting. When they subtract, there is only one operation to do.

Draw students’ attention to the need to align the colons when subtracting, to make sure that they subtract hours from hours and minutes from minutes.

**Exercises:** Subtract to find the elapsed time. Write the answer in words and numbers.

a) 3:25 \( - \) 3:05  b) 4:55 \( - \) 4:24  c) 2:48 \( - \) 2:12

**Answers:** a) 20 minutes, b) 31 minutes, c) 36 minutes

SAY: Lessons start at 8:35. Helen wakes up at 7:15. ASK: How much time elapses from the time Helen wakes up to the start of the lessons? Ask a volunteer to perform the subtraction on a grid on the board, as shown below:

\[
\begin{array}{c}
8 : 35 \\
- 7 : 15 \\
\hline
1 : 20
\end{array}
\]

ASK: How much time elapsed? (1 hour 15 minutes)

**Exercises:** Subtract to find the elapsed time.

a) 12:55 \( - \) 9:15  b) 10:54 \( - \) 6:25  c) 6:58 \( - \) 1:32

**Answers:** a) 3:40, b) 4:29, c) 5:26
Have students solve the exercises below that require finding elapsed time. Solve the first two exercises as a class, then have students work individually. For students who struggle at lining up the times vertically for subtraction, suggest writing the colons first, one above the other.

**Exercises:** Subtract the times to solve the problem.

a) A plane took off at 8:35 p.m. and landed at 10:42 p.m. How long was the flight?

b) Football practice starts at 5:10 p.m. and ends at 6:40 p.m. How long is the practice?

c) A movie starts at 2:10 p.m. and ends at 4:28 p.m. How long is the movie?

d) Joan should read for at least 20 minutes every day. Today, she read from 7:22 p.m. to 7:44 p.m. Did she read enough?

**Bonus:** Movie A lasts from 3:20 p.m. to 5:52 p.m. Movie B lasts from 4:11 p.m. to 6:39 p.m. Which movie is longer? How do you know?

**Answers:** a) 2 hours 7 minutes; b) 1 hour 30 minutes; c) 2 hours 18 minutes; d) She read for 22 minutes, so yes; Bonus: Movie A is 2 hours 32 minutes long, Movie B is 2 hours 28 minutes long, so Movie A is longer.

**Extensions**

1. At various times during the day, ask students to record the time from the classroom analog clock in 12-hour notation. Ask them to calculate the amount of time that passed between each reading of the clock.

   (MP.6)  

2. Solve the problem.
   
a) It takes Tony 1 hour 15 minutes to cook dinner. Dinner should be ready by 6:30 p.m. When should he start cooking?

b) It takes Ms. B. 23 minutes to drive to work. She needs to be at the office at 8:55 a.m. When should she leave home?

c) It takes Lily 1 hour 33 minutes to get to the concert hall. The concert starts at 8:00 p.m., but she wants to be there 10 minutes early. When should she leave home?

**Answers:** a) 5:15 p.m., b) 8:32 a.m., c) 6:17 p.m.

Look for students to include the units a.m. or p.m. in their answers (MP.6).

3. Find the missing numbers.

   a) \[
   \begin{array}{c}
   8 : 3 \\
   - 8 : 9 \\
   \hline
   2 : 3 \\
   \end{array}
   \]

   b) \[
   \begin{array}{c}
   6 : 5 \\
   - 4 : 7 \\
   \hline
   2 : 3 \\
   \end{array}
   \]

   c) \[
   \begin{array}{c}
   5 : 1 \\
   - 5 : 4 \\
   \hline
   7 : 4 \\
   \end{array}
   \]

**Answers:** a) 8:32 − 8:09 = 0:23, b) 6:50 − 4:27 = 2:23, c) 12:51 − 5:47 = 7:04.
Goals
Students will tell time from an analog or digital clock using the format “25 minutes to 5.”

PRIOR KNOWLEDGE REQUIRED
Can tell time to 1 minute
Can read and write time using numbers
Can find elapsed time
Can add using the standard algorithm

MATERIALS
analog clock for demonstration
digital clock for demonstration
2 dice for each pair of students

Review telling time using an analog clock. Show several times on an analog clock and have students tell the time. Examples: 3:00, 3:15, 4:17, 5:38. Have students explain how they tell the time from the clock. Make sure students mention that if the hour hand is between two numbers, we say the number it just passed for the hour. Review both multiplying by 5 and counting by 5s before counting on by 1s to find the number of minutes after the hour.

Time left to the next hour. Explain that when the minute hand has passed 6, there is less time left before the next hour than has passed after the last hour. In this case, people often tell the time by saying what is left until the next hour. For example, if the clock shows 4:58 (show this time on the analog clock), only 2 minutes are left until 5, which is the next hour, so people often say that the time is 2 minutes to 5, rather than 58 minutes after 4.

Show 2:55 on an analog clock. ASK: How many minutes are left to the next hour? (5) What is the next hour? (3) What is the time? (2 minutes to 3) Repeat with 6:59, 10:57, 1:56.

Now show 5:43 on the analog clock. SAY: It is a lot of work to count every minute. ASK: How can we find how many minutes are left to the next hour more quickly? (count by 5s, then count by 1s) If students mention multiplying the number the minute hand points at by 5, remind them that the numbers count the time in the opposite direction from what you need; they count past the last hour so this method will not work for counting the minutes to the next hour.

Trace your finger counterclockwise from 12 to the position of the minute hand, and have students count by 5s and by 1s as a class: 5, 10, 15, 16, 17.
Write on the board:

17 minutes to ____

ASK: What hour should I put in the blank? (6) Why? (the hour hand is between 5 and 6, and we counted the time to the next hour, which is 6)
Write 6 in the blank.

Show 2:32 on the clock. Count the minutes left to the next hour as a class and record them on the board. (5, 10, 15, 20, 25, 26, 27, 28) Again, have students explain how to say the hour, and what the time is. (28 minutes to 3)

Show several different times on the analog clock (with the minute hand in the second half of the clock face) and have students write the time in the format “____ minutes to ____.” Then continue with a few more times, and have students write the time both ways, as “___ minutes to ___” and “___ minutes past ___.” Example: 10:37 is 23 minutes to 11 and 37 minutes past 10. Have volunteers record the times and the answers on the board.

**Using subtraction to tell time to the hour.** Have students count by 5s all the way around a clock face. ASK: How many minutes are in one hour? (60) Ask students to look at the number of minutes in their answers written on the board. Have students add the number of minutes in both ways to say each time. For example, for 10:37 they should make the addition equation $23 + 37 = 60$. ASK: What do the minutes add to? (60) Why does this happen? (because we count the minutes past the hour and the minutes that are left to the next hour, so in total we should get one full hour, 60 minutes)

Show 6:35 on a digital clock. Ask students to write the subtraction sentence to say how many minutes are left to the next hour. ($60 - 35 = 25$)

**PROMPT:** There are 60 minutes in an hour. 35 of them passed. How many are left? Have students write the time in words and numbers, both ways, past the hour and to the next hour. (6:35, 35 minutes past 6, 25 minutes to 7)

**ACTIVITY**

Give each pair of students two dice. Player 1 rolls the dice, adds the numbers, and writes the sum as the hour. The player rolls the dice again, multiplies the numbers, and writes the product as the minutes. Player 2 tells the time two ways: past the hour and to the next hour. Player 1 checks the answers and players switch roles. (For example, roll 2 and 6, 8 hours; roll 3 and 5, so $3 \times 5 = 15$ minutes; the time 8:15 is 15 minutes past 8 and 45 minutes to 9.)

**Quarter to the hour.** Remind students that there is another way to say some special times such as 4:15. ASK: How else can we say this time? (quarter past 4) Remind students that 15 minutes is one quarter of an hour, and 30 minutes is one half of an hour.

Show an analog clock that shows 3:45. ASK: What time is it? (45 minutes past 3, 15 minutes to 4) Explain that if a quarter of an hour is left before 4,
we say that it is “quarter to 4.” Show various “quarter to” times on an analog clock and have students tell the time. First have students only use the “quarter to” format, and then encourage them to say the time in as many different ways as possible. Then include other times, and have students give as many different forms for each as they can. Examples: 5:45, 6:15, 7:20, 8:00, 9:35, 10:57, 11:30.

Discuss how they can tell which times on a digital clock have special ways to say the time. Make sure students recall that times that end with 00 are said as “o’clock,” times that end with 30 are “half past,” times that end with 15 are “quarter past,” and times that end with 45 are “quarter to.” Then show different times on a digital clock and have students say the time in many different ways. Examples: 3:45, 2:25, 7:30, 4:00, 9:15, 10:47, 1:55.

**Extensions**

1. Show the hour hand at various positions between two hours and have students predict where the minute hand will be—closer to 12, 3, 6, or 9? Examples: The hour hand is about halfway between 7 and 8, so the minute hand will be near the 6. The hour hand is just a little after the 4, so the minute hand will be near the 3.

2. A birthday party starts at half past 2 p.m. and ends at quarter to 6 p.m. How long does the party last? Use any tool you think will help.

   **Sample answer:** The party starts at 2:30 p.m. and ends at 5:45 p.m. Subtract:
   
   $\begin{align*}
   5:45 \\
   - 2:30 \\
   \hline
   3:15
   \end{align*}$

   The party lasted for 3 hours 15 minutes or 3 and a quarter hours.

   **NOTE:** Students might use a different tool than subtraction, such as a number line or skip counting.

3. Adam played soccer from quarter past 6 p.m. until 20 minutes to 8 p.m. How long did he play for? How do you know?

   **Answer:** Adam played soccer from 6:15 p.m. to 7:40 p.m. Subtract:
   
   $\begin{align*}
   7:40 \\
   - 6:15 \\
   \hline
   1:25
   \end{align*}$

   Adam played for 1 hour 25 minutes.
Reading Digital Times

02 04 06

09 11 12

minutes past
Time Memory Cards (I)

01:05  5 minutes past 1
02:10

03:12  12 minutes past 3
10 minutes past 2

04:30  30 minutes past 4
05:01

10:02  2 minutes past 10
1 minute past 5
Time Memory Cards (2)

- 06:50  50 minutes past 6
- 07:13
- 11:07  7 minutes past 11
- 11 minutes past 7
- 08:42  42 minutes past 8
- 04:03
- 12:22  22 minutes past 12
- 3 minutes past 9
Time Memory Cards (3)

12:00

11:00

06:00

05:00

04:00

03:00

02:00

01:00

00:00

07:00

08:00
Time Memory Cards (4)

03:00

04:00

09:00

04:00

10:00

05:00

11:00

06:00

12:00
Time Memory Cards (5)

06:05
02:35
08:15
12:11
11:20
03:45
12:25
Time Memory Cards (6)

12:30  half past 12  05:30

03:30  half past 1  half past 5

half past 8

half past 9  half past 4
Time Memory Cards (7)

1 o’clock

01:00

2 o’clock

9 o’clock

02:00

4 o’clock

04:00

5 o’clock

05:00

8 o’clock

08:00
Time Memory Cards (8)

- Quarter past 12

- Quarter past 1

- Quarter past 5

- Quarter past 8

- Quarter past 4

- Quarter past 9

- Quarter past 4
Make Your Own Clock
Empty Clock Faces

![Clock Faces](image-url)
Telling Time (The Second Hand)

The **second hand** is longer and thinner than both the minute and hour hands.

We read seconds the same way we read minutes.

The exact time with seconds is:

```
8:50:07
```

I. Write the time in numbers under the clock.

1. **a)**
2. **b)**
3. **c)**

```
10 11 12
q 9 8 7 6 5 4 3
```

4. **d)**
5. **e)**
6. **f)**

```
10 11 12
q 9 8 7 6 5 4 3
```

II. Draw the hands on the analog clock to show the time.

1. **a)** 3:50:35
2. **b)** 7:02:23
3. **c)** 9:15:05
PS3-5  Using Number Lines

Use this lesson after: 3.2 Unit 5

Standards: 3.MD.A.1, 3.OA.A.3, 3.NBT.A.2

Goals:
Students will use number lines to solve problems.

Prior Knowledge Required:
Can tell and write time to the nearest minute
Can measure time intervals in minutes
Can draw a number line, with increasing numbers from left to right for horizontal number lines, or from bottom to top for vertical number lines
Can use a number line to solve word problems involving addition and subtraction of time intervals
Can skip count
Can add several two-digit numbers

Vocabulary: counting backward, counting forward, decreasing, increasing, minute, number line, vertical, week

Materials:
BLM Number Lines (p. Q-78)
BLM Phone Rings (p. Q-80, see Performance Task)
BLM Clock Problems (pp. Q-81–82, see Performance Task)

Using number lines is easier than counting backward. Write on the board:

36, 31, 26, 21

ASK: Is this sequence increasing or decreasing? (decreasing) PROMPT: Do the numbers get bigger or smaller? (smaller) SAY: The numbers get smaller, so this is a decreasing sequence.
ASK: How much do the numbers get smaller by each time? (5) Ask a volunteer to write the next term in the sequence. (16) ASK: How did you get that? (I counted backward; I did 21 – 5)

Write on the board:

Karen decides to hike from her home to Tea Lake.
On Monday morning, she is 20 miles away from the lake.
She hikes 6 miles each day.
How far from the lake will she be by Wednesday evening?

ASK: What is the farthest she is from the lake? (20 miles) SAY: She started 20 miles away and she is getting closer each day, so 20 miles is the farthest. So, let’s make a number line up to 20.
Number lines are easier to use than counting backward because you can write the numbers going up in order. Draw on the board:

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
```

Point to the dot at 20, and SAY: On Monday, she starts 20 miles away from the lake. Ask different volunteers to show her progress each day. (see picture below)

```
<table>
<thead>
<tr>
<th>Wednesday</th>
<th>Tuesday</th>
<th>Monday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 1 2 3</td>
<td>4 5 6</td>
<td>7 8</td>
</tr>
<tr>
<td>9 10 11</td>
<td>12 13</td>
<td>14 15</td>
</tr>
<tr>
<td>16 17 18</td>
<td>19 20</td>
<td></td>
</tr>
</tbody>
</table>
```

**(MP.6)** SAY: Because Karen is getting closer to the lake each day, you just have to show moving back 6 spaces at a time. You could count back 6 to find out how far she is after each day, but you don’t have to. The number line does the counting back for you, because you’ve already written the numbers in order. ASK: How far will Karen be from the lake by the end of the day Wednesday? (2 miles) If students say just “2,” ASK: Two what? (miles) Point out that in order to answer how far, you have to say whether Karen is 2 inches, 2 feet, 2 yards, or 2 miles from where she wants to be—otherwise, you really don’t know how far she is.

In the exercises below, students will solve real-life word problems by counting backward on a number line.

**(MP.4)** **Exercise:** Complete BLM Number Lines.
**Selected solution:**
3. c)

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

**Answers:** 1. 5 miles; 2. 1 mile; 3. a) 2 miles, b) 6 minutes, c) 5 miles

**Bonus:**
1. A snail crawls 3 inches in an hour. It is 20 inches away from the end of the branch.
   a) How far from the end of the branch will it be after 4 hours of crawling?
   b) A cherry hangs 5 inches before the end of the branch. When will the snail reach the cherry?

2. A messenger pigeon flies 1 mile every 3 minutes.
   a) How long will it take the pigeon to carry a letter to a person who lives 6 miles away?
   b) The message needs to arrive by 9:00 a.m. What time should the pigeon leave?
**Answers:** 1. a) 8 inches, b) after 5 hours; 2. a) 18 minutes, b) 18 minutes to 9 or 8:42
**Jumps in two directions on a number line.** Tell students that Amy is riding an elevator in her apartment building. Draw a vertical number line on the board, as shown below:

```
10 9 8 7 6 5 4 3 2
Ground level = G = 1
Basement = B = 0
```

Tell students that this vertical number line shows the floor numbers in the building. SAY: Amy starts at the second floor. Have a volunteer show where she starts by drawing a dot on the number line. SAY: Amy goes up five floors. Have a volunteer draw an arrow to show where Amy travels to. (to the 7th floor) SAY: Amy now goes down two floors. Have a volunteer draw an arrow to show Amy’s movement. (to the 5th floor) The completed number line should look like this:

```
10 9 8 7 6 5 4 3 2
Ground level = G = 1
Basement = B = 0
```

**Exercises:** Draw a vertical number line in your notebook to answer these questions.

a) Amy starts at the 5th floor. She goes up 4 floors, then down 7 floors. What floor does she end on?

b) Amy starts at the 10th floor. She goes down 6 floors, then up 4 floors. What floor does she end on?

c) Amy starts at the 8th floor. She goes up 2 floors, then down 5 floors. What floor does she end on?

d) Amy is on the 3rd floor. She goes up to the 9th floor. How many floors did she go up?

e) Amy pushes the button to go to the 10th floor. She travels up four floors before getting off. What floor did she start on?

**Bonus:** Amy starts at the 7th floor. She goes down 5 floors, up 6 floors, and then down 3 floors. What floor does she end on?

**Answers:** a) 2nd, b) 8th, c) 5th, d) 6, e) 6th, Bonus: 5th
**Using number lines with unknown numbers.** Write on the board:

Anwar is 2 years older than Megan.
Sara is 7 years older than Megan.
Mark is 3 years younger than Sara.
Who is older, Anwar or Mark? How much older?

Read the problem aloud. SAY: A good way to start a problem like this is to draw a number line, so that you can see how far apart everyone’s ages are on the line. Draw on the board:

```
  |   |   |   |   |   |   |   |   |   |   |   |   |
Megan     Anwar
```

SAY: On the number line, we’ll use one place to the right to mean 1 year older. Look at the first sentence. ASK: Who is older, Anwar or Megan? (Anwar) Underline “Anwar is” and “older than Megan” as shown below:

```
Anwar is 2 years older than Megan.
```

ASK: How much older? (2 years) SAY: That means we should place Anwar two places to the right of Megan. Add “Megan” and “Anwar” to the number line on the board, as shown below:

```
  |   |   |   |   |   |   |   |   |   |   |   |   |
Megan     Anwar
```

SAY: We can’t put numbers on the number line, because we don’t know anyone’s actual age. We just know how they compare to each other—that Anwar is two years older than Megan. We don’t know yet if we need more places on the left or the right, so I’ve written their names in the middle of the number line. If there is not enough room, I might have to erase and redraw the line. Read the second sentence of the problem aloud, and then have a volunteer extend the number line to show Sara’s age, as shown below:

```
  |   |   |   |   |   |   |   |   |   |   |   |   |
Megan     Anwar     Sara
```

Show the two comparisons done so far, using arrows, as shown below:
Read the third sentence of the problem aloud, and have a volunteer show Mark’s age. Show this comparison with an arrow, as shown below:

SAY: Now we can answer the problem. ASK: Who is older, Anwar or Mark? (Mark) How much older? (2 years) Show how you can get from Anwar’s age to Mark’s age with a jump forward of size 2 by tracing the jump with your finger. ASK: Do you know from this picture how old anyone is? (no, we just know how the ages compare)

(MP.1, MP.6) Exercises: Draw a number line to answer the questions.

a) Jake is 3 years older than Sandy. Ron is 5 years older than Sandy. Madison is 4 years younger than Ron. Who is older, Jake or Madison? How much older?

b) Ron, Mark, Sam, and Greg are identical quadruplets. Greg is 19 minutes older than Ron. Mark is 9 minutes younger than Sam. Greg is 6 minutes older than Sam. Who is older, Ron or Mark? How much older?

Bonus: Josh’s birthday is 3 weeks before Gary’s. Sara’s birthday is 4 days after Gary’s birthday. Nina is 2 weeks younger than Sara. Whose birthday is first, Josh or Nina’s? By how many days? Hint: A week has 7 days.

Answers: a) Jake is 2 years older than Madison, b) Mark is 4 minutes older than Ron,

Bonus: Josh’s birthday is 11 days before Nina’s

For extra practice, students can complete the following exercises.

Exercises:
1. David plants four apple trees in a row. The nearest tree is 5 yards from his house. The trees are 2 yards apart. How far away from David’s house is the last tree?

   Hint: Put David’s house at zero on the number line.

   Answer: 11 yards

2. A snail is at the bottom of a well on Monday morning. Every day, the snail climbs 3 feet, and every night it slides back 1 foot. The well is 7 feet deep. On what day does the snail reach the top of the well?

   Solution: Use a number line:
Problem Bank
(MP.1, MP.6) 1. When Alex wakes up, he takes 4 minutes to get dressed, 3 minutes to brush his teeth, 8 minutes to have breakfast, 2 minutes to gather his books, 2 minutes to put on his shoes, and 4 minutes to bike to school. If Alex needs to be at school by 8:30 a.m., what time should he set his alarm for?
Answer: 8:07 a.m.

(MP.1, MP.6) 2. Marco gets up at 8:10 a.m. It takes him 8 minutes to shower and get dressed, 4 minutes to eat breakfast, 2 minutes to gather his books, and 1 minute to walk to the bus stop. If the bus leaves at 8:32 a.m., for how long can Marco play with his sister before he leaves?
Answer: 7 minutes

3. Clara takes 10 minutes to have breakfast, and 10 minutes to get ready for school. It takes her 5 minutes to walk to school. If she has to be at school by 8:30 a.m., what time does Clara have to get up?

a) Use this number line to solve the problem.

```
0         5         10       15        20        25       30
```

b) Solve the problem again using this number line.

```
0         5         10       15        20        25       30
```

(MP.1) c) Did you get the same answer in parts a) and b)?
(MP.5) d) Which way was faster?
(MP.3, MP.5) e) Ravi tries to use this number line to solve the same problem. What difficulty will he have?

```
0         10        20       30        40        50       60
```

Answers: a) 8:05 a.m., b) 8:05 a.m., c) yes, d) part b), e) there won’t be a marking for when Clara gets to school—it will be halfway between two markings

4. Draw a number line that skip counts to solve the problem. Use as few markings as you can.

a) Nina takes 6 minutes to have breakfast and 8 minutes to get ready for school. It takes her 12 minutes to bike to school. If Nina has to be at school by 8:45 a.m., what time does she have to wake up?

b) Nina takes 6 minutes to get ready for school, 9 minutes to eat breakfast, and 3 minutes to walk to the bus stop. If the bus leaves at 8:30 a.m., what is the latest time Nina can wake up?

Answers: a) Skip count by 2s. She has to wake up by 8:19 a.m., b) Skip count by 3s. She can wake up as late as 8:12 a.m.

(MP.1, MP.4) 5. Sara is at soccer practice. She does dribbling exercises for 10 seconds, then rests for 5 seconds. Sara dribbles four times and rests three times. Does the drill take more or less than 1 minute (60 seconds)?

Answer: less, it takes 55 seconds
6. For a school fundraiser, Bo sold 5 more muffins than Mark. Liz sold 3 fewer muffins than Mark. Sharon sold 4 more muffins than Liz. Bo sold 43 muffins. How many did the four students sell altogether?

**Solution:** Liz sold 35, Mark sold 38, Sharon sold 39, and Bo sold 43, so altogether they sold 35 + 38 + 39 + 43 = 155 muffins

**NOTE:** Problem 7 combines number lines with strategic searching from earlier problem-solving lessons.

**MP.1, MP.4** 7. Ethan has 8 more marbles than Grace. Grace has 5 fewer marbles than Gary. Jennifer has 7 more marbles than Gary. Altogether they have 41 marbles.
   a) How many marbles does each friend have?
      Hint: Decide who has the fewest marbles and go through in order assuming that person has 0 marbles, then 1 marble, then 2 marbles, and so on, until you get a total of 41 marbles.
      b) Check your answer to part a) by adding your answers together. Do you get 41? If not, find your mistake.

**Answers:**
   a) Grace has 4 marbles, Gary has 9 marbles, Ethan has 12 marbles, and Jennifer has 16 marbles; 
   b) 4 + 9 + 12 + 16 = 41

**MP.1, MP.4** 8. A painter’s ladder has 12 steps. The painter spills red paint on the ground, and on every second step. He spills blue paint on the ground and on every third step.
   a) Which steps have red and blue paint on them?
   b) Which steps will have no paint on them?

**Answers:**
   a) 6th and 12th; 
   b) 1st, 5th, 7th, and 11th
Number Lines

1. On Tuesday morning, Jim is camping 20 miles away from the next town. He plans to walk 5 miles each day toward the town.
   How far from the town will he be on Thursday evening? _________

2. Clara is playing baseball in a park 16 miles from her home. She decides to jog home. She can jog 5 miles every hour.
   How far from home will she be after 3 hours? _________

3. Draw a number line on the grid for the problem, then solve the problem. The first number line is started for you.
   a) Ravi is 11 miles from home. He is walking home. He can walk 3 miles every hour. How far from home will he be after 3 hours? _________

   b) May is 12 blocks from home. She can walk 2 blocks in a minute. How long will it take her to walk home? ________________

   c) Ray walks 4 miles in the first hour and 3 miles each hour after that. When he starts walking home, he is 15 miles from home.
   How far from home is he after 3 hours? ________________
Performance Task: Clock Problems

Materials:
BLM Phone Rings (p. Q-80)
BLM Clock Problems (pp. Q-81–82)

Preparation for the performance task. Have students complete BLM Phone Rings, in which they solve a multi-part problem in a contextual setting. This will prepare students for the performance task and allow you to gauge students’ ability to do independent work in a multi-part contextual problem. If students are not ready to do such work, wait until later in the year when they have had a chance to build confidence.

Answers: a) 19, 16, 17; b) Tom’s; c) Mark’s; d) yes

When students are ready to do the performance task, tell students to look at the clock in the classroom, and ASK: What time does it say? Then ask if anyone has a watch that shows a different time. How much are they off by? Draw three clocks on the board, showing 3:21, 3:24, and 3:29. SAY: The correct time is 3:24. Have students point to the correct clock. Then point to the other two, and ASK: Is this clock ahead or behind? By how much? (the clock showing 3:21 is behind by 3 minutes and the clock showing 3:29 is ahead by 5 minutes)

Performance Task: Clock Problems. Provide students with BLM Clock Problems. Question 6 provides an opportunity to apply the problem-solving strategy of using a number line. All students might find an answer to Question 6, but students who notice that the strategy can be used will find the problem easier.

Answers: 1. I circled the clock showing 2:19 because it is 4 minutes ahead of the other clock; 2. a) 2:04, b) 1:09, c) 12:16; 3. a) 3:04, b) 1:11, c) 10:26; 4. 5:25; 5. a) 6, b) 9, c) after 20 years, d) 6 × 7 = 42; 6. 9:27
Phone Rings

I. Tom, Mark, and Kate each have a phone. Each phone has a different ring.

- Tom’s phone’s rings last for 3 seconds each with a 1-second pause between rings. His phone takes a message after 5 rings.
- Mark’s phone’s rings last for 4 seconds each with a 2-second pause between rings. His phone takes a message after 3 rings.
- Kate’s phone’s rings last for 2 seconds each with a 1-second pause between rings. Her phone takes a message after 6 rings.

a) Use a number line to show how long each person’s phone rings for from the beginning of the first ring to the end of the last ring.

Tom: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Tom’s phone takes a message after _______ seconds.

Mark: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Mark’s phone takes a message after _______ seconds.

Kate: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Kate’s phone takes a message after _______ seconds.

b) From the beginning of the first ring to the end of the last ring, whose phone ring lasts the longest? __________________________

c) From the beginning of the first ring to the end of the last ring, whose phone ring lasts the shortest? __________________________

d) When Kate is playing outside, it takes her 15 seconds from the time the first ring starts to get to her phone. Will she answer before the phone takes a message?
Clock Problems (I)

The clock in Mr. B’s classroom is always 4 minutes ahead.

1. One clock below is correct and the other one is the clock in Mr. B’s classroom. Circle the clock that is in Mr. B’s classroom.

Explain your choice.

2. The correct time is given. What time does Mr. B’s classroom clock show?
   a) 2:00  
   b) 1:05  
   c) 12:12

3. Mr. B’s classroom clock shows the time given. What is the correct time?
   a) 3:08  
   b) 1:15  
   c) 10:30

4. Mr. B’s classroom clock is shown below.

What is the correct time? ________
Clock Problems (2)

5. Ms. K buys a clock for her classroom. She sets it to the correct time. After 1 year, it is 3 minutes ahead.
   a) How many minutes ahead will the clock be after 2 years? _____
   b) How many minutes ahead will the clock be after 3 years? _____
   c) When will the clock be 60 minutes ahead? _______________
   d) After two years, the principal says she will bring the class 7 balloons for every minute the clock is ahead. How many balloons will the principal bring?

6. Ron’s clock is 5 minutes ahead of Sara’s clock.
   Bob’s clock is 2 minutes behind Sara’s clock.
   John’s clock is 4 minutes ahead of Bob’s clock.
   What time does John’s clock say when Ron’s clock says 9:30?
PS3-6 Using Number Lines with Two Points Given

Use this lesson after: 3.2 Unit 5


Goals:
Students will estimate values on number lines with two points given and check their estimates.
Students will use number lines to solve problems when given only two points.

Prior Knowledge Required:
Can count by 10s
Knows that numbers on number lines get bigger from left to right and bottom to top
Can compare numbers up to 1,000
Can skip count by 2s, 3s, 4s, 5s, and 10s
Can multiply and divide within 100
Can add and subtract within 1,000
Can multiply one-digit multiples of 10 by one-digit numbers
Can interpret division as a missing factor problem

Vocabulary: estimate, number line

Materials:
BLM Feet (p. Q-92, optional)
BLM Hidden Number Lines Game (p. Q-93), one copy cut into number lines for each pair of students
extra opaque paper, such as colored sheets of 8.5 by 11 inch paper
a small paper clip for each pair of students

Review how grouping tens makes counting easy. Draw 10 pairs of feet on the board or copy BLM Feet five times, and alternate the colors as shown below.

ASK: How many toes are in this group? (30) Verify by counting by 1s, but emphasizing the multiples of 10 as you get to them. ASK: Do I need to count by 1s, or is there an easier way? (you can count by 10s instead) Have students count by tens to find how many toes are in other groups.
Estimating numbers on number lines. Underneath the row of toes, draw a number line with markings from 0 to 100 in multiples of 10, as shown below:

![Number line diagram](image)

SAY: Not all numbers are marked on this line, because it would be hard to write all the numbers from 0 to 100 in this small space. ASK: Where would you estimate 81 is on the number line? (right after the 80) PROMPTS: Which two tens is it between? (80 and 90) Is the number closer to 80 or to 90? (80) Is it a lot closer or a little closer to 80? (a lot closer) SAY: 81 is right next to 80. Then have a volunteer mark an estimate for 81 on the number line. Repeat with various numbers—for example, 41, 92, 22, 59, 38, 99—giving prompts as necessary. Continue with numbers that are halfway between two tens—for example, 35, 75, 45, 95—and then with numbers that are only a little closer to one ten than the other—for example, 74, 86, 37, 13.

Activity

(MP.2) Provide each pair of students with the first third of BLM Hidden Number Lines Game and a small paper clip. The first third of the BLM has a pair of number lines, one with only the endpoints 60 and 80 and the other with all the whole numbers from 60 to 80. Ask students to fold along the dotted line and put something opaque, such as a colored sheet of paper, between the two sides of paper. Ask the pairs to hold the number line vertically between them so that Player 1 sees the side with only 60 and 80 marked and Player 2 sees the side with all the numbers marked. Player 2 calls out a number from 60 to 80. Player 1 puts a small paper clip on the number line by estimating where the number should be. Player 2 tells Player 1 how to adjust the estimate by saying “too high” or “too low.” Play until Player 1’s estimate is accurate enough so that Player 2 can see the number inside the paper clip. Player 2 then shows Player 1 their side of the number line to verify the position. Players switch roles and repeat.

When students are comfortable enough estimating on the 60 to 80 number line to only need small adjustments, give each pair of students the number line from 0 to 100 from the BLM and repeat the activity. Finally, give each pair of students the number line from 0 to 1,000 from the BLM and repeat.

(end of activity)

Review counting by 10s starting from any number. SAY: Mary has four stamps. Draw on the board:

![Four stamps](image)

SAY: Stamps come in strips of 10. ASK: When she adds a strip of 10 stamps, how many stamps will she have? (14) Continue drawing on the board:

![Fourteen stamps](image)
Verify that there are 14 stamps by counting them. Repeat with another strip of 10 stamps (there are now 24 stamps), again counting all the stamps, but this time emphasize the numbers 4, 14, and 24 as you count. Next, as a class, count forward by 10s from 4 to 94 (4, 14, 24, and so on), and then count backward by 10s from 94 to 4 (94, 84, 74, and so on).

**Exercises:** Skip count by 10s, forward or backward.

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<tbody>
<tr>
<td>a) from 3 to 43</td>
<td>b) from 37 to 87</td>
<td>c) from 59 to 99</td>
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<tr>
<td>d) from 74 to 44</td>
<td>e) from 69 to 39</td>
<td>f) from 81 to 31</td>
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**Answers:**

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<tr>
<td>a) 3, 13, 23, 33, 43</td>
<td>b) 37, 47, 57, 67, 77, 87</td>
<td>c) 59, 69, 79, 89, 99</td>
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<td>d) 74, 64, 54, 44</td>
<td>e) 69, 59, 49, 39</td>
<td>f) 81, 71, 61, 51, 41, 31</td>
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Remind students that when you skip count forward by 10s, the ones digit stays the same and the number of tens gets bigger. ASK: How many tens are in 134? (13) PROMPT: If you were to make 134 using tens and ones blocks, and you didn’t have any hundreds blocks, how many tens blocks would you need? (13) SAY: You would need 10 tens blocks for the 1 hundred and 3 tens blocks for the 3 tens, so you would need 13 altogether, plus you would have 4 ones blocks. Draw on the board:

![Diagram](10 tens + 3 tens = 13 tens)

SAY: You can get the number of tens in any number by covering up the ones digit. Write on the board:

\[
134 \quad 184
\]

Demonstrate covering up the 4 in 134 to show 13, the number of tens in 134. Ask a volunteer to cover up the 4 in 184 to show the number of tens. (18 tens) SAY: Let’s count by 10s starting from 184. ASK: What comes next? (194) Write “194” on the board beside 184. ASK: What comes next? (204) Write “204” on the board. SAY: 184 has 18 tens, 194 has 19 tens, and 204 has 20 tens. You can keep going by keeping the ones digit the same and adding one more ten each time. Continue writing on the board:

\[
184, \quad 194, \quad 204, \quad 214, \quad 224, \ldots
\]

**Exercises:**

1. Skip count by 10s.

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<tr>
<td>a) from 64 to 114</td>
<td>b) from 79 to 139</td>
<td>c) from 455 to 515</td>
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<tr>
<td>d) from 356 to 316</td>
<td>e) from 819 to 769</td>
<td>f) from 405 to 385</td>
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**Answers:**

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<tbody>
<tr>
<td>a) 64, 74, 84, 94, 104, 114</td>
<td>b) 79, 89, 99, 109, 119, 129, 139</td>
<td>c) 455, 465, 475, 485, 495, 505, 515</td>
</tr>
<tr>
<td>d) 356, 346, 336, 326, 316</td>
<td>e) 819, 809, 799, 789, 779, 769</td>
<td>f) 405, 395, 385</td>
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2. What number did Blanca end at?
   a) She skip counted forward by 10s three times starting at 43.
   b) She skip counted forward by 10s four times starting at 645.
   c) She skip counted forward by 10s five times starting at 87.
   d) She skip counted forward by 10s four times starting at 296.
   Bonus: She skip counted backward by 10s three times from 126.
   Answers: a) 73, b) 685, c) 137, d) 336, Bonus: 96

SAY: Peter skip counted forward by 10s three times and finished at 56. I want to know what number he started at. Draw on the board:

```
  56
```

ASK: How did I know where to write 56? (Peter finished at 56) How did I know to put three markings on the number line before 56? (Peter skip counted 3 times) Point to the third mark and ASK: Did he start here? (no) What number goes here? (46) Write “46” under the mark. Repeat for the next mark (no, 36), then the mark after that (yes, 26). SAY: So Peter started at 26.

**Exercises:** What number did Peter start at?
   a) He skip counted forward by 10s four times and ended at 83.
   b) He skip counted forward by 10s five times and ended at 72.
   c) He skip counted forward by 3s four times and ended at 30.
   d) He skip counted forward by 5s six times and ended at 42.
   Answers: a) 43, b) 22, c) 18, d) 12

**Determining the number of jumps.** Write on the board:

```
Sharon skip counted by 5s from 35 to 50. How many times did she count?
```

Read the problem aloud. SAY: You can solve this problem by drawing a number line that starts at 35, then skip counting by 5s. Draw on the board:

```
  35
```

Have a volunteer mark the numbers by counting by 5s from 35. (40, 45, 50, 55, 60) SAY: The problem says Sharon skip counted to 50. Circle “50” on the number line. ASK: So how many times did she skip count? (3 times)

**Exercises:** How many times did Jayden skip count?
   a) He skip counted by 6s from 30 to 72.
   b) He skip counted by 3s from 16 to 40.
   Bonus: He skip counted by 8s from 133 to 157.
   Answers: a) 7 times, b) 8 times, c) 3 times
Another way to determine the number of jumps. Refer back to the skip counting from 35 to 50 on the board. SAY: Sharon skip counted from 35 to 50. ASK: How much more than 35 is 50? (15) How did you get that? (subtracted 50 − 35) SAY: Sharon skip counted by 5s each time. ASK: How many jumps of five did she need to make 15? (3) SAY: Three jumps of five each make 15. Write on the board:

\[ \_ \times 5 = 15 \]

SAY: You’re really trying to find the missing number in a multiplication sentence. ASK: What could you do instead of multiplication? (division) Write on the board:

\[ 15 \div 5 = \_ \]

SAY: Remember that these two questions have the same answer: a number multiplied by five to equal 15 is the same number that is 15 divided by five.

Exercises:
(MP.1, MP.7) a) Complete the previous exercises by dividing. Make sure you get the same answer both ways.
(MP.5) b) Which way of answering is faster? Explain why it is faster.

Answers: b) Dividing is faster, because you just have to do one thing. Skip counting several times takes more time.

Determining the interval skip counted. Write on the board:

Clara skip counted by the same number three times.
She started at 7 and ended at 25. What number did she skip count by?

Read the problem aloud. SAY: You can solve this problem by drawing a number line. Draw on the board:

\[
\begin{array}{c}
\text{7} \\
\mid \mid \mid \mid \\
\text{25}
\end{array}
\]

(MP.3) SAY: Clara started at seven, so I put seven at the beginning of the number line. ASK: How many jumps should I draw? (3) How do you know? (Clara skip counted 3 times) Draw three tick marks on the number line, and ASK: What number did she end at? (25) Write “25” at the end, as shown below:

\[
\begin{array}{c}
\text{7} \\
\mid \mid \mid \mid \mid \\
\text{25}
\end{array}
\]

(MP.1) SAY: We need to figure out what Clara skip counted by. Have students make predictions. Then ASK: There are a lot of possibilities—how can we try them in an organized way? (try the numbers in order, starting at 1) SAY: Let’s try skip counting by the numbers in order, starting with 1s.
Continue writing on the board:

7 8 9 25

ASK: Do I reach 25 by counting by 1s? (no) SAY: 25 isn’t one more than nine, so this doesn’t work. Have volunteers try skip counting on the number line by 2s (7, 9, 11, 25; doesn’t work), 3s (7, 10, 13, 25; doesn’t work), 4s (7, 11, 15, 25; doesn’t work), 5s (7, 12, 17, 25; doesn’t work), and 6s (7, 13, 19, 25; does work). SAY: So, Clara skip counted by 6s three times, from seven to 25. Leave the number line on the board for later use.

(MP.1) Exercises: John skip counted by the same number three times. What number did he skip count by?
   a) from 13 to 25
   b) from 456 to 462
   c) from 394 to 409
   d) back from 84 to 75

   Answers: a) 4, b) 2, c) 5, d) 3

Another way to solve the same problem. Refer back to the example on the board and
ASK: How many jumps did Clara have to do? (3) How do you know? (the problem says so) How far do we have to jump in total? (18) PROMPT: How far is it from 7 to 25? (18) How do you know? (25 is 18 more than 7, 25 − 7 is 18) SAY: So you need to go 18 units in 3 equal jumps.
ASK: How big should each jump be? (6) How did you get that? (18 ÷ 3 = 6) Ask a volunteer to check that the jumps are 6 units each. (see completed number line below)

SAY: If you know that three jumps make 18, then each jump is 6.

Exercises:
(MP.1, MP.7) 1. a) Complete the previous exercises using this new way. Make sure you get the same answer both ways.
(MP.5) b) Which way is faster? Explain why it is faster.
Answer: b) This new way is faster, because I only had to do one subtraction and one division instead of trying all the numbers in order.
(MP.1, MP.2, MP.7) 2. Complete the skip counting on the number line.

a) 42, 48, 54, 60, 66, 72
b) 42, 52, 62, 72
c) 16, 19, 22, 25, 28, 31, 34, 37, 40
d) 16, 20, 24, 28, 32, 36, 40
e) 16, 22, 28, 34, 40
f) 16, 28, 40

Answers: a) 42, 48, 54, 60, 66, 72; b) 42, 52, 62, 72; c) 16, 19, 22, 25, 28, 31, 34, 37, 40; d) 16, 20, 24, 28, 32, 36, 40; e) 16, 22, 28, 34, 40; f) 16, 28, 40

Skip counting by multiples of 10. Write on the board:

3 × 40 = ______

Have a volunteer fill in the blank. (120) Have another volunteer explain the answer. (3 × 4 is 12, so 3 × 40 is 120) Then have a volunteer write two division equations from the multiplication equation, as shown below:

120 ÷ 3 = 40
120 ÷ 40 = 3

Exercises:

1. Write two division equations from the multiplication equation.
   a) 4 × 60 = 240   b) 70 × 9 = 630   c) 80 × 2 = 160
   Answers: a) 240 ÷ 60 = 4, 240 ÷ 4 = 60; b) 630 ÷ 9 = 70, 630 ÷ 70 = 9; c) 160 ÷ 2 = 80, 160 ÷ 80 = 2

2. Divide by finding the missing number in a multiplication equation.
   a) 60 ÷ 2   b) 80 ÷ 4   c) 60 ÷ 30   d) 80 ÷ 20
   e) 720 ÷ 8   f) 360 ÷ 90   g) 210 ÷ 7   h) 420 ÷ 60
   Answers: a) 30, b) 20, c) 2, d) 4, e) 90, f) 4, g) 30, h) 7

3. Complete the skip counting.
   a) 18, 68, 118, 168, 218
   b) 428, 458, 488, 518, 548, 578
   Answers: a) 18, 68, 118, 168, 218; b) 428, 458, 488, 518, 548, 578
Problem Bank

1. Show your answer to the question on a number line.
   a) Sal has $140. He needs to buy a $200 winter jacket before school starts in 3 weeks. How much money does Sal need to save each week for the 3 weeks?
   b) Kim has $73. She wants to buy a $97 gift for her mother’s birthday in 4 weeks. How much money does Kim need to save each week for the 4 weeks?
   c) Anna has $68. She wants to buy a sweater for $100. She saves $4 a week. How many weeks does Anna need to save money for?
   d) A test has 7 questions. The test starts at 9:00 and will end at 9:30. Josh takes 2 minutes to read all the questions before starting to answer the questions. How much time does Josh have to answer each question?
   e) On a worksheet full of simple questions, Lily takes 5 seconds to do each question. How many questions can she do in 1 minute? Remember: 1 minute = 60 seconds.
   f) A television show ended at 9:25. The television channel then played the same commercial 4 times. The next show started at 9:33. How long was the commercial?
   g) Abdul started his homework at 7:15. It took him 10 minutes for each subject’s homework. He finished at 7:55. How many subjects did Abdul work on?
   h) Gym class starts at 2:05. The class will do several 10-minute activities, one after the other. If the class ends at 2:45, how many activities can the class do?
   i) Science class starts at 3:05. The class will do 5 different activities. Each activity is done for the same amount of time. If class ends at 3:55, how long does the class do each activity?

Selected solution: f) The commercial was 2 minutes long.

Answers: a) $20, b) $6, c) 8 weeks, d) 4 minutes, e) 12, g) 4, h) 4, i) 10 minutes

2. Which of the parts from Problem 1 require finding the number of jumps? Which require finding the length of each jump?

Answers: the number of jumps: c), e), g), h); the length of each jump: a), b), d), f), i)

3. A test has eight questions. Sally takes 5 minutes each for the first three questions and then 8 minutes each for the next five questions. Can Sally finish the test in 1 hour (60 minutes)?

Answer: yes, she finishes in 55 minutes

4. Draw a number line to answer the question.
   a) 3 × 2 = _____  b) 4 × 3 = _____  c) 2 × 7 = _____

Answers: a) 6, b) 12, c) 14
5. Write a multiplication equation from the skip counting.
   a) 0 4 8 12 16 20
   b) 0 3 6 9 12 15 18
   c) 0 40 80 120 160 200 240
   **Answers:** a) $5 \times 4 = 20$, b) $6 \times 3 = 18$, c) $6 \times 40 = 240$

6. Start a number line at 20 and use skip counting to find the answer.
   a) $20 + (5 \times 3) = _____$  
   b) $20 + (3 \times 4) = _____$  
   c) $20 + (2 \times 10) = _____$
   **Answers:** a) skip count from 20 to: 23, 26, 29, 32, 35, so 35; b) skip count from 20 to: 24, 28, 32, so 32; c) skip count from 20 to: 30, 40

**Bonus:** Write an addition and multiplication equation from the number line.
   a) 14 17 20 23 26 29
   b) 14 16 18 20 22 24 26
   c) 14 44 74 104 134 164 194
   **Answers:** a) $14 + (5 \times 3) = 29$, b) $14 + (6 \times 2) = 26$, c) $14 + (6 \times 30) = 194$
Feet
Hidden Number Lines Game

Fold along the dotted line

Fold along the dotted line

Fold along the dotted line

Fold along the dotted line
Creating Number Lines

Use this lesson after: 3.2 Unit 5

Standards: 3.OA.B.5, 3.OA.C.7, 3.NBT.A.2, 3.NBT.A.3

Goals:
Students will use an empty number line to add and subtract two- and three-digit numbers.

Prior Knowledge Required:
Can add and subtract within 1,000
Can draw number lines for skip counting
Can add, subtract, and multiply one-digit numbers using a number line
Can multiply one-digit multiples of ten by one-digit numbers

Vocabulary: empty number line, number line

Materials:
BLM Feet (p. Q-102, optional)

Counting tens, then ones. Draw 10 pairs of feet on the board or copy BLM Feet five times, and alternate the colors as shown below. Starting from the left, ask students to say how many toes are in each group. Use groups that are slightly more than multiples of 10, as shown by the bracket below:

Have a volunteer fill in the blank. (43) Point out that students can count by 10s first and then count by 1s. By counting by as many 10s as possible, students are using as small a number of jumps as they can. Continue with more numbers, such as 31, 52, 61, 73, 33.

Exercises: Count by 10s and then by 1s. Use the fewest number of jumps.
a) from 0 to 32  b) from 0 to 61  c) from 0 to 54
Answers: a) 0, 10, 20, 30, 31, 32; b) 0, 10, 20, 30, 40, 50, 60, 61; c) 0, 10, 20, 30, 40, 50, 51, 52, 53, 54

Sketching number lines. Show students how they can keep track of their counting by using a number line. Write on the board:
SAY: This isn’t a precise number line with, for example, numbers the same distance apart. On this line, 10 and 20 are closer together than 20 and 30, even though they are both 10 apart. But as a sketch of a number line, it is good enough to help us keep track of how we counted.

Exercises:
1. Sketch a number line to show counting by 10s and 1s.
   a) from 0 to 41  
   b) from 0 to 62  
   c) from 0 to 71
   
   **Answers:** a) 0, 10, 20, 30, 40, 41; b) 0, 10, 20, 30, 40, 50, 60, 61, 62; c) 0, 10, 20, 30, 40, 50, 60, 70, 71

2. Sketch a number line to show counting by 100s, 10s, and 1s.
   a) from 0 to 341  
   b) from 0 to 152  
   c) from 0 to 333
   
   **Selected solution:** a) 
   
   **Answers:** b) 0, 100, 110, 120, 130, 140, 150, 151, 152; c) 0, 100, 200, 300, 310, 320, 330, 331, 332, 333

**Counting tens and then counting ones backward.** Write on the board:

Count from 0 to 39. Use only jumps of 10 or 1. Use only 5 jumps.

SAY: Here is a problem I want you to solve. Read the problem aloud. Wait until several students think they have the answer, then SAY: You are allowed to use backward jumps. After allowing students another minute to work, have a volunteer show the answer on the board, as shown below:

Exercises:
1. Use only 6 jumps. Use jumps of 1 or 10, forward or backward.
   a) Count from 0 to 51.  
   b) Count from 0 to 38.  
   c) Count from 0 to 42.
   d) Count from 0 to 49.  
   e) Count from 0 to 27.  
   f) Count from 0 to 33.
   
   **Answers:** a) 0, 10, 20, 30, 40, 50, 51; b) 0, 10, 20, 30, 40, 39, 38; c) 0, 10, 20, 30, 40, 41, 42; d) 0, 10, 20, 30, 40, 49; e) 0, 10, 20, 29, 28, 27; f) 0, 10, 20, 31, 32, 33

2. Use only 6 jumps. Use jumps of 1, 10, or 100, forward or backward.
   a) Count from 0 to 312  
   b) Count from 0 to 114.  
   c) Count from 0 to 499.
   d) Count from 0 to 391  
   e) Count from 0 to 409
   
   **Bonus:** Use jumps of 1, 10, 100, or 1,000 to count from 0 to 2,909.
   
   **Answers:** a) 0, 100, 200, 300, 310, 311, 312; b) 0, 100, 110, 111, 112, 113, 114; c) 0, 100, 200, 300, 400, 500, 499; d) 0, 100, 200, 300, 400, 390, 391; e) 0, 100, 200, 300, 400, 410, 409; 
   
   **Bonus:** 0, 1,000, 2,000, 3,000, 2,900, 2,910, 2,909
Review adding on a number line. Remind students that they can add using a number line. Draw on the board:

\[ 2 + 3 = \underline{\phantom{0}} \]

\[ \begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8
\end{array} \]

ASK: How would you show adding two plus three on the number line? (start at 2 and move 3 places right) Demonstrate doing so.

Using a sketched number line, count on by 10s and 1s to add. Draw on the board:

\[ 38 + 54 = \underline{\phantom{0}} \]

\[ \begin{array}{cccccccc}
38 & 48 & 58 & 68 & 78 & 88
\end{array} \]

SAY: It is a lot of work to draw a number line with 54 markings, so I’m just going to show the markings that I need to solve the problem. I mark 38 because I know that’s my starting number. Instead of counting by 1s, I want to count by a bigger number so that I have less work to do. ASK: What’s an easy number to skip count by? (10) If students say 2 or 5, SAY: Those are easy numbers to count by, but it would still be a lot of work. PROMPT: What’s an easy number to skip count by that would be less work? (10) SAY: I’m going to start by adding 50. ASK: If I want to add 50 to 38, how many times do I have to add 10? (5) Show this on the sketched number line:

\[ \begin{array}{cccccccc}
38 & 48 & 58 & 68 & 78 & 88
\end{array} \]

SAY: I drew five markings because I want to add 10 five times. I’m not worried about making the jumps exactly equally spaced, because I don’t need them to be equally spaced to answer the question. Have a volunteer label the tick marks on the number line, as shown below:
ASK: What is $38 + 50$? (88) Now that I know $38 + 50$, what do I need to do to get $38 + 54$? (add 4 more) Add four jumps of 1 on the number line, but closer together than the tens, and have a volunteer label the tick marks, as shown below:

```
38          48          58       68            78             88   89   90   91   92
```

ASK: What is $38 + 54$? (92) Write “92” in the blank. SAY: Using a number line like this, where you have to keep the numbers in order, but you don’t have to make them the same distance apart, is sometimes called using an empty number line because it starts with no numbers; we could also call the number line a blank number line.

**Exercises:** Starting with an empty number line, show how to add.

- a) $23 + 40$
- b) $31 + 63$
- c) $85 + 46$

**Bonus:**
- d) $128 + 30$
- e) $128 + 314$

**Answers:**
- a) $23, 33, 43, 53, 63$;
- b) $31, 41, 51, 61, 71, 81, 91, 92, 93, 94$;
- c) $85, 95, 105, 115, 125, 126, 127, 128, 129, 130, 131$;
- d) $128, 138, 148, 158$;
- e) $128, 228, 328, 428, 438, 439, 440, 441, 442$

**Using backward jumps to add on a number line.** Write on the board:

```
27 + 39
```

SAY: I want to solve this problem by sketching a number line. Draw a blank horizontal line on the board, and have a volunteer write the number to start with, as shown below:

```
27
```

SAY: Here is a problem. ASK: How can you add 39 using only five jumps of tens and ones? (add 4 tens and move back 1 one) Have a volunteer show it on the sketched number line, as shown below:

```
27         37        47        57   66 67
```

Circle “66.” SAY: Adding 39 is the same as adding 40 and then subtracting one. Write on the board:

