Grade 5
End of Year Teacher Pack

Includes:

- Lesson Plans
- Blackline Masters
- Assessment & Practice Book Answers
- Quizzes and Unit Tests
- Rubrics for Scoring Quizzes and Tests
- Common Core State Standards Correlations
Unit 5 Measurement and Data: US Customary Units

This Unit in Context

In 5.1 Unit 7, students measured length using the same metric units they covered in Grade 4, and they converted between different-sized metric units of length. Students in this unit will measure lengths to the nearest eighth of an inch, just as they did in Grade 4 (4.MD.B.4). Students will also convert between different-sized US customary units of length (5.MD.A.1)—including miles, which they did not cover in Grade 4. Grade 4 students converted from larger to smaller units and were restricted to conversions that resulted in whole number answers; in this grade, students will convert between units in both directions and their conversions can result in decimals or fractions (5.MD.A.1). In higher grades, students will be required to convert between units of area within the same measurement system. For example, in Grade 7, when students study scale drawings (7.G.A.1), they will need to convert between area measurements given in square yards to square feet.

In Grade 4, students measured mass using grams, kilograms, pounds, and ounces. Students converted from kilograms to grams and from pounds to ounces (4.MD.A.1). In 5.1 Unit 7, students measured mass using the metric units (covered in Grade 4) and converted between kilograms and grams in both directions. In this unit, students will measure mass using pounds and ounces, and they will convert between pounds and ounces in both directions (5.MD.A.1).

Starting in Grade 1, students learned to tell time to the hour and half hour (1.MD.B.3) using analog clocks, digital clocks, and words like “o’clock” and “half past.” By Grade 2, students had learned to tell time to the nearest five minutes (2.MD.C.7) and assign times of day to events by placing events on a timeline (which looks like a number line), distinguishing between events that occur in the a.m. and p.m. (2.MD.C.7). In Grade 3, students learned to tell time to the nearest minute (3.MD.A.1) and they learned that they can look at what the clock says when an event starts and when it ends to determine how much time the event takes. Grade 4 students used multiplication and addition to solve problems involving time intervals (4.MD.A.2). They also converted from larger units (including mixed units) of time to smaller units of time (4.MD.A.1). In this unit, students will continue converting between units of time (both from larger to smaller and smaller to larger) in order to solve multistep real-world problems involving elapsed time (5.MD.A.1). Students will also learn about the 24-hour system of recording time and convert between the two systems of recording time.

As students do measurement conversions and use these conversions to solve real-world multistep problems (5.MD.A.1), students will apply many of the skills they reviewed and learned in 5.1: reducing fractions to lowest terms, using division of whole numbers to produce a fraction (5.NF.B.3), converting between mixed numbers and improper fractions (4.NF.B.3c), comparing fractions with different numerators and different denominators (4.NF.A.2), and performing all four operations with multi-digit whole
numbers (4.NBT.B.4, 5.NBT.B.5, 6), decimals (5.NBT.B.4), and fractions (5.NF.A.1, 5.NF.B.4, 7). Note that division of fractions was restricted to dividing fractions by whole numbers or whole numbers by unit fractions. Students will continue to solve real-world problems involving many measurement attributes throughout their school years.

In this unit, students will read and draw line plots. To do so, students will use their knowledge of representing numbers on number lines from Grade 2 (2.MD.B.6), representing fractions on number lines from Grade 3 (3.NF.A.2), and measuring to the nearest eighth of an inch from Grade 4 (4.MD.B.4). They will solve problems with data presented in line plots that require measurement conversions and adding fractions and mixed numbers with different denominators (5.MD.B.2).

Between Grades 3 and 5, the concept of scale on a line plot extends mainly through changes to the horizontal scale: the horizontal scale on a line plot in Grade 3 may be marked off using whole numbers, halves, or fourths (3.MD.B.4); and in Grades 4 and 5, it may be marked off using whole numbers, halves, fourths, or eighths (4.MD.B.4, 5.MD.B.2). This work on data management in Kindergarten through to Grade 5 prepares students for the study of statistics from Grades 6 through to high school.

**Mathematical Practices in This Unit**

In this unit, you will have the opportunity to assess MP1 to MP7. Here are some examples of how students can show that they have met a standard.

**MP2:** In MD5-20 Extension 3, students reason abstractly and quantitatively when they interpret the data shown in a line plot in the context of a real-world situation. They read data from the line plot to create equations and then interpret the results of their equations to solve a real-world problem.

**MP4:** In MD5-15 Extension 3, students model mathematically when they use either a T-table or number lines to model and solve a real-world problem. In MD5-21 Extension 1, students use equations and/or timelines to model and solve a real-world problem involving elapsed time.

**MP6:** In MD5-11 Extension 3, students attend to precision when they measure distances accurately and when they specify the correct units in their measurements.

**MP7:** In MD5-12 Extension 3, students look for and make use of structure when they recognize the expression $57,831 + 45,671$ as a single object, when they notice that the same expression is being multiplied by two different factors, and when they notice that one product is four times as large as another (because the first factor in one is four times as large as in the other while the second factors are equal).
Unit 5 Measurement and Data:
US Customary Units

Introduction

In this unit, students will learn about US customary units of length and mass. They will convert between different units in the US customary system, and will solve problems requiring such conversions. They will also make and read line plots involving fractions of a unit. Students will learn about the 24-hour system of recording time, convert between the two systems of recording time, and solve problems connected to elapsed time.

When solving problems that require conversion between units, students will add, subtract, multiply, and divide numbers, including fractions, mixed numbers, and decimals. Students need to be able to fluently convert between fractions and mixed numbers. They also need to be fluent with multiplying by 2, 3, 4, 8, and 12. Use BLM Patterns in the Multiples of 12 (p. P-55) to reinforce the patterns in the 12 times tables for students struggling with this concept. For details, see the prior knowledge required for each lesson.
MD5-10  Inches

Vendor: Jump Math
Grade: 5
Edition: US

Standards
preparation for 5.MD.B.2

Vocabulary
fraction part
interval
line segment
mixed number
numerator
perimeter
reduced fraction
unit fraction
whole number

Goals
Students will measure and draw line segments and objects of a given length in inches, to the closest eighth of an inch.

Prior Knowledge Required
Understands that we use standard units of the same size to measure
Can measure in standard units such as centimeters
Is familiar with mixed numbers
Can order mixed numbers
Can read and create number lines with mixed numbers
(with denominators 2, 4, and 8)
Can draw a line segment of a given length in whole inches, centimeters,
or millimeters

Materials
rulers
small objects to measure (e.g., thumbtacks, buttons, pattern blocks)

Introduce inches. Remind students that we commonly use inches in the United States as a unit for measuring length. Give students a number of small objects to measure (e.g., thumbtacks, buttons, pattern blocks) that are close to 1 inch long, and several that are exactly 1 inch. Have students use a ruler to check which items are exactly 1 inch long (or wide or thick).

Measuring in inches. Explain that using an inch ruler is similar to using a centimeter ruler: Students can count “hops” or intervals to tell how many inches long an object is. (See Lesson MD5-1 in the TR for AP Book 5.1.) Remind students to align one end of the object to the zero mark on the ruler, which might not be at the very edge of the ruler. Also remind students that line segments are pieces of a straight line that can be measured. Draw a ruler on the board and mark line segments in whole inches beside it. Have students signal the length of the line segment by holding up the number of fingers equal to the length.

Review number lines with fractions. Draw a number line from 0 to 2, as shown in the margin. Leave enough space between the marks to write the mixed numbers. ASK: How many pieces did I divide each whole unit into? (8) Invite volunteers to mark the fractions between 0 and 1 in eighths: 1/8, 2/8, 3/8, etc. Have volunteers rewrite the marks using reduced fractions. Invite other volunteers to mark the rest of the number line, up to 2. Have students draw a number line (as in the margin) from 4 to 5, divide it into eighths, and fill in the mixed numbers between 4 and 5.
Identifying marks for mixed numbers on a number line with only whole numbers labeled. Present several parts of number lines, for example, from 4 to 5, as shown on the previous page. Give students a mixed number, such as 4 3/8, and ask which mark indicates that number. Point to different marks in turn and have students signal thumbs up if this is the right mark, and thumbs down if it is not. Repeat with several mixed numbers.

**NOTE:** If the rulers students are using have markings for sixteenths of an inch, review Extension 1 (on p. P-4) with your students.

Measuring in eighths of an inch. Display the pictures in the margin. Explain that measuring to the closest eighth of an inch is similar to measuring in millimeters. When measuring to the closest millimeter, you count centimeters first and then count millimeters, so the first line segment is 2 cm 3 mm long or 23 mm long. Similarly, you count whole inches first and then eighths after, so the second line segment is 2 3/8 inches long.

**ACTIVITY**

Have students work in pairs. Ask each partner to draw a line segment in their notebook starting at 0 and ending at one of the marks for the eighths of an inch. Ask partners to exchange notebooks and measure the line segment their partner drew to the closest eighth of an inch. Partners should then check each other’s answers.

Measuring to the nearest eighth of an inch. Point out that even though eighths of an inch are rather small, objects often have lengths that are between two eighth-inch marks. Provide several examples of measuring objects to the closest eighth of an inch, and then have students measure actual objects to the closest eighth of an inch. Students can work in pairs, checking each other’s work.

Solving problems with fractions. Review multiplying a fraction by a whole number and dividing a whole number by a unit fraction before assigning the exercises below.

**Exercises**

a) A paperclip is \( \frac{3}{8} \) in long. Three paperclips are laid end-to-end. What is the total length of the paperclips?

b) Six ants each measuring \( \frac{5}{8} \) in are walking in a line. There is no space between them. How long is the line of ants?

c) Ron draws a line segment 3 in long. He stacks \( \frac{1}{4} \) in cubes along the line segment. How many cubes long is the line segment?

**Bonus:** Alice makes a chain from two paperclips each measuring \( \frac{7}{8} \) in long, and three paperclips each measuring \( \frac{3}{8} \) inches long. What is the total length of the chain?
Solutions
a) \(3 \times 3 = 9\), so \(3 \times 3/8 = 9/8\) in \(= 1\ 1/8\) in
b) \(6 \times 5 = 30\), so \(6 \times 5/8 = 30/8\) in \(= 3\ 3/4\) in
c) 4 cubes fit along 1 in, so \(3 \times 4 = 12\) cubes fit along a 3 in line segment
Bonus: \(1\ 7/8\) in \(= 15/8\) in, so two long paper clips measure \(30/8\) in;
\(1\ 3/8\) in \(= 11/8\) in, so three short paper clips measure \(33/8\) in. The total
length is \(30/8 + 33/8 = 63/8\) in \(= 7\ 7/8\) in.

Remind students that perimeter is the distance around a shape before
assigning the AP book pages. Students can use real pennies to check their
answers to Question 7 on AP Book 5.2 p. 83.

Extensions
1. **Identifying marks for halves, quarters, and eighths on a number
   line with sixteenths marked.** Point out that students' rulers have more
   than eight marks between the whole inch marks. ASK: How many parts
   is each inch divided into on your rulers? Give students time to count.
   (16) Remind students that taking half of a fraction means dividing each
   part of the fraction into two parts, so the number of parts doubles. What
   is the double of \(8\)? (16) What is half of \(1/8\)? (1/16)

   Have students identify the numerator in each fraction equation below.

   a) \(\frac{2}{16} = \frac{8}{4}\)  b) \(\frac{12}{16} = \frac{4}{8}\)  c) \(\frac{10}{16} = \frac{8}{16}\)  d) \(\frac{8}{16} = \frac{1}{2}\)

   **Answers:** a) 1, b) 3, c) 5, d) 1

   Explain that, if each inch is divided into 16 parts on the ruler, this means
   that each eighth of an inch is divided into two parts. Since students
   need to measure to the closest eighth of an inch, they need to look at
   the marks for the eighths, not the sixteenths of an inch. Point out that
   the sixteenths are the shortest marks on the ruler. Display an enlarged
   ruler with marks the same as those on the rulers students use. Point
to several marks in turn, and have students identify whether they are
marks for eighths, quarters, or halves of an inch; students can signal
thumbs up or thumbs down. Repeat with just eighths, just quarters,
and just halves. Finally, mark the part of the ruler on the board starting
at 3 inches and ending at 4 inches. Have students identify whether
these marks show 3 1/2 inches, 3 7/8 inches, 3 1/8 inches, etc. Point
to different marks in turn, including the marks for sixteenths, and have
students signal thumbs up or down whether this is the mark for the
measurement you’ve given.

2. Have students make a figure from pattern blocks and find its perimeter.
MD5-11  Feet and Inches
Pages 84–85

STANDARDS
5.MD.A.1, 5.NBT.B.5, 5.NF.B.3

VOCABULARY
conversion
convert
denominator
lowest terms
remainder

Goals
Students will convert between feet and inches.

PRIOR KNOWLEDGE REQUIRED
Can measure length and distance to the closest unit
Can multiply two-digit numbers
Can reduce a fraction to lowest terms
Knows that division of two whole numbers can produce a fraction
Knows that a fraction can be regarded as division

MATERIALS
rulers
yardsticks
a measuring tape for every student
BLM Patterns in the Multiples of 12 (p. P-55)

Review feet. Identify the length and marking of a foot on a ruler, a yardstick, or a measuring tape. Remind students that small distances and lengths of larger objects are measured in feet. Review the short form of the unit: ft.

How many inches are in 1 foot? Give each student or pair of students a measuring tape showing both feet and inches. ASK: How does a measuring tape show feet and inches at the same time? How can you tell how many inches are in a foot? (The answer will depend on the measuring tape used. Some have measurements in feet and in inches written beside the hash mark for feet, and some have only the feet measurement written. If some of the tapes you are using are of the second type, ensure students look at the numbers on both sides of the 1 foot mark, 11 inches before and 13 inches after, so 1 ft = 12 in.)

ACTIVITY
Converting feet to inches using a ruler and a measuring tape. Find an object in the classroom that is an exact number of feet long. If there is no such object in the classroom, create a line of masking tape on the floor that is an exact number of feet long. Have students use a 1 ft ruler to measure the object. Then have them use a measuring tape to measure the same distance in inches and write a conversion statement.

Converting feet to inches. Draw the table in the margin on the board. Have students copy the table, and then use their measuring tapes to fill in the empty column. Extend the table by three rows and have students use the pattern to fill in the table. ASK: What pattern did you use to fill in the right-hand column? (skip counting by 12, starting from 12; students might
also notice that the numbers are multiples of 12) Why did you skip count by 12 and not by any other number? (1 ft = 12 in) How can you get the number in the right-hand column from the number in the same row in the left-hand column? (multiply by 12) Leave the table on the board for further reference.

NOTE: For students who have trouble multiplying by 12, use BLM Patterns in the Multiples of 12 to reinforce the 12s times tables.

Exercises: Multiply by 12 to convert measurements from feet to inches.

a) 10 ft  
   b) 12 ft  
   c) 30 ft  
   d) 80 ft  

Answers: a) 120 in, b) 144 in, c) 360 in, d) 960 in

Review multiplying two-digit and three-digit numbers, and have students convert larger measurements to inches.

Exercises: Convert from feet to inches.

a) 21 ft  
   b) 320 ft  
   c) 150 ft  
   d) 256 ft  

Answers: a) 252 in, b) 3,840 in, c) 1,800 in, d) 3,072 in

Comparing measurements. Remind students that when they compare measurements in different units, they need to first convert the measurements to the same unit. It is usually easier to convert the measurement in larger units to the smaller units.

Before assigning the following exercises, write two numbers, such as 7 and 10, on the board. Ask students to signal which number is larger by showing a “greater than” or “less than” sign as a “V” sign pointing sideways. Repeat with a few other pairs of numbers to practice the signaling. Then present each of the following exercises individually and have students signal the answer.

Exercises: Convert the measurement in feet into inches. Then compare the measurements. Which is larger?

a) 10 ft 100 in  
   b) 12 in 2 ft  
   c) 53 ft 600 in  
   d) 250 in 20 ft  

Bonus: 1,000 ft or 100,000 in

Answers: a) 10 ft = 120 in, so 10 ft > 100 in; b) 12 in < 2 ft; c) 53 ft > 600 in;  
   d) 250 in > 20 ft; Bonus: 1,000 ft < 100,000 in

Converting mixed measurements to measurements in inches. Draw students’ attention to how inches are indicated on a measuring tape for measurements longer than 1 foot. Many measuring tapes give the length in inches, such as 15 inches, instead of (or together with) 1 ft 3 in. Have students think of how to convert a mixed measurement, such as 3 ft 4 in, into a measurement in inches only. They can discuss the ideas in pairs, and then in groups of four. To prompt students to think of the following method, ask them to recall what they did with other units of measurement, such as centimeters and meters.
To convert a mixed measurement such as 3 ft 4 in to inches:

**Step 1**: Convert feet to inches.

\[ 3 \text{ ft} = 3 \times 12 \text{ in} = 36 \text{ in} \]

**Step 2**: Add the leftover inches.

\[ 36 \text{ in} + 4 \text{ in} = 40 \text{ in} \]

So, 3 ft 4 in = 40 in

**Exercises**: Convert the mixed measurement to inches.

a) 7 ft 2 in  
  b) 2 ft 5 in  
  c) 5 ft 7 in  
  d) 38 ft 1 in  
  e) 14 ft 8 in  
  f) 29 ft 10 in  
  
**Bonus**:  
  
1. 50 ft 6 in  
2. 1,000 ft 10 in  
3. 2 ft 9 \( \frac{5}{8} \) in

**Answers**:  
a) 86 in, b) 29 in, c) 67 in, d) 457 in, e) 176 in, f) 358 in,  
Bonus: g) 606 in, h) 12,010 in, i) 33 \( \frac{5}{8} \) in

Encourage students who are struggling to practice the steps separately, and then combine them. Refer them to the conversion table from feet to inches (on p. P-5) as well.

**Converting inches to feet using a table**. SAY: If 1 ft = 12 in, then what fraction of a foot is 1 inch? (one twelfth) Start a table as shown in the margin, and have students help you fill it in, using fractions with the denominator 12. Students should also make the same table in their notebooks. Remind students that fractions can be reduced to lowest terms, and have them reduce the fractions where possible.

**Converting mixed measurements to feet**. Tell students that you would like to convert mixed measurements to measurements in feet. Ask them to think how the table they made might help them to do so. Again, students can discuss the solution in pairs and then in groups of four. Have groups present their solutions. (convert the part in inches to feet using the table, and then write the mixed measurement) Then have students convert the mixed measurements in the exercises above to measurements in feet only. Omit part i) of the Bonus.

**Answers**:  
a) 7 1/6 ft, b) 2 5/12 ft c) 5 7/12 ft, d) 38 1/12 ft, e) 14 2/3 ft,  
f) 29 5/6 ft, Bonus: g) 50 1/2 ft, h) 1,000 5/6 ft

**Converting measurements in inches to mixed measurements**.

Explain that you would like to convert measurements in inches to mixed measurements. ASK: How can I find how many whole feet are in 27 inches? (divide 27 by 12) Write on the board:

\[ 27 \div 12 = ____ \text{ R } ____ \]

Have students raise the number of fingers equal to the number you need to put in the first blank. (2) Repeat with the remainder. (R 3) Point out that this means there are 2 whole feet in 27 inches, but there are 3 leftover inches. This means the mixed measurement is 2 ft 3 in.
NOTE: Students who have trouble with dividing by 12 will benefit from a list of multiples of 12. They can also use skip-counting to divide.

Exercises: Convert to a mixed measurement.

a) 29 in  

b) 75 in  

c) 100 in  

Bonus: 1,000 in

Answers: a) 2 ft 5 in, b) 6 ft 3 in, c) 8 ft 4 in, Bonus: 83 ft 4 in

Using division to convert measurements in inches to feet. Remind students that a fraction is another way to write division. Have them rewrite the division following questions as fractions and reduce to lowest terms. Then review division of whole numbers resulting in mixed numbers. Do the first question as a class and have students practice with the rest.

Exercises: Divide.

a) $28 \div 12$  

b) $73 \div 12$  

c) $42 \div 12$  

d) $39 \div 12$

Sample solution: a) $28 \div 12 = \frac{28}{12} = \frac{7}{3}$, $7 \div 3 = 2$ R 1, so $\frac{7}{3} = 2 \frac{1}{3}$

Answers

b) $73/12 = 6 \frac{1}{12}$, c) $42/12 = 7/2 = 3 \frac{1}{2}$, d) $39/12 = 13/4 = 3 \frac{1}{4}$

SAY: To convert from meters to centimeters, we multiply by 100. To convert from centimeters to meters, we divide by 100. To convert from feet to inches, we multiply by 12. What should we do to convert from inches to feet? (divide by 12) How many feet are in 28 inches? (2 1/3) How do you know? (28 $\div 12 = 2 \frac{1}{3}$, so 28 in $= 2 \frac{1}{3}$ ft)

Exercises: Divide by 12 to convert the measurements in inches to feet. Write your answer as a mixed number, reduced to lowest terms.

a) 86 in  

b) 29 in  

c) 69 in  

d) 460 in

Bonus

e) 1,000 in  

f) 10,000 in

Answers: a) $7 \frac{1}{6}$ ft, b) $2 \frac{5}{12}$ ft, c) $5 \frac{3}{4}$ ft, d) $38 \frac{1}{3}$ ft, Bonus: e) $83 \frac{1}{3}$ ft, f) $833 \frac{1}{3}$ ft

Point out that there is another way to solve the same problems. To convert 28 inches to feet, you can divide $28 \div 12 = 2$ R 4, so $28 \div 12 = 2 \frac{4}{12}$, then reduce the fractional part of the mixed number, so $28 \text{ ft} = 2 \frac{1}{3}$ ft. Have students use this alternative way to check their answers in the exercises above.

Extensions

(MP1)  

1. a) Mateo draws a line segment that is 3 ft long. He measures the line segment using the length of his own feet and finds that the line segment is exactly 5 of his feet long. How many feet long is Mateo’s foot?

Answer: 3/5 ft
b) The school corridor is about \(\frac{22}{5}\) of Mateo’s feet wide. How wide is the corridor?

**Solution:** \(22 \times \frac{3}{5} \text{ ft} = \frac{66}{5} \text{ ft} = 13 \frac{1}{5} \text{ ft}\)

c) Using tape on the floor, make a line segment exactly 1 foot long. Measure your foot against the line segment. Is your foot larger than a foot, smaller than a foot, or exactly 1 foot long?

d) Using tape, make a very long line segment on the floor, and make marks of 0 ft, 1 ft, 2 ft, 3 ft, etc., up to 10 ft. Measure the line segment with your own foot, starting at 0 ft, and try to find a mark on the line for 1 ft or higher that is an exact number of your feet. How many feet long is your foot?

e) Use the length of your foot to estimate distances. For example, measure the width of the school corridor in your feet.

2. Convert the measurement from feet to inches, and then solve the problem.

   a) A snail is sitting on a branch. The snail is 2 feet from the tree trunk. It crawls 3 inches farther away from the trunk. How far from the trunk is the snail now?

   b) Another snail is sitting on the same branch. The snail is 3 feet from the trunk. It crawls 4 inches toward the trunk. How far from the trunk is the second snail now?

   c) How far are the snails from each other?

**Answers:** a) 27 in or 2 1/4 ft, b) 32 in or 2 2/3 ft, c) 5 in

(MP6) 3. a) When would you need to measure length or distance with great precision? Think of some examples.

   b) The rectangle in the margin is a sketch of a tabletop with markings for where table legs will be inserted. Draw the rectangle to size, making sure the measurements are exactly as marked.

   c) Exchange rectangles with a partner and check each other’s measurements.

**Sample answer:** a) When you are assembling furniture, the location of the holes on the different parts needs to match exactly. The carpenter needs to precisely measure the distance from a hole to the end of, say, a shelf. Look for students to accurately convert to an appropriate unit (such as inches), to measure accurately, and to specify the correct units in their measurements (MP6).
MD5-12  Yards
Pages 86–87

STANDARDS
5.MD.A.1, 5.NF.B.3, 5.NF.B.7

VOCABULARY
convert
denominator
improper fraction
interval
numerator
yard (yd)

Goals
Students will convert between yards, feet, and inches.

PRIOR KNOWLEDGE REQUIRED
Is familiar with < and > symbols
Can convert measurements between feet and inches
Can identify multiples of 2, 3, and 9
Can multiply and divide multi-digit numbers and decimals
Can convert between mixed numbers and improper fractions
Can multiply fractions and mixed numbers
Knows that the division of two whole numbers can produce a fraction
Can compare fractions with different denominators

MATERIALS
yardsticks

Introduce yards. Ask students to describe what could be measured in yards. (sample answers: distances, length of sport fields, large swimming pools) Write the word “yard” and the abbreviation “yd” on the board, and explain that these are two ways to write “yard.”

Converting yards to feet. Have students look at a yardstick. Does it have marks for feet and inches? How many feet are in 1 yard? (3) How many inches are in 1 yard? (36) Have students make a conversion table from yards to feet (see example in margin), and extend it to eight columns.
ASK: To change a measurement from yards to feet, what number do you multiply by? (3) Why? (1 yard = 3 feet)

Exercises: Convert the measurement in yards to feet.

a) 12 yd  b) 15 yd  c) 50 yd  d) 58 yd  e) 382 yd

Answers: a) 36 ft, b) 45 ft, c) 150 ft, d) 174 ft, e) 1,146 ft

Converting measurements in yards with mixed numbers to feet. Review converting mixed numbers to improper fractions and multiplying fractions by a whole number. Then show students how to convert a measurement with a mixed number of yards to feet.
ASK: How many feet are in 1 5/8 yards? (see sample solution below)

\[
1 \frac{5}{8} \text{ yd} = \frac{8 + 5}{8} \text{ yd} = \frac{13}{8} \times 3 \text{ ft} = \frac{39}{8} \div 8 = 4 \frac{7}{8} \text{ ft}
\]
ASK: Does the answer make sense? What two whole numbers is 1 5/8 between? (1 and 2) SAY: So 1 5/8 is between 1 yd and 2 yd. What two measurements in feet should the answer in feet be between? (1 \times 3 \text{ ft} = 3 \text{ ft} \\
and 2 \times 3 \text{ ft} = 6 \text{ ft}) Is the answer we got between 3 ft and 6 ft? (yes)

Comparing measurements in yards and feet. Provide students with the following pairs of measurements, one at a time, and have students decide which measurement is larger. Students can signal the answer by pointing to the larger side with their thumbs, or by showing a < or a > sign with their fingers.

Exercises: Which measurement is larger?

a) 20 yd or 50 ft 
   b) 15 yd or 35 ft 
   c) 30 ft or 3 yd
   d) 125 yd or 400 ft 
   e) 2 \frac{3}{4} \text{ yd or 8 ft} 
   f) 12 \frac{1}{8} \text{ yd or 37 ft}

Bonus: 1,000 yd or 10,000 in

Answers: a) 20 yd, b) 15 yd, c) 30 ft, d) 400 ft, e) 2 3/4 yd, f) 37 ft,
Bonus: 1,000 yd

Converting measurements in feet to yards. Draw the following table on the board, and fill in the feet row to 12 ft.

| feet | 1 | 2 | 3 | 4 | ...
|------|---|---|---|---|---
| yards|   |   |   |   |   |

Ask volunteers to fill in the columns that they know from the conversion table from yards to feet. (3 ft = 1 yd, 6 ft = 2 yd, 9 ft = 3 yd, 12 ft = 4 yd)

ASK: From looking at the filled columns of the table, what do you do to convert a measurement in feet to yards? (divide by 3) Have students copy the table, and then fill in the remaining columns as a class. Point out that the answers are often mixed numbers. Do the first exercise as a class, and then have students work individually.

Exercises: Convert to yards.

a) 50 ft 
   b) 40 ft 
   c) 400 ft 
   d) 4 \frac{1}{2} ft

Answers: a) 50 \div 3 = 50/3 \text{ yd} = 16 \frac{2}{3} \text{ yd}, b) 13 \frac{1}{3} \text{ yd}, c) 133 \frac{1}{3} \text{ yd}, 
d) 9/2 \text{ ft} = 3/2 \text{ yd}

Converting yards to inches. Remind students that there are 12 inches in 1 foot. Add a third row, “inches,” to the conversion table from yards to feet above, and have students fill in the table. They will see the pattern from the first three entries, which they know from previous lessons (2 \text{ yd} = 6 \text{ ft} \\
= 72 \text{ in}, 3 \text{ yd} = 9 \text{ ft} = 108 \text{ in}, etc.). Have students add 36 to get each next number in the inch row. Remind students that they got the feet row from the yards row by multiplying by 3, and they can multiply by 12 to get from feet to inches. ASK: What number would you multiply the number of yards by to get the number of inches? (36) How do you know? (3 \times 12 = 36, and multiplying first by 3, then by 12, is the same as multiplying by 36)
Strategies to check multiplication on a calculator. Point out that if you want to convert a measurement from yards to inches, you could use a calculator to multiply by 36. Discuss possible ways to check at a glance that the calculation is correct. For example, draw students’ attention to the fact that all numbers for inches in the table are even. Are all the numbers for inches even in the tables that students produced? Is each next number in the table larger than the previous one? If students are familiar with quick ways to check that a number is a multiple of 3 or 9, they can also check this with the numbers in the table. (A number is a multiple of 3 if the sum of its digits is a multiple of 3. A number is a multiple of 9 if the sum of its digits is a multiple of 9.)

Exercises: Multiply by 36 to convert the measurements from yards to inches.

a) 12 yd  b) 15 yd  c) 40 yd  d) 2 1/2 yd  e) 8 5/8 yd

Answers: a) 432 in, b) 540 in, c) 1,440 in, d) 90 in, e) 310 1/2 in

Real-world application of conversions. Explain that you would like to make new curtains for the classroom windows. Have students measure the width of the windows to the closest inch. Explain that, to produce a curtain that hangs properly, you need three times as much cloth as the width of the windows. How much cloth do you need for each window?

Explain that cloth is usually sold in a roll that is a certain measurement wide (e.g., 36 in, 45 in), and a shopper asks for a length of cloth to be cut from the roll (e.g., 1 ft, 2 ft, 2 1/2 ft). Point out that stores sell cloth by the yard and eighths of a yard, so you can buy 2 3/8 yd or 2 1/2 yd, but you cannot buy a length that is between these two, such as 2 5/12 yd. Imagine that the cloth roll’s width can be used as the curtain height and you like a cloth that costs $14.99 per yd. How much cloth should you buy for the length of the window? About how much money, rounding up to the closest dollar, will the cloth cost? Allow students to problem-solve individually, and then in pairs.

Here are two solutions for a window measuring 43 inches.

The window measures 43 inches, so we need 129 in (43 × 3) of cloth.

129 in = 129/36 yd = 3 7/12 yd

Solution 1: 7/12 is only a little (1/12, less than 1/8) more than 1/2, so we will choose 3 5/8 yd of cloth, which will cost about 3 5/8 × 15, or about $55.

Solution 2: Draw a partial number line to represent the interval between 3 and 4. Show both divisions, into 8ths and into 12ths:
As shown on the number line, $3 \frac{7}{12}$ is between $3 \frac{1}{2}$ and $3 \frac{5}{8}$. We need at least $3 \frac{7}{12}$ yd of cloth, so the number of yards we buy should be the larger of those two numbers, $3 \frac{5}{8}$. We need $3 \frac{5}{8}$ yd of cloth, costing about $55$.

**Extensions**

1. Teach students another method to convert mixed number measurements in yards to feet.

   \[
   3 \frac{5}{8} \text{ yd} = 3 \text{ yd} + \frac{5}{8} \text{ yd} \\
   = 3 \times 3 \text{ ft} + \frac{5}{8} \times 3 \text{ ft} \\
   = 9 \text{ ft} + \frac{15}{8} \text{ ft} \\
   = 9 \text{ ft} + 1 \frac{7}{8} \text{ ft} = 10 \frac{7}{8} \text{ ft}
   \]

   a) Convert the measurements in yards to feet.

   i) $12 \frac{5}{8}$ yd
   ii) $6 \frac{3}{4}$ yd
   iii) $15 \frac{7}{8}$ yd
   iv) $55 \frac{1}{2}$ yd

   b) Convert the answers in part a) to inches.

   **Answers**

   a) i) 37 $\frac{3}{8}$ ft, ii) 20 $\frac{1}{4}$ ft, iii) 47 $\frac{5}{8}$ ft, iv) 166 $\frac{1}{2}$ ft
   b) i) 448 $\frac{1}{2}$ in, ii) 243 in, iii) 571 $\frac{1}{2}$ in, iv) 1,998 in

2. Convert to a single unit: inches, feet, and yards.

   a) 12 yd 2 ft 3 in
   b) 15 yd 1 ft 3 in
   c) 40 yd 1 ft 1 in

   **Answers**

   a) 459 in = 38 $\frac{1}{4}$ ft = 12 $\frac{3}{4}$ yd
   b) 555 in = 46 $\frac{1}{4}$ ft = 15 $\frac{5}{12}$ yd
   c) 1,453 in = 121 $\frac{1}{12}$ ft = 40 $\frac{13}{36}$ yd

   (MP3, MP7)

3. a) Ed and Tina each choose a large number:

   Ed’s number: $2 \times (57,831 + 45,671)$
   Tina’s number: $8 \times (57,831 + 45,671)$

   Whose number is larger? How many times larger? Explain how you know without actually calculating.

   b) In pairs, explain your answers to part a). Do you agree with each other? Discuss why or why not.

   **Answers:** a) The only difference between Ed’s expression and Tina’s expression is the first factor. So Tina’s number is 4 times larger, since 8 is 4 times as large as 2.
**Goals**

Students will convert between miles and yards.

**PRIOR KNOWLEDGE REQUIRED**

- Is familiar with < and > symbols
- Can convert length measurements in the US customary system
- Can multiply and divide multi-digit numbers and decimals
- Can multiply fractions and mixed numbers
- Can convert between improper fractions and mixed numbers
- Can reduce fractions to lowest terms
- Knows that division of two whole numbers can produce a fraction

**Introduce miles.** Ask students to describe what could be measured in miles. (e.g., large distances) Write the word “mile” and the abbreviation “mi” on the board, and explain that these are two ways to write “mile.”

**Converting miles to yards.** Explain that one mile is 1,760 yards long. ASK: What operation did you do to convert yards to feet? (multiplied by 3) Why did you use 3? (1 yd = 3 ft) What should you do to convert miles to yards? (multiply by 1,760 because there are 1,760 yards in a mile)

**Exercises:** Convert from miles to yards.

<table>
<thead>
<tr>
<th>a) 2 mi</th>
<th>b) 5 mi</th>
<th>c) 15 mi</th>
<th>d) 40 mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer</td>
<td>3,520 yd</td>
<td>8,800 yd</td>
<td>26,400 yd</td>
</tr>
</tbody>
</table>

**Converting measurements in miles with mixed numbers to yards.**

ASK: How many yards are in half a mile? (880) How do you know? (1,760 ÷ 2) How many yards are in 1/4 a mile? (440) How do you know? (1,760 ÷ 4 or 1/4 is half of half, so 880 ÷ 2 = 440) How many yards are in 3/4 of a mile? (3 × 440 = 1,320 yd) in 1/8 mi? (220 yd) How do you know? (1,760 ÷ 8 or 440 ÷ 2) Summarize the conversions and keep the following on the board for future reference:

\[
\frac{1}{8} \text{ mi} = 220 \text{ yd} \quad \frac{1}{4} \text{ mi} = 440 \text{ yd} \quad \frac{1}{2} \text{ mi} = 880 \text{ yd}
\]

Review converting mixed numbers to improper fractions and multiplying fractions by a whole number. Then show students how to convert a measurement with a mixed number of miles to yards:

\[
\frac{5}{8} \text{ mi} = \frac{8 + 5}{8} \text{ mi} = \frac{13}{8} \times 1,760 \text{ yd} = 13 \times \frac{1}{8} \text{ mi} = 13 \times 220 \text{ yd} = 2,860 \text{ yd}
\]
ASK: Does the answer make sense? What two whole numbers of miles is the number between? (1 mi and 2 mi) How many yards are in 2 miles? (3,520 yd) Point out that the difference between 1,760 and 3,520 is large, so even though $1,760 < 2,860 < 3,520$, this does not give us a very clear picture of whether the answer is correct. SAY: $1 \frac{5}{8}$ is a little larger than $1 \frac{1}{2}$. How many yards are in $1 \frac{1}{2}$ miles? (2,640) How do you know? ($1 + \frac{1}{2} = 1 \frac{1}{2}$, so $1,760 \text{ yd} + 880 \text{ yd} = 2,640 \text{ yd} = 1 \frac{1}{2} \text{ mi}$) SAY: 2,860 is only a little more than 2,640, so the answer makes sense.

**Exercises:** Convert from miles to yards.

a) $\frac{7}{8} \text{ mi} 

b) 1 \frac{3}{8} \text{ mi} 

c) 2 \frac{1}{4} \text{ mi} 

d) $\frac{7}{10} \text{ mi}$

**Answers:** a) 1,540 yd, b) 2,420 yd, c) 3,960 yd, d) 1,232 yd

**Estimating measurements in miles—about how many yards is that?**

Write on the board:

\[ 5 \text{ mi} \quad 5,000 \text{ yd} \]

ASK: How can you tell which distance is larger? (convert 5 mi to yards) Point out that you do not really need to convert miles to yards to say that 5 miles is longer. ASK: What is longer: 1 mile or 1,000 yards? (1 mile) What is longer: 5 miles or 5,000 yards? (5 miles) Work through the following exercises as a class and discuss different estimation strategies.

**Exercises:** Estimate which distance is greater. Then check your estimate by converting the measurement in miles to yards.

a) 20,000 yd or 2 mi 

b) $\frac{7}{8} \text{ mi}$ or 1,400 yd 

c) 1 \frac{3}{4} \text{ mi} or 3,200 yd

**Sample solutions**

a) 1 mi $< 10,000 \text{ yd}$, so 2 mi $< 20,000 \text{ yd}$; check: 2 mi = 3,520 yd $< 20,000 \text{ yd}$

b) 1/8 mi $> 200 \text{ yd}$, so 7/8 mi $> 1,400 \text{ yd}$; check: 7/8 mi = $7 \times 220 \text{ yd} = 1,540 \text{ yd}$

c) There are several possible ways to estimate: 1 3/4 mi = 7/4 mi, and 1/4 mi is between 400 yd and 500 yd, closer to 400 yd. So 7/4 mi is between 2,800 yd and 3,500 yd, closer to 2,800 yd, so 3,200 yd is larger. Another way to look at this is to say that 3/4 mi = 1/2 mi + 1/4 mi = 880 yd + 440 yd = about 1,300 yd, so 1 3/4 mi is about 1,800 + 1,300 yd = 3,100 yd, and so 3,200 yd is larger. Check: 1 3/4 mi = 7/4 mi = $7 \times 440 \text{ yd} = 3,080 \text{ yd}$, so 3,200 yd is larger.

**Review multiplying decimals.** Remind students that, when multiplying decimals, you multiply the numbers as if they were whole numbers, then add the number of decimal places in the factors, and finally move the decimal point that number of places to the left. Show an example as shown in the margin: 0.15 $\times 1,760$ will have 2 + 0 = 2 decimal places. 15 $\times 1,760 = 26,400$. The decimal point shifts two places to the left, and since 26,400 has zeroes in the tens and ones places, the result is a whole number: 264.00 = 264.
Converting measurements in miles with decimals. ASK: How many yards are in 0.15 miles? How do you know? (0.15 mi = 0.15 × 1,760 yd = 264 yd)

**Exercises:** Multiply by 1,760 to convert miles to yards.

a) 0.4 mi b) 1.1 mi c) 0.8 mi d) 2.5 mi

**Answers:** a) 704 yd, b) 1,936 yd, c) 1,408 yd, d) 4,400 yd

Remind students how to reduce fractions to lowest terms, and then ask them to convert 0.15 to a decimal fraction and reduce it to lowest terms. (15/100 = 3/20) ASK: How could I check the calculation for converting 0.15 miles to yards? (multiply 3/20 by 1,760) Have students perform the multiplication and check the answer obtained earlier. (3/20 × 1,760 yd = 3 × (1,760 ÷ 20) yd = 3 × 88 yd = 264 yd) Repeat with the previous exercises.

**Extensions**

1. 0.1 mi = 0.1 × 1,760 yd = 176 yd. Use this to convert decimal tenths of a mile to yards.

   a) 0.3 mi b) 0.7 mi c) 1.1 mi d) 1.5 mi

   **Answers:** a) 528 yd, b) 1,232 yd, c) 1,936 yd, d) 2,640 yd

   (MP4) [Note: The text contains an image of distance markers on a highway, with place values stacked vertically from top to bottom.

   a) Distance markers on a highway show distance in miles from the beginning of the highway, with place values stacked vertically from top to bottom. What distances do the markers show? (see examples in margin)

   b) A stopping distance for a car is the distance it takes a car to stop from the moment the driver sees the reason to brake. Write the stopping distance in yards for each speed in the table in the margin.

   c) A car is traveling on a road with the distance markers shown in part a). At the first marker, the driver sees a reason to brake. Halfway to the other marker, the car crashes. A police officer is examining the scene of the crash. The speed limit for the road is 70 mi/hr. Was the car traveling at the speed limit or not? Explain.

   **Answers**

   a) 79.8 mi and 80.0 mi

   b) from top to bottom: 25, 39 1/3, 58 1/3, 80, 105

   c) The markers are 80.0 − 79.8 = 0.2 mi apart, so the car traveled 0.2 ÷ 2 = 0.1 mi = 176 yd before crashing. 176 yd > 105 yd, the stopping distance for a car traveling 70 mi/hr, so the car was traveling at a speed higher than the speed limit.
Measurement and Data 5-14 Changing US Customary Units of Length

STANDARDS
5.MD.A.1, 5.NBT.B.5, 5.NBT.B.7, 5.NF.B.3

VOCABULARY
convert
mile (mi)
mixed measurement
perimeter
yard (yd)

Goals
Students will convert between different US customary units of length and solve problems requiring such conversions.

PRIOR KNOWLEDGE REQUIRED
Can convert between feet and inches, yards and feet, and miles and yards, including mixed measurements
Can perform the four basic operations with multi-digit numbers, fractions, mixed numbers, and decimals
Can find the perimeter of a rectangle
Can solve word problems requiring the four operations

Review conversion between all four units learned to date. Start the diagram below on the board by writing the units and drawing the arrows. Have volunteers tell you which operation should be performed for each arrow.

\[ \begin{align*}
\text{in} & \quad \times 12 \quad \times 3 \quad \times 1,760 \\
\text{ft} & \quad \div 12 \quad \div 3 \quad \div 1,760 \\
\text{yd} & \quad \text{mi}
\end{align*} \]

Exercises: Convert.

a) 7 ft to inches  
b) 7 yd to feet  
c) 3 mi to yards  
d) 42 ft to yards  
e) 48 in to feet  
f) 30 in to feet

Answers: a) 84 in, b) 21 ft, c) 5,280 yd, d) 14 yd, e) 4 ft, f) 2 1/2 ft

Converting between units that are not adjacent. ASK: How can you find out how many inches are in 1 yard? (convert yards to feet, and then feet to inches) Have students perform the conversion. Repeat with 2 yards and then 3 yards. SAY: Can I convert 7 yards into inches by multiplying by just one number? (yes) What number should I multiply by? (36) How do you know? (12 \times 3 = 36) How many inches are in 7 yards? (7 yd = 21 ft = 252 in) Then work as a class to convert 2 1/4 yd to inches. (2 1/4 yd \times 36 = 81 in)

Exercises: Multiply by 36 to convert the measurement to inches.

a) 10 yd  
b) 12 yd  
c) \(\frac{3}{2}\) yd  
d) \(\frac{1}{10}\) yd

Answers: a) 360 in, b) 432 in, c) 126 in, d) 67 1/2 in

Now ask students what number they will multiply by to convert a measurement in miles to feet. (1,760 \times 3 = 5,280) Remind students that to multiply a decimal by a whole number, you multiply the numbers as if both were whole numbers, and then shift the decimal point. For example, to multiply 0.3 \times 1,760, you multiply 1,760 \times 3 = 5,280 and move the decimal point one place to the left, because there is only one decimal place in 0.3.
and no decimal places in the whole number 1,760. So, \(0.3 \times 1,760 = 528.0 = 528\). This also means that there are 528 yards in 0.3 miles.

Do the first two exercises below as a class, and then have students work individually.

**Exercises:** Convert to feet.

a) 4 mi  
b) 2.1 mi  
c) 5 mi  
d) 0.25 mi  
e) \(\frac{1}{4}\) mi  
f) \(\frac{3}{8}\) mi  
g) 0.5 mi  
h) \(\frac{1}{10}\) mi

**Answers:** a) 21,120 ft, b) 11,088 ft, c) 26,400 ft, d) 1,320 ft, e) 1,320 ft, f) 7,260 ft, g) 2,640 ft, h) 11,088 ft

ASK: Which of these answers should be the same? Why? (b and h, d and e, because each pair has the same number presented as a decimal and as a fraction or mixed number) Have students check that their answers are the same for these questions. ASK: Which answer can you use to check your answer to part g)? (part c) How do you check that? (0.5 is 10 times smaller than 5, so the answer to part g) should be one tenth of the answer to part c))

Review conversions of mixed measurements to the smaller unit, such as 3 feet 8 inches to inches. (3 ft \(= 3 \times 12\) in \(= 36\) in, so 3 ft 8 in \(= 36\) in + 8 in \(= 44\) in)

**Exercises:** Convert to the smaller unit.

a) 7 ft 5 in  
b) 9 ft 1 in  
c) 3 mi 120 yd  
d) 2 mi 357 ft

**Answers:** a) 89 in, b) 109 in, c) 5,400 yd, d) 10,917 ft

**Review perimeter.** Remind students that the perimeter of a shape is the sum of the lengths of all the sides. As well, remind students that, since rectangles have equal opposite sides, they can find the perimeter by adding width and length and multiplying by 2.

**Exercises:** Find the perimeter of the rectangle.

a) 21 in by 11 in  
b) 2.6 mi by 2.3 mi  
c) 7 1/2 yd by 10 1/4 yd

**Answers:** a) 64 in, b) 9.8 mi, c) 35 1/2 yd

**Finding the perimeter of rectangles with sides in different units.** Draw a rectangle and label two sides as 5 inches and 2 feet. Tell students that you think the perimeter of this rectangle is 14 inches. Is that correct? (no) Have students explain the mistake. (you cannot add feet to inches; you first need to convert the measurements to the same unit) Have students convert both measurements to inches and find the perimeter. (2 ft \(= 24\) in, so (5 in + 24 in) \(\times 2\) = perimeter 58 in) Then have students find the perimeter of a rectangle that measures 5 3/4 in by 1 1/4 ft. (1 1/4 ft = 15 in, so the perimeter is 41 1/2 in)
Exercises

a) Convert the side lengths to the smaller unit. Then find the perimeter of the rectangle.

i) 15 in by 2 ft
ii) \(2 \frac{1}{4} \text{ ft by } 1 \frac{1}{8} \text{ yd}\)

**Bonus**

i) 2.1 mi by 2,000 ft

**Answers:**

i) 15 in by 24 in, perimeter 78 in; ii) 2 1/4 ft by 3 3/8 ft, perimeter 11 1/4 ft; Bonus: 11,088 ft by 2,000 ft, perimeter 26,176 ft

b) Convert the side lengths in part a) to the larger unit. Then find the perimeter of the rectangle.

**Answers:**

i) 1 \frac{1}{4} \text{ ft by } 2 \text{ ft, perimeter 6 } 6 \frac{1}{2} \text{ ft = 78 in; ii) 3/4 yd by 1 1/8 yd, perimeter 3 3/4 yd = 11 1/4 ft; Bonus: 2 1/10 mi by 25/66 mi, perimeter } 1,636/330 \text{ mi} = 26,176 \text{ ft}

b) Convert the side lengths in part a) to the larger unit. Then find the perimeter of the rectangle.

**Answers:**

i) 1 \frac{1}{4} \text{ ft by } 2 \text{ ft, perimeter 6 } 6 \frac{1}{2} \text{ ft = 78 in; ii) 3/4 yd by 1 1/8 yd, perimeter 3 3/4 yd = 11 1/4 ft; Bonus: 2 1/10 mi by 25/66 mi, perimeter } 1,636/330 \text{ mi} = 26,176 \text{ ft}

c) Compare the answers. Did you get the same perimeter from parts a) and b)? If not, find your mistake.

**Solving word problems with conversions.** Work through the following problems as a class. Discuss different ways to solve the problem, including converting to different units.

a) A building has 8 ft ceilings on each floor. The space between the floors and under the roof is 18 in tall. The building has 12 floors. How tall is the building? (114 ft)

b) A backyard is a rectangle that is 34 ft wide and 15 yd deep. A fence goes around two long sides and one short side. How long is the fence? (124 ft)

c) Maria has two boards, 7 ft 9 in long and 3/4 inch thick each. She cuts one board into two equal pieces to be the walls of a bookshelf. She cuts the other board into three equal pieces to be the top, bottom, and middle shelves, as shown in the margin. The middle shelf is the same distance from the top and the bottom. What are the dimensions of the bookshelf? What are the dimensions of each opening between shelves?

**Solution:** 7 ft = 84 in, so 7 ft 9 in = 84 in ÷ 9 in = 93 in. Each board is 93 in long. One board is cut into two pieces: 93 in ÷ 2 = 46 1/2 in. The other board is cut into three pieces: 93 in ÷ 3 = 31 in.

The height of the entire bookshelf equals the length of the longer piece, 46 1/2 in = 3 ft 10 1/2 in. The width of the bookshelf equals the length of the shorter piece together with two widths of the side boards, so 31 in + (2 × 3/4 in) = 31 in + 1 1/2 in = 32 1/2 in = 2 ft 8 1/2 in.

The width of the opening is the same as the length of the smaller piece, so 31 in. The height of the opening is the distance between the shelf boards. The height of the bookshelf (46 1/2 in) is equal to the two openings plus three widths of the board (3 × 3/4 in = 9/4 in = 2 1/4 in). The height of the two openings together is 46 1/2 in − 2 1/4 in = 44 1/4 in. This means the height of the opening is 44 1/4 in ÷ 2 = 22 1/8 in, or 1 ft 10 1/8 in.
Extensions

1. Convert to inches.
   a) 2 mi 2 yd 2 ft 2 in  
   b) 10 mi 10 yd 10 ft 10 in
   
   **Answers:** a) 126,818 in, b) 634,090 in

2. To convert a measurement in smaller units to a mixed measurement, use division with a remainder.
   
   **Example:** Convert 10,000 in to a mixed measurement.
   
   12 inches in 1 ft, so 10,000 in = \(10,000 \div 12 = 833\) ft 4 in
   
   3 ft in 1 yd, so 833 ft = \(833 \div 3 = 277\) yd 2 ft
   
   So 10,000 in = 277 yd 2 ft 4 in.
   
   Convert to a mixed measurement.
   a) 20,000 in  
   b) 100,000 in  
   c) 1,000,000 in
   
   **Answers**  
   a) 555 yd 1 ft 8 in  
   b) 1 mi 1,017 yd 2 ft 4 in  
   c) 15 mi 1,377 yd 2 ft 4 in

(MP1, MP2, MP4)  

3. Ms. Kim earns $57,420 every year at her job. She gets paid twice a month. Last year, Ms. Kim worked exactly 20 days each month for 7.5 hours each day. How much money did she earn per hour? Use equations to model and solve the problem and explain what each part of your model means in the situation.

**Sample solutions**

- Since there are 12 months in a year, Ms. Kim earned \(57,420 \div 12 = 4,785\) each month. Since she worked 20 days each month, she earned \(4,785 \div 20 = 239.25\) each day. Since she worked 7.5 hours each day, she earned \(239.25 \div 7.5 = 31.90\) per hour.

- I found how many hours she worked each month by multiplying \(20 \times 7.5 = 10 \times 15 = 150\) hours. I found how many hours she worked per year by multiplying \(150 \times 12 = 30 \times 5 \times 2 \times 6 = 180 \times 10 = 1,800\) hours. I divided the amount she earns in a year by the number of hours she works in a year to find that she earned \(57,420 \div 1,800 = 31.90\) per hour.

Whole-class follow-up: Present both solutions or have volunteers do so. ASK: Which way is faster? (using multiplication and only doing one division) Why was multiplication faster? (because they were easy numbers to multiply) Is multiplication always faster than division? (no, it’s just because we had easy numbers to multiply) Which fact did you not need to use? (she gets paid twice a month)
Goals

Students will convert between pounds and ounces.

Prior Knowledge Required

Understands the concept of mass
Can perform the four basic operations with multi-digit numbers, fractions, mixed numbers, and decimals
Understands that division can produce a fraction
Can convert between mixed numbers and improper fractions

Materials

objects (including packages) that weigh 1 lb and 1 oz
scales to measure in pounds and ounces

Introduce ounces. Bring in several packages of different weights, including 1 oz. NOTE: use light packages, such as paper or bags, not glass bottles. Have students look at the packages and find the weight. In what units is the weight listed?

Explain that the weight (or mass) of small objects is measured in ounces in the US, and that a short form of ounce is oz. Write the word and the abbreviation on the board, and point out that, unlike with metric units, the short form combines one letter that is part of the word and one that is not.

Have students find the packages that weigh 1 oz. In addition, point out as a reference that a slice of bread weighs about 1 oz, and that a stack of five quarters together weigh almost exactly 1 oz.

ASK: How many quarters make 1 whole? (4) Tell students that a quarter of a small apple weighs about 1 oz. ASK: How much would the whole apple weigh? (4 oz)

Introduce pounds. Remind students that the unit they will use most often for measuring weight is a pound, and that the short form of pound is lb. Write the word and the abbreviation on the board, and explain that both are ways to write “pounds.” You might point out that the abbreviation comes from the Latin word “libra,” meaning scales or balance, and the phrase “libra pondo” or pound weight. Explain that pounds are larger than ounces, and they are used to measure the mass of heavier objects, or when there is no need for great precision. Show students several objects that weigh 1 lb. Have students compare these objects with other objects. For example, is a shoe heavier, lighter than, or about the same as 1 lb?

1 lb = 16 oz. Have students look at the markings on the face of a scale and find out how many ounces are in 1 pound. (16) Write on the board:

1 pound = 16 ounces, 1 lb = 16 oz
**Converting from pounds to ounces.** Start a conversion table as shown:

<table>
<thead>
<tr>
<th>lb</th>
<th>oz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Have students fill in the numbers in the ounces column. **ASK:** How do you know what numbers to write in the “ounces” column? How did you get each new number? If students are adding 16 to the previous row, ask them to look for a pattern: how can we get the number of ounces from the number of pounds? What number would we multiply the number of pounds by to get the number of ounces? (16) **Why?** (because there are 16 oz in 1 lb) Have students extend the table to eight rows and keep it on the board for future reference.

**Exercises:** Multiply by 16 to convert the measurement to ounces.

a) 9 lb  

b) 12 lb  

c) 35 lb  

d) 50 lb

**Answers:** a) 144 oz, b) 192 oz, c) 560 oz, d) 800 oz

Remind students that, to multiply a mixed number by 16, we convert the mixed number to an improper fraction first. Have students tell you what to do next at each step when converting 2 1/4 lb to ounces:

\[
2 \frac{1}{4} \text{ lb} = \frac{9}{4} \text{ lb} \\
= \frac{9}{4} \times 16 \text{ oz} \\
= 9 \times (16 \div 4) \text{ oz} \\
= 9 \times 4 \text{ oz} = 36 \text{ oz}
\]

**Exercises:** Multiply by 16 to convert the measurement to ounces.

a) \( \frac{3}{4} \) lb  

b) \( \frac{1}{8} \) lb  

c) \( \frac{3}{8} \) lb  

d) \( \frac{5}{2} \) lb

**Answers:** a) 12 oz, b) 18 oz, c) 38 oz, d) 88 oz

**Comparing measurements in different units.** Remind students that when they compare two measurements in different units, they need to convert one of the measurements so that the units become the same. For example, 9 oz is smaller than 1 lb, even though 1 is smaller than 9, because 1 lb = 16 oz and 16 > 9.

Write the pairs of measurements in the following exercise one by one on the board, and have students signal the answer by showing the < or the > sign with their fingers.
Exercises: Convert the measurement in pounds to ounces before comparing the measurements. Which is larger?

a) 50 oz or 3 lb    b) 9 lb or 90 oz    c) 1,000 oz or 20 lb

d) \(5 \frac{1}{2}\) lb or 100 oz    e) 100 oz or \(6 \frac{1}{2}\) lb

Bonus: 1,000,000 oz or 60,000 lb

Answers: a) 50 oz, b) 9 lb, c) 1,000 oz, d) 100 oz, e) 6 \(\frac{1}{2}\) lb,
Bonus: 1,000,000 oz

Converting measurements in ounces to pounds. ASK: To convert a measurement in pounds to ounces, you multiply by 16. What should you do to convert a measurement in ounces to pounds? (divide by 16) Remind students that when they divide \(49 \div 16 = 3\), they get a remainder 1. So when converting 49 oz to pounds, we get 3 whole pounds and 1 ounce leftover. We can write the answer as a mixed measurement, 3 lb 1 oz, or, if we want an answer in pounds only, we can write a mixed number of pounds: \(49 \frac{1}{16}\) lb. Write both answers on the board. Repeat with 70 oz. (70 oz = 4 lb 6 oz = 4 \(\frac{3}{16}\) lb = 4 \(\frac{3}{8}\) lb)

Exercises: Convert the measurement to a mixed measurement and then to pounds only.

a) 25 oz    b) 34 oz    c) 60 oz    d) 100 oz

Bonus: 1,000 oz

Answers: a) 1 lb 9 oz, 1 \(\frac{9}{16}\) lb; b) 2 lb 2 oz, 2 \(\frac{1}{8}\) lb; c) 3 lb 12 oz, 3 \(\frac{3}{4}\) lb;
d) 6 lb 4 oz, 6 \(\frac{1}{4}\) lb; Bonus: 62 lb 8 oz, 62 \(\frac{1}{2}\) lb

Converting mixed measurements to ounces. ASK: How can you convert 4 lb 3 oz to a measurement in ounces only? Have students think individually, and then discuss the method in pairs. (convert 4 lb to ounces, 4 lb = 64 oz, and then add the leftover ounces, 64 oz + 3 oz = 67 oz)

Exercises: Convert to ounces.

a) 3 lb 7 oz    b) 2 lb 4 oz    c) 6 lb 8 oz    d) 10 lb 10 oz

Bonus: 100 lb 4 oz

Answers: a) 55 oz, b) 36 oz, c) 104 oz, d) 170 oz, Bonus: 1,604 oz

Converting mixed measurements to pounds. ASK: How can you convert 4 lb 3 oz to a measurement in pounds only? (convert the ounces part to pounds, so 3 oz = 3/16 lb, and write a mixed measurement, so 4 lb 3 oz = 4 \(\frac{3}{16}\) lb) Have students convert the mixed measurements above to pounds, then reduce the answer to lowest terms. Early finishers can convert the answers to ounces and check that the answers coincide with the results of the previous exercise.

