Grade 4

End of Year Teacher Pack

Includes:

- Lesson Plans
- Blackline Masters
- Assessment & Practice Book Answers
- Quizzes and Unit Tests
- Rubrics for Scoring Quizzes and Tests
- Common Core State Standards Correlations
Unit 6  Operations and Algebraic Thinking: Factors

This Unit in Context

In Grade 3, students fluently multiplied within 100 (3.OA.C.7) and they learned to think of division as a missing factor problem (3.OA.B.6). In this unit, students will find the factors of a number first by using rectangles, then by using organized lists, and finally by using long division with remainders (as learned in 4.2 Unit 2)—identifying that a remainder of 0 indicates that the number is a factor. Students will use organized search to find all the factor pairs for numbers within 100 and identify prime and composite numbers within 100, recognizing that 1 is neither prime nor composite. Students will decide whether statements about multiples and factors are true or false, and use counterexamples to show that a statement is false.

Prime numbers play a foundational role in much of mathematics, because every number greater than 1 can be written as a product of prime factors. Students will learn in Grade 6 to find the greatest common factor of two numbers within 100 and the lowest common multiple of two numbers within 12 (6.NS.B.4). Students will also recognize factors in numerical expressions—for example, in the expression 4(3 + 5), both 4 and 3 + 5 are factors (6.EE.A.2b). In Grade 7, students will recognize factors in linear algebraic expressions (7.EE.A.1), and in high school, students will recognize factors in quadratic expressions (HSA.SSE.B.3a) and other polynomials (HSA.APR.B.2).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP.1 to MP.8. Here are some examples of how students can show that they have met a standard.

MP.1: In OA4-43 Extension 6, students make sense of a non-routine problem when they analyze the conditions a number satisfies to find the number. They persevere to solve the problem by making a plan to decide which of the conditions will be easier to start with, before rushing into the solution.

MP.3: In OA4-44 Extension 4, students construct an argument when they explain why any number with an even factor is itself even.

MP.5: In OA4-43 Extension 6, students choose tools strategically to solve a riddle that requires finding a number less than 88 with a given property, when they recognize that a pre-made list of the numbers from 1–100, such as on a hundreds chart, will help them keep track of which numbers they have eliminated.

MP.8: In OA4-42 Extension 5, students notice their repeated reasoning when they explain why a factor of various numbers that is more than half of the number must be the number itself. They use their repeated reasoning to make a general statement.
A context for organizing dimes and nickels. Tell students that you need to program a machine that will sell candy bars for 45¢. Your machine accepts only dimes and nickels. It does not give change. But you have to teach your machine to recognize when it’s been given the correct change. The simplest way to do this is to give your machine a list of which coin combinations to accept and which to reject. So you need to list all combinations of dimes and nickels that add up to 45¢.

Making sure no possibilities are missed. SAY: There are two types of coins, dimes and nickels. The best way to find the possible combinations is to list one of the coin types (say, dimes) in increasing order. You start with no dimes, then one dime, etc. ASK: Where do you stop? PROMPT: How many dimes will be too much? (5 dimes is too much) Demonstrate using the table below.

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

SAY: If there are no dimes, I need 9 nickels because 9 nickels is 45¢. Tell students you want to know how many nickels you need if you have 1 dime. Demonstrate counting on from 10 by fives until you reach 45. You will have 7 fingers up, so 7 nickels are required. Have volunteers demonstrate counting on for the remaining rows.

Exercises: Complete the table to program a candy bar machine.

a) Make 35¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

b) Make 55¢
Decide how many rows to put in the table and complete it.

c) Make 65¢

d) Make 85¢

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bonus
Make 145¢ using dimes and nickels. Put dimes first.

Selected Answers

a) | Dimes | Nickels |
---|-------|---------|
| 0  | 7     |
| 1  | 5     |
| 2  | 3     |
| 3  | 1     |

b) | Dimes | Nickels |
---|-------|---------|
| 0  | 11    |
| 1  | 9     |
| 2  | 7     |
| 3  | 5     |
| 4  | 3     |
| 5  | 1     |

**Reasons to start with the larger denomination.** Remind students that when finding all combinations of nickels and dimes that make 45¢, you started with the dimes and listed all possibilities in order. Point out that you can also start by listing the number of nickels in increasing order.

Begin drawing a table with the “Nickels” heading first, then ASK: What is the biggest number of nickels I need to put in my table? (9) How do you know? (because 10 nickels would be too much) Demonstrate making the table with 10 rows for 0 to 9 nickels. SAY: It looks like there are more combinations now, but some of them won’t work. ASK: If there are no nickels, can I make 45¢ with just dimes? (no) Why not? (you can make only multiples of 10 cents with dimes)

SAY: We could start with nickels and get all the same answers; it’s just more work. It takes fewer dimes than nickels to make 45 cents because dimes are worth more than nickels.

**Exercises:** Start with the larger denomination. Make a list …

a) of dimes and nickels to make 75¢
b) of nickels and pennies to make 34¢
c) of quarters and nickels to make 85¢

**Selected Answers**

b) | Nickels | Pennies |
---|---------|---------|
| 0  | 34      |
| 1  | 29      |
| 2  | 24      |
| 3  | 19      |
| 4  | 14      |
| 5  | 9       |
| 6  | 4       |

c) | Quarters | Nickels |
---|---------|---------|
| 0  | 17      |
| 1  | 12      |
| 2  | 7       |
| 3  | 2       |
Calculating values when the number of coins is constant instead of the value. Tell students that you have 4 coins in your pocket, all either dimes or nickels. Tell students that you want to know the possible total values. Draw the following table on the board:

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
<th>Total value (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ASK: If I have no dimes, how many nickels must I have? (4) PROMPT: How many coins do I have in total? (4) Repeat for 1 dime (3 nickels), 2 dimes (2 nickels), 3 dimes (1 nickel) and 4 dimes (no nickels). Fill in the nickels column as you go. SAY: Now that you know how many of each coin there are, you can figure out the value of the coins. ASK: How much are 0 dimes and 4 nickels worth? (20 cents) Repeat for 1 dime and 3 nickels (25 cents), and so on. When the table is complete, ASK: I have four coins, all dimes or nickels, worth 35 cents—which coins are they? (3 dimes and 1 nickel)

Exercises: Make a table to answer the questions.

a) I have 5 coins, all quarters or dimes, worth 95¢. Which coins are they?

b) I have 4 coins, all quarters or nickels, worth 40¢. Which coins are they?

Answers: a) 3 quarters and 2 dimes, b) 1 quarter and 3 nickels

Tell students that they can answer the same type of question in different contexts. Tell students that birds have two legs and cats have four legs. SAY: There are 4 animals and 10 legs in total, and I want to know how many of each type of animal there are. Demonstrate how to begin the table, then have students complete it individually (answers are in italics):

<table>
<thead>
<tr>
<th>Birds</th>
<th>Cats</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

Students can then answer how many of each type of animal there are (3 birds and 1 cat).

Extensions

1. Dragons come in two varieties: the Three-Headed Fearsome Forest Dragon and the Nine-Headed Horrible Hill Dragon. A mighty and courageous knight is fighting these dragons, and he has slain 5 of them. There are 27 heads in the pile after the battle. How many of each type of dragon did he slay?
Answer: Make a table, starting with the possibilities for nine-headed dragons (0, 1, 2, or 3). No more than three nine-headed dragons are possible because more than 3 would have more than 27 heads. Then complete the possibilities for the number of 3-headed dragons (based on the total being 5). Then calculate the total number of heads based on the number of each kind of dragon:

<table>
<thead>
<tr>
<th>Nine-Headed Horrible Hill Dragons</th>
<th>Three-Headed Fearsome Forest Dragons</th>
<th>Total Number of Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>33</td>
</tr>
</tbody>
</table>

From the table, there were 2 nine-headed dragons and 3 three-headed dragons slain.

2. Find all the whole-number widths and lengths of a rectangle with perimeter 12 units.

You may need to remind students about the definitions of width, length, and perimeter for rectangles. The width is the length of the shortest side, the length is the length of the longest side, and the perimeter is the distance around the rectangle.

Repeat for perimeter 14 and 16. Students might wish to investigate the question: Which rectangle with a given perimeter has the largest area? (either a square or the one closest to a square)

Answers

<table>
<thead>
<tr>
<th>Perimeter 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Perimeter 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Perimeter 16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>
Making rectangles from connecting cubes and recording the results. Demonstrate making some rectangles from connecting cubes:

Then give students 20 connecting cubes and have them make as many different rectangles as they can, using some or all of the cubes. Students should record the results by drawing copies on grid paper.

Making rectangles from a given number of cubes. Tell students to use only four cubes. ASK: How many rectangles can you make using exactly four cubes? Have volunteers show the different rectangles. Repeat for five cubes and six cubes.

Using the number of squares across to record the rectangle. Tell students that instead of drawing the rectangles to record them, they can save time by just writing how many cubes across their rectangles are.
Demonstrate this (bold the top side of the rectangles to emphasize where the numbers come from):

- Four cubes: 1, 2, and 4
- Five cubes: 1 and 5
- Six cubes: 1, 2, 3, and 6

Now ask students to instead write how many cubes down their rectangles are. **ASK:** What do you notice? (I get the same numbers, but in reverse order) **SAY:** It doesn’t matter whether you record the number of cubes across or down—you just have to pick one and use it for all the rectangles you make.

**Introduce the word “factor.”** Tell students that the numbers of squares across in the rectangles are called the factors of the number, so the factors of 4 are 1, 2, and 4. **ASK:** What are the factors of 5? (1 and 5) What are the factors of 6? (1, 2, 3, and 6)

**Exercises:** Make rectangles to find as many factors of 8 as you can.

**Bonus:** Make rectangles to find as many factors of 10 as you can.

**Answers:** 1, 2, 4, 8; Bonus: 1, 2, 5, 10

Tell students that you want to list all the factors of 12, so to make sure that you don’t miss any factors, you’re going to try the numbers in order. **SAY:** Let’s try 1 first. Demonstrate making a single column of 12 cubes. **SAY:** That’s 1 in each row. Now let’s try 2 in each row. Demonstrate putting two in each row at a time until you use all 12 cubes. **ASK:** Is 2 a factor of 12? (yes) Have volunteers try 3 (yes), 4 (yes), and 5 (no).

**Exercises:** Check whether 6, 7, 8, 9, 10, 11, and 12 are factors of 12.

**Answers:** yes, no, no, no, no, no, yes

**SAY:** So far, the factors of 12 we found are 1, 2, 3, 4, 6, and 12.

**ASK:** Can 13 be a factor of 12? (no) **ASK:** Why not? **PROMPT:** Why can’t you use 13 cubes in the first row? (because we only have 12 cubes) Repeat for 14, 15, and 1,742. Point out that no number bigger than 12 can be a factor of 12, so we found all the factors of 12.

**Exercises:** Make rectangles to find the factors of 9. Draw the rectangles that work on grid paper.

**Answers:** 1, 3, 9
Extensions

1. It is recommended to drink about 2 L of water each day. About how many 250 mL glasses of water should you drink in a year? Explain how you figured it out.

**Answer:** It is recommended to drink about 8 glasses a day. I did this by writing $2 \text{ L} = 2,000 \text{ mL}$, so I had to find out how many 250 mL glasses are in 2,000 mL. I skip counted by 250 until I got to 2,000, and the answer was 8. The question isn’t just asking about how many glasses each day, but how many each year, so I multiplied $365 \times 8 = 2,920$. So, it is recommended to drink about 2,920 glasses of water in a year.

Redirecting students: If some students do not remember how many days are in a year, encourage them to ask other students who might know, or to ask you.

2. Have students put the numbers from 1 to 10 in the Venn diagram.

![Venn diagram](image)

**Answer:** Factors of 8: 1, 2, 4, 8; Factors of 10: 1, 2, 5, 10; Factors of both: 1, 2

3. Check whether 4 is a factor of 92 by using base ten blocks (yes). Start with 9 tens blocks and 2 ones blocks. Start making a rectangle with 4 squares across by using 8 of the 9 tens blocks, then trade a ten for 10 ones. Point out that this method is much faster than using 92 ones blocks, even if we had to trade a tens block for ones blocks at the end.

Students can use this method to check whether 3 is a factor of 72 (yes) or 41 (no).

4. Find the numbers between 1 and 20 where you can make a square instead of a rectangle with connecting cubes or grid paper. (1, 4, 9, 16) Tell students that these are called square numbers. When students finish, challenge them to find an organized way of making the squares so that it is easy to find the next square number. For example, students could make squares one layer at a time (the next square number is 25):
Drawing rectangles to check for factors. Tell students that you want to know if 2 is a factor of 8. ASK: What does it mean for 2 to be a factor of 8? (There has to be a rectangle with 8 squares that has 2 squares across.) SAY: You can keep adding rows of 2 to check if 2 is a factor. Demonstrate on the board how to check if 2 is a factor of 8:

2 4 6 8

ASK: Is 2 a factor of 7? (no) Point out that you already passed 7 and didn’t make a rectangle, so 2 is not a factor of 7. Ask a volunteer to draw the next rectangle, with 5 rows of 2 instead of 4 rows of 2. ASK: What is the next number that has 2 as a factor? (10)

Exercises: Draw rectangles with 3 squares in each row. Is 3 a factor of …

a) 10?  b) 11?  c) 12?

Answers: a) no, b) no, c) yes

Drawing one rectangle to check for factors. Draw a rectangle with 4 rows of 3 on the board. Tell students that you want a shortcut way to check which numbers 3 is a factor of. Then point out that instead of drawing a rectangle with only one row, they can just look at the first row. ASK: How many cubes are in the first row? (3) Repeat for the first two rows (now there are 6 cubes), the first three rows (9 cubes), and all four rows (12 cubes).
Summarize by writing the numbers beside the rows:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SAY: 3 is a factor of 3, 3 is a factor of 6, 3 is a factor of 9, and 3 is a factor of 12.

**Exercise:** Draw one rectangle to check if 4 is a factor of 13, 16, 17, 18, and 20.

**Answer:** 4 is a factor of 16 and 20, but not of 13, 17, or 18.

**Skip counting to check for factors.** Point out that what students really did was skip count by 4s to check which numbers 4 is a factor of. SAY: You actually don’t need the rectangle at all. You can just skip count by 4s and check if the number is in the list. Demonstrate checking if 5 is a factor of 26: Skip count: 5, 10, 15, 20, 25, 30. SAY: We passed 26, so 5 is not a factor of 26.

**Exercises:** Skip count to check for factors.

a) Is 5 a factor of 32?  
b) Is 2 a factor of 16?  
c) Is 3 a factor of 25?  
d) Is 4 a factor of 28?

**Answers:** a) no, b) yes, c) no, d) yes

**Using multiplication to check for factors.** Remind students that if they can skip count by 7s to get 28, then they are showing a multiplication. Demonstrate skip counting by 7s to get 28 using your fingers to track how many times you skip counted. Point out that you had 4 fingers up. ASK: What multiplication does this show? (4 \times 7 = 28) SAY: The fact that a whole number times 7 is 28 says that 7 is a factor of 28. Tell students you want to find all the factors of 12 and that you can find them by checking each number in turn. ASK: Does some whole number times 1 equal 12? (yes, 12) Write on the board:

\[ 12 \times 1 = 12 \]

SAY: So 1 is a factor of 12. ASK: Does some whole number times 2 equal 12? (yes, 6) Show this on the board: \( 6 \times 2 = 12 \). Continue in this way. When you get to 5, point out that 5 is not a factor of 12 because no whole number times 5 equals 12. Continue until all numbers up to 12 have been checked. ASK: Can 13 be a factor of 12? (no) How do you know? (because 13 is greater than 12; no whole number times 13 can equal 12)

**Exercise:** Find all the factors of 9.

**Answer:** 1, 3, and 9
Using division to check for factors. Write on the board:

\[ \begin{align*}
15 \div 3 & \quad 16 \div 3 & \quad 17 \div 3 & \quad 18 \div 3 \\
\end{align*} \]

Ask students to answer the four division statements. (5 R 0, 5 R 1, 5 R 2, 6 R 0) Then write on the board:

\[ \begin{align*}
3 \times \_ \_ \_ = 15 \quad & \quad 3 \times \_ \_ \_ = 18 \\
\end{align*} \]

Have volunteers fill in the blanks. ASK: Is 3 a factor of 15? (yes) A factor of 16? (no) How do you know? PROMPT: Is 3 times some whole number equal to 16? (no) Emphasize that 16 $\div 3 = 5 \text{ R } 1$ tells us that 16 is between $5 \times 3$ and $6 \times 3$, so there is no whole number times 3 that equals 16. Repeat for 17 (no) and 18 (yes). ASK: How can you tell from the remainder whether 3 is a factor? (if the remainder is 0, then 3 is a factor; if the remainder is not 0, then 3 is not a factor)

**Exercises:** Write a division equation. Is 3 a factor?

a) 19  

b) 20  

c) 21  

**Bonus:** 602

**Answers:**
a) $19 \div 3 = 6 \text{ R } 1$; no, b) $20 \div 3 = 6 \text{ R } 2$; no, c) $21 \div 3 = 7 \text{ R } 0$, yes; **Bonus:** $602 \div 3 = 200 \text{ R } 2$, no

**Exercises:** Is 4 a factor?

a) 18  

b) 24  

c) 16  

**Bonus:** no

**Using long division to check for factors.** Review long division. Have volunteers guide you through the steps of long division to divide 364 $\div 7$.

ASK: Is 7 a factor 364? (yes) How can you tell? (the remainder is 0)

Students should use grid paper to do the exercises below.

**Exercises:** Is 3 a factor? Use long division to check.

a) 85  

b) 72  

c) 81  

d) 77  

**Bonus**
e) 329  

f) 417  

g) 5,316  

h) 7,136

**Answers:**
a) no, b) yes, c) yes, d) no, **Bonus:** e) no, f) yes, g) yes, h) no

**Choosing between skip counting and long division.** Tell students that you want to check whether 3 is a factor of 145. ASK: Is it a better strategy to skip count or to use long division? Take guesses, then demonstrate trying to skip count. Point out that it is easy to get lost and forget where you are. Then ask a volunteer to demonstrate how much faster it is with long division. Now tell students that you want to check whether 3 is a factor of 14. ASK: Would you use long division or skip counting? (skip counting) Why? If students say because the numbers are so small, challenge them to be more precise. ASK: Would you use long division or skip counting to check whether 3,000 is a factor of 14,000? Why? PROMPT: What number is small, the divisor, 3,000; the dividend, 14,000; or the quotient? (the quotient) SAY: I can count on one hand how many times 3,000 goes into 14,000, so this is easy to check by skip counting.
Exercises: Is 6 a factor?

a) 32  b) 24  c) 78  d) 86

Bonus: 1,926

Answers: a) no, b) yes, c) yes, d) no, Bonus: yes

Extensions

1. Is 3 a factor of 64,752? Use long division to check.

   Answer: yes

2. Find a digit so that 3 is a factor of 2,86   .

   Answer: 2, 5, or 8

3. Ron knows that 3 is a factor of 5,871. What is the next number that 3 is a factor of? Check your prediction using long division.

   Answer: 5,874

4. a) Randi has four textbooks. The table below shows the weight of each book. Randi’s friend broke his arm and needs help carrying his books, so Randi carries both her and her friend’s textbooks on Friday after school. If they both have math, science, and social studies homework for the weekend, how much will all the books weigh that Randi has to carry?

   
<table>
<thead>
<tr>
<th>Textbooks</th>
<th>Subject</th>
<th>Weight (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math</td>
<td>$\frac{9}{8}$</td>
</tr>
<tr>
<td></td>
<td>Science</td>
<td>$\frac{6}{8}$</td>
</tr>
<tr>
<td></td>
<td>Language Arts</td>
<td>$\frac{11}{8}$</td>
</tr>
<tr>
<td></td>
<td>Social Studies</td>
<td>$\frac{5}{8}$</td>
</tr>
</tbody>
</table>

   b) Think of something that weighs about the same as the books that Randi has to carry. Talk with a partner: Do you think Randi will have a hard time carrying all the books?

   Answer: a) Randi’s books weigh $\frac{9}{8} + \frac{6}{8} + \frac{5}{8} = \frac{20}{8}$ pounds. Her friend’s books weigh the same amount, so in total they weigh $\frac{20}{8} + \frac{20}{8} = \frac{40}{8}$ pounds. Since $\frac{40}{8} = 5$, the books weigh 5 pounds.

   Sample answer: b) The books weigh about the same as 5 JUMP Math AP Books; it would be hard to carry for a long distance but a short distance should be okay.
OA4-42  Factor Pairs
Pages 100–101

STANDARDS
4.OA.B.4, 4.NBT.B.6

VOCABULARY
factor
factor pair

Goals
Students will find all factor pairs for a whole number in the range 1–100.

PRIOR KNOWLEDGE REQUIRED
Can determine when a given 1-digit number is a factor of another given number

MATERIALS
BLM When 2 or 5 Is a Factor (p. Q-23)
6 connecting cubes

Introduce factor pairs. Make a rectangle from 6 cubes using 3 rows of 2, but don’t show the students the rectangle. Tell students that you made a rectangle using 6 cubes with 2 cubes across. Have students guess how many cubes down the rectangle is. (3) Then show them the rectangle you made. Tell students that 2 and 3 are called a factor pair of 6 because $2 \times 3 = 6$. Make another rectangle with 20 cubes and 5 cubes across. Have students guess how many cubes down the rectangle is, then show them the rectangle. (4) ASK: What is a factor pair for 20? (5 and 4)

Looking for factor pairs in an organized way. Tell students that you want to find all the factor pairs for 15, so you will list the possible first numbers in order to make sure you don’t miss any. Show the table in the margin but without the answers in italics. ASK: Why did I stop at 15? Why can’t 16 be a factor of 15? (because 16 is more than 15) Why can’t numbers more than 15 be factors of 15? (You can’t have a rectangle with 15 cubes that uses 16 cubes in the first row.)

Go through the first numbers, one at a time. Students can skip count to check for factors.

Exercises: Make a table to find all the factors of the number.

<table>
<thead>
<tr>
<th>First Number</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
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<tr>
<td>6</td>
<td>x</td>
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<tr>
<td>7</td>
<td>x</td>
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<td>9</td>
<td>x</td>
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<td>x</td>
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<tr>
<td>11</td>
<td>x</td>
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<tr>
<td>12</td>
<td>x</td>
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<tr>
<td>13</td>
<td>x</td>
</tr>
<tr>
<td>14</td>
<td>x</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
</tr>
</tbody>
</table>

Answers: a) 1, 2, 3, 4, 6, and 12; b) 1, 2, 7, and 14

Writing in the table only the numbers that are factors. Tell students that instead of listing the numbers that don’t work as factors in the table, they can save time by just writing the numbers that are factors. Show some of the tables done so far using this new shortcut notation:
First Factor | Second Factor
------------|-------------
1           | 15          
3           | 5           
5           | 3           
15          | 1           

First Factor | Second Factor
------------|-------------
1           | 12          
2           | 6           
3           | 4           
4           | 3           
6           | 2           
12          | 1           

First Factor | Second Factor
------------|-------------
1           | 14          
3           | 7           
7           | 2           
14          | 1           

Exercises: List all the factor pairs of the number.

a) 16   b) 21   Bonus: 24

Answers: a) 1 and 16, 2 and 8, 4 and 4, 8 and 2, 16 and 1, b) 1 and 21, 3 and 7, 7 and 3, 21 and 1, Bonus: 1 and 24, 2 and 12, 3 and 8, 4 and 6, 6 and 4, 8 and 3, 12 and 2, 24 and 1

Ask students if they notice any patterns in the tables they made or the ones on the board. To guide students if necessary, you can draw a horizontal line at the point where the numbers start repeating (see the example in the margin). Allow students to articulate any patterns they notice, then point out that the bottom half has the same numbers as the top half; the pairs are the same, just in reverse order. SAY: As soon as we find a number that is already part of a pair, we can stop because the pairs will start repeating.

Remind students that they may need to use long division to look for factors. When the remainder is 0, the number is a factor.

Exercises: Make a table to find all the factor pairs. Stop when you get to a number that is already part of a pair.

a) 17   b) 24   c) 49   d) 64   e) 75

Answers:

a) 1, 17
    17, stop

d) 1, 64
    2, 32
    4, 16
    8, 8, stop

e) 1, 75
    3, 25
    5, 15
    15, stop
NOTE: Students who have trouble identifying when 2 or 5 is a factor of a number will benefit from completing BLM When 2 or 5 Is a Factor. A number with ones digit 0, 2, 4, 6, or 8 has 2 as a factor, while a number with ones digit 0 or 5 has 5 as a factor.

Extensions

1. A number is called perfect if the sum of its factors is double the number. Show that 6 and 28 are perfect numbers.

   Answers: The factors of 6 are 1, 2, 3, 6. They add to \(1 + 2 + 3 + 6 = 12\), which is double 6. The factors of 28 are 1, 2, 4, 7, 14, 28. They add to \(1 + 2 + 4 + 7 + 14 + 28 = 56\), which is double 28.

2. Investigate: When does double a number have twice as many factors as the original number?

   Students can investigate this question by making a table and circling the numbers where double the number has double the number of factors:

<table>
<thead>
<tr>
<th>Number</th>
<th>Factors</th>
<th>(2 \times ) Number</th>
<th>New Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1, 2</td>
<td>4</td>
<td>1, 2, 4</td>
</tr>
<tr>
<td>3</td>
<td>1, 3</td>
<td>6</td>
<td>1, 2, 3, 6</td>
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<td>4</td>
<td>1, 2, 4</td>
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<td>1, 2, 4, 8</td>
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<td>5</td>
<td>1, 5</td>
<td>10</td>
<td>1, 2, 5, 10</td>
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<tr>
<td>6</td>
<td>1, 2, 3, 6</td>
<td>12</td>
<td>1, 2, 3, 4, 6, 12</td>
</tr>
<tr>
<td>7</td>
<td>1, 7</td>
<td>14</td>
<td>1, 2, 7, 14</td>
</tr>
<tr>
<td>8</td>
<td>1, 2, 4, 8</td>
<td>16</td>
<td>1, 2, 4, 8, 16</td>
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<tr>
<td>9</td>
<td>1, 3, 9</td>
<td>18</td>
<td>1, 2, 3, 6, 9, 18</td>
</tr>
<tr>
<td>10</td>
<td>1, 2, 5, 10</td>
<td>20</td>
<td>1, 2, 4, 5, 10, 20</td>
</tr>
</tbody>
</table>

Answer

When the number is odd!
3. To make a factor chain, start with any number from 2 to 20. Add up all the factors of the number, not including the number itself, to get the next number in the chain. Stop when you reach 1 or when you repeat the number you started with. Examples:

1) Start with 10.

\[10 \rightarrow 8 \rightarrow 7 \rightarrow 1, \text{ stop}\]

\[(1 + 2 + 5 = 8) \quad (1 + 2 + 4 = 7) \quad (1 = 1)\]

factors of 10 \quad \text{factors of 8} \quad \text{factors of 7}

other than 10 \quad \text{other than 8} \quad \text{other than 7}

2) Start with 6. The factors are 1, 2, 3, and 6, so add: \(1 + 2 + 3 = 6\).

\[6 \rightarrow 6, \text{ stop}\]

3) Start with 17. The factors are 1 and 17:

\[17 \rightarrow 1, \text{ stop}\]

What starting number gives you the longest chain?

**Answer:** 12 or 20

4. Find the number. Justify your answer.

a) I am a factor of 10. I am more than half of 10.

b) I am a factor of 15. I am more than half of 15.

c) I am a factor of 36. I am more than half of 36.

d) I am a factor of 100. I am more than half of 100.

**Answers:** a) 10, b) 15, c) 36, d) 100, Sample justification: d) the factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, 100, and the only factor more than half of 100 is 100

**NOTE:** Students should complete Extension 4 before doing Extension 5.

(MP3, MP6, MP8)

5. a) Without listing the factors, explain why a factor of 25 that is more than half of 25 must actually be 25. Use the meaning of the word “factor” in your explanation.

b) Would your reasoning from part a) also work for 48? 76? 576?

c) Use your answers to parts a) and b) to make a general statement.

**Answers:** a) If a number is a factor of 25, then something times it is 25. But if the number is more than half of 25, then 2 times it is already more than 25. So the “something” must be 1. So, 1 times the number is 25, so the number is 25; b) Yes, it would work for any number; c) A factor of a number that is more than half of the number must actually be the number.
OA4-43  Prime Numbers and Composite Numbers

Pages 102–103

STANDARDS
4.OA.B.4, 4.NBT.B.6

VOCABULARY
composite
factor
factor pair
multiple
prime
whole number

Goals
Students will determine whether a given whole number in the range 1–100 is prime or composite, and recognize that any whole number is a multiple of each of its factors.

PRIOR KNOWLEDGE REQUIRED
Can determine whether a given 1-digit number is a factor of a given number

A number always has 1 and itself as factors. Remind students that they can use multiplication to check for factors. For example, to check if 2 is a factor of 13, ASK: Is some whole number times 2 equal to 13? ASK: What numbers is 1 a factor of? (all of them; every whole number) PROMPT: A number is always 1 times itself. ASK: What numbers are factors of themselves? (all of them; every whole number)

Smaller numbers can have more factors. Tell students that some numbers have many factors and other numbers have only a few. Remind students that they can check for factors using division. If the remainder is 0, the number is a factor. For example, 5 is a factor of 20 because $20 \div 5 = 4$ R 0, but 5 is not a factor of 21 because $21 \div 5 = 4$ R 1.

Exercises: Which number has more factors?

Bonus
a) 9 or 10  b) 12 or 13  c) 24 or 25  d) 35 or 36  e) 45 or 46

Answers: a) 10, b) 12, Bonus: c) 24, d) 36, e) 45

SAY: Sometimes the greater number has more factors and sometimes the lesser number has more factors.

Introduce prime and composite numbers. Tell students that mathematicians have divided whole numbers into two main categories based on the number of factors. Write on the board:

<table>
<thead>
<tr>
<th>Prime</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 3, 5, 7, 11, 13, 17, 19</td>
<td>4, 6, 8, 9, 10, 12, 14, 15, 16, 18</td>
</tr>
</tbody>
</table>

Exercises: Find the factors of the numbers from 2 to 19 to see if you can determine the rule: What makes a number prime and what makes a number composite?

Bonus: Predict where to put 20, 21, 22, and 23.

Summarize by pointing out that prime numbers have exactly two factors and composite numbers have more than two factors. (20, 21, and 22 are composite; 23 is prime) If there are any factors between 1 and the number, the number is composite; if not, the number is prime.
Exercises: Is the number prime or composite?

a) 20  b) 21  c) 22  d) 23  e) 24  
f) 25  g) 26  h) 27  i) 28  j) 29

Answers: a) composite, b) composite, c) composite, d) prime, 
e) composite, f) composite, g) composite, h) composite, i) composite,  
j) prime

ASK: How many factors does 1 have? (1) Tell students that 1 is neither prime nor composite.

Multiples and factors. Tell students that a whole number is a multiple of each of its factors. SAY: From knowing \(2 \times 3 = 6\), you can say that 2 is a factor of 6 or that 6 is a multiple of 2.

Exercises: Write "multiple" or "factor."

a) 4 is a ________ of 2.  b) 4 is a ________ of 8. 
   c) 3 is a ________ of 1.  d) 1 is a ________ of 3. 
   e) 40 is a ________ of 8.  f) 8 is a ________ of 40.

Answers: a) multiple, b) factor, c) multiple, d) factor, e) multiple, f) factor

NOTE: Some students might confuse the words "multiple" and "factor." You can point out that a number has many more multiples and far fewer factors as a way to remember which word means what.

Checking for multiples. SAY: You can check for multiples the same way you check for factors. ASK: How would you check if 3 is a factor of 17? (check if 3 times something is 17) Would you use skip counting, the 3 times table, or long division? (skip counting or the 3 times table) Tell students that you could do the same thing to check if 17 is a multiple of 3, because they say the same thing. Finding whether 3 is a factor of 17 is the same as finding whether 17 is a multiple of 3. Demonstrate checking by skip counting: 3, 6, 9, 12, 15, 18, so 17 is not a multiple of 3.

Exercises: Skip count to check for multiples.

a) Is 14 a multiple of 2?  b) Is 14 a multiple of 3? 
   c) Is 18 a multiple of 5?  d) Is 18 a multiple of 3?

Answers: a) yes, b) no, c) no, d) yes

Tell students that when the lesser number goes into the greater number many times, it is faster to use long division than skip counting.

Exercises: Use long division to check for multiples.

a) Is 96 a multiple of 4?  b) Is 76 a multiple of 3?

Answers: a) yes, b) no
All even numbers greater than 2 are composite. Remind students that even numbers are multiples of 2. SAY: That means that 2 is a factor of every even number.

Exercises: What factor pairs with 2 to make the number?

a) 12  b) 14  c) 30  d) 48  Bonus: 86,424

Answers: a) 6, b) 7, c) 15, d) 24, Bonus: 43,212

ASk: Is 12 prime or composite? (composite) How do you know? (2 is a factor between 1 and 12) Repeat for 30, 48, and 86,424. (they are all composite because 2 is a factor of each number and each number is greater than 2) ASK: Are there any even numbers that are prime? (yes, 2 is even and prime) How do you know? (it has only 2 factors, 1 and 2)

Refining the search for prime numbers. Tell students that you are thinking of a number that has a factor pair where both numbers are 10 or more.

ASk: What is the smallest number that I could be thinking of? (10 × 10 = 100) To guide students to see this, write on the board:

______ × ______ = ______

Point to the first two blanks and SAY: These two numbers form a factor pair for that one (the third blank). Both of these numbers are 10 or more, so 10 × 10 is the smallest number that can go here (the third blank).

Tell students that every number less than 100, unless it is prime, has to have a factor between 2 and 9. So to check if a number less than 100 is prime, they just have to check if the numbers from 2 to 9 are factors. If none of them are, then the number is prime. As soon as you find a factor other than 1 and itself, you can stop checking because then the number is composite, not prime.

Exercises: Is the number prime or composite? Check for factors 2, 3, 4, 5, 6, 7, 8, and 9.

a) 81  b) 27  c) 29  d) 45  e) 32  f) 31

Answers: a) composite, b) composite, c) prime, d) composite, e) composite, f) prime

Extensions

1. Find the smallest number you can that has 8 factors. (24)

2. Display the numbers from 1 to 12 in a Venn diagram with categories “factors of 3” and “factors of 12.” Which region is empty? Why? (the region with factors of 3 only; there are no factors of 3 that are not factors of 12)

3. Teach students a simplified version of Eratosthenes’ Sieve, which was discovered over 2,000 years ago as a way to list all the prime numbers up to 100. Use BLM Eratosthenes’ Sieve (p. Q-24).
4. **BLM Pascal’s Triangle** (p. Q-25) shows the first 16 rows of Pascal’s triangle. Challenge students to determine the rule with which it was made (1s on the outside, and every other number is the sum of the two above it). Then have students color all the even numbers to see a pattern. Interested students can repeat for multiples of 3, 4, 5, 6, and 7. Challenge very advanced students to determine why the shading patterns always form upside down triangles. Is the sum of two even numbers always even? Is the sum of two multiples of 3 always a multiple of 3?

5. Can a multiple of 4 be odd? Explain. (no)

6. Solve the following riddle. Use any tool you think will help.

   I have 8 factors, but 8 is not a factor. I am less than 88, and 8 is one of my digits. What number am I? (78)

Redirecting students: ASK: What tool could you use to help you remember which numbers from 1-100 you already tried? Students should see that a hundreds chart (e.g., from BLM Hundreds Charts (p. Q-26)) is a good choice. If students are struggling with deciding how to eliminate numbers, encourage them to figure out which clues will eliminate more numbers, and to take care of those first.

7. Arsham made orange juice to have with breakfast. He squeezed 9 oranges, all the same size, to get 558 mL of juice. He shared the juice equally between himself, his sister, and his brother. After he poured the juice, his sister said, "That’s not enough juice!" So he squeezed another orange and gave it all to his sister. Now how much juice does she have?

   Use equations to show your work. Explain what each step means in the situation.

   **Solution:** I did $\frac{558}{3} = 186$, so each person had 186 mL of juice. Each orange makes 62 mL of juice because $\frac{558}{9} = 62$. He squeezed another orange for his sister, so she got 248 mL of juice in total (because $186 + 62 = 248$).
Goals
Students will use counterexamples to show that a statement is false, including statements about prime and composite numbers.

Prior Knowledge Required
Can determine whether a given whole number in the range 1–100 is prime or composite.

Introduce the term “counterexample.” Draw the picture on the board:

Write the statement: All circles are shaded. Have a volunteer circle the picture that shows that the statement isn’t true. Tell students that an example that proves a statement false is called a counterexample to the statement.

Exercises: Write the counterexample to the statement.

a) All squares are shaded.
   A. [ ] B. [ ] C. [ ] D. [ ]

b) All circles are striped.
   A. [ ] B. [ ] C. [ ] D. [ ]

Bonus: All shapes have four sides.
   A. [ ] B. [ ] C. [ ] D. [ ]

Answers: a) C, b) A, Bonus: B

One counterexample is enough to prove a statement false. Tell students that some statements have more than one counterexample, but students only need to find one to prove that a statement is false.

Exercises: Write the first counterexample starting from the left.

A. [ ] B. [ ] C. [ ] D. [ ] E. [ ]

a) All squares are shaded.
b) All squares are small.

Bonus: All squares are small and shaded.

Answers: a) B, b) A, Bonus: A
Determining where to look for counterexamples. Have students list the days of the week and the months of the year in a table. Then write various false statements on the board:

a) All months have 31 days.
b) All days are school days.
c) All months are summer months.
d) All days have an “e” in their spelling.
e) All months have an “e” in their spelling.

Have volunteers tell you counterexamples. Point out that if the sentence is about months, they should look at the list of months for a counterexample. If the sentence is about days, they should look at the list of days for a counterexample.

Review prime and composite numbers.

Exercises:

a) Write all the even numbers from 1 to 20.
b) Write all the odd numbers from 1 to 20.
c) Write all the prime numbers from 1 to 20.
d) Write all the composite numbers from 1 to 20.

Answers: a) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, b) 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, c) 2, 3, 5, 7, 11, 13, 17, 19, d) 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

Finding counterexamples for statements about numbers. For these problems, students have to be careful to look at the list that the statement is about. For example, for part a), students should look at the list of prime numbers, not the list of odd numbers, to find a counterexample.

Exercises: Find a counterexample for each statement for the numbers from 1 to 20.

a) All prime numbers are odd.
b) All odd numbers are prime.
c) All composite numbers are even.
d) All even numbers are composite.

Answers: a) 2, b) 1, 9, or 15, c) 9 or 15, d) 2

Review the terms “digit,” “tens digit,” and “ones digit.” SAY: The number 43 has two digits: the tens digit is 4 and the ones digit is 3.

Exercises: Find a counterexample for each statement.

a) All factors of 12 are even. b) All multiples of 3 are odd.
c) All multiples of 4 have ones digit 4. d) All factors of 4 are even.

Answers: a) 1 or 3, b) 6 or 12 or 18 or ..., c) 8 or 12 or 16 or 20 or 28 or ..., d) 1
Extensions

1. Is there a 2-digit number whose digits add to 6 and is prime?

   **Answer:** Students should list all the 2-digit numbers whose digits add to 6 (15, 24, 33, 42, 51, 60) and check each one in turn. They are all composite, divisible by 3.

2. Find a counterexample for the statement.
   a) All things with legs are animals.
   b) All animals with four legs are dogs.
   c) All words have an "e."

   **Bonus:** Make up your own false sentence, and have a partner find a counterexample.

   **Sample answers:** a) a table or a chair, b) a cat, c) "it"

3. Find a counterexample from the list for the statement.
   a) All the numbers from 20 to 25 are composite.

   20  21  22  23  24  25

   b) All numbers with ones digit 3 are prime.

   3  13  23  33  43  53

   **Answers:** a) 23, b) 33

4. Give three examples of an even number that has an odd factor. Can you find an odd number with an even factor? Explain.

   **Sample answer:** any even number, since 1 is always a factor; no, because any number with an even factor is a multiple of 2; For example, if 6 is a factor, then the number is:

   \[ 6 \times \text{something} = (2 \times 3) \times \text{something} = 2 \times (3 \times \text{something}) \]

5. The number 41 is prime, but its reverse, 14, is composite. Find another number with this property. Explain your strategy to a partner.

   **Sample answers**
   • 19; I tried all the two-digit numbers that are prime in order, and I stopped when I found one whose reverse was composite; 91 is 7 \times 13.
   • 23; I looked for a prime number in the twenties, because I knew the reverse would be an even number and so would be composite

6. a) Name a prime number that has 5 as a factor.

   b) Can a prime number have 8 as a factor? Explain.

   **Answers:** a) 5, b) no, because a prime number only has 1 and itself as factors, so 8 would have to be itself, but 8 is not prime
When 2 or 5 Is a Factor

1. a) Shade the numbers that have 2 as a factor.
   b) What are the ones digits of the numbers you shaded in the third row?

   c) What are the ones digits of the numbers you shaded in any row?

   d) How can you tell whether a number has 2 as a factor without counting up?

2. a) Shade the numbers that have 5 as a factor.
   b) What are the ones digits of the numbers you shaded in the third row?

   c) What are the ones digits of the numbers you shaded in any row?

   d) How can you tell whether a number has 5 as a factor without counting up?

3. Circle the numbers that have 2 as a factor.
   7  38  45  96  187  246  1,980  6,005

4. Circle the numbers that have 5 as a factor.
   10  13  35  42  542  705  3,698  8,940

BONUS ➤ Circle the numbers that have both 2 and 5 as a factor.
   21  42  45  70  236  895  4,800  7,205
Eratosthenes’ Sieve

Eratosthenes was a scholar who lived over 2,000 years ago. He developed a method to identify prime numbers. The following is a simplified version of **Eratosthenes’ Sieve**.

a) Circle the prime numbers between 1 and 10 (2, 3, 5, and 7).

b) Cross out the remaining numbers between 1 and 10 (1, 4, 6, 8, 9, and 10).
   They are not prime.

c) Look for and cross out all the remaining multiples of 2.
   They are not prime.

d) Look for and cross out all the remaining multiples of 3.
   They are not prime.

e) Look for and cross out all the remaining multiples of 4.
   They are not prime. What do you notice?

f) Look for and cross out all the remaining multiples of 5.
   They are not prime.

g) Look for and cross out all the remaining multiples of 6.
   They are not prime. What do you notice?

h) Look for and cross out all the remaining multiples of 7.
   They are not prime.

i) Look for and cross out all the remaining multiples of 8.
   They are not prime. What do you notice?

j) Look for and cross out all the remaining multiples of 9.
   They are not prime. What do you notice?

k) Circle all of the numbers that have not been crossed out.
   They are all prime. You’ve just used Eratosthenes’ Sieve to find all the prime numbers from 1 to 100!

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Pascal's Triangle
Hundreds Charts

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PS4-6  Guessing, Checking, and Revising

Teach this lesson after: 4.2 Unit 6

Standards: 4.OA.A.3, 4.NBT.A.2, 4.NBT.B.5

Goals:
Students will make organized guesses and will use the result of the previous guess to revise their next guess.

Prior Knowledge Required:
Can use organized search
Can round multi-digit whole numbers to any place
Can multiply multi-digit numbers by single-digit numbers
Can compare two multi-digit numbers using place value
Can identify fractions of a number in the range 1–100

Vocabulary: divisor, guess-check-revise, product, remainder, round

Materials:
calculators
BLM Hockey Jerseys (pp. Q-36–37, see Performance Task)

Introduce the guess-check-revise strategy. Hide an object in the room and have a volunteer try to find the object. If the volunteer finds it quickly, play again until it takes a while. When they find it, ASK: What strategy did you use? (sample answer: I guessed and tried again) Play again, but this time tell the volunteer whether they are hot or cold as they try to find the object. Use hints such as “freezing cold” for very far away from the object, “lukewarm” for getting close, and “burning hot” for very close. ASK: What strategy did you use? (sample answer: I guessed and tried again) When you tried again with a hint, was it easier than last time? Why? Tell students that when they have more information about their guess, they can use that information to revise their next guess. Write on the board:

guess-check-revise

SAY: When you play hide-and-seek you are using a guess and check strategy, but when you play with hints such as “burning hot,” “lukewarm,” and “freezing cold,” you are guessing, checking, and revising the next guess. This guess-check-revise strategy is very useful in math.

Make sure everyone has a copy of their JUMP Math AP Book. Tell students to open to page 80 on their first try. Have different volunteers tell you what page number they turned to on their first try. Point out how all the attempts are fairly close to 80. SAY: No one’s first try was page 5 and no one’s first try was page 175. Everyone picked a page pretty close to 80. Now have students use their first guess to make a second guess. ASK: From the first page you turned to, which way in the book should you turn? Should you turn a lot of pages or only a few?
SAY: When you use your first guess to help you make your second guess, you are using the guess-check-revise strategy.

**Review systematic search when two related quantities are changing.** SAY: A farmer has cows and chickens. Jayden counts all the legs and Alice counts all the heads. Write on the board:

    Jayden counts 16 legs.
    Alice counts 6 heads.

ASK: Are there more heads or legs? (legs) Why does that make sense? (because each animal has more legs than heads) SAY: I want to know how many cows and how many chickens there are. Remember, to do this type of problem, you can start by guessing one of the two quantities and go up in order through all the possibilities. Draw on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
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ASK: How can you get the number of chickens from the number of cows? (they add to 6) SAY: There are six heads, so the total number of animals is six. Have a volunteer complete the second column. (6, 5, 4, 3, 2, 1, 0)

**Exercise:** Copy and complete the chart. How many cows and how many chickens are there if there are 16 legs in total?

**Answers:** 12, 14, 16, 18, 20, 22, 24; 2 cows and 4 chickens

Have a volunteer complete the third column of the chart on the board. ASK: If you move down a row, does the total number of legs get bigger or smaller? (bigger) How much bigger? (by 2) SAY: When you start at the top of the table, you have six chickens. When you move down a row, you replace a chicken with a cow, so now you have one cow and five chickens. Every time you replace a chicken with a cow, you replace two legs with four legs, so you have two more legs than before.

**Searching from either direction.** SAY: Jayden and Alice went to another farm that has cows and chickens. Write on the board:

    Jayden counts 36 legs.
    Alice counts 10 heads.

Have a volunteer complete the third column of the chart on the board.
ASK: How many animals are there altogether? (10) How do you know? (the number of heads)
Write on the board:

<table>
<thead>
<tr>
<th>Cows</th>
<th>Chickens</th>
<th>Total Number of Legs</th>
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Have a volunteer complete the second column. (10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0) ASK: If there are
no cows and 10 chickens, how many legs are there? (20) Write “20” in the first row of the third
column. ASK: If there are 10 cows and no chickens, how many legs are there? (40) Write “40” in
the last row of the third column. ASK: Do you think the number of cows in our answer will be
closer to zero or to 10? (10) Why? (the number of legs is closer to 40 than to 20) PROMPT: Is
the actual number of legs closer to 20 or 40? (40) So, is it better to start our guess closer to zero
or to 10? (10) SAY: We could save ourselves a lot of work by starting at 10 cows and zero
chickens and moving up the chart instead of starting at zero cows and 10 chickens. ASK: How
many legs do nine cows have? (36) Write on the board:

36 +

ASK: How many legs does one chicken have? (2) Continue writing on the board:

36 + 2 = 38

Write “38” as the total in the row for 9 cows and 1 chicken. Repeat for the row with 8 cows and
2 chickens. (32 + 4 = 36) SAY: So, eight cows and two chickens have a total of 36 legs. Starting
from 10 cows and searching is a lot less work than starting from zero cows and going all the
way up to eight cows. Leave the chart on the board for later use.

(MP.1, MP.5) Exercises: How many cows and how many chickens are there on the farm?
a) Jayden counts 22 legs. Alice counts 9 heads.
b) Jayden counts 26 legs. Alice counts 7 heads.
c) Jayden counts 32 legs. Alice counts 15 heads.
d) Jayden counts 52 legs. Alice counts 14 heads.
Answers: a) 2 cows, 7 chickens; b) 6 cows, 1 chicken; c) 1 cow, 14 chickens; d) 12 cows,
2 chickens
Refer students to the chart on the board. SAY: You don’t have to start the chart with zero cows and then move up the chart from the end. You can start with 10 cows and move down the chart instead. Write on the board:

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<th>Cows</th>
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<th>Total Number of Legs</th>
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<td>10</td>
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<td>40</td>
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<tr>
<td>9</td>
<td>1</td>
<td>38</td>
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<td>8</td>
<td>2</td>
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**Exercises:** If all the animals are cows, how many legs are there? If all the animals are chickens, how many legs are there?

a) Alice counts 30 heads.  
b) Alice counts 37 heads.  
c) Alice counts 28 heads.  
**Bonus:** Alice counts 1,000 heads.

**Answers:** a) 120, 60; b) 148, 74; c) 112, 56; Bonus: 4,000, 2,000

SAY: Once you know how many legs there are if all the animals are cows and if all the animals are chickens, you can compare those numbers with the total number of legs given. Then you can decide which option to start your search with.

**(MP.1, MP.5) Exercises:** How many cows and how many chickens are there?

a) Jayden counts 114 legs. Alice counts 30 heads.  
b) Jayden counts 140 legs. Alice counts 37 heads.  
c) Jayden counts 60 legs. Alice counts 28 heads.  
**Bonus:** Jayden counts 3,996 legs. Alice counts 1,000 heads.

**Answers:** a) 27 cows, 3 chickens; b) 33 cows, 4 chickens; c) 2 cows, 26 chickens;  
Bonus: 998 cows, 2 chickens

**(Making guesses up or down by 10.** SAY: Now there is a bigger farm with cows and chickens. Write on the board:

- Jayden counts 344 legs  
- Alice counts 100 heads.  

How many cows and chickens are there?

ASK: If all the animals are cows, how many legs are there? (400) If all the animals are chickens, how many legs are there? (200) Is 344 closer to 400 or to 200? (400) Do you think there are more cows or chickens? (cows) SAY: Let’s start the search for the answer with 100 cows and zero chickens. Write on the board:

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<th>Cows</th>
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<th>Total Number of Legs</th>
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<td>97</td>
<td>3</td>
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</table>
Have volunteers fill in the chart. (400, 398, 396, 394) SAY: We’re getting closer to 344 legs, but it’s going to take a while. ASK: How could I make the search go faster? Take students’ suggestions, then SAY: I am going to count by 10s instead of by 1s so that I can find the answer faster. Erase the chart on the board. Draw on the board:

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<th>Cows</th>
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<td>100</td>
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**Exercises:**

a) Complete the chart. (MP.3, MP.7)

b) What 2 tens is the number of cows between? Explain how you know.

**Answers:**

a) 400, 380, 360, 340, 320; b) The number of cows is between 70 and 80, because 70 cows and 30 chickens have a total of 340 legs and 80 cows and 20 chickens have a total of 360 legs.

Further narrowing the search. Write on the board:

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<th>Cows</th>
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<td>80</td>
<td>20</td>
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ASK: Is the actual number of cows closer to 70 or to 80? (70) A lot closer or a little closer? (a lot closer) Why? (because 344 is a lot closer to 340 than to 360) What number should we try next? (71 or 72)

**Exercises:**

1. Complete the chart until the total number of legs is 344.

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**Answer:** 72 cows and 28 chickens
Problem Bank

(MP.1, MP.7) 1. What numbers might I be?
a) When you multiply me by 7, the result is between 500 and 550.
b) When you multiply me by 5, the result is less than 400. When you multiply me by 6, the result is more than 400.

Answers: a) 72, 73, 74, 75, 76, 77, or 78; b) 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, or 79

(MP.1, MP.7) 2. What number am I?
a) Multiply me by 7. Then round to the nearest 10. The result is 260.
b) Multiply me by 88. Then round to the nearest hundred. The result is 5,000.
c) When you multiply me by 800 and then round to the nearest 1,000, the result is 5,000.

Answers: a) 37, b) 57, c) 6

3. What are the numbers?
a) The quotient of two numbers is 8 and their product is 200.
b) The quotient of two numbers is 8 and their product is 20,000.
c) The quotient of two numbers is 8 and their product is 10,952.

Answers: a) 5 and 40, b) 50 and 400, c) 37 and 296

(MP.1, MP.4) 4. A school fundraiser has a bake sale that sells muffins and cake. A muffin costs $2 and a piece of cake costs $3. The bake sale sold 30 items altogether and made $71. How many muffins and how many pieces of cake were sold?

Answer: 19 muffins and 11 pieces of cake

(MP.1, MP.7) 5. Use a calculator to answer these questions. Remember that two whole numbers are consecutive if they differ by 1.
a) Calculate the products.
   i) 1 × 2   ii) 2 × 3   iii) 3 × 4   iv) 4 × 5   v) 5 × 6
b) Is 14 the product of two consecutive whole numbers? Explain how you know.
c) Can 160 be the product of two consecutive whole numbers? Explain how you know.
d) Can 992 be the product of two consecutive whole numbers? Explain how you know.
e) Write 6,972 as a product of two consecutive whole numbers.

Answers: a) i) 2, ii) 6, iii) 12, iv) 20, v) 30; b) no, it is between 3 × 4 and 4 × 5; c) no, it is between 12 × 13 = 156 and 13 × 14 = 182; d) yes, it is 31 × 32; e) 83 × 84

(MP.1, MP.7) 6. Find N so that …
a) (2 × N) + 1 = 177  b) (N × 3) + N = 228  c) (N × 5) + 5 = 320

Bonus: Use a calculator to find N if N × N = 1,849

Answers: a) 88, b) 57, c) 63, Bonus: 43
(MP.1, MP.7) 7. a) Fill in the blanks with a whole number when you can.

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\begin{align*}
\_ \times 1 + 6 &= 30 \\
\_ \times 2 + 6 &= 30 \\
\_ \times 3 + 6 &= 30 \\
\_ \times 4 + 6 &= 30 \\
\_ \times 5 + 6 &= 30 \\
\_ \times 6 + 6 &= 30
\end{align*}
\]

b) Which blanks have a whole number that works? Explain. Hint: Make sure the remainder is less than the divisor.

\[
\begin{align*}
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30 \div \_ &= 2 \text{ R } 6 \\
30 \div \_ &= 3 \text{ R } 6 \\
30 \div \_ &= 4 \text{ R } 6 \\
30 \div \_ &= 5 \text{ R } 6 \\
30 \div \_ &= 6 \text{ R } 6
\end{align*}
\]

c) John divides 45 by a number and gets a remainder of 9. What numbers could he have divided by?

**Answers:**

a) 24, 12, 8, 6, no whole number possible, 4

b) \(30 \div 24 = 1 \text{ R } 6\), \(30 \div 12 = 2 \text{ R } 6\), and \(30 \div 8 = 3 \text{ R } 6\) work here. \(6 \text{ R } 6\) and \(4 \text{ R } 6\) don’t work because dividing by 6 or 4 can’t leave a remainder of 6. \(5 \text{ R } 6\) doesn’t work because 5 is not a factor of 24 (30 − 6).

c) To fill in the blank in \(45 \div \_ = \text{ ? R } 9\), you need \(\_ \times \text{ ?} + 9 = 45\), so \(\_\) is a factor of 36. \(\_\) also must be bigger than 9—otherwise dividing by it can’t get a remainder of 9. So, the numbers he could have divided by are 12, 18, and 36.
Performance Task: Hockey Jerseys

Materials:
BLM Hockey Jerseys (pp. Q-36–37)

Preparation for the performance task. Tell students that the performance task is about hockey jerseys (the team shirts) and how much it costs overall for a team to play ice hockey. Part of the total cost is for the jerseys and part is for renting the rink. The players themselves supply their skates, hockey sticks, and other equipment. The team buys the jerseys, then sends them out to have numbers printed on them. The numbers start at one and go up in order to the number of players on the team. Numbers with two digits cost more than numbers with one digit, because the printing cost is per digit. Tell students that this will be part of what they investigate in the performance task.

Performance Task: Hockey Jerseys. Provide students with BLM Hockey Jerseys. Question 6 is a good opportunity to apply the problem-solving strategy learned in this lesson. Students who have not had the opportunity to do this lesson might find it difficult.

Answers: 1. $360, 2. 21, 3. $63, 4. $200, 5. $623, 6. 17
Hockey Jerseys (1)

In an ice hockey league, each team buys jerseys for their players before the season begins.

1. Jerseys cost $24 each. The Warriors is a team with 15 players. How much does the team need to pay for jerseys?

2. Each team decides to put numbers on the back of each jersey. Each team starts at 1 and numbers the jerseys in order. How many digits will the Warriors need in total?

3. Each digit costs $3 to put on. How much would a team of 15 players have to pay for the digits?

4. It costs the league $1,800 to rent the ice rink for the year. There are 9 teams in the league. How much does each team pay for the rink rental?
Hockey Jerseys (2)

5. What is the team’s total cost, including rink rental, jerseys, and digits?

6. The Athenas is another team in the league. They paid $75 for the digits on their jerseys. How many players are on that team?
Unit 7  Number and Operations—Fractions: Decimals

This Unit in Context

In Grade 3, students started learning about fractions and understanding them as numbers (3.NF.A.2). In 4.2 Unit 4, students continued learning about fractions by comparing fractions with different denominators and adding and subtracting fractions with the same denominator. In this unit, students will be exposed to decimal fractions and learn how our decimal place value system can extend to fractions. In particular, they will use decimal notation for tenths and hundredths (4.NF.C.6). Students likely have already seen decimal numbers in their daily lives through exposure to money, but students in this unit will formally make the connection between decimals and fractions. Students adding tenths and hundredths (4.NF.C.5) will make the connection to adding dimes and pennies. They will start by learning about decimals less than 1, and then use their knowledge of mixed numbers from 4.2 Unit 4 to extend that to decimals greater than 1, writing, for example, 1 3/10 as 1.3. Students will learn to convert between decimal fractions and decimal notation, including numbers written in mixed number or improper fraction format, up to the hundredths (4.NF.C.6).

In Grade 3 and 4.2 Unit 4, students placed fractions on number lines (3.NF.A.2). In this unit, students will place decimals to hundredths on number lines (4.NF.C.6). In Grade 3, students learned that comparisons are valid only when two fractions refer to the same whole—for example, half of one pie could be smaller than a third of another pie, even though 1/2 is a greater fraction than 1/3 (3.NF.A.3d). In this unit, students will compare two decimals to hundredths by reasoning about their size and will recognize that comparisons are valid only when the two decimals refer to the same whole—for example, 0.3 on one number line can be farther right than 0.5 on another number line, even though 0.3 is less than 0.5 (4.NF.C.7). Students will also solve word problems involving simple decimals that use measurement contexts, such as distance or money (4.MD.A.2).

Students will continue to work with decimals throughout the school curriculum. In Grade 5, students will read, write, and compare decimals to thousandths (5.NBT.A.3); they will use place value understanding to round decimals to any place (5.NBT.A.4); and they will add, subtract, multiply, and divide decimals to hundredths (5.NBT.B.7). In Grade 6, students will fluently add, subtract, multiply, and divide multi-digit decimals (6.NS.B.3). Grade 7 will students use decimal notation for any rational number, including both terminating and repeating decimals (7.NS.A.2d), and they will solve multistep real-life mathematical problems involving positive and negative rational numbers in any form (7.EE.B.3). In Grade 8, students will learn that there are numbers that are not rational—in other words, cannot be written as a fraction (8.NS.A.1)—and they will use terminating decimals to approximate irrational numbers (8.NS.A.2).
Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP.1 to MP.7. Here are some examples of how students can show that they have met a standard.

**MP.2:** In NF4-21 AP Book Question 9, students reason abstractly and quantitatively when they shade square grids to decide whether a given number of tenths is greater or smaller than a given number of hundredths.

**MP.3:** In NF4-28 AP Book Question 4, students critique (the fictional) Sarah’s argument that 0.25 is more than 0.3 because more is shaded in one picture compared to another.

**MP.5:** In NF4-22 Extension 4, students choose tools strategically to decide whether 8 is a factor of 1,672 or not. In particular, students choose between base ten blocks, paper and pencil, and mental math.

**MP.6:** In NF4-24 and 25 Extension 3, students attend to precision when they show their work to solve a multi-step problem using clearly labelled pictures.

**MP.7:** In NF4-26 Extension 4, students look for and make use of structure when they decide whether the 100th term of a pattern is even or odd. For example, students use the fact that two odd numbers add to an even number and that an odd and an even number add to an odd number.
Unit 7  Number and Operations—Fractions: Decimals

Students will use decimal notation for decimal fractions and compare and order decimals up to hundredths. Students will also add tenths and hundredths.

Terminology

Do not shorten "decimal point" to "decimal." This creates confusion between two different concepts: decimal (a number) and decimal point (the symbol separating parts of the number). Make sure students use proper terminology.

We often use slashes for fractions (such as 1/2) to save space. Do not display fractions to students this way.

Materials

We suggest having students work in grid paper notebooks. Paper with 1/4-inch grid works well in most lessons. Grid paper is very helpful when comparing decimals using place value. If students do not use grid paper notebooks in general, you will need to have lots of grid paper on hand throughout the unit (for example, from BLM 1 cm Grid Paper (p. U-1)). If students who have difficulties in visual organization will be working without grid paper, they should be taught to draw a grid before starting to work on a problem.
**Goals**

Students will express monetary values in dollar and cent notation.

**PRIOR KNOWLEDGE REQUIRED**

- Can use the cent symbol correctly
- Can use dollar notation correctly for amounts given in whole dollars
- Is familiar with US coins
- Knows the relative values of US coins and $1 bills

**MATERIALS**

- play money: pennies, nickels, dimes, and quarters
- BLM Money-Matching Memory Game (p. R-37)

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**Writing dimes and pennies in cent notation.** Tell students to pretend that there is a vending machine that takes only dimes and pennies. **ASK:** What would you use to buy a toy that costs 38¢? (3 dimes and 8 pennies) **SAY:** Because the dimes are worth 10 cents and the pennies are worth 1 cent, the tens digit tells you how many dimes you need and the ones digit tells you how many pennies you need.

**Exercises:** How much money is there?

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<td>a)</td>
<td>2 dimes and 6 pennies</td>
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<td>b)</td>
<td>8 dimes and 9 pennies</td>
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<tr>
<td>c)</td>
<td>5 dimes and 5 pennies</td>
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<tr>
<td>d)</td>
<td>4 dimes and 1 penny</td>
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**Answers:** a) 26¢, b) 89¢, c) 55¢, d) 41¢

**ACTIVITY 1**

**The Change Machine**

Player 1 is given only nickels and quarters and needs to buy something from a vending machine that only takes dimes and pennies. Player 1 makes an amount using nickels and quarters. Player 2 plays the role of a change machine and changes the amount to dimes and pennies. Players then switch roles.

**Writing dimes and pennies in dollar notation.** Ask a volunteer to write 26¢ in *dollar notation*. If no one knows, write the answer ($0.26) on the board. **SAY:** In dollar notation, the number of dimes goes right after the dot and the number of pennies goes right after that.

**Exercises:** Write the amount in dollar notation.

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<table>
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<tr>
<td>a)</td>
<td>3 dimes and 8 pennies</td>
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<td>b)</td>
<td>4 dimes and 2 pennies</td>
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<td>c)</td>
<td>41¢</td>
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<td>d)</td>
<td>29¢</td>
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<td>e)</td>
<td>35¢</td>
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<td>f)</td>
<td>81¢</td>
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Answers: a) $0.38, b) $0.42, c) $0.41, d) $0.29, e) $0.35, f) $0.81

Now write “8¢” on the board. ASK: How many dimes are in 8 cents? (0)
How many pennies are in 8 cents? (8) Show this on the board by writing
8¢ in dollar notation (see margin).

Exercises: Write the amount in dollar notation.

a) 0 dimes and 6 pennies  b) 3¢
  d) 5 dimes and 0 pennies  e) 40¢
d) 5 dimes and 0 pennies  e) 40¢  f) 90¢

Answers: a) $0.06, b) $0.03, c) $0.07, d) $0.50, e) $0.40, f) $0.90

Writing multiples of a hundred cents in dollar notation. Remind students
that one hundred cents can be written as one dollar. Write on the board:

100¢ = $1  200¢ = ______  1,300¢ = ______

Have volunteers fill in the blanks ($2 and $13) SAY: Two hundred cents
equals two dollars and thirteen hundred cents equals thirteen dollars.
You knock off the last two zeros because you’re dividing by one hundred.

Exercises: Write the amount in dollar notation.

a) 300¢  b) 800¢  c) 1,000¢  d) 1,200¢  e) 38,400¢

Answers: a) $3, b) $8, c) $10, d) $12, e) $384

Converting cent notation to dollar notation. Write on the board:

348¢ = 300¢ + 48¢  1,746¢ = 1,700¢ + 46¢

Ask volunteers to write 300¢, 48¢, 1,700¢, and 46¢ in dollar notation:

= $3 + $0.48  = $17 + $0.46

Show students how to combine them into one single dollar notation:

= $3.48  = $17.46

Point out that the dot is always in front of the last two digits.

Exercises: Write the amount in dollar notation.

a) 156¢  b) 704¢  c) 1,804¢  Bonus: 1,234,567,890¢

Answers: a) $1.56, b) $7.04, c) $18.04, Bonus: $12,345,678.90

Changing dollar notation to cent notation. Write on the board:

$13.85 = 13 dollars 85 cents

ASK: How many cents are in 13 dollars? (1,300) Write on the board:

= 1,300¢ + 85¢

= 1,385¢

Exercises: Write the dollar amount in cent notation.

a) $8.00  b) $17.00  c) $0.28  d) $0.04
  e) $3.54  f) $9.03  g) $30.42  h) $10.05

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Bonus: $10,000,000.00

Answers: a) 800¢, b) 1,700¢, c) 28¢, d) 4¢, e) 354¢, f) 903¢, g) 3,042¢, h) 1,005¢, Bonus: 1,000,000,000¢

Making a given amount of money using the fewest possible coins and bills. ASK: What coins could you use to make 80¢? (8 dimes, 3 quarters and 1 nickel, 3 quarters and 5 pennies, 2 quarters and 3 dimes, and so on) Allow different volunteers to give answers, including repeated answers. Now tell students that you want to make 80¢ using the fewest possible coins. ASK: How can I do that? (3 quarters and 1 nickel) Point out that you need to use as many quarters as you can, because quarters have the greatest value of any coin that is less than 80¢, so you’ll need fewer of them to make 80¢.

Exercises: Make each amount using the fewest coins possible.

a) 71¢ b) 85¢ c) 54¢ d) 26¢ e) 13¢

Answers: a) 2 quarters, 2 dimes, 1 penny, b) 3 quarters, 1 dime, c) 2 quarters, 4 pennies, d) 1 quarter, 1 penny, e) 1 dime, 3 pennies

ASK: What dollar amounts do bills come in? (sample answers: $1, $5, $10, $20)

Exercises: Make each amount using the fewest coins and bills possible.

a) $6 b) $16 c) $3.25 d) $8.34 e) $10.06

Answers: a) $5, $1; b) $10, $5, $1; c) $1, $1, $1, 25¢; d) $5, $1, $1, $1, 25¢, 5¢, 1¢, 1¢, 1¢; e) $10, 5¢, 1¢

Making a given amount of money using a given number of coins and bills. Now tell students that you want to make 80¢ using exactly 5 coins. ASK: How can I do that? (3 quarters and 1 nickel) Write the following problem on the board:

Make $6.30 using exactly 5 coins and bills.

