Grade 6

End of Year Teacher Pack

*Includes:*

- Lesson Plans
- Blackline Masters
- Assessment & Practice Book Answers
- Quizzes and Unit Tests
- Rubrics for Scoring Quizzes and Tests
- Common Core State Standards Correlations
Unit 5  Geometry: Coordinate Grids

This Unit in Context

In Grade 5, students used a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. They understood that the first number indicates how far to travel from the origin in the direction of one axis and that the second number indicates how far to travel in the direction of the other axis, and followed the convention that the names of the two axes and the coordinates correspond—for example, x-axis and x-coordinate, y-axis and y-coordinate (5.G.A.1). They also represented real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpreted coordinate values of points in the context of a given situation (5.G.A.2).

In this unit, students will extend their knowledge of coordinate grids to include all four quadrants. They will learn about horizontal and vertical lines and how to find distance on these lines. Students will also expand their understanding of properties of two-dimensional shapes using coordinate systems, draw polygons in the coordinate plane with given coordinates for the vertices, and use coordinates to find the length of a side that joins points with the same first coordinate or the same second coordinate. Finally, they will apply these techniques in the context of solving real world and mathematical problems (6.G.A.3).

Students will use their knowledge of coordinate grids in Grade 8 to understand the properties of rotations, reflections, and translations (8.G.A.1), and to understand and apply the Pythagorean Theorem (8.G.B.6).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP 1 and MP 3 to MP 7. Here are some examples of how students can show that they have met a standard.

MP 3: In G6-21 Extension 3, students make a conjecture about what the distance will be between any number and its opposite, and construct a viable argument to explain their conjecture.

MP 5: In G6-24 Extension 2, students use tools strategically when they choose between a table or a tape diagram to represent and solve a real-world problem.

MP 6: In G6-22 Extension 3, students attend to precision when they explain what symbols and calculations are correct and incorrect in the question, including why a proportion was not written correctly. In G6-24 Extension 2, students attend to precision when they use clear labels in a table or tape diagram to help solve a real-world problem. In G6-21 Extension 3, students attend to precision when they use the definitions of opposite numbers and absolute value to explain a conjecture.
MP.7: In G6-26 Extension 1, students look for and make use of structure when they recognize that the missing points in a parallelogram are determined by the height (which is in turn determined by dividing the given area by the length of the base) and the property that parallelograms have two pairs of parallel sides. In G6-24 Extension 1, students notice structure when they recognize that a given collection of points forms a shape that can be divided into two rectangles. Students make use of this structure when they calculate the area of the hexagon by adding the areas of the two rectangles.
Unit 5  Geometry: Coordinate Grids

Introduction

In this unit, students will identify and plot points in all four quadrants of the coordinate plane, including points with decimal and fractional coordinates, and find distances between points on the same vertical or horizontal line. Students will also plot polygons and use coordinates to find their area.

Materials

In all lessons of this unit, you will need a pre-drawn grid on the board. If you do not have such a grid, you can photocopy BLM 1 cm Grid Paper (p. S-1) onto a transparency and project it onto the board. This will allow you to draw and erase shapes and lines on the grid without erasing the grid itself.

To save time, students can use ready-made coordinate grids on BLM Coordinate Grids (Four Quadrants) (p. N-31), but be sure that students are also able to draw the grids.

Students will use the cards on BLM Ordered Pair Cards (pp. N-32 and N-33) many times in this unit. To save time and effort, cut out and laminate one set of cards for each pair of students in the class.
Review coordinates in the first quadrant. Draw a coordinate grid (first quadrant only) on the board. Have students copy the grid. Review the vocabulary words that students should already know and the meaning of each word. Remind students how to find the coordinates of a point in the first quadrant and how to find points by their coordinates.

Exercises: Draw a coordinate grid on the board (first quadrant only), and mark the following points: A (1, 1), B (2, 3), C (3, 1), D (0, 4).

1. Find the coordinates of each point on the grid.
2. Mark the following points on the coordinate grid:
   - E (1, 2)  F (3, 4)  G (4, 3)  H (4, 1)  O (0, 0)

Review locating halves and quarters on a number line. Draw a number line from 0 to 5 and ask students where the point 0.5 is located. Invite a volunteer to show the location. Repeat with 1/4, 2.5, 3 3/4, 4 1/2.

Halves and quarters on a coordinate grid. Draw a large coordinate grid (first quadrant only), and mark the point (1/2, 2 1/4) on it. Draw dashed lines from the point to the axes and have students identify the coordinates of the point. Repeat with several other points; then have students draw a grid and mark several points on it.

Exercises
a) Draw a coordinate grid.

b) Mark the points on the coordinate grid.
   - A (1.5, 2)  B (3, 3.5)  C (1/4, 3)  D (4 1/2, 1)  E (0.5, 0.5)  F (4 3/4, 4 1/4)
Review negative numbers on a number line. Remind students that number lines can be extended in both directions.

Introduce the quadrants. Display another coordinate grid. Make sure the axes are centered on the grid so that all four quadrants are visible. Label the axes and the origin. Point out that the axes separate the grid into four parts. Explain that these are called quadrants; ask students what words they know that start with “quad.” (e.g., quadrilateral, quadruple) Point out the connection these words have to the number 4. Show how the quadrants are numbered, from the top right quadrant and continuing in a counterclockwise direction. Explain that we use Roman numerals to number the quadrants. Draw several points in various quadrants and ask students to tell which quadrant the points are in. They can signal the answer by holding up 1 to 4 fingers.

Points in first and third quadrants. Add numbers to the axes. Draw students’ attention to the fact that the points in the first quadrant each have two positive coordinates, so all the points they plotted until now were actually plotted in the first quadrant of the coordinate grid or on the axes.

Now mark the point (−1, −1) and ask students what the coordinates of this point might be. Repeat with (−2, −3). Then give students a coordinate pair, such as (−6, −1), and point at several locations on the grid. Have students signal thumbs up or thumbs down whether the location you are pointing at is the point (−6, −1). Repeat with (−1, −4), (−3, −4), and (−5, −2).

Mark a point in the second quadrant, such as (−3, 4), and show students how to find which quadrant the point is in, depending on the signs of the coordinates—the point is on the negative side of the x-axis and the positive side of the y-axis, so the point is in the second quadrant. Repeat with a point in the fourth quadrant.

Exercise: Hold up 1 to 4 fingers to signal in which quadrant each of these points appear.

(−2, 3) (6, −1) (2, −5) (−3, 2) (4, −6) (2, −2) (−100, 200) (−350, −245) (234, −1,789)

Show students how to find the complete coordinates of a point in any quadrant (number and sign). Then mark several points on the grid and have students determine the coordinates. Then do the reverse. Students can use the grids on BLM Coordinate Grids (Four Quadrants) instead of drawing grids.

Exercise: Mark the points on a coordinate grid.

A (2, 3) B (5, 4) C (−3, −3) D (−4, −5) E (−3, 3) F (3, −3) G (−4, 7) H (3, −6)

Points on axes. Ask students where the point (0, 8) is located. (on the top half of the y-axis) Point out that axes do not belong to any quadrant—they are the border lines. Give pairs of coordinates such as (0, 4) or (−8, 0), and point to different locations on the axes. Students should signal whether the
point you are pointing to is the correct one. Then mark several points on the axes and have students find the coordinates of these points. Students need lots of practice plotting points in different quadrants.

**ACTIVITY**

Students will work in pairs. Each pair of students will need two dice of different colors. One player rolls the dice. Players record the results as a pair of coordinates for the grid, with the signs given by the spinner from BLM Spinner Template (see margin for directions on how to mark up the spinner). For example, if a student had one red die and one blue die, the number on a red die could be the \(x\)-coordinate and the number on the blue die could be the \(y\)-coordinate. Students then spin the spinner and record the signs with the coordinates. Next, students plot on grid paper a point that has that pair of coordinates. Students can check each other’s answers.

**Advanced:** The player then rolls the dice a second time and obtains a second point in the same way. The player joins the points with a line, then has to draw a rectangle with vertical and horizontal sides so that the line is a diagonal of the rectangle. Ask students to look for a pattern in the coordinates of the vertices.

**Extensions**

a) Plot and join the points, in order. Use the same grid for all parts.

   i) \((-1, -2), (-1, 3), (0, 4), (1, 3), (1, -2), (0, -3)\). Join the first point to the last point.

   ii) \((-1, -2), (-2, -3), (-3, -3), (-3, -2), (-1, 0)\)

   iii) \((1, 0), (3, -2), (3, -3), (2, -3), (1, -2)\)

b) What shape did you make?

c) Find the area of the shape.

**Selected answers:** b) rocket, c) 19 square units
Review marking decimals and fractions on a number line. Draw a number line divided into tenths, from $-2.5$ to $3.5$, but mark only the whole numbers. Point to a few locations on the line and have students identify the decimal for each location. Write several decimals and point at locations on the number line. Have students signal whether the location you are pointing at is the decimal in question. Keep the number line on the board.

Review the fact that some fractions are equivalent to decimal tenths. Go through the example below as a class, then locate $1 \frac{2}{5}$ on the number line. Repeat with $-1 \frac{3}{5}$, $-\frac{1}{4}$, $1 \frac{3}{4}$, and $1 \frac{1}{2}$.

\[
\begin{align*}
1 \frac{2}{5} &= 1 \frac{2 \times 2}{10} = 1 \frac{4}{10} \\
&= 1.4
\end{align*}
\]

Draw a number line divided into quarters with increments for half slightly longer than increments for quarters, but shorter than increments for whole numbers, as shown in the margin, and review placing halves and quarters on it, using the same two exercises you did for the decimal number line.

Finally draw a number line from $-3$ to $3$, with $10$ cm between the increments. Show students how to use a ruler to mark the exact location of points such as $2.6$ and $-1.8$.

**Exercises:** Draw a number line with $1$ cm between the increments. Use a centimeter ruler to place the points.

a) $-2.4$  

b) $1.9$  

c) $-0.2$  

d) $0.8$  

e) $3.1$
Review using signs to place points in different quadrants.
Remind students how the quadrants are numbered and what the signs of coordinates for different quadrants are. Point out that for the next question students do not need to know the exact location. They only need to identify the quadrant.

**Exercises:** Identify the quadrant each point is in. (Students can signal the answers.)

A (2.45, 3.97)  B (3 \frac{1}{21}, \frac{33}{298})  C (-\frac{33}{298}, -\frac{33}{298})  D (-4.76, -5.2)

E (-3.03, 3.678)  F (45 \frac{1}{12}, -678 \frac{1}{5})  G (-4.01, 7.999)  H (5 \frac{2}{9}, -623)

Finally, have students write the coordinates for a total of eight points—four in decimals and four in fractions. One point should be in each of the four quadrants, and one point should be on each half of each axis. Students can write coordinates on small cards, writing the quadrant on the back, and exchange cards (see example in the margin). Partners have to identify the quadrant/part of the axis the points are on and can check their answers on the back.

**Extensions**

1. Plot the points on a grid. Use a ruler to join them in order.
   
   a) (-6, 1), (3, 0), (2.5, -1), (0, -1), (-2.5, -0.5), (-3, 0.5). Join the last point to the first one.
   
   b) (0, \frac{1}{3}), (0, 9), (2, 6), (3 \frac{1}{2}, 1 \frac{1}{2}), (0, 1)
   
   c) (0, 1), (-\frac{2}{3}, 2), (-1, 6), (0, 9), (-6, 1), join to (-\frac{2}{3}, 2)

   If you drew all three parts, what did you draw? (sailboat)

2. Predict which point is lower on the y-axis: (0, 2.7) or (0, 2 \frac{3}{4}).
   
   Explain your prediction. Plot the points and check your answer.

3. What is the distance between any number and its opposite? Explain how you know. Use the terms “opposite number” and “absolute value,” and refer to their definitions.

   **Answer:** A number and its opposite are the same distance from 0, but in opposite directions, so the distance between any number and its opposite is twice the distance between the number and 0. And the distance from 0 is the absolute value of the number, so the distance between any number and its opposite is double the absolute value of the number.
Points on a vertical line have the same \( x \)-coordinate. Display a coordinate grid and mark the points \((4, -3), (4, 0), \) and \((4, 2)\). Ask students to identify the coordinates of the points. ASK: What do the coordinates of these points have in common? (they all have 4 for the first coordinate) Remind students that the first coordinate is called the \( x \)-coordinate, and we say that these points have \( x = 4 \). Draw the line through the three points and extend the line all the way up and down the grid. Point to different locations on the line and ask students what the \( x \)-coordinate of the point is. (4)

Draw another vertical line, for example, through \((-2, 3)\). Ask students to predict what the \( x \)-coordinate will be for different points on this line. (−2) Then ASK: Will the point \((-2, -5)\) be on this line? (yes) Have a volunteer mark the point to check. Will the point \((-2, 156)\) be on this line? (yes) You cannot draw the point, so how do you know? (the \( x \)-coordinate is −2) What about \((-3, 3)\)? (no) How do you know? (the \( x \)-coordinate is −3, not −2) Again, have a volunteer mark the point and check. Then present several pairs of coordinates and have students signal whether the point is on the line.

**Exercises:** Are the pairs of points on the same vertical line?

\[
a) \quad (2, 9) \text{ and } (2, 81) \quad \quad b) \quad (-12, -9) \text{ and } (-2, -9) \\
\quad (c) \quad (-4.5, -1) \text{ and } (-4.5, 1.87) \quad \quad d) \quad (53.1, 4) \text{ and } (53.1, 8) \\
\quad e) \quad \left(\frac{1}{2}, 7\right) \text{ and } \left(\frac{1}{2}, -\frac{1}{2}\right) \quad \quad f) \quad \left(-\frac{1}{2}, -\frac{1}{4}\right) \text{ and } \left(-\frac{1}{4}, -\frac{1}{4}\right)
\]

**Answers:** a) yes, b) no, c) yes, d) yes, e) yes, f) no; students can signal the answers

Points on the same horizontal line have the same \( y \)-coordinate. Repeat the first part of the lesson for horizontal lines.
### Exercises: Are the pairs of points on the same horizontal line?

a) (5, 9) and (2, 9)  
b) (−12, −3) and (−2, −3)  
c) (−4, −1.25) and (−4, 1.87)  
d) (53.1, 4.3) and (0.1, 4.3)  
e) \(\left(\frac{1}{2}, -\frac{1}{2}\right)\) and \(\left(3\frac{1}{2}, -\frac{1}{2}\right)\)  
f) \(\left(-\frac{1}{2}, \frac{1}{4}\right)\) and \(\left(\frac{1}{4}, -\frac{1}{2}\right)\)

### Answers: a) yes, b) yes, c) no, d) yes, e) yes, f) no; students can signal the answers

### Distinguishing between horizontal and vertical lines.

Show students how to sketch the location of the points to determine whether they are on the same horizontal line or the same vertical line. For example, are the points \((-3.4, 2)\) and \((-3.4, -1.8)\) on the same horizontal line or the same vertical line? (same vertical line) The first coordinates are the same, so the points are definitely on the same line. To help students quickly figure out whether the line is horizontal or vertical, you can draw a sketch as shown in the margin. The line is vertical because the points have the first coordinates in common.

### Exercises: Are the points on the same horizontal line or the same vertical line? Is the line vertical or horizontal? Sketch the location of the points to check your answers.

a) (5, 2) and (2, 2)  
b) (−12, 3) and (−12, −3)  
c) (−4, −1.25) and (−4, 1.87)  
d) (53.1, −4.3) and (0.1, 5.8)  
e) \(\left(2\frac{1}{2}, -\frac{2}{5}\right)\) and \(\left(3\frac{1}{2}, -\frac{2}{5}\right)\)  
f) \(\left(-\frac{1}{3}, \frac{1}{3}\right)\) and \(\left(\frac{1}{3}, -\frac{1}{3}\right)\)

### Answers: a) yes, horizontal; b) yes, vertical; c) yes, vertical; d) no, neither; e) yes, horizontal; f) no, neither; students can signal the answers

### ACTIVITY

Object of the game: Collect all the cards so that the “horizontal” and “vertical” piles have the same number of pairs.

**Preparation:** Place the laminated cards from BLM Ordered Pair Cards face down in a rectangle.

**Instructions:** Players take turns turning up two cards at a time. If the points given by the coordinates on the cards are on the same horizontal line, the cards are placed in the “horizontal” pile. If the points are on the same vertical line, the cards are placed in the “vertical” pile. Partners check that the decision about the pile is correct. If the points are not on the same vertical or horizontal line, the player turns the cards back down and the next player takes a turn.

**Strategy:** Players should keep track of the number of pairs in each pile as they play. The player can choose to put the pair of cards back in the stack if the cards will increase the wrong pile.
Extensions

1. Review conversion between fractions and decimals before assigning this exercise. Are the points on the same horizontal line or the same vertical line? Is the line horizontal or vertical? Explain how you know.
   a) \((-\frac{1}{2}, -\frac{1}{4})\) and \((-0.5, 11)\)
   b) \((3, -0.25)\) and \((\frac{1}{4}, -\frac{1}{4})\)
   c) \((\frac{2}{5}, 56)\) and \((0.4, -2)\)
   d) \((4.3, -\frac{1}{5})\) and \((4.35, -0.5)\)
   e) \((-3\frac{1}{25}, -3\frac{7}{25})\) and \((-3.25, -3.725)\)
   f) \((-3\frac{1}{25}, -3\frac{7}{25})\) and \((-3.04, -3.725)\)
   g) \((-3\frac{1}{25}, -3\frac{7}{25})\) and \((-3.25, -3.28)\)

   Answers: a) yes, vertical, \(-1/2 = -0.5\); b) yes, horizontal, \(-0.25 = -1/4\); c) no, neither; d) no, neither; e) no, neither; f) yes, vertical, \(-3 \ 1/25 = -3.04\); g) yes, horizontal, \(-3 \ 7/25 = -3.28\)

2. Use BLM Inequalities on a Coordinate Grid (p. N-35) and coordinates to determine whether a point is on the right or on the left of a given vertical line, and above or below a given horizontal line.

   (MP6)

3. Vicky bought 12 bus tickets for $9. She calculated how much 36 bus tickets cost as follows:
   \[
   \frac{12}{9} = \frac{x}{36}, \text{ and } 4 \times 9 = 36, \text{ so } x = 4 \times 12 = 48, \text{ so 36 bus tickets cost$48.}
   \]
   a) Do you agree with Vicky’s answer? Why or why not?
   b) What did Vicky do correctly? What did she do incorrectly?
   c) How much would 36 bus tickets cost? Explain.