Answers

a) 3 \(\frac{7}{16}\) lb, b) 2 \(\frac{1}{4}\) lb, c) 6 \(\frac{1}{2}\) lb, d) 10 \(\frac{5}{8}\) lb, Bonus: 100 \(\frac{1}{4}\) lb
Extensions

**NOTE:** In the extensions below, encourage students to model the situation using equations and to state what their equations mean in the context of the problem.

1. A box of 20 books weighs 34 lb. The empty box weighs 14 oz. How much does one book weigh?

   **Solution:** $34 \times 16 = 544$, so the box of 20 books weighs 544 oz. The 20 books alone weigh $544 - 14 = 530$ oz. So one book weighs $530 \div 20 = 26.5$ oz.

2. A school wants to donate at least 180 lb of canned food to a food bank. The school has collected 272 eleven-ounce cans. Did the school reach its goal? How many more cans than the goal did the school collect, or how many more cans does it need?

   **Answer**
   The school collected $185 \frac{5}{8}$ lb of food ($270 \times 11$ oz $= 2,970$ oz $= 185$ lb 10 oz), so it reached the goal of 180 lb. The school needed 262 eleven-ounce cans to reach the goal ($262 \times 11 = 2,882$), so they collected eight cans more than the goal. Alternatively, students can convert all measurements to ounces: the school wanted to donate 2,880 oz and collected 2,970 oz, so they exceeded the goal by 90 oz, which is eight cans extra. Note that, in this case, students need to realize that if there were nine cans fewer, the goal would not be achieved, as students would collect 99 oz food less (i.e., 2,871 oz), which is less than the goal.

3. Kelly and Zack are each reading the same book that is 216 pages long. Kelly reads 4 pages a day and Zack reads 8 pages a day. If they both start from the beginning of the book on the same day, how many pages farther ahead will Zack be when Kelly has read 80 pages? Show your work with a T-table or number lines.

   **Sample solutions**
   • I used a table to show Kelly and Zack’s reading:

<table>
<thead>
<tr>
<th>Kelly (Add 4)</th>
<th>Zack (Add 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

   To find how many pages Zack has read, multiply the number of pages Kelly has read by 2. When Kelly has read 80 pages, Zack has read 160 pages ($80 \times 2 = 160$). So Zack is 80 pages farther ahead than Kelly ($160 - 80 = 80$).

   • I drew two number lines, one for the amount Kelly reads and one for the amount Zack reads. Both number lines have the same number of jumps, but Zack’s jumps are twice as big so they always go twice as far. When Kelly has read 80 pages, Zack has read 160 pages, so Zack will be 80 pages farther ahead than Kelly ($160 - 80 = 80$).
MD5-16 Word Problems with Mass and Length

Pages 95–96

STANDARDS
5.MD.A.1, 5.NF.A.2, 5.NF.B.3, 5.NF.B.6, 5.NBT.B.5, 5.NBT.B.7

VOCABULARY
convert
denominator
mass
mile (mi)
mixed measurement
numerator
ounce (oz)
perimeter
pound (lb)
yard (yd)

Goals
Students will solve problems requiring conversions between units of mass and units of length in the US customary system.

PRIOR KNOWLEDGE REQUIRED
Can convert between different units of length in the US customary system
Can convert between ounces and pounds
Can perform the four basic operations with multi-digit numbers, fractions, mixed numbers, and decimals
Understands that division can produce a fraction
Can convert between mixed numbers and improper fractions
Can solve word problems using the four operations

NOTE: This lesson serves as a review of the concepts learned in this unit to date. You can use the following problems to supplement those on AP Book 5.2 pp. 95–96.

Exercises
1. a) Find the total mass of the ingredients in the following recipe. Is it more than 2 pounds?

   1 \frac{1}{2} \text{ lb ground meat}
   2 \text{ oz onion}
   4 \text{ oz oatmeal}
   8 \text{ oz ketchup}

   b) The recipe makes six portions. What is the size of one portion?

   Answers: a) 38 oz, yes; b) 6 1/3 oz

2. Kim makes 12 equal-sized sandwiches from a 2 lb loaf of bread and 1 lb of cheese. What is the weight of each sandwich?

   Answer: 4 oz

3. The lesser Egyptian jerboa is a small rodent that weighs about 2 oz and is about 4 in long. The largest rodent in the world, the capybara, weighs about 108 lb and is about 3 ft 10 in long.

   a) How many times as heavy is a capybara than a jerboa?
   b) How many times as long is a capybara than a jerboa?

   Answers
   a) 864 times as heavy
   b) 11 1/2 times as long
Extensions

1. Two squares, each with a perimeter of 2 ft, are put together to make a rectangle with a length of 1 ft. What is the perimeter of the rectangle?

   **Solution**
   The squares have sides of 6 in, so the rectangle is 6 in by 12 in. The perimeter of the rectangle is 36 in = 3 ft = 1 yd.

2. If a snail crawls 2 ft 3 in every day for a whole leap year, how many yards does it crawl in total?

   **Sample solution**
   \[
   2 \text{ ft 3 in} \times 366 = 732 \text{ ft} + 1.098 \text{ in} = 732 \text{ ft} + 91 \frac{1}{2} \text{ ft} = 823 \frac{1}{2} \text{ ft} = 274 \frac{1}{2} \text{ yd}\; \text{or}
   2 \text{ ft 3 in} = 27 \text{ in} \times 366 = 9882 \text{ in} = 274 \frac{1}{2} \text{ yd}.
   \]

   (MP3)

3. Rani and Ted find the product of 7.4 and 8.5. Rani uses the standard algorithm and says the answer is 62.90. Ted uses a calculator and says the answer is 62.9.

   a) Who is correct? Explain.
   b) In pairs, explain your answers in part a). Do you agree with each other? Discuss why or why not.

   **Sample answer:** They are both correct because 62.90 = 62.9.

   **NOTE:** For part b), encourage partners to ask questions to understand and challenge each other’s thinking (MP3)—see p. A-49 for sample sentence and question stems to guide students.
MD5-17 Line Plots

Pages 97–98

STANDARDS
5.MD.B.2, 5.NBT.B.5, 5.NBT.B.7

VOCABULARY
- convert
- frequency
- label
- line plot
- number line
- tally
- template
- title

Goals
Students create and interpret line plots.

PRIOR KNOWLEDGE REQUIRED
- Can create a line plot with whole numbers
- Can solve problems using data from a line plot
- Can multiply multi-digit numbers, including decimals

MATERIALS
- BLM Shoe Length Line Plot (p. P-56)
- rulers

Counting by making tally marks. SAY: When counting an event that occurs a lot, it is easier to count by keeping a tally. Draw four tick marks on the board as you count from one to four:

||||

SAY: When we get to five, we use a horizontal (or diagonal) tick mark.

Draw a line across the first four tick marks:

++++

SAY: If we continue counting, we start another group of five tick marks.

Draw another four tick marks as you count from six to nine:

++++ ||||

SAY: When we get to ten, we use another horizontal (or diagonal) tick mark on the second group of five.

Draw a line across the second group:

++++ ++

SAY: Organizing our counting into groups of five helps us keep track.

Exercises: Count the tick marks by skip counting by five.

a) ++++ ++++ ||

b) ++++ ++++ ++++ ||||

c) ++++ ++++ ++++ ++++ ++++ ++++ ++++ ++++ ||

Answers: a) 12, b) 19, c) 38

Ask students to count how many times you tap the desk with a pencil by using this tally technique. Tap the desk 48 times. Students should draw:

++++ ++++ ++++ ++++ ++++ ++++ ++++ ++++ ++++
### ACTIVITY

Gathering data and creating a line plot.

Draw on the board:

<table>
<thead>
<tr>
<th>Length of Shoe to the Nearest 1/2 Inch</th>
<th>Shoe Length Tallies</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 $\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 $\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 $\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 $\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** You may have to extend this table depending on the lengths of shoes for your class.

Ask students to each find the length in inches of one of their shoes, to the nearest half inch.

After they have measured their shoes, ask students to come up to the board one at a time and each make a tally mark on the chart according to their shoe length. For example, a student with a 8 in shoe makes one mark in that row, a student with a 8 1/2 in shoe makes one mark in that row, etc. Remind them that when they reach four tick marks (|||), the next tally should look like this: ||||. After each group of five, a new group of tally marks should begin.

**ASK:** How can we display the results with a graph? (line plot) **SAY:** Each line plot needs a title, number line, and label. **ASK:** What can we use for the title? (sample answer: “Grade 5 Shoes”). What numbers must we use for the number line? (8, 8 1/2, 9, 9 1/2, etc.) **WHAT** can we use for the label? (shoe lengths)

Display BLM Shoe Length Line Plot using an overhead projector or digital display. **ASK:** How do we show the total for each length in the line plot? (place an X for each person who has that shoe length) **SAY:** The line plot shows the number of people who have each shoe length. The number of times each shoe length appears in the tally chart is called the frequency. **ASK:** Where do we get these frequencies? (from the “Total” column in the tally chart)
Ask for a volunteer to come to the board and place an $\times$ on the line plot for each person who had a shoe length with the first measurement. Repeat for the other lengths, using a different volunteer each time. Once the line plot has been completed, ASK: What was the smallest shoe length? What was the largest shoe length? How many people have a shoe length 10 in or greater? Which shoe length occurred most often? (answers will vary for all questions)

**Using the data to solve problems.** SAY: Shoes that are 10 in long are on sale for $32.99 a pair. SAY: For our class, what is the total cost of buying new shoes for all the students who wear shoes 10 in long?

Students should multiply $32.99 by the number of people wearing 10 in shoes. (see sample grid in margin) Ask the class to perform the multiplication in their notebooks. Remind them that to multiply decimals, first multiply as if they were whole numbers, then count the total number of decimal digits, and move the decimal point in the answer to the left that many places. After they have had enough time, ask for a volunteer to write the calculation on the board.

Present the following table and tell students that a Grade 5 class was asked how many hours of television they each watched in a typical week. The table lists the individual results in hours:

<table>
<thead>
<tr>
<th>11</th>
<th>18</th>
<th>13</th>
<th>14</th>
<th>18</th>
<th>11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>13</td>
<td>11</td>
<td>11</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

Explain that you want students to make a line plot in their notebooks to display this data. They will need to include the title, number line, and label. ASK: What is the smallest number in the table? (11) What is the largest number in the table? (18) When drawing the number line, what numbers should we use? (all numbers from 11 to 18) Remind students to include all the numbers from 11 to 18, even if a number does not show up in the table.

Have students prepare a template for the line plot.

**Exercises**

a) Use the template you prepared to make a line plot to display this TV-watching data. To keep track, cross out each number as you plot it on the line plot.

b) What is the most common number of hours watched per week?

c) How many students watched more than 14 hours per week?

d) For those students who watched 18 hours per week, how many hours per year does each student spend watching TV? Hint: assume 52 weeks in a year.

**Bonus:** For the students in part d), approximately how many full days does each student spend watching TV in a year?
Answers

a) **How Much Television Students Watch in a Week**

<table>
<thead>
<tr>
<th>Watching time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
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</tr>
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<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

b) 11, c) 9, d) $18 \times 52 = 936$ hours per year per student, Bonus: 39 days per student

**Review converting miles to yards.** ASK: How many yards are in a mile? (1,760 yd) What calculation can we perform to find how many yards are in 5 miles? ($5 \times 1,760$)

**Exercises:** Convert the measurement in miles to yards.

a) 3 mi  

b) 5 mi  

c) $\frac{1}{2}$ mi  

d) $\frac{3}{4}$ mi

**Answers:** a) 5,280 yd, b) 8,800 yd, c) 880 yd, d) 1,320 yd

**Review multiplying mixed numbers.** Write on the board:

$1\frac{3}{4} \times 3$

ASK: How can we calculate this by addition? ($1\frac{3}{4} + 1\frac{3}{4} + 1\frac{3}{4}$)

By addition: $1\frac{3}{4} \times 3$

$= 1\frac{3}{4} + 1\frac{3}{4} + 1\frac{3}{4}$

ASK: How can we write 1 3/4 as an improper fraction? (7/4)

$= \frac{7}{4} + \frac{7}{4} + \frac{7}{4}$

$= \frac{21}{4}$

ASK: How can we write 21/4 as a mixed fraction? (5 1/4)

$= 5\frac{1}{4}$

ASK: How can we calculate this by multiplication? (convert 1 3/4 into an improper fraction)

By multiplication: $1\frac{3}{4} \times 3$
ASK: How can we write 1 3/4 as an improper fraction? (7/4)

\[
\frac{7}{4} \times \frac{3}{1}
\]

ASK: How do we multiply the fractions? (multiply the numerators and denominators)

\[
\frac{21}{4} = 5 \frac{1}{4}
\]

**Exercises:** Calculate by addition and then by multiplication.

a) \(\frac{3}{4} \times 3\)  
b) \(1 \frac{1}{4} \times 5\)

**Answers:** a) 2 1/4, b) 6 1/4

**Extension**

(MP1, MP2, MP4) Fifteen students were asked how many board games they have at home. The teacher determined the following facts from the data:

- The smallest number of board games is 3.
- The largest number of board games is 8.
- Two students have 3 board games each, and two students have 8 board games each.
- Five students have fewer than 5 board games each, and five students have more than 6 board games each.
- The number of students who have 6 board games each is twice the number who have 3 board games each.

Use this information to draw a line plot to show the data.

**Answer**

![Line plot of board games owned by students](image-url)
MD5-18 Line Plots with Fractions
Pages 99–100

STANDARDS
5.MD.B.2, 5.NBT.B.7,
5.NBT.A.1, 5.NBT.A.2

VOCABULARY
common denominator
denominator
improper fraction
label
line plot
number line
numerator
tally chart
title

Goals
Students create and interpret line plots with fractions.

PRIOR KNOWLEDGE REQUIRED
Can create a line plot
Can solve problems using data from a line plot
Can multiply a mixed number by a whole number
Can multiply a decimal by a whole number
Can add mixed fractions with different denominators
Can change a decimal greater than one into a mixed fraction

MATERIALS
BLM Drinks Line Plot (p. P-57)

Review the values of coins, cents, and dollars. ASK: Why is 25 cents also called a quarter? (because it is a quarter of a dollar) What fraction of a dollar could be written for 25 cents? ($1/4)

Write on the board:

a) 25 cents = $0.25 = $1/4
b) 50 cents = $ = $
f) 150 cents = $ = $
g) 175 cents = $ = $
h) 200 cents = $ = $

ASK: How many quarters are there in 50 cents? (two quarters)
How do we read the fraction 2/4? (two fourths or two “quarters”)

ASK: How many quarters are there in 75 cents? (three quarters)
How do we read the fraction 3/4? (three quarters)

Remind students that, to change a decimal greater than 1, write the whole number and then write the decimal as a decimal fraction in lowest terms.

Example: $1.25 = 1 + \frac{25}{100} = 1 + \frac{1}{4} = 1 \frac{1}{4}$

Ask for student volunteers to come to the board to fill in the missing money amounts written in dollars and cents and written as fractions. (see answers in the margin)
Interpreting line plots with fractions. Display BLM Drinks Line Plot.
SAY: A class of students gathered information about how much money they each spend in one week on milk, juice, and other drinks at school.

Exercise: Convert each fraction on the number line into dollars and cents.

Answers: $1.00, $1.25, $1.50, $1.75, etc.

ASK: How many students spend exactly $2.25 per week on drinks? (3) For just these three students, what is the total amount of money they spend on drinks per week? ($6.75) Provide two different calculations to get that answer. ($2.25 + $2.25 + $2.25, 3 × $2.25)

ASK: How many students spend less than $2 per day on drinks? (6) How did you get the answer? (count the number of Xs on the line plot to the left of $2 on the number line)

ASK: How many students spend more than $2.50 per week on drinks? (6) How did you get the answer? (count the number of Xs on the line plot to the right of $2 1/2 on the number line)

Adding fractions mentally. SAY: Sometimes when adding fractions, it is possible to save time by using a few tricks.

Write on the board:
\[ 3 \frac{1}{4} + 5 \frac{3}{4} \]

SAY: How do we write these fractions as improper fractions? (13/4 + 23/4) SAY: The denominators are the same. Add the numerators. What do you get? (36/4) What is 36 ÷ 4? (9) Record the whole calculation on the board as you go.

SAY: We can make the calculation easier by rewriting the question. Write:
\[ = 3 + \frac{1}{4} + 5 + \frac{3}{4} \]

SAY: Regroup the numbers to make it easier. Write:
\[ = (3 + 5) + \left( \frac{1}{4} + \frac{3}{4} \right) \]

ASK: Why is this easier? (we can add whole numbers separately and the fractions separately) What do you get when you add the fractions? (1\(\frac{1}{4} + 3\frac{3}{4} = 4\frac{1}{4} = 1\)) Record the rest of the calculation on the board:
\[ = 8 + 1 \]
\[ = 9 \]

Exercises: Add the fractions mentally.

\[ a) \ 6 \frac{3}{8} + 12 \frac{5}{8} \quad b) \ 25 \frac{7}{16} + 32 \frac{9}{16} \quad c) \ 3 \frac{2}{8} + 7 \frac{5}{8} + 14 \frac{1}{8} \]

Answers: a) 19, b) 58, c) 25
SAY: Sometimes we need to add mixed numbers but the denominators are not the same.

Write on the board:

\[ 6 \frac{3}{4} + 12 \frac{1}{2} + 5 \frac{1}{8} \]

ASK: How can we make the denominators the same? (find a common denominator) What is the common denominator here? (8) ASK: How do we rewrite the fractions 3/4 and 1/2 as equivalent fractions with 8 as the new denominator? (6/8, 4/8)

Continue working on the addition question above by walking students through the steps as you go: split each mixed number as the sum of a whole number and a fraction, replace the fraction parts with equivalent fractions, and then regroup the numbers. Write on the board:

\[ = 6 + \frac{3}{4} + 12 + \frac{1}{2} + 5 + \frac{1}{8} \]
\[ = 6 + \frac{6}{8} + 12 + \frac{4}{8} + 5 + \frac{1}{8} \]
\[ = (6 + 12 + 5) + \left( \frac{6}{8} + \frac{4}{8} + \frac{1}{8} \right) \]

SAY: Now, you can add the whole numbers and the fractions separately.


\[ = 23 + \frac{11}{8} \]
\[ = 23 + 1 \frac{3}{8} \]

SAY: Add the whole numbers. Write on the board:

\[ = 24 \frac{3}{8} \]

**Exercises:** Add the whole parts and the fractional parts separately to add the fractions.

a) \[ 1 \frac{3}{4} + 2 \frac{1}{4} \]

b) \[ 7 \frac{3}{4} + 2 \frac{3}{4} \]

c) \[ 7 \frac{3}{8} + 3 \frac{1}{4} \]

d) \[ 14 \frac{3}{8} + 18 \frac{1}{2} + 25 \frac{3}{4} \]

**Answers:** a) 4, b) 10 1/2, c) 10 5/8, d) 58 5/8

**Calculating totals from line plots with fractions mentally.** Refer students to the first exercise on p. P-33 and BLM Drinks Line Plot. SAY: Suppose we need to calculate how much the entire class spends on drinks at school each week. ASK: What fractions can we add to find the total? Remember, each X on the line plot represents how much money one student spends. (1 + 1 1/4 + 1 1/4 + 1 1/2 + 1 3/4 + 1 3/4 + 2 + 2 + ...)

---

**P-34**

Teacher Resource for Grade 5
ASK: How many fractions would we need to add? (18: one for each student, which is one for each X on the line plot) ASK: What are the fractions on the number line written as improper fractions? (1/1, 5/4, 3/2, 7/4, 2/1, 9/4, 5/2, 11/4, 3/1, 13/4, 7/2, 15/4)

ASK: What common denominator can we use when adding these fractions? (4)

ASK: What are the fractions on the number line written as improper fractions but now with the denominator 4? (4/4, 5/4, 6/4, 7/4, 8/4, 9/4, 10/4, 11/4, 12/4, 13/4, 14/4, 15/4)

Write these equivalent fractions on the BLM overhead, just below the existing number line. Circle the numerator for each fraction:

\[
\begin{align*}
4 & \quad 5 & \quad 6 & \quad 7 & \quad 8 & \quad 9 & \quad 10 & \quad 11 & \quad 12 & \quad 13 & \quad 14 & \quad 15 \\
4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4 & \quad 4
\end{align*}
\]

SAY: Remember, when adding fractions with the same denominators, keep the denominator the same and add only the numerators.

ASK: What numerators are being added if we are calculating the total for all students? (4 + 5 + 5 + 6 + 7 + 7 + 8 + 8 + 9 + 9 + 9 + 10 + 11 + 13 + 13 + 14 + 15 + 15)

Ask students to add these mentally, on paper, or add together out loud as a class. ASK: What is the total of the numerators? (168) What is the total when we put the denominator back? (168/4) SAY: Divide 168 by 4 mentally. (42) ASK: How much did the class spend on drinks at school during the week? ($42)

Write on the board:

\[
4 + 5 + 5 + 6 + 7 + 7 + 8 + 8 + 9 + 9 + 9 + \ldots
\]

ASK: Instead of writing 9 + 9 + 9, what multiplication could we have used? (9 \times 3) ASK: How can we rewrite the addition of the numerators using addition and multiplication?

Write the answer on the board:

\[
(4) + (5 \times 2) + (6) + (7 \times 2) + (8 \times 2) + (9 \times 3) + (10) + (11) + (13 \times 2) + (14) + (15 \times 2)
\]

ASK: What are each of the products? Write on the board:

\[
= 4 + 10 + 6 + 14 + 16 + 27 + 10 + 11 + 26 + 14 + 30
\]

SAY: Add to see if the total is the same. (168)

**Extensions**

1. Use the data from **BLM Drinks Line Plot** to find the following.

   a) Assume each student has a weekly allowance of $10. Create a tally chart to show how much money each student has left from $10.00 after spending money on drinks at school.

   b) Create a line plot to display the results from part a).
Answers

a) Money left over after buying drinks ($)  

<table>
<thead>
<tr>
<th>Money ($)</th>
<th>Student Tally</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \frac{1}{4}$</td>
<td>$</td>
</tr>
<tr>
<td>$6 \frac{1}{2}$</td>
<td>$</td>
</tr>
<tr>
<td>$6 \frac{3}{4}$</td>
<td>$</td>
</tr>
<tr>
<td>$7$</td>
<td>$</td>
</tr>
<tr>
<td>$7 \frac{1}{4}$</td>
<td>$</td>
</tr>
<tr>
<td>$7 \frac{1}{2}$</td>
<td>$</td>
</tr>
<tr>
<td>$7 \frac{3}{4}$</td>
<td>$</td>
</tr>
<tr>
<td>$8$</td>
<td>$</td>
</tr>
<tr>
<td>$8 \frac{1}{4}$</td>
<td>$</td>
</tr>
<tr>
<td>$8 \frac{1}{2}$</td>
<td>$</td>
</tr>
<tr>
<td>$8 \frac{3}{4}$</td>
<td>$</td>
</tr>
<tr>
<td>$9$</td>
<td>$</td>
</tr>
</tbody>
</table>

b) Money Left After Buying Drinks

(MP.2) 2. Compare data from two distributions using BLM When Should We Go on Vacation? (p. P-58) and answer the questions on the BLM.

Answers: a) 12; b) 19; c) 5 3/4 in; d) 12; e) 18; f) 8 3/4 in; g) They have the same shape, same number of days with rain, and same number of days without rain; h) The number line for September starts with a higher rainfall, and the amount of rainfall each day is higher overall in September.
Goals
Students will convert between 24-hour format and 12-hour format clock times.
Students will calculate the end time for an event given the start time and elapsed time.

PRIOR KNOWLEDGE REQUIRED
Knows how to tell time using a 12-hour clock format
Knows that 1 h = 60 min
Can convert time from hours to minutes
Is familiar with a.m./p.m. notation

MATERIALS
BLM 24-Hour Clock (p. P-59)
paper fastener

Converting hours to minutes. Remind students that there are 60 minutes in one hour. ASK: How many minutes are in 2 hours? (120) 3 hours? (180) ASK: What is a quick way to find out how many minutes are in 7 hours? (multiply 7 × 60)

Exercises: Find the number of minutes.
 a) 4 hours b) 8 hours c) 11 hours d) 24 hours
Answers: a) 240, b) 480, c) 660, d) 1,440

ASK: What operation should I perform to find the number of minutes in 3 h 15 min? (multiply 3 × 60 and then add 15)

Exercises: Find the number of minutes.
 a) 5 h 12 min b) 7 h 54 min c) 10 h 30 min d) 14 h 10 min
Answers: a) 312, b) 474, c) 630, d) 850

Converting minutes to seconds. ASK: How many seconds are in 1 minute? (60) How many seconds are in 2 minutes? (120) What arithmetic should I perform to find the number of seconds in 3 minutes 12 seconds? (multiply 3 × 60 and then add 12) SAY: Note that this is the same way we converted hours to minutes!

Converting seconds to minutes. ASK: How many minutes are in 120 seconds? (2) How did you calculate this? (divided by 60) NOTE: If students say they skip counted, ask them if there is a way that might be easier.

Ask students to divide by 60 to find the number of minutes in 480 seconds. (8) After they have had time to work on it, ask for a volunteer to come to the board to perform the division using the division algorithm. (see sample answer in the margin)
Write on the board:

\[ 480 \text{ seconds} = 8 \text{ minutes} \]

Ask students to repeat the exercise and find the number of minutes in 563 seconds. After they have had time to work on it, ask for a different volunteer to come to the board to perform the division using the division algorithm. (see sample answer in the margin)

ASK: What is different about the division? (the remainder is not zero) What does the remainder represent? (number of seconds left over) So how many minutes and seconds does 563 seconds equal? (6 min 23 s)

**Exercises:** Convert seconds to minutes by dividing by 60.

**Bonus**

a) 731 s  
   b) 845 s  
   c) 1,928 s  
   d) 15,374 s

**Answers:**

a) 12 min 11 s  
   b) 14 min 5 s  
   Bonus:  
   c) 32 min 8 s  
   d) 256 min 14 s

Using the 24-hour clock format. SAY: To avoid confusion between times like 7:00 a.m. and 7:00 p.m., bus stations, train stations, and airports often use a 24-hour clock format.

Create an overhead transparency or use a digital projector to display **BLM 24-Hour Clock Format** with the hands for the minute and the hour attached with a split pin (see example in margin).

SAY: In the 24-hour clock, we use two scales of numbers for the digits of the hour. We use the outer scale—or ring—from midnight until just before noon, and the inner scale from noon until just before midnight. When using the 24-hour clock format, we always write two digits for the hour. The short forms **a.m.** and **p.m.** are not used in the 24-hour clock format.

Point to the overhead to show where 07:00 and 19:00 appear on the clock. SAY: If it is 7:00 a.m., we use the outer scale of numbers. We write 07:00 and say: “oh-seven-hundred.”

If it is 7:00 p.m., we use the inner scale of numbers. We write 19:00 and say “nineteen-hundred.”

Write on the board:

\[ \begin{align*}
7:00 \text{ a.m.} & \rightarrow 07:00 \\
7:00 \text{ p.m.} & \rightarrow 19:00 
\end{align*} \]

ASK: Is it a.m. or p.m. from 12:00 until 23:59? (p.m.)

ASK: Is it a.m. or p.m. from 00:00 until 11:59? (a.m.)

**Converting times from 1:00 p.m. to 11:59 p.m. to the 24-hour clock format using arithmetic.** Ask volunteers to display the following times on the transparency using the hands. After each time is displayed, ask a different student to state the time using the 24-hour clock format, using the inner ring of numbers for the hour. (see answers in margin)

\[ \begin{align*}
a) & \ 14:15 \\
b) & \ 16:30 \\
c) & \ 21:45 \\
d) & \ 23:30 
\end{align*} \]

\[ \begin{align*}
a) & \ 2:15 \text{ p.m.} \\
b) & \ 4:30 \text{ p.m.} \\
c) & \ 9:45 \text{ p.m.} \\
d) & \ 11:30 \text{ p.m.} 
\end{align*} \]
ASK: If you look at the two scales of numbers on the 24-hour clock, what operation can be done to get each inner scale number from the outer scale number? (add 12)

ASK: Looking at the answers to the exercise above, what is a shortcut for converting times from 1:00 p.m. until 11:59 p.m. into the 24-hour clock format? (add 12 to the hour and don't use p.m.)

**Exercises:** Convert to a 24-hour clock format.

a) 5:30 p.m.  b) 8:15 p.m.  c) 9:25 p.m.  d) 11:47 p.m.

**Answers:** a) 17:30, b) 20:15, c) 21:25, d) 23:47

**Converting times from 13:00 to 23:59 to the 12-hour clock format using arithmetic.** Ask for volunteers to display the following times on the transparency. After each time is displayed, ask a different student to state the time using the 24-hour clock format by using the outer scale for the hour. (see answers in margin)

a) 16:30  b) 19:00  c) 21:40  d) 23:15

ASK: If you look at the two scales or rings of numbers on the 24-hour clock, what operation can be done to get each number on the outer scale from the number on the inner scale? (subtract 12 from the hour and add p.m.)

ASK: Looking at the answers to the exercise above, what is a shortcut for converting times from 13:00 until 23:59 into the 12-hour clock format? (subtract 12 from the hour and use p.m.)

**Exercises:** Convert to a 12-hour clock format.

a) 18:30  b) 20:25  c) 21:30  d) 23:55

**Answers:** a) 6:30 p.m., b) 8:25 p.m., c) 9:30 p.m., d) 11:55 p.m.

**Converting between the 12-hour and 24-hour clock formats for times between 1:00 a.m. and 11:59 a.m.** SAY: Between 1:00 a.m. and 11:59 a.m., the difference between the two formats is that the 24-hour clock format always uses two digits and doesn’t use a.m. Do not add or subtract anything from the hour digits. Ask volunteers to display the following times on the transparency. After each time is displayed, ask a different student to state the time using the 24-hour clock format by using the outer scale for the hour. (see answers in the margin)

a) 10:30  b) 11:15  c) 01:30  d) 08:00

**Exercises** 1. Convert from a 12-hour clock format to a 24-hour clock format.

a) 2:35 a.m.  b) 8:15 a.m.  c) 11:42 a.m.
2. Convert from a 24-hour clock format to a 12-hour clock format.
   a) 02:53   b) 09:18   c) 11:25

   **Answers:** 1. a) 02:35, b) 08:15, c) 11:42; 2. a) 2:53 a.m., b) 9:18 a.m.,
   c) 11:25 a.m.

   **Converting between the 12-hour and 24-hour clock formats for times**
   **between noon and 1 p.m.** SAY: In the 24-hour clock format, the time
   represents how much time has passed *since midnight*. For example: 12:15
   means 12 hours and 15 minutes have passed since midnight. What time is
   it in the 12-hour clock format if 12 hours and 15 minutes have passed since
   midnight? (12:15 p.m.)

   Write on the board:
   
<table>
<thead>
<tr>
<th>12-hour clock format</th>
<th>24-hour clock format</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:15 p.m.</td>
<td>12:15</td>
</tr>
</tbody>
</table>

   SAY: Between noon (12:00 p.m.) and 12:59 p.m., the only difference
   between the two formats is that the 24-hour clock format does not use p.m.

   **Exercises**

   1. Convert to the 24-hour clock format.
      a) 12:02 p.m.   b) 12:48 p.m.

   2. Convert to the 12-hour clock format.
      a) 12:15   b) 12:58

   **Answers:** 1. a) 12:02, b) 12:48; 2. a) 12:15 p.m., b) 12:58 p.m.

   **Converting between the 12-hour and 24-hour clock formats for times**
   **between midnight and 1 a.m.** SAY: In the 24-hour clock format, the time
   represents how much time has passed since midnight. For example: 03:15
   means 3 hours and 15 minutes have passed *since midnight*. What time is
   it in the 12-hour clock format if 3 hours and 15 minutes have passed since
   midnight? (3:15 a.m.)

   Write on the board:
   
<table>
<thead>
<tr>
<th>12-hour clock format</th>
<th>24-hour clock format</th>
</tr>
</thead>
<tbody>
<tr>
<td>3:15 a.m.</td>
<td>03:15</td>
</tr>
</tbody>
</table>

   SAY: In the first hour after midnight, the 24-hour clock format can only note
   zero hours and some minutes after midnight. So the 24-hour clock format
   uses 00 for the hour digits between midnight and 1 a.m. As well, it does not
   use a.m. For example: 12:48 a.m. = 00:48.

   **Exercises**

   1. Convert to the 24-hour clock format.
      a) 12:02 a.m.   b) 12:48 a.m.
2. Convert to the 12-hour clock format.
   a) 00:45  b) 00:12

   **Answers:** 1. a) 00:02, b) 00:48; 2. a) 12:45 a.m., b) 12:12 a.m.

**Calculating the end time.** SAY: If we know the starting time of an event and how long the event lasts, we can calculate the end time of the event.

SAY: Math class starts at 9:35 and lasts for 1 hour and 20 minutes. What time does Math class end? SAY: We can calculate the answer using addition on a grid. We can write 1 hour and 20 minutes as 1:20. Be sure to line up the colons underneath each other, just as we did when we added decimals.

Write on the board:

```
start time 9 : 3 5
+         1 : 2 0
end time  
```

SAY: When we write the elapsed time is 1:20, we mean 1 hour and 20 minutes must pass to reach the end time.

Ask for a volunteer to add the times as if they were whole numbers to calculate the end time. (10:55)

**Exercises:** Calculate the end time.

<table>
<thead>
<tr>
<th>a) start time</th>
<th>b) start time</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 8:35</td>
<td>= 12:15</td>
</tr>
<tr>
<td>elapsed time</td>
<td>elapsed time</td>
</tr>
<tr>
<td>= 3:06</td>
<td>= 3:27</td>
</tr>
</tbody>
</table>

   **Answers:** a) 11:41, b) 15:42

**Review regrouping 60 minutes as one hour.** ASK: How many minutes are in one hour? (60) How many hours are in 78 minutes? (1) How many minutes are left over? (18) How did you calculate the answer 18? (subtract 78 – 60)

Write on the board:

```
Time = 9:78
```

ASK: What is unusual about this time? (the number of minutes is greater than 60) SAY: But we can regroup the minutes as hours; in other words, we can write 78 minutes as 1 hour and 18 minutes. So, 9:78 is really 9:00 plus 1 more hour and 18 minutes. What time is 1 hour and 18 minutes after 9:00? (10:18)

**Exercises:** Regroup 60 minutes as 1 hour. Find the new time.

<table>
<thead>
<tr>
<th>a) 9:83</th>
<th>b) 10:72</th>
<th>c) 14:91</th>
</tr>
</thead>
</table>

   **Answers:** a) 10:23, b) 11:12, c) 15:31

**Calculating the end time by regrouping.** SAY: A music lesson starts at 12:35 and lasts for 1 hour and 30 minutes. What time does the lesson end? SAY: We can calculate the answer using addition. Write the first two rows
below on the board. Then fill in the final two rows after students answer. (see sample answer below)

<table>
<thead>
<tr>
<th>start time</th>
<th>elapsed time</th>
<th>end time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:25</td>
<td>+ 1:30</td>
<td>1:55</td>
</tr>
<tr>
<td>1:36</td>
<td>- 1:40</td>
<td>1:55</td>
</tr>
</tbody>
</table>

Ask for a volunteer to add the start time and elapsed time as if they were whole numbers. (13:65) Ask: What is the end time if we regroup 60 minutes as 1 hour? (14:05) How do we write this time using a.m. or p.m.? (2:05 p.m.)

**Exercises:** Find the end times. Write the end times first in the 24-hour format. Remember to line up the colons just as you aligned the decimal points when we were adding decimals. Then convert your answers to the 12-hour format.

<table>
<thead>
<tr>
<th>Start Time</th>
<th>Elapsed Time</th>
<th>End Time (24-Hour Format)</th>
<th>End Time (12-Hour Format)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 09:25</td>
<td>2:53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 12:25</td>
<td>3:40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) 14:35</td>
<td>2:55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** a) 12:18 = 12:18 p.m., b) 16:05 = 4:05 p.m., c) 17:30 = 5:30 p.m.

**Extensions**

1. Find the number of minutes in the time period.
   a) 3 days  
   b) 4 weeks  
   c) 1 year (not a leap year)

   **Answers:** a) 4,320 minutes, b) 40,320 minutes, c) 525,600 minutes

2. Find the number of minutes in 2 years 15 weeks 2 days and 48 minutes. **NOTE:** the time does not include a leap year.

   **Answer:** 1,205,328 minutes

3. A double bill at a movie theater starts at 18:55. The first movie lasts 1:45. The second movie lasts 2:35. What time does the second movie end?

   **Answer:** 11:15 p.m.

4. Linda got on a plane at 22:15. The plane trip took 3 h 25 min. She calculated she would get off the plane at 25:40 (see margin). At what time did she get off the plane?

   **Answer:** 01:40 or 1:40 a.m. the next day

5. A movie starts at 23:45 and lasts for 2 hours 35 minutes. At what times does it end?

   **Answer:** 02:20 or 2:20 a.m. the next day
MD5-20  Calculating Elapsed Time

Goals
Students will calculate elapsed times and start times by subtracting times written in 24-hour clock format.

STANDARDS
5.MD.A.1

VOCABULARY
elapsed time
end time
start time

PRIOR KNOWLEDGE REQUIRED
Knows how to tell time using a 12-hour clock format
Knows how to tell time using a 24-hour clock format
Knows how to regroup 1 hour as 60 minutes

Calculating elapsed time by subtracting without regrouping. SAY: A television program starts at 9:00 and ends at 11:00. ASK: How long is the program? (2 hours) What calculation can you perform to get the answer? (subtract: 11:00 − 9:00)

Write on the board:

\[
\begin{array}{c}
\text{start time: 14:35 } \\
\text{end time: 17:50 }
\end{array}
\]

Ask a volunteer to fill in the times in the correct positions for subtraction. Remind students that it is important to line up the colons. Write on the board:

\[
\begin{array}{c}
\text{start time: 14:35 } \\
\text{end time: 17:50 }
\end{array}
\]

Ask a volunteer to subtract as if the times were whole numbers. (3:15) SAY: So the elapsed time is 3 hours and 15 minutes.

Exercises: Calculate the elapsed time.

a) 09:15 to 12:28
b) 11:35 to 15:58
c) 08:24 to 13:43

Answers: a) 3:13, b) 4:23, c) 5:19

Regrouping 1 hour as 60 minutes. ASK: How many minutes are in 1 hour? (60) Write on the board:

\[
3:15 = 3 \text{ h } + 15 \text{ min}
\]

SAY: Regroup 1 hour as 60 min.

\[
= 2 \text{ h } + 60 \text{ min } + 15 \text{ min}
\]

ASK: What is 60 + 15? (75)

\[
= 2 \text{ h } + 75 \text{ min}
\]

\[
= 2:75
\]
Exercises: Regroup 1 hour as 60 minutes.

a) 03:25  
b) 07:28  
c) 12:34

Answers: a) 02:85, b) 06:88, c) 11:94

Calculating elapsed time by regrouping 1 hour as 60 minutes.

SAY: Sometimes we need to regroup 1 hour as 60 minutes to calculate the elapsed times. Write on the board:

start time: 14:50   end time: 17:20

Ask a volunteer to fill in the times in the correct positions to subtract. Write on the board:

\[
\begin{array}{c}
\phantom{1}17:20 \\
- \phantom{1}14:50 \\
\hline
\phantom{0}2:30
\end{array}
\]

ASK: What makes subtraction difficult here? (we can’t subtract 2 – 5) SAY: If we regroup 1 hour as 60 minutes, how can we rewrite the end time? (16:80) Write on the board:

\[
\begin{array}{c}
\phantom{1}16:80 \\
- \phantom{1}14:50 \\
\hline
\phantom{0}2:30
\end{array}
\]

Ask a volunteer to subtract the times as if they were whole numbers. (2:30) SAY: So the elapsed time was 2 hours 30 minutes.

Exercises: Calculate the elapsed time. Remember to regroup 1 hour as 60 minutes when needed.

a) 09:45 to 12:25  
b) 10:35 to 14:18  
c) 07:44 to 14:23

Answers: a) 2:40, b) 3:43, c) 6:39

Calculating elapsed time by first converting from a 12-hour clock format.

SAY: It is often easier to calculate elapsed times by first converting from a 12-hour clock format to a 24-hour clock format, especially when the time passing includes noon. Write on the board:

10:35 a.m. to 2:45 p.m.

ASK: What are the times written in a 24-hour clock format? PROMPT: Remember that in the 24-hour clock format, the time represents the amount of time that has passed since midnight. (10:35 and 14:45) Write on the board:

\[
\begin{array}{c}
\phantom{1}14:45 \\
- \phantom{1}10:35 \\
\hline
\phantom{0}4:10
\end{array}
\]

Ask a volunteer to subtract the times as if they were whole numbers. (4:10)
Exercises: Calculate the elapsed time by first converting to a 24-hour clock format. You may have to regroup 1 hour as 60 minutes.

a) 7:10 a.m. to 1:40 p.m.  b) 11:30 a.m. to 4:20 p.m.
c) 9:15 a.m. to 3:23 p.m.  d) 8:24 a.m. to 4:12 p.m.

Answers: a) 6:30, b) 4:50, c) 6:08, d) 7:48

Calculating start times by subtraction. SAY: If we know the end time and elapsed time, we can calculate the start time using the same method we just used. Write on the board:

A hockey game lasts 1:25 and ends at 1:35 p.m. What time did it start?

Draw the grid on the board:

```
|   |   |   |
-  |   |   |
|   |   |   |
```

ASK: How do we write 1:35 p.m. using the 24-hour clock format? (13:35)

Ask the class to copy the grid in their notebooks and perform the subtraction to find the start time. (the game started at 12:10 p.m.)

Write on the board:

```
1 3 : 3 5
- 1 : 2 5
```

```
1 2 : 1 0
```

Write on the board:

A hockey game lasts 1 hour and 35 minutes and ends at 1:25 p.m. What time did it start?

SAY: Although this question is almost the same as the earlier question, the subtraction is more difficult.

ASK: How do you write 1:25 p.m. in the 24-hour clock format? (13:25)

ASK: If we try to subtract 13:25 – 1:35, what problem will we encounter? (we can't subtract 2 – 3) If we regroup 1 hour as 60 minutes, how can we rewrite 13:25? (12:85) Ask a volunteer to come to the board, place the times in the correct positions, and subtract as if the times were whole numbers. (the start time was 11:50 a.m.; see below for sample subtraction grid)

```
1 2 : 8 5
- 1 : 3 5
```

```
1 1 : 5 0
```
Exercises: Find the start time.

a) elapsed time = 2:35  
end time = 1:50 p.m.

b) elapsed time = 14:25  
end time = 11:10 p.m.

Answers: a) 11:15 a.m., b) 8:45 a.m.

Exercises

1. A school trip begins at 9:00 a.m. and lasts 5 h 20 min. What time did it end?

   Answer: 14:20 or 2:20 p.m.

2. Melina’s piano concert takes 3 h 15 min and ends at 2:35 p.m. At what time did it begin?

   Answer: 11:20 or 11:20 a.m.

Extensions

1. Find the elapsed time for an event that starts at 1:55 p.m. and ends at 4:40 p.m.

   Answer: 2 hours 45 minutes

2. Find the start time for an event that ends at 2:30 a.m. and lasts 4 hours and 10 minutes.

   Answer: 10:20 p.m. or 22:20 the previous night

(MP.2) 3. Thirteen people shared some chocolate bars, but some people wanted more chocolate than others. The line plot shows the fraction of a chocolate bar that each person had.

   How many chocolate bars did they share?

   Answers: There are 3 people who had 1 1/8 chocolate bars, 3 people who had 1 1/4, 2 people who had 1 1/2, 4 people who had 1 3/4, and 1 person who had 1 7/8. The total number of chocolate bars can be found using the expression 
   
   \( (3 \times \frac{1}{8}) + (3 \times \frac{1}{4}) + (2 \times \frac{1}{2}) + (4 \times \frac{3}{4}) + \frac{17}{8} = (3 \times \frac{9}{8}) + (3 \times \frac{5}{4}) + (2 \times \frac{3}{2}) + (4 \times \frac{7}{4}) + \frac{15}{8} = 27/8 + 15/4 + 6/2 + 28/4 + 15/8 = 27/8 + 30/8 + 24/8 + 56/8 + 15/8 = 152/8 = 19 \). So they shared 19 chocolate bars.
MD5-21  Time Word Problems
Page 105

STANDARDS
5.MD.A.1

VOCABULARY
time line diagram

Goals
Students will solve problems involving calculating time.

PRIOR KNOWLEDGE REQUIRED
Can calculate the end time given the start time and elapsed time
Can calculate the elapsed time given the start time and end time
Can calculate the start time given the end time and elapsed time

NOTE: Just as with word problems involving addition and subtraction, students need to read word problems involving time carefully to determine which operation is appropriate. For some students, a time line may help them determine whether addition or subtraction is required.

A number line or time line diagram that involves moving to the right will require addition. A number line or time line diagram that involves moving to the left will require subtraction. A number line or time line diagram that involves finding the distance between two places on the line will require subtraction.

Drawing number lines or time lines. Draw the following number line on the board:

```
5 6 7 8 9 10 11 12
```

ASK: If you start at the number 5, how many jumps will it take to get to the number 12? (7) What operation can you perform to get this answer? (12 – 7)

SAY: We can use similar number lines for calculating time. This kind of number line is called a time line diagram.

Draw the following time line on the board:

```
05:00 06:00 07:00 08:00 09:00 10:00 11:00 12:00
```

ASK: If an event starts at 05:00 and ends at 12:00, how much time elapses? (7 hours 0 minutes or 7:00) What operation can you perform to get this answer? (subtraction, 12:00 – 05:00) SAY: We can draw a bracket to represent the elapsed time.

SAY: We can make our diagrams easier to draw if we only include tick marks for the start time and the end time. Draw on the board:

```
05:00 12:00
```

7:00
Practice drawing time line diagrams. SAY: In word problems, sometimes information is missing. In word problems involving time, the start time, end time, or elapsed time can be missing. We can label the missing information with a question mark.

Write on the board:

An event starts at 08:15 and lasts for 2 h 30 min.

SAY: We need to draw a number line to represent this event. ASK: What time should be placed on the left side of the time line? (08:15) What time should be placed under the brackets? (2:30) What should be placed on the right side of the time line? (a question mark; see sample time line below)

\[
\begin{array}{c}
08:15 \\ \text{？} \\ 2:30
\end{array}
\]

NOTE: The following examples of problems require either addition or subtraction of time. Walk your students through each type of question to draw the corresponding time diagrams. Later, students will be asked to perform the appropriate addition or subtraction.

Word problem where the end time is missing. Write on the board:

Phil starts working at his part-time job at 9:20 a.m. and works for 3 hours and 50 minutes. What time did he finish work?

\[
\begin{array}{c}
09:20 \\ \text{？} \\ 3:50
\end{array}
\]

Use addition to find the answer.

Word problem where the start time is missing. Write on the board:

A guitar lesson ends at 1:15 p.m. and is 45 minutes long. What time did the lesson start?

\[
\begin{array}{c}
? \\ 00:45 \\ 13:15
\end{array}
\]

Use subtraction to find the answer.

Word problem where the elapsed time is missing. Write on the board:

Tony's dad starts his holiday shopping at 10:25 a.m. and finishes at 2:10 p.m. How long was he shopping?

\[
\begin{array}{c}
10:25 \\ \text{？} \\ 14:10
\end{array}
\]

Use subtraction to find the answer.
Exercise

Solve each of the problems for which you drew number lines.

Answers: a) 13:10, b) 12:30, c) 3 h 45 min

Extensions

NOTE: In Extensions 1 and 2, encourage students to model the situation using equations, time lines, and addition or subtraction with time, and to state what each part of their model means in the context of the problem.

1. A bike rider starts a charity race at 8:10 a.m. It takes her 40 min to ride 8 miles. If she continues at this pace and wants to ride 32 miles, at what time will she finish the event?

Sample solution: Since she rides 8 miles in 40 minutes, it takes her $\frac{40}{8} = 5$ minutes to ride 1 mile. So 32 miles will take $32 \times 5 = 160$ minutes. $160 \div 60 = 2$ R 40, so it will take her 2 hours and 40 minutes to finish the event. She will finish the event at 8:10 + 2:40 = 10:50 a.m.

2. Fred needs to shovel snow from six driveways between 8:50 a.m. and 11:14 a.m. If he spends the same amount of time on each driveway, at what time will he need to start shoveling the last driveway?

Sample solution: To find the elapsed time between 8:50 a.m. and 11:14 a.m., regroup 1 hour from the finish time into 60 minutes. 10:74 $- 8:50 = 2:24$, so it takes Fred 2 hours and 24 minutes, or $120 \div 24 = 144$ minutes, to shovel 6 driveways. Since it takes Fred 24 minutes to shovel each driveway, he must start shoveling the last driveway at 11:14-24 = 10:44-24 = 10:50. So Fred needs to start shoveling the last driveway at 10:50 a.m.

3. a) What is the smallest number you can create so that each of the following is true?
   - the largest place value is worth 300 times the tens digit
   - the smallest place value is thousandths
   - the thousandths digit is worth one fourth of the hundredths digit
   - each digit is greater than 1

b) In pairs, explain how you know that your answers in part a) are correct. Do you agree with each other? Discuss why or why not.

Answers: a) 6,222.225; b) Since each digit must be greater than 1, the smallest digit in any place is 2. Since the largest place is worth 300 times the tens digit, and the smallest digit for the tens place is 2, the smallest amount the largest digit can be worth is $20 \times 300 = 6,000$. The thousandths digit must be $1/4$ of the hundredths digit, and the smallest digit for the hundredths place is 2, so the thousandths digit must be $0.02 \div 4 = 0.005$. 

Measurement and Data 5-21
**MD5-22 Diagrams and Fractions**

**Goals**
Students will solve multi-step problems with fractions and unit conversions using diagrams.

**PRIOR KNOWLEDGE REQUIRED**
- Can convert between different units of length, mass, time, and money within the same system
- Can perform the four basic operations with multi-digit numbers, fractions, mixed numbers, and decimals
- Understands that division can produce a fraction
- Can convert between mixed numbers and improper fractions
- Can solve word problems using the four operations
- Can draw a diagram to represent parts in a whole

**Identifying parts of a diagram and solving a problem given the diagram.**

**SAY:** Nina has $15. She spends 2/5 of her money on a poster. We want to represent this situation with a diagram.

Draw a long rectangle divided into five equal parts (see example in the margin). Ask a volunteer to shade the part of the diagram that represents the part that was spent on the poster. (two out of five blocks) **ASK:** How much money does each block represent? ($3) How do you know? (there are five blocks and the total is $15, so one block is $15 ÷ 5 = $3) Finish the diagram as shown in the margin. **ASK:** How much money did she spend on the poster? ($6) How do you know? (2/5 = 2 blocks, 2 × $3 = $6) How much money is leftover? ($9) How do you know? ($15 − $6 = 9, or 3 unshaded blocks, 3 × $3 = $9) Emphasize that, even though the first solution is correct and perfectly good, the second solution shows that you can find the leftover in the diagram without calculating how much money was spent on the poster.

Repeat the exercise above with the following problems. To start each problem for students, draw a simple (unlabeled, unshaded) rectangle diagram for the problem. The finished diagrams are shown in the margin.

- **a)** Jake has $20. He spends one fourth of the money on a binder. How much money does he have left? ($15)
- **b)** Kim spends \(\frac{5}{6}\) of her money on lunch. The lunch costs $10. How much money did she have at the beginning? How much money does she have left? ($12 at the beginning, $2 left)
- **c)** Ravi spends \(\frac{2}{3}\) of his money on a book. He has $6 left. How much did the book cost? How much money did he have at the beginning? (the book costs $12, so Ravi had $18 at the beginning)
**Drawing the diagram for a problem and solving the problem.** ASK: How did I know how many blocks should be in each diagram? (the number of blocks is the denominator of the fraction, the number of parts the whole is divided into)

**Exercises:** Draw the diagram and solve the problem.

a) Anna has $20. She spends $\frac{3}{4}$ of her money on a pair of gloves. How much money does she have left?

b) Peter spends $\frac{3}{5}$ of his money on a bus ticket. He has $14 left. How much did the ticket cost?

c) Borana spends $\frac{2}{5}$ of her money on a train ticket that costs $50. How much money did she have before she bought the ticket?

**Answers**

a) $5$

b) $21$

c) $125$

Review long division with many zeros. Solve a few of the following division questions as a class to prepare students for more complicated problems related to money. (see answers in margin)

a) $20.00 \div 8$

b) $10.00 \div 8$

c) $4.00 \div 5$

d) $30.00 \div 8$

**Drawing the diagram for a problem with decimal components and solving the problem.** Remind students that when writing a money amount, we often write $4.00$ instead of $4$. This makes division easier in some cases. For example, if you need to divide $4$ into five parts, you divide using decimals: $4.00 \div 5 = 0.80$. Remind students also that when they multiply decimals, they should work with them as if they were whole numbers, and then shift the decimal point as needed. So, for example, when multiplying $0.85 \times 4$, calculate $85 \times 4 = 340$ and shift the decimal point two spaces to the left to 3.40. ASK: How do we know to shift the decimal point two places to the left? (there are two decimal places in 0.85 and 0 decimal places in 4, and $2 + 0 = 2$ decimal places)

**Exercises:** Solve the problem using a diagram.

a) Mateo has $20. He spends $\frac{3}{8}$ of his money on an airplane model. How much money does he have left?

b) Jennifer spends $\frac{3}{7}$ of her money on a movie ticket. She has $15$ left. How much did the ticket cost?

c) Anwar spends $\frac{5}{7}$ of his money on a bo staff that costs $19.85. How much money did he have initially?
Bonus: Maria spends \(\frac{3}{5}\) of her money on a movie ticket and popcorn. The movie ticket costs $6, and the popcorn costs $4.65. How much money did Maria have initially?

Answers
a) one block $2.50, leftover $12.50  
b) one block $3.75, ticket cost $11.25

Drawing the diagram for a problem that requires conversion and solving the problem. Remind students that there are 16 ounces in 1 pound and 60 minutes in 1 hour. Explain that, in the next set of problems, they will need to convert the measurements to smaller units to solve them.

Exercises: Solve the problem using a diagram.
a) Luis spends \(\frac{1}{6}\) of his lunch break eating, and spends the rest of the time in a library. He is in the library for 1 hour. How long is the lunch break?
b) Grace has 3 lb of meat. She uses \(\frac{3}{8}\) of it for a meat pie. She needs 1.5 lb of meat for meatloaf. Does she have enough meat left?

Bonus: When school ended at 3:30 p.m., Sofia went to a judo class, and then did homework. Judo and homework combined took four fifths of the time between school and dinner. The judo class lasts 50 minutes. Dinner starts at 5:10 p.m. How long did Sofia spend on homework?

Answers
a) one block 12 min, lunch time 72 min  
   1 h 12 min  
   total lunch break

   Time before dinner, 
   1 h 40 min = 100 min

   Judo + homework

b) one block, 6 oz, leftover  
   30 oz > 1.5 lb, Grace has enough meat left

   3 lb = 48 oz

   Pie leftover

Bonus: Total time before dinner: 5:10 – 3:30 = 17:10 – 15:30 = 1:40 or 100 min. One block = 20 min, and judo + homework = 80 min. Judo class = 50 min, so homework took 30 minutes. (see diagram in margin)
Solving problems where an amount is divided and then further divided into unequal parts. Present this situation: Nina has $15, and spends two fifths of her money on a poster. She spends one third of the leftover money on a greeting card.

Point out that the first sentence is the same as the very first situation you dealt with in this lesson (on p. P-50), so we can start with the same diagram as there. Draw the first diagram in the margin. ASK: The leftover is not marked on the diagram, but which part of it shows the leftover? (the unshaded rectangles) Cover up the shaded part and ASK: Which part is 1/3 of the leftover? (one rectangle) Mark that on the diagram as shown in the second picture in the margin.

ASK: How much money does each block represent? ($3) How do you know? (there are five blocks, and the total is $15, so one block is $15 ÷ 5 = $3) How much money did Nina spend on the card? ($3) How much money is left after she bought the poster and the card? ($6) How do you know? ($15 – $6 – $3 = $6; or 2 blocks, 2 × $3 = $6) Emphasize again that the second solution allows you to find the leftover in the diagram without having to calculate what was spent on the poster or on the card.

Repeat with the following problem. Draw the rectangle diagram—but no shading or labels—for the students to start with.

David spends 2/7 of his money on a book, and 3/5 of the leftover on a gift for his mom. He has $10 left. How much did the book cost? ($10) How much money did he have initially? (7 × $5 = $35) How much did the gift cost? (3 × $5 = $15; see completed diagram in margin)

Solving problems by identifying the whole and the parts, and drawing the diagram. ASK: How do you know how many rectangles we should draw in the diagram? (draw the number of parts the whole is divided into) Point out that one sentence in the situation describes which part is the whole, and the other talks about a part of the part (for instance, part of the leftover money). The number of rectangles is the number of parts in the whole, not the leftover.

For each of the following problems, have students signal how many rectangles should be in each diagram.

Exercises: Solve the problem using a diagram.

a) Camille has $30. She spends 5/8 of her money on a hoodie, and 2/3 of the rest on a snack. How much money does she have left? How much did the hoodie cost? How much did the snack cost?

b) Jose picks some apples. He uses 1/6 of the apples to make applesauce and 3/5 of the leftover in an apple crumble. He uses 2 lb 4 oz of apples in the apple crumble. How many ounces of apples did he pick? How many ounces of apples did he have leftover? How many ounces of apples did he use in the applesauce?
c) Alice spends 1/7 of her vacation on a trip. She spends half of the rest of the vacation in a leadership camp, and the remaining 3 weeks and 3 days volunteering at an animal shelter. How long was her trip? How long did she spend in the camp? How long was her vacation?