Tell students that a good strategy is to first find one way—any way—of making $6.30, then decide whether they have too many or too few coins and bills. ASK: What one way do we already know? (find the one with the fewest coins and bills) Have students do so: $5, $1, 25¢, 5¢. Then challenge students to increase the number of coins by 1, but keep the value the same. (replace the quarter and nickel with three dimes)

Exercises

a) Make $2.50 using 6 coins and bills.
b) Make $1.76 using 8 coins and bills.

Answers: a) $1, $1, 25¢, 10¢, 10¢, 5¢; b) $1, 25¢, 10¢, 10¢, 10¢, 10¢, 10¢, 1¢
ACTIVITY 2

Money-Matching Memory Game

Use BLM Money-Matching Memory Game to have students individually play Picking Pairs (see Introduction—Part 2 Unit 4, p. O-1) and Memory (see Introduction—Part 2 Unit 4, p. O-1).

Extensions

1. What coin is being used for skip counting?

   $1.00, _____, _____, _____, $1.20
   $2.00, _____, _____, _____, $3.00

   Answers: 5¢, 25¢

2. What is the greatest number of coins you could use to make $1.34?

   Answer: 134 pennies

3. Make $5.81 using 10 coins and bills, in two different ways.

   Sample answers: $1, $1, $1, $1, 25¢, 25¢, 25¢, 5¢, 1¢; $5, 10¢, 10¢, 10¢, 10¢, 10¢, 10¢, 10¢, 10¢, 1¢; $5, 25¢, 25¢, 25¢, 1¢, 1¢, 1¢, 1¢, 1¢, 1¢

4. a) Convert $8 to cent notation.

   b) Convert 8 m to centimeters.

   c) How are parts a) and b) the same?

   Answers: a) 800¢, b) 800 cm, c) You do the questions the same way, by multiplying both numbers by 100.
Comparing money amounts in cents. SAY: All numbers more than 100 are greater than all numbers less than 100, so numbers with 3 digits are greater than numbers with 2 digits. Write on the board:

2,564¢  917¢

ASK: Which is more money? (2,564¢) How do you know? (it has 4 digits; it is more than 1,000, while 917 has 3 digits, so it is less than 1,000)

Exercises: Which is more money?

a) 80¢ or 205¢  
b) 3,579¢ or 543¢  
c) 344¢ or 7,102¢

Answers: a) 205¢, b) 3,579¢, c) 7,102¢

SAY: When two numbers have different numbers of digits, the one with more digits is greater. When two numbers have the same number of digits, look for the highest place value where they are different. Demonstrate this on the board:

3 2 , 4 1 7 ¢  
3 5 , 1 8 3 ¢

SAY: So 35,183¢ is more money.

Exercises: Which is more money?

a) 274,108¢  
b) 31,763¢  
c) 2,400,381¢

Answers: a) 274,156¢, b) 31,763¢, c) 2,400,381¢

Comparing money amounts given in different units ($ and ¢). Write two money amounts on the board: 234¢ and $2.15. Tell students that you want to know which amount represents more money. ASK: What is different about this problem? (one amount is in dollars; the other is in cents) Ask a volunteer to change the amount in dollar notation to cent notation (215¢), then have another volunteer say which is greater (234¢). ASK: How did changing to cents make it easier to compare them? (if they’re both in cents, just compare the whole numbers)
Exercises: Which is greater?

a) 1,234¢ or $7.56           b) $9,875 or 98,756¢

b) 9¢ or $0.08              d) 40¢ or $0.04

Answers: a) 1,234¢, b) $9,875, c) 9¢, d) 40¢

Writing money amounts in different ways. Tell students that once they know how to write number words, they can write money amounts in words too. Write on the board:

$17.04

ASK: How many dollars are there? (17) How many cents are there? (4)
Tell students that to write the amount in words, they just need to write how many dollars and how many cents. Write on the board:

seventeen dollars and four cents

Point out that they write the words “dollars” and “cents” just like they say it, and the word “and” goes in between.

Exercises: Write the money amount in words.

a) $3.50   b) $5.03   c) $30.18   d) $80.04

Bonus: $10,000,000.35

Answers: a) three dollars and fifty cents, b) five dollars and three cents, c) thirty dollars and eighteen cents, d) eighty dollars and four cents, Bonus: ten million dollars and thirty-five cents

Exercises: Write the money amount in dollar notation.

a) four dollars and forty cents
   b) eighteen dollars and five cents
   c) seventy-three dollars and eighty-four cents
   d) fifty dollars and sixty cents

Bonus: Seven hundred thousand dollars and eight cents

Answers: a) $4.40, b) $18.05, c) $73.84, d) $50.60, Bonus: $700,000.08

Exercises: Write the amounts in cent notation. Then circle the greater amount.

a) ten dollars and thirty-five cents, 1,125¢
   b) three dollars and fifty cents, 305¢
   c) $15, 746¢
   d) sixty-three cents, $6.30

Answers: a) 1,035¢ and 1,125¢, circle 1,125¢, b) 350¢ and 305¢, circle 350¢, c) 1,500¢ and 746¢, circle 1,500¢, d) 63¢ and 630¢, circle 630¢

Present a problem: Jane emptied her piggy bank. She asked her older brother John to help her count the money. He suggested she sort the coins...
and bills by value—a stack of pennies, a stack of nickels, and so on—and count each stack separately. Jane has:

- 19 pennies
- 8 nickels
- 21 dimes
- 5 quarters
- 6 one-dollar bills
- 3 five-dollar bills

How much money is in each stack? Ask students to write the amounts less than a dollar in cent notation and the amounts more than a dollar in dollar notation.

**Answers:** 19¢ in pennies, 40¢ in nickels, $2.10 in dimes, $1.25 in quarters, $6.00 in one-dollar bills, $15.00 in five-dollar bills

**Bonus:** How much money did Jane have in her piggy bank altogether?

**Hint:** Add the dollar amounts separately from the cent amounts.

**Answer:** $24.94

**Extensions**

1. a) I am less than 20¢. You need 6 coins to make me. What am I?
   
   b) I am less than 20¢. You need 5 coins to make me. To double me, you only need three coins. What am I?

   **Answers:** a) 19¢, b) 18¢

2. Without changing to cent notation, compare the money amounts. Did you need to compare the cent amounts or just the whole-dollar amounts?

   a) $3.84, $2.95 
   b) $14.71, $14.36

   **Answers:** a) $3.84 is more than $2.95. Three dollars and anything is more than two dollars and anything, so you do not need to compare the cent amounts. b) The dollar amounts are the same, so you need to compare the cent amounts. Since 71¢ is more than 36¢, $14.71 is more than $14.36.
NF4-21 Tents and Hundredths (Fractions)
Pages 109–110

STANDARDS
4.NF.C.5, 4.NF.C.7

VOCABULARY
dime
equation
equivalent fraction
fraction
hundredth
penny
tenth

Goals
Students will express a fraction with the denominator 10 as an equivalent fraction with the denominator 100, and compare tenths and hundredths.

PRIOR KNOWLEDGE REQUIRED
Knows how many pennies or dimes it takes to make one dollar
Knows that 10 pennies are worth one dime
Knows that different fractions can be equivalent
Can name fractions from area models and number lines

MATERIALS
grid paper
BLM Squares Divided into Hundredths (p. R-38)

Introduce dimes and pennies as fractions of a dollar. ASK: How many dimes is a dollar worth? (10) What fraction of a dollar is a dime? (one tenth) How many pennies is a dollar worth? (100) What fraction of a dollar is a penny? (one hundredth) Write on the board:

A dime is \( \frac{1}{10} \) of a dollar.

A penny is \( \frac{1}{100} \) of a dollar.

ASK: What fraction of a dollar is 2 dimes? (2/10) What fraction of a dollar is 3 pennies? (3/100)

Exercises: What fraction of a dollar is the amount?

a) 5 dimes  
    b) 5 pennies  
    c) 8 dimes  
    d) 17 pennies

Answers: a) 5/10, b) 5/100, c) 8/10, d) 17/100

Using dimes and pennies to make equivalent fractions. ASK: How many pennies are 3 dimes worth? (30) Write on the board:

3 dimes = 30 pennies

Ask a volunteer to write the fraction of a dollar each amount shows.

3 dimes = 30 pennies

\[ \frac{3}{10} = \frac{30}{100} \]

Tell students that 3/10 and 30/100 are equivalent fractions because 3 dimes and 30 pennies are the same amount.
Exercises: Write the equivalent amount, then write a fraction equation.

a) 4 dimes = _____ pennies  
   b) 7 dimes = _____ pennies

Answers: a) 40, \(\frac{4}{10} = \frac{40}{100}\); b) 70, \(\frac{7}{10} = \frac{70}{100}\)

Using pictures to make equivalent fractions. Project a sheet of grid paper onto the board and draw a hundreds square or use BLM Squares Divided into Hundredths. SAY: I want to shade \(\frac{1}{10}\) of the square. ASK: What is an easy way to do that? (shade a row or a column) ASK: How would I show 4 tenths shaded? (shade 4 columns) Shade 4 columns, then ASK: How many hundredths are shaded? (40) Demonstrate counting by tens to count the hundredths: ten, twenty, thirty, forty hundredths. Write on the board:

\[
\frac{4}{10} = \frac{40}{100}
\]

Exercises: Write two equivalent fractions for the picture.

a)  
   b)  
   c)  
   d)  

Answers: a) \(\frac{3}{10} = \frac{30}{100}\), b) \(\frac{7}{10} = \frac{70}{100}\), c) \(\frac{6}{10} = \frac{60}{100}\), d) \(\frac{2}{10} = \frac{20}{100}\)

Using pictures to compare tenths to hundredths. Draw on the board:

Have students name the fraction shaded in both squares. (34/100 and 7/10 or 70/100) Demonstrate how they can count by tens, then by ones, to count the number of hundredths that are shaded in the first square. ASK: Which is more, 34 hundredths or 7 tenths? (7 tenths) How can you tell by the picture? (more is shaded) Which is worth more, 34 pennies or 7 dimes? (7 dimes) SAY: 7 tenths of a dollar is worth more than 34 hundredths of a dollar because 7 tenths is a greater fraction of anything than 34 hundredths is.

Provide students with BLM Squares Divided into Hundredths.

Exercises: Shade and label the fractions. Then compare them.

\[
\frac{24}{100}, \frac{5}{10} \quad \frac{6}{100}, \frac{9}{10} \quad \frac{54}{100}, \frac{3}{10}
\]

Answers: a) \(\frac{24}{100} < \frac{5}{10}\), b) \(\frac{6}{100} < \frac{9}{10}\), c) \(\frac{54}{100} > \frac{3}{10}\)

Comparing tenths and hundredths without pictures. SAY: You can also compare tenths and hundredths by converting the tenths to hundredths. Demonstrate:
ASK: What’s more, 40 hundredths or 71 hundredths? (71 hundredths)

**Exercises**: Compare the fractions by making them both hundredths.

a) \( \frac{35}{100} \) , \( \frac{3}{10} \)  

b) \( \frac{16}{100} \) , \( \frac{7}{10} \)  

c) \( \frac{21}{100} \) , \( \frac{6}{10} \)  

**Answers**: a) \( \frac{35}{100} > \frac{3}{10} \), b) \( \frac{16}{100} < \frac{70}{100} \), c) \( \frac{21}{100} < \frac{60}{100} \)

**Extensions**

(MP.1) 1. Write any number that works.

a) \( \frac{3}{10} < \frac{4}{100} < \frac{4}{10} \)  

b) \( \frac{58}{100} > \frac{9}{10} > \frac{47}{100} \)  

**Answers**: a) any of 31 to 39, b) 5

(MP.1) 2. Write 10 or 100 in each box to make the comparison true.

a) \( \frac{27}{100} < \frac{3}{10} \)  

b) \( \frac{10}{100} = \frac{1}{10} \)  

**Answers**: a) \( \frac{27}{100} < \frac{3}{10} \), b) \( \frac{10}{100} = \frac{10}{10} \)

3. Anna and Bob both made 100-piece puzzles. Anna’s puzzle is 10 by 10. Bob’s puzzle is 5 by 20.

a) What fraction of Anna’s puzzle are border pieces?

b) What fraction of Bob’s puzzle are border pieces?

c) Which puzzle do you think will be easier to put together? Why?

**Answers**: a) \( \frac{36}{100} \), b) \( \frac{46}{100} \), c) Answers may vary depending on puzzle-solving strategies, but students should justify their opinion in terms of which they find easier, when there are many border pieces or when there are many inside pieces.

(MP.7) 4. Find as many whole number answers as you can to fill in the blanks: \( \text{___} \times \text{___} = 240,000 \).

**Sample answer**: \( 1 \times 240,000, 10 \times 24,000, 100 \times 2,400, 1,000 \times 240, \text{or} 10,000 \times 24 \); similarly, you could use \( 24 = 2 \times 12, 3 \times 8, \text{or} 4 \times 6 \).

**NOTE**: Some students might use the associative property to discover other products. For example, \( 240,000 = 100 \times 2,400 = (25 \times 4) \times 2,400 = 25 \times (4 \times 2,400) = 25 \times 9,600 \).
NF4-22 Decimal Tenths and Hundredths
Pages 111–112

STANDARDS
4.NF.C.6, 4.NF.C.7

VOCAULRY
decimal
decimal hundredth
decimal notation
decimal point
decimal tenth
fraction
number line
tenth

Goals
Students will use decimal notation for fractions with denominators 10 and 100, and place decimal hundredths on number lines and order decimal hundredths using a number line.

PRIOR KNOWLEDGE REQUIRED
Knows that, on number lines, greater whole numbers appear to the right of lesser whole numbers
Can name fractions from area models and number lines

MATERIALS
BLM Number Lines Divided into Hundredths (p. R-39)

Introduce decimal tenths. Tell students that the fraction $\frac{1}{10}$ can be represented in various ways. Show three ways on the board:

\[
\frac{1}{10} = 0.1
\]

Point out that each way means 1 part out of 10 equal parts. Tell students that mathematicians have invented a simpler way to write one tenth, called decimal notation. Show this on the board:

\[
\frac{1}{10} = 0.1
\]

SAY: The dot is called a decimal point. People write the number this way because it takes up less space on the page and is easier to write. Ask volunteers to show how they would write 2 tenths (0.2), 3 tenths (0.3), and other numbers up to 9 tenths (0.9) as a decimal. SAY: The 0 before the decimal point tells you that the number is less than 1.

Representing decimal tenths on a number line. Draw a number line from 0 to 1, and ask students to place various decimal tenths on the number line (e.g., 0.8, 0.5, 0.2, 0.7).

Exercises: Write the decimal for each marked point:

\[
\begin{align*}
0 & \quad \star \quad \star \quad \star \quad \star \\
1 & \quad 0 & \quad \star \quad \star \quad \star \quad \star \\
\end{align*}
\]

Answers: 0.3, 0.4, 0.9

Representing decimal tenths using pictures. Draw various shapes on the board, such as circles, squares, or rectangles, and have volunteers represent various numbers given in decimal notation.
a) 0.2   b) 0.3   c) 0.5   d) 0.6

Writing decimal notation for pictures. Now ask students to do the reverse.

Exercises: Write the decimal for the picture.

a)  

b)  

c)  

d)  

Answers: a) 0.8, b) 0.7, c) 0.5, d) 0.8

Introduce decimal hundredths. Tell students that the fraction 1/100 can also be represented in various ways. Show four ways on the board:

\[
\frac{1}{100} \quad \text{one hundredth} \quad 0.01
\]

Point out how one hundredth is written differently from one tenth—there are two digits after the decimal point instead of only one. Ask a volunteer to show how she would write two hundredths as a decimal (0.02), then read it as “zero point zero two.” ASK: How would you write three hundredths as a decimal? (0.03)

Exercises: Write the fraction as a decimal.

a) \( \frac{9}{100} \)  

b) \( \frac{4}{100} \)  

c) \( \frac{8}{100} \)  

d) \( \frac{7}{100} \)  

e) \( \frac{5}{100} \)

Answers: a) 0.09, b) 0.04, c) 0.08, d) 0.07, e) 0.05

Writing 2-digit hundredths. Write on the board:

\[
\begin{align*}
\frac{83}{100} & = 0.83 \\
\frac{49}{100} & = 0.49 \\
\frac{60}{100} & = 0.60
\end{align*}
\]

SAY: To write decimal hundredths, you have to use both places after the decimal point. If there are more than 9 hundredths, you can write the number of hundredths right after the decimal point. Ask volunteers to show how to write various hundredths (28 hundredths, 4 hundredths, 70 hundredths) as decimals (0.28, 0.04, 0.70). Remind volunteers to put in any missing 0s.

Exercises: Write the hundredths as a decimal.

a) 81 hundredths  

b) 30 hundredths  

c) 6 hundredths  

d) \( \frac{9}{100} \)  

e) \( \frac{74}{100} \)  

f) \( \frac{50}{100} \)
Showing decimal hundredths on a number line. Project onto the board BLM Number Lines Divided into Hundredths. Point out how the number line is counting in tens of hundredths. Point to each appropriate mark in turn, and read the marks: 10 hundredths, 20 hundredths, 30 hundredths, and so on to 90 hundredths. SAY: One hundred hundredths is 1 whole. Ask a volunteer to show where 47 hundredths would be. Demonstrate on the number line how to check the volunteer’s answer by counting by tens to 40, then by ones to 47. Write on the board:

A. 0.34  B. 0.87  C. 0.06  D. 0.50

Have volunteers read out loud the number of hundredths (34, 87, 6, 50), then show the points on the number line.

Exercises: Write the decimals that are marked.

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1

Answers: 0.08, 0.30, 0.60, 0.75

Comparing hundredths on number lines. Remind students that when two numbers are placed on a number line, the number to the right is always greater. Demonstrate this with whole numbers. Draw a number line from 0 to 10 on the board and SAY: 6 is to the right of 5 because 6 is greater than 5. Now write on the board:

0.60  0.75

ASK: Which number is greater? (0.75) SAY: 75 hundredths is more than 60 hundredths because 75 of anything is more than 60 of the same thing. Ask: Which symbol goes between the numbers, < or >? Remind students that the bigger (wider) side points toward the bigger (greater) number. Have a volunteer write the correct symbol (<).

Exercise: Write the marked decimals in order from least to greatest.

0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1

Answers: 0.03 < 0.34 < 0.50 < 0.81

Connect decimal notation to dollar notation. Ask: Where have you seen this kind of notation before, with a dot between numbers? (dollar notation) If some students answer less precisely by saying “money notation,” write 34¢ on the board, and point out that there is no dot between numbers. Point out that the dot in dollar notation makes sense because each cent is one hundredth, or 0.01, of a dollar, so 34 cents is 34 hundredths of a dollar, or 0.34 of a dollar.
Practicing word problems.

a) Ahmed lives 0.85 mi from the library and 0.67 mi from school. Does he live closer to the library or to school?

b) Katie jogged 0.96 mi on Monday and 0.48 mi on Tuesday. On which day did she jog farther?

c) Nomi lives 0.76 mi from school and Jacob lives 0.83 mi from school. Who lives closer to school?

Answers: a) school, b) Monday, c) Nomi

Extensions

1. Write the fraction as tenths, then as a decimal.
   a) \( \frac{1}{2} \) b) \( \frac{1}{5} \) c) \( \frac{2}{5} \) d) \( \frac{3}{5} \) e) \( \frac{4}{5} \)
   Answers: a) \( 0.5 \) b) \( 0.2 \) c) \( 0.4 \) d) \( 0.6 \) e) \( 0.8 \)

2. Write the fraction as hundredths, then as a decimal.
   a) \( \frac{3}{20} \) b) \( \frac{7}{50} \) c) \( \frac{12}{50} \) d) \( \frac{8}{25} \) e) \( \frac{9}{20} \)
   Answers: a) \( 0.15 \) b) \( 0.14 \) c) \( 0.24 \) d) \( 0.32 \) e) \( 0.45 \)

3. Have students look for decimals (less than 1) in the media and write the decimals as fractions.

(MP.5) 4. Is 8 a factor of 1,672? Use one or more of the following tools to check: base ten blocks, paper and pencil, or mental math.

Sample solutions
• I used paper and pencil. \( 1,672 \div 8 = 209 \text{ R } 0 \), so 8 is a factor of 1,672
• I used mental math. I did \( 1,600 \div 8 = 200 \) and \( 72 \div 8 = 9 \), so 1,672 \( \div 8 = 209 \); so 8 is a factor.
• I used base ten blocks. I made 8 rows and there were 2 hundreds blocks and 9 ones blocks in each row. There was nothing leftover, so 8 is a factor.

Whole-class follow-up: Have students discuss the strategies in pairs and then take them up as a class. ASK: Which strategy is fastest? (mental math) What strategy would you use to check if 8 is a factor of 1,376? (paper and pencil) Why? (It always works and it’s faster than base ten blocks)
Introduce equivalent tenths and hundredths as fractions and decimals.

Draw on the board:

```
  .
  .
  .
10 ---- 100
  .
  .
```

Ask: How many tenths are shaded? (3) Fill in the first numerator. Say: Each column is one tenth and three of them are shaded. Ask: How many hundredths are shaded? (30) Prompt: How many hundredths are in each column? (10) So there are 10, 20, 30 hundredths shaded. Fill in the second numerator. Ask: How would you write 3 tenths as a decimal? (0.3) How would you write 30 hundredths as a decimal? (0.30) Write on the board:

```
0.3 = 0.30
```

Say: These are equivalent decimals because they are equal to equivalent fractions.

Exercises: Write two equivalent fractions and two equivalent decimals for the amount shaded.

a) b) c) d)

Answers: a) 5/10 = 50/100 = 0.5 = 0.50, b) 2/10 = 20/100 = 0.2 = 0.20, c) 7/10 = 70/100 = 0.7 = 0.70, d) 4/10 = 40/100 = 0.4 = 0.40
Reading decimals. Although it is correct to read 0.7 as “zero point seven,” it is not correct to read 0.70 as “zero point seventy.” Each digit after the decimal point should be read separately, so 0.70 becomes “zero point seven zero.” Always be sure to correct students who read 0.70 as “zero point seventy.” Students who are allowed to do so are more likely to incorrectly believe that 0.70 is greater than 0.8, since 70 > 8. Ask volunteers to use this way to read various decimals: 0.9, 0.09, 0.90, 0.13, 0.31, 0.03, 0.00003, 0.0000700.

Ordering decimals using hundreds squares. Write on the board:

0.8    0.12

Have volunteers read the numbers aloud. Then project onto the board grid paper (or BLM Squares Divided into Hundredths) and draw two blank hundreds squares on the board. Ask volunteers to shade each decimal above. ASK: Which decimal is larger? (0.8) PROMPT: Which square has more shaded?

Provide students with BLM Squares Divided into Hundredths.

Exercises: Shade and label the decimals. Then compare them.

(MP2)

a) 0.4, 0.30  
   b) 0.08, 0.7  
   c) 0.36, 0.4

Answers: a) 0.4 > 0.30, b) 0.08 < 0.7, c) 0.36 < 0.4

Ordering decimals using number lines. Write on the board:

0.60    0.08    0.34

Ask volunteers to read the numbers as decimals (zero point six zero, zero point zero eight, zero point three four), then as hundredths (60 hundredths, 8 hundredths, 34 hundredths). Point out how reading the numbers as hundredths makes it easy to place the numbers on a number line divided into hundredths, since they can just count the hundredths. Project onto the board BLM Number Lines Divided into Hundredths, and demonstrate counting out 60 hundredths by tens. Then have volunteers show where they would place 8 hundredths and 34 hundredths. ASK: How does the number line show you which number is greater? (the greater number is on the right)

Ask a volunteer to write the decimals in order from least to greatest. Point out that the order makes sense because 8 of anything is less than 34 of the same thing, so 8 hundredths is less than 34 hundredths.

Provide students with BLM Number Lines Divided into Hundredths.

(MP2) Exercises: Mark the decimals on one number line, then write the decimals in order from least to greatest.

a) 0.18, 0.65, 0.39  
   b) 0.84, 0.08, 0.40

Answers: a) 0.18 < 0.39 < 0.65, b) 0.08 < 0.40 < 0.84
Writing decimals as fractions. Remind students that fractions with denominator 10 have one digit after the decimal point and fractions with denominator 100 have two digits after the decimal point. For example, $\frac{3}{10}$ is 0.3, but $\frac{3}{100}$ is 0.03.

Exercises: Write the fraction that the decimal represents.

a) 0.4  
b) 0.04  
c) 0.24  
d) 0.5  
e) 0.87

Answers: a) $\frac{4}{10}$, b) $\frac{4}{100}$, c) $\frac{24}{100}$, d) $\frac{5}{10}$, e) $\frac{87}{100}$

Exercises: Write the letters for the incorrect equations. Then write the correct fraction to go with the decimal.

A. $0.03 = \frac{3}{10}$  
  B. $0.4 = \frac{4}{10}$  
  C. $0.05 = \frac{5}{100}$

D. $0.6 = \frac{6}{100}$  
  E. $0.24 = \frac{24}{100}$  
  F. $0.81 = \frac{81}{10}$

Answers: A, D, F; The correct fractions for the decimals are $\frac{3}{100}$ (A), $\frac{6}{10}$ (D), and $\frac{81}{100}$ (F)

Comparing tenths and hundredths by writing both as hundredths. Write the decimals 0.4 and 0.38 on the board. SAY: It is hard to compare 4 tenths to 38 hundredths because tenths are bigger than hundredths. ASK: How many hundredths is 4 tenths equal to? (40) Is it easier to compare 40 hundredths to 38 hundredths? (yes) Why? (because now we are comparing hundredths to hundredths, and 40 of them is more than 38 of them) Show this on the board:

$$0.4 > 0.38 \text{ because } 0.40 > 0.38$$

Exercises: Write both decimals as hundredths, then compare them.

a) 0.3, 0.25  
b) 0.05, 0.4  
c) 0.76, 0.6

Answers: a) 0.30 > 0.25, b) 0.05 < 0.40, c) 0.76 > 0.60

Word problems practice.

a) John ran 0.3 mi on Monday and 0.24 mi on Tuesday. On which day did he run farther?

b) In Tina’s collection, 0.34 of her stamps are US stamps and 0.2 of her stamps are Canadian stamps. Does she have more US or Canadian stamps?

c) The school is 0.5 mi from the library and 0.48 mi from the skating rink. Which is closer to the school, the library or the skating rink?

Answers: a) Monday, b) US, c) the skating rink
Extensions

1. Predict how to write 1/1,000, 1/10,000, and 1/1,000,000,000 as a decimal.

   **Answers:** 0.001, 0.0001, 0.000000001
   (Students might put commas after every 3 place values in accordance with what is done before the decimal point. This is not the convention, but it is not incorrect thinking.)

2. Make up a word problem that requires comparing 0.4 to 0.26. Ask a partner to solve your problem.

3. Represent 6/10 in as many ways as you can.

   **Sample answers:** 0.6, 3/5, 60/100, 0.60, six tenths, three fifths, sixty hundredths. Students may also write “zero point six” and “zero point six zero.” These answers are correct as ways of orally representing these numbers but should not be encouraged as a way of writing the numbers. Students can also draw pictures that show the amount shaded.

4. Explain how you know that 0.7\(\neq\)0.70.

   **Answer:** 0.7\(=\)7/10\(=\)70/100\(=\)0.70

5. Find a fast way to answer the question below.

   Orange juice comes in cases of 24 cans. The school went through 15 cases in December and 18 cases in January. How many more cans of juice did the school go through in January than in December?

   **Solution:** The school went through 3 more cases in January compared to December (because 18 is 3 more than 15), and each case has 24 cans, so I did 24 \(\times\) 3 \(=\) 72. The school went through 72 more cans of juice in January than in December.

   Redirecting students: If students do not see the fast way, encourage them to find the totals for January and December and directly calculate how many more. When they finish, ASK: How does your answer relate to 24? How could you have predicted that?

   Individual or small-group follow-up: If students do not see how their answer relates to 24, encourage them to list all the factor pairs of 72; they should see that 24 \(\times\) 3 is a factor pair. Once they see that 72 is 24 \(\times\) 3, ask them what 24 \(\times\) 3 means in the situation of the problem. (it’s the same as 3 cases)
Combining Tenths and Hundredths
Decimals and Money

Pages 115–117

STANDARDS
4.NF.C.6, 4.NF.C.7

VOCABULARY
decimal
estimate
fraction
hundredth
ones
tens
tenth

Goals
Students will express hundredths in terms of mixed units (tenths and hundredths).

PRIOR KNOWLEDGE REQUIRED
Can write equivalent tenths and hundredths as fractions
Can write equivalent tenths and hundredths as decimals
Can write the value of dimes and pennies as fractions of a dollar
Can name fractions from area models and number lines

MATERIALS
BLM Number Lines Divided into Hundredths (p. R-39)

Using a picture to show a combination of tenths and hundredths. Draw the first picture below on the board:

ASK: How many hundredths are shaded? (30) How many tenths are shaded? (3) Then shade two more hundredths. ASK: Now how many hundredths are shaded? (32) Summarize by saying that 32 hundredths = 3 tenths and 2 more hundredths. Write on the board:

32 hundredths = 3 tenths 2 hundredths

Exercises: Describe the fraction shaded as hundredths and as tenths and hundredths.

Bonus

Answers: a) 64 hundredths = 6 tenths 4 hundredths, b) 47 hundredths = 4 tenths 7 hundredths, Bonus: c) 85 hundredths = 8 tenths 5 hundredths, d) 68 hundredths = 6 tenths 8 hundredths

Relate tenths and hundredths to money. Remind students that a dime is one tenth of a dollar and a penny is one hundredth of a dollar. SAY: “Zero point seven three dollars” can be represented in different ways:
Exercises: Write the amount in three more ways.

a) 8 dimes 5 pennies  
b) 0 dimes 6 pennies  
c) 9 dimes 0 pennies

Answers: a) 8 tenths 5 hundredths, 85 pennies, 85 hundredths,  
b) 0 tenths 6 hundredths, 6 pennies, 6 hundredths,  
c) 9 tenths 0 hundredths, 90 pennies, 90 hundredths

Relate tenths and hundredths to place value. Tell students that just like there is a ones place and a tens place in whole numbers, there is a tenths place and a hundredths place in decimals. Show this on the board:

68 hundredths = 6 tenths 8 hundredths

SAY: The first digit to the right of the decimal point is the number of tenths, and the second digit is the number of hundredths. Write on the board:

\[
\frac{68}{100} = 0.68
\]

Exercises: Describe the hundredths using the three ways shown above (written on the board).

a) 54 hundredths  
b) 8 hundredths  
c) 37 hundredths

Answers: a) 54/100, 5 tenths 4 hundredths, 0.54;  
b) 8/100, 0 tenths 8 hundredths, 0.08;  
c) 37/100, 3 tenths 7 hundredths, 0.37

Relate tenths and hundredths to number lines. Project onto the board BLM Number Lines Divided into Hundredths. Label the tenths as shown.

Exercises: Write the fraction of the distance from 0 to 1 as hundredths and tenths and hundredths.

Use BLM Number Lines Divided into Hundredths to display these exercises.
Answers: a) 9 hundredths = 0 tenths 9 hundredths, b) 28 hundredths = 2 tenths 8 hundredths, c) 52 hundredths = 5 tenths 2 hundredths, d) 70 hundredths = 7 tenths 0 hundredths

Estimating decimals on number lines. Draw on the board:

Tell students you want to know where to place 0.61. ASK: How many tenths are in 61 hundredths? (6) Is 0.61 closer to 6 tenths or 7 tenths? (6 tenths) SAY: The decimal 0.61 is only one more hundredth than 0.6, but 0.7 is ten more hundredths than 0.6. Ask a volunteer to mark where he estimates 0.61 will be on the number line. Repeat for 0.48 (closer to 0.5 than to 0.4) and 0.95 (equally close to 0.9 and 1).

Exercises: Estimate the location of the decimal on a number line divided into tenths from 0 to 1.

a) 0.75  b) 0.37  c) 0.29  d) 0.94

A centimeter is one hundredth of a meter. ASK: How many centimeters are in 1 m? (100) What fraction of a meter is a centimeter? (one hundredth) Write on the board:

1 cm = 0.01 m    5 cm = _____ m    17 cm = _____ m

SAY: 1 cm is one hundredth of 1 m, so 5 cm is 5 hundredths of 1 m. Have a volunteer write the decimal to show this. (0.05) ASK: What fraction of 1 m is 17 cm? (17/100) Ask a volunteer to write the decimal. (0.17)

Exercises: Write the decimal.

a) 37 cm = _____ m    b) 4 cm = _____ m

A centimeter is one hundredth of a meter. ASK: How many centimeters are in 1 m? (100) What fraction of a meter is a centimeter? (one hundredth) Write on the board:

1 cm = 0.01 m    5 cm = _____ m    17 cm = _____ m

SAY: 1 cm is one hundredth of 1 m, so 5 cm is 5 hundredths of 1 m. Have a volunteer write the decimal to show this. (0.05) ASK: What fraction of 1 m is 17 cm? (17/100) Ask a volunteer to write the decimal. (0.17)

Exercises: Write the decimal.

a) 37 cm = _____ m    b) 4 cm = _____ m

A centimeter is one hundredth of a meter. ASK: How many centimeters are in 1 m? (100) What fraction of a meter is a centimeter? (one hundredth) Write on the board:

1 cm = 0.01 m    5 cm = _____ m    17 cm = _____ m

SAY: 1 cm is one hundredth of 1 m, so 5 cm is 5 hundredths of 1 m. Have a volunteer write the decimal to show this. (0.05) ASK: What fraction of 1 m is 17 cm? (17/100) Ask a volunteer to write the decimal. (0.17)

Exercises: Write the decimal.

a) 37 cm = _____ m    b) 4 cm = _____ m

A centimeter is one hundredth of a meter. ASK: How many centimeters are in 1 m? (100) What fraction of a meter is a centimeter? (one hundredth) Write on the board:

1 cm = 0.01 m    5 cm = _____ m    17 cm = _____ m

SAY: 1 cm is one hundredth of 1 m, so 5 cm is 5 hundredths of 1 m. Have a volunteer write the decimal to show this. (0.05) ASK: What fraction of 1 m is 17 cm? (17/100) Ask a volunteer to write the decimal. (0.17)
Answer: The number lines that are used to answer these questions will be identical, except for the scale, and the points will be marked at exactly the same place on each number line:

The scale will be 0 to 10, 0 to 1, or 0 to 0.1, but that is the only difference. In all cases, the number is closer to 0.

3. Lily enters a contest to run as many miles as she can in two weeks. At the end of the contest, she earns two tickets for each mile she runs. She runs 3/4 mile each day for 5 days in the first week and 3 days in the second week. Lily can pick one prize for every 5 tickets she earns. What is the greatest number of prizes she can pick?

Show your work using a picture. Say what each part of your picture means.

Sample solution: I used number lines. The numbers on the number line show the number of miles. The arrows show how far Lily runs each day.

She ran a total of 6 miles, so she gets 12 tickets \( 2 \times 6 = 12 \). She can pick 1 prize for every 5 tickets, so she can get 2 prizes \( 12 \div 5 = 2 \text{ R } 2 \), and she can’t use the leftover 2 tickets.

NOTE: There are different ways students might determine that Lily ran a total of six miles. For example:

- I drew fraction circles. Since 4 fourths make a whole, I grouped them into groups of 4 and there were 6 groups. Each circle is a day and the shaded part of the circle shows the fraction of a mile Lily runs each day. So Lily ran 6 whole miles.

- I drew fraction circles too, but I used different numbers for different days. Each circle is a whole mile, so she runs 6 miles in the 8 days.
Goals
Students will add two fractions, one with denominator 10 and one with denominator 100.

Prior Knowledge Required
Knows that a dime is one tenth of a dollar
Knows that a penny is one hundredth of a dollar
Can add fractions with the same denominator
Can write tenths as equivalent hundredths
Can name fractions from area models and number lines

Materials
grid paper
BLM Squares Divided into Hundredths (p. R-38)

Review equivalent money amounts in dimes and pennies. Remind students about how to write equivalent statements. Write on the board:

\[
\frac{4}{10} = \frac{40}{100} \quad \frac{3}{10} = \frac{30}{100}
\]

Ask a volunteer to fill in the blanks (both 30).

Equivalent money amounts in mixed units (dimes and pennies). Tell students that you have 2 dimes and 3 pennies. ASK: If I were to trade my dimes for pennies, how many pennies would I have? (23 pennies) Write on the board:

2 dimes and 3 pennies is the same amount of money as 23 pennies.

Have volunteers write the fraction of a dollar under the money amounts: (2/10, 3/100, and 23/100). SAY: 2 tenths and 3 hundredths in total is the same as 23 hundredths. Show the addition:

\[
\frac{2}{10} + \frac{3}{100} = \frac{23}{100}
\]

Exercises: Write a fraction equation for the equal money amounts.

a) 3 dimes and 8 pennies = 38 pennies
b) 7 dimes and 4 pennies = 74 pennies
c) 6 dimes and 7 pennies = 67 pennies

Answers: a) \(\frac{3}{10} + \frac{8}{100} = \frac{38}{100}\), b) \(\frac{7}{10} + \frac{4}{100} = \frac{74}{100}\), c) \(\frac{6}{10} + \frac{7}{100} = \frac{67}{100}\)

Using pictures to write fraction additions. Project onto the board a sheet of grid paper (or BLM Squares Divided into Hundredths).
Draw three hundreds squares and shade the first two as shown (the third one is shaded here to show the answer).

\[
\begin{array}{c}
\begin{array}{c}
+ \\
= \\
\end{array}
\end{array}
\]

Ask a volunteer to shade the third square with the total amount shaded in the first two. Have another volunteer write the fraction that each picture shows. Then finish the equation:

\[
\frac{4}{10} + \frac{3}{100} = \frac{43}{100}
\]

SAY: This is just like dimes and pennies. Just as 4 tenths and 3 hundredths of a dollar is the same as 43 hundredths of a dollar, 4 tenths and 3 hundredths of a square is the same as 43 hundredths of the square.

Provide students with BLM Squares Divided into Hundredths.

**Exercises:** Shade the total in a hundreds square. Then write a fraction addition.

a) \[
\begin{array}{c}
+ \\
= \\
\end{array}
\]

**Bonus:** Make up your own similar question using grid paper, and have a partner solve it.

**Answers:** a) \( \frac{5}{10} + \frac{7}{100} = \frac{57}{100} \), b) \( \frac{8}{10} + \frac{1}{100} = \frac{81}{100} \)

**Writing a decimal addition from the equivalent fraction addition.**

Draw on the board:

\[
\begin{array}{c}
+ \\
= \\
\end{array}
\]

Ask a volunteer to shade the total, another volunteer to write a fraction addition from the picture (\( \frac{3}{10} + \frac{5}{10} = \frac{8}{10}, \frac{3}{10} + \frac{5}{100} = \frac{35}{100} \)), and another volunteer to write a decimal addition from the fraction addition (\( 0.3 + 0.5 = 0.8, 0.3 + 0.05 = 0.35 \)).

Provide students with BLM Squares Divided into Hundredths. You can also use the BLM or grid paper to project these pictures onto the board.

**Exercises:** Shade the total. Write a decimal addition.

\[
\begin{array}{c}
+ \\
= \\
\end{array}
\]

**Bonus:**
Tell students that they can write the fraction addition first if it helps.

**Answers:** $0.7 + 0.03 = 0.73$, Bonus: $0.2 + 0.07 = 0.27$

**Adding tenths and hundredths without pictures.** SAY: You can add tenths and hundredths without pictures by using equivalent fractions.

**Exercises:** Write an equivalent fraction with denominator 100.

a) \( \frac{5}{10} \)  
b) \( \frac{9}{10} \)  
c) \( \frac{6}{10} \)  
d) \( \frac{2}{10} \)

**Selected solution:** a) \( \frac{5}{10} \times \frac{10}{10} = \frac{50}{100} \)

**Answers:** b) \( \frac{90}{100} \), c) \( \frac{60}{100} \), d) \( \frac{20}{100} \)

**Exercises:** Add the hundredths.

a) \( \frac{70}{100} + \frac{5}{100} \)  
b) \( \frac{50}{100} + \frac{6}{100} \)  
c) \( \frac{30}{100} + \frac{9}{100} \)

**Answers:** a) \( \frac{75}{100} \), b) \( \frac{56}{100} \), c) \( \frac{39}{100} \)

SAY: Now you can combine the steps to add tenths to hundredths.

**Exercises:** Add the tenths and hundredths.

a) \( \frac{8}{10} + \frac{3}{100} \)  
b) \( \frac{2}{10} + \frac{9}{100} \)  
c) \( \frac{6}{10} + \frac{4}{100} \)

**Selected solution:** a) \( \frac{8}{10} + \frac{3}{100} = \frac{80}{100} + \frac{3}{100} = \frac{83}{100} \)

**Answers:** b) \( \frac{29}{100} \), c) \( \frac{64}{100} \)

**Exercises:** Add mentally.

a) \( \frac{6}{10} + \frac{7}{100} \)  
b) \( \frac{7}{10} + \frac{1}{100} \)  
c) \( \frac{1}{10} + \frac{3}{100} \)

**Answers:** a) \( \frac{67}{100} \), b) \( \frac{71}{100} \), c) \( \frac{13}{100} \)

**Word problems practice.**

a) \( \frac{4}{10} \) of Ahmed’s stamp collection is US stamps and \( \frac{7}{100} \) of the collection is Canadian stamps. What fraction of his stamp collection are the US and Canadian stamps together?

b) The school is \( \frac{6}{10} \) km from Lina’s house. The library is \( \frac{8}{100} \) km farther. How far is the library from Lina’s house?

**Bonus**

c) The school is \( \frac{4}{10} \) km from Jacob’s house. The library is \( \frac{38}{100} \) km farther. How far is the library from Jacob’s house?

d) \( \frac{8}{10} \) of Lina’s collection is US stamps and \( \frac{5}{100} \) of the collection is Canadian stamps. What fraction of her stamps are from other places?

**Answers:** a) \( \frac{47}{100} \), b) \( \frac{68}{100} \) km, Bonus: c) \( \frac{78}{100} \) km, d) \( \frac{15}{100} \)
Extensions

1. Write the missing number.

   a) \( \frac{2}{10} + \frac{3}{10} = \frac{5}{10} \)
   b) \( \frac{2}{10} + \frac{3}{100} = \frac{23}{100} \)
   c) \( \frac{4}{10} + \frac{1}{10} = \frac{9}{10} \)
   d) \( \frac{3}{10} + \frac{10}{100} = \frac{38}{100} \)

   Bonus

   e) \( \frac{6}{10} + \frac{10}{100} = \frac{67}{100} \)
   f) \( \frac{4}{10} + \frac{10}{100} = \frac{67}{100} \)
   g) \( \frac{7}{100} + \frac{10}{100} = \frac{4}{10} \)

   Answers: a) 5, b) 23, c) 5, d) 8, Bonus: e) 7/100, f) 27/100, g) 33/100

2. Make up a word problem that requires adding \( \frac{5}{10} + \frac{6}{100} \).

3. a) Find something that weighs 5 times as much as your pencil.
   b) What tool did you use to answer part a)? Explain your choice.

   Look for students to attend to precision (MP 6) when they perform the measurements necessary to answer the question.

   Whole-class follow-up: Have students discuss with a partner: Why is a scale a better choice than a pan balance? (a scale tells you the measurement in numbers, so it’s easy to multiply by 5 and look for things that weigh that much; to use a pan balance, I would have to have 5 pencils the same weight as mine and that isn’t easy to find) Then take up the answer as a class. ASK: How did you know to multiply the number by 5? (that’s what 5 times as much means)

4. To find the next term in the sequence 1, 1, 2, 3, 5, 8, 13, …, add the previous two terms.

   a) Find the next three terms.
   b) Decide whether the 100th term is even or odd. Explain your strategy.

   Answers: a) 21, 34, 55; b) the pattern of odd and even terms is odd, odd, even, and this keeps repeating because an odd number and an odd number add to an even number. Also, an odd number and an even number add to an odd number. Only the terms in positions that are multiples of 3 (3rd, 6th, 9th, and so on) are even and the rest are odd. Since 100 is not a multiple of 3 (100 ÷ 3 = 33 R 1, so the remainder is not 0), the 100th term will be odd.
NF4-27  Decimals Greater Than 1

Pages 120–121

STANDARDS
4.NF.C.5

VOCABULARY
decimal
decimal point
denominator
digit
equivalent fraction
improper fraction
mixed number
numerator

Goals
Students will convert between mixed numbers, improper fractions, and decimals greater than 1.

PRIOR KNOWLEDGE REQUIRED
Can write tenths and hundredths as decimals
Can write an improper fraction as a mixed number

Review writing fractions as decimals. Remind students that tenths are written with one digit to the right of the decimal point, and hundredths are written with two digits to the right of the decimal point.

Exercises: Write the fraction as a decimal.

a) \( \frac{6}{10} \)  
b) \( \frac{34}{100} \)

c) \( \frac{8}{100} \)  
d) \( \frac{7}{10} \)  
e) \( \frac{7}{100} \)

Answers: a) 0.6, b) 0.34, c) 0.08, d) 0.7, e) 0.07

Writing mixed numbers as decimals. Tell students that mixed numbers can be written as decimals too. Write on the board:

\[ 2 \frac{3}{10} = 2.3 \]

SAY: The whole-number part is written to the left of the decimal point, and the fraction part is written to the right of the decimal point. Ask a volunteer to write the decimal for \( 2 \frac{4}{100} \) (2.04).

Exercises: Write the mixed number as a decimal.

\begin{align*}
a) \quad & \frac{13}{100} \quad \quad \quad b) \quad \frac{74}{100} \\
& \frac{8}{10} \quad \quad \quad c) \quad \frac{6}{100} \quad \quad \quad d) \quad \frac{83}{100}
\end{align*}

Answers: a) 13.74, b) 8.6, c) 9.06, d) 83.05

Writing decimals as mixed numbers. Write on the board:

\[ 3.14 \]

ASK: What is the whole-number part? (3) What is the fractional part? (14 hundredths) Write on the board:

\[ 3.14 = 3 \frac{14}{100} \]

Exercises: Write the decimal as a mixed number.

\begin{align*}
a) \quad & 2.7 \\
b) \quad & 3.07 \\
c) \quad & 4.80 \\
d) \quad & 235.6 \\
e) \quad & 17.9
\end{align*}

Bonus: 3.801

Answers: a) 2 7/10, b) 3 7/100, c) 4 80/100, d) 235 6/10, e) 17 9/10,
Bonus: 3 801/1,000

Writing decimals in words. Tell students that if they can write whole numbers and fractions in words, then they can write decimals in words too.
SAY: Just like a decimal point separates the whole number part and the fractional part, use the word “and” to separate the whole number from the fractional part. Write on the board:

\[ 3 \text{ and } 17 \text{ hundredths} \]

Exercises: Write the missing words.

a) \( 4.08 = \) __________ and eight hundredths
b) \( 17.6 = \) __________ and six tenths
c) \( 16.5 = \) sixteen and __________ tenths
d) \( 3.07 = \) three and __________ hundredths
e) \( 38.14 = \) thirty-eight and __________ hundredths
f) \( 30.8 = \) thirty and eight __________
g) \( 3.08 = \) three and eight __________
h) \( 4.17 = \) four __________ seventeen __________

Answers: a) four, b) seventeen, c) five, d) seven, e) fourteen, f) tenths, g) hundredths, h) and, hundredths

Exercises: Write the decimal in words.

a) \( 3.8 \)  
b) \( 26.09 \)  
c) \( 30.40 \)  
d) \( 41.5 \)

Bonus: \( 3,000,000.45 \)

Answers: a) three and eight tenths, b) twenty-six and nine hundredths, c) thirty and forty hundredths, d) forty-one and five tenths, Bonus: three million and forty-five hundredths

Writing the decimal for the word. Refer to what you wrote on the board above:

\[ 3 \text{ and } 17 \text{ hundredths} \]

SAY: The “and” tells you where to put the decimal point, and the fraction word tells you how many digits to put after the decimal point.

Exercises: Write the decimal.

a) twelve and thirteen hundredths  
b) fifty and three tenths  
c) two and fifty-three hundredths  
d) two hundred and five hundredths  
e) two hundred five and six tenths

Answers: a) 12.13, b) 50.3, c) 2.53, d) 200.05, e) 205.6

Review writing improper fractions as mixed numbers. Remind students that they can change improper fractions to mixed numbers by dividing. Write on the board:

\[ \frac{7}{3} = 2 \frac{1}{3} \text{ because } 7 \div 3 = 2 \text{ R } 1. \quad \frac{24}{10} = 2 \frac{4}{10} \text{ because } 24 \div 10 = 2 \text{ R } 4 \]
Exercises: Write the improper fraction as a mixed or whole number.

a) \( \frac{31}{10} \)  

b) \( \frac{14}{10} \)  

c) \( \frac{852}{100} \)  

d) \( \frac{73}{10} \)  

e) \( \frac{500}{100} \)

Answers: a) 3 1/10, b) 1 4/10, c) 8 52/100, d) 7 3/10, e) 5

Writing improper fractions as decimals. Write on the board: 382/100. Ask a volunteer to change it to a mixed number (3 82/100), then have another volunteer change the mixed number to a decimal (3.82). Write more improper fractions on the board, with denominators 10 and 100 (example: 45/10, 402/100, 637/100), and ask volunteers to do both steps of converting to a decimal (4.5, 4.02, 6.37).

Exercises: Write the improper fraction as a decimal.

a) \( \frac{28}{10} \)  

b) \( \frac{154}{100} \)  

c) \( \frac{769}{100} \)  

d) \( \frac{61}{10} \)  

e) \( \frac{32}{10} \)

Answers: a) 2.8, b) 1.54, c) 7.69, d) 6.1, e) 3.2

Shortcut for converting improper fractions to decimals. Write on the board the answers to the exercises above:

\[
\begin{align*}
\frac{28}{10} &= 2.8 \\
\frac{154}{100} &= 1.54 \\
\frac{769}{100} &= 7.69 \\
\frac{61}{10} &= 6.1 \\
\frac{32}{10} &= 3.2 \\
\end{align*}
\]

SAY: The numerator tells you what number to write and the denominator tells you how many digits go after the decimal point. Write on the board:

\[
\begin{align*}
\frac{384}{100} &= 3.84 \\
\frac{384}{10} &= 38.4 \\
\frac{43}{10} &= 4.3 \\
\end{align*}
\]

Have volunteers show where to put the decimal point. (3.84, 38.4, 4.3)

SAY: The denominator 10 means 1 digit after the decimal point, and the denominator 100 means 2 digits after the decimal point.

Exercises: Write the decimal.

a) \( \frac{497}{100} \)  

b) \( \frac{84}{10} \)  

c) \( \frac{604}{100} \)  

d) \( \frac{307}{10} \)

Bonus: \( \frac{785,234}{10} \)

Answers: a) 4.97, b) 8.4, c) 6.04, d) 30.7, Bonus: 78,523.4

Writing decimals as improper fractions. SAY: You can go the other way too. Write on the board:

\[
314.08 = \frac{31408}{10}
\]

SAY: Write the number without the decimal point as the numerator. Then write the denominator. ASK: Will the denominator be 10 or 100? (100) How do you know? (there are 2 digits after the decimal point)

Exercises: Write the decimal as an improper fraction.

a) 5.4  

b) 6.07  

c) 80.3  

d) 54.76

Answers: 54/10, b) 607/100, c) 803/10, d) 5,476/100
Introduce equivalent tenths and hundredths. Write on the board:

\[
\frac{7}{10} = \frac{70}{100} \quad \text{so} \quad \frac{3}{10} = \frac{30}{100}
\]

SAY: Seven tenths equals seventy hundredths, so three and seven tenths equals three and seventy hundredths. Ask volunteers to write both fractions and both mixed numbers as decimals:

\[
0.7 = 0.70 \quad \text{so} \quad 3.7 = 3.70
\]

Exercises: Write the equivalent decimal hundredths.

a) 4.8  
   b) \( \frac{5}{10} \)  
   c) \( \frac{50}{100} \)  
   d) 8.7  

Answers: a) 4.80, b) 0.50, c) 0.50, d) 8.70

Exercises: Write the equivalent decimal tenths.

a) 5.80  
   b) \( \frac{4}{10} \)  
   c) \( \frac{90}{100} \)  
   d) 174.30  

Answers: a) 5.8, b) 17.4, c) 6.9, d) 174.3

Extensions

(MP.7) 1. Point out the symmetry in the place values on either side of the ones position. For example, in the number 743.61, the place values are as follows:

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
<td>3</td>
<td>.</td>
<td>6</td>
</tr>
</tbody>
</table>

Challenge students to name the place values in the number 3.612349 by using this symmetry. (ones, tenths, hundredths, thousandths, ten thousandths, hundred thousandths, millionths)

NOTE: Some students might look for a “oneths” position, from thinking of the decimal point as the center of symmetry. This might seem natural because the decimal point is the only part of the number that looks different. However, the ones are the basic units and in fact are the basis for the symmetry.

2. Add whole numbers, tenths, and hundredths.

Example: \( 3 + \frac{8}{10} + \frac{5}{100} = 3.85 \)

3. Have students look for decimals greater than 1 in the media and write the decimals as mixed numbers.
Goals
Students will recognize that comparisons are valid only when the two
decimals refer to the same whole.

PRIOR KNOWLEDGE REQUIRED
Can write tenths and hundredths as decimals
Can compare decimal tenths to decimal hundredths
Can compare fractional tenths to fractional hundredths
Knows how many dimes or pennies make a dollar

MATERIALS
grid paper

Introduce the need to compare decimals using the same whole.
Draw on the board:

Ask volunteers to shade, roughly, 0.5 of the first square and 0.3 of the
second square. ASK: Which square has a greater amount shaded?
(the second square) Does that mean that 3 tenths is more than 5 tenths?
(no) Why not? (you can only compare decimals by comparing the amounts
they represent of the same whole)

Exercise: Draw two squares so that 0.5 of one is more than 0.6 of the other.
Sample answer: The two squares above would work: 0.5 of the second
square is more than 0.6 of the first.

Draw on the board:

Point to each of the last two number lines and, in turn, ASK: Is 0.3 to the
right or left of 0.5 on the first number line? (left on first; right on second) SAY:
If you look only at one number line, 0.5 is always to the right of 0.3 because
0.5 is greater than 0.3. But if you look at two number lines, and one number
line is longer than the other, then the rule breaks down. The number 0.3 on
one number line can be farther to the right than 0.5 on another number line.
Students can use grid paper to do the exercises below.

**Exercise:** Draw two number lines so that 0.2 on one number line is farther to the right than 0.3 on the other. Start with the 0 at the same place.

**Review that a penny is a hundredth of a dollar.** **ASK:** How many pennies make a dollar? (100) What fraction of a dollar is a penny? (1/100) Write on the board:

A penny is worth \( \frac{1}{100} \) of a dollar.

A penny is worth 0.01 dollars.

**SAY:** A penny is worth zero point zero one dollars. This is just another way of saying that a penny is one hundredth of a dollar, since a decimal is just another way of writing a fraction. Point out that this is why we write money notation the way we do:

\[
1\text{¢} = 0.01 \quad \text{so} \quad 13\text{¢} = 0.13
\]

**SAY:** One cent equals one hundredth of a dollar, so thirteen cents equals thirteen hundredths of a dollar.

**Exercises:** Write the cents in dollar notation.

a) 3¢  

b) 18¢  

c) 9¢  

d) 50¢  

**Bonus:** 432¢

**Answers:** a) $0.03, b) $0.18, c) $0.09. d) $0.50, Bonus: $4.32

**ASK:** How many dimes make a dollar? (10) How many pennies make a dime? (10) Hold up a meter stick. **ASK:** How many centimeters make a meter? (100) Have students look at a centimeter ruler. **ASK:** How many millimeters make a centimeter? (10) **SAY:** One of ten equal parts is one tenth, and one of a hundred equal parts is one hundredth.

**Exercises:** Write the fraction.

a) A penny is worth _____ of a dime.

b) A penny is worth _____ of a dollar.

c) A dime is worth _____ of a dollar.

d) A centimeter is worth _____ of a meter.

e) A millimeter is worth _____ of a centimeter.

**Bonus:** A millimeter is worth _____ of a meter.

**Answers:** a) 1/10, b) 1/100, c) 1/10, d) 1/100, e) 1/10, Bonus: 1/1,000

**SAY:** Remember that you can write a fraction as a decimal. So a penny is worth zero point one of a dime because the decimal zero point one is equal to the fraction one tenth. Write on the board:

A penny is worth 0.1 dimes.

Point out that students will never see this in money notation because here we are using a dime as the whole instead of a dollar as the whole.
Exercises: Write the decimal.

a) A penny is worth ____ dollars.
b) A dime is worth ____ dollars.
c) A centimeter is worth ____ meters.
d) A millimeter is worth ____ centimeters.

Answers: a) 0.01, b) 0.1, c) 0.01, d) 0.1

SAY: A penny is worth one tenth of a dime. ASK: What fraction of a dime are two pennies worth? (2/10) What decimal is that? (0.2) SAY: Two pennies are worth zero point two dimes.

Exercises: Write the decimal.

a) 8 pennies are worth ____ dimes.
b) 5 pennies are worth ____ dimes.
c) 13 pennies are worth ____ dollars.
Bonus: 13 pennies are worth ____ dimes.

Answers: a) 0.8, b) 0.5, c) 0.13, Bonus: 1.3

Exercises: How many pennies are worth

a) 0.4 dimes? b) 0.9 dimes? c) 0.7 dimes?
d) 0.2 dollars? e) 0.5 dollars? f) 0.35 dollars?

Answers: a) 4, b) 9, c) 7, d) 20, e) 50, f) 35

Exercises: What is worth more, 0.4 dimes or 0.3 dollars? How do you know?

Answer: 0.3 dollars is worth more than 0.4 dimes because 0.3 dollars is equal to 30 pennies and 0.4 dimes is only equal to 4 pennies.

Extensions

1. Write the decimals, then write bigger or smaller.

   A penny is worth _____ dimes or _____ dollars.
   A penny is a bigger decimal of a __________________________ whole.

   Answers: 0.1, 0.01, smaller

2. a) How many minutes are in 0.1 h?

   b) Molly says 0.3 min is longer than 0.1 h, because 0.3 > 0.1. Is her comparison correct? Explain.

   c) Benito says 0.3 h is longer than 0.1 min. Is he correct? Explain.

   Answers: a) 1/10 of 60 min is 6 min; b) no, 0.1 h is 6 min, which is longer than 0.3 min; c) yes, because even 0.1 h is longer than 0.1 min, so 0.3 h is definitely longer than 0.1 min.

   (MP6)  3. Explain why it makes sense to write 14 cents as 0.14 dollars.

   Sample answer: because 14 cents is 14 hundredths of a dollar or 14/100, and the decimal for that is written as 0.14
Goals
Students will extend their understanding of concepts learned so far.

PRIOR KNOWLEDGE REQUIRED
Can change meters to centimeters, dollars to cents, and pounds to ounces
Can add fractions and mixed numbers with the same denominator
Can convert between decimal fractions and decimals

Adding decimals. Tell students that, just like they can add tenths and hundredths as fractions, they can add tenths and hundredths as decimals.

Exercises: Write these equations with decimals instead of fractions.

a) \(3 + \frac{4}{10} = 3.4\)  
b) \(2 + \frac{9}{100} = 2.09\)

c) \(\frac{2}{10} + \frac{3}{10} = \frac{5}{10}\)  
d) \(\frac{9}{10} - \frac{3}{10} = \frac{6}{10}\)

e) \(3\frac{1}{10} + 2\frac{3}{10} = 5\frac{4}{10}\)  
f) \(1\frac{16}{100} + 5\frac{13}{100} = \frac{29}{100}\)

Answers: a) \(3 + 0.4 = 3.4\), b) \(2 + 0.09 = 2.09\), c) \(0.2 + 0.3 = 0.5\),  
d) \(0.9 - 0.3 = 0.6\), e) \(3.1 + 2.3 = 5.4\), f) \(1.16 + 5.13 = 6.29\)

Exercises: Add, then change the answer to a decimal.

a) \(8 + \frac{9}{10}\)  
b) \(6\frac{4}{10} + 2\frac{5}{10}\)  
c) \(3\frac{20}{100} + 8\frac{63}{100}\)

Answers: a) \(8\frac{9}{10} = 8.9\), b) \(8\frac{9}{10} = 8.9\), c) \(11\frac{83}{100} = 11.83\)

Exercises: Add by changing all the decimals to fractions. Change your answer back to a decimal.

a) \(0.3 + 0.5\)  
b) \(0.51 + 0.32\)  
c) \(0.4 + 0.06\)

d) \(3 + 0.8\)  
e) \(4 + 0.05\)  
f) \(6 + 0.48\)

Answers: a) \(0.8\), b) \(0.83\), c) \(0.46\), d) \(3.8\), e) \(4.05\), f) \(6.48\)

Connecting cents and dollars to other units. Display on the board the conversions between units:

\[1 \text{ m} = 100 \text{ cm}\]
\[1 \text{ lb} = 16 \text{ oz}\]

ASK: Which pair is more like cents and dollars—centimeters and meters, or ounces and pounds? (centimeters and meters) Why? (because there are 100 cents in a dollar, just like there are 100 centimeters in a meter)
Exercises: Convert to centimeters.

a) 2 m  

b) 5 m  

Bonus: 23 m

Answers: a) 200 cm, b) 500 cm, Bonus: 2,300 cm

Exercises: Convert to cents.

a) $2  

b) $5  

Bonus: $23

Answers: a) 200¢, b) 500¢, Bonus: 2,300¢

Exercises: Convert to ounces.

a) 2 lb  

b) 5 lb  

Bonus: 23 lb

Answers: a) 32 oz, b) 80 oz, Bonus: 368 oz

ASK: Is it easier to convert meters to centimeters or to convert pounds to ounces? (meters to centimeters) Which is more like converting dollars to cents? (converting meters to centimeters)

Extensions

1. How many times as great as 0.05 is 0.2? Hint: How many hundredths are in each?

   Answer: 0.2 = 20 hundredths and 0.05 = 5 hundredths, so 0.2 is four times as great as 0.05.

2. Write the correct decimal.

   $40 + 3¢ = $ _______ . _______ 

   Answer: $40.03

3. There is 3 L of water in a pitcher. Evan pours the same amount of water into 9 glasses. Afterwards, there is 435 mL of water left in the pitcher. How much water did he pour into each glass?

   Show your work using equations. Say what each equation means in the situation.

   Answer: There is 3,000 mL of water in the pitcher. After he poured the water, there was 435 mL left, so I subtracted 3,000 – 435 = 2,565. Evan must have poured out 2,565 mL of water altogether. Since he poured the same amount into 9 glasses, I did 2,565 ÷ 9 = 285. So, Evan poured 285 mL of water into each glass.
## Money-Matching Memory Game

<table>
<thead>
<tr>
<th>$0.75</th>
<th>75¢</th>
<th>$7.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>750¢</td>
<td>20¢</td>
<td>$0.20</td>
</tr>
<tr>
<td>200¢</td>
<td>$2</td>
<td>$1</td>
</tr>
<tr>
<td>1¢</td>
<td>100¢</td>
<td>$0.01</td>
</tr>
<tr>
<td>$2.02</td>
<td>$2.20</td>
<td>22¢</td>
</tr>
<tr>
<td>202¢</td>
<td>220¢</td>
<td>$0.22</td>
</tr>
</tbody>
</table>
Squares Divided into Hundredths
Number Lines Divided into Hundredths
This Unit in Context

In Grade 3, students estimated and measured lengths in centimeters, meters, inches, feet, and yards (3.MD.A.2). In 4.1 Unit 6, students estimated and measured lengths in centimeters, millimeters, meters, and kilometers, and they converted from larger units to smaller units within the metric units studied (4.MD.A.1). In this unit, students will estimate and measure lengths in inches, feet, and yards, and they will convert from larger units to smaller units, including from mixed units to smaller units, within the US customary units of length studied (4.MD.A.1). Students will measure to the nearest eighth of an inch (4.MD.B.4). In Grade 5, students will convert among different-sized standard measurement units within a given measurement system, including from smaller to larger units and conversions that result in decimals or fractions (5.MD.A.1). In higher grades, students will be required to convert between units of area within the same measurement system. For example, in Grade 7, when students study scale drawings (7.G.A.1), they will need to convert between measurements given in square yards to square feet.

In Grade 3, students were introduced to area as a measurement attribute of plane figures (3.MD.C.5). Just as they did in Grade 3, students will measure in square centimeters, square meters, square inches, and square feet, and they will understand that the area of a rectangle is the same as would be found by multiplying the side lengths (3.MD.C.7a). In Grade 3, students learned to look at division as a missing factor problem (3.OA.B.6); in this unit, students will apply that knowledge to find the width of a rectangle given its area and length (4.MD.A.3).

Students will also find areas of composite shapes and apply the area and perimeter formulas for rectangles in real-world, mathematical problems (4.MD.A.3). Building on this work with area and composite shapes, students in Grade 6 will find the areas of triangles and special quadrilaterals. They will do so by composing and decomposing them into familiar shapes and calculating the area of those shapes (6.G.A.1).

In order to read and draw line plots, students will use their knowledge of representing numbers on number lines from Grade 2 (2.MD.B.6) and representing fractions on number lines from Grade 3 (3.NF.A.2). Between Grades 3 and 5, the concept of scale on a line plot extends mainly through changes to the horizontal scale: the horizontal scale on a line plot in Grade 3 may be marked off using whole numbers, halves, or fourths (3.MD.B.4); and in Grades 4 and 5, it may be marked off using whole numbers, halves, fourths, or eighths (4.MD.B.4 and 5.MD.B.2). This work on data management in Kindergarten through to Grade 5 prepares students for the study of statistics from Grades 6 through to high school.
Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP.1 to MP.7. Here are some examples of how students can show that they have met a standard.

**MP.1:** In MD4-39 Extension 3, students make sense of a non-routine problem that involves comparing the areas of a 4 1/2 cm by 4 1/2 cm square with the area of a 4 cm by 5 cm rectangle. Since students do not learn in Grade 4 to multiply fractions by fractions, they persevere to find a way to solve the problem without computing the areas.

**MP.4:** In MD4-32 AP Book Question 9, students model mathematically a real-world situation when they use the information that the cloth for a curtain needs to be three times as wide as the window to determine the width of the curtain in inches when given the width of the window in feet and inches.

**MP.5:** In MD4-34 Extension 3, students choose tools strategically to find the area of a rectangle given its perimeter and the multiplicative relationship between its length and width.

**MP.6:** In MD4-35 AP Book Question 6. part g), students attend to precision when they describe the difference between perimeter and area.
Unit 8 Measurement and Data: US Customary Units and Area

Introduction

In this unit, students will learn about customary US units of length and about area. As they convert the units of length, students will add, subtract, multiply, and divide numbers, including fractions. For details, see the prior knowledge required for each lesson.

Students will generate data to make line plots and will construct and analyze line plots from already generated data. The data can be given in whole number units or in fractions of a unit (1/2, 1/4, 1/8).
**Goals**

Students will develop understanding of the size of an inch, estimate and measure length in inches, and draw line segments and objects of given length in inches, to the closest inch.