   Sample answers
   a) No, because the cost of 12 tickets is less than $12, so the cost of 36 tickets should be less than $36.
   b) She solved the proportion correctly, and interpreted what x means correctly, but she wrote the proportion wrong in the first place. She should have written \(\frac{12}{9} = \frac{36}{x}\).
   c) To solve the proportion, I solved \(36 \div 12 = 3, \text{ so } x = 3 \times 9 = 27. \) So 36 tickets will cost $27.
Review finding distance between points on the same vertical or horizontal line on a grid. Show a coordinate grid and mark points (2, 4) and (−3, 4). ASK: What is the distance between these points? (5 units) Repeat with points (2, 2) and (−3, 2), then (2, −1) and (−3, −1). SAY: These three pairs of points are directly above each other. How are the coordinates of the points above each other the same? (they have the same x-coordinate) Does the distance between the points in each pair depend on how high on the grid the pair is? (no) Point out that the points in each pair have the same y-coordinate; therefore, they are on the same horizontal line and the distance does not depend on the y-coordinate. The distance between the points on the same horizontal line only depends on the x-coordinates of the points.

Repeat the exercise with three pairs of points on the same vertical line: (−3, −1) and (−3, 4); (0, −1) and (0, 4); and (2, −1) and (2, 4). Emphasize that the distance between points on the same vertical line only depends on the y-coordinates of the points.

**Review finding distance between points on a number line.** Draw a number line from −5 to 5, and remind students how to find the distance between two points on the same side of zero:

- If both numbers are positive, subtract the numbers. The distance between 2 and 5 is 5 − 2 = 3.
- If both numbers are negative, subtract the opposites: −5 − (−2) = 3, so the distance between −5 and −2 is 3.
Remind students that the absolute value of a number is the distance from the number to zero. This means that, using the absolute value, they can rewrite the rules for two positive numbers and two negative numbers as “subtract the absolute values”: \(|−5| − |−2| = 5 − 2 = 3\). Show this using the picture in the margin.

Draw the second picture in the margin. Ask students how they would find the distance between a positive and a negative number, for instance, \(−2\) and \(5\). When two numbers have different signs, they are on different sides of 0. To find the distance between the numbers, add the distances to 0. So the distance from \(−2\) to \(5\) is \(2 + 5 = 7\). In absolute value notation, we write \(|−2| + |5| = 2 + 5 = 7\).

**Exercises:** Find the distance between the numbers.

a) 7 and 13  
 b) \(−12\) and \(−6\)  
 c) \(−11\) and \(−4\)  
 d) \(−3\) and \(6\)  
 e) \(−6\) and \(−1\)  
 f) \(−4\) and \(9\)  
 g) \(−12\) and \(23\)  

**Bonus:** \(−123\) and \(71\)

**Sample solution:** b) \(12 − 6 = 6\) OR \(|−12| − |−6| = 12 − 6 = 6\)

**Answers:** a) 6, b) 6, c) 7, d) 9, e) 5, f) 13, g) 35, Bonus: 194

**Distance to the axis.** Remind students that the coordinates of a point are directly related to the distances from the point to the axes. Draw the point \((1, −5)\) on a coordinate grid, and ASK: What is the distance from this point to the \(x\)-axis? (5) Where do we see that 5 in the coordinates? (the \(y\)-coordinate is \(−5\)) SAY: The absolute value of the \(y\)-coordinate is the distance from the point to the \(x\)-axis. Repeat with the distance to the \(y\)-axis.

**Find distances between points on the coordinate grid.** Draw on the board the rough sketch in the margin. SAY: I sketched the points \((1, −5)\) and \((−3, −5)\) on the same horizontal line. Does it make sense? (yes, the points have the same second coordinate) I sketched them in different quadrants. Does this make sense? (yes, their first coordinates have different signs) Draw an arrow showing the distance from \((−3, −5)\) to the \(y\)-axis, and ask students what the distance is. (3) Repeat with the distance from \((1, −5)\) to the \(y\)-axis. ASK: What is the distance between the points? (4) Repeat with the second picture in the margin, this time showing the distance to the \(x\)-axis.

Repeat with the pictures below.

a) \((-5, 1)\) \((-2, 1)\)  
 b) \((1, 4)\) \((1, 2)\)

Tell students that they will now need to do the sketches themselves. Do the first few pairs as a class, then have students work independently on the rest of the questions.
Exercises: Sketch the pair of points and find the distance.

a) \((-2, -2)\) and \((-2, 4)\)  
b) \((4, -1)\) and \((4, 5)\)

c) \((-4, 1)\) and \((-3, 1)\)  
d) \((5, -2)\) and \((1, -2)\)

e) \((3, 0)\) and \((5, 0)\)  
f) \((-25, -5)\) and \((-25, -1)\)

Before starting the Activity, use fractions with denominators 2, 4, and 5 to review converting fractions to decimals. On the Ordered Pairs Cards that have coordinates with fractions and mixed numbers, struggling students can rewrite those coordinates as decimals.

**ACTIVITY**

**Object of the game:** Students work in pairs to make the “horizontal” and “vertical” sums as close to the same total as possible.

**Preparation:** Stack the laminated cards from BLM Ordered Pairs Cards face down. Select 6 cards and place them on the table, face up. Divide a sheet of paper into two columns. Write the heading “Horizontal” at the top of one column and the heading “Vertical” at the top of the other column.

**Instructions:** Players take turns choosing a pair of cards that have coordinates on the same horizontal or vertical line. If the points given by the coordinates on the card are on the same horizontal line, the cards are placed in the “horizontal” pile. If the points are on the same vertical line, the cards are placed in the “vertical” pile. The player finds the distance between the points on the selected cards and writes it in the appropriate column. As each new pair of cards is added to a pile, players write the distance in the appropriate column and add the distances in that column together. Partners check that the decision about the pile and the calculations are correct. If there are no pairs possible, the player adds cards one at a time until there is a pair. The next player then selects new cards from the main stack so at least six cards are facing up. More cards will be facing up, if there are no pairs. The game continues until the cards in the main stack run out and no more pairs can be made.

**Strategy:** Keep track of the sums as you go. Try to choose a pair that will make the difference between the sums as small as possible.

**Extensions**

1. Find the distance between the points.

a) \((3.124, 0)\) and \((5.876, 0)\)

b) \((4.56, -1.89)\) and \((4.56, 5.243)\)

c) \((-4.008, 1.34)\) and \((-2.87, 1.34)\)

d) \((6.9, -212)\) and \((-3.901, -212)\)
e) \((-234, \frac{45}{7})\) and \((-234, \frac{-5}{7})\)

f) \((-25, \frac{-23}{45})\) and \((-25, \frac{-7}{9})\)

**Answers:** a) 2.752, b) 7.133, c) 1.138, d) 10.801, e) 50 1/7, f) 1 42/45 = 1 14/15

(MP7) 2. Find the perimeter of the polygon.

a) **ABCD:**

\[
A (-2, \frac{3}{8}) \quad B (-2, -\frac{5}{8}) \quad C (\frac{1}{2}, -\frac{4}{8}) \quad D (\frac{1}{2}, \frac{3}{8})
\]

b) **ABCDEF:**

\[
A (23.4, 3.15) \quad B (-6.17, 3.15) \quad C (-6.17, 8.2) \\
D (-12.03, 8.2) \quad E (-12.03, -1.9) \quad F (23.4, -1.9)
\]

**Answers:** a) 27, b) 91.06
Review finding distance between points. Remind students how to identify points on the same line. Remind them that the same x-coordinate means that the line is vertical, and the same y-coordinate means that the line is horizontal. Encourage students to sketch the points to verify whether the line is horizontal or vertical. Remind them also that the absolute values of coordinates of a point give the distance from the point to the axes.

ASK: Are the points \((-2, -3)\) and \((-2, -8)\) on the same line? (yes) Is it a vertical or a horizontal line? (vertical) Draw the sketch at left and ask students to decide which line the points are on. (the line on the left of the axis) Have students signal the answer by holding up their right or left hand. Which quadrant is the point \((-2, -3)\) in? (III) Mark the point on the line. Which quadrant is the other point in? (also III) Is the second point above or below the first point? (below) Mark the second point as well. SAY: I want to find the distance between these points. Should I add or subtract the distances from the x-axis? (subtract) Why? (the points are on the same side of the axis) What is the distance between the points? (5 units)

Repeat with these pairs of points: \((-2, -3)\) and \((-2, 5)\) (add, 8 units), \((1, 1)\) and \((1, 8)\) (subtract, 7 units), \((-23, -7)\) and \((-41, -7)\) (subtract, 18 units).

Identifying rectangles and squares. Ask students to sketch a quadrilateral with two vertical and two horizontal sides. ASK: What special quadrilateral did you draw? (rectangle or square) Could there be any other type of
quadrilateral? (no) Why not? (If the sides of a quadrilateral are either vertical or horizontal, it has four right angles, so it has to be a rectangle. A square is a special case of a rectangle.) How can you decide whether a quadrilateral is a rectangle or a square? (a square has four equal sides) How do you find area of a rectangle or a square? (multiply length by width)

Write these coordinates on the board:

\[
A (2, 3) \quad B (-6, 3) \quad C (-6, 8) \quad D (2, 8)
\]

ASK: Which sides of this quadrilateral are horizontal? \((AB, CD)\) Which sides are vertical? \((BC, AD)\) Have students find the lengths of the sides. \((AB = CD = 8, BC = AD = 5)\) ASK: Is \(ABCD\) a rectangle or a square? (rectangle) Have students plot the points on a grid (they can use the large grid on BLM Coordinate Grids (Four Quadrants) and check the answers. Have students also find the area of the rectangle. (40 square units)

Repeat with \(E (-1, 1), F (2, 1), G (2, -2), H (-1, -2)\) (square, all sides are 3 units long, area = 9 square units) and \(I (-1.5, -1.25), J (4.5, -1.25), K (4.5, -2.75), L (-1.5, -2.75)\) (rectangle, \(IJ = KL = 6, IL = JK = 1.5,\) area = 9 square units).

**Using lengths of parallel sides to distinguish between trapezoids and parallelograms.** Draw on the board the picture in the margin.

ASK: What shape is it? (parallelogram) What are the lengths of the other sides? (25 cm and 15 cm) How do you know? (opposite sides of a parallelogram are equal)

Draw the second picture in the margin and ask students whether this is a parallelogram. (no) How do they know? (the parallel sides are not equal) Emphasize that students have described a method to decide whether a quadrilateral with two parallel sides is a parallelogram: if parallel sides are equal, the quadrilateral is a parallelogram, and if they are not equal, it is a trapezoid. Draw several quadrilaterals, mark one pair of sides as parallel, and write the lengths on the sides. Have students decide whether the quadrilateral is a parallelogram or a trapezoid.

Write these coordinates on the board:

\[
A (2, 4) \quad B (-5, 4) \quad C (-4, -2) \quad D (3, -2)
\]

ASK: Which points are on the same horizontal or vertical line? \((A\) and \(B, C\) and \(D)\) Are these lines horizontal or vertical? (horizontal) SAY: This means \(AB\) and \(CD\) are parallel. Have students find the lengths of the sides \(AB\) and \(CD\). \((AB = 7, CD = 7)\) ASK: Is \(ABCD\) a parallelogram or a trapezoid? (parallelogram) Have students plot the points on a coordinate grid (they can use BLM Coordinate Grids (Four Quadrants) to save time) and check the answers.
Repeat with these quadrilaterals (students should only plot the first quadrilateral):

a) \(E (-2, 3), F (3, 4), G (3, 3), H (-2, 1)\)

b) \(I (3.5, -1.2), J (-2, -1.2), K (-6.25, -2.3), L (-0.75, -2.3)\)

c) \(M (1, 4\frac{3}{8}), N (-2, \frac{7}{8}), O (-2, -1\frac{5}{8}), P (1, 2\frac{1}{8})\)

**Answers:**
a) \(EH = 2, FG = 1,\) vertical, trapezoid; b) \(IJ = 5.5, KL = 5.5,\) horizontal, parallelogram; c) \(ON = 2\frac{1}{2}, MP = 2\frac{1}{4},\) vertical, trapezoid

Make sure students can convert fractions and mixed numbers with denominators 2, 4, and 5 before assigning the Activity below.

**ACTIVITY**

Give each pair of students the laminated cards from BLM Ordered Pair Cards. Have students lay out 8 cards face up. Player 1 picks two cards that are on the same vertical or horizontal line. Player 2 picks two more cards that are on a line parallel to the line Player 1 picked, making sure the four cards do not make a rectangle. More cards can be added to the center if there is no second pair. The four points together make a quadrilateral with two vertical or horizontal sides. Players then determine the length of the side they picked, and decide whether the quadrilateral is a parallelogram or a trapezoid. The player who picked the second pair goes first at the next turn. The goal is to create as many quadrilaterals as possible, while avoiding creating rectangles.

**Area of right triangles on a coordinate grid.** Write these coordinates on the board:

\[A (2.1, 4.4) \quad B (-2, 4.4) \quad C (-2, -2)\]

ASK: Are any of these points on the same horizontal or vertical line? (A and B are on the same horizontal line, B and C are on the same vertical line) Have students sketch the points on a grid without numbers. ASK: What type of triangle do we have? (right triangle) SAY: I would like to find the area of this triangle. How can we do it? (multiply \(AB \times BC\), then divide by 2)

Have students find the lengths of the sides \(AB\) and \(BC\). \((AB = 4.1 \ BC = 6.4)\)

Remind students that when multiplying decimals they can multiply them as whole numbers, then shift the decimal point to the left. How many places do they need to shift the decimal point here? (2) How do you know? (there is 1 decimal place in each number, so we need to shift the point \(1 + 1 = 2\) decimal places) What is the area of the triangle? \((4.1 \times 6.4 \div 2 = 26.24 \div 2 = 13.12)\)

Repeat with these coordinates: \(D (-1.3, -2), E (2.7, -4.8), F (-1.3, -4.8)\).

**Solution**

\(DF\) vertical, \(DF = 2.8,\) \(EF\) horizontal, \(EF = 4,\) so area \(DEF = 2.8 \times 4 \div 2 = 5.6\) square units
Extensions

1. Find the area of the hexagon.

\[ A (23.4, 3.15) \quad B (-6.17, 3.15) \quad C (-6.17, 8.2) \]
\[ D (-12.03, 8.2) \quad E (-12.03, -1.9) \quad F (23.4, -1.9) \]

**Answer:** 208.5145

Look for students to sketch the hexagon by plotting the points, and to notice that the hexagon can be split into two rectangles (MP.7) to find the area.

2. Liz has red, blue, and white paint in the ratio 3 : 2 : 1. She mixes equal parts of all three colors to make light purple paint. If she finishes all her white paint, what is the ratio of red to blue paint that she has left over? Use a T-table or a tape diagram. Use clear labels.

**Sample solutions**

- I used a T-table. I used cups as the unit:

<table>
<thead>
<tr>
<th>Red Paint (cups)</th>
<th>Blue Paint (cups)</th>
<th>White Paint (cups)</th>
<th>Leftover Red Paint (cups)</th>
<th>Leftover Blue Paint (cups)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

  There was always twice as much red paint left over as blue paint, so the ratio of red to blue paint left over is 2 : 1.

- I used a tape diagram:

  Red paint: [ ] [ ] [ ] [ ] [ ]
  Blue paint: [ ] [ ] [ ] [ ] [ ]
  White paint: [ ] [ ] [ ] [ ] [ ] [ ] [ ]

  When I took away all the white paint, I had to remove one box from each bar, so there were two boxes left of the red paint and one box left of the blue paint, so the ratio is 2 : 1.

Whole-class follow-up: Have volunteers show both solutions. **ASK:** Which way was faster? (using tape diagrams) Have students discuss in small groups: What does the tape diagram show you that the T-table does not? (It shows why it’s true. The T-table only shows what happens for given numbers, but not why it is happening. With the tape diagram I can see that when I remove one equal part from three equal parts, I always get two equal parts, and when I remove one equal part from two equal parts, I always get one equal part, so I’m always left with two parts red to one part blue, no matter how much of each color of paint I started with.) Then allow volunteers to share their group’s answers with the class.
Patterns in coordinates of the vertices of a rectangle. Ask students to draw a rectangle with vertical and horizontal sides on a coordinate grid, so that the rectangle is not divided by the axes into equal parts (in other words, the rectangle is not symmetrical relative to the axes). Assign different conditions to different students, such as “draw the rectangle in the first and second quadrants,” to ensure a wide variety of rectangles. Ask students to write the coordinates of the four vertices and look at how the coordinates are the same. (the pairs of adjacent points have one coordinate the same) Have students share the results with partners. Did everyone have the same rectangle? (no) Did everyone get the same result? (yes) Why does each pair of vertices that share a side have the same first or second coordinates? (the sides of the rectangle are either horizontal or vertical, so the vertices are on the same vertical or horizontal line)

Now ask students to share with a partner three of the four vertices of their rectangles. Ask them to try to use the pattern they noticed to find the coordinates of the fourth vertex of the rectangle. Ask several students to explain the solution. Then have students exchange the three vertices with another partner and repeat the exercise.

Drawing rectangles with specific length or width. On the board, draw a line segment with endpoints (−3, 2) and (4, 2). Tell students that this is one side of a rectangle. The rectangle has a width of 3 units. Could this be the width of the rectangle? (no) Why not? (the points are 7 units apart, not 3) Have students copy the points and draw the other two vertices on the rectangle. Have students present the two possible solutions. (above the line segment: (−3, 5) and (4, 5), below the line segment: (−3, −1) and (4, −1))
Repeat the exercise with a rectangle with vertices (1, 1) and (1, 4) and a length of 4 units. Ask students to find the length and the width of the rectangles they drew at the beginning of the lesson, then pair up with a student they have not worked with yet. Students give their partners two adjacent vertices of the rectangle they drew and the length of the perpendicular side. Partners need to find the two potential rectangles.

**Using length of a line segment to predict the other end.** Discuss with students how you could predict the two vertices of the rectangles. For example, in the first rectangle above, you know that the points need to be either above or below the other two points. The new points are on the same vertical lines as the given points. What does this say about their coordinates? (they should have the same $x$-coordinates as the given points)

Write on the board:

\[(−3, \_\_\_\_\,) \text{ and } (4, \_\_\_\_) \text{ or } (−3, \_\_\_\_) \text{ and } (4, \_\_\_\_)\]

ASK: If the point is 3 units above the point $(−3, 2)$, what should its $y$-coordinate be? (5) How do you know? (5 is 3 more than 2) Fill in the blank and repeat with the other point.

Now discuss the situation in which the points are below the given line segment. What is the distance from the points $(−3, 2)$ and $(4, 2)$ to the $x$-axis? (2 units) How do you know? (the distance to the $x$-axis is the absolute value of the $y$-coordinate) Will a rectangle with width of 3 units fit between the line segment and the $x$-axis? (no) Draw a vertical number line and have a volunteer show which point is 3 units below 2. $(−1)$ What are the coordinates of the points 3 units below $(−3, 2)$ and $(4, 2)$? Fill in the second pair of blanks with the answer: $(−3, −1)$ and $(4, −1)$.