```
27 + 40 = 67, so 27 + 39 = 66
```

Leave this statement on the board.
(MP.1, MP.5, MP.7) Exercises: Add using as few jumps as you can. Use only jumps of 10 or 1.

a) 16 + 48  
   b) 38 + 51  
   c) 17 + 48

Answers: a) 16, 26, 36, 46, 56, 66, 65, 64; b) 38, 48, 58, 68, 78, 88, 89; c) 17, 27, 37, 47, 57, 67, 66, 65

(MP.8) SAY: You can add even faster by doing all the tens together and all the ones together. If you can add 40, you can add all the tens together in one jump. Draw on the board:

---

27 40 66 67

---

Exercises: Add 40.

a) 37 + 40     b) 16 + 40     c) 28 + 40     d) 54 + 40

Answers: a) 77, b) 56, c) 68, d) 94

Point to “27 + 40 = 67, so 27 + 39 = 66” on the board from earlier and SAY: If you can add 40, then you can add 39, because 39 is one less than 40.

NOTE: In the exercises below, some students might see how to answer a) and c) immediately by using 27 + 39 = 66.

Exercises: Add 39.

a) 37 + 39     b) 16 + 39     c) 28 + 39     d) 54 + 39

Answers: a) 76, b) 55, c) 67, d) 93

Write on the board:

54 + 28

ASK: What number is close to 28 but easier to add? (30) SAY: So start by adding 30 on a number line. Draw on the board:

---

54 30

---

ASK: What is 54 + 30? (84) Write the sum on the number line. ASK: We added 30 instead of 28 so how do we adjust the answer? (subtract 2) Have a volunteer show that on the number line, as shown below:

---

54 82 84
(MP.1, MP.7) Exercises: Use a small number of easy jumps, forward or backward. Show your jumps by sketching a number line.

a) $57 + 24$  

b) $36 + 48$  

**Bonus:** $143 + 288$

**Answers:** a) 57, 77, 81; b) 36, 86, 84; Bonus: 143, 443, 433, 431

**Counting on from numbers other than zero to subtract.** Write on the board:

$$61 - 39$$

**SAY:** You can use jumps on a number line to see how far apart 39 and 61 are. That tells you the answer to $61 - 39$.

**Exercises:** Use jumps of 10 and 1 to subtract.

a) $61 - 39$

b) $93 - 65$

**Bonus:** Use jumps of 100, 10, and 1 to subtract $892 - 456$.

**Selected solution:**

b) 

| 65 | 75 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 |

There are two jumps of size 10, and eight jumps of size 1, so $93 - 65 = 28$.

**Answers:** a) 22, Bonus: 436

**Problem Bank**

(MP.1) 1. Subtract $83 - 46$ two ways using an empty number line. Make sure you get the same answer both ways.

a) Count up from 46 until you reach 83.

b) Count back 46 from 83.

**Answers:** a) 46, 56, 66, 76, 80, 83, so $83 - 46$ is $10 + 10 + 10 + 4 + 3 = 37$; b) 83, 73, 63, 53, 43, 42, 41, 40, 39, 38, 37, so $83 - 46 = 37$

(MP.2) 2. Without drawing a number line, picture the number line in your head to do these questions. Hint: Do the additions in order.

a) $26 + 50$, $26 + 52$, $26 + 49$

b) $148 + 70$, $148 + 270$, $148 + 273$, $148 + 268$

c) $76 - 40$, $76 - 39$, $76 - 43$

d) $95 - 35$, $93 - 35$, $193 - 35$

**Answers:** a) 76, 78, 75; b) 218, 418, 421, 416; c) 36, 37, 33; d) 60, 58, 158

(MP.3, MP.5) 3. Use counting forward to subtract, then use counting back to subtract. Which way was easier? Explain why it was easier.

a) $132 - 118$  
b) $76 - 4$  
c) $283 - 30$  
d) $283 - 260$

e) $100 - 98$  
f) $100 - 12$  
g) $1,000 - 372$
Answers: a) 14, counting up; b) 72, counting back; c) 253, counting back; d) 23, counting up; e) 2, counting up; f) 88, counting back; g) 628, equally easy, maybe counting up is slightly easier; in a) to f), numbers that are close together are easier by counting up and when the number being subtracted is small, it is easier to count back; in g) they are both about equal amounts of work, but some students may still find counting forward easier

4. Milly adds 57 + 28 by adding the tens first.

a) Add 57 + 28 by adding 3 ones first, then adding the rest. Make sure you get the same answer.
b) Add 57 + 28 by adding all the ones first, then all the tens. Make sure you get the same answer.
c) Add 57 + 28 by adding 30, then subtracting 2. Make sure you get the same answer.

5. Add 28 + 66 using a number line in two different ways.
Sample answers: 28, 88, 89, 90, 91, 92, 93, 94; or 28, 98, 94; or 28, 88, 94; or 28, 30, 90, 94; or 28, 30, 94

6. Add 544 + 388 using a number line in as many ways as you can.
Sample answers: 544, 944, 934, 932; or 544, 550, 600, 900, 904, 924, 930, 932
Feet
PS3-8  Using Structure II

Use this lesson after: 3.2 Unit 5

Standards: 3.OA.B.5, 3.OA.B.6, 3.OA.C.7, 3.OA.D.9, 3.NBT.A.3

Goals:
Students will understand and apply the distributive property to multiplication and division using pictures.

Prior Knowledge Required:
Can multiply by 10
Can apply the commutative property of multiplication
Knows 1, 2, 3, 4, and 5 times tables up to 9 × 5
Can multiply one-digit numbers by 10
Can interpret products in terms of equal groups
Can decompose numbers up to 10 as a sum of smaller numbers, in all possible ways
Can add and subtract two-digit numbers
Can skip count by 10, 100, and 1,000
Can sketch a number line to solve addition and subtraction problems
Understands division as a missing factor problem

Vocabulary: divide, division, group, multiplication, multiply, product, sum

Materials:
chalk in two different colours

NOTE: The following lesson does not involve time, but makes use of number lines so must be used after problem-solving lessons involving number lines.

Review decomposing groups of numbers. Draw on the board:

\[ \begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array} \]

SAY: This is five groups of three. Then draw a vertical line between the first group and the others, as shown below:

\[ \begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array} \] | \[ \begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array} \]
SAY: Just like you can write five as one and four, you can write five groups of three as one group of three and four groups of three. Write on the board (use a different color for the bolded numbers):

\[ 5 = 1 + 4, \text{ so } 5 \text{ groups of } 3 = 1 \text{ group of } 3 + 4 \text{ groups of } 3 \]

**MP.1** Exercises: Use the picture to complete the sentence.

a) 

\[
\text{_____ groups of _____} = \text{ _____ groups of _____} + \text{ _____ groups of _____}
\]

b) 

\[
\text{_____ groups of _____} = \text{ _____ groups of _____} + \text{ _____ groups of _____}
\]

**Bonus:**

\[
\text{_____ groups of _____} = \text{ _____ groups of _____} + \text{ _____ groups of _____}
\]

**Answers:** a) 5, 3, 2, 3, 3; b) 8, 4, 5, 4, 3, 3; Bonus: 6, 124, 4, 124, 2, 124

**Review the distributive property of multiplication.** Remind students that “5 groups of 3” can be written as the product “5 × 3,” so when you add groups of 3, you’re really adding products. Write on the board:

\[
1 + 4 = 5 \\
(1 \times 3) + (4 \times 3) = 5 \times 3
\]

SAY: Instead of using groups, you can use repeated addition to show the same thing. Write on the board:

\[
\underline{\begin{array}{c}
5 \times 3 \\
3 + 3 + 3 + 3 \\
1 \times 3 + 4 \times 3
\end{array}}
\]

SAY: Five threes is the same as 1 three plus 4 more threes.
Exercises: Write $9 \times 8$ as a sum of smaller products.

a) $9 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + 8$
   \[ = \underline{______} + \underline{______} \]

b) $9 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8}$
   \[ = \underline{______} + \underline{______} \]

c) $9 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8 + 8 + 8}$
   \[ = \underline{______} + \underline{______} \]

d) $9 \times 8 = \underline{8 + 8 + 8 + 8 + 8} + \underline{8 + 8 + 8 + 8 + 8}$
   \[ = \underline{______} + \underline{______} \]

Answers: a) $8 \times 8$, $1 \times 8$; b) $7 \times 8$, $2 \times 8$; c) $6 \times 8$, $3 \times 8$; d) $5 \times 8$, $4 \times 8$

Write on the board:

The 8 times table up to $5 \times 8$ is:

\[
\begin{array}{cccc}
8 & 16 & 24 & 32 \\
\end{array}
\]

SAY: Look at your answers to the exercises. ASK: How can you use the 8 times table up to $5 \times 8$ to find $9 \times 8$? (add $5 \times 8 + 4 \times 8$) PROMPT: If you have 5 eights, how many more eights do you need to get 9 eights? (4 more) Circle “32” and “40” and continue writing on the board:

\[
\begin{array}{cccc}
8 & 16 & 24 & 32 & 40 \\
\end{array}
\]

9 \times 8 = 40 + 32 = _____

Point to the equation and the circled numbers as you SAY: Nine eights is 5 eights (point to 40) plus 4 more eights (point to 32). ASK: So what is $9 \times 8$? (72) Write “72” in the blank. SAY: So, knowing the 8 times table up to $5 \times 8$ is enough to know the times table up to $9 \times 8$.

Exercises:
1. The 2 times table up to $5 \times 2$ is: 2, 4, 6, 8, 10. Fill in the blanks.
   a) $8 \times 2 = (5 \times 2) + (\underline{____} \times 2)$
      \[ = 10 + \underline{____} \]
      \[ = \underline{____} \]
   b) $7 \times 2 = (5 \times 2) + (\underline{____} \times 2)$
      \[ = 10 + \underline{____} \]
      \[ = \underline{____} \]
   c) $9 \times 2 = (5 \times 2) + (\underline{____} \times 2)$
      \[ = 10 + \underline{____} \]
      \[ = \underline{____} \]

Answers: a) 3, 6, 16; b) 2, 4, 14; c) 4, 8, 18
2. The 6 times table up to $5 \times 6$ is: 6, 12, 18, 24, 30. Fill in the blanks.

a) $9 \times 6 = (5 \times 6) + (____ \times 6)$
   = ____ + ____
   = ____

b) $6 \times 6 = (5 \times 6) + (____ \times 6)$
   = ____ + ____
   = ____

c) $8 \times 6 = (5 \times 6) + (____ \times 6)$
   = ____ + ____
   = ____

d) $7 \times 6 = (5 \times 6) + (____ \times 6)$
   = ____ + ____
   = ____

**Answers:** a) 4, 30, 24, 54; b) 1, 30, 6, 36; c) 3, 30, 18, 48; d) 2, 30, 12, 42

SAY: As long as you have a times table up to five times the number memorized, you can know the times table up to nine times the number.

**Showing the distributive property on a number line.** SAY: You can show this by sketching it on a number line too. Draw on the board:

\[0\quad 5 \times 6\quad +\quad ____ \times 6 = 7 \times 6\]

ASK: If you have 5 sixes, how many more sixes do you need to make 7 sixes? (2) Write “2” in the bottom blank. ASK: What is $2 \times 6$? (12) Write “12” in the blank above $2 \times 6$ on the number line. ASK: So what is $7 \times 6$? (42) Write “42” in the last blank. SAY: $7 \times 6$ is 12 more than 30, so that’s 42.

**Exercises:**
1. Use the number line to multiply.
   a) \[0\quad 5 \times 6\quad +\quad ____ \times 6 = 8 \times 6\]
   b) \[0\quad 5 \times 8\quad +\quad ____ \times 8 = 9 \times 8\]
   c) \[0\quad 5 \times 8\quad +\quad ____ \times 8 = 6 \times 8\]
   d) \[0\quad 5 \times 7\quad +\quad ____ \times 7 = 8 \times 7\]
**Answers:** a) 18, 48, 3; b) 32, 72, 4; c) 8, 48, 1; d) 21, 56, 3

2. Which two answers in Exercise 1 are the same? Why does this make sense?

**Answer:** Parts a) and c) have the same answer. This makes sense because we are multiplying the same numbers, just in a different order.

**Introduce the distributive property for division.** SAY: Because multiplication and division are related, you can use the same idea to make dividing easier too. Write on the board:

\[16 \div 2\]

SAY: I want to solve 16 ÷ 2, so I need to find out how many twos I need to make 16. Draw on the board:

\[
\begin{array}{cccc}
10 & & & \\
0 & 5 \times 2 & + & ____ \times 2 = _____ \times 2 \\
\end{array}
\]

SAY: I already made 10 from 5 twos. ASK: 16 is how much more than 10? (6) Write “6” in the top blank. ASK: How many twos are in 6? (3) Write “3” in the blank below the 6. SAY: We needed 5 twos for the 10 and 3 twos for the 6. Circle the 5 and 3 in 5 × 2 and 3 × 2. ASK: How many twos do we need altogether for the 16? (8) Write “8” in the last blank. Write on the board:

\[8 \times 2 = 16, \text{ so } 16 \div 2 = 8\]

**Exercises:**

1. Fill in the blanks, then divide.

a) \[
\begin{array}{cccc}
10 & & & \\
0 & 5 \times 2 & + & ____ \times 2 = _____ \times 2 \\
\end{array}
\]

So 18 ÷ 2 = ______

b) \[
\begin{array}{cccc}
30 & & & \\
0 & 5 \times 6 & + & ____ \times 6 = _____ \times 6 \\
\end{array}
\]

So 48 ÷ 6 = ______

c) \[
\begin{array}{cccc}
30 & & & \\
0 & 5 \times 6 & + & ____ \times 6 = _____ \times 6 \\
\end{array}
\]

So 54 ÷ 6 = ______

**Answers:** a) 8, 4, 9, 9; b) 18, 3, 8, 8; c) 24, 4, 9, 9

2. Draw a number line to divide.

a) 56 ÷ 8 \hspace{2cm} b) 72 ÷ 8 \hspace{2cm} c) 42 ÷ 6 \hspace{2cm} d) 42 ÷ 7
Answers: a) 7, b) 9, c) 7, d) 6

3. The 7 times table up to $5 \times 7$ is: 7, 14, 21, 28, 35.
   a) Write 56 as the sum of two smaller multiples of 7, in two different ways.
   b) Divide 56 ÷ 7 using the sums from part a). Make sure you get the same answer both times.
   Sample answers: a) $56 = 21 + 35$ or $56 = 28 + 28$; b) $56 ÷ 7 = 3 + 5 = 8$ or $56 ÷ 7 = 4 + 4 = 8$

Problem Bank
1. Cathy knows that $5 \times 8 = 40$. She calculates $9 \times 8$ by skip counting by 8s past 40:

   40,  48,   56,   64,   72
   5 \times 8   6 \times 8   7 \times 8   8 \times 8   9 \times 8

   Use Cathy’s method to multiply.
   a) Using $5 \times 6 = 30$, skip count to find $6 \times 6$, $7 \times 6$, $8 \times 6$, and $9 \times 6$.
   b) Using $5 \times 7 = 35$, skip count to find $6 \times 7$, $7 \times 7$, $8 \times 7$, and $9 \times 7$.
   c) Using $5 \times 9 = 45$, skip count to find $6 \times 9$, $7 \times 9$, $8 \times 9$, and $9 \times 9$.
   d) Using $5 \times 12 = 60$, skip count to find $6 \times 12$, $7 \times 12$, $8 \times 12$, and $9 \times 12$.
   e) Using $5 \times 21 = 105$, skip count to find $6 \times 21$, $7 \times 21$, $8 \times 21$, and $9 \times 21$.
   Answers: a) 36, 42, 48, 54; b) 42, 49, 56, 63; c) 54, 63, 72, 81; d) 72, 84, 96, 108; e) 126, 147, 168, 189

2. Multiply by 10 by skip counting. Show your skip counting.
   a) $3 \times 10 = ____$  
      skip count: ____, ____, ____
   b) $4 \times 10 = ____$  
      skip count: ____, ____, ____, ____
   c) $7 \times 10 = ____$  
      skip count: ____, ____, ____, ____, ____, ____, ____
   Answers: a) 30, b) 40, c) 70

(MP.7) 3. a) If $8 \times 10$ is 80, what is $10 \times 8$? Explain.
   b) Use $10 \times 8$ and skip counting to find $12 \times 8$.
   c) Use $10 \times 9$ and skip counting to find $13 \times 9$.
   d) Use $10 \times 11 = 110$ and skip counting to find $11 \times 11$.
   Answers: a) 80, because the same two numbers are multiplied; b) 96; c) 117; d) 121

(MP.7) 4. Use 84 = 70 + 14 to divide 84 ÷ 7.
   Answer: 12

(MP.7) 5. a) How does $9 \times 7$ compare to $10 \times 7$? Use the picture to explain.

   $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$

   b) Use $10 \times 7 = 70$ to calculate $9 \times 7$.
   c) Use $10 \times 8 = 80$ to calculate $9 \times 8$.
   Answers: a) $9 \times 7$ is 7 less than $10 \times 7$, b) $70 - 7 = 63$, c) $80 - 8 = 72$
6. a) Use $10 \times 6 = 60$ to calculate $9 \times 6$.
   
   b) Use $5 \times 6 = 30$ to calculate $9 \times 6$.
   
   c) Are your answers to parts a) and b) the same? If not, find your mistake.
   
   **Answers:** a) 60 − 6 = 54, b) 30 + 24 = 54

7. Multiply by 100. Show your skip counting.
   
   a) $3 \times 100 = \_\_\_\_$, skip count: ____, ____, ____
   
   b) $4 \times 100 = \_\_\_\_$, skip count: ____, ____, ____
   
   c) $7 \times 100 = \_\_\_\_$, skip count: ____, ____, ____
   
   **Answers:** a) 300, b) 400, c) 700

8. a) If $8 \times 100$ is 800, what is $100 \times 8$? Explain.
   
   b) Use $100 \times 8$ to find $102 \times 8$.
   
   c) Use $100 \times 9$ to find $103 \times 9$.
   
   **Answers:** a) 800, because you are multiplying the same two numbers; b) 800 + 16 = 816; c) 900 + 27 = 927

9. a) Use $100 \times 2$ to calculate $99 \times 2$.
   
   b) Use $100 \times 3$ to calculate $99 \times 3$.
   
   c) Use $100 \times 4$ to calculate $99 \times 4$.
   
   d) Use $100 \times 5$ to calculate $99 \times 5$.
   
   e) Use $100 \times 8$ to calculate $99 \times 8$.
   
   **Answers:** a) 200 − 2 = 198, b) 300 − 3 = 297, c) 400 − 4 = 396, d) 500 − 5 = 495, e) 800 − 8 = 792

10. a) Use $10 \times 7$ to calculate $9 \times 7$.
   
   b) Use $100 \times 7$ to calculate $99 \times 7$.
   
   c) Use $1,000 \times 7$ to calculate $999 \times 7$.
   
   d) Use $10,000 \times 7$ to calculate $9,999 \times 7$.
   
   e) Use the pattern to predict $99,999,999 \times 7$.
   
   **Answers:** a) 70 − 7 = 63; b) 700 − 7 = 693; c) 7,000 − 7 = 6,993; d) 70,000 − 7 = 69,993; e) 700,000,000 − 7 = 699,999,993
13. Add 46 + 7 in three ways.
   a) Getting to the next ten and adding extra.
   b) Adding the tens and ones separately.
   c) Adding 10 and subtracting 3.
   **Answers:** a) 46 + 4 + 3 = 53, b) 40 + 13 = 53, c) 56 – 3 = 53

14. a) Skip count from 70 by 7s on the number line. Use whatever method you like to add 7 each time.
    
    70
    77
    84
    91
    98
    105
    112

   b) What is 16 × 7? Explain how you know.
   **Answers:** a) 70, 77, 84, 91, 98, 105, 112; b) 112, because it is 6 sevens more than 10 sevens, so it is 16 sevens

15. Multiply using an empty number line. Which two answers are the same? Why?
   a) 9 × 8    b) 8 × 6    c) 7 × 9    d) 8 × 9
   **Answers:** a) 72; b) 48; c) 63; d) 72; the answers to a) and d) are the same because you are multiplying the same two numbers

16. Multiply using an empty number line by splitting into easy jumps.
   a) 7 × 12    b) 7 × 15    c) 7 × 21    d) 7 × 32
   **Sample answers:** a) 84 (70, 84); b) 105 (70, 105); c) 147 (70, 140, 147); d) 224 (70, 140, 210, 217, 224)

17. Multiply in order.
   a) 7 × 10, 7 × 13, 7 × 130, and 7 × 131
   b) 8 × 10, 8 × 20, 8 × 30, 8 × 32, 8 × 320, 8 × 321
   **Answers:** a) 70, 91, 910, 917; b) 80, 160, 240, 256, 2,560, 2,568

18. Use an empty number line to find the missing number.
   a) _____ × 8 = 56    b) _____ × 8 = 96    c) _____ × 8 = 128
   d) _____ × 8 = 184    e) _____ × 8 = 456
   **Answers:** a) 7, b) 12, c) 16, d) 23, e) 57

19. Use your answers to Problem 18 to divide by 8.
   a) 56 ÷ 8 = _____    b) 96 ÷ 8 = _____    c) 128 ÷ 8 = _____
   d) 184 ÷ 8 = _____    e) 456 ÷ 8 = _____
   **Answers:** a) 7, b) 12, c) 16, d) 23, e) 57
(MP.1, MP.7) 20. Divide by using the number line and filling in the blanks.

a) \(56 \div 7\)

\[
\begin{array}{cccc}
& 35 & + & \underline{\quad \times 7} & = 56 \\
0 & 0 & 5 \times 7 & + & \underline{\quad \times 7}
\end{array}
\]

So \(56 \div 7 = \underline{\quad}\)

b) \(91 \div 7\)

\[
\begin{array}{cccc}
& 70 & + & \underline{\quad \times 7} & = 91 \\
0 & 0 & 10 \times 7 & + & \underline{\quad \times 7}
\end{array}
\]

So \(91 \div 7 = \underline{\quad}\)

c) \(126 \div 9\)

\[
\begin{array}{cccc}
& 90 & + & \underline{\quad \times 9} & = 126 \\
0 & 0 & 10 \times 9 & + & \underline{\quad \times 9}
\end{array}
\]

So \(126 \div 9 = \underline{\quad}\)

Answers: a) \(3 \times 7 = 21, \ 56 \div 7 = 8\); b) \(3 \times 7 = 21; \ 91 \div 7 = 13\); c) \(4 \times 9 = 36, \ 126 \div 9 = 14\)
Unit 6 Measurement and Data: Length in US Customary Units

This Unit in Context

In 3.1 Unit 6, students estimated and measured lengths in centimeters and meters. In this unit, students will estimate and measure lengths in inches, feet, and yards. Students will learn about half inches and quarter inches (3.MD.B.4), measure lines and objects to the nearest half of an inch or quarter of an inch, and draw lines of specified lengths. This continues in Grade 4 to eighth inches (4.MD.B.4).

In this unit, students will measure in feet and yards, and by doing so, they prepare for 3.2 Unit 7 when they will measure in square feet and square yards (3.MD.C.6). Students will also begin to do simple measurement conversions from yards to feet by multiplying by 3 (3.OA.C.7). In Grade 4, students will express the measurement of a larger unit in terms of a smaller unit within different systems of units (4.MD.A.1)—not just length, for example. In Grade 5, students will convert among different-sized standard measurement units within a given measurement system, including conversions from smaller to larger units and conversions that result in decimals or fractions (5.MD.A.1).

In this unit, students will solve one- and two-step real-world word problems involving length measurements and the four operations (3.OA.D.8) in preparation for 3.2 Unit 7 when they will solve problems involving area. Students will also organize data in line plots marked in quarter-inch intervals, and answer questions about the data (3.MD.B.4). In order to read and draw line plots, students will use their knowledge of representing numbers on number lines from Grade 2 (2.MD.B.6) and representing fractions on number lines from 3.2 Unit 2 (3.NF.A.2). From Grades 3 to 5, this concept of scale on a line plot is extended mainly through changes to the horizontal scale: the horizontal scale on a line plot in Grade 3 may be marked off using whole numbers, halves, or fourths (3.MD.B.4); in Grades 4 and 5, they may be marked off using whole numbers, halves, fourths, or eighths (4.MD.B.4 and 5.MD.B.2). This work on data management in Kindergarten through to Grade 5 prepares students for the study of statistics from Grades 6 to high school.

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1, MP2, and MP4 to MP6. Here are some examples of how students can show that they have met a standard.

MP1: In MD3-25 Extension 5, students make sense of a non-routine problem when they decide whether it is worth spending time to save money. Students persevere when they reread the problem to understand what it is asking for and when they problem solve to find benchmarks that help them compare the two options.
**MP.5:** In MD3-26 Extension 3, students choose tools strategically and without guidance to solve a word problem involving time intervals.

**MP.6:** In MD3-27 Extension 5, students attend to precision when they include “p.m.” in their answer even though the clock provided does not include that information.
Unit 6 Measurement and Data: Length in US Customary Units

Introduction

In this unit, students will learn to estimate and measure lengths in inches, feet, and yards. They will use their fingers, their arms, and giant steps to estimate lengths in inches, feet, and yards, respectively; they will use rulers, yardsticks, and measuring tapes for more precise measurements of length.

Students will learn about half inches and quarter inches. They will measure lines and objects to the nearest half of an inch or to the nearest quarter of an inch. They will also draw lines of specified lengths.

Students will learn to organize measurements into line plots. They will analyze data by using these line plots.

Students will solve one- and two-step real-world word problems involving length measurements and the four operations.

Fraction notation. We show fractions in two ways in our lesson plans:

- Stacked: $\frac{1}{2}$
- Not stacked: 1/2

Remember to only show students the stacked form when teaching fractions.

Materials. Students will need inch, half-inch, and quarter-inch rulers for measuring objects and lines in these lessons and for the exercises in the AP Book. If you do not have such rulers, you can use the rulers from BLM Inch Rulers, BLM Half-Inch Rulers, and BLM Quarter-Inch Rulers.

In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section V, are used in Unit 6:

- BLM Pattern Blocks (p. V-2)
- BLM Inch Rulers (p. V-3)
- BLM Half-Inch Rulers (p. V-4)
- BLM Quarter-Inch Rulers (p. V-5)
MD3-24 Inches
Pages 138–140

STANDARDS
preparation for 3.MD.B.4
3.MD.C.6

VOCABULARY
centimeters
estimate
inch (in)
length
measure
measurement
to the nearest inch
whole numbers
width

Goals
Students will estimate lengths in inches by using their fingers, measure lengths in inches by using inch rulers, and draw lines of specified lengths by using those rulers.

PRIOR KNOWLEDGE REQUIRED
Understands the concepts of linear measurement and estimation
Understands that we use standard units to measure
Can estimate and measure in centimeters, including measuring to the nearest centimeter
Can draw a line of a given length in centimeters

MATERIALS
inch rulers or BLM Inch Rulers (p. V-3)
objects to measure in whole numbers of inches (pencils sharpened, pieces of yarn, or drinking straws cut to exact lengths)
pattern blocks or shapes cut out from BLM Pattern Blocks (p. V-2, see Extension 1)

Introduce inches. Hold up a pencil that is an exact whole number of inches long (for example, 6 inches), and ASK: If I want to know how long this pencil is, how could I measure it? (measure it with a ruler) Remind students that a centimeter is one standard unit for measuring length and an inch is another that is used in many countries, including the United States. Write on the board:

inch

Show students an inch ruler, and then draw a picture of an inch ruler on the board as shown below:

| 0 inches | 1 | 2 | 3 | 4 |

Explain to students that you made your picture on the board larger than an actual inch ruler so that they can see the picture easily from where they are sitting.

Explain that one inch is longer than one centimeter. Tell students that two of their fingers side by side measure about 1 inch across. Have a couple of volunteers come to the board and place two fingers side by side (such as index and middle finger) starting at the “0” mark of an actual inch ruler to demonstrate. Some students might need to use a different pair of fingers or even three fingers side by side.
ACTIVITY 1

**Personal estimate for measuring an inch.** Give each student an inch ruler or a ruler from BLM Inch Rulers. Students will measure different pairs of their fingers to find out which pair is closest to 1 inch. Tell students that for future measuring, they will use the pair of fingers that is closest to 1 inch to estimate lengths. **NOTE:** Students with small hands might need to use three fingers side by side.

**Review of whole numbers.** Write on the board:

\[
5 \quad 3 \quad 4\frac{1}{2} \quad 3 \quad 4 \quad 17
\]

**ASK:** Which of these numbers have a fractional part? (4\(\frac{1}{2}\), 3/4)

SAY: Remember that numbers with no fractional part are called whole numbers. **ASK:** Which numbers on the board are whole numbers? (5, 3, 17)

**Exercises:** Circle the whole numbers.

a) 15, 4, 3\(\frac{1}{2}\), 68  
b) 0, 42\(\frac{1}{3}\), 13\(\frac{1}{4}\), 56  
c) \(\frac{4}{9}\), 1, 8\(\frac{1}{2}\), 11

**Bonus**

d) 212\(\frac{1}{2}\), 380, 314\(\frac{3}{8}\), 115\(\frac{6}{32}\)  
e) 108, 216, \(\frac{2}{4}\), 17\(\frac{3}{3}\)

**Answers:** a) 15, 4, 68; b) 0, 56; c) 1, 11; Bonus: d) 380; e) 108, 216

**Estimating lengths in inches.** Remind students that estimating the length of an object means trying to describe the length as closely as they can without using something, such as a ruler, to make an exact measurement. SAY: You can use your fingers to estimate lengths in inches.

Have a volunteer estimate the length of the pencil from earlier in the lesson (the pencil that is, for example, exactly 6 inches long) by using the fingers they chose in Activity 1. SAY: When we use our fingers, we are estimating to the nearest inch, which means we want to find the closest whole number of inches. If the student volunteer’s measurement is between 5 inches and 6 inches but closer to 6 inches, SAY: The pencil is between 5 and 6 inches but closer to 6 inches. If the student volunteer’s measurement was between 6 inches and 7 inches but closer to 6 inches, SAY: The pencil is between 6 and 7 inches but closer to 6 inches. Write on the board:

The pencil is about 6 inches long.

Have several different volunteers estimate the length of the pencil in the same way.

Draw lines of various lengths on the board that are an exact whole number of inches long (for example, 4 inches, 10 inches, 1 inch, 7 inches). Have volunteers use their fingers to estimate the lengths. Write the estimate under each line. (about 4 inches, about 10 inches, about 1 inch, about 7 inches)
ACTIVITY 2

**Estimating in inches.** Students work in pairs to estimate the lengths of objects to the nearest inch by using their fingers. Provide each pair of students with objects of lengths in whole numbers of inches—for example, sharpened colored pencils or pieces of yarn or straws cut to an exact length. Students make a table to record their estimates in inches.

**The short form “in” for inches.** SAY: The short form “in” is used for inch or inches. Under the lines drawn earlier on the board, write the first estimate now using the short form “in” and ask volunteers to write the rest (about 4 in, about 10 in, about 1 in, about 7 in). Point out that the letters “i” and “n” make the short form for inch or inches, but they also spell the word “in.” Ask a volunteer to write a sentence on the board that uses both the word “in” and the short form “in” for inches. Then have a volunteer underline the “in” for “inches” and circle the “in” that is a preposition. (see example below)

The pencil in my bag is 5 in long.

Explain to students that when recording many measurements, it is convenient to write the short form “in,” but when writing a measurement in a sentence, it is clearer to spell the full word “inch” or “inches.”

**Counting inches as jumps to find the distance on a ruler.** Draw on the board:

```
0 inches 1 2 3 4
```

SAY: To find how far apart the arrows are, we can jump from 0 to 1 on the ruler, then from 1 to 2, and so on, and count the jumps, just like we did for centimeter rulers. Draw the jumps (as on AP Book 3.2 p. 138). Trace the jumps with your finger. ASK: How long is each jump? (1 inch) How many jumps do you need to make from 0 to 4? (4 jumps) How far apart are the arrows? (4 inches) Change the positions of the arrows to show them above 0 and 2, and repeat the process. (2 jumps, 2 inches)

ASK: Do you need to count jumps to find how far an arrow is from the “0” mark? (no) How can you do it by simply looking at the ruler? (the second arrow points to the answer) SAY: We can count jumps to find a distance, or we can check which number the second arrow points to.
Aligning at zero when measuring. Draw on the board:

```
| 0 inches | 1 | 2 | 3 | 4 |
```

ASK: Are these good ways to measure the length of the line? (no) Why not? (the line must start at the “0” mark) PROMPT: What is the problem with the first example? (the line starts at the edge of the ruler, not the “0” mark) What is the problem with the second example? (the line starts to the right, or after, the “0” mark)

Draw a ruler at least 6 inches long, and draw a line above it from 0 to 6. Have students signal the length of the line in inches. (6) Repeat with other lines that are an exact number of whole inches in length, always starting at the “0” mark on the ruler.

Drawing lines of a given length. Ask students how they would draw a line exactly 4 inches long on a piece of paper. Demonstrate on the board by holding a ruler straight and drawing two dots—one above the 0-inch mark and one above the 4-inch mark. Then demonstrate how to use the ruler to draw a straight line connecting the two dots. Have a volunteer use this method to draw a 5-inch line on the board, and then have another volunteer draw a 6-inch line.

Give each student an inch ruler or a ruler from BLM Inch Rulers for the following exercises.

**Exercises:** Draw a line of the given length.

a) 3 inches  
   b) 1 inch  
   c) 7 inches

Measuring to the closest inch. Remind students that real objects might not be exactly the distance between two whole number marks on a ruler. Draw on the board:

```
| 0 inches | 1 | 2 | 3 | 4 |
```

Trace the line and point to the numbers with your finger as you SAY: This line is between 3 and 4 inches long. ASK: Is the end closer to the mark for 3 inches or to the mark for 4 inches? (4 inches) SAY: We say that this line is about 4 inches long or 4 inches long when measured to the nearest inch.
Repeat with a line that is a bit longer than 5 inches. (the line is about 5 inches long or 5 inches long to the nearest inch) Repeat several times by drawing lines slightly shorter and slightly longer than a whole number of inches. SAY: Remember, when a line or an object is exactly halfway between two numbers, we use the longer measurement. Draw on the board:

| 0 inches | 1 | 2 | 3 | 4 |

SAY: We describe this line as being about 4 inches long or 4 inches when measured to the nearest inch.

Have students use inch rulers or rulers from BLM Inch Rulers for the following exercises.

**Exercises:** About how many inches long is the object?

a) pen  
b) JUMP Math book  
c) desk  
d) eraser

**Sample answers:** a) about 6 in, b) about 11 in, c) about 25 in, d) about 1 in

**ACTIVITY 3**

**Measuring in inches.** Students measure items in the classroom by using an inch ruler or rulers from BLM Inch Rulers. Students should find an item of each of the following lengths:

a) 6 inches  
b) 1 inch  
c) between 2 and 5 inches  
d) more than 6 inches  
e) less than 1 inch  
f) more than 12 inches

**Extensions**

1. Have students use an inch ruler to measure the edges of pattern blocks or shapes cut from BLM Pattern Blocks. They should see that most of the edges of pattern blocks are 1 inch long. Students can then use pattern blocks to measure objects in the classroom in inches.
2. How much longer is the top line than the line underneath?
   a) 
   b) 
   c) 

   **Answers:** a) 2 inches, b) 2 inches, c) 1 inch

3. Measure both lines with an inch ruler. How much longer is the longer line?
   a) 
   b) 
   c) 

   **Answers**
   a) top line: 4 in, bottom line: 2 in, difference: 2 in
   b) top line: 4 in, bottom line: 3 in, difference: 1 in
   c) top line: 4 in, bottom line: 1 in, difference: 4 in

4. Provide students with one-step word problems involving inches and addition, subtraction, multiplication, or division, such as the ones below, or create new ones:
   a) A bamboo plant is 359 inches tall. In 3 months, it grows 12 inches more. How tall is the bamboo plant now?
   b) If the bamboo plant in part a) grows the same amount each month, how many inches does the bamboo grow each month?
   c) Nina is 52 inches tall. Ben is 48 inches tall. How much taller is Nina than Ben?
   d) Alexa’s hair grows 6 inches every year. If she doesn’t cut her hair, how much will it have grown after 4 years?

   **Answers:** a) 371 inches, b) 4 inches, c) 4 inches, d) 24 inches
5. Provide students with multistep word problems involving inches and addition, subtraction, multiplication, or division, such as the ones below, or create new ones:

a) Ben's shoes are 8 inches long. His younger brother's shoes are 1 inch shorter than Ben's. If Ben puts his 2 shoes and his brother's 2 shoes end to end along a wall, how long will the line of shoes be?

b) Ross is 53 inches tall. Rani is 7 inches shorter than Ross, and Rani's sister is 3 inches taller than Rani. How much taller is Ross than Rani's sister?

c) A table tennis paddle is about 10 inches long. Ed fits three table tennis paddles end to end along a shelf. The shelf is 33 inches long. How much longer is the shelf than the line of paddles?

**Answers:** a) $8 - 1 = 7$ inches, $2 \times 8 = 16$ inches, $2 \times 7 = 14$ inches, $16 + 14 = 30$ inches; b) $53 - 7 = 46$ inches, $46 + 3 = 49$ inches, $53 - 49 = 4$ inches; c) $3 \times 10 = 30$ inches, $33 - 30 = 3$ inches
Goals

Students will learn about halves of an inch and measure and draw lines and objects with lengths of half-inch multiples.

Students will measure lines and objects to the nearest half of an inch.

PRIOR KNOWLEDGE REQUIRED

Can measure in inches (to the closest inch)
Can order fractions and mixed numbers (with denominator 2)
Can read and draw number lines with fractions and mixed numbers (with denominator 2)

MATERIALS

half-inch ruler or ruler from BLM Half-Inch Rulers (p. V-4) for demonstration
BLM Half-Inch Rulers (p. V-4)
BLM Measuring in Half Inches (p. R-35)
BLM Half-Inch Measurement Cards (p. R-36)
8 objects of the following lengths in inches: 1, 1 1/2, 2, 2 1/2, 3, 3 1/2, 4, 4 1/2, one set per pair of students

Review fractions (with denominator 2) on number lines. Draw on the board:

```
0 1 2 3
```

ASK: On this number line, how many pieces did I divide each whole into? (2) Draw a fraction strip above the number line, as shown below:

```
0 1 2 3
\[\frac{1}{2}\]
```

ASK: How many pieces are in the fraction strip? (2) How many pieces are shaded? (1) What fraction of the strip is shaded? (1/2) Have a volunteer write the fraction above the fraction strip. Remind students that in the fraction 1/2, the top number is called the numerator and the bottom number is called the denominator. ASK: Is the fraction strip the same length as the distance between 0 and 1 on the number line? (yes) So, what fraction should be written for the mark between 0 and 1? (1/2) Point to the tick mark between 0 and 1, and prompt students by tracing your finger from the tick mark up to the dividing line of the fraction strip. Have a volunteer label the tick mark between 0 and 1 on the number line as 1/2.
Review mixed numbers (with denominator 2) on number lines. Tell students you need to label the mark between 1 and 2 on the number line. Add another fraction strip above the number line and shade the piece up to 1 1/2. The picture should look like this:

![Number line with mixed number]

Point to the tick mark between 1 and 2 on the number line and ASK: How many whole numbers have we counted up to this point? (1) Prompt students by tracing a jump with your finger from 0 to 1. ASK: How much more than 1 whole do we have? (1/2) Prompt students by pointing to the 1, moving your finger to the tick mark between 1 and 2, then pointing to the fraction strip above the “1 1/2” mark. Write on the board beside the number line:

\[ 1 \text{ and } \frac{1}{2} = 1 \frac{1}{2} \]

Have a volunteer write the correct mixed number on the number line below the tick mark between 1 and 2. (1 1/2) ASK: How is the label between 1 and 2 different from the label between 0 and 1? (the label between 0 and 1 is a fraction and the label between 1 and 2 is a mixed number) Remind students that a mixed number has two parts: the whole part and the fraction part. Remind students how to read the whole part on the number line first. For example, 1 1/2 is “one and one half.” Have a volunteer write the label between 2 and 3 (2 1/2).

Draw on the board:

![Number line with mixed number]

Have volunteers write the missing mixed numbers on the number line below the tick marks. (6 1/2, 7 1/2, 8 1/2)

**Exercises:** Write the missing mixed numbers on the number line.

a) ![Number line with mixed number]

b) ![Number line with mixed number]

**Bonus**

![Number line with mixed number]
Answers: a) 5 1/2, 6 1/2; b) 9 1/2, 10 1/2, Bonus: 10 1/2, 11 1/2, 12 1/2, 13 1/2, 14 1/2

Measuring in half inches. Show students a half-inch ruler or a ruler from BLM Half-Inch Rulers, and draw a half-inch ruler on the board. Point out the half-inch markings. ASK: How many parts has each inch been divided into? (2) What fraction is each part? (1/2) Trace your finger along the ruler from the 0 to the “1/2” mark, and SAY: This is one half inch or one half of an inch. Write it as a fraction on the board, as shown below:

\[
\frac{1}{2} \text{ of an inch or half of an inch}
\]

Review the words “half” and “halves.” Trace your finger from 0 to 1/2, and SAY: This is one half of an inch. Then trace your finger from 1/2 to 1, and SAY: This is another half of an inch. Each inch is divided into two halves. Remind students that when they have only one half, they use the word “half,” and when they have two or more halves, they use the word “halves.” Write on the board:

1 half 2 or more halves

Half-inch markings on a ruler. Point to the “1/2” mark on the ruler you drew on the board. SAY: This mark is halfway between 0 and 1. ASK: What number could we write under this mark? (1/2) Write the 1/2 under the mark between 0 and 1. ASK: What mixed number could we write between 1 and 2? (1 1/2) PROMPT: How many whole numbers are there? (1) How many halves are left over? (1/2) Have a volunteer write “1 1/2” under the tick mark between 1 and 2. Extend the ruler drawing to 7 inches, and repeat by having students add the following measurements to the ruler: 2 1/2, 3 1/2, 4 1/2, 5 1/2, and 6 1/2.

Give each student a ruler from BLM Half-Inch Rulers. Have each student write the fractions and mixed numbers under the tick marks between the whole-inch markings. (1/2, 1 1/2, 2 1/2, 3 1/2, 4 1/2, 5 1/2, 6 1/2) NOTE: Since the word “inches” appears after the 0 mark on the ruler, students can either skip writing the label for 1/2 or they can write these fractions in small printing underneath the word “inches.” Explain to students that rulers show the markings for fractions and mixed numbers, but only the whole inches are labeled; writing these fractions and mixed numbers will help students to measure with a half-inch ruler.

Measuring lines with a half-inch ruler. Draw on the board:
SAY: Let’s use the ruler to measure the line. ASK: Is the line more than 3 inches long or less than 3 inches long? (more) How much longer than 3 inches? (half an inch) Prompt students by covering the ruler and line to the left of the 3-inch mark and tracing your finger along the line from the 3-inch to the 3 1/2-inch mark. SAY: So the line is 3 inches and another half of an inch long, or 3 1/2 inches long. Write “3 1/2 inches long” beside the line. Repeat with lines of the following lengths in inches: 2 1/2, 6 1/2, 3, 1/2, 1. For 1/2, write “1/2 an inch long” and for 1, write “1 inch long.”

**Measuring to the nearest half of an inch.** Point out that even though halves of an inch are smaller than whole inches, objects often have lengths that are between two half-inch marks. Draw a ruler with half-inch marks on the board, and draw a line that ends between 2 and 2 1/2 inches, closer to the 2 1/2 mark. Have students think, based on their previous measuring experience, how they should record the length of this line. PROMPT: We want to find the closest half of an inch, or measure to the nearest half of an inch. Have students explain their ideas. (the line is closer to 2 1/2 in than to 2 in, so we should write its length as “about 2 1/2 in” or “2 1/2 inches to the nearest half of an inch”) Repeat with lines that are just over 4 inches, just under 3 1/2 inches, just over 1 1/2 inch, and just under 2 inches. Have students complete BLM Measuring in Half Inches with the rulers they created earlier. (1. b) 2, c) 5 1/2; 2. a) 1 1/2, b) 2 1/2, c) 5, d) 4 1/2)

**ACTIVITY**

**Measuring half inches.** Give each pair of students a set of cards from BLM Half-Inch Measurement Cards and eight objects (for example, sharpened colored pencils) of the following lengths in inches: 1, 1 1/2, 2, 2 1/2, 3, 3 1/2, 4, 4 1/2. Player 1 shuffles the cards and places them face down. Player 2 chooses one card. From the set of objects, Player 2 has to find one that is the length given on the card. Player 2 uses a half-inch ruler or a ruler from BLM Half-Inch Rulers to help identify the correct object. After Player 2 has found the correct object, students switch roles and play again. Students might keep the cards and matched objects as visual reminders of measurements.

**NOTE:** Students will need a half-inch ruler or a ruler from BLM Half-Inch Rulers to complete Question 5 on AP Book 3.2 p. 142.

**Extensions**

1. Bill estimates the length of a pencil by using his fingers. Two of his fingers measure 1 inch across. Bill finds that the pencil is exactly 7 fingers long. He says the pencil is about 4 inches long. Alexa says the pencil is about 3 1/2 inches long. Which answer is closer to the actual length of the pencil? Explain.
Answer: Alexa’s answer is closer. Two of Bill’s fingers measure 1 inch across, so 6 fingers make 3 inches, and the 7th finger makes an extra half of an inch. Bill’s answer would be right if he were measuring to the nearest inch.

2. Have students estimate the lengths of objects to the nearest half of an inch by using their fingers. First, have them use two fingers until there is no space for two more fingers. This is the whole number part in inches. Then, have students use just one finger to see whether they need to add half of an inch to their estimate. Have students check their estimates by using a ruler.

3. Have students count half inches on a half-inch ruler to convert a mixed number to an improper fraction. For example, there are 7 half inches in 3 1/2, so 3 1/2 = 7/2.
   a) 2 1/2   b) 5 1/2   c) 1 1/2   d) 4 1/2
   Answers: a) 5/2, b) 11/2, c) 3/2, d) 9/2

4. Have students count half inches on a half-inch ruler to convert an improper fraction to a mixed number. For example, they count 7 half inches on the ruler to end at the 3 1/2-inch mark, so 7/2 = 3 1/2.
   a) 5/2   b) 9/2   c) 3/2   d) 13/2
   Answers: a) 2 1/2, b) 4 1/2, c) 1 1/2, d) 6 1/2

5. Alice wrote a story and wants to print copies for some friends. She needs to print a total of 40 pages. Company A is a 5-minute walk away and charges 10 cents for each page. Company B is a 15-minute bus ride away and charges 5 cents for each page. A bus ticket costs 30 cents. Use any tools you think will help you answer the question.
   a) How much time would Alice save by going to the closer company?
   b) How much money would Alice save by going to the farther company?
   c) Talk with a partner: Is it worth the extra time to take the bus? Explain.

Sample answers
   a) She spends 10 minutes travelling to and from the closer company (Company A) and 30 minutes travelling to and from the farther company (Company B). She would save 20 minutes (because 30 – 10 = 20) by going to Company A.
   b) She spends 400 cents (40 × 10 = 400) if she goes to Company A. If she goes to Company B, she spends 200 cents (40 × 5 = 200) for the copies plus 60 cents for the bus (both ways), so she spends 260 cents altogether. She would save 140 cents by going to Company B (because 400 – 260 = 140).
   c) Answers will vary.
Look for students to use an appropriate tool to get a sense of how long 20 minutes is and how much 140 cents can buy.

Redirecting students: For part c), encourage students to use benchmarks to help them get a sense of how long 20 minutes is or how much 140 cents can buy.

Individual or small group follow-up: Have students calculate how long recess is using the starting and ending times, or suggest they make 140 cents using coins to get a sense of what they can buy with that much money. If students do not have experience buying things, you may need to provide examples of what they could buy with that amount of money.

Whole-class follow-up: To emphasize the importance of units, ASK: Would you make a different choice if the time saved was 20 hours? What if the money saved was 140 dollars?
Goals

Students will learn about fourths or quarters of an inch and measure and draw lines and objects with lengths that are quarter-inch multiples.

Students will measure lines and objects to the nearest quarter of an inch.

PRIOR KNOWLEDGE REQUIRED

Can measure in half inches (to the nearest half of an inch)
Can order fractions and mixed numbers (with denominator 2 or 4)
Can read and draw number lines with fractions and mixed numbers (with denominator 2)

MATERIALS

quarter-inch ruler or ruler from BLM Quarter-Inch Rulers (p. V-5) for demonstration
BLM Quarter-Inch Rulers (p. V-5)
BLM Measuring in Quarter Inches (p. R-37)
objects to measure to the nearest quarter of an inch, one set per pair of students

Review number lines with multiples of 1/2 marked. Draw on the board:

0 1 2 3

NOTE: Be sure to leave enough space to write multiples of 1/4, which will be added later.

Ask students to remember how they filled in the number line in the last lesson. PROMPT: How many pieces is each whole is divided into? (2) Have volunteers write the missing fractions and mixed numbers for the line above the tick marks. (1/2, 1 1/2, 2 1/2)

Review fractions and mixed numbers (with denominator 2 or 4) on number lines. Add smaller ticks at the midway point between the existing tick marks on the number line. The number line should now resemble a quarter-inch ruler, as shown below:

0 1 2 3
ASK: Now how many pieces is each whole divided into? (4) Have volunteers mark the fractions and mixed numbers with denominator 4 under the number line. The final picture should look like this:

![Number line with fractions and mixed numbers marked]

Remind students that $\frac{1}{2} = \frac{2}{4}$. Point to the halfway mark between 0 and 1. ASK: What do you notice about this mark on the number line? (there are two labels) What are the two labels? (1/2 and 2/4) Is one half the same as two quarters? (yes) Write on the board:

$$\frac{1}{2} = \frac{2}{4}$$

SAY: These two fractions are equivalent. Point to each fraction as you ASK: What is the denominator of 1/2? (2) What is the denominator of 2/4? (4) Which fraction has the smaller denominator, 1/2 or 2/4? (1/2) SAY: We usually write fractions with the smallest denominator we can, so it is better to write 1/2 instead of 2/4. Cross out the label “2/4” and circle the label “1/2” on the number line. Then point to the halfway mark between 1 and 2. ASK: Which label do you think we should use for this mark, 1 1/2 or 1 2/4? (1 1/2) Cross out the label “1 2/4” and circle the label “1 1/2.” Repeat with the halfway mark between 2 and 3. (cross out 2 2/4, circle 2 1/2)

**Exercises:** Write the missing fractions and mixed numbers on the number line. Use the smallest denominator you can.

**a)**

```
2 3 4 5
```

**b)**

```
7 8 9 10
```

**Bonus**

```
13 14 15
```

**Answers:**

**a)** 2 1/4, 2 1/2, 2 3/4, 3 1/4, 3 1/2, 3 3/4, 4 1/4, 4 1/2, 4 3/4;

**b)** 7 1/4, 7 1/2, 7 3/4, 8 1/4, 8 1/2, 8 3/4, 9 1/4, 9 1/2, 9 3/4; Bonus: 12 1/4, 12 1/2, 12 3/4, 13 1/4, 13 1/2, 13 3/4, 14 1/4, 14 1/2, 14 3/4, 15 1/4
Identifying marks for mixed numbers on a number line with only whole numbers labeled. Draw on the board a small number line from 4 to 5. Include tick marks for quarters, but label only the whole numbers. Ask: Where should I add the label 4 3/4? Point to the tick marks in turn, and have students signal thumbs up if it is the right mark and thumbs down if it is not. Repeat with several mixed numbers on number lines starting from different whole numbers.

Measuring in quarter-inches. Show students a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers, and draw on the board:

```
0 inches  1  2
```

Point out the quarter-inch markings. Ask: How many parts has each inch been divided into? (4) So what fraction is each part? (one fourth or one quarter) Trace your finger along the ruler from the “0” to the “1/4” mark and say: This is one fourth of an inch, one quarter inch, or one quarter of an inch. Continue drawing on the board:

```
0 inches  1  2
```

Quarter-inch markings on a ruler. Point to the tick mark for 1/4 inch on the ruler you drew on the board. Ask: What number could we write under this mark? (1/4) Have volunteers fill in the missing fractions and mixed numbers on the ruler under each tick mark. (1/4, 1/2, 3/4, 1 1/4, 1 1/2, 1 3/4, 2 1/4, 2 1/2) Explain to students that they are adding quarter-inch labels to the ruler just as they added quarter-inch labels to a number line.

Give each student a ruler from BLM Quarter-Inch Rulers. Have students write the fractions and mixed numbers under the tick marks that do not have labels. Note: Since the word “inches” appears after the 0 mark on the ruler, students can either skip writing the label for 1/4 and 1/2, or they can write these fractions in small printing underneath the word “inches.” Point out to students that rulers show the marks for fractions and mixed numbers, but only the whole inches are labeled. Writing these fractions and mixed numbers will help students to measure with a quarter-inch ruler.

Measuring lines with a quarter-inch ruler. Draw on the board:

```
0 inches  1  2  3  4
```
SAY: Let’s use the ruler to measure the line. ASK: Is the line more than 2 inches or less than 2 inches? (more) Does the line end at a short tick mark or a long tick mark? (short tick mark) Tell students that if the line ends at a short tick mark, they should count quarters, and if the line ends at a long tick mark without a label, they should count halves. Guide students to count quarters by tracing your finger along the quarter inches from the 2-inch mark to the 2 3/4-inch mark. SAY: So the line is 2 inches and 3/4 of an inch long, or 2 3/4 inches long. Write “2 3/4 inches long” beside the line. Repeat with lines of the following lengths in inches: 2 1/4, 3 1/2, 4, 1 3/4, 1 1/2.

Have students complete BLM Measuring in Quarter Inches. They will need to use either a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers for Question 2. (1. b) 2 1/2, c) 4 3/4; 2. from left vertical line, going clockwise: 1 1/2 in, 1 3/4 in, 1 in, 2 1/4 in, 1 3/4 in, 4 1/2 in)

Measuring to the nearest quarter of an inch. Point out that even though a quarter of an inch is quite small, objects often have lengths that are between two quarter-inch marks. Draw a ruler with quarter-inch marks on the board, and draw a line that ends between 2 and 2 1/4 inches, closer to the 2 1/4 mark. Have students think, based on their previous measuring experience, how they should record the length of this line. PROMPT: We want to find the closest quarter of an inch, or measure to the nearest quarter of an inch. Have students explain their ideas. (the line is closer to 2 1/4 in than to 2 in, so we should write its length as “about 2 1/4 in” or “2 1/4 inches to the nearest quarter of an inch”) Repeat with lines of the following lengths: just over 4 inches, just under 3 1/4 inches, just over 2 1/2 inches, just under 3 1/4 inches, just over 2 1/2 inches, just under 2 3/4 inches.

ACTIVITY

Measuring in quarter inches. Students measure actual objects to the nearest quarter of an inch by using a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers. Students work in pairs and check each other’s work. Students can either find objects in the classroom or you can provide them with a set of objects (such as colored pencils sharpened to different lengths). They should record the names of the objects they measure and their lengths to the nearest quarter of an inch.

NOTE: Students will need a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers to complete Question 6 on AP Book 3.2 p. 145.
Extensions

1. Draw a line above a ruler. Start the line at the "0" mark, and end the line between the 1 1/2 inch and 1 3/4 inch mark, closer to the 1 3/4 mark, as shown below:

   ![Ruler Diagram]

   Have students refer to this drawing to answer the questions.

   a) Raj says the line is about 1 1/2 inches long to the nearest half of an inch. Nina says the line is about 1 3/4 inches long to the nearest quarter of an inch. Explain why they are both right.

   Answers: a) Raj's answer is correct to the nearest half of an inch, because the line is closer to the mark for 1 1/2 inches than to the mark for 2 inches, and Nina's answer is correct to the nearest quarter of an inch, because the line is closer to the mark for 1 3/4 inches than to the mark for 1 1/2 inches; b) closer to 1 3/4 inches

   b) Is the actual length of the line closer to 1 1/2 inches or 1 3/4 inches?

   2. Use the number line to answer the questions.

   ![Number Line Diagram]

   a) What letter is above the mark for 3 \( \frac{3}{4} \)?

   b) What letter is above the mark for 4 \( \frac{3}{4} \)?

   c) What letter is above the mark for 2 \( \frac{1}{4} \)?

   d) What letter is above the mark for 3 \( \frac{1}{2} \)?

   e) What letter is above the mark for 2 \( \frac{3}{4} \)?

   f) What letter is above the mark for 4 \( \frac{1}{2} \)?

   g) What letter is above the mark for 4 \( \frac{1}{4} \)?

   h) What letter is above the mark for 3?  

   i) Write the letters you've written in order. What word do you get?


Measurement and Data 3-26

R-19
3. Morning recess is from 10:05 to 10:25 a.m. Lunch recess is from 11:45 a.m. to 12:30 p.m., and afternoon recess is from 1:50 p.m. to 2:10 p.m. How much longer is lunch recess than the other two recesses put together? Use any tool you think will help. Explain what each step means in the situation.

Answer: Lunch recess is 5 minutes longer than the other two recesses put together.

Look for students to model (MP 4) the amount of time elapsed by choosing a helpful tool (MP 5), such as a number line, a clock, counting up by 5s or 15s, or subtracting times. At each step, look for students to interpret their calculations in the context of the problem (MP 2).

Redirecting students: To help students understand what is being asked, ASK: What do you need to know to answer how much longer something is than two other things put together? (the length of the first thing and the total length of the two other things) To help students pick a tool (MP 5), ASK: What tools can you use to measure time intervals?

If students wrote “a.m.” or “p.m.” as part of the answer, help them interpret the result of their calculation in the context (MP 2). ASK: Is the question asking for a specific time or for an amount of time? Can an amount of time be a.m. or p.m.?

Individual or small-group follow-up: If students do not include the units in their answers (MP 2), ASK: What did you calculate: the number of seconds, minutes, hours, yards, feet, or inches? (minutes) Why is it important to say what units your answer is in? (it tells you what the answer means; 5 minutes longer is a lot different than 5 hours longer or 5 days longer)
MD3-27 Quarters of an Inch and Line Plots

Goals

Students will measure to the quarter inch and read and answer simple questions about line plots of measurements.

Students will complete line plots given the data and the number line.

PRIOR KNOWLEDGE REQUIRED

Can read and draw number lines with fractions and mixed numbers (with denominator 2 or 4)

Can measure by using a quarter-inch ruler

MATERIALS

10 drinking straws cut to the following lengths in inches: 5, 5 1/4, 5 1/4, 5 3/4, 5 3/4, 5 3/4, 5 3/4, 6

 quarter-inch rulers or BLM Quarter-Inch Rulers (p. V-5)

Measuring lengths for line plots. Divide students into small groups (about two or three students in each group), and give each group one of the straws. Ask each group to measure the drinking straw to the nearest quarter of an inch by using a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers. Record the measurements on the board in a table as shown below (the measurements do not need to be in order):

| Length (inches) | 5 1/4 | 5 | 5 3/4 | 5 1/4 | 5 3/4 | 5 | 5 3/4 | 5 1/4 | 6 | 5 3/4 |

Introduce line plots. SAY: We have straws of different lengths. We can organize and show information about those lengths by using a line plot.

SAY: We will begin our line plot by drawing a number line. Draw a number line on the board, but don’t mark any tick marks or write any labels.

ASK: What is the smallest length of straw that we measured? (5 inches)

Prompt students by tracing your finger along the table above to scan for the lowest number or the shortest measurement. Draw a tick mark near the left end of the number line and write “5” underneath the tick mark. ASK: What is the largest number or the longest measurement? (6 inches) Complete the number line with all of the measurements in the table, as shown below:
SAY: The numbers on this number line show lengths in inches. Add the label “Length (inches)” on the board below the number line. SAY: Now we will draw an X above the number line for each measurement. Point to the first measurement in the table from the left side. ASK: What is this measurement? (5 1/4 inches) Have a volunteer come to the board and point to 5 1/4 on the number line. Ask the volunteer to draw an X above 5 1/4, and then cross out the measurement on the table. Continue in the same manner, going through the table from left to right, one measurement at a time. The completed line plot should look like this:

![Line plot with Xs above numbers]

SAY: In our line plot, we drew an X above the numbers on the number line for each measurement. ASK: How many straws are 5 inches long? (2) How can you tell by looking at the line plot? (there are two Xs above 5 on the number line) How many straws are 5 1/4 inches long? (3) PROMPT: How many Xs are above 5 1/4 on the number line? ASK: How many Xs are above 5 1/2 on the number line? (0) So how many straws are 5 1/2 inches long? (0) Keep the line plot on the board for the following exercises.

**Exercises:** Use the line plot on the board to answer the question.

a) How many Xs do you see for 5 3/4? How many straws are 5 3/4 inches long?

b) How many Xs do you see for 6? How many straws are 6 inches long?

**Bonus**

c) How many straws are less than 5 1/2 inches long?

d) How many straws are more than 5 1/2 inches long?

e) What is the total number of straws that we measured?

**Answers:** a) 4, 4; b) 1, 1; Bonus: c) 5; d) 5; e) 10

**Title and label of a line plot.** Pointing to the line plot on the board, SAY: Every line plot needs a title. The title of a line plot tells you what the line plot is about. ASK: What did we measure for this line plot? (lengths of straws) Add the title to the line plot as shown on the next page.
Lengths of Straws

Point to the label "Length (inches)" and SAY: The label of a line plot tells you what the number line shows and the units. This number line shows length, and the unit for measurement is inches. Tell students that when they add a title and label to a line plot, they give information essential for understanding the line plot; even if they looked at this line plot weeks later, they would understand what the Xs and numbers on the line plot meant.

NOTE: Students will need a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers for Questions 1, 3, and 4 on AP Book pp. 146–148.

Extensions

1. Ask some slightly trickier questions about the Length of Straws line plot. For example:
   a) ASK: Which are there more of, straws that are 5 3/4 inches long or straws that are less than 5 3/4 inches long?
   b) Explain that the "most common length" means the length that appears most frequently on the line plot (i.e., has the most Xs), and then ASK: What is the most common length of straw?

   Answers: a) less than 5 3/4 inches, b) 5 3/4 inches

2. Have students work in groups to come up with their own questions about the line plot, exchange them with other groups, and answer the questions.

3. Ask students to explain why it is helpful to write the lengths of the straws on the number line in counting order (5, 5 1/4, 5 1/2, and so on). Ask why the number 5 1/2 is included even though there are no straws that are 5 1/2 inches long.

   Answers: Writing the lengths in counting order on the number line helps us to see the lengths in order of size from smallest to largest and to compare the lengths easily. The number 5 1/2 belongs on the number line even though there are no straws of length 5 1/2 inches because writing the mark for 5 1/2 inches helps us to see that straws of length 5 1/4 and 5 3/4 inches are half an inch apart.
4. Alexa measured 15 pencils and made a line plot. She forgot to mark \( \times \)s for pencils that are 6 inches long.

<table>
<thead>
<tr>
<th>Lengths of Pencils</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \times )</td>
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<tr>
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<td>( \times )</td>
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<tr>
<td>( \times )</td>
</tr>
</tbody>
</table>

Length (inches)

- a) How many \( \times \)s should Alexa mark for 6 inches on the line plot?
- b) Copy and complete the line plot.
- c) Are there more pencils longer than 5 1/2 inches or shorter than 5 1/2 inches long?

**Answers:**
- a) 4
- b) Add 4 \( \times \)s above 6
- c) Longer than 5 1/2 inches

(MP6)

5. When Evan came home after school, the clock looked like this:

He played basketball for 20 minutes, then he went inside and asked for a snack. 8 minutes later, he ate his snack. What time did Evan eat his snack?

**Answer:** 4:10 p.m.

Look for students to tell the time on the clock and to include "p.m." in their answers (MP6).
**Goals**

Students will create line plots by using given measurement data.  
Students will measure to gather the data to create line plots.  
Students will answer questions about the line plots they create.

**PRIOR KNOWLEDGE REQUIRED**

Can read and draw number lines with fractions and mixed numbers (with denominator 2 or 4)  
Can measure by using a quarter-inch ruler

**MATERIALS**

BLM Lengths of Shoes Line Plot (p. R-38)  
quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers (p. V-5)

Creating line plots. SAY: Marta borrows one shoe from each of her 10 friends and measures the length of each shoe to the nearest quarter inch. Draw the table of measurements shown below on the board:

| Length (inches) | 7 3/4 | 7 1/2 | 7 3/4 | 8 | 7 1/4 | 7 1/2 | 8 1/2 | 7 3/4 | 8 | 8 1/2 |

Give students each one half of BLM Lengths of Shoes Line Plot, and ask them to fill in the measurements from the table above on their own copy.

Underneath the table on the board, leave some space (enough for the title and X markings of a line plot) and draw a simple horizontal line to start the number line. Do not make tick marks or add numbers to the number line yet (the line should be long enough to accommodate 7 evenly spaced tick marks). ASK: On the table, what is the smallest length of shoe? (7 1/4 inches) Prompt students by scanning the table from left to right. SAY: We usually start line plots with whole numbers, so we will start at 7 instead of 7 1/4. Make the first tick mark at the far left of the number line and label it as 7. Then add 4 more evenly spaced tick marks and write the labels in quarter-inch increments up to 8. ASK: Are there measurements longer than 8 inches? (yes) What is the longest measurement? (8 1/2 inches) Draw 2 more tick marks, evenly spaced, and add the labels up to 8 1/2. The line plot should look like this:

![Line Plot Diagram]

Have students add numbers to the number line on their copies of BLM Lengths of Shoes Line Plot.
ASK: What should we write as the title for this line plot? (Lengths of Shoes)
PROMPT: What is the line plot about? Write “Lengths of Shoes” as the title of the line plot. ASK: What does the number line show? (length) What units are we using for length? (inches) Prompt students by drawing their attention to the table of measurements. Write the label for the line plot under the number line. The line plot should look like this:

**Lengths of Shoes**

Have students add titles and labels to their own copies of BLM Lengths of Shoes Line Plot.

Point to the first 7 3/4 in the measurement table and ASK: Where should I write the X for this measurement? Point to an incorrect place above the number line and have students signal thumbs up for correct or thumbs down for incorrect. Repeat with one or two more incorrect places before showing the correct spot, above the mark for 7 3/4 on the number line. Repeat with the remaining measurements from the table, showing sometimes 1 or 2 incorrect places before the correct place, and sometimes showing the correct place right away.

The finished line plot should look like this:

**Lengths of Shoes**

Have students complete BLM Lengths of Shoes Line Plot by adding the Xs. ASK: How many shoes are 7 1/2 inches long? (2) How do you know by looking at the line plot? (there are two Xs above the mark for 7 1/2)

Are there any shoes that are 7 inches long? (no) 8 1/4 inches long? (no)

How do you know? (there are no Xs above these marks) What is the most common length of shoe? (7 3/4 inches). How do you know? (because there are more Xs for 7 3/4 than for any other length)
Exercises: Look at the line plot for Lengths of Shoes to answer the question.

a) How many shoes are $8\frac{1}{2}$ inches long? How can you tell by looking at the line plot?

b) How many shoes are 8 inches long?

c) How many shoes are less than $7\frac{3}{4}$ inches long?

d) How many shoes are more than $7\frac{3}{4}$ inches long?

e) How many more shoes are $7\frac{3}{4}$ inches long than $7\frac{1}{4}$ inches long?

Bonus: How many shoes are more than $7\frac{1}{4}$ inches long and less than $8\frac{1}{4}$ inches long?

Answers: a) 2, there are two $\times$'s for $8\frac{1}{2}$ on the line plot; b) 2; c) 3; d) 4; e) 2; Bonus: 7

NOTE: Students will need a quarter-inch ruler or a ruler from BLM Quarter-Inch Rulers to complete Question 3 on AP Book 3.2, p. 151.

Extensions

1. Have each student use a quarter-inch ruler to measure the length of their left hand (from the bottom of their palm to tip of their longest finger) to the nearest quarter of an inch. Ask students to work in groups to record the hand measurements in a table and then make a line plot. Have students come up with and answer questions about their group’s line plot.

2. Ed measures the heights of flowers in his garden and makes the line plot below:

```
7 7\frac{1}{4} 7\frac{1}{2} 7\frac{3}{4} 8 8\frac{1}{4} 8\frac{1}{2}
```

He measures 20 flowers, but he forgets to draw $\times$'s for flowers of heights $7\frac{1}{4}$, 8, and $8\frac{1}{4}$ inches. He also forgets to write a title for the line plot.

a) What title should Ed write for the line plot?
b) There are the same number of flowers with heights 7 1/4, 8, and 8 1/4 inches. How many $\times$s should Ed draw for each of these heights?

c) Copy and complete the line plot.

d) Are most of the flowers taller than 7 3/4 inches or shorter than 7 3/4 inches?

**Answers**

a) Heights of Flowers

b) 4, 20 flowers measured $-8$ marked $= 12$ not marked, $12 \div 3 = 4$ for each height

c) teacher to check

d) taller

(MP2) 3. Nora measures a pencil to the nearest inch and gets 3 inches. Ken measures the same pencil to the nearest quarter inch and gets 2 1/4 inches. Can they both be correct? Explain.

**Answer:** No. If Nora is correct, the pencil is at least 2 1/2 inches. If Ken is correct, the pencil has to be less than 2 1/2 inches.

Look for students to reason about the real-world situation (MP2) by thinking, for example, about how long the pencil needs to be if it is closest to 3 inches when measured to the nearest inch.

Redirecting students: Suggest that students use Nora and Ken’s measurements to decide if the pencil is longer or shorter than 2 1/2 inches.
MD3-29 Feet and Yards
Pages 152–154

STANDARDS
preparation for 3.MD.C.6, 3.OA.A.3

VOCABULARY
feet
to the nearest foot
yard (yd)
yardstick
foot (ft)
measuring tape
ruler
yardstick

Goals
Students will estimate lengths in feet by using their arms and measure
distances in feet by using rulers, yardsticks, or measuring tapes.
Students will estimate distances in yards by using giant steps and will
measure distances in yards by using yardsticks or measuring tapes.

PRIOR KNOWLEDGE REQUIRED
Can estimate lengths in inches by using two fingers
Can multiply by 3
Understands the concepts of linear measurement and estimation
Understands that we use standard units to measure
Can estimate and measure in inches and in meters

MATERIALS
rulers of length 1 foot or longer
measuring tapes that show both inches and feet
yardsticks
masking tape

Introduce feet. Show students the length and marking of a foot on a ruler,
a yardstick, or measuring tape. Explain that a foot is another standard unit
of measurement. Write the word “foot” and the short form “ft” on the board,
and explain that these are two ways to write foot. SAY: The name comes
from the average length of a foot of an adult man, from heel to toe. The
plural of the unit foot for measurement is feet, just as for the body parts,
and we write the short form as “ft” for feet as well. Add “feet” to the board,
as shown below:

foot ft feet

Introduce foot as compared with inch. Remind students that they used
their fingers to estimate small lengths and that two fingers together are
about an inch wide. To estimate how wide a book is, for example, students
could measure the book with their fingers. ASK: Would it be easy to
estimate the length of the blackboard by using your fingers? (no) Why
not? (It is too long.) Point out that, if they want to estimate the length of
the blackboard in feet, they need something that is about a foot long. Their own
feet are probably too small, but they can use their arms instead.
ACTIVITY 1

Developing a personal estimate for 1 foot. Students work in pairs to measure the length of each other’s arms by using a ruler, a yardstick, or a measuring tape. Students measure their arms first from elbow to wrist, then from elbow to knuckles, and then from elbow to fingertips. Which of these three lengths is closest to one foot? Point out that the answer is different for different students, and they can each have their own personal tool for estimating in feet.

Estimating length in feet. Ask a volunteer to explain how to record an estimate of an object’s length in feet when the length is not a whole number. (Round numbers up or down to the closest number and use expressions such as “about 4 feet,” “3 feet to the nearest foot”) Ensure students understand that you use the closest whole number of feet and use the higher number if the length is exactly halfway between the two whole numbers. Have students use their arms to estimate the lengths or widths of objects such as a blackboard, a long desk, a table, and a door. Remind them to use their own personal measurements for feet. Then have students use rulers, yardsticks, or measuring tape to take actual measurements of the objects to the nearest foot.

Checking how many inches are in one foot. Give each student or pair of students a measuring tape showing both feet and inches. Ask: How does the measuring tape show feet and inches at the same time? How can you tell how many inches are in a foot? NOTE: The answer will depend on the measuring tape used. Some have measurements in feet and inches written beside the hash mark for feet, and some have only the feet measurement written for those hash marks. If some of the tapes you are using are of the second type, students will need to look at the numbers on both sides of the 1-foot mark to see the marks for 11 inches before and 13 inches after to conclude that 1 ft = 12 in.

More than or less than one foot. Point at several objects in the classroom, and ask whether they are longer or shorter than 1 foot. Students can signal the answer by showing thumbs up for longer and thumbs down for shorter. Then point to several other objects in the classroom, some clearly shorter than a foot and some longer than a foot. For each one, Ask: Would you measure this in inches or feet? Why? Students should see that for objects smaller than a foot, it is better to measure in inches because a foot is too long. Explain that for objects longer than one foot, it is usually easier and faster to measure in feet. Have a volunteer try to measure in inches (with a ruler or with two fingers) something in the classroom much longer than a foot, such as the side-to-side measurement of a window or a blackboard. Then have the volunteer measure the same thing in feet. Students should see that measuring in feet is much faster and easier.

Introduce yards. Show students a yardstick, and tell them that the length of the yardstick is one yard. Explain that a yard is another unit for measuring length. Point out that a yard is longer than a foot. Ask students to give
examples of what could be measured in yards. (sample answers: distances, sports fields, swimming pools) Write the word “yard” and the abbreviation “yd” on the board, and explain that these are two ways to write “yard.”

SAY: If something measures to 2 or more yards, we can still write “yd” as the short form. Add “yards” to the board as shown below:

yard   yd   yards

ASK: What could you use to estimate in yards? (giant steps) PROMPT: What did you use to estimate meters and centimeters? (giant steps and fingers)

What unit is a yard closer to, a meter or a centimeter? (meter)

ACTIVITY 2

Giant steps are about a yard long. Using masking tape, make two long lines on the floor, 1 yard apart. Students stand with their heels against one of the lines, facing the other line. Students make a step so that the heel of the foot going forward touches the second line. Explain that they just took a step 1 yard in length. Ask students to repeat the step several times and then try to walk forward three yard-long steps. Students should mark the place where they finished and then use a yardstick to measure the distance from the starting line to the mark. How close were they to 3 yards? Discuss whether a giant step is a convenient tool to estimate distances. Explain that when the answer does not have to be exact, we can save time by using giant steps instead of measuring exactly with yardsticks or measuring tape.

More than or less than one yard. Point at several objects or distances in the classroom and ask whether each is longer or shorter than 1 yard (for example, the side-to-side measurement of the blackboard, the height of the door, the distance from one desk to the door). Students can signal the answer by showing thumbs up for longer and thumbs down for shorter.

Converting yards to feet. Have students look at a yardstick. ASK: Does it have marks for feet? (yes) How many feet are in 1 yard? (3) Draw a line on the board 1 yard long by using a yardstick. SAY: This line is 1 yard long. Write the markings for each foot as shown below:

1 ft 1 ft 1 ft

ASK: How many feet will be in 2 yards? (6) Extend the line to show 2 yards long, and mark off the feet so students can see that 2 yards equals 6 feet. ASK: How can we find how many feet are in 3 yards? Students might suggest counting by threes or multiplication by 3.

Exercise: Fill in the measurements in feet.

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Answers: 3, 6, 9, 12, 15

ASK: To change a measurement from yards to feet, what number do you have to multiply by? (3) PROMPT: Look for a pattern in the table. ASK: Why does multiplying by 3 work? (there are 3 feet in 1 yard)

Exercises: How long is the object in feet? Hint: Multiply the length in yards by 3 to find the number of feet.

a) A car, 5 yards long  b) A hallway, 7 yards long  c) A tree, 9 yards tall

Answers: a) 15 ft, b) 21 ft, c) 27 ft

Solving word problems involving length measurements. Write on the board:

Ben walks 312 yards on Monday. He walks 408 yards on Tuesday. How many yards does he walk in total?

Have a volunteer read the question aloud. ASK: How can we find the answer? Should we add, subtract, multiply, or divide? (add) How do you know? (the question asks for the “total” of two distances) Have a volunteer add the distances by lining up the digits vertically and regrouping.

\[ 312 + 408 = 720 \text{ yards} \]

Continue writing on the board:

How much farther does Ben walk on Tuesday than on Monday?

ASK: How do we find the answer? Should we add, subtract, multiply, or divide? (subtract) How do you know? (the question asks “how much farther” so we are looking for the difference) Have a volunteer subtract the distances by lining up the digits vertically and regrouping.

\[ 408 - 312 = 96 \text{ yards} \]

Write on the board:

A small train is made up of 8 cars. Each car is 9 yards long. How long is the train?

ASK: How do we find the answer? Should we add, subtract, multiply, or divide? (multiply) PROMPT: If the train was made up of just 1 car, how long would the train be? (9 yards) If the train had 2 cars? (9 \times 2 = 18 \text{ yards}) 5 cars? (9 \times 5 = 45 \text{ yards}) Have a volunteer write the multiplication on the board for the question about 8 cars. (9 \times 8 = 72 \text{ yards})

Write on the board:

Kelly’s fence is made of 5 parts of equal length. The fence is 35 ft long altogether. How long is each part?
ASK: How do we find the answer? Should we add, subtract, multiply, or divide? (divide) Prompt students by drawing the picture below:

\[ \begin{array}{c}
35 \text{ ft} \\
\hline
? \text{ ft}
\end{array} \]

SAY: The fence is divided into 5 equal parts, so we have to divide the total length of the fence by 5 to find how long each part is. Have a volunteer write the division on the board. \(35 ÷ 5 = 7\) ft

**Exercises:** Add, subtract, multiply, or divide to solve the problem.

1. Jen swims 53 yards on Tuesday and 37 yards on Wednesday.
   a) How many yards did she swim altogether?
   b) How much farther does Jen swim on Tuesday than on Wednesday?
   **Answers:** a) \(53 + 37 = 90\) yd; b) \(53 - 37 = 16\) yd

2. 6 bikes fit end to end along a fence. The bikes take up the entire fence. Each bike is 5 feet long.
   a) How long is the fence in feet?
   b) The fence is divided into 3 sections of equal length. How long is each section?
   **Answers:** a) \(6 \times 5 = 30\) ft, b) \(30 ÷ 3 = 10\) ft

**Extensions**

**NOTE:** For an extra challenge in Extensions 1 to 3, have students attempt part b) without showing them part a).

1. Jon needs 3 yards of a fabric that costs $4 for each foot.
   a) How many feet of the fabric does Jon need?
   b) How much will Jon have to pay?
   **Answers:** a) \(3 \times 3 = 9\) ft, b) \(9 \times 4 = $36\)

2. The outside wall of a store is 72 feet wide. There are parking spots beside the entire wall. Each parking spot is 3 yards wide.
   a) How wide are the parking spots in feet?
   b) How many parking spots fit along the wall?
   **Answers:** a) \(3 \times 3 = 9\) ft, b) \(72 ÷ 9 = 8\) parking spots
3. Ron runs 347 yards on Saturday and 428 yards on Sunday. Sarah runs 800 yards over the weekend.

   a) How far does Ron run over the weekend?
   b) How much farther does Sarah run than Ron?

   Answers: a) \( 347 + 428 = 775 \text{ yd} \), b) \( 800 - 775 = 25 \text{ yd} \)

4. Have students convert measurements given in feet into measurements in yards. Remind students that there are 3 feet in 1 yard, and explain that they will need to divide by 3 to change a measurement from feet into yards.

   a) \( 9 \text{ ft} = \) ___ yd
   b) \( 12 \text{ ft} = \) ___ yd
   c) \( 24 \text{ ft} = \) ___ yd
   d) \( 18 \text{ ft} = \) ___ yd

   **Bonus**
   e) \( 90 \text{ ft} = \) ___ yd
   f) \( 240 \text{ ft} = \) ___ yd

   Answers: a) 3, b) 4, c) 8, d) 6, Bonus: e) 30, f) 80
Measuring in Half Inches

1. How long is the line?
   a) \[4 \frac{1}{2}\] inches
   b) \\
   c) \\

2. Use a ruler to measure the length of the worm.
   a) \\
   b) \\
   c) \\
   d) \\

NAME ___________________________ DATE ____________________
Half-Inch Measurement Cards

1 in

1\(\frac{1}{2}\) in

2 in

2\(\frac{1}{2}\) in

3 in

3\(\frac{1}{2}\) in

4 in

4\(\frac{1}{2}\) in
Measuring in Quarter Inches

1. How long is the line?
   a) \[\frac{3}{4}\] in
   b) \[\text{in}\]
   c) \[\text{in}\]

2. Measure the lines to the nearest quarter of an inch.
# Lengths of Shoes Line Plot

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This Unit in Context

In 3.1 Unit 6, students measured area using square centimeters and square meters. In this unit, students will measure area using square inches, square feet, and square yards (3.MD.C.6). As well, students will review the commutative property of multiplication (3.OA.B.5) in the context of finding the area of a rectangle (3.OA.C.7a).

Students will explore the additive property of area, especially in situations where a shape can be split into non-overlapping rectangles (3.MD.C.7d). They will learn two different ways of finding the area of a rectangle: first by multiplying the length and width of the rectangle directly, and then by splitting the rectangle into two smaller rectangles and finding and adding the areas of the two smaller rectangles. In this way, students will explore the distributive property of multiplication (3.MD.C.7c). Understanding multiplication through this type of area diagram will prepare students for using area models when they multiply two-digit numbers by two-digit numbers in Grade 4 (4.NBT.B.5) and when they multiply linear expressions, such as \((x + 3)(2x + 4)\), in high school (HSA.APR.A.1).

In 3.1 Unit 6, students were introduced to the ideas of perimeter (3.MD.D.8) and area (3.MD.C.5). In this unit, students will take those understandings further by exploring the difference between perimeter and area. They will find rectangles that have the same perimeter but different areas and those that have the same area but different perimeters (3.MD.D.8).

Students will solve real-world word problems relating to area. In particular, students will solve problems that involve arrays of unit squares by using the language of rows and columns, as described in Table 3 of the K–5 Progressions on Operations and Algebraic Thinking (3.OA.D.8).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1 to MP7. Here are some examples of how students can show that they have met a standard.

**MP1**: In MD3-36 Extension 5, students make sense of a non-routine problem by trying different starting numbers. They persevere when they try different approaches until they find one that works.

**MP2**: In MD3-35 Extension 4, students reason abstractly when they recognize the area of Sally’s rectangle is 21 square centimeters and the area of Jack’s rectangle is 21 square inches (because \(3 \times 7 = 21\)). They reason quantitatively when they argue that 21 square inches is bigger than 21 square centimeters because each square inch is bigger than each square centimeter.
MP.4: In MD3-31 Extension 4, students model a real-world situation when they solve a multi-step word problem and show their answer using equations. Students use both multiplication and division to solve the problem, and may also use addition.

MP.7: In MD3-38 Extension 5, students look for and make use of structure when they split a rectangle into two smaller rectangles to find the total area.
Unit 7 Measurement and Data: Area in US Customary Units

Introduction

In this unit, students will measure area using square inches, square feet, and square yards. They will review finding the area of a shape by counting tiles or square units and by using more advanced methods for rectangles: by skip counting rows or columns to find the total number of square units and by multiplying length times width. Students review the commutative property of multiplication in the context of finding the area of a rectangle.

Students will explore the additive property of area, especially in situations where a shape can be split into two non-overlapping rectangles. They will also learn to split a large rectangle into two smaller rectangles to find the area of a large rectangle in two different ways: by multiplying the length and width of the larger rectangle directly, and by finding and adding the areas of the two smaller rectangles. In this way, students will explore the distributive property of multiplication. Students will employ these skills to solve real world problems relating to area.

Students will review perimeter with special attention to the difference between perimeter and area. They will find rectangles that have the same perimeter but different areas and those that have the same area but different perimeters.

NOTE: It is a standard convention to call the length of the longer side of rectangles the “length” and the length of the shorter side the “width.” In this unit, students will continue to use the terms in this way.

Materials. Use a large ruler to draw shapes on the board throughout this unit.

In addition to the BLMs provided at the end of this unit, the following Generic BLMs, found in section V, are used in Unit 7:

BLM 1 cm Grid Paper (p. V-1)
BLM Inch Rulers (p. V-3)
BLM 1 in Grid Paper (p. V-6)
MD3-30  Area in Square Inches

Pages 155–156

STANDARDS
3.MD.C.6

VOCABULARY
area
square inch (in²)
square units

Goals
Students will measure areas of shapes in square inches by counting the number of square inches that cover the surface of the shape.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape
Can measure the area of a shape (in square centimeters) by counting square units

MATERIALS
a set of shapes cut out from BLM Measuring Area in Square Inches (1) (p. S-68)
pattern block squares or squares from BLM 1 in Grid Paper, 10 per student (p. V-6)
inch rulers or rulers from BLM Inch Rulers (p. V-3)
BLM Measuring Area in Square Inches (pp. S-68–69)

Review area. Show students the shapes cut out from BLM Measuring Area in Square Inches (1). ASK: Which of these shapes do you think uses the most amount of paper? Accept all answers and explain that students will find the answer later in the lesson. Remind students that when they want to measure how much paper a shape needs, they want to measure the area of a shape. The number of squares of the same size that covers the shape is the area of the shape. The squares are called square units.

Introduce square inches. Ask students what specific square units they have used to measure areas so far. (square centimeters, square meters) Hold up a pattern block square or a square from BLM 1 in Grid Paper. SAY: This is a pattern block square. It is a square with sides 1 inch long, so we call it a square inch. Draw a square inch on the board using a ruler for accuracy. Have a volunteer measure the sides of the square inch with a ruler to verify they are each 1 inch long. Write “square inch” on the board. ASK: What is the short form for the word inch? (in) Write “in” on the board. SAY: There is a short form for square inch too. We write the letters “in” and then a small “2” at the top. Add the superscript 2 to the “in” on the board as shown below:

square inch  in²

Ask students where they have seen a similar short form. PROMPT: What is the short form for square centimeters and square meters? (cm² and m²) ASK: What do you think the area of this square on the board is in square inches? (1 square inch) Hold up the pattern block square again and SAY: Remember, this is 1 square inch, so this square should cover the square on the board exactly. Hold the pattern block square over the square inch on the board to cover it exactly.
**ACTIVITY 1**

**Measuring the sides of a pattern block square.** Provide each student (or pair of students) with an inch ruler or a ruler from **BLM Inch Rulers**. Give each student at least one pattern block square (or a square from BLM 1 in Grid Paper). Students measure the length of each side of the square using their ruler to make sure each side is really 1 inch long.

**Measuring in square inches using pattern block squares.** SAY: We can use pattern block squares to measure the area of shapes in square inches.

**ACTIVITY 2**

**Measuring area using pattern block squares with gridlines given.** Give each student a copy of **BLM Measuring Area in Square Inches (1)** and 10 pattern block squares or 10 squares from BLM 1 in Grid Paper. Students cover the shapes with their pattern block squares and count how many pattern block squares they use for each shape. SAY: This is the area in square inches. Students write the area of each shape on the BLM. (1. a) 7, b) 10, c) 4, d) 7)

Ask students if they can figure out an easier way to find the area. Remind students that they have measured area before but with centimeters. ASK: Do we need to use the pattern block squares or can we just count the square inches on the shapes? (count the square inches on the shapes) PROMPT: Look at the gridlines that divide the shapes on the BLM.

Have students complete **Question 1** on AP Book 3.2 p. 155.

**ACTIVITY 3**

**Measuring area using pattern block squares without gridlines.** Give each student a copy of **BLM Measuring Area in Square Inches (2)** and 10 pattern block squares or 10 squares from BLM 1 in Grid Paper. On this page of the BLM, the gridlines are not given but tick marks are, so students must be more careful to place their pattern block squares correctly, without overlapping or leaving gaps. Students cover the shapes with their pattern block squares and count how many pattern block squares they used. This is the area in square inches. Students write the area of each shape on the BLM. (2. a) 4, b) 9, c) 6, d) 10)

Ask students if they can figure out another way to find the area without using pattern block squares. Remind students that they have used this method before to measure area. (use an inch ruler to make lines extending from the tick marks to divide the shape into square inches, then count the square inches) Give each student an inch ruler or a ruler from BLM Inch Rulers and have them use their rulers to extend the tick marks to divide each shape on BLM Measuring Area in Square Inches (2) into square inches.
Have students complete **Questions 2–3** on AP Book 3.2 p. 156. Discuss different strategies for solving Question 3. Ensure the following two strategies emerge from the discussion: placing pattern block squares to cover the shapes, and extending tick marks to make grid lines using a ruler and then counting the squares. Encourage students to use the second method for Question 3. Some students might be able to count the squares without extending the grid lines.

**Extensions**

1. Have students use pattern blocks to measure in square inches the areas of different objects with flat surfaces in the classroom, such as books, sheets of paper, desks, and tables. Encourage students to choose objects that are of a size that can be reasonably measured with inches. **ASK:** Which objects are so large that it becomes very difficult to measure them with square inches? (sample answer: the classroom floor) For the smaller objects, how do you measure them if you don’t have enough pattern blocks? **PROMPT:** How did you measure using squares and fingers earlier in the year? (place one pattern block down and then another and so on, then lift those already counted to re-use them to finish covering the area) **ASK:** What do you do if you cannot cover the object with an exact number of square inches? (estimate) Show students how to estimate the remaining area that is not covered.

2. Maria and Ali make paintings on paper. The area of Maria’s painting is 88 in². The area of Ali’s painting is 70 in².
   a) How many more square inches of paper does Maria use than Ali?
   b) How much paper (in square inches) do they use altogether?
   **Answers:** a) 18 in², b) 158 in²

3. Yu and her friends make small wooden shelves. The top of Yu’s shelf has an area of 221 in². The top of Ted’s shelf has an area of 270 in².
   a) What is the total of the area of the tops of both shelves?
   b) Yu, Ted, and Leo paint the top of their shelves using a small can of paint that can cover 810 in². There is exactly enough paint to cover all three shelves. What is the area of the top of Leo’s shelf?
   **Answers:** a) 221 + 270 = 491 in², b) 810 − 491 = 319 in²

4. Give each student 10 pattern block squares or squares from BLM 1 in Grid Paper. Have them make shapes with an area of 7 in². They should try to make as many different shapes as they can with this given area. Have students draw sketches of the shapes they make. Repeat this exercise with different areas that you choose between 1 and 10 in².

5. One yard of cloth costs $6. Ivan pays for cloth with four $20 dollar bills and gets $8 back. How much cloth does he buy? Explain how you know. Show what each step means in the situation.
Sample solution: Ivan pays $80 because $4 \times 20 = 80$. He gets $8$ back, and $80 - 8 = 72$, so the cloth costs $72$. To find out how many yards of cloth he buys, divide $72 \div 6$, which is $(60 \div 6) + (12 \div 6) = 10 + 2 = 12$. Ivan buys 12 yards of cloth.

Look for students to use strategies to help them understand the problem (MP1), such as drawing a picture or otherwise recording given information; to persevere in solving the problem (MP1); to model the situation (MP4) with equations; to interpret the result of each computation in terms of the situation (MP2); and to recognize and use structure (MP7) in multiplying tens ($4 \times 2$ is 8, so $4 \times 20$ is 80) and in dividing $72 \div 6$. 
### MD3-31 Area in Square Feet and Square Yards

**Pages 157–158**

**STANDARDS**

3.MD.C.6

**VOCABULARY**

area
square feet (ft²)
square foot (ft²)
square inch (in²)
square yard (yd²)

**Goals**

Students will learn to use square feet and square yards to measure areas.

Students will solve simple word problems involving square feet and square yards.

**PRIOR KNOWLEDGE REQUIRED**

Understands area as the number of square units that cover a shape

Can measure area of a shape in square inches by counting square units

Can multiply a one-digit number by a multiple of 10

**MATERIALS**

ruler showing inches, at least 1 foot long
scissors
sheets of newspaper larger than 1 ft by 1 ft, one per pair of students
foot-long rulers, yard sticks, or measuring tapes, one per pair of students
masking tape
paper square yard (cut out from wrapping paper or newspaper) for demonstration

**Introduce the unit “square foot.”** Review square inches. Using a ruler, draw a square on the board that is 1 inch by 1 inch. SAY: Each side of this square is 1 inch long. ASK: What do we call this square? (a square inch) What is a short way to write 1 square inch? (1 in²) Write “1 in²” beside the drawing of the square inch.

Use a ruler to draw a square on the board that is 1 foot by 1 foot. SAY: Each side of this square is 12 inches long or 1 foot long. ASK: What do you think this square is called? Explain to students that a square with sides 1 foot long is called a square foot. SAY: If we have more than one of these squares, we say square feet. Tell students that, just like “in²” is a short way to write square inch and square inches, there is also a short way to write square foot and square feet. ASK: What do you think the short form is for writing a square foot or square feet? (ft²) Write “1 ft²” beside the drawing of a square foot. The final picture should look like this:

- 1 in²
- 1 ft²
SAY: Square feet are another standard unit used to measure area. Point out how much larger a square foot is than a square inch. Explain that square feet are used to measure larger areas than square inches.

**ACTIVITY**

**Making a square foot.** Give each pair of students a pair of scissors, a sheet of newspaper larger than 1 ft by 1 ft, and a foot-long ruler or a yard stick. Demonstrate how to use a ruler and pencil to draw a square foot on the paper:

**Step 1:** Open the newspaper spread so that the crease is vertical, as if you are going to read the pages.

**Step 2:** Using the ruler, measure 12 inches down the left edge of the paper, starting from the top edge, and make a tick mark.

**Step 3:** Measure 12 inches down along the crease, from the top edge, and make another tick mark.

**Step 4:** Draw a horizontal line joining the two tick marks, and extend the line so that it is 12 inches long from the left edge to the end.

**Step 5:** Measure 12 inches across the top edge of the paper, starting from the top left corner. Make a tick mark there. Draw a vertical line connecting this tick mark to the end of the 12-inch horizontal line.

Student 1 draws a square foot on the paper. Both students measure the sides of the square to make sure they are each 1 foot long. Student 2 cuts out the square with a pair of scissors. Again, both students check the measurements to make sure it is a square foot.

**Measuring with square feet.** Create a 2 ft by 3 ft rectangle on the floor using masking tape. Have a couple of volunteers measure the area in square feet using as many of the paper squares made in the activity above as they need. Ensure students lay the paper squares with no gaps or overlaps. They should find that 6 square feet cover the rectangle.

ASK: What is the area of this rectangle? (6 ft²)

Show students a square yard cut out of paper, but don’t tell them it is a square yard. ASK: How many square feet do you think would fit on this large sheet of paper? Have a few different volunteers measure the area with their paper squares. ASK: What is the area of this large sheet of paper in square feet? (9 ft²)

**Introduce the unit “square yard.”** Hold up the paper square yard again and ASK: How long do you think the sides of this square are? Lay the sheet of paper on the floor. Have a volunteer measure the sides using a yardstick or measuring tape. SAY: The length of each side is 3 feet, or 1 yard. ASK: What do you think we call a square with sides 1 yard long?
Tell students that a square with sides 1 yard long is called a square yard. Write “square yard” on the board. ASK: What do you think is the short form for writing square yards? (yd²) Continue writing on the board:

square yard       yd²

Point out that a square yard is much larger than a square foot, so we can use square yards to measure very large areas. ASK: Which unit would be easier to use to measure the area of the classroom floor, square feet or square yards? (square yards) Which unit would you use to measure the area of the top of your desk, square feet or square yards? (square feet)

**Word problems with square feet and square yards.** Write on the board:

Madison’s class makes a large poster using 10 square sheets of paper side by side. Each sheet of paper is 1 yd². What is the area of the poster in square yards? Each sheet of paper costs $3. How much money will Madison’s class spend on the paper?

Have a volunteer read the problem. Point to the first question and SAY: To find the area of the poster in square yards, we need to know how many square yards fit on the poster. ASK: How many square sheets of paper did the class use for the poster? (10) Is each sheet 1 yd²? (yes) So what is the area of the poster in square yards? (10 yd²) Write “Area = 10 yd²” beside the first question on the board.

Point to the second question and ASK: What operation do we use to find the cost of all the paper for the poster? (multiplication) What numbers do we multiply? (the number of sheets and the cost of each sheet) How many sheets are there? (10) How much does each sheet cost? (3 dollars) What is 10 × 3? (30) Write “10 × 3 = $30” beside the second question on the board.

Write on the board:

The area of Peter’s carpet is 30 ft². The area of Ravi’s carpet is 42 ft². How much larger is Ravi’s carpet than Peter’s?

Have a volunteer read the problem. ASK: What are we asked to find? (how much larger is Ravi’s carpet than Peter’s) What operation should we use, addition, subtraction, multiplication, or division? (subtraction) PROMPT: We are asked to find the difference between two numbers. Have a volunteer write the subtraction on the board and solve it. (42 − 30 = 12 ft²)

**NOTE:** Students will need to multiply a multiple of 10 by a one-digit number in the next set of exercises and in other exercises throughout the unit. You might need to remind students how to do this (see Lesson NBT3-16).
Exercises: Lani and Jim are putting tiles on their kitchen floors. Each tile is 1 ft². Each tile costs $5.

a) Lani uses 80 tiles on her floor. What is the area of her kitchen floor?

b) The area of Jim’s kitchen floor is 90 ft². How many tiles do Lani and Jim use altogether?

c) How much money does Lani spend on tiles?

d) How much money does Jim spend on tiles?

e) How much money do they spend altogether?

Answers: a) 80 ft², b) 80 + 90 = 170, c) 80 × 5 = $400, d) 90 × 5 = $450, e) 400 + 450 = $850

Extensions

1. Have students use paper square feet to measure the area of objects in the classroom that have flat surfaces and that can be measured easily in square feet, such as desks, tables, the chalkboard, the door, windows, posters on the wall, or a small mat or carpet on the floor. If students did Extension 1 in MD3-30, they should choose different, larger objects to measure. Remind students how to place one square foot down and then another and so on, and then to lift those already counted to re-use them to finish covering the area. You might have students work in pairs. ASK: Which objects are so large that it becomes very difficult to measure with square feet? (sample answers: the door, windows, the chalkboard) What do you do if you cannot cover the object with an exact number of square feet? (estimate) Show students how to estimate the remaining area that is not covered.

2. The area of Ethan’s garden is 6 yd². It takes 9 square feet to cover 1 square yard.

a) What is the area of Ethan’s garden in square feet?

b) The area of Mona’s garden is 8 yd². What is the area of her garden in square feet?

c) How much larger is Mona’s garden than Ethan’s garden in square feet?

Answers: a) 6 × 9 = 54 ft², b) 8 × 9 = 72 ft², c) 72 − 54 = 18 ft²

3. The area of the field at School A is 36 yd². The field is divided into 4 equal sections. The area of the field at School B is 54 yd². The field is divided into 9 equal sections.

a) What is the area of each section of the field at School A?

b) What is the area of each section of the field at School B?

c) How much larger are the sections at School A than at School B?

Answers: a) 36 ÷ 4 = 9 yd², b) 54 ÷ 9 = 6 yd², c) 9 − 6 = 3 yd²
4. Six cars fit end-to-end along a fence. The cars take up the entire fence. Each car is 9 feet long. The fence is divided into 3 equal sections. How long is each section of the fence? Show your answer using equations.

Sample solutions

- There are 6 cars that are each 9 feet long, so I found $6 \times 9 = (5 \times 9) + 9 = 45 + 9 = 54$. The fence is 54 feet long. There are 3 sections, so I found $54 \div 3 = (30 \div 3) + (24 \div 3) = 10 + 8 = 18$. Each section of the fence is 18 feet long.
- Three sections of the fence fit 6 cars, so I found $6 \div 3 = 2$. Each section of the fence fits 2 cars. Each car is 9 feet long, so I found $2 \times 9 = 18$. Two cars together are 18 feet long. So each section of the fence is 18 feet long.

Look for students to use a strategy that helps them understand the problem (MP.1), such as drawing a picture or otherwise recording given information, and to persevere in solving the problem. Look for students to model the situation (MP.4) with multiplication and division, to notice and use structure (MP.7) to help them calculate the answer, and to use the equal sign and brackets (MP.6). For example, $6 \times 9 = 54 \div 3 = 18$ is incorrect, while correct versions include:

- $6 \times 9 = 54$ and $54 \div 3 = 18$
- $6 \times 9 \div 3 = 54 \div 3 = 18$
- $6 \times 9 \div 3 = 6 \times 3 = 18$

Redirecting students: If students struggle with how to begin (MP.1), ASK: What information do you need to find? (how long each section of the fence is) What information do you need to answer that? (how long the whole fence is and how many sections there are)

If students struggle to calculate $54 \div 3$, ask them if they can decompose 54 into smaller numbers to make the problem easier (MP.7).

Whole class follow-up: Have volunteers share their strategies. Then compare the strategies. For example, ASK: How does this person’s picture show that person’s multiplication? How does this person’s picture show that person’s division? If time permits, show a more efficient method that allows students to calculate with smaller numbers.
MD3-32 Finding Area by Skip Counting
Pages 159–160

Goals
Students will skip count rows or columns to find the area of rectangular arrays where each square in the array represents a square inch, foot, or yard. Students will then multiply the number of rows and columns as a faster way to find the area of these rectangular arrays.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape Can measure area of a shape (in square inches, feet, and yards) by counting square units Can skip count as a shortcut for repeated addition Can multiply as a shortcut for repeated addition

Review area. Remind students that when they want to measure how much paper a shape needs, they want to measure the area of a shape. The number of squares of the same size that covers the shape is the area of the shape. The squares are called square units.

Dividing rectangles into arrays of squares. Draw a rectangle on the board that measures four units down and five units across. The actual length of the units you use can be anything convenient. Remind students that you can draw lines to divide the rectangle into an array of squares, where all squares are the same size. Make tick marks along the edges of the rectangle with a ruler and draw lines to connect the tick marks, as shown below:

Say: The number of squares in this array is the area of the rectangle. All you need to do to find the area is to count the squares. Ask a volunteer to count the squares in the array on the board. Ask the volunteer to write the area as the total number of squares on the board beside the array. (Area = 20 units)

Review skip counting rows to find the area of a rectangle. Point to the array on the board and ASK: How can we use skip counting to count the number of squares in this array faster? (count the number of squares in one row and then skip count the number of rows) Point to the first row of the array and ASK: How many squares are in the first row? (5) Add “5” to the picture on the board to the right of the first row. ASK: How many more squares are in the next row? (5 more) So how many squares are in the first two rows? (5 + 5 = 10) Write “5 + 5 = 10” on the board to the right of the
second row. Continue in this manner with the third and fourth row. The final picture should look like this:

\[
\begin{array}{c|c|c|c|c|c|c|c}
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
\hline
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
\hline
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
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5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
\hline
\end{array}
\]

Area = 20 units

SAY: We are adding five each time. This means we can skip count to find the total number of squares in the array, which tells us the area of the rectangle. Let’s pretend each small square is 1 square yard. ASK: What is the area of the rectangle? (20 yd²) Write the area of the rectangle on the board:

Area = 20 yd²

Draw an array of four rows with two squares in each row. ASK: How many squares are in each row? (2) Which number can we skip count by to count the squares? (2s) Have students skip count by 2s as a class and write the number of squares beside each row. (2, 4, 6, 8) SAY: In this picture, each small square is 1 square foot. What is the area of the rectangle? (8 ft²)

**Exercises:** Each small square is 1 ft². Skip count the rows to find the area of the large shape.

a) 

b) 

c) 

**Answers:** a) 30 ft², b) 14 ft², c) 18 ft²

**Skip counting columns to find the area of an array.** Return to the array on the board that is four rows by five columns. Erase all the numbers beside the array but keep the area sentence (Area = 20 yd²). SAY: Another way to count the number of squares quickly is to count the number of squares in one column and then skip count the number of columns. Trace your finger down the first column. ASK: How many squares are in the first column? (4) Are there the same number of squares in each column? (yes) What number do we skip count by as we count along the columns? (4) Write the numbers on the board, as shown below:

\[
\begin{array}{c|c|c|c|c}
4 & 8 & 12 & 16 & 20 \\
\hline
\end{array}
\]

Area = 20 yd²
ASK: How many square units are in the array? (20) Remind students that each square represents 1 square yard. ASK: What is the area of the array? (20 yd²) Point to the sentence “Area = 20 yd²” on the board and ASK: Did we get the same answer by skip counting columns as by skip counting rows? (yes)

**Exercises:** Each small square is 1 in².

1. Skip count the columns to find the area of the large shape.

   a) 
   b) 
   c) 

   **Answers:** a) 35 in², b) 24 in², c) 36 in²

2. Skip count the rows to find the area for each part of Exercise 1. Do you get the same area each time?

   **Answer:** Yes

**Multiplying the number of rows and columns to find the area of an array.**

Draw on the board:

ASK: How many rows are in this array? (3) How many columns are there? (5) SAY: Let’s find the area by skip counting the rows. ASK: How many squares are there in each row? (5) So what number do we skip count by? (5) SAY: How many times do we need to skip count by five? (3 times) Write on the board:

\[ 5 + 5 + 5 = \]

ASK: How can we write this in a shorter way, using multiplication? (3 \( \times \) 5) Write "3 \( \times \) 5" to the right of the equal sign, as shown below:

\[ 5 + 5 + 5 = 3 \times 5 \]

ASK: What does the number 3 show? (the number of rows) What does the number 5 show? (the number of squares in each row) Is this the same as the number of columns? (yes) So we can multiply the number of rows by the number of columns to find the area. Continue writing on the board:

\[ 5 + 5 + 5 = 3 \times 5 = \text{Area} \]

SAY: Let’s pretend each square unit in this array is 1 square inch. ASK: What is the area of the array? (15 in², because 3 \( \times \) 5 = 15 in²)
Exercises: Count the number of rows and the number of columns. Multiply to find the area. Include the units.

a) 1 square unit = 1 yd²  b) 1 square unit = 1 in²  c) 1 square unit = 1 ft²

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Area = ______  Area = ______  Area = ______

Answers: a) 4 rows, 7 columns, Area = 4 × 7 = 28 yd²; b) 6 rows, 3 columns, Area = 6 × 3 = 18 in²; c) 5 rows, 6 columns, Area = 5 × 6 = 30 ft²

Extensions

1. Bo draws lines to divide a rectangle into an array of square inches. Find the missing number.

   a) ____ rows, 7 columns, Area = 42 in²
   b) 9 rows, ____ columns, Area = 72 in²
   c) 10 rows, ____ columns, Area = 80 in²
   d) ____ rows, 9 columns, Area = 81 in²

   Answers: a) 6, b) 8, c) 8, d) 9

(MP.1) 2. Carlos draws lines to divide a rectangle into an array of square inches. The area of the rectangle is 18 in². What are the possible numbers of rows and columns?

   Answers: 1 row, 18 columns; 2 rows, 9 columns; 3 rows, 6 columns; 6 rows, 3 columns; 9 rows, 2 columns; 18 rows, 1 column

(MP.4) 3. Three students make paper rectangular arrays of square inches. Ali’s array has 4 rows and 9 columns. Nina’s array has 5 rows and 8 columns. Roy’s array has 7 rows and 6 columns.

   a) How much paper does each student use?
   b) Who uses the most amount of paper?
   c) Who uses the least amount of paper?
   d) How much paper do Ali, Nina, and Roy use altogether?

   Answers: a) Ali: 4 × 9 = 36 in², Nina: 5 × 8 = 40 in², Roy: 7 × 6 = 42 in²; b) Roy; c) Ali; d) 36 + 40 + 42 = 118 in²
STANDARDS
3.MD.C.7a

VOCABULARY
area
array
column
commutative property
length
product
row
square feet (ft²)
square inches (in²)
square units
square yards (yd²)
width

Goals
Students will find the area of a rectangle with whole number side lengths by tiling it.
Students will show that the area is the same as would be found by multiplying the side lengths.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape
Can find the total number of square units in a rectangular array by skip counting rows, skip counting columns, or multiplying the number of rows and columns

MATERIALS
yardstick
inch rulers or rulers from BLM Inch Rulers (p. V-3)

Tiling a rectangle. Use a yardstick to draw a rectangle on the board that is 3 feet long and 2 feet wide. ASK: How can we find the area of this rectangle in square feet? Ensure students understand that the rectangle must be covered with squares of size 1 ft by 1 ft, and the number of these squares is the area of the rectangle (in square feet). SAY: I’m going to use the yardstick to make tick marks for square feet along the sides of the rectangle. Place the yardstick along the bottom edge of the rectangle, with 0 aligned at the left end of the edge. SAY: I have the 0 mark on the ruler exactly at the start of the line. ASK: Where should I draw the first tick mark on this line? (at the 1-foot mark) PROMPT: Remember, we want to cover this rectangle with squares with each side 1 foot long. Draw the tick mark for 1 foot. ASK: Where should I draw the next tick mark on this line? (at the 2-feet mark) Draw the tick mark. SAY: The next mark should be at the mark for 3 feet, but that mark is right at the end of the line. ASK: How long is this line? (3 feet) Write the length on the board below the bottom edge. Continue with the other sides of the rectangle in any order. The final picture should look like this:

```
   3 feet
  2 feet 2 feet
  3 feet
```

ASK: Now that we have the tick marks, how can we cover the rectangle with square feet? (join the tick marks from side to side) Remind students that they have done this before with square inches. Have a volunteer come to the board and join the tick marks using a yardstick. NOTE: In this way, students are tiling the rectangle—covering a shape with squares without overlap or gaps.
ASK: How can we find the area now? (count the squares) Have a volunteer come to the board and count the squares to find the area. (6 ft²) ASK: Did we turn this rectangle into an array of squares? (yes) Is there a faster way to count the number of squares using the rows and columns? (yes, multiply the number of columns by the number of rows) ASK: How many columns are in this array? (3) How many rows? (2) What is the area? (3 \times 2 = 6 \text{ ft}^2) Write the area on the board.

**Formula for the area of a rectangle.** Point to the rectangle on the board and ASK: What is the length of this rectangle? (3 feet) What is the width of this rectangle? (2 feet) Add labels and arrows to the equation for the rectangle’s area on the board, as shown below:

\[
\text{Area} = 3 \times 2 = 6 \text{ ft}^2
\]

SAY: The length and width of a rectangle tell you how many rows and columns there are if you fill the rectangle with square units. So if you multiply the length and the width of the rectangle, the product is the area. This means you don’t need to draw the square units, you just need to measure the length and width and multiply them. ASK: Did we get the same answer for the area of the rectangle by counting the square units as we did by multiplying the length and width? (yes) Why should we get the same answer? (because both ways tell us how many square units cover the rectangle) Write on the board:

\[
\text{Area of a rectangle} = \text{length} \times \text{width}
\]

ASK: When we are multiplying two numbers, does the order matter? (no) SAY: To find the area of a rectangle, we multiply length times width. ASK: Could we instead multiply width times length? (yes) Would we get the same answer? (yes) Write “= width \times length” on the board, as shown below:

\[
\text{Area of a rectangle} = \text{length} \times \text{width} = \text{width} \times \text{length}
\]

Remind students that the property of multiplication shown above is called the commutative property.

**NOTE:** You might also wish to remind students how the terms “length” and “width” are used: mathematicians call the length of the longer side of a rectangle the length and the length of the shorter side the width. Even with squares, which are rectangles with all four sides equal, we can describe them as having length and width, but the measurements of these are the same. Point out that some people use alternative ways of describing linear measurement. For example, width as the side-to-side measurement of a rectangle even if the side-to-side measurement of the rectangle is longer than the up-to-down measurement. The side-to-side measurement of a television screen is called the width, even though it is usually longer than the up-to-down measurement.

Have students complete **Questions 1–3** on AP Book 3.2 p. 161.
Exploring length times width as rows times columns. Draw on the board:

\[ \text{Area} = \text{___} \times \text{___} = \text{___} \]

**NOTE:** Be sure to keep the drawing proportional.

Explain to students that even though each square unit is smaller than a square yard, each square unit stands for (or shows) one square yard. Fill in the blanks as you ask students the following questions. Point to the shaded row and ASK: How many squares are in this row? (5) Will there be the same number of squares in each row? (yes) How many rows of five squares are there? (4) How do we find the total number of square yards? (multiply the number of rows times how many squares in each row, \(4 \times 5\)) What is \(4 \times 5\)? (20) SAY: So there are 20 square yards covering this rectangle. ASK: What is the area of this rectangle? (20 yd\(^2\)) The text under the picture should look like this:

\[ \text{Area} = \text{4} \times \text{5} = \text{20 yd}^2 \]

ASK: What two numbers do we always need to multiply to find the area of a rectangle? (length and width) If we multiply length and width for this rectangle, what equation do we get? (\(4 \times 5 = 20\) or \(5 \times 4 = 20\)) Is this the same multiplication as we did to find the total number of square yards that cover the rectangle? (yes) So, do we get the same area both ways? (yes)

Repeat with the rectangle below.
Point out to students that this time they are counting columns rather than rows. (3; 4; 3, 4, 12)

Have students complete Question 4 on AP Book 3.2 p. 162.

**Area of rectangles and word problems.** Write on the board:

Mia wants to hang a painting on her wall.
The painting is a rectangle 6 feet high and 2 feet wide.
What is the area of the painting?

Have a volunteer read the problem. Point to the second sentence and SAY: It says the painting is a rectangle. It also says the painting is 6 feet high and 2 feet wide. ASK: Which is the length, 6 feet or 2 feet? (6 feet) How do you know? (because 6 is the longer measurement) Write “length = ___” and “width = ___” on the board and have students signal the correct number for each. (6 ft and 2 ft) ASK: How do we find the area? (multiply length times width) Write “Area = 6 × 2 = ___” on the board and have a volunteer fill in the blanks (including units), as shown below:

\[
\text{length} = 6 \text{ ft} \quad \text{width} = 2 \text{ ft} \quad \text{Area} = 6 \times 2 = \text{12 ft}^2
\]

**Exercises:** Write the length and width of the rectangle. Multiply to find the area. Include the units.

a) A field in a park is a rectangle 6 yards wide and 7 yards long.
What is the area of the field?

b) Jenna’s sheet of paper is a rectangle 8 inches wide and 10 inches long.
What is the area of the sheet of paper?

**Answers:** a) length = 7 yd, width = 6 yd, Area = 7 × 6 = 42 yd²; b) length = 10 in, width = 8 in, Area = 10 × 8 = 80 in²

Have students complete Questions 5–7 on AP Book 3.2 p. 163.
For Question 5, provide inch rulers or **BLM Inch Rulers**.

**Extensions**

1. Rita has a rectangular painting that is 9 inches high. The area of her painting is 81 in².
   a) How many inches wide is the painting?
   b) Is this rectangle a square? How do you know?
   **Answers:** a) 81 ÷ 9 = 9 inches, b) This rectangle is a square because it is 9 inches high and 9 inches wide

2. Ava draws a rectangle. She colors one side of the rectangle blue. The blue side is 6 inches long. The area of the rectangle is 36 in².
   a) Find the measurement of each of the four sides of this rectangle.
   b) Is the rectangle a square?
c) Ben says that the blue side is the length of the rectangle. Ava says the blue side is the width of the rectangle. Who do you think is right? Explain.

Answers
a) 36 ÷ 6 = 6 inches, so each of the four sides is 6 inches long; b) yes; c) Ben and Ava are both right because the length and width are each 6 inches long, so the blue side could be called the length or the width.

3. The area of a rectangle is given, and one side is given. Find the missing side. If both sides are equal, write "square."
   a) Area = 24 ft², length = 8 ft, width = ____
   b) Area = 49 yd², length = ____ , width = 7 yd
   c) Area = 72 in², length = ____ , width = 8 in
   d) Area = 25 in², length = 5 in, width = ____

   Answers: a) 3 ft; b) 7 yd, square; c) 9 in; d) 5 in, square

4. The area of a rectangle is given. The length is a whole number of yards and so is the width. Write the possible lengths and widths. Hint: If the rectangle is a square, then the length and width are equal. If the rectangle is not a square, then the length is a larger measurement than the width.
   a) 6 yd²
   b) 5 yd²
   c) 12 yd²
   d) 9 yd²
   e) 16 yd²
   f) 1 yd²

   Answers
   a) length = 6 yd, width = 1 yd; length 3 yd, width = 2 yd
   b) length = 5 yd, width = 1 yd
   c) length = 12 yd, width = 1 yd; length = 6 yd, width = 2 yd
   length = 4 yd, width = 3 yd
   d) length = 9 yd, width = 1 yd; length = 3 yd, width = 3 yd
   e) length = 16 yd, width = 1 yd; length = 8 yd, width = 2 yd
   length = 4 yd, width = 4 yd; f) length = 1 yd, width = 1 yd

   (MP.2, MP.4, MP.6) 5. There are seven posts in a row. The first post is 5 yards from David’s house. Every post after that is 3 yards farther from the house. How far is the last post from the house? Show your work using equations. Write what each equation means in the situation.

   Answer: 23 yards, sample strategies:
   • If the first post were 3 yards from the house, the last post would be 21 yards away (because 7 × 3 = 21), but the first post is 5 yards away, so the last post is 2 yards farther than that, or 23 yards away (because 21 + 2 = 23)
   • The distance from the first post to the last post is 18 yards (because 6 × 3 = 18), so the distance from the house to the last post is 23 yards (because 18 + 5 = 23).
Look for students to model the situation (MP4) using equations and to use brackets and the equal sign (MP6). For example, “$7 \times 3 = 21 + 2 = 23$” is incorrect, while “$7 \times 3 = 21$ and $21 + 2 = 23$” and “$(7 \times 3) + 2 = 21 + 2 = 23$” are both correct. Look for students to interpret what each equation means in the context of the problem (MP2).

Redirecting students: If students struggle with how to begin (MP1), encourage them to draw a picture of all the posts and the house.
STANDARDS
3.MD.C.7b

VOCABULARY
area
commutative property
equation
length
square feet (ft²)
square inches (in²)
square units
square yards (yd²)
width

Goals
Students will further explore finding the area of a rectangle and the commutative property of multiplication in this context. Students will solve real world problems by finding the area of rectangles.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape
Can find the total number of square units in a rectangular array by skip counting rows, skip counting columns, or multiplying the number of rows and columns
Can find the area of a rectangle by multiplying length and width

MATERIALS
- cut-out array from BLM Array of Square Inches (p. S-70)
- tape
- grid paper or BLM 1 cm Grid Paper (p. V-1)
- rulers

Turning rectangular arrays on their sides to switch rows and columns. Display the 5 inch by 8 inch array cut out from BLM Array of Square Inches on the board using tape (make sure the 8 inch side is vertical). Tell students that this is a rectangle and that lines have been drawn to make an array of square inches covering the rectangle. ASK: Which column is shaded? (the first column) How many columns of square inches are there? (5) How many rows are there? (8) Write the number of rows and columns beside the array on the board, as shown below:

```
5 columns

8 rows

SAY: Let’s find the area of the rectangle by multiplying, starting with the number of rows. ASK: What equation do we get? (8 × 5 = 40 in²)
Write the equation on the board underneath the array:

\[ 8 \times 5 = 40 \text{ in}^2 \]

Number of rows

SAY: Let’s turn the array on its side. Before moving the paper array, trace the outline of the rectangle on the board so that students will be able to compare the shape with the first orientation to the shape with the new orientation. Take the array off the board and turn it clockwise 90° so that what used to be the top side is now on the right. Affix the rectangle to the board to the right side of the original position, which you have traced. The rectangle should now be five rows by eight columns, and the shaded squares should form the top row.

ASK: Did we change the size of the rectangle when we turned it? (no) Should the area be the same as it was before? (yes) Did the number of rows change? (yes) How many rows are there now? (5) Did the number of columns change? (yes) How many columns are there now? (8) Label the sides “8 columns” and “5 rows.” ASK: What happened to the rows and columns when we turned the array on its side? (they switched; the rows became columns and the columns became rows) PROMPT: What happened to the shaded first column? (it is now the shaded first row) SAY: Let’s write an equation for the area. We’ll multiply starting with the number of rows. ASK: What equation do we get? \( 8 \times 5 = 40 \text{ in}^2 \) Write the equation on the board. The final picture should look like this:

\[ 5 \text{ columns} \quad 8 \text{ rows} \quad 8 \times 5 = 40 \text{ in}^2 \]

Number of rows

SAY: We turned the array on its side and we got a different multiplication equation, but the area stayed the same. ASK: When you are multiplying two numbers, does it matter which number you write first? (no) Will you get the same answer either way? (yes) Remind the students that this is a property of multiplication called the commutative property. Leave the 5 by 8 array on the board to use later in the lesson.
ACTIVITY

Students work in pairs. Each student has a sheet of grid paper or BLM 1 cm Grid Paper. Student 1 chooses two different numbers between 1 and 10 as the number of rows and columns of an array. Student 2 draws an array on grid paper using the given number of rows and columns and writes an equation for the area using multiplication, writing the number of rows first. Student 1 then turns the rectangle on its side so that rows and columns switch. Student 1 writes a new equation for the area, writing the number of rows first. Finally, Students 1 and 2 compare their arrays and equations to see that the number of rows and columns switch but the area remains the same.

Exploring rows times columns versus columns times rows. Using a large ruler and any convenient unit of length, draw a rectangle that measures three units down and seven units across. SAY: Remember, to find the area of this rectangle, we can draw lines to cover the rectangle in square units. Demonstrate drawing these lines. The picture should look like this:

```
 | | | | | | | |
 | | | | | | | |
 | | | | | | | |
```

SAY: We made the rectangle into an array of square units. ASK: How many rows of square units are there? (3) How many columns are there? (7) How can we find the total number of square units? (multiply the number of rows by the number of columns or multiply the number of columns by the number of rows) Write both answers on the board, as shown below:

\[
\begin{array}{c}
3 \times 7 = 21 \\
7 \times 3 = 21
\end{array}
\]

Remind students that they can think of this array as three rows with seven squares in each row or as seven columns with three squares in each column. ASK: Do we get the same number of square units both ways? (yes, 21) What is the area of the rectangle? (21 square units) When we find the area of an array of square units, does it matter if we multiply rows times columns or columns times rows? (no)

Have students complete Question 1 on AP Book 3.2 p. 164.

Exploring length times width versus width times length. Return to the 5 by 8 paper array on the board. ASK: What is the length of this array? (8 inches) PROMPT: Count the number of square inches in the shaded row.

ASK: What is the width? (5 inches)
Write on the board:

\[
\text{Area} = \underline{\text{length}} \times \underline{\text{width}}
\]

Have volunteers fill in the blanks. ASK: Are the two equations we get the same as the equations we got when we multiplied rows times columns? (yes) When you find the area of a rectangle, does it matter if you multiply “length times width” or “width times length”? (no) Will you get the same area both ways? (yes)

**Exercises:** Find the area of the rectangle in two different ways.

a) length = 10 in, width = 7 in  
b) length = 8 yd, width = 5 yd  
c) length = 9 ft, width = 4 ft  
d) length = 6 in, width = 2 in

**Answers:**

a) \(10 \times 7 = 70 \text{ in}^2\); \(7 \times 10 = 70 \text{ in}^2\);  
b) \(8 \times 5 = 40 \text{ yd}^2\);  
c) \(9 \times 4 = 36 \text{ ft}^2\); \(4 \times 9 = 36 \text{ ft}^2\);  
d) \(6 \times 2 = 12 \text{ in}^2\)

**Solving word problems using the commutative property of multiplication.** Write on the board:

A garden is in the shape of a rectangle with a length of 7 ft and a width of 5 ft. Find the area of the garden in two different ways.

ASK: What is the length of the rectangular garden? (7 feet) Write “length = 7 ft” on the board. ASK: What is the width? (5 feet) Write “width = 5 ft” on the board. ASK: What two equations can we write for the area? (7 \(\times 5 = 35 \text{ ft}^2\) and 5 \(\times 7 = 35 \text{ ft}^2\)) Have a volunteer write both equations on the board:

\[
\begin{align*}
\text{length} &= 7 \text{ ft} \\
\text{width} &= 5 \text{ ft} \\
\text{Area} &= 7 \times 5 = 35 \text{ ft}^2 \\
\text{Area} &= 5 \times 7 = 35 \text{ ft}^2
\end{align*}
\]

**Exercises:** Find the area in two different ways.

1. Sun’s door is 7 feet tall and 3 feet wide.
   a) Write the length and width of the door.
   b) What is the area of the door?

   **Answers:** a) length = 7 ft, width = 3 ft; b) Area = 7 \(\times 3 = 21 \text{ ft}^2\) or Area = 3 \(\times 7 = 21 \text{ ft}^2\)

2. John buys paper for a painting. The paper measures 6 feet by 5 feet.
   a) Write the length and width of the paper.
   b) What is the area of the paper?
   c) The price of the paper is $4 per square foot. How much did John pay for the paper?
Answers: a) length = 6 ft, width = 5 ft; b) Area = 6 × 5 = 30 ft² or Area = 5 × 6 = 30 ft²; c) John paid 30 × 4 = 120 dollars

Extensions

1. Bo writes equations for the areas of rectangles. He always writes the area on the right side of the equal sign. He says there are always two different multiplication equations to write for the area. Jayden says that sometimes there is only one multiplication equation. Explain why Jayden is correct. Use an example to help explain.

Answer: Equations for area can be length × width = Area or width × length = Area (for example, 7 × 5 = 35 ft² or 5 × 7 = 35 ft²), but Jayden is correct because a rectangle that is a square (for example, 3 × 3 units) will give only one multiplication equation (3 × 3 = 9 square units).

2. Remind students how to get a division equation from a multiplication equation by reading the numbers backward. For example:

\[6 \times 3 = 18\] leads to \[18 \div 3 = 6\]
and
\[3 \times 6 = 18\] leads to \[18 \div 6 = 3\]

For problems in which the area of a rectangle is given, ask students to write a multiplication equation to give the unknown number first. Then read the multiplication equation backward to write a division equation that has the unknown length or width by itself on one side of the equation. Then have them find the unknown number.

a) Area = 42 yd², width = 6 yd, length = ?

b) Area = 90 ft², width = ?, length = 10 ft

c) Area = 56 yd², width = ?, length = 8 yd

d) Area = 48 in², width = 6 in, length = ?

Answers
a) ? × 6 = 42, 42 ÷ 6 = ?, length = 7 yd
b) ? × 10 = 90, 90 ÷ 10 = ?, width = 9 ft
c) ? × 8 = 56, 56 ÷ 8 = ?, width = 7 yd
d) ? × 6 = 48, 48 ÷ 6 = ?, length = 8 in

3. Jennifer draws a rectangle on a piece of paper. She draws lines to divide the rectangle into an array of square inches. There are 4 rows of square inches in the array. Then she turns the sheet of paper on its side. Now there are 7 rows of square inches in the array.

a) How many columns are in the turned array?

b) What is the area of the rectangle?

Answers: a) 4, b) 7 × 4 = 28 in²
4. Hanna has a picture that is 7 inches wide by 4 inches tall. She puts one-inch squares around the outside to make a frame. What is bigger, the frame or the picture? How much bigger?

Sample answer: The frame is made of $6 + 7 + 6 + 7$ or 26 square inches and the picture is made of $4 \times 7$ or 28 square inches, so the picture is 2 square inches bigger.

Redirecting students: Have students draw the picture on grid paper. Encourage them to record how big the picture and the frame are in square inches, so that they can easily compare the sizes.

Individual or small-group follow-up: Ask students who used grid paper if they could have found how many square inches are in the picture using only the numbers given in the problem and without a drawing (yes, the picture is 4 by 7, so multiply $4 \times 7$). Repeat to ask if students could do the same to find how many square inches are in the frame (yes, there are 4 along each side, 7 along the top and bottom, and 4 in the corners, so $4 + 4 + 7 + 7 + 4$).
Goals
Students will find the area of certain shapes by splitting each shape into two rectangles and adding the areas of both rectangles. Students will apply this technique to solve real world problems.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape Can find the area of a rectangle by multiplying length and width

MATERIALS
overhead projector transparency of BLM Adding Areas (p. S-71) BLM Adding Areas (p. S-71) connecting cubes of two different colors, 12 of each color for demonstration and 16 of each color per pair of students erasable markers in two colors

Finding the area of a shape made from two rectangles. Project BLM Adding Areas on the board. Cover the transparency so that only the first row, Question 1.a), is visible. SAY: Each small square is 1 square unit. For this exercise, it doesn’t matter if a square unit stands for a square inch, square centimeter, square foot, square yard, or something else. We’ll just call them square units. ASK: What is the area of rectangle A? (6 square units) Have students count the squares as you point to them, or they could count the number of rows and columns as you point to them and then multiply to find the area. Repeat for rectangle B. (12 square units) Repeat for shape C by counting the squares. (18 square units) Write the areas of each shape in the blanks on the BLM.

Using red connecting cubes, build rectangle A. Have students verify that your model is identical to rectangle A on the BLM, including the area. Use blue connecting cubes to build rectangle B and repeat the process. Then show students how to connect the rectangles you made from cubes to form shape C. (see diagram on next page) Have students verify that the shape made with red and blue cubes is identical to shape C, including the area.
**ACTIVITY**

Students work in pairs, but each student gets a copy of BLM Adding Areas. Provide 12 red connecting cubes and 12 blue connecting cubes for each pair of students. Students repeat Question 1.a) and then answer Questions 1.b), 1.c), and 1.d) in the following way: Student 1 builds rectangle A with red cubes and Student 2 builds rectangle B with blue cubes. Students find the area of their rectangles and record them in the blanks provided on the BLM. Then Students 1 and 2 join their rectangles to form shape C and record the area of shape C. Remind students that the units are “square units.” Tell students that they don’t have to write “square units” in each blank for this activity, or have them write “s.u.” as a short form. (a) 6, 12, 18; b) 6, 5, 11; c) 10, 6, 16; d) 3, 16, 19)

**Dividing or splitting a shape into two rectangles.** Return to the transparency of Question 1.a) from BLM Adding Areas. SAY: You can make shape C by joining rectangle A and rectangle B. ASK: Where can we draw a line on shape C to show how rectangles A and B have been joined? PROMPT: Where can we draw a line on shape C to divide or split that shape into rectangles A and B? Trace vertical lines with your finger in various incorrect places along shape C before tracing the correct line. Ask students to signal thumbs up for the correct line or thumbs down for an incorrect line. Draw a thick line in the correct place on shape C, as shown below:

```
  C

```

Tell students that you will color rectangles A and B using red and blue. Color the outer borders of A in red and those of B in blue. ASK: Where do you see a copy of rectangle A on shape C, to the left of the dividing line or to the right? (to the left) SAY: Let’s use the same color on this part of shape C as on rectangle A. Color the outline of rectangle A in red on shape C. ASK: Where do you see a copy of rectangle B on shape C? (to the right of the dividing line) Color the outline of rectangle B in blue on shape C.

ASK: How could we get the area of shape C from the areas of rectangles A and B? (add them) Have a volunteer write the addition on the board. (12 + 6 = 18 square units) ASK: Do we get the same answer by counting the square units in shape C as by adding the areas of rectangles A and B? (yes) Repeat this process with Questions 1.b), 1.c), and 1.d) from BLM Adding Areas.
Have students complete Questions 1–3 on AP Book 3.2 p. 166. Students can leave off “square units” or write “s.u.” for a short form in the blanks in Question 1.

**NOTE:** Rectangles are described below as shaded and unshaded to allow you to differentiate shapes, but the AP pages refer to gray and white rectangles. Another way of distinguishing shapes that combine is to number them (e.g., Area 1 and Area 2). Ensure that students understand the use of shaded or unshaded, gray or white, and numbering as ways to differentiate the shapes.

### Finding the area of a shape made from two rectangles (with shading).

Draw on the board:

```
<table>
<thead>
<tr>
<th></th>
<th>8 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 ft</td>
<td></td>
</tr>
</tbody>
</table>
```

4 ft

8 ft

10 ft

6 ft

ASK: How can we find the area of the shaded rectangle? (multiply length times width) What is the length of the shaded rectangle? (8 feet) Prompt students by pointing to the shaded rectangle and tracing your finger along the length. ASK: What is the width of the shaded rectangle? (6 feet) Write on the board:

Area of shaded rectangle = \(8 \times 6 = 48 \text{ ft}^2\)

Have a volunteer fill in the blanks. (8 \(\times\) 6 = 48 \(\text{ft}^2\)) Repeat this process with the unshaded rectangle, as shown below:

Area of unshaded rectangle = \(10 \times 4 = 40 \text{ ft}^2\)

ASK: How can we find the area of the larger shape? (add the areas of the shaded and unshaded rectangles) Write on the board:

Area of larger shape = \(48 + 40 = 88 \text{ ft}^2\)

Have a volunteer fill in the blanks. (48 + 40 = 88 \(\text{ft}^2\))

Have students complete Question 4 on AP Book 3.2 p. 167.

### Finding the area of an L-shaped room.

Draw on the board:

```
<table>
<thead>
<tr>
<th></th>
<th>3 yd</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 yd</td>
<td></td>
</tr>
</tbody>
</table>
```

9 yd

5 yd

2 yd

3 yd

ASK: How can we find the area of the L-shaped room? (divide into rectangles and sum the areas) Write on the board:

Area of L-shaped room = \(3 \times 9 = 27 \text{ sq yd}\) + \(5 \times 2 = 10 \text{ sq yd}\)

Have a volunteer fill in the blanks. (27 + 10 = 37 \(\text{sq yd}\))
SAY: This is a picture of an L-shaped room in a house. Pretend we are looking down at the floor of the room. We call it an L-shaped room because the floor of the room is in the shape of the letter “L.” Trace your hand along the shape to show how the shape resembles an L. Point to each labeled side in turn and ASK: How long is this side of the L-shaped room? (clockwise from left: 9 yd, 3 yd, 5 yd, 2 yd) Where can we draw a line going up and down to split this shape into two rectangles? Trace various incorrect vertical lines with your finger before tracing the correct line and have students signal yes or no as you do so. Draw a thick line as shown below:

- Width of 3 yd
- Length of 9 yd
- Width of 5 yd
- Length of 2 yd

SAY: Let’s shade one of the two rectangles so that we can tell them apart more easily. Shade the rectangle on the left, then ask students for the area of each rectangle, and then the area of the entire shape in the same manner as in the previous example, as shown below:

\[
\begin{align*}
\text{Area 1} &= 9 \times 3 = 27 \text{ yd}^2 \\
\text{Area 2} &= 5 \times 2 = 10 \text{ yd}^2 \\
\text{Total area} &= 27 + 10 = 37 \text{ yd}^2
\end{align*}
\]

Have students complete Questions 5–7 on AP Book 3.2 p. 168.

**Extensions**

1. Sam and Milena find the area of an L-shaped room. Sam draws a line to divide the room into two rectangles. Milena draws a different line to split the room into two rectangles.

   **Sam’s Method**
   - Width of 3 yd
   - Length of 8 yd
   - Width of 6 yd
   - Length of 2 yd

   **Milena’s Method**
   - Width of 3 yd
   - Length of 8 yd
   - Width of 6 yd
   - Length of 2 yd

   a) Find the area of the L-shaped room using Sam’s method.

   Hint: To find the length of the top rectangle, you will need to subtract. To find the length of the bottom rectangle, you will need to add.
b) Find the area of the L-shaped room using Milena’s method.

c) Did you get the same area both ways?

d) Which method is easier? Explain.

**Answers**

a) length of top rectangle = 8 – 2 = 6 yd, area of top rectangle = 6 × 3 = 18 yd², length of bottom rectangle = 3 + 6 = 9 yd, area of bottom rectangle = 9 × 2 = 18 yd², total area = 18 + 18 = 36 yd²

b) area of left rectangle = 8 × 3 = 24 yd², area of right rectangle = 6 × 2 = 12 yd², total area = 24 + 12 = 36 yd²

c) yes

d) Milena’s method is easier because you don’t need to add or subtract to find the lengths of the two rectangles

2. Find the area of the shape. Hint: Add the areas of three rectangles. You will need to add or subtract to find some of the lengths.

![Diagram of a shape with dimensions 3 in, 4 in, 2 in, 4 in, 7 in, and 1 in.]

**Answer:** area of left rectangle = 3 × 4 = 12 in²; length of middle rectangle = 4 + 1 = 5 in, area of middle rectangle = 2 × 5 = 10 in²; length of right rectangle = 7 – 4 – 2 = 1 in, area of right rectangle = 1 × 1 = 1 in²; total area = 12 + 10 + 1 = 23 in²

3. Ava has an L-shaped bedroom. She makes a line on the floor using masking tape to divide the room into two rectangular sections. The area of one of the rectangular sections is 72 ft². The area of the whole room is 114 ft².

a) What is the area of the other rectangular section?

b) The length of the other rectangular section is 7 ft. What is the width?

**Answers:** a) 114 – 72 = 42 ft², b) 42 ÷ 7 = 6 ft

(MP2)

4. Sally and Jack both drew rectangles on grid paper that were 3 rows of 7 squares. Sally drew her rectangle on 1 cm grid paper. Jack drew his rectangle on 1 inch grid paper. Whose rectangle is bigger? Explain how you know.

**Answer:** Jack’s rectangle is bigger, because square inches are bigger than square centimeters, and Jack’s rectangle is 21 square inches while Sally’s is 21 square centimeters.

Look for students to use the meaning of the units (MP2): 1 inch is bigger than 1 centimeter, so 1 square inch is bigger than 1 square cm and 21 square inches is bigger than 21 square centimeters.
MD3-36  Splitting Rectangles
Pages 169–171

STANDARDS
3.MD.C.7b, 3.MD.C.7c

VOCABULARY
area
length
product
square feet (ft²)
square inches (in²)
square units
square yards (yd²)
width

Goals
Students will find the area of a large rectangle divided into two smaller rectangles by adding the areas of the smaller rectangles.
Students will find the product of two numbers by using an area model of this kind.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape
Can find the area of a rectangle by multiplying length and width

MATERIALS
overhead projector (optional)
transparency of BLM Array of Square Inches (p. S-70, optional)
grid paper or BLM 1 cm Grid Paper (p. V-1)
connecting cubes of two different colors, 36 of each per pair of students
dice, 1 per pair of students
BLM Areas of Large Rectangles (p. S-72)
BLM Finding Products Using Rectangles (p. S-73)

Showing the product of two numbers as the area of a rectangle.
Write on the board:

$$5 \times 6 = 30$$

ASK: When you multiply 5 × 6, what answer do you get? (30) What is the answer called? (the product) Remind students that whenever two (or more) numbers are multiplied together, the result is called the product of the numbers. ASK: How can we show the product of 5 and 6 as the area of a rectangle? (draw a rectangle with length 6 units and width 5 units)

Use any unit that is convenient to draw on the board:

![Rectangle Diagram]

Leave this picture on the board to use later.

NOTE: As an alternative to drawing the above rectangle on the board, you could use a transparency of BLM Array of Square Inches cut to be 5 × 6 and project it on the board.
SAY: The area of this rectangle shows you the product of 5 and 6. Write “5 × 6 = 30” on the board below the rectangle. Have students use grid paper or BLM 1 cm Grid Paper for the following exercises.

**Exercises:** Draw a rectangle on grid paper to show the multiplication. Find the product.

a) 8 × 4  
b) 5 × 9  
c) 7 × 3

**Answers:** a) length = 8, width = 4, 8 × 4 = 32; b) length = 9, width = 5, 5 × 9 = 45; c) length = 7, width = 3, 7 × 3 = 21

**Rectangles made up of two smaller rectangles.** Return to the rectangle on the board and draw a thick vertical line to split the rectangle into two smaller rectangles and shade the rectangle on the left, as shown below:

```
+---+---+---+---+---+---+---+---+
|   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |
|   |   |   |   |   |   |   |   |
|   |   | 5 |   |   |   |   |   |
|   |   |   |   |   |   |   |   |
+---+---+---+---+---+---+---+---+
```

5 × 6 = 30

SAY: I’ve divided the rectangle into two smaller rectangles. Trace your finger along the top edge of the shaded rectangle and ASK: What is the width of the shaded rectangle? (4 units) Write “4” above this edge. Trace your finger along the top edge of the unshaded rectangle and ASK: What is the width of the unshaded rectangle? (2 units) Write “2” above this edge.

SAY: This picture shows three different rectangles: the shaded rectangle, the unshaded rectangle, and the large rectangle that is made up of the two smaller rectangles. Trace your finger around the edges of each rectangle as you mention it.

Tell students that you would like to know the areas of all three rectangles. SAY: We already know the area of the large rectangle: we multiply the length of the large rectangle, 6 units, by the width of the large rectangle, 5 units. Write “Area of large rectangle =” to the left of the equation “5 × 6 = 30” on the board. You might wish to add “square units” to the right of 30. The equation should look like this:

```
Area of large rectangle = 5 × 6 = 30 square units
```

ASK: How do we find the area of the shaded rectangle? (length × width) What is the length of the shaded rectangle? (5 units) Prompt students by tracing your finger around all edges of the shaded rectangle and then along the length of the shaded rectangle. SAY: Remember, we already found the width of the shaded rectangle. ASK: What is the width? (4 units) Prompt students by tracing your finger along the top edge of the shaded rectangle. Write “Area of shaded rectangle = 5 × 4 =” on the board and have a volunteer fill in the answer. (20 square units) Repeat with the area of the unshaded rectangle. (5 × 2 = 10)
The equations should look like this:

Area of shaded rectangle = 5 × 4 = 20 square units
Area of unshaded rectangle = 5 × 2 = 10 square units

ASK: How can we find the area of the large rectangle from the areas of the shaded and unshaded rectangles? (add) Why does that work? (the two smaller rectangles make up the larger rectangle) Write on the board: “Total Area = ___ + ___ =” and have a volunteer fill in the blanks. (20, 10) Have another volunteer add the areas and write the total. (30 square units) The finished picture should look like this:

Area of large rectangle = 5 × 6 = 30 square units
Area of shaded rectangle = 5 × 4 = 20 square units
Area of unshaded rectangle = 5 × 2 = 10 square units
Total Area = 20 + 10 = 30 square units

Review the two ways to find the area of the large rectangle:

**Method 1:** Multiply the length and width of the large rectangle.

**Method 2:** Find and add the areas of the two smaller rectangles.

**ACTIVITY**

Give each pair of students 36 red and 36 blue connecting cubes and a die. Student 1 rolls the die. The number on the die is the length or width of a rectangle. Student 1 makes a rectangle using red cubes that has the rolled length or width and chooses another number from 1 to 6 for the other measurement. Student 2 does the same with blue cubes, using the same number on the die for the length or width of a rectangle and choosing a number from 1 to 6 for the other measurement. Students record the areas of their rectangles. Then Student 1 and Student 2 join their rectangles to make a larger rectangle and find the area of the larger rectangle using both Method 1 and Method 2 above.

Give each student a copy of BLM Areas of Large Rectangles to complete individually.
Exercises: Complete BLM Areas of Large Rectangles.

Answers: a) $4 \times 2 = 8$, $3 \times 2 = 6$, $8 + 6 = 14$; $4 + 3 = 7$, $2$, $7 \times 2 = 14$; 
b) $4 \times 10 = 40$, $5 \times 10 = 50$, $40 + 50 = 90$; $10$, $4 + 5 = 9$, $9 \times 10 = 90$;  
c) $4 \times 4 = 16$, $4 \times 4 = 16$, $16 + 16 = 32$; $4 + 4 = 8$, $4$, $8 \times 4 = 32$

Finding the product of two numbers by splitting the larger number.
Write on the board:

$$9 \times 15 =$$

Tell students that even though they have not learned how to find $9 \times 15$ directly by multiplying, they can show the product of 9 and 15 by drawing a rectangle. ASK: What is the length of the rectangle? (15 units) What is the width? (9 units) Draw on the board:

$$9 \times 15 =$$

15

9

Explain to students that you did not need to make your rectangle exactly 15 units by 9 units because the picture of the rectangle is just a rough sketch that will help to find the product of 9 and 15. Remind the students that we can show the product of two numbers as the area of a rectangle, just as they did with $5 \times 6 = 30$ at the beginning of this lesson.

SAY: If we are multiplying and one of the numbers is larger than 10, we can split the number to find two smaller products that add to become the larger product. We can show this in our picture. Let’s draw a line to split this large rectangle into two smaller rectangles. We want to make each side of the smaller rectangles 10 units or less. Ask students to find two numbers that are 10 or less and that add to 15. (5 and 10, 6 and 9, 8 and 7) SAY: We could choose any one of these pairs. Let’s choose 10 and 5. Let’s draw a line to divide the length of 15 into two parts: 10 units and 5 units. Continue drawing on the board:

$$9 \times 15 =$$

15

10

5

9
ASK: If we add the areas of the two smaller rectangles, will we get the area of the larger rectangle? (yes) Is the area of the larger rectangle the product of 9 and 15? (yes) Continue writing the equation on the board:

\[ 9 \times 15 = (9 \times \_ \_ \_ ) + (9 \times \_ \_ \_ ) \]

Remind students that brackets tell you which operation to do first. Explain that the first set of brackets gives the area of the left rectangle and the second set of brackets in the equation shows the area of the right rectangle. Point to the left rectangle and SAY: The width of this rectangle is 9 units. ASK: What is the length of this rectangle? (10 units) Write “10” in the first blank in the equation. Point to the right rectangle and SAY: The length of this rectangle is 9 units. ASK: What is the width of this rectangle? (5 units) Write “5” in the second blank. Add an equal sign and two more blanks under the equation on the board, as shown below:

\[ 9 \times 15 = (9 \times 10\,) + (9 \times 5\,) = \_ \_ \_ + \_ \_ \_ \]

ASK: What is 9 \times 10? (90) Write “90” in the first blank. ASK: What is 9 \times 5? (45) Write “45” in the second blank. Write another equal sign underneath the last line and one more blank. Have a volunteer fill in the final blank by adding 90 and 45. The picture should look like this:

\[ 9 \times 15 = (9 \times 10\,) + (9 \times 5\,) = \_ \_ \_ + \_ \_ \_ \]

\[ 90 \quad 45 \]

\[ 9 \quad 0 \]

\[ 4 \quad 5 \]

\[ 135 \]

Give each student a copy of BLM Finding Products Using Rectangles to complete individually.

**Exercises:** Complete BLM Finding Products Using Rectangles.

**Answers:**
1. a) 8, 6; 14; b) 7, 45, 35, 80; c) 10, 3, 80, 24; 104; d) 10, 9; 90, 81, 171

**Extensions**
1. Find the area of the shaded rectangle.

a) Area of large rectangle = 128 in²
Area of unshaded rectangle = 48 in²

b) Area of large rectangle = 288 ft²
Area of unshaded rectangle = 108 ft²

c) Area of large rectangle = 512 yd²
Area of unshaded rectangle = 192 yd²
Answers: a) $128 - 48 = 80$ in², b) $288 - 108 = 180$ ft², c) $512 - 192 = 320$ yd²

2. Ivan’s garden is a rectangle. He splits the garden into two rectangular sections. The larger section is for flowers and the smaller section is for vegetables.

```
5 ft
\_\_\_
3 ft
\_\_\_
```

a) The area of the entire garden is 32 ft². What are the length and width of the entire garden?

b) What is the area of the flower section?

c) What is the area of the vegetable section?

Answers: a) length = 5 + 3 = 8 ft, width = 32 ÷ 8 = 4 ft; b) 4 × 5 = 20 ft²; c) 4 × 3 = 12 ft²

3. Fill in the blanks to find the product.

a) $22 = 10 + 10 + 2$, so

$6 \times 22 = (6 \times 10) + (6 \times 10) + (6 \times 2)$

= ___ + ___ + ___

= ___

b) $24 = 8 + 8 + 8$, so

$7 \times 24 = (7 \times 8) + (7 \times ___) + (7 \times ___)$

= ___ + ___ + ___

= ___

c) $29 = 10 + 10 + 9$, so

$3 \times 29 = (3 \times ___) + (3 \times ___) + (3 \times ___)$

= ___ + ___ + ___

= ___

d) $27 = 9 + 9 + 9$, so

$8 \times 27 = (8 \times ___) + (8 \times ___) + (8 \times ___)$

= ___ + ___ + ___

= ___

Answers

a) 60, 60, 12; 132
b) 8, 8; 56, 56, 56; 168
c) 10, 10, 9; 30, 30, 27; 87
d) 9, 9, 9; 72, 72, 72; 216
4. Draw a rectangle divided into 3 smaller rectangles to show the product for Extension 3.a).

**Answer**

![Diagram of a rectangle divided into smaller rectangles]

5. Jayden has some marbles. He shares the marbles equally between himself, Ron, and Tina. Then Mary joins them. Everyone gives Mary two marbles. Now, they all have the same number of marbles. How many marbles did Jayden start with?

**Sample solution:** Three people gave Mary two marbles, so at the end Mary has 6 marbles. Then, all four people have 6 marbles. So I found $4 \times 6 = 24$. Jayden had 24 marbles to start.

Whole-class follow-up: Have volunteers share their strategies and how they came up with it. For example, how did they know that Mary had six marbles at the end—did they read the question and know it right away or did they try out some numbers first? Have volunteers share their answers. Emphasize the importance of trying numbers to get a feel for the problem (MP.1).
MD3-37  Area and Perimeter (1)

STANDARDS
3.MD.C.7b, 3.MD.D.8

VOCABULARY
area
distance
length
opposite sides
parallelogram
perimeter
product
square feet (ft²)
square inches (in²)
square units
square yards (yd²)
width

Goals
Students will reinforce their understanding of area and perimeter by distinguishing between the two.
Students will learn shortcuts for finding the perimeters of rectangles (including squares) and parallelograms.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape
Can find the area of a rectangle by multiplying length and width
Understands perimeter as the distance around a shape
Can find the perimeter of a shape by adding the side lengths

MATERIALS
BLM Area and Perimeter of Rectangles (p. S-74)

Review area and perimeter of rectangles. Draw on the board:

```
  5 ft

  4 ft
```

ASK: What is the length of the rectangle? (5 feet) What is the width of the rectangle? (4 feet) Ask students if they remember what the word perimeter means. Ensure students understand that perimeter means the distance around a shape. SAY: The perimeter of this rectangle is the distance around the rectangle. ASK: What side lengths do I need to add to find the perimeter of the rectangle? (all four sides)

SAY: Let’s write the lengths of the other two sides in this rectangle. Point to the right edge of the rectangle and ASK: How long is this side? (4 feet) How do you know? (because the opposite side is 4 feet) Write “4 ft” beside the right edge. Point to the bottom edge and ASK: How long is this side? (5 feet) Write “5 ft” under the bottom edge. Write on the board:

```
Perimeter = _____ + _____ + _____ + _____ = _____
```

SAY: Remember, we need to add all four sides of the rectangle to find the perimeter. Have a volunteer fill in the blanks. Prompt the volunteer by tracing your finger around the rectangle counter clockwise, beginning with the left edge to add the side lengths. (4 + 5 + 4 + 5 = 18 feet)

Tell students that you also want to find the area of this rectangle. Ensure students understand that area is the number of square units that cover a
shape. Trace a finger along the edges as you SAY: Perimeter is the distance around a shape. Then shade the area of the shape as you SAY: Area is the number of square units that cover a shape. To find the perimeter, we want to know how long a piece of string that goes around the outside of the rectangle would be, but to find the area, we want to know how many square units will cover the shape.