**VOCABULARY**

- estimate
- inch (in)
- line segment
- perimeter

**STANDARDS**

4.MD.A.1

**PRIOR KNOWLEDGE REQUIRED**

- Understands the concepts of linear measurement and estimation
- Understands that we use standard units of the same size to measure
- Can estimate and measure in standard units such as centimeters
- Can draw a line segment to the closest centimeter

**MATERIALS**

- inch rulers
- collection of objects to use as benchmarks for inches (see below)
- objects to measure
- pattern blocks
- dice

**Introduce inches.** Remind students that an inch is a unit of measuring length used in many countries of the world, including the United States. Have students identify inch marks on their rulers. Write the word “inch” and the abbreviation “in” on the board. Explain that these are the two ways to write “inch.”

Give students a number of small items (e.g., tacks, buttons, pattern blocks, etc.) that are close to 1 inch long, and several that are exactly 1 inch. Have students use a ruler to check which items are exactly 1 inch long (or wide or thick).

**Using fingers for estimating.** Remind students that estimating means guessing the length of an object or distance, producing a number as close to the actual measurement as possible. Explain that students have a convenient tool for estimating length in inches. Two fingers together are about 1 inch wide. Have students measure the width of various pairs of touching fingers to find which pair is closest to 1 inch, and therefore is best for estimating (students with really small hands might try to use three fingers together). Point out that a finger is slightly larger than a centimeter, but 2 fingers might be a little smaller than an inch.

Remind students that they used fingers to estimate length in centimeters. Demonstrate how they could use pairs of fingers to estimate the length of an object (e.g., a book, a pencil, etc.) in inches. Point out that an inch is a unit of measurement that is larger than a centimeter or millimeter. This means that many objects will have a length between the exact numbers of inches. When an object measures, say, between 5 and 6 fingers, we would
estimate the length to the closest inch. The length is closer to 6 fingers (3 inches) than to 4 fingers (2 inches), so we estimate “about 3 in.” If the object measures exactly 5 fingers, we would round up, saying that it is about 3 inches long.

Have students select at least two objects from their desks or backpacks, measure the objects with their fingers, then complete the following sentence for each one:

______________ is about ___ in long.

Measuring in inches. Explain that using an inch ruler is similar to using a centimeter ruler: Students can count the hops to tell how many inches long an object is. (See Lesson MD4-1 in the TR.) Remind students to align one end of the object to the zero mark, which might not be at the very edge of the ruler. Remind students that line segments are pieces of straight line that can be measured. Draw a ruler on the board and mark line segments in whole inches beside it. Have students signal the length of the line segment by holding up the number of fingers equal to the length.

Measuring to the closest inch. Draw students’ attention to the half-inch marks on a ruler. Remind students how they used half-pound marks on a scale, and explain that they can use the half-inch marks in the same way to decide whether the measurement should be rounded to the smaller or to the larger number. Have students use the ruler to measure the items they estimated earlier to the closest inch.

Remind students that the distance around a shape is called its perimeter. Assign pp. 124–125 in the AP book.

Drawing a line segment of given length. Draw a line segment on the board. Invite a volunteer to measure it. Review the steps students used in Lesson MD4-4 in the TR to draw a line segment of a given length in centimeters. Adjust the steps to use inches.

Step 1: Draw a small vertical dash to mark where the line segment starts. Place the ruler so that the dash is at the zero mark.

Step 2: Count forward from 0 by hops (two hops for 2 inches, three hops for 3 inches, etc.). Make sure not to move the ruler.

Step 3: Draw a second vertical dash to mark the other end of the line segment. (For 2 inches, draw the dash at the 2 inch mark.)

Step 4: Draw a line segment connecting the dashes.

Point out that when students are working on Questions 6 and 7 on AP Book 4.2 p. 126, the rulers are already drawn, so Step 1 has to be performed differently: Students need to make the first vertical dash at the 0 mark of the picture of a ruler.

Have students work through Questions 6–8 on AP Book 4.2 p. 126.
**Drawing objects of a given length.** Ask students to draw a spoon that is 4 inches long, then measure the result. Did everyone get the right length on the first try? (no) Did anyone get the right length on the first try? Point out to students that when they need to draw an object of a given length, such as a spoon 4 inches long, it makes sense to make the vertical dashes for the given length first, and then make sure the drawing extends from one dash to the other. Have students practice drawing objects to a given length.

**Exercises:** Draw the object to the given size.

a) a shoe 3 inches long  
b) a tree 5 inches tall  
c) a door 2 inches wide

**Bonus:** Draw a house 4 inches wide and 3 inches tall.

**Activity**

Students work in pairs to practice measuring. Each partner rolls a die, then draws an object of their choice to match that length in inches. Partners exchange drawings and measure them.

**Extensions**

1. Have students make a figure from pattern blocks and find its perimeter. (MP.4)

2. Nina makes orange juice by adding 3 cans of water for each can of orange juice mix. Nina has 5 cans of juice mix with 255 mL each. How much orange juice can she make?

   **Sample answer:** Each can has 255 mL of juice mix in it, and there are 5 cans, so I found $255 \times 5 = 1,025$ mL. She adds 3 cans full of water, so I found $3 \times 1,025 = 3,075$. The amount of liquid is 1,025 mL from the cans of juice mix and 3,075 mL from the water. So in total the amount of orange juice is 4,100 mL (because $1,025 + 3,075 = 4,100$), or 4 L 100 mL.
**MD4-26  Quarters of an Inch**

Pages 127–129

**STANDARDS**

4.NF.B.3, 4.NF.B.4

**VOCABULARY**

denominator
fraction part
inch (in)
line segment
mixed number
number line
quarter

**Goals**

Students will measure and draw line segments and objects of a given length in inches, to the closest quarter of an inch.

**PRIOR KNOWLEDGE REQUIRED**

Can measure in inches (to the closest inch)
Is familiar with mixed numbers
Can order mixed numbers (with denominators 2 and 4)
Can read and create number lines with mixed numbers (with denominators 2 and 4)
Can draw a line segment of given length in whole inches, centimeters, or millimeters

**MATERIALS**

inch rulers or quarter-inch rulers only or BLM Quarter-Inch Rulers
(p. S-48)
objects to measure

---

**Review number lines with fractions.** Draw a number line resembling an inch ruler as shown in the margin. Leave enough space between the marks to write the mixed numbers. ASK: How many pieces did I divide each whole into? (4) Invite volunteers to mark the fractions between 0 and 1: 1/4, 2/4, 3/4. Remind students that there is another way to write 2/4 with a smaller denominator. What is it? (1/2) Erase 2/4 and write 1/2 instead. Then tell students you need to label the markings between 1 and 2 in a similar way. ASK: How should the labels between 1 and 2 be different from the labels between 0 and 1? (the labels between 0 and 1 are fractions and the labels between 1 and 2 are mixed numbers) Invite volunteers to mark the rest of the number line.

Extend the line to 3 and ask students what the marks should be. Then have students draw a number line one unit long that is divided into quarters as shown in the margin. Have students mark the whole numbers as 5 and 6 and fill in the mixed numbers between 5 and 6.

**Identifying marks for mixed numbers on a number line with only whole numbers labeled.** Present several parts of number lines, for example, from 4 to 5. Give students a mixed number, such as 4 1/4, and ask which mark indicates that number. Point to the marks in turn and have students signal thumbs up if this is the right mark, and thumbs down if it is not. Repeat with several numbers.

**Measuring in quarters of an inch.** Remind students that when they measured length in millimeters, they counted in centimeters first to the last full centimeter mark before the length of the object being measured, then...
they counted the millimeters. Explain that you work with inches in a similar way: Count whole inches first, and when the next whole inch goes over the length of the object, count how many quarter-inch pieces you have. For example, if you have three quarter-inch pieces covered, as in the picture in margin, then the fractional part of the measurement is $3 \times \frac{1}{4} = \frac{3}{4}$ of an inch, and the total length is $5\frac{3}{4}$ inches. Point out that the mark for half an inch is longer than the quarter-inch marks. This helps to distinguish between the $\frac{1}{4}$ mark and the $\frac{3}{4}$ mark.

**ACTIVITY**

If available, students use rulers with only quarter-inch marks. If not, they can use BLM Quarter-Inch Rulers. Students work in groups of three. Each student draws a line segment starting at 0 and ending at one of the marks between the whole inch marks. Students then pass their notebook to another student in the group. Next, they measure the line segments their partners drew, and write the length of the line segment. Students then pass the notebook to the same student they did before, so that each student in the group of three gets a new notebook. They check the work of the previous partner before returning the notebook to the owner and starting the cycle again.

Present the following problem: Five tacks each measuring $\frac{3}{4}$ of an inch are laid end to end. What is the total length of the tacks? Have students draw a picture of five tacks laid end to end, to scale, and find the answer. (3 $\frac{3}{4}$ inches)

**Bonus:** How long is the snail in the picture below? Use the picture to find the answer, then write a subtraction equation.

Answer: 2 1/2 in, equation: $3 \frac{1}{4} - \frac{3}{4} = 2 \frac{1}{2}$

**Measuring to the nearest quarter of an inch.** Point out that even though quarters of an inch are smaller than whole inches, objects often have lengths that are between two quarter-inch marks. Draw a ruler with quarter-inch marks on the board, and draw a line segment that ends between 2 1/4 and 2 1/2 inches, closer to the 2 1/4 mark. Have students think, based on their previous measuring experience, how they should record the length of
this line segment. Have students explain their ideas. (the line segment is closer to 2 1/4 in than to 2 1/2 in, so we should record its length as 2 1/4 in) Provide several more examples, then have students measure actual objects to the closest quarter of an inch, using rulers with quarter-inch marks only. Students can work in pairs, checking each other’s work.

When students are working on the questions in the AP book, note that the answer for Question 2 Bonus on AP Book 4.2 p. 127 is “race.”

Extensions

1. Draw three line segments each 3 1/4 inch long. Extend the line segments by the given length. What is the total length of the line segment?
   a) \(\frac{1}{4}\) inch  
   b) \(1 \frac{1}{4}\) inch  
   c) \(\frac{1}{2}\) inch
   
   Hint: \(1/2 = 2/4\)

   Answers: a) 3 1/2 in, b) 4 1/2 in, c) 3 3/4 in

   (MP1, MP4)

2. Don wants to make fruit salad for a party using the following recipe:
   1 1/4 lbs bananas, 2 2/4 lbs oranges, and 1 lb mangoes.

   He will make four times as much fruit salad as the recipe calls for. What is the total amount of fruit salad, in pounds, that Don will make? Show your work.

   Sample answer: I multiplied each ingredient by 4, and then I added the totals: 5 lbs bananas + 10 lbs oranges + 4 lbs mangoes, so Don will make 19 lbs of fruit salad altogether.

   Whole-class follow-up: Take up different strategies that students used. Compare strategies. For example, if some students used a picture, ASK: How does this picture show a pound of fruit? How does this person’s picture show that person’s addition? How does this person’s picture show that person’s multiplication? Which way is fastest? Which way is easiest to understand? Which way is easiest to come up with?

   NOTE: The sample answer above uses structure (MP7), for example, by noticing that each ingredient can be multiplied separately. A student might also notice, without needing to calculate, that the amount of oranges will be double the amount of bananas.
MD4-27  Eighths of an Inch
Pages 130–133

STANDARDS
4.NF.B.3, 4.NF.B.4, preparation for 4.MD.B.4

VOCABULARY
denominator
eighth
fraction part
half
inch (in)
line segment
mixed number
quarter

Goals
Students will measure and draw line segments and objects of a given length in inches, to the closest eighth of an inch.

PRIOR KNOWLEDGE REQUIRED
Can measure in inches (to the closest quarter of an inch)
Can order mixed numbers (with denominators 2, 4, and 8)
Can read and create number lines with mixed numbers (with denominators 2, 4, and 8)
Can draw a line segment of a length in inches, in a mixed number with denominator 2 or 4

MATERIALS
inch rulers
objects to measure
play pennies (optional)

Review number lines with fractions. Draw a number line from 0 to 2, as shown in the margin. Leave enough space between the marks to write the mixed numbers. ASK: How many pieces did I divide each whole into? (8).

Have students draw a number line from 4 to 5, divide it into eighths, and fill in the mixed numbers between 4 and 5.

Identifying marks for mixed numbers on a number line with only whole numbers labeled. Present several parts of number lines, for example, from 4 to 5. Give students a mixed number, such as 4 3/8, and ask which mark indicates that number. You can point to the marks in turn and students can signal thumbs up if this is the right mark, and thumbs down if it is not. Repeat with several numbers.

Measuring in eighths of an inch. Review measuring to the nearest quarter of an inch, and explain that measuring to the closest eighth of an inch is similar—you count whole inches first, then eighths after. Have students use the Activity in Lesson MD4-26 in the TR to practice measuring to the nearest eighth of an inch. This time, students need to draw line segments starting at 0 and ending at one of the marks for the eighths of an inch.

Exercises: Draw a picture to scale, then measure it.

(MP.4)

- A paper clip is 1 1/4 in long. Three paper clips are laid end to end. What is the total length of the paper clips?
- Six ants each measuring 5/8 in long are walking in a line. How long is the line of ants?

NOTE: If students’ rulers have marks for sixteenths of an inch, make sure to teach students to distinguish the marks for eighths before measuring. See Extension.
c) Ron made a chain from two paper clips each measuring 1 7/8 inches long and three paper clips each measuring 1 3/8 inches long. What is the total length of the chain?

**Bonus:** What is the length of the lizard?

Answers: a) 3 3/4 in, b) 3 3/4 in, c) 7 7/8 in, Bonus: 2 2/8 in = 2 1/4 in

**Measuring to the nearest eighth of an inch.** Point out that even though eighths of an inch are rather small, objects often have lengths that are between two eighth-inch marks. Provide several examples of measuring objects to the closest eighth of an inch, then have students measure actual objects to the closest eighth of an inch. Students can work in pairs, checking each other's work.

Students can use play pennies to check their answers to Question 8, part b) on AP Book 4.2 p. 133.

**Extension**

**Identifying marks for halves, quarters, and eighths on a number line with sixteenths marked.** Point out that the rulers students use have more than eight marks between the whole inches. ASK: How many parts is each inch divided into on your rulers? Give students time to count. (16) Write the pattern shown in the margin on the board and have students fill in the first three blanks. ASK: What pattern do you see in the denominators? 
(2, 4, 8) How do we get each next number from the previous one? (double it) Remind students that taking half of a fraction means dividing each part of the fraction into two parts, so the number of parts doubles. What is the double of 8? (16) What should the next fraction be? (1/16)

Explain that if each inch is divided into 16 parts on the ruler, this means that each eighth of an inch is divided into two parts. Since students need to measure to the closest eighth of an inch, they need to look at the marks for the eighths, not the sixteenths of an inch. Point out that the sixteenths are the shortest marks on the ruler. Display an enlarged ruler with marks the same as on the rulers students use. Point to several marks in turn, and have students identify whether they are marks for the eighths, quarters, or halves of an inch (students can show thumbs up or thumbs down). Repeat with just eighths, just quarters, and just halves. Finally, mark the part of the ruler on the board starting at 3 inches and ending at 4 inches. Have students identify whether these marks show 3 1/2, 3 7/8, 3 1/8 inches, etc. You can point to different marks in turn, including the marks for sixteenths, and have students signal thumbs up or down whether this is the mark for the measurement you've given.
MD4-28 Feet
MD4-29 Feet and Inches
Pages 134–135

GOALS
Students will develop an understanding of the size of 1 foot, measure and estimate distances and lengths in feet, and convert feet to inches.

PRIOR KNOWLEDGE REQUIRED
- Can measure length and distance to the closest unit
- Knows that an inch is about 2 fingers wide
- Understands the concept of estimating length and distance
- Can multiply by 12
- Can use place value to perform multiplications such as 50 × 12

MATERIALS
- rulers
- yardsticks
- measuring tape for every student

INTRODUCE FEET. Identify the length and marking of a foot on a ruler, a yardstick, or measuring tape. Explain that a foot is another unit of measurement. Write the word “foot” and the abbreviation “ft” on the board, and explain that these are two ways to write “foot.” Explain that the name comes from the length of a foot of an adult man, from heel to toe. The plural of “foot” is the same as for the part of the body—“feet.”

INTRODUCE THE LENGTH OF ONE FOOT. Remind students that they used their fingers to estimate small lengths, and that 2 fingers together are about as wide as an inch. To check how many inches long something the size of a book is, they could measure the book with their fingers. Would it be convenient to measure something as long as a blackboard the same way? Why not? (it is too long) Point out that if they want to estimate the length of the blackboard in feet, they need something that is about a foot long. Their own feet are probably too small for the task, but they can use their arms.

ACTIVITY
Developing a personal estimate for 1 foot. Students work in pairs, measuring the length of each other’s arms. To make the measurement easier and more precise, have students whose arm is being measured bend their arm in front of them. Students measure the arms first from elbow to wrist, then elbow to knuckles, then elbow to fingertips. Which of these three lengths is closest to a foot? Point out that the answer is different for different students, but this way, they each have their own personal tool for estimating in feet.
Estimating length in feet. Ask a volunteer to explain how to make and record an estimate of an object that has a length between two units. Have students use their arms to measure objects such as a blackboard, a long desk, and a wide shelf to get an estimate of length. Then have students use rulers, yardsticks, or measuring tapes to actually measure the objects to the closest foot.

Checking how many inches are in one foot. Give each student or pair of students a measuring tape showing both feet and inches. ASK: How does a measuring tape show feet and inches at the same time? How can you tell how many inches are in a foot? The answer will depend on the measuring tape used. Some have measurements in feet and inches written beside the hash mark for feet, and some have only the feet measurement written. If some of the tapes you are using are of the second type, students will need to look at the numbers on both sides of the 1 foot mark: 11 inches before, and 13 inches after, so 1 ft = 12 in.

Converting feet to inches. Draw the table at left on the board. Have students copy the table, then use their measuring tapes to fill in the empty column. Extend the table by three rows and have students use the pattern to fill in the table. ASK: What pattern did you use to fill in the right-hand column? (skip counting by 12, starting from 12; students might also notice that the numbers are multiples of 12) Why did you skip count by 12 and not by any other number? (1 ft = 12 in) How can you get the number in the right-hand column from the number in the same row, but in the left-hand column? (multiply by 12) PROMPT: To get the number of millimeters in 7 cm, for example, you multiply by 10. Where does the 10 come from? (10 mm = 1 cm) Leave the table on the board for further reference. Have students multiply by 12 to convert several measurements from feet to inches.

<table>
<thead>
<tr>
<th>feet</th>
<th>inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Exercises

a) 10 ft  
b) 12 ft  
c) 40 ft  
d) 95 ft

Answers: a) 120 in, b) 144 in, c) 480 in, d) 1,140 in

Comparing measurements. Remind students that when they compare measurements in different units, they need to first convert the measurement in the larger unit to the smaller unit.

Provide several pairs of measurements and have students convert the measurement in the larger unit into the smaller unit. Students can then signal which measurement is larger by using their thumbs to point in the direction of the larger measurement.

Exercises

a) 10 ft or 100 in  
b) 12 in or 3 ft  
c) 43 ft or 600 in  
d) 250 in or 21 ft

Bonus: 1,000 ft or 100,000 in

Answers: a) 10 ft > 100 in, b) 12 in < 3 ft, c) 43 ft < 600 in, d) 250 in < 21 ft, Bonus: 1,000 ft < 100,000 in
Comparing measurements between other measurements. Write these pairs of measurements on the board:

- 2 ft and 3 ft
- 3 ft and 4 ft
- 4 ft and 5 ft

ASK: Is 50 inches less than 2 feet? (no) How do you know? (2 ft is 24 in, 50 > 24) Is it less than 3 feet? (no, 3 ft is 36 in) Continue with similar questions until reaching the correct interval. (between 4 ft and 5 ft) Then ask students to write a measurement in each of the intervals on the board. Students can use the table of conversions produced earlier in the lesson for help. Repeat the exercise with 7 feet and the intervals below.

- 70 in and 80 in
- 80 in and 90 in
- 90 in and 100 in

**Answer:** 7 ft = 84 in, so it is in the second interval

**Extensions**

1. a) Using tape, make a line segment on the floor exactly 1 foot long. Measure your foot against the line segment. Is your foot longer than a foot, shorter than a foot, or exactly 1 foot long?

   b) Using tape, make a very long line on the floor, and make marks of 0 ft, 1 ft, 2 ft, 3 ft, etc., up to 10 ft. Measure the line with your feet and try to find a mark on the line that is an exact number of your feet away from the 0 ft mark. Make a table with headings “Distance (my feet)” and “Distance (ft).” See the example in the margin.

   c) Use the table you created to estimate distances. For example, measure the width of the school corridor in your feet.

   **Example:** The school corridor is 22 of my feet wide. I know that 20 of my feet is 12 ft, so the school corridor is about 12 ft wide.

2. a) A snail is sitting on a branch. The snail is 2 feet from the tree trunk. It crawls 3 inches farther away from the trunk. How far from the trunk is the snail now?

   b) Another snail is sitting on the same branch. The snail is 3 feet from the trunk. It crawls 4 inches toward the trunk. How far from the trunk is the second snail now?

   c) How far are the snails from each other?

   **Answers:** a) 27 in, b) 32 in, c) 5 in
MD4-30  Measuring in Feet and Inches
Pages 136–137

STANDARDS
4.NBT.B.5, 4.MD.A.1

VOCABULARY
convert
estimate
foot (ft)
inch (in)
mixed measurement
round

Goals
Students will measure and estimate distances in feet and inches, and convert between mixed measurements and measurements in inches or feet to the closest foot.

PRIOR KNOWLEDGE REQUIRED
Can measure length and distance to the closest eighth of an inch
Can estimate in inches and in feet
Is familiar with mixed measurements in meters and centimeters
Is familiar with < and > signs

MATERIALS
objects to measure
two dice per student
yardsticks or measuring tapes

Compare measuring in feet and inches to measuring in meters and centimeters. Remind students that when they wanted to use meters to measure a large object, such as a blackboard, they got an answer between, say, 2 m and 3 m. SAY: Imagine you need to check whether this blackboard will fit between, for instance, a window and the end of the wall. Would saying that the board is 2 m long to the closest meter help? (no) What would you need to do? (measure the precise length in meters and centimeters) Explain that when working with feet and inches, you need to do exactly the same thing: Measure in feet first, and then measure the leftover in inches. Remind students that a measurement that is a combination of two units, such as 2 m 35 cm or 3 ft 7 in, is called a mixed measurement.

Estimating in feet and inches. Remind students that they can estimate length using their arms and fingers. The length of the arm from elbow to knuckles is about one foot, and the width of two fingers is about one inch. Show students how they could estimate the length of an object, such as the width of a desk. They will need to use their arms to estimate in feet and, when another arm will not fit, they can use their fingers to measure the leftover. Explain that we first write the number of whole feet that fit along the object, such as 3 ft, then the number of inches in the remaining length. Have students estimate several lengths in the classroom in feet and inches, such as the width and the height of their desks and chairs.

Measuring in feet and inches. Demonstrate the process of measuring in feet, then in inches, using a yardstick or a measuring tape. Then have students measure the objects they estimated earlier in feet and inches, to the closest inch. Students can record their work in Questions 2 and 3 on p. 136 of the AP Book.
Point out that sometimes the measurement for an object can have a mixed number in the inches part if we need greater precision than measuring to the nearest inch. For example, a shelf could measure 2 ft 10 3/4 inches wide. Ask students to look at the objects they measured. Were any of the measurements they recorded rounded to the closest inch? Have students measure the same objects again, this time recording the measurement to the closest eighth of an inch.

**Converting mixed measurements to measurements in inches.** Draw students' attention to how inches are indicated on a measuring tape for measurements longer than 1 foot. Many measuring tapes give the length in inches, such as 15 inches, instead of (or together with) 1 ft 3 in. Have students think of how to convert a mixed measurement, such as 3 ft 4 in, into a measurement in inches only. They can discuss the ideas in pairs, then in groups of four. To prompt students to think of the method shown below, ask them to recall what they did with other units of measurement, such as centimeters and meters, or liters and milliliters.

To convert a mixed measurement such as 3 ft 4 in to inches:

**Step 1:** Convert feet to inches.

\[
3 \text{ ft} = 3 \times 12 \text{ in} = 36 \text{ in}
\]

**Step 2:** Add the leftover inches.

\[
36 \text{ in} + 4 \text{ in} = 40 \text{ in}
\]

So, 3 ft 4 in = 40 in

**Exercises:** Convert the mixed measurement to inches.

a) 7 ft 3 in  
b) 2 ft 6 in  
c) 5 ft 8 in  
d) 38 ft 2 in  
e) 14 ft 9 in  
f) 29 ft 11 in  
g) 50 ft 7 in  
h) 1,000 ft 10 in  
i) 2 ft 10 \frac{3}{8} \text{ in}

**Answers:**
a) 87 in, b) 30 in, c) 68 in, d) 458 in, e) 177 in, f) 359 in, 
g) 607 in, h) 12,010 in, i) 34 \frac{3}{8} \text{ in}

Encourage struggling students to practice the steps separately, then combine them. A conversion table from feet to inches may also be helpful.

**Converting a measurement in inches to a measurement in feet, to the closest foot.** Review the notation for “less than” and “greater than” using the signs < and >.

Ask students to make a conversion table from feet to inches, and to extend it to 10 ft. ASK: Which two measurements in feet is 20 inches between? (1 ft and 2 ft) Record that on the board:

\[
1 \text{ ft} = 12 \text{ in and } 2 \text{ ft} = 24 \text{ in, so } 1 \text{ ft} < 20 \text{ in} < 2 \text{ ft}
\]
ASK: Which measurement is 20 inches closer to, 1 ft or 2 ft? (2 ft) How do you know? (20 is closer to 24 than to 12: \(24 - 20 = 4\), \(20 - 12 = 8\))

**Exercises:** In the table, circle the measurement in feet that is closest to the measurement in inches.

- a) 40 in
- b) 80 in
- c) 42 in

**Answers:** a) 3 ft, b) 7 ft, c) exactly in the middle between 3 ft and 4 ft, so round up as is always done in measurement

Have students use the activity below to practice both converting from mixed measurements to inches and from inches to the closest foot. Discuss ways to check the answer before starting the activity (if the number of inches in the mixed measurement is 6 or less, then the measurement is rounded down; if the number of inches is more than 6, then it is closer to the next number of feet, so round up).

**ACTIVITY**

Students work in pairs. Each pair will need two dice. Player 1 rolls the dice twice to obtain a mixed measurement in feet and inches. The total of the first roll is the number of feet. The total of the second roll is the number of inches. If the total of the second roll is 12, students need to add 1 foot to the total of feet, since 12 in = 1 ft. Player 1 then converts the measurement produced this way to a measurement in inches and passes the result to Player 2. Player 2 converts the measurement to feet, to the closest foot, and gives it to Player 1 to check the answer.

Example: First roll 7, second roll 11. Player 1 produces mixed measurement 7 ft 11 in and converts it to 95 inches. Player 2 converts 95 inches to 8 ft. Player 1 checks: 11 > 6, so 7 ft 11 inches is about 8 ft; the conversion is correct.

**Extension**

When would you need to measure length or distance with great precision? Have students think of examples. For example, when you are assembling furniture, the location of the holes on the different parts needs to match exactly. The carpenter needs to precisely measure the distance from a hole to the end of, say, a shelf.
Introduce yards. Ask students to describe what could be measured in yards. (sample answers: distances, length of sport fields, large swimming pools) Write the word “yard” and the abbreviation “yd” on the board, and explain that these are two ways to write “yard.”

ASK: What could you use to estimate in yards? To lead students to the idea of using giant steps, ask students what they used to estimate meters and centimeters. (giant steps and fingers) What unit is a yard close to, a meter or a centimeter? (meter)

ACTIVITY

Giant steps are about a yard long. Using tape, make two long lines on the floor, 1 yard apart. Students stand with their heels against one of the lines, facing the other line. Students make a step so that the heel of the foot going forward touches the second line. Explain that this way, the step students make will be exactly 1 yard long. Ask students to make the step that wide several times, then try to walk forward three steps of about 1 yard each. Students mark the place where they finished, then use a yardstick to measure the distance from the starting line to the mark. How close were they to 3 yards?

Discuss whether a giant step is a convenient tool to estimate distances. (A giant step is a good tool to estimate distances of a few yards, but it would be inconvenient to use to estimate a distance of 100 yards, for example. It will also not work well for distances smaller than 1 yard.) What ways are better to estimate longer distances?

Students estimate and measure the length of the classroom in giant steps. They record the answers in Question 1 on AP Book 4.2 p. 138.
Estimating yards and feet. Point at several objects in the classroom, and ask whether they are longer or shorter than 1 yard. Students can signal the answer by showing thumbs up for longer and thumbs down for shorter. Ask students whether the same objects are longer or shorter than 1 foot.

Converting yards to feet. Have students look at a yardstick. Does it have marks for feet and inches? How many feet are in 1 yard? (3) How many inches are in 1 yard? (36) Have students make a conversion table from yards to feet (see margin), and extend it to 8 rows. ASK: To change a measurement from yards to feet, what number do you multiply by? (3) Why? (there are 3 feet in 1 yard)

Exercises: Convert yards to feet.

a) 12 yd  b) 15 yd  c) 40 yd  d) 68 yd  e) 482 yd

Answers: a) 36 ft, b) 45 ft, c) 120 ft, d) 204 ft, e) 1,446 ft

Comparing measurements in yards and feet. Provide several pairs of measurements, as below, and have students decide which measurement is larger. Students can signal the answer by pointing to the larger side with their thumbs, or by showing a < or a > sign with their fingers.

Exercises

a) 10 yd or 25 ft  b) 12 yd or 35 ft  c) 40 yd or 4 yd
d) 25 yd or 100 ft  e) 135 yd or 400 ft  f) 2,357 yd or 8,000 ft

Bonus: 1,000 yd or 10,000 in

Answers: a) 10 yd, b) 12 yd, c) 40 ft, d) 100 ft, e) 135 yd, f) 8,000 ft, Bonus: 1,000 yd

Finding measurements between other measurements. Write these pairs of measurements on the board:

2 yd and 3 yd  3 yd and 4 yd  4 yd and 5 yd

Ask students to provide a measurement in feet within each of the intervals. They can use the table of conversions produced earlier in the lesson for help.

ASK: What do you notice about the numbers in the column on the right side of the conversion table? (all the numbers are multiples of 3) If you multiply by 3 to get measurements in feet from measurements in yards, how can you get measurements in yards from measurements in feet? (divide by 3)

SAY: I want to convert 10 ft to yards, and I want the answer to the closest yard. What is 10 ft ÷ 3? (3 R 1) How many whole yards are in 10 feet? (3 yd) How many feet are in 4 yards? (12 ft) Is 10 feet closer to 3 yards or to 4 yards? (10 feet is a little more than 3 yards) Repeat with 35 feet.
Converting yards, feet, and inches. Remind students that there are 12 inches in 1 foot. Add a third column, “inches,” to the conversion table, and have students fill in the table. They can see the pattern from the first three entries, which they know from previous lessons (2 yd = 6 ft = 72 in, 3 yd = 9 ft = 108 in), and can add 36 to get each next number in the column. Remind students that they got the feet column from the yards column by multiplying by 3, and they can multiply by 12 to get from feet to inches. ASK: What number would you multiply the number of yards by to get the number of inches? (36) How do you know? (3 × 12 = 36, and multiplying first by 3, then by 12, is the same as multiplying by 36)

Point out that if you want to convert a measurement from yards to inches, you could use a calculator to multiply by 36. Discuss possible ways to check at a glance that the calculation is correct. For example, draw students’ attention to the fact that all numbers in the table are even. Are all the numbers in the tables students produced even? Is each next number in the table larger than the previous one? If students are familiar with quick ways to check that a number is a multiple of 3 or 9, they can also check that the numbers in the table are multiples of 3 and multiples of 9. (A number is a multiple of 3 if the sum of its digits is a multiple of 3. The same rule applies to 9.)

Have students convert several measurements from yards to inches by multiplying by 36.

Exercises
a) 12 yd  
b) 15 yd  
c) 40 yd  
d) 28 yd  
e) 82 yd

Answers: a) 432 in, b) 540 in, c) 1,440 in, d) 1,008 in, e) 2,952 in

Extensions
1. Convert to inches.
   a) 12 yd 2 ft 5 in  
   b) 15 yd 1 ft 3 in
   c) 40 yd 2 ft 1 in  
   d) 28 yd 2 ft 9 in

   Answers: a) 461 in, b) 555 in, c) 1,465 in, d) 1,041 in

2. Fifty-six people go canoeing. Each canoe holds 3 people. They rent only as many canoes as they need to. Each canoe costs $34 to rent for the day. How much do all the canoes cost?

   Show your work using equations. Say what each equation means in the situation.

   Answer: 56 ÷ 3 = 18 R 2, so they need to rent 19 canoes. 
   19 × 34 = 646, so all the canoes cost $646.
Goals

Students will solve problems involving measurements in yards, feet, and inches, including problems requiring converting larger units into smaller units.

PRIOR KNOWLEDGE REQUIRED

Can convert measurements from feet and yards to smaller units
Can convert a mixed measurement to inches
Can find the perimeter of a rectangle
Can solve word problems requiring four operations

Review conversion between all three units learned to date. Have students make conversion tables from yards to feet, then from feet to inches. Remind students that they can multiply by 3 to convert yards to feet, and by 12 to convert feet to inches. Students can also multiply by 36 to convert yards to inches directly.

Exercises:

Convert the measurement.

a) 12 ft to inches  
b) 7 yd to feet  
c) 10 yd to inches 
d) 23 ft to inches  
e) 47 ft to inches  
f) 75 yd to feet

Answers: a) 144 in, b) 21 ft, c) 360 in, d) 276 in, e) 564 in, f) 225 ft

Review conversions of mixed measurements to inches only, such as 3 feet 8 inches to inches. (44 in)

Exercises: Convert to inches.

a) 7 ft 5 inches  
b) 9 ft 1 in  
c) 35 ft 2 in

Answers: a) 89 in, b) 109 in, c) 422 in

Review perimeter. Remind students that the perimeter of a shape is the sum of the lengths of all the sides. As well, remind students that since rectangles have equal opposite sides, they can find the perimeter by adding width and length and multiplying by 2.

Exercises: Find the perimeter of the rectangle.

a) 21 in by 11 in  
b) 26 ft by 23 ft  
c) 7 yd by 10 yd

Answers: a) 64 in, b) 98 ft, c) 34 yd

ASK: Which rectangle, the second or the third, has the larger perimeter? (the third) How do you know? (the perimeter of the third rectangle is 34 yd = 102 ft, which is larger than 98 ft)

Solving problems without conversions. Have students solve several simple problems that do not require conversions between units. Keep the problems below on the board so you can compare them later with similar problems that require conversions.
Solving problems with conversions. Present problem a) below and discuss with students how it is similar to problem a) above. How is it different? (some of the heights are in feet, some are in inches, and some are mixed measurements) How will finding the solution be different? (need to convert all the measurements to the same unit) Have students convert the measurements to inches, then solve the problem. Repeat with the rest of the problems.

a) A table is 3 ft tall, and a stool is 2 ft tall. George is 4 ft tall. He places the stool on top of the table and climbs onto the stool. How high will his head be? Can he stand upright if the ceiling is 8 ft high?

b) A building has 8 ft ceilings on each floor. The space between the floors and under the roof is 2 ft high. See example in margin at left. The building has 10 floors. How tall is the building?

c) A board is 75 in long. Maria cuts it into three equal shelves. How long is each shelf?

d) A backyard is a rectangle that is 12 yd wide and 15 yd deep. A fence goes around two long sides and one short side. How long is the fence?

Answers: a) 9 ft, George cannot stand upright; b) 100 ft; c) 25 in; d) 42 yd

Exercises

a) A newborn black bear cub is about 6 inches long. An adult male bear is about 12 times as long as a newborn cub. An adult female bear is about 10 times as long as a newborn cub. How long are the adult male and female black bears? How much longer is the male than the female?

b) The archaeopteryx dinosaur was about 11 in tall. Argentinosaurus was 76 times as tall. About how many feet tall was argentinosaurus? Seismosaurus was about 84 ft tall. How many feet taller than argentinosaurus was seismosaurus?
c) Two walls are 7 ft 4 in and 10 ft 8 in long. Ryan wants to decorate the walls with tiles that are 8 inches long. He will place 7 horizontal rows of tiles along each wall. How many tiles will he need?

d) A board is 7 ft 5 in long. Ron wants to use it to make the longest possible four equal shelves, in a whole number of inches. How long will each shelf be? How much wood will remain?

e) i) Julie needs 8 pieces of rope to put up a tarp to keep her tent dry. The ropes each need to be at least 7 ft 4 in long. How much rope does Julie need?

ii) Julie has 60 ft of rope. In addition to the 8 pieces of rope for the tarp, she needs at least 15 inches of rope to make a makeshift sunglasses holder. Does she have enough rope? How long will the sunglasses holder be?

Answers: a) male: 72 in, female: 60 in, the male is 12 in longer than the female; b) argentinosaurus is about 70 ft tall, seismosaurus is about 14 ft taller than argentinosaurus; c) 189 tiles; d) 22 in, 1 in of wood left over; e) i) 7 ft 4 in = 88 in, so Julie needs 704 in of rope; ii) 60 ft = 720 in, 704 in + 15 in = 719 in, so Julie has enough rope, and the eyeglasses holder will be 16 in long

Extension

Answer the question using a clock, yardstick, calendar, or money. Say how you used the tool that you chose to answer the question. Check your answer using paper and pencil.

a) 12 × 3  
   b) 12 × 5  
   c) 15 × 4  
   d) 25 × 4

Bonus: 7 × 52

Sample solutions

a) A yard is 3 feet and a foot is 12 inches; a yardstick shows 36 inches, so 12 × 3 = 36;

b) An hour has 60 minutes, and it also has 12 × 5 minutes, by looking at a clock, so 12 × 5 = 60;

c) An hour has 60 minutes, and each quarter of an hour is 15 minutes, so 4 × 15, or 15 × 4, is 60;

d) There are 4 quarters in a dollar, and there are 100 cents in a dollar, and since a quarter is 25 cents, 4 × 25, or 25 × 4, is 100;

Bonus: There are 7 days in a week and about 52 weeks in a year, so about 7 × 52 days in a year. There are 365 days in a year, but the product of 7 and 52 has ones digit 4, so I will guess that 7 × 52 = 364.
STANDARDS
preparation for 4.MD.A.3

VOCABULARY
area
perimeter
square centimeter (cm²)

Goals
Students will find the area of shapes in square centimeters by counting squares.

PRIOR KNOWLEDGE REQUIRED
Can use a ruler to draw line segments
Can find the perimeter of shapes on grids

MATERIALS
two grid-paper rectangles for every student
tape
glue (optional)
scissors
paper
meter stick
grid paper
pattern block squares or connecting cubes (optional)

Introduce area. Give students two identical grid-paper rectangles. Ask students to fold one of their rectangles in half, then cut along the fold to make two smaller rectangles (vertically or horizontally). Ask students to rearrange the pieces to make a different rectangle than the one they started with and glue or tape the new rectangle onto white paper. Invite volunteers to tape the original rectangle and the two possible rearranged rectangles onto the board, so that students can see all three shapes together. ASK: Which shape is larger? How is it larger? Did one shape use more paper than the other? Did they use the same amount of paper? Explain that the amount of paper you need to make the shape is the shape’s area.

ASK: When you cut the rectangle in two and rearranged the pieces, did you change the amount of paper you were using? (no) Does your shape have the same shape as the rectangle I gave you? (no) Does it have the same area? (yes) How do you know? (the same amount of paper was used to make it)

Introduce squares as units of area. Draw two rectangles as shown in the margin. ASK: Which rectangle has a larger area? PROMPT: Imagine these are two pieces of cake. Which one has more cake? Explain that squares that cover the shape are used as units to measure area. Point out that units should cover the shape without gaps or overlaps. Draw several shapes made of squares (see examples in the margin). Ask students to count the number of squares in each shape and write it as “[6] square units.”
Determining area of shapes on grids. Draw several rectangles and mark their sides at regular intervals, as shown in the margin. Ask volunteers to divide the rectangles into squares by using a meter stick to join the marks. Ask students to find the area of these rectangles.

Review perimeter. Remind students that the distance around a shape is called perimeter and that, to find it, you add the lengths of the sides. It helps to write the side lengths directly on the sides to make sure you do not miss any sides. Have students find the perimeter of the rectangles and other shapes that were drawn earlier in the lesson.

**ACTIVITY**

Students work in pairs. One student draws a shape on grid paper, and the other student calculates the area and the perimeter.

Introduce square centimeters. Explain that exercises in the AP books use squares that have all sides equal to 1 cm. They are called “square centimeters.” Write “square centimeter” and “cm²” on the board, and explain that these are two ways to write square centimeters.

Draw a rectangle 2 by 3 squares and tell students that these squares are each 1 cm by 1 cm. What is the length of each side? Have students copy the rectangle and write the lengths on the sides. Then ask students to find the area and perimeter of the rectangle. **ASK:** What units is perimeter measured in? **SAY:** Perimeter is measured in centimeters, not square centimeters. Emphasize that these are different (though connected) units.

Students who have trouble with Question 4 on p. 142 of the AP Book can use pattern block squares or connecting cubes to solve the problems.

**Extensions**

1. On grid paper or a geoboard, make as many shapes as possible with an area of 6 squares. For a challenge, try making shapes that have at least one line of symmetry. For instance, the shapes in the margin have an area of 6 square units and one line of symmetry.

2. Mario says that shape A at left has an area of 4 squares and shape B has an area of 3 squares, so shape A has a larger area than shape B. Explain his mistake.

**Answer:** 2. Each square in shape B is larger than each square in shape A, and three larger squares can have a larger total area than four smaller squares.
**MD4-34 Area of Rectangles**

**STANDARDS**

4.NBT.B.5, 4.MD.A.1, 4.MD.A.2

**VOCABULARY**

area
centimeter (cm)
foot (ft)
inch (in)
kilometer (km)
meter (m)
millimeter (mm)
square centimeter (cm²)
square foot (ft²)
square inch (in²)
square kilometer (km²)
square meter (m²)
square millimeter (mm²)
square yard (yd²)
yard (yd)

**Goals**

Students will find the area of rectangles and develop the formula for area of rectangles.

**PRIOR KNOWLEDGE REQUIRED**

Knows the relative size of units of length measurement within the metric and customary US systems
Can use a ruler to draw and measure line segments
Can use multiplication to find the number of objects in an array
Can multiply and divide mentally up to 12 × 12
Can multiply up to 4-digit numbers by 1-digit numbers
Can multiply two 2-digit numbers

**MATERIALS**

- 1 cm grid paper
- rulers
- pattern block square
- centimeter cube
- 1 ft floor tile (optional)
- wrapping paper or old newspapers
- scissors
- calculators

**Introduce rectangles as arrays of squares.** Draw a 3 × 4 array of dots on the board. ASK: How many dots are in the array? How did you count the dots? Have students write the corresponding multiplication statement.

Now draw a 3 × 4 rectangle on a grid on the board, and have students copy it on grid paper and write the length and width of the rectangle. ASK: How are the rectangle and the array the same? What multiplication statement gives us the area of the rectangle? To prompt students to see the answer, draw a dot in each square of the array. Ask students to write the multiplication statement for the area of the rectangle.

Draw several rectangles on a grid. Ask students to write the length and the width and calculate the area.

**Determining the formula for area of rectangles.** On the board, draw a rectangle not on a grid and tell students that it is 50 squares long and 30 squares wide. What is the area of this rectangle? (1,500) How did you find the answer? (50 × 30 = 1,500) Explain that because area is measured in squares, we can think about any rectangle as being made of squares.

Draw a rectangle 25 cm by 20 cm. Use this opportunity to review using a ruler to draw line segments of a given length by having students tell
you what to do next. Write the length and the width beside the sides of
the rectangle. SAY: The rectangle is 25 cm long. How many 1 cm squares
long is it? (25) How many 1 cm squares wide is it? (20) How many 1 cm
squares are needed to cover the rectangle? (500) Have students write the
multiplication statement. (25 × 20 = 500) What is the area of the rectangle?
(500 cm²) Did we divide the rectangle into squares to find the answer? (no)
How did we find the answer? (multiplied length by width) Do you think
this method will work to find the area of any rectangle? (yes) Summarize
on the board:

Area of rectangle = length × width

**ACTIVITY 1**

**Finding areas of rectangles.** Students work in pairs to draw various
rectangles with length and width in whole centimeters, then exchange
notebooks. Partners then measure the sides of the rectangles,
record their length and width, and find the area. Partners check each
other’s work.

**Introduce different units.** Explain that just as length is measured in
different units, so is area. Square centimeters are squares with length and
width 1 cm. ASK: What other square units can you think of? Write the list
students suggest on the board. Add units as necessary until you have
at least the units shown in the vocabulary list. Introduce the short forms
of the units.

Ask students if they know of anything that is exactly 1 square unit of some
kind. If the following examples are not offered, point them out and, if
possible, show them: a side of a centimeter cube is 1 cm², a pattern block
square is 1 in², some floor tiles are 1 ft².

**ACTIVITY 2**

**Making and comparing unit squares.** Assign different square units
(1 cm², 1 in², 1 ft², 1 yd², 1 m²) to students and have them use wrapping
paper or old newspapers to make and label squares of the given size.
Have students who are working on the small units (square inch, square
centimeter, square foot) make several squares, while their peers cut
out large squares. Students compare and order the units in groups.
Will a 3 ft by 2 ft rectangle fit inside 1 square yard? Will a 5 cm by 4 cm
rectangle fit inside 1 square meter? Students can also try to draw
rectangles given in small units on the rectangles representing larger
units to see if they fit inside. For example, will an 11 in by 7 in rectangle
fit inside 1 square foot?
Finding the area of rectangles using the formula. On the board, draw the picture in the margin. Ask students to copy the dimensions of the rectangle. Demonstrate how to use a formula to record the process of finding the area, as shown below.

\[
\begin{align*}
\text{Length} & = 10 \text{ cm} \\
\text{Width} & = 5 \text{ cm} \\
\text{Area} & = \text{length} \times \text{width} \\
& = 10 \text{ cm} \times 5 \text{ cm} \\
& = 50 \text{ cm}^2
\end{align*}
\]

Remind students to write the units for each measurement. In the first three exercises below, provide a picture of a rectangle with the length and the width. For the rest of the exercises, just write the length and width.

**Exercises:** Use the formula to find the area.

a) length 5 m, width 4 m  
b) length 6 km, width 7 km  
c) length 20 cm, width 15 cm  
d) length 5 in, width 8 in  
e) length 6 ft, width 11 ft  
f) length 12 yd, width 4 yd  
g) length 56 cm, width 20 cm  
h) length 42 yd, width 42 yd  
i) length 16 ft, width 11 ft  
j) length 132 m, width 4 m  
k) length 518 mm, width 9 mm  
l) length 472 in, width 8 in  

**Bonus:** length 2,121,000 cm, width 400 cm

**Answers:** a) 20 m², b) 42 km², c) 300 cm², d) 40 in², e) 66 ft², f) 48 yd²,  
g) 1,120 cm², h) 1,764 yd², i) 176 ft², j) 528 m², k) 4,662 mm², l) 3,776 in²,  
Bonus: 848,400,000 cm²

**Extensions**

1. a) Draw a rectangle so that the number representing the area (in square centimeters) is the same as the number representing the perimeter (in centimeters).

b) Convert the measurements of the sides to millimeters. Find the area and perimeter of the rectangle with the new measurements. Is the area in square millimeters still equal to the perimeter in millimeters?

**Sample answers:** a) \( 18 = 3 \times 6 \) (area) = \( 2 \times 3 + 2 \times 6 \) (perimeter), or \( 16 = 4 \times 4 \) (area and perimeter); b) 30 mm by 60 mm, so area = 1,800 mm² and perimeter = 180 mm; or, 40 mm by 40 mm, area = 1,600 mm² and perimeter = 160 mm; the area is not equal to the perimeter

**NOTE:** This exercise shows that the numbers representing area and perimeter can be identical in one set of units and different in another set of units, even though the rectangles do not change. This means that area and perimeter cannot be compared because the units they are measured in are incomparable.
2. **Acres.** Ask students what units the area of plots of land are measured in. Explain that an acre is not a square unit. It is a rectangle 22 yd wide by 220 yd long.

   a) How many square yards are in 1 acre?
   
   b) 1 yd = 3 ft. How many square feet are in one square yard?
   
   c) How many square feet are in 1 acre?

   Have students use a calculator for the next problems. Remind students to estimate their answer first to catch possible mistakes when punching in the numbers.

   d) 1 square mile = 640 acres. Use a calculator to find how many square yards and square feet are in 1 square mile.
   
   e) 1 mi = 1,760 yd. How many square yards are in 1 square mile? Did you get the same answer as in part d)? If not, find your mistake.

   **Answers:** a) 4,840 yd²; b) 9 ft²; c) 43,560 ft²; d) 3,097,600 yd², 27,878,400 ft²; e) 3,097,600 yd²

3. The perimeter of a rectangle is 36 cm. The length of the rectangle is 5 times as long as the width. What is the area of the rectangle? Use any tool you think will help.

   **Sample solutions**
   - I used a table, and listed widths and lengths until they added to 18, (1, 5; 2, 10; 3, 15) because that's half the perimeter. So the width is 3 cm, the length is 15 cm, and the area is 45 cm², because $3 \times 15 = 45$.
   - I used grid paper, and drew rectangles that are 5 times as long as they are wide. Then I calculated the perimeter and found that the 3 cm by 15 cm rectangle has perimeter 36 cm. Its area is 45 cm².
   - I did it the same way, but I made the rectangles from connecting cubes.

4. Jessica divides 583 ÷ 6 and gets 97 R 1.

   a) Find the smallest multiple of 6 that is larger than 583. Explain how you know.

   b) Is 6 a factor of your answer to part a)? Explain how you know.

   **Answers:** a) 588, because 583 is 97 groups of 6 with 1 left over. When you add 5 more, you get another group of 6. So I just added 583 + 5 and I got 588; b) yes, because “588 is a multiple of 6” means the same thing as “6 is a factor of 588.”
MD4-35 Problems with Area and Perimeter of Rectangles

STANDARDS
4.NBT.B.5, 4.MD.A.1, 4.MD.A.2

VOCABULARY
area
centimeter (cm)
equation
foot (ft)
inch (in)
kilometer (km)
meter (m)
millimeter (mm)
perimeter
square centimeter (cm²)
square foot (ft²)
square inch (in²)
square kilometer (km²)
square meter (m²)
square millimeter (mm²)
square yard (yd²)
unknown
yard (yd)

Goals
Students will solve problems involving area and perimeter of rectangles.

PRIOR KNOWLEDGE REQUIRED
Knows the relative size of units of length measurements within the metric and customary US systems
Can find the perimeter and area of a rectangle
Can multiply and divide mentally up to 12 × 12
Can multiply and divide up to 4-digit numbers by 1-digit numbers
Can multiply two 2-digit numbers
Can solve an equation of type a × x = b

MATERIALS
BLM Rectangles (p. S-49)

Review finding the area of rectangles. Draw a few rectangles on the board. Write the dimensions beside them and have students find the areas. Repeat with some problems in which you only write the length and the width. Then ask students to explain what they did, and summarize on the board:

Area of rectangle = length × width

Finding the missing dimension in a rectangle. Tell students that you want to find the length of a rectangle with area 24 cm² and width 4 cm. Explain that you want to organize what you know and what you need to find. Show how to do it as shown in the margin. ASK: What could I write instead of the length? Remind students that in word problems, we often use a letter for the unknown number. Explain that you will use the letter \( l \) as short form of “length.” Write “\( l \) cm” in the blank. ASK: What is the area of a rectangle with length \( l \) cm and width 4 cm? \( (l \times 4) \) What equation does this make? \( (l \times 4 = 24) \)

Remind students that to solve this multiplication equation they can, for example, think what number multiplied by 4 produces 24, or use an equation from the same fact family, so that the letter \( l \) is by itself, \( 24 ÷ 4 = l \). ASK: What is \( l \) equal to? (6)

Repeat with rectangle with area 24 in² and length 8 in. Use \( w \) for the unknown number. Then have students practice.

Exercises: Find the missing length or width.

a) length 7 m, width \( w \) m, area 42 m²
b) length 5 cm, width \( w \) cm, area 45 cm²
c) length \( l \) km, width 9 km, area 72 km²
d) length \( l \) in, width 11 in, area 121 in²
e) length 12 ft, width \( w \) ft, area 72 ft²
f) length 5 cm, area 20 cm², find the width

g) width 2 m, area 24 m², find the length

h) width 5 ft, area 625 ft², find the length

i) width 7 yd, area 8,484 yd², find the length

**Bonus:** A square has area 16 km². What is its width? Hint: What can you say about the length and the width of a square? What number multiplied by itself equals 16?

**Answers:**  
a) 6 m,  
b) 9 cm,  
c) 8 km,  
d) 11 in,  
e) 6 ft,  
f) 4 cm,  
g) 12 m,  
h) 125 ft,  
i) 1,212 yd,  
Bonus: 4 km

**Review perimeter.** Remind students that perimeter is the distance around the shape. Draw several rectangles on the board and have students find the perimeter. Discuss efficient ways to find the perimeter of a rectangle using the fact that a rectangle has equal opposite sides. (add length and width and multiply by 2; double the length and the width then add them together)

Draw students’ attention to the units in which perimeter is measured. Are these the same units as for area? (no, these are units of length) Have students find the perimeter of the rectangles in the exercises above.

**Answers:**  
a) 26 m,  
b) 28 cm,  
c) 34 km,  
d) 44 in,  
e) 36 ft,  
f) 18 cm,  
g) 28 m,  
h) 260 ft,  
i) 2,438 yd,  
Bonus: 16 km

**Comparing area and perimeter.** Draw several rectangles on a grid with the dimensions below, or display BLM Rectangles.

\[
\begin{array}{c@{}c@{}c@{}c@{}c@{}c}
& A & 4 \times 6, & B & 5 \times 5, & C & 6 \times 3, & D & 11 \times 1, & E & 3 \times 8 \\
\end{array}
\]

Have students fill in the table below. Ask students to list the rectangles from least to greatest by area. Then ask them to list the rectangles from least to greatest by perimeter.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Length</th>
<th>Width</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>6</td>
<td>24</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>5</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>3</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>11</td>
<td>1</td>
<td>11</td>
<td>24</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>8</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>

**Answers:** Rectangles ordered from least to greatest area: D, C, A, E, B or D, C, E, A, B; from least to greatest perimeter: C, A, B, E, D or C, B, A, E, D

ASK: Are your lists the same? (no) Does the rectangle with the greatest area also have the greatest perimeter? (no) Does the rectangle with the smallest perimeter also have the smallest area? (no) Which rectangles have the same area? (A, E) Do they have the same perimeter? (no) Which rectangles have the same perimeter? (A, B) Do they also have the same area? (no) Emphasize that when one shape has an area larger than the other, you cannot tell which shape will have greater perimeter until you...
calculate it. The same is true the other way around: When one shape has
greater perimeter than another, you still need to calculate which shape will
have greater area. Area and perimeter do not depend on each other, and
are measured in different units.

**Using area and perimeter.** Discuss situations in which you would need
to find the area or perimeter of a shape. For example, when installing a
hardwood floor, you would need to know how much flooring to buy. Would
you need to find the area or the perimeter of the room you are renovating?
(area) If you need to find the area or the perimeter of the room you are renovating?
(perimeter) If you need to install quarter round around the room (explain what
quarter round is), will you need to know area or perimeter of the room?
(area) If you are planning a race around a city park, do you need
to know the area or the perimeter of the park? (perimeter) If you need to
decide how much fertilizer to put in your garden, do you need to know
the area or the perimeter of the garden? (area) If you need to put a fence
around the garden, do you need to know the area or the perimeter of the
garden? (perimeter) Encourage students to think of examples of situations
in which they would need to find area or perimeter.

**Extensions**

1. On a grid, draw a square with sides 7 units long.
   a) Find the area and the perimeter of the square.
   b) Inside the square, draw a shape that has an area smaller than the
      area of the square and a perimeter larger than the perimeter of
      the square. There are many possible answers.

   **Answers:**
   a) area = 49 square units, perimeter = 28 units; b) for the
   shape in the margin: area = 17 square units, perimeter = 36 units

2. Draw two rectangles that have an area of 20 cm$^2$ but different
    perimeters. Calculate their perimeters.

   **Sample answer:** 4 cm × 5 cm, perimeter 18 cm; and 2 cm × 10 cm,
   perimeter 24 cm

3. a) Draw a square with an area of 9 in$^2$. Calculate the perimeter.
   b) Draw a non-square rectangle with the same perimeter as the square
      in part a). Find its area.
   c) Which of your shapes has a greater area?
   d) Try to find a different answer in part b). Did your answer to part
      c) change?

   **Answers:**
   a) 3 in by 3 in, perimeter 12 in; b) sample answers: 2 in by
   4 in, area 8 in$^2$, 1 in by 5 in, area 5 in$^2$; c) the square has a greater area;
   d) no, the square will always have the greatest area
MD4-36  Area (Advanced)
Pages 147–148

STANDARDS
4.NBT.B.5, 4.MD.A.1, 4.MD.A.2

VOCABULARY
area
centimeter (cm)
equation
foot (ft)
inch (in)
kilometer (km)
meter (m)
millimeter (mm)
perimeter
square centimeter (cm²)
square foot (ft²)
square inch (in²)
square kilometer (km²)
square meter (m²)
square millimeter (mm²)
square yard (yd²)
yard (yd)

Goals
Students will find areas of composite shapes by decomposing them into rectangles.

PRIOR KNOWLEDGE REQUIRED
Knows that area is additive
Knows the relative size of units of length measurements within the metric and customary US systems
Can find area and perimeter of a rectangle
Can multiply and divide mentally up to 12 × 12
Can multiply and divide up to 4-digit numbers by 1-digit numbers
Can multiply two 2-digit numbers
Can solve an equation of type \(a \times x = b\)

MATERIALS
grid paper
coloring pencils

Finding the area of shapes composed of two rectangles (on a grid).
On a grid, draw several shapes composed of two rectangles. Have students draw the line that divides the composite shape into the two simple ones. Then, ask students to find the area of each shape.

Exercises: Find the area.

a)  

b)  

c)  

Point out that part a) has two different solutions. Encourage students to find both, and have volunteers show both solutions (1 × 4 + 1 × 2 and 1 × 2 + 2 × 2).

Answers: a) 6 square units, b) 10 square units, c) 11 square units

Finding the area of shapes composed of two rectangles (not on a grid).
Draw several shapes composed of two rectangles not on a grid, and mark the dimensions on four of the sides, as shown in the Exercises below.
Ask students to shade each rectangle with its own color, then circle the dimensions that belong to the rectangle in the same color. Have students find the area of each rectangle and add the areas of the rectangles in each shape to find the total area.
Exercises

a)  
\[
\text{Area} = (2 \text{ yd} \times 3 \text{ yd}) + (1 \text{ yd} \times 2 \text{ yd}) = 6 \text{ yd}^2 + 2 \text{ yd}^2 = 8 \text{ yd}^2
\]

b)  
\[
\text{Area} = (8 \text{ ft} \times 5 \text{ ft}) + (7 \text{ ft} \times 4 \text{ ft}) = 40 \text{ ft}^2 + 28 \text{ ft}^2 = 68 \text{ ft}^2
\]

c)  
\[
\text{Area} = (5 \text{ cm} \times 20 \text{ cm}) = 100 \text{ cm}^2
\]

Bonus

\[
\text{Area} = (6 \text{ cm} \times 23 \text{ cm}) + (6 \text{ cm} \times 8 \text{ cm}) = 138 \text{ cm}^2 + 48 \text{ cm}^2 = 186 \text{ cm}^2
\]

Answers: a) 8 yd², b) 68 ft², c) 130 cm², Bonus: 193 cm²

Show students the shape in the margin. ASK: Is 7 cm the length of the short side of the shaded rectangle (trace it with your finger)? (no) Is it the length of the short side of the unshaded rectangle? (no) Point out that 7 cm is the length of the combined side, so 7 cm should be the sum of the lengths of two other sides. ASK: What is the length of the short side of the unshaded rectangle? (3 cm) How do you know? (7 cm – 4 cm = 3 cm) Finally, have students find the areas of both rectangles and add them to find the total area of the shape. (5 cm × 3 cm + 11 cm × 4 cm = 15 cm² + 44 cm² = 59 cm²)

Present the second shape at left, and ask students to find the length of the longer side of the shaded rectangle. (21 in) Have students find the area of the shape. (7 in × 10 in + 9 in × 21 in = 70 in² + 189 in² = 259 in²) Then have students work individually to practice finding the area of composite shapes.

Exercises

a)  
\[
\text{Area} = (2 \text{ m} \times 2 \text{ m}) + (3 \text{ m} \times 1 \text{ m}) = 4 \text{ m}^2 + 3 \text{ m}^2 = 7 \text{ m}^2
\]

b)  
\[
\text{Area} = (4 \text{ ft} \times 3 \text{ ft}) + (2 \text{ ft} \times 2 \text{ ft}) = 12 \text{ ft}^2 + 4 \text{ ft}^2 = 16 \text{ ft}^2
\]
Present the shape in the margin, and shade one of the rectangles as shown in the first picture. Ask students to identify the side lengths of the shaded rectangle. Point to each label in order, and have students signal thumbs up if the label shows the length of a side of the shaded rectangle or thumbs down if it does not. Circle the labels for the shaded rectangle, then repeat with the side lengths of the unshaded rectangle. Then present the same shape with the same labels, but broken into two rectangles in a different way (as in the second picture) and repeat the exercise. Point out that, in each case, students need to use four of the six labels to find the area of the shape. The remaining two labels are unused, and the unused labels differ in each case. Have students use both ways to find the area of the shape. (89 cm²) Did they get the same answer?

Have students find the area of the shapes below. In the first two shapes, all side lengths are given. In the rest of the shapes, only four side lengths are given.

**Exercises:** Find the area.

**a)** 8 ft 7 ft 4 ft 7 ft 12 ft 14 ft

**b)** 6 in 5 in 5 in 16 in 22 in 10 in

**c)** 12 m 15 m 12 m 12 m 23 m 5 m

**d)** 3 ft 5 ft 4 ft 5 ft

**Answers:** a) 140 ft², b) 140 in², c) 456 m², d) 57 ft²

**Bonus:** Find a different way to split these shapes into rectangles and use them to find the area. Did you get the same answers as before?

**Finding the perimeter of a rectangle given the area and side.** Review finding the length or the width of a rectangle from the area and the other dimension. For example, have students find the width of a rectangle with an area of 14 ft² and a length of 7 ft. (2 ft) Remind students that the perimeter of a shape is the distance around it, and have them find the perimeter of the rectangle. (18 ft)
Exercises: Find the missing length and the perimeter of the rectangle.

a) \( \text{area} = 20 \text{ cm}^2, \text{width} = 4 \text{ cm} \)

b) \( \text{area} = 20 \text{ cm}^2, \text{width} = 2 \text{ cm} \)

Answers: a) length 5 cm, perimeter 18 cm; b) length 10 cm, perimeter 24 cm

Finding the area of a rectangle given the perimeter and side. Discuss ways of finding the perimeter of a rectangle. (add all four sides; add two sides and double; double length and width, then add them) Draw a rectangle on the board and mark one side as 5 m. SAY: The perimeter of this rectangle is 22 m. What are the other sides of the rectangle? Have students present multiple solutions. To prompt students, have them first find the length of the opposite side. SAY: These two sides together add to 10 m. What do the other two sides add to? (12 m) How do you know? (22 m – 10 m = 12 m) How long is each side? (6 m) How do you know? (they are equal and add to 12 m, so each is 12 m ÷ 2 = 6 m) Another way of thinking about the perimeter is: Two touching sides should be half of 22, so two touching sides add to 11 m. One of the sides is 5 m. How long is the other side? (6 m)

Then have students find the area of the rectangle. (5 m × 6 m = 30 m²)

Exercises: Find the missing length, then find the area of the rectangle.

a) perimeter 20 cm, width 4 cm  
b) perimeter 20 cm, width 2 cm

Answers: a) length 6 cm, area 24 cm²; b) length 8 cm, area 16 cm²

Finally, draw a square and tell students that its perimeter is 24 cm. How long is each side? (6 cm) How do you know? (the sides are equal, so perimeter is 24 cm ÷ 4 = 6 cm) What is the area of the square? (36 cm²)

Extension

Grace wants to paint a wall that is 8 feet high and 11 3/4 feet wide. She needs 9 mL of paint per square foot. How much paint does she need? Is that more or less than a liter?

Sample solution: I drew a picture of the wall (shown in margin) and split it into two smaller rectangles. The first rectangle has area 88 ft² and the second has area 6 ft² (because $8 \times \frac{3}{4} = 24$). I found $88 + 6 = 94$, so the total area of the wall is 94 ft². Grace needs 9 mL of paint per square foot. Now, $94 \times 9 = 846$, so she needs 846 mL. This is less than 1 L because 846 is less than 1,000.

Whole-class follow-up: Some students may draw a model (such as fraction circles or a number line) to calculate $8 \times 3/4$. Take up various strategies and then compare them with questions such as: How does this person’s number line show that person’s equation?
**MD4-37 Problems and Puzzles**

**Page 149**

**STANDARDS**
4.NBT.B.5, 4.MD.A.1, 4.MD.A.2

**VOCABULARY**
- area
- centimeter (cm)
- foot (ft)
- inch (in)
- kilometer (k)
- meter (m)
- millimeter (mm)
- perimeter
- square centimeter (cm²)
- square foot (ft²)
- square inch (in²)
- square meter (m²)

**Goals**
Students will solve problems connected to the area and perimeter of rectangles.

**PRIOR KNOWLEDGE REQUIRED**
- Knows that area is additive
- Knows the relative size of units of length measurements within the metric and customary US systems
- Can find the area and perimeter of a rectangle
- Can multiply and divide mentally up to 12 × 12
- Can multiply and divide up to 4-digit numbers by 1-digit numbers
- Can multiply two 2-digit numbers
- Can convert between dollar and cent notations

This lesson is a review of area. Work through the problems below as a class before assigning AP Book 4.2 p. 149. Review dollar and cent notation for money along the way.

1. A room has the shape as shown in the margin.
   a) Split the shape into two rectangles. Then find its area.
   b) Carpet costs $15.00 per square meter. How much will carpet cost for the room?
   c) Find the perimeter of the room.
   d) Carpet tack strips need to be attached along all the walls to hold the carpet in place. Tack strips cost 18¢ for 1 m. How much will the tack strips cost for the room?
   **Answers:** a) 24 m², b) $360, c) 22 m, d) $3.96

2. Mario’s backyard is a square 90 ft long. The house is along one side of the square, as shown in the margin.
   a) How many square feet of grass does Mario need to cover the backyard?
   b) Grass seed with fertilizer costs 36¢ for 100 ft². How much will the seed for the backyard cost?
   c) Mario wants to fence the backyard. How much fencing does he need?
   d) Fencing costs $9.00 for 1 ft. How much will the fence cost?
   **Answers:** a) 8,100 ft², b) $29.16, c) 270 ft, d) $2,430

---

**Measurement and Data 4-37**

S-35
3. Use the table below to find all possible rectangles with sides in whole inches and an area of 24 in². Then find the perimeter of the rectangles. Which rectangle has the smallest perimeter?