Repeat the argument with points $(−3, −1)$ and $(−3, 2)$ and a rectangle with a length of 5 units, and another rectangle with the same two points and a length of 2 units.

**Drawing parallelograms given three vertices.** Write on the board:

\[A (−4, 6), B (2, 6), C (4, −1), \text{ and } D (\_\_\_, \_\_\_)\]

Explain that these are the vertices of parallelogram $ABCD$. ASK: What are the sides of parallelogram $ABCD$? $(AB, BC, CD, \text{ and } AD)$ Which sides are given when we have points $A, B$, and $C$? $(AB, BC)$ Are any of these sides horizontal or vertical? $(AB \text{ is horizontal})$ Will there be any other horizontal sides? (yes) Which one? $(CD)$ What does this tell us about the coordinates of $D$? (the second coordinate of $D$ is the same as the second coordinate of $C$) Fill in the second blank with $−1$.

Sketch two axes on the board, without the numbers. Ask volunteers to mark the approximate locations of points $A, B, \text{ and } C$ (see margin). ASK: About where should point $D$ be? Should it be to the right of $C$ or to the left of $C$? (to the left) Point out that a point to the right of $C$ would produce a parallelogram (sketch it) but, since we name shapes by going around the shape in order, this parallelogram would be $ABDC$, not $ABCD$. This means that the point $D$ should be to the left of $C$. 
ASK: How can we find the first coordinate of D? What do we know about the lengths of $AB$ and $CD$? (they are the same) How long is $AB$? (6 units) How do you know? ($|−4| + |2| = 4 + 2 = 6$) This means $D$ is 6 units away from $C$. How far from the $y$-axis is $C$? (4 units) Will $D$ be in the same quadrant as $C$? (no) Add numbers to the horizontal axis so that $C$ is at 4 and have students count 6 units to the left of 4. What should the first coordinate of $D$ be? ($−2$) Fill in the remaining blank.

Repeat the exercise with parallelogram $EFGH$ with points $E$ ($−3, 2$), $F$ ($−3, −3$), and $G$ ($−0.5, 1$).

**Answer:** $H$ ($−0.5, 0$)

For practice, students can each draw a parallelogram, then give a partner three out of the four vertices. The partner will need to find the fourth vertex of the parallelogram.

**Extensions**

1. $ABC$ is a right triangle with a vertical side of 6 units. Predict the coordinates of vertex $C$ if $A$ is ($−3, 1$) and $B$ is ($2, 1$). How many answers are there?

**Answer:** 4 possible points: ($−3, 7$), ($−3, −5$), ($2, 7$), and ($2, −5$)

(MP.4) 2. Ed owns a company and he is asking for investments. Which of these offers values the company the most?

- Alice offers $35,000 for 20% of the company
- Karen offers $27,000 for 15% of the company
- Jon offers $50,000 for 40% of the company

**Solution:** Alice’s offer means that she thinks that 20% of the company is worth $35,000, so that means that Alice thinks that $35,000 is 20% of the value of the company, so I solved $35,000 = 20\% \text{ of what number and got } 175,000$. Karen’s offer values the company at $180,000, and Jon’s offer values the company at $125,000. So Karen’s offer values the company the most.
Distance between parallel lines. On a coordinate grid, draw two lines:
\( y = -1 \) and \( y = -3 \). ASK: What is the distance between these two lines? (2 units) How do you find the distance between parallel lines? Remind students that to find the distance between two horizontal lines, they draw a vertical line and measure the distance between the points where the vertical line intersects the horizontal lines. Remind students that the distance does not depend on where it is measured.

Explain that you would like to have a method of using coordinates to find the distance between two horizontal (or vertical) lines, so that you would not need to draw the lines in the future. Draw a vertical line, such as \( x = 1 \), and mark the intersection points with the lines. Ask students to determine the coordinates of the points. \((1, -1)\) and \((1, -3)\) ASK: How do you use coordinates to find the distance between the points? (subtract the absolute values of the \( y \)-coordinates) Choose a different vertical line, such as \( x = 4 \). Repeat finding the distance between the lines. ASK: What do all points on the upper line have in common? (the \( y \)-coordinate is always \(-1\)) What do all points on the lower line have in common? (the \( y \)-coordinate is always \(-3\)) How do you use coordinates to find the distance between these two lines? (subtract the absolute values of \( y \)-coordinates of any two points on the lines)

Draw two horizontal lines between the gridlines, and tell students that one of them passes through point \((1, 2.3)\) and the other passes through point \((2, 4.5)\). What is the distance between these two lines? Have students write a subtraction sentence. \((4.5 - 2.3 = 2.2)\)

Now draw two lines, \( y = -1 \) and \( y = 3 \). Ask students to explain how they would use coordinates to find the distance between these two lines. (add the absolute values of the \( y \)-coordinates; because these two lines are on different sides of the \( x \)-axis, the \( y \)-coordinates have different signs) Then ask students to find the distance between the horizontal lines through points \((2.5, 1)\) and \((3.4, -2.1)\). \((1 + 2.1 = 3.1 \text{ units})\)
Repeat the discussion for vertical lines. Have students find the distance between vertical lines through these points:

a) \((-2, 1)\) and \((-4, 1)\)  
b) \((-2, 3)\) and \((-5, 1)\)  
c) \((2, 1)\) and \((-3, 6)\)  
d) \((-4.75, 2)\) and \((2.11, -3)\)  

**Answers:** a) 2, b) 3, c) 5, d) 6.86

**Finding bases and heights of parallelograms and trapezoids.** Explain that you would like to be able to find areas of parallelograms and trapezoids on a coordinate grid. Remind students that to find the area, you need to know the length of the base (or bases) and the height of the shape. The height of a shape is the distance between the bases, so distance between parallel lines comes in handy.

Tell students that the points \(A(-3, 1), B(-1, 5), C(4, 5),\) and \(D(2, 1)\) are vertices of a parallelogram. ASK: Does this parallelogram have horizontal or vertical sides? (horizontal) Which ones? \((BC)\) and \((AD)\). What is the length of these horizontal sides? \((5 \text{ units})\) How do you know? \((4 + |-1| = 5 \text{ or } |3| + 2 = 5)\). What is the height of the parallelogram? \((4 \text{ units})\) How do you know? \((5 - 1 = 4)\)

Have students draw the parallelogram and check their answers.

Repeat with a trapezoid: \(E(-2, -1), F(-2, 4), G(1, 5),\) and \(H(1, 1)\). It has vertical bases \(EF = 5 \text{ units}\) and \(GH = 4 \text{ units}\), and a height of 3 units.

**Exercises:** Find the length of the vertical or the horizontal sides and the height.

a) \(A(1.5, 3), B(-2.5, -1), C(-2.5, -5),\) and \(D(1.5, -1)\)

b) \(A(-4.5, -0.5), B(-1, 3.25), C(1, 3.25),\) and \(D(1.5, -0.5)\)

**Answers:** a) \(BC = 4\), vertical, \(AD = 4\), vertical, height 4;  
b) \(BC = 2\), horizontal, \(AD = 6\), horizontal, height 3.75

**Finding area of parallelograms and trapezoids.** Review the formulas for finding the area of a trapezoid and the area of a parallelogram. Point out that it is essential to know whether the shape is a parallelogram or a trapezoid to know which formula to use. Review how to decide whether a quadrilateral with two vertical or two horizontal sides is a trapezoid or a parallelogram. (If the side lengths are equal and parallel, it is a parallelogram; if they are parallel but unequal, it is a trapezoid.) Point out that since you need to find the length of the base or bases, this does not create any additional work.

**Exercise:** Find the area of the shapes from the previous exercise.

**Answers:** a) 16 square units, b) 11.25 square units

Before assigning the following exercises, review changing mixed numbers to fractions, multiplying fractions, and dividing fractions by whole numbers as needed. In part c), have students convert decimals into fractions before dividing by 2 to find the area.

**Exercises:** Find the length of the base or bases and the height of \(ABCD\). Then find the area.
a) \( A (2.3, 3), B (-2.5, 1), C (-2.5, -5), D (2.3, -1) \)

b) \( A (-2, 1\frac{1}{3}), B (-\frac{1}{3}, 1\frac{1}{3}), C (1\frac{1}{3}, -1\frac{1}{4}), D (-\frac{1}{3}, -1\frac{1}{4}) \)

c) \( A (2.3, 1), B (1.8, 2.5), C (-1.2, 2.5), D (-1.2, 1) \)

Answers:
a) \( BC = 6, AD = 4, \) height = 4.8, trapezoid, area = 24 square units; b) \( AB = 1\frac{2}{3}, CD = 1\frac{2}{3}, \) height = 2\(\frac{3}{4}\), parallelogram, area = \(\frac{55}{12} = 4\frac{7}{12}\) square units; c) \( BC = 3, AD = 3.5, \) height = 1.5, trapezoid, area = \(\frac{39}{8} = 4\frac{7}{8}\) square units

**ACTIVITY**

Have students create three quadrilaterals using the rules for the Activity in Lesson G6-24. Have them find the areas of the quadrilaterals. Partners should check each other’s answers.

**Extensions**

(MP1, MP7)

1. The points \( A (-1, 1), B (4, 1), C, \) and \( D, \) when joined in order, make a parallelogram with area 20 square units. Describe all possible pairs of points \( C \) and \( D. \)

   *Answers:* The length of \( AB \) is the distance between 4 and -1, which is 5 units, so the height of the parallelogram must be 4 units (because \( 5 \times 4 = 20 \) and the area is 20 square units); \( CD \) must be parallel to \( AB \) so it too is horizontal, and it must be 4 units above or below \( AB, \) so the \( y \) coordinate is either 5 or -3. The points can be \((-1, 5)\) and \((4, 5), (-2, 5)\) and \((3, 5), \) or any points on the line \( y = 5 \) that are 5 units apart, or any points on the line \( y = -3 \) that are 5 units apart. I know that \( CD \) has length 5 because opposite sides of a parallelogram are equal.

2. **Area of triangles.** Remind students that the height of a triangle is the distance from a vertex to the opposite side (called the base), measured along a perpendicular to the base. Point out that you can find the height of a triangle the same way you find the height of a parallelogram. For example, if your triangle has vertices \( A (-1, 1), B (4, 1), \) and \( C (2, 4), \) the side \( AB \) is horizontal because all points on it have \( y = 1. \) So the distance from \( C \) to line \( AB \) is \( 4 - 1 = 3. \)

   Review the formula for finding the area of a triangle, then have students find the area of triangle \( ABC \) above:

   \[
   \text{Base } AB = 5 \\
   \text{Height} = 3 \\
   \text{Area} = (3 \times 5) \div 2 = 15 \div 2 = 7.5
   \]

   Have students find the area of each triangle.

   a) \( A (-1, 1), B (4, 1), C (3, -5) \)
   
b) \( A (-2, 1), B (-2, 3), C (-7.5, 6) \)

   *Answers:* a) base \( AB = 5, \) height = 6, area = 15 square units; b) base \( AB = 2, \) height = 5.5, area = 5.5 square units
**Goals**

Students will perform and describe reflections of points on a coordinate grid through axes.

**PRIOR KNOWLEDGE REQUIRED**

Can plot points, including decimals and fractions, in all four quadrants of a coordinate grid

Can identify when two points are on the same vertical or horizontal line

Knows that opposite numbers have the same absolute value but differ in sign, and are the same distance from zero on a number line

Knows that a variable represents a number

**MATERIALS**

BLM Coordinate Grids (Four Quadrants) (p. N-31)

one set of laminated BLM Ordered Pair Cards (pp. N-32 and N-33) for each pair of students

---

**Introduce the new vocabulary.** Draw a picture with a *line of symmetry*, such as a smiley face, and draw the line of symmetry in it. Remind students what a line of symmetry is. Draw the picture at left and explain that when you want to draw the remaining half of the picture, a mathematician would say that you want to *reflect* the picture through the vertical line. The vertical line is called the *mirror line*. The missing half of the face you will be drawing when performing this *reflection* is called the *image*.

**Reflecting points through a line.** Teach students the steps for reflecting a point through a line. Work on a pre-drawn grid and use a grid line as the mirror line. Label the point you are reflecting A.

**Step 1:** Draw a line perpendicular to the mirror line through A. Extend the perpendicular beyond the mirror line.

**Step 2:** Measure the distance from A to the mirror line along the perpendicular line.

**Step 3:** Construct point A' on the perpendicular line so that the distance from A' to the line is the same as the distance from A to the line.

Have students practice this construction with several points on a grid, and with horizontal and vertical mirror lines. Make sure mirror lines always coincide with the grid lines, but include points that are not on the grid lines.

**Reflecting points through the horizontal axis.** Display a coordinate grid. Tell students you want to use the horizontal axis as the mirror line. Ask them to copy the table below. Have them reflect the points through the x-axis and
fill in the coordinates of the images. They can work on **BLM Coordinate Grids (Four Quadrants)** to save time.

<table>
<thead>
<tr>
<th>Point</th>
<th>A (2, 3)</th>
<th>B (4, 1)</th>
<th>C (−1, 4)</th>
<th>D (3, −5)</th>
<th>E (−5, −3)</th>
<th>F (0, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** $A' (2, −3), B' (4, −1), C' (−1, −4), D' (3, 5), E' (−5, 3), F' (0, −2)$

Ask students to look at the coordinates of points and their images. Which coordinate is the same in the point and in the image? ($x$-coordinate) Why did that happen? **PROMPT:** The original point and the image are on the same line. Is that line horizontal or vertical? (vertical) Which coordinate changed? ($y$-coordinate) How did it change? (the number part stays the same, the sign changes; the $y$-coordinate of the image is the opposite of the $y$-coordinate of the original point) Can you predict the coordinates of the point $(4, 3)$ when reflected through the $x$-axis? $(4, −3)$

**Exercises:** Predict the coordinates of the points when reflected through the $x$-axis.

$E (8, 12), F (1.5, 3.89), G (−3.1, 4.3), H (\frac{1}{2}, −2\frac{1}{4}), K (−128, −312)$

**Answers:** $E' (8, −12), F' (1.5, −3.89), G' (−3.1, −4.3), H' (\frac{1}{2}, 2\frac{1}{4}), K' (−128, 312)$

Encourage students to sketch the points and the images (using a pair of axes without increments and numbers) to see which quadrant they expect the reflection to be in, and to check that their prediction makes sense.

Then ask students to predict the coordinates of the point $(1, 0)$ when reflected through the $x$-axis. **PROMPT:** What is the opposite of 0? (0, so the image is $(1, 0)$ again) Have students graph the point. Emphasize that points on the mirror line stay the same when reflected.

**Reflecting points through a vertical axis.** Repeat the above for reflection through the vertical axis, using the same points as in the exercise above.

**Exercises:** Predict the coordinates of the points when reflected through the $y$-axis.

$E (8, 12), F (1.5, 3.89), G (−3.1, 4.3), H (\frac{1}{2}, −2\frac{1}{4}), K (−128, −312)$

**Answers:** $E' (−8, 12), F' (−1.5, 3.89), G' (3.1, 4.3), H' (−1/2, −2 1/4), K' (128, −312)$

**Identifying points that are reflections in the axes.** Tell students that they have predicted coordinates of reflected points. You want them now to try to do the opposite. Ask students to tell whether the points are reflections of each other through an axis and, if so, which axis was used as the mirror line.

Write the coordinates of these three points on the board:

$A (−2, 3), B (−2, 2), C (2, 3)$
ASK: Which points are on the same line? (A and B, A and C) Which points are reflections of each other through an axis? (A and C) How do you know? (their coordinates are almost the same, only the first coordinates differ by sign) Which axis is that? (y-axis) Have students sketch the points on a grid without numbers to check that the answer makes sense.

**ACTIVITY 1**

*Object of the game:* Collect all the cards so that the “x-axis” and “y-axis” piles have the same number of pairs.

*Preparation:* Lay the laminated cards from **BLM Ordered Pair Cards** face down in a rectangle.

*Instructions:* The player turns over two cards. The player then determines whether the points given by the coordinates on the card are reflections of each other through an axis. If yes, the player determines whether the point is reflected through the x-axis or the y-axis and places the cards on the appropriate pile. The player can choose to put the pair of cards back in the stack if the cards will increase the wrong pile. The player asks a partner to check that the decision about the pile is correct. If the points are not reflections through an axis, the player turns the cards back and the next player takes a turn.

**Reflecting points through both axes (advanced).** Ask students to copy the table below. Have them reflect the points through the x-axis, then through the y-axis, and then fill in the coordinates of the images.

<table>
<thead>
<tr>
<th>Point</th>
<th>A (3, 4)</th>
<th>B (-2, 5)</th>
<th>C (4, -1)</th>
<th>D (-1, -4)</th>
<th>E (0, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Answers:** A’ (-3, -4), B’ (2, -5), C’ (-4, 1), D’ (1, 4), E’ (0, -3)

Add a row labeled “Image 2” to the table. Ask students to reflect the same points, this time through the y-axis first, then through the x-axis.

Have students record the coordinates of the images in the third row. Ask volunteers to provide the answers.

**Answers:** A’ (-3, -4), B’ (2, -5), C’ (-4, 1), D’ (1, 4), E’ (0, -3)

Ask students to compare the answers in the second and third rows. Why are the results identical? Ask students to explain using what they learned about the coordinates during the lesson. They can try to write an answer individually, then work in pairs, then in groups of four to come up with the best-formulated common answer. (reflect through the x-axis, the second coordinate changes sign; reflect through the y-axis, the first coordinate changes sign; the signs of both coordinates change, and the order in which you change them does not matter)
Exercises: Predict the coordinates of the points when reflected through one axis, then through the other axis.

\[E(4, 3) \quad F(-3.66, 4.3) \quad G\left(\frac{2}{3}, -3\frac{5}{8}\right) \quad H(-432, -960) \quad I(6, y) \quad J(x, y)\]

Answers: \[E'(-4, -3), F'(3.66, -4.3), G'(\frac{-2}{3}, 3\frac{5}{8}), H'(432, 960), I'(-6, -y), J'(x, -y)\]

ACTIVITY 2

Lay the laminated cards from BLM Ordered Pairs Cards face down in a rectangle. Turn over two cards at a time. Two cards match if they are reflections in both axes. The goal of the game is to collect all the matching pairs.