SAY: We already know that the perimeter is 18 feet. ASK: How do we find the area? (multiply length and width) Write on the board:

\[ \text{Area} = \_ \times \_ = \_ \]

Have a volunteer fill in the blanks. \((5 \times 4 = 20 \text{ ft}^2)\) Point out that for the area of a rectangle, you multiply two numbers, but for the perimeter of a rectangle, you add four sides. Refer back to the earlier addition for perimeter. Have students compare the perimeter and area of the rectangle.

ASK: What are the units for the perimeter? (feet) What are the units for the area? (square feet) Are the perimeter and area the same number or different numbers? (different numbers) Point out that even if the perimeter and area were the same number, they would still be very different because perimeter is measured in units of length (such as feet), but area is measured in square units (such as square feet).

Give each student a copy of BLM Area and Perimeter of Rectangles to complete individually.

**Exercises:** Complete BLM Area and Perimeter of Rectangles.

**Answers**

1. a) 4, 8; 24; 4, 8, 32
   
   b) 9 yd, 8 yd; 9, 8, 9, 8; 34; 9, 8, 72
   
   c) 8 ft, 8 ft; 8 + 8 + 8 + 8; 32 ft; 8 \times 8 = 64 \text{ ft}^2
   
   d) 2 yd, 10 yd; 2 + 10 + 2 + 10; 24 yd; 2 \times 10 = 20 \text{ yd}^2

**Using a shortcut for perimeter of rectangles and parallelograms.** Return to the picture of the 4 ft by 5 ft rectangle on the board. Point to the equation for the perimeter and SAY: We added the side lengths in order, going around the rectangle. ASK: Could we add the side lengths in a different order? (yes) Would we get the same answer? (yes) Write on the board:

\[ \text{Perimeter} = 5 + 5 + 4 + 4 \]

ASK: Could we add the side lengths in this order? (yes) Add brackets as shown below:

\[ \text{Perimeter} = (5 + 5) + (4 + 4) \]

SAY: Now we've written brackets to show adding the long sides first and then the short sides. ASK: Will we get the same answer if we add the side lengths in this order? (yes) SAY: We can also use multiplication inside each pair of brackets. ASK: How can we write 5 + 5 using multiplication? (2 \times 5) How can we write 4 + 4 using multiplication? (2 \times 4) Write "= (2 \times 5) + (2 \times 4)" under the previous equation on the board. Have students signal the
answers to the following two questions. ASK: What is $2 \times 5$? (10) What is $2 \times 4$? (8) Continue writing on the board:

$$\text{Perimeter} = (5 + 5) + (4 + 4)$$
$$= (2 \times 5) + (2 \times 4)$$
$$= 10 + 8$$
$$= \underline{18} \text{ ft}$$

Have a volunteer fill in the blank. (18) ASK: Did we get the same perimeter as before? (yes) Leave the picture on the board.

Draw on the board:

![Parallelogram Diagram]

Remind students that this shape is called a parallelogram. SAY: A parallelogram has four sides, just like a rectangle, and the sides of a parallelogram that are opposite are equal in length. In this shape, the opposite lengths are 7 inches and the opposite widths are 3 inches. ASK: Can we find the perimeter of the parallelogram in the same way as we found the perimeter of the rectangle? (yes) Write on the board:

$$\text{Perimeter} = (7 + 7) + (3 + 3)$$
$$= (2 \times \underline{7}) + (2 \times \underline{3})$$
$$= \underline{14} + \underline{6}$$
$$= \underline{20} \text{ in}$$

Have a volunteer fill in the blanks. (7, 3; 14, 6; 20) Point out that to find the perimeter of a rectangle or a parallelogram, you really only need two numbers—the length of the longer sides and the length of the shorter sides. You don’t even need a picture of the shape. Write on the board:

Long sides are 8 yards each and short sides are 7 yards each.

$$\text{Perimeter} = (2 \times \underline{8}) + (2 \times \underline{7})$$
$$= \underline{16} + \underline{14}$$
$$= \underline{30} \text{ yd}$$

Have students signal the answers to the following questions. ASK: What number do I write in the first blank? (8) What number do I write in the second blank? (7) Write “8” in the first blank and “7” in the second blank. Have a volunteer fill in the remaining blanks. (16, 14; 30)
Exercises: Find the perimeter of the rectangle or parallelogram.

a) 5 yd
   3 yd
   3 yd
   5 yd

   Perimeter = \((2 \times ___) + (2 \times ___)\)
   = ___ + ___
   = ___ yd

b) 9 ft
   9 ft
   6 ft
   6 ft

   Perimeter = \((2 \times ___) + (2 \times ___)\)
   = ___ + ___
   = ______

Bonus

Find the perimeter of a parallelogram where each side is 10 yards long.

Perimeter = \((2 \times ___) + (2 \times ___)\)
= ___ + ___
= ______

Answers
a) 5, 3; 10, 6; 16;
b) 9, 6; 18, 12; 30 ft;
Bonus: 10, 10; 20, 20; 40 yd

Using another shortcut for the perimeter of a rectangle. Return to the picture of the 4 ft by 5 ft rectangle on the board. SAY: Let’s see if we can find the perimeter in a different way. Write on the board:

\[
\text{Perimeter} = 4 + 5 + 4 + 5
\]

SAY: Let’s insert brackets to show which numbers we will add first. Remember, the order in which we add the numbers doesn’t change the answer. Add brackets as shown below:

\[
\text{Perimeter} = (4 + 5) + (4 + 5)
\]

ASK: What do you notice about the additions in the two sets of brackets? (they’re the same) PROMPT: What is the addition in the first set of brackets? (4 + 5) What is the addition in the second set of brackets? (4 + 5) Are they the same? (yes) SAY: Remember, when you’re adding things that are the same, you can write that as multiplication. Add to the equation on the board:

\[
\text{Perimeter} = (4 + 5) + (4 + 5)
\]

\[
= 2 \times (4 + 5)
\]

ASK: Which operation do we do first, multiplication or addition? (addition) Why? (because the brackets tell us what to do first) What is 4 + 5? (9)
Write "9" under "(4 + 5)" on the board. Copy "= 2 ×" in front of the 9, and ASK: What is 2 × 9? (18) Write "= 18" on the next line. The final equation should look like this:

Perimeter = (4 + 5) + (4 + 5)
= 2 × (4 + 5)
= 2 × 9
= 18

Point out that, in the previous set of exercises, students multiplied the length and the width by 2 first and then added, but in this method, you add the length and width first and then multiply by 2.

NOTE: Before students continue with perimeter, give them some practice writing addition as multiplication with the following exercises. Demonstrate Exercise 1 before having students complete the remaining exercises individually.

Exercises

1. Fill in the blank to write the addition as a multiplication.
   a) 6 + 6 = 2 × ____
   b) 100 + 100 = 2 × ____
   c) n + n = 2 × ____
   d) (6 + 5) + (6 + 5) = 2 × ____
   e) (9 + 8) + (9 + 8) = 2 × ____

   **Answers:** a) 6, b) 100, c) n, d) (6 + 5), e) (9 + 8)

2. Write the addition as a multiplication.
   a) 8 + 8 = ____
   b) 999 + 999 = ____
   c) a + a = ____
   d) (10 + 3) + (10 + 3) = ____
   e) (15 + 12) + (15 + 12) = ____

   **Bonus**
   f) (13 + 20 + 7) + (13 + 20 + 7) = ____
   g) (45 + 15 + 9) + (45 + 15 + 9) + (45 + 15 + 9) = ____

   **Answers:** a) 2 × 8, b) 2 × 999, c) 2 × a, d) 2 × (10 + 3),
   e) 2 × (15 + 12), Bonus: f) 2 × (13 + 20 + 7), g) 3 × (45 + 15 + 9)
3. Fill in the blanks to find the perimeter of the rectangle or parallelogram.

a) 
\[ \text{Perimeter} = (2 \times \_ ) + (2 \times \_ ) \]
\[ = \_ + \_ \]
\[ = \_ \text{ yd} \]

b) 
\[ \text{Perimeter} = (2 \times \_ ) + (2 \times \_ ) \]
\[ = \_ + \_ \]
\[ = \_ \text{ yd} \]

**Bonus**

Find the perimeter of a rectangle where each side is 20 yards long.

\[ \text{Perimeter} = 2 \times (\_ + \_ ) \]
\[ = 2 \times \_ \]
\[ = \_ \]

**Answers:** a) 6, 2; 8; 16; b) 1, 4; 5; 10 ft; Bonus: 20, 20; 40; 80 yd

**Using a shortcut for perimeter of a square.** Draw on the board:

\[ \begin{array}{c}
| 3 \text{ ft} | 3 \text{ ft} | 3 \text{ ft} | 3 \text{ ft} \\
\end{array} \]

\[ \text{Perimeter} = \_ + \_ + \_ + \_ \]

SAY: This is a square. All four sides are the same length. ASK: What number do we write in each of the blanks to find the perimeter? (3) Write “3” in each of the blanks. SAY: We are adding 3 four times. ASK: How can we use multiplication to write this in a shorter way? (4 \times 3) Write “= 4 \times 3” and then have a volunteer write the answer. (12 ft) The equation should look like this:

\[ \text{Perimeter} = 3 + 3 + 3 + 3 \]
\[ = 4 \times 3 \]
\[ = 12 \text{ ft} \]

Point out that to find the perimeter of a square, all you need is the length of one side. You don’t even need a picture of the shape. Write on the board:

Each side of a square is 5 inches long. Find the perimeter.

\[ \text{Perimeter} = 4 \times \_ = \_ \]

ASK: What number goes in the first blank? (5) PROMPT: What is the length of each side of the square? (5 in) ASK: What is the perimeter? (20 in) PROMPT: What is 4 \times 5? (20)
Exercises

1. Find the perimeter of the square.
   a) 2 in
      \[
      \text{Perimeter} = 4 \times 2 = 8 \text{ in}
      \]
   b) 1 ft
      \[
      \text{Perimeter} = 4 \times 1 = 4 \text{ ft}
      \]

Bonus

c) Each side is 8 yards long.
   \[
   \text{Perimeter} = 4 \times 8 = 32 \text{ yd}
   \]
d) Each side is 10 inches long.
   \[
   \text{Perimeter} = 4 \times 10 = 40 \text{ in}
   \]
e) Each side of a square has an unknown side length. Complete the equation for the perimeter of the square. Use \( s \) for the unknown side length.
   \[
   \text{Perimeter} = 4 \times s
   \]

Answers: a) 2, 8; b) 1, 4; Bonus: c) 8, 32 yd; d) 4 \times 10 = 40 \text{ in}; e) 4, \( s \)

Extensions

(MP6) 1. Len calculates the area of the rectangle below.

\[
\text{Area} = 4 \times 5 \times 4 \times 5 = 20 \times 4 \times 5 = 80 \times 5 = 400 \text{ ft}^2
\]

a) Explain Len’s mistake. Use the words “length” and “width.”
b) What should the area be?

Answers
a) Len multiplied the lengths of all four sides instead of just multiplying length and width. To find the area of a rectangle, you multiply two numbers, length and width.
b) The area is \( 4 \times 5 = 20 \text{ ft}^2 \).
2. Anika draws squares of different sizes. She makes a table to write the perimeter and area of each square. Complete the table.

<table>
<thead>
<tr>
<th>Length of one side</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 1 in</td>
<td>$4 \times 1 = 4$ in</td>
<td>$1 \times 1 = \text{in}^2$</td>
</tr>
<tr>
<td>b) 2 in</td>
<td>$4 \times 2 = 8$ in</td>
<td>$2 \times 2 = \text{in}^2$</td>
</tr>
<tr>
<td>c) 3 in</td>
<td>$4 \times 3 = 12$ in</td>
<td>$3 \times 3 = \text{in}^2$</td>
</tr>
<tr>
<td>d) 4 in</td>
<td>$4 \times 4 = 16$ in</td>
<td>$4 \times 4 = \text{in}^2$</td>
</tr>
<tr>
<td>e) 5 in</td>
<td>$4 \times 5 = 20$ in</td>
<td>$5 \times 5 = \text{in}^2$</td>
</tr>
<tr>
<td>f) 6 in</td>
<td>$4 \times 6 = 24$ in</td>
<td>$6 \times 6 = \text{in}^2$</td>
</tr>
<tr>
<td>g) 7 in</td>
<td>$4 \times 7 = 28$ in</td>
<td>$7 \times 7 = \text{in}^2$</td>
</tr>
<tr>
<td>h) 8 in</td>
<td>$4 \times 8 = 32$ in</td>
<td>$8 \times 8 = \text{in}^2$</td>
</tr>
<tr>
<td>i) 9 in</td>
<td>$4 \times 9 = 36$ in</td>
<td>$9 \times 9 = \text{in}^2$</td>
</tr>
<tr>
<td>j) 10 in</td>
<td>$4 \times 10 = 40$ in</td>
<td>$10 \times 10 = \text{in}^2$</td>
</tr>
</tbody>
</table>

**Answers:**

a) 1; b) 8; 4; c) 3, 12; 3, 3, 9; d) 4, 16; 4, 4, 16; e) 5, 20; 5, 5, 25; f) 6, 24; 6, 6, 36; g) 7, 28; 7, 7, 49; h) 8, 32; 8, 8, 64; i) 9, 36; 9, 9, 81; j) 10, 40; 10, 10, 100

3. Jayden draws a square. The perimeter (in inches) is the same number as the area (in square inches). What is the length of each side of the square?

**Answer:** The length of each side is 4 inches. Perimeter $= 4 \times 4 = 16$ in and area $= 4 \times 4 = 16 \text{in}^2$.

4. Look at the table in Extension 2. Compare the perimeter (in inches) and area (in square inches).

a) For which square(s) is the perimeter a larger number than the area?

b) For which square(s) is the perimeter a smaller number than the area?

c) For which square(s) is the perimeter the same number as the area?

**Answers:**

a) squares with side lengths of 1, 2, and 3 inches;

b) squares with side lengths more than 4 inches;

c) the square with side length 4 inches
Goals

Students will find missing side lengths in rectangles given the area or perimeter and one side length.

Prior Knowledge Required

Understands area as the number of square units that cover a shape
Can find the area of a rectangle by multiplying length and width
Understands perimeter as the distance around a shape
Can find the perimeter of a shape by adding the side lengths

Review whole numbers. Write on the board:

10 3/4 1 1/2 15 4 1/4 1 2/3 9 14 3/4

Ask students to signal whether each number is a whole number or not.
(yes, no, no, yes, no, yes, no, yes, yes, no)

Review perimeter and area. Draw on the board:

Ask students what it means to find the perimeter of the rectangle. (perimeter is the distance around the rectangle) Trace a finger around the perimeter to demonstrate. Ask students what it means to find the area of the rectangle. (area is the number of square units that cover the rectangle)

Review lengths and widths. Remind students about the terms "length" and "width." Write "2 ft" beside the left edge of the rectangle on the board. Point to the right edge of the rectangle and ASK: What is the length of this side of the rectangle? (2 ft) PROMPT: Opposite sides of a rectangle are equal in length. SAY: We know that two opposite sides of the rectangle are each 2 feet long. But we can see that the other two sides are even longer. Point to the top and bottom edges of the rectangle and SAY: The two longer sides are sometimes called the lengths of the rectangle. Write "lengths" on the board. We can then call these two shorter sides the widths. Write "widths" on the board. SAY: We use the plural, lengths and widths, when we talk about the actual sides, but we use the singular, length and width, when we talk about the measurements of the sides.

What do the lengths and widths add to? ASK: What do the widths add to in this rectangle? (2 + 2 = 4 ft) SAY: Suppose we know that the perimeter is 10 feet. Write on the board:

Perimeter = 10 ft
The widths add to 4 ft.
ASK: What will the missing lengths add to? (6) How do you know? 
(10 − 4 = 6) Explain that because the perimeter is 10 feet, the missing lengths have to make up the remaining 6 feet. Write on the board:

Perimeter = 10 ft
The widths add to 4 ft.
The missing lengths are 10 − 4 = 6 ft altogether.

**Exercises:** Use subtraction to find out how much the missing lengths add to.

a)  
\[
\begin{array}{c}
3 \text{ yd} \\
3 \text{ yd}
\end{array}
\]
Perimeter = 20 yd
The widths add to ____ yd.
The missing lengths are
20 − ____ = ____ yd altogether.

b)  
\[
\begin{array}{c}
4 \text{ in} \\
4 \text{ in}
\end{array}
\]
Perimeter = 26 in
The widths add to ____ in.
The missing lengths are
26 − ____ = ____ in altogether.

**Answers:** a) 6; 6, 14 b) 8; 8, 18

**Finding missing sides in a rectangle when the perimeter is given.**

Write on the board:

Carol draws a rectangle that has a perimeter of 18 in. Each side is a whole number of inches long. How long are the sides?

Tell students that you will try different whole numbers for the widths and figure out what the lengths would need to be in Carol’s rectangle. Draw on the board:

\[
\begin{array}{c}
\text{Perimeter = 18 in}
\end{array}
\]

SAY: We know that the perimeter of Carol’s rectangle is 18 in. We want to find the possible measurements for the lengths and widths. Explain to students that you drew a rectangle on the board to help with this question, but the picture is just a rough sketch. Measuring the sides of this picture won’t help to find the answer, because the picture might not be exactly right.

SAY: We will try different widths. Let’s start with 1 inch first. Write “1 in” beside the left edge and the right edge of the rectangle. Continue writing on the board, as shown on the next page.
Perimeter $= 18$ in

The widths add to ____ in.
The missing lengths are
$18 - ____ = ____$ in altogether.
Each length is ____ $\div 2 = ____$ in.

Point to each of the short edges and ASK: What do the widths add to?
$(1 + 1 = 2)$ Write “2” in the first blank. SAY: We need to subtract 2 inches from 18 inches to find how much the lengths add to. Write “2” in the second blank and ASK: What is $18 - 2$? $(16)$ Write “16” in the next blank.

Point to the long sides of the rectangle and ASK: If the two long sides add to 16 inches, how do we find the measurement of each length?
$(16 \div 2 = 8)$ PROMPT: The two long sides are opposite and have the same length. Write “16” and “8” in the remaining blanks. ASK: So, if the short sides in the rectangle are 1 inch each, how long are the long sides? (8 inches each) Write “8” in above the top edge and below the bottom edge of the rectangle. Have a volunteer add the measurements of the four sides of the rectangle to verify that you get a perimeter of 18 inches.
$(1 + 8 + 1 + 8 = 18)$

Repeat this process with a rectangle that has the same perimeter but a width of of 2 inches. (length = 7 inches) Then ask students to complete the following exercises to try other numbers for the same shape.

**Exercises**

1. Fill in the blanks. Find the missing lengths of the rectangle.

   a)  
   
   $\begin{array}{c}
   \text{Perimeter } = 18 \text{ in} \\
   \text{The widths add to ____ in.} \\
   \text{The missing lengths are} \\
   18 - ____ = ____ \text{ in altogether.} \\
   \text{Each length is ____ } \div 2 = ____ \text{ in.}
   \end{array}$

   b)  
   
   $\begin{array}{c}
   \text{Perimeter } = 18 \text{ in} \\
   \text{The widths add to ____ in.} \\
   \text{The missing lengths are} \\
   18 - ____ = ____ \text{ in altogether.} \\
   \text{Each length is ____ } \div 2 = ____ \text{ in.}
   \end{array}$

   **Answers:** a) 6; 6, 12; 12, 6; b) 8; 8, 10; 10, 5

**Bonus:** The perimeter of a rectangle is 18 inches.

a) Can the widths be 5 inches each? Explain.

b) Can the lengths be 10 inches each? Explain.
Answers
a) No. If the short sides were 5 inches each, the missing lengths would be 4 inches each. The missing sides are supposed to be longer than 5 inches each, so 4 inches does not work.
b) No. The perimeter is 18 inches but long sides of 10 inches each would add to more than the perimeter of the rectangle; $2 \times 10 = 20$.

Using another method to find the missing sides in a rectangle when the perimeter is given. Tell students there is another method for finding the missing sides in a rectangle if they know one side and the perimeter. Draw on the board:

```
4 in
```

SAY: We know that the width of the rectangle is 4 inches. One way to find the perimeter of a rectangle is to add the side lengths in order, going around the rectangle. First let’s label all four sides. Let’s write $L$ for the unknown length. Pointing to the top edge, ASK: How long is this side? (we don’t know) What should we write? ($L$) Write “$L$” above the top edge. Continue in the same manner for the right edge (4 in) and the bottom edge ($L$). Write “Perimeter =” on the board. Ask students to say out loud the lengths of the edges as you fill in the equation to find the perimeter, as shown below:

$\text{Perimeter} = 4 + L + 4 + L$

SAY: Let’s use brackets to show adding the first two sides and the last two sides separately. Add the brackets on the board, as shown below:

$\text{Perimeter} = (4 + L) + (4 + L)$

Students might recognize this order from the previous lesson. ASK: What do you notice about the additions in the two sets of brackets? (they are the same) Trace your finger along the left edge and then along the top edge, and SAY: If we add these two sides, we get the same result as adding the other two sides. Trace the left and top edges again, and SAY: These two sides add to half of the perimeter, and adding the other two sides gives the other half of the perimeter. So adding the length and the width gives you half of the perimeter.

SAY: Let’s say the perimeter is 20 inches. ASK: How do we find what is half of the perimeter? ($20 \div 2 = 10$ in) Write this on the board. SAY: So the length and width will add to 10. Write “length + width = 10 in” on the board. Ask students what the length is and what the width is. ($L, 4$) Write these values in the next line, as shown below:

$20 \div 2 = 10 \text{ in}$

$\text{length} + \text{width} = 10 \text{ in}$

$L + 4 = 10 \text{ in}$
ASK: How do we find the missing length, $L$? ($10 - 4 = 6$ in) Write on the board:

Each missing length is $\square - \square = \square$ in

Have students signal the numbers that belong in the blanks. ($10 - 4 = 6$)

**Exercises:** Fill in the blanks to find the missing lengths.

a) Perimeter = 18 ft

$$\begin{array}{c}
3 \text{ ft} \\
3 \text{ ft}
\end{array}$$

Half of the perimeter is $18 \div 2 = \square$ ft.

Each length is $\square - 3 = \square$ ft.

b) Perimeter = 10 yd

$$\begin{array}{c}
1 \text{ yd} \\
1 \text{ yd}
\end{array}$$

Half of the perimeter is $10 \div 2 = \square$ yd.

Each length is $\square - 1 = \square$ yd.

**Answers:** a) 9; 9, 6; b) 5; 5, 4

**Finding missing sides in a rectangle when the area is given.** Write on the board:

The area of a rectangle is 28 square yards.

The width is 4 yards.

Find the length.

SAY: Now we are given the area of a rectangle rather than the perimeter. We are also given the width. We need to find the length. Let’s draw a picture of the rectangle. Draw on the board:

$$\begin{array}{c}
4 \text{ yd} \\
4 \text{ yd}
\end{array}$$

Area $= 28 \text{ yd}^2$

SAY: Let’s write an equation for the area of this rectangle. Let’s write a question mark (?) for the unknown length. Remember, for a rectangle, length times width gives you the area. Write on the board

$$\text{length} \times \text{width} = \text{area}$$

ASK: What is the length of this rectangle? (we don’t know) Write “? $\times$” on the board. ASK: What is the width of the rectangle? (4 yd) Write “4 =” on the board. ASK: What is the area of the rectangle? (28 square yards) Write “28 $\text{yd}^2$” on the board.

The equation should look like this:

$$\text{length} \times \text{width} = \text{area}$$

$$? \times 4 = 28 \text{ yd}^2$$

ASK: What division equation will give the unknown length? ($28 \div 4 = ?$)

Prompt students by reminding them how to read a multiplication equation backward to get a division equation. ASK: What is $28 \div 4$? (7) So what
is the missing length of the rectangle? (7 yards) Write the answer on the board, as shown below:

\[
28 \div 4 = ? \\
? = 7
\]

The missing length is 7 yards.

Write “7” on the top and bottom edges of the rectangle. ASK: How can we check that we got the right number for the length? (multiply 4 and 7 and check if you get the given area, 28) What is \(4 \times 7\)? (28)

Repeat this process for a rectangle with area 24 ft\(^2\) and length 8 ft.
\[
(8 \times ? = 24 \text{ ft}^2, 24 \div 8 = ?, ? = 3 \text{ ft})
\]

Write on the board:

The area of the top of Betty’s kitchen table is 15 ft\(^2\).
The table is 3 feet wide. How long is the table?

ASK: What is the area of the table? (15 ft\(^2\)) Write “Area = 15 ft\(^2\)” on the board. ASK: What is the width of the table? (3 ft) Write “width = 3 ft.” ASK: How long is the table? (we don’t know) Write “length = ?” ASK: How do we find the unknown length? (15 \div 3 = 5 \text{ feet}) PROMPT: What is the equation for the area of a rectangle? (Area = length \times width) Have a volunteer write the equation on the board and another volunteer write a short sentence to answer the question as shown below:

\[
\text{Area} = 15 \text{ ft}^2 \\
\text{width} = 3 \text{ ft} \\
\text{length} = ? \\
15 \div 3 = 5 \text{ feet}
\]

The length of the table is 5 feet.

**Exercises:** Find the missing length or width of the rectangle.

a) In a board game with fake money, each dollar bill has one side that is 3 inches long. Each bill has an area of 18 square inches. What is the length of each bill?

b) The front cover of an art book is 10 inches high. The area of the cover is 80 square inches. What is the width of the cover?

**Bonus:** Anwar draws a square on the school playground using chalk. The area of the square is 9 yd\(^2\). What is the length of each side of the square? Hint: Try guessing different side lengths between 1 and 5 yd. Which side length gives you 9 yd\(^2\)?

**Answers:** a) \(18 \div 3 = 6 \text{ inches}\); b) \(80 \div 10 = 8 \text{ inches}\); Bonus: a square has 4 equal sides, and \(3 \times 3 = 9\), so each side is 3 yards long
Extensions

1. The perimeter of a parallelogram is 20 ft. Each of the long sides is 7 ft long. How long is each short side?
   **Sample solution:** The long sides add to 14 ft. The short sides are 20 – 14 = 6 ft altogether. The length of each short side is 6 ÷ 2 = 3 ft.

2. The length of a rectangle is 9 yd. The area is 45 yd². What is the perimeter of the rectangle?
   **Solution:** Width = area ÷ length, so 45 ÷ 9 = 5 yd. The perimeter of the rectangle is (2 × 9) + (2 × 5) = 18 + 10 = 28 yd or 9 + 5 + 9 + 5 = 28 yd.

3. The perimeter of a square is 36 ft.
   a) What is the length of each side of the square?
   b) Find the area of the square.
   **Solutions:** a) each side of the square is 36 ÷ 4 = 9 ft, b) the area of the square is 9 × 9 = 81 ft²

4. A school field is in the shape of a rectangle. The length of the field is 10 yd. The area of the field is 90 yd². Eva walks around the edges of the field. How many yards does she walk?
   **Solution:** The field is 90 ÷ 10 = 9 yd wide, so Eva walks (2 × 10) + (2 × 9) = 20 + 18 = 38 yd or 10 + 9 + 10 + 9 = 38 yd.

5. Lewis wants to tile the floor shown below. Tiles are 1 foot by 1 foot and come in packs of 10. Lewis already has 3 packs. How many more packs does Lewis need to buy?
   **Sample solution:** Divide the floor into two rectangles, a 23 ft by 4 ft rectangle and an 8 ft by 1 ft rectangle. To find the total area, find (23 × 4) + (8 × 1). I know that 23 × 4 is two less 4s than 25 × 4 and I know that 25 × 4 is 100 because 4 quarters make 100 cents, so 23 × 4 is 8 less than 100. So, (23 × 4) + 8 = 100. Lewis needs 100 tiles. He already has 30 tiles (because he has 3 packs of 10 and 3 × 10 = 30), so he needs 70 more tiles. That means he needs 7 more packs of 10, because 70 ÷ 10 = 7.
Redirecting students: There are many different parts to this problem and different students will struggle with different parts, for instance:

- understanding what the question is asking (MP1)
- deciding how to divide the area into smaller parts (MP7)
- calculating the total number of tiles Lewis needs to use (MP4, MP7)
- calculating the number of tiles Lewis already has (MP4)
- calculating the number of tiles that Lewis still needs (MP4)
- calculating the number of packs of 10 that Lewis needs to buy (MP4)

You might encourage students who are struggling with different parts to work together.

**NOTE:** The sample solution above also makes use of MP2 by using the context of money to calculate $23 \times 4$, but many students will not use this strategy.
Goals
Students will draw rectangles with equal areas but different perimeters.
Students will also draw rectangles with equal perimeters but different areas.

PRIOR KNOWLEDGE REQUIRED
Understands area as the number of square units that cover a shape
Can find the area of a rectangle by multiplying length and width
Understands perimeter as the distance around a shape
Can find the perimeter of a shape by adding the side lengths

MATERIALS
grid paper or BLM 1 cm Grid Paper (p. V-1)
overhead projector
transparency of BLM 1 cm Grid Paper (p. V-1)
erasable markers

Review the three methods for finding perimeter of a rectangle. Draw on the board:

width
length
length
width

Remind students about the three different ways they have been taught to find the perimeter of a rectangle:

Method 1: Add the four sides in order going around the rectangle.
Perimeter = length + width + length + width

Method 2: Double the lengths, double the widths, and then add.
Perimeter = (2 × length) + (2 × width)

Method 3: Add the length and width, and then double.
Perimeter = 2 × (length + width)

Identifying rectangles with the same perimeter, but different areas.
Draw two rectangles on the board, as shown on the following page (the rectangles are only sketches; they do not need be perfectly proportional).
SAY: Let’s find the perimeter and area for the rectangles. Draw on the board:

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: First, let’s find the missing side measurements for rectangle A. Remember that opposite sides of a rectangle have equal measurements. Point to the bottom edge of rectangle A and ASK: How long is this side? (5 ft) Write “5” in the blank. Point to the right edge and ASK: How long is this side? (3 ft) Write “3” in the blank. ASK: How do we find the perimeter? (3 + 5 + 3 + 5 = 16 ft) Have a volunteer write the answer in the table for the perimeter of rectangle A. NOTE: If students are comfortable with the other two methods of finding perimeter reviewed at the beginning of this lesson, you can have them record the answer using one of those methods. ((2 × 5) + (2 × 3) = 16 ft or 2 × (5 + 3) = 16 ft)

ASK: How do we find the area of rectangle A? (3 × 5 = 15 ft² or 5 × 3 = 15 ft²) Have a volunteer write the answer in the table for the area of rectangle A. Repeat for rectangle B. The finished table should look like this:

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3 + 5 + 3 + 5 = 16 ft</td>
<td>3 × 5 = 15 ft²</td>
</tr>
<tr>
<td>B</td>
<td>2 + 6 + 2 + 6 = 16 ft</td>
<td>2 × 6 = 12 ft²</td>
</tr>
</tbody>
</table>

ASK: What do you notice about the perimeters of rectangles A and B? (they’re the same) What about the areas of rectangles A and B? (they are different) SAY: Two rectangles can have the same perimeter, but different areas.

Identifying rectangles with the same area, but different perimeters.
ASK: Do you think it’s possible for two rectangles to have the same areas, but different perimeters? (yes) Draw another rectangle on the board beside rectangles A and B:
Extend the table to include a row for rectangle C. Have students find the missing sides of rectangle C and then its perimeter and area. (3 ft, 4 ft, 14 ft, 12 ft$^2$) Have a volunteer fill in the perimeter and area of rectangle C in the table, as shown below:

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$3 + 5 + 3 + 5 = 16$ ft</td>
<td>$3 \times 5 = 15$ ft$^2$</td>
</tr>
<tr>
<td>B</td>
<td>$2 + 6 + 2 + 6 = 16$ ft</td>
<td>$2 \times 6 = 12$ ft$^2$</td>
</tr>
<tr>
<td>C</td>
<td>$4 + 3 + 4 + 3 = 14$ ft</td>
<td>$4 \times 3 = 12$ ft$^2$</td>
</tr>
</tbody>
</table>

ASK: Which two rectangles have the same area? (B and C) Do rectangles B and C have the same perimeter? (no) SAY: So rectangles B and C have the same area, but different perimeters.

Have students complete Question 1 on AP Book 3.2 p. 175.

**ACTIVITY 1**

**Creating a rectangle with a given perimeter.** Using grid paper or BLM 1 cm Grid Paper, students try different ways to draw a rectangle with a perimeter of 12 units. They should record the length and width of their rectangle, and find the area. Students who finish early can try to draw one or two different rectangles with a perimeter of 12 units, and record the length, width, and area of the rectangles. Have volunteers sketch their rectangles on the board, labeling the lengths and widths. Have other volunteers verify that the perimeter for each rectangle is 12 units and calculate the area.

**Drawing rectangles with the same perimeter, but different areas.** Project BLM 1 cm Grid Paper on the board. Tell students that you want to draw as many rectangles on the grid with a perimeter of 12 units as you can, just like in the activity. ASK: When we’re drawing rectangles on the grid, what is the smallest width in whole numbers a rectangle could have? (1 unit) Write on the board:

\[
\text{width} = 1 \text{ unit}
\]

SAY: The width is 1 unit and the perimeter is 12 units. Adding width and length will give half the perimeter of a rectangle. ASK: How do we find half the perimeter of the rectangle? ($12 \div 2$) What is $12 \div 2$? (6) Write on the board:

\[
\text{length} + \text{width} = 6
\]

\[
L + 1 = 6
\]

SAY: I wrote $L$ for the missing length. ASK: How do I find the missing length? ($6 - 1 = 5$ units) SAY: So if the width of the rectangle is 1 unit, the length must be 5 units to make a perimeter of 12 units. Draw this rectangle on the grid.
ASK: How can we check to make sure this rectangle has a perimeter of 12 units? (add the four sides) How long is each side? (1, 5, 1, 5) Write the measurements beside each side on the board. ASK: What is the perimeter of the rectangle? (1 + 5 + 1 + 5 = 12 units) Write the calculation under the rectangle.

SAY: We’ve drawn a rectangle with a perimeter of 12 units that has a width of 1 unit. ASK: What’s the next smallest width in whole numbers a rectangle on the grid could have? (2 units) Repeat the process above to find the length, draw the rectangle, and have a volunteer verify that the perimeter is 12 units. (length = 4 units) Repeat with the width 3 units. (length = 3 units) The finished rectangles are shown below:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
<td>3</td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

ASK: What is special about the third rectangle we drew? (all four sides are the same) What do we call a rectangle where all four sides are the same? (a square)

ASK: What is the area of the first rectangle? (1 × 5 = 5 square units) Write the equation under the rectangle. Repeat for the second rectangle and the third rectangle. (2 × 4 = 8 square units; 3 × 3 = 9 square units) ASK: Do the rectangles have the same area? (no) Do the rectangles have the same perimeter? (yes, 12 units) SAY: So these are three rectangles with the same perimeter, but different areas.

**ACTIVITY 2**

*Rectangles with the same perimeter but different areas.* Students work in pairs. Student 1 draws a rectangle with a perimeter of 16 units on grid paper or BLM 1 cm Grid Paper. Student 2 checks that the rectangle’s perimeter is 16 units and then finds the area of the rectangle. Then student 2 draws a different rectangle with a perimeter of 16 units. Student 1 checks that this new rectangle also has a perimeter of 16 units and then finds the area of the rectangle.

*Drawing rectangles with the same area, but different perimeters.* Tell students that you will now try to draw different rectangles on a grid with an area of 12 square units. Write on the board:

\[ \text{length} \times \text{width} = 12 \text{ square units} \]

ASK: What’s the smallest width in whole numbers a rectangle on the grid can have? (1 unit) Write “1” underneath “width.” SAY: To check what the length will be, we have to count by 1s until we get to 12. Count by 1s together as a class, holding up your fingers to keep count. After you reach
10 on the second hand, use the other hand to show two more fingers to reach 12. ASK: How many fingers did we raise? (12) Write “12” under “length.” ASK: Does $12 \times 1 = 12$? (yes) NOTE: Grade 3 students are not expected to know their 12 times tables, so remind students that 1 times any number gives that same number. Complete the equation on the board:

\[
\text{length} \times \text{width} = 12 \text{ square units}
\]

\[
12 \times 1 = 12
\]

SAY: Let’s try a width of 2 units. Write “2” under the 1 on the board. SAY: To find out what the length will be when the width is 2 units, we need to count by 2s until we reach 12. Count by 2s together as a class, holding up your fingers to keep count. ASK: How many fingers did we raise? (6) Write “6” under “length.” SAY: Let’s check our answer. ASK: Does $6 \times 2 = 12$? (yes) Complete the equation on the board:

\[
\text{length} \times \text{width} = 12 \text{ square units}
\]

\[
6 \times 2 = 12
\]

SAY: I want to rush to the larger numbers. Let’s try a width of 5 units. Write “5” under the 2 on the board. SAY: Remember, we need to count by 5s and see if we get to 12. Count by 5s together as a class, holding up your fingers to keep count. Stop when you say “15.” ASK: Did we pass 12 already? (yes) So that means the width cannot be 5 units. No whole number times 5 will equal 12. No matter what whole number we write for the length, if the width is 5, we won’t get an area of 12 square units. Cross out the 5 on the board. SAY: Let’s try a width of 3 units. Repeat the above process with a width of 3 units. (4 $\times$ 3 = 12) The picture should look like this:

\[
\text{length} \times \text{width} = 12 \text{ square units}
\]

\[
6 \times 2 = 12
\]

SAY: We found three different rectangles. Let’s draw these rectangles on a grid. Project a transparency of BLM 1 cm Grid Paper on the board. Draw the three rectangles shown below on the grid:

Have volunteers write the perimeter of each rectangle on the board. (see answers on the following page)
Perimeter of A = 1 + 12 + 1 + 12 = 26 units
Perimeter of B = 2 + 6 + 2 + 6 = 16 units
Perimeter of C = 3 + 4 + 3 + 4 = 14 units

Exercises

1. Draw two different rectangles with the given area on grid paper. Find the perimeter of each rectangle.

   a) 6 square units   b) 8 square units
   c) 9 square units   d) 10 square units

   Answers
   a) width = 1 unit, length = 6 units, perimeter = 14 units; width = 2 units, length = 3 units, perimeter = 10 units
   b) width = 1 unit, length = 8 units, perimeter = 18 units; width = 2 units, length = 4 units, perimeter = 12 units
   c) width = 1 unit, length = 9 units, perimeter = 20 units; width = 3 units, length = 3 units, perimeter = 12 units
   d) width = 1 unit, length = 10 units, perimeter = 22 units; width = 2 units, length = 5 units, perimeter = 14 units

   Bonus: Draw three different rectangles with an area of 16 square units on grid paper. Find the perimeter of each. Hint: One of the rectangles will be a square.

   Answer: width = 1 unit, length = 16 units, perimeter = 34 units; width = 2 units, length = 8 units, perimeter = 20 units; width = 4 units, length = 4 units, perimeter = 16 units

Extensions

1. How many different rectangles can you draw on a grid with an area of 24 square units? Find the perimeter of each rectangle.

   Answers: four: width = 1 unit, length = 24 units, perimeter = 50 units; width = 2 units, length = 12 units, perimeter = 28 units; width = 3 units, length = 8 units, perimeter = 22 units; width = 4 units, length = 6 units, perimeter = 20 units

2. Sara draws three rectangles on a grid. She labels the rectangles A, B, and C. Each rectangle has a perimeter of 18 units. The width of rectangle A is 2 units, the width of rectangle B is 3 units, and the width of rectangle C is 4 units.

   a) Draw rectangles A, B, and C.
   b) What are the lengths of rectangles A, B, and C?
   c) Find the area of rectangles A, B, and C.
   d) Write the rectangles in order from shortest width to longest width.
   e) Write the rectangles in order from smallest area to largest area.
Selected answers
b) length of A = 7 units, length of B = 6 units, length of C = 5 units
c) area of A = 14 square units, area of B = 18 square units, area of C = 20 square units
d) A, B, C
e) A, B, C

3. Carlos and Mona each draw a rectangle on a grid. The area of each rectangle is 18 square units, and the width of each rectangle is longer than 1 unit. The perimeter of Mona’s rectangle is larger than the perimeter of Carlos’ rectangle. Find the length and width of each rectangle. Hint: First try to see how many rectangles you can make on a grid so that the width is greater than 1 unit and the area is 18 square units.

Answer: Carlos’ rectangle has a width of 3 units and a length of 6 units. Mona’s rectangle has a width of 2 units and a length of 9 units.

(MP3, MP5) 4. a) Find $15 \times 8$. Use any tool you want.

b) Explain your strategy to a partner and why it works. Do you agree with each other? Discuss why or why not.

c) Talk with a partner. Compare the following strategies:

- I used a clock. I know that $15 \times 4$ is 60 because there are 4 groups of 15 in 60, and $15 \times 8$ is twice as much as $15 \times 4$, so $15 \times 8$ is 120.

- I used paper and pencil. $15 \times 8 = (3 \times 5) \times 8 = 3 \times (5 \times 8) = 3 \times 40 = 120$.

How are they the same? How are they different?

Selected sample answers: c) same: they both use easier products, different: one uses something in the real world, the other just uses math; the first way uses that $15 \times 4$ is easy because it occurs in real life on clocks, the second way uses that multiplying tens is easy.

In part b), encourage partners to ask questions to understand and challenge each other’s reasoning (MP3)—see p. A-49 for sample sentence and question stems to guide students.
Goals

Students will review their work on area and perimeter in this unit and apply their knowledge to solve problems involving area and perimeter.

PRIOR KNOWLEDGE REQUIRED

Understands area as the number of square units that cover a shape
Can find the area of a rectangle by multiplying length and width
Understands that area is additive
Understands perimeter as the distance around a shape
Can find the perimeter of a shape by adding the side lengths
Can find the missing length or width of a rectangle given area and one side

Review finding the area of shapes made from two rectangles. Draw on the board:

```
3 yd
\[ \text{6 yd} \quad 5 \text{ yd} \quad 3 \text{ yd} \]
```

ASK: How do we find the area of this L-shaped garden? (add the area of the two rectangles) Prompt the students by pointing out the dashed line that divides the large shape into two rectangles. Shade the left rectangle. SAY: Let’s find the area of the shaded rectangle first. ASK: What is the length of the shaded rectangle? (6 yd) What is the width? (3 yd) How do we find the area of the first rectangle? (multiply length times width, 6 \( \times \) 3) Write “Area 1 =” and have a volunteer complete the calculation for the area of the first rectangle. (6 \( \times \) 3 = 18 yd\(^2\)) Pointing to the unshaded rectangle, ASK: What is the length of this second rectangle? (5 yd) What is the width? (3 yd) How do we find the area of this unshaded rectangle? (multiply length times width, 5 \( \times \) 3) Write “Area 2 =” and have another volunteer complete the calculation for the area of the second rectangle. (5 \( \times \) 3 = 15 yd\(^2\)) ASK: How do we find the total area of the garden? (add the areas of the two rectangles) Write “Total area =” and have a third volunteer complete the calculation. They will need to write the numbers vertically and add by regrouping. (18 + 15 = 33 yd\(^2\))

For the following exercises, warn students to pay attention to the units.
Exercises: Find the area of the L-shaped garden. Include the units.

a) 4 ft  
10 ft  
8 ft  
3 ft  
Area 1 =  
Area 2 =  
Total area =

b) 9 m  
5 m  
6 m  
7 m  
Area 1 =  
Area 2 =  
Total area =

Answers: a) \(10 \times 4 = 40 \text{ ft}^2\), \(8 \times 3 = 24 \text{ ft}^2\), \(40 + 24 = 64 \text{ ft}^2\);
b) \(5 \times 9 = 45 \text{ m}^2\), \(6 \times 7 = 42 \text{ m}^2\), \(45 + 42 = 87 \text{ m}^2\)

Review finding the area of a large rectangle in two different ways.

Draw on the board:

SAY: Let’s find the area of the large rectangle in two different ways. Point to the bottom edge of the large rectangle and ASK: What is the length of the large rectangle? (7 inches) Point to the left edge of the large rectangle and ASK: What is the width of the large rectangle? (3 inches) What do we get if we multiply the length and width? (the area of the large rectangle) Write “Area = 7 \times 3 =” and have a volunteer complete the equation. (21 in\(^2\))

ASK: What is another way we could find the area of the large rectangle? (add the areas of the two smaller rectangles) Write on the board:

\[
\text{Area} = (3 \times \underline{\quad}) + (3 \times \underline{\quad})
\]

\[= \underline{\quad} + \underline{\quad}
\]

\[= \underline{\quad}
\]

SAY: One side of each rectangle is 3 inches long. We just need to fill in the other side. Point to the first set of brackets and SAY: We will write the area of the first rectangle in these brackets. Trace your finger around the three by five rectangle and SAY: The width of this rectangle is 3 inches. What is the length? (5 inches) Write “5” in the first blank on the board. Trace your finger around the three by two rectangle and SAY: The length of this rectangle
is 3 inches. What is the width? (2 inches) Write “2” in the second blank.
ASK: What is 3 × 5? (15) Write “15” in the third blank. ASK: What is 3 × 2? (6) Write “6” in the fourth blank. Have a volunteer add 15 and 6 to fill in the final blank. (21 in²) ASK: Did we get the same area both ways? (yes)

**Exercises:** Find the area of the large rectangle in two different ways. Include the units in your final answer.

![Diagram of a rectangle divided into two smaller rectangles](image)

**Answer:** 50 cm²; 7, 3; 35, 15; 50 cm²

**Bonus:** Does 4 × 7 = (4 × 5) + (4 × 2)? Explain using the picture.

**Answers:** Yes. The large rectangle in the picture shows the product 4 × 7. The large rectangle is made up of two smaller rectangles, to the left and right of the dashed line. The left rectangle shows the product 4 × 5, and the right rectangle shows the product 4 × 2.

**Word problems with area and perimeter.** Write on the board:

Keesha wants to put tiles on her kitchen floor.
Her kitchen floor is a rectangle 9 ft long.
The area of the kitchen floor is 72 ft²

What is the width of the kitchen floor?

ASK: What is the length of the kitchen floor? (9 ft) What is the area? (72 ft²) Do we know the width? (no) How do we find the width? (72 ÷ 9) What is the width? (8 ft) Remind students that to find the missing side measurement of a rectangle, they need to divide the area by the known side measurement.

Continue writing on the board:

The width is 72 ÷ 9 = 8 ft.

What is the perimeter of the kitchen floor?
Tell students it will be helpful to draw a picture of the rectangle. The picture does not need to be perfect. Draw on the board:

```
9 ft
8 ft
___ ft
___ ft
```

SAY: I drew the length of 9 feet on the top side and the width of 8 feet on the left side. Point to the right edge and SAY: This side is opposite to the width that is 8 ft. ASK: How long is this side? (8 ft) Write “8 ft” in the blank. Point to the bottom edge and ASK: How long is this side? (9 ft) Write “9 ft” in the blank. ASK: How do we find the perimeter? (8 + 9 + 8 + 9 or (2 × 9) + (2 × 8)) Have a volunteer write the calculation and the answer for the perimeter on the board, as shown below:

Perimeter = 8 + 9 + 8 + 9 = 34 ft

Write on the board:

```
Each tile is 1 square foot. How many tiles will Keesha need?

ASK: What is the area of the kitchen floor? (72 ft²) So how many square foot tiles will it take to cover the floor? (72) Write on the board:

Keesha will need 72 tiles.

Each tile costs $4. How much money will Keesha spend?

ASK: Since each tile costs 4 dollars, how can we find the cost of all 72 tiles? (4 × 72) Write “4 × 72” on the board. Remind students that when they don’t know how to do a multiplication, they can use repeated addition. ASK: How can we write 4 × 72 using repeated addition? (72 + 72 + 72 + 72) Complete the equation on the board as shown below:

4 × 72 = 72 + 72 + 72 + 72

Have a volunteer do the addition on the board by lining up the digits vertically and using regrouping. (288) Write on the board:

Keesha will spend $288.
```

**Exercises:** Matt’s garden is a rectangle 6 yards long. The area of the garden is 30 yd².

a) How wide is the garden?

b) What is the perimeter of the garden?

c) Matt divides the garden into three equal sections. What is the area of each section?
d) Sofia’s garden is a rectangle 8 yards long and 6 yards wide. How much larger is the area of Sofia’s garden?

e) Sofia’s plants flowers in her garden. She plants 3 flowers in each square yard. How many flowers does she plant altogether?

Bonus: Matt plants flowers in his garden. He plants 8 flowers in each square yard of the garden. How many flowers do Matt and Sofia plant in total?

Answers: a) $30 \div 6 = 5$ yards wide; b) $6 + 5 + 6 + 5 = 22$ yards; c) each section has an area of $30 \div 3 = 10$ yd$^2$; d) area of Sofia’s garden $= 8 \times 6 = 48$ yd$^2$, $48 - 30 = 18$ yd$^2$, so the area of Sofia’s garden is 18 yd$^2$ larger; e) $48 \times 3 = 48 + 48 + 48 = 144$ flowers altogether; Bonus: Matt: $30 \times 8 = 240$ flowers, $240 + 144 = 384$ flowers altogether

Extensions

1. Find the product of two numbers where one number is larger than 10. Hint: You can break the larger number into two numbers less than (or equal to) 10. Draw a sketch of the rectangle to help find the answer.

   a) $6 \times 15$

   b) $9 \times 18$

   c) $8 \times 12$

   d) $4 \times 17$

   e) $7 \times 13$

   f) $5 \times 20$

   Sample solutions:

   a) $6 \times 15 = (6 \times 10) + (6 \times 5) = 60 + 30 = 90$

   b) $9 \times 18 = (9 \times 9) + (9 \times 9) = 81 + 81 = 162$

   c) $8 \times 12 = (8 \times 6) + (8 \times 6) = 48 + 48 = 96$

   d) $4 \times 17 = (4 \times 10) + (4 \times 7) = 40 + 28 = 68$

   e) $7 \times 13 = (7 \times 10) + (7 \times 3) = 70 + 21 = 91$

   f) $5 \times 20 = (5 \times 10) + (5 \times 10) = 50 + 50 = 100$

   (MP2) 2. Draw a picture to show that $(6 \times 5) + (6 \times 2) = 6 \times (5 + 2)$. Explain how your picture shows that this equation is true.

   Sample answer: The length of the large rectangle in the picture below is $5 + 2 = 7$ units. To find the area of the large rectangle, add the areas of the two smaller rectangles $(6 \times 5) + (6 \times 2)$, or multiply the length and width of the large rectangle, $6 \times 7$. 

![Diagram of a rectangle split into smaller rectangles showing the calculation of $6 \times 5 + 6 \times 2 = 6 \times 7$.]
3. Fill in the missing number.
   a) \[3 \times 5 = (\_ \times 4) + (3 \times 1)\]
   b) \[6 \times 7 = (6 \times 4) + (6 \times \_)\]
   c) \[\_ \times 6 = (7 \times 2) + (7 \times 4)\]
   d) \[9 \times 8 = (9 \times \_) + (9 \times 5)\]
   e) \[8 \times \_ = (8 \times 6) + (8 \times 4)\]
   f) \[10 \times \_ = (10 \times 8) + (10 \times 9)\]

   **Answers:** a) 3, b) 3, c) 7, d) 3, e) 10, f) 17

4. The shaded area in the picture is 15 ft². What is the total area of the shape?

   **Solution:** The shaded area is made of 3 equal squares. So each square has the area \(15 \div 3 = 5\) ft². The whole shape is made up of 4 of these small squares, so the total area is \(4 \times 5 = 20\) ft².
Measuring Area in Square Inches (I)

I. Measure the area in square inches using pattern block squares.

a) 

Area = _____ in²

b) 

Area = _____ in²

c) 

Area = _____ in²

d) 

Area = _____ in²
Measuring Area in Square Inches (2)

2. Measure the area in square inches using pattern block squares.

a) 

[Diagram of a shape]

Area = _____ in²

b) 

[Diagram of a shape]

Area = _____ in²

c) 

[Diagram of a shape]

Area = _____ in²

d) 

[Diagram of a shape]

Area = _____ in²
Array of Square Inches
Adding Areas

I. Find the area of each shape in square units.

a) A.  
   
   Area of A = ____

b) A.  
   
   Area of A = ____

c) A.  
   
   Area of A = ____

d) A.  
   
   Area of A = ____
Areas of Large Rectangles

I. Find the area of the large rectangle in two different ways.

a)  

\[
\begin{align*}
\text{Gray area} &= \_ \times \_ = \_ \text{ in}^2 \\
\text{White area} &= \_ \times \_ = \_ \text{ in}^2 \\
\text{Total area} &= \_ + \_ = \_ \text{ in}^2
\end{align*}
\]

\[
\begin{align*}
\text{Length of large rectangle} &= \_ + \_ = \_ \text{ in} \\
\text{Width of large rectangle} &= \_ \text{ in} \\
\text{Area of large rectangle} &= \_ \times \_ = \_ \text{ in}^2
\end{align*}
\]

b)  

\[
\begin{align*}
\text{Gray area} &= \_ \times \_ = \_ \text{ yd}^2 \\
\text{White area} &= \_ \times \_ = \_ \text{ yd}^2 \\
\text{Total area} &= \_ + \_ = \_ \text{ yd}^2
\end{align*}
\]

\[
\begin{align*}
\text{Length of large rectangle} &= \_ \text{ yd} \\
\text{Width of large rectangle} &= \_ + \_ = \_ \text{ yd} \\
\text{Area of large rectangle} &= \_ \times \_ = \_ \text{ yd}^2
\end{align*}
\]

c)  

\[
\begin{align*}
\text{Gray area} &= \_ \times \_ = \_ \text{ ft}^2 \\
\text{White area} &= \_ \times \_ = \_ \text{ ft}^2 \\
\text{Total area} &= \_ + \_ = \_ \text{ ft}^2
\end{align*}
\]

\[
\begin{align*}
\text{Length of large rectangle} &= \_ + \_ = \_ \text{ ft} \\
\text{Width of large rectangle} &= \_ \text{ ft} \\
\text{Area of large rectangle} &= \_ \times \_ = \_ \text{ ft}^2
\end{align*}
\]
Finding Products Using Rectangles

I. Fill in the blanks to find the product.

a) \[2 \times 7 = (2 \times 4) + (2 \times 3)\]
\[= \quad + \quad\]
\[= \quad\]

b) \[5 \times 16 = (5 \times q) + (5 \times 7)\]
\[= \quad + \quad\]
\[= \quad\]

c) \[8 \times 13 = (8 \times 3) + (8 \times 10)\]
\[= \quad + \quad\]
\[= \quad\]

d) \[q \times 1q = (q \times q) + (q \times 1)\]
\[= \quad + \quad\]
\[= \quad\]
Area and Perimeter of Rectangles

I. Fill in the blanks. Find the perimeter and the area.

a) 8 in  
   \[ \text{Perimeter} = 4 + 8 + _____ + _____ = _____ \text{ in} \]
   \[ \text{Area} = _____ \times _____ = _____ \text{ in}^2 \]

b) 8 yd  
   \[ \text{Perimeter} = _____ + _____ + _____ + _____ = _____ \text{ yd} \]
   \[ \text{Area} = _____ \times _____ = _____ \text{ yd}^2 \]

c) 8 ft  
   \[ \text{Perimeter} = _____ = _____ \]
   \[ \text{Area} = _____ \]

d) 0.875 in  
   \[ \text{Perimeter} = _____ = _____ \]
   \[ \text{Area} = _____ \]
PS3-9 Making a Simpler Problem

Use this lesson after: 3.2 Unit 7

Standards: 3.OA.B.6, 3.OA.D.8, 3.NBT.A.2, 3.NBT.A.3, 3.NF.A.1

Goal:
Students will, given a problem, make a simpler problem and use the solution to the simpler problem to solve the harder problem.

Prior Knowledge Required:
Can add and subtract two-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can represent fractions by using lengths and areas

Vocabulary: add, perimeter, subtract

Materials:
two pre-made sticks of different lengths and different colors
BLM Fraction Strips and Circles (p. S-82, see Problem Bank 12)
scissors (see Problem Bank 13)
BLM Planting a Flower Garden (pp. S-84–85, see Performance Task)

Using a given simpler problem to help solve a harder problem. Write on the board:

There are 300 people in line. How many people are behind the 7th person?

(MP.3, MP.6) ASK: What makes this problem hard? (students might say because 300 is a lot of people) Would it be easier if I asked how many people are behind the 299th person in line? (yes, there is only 1 person) SAY: So, it is not how big 300 is that makes this problem hard. ASK: Can you say what exactly makes it hard? (7 and 300 are far apart) Write on the board:

There are 8 people in line. How many people are behind the 7th person?

SAY: I'm going to draw the 8 people. Draw on the board:

Front of line: ☺☺☺☺☺☺☺

Have a volunteer circle the 7th person in line. ASK: How many people are behind the 7th person? (1) SAY: You don't have to draw happy faces. You could just draw dots. Draw on the board:

● ● ● ● ● ● 7th
Exercises: Draw a picture to show your answer.
a) There are 6 people in line. How many people are behind the 5th person?
b) There are 5 people in line. How many people are behind the 3rd person?
c) There are 9 people in line. How many people are behind the 4th person?
Answers: a) 1, b) 2, c) 5

(MP.8) ASK: For exercises like the ones you just did, how can you get the answer from the two numbers given? (subtract) PROMPT: What operation can you do? (subtract) Draw on the board:

```
   ● ● ● ● ●
```

SAY: This picture shows there are 9 people in line, and I want to know how many people are after the 5th person. ASK: Where are the dots that represent the people who are after the 5th person? (to the right of the line) SAY: So I have to subtract all the dots that are before the line. ASK: How many people are before the line? (5) SAY: So you subtract 9 − 5 to get how many people are after the 5th person. ASK: What is 9 − 5? (4) SAY: So there are four people after the 5th person. Now you know that you can do any problem like this with subtraction.

Exercises:
a) There are 300 people in line. How many people are behind the 12th person?
b) There are 487 people in line. How many people are behind the 30th person?
Bonus: There are 3,459 people in line. How many people are behind the 1,459th person?
Answers: a) 300 − 12 = 288; b) 487 − 30 = 457; Bonus: 3,459 − 1,459 = 2,000

ASK: How did solving the easier problems make it easier to solve the harder problems? (doing so told me that the right approach is to subtract: number of people in line − the position of the person in line)

Off-by-one errors. Tell students that you are waiting in line to get on a rollercoaster ride. You are 37th in line and you see your friend who is 7th in line. ASK: How many people are between my friend and me? Note various guesses. Most students will likely guess 37 − 7 = 30. If they do, SAY: That answer is close, but not quite right. Let’s draw a simple picture using smaller numbers to see what is going on. Write on the board:

```
Front of line: ● ● ● ● ● ● ● ● ● ●
```

SAY: Each dot represents a person. ASK: How many dots did I draw? (9) SAY: For this simpler problem, suppose I am 9th in line. Have a volunteer circle the last dot. SAY: My friend is 5th in line. Have another volunteer circle the 5th dot. Label the dots, as shown below:

```
Front of line: ● ● ● ● ● ● ● ● ● 5th 9th
```

ASK: How many people are between the 5th and 9th person? (3) Is that equal to 9 − 5? (no) SAY: It is close, but not quite equal.
**Exercises:** Draw a picture to decide how many people are between the given positions.

a) the 7th and 8th person  
b) the 7th and 9th person  
c) the 7th and 10th person  
d) the 7th and 11th person  
e) the 7th and 12th person  
f) the 7th and 37th person

**Answers:** a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the right answer? (no) Did it give close to the right answer? (yes) How can you get the number of people between two people given their positions in line? (subtract the smaller position from the other and then subtract 1 from the difference) How many people are between the 37th person and the 7th person in the rollercoaster line? (29, because $37 - 7 = 30, 30 - 1 = 29$) How did starting with smaller numbers help? (answers will vary) 

SAY: Sometimes, it is easier to start by using smaller numbers than what is given in the problem. Then you will see patterns and how to solve the harder problem. Now that you know the pattern for finding the number of people between any two positions, you can use that method with any numbers.

**Exercises:** How many people are in line between the given positions?

a) the 8th and 78th person  
b) the 314th and 1,000th person  
c) the 492nd and 613th person

**Answers:** a) 69, b) 685, c) 120

**Making the problem easier by finding what is relevant.** SAY: Sometimes making the problem easier has nothing to do with using smaller numbers and finding a pattern. Sometimes all you need to do is eliminate information that’s not relevant, and moving objects around can help with that.

Affix pre-made sticks of different colors and different lengths to the board, end to end. Label one length and their combined length. The following is an example for 8 cm and 12 cm but your sticks can be other lengths:

![Diagram](image)

Tell students that all the measurements are in centimeters. ASK: How long is the second stick? (12 cm) SAY: It is easy to see with sticks, but now I’m going to move these sticks around. Slide the gray stick down and draw the lines around it, as shown below:

![Diagram](image)
ASK: How did I move the sticks? (you slid one of them down) SAY: This now looks like a problem to do with shapes and the lengths of missing sides. There's a lot of extra information in this second problem compared with the first problem, so it looks harder, but it actually has exactly the same answer as the other one. The total length of the two sticks is still 20 cm—I just slid one of the sticks down so that they are not side by side anymore.

Exercises: Find what the ? stands for by pretending the sticks are side by side.

a)                                    b)                              
11  ?                                    15  ?
18

c)                                    d)                              
15  25                                    683
?  ?                                    315  ?

Bonus:  
e)                                    f)                              
12  21                                    8  9
16  ?                                    8  ?
?                                   12

Answers: a) 7, b) 9, c) 40, d) 368, Bonus: e) 17, f) 13

SAY: By pretending that the sticks were side by side, you turned the problem into an easier problem.
Making the problem easier by emphasizing what is relevant. SAY: We can look at a problem and focus on what matters most. For example, if you need to find a vertical edge—straight up and down—then bold all the vertical lines. If you need to find a horizontal edge, bold all the horizontal lines.

**Exercises:** Find what the ? stands for by making the problem into an easier problem.

a) ![Diagram](a)  

b) ![Diagram](b)  

**Answers:** a) bold vertical, ? = 5; b) bold horizontal, ? = 12

Point out to students that by bolding the horizontal or vertical lines, they changed the problem into an easier problem.

Finding perimeter by finding missing side lengths. Remind students that, to find the perimeter of a shape, they have to add up the lengths of all the outside edges.

**Exercises:** Find the perimeter by finding missing sides, then adding all the sides.

a) ![Diagram](a)  

b) ![Diagram](b)  

**Answers:** a) 48, b) 30

Problem Bank

(MP.1) 1. How many posts are needed to build the fence?

a) A fence 38 m long made with posts 1 m apart.

b) A fence 50 m long made with posts 5 m apart.

c) A fence 60 m long made with posts 3 m apart.

**Answers:** a) 39, b) 11, c) 21

2. Ken wants to cut a rope into 20 equal parts. How many cuts does he need to make?

**Answer:** 19
(MP.7, MP.8) 3. A teacher tells the class to read pages for homework.
a) How many pages do students read if they are assigned pages 87 and 88?
b) How many pages do students read if they are assigned pages 87, 88, 89, and 90?
c) How many pages do students read if they are assigned pages 87 to 91?
d) How many pages do students read if they are assigned pages 87 to 92?
e) How can you get the number of pages from the two page numbers given? Hint: Compare your answers to the result of subtracting the two numbers.
f) How many pages do students read if they are assigned pages 87 to 104?
Answers: a) 2, b) 4, c) 5, d) 6, e) subtract and add 1, f) 18

(MP.4, MP.7) 4. Ben reads every night at home. How many pages does he read?
a) from pages 352 to 386  b) from pages 298 to 314  c) from pages 408 to 451
Answers: a) 35, b) 17, c) 44

(MP.1) 5. Ava reads pages 354 to 412 except for pages 363 to 389, which have illustrations only. How many pages does Ava read?
Answer: 32

(MP.1) 6. How many whole numbers are greater than 11 and less than 45?
Answer: 33

(MP.1, MP.4) 7. When everyone in Tom’s class stood in line, Tom was 12th in line and 15th from the end of the line. How many people are in the class?
Answer: 26

(MP.1, MP.4) 8. Make several easier problems until you see the pattern to help you do the harder problem.
a) A fence is made using 42 posts, each 1 meter apart. How long is the fence?
b) A fence is made using 34 posts, each 2 meters apart. How long is the fence?
Answers: a) 41 meters, b) 66 meters

(MP.1, MP.4) 9. Jen builds a fence for a square garden with posts 1 m apart, including a post at each corner. How many posts does she need for a garden of the given size?
a) 10 m by 10 m
Hint: Start with a garden that is 1 m by 1 m and then move on to 2 m by 2 m, 3 m by 3 m, and so on.
b) 20 m by 20 m
Answers: a) 40, b) 80

(MP.1, MP.4) 10. Ravi builds a fence around a square field that is 20 m by 20 m. He uses a post at each corner.
a) How many posts are needed if the posts are 1 m apart?
b) How many posts are needed if the posts are 2 m apart?
c) How many posts are needed if the posts are 4 m apart?
d) How many posts are needed if the posts are 5 m apart?
Answers: a) 80, b) 40, c) 20, d) 16
11. Each line segment of the path below has a length of 1 meter. What is the path’s total length?

Solution: 18 vertical meters plus 17 horizontal meters = 35 meters altogether

12. Cut out the strips and circles from BLM Fraction Strips and Circles (you may cut the line down to the center of the circles).
   a) Use folding to check that one fifth of strip A is shaded.
   b) Use folding to check that two fifths of strip B is shaded.
   Hint: Use your answers to parts a) and b) to help you determine a strategy for parts c) and d).
   c) Use folding to check that one fifth of circle C is shaded.
   d) Use folding to check that two fifths of circle D is shaded.
Fraction Strips and Circles

A

B

C

D
Performance Task: Planting a Flower Garden

Materials:
BLM Planting a Flower Garden (pp. S-84–85)

Preparation for the performance task. Write on the board:

There are 7 trees in 4 rows. How many trees are there altogether?

ASK: How can you solve this problem? (multiply 4 × 7) Erase the 7 and write “18” in its place.
ASK: How can you solve this problem? (multiply 4 × 18) SAY: But I don’t have the 18 times table memorized. What else can I do instead of multiply? (18 + 18 + 18 + 18) PROMPT: What is 4 × 18 short for? Write the addition vertically on the board and have a volunteer do the addition, as show below:

\[
\begin{array}{c}
3 \\
18 \\
18 \\
18 \\
+ 18 \\
72
\end{array}
\]

SAY: Another way to make a problem easier is to turn it into one that you already know how to do. If you find one of the questions difficult, but you need the answer to do the next question, guess an answer and use it in the next question. You can still get the next question right based on your answers to the other question, even if your answer to the other question is wrong.

Performance Task: Planting a Flower Garden. Provide students with BLM Planting a Flower Garden. Question 6 provides an opportunity to apply the problem-solving strategy of making an easier problem. All students might find an answer to Question 6, but students who notice that the strategy can be used will find the problem easier.

Answers: 1. 50 feet; 2. $150; 3. $100; 4. $251; 5. 4; 6. 19, I subtracted 20 − 1; 7. 76
Planting a Flower Garden (I)

Ivan has a flower bed. His flower bed is 5 ft wide by 20 ft long.

1. Ivan wants to put a fence around his flower bed to keep his pet dog out. How much fencing does he need?

2. If fencing costs $3 for each foot, how much will the fence cost?

3. Ivan needs to add more soil to his garden. The added soil will cost $1 for each square foot. How much will the soil cost?

4. Ivan will grow the flowers from seeds. He buys the seeds for $1. How much will Ivan’s total cost for the flower bed be?
Planting a Flower Garden (2)

5. To plant his flower bed, Ivan will:
   • plant each flower 1 foot apart,
   • start planting 1 foot from each edge of his flower bed.

   His flower bed is 5 feet wide and 20 feet long.

   How many rows of flowers can Ivan plant?