<table>
<thead>
<tr>
<th>Length (in)</th>
<th>Width (in)</th>
<th>Area (in²)</th>
<th>Perimeter (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answers: 1 × 24, perimeter 50 in.; 2 × 12, perimeter 28 in; 3 × 8, perimeter 22 in; 4 × 6, perimeter 20 in; rectangle with smallest perimeter is 4 in × 6 in, perimeter 20 in

Extensions

(MP3) 1. In pairs, decide if the statement is true and explain why or why not. Do you agree with each other? Discuss why or why not.
   a) All rectangles with the same area have the same perimeter.
   b) All rectangles with the same perimeter have the same area.

Sample answers: a) No; the rectangles 2 in by 12 in and 4 in by 6 in have the same area (24 in²) but different perimeters (28 in and 20 in);
   b) No; the rectangles 4 in by 6 in and 7 in by 3 in both have perimeter 20 in, but different areas (24 in² and 21 in²)

NOTE: Encourage partners to ask questions to understand and challenge each other’s thinking (see p. A-49 for sample sentence and question stems).

(MP1, MP2, MP4) 2. Jayden’s garden is a rectangle that is 12 feet long and 6 feet wide. He wants to make it twice as big, but his yard is only 8 feet wide. If he uses the whole width of his yard to make his new garden, how long does he need to make the garden?

Show your work using equations. Say what each equation means in the situation.

Answer: The area of Jayden’s garden is 72 ft² (because 12 × 6 = 72). He wants to make his new garden 8 feet wide with an area of 144 ft² (because 2 × 72 = 144). Then the length has to be 18 feet (because 144 ÷ 8 = 18).
Goals
Students will create and interpret line plots that have whole number data values.

VOCABULARY
- Data
- Label
- Line plot
- Number line
- Title

PRIOR KNOWLEDGE REQUIRED
Can use a number line to subtract

MATERIALS
- Sticky notes

Making and reading line plots. Draw this number line on the board:

Have students write their favorite number from 1 to 10 on a sticky note, then place it on the board above their favorite number. If their sticky notes are messy, making it difficult to answer questions, try asking a question such as: What is the most popular number? Then organize their sticky notes and explain that doing so will make it easier to answer questions. If the sticky notes are already neatly displayed, compliment students on how easy it is to answer questions because of it.

Continue asking questions about the line plot. ASK: What is the least popular number? How many people chose 3 as their favorite number? How many people chose 8 as their favorite number? Did more people choose 3 or 8? Do people seem to prefer smaller numbers or bigger numbers?

The importance of number lines. SAY: Using a number line on the bottom, with the numbers in order and equally spaced, makes it easy to answer a lot of questions. You didn’t have to look for where the 3 is because it’s where you expect it to be. When data is counted using a number line, it is called a line plot.

Choosing the number line. Tell students that you want to make a line plot to show the lengths of students’ first names. Draw this number line on the board:

ASK: Is there anyone in the class with 0 letters in their name? (no) 1 letter? Continue until you find the shortest name in the class. Determine who has the longest name by continually asking who has a longer name than the last person. SAY: Now that I know the smallest number of letters (say, 3) and the largest number of letters (say, 11), I can make the number line:
Have students write their name and the number of letters on a sticky note. Students can then make a line plot as before. Keep this line plot on the board for later use. Do not add the title or label yet.

ASK: How many people have at least 6 letters in their name? Have a volunteer circle the two longest names. ASK: How many letters are in the two longest names combined? How much longer is the longest name than the shortest name? Point out that the answer to the last question can be seen on the number line. It is the distance between the smallest and largest values: $11 - 3 = 8$.

3 4 5 6 7 8 9 10 11

Exercises

**Length of Students’ Names**

a) How many people have at least 8 letters in their name?
b) How many letters does the longest name have?
c) How much longer is the longest name than the shortest name?
d) How many people have 5-letter names? How many letters altogether are in 5-letter names?
e) Are there more letters altogether in 6-letter names or in 8-letter names?

**Bonus:** It costs 10¢ a letter to put your name on a T-shirt. How much would it cost to put everyone’s name on the T-shirt?

**Answers:** a) 3; b) 10; c) $10 - 3 = 7$; d) 3, 15; e) in 6-letter names, because $3 \times 6 = 18$ is more than $2 \times 8 = 16$; Bonus: $7.00$: add up the data values by column on the line plot: $3 + 8 + 15 + 18 + 16 + 10 = 70$ letters, which cost $70 \times 10¢ = 700¢ = $7.00$

**Putting given data into a line plot.** Tell students that you asked several friends how many siblings they have. Here is the data you recorded:

0 4 2 1 0 1
1 1 0 1 2 2

ASK: How many friends did I ask? (12) What numbers should I put on the number line? (0, 1, 2, 3, and 4) Ask a volunteer to draw the number line:

0 1 2 3 4

Put an $\times$ above the 0 and SAY: I can use $\times$s instead of sticky notes to record the data. Cross out the first 0 and SAY: I can cross out the numbers I’ve already recorded so that I don’t miss any or record any twice.
Continue recording the first row of data, then have volunteers finish recording the second row:

<table>
<thead>
<tr>
<th>0</th>
<th>4</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Point out that the 3 is still marked on the number line even though there are no people with 3 siblings in the group you asked. You have to put all the numbers in between the data values. (see margin)

**Exercises:** Make a line plot for the number of siblings.

1. 0 0 1 1 1 1 1 2 2 2 2 2 5 (0 1 2 3 4 5)
2. 0 2 2 2 2 3 3 4 4 4 5 5 (0 1 2 3 4 5)

**Answers**

1. a) 
   ![Line plot](image)
   
2. b) 
   ![Line plot](image)

**Checking your work.** Tell students that they can double-check that they entered the correct data by counting the Xs they marked and counting the data values. There should be one X for each data value, so there should be the same number of each. Demonstrate with the “siblings” example above, and have students check their work from the exercises above.

**The importance of including a title and label.** Look at all the line plots already on the board. For each one in turn, ask if anyone remembers what the numbers represent. Tell students that when they add a title and a label, they can look at the line plot weeks later and still remember what it was for, and what it measured. Say: The title tells you what it is about and the label tells you what the numbers measured. Go through each line plot in turn and together construct a title and label:

1. Title: Our Class’s Favorite Numbers
   Label: Number

2. Title: Length of Students’ Names in Our Class
   Label: Number of Letters

Point out that in the first case, the label was just “number” because it wasn’t counting anything. But, in the second example, it is counting something, so we say what it is counting. The title includes who or what the information is about.

**Extension**

On BLM Comparing Heights (p. S-50), students compare two line plots to draw conclusions about how students’ heights change from age 8 to age 10.
MD4-39 Fractions on Line Plots

Goals
Students will make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Students will solve problems involving subtraction of fractions by using the number line on a line plot.

PRIOR KNOWLEDGE REQUIRED
Can read and interpret line plots
Can measure to the nearest eighth of an inch
Can subtract fractions using a number line

MATERIALS
ruler
BLM Animals (p. S-51)
meter stick

Review line plots. Remind students that a line plot has to have a title, a number line, and a label saying what the number line is measuring. For example, if it is measuring letters in a name, we write a label for that. If it is measuring the number of siblings, we write a label for that.

Reading line plots with fractional entries. Show students the following line plot.

Height of Players on a Grade 4 Basketball Team

Tell students that the label is not complete—something more needs to be added. ASK: What is missing? (the units) PROMPTS: Can a basketball player be 55 feet tall? (no) Point out that a female adult is between 5 ft and 6 ft tall, and that 55 ft is about the height of a five-story building. Can a basketball player be 55 centimeters tall? (no) Show a meter stick and where 55 cm would reach. SAY: For the line plot to make sense, we need to write what unit we’re using to measure with. The units go in brackets. Write “inches” in the brackets. ASK: How many players are on the team? (14) How can you tell? (there are 14 Xs) How tall is the tallest person on the team? (56 1/2 in) How tall is the shortest person on the team? (52 in) Tell students that the basketball team decided to take a team-building trip to an amusement park.
**Exercises:** How many people on the basketball team can go on each ride at the amusement park?

a) Ride 1: You have to be 53 inches tall.

b) Ride 2: You have to be 54 inches tall.

c) Ride 3: You have to be 57 inches tall.

**Bonus:** Seven people from the team can go on a ride. What is the height restriction for that ride?

**Answers:** a) 11, b) 9, c) 0, Bonus: 55 in

**Using the number line on a line plot to subtract fractions.** Say: You can also find out information that is not directly on the line plot. Ask: How much taller is the tallest person than the shortest person? (4 1/2 inches) How did you get that from the line plot? (subtracted 56 1/2 − 52) Point out that they don’t have to subtract, they can just look at their distance apart on the number line. Draw on the board:

![Number Line Example](image)

Say: Each space on the number line is one-half inch, and there are nine half-inches from the smallest height to the largest height that appears in the line plot. So the difference between the smallest and largest heights is 9 half-inches. There are 2 half-inches in one whole inch, so 9 half-inches is 4 1/2 whole inches. Write on the board:

\[
9 \div 2 = 4 \text{ R } 1 \quad \text{so} \quad \frac{9}{2} = \frac{4}{2} + \frac{1}{2}
\]

**Exercises:** How much taller is the tallest person than the shortest person?

a) ![Height Example](image)

b) ![Height Example](image)

Say: Now each space is a quarter of an inch. (Use the same data as in part a) and just change the numbers.)
SAY: Now each space is an eighth of an inch. (Use the same data as in part b) and just change the numbers.)

\[ \begin{align*}
(49) & \quad (49) & \quad (49) & \quad (50) & \quad (50) & \quad (50) & \quad (50) & \quad (50) & \quad (51) & \quad (51) \\
1 & \quad \frac{1}{4} & \quad \frac{1}{2} & \quad \frac{3}{4} & \quad 1 & \quad \frac{1}{4} & \quad \frac{3}{4} & \quad \frac{3}{4} & \quad 1 & \quad \frac{1}{2} & \quad \frac{1}{4} & \quad \frac{1}{2} & \quad \frac{1}{4} \\
\end{align*} \]

Height (inches)

Answers: a) 5/2 in = 2 1/2 in, b) 9/2 in = 4 1/2 in, c) 5/4 in = 1 1/4 in, d) 9/8 in = 1 1/8 in

Creating line plots with fractions on the number line. A pet store asked customers the age of their dogs to the nearest 1/4 year. Here are the answers:

1 1/4 1 1/2 1 1/4 2 1 1/4 1 1/2 3 1/4 2 1/2 2 1/4 2 1/2

Ask volunteers to circle the ages of the youngest and oldest dogs. (1 1/4 and 3 1/4) ASK: How is that information helpful for making a line plot? (you know to draw the number line from 1 1/4 to 3 1/4) Demonstrate doing so, then have students copy the number line and make the line plot. Students can then answer questions about their line plot.

Exercises

a) How many dogs are at least two years old?
b) What is the most common age of the dogs?
c) How much older is the oldest dog than the youngest dog?

Answers: a) 5, b) 1 1/4 years old, c) 2 years

Measuring to create line plots. Provide students with BLM Animals. Review measuring to the nearest eighth of an inch with individual students as needed. Have students measure the animal pictures, make a line plot of the measurements, and then answer the questions on the BLM.

Extensions

1. Explain why units are important to include when labeling a line plot.
2. The two line plots below show the same data. Why do they look so different?

**Time to Run 250 Meters**

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
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**Time to Run 250 Meters**

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</tbody>
</table>

**Sample answers:**

1. Including units makes the line plot make sense. When units are not included, the data values are meaningless—you cannot see how large the difference is between the data values. A lot of information is lost.

2. The first line plot is in minutes, and the second line plot is in hours. Since the data values are at most 1 minute, they all fall in the same column on the second line plot. The time intervals on the second graph are too large to show any differences in the data values.

3. A square has sides 4 1/2 cm by 4 1/2 cm. A non-square rectangle has width 4 cm and the same perimeter as the square. Which shape has the larger area? Use any tool you think will help.

**Sample answers**

- I drew both shapes on grid paper. The square has perimeter 18 cm, so the non-square rectangle does too. Then halfway around must be 9 cm, so the length is 5 cm. The area of this rectangle is easy to find: I just found $4 \times 5 = 20$, so its area is 20 cm$^2$. The square is harder, but I looked at the grid paper and counted 16 whole squares, 8 half squares, and 1 quarter square, so the total area is:
  \[16 + \left(\frac{1}{2} \times 8\right) + \frac{1}{4} = 16 + 4 + \frac{1}{4} = 20\frac{1}{4} \text{ cm}^2.\]
  The square has larger area.

- I drew both shapes on grid paper and cut them both out. I placed one shape on top of the other, and the square had more paper sticking out than the non-square rectangle, so the square is bigger.

- I made four copies of each shape, and made a bigger rectangle using a two by two array of the four shapes. Four copies of the square has area 81 cm$^2$, using $9 \times 9 = 81$, while four copies of the non-square rectangle had area 80 cm$^2$, using $8 \times 10 = 80$. So, the square is bigger.
MD4-40  Line Plots (Advanced)

STANDARDS
4.NF.B.3, 4.MD.B.4

VOCABULARY
- denominator
- eighth
- foot (ft)
- fourth
- fraction
- gram
- half
- inch (in)
- line plot
- numerator
- quarter

Goals
Students will solve problems involving addition and subtraction of fractions by using information presented in line plots.

PRIOR KNOWLEDGE REQUIRED
Can add fractions with the same denominator
Can multiply a fraction by a whole number
Can subtract fractions on a number line
Can read and interpret line plots

MATERIALS
- a very thin book, less than an eighth of an inch thick, for yourself
- one book for every student (varying thicknesses)
- sticky notes

BLM How Long? (p. S-52)

Have each student pick a favorite book from a class library. Pick a book for yourself that is less than an eighth of an inch thick. Tell students you would like them to measure how thick their books are. ASK: Why would it be hard to measure a book to the nearest inch? (most books will be less than an inch thick, so the only two answers will be 0 or 1) Ask students to measure their books to the nearest quarter of an inch, and write their measurement on a sticky note. Draw this number line near the bottom of the board:

0 1/4 2/4 3/4 1

ASK: What unit are we using? (inches) Point to the “1” mark and SAY: 1 means 1 inch. We’re using inches as the unit, but we’re measuring to the nearest quarter of an inch. Write “inches” in the brackets.

Have a volunteer measure the thickness of your book. Hold the book tightly closed while the volunteer measures it. Ask the volunteer: Is it closer to 0 inches thick or a quarter of an inch thick? (0 inches thick) Have the volunteer place a sticky note on the board with the measurement “0.” SAY: It might look strange to see a book marked as 0 inches—it looks as though the book isn’t thick at all! But that’s just because the measurement is not exact—we are only measuring to the nearest quarter of an inch.

Have students make the line plot with their sticky notes. When they finish, tell students that you want to know how much space the two thickest
books will take on the shelf. **ASK:** How could you answer that from the line plot? (find the two right-most data values and add them) Remind students that they know how to add fractions with the same denominator and demonstrate an example. For example, if the two thickest books are 3/4 in thick and 2/4 in thick, write on the board:

\[
\frac{3}{4} + \frac{2}{4} = \frac{5}{4}
\]

**SAY:** So you would need 5/4 in for the two thickest books.

**Exercises:** Use the line plot to answer the questions.

**Thickness of Books**

<table>
<thead>
<tr>
<th>Thickness (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1/4 2/4 3/4 1</td>
</tr>
</tbody>
</table>

a) About how much space would the two thickest books take up on a shelf if they were placed beside each other?

b) About how much space would the rest of the books take up?

c) What takes up more space, the two thickest books or the rest of the books?

d) About how much thicker is the thickest book than the thinnest book?

e) How much space on the shelf would all the books take up if they were placed beside each other?

**Answers:** a) 1 3/4 in, b) 11/4 in = 2 3/4 in, c) the rest of the books, d) 3/4 in, e) 4 2/4 in = 4 1/2 in

Take up the answers, and show students a quick way to add the data values. **SAY:** You can add the data values one column at a time. **ASK:** How many books are 1/4 in thick? (3) How much space do they take up altogether? (3/4 in) Write on the board:

\[
\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4} = \frac{3}{4}
\]

Repeat for the 2/4 column. (4 \times 2/4 = 8/4 = 2) So altogether, the books take up \( \frac{3}{4} \text{ in} + 2 \text{ in} + \frac{3}{4} \text{ in} + 1 \text{ in} = 4 \frac{2}{4} \text{ in}. \)
Changing the denominators to make them equal. Draw on the board:

**Thickness of Books**

0 1 2 3 4

Thickness (inches)

Tell students that the fractions on a line plot are not always given with the same denominator, but they are always easy to change to the same denominator. SAY: Zero is zero quarters, one quarter is one quarter. ASK: How many quarters are in one half? (2) Fill in the first three numerators and have a volunteer finish the numerators. ASK: How many books are 2/4 inches thick? (3) How much space do they take up? (6/4 in) Write on the board:

\[
\frac{2}{4} + \frac{2}{4} + \frac{2}{4} = \frac{3}{4} \times \frac{2}{4} = \frac{6}{4}
\]

**Exercises**

a) About how much space do you need for all the 3/4-inch books?

b) The thinnest books in the library are a collection of eight books on natural disasters. Which columns are these books in? Circle the Xs representing these books. How much space do these books take up when placed together?

c) About how much space do you need for the two thickest books?

d) Do you need more space for the eight thinnest books or the two thickest books?

**Answers:** a) 4 2/4 in; b) the books are in the 0 in, 1/4 in, 1/2 in, and 3/4 in columns, 2 2/4 in; c) 2 2/4 in; d) you need the same amount of space for each

**Exercise:** Eight ants are carrying food. About how much weight are they carrying altogether?

\[
\frac{1}{4} + \frac{2}{4} + (3 \times \frac{3}{4}) + (2 \times 1) = \frac{1}{4} + \frac{2}{4} + \frac{9}{4} + 2 = \frac{12}{4} + 2 = 3 + 2 = 5 \text{ grams}
\]
ACTIVITY

(MP4, MP6) Measuring multiple times to avoid errors. Tell students that scientists will often take many measurements of the same thing as a way to double check the measurements. If many people get the same result, it’s more likely to be right. Provide students with BLM How Long?

Have students measure how long the picture is to the nearest eighth of an inch and write their measurement on a sticky note. Draw a number line from 4 6/8 in to 5 6/8 in, and have students place their sticky note above their measurement. Then, have students copy the line plot onto the BLM in the space provided and answer the questions on the BLM.

Extensions

1. Provide students with BLM Coins (p. S-53). Students will problem solve to determine the thickness of the coins by first determining how many of each coin there are, then using the line plot.

2. A grocery store is having a sale on ham. Each customer is allowed to buy up to 800 g of ham at the price of $1 per 100 g. Any extra ham costs $2 per 100 g. How much will 2 kg of ham cost?

   Answer: 2 kg = 2,000 g and you can get 800 g for $8. There’s still 2,000 – 800 = 1,200 g to buy at $2 per 100 g, so that will cost $24. The total cost will be $24 + $8 = $32.

3. Anna is making a patio with an area of 90 ft². She is using 1 foot square tiles to make the patio. She wants the patio to be a rectangle. She wants the patio to be about twice as long as it is wide. What is the best way she can build the patio?

   Solution: I found the possible factor pairs of 90 in order to find the rectangles. They are: 1 × 90, 2 × 45, 3 × 30, 5 × 18, 6 × 15, and 9 × 10. The closest one to being twice as long as it is wide is the rectangle that is 6 feet wide and 15 feet long.
Quarter-Inch Rulers

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7

0 inches 1 2 3 4 5 6 7
Rectangles

A

B

E

C

D

NAME ___________________________ DATE _____________
Comparing Heights

Heights of 8-Year-Olds

Heights of 10-Year-Olds

1. Who is generally taller, 8-year-olds or 10-year-olds?
   How do the line plots show this?

2. Are all 10-year-olds taller than all 8-year-olds?
   Choose two data values to justify your answer.
   There is an 8-year-old who is _______ inches tall.
   There is a 10-year-old who is only _______ inches tall.
   The 8-year-old is _______ inches taller than the 10-year-old.

3. Draw a line plot of what the heights might be of fifteen 9-year-olds.

NAME ___________________________ DATE ___________
Animals

1. a) Measure the width of each picture to the nearest quarter of an inch.

b) Make a line plot from your results.

Title: ________________________________

Number Line: 1 1\(\frac{1}{4}\) 1\(\frac{1}{2}\) 1\(\frac{3}{4}\) 2 2\(\frac{1}{4}\) 2\(\frac{1}{2}\) 2\(\frac{3}{4}\) 3

Label: _________________________________

c) How wide is the widest picture? __________

d) How wide is the narrowest picture? __________

e) How much wider is the widest picture than the narrowest picture? __________
How Long?

1. Measure how long the picture is, to the nearest eighth of an inch.

   ![Image of an airplane]

   a) Make a line plot from your class results.

   Title: __________________________

   Number Line:

   ![Line plot with numbers 4 to 8 in eighths]

   Label: __________________________

   b) What was the shortest measurement? __________

   c) What was the longest measurement? __________

   d) What was the most common measurement? __________

   e) How much longer was the longest measurement than the shortest measurement? __________

   f) What would you guess is the correct measurement of the picture’s length? __________

   Why? __________________________
1. a) Milena has pennies, nickels, dimes, and quarters in her pocket. Write the units you think were used in these line plots (cm or mm).
   
   i) **Width of Coins in Milena’s Pocket**

   ![Width of Coins Line Plot]

   ii) **Thickness of Coins in Milena’s Pocket**

   ![Thickness of Coins Line Plot]

   b) Use what you know about coins to order them from smallest width to largest width.

   ________________________________ , ________________________________, ________________________________, ________________________________

   c) How many quarters does Milena have? _________

   Circle the Xs on both line plots above that show the quarters.

   d) Complete the table. Write the units in the brackets.

<table>
<thead>
<tr>
<th>Width ( )</th>
<th>Penny</th>
<th>Nickel</th>
<th>Dime</th>
<th>Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness ( )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   e) Explain how you knew which coin had which thickness.

   ________________________________

   f) Finish the line plot. **Value of Coins in Milena’s Pocket**

   ![Value of Coins Line Plot]

   g) How much money is in Milena’s pocket? _________
Discovering patterns in sums.

(MP.7) Exercises: Add and then find a pattern. What will be the next addition equation?

1 + 2 + 3 = ____
2 + 3 + 4 = ____
3 + 4 + 5 = ____
4 + 5 + 6 = ____

Answers: 6, 9, 12, 15; 5 + 6 + 7 = 18

ASK: How did the answers in the exercises change? (they increased by 3) How did the additions change? (they started with a number 1 greater each time) Give students 12 counters each. Tell them to arrange the counters to show 3 + 4 + 5. Students should make three groups—a group of three, a group of four, and a group of five. SAY: Now I want you to show 4 + 5 + 6 instead of 3 + 4 + 5. ASK: How many more counters do I need to give you? (3) How do you know? (because the sum increases by 3 each time) Give students three more counters each. Challenge them to add the three counters, without moving the original counters to show 4 + 5 + 6. (add 1 more to each group)
Draw on the board:

\[ \begin{array}{ccc}
\bullet & \circ & \circ \\
\circ & \circ & \circ \\
\circ & \bullet & \circ
\end{array} \]

SAY: I had a picture showing 3 + 4 + 5, then I added one more dot to each group, so now the picture shows 4 + 5 + 6. I needed to add three dots to do this, because there were three groups.

**Exercises:**
a) If you have dots showing 8 + 9 + 10 + 11, how many more dots would you need to show 9 + 10 + 11 + 12?  
b) If you have dots showing 5 + 7 + 9, how many more dots would you need to show 6 + 8 + 10?  
**Answers:** a) 4, b) 3

**Patterns using the times tables.** Write on the board:

\[
\begin{array}{l}
1 + 2 + 3 = 6 = 3 \times _____ \\
2 + 3 + 4 = 9 = 3 \times _____ \\
3 + 4 + 5 = 12 = 3 \times _____ \\
4 + 5 + 6 = 15 = 3 \times _____ \\
\end{array}
\]

Ask volunteers to fill in the blanks. (2, 3, 4, 5) SAY: All these additions are three times a number. ASK: How can you know what number to multiply three by from just looking at the sum or addition? (it’s the middle number) PROMPT: Where can you see the underlined number in the addition? Have volunteers circle where the underlined number is in each addition. PROMPT: Is it always in the same place? (yes) Can you predict where the number will be for the next sum? (yes, the middle number) SAY: When you can write the numbers in the sequence as a product, you can predict any term without having to find all the terms in between. Write on the board:

\[
\begin{array}{cccccccc}
3 & \times & 2 & = & 6 & 3 & \times & 3 & = & 9 & 3 & \times & 4 & = & 12 & 3 & \times & 5 & = & 15 & 3 & \times & _____ & = & ____ & 3 & \times & ____ & = & ____ \\
\end{array}
\]

Have a volunteer fill in the blanks. (10, 30; 100, 300) Have students check the answers by doing the addition and the multiplication.

**(MP.8) Exercises:** Add the numbers and then multiply the middle number by 3. Did you get the same answer?  
a) 16 + 17 + 18  
b) 29 + 30 + 31  
c) 142 + 143 + 144  
**Bonus:** 65,843 + 65,844 + 65,845  
**Answers:** a) 51, 51, yes; b) 90, 90, yes; c) 429, 429, yes; Bonus: 197,532, 197,532, yes
Using models to understand the pattern. Have students use 12 counters to again show $3 + 4 + 5$. Challenge them to rearrange the counters to show $3 \times 4$ by moving only one counter. (move 1 counter from the group of 5 to the group of 3; this makes 3 groups of 4) When all students have found the answer, have a volunteer demonstrate it on the board:

- Initial arrangement: \[ \begin{array}{cccc} \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \end{array} \]
- After rearrangement: \[ \begin{array}{cccc} \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \end{array} \]

Repeat with $2 + 3 + 4$ and have students change it to $3 \times 3$ by moving only one counter. (move 1 counter from the group of 4 to the group of 3)

**(MP.1, MP.2, MP.7) Exercises:** Draw a picture to show why the sum equals the product.  
(a) $5 + 6 + 7 = 3 \times 6$  
(b) $7 + 8 + 9 = 3 \times 8$

SAY: When you can see a pattern and understand the reasons for it, you can sometimes see many other patterns.

**(MP.1, MP.2, MP.7) Exercises:** Draw a picture or use counters to show the addition. Then …  
(a) move two dots to show that $2 + 4 + 6 = 3 \times 4$.  
(b) move three dots to show that $2 + 5 + 8 = 3 \times 5$.  
(c) move three dots to show that $4 + 5 + 6 + 7 + 8 = 5 \times 6$.  
**Bonus:** move six dots to show that $1 + 2 + 3 + 4 + 5 + 6 + 7 = 7 \times 4$.

Using area models to discover patterns. Write on the board:

- $1 =$  
- $1 + 2 =$  
- $1 + 2 + 3 =$  
- $1 + 2 + 3 + 4 =$  
- $1 + 2 + 3 + 4 + 5 =$

Fill in the blanks as volunteers tell you the sums. (1, 3, 6, 10, 15) SAY: The gaps increase because that’s how we made the sequence, but I want to know if there is a way to get an expression that will help me find any term. One way to think of the sums is as an area. Project grid paper or BLM 1 cm Grid Paper onto the board and draw the following shape on the board:
SAY: Let’s count the squares inside the shape to find the area. ASK: How many squares are in the first row? (1) In the second row? (2) Third row? (3) Fourth row? (4) Fifth row? (5) SAY: So, we can add all these together to find the total. Write on the board:

\[
\text{Area} = 1 + 2 + 3 + 4 + 5 = 15 \text{ square units}
\]

**(MP.1, MP.4) Exercises:**
1. Write the area as a sum by adding the number of squares in each row.
   a)  b)  c)

   ![Images of shapes](images)

   **Answers:** a) 3 + 4 + 5, b) 1 + 2 + 3 + 4, c) 3 + 5 + 7

**NOTE:** Provide students with grid paper or BLM 1 cm Grid Paper for the following exercises.

**(MP.1, MP.2)** 2. Draw an area model for the expression.
   a) 2 + 3 + 4  b) 4 + 5 + 6 + 7  c) 2 + 5 + 8

   **Answers:**
   a)  b)  c)
   ![Images of shapes](images)

   Have students draw two identical shapes like the one on the board on grid paper for 1 + 2 + 3 + 4 + 5 and then cut them out. Challenge students to use their two identical shapes to find twice the area and then divide by two to find the actual area. Encourage students to play with rearranging the shapes into an easy shape to find the area of the single shape. When students are finished, draw on the board:

\[
(1 + 2 + 3 + 4 + 5) \times 2 = 5 \times 6
\]
(MP.1, MP.2, MP.7) Exercises: Use two copies of a shape to make a rectangle. Then write the multiplication.

a) \((1 + 2 + 3) \times 2 = \_\_\_ \times \_\_\_
\)

b) \((3 + 5 + 7) \times 2 = \_\_\_ \times \_\_\_
\)

c) \((1 + 2 + 3 + 4) \times 2 = \_\_\_ \times \_\_\_
\)

d) \((2 + 5 + 8) \times 2 = \_\_\_ \times \_\_\_
\)

Bonus: Which two questions have the same answer? Why does that make sense?

Answers: a) 3 \times 4; b) 3 \times 10; c) 4 \times 5; d) 3 \times 10; Bonus: parts b) and d), 3 + 5 + 7 = 2 + 5 + 8, so multiplying both by 2 gets the same answer.

Using layers instead of rows in an area model. Draw on the board:

\[
\begin{array}{cccccccc}
\text{1} & \text{4} & \text{7} & \text{3} & \text{6} & \text{7} & \text{10} & \text{9} \\
\end{array}
\]

ASK: How does the picture show 1 + 3 + 5 + 7 + 9? (the layers have 1, 3, 5, 7, and 9 squares)

PROMPT: Where do you see the numbers one, three, five, seven, and nine in the picture?

ASK: How does the picture show 5 \times 5? (there are 5 rows of 5 squares)

SAY: When you have two ways to show the number represented, you can write an equation. Continue writing on the board:

\[1 + 3 + 5 + 7 + 9 = 5 \times 5\]

(MP.1, MP.2) Exercises:

1. Draw a picture using layers to write the sum as a product.

a) 1 + 3
b) 1 + 3 + 5
c) 1 + 3 + 5 + 7
d) 1 + 3 + 5 + 7 + 9 + 11

Bonus: Draw a picture to show that 2 + 4 + 6 + 8 + 10 = 5 \times 6.

Selected Solution: d) 6 \times 6

Answers: a) 2 \times 2, b) 3 \times 3, c) 4 \times 4

(MP.1, MP.2, MP.3) 2. Draw a picture using rows instead of layers to show that \((1 + 3 + 5 + 7) \times 2 = 4 \times 8\). Does this match the answer you got in Exercise 1.c)?

Selected answer: Yes, because 4 \times 8 = 4 \times 4 \times 2.
Finding a pattern in the ones digits of products.

**(MP.8) Exercises:** Multiply each pair of numbers. Circle the ones digits. What do you notice?

\[
\begin{array}{cccc}
4 & 14 & 34 & 24 & 74 \\
\times 3 & \times 23 & \times 83 & \times 13 & \times 63 \\
\end{array}
\]

**Answers:** 12; 322; 2,822; 312; 4,662; they are the same each time

Have volunteers write their answers for the previous exercises on the board. Point to the ones digits in each answer, and **ask:** Why are the ones digits always the same in the answers? (because the ones digits of the numbers being multiplied are always the same, 3 and 4)

**SAY:** To find the ones digit of a product, you just have to multiply the ones digits and take the ones digit of the answer. In each pair of numbers, the ones digits multiply to 12, so all the products have ones digit 2.

**(MP.7) Exercises:** Mentally determine the ones digit of the multiplication.

a) \(243 \times 117\)  
b) \(614 \times 516\)  
c) \(235 \times 912\)  
d) \(817 \times 4,367\)  

**Bonus:**
e) \(11 \times 22 \times 33 \times 44 \times 55\)  
f) \(31 \times 71 \times 81 \times 21 \times 51\)  
g) \(238 \times 342 \times 673 \times 501 \times 704\)  

**Answers:** a) 1, b) 4, c) 0, d) 9, Bonus: e) 0, f) 1, g) 2

**(MP.1, MP.7) Using structure to explain the pattern.** Challenge students to draw an area model for \(14 \times 23\) and use the area model to explain why the only part of the multiplication that contributes to the ones digit is when they multiplied four times three. When students finish, draw on the board:

\[
\begin{array}{cccc}
14 & \times 23 & 20 & + 3 \\
10 & 10 \times 20 & 10 \times 3 & 4 \times 20 \\
+ & 4 & + 4 \times 3 \\
\end{array}
\]

**These end in 0 so they don’t contribute to the ones digit.**

**SAY:** Three of the four products end in zero. Only four times three contributes to the ones digit. So the ones digit is two because the ones digit of 12 is two.
Problem Bank

1. a) Use grid paper to make a rectangle from two copies of a shape with area $8 + 9 + 10 + 11 + 12$.
b) What is the area of the rectangle?
c) What is the area of the original shape?
d) What rectangle has the same area as two shapes with area $1 + 2 + 3 + \ldots + 7$?
e) What rectangle has the same area as two shapes with area $1 + 2 + 3 + \ldots + 12$?
f) How can you get your answer to part e) from your answers to parts b) and d)? Write an equation that shows this equality.
g) Use your equation in part f) to mentally calculate $12 \times 13$.

Answers:

a) 

b) 100 square units
c) 50 square units
d) $7 \times 8$
e) $12 \times 13$
f) $(1 + 2 + 3 + 4 + 5 + 6 + 7) + (8 + 9 + 10 + 11 + 12) = 1 + 2 + 3 + \ldots + 12$, so $12 \times 13 = (7 \times 8) + (5 \times 20)$
g) 156

2. a) Fill in the blanks.

i) $1 + 2 + 3 = (\_ \times \_ ) + 2$
ii) $1 + 2 + 3 + 4 = (\_ \times \_ ) + 2$
iii) $1 + 2 + 3 + 4 + 5 = (\_ \times \_ ) + 2$
iv) $1 + 2 + 3 + 4 + 5 + 6 = (\_ \times \_ ) + 2$
v) $1 + 2 + 3 + 4 + 5 + 6 + 7 = (\_ \times \_ ) + 2$

b) Look for a pattern in your answers to part a). Predict: $1 + 2 + 3 + \ldots + 20$

Answers: a) i) 3, 4; ii) 4, 5; iii) 5, 6; iv) 6, 7; v) 7, 8
b) $(20 \times 21) + 2 = 210$
c) $(100 \times 101) + 2 = 5,050$

3. a) Draw a picture with layers to show that $2 + 4 + 6 + 8 + 10 = 5 \times 6$.
b) Draw a picture with rows to show that $2 + 4 + 6 + 8 + 10 = 5 \times 6$.
c) Draw a picture with rows to show that $2 \times (1 + 2 + 3 + 4 + 5) = 5 \times 6$.
d) Explain how you could have predicted that $2 + 4 + 6 + 8 + 10 = 2 \times (1 + 2 + 3 + 4 + 5)$.
e) Calculate $1 + 2 + 3 + 4 + 5$ using $5 \times 6 = 30$. 
Answers:

a) ![Diagram](image)

b) ![Diagram](image)

c) ![Diagram](image)

d) When you double each term, you double the total

e) $30 \div 2 = 15$

(MP.1, MP.2, MP.7) 4. Draw a picture to show that $2 + 4 + 6 + 8 + 10 + 12 = 6 \times 7$.

Answer:

![Diagram](image)

(MP.1, MP.2, MP.3, MP.7) 5. a) Draw a picture for the sum and move one dot to show that the sum is equal to the product.

i) $3 + 5 = 2 \times 4$  
ii) $5 + 7 = 2 \times 6$  
iii) $6 + 8 = 2 \times 7$

b) Draw a picture for the sum and move two dots to show that the sum is equal to the product.

i) $1 + 5 = 2 \times 3$  
ii) $2 + 6 = 2 \times 4$  
iii) $3 + 7 = 2 \times 5$

c) Make a conjecture: How can you get the sum of two numbers from the number that is halfway between them?

d) Use a number line to find the number halfway between the two numbers. Then check your conjecture from part c).

i) 1 and 5  
ii) 2 and 7  
iii) 11 and 16
Selected answers:

a) i) ● ● ● ● ○
   ● ● ● ● ● ● ○

b) i) ●
   ○ ○ ○ ○ ○
   ● ● ● ● ●

c) Double the number that is halfway between them

d) iii) 11 12 13 14 15 16

\[ 2 \times 13 \frac{1}{2} = 2 \times \frac{27}{2} = 27 \text{ and } 11 + 16 = 27, \text{ so the conjecture works} \]

(MP.1, MP.2) 6. Draw a picture to show that 10 + 11 + 12 + 13 is 4 more than 9 + 10 + 11 + 12.
Answer:

(MP.7) 7. If 33 + 158 is 191, what is 34 + 159?
Answer: 193

(MP.7) 8. If 375 + 406 = 781, what is 378 + 409?
Answer: 787

(MP.3, MP.7) 9. If 74 + 75 + 76 + 77 is 302, what is 75 + 76 + 77 + 78? How do you know?
Answer: 306, because if you were to draw rows of 74, 75, 76, and 77, and then add 1 to each of the four rows, you would get rows of 75, 76, 77, and 78

(MP.7) 10. If 44 + 45 + 46 + 47 = 182, what is \(44 \frac{1}{2} + 45 \frac{1}{2} + 46 \frac{1}{2} + 47 \frac{1}{2}\)?
Answer: 184
1 cm Grid Paper
PS4-8 Using a Diagram

Teach this lesson after: 4.2 Unit 8

Standards: 4.NBT.B.5, 4.NBT.B.6

Goals:
Students will create number lines to solve problems involving multiplication of four-digit numbers by one-digit numbers.

Prior Knowledge Required:
Can multiply by 10
Can apply the commutative property of multiplication
Knows the times tables up to 10 × 10
Can multiply whole numbers by 10, 100, and 1,000
Can interpret products in terms of repeated addition
Can multiply one-digit whole numbers by multiples of 10 in the range 10–90
Can add and subtract multi-digit numbers
Understands division as a missing factor problem

Vocabulary: division, multiplication, multiply, number line, product, sum

Materials:
BLM Posters (pp. S-74–75, see Performance Task)

Review the distributive property. Write on the board:

\[
\begin{align*}
5 + 2 &= 7 \\
3 + 3 + 3 + 3 + 3 + 3 + 3 &= (5 \times 3) + (2 \times 3) = 7 \times 3
\end{align*}
\]

SAY: Just like five plus two is seven, five 3s plus two 3s is seven 3s.

(MP.7) Exercises: Write 12 × 8 as a sum of smaller products.

a) \(12 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8}\)

\[= \underline{_______} + \underline{_______}\]

b) \(12 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8 + 8}\)

\[= \underline{_______} + \underline{_______}\]

c) \(12 \times 8 = \underline{8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8} + \underline{8 + 8 + 8}\)

\[= \underline{_______} + \underline{_______}\]
d) \[ 12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 \]
\[ = \underline{\hspace{1.5cm}} + \underline{\hspace{1.5cm}} \]

e) \[ 12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 \]
\[ = \underline{\hspace{1.5cm}} + \underline{\hspace{1.5cm}} \]

f) \[ 12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 \]
\[ = \underline{\hspace{1.5cm}} + \underline{\hspace{1.5cm}} \]

Answers: a) \((11 \times 8) + (1 \times 8)\), b) \((10 \times 8) + (2 \times 8)\), c) \((9 \times 8) + (3 \times 8)\), d) \((8 \times 8) + (4 \times 8)\),
e) \((7 \times 8) + (5 \times 8)\), f) \((6 \times 8) + (6 \times 8)\)

(MP.5) Ask: Which way of separating 12 eights into two smaller numbers of eight would be easiest to use to calculate \(12 \times 8\) mentally? (split 12 into 10 and 2)

SAY: Multiplying by 10 is easy to do, and the result is easy to add. Write on the board:

\[ 10 \times 8 = 80 \quad 2 \times 8 = 16 \quad \text{so } 12 \times 8 = 80 + 16 = 96 \]

SAY: You would get the same answer using any other way, but using the 10 times table is easiest.

Showing the distributive property on a number line. Show students how they can keep track of the multiplication in parts by sketching a number line. Draw on the board:

\[ 14 \times 7 \]
\[ 70 + \underline{\hspace{1.5cm}} = \underline{\hspace{1.5cm}} \]
\[ 0 \quad 10 \times 7 \quad + \quad 2 \times 7 = 12 \times 8 \]

SAY: This isn’t a precise number line because I didn’t try to make the numbers the right distance apart. But the sketch is good enough to help us keep track of the numbers that we are adding. You can use this method to help you multiply.

Draw on the board:

\[ 14 \times 7 \]
\[ 70 + \underline{\hspace{1.5cm}} = \underline{\hspace{1.5cm}} \]
\[ 0 \quad 10 \times 7 \quad + \quad \underline{\hspace{1.5cm}} \times 7 = 14 \times 7 \]

SAY: For the multiplication \(14 \times 7\), I started with 10 sevens, because that’s easy to multiply.

Ask: How many more sevens do I need? (4) Prompt: I need 14 sevens altogether. Write “4” in the bottom blank. Ask: What is \(4 \times 7\)? (28) Write “28” in the blank above “\(4 \times 7\).” Ask: So, what is \(14 \times 7\)? (98) Write “98” in the final blank.
(MP.1, MP.2, MP.7) Exercises:
1. Use the diagram to multiply.
   a) $13 \times 6$
   
   \[
   \begin{array}{c}
   \underline{60} \\
   0 \quad 10 \times 6
   \end{array} + \underline{____ \times 6} = \underline{13 \times 6}
   
   b) $14 \times 8$
   
   \[
   \begin{array}{c}
   \underline{80} \\
   0 \quad 10 \times 8
   \end{array} + \underline{____ \times 8} = \underline{14 \times 8}
   
   c) $17 \times 8$
   
   \[
   \begin{array}{c}
   \underline{80} \\
   0 \quad 10 \times 8
   \end{array} + \underline{____ \times 8} = \underline{17 \times 8}
   
   d) $16 \times 7$
   
   \[
   \begin{array}{c}
   \underline{70} \\
   0 \quad 10 \times 7
   \end{array} + \underline{____ \times 7} = \underline{16 \times 7}
   
   Answers: a) $3 \times 6 = 18$, so $13 \times 6 = 78$; b) $4 \times 8 = 32$, so $14 \times 8 = 112$; c) $7 \times 8 = 56$, so $17 \times 8 = 136$; d) $6 \times 7 = 42$, so $16 \times 7 = 112$

(MP.7) 2. Fill in the blanks to multiply.
   a) $13 \times 7 = (10 \times 7) + (____ \times 7)$
      
      \[
      \begin{array}{c}
      \underline{70} \\
      0 \quad 10 \times 7
      \end{array} + \underline{____ \times 7} = \underline{13 \times 7}
      
   b) $18 \times 4 = (10 \times 4) + (____ \times 4)$
      
      \[
      \begin{array}{c}
      \underline{40} \\
      0 \quad 10 \times 4
      \end{array} + \underline{____ \times 4} = \underline{18 \times 4}
      
   c) $16 \times 9 = (10 \times 9) + (____ \times 9)$
      
      \[
      \begin{array}{c}
      \underline{90} \\
      0 \quad 10 \times 9
      \end{array} + \underline{____ \times 9} = \underline{16 \times 9}
      
   Answers: a) $3, 70 + 21, 91$; b) $8, 40 + 32, 72$; c) $6, 90 + 54, 144$

Review multiplying tens. SAY: Remember, if you can multiply $7 \times 3$, then you can multiply $70 \times 3$; the answer is just 10 times as much. Write on the board:

$7 \times 3 = 21$, so $70 \times 3 = 210$

Exercises: Multiply.
   a) $60 \times 4$  b) $60 \times 8$  c) $90 \times 5$  d) $50 \times 4$

Bonus: Multiply in order: $8 \times 4, 80 \times 4, 800 \times 4, 800 \times 40$

Answers: a) 240; b) 480; c) 450; d) 200; Bonus: 32, 320, 3,200, 32,000
Multiplying two-digit numbers by one-digit numbers using a number line. Write on the board:

\[ 78 \times 3 \]

SAY: You can calculate \( 78 \times 3 \) by multiplying parts of \( 78 \times 3 \) separately. Let’s split 78 into 70 + 8. Draw on the board:

\[
\begin{array}{c}
0 & \quad 70 \times 3 \quad & \quad 8 \times 3 \quad = \quad 78 \times 3 \\
\end{array}
\]

ASK: What is \( 70 \times 3 \)? (210) Write “210” above the “\( 70 \times 3 \)” on the number line. ASK: What is \( 8 \times 3 \)? (24) Write “+ 24 =” above “\( 8 \times 3 \)” on the number line, as shown below:

\[
\begin{array}{c}
0 & \quad 70 \times 3 \quad & \quad 8 \times 3 \quad = \quad 78 \times 3 \\
210 & \quad + \quad 24 & \quad = \\
\end{array}
\]

ASK: So, what is \( 78 \times 3 \)? (234) Write “234” above “\( 78 \times 3 \)” on the number line. SAY: Keeping track of the numbers you’re adding on a number line picture means you don’t have to remember them mentally, and that makes adding the numbers easier.

(MP.1, MP.2, MP.7) Exercises: Sketch a number line to help you multiply.

a) \( 64 \times 3 \)  

b) \( 36 \times 5 \)  

c) \( 87 \times 2 \)  

d) \( 48 \times 7 \)

Selected solution:

a) \[
\begin{array}{c}
0 & \quad 60 \times 3 \quad & \quad 4 \times 3 \quad = \quad 64 \times 3 \\
180 & \quad + \quad 12 & \quad = \quad 192 \\
\end{array}
\]

Answers: b) \( (30 \times 5) + (6 \times 5) = 150 + 30 = 180 \), c) \( 7 \times 2 = (80 \times 2) + (7 \times 2) = 160 + 14 = 174 \), d) \( (40 \times 7) + (8 \times 7) = 280 + 56 = 336 \)

Introduce the distributive property for division. SAY: Because multiplication and division are related, you can use the same idea to make dividing easier too. Draw on the board:

\[
\begin{array}{c}
0 & \quad 10 \times 2 \quad & \quad + \quad \_ \times 2 \quad = \quad 26 \\
20 & \quad + \quad \_ \times 2 \quad = \quad \_ \times 2 \\
\end{array}
\]

SAY: I want to divide 26 by 2, so I need to find out how many twos I need to make 26. Point to the “\( \_ \times 2 \)” under the 26 and SAY: I need to find out what times two is 26. I already have 20 from \( 10 \times 2 \). ASK: How much more is 26 than 20? (6) Write “6” in the top blank. ASK: How many twos are in six? (3) Write “3” in the blank below “6.” SAY: We needed 10 twos and 3 more twos to make 26. ASK: How many twos do we need altogether? (13) Write “13” in the last blank.
Write on the board:

\[ 13 \times 2 = 26, \text{ so } 26 \div 2 = 13 \]

**Exercises:** Fill in the blanks and then use the sketched number line to divide.

a) \( 24 \div 2 \)

\[
\begin{align*}
0 & \quad 10 \times 2 \\
20 & \quad + \quad ____ \times 2 \quad = \quad 24
\end{align*}
\]

b) \( 28 \div 2 \)

\[
\begin{align*}
0 & \quad 10 \times 2 \\
20 & \quad + \quad ____ \times 2 \quad = \quad 28
\end{align*}
\]

c) \( 48 \div 4 \)

\[
\begin{align*}
0 & \quad 10 \times 4 \\
40 & \quad + \quad ____ \times 4 \quad = \quad 48
\end{align*}
\]

d) \( 39 \div 3 \)

\[
\begin{align*}
0 & \quad 10 \times 3 \\
30 & \quad + \quad ____ \times 3 \quad = \quad 39
\end{align*}
\]

**Answers:** a) 4, 2, 12, so \( 24 \div 2 = 12 \); b) 8, 4, 14, so \( 28 \div 2 = 14 \); c) 8, 2, 12, so \( 48 \div 4 = 12 \); d) 9, 3, 13, so \( 39 \div 3 = 13 \)

**Say:** You can divide bigger numbers this way, too. You will just have more to keep track of on the number line picture. Draw on the board:

\[ 76 \div 2 \]

**Say:** When you’re dividing by 2, you can count by 20s until you get close to the number you’re dividing, in this case 76. You count by 20s because that’s ten times two. Show the skip counting on the board:

\[ 76 \div 2 \]

**Say:** Adding another 20 would be more than 76, so we can stop skip counting by 20s at 60. **Ask:** From 60, how much more do we need to get 76? (16) 16 is what times two? (8)
Continue writing on the board:

\[ \frac{76}{2} \]

\[ \begin{array}{cccc}
10 \times 2 & 10 \times 2 & 10 \times 2 & 8 \times 2 = 16 \\
0 & 20 & 40 & 60 & 76
\end{array} \]

So \( ____ \times 2 = 76 \) and \( 76 \div 2 = ____ \)

SAY: So, now it’s just a matter of totalling what we multiplied by. Circle the three 10s and the 8 above the number line. SAY: We are adding 10 twos, then 10 more, then 10 more, then 8 more.

ASK: So, how many twos did we add altogether? (38) Write “38” in the first blank. ASK: So, what is \( 76 \div 2 \)? (38) Write “38” in the second blank.

**(MP.1, MP.2, MP.7) Exercises:** Show counting by 20s on a number line and then divide.

a) \( 52 \div 2 \)  
b) \( 74 \div 2 \)  
c) \( 68 \div 2 \)  
d) \( 90 \div 2 \)

**Answers:** a) 26, b) 37, c) 34, d) 45

Write on the board:

\[ \frac{285}{5} \]

ASK: What can you start skip counting by to divide by 5? (50s) Why? (50 is \( 10 \times 5 \)) Show the skip counting by 50s on a number line, as shown below:

\[ \begin{array}{cccc}
0 & 50 & 100 & 150 & 200 & 250 & 285
\end{array} \]

SAY: Skip counting one more time would pass 285, so we can stop skip counting at 250.

ASK: From 250, how much more do we need to get to 285? (35) Continue writing on the board:

\[ \begin{array}{cccc}
10 \times 5 & 10 \times 5 & 10 \times 5 & 10 \times 5 & 10 \times 5 & 7 \times 5 = 35 \\
0 & 50 & 100 & 150 & 200 & 250 & 285
\end{array} \]

So \( ____ \times 5 = 285 \) and \( 285 \div 5 = ____ \)

Circle the five 10s and the 7 in the picture. SAY: There are 5 tens and a seven. ASK: What number did we multiply by five altogether? (57) Write “57” in both blanks.

**(MP.1, MP.2, MP.7) Exercises:** Sketch a number line to divide. Check your answer using long division.

a) \( 190 \div 5 \)  
b) \( 136 \div 4 \)  
c) \( 364 \div 7 \)

**Answers:** a) 38, b) 34, c) 52
Problem Bank

(MP.1, MP.2, MP.7) 1. Find $678 \div 2$ by counting by 200s and then by 20s. Show your skip counting on a sketched number line.

Solution:

```
<table>
<thead>
<tr>
<th>100 × 2</th>
<th>100 × 2</th>
<th>100 × 2</th>
<th>10 × 2</th>
<th>10 × 2</th>
<th>10 × 2</th>
<th>9 × 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>620</td>
<td>640</td>
<td>660</td>
</tr>
</tbody>
</table>
```

$339 \times 2 = 678$, so $678 \div 2 = 339$

(MP.3, MP.7) 2. a) How does $99 \times 2$ compare to $100 \times 2$? Explain.
b) Use $100 \times 2$ to calculate $99 \times 2$.
c) Use $100 \times 3$ to calculate $99 \times 3$.
d) Use $100 \times 17$ to calculate $99 \times 17$.
e) Use $1,000 \times 2$ to calculate $999 \times 2$.
f) Use $1,000 \times 3$ to calculate $999 \times 3$.
g) Use $1,000 \times 49$ to calculate $999 \times 49$.

**Answers:**
a) $99 \times 2$ is one fewer 2 than $100 \times 2$, so it is 2 less than $100 \times 2$; b) $200 - 2 = 198$; c) $300 - 3 = 297$; d) $1,700 - 17 = 1,683$; e) $2,000 - 2 = 1,998$; f) $3,000 - 3 = 2,997$; g) $49,000 - 49 = 48,951$

(MP.8) 3. a) Calculate each product.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>99</td>
<td>7</td>
</tr>
<tr>
<td>999</td>
<td>7</td>
</tr>
<tr>
<td>9,999</td>
<td>7</td>
</tr>
</tbody>
</table>

b) Use the pattern in part a) to calculate $999,999,999 \times 7$.
c) Check your answer to part b) by doing the multiplication.

**Answers:**
a) 63, 693, 6,993, 69,993; b) 6,999,999,993

(MP.7) 4. Multiply in order.

a) $7 \times 10, 7 \times 13, 7 \times 130, 7 \times 131$
b) $8 \times 10, 8 \times 20, 8 \times 30, 8 \times 32, 8 \times 320, 8 \times 321$

**Answers:**
a) 70, 91, 910, 917; b) 80, 160, 240, 256, 2,560, 2,568

(MP.1, MP.2, MP.7) 5. a) Multiply $4,126 \times 3$ using a number line.

```
<table>
<thead>
<tr>
<th>4,000 × 3</th>
<th>100 × 3</th>
<th>20 × 3</th>
<th>6 × 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>800</td>
<td>50</td>
<td>2</td>
</tr>
</tbody>
</table>
```

b) Sketch a number line to multiply.

i) $852 \times 7$

ii) $613 \times 9$

iii) $4,444 \times 4$

**Bonus:** $312,403 \times 2$

**Selected solution:**

b) i)

```
<table>
<thead>
<tr>
<th>5,600</th>
<th>350</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>800</td>
<td>50</td>
</tr>
</tbody>
</table>
```

**Answers:**
a) 12,378; b) ii) 5,517, iii) 17,776, Bonus: 624,806
(MP.1, MP.2, MP.7) 6. Find the missing number two ways. Make sure you get the same answer both ways.
a) \( ? \times 6 = 192 \)

\[
\begin{array}{cccc}
10 \times 6 & 10 \times 6 & 10 \times 6 & 2 \times 6 \\
0 & 60 & 120 & 180 192
\end{array}
\]

\(? = \underline{\underline{10}} + \underline{\underline{10}} + \underline{\underline{10}} + \underline{\underline{2}} = \underline{\underline{32}} \) or \( \underline{\underline{40}} \)

\[
\begin{array}{cccc}
40 \times 6 = 240 \\
0 & 192 & 210 & 240
\end{array}
\]

\(? = \underline{\underline{30}} - \underline{\underline{18}} - \underline{\underline{12}} = \underline{\underline{32}} \)

b) \( ? \times 8 = 312 \)

\[
\begin{array}{cccc}
30 \times 8 & 9 \times 8 \\
0 & 240 & 312
\end{array}
\]

\(? = \underline{\underline{30}} + \underline{\underline{9}} = \underline{\underline{39}} \)

\[
\begin{array}{cccc}
40 \times 8 & 1 \times 8 \\
0 & 312 & 320
\end{array}
\]

\(? = \underline{\underline{40}} - \underline{\underline{1}} = \underline{\underline{39}} \)

c) Use your answers to parts a) and b) to divide.
i) \( 192 \div 6 \) ii) \( 312 \div 8 \)
d) What multiple of 10 would you put in the blank so that \( 6 \times \underline{\underline{\text{____}}} \) is closest to 294?
e) Sketch a number line to divide 294 \div 6.

**Answers:**
a) \( 10 + 10 + 10 + 2 = 32 \) or \( 40 - 5 - 3 = 32 \); b) \( 30 + 9 = 39 \) or \( 40 - 1 = 39 \);
c) i) 32, ii) 39; d) 50; e) 49
Performance Task: Posters

Materials:
BLM Posters (pp. S-74–75)

Preparation for the performance task. Tell students that the performance task involves the following situation: students are making posters for parents to come and see in a poster show. Each student makes a poster on a large sheet of paper. Draw on the board:

![Diagram of poster dimensions]

17 in  
 22 in

Have students show with their hands about how wide and how long the posters are. Attach some regular sheets of paper to the board, with some really close together and others really far apart. ASK: Is this a good way to display the posters? (no) Why not? (for example, it doesn’t look very good) Tell students that the performance task is partly about decorating posters to make them look good and partly about how to display them to make them look good. Tell students that one of the things they will need to remember how to do is to convert feet to inches.

ASK: How many inches are in one foot? (12) In two feet? (24) Three feet? (36) SAY: You can always skip count by 12 to get the number of inches if you know the number of feet.

Performance Task: Posters. Provide students with BLM Posters. Question 6 is a good opportunity for students to apply the strategy of using a diagram, specifically a number line. Question 7 is another opportunity to apply the strategy of using a diagram, similar to Lesson OA4-33. Students who have not done the relevant problem-solving lesson or Lesson OA4-33 may find those questions difficult.

Answers: 1. 78 in; 2. 2,480 in; 3. 200 × 12 = 2,400 in, no; 4. 15 in; 5. from the edge of one poster to the next, 13 in; 6. 4; 7. 1 1/2 in
Posters (1)

Your class is holding a poster show for parents to come and see some of your class artwork. The posters will be hung in the school hallway all along the hallway. The posters are 17 inches wide by 22 inches tall. Each student can decorate the outside edges of their poster with yarn.

1. How much yarn do you need to decorate the outside edges of your poster?

2. Each student takes an extra 2 inches of yarn, to make sure they have enough. How much yarn, in inches, will a class of 31 students need?

3. A ball of yarn has 200 feet of yarn. Is that enough for the whole class?
Posters (2)

4. You put two pins near the top of the poster, 1 inch from each edge. How far apart are the pins?

![Diagram of a poster with two pins 1 inch from each edge, 22 inches high by 17 inches wide.]

5. You decide to place all the pinholes the same distance apart. How far apart will the posters be?

![Diagram of two posters with pinholes spaced evenly apart.]

6. Doors along the hallway are 9 feet apart. How many posters can you put between each pair of doors?

7. If you want to center the posters so that the posters closest to the doors are the same distance from the door, how far from the door should you make the first pinhole?

![Diagram showing the placement of posters and pinholes centered behind the doors.]

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Blackline Master — Problem-Solving Lessons — Teacher’s Guide for Grade 4
PS4-9  Making a Simpler Problem

Teach this lesson after: 4.2 Unit 8

Standards: 4.NBT.B.4, 4.NF.B.3c, 4.NF.B.4, 4.MD.A.1

Goal:
Students will, when given a problem, make a simpler problem and use the solution to the simpler problem to help solve the original problem.

Prior Knowledge Required:
Can add and subtract two-digit numbers
Can multiply 2 two-digit numbers
Can use long division to divide numbers up to four digits by one-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can represent fractions by using lengths and areas
Can add and subtract fractions and mixed numbers with like denominators
Can multiply a fraction and a whole number

Vocabulary: area, fraction, horizontal, inch, perimeter, vertical

Materials:
chalk, two different colors
BLM Fraction Strips and Circles (p. S-87, see Problem Bank 16)
scissors (see Problem Bank 16)
BLM Flower Garden (pp. S-90–91, see Performance Task)

Using whole numbers instead of fractions to make the problem easier. Draw on the board:

\[ \begin{array}{c}
\text{20}\hline
\end{array} \]
\[ 8 \quad \frac{1}{4} \quad ? \]

SAY: I have two sticks. I know one stick’s length and the total length, but I want to know the length of the second stick. All measurements are in inches. ASK: What makes this problem hard? (the fractions) SAY: Let’s solve the problem by looking at a simpler question with whole numbers first. Leave the above diagram on the board for later use. Draw on the board:

\[ \begin{array}{c}
\text{20}\hline
\end{array} \]
\[ 8 \quad ? \]
ASK: How long is the second stick? (12 inches) If students just say 12, PROMPT: 12 what? (inches) ASK: How did you get that? (subtracted $20 - 8$) SAY: So, you can do the problem the same way when one of the lengths is a fraction. Instead of subtracting $20 - 8$, now you are subtracting $20 - 8 \frac{1}{4}$. Draw on the board:

$$\begin{array}{c}
8 \frac{1}{4} & 9 & 20 \\
\end{array}$$

ASK: What fraction will take you from $8 \frac{1}{4}$ to the next whole number, 9? (3/4) Write “3/4” above the first arrow. ASK: What whole number will take you from 9 to 20? (11) Write “+ 11” above the second arrow, as shown below:

$$\begin{array}{c}
\frac{3}{4} & + & 11 \\
8 \frac{1}{4} & 9 & 20 \\
\end{array}$$

SAY: So, the second stick is $11 \frac{3}{4}$ inches long.

**Exercise:** All lengths are in inches. Find the missing length. Hint: Make the problem simpler, with whole numbers first.

a) $$\begin{array}{c}
\quad & \quad & ? \\
\frac{3}{4} & \quad & \frac{5}{4} \\
\end{array}$$

b) $$\begin{array}{c}
\quad & \quad & ? \\
\frac{15}{4} & \frac{12}{4} \\
\end{array}$$

Answers: a) 8 3/4 in, b) 2 2/4 in

**Using smaller numbers to make the problem easier.** Draw on the board:

$$\begin{array}{c}
32,415 & \quad & 83,122 \\
\quad & \quad & 100,000 \\
\end{array}$$

SAY: Again, all lengths are in inches. ASK: What makes this problem hard? (the numbers are large and the sticks are arranged differently) SAY: Let’s see if we can use smaller numbers to help us decide what to do.
Draw on the board:

![Diagram showing overlapping lengths](image)

ASK: How would you solve this problem? (8 + 3 is 11, and the whole thing is 10, so the overlapping part is 11 − 10 = 1 inch) SAY: You can solve the problem with bigger numbers in exactly the same way. Have a volunteer complete the original problem on the board. (32,415 + 83,122 = 115,537, so the overlapping part is 115,537 − 100,000 = 15,537)

**(MP.1, MP.6)** **Exercises:** All lengths are in inches. Find the length of the overlapping part.

a) ![Diagram showing overlapping lengths](image)

b) ![Diagram showing overlapping lengths](image)

**Answers:** a) 1,494,387 in, b) $\frac{4}{5}$ in

**Making easier problems by moving objects around.** Point to the first example on the board. SAY: Now I'm going to move these sticks around. Draw on the board:

![Diagram showing overlapping lengths](image)

ASK: How did I move the sticks? (you slid one of them down) What else is different about this picture? (there are extra lines) SAY: This now looks like a problem about perimeter and the lengths of missing sides. There’s a lot of extra information in this second problem compared with the first problem, so it looks harder, but it actually has exactly the same answer as the first one. The total length of the two sticks is still 20 inches—I just slid one of the sticks down so that they are not side by side anymore.
(MP.1, MP.6) Exercises: All lengths are in inches. Find the missing side length.

a) 
\[ \begin{array}{c}
32,468 \\
56,219 \\
? 
\end{array} \]

b) 
\[ \begin{array}{c}
15 \frac{3}{8} \\
? 
\end{array} \]

c) 
\[ \begin{array}{c}
\frac{11}{8} \\
21 \\
? 
\end{array} \]

d) 
\[ \begin{array}{c}
2,456 \\
3,807 \\
? 
\end{array} \]

e) 
\[ \begin{array}{c}
3 \\
\frac{4}{2} \\
13 \\
? 
\end{array} \]

f) 
\[ \begin{array}{c}
11 \\
\frac{8}{3} \\
9 \frac{1}{3} \\
? 
\end{array} \]

Answers: a) 23,751 in; b) 8 6/8 in or 8 3/4 in; c) 32/8 in or 4 in; d) 3,464 in; e) 5 1/2 in; f) 7 in

Making the problem easier by emphasizing what is relevant. SAY: If you have a problem that shows more information than you need in order to solve it, you can just focus on what is necessary. If the side length that you’re asked to find is vertical—straight up and down—then bold all the vertical lines. If it is horizontal, bold the horizontal lines.

(MP.1, MP.6) Exercises: All lengths are in inches. Find the missing side length.

a) 
\[ \begin{array}{c}
14 \frac{2}{5} \\
13 \frac{3}{5} \\
11 \frac{2}{5} \\
? 
\end{array} \]

b) 
\[ \begin{array}{c}
958 \\
578 \\
487 \\
? 
\end{array} \]
Bonus:

Answers: a) 5 2/5 in; b) 1,300 in; Bonus: 4 1/4 in

Point out to students that by bolding the horizontal or vertical lines, they were changing the problem into an easier problem.

Finding perimeter by finding missing side lengths. Remind students that to find the perimeter of a shape, they have to add up the lengths of all the outside edges.

(MP.1, MP.4, MP.6) Exercises: All measurements are in feet. Find the perimeter.

Answers: a) 438 ft; b) 215,398 ft

Finding perimeter without knowing all the side lengths. Draw on the board:

SAY: All measurements are in centimeters. Point to the edge on the right and ASK: What is this length? (3 cm) Point to the bottom two horizontal edges and ASK: What might these be? (1 and 5, 2 and 4, or 3 and 3) SAY: We don’t know for sure what the missing horizontal edges are, but let’s find the perimeter using different possibilities.
(MP.6) Exercises: All lengths are in centimeters. Find the perimeter.

a)

b)

c)

Answers: a) 18 cm, b) 18 cm, c) 18 cm

ASK: Did using different possibilities change the answer to the perimeter? (no) Why not? (because the two bottom edges always add to 6, so it didn't change the total) SAY: The bottom numbers always add to 6 because they have to add to the same as the top edge. So, the total perimeter didn’t change.

Using different colors for the horizontal and vertical edges, draw on the board:

SAY: Let’s go back to the original picture. There are two kinds of edges in this shape—horizontal edges, which go sideways, and vertical edges, which go straight up and down. ASK: How long is the top edge? (6 cm) How long are the two bottom edges put together? (6 cm) How do you know? (because if you lined up the two bottom edges together, they would match the top one) How long are the two vertical edges on the left side of the shape? (2 cm and 1 cm) How long is the edge on the right? (3 cm) How do you know? (because it’s the same as the two other vertical edges put together) Write on the board:

Horizontal edges add to ____  Vertical edges add to ____

Perimeter is ____ + ____ = ____

Have volunteers fill in the blanks. (12, 6, 12 + 6 = 18)
(MP.1, MP.3, MP.4, MP.6) Exercises: All measurements are in centimeters. Find the perimeter.

a) ![Diagram of a shape with measurements]
   b) ![Diagram of a shape with measurements]

Answers: a) 8,100 cm; b) 1,204,000 cm

Problem Bank
1. A teacher asks students to read some pages of a book for homework. Write the page numbers down, then count them to find out how many pages the students have to read in total.
   a) Read pages 3 to 6.
   b) Read pages 5 to 10.
   c) Read pages 2 to 7.
   d) Read pages 1 to 8.
   e) Read pages 6 to 11.

   Answers: a) 3, 4, 5, 6; 4 pages; b) 5, 6, 7, 8, 9, 10; 6 pages; c) 2, 3, 4, 5, 6, 7; 6 pages; d) 1, 2, 3, 4, 5, 6, 7, 8; 8 pages; e) 6, 7, 8, 9, 10, 11; 6 pages

(MP.8) 2. a) Complete the chart.