Answers

a) one block $3.75, leftover $3.75, hoodie $18.75, snack $7.50
b) one block 12 oz, total picked
72 oz = 4 1/2 lb, leftover 24 oz = 1 1/2 lb, applesauce 12 oz = 3/4 oz

c) one block 8 days, trip 8 days, camp 24 days, vacation 56 days = 7 weeks

Extension

(MP1, MP4) Madison spends the money she was given for her birthday in this way: she donates 1/10 to charity, and places 1/3 of the remainder in her savings account. One sixth of what is left is a gift card to an ice cream parlor. She uses the rest of the money to buy three books for $7.99 each, a T-shirt for $3.99, and a basketball for $29.79. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each other part (donation, savings, gift card)?

Answer: total purchases $57.75, total money given $115.50, charity $11.55, savings $34.65, gift card $11.55 (see sample diagram below)
Patterns in the Multiples of 12

The multiples of 12 are the numbers you say when you count by 12s: 0, 12, 24, 36, ....

1. a) Write the first five multiples of 12 in the first column of the table. Write the next ten multiples of 12 in the second and third columns.

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<th>0</th>
<th>6</th>
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<td></td>
<td>5</td>
<td>10</td>
<td>12</td>
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</tbody>
</table>

b) What pattern do you see in the ones digits in each column?

REMINDER To find the number of tens, cover the ones digit.
Example: The number of tens in 168 is 16.

c) What pattern do you see in the number of tens in each column?

d) What pattern do you see in the first row of the table?
The numbers in the first row are multiples of ___.

e) Use the patterns you found to write the next ten multiples of 12.

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</table>

f) Are all multiples of 12 even? Explain.

BONUS Add the digits in each multiple of 12 on this page. The answers are all multiples of the same number. What number is that? _____
Shoe Length Line Plot

Grade 5 Shoes

8 8 1/2 9 9 1/2 10 10 1/2 11 11 1/2 12 12 1/2

Shoe Length (in)
Drinks Line Plot

Student Spending on Drinks per Week

Money ($)
When Should We Go on Vacation?

1. A family is trying to decide which month to visit their favorite vacation spot.

<table>
<thead>
<tr>
<th>Rainfall per Day in June</th>
<th>Rainfall per Day in September</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 3 1 5 3 7</td>
<td>3 1 5 3 7 1 1</td>
</tr>
<tr>
<td>8 4 8 2 8 4 8</td>
<td>8 2 8 4 8 1 1/8</td>
</tr>
</tbody>
</table>

   Amount of Rainfall (in)

   a) How many days did it rain in June? _________________

   b) How many days had less than 1/4 inch of rain in June? _________________

   c) What was the total rainfall in June? _________________

   d) How many days did it rain in September? _________________

   e) How many days had less than 3/8 inch of rain in September? _________________

   f) What was the total rainfall in September? _________________

   g) What is similar about the two line plots? ______________________________________

   h) What is different about the two line plots? ______________________________________

   i) Which month should the family go on vacation? Explain.

   ______________________________________

   ______________________________________

   ______________________________________
24-Hour Clock

midnight

Hour hand

Minute hand
Unit 6 Measurement and Data: Area and Volume

This Unit in Context

Students studied length as a measurement attribute beginning in Kindergarten and began measuring length in Grade 1. In Grade 3, students began to measure areas, including measuring the area of a rectangle by multiplying the side lengths (3.MD.C.7a, b). In this unit, students will review finding the area of a rectangle and then apply Grade 5-level operations such as multiplying fractions by fractions (5.NF.B.4), which they learned in 5.2 Unit 2, and multiplying decimals by decimals (5.NBT.B.4), which they learned in 5.2 Unit 4, when finding the areas of rectangles that have fractional and decimal lengths and widths. Students will stretch their understanding of area in Grade 6 to include areas of non-rectangular shapes—such as triangles, parallelograms, trapezoids, and composite shapes made up of straight sides (6.G.A.1)—and in Grade 7 to include areas of circles (7.G.B.4).

Students in this unit will expand the concept of area as an attribute of two-dimensional space (3.MD.C.5) to the concept of volume as an attribute of three-dimensional space (5.MD.C.3). They will measure volume by counting unit cubes (5.MD.C.4), just as they measured area in Grade 3 by counting unit squares (3.MD.C.6). Students will employ much the same approach to calculate volumes as they used to calculate areas: just as the area of a rectangle can be found by multiplying the side lengths (3.MD.C.7a), the volume of a rectangular prism can be found by multiplying the edge lengths (5.MD.C.5a). In the same way students learned in Grade 3 to solve problems involving areas of rectangles in real-world contexts (3.MD.C.7b), in this unit students will learn to solve problems involving volumes of right rectangular prisms in real-world contexts (5.MD.C.5b). Students will also learn that volume is additive (5.MD.C.5c), just as they learned in Grade 3 that area is additive (3.MD.C.7d). In Grade 6, students will find volumes of right rectangular prisms that have fractional edge lengths (6.G.A.2). In Grade 8, students find volumes of cones, cylinders, and spheres (8.G.C.9).

In Grades 1 and 2, students understood capacity as how much a container can hold. In Grade 3, students were introduced to liquid volume (3.MD.A.2) and measured liquid volume using milliliters and liters. Students learned to distinguish between the volume of a liquid in a container and the capacity of the container, and they solved word problems involving liquid volume (3.MD.A.2). In Grade 4, students converted from liters to milliliters (4.MD.A.1) and they converted from mixed units in liters and milliliters to milliliters. Earlier in Grade 5, students converted between different-sized measurement units within a given measurement system. In this unit, students will convert between liters and milliliters in both directions, and they will recognize that a milliliter is equivalent to a cubic centimeter. Students will also convert (in both directions) between fluid ounces and cups, and between cups, pints, quarts, and gallons. Students will use these conversions to solve multistep problems (5.MD.A.1), adding, subtracting, multiplying, and dividing numbers, including fractions and decimals.
Therefore, students will need to be able to convert between mixed numbers and improper fractions, and fluently multiply fractions, mixed numbers (5.NF.B.4, 6), and decimals (6.NBT.B.7).

**Mathematical Practices in This Unit**

In this unit, you will have the opportunity to assess MP1 and MP3 to MP8. Here are some examples of how students can show that they have met a standard.

**MP.1:** In part b) of the extension in MD5-26, students make sense of a non-routine problem when they find an organized method to check the possible combinations for the length, width, and height of a rectangular stack of 48 cubes. They persevere to solve the problem when they make use of the structure of rectangular prisms and factors of 48 to narrow down their search to find all possible combinations.

**MP.3:** In MD5-30 Extension 4, students critique the reasoning in the question and explain the flaw in the argument.

**MP.4:** In MD5-29 Extension 2, students model mathematically when they decide what information in a real-world situation they need to use to represent the problem and solve it, and what information they can disregard.

**MP.5:** In MD5-37 Extension 3, students use appropriate tools strategically when they choose, for example, base ten blocks or a number line to explain how a simple product can be used to calculate two related complex products.

**MP.8:** In MD5-38 Extension 5, students look for and express regularity in repeated reasoning when they notice a pattern in their calculations from the previous extension and write a general equation for finding the height of a rectangular prism with a square base given the volume and one side length of the square base.
Unit 6 Measurement and Data: Area and Volume

Introduction

In this unit, students will learn about area and volume. As they find the areas of rectangles and shapes made from rectangles, students will need to multiply fractions. When solving problems that require conversion between units, students will add, subtract, multiply, and divide numbers, including fractions and decimals. Therefore, they need to be able to convert between mixed numbers and improper fractions, and fluently multiply fractions, mixed numbers, and decimals. For details, see the prior knowledge required for each lesson.

Capacity vs. Volume

*Volume* and *capacity* are concepts that are sometimes interchanged. *Volume* is the amount of space taken up by a three-dimensional object, and *capacity* is defined as how much a container can hold. While volume is measured in cubic units—such as cubic centimeters (cm³) or cubic feet (ft³)—capacity is measured in fluid ounces (fl oz), cups (c), gallons (gal), liters (L), etc. Units of capacity are also used to measure the volume of any pourable substance, such as water. Liters are not strictly part of the metric system; the SI or International System of Units has basic units and derivatives and steers clear of multiple units for the same thing. However, liters are compatible with the use of the metric system, specifically for the volume of pourable substances. Liters are commonly used and appealing because they are human-sized, not too large and not too small.

Materials

Throughout the unit, collect various sizes of boxes (such as shoe boxes, small food and medication packaging boxes, facial tissue boxes) so that students can measure and calculate their volumes in Lessons MD5-29 and MD5-31. Invite students to add to your collection of boxes by bringing some in from home. In Lesson MD5-34, you will also need a rectangular aquarium.
MD5-23 Area of Rectangles
Pages 109–110

STANDARDS
preparation for 5.NF.B.4b

VOCABULARY
area
column
row
square centimeter (cm²)
square foot (ft²)
square inch (in²)
square kilometer (km²)
square meter (cm²)
square millimeter (mm²)
square yard (yd²)

Goals
Students will find the area of rectangles with whole-number sides and review units of area learned in previous grades.

PRIOR KNOWLEDGE REQUIRED
Knows the relative size of units of length measurement within the metric and customary US system
Can use multiplication to find the number of objects in an array
Can multiply and divide mentally up to 12 × 12
Can multiply two 2-digit numbers

MATERIALS
meter sticks or yard sticks
pattern block squares
1 cm cube
1 ft square floor tile
old newspapers or wrapping paper
rulers

Review counting squares in a rectangular array. Remind students that, when they have an array of rows and columns of the same length, they can use skip counting, repeated addition, or, more efficiently, multiplication to find the total number of squares. To find the number of squares in an array such as the one in the margin, students usually multiply the number of squares along the length of the array (5) by the number of squares along the width of the array (4) for the total (20).

Introduce square centimeters as units of area. Explain that squares that have sides of 1 cm each are called square centimeters, and that we write 1 cm² for 1 square centimeter. Explain that, when we cover a flat shape with squares of the same size so that there are no gaps or overlaps, the number of squares needed to cover the shape is called its area. A square centimeter is then a unit for measuring area.

Draw a rectangle and a square as shown in the margin. ASK: Which shape has a larger area? How do you know? (the square is covered by 4 squares and the rectangle is covered by 3 squares, so the square has a larger area)

Determining area of rectangles covered by squares. On the board, draw several rectangles and mark their sides at regular intervals, as shown in the margin, so that students will be able to extend the marks into squares. Ask volunteers to divide the rectangles into squares by using a meter stick or a yardstick to join the marks. Ask students to find the area of the rectangles.
Draw a rectangle on the board as shown in the margin. Tell students that the squares represent square centimeters since actual square centimeters are too small to draw on the board. Explain that you want to find the area of the rectangle, which means counting the total number of squares needed to cover the rectangle without gaps or overlaps. How can you do it without drawing all the squares? (multiply the number of rows by the number of columns, or multiply the number of squares along the length by the number of squares along the width) ASK: How many squares fit along the length, the longer side, of the rectangle? (6) How many squares fit along the width of the rectangle, the shorter side? (4) How many columns will there be? (6) How many squares will there be in each column? (4) What is the total number of squares? (6 \times 4 = 24 squares) What is the area of the rectangle? (24 cm²)

Emphasize that finding area is counting squares; the trick is to do it efficiently. In a rectangle, the squares are arranged in an array, so we multiply the number of columns by the number of rows.

**Determining the formula for area of rectangles.** Draw another rectangle and mark its sides as 5 cm and 3 cm. ASK: How many 1 cm squares will fit along the length of the rectangle? (5) How do you know? (the rectangle is 5 cm long) How many squares will fit along the width of the rectangle? (3) How many square centimeters are needed to cover the entire rectangle? (15) How do you know? (3 rows of 5 squares, or 5 columns of 3 squares, make an array of 15 squares) What is the area of the rectangle? (15 cm²)

Explain that, because area is measured in squares, we can think about any rectangle as being made of squares. ASK: Did we divide the rectangle into squares to find the answer? (no) How did we find the answer? (we multiplied length by width) Do you think this method will work to find the area of any rectangle? (yes) Summarize on the board:

\[
\text{Area of rectangle} = \text{length} \times \text{width}
\]

**Finding areas of rectangles.** Have students work in pairs. Ask each student to draw various rectangles, label the length and width of the rectangles in whole centimeters (not necessarily to scale), and then exchange notebooks. Then ask partners to find the area of the rectangles and check each other’s work.

**Introduce different units.** Explain that, just as length is measured in different units, so is area. Square centimeters are squares with length and width each of 1 cm. ASK: What other square units can you think of? Write the students' suggestions on the board. Add units as necessary until you have at least the units shown below. Introduce the short forms of the units.

- square inch (in²)
- square foot (ft²)
- square yard (yd²)
- square millimeter (mm²)
- square centimeter (cm²)
- square meter (m²)
- square kilometer (km²)
Ask students if they know of anything that is exactly 1 square unit of some kind. If the following examples are not offered, point them out and, if possible, show them: a face of a centimeter cube is 1 cm², a pattern block square is 1 in², and some floor tiles are 1 ft².

**ACTIVITY**

*Making and comparing unit squares.* Assign different square units (1 cm², 1 in², 1 ft², 1 yd², 1 m²) to students and have them use wrapping paper or old newspapers to make and label squares of the given size. Have students who are working on the small units (square inch, square centimeter, square foot) make as many squares of the same size as they can, while their peers create single large squares.

Have students work in groups to compare and order the units by trying to place one unit on top of the other. Then have them make rectangles from the squares they created to answer questions such as the following:

- Will a rectangle that is 5 cm by 4 cm fit inside 1 square meter? (yes)
- Will a rectangle that is 3 ft by 2 ft fit inside 1 square yard? (yes)
- Will a rectangle that is 4 ft by 1 ft fit inside 1 square yard? (no)
- Does a rectangle that is 4 ft by 1 ft have a larger area than 1 square yard? (no) How can you show that with unit squares? (a rectangle 4 ft by 1 ft is made of 4 square feet, which all fit together inside a square yard)

Students can also use a ruler to draw rectangles in small units on the rectangles representing larger units to see if they fit inside. They might ask, for example: Will a rectangle that is 7 mm by 9 mm fit inside 1 cm²? (yes) Will a rectangle that is 11 in by 7 in fit inside 1 square foot? (yes) How about a rectangle that is 13 in by 7 in? (no) Have students try to cover the part of the rectangle that “sticks out” from the 1 ft square in square inches, and then see that the leftover part fits inside the 1 ft square together with the rest of the rectangle. Have students conclude that a rectangle measuring 13 in by 7 in has an area smaller than 1 ft².

**Finding area of rectangles in different units.** Work through the following problem as a class. Ensure students pay attention to the numbers as well as the units in part b).

a) Calculate the area of each rectangle. Include the units.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>D</th>
<th>R</th>
<th>E</th>
<th>V</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3 km</td>
<td>8 mm</td>
<td>15 cm</td>
<td>13 cm</td>
<td>21 m</td>
</tr>
<tr>
<td>Width</td>
<td>2 km</td>
<td>7 mm</td>
<td>6 cm</td>
<td>7 cm</td>
<td>15 m</td>
</tr>
</tbody>
</table>

(area: 6 km², 56 mm², 90 cm², 91 cm², 315 m²)
b) Jake thinks that rectangle O fits inside rectangle D. Is he correct? Explain. (yes, because 21 m and 15 m are both smaller than 1 km; 1 km = 1,000 m, so this rectangle will fit inside a rectangle that is 3 km long and 2 km wide)

c) List the rectangles from greatest area to least area. What state capital does your answer spell? (Dover)

**Bonus:** For what state is it the capital? (Delaware)

**Extensions**

1. On grid paper or a geoboard, make as many shapes as possible with an area of 6 squares. For a challenge, try making shapes that have at least one line of symmetry—a line that divides a shape into two identical parts that are mirror images of each other. For instance, the shapes in the margin each have an area of 6 square units and one line of symmetry, shown as a thin line.

2. Mario says that shape A in the margin has an area of 4 squares and shape B has an area of 3 squares, so shape A has a larger area than shape B. Explain his mistake.

   **Answer:** Each square in shape B is larger than each square in shape A, and three larger squares can have a larger total area than four smaller squares.

3. Explain that perimeter is the distance around a shape. Students should be familiar with this concept from earlier grades. Have them investigate connections between area and perimeter of rectangles using **BLM Area and Perimeter** (p. Q-66).
MD5-24  Area of Rectangles with Fractions

Pages 111–112

STANDARDS
5.NF.B.4b

VOCABULARY
area
denominator
formula
fraction
numerator
square centimeter (cm²)
square foot (ft²)
square inch (in²)
square kilometer (km²)
square meter (cm²)
square millimeter (mm²)
square yard (yd²)

Goals
Students will find the area of rectangles with fractional sides.

PRIOR KNOWLEDGE REQUIRED
Can multiply numbers including multi-digit whole numbers, fractions, and mixed numbers
Knows what area is
Can find the area of a rectangle
Knows that division of two whole numbers can produce a fraction

MATERIALS
rulers

NOTE: This lesson assumes that students will use quarter-inch grid paper notebooks.

Review multiplying fractions. Draw a square on the board and explain that this is one whole. Divide the square into three equal columns. ASK: What fraction does each column represent? (1/3) Then divide the square into four rows and ASK: What fraction of the whole does each row represent? (1/4) Invite a volunteer to outline 2/3 and to shade 3/4, as shown in the margin. ASK: How can you show multiplying 3/4 × 2/3 using this picture? (3/4 × 2/3 = 3/4 of 2/3, so the part that is both shaded and outlined in the picture is 3/4 × 2/3) ASK: How many parts is the whole square divided into? (4 × 3 = 12) How many parts are both shaded and outlined? (3 × 2 = 6) What is 3/4 × 2/3? (6/12) Show the equation in the margin to summarize. Remind students that, when multiplying fractions, they multiply the numerator by the numerator and the denominator by the denominator.

Area of rectangles that are parts of a unit square. ASK: What fraction of the square (in the margin and on the board) is one small rectangle? (1/12) How do you know? (the square is divided into 12 equal pieces) SAY: Imagine this square has a length and a width of 1 inch. Label the sides of the square as 1 in. ASK: What is the area of the square? (1 in²) What is the area of the small rectangle? (1/12 in²) What are the length and the width of the rectangle? (1/3 in by 1/4 in) How do you find the area of a rectangle (multiply length by width) What do you get if you multiply 1/3 in by 1/4 in? (1/3 in × 1/4 in = 1/12 in²)?

On grid paper, have students draw 1 in squares and repeat the discussion above. (Four small squares fit along the edge of a large square. There are 16 small squares in a square inch, so each small square has area 1/16 in² and length and width 1/4 of an inch, so length × width = 1/4 in × 1/4 in = 1/16 in².) Emphasize that both methods—dividing the unit by the number of small squares and multiplying length by width—give the same answer.
Area of rectangles on quarter-inch grid. Ask students to draw in their notebooks a rectangle that is 8 squares wide and 12 squares long.

ASK: What is the area of the rectangle in squares? (8 × 12 = 96 squares)

ASK: If each square has an area of 1/16 in², what is the area of the rectangle? Have students do the calculation. (96 × 1/16 = 96/16 in² = 6 in²) Remind them that each grid square is 1/4 inch long and 1/4 inch wide. ASK: What are the length and the width of the rectangle you drew? (3 in × 2 in) What is its area in inches? (3 in × 2 in = 6 in²) Did you get the same answer both ways? (yes)

How many squares are needed to cover this rectangle? Draw a rectangle on the board and label the sides as 3 1/2 in and 2 1/4 in. SAY: I want to divide this rectangle into squares. Will 1 in squares work? (no) Why not? (the rectangle sides are not in whole inches, so there will be gaps left) Will 1/2 in squares work? (no, because the width has a fractional part that is smaller than 1/2 in, and there will be a gap left) Ask students to use a ruler to draw a square that is 1/2 in long and 1/2 in wide so that the sides of the square run along the grid lines in their notebooks. How many grid squares long is the square? (2) Then ask them to draw the rectangle (3 1/2 in by 2 1/4 in) to scale. Have students check that it is impossible to divide the rectangle into 1/2 in squares.

Ask students to cover up the rectangle they just drew. SAY: I would like to be able to tell, without looking at the rectangle you drew, how many 1/4 in grid squares you will need to cover it. How can you tell how many 1/4 in grid squares long your rectangle will be without looking at the rectangle? (divide 3 1/2 by 1/4) Have students write the division statement and find the answer. (3 1/2 ÷ 1/4 = 7/2 ÷ 1/4 = 7/2 × 4 = 7 × (4 ÷ 2) = 7 × 2 = 14)

Repeat with the width. (2 1/4 ÷ 1/4 = 9/4 ÷ 1/4 = 9/4 × 4 = 9) Have them find the total number of squares. (14 × 9 = 126) Then have students uncover the rectangle and check the answer. Finally, have them find the area of the rectangle two ways: by multiplying the number of grid squares by the area of one grid square (126 × 1/16 in² = 63/8 in² = 7 7/8 in²) and by multiplying the length and the width of the rectangle directly. (3 1/2 in × 2 1/4 in = 7/2 in × 9/4 in = 63/8 in² = 7 7/8 in²)

Using a formula to find the area of rectangles with fractional sides. Explain that a formula is a short way to write the instructions for how to calculate something. Remind students of the formula for the area of a rectangle (length × width), and ASK: Do you think this formula works for fractional lengths too? (yes) Point out that students have actually checked the formula several times, so they now can use the formula without dividing a rectangle into unit squares.

Review converting mixed numbers to improper fractions before assigning the exercises.
Exercises: Find the area of the rectangle.

a) length 3 ft, width \( \frac{7}{8} \) ft

b) length \( \frac{3}{8} \) m, width \( \frac{3}{4} \) m

c) length \( 2\frac{1}{2} \) yd, width \( \frac{5}{8} \) yd

d) length \( 4\frac{3}{10} \) cm, width \( 2\frac{7}{10} \) cm

Answers

a) \( \frac{21}{8} \) ft\(^2\) = \( 2\frac{5}{8} \) ft\(^2\)
b) \( \frac{9}{32} \) m\(^2\)
c) \( \frac{25}{16} \) yd\(^2\) = \( 1\frac{9}{16} \) yd\(^2\)
d) \( \frac{1,161}{100} \) cm\(^2\)

Extensions

1. Find the area of the rectangle.

a) length 3 m, width 0.7 m

b) length 3.8 m, width 3.4 m

c) length 2.55 km, width 0.8 km

d) length 4.4 cm, width 2.75 cm

Answers: a) 2.1 m\(^2\), b) 12.92 m\(^2\), c) 2.04 km\(^2\), d) 12.1 cm\(^2\)

2. a) 1 yd = 3 ft. How many square feet are in one square yard?

b) Convert the dimensions of the rectangle to feet and find the area of the rectangle.

i) 12 ft by 5 yd    ii) 10 ft by 2 yd    iii) 7 ft by 5 \( \frac{1}{2} \) yd

c) Convert the dimensions of the rectangle to yards and find the area of the rectangle.

d) Use your answer in part a) to check that your answers in parts b) and c) are the same. If they are not the same, find your mistake.

Answers

a) 1 yd\(^2\) = 9 ft\(^2\)
b) i) 180 ft\(^2\), ii) 60 ft\(^2\), iii) 115 1/2 ft\(^2\)
c) i) 20 yd\(^2\), ii) 6 2/3 yd\(^2\), iii) 12 5/6 yd\(^2\)
MD5-25 Problems with the Area of Rectangles
Pages 113–114

STANDARDS
5.NF.B.4b, 5.NBT.B.7

VOCABULARY
adjacent
area
decimal
decimal place
decimal point
denominator
factor
fraction
numerator
perimeter
square centimeter (cm²)
square foot (ft²)
square inch (in²)
square kilometer (km²)
square meter (m²)
square millimeter (mm²)
square yard (yd²)

Goals
Students will solve problems connected to the area of rectangles.

PRIOR KNOWLEDGE REQUIRED
Can multiply and divide, including multi-digit whole numbers, decimals, fractions, and mixed numbers
Can find the area of a rectangle with fractional sides using the formula for the area of a rectangle
Knows that division of two whole numbers can produce a fraction
Can solve an equation $a \times x = b$
Knows that a variable can replace a number
Knows that area is additive
Can find the perimeter of a rectangle

Review using a formula for area of a rectangle. Remind students that they can find the area of a rectangle by multiplying length by width. Since order does not matter in multiplication, they can write the factors in any order. Remind them also that we often use letters instead of whole words in formulas, so instead of writing “Area = length $\times$ width = width $\times$ length,” we can write “Area = $\ell \times w = w \times \ell$.” Write both versions of the formula on the board.

Show students how to record the information in a problem and the solution. Explain that it makes it easier to check your own work and to check the work of other people if all the information is written out neatly. For example, show students how to record their work when finding area using a formula:

Length = 2.8 m
Width = 1.5 m
Area = $\ell \times w = 2.8 \text{ m} \times 1.5 \text{ m}$
= 4.20 m² = 4.2 m²

Review decimals. This example gives you the opportunity to review multiplying decimals. Remind students that we multiply decimals as if they were whole numbers and then shift the decimal point. We add the number of decimal places in the factors to determine the number of decimal places in the product. Sometimes, as in the example above, the last digit in the product is zero. Since it is in the hundredth place, we do not need to write it, so the area is 4.2 m² rather than 4.20 m².

Exercises: Find the area of the rectangle. Show your calculations.

a) length 2 m, width 0.6 m    b) length 2.8 cm, width 0.4 cm

Answers: a) 1.2 m², b) 1.12 cm², c) 1.16 km², d) 30.6 mm²
When students have finished, have them check their work with a partner. Point out that, if students have written the information and the calculations neatly, they will easily see not only if the answers are different, but also why they are different.

**Using equations for the area of a rectangle to find the missing dimension.** Remind students that they can use a letter for an unknown number in a problem. For example, a rectangle has a length of 7 cm and width of \( w \) cm. The rectangle's area is 35 cm\(^2\). Have students record this information. SAY: In this case, we can write an equation for the area: \( w \times 7 = 35 \). Write the equation on the board and have students copy it.

**ASK:** What number will make the equation true? (5) **How do you know?** (5 \( \times 7 = 35 \)) Remind students that multiplication and division are related, so if \( 5 \times 7 = 35 \), there is an equation from the same fact family that has 5 as the answer. **ASK:** What equation is that? (35 \( \div 7 = 5 \)) Remind students that they've learnt to write an equation where the unknown number is by itself, and that's what they could do to solve this equation. Write the solution on the board:

\[
\begin{align*}
  w \times 7 &= 35 \\
  w &= 35 \div 7 \\
  &= 5
\end{align*}
\]

**ASK:** What units will the answer be in? (cm) **How do you know?** (the area is in cm\(^2\) and the length is in centimeters, so the width has to be in centimeters) Add the units to the answer: \( w = 5 \text{ cm} \).

**Exercises:** Write an equation for the area of the rectangle. Then find the unknown length.

- **Bonus**
  - a) \( \text{Width} = 5 \text{ cm} \) \( \text{Length} = \ell \text{ cm} \) \( \text{Area} = 55 \text{ cm}^2 \)
  - b) \( \text{Width} = 6 \text{ km} \) \( \text{Length} = \ell \text{ km} \) \( \text{Area} = 96 \text{ km}^2 \)
  - c) \( \text{Width} = \ell \text{ m} \) \( \text{Length} = \ell \text{ m} \) \( \text{Area} = \ell \text{ m}^2 \)

**Answers:** a) 11 cm, b) 16 km, Bonus: 45 m

Review dividing decimals and fractions by a whole number.

**Exercises:** Write an equation for the area of the rectangle. Then find the unknown length or width.

- **Bonus**
  - a) \( \text{Width} = 5 \text{ m} \) \( \text{Length} = \ell \text{ m} \) \( \text{Area} = 41.25 \text{ m}^2 \)
  - b) \( \text{Width} = w \text{ in} \) \( \text{Length} = 7 \text{ in} \) \( \text{Area} = \frac{3}{4} \text{ in}^2 \)
  - c) \( \text{Width} = 3 \text{ ft} \) \( \text{Length} = \ell \text{ ft} \) \( \text{Area} = 20 \text{ ft}^2 \)

**Bonus:** \( \text{Width} = w \text{ yd}, \text{Length} = 6 \text{ yd}, \text{Area} = \frac{1}{2} \text{ yd}^2 \)

**Answers:** a) 8.25 m, b) 2 1/4 in, c) 20/3 ft = 6 2/3 ft,
Bonus: 21/12 yd = 7/4 yd = 1 3/4 yd
Shapes composed of two rectangles. Draw the shapes composed of two rectangles from the Exercises below and mark the dimensions on four of the sides. Ask students to copy the shapes and dimensions, shade each rectangle with its own color, and then circle the dimensions that belong to the rectangle in the same color.

Exercises: Find the area of each rectangle. Add the areas of the rectangles in each shape to find the total area of the shape.

a)  
\[
\begin{array}{c}
3 \text{ yd} \\
2 \text{ yd} \\
1 \text{ yd}
\end{array}
\]

b)  
\[
\begin{array}{c}
7 \text{ ft} \\
6 \text{ ft} \\
4 \text{ ft}
\end{array}
\]

Answers: a) 8 yd², b) 49 ft², c) 1.3 m², Bonus: 194.6 cm²

Determining the missing side lengths to find area. Draw the shape in the margin on the board. ASK: Is 7 cm the length of the short side of the shaded rectangle? Trace it with your finger. (no) Is it the length of the short side of the unshaded rectangle? (no) Point out that 7 cm is the length of one side of the unshaded rectangle and one side of the shaded rectangle combined—in other words, 7 cm is the sum of the two side lengths.

ASK: What is the length of the short side of the unshaded rectangle? (3 cm) How do you know? (7 cm − 4 cm = 3 cm) Finally, have students find the areas of both rectangles and add them to find the total area of the shape. (5 cm × 3 cm = 15 cm², 11 cm × 4 cm = 44 cm², total area = 59 cm²)

Bonus  
\[
\begin{array}{c}
5 \text{ cm} \\
6 \text{ cm}
\end{array}
\]

\[
\begin{array}{c}
5 \text{ cm} \\
23 \text{ cm}
\end{array}
\]

Draw the second shape in the margin on the board, and ask students to find the length of the longer side of the shaded rectangle. (10 in + 11 in = 21 in) Have students find the area of the shape. (7 in × 10 in = 70 in², 9 in × 21 in = 189 in², total area = 259 in²)
Exercises: Find the area.

a) \[
\begin{array}{c}
2 \text{ m} \\
\hline
2 \text{ m} \\
\hline
3 \text{ m} \\
\hline
1 \text{ m}
\end{array}
\]

b) \[
\begin{array}{c}
2 \text{ ft} \\
\hline
4 \frac{1}{8} \text{ ft} \\
\hline
3 \text{ ft} \\
\hline
1 \frac{7}{8} \text{ ft}
\end{array}
\]

Answers: a) 9 m\(^2\), b) 10 1/8 ft\(^2\)

Finding the area two ways by dividing the shape differently. On the board, draw the first shape shown in the margin. Shade one of the rectangles as shown. Ask students to identify the side lengths of the shaded rectangle. Point to each label in order, and have students signal thumbs up if the label shows the length of a side of the shaded rectangle and thumbs down if it does not. Circle the labels for the shaded rectangle, and then repeat with the side lengths of the unshaded rectangle. Then present the same shape with the same labels but broken into two rectangles in a different way (as in the second picture in the margin) and repeat the exercise. Point out that, in each case, students need to use four of the six labels to find the area of the shape. The remaining two labels are unused, and the unused labels differ in each case. Have students find the area of the shape both ways. (89 cm\(^2\)) Did they get the same answer? If not, ask them to find their mistakes.

Exercises: Divide the shape into two rectangles. Then find the area of the shape.

a) \[
\begin{array}{c}
12 \text{ m} \\
\hline
23 \text{ m} \\
\hline
15 \text{ m} \\
\hline
12 \text{ m}
\end{array}
\]

b) \[
\begin{array}{c}
8 \text{ ft} \\
\hline
13 \frac{1}{2} \text{ ft} \\
\hline
12 \text{ ft} \\
\hline
6 \frac{1}{2} \text{ ft}
\end{array}
\]

Answers: a) 456 m\(^2\), b) 134 ft\(^2\)

Bonus: Find a different way to split these shapes into rectangles and find the area the new way. Did you get the same answer as before?

Review perimeter. Remind students that the perimeter of a shape is the distance around the shape. To find it, you add the measurements of all sides. As well, remind students that, since a rectangle has equal opposite sides, students can find the perimeter of a rectangle by adding length and width and multiplying by 2.

Exercises: Find the perimeter of the rectangle with the following length and width.

a) 21 in by 11 in  
 b) 26 ft by 23 ft  
 c) 7 yd by 10 yd

Answers: a) 64 in, b) 98 ft, c) 34 yd
Draw a rectangle on the board and mark one side as 5 m. SAY: The perimeter of this rectangle is 22 m. What are the other sides of the rectangle? Have students present multiple solutions. To prompt students, have them first find the length of the opposite side. SAY: These two sides together add to 10 m. (5 m + 5 m = 10 m) What do the other two sides add to? How do you know? (22 m − 10 m = 12 m) How long is each side? (6 m) How do you know? (they are equal and add to 12 m, so each is 12 m ÷ 2 = 6 m) Another way of thinking about the perimeter is that two adjacent sides should be half of 22, so two adjacent sides add to 11 m. One of the sides is 5 m. How long is the other side? (6 m) What is the area of the rectangle? (5 m × 6 m = 30 m²)

**Exercise:** Find the missing length of the rectangle. Then, find the area.

a) perimeter 20 cm, width 4 cm  
   b) perimeter 20 cm, width 2 cm

**Answers:** a) length 6 cm, area 24 cm²; b) length 8 cm, area 16 cm²

**Extensions**

1. a) Draw a rectangle so that the number representing the area (in cm²) is the same as the number representing the perimeter (in cm).
   b) Convert the measurements of the sides to millimeters. Find the area and perimeter of the rectangle with the new measurements. Is the number representing the area in mm² still equal to the number representing the perimeter in mm?

   **Sample answer:** a) \(18 = 3 \times 6\) (area) = \(2 \times 3 + 2 \times 6\) (perimeter)  
                      or \(16 = 4 \times 4\) (area and perimeter);  
                      b) \(30 \text{ mm} \times 60 \text{ mm}, \text{ so area} = 1,800 \text{ mm}^2\) and \(\text{perimeter} = 180 \text{ mm}\).  
   Or: \(40 \text{ mm} \times 40 \text{ mm}, \text{ area} = 1,600 \text{ mm}^2, \text{ perimeter} = 160 \text{ mm}\). The numbers are different.

   **NOTE:** This exercise shows that the numbers representing area and perimeter can be identical in one set of units and different in another set of units, even though the rectangles do not change. This means that area and perimeter simply cannot be compared; the units they are measured in are incomparable.

2. **Acres.** Ask students if anyone knows what unit the area of a plot of land is measured in. Students might suggest acres or hectares. Explain that an acre is a commonly used unit for measuring land area, but it is not a square unit. It is a rectangle 22 yd by 220 yd long. A hectare (ha) equals 10,000 m².
   a) How many square yards are in 1 acre?
   b) 1 yd = 3 ft. How many square feet are in one square yard?
   c) How many square feet are in 1 acre?

   Use a calculator for the next problems. Remember to estimate your answer first to catch possible mistakes when punching in the numbers!
d) 1 square mile = 640 acres. Use a calculator to find how many square yards and square feet are in one square mile.

e) 1 mi = 1,760 yd. How many square yards are in 1 square mile? Did you get the same answer as in d)? If not, find your mistake.

**Answers**

- a) 4,840 square yards
- b) 9 square feet
- c) 43,560 square feet
- d) 1 square mile = 3,097,600 square yards
- e) 1 square mile = 3,097,600 square yards

3. On a grid, draw a square with sides 7 units long.

a) Find the area and the perimeter of the square.

b) Inside the square, draw a shape that has an area smaller than the area of the square and perimeter larger than the perimeter of the square. There are many possible answers.

c) If the 7 by 7 square has an area of 1 in², what is the area and perimeter of the shape you drew in part b)?

d) In pairs, explain your answers to part c). Do you agree with each other? Discuss why or why not.

**Sample answers**

- b) The shape below is one possibility, with area = 17 square units and perimeter = 36 units

- c) Since the large square has area 1 in², each of its sides has length 1 in. So the length of one of the small grid squares is 1/7 in and the area is 1/7 in × 1/7 in = 1/49 in². For any shape drawn in the square, find the perimeter by multiplying the number of unit side lengths by 1/7, and find the area by counting the number of small grid squares and multiplying by 1/49 in².

**Answer:** a) area = 49 square units, perimeter = 28 units
MD5-26  **Stacking Blocks**

Pages 115–116

**STANDARDS**

5.MD.C.4

**VOCABULARY**

column  
height  
horizontal  
layer  
row  
vertical

**Goals**

Students will find the number of cubes in a rectangular stack and develop the formula length × width × height for the number of cubes in a stack.

**PRIOR KNOWLEDGE REQUIRED**

Can find the area of a rectangle using the formula for the area of a rectangle

**MATERIALS**

connecting cubes

---

**Counting cubes in a stack using horizontal layers.** Build a 2 × 4 “rectangle” using connecting cubes, as shown in the margin. ASK: How many cubes are in this rectangular layer? (8) Have students explain how they calculated the number of cubes—did they count the cubes one by one or did they count them another way? Tell students that you can use addition or multiplication to count the total number of cubes:

\[2 + 2 + 2 + 2 = 8\]  (2 cubes in each of 4 columns)

\[2 \times 4 = 8\]  (2 rows of 4 cubes OR length × width)

Add another layer of cubes to the layer so that you have a stack of 2 layers 2 cubes high, as shown in the margin. ASK: How many cubes are in this stack of cubes now? (16) Prompt students to use the fact that the stack has two horizontal layers. Remind them that they already know the number of cubes in one layer. Ask students to write the addition and multiplication equations for the number of cubes in the stack using layers. (8 + 8 = 16; 2 × 8 = 16)

Add a third layer to the stack and repeat. (8 + 8 + 8 = 24; 3 × 8 = 24)

**Counting cubes in a stack using a vertical layer.** Invite students to look at this last stack and calculate the number of cubes by adding vertical layers instead of horizontal layers (see example in margin). ASK: How many cubes are at the end of the stack? (3 × 2 = 6) Separate the stack into four vertical layers. ASK: How many cubes are in each vertical layer? (6) How many vertical layers are in the stack? (4) Invite volunteers to write the addition and multiplication statements for the total number of cubes using the number of cubes in the vertical layer. ASK: Does this method produce a different result than the previous method? (no) Why should we expect the same answer? (it is the same stack of cubes, only counted differently)

**Determining the number of cubes in a stack as a product of length, width, and height.** Remind students that, to find the number of squares in a rectangle, they can multiply length by width. Now they have a three-dimensional stack, but it is very similar to a rectangle. Remind students
that the vertical dimension in 3-D objects is called the *height*. Display the picture in the margin and identify the length, width, and height of the stack. Then write the following equation on the board and use the terms “length,” “width,” and “height” to label the multiplication statement that gives the number of cubes in the stack:

\[
4 \times 2 \times 3 = 24
\]

**length**  **width**  **height**

Draw several stacks on the board and ask students to find the number of cubes in each stack by multiplying length, width, and height. Remind students that order does not matter in multiplication, so the number of cubes can be found in other ways, such as height \( \times \) length \( \times \) width or width \( \times \) length \( \times \) height.

### ACTIVITY

Students work in pairs. Each student creates several rectangular stacks of connecting cubes, and then exchange stacks with their partner. Ask students find the number of cubes in the stacks created by their partner by multiplying length, width, and height.

Discuss how the dimensions in the formula length \( \times \) width \( \times \) height are related to the layers in a stack. ASK: What part of the formula gives you the number of cubes in one horizontal layer? (length \( \times \) width) Then you multiply by the number of horizontal layers, which is height. Repeat with a vertical layer (width \( \times \) height).

### Extension

(MP1, MP7)

Use connecting cubes.

a) How many ways can you build a rectangular stack of 36 cubes with a height of 3 cubes?

b) How many ways can you build a rectangular stack with 48 cubes?

**Answers**

a) 3 ways; dimensions in order of length \( \times \) width \( \times \) height: 12 \( \times \) 1 \( \times \) 3, 6 \( \times \) 2 \( \times \) 3, 4 \( \times \) 3 \( \times \) 3

b) 9 combinations of dimensions (with different arrangements of the same dimensions possible): 48 \( \times \) 1 \( \times \) 1, 24 \( \times \) 2 \( \times \) 1, 16 \( \times \) 3 \( \times \) 1, 12 \( \times \) 4 \( \times \) 1, 12 \( \times \) 2 \( \times \) 2, 8 \( \times \) 3 \( \times \) 2, 8 \( \times \) 6 \( \times \) 1, 6 \( \times \) 4 \( \times \) 2, 4 \( \times \) 4 \( \times \) 3

Redirecting students: If students struggle to find all the combinations, ASK: How can you be organized to make sure you check all combinations without repeating any? (for example, list possibilities in increasing order) How can you simplify your work? (start by finding the possible heights, then find the possible lengths and widths given the height) Which numbers don’t need to be checked for the dimensions? (numbers that are not factors of the total number of cubes)
MD5-27 Volume
Pages 117–119

STANDARDS
5.MD.C.3a, 5.MD.C.3b, 5.MD.C.4, 5.MD.C.5a, 5.MD.C.5b

VOCABULARY
area
cubic centimeter (cm³)
cubic foot (ft³)
cubic inch (in³)
cubic kilometer (km³)
cubic meter (m³)
cubic millimeter (mm³)
cubic yard (yd³)
formula
height
rectangular prism
volume

Goals
Students will develop and use the formula Volume = length × width × height for the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED
Can find the area of a rectangle using the formula
Can find the number of blocks in a rectangular stack
Can multiply two-digit numbers
Is familiar with units for measuring length and can convert between the units within the same system

MATERIALS
connecting cubes
BLM Cube Skeleton (p. Q-67)
old newspapers or wrapping paper
tape
centimeter cubes for demonstration
cubic inch cubes constructed from the net on BLM Cubic Inch (p. Q-68)
prism sample, such as a facial tissue box

Introduce volume. Remind students that area is the amount of two-dimensional (flat) space a shape takes up. We measure area in square units—squares that have all sides 1 length unit long.

Hold up two shapes made from connecting cubes, one made from 3 cubes and another made from 4 cubes. ASK: Which shape takes up more three-dimensional space? (the shape made from 4 cubes) How do you know? (it is made from more cubes)

Explain that the space taken up by a box or a cupboard or any other three-dimensional object—in other words, an object that has length, width, and height—is called volume. We measure volume in cubic units: units that are cubes.

Introduce standard cubic units. Draw several cubes on the board and mark the sides as 1 cm for one cube, 1 in for another cube, 1 m for another cube, and so on. Explain that these represent different cubic units: cubic centimeter, cubic inch, cubic meter, etc. Show the abbreviated form for each unit:

- cubic centimeter (cm³)
- cubic inch (in³)
- cubic meter (m³)
ACTIVITY

Divide students into three groups and assign a unit to each group: ft³, yd³, or m³. Have students roll old newspapers or wrapping paper into tubes slightly longer than the unit they were assigned (to allow for binding at the ends). Mark the unit on each tube as shown in the margin. Students might need to combine several newspapers to produce a longer tube. Have each group produce 12 tubes. Give each group tape and BLM Cube Skeleton and have them create skeletons of their assigned units to scale (see sample cube skeleton in margin).

Comparing cubic units. NOTE: To prepare, have a centimeter cube to show 1 cm³ and create a cube from BLM Cubic Inch to show 1 in³. Compare the relative sizes of the units with students: cubic inch, cubic centimeter, cubic foot, cubic yard, and cubic meter. Point out that cubic meters and cubic yards are very large. For example, to fill an aquarium with a volume of one cubic meter or one cubic yard, you would need about 100 large pails of water!

Finding the volume of rectangular prisms. Explain that a threedimensional object, such as a cupboard or a box, has length, width, and height. Show a 3 × 4 × 2 stack of centimeter cubes, and review how to find the number of cubes in a stack. (multiply length, width, and height) ASK: How many cubes are in the stack? (3 × 4 × 2 = 24) Explain that the volume of this stack is 24 cubic centimeters.

Draw several stacks of cubes on the board and explain that the cubes in each stack are unit cubes. Mark the size of each cube drawing, using different units for different pictures. Have students find the volume of each stack.

On the board, draw the rectangular prism shown in the margin. Explain that mathematicians call rectangular boxes rectangular prisms. ASK: What is the length of this prism? (5 m) What is the width of this prism? (3 m) What is the height of this prism? (4 m) Record the information on the board:

- Length = 5 m
- Width = 3 m
- Height = 4 m

SAY: Imagine this is a container, and I want to pack it with cubes that each measure 1 m by 1 m by 1 m. How many cubes would fit along the length of this container? (5) Width? (3) Height? (4) How many cubes in total would fit in the container? (60) How do you know? (5 × 3 × 4 = 60; volume = length × width × height) Have students write the multiplication statement. What is the volume of the prism? (60 m³)

Point out that students found the volume of the prism even though you did not draw the cubes that filled it. ASK: What did you do to find the volume? (multiplied the dimensions: length × width × height) Write the formula Volume = length × width × height on the board, and explain that it is also convenient to use the short form V = l × w × h.
On the board, draw the prism in the margin. Have students write the multiplication statement for its volume. Explain that, when finding volume, it helps to write the units for each measurement in the multiplication statement to prevent mistakes. For example, for this prism, the multiplication statement should be:

\[
\text{Volume} = 6 \text{ in} \times 2 \text{ in} \times 3 \text{ in} \\
= 36 \text{ in}^3
\]

Explain that writing the units in the equation helps to see that there are three dimensions. Explain also that a few lessons later, students will see prisms where the dimensions for one prism are given in different units, so writing the units in the equations will be even more important.

Point out that the raised 3 that appears in the cubic units is the number of length measurements that were multiplied together to make the cubic unit: you multiplied three measurements—length, width, and height—to get the volume. All three measurements were in inches, so the result is in cubic inches. Point to the raised 3 in “\(\text{in}^3\)”. Point out that, in area, we also use a raised number in the units. In units of area, we multiply two dimensions—length and width—and the result is square units, for example, square inches. Write “\(\text{in}^2\)” on the board and point to the raised 2.

ASK: Where have you seen a similar notation with numbers? (in exponents) Point out the similarity:

\[
2 \times 2 \times 2 = 2^3 \\
= 8
\]

\[
2 \text{ in} \times 2 \text{ in} \times 2 \text{ in} = 8 \text{ in}^3
\]

**Exercises:** Find the volume of the rectangular prism. Include the units in your calculation and answer.

a) 10 cm, 6 cm, 2 cm  

b) 8 ft, 2 ft, 7 ft

c) 4 km, 7 km, 3 km  

d) 15 m, 4 m, 3 m

e) length 11 in, width 6 in, height 11 in  

f) length 3 ft, width 3 ft, height 3 ft  

g) length 12 in, width 12 in, height 12 in  

h) length 100 cm, width 100 cm, height 100 cm

**Answers:** a) 120 cm³, b) 112 ft³, c) 84 km³, d) 180 m³, e) 726 in³, f) 27 ft³, g) 1,728 in³, h) 1,000,000 cm³
(MP5) **Bonus:** Use some of your answers above to convert these units.

a) \(1 \text{ yd}^3 = \underline{27} \text{ ft}^3\)  
b) \(1 \text{ m}^3 = \underline{1,000,000} \text{ cm}^3\)  
c) \(1 \text{ ft}^3 = \underline{1,728} \text{ in}^3\)

**Answers:** a) \(1 \text{ yd}^3 = 27 \text{ ft}^3\),  
b) \(1 \text{ m}^3 = 1,000,000 \text{ cm}^3\),  
c) \(1 \text{ ft}^3 = 1,728 \text{ in}^3\)

**Show that volume does not depend on orientation.** Show students a prism that has different measurements for length, width, and height, such as a tissue box. Write its dimensions to the nearest centimeter on the board. Have students find the volume of the prism. Then turn the prism.

ASK: Did the height of the prism change? (yes) The length? (yes) The width? (yes) Do you expect the volume to change? (no) Why not? (rotating the box does not change the volume, the space that the prism takes up; the dimensions are the same but we multiply them in a different order)

**Extensions**

1. Find the volume of the shape in the margin.

   **Answer:** \((3 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm}) + (3 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm}) = 48 \text{ cm}^3\)

2. a) Choose a cubic unit: \(\text{m}^3\), \(\text{yd}^3\), or \(\text{ft}^3\). Guess the volume of the classroom in the unit you chose.

   b) Estimate the length, width, and height of the classroom in the matching linear unit. For example, if you chose \(\text{m}^3\), estimate in meters.

   c) Measure the length, width, and height of the classroom in your unit. Then calculate the volume of the classroom. How close was your guess?

   Look for students to choose appropriate tools (MP5) to measure the dimensions of the classroom, such as yard sticks, meter sticks, and masking tape to mark partial measurements.

3. During an excavation of an old military fort from the War of 1812, an underground ammunition storage room was discovered. The room is 10 ft long, 6 ft wide, and 4 ft deep. Several ammunition crates measuring 2 ft by 1 ft by 1 ft each were found in the room. What is the greatest number of crates that would fit in the room?

   **Answer:** The 2 ft edge can be placed along any side of the storage room. One way to do so would be to place 5 crates along the length of the storage room, 6 along the width, and 4 along the height. This means 120 crates \((5 \times 6 \times 4 = 120)\) could fit in the room.
NOTE: Before the lesson, prepare a cube skeleton from toothpicks and modeling clay. Cut out two paper squares and four transparency squares to act as faces. Label the paper squares “bottom face” and “back face,” and the transparency squares “front face,” “top face,” “left-side face,” and “right-side face.” Also, cut out the net from BLM Net of Rectangular Prism.

Introduce faces, vertices, and edges. Explain that the flat surfaces of 3-D shapes are called faces. Hold up a box and have students count how many faces it has. (6)

Explain that the line segments along which faces meet are called edges. Run your finger along the edges of the box. Explain that the corners of the shape, where edges meet, are called vertices. Each corner is one vertex. Show the vertices on the box.

Introduce the names of different faces. Display the cube skeleton and add the square faces one at a time, emphasizing each face’s position and name—that is, bottom face and back face and then front face, top face, left-side face, and right-side face.

Identifying hidden edges. Trace the edges of the cube with your finger one at a time and ASK: Can you see this edge directly or only through the transparent face? If the transparent faces were made of paper, would you see this edge? Students can signal the answer with thumbs up or thumbs down. Explain that the edges that would be invisible if all the faces were non-transparent are called the hidden edges.

Draw a picture of a skeleton of a cube. Have a volunteer show the vertices of the actual cube. Point out that the vertices are the only places where the
edges of the cube meet. Explain to students that hidden edges are often drawn using dashed lines. Have volunteers draw the edges, showing the hidden edges as dashed lines, as shown in the margin. Point out that, in the picture on the board, the dashed lines and the solid lines seem to intersect but the edges don’t intersect in this way on the actual cube. In other words, where the arrows point in the diagram below, there is no vertex.

Not a vertex

Introduce nets. Explain to students that a net of a 3-D shape is a pattern that can be folded to make the shape. Show a cut-out from BLM Net of Rectangular Prism and show how it folds to produce a prism.

Ahead of time, label faces of a box with the names of the faces. Use the box to demonstrate how to make a net of it by tracing the bottom face onto paper, then rolling the box so that the front face can be traced onto the paper adjacent to the first face, then the top face, and then the back face. Then roll the prism to the sides and trace the side faces, so that each side face attaches to a bottom, front, top, or back face. Point out that you could attach the side faces at different places, depending on the face the prism was standing on before you rolled it sideways. Also, point out that both side faces do not have to be attached to the same face. For example, all three nets below fold into the same box.

ACTIVITY

Producing a net for a rectangular prism. Give each student a different box and have them produce a net the same way you produced the net for a box. Have them fold the net and check that it creates a copy of the box, and then unfold it.

Have students assemble in groups of four, exchange boxes, and mix up their nets. Have students find the net for the box that they now have. Discuss matching strategies. For example, students can try to identify the number of square faces the box has, and look for a net with the same number of square faces.
Labeling faces on a net. Return to the box you used earlier. Place the box on the net so that the bottom face touches the net, and label the faces on the net following the order you traced them in, as shown in the margin. Have students label the faces on the nets they produced. Discuss how some faces are exactly the same—for example, the top face and the bottom face are exactly the same. Which other faces match exactly? (back and front, left side and right side)

On the board, sketch a box as shown in the margin. ASK: What are the dimensions of the top face? (6 in by 4 in) Have students sketch a rectangle that is 6 squares long and 4 squares wide on grid paper and label it “top face.” ASK: What faces touch the top face on the prism? (front, back, and both side faces) What face does not touch the top face? (bottom face) If I now draw an identical rectangle right beside the top face and label it “bottom face,” would that be correct? (no) I want to draw the front face now. Where should I attach it? (to the top or to the bottom edge of the top face) What should the length of the front face rectangle be? (6 squares) The width? (2 squares) How do you know? (The front face dimensions are the length and the height of the prism.) Continue with the other faces.

Exercises: On grid paper, draw a net for the prism and label the faces. Use 1 square of the grid for 1 cm².

a)  

b)  

Answers

a)  

b)  

Measurement and Data 5-28  Q-23
Extensions

1. a) Which of these nets will fold into a cube?

   i)  

   ii)  

   iii)  

   iv)  

   b) Use six squares to make a different net for a cube.

   c) Use six squares to make a different figure that will not be a net for a cube.

   Answers: a) i) no, ii) yes, iii) yes, iv) no

   Sample answers

   b)  

   c)  

2. a) A store sold 332 comics on Monday and 487 comics on Tuesday. The store was open for 8 hours on Monday and 10 hours on Tuesday. Each comic cost $6.32, including tax. What was the total cost of the comics that the store sold on both days?

   b) Which facts did you not need to use in part a)? Explain.

   Sample solutions

   a) The total comics sold on both days was $332 + 487 = 819$. Since each comic costs $6.32, the total cost of the comic books is $819 \times 6.32 = 5,176.08$.

   b) I did not need to use the fact that the store opened for 8 hours on Monday and 10 hours on Tuesday, since this information does not change the total cost.

   Look for students to use the standard algorithm to multiply 3-digit by 3-digit numbers, or to break up one of the numbers to create simpler products.
MD5-29 Volume and Area of One Face
Pages 122–123

STANDARDS
5.MD.C.5a, 5.MD.C.5b

VOCABULARY
back, bottom, front, left side, right side, top face

NOTE: In rectangular prisms, any pair of opposite faces can be considered bases. Since in Grade 5 students only deal with rectangular prisms, we have not introduced the term “base.” Instead, students should develop an understanding that they can find the volume of a rectangular prism by multiplying the area of any face by the dimension that is not one of the dimensions of that same face.

Review the names of faces and the fact that opposite faces of a rectangular prism match. Show students a rectangular prism. ASK: In a rectangular box, or in a rectangular prism, can the top face be larger than the bottom face? (no) Can the bottom face be larger than the top face? (no) Can they have different shapes? (no) The top face and the bottom face are always the same rectangles. If some students have difficulty seeing that the top face and the bottom face match exactly, give them a box, have them trace the bottom face of the box, and check that the top face matches the bottom face exactly. Repeat with other pairs of opposite faces.

Review the formula \( V = \ell \times w \times h \). Ask students how they can find the volume of their boxes. ASK: What do you need to measure? (length, width, height) Remind students that a formula is a short way to write the instructions for how to calculate something. ASK: What is the formula for the volume of a rectangular prism? (Volume = length \times width \times height) How do we write it the short way? \( V = \ell \times w \times h \) Write both formulas on the board.

Remind students that they used a formula to find areas of rectangles and how they recorded the solution. For example, write the solution for a problem “Find the area of a rectangle with the length 3 cm and the width 2 cm” on the board, as shown on the next page. Keep the formula on the board for future use.

Goals
Students will develop and use the formula Volume = area of horizontal face \( \times \) height for the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED
Can use the formula \( V = \ell \times w \times h \) to find the volume of a rectangular prism
Can multiply and divide multi-digit whole numbers

MATERIALS
rectangular box for each student
box with faces labeled with their names for demonstration
colored marker

STANDARDS
5.MD.C.5a, 5.MD.C.5b

VOCABULARY
back, bottom, front, left side, right side, top face
cubic centimeter (cm³)
cubic foot (ft³)
cubic inch (in³)
cubic kilometer (km³)
cubic meter (m³)
cubic millimeter (mm³)
cubic yard (yd³)
edge
face
formula
height
horizontal face
rectangular prism
vertex, vertices
volume
Length = 3 cm
Width = 2 cm
Area = length \times width
Area = 3 \text{ cm} \times 2 \text{ cm} = 6 \text{ cm}^2

Demonstrate how to record the solution to the problem: “Find the volume of a rectangular prism with length 51 mm, width 33 mm, and height 20 mm.” (Volume = 33,660 mm$^3$)

Give each student a rectangular box. Have students measure and record the dimensions of the boxes to the nearest millimeter. Then have them find the volume of their boxes.

**Estimating to check calculations.** If students use a calculator to solve the problem, they need to estimate the result before punching in the numbers. For example, in the prism above, the volume should be about $50 \text{ mm} \times 30 \text{ mm} \times 20 \text{ mm} = 30,000 \text{ mm}^3$, and, since we rounded down both the length and the width, the actual number should be larger.

**Develop the formula** $\text{Volume} = \text{area of horizontal face} \times \text{height}$. Write on the board the formula for the volume of a rectangular prism:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

Ask students whether they see the formula for the area of a rectangle hidden in that formula. ($\text{length} \times \text{width}$) Draw the picture in the margin and

ASK: Which face or faces of the prism have the same length and width as the prism itself? (top face and bottom face) Explain that we can describe both the top and the bottom faces as horizontal faces.

Write on the board:

$$\text{Volume} = \frac{\text{area of horizontal face}}{\text{height}} \times \text{height}$$

Explain that we now have a new formula: $\text{Volume} = \text{area of horizontal face} \times \text{height}$. Write that formula on the board as well.