Ask students to predict the result when you take point \((3, 5)\) and reflect it three times: first through the horizontal axis, then through the vertical axis, then through the horizontal axis again. \((-3, 5)\) ASK: What reflection through just one axis would give you the same result? (reflection through the \(y\)-axis) Why did the reflections through the \(x\)-axis not result in different coordinates? (because you changed the sign of the second coordinate twice—to the opposite sign and back again) Emphasize that what happens is not just a change of the sign: You change the coordinate to the opposite number, then change that number to the opposite number. The opposite of the opposite number is the same number you started with. So the second coordinate is \(-(-5)\), which is indeed equal to 5.

Extensions

1. a) Draw a horizontal mirror line that is not an axis on a coordinate grid. Draw three points above the line and three points below the line. Reflect the points through the mirror line. Compare the coordinates of the points and the images. Which coordinate changes? Which coordinate stays the same?

Answer: \(x\)-coordinate stays the same, \(y\)-coordinate changes

b) Draw a vertical mirror line that is not an axis on a coordinate grid. Draw three points to the left of the line and three points to the right of the line. Reflect the points through the mirror line. Compare the coordinates of the points and the images. Which coordinate changes? Which coordinate stays the same?

Answer: \(y\)-coordinate stays the same, \(x\)-coordinate changes

c) Draw a triangle with vertices \(A\ (1, 1), B\ (4, 2),\) and \(C\ (3, 3)\). Draw a vertical mirror line through the points \((5, 0)\) and \((5, 5)\). Reflect the triangle through the mirror line. What are the coordinates of the vertices of the image?

Answer: \(A'\ (9, 1), B'\ (6, 2),\) and \(C'\ (7, 3)\)
d) If you want to reflect a point through a line so that its x-coordinate doesn’t change, which type of line should you reflect it in, horizontal or vertical?

Answer: horizontal

2. On a coordinate grid, draw a line through points (0, 0) and (1, 1). Use this line as a mirror line. Draw several points in different quadrants and reflect them through the mirror line. Use a protractor or a set square to draw the perpendicular lines. Compare the coordinates of the points and the images. What do you notice? (the coordinates switch places)

3. Marissa reflected a point on the x-axis through the y-axis. The point coordinates did not change. What are the coordinates of Marissa’s point?

Answer: (0, 0)

(MP6) 4. a) Draw a triangle in the first quadrant that has one side on the y-axis. Reflect it on the y-axis.

b) Use the words “image,” “reflection,” and “line of symmetry” to describe your drawing in part a).

Selected sample answer: b) The triangle in the second quadrant is the image of the original triangle in the first quadrant, after a reflection in the y-axis. The original triangle and the image together form a new larger shape that is symmetrical. The y-axis is a line of symmetry for the larger shape.
G6-29 Applications of Coordinate Grids
Pages 108–109

STANDARDS
6.NS.C.6

VOCABULARY
axes
coordinate grid
horizontal
vertical
x-axis
x-coordinate, first coordinate
y-axis
y-coordinate, second coordinate

Goals
Students will solve problems using coordinate grids.

PRIOR KNOWLEDGE REQUIRED
Can plot points, including decimals and fractions, in all four quadrants of a coordinate grid
Can identify when two points are on the same vertical or horizontal line and find the distance between such points
Knows that opposite numbers have the same absolute value but differ in sign and are the same distance from zero on a number line

MATERIALS
BLM Washington and Oregon (p. N-36)
BLM DeepSea Challenger Ascent (p. N-37)

Use BLM Washington and Oregon to review concepts learned in this unit. Answer the questions below as a class and have students explain the solution.

Exercises
1. What are the coordinates of the following cities?
   a) Spokane, WA  b) Aberdeen, WA  c) Eugene, OR
   d) Portland, OR  e) Yakima, WA

2. What city is located at each point?
   a) (0.4, 3)  b) (−0.5, −4)  c) (0.25, 2.5)  d) (−1/3, −2/3)

   Answers: 1. a) (5, 3), b) (−1, 2), c) (−0.5, −2), d) (0, 0), e) (2, 1.5); 2. a) Seattle, WA, b) Grants Pass, OR, c) Tacoma, WA, d) Salem, OR

   Explain that the directions North, East, South, and West are marked on the map with letters N, E, S, and W. Ask students to show with their fingers which direction on the map they would move if they were looking for a point that is 1 square north (up), 100 miles west (left), 20 miles south (down), or 50 miles east (right). Explain that each square on the map is 50 miles long and 50 miles wide. ASK: If you move 2 squares north, how many miles North are you moving? (100 mi) If you move 150 miles south, how many squares are you moving? (3 squares) If you move 2.5 squares west, how many miles are you moving? (125 mi) How do you know? (2.5 \times 50 \text{ mi} = 125 \text{ mi})

3. What is located at each point?
   a) 6 squares west of Spokane, WA
   b) 100 miles east of Aberdeen, WA
c) 50 miles west of Willamette National Forest  
d) 100 miles south of Eugene, OR  

4. How many squares from Grants Pass, OR, is Hart Mountain National Antelope Refuge?  

5. A plane flew from Seattle to Spokane. How far did it fly?  

6. A helicopter set out from Grants Pass, OR, flew to Eugene, OR, then to Willamette National Forest. How far did it fly?  

**Answers:** 3. a) Olympic National Park; b) Mt. Rainier National Forest; c) Eugene, OR; d) Grants Pass, OR; 4. 3.5 squares; 5. 230 mi; 6. 150 mi

**Using a graph to solve a problem.** Display **BLM DeepSea Challenger Ascent** photocopied onto a transparency. Point out the increments that show the round hours. What is the length of each interval on the horizontal axis? (10 min) How do you know? (each hour is divided into 6 parts, an hour is 60 minutes long, so $60 \div 6 = 10$ min) Repeat with the vertical axis. (each interval is 5,000 ft)  

Explain that you have some data about the DeepSea Challenger submersible during its record-setting dive to the deepest part of the Pacific Ocean, which is called the Challenger Deep. Explain that at 10:50 a.m., the submersible was at $-35,600$ feet and at 11:45 a.m., it was at elevation $-7,000$ feet. Record the data in a table. Have volunteers mark the data on the graph. Explain that you want to assume that the submersible was rising at the same rate during this time and join the points with a straight line.  

**ASK:** How long did the part of the ascent shown in the graph take? (55 min) How far did the submersible rise? ($35,600 - 7,000 = 28,600$ ft) Explain that you want to know how far the submersible rises in 10 minutes. Have students check that on the graph. (about 5,000 ft) How did you find the answer? (check different intervals) Point out that the result obtained from the graph is an estimate, because the scale of the graph is too large. You would need a computation to get a more accurate result. Remind students that a ratio table can be used to find the answer and find the answer as a class (see the ratio table in the margin).  

Have students check what the elevation of the submarine was at 11:00 a.m. (about $-31,000$ ft) Then work as a class to find the exact elevation at 11:00 a.m. At 10:50 a.m., the submersible was 35,600 ft below sea level, and it ascended 5,200 ft in 10 minutes. Students can think in terms of depth: $35,600 - 5,200 = 30,400$ ft, so the submersible was at $-30,400$ ft at 11:00 a.m.  

**Extension**  
At what time, to the closest minute, was the DeepSea Challenger at an elevation of $-10,000$ feet?  

**Answer:** 11:39 a.m.
## Ordered Pair Cards (1)

<table>
<thead>
<tr>
<th>(-5, 4)</th>
<th>(5, -4)</th>
<th>(-5, -4)</th>
<th>(5, 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5, -2)</td>
<td>(1, 4)</td>
<td>(-1, 4)</td>
<td>(-2.5, 3)</td>
</tr>
<tr>
<td>(2.5, 3)</td>
<td>(-2.5, -3)</td>
<td>(-2.5, 1.2)</td>
<td>(2.5, 1.2)</td>
</tr>
<tr>
<td>(-1 3/4, -3)</td>
<td>(1, 2 1/5)</td>
<td>(-1, 2 1/5)</td>
<td>(-3.4, 2)</td>
</tr>
</tbody>
</table>
## Ordered Pair Cards (2)

<table>
<thead>
<tr>
<th>(-2.5, -1.2)</th>
<th>(2.5, -1.2)</th>
<th>(5, 2)</th>
<th>(5, -2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1 3/4, -3)</td>
<td>(1, -2 1/5)</td>
<td>(-1, -2 1/5)</td>
<td>(3.4, 2)</td>
</tr>
<tr>
<td>(1, -4)</td>
<td>(-1, -4)</td>
<td>(-5, 2)</td>
<td>(1 3/4, 3)</td>
</tr>
<tr>
<td>(-1 3/4, 3)</td>
<td>(3.4, -2)</td>
<td>(2.5, -3)</td>
<td>(-3.4, -2)</td>
</tr>
</tbody>
</table>
Spinner Template
Inequalities on a Coordinate Grid

1. Plot the point \( P (3, 2) \). Draw a vertical line \( v \) through \( P \).
   a) Mark three points to the right of line \( v \). Write the \( x \)-coordinates for the points.
      \( x = \) ______   \( x = \) ______   \( x = \) ______
      Which inequality applies to all three points? \( x < 3 \) or \( x > 3 \)
   b) Mark one point in each quadrant to the left of line \( v \). Write the \( x \)-coordinates for the points.
      \( x = \) ______   \( x = \) ______   \( x = \) ______   \( x = \) ______
   c) Circle the inequality that applies to all four points.
      \( x < 3 \) or \( x > 3 \)
   d) Draw a check mark for each point in the correct row. Write an inequality or equation to explain your choice.

<table>
<thead>
<tr>
<th>Point</th>
<th>( R (2, 2) )</th>
<th>( T (5, -2) )</th>
<th>( S (-2, 3) )</th>
<th>( U (3, -3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>To the left of line ( v )</td>
<td>( \sqrt{2} &lt; 3 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>To the right of line ( v )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On line ( v )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plot the points to check your answers.

2. Plot the point \( P (2, 1) \). Draw a horizontal line \( h \) through \( P \).
   a) Mark three points above line \( h \). Write the \( y \)-coordinates for the points.
      \( y = \) ______   \( y = \) ______   \( y = \) ______
      Circle the inequality that applies to all three points. \( y < 1 \) or \( y > 1 \)
   b) Mark one point in each quadrant below line \( h \). Write the \( y \)-coordinates for the points.
      \( y = \) ______   \( y = \) ______   \( y = \) ______   \( y = \) ______
      Circle the inequality that applies to all three points. \( y < 1 \) or \( y > 1 \)
   c) Draw a check mark for each point in the correct row. Write an inequality or equation to explain your choice.

<table>
<thead>
<tr>
<th>Point</th>
<th>( R (4, 5) )</th>
<th>( T (3, -2) )</th>
<th>( S (-2, 4) )</th>
<th>( U (-3, -1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Above line ( h )</td>
<td>( \sqrt{5} &gt; 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below line ( h )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On line ( h )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plot the points to check your answers.
Washington and Oregon
DeepSea Challenger Ascent

<table>
<thead>
<tr>
<th>Time</th>
<th>Elevation (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00</td>
<td>Sea level</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>12:00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5,000</td>
</tr>
<tr>
<td></td>
<td>-10,000</td>
</tr>
<tr>
<td></td>
<td>-15,000</td>
</tr>
<tr>
<td></td>
<td>-20,000</td>
</tr>
<tr>
<td></td>
<td>-25,000</td>
</tr>
<tr>
<td></td>
<td>-30,000</td>
</tr>
<tr>
<td></td>
<td>-35,000</td>
</tr>
<tr>
<td></td>
<td>-40,000</td>
</tr>
</tbody>
</table>
Unit 6  Expressions and Equations:  
Algebraic Equations

This Unit in Context

In 6.2 Unit 3, students learned to create and solve equations to find answers to real world problems. In this unit, they will extend their knowledge of solving equations with rational coefficients and investigate how the distributive property can be used to find equivalent expressions (6.EE.A.3). They will analyze the relationship between dependent and independent variables using graphs and tables, and relate these to the equations that define the relationship between the variables (6.EE.C.9). They will also solve inequalities and recognize that, while the equations they have solved thus far have only one solution, solving inequalities leads to infinitely many solutions (6.EE.B.8).

In Grade 7, students will become fluent in solving equations of the form \( px + q = r \) (7.EE.B.4a). In Grade 8, students will solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms (8.EE.C.7b).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1, MP3, and MP5 to MP8. Here are some examples of how students can show that they have met a standard.

MP1: In EE6-18 Extension 3, students make sense of a challenging problem when they find a way to relate 785, the sum of five consecutive numbers, to a multiple of 5 times an unknown quantity. Students persevere when they try to look for a pattern or a different way of representing their sum in order to determine the unknown quantity and the five numbers.

MP3: In EE6-17 Extension 1, students construct a viable argument when they use the distributive property or the definition of multiplication as repeated addition to explain why \( 7y + 2y = 9y \). Students then analyze and critique the argument of a partner.

MP5: In EE6-21 Extension 3, students use appropriate tools strategically when they choose a tool (such as a T-table, equations without variables, or equations with variables) to model and solve a real-world problem.

MP7: In EE6-20 Extension 3, students look for and make use of structure when they use an expression with one variable to write the sum of three consecutive numbers, and when they notice that they can rearrange the sum and rewrite it as an equivalent expression to show that it is a multiple of 3.
Using a table to compare the value of two expressions. Remind students that to evaluate an algebraic expression, they have to replace the variables with numbers.

Create the table in the margin on the board, but fill in only the headings and the first column. Ask a volunteer to come and complete the table. ASK: Do $2n$ and $n + n$ have equal values for every $n$ in the table? (yes)

Exercises: Complete the table to show that two expressions have the same values for every $n$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$2n$</th>
<th>$n + n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2(1)$</td>
<td>$1 + 1 = 2$</td>
</tr>
<tr>
<td>2</td>
<td>$2(2)$</td>
<td>$2 + 2 = 4$</td>
</tr>
<tr>
<td>3</td>
<td>$2(3)$</td>
<td>$3 + 3 = 6$</td>
</tr>
<tr>
<td>4</td>
<td>$2(4)$</td>
<td>$4 + 4 = 8$</td>
</tr>
</tbody>
</table>

Answers: a) 0, 0; 2, 2; 4, 4; 6, 6; b) 3, 3; 6, 6; 9, 9; 12, 12

Using area to compare two expressions. Draw a rectangle with length 4 and width 1 on the board.

ASK: What is the area of the rectangle? (4) How do you know? (because $4 \times 1$ is equal to 4) Draw another rectangle with the same length and width beneath the first one. ASK: What is the area of two rectangles? (8) How do you know? (4 + 4 = 8) SAY: Now I would like to join two rectangles and make a bigger rectangle. Draw this bigger rectangle, as shown in the margin.

ASK: What is the length of the new, big rectangle? (4) What is the width? (2) How can we find the area of the big rectangle? (2 × 4 = 8) Is this area equal to the area of the two smaller rectangles? (yes) Write on the board:

$4 + 4 = 2 \times 4$

Exercises: Write an addition equation and a multiplication equation for the area of each figure.
Using area to write equivalent expressions. Explain to students that sometimes the dimensions of a rectangle are not given. Draw the rectangle in the margin on the board and explain to students that this rectangle is a strip of $n$ squares whose length and width are equal to 1. SAY: The length of the rectangle is $n$, so the area of the rectangle is $1 \times n$. ASK: What is 1 times any number? (the same number) SAY: So the area of the rectangle is $n$.

Draw a second rectangle below the first one and SAY: The area of the two rectangles is $n + n$. Write $n + n$ on the board and SAY: If I join the two rectangles, then the new, bigger rectangle has the same length but its width is 2. Draw the bigger rectangle to the right of the two separate rectangles, as in the exercises below. ASK: What is the area of the big rectangle? (2 times $n$) SAY: The two figures have equal area for all $n$'s, so the expressions $n + n$ and $2 \times n$ are equal for all $n$'s; mathematicians call them equivalent expressions.

Have students use the areas of rectangles to write more equivalent expressions. In all cases, give students the figures and ask them to write the expressions.

**Exercises:** Use the figures to write equivalent expressions.

**Answers:** a) $6 + 6 + 6 = 4 \times 6$, b) $4 + 4 = 4 \times 2$

Using properties of operations to write equivalent expressions.

Review the commutative property of operations and explain to students that $n + 2 = 2 + n$, which they showed using figures in part b) of the previous exercises, is the commutative property applied to expressions. Write on the board some more examples of the commutative property of addition and of multiplication. Examples:

- $n + 3 = 3 + n$
- $5 + x = x + 5$
- $4 \times n = n \times 4$
- $2x = x \times 2$

Review the distributive property of multiplication over addition and subtraction. Write some example of this property on the board, but use only numbers (no variables). Explain to students that the distributive property holds for all numbers, so instead of numbers they can use variables. For instance,
in the equation $2 \times (3 + 4) = 2 \times 3 + 2 \times 4$, if we replace the number 3 with a variable, $n$, we have $2 \times (n + 4) = 2 \times n + 2 \times 4$. SAY: Using order of operations we can rewrite this equation as follows:

$$2 \times (n + 4) = 2n + 8$$

**Exercises:** Use the distributive property to write an equivalent expression.

a) $3 \times (x + 4)$  
b) $2 \times (3 + x)$  
c) $(x - 3) \times 4$  
**Bonus:** $(4 - 2) \times x$

**Answers:** a) $3x + 12$, b) $6 + 2x$, c) $4x - 12$, Bonus: $4x - 2x = 2x$

**Factoring equations.** Ask students to fill in the blanks.

1. a) $3n + 6 = 3 (\_ \_ \_ + \_ \_ \_)$  
b) $5x + 25 = 5 (\_ \_ \_ + \_ \_ \_)$  
c) $4y + 12$

**Answers:** a) $3(n + 2)$, b) $5(x + 5)$, c) $4(y + 3)$

2. a) $6n + 9 = 3 (\_ \_ \_ + \_ \_ \_)$  
b) $4x + 10 = 2 (\_ \_ \_ + \_ \_ \_)$  
c) $10n + 35 = 5 (\_ \_ \_ + \_ \_ \_)$

**Answers:** a) $3(2n + 3)$, b) $2(2x + 5)$, c) $5(2n + 7)$

Remind students that the numbers 3 and 6 are factors of 18 because they multiply to give 18. When you rewrite an expression such as $6x + 15$ in the form $3(2x + 5)$, this process is called factoring the expression.

Even though $2x + 5$ is the sum of two terms, it is also a factor of the larger expression $3(2x + 5)$. $3$ is the other factor of $3(2x + 5)$. Ask students to name the factors of expressions such as $5(3x + 7)$. ($5$ and $3x + 7$)

**Using greatest common factors to multiply.** Remind students how to find the greatest common factor of two numbers. Example: Find the greatest common factor of 12 and 18. The factors of 12 are 1, 2, 3, 4, 6, and 12, and the factors of 18 are 1, 2, 3, 6, 9, and 18. The greatest number on both lists is 6, so 6 is the greatest common factor (GCF) of 12 and 18.