<table>
<thead>
<tr>
<th>Pages to Read</th>
<th>How Many Pages</th>
<th>(Largest Page) – (Smallest Page)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) 1, 2, 3, 4, 5</td>
<td>5</td>
<td>5 – 1 = 4</td>
</tr>
<tr>
<td>ii) 2, 3, 4, 5, 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii) 3, 4, 5, 6, 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) 4, 5, 6, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>v) 5, 6, 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vi) 5, 6, 7, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vii) 5, 6, 7, 8, 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>viii) 5, 6, 7, 8, 9, 10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) How can you get the number of pages to read from the result of the subtraction?

   Answers: a) ii) 5, 6 – 2 = 4; iii) 5, 7 – 3 = 4; iv) 5, 8 – 4 = 4; v) 3, 7 – 5 = 2; vi) 4, 8 – 5 = 3; vii) 5, 9 – 5 = 4; viii) 6, 10 – 5 = 5; b) The number of pages is 1 more than the result of the subtraction.

(MP.4, MP.7) 3. A teacher tells his class to read the given pages in a textbook for homework. How many pages does the class have to read?
   a) from 320 to 387
   b) from 352 to 386
   c) from 298 to 314
   d) from 408 to 451

   Answers: a) 68, b) 35, c) 17, d) 44
(MP.1, MP.4, MP.7) 4. What was the last page that Roy read?
   a) Roy read 5 pages, starting at page 263.
   b) Roy read 156 pages, starting at page 24.
   **Answers:** a) 267, b) 179

(MP.1, MP.4, MP.7) 5. A teacher tells her class to read from pages 354 to 412 but skip pages 363 to 389. How many pages does the class have to read?
   **Answer:** 32

(MP.1, MP.7) 6. How many whole numbers are greater than 11 and less than 45?
   **Answer:** 33

(MP.1, MP.4, MP.7) 7. When everyone in Tom’s class stood in line, Tom was 12th in line and 15th from the end of the line. How many people are in Tom’s class?
   **Answer:** 26

(MP.1, MP.4, MP.7) 8. There are 800 people in line. How many people are behind the 12th person?
   **Answer:** 788

(MP.1, MP.4, MP.7) 9. There is a long line-up at a roller coaster. Sun is 8th in line and Mark is 78th in line. How many people are between Sun and Mark in the line-up?
   **Answer:** 69

(MP.1, MP.4, MP.6, MP.7) 10. Make several easier problems until you see how to do the harder problem.
   a) A fence is made using 42 posts, each 1 meter apart. How long is the fence?
   b) A fence is made using 34 posts, each 2 meters apart. How long is the fence?
   **Answers:** a) 41 m, b) 66 m

(MP.1, MP.4, MP.7, MP.8) 11. How many posts are needed to make the fence?
   a) A fence is 38 meters long with posts 1 meter apart.
   b) A fence is 50 meters long with posts 2 meters apart.
   c) A fence is 63 meters long with posts 3 meters apart.
   d) A fence is 2,156 meters long with posts 4 meters apart.
   **Answers:** a) 39, b) 26, c) 22, d) 540

(MP.1, MP.4, MP.7) 12. A fence for a square yard is made with posts 1 meter apart, including a post at each corner. How many posts are needed for a field that is …
   a) 10 m by 10 m?    **Hint:** Start with 1 m by 1 m, then move on to 2 m by 2 m, 3 m by 3 m, etc.
   b) 20 m by 20 m?
   **Answers:** a) 40, b) 80
NOTE: For the following problems, encourage students to predict each answer before checking.

(MP.1, MP.4, MP.7) 13. A field is a square 20 m by 20 m. How many posts are needed if the posts are ...
   a) 1 m apart?
   b) 2 m apart?
   c) 4 m apart?
   d) 5 m apart?
   e) 40 cm apart?
   **Answers:** a) 80, b) 40, c) 20, d) 16, e) 200

(MP.1, MP.4) 14. In 1993, an artist named Manfred Laber started a piece of public art called The Time pyramid. Every 10 years, a cube is added to the pyramid. This will continue until 120 cubes are placed. In what year will the artwork be finished?
   **Answer:** 3183

(MP.1, MP.4, MP.8) 15. The sides of a square are made of 652 dots (like in the sketch below, but with more dots). Each side has the same number of dots. How many dots are on each side?

Answer: 164

(MP.1) 16. Cut out the strips and circles from BLM Fraction Strips and Circles (you may cut the line down to the center of the circles).
   a) Look at the strip of paper that is partly shaded. Sara thinks that the amount shaded is one fifth. Is she correct? Use folding to check your answer.
   b) Estimate two fifths of the blank strip of paper. Color to show your estimate. Use folding to check your estimate.
   c) Look at the circle that is partly shaded. John thinks that the amount shaded is one fifth. Is he correct? Use folding to check your answer.
   d) Estimate two fifths of the blank circle. Color to show your estimate. Use folding to check your estimate.

(MP.1, MP.6) 17. On this crooked path, each line segment is 1 meter long. What is the total length of the path?

**Solution:** 18 vertical meters plus 17 horizontal meters = 35 meters altogether
18. All measurements are in feet.
   a) Add the horizontal edges and the vertical edges together to find the perimeter.

   \[ \text{Perimeter} = 3 + 8 + 1 + 5 = 17 \text{ ft} \]

   b) Is there enough information to find the area of this shape? Explain.
   **Answers:** a) 34 ft; b) no, because to find the area you need the measurements of each part

19. Each shape was made by placing a small square on top of a large square. All measurements are in centimeters.
   a) Find the perimeter of each shape.
      i) ![Shape 1]
      ii) ![Shape 2]
      iii) ![Shape 3]
      iv) ![Shape 4]

   b) Make a table with headings “Size of Smaller Square,” and “Total perimeter.” Use the pattern from part a) to solve the problems.
      i) A square has side length 11 cm. A smaller square with side length 5 cm is placed on top of it. What is the perimeter of the resulting shape?
      ii) A square has side length 11 cm. A smaller square is placed on top of it. Together they have a perimeter of 58 cm. What is the side length of the smaller square?
   **Answers:** a) i) 46 cm, ii) 48 cm, iii) 50 cm, iv) 52 cm; b) i) 54 cm, ii) 7 cm

20. a) Convert the measurement in feet to inches. Hint: 1 ft = 12 in
   i) 3 feet = _____ inches  ii) 10 feet = _____ inches  iii) 5 feet = _____ inches
   b) Find the perimeter, in inches.
      i) ![Shape 5]
      ii) ![Shape 6]
      iii) ![Shape 7]
      iv) ![Shape 8]

   **Answers:** a) i) 36, ii) 120, iii) 60; b) i) 160 in, ii) 764 in, iii) 294 in, iv) 274 in
Fraction Strips and Circles

[Diagram showing fraction strips and circles]
Performance Task: Flower Garden

Materials:
BLM Flower Garden (pp. S-90–91)

Performance Task: Flower Garden. Provide students with BLM Flower Garden. Question 6 is a good opportunity to apply the problem-solving strategy learned in this lesson. Students who have not had the opportunity to do this lesson will find that question difficult.

Answers: 1. 20 m; 2. $60; 3. $252; 4. 13 bags; 5. 60 + 252 + 13 = $325; 6. 6 rows of 14 flowers each, or 84 flowers
Kyle has a flower garden. His flower garden is 3 meters wide by 7 meters long.

1. He wants to put a fence around his garden to keep animals away from his plants. How many meters of fencing does Kyle need?

2. If fencing costs $3 per meter, how much will the fence cost?

3. Kyle needs to add more soil to his garden. The soil will cost $12 for each square meter. How much will the soil cost?
Flower Garden (2)

4. Kyle decides to plant 115 flowers in the garden. Flower seeds cost $1 per bag. Each bag holds 9 seeds. How many bags does Kyle need to buy?

5. What is Kyle’s total cost for the flower garden?

6. After planting his garden this year, Kyle sees new instructions for planting a flower garden:
   
   Plant each flower 50 cm apart.
   
   You can start planting 25 cm from each edge of the garden.
   
   If he follows these instructions next year, what is the greatest number of flowers he can plant?
This Unit in Context

Students have seen many measurement attributes by this point: length, starting in Grade 1 (1.MD.A.2); time, starting in Grade 1 (1.MD.B.3); money values, starting in Grade 2 (2.MD.C.8); and time intervals (3.MD.A.1), liquid volumes (3.MD.A.2), masses (3.MD.A.2), and areas (3.MD.C.6), starting in Grade 3. In this unit, students are introduced to measuring angles—the amount of rotation from one ray to another (4.MD.C.5)—and degrees as the units of measurement for angles (4.MD.C.6).

In Kindergarten and Grade 1, students gained experience with angles by composing shapes to make new shapes (K.G.B.6)—for example, students might check to see if one angle fits inside an open space. Students also began in Grades 1 and 2 to recognize when a corner of a shape is like the corner of a square (in other words, a right angle) referring to these as “square corners.” In Grade 3, students learned that an angle is the space between two straight sides that meet at a vertex and that an angle is a right angle if a square corner fits it exactly.

Just as students learned in Grade 2 that lengths can be added (2.MD.B.5) and in Grade 3 that areas can be added (3.MD.C.7), students will learn in this unit that angles can be added (4.MD.C.7). Students will distinguish between lines, line segments, and rays, and they will learn that angles are formed wherever two rays share a common endpoint (4.MD.C.5).

Students learned in 4.2 Unit 4 how to find a fraction of a whole number. In this unit, students will learn that the unit of measurement, a degree, is the angle that rotates 1/360 of a circle (4.MD.C.5), and they will determine fractions of 360° in order to find the angle covered by a given fraction of a circle. Students will also learn that a right angle is equal to 90°, which is a quarter turn of a circle. They will classify angles as obtuse if they are greater than 90° and acute if they are less than 90° (4.G.A.1). Students will measure the angles in polygons and recognize right triangles as a new category (4.G.A.2).

Further along in the curriculum, students in Grade 5 will use their ability to measure angles when they learn that opposite angles in a parallelogram are equal and that rhombuses have perpendicular diagonals (5.G.B.4).

In Grade 7, students will construct triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle (7.G.A.2). They also will solve simple equations for an unknown angle in a figure, using facts about supplementary, complementary, vertical, and adjacent angles (7.G.B.5). In Grade 8, students will study the angles created when parallel lines are cut by a transversal, and they will learn the angle-angle criterion for similarity of triangles (8.G.A.5).
Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1 to MP6 and MP8. Here are some examples of how students can show that they have met a standard.

**MP2:** In G4-22 Extension 3, students reason abstractly and quantitatively when they fit two pattern block triangles into one hexagon angle and then use that to decide the angle in a hexagon from the known angle in the pattern block triangle.

**MP3:** In the extension in G4-15, students critique the argument that because one angle is drawn completely inside another (but without matching up the vertices), that the angle is smaller than the other angle.

**MP6:** In G4-18 AP Book Question 2, students attend to precision when they measure angles in order to notice a pattern. Students who do not measure precisely will not be able to notice the pattern.

**MP8:** In G4-18 AP Book Question 2, students notice regularity when they measure several angles and the small angles that make up each angle, and then use the result to decide how to get the measure of the whole angle from the measures of the small angles.
Introduce points and line segments. Draw a dot on the board and explain to students that in geometry we deal with different objects. One of these objects is called a point. To distinguish different points we give them names, such as A, P, or X. We use capital letters to name points. Write the letter A beside the dot you drew and explain that this will be point A.

Draw a line segment on the board using a ruler. Explain that a line segment is a straight path between two points, called endpoints. We can measure the length of a line segment. We draw little dots at the ends of a line segment to mark the endpoints. Emphasize that in geometry it is important to draw pictures accurately and precisely, so when students are asked to draw something, they should use a ruler.

Review the terms “horizontal” and “vertical.” Have students draw a horizontal line segment on grid paper, then mark and label a point on it. Ask students to place the point so that it is at about the midpoint of the line segment. Repeat with a point that is closer to one of the endpoints. Repeat with a vertical line segment. Then ask students to mark another point that is not on the line segment. Finally, have students draw a line segment that is neither horizontal nor vertical.

Introduce lines. Explain that a line extends in a straight path forever in two directions. It has no endpoints. We draw lines with little arrows at both ends to emphasize that we can extend them in both directions.

Draw a few line segments and lines on the board and have students identify each as either a line or a line segment. Have students draw a horizontal, a vertical, and a slanted line segment. Make sure students draw little dots at the endpoints. Then ask students to draw a horizontal, a vertical, and a slanted line. Make sure students draw little arrows at the ends. Have students exchange their notebooks and extend each other’s lines in both directions.
Give students who need additional practice questions like the following drawn on grid paper:

Extend each line using a ruler. Is the point on the line?

**Answers:** a) yes, b) no

![Diagram](image1)

**Bonus:** Draw a letter that does not have curved parts (Examples: W, E) and mark each part of the letter as a line. Extend each line. Do some letters produce similar patterns? (X and V, N and Z, for example)

**ASK:** Can we measure the length of a line? Draw a line, and measure the picture. **ASK:** Can I say that the length of the line is [20 cm]? Have several students explain their answers. Extend the line and **ASK:** Is it the same line? (yes) Is it still 20 cm long? (no) Point out that if it is the same line, it should have the same length; it cannot be both 20 cm long and 25 cm long. Explain that we cannot measure the length of the line because every time we extend it, the length changes. We can measure the length of a line segment or a picture of a line, but not the length of a line itself.

**Introduce rays.** Ask students where they have heard the word “ray.” Point out that a ray, such as a ray of light, has a beginning but no end—it goes on and on. Explain that in mathematics a ray is part of a line (so it is straight, just like a ray in real life), and it has one endpoint (so it has a beginning, also like a ray in real life). A ray extends forever in the other direction. We draw a dot to show the endpoint and an arrow on the other end to show that it can be extended as much as needed. Draw a few rays pointing in different directions and have students identify which end can be extended. Then ask students to copy the rays, exchange their notebooks with a partner, and extend the rays their partners drew.

Draw a few lines, line segments, and rays; point to each in turn; and have students signal which is which. (For example, students can point both thumbs outward to show a line, inward to show a line segment, and to one side to show a ray.)

Draw a line segment and a point $A$ such that when the line is extended it passes through the point $A$:

![Diagram](image2)

**ASK:** Is this point on the line segment? (no) Change the line segment into a line by replacing the endpoints with arrows. **ASK:** Does it look like this point might be on the line? Can we tell for sure from the picture? (no) How can we check if the point is on the line? (extend the line) Invite a volunteer to extend the line and check that the point is indeed on the line.
Draw a line segment and a point \( B \) to the right of it, as before, but do not mark either endpoints or arrows on the ends. Tell students that you want to make this into a picture of a ray. How can you do that? Take suggestions. Then draw an arrow on the left end and a dot on the right end to make a ray. Is the point \( B \) on the ray? (no) What if we extend the ray? Will the point then be on the ray? (no) Demonstrate that when you extend the ray, only the end with the arrow can be extended, so the point \( B \) is not on the ray. Now reverse the position of the dot and the arrow, so that the ray points in the opposite direction. Point out that this is a different ray, because it points to the opposite direction. Is the point \( B \) on this new ray? (yes) How can we check? (by extending the ray) Extend the ray to demonstrate that the point is now on the ray. Emphasize that the fact that the ray points in a different direction makes the rays different, even if they are parts of the same line. In fact, any point splits a line into two rays going in opposite directions (show a picture of a line with a point on it and emphasize each ray with a different color).

Have students practice drawing and extending lines, line segments, and rays using Questions 1 through 10 on AP Book 4.2 pp. 155–156.

**Introduce intersection points.** Explain that a point where lines, line segments, or rays meet is called an *intersection point*. Draw a pair of intersecting lines as shown at left and mark the intersection point with a small dot. Draw a few other intersecting objects and invite volunteers to mark the intersection point. **Exercises:** a pair of intersecting rays, a line intersecting a line segment at a point that is not an endpoint, a ray intersecting a line

Explain that the endpoints of a line segment or the endpoint of a ray can also be intersection points. Draw a few examples of objects that intersect at an endpoint or endpoints and have volunteers identify the intersection point. **Exercises:**

Draw a line. Tell students that there is a point \( A \) on the line. Point to a few locations on the line and explain that the point could be any of these. To make it clear which point we are talking about, we need to draw a dot. Then draw a pair of intersecting lines, but do not mark the intersection point with a dot. **ASK:** Can you tell where the intersection point of these two lines is? (yes) Explain that there can be no confusion about where exactly the point is in this case, so there is no need to mark it with a dot. Draw several examples of intersecting objects, this time without dots marking the intersection point, and invite volunteers to circle the intersection points. Have students practice identifying intersection points using Question 11 on AP Book 4.2 p. 157. Then have students draw intersecting objects (Question 12), and have partners exchange student books and circle the intersection points.
**Intersecting lines, line segments, and rays.** Draw a pair of lines that will intersect if extended and ask if these lines intersect. Invite a volunteer to extend the lines to check. Then draw a pair of line segments in the same position. Do they intersect? (no) Can we extend them? (no) Repeat with the pictures below, but this time have students predict whether the objects intersect prior to checking the intersection. Students can signal the answers with thumbs up and thumbs down. **Exercises:**

a) ![Image of lines](image1.png) 

b) ![Image of lines](image2.png) 

c) ![Image of lines](image3.png) 

d) ![Image of lines](image4.png) 

e) ![Image of lines](image5.png) 

f) ![Image of lines](image6.png) 

g) ![Image of lines](image7.png) 

h) ![Image of lines](image8.png) 

**Answers:** a) no, b) yes, c) yes, d) yes, e) no, f) no, g) yes), h) no

(MP6) **Exercise:** Imagine extending the line and the rays in the directions possible (shown by the arrowheads).

- ![Image of rays and lines](image9.png)

Decide which rays and line segments intersect the line. Extend the objects you can to show the intersection points or explain why you think the object does not intersect the line. Use the words “intersect,” “line,” “ray,” and “line segment.”

**Selected answers:** Ray A will never intersect the line (G) because it only extends up and it would have to extend down to intersect the line. The line segments B and D do not intersect the line because B is above the line and D is below it and neither can be extended; the ray F and the line segment C both meet the line.

**Extension**

(MP4) **Exercise:** A book is about 33 cm long and 25 cm wide. How many of these books will you need to cover an area of about 1 m²?

**Solution:** 3 books will fit across the length of 1 meter and 4 books will fit across the width of 1 meter. I drew an array and saw that 3 × 4 books fit into 1 square meter.
**Introduce angles.** Remind students that rays are parts of lines that have one endpoint and can be extended as much as needed at the other end. Explain that in mathematics, when two rays have a common endpoint, they make an angle. Draw two rays with a common endpoint. Explain that when we deal with angles, the common endpoint of the rays is very easy to see, so there is no need to draw a dot to show it. Draw a few pictures of pairs of rays as in Question 1 on AP Book 4.2 p. 158, and have students show thumbs up if the picture shows an angle and thumbs down if it does not. Avoid pictures like the one at left, since you did not talk about one ray being part of another ray.

Explain that the rays are called the arms of the angle, and the common endpoint is called the vertex of the angle. Have students identify the vertex and extend the arms of a few angles by completing Question 2 on AP Book 4.2 p. 158.

**Compare angles.** Students often mistake the area between the arms of an angle for the angle. To help dispel this misconception (and to reinforce the correct concept of an angle), tell students that angles are like corners, and the sharper the corner, the smaller the angle. Ahead of time, make keys like the one shown at left from old postcards. Give each student a key and ask students to run their fingers over the corners to identify the sharpest one. Then ask students to order the corners A, B, and C from sharpest to bluntest. Students could then trace the corners of the key to draw the corresponding angles.
Tell students that we can compare angles the way we compare corners:

Draw two angles as shown at left on the board. Ask: Which corner is sharper? (the one on the left) Explain that the angle is the region between the arms. Shade the region to emphasize it. Point out that the angle on the left will fit inside the angle on the right. Using paper, trace the smaller angle and cut it out. Show how the tracing fits inside the other angle by placing the cutout so that the vertices of the two angles coincide and their horizontal arms are aligned. Explain that the sharper corner means that the angle is smaller; the angle on the left is smaller than the angle on the right.

Now draw the second pair of angles at left. Ask: Which corner is sharper? (the one on the left) Again, shade the region between the arms. Explain that even though the shaded region in the angle on the left looks larger, this does not mean that the angle is larger. Remind students that the sides of the angle are rays, so they can be extended. Have a volunteer extend the arms of the angle on the right. Shade the region between the extended arms. Point out that just as we do not change a ray by extending it, we do not change an angle by extending its arms. The corner does not become sharper (or less sharp) after we extend the arms. Use the tracing and cutting method above to show that the angle on the left is smaller than the other one, and emphasize that the sharper the corner, the smaller the angle.

The size of an angle is the amount of rotation between the arms.

Explain that in mathematics the size of an angle is not defined in terms of sharpness, because it is hard to attach a number to sharpness; it is hard to measure how sharp something is. Instead, the size of an angle is measured in terms of rotation between the angle’s arms. Show a pair of scissors. Point out that the legs of the scissors rotate around a peg. Hold the scissors such that one leg is horizontal at all times. Open the scissors a little bit, then open them more and more to show how the space between the arms increases as the top leg rotates away from the horizontal leg. Trace a small angle on the board, then open the scissors wider than that angle, and show how the smaller angle on the board “fits inside” the larger angle of the open scissors. Emphasize that the more you open the scissors, the more you turn the legs, and the larger the angle becomes.

Draw the following picture to illustrate what you mean by smaller and larger angles.

![Smaller Larger](image)

When the angle is small, the arms are only open a little bit. When the angle is large, the arms are open a lot.

With a piece of chalk you can demonstrate how much an angle’s arm rotates. Draw a ray on the chalkboard, then rest the chalk along the ray’s length. Fix the chalk to the endpoint of the ray and rotate the free end around the endpoint to any desired position.
Comparing and ordering angles.

**ACTIVITY 1**

On the school playground, have students work in pairs. Each student draws two angles in chalk on the pavement such that one angle is smaller, or “sharper,” than the other. Ask partners to mark their angles with different letters (A, B, C, D) so that they can order them later. Then ask each student to stand on the vertex of one of their angles, lock their hands together, and stretch their arms out in front of them in line with one of the arms of the angle (their arms will be pointing in the direction of that arm). Have students turn, or rotate, their bodies to make their arms line up with the other arm of the angle. Repeat with a different angle. **ASK:** For which angle did you need to turn more? Remind students that the size of the angle is the amount of rotation from one arm to the other, so the angle that required more rotation (a larger turn) is the larger angle. Have students individually compare their angles and then order the four angles they produced as a pair from smallest to largest.

Draw several pairs of angles on the board and have students decide which angle is larger. **Exercises:**

a) ![Angle A](image)

b) ![Angle B](image)

c) ![Angle C](image)

d) ![Angle D](image)

e) ![Angle E](image)

f) ![Angle F](image)

g) ![Angle G](image)

h) ![Angle H](image)

i) ![Angle I](image)

See also Questions 3 through 5 on AP Book 4.2 pp. 158–159. Finally, draw the angles below on the board and ask students to order them from smallest to largest.
Acute and obtuse angles. Review right angles with students. Remind students that square corners are called right angles in mathematics, and that a convenient way to check whether an angle is a right angle is to compare it to a corner of a regular sheet of paper. If an angle is not a right angle, we can tell whether it is larger than a right angle or smaller than a right angle. Have students compare a few angles to a corner of a sheet of paper and tell whether they are larger or smaller than a right angle.

**ACTIVITY 2**

Use an object with a square corner (such as an index card or a rectangular sheet of paper) to check whether objects in the classroom have angles greater than, smaller than, or equal to a right angle. Possible angles to check: corners of a desk, window corners, corners of base ten materials and pattern blocks, angles made by a door and a wall.

Explain that angles that are smaller than a right angle are called acute angles, and angles that are larger than a right angle are called obtuse angles. You might point out the connection with sharp and blunt corners: acute is a synonym of sharp, and obtuse is a synonym of blunt. Draw a few angles and have students identify them as acute or obtuse. Students can signal their answers by making the letters A or O with their hands.

Have students identify examples of acute and obtuse angles in the classroom (e.g., corners of garbage bins, pattern blocks, the angle that a door makes with a wall). Then have students draw acute and obtuse angles, or make them on a geoboard.

**Extension**

In the picture, shown in the margin, Mandy says that angle B is smaller than angle A because angle B fits inside angle A. Do you agree with Mandy? Explain why or why not.

**Answer:** I disagree.

**Sample explanations**

- To compare the angles properly, you need to put one vertex on top of the other and turn the angles to match up one of the arms. Then you can check to see which angle fits inside the other to find out which angle is smaller.
- When you extend the rays of B, they go outside angle A, so B doesn’t really fit inside A.
Review comparing angles and size of angles. Draw several pairs of angles on the board (see exercises below) and ask students to tell which angle in each pair is larger. Ask students to justify their answer. Remind students that the size of the angle is the amount of rotation from one arm to the other, and that the arms of an angle can easily be extended (because they are rays), so their size in the picture does not matter. Exercises:

a)  

b)  

c)  

Then present the following pair of angles (30° and 35°; do not provide the measurements), and ask students which angle is larger. You might call a vote.

Then present the following pair of angles (30° and 35°; do not provide the measurements), and ask students which angle is larger. You might call a vote.

Explain to students that because the angles are very close in size, it is hard to compare them by sight alone.

Review the need for standard units to measure length. For example, if the engineer who designed a product sends information to a worker who will make it, that information must include the product’s exact dimensions. An engineer cannot say “make a table that will be as wide as an arm span,” because that description will produce a few very different tables, depending on the arm span of the maker!
Degrees. Remind students that the size of an angle is how much one arm turns to become the other arm. ASK: How can we measure how much something turns? Make a full turn (spin around) and ask students to describe what you did. Show a half turn and a quarter turn. Can students tell what part of a turn each of the angles you drew before is just by looking at them? (no) Even if you created those angles on the floor (with tape, for example), it is hard to compare angles by turning your body, especially if they are close in size.

Explain that a degree is a unit for measuring angles. Have students look at the picture of a one-degree angle in the student book and discuss why the arms are drawn so long (it would be hard to see the opening between the arms otherwise). Explain that a right angle measures 90 degrees. Have students complete Question 1 on AP Book 4.2 p. 160.

Introduce the notation for degrees. Explain that writing the word “degrees” takes time, and people often use a symbol instead. The symbol is a small raised circle that is written after the number. For example, the measure of a right angle is written as 90°. Have students write the angle measures in Question 1 using the degree symbol (°).

Acute and obtuse angles. Review the meaning of “acute” (smaller than a right angle) and “obtuse” (larger than a right angle). Point out that in the measurement of length, if a pencil is longer than an eraser, the measurement for the pencil will be larger than the measurement for an eraser. Draw a red right angle, a blue acute angle, and a black obtuse angle on the board. ASK: Which angle is the largest? (the black angle) Is it an acute angle, an obtuse angle, or a right angle? (obtuse) Which angle is the smallest? (the blue angle) Is it an acute angle, an obtuse angle, or a right angle? (acute) Which angle is the right angle? (the red angle) Explain that since the red angle is larger than the blue angle, this means that the degree measure of the red angle will be larger than the degree measure of the blue angle.

ASK: How does an acute angle compare to a right angle? (it is smaller—less than 90°) How does an obtuse angle compare to a right angle? (it is larger—greater than 90° but less than 180°) Write a few angle measures (such as 89°, 34°, 142°, etc.) and have students signal whether these angles are acute or obtuse. (Students can signal thumbs up for obtuse and thumbs down for acute, or they can make the letters A and O with their hands.) Repeat with angles drawn on the board. For each angle, have students also say whether they expect the measure of each angle to be more than 90° or less than 90°.

Introduce protractors. Draw a pair of line segments of almost identical length, one horizontal and one vertical. Ask students how they would tell which one is longer. (measure them with a ruler) In this case, we need a tool to measure and thus compare line segments, and the tool we use is a ruler. When angles are similarly positioned in different orientations, we also need a tool to measure and compare them. Explain that the tool for measuring angles is called a protractor.
Two scales of a protractor. Have students examine their protractors and say how they are similar to rulers and how they are different. Draw attention to the fact that a protractor has two scales, while a ruler often has only one scale. (If students have rulers with two scales on them, they should note that the units in each scale are different, whereas the units in each scale on the protractor are the same.) Explain that having two identical scales going in different directions allows you to measure the angles from both sides, but this also means that you need to decide which scale you will use each time.

Display the first picture below and explain that this protractor is like the protractor in Question 5 on AP Book 4.2 p. 161. ASK: Is this angle an acute angle or an obtuse angle? (acute) Circle both numbers that the arm of the angle passes through. Which one is the answer? (30°) How do you know? (the angle is acute, so the answer must be less than 90°) Repeat with the second picture below. Then have students complete Questions 5 and 6 on AP Book 4.2 pp. 161–162.

Placing protractors on angles. Point out the base line and the origin on a large protractor or on a picture of a protractor on the board. Have students find the base line and the origin on their protractors. Demonstrate how to place a protractor correctly, so that one arm of the angle lines up with the base line and the vertex is at the origin. Point out that this is similar to placing a ruler with the zero at the beginning of the object we are measuring. Have students draw an acute angle in their notebooks and ask them to place their protractors correctly. Circulate in the classroom to check that all students have done so. Then have students measure the angle they drew. Repeat with an obtuse angle. Have students swap notebooks with a partner and measure each other's angles to check their work.

When students have completed all of the questions for this lesson in the student book, SAY: Captain Flint, a bloodthirsty pirate, has hidden part of his treasure in our classroom. The clue to its hiding place is in your student book. Point out that each angle in Questions 6 and 8 on AP Book 4.2 pp. 161 and 163 is labeled with an uppercase letter. Students should arrange the angles from smallest to largest. When they write the letters according to the order of the angles, they will get a phrase that tells where the treasure is hidden. (Answer: Under the window)

You could create different clues by assigning different letters to the parts of these questions and having students order them a different way (Examples: order the angles from largest to smallest; order only the angles larger than or smaller than a certain measurement). You could also hide actual small treasures around the classroom to make deciphering the clues more fun.
Extensions

1. a) Use a ruler to draw a square on grid paper. Then draw the line segments joining the opposite vertices of the square. These line segments are called diagonals. Label the point where the diagonals intersect $P$.

b) Measure the angles around the point $P$. What do you notice?

**Answer:** b) all the angles are right angles

2. Have students copy the 40° angle from AP Book p. 160 Question 1 part b) onto tracing paper. Have students predict which of the other angles on that same page are more than 40° and then check using the tracing paper.

3. Thirteen people shared some chocolate bars, but some people wanted more chocolate than others. The line plot shows the fraction of a chocolate bar each person ate. How many chocolate bars were shared?

   ![Line plot](image)

   **Answer:** The total shared is 48 eighths, which is equal to 6 full chocolate bars.

   Whole-class follow-up: Take up various strategies for finding the total, such as using equations, drawings of chocolate bars divided into eighths, or fraction circles. Compare the approaches: How does one student’s pictures show another student’s equations? Which way is easier to understand? Which way is faster? How are these two pictures (or these two equations) the same or different?

4. Aputik is fixing up her basement bathroom. She is going to need a total of 28 tiles and 6 yards of wood. Tiles come in packs of 5 and cost $8 per pack. Wood is sold by the foot and costs $7 per foot. How much will it cost Aputik to fix up her basement bathroom?

   **Answer:**
   
   \[ 28 \div 5 = 5 R 3, \text{ so she needs to buy 6 packs of tiles. Each pack costs } 8, \text{ so the cost of tiles is } 48 \text{ (because } 6 \times 8 = 48). \text{ Aputik needs 6 yards of wood and each yard is 3 feet, so she needs 18 feet of wood (because } 6 \times 3 = 18). \text{ Wood costs } 7 \text{ per foot, so the cost of wood is } 126 \text{ (because } 8 \times 7 = 126). \text{ Then I found } 48 + 126 = 174. \text{ It will cost Aputik } 174 \text{ to fix up her basement bathroom.} \]
Review measuring angles with a protractor. Make sure every student knows how to choose the correct scale on the protractor when measuring an angle, by deciding whether the angle is obtuse or acute and choosing the measure that is respectively larger than 90° or smaller than 90°.

Have students draw one acute angle and one obtuse angle in a partner’s notebook and measure the angles their partners drew.

Drawing angles. Model drawing angles step by step (see AP Book 4.2 p. 164). Emphasize the correct positioning of the protractor. Have students complete Questions 1 and 2 on AP Book 4.2 p. 164 and check that they use protractors correctly (you can photocopy BLM Protractors onto transparencies to create protractors). You can use Question 1 for diagnostic assessment. In 1 a), the correct answer is C, and in 1 b) it is A. The wrong answers correspond to common mistakes in placing the protractor:

a)  A, D – the base line of the protractor is not aligned with the arm (it is rotated)  
    B – the arm is not aligned with the base line (students using BLM Protractors have aligned the arm with the bottom edge of the protractor)

b)  C – the base line of the protractor is not aligned with the arm (it is rotated)  
    B – the arm is not aligned with the base line (students using BLM Protractors have aligned the arm with the bottom edge of the protractor)  
    D – the wrong scale was chosen
Have students practice drawing angles, starting with measures that are multiples of 10 (Examples: 20°, 50°, 140°). Continue to measures that are multiples of 5 (Examples: 35°, 95°) and then all measures in between (Examples: 47°, 121°). The Activity below also provides practice drawing angles.

**ACTIVITY**

Students will need a die, a protractor, a sheet of paper, and a pencil. Draw a starting line on the paper. Roll the die and draw an angle with the measure given by the die. Use the starting line as your base line and draw the angle counterclockwise. Label your angle with its degree measure. For each next roll, draw a new angle in the counterclockwise direction using the arm drawn on the previous roll as the base line. The measure of the new angle is the sum of the result of the die and the measure of the angle on the previous roll. Stop when there is no room to draw an angle of the size given by the roll. For example, if the first three rolls are 4, 5, and 3, the picture will be as shown in the margin.

**Extensions**

1. Create a triangle by following these steps:
   a) Draw a base line segment of the given length below.
   b) At each endpoint of the line segment, draw an angle of the given size.
      i) \[40°\] \[5\text{ cm}\] \[70°\]
      ii) \[50°\] \[6\text{ cm}\] \[40°\]

2. Draw a square using a ruler and a protractor.

3. Edmond says “Since there are 1,000 milliliters in a liter, milliliters must be bigger than liters.” Do you agree with Edmond’s reasoning? Why or why not?

   **Sample answer:** I disagree, because for 1,000 milliliters to fit into one liter, a milliliter has to be smaller than a liter.
**G4-18 Adding Angles**

**STANDARDS**
4.MD.C.7

**VOCABULARY**
angle
arm
degree
protractor
ray
vertex

**Goals**
Students will recognize angle measure as additive and will find unknown angles.

**PRIOR KNOWLEDGE REQUIRED**
Can identify and draw rays
Understands that a ray can be extended
Can identify and draw right, acute, and obtuse angles
Knows that the size of an angle is a measure of rotation from one arm to the other
Can measure angles with a protractor
Knows that a variable can replace a quantity

**MATERIALS**
a clock with a second hand
BLM Clocks (p. T-32)
BLM Angle Measures Are Additive (p. T-33)
protractors

**Angle measures are additive.** Show a large clock with a second hand. Draw students’ attention to the 1-minute marks. How long does it take a second hand to turn from one such mark to the other? (1 second) So you can call the space between the marks a 1-second interval. Point out that the hands do not move in a straight line, they rotate. So a 1-second interval is actually an angle. Where is the vertex of that angle? (at the center of the clock, where the hands are attached)

Ask students to start clapping when the second hand is exactly at 12 and to stop when the second hand is exactly at 2. Then ASK: How many seconds did you clap for? (10) How do you know? (the hand turned from 12 to 2—this is ten 1-second intervals) Then ask students to start tapping a hand on a leg when the second hand is at 2 and to stop tapping when it is at 3. ASK: How many seconds did you tap for? How do you know? Now repeat the two actions in succession: start clapping when the hand is at 12, stop clapping and start tapping when the hand is at 2, and stop tapping when the hand is at 3. ASK: How many seconds passed? How many 1-second intervals did the second hand turn? (15) How did you get 15 from 10 (the clapping interval) and 5 (the tapping interval)? Do you think the same will happen with angles?

Have students draw all three positions of the second hand on a blank clock face (they could use BLM Clocks) and measure the angles the hands turned. Then ask them to check whether the angle that the clock hand traced when turning from 12 to 3 is equal to the sum of the angles between 12 and 2 and between 2 and 3.
Have students draw an angle, divide it into two unequal parts, and measure the small angles and the whole angle. What do they notice about the measures of the two smaller angles and the measure of the whole angle? (the measures of the small angles add to the measure of the whole angle)

Did everyone get the measure of the whole angle equal to the sum of the measures of the smaller angles? Have students who did not get this result measure the angles again to correct their mistake. Did everyone draw the same whole angle? (no) Will any angle that is made of two smaller angles measure the same as the sum of the two smaller angles? (yes)

Present the pictures below, and have students find the measure of the large angle from the measures of the smaller angles.

Reverse the task: Provide the measures of the larger angle and one of the smaller angles, and have students find the measure of the second small angle. Solve the first example below as a class, then have students work individually.

Angles are not additive if there are gaps or overlaps. Display the first picture below (both pictures are on BLM Angle Measures Are Additive) and say that you think the large angle will be $75^\circ + 60^\circ = 135^\circ$. Ask a volunteer to come and measure the large angle. (You can also distribute the BLM and have all students measure the angles.)
Ask students to work in pairs to explain your mistake to each other and to come up with a common explanation about the problem. You can put pairs into groups of four to further refine the explanation. Repeat with the second picture (you “think” that the large angle is 90°).

**Extensions**

1. The large angle measures 90°. The small angles are the same size. What are the unknown angles? Explain how you know.

   a) 45°; the two angles are equal and add to 90°, so each angle is half of 90°. Since \(90 ÷ 2 = 45\), each angle is 45°.

2. Semaphore flags are used to send signals at sea.

   a) Find the angle between flags in each letter. (Hint: Use your answer to Question 1 a) to help.)

   b) Using a protractor, construct the sequence of flags for the word “BAGS.”

3. a) Draw two intersecting lines. Measure all the four angles that meet at the intersection. Are there any equal angles?

   b) Draw more pairs of lines that intersect. Which angles are equal?

   c) In pairs, explain the result you found in part b). Do you agree with each other? Discuss why or why not.

   **Bonus:** Draw three intersecting lines and look for angles that are equal. Explain what you find.

   **Answers:** a) when you go around the point of intersection in order, every second angle is equal; b) when you go around in order, every second angle is equal, just like in part a); c) when you go around in order, the two angles next to each other add to 180°. For example, if one angle is 30°, then both the angles beside it are 150°; Bonus: the angles opposite each other are equal

   **NOTE:** In part c), encourage partners to ask questions to understand and challenge each other’s thinking (see p. A-49 for sample sentence and question stems).
Writing equations for angle problems. Present the two pictures below and have students find the measure of the unknown angle.

a) 

\[
\begin{align*}
60^\circ & \quad 15^\circ \\
\quad & \quad \\
? & 
\end{align*}
\]

b) 

\[
\begin{align*}
60^\circ & \quad 75^\circ \\
\quad & \\
? & 
\end{align*}
\]

Compare the two tasks to solving these two problems:

\[
60 + 15 = \underline{75} \quad \text{and} \quad 60 + \underline{____} = 75
\]

ASK: How are these two addition equations different? How are they the same as the problems with the angles? (If we write the measures in each angle problem as an addition equation, leaving a blank for the angle we do not know, we will get addition equations like these.) Remind students that mathematicians often use variables instead of the blanks, and have students rewrite both problems with variables. Encourage the use of different symbols and letters for variables. Finally, explain that it is convenient to use a variable instead of a question mark to label the unknown angle in the picture as well, and replace the question mark with the letter \(x\) to illustrate. Use \(60 + x = 75\) to review solving equations of this sort. Present the problems below and have students write and solve the corresponding equations.
Each student will need a pair of dice (preferably two different colors) and a protractor. Students will work individually to create a problem, then exchange notebooks with a partner and solve the problem their partner created. Then they will exchange notebooks again to check each other’s work.

Students roll the dice twice. They multiply the number on the red die by 25 and the number on the blue die by 2 and add the products. (Students who roll two equal numbers must roll again.) The numbers students obtain from the two rolls are the measures of two angles. Emphasize that since the numbers are angle measures, it is important to indicate the units, so a degree symbol should be added to the measures. Students need to draw two angles sharing an arm and a vertex such that the smaller angle is part of the larger angle. Example:

First roll: red 3, blue 5. Number: \((3 \times 25) + (5 \times 2) = 75 + 10 = 85\)

Second roll: red 5, blue 2. Number: \((5 \times 25) + (2 \times 2) = 125 + 4 = 129\)

The picture is shown in margin.

Model the complete activity (rolling the dice, finding the angles, and drawing them) at least twice before students work in pairs.

Before students complete the questions in the student book, remind them that square corners are used to mark right angles.

**Extension**

What is the size of the angles below? Explain how you know.

### Question 1, part c)

Sample answer: a) Draw a horizontal ray as shown. Now the whole angle measures 130° and the angle between the new ray and the other arm is 60°. So the given angle is \(130° - 60° = 70°\).
A full turn is a circle. Make a full turn while holding your arm out. ASK: What geometric shape did my arm draw? (a circle) Turn again to make sure all students see that your arm does make a circle. Draw the picture at left to illustrate.

A full turn is 360°. Give each student a paper circle (e.g., from BLM Circles) and ask them to fold the circle in half, so that the halves match exactly. Then ask them to fold the circle again, so that the halves of the folded circle match exactly. (The circle will be folded in quarters.) Have students unfold the circle. ASK: What fraction of a circle is each part? (a quarter) How many right angles do you see in your folded circle? Have students check that the angles they created by folding are indeed right angles by comparing them to a corner of a sheet of paper. Remind students that a right angle measures 90°. ASK: If a right angle is 90°, how many degrees are in the whole circle? (360°) How do you know? (90° × 4 = 360°)

ACTIVITY

Have students stand up and turn right or left a certain number of degrees. Examples: Turn left 90°. Turn right 180°. Turn right 360°. (Use only 90°, 180°, and 360° for this Activity.) To help students visualize the angles they are making, they can stretch their arms out ahead of them and lock their hands together (to act as a pointer).
Straight angles. Remind students that an obtuse angle can measure between 90° and 180°. ASK: What would an angle of 180° look like? To lead students to the answer, have them look at the circles they folded earlier. Ask them to draw a ray along the line of one of the folds, starting at the point where the folds meet (the center of the circle). Have students write 0° beside the ray. Then ask them to mark each right angle with a small square. Ask them to draw another ray so that the rays make an angle that is 90°. Ask students to draw an arc with an arrow to show the rotation from the first ray to the second ray. ASK: If you rotate the second ray another 90°, what angle will be between the first and second rays? How do you know? (90° + 90° = 180°) Ask students to draw the ray that shows the rotation. Where is that ray? (along the same fold as the 0° ray, in the opposite direction) Summarize: A 180° angle looks like two rays pointing in opposite directions or like a line with a vertex drawn on it.

Review fractions of a circle. Draw several circles divided into equal parts and shade part of each circle. Have students identify the fraction of the circle that is shaded. Start with unit fractions, and continue to other fractions. Remind students that any fraction can be obtained from unit fractions, e.g.,

\[
\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 3 \times \frac{1}{5}
\]

Review finding a fraction of a number. To find, say, one third of a number, you divide the number by 3, because you are interested in one of three equal parts. For example, if you have 300 jelly beans, and you divide them into three equal parts, each part is one third of the total. So one third of 300 is 300 ÷ 3 = 100. If you need to find, say, two thirds of 300, you need to recall that 2/3 = 2 × 1/3, and so two thirds of 300 is 2 × 100 = 200. Have students practice finding fractions of 360 by using this method. Exercises:

a) 1/2 of 360  b) 1/3 of 360  c) 2/3 of 360  d) 1/4 of 360  

f) 1/12 of 360  g) 7/12 of 360  h) 3/5 of 360

Answers: a) 180, b) 120, c) 240, d) 90, e) 270, f) 30, g) 210, h) 216

Finding angles as fractions of a circle. Remind students that a full turn is 360° and 1° is 1/360 of a full turn. If available, show students a compass and demonstrate how you draw a circle with a compass by rotating it around a fixed point. Explain that it is convenient to describe angles as parts of a circle. Display a circle divided into six equal parts with two parts shaded (you could photocopy BLM Large Fractions onto a transparency) and have students identify which fraction of the circle is shaded. (2/6) Extend the sides of the shaded part to create an angle (as in the box at the top of AP Book 4.2 p. 170). ASK: If the shaded part is 2/6 of a full circle, then what part of a full turn is this angle? (2/6) How could we find the measure of this angle without using a protractor? (it is 2/6 of 360°) Have students find 2/6 of 360° (120°) and then invite a volunteer to check the answer by measuring the angle. Repeat with a second circle divided into nine equal parts with 4 parts shaded. (4/9 of the full circle, 160°) Have students work through Questions 6 and 7 on AP Book 4.2 p. 170.
**Bonus**

**a)**

What fraction of the circle is shaded? ______

What is the angle?

\[ \text{of } 360^\circ = \underline{\hspace{2cm}} \]

**b)**

What fraction of the circle is shaded? ______

What is the angle?

\[ \text{of } 360^\circ = \underline{\hspace{2cm}} \]

**NOTE:** The following paragraph assumes that the class has a clock whose minute hand rotates continuously as opposed to jumping from position to position once a minute.

**Angles and clocks.** Ask students what familiar object in the classroom they can think of that rotates all the time. Point to an analog clock and ASK: How many minutes are in 1 hour? How many seconds are in 1 minute? How many seconds are in 60 minutes? (3,600) How do you know? (60 \times 60 = 3,600) A minute hand makes one full turn in 1 hour (or 60 minutes, or 3,600 seconds). So, in 3,600 seconds, it makes a 360° turn.

How many seconds does it take for a minute hand to make a turn of 1°? (10 seconds) Point out that if you draw a ray tracing the minute hand at one moment, then wait a little and look at it again, you will see that the hand has rotated. There will now be an angle between the tracing and the hand.

The more time that passes from the moment you traced the hand to the moment you compare the positions, the larger the angle. In 10 seconds, the minute hand makes a 1° angle. How long does it take for the minute hand to make a 2° angle? (20 seconds) A 3° angle? (30 seconds) A 10° angle? (100 seconds) How do you know? (multiply the number of degrees by 10, because each 10 seconds adds another 1°; PROMPT: How many degrees are added to the angle in each 10 seconds?) How long does it take for a minute hand to make a 12° angle? (120 seconds) How many minutes does it take for a minute hand to make an angle of 12°? (2 minutes)

**Word problems with sprinklers.** Remind students that some water sprinklers turn the same way as the hands on a clock. ASK: What is the difference between the rotation of a clock hand and that of a water sprinkler? (a clock hand always turns in the same direction and at the same speed, and it does not stop; a water sprinkler can turn in both directions and stops occasionally) Tell students that a sprinkler turned an angle of 60°, then stopped for 10 seconds, then continued for another 20° in the same direction before stopping again. ASK: How much did the sprinkler rotate in total? (80°) How do you know? (As the sprinkler rotates in the same direction, it makes a larger angle. So we add the angles: 60° + 20° = 80°.) Have students solve more such problems. Examples:
a) A water sprinkler turns 34°, stops, then turns another 28° in the same direction. What is the total angle that the sprinkler made?

b) A sprinkler turns 72°, stops, turns another 37° in the same direction, stops again, then rotates another 12° in the same direction. What is the total angle that the sprinkler covered?

SAY: A sprinkler turns 60°, stops, then starts turning in the opposite direction. It covers 45° in the opposite direction. What is the angle between the sprinkler’s final position and its initial position? How do you know? (60° − 45° = 15°; you subtract because the sprinkler turns in the opposite direction) Again, give students practice. Exercises:

a) A sprinkler turns 90°, stops, then starts turning in the opposite direction. It stops after turning 33°. How much more does it need to turn to get back to its initial position?

b) A sprinkler turns 120°, stops, then starts turning in the opposite direction. It stops 42° before reaching its initial position. What angle did it cover after the first stop?

Bonus: The time is 5:45. Carmelita thinks that the angle between the hands is 90°. Explain why she cannot be right. (the hour hand rotates together with the minute hand)

Extensions

1. Have students complete BLM Angles and Clocks (p. T-36).

2. What is the angle between the minute and the hour hands at 12:24? 13:36? 15:48? (Draw the hands first!)

To guide students to the method for finding the answer, draw an analog clock that shows 3:00 on the board. Ask students what type of angle the hands create. What is the measure of that angle?

Draw a line from the center of the clock to each of the hour marks to divide the clock into 12 equal parts. Have students identify the angle between a pair of adjacent lines. (PROMPT: What fraction of the circle is it?) If the time is 1:00, what is the measure of the angle between the hands? (1/12 of 360°, so 30°) Do you need a protractor to find the answer? (no) Ask volunteers to write the angle measures for each hour from 1:00 to 6:00. (PROMPT: What fraction of a whole circle is the part between the hands?) Which number do they skip count by? (30)

An hour is 60 minutes and a whole circle is 360°. What angle does the minute hand cover every minute? (6°) How long does it take the hour hand to cover that many degrees? How do you know? (The minute hand travels all the way around the circle in 1 hour, but it takes the hour hand 12 hours to do the same. The hour hand covers only 1/12 of the full circle in 1 hour. If it takes the minute hand 1 minute to cover 6°, it takes the hour hand 12 minutes to do the same.)

If the time is 12:12, where do the hour hand and the minute hand point?
Draw a clock face and a vertical ray from the center through 12. Mark the angle between the vertical ray and the position of the hour hand. What angle does the hour hand make with the vertical ray? (6°) Repeat with the minute hand using a separate clock face. Finally, combine the pictures as shown at left. Have students write and solve an equation to find the missing angle, x. **Solution:** The hour hand points at one minute and the angle that it makes with the vertical line is 6°. The minute hand points at 12 minutes and the angle that it makes with the vertical line is $12 \times 6 = 72°$. The equation is $6° + x = 72°$, so angle $x = 72° - 6° = 66°$.

3. The minute hand on a clock moves from the 4 to the 11. Measure the angle it turned in degrees. Explain your strategy.

**Answer:** 210°

**Sample strategies**
- When the minute hand moves from one number to the next, it moves 30° (because $360° \div 12 = 30°$), so when it moves from 4 to 11, it moves 7 times as much, so 210°.
- When the minute hand moves from the 4 to 10, it moves 180°. When it moves from 10 to 11, it moves another 30° (because $90° \div 3$), so it moves a total of 210° (because $180° + 30° = 210°$).
- I subtracted the amount it turns from 11 to 4, which is $5 \times 30$, or 150, degrees. So, I found $360° - 150° = 210°$, so it moved 210°.

Whole-class follow-up: Take up various strategies and have pairs explain the strategy in their own words and then discuss: Does the strategy work? Then compare the strategies in the whole-group setting. **ASK:** Which strategy is fastest? Which strategy is easiest to understand? Which strategy is easiest to come up with?

4. Kathy makes a shape, shown in the margin, from pattern block triangles. She says all the angles in the triangles are 60° because 6 of them in the middle make a full circle. With a partner, explain what she means by that. Is she right? Use the words “vertex,” “angle” and “full circle.”

**Answer:** The 6 inside angles around the middle vertex make a full circle, and they are all equal. A full circle is 360°, so each angle is 60°, since $6 \times 60° = 360°$. So, Kathy is right.

Encourage partners to ask questions to understand and challenge each other’s thinking (see p. A-49 for sample sentence and question stems).
Review acute and obtuse angles. Acute angles measure less than 90°, and obtuse angles measure between 90° and 180°. Have students identify a few angles drawn on the board as acute or obtuse (students can signal the answers) and sketch an example of an acute and an obtuse angle.

Estimating angles. Remind students how they used their fingers and arm spans as benchmarks to estimate length. For example, a finger is about 1 cm wide, a hand with fingers spread out slightly is about 10 cm wide, a giant step is about 1 m long.

Show students a piece of paper that does not have corners (it is in the shape of a circle or an oval), and ask them how they could make a right angle from this piece of paper. Have students cut a large circle out of a piece of paper (the circle doesn’t have to be perfect; students could use BLM Circles) and use it to test their ideas. To prompt students to see the solution, remind them about how they folded circles to figure out the number of degrees in a full turn. After students have figured out that they need to fold the shape, then fold it a second time so that the crease falls on itself, have them unfold the paper and ASK: How many degrees are in a full circle? How many degrees are in a right angle? How does your folding show that? (There are 4 right angles in a circle, and there are 90° in a right angle, so each full turn measures $90° \times 4 = 360°$.) Point out that students have made a right angle.

Give students a second piece of paper that has no corners and is neither circular nor oval (see example at left). Explain that this shape does not have...
a center, but the same folding will still produce a right angle. The first fold can be anywhere, but it is important to fold the shape a second time so that the crease folds exactly onto itself. Have students fold the shape twice to see that they can produce a right angle.

Have students refold their circles (which are more convenient to use because there is about the same distance from the center to the edge in any direction), then fold them again, creating wedges of 1/8 of a circle. Then have them unfold the paper completely. ASK: How many equal parts have you created? (8) How many degrees are in each angle you created? How do you know? (each angle is 1/8 of 360° = 360° ÷ 8 = 45°; each angle is half a right angle, so it is 90° ÷ 2 = 45°) Have students fold their circles in half along one of the existing folds and write 0° beside the fold. ASK: What should we mark each of the creases? Lead students to the idea of marking the crease closest to 0° as 45° and the next one (moving in the same direction) as 90° (because it makes a right angle with the fold, and because it makes two 45° angles). What should the third crease be marked? (90° + 45° = 135°, or 45° × 3 = 135°, because there are three 45° angles between the fold and that crease) Finish with the fourth fold. (45° × 4 = 180° or 360° ÷ 2 = 180°, because this is half the whole circle) Explain to students that they have created a tool to help them estimate angles. How is this tool similar to a protractor? How is it different? (This tool is similar to a protractor in shape. It allows you to measure and estimate some angles, but it is made of paper and does not allow you to take precise measurements. On the other hand, it is easy to make: you can use any piece of paper and you don’t need any other tools.)

Draw several angles on the board and show students how to estimate the angles using the folded paper. See example in margin (also used in the Extension).

In this example, because the angle is between 90° and 135°, the measure should be between these two numbers. Have students suggest estimates, then measure the angle to check the estimate. Review how to use a protractor to measure an angle.

**Angles in a shape.** Draw a triangle on the board. Explain to students that we can regard the sides of a shape as rays that meet at the vertex of the shape. The measure of an angle is the amount of rotation between one arm and the other, so we can think of an angle in a shape as one side of the shape being turned through the shape. Extend two sides of the triangle to create an angle and draw an arc to emphasize the angle between the sides. Point out that the triangle is inside the angle—it is part of the space between the arms. Have a volunteer measure the angle.

Display a pre-drawn grid on the board and draw a triangle as shown at left. Have students copy it into their notebooks.

Mark one of the angles in the triangle with an arc and say that you want to measure it. Have students estimate the angle using folded circles. Then demonstrate trying to measure the angle and encountering a problem (the arms are too short). Ask students to describe the problem and to offer a
solution. If necessary, remind them that they have already encountered angles with short arms. What did they do before measuring these angles? (extended the arms) Invite volunteers to extend the arms and measure the angle. Repeat with the other angles of the triangle.

Have students work through Questions 1–4 on AP Book 4.2 pp. 171–172. Students can estimate the angles in Questions 2, 3, and 4 before actually measuring them.

**Bonus:** If origami toys are available, have students estimate and measure angles between the creases on an origami toy.

**Right triangles.** Explain to students that a triangle that has a right angle is called a *right-angled triangle* or simply a *right triangle*. Draw a triangle on the board and point to each angle, one at a time. Have students decide whether the angle looks like a right angle. Invite volunteers to check the angles using a corner of a sheet of paper or the folded circles students used earlier. Is the triangle a right triangle? Repeat with other triangles in different positions. Then have students say whether a triangle could be a right triangle or not without checking the angles separately. If students think the triangle could be a right triangle, ask them which angle should be checked (students can answer by signaling).

Give students several paper triangles (or have them cut out the triangles on BLM Triangles for Sorting) and have them sort the triangles into right triangles and not right triangles. Have students check whether the sorting is correct by checking the angles that might be the right angles using a piece of paper or folded circles. Then have students complete the Student Book pages for this lesson.

**Extension**

(MP5, MP6) Draw an angle you think will be closer to 180° than to 90°. Check your prediction. Explain how you know whether the angle is closer to 90° or to 180°. Use any tool you think will help.

**Sample answer**

- I used a protractor and measured the angle to be 170° and that is only 10° away from 180°, but 80° away from 90°, so the angle I drew is closer to 180° than to 90°.
- I used grid paper and drew an angle that was halfway between 90° and 180° and that made it easy to tell that my angle was closer to 180° than to 90°.

**NOTE:** If using a protractor, students attend to precision (MP6) when they place the origin of the protractor at the vertex of their angle; if using grid paper, they attend to precision when they draw the angle halfway between 90° and 180°. Students who argue that an angle is closer to 180° than to 90° when the number is closer to 180 than it is to 90 are reasoning abstractly and quantitatively (MP2).
Review perpendicular and parallel lines. Remind students that lines that make a right angle are called perpendicular and lines that never meet, even if extended, are called parallel. Review with students how to mark parallel sides with arrows and right angles with small squares. Have students identify parallel sides and perpendicular adjacent sides in shapes. Draw the shape at left with one right angle (mark it) and ASK: How many right angles are in this shape? (1) How many pairs of perpendicular sides does this shape have? (1) Repeat with a shape that has two right angles (such as a right trapezoid). Then distribute BLM Sorting Polygons and have students sort the shapes into the tables. They should do shapes A through F first, and then the rest. **Answers:**

<table>
<thead>
<tr>
<th>Property</th>
<th>Shapes with Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>No parallel sides</td>
<td>B, C, E, I, K</td>
</tr>
<tr>
<td>One pair of parallel sides</td>
<td>H, J</td>
</tr>
<tr>
<td>Two pairs of parallel sides</td>
<td>A, G, L</td>
</tr>
<tr>
<td>More than two pairs of parallel sides</td>
<td>D, F</td>
</tr>
<tr>
<td>No perpendicular sides</td>
<td>C, D, E, G, H</td>
</tr>
<tr>
<td>One pair of perpendicular sides</td>
<td>B, I, K</td>
</tr>
<tr>
<td>Two pairs of perpendicular sides</td>
<td>F, J</td>
</tr>
<tr>
<td>More than two pairs of perpendicular sides</td>
<td>A, L</td>
</tr>
</tbody>
</table>
Review Venn diagrams. Draw an empty Venn diagram on the board and have students identify what it is. Add the property labels “Has two pairs of parallel sides” and “Has more than two pairs of perpendicular sides” and ask students to name all the shapes from the collection on BLM Sorting Polygons that go in each circle. Then ask students which shapes appear in both groups (A and L). Remind students that shapes that are in neither group go outside both circles, in the rectangle around them. Finally, have students help you fill the Venn diagram.

ASK: Which shapes are in the central area of the Venn diagram? (A, rectangle, and L, square) Ask students to draw another rectangle (M) and another square (N) on grid paper. Where will they go on this Venn diagram? (in the central area) Did everyone draw exactly the same square? (no) Exactly the same rectangle? (no) Did everyone get the same answer? (yes) The fact that everyone got the same answer means that any rectangle and any square belong in the central area of the Venn diagram. Why would that happen? (Any rectangle or square would have two pairs of parallel sides. Because these shapes have right angles, they also have pairs of perpendicular sides. These are properties of all squares and all rectangles.)

NOTE: Shapes with perpendicular sides that aren’t connected by right angles (e.g., regular octagons, which have 8 pairs of perpendicular sides but no right angles) have purposely been excluded from the collection on the BLM. Do not include such shapes in your discussion.

Have students fill in the table below (answers are in italics) and create a Venn diagram (see answer in margin) for another pair of properties using the shapes from the collection on the BLM. Again, students should sort shapes A through F first and then the rest of the shapes (including M and N).

| At least two pairs of parallel sides | A, D, F, G, L, M, N |
| No perpendicular sides              | C, D, E, G, H       |

ASK: In which area of the diagram are A and L now? (in one of the groups) What about M and N? (in the same group as A and L) Do all rectangles and squares fall into this area? (yes) Why? (all rectangles and squares have two pairs of parallel sides as well as some perpendicular sides, so they fit in one group but not in the other). Remind students that a right triangle is a triangle that has a right angle. Ask them to draw a right triangle on grid paper. Where would the triangle fit in the second Venn diagram? (outside both groups) Repeat with a different right triangle. Did everyone draw the same triangle? (no) Did all the right triangles fall into the same part of the diagram? (yes) Why did this happen? (PROMPT: Can a triangle have parallel sides? Have students draw two parallel lines and try to make a triangle using these two lines. Is it possible to make a triangle with these two sides?) Why do right triangles fall outside of the group “No perpendicular sides”? (any right triangle has a right angle, so it has 1 pair of perpendicular sides) Repeat this exercise with a pair of triangles that are not right triangles. This time the triangles will be placed inside the group “No perpendicular sides.”
Sorting shapes by the presence of an angle of a specific size. Review drawing angles using a protractor. Ask students to draw a horizontal ray and use it to draw an angle measuring 60°. Repeat with two more rays that are not horizontal and point in different directions. Give students pattern blocks, one of each kind, and have them sort the blocks into two groups: shapes that have a 60° angle and shapes that do not. Repeat the whole process with angles measuring 120°.

Extensions
1. **A perpendicular through a point.** Have students do Question 1 on BLM Perpendicular Line Through a Point (p. T-40). Then point out that there are many lines that go through a point and there can be many lines perpendicular to a given line (thick line in the pictures on the BLM), but there is only one line that is perpendicular to a given line and passes through a particular point.

   **Constructing a perpendicular through a point.** Demonstrate using a protractor to construct a line perpendicular to a given line through a point that is not on the given line. Use the steps given on the BLM. Emphasize the correct position of the protractor (the given line should pass through the origin and through the 90° mark). Have students practice the construction by completing BLM Perpendicular Line Through a Point.

2. Avril makes a shape, shown in the margin, from pattern block rhombuses. What are all the angles in the rhombus? How do you know?

   **Answer:** There are two large angles and two small angles in each rhombus. From the picture, six of the small angles make a full circle. A full circle is 360°, so each small angle is 60°, because \(360 \div 6 = 60\); three of the large angles make a full circle, so each large angle is 120°, because \(360 \div 3 = 120\).

3. Without measuring, determine all the angles in all the pattern block shapes. Explain how you know.

   **Answer:** A triangle has all angles 60° because 6 triangles fit together to make a full circle. A hexagon has all angles 120° because two triangles fit into each corner. A blue rhombus has two 60° angles and two 120° angles. In a tan rhombus, two of the small angles fit into a triangle angle, so the two small angles are 30° angles. Five of the small angles fit into a large angle, so the large angles are 150°. A trapezoid has two angles equal to a triangle angle, which is 60° and two angles equal to two triangle angles, so 120°.
Clocks
Angle Measures Are Additive

75°

60°

50°

40°

50°

40°
Large Fractions
Circles
Angles and Clocks

REMINDER ▶ There are 360° in a full turn.

1. In 5 minutes, a minute hand turns from 12 to 1.
   a) What fraction of 360° did the minute hand turn?  
   b) By what angle did the minute hand turn? ______
   c) By what angle does the minute hand turn in 10 minutes? ______
   d) By what angle does the minute hand turn in 1 minute? ______
   e) 1 min = 60 s. By what angle does the minute hand turn in 10 seconds? ______
   f) How many 10-second periods are in 5 minutes? ______
   g) How many degrees does the minute hand turn in 5 minutes? ______
   h) Are your answers in parts b) and g) the same? ______ If not, find your mistake.

2. a) Use a ruler to extend the clock hands.  
   Then measure the angle between the hands.
   b) Compare the angle you found to the angle in Question 1 b).
      How are the angles the same? _______________________
      How are the angles different? _______________________
   c) Find the angles made by the hands of the clock without measuring!
      i) ______
      ii) ______
      iii) ______

BONUS ▶

3. An hour hand moves from 12 to 1 in 60 minutes.
   a) By what angle does an hour hand turn in 60 minutes? ______°
   b) By what angle does an hour hand turn in 12 minutes? ______
   c) By what angle does a minute hand turn in 12 minutes?  
      Hint: Use your answer to Question 1 c). ______
   d) Find the angle between the hands at 12:12. ______
      Extend the hands in the picture and measure the angle to check your answer.
Reflexive Angles as Fractions of a Circle

1. a) What fraction of the circle is shaded? 
What is the angle? \( \frac{4}{5} \) of 360° = 

b) What fraction of the circle is shaded? 
What is the angle? 

What fraction of the circle is shaded? 
What is the angle? 

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Triangles for Sorting
Sorting Polygons

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<td></td>
</tr>
</tbody>
</table>
Perpendicular Line Through a Point

1. a) Trace the lines that are perpendicular to the thick line in red.
   b) Trace the lines that pass through point $P$ in blue.
   c) Mark the lines that are perpendicular to the thick line and pass through the point $P$ with an “$\times$”.

   ![Diagram of perpendicular lines through point P](image)

To draw a line perpendicular to the thick line through point $P$:

**Step 1** Align the line from the origin to the $90^\circ$ mark with the thick line.

**Step 2** Without turning the protractor, move it so that the straight side of the protractor touches the point $P$.

**Step 3** Draw a straight line through $P$ using the straight side of protractor as a ruler.

2. Draw a line perpendicular to the given line that passes through $P$.

   a) ![Diagram of line perpendicular to given line through P](image)
   b) ![Diagram of line perpendicular to given line through P](image)
   c) ![Diagram of line perpendicular to given line through P](image)
   d) ![Diagram of line perpendicular to given line through P](image)
PS4-10 Choosing Strategies

Teach this lesson after: 4.2 Unit 9


Goals:
Students will solve problems and puzzles using any of the problem-solving strategies studied so far in the Grade 4 problem-solving lessons.

Prior Knowledge Required:
Can round whole numbers to any place
Can add and subtract within 1,000
Can multiply and divide within 100
Can find the area and perimeter of a rectangle with given side lengths
Can solve multi-step problems
Can multiply simple multiples of 10 by one-digit numbers
Can find all factor pairs of a number from 1 to 100

Vocabulary: decimal, decimal point, prime number

Materials:
grid paper or BLM 1 cm Grid Paper (p. T-48, see Problem Bank 22)
scissors
BLM Electric! (pp. T-51–53, see Performance Task)

NOTE: The following Problem Bank questions reflect all the problem-solving strategies used in the problem-solving lessons for Grade 4. Choose among the following questions based on which problem-solving lessons you have taught.

Problem Bank
(MP.1, MP.7) 1. I am a decimal with two digits after the decimal point. What number am I?
   a) I am less than 0.1. My hundredths digit is 7.
   b) I am between 0.4 and 0.5. My digits add to 8.
   c) I am less than 1. My tenths digits and hundredths digits are equal. My digits add to 12.
   d) I am between 1 and 10. All my digits are equal. My digits add to 9.
   Answers: a) 0.07, b) 0.44, c) 0.66, d) 3.33

(MP.7, MP.8) 2. Add mentally: 1 + 11 + 111 + 1,111 + 11,111 + 111,111 + 1,111,111
Solution: In the ones place, there are seven 1s, in the tens place, there are six 1s, in the hundreds, there are five 1s, and so on; in the millions place, there is one 1, so the sum is 1,234,567.
(MP.7) 3. A pile of coins has exactly 7 pennies and an unknown number of quarters, nickels, and dimes. Which of these can be the total value of the coins: $7.30, $7.31, $7.32, $7.33, or $7.34?