**Using the formula to find the volume.** Solve the first exercise as a class, and then have students work individually.

**Exercises:** Find the volume.

a) $\text{16 in}^2$ b) $\text{18 cm}^2$ c) height 5 m, area of top face 19 m$^2$ d) height 12 ft, area of bottom face 2,250 ft$^2$

**Answers:** a) 32 in$^3$, b) 54 cm$^3$, c) 95 m$^3$, d) 27,000 ft$^3$
Units in the formula. Point out that area is given in square units. Square units mean that there were two length units multiplied to get the square unit, which we can see in the raised 2 in square units—for example, m². In volume, we need to multiply three length units, so we need to multiply area by another quantity in length units—height—to get the volume:

\[ \text{Volume} = \text{area of horizontal face} \times \text{height} \]

\[ \text{m}^3 = \text{m}^2 \times \text{m} \]

Finding volume in other ways. Draw the picture in the margin on the board. Ask students how they can find the volume of the prism. (multiply 25 cm² × 4 cm) Confirm that this is the correct answer. ASK: Do you know the height? (no) Do you know the area of the top face or the bottom face? (no) Then how do you know that this product is the volume? If the following two explanations do not arise, lead students to think about them:

1. If you turn this prism so that the front face becomes a horizontal face, the given length becomes the height, and the formula works. To illustrate this, use a box with the faces labeled on it, trace the width (the edge marked as 4 cm) with a colored marker, and have a volunteer rotate the prism so that the front face becomes the bottom or the top face. What dimension did the colored edge become? (height)

2. An alternative is that students can think of the following: in having the height and the area of the front face, they have all the information needed for calculating the volume. The only difference is that they multiply the dimensions in a different order.

\[ \text{Volume} = \text{length} \times \text{width} \times \text{height} \]

Area of front face = 25 cm²

Exercises: Find the volume of the prism.

a) length 8 in  
area of right-side face 12 in²  
b) width 22 m  
area of front face 33 m²  
c) area of right-side face 94 cm², length 5 cm  
d) area of front face 23 ft², width 6 ft

Answers: a) 96 in³, b) 726 m³, c) 470 cm³, d) 138 ft³

Measurement and Data 5-29
NOTE: Question 3 on AP Book 5.2 p. 122 shows a different way of developing the same formula for the volume of a rectangular prism, emphasizing the equality between the height and the number of horizontal layers in the prism. You might want to point out this connection to the students when they have finished working on the AP Book questions for this lesson.

Extensions

1. Find the volume of the prism or explain why you cannot.
   
   a) height 9 in
   area of front face 18 in²

   b) width 13 m
   area of left-side face 39 m²

   c) area of top face 4 cm²
   length 2 cm

   d) area of front face 225 ft²
   width 15 ft

   Sample solution: a) The measurements given are the front face (so length and height) and the height again, but not a third dimension, so we cannot calculate volume.

   Answers: b) cannot calculate, c) cannot calculate, d) 3,375 ft³

   (MP1, MP4)  

2. a) Mandy coaches a baseball team. She had a pizza party for her team and spent $95.95 for 5 pizzas. Each pizza cost the same amount. Three of the pizzas were vegetarian. One of the vegetarian pizzas had pineapple on it. How much did the vegetarian pizzas cost?

   b) Which facts did you not need to use in part a)? Explain.

   Sample solutions
   a) $95.95 ÷ 5 = 19.19$, so each pizza costs $19.19. Since three of the pizzas were vegetarian, those pizzas cost $19.19 × 3 = $57.57.
   b) I did not need to use the fact that one of the vegetarian pizzas had pineapple on it, since this information does not change the total cost of the vegetarian pizzas.
Review finding the missing length given the area of a rectangle. Remind students that the area of a rectangle is the product of its length and width, or area = length × width. Draw a rectangle and label the width as 5 in and area as 35 in². Explain that you want to find the length of the rectangle. Have students explain how to find the length. (the area is 5 in × length = 35 in², so 35 in² ÷ 5 in = 7 in)

Remind students that it is important to include the units in their calculations, and even more important to include them in the answers.

**Exercises:** Find the missing width or length.

a) length 9 m, area 45 m²  b) width 6 ft, area 72 ft²

b) length 8 yd, area 48 yd²  d) width 8 cm, area 176 cm²

**Bonus:** length 3,500 ft, area 70,000 ft²

**Answers:** a) 5 m, b) 12 ft, c) 6 yd, d) 22 cm, Bonus: 20 ft

**Finding length of edges from a sketch.** On the board, draw the picture in the margin. Point at the edges one at a time and have students raise the number of fingers equal to the length of the edge. For the first few edges that are not directly labeled, have a volunteer explain how they know the length. Repeat with the two prisms below.
Review finding the volume of rectangular prisms. Remind students of the formulas for volume: Volume = area of the horizontal face × height and \( V = l \times w \times h \). Draw the picture below on the board for reference.

Finding the volume of rectangular prisms when some of the dimensions are not known. Draw the picture below on the board:

Have students identify the missing linear dimension. PROMPT: Which dimensions are you given: length, width, or height? (width and height) ASK: What is the width of the prism? (1 cm) What is the height of the prism? (2 cm) ASK: For which face are you given the area? (front face) Which dimensions do you multiply to find the area of the front face? (length and height) What is the missing length? (4 cm) Have students find the volume of the prism two ways, by multiplying all the dimensions and by multiplying the area of the front face by the width. Remind students that they should get the same answer (8 cm³) both ways.

Exercises

a) Find the missing dimension of the prism.

b) Find the volume of the prism.

Answers: a) width = 35 cm, b) volume = 29,400 cm³

Activity

Ask students to imagine, sketch, and write the dimensions for a prism. (Provide students who have trouble sketching a prism with BLM Pictures of Rectangular Prisms.) Then ask students to choose one face of the prism to shade. What is the area of the shaded face? Ask students to mark the area on the face and erase one of the dimensions for that face. Then ask students to form pairs, exchange prisms, and calculate the volume of the prism. Players can then repeat the activity for new prisms.

NOTE: An advanced variation of this activity is to find the missing dimension of the prism.
Find the area of the base or the height given the volume. Tell students you want to make a box that is 30 cm long and 20 cm wide. You need your box to have a volume of 21,000 cm³. Record the data on the board. ASK: What height should the box be? (35 cm) Let students think about how to solve this problem and encourage multiple solutions.

Emphasize the importance of writing the units in the answer and checking that the answer makes sense: when you are looking for height, you divide the volume (in cubic centimeters) by the area of the face (in square centimeters), so the answer should be in centimeters. Students who have trouble with the units can benefit from the following exercises.

**Exercises:** Write the missing unit.

a) $5 \text{ cm} \times 6 = 30 \text{ cm}^3$  
   b) $5 \text{ in} \times 6 \text{ in}^2 = 30 \text{ in}^3$

c) $5 \text{ ft} \times 6 \text{ ft} = 30 \text{ ft}^2$  
   d) $5 \text{ m}^2 \times 6 = 30 \text{ m}^3$

**Answers:** a) cm², b) in³, c) ft², d) m

Then have students practice finding the missing dimension with the exercises below. Students can use a method of their choice to solve the problem. Encourage multiple solutions, and encourage students to check their answers by substitution.

**Exercises:** Solve the problem.

a) area of top face 3 m², volume 12 m³, height = ?

b) area of top face 30 cm², volume 450 cm³, height = ?

c) height 3 ft, volume 51 ft³, area of top face = ?

d) length 5 in, width 3 in, volume 45 in³, height = ?

e) length 8 in, height 3 in, volume 72 in³, width = ?

**Bonus:** The volume of the combined shape in the margin is 20 cm³. What is the missing height?

**Answers:** a) 4 m, b) 15 cm, c) 17 ft², d) 3 in, e) 3 in, Bonus: 3 cm

**Review nets.** Show a rectangular prism with labeled faces. Sketch a net for it. Place the prism with the bottom face down, and point to different faces on the net. Have students signal thumbs up if this is one of the horizontal faces and thumbs down if it is not. Label the top and the bottom faces on the net.

**Identifying dimensions from the net.** ASK: Which dimensions do you multiply to get the area of a horizontal face? (length and width) Point to different edges on the net in turn, and ask students to signal thumbs up if the edge shows the **length** and thumbs down if it does not. Repeat with **width**. Finally, label length and width on the sides of the bottom and top faces of the net.
ASK: Which edges on the net show the height of the prism? Point to edges one by one and have students signal thumbs up if they show the height and thumbs down if they do not.

Now show a net on a grid, with height, length, and width all different. Label one of the faces as the bottom face and have students identify the length, width, and height of the prism. Finally, ask them to find the volume of the prism.

**Extensions**

1. Use 30 connecting cubes to make as many different rectangular prisms as possible, using all of the cubes.

   **Answer:** There are five possible prisms: $1 \times 1 \times 30$, $1 \times 2 \times 15$, $1 \times 3 \times 10$, $1 \times 5 \times 6$, $2 \times 3 \times 5$

2. **Sketching cubes and rectangular prisms.** Show students how to sketch a cube, and then have them sketch a cube themselves.

   **Step 1:** On grid paper, draw a square that will become the front face.

   **Step 2:** Draw another square of the same size, so that the center of the first square is a vertex of the second square.

   **Step 3:** Join the vertices with lines as shown.

   **Step 4:** Turn the lines that represent hidden edges into dashed lines by erasing a few parts of those lines.

   ![Sketching cubes and rectangular prisms](image)

   On the board, sketch the two diagrams in the margin. ASK: Which dimension is different in these two shapes? (length) How is Step 2 performed differently in each drawing? (In the second cube, the square that is the back face is farther apart from the square that is the front face. In the drawings, the back face’s bottom left vertex is closer to the bottom left vertex of the front face in the shorter shape, and farther from that vertex in the longer shape.)

   ASK: What would you do in Step 2 to draw a very long rectangular prism? Invite a volunteer to draw it. If the drawing is difficult to understand (i.e., if edges overlap), suggest to students that the vertex of the back face should not sit on the diagonal (see sample in margin).

   Have students sketch rectangular prisms.
3. A wealthy king ordered a new treasure chest. The old chest is 6 in long, 4 in wide, and 5 in high. The king asked four carpenters to each make a chest. Each carpenter changed the dimensions:

1st carpenter: twice as long, but the same width and height
2nd carpenter: twice as high, but the same width and length
3rd carpenter: twice as long and twice as wide, but half the height
4th carpenter: the same height, but a different length and width, which makes the area of the new chest’s bottom twice the area of the old chest’s bottom

a) Which carpenter made the chest with the largest volume? Explain.

b) The fourth carpenter’s chest has length and width in whole inches. What might the dimensions of the new treasure chest be?

Answers
a) All new chests have a volume of 240 in$^3$, which is twice the volume of the old chest, 120 in$^3$.
b) The area of the base of the old chest is 6 in $\times$ 4 in = 24 in$^2$. So the area of the base of the new chest is 48 in$^2$. If both the length and the width of the fourth carpenter’s chest are different from those of the old chest, the chest might be 3 in wide and 16 in long. Any other pair of whole numbers that multiply to 48 would also work (e.g., 8 in $\times$ 6 in, 4 in $\times$ 12 in).

4. Tony says, “Since there are 1,000 milliliters in a liter, milliliters must be bigger than liters.” Explain why you agree or disagree with Tony’s reasoning.

Sample answer: I disagree, because for 1,000 milliliters to fit into one liter, a milliliter has to be smaller than liters.
Volume of Shapes Made from Rectangular Prisms

STANDARDS
5.MD.C.5c

VOCABULARY
area
back, bottom, front,
left side, right side,
top face
cubic centimeter (cm³)
cubic foot (ft³)
cubic inch (in³)
cubic kilometer (km³)
cubic meter (m³)
cubic millimeter (mm³)
cubic yard (yd³)
edge
face
height
net
vertex
volume

Goals
Students will find the volume of shapes made from rectangular prisms.

Prior Knowledge Required
Can find the volume of rectangular prisms using the formulas

Materials
connecting cubes
two boxes glued together
two boxes per student, such as shoe boxes, small food packages,
medication packaging boxes, and facial tissue boxes

Volume is additive. Show students two prisms made from connecting cubes and have students find the length, width, height, and volume of the prisms. Then place one prism on top of the other. ASK: How can you find the volume of this new shape? (add the volumes of the two prisms) What is the volume? Have students add the volumes of the two prisms. Combine the prisms in a different way and repeat. Add a third prism and repeat.

Then show students a shape made from two prisms, such as the one in the margin, and have students find the volume. Ask them to describe how they did it. Point out again that the best way to find the volume is to separate the shape into two prisms and find the volume of each by multiplying the dimensions.

Finding the volume of shapes made from two rectangular prisms.
Show students two boxes glued together so that some faces and edges are the continuation of others (see sample in margin). Trace the edges of the combined shape, one at a time, and ask students if the length of this edge is the one they need to know to find the total volume of the shape. Have students signal the answer with thumbs up or thumbs down. Tell them the measurements (to the closest centimeter) of the dimensions they think are necessary, and have them find the volume of the whole shape.

Activity
Give each student two boxes, such as shoe boxes, small food and medication packaging boxes, or facial tissue boxes. Have students measure their boxes to the closest centimeter and find the volumes. Have students glue their boxes into a combined shape, and then exchange shapes with a partner. Ask students to find the volume of their partner’s combined shape. Then ask pairs to compare answers. Pairs that finish early can add a third box to their shapes.
Finding missing dimensions. Review finding the area of a shape made from rectangles. Draw the shape in the margin on the board, and ask students how they could find its area. (split the shape into two rectangles)

Have a volunteer draw the line that splits the two rectangles and then ask students how they can find the sides of the rectangles. (add or subtract the given dimensions) Invite another volunteer to divide the shape a different way, and have students find the side dimensions of the rectangles again.

Have students find the area both ways. \((12\,\text{cm} \times 26\,\text{cm} = 312\,\text{cm}^2, \quad 27\,\text{cm} \times 11\,\text{cm} = 297\,\text{cm}^2, \quad 312\,\text{cm}^2 + 297\,\text{cm}^2 = 609\,\text{cm}^2\) and \((12\,\text{cm} \times 15\,\text{cm} = 180\,\text{cm}^2, \quad 39\,\text{cm} \times 11\,\text{cm} = 429\,\text{cm}^2, \quad 180\,\text{cm}^2 + 429\,\text{cm}^2 = 609\,\text{cm}^2\)

Explain that the situation with 3-D shapes is often similar. Display the picture in the margin and invite a volunteer to separate the shape into two rectangular prisms. Shade one of the prisms, as shown in the second diagram in the margin. Have students identify the dimensions of one prism and then the other prism. ASK: How did you find the width of the prisms? If the separation produced by the students is as shown, the width of the gray prism is labeled. ASK: How can you tell from the picture that both prisms have the same width? (The top-left edge of the gray prism goes all the way across the white prism. Students are likely to show the edge, not name it.) Finally, have students list the dimensions of the prisms and find the volume. (see answers below)

**White prism:**
- **Length:** 7 cm
- **Width:** 3 cm
- **Height:** 11 cm
- **Volume:** 231 cm³

**Gray prism:**
- **Length:** 8 cm
- **Width:** 3 cm
- **Height:** 5 cm
- **Volume:** 120 cm³

**Total volume of shape:** 351 cm³

Then have students split the shape a different way and find the volume. (split into a bottom prism and top prism; the dimensions are: length 15 cm, width 3 cm, and height 5 cm for a volume of 225 cm³; length 7 cm, width 3 cm, height 6 cm, for a volume of 126 cm³; total volume is 351 cm³)

Remind students to expect the same answer, and if they did not get the same answer both ways, they should look for their mistake.

**Identifying edges of the same length in shapes made from prisms.** Display the shapes below, one at a time. Point to edges one at a time and have students signal thumbs up if the edge is the same length as the thickened edge in the picture, and thumbs down if it is not.
The edges that are the same length are shown as additional thickened edges below:

![Thickened Edges](image-url)

The dimensions for the first of the two shapes displayed above are provided in the following exercise.

**Exercises:** Find the volume of the shape.

a) \[33 \text{ in} \times 30 \text{ in} \times 25 \text{ in} = 24,750 \text{ in}^3\]

b) \[33 \text{ m} \times 13 \text{ m} \times 12 \text{ m} = 5,148 \text{ m}^3\]

\[40 \text{ in} \times 40 \text{ in} \times 40 \text{ in} = 64,000 \text{ in}^3\]

\[15 \text{ m} \times 13 \text{ m} \times 11 \text{ m} = 2,145 \text{ m}^3\]

\[24,750 \text{ in}^3 + 64,000 \text{ in}^3 = 88,750 \text{ in}^3\]

\[5,148 \text{ m}^3 + 2,145 \text{ m}^3 = 7,293 \text{ m}^3\]

**Bonus**

\[45 \text{ ft} \times 45 \text{ ft} \times 45 \text{ ft} = 91,125 \text{ ft}^3\]

\[60 \text{ ft} \times 55 \text{ ft} \times 61 \text{ ft} = 201,300 \text{ ft}^3\]

\[60 \text{ ft} \times 25 \text{ ft} \times 40 \text{ ft} = 60,000 \text{ ft}^3\]

\[91,125 \text{ ft}^3 + 201,300 \text{ ft}^3 + 60,000 \text{ ft}^3 = 352,425 \text{ ft}^3\]

**Extensions**

a) Find the length of the thickened edge of the figure in the margin.

b) Find the volume of the shape.

**Solutions**

a) Total width of the shape = 35 ft + 47 ft = 82 ft, so the length of the thickened edge is 82 ft – 55 ft = 27 ft.

b) \[50 \text{ ft} \times 35 \text{ ft} \times 50 \text{ ft} = 87,500 \text{ ft}^3\]

\[60 \text{ ft} \times 55 \text{ ft} \times 60 \text{ ft} = 198,000 \text{ ft}^3\]

\[60 \text{ ft} \times 27 \text{ ft} \times 40 \text{ ft} = 64,800 \text{ ft}^3\]

\[87,500 \text{ ft}^3 + 198,000 \text{ ft}^3 + 64,800 \text{ ft}^3 = 350,300 \text{ ft}^3\]
MD5-32 Problems with Volume
Pages 128–129

STANDARDS
5.MD.C.5c

VOCABULARY
back, bottom, front,
left side, right side,
top face
cubic centimeter (cm³)
cubic foot (ft³)
cubic inch (in³)
cubic kilometer (km³)
cubic meter (m³)
cubic millimeter (mm³)
cubic yard (yd³)
edge
face
height
net
vertex, vertices
volume

Goals
Students will solve real-world and mathematical problems related to the volume of rectangular prisms.

PRIOR KNOWLEDGE REQUIRED
Can find the volume of rectangular prisms using formulas
Can find a missing dimension given the volume
Can draw a net for a rectangular prism
Can identify dimensions of a prism from a net
Can find the volume of shapes made from rectangular prisms
Can multiply fractions and mixed numbers by whole numbers
Can convert from larger units to smaller units within the same measurement system, including fractions, mixed numbers, and decimals
Can convert a mixed measurement to a smaller unit
Can multiply and divide multi-digit numbers, including decimals

MATERIALS
scissors
BLM Buildings and Ground Floor Plans (p. Q-71) photocopied onto a transparency
cardboard box to pack books into (see Activity on p. Q-40)

Review converting units of length. Draw the diagrams below on the board. Ask volunteers to help you fill in the multiplication factors above the arrows. (× 1,000 km to m, × 100 m to cm, × 10 cm to mm, × 1,760 mi to yd, × 3 yd to ft, × 12 ft to in)

Exercises: Convert.

a) 1.2 m to cm  
b) 3.87 km to m  
c) 7.6 cm to mm

d) 7 ft to in  
e) 3 1/4 ft to in  
f) 6 2/3 yd to ft

Answers: a) 120 cm, b) 3,870 m, c) 76 mm, d) 84 in, e) 39 in, f) 20 ft

Review converting mixed measurements to smaller units. Remind students to convert the part in larger units to smaller units and then add the leftover in the smaller unit. For example, 3 ft 5 in = 3 × 12 in + 5 in = 36 in + 5 in = 41 in.
Exercises: Convert.

a) 2 m 45 cm to cm  
   b) 3 m 8 cm to cm  
   c) 4 ft 2 in to in

Answers: a) 245 cm, b) 308 cm, c) 50 in

Finding the volume of 3-D shapes with dimensions in different units.
Sketch the prism below on the board:

![Diagram of a rectangular prism with dimensions labeled: 1 m 25 cm, 1.3 m, 70 cm.]

Discuss with students how to find the volume of the prism. Lead them to the idea of converting all measurements to the same unit before finding the volume. Emphasize how writing the units (as well as the numbers) when substituting the measurements in the formula helps avoid multiplying different units, such as meters and centimeters.

\[
\text{Length} = 1 \text{ m } 25 \text{ cm} = 125 \text{ cm} \\
\text{Width} = 1.3 \text{ m} = 130 \text{ cm} \\
\text{Height} = 70 \text{ cm} \\
\text{Volume} = \ell \times w \times h \\
= 125 \text{ cm} \times 130 \text{ cm} \times 70 \text{ cm} \\
= 1,137,500 \text{ cm}^3
\]

Exercises: Convert the measurements to the same smaller unit. Find the volume of the rectangular prism.

a) length 3.2 cm, width 11 mm, height 1 cm  
   b) length 2 ft 3 in, width 15 in, height 4 in  
   c) length 2 1/2 ft, width 17 in, height 3 ft  
   d) length 3 yd, width 5 ft, height 35 in

Bonus: length 2 km, width 2 m, height 50 cm

Answers: a) 3,520 mm\(^3\), b) 1.620 in\(^3\), c) 18,360 in\(^3\), d) 226,800 in\(^3\), Bonus: 2 km = 2,000 m = 200,000 cm, 2 m = 200 cm, so the volume is 200,000 cm \(\times\) 200 cm \(\times\) 50 cm = 2,000,000,000 cm\(^3\)

Finding a prism given its volume. Ask students to find the volume of a rectangular prism 1.2 m long, 30 cm wide, and 25 cm tall. (90,000 cm\(^3\))
Then explain that you want to make a different rectangular prism with the same volume. If you want this prism to be 50 cm tall, what could the other two dimensions be? Give students time to work alone, then in pairs, and then in groups of four to solve. Encourage multiple solutions. Make sure the following two solutions arise, and present the missing one if needed.
1. The area of the horizontal face of this different prism would be $90,000 \text{ cm}^3 \div 50 \text{ cm} = 1,800 \text{ cm}^2$. Then find a pair of numbers that multiplies to $1,800 \text{ cm}^2$, e.g., $100 \text{ cm} \times 18 \text{ cm}$, or $90 \text{ cm} \times 20 \text{ cm}$.

2. Note that the new prism is twice as tall as the old prism. In order to keep the volumes the same, halve another dimension of the old prism. So a prism with length 1.2 m, width 15 cm, and height 50 cm will work.

ASK: What if I want the prism to be 50 cm tall and 25 cm wide, but have the same volume as earlier? Again, give students time to work independently and in pairs. (divide the area of the horizontal face 1,800 cm$^2$ by 25 cm, giving the length 72 cm)

Review nets of rectangular prisms. Remind students what a net looks like, label the bottom face of the net, and invite volunteers to label the rest of the faces. Then draw a net as shown in the margin. Explain that this is a net of a box without a lid. ASK: Which face is the bottom face? How do you know? (the face in the center is the bottom face and the rest of the faces are attached to it)

ASK: When you know the bottom face, how can you tell what the length and the width of the box are? (the dimensions of the bottom face are the length and the width of the whole prism) What are the length and the width of this box? (4 units by 3 units) What is the height of the prism? (2 units) Have a volunteer indicate an edge of the net that shows the height of the prism. Have students find the volume of the prism. ($4 \times 3 \times 2 = 24 \text{ units}^3$)

Exercise: Use the net to find the dimensions of the box without a lid. Then find the volume.

a) 

b) 

Answers: a) length 6 units, width 4 units, height 2 units, volume 48 units$^3$; b) length 4 units, width 2 units, height 4 units, volume 32 units$^3$

Have students copy the nets on a sheet of grid paper, cut them out, and fold them to check their answers.
Finding the volume of a shape made from prisms. Show the top half of the BLM Buildings and Ground Floors and explain that you would like to build a model of a building. You might want to point out that architectural offices sometimes need such models to show a client or the general public what a building will look like. Explain that the building model will show a main part, 40 cm tall, and a smaller addition, 25 cm tall. The picture on the left side of the BLM shows the ground floor plan. Ask a volunteer to shade the part that is the main building on the ground floor plan. (see sample below)

Ask students to determine the dimensions of both prisms that make up the model and to find the volume of both parts and the total. (main building: 10 cm × 20 cm × 40 cm, volume = 8,000 cm³; addition: 10 cm × 10 cm × 25 cm, volume = 2,500 cm³, total volume = 10,500 cm³)

Repeat the exercise with the bottom half of the BLM Buildings and Ground Floor Plans. Explain that this time the building model consists of the main tower and four small towers. You can see only two of the small towers in the picture of the building because the main tower blocks the other two. The small towers are 50 cm tall in the model, and the main tower is 70 cm taller. (main tower: 35 cm × 35 cm × 120 cm, volume = 147,000 cm³; each small tower: 5 cm × 5 cm × 50 cm, volume = 1,250 cm³ × 4 towers = 5000 cm³; total volume for main tower and 4 small towers = 152,000 cm³)

**ACTIVITY**

Show students a cardboard box and tell them that you want to pack some books—for example, the student AP books—into it. How many books will fit into the box? Explain that first you want to get an estimate. Have volunteers measure the box to the closest centimeter and find its volume. Then have volunteers measure the AP books to the closest centimeter and find their volume. Have them find the estimate for the number of books using division.

Now invite volunteers to actually pack the books into the box. Is there any leftover space? Can you pack a book into that leftover space? Can you fill the leftover space completely, without cutting books into pieces? Students should see that actually packing the books does not allow the same number of books going into the box as the estimate indicated because of the leftover space.
Extensions

(MP.1, MP.4) 1. A skyscraper is a rectangular prism, with all floors exactly the same. The volume of the skyscraper is 588,000 m³, and it is 280 m tall. The distance between the floors of the skyscraper (from the ceiling of one floor to the ceiling of the floor above it) is 4 m.

   a) How many floors tall is the skyscraper?
   
   b) Carpet needs to be installed on half of the total area of the floors of the skyscraper. Installing carpet costs $20 per square meter. How much will installing the carpet in the whole building cost?

Solutions

   a) Since the skyscraper is 280 m tall and each floor is 4 m high, there are 280 ÷ 4 = 70 floors.
   
   b) Divide the volume of the skyscraper by the height to get the area of each floor: 588,000 m³ ÷ 280 m = 2,100 m², so the total area for 70 floors is 2,100 m² × 70 = 147,000 m². Half of the total area is 147,000 m² ÷ 2 = 73,500 m². To find the cost of installing the carpet, multiply 73,500 × 20 = 1,470,000. So the total cost is $1,470,000.

2. a) Find the volume of a rectangular prism 1.25 m long, 28 cm wide, and 25 cm tall.
   
   b) To make a different rectangular prism with the same volume, but height 50 cm, what could the other two dimensions be?

Answers: a) 87,500 cm³, b) sample answer: 1.25 m, 14 cm
MD5-33  **Liquid Volume**
Pages 130–131

**STANDARDS**
5.MD.A.1, preparation for 5.MD.C.5

**VOCABULARY**
appropriate
gram (g)
kilogram (kg)
liter (L)
milliliter (mL)
volume

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**Goals**
Students will convert between liters and milliliters using tables and place value arguments.

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**PRIOR KNOWLEDGE REQUIRED**
- Can multiply and divide a decimal by 1,000
- Can convert between kilograms and grams, meters and millimeters, or meters and kilometers

**MATERIALS**
cup or mug
container for 1 L
funnel
large pan, bowl, or tub to contain any spills
water or other pourable substance
other containers, including empty medicine bottles

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**Introduce liquid volume.** Explain that, when students first looked at the volume of a box, they imagined the box packed with cubes. Show students a cup or a mug and tell them that you would like to find its volume. ASK: Should I fill the cup with cubes? (no) Why not? (there will be many gaps between the cubes) Should I measure the length, width, and height of the cup and multiply those measurements? (no) Why not? (there is no clear length and width, and the formula only works for rectangular prisms, not for round objects such as cups) Explain that the way to measure the volume of the cup is to fill it with something that fills the cup without gaps, such as rice, sand, sugar, or water, and then pour the contents of the cup into a container with a volume that can be measured.

**Introduce liters.** Explain that the volume of liquids can be measured in different units. One of the common units is a liter. Write the word “liter” and its abbreviation “L” on the board and explain that these are both ways to write liter. Show students a container holding 1 L of water, and explain that this is a liter.

Hold up several containers, one at a time, and have students signal thumbs up if they think the container can hold more than 1 L, thumbs down for less than 1 L, and thumbs to side if they think it is about 1L. If possible, have volunteers pour the water from your 1 L container into the containers to check their estimates. You will need a funnel and a large pan, bowl, or tub to contain spills.

**Discuss with students whether one liter is a large quantity of liquid.** Is one liter enough to water the plants in your home? (probably, yes) In your garden? (no) Is one liter enough to take a bath or a shower? To wash the dishes? To fill an aquarium? (MP.2)
Introduce milliliters. Explain that for quantities of liquid smaller than 1 L, there is another unit: milliliters. Milliliters are very small—for example, there are about 5 milliliters in one teaspoon. Introduce the short form of the unit (mL) as well.

**ACTIVITY**

**Developing the sense of the size of containers.** Give each student one empty medicine bottle. Have students find the labels that show the volume of their bottle and round the amount to the closest 10 mL. Have students work in pairs, and guess the volume of their partner’s bottle rounded to the nearest 10 mL. Students can provide hints (too high or too low) and then, when both partners have correctly guessed the capacity, change partners.

Most appropriate unit. Review the term “appropriate”—specifically, an appropriate unit is the unit that gives the most convenient number for the measurement. For example, it is more convenient to measure the volume of a bathtub in liters than in milliliters because the number of milliliters would be really large. PROMPT: A teaspoon contains 5 mL of water. How many teaspoons would we need to fill a bathtub? (It’s hard to tell!) As well, most people do not need to know the volume of a bathtub with great precision. In contrast, if you need to give medication to your pet, you need to know the volume very precisely to avoid over-dosing or under-dosing your pet, so you will measure the medication in milliliters.

Take some of the containers used during the lesson and ask students to decide which unit—liter or milliliter—is more appropriate for measuring how much the container can hold. Write both units on the board and have students point to the correct unit to signal their answers.

1 L = 1,000 mL. Write on the board:

1 meter = 1,000 millimeters 1 m = 1,000 mm

Invite volunteers to circle the common parts in the words “meter” and “millimeter.” Ask students to guess what the part “milli” means. Explain that “milli” means 1,000 in Latin. Explain to students that the prefix “kilo” is used to create large units, and “milli” is used to create small units. So when they see a “milli” in a measurement unit, they know right away that there are 1,000 smaller units (“milli”-units) in the large unit. Write on the board:

1 liter = ______ milliliters 1 L = ______ mL

ASK: What number goes in the blanks? (1,000)

Converting liters to milliliters. Draw a conversion table on the board as shown in the margin, and have students fill in the milliliters column. Ask students to look for regularity: How can we get the number of milliliters from the number of liters? What number would we multiply the number of liters
by to get the number of milliliters? (1,000) Why? (because there are
1,000 mL in 1 L) Remind students that this is very similar to getting grams
from kilograms, or meters from kilometers, or millimeters from meters.

**Exercises:** Multiply by 1,000 to convert the measurement to milliliters.

a) 9 L  b) 18 L  c) 42 L  d) 100 L  e) 394 L  f) 1,000 L

**Answers:** a) 9,000 mL, b) 18,000 mL, c) 42,000 mL, d) 100,000 mL,
e) 394,000 mL, f) 1,000,000 mL

**Compare measurements in different units.** Remind students that when
they compare two measurements in different units, they need to convert one
of the measurements so that the units become the same. Ask students to
give an example of how they did it with other units. Have students convert the
measurement in liters to milliliters before they compare the measurements.
If you present the problems one by one, students can point to the larger
measurement to signal the answer.

**Exercises:** Which measurement is larger?

a) 473 mL or 4 L b) 73 L or 7,300 mL c) 25,678 mL or 25 L

**Bonus:** d) 456,654 mL or 400 L e) 1,000,000 mL or 10,000 L

**Answers:** a) 4 L, b) 73 L, c) 25,678 mL, Bonus: d) 456,654 mL, e) 10,000 L

**Convert milliliters to liters.** ASK: If 1 L = 1,000 mL, what fraction of a
liter is 1 milliliter? (one thousandth) Remind students that thousandths
are written with three digits after the decimal point. Write on the board:

\[ 1 \text{ L} = 1,000 \text{ mL}, \text{ so } 1 \text{ mL} = 0.001 \text{ L} \]

ASK: How many thousandths of 1 L is 2 mL? (2 thousandths) Make a table
as shown and ask students to fill it in up to 10 mL.

<table>
<thead>
<tr>
<th>mL</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remind students that they can remove zeros to the right of the non-zero
digits after the decimal point, so 0.010 L can be written as 0.01 L.

**Exercises:** Convert to liters.

a) 9 mL = _____ thousandths of 1 L = _____ L
b) 17 mL = _____ thousandths of 1 L = _____ L
c) 324 mL d) 607 mL e) 500 mL f) 250 mL

**Answers:** a) 9, 0.009 L; b) 17, 0.017 L; c) 0.324 L; d) 0.607 L;
e) 0.500 L = 0.5 L; f) 0.250 L = 0.25 L

Reverse the task. ASK: How many milliliters are in 0.748 L? (748 mL) How
many thousandths are in 0.06? (60) How do you know? (0.06 = 0.060)
How many milliliters are in 0.06 L? (60 mL)
Exercises: Convert liters to milliliters.

a) 0.502 L  
b) 5.171 L  
c) 3.231 L  
d) 14.627 L  
e) 5.003 L  
f) 0.006 L  
g) 0.8 L  
h) 20.21 L

Answers: a) 502 mL, b) 5,171 mL, c) 3,231 mL, d) 14,627 mL, e) 5,003 mL, f) 6 mL, g) 800 mL, h) 20,210 mL

Extensions

1. a) How many teaspoons are needed to fill a 1 L bottle of water?  
   (Remember: 1 teaspoon = 5 mL)
   
   Answer: 200
   
b) A bathtub can hold 220 L of water. How many teaspoons would you need to fill the bathtub?
   
   Answer: 44,000
   
c) A cup holds 250 mL of water. How many cups will you need to fill the bathtub from part b)?
   
   Answer: 880
   
d) It takes 45 seconds to fill a cup with water in a bathroom sink, empty it into the bathtub, and return to the sink. If you want to fill the bathtub in part b) with cups, how much time will it take? Express your answer in seconds, minutes, and hours.
   
   Answer: 880 × 45 sec = 39,600 sec = 660 min = 11 h

2. Measuring volume. Show students measuring cups of different sizes and draw their attention to the marks. Point out that 1 mL is a small unit, so when a liquid needs to be measured to the closest milliliter, you need to use a small measuring cup or a graduated cylinder that can hold a very small amount of liquid. Demonstrate using a measuring cup to measure the volume of a container by filling a container with water, then pouring the water into a measuring cup. Choose a container that holds an amount of liquid that is not a round number so that the water level is between adjacent marks on the measuring cup. Remind students that they need to look at the mark that is closest to the water level. Point out that the measurement produced in this way is approximate.

   Give students a variety of containers, and have them estimate and then measure their volume in milliliters. Students will need funnels and a large bowl or tub to work over to contain any spills.
MD5-34 Changing Units of Liquid Volume
Pages 132–133

STANDARDS
5.MD.A.1, 5.NBT.B.7

VOCABULARY
gram (g)
kilogram (kg)
liter (L)
milliliter (mL)
operation

Goals
Students will convert between liters and milliliters and solve problems involving converting units of capacity.

PRIOR KNOWLEDGE REQUIRED
Can multiply and divide a decimal by 1,000
Knows that capacity is measured in liters and milliliters
Knows that 1 L = 1,000 mL
Can convert between kilograms and grams, meters and millimeters, or meters and kilometers

Review multiplying and dividing decimals and whole numbers by 1,000.
Remind students that they shift the decimal point three places to the right to multiply by 1,000 and three places to the left to divide by 1,000. Remind them that they can write zeros after the decimal point, and the number will not change. For example, $6.7 = 6.70 = 6.700$. ASK: Why are these numbers equal? (the decimal part is $7/10 = 70/100 = 700/1,000$) As well, remind students that, even though we do not write a whole number with a decimal point, we can still add the decimal point without changing the number. For example, $12 = 12.000$. Use the questions below as a test to make sure all students can perform the multiplication and division required.

Exercises: Calculate.

a) $247 \div 1,000$

b) $34 \div 1,000$

c) $2 \div 1,000$

d) $4,377 \div 1,000$

e) $0.004 \times 1,000$

f) $4.356 \times 1,000$

g) $1.79 \times 1,000$

h) $0.07 \times 1,000$

i) $0.3 \times 1,000$

Answers

a) 0.247, b) 0.034, c) 0.002, d) 4.377, e) 4, f) 4,356, g) 1.790, h) 70, i) 300

Use multiplication to convert from liters to milliliters (decimals). Remind students that, to convert from kilometers to meters, they multiplied by 1,000. ASK: What other conversions require multiplying by 1,000? (kilograms to grams, meters to millimeters) Why do you multiply by 1,000? (there are 1,000 meters in a kilometer, etc.) How will you convert a measurement in liters to milliliters then? (multiply by 1,000)

For example, convert 0.12 L to mL:

$0.12 \text{ L} = 0.120 \text{ L} = 0.120 \times 1,000 \text{ mL} = 120 \text{ mL}$

Move the decimal point three places to the right.
Exercises: Convert to milliliters.

a) 7.239 L  b) 10.825 L  c) 0.002 L  d) 0.04 L

e) 0.063 L  f) 0.41 L  g) 10.89 L  h) 2.3 L

Answers: a) 7,239 mL, b) 10,825 mL, c) 2 mL, d) 40 mL, e) 63 mL, f) 410 mL, g) 10,890 mL, h) 2,300 mL

ASK: If we multiply by 1,000 to convert from liters to milliliters, what operation would you use to convert from milliliters to liters? (divide by 1,000) Point out that this conversion is very similar to converting from grams to kilograms, from meters to kilometers, and from millimeters to meters.

Exercises: Convert to liters.

a) 7,253 mL  b) 739 mL  c) 75 mL  d) 8 mL

e) 14,105 mL  f) 500 mL  g) 30 mL  h) 32,200 mL

Answers: a) 7.253 L, b) 0.739 L, c) 0.075 L, d) 0.008 L, e) 14.105 L, f) 0.5 L, g) 0.03 L, h) 32.2 L

Mixed measurements. Remind students that they wrote mass as a mixed measurement: for example, 3 kg 456 g. Remind students how they converted mixed measurements to grams, as in the margin.

ASK: Would the same method work for liters and milliliters? (yes) Why does it make sense? (we used a similar method for other units, such as centimeters and meters)

Convert mixed measurements to milliliters. Do the first three exercises below as a class, and then have students do the remaining exercises individually, working on a grid.

Exercises: Convert to milliliters.

a) 2 L 345 mL  b) 17 L 67 mL  c) 4 L 8 mL  d) 2 L 371 mL

e) 45 L 604 mL  f) 658 L 400 mL  g) 8 L 75 mL  h) 30 L 5 mL

Bonus: i) 100 L 100 mL  j) 1,000 L 10 mL

Answers: a) 2,345 mL, b) 17,067 mL, c) 4,008 mL, d) 2,371 mL, e) 45,604 mL, f) 658,400 mL, g) 8,075 mL, h) 30,005 mL, Bonus: i) 100,100 mL, j) 1,000,010 mL

Converting measurements in liters to mixed measurements. Explain that in a measurement in liters, such as 6.527 L, the whole part shows the liters, and the decimal part shows the number of milliliters, so 6.527 L = 6 L 527 mL.

Exercises: Convert to a mixed measurement.

a) 6.998 L  b) 4.708 L  c) 2.039 L  d) 3.007 L

Answers: a) 6 L 998 mL, b) 4 L 708 mL, c) 2 L 39 mL, d) 3 L 7 mL
SAY: A student I know thinks that 2.5 L is 2 L 5 mL. Is he correct? (no) Have a volunteer explain the mistake. (1 mL is one thousandth of a liter, so 2 L and 5 mL is 2.005 mL, not 2.5 mL) ASK: What do you need to do to convert 2.5 L to mL? (write 2.5 with digits in the thousandths place, 2.5 = 2,500, note the 2 as liters or 2 L, and then move the decimal point right three places for the milliliters) Have students rewrite the measurement and then perform the conversion. (2.5 L = 2 L 500 mL)

**Exercises:** Expand the measurement to the thousandths place. Convert to a mixed measurement.

a) 6.9 L  

b) 4.7 L  

c) 2.19 L  

d) 3.87 L

**Answers:** a) 6 L 900 mL, b) 4 L 700 mL, c) 2 L 190 mL, d) 3 L 870 mL

**Converting measurements in milliliters to mixed measurements.** Ask students to think about how to convert 3,046 mL to a mixed measurement. One possible way is to convert the measurement to liters (3.046 L), then write the whole part as liters and the decimal part as milliliters. (3 L 46 mL)

Point out that we do not need to write the intermediate step; when you convert to liters, you divide by 1,000 by shifting the decimal point three places to the left. So instead of shifting the decimal point, we could take the hundreds, tens, and ones to be the number of milliliters, and the number of thousands to be the number of liters. Demonstrate with 34,678 mL = 34 L 678 mL.

**Exercises:** Convert to a mixed measurement.

a) 81,032 mL  

b) 7,643 mL  

c) 35,650 mL  

d) 200,002 mL

**Bonus:** 87,654,321 mL

**Answers:** a) 81 L 32 mL, b) 7 L 643 mL, c) 35 L 650 mL, d) 200 L 2 mL,  
**Bonus:** 87,654 L 321 mL

**Word problems with conversions.** Solve the following problems as a class in two ways: by converting all the measurements in liters to milliliters and working with milliliters, and by converting all the measurements to liters and working with decimals. Remind students that they need to get the same answer both ways, and if the answers are not the same, they need to look for mistakes.

1. a) To make juice from concentrate, you need to mix a 355 mL can of concentrate with three cans of water. How much juice do you get? (355 mL × 4 = 1,420 mL = 1.42 L)

b) Ron makes a fruit drink by mixing juice from concentrate (one 355 mL can and 3 cans of water) with a 946 mL bottle of cranberry juice and 1.5 L bottle of ginger ale. How much fruit drink does Ron mix? (3,866 mL = 3.866 L)

c) Ron plans that each of 12 people at a party needs at least one 300 mL glass of fruit drink. Did he make enough? (12 × 300 mL = 3,600 mL = 3.6 L < 3.866 L; yes, Ron made enough fruit drink)
2. A pack of six bottles of apple juice, each 300 mL, costs $3.99 per pack.
A pack of eight apple juice boxes, each 125 mL, costs $2.29.

   a) How much juice is in each pack? Write your answer in two ways:
in milliliters and in liters. (6 bottles contain 1,800 mL = 1.8 L juice,
8 boxes contain 1,000 mL = 1 L juice)

   b) Which contains more juice, five packs of six juice bottles or nine
packs of eight juice boxes? (both are 9 L)

   c) What costs more, five packs of six bottles or nine packs of eight
juice boxes? (five packs of six bottles cost $19.95, nine packs of
eight juice boxes cost $20.61)

   d) Which way of buying the juice is cheaper by volume? Explain.
(buying the juice in bottles is cheaper by volume)

**Extension**

1. a) Show students two containers, one that can hold 500 mL and
another that can hold 300 mL. ASK: How could you use only these
containers to measure 200 mL of water? Have students develop a
solution mentally or on paper. They can use the containers to check
their solution afterward. (You could make the requisite containers by
filling two 1-quart juice containers with the different amounts of water,
marking the water level, and cutting the containers at that height.)

   **Solution:** Fill the 500 mL container with water. Fill the 300 mL
   container with water from the 500 mL container. There will be
   200 mL of water left in the larger container.

   b) Use the containers from part a) to measure 400 mL of water.

   **Solution:** Use the method from part a) to measure 200 mL of water.
Pour the 200 mL of water into the 300 mL container so that you
have only 100 mL of room in the small container. Fill the 500 mL
container. Pour water from the 500 mL container into the 300 mL
container until the 300 mL container is full. There will be 400 mL
of water left in the 500 mL container.

   c) Use containers that can hold 1 L, 250 mL, and 100 mL to measure
out 750 mL in three different ways.

   **Solution:** 1 L − 250 mL = 750 mL, 3 × 250 mL = 750 mL,
5 × 100 mL + 250 mL = 750 mL
MD5-35  Filling Containers
Pages 134–136

STANDARDS
5.MD.A.1, 5.MD.C.5b, 5.MD.C.5c, 5.NBT.B.5

VOCABULARY
capacity
cubic centimeter (cm³)
cubic decimeter (dm³)
cubic meter (m³)
displace
liter (L)
milliliter (mL)
volume

Goals
Students will recognize the connection between milliliters, liters, and cubic units, and will solve problems involving capacity and volume.

PRIOR KNOWLEDGE REQUIRED
- Can multiply and divide a decimal by 1,000
- Knows that capacity is measured in liters and milliliters
- Knows that 1 L = 1,000 mL
- Knows that 1 dm = 10 cm
- Can convert metric units of length and units of capacity
- Can find the volume of a rectangular prism
- Can find the missing dimension given the volume of a prism and other dimensions

MATERIALS
- rectangular aquarium
- cube with 10 cm sides (e.g., a thousands block)
- large graduated pitcher
- water
- ruler
- smaller graduated pitchers for each student
- small toys (heavy enough to not float in water)

Introduce capacity. Explain that the amount of liquid (or sand, rice, or anything else that can be poured) a container can hold is called the capacity of the container. Capacity is often measured in liters and milliliters. Remind students that, when they found the volume of boxes, they found that volume in cubic units, such as cubic centimeters. If available, show students a rectangular aquarium. Explain that we can find the volume of the inside of the aquarium in cubic units. But we also can pour water into it and find its capacity—or the volume of the liquid—in milliliters or liters. Explain that there is a connection between these two types of units.

Review decimeters and cubic decimeters. Remind students that 1 decimeter is a unit of length equal to 10 centimeters.

A cube that has sides of 1 decimeter (1 dm) has a volume of 1 dm³. Show students a cube with sides 1 dm and explain that this is a cubic decimeter. Write on the board:

\[ 1 \text{ dm} = 10 \text{ cm} \]
\[ 1 \text{ dm} \times 1 \text{ dm} \times 1 \text{ dm} = 1 \text{ dm}^3 \]

Introduce the connection between dm³ and liters. If available, show students a large graduated pitcher with water in it so that at least 1 L can be added to it. Have a volunteer say how much water is in the pitcher.
(say, 2 L) Place the 1 dm cube into the pitcher and have students observe that the water level rises. Explain that we say that the cube *displaces* some water in the pitcher—in other words, it takes the place of the water. The volume of water the cube displaces equals the volume of the cube. What is the total volume of the liquid and the cube in the pitcher? Have a volunteer check. (3 L) ASK: How much water did the cube displace? (1 L) What is the volume of the cube in liters? (1 L) Write on the board:

\[ 1 \text{ dm}^3 = 1 \text{ L} \]

Point out that both cubic decimeters and liters are measurements of volume; however, cubic decimeters are metric units and liters are equal to the metric units. The volume of liquids and capacity are usually measured in liters, and geometric volume is usually measured in cubic units.

**Introduce the connection between cm\(^3\) and mL.** Remind students that a milliliter is *one thousandth* of a liter, so there are 1,000 mL in a liter. Write on the board:

\[ 1 \text{ L} = 1,000 \text{ mL} \]

Draw a cube on the board and mark its length, width, and height as 1 dm = 10 cm. Ask students to find the volume of the cube in cubic centimeters. Write the calculation on the board:

\[ 1 \text{ dm}^3 = 10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} = 1,000 \text{ cm}^3 \]

Write on the board:

\[ \begin{align*}
1 \text{ dm}^3 &= 1,000 \text{ cm}^3 \\
1 \text{ L} &= 1,000 \text{ mL}
\end{align*} \]

ASK: How many milliliters are in 1 cubic centimeter? (1) How do you know? (1,000 mL = 1,000 cm\(^3\), so 1 mL and 1 cm\(^3\) should be the same thing)

**Finding volume of an aquarium.** Draw a prism on the board, and mark the sides as 5 dm, 6 dm, and 4 dm. Ask students to find the volume of the prism in cubic decimeters. \((5 \text{ dm} \times 6 \text{ dm} \times 4 \text{ dm} = 120 \text{ dm}^3)\) Then tell them that this prism is an aquarium, and ask them how much water will fit into the aquarium. \((120 \text{ L})\)

Ask students to convert the dimensions of the prism into centimeters and find the volume in cubic centimeters and in milliliters. \((50 \text{ cm} \times 60 \text{ cm} \times 40 \text{ cm} = 120,000 \text{ cm}^3 = 120,000 \text{ mL})\) Do the answers match? (yes, 120 L = 120,000 mL)

Invite a volunteer to measure the inner dimensions of the rectangular aquarium you showed students at the beginning of the lesson. Ask them to find the volume of the aquarium in cubic centimeters. Then ASK: How many milliliters of water will fit into the aquarium? (the number of milliliters will be the same number as the volume in cm\(^3\)) Have them convert the answer to liters.
Pour several liters of water into the aquarium using a pitcher. Have a volunteer use a ruler to measure the height in centimeters the water reaches, and have students find the volume of water in the aquarium in cubic centimeters. Again, have students convert the answer to milliliters and then to liters. Is the answer close to the amount of water you poured in? The answer is likely not to be exactly the same, so discuss why there might be the difference. (The measurements are only approximations. The exact height might have been, say, 12.3 cm, which we rounded to 12 cm. The same applies to length and width.)

**Finding the height of water.** Review finding the missing dimension when given the volume of a prism. For example, if a prism has a volume of 120 cm$^3$, and it is 10 cm long and 6 cm wide, what is its height? Remind students of the formula Volume = length $\times$ width $\times$ height and that length and width give the area of the bottom or the top face. So, in the example, $10 \text{ cm} \times 6 \text{ cm} = 60 \text{ cm}^2$, the height is $120 \text{ cm}^3 \div 60 \text{ cm}^2 = 2 \text{ cm}$.

Draw another prism on the board and mark the length and the width as 75 cm and 40 cm. Tell students that this is an aquarium, and you want to pour, say, three pails equal to 30 liters of water into it. How high do you think the water will be? Have students think how they can find the height and discuss the potential solution in pairs. Go through the solution as a class:

\[
\text{Volume of water} = 30 \text{ L} = 30,000 \text{ mL} = 30,000 \text{ cm}^3
\]

\[
\text{Area of bottom face} = 75 \text{ cm} \times 40 \text{ cm} = 3,000 \text{ cm}^2
\]

\[
\text{Height} = \frac{30,000 \text{ cm}^3}{3,000 \text{ cm}^2} = 10 \text{ cm}
\]

Point out that this is very little water for such an aquarium—it is only 10 cm of water, and aquariums usually have water levels much higher than that!

Repeat the exercise with an aquarium of the size you might see, say, in a doctor’s office. For example: an aquarium measures 1.5 m long, 60 cm wide, and contains 819 L of water (including sand, decorations, and fish). What is the height of the water? This time students will need to convert 1.5 m to centimeters. (height $= 91 \text{ cm}$)

**Exercise:** An aquarium that is 1.4 m long and 50 cm wide contains 770 L of water including sand, decorations, and fish. How high is the water in the aquarium?

**Answer:** 110 cm $= 1.1 \text{ m}$

**Using displacement of water to find volume.** Explain that displacement allows you to find the volume of objects that are not rectangular prisms. Pour some water into a graduated pitcher and have a volunteer check how much water is in the pitcher. Place an object with a complicated shape, such as a toy, into the water and have another volunteer check the level of the water. SAY: The water level now shows the volume of the water plus the volume of the toy. How can we find the volume of the toy? Have students write the subtraction sentence and find the volume of the toy.
ACTIVITY

Give each student a small toy and a small graduated pitcher. Have students use displacement to find the volume of the toy. Then have students form pairs, and exchange toys with their partner. Have students find the volume of their partner’s toy, and then compare answers.

Extensions

1. How can we “count” water? Read students the chapter on capacity from Anno’s Math Games II by Mitsumasa Anno.

2. Tom measured the dimensions of a 2 L juice carton and calculated the volume to be 200 cm³. Is his answer correct? Explain how you know.

   Answer: No; 2 L = 2,000 mL = 2,000 cm³, so Tom’s answer is incorrect.

3. Explore cubic kilometers.

   a) How many cubic meters are in 1 km³?
      
      Answer: 1,000,000,000 m³

   b) The volume of water in Lake Michigan is 4,920 km³. Find the volume of Lake Michigan in cubic meters and in liters. How many zeroes are in each of these numbers?

      Answer: 4,920 km³ = 4,920,000,000,000 m³ = 4,920,000,000,000 L; 10 zeroes for m³; 13 zeroes for L

   c) The tower of the Aon Center in Chicago, IL, is a rectangular prism 342 m tall, 59 m long, and 59 m wide. What is the volume of the tower? Round your answer to the nearest hundred thousand cubic meters. (NOTE: Students may remember that the Aon Center tower was also considered on AP Book 5.2 p. 119, but measured in feet.)

      Answer: Aon Center tower volume = 1,190,502 m³ ≈ 1,200,000 m³

   d) Estimate the volume of Lake Michigan in Aon Center towers. Use your answer from part c) to estimate.

      Answer: about 4,100,000 Aon Center towers

(MP7) 4. Find a fast way to calculate \( \frac{4}{5} \times 3.2 \times \frac{15}{12} \). Do not use a calculator.

Sample answers

• I changed the product to \((4/5 \times 15/12) \times 3.2\), since you can multiply the same three numbers in any order. \(4/5 \times 15/12 = 60/60 = 1\), and \(1 \times 3.2 = 3.2\).

• I changed the product to \((4/5 \times 15/12) \times 3.2\), since you can multiply the same three numbers in any order, then I changed the product to \((4/5 \times 5/4) \times 3.2\). \(4/5 \times 5/4 = 20/20 = 1\), and \(1 \times 3.2 = 3.2\).
Cups and Fluid Ounces

STANDARDS
5.NF.A.1, 5.NF.B.4, 5.MD.A.1

VOCABULARY
capacity
cup (c)
denominator
fluid ounce (fl oz)
mixed number
numerator
remainder

Goals
Students will convert between cups and fluid ounces.

PRIOR KNOWLEDGE REQUIRED
Can add fractions and mixed numbers with different denominators
Can multiply and divide fractions and mixed numbers by whole numbers
Can divide whole numbers to produce a mixed number
Is familiar with measuring capacity
Can convert US customary units of length

Review adding fractions and mixed numbers with different denominators. Remind students that, to add fractions with different denominators, they need to change them to a common denominator.

Exercises: Add.

a) \( \frac{1}{2} + \frac{3}{4} \)
b) \( \frac{3}{4} + \frac{5}{8} \)
c) \( \frac{3}{4} + \frac{1}{2} + \frac{7}{8} \)

Answers: a) 5/4, b) 11/8, c) 17/8

Review converting improper fractions to mixed numbers. Have students convert the answers to the previous exercises to mixed numbers. (a) 1 1/4, b) 1 3/8, c) 2 1/8)

Review adding mixed numbers by adding the whole parts and the fractional parts separately, and then regrouping the fractional part if necessary.

Exercises: Add.

a) \( \frac{1}{2} + 3 \frac{3}{4} \)
b) \( 2 + \frac{1}{4} + 3 \frac{5}{8} \)
c) \( \frac{3}{4} + 2 \frac{1}{2} + \frac{7}{8} \)

Answers: a) 4 1/4, b) 5 7/8, c) 6 1/8

Introduce cups. Show students the recipe below.

Blueberry smoothie (2 servings)

1 cup blueberries
1/2 cup yogurt
3/4 cup milk
1 tablespoon honey

Point out that the measurements in this recipe are not given in liters or milliliters. Three ingredient measurements in the recipe are given in cups. Explain that cups are one of the standard units of measurements commonly used in the United States, and that the short form for it is c. Have students
total the measurements in the recipe above, but disregarding the honey because it is a small amount. \((1 + 1/2 + 3/4 = 2 1/4 \text{ c})\) Then have them find the size of one serving. \((1 1/8 \text{ c})\)

**Introduce fluid ounces.** ASK: What other units do you often see in recipes? (ounces, pounds) Explain that when a liquid or another pourable substance, such as flour or sugar, is measured, “ounces” usually mean “fluid ounces.” The short form for fluid ounces is fl oz. Write the name of the unit and the abbreviation on the board. Explain that there are 8 fluid ounces in one cup. Write on the board:

\[
1 \text{ c} = 8 \text{ fl oz}
\]

Start a conversion table as in the margin, and have students help you fill it in. Then ASK: To convert a measurement in cups to fluid ounces, what number will you multiply by? (8)

**Exercises:** Multiply by 8 to convert the measurement to fluid ounces.

<table>
<thead>
<tr>
<th>c</th>
<th>fl oz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

**Answers:** a) 72 fl oz, b) 96 fl oz, c) 296 fl oz, d) 360 fl oz, e) 800 fl oz, f) 8,000 fl oz

**Review multiplying a fraction and a mixed number by a whole number.** Point out that, to convert a measurement such as \(3/4 \text{ c} \) or \(2 1/4 \text{ c}\) to fluid ounces, students need to multiply the number of cups by 8. Remind students that, to multiply a fraction by a whole number, we think of \(3/4 \times 8\) as \(3/4\) of 8, meaning one quarter taken three times or one quarter times three. A quarter of 8 is 2, so \(3/4\) of 8 is \(3 \times 2 = 6\). Draw the picture provided below to illustrate the process, and also write on the board the actual calculation students need to do:

\[
\frac{1}{4} \text{ of } 8 = \frac{3}{4} \times 8 = 3 \times \left(\frac{8}{4}\right) = 3 \times 2 = 6
\]

**Exercises:** Multiply.

| a) \(\frac{1}{4} \times 8\) | b) \(\frac{3}{8} \times 8\) | c) \(\frac{3}{4} \times 16\) | d) \(\frac{1}{2} \times 32\) |

**Answers:** a) 2, b) 3, c) 12, d) 16

Write on the board:

\[
2 \frac{1}{4}
\]

Remind students that, to multiply a mixed number by a whole number, they need first to convert the mixed number to an improper fraction. For example, \(2 1/4 = 9/4\), where 9 comes from multiplying the denominator by the whole
part and adding the numerator of the fractional part: \(2 \times 4 + 1 = 9\).