**Exercises:** In each expression, find the GCF of the two numbers. Use the GCF to factor the expression.

a) $6x + 9$  
b) $12y + 30$  
c) $15n - 25$  
d) $21x - 49$

**Answers:** a) the GCF of 6 and 9 is 3: $3(2x + 3)$, b) the GCF of 12 and 30 is 6: $6(2y + 5)$, c) the GCF of 15 and 25 is 5: $5(3n - 5)$, d) the GCF of 21 and 49 is 7: $7(3x - 7)$

**The associative property.** The associative property of addition and multiplication states that you can add or multiply more than two numbers in any order, e.g.,

$$(3 \times 4) \times 5 = 3 \times (4 \times 5)$$

$$(2 + 6) + 8 = 2 + (6 + 8)$$
Exercises
1. Use the associative property to simplify the expression and write an equivalent expression.
   a) \(2 \times (5x)\)  
   b) \(3 \times (5y)\)  
   c) \(4 \times (3n) - 3 \times (2n)\)  
   d) \((3x) \times 2 + 3 \times (2x)\)

2. Find the equivalent expression mentally.
   a) \(2 \times (p \times 3)\)  
   b) \(3 \times (7m) \times 2\)  
   c) \(3 \times (2x) + (4x) \times 5\)  
   Bonus: \((8x) \times 3 - 4 \times (6x)\)

Answers: 1. a) \(2 \times (5x) = (2 \times 5)x = 10x\), b) \(15y\), c) \(6n\), d) \(12x\),
   2. a) \(6p\), b) \(42m\), c) \(26x\), Bonus: \(0\)

Extensions
1. Use the distributive property and order of operations to simplify the expression and write an equivalent expression.
   a) \(2 \times (x + 5) + 3 \times (x + 7)\)  
   b) \((x - 4) + 2 \times (x + 1)\)  
   c) \((x + 2) \times 3 - 2 \times (x + 2)\)  
   d) \(4 \times (x - 3) + 2 \times (x + 6)\)

Answers: a) \(5x + 31\), b) \(3x - 2\), c) \(x + 2\), d) \(6x\)

(MP5, MP8)  2. Write an equivalent expression without brackets. Use any tool you think will help. Explain how you got your answer.
   a) \(3(2x + 5)\)  
   b) \(2(4x + 7)\)  
   c) \(5(8x - 3)\)  
   d) \(a(bx + c)\)

Selected sample solutions: a)
- I wrote \(3(2x + 5)\) as \(2x + 5 + 2x + 5 + 2x + 5\), which is \(6x + 15\).
- I drew a picture. I used a circle for the unknown quantity and a stick for a 1. So \(2x + 5\) is shown by 2 circles and 5 sticks. I drew it 3 times.
- There are 6 circles and 15 sticks, so that is 6 copies of the unknown \(x\) and 15 ones, so I got \(6x + 15\).
- I used the distributive property. I know I can do that because a variable just represents a number, so I can do the same thing as though I was multiplying 3 by the sum of two numbers. I multiplied each term of \(2x + 5\) by 3 to get \(6x + 15\).

Answers: b) \(8x + 14\), d) \(abx + ac\)
Coefficient. The number that appears in front of a variable in an expression is called a coefficient. In the expression 7y, the number 7 is the coefficient.

Exercises: Circle the coefficients in the following equations.

a) 9x  
b) .07y + 6k  
c) −2x − \frac{1}{2}y

Answers: a) 9x, b) .07y + 6k, c) −2x − \frac{1}{2}y

In an expression, a term that has no variable is called a constant term, because its value does not change, even if the value of the variable changes. In the expression 2x + 3y + 5, 5 is a constant term.

Multiplication as a short form for addition. Remind students that the expression 3×5 is short for repeated addition: 3×5 = 5 + 5 + 5. Similarly, 3x is short for x + x + x.

Exercises

1. Write the expression as repeated addition.
   a) 5n  
b) 3x  
c) 4x  
d) 7y

2. Write the sum as a product.
   a) x + x  
b) n + n + n  
c) x + x + x + x + x  
d) m + m + m + m + m + m

Answers: 1. a) n + n + n + n + n, b) x + x + x, c) x + x + x + x, d) y + y + y + y + y + y; 2. a) 2x, b) 3n, c) 5x, d) 6m

Grouping x’s together to simplify an expression. Challenge students to write 2x + 3x as a single term.

Answer: 2x + 3x = \frac{x + x}{2x} + \frac{x + x + x}{3x} = 5x

Explain that students can do this because all the x’s represent the same number. Show students how to verify that 2x + 3x and 5x are equal for various values of x.
Values of \( x \) & \( 2x + 3x \) & \( 5x \) & Result
\hline
\( x = 1 \) & \( 2(1) + 3(1) = 2 + 3 = 5 \) & \( 5(1) = 5 \) & \( 2x + 3x = 5x \) for \( x = 1 \) \\
\( x = 2 \) & \( 2(2) + 3(2) = 4 + 6 = 10 \) & \( 5(2) = 10 \) & \( 2x + 3x = 5x \) for \( x = 2 \) \\
\( x = 3 \) & \( 2(3) + 3(3) = 6 + 9 = 15 \) & \( 5(3) = 15 \) & \( 2x + 3x = 5x \) for \( x = 3 \) \\
\( x = 4 \) & \( 2(4) + 3(4) = 8 + 12 = 20 \) & \( 5(4) = 20 \) & \( 2x + 3x = 5x \) for \( x = 4 \) \\

Have students substitute \( x = 5 \) and \( x = 6 \) into the two expressions to verify equality. Explain that the expressions are equal simply because 2 of anything plus 3 of the same thing is 5 of the same thing:

- \( 2 \) ones + \( 3 \) ones = \( 5 \) ones
- \( 2 \) twos + \( 3 \) twos = \( 5 \) twos
- \( 2 \) threes + \( 3 \) threes = \( 5 \) threes
- \( 2 \) \( x \)'s + \( 3 \) \( x \)'s = \( 5 \) \( x \)'s

ASK: How many \( x \)'s are in \( 3x + 4x \)? \( 7 \) So \( 3x + 4x = 7x \). Explain that grouping all the \( x \)'s together is called simplifying. Point out to students that to simplify \( 3x + 4x \), they can add the coefficients of the two terms (3 and 4) to find the coefficient of the answer (7). Then have students simplify expressions.

**Exercises**: Simplify the expression.

a) \( 8x + 2x \)  
  b) \( 9x + 4x + 3x \)

c) \( 3x + 3x + 4x \)  
  **Bonus**: \( x + 2x + 3x + 4x + 5x \)

**Answers**: a) 10x, b) 16x, c) 10x, Bonus: 15x

**Canceling**. Review undoing one operation. ASK: How could you undo “add \( x \)” (subtract \( x \)) SAY: Since subtracting \( x \) undoes adding \( x \), I can cancel \( x - x \). Write on the board \( x + x - x - x \), and SAY: We can cancel these also; in other words, we can cancel \( 2x - 2x \). ASK: What about \( 5x - 3x \)? Ask a volunteer to come and rewrite the expression as the sum of individual variables, and then cancel to find the difference.

\[ 5x - 3x = x + x + x + x - x - x = x + x = 2x \]

**Exercises**

Rewrite the expression as a sum of individual variables and then cancel.

a) \( 8x - 4x \)  
  b) \( 3x - 2x \)

c) \( 6x - 3x \)  
  d) \( 9x - 5x \)

e) \( 7x - 2x \)  
  f) \( 8x - 6x \)

**Answers**: a) 4x, b) \( x \), c) 3x, d) 4x, e) 5x, f) 2x
Then have students write the answers to both the algebraic subtractions and the corresponding numerical subtractions below, and compare the answers. What do students notice?

a) $7x - 2x = \underline{5x}$ and $7 - 2 = \underline{5}$

b) $3x - x = \underline{2x}$ and $3 - 1 = \underline{2}$

c) $6x - 4x = \underline{2x}$ and $6 - 4 = \underline{2}$

d) $8x - 2x = \underline{6x}$ and $8 - 2 = \underline{6}$

e) $6x - x = \underline{5x}$ and $6 - 1 = \underline{5}$

f) $8x - 2x + 3x = \underline{9x}$ and $8 - 2 + 3 = \underline{9}$

g) $7x + 3x - 4x - 2x = \underline{6x}$ and $7 + 3 - 4 - 2 = \underline{6}$

Sample answer: a) $7x - 2x = 5x$ and $7 - 2 = 5$, can add or subtract just the coefficients.

Exercises

1. Add or subtract without writing the expression as a sum of individuals $x$’s.

   a) $10 - 3 = \underline{7}$ so $10x - 3x = \underline{7x}$
   b) $8 - 5 = \underline{3}$ so $8x - 5x = \underline{3x}$
   c) $7 - 4 = \underline{3}$ so $7x - 4x = \underline{3x}$
   d) $9 - 4 = \underline{5}$ so $9x - 4x = \underline{5x}$
   e) $6 - 2 = \underline{4}$ so $6x - 2x = \underline{4x}$
   f) $5 - 3 = \underline{2}$ so $5x - 3x = \underline{2x}$
   g) $8 - 3 + 1 = \underline{6}$ so $8x - 3x + x = \underline{6x}$
   h) $7 - 4 + 2 - 3 = \underline{2}$ so $7x - 4x + 2x - 3x = \underline{2x}$

Answers: a) 7, 7x; b) 3, 3x; c) 3, 3x; d) 5, 5x; e) 4, 4x; f) 2, 2x; g) 6, 6x; h) 2, 2x

2. Rewrite the expression so that there is only one $x$.

   a) $6x - 5x$
   b) $6x - 4x$
   c) $6x - x$
   d) $8x - 3x + 2x$
   e) $9x - 2x + 3x$
   f) $7x - 2x + 4x$
   g) $8x - 5x + 2x$
   h) $7x - x + 2x$

Sample answer: e) $9x - 2x + 3x = 10x$ because $9 - 2 + 3 = 10$

Bonus: $9x - 8x + 7x - 6x + 5x - 4x + 3x - 2x + x$ (Students might notice that each pair of consecutive terms ($9x - 8x$, $7x - 6x$, etc.) just makes $x$, so the answer is $x + x + x + x + x = 5x$.)

Using canceling to solve equations. Exercises: Give students equations with one addition or subtraction. Have students describe what was done to the variable $x$, and how to undo that operation to find the original $x$.

   a) $x + 3 = 12$
   b) $5 + x = 12$
   c) $4 = x - 3$
   d) $x - 13 = 9$

Sample answer: a) 3 was added to $x$, so subtract 3 from 12 to get $x$:
$x = 12 - 3 = 9$
When students are comfortable solving for $x$ this way, relate this method to canceling:

\[
\begin{align*}
x + 3 &= 12 \\
x + 3 - 3 &= 12 - 3 \quad &\text{(if $x + 3$ and 12 are the same number, then subtracting 3 from both gives the same number; this step preserves equality)} \\
x &= 9 \quad &\text{(by canceling $3 - 3$, only $x$ remains)}
\end{align*}
\]

Explain that you subtract 3 from both sides because you want to undo the adding of 3 and get back to where you started.

**Exercises:** Use canceling to solve the equation.

a) $5 + x = 32$  
   b) $31 = x + 13$  
   c) $19 + x = 45$  
   d) $x + 22 = 33$

**Answers:**

a) $x = 32 - 5 = 27$,  
   b) $x = 31 - 13 = 18$,  
   c) $x = 45 - 19 = 26$,  
   d) $x = 33 - 22 = 11$

**Extensions**

1. a) Explain why $7y + 2y = 9y$.
   
   b) In pairs, explain your arguments for part a). Do you agree with each other? Discuss why or why not.

**Sample solutions:**

a) 
   - $7y + 2y = (y + y + y + y + y + y + y) + (y + y)$, so there are 9 $y$’s altogether.
   - I tried different numbers for $y$. For example, when $y = 3$, the equation says that $7 \times 3 + 2 \times 3 = 9 \times 3$, and you can write the left side as $(3 + 3 + 3 + 3 + 3 + 3 + 3) + (3 + 3)$, so there are seven 3s plus two more 3s, which is nine 3s altogether. That works for any number, not just 3.

**NOTE:** For part b), encourage partners to ask questions to understand and challenge each other’s thinking (MP3)—see p. A-49 for sample sentence and question stems to guide students.

2. Solve.

   a) $x + \frac{1}{2} = 0.6$  
   
   b) $\frac{2}{3}x = 0.4$

**Hint:** Change all decimals to fractions.

**Solutions**

a) $x = \frac{6}{10} - \frac{5}{10} = \frac{1}{10} = 0.1$,  
   b) $x = \frac{4}{10} \times \frac{3}{2} = \frac{12}{20} = \frac{60}{100} = 0.6$
**Goals**

Students will solve word problems involving equations by translating phrases and sentences into expressions and equations.

**PRIOR KNOWLEDGE REQUIRED**

- Can evaluate expressions
- Knows that in an equation the variable is unknown
- Can solve simple one-step addition and subtraction equations

**Associating phrases with operations.** Write the phrases from the box in AP Book 6.2 p. 114 (increased by, sum, more than, etc.) on the board out of order. Have students decide which operation each phrase makes them think of. Students can then create a table with headings “add,” “subtract,” “multiply,” and “divide,” and sort each phrase into the correct column.

**Exercises:** Translate the phrase into an expression.

- a) 5 more than a number
- b) 5 less than a number
- c) 5 times a number
- d) the product of a number and 5
- e) a number reduced by 5
- f) a number divided by 5
- g) 5 divided into a number
- h) 5 divided by a number
- i) a number divided into 5
- j) a number decreased by 5
- k) a number increased by 5
- l) the sum of a number and 5
- m) 5 fewer than a number
- n) the product of 5 and a number
- **Bonus:** a number multiplied by 3, then increased by 5

**Answers:**

- a) $x + 5$
- b) $x - 5$
- c) $5x$
- d) $5(x \div 5)$
- e) $x - 5$
- f) $x \div 5$
- g) $5 \div x$
- h) $x \div 5$
- i) $5(x \div 5)$
- j) $x - 5$
- k) $x + 5$
- l) $x + 5$
- m) $x - 5$
- n) $5x$, Bonus: $3x + 5$

**Translating sentences into equations.** Once students can reliably translate phrases into expressions, it is easy to translate sentences into equations: simply replace “is” with the equal sign (=) and replace the two phrases separated by “is” with the appropriate expressions. Give sentences in which one of the phrases is just a number (e.g., Three times a number is 12). **Exercises:** Write an equation for the sentence.

- a) 5 times a number is 20
- b) 5 fewer than a number is 7

**Answers:**

- a) $5x = 20$
- b) $x - 5 = 7$

**Bonus:** Use sentences in which both of the phrases are not directly numbers. For example: Three times a number is the number increased by 8. ($3x = x + 8$)
**Solving word problems.** Have students translate sentences into equations and then solve the equations. Use sentences that produce equations that can be solved in one step and include a variable on one side only (e.g., $3x = 15$, not $3x + 5 = 20$ and not $3x + 5 = 2x + 8$).

Show students how to translate a word problem into an equation. For example: Carl has 12 stickers. He has twice as many stickers as John has. How many stickers does John have? Solution: SAY: Let $x$ stand for the number of stickers that John has, since that is the unknown that we want to find. Let’s try to find two ways of writing how many stickers Carl has so that we can write an equation:

\[
\text{number of stickers Carl has} = 2 \times \text{number of stickers John has}
\]

\[
12 = 2 \times x
\]

\[
12 \div 2 = 2x \div 2
\]

\[
x = 6, \text{ so John has 6 stickers.}
\]

**Exercises**

a) Katie has 10 stickers. She has 3 fewer stickers than Laura. How many stickers does Laura have?

**Solution:** Let $n$ be the number of stickers that Laura has, the unknown we are looking for. Try to find two ways of writing how many stickers Katie has. Katie has 3 fewer stickers than the number Laura has, so Katie has $n - 3$ stickers. But Katie has 10 stickers. So $n - 3 = 10$ and $n = 10 + 3 = 13$.

b) Shareen has 12 stickers. She has 3 times as many stickers as Tyler does. How many stickers does Tyler have? (4)

c) Melissa has 12 stickers. She has 3 more stickers than Emmett does. How many stickers does Emmett have? (9)

d) Henry has 12 stickers. He has half as many stickers as Russell does. How many stickers does Russell have? (24)

**Changing the context in word problems. Exercises:** Have students solve the three problems below, and then discuss how they are similar and how they are different.

a) Bilal has 20 stickers. He has 5 times as many stickers as Omar. How many stickers does Omar have?

b) Lilly is 20 years old. She is 5 times older than Jacinta. How old is Jacinta?

c) Trevor walked 20 km. He walked 5 times further than Rebecca. How far did Rebecca walk?

Challenge students to make up their own contexts for the same numbers.
Extra practice with word problems.

a) Samuel runs 600 m each day. He runs 3 laps each day. How long is each lap?

b) Gabe runs 800 m each day. He runs 40 m more than Ahmed. How far does Ahmed run?

c) Jane has $3 in coins, all dimes and quarters. She has 21 coins in all. How many of each coin does she have? (Tell students who need guidance to let the number of quarters be $x$.)

i) Write an expression for the number of dimes.

ii) Write an expression for the value of the quarters, in cents.

iii) Write an expression for the value of the dimes, in cents.

iv) Explain why your answer to iii) is the same as $10 \times 21 - 10x$.

v) Use the expressions in ii) and iv) to write another way of finding the total value of the coins, in cents.

vi) We are given the total value of the coins: $3 or 300¢. Write an equation using this information and the expression for the total value of the coins from v).

vii) Solve the equation.

viii) Verify your answer by totaling the value of the coins you have from part vi).

Answers: a) 200 m; b) 760 m; c) i) $21 - x$, ii) $25x$, iii) $10(21 - x)$, iv) this is the distributive property, v) $25x + 10 \times 21 - 10x = 15x + 210$, vi) $15x + 210 = 300$, vii) $x = 6$, viii) 6 quarters = $1.50 and 15 dimes = $1.50, so altogether we have $3 and 6 + 15 = 21 coins.

The bonus question on AP Book 6.2 p. 115 and the extensions below involve two-step problems. To solve them, students can either use guess and check or they can use preserving equality twice. Example: $2x + 5 = 23$.

Solution: $2x + 5 - 5 = 23 - 5$

$2x = 18$

$x = 18 \div 2 = 9$

Extensions

Two numbers are consecutive if one is the next number after the other. Example: 6 and 7 are consecutive numbers because 7 is the next number after 6.