Answer: $7.32

(MP.7, MP.8) 4. The key with the digit 4 on your calculator isn’t working. What could you press to find …

a) 214 + 63  b) 241 + 63  c) 841 + 34  d) 34 × 15  e) 42 × 8

Sample answers: a) 210 + 67, b) 200 + 100 + 3 + 1, c) 800 + 70 + 5, d) 33 × 15 + 15, e) 32 × 8 + 10 × 8

(MP.7) 5. Fill in the blank.

a) \((83 \times 2) + (83 \times 4) = 83 \times ____\)

b) \((83 \times 41) + (2 \times 41) = ____ \times 41\)

c) \((72 \times 41) + (72 \times 3) + (3 \times 39) + (3 \times 5) = 75 \times ____\)

Answers: a) 6, b) 85, c) 44

(MP.8) 6. Add: 98 + 98 + 98 + 98 + 98.

Answer: 490

(MP.8) 7. Add: \((35 + 35 + 35 + 35 + 35 + 35) + (65 + 65 + 65 + 65 + 65 + 65)\).

Answer: 600

(MP.3) 8. Remember: A prime number is a number greater than 1 that has exactly two factors, 1 and itself.

a) How many prime numbers have a ones digit of 2?

b) Can a prime number have the ones digit 6? Explain.

Answers: a) one number: 2; b) no, because any number with the ones digit 6 is even and the only prime number that is even is 2

(MP.1, MP.7) 9. A number rounds to 500 when rounded to the nearest hundred and 450 when rounded to the nearest ten. The digits add to 12. What number is it?

Answer: 453

(MP.1, MP.7) 10. \(3131 = 31 \times ____\)

Answer: 101

(MP.7) 11. \(2 \times 3 \times 4 \times 5 \times 6 = 3 \times 4 \times 5 \times 6 \times ____\)

Answer: 2

(MP.1, MP.7) 12. John and Mary’s ages add to 32. What will they add to 3 years from now?

Answer: 38

(MP.1, MP.5, MP.7) 13. What number between 421 and 469 is divisible by 3 and 10?

Solution: Start with the multiples of 10 between 421 and 469: 430, 440, 450, 460. Divide each by 3, using long division: \(430 \div 3 = 143 \text{ R } 1, 440 \div 3 = 146 \text{ R } 2, 450 \div 3 = 150 \text{ R } 0, 460 \div 3 = 153 \text{ R } 1\). The number is 450.
14. I have 4 digits. The product of my digits is 60. What is the largest possible number I can be? Hint: Remember that 1 can be a digit too.
Answer: 6,521

15. Rick has 85 marbles and Glen has 92 marbles. Can Glen give some marbles to Rick so that they have the same number of marbles? Explain.
Answer: No. Glen has 7 more marbles than Rick, and since 7 is not a multiple of 2, Glen can’t give Rick half of the extra marbles.

16. Sam has 5 apples. Hanna has 8 apples. Ava has 11 apples.
a) How many apples do they have altogether?
b) They decide to share the apples equally. How many apples should each person get?
c) Who doesn’t need to give or receive any apples?
d) How many apples do the other two people need to give or receive?
Answers: a) 24, b) 8, c) Hanna, d) Ava needs to give away 3 apples and Sam needs to receive 3 apples

17. There are three sets of numbers.
Set A: 2, 3, 5
Set B: 4, 5, 6
Set C: 1, 8, 11
a) What is the sum of the numbers in each set?
b) Jane traded exactly two numbers between groups. When she was done, all the groups had the same sum.
i) Which set did she leave alone? How do you know?
ii) What two numbers did she trade?
Answers: a) A: 10, B: 15, C: 20; b) i) Jane must have left Group B alone because it has the number in the middle. If one sum increased and another decreased, it must be the one in the middle that stayed the same. ii) 3 and 8

18. Clara and John play a game. The rules are that Player 1 rolls three dice and then Player 2 rolls three dice. They both win when they get the same total. To help them get the same total, players are allowed to trade exactly one die for one die. For example: Player 1 rolls 2, 3, 5 and Player 2 rolls 1, 5, 6. Player 1’s total is 10 and Player 2’s total is 12. Player 1 can trade the 5 for Player 2’s 6 so that they each get the same total of 11.
a) Clara rolls 2, 3, 6. Glen rolls 4, 5, 6.
i) What do Clara’s dice add to?
ii) What do Glen’s dice add to?
iii) How far apart are their totals?
iv) When they trade, who should give away a bigger number? Why?
v) If Glen and Clara trade Clara’s 3 for Glen’s 4, what are their new totals?
vi) If Glen and Clara trade Clara’s 3 for Glen’s 5, what are their new totals?
vii) If Glen and Clara trade Clara’s 2 for Glen’s 4, what are their new totals?
viii) If Glen and Clara trade Clara’s 2 for Glen’s 6, what are their new totals?
b) When Clara and Glen won, how far apart were the numbers they traded?
c) Clara and Glen roll the given numbers. Help them win.
i) Clara: 4, 6, 6 Glen: 1, 2, 5
ii) Clara: 3, 3, 5 Glen: 1, 2, 4
iii) Clara: 2, 3, 6 Glen: 4, 4, 5
d) Clara rolls 1, 4, 5. Glen rolls 2, 3, 6.
   i) What do Clara’s dice add to?
   ii) What do Glen’s dice add to?
   iii) How far apart are their totals?
   iv) When they trade, who should give away a bigger number? Why?
   v) If Glen and Clara trade Clara’s 1 for Glen’s 2, what are their new totals?
   vi) If Glen and Clara trade Clara’s 5 for Glen’s 6, what are their new totals?
   vii) If Glen and Clara trade Clara’s 1 for Glen’s 3, what are their new totals?

e) Clara rolls 1, 2, 6. Glen rolls 1, 6, 6. Can Clara and Glen win? Explain.

f) Clara rolls 1, 1, 3. Glen rolls 1, 6, 6. Can Clara and Glen win? Explain.

**Answers:**

a) i) 11; ii) 15; iii) 4; iv) Glen, because his sum is greater; v) Clara has 12 and Glen has 14;
   vi) Clara has 13 and Glen has 13; vii) Clara has 13 and Glen has 13; viii) Clara has 15 and
   Glen has 11

b) 2

c) i) trade Clara’s 6 for Glen’s 2, ii) trade Clara’s 3 for Glen’s 1, iii) trade Clara’s 3 for Glen’s 4
   d) i) 10; ii) 11; iii) 1; iv) Glen, because his dice add to more; v) Clara has 11 and Glen has 10;
   vi) Clara has 11 and Glen has 10; vii) Clara has 12 and Glen has 9; viii) no, because no matter
   what they trade, if Glen gives away a bigger number than Clara, Clara’s total will be bigger than
   Glen’s because they were only 1 apart to begin with

e) no, because their totals are 4 apart, so they would have to trade numbers that are 2 apart to
   win, but there are no such numbers

   f) no, because their totals are 8 apart, so they would have to trade numbers that are 4 apart to
   win, but there are no such numbers

(MP.1, MP.7) 19. You can find the mirror image of a number by placing a mirror to the right of
the number.

a) The dashed line is a mirror. Draw the mirror image of each digital clock digit:

```
  0  1  2  3  4
```

```
  5  6  7  8  9
```

b) Which digits, when written like on a digital clock, have a mirror image that is also a digit?

c) What is the mirror image of each number?
   i) 202    ii) 11    iii) 55    iv) 218

   d) What can the two numbers be if a number and its mirror image ...  
      i) add to 7    ii) add to 2    iii) multiply to 10    iv) add to 99
      v) add to 909  vi) add to 9,009  vii) add to 9,999  viii) add to 50

   **Bonus:**
      ix) add to 5,000  x) are the same
Answers: a) \(0, 5, 2, 7, 8, 2\)
b) 0, 1, 2, 5, and 8
c) i) 505, ii) 11, iii) 22, iv) 815
d) i) 2 and 5; ii) 1 and 1; iii) 2 and 5; iv) 18 and 81; v) 108 and 801; vi) 1,008 and 8,001; vii) 1,818 and 8,181 or 1,188 and 8,811; viii) 25 and 25; Bonus: ix) 2,185 and 2,815; x) 0, 1, 8, 25, 52, 205, 502, 215, 512, 285, 582, 2255, 5522, and many more

20. Pens and pencils both cost a whole number of dollars. Three pens and two pencils cost $34. Two pens and three pencils cost $31.

(MP.3) a) If three pens and two pencils cost more than two pens and three pencils, what costs more, a pen or a pencil? Explain.

(MP.1, MP.2, MP.4) b) How much does each pen and each pencil cost? Use the information from part a) to make sure your answer makes sense.

**Answer:** a) a pen costs more because, compared to buying two pens and two pencils, an extra pen adds a greater cost than an extra pencil

**Sample solution:** b) Start the cost of pencils at $1 each in the equation 3 pens and 2 pencils cost $34, and continue raising the cost of each pencil until the cost of a pen becomes less than the cost of a pencil.

<table>
<thead>
<tr>
<th>Cost of 1 pencil</th>
<th>Cost of 2 pencils</th>
<th>Cost of 3 pens</th>
<th>Cost of 1 pen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$2</td>
<td>$34 - $2 = $32</td>
<td>X</td>
</tr>
<tr>
<td>$2</td>
<td>$4</td>
<td>$34 - $4 = $30</td>
<td>$10</td>
</tr>
<tr>
<td>$3</td>
<td>$6</td>
<td>$34 - $6 = $28</td>
<td>X</td>
</tr>
<tr>
<td>$4</td>
<td>$8</td>
<td>$34 - $8 = $26</td>
<td>X</td>
</tr>
<tr>
<td>$5</td>
<td>$10</td>
<td>$34 - $10 = $24</td>
<td>$8</td>
</tr>
<tr>
<td>$6</td>
<td>$12</td>
<td>$34 - $12 = $22</td>
<td>X</td>
</tr>
<tr>
<td>$7</td>
<td>$14</td>
<td>$34 - $14 = $20</td>
<td>X</td>
</tr>
<tr>
<td>$8</td>
<td>$16</td>
<td>$34 - $16 = $18</td>
<td>$6</td>
</tr>
</tbody>
</table>

We could keep going, but we don’t need to because we have got to the point where pencils cost $8 each and pens cost $6 each, but we need a pencil to cost less than a pen. So, there are only two possibilities to check with the second equation: 2 pens and 3 pencils cost $31. Looking at the first possibility, if 1 pencil costs $2 and 1 pen costs $10, then 2 pens and 3 pencils cost $26; in the second possibility, if 1 pencil costs $5 and 1 pen costs $8, then 2 pens and 3 pencils cost $31. So 1 pencil costs $5 and 1 pen costs $8.

(MP.1, MP.2, MP.4) 21. Shirts and crayons each cost a whole number of dollars.
Anwar paid $30 for 3 shirts and 2 crayons.
Liz paid $23 for 1 shirt and 5 crayons.
How much does each shirt and each crayon cost?

**Answer:** each crayon costs $3 and each shirt costs $8
(MP.1, MP.7) 22. A pentomino is made of 5 squares in the same way a domino is made of 2 squares. The picture below shows all 12 pentominoes. Give students grid paper or BLM 1 cm Grid Paper and have them create and cut out the pentominoes.

a) What is the total area of all 12 pentominoes?
b) For your answer to part a), what pairs of numbers multiply to the number?
c) For which pairs of numbers from part b) can you make the 12 pentominoes into a rectangle with those dimensions? You will need to use the pentominoes you created and cut out.

Answers: a) 60 units$^2$; b) $1 \times 60, 2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$; c) $3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$

(MP.7) 23. a) When Yu multiplies 2 one-digit numbers, the answer has ones digit 7. What might the 2 one-digit numbers be? List all possible answers.
b) In part a), how could you have predicted that all the answers would be odd?
c) AB is a two-digit number so that AB $\times$ BA has the ones digit 7 and is less than 2,000. What are the numbers AB and BA?
Selected solution: a) Solution 1: Try all the possible first numbers in order, and then decide what the second number has to be: 1 and 7, 3 and 9.
Solution 2: Try all the possible answers in order: 7, 17, 27, 37, 47, 57, 67, 77 (87 is bigger than 9 × 9, so we can stop). Only 7 = 1 × 7 and 27 = 3 × 9 are in the times tables
Answers: b) We know that an even number multiplied by any number equals an even number; because the answer has the ones digit 7, we can rule out even numbers and predict that the numbers multiplied must be odd; c) 17 × 71

(MP.1) 24. Solve this problem by working backward. \( \frac{1}{2} \) of \( \frac{2}{3} \) of \( \frac{3}{4} \) of \( \frac{4}{5} \) of 30 is ______.
Solution: 6, because 4/5 of 30 is 24, 3/4 of 24 is 18, 2/3 of 18 is 12, and 1/2 of 12 is 6
1 cm Grid Paper
Performance Task: Electric!

Materials:
BLM Electric! (pp. T-51–53)

Preparation for the performance task. Write on the board:

\[ 3 \times 4 = \underline{\hspace{2cm}} \quad 30 \times 4 = \underline{\hspace{2cm}} \]

Have volunteers fill in the blanks. (12, 120) ASK: How can you use \(3 \times 4\) to get \(30 \times 4\)? (multiply the answer by 10)

SAY: You can do the same thing with two-digit numbers. Write on the board:

\[
\begin{array}{c}
31 \\
\times 14 \\
\end{array}
\quad \text{so } 31 \times 140 = \underline{\hspace{2cm}}
\]

Have a volunteer do the first multiplication and then another volunteer do the second. (434; 4,340) ASK: How can you use \(31 \times 14\) to get \(31 \times 140\)? (multiply the answer by 10)

Tell students that they are going to do a performance task about electricity. SAY: Different appliances use different amounts of power from electricity. ASK: What do you think uses more electricity, a ceiling fan or an air conditioner? (an air conditioner)

SAY: Just like length can be measured using feet and inches, electricity can be measured using watts and amps. Write on the board:

\[
1 \text{ foot} = 12 \text{ inches} \\
A 1 \text{ amp appliance uses 120 watts}
\]

ASK: How can you convert five feet to inches? (multiply 5 by 12) Write on the board:

\[ 5 \times 12 = 60, \text{ so } 5 \text{ feet} = 60 \text{ inches} \]

ASK: How can you convert five amps to watts? (multiply 5 by 120) Write on the board:

\[ 5 \times 120 = 600, \text{ so a 5 amp appliance uses 600 watts} \]
Draw on the board:

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blender</td>
<td>360</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1,200</td>
</tr>
<tr>
<td>Electric can opener</td>
<td>150</td>
</tr>
<tr>
<td>Kettle</td>
<td>1,800</td>
</tr>
<tr>
<td>Microwave</td>
<td>1,080</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>720</td>
</tr>
<tr>
<td>Coffee maker</td>
<td>960</td>
</tr>
</tbody>
</table>

SAY: These are some typical appliances in a home and how much power they might use.
ASK: Which appliance uses the most power? (the kettle) SAY: You don’t use a kettle for a long time, but when you do use it, it uses a lot of electricity. A kettle will use less electricity in a year than a microwave, because you use it for less time. ASK: Which appliance uses the least amount of electricity? (the can opener) Which appliance do you think will use the most electricity in a year? (the refrigerator) PROMPTS: Which one gets used the most? Which one is almost always running?

Tell students that the way electricity works in a house is that each outlet has a maximum number of watts that you can use at one time. For example, in some houses, if you are using a kettle, you can’t use a microwave plugged into the same outlet at the same time—you would have to plug it in somewhere else. The performance task that follows investigates that kind of situation.

NOTE: The word “outlet” is being used imprecisely here; the precise word that should be used is “circuit,” but we use outlet here since students will be more familiar with the word and also because the concept of a circuit is not necessary for this task.

Performance Task: Electric! Provide students with BLM Electric! On the BLM, Question 4 and the Bonus question are good opportunities for students to apply the guess-check-revise and using structure problem-solving strategies learned this year. If students have not done these problem-solving lessons, they will likely find these questions difficult.

Answers:
1. a) kettle; b) i) yes, ii) no, iii) yes
2. a) 1,560 watts; b) 600 watts; c) any one of the lamp, TV, or stereo with the heater, or the television and the lamp with the heater
3. 120 more watts because 1 amp is the same as 120 watts
4. 9 amps
Bonus: 1,001 amps
Electric! (1)

Electricity is needed to power an appliance. The amount of power an appliance uses is given in watts.

These are some typical appliances in a kitchen and how much power they might use.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric can opener</td>
<td>150</td>
</tr>
<tr>
<td>Blender</td>
<td>360</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>720</td>
</tr>
<tr>
<td>Coffee maker</td>
<td>960</td>
</tr>
<tr>
<td>Microwave</td>
<td>1,080</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>1,200</td>
</tr>
<tr>
<td>Kettle</td>
<td>1,800</td>
</tr>
</tbody>
</table>

1. The number of watts used by an outlet should not pass the outlet’s capacity. One outlet in the kitchen has a capacity of 1,620 watts.

   a) Which appliance can never be used in that outlet?

   b) Can the given appliances be used in the outlet at the same time?
      i) the blender and dishwasher
      ii) the microwave and refrigerator
      iii) can opener, blender, and coffee maker
Electric! (2)

2. Sometimes the capacity of an outlet is given in amps. In a typical home, to get the number of watts from the number of amps, multiply by 120.

These are some typical appliances in a living room and how much power they might use.

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Watts Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamp</td>
<td>72</td>
</tr>
<tr>
<td>Television</td>
<td>240</td>
</tr>
<tr>
<td>Stereo</td>
<td>360</td>
</tr>
<tr>
<td>Window air conditioner</td>
<td>960</td>
</tr>
<tr>
<td>Heater</td>
<td>1,200</td>
</tr>
</tbody>
</table>

a) An outlet in the living room has a capacity of 13 amps. How many watts can be used at the outlet?

b) The outlet from part a) is already being used by a window air conditioner. How many additional watts can be used at the outlet?

c) In the winter, a heater is plugged into the living room outlet instead of the air conditioner. Which of these items can be used in that outlet at the same time as the heater? List all possible combinations.

   A. Television   B. Lamp   C. Stereo
Electric! (3)

3. How many more watts could an outlet with a capacity of 10 amps use than an outlet with a capacity of 9 amps? Explain how you know.

4. An outlet has a capacity that is a whole number of amps. The number of watts it can use, to the nearest 100, is 1,100. What is the outlet’s capacity in amps?

BONUS ► In a building, 120,120 watts are being used at the same time. How many amps is that?
1. a) 0.29
   
<table>
<thead>
<tr>
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<th>P</th>
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<tbody>
<tr>
<td>0</td>
<td>29</td>
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<tr>
<td>1</td>
<td>19</td>
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<tr>
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   b) 0.45
   
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<th>N</th>
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<tr>
<td>0</td>
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c) 0.24
   
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d) 0.35
   
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2. 0.55

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<td>6</td>
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<td>2</td>
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   He stopped at 2 quarters because that equals 0.50 and an additional one would make more than 0.55.

3. a) 0.13
   
<table>
<thead>
<tr>
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<th>P</th>
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4. a) 0.32
   
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   b) 0.30
   
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5. a) 0.90
   
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   b) 0.35
   
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<tr>
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<td>1</td>
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   c) 0.45
   
<table>
<thead>
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<th>D</th>
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   d) 0.30
   
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   1 dog and 1 bird.

2. 0.55

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   He stopped at 2 quarters because that equals 0.50 and an additional one would make more than 0.55.

3. a) 0.13
   
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   He stopped at 2 quarters because that equals 0.50 and an additional one would make more than 0.55.
4. a) \(10 \times 1 = 10\)
   \(5 \times 2 = 10\)
   \(X \times 3 = 10\)
   \(X \times 4 = 10\)
   \(2 \times 5 = 10\)
   \(X \times 6 = 10\)
   \(X \times 7 = 10\)
   \(X \times 8 = 10\)
   \(X \times 9 = 10\)
   \(1 \times 10 = 10\)

b) Factors of 10 are 1, 2, 5, and 10.

5. a) \(15 \div 3 = 5\text{ R }0\) or
   \(15 \div 5 = 3\text{ R }0\)
   
   b) \(24 \div 4 = 6\text{ R }0\) or
   \(24 \div 6 = 4\text{ R }0\)
   
   c) \(16 \div 2 = 8\text{ R }0\) or
   \(16 \div 8 = 2\text{ R }0\)
   
   d) \(30 \div 6 = 5\text{ R }0\)

6. a) \(11 \div 3 = 3\text{ R }2\); No
   
   b) \(18 \div 3 = 6\text{ R }0\); Yes
   
   c) \(21 \div 3 = 7\text{ R }0\); Yes
   
   d) \(16 \div 3 = 5\text{ R }1\); No
   
   e) \(14 \div 3 = 4\text{ R }2\); No
   
   f) \(27 \div 3 = 9\text{ R }0\); Yes

7. a) \(11 \div 6 = 1\text{ R }5\)

b) \(1 \div 4 = 0\text{ R }1\)

8. a) \(1 \div 2 = 0\text{ R }1\)

b) \(2 \div 3 = 0\text{ R }2\)

f) \(1 \div 3 = 0\text{ R }2\)

9. Teacher to check explanations.
   a) Yes.
   b) No.

AP Book OA4-42

1. a) Alana does not try 11 as a first number because it is larger than 10 and so cannot be a factor of 10.
   
   b) 1, 2, 5, and 10
   
   c) 1, 10, 2, 5, 10, 2

2. a) 12

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BONUS

72

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4. d) 42

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4. e) 99

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11 stop

4. f) 100

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stop

6. a) No: 8, 16, 24, 32, 40

b) No: 9, 18, 27, 36, 45

c) Yes: 7, 14, 21, 28, 35

d) Yes: 6, 12, 18, 24

7. a) 1 4

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Yes.

b) 1 9

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No.

c) 2 3

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Yes.

8. a) i) 2 and 3

ii) 2 and 10

iii) 2 and 32

BONUS

2 and 24, 213

b) All even numbers except 2 are composite because 2 is always a factor.

BONUS

72

2 and 24, 213

b) All even numbers except 2 are composite because 2 is always a factor.

AP Book OA4-43

page 102

1. 5: 1, 5

6: 1, 2, 3, 6

7: 1, 7

8: 1, 2, 4, 8

9: 1, 3, 9

10: 1, 2, 5, 10

11: 1, 11

12: 1, 2, 3, 4, 6, 12

13: 1, 13

14: 1, 2, 7, 14

15: 1, 3, 5, 15

16: 1, 2, 4, 8, 16

17: 1, 17

18: 1, 2, 3, 6, 9, 18

19: 1, 19

2. a) Teacher to check circles.

b) 2

3. 1 has 1 factor.

Neither.

4. a) Check circles: 13, 17, 19

b) Answers may vary. Sample answer:

It is faster to show that a composite number is composite because finding a third factor is enough. Showing that a prime number is prime requires checking every number up to that number.

5. a) 10 is a multiple of two and 2 is a factor of 10.
Numbers and Operations—Fractions:
Decimals – AP Book 4.2: Unit 7

1. b) 5¢ = $0.05
c) 43¢ = $0.43
d) 87¢ = $0.87
e) 54¢ = $0.54
f) 9¢ = $0.09
g) 2¢ = $0.02
h) 75¢ = $0.75
i) 1¢ = $0.01
2. b) 0 4 7 $0.47
c) 3 2 5 $3.25
d) 0 0 3 $0.03
e) 28 1 6 $28.16
3. a) 300¢
b) 60¢
c) 9¢
d) 100¢
e) 700¢
f) 1,200¢
g) 1,500¢
h) 199¢
i) 151¢
j) 98¢
k) 3¢
l) 8¢
4. b) $1.03
c) $2.16
d) $3.75
e) $3.00
f) $0.04
g) $0.07
h) $0.90
i) $6.00
j) $0.99
k) $12.00
l) $16.04
5. b) $20 55¢ $20.55
c) $30 7¢ $30.07
d) $26 11¢ $26.11
6. Quarters
7. Teacher to check drawings. Three $1 bills + two 25¢ coins.
8. Five $1 bills and one quarter. One $5 bill and five nickels.

1. a) 30
   b) 30
   c) 60
   d) 34

2. b) 7 dimes = 70 pennies
   c) 8 dimes = 80 pennies
   d) 5 dimes = 50 pennies

3. b) $0.20
   c) $0.60
   d) $0.04
   e) $0.13
   f) $0.25
   g) $0.25
   h) $0.75
   i) $0.80
   j) $1.00
   k) $5.00
   l) $7.00

4. The greater amount is 168¢ because in dollar notation, it is $1.68 and that is greater than $1.65.

1. a) $4
   b) $3 or $0.3
   c) $8
   d) $5

2. b) 70¢ = $0.70
   c) 80¢ = $0.80
   d) 50¢ = $0.50

3. b) 6 6 60 100
   c) 9 9 90 100
   d) 3 3 30 100

4. Answers may vary. Sample explanation: No, Sarah is not correct because while 37 coins is a larger number of coins, the value of 37 pennies is less than 5 dimes. In cent notation, it is clear that 37¢ (37 pennies) is less than 50¢ (5 dimes).

5. Teacher to check shading.

6. Teacher to check shading and circling.

7. Explanations may vary. Sample answer: Rocco is incorrect because while 17 is greater than 8, 17/100 is less than 8/10 because when they are compared in hundredths, 17/100 is less than 80/100.

1. b) 30
   c) 60
   d) 2

2. b) 0.7
   c) 0.6
   d) 0.9

3. Teacher to check shading of fractions.

4. Teacher to check number lines.

5. b) 46
   c) 18
   d) 50
   e) 47

6. b) 8 80 100

7. a) $36
   b) $0.09
7. Teacher to check number line.
   b) 0.06 < 0.24 < 0.45 < 0.70

**AP Book NF4-23**

**page 113**

1. Teacher to check shading.
   b) \( \frac{90}{100} \) 0.90
   c) \( \frac{60}{100} \) 0.60

2. a) ii) \( \frac{7}{10} \) 0.70
   iii) \( \frac{4}{10} \) 0.40

b) 0.20 < 0.40 < 0.70

3. A 2 tenths
   20 hundredths
   0.2 = 0.20

   B 6 tenths
   60 hundredths
   0.6 = 0.60

   C 7 tenths
   70 hundredths
   0.7 = 0.70

4. a) Teacher to check number line.
   0.05 < 0.27 < 0.40

   b) Teacher to check number line.

5. a) 0.7 = \( \frac{7}{10} \) 0.70

b) 0.48 = \( \frac{48}{100} \)

   c) 0.09 = \( \frac{9}{100} \)

   d) 0.3 = \( \frac{30}{100} \)

6. a) 0.6 = 0.60

   b) 0.77

   c) 0.3 = 0.30

   d) 0.09

**AP Book NF4-24**

**page 115**

1. b) 47 hundredths = 4 tenths 7 hundredths
   \( \frac{47}{100} \) = 0.47

   c) 76 hundredths = 7 tenths 6 hundredths
   \( \frac{76}{100} \) = 0.76

   d) 87 hundredths = 8 tenths 7 hundredths
   \( \frac{87}{100} \) = 0.87

2. b) 2 tenths, 8 hundredths
   \( \frac{28}{100} \) = 0.28

   c) 4 tenths, 1 hundredth
   \( \frac{41}{100} \) = 0.41

   d) 6 tenths, 0 hundredths
   \( \frac{60}{100} \) = 0.60

   f) 0 tenths, 2 hundredths
   \( \frac{2}{100} \) = 0.02

3. b) 8 tenths, 3 hundredths
   = 83 hundredths

   c) 7 tenths, 0 hundredths
   = 70 hundredths

   d) 0 tenths, 2 hundredths
   = 2 hundredths

4. A 3 tenths, 7 hundredths
   = 37 hundredths

   B 8 tenths, 4 hundredths
   = 84 hundredths

5. Teacher to check number lines.

6. b) 58 cm = 0.58 m
   \( \frac{58}{100} \) m

**AP Book NF4-25**

**page 117**

1. b) 0.62
   6 dimes 2 pennies
   62 pennies
   62 hundredths

   c) 0.48
   4 dimes 8 pennies
   48 pennies
   48 hundredths

   d) 0.03
   0 dimes 3 pennies
   3 pennies
   3 hundredths

   e) 0.09
   0 dimes 9 pennies
   0 tenths, 9 hundredths

   f) 0.19
   1 dime 9 pennies
   1 tenth, 9 hundredths

2. b) 0.6
   6 dimes 0 pennies
   6 tenths 0 hundredths
   60 pennies
   60 hundredths

   c) 0.8
   8 dimes 0 pennies
   8 tenths 0 hundredths
   80 pennies
   80 hundredths

3. 0.3
   3 dimes 0 pennies
   3 tenths 0 hundredths

   0.18
   1 dime 8 pennies
   1 tenth 8 hundredths

   0.18
   18 pennies
   18 hundredths

   Circle 0.3

4. 0.32 = \( \frac{32}{100} \)

   0.5 = \( \frac{50}{100} \)

   Will is incorrect because 50 is greater than 32.

**AP Book NF4-26**

**page 118**

1. b) \( \frac{3}{10} + \frac{4}{100} = \frac{34}{100} \)

   c) \( \frac{2}{10} + \frac{5}{100} = \frac{25}{100} \)

2. Teacher to check shading.
   b) \( \frac{8}{10} + \frac{7}{100} = \frac{87}{100} \)

   c) \( \frac{6}{10} + \frac{2}{100} = \frac{62}{100} \)

   d) \( \frac{7}{10} + \frac{6}{100} = \frac{76}{100} \)

3. Teacher to check shading.
   b) 0.4 + 0.3 = 0.7

   c) 0.40 + 0.09 = 0.49

4. a) \( \frac{80}{100} \)

   b) \( \frac{70}{100} \)

   c) \( \frac{40}{100} \)

   d) \( \frac{30}{100} \)

5. a) 0.23

   \( \frac{23}{100} \)
Numbers and Operations—Fractions: Decimals – AP Book 4.2: Unit 7

(continued)

2. a) $\frac{10}{100}$
   b) $\frac{100}{100}$
   c) $\frac{100}{100}$
   d) $\frac{6.2}{10} = 64.2$
   e) $\frac{5.64}{100} = 5.64$
   f) $\frac{42.8}{100} = 42.08$

3. a) $\frac{381}{100}$
   b) $\frac{6.9}{10}$
   c) $\frac{7.4}{100}$
   d) $\frac{18.15}{100}$
   e) $\frac{13.4}{10}$
   f) $\frac{17.6}{100}$
   g) $\frac{193.45}{100}$
   BONUS
   $\frac{10,000}{10,000}$

8. a) $\frac{38}{100}$
   b) $\frac{708}{100}$
   c) $\frac{860}{100}$
   d) $\frac{6.004}{100}$
   e) $\frac{708}{10}$
   f) $\frac{17.5}{10}$
   g) $\frac{3.189}{100}$
   h) $\frac{904}{10}$
   BONUS
   $\frac{471,000}{471,000}$

4. a) hundredths
   b) tenths
   c) hundredths
   BONUS
   $\frac{74}{100}$

5. a) seven and four tenths
   b) four and nine hundredths
   c) 74.11
   d) 20.4
   BONUS
   $\frac{99}{100}$ km

No, he did not bike more than a kilometer.

AP Book NF4-27
page 120
1. a) 3.4
   b) 12.5
   c) 8.45
   d) 46.03

7. c) $\frac{41}{10} = 4.1$

BONUS

AP Book NF4-28
page 122
1. a) $\frac{1}{10}$
   b) $\frac{1}{100}$

2. a) 5
   b) 50
   c) Naomi

3. a) 4
   b) 25
   c) Rashida
   BONUS
   $\frac{471,000}{471,000}$

4. Sample answer: Sarah is not correct. Although 0.25 appears to be more because more is shaded, it is 0.25 of 100 (hundredths), while 0.3 is 0.30 in hundredths, which is greater.

5. Sample answer: Yes, 0.3 of a square can be more than 0.4 of another square if the two squares are different sizes.

6. Sample answer: Yes, 0.3 on one number line can be farther right than 0.5 on another number line, depending on the size and spacing of either number line.

AP Book NF4-29
page 123
1. a) $\frac{28}{10} + 0.8 = 2.8$
   b) $\frac{5}{10}$
   c) $\frac{6.7}{10} = 6.70$
   d) $\frac{9.53}{100} = 0.953$
2. 23 tenths 14 tenths

So $2.3 + 1.4$
$= 37$ tenths
$= \frac{37}{10}$
$= 3.7$

So $2.3 - 1.4$
$= 9$ tenths
$= \frac{9}{10}$
$= 0.9$

3. a) Answer may vary.
   b) Answers may vary.
   c) Teacher to check.

4. b) $rac{300}{308}$
   c) $rac{48}{56}$
   d) Centimeters are more like cents because there are 100 centimeters in a meter just like there are 100 cents in a dollar. Ounces are less like cents because there are 16 ounces in a pound.

5. $2 + 0.8 + 2.09 = 4.89$
   Gia biked 4.89 km altogether.

6. $\frac{24}{100}$
Measurement and Data:  
US Customary Units and Area – AP Book 4.2: Unit 8

AP Book MD4-25  
page 124

1. a) Answers may vary: 5–8 inches
   b) Answers may vary: 5–9 inches

2. Answers may vary; teacher to check.

3. a) 3 inches
    b) 5 inches
    c) 2 inches

BONUS

3 inches

4. a) 4 inches
    b) 2 inches
    c) 3 inches

5. a) A: 2 in by 6 in
    B: 4 in, 3 in, 5 in
   b) A: 16 inches
    B: 12 inches

6. Teacher to check.

7. Teacher to check.

8. Teacher to check.

9. Teacher to check.

10. Teacher to check.

AP Book MD4-26  
page 127

1. a) \( \frac{2}{2} \), \( \frac{4}{2} \), \( \frac{2}{4} \)
   b) \( \frac{4}{1} \), \( \frac{4}{1} \), \( \frac{2}{2} \)
   c) \( \frac{3}{4} \), \( \frac{1}{4} \), \( \frac{1}{4} \)
   d) \( \frac{1}{2} \)
   e) \( \frac{3}{4} \)
   f) \( \frac{2}{4} \), or \( \frac{1}{2} \)
   g) \( \frac{2}{4} \)
   h) \( \frac{1}{4} \)
   i) \( \frac{1}{2} \)
   j) \( \frac{3}{4} \)
   k) \( \frac{3}{4} \)

2. Teacher to check circles.

BONUS

RACE

3. a) \( \frac{3}{4} \)
    b) \( \frac{3}{4} \)
    c) \( \frac{3}{4} \)
    d) \( \frac{1}{2} \)
    e) \( \frac{4}{1} \)

4. a) \( \frac{1}{2} \)
    b) \( \frac{1}{4} \)
    c) \( \frac{3}{4} \)
    d) \( \frac{1}{2} \)
    e) \( \frac{4}{1} \)

5. a) A: 2 in by 6 in
    B: 4 in, 3 in, 5 in
   b) A: 16 inches
    B: 12 inches

BONUS

No, the pencil is not exactly 4 inches long because the end of it (the eraser) is not lined up with the ‘0’ line.

6. Answers may vary.

7. a) Length: 2 in
    Width: \( \frac{1}{4} \) in
    Diagonal: \( \frac{1}{4} \) in
   b) Length: \( \frac{1}{2} \) in
    Width: 1 in
    Diagonal: \( \frac{1}{2} \) in

BONUS

AP Book MD4-27  
page 130

1. Teacher to check.

2. Teacher to check.

3. b) \( \frac{7}{8} \)
    c) \( \frac{7}{8} \)
    d) \( \frac{3}{8} \)
    e) \( \frac{5}{8} \)
    f) \( \frac{2}{8} \)
    g) \( \frac{5}{8} \)
    h) \( \frac{2}{4} \)
    i) \( \frac{1}{8} \)
    j) \( \frac{1}{8} \)
    k) \( \frac{1}{4} \)

4. a) \( \frac{3}{8} \)
    b) \( \frac{7}{8} \)
    c) \( \frac{5}{8} \)
    d) \( \frac{1}{4} \)

5. Teacher to check.

6. Teacher to check.

7. Teacher to check.

8. a) i) \( \frac{2}{8} \)
    ii) \( \frac{3}{8} \)
    iii) \( \frac{4}{8} \)
   b) i) \( \frac{1}{4} \)
    ii) \( \frac{3}{8} \)
    iii) \( \frac{1}{2} \)
   c) 16 pennies

9. Answers may vary.

10. Teacher to check.

AP Book MD4-28  
page 134

1. Circle desk and bus.
   Cross out ant.

2. a) Answers may vary.
   b) 3 ft

3. Answers may vary; teacher to check.

4. a) inch
   b) foot
   c) inch
   d) foot
   e) inch

5. a) inch
b) foot
c) inch
d) foot

6. Answers may vary; teacher to check.

AP Book MD4-29
page 135
1. 12, 24, 36, 48, 60, 72, 84, 96
2. 12
3. a) ft in

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<td>132</td>
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</tbody>
</table>

b) ft in

| 20 | 240|
| 25 | 300|
| 30 | 360|

c) ft in

| 50 | 600|
| 100| 1200|
| 120| 1440|

4. b) 72 in, circle 83 in.
c) 60 in, circle 5 ft.
d) 132 in, circle 11 ft.
e) 120 in, circle 10 ft.
f) 240 in, circle 2,000 in.
g) 360 in, circle 1,500 in.
h) 600 in, circle 50 ft.
i) 1,200 in, circle 1,212 in.
5. Answers may vary; teacher to check.
6. Answers may vary; teacher to check.

AP Book MD4-30
page 136
1. a) 1 ft 6 in
   b) 1 ft 11 in
   c) 2 ft 4 in
2. a) Answers may vary; teacher to check.
b) Answers may vary; teacher to check.
3. Answers may vary; teacher to check.
4. a) 2 ft
   b) 24 inches. The snake is 24 inches because 1 ft = 12 inches, so 2 ft = 24 inches
5. b) 5 ft = 60 inches
   so 5 ft 7 in
   = 60 + 7 in
   = 67 in
   c) 4 ft = 48 inches
   so 4 ft 2 in
   = 48 + 2 in
   = 50 in
d) 2 ft = 24 inches
   so 2 ft 10 in
   = 24 + 10 in
   = 34 in
e) 6 ft = 72 inches
   so 6 ft 1 in
   = 72 + 1 in
   = 73 in
   f) 10 ft = 120 inches
   so 10 ft 11 in
   = 120 + 11 in
   = 131 in
   BONUS
g) 100 ft = 1,200 inches
   so 100 ft 10 in
   = 1,200 + 10 in
   = 1,210 in
   h) 20 ft = 240 inches
   so 20 ft 5 3/4 in
   = 240 + 5 3/4 in
   = 245 3/4 in
6. a) 2 3 4 5 6
   24 36 48 60 72
   7 8
   84 96
   b) ii) 2 ft < 35 in < 3 ft
   iii) 3 ft < 47 in < 4 ft
   iv) 7 ft < 89 in < 8 ft
c) Teacher to check circles.
d) ii) 35 inches is about 3 ft
   iii) 47 inches is about 4 ft
   iv) 89 inches is about 7 ft
   BONUS
   88 1/2 inches is closer to 7 ft.

AP Book MD4-31
page 138
1. Answers may vary; teacher to check.
2. Answers may vary; teacher to check.
4. 2 3 4 5 6 7 8
   6 9 12 15 18 21 24
5. 3
6. a) yd ft

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<td>10</td>
<td>30</td>
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<tr>
<td>11</td>
<td>33</td>
</tr>
</tbody>
</table>

b) yd ft

| 20 | 60 |
| 25 | 75 |
| 30 | 90 |

c) yd ft

| 32 | 96 |
| 57 | 171 |
| 100| 300 |

7. b) 15 ft, circle 30 ft.
c) 27 ft, circle 9 yd.
d) 30 ft, circle 36 ft.

AP Book MD4-32
page 140
1. a) Teacher to check marks on number line.
   b) Teacher to check marks on number line.
c) BALD EAGLE
2. a) i) 18 yd
   ii) 22 yd
   iii) 11 yd
   b) i) 54 ft
   ii) 66 ft
   iii) 33 ft
3. Tiger = 7 ft = 84 inches
   Cobra = 100 inches
   The cobra is longer.
4. a) Branch: 30 in
   Trunk: 2 ft = 24 in
   The snail crawled 6 in farther along the trunk than along the branch.
b) 30 + 24 = 54 in in total.
1 yd = 3 ft = 36 in.
Yes, it crawled more than a yard.

5. a) 6 ft
     b) 9 ft
     c) 88 inches
        About 7 ft long.

6. a) 630 – 555 = 75 ft
     b) About 9 times.

7. 6 ft × 4 = 24 ft
8. a) 84 inches
     b) 84 ÷ 3 = 28
9. a) 3 ft 4 in
     = 36 in + 4 in
     = 40 inches
     b) 3 × 40 = 120 inches

BONUS
120 ÷ 12 = 10 ft

AP Book MD4-33
page 142
1. a) 12 cm²
     b) 7 cm²
     c) 11 cm²
2. a) 4 cm²
     b) 8 cm²
     c) 3 cm²
3. A: 12 cm²
   B: 4 cm²
   C: 8 cm²
4. Teacher to check.

AP Book MD4-34
page 143
1. a) 3 × 5 = 15
     b) 2 × 8 = 16
     c) 2 × 3 = 6
     d) 3 × 3 = 9
2. a) 4 × 4 = 16
     c) 2 × 4 = 8

d) 6 × 2 = 12

3. b) 7 × l = 21 m²
    l = 21 ÷ 7
    l = 3
    c) 10 × l = 40 in²
       l = 40 ÷ 10
       l = 4
4. a) 24 + 3 = 8
     b) 10 + 5 = 2
     c) x² = 9
        x = 3
5. a) 2 × 6 = 12
     1 × 12 = 12
     b) Teacher to check.

6. a) B Perimeter:
     2 + 3 + 2 + 3 = 10 cm
     B Area:
     2 × 3 = 6 cm²
     C Perimeter:
     3 + 4 + 3 + 4 = 14 cm
     C Area:
     3 × 4 = 12 cm²
     D Perimeter:
     5 + 2 + 5 + 2 = 14 cm
     D Area:
     2 × 5 = 10 cm²
     E Perimeter:
     2 + 7 + 2 + 7 = 16 cm
     E Area:
     2 × 7 = 14 cm²
     F Perimeter:
     2 + 4 + 2 + 4 = 12 cm
     F Area:
     2 × 4 = 8 cm²
     b) No
     c) C and D
     d) E, A, D, C, F, B
     e) A, E, C, D, F, B
     f) No
     g) Answers may vary;
        teacher to check.
        Sample answer:
        Perimeter is the outline
        of a shape, found by
        adding the length of
        each side. Area is the
        total size of the shape,
        found by multiplying
        length by width. Shapes
        with the same perimeter
        may have different
        areas, and shapes with
        the same area may
        have different
        perimeters.

AP Book MD4-35
page 145
1. a) 3 × 6 = 18 m²
     b) 2 × 9 = 18 m²
     c) 6 × 8 = 48 cm²
2. a) w × 2 = 12 cm²
    w = 12 ÷ 2
    w = 6
    c) w × 6 = 24 cm²
    w = 24 ÷ 6
    w = 4
3. b) 7 × l = 21 m²
    l = 21 ÷ 7
    l = 3
    c) 10 × l = 40 in²
       l = 40 ÷ 10
       l = 4
4. a) 24 + 3 = 8
     b) 10 + 5 = 2
     c) x² = 9
        x = 3
5. a) 2 × 6 = 12
     1 × 12 = 12
     b) Teacher to check.

6. a) B Perimeter:
     2 + 3 + 2 + 3 = 10 cm
     B Area:
     2 × 3 = 6 cm²
     C Perimeter:
     3 + 4 + 3 + 4 = 14 cm
     C Area:
     3 × 4 = 12 cm²
     D Perimeter:
     5 + 2 + 5 + 2 = 14 cm
     D Area:
     2 × 5 = 10 cm²
     E Perimeter:
     2 + 7 + 2 + 7 = 16 cm
     E Area:
     2 × 7 = 14 cm²
     F Perimeter:
     2 + 4 + 2 + 4 = 12 cm
     F Area:
     2 × 4 = 8 cm²
     b) No
     c) C and D
     d) E, A, D, C, F, B
     e) A, E, C, D, F, B
     f) No
     g) Answers may vary;
        teacher to check.
        Sample answer:
        Perimeter is the outline
        of a shape, found by
        adding the length of
        each side. Area is the
        total size of the shape,
        found by multiplying
        length by width. Shapes
        with the same perimeter
        may have different
        areas, and shapes with
        the same area may
        have different
        perimeters.

7. a) Area
    b) Perimeter
    c) Area
    d) Perimeter

AP Book MD4-36
page 147
1. a) i) A: 8
     B: 4
     C: 12
      ii) A: 16
         B: 2
         C: 18
     iii) A: 4
         B: 12
         C: 16
    b) Teacher to check.

2. Teacher to check separate rectangles.
   a) Total: 44 m²
   b) Total: 52 cm²
   c) Total: 37 in²

3. a) 3 stories
    b) 40 m

4. a) 4 m
   Total: 28 m²
   b) 7 ft, 3 ft
   Total: 30 ft²

5. a) 4 cm
    b) 3 cm

6. a) L = 3 cm
    A = 6 cm²
    b) L = 5 cm
    A = 20 cm²

7. a) 9 cm²
    b) 25 cm²
### Measurement and Data:
#### US Customary Units and Area – AP Book 4.2: Unit 8

**AP Book MD4-37**

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<td>1. Teacher to check.</td>
</tr>
<tr>
<td>2. a) 20</td>
</tr>
<tr>
<td>b) $33 - 20 = 13$ (Subtract number of shaded squares from total number of squares to find unshaded units.)</td>
</tr>
<tr>
<td>3. a) Teacher to check.</td>
</tr>
<tr>
<td>b) 4 ft</td>
</tr>
<tr>
<td>c) $12 \times 3 = 36$</td>
</tr>
<tr>
<td>d) Area = $2 \times 4 = 8 \text{ ft}^2$</td>
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<tr>
<td>$8 \times 2\frac{1}{2} = 16\frac{1}{2}$</td>
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<tr>
<td>4. Teacher to check.</td>
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<tr>
<td>5. Teacher to check.</td>
</tr>
<tr>
<td>6. a) Teacher to check.</td>
</tr>
<tr>
<td>b) 4 squares</td>
</tr>
<tr>
<td>c) Teacher to check.</td>
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<tr>
<td>d) Teacher to check.</td>
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**AP Book MD4-39**

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<tbody>
<tr>
<td>1. a) 4</td>
</tr>
<tr>
<td>b) $5 \frac{1}{2}$</td>
</tr>
<tr>
<td>c) $3 \frac{1}{2}$</td>
</tr>
<tr>
<td>d) Teacher to check.</td>
</tr>
<tr>
<td>2. a) 3</td>
</tr>
<tr>
<td>b) $\frac{3}{4}$</td>
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<tr>
<td><strong>BONUS</strong></td>
</tr>
<tr>
<td>3:15 p.m.</td>
</tr>
<tr>
<td>c) $\frac{1}{4}$</td>
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<tr>
<td><strong>BONUS</strong></td>
</tr>
<tr>
<td>12:05 p.m.</td>
</tr>
<tr>
<td>d) $\frac{1}{2}$</td>
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<tr>
<td>e) 5</td>
</tr>
<tr>
<td>3. a) $\frac{7}{8}$</td>
</tr>
<tr>
<td>b) $\frac{4}{8}$</td>
</tr>
<tr>
<td>c) 4</td>
</tr>
<tr>
<td>d) $\frac{1}{8}$</td>
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**AP Book MD4-38**

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<td>b) 5</td>
</tr>
<tr>
<td>c) 3</td>
</tr>
<tr>
<td>d) 2</td>
</tr>
<tr>
<td>2. Teacher to check.</td>
</tr>
<tr>
<td>3. a) 10</td>
</tr>
<tr>
<td>b) 12</td>
</tr>
<tr>
<td>4. a) Zoe, 3 letters</td>
</tr>
<tr>
<td>b–d) Teacher to check.</td>
</tr>
<tr>
<td>5. a) 10 letters</td>
</tr>
<tr>
<td>b) 5</td>
</tr>
<tr>
<td>c) 4</td>
</tr>
<tr>
<td>d) No</td>
</tr>
<tr>
<td>e) No</td>
</tr>
<tr>
<td>f) 7</td>
</tr>
</tbody>
</table>

**AP Book MD4-40**

<table>
<thead>
<tr>
<th>Page 154</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a) Teacher to check.</td>
</tr>
<tr>
<td>b) Basket: 4 lb</td>
</tr>
<tr>
<td>c) Backpack: 1 lb</td>
</tr>
<tr>
<td>d) 3 lbs</td>
</tr>
<tr>
<td>e) 4 times</td>
</tr>
<tr>
<td><strong>BONUS</strong></td>
</tr>
<tr>
<td>Tarek is wrong because although no food can weigh 0 pounds, it is possible for something to weigh closer to 0 than one eighth.</td>
</tr>
</tbody>
</table>
1. a) 2  
b) 1  
c) 0  
2. a) Line segment  
b) Line  
c) Ray  
d) Ray  
e) Line segment  
f) Line  
3. a) A  
b) A, C  
c) A, C  
4. Teacher to check:  
a)  
b)  
c)  
5. Teacher to check. c) has two answers because ray can have endpoint or arrow on either end.  
6. Answers will vary; teacher to check.  
Sample answers:  
a)  
b)  
c)  
7. Teacher to check.  
Sample answers:  
a)  
b)  
8. Teacher to check. Sample answers:  
a)  
b)  
c) Yes  
d) No  
9. a) C  
b) B  
c) B  
10. Teacher to check.  
11. b)  
12. Teacher to check. Sample answers:  
a)  
b)  
c)  
d)  
13. a) Yes  
b) Yes  
14. e) Obtuse  
f) Acute  
g) Acute  
h) Acute  
15. a) Obtuse  
b) Obtuse  
c) Acute  
d) Acute  
e) Obtuse  
f) Acute  
16. a) Acute  
b) Obtuse  
c) Acute  
d) Acute  
e) Obtuse  
f) Acute  
17. b)  
c)  
d)  
e)  
f)  
g)  
h)  
18. a) B
b) A: Bottom arm of angle is not lined up with 0° line on protractor; arm should line up with the 0° base line, and vertex should be at origin of protractor.
C: Bottom arm of angle is lined up with 10° measure, not lined up with 0° base line.

8. a) 30°
   b) 43°
   c) 130°
   d) 128°
   e) 105°
   f) 56°
9. a) 136°
   b) 43°
   c) 130°
   d) 128°
10. a) Teacher to check.
    b) Teacher to check.
11. Teacher to check.

AP Book G4-17
page 164
1. a) C
   b) A
2. a) Teacher to check.
   Sample answer:
   b)
   c)
3. a) Teacher to check.
   Sample answer:
   b)

AP Book G4-18
page 165
1. a) X: 20° Y: 70°
   b) X: 20° Y: 110°
2. a) ii) X: 45° Y: 85°
    Whole: 130°
   iii) X: 40° Y: 20°
    Whole: 60°
   iv) X: 45° Y: 85°
    Whole: 130°
   b) You can add the two smaller angles to get the measure of the whole angle.
3. a) 10° + 40° = 50°
   b) 35° + 30° = 65°
   c) 50° + 60° = 110°
   d) 10° + 25° + 55° = 90°
4. b) x = 120° − 90° = 30°
   c) x = 80° − 48° = 32°
   d) x = 150° − 55° = 95°
5. a) 120°

AP Book G4-19
page 167
1. a) i) x + 55° = 90°
    x = 90° − 55°
   ii) x + 50° = 150°
    x = 150° − 50°
   iii) x + 52° = 103°
    x = 103° − 52°
   iv) x + 42° = 123°
    x = 123° − 42°
   b) i) x + 55° = 90°
    x = 90° − 55°
    x = 35°
   ii) x + 50° = 150°
    x = 150° − 50°
    x = 100°
   iii) x + 52° = 103°
    x = 103° − 52°
    x = 51°
   iv) x + 42° = 123°
    x = 123° − 42°
    x = 81°
b) Dan’s mistake is that the measures of the angles overlap over a angle in between and both 80° and 60° measures included this smaller third angle, adding that angle twice; measuring the whole angle, the correct measure is 120°
6. a) x = 120°
   b) Marina’s mistake is that adding the 60° and 40° omitted the third angle in between (which is 20°); measuring the whole angle, the correct measure is 120°
7. a) x = 5°
   b) 160° − 50° − 80° = 30°

BONUS
x + 41° + 27° = 90°
x = 22°
5. 25° + x = 60°
x = 60° − 25°
x = 35°
The sprinkler turned 35° after the stop.

AP Book G4-20
page 169
1. a) 4
   b) 90°
   c)
   d) 4 × 90° = 360°
There are 360° in a full turn.
2. a) 6 × 10 seconds = 60 seconds to rotate 6°
   b) 30 × 10 seconds = 300 seconds to rotate 30°
   That is 5 minutes.
   c) 360 × 10 seconds = 3,600 seconds to rotate 360°
   That is 60 minutes.
d) 1 hour = 60 min
   60 min = 3,600 seconds.
   Minute hand turns 1° in 10 seconds, thus 3,600 ÷ 10 = 360°. The minute hand turns one full circle in an hour.
   Yes, answer in c) makes sense because with 1° turn = 10 seconds, to turn a full hour (3,600 seconds), the minute hand will turn 360°.

3. 57 turns
4. a) 28° + 15° = 43°
   b) 150° − 73° = 77°
5. a) \(\frac{1}{10}\) of 360
   = 360 ÷ 10
   = 36
   \(\frac{3}{10}\) of 360
   = 3 × 36
   = 108
   b) \(\frac{1}{5}\) of 360
   = 360 ÷ 5
   = 72
   \(\frac{2}{5}\) of 360
   = 2 × 72
   = 144
6. Student to check angle using protractor.
7. a) \(\frac{1}{5}\)
   \(\frac{1}{5}\) of 360° = 72°
   b) \(\frac{2}{5}\)
   \(\frac{2}{5}\) of 360° = 144°
   c) \(\frac{1}{10}\)
   \(\frac{1}{10}\) of 360° = 36°
   d) \(\frac{3}{10}\)
   \(\frac{3}{10}\) of 360° = 108°
8. a) 63°, 90°, 27°
   b) 57°, 50°, 73°
   c) Right
   d) 118°
   e) 52°
   f) 131°
   4. a) 45°, 90°, 45°
   b) 42°, 112°, 26°
   5. a) Right
   b) Not right
   c) Right
   d) Not right
   e) Not right
   6. a) Teacher to check.

BONUS

f) Middle portion of the Venn diagram; has both parallel and perpendicular sides
   g) No, there cannot be a triangle that goes into the central area of the Venn diagram because although a triangle can have perpendicular sides (right triangle), it cannot have parallel sides.

2. a) Teacher to check.
   Sample answers:
   b) Teacher to check.
   c) Teacher to check.
   4. a) Teacher to check.

Answers may vary; teacher to check.

f) Yes, all went into the central area of having at least a right angle and at least an acute angle.
   g) No, because if a triangle has a right angle, its other two angles must both be acute (because all angles of a triangle must add to 180°).
1. The multiples of 5 are 0, 5, 10, …
   a) Write the multiples of 5 from 0 to 25.

   
   
   
   
   
   
   
   
   

   b) Describe the pattern in the ones digits of the numbers.

   
   

   c) Circle the multiples of five.

   127  520  615  403  5,058

2. Determine the gaps between numbers, and then extend the sequence.

   100, 90, 81, 73, _____, _____

BONUS ►

   Figure 1

   Figure 2

   Figure 3

   Figure 4

   a) Fill in the T-table.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Number of White Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   b) Predict the number of white triangles needed to make Figure 7. Explain.
Unit 1: Operations and Algebraic Thinking

Quiz (Lessons 26 to 27)

1. a) 15
   20
   25
   b) 0, 5 repeat
   c) 520
   615
   2. 66, 60

**BONUS**

a)

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

b) $12 + 3 + 3 = 18$
1. a) Draw a picture using dots and circles. Share the dots as equally as possible among the circles. 
Show any remainders outside the circles.
   i) 9 dots in 2 circles
   ii) 11 dots in 3 circles

   b) Write a division equation including remainders for each picture you drew in part a).
   i) $9 \div 2 = \underline{\hspace{2cm}} \text{ R } \underline{\hspace{1cm}}$
   ii) $11 \div 3 = \underline{\hspace{2cm}} \text{ R } \underline{\hspace{1cm}}$

2. Lynn has 8 cookies and wants to make bags with 3 cookies in each.

   Draw a number line picture to show the division.

   How many bags can she make? _______
   How many cookies will be left over? _______

3. Divide.
   a) $11 \div 4 = \underline{\hspace{2cm}}$
   b) $52 \div 10 = \underline{\hspace{2cm}}$

4. For each picture, write a division equation and an equation with multiplication and addition.
   a) $\begin{array}{cc}
   \bullet & \bullet \\
   \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
   \end{array}$
   b) $\begin{array}{cc}
   \bullet & \bullet & \bullet & \bullet & \bullet & \bullet
   \end{array}$

   __________________________________________________________________________
   __________________________________________________________________________

5. Pam says that $13 \div 2 = 5 \text{ R } 3$. What is wrong with Pam’s equation? How would you rewrite it?

6. Divide.
   $40,000 \div 2 = \underline{\hspace{2cm}}$
   $160,000,000 \div 4 = \underline{\hspace{2cm}}$
Unit 2: Number and Operations in Base Ten

Quiz (Lessons 40 to 43)

1. a) i) 
   
   ii) 

   b) i) $9 ÷ 2 = 4 R 1$
   
   ii) $11 ÷ 3 = 3 R 2$

2. 2 bags, 2 left over

3. a) $11 ÷ 4 = 2 R 3$
   
   b) $52 ÷ 10 = 5 R 2$

4. a) $8 ÷ 3 = 2 R 2$
   
   $(2 \times 3) + 2 = 8$
   
   b) $7 ÷ 2 = 3 R 1$
   
   $(3 \times 2) + 1 = 7$

5. In $13 ÷ 2 = 5 R 3$, the remainder is bigger than size of the groups.
   Another group of 2 could be made. $13 ÷ 2 = 6 R 1$

6. a) 20,000

BONUS 40,000,000
1. Carry out all steps of long division.
   a) \[
   \begin{array}{c|c}
   3 & 8 \ \underline{-} \\
   \hline
   & 3 \\
   \hline
   & 
   \end{array}
   \]
   b) \[
   \begin{array}{c|c}
   4 & 9 \ \underline{-} \\
   \hline
   & 2 \\
   \hline
   & 
   \end{array}
   \]
   c) \[
   \begin{array}{c|c}
   3 & 7 \ \underline{-} \\
   \hline
   & 2 \\
   \hline
   & 
   \end{array}
   \]
   d) \[
   \begin{array}{c|c}
   5 & 8 \ \underline{-} \\
   \hline
   & 7 \\
   \hline
   & 3 \\
   \hline
   & 2 \\
   \hline
   & 
   \end{array}
   \]

2. Carry out all steps of long division.
   a) \[
   \begin{array}{c|c}
   5 & 7 \ \underline{-} \\
   \hline
   & 3 \\
   \hline
   & 2 \\
   \hline
   & 1 \\
   \hline
   & 
   \end{array}
   \]
   b) \[
   \begin{array}{c|c}
   3 & 9 \ \underline{-} \\
   \hline
   & 9 \\
   \hline
   & 6 \\
   \hline
   & 3 \\
   \hline
   & 
   \end{array}
   \]

3. A square park has a perimeter of 596 ft. How long is each side of the park?

4. Divide mentally.
   a) \[
   \begin{array}{c|c}
   3 & 9 \ \underline{-} \\
   \hline
   & 6 \\
   \hline
   & 
   \end{array}
   \]
   b) \[
   \begin{array}{c|c}
   2 & 4 \ \underline{-} \\
   \hline
   & 2 \\
   \hline
   & 
   \end{array}
   \]
   c) \[
   \begin{array}{c|c}
   4 & 8 \ \underline{-} \\
   \hline
   & 4 \\
   \hline
   & 2 \\
   \hline
   & 0 \\
   \hline
   & 
   \end{array}
   \]
   d) \[
   \begin{array}{c|c}
   2 & 8 \ \underline{-} \\
   \hline
   & 4 \\
   \hline
   & 2 \\
   \hline
   & 0 \\
   \hline
   & 
   \end{array}
   \]
   e) \[
   \begin{array}{c|c}
   3 & 3 \ \underline{-} \\
   \hline
   & 6 \\
   \hline
   & 3 \\
   \hline
   & 2 \\
   \hline
   & 1 \\
   \hline
   & 
   \end{array}
   \]
   f) \[
   \begin{array}{c|c}
   2 & 2 \ \underline{-} \\
   \hline
   & 4 \\
   \hline
   & 2 \\
   \hline
   & 0 \\
   \hline
   & 
   \end{array}
   \]

BONUS:

f) \[
   \begin{array}{c|c}
   2 & 2 \ \underline{-} \\
   \hline
   & 4 \\
   \hline
   & 2 \\
   \hline
   & 0 \\
   \hline
   & 
   \end{array}
   \]
Unit 2: Number and Operations in Base Ten

Quiz (Lessons 44 to 48)

1. a)  
   \[
   \begin{array}{c}
   3) 83 \\
   - 6 \\
   \hline
   23 \\
   - 21 \\
   \hline
   2 \\
   \end{array}
   \]

   b)  
   \[
   \begin{array}{c}
   4) 92 \\
   - 8 \\
   \hline
   12 \\
   - 12 \\
   \hline
   0 \\
   \end{array}
   \]

   c)  
   \[
   \begin{array}{c}
   3) 72 \\
   - 6 \\
   \hline
   12 \\
   - 12 \\
   \hline
   0 \\
   \end{array}
   \]

   d)  
   \[
   \begin{array}{c}
   5) 87 \\
   - 5 \\
   \hline
   37 \\
   - 35 \\
   \hline
   2 \\
   \end{array}
   \]

   2. a)  
   \[
   \begin{array}{c}
   5) 738 \\
   - 5 \\
   \hline
   23 \\
   - 20 \\
   \hline
   38 \\
   - 35 \\
   \hline
   3 \\
   \end{array}
   \]

   b)  
   \[
   \begin{array}{c}
   3) 979 \\
   - 9 \\
   \hline
   07 \\
   - 6 \\
   \hline
   19 \\
   - 18 \\
   \hline
   1 \\
   \end{array}
   \]

   3. 149 ft

   BONUS  
   e) 12,322
   \[
   \begin{array}{c}
   3) 12322 \\
   - 3696 \\
   \hline
   8246 \\
   - 8246 \\
   \hline
   0 \\
   \end{array}
   \]

   f) 102,244
   \[
   \begin{array}{c}
   2) 102244 \\
   - 20448 \\
   \hline
   8004 \\
   - 8004 \\
   \hline
   0 \\
   \end{array}
   \]
Unit 2: Number and Operations in Base Ten

Test (Lessons 40 to 51)

1. Divide. Show remainders.
   a) \(24 \div 5 = \) \__________
   b) \(12 \div 4 = \) \__________
   c) \(13 \div 2 = \) \__________
   d) \(23 \div 3 = \) \__________
   e) \(18 \div 6 = \) \__________
   f) \(41 \div 4 = \) \__________

2. Divide mentally.
   a)
   b)
   c)
   d)

3. a) Draw a picture using dots and circles. Share the dots as equally as possible among the circles. Show any remainders outside the circles.
   i) 7 dots in 2 circles
   ii) 11 dots in 4 circles

   b) Write a division equation including remainders for the pictures you drew in part a).
   i) \(7 \div 2 = \) \________ R \________
   ii) \_____________________________

4. Carry out all steps of long division.
   a)
   b)
   c)
   d)
5. Helen paid $154 for 7 shirts. How much did each shirt cost?

6. Write the division equation. Then answer the question.
   a) Andrew has $11. A pack of trading cards is $2.
      How many packs of trading cards can he buy? ______
      \[ \underline{\text{____}} \div \underline{\text{____}} = \underline{\text{____}} \text{ R } \underline{\text{____}} \]
   b) There are 17 candies to share equally with 3 friends.
      How many candies does each friend get? ______
      \[ \underline{\text{____}} \div \underline{\text{____}} = \underline{\text{____}} \text{ R } \underline{\text{____}} \]
   c) Each car in an amusement park ride holds 4 people.
      14 people are getting on the ride.
      \[ \underline{\text{____}} \div \underline{\text{____}} = \underline{\text{____}} \text{ R } \underline{\text{____}} \]
      How many cars are used? ______

7. a) Repeat the core until you find the 6th term. The blocks are colored red (R), yellow (Y), and blue (B).
   
   i) \[ \begin{array}{cccc} \text{R} & \text{Y} & \text{ } & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \end{array} \]
   
   ii) \[ \begin{array}{cccc} \text{B} & \text{R} & \text{Y} & \text{ } \\ \text{ } & \text{ } & \text{ } & \text{ } \end{array} \]
   
   b) Use division to predict the color of the 20th block from the patterns in part a).
      i) \[ \underline{\text{____}} \div \underline{\text{____}} = \underline{\text{____}} \text{ R } \underline{\text{____}} \]
      ii) \[ \underline{\text{____}} \div \underline{\text{____}} = \underline{\text{____}} \text{ R } \underline{\text{____}} \]
      20th block color: ________
      20th block color: ________

BONUS ▲

Use division to predict the color of the 201st block from the pattern with the core RYBB.
Unit 2: Number and Operations in Base Ten

Test (Lessons 40 to 51)

ADVANCED

8. How many squares across is each rectangle? Complete the division statement.
   a)  
   
   2 8 + 2 = ______
   
   b)  
   
   3 9 + 3 = ______

9. Decide how many squares across you need to make the rectangle. Write the division statement.
   a)  
   
   4 100 ÷ 4 = ______
   
   b)  
   
   5 60 ______

10. Divide by splitting the number.
    a) 76 ÷ 2
    
    76 ÷ 2 = _____________
    
    b) 78 ÷ 3
    
    78 ÷ 3 = _____________

11. Write the division equation. Then answer the questions.
    a) Sarah has $20. A raffle ticket costs $3.
       How many tickets cards can she buy? ______
       How much money is left over? ______
       ____ ÷ ____ = ____ R ____

    b) Justin is moving 17 boxes. He can carry at most 3 boxes on each trip.
       How many trips does he make? ______
       How many boxes does he carry on the last trip? ______
       ____ ÷ ____ = ____ R ____
Test (Lessons 40 to 51)

1. a) \(24 ÷ 5 = 4 R 4\)
   b) \(12 ÷ 4 = 3 R 0\)
   c) \(13 ÷ 2 = 6 R 1\)
   d) \(23 ÷ 3 = 7 R 2\)
   e) \(18 ÷ 6 = 3 R 0\)
   f) \(41 ÷ 4 = 10 R 1\)

2. a) 
   b) 
   c) 
   d) 

3. a) i) \(7 ÷ 2 = 3 R 1\)
   ii) \(11 ÷ 4 = 2 R 3\)
   b) i) \(7 + 2 = 3 R 1\)
   ii) \(11 + 4 = 2 R 3\)

4. a) 
   b) 

5. \(154 ÷ 7 = 22\)
   Each shirt cost $22.