Then we can multiply by 8:

\[
2 \frac{1}{4} \times 8 = \frac{9}{4} \times 8
= 9 \times (8 \div 4)
= 9 \times 2 = 18
\]

**Exercises:** Multiply by 8 to convert cups to fluid ounces.

a) \(2 \frac{1}{4}\) c  

b) \(1 \frac{3}{8}\) c  

c) \(2 \frac{3}{4}\) c  

d) \(3 \frac{1}{2}\) c

**Answers:** a) 18 fl oz, b) 11 fl oz, c) 22 fl oz, d) 28 fl oz

**Compare measurements in different units.** Remind students that, when they compare two measurements in different units, they need to convert one of the measurements so that the units become the same. For example, 7 fl oz is smaller than 1 c, even though 1 is smaller than 7, because 1 c = 8 fl oz and 8 > 7. Present the following exercises one by one and have students point to the larger measurement to signal the answer.

**Exercises:** Convert the measurement in cups to fluid ounces, and then compare the measurements. What is the larger measurement?

a) 50 fl oz or 6 c  

b) 8 c or 80 fl oz  

c) 50 fl oz or 10 c  

d) 45 c or 300 fl oz  

**Bonus:** 1,000,000 fl oz or 100,000 c

**Answers:** a) 50 fl oz, b) 80 fl oz, c) 10 c, d) 45 c, Bonus: 1,000,000 fl oz

**Converting the measurements in fluid ounces to cups.** SAY: If you multiply by 8 to convert a measurement in cups to fluid ounces, what would you do to convert a measurement in fluid ounces to cups? (divide by 8) Remind students that when dividing, for example, 25 by 8, you might get a remainder and this remainder needs to be divided further, giving a fractional part. This means the answer to a division question is often a mixed number. So, for example, \(25 \div 8 = 3 \text{ R } 1\), so \(25 \div 8 = 3 \frac{1}{8}\). This means that 25 fl oz = 3 1/8 c.

**Exercises:** Divide by 8 to convert the measurement in fluid ounces to cups. Reduce the answer to lowest terms.

a) 28 fl oz  

b) 31 fl oz  

c) 54 fl oz  

d) 78 fl oz

**Answers:** a) 3 1/2 c, b) 3 7/8 c, c) 6 3/4 c, d) 9 3/4 c

Solve the following problem as a class.

Leo wants to make a berry smoothie. The recipe calls for 6 fl oz of yogurt, 1 fl oz of honey, 1 1/2 cups of berries, 3/4 cups of milk, and 1 3/4 cups of ice.

a) Leo needs to put the ingredients in a blender. The blender can hold 4 cups. Is the blender large enough for all the recipe’s ingredients? (no, the ingredients total 4 7/8 c or 39 fl oz, which is more than 4 c or 32 fl oz)
b) The recipe makes three servings. How large is each serving? (1 serving = 13/8 c = 1 5/8 c = 13 fl oz, or 39/3 fl oz = 13 fl oz = 1 5/8 c)

c) Leo wants to serve the smoothie in three large glasses that each holds 15 fl oz. How much empty space will be left in each glass? (2 fl oz)

Extensions

1. Many recipes mention teaspoons (tsp) and tablespoons (tbsp) as capacity measurements.

   a) There are 2 tablespoons in a fluid ounce and 3 teaspoons in a tablespoon. Complete the equations.

   \[1 \text{ tbsp} = \frac{1}{2} \text{ fl oz} \quad 1 \text{ tsp} = \frac{1}{6} \text{ fl oz}\]

   **Answer:** a) 1/2, 1/6

   (MP4)

   b) What is the total amount of the ingredients in the following recipe? Will a 1/2-gallon tray be enough to contain the ingredients? Hint: Convert all measurements to fluid ounces.

   \[
   \begin{align*}
   1 \text{ c butter} & \quad 12 \text{ fl oz milk} & \quad 6 \text{ tbsp water} \\
   2 \text{ c pumpkin puree} & \quad 2 \text{ eggs} = 3 \frac{1}{2} \text{ fl oz} & \quad \frac{1}{2} \text{ tsp salt} \\
   2 \frac{1}{2} \text{ c flour} & \quad \frac{1}{2} \text{ tsp spices} \\
   \frac{3}{4} \text{ c sugar} & \quad &
   \end{align*}
   \]

   **Answer:** 68 5/6 fl oz, no

2. A rectangular prism has two faces that are 5 cm by 4 cm, two faces that are 5 cm by 3 cm, and two faces that are 4 cm by 3 cm.

   a) Vicky says you can find the volume of the prism with the expression \[2 \times (5 \times 4) + 2 \times (5 \times 3) + 2 \times (4 \times 3).\] Do you agree? Explain.

   b) Find the volume of the prism. Explain how you know.

   **Answers**

   a) I disagree: Vicky is adding the area of the faces, not multiplying the area of a base times the height

   b) the length, width, and height of the prism must be 5 cm, 4 cm, and 3 cm, so Volume = 5 \times 4 \times 3 = 60 \text{ cm}^3
MD5-37 US Customary Units of Capacity
Pages 139–141

STANDARDS
5.NF.A.1, 5.NF.B.4, 5.MD.A.1

VOCABULARY
- capacity
- cup (c)
- denominator
- fluid ounce (fl oz)
- gallon (g)
- mixed number
- numerator
- pint (pt)
- quart (qt)
- remainder

Goals
Students will convert between different US customary units used to measure capacity and solve problems requiring such conversions.

PRIOR KNOWLEDGE REQUIRED
- Can convert between mixed numbers and improper fractions
- Can multiply and divide fractions and mixed numbers by whole numbers
- Is familiar with measuring capacity
- Can convert US customary units of length
- Can convert between cups and fluid ounces
- Can divide whole numbers to produce a mixed number
- Can add and subtract fractions and mixed numbers with different denominators

MATERIALS
- containers with the capacities of 1 cup, 1 pint, 1 quart, and 1 gallon
- BLM Unit Spinner (p. Q-72)

Introduce pints. Remind students that cups are a common unit to measure volumes of liquids and capacities of containers. Ask students to think about what other units they have seen on various liquid containers. (gallons, quarts, pints) Explain that a pint is a larger unit of capacity, equal to 2 cups. Ask students to think where they have seen measurements in pints (e.g., on milk containers). Write on the board:

1 pint = 2 cups  1 pt = 2 c

Explain that “pt” is a short way to write pints, and have a volunteer circle the letters “p” and “t” in the word “pint.” Show a 1 pt container and explain that this is how large a pint is. Keep the container displayed for future reference, together with a 1 cup container.

Converting between pints and cups. ASK: How would you convert a measurement in pints to cups? (multiply by 2) Remind students how to multiply a mixed number by a whole number, and work through the first three exercises below as a class. Students should do the rest individually.

Exercises: Convert to cups.

a) 4 pt  b) 2 1/2 pt  c) 3 3/4 pt  d) 7 pt

e) 11 pt  f) 4 1/2 pt  g) 5 5/8 pt  h) 7 3/4 pt

Answers: a) 8 c, b) 5 c, c) 7 1/2 c, d) 14 c, e) 22 c, f) 9 c, g) 1 1/4 c, h) 15 1/2 c
ASK: How would you convert a measurement in cups to pints? (divide by 2) Remind students that, when they have a remainder in division, they need to divide this remainder further to obtain a fractional part, so the answer is a mixed number. For example, how many pints are in 25 cups? \((25 \div 2 = 12 \text{ R } 1, \text{ so } 25 \text{ c } = 12 \frac{1}{2} \text{ pt})\) Also remind students that sometimes (e.g., in part h) below) they need to convert the mixed number to an improper fraction before dividing. Again, work through the first three exercises below as a class, and have students do the rest individually.

**Exercises:** Convert cups to pints.

a) 4 c  
   b) 5 c  
   c) 2 \(\frac{1}{2}\) c  
   d) 10 c  
   e) 11 c  
   f) \(\frac{3}{4}\) c  
   g) 2 \(\frac{1}{4}\) c  
   h) 3 \(\frac{3}{8}\) c  

**Answers:** a) 2 pt, b) 2 \(\frac{1}{2}\) pt, c) 1 \(\frac{1}{4}\) pt, d) 5 pt, e) 5 \(\frac{1}{2}\) pt, f) 3/8 pt, g) 1 \(\frac{1}{8}\) pt, h) 3 \(\frac{3}{8}\) c = \((24 + 3)/8\) c = 27/8 c = 27/16 pt

Introduce quarts. Explain that the next largest unit after pints is a quart, written in short as “qt.” Show a 1 qt container, such as a milk carton. Add it to the display and keep for future reference. Explain that there are two pints in a quart. ASK: How many cups are in 1 quart? (4) What number will you multiply by to convert from quarts to pints? (2) From quarts to cups? (4) Summarize by drawing the diagram below on the board, leaving room on both sides to add another unit (gallons and fluid ounces). Keep the diagram on the board for future reference.

\[
\begin{array}{ccc}
\times 2 & \times 2 \\
\text{qt} & \text{pt} & \text{c}
\end{array}
\]

Again, do the first three questions in each exercise as a class, and then have students work individually.

**Exercises**

1. Multiply by 2 to convert to pints.
   a) 6 qt  
   b) \(\frac{3}{2}\) qt  
   c) \(\frac{3}{4}\) qt  
   d) 8 qt  
   e) 21 qt  
   f) \(\frac{4}{4}\) qt  
   g) \(\frac{7}{8}\) qt  
   h) \(\frac{7}{2}\) qt

**Answers:** a) 12 pt, b) 7 pt, c) 5 \(\frac{1}{2}\) pt, d) 16 pt, e) 42 pt, f) 8 \(\frac{1}{2}\) pt, g) 1 \(\frac{3}{4}\) pt, h) 15 pt

2. Multiply by 4 to convert to cups.
   a) 4 qt  
   b) \(\frac{1}{2}\) qt  
   c) \(\frac{3}{8}\) qt  
   d) 6 qt  
   e) 13 qt  
   f) \(\frac{2}{2}\) qt  
   g) \(\frac{3}{8}\) qt  
   h) \(\frac{3}{4}\) qt

**Answers:** a) 16 c, b) 6 c, c) 13 \(\frac{1}{2}\) c, d) 24 c, e) 52 c, f) 10 c, g) 1 \(\frac{1}{2}\) c, h) 23 c
3. Divide by 2 to convert to quarts.

a) 10 pt  

b) 17 pt  

c) 3 \(\frac{3}{4}\) pt  

d) 16 pt  

e) 15 pt  

f) 5 \(\frac{1}{2}\) pt  

g) 3 \(\frac{3}{4}\) pt  

h) 7 \(\frac{1}{4}\) pt  

Answers: a) 5 qt, b) 8 1/2 qt, c) 15/8 qt = 1 7/8 qt, d) 8 qt, e) 7 1/2 qt, f) 2 3/4 qt, g) 3/8 qt, h) 3 5/8 qt

Introduce gallons. Explain that a gallon is the largest of the common US units to measure volume of liquids or capacity of containers. Have students think about what they often see measured in gallons, or where they often see gallons mentioned. (gas, water, car efficiency measured in miles per gallon, etc.) Show students a gallon container and add it to the display. Explain that 1 gallon equals 4 quarts. How many pints is that? (8) How many cups is that? (16) Add gallons to the diagram as shown in the margin. Again, do some of the exercises below as a class, and have students work individually.

Exercises

1. Multiply by 4 to convert to quarts.

a) 6 gal  

b) \(\frac{5}{8}\) gal  

c) \(1 \frac{1}{2}\) gal  

d) 8 gal  

e) 19 gal  

f) \(4 \frac{1}{4}\) gal  

g) \(\frac{1}{8}\) gal  

h) \(3 \frac{1}{2}\) gal  

Answers: a) 24 qt, b) 2 1/2 qt, c) 6 qt, d) 32 qt, e) 76 qt, f) 17 qt, g) 2 qt, h) 14 qt

2. Multiply by 16 to convert to cups.

a) 3 gal  

b) \(1 \frac{1}{4}\) gal  

c) \(2 \frac{3}{8}\) gal  

d) \(3 \frac{1}{2}\) gal  

e) \(\frac{5}{8}\) gal  

Answers: a) 48 c, b) 20 c, c) 38 c, d) 56 c, e) 10 c

3. Divide by 16 to convert to gallons.

a) 10 c  

b) 17 c  

c) \(23 \frac{1}{2}\) c  

d) 120 c  

Answers: a) 5/8 gal, b) 1 1/16 gal, c) 1 15/32 gal, d) 7 1/2 gal

Review fluid ounces. Draw students’ attention to the diagram with the other four units of capacity and ASK: What unit that we recently learned about is missing here? (fluid ounces) Add fl oz to the diagram that you drew earlier, and add the arrow between cups and fluid ounces. ASK: What number should I multiply by to get from cups to fluid ounces? (8) Go through each unit in the diagram and check how many fluid ounces are in 1 unit. (16 fluid ounces in a pint, 32 in a quart, 128 in a gallon) Then have students convert the measurements they converted from gallons to cups (Exercise 2 above) to fluid ounces, and check the answers by direct conversion from gallons to fluid ounces. (a) 384 fl oz, b) 160 fl oz, c) 304 fl oz, d) 448 fl oz, e) 80 fl oz)
Before having students do the activity below, review the diagram for converting units. Remind students that, when they try to convert, for example, from gallons to pints, they can think of the conversion as converting to quarts first (by multiplying by 4) and then converting to pints (by multiplying the result by 2). The total factor is 8. ASK: What number would you multiply by to convert from gallons to cups? (16) How would you convert gallons to fluid ounces? (multiply by 128) What number would you divide by to convert from cups to gallons? (16) How can you convert from fluid ounces to gallons? (divide by 128) PROMPTS: Are gallons larger than fluid ounces or smaller than fluid ounces? (larger) Do you need more or fewer of them? (fewer) Will you multiply or divide the number of fluid ounces? (divide) By how much? (128, the number of fluid ounces in a gallon)

**ACTIVITY**

Provide students with BLM Unit Spinner. Ask students to work in pairs. Players should take turns choosing a number and spinning the spinner to choose what unit-to-unit conversion will be required. They can then work as a pair to solve the problem. Then the other player can choose the number and spin the spinner.

Advanced variation: players use the second spinner on the BLM to decide whether the given measurement should be a whole number, a fraction, or a mixed number.

**Problems that require conversions.** Work through the problems below as a class.

1. There will be 15 people at a party. Each might drink 3 cups of juice.
   a) How many cups of juice are needed for the party? (45 cups)
   b) To make juice from a can of frozen juice concentrate, you need to mix the contents of the can with 3 cans of water. The can contains 12 fl oz of concentrate. How many cups of juice does one can of frozen juice concentrate make? (48 fl oz = 6 cups)
   c) How many cans of concentrate do you need to buy for the party? (45 ÷ 6 = 7 R 3, so you need to buy 8 cans of juice concentrate)

2. Each person on a hiking trip needs 15 cups of water a day. How many cups of water does a family of four need on a two-day trip? How many gallons of water should they have for their trip? (120 c, 7 1/2 gal)

3. A dessert recipe asks for 6 fl oz of lime jelly, a 6 fl oz can of crushed pineapple, and 2 cups of cottage cheese.
   a) How much dessert will the recipe make? (28 fl oz or 3 1/2 c)
   b) The recipe serves six. How large is each portion? (4 2/3 fl oz)
Extensions

(MP4) 1. A recipe for home-made strawberry yogurt asks for 2 cups of strawberries, 1/2 gal of milk, and 2 fl oz of yogurt starter. How many full 4 fl oz portions does the recipe make?

Answer: 20 portions (82 fl oz, so 20 × 4 fl oz, plus 2 fl oz extra)

(MP2) 2. A tap leaks 1 drop of water every second. There are 480 drops of water in 1 fluid ounce.

a) If the tap continues to leak for a whole week, how much water is lost?

b) Of the amount of water lost in a week, how many whole gallons are there? Of the remaining amount lost, how many whole quarts, whole pints, whole cups, and whole fluid ounces are there?

c) Express the answer to part a) as a mixed measurement.

Solutions
a) 60 drops in 1 minute, 3,600 drops in 1 hour, 86,400 drops per day, or 604,800 drops per week; 604,800 ÷ 480 = 1,260 fl oz in a week; 1,260 fl oz = 1,260 ÷ 128 = 9 108/128 = 9 27/32 gal in a week
b) 9 gal, 3 qt, 0 pt, 1 c, and 4 fl oz

c) 9 gal 3 qt 1 c 4 fl oz

(MP3, MP5) 3. Explain how you can use the answer to 921 × 7 to find the product of 921 and 70 and the product of 9.21 and 70. Use one or more of these tools: an array, a number line, base ten blocks, equations with brackets.

Sample answers
• I used base ten blocks and smaller numbers. If you make 3 groups of 7 ones blocks, you get 21 ones blocks. If you replace all the ones blocks with tens blocks, you get 3 × 70 = 21 tens blocks, or ten times more, so you can just add a zero to 21 to get the answer. The same rule works for 921 × 70: add a zero at the end of the answer to 921 × 7. To find 9.21 × 70, move the decimal in the product two places to the left.
• I used two number lines. Skip counting by 70 is ten times as much as skip counting by 7, and counting by 10s is like counting by 1s but with a zero at the end. To count by 70s, say every seventh number when counting by 10s. To count by 7s, say every seventh number when counting by 1s. To find 9.21 × 70, move the decimal in the product of 921 and 70 two places to the left.
• I wrote 921 × 70 as 921 × (7 × 10). That's the same as (921 × 7) × 10, so you just have to multiply the answer to 921 × 7 by 10 to get the answer. 9.21 × 70 = (921/100) × 70 = (921 × 70) ÷ 100.
Connection between gallons and cubic inches. Remind students that there is a connection between cubic centimeters and milliliters, and between cubic decimeters and liters. Note the following on the board:

\[ 1 \text{ cm}^3 = 1 \text{ mL} \]
\[ 1 \text{ dm}^3 = 1,000 \text{ cm}^3 = 1 \text{ L} \]

Explain that there is also a connection between cubic units in the US customary system and the measurements of liquid volume:

1 gallon = 231 in\(^3\).

Draw a cube on the board and tell students that this cube is 1 ft long, 1 ft wide, and 1 ft tall. Have students find the volume of the cube in cubic inches. (1,728 in\(^3\)) ASK: Is 1,728 in\(^3\) more than 1 gallon? (yes) If this cube were an aquarium, how can you find how many gallons would be needed to fill it? (divide 1,728 \(\div\) 231)

**Exercises:** Find the volume of the aquarium in cubic inches. Then find its capacity in gallons.

a) 6 in by 7 in by 11 in  

b) 15 in by 14 in by 1 ft 10 in

**Answers:** a) 462 in\(^3\), 2 gal; b) 4,620 in\(^3\), 20 gal

ASK: How many fluid ounces are in 1 gallon? (128) Which fraction of a gallon is 1 cubic inch? (1/231) Which fraction of a gallon is 1 fluid ounce? (1/128) What is larger, 1 in\(^3\) or 1 fl oz? (1 fl oz) How do you know? (231 > 128, so 1/128 > 1/231, so 1 fl oz > 1 in\(^3\))

The rest of the lesson is cumulative review. Here are some additional exercises.
Exercises

1. An aquarium is 1.4 m long, 70 cm wide, and 1 m 5 cm tall.
   a) What is the capacity of the aquarium?
   b) The aquarium is filled with water, equipment, fish, etc. The total volume of the contents is 980 L. What height does the water reach? How far is the water from the top of the aquarium?
   Answers: a) 1,029 L; b) water level = 1 m height, 5 cm from the top

2. A lawn is a rectangle 12 ft 6 in wide and 25 ft long.
   a) What is the area of the lawn in square feet? In square inches?
   b) Imagine a prism that is 1 in tall and its bottom face is the lawn. What is the volume of the prism?
   c) In a week, the lawn needs an amount of water equal to the capacity of the prism in part b). How many gallons of water (to the closest gallon) does the lawn need in a week?
   Answers: a) 312 1/2 ft², 45,000 in²; b) 45,000 in³, c) 195 gal

3. A café sells 150 cups of 8 fl oz coffee, 200 cups of 12 fl oz, and 150 cups of 16 fl oz.
   a) How much coffee in total did the café sell?
   b) A small cup of coffee costs $1.95, a medium cup costs $2.45, and a large cup costs $2.95. How much money did the café charge for the total amount of coffee sold?
   c) The café makes coffee in containers of a quarter gallon each. How many containers did they make?
   d) It costs about $0.70 to make one container of coffee. How much will it cost to make the total amount of coffee the café sold?
   e) How much profit did the café make from coffee?
   Answers: a) 6,000 fl oz = 46 112/128 gal = 46 7/8 gal, b) $1,225, c) 187.5 containers, d) $132.25, e) $1,093.75

Extensions

(MP1, MP7) 1. Find the area of the shape below.

```
10 in
6 in
4 in
5 in
8 1/4 in
3 in
```

Answer: 90 1/4 in²
2. Project: Volumes of lakes
   a) Use the Internet to identify the largest lake by volume on each continent. Express the volume of each lake in as many different units as you can.
   b) Order the lakes by volume.
   c) Use the Internet to find the area of each lake.
   d) Order the lakes by area. Is the order the same for area as for volume?
   e) Why do some lakes have a larger volume than others that have a larger area?
   f) Pick two of the lakes. By volume, how many times as large is one lake compared with the other lake?

3. Counting on Frank, by R. Clement.
   Read the book with students. Discuss the reasonableness of some of Frank’s estimates. For example, using the volume of a pea (about 1/2 cm$^3$), estimate the volume of peas the book suggests. Then estimate the number of peas needed to fill a kitchen.
   As you and the students will discover, the estimates in the book are unreasonable.

4. Find the height of the square-based rectangular prism with volume $V$.
   a) $V = 36 \text{ m}^3$
   b) $V = 80 \text{ ft}^3$
   c) $V = 128 \text{ cm}^3$
   \textbf{Answers:} a) 4 m, b) 5 ft, c) 2 cm

5. Write a general rule about how you could find the height of a rectangular prism with volume $V$ that has a square base with side length $L$.
   \textbf{Answer:} Height = $\frac{V}{L \times L}$
Area and Perimeter

1. a) Measure the length and the width of each rectangle in centimeters. Find the perimeter and area of each rectangle. Write the answers in the table.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$3 \text{ cm} + 5 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} = 16 \text{ cm}$</td>
<td>$3 \text{ cm} \times 5 \text{ cm} = 15 \text{ cm}^2$</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
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<td>C</td>
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<td>F</td>
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</tr>
</tbody>
</table>

b) Shape E has a greater perimeter than shape A. Does it also have a greater area? _____
c) Name two rectangles that have the same perimeter and different areas. _____ and _____
d) Write the shapes in order from greatest perimeter to least perimeter. __________________
e) Write the shapes in order from greatest area to least area. ___________________
f) Are the orders in parts d) and e) the same? _____

2. Find the area of the rectangle using the clues.
   a) Width = 2 cm, perimeter = 10 cm
   b) Length = 5 cm, perimeter = 18 cm

   Area = ____________    Area = ____________

3. On grid paper, draw a rectangle with the given area and perimeter.
   a) Area = 10 square units
   Perimeter = 14 units
   b) Area = 8 square units
   Perimeter = 12 units
Cube Skeleton

You will need 12 tubes and tape to make a skeleton of a cube.

1. Use tape to bind tubes together as shown.

2. Make two squares with the tubes. Make sure the squares are 1 unit long and 1 unit wide on the inside.

3. Bind the four leftover tubes to one of the squares as shown.

4. Add the other square to the top.
Cubic Inch

1 in³
Net of Rectangular Prism
Pictures of Rectangular Prisms
Buildings and Ground Floor Plans

Ground Floor Plan 10 cm

Ground Floor Plan 10 cm

Ground Floor Plan 10 cm

Ground Floor Plan 10 cm

40 cm
25 cm

50 cm
70 cm

5 cm
15 cm
Unit Spinner

- Gallons to cups
- Fluid ounces to cups
- Cups to fluid ounces
- Cups to gallons
- Fluid ounces to gallons
- Gallons to fluid ounces

- Fraction
- Whole number
- Mixed number
**PS5-8 Making a Simpler Problem**

**Teach this lesson after:** 5.2 Unit 6

**Standards:** 5.NBT.B.6, 5.NBT.B.7, 5.NF.A.1, 5.NF.B.4

**Goals:**
Students will, when given a problem, make a simpler problem and use the solution to the simpler problem to help solve the original problem.

**Prior Knowledge Required:**
- Can do operations on decimals
- Can find the perimeter of a shape by adding the side lengths
- Can identify patterns in sequences that increase by the same amount
- Can write an expression for a given term in a pattern

**Vocabulary:** area, horizontal, length, perimeter, vertical

**Materials:**
- BLM Fraction Strips and Circles (p.Q-83, see Problem Bank 7)
- BLM Class Art Show (pp. Q-86–87, see Performance Task)

**Using a given simpler problem to help solve a harder problem.** Write on the board:

There are 300 people in line. How many people are after the 12th person?

ASK: What makes this problem hard? (300 is a big number) Would it be easier if I asked how many people are after the 299th person in line? (yes) SAY: So, it’s not exactly how big 300 is that makes this problem hard. ASK: Can you find a more precise way to say what makes it hard? (12 and 300 are far apart)

**Exercises:** Answer the question. Hint: Use subtraction.

a) There are 13 people in line. How many people are after the 12th person?

b) There are 5 people in line. How many people are after the 3rd person?

c) There are 300 people in line. How many people are after the 12th person?

**Bonus:** There are 3,459 people in line. How many people are after the 1,459th person?

**Answers:** a) 1; b) 2; c) 288; Bonus: 2,000

ASK: How did solving the easier problems make it easier to solve the harder problems? (it told me that the right approach is to subtract: number of people in line − position of the person in line)

**Off-by-one errors.** Tell students that you are waiting in line to get on a roller coaster. You are 37th in line and you see your friend, who is 7th in line. ASK: How many people are between us? (30)
Draw on the board:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>37</td>
</tr>
</tbody>
</table>

SAY: Thirty is a good guess because, on a number line, the length of the part of the line between 7 and 37 is 30. Add 8 to the number line. ASK: If you are 7th in line and I’m 8th in line, how many people are between us? (none) SAY: But eight minus seven is one, not 0, so although subtracting gets us close to the right answer, it’s not exactly right. Let’s try to figure out what is going on.

**Exercises:** How many people are in between? Hint: Use a number line to solve.

a) the 7th and 8th person  
b) the 7th and 9th person  
c) the 7th and 10th person  
d) the 7th and 11th person  
e) the 7th and 12th person  
f) the 7th and 37th person  

**Answers:** a) 0, b) 1, c) 2, d) 3, e) 4, f) 29

ASK: Did subtracting give exactly the right answer? (no) Did it give close to the right answer? (yes) How can you get the number of people between two people given their positions in line? (the number is off by 1, so I can find the difference between the positions and then subtract 1 more) How many people are between the 37th person and the 7th person? (29, one less than 30) How did starting with smaller numbers help? (it makes the problem easier) SAY: Sometimes, it’s easier to start by using smaller numbers than what is given in the problem. Then you will see things that help you solve the harder problem. Now that you know the pattern, you can find the number of people between any two positions.

**Exercises:** How many people are in line between?

a) the 8th and 78th person  
b) the 314th and 1,000th person  
c) the 492nd and 613th person  

**Answers:** a) 69, b) 685, c) 120

Write on the board:

A teacher tells her class to read pages 287 to 304 for homework. How many pages is that?

Have volunteers give you similar, simpler problems that you could solve first. (for example, make the numbers smaller) Write down all the volunteers’ suggestions on the board.
Exercises: Solve all the simpler problems on the board. Do you see a pattern in your answers?

Answers: In all cases, you can find the number of pages by subtracting the smaller number from the bigger number and then adding 1.

SAY: Now that you know the pattern, you can solve any problem of the same type.

Exercises: A teacher tells her class to read pages in a textbook for homework. How many pages of reading do the students need to do?

a) from 352 to 386
b) from 298 to 314
c) from 408 to 451

Answers: a) 35, b) 17, c) 44

Tell students that it can be helpful to examine the simpler problems in an organized way. Refer students to the problem about reading from pages 287 to 304. Write on the board:

<table>
<thead>
<tr>
<th>Read Pages</th>
<th>How Many Pages?</th>
</tr>
</thead>
<tbody>
<tr>
<td>287 to 288</td>
<td>2</td>
</tr>
<tr>
<td>287 to 289</td>
<td>3</td>
</tr>
<tr>
<td>287 to 290</td>
<td>4</td>
</tr>
<tr>
<td>287 to 291</td>
<td>5</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>287 to 304</td>
<td>?</td>
</tr>
</tbody>
</table>

SAY: By being organized, you might find the pattern sooner. Patterns can be easier to see when you have something organized to look at, like a table or a diagram.

Exercises: Make several simpler problems until you see the pattern to do the harder problem. Organize the simpler problems.

a) A fence is made using 42 posts, each 1 meter apart. How long is the fence?
b) A fence is made using 34 posts, each 2 meters apart. How long is the fence?

Answers: a) 41 m, b) 66 m

SAY: You can extend this type of problem to fences that go all the way around a field.

Exercises: A fence for a square yard is made with posts 1 m apart, including a post at each corner. How many posts are needed for a field that is …

a) 10 m by 10 m?  Hint: Start with a yard that is 1 m by 1 m and then move on to 2 m by 2 m, 3 m by 3 m, and so on.
b) 20 m by 20 m?

Answers: a) 40, b) 80

NOTE: For the following exercises, encourage students to predict the answer before checking.
Exercises: A square field is 20 m by 20 m. How many posts are needed if the posts are ...

a) 1 m apart?
b) 2 m apart?
c) 4 m apart?
d) 5 m apart?

Bonus: 40 cm apart?

Answers: a) 80, b) 40, c) 20, d) 16, Bonus: 200

Using whole numbers instead of decimals or fractions to make the problem easier. Draw on the board:

Tell students that you have two sticks. You know one stick’s length and the total length, but you want to know the length of the second stick. ASK: What makes this problem hard? (there are fractions) SAY: Let’s solve the problem approximately with whole numbers first. Erase the fraction part of the numbers on the board, as shown below:

ASK: Approximately how long is the second stick? (11) How did you get that? (17 – 6 = 11)

SAY: So you can do the problem the same way when the lengths include fractions. Instead of subtracting 17 – 6, you subtract 17 3/5 – 6 2/3. We will do it in steps by finding the fraction that takes 6 2/3 to the next whole number, then finding the difference in the whole numbers, and then finding the remaining fraction that takes us to 17 3/5. Draw on the board:

Have a volunteer demonstrate how to finish finding the answer, as shown below:

\[
\frac{1}{3} + 10 + \frac{3}{5} = \frac{5}{15} + 10 + \frac{9}{15} = 10 + \frac{14}{15} = 10 \frac{14}{15}
\]
Exercises: Find the missing length.

a) \[ \frac{2}{3} \quad \frac{4}{5} \]

b) \[ 17 \frac{3}{8} \]

Bonus:

\[ \frac{4}{3} \quad \frac{6}{4} \]

Answers: a) 7 1/12, b) 5 39/40, Bonus: 1 11/12

SAY: You can do the same thing with decimals.

Exercises: Find the missing length.

a) \[ 3.2 \quad 5.68 \]

b) \[ 15.34 \]

Bonus:

\[ 4.2 \quad 6.5 \]

Answers: a) 8.88, b) 2.58, Bonus: 2.7

Focusing only on relevant information to make a problem simpler. Remind students about the example of 6 2/3 + ? = 17 3/5 from earlier. SAY: I’m going to move these sticks around.
Draw on the board:

![Diagram of sticks]

ASK: How did I move the sticks? (slid one of them down) SAY: There’s a lot of extra information in this second problem, so it looks harder, but it actually has exactly the same answer as the other one. The total length of the two sticks at the bottom is still $17 \frac{3}{5}$, they are just not side by side anymore.

**Exercises:**

a) Find what the ? stands for by making the problem into a simpler problem.

i) 

![Simpler problem 1]

ii) 

![Simpler problem 2]

iii) 

![Simpler problem 3]

iv) 

![Simpler problem 4]

b) If the edge you are asked to find is vertical, bold all the vertical lines. If it is horizontal, bold all the horizontal lines. Then find what the ? stands for by making the problem into a simpler problem.

i) 

![Problem with vertical and horizontal lines]

ii) 

![Problem with vertical and horizontal lines]
Answers: a) i) 7, ii) 9 1/8, iii) 4.73, iv) 1.05; b) i) 4 7/12, ii) 4.45

Point out to students that by bolding the horizontal or vertical lines in part b), they changed the problem into a simpler problem by focusing only on the information that matters to the problem.

Remind students that to find the perimeter of a shape, we add up the lengths of all the sides.

Draw on the board:

![Diagram of a shape]

SAY: I want to find the perimeter of this shape. It looks like a hard problem, because there are a lot of missing side lengths. Ask a volunteer to mark three sides that you do not know the length of. (the two bottom horizontal sides and the right side) SAY: There are two kinds of sides in this shape: horizontal sides and vertical sides.

ASK: How long is the top side? (20) How long are the two bottom sides put together? (20) How do you know? (put together they are the same length as the top side) How long are the two sides on the left of the shape? (5 and 3) How long is the side on the right? (8) How do you know? (it’s the same as the two left sides put together) Write on the board:

\[
\text{Horizontal edges add to } \underline{\phantom{0000}} \quad \text{Vertical edges add to } \underline{\phantom{0000}}
\]

\[
\text{Perimeter is } \underline{\phantom{0000}} + \underline{\phantom{0000}} = \underline{\phantom{0000}}
\]

Have volunteers fill in the blanks. (40, 16, 40 + 16 = 56)

Exercises: Find the perimeter of the shape.

a) \[
\begin{array}{c}
3 \\
5 \\
13 \\
8
\end{array}
\]

b) \[
\begin{array}{c}
5.4 \\
4.6 \\
5.7 \\
5.8 \\
9.38 \\
8
\end{array}
\]

Answers: a) 48, b) 56.2

SAY: You can find the area of these shapes by dividing them into rectangles, finding the area of each rectangle, and adding them together.
**Exercises:**
1. Find the area of each shape from the previous exercises.
**Answers:** a) 128, b) 132.008

2. a) Find the perimeter.

```
3   8   1
  5
```

**Bonus:** Is there enough information to find the area of this shape? Explain.
**Answers:** a) 34; Bonus: no, because to find the area you need the measurements of each part

**Problem Bank**
1. When everyone in Tom’s class stands in line, Tom is 14th in line and 11th from the end of the line. How many people are in the class?
**Answer:** 24

2. There are 126 people in line. How many people are behind the 94th person?
**Answer:** 32

3. Make several simpler problems until you see how to do the harder problem.
   a) A fence is made using 53 posts, each 3 meters apart. How long is the fence?
   b) A fence is made using 61 posts, each 2.5 meters apart. How long is the fence?
   **Answers:** a) 156 m, b) 150 m

4. How many posts are needed to make the fence?
   a) A fence is 47 meters long with posts at 1 meter intervals.
   b) A fence is 100 meters long with posts at 2.5 meter intervals.
   c) A fence is 84 meters long with posts at 3.5 meter intervals.
   **Answers:** a) 48, b) 41, c) 25

5. A fence for a square garden is made with posts 1.5 m apart, including a post at each corner. How many posts are needed for the garden? Hint: Start with a yard that is 1.5 m by 1.5 m and then move on to 3 m by 3 m, 4.5 m by 4.5 m, and so on.
   a) The garden is 12 m by 12 m.
   b) The garden is 21 m by 21 m.
   **Answers:** a) 32, b) 56

6. Predict each answer before checking. A field is a square 30 m by 30 m. How many posts are needed if the posts are ...
   a) 1 m apart?  b) 2 m apart?
   c) 1.5 m apart?  d) 2.5 m apart?
   **Bonus:** 60 cm apart?
   **Answers:** a) 120, b) 60, c) 80, d) 48, Bonus: 200
7. Cut out the strips and circles from **BLM Fraction Strips and Circles** (you may cut the line down to the center of the circles). Estimate to color the given amount. Use folding to check your estimate.

a) one fifth of a strip of paper, starting from the left
b) two fifths of a strip of paper, starting from the left

Hint: Use your answers to parts a) and b) to help you determine a strategy for parts c) and d).

Hold the circle so that the cut line is at the top.

8. Each line segment is 1.2 meters long. What is the total length of this path?

![Path Diagram]

**Answer:** 42 m

9. a) Find the perimeter.

![Perimeter Diagram]

b) Is there enough information to find the area of this shape? Explain.

**Answers:** a) 36.6, b) no, we don’t have the side length for the small rectangles

10. All the measurements are in yards. Find the perimeter.

![Perimeter Diagram (in fractions)]

**Answer:** 53 1/30 or 53.03333 yards
11. a) A foot is 12 inches. Convert the measurement.
   i) 3 feet = ____ inches  
   ii) 10 feet = ____ inches  
      **Bonus:** 48 inches = ____ feet
b) Find the perimeter in inches.
   i) 
   ![Diagram](image1)
      26 in
   ii) 
   ![Diagram](image2)
      85 in
   iii) 
   ![Diagram](image3)
      4 \(\frac{3}{5}\) ft
   iv) 
   ![Diagram](image4)
      2 ft
   
   **Answers:** a) i) 36, ii) 120, Bonus: 4; b) i) 122 in, ii) 742 in, iii) 287.4 in, iv) 222.4 in

12. In the figures below, each square has a side length of 1.5 m.
   a) Complete the table for the figures.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   ![Figure](image)
   b) What is the perimeter of the 10th figure?
   c) Which figure has perimeter 72?
   **Answers:** a) 12, 18, 24; b) 60; c) 12th figure
Fraction Strips and Circles
Performance Task: Class Art Show

Materials:
BLM Class Art Show (pp. Q-86–87)

Performance Task: Class Art Show. Give students BLM Class Art Show. Tell students the context for the performance task: You are holding an event for parents to come and see posters created by your class. The posters will be hung in the school hallway. Tell students to use diagrams in their answers.

Answers: 1. a) 4 1/2 inches, b) 61 1/2 inches; 2. a) 20 1/2 inches, b) 7 posters
Class Art Show (1)

1. Each student makes a poster that is 11 inches tall and $8\frac{1}{2}$ inches wide. You put two pins in the poster 1 inch below the top of the poster and 1 inch from each edge:

![Diagram of a poster with pins at the corners and 1 inch below the top]

The center of the posters should be at eye level so that they can be seen easily. For adults, eye level is about 57 inches above the floor.

a) How high is each pinhole above the level of the center of the poster?

b) How high above the floor is each pinhole?
Class Art Show (2)

2. Two doors along the school hallway are 7 feet apart. Posters on the wall between the doors should be 3 inches apart from each other. The posters should also be centered so that the posters at the ends of the hallway are each the same distance from a door.

   a) If you put 4 posters between the two doors, how far will the posters nearest the doors be from the door’s edge?

   b) If the posters are 3 inches apart and the end posters are equal distances from the doors, what is the maximum number of posters you can hang on each wall?
PS5-9 Using Tape Diagrams

Teach this lesson after: 5.2 Unit 6

Standards: 5.NF.A.1, 5.NF.A.2, 5.NF.B.4a

Goals:
Students will use tape diagrams to solve multistep word problems involving all four operations and fractions.

Prior Knowledge Required:
Is familiar with tape diagrams
Can add and subtract fractions with like and unlike denominators

Materials:
BLM Swimming Pool (pp. Q-96–97, see Performance Task)

Vocabulary: tape diagram

(MP.1) Identifying parts of a diagram and solving a problem given the diagram. SAY: Marta had 35 dollars and spent 3/5 of her money on a shirt. We are going to represent this problem with a diagram. Draw a long rectangle divided into five equal parts. Ask a volunteer to shade the part of the diagram that represents the part that was spent on the shirt. (3 blocks) ASK: How much money does each block represent? ($7) How do you know? (there are 5 blocks and the total is $35, so one block is 35 ÷ 5 = $7) Finish the diagram as shown below:

$35

|$7 |

shirt

ASK: How much money did Marta spend on the shirt? ($21) How do you know? (3 × 7 = $21)
How much money does she have left? ($14) How do you know? (35 − 21 = $14; or 2 unshaded blocks, 2 × 7 = $14) Point out that, even though the first solution is correct, the second solution shows that you can find the leftover in the diagram without calculating how much money was spent on the shirt.

Exercises: Draw a diagram to solve the problem.

a) Jane had $36. She spent \( \frac{3}{4} \) of her money on a pair of shoes. How much money does she have left?

b) John spent \( \frac{2}{5} \) of his money on a toy. He has $15 left. How much did the toy cost?

c) Nancy spent \( \frac{2}{5} \) of her money on a poster that cost $8. How much money did she have before she bought the poster?
Solutions:

d)  $36

\[ \begin{array}{c}
\text{shoes} \\
\text{leftover} = 9
\end{array} \]

e)  $15

\[ \begin{array}{c}
\text{toy} \\
\text{= } 10
\end{array} \]

f)  total before = $20

\[ \begin{array}{c}
\text{poster}
\end{array} \]

SAY: Now we are going to solve the same kind of problem but with decimals.

Exercise: Kate spent $\frac{2}{5}$ of her money on a shirt and a hat. The shirt cost $18 and the hat cost $5.50. How much money did Kate have at first?

Solution: The shirt and hat together cost $23.50, so each block is $23.50 \div 2 = $11.75.

Five blocks together is $5 \times $11.75 = $58.75

total = $58.75

shirt + hat = $23.50

Solving problems where an amount is divided and then further divided into unequal parts. Write on the board:

Cam has some eggs. He uses $\frac{3}{7}$ of them to make pancakes and $\frac{1}{2}$ of the remainder to make sandwiches. Now Cam has 6 eggs left. How many eggs did Cam use to make pancakes? How many eggs did Cam have at first?

Point out that the first two sentences are similar to the problem you just did, so to solve this problem we can start with the same type of diagram as earlier. Draw on the board:

\[ \begin{array}{c}
\text{pancakes}
\end{array} \]

ASK: Which part of the diagram shows the leftover? (the unshaded squares) Cover up the shaded part and ASK: Which part is half of the leftover? (2 squares) Mark that on the diagram, as shown below:

\[ \begin{array}{c}
\text{pancakes} \\
\text{sandwiches}
\end{array} \]

ASK: How many eggs are left after he made the pancakes and sandwiches? (6) So how many eggs does each block represent? (3) How do you know? (6 ÷ 2 = 3) How many eggs did Cam
use for the pancakes? (9) How many eggs did Cam have initially? (21) How do you know? (7 blocks, \(7 \times 3 = 21\))

Write on the board:

Raj had 30 stickers. He gave \(\frac{2}{5}\) of his stickers to his brother and \(\frac{1}{2}\) of the rest to his friend. How many stickers did Raj’s brother get? How many stickers are left?

\[
\text{total stickers} = 30
\]

\[
\begin{array}{c}
\text{brother} \\
\end{array}
\]

ASK: How many blocks are there in total? (5) How many stickers does each block represent? (6) How do you know? (30 ÷ 5 = 6) How many blocks did Raj’s brother get? (2) So how many stickers did Raj’s brother get? (12) SAY: Raj’s friend got half of the rest. Draw a dashed line to show Raj’s friend’s stickers, as shown below:

\[
\begin{array}{c}
\text{total stickers} = 30 \\
\text{brother} \\
\text{friend}
\end{array}
\]

ASK: How many blocks did Raj’s friend get? (one and a half) How many stickers does each half block represent? (3) How do you know? (each block represents 6 so half represents 3) How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

Explain to students that it is sometimes easier to take away the first part of the question and then continue with the rest of the problem. SAY: At the beginning of the question, we know Raj has 30 stickers and he gave \(\frac{2}{5}\) of his stickers to his brother. Draw the original tape diagram on the board again:

\[
\text{total stickers} = 30
\]

\[
\begin{array}{c}
\text{brother}
\end{array}
\]

ASK: How many stickers does each block represent? (6) How many are left? (18) What fraction of the leftover goes to Raj’s friend? (half) Draw on the board:

\[
\begin{array}{c}
\text{stickers left} = 18 \\
\text{friend}
\end{array}
\]
Point to the new tape diagram and ASK: How many stickers does each block represent? (9)
How many stickers did Raj’s friend get? (9) How many stickers are left? (9)

**Exercises:** The next time Raj has stickers, he decides to give \( \frac{2}{5} \) of his 30 stickers to his brother and \( \frac{5}{6} \) of the remainder to his friend.

a) How many stickers did Raj’s friend get?
b) How many stickers are left?

**Solution:**

\[
\begin{align*}
\text{total stickers} &= 30 \\
\text{brother} &= 12 \\
\text{remainder} &= 18 \\
\text{friend} &= 15 \\
\text{left} &= 3
\end{align*}
\]

**Solving problems backward.** Write on the board:

Emma has some stickers. She colors \( \frac{1}{4} \) of them red and \( \frac{2}{5} \) of the remainder green.

If Emma doesn’t color 9 stickers, how many stickers does Emma have in total?

Explain to students that, like in the previous problem, they can draw tape diagrams, one for each step of the problem. Draw on the board:

\[
\begin{align*}
\text{total stickers} &= ? \\
\text{red} &= ? \\
\text{stickers left} &= ? \\
\text{sticker left} &= ? \\
\text{green} &= ? \\
\text{not colored} &= 9
\end{align*}
\]

SAY: In the diagram on the left, all parts are unknown. ASK: Can I start with the diagram on the left? (no) SAY: Look at the diagram on the right. ASK: How many stickers are not colored? (9)
How many stickers does each block represent? (3) How do you know? (9 ÷ 3 = 3) How many blocks are green? (2 × 3 = 6) Erase the question mark beside “green” and write “6” in its place, as shown below:

\[
\begin{align*}
\text{total stickers} &= ? \\
\text{red} &= ? \\
\text{stickers left} &= ? \\
\text{green} &= 6 \\
\text{not colored} &= 9
\end{align*}
\]

Point to the diagram on the right and ASK: How many stickers are shown here in total? (9 + 6 = 15)
So how many are left after Emma colors some red? (15) Erase the question mark beside “stickers left” in both diagrams and write “15” in its place, as shown below:

<table>
<thead>
<tr>
<th>Total Stickers</th>
<th>Stickers Left = 15</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
</tr>
<tr>
<td>Red = ?</td>
<td>Green = 6</td>
</tr>
<tr>
<td>Stickers Left = 15</td>
<td>Not Colored = 9</td>
</tr>
</tbody>
</table>

Ask a volunteer to solve the diagram on the left and find the total numbers of stickers. (20)

**Exercises:**

1. Mike spent \(\frac{3}{5}\) of his money on a book and \(\frac{3}{4}\) of the remainder on some music. Mike has $4 after he paid for the book and the music. How much money did Mike have initially?

   **Solution:**

   - Initial money = $40
   - Left after book = $16
   - Book = $24
   - Left after music = $4
   - Music = $12

   **Answer:** 18 ounces

2. Sal has 5 lb of flour. He uses \(\frac{3}{8}\) of it to make dumplings, and \(\frac{16}{25}\) of the rest to bake a loaf of bread. How many ounces of flour are left? Remember: 1 lb = 16 oz.

   **Answer:** 18 ounces

**Problem Bank**

1. Zara received some money for her birthday. She donated \(\frac{1}{5}\) to charity, and she saved \(\frac{2}{3}\) of the remainder in her savings. Of what was left, \(\frac{1}{4}\) was a gift card to an ice-cream store. She used the rest of the money to buy 3 books for $6.99 each, a T-shirt for $5.99, and a basketball for $7.99. How much money did she spend on her purchases? How much money was she given altogether? How much money was in each other part (donation, savings, gift card)?

   **Answers:** the books are $6.99 each, the T-shirt is $5.99, the basketball is $7.99, so $34.95 for purchases plus $11.65 for the ice cream parlor gift card; $93.20 went to savings; $34.95 was given to charity; total birthday gift was $174.75; Zara had $139.80 after making the donation and $46.60 after putting money in her savings

2. Kate reads 10 pages of a book on Saturday and she reads \(\frac{3}{4}\) of the rest of the book on Sunday. She still has 17 pages to read. How many pages are in the book?

   **Answer:** 78 pages
3. A convenience store has some ice cream treats. It sells \( \frac{2}{5} \) of them on Friday, \( \frac{1}{4} \) of the remainder of Saturday, and \( \frac{2}{3} \) of the rest on Sunday. The store has 30 ice cream treats left by the end of Sunday.

a) How many ice cream treats did the store have initially?

b) On which day did the store sell the most ice cream treats?

**Answers:** a) 200; b) Friday, 80
Performance Task: Swimming Pool

Materials:
BLM Swimming Pool (pp. Q-96–97)

(MP.1) Performance Task: Swimming Pool. Give students BLM Swimming Pool. Tell students that they will be doing a performance task involving a swimming pool and volume.

Answers: 1. 1,440 ft³; 2. 10,800 gallons; 3. 6.48 oz; 4. $0.50; 5. a) $12, b) $40, c) $16, d) 2 boxes
Swimming Pool (1)

Jay’s pool is in the shape of a rectangular prism. It is 24 feet long, 12 feet wide, and 5 feet deep.

1. Find the capacity of the pool in cubic feet.

2. Each cubic foot is about 7.5 gallons. About how many gallons of water are in the pool when it is full?

3. Jay disinfects the water in the pool every day by adding 0.0006 ounces of chlorine for each gallon of water in the pool. How many ounces of chlorine does Jay need to put in the pool when the pool is full?
Swimming Pool (2)

4. Each box of the chlorine weighs 1 pound and costs $8.00. How much does each ounce of the chlorine cost?

5. Jay spent \( \frac{2}{5} \) of his money at a pool store to buy some chlorine and half of the remainder on a pool toy. Jay has $12.00 after he paid for the chlorine and the pool toy.
   a) How much did Jay pay for the pool toy?

   b) How much money did Jay have when he went to the store?

   c) How much did Jay pay for the chlorine?

Unit 7  Geometry: 2-D Shapes

This Unit in Context

In Kindergarten, students learned how to identify basic shapes (circles, squares, triangles, and rectangles), regardless of their orientation or size (K.G.A.2), by focusing on attributes such as sides and corners (K.G.B.4). In Grade 1, students learned to distinguish between defining attributes and non-defining attributes of shapes, and they built and drew shapes (triangles, squares, rectangles, and circles) based on defining attributes (1.G.A.1). In Grade 2, students extended this work to include quadrilaterals, pentagons, hexagons, and cubes (2.G.A.1). Grade 3 students learned that different shapes can share attributes and that these shared attributes can define categories of shapes. For example, rectangles and rhombuses share the attribute of having four sides, and “having four sides” defines the category of “quadrilateral”—of which rectangles and rhombuses are two subcategories (3.G.A.1). In Grade 4, this work was extended further to include such properties as the number of right angles and the existence of parallel sides (4.G.A.2). In this unit, students will classify two-dimensional shapes in a hierarchy based on properties (5.G.B.4), such as “all squares are rhombuses,” “all rhombuses are parallelograms,” “all parallelograms are quadrilaterals,” and “all quadrilaterals are polygons.”

As students classify shapes based on their properties, they will need to measure angles (4.MD.C.5c), lengths of sides, and diagonals of polygons (4.MD.C.5c) to the nearest millimeter, both of which they learned to do in Grade 4. For both quadrilaterals and triangles, various properties concerning length of sides, size of angles, length of diagonals and angles between them, define the categories and hierarchies of a shape. For example, students will use their ability to measure angles when they learn that opposite angles in a parallelogram are equal and that rhombuses have perpendicular diagonals. Furthermore, students will learn that, because rhombuses are a subcategory of parallelograms, all the properties of parallelograms apply to rhombuses (5.G.B.3). In Grade 6, when finding areas of quadrilaterals (6.G.A.1), students will see how the general formulas for the area of shapes of a certain category also work for subcategories and transform when the special properties of subcategories apply. Students will use reasoning of the same sort in high school geometry when they use properties of isosceles triangles to make conclusions about equilateral triangles. For example, once students learn that the base angles of an isosceles triangle are equal, they can deduce that all angles of an equilateral triangle are equal (HSG.CO.C.10).
Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1, MP3, and MP5 to MP8. Here are some examples of how students can show that they have met a standard.

**MP3:** In G5-17 Extension 2, students construct a viable argument when they use the definition of a rhombus and a kite to explain why all rhombuses are kites, and when they analyze and critique the argument of a partner. In G5-12 Extension 2, students construct a viable argument when they explain why an equilateral triangle must be isosceles and acute, and when they analyze and critique the argument of a partner.

**MP5:** In G5-18 and 19 Extension 2, students use appropriate tools strategically when they recognize that, for example, connecting cubes are a good tool to make simpler versions of a problem, a T-table is a good tool to organize a search for a correct answer, and a pan balance would not be helpful in this situation.

**MP6:** In G5-10 Extension 2, students attend to precision when they measure their JUMP Math AP Book and other objects accurately, expressing their measurements with the appropriate units. In G5-17 Extension 2, students attend to precision when they use clear definitions to explain why a rhombus must be a kite.
Unit 7  Geometry: 2-D Shapes

Introduction

In this unit, students will classify shapes based on their properties. To find the length of sides and diagonals of polygons, students will need to measure distances to the closest millimeter. Review this skill before starting the unit.

Materials

Students will need to sort paper polygons in a variety of ways, in particular by making Venn diagrams. Cut a ball of yarn into pieces about 60 cm (2 ft) long, and tie them into loops. Students can use these yarn circles as group boundaries on Venn diagrams. You will also need to provide index cards or small pieces of paper to label the groups.

Students will also use protractors frequently in this unit. If commercial protractors are unavailable, photocopy BLM Protractors (p. R-53) onto a transparency and cut it into eight separate protractors.

Trapezoids and Parallelograms

Note that there are two possible definitions of a trapezoid:

• a quadrilateral with exactly one pair of parallel sides
• a quadrilateral with at least one pair of parallel sides

When using the first definition, parallelograms are different from trapezoids, but when using the second definition, all parallelograms are trapezoids. Both definitions are legitimate; we use the first definition.

Learning Geometric Terms

Most students find elementary geometry easy; what they find hard is all the new terminology. The most important terms are:

• side
• vertex, vertices
• right angle
• acute angle
• obtuse angle
• parallel
• perpendicular
• polygon
• triangle
• quadrilateral
• pentagon
• hexagon
• equilateral
• scalene
• isosceles
• rectangle
• square
• rhombus
• parallelogram
• trapezoid

Students won’t be able to use the relevant terms well if they can’t spell them. Therefore, we recommend that you teach the spelling of geometric terms during spelling lessons.

Another way to help students learn these terms is to use the cards on BLM Geometric Terms (pp. R-54–55). Have students match cards with the different representations of the same concept or play a memory game: a player lays out the cards face down and turns over pairs at random. If the pair matches, the player collects the pair. If not, the player turns the cards over.
**G5-6 Angles**

**Pages 144–145**

**STANDARDS**
preparation for 5.G.B.3, 5.G.B.4

**VOCABULARY**
angle arc arm endpoint line line segment point ray right angle rotation vertex vertices

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**Goals**
Students will identify and draw line segments, lines, rays, and angles.

**PRIOR KNOWLEDGE REQUIRED**
Can use a ruler to draw lines
Can identify right angles

**MATERIALS**
- rulers
- a large pair of scissors
- a smaller pair of scissors, with short blades
- chalk
- rectangular sheets of paper or index cards

**Introduce points and line segments.** Draw a dot on the board and SAY: In geometry, we deal with different objects. The object I drew is called a point. Draw a line segment on the board using a ruler. SAY: A line segment is a straight path between two points, called endpoints. We can measure the length of a line segment. We draw little dots at the ends of a line segment to mark the endpoints. Emphasize that, in geometry, it is important to draw pictures accurately and precisely, so when students are asked to draw something, they should use a ruler.

**Introduce lines.** Explain that a line extends in a straight path forever in two directions. It has no endpoints. We draw lines with little arrows at both ends to emphasize that we can extend them in both directions.

Draw a few lines and line segments on the board and have students identify each as either a line or a line segment. Students can signal thumbs outward if the object is a line and thumbs inward if it is a line segment. Have students draw a horizontal, a vertical, and a slanted (neither horizontal nor vertical) line segment. Make sure students draw little dots at the endpoints. Then ask students to draw a horizontal, a vertical, and a slanted line. Make sure students draw little arrows at the ends of the lines. Have students exchange their notebooks with a partner and extend each other’s lines in both directions.

Point out that we cannot measure the length of a line because every time we extend it, the length changes. We can measure the length of a line segment or a picture of a line, but not the length of a line itself.

**Introduce rays.** Ask students where they have heard the word “ray.” Point out that a ray, such as a ray of sunlight or another light, has a beginning but no end—it goes on and on. Explain that, in mathematics, a ray is part of a line (so it is straight, just like a ray in real life), and it has one endpoint...
(so it has a beginning, also like a ray in real life). A ray extends forever in the other direction from the endpoint. We draw a dot to show the endpoint and an arrow on the other end to show that it can be extended as much as needed. Draw a few rays pointing in different directions and have students identify which end can be extended. Students can point their thumbs in the direction of the end that can be extended.

Then ask students to draw three rays pointing in different directions, exchange their notebooks with a partner, and extend the rays their partners drew.

Draw a few lines, line segments, and rays on the board. Point to each in turn, and have students signal which is which. Students can point both thumbs outward to show a line, inward to show a line segment, and to one side to show a ray.

**Introduce angles.** Explain that, in mathematics, when two rays have a common endpoint, they make an **angle**. Draw two rays with a common endpoint. Explain that, when we deal with angles, the common endpoint of the rays is very easy to see, so there is no need to draw a dot to show it. Draw a few pictures of pairs of rays as in **Question 3** on AP Book 5.2 p. 144, and have students signal thumbs up if the picture shows an angle and thumbs down if it does not. Avoid pictures like the one in the margin.

Explain that the rays are called the **arms** of the angle, and the common endpoint is called the **vertex** of the angle. The plural of **vertex** is **vertices**. Have students draw a few angles, exchange notebooks, and identify the vertex and extend the arms of the angles drawn by their partners.