1. Fill in the blank.

a) 4 and ___ are consecutive. b) 7 and ___ are consecutive.

c) $x, x + 1$, and ___ are consecutive.
2. Solve the problem. The sum of two consecutive numbers is 15. What are the numbers?

   a) Use a T-table to solve the problem. List all possible pairs of consecutive numbers, in order, and find the sums. Stop when you reach 15.

   b) Use algebra to write an equation for the problem.

      i) If the smallest of the consecutive numbers is \( x \), write a formula for the other number.

         \[
         \text{smallest number} = \underline{\hspace{2cm}} \quad \text{next number} = \underline{\hspace{2cm}}
         \]

      ii) Write an equation using the given information: The sum of two consecutive numbers is 15.

         \[
         \underline{\hspace{2cm}} = 15
         \]

   **Bonus:** Group the \( x \)'s together, then solve for \( x \). What are the two numbers?

   (MP1, MP5)

3. Josh says that five consecutive numbers add to 785. Rani says: That means \( 5x = 785 \). What is the "\( x \)" that Rani is referring to? What are the five numbers? Use any tool you think will help.

   **Sample solutions**

   - I used a table:

     | Two Consecutive Numbers | Their Sum  |
     |-------------------------|-----------|
     | 1, 2                    | 1 + 2 = 3 |
     | 2, 3                    | 2 + 3 = 5 |
     | 3, 4                    | 3 + 4 = 7 |
     | 4, 5                    |           |
     | 5, 6                    |           |

     I noticed that the sum was always 5 times the middle number. (I knew this pattern continued because I noticed that from one row to the next the sum always increases by 5 as 1 is added to five numbers, and the sum divided by 5 always increases by 1.) So the "\( x \)" that Rani is referring to must be the middle number. The middle number is \( 785 \div 5 = 157 \), and the five consecutive numbers are 155, 156, 157, 158, and 159.

   - I used variables. I let \( s \) be the smallest number and found that

     \[
     785 = s + (s + 1) + (s + 2) + (s + 3) + (s + 4) = (s + s + s + s + s) + (1 + 2 + 3 + 4) = 5s + 10 = 5(s + 2).
     \]

     So the "\( x \)" that Rani is referring to must be two more than the smallest number, which is the middle number. I found \( 785 \div 5 = 157 \), so the smallest number must be 155 and the five consecutive numbers are 155, 156, 157, 158, and 159.
EE6-19  Graphs and Equations

Pages 116–117

STANDARDS
6.EE.C.9

VOCABULARY
coordinates
graph
input
output
tables

Goals
Students will find rules for linear graphs.

PRIOR KNOWLEDGE REQUIRED
Can identify and plot points on coordinate grids
Can make a table to record ordered pairs
Can find rules for tables

MATERIALS
dice of different colors

Review coordinate grids. Remind students of how the columns and rows are numbered in coordinate grids. Draw a coordinate grid on the board, and invite volunteers to locate points by their coordinates and to give the coordinates of various points on the grid. Remind them that the first number in the coordinate pair is the number of the column and the second number is the number of the row. For extra practice, have students do the activity.

ACTIVITY
Students will need a pair of dice of different colors (for example, blue and red) and a coordinate grid. Player 1 rolls the dice so that Player 2 does not see the result. He marks a dot on the coordinate grid, where the coordinates of the dot are given by the dice: the blue die gives the first number and the red die gives the second number. Player 2 has to write the coordinates of the dot.

Finding rules for linear graphs. Draw a line on the grid so that it passes through several grid vertices. (For example, draw a line that passes through the points (1, 1) and (4, 7).) Invite volunteers to mark points on the line and to write their coordinates on the board. ASK: What is the second coordinate of a number with first coordinate 11? How can you find the answer to this question? (you can extend the line on the grid)

How would you find the second coordinate of a number with first coordinate 100? Suggest to students that they try to derive an equation or formula using a T-table, as they did earlier (see AP Book 6.1 p. 74). Ask students how they could make a T-table from the graph. Students might suggest making a T-table with headings “first number” and “second number.” Invite a volunteer to make such a table. Let students practice making T-tables from several different graphs. They should mark points on the line, list their coordinates, and make a T-table with headings as above.
Exercises

1. On grid paper, make a T-table and graph the rule “Multiply by 2 and add 3.”

2. Mark four points on the line segment. Write a list of ordered pairs, complete the T-table, and write a rule that tells you how to calculate the output from the input.

Answers

1.  

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Rule: $3 \times \text{Input} = \text{Output}$

Extension

Wilson has $30. For every book he reads, his mother gives him $5.

a) Create a T-table that shows the number of books as input and the amount of money as output.

b) Write a rule for the amount of money Wilson has after reading $n$ books.

c) How many books does Wilson need to read to get $50? $100? $1,000?
EE6-20  Dependent and Independent Variables
Pages 118–119

STANDARDS
6.EE.C.9

VOCABULARY
coordinates
dependent variable
graph
independent variable
input
output

Goals
Students will analyze line graphs.

Prior Knowledge Required
Can identify and plot points on coordinate grids
Can read and create line graphs

Interpreting graphs. Draw the graph in the margin on a coordinate grid on the board. Explain to students that this graph represents the cost of parking a car in a lot. How much would it cost to leave the car in the lot for 1 hour? For 2 hours? For half an hour?

ASK: How much will you pay for entering the parking lot, even before you park the car there? ($3.00) How much does each additional hour cost? ($2.00) How do you know? Does the hourly charge vary? (no) As a challenge, ask students to give a rule that allows you to calculate the cost of parking. (For example, “$3 for parking plus $2 for each hour up to 3 hours, up to a maximum of 3 hours (or $9).” Or, in a more mathematical way, “If you park for less than 3 hours, multiply the time by $2 and add $3. If you park for 3 hours or more, the price is $9.”)

Explain that a nearby parking lot charges $2.50 per hour. What is the mathematical rule for the cost of parking there? Ask students to write ordered pairs for time and cost for the second parking lot, and ask them to plot the graph for the second rule. ASK: Which parking lot will be cheaper to stay in for 2 hours? (the second one because it’s $5) For 4 hours? (the first one because the cost doesn’t increase after 3 hours) (The table in the margin summarizes the answers students should get from their graphs.)

For a challenge, students might plot the cost of a third parking lot that charges $0.50 for any time less than 1 hour and $4.50 per hour after that.

a) Which of the three parking lots is best for parking times of 1, 2, and 3 hours?

b) Joe and Zoe left their cars at parking lots 2 and 3, respectively. They parked for the same amount of time and paid the same amount of money. How long did they park for?

Dependent and independent variables. Ask a volunteer to come and write the equation for the second parking lot ($2.50 per hour). Tell the volunteer to use the variables c for cost and t for time. (c = 2.5t) ASK: What variable in this equation is the input? (t) SAY: You can park in the parking lot for any number of hours you want, but the cost depends on the duration of park-time, so c changes depending on t. SAY: Mathematicians name the input the dependent variable and the output the independent variable. In this example, the dependent variable is t and the independent variable is c.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Cost at Parking Lot 1</th>
<th>Cost at Parking Lot 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$7</td>
<td>$5</td>
</tr>
<tr>
<td>4</td>
<td>$9</td>
<td>$10</td>
</tr>
</tbody>
</table>
Exercises: Let students analyze another graph that shows the motion of two objects over time. Use the situation below, or create your own.

A boat leaves port at 9:00 a.m. and travels at a steady speed. Some time later, a man jumps into the water and starts swimming in the same direction as the boat.

a) How many minutes passed between the time the boat left port and the time the man jumped into the water? (15 minutes) When did the man jump into the water?

b) How far from the port was the man at 9:15 a.m.?

c) When did the boat overtake the man?

d) How far can the boat travel in 1 hour?

e) How long does it take the man to swim 1 kilometer?

f) i) How far did the man swim before he was taken aboard?

ii) How can you see from the graph when the man was picked up by the boat?

g) Did the boat continue traveling when it met the man or did it stay at the same place for some time? How do you know?

h) The boat docked at another port 9 km from the starting point. At what time did this happen?

Answers: a) 9:15 a.m., b) 3 km, c) 10 a.m., d) 6 km, e) 15 minutes, f) i) 3 km, ii) The two lines intersect where the boat and the man meet. There is only one line on the graph after that time, which means the boat and the man are traveling together., g) The graph is horizontal, which means the distance didn’t change over time., h) 9 a.m. + 105 minutes = 10:45 a.m.

Extension

Write the sum of three consecutive numbers in a way that makes it easy to show that it is always a multiple of 3. Say what your variable represents. Explain how your expression shows that the sum is always a multiple of 3.

Sample solution: a) Let \( x \) be the smallest of the three numbers, then the sum of the three numbers is \( x + x + 1 + x + 2 = (x + x + x) + (1 + 2) = 3x + 3 \) which is always a multiple of 3, because it is the sum of two multiples of 3 (or because it is equal to \( 3(x + 1) \), which is also a multiple of 3).

Whole-class follow-up: Have students discuss with a partner how writing the expressions in different formats made the task easier.
Introduction to Inequalities

STANDARDS
6.EE.B.8

VOCA-BULARY
equal
greater than (>)
inequality
less than (<)
solution for an inequality

Goals
Students will write inequalities to represent conditions and will also learn how to show inequalities on a number line diagram.

Prior Knowledge Required
Can compare numbers
Is familiar with number lines
Knows what algebraic expressions and equations are

Numerical inequalities. Review comparing two numbers. Explain to students that the result of any comparison between two numbers is “greater than” or “less than” or “equal.” Remind students that the sign “<” is read “less than” and “>” is read “greater than.” Give students practice reading some inequalities, for instance, 12 < 17 is read “12 is less than 17” and 8 > 2 is read “8 is greater than 2.” SAY: We can use a number line to verify numerical inequality. For example, if a number is to the left of 5, then it is less than 5, and if a number is to the right of 5, then it is greater than 5. Draw the number line in the margin on the board and SAY: 7 is greater than 5 because it’s to the right of 5.

Explain to students that every inequality can be written in two ways. For instance, the inequality 5 < 7 implies 7 > 5 because if 7 is to the right of 5, then 5 is to the left of 7 (or, if 5 is less than 7, then 7 is greater than 5).

Exercises: Write the numerical inequality in another way.

a) 3 < 6  b) -1 > -4  c) 12 < 21  d) -87 > -96

Answers: a) 6 > 3, b) -4 < -1, c) 21 > 12, d) -96 < -87

Introducing algebraic inequalities. Remind students of numerical equations and algebraic equations. Write on the board: 2 + 3 = 5 and say “this is a numerical equation,” then write x + 3 = 5 and continue: “but this is an algebraic equation.” Explain to students that there are both numerical inequalities and algebraic inequalities, just as there are numerical equations and algebraic equations. Write on the board the inequalities 5 < 7 and 5 < x, and identify them as a numerical inequality and an algebraic inequality, respectively. SAY: A solution for an algebraic inequality is a number that make the inequality true. For example, 6 is a solution for the inequality 5 < x because 5 < 6 is a true statement, but 3 is not a solution because 5 < 3 is not true. Emphasize that 5 is not a solution for the inequality 5 < x because 5 is not less than 5 (5 is equal to 5). Explain to students that an inequality has infinitely many solutions. For example 6, 7, 8, 5.2, 9 1/2, and 100 are different solutions for the inequality x > 5, but they are not all the solutions.
Exercise: Connect each inequality on top to its solutions on the bottom.

\[ x > 15 \quad x < 0 \quad -4 > x \]

17    22    -4    16    -7    10

Answer

\[ x > 15 \quad x < 0 \quad -4 > x \]

Showing algebraic inequalities on a number line diagram. SAY: We can use number lines to show algebraic inequality. For example, all numbers to the right of 5 are greater than 5, so to show the inequality \( 5 < x \) on a number line, we have to mark all numbers to the right of 5. The thick black part of the number line in the margin shows all numbers greater than 5. Five itself is not a solution for the inequality and that's why we put a "hole" or circle at 5.

Exercises: Write an inequality for the number line diagram.

a) \[ 0 < x \quad x > 3 \]

b) \[ 4 < x \quad x > 7 \]

c) \[ -3 < x \quad x > 0 \]

d) \[ -2 < x \quad x > 0 \]

Explain to students that they can show the solutions of an inequality on a given number line. Draw the number line in the margin on the board and say that you want to show the solutions of the inequality \( x < 14 \) on this number line. SAY: First we have to decide whether the solutions are to the left of 14 or to the right. ASK: What numbers are less than 14, the numbers to the left of 14 or the numbers to the right of 14? (to the left) Draw a thick line (see the margin) to show the solution for \( x < 14 \).

Exercises: Draw a thick line to show the solution for the inequality on the given number line.

a) \[ x > 10 \]

b) \[ x < 60 \]

c) \[ 45 > x \]

d) \[ -4 < x \]
Inequalities as conditions. Start with an example. SAY: Tom is 12 years old and Jack is his younger brother. SAY: The inequality $x < 12$ can show just a part of the story. Explain to students that if $x$ shows Jack’s age, then $x$ can be any number less than 12, but it cannot be negative. For example, $-3$ is a solution for the inequality $x < 12$ but we cannot say Jack is $-3$ years old! So we need an extra condition on $x$. Write on the board “$x > 0$” and tell students this inequality is a condition that we add to express the solution more precisely.

Exercises: Write an inequality for the story. Let $x$ represent the unknown. If the solution is always a positive number, write “and $x > 0$."

a) Sasha weighs 90 pounds. Sasha’s sister weighs less than Sasha.

b) Mike has $19 and Sam has more money than Mike.

Answers: a) $x < 90$ and $x > 0$, b) $x > 19$

Extensions

1. Write an inequality for the story. Let $x$ represent the unknown. If the solution is always a positive number, write “and $x > 0$."

   a) Three times a number is greater than 15.

   b) Amanda’s father weighs 180 pounds. Amanda weighs less than half of her father’s weight.

   Answers: a) $3x > 15$, b) $x < \frac{180}{2}$ or $x < 90$ and $x > 0$

2. Draw a thick line to show the solution for each inequality on the given number line diagram.

   a) $x < 20$ and $x > 0$

   b) $x < 5$ and $x > -5$

   Answers

   a)  
   b)  

Expressions and Equations 6-21

O-19
3. A plant is 15 cm tall and grows 4 cm each week. In how many weeks from now will the plant be 95 cm tall? Use any tool you think will help.

Sample solutions
• I made a T-table:

<table>
<thead>
<tr>
<th>Weeks from Now</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
</tr>
</tbody>
</table>

I noticed that after 5 weeks it was 20 cm taller; that makes sense because it grows 4 cm each week. After reaching 35 cm, it will grow 60 more cm, so it will grow 20 cm three times, which means it needs another 15 weeks. So the plant will be 95 cm tall in 20 weeks.

• I found $95 - 15 = 80$. That's how many cm the plant grows to reach 95 cm. I know that it grows 4 cm each week, so I did $80 \div 4 = 20$ to find the number of weeks it takes to grow 80 cm. So the plant will be 95 cm tall in 20 weeks.

• I used variables. I let $x$ be the number of weeks from now, and I know that the height of the plant for any $x$ is $15 + 4x$. Since I want to know what $x$ is when the height is 95, I wrote $15 + 4x = 95$. I solved the equation in steps: $4x = 95 - 15$, $4x = 80$, $x = 20$. So the plant will be 95 cm tall in 20 weeks.

Whole-class follow-up: Have several volunteers present solutions. For each solution, ask the rest of the class to discuss in pairs the following questions: Is the argument correct? Is there a part of the argument they do not understand? Could the argument be improved to make it clearer? Then compare the solutions. Which solution was the fastest? How are the solutions the same? How are they different?

4. Mindy solved $83x = 1,328$ and got $x = 16$.

a) Check if Mindy is correct.

b) Use the solution to the equation Mindy solved to solve these equations. Explain how you came up with your answer.

i) $5(83x) = 5(1,328)$

ii) $83x + 2 = 1,330$

iii) $83(x + 5) = 1,328$

Answers: a) Mindy is correct, because when I multiplied $83 \times 16$, I got 1,328. b) i) $x = 16$ because multiplying both sides by 5 doesn’t change the value of $x$ that makes both sides equal. ii) $x = 16$, because adding 2 to both sides doesn’t change the value of $x$ that makes both sides equal. iii) From Mindy’s answer, $83 \times 16 = 1,328$, so $x + 5$ must be 16 and that means $x = 11$
PS6-8  Using Logical Reasoning

Teach this lesson after: 6.2 Unit 6

Standards: 6.EE.A.2, 6.EE.B.5, 6.EE.B.7, 6.EE.B.8

Goals:
Students will use logical reasoning to solve balance problems involving equations and inequalities.

Prior Knowledge Required:
Can use variables to represent an unknown value
Can solve a simple equation to find an unknown value
Understands balance models
Can substitute numbers for unknowns in an expression or equation
Can check whether a number solves an equation or inequality
Can use variables to represent an unknown quantity
Can use the inequality signs, including the “or equal to” signs
Can convert between units of weight
Can work with fractions

Vocabulary: balance, coefficient, equality, equation, expression, inequality, variable

Materials:
pan balance
10 identical objects (e.g., apples)
paper bags and counters
BLM Solving Equations with Balances (pp. O-28–30)

Review pan balances and equations. Show students a pan balance. Place the same number of identical objects, such as apples, on both pans, and show that the pans balance. Remind students that when the pans, or scales, are balanced, it means the same number of apples are on each pan. Draw on the board:

\[
\begin{align*}
\text{Apples} & \quad \text{Apples} \\
\text{Bowls} & \quad \text{Bowls}
\end{align*}
\]

SAY: Both balances show the same equality, but the drawing on the right is easier to draw. Remind students that we can use variables to represent numbers we do not know. Draw on the board:

\[
\begin{align*}
\text{Apples} & \quad \text{Apples} \\
\text{Bowls} & \quad \text{Bowls}
\end{align*}
\]
Ask students to write an expression for the number of apples on the side with the paper bag.

(x + 2) Remind students that the parts of the equation on either side of the equal sign are called the sides of the equation. SAY: Each pan of the balance becomes a side in the equation, so the “balance” becomes x + 2 = 5. Point to the balance and SAY: Removing the same number of apples from each pan keeps the sides balanced. ASK: How many apples should I take from each pan? (2) Cross out two apples from each pan, as shown below:

ASK: How many apples are left in the right pan? (3) How many apples are in the bag? (3) Write “x = 3” underneath the picture.

Exercise: Complete BLM Solving Equations with Balances.