6. a) 5
   b) \(11 ÷ 2 = 5 R 1\)
   c) \(17 ÷ 3 = 5 R 2\)
   d) \(14 ÷ 4 = 3 R 2\)

7. a) i) \(20 ÷ 2 = 10 R 0\)
   ii) \(20 ÷ 3 = 6 R 2\)
   b) i) \(20 + 2 = 10 R 0\)
   ii) \(20 + 3 = 6 R 2\)

8. a) 
   b) \(3 + 3 = 6 R 2\)

9. a) \(25 \div 4 = 6 R 1\)
   \(100 + 4 = 25\)
   b) \(60 \div 5 = 12\)
   \(60 + 5 = 12\)

10. a) \(\frac{35}{3} + 3 = 14 R 2\)
    \(76 + 2 = 38\)
    b) \(\frac{20}{6} + 6 = 6 R 2\)
    \(78 ÷ 3 = 26\)

11. a) \(20 ÷ 3 = 6 R 2\)
   6 tickets
   2 dollars
   b) \(17 ÷ 3 = 5 R 2\)
   6 trips
   2 boxes

BONUS
\(201 ÷ 4 = 50 R 1\)
\(201 ÷ 3 = \text{Red (R)}\)

8. a) 
   b) \(8 ÷ 2 = 4\)
1. Solve the equation. Show your work.
   
   a) \(8 + \underline{} = 13\)  
   b) \(4 \times w = 12\)

2. Write and solve the equation for the problem.
   
   a) 25 children altogether  
   \hspace{0.5cm} 11 are boys  
   \hspace{0.5cm} \underline{w} \hspace{0.5cm} \text{are girls}  
   b) There are 12 reading books on the bookshelf.  
   \hspace{0.5cm} \text{There are 7 fewer coloring books than reading}  
   \hspace{0.5cm} \text{books on the bookshelf. How many coloring}  
   \hspace{0.5cm} \text{books are there?}

3. Solve the problem in two steps.
   
   Lorna has 14 stickers with animals and some stickers with plants. She has 5 more  
   stickers with animals than stickers with plants. How many stickers does Lorna have  
   altogether?

BONUS

Phil collects quarters, dimes, and nickels. He has 40 coins in his collection. He has  
18 quarters and 15 dimes. How many more dimes than nickels does Phil have?
Unit 3: Operations and Algebraic Thinking

Quiz (Lessons 28 to 33)

1. a) 5, solution will vary, teacher to check
   b) Sample solution:
      \[ w = 12 ÷ 4 \]
      \[ w = 3 \]

2. a) 25 – 11 = w
    \[ w = 14 \]
   b) 12 – 7 = w
    \[ w = 5 \]

3. Sample solution:
   14 – 5 = 9 stickers with plants
   14 + 9 = 23
   Lorna has 23 stickers altogether.

BONUS

Phil has 8 more dimes than nickels.
1. Solve the equation.
   
   a) \( w + 7 = 26 \)
   b) \( b \times 3 = 27 \)
   
   \( w = \underline{_______} \)  \( b = \underline{_______} \)

2. Write and solve the equation for the problem.

   There are 14 red cars. There are 7 fewer red cars than blue cars. How many blue cars are there?

3. Draw a model for the story then solve the problem.

   a) Lina is four times as old as Jordan. Lina is 6 years older than Jordan. How old are Lina and Jordan?

   b) There are 42 people on a school bus. There are five times as many children as there are adults. How many children and how many adults are on the bus?

4. There are 6 cars used to transport 30 people. How many people are in each car? Write an equation then solve it.
5. Burj Khalifa in Dubai is the tallest building in the world. Use the diagram to find the number of floors between the observation deck and the top floor.

![Diagram of Burj Khalifa showing floors]

6. An apple slicer cuts apples into 6 equal pieces. There are 5 apples and 7 people sharing the apples. Each person gets the same number of pieces. How many pieces of apple does each person get?

BONUS
Miranda is arranging floor tiles. She uses 6 red tiles. There are 10 fewer red tiles than blue tiles and twice as many white tiles than the sum of red and blue tiles.

a) How many tiles of each color does Miranda use?

b) How many tiles does she use in total?
Unit 3: Operations and Algebraic Thinking

Test (Lessons 28 to 38)

ADVANCED

7. Tanner and Jessica are organizing their rooms. Tanner has collected three times as many books as Jessica. Jessica has 12 fewer books than Tanner.

   a) How many books do Tanner and Jessica each have?

   b) How many books do they have in total?

8. A choir has 18 children who are in 3rd Grade, which is 10 more than the number of children who are in 4th Grade. How many children in 3rd and 4th Grades are in the choir?

BONUS

There are twice as many orange pens as red pens. There are 8 more black pens than red pens. There are 4 red pens. How many pens are there altogether?
Unit 3: Operations and Algebraic Thinking

Test (Lessons 28 to 38)

1. a) \( w = 19 \)
   b) \( b = 9 \)
2. \( w = 14 + 7 \)
   \( w = 21 \)
3. a) Lina is 8 years old.
    Jordan is 2 years old.
   b) 7 adults and 35 children
4. \( 30 ÷ 6 = w \)
   5 people in each car
5. 46 floors
6. 4 pieces
   BONUS 6 red, 16 blue, 44 white
   Miranda uses 66 tiles in total.
   ADVANCED
7. a) Tanner: 18, Jessica: 6
   b) 24 total
8. 26 children in 3rd and 4th grade
   BONUS 24 altogether (4 red, 8 orange, 12 black)
Unit 4: Number and Operations—Fractions

Quiz (Lessons 1 to 5)

1. The pictures below show fractions.
   a) Circle the picture that shows $\frac{2}{5}$.
   
   ![Fraction pictures]

   b) Explain how you know the fraction is $\frac{2}{5}$.

2. a) Shade the amount then circle the greater fraction.
   b) Use $<$ or $>$ to complete the statement.

   ![Fraction comparison]

3. Use the picture to find equivalent fractions.
   a) $\frac{3}{6} = \frac{9}{9}$
   b) $\frac{2}{3} = \frac{9}{9}$

   **BONUS** $1 = \frac{9}{9}$

4. Circle the greater fraction in the pair.
   a) $\frac{1}{4}$ or $\frac{1}{2}$
   b) $\frac{2}{9}$ or $\frac{2}{12}$

   **BONUS** $\frac{108}{1,000}$ or $\frac{108}{10,000}$
Unit 4: Number and Operations—Fractions

Quiz (Lessons 1 to 5)

1. a) 
   
   b) The container is divided into 5 equal parts and 2 of the parts are shaded.

2. a) 
   
   b) \( \frac{4}{5} > \frac{3}{4} \)

3. a) 
   
   b) \( \frac{6}{9} \)

   BONUS \( \frac{9}{9} \)

4. a) 
   
   b) \( \frac{2}{9} \)

   BONUS \( \frac{108}{1,000} \)
Unit 4: Number and Operations—Fractions  

Quiz (Lessons 6 to 11)

1. Write an equivalent fraction for the picture.
   a)  
   b)  

2. Use multiplication to find the equivalent fraction.
   a)  
   b)  
   c)  

3. a) Rewrite the fractions so they have the same denominator.
   b) Write < or > to show the greater fraction.
   c) Explain how you know which fraction is greater.

4. Add the fractions.
   a)  
   b)  

5. Subtract the fractions.
   a)  
   b)  

Sample Unit Quizzes and Tests for AP Book 4.2
Unit 4: Number and Operations—Fractions

Quiz (Lessons 6 to 11)

ADVANCED

6. Write “more” or “less.”

a) \( \frac{10}{7} \) is _________ than 1. \\
   \( \frac{8}{9} \) is _________ than 1.

b) \( \frac{13}{30} \) is _________ than \( \frac{1}{2} \). \\
   \( \frac{7}{12} \) is _________ than \( \frac{1}{2} \).

So \( \frac{10}{7} \) is _________ than \( \frac{8}{9} \).

So \( \frac{13}{30} \) is _________ than \( \frac{7}{12} \).

7. Write > or <.

a) \( \frac{1}{2} \) _________ \( \frac{6}{10} \) \\
b) \( \frac{1}{2} \) _________ \( \frac{7}{15} \) \\
c) \( \frac{6}{10} \) _________ \( \frac{7}{15} \)

8. What fraction of the whole is a single square?

a) i) \\
    ii) \\
    iii) \\
    iv)

b) Write “bigger” or “smaller.”

The square is a ________________________ fraction of a bigger whole.
Unit 4: Number and Operations—Fractions

Quiz (Lessons 6 to 11)

1. a) \[ \frac{4}{10} \]
   b) \[ \frac{6}{9} \]
2. a) \[ \frac{3}{12} \]
   b) \[ \frac{8}{20} \]
   c) \[ \frac{10}{45} \]
3. a) \[ \frac{36}{45} \] and \[ \frac{35}{45} \]
   b) \[ \frac{4}{5} > \frac{7}{9} \]
   c) When both denominators are the same, the numerator of 4/5 is larger, meaning that there are more pieces shaded.
4. a) \[ \frac{3}{5} \]
   b) \[ \frac{4}{6} \]
BONUS \[ \frac{90}{100} \]
5. a) \[ \frac{3}{5} \]
   b) \[ \frac{4}{6} \]
BONUS \[ \frac{23}{72} \]
6. a) more; less; more
   b) less; more; less
7. a) <
   b) >
   c) >
8. a) i) \[ \frac{1}{2} \]
   ii) \[ \frac{1}{3} \]
   iii) \[ \frac{1}{4} \]
   iv) \[ \frac{1}{5} \]
   b) smaller
Unit 4: Number and Operations—Fractions

Quiz (Lessons 12 to 14)

1. a) One circle is one whole. Shade in the correct number of pieces to show \( \frac{23}{3} \).

   ![Shaded Circles]

b) Write \( 3 \frac{2}{3} \) as an improper fraction.

2. Circle groups that equal 1. Then write the improper fraction as a mixed number.

   \[
   \frac{9}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}
   \]

3. Write the improper fraction as a mixed number by dividing.

   \[
   \frac{23}{5} \div \frac{1}{2} = \frac{46}{5} \quad \text{R} \quad \frac{1}{5}
   \]

   so \( \frac{23}{5} = \)

4. Add or subtract. Write your answer as an improper fraction.

   a) \( \frac{5}{4} + 2 \frac{1}{4} = \)

   b) \( \frac{5}{6} - 1 \frac{4}{6} = \)  

BONUS \( \frac{9}{10} - \frac{30}{10} = \)

5. Karl has \( 4 \frac{3}{4} \) cups of milk. He uses \( 1 \frac{1}{4} \) cups of milk making hot chocolate. How much milk does Karl have left? Write answer as a mixed number.
1. a) \[ \begin{array}{cccccc}
\text{□□□□□□□□□□□}
\end{array} \]
   b) \[ \frac{11}{3} \]

2. \[ \frac{41}{2} \]

3. \[ 23 \div 5 = 4 \text{ R } 3 \]
   So \[ 4\frac{3}{5} \]

4. a) \[ \frac{14}{4} \]
   b) \[ \frac{21}{6} \]

   **BONUS**
   \[ \frac{443}{10} \]

5. \[ 3\frac{1}{2} \]
1. What fraction of the animals are cats?

2. Write the correct sign <, >, = between the pair of fractions.
   a) $\frac{3}{8}$  $\square$  $\frac{3}{5}$
   b) $\frac{5}{6}$  $\square$  $\frac{3}{6}$
   c) $\frac{1}{2}$  $\square$  $\frac{4}{8}$
   d) $\frac{2}{5}$  $\square$  $\frac{4}{9}$

   BONUS: $\frac{150}{200}$  $\square$  $\frac{300}{400}$

3. Write the addition equation.

   $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

4. Write $\frac{3}{4}$ as an improper fraction.

5. Write $\frac{8}{3}$ as a mixed number.

6. Add or subtract. Write your answer as an improper fraction.
   a) $\frac{1}{3} + \frac{4}{3} =$
   b) $\frac{9}{4} - \frac{2}{4} =$
   c) $\frac{6}{8} + \frac{5}{8} =$
   d) $\frac{10}{7} - \frac{2}{7} =$
7. Add or subtract. Write your answer as a mixed number.
   a) \(1\frac{1}{5} + 2\frac{3}{5} = \)
   b) \(2\frac{3}{5} + 1\frac{4}{5} = \)
   c) \(3\frac{7}{8} - 2\frac{4}{8} = \)
   d) \(5\frac{1}{3} - 1\frac{2}{3} = \)

8. Multiply.
   a) \(4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \)
   b) \(5 \times \frac{2}{9} = \)

9. Six friends are picking strawberries. Each person carries a small basket that fits \(\frac{5}{8}\) of a pound of strawberries.
   a) If everyone fills their basket, how many pounds of strawberries can they pick altogether?
   b) What two whole numbers is your answer between?
       ________ and ________

10. Four people each picked \(\frac{2}{9}\) of a pound of cherries. Did they pick more or less than a pound of cherries altogether? How do you know?
11. A shelf can hold a maximum of exactly 5 fat books or a maximum of exactly 8 skinny books, as shown in the picture below.

![Diagram of shelf with fat and skinny books]

a) Can 3 fat books and 4 skinny books fit on one shelf? _________

b) Is $\frac{3}{5} + \frac{4}{8}$ greater than 1? How do you know?

**BONUS** ► Add.

$$\frac{1}{11} + \frac{2}{11} + \frac{3}{11} + \frac{4}{11} =$$
Unit 4: Number Operations—Fractions

Test (Lessons 1 to 18)

ADVANCED

12. a) Match the fraction with the description:

A. \( \frac{18}{15} \)  
B. \( \frac{16}{32} \)  
C. \( \frac{3}{8} \)  
D. \( \frac{3}{5} \)  
E. \( \frac{9}{9} \)

Less than \( \frac{1}{2} \) ______

Equal to \( \frac{1}{2} \) ______

More than \( \frac{1}{2} \) and less than 1 ______

Equal to 1 ______

More than 1 ______

b) Arrange the fractions from least to greatest:

\( \frac{3}{5}, \frac{3}{8}, \frac{9}{9}, \frac{16}{32}, \frac{18}{15} \)

<  <  <  <

13. Tia and Luciano drew two different diagrams to compare \( \frac{5}{8} \) and \( \frac{3}{5} \). Which diagram is better to compare the fractions? Explain.

Tia's diagram:

Luciano's diagram:
1. \( \frac{6}{9} \)
2. a) <  
   b) >  
   c) =  
   d) <  
   **BONUS**  
   =
3. \( \frac{3}{5} + \frac{4}{5} = \frac{7}{5} \)
4. \( \frac{23}{4} \)
5. \( \frac{2\frac{2}{3}}{} \)
6. a) \( \frac{5}{3} \)
   b) \( \frac{7}{4} \)
   c) \( \frac{11}{8} \)
   d) \( \frac{8}{7} \)
7. a) \( \frac{3\frac{4}{5}}{} \)
   b) \( \frac{4\frac{2}{5}}{} \)
   c) \( \frac{1\frac{3}{8}}{} \)
   d) \( \frac{3\frac{2}{3}}{} \)
8. a) \( \frac{12}{5} \)
   b) \( \frac{10}{9} \)
9. a) \( \frac{30}{8} \)
   b) 3 and 4
10. They picked \( \frac{8}{9} \) of a pound, which is less than 1 pound, because \( \frac{8}{9} < 1 \).
11. a) No.  
    b) Yes, because the books fill up more than one whole shelf.

**BONUS**

10  
11

**ADVANCED**

12 a) C, B, D, E, A  
    b) \( \frac{3}{8} < \frac{16}{32} < \frac{3}{5} \)  
    \( \frac{9}{9} < \frac{18}{15} \)
13 Luciano’s is better, because he drew the wholes the same length, (so the one with more shaded, \( \frac{5}{8} \), shows the bigger fraction.)
1. Circle the correct mass or capacity for the item.
   a) 5 g or 5 kg
   b) 30 oz or 30 lb
   c) 200 mL or 200 L
   d) 355 mL or 355 L

2. Convert the measurement in grams to a mixed measurement.
   a) 7,128 g = _____ kg ______ g
   b) 45,037 g = _____ kg ______ g

3. Convert the measurement in pounds to a measurement in ounces.
   a) 2 lb = _______________ oz
   b) 100 lb = _______________ oz

4. Convert the mixed measurement to a measurement in grams.
   a) 4 kg 135 g
   b) 5 kg 23 g

5. Convert the mixed measurement to a measurement in millilitres.
   a) 3 L 243 mL
   b) 8 L 7 mL

7. Walter changes the water in the fish tanks at the pet store where he works. The volume of water he changes each day is shown in the chart:
   a) What is the total volume of water measured in mL? __________
   b) What is the total volume of water measured in L? _______

<table>
<thead>
<tr>
<th>Day</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>250 mL</td>
</tr>
<tr>
<td>Tuesday</td>
<td>500 mL</td>
</tr>
<tr>
<td>Wednesday</td>
<td>250 mL</td>
</tr>
<tr>
<td>Thursday</td>
<td>500 mL</td>
</tr>
<tr>
<td>Friday</td>
<td>500 mL</td>
</tr>
<tr>
<td>Total</td>
<td>60 L</td>
</tr>
</tbody>
</table>

8. A gym orders 80 barbells for a weight-lifting class. Each barbell weighs \(\frac{3}{4}\) lb. What is the total weight of the barbells?

9. A 3 L pail is used to pour water into a 60 L tub. How many pails are needed to fill the tub?

10. Jane has a one-liter bottle of milk. If she drinks 340 mL, how much milk is left in the bottle?

11. A box contains chocolate bars having a total mass of 800 g. The box can hold a maximum of 1 kg of chocolate bars.
   a) What mass of chocolate bars is needed to fill the box?
   b) One chocolate bar has a mass of 40 g. How many more chocolate bars are needed to fill the box?

BONUS ►
One milliliter of water has a mass of one gram. Find the mass of water in a container that holds 3 L 250 mL of water.
1. a) 5 g  
   b) 30 lb  
   c) 200 L  
   d) 355 mL  
2. a) 7 kg 128 g  
   b) 45 kg 37 g  
3. a) 32 oz  
   b) 1,600 oz  
4. a) 4,135 g  
   b) 5,023 g  
5. a) 3,243 mL  
   b) 8,007 mL  
6. Ranjit  131 oz  
   Rohan  8 × 16 = 128 oz  
   Ranjit is heavier.  
7. a) 2,000 mL  
   b) 2 L  
8. 60 lb  
9. 20  
10. 660 mL  
11. a) 200 g  
    b) 5 bars  
   BONUS  
   3,250 g
1. Find the number of pennies or nickels you need to make the amount.
   a) 19¢
      | Nickels | Pennies |
      | 0       | 1       |
      | 2       | 3       |
   b) 35¢
      | Dimes   | Nickels |
      | 0       | 1       |
      | 2       | 3       |

2. Find the number of pennies, nickels, dimes, or quarters you need to make the amount.
   Hint: You may not need to use all of the rows.
   a) 43¢
      | Dimes | Pennies |
      |       |         |
      |       |         |
      |       |         |
   b) 55¢
      | Quarters | Nickels |
      |          |         |
      |          |         |
      |          |         |

3. A ladybug has 6 legs. A cat has 4 legs. Three of these creatures have a total of 16 legs. How many of each creature are there?

<table>
<thead>
<tr>
<th>Ladybugs</th>
<th>Cats</th>
<th>Total Number of Legs</th>
</tr>
</thead>
</table>

4. Skip count to check for factors.
   a) Is 4 a factor of 14? ______, _____, _____, _____, _____, ____
   b) Is 3 a factor of 15? _____, _____, _____, _____, _____, ____
   c) Is 30 a factor of 80? _____, _____, _____, _____, _____, ____
Unit 6: Operations and Algebraic Thinking

Test (Lessons 39 to 44)

5. Divide to check for factors.
   a) \(11 \div 2 = \) _____ R _____ Is 2 a factor of 11? _____
   b) \(24 \div 6 = \) _____ R _____ Is 6 a factor of 24? _____

6. Find all the factor pairs of the number. Stop when you get a number that is already part of a pair. There might be more rows than you need.
   a) 16
      | First Factor | Second Factor |
      |--------------|--------------|
      |              |              |
      |              |              |
      |              |              |
      |              |              |
   b) 24
      | First Factor | Second Factor |
      |--------------|--------------|
      |              |              |
      |              |              |
      |              |              |
      |              |              |

7. Circle the prime numbers. 3, 4, 5, 7, 8, 11, 12, 13, 15, 19, 21, 23, 25

8. Is 25 a multiple of 6? _____
   Skip count to check: _____, _____, _____, _____, _____, _____

9. Use long division to check for multiples.
   a) 82
      \[\begin{array}{c|c|c|c|c}
      \hline
      3 & 8 & 2 \\
      \hline
      \hline
      \hline
      \hline
      \end{array}\]
   Is 82 a multiple of 3? _____

   b) 68
      \[\begin{array}{c|c|c|c|c}
      \hline
      4 & 6 & 8 \\
      \hline
      \hline
      \hline
      \hline
      \end{array}\]
   Is 68 a multiple of 4? _____

**BONUS**
Find the number between 20 and 30 which is a multiple of both 6 and 8.
ADVANCED

Thomas believes all composite numbers are even. Circle the first number in the list which shows his claim is not true: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21
### Unit 6: Operations and Algebraic Thinking

#### Test (Lessons 39 to 44)

1. a) Nickels | Pennies
   0 | 19
   1 | 14
   2 | 9
   3 | 4

   b) Dimes | Nickels
   0 | 7
   1 | 5
   2 | 3
   3 | 1

2. a) Dimes | Pennies
   0 | 43
   1 | 33
   2 | 23
   3 | 13
   4 | 3

   b) Quarters | Nickels
   0 | 11
   1 | 6
   2 | 1

3. 2 ladybugs, 1 cat

4. a) no
   b) yes
   c) no

5. a) no; $11 \div 2 = 5 \text{ R } 1$
   b) yes; $24 \div 6 = 4 \text{ R } 0$

6. a) 1st factor | 2nd factor
   1 | 16
   2 | 8
   4 | 4

   b) 1st factor | 2nd factor
   1 | 24
   2 | 12
   3 | 8
   4 | 6

7. 3, 5, 7, 11, 13, 19, 23

8. no

9. a) No; $82 \div 3 = 27 \text{ R } 1$
   b) Yes; $68 \div 4 = 17$

#### BONUS

24

#### ADVANCED

9
1. Complete the table by writing in cent or dollar notation.

<table>
<thead>
<tr>
<th>Cent Notation</th>
<th>Dollar Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>312¢</td>
<td>$0.76</td>
</tr>
</tbody>
</table>

2. Write the fraction that the picture shows. Then write the decimal.

a)

Fraction: 
Decimal: 

b)

Fraction: 
Decimal: 

c)

Fraction: 
Decimal: 

3. Show the decimals on the number line.

A. 0.80  B. 0.45  C. 0.09

4. Write the decimal as a fraction with denominator 10, then as a fraction with denominator 100.

a) 0.5  b) 0.8

BONUS

Write the fraction that the picture shows. Then write the decimal for it.

Fraction: 
Decimal:  

Name: ________________________
Date: ________________
Unit 7: Number and Operations—Fractions

Quiz (Lessons 19 to 23)

1. a) $3.12
   b) 76¢

2. a) \(\frac{3}{10}, 0.3\)
   b) \(\frac{6}{10}, 0.6\)
   c) \(\frac{50}{100}, 0.50\)

3. 

   

4. a) \(\frac{5}{10}, \frac{50}{100}\)
   b) \(\frac{8}{10}, \frac{80}{100}\)

BONUS

   \(\frac{64}{100}, 0.64\)
1. For points A and B, write how many tenths and how many hundredths.

   [Number line diagram]

   A. _______ tenths  _______ tenths
   B. _______ hundredths  _______ hundredths

2. Estimate and mark the location of the decimals on the number line.

   A. 0.73  B. 0.08  C. 0.39

   [Number line from 0 to 1]

3. What number is greater? Write < or >. Then explain how you know.

   0.4  ____  0.38

4. Add the fractions.

   a) \( \frac{8}{10} + \frac{7}{100} = \)  
   b) \( \frac{4}{10} + \frac{9}{100} = \)  
   BONUS \( \frac{3}{10} + \frac{21}{100} = \)

5. Write the equivalent decimal or words.

   a) forty-three and twenty-seven hundredths
   b) 11.6

   BONUS \( 8,000.05 \)
6. Complete the table.

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Mixed Number</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td>$32\frac{7}{10}$</td>
</tr>
<tr>
<td>b)</td>
<td>4.21</td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td>$\frac{603}{100}$</td>
</tr>
</tbody>
</table>

7. Tyrell spent $\frac{3}{10}$ of a dollar on marbles and $\frac{4}{10}$ of a dollar on candy. What fraction of a dollar did he spend altogether?

8. Describe the decimal in two ways.
   a) $0.83 = \_\_\_\_ \text{tenths} \_\_\_\_ \text{hundredths}$
   b) $0.04 = \_\_\_\_ \text{tenths} \_\_\_\_ \text{hundredths}$

9. Circle the equalities that are incorrect.

   $0.3 = \frac{3}{100}$
   $0.27 = \frac{27}{100}$
   $0.05 = \frac{5}{100}$
   $0.07 = \frac{7}{10}$

BONUS ▶ Add. Hint: change the decimals to fractions.

   $0.3 + 0.5 =$
10. Kamali draws a model of 0.35 and a model of 0.2. She then writes 0.35 < 0.2. Explain Kamali’s mistake.

Kamali’s models

0.35

0.2
1. **A.** 2 tenths, 20 hundredths  
   **B.** 6 tenths, 60 hundredths

2.  

   \[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1]\n
   A\n
3. 0.4 > 0.38 because 0.4 is 0.40 (40 hundredths) and 40 hundredths are larger than 38 hundredths.

4.  
   a) \[
   \frac{87}{100}
   \]
   b) \[
   \frac{49}{100}
   \]

   **BONUS**  
   \[
   \frac{51}{100}
   \]

5.  
   a) 43.27
   b) Eleven and six tenths  

   **BONUS**  
   Eight thousand and five hundredths

6.  

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal</td>
<td>Mixed Number</td>
<td>Improper Fraction</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
   | a) 32.7 | 32 \[
   \frac{7}{10}
   \] | 327 \[
   \frac{10}{10}
   \] |
   | b) 4.21 | 4 \[
   \frac{21}{100}
   \] | 421 \[
   \frac{100}{100}
   \] |
   | c) 6.03 | 6 \[
   \frac{3}{100}
   \] | 603 \[
   \frac{100}{100}
   \] |

7.  

   \[
   \frac{7}{10}
   \]

8.  
   a) 8 tenths 3 hundredths, 83 hundredths
   b) 0 tenths 4 hundredths

9.  

   \[
   0.3 = \frac{3}{100}, 0.07 = \frac{7}{10}
   \]

   **BONUS**  
   0.8

10. Kamali’s models are not the same size, so 0.2 appears to be bigger than 0.35, but 0.35 is larger than 0.2.
1. Measure the length of the object.
   a) [Image of a screwdriver] ___________ in  
   b) [Image of a key] ___________ in

2. Circle the mark for the mixed number or fraction on the ruler.
   a) \( \frac{5}{4} \)  
   b) \( \frac{1}{4} \)

3. Measure the length of the line segment.
   a) [Image of a line segment] ___________ in  
   b) [Image of another line segment] ___________ in

4. Measure each side of the rectangle shown to the nearest \( \frac{1}{4} \) inch. Then find the perimeter.
   length = ___________ in  
   width = ___________ in

   Perimeter =
Unit 8: Measurement and Data

Quiz (Lessons 25 to 29)

5. Convert from feet to inches.
   
   a) 2 feet = _____ inches  
   b) 7 feet = _____ inches

6. Gertrude is 4 feet tall. Her friend Gabby is 51 inches tall. How much taller is Gabby?

7. Wajid measured the length of a pencil to be approximately 2 1/2 inches as shown in the diagram. Explain why he is incorrect.

BONUS ▶️

On some rulers, an inch is divided into 16 equal parts. Circle the mark for the measurement 1 3/16 inches.
Unit 8: Measurement and Data

Quiz (Lessons 25 to 29)

1. a) 3 in  
   b) 2 in  

2. a) 

   b) 

3. a) \(\frac{3}{4}\) in  
   b) \(\frac{7}{8}\) in  

4. Length = \(2\frac{3}{4}\) in  
   Width = \(1\frac{1}{4}\) in  
   
   \(P = \frac{1}{4} + \frac{1}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}\)  
   = 8

5. a) 24 in  
   b) 84 in  

6. 3 in  

7. The pencil is not placed at the zero mark on the ruler.

BONUS

```
0   1   2
```

Unit 8: Measurement and Data
Test (Lessons 25 to 32)

Name: ________________________ Date: __________________

1. Circle the unit that is more appropriate for measuring the length of the object.
   a) __________ b) __________ c) __________
      inch foot   inch foot   inch foot

2. Draw a second arrow to show where a line segment of given length would end.
   a) 2 inches
      __________
      0 inch 1 2
   b) 1 3/4 inches
      __________
      0 inch 1 2
   c) 1 1/2 inches
      __________
      0 inch 1 2
   d) 2 3/8 inches
      __________
      0 inch 1 2

3. Use a ruler to find the lengths of the sides of the pentagon to the nearest 1/8 inch.
   Then find the perimeter.

4. Convert the measurement to the indicated units.
   a) 2 ft = _______ in     b) 4 yd = ______ ft     c) 3 ft 8 in = _____ in + _____ in
      = ______ in
   BONUS ► d) 100 ft = _______ in     e) 1,000 yd = _______ ft
Unit 8: Measurement and Data

Test (Lessons 25 to 32)

5. Complete the table.

<table>
<thead>
<tr>
<th></th>
<th>1 ft</th>
<th>2 ft</th>
<th>3 ft</th>
<th>4 ft</th>
<th>5 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. For the measurement in inches, write the pair of measurements in feet or yards that it is between. Hint: Use the tables in Question 5.

a) _____ ft < 32 in < _____ ft

b) _____ yd < 13 ft < _____ yd

7. Freddie needs 4 yards of chain, but the hardware store only has 10 feet in stock. How many more feet of chain does Freddie need?

8. A rope is 14 yards long. It needs to be divided into 6 equal parts.

   a) How long is the rope when measured in feet? _______________________

   b) How long should each of the equal parts be? _______________________

9. A snail crawled a few inches each day as shown in the table. What is the total distance traveled for the week when measured in feet?

<table>
<thead>
<tr>
<th>Day</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>15 in</td>
</tr>
<tr>
<td>Tuesday</td>
<td>38 in</td>
</tr>
<tr>
<td>Wednesday</td>
<td>19 in</td>
</tr>
<tr>
<td>Thursday</td>
<td>17 in</td>
</tr>
<tr>
<td>Friday</td>
<td>31 in</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

10. When a giraffe is born, it is already 6 feet tall. Baby giraffes can grow 2 inches in a single day. If this growth pattern continues for 12 days, how tall will the giraffe be when measured in feet?

BONUS ▶

Place the measurements in order from least to greatest: 2 yards, 5 feet, 48 inches. Hint: convert them all to a common unit of measurement first.

_________ < _________ < ___________
1. a) foot  
b) inch  
c) foot  
2. a)  
   
   
   
   b)  
   
   
   
   c)  
   
   
   
   d)  
   
   
   
   3. Sides: 1 in, $\frac{5}{8}$ in, $\frac{7}{8}$ in, $\frac{9}{8}$ in, $2\frac{1}{8}$ in, $2\frac{3}{8}$ in  

   Perimeter = 7 in  

4. a) 24 in  
   b) 12 ft  
   c) 44 in  

   BONUS  
   d) 1,200 in  
   e) 3,000 ft  

5. ft: 1 2 3 4 5  

   in: 12 24 36 48 60  

   yd: 1 2 3 4 5  

   ft: 3 6 9 12 15  

6. a) 2 ft < 32 in < 3 ft  
   b) 4 yd < 13 ft < 5 yd  

7. 2 ft  

8. a) 42 ft  
   b) 7 ft  

9. Total = 120 in  
   = 10 ft  

10. Height = 8 ft  

   BONUS 48 in < 5 ft < 2 yd
Unit 8: Measurement and Data

Quiz (Lessons 33 to 37)

Name: ________________________  Date: ________________

1. Find the area of the figure in square centimeters.
   a) 
   b) ___ cm² ___ cm²

2. Find the area of the rectangles in square centimeters.

   A. Area = ______ cm²
   B. Area = ______ cm²
   C. Area = ______ cm²

3. Write the number of boxes along the length and width for each rectangle. Then write a multiplication equation for the area of the rectangle measure in square units.
   a) 
   b) 

   Length = _____ units  Width = _____ units  Area = _____ × _____ = _____ sq. units
   Length = _____ units  Width = _____ units  Area = _____ × _____ = _____ sq. units
4. Ingrid has a rectangle with width 3 in and length 4 in. Marek has a rectangle with width 4 in and length 3 in. Do the rectangles have the same area? Explain.

5. A rectangle with area 24 cm² has a width of 4 cm. What is the length of the rectangle?

6. Luigi's vegetable garden has length 20 ft and width 10 ft.
   a) What is the area of the garden?
      \[ \text{Area} = \text{___ ft} \times \text{___ ft} = \text{___ ft}^2 \]
   b) If Luigi changes the width to 15 ft, what is the new area of the garden?
      \[ \text{Area} = \text{___ ft} \times \text{___ ft} = \text{___ ft}^2 \]
   c) How many square feet of area would Luigi add by changing the width to 15 ft?

7. Saul’s backyard is in the shape of a square with each side having a length of 30 m.
   a) Saul wants to fence in 3 sides of his yard. How many meters of fencing will he need?
      _______________________
   b) Fencing costs $100 per meter. What will the cost of the fence be? _______________________

BONUS

Find the lengths and widths of 3 different rectangles which will have an area of 12 yd².

<table>
<thead>
<tr>
<th>Length (yd)</th>
<th>Width (yd)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unit 8: Measurement and Data

Quiz (Lessons 33 to 37)

1. a) 10 cm²
   b) 11 cm²
2. a) 15 cm²
   b) 3 cm²
   c) 8 cm²
3. a) 8 sq. units
   b) 21 sq. units
4. The rectangles have the same area since $4 \times 3 = 3 \times 4$
5. Length = $24 / 4 = 6$ cm
6. a) 200 ft²
   b) 300 ft²
   c) 100 ft²
7. a) 90 m
   b) $9,000$

BONUS
L = 1 yd  W = 12 yd
L = 2 yd  W = 6 yd
L = 3 yd  W = 4 yd
1. Find the area of the figure in square centimeters.
   a) 
   b) 
   ____ cm²

2. Write a multiplication statement that tells you how many squares are in the rectangle.
   a) 
   b) 
   ___________________  ___________________

3. Find the area of the rectangle. Include the units.
   a) 
   b) 
   ________  ________

4. A rectangle has area 28 ft² and length 7 ft. Find the width of the rectangle. Explain your answer.
5. Measure the length and width of the rectangle to the nearest centimeter. Find the perimeter and area for each rectangle.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. In gym class, students jogged for 15 minutes. The distances run are shown in the table.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>1$\frac{1}{8}$</td>
<td>1$\frac{2}{8}$</td>
<td>1$\frac{1}{8}$</td>
<td>1$\frac{1}{8}$</td>
<td>1$\frac{1}{8}$</td>
<td>1$\frac{2}{8}$</td>
<td>1$\frac{2}{8}$</td>
<td>1$\frac{5}{8}$</td>
<td>1$\frac{6}{8}$</td>
<td>1$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

a) Complete the number line on the line plot.

b) Build a line plot with the data.
c) How much farther was the longest distance run than the shortest distance? _______

**BONUS** A rectangle with length 3 cm has an area of 12 cm$^2$. Find the perimeter of the rectangle in centimeters.
Unit 8: Measurement and Data

Test (Lessons 33 to 40)

ADVANCED

7. A room has the shape as shown in the diagram.
   a) Draw a line to form two rectangles.
   b) Find the area of each rectangle.
      Area 1 = _____ m \times _____ m = _____ m^2
      Area 2 = _____ m \times _____ m = _____ m^2
   c) Find the total area of the room.
   d) Carpeting costs $10 per square meter. What is the total cost of carpeting the room?

8. The distance of a horse race is measured in miles.
   The line plot shows lengths of different horse races.
   a) Rewrite the lengths of the horse races so that the denominators are all the same.
   b) What distance is the most common for a horse race?
   c) Charlie the horse only ran the three longest races last year. What was the total distance Charlie ran?
Unit 8: Measurement and Data

Test (Lessons 33 to 40)

1. a) 13 cm²
   b) 15 cm²

2. a) 3 \times 4 = 12
   b) 8 \times 3 = 24

3. a) 6 m²
   b) 8 ft²

4. Divide the area by the length. Width = \frac{28}{7} = 4 ft

5. A \quad P = 8 + 2 + 8 + 2 = 20 cm
   \quad A = 8 \times 2 = 16 cm²
   B \quad P = 3 + 3 + 3 + 3 = 12 cm
   \quad A = 3 \times 3 = 9 cm²

6. a), b)

```
+---+---+---+---+---+---+---+---+
| x | x | x | x | x | x | x | x |
+---+---+---+---+---+---+---+---+
```

6. c) \frac{5}{8} of a mile

BONUS \quad P = 14 cm

ADVANCED

7. b) \quad \text{Area 1} = 10 m²
   \quad \text{Area 2} = 6 m²
   c) \quad \text{Area} = 16 m²
   d) \quad $160

8. a) \[
\begin{array}{ccccccc}
9 & 10 & 11 & 12 & 13 & 14 \\
8 & 8 & 8 & 8 & 8 & 8 \\
\end{array}
\]

8. b) \frac{5}{4} mi

8. c) \frac{40}{8} mi or 5 mi
Unit 9: Geometry

Quiz (Lessons 14 to 17)

1. Identify the picture as a line, line segment, or ray.
   a) ____________________  
   b) ____________________  
   c) ____________________

2. Extend the lines and rays. Circle the intersection points.
   a) ____________________  
   b) ____________________  
   c) ____________________

3. Identify the angle as acute or obtuse.
   a) ____________________  
   b) ____________________  
   c) ____________________  
   d) ____________________

4. Identify the angle as acute or obtuse. Then write the measure of the angle.
   a) ____________________  
   b) ____________________

5. Finish drawing the angle.
   a) 60°
   b) 140°
Unit 9: Geometry

Quiz (Lessons 14 to 17)

1. a) ray
   b) line segment
   c) line

2. a) ![Diagram A]
   b) ![Diagram B]
   c) ![Diagram C]

3. a) obtuse
   b) acute
   c) obtuse
   d) acute

4. a) acute, $50^\circ$
   b) obtuse, $120^\circ$

5. a) ![Diagram D]
   b) ![Diagram E]
1. Mark each right angle in the shape with a small square, each acute angle with an A, and each obtuse angle with an O.

   a) ![Diagram](image1)
   
   b) ![Diagram](image2)
   
   c) ![Diagram](image3)

2. Extend the line or ray where possible. Name the points that are on the line or ray.

   a) ![Diagram](image4)
   
   b) ![Diagram](image5)
   
   c) ![Diagram](image6)

3. Finish drawing the angle.

   a) ![Diagram](image7)
   
   b) ![Diagram](image8)

4. Fill in the blanks.

   a) ![Diagram](image9)

   What fraction of the circle is shaded? _____  What fraction of the circle is shaded? _____

   What is the angle? $\square$ of $360^\circ$= ____

   What is the angle? $\square$ of $360^\circ$= ____
5. a) Mark acute angles with an A, obtuse angles with O, and right angles with a small square.

![Diagram with different shapes marked]

b) Sort the shapes into the Venn diagram below.

![Venn diagram with categories: At least one right angle and At least one acute angle]

6. In the picture the whole angle is 140°. Andrew says that the angle $x$ is $140° - 130° = 10°$. Explain Andrew’s mistake.

![Diagram with angles 140°, 100°, and 30°]

7. Find the measure of the whole angle by adding the measures of the small angles.

a) $\text{the whole angle} = \phantom{123} + \phantom{123} = \phantom{123}$

b) $\text{the whole angle} = \phantom{123} + \phantom{123} = \phantom{123}$

8. A clock’s hour hand turns 1° every 2 minutes.

a) How many degrees has it turned in 20 minutes? ______

b) How many minutes does it take to turn 60 degrees? ______
   How many hours is that? ______

**BONUS** ▶ How many minutes will it take for an hour hand to turn 180 degrees? ______
   How many hours is that? ______

<table>
<thead>
<tr>
<th>Hours</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
ADVANCED

9. a) Write an equation to find $x$.
   i) The whole angle is $130^\circ$.
   ![Diagram]

   b) Solve your equations from part a) to find $x$.
   i) 

ii) The whole angle is $111^\circ$.
   ![Diagram]

   i) 

10. a) Write an equation to find $x$.
   i) The whole angle is $170^\circ$.
   ![Diagram]

   b) Solve your equations from part a) to find $x$.
   i) 

ii) The whole angle is $160^\circ$.
   ![Diagram]

   i) 

BONUS ▶ Write and solve an equation to find $x$.
The whole angle is $130^\circ$.
![Diagram]
1. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]

2. a) 
   ![Diagram]
   b) 
   ![Diagram]
   c) 
   ![Diagram]

3. a) 
   ![Diagram]
   b) 
   ![Diagram]

4. a) \( \frac{2}{5}, 120^\circ \)
   b) \( \frac{1}{5}, 72^\circ \)

5. a) 
   ![Diagram]
   b) 
   ![Diagram]

6. Andrew should only have subtracted 100°. The 100° and 30° angles overlap.

7. a) 90°
   b) 150°

8. a) 10°
   b) 120 minutes
   2 hours (use the table to skip count the number of minutes in 2 hours)
   
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>360</td>
</tr>
<tr>
<td>7</td>
<td>420</td>
</tr>
</tbody>
</table>

   BONUS 360 minutes
   6 hours

ADVANCED

9. a) i) \( x = 130^\circ - 50^\circ \)
    ii) \( x = 111^\circ - 31^\circ \)
   b) i) \( x = 80^\circ \)
    ii) \( x = 80^\circ \)

10. a) i) \( x = 170^\circ - 55^\circ - 45^\circ \)
    ii) \( x = 160^\circ - 40^\circ - 50^\circ \)
   b) i) \( x = 70^\circ \)
    ii) \( x = 70^\circ \)

BONUS \( x = 130^\circ - 90^\circ \)
   \( x = 40^\circ \)
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 1: Operations and Algebraic Thinking

**Quiz (Lessons 26 to 27), p. W-1**

**Common Core State Standards Emphasized:** 4.OA.C.5

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>15, 20, 25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>The pattern in the ones is 0, then 5, then repeat.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles both correct answers</td>
<td>520, 615</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles one correct answer</td>
<td>(0.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Gives correct gaps</td>
<td>–10, –9, –8, –7, –6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly extends sequence</td>
<td>66, 60</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>a) Gives correct answer</td>
<td>3 6</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>18</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>12 + 3 + 3 = 18</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /5
### Rubric for Unit 1: Operations and Algebraic Thinking
**Quiz (Lessons 26 to 27), p. W-1**

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.C.5</td>
<td></td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

1, 2, Bonus

### Comments

Name: ____________________
### Scoring Guides for Sample Unit Quizzes and Tests

#### Unit 2: Number and Operations in Base Ten

**Quiz (Lessons 40 to 43), p. W-3**

**Common Core State Standards Emphasized:** 4.NBT.B.6

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) i) Draws correct picture</td>
<td><img src="image" alt="Picture" /></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ii) Draws correct picture</td>
<td><img src="image" alt="Picture" /></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) i) Gives correct answer</td>
<td>4, 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>$11 \div 3 = 3 \text{ R } 2$</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>2</td>
<td>Correctly uses the number line</td>
<td><img src="image" alt="Number Line" /></td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>2, 2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>2 R 3</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>5 R 2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives a correct division equation</td>
<td>$8 \div 2 = 3 \text{ R } 2$ or $8 \div 3 = 2 \text{ R } 2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives a correct multiplication and addition equation</td>
<td>$(2 \times 3) + 2 = 8$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives a correct division equation</td>
<td>$7 \div 3 = 2 \text{ R } 1$ or $7 \div 2 = 3 \text{ R } 1$</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td>Gives a correct multiplication and addition equation</td>
<td>$(3 \times 2) + 1 = 7$</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>5</td>
<td>Correctly explains mistake</td>
<td>The remainder is bigger than the size of the groups. Another group of 2 could be made.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>$13 \div 2 = 6 \text{ R } 1$</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>6</td>
<td>Gives correct answer</td>
<td>20,000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>40,000,000</td>
<td>yes / no</td>
<td>1</td>
</tr>
</tbody>
</table>

**Total Points** | /15
## Scoring Guides for Sample Unit Quizzes and Tests

### Unit 2: Number and Operations in Base Ten

(continued)

**Quiz (Lessons 44 to 48), p. W-5**

**Common Core State Standards Emphasized:** 4.NBT.A.1, 4.NBT.B.6, 4.MD.A.2

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1 a)      | Correctly carries out all steps | $\begin{array}{c}
7 \\
3 \\
5 \\
2 \\
- \\
\hline \\
9 \\
3 \\
5 \\
2 \\
- \\
\hline \\
2 \\
1 & 2 \end{array}$ | 2 | (1) |
| 1 b)      | Correctly carries out all steps | $\begin{array}{c}
7 \\
3 \\
5 \\
2 \\
- \\
\hline \\
9 \\
3 \\
5 \\
2 \\
- \\
\hline \\
2 & 2 \end{array}$ | 2 | (1) |
| 1 c)      | Correctly carries out all steps | $\begin{array}{c}
7 \\
3 \\
5 \\
2 \\
- \\
\hline \\
9 \\
3 \\
5 \\
2 \\
- \\
\hline \\
2 & 2 \end{array}$ | 2 | (1) |
| 1 d)      | Correctly carries out all steps | $\begin{array}{c}
7 \\
3 \\
5 \\
2 \\
- \\
\hline \\
9 \\
3 \\
5 \\
2 \\
- \\
\hline \\
2 & 2 \end{array}$ | 2 | (1) |
| 2 a)      | Correctly carries out all steps | $\begin{array}{c}
4 \\
3 \\
1 \\
- \\
\hline \\
8 \\
7 \\
1 \\
- \\
\hline \\
2 & 2 \end{array}$ | 2 | (1) |
| 2 b)      | Correctly carries out all steps | $\begin{array}{c}
4 \\
3 \\
1 \\
- \\
\hline \\
8 \\
7 \\
1 \\
- \\
\hline \\
2 & 2 \end{array}$ | 2 | (1) |
| 3         | Gives correct answer with units | 149 ft | 1 | /1 |
| 4 a)      | Gives correct answer | 32 | 1 | |
| 4 b)      | Gives correct answer | 21 | 1 | |
| 4 c)      | Gives correct answer | 21 | 1 | |
| 4 d)      | Gives correct answer | 412 | 1 | |
| Bonus: e) | Gives correct answer | 12,322 | yes / no | |
| Bonus: f) | Gives correct answer | 102,244 | yes / no | /4 |

**Total Points** /17
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 2: Number and Operations in Base Ten

**Test (Lessons 40 to 51), p. W-7**

**Common Core State Standards Emphasized:** 4.NBT.A.1, 4.NBT.B.6, 4.MD.A.2, 4.OA.A.3, 4.OA.C.5

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>4 R 4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>3 R 0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>6 R 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>7 R 2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>3 R 0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>10 R 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>23</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>12</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>34</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>311</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) i) Draws correct picture</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Draws correct picture</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) i) Gives correct answer</td>
<td>3, 1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>11 ÷ 4 = 2 R 3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Correctly carries out all steps</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly carries out all steps</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly carries out all steps</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Correctly carries out all steps</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Makes one computational error</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 2: Number and Operations in Base Ten**

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5</strong></td>
<td>Gives correct answer</td>
<td>$22</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>a) Gives correct division equation</td>
<td>$11 ÷ 2 = 5 R 1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct division equation</td>
<td>$17 ÷ 3 = 5 R 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct division equation</td>
<td>$14 ÷ 4 = 3 R 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>7</strong></td>
<td>a) i) Gives correct answer</td>
<td>R Y R Y</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>B R Y</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) i) Gives correct division</td>
<td>$20 ÷ 2 = 10 R 0$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct division</td>
<td>$20 ÷ 3 = 6 R 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>R</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bonus</strong></td>
<td>Gives correct division</td>
<td>$201 ÷ 4 = 50 R 1$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>R</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td>a) Gives correct answer</td>
<td>4, 4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>8</strong></td>
<td>b) Gives correct answer</td>
<td>3, 3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td>a) Gives correct answer</td>
<td>25, 25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>9</strong></td>
<td>b) Gives correct answer</td>
<td>$12, 60 ÷ 5 = 12$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td>a) Correctly divides the parts</td>
<td>35, 3</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly divides the parts</td>
<td>20, 6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>26</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td>a) Gives correct division</td>
<td>$20 ÷ 3 = 6 R 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>6, $2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct division</td>
<td>$17 ÷ 3 = 5 R 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>6, 2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /35
Rubric for Unit 2: Number and Operations in Base Ten
Test (Lessons 40 to 51), p. W-7

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and</td>
<td>1, 2, 3, 4, 8 (Advanced),</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately</td>
</tr>
<tr>
<td>one-digit divisors, using strategies based on place value, the properties of operations,</td>
<td>9 (Advanced), 10 (Advanced)</td>
<td></td>
<td></td>
<td></td>
<td>and independently.</td>
</tr>
<tr>
<td>and/or the relationship between multiplication and division. Illustrate and explain the</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>calculation by using equations, rectangular arrays, and/or area models.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.MD.A.2 Use the four operations to solve word problems involving distances, intervals of</td>
<td>5, 6.a), 11.a) (Advanced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>time, liquid volumes, masses of objects, and money, including problems involving simple</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fractions or decimals, and problems that require expressing measurements given in a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>larger unit in terms of a smaller unit. Represent measurement quantities using diagrams</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>such as number line diagrams that feature a measurement scale.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number</td>
<td>6.b), 6.c), 11.b) (Advanced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>answers using the four operations, including problems in which remainders must be</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interpreted. Represent these problems using equations with a letter standing for the</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unknown quantity. Assess the reasonableness of answers using mental computation and</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>estimation strategies including rounding.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Name: ____________________
## Rubric for Unit 2: Number and Operations in Base Ten
### Test (Lessons 40 to 51), p. W-7

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</td>
<td>7, Bonus</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

### Comments

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### Scoring Guides for Sample Unit Quizzes and Tests

#### Unit 3: Operations and Algebraic Thinking

**Quiz (Lessons 28 to 33), p. W-11**

Common Core State Standards Emphasized: 4.OA.A.3

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>$\Box = 5$</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$w = 3$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct equation</td>
<td>$25 - 11 = w$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$w = 14$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct equation</td>
<td>$12 - 7 = w$</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$w = 5$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Correctly finds number of stickers with plants</td>
<td>$14 - 5 = 9$</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>Correctly finds total number of stickers</td>
<td>$14 + 9 = 23$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bonus</td>
<td>Correctly finds number of nickels</td>
<td>$40 - 18 - 15 = 7$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly finds how many more dimes than nickels</td>
<td>$15 - 7 = 8$</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

| Total Points | /8 |
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 3: Operations and Algebraic Thinking

(continued)

<table>
<thead>
<tr>
<th>Test (Lessons 28 to 38), p. W-13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Core State Standards Emphasized: 4.OA.A.1, 4.OA.A.2, 4.OA.A.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>( w = 26 - 7 ) ( w = 19 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>( b = 27 + 3 ) ( b = 9 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Gives correct equation</td>
<td>( 14 + 7 = w ) ( w = 21 )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Draws a correct model</td>
<td>Lina is 8 and Jordan is 2.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws a correct model</td>
<td>There are 7 adults and 35 children.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gives correct equation</td>
<td>( 6 \times w = 30 ) ( w = 30 \div 6 ) ( w = 5 )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer</td>
<td>( 163 - 41 - 76 = 46 \text{ floors} )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Gives correct number of pieces in total</td>
<td>( 5 \times 6 = 30 ) ( 30 \div 7 = 4 \text{ pieces each with 2 left over} )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct number of pieces per person</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>a) Gives correct number of red tiles</td>
<td>6</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct number of blue tiles</td>
<td>16</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct number of white tiles</td>
<td>44</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>66</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Gives correct answer</td>
<td>Tanner has 18 and Jessica has 6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>b) Gives correct answer</td>
<td>( 18 + 6 = 24 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct number of students in Grade 3</td>
<td>18</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Gives correct number of students in Grade 4</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct total</td>
<td>26</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct answer</td>
<td>4 red, 8 orange, 12 black, 24 total</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total Points \( /13 \)
### Rubric for Unit 3: Operations and Algebraic Thinking
Test (Lessons 28 to 38), p. W-13

#### Level 1
Can answer few, if any, questions accurately and independently.

#### Level 2
Can answer some questions accurately and independently.

#### Level 3
Can answer most questions accurately and independently.

#### Level 4
Can answer all or almost all questions, including bonuses, accurately and independently.

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</td>
<td>3, Bonus (p. W-14), 7 (Advanced), Bonus (Advanced, p. W-15)</td>
</tr>
<tr>
<td>4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.</td>
<td>2, 4, 5, 6, 8 (Advanced)</td>
</tr>
</tbody>
</table>

#### Comments
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 4: Number and Operations—Fractions

#### Quiz (Lessons 1 to 5), p. W-17

Common Core State Standards Emphasized: 4.NF.A.2

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Circles correct picture</td>
<td>middle picture</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>The container is divided into 5 equal parts and 2 of the parts are shaded.</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly shades amount in top picture Correctly shades amount in bottom picture Circles correct answer</td>
<td>4 boxes 3 boxes 4/5</td>
<td>0.5 0.5 1</td>
<td>3/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>&gt;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>9</td>
<td>yes / no</td>
<td>1/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Circles correct answer</td>
<td>1/2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>2/9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Circles correct answer</td>
<td>108/1,000</td>
<td>yes / no</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td><strong>/9</strong></td>
<td><strong>/9</strong></td>
<td><strong>/9</strong></td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 4: Number and Operations—Fractions

(continued)

**Quiz (Lessons 6 to 11), p. W-19**

**Common Core State Standards Emphasized:** 4.NF.A.1, 4.NF.A.2, 4.NF.B.3a, 4.NF.B.3b, 4.NF.B.3d

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>(\frac{4}{10})</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(\frac{6}{9})</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>3</td>
<td>1</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>8</td>
<td>1</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>10</td>
<td>1</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>(\frac{36}{45}, \frac{35}{45})</td>
<td>1</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>&gt;</td>
<td>1</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct explanation</td>
<td>When both denominators are the same, the numerator of (\frac{4}{5}) is larger, meaning that there are more pieces shaded.</td>
<td>1</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>(\frac{3}{5})</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(\frac{4}{6})</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>(\frac{90}{100})</td>
<td>yes / no</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>(\frac{3}{5})</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(\frac{4}{6})</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>(\frac{23}{72})</td>
<td>yes / no</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>Advanced</td>
<td>6</td>
<td>a) Gives correct answer</td>
<td>more, less, more</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>less, more, less</td>
<td>1</td>
<td>(\frac{1}{2})</td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 4: Number and Operations—Fractions**

(continued)

<table>
<thead>
<tr>
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<th>Number of Points</th>
<th>Total Points</th>
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</thead>
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<tr>
<td><strong>Advanced 7</strong></td>
<td>a) Gives correct answer</td>
<td>&lt;</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>&gt;</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>&gt;</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>Advanced 8</strong></td>
<td>a) i) Gives correct answer</td>
<td>1/2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>1/3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>iii) Gives correct answer</td>
<td>1/4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>iv) Gives correct answer</td>
<td>1/5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>smaller</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td><strong>Total Points</strong></td>
<td></td>
<td></td>
<td></td>
<td>/22</td>
</tr>
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</table>
### Quiz (Lessons 12 to 14), p. W-22

Common Core State Standards Emphasized: 4.NF.B.3c, 4.NF.B.3d

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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Shades correct number of pieces</td>
<td>![Shades correct number of pieces]</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{11}{3}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Correctly circles groups of 2 halves</td>
<td>4 groups $\frac{41}{2}$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gives correct division</td>
<td>$23 \div 5 = 4 \text{ R } 3$ $\frac{43}{5}$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>$\frac{14}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{21}{6}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>$\frac{443}{10}$</td>
<td>yes / no</td>
<td>/2</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer</td>
<td>$\frac{32}{4}$ or $\frac{31}{2}$</td>
<td>1</td>
<td>/1</td>
</tr>
</tbody>
</table>

Total Points /9
Scoring Guides for Sample Unit Quizzes and Tests
Unit 4: Number and Operations—Fractions

Test (Lessons 1 to 18), p. W-24

**Common Core State Standards Emphasized:** 4.NF.A.1, 4.NF.A.2, 4.NF.B.3a, 4.NF.B.3b, 4.NF.B.3c, 4.NF.B.3d, 4.NF.B.4a, 4.NF.B.4b, 4.NF.B.4c

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives correct answer</td>
<td>$\frac{6}{9}$</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>&lt;</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>&gt;</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>=</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>&lt;</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>=</td>
<td>yes / no</td>
<td>/4</td>
</tr>
<tr>
<td>3</td>
<td>Gives correct answer</td>
<td>$\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct answer</td>
<td>$\frac{23}{4}$</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer</td>
<td>$2\frac{2}{3}$</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>$\frac{5}{3}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{7}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$\frac{11}{8}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>$\frac{8}{7}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td>$\frac{3}{4} + \frac{4}{5}$</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{4}{5} + \frac{2}{5}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$\frac{3}{8}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>$\frac{3}{2} + \frac{2}{3}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>$\frac{12}{5}$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{10}{9}$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 4: Number and Operations—Fractions**

(continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>a) Gives correct answer</td>
<td>$6 \times \frac{5}{8} = \frac{30}{8}$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>3, 4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Gives correct answer</td>
<td>$4 \times \frac{2}{9} = \frac{8}{9}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>They picked less than 1 pound because $\frac{8}{9} &lt; 1$.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>a) Gives correct answer</td>
<td>no</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>yes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>The books fill up more than one whole shelf.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>$\frac{10}{11}$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Correctly matches all 5 fractions and descriptions</td>
<td>C, B, D, E, A</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Correctly matches 1, 2, or 3 fractions and descriptions</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Writes all five numbers in correct order</td>
<td>$\frac{3}{8}, \frac{16}{32}, \frac{3}{9}, \frac{9}{18}, \frac{15}{15}$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly writes first and/or last number(s), with some mistakes in the middle</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct explanation</td>
<td>Luciano’s is better because he drew the wholes the same length, so the one with more shaded $\frac{5}{8}$ shows the bigger fraction.</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total** /25
## Rubric for Unit 4: Number and Operations—Fractions

### Name: ____________________

Test (Lessons 1 to 18), p. W-24

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
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<tbody>
<tr>
<td>4.NF.A.2</td>
<td></td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td><strong>4.NF.A.2</strong></td>
<td><strong>2, 11, 12 (Advanced), 13 (Advanced)</strong></td>
<td>2, 11, 12 (Advanced), 13 (Advanced)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>4.NF.B.3a</strong></td>
<td></td>
<td>6, Bonus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4.NF.B.3b</strong></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4.NF.B.3c</strong></td>
<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Common Core State Standard Assessed by Question(s) …**

- **4.NF.A.2**: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.
- **4.NF.B.3a**: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **4.NF.B.3b**: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$. 
- **4.NF.B.3c**: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.B.4a</td>
<td>Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).</td>
<td>8.a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.NF.B.4b</td>
<td>Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)</td>
<td>8.b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.NF.B.4c</td>
<td>Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</td>
<td>9, 10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
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Comments
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
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<tbody>
<tr>
<td>1</td>
<td>a) Circles correct answer</td>
<td>5 g</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>30 lbs</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>200 L</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Circles correct answer</td>
<td>355 mL</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>7 kg 128 g</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>45 kg 37 g</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>32</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>1,600</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>4,135</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>5,023</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>3,243</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>8,007</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>6</td>
<td>Gives correct conversion for Rohan's weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>8 × 16 = 128 oz</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>Ranjit is heavier.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>131 oz &gt; 128 oz</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td>2,000 mL</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>2 L</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>8</td>
<td>Gives correct answer</td>
<td>80 × $\frac{3}{4} = 60$ lbs</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>9</td>
<td>Gives correct answer</td>
<td>60 ÷ 3 = 20 pails</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>10</td>
<td>Gives correct answer</td>
<td>1,000 – 340 = 660 mL</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>11</td>
<td>a) Gives correct answer</td>
<td>1,000 – 800 = 200 g</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>200 ÷ 40 = 5 bars</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>3 L 250 mL = 3,250 mL or 3,250 g</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points**: /22
Rubric for Unit 5: Measurement and Data
Test (Lessons 18 to 24), p. W-29

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.A.1</td>
<td>Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</td>
<td>1, 2, 3, 4, 5, Bonus</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
</tr>
<tr>
<td>4.MD.A.2</td>
<td>Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</td>
<td>6, 7, 8, 9, 10, 11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments

Name: ____________________
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 6: Operations and Algebraic Thinking

**Test (Lessons 39 to 44), p. W-32**

**Common Core State Standards Emphasized:** 4.OA.B.4, 4.NBT.B.6

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>19, 14, 9, 4</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>7, 5, 3, 1</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives 4 or 5 correct answers</td>
<td>0, 43</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives 1, 2, or 3 correct answers</td>
<td>1, 33</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2, 23</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3, 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4, 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives 3 correct answers</td>
<td>0, 11</td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>Gives 1 or 2 correct answers</td>
<td>1, 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2, 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Correctly fills in table</td>
<td>1, 1, 10</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>1, 2, 14</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2, 1, 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Skip counts correctly</td>
<td>4, 8, 12, 16</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Skip counts correctly</td>
<td>3, 6, 9, 12, 15</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Skip counts correctly</td>
<td>30, 60, 90</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer to division</td>
<td>5 R 1</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to factor question</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer to division</td>
<td>4 R 0</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to factor question</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>a) Gives all 3 factor pairs</td>
<td>1, 16</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives 1 or 2 factor pairs</td>
<td>2, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4, 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives all 4 factor pairs</td>
<td>1, 24</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives 1, 2, or 3 factor pairs</td>
<td>2, 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3, 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4, 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Circles all 7 prime numbers</td>
<td>3, 5, 7, 11, 13, 19, 23</td>
<td>2</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Circles 4, 5, or 6 prime numbers</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles 1, 2, or 3 prime numbers</td>
<td></td>
<td>(0.5)</td>
<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Gives correct answer</td>
<td>no</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Skip counts correctly</td>
<td>6, 12, 18, 24, 30</td>
<td></td>
<td>/2</td>
</tr>
<tr>
<td>9</td>
<td>a) Gives correct answer to long division</td>
<td>27 R 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to multiple question</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer to long division</td>
<td>17 R 0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to multiple question</td>
<td>yes</td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>24</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct answer</td>
<td>9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Points</td>
<td></td>
<td></td>
<td>/24</td>
</tr>
</tbody>
</table>
Rubric for Unit 6: Operations and Algebraic Thinking
Test (Lessons 39 to 44), p. W-32

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.B.4</td>
<td>Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.</td>
<td>4, 5, 6, 7, 8, 9, Bonus, Advanced</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
</tr>
</tbody>
</table>

Comments
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 7: Number and Operations—Fractions**

**Quiz (Lessons 19 to 23), p. W-36**

**Common Core State Standards Emphasized:** 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>$3.12</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>0.76¢</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct fraction</td>
<td>$\frac{3}{10}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct fraction</td>
<td>$\frac{6}{10}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct fraction</td>
<td>$\frac{50}{100}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.50</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td>3</td>
<td>Correctly positions all 3 decimals</td>
<td>Order: C, B, A</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly positions 1 or 2 decimals</td>
<td>(1)</td>
<td>/2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct fraction with denominator 10</td>
<td>$\frac{5}{10}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct fraction with denominator 100</td>
<td>$\frac{50}{100}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct fraction with denominator 10</td>
<td>$\frac{8}{10}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct fraction with denominator 100</td>
<td>$\frac{80}{100}$</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct fraction</td>
<td>$\frac{64}{100}$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.64</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /14
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 7: Number and Operations—Fractions

Test (Lessons 19 to 29), p. W-38

<p>| Common Core State Standards Emphasized: 4.NF.C.5, 4.NF.C.6, 4.NF.C.7 |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1        | Gives correct answer for A  
Gives correct answer for B | 2, 20  
6, 60 | 1 | 1 /2 |
| 2        | Correctly estimates the position of A  
Correctly estimates the position of B  
Correctly estimates the position of C | right of 0.7  
left of 0.1  
left of 0.4 | 1 | 1 /3 |
| 3        | Gives correct answer  
Gives correct explanation | >  
0.4 is 0.40 and 40 hundredths are larger than 38 hundredths. | 1 | 1 /2 |
| 4        | a) Gives correct answer | $\frac{87}{100}$ | 1 |  |
|          | b) Gives correct answer | $\frac{49}{100}$ | 1 |  |
|          | Bonus: Gives correct answer | $\frac{51}{100}$ | yes / no | 1 /2 |
| 5        | a) Gives correct answer | 43.27 | 1 |  |
|          | b) Gives correct answer | eleven and six tenths | 1 |  |
|          | Bonus: Gives correct answer | eight thousand and five hundredths | yes / no | 1 /2 |
| 6        | a) Gives two correct answers  
Gives one correct answer | 32.7, $\frac{327}{10}$ | 2 | (1) |
|          | b) Gives two correct answers  
Gives one correct answer | 4.21, $\frac{421}{100}$ | 2 | (1) |
|          | c) Gives two correct answers  
Gives one correct answer | 6.03, $\frac{3}{100}$ | 2 | (1) /6 |
| 7        | Gives correct answer | $\frac{7}{10}$ | 1 | 1 /1 |
| 8        | a) Gives correct answer | 8, 3, 83 | 1 |  |
|          | b) Gives correct answer | 0, 4, 4 | 1 |  |
| 9        | Circles both incorrect equalities  
Circles one incorrect equality | 1st and 4th | 2 | (1) /2 |
## Unit 7: Number and Operations—Fractions

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>0.8</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct explanation</td>
<td>Kamali’s models are not the same size, so 0.2 appears to be bigger than 0.35, but 0.35 is larger than 0.2.</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Total Points /22
### Rubric for Unit 7: Number and Operations—Fractions

Test (Lessons 19 to 29), p. W-38

<table>
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<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.C.5</td>
<td>Expression a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</td>
<td>1, 4, 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.NF.C.6</td>
<td>Use decimal notation for fractions with denominators 10 or 100. For example, rewrite $0.62$ as $62/100$; describe a length as $0.62$ meters; locate $0.62$ on a number line diagram.</td>
<td>6, 8, 9, Bonus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.NF.C.7</td>
<td>Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $&gt;$, $=$, or $&lt;$, and justify the conclusions, e.g., by using a visual model.</td>
<td>2, 3, 10 (Advanced)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Comments
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 8: Measurement and Data

**Quiz (Lessons 25 to 29), p. W-42**

**Common Core State Standards Emphasized:** 4.MD.A.1, 4.NF.B.3, 4.NF.B.4, 4.NBT.B.5

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>3</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>a) Circles correct mark</td>
<td>[Image]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Circles correct mark</td>
<td>[Image]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>$\frac{3}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{7}{8}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gives correct length</td>
<td>$2\frac{3}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct width</td>
<td>$1\frac{1}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter with units</td>
<td>8 in</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>24</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>84</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Gives correct conversion</td>
<td>4 ft = 48 in</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>Gabby is 3 in taller.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Gives correct answer</td>
<td>The pencil is not placed at the zero mark on the ruler.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Circles correct mark</td>
<td>[Image]</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /14
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 8: Measurement and Data**

(continued)

Test (Lessons 25 to 32), p. W-45

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Circles correct answer</td>
<td>foot</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>inch</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>foot</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Correctly measures sides in inches</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly gives perimeter with units</td>
<td></td>
<td>1 /2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>24</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>12</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>36, 8, 44</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: d) Gives correct answer</td>
<td>1,200</td>
<td>yes / no</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Bonus: e) Gives correct answer</td>
<td>3,000</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>24, 36, 48, 60</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>6, 9, 12, 15</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>2, 3</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>4, 5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Gives correct conversion</td>
<td>4 yd = 12 ft</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He needs 2 more feet</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer with units</td>
<td>$14 \times 3 = 42$ ft</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>$42 \div 6 = 7$ ft</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Gives correct total in inches</td>
<td>120 in</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct total in feet</td>
<td>10 ft</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Gives correct growth after 12 days</td>
<td>$2 \times 12 = 24$ in= 2 ft</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>8 ft tall</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus</td>
<td>Gives correct answer with units</td>
<td>48 in, 5 ft, 2 yd</td>
<td>yes / no</td>
</tr>
</tbody>
</table>

**Total Points** 24
# Rubric for Unit 8: Measurement and Data

Test (Lessons 25 to 32), p. W-45

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.A.1</td>
<td>1, 4, 5, 6, Bonus (p. W-46)</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

4.MD.A.1
Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …

4.MD.A.2
Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Comments
### Quiz (Lessons 33 to 37), p. W-48

**Common Core State Standards Emphasized:** 4.MD.A.1, 4.MD.A.2, 4.NBT.B.5

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>10</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>11</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct answer for A</td>
<td>15</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for B</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for C</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer for length and width</td>
<td>4, 2</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for area</td>
<td>$4 \times 2 = 8$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for length and width</td>
<td>7, 3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for area</td>
<td>$7 \times 3 = 21$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gives correct explanation</td>
<td>Yes, since $4 \times 3 = 3 \times 4$.</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer with units</td>
<td>$24 + 4 = 6 \text{ cm}$</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>20, 10, 200</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>20, 15, 300</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td>100 ft$^2$</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer with units</td>
<td>$30 + 30 + 30 = 90 \text{ m}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$9,000$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives three correct dimensions</td>
<td>1, 12, 2, 6, 3, 4</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points**  /16
## Scoring Guides for Sample Unit Quizzes and Tests

### Unit 8: Measurement and Data

(continued)

**Test (Lessons 33 to 40), p. W-51**

**Common Core State Standards Emphasized:** 4.MD.A.1, 4.MD.A.2, 4.MD.B.4, 4.NBT.B.5, 4.NF.B.3

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>13</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>15</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>$4 \times 3 = 12$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$3 \times 8 = 24$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer with units</td>
<td>6 m$^2$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>8 ft$^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Gives correct answer with units</td>
<td>4 ft</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>Divide the area by the length.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Measures the length and width of A correctly</td>
<td>8 cm, 2 cm</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter of A with units</td>
<td>20 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct area of A with units</td>
<td>16 cm$^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Measures the length and width of B correctly</td>
<td>3 cm, 3 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter of B with units</td>
<td>12 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct area of B with units</td>
<td>9 cm$^2$</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td>6</td>
<td>a) Correctly completes number line</td>
<td>1, 2, 3, 4, 5, 6, 7</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Correctly builds line plot</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$\frac{5}{8}$ of a mile</td>
<td>yes / no</td>
<td>yes / no</td>
</tr>
<tr>
<td>Bonus</td>
<td>Correctly finds width</td>
<td>$12 + 3 = 4$ cm</td>
<td>1</td>
<td>yes / no</td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter</td>
<td>14 cm</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Correctly makes two rectangles</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>b) Gives correct answer for vertical division</td>
<td>$5 \times 2 = 10$ and $2 \times 3 = 6$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or Gives correct answer for horizontal division</td>
<td>$2 \times 2 = 4$ and $3 \times 4 = 12$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td>16 m$^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>$160$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Gives correct answer</td>
<td>9, 10, 11, 12, 13, 14</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>b) Gives correct answer</td>
<td>$\frac{5}{4}$ mi</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$\frac{13}{8} + \frac{13}{8} + \frac{14}{8} = \frac{40}{8} = 5$ mi</td>
<td>1</td>
<td>/17</td>
</tr>
</tbody>
</table>

**Total Points**
### Rubric for Unit 8: Measurement and Data

Test (Lessons 33 to 40), p. W-51

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
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<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.B.4</td>
<td></td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</td>
<td>6, 8 (Advanced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 9: Geometry

**Quiz (Lessons 14 to 17), p. W-55**

**Common Core State Standards Emphasized:** 4.G.A.1, 4.MD.C.5, 4.MD.C.6

<table>
<thead>
<tr>
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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>ray</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>line segment</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>line</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly extends lines and rays</td>
<td>Correctly circles intersection point</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>b) Correctly extends lines and rays</td>
<td>Correctly circles intersection point</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>c) Correctly extends lines and rays</td>
<td>Correctly circles intersection point</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>obtuse</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>obtuse</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Correctly identifies angle</td>
<td>Gives correct measurement</td>
<td>acute</td>
<td>50°</td>
</tr>
<tr>
<td></td>
<td>b) Correctly identifies angle</td>
<td>Gives correct measurement</td>
<td>obtuse</td>
<td>120°</td>
</tr>
<tr>
<td>5</td>
<td>a) Draws correct angle</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws correct angle</td>
<td></td>
<td>1</td>
<td>/2</td>
</tr>
</tbody>
</table>

**Total Points** /16
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Marks all 4 angles correctly</td>
<td>![Diagram A]</td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>Marks 2 or 3 angles correctly</td>
<td>![Diagram B]</td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>b) Marks all 4 angles correctly</td>
<td>![Diagram C]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Marks 2 or 3 angles correctly</td>
<td>![Diagram D]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Marks all 3 angles correctly</td>
<td>![Diagram E]</td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td>Marks 2 angles correctly</td>
<td>![Diagram F]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly extends the line</td>
<td>![Diagram G]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>![Diagram H]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly extends the line</td>
<td>![Diagram I]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>![Diagram J]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly extends the ray</td>
<td>![Diagram K]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>![Diagram L]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Draws correct angle</td>
<td>![Diagram M]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws correct angle</td>
<td>![Diagram N]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer for fraction</td>
<td>$\frac{2}{5}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for angle</td>
<td>$120^\circ$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for fraction</td>
<td>$\frac{1}{5}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for angle</td>
<td>$72^\circ$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Correctly marks all 21 angles</td>
<td>![Diagram O]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly marks 16–20 angles</td>
<td>![Diagram P]</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly marks 11–15 angles</td>
<td>![Diagram Q]</td>
<td>(0.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly sorts all 5 shapes</td>
<td>![Diagram R]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly sorts 3 or 4 shapes</td>
<td>![Diagram S]</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly sorts 1 or 2 shapes</td>
<td>![Diagram T]</td>
<td>(0.5)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Gives correct explanation</td>
<td>He should only have subtracted $100^\circ$. The $100^\circ$ and $30^\circ$ angles overlap.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td>$30, 60, 90^\circ$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$55, 95, 150^\circ$</td>
<td>1</td>
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</tr>
</tbody>
</table>
### Question How to Score Answer Number of Points Total Points

**8**
- a) Gives correct answer $10^\circ$ 1
- b) Gives correct answer 120 min, 2 hr 1

**Bonus**
- Gives correct answer 360 min, 6 hrs yes / no

**Advanced**
- **9**
  - a) i) Gives correct answer $x = 130 - 50$ 1
  - ii) Gives correct answer $x = 111 - 31$ 1
- b) i) Gives correct answer $x = 80^\circ$ 1
  - ii) Gives correct answer $x = 80^\circ$ 1

**Advanced**
- **10**
  - a) i) Gives correct answer $x = 170 - 55 - 45$ 1
  - ii) Gives correct answer $x = 160 - 40 - 50$ 1
  - b) i) Gives correct answer $x = 70^\circ$ 1
  - ii) Gives correct answer $x = 70^\circ$ 1

**Advanced**
- **Bonus**
- Gives correct answer $x = 130 - 90$ 1
  - $x = 40^\circ$ yes / no

**Total Points** /24
Rubric for Unit 9: Geometry
Test (Lessons 14 to 22), p. W-57

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.G.A.1</td>
<td>1, 2, 5.a)</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>4.G.A.2</td>
<td>5.b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.MD.C.5</td>
<td>4</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>4.MD.C.6</td>
<td>3</td>
<td></td>
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</tr>
</tbody>
</table>

- **4.G.A.1**: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- **4.G.A.2**: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- **4.MD.C.5**: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
- **4.MD.C.6**: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
### Rubric for Unit 9: Geometry
Test (Lessons 14 to 22), p. W-57

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.C.7</td>
<td></td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td><strong>Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</strong></td>
<td><strong>6, 7, 9 (Advanced), 10 (Advanced), Bonus (Advanced, p. W-59)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.OA.A.3</td>
<td></td>
<td><strong>8, Bonus (p. W-58)</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</strong></td>
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</tr>
</tbody>
</table>

### Comments
Grade 4 Common Core State Standards Curriculum Correlations

NOTE: The italicized gray JUMP Math lessons contain prerequisite material for the Common Core standards.