**The size of an angle is the amount of rotation between the arms.** Explain that, in mathematics, the size of an angle measures how much you need to turn one arm to get the other arm. In other words, the size of an angle is measured in terms of the rotation between the angle’s arms. Show a pair of scissors. Point out that the blades of the scissors rotate around a peg. Hold the scissors so that one blade is horizontal at all times. Open the scissors a little bit and then open them more and more to show how the angle increases as the top blade rotates away from the horizontal blade. Trace a small angle on the board and open the scissors wider than that angle; then show how the smaller angle on the board “fits inside” the larger angle of the open scissors. Emphasize that the more you open the scissors, the more you turn the blades, and the larger the angle becomes.

Draw the following picture (without the arcs) to illustrate what you mean by smaller and larger angles.

**SAY:** When the angle is small, the arms are only open a little bit. When the angle is large, the arms are open a lot. Explain that we draw an **arc** (a part
of a circle around the vertex of an angle) to show how much the arms open or how much we turn one arm to get to the other. Add the arcs to the picture.

**ACTIVITY 1**

Moving to compare and order angles. Have students work in pairs on the school playground. Each student draws two angles in chalk on the pavement so that one angle is smaller than the other. Partners mark their angles with different letters (A, B, C, D) so that they can order them later. Students stand on the vertex of one of their angles, lock their hands together, and stretch both arms out in front of them in line with one of the arms of the angle (with their arms pointing in the direction of the angle arm). Students turn, or rotate, their bodies to line up both arms with the other arm of the angle. Repeat with a different angle. ASK: For which angle did you need to turn more?

Remind students that the size of the angle is the amount of rotation from one arm to the other, so the angle that required more rotation (a larger turn) is the larger angle. Students individually compare their angles and then order the four angles they produced in pairs from smallest to largest.

Size of the angle does not depend on the length of the arms in the picture of the angle. Draw the first pair of angles in the margin and explain that these angles are the same size, even though one of them looks larger. In other words, you need to turn one arm the same amount of rotation to get to the other arm. You can demonstrate this with two pairs of scissors, one with long blades and one with short blades; show that you open the scissors the same amount to show both angles. Point out that, just as we do not change a ray by extending it, we do not change an angle by extending its arms. Extend the arms on the smaller picture to show that the angles are exactly the same.

Now draw the second pair of angles in the margin. ASK: Which angle requires you to turn more from one arm to the other? Which angle is larger? (the one on the left) Explain that even though the arc in the angle on the right looks larger, this does not mean that the angle is larger. Emphasize that the sides of the angle are rays, so they can be extended, and that does not change the angle. Have a volunteer extend the arms of the angle on the left. Students should clearly see that the angle on the left is larger. Repeat the discussion with the third pair of angles in the margin.

Draw several pairs of angles, one angle at a time, on the board. For each pair, have students decide which angle is larger, the angle on the left or the angle on the right.

**Exercises:** Point your thumbs toward the larger angle.

a)  

b)  

c)  

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R-4  

Teacher Resource for Grade 5
NOTE: Students often mistake the area between the arms of an angle for the size of the angle. To help students who have this misconception, remind them that the angle is how much we need to turn from one arm of the angle to the other. Have them place a pencil on one of the arms of the angle and rotate it to get to the other arm. Repeat with the second angle. The angle that needs more rotation is the larger angle.

Review right angles. Remind students that square corners are called right angles in mathematics. To check whether an angle is a right angle, compare it to a corner of a regular sheet of paper. If an angle is not a right angle, we can tell whether it is larger than a right angle or smaller than a right angle. Have students compare a few angles to a corner of a sheet of paper and tell whether the angles are larger or smaller than a right angle.

ACTIVITY 2

Students use an object with a square corner (such as an index card or a rectangular sheet of paper) to check whether objects in the classroom have angles greater than, smaller than, or equal to a right angle. Possible angles to check include: corners of a desk, window corners, corners of base ten materials and pattern blocks, angles made by a door and a wall.

Extension

Sometimes a ray can be part of a different ray. Draw a ray in black and mark a red point on it. Now there are two overlapping rays in the picture: one ray has a black endpoint and the other ray has a red endpoint (see example in the margin). The ray with the red endpoint is part of the other ray. Add a blue point between the red point and the black point. ASK: Is this point on the ray with the black endpoint? (yes) Is it on the ray with the red endpoint? (no)

Draw the picture in the margin on the board. Explain that a student you know thinks that the solid line is part of the dashed line. Is that correct? (no) Why not? (a line has no endpoint, so the parts of the line that have different arrows are in fact the same line)
### G5-7  Sides and Vertices of 2-D Shapes
**Pages 146–147**

#### STANDARDS
preparation for 5.G.B.3, 5.G.B.4

#### VOCABULARY
- endpoints
- hexagon
- line segment
- pentagon
- **polygon**
- quadrilateral
- **side**
- three-dimensional, 3-D
- **triangle**
- two-dimensional, 2-D
- **vertex**, vertices

#### Goals
Students will identify polygons, sides, and vertices and will classify polygons according to the number of sides.

#### PRIOR KNOWLEDGE REQUIRED
Can identify a line segment

#### MATERIALS
- various polygons from *BLM Polygons* (pp. R-56–57)
- cube
- geoboards (optional)
- scissors

**Introduce sides and vertices.** Draw and label a polygon with the words **sides** and **vertex/vertices**. Remind students of what a side and vertex are. Show students how to count sides (marking each side as you count) and vertices (circling each as you count). Draw several more polygons and have students count the sides and the vertices of several polygons. Be sure that all students are counting sides and vertices properly so they don’t miss or count any twice. Ask students if they can see a pattern between the number of vertices and the number of sides.

**Polygons.** Explain that flat shapes such as squares, triangles, and circles are called **two-dimensional**, or 2-D for short. That is because they have two dimensions: length and width. Shapes such as cubes (show a cube) have length, width, and height, so they have three dimensions and are called **three-dimensional** or 3-D.

Divide the board in two. On one side, draw a variety of polygons, including both regular polygons (shapes with equal sides and equal angles) and irregular polygons. Label the shapes “Polygons”. Explain that **polygons** are 2-D shapes with sides that are line segments, so the sides are straight. Sides of polygons only meet at vertices, and exactly two sides meet at each vertex. Sides of polygons do not intersect other than at endpoints. Explain that “poly” means *many*, and “-gon” means *angle* or *vertex*. Then ask students what the word “polygon” might mean. (many vertices)

Label the other side of the board “Not Polygons.” Draw a pentagon with a little part of one side missing on this side and ASK: How is this different from the polygons? (there are two vertices where there is only one side) Repeat with the other examples in the margin.
Draw several more shapes and ask students to decide whether each is a polygon or not. Include shapes that have indentations (see example below). Have students point to the correct side of the board to indicate whether a shape is a polygon or not.

![indentation]

**ACTIVITY**

**Sorting polygons by the number of sides.** Give students copies of BLM Polygons. Students count the sides or the polygons and sort the shapes by the number of sides. (Help students who are struggling by providing group names, such as 3 sides, 4 sides, 5 sides, 6 sides, more than 6 sides.) After giving students time to work, make a chart on the board with columns labeled according to the number of sides and display larger copies of the same shapes. Students show in which column each shape belongs by raising the number of fingers equal to the number of sides.

**Introduce the names of polygons.** Explain that mathematicians give special names to polygons according to the number of sides and vertices they have. Present the names (see vocabulary) on cards, explain each one (e.g., triangles have three straight sides, quadrilaterals have four straight sides, pentagons have five straight sides, hexagons have six straight sides), and invite volunteers to assign each card as the label for a column. Students may notice that the column for shapes with four sides, labeled “quadrilateral,” includes shapes that they would call “squares” and “rectangles.” Explain that, just as more than one word can be used to describe a student (e.g., boy/girl, child, person), many shapes have more than one name. Tell students that squares and rectangles are special types of quadrilaterals.

Draw a pentagon on the board. ASK: How many sides does this polygon have? What is the name for this polygon? How do you know that this is a pentagon? If students do not say that it has five sides and five vertices and all sides are straight, draw another pentagon (different size, color, or pattern) and ask what the two pentagons have in common. Repeat with a triangle and a quadrilateral that is neither a rectangle nor a square (e.g., a parallelogram, rhombus, or trapezoid).

Draw a group of shapes on the board, label them with letters, and ask students to count the sides and sort the shapes into the table below. Include shapes with curved sides, such as a quarter of a circle, which should be sorted into the “other” column.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Quadrilateral</th>
<th>Pentagon</th>
<th>Hexagon</th>
<th>Other</th>
</tr>
</thead>
</table>

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Geometry 5-7

R-7
**Exercise:** On grid paper, draw two different polygons of the given type. Explain how the polygons you drew are different. **NOTE:** This exercise can also be done on a geoboard.

a) triangle b) quadrilateral c) pentagon d) hexagon

**NOTE:** This exercise allows you to see students’ level of mathematical thinking. Students who describe shapes in non-geometrical terms (such as “This triangle is more pointy” or “This shape is turned on the side”) need to engage in sorting shapes by their geometric properties. Have them start from actual shapes, as opposed to pictures of shapes, to understand that the orientation or position of a shape does not matter. Draw their attention to the geometric properties and language as the unit progresses. Repeat this exercise several times throughout the unit to track progress.

**Exercises:** Draw a figure that is not a polygon with the given features:

a) 3 sides b) 4 vertices c) 2 sides d) 0 vertices

**Bonus**

e) 2 curved sides and 3 straight sides
f) 2 straight sides and 3 curved sides

**Sample answers**

a) ![Shape 1]
   b) ![Shape 2]
   c) ![Shape 3]
   d) ![Shape 4]

**Bonus**

e) ![Shape 5]
   f) ![Shape 6]

Have students who need additional practice do the questions below.

**Exercises**

a) Draw a polygon with 7 sides.
b) Draw a quadrilateral. How many vertices does it have?
c) Draw a shape that is not a polygon and explain why it is not a polygon.

**Sample answers**

a) ![Shape 7]
   b) 4 vertices
   c) This shape has one curved side and no vertices.
Extensions

1. Draw a large polygon on a sheet of paper and cut it out. Count the sides of the paper polygon. Count the vertices. Cut off one of the vertices. Count the sides and vertices again. Cut off another vertex. Repeat the count. Do you notice a pattern?

   **Answer:** Every time you cut off a vertex, you create two new vertices joined by a new edge. The total number of sides increases by one and the total number of vertices also increases by one.

2. Is it possible to draw a polygon with exactly two sides? Discuss in pairs why this is impossible. Debrief as a class.

   **Sample answer:** Two straight sides can meet at a common vertex but they won’t meet at the other ends. Those ends will be loose ends. As a result, polygons need three or more straight sides.
**Goals**

Students will measure and create angles using protractors.

**PRIOR KNOWLEDGE REQUIRED**

- Can draw and identify rays
- Can identify right angles
- Knows that the size of an angle is a measure of rotation from one arm to the other
- Understands the concept of measurement

**MATERIALS**

- protractors
- rulers

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**Review comparing angles and size of angles.** Remind students that the size of an angle is how much one arm turns to get to the other arm. You can easily extend the arms of an angle (because they are rays), so the length of the arms in the picture does not matter.

**Exercises:** Point your thumb toward the larger angle in the pair.

a)  

b)  

c)  

Ask volunteers to justify their answer for each pair of angles in the exercise above.

Present the pair of angles in the margin (which have the angles 30° and 35°, but do not provide the measurements), and ask students which angle is larger. Have students discuss in pairs and then in groups of four how to check which angle is larger. The idea of measuring the angles is likely to arise.

**Degrees.** ASK: How can we measure how much something turns? With your arms stretched forward and hands locked together, spin your body around a full turn and ask students to describe what you did. Show a half turn and then a quarter turn. Refer to the angles you drew on the board. ASK: Can you tell what part of a turn each of the angles is just by looking at them? (no) Even if you created those angles on the floor (with tape, for example), it is hard to compare angles by turning your body, especially if they are close in size.
Explain that a *degree* is a unit for measuring angles. Have students look at the picture of a one-degree angle on AP Book 5.2 p. 148 and discuss why the arms are drawn so long. (otherwise it would be hard to see the opening between the arms) Explain that a right angle measures 90 degrees. Have students complete Question 1 on AP Book 5.2 p. 148.

**Introduce the notation for degrees.** Explain that writing the word "degrees" takes time, so people often use a symbol instead. The symbol is a small raised circle that is written after the number. For example, the measure of a right angle is written as 90°. Have students write the angle measures in Question 1 using the degree symbol (°).

**Introduce acute and obtuse angles.** Explain that angles that are smaller than a right angle are called *acute angles*, and angles that are larger than a right angle are called *obtuse angles*. You might point out the connection with sharp and blunt corners: *acute* is a synonym for sharp, and *obtuse* is a synonym for blunt. Draw a few angles on the board and have students identify them as acute or obtuse. Students can signal their answers by making the letters A or O with their hands.

Point out that when measuring length, if a pencil is longer than an eraser, the measurement for the pencil will be larger than the measurement for an eraser. Draw a red right angle, a blue acute angle, and a black obtuse angle on the board. ASK: Which angle is the largest? (the black angle) Is it an acute angle, an obtuse angle, or a right angle? (obtuse) Which angle is the smallest? (the blue angle) Is it an acute angle, an obtuse angle, or a right angle? (acute) Which angle is the right angle? (the red angle) Explain that since the red angle is larger than the blue angle, this means that the degree measure of the red angle will be larger than the degree measure of the blue angle.

ASK: How does an acute angle compare to a right angle? (the acute angle is smaller than the right angle—less than 90°) How does an obtuse angle compare to a right angle? (the obtuse angle is larger—greater than 90° but less than 180°) Write a few angle measures (such as 89°, 34°, 142°, etc.) and have students signal whether these angles are acute or obtuse. Students can signal thumbs up for obtuse and thumbs down for acute, or they can make the letters A and O with their hands. Repeat with angles drawn on the board. For each angle, have students also say whether they expect the measure of each angle to be more than 90° or less than 90°.

**Introduce protractors.** Draw a pair of line segments of almost identical length, one horizontal and one vertical. Ask students how they would tell which one is longer. (measure them with a ruler) In this case, we need a tool to measure and compare line segments, and the tool we use is a ruler. When angles are similarly positioned in different orientations, we also need a tool to measure and compare them. Tell students that the tool for measuring angles is called a protractor.
Two scales of a protractor. Have students examine their protractors and say how they are similar to rulers and how they are different. Draw attention to the fact that a protractor has two scales, while a ruler often has only one scale. (If students have rulers with two scales on them, they should note that the units in each scale are different, whereas the units in each scale on the protractor are the same.) Explain that having two identical scales going in different directions allows you to measure the angles from both sides, but this also means that you need to decide which scale you will use each time.

Display the picture below on the left and explain that this protractor is like the protractor in Question 5 on AP Book 5.2 p. 149. ASK: Is this angle an acute angle or an obtuse angle? (acute) Circle the numbers that the arm of the angle passes through. Which one is the answer? (30°) How do you know? (the angle is acute, so the answer must be less than 90°) Repeat with the second picture below. Then have students complete Questions 5 and 6 on AP Book 5.2 pp. 149–150.

Point out that there is another way to check that you are using the correct scale. The correct scale starts with 0 on the arm of the angle that lines up with the base line of the protractor.

Placing protractors on angles. Point out the base line and the origin on a large protractor or on a picture of a protractor on the board. Have students find the base line and the origin on their protractors. Demonstrate how to place a protractor correctly, so that the base line lines up with one arm of the angle and the origin is at the vertex. Point out that this is similar to placing a ruler with the 0 at the beginning of the object you are measuring. Have students draw an acute angle in their notebooks and ask them to place their protractors correctly. Circulate in the classroom to check that all students have done so. Then have students measure the angle they drew. Repeat with an obtuse angle. Have students exchange notebooks with a partner and measure each other’s angles to check their work.

Extensions

1. a) Find two letters of the alphabet that have four right angles.

   b) Find a letter of the alphabet that has three right angles.

   c) Find at least three letters that have an acute angle. Mark the acute angles with an arc.

   d) Find three letters that have an obtuse angle. Mark the obtuse angles with a double arc.

2. a) Use a ruler to draw a square on grid paper. Then draw the line segments joining the opposite vertices of the square. These line segments are called **diagonals**. Label the point where the diagonals intersect P. See example in the margin.

   b) Measure the angles around the point P. What do you notice?

   **Answer:** b) All the angles are right angles.

3. Use tracing paper to compare angles. Follow these steps to create the tracing of an angle (see example in the margin), particularly when the arms are faint and not easily visible:

   a) Mark the vertex of the angle you want to copy with a dot and write “v” beside it.

   b) Put a bold dot on each of the arms and write “a” beside it.

   c) Place the tracing paper over the angle so that it covers all the dots. Can you see the dots through the paper?

   d) Copy the dots and the letters onto the tracing paper.

   e) Use a ruler to join the dot labeled “v” to each dot labeled “a.” Do not join the dots labeled “a” to each other!

Have students draw an angle and exchange notebooks with partners, create a tracing of their partner’s angle, then compare the angles using the tracing. Remind students to place the tracing on the angle they are comparing it to so that the vertices and one arm of both angles match exactly.
STANDARDS
preparation for 5.G.B.3, 5.G.B.4

VOCABULARY
acute angle
angle
arc
arm
base line
obtuse angle
ray
right angle
scale
side
vertex

Goals
Students will measure angles in geometric shapes and draw angles using a protractor.

PRIOR KNOWLEDGE REQUIRED
Can use a ruler to draw rays
Can identify right, acute, and obtuse angles

MATERIALS
protractors
rulers
pair of dice

Review measuring angles with a protractor. Remind students that acute angles measure less than 90°, and obtuse angles measure between 90° and 180°. Draw a few angles on the board and ask students to identify them as acute or obtuse. Students can signal their answers by making an “A” or an “O” with their fingers. Then ask students to sketch an example of an acute and an obtuse angle.

Make sure every student knows how to choose the correct scale on the protractor when measuring an angle. They can decide whether the angle is obtuse or acute and choose the measure that is respectively larger than 90° or smaller than 90°. Or they can use the scale where the 0 aligns with one of the arms of the angle.

Have students draw one acute angle and one obtuse angle in a partner’s notebook, exchange notebooks, and then measure the angles their partners drew.

Angles in a polygon. Draw a triangle on the board. Explain to students that we can measure angles inside the shapes. The measure of an angle inside the shape is the amount of rotation between one arm (one side of the shape) and another arm (a different side of the shape). Draw an arc to emphasize the angle between the sides. Have students draw a large triangle in their notebooks, measure all three angles in the triangle, then exchange notebooks and check their partner’s answers.

Point out that the sides of a shape are often too short to conveniently measure the angle. But the arms of an angle are actually rays starting at the vertex of the shape, so you can extend these rays. Extend two sides of the triangle you drew on the board. Point out that the triangle is inside the angle—it is part of the space between the arms. Have a volunteer measure the angle.
Display a pre-drawn grid on the board and draw a triangle as shown in the margin. Have students copy it into their notebooks. Then ask them to extend the arms and measure the angles in the triangle. (52°, 64°, 64°)

**Drawing angles.** Model drawing angles step by step (see Questions 4 and 5 on AP Book 5.2 p. 152). Emphasize the correct positioning of the protractor. Have students complete Questions 4 and 5 and check that they use protractors correctly. You can use Question 4 for diagnostic assessment. The following table shows the correct answers and how the wrong answers correspond to common mistakes in placing the protractor:

<table>
<thead>
<tr>
<th></th>
<th>4. a)</th>
<th>4. b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Error Description</th>
<th>4. a)</th>
<th>4. b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrong: the base line of the protractor is not aligned; it is rotated.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Wrong: the arm is not aligned with the base line; students have aligned the arm with the bottom edge of the protractor instead.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Wrong: the wrong scale was chosen.</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

Have students practice drawing angles starting with measures that are multiples of 10, such as 20°, 50°, and 140°. Continue to measures that are multiples of 5, such as 35° and 95°, and then all measures in between, such as 47° and 121°. The Activity below also provides practice drawing angles.

**ACTIVITY**

Students will need a ruler, pair of dice, a protractor, a sheet of paper, and a pencil. Draw a ray on the paper. Roll the dice and draw an angle with the measure given by the numbers on the dice combined. Use the ray as one of the arms and draw the angle counter-clockwise. Label your angle with its degree measure. For each next roll, draw a new angle in a counter-clockwise direction and using the arm drawn on the previous roll as the new base line. The measure of the new angle is the sum of the result of the dice and the measure of the angle on the previous roll. Stop when there is no room to draw an angle of the size given by the roll. For example, if the first three rolls are $1 + 3 = 4$, $2 + 3 = 5$ added to 4 to give 9, and $1 + 2 = 3$ added to 9 to give 12, the picture will be as shown in the margin.
Extensions

1. Draw a triangle by following these steps:
   a) Draw a base line segment of the given length.
   b) At the endpoint on the left, rotate counter-clockwise to draw a second line at the given angle.
   c) At the endpoint on the right, rotate clockwise to draw a third line at the given angle.

<table>
<thead>
<tr>
<th>Base line segment length</th>
<th>Angle at left</th>
<th>Angle at right</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) 5 cm</td>
<td>40°</td>
<td>70°</td>
</tr>
<tr>
<td>ii) 8 cm</td>
<td>55°</td>
<td>45°</td>
</tr>
<tr>
<td>iii) 3 cm</td>
<td>50°</td>
<td>120°</td>
</tr>
</tbody>
</table>

2. A square has 4 equal sides and 4 right angles. Draw a square using a ruler and a protractor.
Introduce the terms “group” and “property.” Explain that when you sort objects or shapes that have the same common feature, these objects together make a group. All objects in a group have the feature, and the objects outside the group do not. The common feature is called a property. For example, being a toy or being red might be a property of some objects. Being a triangle is a property of shapes.

Sorting geometric shapes. Draw several shapes and label them with letters:

A.  
B.  
C.  
D.  
E.  
F.  

Then draw the diagram in the margin on the board. Explain that you want to sort the pictures you drew using a diagram, but you need a title for the diagram. All the pictures should be marked on the diagram, so you need a word that describes the six pictures you have drawn. One such word is shapes. Explain that the oval could show a group of all triangles. ASK: Which shapes belong inside the group? (C, E, F) Which shapes belong outside the group? (A, B, D) Explain that everything in the diagram is a shape, but the shapes noted in the diagram as a group are all triangles.

Exercise: Sort the shapes above. Signal thumbs up if the shape is inside the group and thumbs down if the shape is outside the group.
a) Shapes
   Polygons

b) Shapes
   Pentagons

**Answers:**
a) inside: A, C, D, E, F; b) all shapes outside the group

**ASK:** Why is the group empty when it’s labeled “pentagons”? (there are no pentagons in the collection of shapes)

**ASK:** How else could you group the shapes? (dark shapes inside the group, light shapes outside the group; circles inside the group, everything else outside the group; etc.)

**ASK:** What other property could you use to label the group so that it remains empty? (dark circles, shapes with two obtuse angles, etc.) Remind students that the group label should reflect the fact that the entire collection consists of shapes, so “rockets” isn’t a good choice even though the oval would still be empty. Keep the shapes on the board for later use.

**ACTIVITY**

Use masking tape to create a large circle on the floor. Have students who are 11 years old stand inside the circle. Label the circle “11 years old.” Repeat with several examples such as wearing yellow, girl, wearing blue, or takes the school bus. Have students stand inside the circle when the property applies to them and outside the circle when it does not.

Introduce Venn diagrams. Clearly label one yarn circle “pens” and another “blue,” and place the circles side by side without overlapping. Ask students to assign several colored pens (black and red) and pencils (blue, red, and yellow) to the proper position—inside one of the groups or outside both of them. Then show a blue pen and explain that it belongs in both groups. Allow students to problem-solve a way to move the yarn circles so that the blue pen is placed in both groups at the same time. Have a volunteer move the yarn circles. Students might suggest overlapping the circles, placing one circle within the other, or placing the circles side by side. Note that overlapping is correct. If students do not overlap circles, show them the implications of their ideas (for example, if students place one circle within another, what will they do with a pen of a different color?) Have students adjust the “pens” and “blue” circles so that they overlap.

Return to the collection of shapes used previously. Then draw the diagram in the margin. Explain that this picture is called a Venn diagram. **ASK:** Why do the two groups overlap? Have a volunteer shade the overlapping region of the ovals and ask the class which letters go in that region. (C, F) Have a second volunteer shade the area outside both groups. **ASK:** Which letters go in this region? (A, B) Point out that A and B are shapes that are neither dark, nor triangles. Finally, ask students where to place the remaining two shapes. (D dark, E triangles)
**Exercises:** Show the region in which to put each shape (shapes A to F, on page R-17) with your thumbs. Point your thumbs inward for the overlap between the groups, a thumb to the left or right for one group or the other, and thumbs down for outside both groups.

a) Group 1: Light; Group 2: Quadrilaterals
b) Group 1: Polygons; Group 2: Triangles
c) Group 1: Polygons; Group 2: Circles

**Answers**

- **Shapes**
  - **a)**
    - Shapes
    - **Light**
    - **Quadrilaterals**
    - **b)**
    - Shapes
    - **Polygons**
    - **Triangles**
  
**Empty regions in Venn diagrams.** Point out that, in some diagrams, some regions stay empty. Look together at the diagram for c). **ASK:** How many empty regions are there? (2: the overlapping region for polygons and circles, and the area outside the ovals) How can you describe the shapes that should be outside both groups? (not circles and not polygons) Ask students to draw a shape that is not a polygon, but not a circle. See examples in the margin. **ASK:** Can you think of a shape that would be in the central region of the Venn diagram? What properties should it have? (it should be a polygon and a circle at the same time) Have students think for some time to arrive at the conclusion that a shape like that does not exist. Ask them to explain why there cannot be a shape that is both a polygon and a circle. (a polygon has only straight sides, a circle has only one curved side; a polygon has vertices, a circle has no vertices; etc.)

**Sorting numbers into Venn diagrams.** Draw a Venn diagram on the board with the title “Numbers” and the groups “Even” and “Greater than 7.” Have students sort the numbers 2, 3, 18, and 29 into the diagram. Then **ASK:** Where would you place the number 6.8? Is it greater than 7? (no) Remind students that only whole numbers can be described as even and odd; decimals and fractions are neither even nor odd. Where should 6.8 be in the Venn diagram? (outside both circles) Add 6.8 to the diagram and have students add another number to each region of the Venn diagram. Change the properties to “Even” and “Odd,” and have students re-sort the numbers. Is there an empty region? (yes) Which region is empty? (the part inside both groups) Have students try to add a number to each region,
including the empty region. ASK: Is there a region that you cannot find a number to add to? (yes) Why does this happen? (there are no numbers that are both even and odd)

**Sorting polygons using geometric properties.** Review acute, right, and obtuse angles. Draw a right trapezoid and have students identify how many angles of each type it has. (one acute, one obtuse, two right)

Have students cut out copies of the shapes from **BLM Polygons for Venn Diagrams**. They will not use the circle on the BLM at this point, but they will need it later in the lesson. Students can use yarn circles to act as group boundaries on the Venn diagram and index cards to label properties and the title. Have students create a Venn diagram titled “Polygons” with properties “at least one obtuse angle” and “quadrilateral.” Draw the same diagram on the board (see example in the margin). Discuss the meaning of the label for the first group. If a shape has one obtuse angle, does it belong to the group? (yes) If it has two obtuse angles? (yes) Which shapes do not belong in the group? (shapes without obtuse angles)

Go through the shapes below one by one and have students place them in the correct region.

Create a new Venn diagram by repeating the task but changing “at least one obtuse angle” to “exactly one obtuse angle.” See the answer in the margin.

**Introduce diagrams with groups inside groups.** Remind students that earlier in the lesson they sorted the shapes using the groups “polygons” and “triangles.” ASK: Were there empty regions on that diagram? (yes) Can there be a polygon that is not a triangle? (yes) Have students give examples of such polygons. ASK: Can there be a triangle that is not a polygon (no) Why not? (triangles are special type of polygons; any triangle is a polygon) Explain that, in this case, it makes more sense to draw the Venn diagram so there is no empty region, placing the oval for triangles completely inside the oval for polygons. You can also make a rounded rectangle instead of a circle. See example below. Draw the Venn diagram on the board, leaving room to add more groups later.

From **BLM Polygons for Venn Diagrams**, add a circle (H) to the collection of shapes used earlier and have students sort the shapes again.
SAY: Some Venn diagrams have more than two groups. I want to add “quadrilaterals” to the Venn diagram. Where should it be? (inside “polygons”) Should it overlap with “triangles”? (no) Why not? (there are no triangles that are also quadrilaterals) Add the group “quadrilaterals” to the Venn diagram, and then have students move the shapes that need to be placed into the new group. Repeat with “pentagons.” See answer below.

![Venn diagram]

ASK: Which region is empty? (inside “polygons” but outside the three smaller groups—Triangles, Quadrilaterals, and Pentagons) Have a volunteer shade that region. Then ask students to produce a shape that would fit in that region. (a polygon with more than five sides, such as a hexagon) Finally, draw a shape with four sides with some curved sides (see example in margin). Ask students to identify where this shape should be placed and explain their reasoning to each other. (outside “polygons” because it has curved sides, and so is not a polygon)

**Bonus:** Draw another shape in every region of the last Venn diagram.

**Extensions**

1. This lesson provides a good opportunity for you to tie in ideas learned in other subjects. For example, have students sort words in different ways, such as by their first or last letters, their initial sounds, the number of syllables, what each word rhymes with, city names starting with …, state names, food groups, etc. Start with four words and then add four more words to be sorted into the same groups. Encourage students to suggest words and their placement in the Venn diagram.

2. a) Find something that has a volume of about one third the volume of your JUMP Math AP Book.

b) What tools did you use? Explain your choice.

**Sample solutions**

b) • I used a ruler to measure the length, width, and height of my book and of the object. I measured to the nearest millimeter to make sure I was accurate.
• I used a ruler to measure the length, width, and height of my book, but the object I picked wasn’t a rectangular prism, so I put my object in a graduated cylinder with water in it to see how much the water level rose. I converted from mL to cm³ to calculate the volume of the object.
Introduce hash marks for equal sides. Remind students that in the previous lesson, they classified polygons by the number of sides. Draw the two pentagons shown in the margin and explain that you need a way to describe how these two polygons are different. Is the number of sides going to help you with that? (no, they are both pentagons) The number of vertices? The number of right angles? (no, they both have two right angles) Have students try to describe how the shapes are different, and lead them to the idea that one of the shapes has pairs of equal sides, but the other does not.

Explain that sometimes you need to know that two sides are equal even though you do not need to know the exact length. In such cases, we use hash marks to show equal sides. Show how this is done, marking the equal sides on one of the pentagons as shown in the margin. Explain that we use matching hash marks on a shape when we want to show that some sides have an equal length (each marked with one hash mark) and some other sides have a different but equal length (each with two hash marks), and so on.

Identifying shapes with equal sides. Review measuring to the closest millimeter. Divide students into groups of four and give each group the cards from BLM Polygons. Have students divide the cards between themselves so that each student has at least three different types of polygons. Have students predict which sides of the polygons might be
equal, then measure these sides and use hash marks to label equal sides. They can also fold some of the cards to check for equal sides. Then give students more cards from BLM Polygons, and have them copy the hash marks from the cards their peers measured.

**Introduce equilateral shapes.** Write the term *equilateral* on the board and circle the parts “equi” and “lateral.” **ASK:** Which words do you know that have one of these parts? (equal, equality, equation; quadrilateral) What does the part “equi” mean? (equal) What does the part “lateral” mean? (sides) Explain that *equilateral* means “equal sides” so we say that shapes with all sides the same length are *equilateral* shapes. In a similar way, “quadri” means four and *quadrilateral* is a shape with four sides.

Draw a square and mark all sides as equal. Explain that the shape is an equilateral quadrilateral. Then draw a rectangle, and ask students how you should mark equal sides on a rectangle. (See example in the margin.) **ASK:** Is this shape equilateral? (no) Why not? (each side is equal to some other side, but the sides have different lengths; there are longer sides and shorter sides)

**Exercise:** Sort the polygons from BLM Polygons into equilateral and not equilateral.

**Answer:** Equilateral: 1, 2, 11, 13, 14, 15, 18

**ACTIVITY 1**

**Creating equilateral polygons.** Have students use toothpicks of equal length to create equilateral polygons. Ask students to try to create different shapes of the same type. How many different equilateral triangles can they make? (only one) How many different quadrilaterals? pentagons? hexagons? To prompt students to create different shapes, ask them to make a quadrilateral with at least one acute angle, a pentagon with a right angle, a hexagon with at least one acute angle and at least one obtuse angle, and so on.

**Bonus:** Create a hexagon with an indentation.

**Introduce notation for equal angles.** Review measuring angles with a protractor. Explain that we use arcs and double or triple arcs to show angles that are equal. Draw the picture in the margin on the board and explain that, in this quadrilateral, there are two sets of angles: two acute angles and two obtuse angles. Obviously, the acute angles are not the same size as the obtuse angles. The matching single arcs show that the obtuse angles are equal, and the matching double arcs show that the acute angles are equal. Point out that when two angles are marked with a square, this means they are both right angles, so they are both 90°. This means they are also equal.

**Identifying equal angles.** Ask students to work in groups to examine some polygons from BLM Polygons for equal and unequal angles, and then share the results with the class. Folding the shapes to check the angles will
work well for some shapes (1–7, 11, 13–15, 17, 18, 20, 21, 24); for others, students will need to check the angles with protractors. Have students mark equal angles on all the shapes using the combined work of their peers.

**Exercise:** Sort the polygons from BLM Polygons into shapes that have all angles equal and shapes that have different angles.

**Answer:** Shapes with all angles equal: 1, 2, 5, 13, 14, 15

**ACTIVITY 2**

**Creating polygons with equal angles.** Give each student three pipe-cleaners and ask them to cut the pipe-cleaners into three approximately equal pieces. Have them bend three pieces of a pipe-cleaner so that they create an angle of 60°, with one arm about twice as long as the other arm. See example in margin. Ask them to place the bended pipe-cleaner pieces together to create a triangle, using the angles as vertices and parts of the sides. The pipe-cleaner pieces should overlap to create sides of different lengths, and students can use tape to create proper edges afterwards.

Ask students to try to create different triangles. Did they manage to create triangles of different shapes or are they all different sizes, but the same shape? (same shape) Ask them to tape the arms of the angles together to create proper polygons.

Repeat the activity with each student using six pipe-cleaner pieces and angles of 120°. Move the pipe-cleaner pieces to create different hexagons. Try to make hexagons without equal sides. Have students tape the arms of the angles together.

Display the hexagons that students produced. Did everyone produce the same hexagon? (no) Are they just different sizes, as with the triangles? (no, there are different shapes) Ensure students understand that having all angles equal does not fix a polygon unless it is a triangle.

**Sorting polygons using a Venn diagram.** Give students yarn circles and index cards. Ask students to create a Venn diagram labeled “Polygons” with the groups “equilateral” and “all angles equal.”

**Exercise:** Sort the cards from BLM Polygons into this Venn diagram.

**Answers:** equilateral and all angles equal: 1, 2, 13, 14, 15; equilateral only: 11, 18; all equal angles only: 5; the rest of the shapes are outside both circles.

**Introduce regular shapes.** Explain that shapes that have all sides the same length and all angles equal are called regular. Have students identify regular shapes in their collection from BLM Polygons. Where are these shapes on the Venn diagram? (in the equilateral and all angles equal region) Point out that some shapes are equilateral but not all angles are equal, and some shapes have all angles equal but are not equilateral. Point these shapes out on the Venn diagram above.
Draw a Venn diagram with the groups “equilateral” and “regular.” Ask students to think how a shape in each region of this diagram will look. Ask: Which region will always be empty? (regular, but not equilateral) What properties should the shape in this region have? (regular, but not equilateral) Explain that because regular shapes always have equal sides, they have to be equilateral. This means that we can draw a Venn diagram in a better way, so that “regular” shapes are completely inside the “equilateral” group, just as “triangles” were always inside the “polygons” group. Show the diagram in the margin as an example.

**Extensions**

1. Sort the polygons from BLM Polygons using a Venn diagram with the groups given below and underlined. (The answers are given below at right. **NOTE:** The shapes that are not listed are outside both groups.)

<table>
<thead>
<tr>
<th>Venn Diagram Regions</th>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Equilateral (only)</td>
<td>1, 11, 13, 14, 15</td>
</tr>
<tr>
<td>At least two right angles (only)</td>
<td>5, 6, 19, 20, 22, 24</td>
</tr>
<tr>
<td>Overlap</td>
<td>2, 18</td>
</tr>
<tr>
<td>b) All angles equal (only)</td>
<td></td>
</tr>
<tr>
<td>At least two equal sides (only)</td>
<td>3, 6, 8, 11, 17, 18, 20, 21, 24</td>
</tr>
<tr>
<td>Overlap</td>
<td>1, 2, 5, 13, 14, 15</td>
</tr>
<tr>
<td>c) At least two right angles (only)</td>
<td>19, 22</td>
</tr>
<tr>
<td>At least two equal sides (only)</td>
<td>1, 3, 8, 11, 13, 14, 15, 17, 21</td>
</tr>
<tr>
<td>Overlap</td>
<td>2, 5, 6, 18, 20, 24</td>
</tr>
<tr>
<td>d) Quadrilateral (only)</td>
<td>3, 5, 6, 8, 9, 11, 12, 24</td>
</tr>
<tr>
<td>Regular (only)</td>
<td>1, 13, 14, 15</td>
</tr>
<tr>
<td>Overlap</td>
<td>2</td>
</tr>
</tbody>
</table>

2. a) What does the term “equiangular” mean? Hint: Divide the word into two parts similar to “equilateral.”

b) Sort the shapes on BLM Polygons on a Venn diagram with the groups “equilateral” and “equiangular.”

c) Ron thinks that any equiangular triangle is also equilateral. Check his idea: Draw a line segment. Then draw an angle of 60° at each end and create a triangle. Measure the sides and the angles. What do you notice? Repeat with another triangle starting with a line segment of a different length. Is Ron correct?

**Answers:** a) equal angles, so equiangular shapes are shapes with all angles equal; b) equilateral and equiangular: 1, 2, 13, 14, 15; equilateral only: 11, 18; equiangular only: 5; the rest of the shapes are outside both circles; c) Ron is correct.
Introduce classification of triangles by angles. Draw a few different triangles on the board. Number the angles in each as 1, 2, and 3. Have students identify the largest angle in each triangle by raising the number of fingers equal to the number in the angle. Circle the label for the largest angle and ask students to identify the angle as acute, obtuse, or right angle.

Explain that triangles are classified by the size of the largest angle. Triangles in which the largest angle is acute are called acute-angled or acute triangles; triangles in which the largest angle is a right angle are called right-angled or simply right triangles; and triangles in which the largest angle is obtuse are called obtuse-angled or obtuse triangles. Have students identify each triangle above as acute, right, or obtuse.

Draw a triangle as shown in the margin. Ask students to identify the largest angle. (1) ASK: What type of triangle is this? Students are likely to say this is a right triangle. ASK: How can we check without using a protractor? (compare the angle to a corner of a sheet of paper) Invite a volunteer to check. The triangle is an acute triangle, with the largest angle of 85°.
their ideas. To prompt students to see the solution, remind them that there are 360° in a full turn, and they need 90°. ASK: What fraction of the whole turn is a right angle? (a quarter) How can you fold the paper into four equal parts? (fold the shape in half, and then fold it a second time so that the crease is folded against itself) Point out that students have made a right angle, which they can use to distinguish between right, acute, and obtuse angles. All they need to do is compare the angle in question to the right angle they created.

2. Sorting triangles. Give students BLM Triangles for Sorting and have them cut out the triangles. Ask students to sort them into acute, right, and obtuse triangles. Students should check the angles that are close to a right angle using a sheet of paper or the right angle benchmark created in Activity 1.

3. Creating triangles. Have students create two different triangles of the given type on a geoboard or on grid paper. Ask them to explain how each pair of triangles is different.
   a) acute triangle  
   b) obtuse triangle  
   c) right triangle

Introduce classification by side lengths. Explain that another way to classify triangles is by using side lengths. Triangles with at least two equal sides are called isosceles triangles. Triangles that have no equal sides are called scalene triangles.

Have students classify the triangles cut out from BLM Triangles for Sorting by the number of equal sides. Point out that measuring all the sides in the triangles will take a lot of time. Instead, students can identify sides that look equal, and then fold the triangles so that the sides they are checking line up together. Are they equal? Show students how to fold the triangles to check for equal sides.

**Exercise:** Sort the triangles from BLM Triangles for Sorting into isosceles and scalene.

Introduce equilateral triangles. Remind students that shapes with all sides equal are called equilateral. So triangles with three equal sides are called equilateral triangles. ASK: Where would you look for equilateral triangles: among the isosceles or among the scalene triangles? (isosceles) Have students find the equilateral triangles and place them as a subgroup of isosceles triangles.

**Answers**
Isosceles but not equilateral: C, F, I, L, M
Equilateral: B, E

**Isosceles triangles also have equal angles.** When students have finished sorting the triangles for sides, ask them to look at the isosceles triangles. Ask them to fold the triangles to show the equal sides. Point out that this
folding also compares two of the angles in the triangle. ASK: What do you notice about these angles? (they are equal) What do you think happens with equilateral triangles? Are there also equal angles? How many? Have students explain the prediction. (All three angles are equal because when you fold the triangle, you see that two of the angles are equal. You can fold an equilateral triangle and show that the sides are equal in different ways so that any two angles match.) After students have made the prediction, have them check how many equal angles equilateral triangles have. When students realize that equilateral triangles also have all angles the same size, ASK: What do we call polygons that have equal sides and equal angles? (regular) Point out that this means that all equilateral triangles are also regular triangles.

Classify triangles using both classifications. Explain that you can label triangles using both classifications, according to the largest angle and by the number of equal sides. For example, if you fold a rectangular sheet of paper along the diagonal, you get a right scalene triangle. Have students use the triangles they classified earlier for the next exercise.

Exercise: Hold up a triangle of the given type.

- a) right isosceles triangle
- b) obtuse scalene triangle
- c) acute isosceles triangle
- d) acute equilateral triangle
- e) right scalene triangle
- f) obtuse isosceles triangle

Remind students that a Venn diagram can group shapes that have two properties by showing shapes that have one property or the other property, both properties, or neither property. Remind them that shapes that have neither property are placed outside of both groups. Write on the board these properties for use on Venn diagrams:

<table>
<thead>
<tr>
<th>Group 1:</th>
<th>Group 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute</td>
<td>Scalene</td>
</tr>
<tr>
<td>Right</td>
<td>Isosceles</td>
</tr>
<tr>
<td>Obtuse</td>
<td>Equilateral</td>
</tr>
</tbody>
</table>

Ask students to pick a property from each column and make a Venn diagram using both properties. Point out that, in some cases, a Venn diagram will have an empty region. Also, remind them that equilateral triangles are also isosceles, so when they are sorting triangles using “isosceles,” equilateral triangles should be within this group.

When students finish, ask them to try to draw a triangle in each region of the Venn diagram. If they cannot manage to produce a triangle in one of the regions, ask them to explain to a partner what problems they encountered and ask them to think together whether a triangle in that region is possible.

Triangles cannot have two right or obtuse angles. Ask students to pick a few triangles and look at the angles that are not the largest angle. ASK: Are these angles right, obtuse, or acute? (acute) Explain that, to draw a triangle, start with a line segment and then draw an angle using each endpoint as a vertex. Extend the arms of both angles (which are rays) as much as needed.
until you have created a triangle. Demonstrate how to do this by starting with a line segment and drawing an angle of $40^\circ$ at one endpoint and an angle of $110^\circ$ at the other endpoint.

Divide students into three groups and ask them to try to draw the following:

- Group 1 – a triangle with two right angles
- Group 2 – a triangle with two obtuse angles
- Group 3 – a triangle with one right and one obtuse angle

Have them discuss what problem they encounter, and then explain the problem to the whole class. (The rays will never intersect, so no triangle is possible.)

**Review measuring angles in a polygon before assigning the AP pages.**

Remind students how to place a protractor correctly and that, when they measure an angle in a polygon, they measure the rotation inside the shape. Remind them also that they can extend the sides of the shape—as rays—to make measuring angles easier and more precise. Remind them how to choose the scale on a protractor depending on whether the angle they are measuring is acute or obtuse, or by checking that the 0 aligns with one arm. Have students draw a triangle using a ruler, exchange notebooks with a partner, and measure the angles in the triangle the partner created.

**Extensions**

1. Make triangles with the given number of toothpicks.
   a) Can you make a triangle using 3 toothpicks? How many?
   b) Can you make a triangle using 4 toothpicks? 5 toothpicks?
   c) Make two different triangles with 7 toothpicks.
   d) How many different triangles can you make from 8 toothpicks? 9 toothpicks? What types of triangles are they?

**Answers**

a) 1
   b) 4: no
   c) 5:
   d) 8: 1 triangle, isosceles, with sides 3, 3, and 2 toothpicks long
   9: 3 triangles: one equilateral with sides 3 toothpicks long; one isosceles with sides 1, 4, and 4 toothpicks long; one scalene triangle with sides 2, 3, and 4 toothpicks long
2. a) Draw a triangle in each cell of the table using square grid paper or isometric grid paper (from BLM Isometric Grid Paper, p. R-60).

<table>
<thead>
<tr>
<th></th>
<th>acute</th>
<th>right</th>
<th>obtuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalene</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isosceles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Where in this table are equilateral triangles?

c) What do you know about the angles of equilateral triangles?

d) Can a triangle have three right angles? three obtuse angles?

e) Use your answers in parts c) and d) to explain your answer in part b).

f) In pairs, share your explanations from part e). Do you agree with each other? Discuss why or why not.

**Answers:**
b) equilateral triangles are all isosceles acute, c) equilateral triangles have all angles equal, d) no and no, e) All angles in an equilateral triangle are equal, so they all have to be acute angles. This means that an equilateral triangle is always an acute triangle.

3. Regular polygons are equilateral and equiangular (have equal sides and equal angles). Can a triangle be equilateral but not equiangular? (no) This means all equilateral triangles are also regular. Unlike other polygons, triangles cannot be equilateral but not regular, or equiangular but not regular.

4. John and Mark both have aquariums that are 1 m high. The length and width of John’s aquarium are equal. In Mark’s aquarium, the length is 1 cm longer than the length of John’s, and the width is 1 cm shorter than the width of John’s.

Whose aquarium can hold more? Do you think the answer will always be the same? Explain why or why not. Use one or more of these tools: arrays, number lines, a T-table, equations, pictures.

**Sample answers**
Since the height is the same in both aquariums, I only looked at the area of the bases.

- I tried many examples and listed them in a T-table. I saw that John’s aquarium always has area 1 cm² more than Mark’s, so John’s aquarium will always hold more.
- I used arrays for the dimensions of the bases. I noticed that every time I started with a square array and then took away one row and added one column, I was adding one less than I was taking away. So John’s aquarium will always have a larger base and will always hold more.
G5-13  Parallel Lines

Pages 160–161

STANDARDS
5.G.B.3, 5.G.B.4

VOCABULARY
angle
arm
hash mark
line
parallel lines
perpendicular lines
protractor
ray
right angle

Goals
Students will identify and construct parallel lines.

PRIOR KNOWLEDGE REQUIRED
- Can measure and draw angles using a protractor
- Can measure line segments to the closest millimeter
- Can identify right angles
- Can identify straight lines
- Knows that hash marks and arcs are used to label equal sides and equal angles

MATERIALS
rulers
rectangular sheet of paper for each student
protractors
geoboards

Lines that are the same distance away at all points. Tell students that an engineer wants to build a bridge over a river. Draw a picture of the river, as shown in the margin, on the board. The engineer wants the bridge to be as short as possible. Where should the engineer build the bridge? Have students come to the board and show where they would build a bridge. Have students measure the width of the river at different points.

Repeat with another picture of a river with a width that varies. Finally, present a “river” with straight, parallel banks. Have students measure its width in different places. ASK: What do you notice about the distance between the banks? (It is always the same) Is there one place that is better than all others to build a bridge? (No)

Introduce parallel lines. Explain that lines that look like the banks of the last river are called parallel lines. Parallel lines are straight lines that are always the same distance apart. Like railroad tracks, they never meet. Ask students where they can see parallel lines in the classroom. (shelves, table sides or legs, the lines where the walls meet, etc.) Draw several parallel lines on the board (see example in the margin). Show how to mark parallel lines with arrows. Note also how you can use different arrows to mark different sets of parallel lines—a single arrow on a first set of parallel lines, double arrows on a second and different set of parallel lines, etc. Students might note that this is similar to how hash marks and arcs are used.

Draw a pair of lines on the board that are not parallel but will intersect if extended. ASK: Could a train go along a pair of tracks like this? (No) Why not? (They are getting closer together, and the wheels of a train do not get closer together) Explain that lines can be extended in either direction, and
parallel lines will never meet, even if extended. Invite volunteers to extend the lines so that they clearly intersect. ASK: Are these lines parallel? (no)

Discuss with students how to draw parallel lines. One strategy could be to draw a line using a ruler, then slide the ruler without turning it and draw another line. Point out that, just as railroad tracks go in the same direction, parallel lines also go in the same direction. So if one parallel line on grid paper goes 3 squares up and 4 squares left, the other parallel line will also go 3 squares up and 4 squares left.

Ask students to draw a horizontal line on grid paper and to draw another line parallel to it. Ask them to repeat this with a vertical line and then a diagonal line. Mark the parallel lines with arrows. On a grid on the board, draw a right-angled triangle that is not isosceles (sides of 4 and 3 squares, as shown in the margin). Have students copy the triangle and draw lines parallel to each of the sides. The purpose of this exercise is to draw a line parallel to a diagonal line that does not pass through every point of intersection on the grid.

Draw several shapes that contain parallel sides on the board and ask volunteers to mark pairs of parallel sides. NOTE: it is important to have parallel lines with different slopes and of different lengths; use various trapezoids, parallelograms, and hexagons, not only rectangles and parallelograms.

Introduce perpendicular lines. Explain that, in mathematics, lines that intersect at a right angle are called perpendicular. To make sure that students remember the term, say “parallel” and “perpendicular” several times, and have them show parallel or perpendicular lines with their arms. You can also hold up a pen in various positions and have students show a pen perpendicular or parallel to your pen, depending on the word you say.

**ACTIVITY**

**Using perpendicular lines to create and identify parallel lines.**

Explain that we can use perpendicular lines to draw and to identify parallel lines. Give each student a rectangular sheet of paper and ask them to fold the sheet in half so that the opposite sides match exactly. Have them fold the sheet again in the same direction so that the first fold exactly matches the edges of the paper. Ask students to unfold the sheet. Explain that they have created three parallel folds.

Have students use a protractor to draw a line that is at right angles to one of the folds and extend it so that it meets the two other folds. Ask them to mark the angle between the first fold and the drawn line as a right angle. Have students measure each angle this new line creates with the two other folds. Students should notice that the other two folds are also at right angles, so they are perpendicular to the drawn line. The three folds are all parallel.
Discuss the findings in the activity. Did everyone get the same results? Remind students that, in the last lesson, one group tried to draw a triangle with two right angles. They could not do this because the rays did not intersect. Draw a horizontal line on the board and two vertical lines intersecting it (as shown in the margin). ASK: If I extend the lines upward, will they ever intersect? (no) If they did, what triangle would that create? Extend the lines but bending them slightly so that they intersect at a point far up. Emphasize that this “triangle” would have two right angles, which is impossible. Repeat by extending the lines downward. Explain that this means that you have drawn two parallel lines using right angles.

**NOTE:** Students who have trouble with Question 6 on AP Book 5.2 p. 161 can try this question on a geoboard.

### Extensions

1. Draw a polygon that has:
   a) more than 1 pair of parallel sides
   b) more than 2 pairs of parallel sides
   c) 3 parallel sides

   **Sample answers**

   ![Sample answers](image)

   (MP5, MP6)

2. Pick two containers that look like one has a capacity of about one liter more than the other.
   a) Measure the capacities of each container to check.
   b) Express the capacities of each container in L and mL. Explain to a partner how you converted between L and mL. Use words like place value, decimal, and powers of ten.

   Look for students to understand that capacity refers to the amount of liquid that a container can hold and to choose an appropriate measuring tool (MP5), such as a graduated beaker. Look for them to attend to precision (MP6) when measuring (for example, fill the container completely, avoid spillage, wait until the water settles in the container to record the measurement, look at a graduated beaker at eye level). Finally, look for students to use math terms precisely (MP6) when explaining how to convert between L and mL.
G5-14 Trapezoids and Parallelograms
Pages 162–163

Goals
Students will identify parallel sides in quadrilaterals and distinguish between parallelograms and trapezoids using the lengths of parallel sides.

STANDARDS
5.G.B.3, 5.G.B.4

VOCABULARY
acute
angle
kite
line
obtuse
parallelogram
parallel lines
perpendicular lines
protractor
right angle
trapezoid

PRIOR KNOWLEDGE REQUIRED
Can measure and draw angles using a protractor
Can measure line segments to the closest millimeter
Can identify right angles
Can identify lines as parallel by visual assessment
Knows the notation for sides of equal length
Knows the notation for parallel lines

MATERIALS
rulers
protractors
cards from BLM Quadrilaterals (p. R-61)
toothpicks
scissors
three 8 in long drinking straws for each student

Drawing and identifying parallel lines. Summarize the steps to draw parallel lines:

Step 1: Draw one line.

Step 2: Draw a second line that is perpendicular to the first (to make a right angle).

Step 3: Draw a third line that is perpendicular to the first (to make a right angle).

Which of the three lines are parallel? (second and third lines)

Exercises

a) On grid paper in your notebook, draw a pair of parallel lines that are not parallel to the grid lines. Erase the perpendicular line you used to make them. Do not label the lines as parallel.

b) Draw a pair of perpendicular lines that are not parallel to the grid lines. Then draw a third line that meets one of the other lines at the angle of 88°. Erase the line that meets the other two lines.
Discuss how to check if two lines are parallel. Summarize the steps:

**Step 1:** Draw a new line perpendicular to one of the lines. Extend the new line so that it intersects both lines.

**Step 2:** Measure the angle between the line you drew and the second line.

**Step 3:** If the angle is 90°, the original two lines are parallel. If the angle is not a right angle, the original lines are not parallel.

Have students exchange notebooks with a partner. Ask them to use the two pairs of lines drawn in the previous exercise to practice the steps above. Have students label the parallel lines and check the answers of their partners.

**Introduce parallelograms and trapezoids.** Explain that quadrilaterals can be classified by how many pairs of parallel sides they have. Can a quadrilateral have three parallel sides? (no) How many pairs of parallel sides can a quadrilateral have? (0, 1, or 2) Explain that quadrilaterals that have exactly one pair of parallel sides are called trapezoids, and quadrilaterals that have two pairs of parallel sides are called parallelograms. Draw a few examples of each kind on the board. Point out that quadrilaterals that have no parallel sides do not have a special name. Point out the difference between this classification and the classification of triangles: triangles can be acute, obtuse, or right and this classification covers all possible triangles; the classification for quadrilaterals does not cover all possible quadrilaterals—there are plenty of quadrilaterals that are neither parallelograms nor trapezoids.

Remind students that we use arrows to show parallel sides, and when there are several groups of parallel sides, we use different numbers of arrows for each group. How many groups of parallel sides will a parallelogram have? (2) A trapezoid? (1)

**ACTIVITY 1**

**Classifying quadrilaterals by the number of parallel sides.**

Divide students into groups of four and give them cards from BLM Quadrilaterals. Have students use a visual assessment to identify sides that might be parallel in each quadrilateral. Then have students work individually using the three-step process above from the previous exercise to check whether the sides are parallel. To save time, within each group of four, different pairs of students could check different polygons, compare the results with other pairs who checked the same, and then share the results with the other pair in their group.

Have students sort the quadrilaterals by the number of parallel sides. How many trapezoids do they have? (4: numbers 2, 4, 5, 8) How many parallelograms? (6: numbers 1, 3, 6, 7, 9, 12) How many other quadrilaterals? (2: numbers 10, 11)
Investigating lengths of parallel sides in parallelograms and trapezoids.
Ask students to pick two parallelograms and two trapezoids from BLM Quadrilaterals, ensuring students pick different shapes so that all 10 special quadrilaterals on the BLM are covered. Have students measure the parallel sides on the quadrilateral. Which quadrilaterals have equal parallel sides? (parallelograms) Do any trapezoids have equal parallel sides? (no) Point out that trapezoids might have equal opposite sides, as the trapezoid on card 2 on BLM Quadrilaterals, but these sides are not parallel.

Distinguishing between parallelograms and trapezoids using the length of parallel sides. Explain that checking the length of parallel sides gives us a way to distinguish between parallelograms and trapezoids. Draw the polygon in the margin on the board. ASK: Is this a parallelogram or a trapezoid? (trapezoid) How do you know? (the opposite sides are parallel but not the same length) Emphasize that, if this were a parallelogram, the opposite sides would be parallel and the same length.

Point out that this way to distinguish between parallelograms and trapezoids is very convenient when using grid paper: if two sides of a quadrilateral run along the grid lines, check if they are the same length. If they are the same, the quadrilateral is a parallelogram. If not, the quadrilateral is a trapezoid.

Ask students to use the fact that the opposite sides of a parallelogram are equal lengths to produce different quadrilaterals on grid paper.

Exercise: On grid paper, draw a quadrilateral with the given properties.

a) a parallelogram
b) a trapezoid
c) a quadrilateral that is not a parallelogram and not a trapezoid
d) a trapezoid with two right angles
e) a parallelogram with at least one right angle
f) a trapezoid with two equal sides sharing a vertex

Bonus: a trapezoid with two obtuse angles that do not share a side

Sample answers
**Bonus**

Is any quadrilateral with opposite equal sides a parallelogram?

SAY: A parallelogram has opposite equal sides. Does this work the other way around? Is any quadrilateral with opposite equal sides a parallelogram? Use the activity below to investigate.

**ACTIVITY 2**

Give students toothpicks of different lengths or give them each three 8-inch drinking straws and ask them to cut them into lengths of 5.5 in and 2.5 in, 5.5 in and 2.5 in, and 5 in and 3 in. Ask students to try to create a quadrilateral using two straws that are 2.5 in long as opposite sides and straws that are 3 in long and 5.5 in long. ASK: What kind of quadrilateral can you produce? Can you make a parallelogram? (no) Can you make a trapezoid? (yes) Can you make a quadrilateral that is neither parallelogram nor trapezoid? (yes)

Repeat with two pairs of straws of equal length (e.g., 2.5 in, 2.5 in, 5.5 in, and 5.5 in). Remind students that equal lengths of straws should be opposite sides. ASK: Can you make a trapezoid? (no) Can you make a parallelogram? (yes) Can you make a quadrilateral that is neither a parallelogram nor a trapezoid? (no) Explain that a quadrilateral with two pairs of equal opposite sides is always a parallelogram.