Answers:
1. b) 2x = 6, c) 3x = 6, d) x + 1 = 3, e) 2x + 1 = 5, f) 3x + 4 = 7
2. a) 4 circles, 2x + 1 = 5, 2x = 4; b) 3 circles, 3x + 2 = 5, 3x = 3; c) 2 circles, x + 4 = 6, x = 2
3. I determined how many circles to draw by taking the same number of apples from the right side as were taken from the left side. You can use the first equation to find the second equation by subtracting the number that does not have a variable from both sides.
4. a) 3 circles, 2x = 6, x = 3; b) 2 circles, 3x = 6, x = 2; c) 2 circles, 5x = 10, x = 2
5. I determined how many circles to draw by dividing both sides of the equation by the number of bags. You can find the second equation by dividing both sides of the equation by the coefficient of x (i.e., the number of bags).
6. Scale A: 3x + 2 = 8; Scale B: 6 circles, 3x = 6; Scale C: 2 circles, x = 2
7. Teacher to check.
8. a) 3, b) 2
9. a) 3 circles, b) 2 circles

Writing inequalities from models. Draw on the board:

SAY: This picture represents a balance with a bag on one side and 3 apples on the other side. ASK: Is the weight of the bag equal to 3 apples? (no) How do you know? (the balance is tipped to one side) Which is heavier, the bag or the 3 apples? (the bag) How do you know? (the left side with the bag is tipped down) SAY: We do not know the exact number of apples in the bag, so let’s say there are x apples in it. Write “x” in the bag. SAY: We cannot write an equation to represent this model since the two sides are not equal, but we can write an inequality. ASK: What inequality can we write to show that there are more than 3 apples in the bag? (x > 3)
Beside the diagram, write on the board:

\[ x \] represents the inequality \( x > 3 \)

Draw three more circles on the left side of the balance, as shown below:

\[ \]

ASK: What expression represents the number of apples in the left side of the balance? \((x + 3)\)
And the right side? \((5)\)
Which side has more apples? (the left side has more than the right side)
SAY: You can write the inequality like this. Write on the board:

\[ x + 3 > 5 \]

Draw on the board:

\[ \]

ASK: What expression can you write for the left side? \((2x + 1)\)
The right side? \((7)\)
And all together as an inequality? \((2x + 1 < 7)\)

**Exercises:** Write an inequality for the balance.

a) 

b) 

c) 

d) 

**Answers:** a) \(5 > x + 2\), b) \(2x + 2 < 6\), c) \(2x + 4 < 8\), d) \(3x > 6\)
Using the balance model to solve addition inequalities. Draw on the board:

![Balance Model Diagram]

ASK: What inequality can we write? \((x + 2 > 6)\). If I remove one circle from each side, will the left side still be heavier? (yes) Erase one circle from each side. ASK: If I remove another circle from each side, will the left side still be heavier? (yes) Erase one more circle from each side. SAY: Now I have \(x\) by itself, so \(x > 4\). Have students write the original inequality \((x + 2 > 6)\) and the new inequality \((x > 4)\), one below the other. SAY: If the number of apples in the bag plus two apples is more than six apples, then the number of apples in the bag is greater than 4. Repeat with a few different examples (use only one bag with \(x\) apples in it per example).

**Exercises:** Write the inequality. Remove the appropriate number of apples from both sides and write the new inequality.

- **a)** [Diagrams A]
- **b)** [Diagrams B]
- **c)** [Diagrams C]
- **d)** [Diagrams D]

**Answers:**
- **a)** \(5 > x + 2, 3 > x\)
- **b)** \(x + 3 < 5, x < 2\)
- **c)** \(x + 4 < 8, x < 4\)
- **d)** \(x + 1 > 6, x > 5\)

(MP.2) Using the balance model to solve multiplication inequalities. Draw on the board:

![Balance Model Diagram]

ASK: What can we say about the objects on the balance? (2 bags each with \(x\) apples weigh more than 6 apples) How many apples would there be in each bag if the sides were balanced? (3) How do you know? (divide 6 into 2 equal groups, \(6 ÷ 2 = 3\)) Ask a volunteer to group the 6 apples into 2 equal bags to check the answer, as shown below:
SAY: But the number of apples is not equal on both sides! Two bags with \( x \) apples in each weigh more than two bags with three apples each. So, one bag with \( x \) apples has more apples than one bag with three apples. We write this inequality as \( x > 3 \).

Repeat the problem with 4 bags that weigh more than 20 apples, 5 bags that weigh less than 10 apples, and 2 bags that weigh more than 8 apples. Students can signal the number of apples that would be in each bag if the weights were equal. (5, 2, 4) Have students write the new inequalities. (\( x > 5, \, x < 2, \, x > 4 \))

Write on the board:

\[
x - 5 < 12
\]

(MP.5) ASK: Can you use the balance model to solve this problem? (no) Why not? (you cannot draw “subtract” 5 apples) Write on the board:

\[
x + 5 \leq 9
\]

ASK: Can you use the balance model to solve this problem? (no) Why not? (you cannot draw a balance that shows that \( x + 5 \) could be less than or equal to 9) SAY: Models are good to help us get an idea of the problem, but we cannot use them for all types of problems. And sometimes they take a long time to draw.

(MP.2) **Solving multiplication inequalities without using the balance model.** SAY: We can write an inequality without drawing a picture, and just imagine the picture. Write on the board:

\[
3x > 12
\]

SAY: This means that 3 bags with \( x \) apples in each weigh more than 12 apples. ASK: What mathematical operation describes splitting up the 12 apples into 3 bags? (division) SAY: You can write a new inequality if you divide the weights on each side by 3. We divide by 3 because, in the picture, there would be 3 bags on the left side. Algebraically, 3 is the coefficient of \( x \). ASK: What is the new inequality? (\( x > 4 \)) Write “\( x > 4 \)” on the board under \( 3x > 12 \).

SAY: When \( x \) has coefficient 1, you can read the solution to the inequality. Point to \( x > 4 \) and SAY: This means that each bag has more than 4 apples.

Write on the board:

\[
2x > 10 \hspace{1cm} 3x < 9 \hspace{1cm} 28 > 4x \hspace{1cm} 12 < 6x
\]

For each inequality, have students signal what number they should divide both sides by so that \( x \) has coefficient 1. (2, 3, 4, 6) Have volunteers write the new inequalities on the board underneath each corresponding inequality. (\( x > 5, \, x < 3, \, 7 > x, \, 2 < x \))

**Exercises:** Divide both sides by the coefficient. Write the new inequality.

a) \( 5x < 30 \) \hspace{1cm} b) \( 8n > 32 \) \hspace{1cm} c) \( 14n > 28 \) \hspace{1cm} d) \( 15p < 45 \)

**Answers:** a) divide by 5, \( x < 6 \); b) divide by 8, \( n > 4 \); c) divide by 14, \( n > 2 \); d) divide by 15, \( p < 3 \)
Problem Bank
1. Two squares and 3 triangles balance 3 squares and 1 triangle. How many triangles does 1 square balance?
Solution: Remove objects that are common to both sides:
\[ \begin{array}{c}
\square \quad \triangle \\
\square \quad \square \quad \square \quad \triangle
\end{array} \quad \text{so} \quad \begin{array}{c}
\triangle \\
\square \quad \square
\end{array} \]
2. Two bricks and a five-ounce weight on one side of a balance will balance with three-quarters of a pound and one brick. What is the brick’s weight in pounds? Hint: 1 pound = 16 ounces.
Answer: 7 ounces
3. One square, 2 circles, and 6 triangles balance 3 squares and 5 circles. Two squares and a circle balance 3 triangles. How many squares does 1 circle balance?
Bonus: How many squares does a triangle balance?
Solutions: Since 3 triangles balance 2 squares and 1 circle, you can replace the 6 triangles on the first balance with 4 squares and 2 circles. Remove the same number of circles and squares on both sides, and you get a circle balancing 2 squares; Bonus: Replace the circle from the second balance with 2 squares. Four squares balance 3 triangles, so 1 triangle balances 4/3 squares, or 1 1/3 squares.
4. Write the inequality. Remove the same weight from both sides. Then, write a new inequality. Hint: You can cross out whole bags.
a) \[ x + 5 > 2x + 1, \quad 4 > x \]
b) \[ x + 4 < 3x + 2, \quad 2 < 2x, \quad 1 < x \]
Answers: a) \( x + 5 > 2x + 1, \quad 4 > x \); b) \( x + 4 < 3x + 2, \quad 2 < 2x, \quad 1 < x \)
5. You solved \( x + 6 < 13 \) by subtracting 6 from both sides, like this:
\[
\begin{align*}
x + 6 &< 13 \\
-6 &\quad -6 \\
x &< 7
\end{align*}
\]
To solve \( x - 6 < 13 \), you add 6 to both sides, like this:
\[
\begin{align*}
x - 6 &< 13 \\
+6 &\quad +6 \\
x &< 19
\end{align*}
\]
Add the same number to both sides to solve the inequality.
a) \( x - 12 < 18 \) \quad b) \( x - 3 > 11 \) \quad c) \( 15 > x - 4 \) \quad d) \( 21 < x - 8 \)
Answers: a) add 12, \( x < 30 \); b) add 3, \( x > 14 \); c) add 4, \( 19 > x \); d) add 8, \( 29 < x \)
6. You solved $3x > 12$ by dividing both sides by 3 to get $x > 4$. To solve $\frac{x}{3} > 12$, you multiply both sides by 3 to get $x > 36$.

Multiply both sides by the same number to solve the inequality.

a) $\frac{x}{5} < 4$  
   b) $\frac{x}{3} > 6$  
   c) $\frac{x}{10} < 5$  
   d) $6 \frac{x}{2}$

**Answers:** a) multiply by 5, $x < 20$; b) multiply by 3, $x > 18$; c) multiply by 10, $x < 50$; d) multiply by 2, $12 > x$

7. Marla works a 12-hour shift. This shift is longer than the 2 identical shifts she worked last week. What can you say about the length of each shift she worked last week?

**Answer:** each shift was less than 6 hours long

8. Jayden has a big bottle that holds one liter of water. This bottle holds more than three identical smaller bottles together. What can you say about the capacity of each small bottle?

**Answer:** each is less than $\frac{1}{3}$ liter

9. In 5 years, everyone in Wendy’s family will be 18 years or older. What can you say about the ages of everyone in Wendy’s family right now?

**Answer:** everyone is 13 years or older
Solving Equations with Balances (1)

Each circle represents one apple. Each bag has the same number of apples in it.

1. Write an equation showing that both sides of the scale are balanced.

   a) \[ x + 2 = 5 \]
   
   b) 
   
   c) 

   d) 
   
   e) 
   
   f) 

2. Draw the circles on the right side of the balance. Write an equation for each balance.

   a) 
   
   b) 
   
   c) 

   so

   so

   so

   so

   so
Solving Equations with Balances (2)

3. In Question 2, how did you determine how many circles to draw? How can you use the first equation to find the second equation?

4. Draw the circles on the right side of the balance. Write an equation for each balance.
   a) 
   ![Image of balance with circles and equation]
   
   b) 
   ![Image of balance with circles and equation]
   
   c) 
   ![Image of balance with circles and equation]

5. In Question 4, how did you determine how many circles to draw? How can you use the first equation to find the second equation?

6. Scale A is balanced. Draw circles so that Scales B and C are balanced too. Write the equation for each scale.
   Scale A 
   ![Image of balance with circles and equation]
   
   Scale B 
   ![Image of balance with circles and equation]
   
   Scale C 
   ![Image of balance with circles and equation]
Solving Equations with Balances (3)

7. Draw a model to solve $2x + 5 = 9$. Does this type of model help you solve the equation $2x - 5 = 9$?

8. How many circles balance a triangle?
   a) 
   b) 

9. Scales A and B are balanced. Draw the number of circles needed to balance Scale C.
   a) 
   b)
This Unit in Context

Students were first introduced to categorical data in Kindergarten, where they classified data into categories and counted the numbers in each category. In Grade 1, they organized, represented, and interpreted data in up to three categories (1.MD.C.4). In Grades 2 and 3, they drew scaled picture graphs and scaled bar graphs to represent data sets in several categories (3.MD.B.3). And in Grades 4 and 5, students made line plots to display data sets of measurements in fractions of a unit, such as 1/2, 1/4, 1/8 (5.MD.B.2). In this unit, students will display numerical data using dot plots (6.SP.B.4) and will use mean, median, and range to summarize the sets of data with single numbers (6.SP.B.5c).

In Grade 7, students will extend their knowledge of statistics by using random samplings to draw inferences about single populations (7.SPA.1), and draw informal comparative references about two populations (7.SPB.3). In Grade 8, their study of statistics will deal with bivariate data—in other words, data that can be categorized according to two attributes (8.SPA.1).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP2 to MP4 and MP6 to MP8. Here are some examples of how students can show that they have met a standard.

**MP2:** In SP6-4 Extension 3, students reason abstractly and quantitatively when they create a story problem for a given data set to represent and explain the terms “data point” and “mean” and when they calculate the mean and explain what it represents in the context of the story problem.

**MP4:** In SP6-2 Extension 2, students model mathematically when they use a table to represent and solve a non-routine, real-world problem.

**MP6:** In SP6-2 Extension 1, students attend to precision when they use the definitions of “median” and “range” to explain why adding the same number to each data point in a data set changes the median but not the range. In SP6-2 Extension 2, students attend to precision when they clearly label the headings of a table to solve a real-world problem.

**MP8:** In SP6-1 Extension 3, students look for regularity in the repeated reasoning of calculating how the mean changes when a number is added to each data value in a data set, and they express this regularity when they write a rule and explain why it works.
Introduce mean. SAY: 5 people want to evenly share the plums they picked. People with a lot could give some to people with fewer until everyone has the same number, or someone could deal them out like cards. Mathematicians call this finding the mean or average number of plums. Draw the table below:

<table>
<thead>
<tr>
<th>Plums Picked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drew</td>
</tr>
<tr>
<td>Brian</td>
</tr>
<tr>
<td>Cindy</td>
</tr>
<tr>
<td>Kara</td>
</tr>
<tr>
<td>Erin</td>
</tr>
</tbody>
</table>

SAY: Each number is called a data point and a group or collection of data points is called a data set or set of data. The data points in a set all refer to the same thing; in this case, 4, 2, 6, 1, and 2 all refer to the number of plums each person picked.

Two methods to find the mean without calculations.
Method 1a. SAY: I will use 5 groups of blocks to represent the plums each person picked (see margin). Have a volunteer move blocks until the groups are equal. ASK: How many blocks are in each group? (3) SAY: If the plums were shared evenly, everyone would get 3 plums, so the mean is 3.

Method 1b. Explain that since we don’t always have blocks, we can also use drawings to find the mean. Draw the first picture of stacked beads shown in the margin. SAY: The numbers along the top (which match the number of beads in each stack) are the plum data and show how many plums each person picked. There are, for example, 4 beads on the stack...
above Drew’s name to show that he picked 4 plums. We find the mean as we did with blocks: We move beads, one by one, between stacks, until the stacks are equal, from the stack(s) with the most beads to the stack(s) with the fewest. We cross out the bead we are moving, and draw in and shade a new bead. Change your drawing to look like the top picture in the margin, explaining the changes to the beads and numbers. Move one bead at a time until your drawing looks like the next picture in the margin.

**Exercise:** Give students BLM Moving Beads to Find the Mean. Have students move beads to find the mean using Method 1b.

**Answers**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean: 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean: 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean: 4</td>
<td></td>
</tr>
</tbody>
</table>

Method 2. Spread 5 sheets of paper, one for each friend, on a table. Put 15 blocks in a pile. Ask a volunteer to distribute the blocks evenly on the 5 sheets. ASK: How many blocks are on each sheet? (3) Is this the answer we got before? (yes)

**ACTIVITY 1**

Put students into groups of 4. Give each group 28 counters and 7 sheets of paper. Have them find the mean of the 4 sets shown in the margin using Method 2. They should work through the sets one set at a time, each student doing one set. If there is time remaining, they can work through the sets again, each student doing a different set this time. Have students record their answers by writing the set and its mean; for example, (3, 1, 2, 2, 8, 8) mean = 4. **NOTE:** The number of sheets must always equal the number of data points in the set.

**Calculating the mean.** Get 5 volunteer “plum pickers.” Give them blocks to represent the plums they picked (4, 2, 6, 1, 2). Write on the board:

\[
\begin{align*}
\text{___} + \text{___} + \text{___} + \text{___} + \text{___}
\end{align*}
\]

One by one, in any order, have students give you all their plums. Write the numbers in the blanks. ASK: How many blocks do I have now? (15) Write “total = 15.” SAY: Now I’d like to give them out, evenly. ASK: Between how many people will I divide them? (5) Write “divide by 5.” ASK: How do we use 15 and 5 to find the mean? (15 ÷ 5) Write “15 ÷ 5 = ____.” ASK: What is the mean? (3) Write it in the blank. SAY: Again, the mean is 3. Repeat with the data set from Exercise a) in BLM Moving Beads to Find the Mean. Have students do part b) and part c) and check the answers for their calculations against their BLM diagrams.
Exercises: Calculate the mean. Signal your answers with your fingers.

a) 4, 7, 2, 1, 1  
b) 10, 12, 3, 7

Bonus: 0, 8, 15, 16, 16, 19, 24

Answers: a) 3, b) 8, Bonus: 14

Showing the mean with a horizontal line. Write “3, 7, 5” on the board. Have students find the mean by calculating and by drawing.

1. Calculating: \[ \text{mean} = \frac{\text{total}}{\text{number of data points in the set}} = \frac{15}{3} = 5 \]

2. Drawing: Project grid paper on the board. Draw the top picture shown in the margin on the grid. Have students draw it on grid paper and move beads until they have found the mean. Then move beads on the projection. Do not draw the horizontal line. SAY: This is what your drawings should look like now. (see middle picture in margin) SAY: If we were using real beads rather than a drawing, the result would look a bit different. Draw the bottom picture shown in the margin but without the horizontal line. SAY: All 3 stacks would have 5 beads, meaning that the mean is 5. Draw the mean line on the bottom picture. SAY: The line shows that the mean is at 5, just as you calculated earlier. Add the mean line to the middle picture. SAY: This is the same mean line, but it looks quite different in this picture. ASK: Which stack did you take beads from? (7) Which did you move beads to? (3) Point to the middle picture. SAY: So the tall stack had 2 beads above the mean line and the short stack had 2 spaces below it. ASK: Did you move those 2 beads from the tall stack into the spaces in the short stack? (yes)

How you know if the mean line is in the correct place. Write “3, 0, 4, 6, 2” on the board. Have students calculate the mean and signal the answer. (3)

Project BLM Stacks on the board. Draw the picture shown in the margin but without Xs. ASK: From your calculation, where does the mean line go? (3). Draw it and then draw the 4 Xs. SAY: Just as there were empty spaces below the mean line in the short stack in the previous example, there are empty spaces below the mean line here too. The Xs show those empty spaces. ASK: Are there the same number of beads above the mean line as spaces below it? (yes) Could we fill up all the spaces below the mean line with the beads that are above it? (yes) SAY: When there are exactly the correct number of beads above the mean line to fill up all the spaces below the mean line, we know the line is in the right position to show the mean.