**Domain**

OA  Operations and Algebraic Thinking  
NBT  Number and Operations in Base Ten  
NF  Number and Operations—Fractions  
MD  Measurement and Data  
G  Geometry

**Cluster**

### 4.OA  Operations and Algebraic Thinking

<table>
<thead>
<tr>
<th>4.OA.A</th>
<th>Use the four operations with whole numbers to solve problems.</th>
<th>JUMP Math Grade 4 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.A.1</td>
<td>Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 1  OA4-9 to 11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 1  OA4-12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 3  OA4-35 to 38</td>
</tr>
<tr>
<td>4.OA.A.2</td>
<td>Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 4  NBT4-36, 39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 2  OA4-21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 2  OA4-24, 25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 6  MD4-2 to 9, 16, 17</td>
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<tr>
<td></td>
<td></td>
<td>2 3  OA4-34 to 38</td>
</tr>
<tr>
<td>4.OA.A.3</td>
<td>Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td>Part  Unit  Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 1  OA4-1 to 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 1  OA4-7, 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 3  OA4-17</td>
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<tr>
<td></td>
<td></td>
<td>1 5  OA4-18 to 21, 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 5  OA4-24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 2  NBT4-22</td>
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<tr>
<td></td>
<td></td>
<td>1 2  NBT4-23, 24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 6  MD4-7, 11, 12, 15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 2  NBT4-40, 41</td>
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<td>2 2  NBT4-49, 50</td>
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<td>2 9  G4-19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 3  OA4-28 to 31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 3  OA4-32, 33, 35 to 38</td>
</tr>
<tr>
<td>4.OA</td>
<td>Operations and Algebraic Thinking</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------</td>
<td></td>
</tr>
<tr>
<td>4.OA.B</td>
<td>Gain familiarity with factors and multiples.</td>
<td></td>
</tr>
<tr>
<td>4.OA.B.4</td>
<td>Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.OA</th>
<th>Operations and Algebraic Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.C</td>
<td>Generate and analyze patterns.</td>
</tr>
<tr>
<td>4.OA.C.5</td>
<td>Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</td>
</tr>
</tbody>
</table>
# 4.NBT  Number and Operations in Base Ten

## 4.NBT.A  Generalize place value understanding for multi-digit whole numbers.

<table>
<thead>
<tr>
<th>4.NBT.A.1</th>
<th>Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NBT.A.2</td>
<td>Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $&gt;$, $=$, and $&lt;$ symbols to record the results of comparisons.</td>
</tr>
<tr>
<td>4.NBT.A.3</td>
<td>Use place value understanding to round multi-digit whole numbers to any place.</td>
</tr>
</tbody>
</table>

## 4.NBT.B  Use place value understanding and properties of operations to perform multi-digit arithmetic.

| 4.NBT.B.4 | Fluently add and subtract multi-digit whole numbers using the standard algorithm. |
| 4.NBT.B.5 | Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
| 4.NBT.B.6 | Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
### 4.NF Number and Operations—Fractions

#### 4.NF.A Extend understanding of fraction equivalence and ordering.

| 4.NF.A.1 | Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. |
| Part | Unit | Lessons |
| 2 | 4 | NF4-1, 2, 4 NF4-6 |

| 4.NF.A.2 | Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. |
| Part | Unit | Lessons |
| 2 | 4 | NF4-1, 2 NF4-3 NF4-4, 5 NF4-7 to 9 |

#### 4.NF.B Build fractions from unit fractions.

| 4.NF.B.3 | Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$. |
| Part | Unit | Lessons |
| 2 | 4 | NF4-10, 11, 17 |
| 2 | 9 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |

| 4.NF.B.3a | Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. |
| Part | Unit | Lessons |
| 2 | 4 | NF4-12 to 14 NF4-15 NF4-16, 18 |
| 2 | 9 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |

| 4.NF.B.3b | Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2 1/8 = 1 + 1/8 = 8/8 + 8/8 + 1/8$. |
| Part | Unit | Lessons |
| 2 | 4 | NF4-11, 17, 18 |
| 2 | 9 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |

<p>| 4.NF.B.3c | Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. |
| Part | Unit | Lessons |
| 2 | 4 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |</p>
<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
<th>JUMP Math Grade 4 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.B.3d</td>
<td>Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</td>
<td>Part</td>
</tr>
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<tr>
<td>4.NF.B.4</td>
<td>Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</td>
<td>JUMP Math Grade 4 Lessons</td>
</tr>
<tr>
<td>4.NF.B.4a</td>
<td>Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \left(\frac{1}{4}\right)$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \left(\frac{1}{4}\right)$.</td>
<td>Part</td>
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<tr>
<td>4.NF.B.4b</td>
<td>Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times \left(\frac{2}{5}\right)$ as $6 \times \left(\frac{1}{5}\right)$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \left(\frac{a}{b}\right) = (n \times a)/b$.)</td>
<td>Part</td>
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<tr>
<td>4.NF.B.4c</td>
<td>Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</td>
<td>Part</td>
</tr>
<tr>
<td></td>
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</tbody>
</table>

**4.NF** Number and Operations—Fractions

4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. |

Part | Unit | Lessons |
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>NF4-21, 26, 27</td>
</tr>
</tbody>
</table>

4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Part | Unit | Lessons |
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>NF4-19, NF4-22, 24, 25</td>
</tr>
</tbody>
</table>

4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

Part | Unit | Lessons |
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>NF4-20, NF4-21 to 25, 28, 29</td>
</tr>
</tbody>
</table>
4.MD  Measurement and Data

4.MD.A  Solve problems involving measurement and conversion of measurements.

4.MD.A.1  Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

4.MD.A.2  Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.A.3  Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

4.MD  Measurement and Data

4.MD.B  Represent and interpret data.

4.MD.B.4  Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.
### 4.MD Measurement and Data

<table>
<thead>
<tr>
<th>4.MD.C</th>
<th>Geometric measurement: understand concepts of angle and measure angles.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.C.5</td>
<td>Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</td>
</tr>
<tr>
<td>4.MD.C.5a</td>
<td>An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.</td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4.MD.C.5b</td>
<td>An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees.</td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4.MD.C.6</td>
<td>Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</td>
</tr>
<tr>
<td>Part</td>
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<td>4.MD.C.7</td>
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</tr>
<tr>
<td>Part</td>
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</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>
## 4.G  **Geometry**

<table>
<thead>
<tr>
<th>4.G.A</th>
<th>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.G.A.1</td>
<td>Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</td>
</tr>
<tr>
<td></td>
<td>JUMP Math Grade 4 Lessons</td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4.G.A.2</td>
<td>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</td>
</tr>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4.G.A.3</td>
<td>Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</td>
</tr>
<tr>
<td></td>
<td>Part</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Find the number of pennies or nickels you need to make the amount.

   a) 19¢  
      | 0 | 1 | 2 | 3 |
      | 0 |   |   |   |
   b) 35¢  
      | 0 | 1 | 2 | 3 |
      | 0 |   |   |   |

2. Find the number of pennies, nickels, dimes, or quarters you need to make the amount.  
   Hint: You may not need to use all of the rows.

   a) 43¢  
      | 0 | 1 | 2 | 3 |
      | 0 |   |   |   |
   b) 55¢  
      | 0 | 1 | 2 | 3 |
      | 0 |   |   |   |

3. A ladybug has 6 legs. A cat has 4 legs. Three of these creatures have a total of 16 legs. How many of each creature are there?

<table>
<thead>
<tr>
<th>Ladybugs</th>
<th>Cats</th>
<th>Total Number of Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Skip count to check for factors.  
   **Skip Counting**

   a) Is 4 a factor of 14?  
      ______  
      _______, _______, _______, _______, ______
   b) Is 3 a factor of 15?  
      ______  
      _______, _______, _______, _______, ______
   c) Is 30 a factor of 80?  
      ______  
      _______, _______, _______, _______, ______
Unit 6: Operations and Algebraic Thinking

Test (Lessons 39 to 44)

5. Divide to check for factors.
   a) \(11 \div 2 = \underline{_____} \text{ R } \underline{____}_\)  Is 2 a factor of 11? \underline{____}_
   b) \(24 \div 6 = \underline{_____} \text{ R } \underline{____}_\)  Is 6 a factor of 24? \underline{____}_

6. Find all the factor pairs of the number. Stop when you get a number that is already part of a pair. There might be more rows than you need.
   a) 16
      \[
      \begin{array}{|c|c|}
      \hline
      \text{First Factor} & \text{Second Factor} \\
      \hline
      \hline
      \hline
      \hline
      \hline
      \hline
      \end{array}
      \]
   b) 24
      \[
      \begin{array}{|c|c|}
      \hline
      \text{First Factor} & \text{Second Factor} \\
      \hline
      \hline
      \hline
      \hline
      \hline
      \hline
      \end{array}
      \]

7. Circle the prime numbers. 3, 4, 5, 7, 8, 11, 12, 13, 15, 19, 21, 23, 25

8. Is 25 a multiple of 6? \underline{____}_
   Skip count to check: \underline{_____}, \underline{_____}, \underline{_____}, \underline{_____}, \underline{_____}, \underline{_____}

9. Use long division to check for multiples.
   a) \[
   \begin{array}{c|cc}
   \hline
   3 & 8 & 2 \\
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \]
   Is 82 a multiple of 3? \underline{____}_
   b) \[
   \begin{array}{c|cc}
   \hline
   4 & 6 & 8 \\
   \hline
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \]
   Is 68 a multiple of 4? \underline{____}_

BONUS:
Find the number between 20 and 30 which is a multiple of both 6 and 8.
Unit 6: Operations and Algebraic Thinking
Test (Lessons 39 to 44)

ADVANCED

Thomas believes all composite numbers are even. Circle the first number in the list which shows his claim is not true: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21
Unit 6: Operations and Algebraic Thinking

Test (Lessons 39 to 44)

1. a) Nickels  Pennies
   0   19
   1   14
   2   9
   3   4

   b) Dimes  Nickels
   0   7
   1   5
   2   3
   3   1

2. a) Dimes  Pennies
   0   43
   1   33
   2   23
   3   13
   4   3

   b) Quarters  Nickels
   0   11
   1   6
   2   1

3. 2 ladybugs, 1 cat

4. a) no
   b) yes
   c) no

5. a) no; $11 \div 2 = 5 \text{ R } 1$
   b) yes; $24 \div 6 = 4 \text{ R } 0$

6. a) $1^{\text{st}}$ factor  $2^{\text{nd}}$ factor
   1   16
   2   8
   4   4

   b) $1^{\text{st}}$ factor  $2^{\text{nd}}$ factor
   1   24
   2   12
   3   8
   4   6

7. 3, 5, 7, 11, 13, 19, 23

8. no

9. a) No; $82 \div 3 = 27 \text{ R } 1$
   b) Yes; $68 \div 4 = 17$

BONUS
   24

ADVANCED
   9
1. Complete the table by writing in cent or dollar notation.

<table>
<thead>
<tr>
<th>Cent Notation</th>
<th>Dollar Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 312¢</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>$0.76</td>
</tr>
</tbody>
</table>

2. Write the fraction that the picture shows. Then write the decimal.

a) \[ \frac{1}{2} \] Fraction: \[ \frac{1}{2} \] Decimal:

b) \[ \frac{3}{4} \] Fraction: \[ \frac{3}{4} \] Decimal:

c) \[ \frac{7}{8} \] Fraction: \[ \frac{7}{8} \] Decimal:

3. Show the decimals on the number line.

A. 0.80  B. 0.45  C. 0.09

4. Write the decimal as a fraction with denominator 10, then as a fraction with denominator 100.

a) 0.5  b) 0.8

BONUS

Write the fraction that the picture shows. Then write the decimal for it.

Fraction: \[ \frac{5}{8} \]

Decimal: \[ 0.625 \]
Unit 7: Number and Operations—Fractions

Quiz (Lessons 19 to 23)

1. a) $3.12  
   b) 76¢

2. a) \( \frac{3}{10} \), 0.3  
   b) \( \frac{6}{10} \), 0.6  
   c) \( \frac{50}{100} \), 0.50

3.

4. a) \( \frac{5}{10} \), 0.50  
   b) \( \frac{8}{10} \), 0.80

BONUS

\( \frac{64}{100} \), 0.64
1. For points A and B, write how many tenths and how many hundredths.

\[ \begin{align*}
A & : \quad \text{_________ tenths} & \text{_________ hundredths} \\
B & : \quad \text{_________ tenths} & \text{_________ hundredths}
\end{align*} \]

2. Estimate and mark the location of the decimals on the number line.

\[ \begin{align*}
A & : \quad 0.73 \\
B & : \quad 0.08 \\
C & : \quad 0.39
\end{align*} \]

3. What number is greater? Write < or >. Then explain how you know.

\[
0.4 \quad \square \quad 0.38
\]

4. Add the fractions.

\[ \begin{align*}
a) \quad \frac{8}{10} + \frac{7}{100} & = \\
b) \quad \frac{4}{10} + \frac{9}{100} & = \\
\text{BONUS} \quad \frac{3}{10} + \frac{21}{100} & =
\end{align*} \]

5. Write the equivalent decimal or words.

\[ \begin{align*}
a) \quad \text{forty-three and twenty-seven hundredths} & \quad \text{_________} \\
b) \quad 11.6 & \quad \text{________________________}
\end{align*} \]

\[ \text{BONUS} \quad 8,000.05 \quad \text{________________________} \]
6. Complete the table.

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Mixed Number</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td></td>
<td>$32\frac{7}{10}$</td>
</tr>
<tr>
<td>b)</td>
<td>4.21</td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td></td>
<td>$\frac{603}{100}$</td>
</tr>
</tbody>
</table>

7. Tyrell spent $\frac{3}{10}$ of a dollar on marbles and $\frac{4}{10}$ of a dollar on candy. What fraction of a dollar did he spend altogether?

8. Describe the decimal in two ways.
   a) $0.83 = \text{____ tenths ____ hundredths}$
   b) $0.04 = \text{____ tenths ____ hundredths}$
      $= \text{____ hundredths}$
      $= \text{____ hundredths}$

9. Circle the equalities that are incorrect.
   
   $0.3 = \frac{3}{100}$  $0.27 = \frac{27}{100}$  $0.05 = \frac{5}{100}$  $0.07 = \frac{7}{10}$

BONUS ► Add. Hint: change the decimals to fractions.

   $0.3 + 0.5 =$
10. Kamali draws a model of 0.35 and a model of 0.2. She then writes 0.35 < 0.2. Explain Kamali’s mistake.

Kamali’s models

0.35

0.2
1. A. 2 tenths, 20 hundredths  
   B. 6 tenths, 60 hundredths

2. \[ B \quad C \quad A \]

3. 0.4 > 0.38 because 0.4 is 0.40 (40 hundredths) and 40 hundredths are larger than 38 hundredths.

4. a) \[
\begin{array}{c}
\frac{87}{100}
\end{array}
\]
   b) \[
\begin{array}{c}
\frac{49}{100}
\end{array}
\]

**BONUS**

5. a) 43.27
   b) Eleven and six tenths

**BONUS** Eight thousand and five hundredths

6. | Decimal | Mixed Number | Improper Fraction |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 32.7</td>
<td>32 \frac{7}{10}</td>
<td>327 \frac{7}{10}</td>
</tr>
<tr>
<td>b) 4.21</td>
<td>4 \frac{21}{100}</td>
<td>421 \frac{21}{100}</td>
</tr>
<tr>
<td>c) 6.03</td>
<td>6 \frac{3}{100}</td>
<td>603 \frac{3}{100}</td>
</tr>
</tbody>
</table>

7. \[
\begin{array}{c}
\frac{7}{10}
\end{array}
\]

8. a) 8 tenths 3 hundredths, 83 hundredths
   b) 0 tenths 4 hundredths

9. \[
0.3 = \frac{3}{100}, \quad 0.07 = \frac{7}{10}\]

**BONUS** 0.8

10. Kamali’s models are not the same size, so 0.2 appears to be bigger than 0.35, but 0.35 is larger than 0.2.
1. Measure the length of the object.
   a) __________ in  
   b) __________ in

2. Circle the mark for the mixed number or fraction on the ruler.
   a) $5 \frac{1}{4}$  
   b) $\frac{1}{4}$

3. Measure the length of the line segment.
   a) __________ in  
   b) __________ in

4. Measure each side of the rectangle shown to the nearest $\frac{1}{4}$ inch. Then find the perimeter.
   length = __________ in  
   width = __________ in
   Perimeter =
5. Convert from feet to inches.
   a) 2 feet = _____ inches  
   b) 7 feet = _____ inches

6. Gertrude is 4 feet tall. Her friend Gabby is 51 inches tall. How much taller is Gabby?

7. Wajid measured the length of a pencil to be approximately $2\frac{1}{2}$ inches as shown in the diagram. Explain why he is incorrect.

BONUS ▶

On some rulers, an inch is divided into 16 equal parts. Circle the mark for the measurement $1\frac{3}{16}$ inches.
Unit 8: Measurement and Data

Quiz (Lessons 25 to 29)

1. a) 3 in  
   b) 2 in

2. a) 

   ![Image of a ruler showing measurements from 5 to 6 with ticks at 5.5 and 6.5]

   b) 

   ![Image of a ruler showing measurements from 0 to 1 with ticks at 0.5 and 1.5]

3. a) $\frac{3}{4}$ in  
   b) $\frac{7}{8}$ in

4. Length = $2\frac{3}{4}$ in  
   Width = $1\frac{1}{4}$ in  
   
   $P = \frac{1}{4} + \frac{1}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$
   
   $= 8$

5. a) 24 in  
   b) 84 in

6. 3 in

7. The pencil is not placed at the zero mark on the ruler.

BONUS

![Image of a ruler showing measurements from 0 to 2 with ticks at 0.5 and 1.5]
1. Circle the unit that is more appropriate for measuring the length of the object.

   a)  
   
   b)  
   
   c)  
   

2. Draw a second arrow to show where a line segment of given length would end.

   a) 2 inches  
   
   b) $\frac{3}{4}$ inches  
   
   c) $1\frac{1}{2}$ inches  
   
   d) $2\frac{3}{8}$ inches  
   

3. Use a ruler to find the lengths of the sides of the pentagon to the nearest $\frac{1}{8}$ inch. Then find the perimeter.

4. Convert the measurement to the indicated units.

   a) 2 ft = _____ in  
   b) 4 yd = _____ ft  
   c) 3 ft 8 in = _____ in + _____ in = _____ in  

   BONUS  d) 100 ft = _____ in  
   e) 1,000 yd = _____ ft
Unit 8: Measurement and Data

Test (Lessons 25 to 32)

5. Complete the table.

<table>
<thead>
<tr>
<th></th>
<th>1 ft</th>
<th>2 ft</th>
<th>3 ft</th>
<th>4 ft</th>
<th>5 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 in</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1 yd</th>
<th>2 yd</th>
<th>3 yd</th>
<th>4 yd</th>
<th>5 yd</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. For the measurement in inches, write the pair of measurements in feet or yards that it is between. Hint: Use the tables in Question 5.

   a) _____ ft < 32 in < _____ ft
   b) _____ yd < 13 ft < _____ yd

7. Freddie needs 4 yards of chain, but the hardware store only has 10 feet in stock. How many more feet of chain does Freddie need?

8. A rope is 14 yards long. It needs to be divided into 6 equal parts.

   a) How long is the rope when measured in feet? _______________________
   b) How long should each of the equal parts be? _______________________

9. A snail crawled a few inches each day as shown in the table. What is the total distance traveled for the week when measured in feet?

<table>
<thead>
<tr>
<th>Day</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>15 in</td>
</tr>
<tr>
<td>Tuesday</td>
<td>38 in</td>
</tr>
<tr>
<td>Wednesday</td>
<td>19 in</td>
</tr>
<tr>
<td>Thursday</td>
<td>17 in</td>
</tr>
<tr>
<td>Friday</td>
<td>31 in</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

10. When a giraffe is born, it is already 6 feet tall. Baby giraffes can grow 2 inches in a single day. If this growth pattern continues for 12 days, how tall will the giraffe be when measured in feet?

BONUS

Place the measurements in order from least to greatest: 2 yards, 5 feet, 48 inches. Hint: convert them all to a common unit of measurement first.

   ________ < ________ < ________
1. a) foot
   b) inch
   c) foot

2. a) 
   b) 
   c) 
   d) 

3. Sides: 1 in, \( \frac{5}{8} \) in, \( \frac{7}{8} \) in
   
   Perimeter = 7 in

4. a) 24 in
   b) 12 ft
   c) 44 in

   **BONUS**
   d) 1,200 in
   e) 3,000 ft

5. ft: 1 2 3 4 5
   In: 12 24 36 48 60
   yd: 1 2 3 4 5
   ft: 3 6 9 12 15

6. a) 2 ft < 32 in < 3 ft
    b) 4 yd < 13 ft < 5 yd

7. 2 ft

8. a) 42 ft
    b) 7 ft

9. Total = 120 in
    = 10 ft

10. Height = 8 ft

   **BONUS** 48 in < 5 ft < 2 yd
Unit 8: Measurement and Data

Quiz (Lessons 33 to 37)

1. Find the area of the figure in square centimeters.
   a) __ cm²
   b) __ cm²

2. Find the area of the rectangles in square centimeters.
   A. Area = ______ cm²
   B. Area = ______ cm²
   C. Area = ______ cm²

3. Write the number of boxes along the length and width for each rectangle. Then write a multiplication equation for the area of the rectangle measure in square units.
   a) Length = _____ units
      Width = _____ units
      Area = _____ × _____
      = _____ sq. units
   b) Length = _____ units
      Width = _____ units
      Area = _____ × _____
      = _____ sq. units
4. Ingrid has a rectangle with width 3 in and length 4 in. Marek has a rectangle with width 4 in and length 3 in. Do the rectangles have the same area? Explain.

5. A rectangle with area 24 cm\(^2\) has a width of 4 cm. What is the length of the rectangle?

6. Luigi's vegetable garden has length 20 ft and width 10 ft.
   a) What is the area of the garden?
   
   Area = ____ ft \(\times\) _____ ft = _____ ft\(^2\)
   
   b) If Luigi changes the width to 15 ft, what is the new area of the garden?
   
   Area = ____ ft \(\times\) _____ ft = _____ ft\(^2\)
   
   c) How many square feet of area would Luigi add by changing the width to 15 ft?

7. Saul’s backyard is in the shape of a square with each side having a length of 30 m.
   
   a) Saul wants to fence in 3 sides of his yard. How many meters of fencing will he need? _______________________
   
   b) Fencing costs $100 per meter. What will the cost of the fence be? _______________________

**BONUS**

Find the lengths and widths of 3 different rectangles which will have an area of 12 yd\(^2\).

<table>
<thead>
<tr>
<th>Length (yd)</th>
<th>Width (yd)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unit 8: Measurement and Data

Quiz (Lessons 33 to 37)

1. a) 10 cm²  
   b) 11 cm²
2. a) 15 cm²  
   b) 3 cm²  
   c) 8 cm²
3. a) 8 sq. units 
   b) 21 sq. units
4. The rectangles have the same area since \(4 \times 3 = 3 \times 4\)
5. Length = \(24 \div 4 = 6\) cm
6. a) 200 ft²  
   b) 300 ft²  
   c) 100 ft²
7. a) 90 m 
   b) $9,000

BONUS
L = 1 yd  W = 12 yd
L = 2 yd  W = 6 yd
L = 3 yd  W = 4 yd
Unit 8: Measurement and Data
Test (Lessons 33 to 40)

1. Find the area of the figure in square centimeters.
   a)  
   ![](image1)
   _____ cm²
   
   b)  
   ![](image2)
   _____ cm²

2. Write a multiplication statement that tells you how many squares are in the rectangle.
   a)  
   ![](image3)
   
   b)  
   ![](image4)
   
   _______________  _______________

3. Find the area of the rectangle. Include the units.
   a)  
   ![](image5)
   
   b)  
   ![](image6)
   
   _______________

4. A rectangle has area 28 ft² and length 7 ft. Find the width of the rectangle. Explain your answer.
5. Measure the length and width of the rectangle to the nearest centimeter. Find the perimeter and area for each rectangle.

![Rectangles A and B]

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. In gym class, students jogged for 15 minutes. The distances run are shown in the table.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

a) Complete the number line on the line plot.

![Number Line Plot]

b) Build a line plot with the data.

c) How much farther was the longest distance run than the shortest distance? _______

**BONUS** A rectangle with length 3 cm has an area of 12 cm$^2$. Find the perimeter of the rectangle in centimeters.
ADVANCED

7. A room has the shape as shown in the diagram.
   a) Draw a line to form two rectangles.
   b) Find the area of each rectangle.
      \[
      \text{Area 1} = \underline{\hspace{2cm}} \text{m} \times \underline{\hspace{2cm}} \text{m} = \underline{\hspace{2cm}} \text{m}^2
      \]
      \[
      \text{Area 2} = \underline{\hspace{2cm}} \text{m} \times \underline{\hspace{2cm}} \text{m} = \underline{\hspace{2cm}} \text{m}^2
      \]
   c) Find the total area of the room.
   d) Carpeting costs $10 per square meter. What is the total cost of carpeting the room?

8. The distance of a horse race is measured in miles. The line plot shows lengths of different horse races.
   a) Rewrite the lengths of the horse races so that the denominators are all the same.
   b) What distance is the most common for a horse race?
   c) Charlie the horse only ran the three longest races last year. What was the total distance Charlie ran?
1. a) 13 cm²  
b) 15 cm²  
2. a) 3 × 4 = 12  
b) 8 × 3 = 24  
3. a) 6 m²  
b) 8 ft²  
4. Divide the area by the length. Width = 28/7 = 4 ft  
5. A  
P = 8 + 2 + 8 + 2 = 20 cm  
A = 8 × 2 = 16 cm²  
B  
P = 3 + 3 + 3 + 3 = 12 cm  
A = 3 × 3 = 9 cm²  
6.  
a), b)  

![Distances Run in Gym Class](image)  
c) \( \frac{5}{8} \) of a mile  

**BONUS**  

P = 14 cm  

**ADVANCED**  

7. b) Area 1 = 10 m²  
Area 2 = 6 m²  
c) Area = 16 m²  
d) $160  
8. a)  

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

b) \( \frac{5}{4} \) mi  
c) \( \frac{40}{8} \) mi or 5 mi
Unit 9: Geometry

Quiz (Lessons 14 to 17)

1. Identify the picture as a line, line segment, or ray.
   a) __________________  b) __________________  c) __________________

2. Extend the lines and rays. Circle the intersection points.
   a) __________________  b) __________________  c) __________________

3. Identify the angle as acute or obtuse.
   a) __________________  b) __________________  c) __________________  d) __________________

4. Identify the angle as acute or obtuse. Then write the measure of the angle.
   a) __________________  b) __________________

5. Finish drawing the angle.
   a) 60°  b) 140°
Unit 9: Geometry

Quiz (Lessons 14 to 17)

1. a) ray
   b) line segment
   c) line

2. a)

   b)

   c)

3. a) obtuse
   b) acute
   c) obtuse
   d) acute

4. a) acute, 50°
   b) obtuse, 120°

5. a)

   b)
1. Mark each right angle in the shape with a small square, each acute angle with an A, and each obtuse angle with an O.

   a)  
   b)  
   c)  

2. Extend the line or ray where possible. Name the points that are on the line or ray.

   a)  
   b)  
   c)  

3. Finish drawing the angle.

   a)  
   b)  

4. Fill in the blanks.

   a)  
   b)  

   What fraction of the circle is shaded? ____  
   What fraction of the circle is shaded? ____

   What is the angle? _______ of 360°= ____  
   What is the angle? _______ of 360°= ____
5. a) Mark acute angles with an A, obtuse angles with O, and right angles with a small square.

b) Sort the shapes into the Venn diagram below.

b) Sort the shapes into the Venn diagram below.

6. In the picture the whole angle is 140°. Andrew says that the angle \( x \) is \( 140° - 130° = 10° \). Explain Andrew’s mistake.

7. Find the measure of the whole angle by adding the measures of the small angles.
   a) \[ \text{the whole angle} = \_\_\_\_ + \_\_\_\_ = \_\_\_\_ \]
   b) \[ \text{the whole angle} = \_\_\_\_ + \_\_\_\_ = \_\_\_\_ \]

8. A clock’s hour hand turns 1° every 2 minutes.
   a) How many degrees has it turned in 20 minutes? ______
   b) How many minutes does it take to turn 60 degrees? ______
      How many hours is that? ______

   **BONUS**
   How many minutes will it take for an hour hand to turn 180 degrees? ______
   How many hours is that? ______
ADVANCED

9. a) Write an equation to find $x$.
    i) The whole angle is $130^\circ$.
      \[
      \begin{align*}
        x + 50^\circ &= 130^\circ \\
        x &= 80^\circ
      \end{align*}
      \]
    ii) The whole angle is $111^\circ$.
      \[
      \begin{align*}
        31^\circ + x &= 111^\circ \\
        x &= 80^\circ
      \end{align*}
      \]

    b) Solve your equations from part a) to find $x$.
    i) $x = 80^\circ$
    ii) $x = 80^\circ$

10. a) Write an equation to find $x$.
    i) The whole angle is $170^\circ$.
      \[
      \begin{align*}
        55^\circ + 45^\circ + x &= 170^\circ \\
        100^\circ + x &= 170^\circ \\
        x &= 70^\circ
      \end{align*}
      \]
    ii) The whole angle is $160^\circ$.
      \[
      \begin{align*}
        40^\circ + 50^\circ + x &= 160^\circ \\
        90^\circ + x &= 160^\circ \\
        x &= 70^\circ
      \end{align*}
      \]

    b) Solve your equations from part a) to find $x$.
    i) $x = 70^\circ$
    ii) $x = 70^\circ$

BONUS ► Write and solve an equation to find $x$.
The whole angle is $130^\circ$.
\[
\begin{align*}
  x + 45^\circ &= 130^\circ \\
  x &= 85^\circ
\end{align*}
\]
Unit 9: Geometry
Test (Lessons 14 to 22)

1. a) 

b) 

c) 

2. a) 

b) 

c) 

3. a) 

b) 

4. a) \[ \frac{2}{6}, 120^\circ \] 

b) \[ \frac{1}{5}, 72^\circ \] 

5. a) 

b) 

6. Andrew should only have subtracted 100°. The 100° and 30° angles overlap.

7. a) 90° 

b) 150° 

8. a) 10° 

b) 120 minutes 

2 hours (use the table to skip count the number of minutes in 2 hours)

<table>
<thead>
<tr>
<th>1</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
</tr>
<tr>
<td>6</td>
<td>360</td>
</tr>
<tr>
<td>7</td>
<td>420</td>
</tr>
</tbody>
</table>

BONUS 360 minutes 6 hours

ADVANCED

9. a) 

i) \[ x = 130^\circ - 50^\circ \] 

ii) \[ x = 111^\circ - 31^\circ \] 

b) 

i) \[ x = 80^\circ \] 

ii) \[ x = 80^\circ \] 

10 a) 

i) \[ x = 170^\circ - 55^\circ - 45^\circ \] 

ii) \[ x = 160^\circ - 40^\circ - 50^\circ \] 

b) 

i) \[ x = 70^\circ \] 

ii) \[ x = 70^\circ \] 

BONUS \[ x = 130^\circ - 90^\circ \] 

\[ x = 40^\circ \]
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 6: Operations and Algebraic Thinking**

**Test (Lessons 39 to 44), p. W-32**

**Common Core State Standards Emphasized:** 4.OA.B.4, 4.NBT.B.6

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>19, 14, 9, 4</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>7, 5, 3, 1</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives 4 or 5 correct answers</td>
<td>0 43</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives 1, 2, or 3 correct answers</td>
<td>1 33</td>
<td>1</td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 23</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives 3 correct answers</td>
<td>0 11</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Gives 1 or 2 correct answers</td>
<td>1 6</td>
<td></td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 1</td>
<td></td>
<td>/3</td>
</tr>
<tr>
<td>3</td>
<td>Correctly fills in table</td>
<td>1 1 10</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>1 2 14</td>
<td></td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 1 16</td>
<td></td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Skip counts correctly</td>
<td>4, 8, 12, 16</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Skip counts correctly</td>
<td>3, 6, 9, 12, 15</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Skip counts correctly</td>
<td>30, 60, 90</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer to division</td>
<td>5 R 1</td>
<td>0.5</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to factor question</td>
<td>no</td>
<td></td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer to division</td>
<td>4 R 0</td>
<td>0.5</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to factor question</td>
<td>yes</td>
<td></td>
<td>/2</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives all 3 factor pairs</td>
<td>1 16</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives 1 or 2 factor pairs</td>
<td>2 8</td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 4</td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>b) Gives all 4 factor pairs</td>
<td>1 24</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives 1, 2, or 3 factor pairs</td>
<td>2 12</td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 8</td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 6</td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td>7</td>
<td>Circles all 7 prime numbers</td>
<td>3, 5, 7, 11, 13, 19, 23</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Circles 4, 5, or 6 prime numbers</td>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Circles 1, 2, or 3 prime numbers</td>
<td></td>
<td></td>
<td>(0.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>/2</td>
</tr>
</tbody>
</table>
Scoring Guides for Sample Unit Quizzes and Tests
Unit 6: Operations and Algebraic Thinking

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Gives correct answer</td>
<td>no 6, 12, 18, 24, 30</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Skip counts correctly</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>a) Gives correct answer to long division</td>
<td>27 R 1 no</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to multiple question</td>
<td>yes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer to long division</td>
<td>17 R 0 yes</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer to multiple question</td>
<td>yes</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>24 yes / no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct answer</td>
<td>9</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /24
### Rubric for Unit 6: Operations and Algebraic Thinking

**Test (Lessons 39 to 44), p. W-32**

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.</td>
<td>4, 5, 6, 7, 8, 9, Advanced</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

**Comments**


### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 7: Number and Operations—Fractions

**Quiz (Lessons 19 to 23), p. W-36**

**Common Core State Standards Emphasized:** 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

<table>
<thead>
<tr>
<th>Question</th>
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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>$3.12</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>0.76¢</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct fraction</td>
<td>(\frac{3}{10})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct fraction</td>
<td>(\frac{6}{10})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct fraction</td>
<td>(\frac{50}{100})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.50</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td>3</td>
<td>Correctly positions all 3 decimals</td>
<td>Order: C, B, A</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly positions 1 or 2 decimals</td>
<td></td>
<td>(1)</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct fraction with denominator 10</td>
<td>(\frac{5}{10})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct fraction with denominator 100</td>
<td>(\frac{50}{100})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct fraction with denominator 10</td>
<td>(\frac{8}{10})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct fraction with denominator 100</td>
<td>(\frac{80}{100})</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct fraction</td>
<td>(\frac{64}{100})</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct decimal</td>
<td>0.64</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /14
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 7: Number and Operations—Fractions

**Test (Lessons 19 to 29), p. W-38**

**Common Core State Standards Emphasized:** 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives correct answer for A &lt;br&gt; Gives correct answer for B</td>
<td>2, 20 &lt;br&gt; 6, 60</td>
<td>1 &lt;br&gt; 1</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>Correctly estimates the position of A &lt;br&gt; Correctly estimates the position of B &lt;br&gt; Correctly estimates the position of C</td>
<td>right of 0.7 &lt;br&gt; left of 0.1 &lt;br&gt; left of 0.4</td>
<td>1 &lt;br&gt; 1 &lt;br&gt; 1</td>
<td>/3</td>
</tr>
<tr>
<td>3</td>
<td>Gives correct answer &lt;br&gt; Gives correct explanation</td>
<td>&gt; 0.4 is 0.40 and 40 hundredths are larger than 38 hundredths.</td>
<td>1 &lt;br&gt; 1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>$\frac{87}{100}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{49}{100}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>$\frac{51}{100}$</td>
<td>yes / no</td>
<td>/2</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>43.27</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>eleven and six tenths</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>eight thousand and five hundredths</td>
<td>yes / no</td>
<td>/2</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives two correct answers &lt;br&gt; Gives one correct answer</td>
<td>$32.7, \frac{327}{10}$</td>
<td>2 &lt;br&gt; (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives two correct answers &lt;br&gt; Gives one correct answer</td>
<td>$4, 21 \frac{21}{100}$, $421 \frac{100}{100}$</td>
<td>2 &lt;br&gt; (1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives two correct answers &lt;br&gt; Gives one correct answer</td>
<td>$6.03, \frac{6}{100}$, $\frac{3}{100}$</td>
<td>2 &lt;br&gt; (1)</td>
<td>/6</td>
</tr>
<tr>
<td>7</td>
<td>Gives correct answer</td>
<td>$\frac{7}{10}$</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>8, 3, 83</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>0, 4, 4</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>9</td>
<td>Circles both incorrect equalities &lt;br&gt; Circles one incorrect equality</td>
<td>1st and 4th</td>
<td>2 &lt;br&gt; (1)</td>
<td>/2</td>
</tr>
</tbody>
</table>
## Scoring Guides for Sample Unit Quizzes and Tests

**Unit 7: Number and Operations—Fractions**

(continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>0.8</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Gives correct explanation</td>
<td>Kamali’s models are not the same size, so 0.2 appears to be bigger than 0.35, but 0.35 is larger than 0.2.</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points /22**
### Rubric for Unit 7: Number and Operations—Fractions
#### Test (Lessons 19 to 29), p. W-38

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.C.5</td>
<td>1, 4, 7</td>
</tr>
<tr>
<td>Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.</td>
<td></td>
</tr>
<tr>
<td>4.NF.C.6</td>
<td>6, 8, 9, Bonus</td>
</tr>
<tr>
<td>Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</td>
<td></td>
</tr>
<tr>
<td>4.NF.C.7</td>
<td>2, 3, 10 (Advanced)</td>
</tr>
<tr>
<td>Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual model.</td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 8: Measurement and Data

**Quiz (Lessons 25 to 29), p. W-42**

**Common Core State Standards Emphasized:** 4.MD.A.1, 4.NF.B.3, 4.NF.B.4, 4.NBT.B.5

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>3</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Circles correct mark</td>
<td>1</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct mark</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>$\frac{3}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$\frac{7}{8}$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct length</td>
<td>$2\frac{3}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct width</td>
<td>$1\frac{1}{4}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter with units</td>
<td>8 in</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>24</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>84</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>6</td>
<td>Gives correct conversion</td>
<td>$4\text{ ft} = 48\text{ in}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>Gabby is 3 in taller.</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>7</td>
<td>Gives correct answer</td>
<td>The pencil is not placed at the zero mark on the ruler.</td>
<td>1</td>
<td>/1</td>
</tr>
<tr>
<td>Bonus</td>
<td>Circles correct mark</td>
<td></td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /14
### Test (Lessons 25 to 32), p. W-45

**Common Core State Standards Emphasized:** 4.MD.A.1, 4.MD.A.2, 4.NF.B.3, 4.NF.B.4, 4.NBT.B.5

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Circles correct answer</td>
<td>foot</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Circles correct answer</td>
<td>inch</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Circles correct answer</td>
<td>foot</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Draws correct answer</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Draws correct answer</td>
<td></td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>3</td>
<td>Correctly measures sides in inches</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly gives perimeter with units</td>
<td></td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>24</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>12</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>36, 8, 44</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus: d) Gives correct answer</td>
<td>1,200</td>
<td>yes / no</td>
<td>/3</td>
<td></td>
</tr>
<tr>
<td>Bonus: e) Gives correct answer</td>
<td>3,000</td>
<td>yes / no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>24, 36, 48, 60</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>6, 9, 12, 15</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>6</td>
<td>a) Gives correct answer</td>
<td>2, 3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>4, 5</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>7</td>
<td>Gives correct conversion</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td></td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>8</td>
<td>a) Gives correct answer with units</td>
<td>$14 \times 3 = 42$ ft</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>$42 \div 6 = 7$ ft</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>9</td>
<td>Gives correct total in inches</td>
<td>120 in</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct total in feet</td>
<td>10 ft</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>10</td>
<td>Gives correct growth after 12 days</td>
<td>$2 \times 12 = 24$ in = 2 ft</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>8 ft tall</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>48 in, 5 ft, 2 yd</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /24
# Rubric for Unit 8: Measurement and Data

**Test (Lessons 25 to 32), p. W-45**

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.A.1</td>
<td>Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</td>
<td>1, 4, 5, 6, Bonus (p. W-46)</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
</tr>
<tr>
<td>4.MD.A.2</td>
<td>Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</td>
<td>2, 3, 7, 8, 9, 10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**

Name: ____________________
### Question How to Score Answer Number of Points Total Points

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a) Gives correct answer</td>
<td>10</td>
<td>1</td>
<td>20</td>
<td>10</td>
<td>200</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>11</td>
<td>1</td>
<td>15</td>
<td>3</td>
<td>8</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Gives correct answer for length and width</td>
<td>4, 2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for area</td>
<td>4 × 2 = 8</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for length and width</td>
<td>7, 3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for area</td>
<td>7 × 3 = 21</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>Yes, since 4 × 3 = 3 × 4.</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>24 ÷ 4 = 6 cm</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Gives correct answer</td>
<td>20, 10, 200</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>20, 15, 300</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td>100 ft²</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Gives correct answer with units</td>
<td>30 + 30 + 30 = 90 m</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$9,000</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives three correct dimensions</td>
<td>1</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
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</table>

**Total Points** /16
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 8: Measurement and Data

**Test (Lessons 33 to 40). p. W-51**

**Common Core State Standards Emphasized:** 4.MD.A.1, 4.MD.A.2, 4.MD.B.4, 4.NBT.B.5, 4.NF.B.3

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>13</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>15</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>$4 \times 3 = 12$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>$3 \times 8 = 24$</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer with units</td>
<td>$6 \text{ m}^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer with units</td>
<td>$8 \text{ ft}^2$</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct answer with units</td>
<td>4 ft</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td>Divide the area by the length.</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>5</td>
<td>Measures the length and width of A correctly</td>
<td>[8 \text{ cm}, 2 \text{ cm}]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter of A with units</td>
<td>20 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct area of A with units</td>
<td>$16 \text{ cm}^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Measures the length and width of B correctly</td>
<td>[3 \text{ cm}, 3 \text{ cm}]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter of B with units</td>
<td>12 cm</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct area of B with units</td>
<td>$9 \text{ cm}^2$</td>
<td>1</td>
<td>1/6</td>
</tr>
<tr>
<td>6</td>
<td>a) Correctly completes number line</td>
<td>1, 2, 3, 4, 5, 6, 7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly builds line plot</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>[\frac{5}{8} \text{ of a mile}]</td>
<td>1</td>
<td>1/3</td>
</tr>
<tr>
<td>Bonus</td>
<td>Correctly finds width</td>
<td>$12 + 3 = 4 \text{ cm}$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct perimeter</td>
<td>$14 \text{ cm}$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Correctly makes two rectangles</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>b) Gives correct answer for vertical division</td>
<td>$5 \times 2 = 10$ and $2 \times 3 = 6$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or Gives correct answer for horizontal division</td>
<td>$2 \times 2 = 4$ and $3 \times 4 = 12$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td>$16 \text{ m}^2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>$160$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>a) Gives correct answer</td>
<td>9, 10, 11, 12, 13, 14</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>b) Gives correct answer</td>
<td>[\frac{5}{4} \text{ mi}]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>[\frac{13}{8} + \frac{13}{8} + \frac{14}{8} = \frac{40}{8} = 5 \text{ mi}]</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** | 1/17 |
### Rubric for Unit 8: Measurement and Data

Test (Lessons 33 to 40), p. W-51

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
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</thead>
<tbody>
<tr>
<td>4.MD.B.4</td>
<td></td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td><strong>Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</strong></td>
<td>6, 8 (Advanced)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Comments
Scoring Guides for Sample Unit Quizzes and Tests
Unit 9: Geometry

<table>
<thead>
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<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>ray</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>line segment</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>line</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td></td>
<td>/3</td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly extends lines and rays</td>
<td>line</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly circles intersection point</td>
<td>line</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly extends lines and rays</td>
<td>line</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly circles intersection point</td>
<td>line</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly extends lines and rays</td>
<td>line</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly circles intersection point</td>
<td>line</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td></td>
<td>/3</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>obtuse</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>obtuse</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td>4</td>
<td>a) Correctly identifies angle</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct measurement</td>
<td>line</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly identifies angle</td>
<td>obtuse</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct measurement</td>
<td>line</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td>5</td>
<td>a) Draws correct angle</td>
<td>line</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Draws correct angle</td>
<td>line</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Total Points</strong></td>
<td></td>
<td></td>
<td>/16</td>
</tr>
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</table>

### Scoring Guidelines for Sample Unit Quizzes and Tests

**Unit 9: Geometry**

(continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Marks all 4 angles correctly</td>
<td><img src="image1.png" alt="Image" /></td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Marks 2 or 3 angles correctly</td>
<td><img src="image2.png" alt="Image" /></td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>b) Marks all 4 angles correctly</td>
<td><img src="image3.png" alt="Image" /></td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Marks 2 or 3 angles correctly</td>
<td><img src="image4.png" alt="Image" /></td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>c) Marks all 3 angles correctly</td>
<td><img src="image5.png" alt="Image" /></td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Marks 2 angles correctly</td>
<td><img src="image6.png" alt="Image" /></td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly extends the line</td>
<td><img src="image7.png" alt="Image" /></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>A</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly extends the line</td>
<td><img src="image8.png" alt="Image" /></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>A, B</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly extends the ray</td>
<td><img src="image9.png" alt="Image" /></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Draws correct angle</td>
<td><img src="image10.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Draws correct angle</td>
<td><img src="image11.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer for fraction</td>
<td><img src="image12.png" alt="Image" /></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for angle</td>
<td>$\frac{2}{5}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for fraction</td>
<td><img src="image13.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for angle</td>
<td>$120^\circ$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer for fraction</td>
<td><img src="image14.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for angle</td>
<td>$\frac{1}{5}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer for fraction</td>
<td><img src="image15.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for angle</td>
<td>$72^\circ$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Correctly marks all 21 angles</td>
<td><img src="image16.png" alt="Image" /></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly marks 16–20 angles</td>
<td><img src="image17.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly marks 11–15 angles</td>
<td><img src="image18.png" alt="Image" /></td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly sorts all 5 shapes</td>
<td><img src="image19.png" alt="Image" /></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly sorts 3 or 4 shapes</td>
<td><img src="image20.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly sorts 1 or 2 shapes</td>
<td><img src="image21.png" alt="Image" /></td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Gives correct explanation</td>
<td><img src="image22.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>He should only have subtracted 100°. The 100° and 30° angles overlap.</td>
<td><img src="image23.png" alt="Image" /></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer</td>
<td><img src="image24.png" alt="Image" /></td>
<td>30, 60, 90°</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td><img src="image25.png" alt="Image" /></td>
<td>55, 95, 150°</td>
<td>1</td>
</tr>
</tbody>
</table>
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 9: Geometry

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>a) Gives correct answer</td>
<td>10°</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>120 min, 2 hr</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>360 min, 6 hrs</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) i) Gives correct answer</td>
<td>(x = 130 - 50)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>(x = 111 - 31)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) i) Gives correct answer</td>
<td>(x = 80°)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>(x = 80°)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) i) Gives correct answer</td>
<td>(x = 170 - 55 - 45)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>(x = 160 - 40 - 50)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) i) Gives correct answer</td>
<td>(x = 70°)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) Gives correct answer</td>
<td>(x = 70°)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Advanced</td>
<td>Bonus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>(x = 130 - 90) (x = 40°)</td>
<td>yes / no</td>
<td></td>
</tr>
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</table>

**Total Points** /24
### Rubric for Unit 9: Geometry

Test (Lessons 14 to 22), p. W-57

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.G.A.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</td>
<td>1, 2, 5.a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.G.A.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</td>
<td>5.b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.MD.C.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.MD.C.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Rubric for Unit 9: Geometry
Test (Lessons 14 to 22), p. W-57

### Common Core State Standard

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</td>
<td>6, 7, 9 (Advanced), 10 (Advanced), Bonus (Advanced, p. W-59)</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td>8, Bonus (p. W-58)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Comments
Grade 4 Common Core State Standards Curriculum Correlations

NOTE: The italicized gray JUMP Math lessons contain prerequisite material for the Common Core standards.

**Domain**
- OA Operations and Algebraic Thinking
- NBT Number and Operations in Base Ten
- NF Number and Operations—Fractions
- MD Measurement and Data
- G Geometry

**Cluster**

### 4.OA Operations and Algebraic Thinking

#### 4.OA.A

**4.OA.A.1** Use the four operations with whole numbers to solve problems.

Interpret a multiplication equation as a comparison, e.g., interpret \(35 = 5 \times 7\) as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OA4-9 to 11, OA4-12</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>OA4-35 to 38</td>
</tr>
</tbody>
</table>

**4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>NBT4-36, 39</td>
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<tr>
<td>1</td>
<td>2</td>
<td>OA4-21, OA4-24, 25</td>
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<tr>
<td>1</td>
<td>6</td>
<td>MD4-2 to 9, 16, 17</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>OA4-34 to 38</td>
</tr>
</tbody>
</table>

**4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OA4-1 to 3, OA4-7, 8</td>
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<td>1</td>
<td>3</td>
<td>OA4-17</td>
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<tr>
<td>1</td>
<td>5</td>
<td>OA4-18 to 21, 23, OA4-24</td>
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<tr>
<td>1</td>
<td>2</td>
<td>NBT4-22, NBT4-23, 24</td>
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<tr>
<td>1</td>
<td>6</td>
<td>MD4-7, 11, 12, 15</td>
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<td>2</td>
<td>NBT4-40, 41, NBT4-49, 50</td>
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<tr>
<td>2</td>
<td>9</td>
<td>G4-19</td>
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<tr>
<td>2</td>
<td>3</td>
<td>OA4-28 to 31, OA4-32, 33, 35 to 38</td>
</tr>
</tbody>
</table>
## 4.OA  Operations and Algebraic Thinking

### 4.OA.B  Gain familiarity with factors and multiples.

4.OA.B.4  Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>OA4-39 to 44</td>
</tr>
</tbody>
</table>

### 4.OA.C  Generate and analyze patterns.

4.OA.C.5  Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OA4-1 to 3 OA4-4 to 8</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>OA4-26, 27</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NBT4-51</td>
</tr>
</tbody>
</table>
## 4.NBT  Number and Operations in Base Ten

### 4.NBT.A  Generalize place value understanding for multi-digit whole numbers.

| 4.NBT.A.1 | Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that \( 700 ÷ 70 = 10 \) by applying concepts of place value and division. |

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NBT4-2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>NBT4-29</td>
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<td>2</td>
<td>NBT4-47</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-24</td>
</tr>
</tbody>
</table>

| 4.NBT.A.2 | Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using \( >, =, \) and \( < \) symbols to record the results of comparisons. |

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NBT4-1 to 9</td>
</tr>
</tbody>
</table>

| 4.NBT.A.3 | Use place value understanding to round multi-digit whole numbers to any place. |

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>OA4-13 to 16</td>
</tr>
</tbody>
</table>

## 4.NBT  Number and Operations in Base Ten

### 4.NBT.B  Use place value understanding and properties of operations to perform multi-digit arithmetic.

| 4.NBT.B.4 | Fluently add and subtract multi-digit whole numbers using the standard algorithm. |

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OA4-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>NBT4-10 to 13 NBT4-14 to 21, 25 to 27</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>MD4-7, 14, 17</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-19, 20, 22 to 24</td>
</tr>
</tbody>
</table>

| 4.NBT.B.5 | Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>NBT4-28 NBT4-30 NBT4-31 NBT4-32 to 39</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-19, 20, 22, 24</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-28 to 32, 34 to 37</td>
</tr>
</tbody>
</table>

| 4.NBT.B.6 | Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>OA4-18 to 20, 22, 23, OA4-24</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NBT4-40 to 42 NBT4-43 to 48</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-22</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>OA4-41 to 43</td>
</tr>
</tbody>
</table>
### 4.NF  Number and Operations—Fractions

#### 4.NF.A  Extend understanding of fraction equivalence and ordering.

<table>
<thead>
<tr>
<th>4.NF.A.1</th>
<th>Explain why a fraction (a/b) is equivalent to a fraction ((n \times a)/(n \times b)) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.NF.A.2</th>
<th>Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual fraction model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

#### 4.NF.B  Build fractions from unit fractions.

| 4.NF.B.3 | Understand a fraction \(a/b\) with \(a > 1\) as a sum of fractions \(1/b\). |
| 4.NF.B.3a | Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. |
| 4.NF.B.3b | Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* \(3/8 = 1/8 + 1/8 + 1/8\); \(3/8 = 1/8 + 2/8\); \(2 1/8 = 1 + 1/8 + 8/8 + 8/8 + 1/8\). |
| 4.NF.B.3c | Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. |
| Part | Unit | Lessons |
| 2 | 4 | NF4-10, 11, 17 |
| 2 | 9 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |

| Part | Unit | Lessons |
| 2 | 4 | NF4-11, 17, 18 |
| 2 | 9 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |

| Part | Unit | Lessons |
| 2 | 4 | NF4-12 to 14 NF4-15 NF4-16, 18 |
| 2 | 9 | G4-20 |
| 2 | 8 | MD4-26, 27, 39, 40 |
### 4.NF.B.3d
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>NF4-11, 14</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-24</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-26, 27, 39, 40</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-20</td>
</tr>
</tbody>
</table>

### 4.NF.B.4
Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

#### 4.NF.B.4a
Understand a fraction \( a/b \) as a multiple of \( 1/b \).
For example, use a visual fraction model to represent \( 5/4 \) as the product \( 5 \times (1/4) \), recording the conclusion by the equation \( 5/4 = 5 \times (1/4) \).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-24</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-26, 27</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-20</td>
</tr>
</tbody>
</table>

#### 4.NF.B.4b
Understand a multiple of \( a/b \) as a multiple of \( 1/b \), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \( 3 \times (2/5) \) as \( 6 \times (1/5) \), recognizing this product as \( 6/5 \). (In general, \( n \times (a/b) = (n \times a)/b \).)

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-24</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-26, 27</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-20</td>
</tr>
</tbody>
</table>

#### 4.NF.B.4c
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat \( 3/8 \) of a pound of roast beef, and there will be \( 5 \) people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-24</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-26, 27</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-20</td>
</tr>
</tbody>
</table>

### 4.NF.C
#### 4.NF.C.5
Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express \( 3/10 \) as \( 30/100 \), and add \( 3/10 + 4/100 = 34/100 \).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>NF4-21, 26, 27</td>
</tr>
</tbody>
</table>

#### 4.NF.C.6
Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>NF4-19, NF4-22, 24, 25</td>
</tr>
</tbody>
</table>

#### 4.NF.C.7
Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>NF4-20, NF4-21 to 25, 28, 29</td>
</tr>
</tbody>
</table>
### 4.MD Measurement and Data

#### 4.MD.A Solve problems involving measurement and conversion of measurements.

<p>| 4.MD.A.1 | Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>MD4-1, MD4-2, 5 to 9, 14, 16, 17</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD4-18 to 23</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-25, 28 to 32, 34 to 37</td>
</tr>
</tbody>
</table>

<p>| 4.MD.A.2 | Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OA4-9 to 11</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>MD4-3, 4, 8, 9 MD4-13 MD4-15 to 17</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>NBT4-11, 12, 13</td>
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<td>NBT4-45, 46</td>
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<td>MD4-19, 20, 22, 24</td>
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<td>8</td>
<td>MD4-32, 34 to 37</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-20</td>
</tr>
</tbody>
</table>

<p>| 4.MD.A.3 | Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>MD4-10 MD4-11, 12</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-33</td>
</tr>
</tbody>
</table>

### 4.MD Measurement and Data

#### 4.MD.B Represent and interpret data.

<p>| 4.MD.B.4 | Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
<td>MD4-27 MD4-38, 39, 40</td>
</tr>
<tr>
<td>4.MD</td>
<td>Measurement and Data</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>4.MD.C</td>
<td>Geometric measurement: understand concepts of angle and measure angles.</td>
<td></td>
</tr>
<tr>
<td>4.MD.C.5</td>
<td>Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</td>
<td></td>
</tr>
<tr>
<td>4.MD.C.5a</td>
<td>An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-15, 20</td>
</tr>
<tr>
<td>4.MD.C.5b</td>
<td>An angle that turns through ( n ) one-degree angles is said to have an angle measure of ( n ) degrees.</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-15, 20</td>
</tr>
<tr>
<td>4.MD.C.6</td>
<td>Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-16, 17, 21</td>
</tr>
<tr>
<td>4.MD.C.7</td>
<td>Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</td>
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</tr>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>G4-18, 19</td>
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<tr>
<td>4.G</td>
<td>Geometry</td>
<td></td>
</tr>
<tr>
<td>------</td>
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<td></td>
</tr>
<tr>
<td>4.G.A</td>
<td>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.G.A.1</th>
<th>Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</th>
</tr>
</thead>
<tbody>
<tr>
<td>JUMP Math Grade 4 Lessons</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
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<tr>
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</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.G.A.2</th>
<th>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</th>
</tr>
</thead>
<tbody>
<tr>
<td>JUMP Math Grade 4 Lessons</td>
<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
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<td>2</td>
<td>9</td>
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</table>

<table>
<thead>
<tr>
<th>4.G.A.3</th>
<th>Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</th>
</tr>
</thead>
<tbody>
<tr>
<td>JUMP Math Grade 4 Lessons</td>
<td></td>
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<tr>
<td>Part</td>
<td>Unit</td>
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