**Extension**

Use the same straws as in Activity 2. Can you use two different pairs of equal-length straws to make a quadrilateral that is neither a parallelogram nor a trapezoid? Hint: Have the straws of the same length share a vertex. A quadrilateral with equal adjacent sides—that is, sides sharing a vertex—and that has no indentation is called a *kite*, like a flying kite. Ordinary flying kites usually do not have any parallel sides. You will note later, in the lesson plan for G5-17, that mathematical kites also include rhombuses. Rhombuses do have parallel sides, but flying kites do not.
Review distinguishing between a parallelogram and a trapezoid.
Remind students that when they know that a quadrilateral has a pair of parallel sides, they can check the length of the parallel sides to decide whether the quadrilateral is a parallelogram or a trapezoid. If the parallel sides are equal, the quadrilateral is a parallelogram. If they are not equal, it is a trapezoid.

ACTIVITIES 1–2

1. Investigating equal sides in quadrilaterals. Point out that sometimes a quadrilateral can have more than two equal sides. Give students three straws. Have them cut two of the straws into halves and the third straw into two unequal parts. Ask them to try to produce a quadrilateral using three equal straws and one shorter straw. ASK: Can you make a parallelogram? (no) A trapezoid? (yes) A quadrilateral that is neither? (yes) Repeat with:
   - three equal straws and a longer straw
   - two equal straws and two other straws of different lengths
   - four equal straws

In the last case, students can only produce parallelograms.
2. a) **Comparing opposite angles in trapezoids and parallelograms.** Ask students to work in pairs. Assign half the pairs some polygons from **BLM Quadrilaterals** and half the pairs the other polygons on the BLM. Ask the students to identify pairs of opposite angles that might be equal on their shapes. Have students fold the shapes or measure the angles with protractors to check for equal angles. Point out that folding will work only in some cases. Have pairs check their results with other pairs assigned the same polygons, and then share their results with a pair that did the other polygons.

b) **Sorting quadrilaterals.** Have students sort the quadrilaterals using the groups given below, on the left. The answers are given below, at right.

<table>
<thead>
<tr>
<th>Two pairs of opposite equal angles</th>
<th>1, 3, 6, 7, 9, 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>One pair of opposite equal angles</td>
<td>10</td>
</tr>
<tr>
<td>No opposite equal angles</td>
<td>2, 4, 5, 8, 11</td>
</tr>
</tbody>
</table>

Discuss the results of Activity 2. **ASK:** Where are all the parallelograms? (in the first group) Are there any other quadrilaterals in the first group? (no) What conclusion can you make about the opposite angles of parallelograms? (opposite angles of parallelograms are always equal) Point out that the fact that no other shapes are in this row also could mean that a quadrilateral with two pairs of equal opposite angles is always a parallelogram. Mathematicians have proved that this is true.

**ASK:** Which group or groups are all the trapezoids in? (the last row) What does this say about trapezoids? (opposite angles of trapezoids are different) **SAY:** A student I know thinks that all quadrilaterals that have different opposite angles are trapezoids. Is the student correct? (no) Have students explain how they know. (quadrilateral number 11 has different opposite angles, but it is not a trapezoid) Finally, ask what type are the quadrilaterals in the middle group. (not parallelograms, not trapezoids)

**ACTIVITY 3**

Ask students to cut out a long strip of paper about 3 cm wide. Students work in pairs to play a game. Player 1 places several quadrilaterals from **BLM Quadrilaterals** in a row and covers them with the strip so that only two vertices and parts of the sides are visible. Player 1 ensures the number on the card is not visible. Player 2 identifies the partially hidden quadrilaterals as trapezoids or parallelograms based on the size of the visible angles.
Introduce diagonals. Explain that a line that joins the opposite vertices of a quadrilateral is called a diagonal. ASK: How many diagonals does each quadrilateral have? (2) Draw a quadrilateral and draw a diagonal in it. ASK: If you cut a quadrilateral along a diagonal, what polygons do you get? (2 triangles)

ACTIVITY 4

Cutting quadrilaterals along diagonals. Students will each need two copies of BLM Quadrilaterals and they will need to save their copies for later use, after this Activity. Have them cut out copies of quadrilaterals 1–5 and 10–12. Have students draw a different diagonal on each copy of each quadrilateral and then fold each quadrilateral along the diagonal they drew. ASK: On which quadrilaterals do the triangles match exactly when folded? (both copies of 3, one copy of 10) Then ask students to cut the quadrilaterals along the fold and try to place the triangles one on top of the other in a different way. Can they find a way for the triangles to match exactly? In which quadrilaterals do the triangles match exactly on both copies? (1, 3, 12) For which quadrilaterals are the triangles different on both copies? (2, 4, 5, 11)

Comparing quadrilaterals using diagonals. Ask students to reassemble the original collection of quadrilaterals from Activity 4—in other words, the copies of BLM Quadrilaterals, both those cut up and those not cut up. ASK: Which of these are parallelograms? (1, 3, 6, 7, 9, 12) When you cut a parallelogram along a diagonal, are the triangles exactly the same or different? (the same with both diagonals) When you cut a trapezoid along a diagonal, are the triangles exactly the same or different? (different) Remind students that the opposite angles of trapezoids are different, so they could expect the triangles to be different because these triangles contain opposite angles of a trapezoid.

Extensions

1. If students are familiar with kites—the mathematical shape as discussed on page R-37, rather than the ordinary flying kites—ask them to think which category (no equal opposite angles, one pair of opposite equal angles, two pairs of opposite equal angles) kites belong to. Have them explain their answer.

   Answer: Kites have at least one pair of opposite equal angles. One of the diagonals splits a kite into two identical triangles, so the angles opposite that diagonal are equal.

2. Cut out the triangles from BLM Triangles for Making Quadrilaterals (p. R-62). Combine the gray triangle with one of the other triangles in different ways so that they share a side—in other words, two sides line up to form the diagonal for a new shape.
a) Which triangle combined with the gray triangle allows you to make a parallelogram? Make more than one parallelogram from this pair of triangles. Make a quadrilateral that is not a parallelogram.

b) Use two different right triangles to produce a quadrilateral that has one pair of equal opposite angles and that is not a kite.

c) Find all triangles that allow you to make a trapezoid with the gray triangle. What kind of trapezoid have you produced? Compare the angles in these triangles with the angles in the gray triangle. What do you notice?

d) Combine the two triangles you picked in part c). Can you make a trapezoid from these two triangles? Is that a trapezoid of the same type as in part c)?

Answers

a) A, you can make three different parallelograms (one of them a rectangle) and a kite.

c) Triangles E and F allow you to make a right trapezoid with the gray triangle. They both have at least one angle that is the same as the smallest angle of the gray triangle.

d) Triangles E and F together can produce a trapezoid, but it is not a right trapezoid.

NOTE: Assign Extension 3 only after students have finished Question 5 on AP Book 5.2 p. 165.

3. a) On grid paper, draw two copies of a trapezoid with two right angles. Draw a different diagonal in each of the copies of the trapezoid.

b) Describe the triangles you got in each picture.

c) Could the quadrilateral Jake got in Question 5 d) be a trapezoid?

Answer: c) yes

(MP.3)

4. A rectangular prism has 6 faces as shown in the margin.

a) Peter says, "There are 27 squares on the base, so the area of the base is 27 square units." Is he correct? Explain.

b) What is the volume of the prism? Explain how you know.

Answers

a) I disagree, because the base is not made of squares of the same size, and to measure area or volume, your units must be the same size.

b) Each side of each face is 3 large square units or 6 small square units, so the volume of the prism is $3 \times 3 \times 3 = 27$ large cubes or $6 \times 6 \times 6 = 216$ small cubes.
Introduce rectangles. Explain that quadrilaterals with four right angles are called rectangles. Have students decide which quadrilaterals on BLM Quadrilaterals are rectangles. (1 and 7)

Review properties of parallel lines intersected by a third line. Remind students that they used right angles to construct a pair of parallel lines. Draw a pair of parallel lines on the board. Review the steps for checking that lines are parallel, illustrating as you go: draw a line that meets one of the lines at a right angle, extend it to meet the other line, measure the angle between the second line and the line you drew to confirm the angle is a right angle, and thus confirm the original pair of lines are parallel.

A rectangle is a parallelogram. Draw a rectangle using a different color for each side and label the vertices. Ask students to explain why the longer sides are parallel. Students should use the colors of the sides to identify lines, and the labels for the vertices to identify angles. Their answers will refer to the chosen colors and vertices, but should mention that rectangles have adjacent right angles, thus the lines perpendicular to the same side must be parallel. Repeat with the two shorter sides. ASK: Will this explanation work for any rectangle? (yes) How many pairs of parallel sides does a rectangle have? (2) Is it a parallelogram or a trapezoid? (parallelogram) Can a rectangle be a trapezoid? (no) Can a rectangle be not a parallelogram and not a trapezoid? (no)
Properties of a parallelogram that apply to a rectangle. Ask students to list all the properties of a parallelogram. Then have them compare the lists in pairs or groups of four and come up with combined lists. Properties should include: four sides in total, two pairs of parallel sides, equal opposite sides, equal opposite angles, both diagonals split a rectangle into two identical triangles. Before reviewing the lists, have students draw two identical rectangles. For each property on the list, ASK: Is that true for a rectangle? (yes) Why? (A rectangle is a parallelogram, so all properties of a parallelogram also apply to a rectangle.) Have students check that the properties are true for the rectangle they drew. Finally, confirm that everyone saw that the properties for parallelograms apply to their rectangles. Point out that students had many different rectangles, but the properties were true for all of them.

Rectangles have equal diagonals. Ask students to measure both diagonals in the rectangles they drew. ASK: What do you notice? (the diagonals are equal) Did everyone get the same result? (yes) Did you have the same rectangle? (no) Ask students to draw a parallelogram that is not a rectangle and measure its diagonals. Are they equal? (no) Did everyone have the same parallelogram? (no) Did anyone get equal diagonals? (no) Point out that a special property of rectangles is that they have equal diagonals.

Other quadrilaterals with equal diagonals. Ask students to draw a trapezoid and draw diagonals in it. Ask them to measure the diagonals. ASK: Did anyone get a trapezoid with equal diagonals? Ask students with trapezoids with equal diagonals to draw their trapezoid on the board. How are these trapezoids special? PROMPT: Does this trapezoid have any equal sides or equal angles? Point out that such a trapezoid is called isosceles: it has equal opposite non-parallel sides.

ACTIVITIES 1–2

1. Creating quadrilaterals with equal diagonals. Give each student two toothpicks, drinking straws, or pieces of pipe cleaner of the same length to act as diagonals of quadrilaterals. Students place the toothpicks so that they intersect and then draw to create a quadrilateral so that the ends of the toothpicks are the vertices of a quadrilateral. Students try to make additional, different quadrilaterals in this way, including a rectangle, a trapezoid, and a quadrilateral that is neither.

Sample answers

rectangle

trapezoid

other quadrilateral
ASK: How should the toothpicks be placed to make the diagonals for a rectangle? (they should intersect exactly halfway between the endpoints) How should you place the toothpicks to make the diagonals for a trapezoid? (they should intersect at the same distance from one of the endpoints, but not in the middle)

Bonus: How should you place the toothpicks to make the diagonals for a square? (at a right angle, intersecting exactly halfway between the endpoints; see margin)

2. Have students sort the polygons from BLM Quadrilaterals into a Venn diagram with the groups “Parallelograms” and “Equal diagonals” and using yarn circles and index cards. (See answer below.) ASK: Where are the rectangles in the Venn diagram? (in the central region) Have students draw a shape as an example for each region of the Venn diagram.

Sum of the angles in a quadrilateral. Have students draw two different quadrilaterals. Ask them to explain how the quadrilaterals are different. Look for the use of geometric properties, such as the number of parallel sides, equal sides and angles, angles of a specific type (right, acute, or obtuse), and the use of proper mathematical language.

Ask students to cut out one of the quadrilaterals and tear off the corners. Then ask them to place the corners together, so that the vertices meet and there is no overlap, as shown in the margin. Remind students that there are 360° in a full turn, so the sum of the angles around a point is 360°. ASK: What is the sum of the angles in your quadrilateral? (360°) Did everyone have the same quadrilateral? (no) Did everyone get the same result? (yes) Should that be true for any quadrilateral? (yes) Point out that this fact is certainly true for a rectangle: a rectangle has four right angles, so the sum of its angles is $4 \times 90° = 360°$.

Investigating the number of right angles in a quadrilateral. ASK: Can a quadrilateral have just one right angle? (yes) Ask students to draw a quadrilateral with a single right angle using a ruler and a protractor or a square corner. Repeat with exactly two right angles. ASK: Can a quadrilateral have exactly three right angles? Have students try to draw a quadrilateral like that and then ask them to explain why this is impossible. (The sum of the angles in a quadrilateral is 360°. If a quadrilateral has three right angles, their sum is 270°, so the fourth angle should be $360° - 270° = 90°$. This means the fourth angle must also be a right angle.)
Ask students to look at the quadrilateral with exactly two right angles. Did anyone draw a trapezoid? A parallelogram? (no) A quadrilateral that is neither? Ask students to draw a quadrilateral with exactly two right angles of a different type from what they already drew. PROMPT: Do the right angles in your quadrilateral share a side? If yes, try to make them opposite. If no, try to make them share a side.

Ask again if anyone managed to draw a parallelogram with exactly two right angles. Have students think why this is impossible. To help their thinking, point out that there are two cases: the right angles share a side, or the right angles are opposite. Have students roughly draw both situations (see example in the margin), without using a ruler, but marking the right angles with a small square and labelling equal angles with the same number of arcs.

ASK: What do you know about the opposite angles of a parallelogram? (they are equal) If the right angles share a side, what can you tell about the other two angles? (each of them is opposite a right angle, so the third and fourth angles must also be right angles) This means that a parallelogram with two right angles that share a side has to be a rectangle.

ASK: If the right angles of a parallelogram are opposite, what do their measures add to? (180°) How much is left for the other two angles? (180°) What should be the size of the other two angles? (90° each) Summarize that this means that the parallelogram has to be a rectangle in this case as well.

**Extensions**

1. If you make a shape with toothpicks as the diagonals, as in Activity 1, how do you need to place the toothpicks to make a kite with two equal diagonals?

2. Cut a rectangle along a diagonal. What different geometric shapes can you make by re-positioning the triangles so that they share a side?

**Answers**

1. The toothpicks need to make a right angle, so the intersection point should be exactly halfway between the endpoints of one of the toothpicks.

2. 2 parallelograms; 1 kite; 2 right triangles; 1 rectangle
**G5-17 Rhombuses**
Pages 168–169

STANDARDS
5.G.B.3, 5.G.B.4

VOCABULARY
- acute
- angle
- degree
- diagonals
- equilateral
- hash mark
- isosceles
- obtuse
- parallelogram
- parallel lines
- perpendicular
- property
- protractor
- rectangle
- rhombus
- right angle
- scalene
- square
- trapezoid
- Venn diagram
- vertex, vertices

**Goals**
Students will compare properties of rhombuses and squares to properties of other quadrilaterals.

**PRIOR KNOWLEDGE REQUIRED**
- Can measure and draw angles using a protractor
- Can measure line segments to the closest millimeter
- Can identify right angles
- Can identify parallel lines
- Knows the notation for equal angles and equal sides
- Knows the notation for parallel lines and right angles
- Can classify triangles based on side lengths or sizes of angles
- Can identify triangles that are exactly the same

**MATERIALS**
- BLM Rhombus or Not? (p. R-63)
- scissors
- protractor
- rulers
- yarn circles
- index cards

**Review equilateral shapes.** Remind students that we use hash marks to show that sides are equal in length, and that we call polygons with all sides the same length *equilateral*. Remind them that an equilateral shape does not need to have all angles equal; it might even have indentations. Remind them how they created equilateral polygons from toothpicks, and ask students to recall a few shapes they created, or display the examples of equilateral shapes shown in the margin.

**Introduce rhombuses.** Explain that an equilateral parallelogram is called a *rhombus*. Have students cut out the quadrilaterals on BLM Rhombus or Not? and sort them into rhombuses and not rhombuses. They can fold the shapes to compare the sides. (A, B, and C are rhombuses)

**Exercise:** Copy the parallelogram on grid paper. Measure the sides to the nearest millimeter to determine if it is a rhombus.
Review properties of parallelograms and check that they all apply to rhombuses. Use the same approach as in the previous lesson: Ask students to list the properties of parallelograms individually and combine the lists. The list should include at least these properties: four sides, four vertices, two pairs of parallel sides, opposite sides are equal, opposite angles are equal, both diagonals split the parallelogram into identical triangles. ASK: Should we expect all these properties to apply to rhombuses? (yes) Why? (because all rhombuses are parallelograms) Have students check the properties one by one, using cut-out copies of the rhombuses from BLM Rhombus or Not? Students should use folding to check that opposite angles are equal.

Review classification of triangles. Remind students that triangles with at least two equal sides are called isosceles, and triangles with three equal sides are called equilateral. ASK: What are triangles that have no equal sides called? (scalene) What are other ways to classify triangles? (by the size of the largest angle: acute, right, and obtuse triangles)

A diagonal separates a rhombus into isosceles triangles. Draw students’ attention to the fact that, when they were folding a rhombus to compare opposite angles, they were actually folding it along a diagonal. Draw a rhombus on the board. ASK: How can I show on the picture that this is a
rhombus? (mark parallel and equal sides) Mark the equal sides with hash marks and then draw a diagonal. ASK: What kind of triangles are these? (isosceles) How can you show that by folding? (fold each triangle again so all four sides coincide) Have students check that all the triangles that result are isosceles.

ASK: When you cut a parallelogram by a diagonal, do you also produce two identical triangles? (yes) Draw a parallelogram that is not a rhombus and draw a diagonal. ASK: What type of triangle are these? (scalene) Will these triangles match exactly if you fold a parallelogram along a diagonal? (no) Have students check using the parallelogram that is not a rhombus (polygon F) from BLM Rhombus or Not?

Have students fold each rhombus once and identify what type of triangle they get. Then have them fold the rhombus a different way and identify the triangles again. (In rhombuses A and C, they get a pair of isosceles acute triangles one way and a pair of isosceles obtuse triangles the other way. In rhombus B, they get two right isosceles triangles both ways.)

**Diagonals of a rhombus are perpendicular.** Ask students to look at the rhombuses they folded. ASK: What angle do the diagonals make? (right angle) How do you know? PROMPT: Are these angles the same? What do they add to? What is the sum of the angles around a point, as in a full turn? (When you fold a rhombus twice along the diagonals, you see that all four angles are the same. They add to 360°, so each angle is 360° ÷ 4 = 90°.)

What do we call two lines that meet at a right angle? (perpendicular lines) Summarize: The diagonals of a rhombus are perpendicular.

Have students check whether other quadrilaterals from BLM Rhombus or Not? have perpendicular diagonals. ASK: Are all quadrilaterals with perpendicular diagonals rhombuses? (no, quadrilateral E is not a rhombus, but it has perpendicular diagonals) Finally, sort the six quadrilaterals into a Venn diagram with the groups “Parallelograms” and “Perpendicular diagonals.”

**Introduce squares.** Remind students that squares are shapes with four equal sides and four right angles. This means that squares are equilateral rectangles. ASK: Is a square also a rhombus? How do you know? PROMPT: A rhombus is an equilateral parallelogram. Is a rectangle a parallelogram? (yes) Is a square equilateral? (yes) Is a square a rhombus? (yes)

**Exercise:** Compare the angles of the quadrilaterals on BLM Rhombus or Not? to a right angle using a protractor. Which quadrilateral is a square? (B)

Draw a Venn diagram with the groups “rhombuses” and “rectangles” on the board. ASK: In which region of the Venn diagram do all squares appear? (in the central region) Can there be anything else in the central region? (no) Why not? (a rectangle that is a rhombus is an equilateral rectangle, so it is a square)

Have students sketch a shape as an example in each region of the Venn diagram.
ACTIVITY

Give students four yarn circles and have them make index cards with a title “Quadrilaterals” and the labels “Parallelograms,” “Trapezoids,” “Rhombuses,” and “Rectangles.” Explain that you want students to place these circles to make a Venn diagram for all quadrilaterals. Have them think how to place the circles. Write these questions on the board for students to consider:

- Should some groups overlap?
- Are there groups that do not overlap? Which ones?
- Are there groups that are completely one inside another group? Which ones?

Have students compare answers in pairs and come up with a common Venn diagram. See the sample diagram in the margin. Then have students sketch an example shape in each region of the Venn diagram they created.

Extensions

NOTES: Extensions 1 and 2 should be done in order.

1. Kites are quadrilaterals with two pairs of equal sides, with equal sides sharing a vertex, and with no indentation. Decide if the statement is true or false. If it is false, draw a shape to illustrate how the statement is false—in other words, a counterexample.

   a) All kites are rhombuses.
   b) Some kites are rhombuses.
   c) All rhombuses are kites.

   Answers: a) false, see example in margin; b) true; c) true

2. In pairs, explain why all rhombuses are kites using the definitions of the shapes. Do you agree with each other’s reasoning? Discuss why or why not.

   Sample answer: Kites are quadrilaterals with two pairs of equal sides that share a vertex and with no indentation. Since rhombuses have 4 equal sides, they are quadrilaterals with two pairs of equal sides. Since all four sides in a rhombus are equal, then the equal sides share a vertex. And since rhombuses are parallelograms, they have no indentation. So all rhombuses are kites.

   Encourage partners to ask questions to understand and challenge each other’s thinking (MP3) and use of math words (MP6)—see p. A-49 for sample sentence and question stems to guide students.
Comparing triangles. Present an isosceles obtuse triangle (A) and a scalene acute triangle (B), as shown in the margin. Have students identify the type of both triangles. As a class, fill in the table below. Students can signal the answer by either raising the correct number of fingers or showing thumbs up for yes and thumbs down for no.

### Property

<table>
<thead>
<tr>
<th>Property</th>
<th>A</th>
<th>B</th>
<th>Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertices</td>
<td>3</td>
<td>3</td>
<td>✓</td>
</tr>
<tr>
<td>Number of sides</td>
<td>3</td>
<td>3</td>
<td>✓</td>
</tr>
<tr>
<td>Number of pairs of parallel sides</td>
<td>0</td>
<td>0</td>
<td>✓</td>
</tr>
<tr>
<td>Number of equal sides</td>
<td>2</td>
<td>0</td>
<td>X</td>
</tr>
<tr>
<td>Number of right angles</td>
<td>0</td>
<td>0</td>
<td>✓</td>
</tr>
<tr>
<td>Number of acute angles</td>
<td>2</td>
<td>3</td>
<td>X</td>
</tr>
<tr>
<td>Number of obtuse angles</td>
<td>1</td>
<td>0</td>
<td>X</td>
</tr>
</tbody>
</table>

Ask students which rows will have the checkmark in the “Same?” column for any pair of triangles. (the first three: number of vertices, sides, pairs of parallel sides) Explain that you want to draw a triangle that will be as different from these two triangles A and B as possible. The first three rows will still be the same, but you want to try to make a triangle that is different from the other two triangles in the other rows. You will not manage to make all the rows different from both triangles A and B. For example, you cannot make a triangle that will be different from both A and B in the last row,
because a triangle can have only one or zero obtuse angles; there are no other options.

Add a column to the table and ask students to each draw a different triangle and fill in the column for it. In how many rows is their triangle different from triangle A? from triangle B? Have several volunteers present their answers on the board, and have the class vote on whether the answers are correct.

**Comparing quadrilaterals.** Use the same approach as with triangles A and B on p. R-50 to compare the quadrilaterals C and D in the margin using the table below. Have a volunteer draw in the diagonals on the shapes for the last row of the table.

<table>
<thead>
<tr>
<th>Property</th>
<th>C</th>
<th>D</th>
<th>Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertices</td>
<td>4</td>
<td>4</td>
<td>✓</td>
</tr>
<tr>
<td>Number of sides</td>
<td>4</td>
<td>4</td>
<td>✓</td>
</tr>
<tr>
<td>Number of pairs of parallel sides</td>
<td>2</td>
<td>1</td>
<td>✗</td>
</tr>
<tr>
<td>Are opposite sides equal?</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Is the polygon equilateral?</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Number of right angles</td>
<td>0</td>
<td>2</td>
<td>✗</td>
</tr>
<tr>
<td>Number of acute angles</td>
<td>2</td>
<td>1</td>
<td>✗</td>
</tr>
<tr>
<td>Number of obtuse angles</td>
<td>2</td>
<td>1</td>
<td>✗</td>
</tr>
<tr>
<td>Number of pairs of opposite equal angles</td>
<td>2</td>
<td>0</td>
<td>✗</td>
</tr>
<tr>
<td>Are the diagonals perpendicular?</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
</tbody>
</table>

Point out that these two quadrilaterals are as different as they can be: all rows but the first two have an ✗ in the last column! Discuss which other properties of quadrilaterals are not in the table. (equal diagonals, type of triangles into which diagonals split the quadrilateral and whether these triangles are the same, whether equal angles share a side, etc.) Make a list of those properties on the board. Then go through all the special quadrilaterals studied in this unit, and have students show thumbs up if the property is true for all such quadrilaterals and thumbs down if it is not true or true for only some of them. For example, when going through properties of rectangles, students should show thumbs up for equal diagonals and thumbs down for perpendicular diagonals, because only some rectangles (squares) have perpendicular diagonals.

Similar to what they did with triangles, ask students to draw another quadrilateral of their choice and describe how the new one differs using both the table above and the added list.

**Comparing polygons.** As with triangles and quadrilaterals, present a pair of polygons, such as a regular hexagon and a pentagon with three right angles (see example in margin), and fill in the table below. Remind students that regular polygons are equilateral and have all angles equal.
<table>
<thead>
<tr>
<th>Property</th>
<th>E</th>
<th>F</th>
<th>Same?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertices</td>
<td>6</td>
<td>5</td>
<td>×</td>
</tr>
<tr>
<td>Number of sides</td>
<td>6</td>
<td>5</td>
<td>×</td>
</tr>
<tr>
<td>Number of pairs of parallel sides</td>
<td>3</td>
<td>1</td>
<td>×</td>
</tr>
<tr>
<td>Is the polygon equilateral?</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Number of right angles</td>
<td>0</td>
<td>3</td>
<td>×</td>
</tr>
<tr>
<td>Number of acute angles</td>
<td>0</td>
<td>0</td>
<td>✓</td>
</tr>
<tr>
<td>Number of obtuse angles</td>
<td>6</td>
<td>2</td>
<td>×</td>
</tr>
<tr>
<td>Is the polygon regular?</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
</tbody>
</table>

Use page 172 from AP Book 5.2 *(G5-19 Problems and Puzzles)* as a cumulative review of the unit.

**Extensions**

1. Are all shapes of this type equiangular (have equal angles)?
   - a) parallelogram
   - b) rhombus
   - c) rectangle
   - d) trapezoid
   - e) kite
   - f) square

   **Answers:** a) no, b) no, c) yes, d) no, e) no, f) yes

2. Nancy has one large cubic box and one small cubic box. To measure the volume of each, she fills them with cubic blocks, each measuring 1 ft³. She uses 91 blocks altogether and the blocks fill the boxes perfectly. How much larger is the larger box than the smaller box?

   Use one or more of these tools: a number line, a pan balance, base ten blocks, a ruler, a T-table, connecting cubes, grid paper, pattern blocks, pencil and paper sketches.

   **Sample solution:** I used a T-table to list the possible sizes of the boxes: a box with side length 1 ft has volume 1 ft³, a box with side length 2 ft has volume 8 ft³, and so on. The possible volumes in ft³ are 1, 8, 27, and 64 (I stopped there because 125 is too big). I looked for two volumes that add to 91 and got 27 and 64. So the smaller box has dimensions 3 ft × 3 ft × 3 ft and volume 27 ft³, and the larger box has dimensions 4 ft × 4 ft × 4 ft and volume 64 ft³. The larger box is 64 − 27 = 37 ft³ larger than the smaller box.
Protractors
### Geometric Terms (1)

<table>
<thead>
<tr>
<th>Term</th>
<th>Diagram</th>
<th>Term</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>right angle</td>
<td><img src="image" alt="Right Angle" /></td>
<td>equilateral polygon</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>obtuse angle</td>
<td><img src="image" alt="Obtuse Angle" /></td>
<td>triangle</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>acute angle</td>
<td><img src="image" alt="Acute Angle" /></td>
<td>pentagon</td>
<td><img src="image" alt="Pentagon" /></td>
</tr>
<tr>
<td>scalene triangle</td>
<td><img src="image" alt="Scalene Triangle" /></td>
<td>hexagon</td>
<td><img src="image" alt="Hexagon" /></td>
</tr>
<tr>
<td>isosceles triangle</td>
<td><img src="image" alt="Isosceles Triangle" /></td>
<td>quadrilateral</td>
<td><img src="image" alt="Quadrilateral" /></td>
</tr>
</tbody>
</table>
Geometric Terms (2)

- parallelogram
- square
- rectangle
- rhombus
- right triangle
- obtuse triangle
- acute triangle
- trapezoid
- parallel lines
- perpendicular lines
Polygons (1)

1. Triangle
2. Square
3. Trapezoid
4. Scalene Triangle
5. Rectangle
6. Parallelogram
7. Isosceles Triangle
8. Rhombus
9. Kite
10. Obtuse Triangle
11. Rhombus
12. Scalene Triangle
Polygons for Venn Diagrams

A

B

C

D

E

F

G

H
Triangles for Sorting

A  B  C
D  E
F  G
H
I
J
K
L
M
N
O
P
Quadrilaterals
Triangles for Making Quadrilaterals
Rhombus or Not?

A  

D

B

E

C

F
PS5-10 Choosing Strategies

Teach this lesson after: 5.2 Unit 7

Standards: 5.NBT.B.5, 5.NBT.B.6, 5.NBT.B.7, 5.NF.A.2, 5.NF.B.4

Goals:
Students will solve a variety of applied problems, across the Grade 5 curriculum, using a variety of problem-solving strategies such as using structure, making diagrams, and making a simpler problem.

Prior Knowledge Required:
Can add and subtract two-digit numbers
Can multiply 2 two-digit numbers
Can use long division to divide up to four-digit numbers by one- or two-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can identify patterns in sequences that increase by the same amount
Can represent fractions by using lengths and areas
Can add and subtract fractions and mixed numbers with unlike denominators
Can multiply fractions
Can add, subtract, and multiply decimals to hundredths
Can write an expression for a given term in a pattern
Can evaluate expressions at a given value for the variable

Materials:
BLM Banquet Hall (pp. R-72–74, see Performance Task)

NOTE: The following Problem Bank questions reflect all the problem-solving strategies used in the problem-solving lessons for Grade 5. Choose among the following questions based on which problem-solving lessons you have taught.

Problem Bank
1. Introduce the buckets problem. Write on the board:

   ![Diagram: 5 L and 2 L buckets]

   You have a 5 L bucket, a 2 L bucket, and you are beside a river.

   ASK: If you have a five-liter bucket and a two-liter bucket, how can you fill the five-liter bucket with three liters of water? (fill the 5 L bucket and pour as much as possible into the 2 L bucket; when the 2 L bucket is full, the 5 L bucket has 3 L in it)
Model this on the board with counters representing liters, as shown below:

![Diagram of buckets](image)

SAY: Start with the two empty buckets again. ASK: How can you fill the five-liter bucket with four liters? (fill the 2 L bucket and pour the water into the 5 L bucket and then repeat) Have volunteers show this process on the board with counters representing liters. ASK: When the five-liter bucket has four liters in it, how can you fill the two-liter bucket with one liter? (fill the 2 L bucket with water and pour all that you can into the 5 L bucket until the 5 L bucket is full; the amount left in the 2 L bucket is 1 L) Again, have volunteers show this process on the board using counters.

Write on the board:

You have a 4 L bucket, a 9 L bucket, and you are beside a river.

SAY: Suppose you want to fill the nine-liter bucket with six liters. ASK: Why would it help to start with one liter in the four-liter bucket? (you can fill the 9 L bucket and then pour into the 4 L bucket until the 4 L bucket is full; that means you poured 3 L from the 9 L bucket into the 4 L bucket to leave 6 L in the 9 L bucket)

a) How would you fill the 4 L bucket with 1 L so that you can still follow the steps to get 6 L in the 9 L bucket?

b) You have a 3 L bucket, a 5 L bucket, and you are beside a river. How can you get exactly 4 L of water from the river?

**Solutions:**
a) Fill the 9 L bucket and then pour 4 L into the 4 L bucket. Empty the 4 L bucket into the river and pour another 4 L from the 9 L bucket into the 4 L bucket. The 9 L bucket now has 1 L. Empty the 4 L bucket and pour the 1 L from the 9 L bucket into the 4 L bucket. The 4 L bucket now has 1 L. Now you can follow the steps to get 6 L in the 9 L bucket.
b) Fill the 5 L bucket and fill the 3 L bucket. Empty the 3 L bucket and pour the remaining 2 L from the 5 L bucket into the 3 L bucket. There is now 2 L in the 3 L bucket and the 5 L bucket is empty. Fill the 5 L bucket and pour 1 L into the 3 L bucket so that the 3 L bucket is full. The 5 L bucket now has 4 L in it. Visual representation of solution:
2. You have an 8 L bucket, a 5 L bucket, and a 3 L bucket. There is no river nearby. The 8 L bucket is full and the other two buckets are empty. How can you pour half of the water from the 8 L bucket into the 5 L bucket, so that 4 L is in the 5 L bucket and 4 L is in the 8 L bucket? 

**Solution:** Fill the 5 L bucket with water from the 8 L bucket, leaving 3 L in the 8 L bucket. Then fill the 3 L bucket with water from the 5 L bucket, leaving 2 L in the 5 L bucket. Fill the 8 L bucket with water from the 3 L bucket, and then pour the 2 L from the 5 L bucket into the 3 L bucket. There will 6 L in the 8 L bucket, nothing in the 5 L bucket, and 2 L in 3 L bucket. Fill the 5 L bucket with water from the 8 L bucket, and then fill the 3 L bucket with water from the 5 L bucket. There will 1 L in the 8 L bucket, 4 L in the 5 L bucket, and 3 L in the 3 L bucket. Pour the water from the 3 L bucket into the 8 L bucket and you will have 4 L in the 8 L bucket, 4 L in the 5 L bucket, and the 3 L bucket will be empty. Visual representation of solution: 

\[
\begin{align*}
8,0,0 \rightarrow & 3,5,0 \rightarrow 3,2,3 \rightarrow 6,2,0 \rightarrow 6,0,2 \rightarrow 1,5,2 \rightarrow 1,4,3 \rightarrow 4,4,0 \\
\end{align*}
\]

3. It is 7:34 a.m. on Wednesday. What day and what hour was it 9 hours and 45 minutes ago? 

**Solution:** 9 hours ago it was 10:34 p.m. on Tuesday. 45 minutes before that it was 9:49 p.m. on Tuesday.

4. The heartbeat of an adult is around 70 beats per minute. How many times does the heart beat in a year? Hint: A year to the nearest minute is 365 days and 5 hours and 49 minutes. 

**Solution:** The number of minutes in a year is \(365 \times 24 \times 60 + 5 \times 60 + 49 = 525,949\), so the number of beats per year is about \(70 \times 525,949 = 36,816,430\).

5. Color the shape to show the product and then find the product. 

![Shape A](image1.png)  
![Shape B](image2.png)

**Answers:** a) 4, b) 8

6. Maria reads \(\frac{3}{4}\) of a book and Bev reads \(\frac{1}{2}\) of another book. Bev says she read more pages than Maria. Is that possible? Explain with an example. 

**Sample answer:** Yes, it is possible. For example, if Bev’s book was 200 pages long and Maria’s book was only 100 pages long, then Bev read 100 pages and Maria only read 75 pages.

7. Explain why \(\frac{1}{3} \div 4\) and \(\frac{1}{4} \div 3\) are equal. 

**Answer:** \(1/3 \div 4 = 1/(3 \times 4)\) and \(1/4 \div 3 = 1/(4 \times 3)\). Since \(3 \times 4 = 4 \times 3\), these are equal.
8. Write four subtraction or addition equations using the numbers in the table below. Use each number only once. Color the 3 numbers for each equation with the same color.

Example: \[ \frac{3}{8} - \frac{1}{8} = \frac{2}{8} \text{ or } 1\frac{1}{4} \]

Use each number only once. Color the 3 numbers for each equation with the same color.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>6(\frac{1}{2})</td>
<td>8(\frac{1}{8})</td>
<td>2(\frac{4}{5})</td>
<td>10(\frac{1}{6})</td>
</tr>
<tr>
<td>9(\frac{3}{8})</td>
<td>1(\frac{5}{6})</td>
<td>1(\frac{1}{4})</td>
<td>12</td>
</tr>
<tr>
<td>2(\frac{1}{5})</td>
<td>3(\frac{1}{2})</td>
<td>6(\frac{3}{10})</td>
<td>4(\frac{3}{10})</td>
</tr>
</tbody>
</table>

**Answer:**

<p>| | | | |</p>
<table>
<thead>
<tr>
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</tr>
<tr>
<td>9(\frac{3}{8})</td>
<td>1(\frac{5}{6})</td>
<td>1(\frac{1}{4})</td>
<td>12</td>
</tr>
<tr>
<td>2(\frac{1}{5})</td>
<td>3(\frac{1}{2})</td>
<td>6(\frac{3}{10})</td>
<td>4(\frac{3}{10})</td>
</tr>
</tbody>
</table>

9. Bo wants to add 345 + 687 on his calculator, but he accidentally presses “×” instead of “+”. What could he press next so that he doesn’t have to re-enter 345?

**Answer:** 1 \((345 \times 1 + 687)\)

10. Put the decimal point in the appropriate position to make the data realistic.

- a) height of a room: 27 m
- b) gas in a car tank: 245 L
- c) length of a pencil: 145 cm
- d) patient’s body temperature: 1007°F

**Answers:** a) 2.7 m, b) 24.5 L, c) 14.5 cm, d) 100.7°F

11. How many shaded and unshaded triangles would be in Figure 10? Hint: First look at the number of shaded triangles only.

**Solution:** The sequence for shaded triangles is: 1, 3, 6, 10, … The sequence does not have a common gap, but the tenth term is \(1 + 2 + 3 + 4 + \ldots + 10\). Find the answer by adding:

\[
\begin{align*}
1 &+ 2 + 3 + 4 + 5 \\
&+ 10 + 9 + 8 + 7 + 6 \\
&+ 11 + 11 + 11 + 11 + 11
\end{align*}
\]

The total for shaded triangles in Figure 10 is 55. The sequence for the unshaded triangles is 0, 1, 3, 6, 10, … Figure 10 has the same number of unshaded triangles as Figure 9 has shaded triangles, which is 10 less than 55, so the total for unshaded triangles in Figure 10 is 45.
12. The number on top is equal to the sum of two numbers on the bottom. Find the missing number.

a) \[
\begin{array}{c}
5 \\
3
\end{array}
\]  

b) \[
\begin{array}{c}
9 \\
2
\end{array}
\]  

c) \[
\begin{array}{c}
3.7 \\
1.4
\end{array}
\]  

d) \[
\begin{array}{c}
5.2 \\
3.4
\end{array}
\]  

Bonus: Guess the top number before you complete the picture.

e) \[
\begin{array}{c}
4 \\
2 \\
2 \\
4
\end{array}
\]  

f) \[
\begin{array}{c}
2.2 \\
1.1 \\
1.1 \\
1.1 \\
1.1
\end{array}
\]  

Answers: a) 8, b) 7, c) 5.1, d) 1.8, Bonus: e) 20, f) 8.8
Performance Task: Banquet Hall

Materials:
BLM Banquet Hall (pp. R-72–74)

Performance Task: Banquet Hall. Give students BLM Banquet Hall. Tell students that this performance task involves hosting an event at a banquet hall.

Selected answers: 1. a) 6, 10, 14, b) 22, c) 7; 2. a) i) 33 1/2 ft, 28 people, ii) 40 1/2 ft, 32 people; b) i) 38 people; Bonus a) 40, c) 9, $40.50
Banquet Hall (1)

1. Sara is preparing for a banquet. She puts tables together in three different ways:

   ![Table Diagram]

   a) Use the picture to fill in the table below and find how many people can fit around one, two, or three joined tables.

<table>
<thead>
<tr>
<th>Number of Joined Tables</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

   b) If Sara puts 5 tables in a row, how many people will fit around the table?

   c) Sara wants to make one long table for 30 people. How many tables will she need to join together to fit that many people?
Banquet Hall (2)

2. Each table is 5 ft long and there needs to be $3\frac{1}{2}$ ft of space between the table groupings.
   
a) How much space is required for the arrangement of 6 tables? How many people can sit in this arrangement?
   
i) Draw a picture to show how 6 tables can be arranged. How much space is required? How many people can sit in this arrangement?
   
   ii) Draw a picture to show how 3 joined tables (7 tables in total) can fit in the banquet hall. How many people can sit in this arrangement?

b) The banquet hall is 55 ft long and only fits one row of tables. There needs to be 3 ft of space from the ends of the table to the walls of the banquet hall.
   i) Draw a picture to show how 4 separate tables and 3 joined tables (7 tables in total) can fit in the banquet hall. How many people can sit in this arrangement?
   
   ii) Draw a picture to show how one long table made from 10 joined tables cannot fit in the banquet hall.
   
   iii) Draw a picture to show how 7 separate tables cannot fit in the banquet hall.
Banquet Hall (3)

**BONUS** Sara has a party for 40 people. She wants to have her party in the 55 ft long banquet hall that only fits one row of tables. Remember: Each table is 5 ft long, there needs to be 3\(\frac{1}{2}\) ft of space between the table groupings, and there needs to be 3 ft of space from the ends of the table to the walls of the banquet hall.

a) How many people can sit around 5 separate tables and 2 joined tables (7 tables in total)?

b) Draw a picture to show that the arrangement in part a) cannot fit in the banquet hall.

c) Find the smallest number of tables that Sara needs to rent for her party. The cost of renting a table is $4.50. How much does she have to pay in total to rent the tables?
1 cm Grid Paper

[1 cm grid paper with black lines]
Dark 1 cm Grid Paper
Answer Keys for AP Book 5.2
iv) \( \frac{15,840}{10} = 1,584 \)

v) \( \frac{21,120}{10} = 2,112 \)

vi) \( \frac{26,400}{10} = 2,640 \)

b) ii) 176

iii) 528

iv) 1,584

v) 2,112

vi) 2,640

c) Yes.

d) Teacher to check.

6. a) 2,200

b) \( \frac{3}{16} \)

c) 2,090

d) 110 yards

AP Book MD5-14

page 90

1. a) i) 15 ft = 180 in

ii) 30 ft = 360 in

iii) 75 ft = 900 in

iv) 120 ft = 1,440 in

b) 36 (3 \times 12)

c) i) 432 in

ii) 720 in

iii) 18 in

iv) 18 in

d) iii) and iv) They should be the same because the fraction is equivalent to 0.5

e) Divide by 36

f) i) \( \frac{3}{4} \)

ii) \( \frac{1}{4} \)

iii) \( \frac{2}{4} \)

iv) \( \frac{1}{3} \)

2. a) i) 1,760 yd

= 5,280 ft

b) i) 3,520 yd

= 10,560 ft

ii) 17,600 yd

= 52,800 ft

iii) 176 yd = 528 ft

b) 5,280

c) i) 26,400

ii) 7,920

iii) 660

iv) 3,960

v) 7,920

vi) 3,960

d) iv) and vi) are the same because the fraction is equivalent to 0.75

AND ii) and v) are the same because the fraction is equivalent to 0.5

c) Yes.

d) Teacher to check.

AP Book MD5-14

page 93

1. a) i) 2 oz, 10 oz

b) 4 oz, 12 oz

c) 8 oz, 24 oz

d) 16 oz, 48 oz

e) 32 oz, 96 oz

f) 64 oz, 192 oz

2. 304.6875 lb (304.69 lb)

3. a) i) 1 mi = 1,760 yd = 5,280 ft = 63,360 in

b) 63,360

c) i) 190,080

ii) 633,600

iii) 6,336

4. a) i) 15 ft, 4 yd, 54 ft

ii) 6 ft, 3\( \frac{1}{3} \) yd, 32 ft

iii) 16.5 ft, 5\( \frac{2}{3} \) yd, 67 ft

b) i) 18 yd

ii) 10\( \frac{2}{3} \) yd

iii) 22\( \frac{1}{3} \) yd

c) Yes.

5. 51 inches

d) \( \frac{4\frac{1}{4}}{12} \) ft

f) \( \frac{5}{12} \) yd

6. a) \( \frac{6}{3} \) ft

b) \( \frac{13}{2} \) ft

7. a) 20,320 ft

14,481 ft

13,796 ft

b) 5,839 ft and

6,524 ft taller

c) 22,841 ft

d) More than 4 miles
tall, less than 5 miles tall.

Teacher to check explanation.

8. a) 88 inches

b) No; each piece is 22 inches and 2 ft is 24 inches, so each

piece is less than 2 ft long

c) 22 inches

d) iv) and vi) are the same because the

fraction is equivalent to 0.75

AND ii) and v) are the same because

the fraction is equivalent to 0.5

9. Width: 26.75

Height: 12.25

10. a) 5,300

b) 10,600

c) 2.25 miles

d) Yes

AP Book MD5-15

page 95

1. a) 2 oz, 10 oz

b) 4 oz, 12 oz

c) 8 oz, 24 oz

d) 16 oz, 48 oz

e) 32 oz, 96 oz

f) 64 oz, 192 oz

2. b) \( \frac{9}{16} \)

3. b) \( \frac{1}{4} \)

c)\( \frac{1}{2} \)

d)\( \frac{1}{8} \)

4. b) \( \frac{9}{16} \)

5. \( \frac{9}{16} \)

6. \( \frac{7}{4} \)

7. \( \frac{3}{4} \)

8. \( \frac{5}{2} \)

9. b) 112 oz

112 + 7 = 119 oz

c) 144 oz

144 + 4 = 148 oz

d) 64 oz

64 + 2 = 66 oz

e) 32 oz

32 + 12 = 44 oz

f) 112 oz

112 + 8 = 120 oz

10. b) \( \frac{9}{16} \)

112 oz

112 + 8 = 120 oz

c) 144 oz

144 + 4 = 148 oz

d) 64 oz

64 + 2 = 66 oz

e) 32 oz

32 + 12 = 44 oz

f) 112 oz

112 + 8 = 120 oz

AP Book MD5-16

page 95

1. a) 225 lb

b) 20.55 lb

c) 304.6875 lb (304.69 lb)
3. 14.34375 lb (14.34 lb)
4. a) 43 oz  
b) 1,403 oz  
c) 40 in
5. 9.25 oz
6. a) They cost the same amount ($36)  
b) 8 large boxes  
c) Large
7. B-brand is cheaper because it is 0.2/oz, whereas A-brand is 0.215/oz
8. 8 ft = 96 inches  
You should buy the shelf without the extension because 80 inches + 20 inches exceeding the 96 inches (8 ft) ceiling height.
9. a) \( \frac{96}{8} \)  
b) No, because 8ft = 96 inches, so the bunk bed would be too high for her room and she would not be able to sit up on the top bunk.  
c) Yes, because 9 ft = 108 inches and sitting on the top bunk, his height would be about 101 inches. The bunk bed would fit in Ben’s room.
10. a) 6,510 ft  
= 1.23 mi  
Yes, it is longer than a mile.  
b) 18,290 ft  
= 3.46 mi  
BONUS  
About 21 minutes

AP Book MD5-17

1. a) 3  
b) 5 in  
c) 15  
d) i) 76 in  
ii) 6.3 ft  
2. Teacher to check line plot.  
a) 53 in  
b) 61 in  
3. a) 4 mi  
b) 8,800 yds  
c) 10,560 yds  
4. a) 304 oz  
b) 20  
c) 320 lbs  
d) Teacher to check.  
e) Answers will vary.

AP Book MD5-18

1. a) 15  
b) 8  
c) 660 yd  
d) 6  
2. a) \( \frac{7}{2} \)  
b) 3  
c) \( \frac{1}{8} \)  
d) 114 oz  
3. a) 12  
b) Yes  
c) 186 \( \frac{3}{4} \)  
4. a) 12  
b) Teacher to check.  
c) 16  
d) 51  
e) \( 12 \frac{3}{4} \)  

AP Book MD5-19

1. b) 194  
c) 632  
2. b) 4 min 32 s  
c) 7 min 36 s  
3. a) 16:15  
b) 19:20  
c) 23:54  
d) 09:45  
e) 11:54  
f) 00:25  
4. a) 4:30 p.m.  
b) 2:48 p.m.  
c) 11:15 p.m.  
d) 3:35 a.m.  
e) 12:15 p.m.  
f) 12:58 a.m.  
5. a) 16:35  
b) 11:55  
c) 13:49  
6. b) 10:12  
c) 14:26  
d) 15:37  
e) 08:25  
f) 13:39  
7. a) ii) 13:29  
iii) 13:11  
iv) 15:15  
v) 15:06  
vi) 16:40  
b) ii) 1:29 p.m.  
iii) 1:11 p.m.  
iv) 3:15 p.m.  
v) 3:06 p.m.  
vi) 4:40 p.m.  
8. It ended at 13:50, which is 1:50 p.m.

AP Book MD5-20

1. a) 5:06  
b) 13:15  
c) 11:13  
d) 12:26  
e) 3:25  
f) 2:05  

AP Book MD5-21

1. 11:55 a.m.  
2. 2:10 p.m.  
3. 2:18  
4. 3:53  
5. a) 6 hours  
b) $65  
6. a) Monday: 3 h 15 min  
Tuesday: 2 h 30 min  
Wednesday: 3 h 30 min  
Thursday: 7 h 25 min  
Friday: 7 h 35 min  
b) 24 h 15 min  
c) 97 \( \frac{3}{4} \)  
d) $291  
e) 24.25  
f) $291  
g) Yes.

AP Book MD5-22

1. a) \( \frac{30 + 4 = 7.5}{3 \times 7.5 = 22.50} \)
b) 28 ÷ 7 = 4
   4 × 4 = $16.00

c) $4.50 × 4 = $18.00

d) 10.50 ÷ 2 = 5.25
   3 × 5.25 = $15.75

e) 6.75 ÷ 3 = 2.25
   4 × 2.25 = $9.00

f) 12.50 ÷ 5 = 2.5
   7 × 2.5 = $17.50

2. a) $12.28

b) $30.70

3. Teacher to check labels.

4. Teacher to check.

5. Teacher to check.

6. Teacher to check diagrams and labels.

   a) one block: $3.75
      Josh has $3.75 left.

   b) one block: 10 min
      Ava spent 30 minutes playing tag.

   c) $7.00

   d) Beginning: $28.50
      Book: $5.70

   e) 15 oz

   f) 35 days = 5 weeks
1. b) \[ 5 + 5 = 10 \] \[ 2 \times 5 = 10 \] 
   c) \[ 5 + 5 + 5 = 15 \] \[ 3 \times 5 = 15 \] 
2. b) Width: 4 
   Length: 5 
   \[ 4 \times 5 = 20 \] 
   c) Width: 3 
   Length: 7 
   \[ 3 \times 7 = 21 \] 
3. a) \[ 4 \text{ cm}^2 \] 
   b) \[ 6 \text{ cm}^2 \] 
   c) \[ 8 \text{ cm}^2 \] 
4. b) square feet 
   c) square inch 
5. b) \[ 24 \text{ ft}^2 \] 
   c) \[ 24 \text{ in}^2 \] 
6. a) E: \[ 24 \text{ m}^3 \] 
   P: \[ 56 \text{ cm}^3 \] 
   O: \[ 36 \text{ cm}^2 \] 
   T: \[ 48 \text{ mm}^2 \] 
   A: \[ 30 \text{ km}^2 \] 
   K: \[ 40 \text{ m}^2 \] 
   b) TOPEKA, Kansas 
7. b) \[ 9 \times 12 = 108 \text{ m}^2 \] 
   c) \[ 8 \times 16 = 128 \text{ cm}^2 \] 
   d) \[ 11 \times 27 = 297 \text{ in}^2 \] 
   e) \[ 468 \text{ ft}^2 \] 
   f) \[ 396 \text{ yd}^2 \] 
8. a) \[ 12 \times 25 = 300 \text{ ft}^2 \] 
   b) \[ $4.20 \] 

AP Book MD5-24

1. a) \[ \frac{1}{4} \text{ in}^2 \] 
   b) \[ \frac{1}{9} \text{ in}^2 \] 
   c) \[ \frac{1}{16} \text{ in}^2 \] 
6. a) Length: 7 
   b) Width: 5 
   c) 35 squares 
   Area: \[ 35 \times \frac{1}{16} = \frac{35}{16} \text{ in}^2 \] 
   d) Yes 
7. a) Area: \[ 2 \times \frac{3}{4} = \frac{3}{2} \text{ ft}^2 \] 

AP Book MD5-25

1. a) \[ 18 \text{ m}^2 \] 
   b) \[ 18 \text{ km}^2 \] 
   c) \[ 52 \text{ cm}^2 \] 
   d) \[ 19.8 \text{ km}^2 \] 
   e) \[ 39.06 \text{ mm}^2 \] 
   f) \[ 34.3 \text{ m}^2 \] 
2. b) \[ w \times 2 = 12 \] 
   w = 12 ÷ 2 
   \[ w = 6 \] 
   c) \[ w \times 6 = 24 \] 
   w = 24 ÷ 6 
   \[ w = 4 \] 
3. a) \[ 5 \times l = 30.5 \] 
   l = 6.1 
   b) \[ 7 \times l = 43.47 \] 
   l = 6.21 
   c) \[ 10 \times l = 167.8 \] 
   l = 16.78 
4. a) \[ \frac{3}{8} \text{ ft} \] 
   b) \[ 0.625 \text{ cm} \] 
   c) \[ 4 \text{ cm} \] 
5. Teacher to check lines. 
   a) Total area: \[ 52 \text{ cm}^2 \] 
   b) Total area: \[ 48.4 \text{ cm}^2 \] 
   c) Total area: \[ 38 \text{ in}^2 \] 
   d) Total area: \[ 11 \frac{23}{32} \text{ ft}^2 \] 
6. a) \[ 3 \times 2 = 4 = 24 \] 
   b) \[ 2 \times 3 = 6 \] 
   c) \[ 4 \times 3 = 12 \] 
   d) \[ 3 \times 4 = 12 \] 
   e) \[ 3 \times 4 = 2 = 24 \] 
7. Teacher to check; explanations may vary. 
   Sample answer: Both have the same number of blocks because the three numbers to be multiplied (width, length, and height) are the same, regardless of order of multiplication.

AP Book MD5-27

1. a) 5 
   b) 9 
   c) 10 
2. a) 7 cm³ 
   b) 15 cm³ 
   c) 11 cm³
d) 30 ft³

e) 12 in³

f) 24 m³

g) 36 m³

h) 40 cm³

i) 48 in³

3. a) i) 18 cm³

ii) 16 km³

iii) 36 yd³

b) i) Length: 3 cm

Width: 2 cm

Height: 3 cm

Volume: 18 cm³

ii) Length: 4 km

Width: 2 km

Height: 2 km

Volume: 16 km³

iii) Length: 4 yd

Width: 3 yd

Height: 3 yd

Volume: 36 yd³

c) Yes.

4. a) Length: 15

Width: 10

Height: 8

b) 1,200
c) 1,200 cubes = 1,200 cm³
d) 15 × 10 × 8 = 1,200 cm³
e) Yes

f) Length: 4 in

Width: 4 in

Height: 3 in

Volume: 48 in³

5. a) 900 m³

b) 20,250 ft³

c) 10,800 cm³
d) 69,000 in³

6. a) width × height

b) 60 cm³

i) 400 ft³

BONUS

7. a) Yes

b) Yes

c) i) 120 cm³

ii) 480 ft³

iii) 560 mm³

AP Book MD5-28

page 120

1. Teacher to check.

2. Teacher to check.

3. Teacher to check.

4. Teacher to check.

5. B, C, A

6. Teacher to check.

AP Book MD5-29

page 122

1. Teacher to check.

2. 12 cm²

3. Length: 15

Width: 10

Height: 8

b) 1,200
c) 1,200 cubes = 1,200 cm³
d) 15 × 10 × 8 = 1,200 cm³
e) Yes

f) Length: 4 in

Width: 4 in

Height: 3 in

Volume: 48 in³

AP Book MD5-30

page 124

1. a) 5 m

b) 4 in

c) 8 cm

2. Teacher to check.

3. a) 5 m

b) 6 in

c) 7 mm

4. a) 120 m³

b) 72 in³

c) 252 mm³

5. a) 25 cm²

b) 10 m²

c) 8 in²

6. a) 6 km

b) 7 ft

c) 5 yd
d) 2 m

e) 12 mm

f) 9 in

7. Teacher to check labels.

b) Length: 6

Width: 2

Height: 3

Volume: 36 units³

AP Book MD5-32

page 128

1. a) 1,080 cm³

b) 9 cm

c) 12 cm long, 10 cm wide, and 9 cm tall

2. Length: 5 units

Width: 4 units

Height: 3 units

Volume: 60 units³

3. Box A

4. a) 1,232 cm³
Measurement and Data: Area and Volume – AP Book 5.2: Unit 6

(continued)

b) 45,000 cm$^3$
c) 36 books
d) i) 15
   ii) 3 sideways and 1
       at the end of each
       stack.
   iii) 35
e) Answers may vary,
teacher to check.