6. How many flowers can Ivan plant in each row? Explain how you got your answer.

This Unit in Context

Using the three-step progression learned in Grades 1 and 2 for measuring length (1.MD.A.1, 2 and 2.MD.A.1) and in earlier Grade 3 units for measuring time and area (3.MD.A.1 and 3.MD.C.5, 6), students will be introduced to liquid volume (3.MD.A.2) through direct comparison, then indirect comparison, and then measurement. Students will also be introduced to mass (3.MD.A.2), first using direct comparison, and then measurement.

Students will measure liquid volume using milliliters (mL) and liters (L), presented in a vertical number line format. This draws both on knowledge from Grade 2 of representing numbers on number lines (2.MD.B.6) and from 3.1 Unit 4 when they saw number lines presented using skip counting intervals. Students will learn to distinguish between the volume of a liquid in a container and the capacity of the container. They will also measure the mass of an object in grams and kilograms, and will use both balances and scales to explore mass. Finally, students will solve word problems involving volume and mass and use drawings to represent problems (3.MD.A.2).

Students will be introduced to more measurement attributes in later grades, such as angles in Grade 4 (4.MD.C.5) and volume of solid objects in Grade 5 (5.MD.C.3, 4). As the years progress, students will continually work with these attributes in more complicated ways. For example, in Grade 8, students will learn to find the volume of cones, cylinders, and spheres (8.G.C.9).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1 to MP8. Here are some examples of how students can show that they have met a standard.

MP3: In MD3-41 Extension 3, students make and investigate a conjecture about how the perimeters of a sequence of rectangles change.

MP7: In MD3-42 Extension 3, students look for and make use of structure to check if the perimeter is 18 craft sticks—for example, rectangles have opposite sides equal.

MP8: In MD3-41 Extension 3, students notice regularity when they recognize that they never change the perimeter when they successively build rectangles by adding a column and taking away a row.
Unit 8 Measurement and Data: Volume and Mass

Introduction

In this unit, students will learn about volume and mass. They will distinguish between volume and capacity using liters and milliliters. They will measure the mass of an object in grams and kilograms, and will use both balances and scales to explore mass. They will solve word problems involving volume and mass, and use drawings to represent problems.

NOTE: Students should be familiar with volumes in US customary units, such as 1 cup, 1 teaspoon, and 1 quart. These measurements are mentioned and demonstrated but are not examined in this unit.

Materials. This unit requires many materials. Gathering and preparing them will take extra time.

For the lessons dealing with liquid volume, the following is an overview of materials needed. Specific needs are listed for individual lessons.

• access to a sink and water
• funnel, tray, water jug
• many containers of various sizes
• graduated measuring cups

NOTE: If a 1 L bottle is difficult to find, you might substitute a bottle large enough to hold the contents of two small plastic water bottles, with a capacity of 16.9 oz. or 500 mL each. If a 5 L water jug is not available, a 2-gallon jug can be substituted. To help find other containers needed in this unit, following are some capacities in US customary units and their equivalent capacities in milliliters:

<table>
<thead>
<tr>
<th>US Capacity</th>
<th>Milliliters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 gallon</td>
<td>≈ 3,785 mL</td>
</tr>
<tr>
<td>1 quart</td>
<td>≈ 946 mL</td>
</tr>
<tr>
<td>1 pint</td>
<td>≈ 473 mL</td>
</tr>
<tr>
<td>1 cup</td>
<td>≈ 237 mL</td>
</tr>
<tr>
<td>1 12 oz soda can</td>
<td>≈ 355 mL</td>
</tr>
<tr>
<td>1 16.9 oz water bottle</td>
<td>≈ 500 mL</td>
</tr>
</tbody>
</table>

For the lessons dealing with mass, the following is an overview of materials needed. Specific needs are listed for individual lessons.

• balances to determine if two masses are in balance or not
• scales to measure mass in grams and in kilograms
• weights of various masses including 1 g, 2 g, and 5 g

NOTE: Ensure that students understand how a balance differs from the more familiar scale. You might explain to students that a balance has two trays and resembles a seesaw. If the mass on each tray is the same, the trays balance. If the tray on one side has more mass, that tray goes down and the other tray goes up.
Goals
Students will compare liquid volumes using both direct and indirect methods.

PRIOR KNOWLEDGE REQUIRED
Can compare numbers or things using the words “more” or “less”

MATERIALS
access to a sink and water
jug for filling containers (optional)
tray (optional)
funnel (optional)
marker or pencil that can be erased
containers that are clear or translucent:
  • identical pairs in 4 different sizes and shapes, including a pair of matching thin bottles
  • 2 containers with the same shape and different heights and/or widths
  • 2 containers with very different shapes, such as a drinking glass and a bottle
  • 6 large containers of different sizes
  • 10 clear plastic cups of identical size
2 identical graduated, clear measuring cups (for example, each with a capacity of 2 cups or 500 mL)
2 large graduated, clear measuring cups (for example, each with a capacity of 5 cups)

Comparing volumes directly with identical containers. SAY: A liquid is something that flows freely and can be poured. Milk and water are examples of liquids. We use the word volume to describe how much of the liquid there is.

NOTE: When filling containers, you may want to use a jug of water and a funnel and then place the containers to be filled on a tray to avoid spilling.

Select two identical thin containers (such as water bottles) and place them at the front of the classroom on a desk. Pour water into one of the containers until it is about half full, as shown below:
Have a volunteer use a marker or pencil to draw a horizontal line on the outside of the container to mark the level of the water. Mark the second container at the same height. Have a different volunteer pour water into the second container until it reaches the mark on that container. Ask: What can you say about the volume of water in the two containers? (It is the same) How can you tell? (The lines are at the same height on the bottles) The containers should look like this:

![Image of two containers with water levels marked]

Erase the line on the second container. Add more water to the second container until it is about three-quarters full. Have a volunteer redraw the line on the second container to indicate the new level of the water. Say: The volume of water in the containers was the same, but then I poured more water into the second container. Ask: Which container has more liquid? (The second container) How can you tell from the lines on the outside of the container? (The mark for the liquid level on the second container is higher than the mark on the first container) The containers should look like this:

![Image of two containers with different water levels]

Point to the pair of containers and ask: Which container has less liquid? (The first) How can you tell from the lines on the outside of the containers? (The mark for the liquid level on the first container is lower than the mark on the second container)

Repeat three times with pairs of identical containers of other sizes.

**Comparing volumes directly with different-sized containers.** Select two containers that have roughly the same shape, but have very different heights and/or widths. Place them at the front of the classroom on a desk. Pour water into the smaller container until it is about half full. Ask a volunteer to use a marker to mark the level of the water on the smaller container, as shown below:

![Image of a tall and a short container with marked water levels]
Mark the second, larger container at the same height as the first. Empty the contents of the first container into the second container and refill the first container until the water reaches the level marked on it before. SAY: I poured the water from the first container into the second container and then refilled the first container, so we know that the second container has the same amount of liquid as the first container. Now we are going to pour more water into the second container until the water level reaches the same height as marked on the first container. Have a volunteer pour more water into the second container until the water levels are at the same height, as shown below:

ASK: Which container has more liquid? (the second one) How do you know? (because the first and second container had the same amount of liquid and then we added lots more water to the second container) SAY: For the first two containers, when the marks were at the same height, the containers had the same amount of liquid. ASK: What is different in this situation? (the containers were the same size before, and now the second container is much bigger)

Have students do Questions 1–2 on AP Book 3.2 p.179.

Comparing volumes indirectly using a pair of identical containers. In advance, select two containers that have completely different shapes. For example, one container might be a wide drinking glass and the other, a thin bottle.

SAY: It is difficult to tell which container has more liquid when the containers are not identical. Place the two containers at the front of the classroom on a desk. Pour some water into each of them, as shown below. The exact amount of water is not important.

Point to the containers and ASK: Are these two containers identical? (no) SAY: It would be easier to compare the amount of liquid in the containers if the containers were identical. Since they are not, we will compare the liquid in these two containers using two new containers that are identical. Take two identical empty containers and place them between the two containers with water, as shown on the following page.
Ask two volunteers to each pour the water from one container into the empty container beside it, as shown below:

Ask two other volunteers to each mark the liquid level in the identical containers, as shown below:

ASK: Which of the identical containers has more liquid? (in the picture above, it is the left container) How can you tell? (the mark on the left container is higher than the mark on the right container) So which of the original containers had more liquid? (the one on the left)

Have students do Question 3 on AP Book 3.2 p.179. Point out that the picture to the left of the arrows shows two different containers with liquid and the picture on the right shows what happens when the liquid has been poured into two identical containers.

**Comparing volumes indirectly using containers with a known volume.**

In advance, select two large, empty containers that are not identical. Using identical clear plastic cups, fill the first container with five cups of water and the second container with three cups of water. Place the containers at the front of the classroom on a desk.

SAY: We can compare volumes by finding how many cups of liquid are in each container. Place several empty cups beside each container, as shown below:
NOTE: At this point, the cups shown are identical cups of any size rather than measuring cups of 1 cup each in US customary units. Later in this lesson, students will use cup measurements in US customary units.

Have a volunteer pour all the contents of the first container into the adjacent cups. Have a different volunteer do the same with the second container, as shown below:

![Diagram of pouring from one container to another](image)

ASK: How many cups of liquid were in the container on the left? (5) How many were in the container on the right? (3) Which container had more liquid? (the one with 5, on the left)

Ask students to do Question 4 on AP Book 3.2 p. 180. Point out that the picture on the left shows one container with liquid and the picture on the right shows what happens when the liquid from that container has been poured into cups of an identical size.

**Reading volumes using a graduated container such as a measuring cup.** Pour exactly 1 cup of water in US customary units into a clear measuring cup. SAY: This is a measuring cup. To find the volume of water in this measuring cup, bend down so that your eye is at the same level as the top of the water. Bend down to demonstrate. SAY: Look at the top of the water and see how the water level lines up to a measurement marked on the side. ASK: What is the volume of water in this measuring cup? (1 cup)

Practice reading the measurements for different volumes of water with students until they are comfortable reading the volume on the measuring cup.

**Comparing volumes indirectly using a graduated container such as a measuring cup.** Select two new, large, non-identical containers. Use a large graduated measuring cup to add 4 cups of water to the first container and 2 cups of water to the second container. **NOTE:** If you do not have measuring cups this large, adjust the amounts of liquid. Place a measuring cup beside each container, as shown below:

![Diagram of measuring cups](image)

SAY: We can compare volumes using a measuring cup. Have a volunteer pour the contents of the first container into a measuring cup. Have a different volunteer do the same with the second container and the other measuring cup, as shown on the following page.
SAY: Look at the measurements marked on the first measuring cup.
ASK: What number does the liquid line up to on the side? (4) How many cups of liquid were in the first container? (4)
SAY: Look at the measurements marked on the second measuring cup. ASK: What number does the liquid line up to on the side? (2) How many cups of liquid were in the second container? (2) Which container had more liquid? (the first) How can you tell? (4 is more than 2)

Have students do Question 5 on AP Book 3.2 p. 180.

Extensions

1. Container A has more liquid than Container B. Container C has more liquid than Container A. What is the order of the containers from the smallest amount of liquid to the largest amount of liquid?
   
   Answer: B, A, C

2. Sam has 8 identical plastic cups filled with water and 2 containers, A and B. He pours one entire cup at a time and pours all 8 cups into the containers. Container A has more water than Container B. Use the table to list all the possible volumes the two containers could be holding.

<table>
<thead>
<tr>
<th>Container A</th>
<th>Container B</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Answers
3. a) Start by drawing a square on grid paper so that it has 6 rows and 6 columns. Take away a row, then add a column. Repeat until you have 4 shapes altogether.

b) Does the perimeter get bigger, smaller, or stay the same? Explain.

**Answers**

a) 

b) The perimeter starts at \(6 + 6 + 6 + 6 = 24\), then becomes \(5 + 7 + 5 + 7 = 24\), then becomes \(8 + 4 + 8 + 4 = 24\), then \(9 + 3 + 9 + 3 = 24\). The length plus the width stays the same, because the length always becomes 1 bigger and the width always becomes 1 smaller, so the total also stays the same. The perimeter is twice the total, so it stays the same too.

Redirecting students: For part b), encourage students to keep track of not only each perimeter but also how it was calculated. Explain that recording how they found each perimeter helped them understand why the perimeter stays the same. If they keep track in an organized way, they can notice more patterns.
MD3-42  Liters and Milliliters
Pages 181–182

STANDARDS
3.MD.A.2

VOCABULARY
capacity
cup
liquid
liter (L)
measuring cup
milliliter (mL)
quart
tea spoon
volume

Goals
Students will distinguish between capacity and volume.
Students will use liters and milliliters to measure and estimate the
volume of liquids and the capacity of containers.

PRIOR KNOWLEDGE REQUIRED
Can read the level of liquid in a graduated container
Understands what volume is

MATERIALS
access to a sink and water
jug for filling containers (optional)
tray (optional)
funnel (optional)
marker
graduated 2-cup measuring cup
graduated 1-liter measuring cup
empty 1-quart milk container
5 L clear water jug
5 water bottles each with 1 L capacity
3 bottles with 50 mL, 500 mL, and 5 L capacities

NOTE: For Questions 7–8 on AP Book 3.2 p. 182, students will need the
following containers to measure capacity in mL and L: glass, bowl, water
bottle, sink, bucket, large jug.

Comparing the capacity of containers. Pass around a 2-cup measuring
cup so students can see the graduated scale used for measuring. ASK: How
many cups of water are needed to fill the cup? (2) SAY: The capacity
of a container is how much liquid it can hold. The capacity of this measuring
cup is 2 cups. ASK: Would a bathtub have more capacity or less capacity than
this measuring cup? (more) Why? (a bathtub can hold more water)

Exercises: Circle the container with greater capacity.

a) glass, sink
b) bathtub, swimming pool
c) teaspoon, pot

Answers: a) sink, b) swimming pool, c) pot

Using liters to measure capacity and volume. SAY: In many places in the
world, liters are used to measure capacity and volume. One liter is a little
larger than 1 quart. Pour 1/4 cup of water into the 2-cup measuring cup.
Hold up an empty 1-quart milk container and the measuring cup and
SAY: A liter is about the same as 1 quart plus 1/4 of a cup.
Write on the board:

1 liter (1 L)

SAY: The short form for liter is “L.” Write on the board:

<table>
<thead>
<tr>
<th>swimming pool</th>
<th>small paint can</th>
<th>bathtub</th>
<th>barrel</th>
</tr>
</thead>
<tbody>
<tr>
<td>large soda bottle</td>
<td>gas tank</td>
<td>teaspoon</td>
<td></td>
</tr>
</tbody>
</table>

Point to each word or phrase and have students signal thumbs up (yes) or thumbs down (no) to indicate whether the container has a capacity of about 1 L. (yes: small paint can, large soda bottle)

**Distinguishing between capacity and volume.** Hold up an empty, clear 5 L water jug. SAY: Let’s find the capacity of the jug. Have five volunteers each fill a 1 L water bottle. Ask the first volunteer to pour the contents of their 1 L water bottle into the 5 L jug. Have another volunteer use a marker to mark the level of the water in the jug. Repeat with the other four volunteers’ 1 L water bottles. When you are done, you will have a jug that can be used as a measuring jug, as shown below:

![Jug with marked levels]

ASK: What is the capacity of the jug? (5 L) Empty the jug. Fill the jug to the 2 L mark, as shown below:

![Jug with 2 L mark]

SAY: The capacity of this jug is 5 L. This jug can hold a maximum of 5 L of liquid. The volume of the water is the actual amount of space the water takes up. ASK: What is the volume of the water in the jug? (2 L)

Draw on the board:

![Jug filled to 2 L mark]

ASK: What is the capacity of the jug in the picture? (5 L) SAY: The jug is not full. ASK: What is the volume of the water in the jug? (4 L)
Exercises: Find the capacity of the container and the volume of the liquid.

\[
\begin{align*}
\text{a)} & \quad & 5 \text{ L} & \quad & 4 \text{ L} & \quad & 3 \text{ L} & \quad & 2 \text{ L} & \quad & 1 \text{ L} \\
\text{b)} & \quad & 3 \text{ L} & \quad & 2 \text{ L} & \quad & 1 \text{ L} \\
\text{c)} & \quad & 8 \text{ L} & \quad & 6 \text{ L} & \quad & 4 \text{ L} & \quad & 2 \text{ L}
\end{align*}
\]

Answers: a) capacity 5 L, volume 1 L; b) capacity 3 L, volume 2 L; c) capacity 8 L, volume 6 L

Using milliliters to measure volumes less than one liter. SAY: Sometimes we need containers for volumes much less than 1 L. A liter can be divided into 1,000 smaller units. Each smaller unit is called 1 milliliter. Write on the board:

1 milliliter (1 mL)

SAY: The short form for milliliter is "mL," a small "m" and a capital "L."

Write on the board:

A teaspoon can hold about 5 mL of liquid.

Read the sentence aloud. Write on the board:

lake swimming pool tablespoon
soup ladle bathtub small yogurt container

Ask volunteers to come up to the board and circle the containers that would hold much less than 1 L. (tablespoon, soup ladle, yogurt container)

Exercises: Find the capacity of the container and the volume of the liquid.

\[
\begin{align*}
\text{a)} & \quad & 200 \text{ mL} & \quad & 150 \text{ mL} & \quad & 100 \text{ mL} & \quad & 50 \text{ mL} \\
\text{b)} & \quad & 60 \text{ mL} & \quad & 40 \text{ mL} & \quad & 20 \text{ mL} \\
\text{c)} & \quad & 500 \text{ mL} & \quad & 300 \text{ mL} & \quad & 100 \text{ mL}
\end{align*}
\]

Answers: a) capacity 200 mL, volume 100 mL; b) capacity 60 mL, volume 20 mL; c) capacity 500 mL, volume 300 mL

Estimating the capacity of containers. Select three containers that have capacities of 50 mL, 500 mL, and 5 L. Label the 50 mL container as Container A, the 500 mL container as Container B, and the 5 L container as Container C. Place the containers at the front of the classroom on a desk.

ASK: Which container is larger than a quart of milk? (Container C) Is the capacity of Container C smaller than or larger than 1 L? (larger) SAY: The capacity of Container C is larger than 1 L but less than 10 L. Ask students to estimate the capacity in liters by signaling their estimate with their fingers. Fill the 1 L measuring cup with water and pour it into Container C.
Repeat and count the total number of times to find the capacity of Container C. (5 L)

ASK: Which container has a capacity that is much less than 1 L? (Container A) Should we measure the capacity using milliliters or liters? (mL) SAY: The capacity is much less than 100 mL. Write the numbers 10, 20, 30, 40, 50, 60, 70, 80, and 90 on the board and point to the numbers in turn as you ask students to estimate the capacity in milliliters.

Fill Container A with water and pour it into the 1 L measuring cup to find the capacity of the container. (50 mL)

Repeat the process of estimating and then measuring with Container B. (500 mL)

Extensions

1. Container A has a capacity of 6 liters. Container B has a capacity of 2 liters. Cara fills Container A with water. Then she pours the water from Container A into Container B until Container B is full. How much water is left in Container A?

   Answer: 4 L

2. Container A has a capacity of 5 L. Container B has a capacity of 3 L.

   a) How can you measure exactly 2 L of water using just these two containers? Hint: See Extension 1.

   b) Empty Container B. Pour the contents of Container A into Container B. How much water is in Container B?

   c) Fill Container A. How much water is in Container A? How much water is in Container B? How can you leave exactly 4 L in Container A?

   Answers
   a) Fill Container A. Pour the water from Container A into Container B until Container B is full. Container B now has 3 L water. Container A has 5 − 3 = 2 L left.
   b) 2 L
   c) 5 L; 2 L; pour water from Container A into Container B until Container B is full, 4 L will remain in Container A

   (MP4, MP5, MP7)

3. Sandy has toy animals and she wants to make a fence for a pretend farm, to keep all the animals inside. She will use craft sticks to make the fence and she wants it to be in the shape of a rectangle. She has 18 craft sticks. What is the biggest area she can make for the farm? Use any tool that you think will help you to solve the problem.

   Answer: 20 square craft stick lengths
Look for students to recognize structure (MP.7)—for example, that rectangles have opposite sides equal—and use it to calculate the perimeter. Look for students to select an appropriate tool (MP.5) such as craft sticks, grid paper, or a picture with labeled side lengths. Look for students to model (MP.4) the situation by drawing all the possible rectangles that can be made with 18 craft sticks.

Redirecting students: Some students may benefit from using actual craft sticks to make the different rectangles. Encourage students to draw copies of their rectangles on grid paper and record the different areas.

Individual or small-group follow-up: If students used craft sticks to help them find the area, ASK: Is there a way you could have found all these rectangles without actually making them? If students struggle, highlight the different rectangle widths and ASK: Would you miss any rectangles if you started at width 1 and went up in order? (no)
Goals
Students will solve one-step and two-step word problems involving capacity and volume.

PRIOR KNOWLEDGE REQUIRED
Can add and subtract two 2-digit numbers
Can multiply two 1-digit numbers
Can multiply multiples of 10 by 1-digit numbers

Solving capacity and volume problems involving addition and subtraction. Write on the board:

A container has a capacity of 20 L. The volume of water in the container is 14 L. How much more water can be poured into the container?

SAY: Some people find word problems easier to solve if they draw a picture. Draw on the board:

ASK: What is the maximum volume of water that can be poured into the container? (20 L) How do you know? (the capacity is 20 L) Label the top of the container with “20 L.”

ASK: What is the volume of water? (14 L) Have a volunteer draw the level of water in the container. Label the water level with “14 L.” The picture should look like this:

ASK: What operation can we perform to find how much more water can be added to fill the container to its capacity? (subtraction) What is 20 L – 14 L? (6 L)

Exercises
1. Find the amount of water that can be added to fill the container to its capacity.
   a) 
   b)
Answers: a) 3 L, b) 100 mL

2. Blanca has 2 containers that are partly full. She can add 325 mL more water. Will she have enough to fill both containers to their capacities? Explain.

\[
\begin{array}{c}
\text{500 mL} \\
\text{300 mL}
\end{array}
\quad
\begin{array}{c}
\text{400 mL} \\
\text{250 mL}
\end{array}
\]

Answer: No. She needs \((500 - 300) + (400 - 250) = 200 + 150 = 350\) mL.

Solving capacity and volume problems involving multiplication. Write on the board:

Anwar has a rain barrel in his backyard. The barrel collects 8 L of rain each day for 4 days. What is the total amount of rain collected in the barrel?

SAY: Sometimes it helps to draw a picture. Draw on the board:

\[
\begin{array}{c}
\text{8 L}
\end{array}
\]

Have a volunteer draw three more bars to represent the rainfall for the other days, as shown below:

\[
\begin{array}{c}
\text{8 L} \\
\text{8 L} \\
\text{8 L}
\end{array}
\]

ASK: What addition can we use to find the total amount of rainfall? \((8 + 8 + 8 + 8)\) Write on the board:

\[
8 + 8 + 8 + 8
\]

ASK: What is the total amount of rainfall? (32 L) What multiplication can we use instead of the addition? \((4 \times 8)\) SAY: We are adding four 8s. Continue writing on the board:

\[
8 + 8 + 8 + 8 = 4 \times 8 = 32
\]
Solving capacity and volume problems involving two-step word problems. Write on the board:

Clara has a 100 mL bottle full of medicine. She has to take 7 mL of medicine each day. What volume of medicine will be left in the bottle after 6 days?

SAY: Clara takes 7 mL of medicine each day for 6 days. ASK: What addition can we use to find the total volume of medicine she takes? \((7 + 7 + 7 + 7 + 7 + 7)\) What multiplication can we use instead of the addition? \((6 \times 7)\)

Write on the board:

\[
7 + 7 + 7 + 7 + 7 + 7 = 6 \times 7
\]

ASK: What is \(6 \times 7\)? (42) Draw on the board:

![42 mL](image)

SAY: The top part of the bottle shows the medicine that is gone, which we just figured out is 42 mL. ASK: What is the capacity of the bottle? (100 mL) What operation can we perform to find how much liquid is left? (subtraction) What is \(100 - 42\)? (58) SAY: So the medicine left in the bottle has a volume of 58 mL.

Exercises

1. Ethan has a 750 mL bottle of orange juice. Each day, he drinks 100 mL. How much orange juice will be left in the bottle after 6 days?

   **Answer:** \(750 - (100 \times 6) = 150\) mL

2. Ben's cake recipe needs 5 mL of vanilla flavoring for one cake. The bottle of vanilla holds 250 mL. How many cakes can Ben make with this bottle of flavoring?

   **Answer:** 50 cakes: \(250 \div 5 = 50\), or \(5 \times 50 = 250\) mL

Extensions

(MP:1)  1. Sally wants to mix enough salad dressing for dinner for the entire week. She needs 30 mL of oil and 10 mL of vinegar for each dinner.

   a) What is the total volume of oil needed for 7 days?

   b) What is the total volume of vinegar needed for 7 days?

   c) Sally has a choice of 3 containers to store the salad dressing: 250 mL, 500 mL, or 1,000 mL. Which container is best for storing the salad dressing for the week? Explain.
**Answers:** a) 210 mL; b) 70 mL; c) 500 mL, because the total volume of oil and vinegar for the week is $210 + 70 = 280$ mL

2. A small children’s pool has a capacity of 72 L. The garden hose can pour 9 L of water into the pool every minute.
   a) How long will it take to fill the pool using the hose?
   b) After it is filled, the pool starts to leak. It loses 1 L of water every minute. How long will it take for all the water to leak out of the pool?
   c) How long will it take to refill the pool if it is still leaking 1 L of water every minute?

**Answers:** a) 8 minutes, b) 72 minutes, c) 9 minutes

3. Tell students the following story:

Ben is making a doll house for his cousin. He wants to paint a wall that has a door and a window. He doesn’t want to paint the door and he doesn’t want to paint the window. Each square inch of wall uses 1 mL paint. The wall is 9 inches wide by 8 inches high. The window is 4 inches wide by 2 inches high. The door is 3 inches wide by 7 inches high. How much paint will he need?

Before asking students to start the problem, ask them to show with their hands about how wide they think the wall is. Then have them check their estimates using a ruler. Ask students if they think this is a reasonable width for the wall of a doll house. Then have them draw a picture that will help them solve the problem (MP1). Encourage students to only include math information in their picture (MP4). Emphasize that they should not draw any details that they don’t need. Encourage students to compare pictures with a partner and make any changes if they want. Then have students solve the problem individually.

**Answer:** 43 mL

If students do not have the multiplication facts memorized, this problem will require noticing structure (MP7), for example:

- calculating the product $8 \times 9$ and using it to find the total area of the wall
- calculating $2 \times 4$ and $7 \times 3$ and using the results to find the areas of the window and the door
- recognizing how to calculate the area of the wall that needs painting (subtract both $2 \times 4$ and $7 \times 3$ from $8 \times 9$, or add $2 \times 4$ and $7 \times 3$ and subtract the total from $8 \times 9$)
- recognizing that the area they calculated is in square inches
- recognizing that the number of square inches tells you the amount of paint, in mL

Encourage students who struggle with different parts to work together.
Goals
Students will compare masses of objects using a balance.

PRIOR KNOWLEDGE REQUIRED
Can compare numbers using “more than” or “less than”

MATERIALS
balance
objects to compare mass: eraser, textbook, pencil, notebook, and other objects of contrasting mass
several weights of 1 g each (optional)

Compare the masses of objects without using a balance or scale. Write on the board:

mass

SAY: Mass is the amount of matter in an object. The heavier the object, the greater the mass. The lighter the object, the less the mass. Place an eraser and a textbook at the front of the classroom on a desk. ASK: Which object is heavier? (the textbook) Which has more mass? (the textbook) Write on the board:

plane   pencil
desk    car
feather house
bed     paper clip
Earth   building

For each pair of objects, have a volunteer come to the board and circle the object with more mass and underline the object with less mass. (objects with more mass: plane, car, house, bed, Earth)

Comparing the mass of objects using a balance. Place a balance at the front of the classroom on a desk. SAY: We can use a balance to compare masses. When there is nothing on either side of the balance, the trays should be at the same height. Check that the trays are level.

Place an eraser on one side of the balance. ASK: What happened to the trays? (the tray with the eraser went down and the other tray went up) Hold a small textbook in the air. Before you place it on the empty tray of the balance, ASK: What do you think will happen? (the tray with the textbook will go down and the tray with the eraser will go up) Place the textbook on the empty tray of the balance. ASK: Which object has more mass? (the textbook)
Comparing the mass of objects using a picture of a balance. Draw on the board:

ASK: Which is lighter, the truck or the nickel? (the nickel) How can you tell by looking at the balance? (the tray for the nickel is higher) Repeat with pictures of various pairs of objects where one of the objects is much heavier than the other. Have students do Questions 1–4 on AP Book 3.2 pp. 185–186.

Comparing the mass of objects indirectly when the balance is level. Select items of different masses. Place one heavier object on one side of the balance. Place several lighter objects on the other side of the balance until the balance is level. For example, place one eraser on one side of the balance. On the other side, place several pencils until the balance is level. Suppose it takes three pencils until the balance is level. SAY: The balance is level. So one eraser has the same mass as three pencils. ASK: Which object has less mass, one pencil or one eraser? (one pencil) How do you know? (it took 3 pencils to have the same mass as 1 eraser, so each pencil has less mass than 1 eraser) Repeat with other pairs of items.

Have students do Question 5 on AP Book 3.2 p. 186. You might want to point out to students that the left side of each picture shows two objects to compare and the right side shows what needs to happen to make the balance level.

Comparing the masses of two objects indirectly using a third object. Select a third object such as a 1 g weight. NOTE: If several of these weights are not available, you can use several identical light objects, such as a box of new erasers. Select two notebooks of different masses. Place one of the notebooks on the left side of the balance. Place enough of the 1 g weights or the identical erasers on the right side to make the balance level.

ASK: How many of the weights did it take to make the balance level? (sample answer: 3) Repeat with the other notebook. ASK: How many of the weights did it take to make the balance level? (sample answer: 4) Which notebook needed more of the weights to make the balance level? (the second one) Which notebook had more mass? (the second one) Repeat with other pairs of objects.

Have students do Question 6 on AP Book 3.2 p. 186.
Extensions

1. Object A has the same mass as 2 erasers. Object B has the same mass as 6 erasers.
   
a) If Object A and Object B are placed on the same balance, will the balance be level? Explain.

b) How many of Object A will be needed to make the balance level?

Answers: a) no, Object B has more mass so Object B’s side of the balance will be lower; b) 3

2. Object A has the same mass as 15 erasers and 20 pencils. Object B has the same mass as 3 erasers and 4 pencils.
   
a) If Object A and Object B are placed on the same balance, will the balance be level? Explain.

b) How many of Object B will be needed to make the balance level?

Answers: a) no, Object A has more mass so Object A’s side of the balance will be lower; b) 5

3. There are 35 students in Ms. M’s class. Ms. M has a bucket of 300 ones blocks and 200 tens blocks. She gives each group of 5 students 3 tens blocks and 9 ones blocks. How many ones blocks are still in the bucket? Use equations to show your work. Show what each step means in the situation.

Solution: The question is asking about ones blocks, not tens blocks, so we don’t need to know anything about the tens blocks. 35 ÷ 5 = 7, so Ms. M gives out 9 ones blocks to each of 7 groups. 7 × 9 = 63, so she gives out 63 ones blocks. 300 − 63 = 237, so there are still 237 ones blocks in the bucket.

Look for students to recognize that the question is asking about only ones blocks, but if students start by recording all the given information, look for them to recognize which part is needed to solve the problem (MP.1). Look for students to model (MP.4) the situation with equations, to interpret the result of each computation in terms of the situation (MP.2), and to persevere in solving the problem (MP.1).

Small-group or individual follow-up: Some students will draw tens and ones blocks and find at the end that they only needed to draw ones blocks. Give them time to recognize this on their own, then ASK: Could you have done the problem without using the tens blocks at all? (yes) Why is that? (the question only asks about ones blocks) Emphasize that it is important to read the question first because that tells you what information you need to use.
Goals

Students will use grams and kilograms to measure the mass of objects using a scale.

PRIOR KNOWLEDGE REQUIRED

Can read the display on a scale in grams or kilograms

MATERIALS

- paper clips
- nickels
- 1 L soda bottle filled with water
- scale measuring in grams
- scale measuring in kilograms
- a full backpack
- balance
- one 1-gram weight, two 2-gram weights, one 5-gram weight

NOTE: For Questions 3–4 on AP Book 3.2 p. 187, students will need the following objects to measure mass in grams and kilograms: 25¢ coin, notebook, calculator, pen, cell phone, scissors, stack of books, backpack, laptop, and desk.

Identifying objects that have a mass of about 1 gram. Give a paper clip to each student. SAY: The mass of a paper clip is about 1 gram. The short form for “gram” is “g.” Write on the board:

Mass is about 1 gram (1 g)

Give a nickel to each student. ASK: Does a nickel have more mass than a paper clip? (yes) SAY: A nickel has a mass of about 5 grams. Write on the board:

5¢ Mass is about 5 grams (5 g)

Ask the class to think of some other objects that have about the same mass as a paper clip. (thumbtack, sheet of paper, ticket, tab on a soda can)

Identifying objects that have a mass of about 1 kilogram. SAY: Some objects are much heavier than a paper clip, so they have a mass much greater than 1 g. Hold up a 1 L soda bottle filled with water. SAY: A plastic 1 L bottle, which is about the size of a large soda bottle, has a mass of 1 kilogram. There are 1,000 grams in a kilogram. Write on the board:

1 kilogram (kg) = 1,000 grams (g)

Write on the board:

- textbook
- car
- truck
- laptop
Have a volunteer circle the names of the objects on the board that have a mass of about 1 kg. (textbook, laptop)

Have students do Questions 1–2 on AP Book 3.2 p. 187.

**Estimating mass and then measuring using a scale.** Hold up a pencil. ASK: Does this have more mass or less mass than a 1 L bottle of water? (less) Should I use grams or kilograms to estimate the mass? (grams) Why? (the pencil has less mass than a 1 L bottle of water, which has a mass of about 1 kilogram) SAY: A nickel has a mass of about 5 grams. Ask students to signal their estimate for the mass of a pencil in grams using their fingers (hold up four fingers for 4 g, seven fingers for 7 g, and so on).

SAY: A scale is used to find the mass of an object in grams or kilograms. Use a scale to measure the actual mass of the pencil and discuss whether the students’ estimates were close.

Have students do Question 3 on AP Book 3.2 p. 187. After they estimate the mass of each object, they should find the actual masses using a scale either as a class or in groups.

Hold up a backpack that is full. ASK: Does this have more mass or less mass than a 1 L bottle of water? (more) Should I use grams or kilograms to estimate the mass? (kilograms) Why? (the backpack has more mass than a 1 L bottle of water, which has a mass of about 1 kg) Ask students to signal their estimate of the mass of the backpack in kilograms using their fingers (hold up two fingers for 2 kg, five fingers for 5 kg, and so on).

Use a scale to measure the actual mass of the backpack and discuss whether the students’ estimates were close.

Have students do Question 4 on AP Book 3.2 p. 187. After they estimate the mass of each object, they should find the actual masses using a scale either as a class or in groups. Have students do Question 5 on AP Book 3.2 p. 188 to consider the better unit for measuring the mass of each object.

**Determining the missing mass.** Place two 2-gram weights and one 1-gram weight on the left side of a balance. The left side of the balance should go down. ASK: What operation can we use to find the missing mass? (addition: \(2 + 2 + 1 = 5\) g) What single weight can we place on the right side to make the balance level? (5 g) Place a 5-gram weight on the right side to make the balance level. (see picture below)
Draw on the board:

![Balance Diagram]

ASK: What is the total mass on the left side of the balance? (7 kg) What operation can we perform to find the mass that needs to be added on the right side to make the balance level? (subtraction: \(7 - 3 = 4\) kg)

Have students do Questions 6–8 on AP Book 3.2 p. 188.

**Extensions**

1. The balance is level and each unknown weight has the same mass. Find the unknown mass.
   
   a) ![Balance Diagram]
   
   Answers: a) 3 kg, b) 2 kg
   
   2. An elevator has a maximum load of 1,000 kg. Six people with a mass of 70 kg each are in the elevator. How many more people can get in the elevator if each new person has a mass of 80 kg?

   **Answer:** 7 people; the first 6 people have a mass of \(6 \times 70 = 420\) kg; \(1,000 - 420 = 580\) kg; only 7 people who each weigh 80 kg can get on the elevator (\(7 \times 80 = 560\) kg, \(420 + 560 = 980\) kg).

   (MP4, MP6, MP7)

   3. There are 28 students in the class. Each student has 5 blocks. How many more blocks would you need for everyone to have 7 blocks?

   **Sample solutions**
   - You need 28 \(\times\) 7 blocks for everyone to have 7 blocks, so that is \((20 \times 7) + (8 \times 7) = 140 + 56 = 196\). You need 28 \(\times\) 5 blocks for everyone to have 5 blocks, so that is \((20 \times 5) + (8 \times 5) = 100 + 40 = 140\). You need 56 more blocks (because 196 – 140 = 56).
- You have to give everyone 2 more blocks and there are 28 students, so the class needs \(2 \times 28\) more blocks. \(2 \times 28 = (2 \times 20) + (2 \times 8) = 40 + 16 = 56\), so you need 56 more blocks.

Look for students to use the equal sign and the brackets (MP.6), to calculate appropriate expressions (MP.4), and to notice and make use of structure (MP.7)—for example, splitting 28 into 20 + 8 to find \(28 \times 7\) or recognizing that they can find how many more blocks each person needs and multiply that by 28.

Individual or small-group follow-up: If students use the first sample solution above, encourage them to look back at their answer to see if there is a faster way to do the problem. ASK: How does the answer compare to 28? Could you have predicted that?

Whole-class follow-up: Take up both solutions provided, starting with the conceptually easier first solution, and move to the faster second solution.
**Goals**

Students will solve word problems involving mass.

**PRIOR KNOWLEDGE REQUIRED**

- Knows that the mass of a 1 L bottle of water is about 1 kg
- Knows that the mass of a paper clip is about 1 g

**Solving word problems involving mass.** Write on the board:

A serving of soup is 250 mL and has 20 g of fat and 4 g of sugar. John eats 3 servings of soup. What is the total volume of soup that he eats?

Ask a volunteer to read the question out loud. ASK: How many milliliters are in one serving? (250 mL) How can we use addition to find the total volume in three servings? (250 + 250 + 250) Write on the board:

\[
\begin{align*}
250 \\
250 \\
+ 250 \\
\end{align*}
\]

Have a volunteer find the total on the board. (750 mL) Write on the board:

How many grams of fat and sugar does John eat?

ASK: How many grams of fat are in one serving? (20) How can we use addition to find the total number of grams of fat in three servings? (20 + 20 + 20) Write on the board:

\[
\begin{align*}
20 \\
20 \\
+ 20 \\
\end{align*}
\]

Have a volunteer find the total on the board. (60 g) Write on the board:

\[
20 + 20 + 20
\]

ASK: How many 20s are being added? (3) What multiplication sentence can we use instead of the addition sentence? (3 \times 20) What is 3 \times 2? (6) What is 3 \times 20? (60) How many grams of fat does John eat? (60 g)

Continue writing on the board:

\[
\begin{align*}
20 + 20 + 20 \\
= 3 \times 20 \\
= 60 \\
\end{align*}
\]

Repeat the process to find the total grams of sugar in three servings. (4 + 4 + 4 = 3 \times 4 = 12 g)
SAY: John eats 60 g of fat in three servings of soup. Write on the board:

The recommended maximum number of grams of fat per day for a Grade 3 student is 65 g. How many more grams of fat can John eat without going over the maximum?

Ask a volunteer to read the question out loud. ASK: How many more grams of fat can John eat without going over the maximum? (5 g) How did you find the answer? (65 − 60 = 5)

Write on the board:

The recommended maximum number of grams of sugar per day is 16 g. How many more grams of sugar can John eat without going over the maximum?

Ask a volunteer to read the question out loud. SAY: John eats 12 g of sugar in three servings of soup. ASK: How many more grams of sugar can John eat without going over the maximum? (4 g) How did you find the answer? (16 − 12 = 4)

**Extensions**

1. The high school weight room has weights that have a mass of 2 kg, 5 kg, 10 kg, and 20 kg. The gym teacher counts the number of each weight and records it in a table:

<table>
<thead>
<tr>
<th>Mass of Each Weight (kg)</th>
<th>Number of Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

a) Find the total mass of all the weights.

b) The gym teacher wants to have a total of 1,000 kg in weights. How many more kilograms does he need?

c) How many of each weight will the school need to get to 1,000 kg?

**Answers**

a) \((7 \times 2) + (8 \times 5) + (10 \times 10) + (30 \times 20) = 14 + 40 + 100 + 600 = 754\) kg

b) \(1,000 - 754 = 246\) kg

**Sample answer:** c) ten 20 kg weights, four 10 kg weights, three 2 kg weights.
NOTE: Extension 2 will be easier if students have first done Extension 3 of OA3-39.

2. John wants to buy tiles for his rectangular kitchen floor. He bought 20 tiles and found that 15 of them fit across one side. He found that 12 of them fit across the other side. How many more tiles does John need to buy? Explain how you know.

Answers: 15 tiles fit across one side and 12 fit across the other, so $15 \times 12 = 180$ tiles fit on the floor in total. He already has 20 tiles, so he needs to buy 160 more.

Look for students to persevere in understanding and solving the problem (MP.1), to model it mathematically (MP.4) with a diagram or number sentences, and to recognize and use structure (MP.7)—for example, the large area can be split into smaller areas with numbers that students know how to multiply, and then you can add the smaller areas.

Redirecting students: If students struggle with how to begin, ASK: What do you need to know to find out how many tiles John still needs to buy? (how many he already has and how many will cover the floor) Are you given any of that information? (he has 20 already, but we don’t know how many tiles cover the floor) Suggest that students start by figuring out how many tiles are needed to cover the floor. Then encourage them to reread the problem to find the next step.

If students struggle with calculating $12 \times 15$, encourage them to draw a picture of the floor and the tiles on grid paper. Encourage students not to count the squares, because that would take too long. The challenge is to find a faster way.

Whole-class follow-up: Take up various strategies for calculating $12 \times 15$. Sample strategies include breaking the rectangle into:

- nine 4 by 5 rectangles
- a 10 by 15 rectangle and a 2 by 15 rectangle
- a 12 by 10 rectangle and a 12 by 5 rectangle
- several randomly chosen smaller rectangles and then add all the areas

Discuss each strategy. Does it work? Is it fast?

If students give you the answer 180, ASK: Is that the answer to the problem? (no) How do you know? (he already has 20, so he only needs to buy 160 more)
PS3-10 Choosing Strategies

Use this lesson after: 3.2 Unit 8


Goals:
Students will decide for one-step word problems what numbers make sense from a given choice. Students will solve multi-step word problems using the four operations, including in measurement contexts. Students will solve problems and puzzles using any of the problem-solving strategies studied so far.

Prior Knowledge Required:
Can round whole numbers to the nearest 10 or 100
Can add and subtract within 1,000
Can multiply and divide within 100
Can find the area and perimeter of a rectangle with given side lengths
Can solve multistep problems
Can multiply simple multiples of 10 by one-digit numbers

Materials:
BLM Number Chains (pp. T-35–37, see Problem Bank 6)
grid paper or BLM 1 cm Grid Paper (p. T-41, see Performance Task)
BLM Jigsaw Puzzles (pp. T-42–44, see Performance Task)

Finding numbers that make sense in a context. Write on the board:

____ people each ate _____ slices of pizza.
Altogether they ate ____ slices of pizza.

SAY: I’m going to fill in some numbers and you tell me if they make sense. Fill in the blanks as shown below:

____3__ people each ate ____2__ slices of a pizza for lunch.
Altogether they ate ____10__ slices of pizza.

ASK: Do these numbers make sense? (no) Why not? (because 3 people eating 2 slices each would only make 6 slices altogether) Challenge students to change one number only to make it true. (change the 3 to a 5; change the 10 to a 6) Put the original numbers back in the blanks and ask if there is another way the original numbers can be changed to make it true.
**Exercises:** Which two numbers make sense?

a) _____ people went in each car. Choose from: 3, 4, 5
   There were _____ cars.
   Altogether there were 12 people.

b) Three people went apple picking. Choose from: 50, 150, 200
   Each person picked _____ apples.
   Altogether they picked _____ apples.

**Answers:** a) 3 and 4 or 4 and 3, b) 50 and 150

Now change the numbers in the blanks in the example to make sense mathematically, but not contextually, as shown below:

____7____ people each ate ____50____ slices of pizza for lunch.
   Altogether they ate ____350____ slices of pizza.

**ASK:** Do these numbers make sense? (no) **SAY:** But 7 × 50 is 350. **ASK:** Why don’t these numbers make sense? (because no one can eat 50 slices of pizza for lunch) **PROMPT:** Can someone eat 50 slices of pizza for lunch? **SAY:** So, the numbers multiply to the answer 350 but the numbers do not make sense in the context of the problem.

**Exercises:** Which two numbers make the most sense from 5, 20, and 80?

a) _____ people went in each car.
   There were 4 cars.
   Altogether there were _____ people.

b) _____ people went in each bus.
   There were 4 buses.
   Altogether there were _____ people.

c) There are 4 floors in the building.
   Each floor needs _____ steps.
   Altogether, _____ steps are needed.

d) Morning recess lasted _____ minutes.
   Kate spent the recess doing skipping, talking to friends, playing tag, and playing soccer, each for _____ minutes.

**Answers:** a) 5 and 20, b) 20 and 80, c) 20 and 80, d) 20 and 5

**Choosing between problem-solving strategies.** Have students brainstorm the different types of problem-solving strategies they have learned so far this year. Write a list of strategies on the board such as: use smaller numbers, use a number line, use systematic search.

**NOTE:** The following examples reflect all the problem-solving strategies used in the problem-solving lessons for Grade 3. Choose among the examples based on which problem-solving lessons you have taught.
Write on the board:

Sam has 35 marbles and Kim has 63 marbles.  
How many more marbles does Kim have than Sam?

ASK: How would you solve this problem? (use subtraction) Do you need to check using smaller numbers that subtraction is the right thing to do here? (no) SAY: Questions like “how many more” can always be done using subtraction, so you don’t need to use smaller numbers to check the strategy. ASK: Could you also use a number line to solve this problem? (yes)

**Exercises:** Use a number line to decide how many more marbles Kim has than Sam.

a) Sam has 35 marbles and Kim has 63 marbles.
b) Sam has 288 marbles and Kim has 335 marbles.
c) Sam has 593 marbles and Kim has 818 marbles.

**Answers:** a) 28, b) 47, c) 225

Write on the board:

Sam has 537 marbles and Kim has 545 marbles.
Kim wants to give some to Sam so that they have the same number of marbles.  
How many marbles should Kim give to Sam?

Read the problem aloud, then ASK: Would using smaller numbers help here? (yes) Why? (because it is not obvious how to start the problem) SAY: When you’re not sure how to start the problem and one of the things making the problem difficult is the large numbers, you can try using smaller numbers. Write on the board:

Sam has 3 marbles and Kim has 7 marbles.

SAY: Suppose Kim gives Sam 1 marble. ASK: Now how many marbles does Kim have? (6) Now how many does Sam have? (4) Continue writing on the board:

Sam has 3 marbles and Kim has 7 marbles.

4                                    6

SAY: Suppose Kim gives Sam another marble. ASK: Now how many does Kim have? (5) And Sam? (5) Continue writing on the board:

Sam has 3 marbles and Kim has 7 marbles.

4                                    6
5                                    5

SAY: Now they have the same number of marbles. Write on the board:

Kim has _____ more marbles than Sam.  
She gives _____ marbles to Sam so they have the same amount.
Have volunteers fill in the blanks. (4, 2)

(MP.8) Exercises: How many more marbles does Kim have than Sam? How many should Kim give to Sam so they have the same amount?

a) Sam has 6 marbles and Kim has 14.
b) Sam has 5 marbles and Kim has 11.
c) Sam has 8 marbles and Kim has 12.
d) Sam has 9 marbles and Kim has 19.

Answers: a) 8 more, give 4; b) 6 more, give 3; c) 4 more, give 2; d) 10 more, give 5

ASK: Do you see a pattern in your answers? (yes) Challenge students to describe the pattern.
(the number Kim needs to give is always half the difference between what Sam and Kim have)

(MP.7) Have students solve the original problem on the board. (4)

(MP.7) Exercise: Yu has 352 marbles and David has 368 marbles. How many marbles does David have to give Yu so that they have the same number of marbles?

Answer: 8

Multi-step word problems practice.

(MP.1, MP.4) Exercises:

a) Mandy needs to be at school in one quarter of an hour. She bikes 1 block every 2 minutes. School is 8 blocks away. Will she be on time?
b) Kim buys 8 books that cost $8 each. Ben buys 9 books that cost $7 each. Who spent more money on books? How much more?
c) Roy swam 8 laps in 48 seconds. John swam 6 laps in 42 seconds. Who swam each lap faster?
d) Mindy has three dogs. All the dogs weigh the same amount. When Mindy weighs herself, the scale says 26 kg. When she holds all three dogs, the scale says 38 kg. How much does each dog weigh?
e) Ross has six pails that hold 5 L of water and three pails that hold 4 L of water. He fills them all up and pours them into a 50 L pail. He wants to fill the 50 L pail completely. How much more water does he need to add?

(MP.3) f) Marco weighs 23 kg and his sister weighs 19 kg. Their dad weighs 42 kg. Marco and his sister go to one side of a seesaw and their dad sits on the other side. Will the seesaw be easy to balance? Explain.
g) Carlos bought 3 large pizzas for $18. Sara bought 6 small pizzas for $24. Rani wants to buy 1 large pizza and 1 small pizza. How much will that cost?
h) A store sold 318 bananas, 396 apples, 203 oranges, 132 carrots, 516 potatoes, and 88 yams. Estimate how many more fruits (bananas, apples, and oranges) the store sold than vegetables (carrots, potatoes, and yams).

Answers: a) no; b) Kim spent $1 more; c) Roy; d) 4 kg; e) 8 L; f) yes, because 23 kg + 19 kg = 42 kg, so Marco and his sister weigh the same together as their dad weighs by himself; g) $10; h) estimate as about 300 + 400 + 200 = 900 fruits and about 100 + 500 + 100 = 700 vegetables, so the store sold about 200 more fruits than vegetables
Problem Bank

1. What number am I?
   a) When you round me to the nearest 10, you get 50. The sum of my digits is 11.
   b) When you round me to the nearest 10, you get 30. The sum of my digits is 8. I am even.
   **Answers:** a) 47, b) 26

(MP.3) 2. Ray has 85 marbles and Sal has 92 marbles. Can Sal give some marbles to Ray so that they have the same number of marbles? Explain.
   **Answer:** No, Sal has 7 more marbles than Ray, but 7 is not a multiple of 2, so Sal can’t give Ray half of the extra marbles.

3. Nina has 5 apples. Alice has 8 apples. Sandy has 11 apples.
   a) How many apples do they have altogether?
   b) They decide to share the apples equally. How many apples should each person get?
   c) Who doesn’t need to give or receive any apples?
   d) How many apples do the other two people need to give or receive?
   **Answers:** a) 24, b) 8, c) Alice, d) Sandy needs to give away 3 apples and Nina needs to receive 3 apples

4. a) Add the numbers in each group.

   \[
   \begin{array}{ccc}
   A. & B. & C. \\
   3 & 4 & 5 \\
   1 & 9 & 10 \\
   1 & 1 & 14
   \end{array}
   \]

   b) Switch exactly two numbers so that all three groups have the same total.
   **Answers:** a) A: 12, B: 20, C: 16; b) switch 5 and 9 between Groups A and B so all have 16

5. Use the 3s chart and the 9s chart to answer the questions.

   \[
   \begin{array}{ccc}
   3 & 6 & 9 \\
   12 & 15 & 18 \\
   21 & 24 & 27
   \end{array}
   \]

   \[
   \begin{array}{ccc}
   9 & 18 & 27 \\
   36 & 45 & 54 \\
   63 & 72 & 81
   \end{array}
   \]

   a) Use the 9s chart to divide.
   \[
   \begin{array}{ccccccc}
   54 \div 9 & 36 \div 9 & 72 \div 9 & 63 \div 9 & 45 \div 9 & 81 \div 9
   \end{array}
   \]

   b) Explain how you used the 9s chart to divide.
   c) Look at the numbers in the first row of the 9s chart. Where do they appear in the 3s chart? Why does this make sense?
   **Answers:** a) 6, 4, 8, 7, 5, 9; b) The answer of the number divided by 9 is the position of the number in the 9s chart, as in 54 is in the 6th position for \(6 \times 9 = 54\), \(54 \div 9 = 6\), and so on; c) they are in the third column. This makes sense because: 9 is \(1 \times 9\), which \(3 \times 3\). All the numbers in the first row of the 9s chart increase by 9, as they do in each column of the 3s chart; alternatively, use the associative property: \(18 = 2 \times 9 = 2 \times (3 \times 3) = (2 \times 3) \times 3 = 6 \times 3\), so it is in the 6th position of the 3s chart.
6. Provide students with **BLM Number Chains**, which challenges students to discover a pattern and to make their own number chain that will have the same property. Students will need to recognize what stays the same and what changes from one number chain to the next in order to create their own. This task also allows students to practice the multiplication and division they did throughout the year in the context of an interesting puzzle.

**Answers:**
1. a) 5, 25, 5, 1; b) 6, 30, 10, 2; c) 7, 35, 15, 3; d) 10, 50, 30, 6
2. The ending number always equals the starting number
3. Answers will vary
4. The ending number will always equal the starting number
5. a) − 10, + 5; b) subtraction, because all the other number chains did; c) 10, because that’s 2 × 5; d) division, because all the other number chains did; e) 5, because Step 2 used 5
6. a) 6, 30, 20, 4; b) yes, I got the number I started with, which is what I expected.

7. a) Add a square to the figure so that the perimeter increases.
   b) Add a square to the figure so that the perimeter decreases.
   c) Add a square to the figure so that the perimeter stays the same.

**Sample answers:**

![Sample answer images here]

8. The capacities of the first two pails are shown. Predict the capacity of the third pail.

   a) 10 L                 20 L                                 ?
   b) 30 L
   c) 300 L
      100 L

**Sample answers:** a) 50 L, b) 20 L, c) 400 L
Number Chains (I)

1. Look at the number chain.

\[
\begin{array}{c}
1 & + 4 & \times 5 & - 20 & \div 5 \\
\end{array}
\]

starting number

a) Start with 1. What number do you finish with?

\[
\begin{array}{c}
1 & + 4 & \times 5 & - 20 & \div 5 \\
\end{array}
\]

b) Start with 2. What number do you finish with?

\[
\begin{array}{c}
2 & + 4 & \times 5 & - 20 & \div 5 \\
\end{array}
\]

c) Start with 3. What number do you finish with?

\[
\begin{array}{c}
3 & + 4 & \times 5 & - 20 & \div 5 \\
\end{array}
\]

d) Start with 6. What number do you finish with?

\[
\begin{array}{c}
6 & + 4 & \times 5 & - 20 & \div 5 \\
\end{array}
\]

2. Look at your answers to Question 1. How can you get the ending number from the starting number?
Number Chains (2)

3. For each number chain, pick a starting number and find the ending number.

a) 
\[ \begin{align*}
\text{starting number} & \quad +4 \quad \times6 \quad -24 \quad \div6 \\
\text{ending number} &
\end{align*} \]

b) 
\[ \begin{align*}
\text{starting number} & \quad +3 \quad \times7 \quad -21 \quad \div7 \\
\text{ending number} &
\end{align*} \]

c) 
\[ \begin{align*}
\text{starting number} & \quad +5 \quad \times6 \quad -30 \quad \div6 \\
\text{ending number} &
\end{align*} \]

4. Look at your answers to Question 3. How can you get the ending number from the starting number?
Number Chains (3)

5. a) Finish the chain so that the chain has the same property as all the other ones.

\[
\begin{array}{cccc}
\text{Step 1} & \text{Step 2} & \text{Step 3} & \text{Step 4} \\
\square & +2 & \times 5 & \square \\
\text{starting number} & \square & \square & \square \\
\text{ending number} & \square & \square & \square \\
\end{array}
\]

b) What operation did you use for Step 3? _______________

Why? _______________________________________________________________________________

c) What number did you use for Step 3? _____

Why? _______________________________________________________________________________

d) What operation did you use for Step 4? _______________

Why? _______________________________________________________________________________

e) What number did you use for Step 4? _____

Why? _______________________________________________________________________________

6. a) Use the starting number 4 on the chain you created in Question 5.

\[
\begin{array}{cccc}
\text{Step 1} & \text{Step 2} & \text{Step 3} & \text{Step 4} \\
4 & +2 & \times 5 & \square \\
\text{starting number} & \square & \square & \square \\
\text{ending number} & \square & \square & \square \\
\end{array}
\]

b) Did you get the ending number you expect? Explain.
Performance Task: Jigsaw Puzzles

Materials:
grid paper or **BLM 1 cm Grid Paper** (p. T-41)
**BLM Jigsaw Puzzles** (pp. T-42–44)

**Preparation for the performance task.** In the performance task, students explore what the area and perimeter of a rectangle mean in the context of solving jigsaw puzzles. To prepare students, start with a classroom discussion by brainstorming what makes a puzzle easier or harder to solve. **PROMPTS:** What is easier, a 50-piece puzzle or a 500-piece puzzle? (the 50-piece puzzle) What about the picture on the front of the puzzle box—can the picture itself make the puzzle easier or harder? (if the puzzle consists of very different-looking parts, the picture will make the puzzle easier than if it consists of very similar-looking parts) How do outside pieces provide a hint for where a piece goes? (they have one or two sides that are straight) **SAY:** If everything else about two puzzles are the same, but one puzzle has more outside pieces, that one will probably be easier to solve. Draw on the board:

\[ \begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
8 & 9 & 10 & 11 & 12 & 13 & 14 \\
15 & 16 & 17 & 18 & 19 & 20 & 21 \\
22 & 23 & 24 & 25 & 26 & 27 & 28 \\
\end{array} \]

**SAY:** This picture represents a puzzle. It has four rows of seven pieces each. **ASK:** How many pieces are in the puzzle altogether? (28) **Write on the board:**

\[ 4 \times 7 = 28 \quad \text{or} \quad 7 + 7 + 7 + 7 = 28 \]

**SAY:** Now I want to count the outside pieces. Demonstrate doing so, by writing the numbers on the pieces as you count around the outside of the rectangle (there are 18 outside pieces).

For the following exercises, provide students with grid paper or **BLM 1 cm Grid Paper**.

**Exercises:** Copy the rectangle onto grid paper. Count the number of outside pieces.

a)

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
8 & 9 & 10 & 11 & 12 & 13 & 14 \\
15 & 16 & 17 & 18 & 19 & 20 & 21 \\
22 & 23 & 24 & 25 & 26 & 27 & 28 \\
\end{array}
\]

b)

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
8 & 9 & 10 & 11 & 12 & 13 & 14 \\
15 & 16 & 17 & 18 & 19 & 20 & 21 \\
22 & 23 & 24 & 25 & 26 & 27 & 28 \\
\end{array}
\]

**Answers:** a) 18, b) 18
Performance Task: Jigsaw Puzzles. Have students complete BLM Jigsaw Puzzles.

Answers:
1. a) 6, b) 9, c) 24, d) 20
2. a) 6 in², b) 9 in², c) 24 in², d) 20 in²
3. The area in square inches is the number of pieces
4. b) 12, c) 18, 22; d) 14, 18
5. The number of outside pieces is the perimeter in inches less 4
6. a) 240, b) 30 + 8 + 30 + 8 − 4 = 72
7. a) 240, b) 60 + 4 + 60 + 4 − 4 = 124
8. I predict that Puzzle A is harder because they both have the same total number of pieces but Puzzle A has fewer outside pieces.
I cm Grid Paper
Jigsaw Puzzles (I)

All the puzzles are made using 1-inch by 1-inch puzzle pieces.

1. How many pieces are in the puzzle?
   a) the rectangle is 2 in by 3 in
      
      
   b) the rectangle is 3 in by 3 in
      
      
   c) the rectangle is 3 in by 8 in
      
      
   d) the rectangle is 4 in by 5 in
      
      
2. Find the area of each rectangle in Question 1. Make sure to include the units.
   a) _______    b) _______    c) _______    d) _______

3. Look at your answers to Questions 1 and 2. How can you get the number of pieces from the area of the rectangle, given in square inches?
4. Complete the table.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Number of Outside Pieces in the Puzzle</th>
<th>Perimeter of Rectangle (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 2 inches by 3 inches</td>
<td>6</td>
<td>$2 + 3 + 2 + 3 = 10$</td>
</tr>
<tr>
<td><img src="image1.png" alt="Rectangle" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 3 inches by 3 inches</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Rectangle" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 3 inches by 8 inches</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Rectangle" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) 4 inches by 5 inches</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Rectangle" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Look at your answers to Question 3. How can you get the number of outside pieces from the perimeter of the rectangle given in inches?
Jigsaw Puzzles (3)

6. Puzzle A is a rectangle 30 inches by 8 inches.
   a) How many pieces does Puzzle A have altogether?

   b) How many outside pieces does it have?

7. Puzzle B is a rectangle 60 inches by 4 inches.
   a) How many pieces does Puzzle B have altogether?

   b) How many outside pieces does it have?

8. Which puzzle do you predict is harder, Puzzle A or Puzzle B? Explain.
Unit 9 Measurement and Data: Graphs

This Unit in Context

In Kindergarten, students classified objects into given categories, counted the numbers of objects in each category, and sorted the categories by count (K.MD.B.3). Students’ work in Kindergarten and in Grade 1 on counting and comparing numbers allowed them to answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another (1.MD.C.4). They used that knowledge to read the answers to those questions off of a chart or picture graph. In Grade 2, students used picture graphs, tally charts, and bar graphs to display and interpret information (2.MD.D.10), though the scale on bar or picture graphs was restricted to a single-unit scale. In this unit, this concept extends to scales where each square in a bar graph or picture in a picture graph might represent more than one object (3.MD.B.3). Students will use their previous knowledge of line plots from 3.2 Unit 6 to compare picture graphs, bar graphs, and line plots.

Presenting and reading information off scaled picture graphs or bar graphs requires constant use of multiplication and division, providing a context for the use of these operations and improving students’ fluency in them (3.OA.C.7).

In Grade 2, students solved “put together,” “take apart,” and “compare” problems that were posed through graphical information. In this unit, students will solve one- and two-step word problems, including comparison problems (3.MD.B.3), using information presented in scaled bar graphs. Students continue to solve multistep real-world word problems throughout the curriculum, as mentioned in 3.1 Unit 4.

This work on data management in Kindergarten through Grade 5 prepares students for the study of statistics from Grades 6 to high school. In Grade 6, students extend the concept of graphs to include histograms and box plots (6.SP.B.4). And in Grade 8, students extend the concept of graphs to scatter plots for the study of bivariate data (8.SP.A.1).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP.1, MP.2, and MP.4 to MP.7. Here are some examples of how students can show that they have met a standard.

MP.1: In MD3-51 Extension 4, students make sense of a non-routine problem when they recognize what information is needed to solve it and what information is unnecessary.

MP.5: In MD3-50 Extension 4, students select tools strategically when they pick a measuring tool, such as a graduated beaker, to measure capacities.

MP.6: In MD3-50 Extension 4, students attend to precision when they measure the capacities of two containers. To verify their prediction, they measure accurately by filling each container completely, avoiding spillage,
and waiting for the water to settle before recording their measurement. In MD3-51 Extension 4, students draw a picture to solve a word problem and attend to precision by labeling what each symbol they use means.

**MP.7:** In MD3-53 Extension 4, students look for and make use of structure when they multiply by tens (for example, to find $3 \times 80$, they can use $3 \times 8$) to solve a multi-step problem.
Unit 9 Measurement and Data: Graphs

Introduction

In this unit, students will learn to read and interpret information presented in the forms of picture graphs and bar graphs. They will analyze the data presented in the graphs to solve one- and two-step word problems. Students will also explore scales and scaled picture graphs and bar graphs. They will use multiplication and division to find the numbers represented on these scaled graphs.

Students will also learn to create their own scaled picture graphs and bar graphs from given data as well as data collected by the students themselves. Finally, students will compare three different types of graphs: picture graphs, bar graphs, and line plots (previously studied in Unit 6). They will examine the three types of graphs to see what information is represented, what is not, how easy it is to find information on each type, and the advantages of each type.

Materials. In several lessons you will need to show students graphs. Most of the graphs in this unit can be found on BLMs. You can photocopy the relevant BLMs onto transparencies and display them using an overhead projector.

Some of the extensions suggest drawing graphs from data students collect. Depending on the level of students who do the extension, they can either use the templates on BLM Picture Graph Templates or BLM Picture and Bar Graph Templates, or produce a graph on grid paper in their notebooks. The last option takes more time and requires organizational skills on the part of the student.

In addition to the BLMs provided at the end of this unit, the following Generic BLM, found in section V, is used in Unit 9:

BLM 1 cm Grid Paper (p. V-1)
Goals

Students will read and draw picture graphs, each with one symbol representing one item. Students will solve problems about the data presented in picture graphs.

Prior Knowledge Required

Can solve one- and two-step “how many more” or “how many less” word problems
Can read data from a table

Materials

20 connecting cubes in 3 colors for each student
chalk in 3 different colors
BLM Picture Graph Templates (p. U-49)
grid paper or BLM 1 cm Grid Paper (p. V-1, see Extensions 2–4)

Creating a concrete graph. Give each student 20 connecting cubes in 3 different colors. Students can have different combinations of the 3 colors and can have a few more or less than 20. Ask students to sort the cubes by color. SAY: I want to see which color you have the most of. ASK: How can you show me that quickly? Have students link the cubes into trains, or linked lines, of the same color so that they can easily compare the lengths. Have students place the trains side by side, so that the difference in length is easy to see.

The need for a common starting line. Show three trains of connecting cubes of different colors side by side without one common starting line, as shown below:

```
  O O O O O
O O O O O O O
      O O O
```

Compare each color to another color. ASK: Is it easy to see that there are more light gray cubes than white cubes? (yes) What about dark gray cubes and white cubes—can you compare them easily? (no) How should the trains be arranged so that we can clearly see the difference in length between all three colors? (the ends of all the trains should line up on one side) Invite a volunteer to rearrange your cube trains so they all line up on the left side. Have students rearrange their own cube trains if necessary.

ASK: Which color do you have the most of? (for example, red) How do you know? (the train of red cubes sticks out the farthest)
The need for matching. Use three different colors of chalk to draw on the board:

SAY: This is a picture of my cubes. ASK: Does this picture make it easy to see which color of cubes I have the most of? (no) Why not? (sample answer: some of the cubes are spread out) Encourage different explanations. ASK: Is it easy to see how many more light gray cubes than dark gray cubes I have? (no) Why not? (the dark gray cubes are spread out) Repeat with the dark gray and white cubes. ASK: How could we draw the cubes on grid paper so that we can see where there are more cubes? (put each cube in one grid square) PROMPT: Think of buddies. Buddies work in pairs. Can cubes go in pairs, too? How could you order the cubes on grid paper so that we see the pairs?

Draw a 3 by 6 grid on the board and ask volunteers to draw the cubes on the grid, starting from the left with each color. Ask them to make the squares that show the cubes smaller than the grid squares. The final picture should look like this:

ASK: Is it easy now to see which color we have more of? (yes) Point out how putting each square inside a grid square helps to organize the picture.

Introduce “graph.” SAY: The picture you have created is called a graph. A graph is a way of ordering and showing information that is easy to see and compare. Write “graph” on the board and have students read the word. Draw their attention to the fact that the “f” sound at the end of the word is written as “ph.”

Title, labels, and data. Have students each draw a similar graph for their own collection of cubes. SAY: We sorted the cubes by color. Add the title and labels to the graph on the board, as shown below:

Colors of Cubes

White

Light gray

Dark gray

Explain that all graphs need a title, or name, so that everyone knows what the graph represents. Labels give more detail about parts of the graph—for
example, what each row represents. Explain that, with the labels on the graph, you could draw all the cubes using the same color, and you could still tell how many cubes of each color there are.

Explain that information in a graph is called data. SAY: This graph shows 3 facts: I have 5 white cubes, 6 light gray cubes, and 4 dark gray cubes. These facts are the data in this graph. We can say that the graph shows 3 data items or 3 pieces of data.

**Graphs can be horizontal or vertical.** Explain that graphs can be created by arranging data in rows or columns. The columns need to have a common starting line just like the trains and rows did—in other words, the columns need to line up at the bottom. Draw a 6 by 3 grid on the board. Invite a volunteer to rearrange the cubes so that it is set up like a graph with columns instead of rows, as shown below.

Graphs can be used to display survey results. Explain that sometimes people need to know what other people think or like to do. SAY: For example, you might want to know how many people are going to vote for somebody in an election. In this case, we ask a question and give some answers to choose from. For example, will you vote for Candidate A or Candidate B, or you have not decided yet? Explain that asking many people a question with prepared answers is called a survey. Explain that you are going to conduct a survey. Your question is: What is your favorite season? SAY: There are four seasons, so my possible answers are winter, spring, summer, and fall. Write the four seasons on the board in a row. Have students suggest a title for the graph. (for example, Favorite Season) Write the title on the board above the seasons. Invite students to stand in a line in front of their favorite season.

**Introduce picture graphs.** SAY: Suppose you want to show the results of this survey at home but the whole class cannot line up every time you want to discuss the data. One way you could show the results would be to write down the names of all the people who voted for each season. But what if it does not really matter who likes which season the most? Suppose you only want to know the number of people that like each season. Explain that there is a way to show this information visually, without writing all the names. For example, you could draw a stick figure or a smiley face for each person in the class. Draw lines on the board to help students organize the graph, as shown on the next page. Extend the table to as many rows as needed.
### Favorite Season

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>Spring</td>
<td>Summer</td>
<td>Fall</td>
</tr>
</tbody>
</table>

Have each student draw a smiley face above the name of the season they chose earlier (they cannot change their answer). Ask them to draw only one smiley face in each box of the table so that the picture is organized, the same as with the the pictures of cubes. Explain that what they have created is called a *picture graph*. Write “picture graph” on the board.

**Analyzing a picture graph.** ASK: Which season is liked by the largest number of people? Which season is liked by the smallest number of people? How do you know? (answers will vary depending on class results) How does our graph make this easy to see? (the column for that season is the tallest) How many people like summer? How many people prefer spring? How can we see that? (count the number of smiley faces in the column for that season)

**How many more/fewer?** Help students compare the actual numbers for two of the seasons with “more.” **NOTE:** Your questions and the answers will vary depending on your class results.

For example, ASK: How many people voted for winter? How many people voted for spring? How many more people voted for spring than for winter? How can you find out? (subtract the number for winter from those for spring, or count the number of extra smiley faces after winter and spring are paired up) How can you see on the graph that more people voted for spring than for winter? (the spring column is taller than the winter column, or the spring column has more smiley faces) Have a volunteer draw circles around pairs of votes for spring and votes for winter. ASK: Are there any spring votes left that are not circled? How many? Is the number of spring votes that were left out after pairing the same as the result of the subtraction? How does the graph make the difference between votes for spring and winter easy to see? (the result of subtraction is the number of people that were not circled; it is also the number of smiley faces that go past the shorter row) Repeat this line of questioning for two other seasons but this time ask for the number that is less than another—for example, how many fewer people voted for fall than for summer?

**NOTE:** As students use the phrases more than and less than and fewer than, they might use the last two interchangeably. “Fewer than” is typically used to compare things that can be counted (for example, votes, books, students, leaves) and “less than” is used to compare things that cannot be counted (for example, water, sun). Model the distinction in language for students, but allow them to use the phrases interchangeably for now.
Explain that the phrase *most popular* in surveys mean that the largest number of people voted for that answer. *ASK:* What is the most popular season? How do you know? (it has the tallest column) What is the *least popular* season? How do you know? (it has the shortest column)

**There should be no “gaps” in the data.** Draw on the board:

<table>
<thead>
<tr>
<th>Students’ Birthplace</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Born in the US</td>
<td>☺☺☺☺☺☺</td>
<td>☺☺☺☺☺</td>
<td>☺☺☺☺</td>
<td>☺☺☺☺</td>
<td>☺☺☺☺</td>
</tr>
<tr>
<td>Born outside the US</td>
<td>☺☺☺☺</td>
<td>☺☺☺</td>
<td>☺☺</td>
<td>☺</td>
<td>☺</td>
</tr>
</tbody>
</table>

Explain that the picture graph shows in rows the number of students in a class that were born in the United States and outside the United States. *SAY:* The faces to represent each person start at the same place, but the ones in the top row extend farther than the ones in the bottom row. So I think more students were born in the United States than outside the United States. Is that correct? (no) *Have a volunteer explain your mistake.* (there are gaps between the face pictures) Draw the graph correctly, with no gaps between the faces. Remind students that the rows must line up at one end and extend in a line of pictures without gaps.

**Introduce symbols.** Explain that people usually use very simple pictures of objects in picture graphs. These simple pictures are called *symbols* and are very easy to draw. Write the word “symbol” on the board. *Ask students to think about what symbols they could use to represent people, flowers, snacks, books, and other items in a picture graph.* Explain that we usually write what the symbol means on the graph. Write the following label underneath the picture graph:

☺ = 1 student

**Creating picture graphs when given numerical data.** Give students *BLM Picture Graph Templates.* Have students create picture graphs for the numerical data given in the exercises below. Students can use stick figures, smiley faces, circles, or other symbols of their choice to represent the data. Alternatively, ask a question, have students vote on the answers, and record the class data on the board. Before assigning the exercises, *tell students to use the template with two rows for part a), the template with three rows for part b), and the template with five rows for part c).*

**Exercises:** Use the data to draw a picture graph.

a) **Soccer Players in Mr. F.’s Class**

- **Likes to Play Soccer**
- **Number of Students**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>9</td>
</tr>
<tr>
<td>No</td>
<td>7</td>
</tr>
</tbody>
</table>
b) **Shoes We Are Wearing Today**

<table>
<thead>
<tr>
<th>Type of Shoe</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandals</td>
<td>7</td>
</tr>
<tr>
<td>Sneakers</td>
<td>8</td>
</tr>
<tr>
<td>Dress Shoes</td>
<td>3</td>
</tr>
</tbody>
</table>

c) **Eye Colors in Ms. K.’s Class**

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>2</td>
</tr>
<tr>
<td>Brown</td>
<td>6</td>
</tr>
<tr>
<td>Gray</td>
<td>7</td>
</tr>
<tr>
<td>Green</td>
<td>2</td>
</tr>
</tbody>
</table>

Ask questions that require students to read their picture graphs, or to interpret and summarize them. Encourage students to solve the questions in different ways to check the answers. For example: How many more students in Ms. K.’s class have brown eyes than blue eyes? (4; there are 4 more symbols in the row for brown eyes than in the row for blue eyes or $6 - 2 = 4$) How many fewer students wear dress shoes than sandals? (3; $7 - 3 = 4$ or there are 3 more symbols in the sandals row) For which eye colors is the number of students the same? (blue and green) How do you know? (the rows are the same length)

Ask questions that require addition and subtraction, and have them write addition or subtraction sentences for the answer. For example: How many students in total were surveyed about shoes? ($7 + 8 + 3 = 18$) How many students have blue, green, or gray eyes? ($2 + 2 + 7 = 11$) How many more students have blue, green, and gray eyes than have brown eyes? ($11 - 6 = 5$)

**Adding missing information.** Present a more complicated situation with the graph from part c) in the exercise above. SAY: There are 22 students in Ms. K.’s class. The row for students with hazel eyes is missing. Hazel eyes are similar to brown eyes, but they are lighter, and change color a little when you wear different colors. Ask students to add a row to that graph for the students with hazel eyes. Have students write an addition equation for the total number of students that are in the graph. ($2 + 6 + 7 + 2 = 17$) ASK: How many students have hazel eyes? ($22 - 17 = 5$) Have them add five symbols to the row for hazel eyes.

Write on the board:

There are 25 students in Ron’s class. 5 students walk to school. 2 fewer students bike to school. 3 more students take the bus than walk. The rest of the students get a car ride to school.
Have students write an addition or a subtraction sentence to find the number of symbols that belong in each row. Then have them create a picture graph to present the information. Remind students to give the graph a title and labels. (see answers below)

Ways to get to school:
Walk: 5
Bike: 5 – 2 = 3
Bus: 5 + 3 = 8
Bus, bike, or walk: 5 + 3 + 8 = 16
Car: 25 – 16 = 9

<table>
<thead>
<tr>
<th>Ways to Get to School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
</tr>
<tr>
<td>Bike</td>
</tr>
<tr>
<td>Bus</td>
</tr>
<tr>
<td>Car</td>
</tr>
</tbody>
</table>

😊 = 1 student

Extensions

1. Explain that sometimes people use the same symbol in all rows of the graph and sometimes they use different symbols. Draw the graph below on the board, and explain that it shows how many times during the week students have different after-school classes:

<table>
<thead>
<tr>
<th>After-School Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Art</td>
</tr>
<tr>
<td>Music</td>
</tr>
<tr>
<td>Soccer</td>
</tr>
</tbody>
</table>

SAY: I think that there are more art classes during the week than music classes or soccer classes. ASK: Is that correct? (no) Why not? (there are less symbols for art than for music and soccer) SAY: I think there are more soccer classes than music classes. ASK: Is that correct? (no) Why not? (there are less symbols for soccer than for music) Discuss with students why you might be making mistakes. (the paintbrushes are longer; the soccer balls aren’t lined up with the other symbols) ASK: How could we redraw the picture graph to make it easier to read? (make the symbols the same size and line them up; use different symbols that are all the same size; use the same symbol in every row) Have students redraw the picture graph using one or two of the suggestions made.

NOTE: If students do not have grid paper, they can use BLM 1 cm Grid Paper for Extensions 2–4.
2. Check what shoes 10 of your classmates are wearing today.
   a) How many people are wearing sneakers?
   b) How many people are wearing sandals?
   c) What other kinds of shoes are people wearing today?
   d) Use grid paper to create a picture graph showing the types of shoes the 10 students are wearing.

3. Change the Ways to Get to School picture graph from the lesson into a picture graph that shows the data in columns instead of rows.

**Answer**

<table>
<thead>
<tr>
<th>Ways to Get to School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
</tr>
<tr>
<td>🧟</td>
</tr>
<tr>
<td>🧟</td>
</tr>
<tr>
<td>🧟</td>
</tr>
<tr>
<td>🧡</td>
</tr>
<tr>
<td>🧡</td>
</tr>
</tbody>
</table>

= 1 student

4. In November, there was 1 day of snow. In December, there were 8 more days with snow than in November. In January, there were 2 fewer days of snow than in December. In February, there were 3 fewer days of snow than in December. Use grid paper to make a picture graph that shows the number of snow days from November to February.

**Answer**

<table>
<thead>
<tr>
<th>Number of Snow Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>November</td>
</tr>
<tr>
<td>December</td>
</tr>
<tr>
<td>January</td>
</tr>
<tr>
<td>February</td>
</tr>
</tbody>
</table>

= 1 day of snow
Goals

Students will read and draw scaled picture graphs.

PRIOR KNOWLEDGE REQUIRED

Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a picture graph with one symbol representing one item
Can multiply numbers up to 10 × 10
Can multiply one-digit numbers by a multiple of 10
Can divide by one-digit numbers

MATERIALS

BLM Picture Graph Templates (p. U-49)

Introduce scaled picture graphs. Tell students that you have a garden and you made a picture graph to show how many flowers are in it. Draw on the board:

Flowers in My Garden

Daffodils 🌸🌸🌸🌸
Buttercups 🌻🌻🌻🌻🌻
Daisies 🌺🌺🌺

ASK: How many daisies are in my garden? (3) How do you know? (there are 3 flowers in the daisy row) SAY: But there is something I did not tell you. Each flower in the picture stands for more than one flower. Each flower could mean any number of actual flowers, but it’s always the same number for each symbol on the graph. For example, if the first symbol means 2 flowers, then all the symbols on the graph mean 2 flowers.

ASK: If each symbol means only 1 flower, how many flowers do I have in my garden altogether? (12) How do you know? (add the flowers in each row) Have students write an addition sentence to show the answer. (4 + 5 + 3 = 12) ASK: If each symbol means 2 flowers, how many daffodils do I have? (8) How do you know? Allow several students to explain how they found the answer, to illustrate the different strategies. (counting by 2s or multiplying 4 × 2 = 8) Repeat with buttercups and daisies. Have students count by 2s and write the multiplication equation for each row. (buttercups: 5 × 2 = 10, daisies: 3 × 2 = 6) ASK: If each symbol means 2 flowers, how many flowers do I have in total? (8 + 10 + 6 = 24) Repeat with each symbol meaning 3 flowers, then 5 flowers, and then 10 flowers. (12 daffodils, 15 buttercups, 9 daisies, 36 flowers altogether; 20 daffodils,
25 buttercups, 15 daisies, 60 flowers altogether; 40 daffodils, 50 buttercups, 30 daisies, 120 flowers altogether)

As a challenge, tell students that you have 20 buttercups in your garden and have them figure out what the symbol means. (4 flowers) Have them explain how they found the answer. Encourage students to think of multiple strategies. (strategies include division: $20 \div 5 = 4$, and writing an equation with a missing number and solving for the missing number: $5 \times \_ \_ = 20$)

Explain that, to avoid confusion about the number of items shown on the picture graph, people use a scale: they write what the symbol means on the graph. Add the scale below to the picture graph:

\[ \star = 10 \text{ flowers} \]

**Drawing symbols to represent numbers when given a scale.** Remind students that symbols on a picture graph are usually simple so that it is easy to draw many symbols. SAY: A circle means 2 flowers. Write on the board:

\[ \bigcirc = 2 \text{ flowers} \]

ASK: How many circles do you need to draw for 6 flowers? (3) How do you know? Have volunteers explain the different methods. (counting by 2s and drawing 1 circle for each number until you reach 6; dividing $6 \div 2 = 3$; writing a multiplication equation with a missing number, $\_ \times 2 = 6$, and using your knowledge of the times table to fill in the missing number) Make sure all three of these methods are mentioned.

**Exercises:** Look at the scale and draw symbols to show each number.

a) $\bigcirc = 3 \text{ people}$  
  12 people $=$

b) $\bigcirc = 5 \text{ people}$  
  10 people $=$

c) $\bigstar = 10 \text{ people}$  
  20 people $=$

**Answers**

a) 12 people = $\bigcirc \bigcirc \bigcirc \bigcirc$, 15 people = $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$

b) 10 people = $\bigcirc \bigcirc \bigcirc$, 20 people = $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$

c) 20 people = $\bigstar \bigstar$, 40 people = $\bigstar \bigstar \bigstar \bigstar $

**Redrawing picture graphs with a new scale.** If the answers from the exercises above are not on the board, add them. Then circle the two ways 20 people are represented in parts b) and c). Point to the examples and SAY: In part b), 20 people are shown with 4 symbols and in part c) 20 people are shown with 2 symbols. ASK: Why is that? (the scales are different) SAY: This means that the same information can be shown on picture graphs with different scales. The picture graphs will look different but they will mean the same thing.
Draw on the board:

**Book Types Sold at a Book Fair**

<table>
<thead>
<tr>
<th>Type of Book</th>
<th>Number of Books</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fantasy</td>
<td></td>
</tr>
<tr>
<td>Mystery</td>
<td></td>
</tr>
<tr>
<td>Non-fiction</td>
<td></td>
</tr>
<tr>
<td>Picture</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type of Book</th>
<th>Number of Books</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fantasy</td>
<td></td>
</tr>
<tr>
<td>Mystery</td>
<td></td>
</tr>
<tr>
<td>Non-fiction</td>
<td></td>
</tr>
<tr>
<td>Picture</td>
<td></td>
</tr>
</tbody>
</table>

= 5 books

Ask students to help you fill in the table. Point out that the scale for the picture graph is 1 symbol meaning 5 books. (fantasy = 20, mystery = 30, non-fiction = 10, picture = 30)

SAY: I see that all the numbers in the table are the numbers we say when we count by tens. ASK: What do we call such numbers? (multiples of 10) SAY: This means that the same information with a scale of 1 symbol meaning 5 books can also be shown in a new picture graph with 1 symbol meaning 10 books.

Distribute **BLM Picture Graph Templates**. ASK: How many different types of books do we have in this picture graph? (4) Can you use the table at the top of the page for this picture graph? (no) Why not? (It has only 2 rows.) Which table can you use? (the third or the fourth)

SAY: The graph we are going to draw uses the same data as the graph on the board. So we can use the same title as the graph on the board. Have students fill in the title. (Book Types Sold at a Book Fair)

Discuss what symbol students can use for their picture graph. Remind them that the symbol should be easy to draw, such as two rectangles together (□□) to represent the open pages of a book, or a square to represent the shape of the book. Have students write the scale below their picture graph (the symbol they chose and “ = 10 books”).

Have students fill in the types of books on the graph, one type per row. ASK: If there were 20 fantasy books sold during the book fair and we are using a scale of 1 book symbol = 10, how many symbols should you draw to represent that number? (2) How do you know? (20 ÷ 10 = 2 or ___ × 10 = 20, the missing number is 2) Add a column to the table on the board called “Equation” and have volunteers fill in the division equation or the multiplication equation that helps them to find how many symbols they should draw on the picture graph. (see finished table on the next page)
<table>
<thead>
<tr>
<th>Type of Book</th>
<th>Number of Books</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fantasy</td>
<td>20</td>
<td>$20 \div 10 = 2$</td>
</tr>
<tr>
<td>Mystery</td>
<td>30</td>
<td>$3 \times 10 = 30$</td>
</tr>
<tr>
<td>Non-fiction</td>
<td>10</td>
<td>$10 \div 10 = 1$</td>
</tr>
<tr>
<td>Picture</td>
<td>30</td>
<td>$30 \div 10 = 3$</td>
</tr>
</tbody>
</table>

After a volunteer fills in a row of the table on the board, have students draw the appropriate number of symbols in the proper row of their picture graphs. Point out that the grid on the picture graph makes it easy to keep the picture graph organized; students need to draw each symbol in a separate box in each row without leaving empty boxes in between. The finished picture graph will look like this:

**Book Types Sold at a Book Fair**

<table>
<thead>
<tr>
<th>Type of Book</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fantasy</td>
<td>![Symbol] ![Symbol]</td>
</tr>
<tr>
<td>Mystery</td>
<td>![Symbol] ![Symbol] ![Symbol]</td>
</tr>
<tr>
<td>Non-fiction</td>
<td>![Symbol] ![Symbol] ![Symbol]</td>
</tr>
<tr>
<td>Picture</td>
<td>![Symbol] ![Symbol] ![Symbol]</td>
</tr>
</tbody>
</table>

= 10 books

**Comparing two picture graphs.** ASK: On the first picture graph, how many mystery books were sold at the fair? (30) On the second picture graph, how many mystery books were sold at the fair? (30) So, do the two graphs present the same data? (yes) Do they look exactly the same? (no) How are they different? (they have a different number of symbols in the same row; they have different scales) Emphasize that the different scales are the real reason the graphs look different. ASK: How many fantasy and mystery books were sold at the fair? (50) Discuss various ways to find the answer. (add the numbers from the table; add the number of symbols in the first two rows and then multiply by 5 or 10, depending on the scale) Make sure both methods are mentioned.

**Using a half symbol.** SAY: Suppose there was another type of book at the fair, craft books. There were 5 craft books sold at the fair. I would like to add this information to both graphs. Have a volunteer come to the board and add a craft row to the graph (where the symbol means 5 books) and add 1 symbol in the new row. SAY: But on the second picture graph, where the symbol means 10 books, 5 books is fewer books than what one symbol means. Have students think of how they could represent 5 books in the second graph. (draw half of a symbol; use a smaller symbol; write something like a fraction, with the symbol as a numerator and 2 in the denominator) Explain that in such cases we can use half of a symbol.

Remind students that to find half of a pizza, they divide the pizza into 2 equal parts. ASK: Can we divide a group of 10 books into 2 equal parts? (yes) If possible, take 10 identical books (such as JUMP Math AP Books)
and have a volunteer divide them into 2 equal piles. **ASK:** How many books are in each pile? (5) **How could you find out mathematically how many books should be in each pile? (divide; 10 ÷ 2 = 5)** Have students write the division sentence. Explain that, on the second picture graph, 1 whole symbol will still mean 10 books and half of a book symbol will mean 5 books. Draw half of a book symbol on the board. Have students add a “Craft” row to the second picture graph and draw half a symbol in the row.

**Draw on the board:**

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Division Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 ÷ 2 = 5</td>
</tr>
</tbody>
</table>
```

**SAY:** Imagine that I am going to draw a picture graph that shows the results of a survey about people’s favorite color. I am going to use a circle for a symbol. **ASK:** If a whole circle means 10 people, how much would a half circle mean? (5 people) **How do you know? (10 ÷ 2 = 5)** P**ROMPT:** This situation is very similar to the last picture graph, where one book symbol meant 10 books and half of the symbol meant 5 books, which we found by dividing 10 by 2. **SAY:** For our survey about favorite colors, let’s fill in the table. Fill in the first row of the table, as shown below:

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Division Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 ÷ 2 = 5</td>
</tr>
</tbody>
</table>
```

Emphasize that we need to divide the number the whole circle means by 2 to get the meaning of half a circle.

**SAY:** Now we are going to use a new scale. **ASK:** If a whole circle means 6 people, how many people does half of the circle mean? (3) **How do you know? (6 ÷ 2 = 3)** Have a volunteer add a row to the table and fill in the row for 6. Add more rows to the table and repeat with other numbers. (see below for possible numbers and answers)

```
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Division Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
<td>6 ÷ 2 = 3</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>8 ÷ 2 = 4</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>16 ÷ 2 = 8</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>20 ÷ 2 = 10</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>12 ÷ 2 = 6</td>
</tr>
</tbody>
</table>
```

Students can signal the numerical answers and volunteers can write the division sentence for each row. Leave the table on the board.

**Translating groups of symbols with a half symbol into numbers. Draw on the board:**

```
= 10 people
```

Point to the 4 and a half symbols on the board and **SAY:** Sometimes you see something like this on picture graphs. If one circle means 10 people,
how can we find out how much this means? Cover the half circle so only the 4 circles show. ASK: How much does this mean? (40 people) Write “40” underneath the 4 circles. Then cover the 4 circles so only the half circle shows. ASK: How much does this mean? (5 people) ASK: What is 10 divided by 2? Write “5” underneath the half circle. SAY: So we have 4 circles and a half circle. How many people is that altogether? (45) Add the addition sign and finish the equation, as shown below:

\[40 + 5 = 45\]

Have students start a table as shown below:

<table>
<thead>
<tr>
<th>10</th>
<th>40</th>
<th>5</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 × 4 = 40</td>
<td>5</td>
<td>40 + 5 = 45</td>
</tr>
</tbody>
</table>

Change the scale to 6 people and repeat the exercise above. Have students record the numbers in the table. Then continue with the scales 8 and 20. The finished table is shown below:

<table>
<thead>
<tr>
<th>10</th>
<th>40</th>
<th>5</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 × 4 = 40</td>
<td>5</td>
<td>40 + 5 = 45</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>32</td>
<td>4</td>
<td>36</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

Showing numbers using groups with a half symbol. ASK: If one circle represents 10 people, how will you show 30 people? (3 circles) How do you know? (count by 10s until you reach 30; look for the missing number in \[\_ \times 10 = 30\]; divide 30 ÷ 10 = 3) How will you show 40 people? (4 circles) How will you show 35 people? (3 and a half circles) How do you know? (35 is halfway between 30 and 40 or 3 circles means 30 and 4 circles means 40 so to show 35 we need half a circle) Draw 3 circles and a half circle and have students check the answer using the method of the table above.

**Exercises:** One circle means 10 people. Draw circles to represent the number.

a) 25
b) 45
c) 15

**Answers**
a) 

b) 

c) 

ASK: If a whole circle means 2 people, what does a half circle mean? (1 person)
**Exercises:** One circle means 2 people. Draw circles to represent the number.

a) 5  

b) 15  

c) 11  

**Answers**

a) ![Circles](image1)  

b) ![Circles](image2)  

c) ![Circles](image3)

**Extensions**

1. Discuss what is wrong with the following picture graph:

   **Favorite Sport of Students in Class A**

<table>
<thead>
<tr>
<th>Sport</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer</td>
<td>🎾♥️♥️♥️♥️♥️</td>
</tr>
<tr>
<td>Hockey</td>
<td>🎾😊😊😊😊</td>
</tr>
<tr>
<td>Basketball</td>
<td>🎾😊😊😊😊</td>
</tr>
</tbody>
</table>

   Tell students that one happy face represents one student who picked that sport as their favorite. ASK: Which sport is the most popular? (soccer) Which sport has the longest row of faces? (soccer) Why is it easier to read the picture graph when all the faces are the same size? (look for the longest row) Have students redraw the picture graph correctly.

2. Ms. Smith’s class chose their favorite drinks. The possible answers were milk, orange juice, apple juice, and water. Draw a picture graph to show the data. Use 🎃 for 2 people.

   a) Apple juice was the most popular. 12 people chose it. Show this on the picture graph.

   b) 7 fewer people chose milk than apple juice. Show this on the picture graph.

   c) There are 25 students in the class. How many students chose orange juice or water?

   d) 2 more people chose orange juice than water. Show this on the picture graph.

**Answers**

<table>
<thead>
<tr>
<th>Favorite drinks</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple juice</td>
<td>🎃♥️♥️♥️♥️♥️</td>
</tr>
<tr>
<td>Milk</td>
<td>🎃😊😊</td>
</tr>
<tr>
<td>Orange juice</td>
<td>🎃😊</td>
</tr>
<tr>
<td>Water</td>
<td>🎃😊</td>
</tr>
</tbody>
</table>

= 2 people
NOTE: Give students two sheets of construction paper for Extension 3 and tell them they can use whatever else they need to do the problem.

(MP5, MP6, MP7) 3. a) Cut out two rectangles: one that is 6 inches by 10 inches and one that is 8 inches by 8 inches. Use your rectangles to make two paper airplanes.

b) Predict which airplane is heavier. Explain how you made your prediction.

c) Check your prediction from part b).

Selected answer: b) Use the area to decide which airplane used more paper. The area of the rectangles are $6 \times 10 = 60$ and $8 \times 8 = 64$, so the airplane made from the 8 inch by 8 inch rectangle used more paper and so should be heavier.

Look for students to cut out their rectangles with precision (MP6). Strategies might include using 1 inch grid paper to do this accurately or using a ruler to make sure the lines aren’t crooked compared to edges of the paper.

Look for students to recognize that they can use the area of the paper to predict which paper airplane will be heavier (MP7) and to recognize that they can use a scale to check their answer (MP5). Look for students to use the scale to measure accurately (MP6); they need to ensure that the scale starts at zero, wait until the numbers on the scale stop changing before recording the measurement, and state the units with which they measured (MP6).

Redirecting students: You may need to help some students get started. ASK: Can you tell which piece of paper was heavier before you made it into an airplane? (the one with the bigger area) Would the mass change after making it into an airplane? (no) PROMPT: Did you change the amount of paper it uses? (no)
MD3-49 Creating Picture Graphs
Pages 196–197

STANDARDS
3.MD.B.3, 3.OA.A.3, 3.OA.D.8

VOCABULARY
data
graph
labels
picture graph
scale
survey
symbol
title

Goals
Students will read and draw scaled picture graphs.
Students will solve problems using data from picture graphs.

PRIOR KNOWLEDGE REQUIRED
Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a scaled picture graph
Can multiply numbers up to 10 × 10
Can multiply one-digit numbers by a multiple of 10
Can divide by one-digit numbers (e.g., by finding the missing number in a times fact)

MATERIALS
BLM Picture Graph Templates (p. U-49)
30 connecting cubes of 5 colors (blue, green, red, brown, yellow) for each pair of students
materials to conduct a survey: world map, cardboard, string, and clothespins, or jars, labels, and tokens (see Extension 2)
grid paper or BLM 1 cm Grid Paper (p. V-1, see Extension 2)

Review creating picture graphs, including using a half symbol. Remind students that a symbol on a picture graph can mean more than one item. Write on the board:

😊 = 10 people
30 people =
40 people =
5 people =
35 people =

ASK: How can we show 30 on a picture graph with the scale given? (3 smiley faces) How do you know? (3 × 10 = 30, or 30 ÷ 10 = 3) Have a volunteer fill in the answer on the board. ASK: How can we show 40? (4 smiley faces) How do you know? (4 × 10 = 40, or 40 ÷ 10 = 4) Have a volunteer fill in the answer on the board. ASK: How can we show 5? (half of a smiley face) How do you know? (10 ÷ 2 = 5, 5 is half of 10, two groups of 5 objects make 10) Make sure all three methods are discussed. Have a volunteer fill in the answer on the board. ASK: How can we show 35? (3 full faces and 1 half of a smiley face) ASK: How do you know? (35 is exactly half way between 30 and 40, 35 is 30 ÷ 5) Have a volunteer fill in the answer on the board. ASK: How can you show 15? (one full face and 1 half of a smiley face)
**ACTIVITY**

Give each student a copy of **BLM Picture Graph Templates** and give each pair of students a collection of 30 connecting cubes in any combination of the following colors: blue, green, red, brown, and yellow. **NOTE**: These should be random, unmatched collections but each student pair should have at least one cube of every color. Have students sort the cubes by color and then work independently to create a picture graph of their collection on the last picture graph template on the BLM using the scale 1 square = 2 cubes. Each pair should compare the two graphs for their cube collection. Keep these graphs for the next lesson.

**Answering questions using a picture graph.** Write on the board:

a) For which color are there the most cubes?

b) For which color are there the least cubes?

c) How many red and blue cubes do you have altogether?

d) How many cubes that are not green do you have?

e) How many cubes do you have in total?

Have students answer the questions about their own graphs.

Tell students that you also have a collection of cubes and made a graph from it. Draw on the board:

<table>
<thead>
<tr>
<th>Colors of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
</tr>
<tr>
<td>Green</td>
</tr>
<tr>
<td>Red</td>
</tr>
<tr>
<td>Brown</td>
</tr>
<tr>
<td>Yellow</td>
</tr>
</tbody>
</table>

![Symbols representing cubes](image)

□ = 2 cubes

SAY: This picture graph is not finished, but let’s look at the data on it so far. **ASK**: How many more red cubes than green cubes do I have? (5) How do you know? (there are 6 green cubes and 11 red cubes, $11 - 6 = 5$; the red row has 2 and a half symbols more, which represents 5 cubes) Make sure both answers are discussed.

**ASK**: How many fewer blue cubes than red cubes do I have? (2) How do you know? (there are 9 blue cubes and 11 red cubes, $11 - 9 = 2$; there is one more square in the red column)

**SAY**: I have 3 more green cubes than brown cubes. **ASK**: How many brown cubes do I have? (3) How do you know? ($6 - 3 = 3$) How can I show this...
on the picture graph? (draw one full square and one half square in the row for brown) Have a volunteer add the squares on the board.

SAY: There are 38 cubes in my collection. ASK: How many yellow cubes do I have? (9) How do you know? (the total number of cubes shown on the picture graph is $9 + 6 + 11 + 3 = 29$ cubes, and $38 - 29 = 9$) How many squares should I draw in my picture graph to show this? (4 full squares and one half square) Invite a volunteer to add this to the picture graph on the board.

Choosing a scale for data. Explain that sometimes students will need to decide what scale to use for data. SAY: Suppose you surveyed 200 people about their favorite team sport and gave them 3 answers to choose from: baseball, soccer, and football. In the survey, 100 people chose baseball, 45 chose soccer, and 55 chose football. Write on the board:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball</td>
<td>100</td>
</tr>
<tr>
<td>Soccer</td>
<td>45</td>
</tr>
<tr>
<td>Football</td>
<td>55</td>
</tr>
</tbody>
</table>

ASK: If I used a scale of one symbol representing two people, how many symbols would I need to show 100 people? (50) Have students try to count by 2s to reach 50. When they see that they do not have enough fingers on their hands to keep track, ASK: Does it make sense to use 2s, or should we count by a bigger number? Students can repeat the counting by 3s and by 5s. ASK: What number should we count by? (10s) Have students count by 10s to see that they need 10 symbols to represent the number of people who chose baseball.

ASK: Can we represent 45 or 55 people using the scale where one symbol is equal to 10 people? (yes) How would you represent these numbers? (4 and a half symbols for 45, 5 and a half for 55) SAY: So 10 seems like a reasonable scale in this case. Point out that if you had, say, 42 people and 58 people choosing soccer and football, you would have trouble using 10 because you would not be able to use halves to show 2 people and 8 people.

Change the numbers in the table to 15, 6, and 9. ASK: Is 10 a good scale for these new numbers? (no) Why not? (6 and 9 cannot be shown with a whole symbol or with half a symbol) SAY: Let’s try 1 symbol equals 2 people. Have students say how many symbols they would use for each number. (7 1/2, 3, 4 1/2) SAY: I think we say 15, 6, and 9 when we count by something other than 2. ASK: What number can we count by? (3) Have students say how many symbols they should draw for each of the numbers with the scale 1 symbol = 3 people. (5, 2, 3)

SAY: I am going to write groups of numbers. Each group of numbers is data. We need to show this data on a picture graph. Students can signal the answers for each part in the following exercises.
Exercises: Which scale should we use: 1 symbol equals 2, 3, or 5?

a) 6, 10, 8  
b) 6, 10, 7  
c) 15, 20, 10  
d) 9, 12, 21

Bonus: 18, 15, 9, 21, 27, 30

Answers: a) 2, b) 2, c) 5, d) 3, Bonus: 3

Review names of polygons and counting sides. In order to complete Question 1 on AP Book 3.2 p. 196, students need to review names of polygons and counting sides. Write on the board:

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name of Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
</tr>
</tbody>
</table>

Have volunteers draw examples for each type of polygon above. Then count the sides in each shape as a class to make sure the examples are drawn correctly. Encourage students to create examples of shapes that look different, with sides of different lengths and indentations as well as shapes with all equal sides.

Looking at shortened labels. Ask students to look at Question 3 on AP Book 3.2 p. 197. Point to the labels on the table and on the picture graphs. Ask them what grades they have in their school. (Grades 1, 2, 3, 4, 5, and so on) Ask them what they think “K” means. Guide them to the conclusion that “K” is a short form for “Kindergarten” that is used because writing the full word would be too long for a label.

Extensions

1. Make a picture graph for the letters in “Mississippi.” Use each letter as its own symbol in the graph.

Answer

Letters in Mississippi

<table>
<thead>
<tr>
<th>M</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>S</td>
<td>S S S S</td>
</tr>
<tr>
<td>P</td>
<td>P</td>
</tr>
</tbody>
</table>
2. In advance, prepare materials to conduct a survey and record the results. On a large piece of cardboard, write the title and labels shown below, and attach strings that hang down from each label. In the survey, students attach a clothespin to the appropriate string to show their answers.

<table>
<thead>
<tr>
<th>Our Birthplaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
</tr>
<tr>
<td>North America</td>
</tr>
<tr>
<td>South America</td>
</tr>
<tr>
<td>Africa</td>
</tr>
<tr>
<td>Asia</td>
</tr>
<tr>
<td>Europe</td>
</tr>
<tr>
<td>Australia</td>
</tr>
</tbody>
</table>

Alternatively, prepare plastic cups or jars and make labels on the board to match the headings in the picture above. In the survey, students put a token, such as a craft stick or a counter, into the jar to show their answers.

Tell students you are going to conduct a survey to find out where they were born. Distribute a clothespin or token to each student. Show a world map, point each continent out, show where the United States is, and show the areas of North America that are not the United States. Point out that Australia is both a country and a continent, and that the continent of Antarctica is missing because nobody lives there.

ASK: Where were you born? Have students record their answers by each attaching one clothespin to the appropriate string or putting one token in the appropriate cup or jar. Help students that know the country of their birth but are not sure which continent it is on. Have volunteers count the clothespins or tokens and record the answers in a table. Have students make a picture graph on grid paper or BLM 1 cm Grid Paper to show the results.

ASK: Where was the largest number of people from our class born? Where was the smallest number of people in our class born? How do you know? How many people were born in the United States? Make a comparison based on your class results. For example: How many people were born in Asia? How many more people were born in the United States than in Asia?

(MP2, MP4)

3. A box containing 8 pairs of scissors weighs 410 g. The empty box weighs 250 g. How much does each pair of scissors weigh? Use equations to show how you got your answer.

Answer: The total weight of the scissors is 160 g (because 410 − 250 = 160) and there are eight pairs of scissors, so each pair weighs 20 g (because 160 ÷ 8 = 20).
Look for students to model with equations (MP.4); to recognize that they need to find the total weight of all the scissors without the box before they can find the weight of one pair of scissors; and to show what each step means in the situation (MP.2).

Redirecting students: Encourage students to draw a picture of the situation. Recommend that they only draw the math details. For example, students can draw lines for scissors—they do not need to draw details on the scissors. Encourage students to draw a picture of all the objects that together weigh 410 g (a box and 8 pairs of scissors) and a picture of the object they need to weigh (1 pair of scissors). Ask students if there is an in-between weight that they can find to help them answer the question (the weight of 8 scissors without the box).

Individual or small-group follow-up: Some students may need to be told the in-between step of finding the weight of the scissors without the box. If so, ask them to reflect on how it helped them to solve the problem.
Goals

Students will read bar graphs and solve problems using information presented in bar graphs.

PRIOR KNOWLEDGE REQUIRED

Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a picture graph with one symbol representing one item
Can multiply numbers up to $10 \times 10$
Can divide by one-digit numbers (e.g., by finding the missing number in a times fact)

MATERIALS

overhead projector
transparency of BLM Colors of Cubes (p. U-50)
transparency of BLM Snacks Bar Graphs (p. U-51)
erasable markers of different colors (blue, green, red, black)

Review picture graphs. Remind students that, during the previous lesson, they sorted cubes by color and created picture graphs to show their collections of cubes. Remind them that they used squares to represent the cubes. Review what features those picture graphs have: They all have titles, symbols (squares), labels showing the color of each cube, a grid on which the results are shown, and a scale.

Introduce bar graphs. Explain that, to make graphs simple, you can join the squares on graphs into columns or rows of squares. These columns or rows of squares, joined together, are called bars. Explain that in this lesson, you will use one grid square for one cube, unlike the previous lesson. Project the transparency of BLM Colors of Cubes on the board. The graphs show the following data:

<table>
<thead>
<tr>
<th>Colors of Cubes</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>4</td>
</tr>
<tr>
<td>Green</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>5</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
</tr>
</tbody>
</table>

Explain that both graphs show the same data, or the same collection of cubes. SAY: The graph on the left is a picture graph, like the ones you have looked at and created in the last few lessons. The graph on the right is...
called a **bar graph**, because it shows the data in bars. The blocks in each bar are usually squares, but sometimes there is not enough space to make the blocks square, so people use rectangles as well. In this bar graph, each square block in each bar means 1 cube. The bar for blue cubes is 4 blocks long, so we know that there are 4 blue cubes in the collection. **ASK**: How many green cubes are there? (3) How do you know? (the bar is 3 blocks long) How many red cubes are there? (5) How many yellow cubes? (4)

**Introduce vocabulary.** Ask students what the two graphs have that is the same, besides showing the same data. (the title, the labels) Circle the title in both graphs using a red marker, and write “title” beside the graphs in red. Circle the labels that are shared in both graphs using a blue marker and write “labels” in blue. Explain that a bar graph has more labels than a picture graph. All other markings in words (not numbers) on a bar graph are also called labels. Circle the rest of the labels in blue.

Point out the general organization of the bar graph. Trace the axes and explain that these two lines make an L-shape and the lines are called axes. Explain that when we talk about one axis, we call it an **axis** but when there are two of them, we call them axes. Write both words on the board and trace the axes on the bar graph with a black marker. Have a volunteer underline the part that is different in the two words. (the third letter)

Explain that axes and their labels help us understand what we are looking at on bar graphs. **SAY**: One of the lines has numbers. Cover the rest of the graph so that only the bottom axis and the numbers are visible. **ASK**: What do you call a line and numbers in counting order under it? (number line) What other type of graph has a number line? (line plot) **SAY**: The numbers on the number line are called a **scale**. Point out that a picture graph also has a scale, something that tells you how many pieces of data each symbol means. Explain that a scale in a bar graph plays a similar role, but you will talk about that more in the next lesson. Circle the scales on both graphs with a green marker and write the word “scale” in green beside the graphs.

Draw students’ attention to the fact that the bars on a bar graph usually have spaces between them. Explain that this makes a bar graph easier to read. Space is usually added on either side of each bar, including the bar closest to the number line.

**Reading a bar graph.** Project the Favorite Snacks graph from **BLM Snacks Bar Graphs** on the board. The graph shows the following data

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>5</td>
</tr>
<tr>
<td>Bagels</td>
<td>4</td>
</tr>
<tr>
<td>Fruit</td>
<td>7</td>
</tr>
<tr>
<td>Cheese</td>
<td>0</td>
</tr>
</tbody>
</table>
Explain that the graph shows most of the results of students voting for their favorite out of 4 snacks. Look at the parts of this bar graph. Point out to students that the title tells us what the graph shows, the label at the bottom (Snack Type) describes what the bars show, and the labels for each bar give detail about what the bar shows.

Have students signal the numerical answers for the next questions.

ASK: How many students chose muffins as their favorite snack? (5) How many students chose fruit? (7) How many students voted for baked snacks? (9) How do you know? (muffins and bagels are both baked snacks, $5 + 4 = 9$)

SAY: We have some more information to add. Four more students voted for fruit than for cheese. How many students voted for cheese? (3) How do you know? ($7 - 4 = 3$) Invite a volunteer to shade the blocks of the bar for cheese and have the class signal if they agree with what was done. Emphasize that the blocks should be shaded together and that the bar should start at the bottom of the graph, where the rest of the bars start, and above the label “Cheese.”

ASK: How many students voted in total? (19) How do you know? ($5 + 4 + 7 + 3 = 19$) What is the most popular snack of the four, or the snack that was chosen the most number of times? (fruit) What is the least popular snack, or the snack that was chosen the smallest number of times? (cheese)

**Most popular and least popular, most common and least common.** Point out that you probably eat your favorite snack many days but you probably eat other snacks as well. For example, if your favorite snack is fruit, you still might have a muffin for a snack sometimes. If you survey the same people about what snack they had today, you may get a different graph. Project the Snacks Eaten Today graph from BLM Snacks Bar Graphs on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Snack Type</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffins</td>
<td>2</td>
</tr>
<tr>
<td>Bagels</td>
<td>7</td>
</tr>
<tr>
<td>Fruit</td>
<td>6</td>
</tr>
<tr>
<td>Vegetables</td>
<td>3</td>
</tr>
</tbody>
</table>

ASK: Which bar on the bar graph is the tallest now? (bagels) Is that the favorite snack shown on the other snack graph? (no) Explain that the phrase *most common* means the one that happens most often or has the greatest number. ASK: So, was the bagel the most common snack today for this group of people? (yes) ASK: What is the *least common* snack on this graph? (muffins)

Explain that we describe something as most common or least common when there is no choice involved, such as when we are not picking favorites. SAY: We can choose our favorite snack, but the snack we have today does not have to be the favorite one. We might even not have any
choice about the snack we have today. ASK: What other things do we have no choice about? (sample answers: hair color, eye color, height, types of flowers that grow where we live, the weather where we live) To check and reinforce students’ understanding of these concepts, have them do the exercises below.

Exercises: Write “most popular” or “most common” to complete the sentence.

a) The New York Yankees is the _____ sports team in New York.
b) The _____ leaves in our garden are maple leaves.
c) I have to buy ice cream for a class party. I buy the _____ flavor.
d) Most students in the class have black hair. Black is the _____ hair color in the class.

Sample answers: a) most popular, b) most common, c) most popular, d) most common

Solving problems using multiplication with information from a bar graph. Explain that the snacks in the bar graph are the options that the school council plans to sell during a school fair. Project the Favorite Snacks graph from BLM Snacks Bar Graphs on the board. Write on the board:

- Muffin: 80¢
- Bagel: 50¢
- Fruit: 30¢
- Cheese stick: 20¢

SAY: The school council is going to sell the snacks at these prices at the fair. Remind students that the symbol after the numbers means that the price is given in cents. ASK: If the same numbers of snacks are sold at the school fair as are shown on the Favorite Snacks bar graph, how many cents will the muffins for the whole class be sold for? (400¢) How do you know? (5 \times 80 = 400) Write the multiplication equation on the board and remind students that, to multiply 5 \times 80, they can think of the multiplication this way: 5 \times 8 \text{ tens} = 40 \text{ tens} = 400. As a shortcut, they can multiply the one-digit numbers and then write a zero at the end of the result to shift the place values.

SAY: I want to know what will sell for more money, all the fruit or all the bagels. ASK: How can we find out? (find the total price for bagels and the total price for fruit, then compare) Have students write multiplication sentences to find out how much each type of snack will sell for. (bagels: 4 \times 50 = 200¢, fruit: 7 \times 30 = 210¢, cheese: 3 \times 20 = 60¢)

ASK: What will sell for more, the fruit or the bagels? (the fruit) How much will the snacks sell for in total? (400 + 200 + 210 + 60 = 870¢) Have students write the addition sentence. SAY: A dollar is 100 cents. How many cents are in 2 dollars? (400) In 3 dollars? (300) In 4 dollars? (400) SAY: I want to know how many dollars I would need to buy all the snacks at the fair. ASK: Whole dollars always give me multiples of what number?
(100) SAY: If I round down, I will get the number that is smaller than what I need. Then I will not be able to pay for all the food. ASK: What is the next multiple of 100 after 870? (900) How many dollars is that? (9 dollars) SAY: This means I need 9 dollars to pay for all the food.

ASK: What if each bagel sells for 55¢? I do not know how to multiply $4 \times 55!$ How else can I find the answer? (repeated addition, or double twice) PROMPT: Could I use doubling? Have students double twice to find the price of the 4 bagels if they cost 55¢ each. (double of 55 is 110, double of 110 is 220)

**Extensions**

1. Present the graph below:

   ![Graph of Our Favorite Breakfast](image)

   ASK: How many students picked eggs as their favorite breakfast? (0) Point out that an empty or missing bar is not a mistake, it just shows that zero people chose that answer. SAY: My favorite breakfast is fruit. How can we mark my answer on the graph? If the following answers do not arise, explain that you can either add a bar for fruit, or add your answer in the column labeled “Other.” Point out that the “Other” bar is used when there are many answers that few people give; it is easier to group all these answers together.

2. The bar graph shows a word we use in math. The height of the bar tells you how many times to use the letter that labels the bar. Write the letters you will use to make the word. Then rearrange the letters to make a word. An example is shown on the following page.
Letters: e e e l n v      Word: eleven

Hint: a) and c) are number words.

a) 4 3 2 1 0 e i n

b) 4 3 2 1 0 a d e n

c) 4 3 2 1 0 e n s t v

Answers: a) nine, b) addend, c) seventeen

3. Redraw the Snacks Eaten Today graph with horizontal bars instead of vertical bars.

(MP5, MP6)

4. Pick two containers that look like one has a capacity of about a litre more than the other. Then measure the capacities of each to check.

Look for students to understand that capacity refers to the amount of liquid a container can hold and to choose an appropriate measuring tool (MP5), such as a graduated beaker. Look for students to attend to precision (MP6) when measuring: they should fill the container completely, avoid spillage, wait until the water settles before recording the measurement, and read the measurement at eye level if using a graduated beaker.
**MD3-51 Bar Graphs**

**STANDARDS**
3.MD.B.3, 3.OA.D.8

**VOCABULARY**
- axis
- bar
- bar graph
- data
- graph
- half
- labels
- least common
- most common
- picture graph
- scale
- symbol
- title

**Goals**
Students will read and draw scaled bar graphs and solve problems using information presented in scaled bar graphs.

**PRIOR KNOWLEDGE REQUIRED**
- Can solve one- and two-step “how many more” and “how many less” word problems
- Can read data from a table
- Can read and draw a bar graph with one symbol representing one item
- Can read and draw a scaled picture graph
- Can skip count by 2s, 3s, 4s, 5s, and 10s
- Can multiply numbers up to $10 \times 10$
- Can divide by one-digit numbers (e.g., by finding the missing number in a multiplication fact)

**MATERIALS**
- overhead projector
- transparency of BLM Picture and Bar Graph Templates (p. U-52)
- BLM Picture and Bar Graph Templates (p. U-52)
- transparency of BLM Bar Graphs for Display (p. U-53)
- erasable markers

Review scaled picture graphs. Draw on the board:

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td>2</td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>8</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>14</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>10</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td>4</td>
</tr>
</tbody>
</table>

Explain that the table shows the times when students in a Grade 3 class go to bed. A student that goes to bed at 8:45 p.m. is counted with the students in the row that says 8:30 p.m.–8:59 p.m.

SAY: I would like to draw a picture graph to show this data. I would like to use rows of symbols for this graph. Project the left half of BLM Picture and Bar Graph Templates on the board and give each student a copy of the BLM so that they can draw the same graph. Have students suggest a title for the graph (such as Bedtimes of Grade 3 Students) and write it at the top of the graph. Together with students, fill in the labels on the picture graph. (Bedtimes, Before 8:00 p.m., 8:00 p.m.–8:29 p.m., 8:30 p.m.–8:59 p.m.,...
9:00 p.m.–9:29 p.m., 9:30 p.m. or later) NOTE: Leave the line below the picture graph empty; students will use it later to note the scale.

ASK: What is the most common time for students to go to bed?

(8:30 p.m.–8:59 p.m.) How many students go to bed in that time? (14)

How many grid squares do we have in each row of the table? (8) Is that enough for 14 symbols? (no) What can we do to show 14 students? (use a scale, use 1 symbol for more than 1 student) What symbol can we use to show students? (sample answers: smiley face, pillow) Record the options students suggest and have the class vote on a symbol.

ASK: What is a scale on a picture graph? (the part that says how many things one symbol represents) SAY: We need to decide what scale we are going to use. ASK: Are all the numbers in the table multiples of the same number? (yes, 2) PROMPT: Is there a number that we can skip count by and we will say all the numbers? ASK: How many symbols would we use for the largest number? (7) How do you know? (14 ÷ 2 = 7) Is there enough space to show 7 symbols? (yes, there are 8 grid squares and 8 is more than 7) SAY: 2 students for 1 symbol is a good scale for this data. Add the scale to the template on the empty line below the graph, and have students do the same on their graphs.

SAY: To find how many symbols to draw for each number, we need to divide the number of students by the number each symbol means. So we divide each number by 2 to find how many symbols we will use. Add a “Number of Symbols” column to the table on the board. Have students write division sentences for each number of students per bedtime and have volunteers write the division sentences in the new column on the board. Then have students draw the symbols for each row on their picture graphs. The completed table, to be kept on the board for further use, and picture graph are shown below:

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Number of Students</th>
<th>Number of Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td>2</td>
<td>2 ÷ 2 = 1</td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>8</td>
<td>8 ÷ 2 = 4</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>14</td>
<td>14 ÷ 2 = 7</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>10</td>
<td>10 ÷ 2 = 5</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td>4</td>
<td>4 ÷ 2 = 2</td>
</tr>
</tbody>
</table>

Bedtimes for Grade 3 Students

<table>
<thead>
<tr>
<th>Bedtime</th>
<th>Number of Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td>☻</td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td>☻☻☻☻☻☻</td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td>☻☻☻☻☻☻☻</td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td>☻☻☻☻☻</td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td>☻☻☻</td>
</tr>
</tbody>
</table>

= 2 students
Introduce scaled bar graphs. Project the right half of BLM Picture and Bar Graph Templates on the board. Explain that you would like to draw a bar graph for the same data as in the picture graph above. Pointing to the bar graph template, SAY: I would like to use this table but I do not have enough room to use 1 grid square, or 1 block, for 1 student. Explain that you can do with bar graphs exactly the same thing you’ve done for picture graphs: you can use 1 block to mean more than 1 person. SAY: On this graph we will use 1 block for 2 people. To show this, we will mark the number line using skip counting by 2. Demonstrate how to mark the number line using the template on the right half of BLM Picture and Bar Graph Templates and have students do the same on their copies of the template.

ASK: What else should be marked on a bar graph? (title, labels) Have students mark the title and the labels on their own graphs, and have volunteers add them to the template on the board. Then explain that the division sentences in the table you created earlier actually give you the length of each bar in blocks. Change the header in the third column of the table from “Number of Symbols” to “Length of Bar.” For example, the top bar, showing students who go to bed “Before 8:00 p.m.”, will be 1 block long. Shade the “Before 8:00 p.m.” bar on the board, and have students do the same on their graphs and have volunteers do the same on the board. The finished graph should look like this:

Bedtimes for Grade 3 Students

<table>
<thead>
<tr>
<th>Bedtimes</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 8:00 p.m.</td>
<td></td>
</tr>
<tr>
<td>8:00 p.m.–8:29 p.m.</td>
<td></td>
</tr>
<tr>
<td>8:30 p.m.–8:59 p.m.</td>
<td></td>
</tr>
<tr>
<td>9:00 p.m.–9:29 p.m.</td>
<td></td>
</tr>
<tr>
<td>9:30 p.m. or later</td>
<td></td>
</tr>
</tbody>
</table>

Analyzing a bar graph with horizontal bars. Remove the data table from the board and ask students to fold the BLM so that they cannot see the picture graph they created. ASK: What is the most common time for students to go to bed? (8:30 p.m.–8:59 p.m.) How can you see that from the bar graph? (the bar for that time is the longest) What is the least common time? (before 8:00 p.m.) How can you see that from the graph? (the bar is the shortest) How many more students go to bed between 8:00 p.m. and 8:29 p.m. than go to bed before 8:00 p.m.? (6 more) Discuss different ways to find the answer. (the longer of the two bars is 3 blocks longer than the shorter bar, and $3 \times 2 = 6$; count the blocks of each bar,
subtract, and multiply the difference using the scale: 7 blocks – 1 block = 6 blocks; 6 × 2 = 12; find the number each bar represents and subtract the numbers: 14 – 2 = 12) Ask students to find the total number of students in the bar graph, and again discuss different ways to find the answer. (add the number of blocks in the bars: 1 + 4 + 7 + 5 + 2 = 19 and then double: 2 × 19 = 38; find all the numbers for the individual bars and add them: 2 + 8 + 14 + 10 + 4 = 38) Ask students to find the answer both ways and check that the answers are the same.

**Deciding what number a scale skip counts by.** SAY: Just as a symbol can mean different numbers on picture graphs, people use different numbers to skip count on scales of bar graphs. Draw on the board:

```
0 4 8 12 16 20
```

SAY: Imagine this is the scale of a bar graph. ASK: What number does the scale skip count by? (4) How do you know? (it is the first number after zero) As a class, skip count by 4s to check that the numbers you say when skip counting by 4s are the numbers on the scale.

**Exercises:** What number does the scale skip count by?

```
a) 0 5 10 15 20 25 30
```

```
b) 0 10 20 30 40 50
```

```
c) 0 3 6 9 12 15 18
```

```
d) 0 20 40 60 80 100
```

**Answers:** a) 5, b) 10, c) 3, d) 20

**Some bar graphs use shortcuts in labels.** Project the Number of Snow Days in Rangeley, ME graph from BLM Bar Graphs for Display on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Months</th>
<th>Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct–Nov</td>
<td>9</td>
</tr>
<tr>
<td>Dec–Jan</td>
<td>21</td>
</tr>
<tr>
<td>Feb–Mar</td>
<td>27</td>
</tr>
<tr>
<td>Apr–May</td>
<td>2</td>
</tr>
</tbody>
</table>

Point to the horizontal axis and SAY: The horizontal axis shows the time in periods of 2 months so that we do not have too many bars on the graph. So, the first bar shows how many snow days there were in October and November together. ASK: What does the second bar show? (the number of snow days in December and January together) The third bar? (the number of snow days in February and March together)

Now examine the vertical axis. Point out that the longest bar goes to the top of the grid, but the scale does not have a number there. ASK: What is the last number that is written on the scale? (24) What number does the scale
skip count by? (3) If you skip count by 3s, what is the next number after 24?
(27) SAY: Sometimes there is no space to write all the numbers on a graph, but if you know what number the scale skip counts by, you can always figure out the missing numbers.

**Exercises:** Fill in the missing numbers on the scale.

a) [ | | | | | | ]

b) [ | | | | | | ]

d) [ | | | | ]

d) [ | | | | | | ]

**Bonus:** [ | | | | ]

**Answers:** a) 10, 25; b) 8, 12, 20; Bonus: 20, 60, 100

**Analyzing a bar graph with vertical bars.** ASK: On the bar graph for snow days in Rangeley, what does each block in the bars mean? (3 days of snow) How can we find out how many days of snow each bar shows? (multiply the number of blocks in the bar by 3) Draw on the board:

<table>
<thead>
<tr>
<th>Months</th>
<th>Height of Bar (blocks)</th>
<th>Multiplication</th>
<th>Number of Snow Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct–Nov</td>
<td>3</td>
<td>$3 \times 3 = 9$</td>
<td>9</td>
</tr>
<tr>
<td>Dec–Jan</td>
<td>7</td>
<td>$7 \times 3 = 21$</td>
<td>21</td>
</tr>
<tr>
<td>Feb–Mar</td>
<td>9</td>
<td>$9 \times 3 = 27$</td>
<td>27</td>
</tr>
<tr>
<td>Apr–May</td>
<td>2</td>
<td>$2 \times 3 = 6$</td>
<td>6</td>
</tr>
</tbody>
</table>

Have students copy the table and fill it in for the other three bars. Have volunteers write the answers on the board. The completed table should look like this:

ASK: How many more snow days are there in December and January than in October and November? (12) How do you know? ($21 - 9 = 12$) How can you see that on the bar graph? (the bar for December and January is 4 blocks longer, and $4 \times 3 = 12$) How many fewer snow days are there in April and May than in February and March? (21) How do you know? ($27 - 6 = 21$) Again, have students check that they get the same answer working directly from the bar graph.

ASK: Are there more snow days from the beginning of October to the end of January or from the beginning of February to the end of May? (from February to May) How do you know? ($9 + 21 = 30$ is less than $27 + 6 = 33$; or $3 + 7 = 10$ blocks is less than $9 + 2 = 11$ blocks) Have students check the answer both ways. ASK: How many more? (3 days more) Discuss which way students prefer to find the answers and why. Some students might prefer to work with the numbers because it is more familiar; others might...
prefer to work with the bar graph because the numbers are smaller and the multiplication at the end is easier.

ASK: If we had a bar in this graph for June–July or for August–September, what would the bars show? (there would be no bar) Why would the bars have height 0? (it does not usually snow in Maine in the summer)

**Extensions**

1. The bar graph shows the coins in Jayden’s pocket.

```
Coins in Jayden’s Pocket

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pennies</td>
<td>2</td>
</tr>
<tr>
<td>Nickels</td>
<td>4</td>
</tr>
<tr>
<td>Dimes</td>
<td>6</td>
</tr>
<tr>
<td>Quarters</td>
<td>8</td>
</tr>
</tbody>
</table>
```

   a) For each type of coin, how much money does Jayden have?
   
   b) How much money does Jayden have in total?

**Answers**

   a) 10¢ in pennies, 40¢ in nickels, 80¢ in dimes, and 50¢ in quarters
   
   b) 180¢ or $1.80

2. Sharon made a bar graph showing coins in her piggy bank, but forgot to mark the scale. She has 50 pennies.

```
Coins in Sharon’s Piggy Bank

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pennies</td>
<td>50</td>
</tr>
<tr>
<td>Nickels</td>
<td>2</td>
</tr>
<tr>
<td>Dimes</td>
<td>6</td>
</tr>
<tr>
<td>Quarters</td>
<td>8</td>
</tr>
</tbody>
</table>
```

(MP.1, MP.4)
a) What number did she skip count by for the scale?
b) How many coins of each type does she have?
c) How much money does she have in her piggy bank in total?

Answer: a) 10s; b) 50 pennies, 40 nickels, 40 dimes, and 10 quarters; c) \[50c + 200c + 400c + 250c = 900c = \$9.00\]

3. Marco drew a bar graph of the coins in his piggy bank, but he forgot to draw the scale. He has 80 coins in total.

 Coins in Marco's Piggy Bank

<table>
<thead>
<tr>
<th>Coins</th>
<th>Number of Coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pennies</td>
<td></td>
</tr>
<tr>
<td>Nickels</td>
<td></td>
</tr>
<tr>
<td>Dimes</td>
<td></td>
</tr>
<tr>
<td>Quarters</td>
<td></td>
</tr>
</tbody>
</table>

Number of Coins

a) How many coins of each type does he have?
b) What number did he skip count by for the scale?
c) How much money does he have in his piggy bank in total?

Answers: a) Marco has 20 coins of each of the 4 types, b) 5s, c) \[20c + 100c + 200c + 500c = 820c = \$8.20\]

(MP.1, MP.4, MP.6)

4. Read the following problem:

Vicky has 5 boxes. Each box weighs 400 g. She puts 9 glue sticks in each box. Now each box weighs 580 g. Then she realizes she can’t close the boxes. So she takes one glue stick out of each box. Now how much does each box weigh?

a) Draw a picture and use equations to show your answer.
b) What fact did you not need to use?

Answers: a) each box weighs 560 g, b) Vicky has 5 boxes

Sample solution

a) I used \[\square\] for a box and \[\square\] for a glue stick.

<table>
<thead>
<tr>
<th>What I know:</th>
<th>What I need to find:</th>
</tr>
</thead>
</table>
| 580 g        | 400 g                | ? g

U-36 Teacher Resource for Grade 3
Together, the 9 glue sticks weigh 180 g (because $580 - 400 = 180$), so each glue stick weighs 20 g (because $180 ÷ 9 = 20$). When you remove one glue stick, you remove 20 g, so the new weight of the box is 560 g (because $580 - 20 = 560$).

Look for students to recognize what information is needed to solve the problem (MP.1). Look for students to draw a picture of the objects that together weigh 580 g and a picture of the objects needed to find the final answer. Look for students to draw simple pictures that isolate mathematical information (MP.4), to use correct equations (MP.4), and to clearly label what the symbols they used mean (MP.6).
Goals
Students will read and draw scaled bar graphs, including graphs where the bars end between grid lines.
Students will solve problems using data presented in scaled bar graphs.

PRIOR KNOWLEDGE REQUIRED
Can solve one- and two-step “how many more” and “how many fewer” word problems
Can read data from a table
Can read and draw a scaled bar graph
Can read and draw a scaled picture graph with half symbols
Can multiply numbers up to 10 × 10
Can divide by one-digit numbers (e.g., by finding the missing number in a multiplication fact)
Can find half of a number by dividing by 2

MATERIALS
BLM Winter Graphs (pp. U-54–55)
overhead projector
transparency of BLM Favorite Winter Activities (p. U-56)
pictures of bar graphs (e.g., from magazines) that are not drawn on grids (optional)
transparency of BLM Bar Graphs for Display (p. U-53)
text paragraphs (see Extension 1)
selection of books popular in the class (see Extension 2)

Graphs with bars that end between grid lines. Distribute BLM Winter Graphs (1). Project Graph 1 from BLM Favorite Winter Activities on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building snow forts</td>
<td>0</td>
</tr>
<tr>
<td>Building snowmen</td>
<td>30</td>
</tr>
<tr>
<td>Skiing</td>
<td>60</td>
</tr>
<tr>
<td>Sledding</td>
<td>50</td>
</tr>
<tr>
<td>Snowshoeing</td>
<td>0</td>
</tr>
</tbody>
</table>

SAY: A nature park asked 200 visitors to vote on their favorite activity that the park offers during winter. The graph shows some of the results of the vote. ASK: What number does the scale on the graph skip count by? (10)
Draw on the board:

<table>
<thead>
<tr>
<th>Winter Activity</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building snow forts</td>
<td></td>
</tr>
<tr>
<td>Building snowmen</td>
<td></td>
</tr>
<tr>
<td>Skiing</td>
<td></td>
</tr>
<tr>
<td>Sledding</td>
<td></td>
</tr>
<tr>
<td>Snowshoeing</td>
<td></td>
</tr>
</tbody>
</table>

Have students copy the table and fill in the rows for Building snowmen, Skiing, and Sledding using the data from the bar graph on the BLM. (30, 60, 50) Have a volunteer fill in these rows on the board. SAY: Two hundred people answered the question. ASK: How many answers does the graph show? (140) How do you know? (30 + 60 + 50 = 140) How many answers does the graph not show? (60) How do you know? (200 – 140 = 60)

SAY: We need to show information for two more activities on the bar graph. In the survey, 45 people chose building snow forts as their favorite activity. Remind students that, on picture graphs, they sometimes had to show a number that was not a multiple of the number each symbol meant (for example, 1 symbol meant 10, and they needed to represent 45). Have students explain what they did in such a case. (used whole and half symbols) ASK: How many symbols do you need to mean 40? (4 symbols) 50? (5 symbols) 45? (4 full symbols and one half symbol) How do you know? (45 is exactly halfway between 40 and 50)

ASK: Does the number 45 appear on the scale of the bar graph? (no) Where on the number line should 45 be? (between 40 and 50) Ask a volunteer to show where 45 is on the number line. Explain that, similar to symbols on picture graphs, the bars on a bar graph do not have to end exactly on a grid line to show a full block. For example, we can draw a bar that ends where the number 45 should be on the number line. Add 45 to the table for building snow forts and add a bar that ends at 45 for building snow forts on the bar graph. Have students write the number in the table and draw the bar on their own graphs.

ASK: How many more people chose sledding than building snow forts? (5) How can you see that from the graph? (the bar for sledding is a half block longer than snow forts) How many fewer people voted for building snowmen than for building snow forts? (15) Discuss which way makes it easier to find the answer to the second question. (subtract the numbers on the table: 45 – 30 = 15; count the blocks on the bar graph) The bar for building snow forts is one whole and one half block longer. One whole block means 10 people and a half block means 5 people, so together the difference is 10 + 5 = 15 people.

SAY: The rest of the people in this survey voted for snowshoeing as their favorite activity. ASK: How many people voted for snowshoeing? (15) How do you know? (60 – 45 = 15; add all the numbers for the other columns
and subtract from 200, so \(200 - 185 = 15\) Add the number 15 to the table, add the bar for snowshoeing on the board, and have students do the same to their tables and graphs.

**Order of categories in a graph.** Project Graph 2 from BLM Favorite Winter Activities on the board. ASK: Does this graph have the same title as the other graph? (yes) The same scale? (yes) The same labels? (yes) What is different between the two graphs? (the order in which the labels for activities are written and the order of the bars on the graph) Have students check that the graphs show the same number of people for each activity. For example, has the number of people who like skiing changed from the other bar graph or stayed the same? (stayed the same) ASK: Do the graphs show the same data? (yes) Why do they look different? (in the second bar graph, the bars have changed order so that they go from longest bars at the top to shortest bars at the bottom)

ASK: Which activity was the most popular? (skiing) Which activity was the least popular? (snowshoeing) Which graph makes that easier to see? (Graph 2) SAY: We say that, on Graph 2, the activities were sorted from most popular to least popular. On Graph 1, the activities were also organized in a special way, but the order has nothing to do with numbers. ASK: Can you guess what the order is? (the activities are in alphabetical order) PROMPT: Look at the first two letters of each word. What do you notice?

**Redrawing a bar graph with a different scale.** Project Graph 1 from BLM Favorite Winter Activities on the board. SAY: I would like to show the same data using a different scale. Distribute BLM Winter Graphs (2). Have students fill in the title and labels (students can choose the order in which to write the labels). SAY: I want to use counting by 5s on this graph. ASK: What number do we start counting with? (0) Point out that 0 should be in the first blank from the left, the blank that is under the vertical axis. Have students fill in the zero and the rest of the numbers on the scale. ASK: What is the last number you filled in? (60)

SAY: There are 15 people who chose snowshoeing. ASK: How long should the bar be? (3 blocks) How do you know? (15 \(\div\) 5 = 3) Have students fill in the bar on their graphs. Before repeating with the bar for building snowmen, ASK: Do you need to count the blocks when you fill in the bar or is there an easier way to find where the bar should end? (when you know the number the bar should extend to, look to the scale to find the correct number and then look directly above to where the bar should end) Trace your finger along the scale to find 30 and then up from 30, along a grid line, to show where the bar must end. ASK: How many people answered that they like sledding? (50) Have students trace their fingers from 50 on the scale and along the line on their new graphs to mark the place where the bar for sledding should end. Have them draw the bar for sledding.