5. a) 22,326 ft$^2$
b) 14,884 ft$^2$
c) 19,870,140 ft$^3$

6. a) No, Sofia is not
correct because not
all three units of
measure (length,
width, and height)
are in the same
units (cm, m, etc)
b) 80 × 100 × 50
   = 400,000 cm$^3$

c) 792,484 cm$^3$
d) 2,016 in$^3$

BONUS
Yes, because 1,000 m ×
1 m × 0.01 m = 10 m$^3$

AP Book MD5-33
page 130
1. Teacher to check.
2. a) mL
   b) L
   c) mL
   d) L
3. a) 300 mL
   b) 200 L
   c) 14,000 L
   d) 330 mL

AP Book MD5-34
page 132
1. a) 590 mL
   b) 2,540 mL
   c) 20 mL
   d) 4 mL
   e) 1,759 mL
   f) 1,040 mL
   g) 24,700 mL
2. a) 0.339 L
   b) 0.393 L
   c) 9.083 L

3. a) Teacher to check.
   b) Teacher to check.
4. b) 3 L 247 mL
c) 4 L 27 mL
d) 5 L 820 mL
e) 5 L 8 mL
f) 12 L 750 mL
g) 2 L 700 mL
h) 58 L 100 mL

5. a) 5 L 217 mL
c) 4 L 367 mL
d) 4 L 81 mL
e) 7 L 6 mL
f) 44 L 300 mL

6. a) 4L = 4,000 mL
   b) 4,510 mL
   c) 9 L = 9,000 mL
   d) 2,128 mL
e) 9 L = 9,000 mL
   f) 7 L 7,000 mL
   g) 7,002 mL

7. 300 mL × 5 = 1,500 mL
   = 1.5 L

8. a) 4
   b) 10
   c) 5

9. 3,060 mL
   3.06 L
   3 L 60 mL

10. 822.5 L

11. a) 2 packs of six cans
   b) One 1.89 L bottle
       costs more

AP Book MD5-35
page 134
1. a) 1 cm × 1 cm ×
   10 cm = 1,000 cm$^3$
b) 1 dm$^3$ = 1,000 cm$^3$
c) 1 L = 1,000 mL
   1 cm$^3$ = 1 mL

2. a) 4 L
   b) 450 mL
   c) 330 cm$^3$
d) 1.89 dm$^3$

3. b) Length = 37 cm
   Width = 20 cm
   Height = 20 cm
   Volume = 14,800 cm$^3$
   Capacity = 14,800 mL
   c) Length = 5 dm
   Width = 3 dm
   Height = 3 dm
   Volume = 45 dm$^3$
   Capacity = 45 L
   d) Length = 16 dm
   Width = 4 dm
   Height = 8 dm
   Volume = 512 dm$^3$
   Capacity = 512 L

4. a) Volume = 60,000 cm$^3$
   = 60,000 mL
   b) Length = 5 dm
   Width = 3 dm
   Height = 4 dm
   c) Volume = 60 dm$^3$
   = 60 L
   d) Yes, because the relationship
   between cm and dm
   is equivalent to the relationship
   between mL and L.
5. a) Length = 100 cm
   Width = 60 cm
   Height = 33 cm
   b) Volume = 198,000 cm³
      = 198,000 mL
   c) 198 L
5. a) Length = 100 cm
   Width = 60 cm
   Height = 33 cm
   b) Volume = 198,000 cm³
      = 198,000 mL
   c) 198 L
6. a) Volume = 36 cm³
      Height: 6 cm
   b) Volume = 320 dm³
      Height = 4 dm
   c) Volume = 90 cm³
      Height = 5 cm
7. a) Volume = 36 cm³
   Height: 6 cm
   b) Volume = 320 dm³
   Height = 4 dm
   c) Volume = 90 cm³
   Height = 5 cm
8. a) 5 R 5
   b) 5 R 6
   c) 0 R 6
   d) 1 R 4
9. 1 1/2 c

BONUS Teacher to check circling.
   a) 120 fl oz
   b) 80 fl oz
   c) 400 fl oz
4. Teacher to check circling.
   a) Length = 100 cm
   Width = 60 cm
   Height = 33 cm
   b) Volume = 198,000 cm³
      = 198,000 mL
   c) 198 L
   BONUS a) Teacher to check.
   b) 7 cups
5. a) Volume = 36 cm³
      Height: 6 cm
   b) 5 fl oz
   c) 4 fl oz
   6. a) 9 fl oz
   b) 20 fl oz
   c) 14 fl oz
7. a) Volume = 36 cm³
   Height: 6 cm
   b) 32 c
   c) 192 c
   d) 1,280 fl oz
8. a) 10 c
   b) 24 c
   c) 34 c
9. a) 48 fl oz
   b) 160 fl oz
   c) 384 fl oz
10. a) 8 L = 8,000 cm³
    Height = 8 cm
   b) 9,000 cm³
   c) 1,000 cm³
11. a) 1 m³
    = 100 cm × 100 cm
    = 1,000,000 cm³
    = 1,000 L
   b) 1,800,000 L
   c) Teacher to check.
BONUS Teacher to check.

AP Book MD5-36
page 137
1. a) 4 3/16
   No
   b) 7 7/8
   Yes
2. 16 24 32 40 48 56 64
   Multiply by 8
3. a) 80 fl oz
   b) 96 fl oz
   c) 200 fl oz
4. Teacher to check circling.
   a) 120 fl oz
   b) 80 fl oz
   c) 400 fl oz

AP Book MD5-37
page 139
1. b) 10 c
   c) 24 c
   d) 1 c
   e) 11 c
   f) 21 c
2. b) 6 pt
   c) 15 pt
   d) 2.5 pt
   e) 4.5 pt
   f) 22.5 pt
3. b) 6 pt = 12 c
   c) 12 pt = 24 c
4. b) 12.5 qt
   c) 7.5 qt

BONUS a) Teacher to check.
   b) 7 cups
5. a) Teacher to check.
   b) 16
   c) 128
7. a) 32 c
   b) 384 fl oz
   c) 192 c
   d) 1,280 fl oz
8. a) 10 c
   b) 24 c
   c) 34 c
9. a) 48 fl oz
   b) 160 fl oz
   c) 384 fl oz
10. a) 8 L = 8,000 cm³
    Height = 8 cm
   b) 9,000 cm³
   c) 1,000 cm³
11. a) 1 m³
    = 100 cm × 100 cm
    = 1,000,000 cm³
    = 1,000 L
   b) 1,800,000 L
   c) Teacher to check.
BONUS Teacher to check.

AP Book MD5-38
page 142
1. a) 217.5 m²
   b) $935.25
   c) 42.4 m
   d) $1,187.20
   e) $2,122.45
2. a) 28 m
   b) 123,500 m³
3. 3 ft × 3 ft
4. 48 in³
5. 32 fl oz or 4 c or $\frac{1}{4}$ gal
6. a) 37,000 cm³
    = 37,500 mL
   b) 37,500 mL = 37.5 L
    = 37.5 kg + 10.5 kg
    = 48 kg
   c) Ron should move the aquarium when it is empty because it is lighter without the water.
7. a) Less than 1 fl oz.
   Teacher to check explanation.
   b) 1728
   c) More than 1 gal.
   Teacher to check explanation.
8. 7,392 in³
   = 32 gal
9. a) 612,000 cm³
    = 612,000 mL or 612 L
   b) 75 cm
10. a) 308 ft²
    b) 144
    c) 44,352 in²
    d) 44,352 in²
   c) 192 gal
11. a) 3,200 fl oz
    b) $427
    c) $55
    d) $372
**Geometry: 2-D Shapes – AP Book 5.2: Unit 7**

**AP Book G5-6**

**page 144**

1. a) line segment  
   2 endpoints  
   b) line  
   0 endpoints  
   c) ray  
   1 endpoint  
2. Teacher to check.
3. Teacher to check.
4. Teacher to check.
5. a) Left  
   b) Left  
   c) Left  
   d) Left  
6. a) Same  
   b) Not the same  
7. a) Left  
   b) Left  
8. a) Teacher to check.  
   b) No

**AP Book G5-7**

**page 146**

1. b) 3 sides  
   3 vertices  
   c) 6 sides  
   6 vertices  
   d) 5 sides  
   5 vertices  
   e) 6 sides  
   6 vertices  
   f) 8 sides  
   8 vertices  
   g) 8 sides  
   8 vertices  
   BONUS 11 sides  
   11 vertices  
2. C  
   It is not a polygon because not all sides are line segments.
3. b) 6 sides  
   hexagon  
   c) 3 sides  
   triangle

**AP Book G5-8**

**page 148**

1. b) 30 degrees  
   c) 80 degrees  
2. b) more than 90°  
   c) less than 90°  
   d) more than 90°  
   e) more than 90°  
   f) less than 90°
3. a) acute  
   b) obtuse  
   c) obtuse  
   d) obtuse  
   e) acute  
   f) acute
4. a) acute  
   b) obtuse  
   c) acute  
   d) obtuse  
   e) obtuse  
   f) acute
5. b) acute  
   12°  
   c) obtuse  
   120°
6. a) acute  
   50°  
   b) acute  
   30°

**AP Book G5-9**

**page 151**

1. Teacher to check.
2. a) 90°  
   35°  
   55°  
   b) 65°  
   75°  
   40°
3. a) 108°  
   b) 118°  
   c) 120°  
   d) 52°
4. a) C  
   b) A
5. Teacher to check.
6. Teacher to check.
7. Teacher to check.

**AP Book G5-10**

**page 153**

1. b) C, D, E  
   c) A, C, D, F
2. a)
3. a)  
   b)  
   c)  
   115°  
   d) acute  
   45°  
   e) obtuse  
   153°  
   f) obtuse  
   105°

b) Teacher to check.
4. a)  
   i)  
   3  
   3
5. a)  
   b)  
   c)  
   2
   B  
   D  
   F  
   A  
   C  
   E  
   D  
   A
6.  
   B  
   H  
   C  
   D  
   A  
   E  
   F  
   G
1. a) 30mm, 30 mm, 25 mm, 25 mm
   b) 20 mm, 21 mm, 21 mm, 27 mm

2. a) 27 mm
   b) 20 mm
   c) 40 mm, 20 mm
   d) 22 mm

3. a) 60°
   b) 90°
   c) 120°
   d) 95°, 85°

4. a) Teacher to check: A, B, G
   b) All angles equal: A, B, C, G
      Equilateral: A, B, E, F, G
   c)
   d) The overlap.
   e) Teacher to check.

AP Book G5-12
page 157

1. a) 90° 52°
   b) 20° 118° 42°

2. Teacher to check.

3. b) acute
c) right
d) obtuse

4. a) acute
   b) right
c) acute
d) obtuse

5. Teacher to check.

6. All three angles in an acute triangle are acute.

7. a) isosceles
   b) equilateral
c) isosceles
d) scalene

8. a) i) scalene
   ii) isosceles
   iii) equilateral
   iv) isosceles

   b) i) right-angle
      ii) right-angle
      iii) acute
      iv) acute

9. a) Teacher to check.
b) Acute: A, D, F
    Right: C, G
    Obtuse: B, E
    Equilateral: D
    Isosceles: A, E, G
    Scalene: B, C, F

c) i)
   j)

   d) Teacher to check.

10. a) Acute
    b) No, because all three angles of a triangle must add up to 180° so no two angles can be 90°, nor can two angles be obtuse in the same triangle.

AP Book G5-13
page 160

1. Teacher to check.

2. a) No
   b) Yes
   c) No
   d) Yes
   e) Yes
   f) Yes
   g) Yes
   h) No

3. Teacher to check.

4. Teacher to check.

5. Teacher to check.

6. a) Teacher to check drawings.
   i) 2
   ii) 3
   iii) 0

   b) Teacher to check drawings.

7. a) Teacher to check.
b) 90°
c) Teacher to check, answers may vary.
d) You can use right angles to draw parallel lines because they can check to see if a perpendicular line is crossing two parallel lines.

AP Book G5-14
page 162

1. Teacher to check.

2. a) parallelogram
   b) trapezoid
   c) trapezoid
   d) parallelogram

3. a)

   b) Trapezoids have only 1 pair of parallel sides, while parallelograms have 2 pairs. A quadrilateral cannot be both.

   c) Parallelograms have equal length parallel sides, whereas trapezoids do not.

5. Ron is incorrect because equal opposite side lengths does not mean that the sides are parallel. Also, the two sides that are parallel are not of equal length. It is a trapezoid.

AP Book G5-15
page 164

1. a) C, G
   b) Teacher to check.

c) None B, D
   2 A
   3 E, I
   4 F
   2 pairs C, G, H
   d) 4 or 2 pairs
2. a) Yes
   b) No
3. a) i) 120°, 60°
     120°, 60°
   ii) 108°, 72°
     108°, 72°
   iii) 90° 90° 90° 90°
     b) Opposite angles of a parallelogram are equal.
     c) No.
4. P, P, T, T, P
5. a) Teacher to check.
     b) Circle i), ii), and v)
     c) Parallelograms
d) No, opposite angles are not equal.
6. Teacher to check.

AP Book G5-16
page 166
1. Teacher to check.
   Circle a, b, e.
2. a) Right-angled
   b) No
3. a) They would not be right-angled triangles.
   c) Teacher to check.
4. Teacher to check.
b) C
5. a) No, opposite angles are not equal.

AP Book G5-17
page 168
1. a) i) rhombus
    ii) no
6. a) Teacher to check; explanations may vary.
    Sample answer: All rectangles are also parallelograms because rectangles have two pairs of equal length, parallel sides.
    b) Teacher to check; explanations may vary. Sample answer: The length of the other two sides are 6 cm and 15 cm because a rectangle is also a parallelogram, and they have equal length opposite sides.
7. a) Yes, because angles in a quadrilateral add up to 360°. All quadrilaterals with 4 equal angles will have 90°, making it a rectangle.
    b) All angles in a quadrilateral add up to 360°, so with three exact angles at 90°, the fourth angle must also be 90°.

BONUS Teacher to check.
Sample answer: Any parallelogram that has at least 2 right angles has to be a rectangle because of the number of degrees needed to add up to the required 360°.

AP Book G5-18
page 170
1. a) i) rhombus
    iii) rhombus
    b) Yes

AP Book G5-19
page 172
1. Teacher to check.
2. Teacher to check.
3. a) Quadrilateral
   b) Quadrilateral
   c) Right-angled

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Answer Keys for AP Book 5.2 T-23
c) Quadrilateral
   Parallelogram
   Rhombus
   Square
   Rectangle

d) Quadrilateral
   Parallelogram
   Rectangle

4. Teacher to check.
Unit 5: Measurement and Data

Quiz (Lessons 10 to 13)

1. Measure the line segment to the closest eighth of an inch.
   a) ________________ in
   b) ________________ in

2. Convert the measurement.
   a) 8 ft to inches
   b) 56 in to feet
   c) 3 ft 9 in to inches
   d) \(2\frac{1}{2}\) yd to feet
   e) 29 ft to yards
   f) \(3\frac{3}{4}\) mi to yards

BONUS

The first indoor hockey game was played at the Victoria Skating Rink in Montreal, Canada. The rink was a rectangle that was 68 yd long and 80 ft wide.

a) What is the length of the rink in feet?

b) What is the perimeter of the rink?
Unit 5: Measurement and Data

Quiz (Lessons 10 to 13)

1. a) \( \frac{3}{8} \)
   b) \( 2 \frac{4}{8} \)

2. a) \( = 8 \times 12 \text{ in} \)
   \( = 96 \text{ in} \)
   b) \( = 48 \text{ in} + 8 \text{ in} \)
   \( = 4 \text{ ft} 8 \text{ in} \)
   c) \( = 36 \text{ in} + 9 \text{ in} \)
   \( = 45 \text{ in} \)
   d) \( = 2.5 \times 3 \text{ ft} \)
   \( = 7.5 \text{ ft} \)
   e) \( = 27 \text{ ft} + 2 \text{ ft} \)
   \( = 3 \text{ yd} 2 \text{ ft} \)
   f) \( = \frac{3}{4} \times 1,760 \text{ yd} \)
   \( = 1,320 \text{ yd} \)

BONUS

a) \( 68 \text{ yd} = 68 \times 3 \text{ ft} \)
   \( = 204 \text{ ft} \)

b) \( 2 \times 204 \text{ ft} + 2 \times 80 \text{ ft} \)
   \( = 408 \text{ ft} + 160 \text{ ft} \)
   \( = 568 \text{ ft} \)
1. The upper bunk of a bunk bed is at the height $68 \frac{3}{8}$ inches. If Kim sits up in bed, she sits 32 inches tall.
   a) If Kim sits on the upper bunk, what height will her head reach?
   
   b) The ceiling in Kim’s bedroom is 8 ft high. Is the bunk bed a good choice for Kim’s room? Explain.

2. Convert.
   a) 5 lb to ounces
   b) 3 lb 6 oz to ounces

3. A muffin weighs $3 \frac{1}{2}$ oz. A café sold 1,000 muffins. How many pounds of muffins did the café sell?

**BONUS** The café used 11 oz of flour for each 10 muffins. How many pounds of flour did it use?
1. a) \[ \frac{3}{8} \text{ in} + 32 \text{ in} = 100 \frac{3}{8} \text{ in} \]

b) \[ 8 \text{ ft} = 8 \times 12 \text{ in} = 96 \text{ in} \]

The bunk bed is not a good choice because when Kim sits up, her height is greater than 96 in.

2. a) \[ 5 \times 16 \text{ oz} = 80 \text{ oz} \]

b) \[ 3 \times 16 \text{ oz} + 6 \text{ oz} = 54 \text{ oz} \]

3. \[ 1,000 \times \frac{3}{2} \text{ oz} = 3,500 \text{ oz} \]

\[ 3,500 \text{ oz} \div 16 = 218 \text{ lb} \]

So 218 lb of muffins were sold.

**BONUS**

1,000 muffins =
100 groups of
10 muffins
100 \times 11 \text{ oz} =
1,100 \text{ oz}

1,100 \text{ oz} \div 16 = 68 \text{ lb}

So 68 lb 12 oz of flour was used.
1. Tony exercised from 10:45 a.m. until 1:20 p.m.
   a) Convert both times into 24 h format.
      10:45 a.m. = ______________
      1:20 p.m. = ______________
   b) Use the grid to find how long he exercised.
      You may have to regroup 1 hour as 60 min.

2. A yoga instructor teaches a class from 6:45 p.m. to 8:15 p.m. each night from Monday to Saturday.
   a) Use the grid to find how long each class is.
   b) How many minutes does she teach each day?
   c) How many hours does she teach each week?
   d) The instructor gets paid $24 per hour. Use the grid to find how much she is paid each week.

3. Draw a diagram to solve the problem.

   Sam had 5 lb of flour. He used \( \frac{5}{8} \) of it to make dumplings, and one third of the rest to bake a loaf of bread. How many ounces of flour are left?
4. A hardware store employee is placing bolts of different weights in a box. The box can hold at most 5 lb.
   a) Can the employee place 15 bolts that weigh 6 oz each in the box? Explain.

   b) The line plot shows the masses of bolts placed in the box. How many bolts weigh more than \( \frac{1}{2} \) lb? _________

   c) What is the total mass of the bolts in the box?

5. A board is 11 ft 6 in long.
   a) The board is cut into 4 equal pieces. Is each piece more than 1 yard long? Estimate and explain your answer.

   b) How many inches long is the board?

   c) How long is each piece? Do your calculations on the grid shown.

BONUS ▶ How many seconds are in 1,000 hours?
Unit 5: Measurement and Data

Test (Lessons 10 to 22)

1. a) 10:45, 13:20
   b) Tony exercised for 2 hours and 35 minutes.

2. a) 1 h 30 min
   b) 90 min
   c) 9 h
   d) $216

3. Teacher to check diagram
   (1 block = 10 oz).
   There are 20 oz of flour left.

4. a) 5 \times 16 \text{ oz} = 80 \text{ oz}
    15 \times 6 \text{ oz} = 90 \text{ oz}
    So the employee cannot place 15 bolts
    that weigh 6 oz each in the box.
   b) 4
   c) \[
   \frac{1}{8} + \frac{4}{8} + \frac{12}{8} + \frac{10}{8} \\
   + \frac{6}{8} + \frac{7}{8} = \frac{40}{8} \\
   = 5 \text{ lb}
   \]

5. a) 4 pieces of 1 yd
    = 4 yd
    4 yd = 12 ft
    12 ft is greater than
    11 ft 6 in so the
    pieces are shorter
    than 1 yd each.
   b) 11 \times 12 \text{ in} + 6 \text{ in}
    = 138 \text{ in}
   c) 34 \frac{1}{2} \text{ in}

BONUS

1,000 \times 60 \text{ min}
= 60,000 \text{ min}
60,000 \text{ min} \times 60 \text{ sec}
= 3,600,000 \text{ sec}
1. Find the area of the rectangle using the length and the width. Include the units.
   a) Length = 8 m  
      Width = 7 m  
      Area = \( \, \times \, \)  
      =  
   b) Length = 33 ft  
      Width = 12 ft  
      Area = \( \, \times \, \)  
      =  

2. Each square has sides measuring \( \frac{1}{4} \) in. Write the length and the width of the rectangle. Then find the area of the rectangle two ways. Write the answer as an improper fraction in lowest terms.
   Area = _______ squares  
   = _______ \times \frac{1}{16} \text{ in}^2 = \boxed{\phantom{0}} \text{ in}^2  
   Area = \boxed{\phantom{0}} \text{ in} \times \boxed{\phantom{0}} \text{ in} = \boxed{\phantom{0}} \text{ in}^2  

3. Write an equation for the area of the rectangle. Then find the unknown length.
   a) Width = 3 cm  
      Length = \( \ell \) cm  
      Area = 27 cm²  
   b) Width = 10 m  
      Length = \( \ell \) m  
      Area = 58 m²  

4. Draw a line to divide the shape into two rectangles. Use the areas of the rectangles to find the total area of the shape.
   Area of rectangle 1 =  
   Area of rectangle 2 =  
   Total area =  

**BONUS** A square has an area of 36 cm². What is the width of the square? __________
Unit 6: Measurement and Data

Quiz (Lessons 23 to 25)

1. a) Area = $8 \text{ m} \times 7 \text{ m}$
   $= 56 \text{ m}^2$

   b) Area = $33 \text{ ft} \times 12 \text{ ft}$
   $= 396 \text{ ft}^2$

2. Area = 42 squares
   $= 42 \times \frac{1}{16} \text{ in}^2 = \frac{42}{16} \text{ in}^2$
   $= \frac{21}{8} \text{ in}^2$

   Area = $\frac{7}{4} \text{ in} \times \frac{6}{4} \text{ in}$
   $= \frac{42}{16} \text{ in}^2 = \frac{21}{8} \text{ in}^2$

3. a) $3 \text{ cm} \times \ell \text{ cm} = 27 \text{ cm}^2$
   \[ \ell = \frac{27}{3} \]
   \[ \ell = 9 \text{ cm} \]

   b) $10 \text{ m} \times \ell \text{ m} = 58 \text{ m}^2$
   \[ \ell = \frac{58}{10} \]
   \[ \ell = 5.8 \text{ m} = 5\frac{4}{5} \text{ m} \]

4. Teacher to check line.

   Area of rectangle 1 =
   $1\frac{1}{4} \text{ ft} \times 2 \text{ ft} = \frac{5}{4} \text{ ft} \times \frac{2}{1} \text{ ft}$
   $= \frac{10}{4} \text{ ft}^2 = \frac{5}{2} \text{ ft}^2$

   Area of rectangle 2 =
   $1 \text{ ft} \times 2\frac{1}{2} \text{ ft} = 1 \text{ ft} \times \frac{5}{2} \text{ ft}$
   $= \frac{5}{2} \text{ ft}^2$

   Total area =
   $\frac{5}{2} \text{ ft}^2 + \frac{5}{2} \text{ ft}^2 = \frac{10}{2} \text{ ft}^2 = 5 \text{ ft}^2$

BONUS

6 cm
1. Find the area of the rectangle.
   
   a) Length = 10.1 cm, width = 6 cm  
   b) Length = $\frac{3}{4}$ yd, width = $\frac{1}{2}$ yd
   
   Area = ______________________  Area = ______________________

2. Find the volume of the stack made from unit cubes. Include the units in your answer.
   
   a)  
   Volume = __________  b) 
   Volume = _______________

3. Find the volume of the prism.
   
   a)  
   Length = ______  Width = ______  Height = ______
   Volume = ______________________
   = __________
   b)  
   Length = ______  Width = ______  Height = ______
   Volume = ______________________
   = __________

4. Draw a net of the prism on the grid below and label each face. Each square on the grid represents 1 cm.
Unit 6: Measurement and Data

Test (Lessons 23 to 32)

5. A bookshelf is 1.75 m tall, 85 cm wide, and 40 cm deep.

   a) \(1.75 \times 40 \times 85 = 5,950\), so Sam thinks the volume of the refrigerator is 5,950 cm\(^3\). Is he correct? Explain.

   b) How can you fix Sam's answer to find the correct volume of the refrigerator?

6. Show how to divide the shape into rectangular prisms. Use the volumes of the prisms to find the total volume of the shape.

   Volume A = ______________________ = ____________
   Volume B = ______________________ = ____________
   Total volume = ____________

BONUS ➤ Find the volume of the shape.

ADVANCED

7. Find the area of the shaded face.

   a) Volume = 240 cm\(^3\)

   Area of shaded face = _______

   b) Volume = 81 m\(^3\)

   Area of shaded face = _______
Unit 6: Measurement and Data

Test (Lessons 23 to 32)

1. a) 60.6 cm$^2$
   b) $\frac{3}{8}$ yd$^2$

2. a) 12 in$^3$
   b) 24 cm$^3$

3. a) Length = 2 ft
    Width = 3 ft
    Height = 5 ft
    Volume
    $= 2 \text{ ft} \times 3 \text{ ft} \times 5 \text{ ft}$
    $= 30 \text{ ft}^3$
   b) Length = 11 m
    Width = 5 m
    Height = 6 m
    Volume
    $= 11 \text{ m} \times 5 \text{ m} \times 6 \text{ m}$
    $= 330 \text{ m}^3$

4. Teacher to check net.

5. a) No. The units were not the same for each measurement.
   b) Multiply Sam’s answer by 100 (Sam forgot to change 1.75 m to 175 cm).
    Volume = 595,000 cm$^3$

6. Volume A
   $= 20 \text{ in} \times 32 \text{ in} \times 20 \text{ in}$
   $= 12,800 \text{ in}^3$
   Volume B
   $= 85 \text{ in} \times 35 \text{ in} \times 20 \text{ in}$
   $= 59,500 \text{ in}^3$
   Total volume = 72,300 in$^3$

BONUS

Volume A
$= 10 \text{ m} \times 10 \text{ m} \times 15 \text{ m}$
$= 1,500 \text{ m}^3$
Volume B
$= 10 \text{ m} \times 10 \text{ m} \times 8 \text{ m}$
$= 800 \text{ m}^3$
Volume C
$40 \text{ m} \times 10 \text{ m} \times 7 \text{ m}$
$= 2,800 \text{ m}^3$
Total volume = 5,100 m$^3$

ADVANCED

7. a) 20 cm$^2$
   b) 9 m$^2$
Unit 6: Measurement and Data

Test (Lessons 33 to 38)

1. Convert the measurements.
   a) 281 mL = ______ thousandths of 1 L = ______ L
   b) 65 mL = _____ thousandths of 1 L = ______ L
   c) 0.3 L = ______ thousandths of 1 L = ______ mL
   d) 0.007 L = ______ thousandths of 1 L = ______ mL
   e) 4 mL = ______ L
   f) 54,231 mL = ______ L
   g) 0.04 L = ______ mL
   h) 6.07 L = ______ mL

2. To make fruit punch you need:
   970 mL of apple juice
   340 mL of cranberry juice
   200 mL of mango juice
   1.5 L of lemonade

Which jar is better for mixing the punch: a 3 L jar or a 4 L jar? Explain.

3. Find the volume and the capacity of the aquarium. Include the units!

   a) Volume = ____________  Volume = ___________
      Capacity = __________   Capacity = __________

   b) Volume = ____________  Volume = ___________
      Capacity = __________   Capacity = __________
4. An aquarium has a length of 50 cm and a width of 30 cm.
   a) The water in the aquarium is 16 cm high. How much water is in the aquarium? Use the grid on the right for rough work.
      ______________ mL = _______ L
   b) Ava added 4.5 L of water to the aquarium. What is the new height of the water in the aquarium?

5. Convert the measurements.
   a) 1 gal = _____ qt  b) 1 qt = ____ pt  c) 1 pt = ____ c
d) 1 c = _____ fl oz  e) 1 gal = ____ c  f) 1 gal = _____ fl oz
g) $\frac{3}{2}$ c = ______ fl oz  h) 6 fl oz = ____ c  i) 18 fl oz = _____ c
   j) $\frac{3}{8}$ gal = ______ c  k) $1\frac{1}{4}$ gal = _____ qt  l) $1\frac{1}{4}$ gal = _____ c

BONUS ►
   m) 44 c = _____ gal  n) 160 fl oz = _____ gal  o) 12 qt = ____ pt = ____ c
1. a) 281 thousandths of 1 L = 0.281 L  
   b) 65 thousandths of 1 L = 0.065 L  
   c) 300 thousandths of 1 L = 300 mL  
   d) 7 thousandths of 1 L = 7 mL  
   e) 4 mL = 0.004 L  
   f) 54,231 mL = 54.231 L  
   g) 0.04 L = 40 mL  
   h) 6.07 L = 6,070 mL  
   
2. 970 mL = 0.970 L  
   340 mL = 0.340 L  
   200 mL = 0.200 L  
   1.5 L = 1.500 L  
   Total = 3.010 L  
   This is greater than 3 L  
   so a 4 L jar is better for  
   mixing the punch.  
   
3. a) Volume  
   = 31,250 \text{ cm}^3  
   Capacity = 31.25 L  
   b) Volume  
   = 58,500 \text{ cm}^3  
   Capacity = 58.5 L  
   
4. a) 24,000 mL = 24 L  
   b) 4.5 L = 45,000 mL  
   45,000 ÷ 1,500  
   = 30  
   So the new height  
   of the water is  
   30 cm.  
   
5. a) 4 qt  
   b) 2 pt  
   c) 2 c  
   d) 8 fl oz  
   e) 16 c  
   f) 128 fl oz  
   g) 28 fl oz  
   h) \frac{3}{4} \text{ c}  
   i) \frac{2}{4} \text{ c}  
   j) 6 c  
   k) 5 qt  
   l) 20 \text{ c}  
   m) 2\frac{3}{4} \text{ gal}  
   n) 1\frac{1}{4} \text{ gal}  
   o) 24 \text{ pt} = 48 \text{ c}
Unit 7: Geometry
Quiz (Lessons 6 to 9)

1. Count the sides. Then name the polygon.
   a) ____________ sides
   b) ____________ sides
   c) ____________ sides
   d) ____________ sides

2. Identify the angle as acute or obtuse. Then measure the angle using a protractor.
   a) The angle is ___________________.
      The angle measures ______.
   b) The angle is ___________________.
      The angle measures ______.

3. Finish drawing an angle that measures 135°.

4. Extend the arms and measure the angle marked with an arc in the triangle.

   BONUS► Measure the other two angles in the triangle. Add all the three angles. What is the sum of the angles in a triangle?
Unit 7: Geometry

Quiz (Lessons 6 to 9)

1. a) 4 sides
   quadrilateral
b) 3 sides
   triangle
c) 6 sides
   hexagon
d) 5 sides
   pentagon

2. a) obtuse
   125°
b) acute
   57°

3. Teacher to check angle.

4. The angle marked with an arc is 45°

   **BONUS**
   
   \[45° + 45° + 90° = 180°\]
1. Use the shapes below to complete the Venn diagram.

   A  B  C  D
   E  F  G  H

2. a) Sort the polygons into the Venn diagram.

   J  K  L  M

b) Shade the region of the Venn diagram where all regular shapes should be.

3. Classify the triangles by sides and angles. Example: right scalene triangle.

   a)  b)  c)  d)

   __________________  __________________  __________________  __________________

   __________________  __________________  __________________  __________________

BONUS► Which of these triangles does not exist? _____
A: right isosceles triangle   B: obtuse isosceles triangle   C: obtuse right triangle
1. Shapes: C E G
   Polygons: D
   Triangles: B F
   Quadrilaterals: A H
2. a) Polygons: L
   Equilateral: M
   All angles equal: J
   Region where ovals overlap: K
   b) Students should shade the region where the ovals overlap.
3. a) acute isosceles
   b) acute equilateral
   c) right scalene
   d) obtuse scalene
   BONUS
   C
1. Use a protractor to check the triangle for right angles. Use a ruler to check it for equal sides. Then identify the triangle.

2. Use a ruler and a protractor to check which sides of the quadrilateral are parallel. Then identify the quadrilateral.

3. a) Some parallel sides are not marked. Mark them with arrows.
   
   ![Arrows on polygons](image)

   b) Sort the polygons into the Venn diagram.

   c) Shade the region of the Venn diagram where all rhombuses should be.

   d) Draw a star in the region where all trapezoids should be.

4. A diagonal splits a quadrilateral into two triangles. What do you know about these two triangles? Mark ✓ if it is always true, ★ if it is never or only sometimes true.

<table>
<thead>
<tr>
<th></th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>The triangles are identical</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The triangles are isosceles triangles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The triangles are right triangles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. a) Check the angle between the diagonals in each quadrilateral. Mark the right angles. Then fill in the Venn diagram.

b) Which of the quadrilaterals are rhombuses? ________________

c) Where should be the squares in the Venn diagram?

6. Is any square also a parallelogram? Explain.

**BONUS** Can you draw a triangle with two right angles? Explain, using a picture.
Unit 7: Geometry

Test (Lessons 6 to 19)

1. right scalene triangle
2. trapezoid
3. a) Shapes A and B are missing parallel marks.
   b) Polygons = F
      Equilateral = D
      Parallelogram = A C
      Region where ovals overlap = B E
   c) The region where the ovals overlap should be shaded.
   d) A star should be drawn in the Polygon region.
   √ ✓ ✓ x
   x ✓ √ x
   x ✓ x x
5. a) Quadrilaterals =
      A B G
      Parallelograms = E F
      Perpendicular diagonals = C
      Region where ovals overlap = D H
   b) D H
   c) At the intersection of parallelograms and perpendicular diagonals.
6. Yes. A parallelogram is a quadrilateral that has opposite sides parallel.
   A square is a quadrilateral because it has 4 sides. Its opposite sides are parallel.
BONUS
   If you draw two right angles, the lines will be parallel and never meet. So you cannot draw a triangle with two right angles.
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>(\frac{3}{8})</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(\frac{4}{8})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>= 8 \times 12 \text{ in} = 96 \text{ in}</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>= 48 \text{ in} + 8 \text{ in} = 4 \text{ ft 8 in}</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>= 36 \text{ in} + 9 \text{ in} = 45 \text{ in}</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>= 2.5 \times 3 \text{ ft} = 7.5 \text{ ft}</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>= 27 \text{ ft} + 2 \text{ ft} = 3 \text{ yd 2 ft}</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>= \frac{3}{4} \times 1,760 \text{ yd} = 1,320 \text{ yd}</td>
<td>1</td>
<td>6/</td>
</tr>
<tr>
<td>Bonus</td>
<td>a) Gives correct answer</td>
<td>68 \text{ yd} = 68 \times 3 \text{ ft} = 204 \text{ ft}</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>2 \times 204 \text{ ft} + 2 \times 80 \text{ ft} = 408 \text{ ft} + 160 \text{ ft} = 568 \text{ ft}</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** 8
### Quiz (Lessons 14 to 16), p. U-28

**Common Core State Standards Emphasized:** 5.MD.A.1, 5.NBT.B.5, 5.NBT.B.7, 5.NF.A.2, 5.NF.B.3, 5.NF.B.4, 5.NF.B.6

<table>
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<tr>
<th>Question</th>
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<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>[ \frac{68}{8} \text{ in} + 32 \text{ in} = \frac{100}{8} \text{ in} ]</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>8 ft = 8 \times 12 \text{ in} = 96 \text{ in} \text{ The bunk bed is not a good choice because when Kim sits up, her height is greater than 96 in.}</td>
<td>1.5 (0.5)</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>5 \times 16 \text{ oz} = 80 \text{ oz}</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>3 \times 16 \text{ oz} + 6 \text{ oz} = 54 \text{ oz}</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>3</td>
<td>Gives correct answer</td>
<td>1,000 \times 3.5 \text{ oz} = 3,500 \text{ oz} \text{ 3,500 oz ÷ 16 = 218 lb 218 lb of muffins were sold.}</td>
<td>2</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>1,000 muffins = 100 groups of 10 muffins \text{ 100 \times 11 oz = 1,100 oz 1,100 oz ÷ 16 = 68 lb 68 lb 12 oz of flour was used.}</td>
<td>yes / no</td>
<td>/2</td>
</tr>
</tbody>
</table>

**Total Points** /7
### Scoring Guides for Sample Unit Quizzes and Tests

#### Unit 5: Measurement and Data

*(continued)*

#### Test (Lessons 10 to 22), p. U-30

**Common Core State Standards Emphasized:** 5.MD.A.1, 5.MD.B.2, 5.NBT.B.5, 5.NBT.B.7, 5.NF.A.1, 5.NF.A.2, 5.NF.B.3, 5.NF.B.4, 5.NF.B.6, 5.NF.B.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 a)</td>
<td>Gives correct answer</td>
<td>10:45, 13:20</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1 b)</td>
<td>Gives correct answer</td>
<td>Tony exercised for 2 hours and 35 minutes</td>
<td>2 (0.5)</td>
<td>2/3</td>
</tr>
<tr>
<td>2 a)</td>
<td>Gives correct answer</td>
<td>1 h 30 min</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2 b)</td>
<td>Gives correct answer</td>
<td>90 min</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2 c)</td>
<td>Gives correct answer</td>
<td>9 h</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2 d)</td>
<td>Gives correct answer</td>
<td>$216</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gives correct answer</td>
<td>Teacher to check diagram (1 block = 10 oz). There are 20 oz of flour left.</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4 a)</td>
<td>Gives correct answer</td>
<td>$5 \times 16 \text{ oz} = 80 \text{ oz}$ $15 \times 6 \text{ oz} = 90 \text{ oz}$ The employee cannot place 15 bolts that weigh 6 oz each in the box.</td>
<td>2 (1)</td>
<td></td>
</tr>
<tr>
<td>4 b)</td>
<td>Gives correct answer</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4 c)</td>
<td>Gives correct answer</td>
<td>$\frac{1}{8} + \frac{4}{8} + \frac{12}{8} + \frac{10}{8} + \frac{6}{8} + \frac{7}{8}$ $= \frac{40}{8} = 5 \text{ lb}$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5 a)</td>
<td>Gives correct answer</td>
<td>4 yd = 12 ft 12 ft is greater than 11 ft 6 in so the pieces are shorter than 1 yd each.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5 b)</td>
<td>Gives correct answer</td>
<td>$11 \times 12 \text{ in} + 6 \text{ in}$ $= 138 \text{ in}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5 c)</td>
<td>Gives correct answer</td>
<td>34.5 in</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>$1,000 \times 60 \text{ min}$ $= 60,000 \text{ min}$ $60,000 \text{ min} \times 60 \text{ sec}$ $= 3,600,000 \text{ sec}$</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** 21
<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
<td>2, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Rubric for Unit 5: Measurement and Data
Test (Lessons 10 to 22), p. U-30 (continued)

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1 Can answer few, if any, questions accurately and independently.</th>
<th>Level 2 Can answer some questions accurately and independently.</th>
<th>Level 3 Can answer most questions accurately and independently.</th>
<th>Level 4 Can answer all or almost all questions, including bonuses, accurately and independently.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 &lt; 1/2.</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 6: Measurement and Data

**Quiz (Lessons 23 to 25), p. U-33**

**Common Core State Standards Emphasized:** 5.NF.B.4b, 5.NBT.B.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>Area = 8 m × 7 m = 56 m²</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>Area = 33 ft × 12 ft = 396 ft²</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Gives correct answer</td>
<td>Area = 42 squares = 42 × 1(\frac{1}{16}) in² = 42(\frac{1}{16}) in²</td>
<td>2 (1)</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer but does not reduce to lowest terms</td>
<td>Area = (\frac{7}{4}) in × (\frac{6}{4}) in = 42(\frac{1}{16}) in² = 21(\frac{1}{8}) in²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>3 cm × (\ell) cm = 27 cm² (\ell = \frac{27}{3}) (\ell = 9) cm</td>
<td>2 (0.5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation but wrong answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>10 m × (\ell) m = 58 m² (\ell = \frac{58}{10}) (\ell = \frac{8}{5}) m</td>
<td>2 (0.5)</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct equation but wrong answer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Draws line to divide the shape.</td>
<td>Area of rectangle 1 = (\frac{1}{4}) ft × 2 ft = (\frac{5}{4}) ft × (\frac{2}{1}) ft = (\frac{10}{4}) ft² = (\frac{5}{2}) ft²</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct area for both rectangles</td>
<td>Area of rectangle 2 = 1 ft × (\frac{5}{2}) ft = (\frac{5}{2}) ft²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct total area</td>
<td>Total area = (\frac{5}{2}) ft² + (\frac{5}{2}) ft² = (\frac{10}{2}) ft² = 5 ft²</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>6 cm</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points** /13
Scoring Guides for Sample Unit Quizzes and Tests
Unit 6: Measurement and Data

Test (Lessons 23 to 32), p. U-35

Common Core State Standards Emphasized: 5.MD.C.3a, 5.MD.C.3b, 5.MD.C.4, 5.MD.C.5a, 5.MD.C.5b, 5.MD.C.5c, 5.NF.B.4b, 5.NBT.B.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>60.6 cm²</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(\frac{3}{8}) yd²</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>12 in³</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>24 cm³</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>Length = 2 ft, Width = 3 ft, Height = 5 ft, Volume = (2 \times 3 \times 5 = 30 \text{ ft}^3)</td>
<td>3</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>Length = 11 m, Width = 5 m, Height = 6 m, Volume = (11 \times 5 \times 6 = 330 \text{ m}^3)</td>
<td>3</td>
<td>(2)</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct answer</td>
<td>[Diagram of a box with dimensions]</td>
<td>2</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>No. The units were not the same for each measurement</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>1.75 m = 175 cm, Volume = 595,000 cm³</td>
<td>2</td>
<td>/3</td>
</tr>
<tr>
<td>6</td>
<td>Gives correct answer</td>
<td>Volume A = 20 in × 32 in × 20 in = 12,800 in³, Volume B = 85 in × 35 in × 20 in = 59,500 in³, Total volume = 72,300 in³</td>
<td>4</td>
<td>(2)</td>
</tr>
</tbody>
</table>
### Question How to Score Answer Number of Points Total Points

| Bonus | Gives correct answer | Volume A  
= 10 m × 10 m × 15 m  
= 1,500 m³  
Volume B  
= 10 m × 10 m × 8 m  
= 800 m³  
Volume C  
= 40 m × 10 m × 7 m  
= 2,800 m³  
Total volume  
= 5,100 m³ | yes / no |
|---|---|---|
| Advanced  
7 | a) Gives correct answer | 20 cm² | 2 |
| | b) Gives correct answer | 9 m² | 2 |
| Total Points | /19 |
## Rubric for Unit 6: Measurement and Data

### Test (Lessons 23 to 32), p. U-35

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.C.3a</td>
<td>A cube with side length 1 unit, called a “unit cube,” is said to have &quot;one cubic unit&quot; of volume, and can be used to measure volume.</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.C.3b</td>
<td>A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.C.4</td>
<td>Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</td>
<td>2, 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.C.5a</td>
<td>Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</td>
<td>2, 7 (Advanced)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.C.5b</td>
<td>Apply the formulas ( V = l \times w \times h ) and ( V = b \times h ) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</td>
<td>3, 7 (Advanced)</td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>

Name: ____________________
<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.C.5c</td>
<td>6, Bonus</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NF.B.4b</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NBT.B.7</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 6: Measurement and Data

**Test (Lessons 33 to 38), p. U-38**

**Common Core State Standards Emphasized:** 5.MD.A.1, 5.MD.C.5, 5.MD.C.5b, 5.MD.C.5c, 5.NBT.B.5, 5.NBT.B.7, 5.NF.A.1, 5.NF.B.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>281, 0.281</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>65, 0.065</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>300, 300</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>7, 7</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>0.004</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>54.231</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>g) Gives correct answer</td>
<td>40</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>h) Gives correct answer</td>
<td>6,070</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct answer</td>
<td>970 mL = 0.970 L</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Gives answer with no explanation</td>
<td>340 mL = 0.340 L</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200 mL = 0.200 L</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5 L = 1.500 L</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total = 3.010 L</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>This is greater than 3 L so a 4 L jar is better for mixing the punch.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct volume</td>
<td>Volume = 31,250 cm³</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct capacity</td>
<td>Capacity = 31.25 L</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct volume</td>
<td>Volume = 58,500 cm³</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct capacity</td>
<td>Capacity = 58.5 L</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>24,000, 24</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>4.5 L = 45,000 mL</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>45,000 ÷ 1,500 = 30</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The new height of the water is 30 cm.</td>
<td></td>
</tr>
</tbody>
</table>

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### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 6: Measurement and Data**

(continued)

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>4</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>8</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>16</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f) Gives correct answer</td>
<td>128</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g) Gives correct answer</td>
<td>28</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>h) Gives correct answer</td>
<td>$\frac{3}{4}$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>i) Gives correct answer</td>
<td>$2^{\frac{1}{4}}$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>j) Gives correct answer</td>
<td>6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>k) Gives correct answer</td>
<td>5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>l) Gives correct answer</td>
<td>20</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Bonus: m</td>
<td>Gives correct answer</td>
<td>$2^{\frac{3}{4}}$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Bonus: n</td>
<td>Gives correct answer</td>
<td>$\frac{1}{4}$</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td>Bonus: o</td>
<td>Gives correct answer</td>
<td>24, 48</td>
<td>yes / no</td>
<td>/6</td>
</tr>
</tbody>
</table>

Total Points /20
# Rubric for Unit 6: Measurement and Data
Test (Lessons 33 to 38), p. U-38

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.A.1</td>
<td>1</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.C.5</td>
<td>2, 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.MD.C.5b</td>
<td>3, 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apply the formulas (V = l \times w \times h) and (V = b \times h) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.NBT.B.5</td>
<td>3, 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Name: ____________________
### Common Core State Standard

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
</table>
| 5.NBT.B.7  
*Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.* | 4                       |         |         |         |         |
| 5.NF.B.4  
*Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.*     | 5                       |         |         |         |         |

**Comments**
### Quiz (Lessons 6 to 9), p. U-41

**Common Core State Standards Emphasized:** preparation for 5.G.B.3, 5.G.B.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer</td>
<td>4, quadrilateral</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>3, triangle</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>6, hexagon</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>5, pentagon</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Correctly identifies angle</td>
<td>obtuse</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct angle measure</td>
<td>125°</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly identifies angle</td>
<td>acute</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct angle measure</td>
<td>57°</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gives correct answer</td>
<td>Teacher to check</td>
<td>2</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct answer</td>
<td>45°</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Correctly measures the angles</td>
<td>45°, 45°</td>
<td>yes / no</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td>Correctly adds the angles</td>
<td>45° + 45° + 90°</td>
<td>yes / no</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 180°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Points</td>
<td></td>
<td>12</td>
<td></td>
</tr>
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</table>

### Quiz (Lessons 10 to 12), p. U-43

**Common Core State Standards Emphasized:** 5.G.B.3, 5.G.B.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gives correct answer for 3 categories</td>
<td></td>
<td>(2)</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 2 categories</td>
<td></td>
<td></td>
<td>/4</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 1 category</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>Polygons: L Equilateral: M All angles equal: J Overlap: K</td>
<td>4 (3)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 3 categories</td>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 2 categories</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 1 category</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Shades correct region</td>
<td>where the ovals overlap</td>
<td>1</td>
<td>/5</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>acute isosceles</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>acute equilateral</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>right scalene</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>obtuse scalene</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>C</td>
<td>yes / no</td>
</tr>
<tr>
<td></td>
<td>Total Points</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 5.2

U-81
### Test (Lessons 6 to 19), p. U-45

Common Core State Standards Emphasized: 5.G.B.3, 5.G.B.4

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives correct answer</td>
<td>right scalene triangle</td>
<td>2</td>
<td>/2</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct answer</td>
<td>trapezoid</td>
<td>2</td>
<td>/2</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>Shapes A and B are missing parallel marks</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>Polygons: F Equilateral: D Parallelogram: A, C Overlap: B, E</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Shades correct region</td>
<td>where the ovals overlap</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Draws star in correct region</td>
<td>in the Polygon region</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct answer</td>
<td>First row: ✓ ✓ ✓</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 2 rows</td>
<td>Second row: × × ✓ ×</td>
<td>(2)</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 1 row</td>
<td>Third row: × ✓ ✓ ×</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a) Gives correct answer</td>
<td>Quadrilaterals: A, B, G Parallelograms: E, F Perpendicular diagonals: C Overlap: D, H</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>D, H</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>In the overlap of parallelograms and perpendicular diagonals.</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td>6</td>
<td>Gives correct answer</td>
<td>Yes. A parallelogram is a quadrilateral that has opposite sides parallel. A square is a quadrilateral because it has 4 sides. Its opposite sides are parallel</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td>Bonus</td>
<td>Gives correct answer</td>
<td>No. If you draw two right angles, the lines will be parallel and never meet.</td>
<td>yes / no</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points /16**
## Rubric for Unit 7: Geometry
Test (Lessons 6 to 19), p. U-45

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.G.B.3</td>
<td>3, 5, 6</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>Understood that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</td>
<td>3, 5, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.G.B.4</td>
<td>3, 4, 5</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>Classify two-dimensional figures in a hierarchy based on properties.</td>
<td>3, 4, 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
Grade 5 Common Core State Standards Curriculum Correlations

NOTE: The italicized gray JUMP Math lessons contain prerequisite material for the Common Core standards.

D Domain
- OA Operations and Algebraic Thinking
- NBT Number and Operations in Base Ten
- NF Number and Operations—Fractions
- MD Measurement and Data
- G Geometry

C Cluster

5.OA  Operations and Algebraic Thinking

5.OA.A  Write and interpret numerical expressions.

<p>| 5.OA.A.1 | Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>OA5-6, 7, 12</td>
</tr>
</tbody>
</table>

<p>| 5.OA.A.2 | Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 ÷ 921) is three times as large as 18932 ÷ 921, without having to calculate the indicated sum or product. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NBT5-7</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>OA5-8 OA5-9 to 12</td>
</tr>
</tbody>
</table>

5.OA  Operations and Algebraic Thinking

5.OA.B  Analyze patterns and relationships.

<p>| 5.OA.B.3 | Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OA5-1 to 3 OA5-4, 5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>NBT5-16 NBT5-25</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>G5-5</td>
</tr>
</tbody>
</table>
### 5.NBT  Number and Operations in Base Ten

#### 5.NBT.A  Understand the place value system.

<table>
<thead>
<tr>
<th>5.NBT.A.1</th>
<th>Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NBT.A.2</td>
<td>Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</td>
</tr>
<tr>
<td>5.NBT.A.3</td>
<td>Read, write, and compare decimals to thousandths.</td>
</tr>
<tr>
<td>5.NBT.A.3a</td>
<td>Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000).</td>
</tr>
<tr>
<td>5.NBT.A.3b</td>
<td>Compare two decimals to thousandths based on meanings of the digits in each place, using &gt;, =, and &lt; symbols to record the results of comparisons.</td>
</tr>
<tr>
<td>5.NBT.A.4</td>
<td>Use place value understanding to round decimals to any place.</td>
</tr>
</tbody>
</table>

| JUMP Math Grade 5 Lessons |
| --- | --- | --- |
| Part | Unit | Lessons |
|  |  |  |
| 1 | 2 | NBT5-1, 12 |
| 1 | 3 | NBT5-17, 22 |
| 1 | 6 | NBT5-38, NBT5-39 to 45, 53 |
| 1 | 7 | MD5-1, 2, 4, 5, 7 |
| 2 | 5 | MD5-18 |
|  |  |  |
| 1 | 3 | NBT5-15, 17, 22 |
| 1 | 4 | NBT5-33 |
| 1 | 6 | NBT5-51 to 53 |
| 1 | 7 | MD5-3 to 5, 7 |
| 2 | 4 | NBT5-60 |
| 2 | 5 | MD5-18 |
|  |  |  |
| 1 | 2 | NBT5-1 to 3 |
| 1 | 6 | NBT5-38 to 45, 53 |
| 2 | 4 | NBT5-55 |
|  |  |  |
| 1 | 2 | NBT5-1 to 3 |
| 1 | 6 | NBT5-38 to 45, 53 |
| 2 | 4 | NBT5-55 |
|  |  |  |
| 1 | 2 | NBT5-9, NBT5-10, 11 |
| 1 | 6 | NBT5-49, 50 |
### 5.NBT Number and Operations in Base Ten

#### 5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

<table>
<thead>
<tr>
<th>5.NBT.B.5</th>
<th>Fluently multiply multi-digit whole numbers using the standard algorithm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NBT.B.6</td>
<td>Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
</tr>
<tr>
<td>5.NBT.B.7</td>
<td>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
</tr>
</tbody>
</table>

#### JUMP Math Grade 5 Lessons

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>NBT5-13, 14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NBT5-15 to 24, 26</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>NBT5-54, 56, 57</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-11, 14, 16, 17</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-35</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>NBT5-27 to 33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NBT5-34 to 37</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>NBT5-58, 59, 62 to 66</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>NBT5-4</td>
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<tr>
<td></td>
<td></td>
<td>NBT5-5, 6, 8, 12</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>NBT5-46 to 48</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>MD5-6 to 9</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>NBT5-54 to 67</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-13, 14, 16 to 18, 22</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-25, 34, 38</td>
</tr>
</tbody>
</table>
### 5.NF  Number and Operations—Fractions

#### 5.NF.A  Use equivalent fractions as a strategy to add and subtract fractions.

**5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

<table>
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<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>NF5-1 to 5, NF5-6, 7, NF-8 to 12, NF5-13 to 16, 19</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-36 to 38</td>
</tr>
</tbody>
</table>

**5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

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</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>NF5-1 to 6, 8 to 16, 19</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NF5-31 to 33</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-16</td>
</tr>
</tbody>
</table>

#### 5.NF.B  Apply and extend previous understandings of multiplication and division.

**5.NF.B.3** Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>NF5-19</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NF5-20</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-11, 12, 14 to 16</td>
</tr>
</tbody>
</table>

**5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

**5.NF.B.4a** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

<table>
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<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>NF5-8, 17, 18</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NF5-21 to 23, 25</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-13, 15</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-36 to 38</td>
</tr>
<tr>
<td>5.NF.B.4b</td>
<td>Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
<td></td>
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<tr>
<td>---</td>
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<td></td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
<td>Lessons</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>NF5-8, 17, 18</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NF5-21 to 23, 25</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-13, 15</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-23, MD5-24, 25, 36 to 38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5.NF.B.5</th>
<th>Interpret multiplication as scaling (resizing), by:</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NF.B.5a</td>
<td>Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</td>
</tr>
<tr>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
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<tr>
<td>2</td>
<td>5</td>
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</tbody>
</table>

| 5.NF.B.5b | Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence \( \frac{a}{b} = \left( \frac{n}{n} \times \frac{a}{b} \right) \) to the effect of multiplying \( \frac{a}{b} \) by 1. |
| Part | Unit | Lessons |
| 2 | 2 | NF5-24 |
| 2 | 5 | MD5-13 |

| 5.NF.B.6 | Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. |
| Part | Unit | Lessons |
| 1 | 5 | NF5-18 |
| 2 | 1 | G5-4 |
| 2 | 2 | NF5-26, 29, 30, 33 |
| 2 | 3 | OA5-13 |
| 2 | 5 | MD5-13, 16, 22 |

| 5.NF.B.7 | Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. |
| 5.NF.B.7a | Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. \( \text{For example, create a story context for } \frac{1}{3} \div 4, \text{ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that } \frac{1}{3} \div 4 = \frac{1}{12} \text{ because } (1/12) \times 4 = 1/3. \) |
| Part | Unit | Lessons |
| 2 | 2 | NF5-27, 28 |
| 2 | 5 | MD5-12 |

| 5.NF.B.7b | Interpret division of a whole number by a unit fraction, and compute such quotients. \( \text{For example, create a story context for } 4 \div \frac{1}{5}, \text{ and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that } 4 \div \frac{1}{5} = 20 \text{ because } 20 \times \frac{1}{5} = 4. \) |
| Part | Unit | Lessons |
| 2 | 2 | NF5-27, 28 |
| 2 | 5 | MD5-12 |
5.NF.B.7c  Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

<table>
<thead>
<tr>
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<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>NF5-27, 28, 33</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-12</td>
</tr>
</tbody>
</table>
### 5.MD Measurement and Data

<table>
<thead>
<tr>
<th>5.MD.A</th>
<th>Convert like measurement units within a given measurement system.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.A.1</td>
<td>Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>MD5-1 to 9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-11 to 16, 19 to 22</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-33 to 38</td>
</tr>
</tbody>
</table>

### 5.MD Measurement and Data

<table>
<thead>
<tr>
<th>5.MD.B</th>
<th>Represent and interpret data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.B.2</td>
<td>Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>MD5-10, MD5-17, 18</td>
</tr>
</tbody>
</table>

### 5.MD Measurement and Data

<table>
<thead>
<tr>
<th>5.MD.C</th>
<th>Geometric measurement: understand concepts of volume.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.C.3</td>
<td>Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</td>
</tr>
</tbody>
</table>

<table>
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<tr>
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<th>Lessons</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-27</td>
</tr>
</tbody>
</table>

| 5.MD.C.3a | A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. |
| 5.MD.C.3b | A solid figure which can be packed without gaps or overlaps using \( n \) unit cubes is said to have a volume of \( n \) cubic units. |

| 5.MD.C.4 | Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. |

| 5.MD.C.5 | Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. |

| 5.MD.C.5a | Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. |

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</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>MD5-27, 29, MD5-33, MD5-38</td>
</tr>
</tbody>
</table>
5.MD.C.5b  Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.  

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 2    | 6    | MD5-27, 29, 30  
|      |      | MD5-33  
|      |      | MD5-35, 38  |

5.MD.C.5c  Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.  

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 2    | 6    | MD5-31, 32  
|      |      | MD5-33  
|      |      | MD5-35, 38  |
## 5.G Geometry

### 5.G.A Graph points on the coordinate plane to solve real-world and mathematical problems.

<table>
<thead>
<tr>
<th>Part</th>
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<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>G5-1, 2, 5</td>
</tr>
</tbody>
</table>

#### 5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., \(x\)-axis and \(x\)-coordinate, \(y\)-axis and \(y\)-coordinate).

#### 5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>G5-3, 4</td>
</tr>
</tbody>
</table>

### 5.G.B Classify two-dimensional figures into categories based on their properties.

#### 5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>G5-6 to 9, G5-10 to 19</td>
</tr>
</tbody>
</table>

#### 5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>G5-6 to 9, G5-11 to 14, 16 to 19</td>
</tr>
</tbody>
</table>