ASK: How many blocks long is the bar you drew? (10 blocks) What division sentence would you use to find the length of the bar? (50 \(\div\) 5 = 10) Do we get the same length both ways? (yes) Repeat with the bar for building snow forts. (45 people, 9 blocks)
ASK: Can we use division to find the length of the bar for skiing using division? (no) Why not? (we don’t know how to do $60 \div 5$) Can we use the other method, following grid lines from the scale? (yes) Have students draw the bar for skiing on their graphs. ASK: How many blocks long is the bar? (12 blocks) SAY: If we drew the bar correctly, $12 \times 5$ should be equal to 60. ASK: How can we find out how much $12 \times 5$ is? Encourage students to come up with different methods to do the calculation or to explain how they know $12 \times 5 = 60$. (use repeated addition; use the distributive property: $12 \times 5 = (10 \times 5) + (2 \times 5)$; double $6 \times 5$; start at $5 \times 10$ and add on 5 two additional times; use the fact that there are 60 minutes in an hour and 12 divisions of 5 minutes on a clock; make the bar for skiing two blocks longer than the bar for sledding)

Have students compare the two graphs. ASK: Which graph takes more space? (the graph with the scale counting by 5s) Which graph has no bars that end between the grid lines? (the graph with the scale counting by 5s) On which graph was it easier to draw the bar for skiing? On which graph was it easier to draw the bar for building snowmen? On which graph is it easier to see how many more people voted for skiing than for sledding? How many fewer people voted for snowshoeing than for building snowmen? Have students explain their opinions.

**Reading graphs that are not on a grid.** Explain that sometimes graphs are not drawn on a grid. If possible, show pictures from magazines or from the Internet that show bar graphs without a grid. Explain that, in this case, we often do not see the blocks of the bars and have to imagine the lines that go from the ends of the bars to the scale. Project the Sam’s Reading on Weekdays graph from **BLM Bar Graphs for Display** on the board. The graph shows the following data:

<table>
<thead>
<tr>
<th>Days of Week</th>
<th>Number of Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>12</td>
</tr>
<tr>
<td>Tuesday</td>
<td>10</td>
</tr>
<tr>
<td>Wednesday</td>
<td>4</td>
</tr>
<tr>
<td>Thursday</td>
<td>8</td>
</tr>
<tr>
<td>Friday</td>
<td>24</td>
</tr>
</tbody>
</table>

Have students try to explain what the graph shows. (how much Sam reads during weekdays) Point out that the days of the week are too long to write in full, so we use short forms for the labels. ASK: What is the short form for Tuesday? (Tue) What is the short form for Thursday? (Thu) Explain that sometimes we use just the first letter as a label. ASK: Why are we not using just the first letter to show days of the week? (Tuesday and Thursday would both be shown with a T, and it would be hard to see which bar belongs to which day)
ASK: What number does the scale skip count by? (4) How many pages did Sam read on Monday? (12) How do you know? (the bar for Monday ends right at the line that we can trace directly to 12) Repeat with Wednesday and Friday. (4, 24) ASK: Is there a bar that does not end at a line that extends to one of the numbers on the scale? (yes, the bar for Tuesday) ASK: Between which two lines does the bar end? (8 and 12) Does it end closer to one of the lines? (no) Does the bar end at a point halfway between 8 and 12? (yes) SAY: So the bar is showing that the number is halfway between 8 and 12. ASK: What number is that? (10)

To prompt students, draw a number line from 8 to 12, and make hops from both sides at the same time, as shown below:

![Number line with hops](image)

**Exercises**

a) How many more pages did Sam read on Friday than on Monday?

b) How many fewer pages did Sam read on Wednesday than on Tuesday?

c) How many pages in total did Sam read during the weekdays?

**Bonus:** Sam’s teacher says that Sam should read at least 10 pages every day. On which days did Sam read enough?

**Answers:** a) 12; b) 6; c) 58; Bonus: Monday, Tuesday, Friday

Have students do Questions 1–5 on AP Book 3.2 pp. 203–206. **NOTE:** For students who struggle with using only the first letters of the dog breeds in Question 3, they can write the full names of the breeds vertically or diagonally underneath the graphs.

**Extensions**

1. Give students a paragraph of text (for example, from a favorite story read in class or from another subject) and ask them each to create a bar graph that shows the number of words on each line.

2. Ask students to name some of their favorite authors. To prompt students, have a selection of books that are popular during read-alouds and independent reading time on hand to refer to. Then select the top 3 authors as well as “Other” for a total of 4 categories. Explain to students that they are to each vote for only one favorite author, and then ask students to vote by raising their hand as you say the categories one at a time. Record the votes. Have students make a bar graph to show the results.
3. Look at the picture graph.

**Snacks We Choose**

<table>
<thead>
<tr>
<th>Snack</th>
<th>🍎 🍎 🍎</th>
<th>🍎 🍎 🍎 🍎</th>
<th>🍊 🍊 🍊</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>🍎 🍎 🍎</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banana</td>
<td>🍎 🍎 🍎 🍎</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orange</td>
<td>🍊 🍊 🍊</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

= 4 people

Fill in the blank to make the sentence true:

Ed counted the total number of people who chose apples and __________. He counted 28 people altogether.

**Sample solutions**

- The number of people who chose apples is $3 \times 4 = 12$, so 16 people must have chosen the other snack (because $28 - 12 = 16$). That means there are 4 faces in the picture graph for other snack (because $16 \div 4 = 4$), so the other snack is oranges. Write "oranges" in the blank.

- Each face represents 4 people, so to get 28 people altogether, the picture graph must show 7 faces for apples and the other snack combined (because $28 \div 4 = 7$). Apples show 3 faces, so the other snack must show 4 faces. The other snack is oranges. Write "oranges" in the blank.

Whole-class follow-up: Show students the two solutions provided and ask them to compare the solutions with a partner. Which is faster? Which is easier to understand? Which is easier to come up with?
Goals
Students will review and compare picture graphs, bar graphs, and line plots

PRIOR KNOWLEDGE REQUIRED
Can solve one- and two-step “how many more” and “how many less” word problems
Can read data from a table
Can read and draw a scaled bar graph and picture graph
Can read and draw a line plot with a number line divided into quarters
Can multiply numbers up to \(10 \times 10\)
Can divide by one-digit numbers (e.g., by finding the missing number in a multiplication fact)
Can find half of a number by dividing by 2

MATERIALS
overhead projector
transparency of BLM Tree Cone Graphs (p. U-57)
leaves from different trees, 5 per group of three students
BLM Picture and Bar Graph Templates (p. U-52)
poster of trees and leaves (optional)
BLM Comparing Graphs (p. U-58, optional)
a coin (see Extension 2)
a shoebox (see Extensions 2 and 3)
2 dice (see Extension 3)

Comparing visual features of picture graphs, bar graphs, and line plots. Explain that some students went to a park and collected some cones from different kinds of trees. Then the students made graphs showing their findings. Project BLM Tree Cone Graphs on the board.

Have students signal their answers for the next questions by raising the number of fingers that corresponds to the number of the graph, or thumbs up for “yes” and thumbs down for “no.”

ASK: Which graph is a line plot?
(3) Which graph is a picture graph? (1) Which graph is a bar graph? (2) Do all three graphs have a title? (yes) Do all three graphs have labels? (yes) Do all three graphs have a number line? (no) Which graph does not have a number line? (1) Do all three graphs have a vertical axis? (no) Which graph has a vertical axis? (2) Do all three graphs use symbols? (no) Point out that line plots use \(\times\)s, which are easy-to-draw symbols. This means that the picture graph and the line plot use symbols, and the bar graph is the only one of the three graph types that does not use symbols. ASK: What symbols do the other two graphs use? (the picture graph uses circles, the line plot uses \(\times\)s) Point out to students how, on the bar graph, the label
for Eastern white pine had to be shortened, just as other labels have been shortened in previous lessons.

Ask students to explain how the number lines are different in the bar graph and the line plot. (in a bar graph, the number line can be horizontal or vertical, and it has whole numbers: in this bar graph, the number line is vertical and skip counts by 2s; in a line plot, the number line is always horizontal: this line plot shows numbers from 2 to 4 and is divided into quarters, so the number line shows mixed numbers) Make sure all of the points are raised.

**Comparing ways the data is presented in the graphs.** ASK: What does the picture graph show? (what types of cones students collected and the number of each) Can we tell from the bar graph what types of cones students collected? (yes) Can we tell that from the line plot? (no) Why not? (the line plot shows the lengths of the cones and the numbers of cones with those lengths, not the types) How else could students have sorted the cones? (sample answers: by who found what number, by what number of cones is short vs long or wide vs narrow) What type of graph could they use to present the data? (answers will vary)

ASK: How many cones did students collect in total? (16) How can you see that from each type of graph? Have students share different ways to answer this question for different graphs. (count the Xs on the line plot; find how many cones of each type was collected from the bar graph or the picture graph and add the numbers) Students can also count the symbols in the picture graph and multiply by the scale (2), but they need to figure out what to do with half symbols. To prompt them, remind them that two halves make one whole, so they can count the two half symbols as one whole symbol, and then conclude that there are 8 full symbols in the picture graph. 8 × 2 = 16, so students collected 16 cones. Students can also use a similar method with the bar graph.

**Answering questions about the data in graphs.** ASK: How many more red spruce cones than Eastern white pine cones did students collect? (4) Which graph can you use to answer the question? (picture graph or bar graph) Have students explain the different ways to find the difference from the bar graph and the picture graph. ASK: How many fewer 3-inch-long cones than 2-inch-long cones did students collect? (5) Which graph did you use to answer the question? (line plot) How can you find the answer on the graph? (there are 6 Xs in the 2-inch column, and 1 X in the 3-inch column, 6 − 1 = 5)
**ACTIVITY**

Bring in five leaves from different trees per student or, if possible, have students collect them. Divide students into groups of three. Have each group put the leaves together, look at and discuss the leaves, and then make graphs showing their findings. In each group, one student should make a picture graph, another should make a bar graph, and the third should make a line plot, similar to the graphs on AP Book 3.2 p. 207. Provide templates for graphs from BLM Picture and Bar Graph Templates to students who struggle with organizing their graphs. If available, display a poster about local trees and their leaves, and help students who struggle with identifying the leaves. Then ask the same kind of questions as in Question 2 on AP Book p. 208 and have students answer them using the different graphs they created.

Have students do Questions 1–3 on AP Book 3.2 pp. 207–208. For Questions 1 and 2, students must compare the three types of graphs; to make comparing easier, and so that students can annotate the graphs, you could give students copies of BLM Comparing Graphs. For Question 3, you might remind students that sometimes there is not enough space on a graph to put a full name (for example, for a type of book, a breed of dog, or the name of a week day or a month). Ask students if they know what “sci-fi” means. Prompt them to understand that it means “science fiction,” which is spelled out in full in Question 3.b). Write “science fiction” and “sci-fi” on the board and have a volunteer circle the common parts in both expressions.

**Extensions**

1. The table shows the length of cones for the types of trees discussed in the lesson:

<table>
<thead>
<tr>
<th>Tree</th>
<th>Length of Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red pine</td>
<td>from $\frac{1}{2}$ in to $2\frac{1}{2}$ in</td>
</tr>
<tr>
<td>Eastern white pine</td>
<td>from 4 in to 8 in</td>
</tr>
<tr>
<td>Red spruce</td>
<td>from $1\frac{1}{4}$ in to 2 in</td>
</tr>
<tr>
<td>Balsam fir</td>
<td>from $1\frac{1}{2}$ in to 4 in</td>
</tr>
</tbody>
</table>

Use the line plot and one of the other two graphs shown in the lesson to answer the question.

a) What type of cone is the longest?

b) One of the cones is 3 inches long. What type of tree could it belong to?
c) How many Eastern white pine cones did the students collect? Which X shows that cone on the line plot? How long is that cone?

d) Which column contains all the red spruce cones?

e) How many balsam fir cones did the students collect? How long are these cones? Hint: You can only figure out the lengths of some of them.

Answers
a) Eastern white pine
b) Balsam fir

c) Students collected only 1 Eastern white pine cone. Its length is a number between 4 inches and 8 inches. The line plot shows only cones that are up to 4 inches long, so the single X in the column for 4 inches has to be the X for the Eastern white pine cone. The cone is 4 inches long.

d) Red spruce cones are up to 2 inches long. All 5 of the red spruce cones have to be in the column for cones that are 2 inches long.

e) Students collected 4 Balsam fir cones and these cones vary in length. The 4-inch long cone is an Eastern white pine cone, so the rest of the cones are shorter than 4 inches. Only Balsam fir cones can be from 3 inches to 3 3/4 inches long, and there are 4 Xs for these cones on the graph. So these 4 marks belong to the Balsam fir cones. The lengths of the Balsam fir cones are: one cone 3 inches long, two cones 3 1/4 inches long, and one cone 3 3/4 inches long.

2. Give each student a coin and a shoebox. Have them toss the coin 20 times to find if the coin turns heads or tails. Students should toss the coin over the shoebox to keep it from rolling away. Have students tally the results for the tosses using a table with headings "Heads" and "Tails." Have them make a bar graph or a picture graph to show the results of the tossing. Students who struggle with organizing graphs can use BLM Picture and Bar Graph Templates.

3. Give students two dice each and a shoebox. Have them conduct this experiment: toss the dice at the same time and add the results. Students should toss the dice over the shoebox to keep them from rolling away. Students then draw a line plot to show the sums on the dice using a number line from 2 to 12. For example, they will plot an X above 8 on the number line for a roll of 3 and 5, plot an X above 10 for a roll of 4 and 6, and so on. Students can work in pairs using two pairs of dice and draw only one graph. Each student should toss their dice 10 to 15 times, so that the line plot contains 20 to 30 Xs. Both students in a pair should have two dice to save time.

Compare the most common results for different pairs of students. Explain that 6, 7, and 8 come out more often than other numbers.

ASK: Why could that be? PROMPT: How many addition sentences can you write that add to 2? (three: $0 + 2$, $1 + 1$, $2 + 0$) Which of them can be rolled on a pair of dice? (only $1 + 1$) How many ways can we
roll 2? (1) Repeat with 6, 7, and 8 so students can see how there are many more combinations that can make up those numbers.

(MP.1, MP.4, MP.7) 4. Jen makes orange juice from oranges that are all the same size. She squeezes the juice from 8 oranges into a measuring cup, as shown below:

![Measuring cup](image)

She shares the juice equally between herself, her sister, and her brother, but her sister wanted more juice. So she squeezes the juice from another orange and gives it all to her sister. Now how much orange juice does her sister have? Use equations to show your work. Explain what each step means in the situation.

**Solution:** Each marking on the measuring cup shows 20 mL (because \(100 \div 5 = 20\), find this by skip counting or using \(5 \times 20 = 100\)). So Jen starts with 480 mL of juice (because \(500 - 20 = 480\)). She shares the juice equally between 3 people, so divide 480 \(\div\) 3. I know 240 \(\div\) 3 = 80 because \(3 \times 80 = 240\), so double 80 to get 480 \(\div\) 3 = 160. Now, she squeezes another orange for her sister. Since 8 oranges made 480 mL of juice, 1 orange will make 60 mL of juice (because \(8 \times 60 = 480\)). So her sister now has 220 mL of juice (because 160 + 60 = 220).

Look for students to persevere (MP.1) in making sense of the problem by reading it several times and recording new information each time; to persevere by checking if each step they take makes sense; to model (MP.4) using correct expressions; and to look for and make use of structure (MP.7) when necessary, such as when multiplying tens.

Redirecting students: Tell students that this is a very hard problem that gives lots of information. Encourage them to find and record all the information that they can from the story, going one piece of information at a time.

Individual or small-group follow-up: Encourage students to answer the following questions:

- How much does each marking on the measuring cup show?
- How much orange juice is in the measuring cup?
- How much orange juice did Jen get from each orange?
- How much orange juice did each person get to start?

Once students answer at least some of those questions, have them reread the problem and determine what else they need to know to solve the problem.
Picture Graph Templates

Title: ___________________________

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</tbody>
</table>
Colors of Cubes

<table>
<thead>
<tr>
<th>Colors of Cubes</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>4</td>
</tr>
<tr>
<td>Green</td>
<td>3</td>
</tr>
<tr>
<td>Red</td>
<td>5</td>
</tr>
<tr>
<td>Yellow</td>
<td>3</td>
</tr>
</tbody>
</table>

= 1 cube
Snacks Bar Graphs

Snacks Eaten Today

Favorite Snacks

Snack Type

Number of Students

Muffins

Bagels

Fruit

Vegetables

Number of Students

Muffins

Bagels

Fruit

Cheese

Number of Students

0  1  2  3  4  5  6
Picture and Bar Graph Templates
Bar Graphs for Display

Sam's Reading on Weekdays

<table>
<thead>
<tr>
<th>Days of Week</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pages</td>
<td>24</td>
<td>20</td>
<td>16</td>
<td>12</td>
<td>8</td>
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</tbody>
</table>

Number of Snow Days in Rangeley, ME

<table>
<thead>
<tr>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Number of Days</td>
<td>24</td>
<td>21</td>
<td>18</td>
<td>15</td>
<td>12</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Blackline Master — Measurement and Data — Teacher Resource for Grade 3

U-53
Winter Graphs (I)

Favorite Winter Activities

- Building snow forts
- Building snowmen
- Skiing
- Sledding
- Snowshoeing

0 10 20 30 40 50
Number of People
Winter Graphs (2)
Favorite Winter Activities

Graph 1

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building snow forts</td>
<td>50</td>
</tr>
<tr>
<td>Skiing</td>
<td>40</td>
</tr>
<tr>
<td>Building snowmen</td>
<td>30</td>
</tr>
<tr>
<td>Sledding</td>
<td>20</td>
</tr>
<tr>
<td>Snowshoeing</td>
<td>10</td>
</tr>
</tbody>
</table>

Graph 2

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skiing</td>
<td>40</td>
</tr>
<tr>
<td>Building snowmen</td>
<td>30</td>
</tr>
<tr>
<td>Building snow forts</td>
<td>50</td>
</tr>
<tr>
<td>Sledding</td>
<td>20</td>
</tr>
<tr>
<td>Snowshoeing</td>
<td>10</td>
</tr>
</tbody>
</table>
Tree Cone Graphs

Graph 1

<table>
<thead>
<tr>
<th>Types of Cones</th>
<th>Red pine</th>
<th>E. wh. pine</th>
<th>Red spruce</th>
<th>Balsam fir</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>○</td>
<td>○</td>
<td></td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

○ = 2 cones

Graph 2

Number of Cones

<table>
<thead>
<tr>
<th>Types of Cones</th>
<th>Red pine</th>
<th>E. wh. pine</th>
<th>Red spruce</th>
<th>Balsam fir</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
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<td>6</td>
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<td>4</td>
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<td>2</td>
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<tr>
<td>0</td>
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</tbody>
</table>

Tree

Graph 3

Lengths of Cones

<table>
<thead>
<tr>
<th>Length (inches)</th>
<th>2</th>
<th>2 1/4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 1/2</td>
<td>2 3/4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3 1/4</td>
</tr>
<tr>
<td></td>
<td>3 1/2</td>
<td>3 3/4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Blackline Master — Measurement and Data — Teacher Resource for Grade 3

U-57
Comparing Graphs

Karen's graph:

Leaves Collected
- Beech = 2 leaves
- Elm
- Willow

Sal's graph:

Lengths of Leaves
- 2 leaves
- 3 leaves
- 4 leaves
- 5 leaves

Type of Leaf
- Beech
- Elm
- Willow

Number of Leaves
- 0
- 1
- 2
- 3
- 4

Yu's graph:

Leaves Collected
- Beech
- Elm
- Willow
Pattern Blocks

- Triangles
- Squares
- Parallelograms
- Hexagons

V-2 Blackline Master — Generic — Teacher Resource for Grade 3
Inch Rulers

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7
Half-Inch Rulers

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7
Quarter-Inch Rulers
I in Grid Paper
Number and Operations in Base Ten: Rounding and Estimating – AP Book 3.2: Unit 4

AP Book NBT3-16

1. b)  
   \(2 \times 4 \text{ tens} = 8 \text{ tens} = 80\)

c)  
   \(2 \times 3 \text{ tens} = 6 \text{ tens} = 60\)

d)  
   \(5 \times 2 \text{ tens} = 10 \text{ tens} = 100\)

e)  
   \(3 \times 3 \text{ tens} = 9 \text{ tens} = 90\)

2. b)  
   \(7 \times 7 \text{ tens} = 49 \text{ tens} = 490\)

c)  
   \(6 \times 7 \text{ tens} = 42 \text{ tens} = 420\)

d)  
   \(8 \times 6 \text{ tens} = 48 \text{ tens} = 480\)

e)  
   \(4 \times 9 \text{ tens} = 36 \text{ tens} = 360\)

3. b)  
   \(8, 80\)

c)  
   \(9, 90\)

d)  
   \(28, 280\)

e)  
   \(16, 160\)

f)  
   \(16, 160\)

g)  
   \(10, 100\)

h)  
   \(18, 180\)

i)  
   \(35, 350\)

4. b)  
   \(120\)

c)  
   \(240\)

d)  
   \(350\)

e)  
   \(360\)

f)  
   \(420\)

g)  
   \(640\)

h)  
   \(540\)

i)  
   \(720\)

5. Add a 0 to the answer. 
   \(2 \times 40 = 80\)

6. Museum A: \(4 \times 60 = 240\)
   Museum B: \(8 \times 40 = 320\)
   Museum B has more shells.

AP Book NBT3-17

1. b)  
   \(8\)

c)  
   \(80\)

d)  
   \(100\)

e)  
   \(90\)

2. b)  
   \(8\)

c)  
   \(20\)

BONUS  
   \(110\)

d)  
   \(70\)

e)  
   \(460\)

f)  
   \(700\)

g)  
   \(690\)

h)  
   \(970\)

3. b)  
   \(60, 70\)

c)  
   \(10, 20\)

d)  
   \(70, 80\)

e)  
   \(0, 10\)

f)  
   \(30, 40\)

4. Teacher to check arrows.
   b)  
   closer to 50

c)  
   closer to 80

d)  
   closer to 90

e)  
   closer to 30

f)  
   closer to 70

5. a)  
   \(2, 3, 4\)

b)  
   \(6, 7, 8\)

c)  
   They are the same distance between the multiple of 10 that comes before and the multiple of 10 that comes after.

6. The following numbers should be circled.
   b)  
   \(30\)

c)  
   \(60\)

d)  
   \(20\)

e)  
   \(80\)

f)  
   \(30\)

7. Teacher to check arrows.
   b)  
   closer to 190

c)  
   closer to 250

d)  
   closer to 470

8. The following numbers should be circled.
   b)  
   \(400\)

c)  
   \(300\)

d)  
   \(600\)

e)  
   \(900\)

f)  
   \(100\)

9. b)  
   \(100, 200\)

c)  
   \(800, 900\)

d)  
   \(600, 700\)

e)  
   \(900, 1000\)

f)  
   \(400, 500\)

g)  
   \(300, 400\)

h)  
   \(500, 600\)

10. b)  
   \(900\)

c)  
   \(400\)

d)  
   \(500\)

e)  
   \(500\)

f)  
   \(700\)

g)  
   \(700\)

h)  
   \(200\)

11. a)  
   \(320, 300\)

b)  
   \(290, 300\)

BONUS  
   j)  
   \(890, 900\)

k)  
   \(990, 1000\)

12. 290 baseball cards
Number and Operations in Base Ten: Rounding and Estimating – AP Book 3.2: Unit 4

12. 300 stamps

AP Book NBT3-19

Page 98

1. b) 20
c) 70
e) 170
f) 320
g) 260
h) 790
i) 670
j) 530
k) 990
l) 830

2. b) 900
c) 700
d) 500
e) 100
f) 200
g) 300
h) 700
i) 500

3. b) 20
   + 70
   90
c) 50
   − 10
   40

4. b) 70 – 30 = 40
c) 40 + 30 = 70
d) 80 – 60 = 20
e) 40 + 20 = 60
f) 50 + 50 = 100
g) 60 + 30 = 90
h) 80 + 20 = 100
i) 80 + 10 = 90

5. b) 200
   + 700
   900
c) 500
   − 100
   400
d) 600
   + 300
   900

6. a) 500 + 200 = 700
   b) 600 – 300 = 300
c) 200 + 800 = 1000
d) 900 – 400 = 500
e) 400 – 200 = 200
   f) 400 – 300 = 100
g) 900 – 200 = 700
   h) 300 + 100 = 400
   i) 600 + 300 = 900

7. a) 30 + 20 = 50 coats
   b) 90 + 20 = 110 coats

8. a) 60 – 40 = 20
    b) 80 – 70 = 10

9. a) 300 + 500
   = 800 books
   b) 600 – 500
   = 100 books
   c) 300 + 500 + 600
   = 1400 books

AP Book NBT3-20

Page 101

1. b) hundreds
c) ones
d) thousands
e) thousands
f) hundreds

2. b) hundreds
c) tens
d) ones
e) tens
f) thousands

3. Th H T O
   b) 9 0 2 1
   c) 0 4 8 5
d) 0 0 3 6
e) 3 2 2 1
   f) 5 6 0 2

4. b) 1
   c) 5
   d) 30
   e) 200
   f) 800
   g) 452
   h) 779

5. b) 300
   c) 30
   d) 300
   e) 30
   f) 3000
   g) 300
   h) 3
   i) 30

6. Estimate: 200 + 100 + 400
   = 700
   This is less than 1,000, so his sum is not reasonable.

BONUS

a) 5,996
b) 8,095
c) 3,611
d) 1,414
e) 8,591
f) 2,698

AP Book NBT3-21

Page 103

1. a) 1,296
   b) 1,037
   c) 1,390
   d) 1,187
   e) 1,198
   f) 936
   g) 1,076
   h) 1,829
   i) 1,427

2. a) 790
   b) 1,214
   c) 1,370

3. a) 1,123
   b) 1,142
   c) 1,172
   d) 1,192

4. a) 792
   b) 1,559
   c) 1,251

5. a) 300
   b) 800
   c) 1,300
   d) 823
   e) 1,279

AP Book NBT3-22

Page 105

1. 400 + 200 = 600
   No

2. a) The following equations should be circled:
    244 + 382 = 526
    357 – 283 = 174
    586 + 394 = 970
    875 – 538 = 137
   b) Answers will vary.

   Sample answer:
   I rounded to the nearest ten to check if the answer is reasonable.

3. a) 574
   b) 542
   c) 86
   d) 117
BONUS

e) Henry and Kayla:
\[228 + 227 = 455\]
Robin and Kathy:
\[346 + 315 = 661\]
Robin and Kathy have more cards.
f) \[661 - 455 = 206\]

4. Estimate:
\[150 + 80 = 230\]
\[300 - 230 = 70\text{ oranges}\]
Actual:
\[152 + 75 = 227\]
\[300 - 227 = 73\text{ oranges}\]

5. a) \[40 + 40 = 80\text{ students}\]
b) \[38 + 44 = 82\text{ students}\]
c) Answers will vary. Sample answer:
\[60 - 38 = 22\text{ more students}\]
\[60 - 44 = 16\text{ more students}\]
\[22 + 16 = 38\]
So, 38 students can fit on the two buses.

6. \[763 + 132 = 895\]
\[932 - 895 = 37\text{ miles by bus}\]

7. a) \[30 + 60 = 90\]
\[100 - 90 = 10\text{ more insects}\]
b) \[27 + 56 = 83\]
\[100 - 83 = 17\text{ more insects}\]

8. a) Sample answer:
\[6\text{ trays of } 4\text{ plants} = 6 \times 50 = 300\$
\[4\text{ trays of } 6\text{ plants} = 4 \times 60 = 240\$
The cheapest way to buy 24 plants is to buy 4 trays of 6 plants.
b) Sample answer: multiplication

9. a) Estimate:
\[150 + 60 = \$210\]
Actual:
\[150 + 62 = \$212\]
Measurement and Data: Time – AP Book 3.2: Unit 5

**AP Book MD3-10**

**page 107**

1. b) 12:20  
   c) 1:03  
2. b) 20 minutes past 10  
   c) 23 minutes past 1  
   d) 35 minutes past 8  
   e) 40 minutes past 2  
   f) 9 minutes past 6  
3. b) 04:15  
   c) 03:08  
   e) 12:12  
   f) 11:09  
   g) 02:23  
   h) 06:30  
   i) 02:01

**AP Book MD3-11**

**page 108**

1. a) 2, 5, 6, 8, 9, 11  
   b) 12, 3, 6, 9  
   c) 1, 2, 4, 5, 7, 8, 10, 11  
   d) 2, 3, 4, 5, 8, 9, 10, 11  
   e) 12, 1, 2, 4, 5, 6, 7, 8, 10, 11  
   f) 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11  
2. b) hour  
   c) hour  
   d) minute  
   e) hour  
   f) minute  
   g) minute  
   h) hour  
   i) minute  
   j) hour  
   k) minute  
   l) minute  
3. Sample answer:  
   An analog clock and a number line both have numbers in counting order.  
   A number line continues to any number. An analog clock stops at 12 and goes to 1 again, in a circle.

**AP Book MD3-12**

**page 110**

1. a) no  
   b) yes  
2. b) 7:00, 7 o’clock  
   c) 10:00, 10 o’clock  
   d) 1:00, 1 o’clock  
   e) 5:00, 5 o’clock  
   f) 12:00, 12 o’clock  
3. b) 04:15  
   c) 03:08  
   e) 12:12  
   f) 11:09  
   g) 02:23  
   h) 06:30  
   i) 02:01

**AP Book MD3-13**

**page 112**

1. b) 15  
   c) 50  
   d) 35  
   e) 30  
   f) 45  
2. When the minute hand moves from one number to the next, 5 minutes have passed. Since the minute hand points to 5, 25 minutes have passed. The time is 9:25.

**AP Book MD3-14**

**page 115**

1. b) 2:15  
   c) 7:25  
   d) 9:05  
   e) 4:50  
   f) 12:30  
2. b) 03:00  
   c) 11:20  
   d) 08:35  
   e) 10:10  
   f) 05:45

**BONUS**

12 × 5 = 60, so 60 minutes have passed.
3. b) 1:25, twenty-five minutes past one
c) 7:15, fifteen minutes past seven
d) 4:05, five minutes past four
e) 11:50, fifty minutes past eleven
f) 10:20, twenty minutes past ten
g) 8:30, thirty minutes past ten
h) 3:35, thirty-five minutes past three
i) 12:00, 12 o’clock

BONUS
a)

07:05
b) 7:05, five minutes past seven

AP Book MD3-15

page 117
1. a) 1, 1:30
   b) 6, 6:30
   c) 11, 11:30
   d) 4, 4:30
2. a) 8:30
   b) 6:30
   c) 10:30
   d) 12:30
3. b) half past 9
   c) half past 12
   d) half past 1
   e) half past 10
   f) half past 5
4. a) \(\frac{1}{4}\)
b) \(\frac{1}{2}\)
c) \(\frac{1}{4}\)

BONUS

(4 \times 5) + 4 = 24

5. a)

\begin{align*}
b) & \quad \frac{1}{4} \\
c) & \quad \frac{1}{4}
\end{align*}

6. a) quarter past 4
   b) quarter past 9
   c) quarter past 3

BONUS

a) half past eleven, thirty minutes past eleven, 11:30
   b) four o’clock, 4:00
   c) quarter past nine, fifteen minutes past nine, 9:15

AP Book MD3-16

page 120
1. b) 11
   c) 19
   d) 33
   e) 42
   f) 28

BONUS

(4 \times 5) + 4 = 24

2. b) 6:03
   c) 3:43
   d) 11:11
   e) 4:36
   f) 7:22

BONUS

3. b) 30
   c) 20
   d) 25
   e) 35
   f) 30

AP Book MD3-17

page 122
1. a) 15
   b) 25
   c) 50

2. b)

AP Book MD3-18

page 125
1. a) 15 minutes
   b) 25 minutes
   c) 45 minutes
2. a) 13 minutes
   b) 17 minutes
   c) 18 minutes
   d) 13 minutes

BONUS

26 minutes

AP Book MD3-19

page 127
1. a) 3 minutes
   b) 2 minutes
   c) 4 minutes
   d) 4 minutes
   e) 1 minute
   f) 3 minutes

BONUS

1 + 35 + 2 = 38

7. a) 7:45
   b) 7:00

BONUS

a) 30 minutes
   b) 8:15

AP Book MD3-15

page 117
1. a) 1, 1:30
   b) 6, 6:30
   c) 11, 11:30
   d) 4, 4:30
2. a) 8:30
   b) 6:30
   c) 10:30
   d) 12:30
3. b) half past 9
   c) half past 12
   d) half past 1
   e) half past 10
   f) half past 5
4. a) \(\frac{1}{4}\)
b) \(\frac{1}{2}\)
c) \(\frac{1}{4}\)

BONUS

(4 \times 5) + 4 = 24

5. a)

\begin{align*}
b) & \quad \frac{1}{4} \\
c) & \quad \frac{1}{4}
\end{align*}

6. a) quarter past 4
   b) quarter past 9
   c) quarter past 3

BONUS

a) half past eleven, thirty minutes past eleven, 11:30
   b) four o’clock, 4:00
   c) quarter past nine, fifteen minutes past nine, 9:15

AP Book MD3-16

page 120
1. b) 11
   c) 19
   d) 33
   e) 42
   f) 28

BONUS

(4 \times 5) + 4 = 24

2. b) 6:03
   c) 3:43
   d) 11:11
   e) 4:36
   f) 7:22

BONUS

3. b) 30
   c) 20
   d) 25
   e) 35
   f) 30

AP Book MD3-17

page 122
1. a) 15
   b) 25
   c) 50

2. b)

AP Book MD3-18

page 125
1. a) 15 minutes
   b) 25 minutes
   c) 45 minutes
2. a) 13 minutes
   b) 17 minutes
   c) 18 minutes
   d) 13 minutes

BONUS

26 minutes

AP Book MD3-19

page 127
1. a) 3 minutes
   b) 2 minutes
   c) 4 minutes
   d) 4 minutes
   e) 1 minute
   f) 3 minutes

BONUS

1 + 35 + 2 = 38

7. a) 7:45
   b) 7:00

BONUS

a) 30 minutes
   b) 8:15
4. a) 2:33  
b) 5:23  
c) 12:18
5. a) 3:30  
b) 8:38  
c) 9:47
6. 47 minutes

### AP Book MD3-20

**page 129**

1. a) a.m.  
b) a.m.  
c) p.m.  
d) p.m.  
e) p.m.  
f) a.m.
2. a) 9:54 p.m.  
b) 12:22 p.m.  
c) 8:40 a.m.  
d) 8:30 a.m.  
e) 5:15 p.m.
3. a) 3:15  
b) 10:12  
c) 4:09  
d) 0:32  
e) 0:05  
f) 3:00

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.

3. a) 6:50  
b) 4:25  
c) 10:15  
d) 8:45  
e) 9:31  
f) 11:59

**BONUS**

g) 9:00 a.m.  
h) 12:12 a.m.
Measurement and Data: Length in US Customary Units – AP Book 3.2: Unit 6

AP Book MD3-24
page 138
1. Answers will vary. Teacher to check.
2. a) 2
   b) 1
   c) 5
   BONUS
   2
3. b) 3 in
   c) 5 in
4. a) 2 in
   b) 5 in
   c) 3 in
   d) 4 in
5. 2 in
   5 in
   6 in
   BONUS
   Teacher to check.
6. Teacher to check.
7. Teacher to check.
8. Teacher to check.

AP Book MD3-25
page 141
1. b) \(\frac{1}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\)
   c) \(\frac{1}{2}, \frac{5}{2}, \frac{6}{2}\)
   d) \(\frac{8}{2}, \frac{9}{2}, \frac{10}{2}\)
   BONUS
   \(\frac{1}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}\)
2. b) \(\frac{1}{2}\)
   c) \(\frac{2}{2}\)
   d) \(\frac{1}{2}\)
3. a) \(\frac{1}{2}\)
   b) \(\frac{2}{2}\)
   c) \(\frac{1}{2}\)
   d) \(\frac{1}{2}\)
4. a) 1
   b) \(\frac{1}{2}\)
   c) \(\frac{1}{4}\)

AP Book MD3-26
page 143
1. b) \(\frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{4}{4}\)
   c) \(\frac{6}{4}, \frac{6}{4}, \frac{6}{4}, \frac{7}{4}\)
   d) \(\frac{10}{4}, \frac{10}{4}, \frac{10}{4}, \frac{11}{4}\)
   e) \(\frac{15}{4}, \frac{15}{4}, \frac{15}{4}, \frac{16}{4}\)
   f) \(\frac{19}{4}, \frac{19}{4}, \frac{19}{4}, \frac{20}{4}\)
   BONUS
   \(\frac{2}{4}, \frac{2}{4}, \frac{2}{4}, \frac{2}{4}\)
2. Teacher to check.
3. b) \(\frac{1}{2}\)
   c) \(\frac{1}{2}\)
   d) \(\frac{1}{2}\)
4. a) \(\frac{3}{4}\)
   b) \(\frac{1}{2}\)
   c) \(\frac{1}{2}\)
   d) \(\frac{1}{2}\)
5. a) \(\frac{3}{4}\)
   b) \(\frac{3}{4}\)
   c) \(\frac{3}{4}\)
   d) \(\frac{3}{4}\)
   BONUS
   0

AP Book MD3-27
page 146
1. a) Teacher to check.
b) \(\frac{13}{2}, \frac{13}{4}, \frac{3}{4}, \frac{14}{4}\)
c) Teacher to check.
d) 2
2. a) \(\frac{1}{2}\)
b) \(\frac{1}{2}\)
c) \(\frac{1}{2}\)
d) \(\frac{1}{2}\)
BONUS
  6
3. a) \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\)
   b) \(\frac{3}{4}\), \(\frac{4}{4}\), \(\frac{4}{4}\), \(\frac{4}{4}\)
   c) \(\frac{3}{4}\)
   BONUS
   6
4. a) \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\)
   b) \(\frac{1}{4}\)
   c) \(\frac{1}{4}\)
   d) \(\frac{1}{4}\)
   BONUS
   6
5. a) \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\)
   b) \(\frac{1}{4}\)
   c) \(\frac{1}{4}\)
   d) \(\frac{1}{4}\)

AP Book MD3-28
page 149
1. a) Teacher to check.
b) \(\frac{13}{2}, \frac{13}{4}, \frac{3}{4}, \frac{14}{4}\)
c) Teacher to check.
d) 2
2. a) Lengths of Pencils
   Length (inches)
b) \(\frac{5}{4}, \frac{5}{4}, \frac{5}{4}, \frac{6}{4}\)
   c) Teacher to check.
d) 0
3. a) \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\)
   b) \(\frac{3}{4}\), \(\frac{4}{4}\), \(\frac{4}{4}\), \(\frac{4}{4}\)
   c) \(\frac{3}{4}\)
   BONUS
   6
4. a) \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\), \(\frac{1}{4}\)
   b) \(\frac{1}{4}\)
   c) \(\frac{1}{4}\)
   d) \(\frac{1}{4}\)
   BONUS
   6
f) Teacher to check.

**BONUS**

3

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**AP Book MD3-29**

**page 152**

1. Circle: desk, whale, bicycle
   Cross out: apple, ant

2. Answers may vary.
   Teacher to check.

3. b) foot
c) inch
d) foot

4. Answers may vary.
   Teacher to check.

5. a) 6, 9, 12, 15, 18, 21, 24
   b) 3
   There are 3 feet in 1 yard, so multiply by the number of yards to find the total number of feet.

6. a) 27 ft
   b) 3 ft

7. a) 413 yards
   b) 55 yards

8. a) 20 in
   b) 32 in

9. 80 yd

**BONUS**

240 ft

10. a) 18 ft
    b) 5 ft

11. 24 ft

12. a) 72 yd
    b) 18 yd
Measurement and Data:
Area in US Customary Units – AP Book 3.2: Unit 7

W-20 Answer Keys for AP Book 3.2

AP Book MD3-30
page 155
1. a) 6
   b) 7
   c) 10
2. a) 4 in
   b) 6 in
3. Area of A = 4 in
   Area of B = 3 in
   Area of C = 7 in

AP Book MD3-31
page 157
1. a) 6 ft
   b) 12 ft
   c) 10 ft
2. a) 15
   b) 81 + 15 = 96 tiles
   c) 66 tiles
3. Area of A = 6 yd
   Area of B = 6 yd
   Area of C = 10 yd
4. a) 9 yd
   b) $36
   BONUS
     $60

AP Book MD3-32
page 159
1. a) 4, 8, 12
   Area = 12 yd
   b) 6, 12, 18
   Area = 18 yd
   c) 10 ft
2. a) 2, 4, 6, 8, 10
   Area = 10 ft
   b) 3, 6, 9, 12, 15, 18, 21
   Area = 21 ft
3. b) 6 columns
   3 rows
   6 × 3 = 18 ft
   c) 8 columns
   5 rows
   8 × 5 = 40 in
   d) 7 columns
   4 rows
   7 × 4 = 28 ft

AP Book MD3-33
page 161
1. a) 20 ft
   b) 18 in
   c) 12 yd
   d) 28 ft
   e) 18 in
2. b) 6 × 3 = 18 in
   c) 4 × 3 = 12 yd
   d) 7 × 4 = 28 ft
   e) 9 × 2 = 18 in
3. Area of A = 4 in
   Area of B = 3 in
   Area of C = 7 in

AP Book MD3-34
page 164
1. a) 5 × 4 = 20
   b) 4 × 8 = 32
   4 × 4 = 20
   7 × 4 = 28 ft

AP Book MD3-35
page 166
1. a) Area of A = 9
   Area of B = 4
   Area of C = 13
   b) Area of A = 6
   Area of B = 8
   Area of C = 14
   c) Area of A = 4
   Area of B = 4
   Area of C = 8
   d) Area of A = 8
   Area of B = 12
   Area of C = 20
   e) Area of A = 9
   Area of B = 3
   Area of C = 27 yd
   f) Area of A = 9
   Area of B = 3
   Area of C = 27 yd
   g) 7 × 3 = 21
   3 × 7 = 21
   d) 6 × 9 = 54
   9 × 6 = 54
   e) 4 × 3 = 12
   3 × 4 = 12
   3. 10 × 4 = 40
   4 × 10 = 40
   4. a) 9 × 7 = 63 yd
      7 × 9 = 63 yd
      b) $63
      5. Yes. Multiplying length times width results in the same answer as multiplying width times length.

AP Book MD3-36
page 169
1. Teacher to check drawings.
   a) 30
   b) 28
2. a) Gray = 6 × 4 = 24
   White = 4 × 4 = 16
   Total = 24 + 16 = 40
   b) Gray = 9 × 6 = 54 yd
   White = 9 × 6 = 54 yd
   Total = 54 + 54 = 108 yd
   c) $60
3. a) Gray = 4 × 2 = 8
   White = 3 × 2 = 6
   Total = 8 + 6 = 14
   Length = 4 + 3 = 7
   Width = 2
   Area = 7 × 2 = 14
   b) Gray = 9 × 6 = 54 yd
   White = 9 × 3
   = 27 yd
   Shape = 54 + 27 = 81 yd
   c) Gray = 2 × 8 = 16 ft
   White = 8 × 4 = 32 ft
   Shape = 16 + 32 = 48 ft
   d) Gray = 3 × 9 = 27 in
   White = 7 × 5
   = 35 in
   Shape = 27 + 35 = 62 in
   e) Area 1 = 14 yd
   Area 2 = 15 yd
   Total area = 29 yd
   b) Area 1 = 40 yd
   Area 2 = 35 yd
   Total area = 75 yd
   c) Area 1 = 24 yd
   Area 2 = 16 yd
   Total area = 40 yd
   b) Area 1 = 42 yd
   Area 2 = 20 yd
   Total area = 62 yd
   7. 62 × 2 = 124 flowers
b) Gray = 9 × 2 = 18
White = 9 × 3 = 27
Total = 18 + 27 = 45
Length = 9
Width = 2 + 3 = 5
Area = 9 × 5 = 45

4. 80 + 48
= 128

5. a) (4 × 5) + (4 × 3)
= 20 + 12
= 32
b) (9 × 10) + (9 × 7)
= 90 + 63
= 153
c) (10 × 8) + (10 × 10)
= 80 + 100
= 180

AP Book MD3-37
page 172
1. a) 3 + 7 + 3 + 7
= 20
b) 8, 10
10 × 8
= 80
c) 5, 5
5 + 5 + 5 + 5
= 20
d) 5, 5
5 × 5
= 25
2. a) (2 × 7) + (2 × 4)
= 14 + 8 = 22
b) (2 × 8) + (2 × 6)
= 16 + 12 = 28 ft
3. a) 7 ft, 7 ft
P = 4 × 7 = 28 ft
A = 7 × 7 = 49 ft²
b) 9 in, 9 in
P = 4 × 9 = 36 in
A = 9 × 9 = 81 in²

AP Book MD3-38
page 174
1. 4
10 − 4 = 6
6 + 2 = 3

2. 35 ÷ 7 = 5
5 ft
3. 80 + 10 = 8 yd
4. 54 + 9 = 6 ft

BONUS

AP Book MD3-40
page 177
1. 36
(4 × 6) + (4 × 3)
= 24 + 12
= 36
2. a) Area 1 = 10 yd²
Area 2 = 8 yd²
Total = 18 yd²
b) Area 1 = 21 m²
Area 2 = 12 m²
Total = 33 m²
3. a) 5 × 8 = 40 yd²
b) 40 ÷ 4 = 10 yd²
c) 7 × 9 = 63 yd²
d) 63 − 40 = 23 yd²
4. a) 63 + 9 = 7 ft
b) 8 × 8 = 64 ft²
c) no
5. a) 56 ÷ 7 = 8 ft
b) 7 + 8 + 7 + 8 = 30 ft
c) 56 tiles
d) 3 × 56 = $168
6. a) (6 × 10) + (3 × 4)
= 60 + 12
= 72 m²
b) 2 × 72 = $144
7. a) Eva: 3 × 7 = 21 in²
Zack: 4 × 6 = 24 in²
24 − 21 = 3 in²
b) 24 + 21 = 45 in²

BONUS

yes
Measurement and Data:
Volume and Mass – AP Book 3.2: Unit 8

AP Book MD3-41
page 179

1. Circle the following:
   a) bottle on the right
   b) bottle on the left
   c) bottle on the right

2. Circle the following:
   a) glass on the right
   b) glass on the left
   c) glass on the right

3. Circle the following:
   a) glass on the right
   b) glass on the right
   BONUS
   bottle on the right

4. Circle the following:
   a) bottle
   b) vase

5. Circle the following:
   a) thermos on the left
   b) can on the right
   BONUS
   kettle on the left

AP Book MD3-42
page 181

1. Circle the following:
   a) milk carton
   b) bathtub
   c) swimming pool
   d) mug

2. Circle milk carton and water bottle

3. a) 4
   b) 3
   c) 5
   BONUS
   180 + 30 = 6 days

AP Book MD3-43
page 183

1. 20 \times 3 = 60 L
2. 75 + 15 = 90 mL
3. a) 10 \times 2 = 20 mL
   b) 20 \times 4 = 80 mL
4. 300 \times 3 = 900 mL
5. a) 6 \times 7 = 42 L
   b) Yes. The 42 L needed for the week is more than the 40 L capacity of the gas tank.
6. 5 - 3 = 2 L
7. a) 30 + 20 = 50 L
   b) 120 - 50 = 70 L
8. a) 180 - (30 \times 2)
   = 180 - 60
   = 120 L
   b) 180 - (30 \times 4)
   = 180 - 120
   = 60 L
   BONUS
   180 + 30 = 6 days

AP Book MD3-44
page 185

1. Circle the following:
   a) elephant
   b) car
   c) book
2. Circle the following:
   a) scissors
   b) calculator
   c) chair
3. Circle the following:
   a) watermelon
   b) grapes
   c) phone
   d) football
   e) book
   f) orange

6. 40 - 30 = 10 mL
7. Answers will vary.
   Teacher to check.
8. Answers will vary.
   Teacher to check.

AP Book MD3-45
page 187

1. Circle cherry, movie ticket, and thumbtack
2. Circle book, baseball bat, and milk carton
3. Answers will vary.
   Teacher to check.
4. Answers will vary.
   Teacher to check.
5. Circle the following:
   a) kg
   b) g
6. a) 15 kg
   b) 6 g
   BONUS
   100 \times 5 = 500 g
7. a) 30 + 6 = 5 grains
   b) 5 \times 8 = 40 grains
   BONUS
   5 \times 100 = 500 grains
8. a) 200 \times 2 = 400 ants
   b) 200 \times 3 = 600 ants
   BONUS
   800 + 200 = 4 g
9. 20 + 20 + 10 + 10 + 10
   = 70 kg
10. 28 + 23 = 51 kg
    51 - 27 = 24 kg
    Yu should have a mass of 24 kg.
11. a) 27 - 23 = 4 kg
    b) 20 \times 4 = $80
Measurement and Data:
Graphs – AP Book 3.2: Unit 9

AP Book MD3-47
page 191
1. a) 4, 5, 4
   b) July
   c) June and August
   d) 4
   e) 30 – 4 = 26
   f) Teacher to check.
g) April
   h) July
2. a) at home, 4
   b) no
   c) Teacher to check
   d) Teacher to check
   e) picture graph.
   f) Teacher to check
   g) picture graph.

AP Book MD3-49
page 196
1. a) Teacher to check
   b) Teacher to check
   c) Teacher to check
   d) Teacher to check
   e) triangle
   f) hexagon
3. a) Teacher to check
   b) football
   c) It has the most circles.
   d) Teacher to check
   e) 2
   f) 15
4. 1, 3, 10, 7, 6, 100
5. a) 4
   b) 20
   c) 1
   d) 5
   e) 25
   b) Answers will vary.
   c) Teacher to check.
   d) Teacher to check
   e) triangle
   f) hexagon
   e) 20
   f) 15
   2. Circle the following:
   b) ★ = 10
   c) ★ = 3
   d) ★ = 5
3. Teacher to check.
4. a) 10
   b) 25
   c) Teacher to check.

AP Book MD3-50
page 198
1. a) 4
   b) 2
   c) 3
   b) catfish
   c) perch
   d) 12
2. a) 4
   b) 6
   c) Teacher to check
   d) 14
   e) 18
3. a) Teacher to check
   b) sparrow
      The bar for sparrows is the longest.
   c) 6
   d) 27
   e) 170 g
   f) all the sparrows
   g) no
   h) 180 g
   i) 320 g
   BONUS
   940 g

AP Book MD3-51
page 200
1. a) apple
   b) mango
   c) apple and orange
   d) 3
   e) 5
   c) Plane:
   4, 4 × 3 = 12, 12
   Train:
   3, 3 × 3 = 9, 9
   d) Cycling:
   25, 25 ÷ 5 = 5, 5
   Hiking:
   15, 15 ÷ 5 = 3, 3
   Swimming:
   50, 50 ÷ 5 = 10, 10
   e) Teacher to check.
   4. a) Teacher to check.
   b) February
   c) December, March
   d) 6
5. a) 9 inches
   b) 54 inches
   c) July, August, and September.
      It doesn’t snow in the summer.

AP Book MD3-52
page 203
1. a) 4
   b) 8
   c) 3
   d) 3
   e) 18
   f) 9
   g) 10
   h) 180 g
   i) 320 g
   BONUS
   940 g

AP Book MD3-53
page 206
1. a) Apple
   b) Mango
   c) Apple and orange
   d) 3
   e) 5
   c) Plane:
   4, 4 × 3 = 12, 12
   Train:
   3, 3 × 3 = 9, 9
   d) Cycling:
   25, 25 ÷ 5 = 5, 5
   Hiking:
   15, 15 ÷ 5 = 3, 3
   Swimming:
   50, 50 ÷ 5 = 10, 10
   e) Teacher to check.
   4. a) Teacher to check.
   b) February
   c) December, March
   d) 6
5. a) 9 inches
   b) 54 inches
   c) July, August, and September.
      It doesn’t snow in the summer.

AP Book MD3-54
page 209
1. a) 4
   b) 8
   c) 3
   d) 3
   e) 18
   f) 9
   g) 10
   h) 180 g
   i) 320 g
   BONUS
   940 g

AP Book MD3-55
page 212
1. a) Apple
   b) Mango
   c) Apple and orange
   d) 3
   e) 5
   c) Plane:
   4, 4 × 3 = 12, 12
   Train:
   3, 3 × 3 = 9, 9
   d) Cycling:
   25, 25 ÷ 5 = 5, 5
   Hiking:
   15, 15 ÷ 5 = 3, 3
   Swimming:
   50, 50 ÷ 5 = 10, 10
   e) Teacher to check.
   4. a) Teacher to check.
   b) February
   c) December, March
   d) 6
5. a) 9 inches
   b) 54 inches
   c) July, August, and September.
      It doesn’t snow in the summer.

Answer Keys for AP Book 3.2
W-23
Measurement and Data: Graphs – AP Book 3.2: Unit 9

3. a) Jake: in alphabetical order
   Hanna: from most common to least common

d) fish
   Hanna’s graph

<table>
<thead>
<tr>
<th>Breed</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beagle</td>
<td>20</td>
</tr>
<tr>
<td>Collie</td>
<td>70</td>
</tr>
<tr>
<td>Dalmatian</td>
<td>50</td>
</tr>
<tr>
<td>Husky</td>
<td>55</td>
</tr>
<tr>
<td>Pug</td>
<td>15</td>
</tr>
</tbody>
</table>

b) 10

c) yes

d) 7

e) Teacher to check.

f) no

g) Graph 2

h) collie
   Graph 1

i) pug
   Graph 2

4. a) 

<table>
<thead>
<tr>
<th>Type</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cereal</td>
<td>5</td>
</tr>
<tr>
<td>Eggs</td>
<td>4</td>
</tr>
<tr>
<td>Pancakes</td>
<td>8</td>
</tr>
<tr>
<td>Toast</td>
<td>2</td>
</tr>
<tr>
<td>Waffles</td>
<td>5</td>
</tr>
</tbody>
</table>

b) 3

c) 1

5. a) 5

b) 10

c) 85

AP Book MD3-53
page 207

1. a) picture graph, bar graph, line plot

<table>
<thead>
<tr>
<th>T</th>
<th>B</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>✗</td>
<td>✓</td>
<td>✓</td>
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<td>✗</td>
<td>✓</td>
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<tr>
<td>V</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Sc</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sy</td>
<td>✓</td>
<td>✗</td>
</tr>
</tbody>
</table>
1. Multiply.
   a) $3 \times 4 = \underline{\quad}$  
   b) $8 \times 5 = \underline{\quad}$  
   c) $9 \times 7 = \underline{\quad}$  
   $3 \times 40 = \underline{\quad}$  
   $8 \times 50 = \underline{\quad}$  
   $9 \times 70 = \underline{\quad}$  
   d) $7 \times 40 = \underline{\quad}$  
   e) $6 \times 60 = \underline{\quad}$  
   f) $30 \times 9 = \underline{\quad}$

2. Round to the nearest ten.
   a) 34  
   b) 88  
   c) 65  
   d) 237  
   e) 542  
   f) 603  

3. Round to the nearest hundred.
   a) 234  
   b) 808  
   c) 450  
   d) 23  
   e) 984  
   f) 63  

4. Estimate by rounding each number to the nearest ten.
   a) $45 + 32$  
   b) $86 - 68$  
   c) $142 - 76$  

5. Zack has 370 stamps, and Tina has 423 stamps.
   a) How many more stamps does Tina have?

   \[ \underline{\quad} \]

   b) Estimate to check your answer by rounding to the nearest ten.

   \[ \underline{\quad} \]
Unit 4: Number and Operations in Base Ten

Quiz (Lessons 16 to 19)

1. a) 12
   120
b) 40
   400
c) 63
   630
d) 280
e) 360
f) 270

2. a) 30
b) 90
c) 70
d) 240
e) 540
f) 600

3. a) 200
b) 800
c) 500
BONUS
d) 0
e) 1,000
f) 100

4. a) 50 + 30 = 80
b) 90 – 70 = 20
c) 140 – 80 = 60

5. a)  

<table>
<thead>
<tr>
<th>3</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Tina has 53 more stamps than Zack.
b) 420 – 370 = 50
Unit 4: Number and Operations in Base Ten  

Name: ______________________  

Quiz (Lessons 20 to 22)  

Date: ________________  

1. Underline the digit 4, then write its place value.  
   a) 1,345 ____________  
   b) 4,780 ____________  
   c) 7,084 ____________  

2. What does the digit 5 stand for in the number?  
   a) 3,504  
   b) 8,958  
   c) 5,987  

3. Add the numbers. You might need to regroup.  
   a)  
   +  

BONUS► Circle the equations that are not correct. Explain how you found the answer.  
   223 + 772 = 835  
   875 − 234 = 641  
   692 − 138 = 464  

________________________________________________________  

________________________________________________________  

________________________________________________________
Unit 4: Number and Operations in Base Ten

Quiz (Lessons 20 to 22)

1. a) tens
   b) thousands
   c) ones

2. a) 500
   b) 50
   c) 5,000

3. a) 
   \[
   \begin{array}{c}
   2 \quad 5 \quad 3 \\
   + \quad 8 \quad 2 \quad 4 \\
   \hline
   1 \quad 0 \quad 7 \quad 7 \\
   \end{array}
   \]

   b) 
   \[
   \begin{array}{c}
   7 \quad 4 \quad 5 \\
   + \quad 4 \quad 3 \quad 8 \\
   \hline
   1 \quad 1 \quad 8 \quad 3 \\
   \end{array}
   \]

   c) 
   \[
   \begin{array}{c}
   4 \quad 8 \quad 6 \\
   + \quad 9 \quad 1 \quad 9 \\
   \hline
   1 \quad 4 \quad 0 \quad 5 \\
   \end{array}
   \]

BONUS

223 + 772 = 835 and
692 – 138 = 464
should be circled.
Answers will vary.
Sample answer:
200 + 800 = 1,000, so 835
is too small.
900 – 200 = 700, so 641 is
close.
700 – 100 = 600, so 464 is
too small.
1. Multiply.
   a) $30 \times 8 = \underline{\hspace{2cm}}$
   b) $9 \times 50 = \underline{\hspace{2cm}}$
   c) $8 \times 70 = \underline{\hspace{2cm}}$

2. Round to the nearest ten.
   a) 39  
   b) 85  
   c) 61  
   d) 334 
   e) 507 
   f) 796 

3. Round to the nearest hundred.
   a) 671 
   b) 329 
   c) 850 
   d) 993 
   e) 84  
   f) 3   

4. Estimate by rounding each number to the nearest hundred.
   a) $450 + 347$  
   b) $806 - 618$ 
   c) $647 - 239$ 

5. There are 58 fiction books and 63 non-fiction books in the library.
   a) Estimate the total number of books in the library.

   ____________________________________________

   b) Find the exact number of books in the library.

   ____________________________________________
6. Write the value of each digit in the number 6,345.
   a) 6 stands for ______________.  
b) 3 stands for ______________.  
c) 4 stands for ______________.  
d) 5 stands for ______________.

7. Add the numbers. You might need to regroup.
   a)  
      \[
      \begin{array}{c}
        2 \ 5 \ 9 \\
        + \ 9 \ 7 \ 4
      \end{array}
      \]
   b)  
      \[
      \begin{array}{c}
        6 \ 4 \ 5 \\
        + \ 3 \ 9 \ 8
      \end{array}
      \]
   c)  
      \[
      \begin{array}{c}
        7 \ 8 \ 4 \\
        + \ 9 \ 1 \ 9
      \end{array}
      \]

8. There are 732 cars in the West parking lot. There are 586 cars in the East parking lot.
   a) Estimate the total number of cars in the parking lots.
      ____________________________ ____________________________
   b) Find the exact number of cars in the parking lots.

   c) The West parking lot can hold 800 cars. The East parking lot can hold 600 cars. How many empty spaces are in both lots altogether?

9. Emma says 272 + 97 + 398 will be greater than 1,000. Is this reasonable? Explain.
   ___________________________________________________
   ___________________________________________________
### Unit 4: Number and Operations in Base Ten

#### Test (Lessons 16 to 22)

1. a) 240  
   b) 450  
   c) 560  
2. a) 40  
   b) 90  
   c) 60  
   d) 330  
   e) 510  
   f) 800  
3. a) 700  
   b) 300  
   c) 900  
   **BONUS**  
   d) 1,000  
   e) 100  
   f) 0  
4. a) 500 + 300 = 800  
   b) 800 − 600 = 200  
   c) 600 − 200 = 400  
5. a) 60 + 60 = 120  
   b) 58 + 63 = 121  
6. a) 6,000  
   b) 300  
   c) 40  
   d) 5  
7. a)  
    \[
    \begin{array}{c}
    1 \\
    2 \\
    + 9 \\
    1 \\
    \end{array}
    \begin{array}{c}
    5 \\
    74 \\
    33 \\
    \end{array}
    
   b)  
    \[
    \begin{array}{c}
    1 \\
    6 \\
    + 3 \\
    1 \\
    \end{array}
    \begin{array}{c}
    4 \\
    98 \\
    43 \\
    \end{array}
    
   c)  
    \[
    \begin{array}{c}
    1 \\
    7 \\
    + 9 \\
    1 \\
    \end{array}
    \begin{array}{c}
    8 \\
    19 \\
    03 \\
    \end{array}
    
8. a) 700 + 600 = 1,300  

b)  
    \[
    \begin{array}{c}
    1 \\
    3 \\
    + 5 \\
    1 \\
    \end{array}
    \begin{array}{c}
    7 \\
    86 \\
    31 \\
    8 \\
    \end{array}
    
   c) 600 + 800 = 1,400  
    \[
    \begin{array}{c}
    1 \\
    3 \\
    + 1 \\
    8 \\
    \end{array}
    \begin{array}{c}
    9 \\
    18 \\
    31 \\
    2 \\
    \end{array}
    
There are 82 empty spaces.

9. 300 + 100 + 400 = 800  
   800 is much less than 1,000, so Emma’s estimate is unreasonable.
Unit 5: Measurement and Data
Quiz (Lessons 10 to 14)

1. Write the time in words and numbers.
   a) [Image of 04:45]
   b) [Image of 11:58]
   c) [Image of 01:03]

2. How is an analog clock face the same as a number line? How is it different from a number line?
   __________________________________________________________
   __________________________________________________________
   __________________________________________________________

3. What time is it? Write the time in numbers. Then write the time in words and numbers.
   a) [Image of clock showing 09:45]
   b) [Image of clock showing 03:06]
   c) [Image of clock showing 08:23]

   ________
   ________
   ________

   __________________________________________________________
   __________________________________________________________
   __________________________________________________________

BONUS► Write the multiplication equation for the number of minutes in Question 3.
   a) ___________________
   b) ___________________
   c) ___________________
Unit 5: Measurement and Data

Quiz (Lessons 10 to 14)

1. a) 45 minutes past 4
   b) 58 minutes past 11
   c) 3 minutes past 1

2. An analog clock and a number line both have numbers in counting order. A number line continues to any number. An analog clock stops at 12 and goes to 1 again, in a circle.

3. a) 8:30
   30 minutes past 8
   b) 8:15
   15 minutes past 8
   c) 9:25
   25 minutes past 9

BONUS

a) \(6 \times 5 = 30\)
b) \(3 \times 5 = 15\)
c) \(5 \times 5 = 25\)
Unit 5: Measurement and Data

Quiz (Lessons 15 to 19)

1. Write the exact time.
   a)  
   ______ : ______  
   b)  
   ______ : ______  
   c)  
   ______ : ______

2. Find the elapsed time.
   a) from 11:25 to 11:45  
   b) from 6:05 to 6:22  
   c) from 10:10 to 10:34

   ______________  
   ______________  
   ______________

   How long did she skate for? ____________________

BONUS► Write the time in as many ways as you can.
   a)  
   __________________  
   __________________  
   __________________
   b)  
   __________________  
   __________________  
   __________________
   c)  
   __________________  
   __________________  
   __________________
Unit 5: Measurement and Data

Quiz (Lessons 15 to 19)

1. a) 6:03 or 06:03
   b) 1:28 or 01:28
   c) 8:50 or 08:50

2. a) 20 minutes
   b) 17 minutes
   c) 24 minutes

3. 35 minutes

BONUS
   a) 8:30
      half past 8
      30 minutes past 8
   b) 9 o'clock
      9:00
   c) 2:15
      quarter past 2
      15 minutes past 2
Unit 5: Measurement and Data

Quiz (Lessons 20 to 23)

1. Write the exact time. Use “a.m.” or “p.m.”
   a) Dad wakes up  
   b) School ends  
   c) Karate practice ends

   at _______________  at _______________  at _______________

2. Fill in the missing times on the timeline.

   5:00 p.m. ____________ ____________ ____________ 6:00 p.m.

3. Add or subtract the times to solve.
   a) A birthday party starts at 4:30 p.m. and lasts for
      3 hours 20 minutes. When does the party end?
      The party ends at _________________.

   b) Hockey practice starts at 5:15 p.m. and ends at
      5:50 p.m. How long does the practice last?
      The practice lasts for _________________.

BONUS► It takes Jay 2 minutes to read 1 page.
   He starts reading at 8:08 a.m. and reads
   12 pages. When does he finish reading?
   Jay finishes reading at _________________.

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4. How many minutes are left before 7 o’clock? Write the subtraction equation.

a) 6:45  
b) 6:52  
c) 6:33
Unit 5: Measurement and Data

Quiz (Lessons 20 to 23)

1. a) 6:03 a.m.
   b) 2:20 p.m.
   c) 7:22 p.m.

2. 5:15 p.m., 5:30 p.m., 5:45 p.m.

3. a) 4:30 + 3:20
   = 7:50 p.m.
   b) 5:50 − 5:15
   = 35 minutes

BONUS
8:08 + 0:24
= 8:32 a.m.

ADVANCED
4. a) 7:00 − 6:45
   = 15 minutes
   b) 7:00 − 6:52
   = 8 minutes
   c) 7:00 − 6:33
   = 27 minutes
Unit 5: Measurement and Data
Test (Lessons 10 to 23)

1. Write the time in words and numbers.
   a) \[02:17\]  
   b) 
   c) 

2. Use a clock face to solve the problem.
   a) A TV show starts at 7:25 p.m. and ends at 7:55 p.m.
      How long is the show? _______________
   b) A math lesson starts at 10:05 a.m. and lasts 45 minutes.
      When does the lesson end? ______________

3. Find the elapsed time.
   a) from 8:10 to 8:25  
   b) from 6:15 to 6:55  
   c) from 5:20 to 5:57  

4. Write a.m. or p.m.
   Ron goes to bed at 9:45 ______ and gets up at 7:05 ______.
Unit 5: Measurement and Data

Test (Lessons 10 to 23)

5. Count by 5s to find the elapsed time. Write the multiplication equation.

a) from 3:20 to 3:50  
   _____ × 5 = _____

b) from 9:10 to 9:25  
   _____ × 5 = _____

c) from 1:05 to 1:55  
   _____ × 5 = _____

6. Finish the picture and find the elapsed time.

a) from 12:10 to 12:58  
   + = __________

b) from 7:45 to 8:03  
   + = __________

7. Liz thinks the time is 7:10 because the minute hand points to 10. Explain her mistake.

   ________________________________
   ________________________________

8. Add or subtract the times to solve.

a) A birthday party starts at 4:30 p.m. and ends at 7:45 p.m. How long does the party last?
   The party lasts _______________.

b) A meatloaf needs to bake for 1 hour 20 minutes. Greg put the meatloaf in the oven at 1:12 p.m.
   When will the meatloaf be ready?
   The meatloaf will be ready at __________.

BONUS ► Art class starts at quarter past 6 in the evening.
   It lasts one and a half hours.
   When does it end? _______________
ADVANCED

9. Write the time as many ways as you can.

a) 07:15  
________________  ______ __________  __ ______________
________________  ______ __________  __ ______________
________________  ______ __________  __ ______________
________________  ______ __________  __ ______________

b)  

________________
________________
________________
________________

c) 04:45  
________________  ______ __________  __ ______________
________________  ______ __________  __ ______________
________________  ______ __________  __ ______________
________________  ______ __________  __ ______________
Unit 5: Measurement and Data

Test (Lessons 10 to 23)

1. a) 17 minutes past 2
   b) 11 minutes past 10
   c) 42 minutes past 4

2. a) 30 minutes
   b) 10:50 a.m.

3. a) 15 minutes
   b) 40 minutes
   c) 37 minutes

4. p.m., a.m.

5. a) $6 \times 5 = 30$
   30 minutes
   b) $3 \times 5 = 15$
   15 minutes
   c) $10 \times 5 = 50$
   50 minutes

6. a) $40 + 8 = 48$
   12:10, 12:50, 12:58
   b) $15 + 3 = 18$
   7:45, 8:00, 8:03

7. When the minute hand points to 10, $10 \times 5 = 50$ minutes have passed. The time is 7:50.

8. a) $7:45 - 4:30 = 3:15$
   3 hours and 15 minutes
   b) $1:12 + 1:20 = 2:32$
   2:32 p.m.

BONUS
   7:45 p.m.

ADVANCED

9. a) 7:15
   15 minutes past 7
   quarter past 7
   b) 12:30
   30 minutes past 12
   half past 12
   c) 4:45
   45 minutes past 4
   quarter to 5
Unit 6: Measurement and Data

Name: ______________________  
Date: ________________

Quiz (Lessons 24 to 26)

1. Measure the distance between the arrows.
   a) [Image of arrows]  
      0 inches 1 2 3 4 5  ____ inches
   b) [Image of arrows]  
      0 inches 1 2 3 4 5  ____ inches

2. Measure the length of the line or object. Write “in” for inches.
   a) [Image of line]  
      0 inches 1 2 3 4 5  ____
   b) [Image of marker]  
      0 inches 1 2 3 4 5  ____

3. Use a ruler to measure the length of the line or object.
   a) [Image of line]  
      ____
   b) [Image of pen]  
      ____

4. Draw a line of the given length. Use a ruler to make your line straight. Start at the “0” mark.
   a) 3 inches [Image of ruler]  
      0 inches 1 2 3 4 5
   b) 1 inch [Image of ruler]  
      0 inches 1 2 3 4 5
5. Write the missing fractions or mixed numbers on the number line.

a) \[ \text{\[0\]} \quad \text{\[1\]} \quad \text{\[2\]} \]
b) \[ \text{\[7\]} \quad \text{\[8\]} \quad \text{\[9\]} \]

6. How long is the line?

a) \[ \text{\[0\text{ inches}\]} \quad \text{\[1\]} \quad \text{\[2\]} \]

b) \[ \text{\[0\text{ inches}\]} \quad \text{\[1\]} \quad \text{\[2\]} \]

7. About how long is the line?

a) \[ \text{\[0\text{ inches}\]} \quad \text{\[1\]} \quad \text{\[2\]} \]

b) \[ \text{\[0\text{ inches}\]} \quad \text{\[1\]} \quad \text{\[2\]} \]

8. Write the missing mixed numbers on the number line. Use the smallest denominator you can.

a) \[ \text{\[1\]} \quad \text{\[1\frac{1}{2}\]} \quad \text{\[2\]} \]
b) \[ \text{\[3\]} \quad \text{\[3\frac{1}{4}\]} \quad \text{\[4\]} \]

9. Circle where the given mixed number belongs on the number line.

a) \[ \text{\[2\]} \quad \text{\[3\]} \]
b) \[ \text{\[5\]} \quad \text{\[6\]} \]
10. What is the length of the line or object?

a) [Scale Diagram]

\[ \text{in} \]

b) [Scale Diagram]

\[ \text{in} \]

11. What is the length of the object to the nearest quarter of an inch?

a) [Scale Diagram]

\[ \text{in} \]

b) [Scale Diagram]

\[ \text{in} \]

**BONUS** Charlie has two worms.
Each worm is \(1\frac{1}{2}\) inches long.
He places them one after the other.
What is the total length of the two worms?

\[ \text{in} \]
Unit 6: Measurement and Data

Quiz (Lessons 24 to 26)

1. a) 2
   b) 4
2. a) 5 in
   b) 4 in
3. a) 2 in
   b) 3 in
4. Teacher to check.
5. a) \( \frac{1}{2}, \frac{1}{2} \)
   b) \( \frac{1}{2}, \frac{8}{2} \)
6. a) \( \frac{1}{2} \)
   b) \( \frac{2}{2} \)
7. a) 1
   b) \( \frac{1}{2} \)
8. a) \( \frac{1}{4}, \frac{1}{4} \)
   b) \( \frac{3}{2}, \frac{3}{4} \)
9. Teacher to check.
10. a) \( \frac{2}{4} \)
   b) \( \frac{3}{4} \)
11. a) \( \frac{1}{4} \)
   b) \( \frac{1}{2} \)

BONUS

3
1. The students in Clara’s class measure the heights of the school basketball players to the nearest quarter of an inch. Clara records the heights in a table.

<table>
<thead>
<tr>
<th>Height (inches)</th>
<th>66</th>
<th>$66 \frac{3}{4}$</th>
<th>$66 \frac{1}{4}$</th>
<th>$66 \frac{1}{2}$</th>
<th>$66 \frac{3}{4}$</th>
<th>67</th>
<th>$66 \frac{1}{4}$</th>
<th>$67 \frac{1}{4}$</th>
<th>67</th>
<th>$66 \frac{3}{4}$</th>
</tr>
</thead>
</table>

a) Draw a box around the label for the line plot.

b) Finish the number line for the line plot.

c) Draw an $\times$ in the line plot for each of the heights in Cindy’s table. Cross out each height after you draw an $\times$ for it on the line plot.

Heights of Basketball Players

$$\begin{array}{cccccccc}
66 & 66 \frac{1}{4} & 66 \frac{3}{4} & 67 & 66 \frac{1}{4} & 67 \frac{1}{4} & 67 & 66 \frac{3}{4}
\end{array}$$

Height (inches)

$66 \quad \frac{1}{4}$

Heights of Basketball Players

d) Use the line plot to answer the questions.

Number of players whose height is $66 \frac{3}{4}$ inches _____

Number of players whose height is $66 \frac{1}{4}$ inches _____

Height of the tallest player in inches. ______

BONUS $\uparrow$ How many players are less than $66 \frac{1}{4}$ inches tall? _____

BONUS $\uparrow$ How many players are more than 67 inches tall? _____
Unit 6: Measurement and Data

Quiz (Lessons 27 to 29)

2. Circle the unit you would use to measure the length.

a) \[\text{inch}\]  b) \[\text{inch}\]  c) \[\text{inch}\]
\[\text{foot}\]  \[\text{foot}\]  \[\text{foot}\]

3. Fill in the measurements in feet.

<table>
<thead>
<tr>
<th>(\text{yd})</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{ft})</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. When Sally’s teacher runs, each of her steps is 2 yards.
   
a) How many yards does she run if she takes 5 steps? ___________________
   
b) How many feet does she run if she takes 5 steps? ___________________
   
**BONUS** How many yards does Sally’s teacher run if she takes 100 steps?

__________________
Unit 6: Measurement and Data

Quiz (Lessons 27 to 29)

1. a) Teacher to check.
   b) \(\frac{1}{2}, \frac{3}{4}, 67, \frac{3}{4}\)
   c) Teacher to check.
   d) \(3\)
   \[\frac{2}{4}\]
   \[\frac{67}{4}\]
   **BONUS**
   1
   1

2. a) foot
   b) inch
   c) inch

3. | yd | 2 | 5 | 7 | 9 |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>ft</td>
<td>6</td>
<td>15</td>
<td>21</td>
<td>27</td>
</tr>
</tbody>
</table>

4. a) \(2 \times 5 = 10\) yd
   b) \(10 \times 3 = 30\) ft
   **BONUS**
   \(100 \times 2 = 200\) yd
Unit 6: Measurement and Data

Test (Lessons 24 to 29)

1. Measure the length. Write “in” for inches.
   a)  
   
   b)  

2. Use a ruler to measure the length of the object. Write “in” for inches.
   a)  
   
   b)  

3. Draw a line of the given length. Use a ruler to make your line straight. Start at the “0” mark.
   a) 2 inches  
   
   b) 4 inches  

BONUS► Draw a line 3 inches long. Start at the “1” mark.
4. Write the missing mixed numbers on the number line.
   a) \[4 \quad 5 \quad 6\]  
   b) \[8 \frac{1}{2} \quad 10\]
5. Find the length of the line to the nearest half of an inch.
   a) \[\text{in} \]
   b) \[\text{in} \]
6. Write the missing fractions or mixed numbers on the number line.
   a) \[0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5\]  
6. Write the missing fractions or mixed numbers on the number line.
   a) \[0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5\]  
7. Find the length of the line to the nearest quarter of an inch.
   a) \[\text{in} \]
   b) \[\text{in} \]
8. a) Measure the length of the pencils to the nearest quarter of an inch.

b) Underline the title of the line plot.

c) Draw a box around the label for the line plot.

d) Draw an × on the line plot for each pencil length.

e) What is the length of the longest pencil?

f) How many more pencils are \(\frac{3}{2}\) inches long than \(2\frac{3}{4}\) inches long? _____

g) How can you tell from the line plot that there were no pencils of length \(2\frac{1}{4}\) inches?
Unit 6: Measurement and Data

Test (Lessons 24 to 29)

9. Circle the better estimate for the length of the object.

a) 15 inches  b) 15 feet  c) 3 inches  d) 3 feet  e) 1 inch  f) 1 foot

10. John’s bedroom is 4 yards long.

a) What is the length of his bedroom in feet?

b) Freda’s bedroom is 10 feet long. How much longer is John’s bedroom?

11. Mr. Field likes to walk back and forth when he is teaching. He walks 5 yards to the back of the classroom, and 5 yards to the front of the classroom.

a) How many yards does he walk altogether if he walks to the back of the classroom and then to the front of the classroom?

b) How many feet does he walk altogether?

BONUS► If he walks back and forth 3 times, how many feet does he walk in total?
Unit 6: Measurement and Data

Test (Lessons 24 to 29)

1. a) 3 in  
   b) 2 in  
2. a) 3 in  
   b) 4 in  
3. Teacher to check.  
4. a) $\frac{4}{2}, \frac{5}{2}$  
   b) 8, 9, $\frac{9}{2}$  
5. a) $\frac{4}{2}$  
   b) $3\frac{1}{2}$  
6. a) $\frac{1}{2}, \frac{1}{2}, \frac{3}{4}$  
   b) 7, $\frac{7}{2}, \frac{7}{4}$  
7. a) $2\frac{3}{4}$  
   b) $\frac{4}{1}$  
8. a) $2, 2\frac{1}{2}, 2, 2\frac{1}{2}, 2\frac{1}{2}, 2\frac{3}{4}$  
   b) Teacher to check.  
   c) Teacher to check.  
   d) Teacher to check.  
   e) $2\frac{3}{4}$  
   f) 2  
   g) There are no $\times$s above $2\frac{1}{4}$ inches on the line plot.  
9. a) 15 feet  
   b) 3 inches  
   c) 1 inch  
10. a) $4 \times 3 = 12$ ft  
     b) $12 - 10 = 2$ ft  
11. a) $5 + 5 = 10$ yd  
     b) $10 \times 3 = 30$ ft  

BONUS  
$30 \times 3 = 90$ ft
Unit 7: Measurement and Data

Quiz (Lessons 30 to 34)

1. Each small square is a picture of 1 in². Count the number of squares to find the area.

   a) 
   
   Area = _____ in²

   b) 
   
   Area = _____ in²

2. The entrance to a house has gray and white tiles on the floor, as shown in the picture. Each tile is 1 ft².

   a) Count the white squares to find the area of the white tiles.
   
   Area = _____ ft²

   b) Count the gray squares to find the area of the gray tiles.
   
   Area = _____ ft²

   c) Find the total area of the entrance floor. _______________________ ft²

3. Each small square is 1 yd². Use skip counting to find the area.

   a) 
   
   Area = _____ yd²

   b) 
   
   Area = _____ yd²
4. Each small square is 1 yd². Count the number of columns and the number of rows. Multiply to find the area.

___ columns

___ rows

Area = _____ × _____ = _____ yd²

5. Multiply the length and width to find the area.

Area = _____ × _____ = _____ in²

6. A classroom has a length of 7 yards and a width of 5 yards. Find the area of the classroom in two different ways.

a) Area = length × width

   = _____ × _____

   = _____ yd²

b) Area = width × length

   = _____ × _____

   = _____ yd²

7. Phil’s bathroom has a length of 4 yards and a width of 2 yards.

   a) Find the area of the bathroom.

      Area = ________________________________

   b) Phil wants to tile the bathroom. The tiles cost $9 per square yard. Find the cost of tiling the bathroom.

      Cost = ________________________________

   **BONUS** A more expensive tile costs $20 per square yard. Find the cost of tiling Phil’s bathroom with the expensive tile.

      Cost = ________________________________
Unit 7: Measurement and Data

Quiz (Lessons 30 to 34)

1. a) 6
   b) 7
2. a) 7
   b) 8
   c) 7 + 8 = 15
3. a) 5, 10, 15, 20
   b) 3, 6, 9, 12, 15
4. a) 6
   4
   6 × 4 = 24
5. 3, 5, 15
6. a) 7, 5
   35
   b) 5, 7
   35
7. a) 4 × 2 = 8 yd²
   b) 8 × 9 = $72
   **BONUS**
   8 × 20 = $160
1. Find the areas of the gray and white rectangles. Add the areas of the rectangles to find the total area of the shape.

Gray area = \(____ \times ____\) = \(____ \text{ ft}^2\)

White area = \(____ \times ____\) = \(____ \text{ ft}^2\)

Area of shape = \(____ + ____\) = \(____ \text{ ft}^2\)

2. Draw a line to divide the L-shaped room into two smaller rectangles. Then find the area of the room.

Area 1 = \(____ \times ____\) = \(____ \text{ yd}^2\)

Area 2 = \(____ \times ____\) = \(____ \text{ yd}^2\)

Total area = \(____ + ____\) = \(____ \text{ yd}^2\)

3. Use the picture to multiply 7 and 15.

\[7 \times 15 = (7 \times ____) + (7 \times ____)
= ____ + ____
= ____\]

4. Fill in the blanks to find the perimeter.

a) Perimeter = \((2 \times ____) + (2 \times ____)\)
= \(____ + ____\)
= \(____ \text{ in}\)

b) Perimeter = \(4 \times ____\)
= \(____ \text{ yd}\)
5. Fill in the missing blanks. Then find the area and the perimeter.

Perimeter = _____ + _____ + _____ + _____
= _____ in

Area = _____ × _____
= _____ in²

6. Tim’s father built a hockey rink in the shape of a rectangle. He put up a fence around the rink. The length of the rink is 9 yards. The width of the rink is 4 yards.

a) How many yards of fencing did he need altogether?

_____________________________________

b) What is the area of the rink?

_____________________________________

**BONUS** Lisa’s father is building a similar rink. It has the same area but is in the shape of a square. Find the length and width of the rink.

_____________________________________

---

**Sample Unit Quizzes and Tests for AP Book 3.2**
Unit 7: Measurement and Data

Quiz (Lessons 35 to 40)

1. 2 × 3 = 6
2 × 4 = 8
6 + 8 = 14

2. 7 × 4 = 28
6 × 5 = 30
28 + 30 = 58
or
4 × 2 = 8
10 × 5 = 50
8 + 50 = 58

3. (7 × 10) + (7 × 5)
= 70 + 35
= 105

4. a) (2 × 5) + (2 × 3)
= 10 + 6
= 16
b) 4 × 6
= 24

5. 5 + 8 + 5 + 8
= 26
5 × 8
= 40

6. a) 4 + 9 + 4 + 9 = 26 yd
b) 4 × 9 = 36 yd²

**BONUS**

6 × 6 = 36 yd²
length = 6 yd
width = 6 yd
Unit 7: Measurement and Data

Test (Lessons 30 to 40)

1. Each small square is a picture of 1 yd². Count the number of squares to find the area.

   a) [Diagram of a 5x5 grid of small squares]  
   Area = _____ yd²

   b) [Diagram of a 4x3 grid of small squares]  
   Area = _____ yd²

2. The 4 paddles on a windmill are made of tiles. Each tile has an area of 1 ft².

   a) Count the white squares to find the area of the white tiles.
   Area = _____ ft²

   b) Count the gray squares to find the area of the gray tiles.
   Area = _____ ft²

   c) Find the total area of the paddles. _______________________ ft²

3. Each small square is 1 yd². Use skip counting to find the rectangle’s area.

   a) [Diagram of a 6x4 grid of small squares]  
   Area = _____ yd²

   b) [Diagram of a 7x5 grid of small squares]  
   Area = _____ yd²
4. Each small square is 1 in². Count the number of columns and the number of rows. Multiply to find the area.

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</tbody>
</table>

_____ columns
_____ rows 
Area = _____ × _____ = _____ in²

5. Multiply the length and width to find the area. Include the units.

![Rectangle Diagram]

Area = _____ × _____ = ______

6. Annie’s mother is building a patio made of patio stones that have an area of 1 yd² each. The patio is 9 yards long and 5 yards wide.

a) Find the area of the patio. 
Area = _____ × _____ = ______ yd²

b) How many patio stones will she need? ______________________

7. Gloria wants to wallpaper two of the walls in her room. The first wall is 10 feet long and 8 feet high. The second wall is 9 feet long and 8 feet high.

a) What area of wallpaper will she need to cover the first wall?
____________________________________________

b) What area of wallpaper will she need to cover the second wall?
____________________________________________

c) What is the total area of wallpaper that Gloria will need?
____________________________________________

d) Gloria bought 160 ft² of wallpaper. Will she have enough? Explain.
____________________________________________

________________________________________________________________________
Unit 7: Measurement and Data
Test (Lessons 30 to 40)

8. Draw a line to divide the L-shaped room into two smaller rectangles. Then find the area of the room.

Area 1 = _____ × _____ = _____ yd²
Area 2 = _____ × _____ = _____ yd²
Total = _____ + _____ = _____ yd²

9. Use the picture to multiply 5 and 14.

\[ 5 \times 14 = (5 \times \underline{\hspace{2cm}}) + (5 \times \underline{\hspace{2cm}}) \]
\[ = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \]
\[ = \underline{\hspace{2cm}} \]

10. Fill in the blanks to find the perimeter.

a) Perimeter = (2 × _____) + (2 × _____)
\[ = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \]
\[ = \underline{\hspace{2cm}} \text{ in} \]

b) Perimeter = 4 × _____ yd
\[ = \underline{\hspace{2cm}} \text{ yd} \]
11. Tim is building a garden that has an area of 36 yd$^2$.

   a) If the width is 4 yd, what is the length of the garden?

   ________________________________

   b) What is the perimeter of the garden?

   ________________________________

   c) Tim bought 25 yd of wood to make a border for the garden. Does he have enough wood? Explain.

   ________________________________

   ________________________________

   **BONUS** The wood for the border costs $4 per yard. How much will it cost Tim to make the whole border?

   ________________________________

   ________________________________
Unit 7: Measurement and Data

Test (Lessons 30 to 40)

1. a) 12
   b) 9
2. a) 22
   b) 18
   c) 22 + 18 = 40
3. a) 5, 10, 15
   b) 4, 8, 12, 16, 20, 24
   24
4. 6
   3
   6 × 3 = 18
5. 6 × 9 = 54 ft²
6. a) 9 × 5 = 45
   b) 45
7. a) 10 × 8 = 80 ft²
   b) 9 × 8 = 72 ft²
   c) 80 + 72 = 152 ft²
   d) Yes. Gloria will have enough because 160 is more than 152.
8. 2 × 3 = 6
   6 × 2 = 12
   6 + 12 = 18
   or
   2 × 5 = 10
   2 × 8 = 16
   10 + 8 = 18
9. (5 × 10) + (5 × 4)
   = 50 + 20
   = 70
10. a) (2 × 7) + (2 × 4)
    = 14 + 8
    = 22
    b) 8
    32
11. a) 36 ÷ 4 = 9 yd
    b) (2 × 4) + (2 × 9)
    = 8 + 18
    = 26 yd
    c) No. 25 is less than 26.
BONUS
    26 × 4
    = 26 + 26 + 26 + 26
    = $104
Unit 8: Measurement and Data

Quiz (Lessons 41 to 43)

1. Circle the container with less liquid.
   a) ![Container A]
   b) ![Container B]
   c) ![Container C]

2. Circle the container with more liquid.
   a) ![Container A]
   ![Container B]
   ![Container C]
   b) ![Container A]
   ![Container B]
   ![Container C]

3. Circle the container that holds less.
   a) ![Container A]
   ![Container B]
   ![Container C]
   b) ![Container A]
   ![Container B]
   ![Container C]
4. Circle the container with less capacity.
   a) ![Image]
   b) ![Image]
   c) ![Image]

5. Circle the containers that have a capacity of about 1 L each.
   ![Image]

6. Circle the objects that have a capacity of less than 1 L each.
   ![Image]

7. Find the capacity of the container and the volume of the liquid.
   a) 
   ![Image]
   Capacity = ________ L
   Volume = ________ L
   
   b) 
   ![Image]
   Capacity = ________ mL
   Volume = ________ mL

8. There are about 5 mL in a teaspoon of water.
   a) How many mL of water are in 2 teaspoons? ____________________________
   
   b) A container has a capacity of 20 mL. How many teaspoons are needed to fill the container? ________________________________

   **BONUS** How many teaspoons are needed to fill a 50 mL bottle? ________________
1. Circle the following:
   a) bottle on the right
   b) glass on the left
   c) container on the right

2. Circle the following:
   a) glass on the left
   b) beaker on the right

3. Circle the following:
   a) first beaker
   b) second container

4. Circle the following:
   a) milk carton
   b) soda can
   c) juice bottle

5. Circle bottle, milk carton

6. Circle mug, spoon, glass

7. 
   a) 5
   4
   b) 1,000
   600

8. 
   a) \(2 \times 5 = 10\) mL
   b) \(20 \div 5 = 4\) teaspoons

   **BONUS**
   \(50 \div 5 = 10\) teaspoons
Unit 8: Measurement and Data
Quiz (Lessons 44 to 46)

1. Circle the object with more mass.
   a)      b)      c) 

2. Circle the lighter object.
   a)      b)      c) 

3. Circle the heavier object. Check the balance beside the objects.
   a)      b) 

4. Circle the lighter object. Check the balances beside the objects.
   a)      b)
5. Circle which unit is better for measuring the mass of the object.

a) [Motorcycle]  
   g  kg

b) [Butterfly]  
   g  kg

c) [Person Running]  
   g  kg

6. Write the missing mass needed to make the balance level.

a) [Balance Scale]  
   ? = _____

b) [Balance Scale]  
   ? = _____

7. A shipping company has 2 different sized boxes. There are 3 boxes of size A and 4 boxes of size B

   a) What is the total mass of boxes of size A?
      ______________________________________________

   b) What is the total mass of boxes of size B?
      ______________________________________________

   c) What is the total mass of all the boxes?
      ______________________________________________

8. A nickel has a mass of 5 g.

   a) What is the mass of 2 nickels? ______________________

   b) How many nickels have a mass of 35 g altogether? ______________________

   BONUS► What is the mass of 20 nickels? ______________________
Unit 8: Measurement and Data

Quiz (Lessons 44 to 46)

1. Circle the following:
   a) house
   b) book
   c) phone

2. Circle the following:
   a) cherry
   b) pencil
   c) paperclip

3. Circle the following:
   a) cylinder
   b) phone

4. Circle the following:
   a) cylinder
   b) glasses

5. Circle the following:
   a) kg
   b) g
   c) kg

6. a) 7 g
   b) 21 kg

7. a) $3 \times 5 = 15$ kg
   b) $4 \times 2 = 8$ kg
   c) $15 + 8 = 23$ kg

8. a) $2 \times 5 = 10$ g
   b) $35 + 5 = 7$ nickels

BONUS
   $20 \times 5 = 100$ g
1. Circle the container with more liquid.
   a) ![Container A]
   b) ![Container B]
   c) ![Container C]

2. Circle the container with less liquid.
   a) ![Container A]
   b) ![Container B]

3. Circle the container that holds more.
   a) ![Container A]
   b) ![Container B]

   ![Equal Containers]
4. Circle the better estimate for the volume of the object.
   a) glass of milk       200 mL 200 L
   b) bathtub full of water 100 mL 100 L
   c) wading pool full of water 900 mL 900 L

5. Find the volume of the liquid and the capacity of the container.
   a) 
   ![Diagram of a container with markings]
   Volume = ___________ L
   Capacity = ___________ L
   b) 
   ![Diagram of a container with markings]
   Volume = ___________ mL
   Capacity = ___________ mL

6. Circle the object with less mass.
   a) 
   ![Images of a dog, a shuttlecock, and a bicycle]
   b) 
   ![Images of a school bus and an eraser]
   c) 
   ![Images of a pencil, a paper clip, and a football]

7. Circle the heavier object.
   a) 
   ![Images of a paper clip and a stack of papers]
   b) 
   ![Images of a paper clip and a stack of papers]
   c) 
   ![Images of a football and a pair of glasses]

8. Circle the lighter object. Check the balance beside the objects.
   a) 
   ![Images of a cube and two smaller cubes]
   ![Balance scale with objects]
   b) 
   ![Images of a bottle of glue and a pair of glasses]
   ![Balance scale with objects]
9. Circle the heavier object. Check the balances beside the objects.

a) 

b) 

10. Circle the unit that is better for measuring the mass of the object.

a) truck  g  kg  
b) notebook  g  kg  
c) bed  g  kg

11. The capacities of two tubs are shown in the picture. Sheila filled Tub A with water 3 times and poured all the water into Tub B.

a) What is the total volume of water she poured into Tub B?

_____________________________________________________

b) How much more water can be poured into Tub B to fill it to its capacity?

_____________________________________________________

c) How many more times will Sheila need to fill Tub A in order to fill Tub B to its capacity?

_____________________________________________________

12. A cough syrup bottle has 80 mL of cough syrup. Peter has to take 5 mL of cough syrup 2 times each day.

a) How much cough syrup will he take in one day? 

b) How many days can Peter take the cough syrup until the bottle is empty?
13. John wants to find the mass of a bag of flour. He has a mass of 52 kg. When he holds the bag of flour and stands on a scale, the total mass is 63 kg. What is the mass of the bag of flour?

14. A shipping company ships a package from New York to San Francisco. They charge $3 per kg.
   a) What is the charge for 5 kg?
   _______________________________________________________________________
   b) How many kg can be shipped for $21?
   _______________________________________________________________________

15. A 5 mL bottle of liquid has a mass of about 26 g.
   a) What is the volume of 2 bottles of the liquid?
   _______________________________________________________________________
   b) What is the mass of 2 bottles of the liquid?
   _______________________________________________________________________

BONUS► Jenny’s dog eats 100 g of dog food each day.
   a) How many grams of food does the dog eat in 2 days?
   _______________________________________________________________________
   b) How many days will it take the dog to eat 800 g of food?
   _______________________________________________________________________
1. Circle the following:
   a) bottle on the left
   b) bottle on the right
   c) beaker on the left
2. Circle the following:
   a) glass on the left
   b) glass on the right
3. Circle the following:
   a) glass
   b) vase
4. Circle the following:
   a) 200 mL
   b) 100 L
   c) 900 L
5. a) 150
   200
   b) 250
   500
6. Circle the following:
   a) shuttlecock
   b) bicycle
   c) paperclip
7. Circle the following:
   a) phone
   b) stapler
   c) soccer ball
8. Circle the following:
   a) cylinder
   b) glasses
9. Circle the following:
   a) cylinder
   b) laptop
10. Circle the following:
    a) kg
    b) g
    c) kg
11. a) 3 \times 5 = 15 L
    b) 35 - 15 = 20 L
    c) 20 + 5 = 4 times
12. a) 2 \times 5 = 10 mL
    b) 80 + 10 = 8 days
13. 63 - 52 = 11 kg
14. a) 3 \times 5 = $15
    b) 21 \div 3 = 7 kg
15. a) 2 \times 5 = 10 mL
   b) 26 + 26 = 52 g

BONUS
   a) 2 \times 100 = 200 g
   b) 800 \div 100 = 8 days
1. Use the picture graph to answer the questions.

**Number of Snow Days**

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Snow Days</th>
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<tbody>
<tr>
<td>November</td>
<td>✘ ✘</td>
</tr>
<tr>
<td>December</td>
<td>✘ ✘ ✘ ✘</td>
</tr>
<tr>
<td>January</td>
<td>✘ ✘ ✘ ✘ ✘ ✘ ✘ ✘ ✘</td>
</tr>
<tr>
<td>February</td>
<td>✘ ✘ ✘</td>
</tr>
<tr>
<td>March</td>
<td>✘ ✘ ✘</td>
</tr>
</tbody>
</table>

✦ = 2 days

a) Fill in the table.

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Snow Days</th>
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<tbody>
<tr>
<td>November</td>
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</tr>
<tr>
<td>December</td>
<td></td>
</tr>
<tr>
<td>January</td>
<td></td>
</tr>
<tr>
<td>February</td>
<td></td>
</tr>
</tbody>
</table>

b) There were 3 snow days in March. Show this on the picture graph.

c) How many more snow days were there in December than in March? ______

d) How many fewer snow days were there in February than in January? ______

e) Which month had the most number of snow days? ________________

f) Which month had the fewest number of snow days? ________________

g) How many snow days were there in total from November to March?

__________________________________________

**BONUS**

► If ☺ means 20 people, what does ☺☺☺☺☺☺☺☺ mean?

Write a multiplication equation.

__________________________________________
1. a) 

<table>
<thead>
<tr>
<th>Number of Snow Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

b) \( \times \rightarrow \)
c) 5
d) 5
e) January
f) March
g) \[ 4 + 8 + 11 + 6 + 3 \]

\[ = 32 \]

BONUS

\[ 8 \times 20 = 160 \text{ people} \]
1. Use the bar graph to answer the questions.
   a) Fill in the table.

<table>
<thead>
<tr>
<th>Type of Fish</th>
<th>Number of Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric eel</td>
<td></td>
</tr>
<tr>
<td>Lionfish</td>
<td></td>
</tr>
<tr>
<td>Seahorse</td>
<td></td>
</tr>
</tbody>
</table>

   b) How many electric eels, lionfish, and seahorses are there in total? ________________

   c) There are 40 fish at the zoo in total.
      How many of them are archerfish? ________________________
      Draw a bar for them.

   d) How many more seahorses than electric eels are there? ________________

   e) How many fewer lionfish than seahorses are there? ________________

   **BONUS** An electric eel weighs about 20 kg. How much do all the eels at the zoo weigh?
   ____________________________________________________________________________
Unit 9: Measurement and Data

Quiz (Lessons 50 to 53)

1. a) 

<table>
<thead>
<tr>
<th>Number of Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>18</td>
</tr>
</tbody>
</table>

b) $4 + 10 + 18 = 32$

c) $40 - 32 = 8$

d) $18 - 4 = 14$

e) $18 - 10 = 8$

BONUS

$4 \times 20 = 80$ kg
1. a) There are 31 shapes in the picture. Count the number of each shape. Fill in the table.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Number of Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
<td></td>
</tr>
<tr>
<td>Quadrilateral</td>
<td></td>
</tr>
<tr>
<td>Pentagon</td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td></td>
</tr>
</tbody>
</table>

b) Draw a bar graph to show how many shapes are in the picture.

<table>
<thead>
<tr>
<th>Shapes in the Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
</tr>
<tr>
<td>Triangle</td>
</tr>
<tr>
<td>Quadrilateral</td>
</tr>
<tr>
<td>Pentagon</td>
</tr>
<tr>
<td>Hexagon</td>
</tr>
</tbody>
</table>

Number of Shapes

0 2 4 6 8 10

How many more quadrilaterals than triangles are in the picture? _________

How does your bar graph show this? ______________________________

____________________________________________________________

____________________________________________________________

d) How many more polygons than circles are in the picture? ______________
2. Use the picture graph to answer the question.
   a) Fill in the table.

<table>
<thead>
<tr>
<th>State</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arizona</td>
<td></td>
</tr>
<tr>
<td>Nevada</td>
<td></td>
</tr>
<tr>
<td>Oregon</td>
<td></td>
</tr>
<tr>
<td>Utah</td>
<td></td>
</tr>
</tbody>
</table>

   b) 5 fewer students visited Washington than Arizona.

   How many students visited Washington? ____________
   Show these students on the picture graph.

   c) How many more students visited Nevada than Oregon? ______

   **BONUS**► In the picture graph above there are 3 more symbols for Nevada than for Arizona. Ron thinks that 3 more students visited Nevada than Arizona. Explain his mistake.

   _______________________________________________________
   _______________________________________________________
   _______________________________________________________

   d) Use the table to show the same data on the picture graph below.

<table>
<thead>
<tr>
<th>States Visited by Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arizona</td>
</tr>
<tr>
<td>Nevada</td>
</tr>
<tr>
<td>Oregon</td>
</tr>
<tr>
<td>Utah</td>
</tr>
<tr>
<td>Washington</td>
</tr>
</tbody>
</table>

   ☀️ = 10 students

   ○ = 5 students
1. a) 

<table>
<thead>
<tr>
<th>Number of Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

b) Teacher to check.

c) 1
The bar for quadrilaterals is half a block longer than the bar for triangles. One block is 2, so half a block is one.

d) $26 - 5 = 21$

2. a) 

<table>
<thead>
<tr>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

b) 15

BONUS
Each symbol means 10 people, not 1 person, so 3 symbols means $3 \times 10 = 30$ people.

d) Teacher to check.
## Scoring Guides for Sample Unit Quizzes and Tests

### Unit 4: Number and Operations in Base Ten

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**Quiz (Lessons 16 to 19), p. X-39**

**Common Core State Standards Emphasized:** 3.NBT.A.1, 3.NBT.A.3, 3.OA.D.8

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>12, 120</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>40, 400</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>63, 630</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>280</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>360</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>270</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td><strong>6</strong></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>30</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>90</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>70</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>240</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>540</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>600</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td><strong>6</strong></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>200</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>800</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>500</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: d) Gives correct answer</td>
<td>0</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: e) Gives correct answer</td>
<td>1,000</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: f) Gives correct answer</td>
<td>100</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td><strong>3</strong></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Rounds correctly</td>
<td>50 + 30</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>80</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Rounds correctly</td>
<td>90 – 70</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>20</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Rounds correctly</td>
<td>140 – 80</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>60</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td><strong>6</strong></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>53</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Rounds correctly</td>
<td>420 – 370</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td><strong>3</strong></td>
<td></td>
</tr>
</tbody>
</table>

---

**Total Points**          | **24**
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>tens</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>thousands</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>ones</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>500</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>5,000</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>1,077</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>1,183</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>1,405</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>Bonus</td>
<td>Circles 223 + 772 = 835 and gives a correct explanation</td>
<td>Sample answer: 200 + 800 = 1,000, so 835 is too small.</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Doesn't circle 875 − 234 = 641 and gives a correct explanation</td>
<td>Sample answer: 900 − 200 = 700, so 641 is close.</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles 692 − 138 = 464 and gives a correct explanation</td>
<td>Sample answer: 700 − 100 = 600, so 464 is too small.</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total Points</td>
<td>/9</td>
</tr>
</tbody>
</table>
Scoring Guides for Sample Unit Quizzes and Tests
Unit 4: Number and Operations in Base Ten

Test (Lessons 16 to 22), p. X-43

Common Core State Standards Emphasized: 3.NBT.A.1, 3.NBT.A.3, 3.OA.D.8

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>240</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>450</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>560</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>40</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>90</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>60</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>330</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>510</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>800</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>700</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>300</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>900</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: d) Gives correct answer</td>
<td>1,000</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: e) Gives correct answer</td>
<td>100</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: f) Gives correct answer</td>
<td>0</td>
<td>yes / no</td>
<td>/3</td>
</tr>
<tr>
<td>4</td>
<td>a) Rounds correctly</td>
<td>500 + 300</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>800</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Rounds correctly</td>
<td>800 − 600</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>200</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c) Rounds correctly</td>
<td>600 − 200</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct estimate</td>
<td>400</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>60 + 60 = 120</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>58 + 63 = 121</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>6,000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>300</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>40</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td>1,233</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>1,043</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>1,703</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------------------------------</td>
<td>-----------------------</td>
<td>------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>700 + 600 = 1,300</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>1,318</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>1,400 – 1,318 = 82</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Gives correct estimate</td>
<td>300 + 100 + 400 = 800</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct conclusion</td>
<td>The sum will not be greater than 1,000.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total Points</td>
</tr>
</tbody>
</table>
### Rubric for Unit 4: Number and Operations in Base Ten

Test (Lessons 16 to 22), p. X-43

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.NBT.A.1</td>
<td>2, 3, 4, 6</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Use place value understanding to round whole numbers to the nearest 10 or 100.</em></td>
<td>2, 3, 4, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.NBT.A.3</td>
<td>1</td>
<td>Can answer some questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.</em></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.OA.D.8</td>
<td>5, 8, 9</td>
<td>Can answer most questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</em></td>
<td>5, 8, 9</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Comments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
### Quiz (Lessons 10 to 14), p. X-46

<table>
<thead>
<tr>
<th>Common Core State Standards Emphasized: 3.MD.A.1, 3.OA.A.3, 3.OA.C.7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question</strong></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Bonus</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Total Points</strong></td>
</tr>
</tbody>
</table>
## Quiz (Lessons 15 to 19), p. X-48

### Common Core State Standards Emphasized:
3.MD.A.1, 3.OA.A.3, 3.OA.C.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>6:03 or 06:03</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>1:28 or 01:28</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>8:50 or 08:50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>20 minutes</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>17 minutes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>24 minutes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gives correct answer</td>
<td>35 minutes</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>Bonus</td>
<td>a) Gives at least 2 different ways</td>
<td>8:30, half past 8, 30 minutes past 8</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives 2 different ways</td>
<td>9:00, 9 o’clock</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives at least 2 different ways</td>
<td>2:15, quarter past 2, 15 minutes past 2</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points**: /7
### Quiz (Lessons 20 to 23), p. X-50

**Common Core State Standards Emphasized:** 3.MD.A.1, 3.OA.D.9

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1 | a) Gives correct answer  
Correctly choses between a.m. and p.m. | 6:03 a.m. | 0.5 | 0.5 |
|   | b) Gives correct answer  
Correctly choses between a.m. and p.m. | 2:20 p.m. | 0.5 | 0.5 |
|   | c) Gives correct answer  
Correctly choses between a.m. and p.m. | 7:22 p.m. | 0.5 | 0.5 |
|   |   | | /3 | |
| 2 | Gives all three correct times with a.m. or p.m.  
Gives one or two correct times with a.m. or p.m. | 5:15 p.m., 5:30 p.m., 5:45 p.m. | 2 | (1) /2 |
| 3 | a) Shows correct work  
Gives correct answer with a.m. or p.m. | 4:30 + 3:20 = 7:50 p.m. | 1 | 1 |
|   | b) Shows correct work  
Gives correct answer | 5:50 – 5:15 = 35 minutes | 1 | 1 |
|   | Bonus: Shows correct work  
Gives correct answer with a.m. or p.m. | 8:08 + 0:24 = 8:32 a.m. | yes / no | /4 |
| Advanced 4 | a) Gives correct answer | 7:00 – 6:45 = 15 minutes | 1 | |
|   | b) Gives correct answer | 7:00 – 6:52 = 8 minutes | 1 | |
|   | c) Gives correct answer | 7:00 – 6:33 = 27 minutes | 1 | |

**Total Points** /9
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 5: Measurement and Data

(continued)

<table>
<thead>
<tr>
<th>Test (Lessons 10 to 23), p. X-53</th>
</tr>
</thead>
</table>

**Common Core State Standards Emphasized:** 3.MD.A.1, 3.OA.A.3, 3.OA.C.7, 3.OA.D.9

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>17 minutes past 2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>11 minutes past 10</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>42 minutes past 4</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>30 minutes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>10:50 a.m.</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>15 minutes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>40 minutes</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>37 minutes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gives two correct answers</td>
<td>p.m., a.m.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives one correct answer</td>
<td>(0.5)</td>
<td>(0.5)</td>
<td>/1</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct multiplication equation</td>
<td>6 × 5 = 30</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>30 minutes</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct multiplication equation</td>
<td>3 × 5 = 15</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>15 minutes</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct multiplication equation</td>
<td>10 × 5 = 50</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>50 minutes</td>
<td>0.5</td>
<td>/3</td>
</tr>
<tr>
<td>6</td>
<td>a) Correctly writes times on picture</td>
<td>12:10, 12:50, 12:58</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>40 + 8 = 48</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly writes times on picture</td>
<td>7:45, 8:00, 8:03</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>15 + 3 = 18</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Gives correct answer</td>
<td>When the minute hand points to 10, 10 × 5 = 50 minutes have passed. The time is 7:50.</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
<td>--------</td>
<td>------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>8</td>
<td>a) Shows correct work&lt;br&gt;Gives correct answer</td>
<td>7:45 – 4:30&lt;br&gt;3 hours and 15 minutes</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Shows correct work&lt;br&gt;Gives correct answer</td>
<td>1:12 + 1:20&lt;br&gt;2:32 p.m.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Bonus: Shows correct work&lt;br&gt;Gives correct answer</td>
<td>6:15 + 1:30&lt;br&gt;7:45 p.m.</td>
<td>yes / no</td>
<td>/4</td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Gives 3 different ways&lt;br&gt;Gives 1 or 2 different ways</td>
<td>7:15, 15 minutes past 7, quarter past 7</td>
<td>2&lt;br&gt;(1)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>b) Gives 3 different ways&lt;br&gt;Gives 1 or 2 different ways</td>
<td>12:30, 30 minutes past 12, half past 12</td>
<td>2&lt;br&gt;(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives 3 different ways&lt;br&gt;Gives 1 or 2 different ways</td>
<td>4:45, 45 minutes past 4, quarter to 5</td>
<td>2&lt;br&gt;(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td><strong>/21</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Rubric for Unit 5: Measurement and Data

Test (Lessons 10 to 23), p. X-53

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.MD.A.1</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, Bonus</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

**Comments**
### Scoring Guides for Sample Unit Quizzes and Tests
**Unit 6: Measurement and Data**

#### Quiz (Lessons 24 to 26), p. X-57

**Common Core State Standards Emphasized:** 3.MD.B.4, 3.NF.A.2a, 3.NF.A.2b

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>2</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>4</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer with units</td>
<td>5 in</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>4 in</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer with units</td>
<td>2 in</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>3 in</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Draws correct line</td>
<td>3 inches long</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Draws correct line</td>
<td>1 inch long</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>(1 \frac{1}{2}, 1 \frac{1}{2})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(7 \frac{1}{2}, 8 \frac{1}{2})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>(1 \frac{1}{2})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(2 \frac{1}{2})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td>1</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(1 \frac{1}{2})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>(1 \frac{1}{4}, 1 \frac{3}{4})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(3 \frac{1}{2}, 3 \frac{3}{4})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>9</td>
<td>a) Correctly circles position on number line</td>
<td>1</td>
<td>/2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly circles position on number line</td>
<td>1</td>
<td>/2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>a) Gives correct answer</td>
<td>(2 \frac{1}{4})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(3 \frac{1}{4})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>11</td>
<td>a) Gives correct answer</td>
<td>(1 \frac{1}{4})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(2 \frac{1}{2})</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>3</td>
<td>yes / no</td>
<td>/22</td>
</tr>
</tbody>
</table>
## Quiz (Lessons 27 to 29), p. X-61

**Common Core State Standards Emphasized:** 3.MD.B.4, 3.NF.A.2a, 3.NF.A.2b, 3.OA.A.3

<table>
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<th>Question</th>
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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Draws box around label</td>
<td>Height (inches)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly labels number line</td>
<td>$66\frac{1}{2}, 66\frac{3}{4}, 67, 67\frac{1}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly draws data in line plot</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>Gives correct answer</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$67\frac{1}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td></td>
<td>1</td>
<td>yes / no</td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td></td>
<td>1</td>
<td>yes / no</td>
</tr>
<tr>
<td>2</td>
<td>a) Circles correct answer</td>
<td>foot</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>inch</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>inch</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Gives 4 correct answers</td>
<td>$6, 15, 21, 27$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Gives 1, 2, or 3 correct answers</td>
<td></td>
<td>(1)</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>$5 \times 2 = 10$ yards</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$10 \times 3 = 30$ feet</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>$100 \times 2 = 200$ yards</td>
<td>yes / no</td>
<td>2</td>
</tr>
</tbody>
</table>

**Total Points** 13
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer with units</td>
<td>3 in 1/2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>2 in 1</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer with units</td>
<td>3 in 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>4 in 1</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>a) Draws correct line</td>
<td>2 inches 1/2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws correct line</td>
<td>4 inches 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Draws correct line</td>
<td>3 inches starting at 1 inch yes / no</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>(\frac{4}{2}, \frac{5}{2}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>8, 9, (9\frac{1}{2}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>(\frac{4}{2}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(3\frac{1}{2}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>7, (7\frac{1}{2}, 7\frac{3}{4}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td>(2\frac{3}{4}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(\frac{4}{4}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>(2, \frac{2}{2}, 2, \frac{2}{2}, \frac{2}{2}, \frac{2}{2}, \frac{3}{4}) 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Underlines title</td>
<td>Lengths of Pencils 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Draws box around label</td>
<td>Length (inches) 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Correctly draws data in line plot</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>(2\frac{3}{4}) in 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>2 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>g) Gives correct answer</td>
<td>There are no Xs above (2\frac{1}{4}) inches on the line plot. 1</td>
<td>1/2</td>
<td></td>
</tr>
</tbody>
</table>

Test (Lessons 24 to 29), p. X-64

### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 6: Measurement and Data**

---

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>a) Circles correct answer</td>
<td>15 feet</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>3 inches</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>1 inch</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>a) Gives correct answer with units</td>
<td>4 × 3 = 12 feet</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>12 − 10 = 2 feet</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>a) Gives correct answer</td>
<td>5 + 5 = 10 yards</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>10 × 3 = 30 feet</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>30 × 3 = 90 feet</td>
<td>yes / no</td>
<td>/2</td>
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</table>

**Total Points** /28
<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1 Can answer few, if any, questions accurately and independently.</th>
<th>Level 2 Can answer some questions accurately and independently.</th>
<th>Level 3 Can answer most questions accurately and independently.</th>
<th>Level 4 Can answer all or almost all questions, including bonuses, accurately and independently.</th>
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</thead>
<tbody>
<tr>
<td>3.MD.B.4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.NF.A.2a</td>
<td>4, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.</td>
<td>4, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.NF.A.2b</td>
<td>6, 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</td>
<td>6, 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 7: Measurement and Data**

**Quiz (Lessons 30 to 34), p. X-69**

**Common Core State Standards Emphasized:** 3.MD.C.5a, 3.MD.C.5b, 3.MD.C.6, 3.MD.C.7a, 3.MD.C.7b

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>7</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>8</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>15</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Skip counts correctly</td>
<td>5, 10, 15, 20</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>20</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Skip counts correctly</td>
<td>3, 6, 9, 12, 15</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>15</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct number of columns</td>
<td>6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct number of rows</td>
<td>4</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>6, 4, 24</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer</td>
<td>3, 5, 15</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>7, 5, 35</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>5, 7, 35</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer with units</td>
<td>$4 \times 2 = 8 \text{ yd}^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$8 \times 9 = 72$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>$8 \times 20 = 160$</td>
<td>yes / no</td>
<td>/2</td>
</tr>
</tbody>
</table>

**Total Points** /14
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 7: Measurement and Data

(continued)

|----------------------------------|---------------------------------------------------------------------------------------------------|

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives correct answer for Gray area</td>
<td>$2 \times 3 = 6$</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for White area</td>
<td>$2 \times 4 = 8$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for Area of shape</td>
<td>$6 + 8 = 14$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

| 2        | Draws a vertical line to divide the L-shape | $7 \times 4 = 28$ | 0.5 | /2 |
|          | Gives correct answer for Area 1 (left) | $6 \times 5 = 30$ | 0.5 | |
|          | Gives correct answer for Area 2 (right) | $28 + 30 = 58$ | 0.5 | |

| 3        | Gives correct answer | $(7 \times 10) + (7 \times 5) = 70 + 35$ | 2 | /2 |
|          | | $= 105$ | |

| 4        | a) Gives correct answer | $(2 \times 5) + (2 \times 3) = 10 + 6$ | 2 | |
|          | | $= 16$ | |
|          | b) Gives correct answer | $6, 24$ | 1 | /3 |

| 5        | Gives correct answer for Perimeter | $8 + 5 + 8 + 5 = 26$ | 1 | /2 |
|          | Gives correct answer for Area | $8 \times 5 = 40$ | 1 | |

| 6        | a) Gives correct answer with units | $9 + 4 + 9 + 4 = 26 \text{ yd}$ | 1 | |
|          | b) Gives correct answer with units | $4 \times 9 = 36 \text{ yd}^2$ | 1 | |
|          | Bonus: Gives correct answer | $6 \times 6 = 36 \text{ yd}^2$ length = 6 yd width = 6 yd | yes / no | /2 |

| Total Points | /13 |
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 7: Measurement and Data

(continued)

Test (Lessons 30 to 40), p. X-75

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>12</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>22</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>18</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>40</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>3</td>
<td>a) Skip counts correctly</td>
<td>5, 10, 15</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>15</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Skip counts correctly</td>
<td>4, 8, 12, 16, 20, 24</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>24</td>
<td>0.5</td>
<td>1/2</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct number of columns</td>
<td>6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct number of rows</td>
<td>3</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>6, 3, 18</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer with units</td>
<td>6, 9, 54 ft²</td>
<td>1</td>
<td>1/1</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>9, 5, 45</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>45</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer with units</td>
<td>10 × 8 = 80 ft²</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>9 × 8 = 72 ft²</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td>80 + 72 = 152 ft²</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>Yes, because 160 is more than 152.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Draws a horizontal line to divide the L-shape</td>
<td>6 × 2 = 12</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for Area 1 (top)</td>
<td>2 × 3 = 6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for Area 2 (bottom)</td>
<td>12 + 6 = 18</td>
<td>0.5</td>
<td>1/2</td>
</tr>
<tr>
<td>9</td>
<td>Gives correct answer</td>
<td>(5 × 10) + (5 × 4)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 50 + 20 = 70</td>
<td></td>
<td></td>
<td>1/2</td>
</tr>
<tr>
<td>10</td>
<td>a) Gives correct answer</td>
<td>(2 × 7) + (2 × 4)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 14 + 8 = 22</td>
<td></td>
<td></td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>8, 32</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>a) Gives correct answer with units</td>
<td>36 ÷ 4 = 9 yd</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>(2 × 4) + (2 × 9)</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>= 26 yd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>No, 25 is less than 26.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>26 × 4 = $104</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

Total Points /26
### Rubric for Unit 7: Measurement and Data

**Test (Lessons 30 to 40), p. X-75**

<table>
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<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1: Can answer few, if any, questions accurately and independently.</th>
<th>Level 2: Can answer some questions accurately and independently.</th>
<th>Level 3: Can answer most questions accurately and independently.</th>
<th>Level 4: Can answer all or almost all questions, including bonuses, accurately and independently.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.MD.C.6</td>
<td>1, 2, 3</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.MD.C.7b</td>
<td>4, 5, 6, 7, 11.a)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.MD.C.7c</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.MD.C.7d</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Rubric for Unit 7: Measurement and Data
Test (Lessons 30 to 40), p. X-75

<table>
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<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
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<tbody>
<tr>
<td>3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</td>
<td>10, 11.b), 11.c), Bonus</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
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Comments
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 8: Measurement and Data

#### Quiz (Lessons 41 to 43), p. X-80

<table>
<thead>
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<th>Question</th>
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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Circles correct answer</td>
<td>bottle on right</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>glass on left</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>container on right</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Circles correct answer</td>
<td>glass on left</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>beaker on right</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Circles correct answer</td>
<td>first beaker</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>second container</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Circles correct answer</td>
<td>milk carton</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>soda can</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>juice bottle</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Circles 2 correct answers</td>
<td>bottle, milk carton</td>
<td>2</td>
<td>(1) /2</td>
</tr>
<tr>
<td></td>
<td>Circles 1 correct answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Circles 3 correct answers</td>
<td>mug, spoon, glass</td>
<td>2</td>
<td>(1) /2</td>
</tr>
<tr>
<td></td>
<td>Circles 1 or 2 correct answers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct capacity</td>
<td>5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct volume</td>
<td>4</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct capacity</td>
<td>1,000</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct volume</td>
<td>600</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>$2 \times 5 = 10 \text{ mL}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$20 \div 5 = 4 \text{ teaspoons}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>$50 \div 5 = 10 \text{ teaspoons}$</td>
<td>yes / no</td>
<td>/2</td>
</tr>
</tbody>
</table>

**Total Points** | /18
## Scoring Guides for Sample Unit Quizzes and Tests

### Unit 8: Measurement and Data

(continued)

#### Quiz (Lessons 44 to 46), p. X-83

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>a) Circles correct answer</td>
<td>house</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>book</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>phone</td>
<td>1</td>
<td><em>1/3</em></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>a) Circles correct answer</td>
<td>cherry</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>pencil</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>paper clip</td>
<td>1</td>
<td><em>1/3</em></td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>a) Circles correct answer</td>
<td>cylinder</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>phone</td>
<td>1</td>
<td><em>1/2</em></td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>a) Circles correct answer</td>
<td>cylinder</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>glasses</td>
<td>1</td>
<td><em>1/2</em></td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>a) Circles correct answer</td>
<td>kg</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>g</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>kg</td>
<td>1</td>
<td><em>1/3</em></td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>a) Gives correct answer with units</td>
<td>7 g</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>21 kg</td>
<td>1</td>
<td><em>1/2</em></td>
</tr>
<tr>
<td><strong>7</strong></td>
<td>a) Gives correct answer with units</td>
<td><em>3 × 5 = 15 kg</em></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td><em>4 × 2 = 8 kg</em></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td><em>15 + 8 = 23 kg</em></td>
<td>1</td>
<td><em>1/3</em></td>
</tr>
<tr>
<td><strong>8</strong></td>
<td>a) Gives correct answer with units</td>
<td><em>2 × 5 = 10 g</em></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td><em>35 ÷ 5 = 7 nickels</em></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer with units</td>
<td><em>20 × 5 = 100 g</em></td>
<td>yes / no</td>
<td><em>1/2</em></td>
</tr>
</tbody>
</table>

**Total Points** | **/20**
**Scoring Guides for Sample Unit Quizzes and Tests**  
**Unit 8: Measurement and Data**

(continued)

**Test (Lessons 41 to 46), p. X-86**  
**Common Core State Standards Emphasized:** 3.MD.A.2

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Circles correct answer</td>
<td>bottle on left</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>bottle on right</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>beaker on left</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Circles correct answer</td>
<td>glass on left</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>glass on right</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Circles correct answer</td>
<td>glass</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>vase</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Circles correct answer</td>
<td>200 mL</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>100 L</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>900 L</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct volume</td>
<td>150</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct capacity</td>
<td>200</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct volume</td>
<td>250</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Gives correct capacity</td>
<td>500</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>a) Circles correct answer</td>
<td>shuttlecock</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>bicycle</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>paper clip</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Circles correct answer</td>
<td>phone</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>stapler</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>soccer ball</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>a) Circles correct answer</td>
<td>cylinder</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>glasses</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>a) Circles correct answer</td>
<td>cylinder</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>laptop</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>a) Circles correct answer</td>
<td>kg</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>g</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>kg</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>a) Gives correct answer with units</td>
<td>$3 \times 5 = 15$ L</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>$35 \div 15 = 20$ L</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$20 \div 5 = 4$ more times</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>12</td>
<td>a) Gives correct answer with units</td>
<td>$2 \times 5 = 10$ mL</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$80 \div 10 = 8$ days</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Gives correct answer with units</td>
<td>$63 \div 52 = 11$ kg</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------------------------------</td>
<td>---------------------</td>
<td>------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>14</td>
<td>a) Gives correct answer</td>
<td>$3 \times 5 = 15$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>15</td>
<td>b) Gives correct answer with units</td>
<td>$21 \div 3 = 7$ kg</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>15</td>
<td>a) Gives correct answer with units</td>
<td>$2 \times 5 = 10$ mL</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>15</td>
<td>b) Gives correct answer with units</td>
<td>$26 + 26 = 52$ g</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>Bonus</td>
<td>a) Gives correct answer with units</td>
<td>$2 \times 100 = 200$ g</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$800 \div 100 = 8$ days</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td><strong>/37</strong></td>
<td></td>
</tr>
</tbody>
</table>
### Rubric for Unit 8: Measurement and Data
Test (Lessons 41 to 46), p. X-86

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.MD.A.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, Bonus</td>
</tr>
</tbody>
</table>

| Level 1 | Can answer few, if any, questions accurately and independently. |
| Level 2 | Can answer some questions accurately and independently. |
| Level 3 | Can answer most questions accurately and independently. |
| Level 4 | Can answer all or almost all questions, including bonuses, accurately and independently. |

**Comments**
# Scoring Guides for Sample Unit Quizzes and Tests

## Unit 9: Measurement and Data

### Quiz (Lessons 47 to 49), p. X-91

**Common Core State Standards Emphasized:** 3.MD.B.3, 3.OA.A.3, 3.OA.D.8

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1        | a) Gives correct answer for all 4 months  
              Gives correct answer for 1, 2, or 3 months | 4  
              8  
              11  
              6 | 2  
              (1) |   |
|          | b) Draws correct answer on graph | ✓ - ✓ | 1 |   |
|          | c) Gives correct answer | 5 | 1 |   |
|          | d) Gives correct answer | 5 | 1 |   |
|          | e) Gives correct answer | January | 1 |   |
|          | f) Gives correct answer | March | 1 |   |
|          | g) Gives correct answer | $4 + 8 + 11 + 6 + 3 = 32$ days | 1 |   |
| Bonus    | Gives correct answer | $8 \times 20 = 160$ people | yes / no |   |

**Total Points** /8

### Quiz (Lessons 50 to 53), p. X-93

**Common Core State Standards Emphasized:** 3.MD.B.3, 3.OA.A.3, 3.OA.D.8

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1        | a) Gives correct answer for all 3 fish  
              Gives correct answer for 1 or 2 fish | 4  
              10  
              18 | 2  
              (1) |   |
|          | b) Gives correct answer | $4 + 10 + 18 = 32$ | 1 |   |
|          | c) Gives correct answer  
              Draws bar correctly on graph | $40 - 32 = 8$  
              2 squares | 1  
              1 |   |
|          | d) Gives correct answer | $18 - 4 = 14$ | 1 |   |
|          | e) Gives correct answer | $18 - 10 = 8$ | 1 |   |
| Bonus    | Gives correct answer | $4 \times 20 = 80$ kg | yes / no |   |

**Total Points** /7
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 9: Measurement and Data**

### Test (Lessons 47 to 53), p. X-95

**Common Core State Standards Emphasized:** 3.MD.B.3, 3.OA.A.3, 3.OA.D.8

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer for all 5 shapes</td>
<td></td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 3 or 4 shapes</td>
<td></td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 1 or 2 shapes</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws bar graph correctly for all 5 shapes</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Draws bar graph correctly for 3 or 4 shapes</td>
<td></td>
<td>(2)</td>
<td></td>
</tr>
</tbody>
</table>
|          | Draws bar graph correctly for 1 or 2 shapes | | (1) | |<br />
|          | c) Gives correct answer | | 1 | |
|          | Gives correct explanation | | 1 | |
|          | d) Gives correct answer | 26 – 5 = 21 | 1 | /9 |
| 2        | a) Gives correct answer for all 4 states | | 2 | |
|          | Gives correct answer for 1, 2, or 3 states | | (1) | |
|          | b) Gives correct answer | | 1 | |
|          | Draws picture correctly on graph | | 1 | |
|          | c) Gives correct answer | | 1 | |
|          | Bonus: Gives correct answer | | yes / no | |
|          | d) Draws correct picture graph for all 5 states | | 3 | |
|          | Draws correct picture graph for 3 or 4 states | | (2) | |
|          | Draws correct picture graph for 1 or 2 states | | (1) | |<br />
|          | **Total Points** | | /17 | |
## Rubric for Unit 9: Measurement and Data

### Test (Lessons 47 to 53), p. X-95

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.MD.B.3 \n<em>Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step &quot;how many more&quot; and &quot;how many less&quot; problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</em></td>
<td>1, 2, Bonus</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

### Comments

Name: ____________________
Grade 3 Common Core State Standards Curriculum Correlations

NOTE: The italicized gray JUMP Math lessons contain prerequisite material for the Common Core standards.

D Domain

OA Operations and Algebraic Thinking
NBT Number and Operations in Base Ten
NF Number and Operations—Fractions
MD Measurement and Data
G Geometry

C Cluster

3.OA Operations and Algebraic Thinking

3.OA.A Represent and solve problems involving multiplication and division.

3.OA.A.1 Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$.

3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
### 3.OA.A.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations:

- \(8 \times ? = 48\)
- \(5 = \_ \div 3\)
- \(6 \times 6 = ?\)

### 3.OA
**Operations and Algebraic Thinking**

#### 3.OA.B
Understand properties of multiplication and the relationship between multiplication and division.

**3.OA.B.5**
Apply properties of operations as strategies to multiply and divide. Examples: If \(6 \times 4 = 24\) is known, then \(4 \times 6 = 24\) is also known. (Commutative property of multiplication.) \(3 \times 5 \times 2\) can be found by \(3 \times 5 = 15\), then \(15 \times 2 = 30\), or by \(5 \times 2 = 10\), then \(3 \times 10 = 30\). (Associative property of multiplication.) Knowing that \(8 \times 5 = 40\) and \(8 \times 2 = 16\), one can find \(8 \times 7\) as \(8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56\). (Distributive property.)

**3.OA.B.6**
Understand division as an unknown-factor problem. For example, find \(32 \div 8\) by finding the number that makes 32 when multiplied by 8.

**JUMP Math Grade 3 Lessons**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>OA3-24</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>OA3-28 to 30, 34, 35</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>OA3-63</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>OA3-52</td>
</tr>
</tbody>
</table>

#### 3.OA.C
Multiply and divide within 100.

**3.OA.C.7**
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that \(8 \times 5 = 40\), one knows \(40 \div 5 = 8\)) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

**JUMP Math Grade 3 Lessons**

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>OA3-17 to 19</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>OA3-26 to 28, 30, 32 to 36</td>
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<tr>
<td>1</td>
<td>8</td>
<td>OA3-51, 55</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>OA3-59, 63</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD3-13 to 15</td>
</tr>
</tbody>
</table>
### 3.OA Operations and Algebraic Thinking

#### 3.OA.D Solve problems involving the four operations, and identify and explain patterns in arithmetic.

| 3.OA.D.8 | Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |
| 3.OA.D.9 | Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* |

<table>
<thead>
<tr>
<th>JUMP Math Grade 3 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
</tr>
<tr>
<td>1</td>
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| Part | Unit | Lessons |
| 1    | 1    | OA3-1, 3           |
|      |      | OA3-2, 4 to 7      |
| 1    | 2    | NBT3-8             |
| 1    | 3    | OA3-9, 10          |
| 1    | 4    | OA3-15, 17 to 19, 22, 23 |
| 1    | 5    | OA3-26, 31 to 35   |
| 2    | 5    | MD3-21 to 23       |
### 3.NBT Number and Operations in Base Ten

<table>
<thead>
<tr>
<th>3.NBT.A</th>
<th><strong>Use place value understanding and properties of operations to perform multi-digit arithmetic.</strong></th>
<th>JUMP Math Grade 3 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.NBT.A.1</td>
<td>Use place value understanding to round whole numbers to the nearest 10 or 100.</td>
<td><strong>Part</strong></td>
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<tr>
<td>3.NBT.A.2</td>
<td>Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</td>
<td><strong>Part</strong></td>
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<tr>
<td>3.NBT.A.3</td>
<td>Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.</td>
<td><strong>Part</strong></td>
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<td>2</td>
</tr>
</tbody>
</table>
# 3.NF  Number and Operations—Fractions

<table>
<thead>
<tr>
<th>3.NF.A</th>
<th>Develop understanding of fractions as numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.NF.A.1</strong></td>
<td>Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.</td>
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<td>JUMP Math Grade 3 Lessons</td>
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<td>Part</td>
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<tr>
<td><strong>3.NF.A.2</strong></td>
<td>Understand a fraction as a number on the number line; represent fractions on a number line diagram.</td>
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<td>JUMP Math Grade 3 Lessons</td>
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<td>6</td>
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<tr>
<td><strong>3.NF.A.3</strong></td>
<td>Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</td>
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<td>JUMP Math Grade 3 Lessons</td>
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<td>Part</td>
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<tr>
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<td>6</td>
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<tr>
<td><strong>3.NF.A.3a</strong></td>
<td>Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</td>
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<td>Part</td>
<td>Unit</td>
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<tr>
<td><strong>3.NF.A.3b</strong></td>
<td>Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</td>
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<td>Part</td>
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<tr>
<td><strong>3.NF.A.3c</strong></td>
<td>Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.</td>
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<td>Part</td>
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<tr>
<td><strong>3.NF.A.3d</strong></td>
<td>Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $&gt;$, $=,$ or $&lt;$, and justify the conclusions, e.g., by using a visual fraction model.</td>
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<tr>
<td>Part</td>
<td>Unit</td>
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</tbody>
</table>
### 3.MD  **Measurement and Data**

#### 3.MD.A  **Solve problems involving measurement and estimation.**

<table>
<thead>
<tr>
<th>3.MD.A.1</th>
<th>Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
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<td>5</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>3.MD.A.2</th>
<th>Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</th>
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</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
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<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

#### 3.MD.B  **Represent and interpret data.**

<table>
<thead>
<tr>
<th>3.MD.B.3</th>
<th>Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</th>
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</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
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<td>2</td>
<td>9</td>
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</table>

<table>
<thead>
<tr>
<th>3.MD.B.4</th>
<th>Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</th>
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</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
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</tbody>
</table>

#### 3.MD.C  **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**

<table>
<thead>
<tr>
<th>3.MD.C.5</th>
<th>Recognize area as an attribute of plane figures and understand concepts of area measurement.</th>
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</thead>
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<table>
<thead>
<tr>
<th>3.MD.C.5a</th>
<th>A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
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<tr>
<td>1</td>
<td>6</td>
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<td>7</td>
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<table>
<thead>
<tr>
<th>3.MD.C.5b</th>
<th>A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</th>
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</thead>
<tbody>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
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### 3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>MD3-3</td>
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<tr>
<td></td>
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<td>MD3-6 to 9</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD3-24, 29</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>MD3-30, 31, 40</td>
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</tbody>
</table>

### 3.MD.C.7 Relate area to the operations of multiplication and addition.

#### 3.MD.C.7a Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

<table>
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<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>MD3-9</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>MD3-32, 33, 40</td>
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</tbody>
</table>

#### 3.MD.C.7b Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>MD3-34, 36 to 40</td>
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</tbody>
</table>

#### 3.MD.C.7c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

<table>
<thead>
<tr>
<th>Part</th>
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<th>Lessons</th>
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<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>MD3-36, 40</td>
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#### 3.MD.C.7d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

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<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>MD3-7</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>MD3-35, 40</td>
</tr>
</tbody>
</table>

### 3.MD Measurement and Data

#### 3.MD.D Geometric measurement: recognize perimeter.

#### 3.MD.D.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>MD3-4, 5</td>
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<tr>
<td>2</td>
<td>7</td>
<td>MD3-37 to 39</td>
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</tbody>
</table>
### 3.G. Geometry

<table>
<thead>
<tr>
<th>3.G.A</th>
<th>Reason with shapes and their attributes.</th>
<th>JUMP Math Grade 3 Lessons</th>
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</thead>
<tbody>
<tr>
<td>3.G.A.1</td>
<td>Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</td>
<td>Part</td>
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<tr>
<td>3.G.A.2</td>
<td>Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</td>
<td>Part</td>
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