Project BLM Stacks. Draw the picture shown in the margin. SAY: Last time we used a solid line for the mean because we knew what it was from your calculations. Now I’m making it dotted to show that it may not be in the correct position; after all, we haven’t calculated the mean this time. ASK: How many beads are above the dotted line? (4) How many spaces are below it? (9) Could the beads that are above the dotted line exactly fill all the spaces below it? (no) SAY: So that tells us that the dotted line isn’t in the correct position.
position to show the mean. SAY: Let’s move the line down one and count everything again. Move the line down and erase any Xs that are above it. (see margin) ASK: How many beads are above the line? (6) How many spaces are below it? (6) Are there the correct number of beads above the line to fill up the spaces below it? (yes) So is the line where it should be to show the position of the mean? (yes)

**ACTIVITY 2**

Give students BLM Finding the Correct Position for the Mean Line. Instruct them to draw Xs in the spaces under the line, write the number of spaces that are below the line, and write the number of beads that are above the line. Ask students to circle the one picture in each row that has the line in the correct position. In the last row, they will need to use a ruler to draw the lines. Students should check their answers by calculating the mean of each set.

**But what is the mean for?** Hold up a pencil and ask students to describe it. Prompt with questions about color, length, uses, and so on. Explain that we can’t describe everything about it, but a person who couldn’t see it would likely know it was a pencil. SAY: Just as we can describe pencils (or shoes, or animals, or anything), we can describe sets of data, and the mean is part of that description. It tells us about the *center* of the set. Explain that the mean is always useful to know, but it is especially useful for very large sets of data. Let’s say, for example, that we know (from asking 1,000 people) that the mean age when a child loses her first tooth is 6. If the mother of a 1-year-old who has just lost his first tooth knows this mean, she may decide to take the child to a doctor to see if everything is okay.

**The mean in context: comparing the mean with data points from the set.** Explain that sometimes we need to be able to compare the mean of a set of data with one or more of the data points in the set. SAY: For example, let’s say the mean weight of newborn babies is 7 pounds. ASK: Is a 2-pound newborn below the mean weight, equal to it, or above it? (below) What about a 6-pound baby? (below) Which is further from the mean? (the 2-pound baby) SAY: Doctors know that babies who weigh a lot less than the mean often need extra care, so knowing the mean weight of newborns helps us decide which babies might need a little extra care.

**The mean in context: comparing means with each other.** Explain that sometimes it’s useful to compare means with each other. Draw the table shown in the margin on the board. SAY: There are many kinds of large, medium-sized, and small cars. The table shows the mean cost of driving each size of car 100 miles. Though people may not know exactly how much they drive their own cars, knowing these means can give us an idea of how much money we could save if we drove a smaller car. Have students do Question 10 on AP Book 6.2 p. 125.

<table>
<thead>
<tr>
<th>Mean Cost of Driving 100 Miles</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Large car</td>
<td>$45</td>
</tr>
<tr>
<td>Medium car</td>
<td>$35</td>
</tr>
<tr>
<td>Small car</td>
<td>$25</td>
</tr>
</tbody>
</table>
Extensions

1. Find the mean.
   
   a) 4, 2, 5, 1  
   b) 7, 0, 8  
   c) 4, 2, 3, 6, 5  
   d) 20, 40, 10, 30  
   e) 18, 0, 23, 32, 27  
   f) 5, 6, 18, 4, 13, 2, 22  

   **Answers:** a) 3, b) 5, c) 4, d) 25, e) 20, f) 10

2. a) Find the mean of 8, 4, 8, 7, and 3.
   
   b) Is 7 above or below the mean?

   **Answers:** a) 6, b) 7 is above the mean

3. a) Find the mean of 0, 0, 1, 4, 6, 8, 8, 10, 11, and 12.
   
   b) Add 2 to each data point from part a). Re-calculate the mean. How did it change?
   
   c) Add 5 to each data point from part a). How does the mean change?
   
   d) Write a rule to describe how the mean changes when you add the same number to each data point. Explain why your rule works.

   **Answers:** a) 6; b) 8; the mean increased by 2; c) the mean increases by 5; d) If you add a number \( n \) to each data point in a data set, the mean increases by \( n \). The rule works for any number that you add, because when you always add the same number to each data value, and there are, for example, 10 data values, you increase the sum by 10 times the number, but then you divide by 10 to get the mean, so you are increasing the mean by the number you added to each data point. Explain why your rule works.

   **(MP3, MP6)**

4. a) Calculate the mean of each set.
    
    i) 7, 2, 15  
    ii) 7, 2, 15, 0

   b) Explain why the mean of i) is greater than the mean of ii).
   
   c) In pairs, explain your answers to part b). Do you agree with each other? Discuss why or why not.

   **Answers:** a) i) 8, ii) 6; b) Both totals are 24 but in part i), 24 has to be divided into 3 groups whereas in part ii), it has to be divided into 4 groups. The fewer groups there are, the more there will be in each group, so the greater the mean.

   **NOTE:** For part c), encourage partners to ask questions to understand and challenge each other’s thinking (MP3) and use of math words (MP6)—see p. A-49 for sample sentence and question stems to guide students.

   Redirecting students: If students struggle to explain why the mean in i) is greater than the mean in ii), encourage them to make up story problems using those numbers (MP2). ASK: Can you see why the mean has to be greater in your real-world examples? For example, students might use test scores to see that if they miss a test, they will get a higher grade if their teacher doesn’t count the test than if the teacher counts the test as 0.

---

**Statistics and Probability 6-1**
SP6-2 Mean, Median, and Range

Pages 126–127

STANDARDS
6.SP.A.2, 6.SP.A.3,
6.SP.B.5a, 6.SP.B.5b,
6.SP.B.5c

VOCABULARY
center position
median
range

Goals
Students will find the mean, median, and range of a set of data and investigate the relationships between them.

PRIOR KNOWLEDGE REQUIRED
Can identify the highest and lowest numbers in a set of data
Can compare and order whole numbers
Can solve a simple equation to find an unknown value

MATERIALS
Five books with different publishing dates
BLM The Mean and Median When the Highest Value Increases (p. P-22)
BLM Mean and Median (p. P-23)

ACTIVITY 1

Invite 9 volunteers to be a set of data. Ask them to order themselves from shortest to tallest. SAY: Now that these students are in order of height, we can say that there is 1 person in the center position. ASK: Who is in the center? Get 5 books and tell students you are going to put them in order by publishing data. SAY: Again, there is 1 book in the center. Ask which one. Put students into groups of 4 or 5, and have each group collect a set of things that can be ordered. They should only create sets with an odd number of things in them. Make clear that all the things in a set have to be of the same type and have to have some characteristic that can be put in numerical order. Give them suggestions if needed (for example, shoe size, hair length, book thickness or page count, the number of letters in words). One at a time, each group shows its set to the class with the things in it in order so that the rest of the class can clearly pick out the one in the center position.

Introduce median. SAY: When we put sets of data in order, as we have been doing, we call the one in the center position the median. Write the set shown in the margin on the board. ASK: Are the numbers in the set in order? (yes) SAY: 14 is in the center so it is the median. Underline 14.

Being organized to order data. Write the set shown in the margin on the board. SAY: We’ll order these numbers from least to greatest in an organized way. ASK: What is the smallest number? (4) Write “4” under 23 and cross out 4 in the original set. SAY: We cross out 4 to show we’ve used it. ASK: What’s next? (8) Write “8” beside the 4 and cross it out in the original set. Continue until the set is in order.

6, 8, 11, 14, 19, 26, 27

23, 11, 9, 16, 27, 4, 20, 12, 8
Exercises: Order the numbers using the method just demonstrated.

a) 16, 12, 23, 42, 7, 33, 10, 41  
   b) 203, 322, 230, 302, 233, 320

Answers

a) 7, 10, 12, 16, 23, 33, 41, 42  
   b) 203, 230, 233, 302, 320, 322

Being organized to find the median. SAY: Let’s find the median of the set we ordered earlier: 4, 8, 9, 11, 12, 16, 20, 23, 27. We work inward from the ends. Cross out 4 and 27, then 8 and 23, and so on, until only 12 is left.

SAY: So 12 is the median. Underline 12. Write the set shown in the margin on the board. SAY: Sets with an odd number of data points always have one number in the center position. Underline 9. SAY: Let’s find the median of a set with an even number of data points. Write the set shown in the margin on the board. Cross out 2, then 8. ASK: Why don’t we cross out 3 and 7 now? (there will be no numbers left) SAY: Sets with an even number of data points have a median that is halfway between the two center numbers.

Underline 3 and 7. Write the numbers from 3 to 7 (see margin). SAY: Now we cross out 3 and 7, then 4 and 6; the median is 5. Underline 5. SAY: The median number doesn’t need to be in the set. Write the following pairs of numbers on the board. ASK: What number is halfway between 12 and 14? (13) 5 and 7? (6) 6 and 10? (8) 13 and 19? (16) (Students who find this hard can write all the numbers between the pair.)

Exercises: Order the numbers and then find the median.

a) 4, 7, 2, 1, 5  
   b) 9, 12, 3, 7  
   c) 19, 1, 26, 4, 28, 17

Bonus: 320, 602, 362, 630, 302, 360, 620, 203

Answers: a) 4, b) 8, c) 18, Bonus: 361

ACTIVITY 2

Invite 9 volunteers to be a set of data. Have them order themselves from shortest to tallest. Who is the median? Now have 1 volunteer sit down, and ask the class how they can find and represent the median. (Students might suggest making a mark on the board to represent the height halfway between the 2 middle students, or they might suggest choosing another student whose height is roughly halfway between the 2 middle students to be the median.)

Mean and median as centers of data sets. SAY: The mean and median both tell us about the center of a set, but we find them in different ways: one using calculations, the other using ordering. Write “14, 2, 5” on the board, and have students find the mean and median.

Exercises: Find the mean and median.

a) 11, 1, 15  
   b) 1, 17, 6, 8

Bonus: 108, 106, 110, 108
Answers: a) mean = 9, median = 11; b) mean = 8, median = 7; Bonus: mean = 108, median = 108

Extreme values change the mean, not the median. SAY: When basketball player, Sasha, practices her free throws, she usually scores 10 out of 10 baskets. Write the table on the board:

Sasha’s Usual Scores

<table>
<thead>
<tr>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

SAY: These are usual scores. Have students find the mean (10) and median (10). Write the table on the board:

Sasha’s Scores Yesterday

<table>
<thead>
<tr>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

SAY: These are her scores from yesterday’s practice. Have students find the mean (8) and median (10). ASK: Why is the mean less in the second set? (0 made it go down) SAY: The 0 didn’t change the median because 10 stayed in the center position. Explain that in sets, unusual numbers like this 0 change the mean, but not the median. SAY: The median, which is 10, is much more like Sasha’s usual score than the mean, which is 8, so we may decide to think of the set’s center as being its median. The mean and median are both types of centers, but sometimes it makes more sense to use one than the other.

Exercise: Have students complete BLM The Mean and Median When the Highest Value Increases.

Answers: a) median = 1, mean = 2; b) median = 1, mean = 3; c) median = 1, mean = 3; d) no; e) yes

Project BLM Mean and Median on the board. Plot the first set on the number line (see margin). SAY: The largest number, 3, is close to 1 and 2. Plot each set noting the largest number’s gradual movement away from 1 and 2. Have students signal the medians (2), then the means (2, then 3, then 4, then 5, then 6). ASK: Did the median change? (no) Did the mean change? (yes) SAY: The median never changed because 2 stayed in the center position. The mean increased from set to set because the totals in the addition step increased from set to set.

Introduce range. Draw the table on the board:

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Highest Mark</th>
<th>Lowest Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>14</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Class 2</td>
<td>14</td>
<td>16</td>
<td>13</td>
</tr>
</tbody>
</table>
SAY: Two teachers gave their classes the same test out of 20. The classes had the same average mark, 14. However, in Class 1, the highest and lowest marks were 16 and 13, whereas in Class 2, the highest and lowest marks were 20 and 5. Discuss why a teacher might want to know this information. (Possible discussion ideas: the smaller range suggests that all students learned roughly the same body of material) Explain that when we look at how far apart the highest and lowest values are, we are talking about the range. Write “12, 16, 14, 11” on the board. ASK: What is the highest value? (16) The lowest value? (11) Underline 16 and 11. Write the equation shown in the margin on the board. SAY: We find the range by subtracting the lowest value from the highest value. Write 16 and 11 in the blanks. ASK: What is the range? (5)

Exercises: Underline the highest and lowest values. Find the range.

a) 9, 2, 5 b) 22, 26, 17, 14 c) 410, 140, 104, 401

Bonus: 473, 1,347, 374, 743, 1,374, 437, 734

Answers: a) 9, 2, 5, and 9 – 2 = 7; b) 22, 26, 17, 14, and 26 – 14 = 12; c) 410, 140, 104, 401, and 410 – 104 = 306; Bonus: 374, 1,347, 473, 743, 1,374, 437, 734, and 1,374 – 374 = 1,000

Finding the highest value given the lowest value and range. Write the set and range shown in the margin on the board. SAY: In this set, the highest value is missing. We can use the range and lowest value to find it. Write the equation shown in the margin on the board. ASK: What is the lowest value? (4) And the range? (10) SAY: Adding 10, which is the range, to the lowest value, which is 4, gives us the highest value. ASK: What is the highest value? (14)

Exercises: Find the highest value.

a) 11, 14, 16, ___ range = 8 b) 25, 28, 28, 33, ___ range = 12

Answers: a) ___ – 11 = 8, highest = 19; b) ___ – 25 = 12, highest = 37

Increasing the highest number in a set. SAY: Let’s see what happens to the range of a set when we increase the highest number. Draw the table below with headings but no numbers on the board.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Range</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 3, 5, 8</td>
<td>8 – 2 = 6</td>
<td>4</td>
</tr>
<tr>
<td>2, 3, 5, 18</td>
<td>8 – 2 = ___</td>
<td>4</td>
</tr>
</tbody>
</table>

Write “2, 3, 5, 8.” ASK: How do we find the range? (8 – 2) Write “8 – 2 = 6.” SAY: Let’s add 10 to the highest value to see what happens to the range. Write “2, 3, 5, 18.” ASK: How do we find the range? (18 – 2) Write “18 – 2 = ___.” ASK: What is the new range? (16) What did adding 10 to the highest value do to the range? (it went up by 10) What is the median of the top set? (4) And the bottom set? (4) Did adding 10 to the highest value change the median? (no)
Exercises: Find the range and median. Add 10 to the highest value. Find the range and median again.

a) 13, 18, 20, 32, 45  
b) 6, 23, 27, 68

Answers: a) range = 32, median = 20, new set is 13, 18, 20, 32, 55, new range = 42, new median = 20; b) range = 62, median = 25, new set is 6, 23, 27, 78, new range = 72, new median = 25

Extensions

(MP3, MP6) 1. a) Find the median and the range for each set:
   
   i) 2, 3, 5, 7, 9, 10  
   ii) 12, 16, 19, 22, 26, 26, 26

   b) Add 4 to each data point in i) and ii). Find the new median and range.

   c) Why did adding 4 to each data point change the median but not the range?

   d) In pairs, explain your answers to part c). Do you agree with each other? Discuss why or why not.

   Answers: a) i) median = 6, range = 8; ii) median = 24, range = 14;
   
b) i) median = 10, range = 8; ii) median = 28, range = 14;
   
c) Every data point in the set changed, so the median—the number in the center—also changed. Although the data points changed, they all changed by the same amount and the difference between the highest and lowest values didn't change, so the range didn't change either.

   NOTE: For part d), encourage partners to ask questions to understand and challenge each other’s thinking (MP3) and use of math words (MP6)—see p. A-49 for sample sentence and question stems to guide students.

2. Write three different sets that have median 9 and range 5.

   Sample answers: 8, 9, 13; 5, 9, 10; 7, 8, 9, 10, 12; 9, 9, 9, 9, 10, 11, 14; 8, 8, 8, 8, 8, 10, 10, 10, 10, 10, 13

(MP4, MP6) 3. a) When Peter turned 4 years old, he was 99 cm tall. He grew 7 cm each year since then. Peter is at a fair and he just went on a ride that requires each person to be more than 120 cm tall. What might his age be? Make a table to solve the problem.

   b) Let a be Peter's age. Write an inequality for a.

   Answers: a) Draw a table with headings “Peter’s age (years)” and “Peter’s height (cm).” From the table, Peter’s height is more than 120 cm when he is more than 7 years old. b) a > 7.
SP6-3 Frequency
Page 128

STANDARDS
6.SPA.2, 6.SPA.3, 6.SPB.5a, 6.SPB.5c

VOCABULARY
cell
data value
frequency
tally
tally chart

Goals
Students will distinguish between the value of a data point and its frequency, and calculate the mean using frequency.

Prior Knowledge Required
Can calculate the mean of a set of data

Introduce frequency. SAY: Frequency is the number of times something happens. In a set, the word “frequency” means the number of times a value occurs. Write “7, 7, 4, 4, 7, 6, 5, 5” on the board. SAY: The set has 8 data points but only 4 different data values: 4, 5, 6, and 7. There are three 7s so we say “the frequency of 7 is 3.” ASK: What is the frequency of 4? (2) And 5? (2) And 6? (1)

Organizing data in a tally chart or frequency chart. On the board, draw the chart shown in the margin, with headings but without data. SAY: This is a tally chart or frequency chart. Fill in the Data Value column as shown. Explain that we put the values from the set into the first column. SAY: Now let’s put the data into the chart. ASK: What is the first data point in the set? (7) Mark one tally in the tally cell for 7. Cross out the first 7 in the set. SAY: I crossed out the 7 to remind myself that I’ve used it already. ASK: What number is next? (7) Mark a second tally beside the first one. Cross out the second 7. Have students help you fill in the remaining data. SAY: Now let’s complete the frequency column. We write “2” in the frequency cell for 4 because there are two 4s. ASK: What do we write in the frequency cell for 5? (2) And 6? (1) What about 7? (3) Write a new set on the board: 4, 7, 4, 4, 7. ASK: What do you notice about the highest and lowest values of these two sets? (they’re the same) Explain that you want to make another chart to record this data. Draw the chart below on the board:

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

Write the headings and fill in the first column. SAY: In the Data Value column, we write all numbers within the range. In other words, we start from 4 and write all the numbers up to and including 7. Now let’s fill in the tally and frequency columns. ASK: What is the first data point in the set? (4) How do we show that in the tally column? (mark one tally beside 4) Cross out the first 4. ASK: What comes next? (7) Have students help you fill in the remainder of the chart. Tell students that for the next set of exercise questions, they will need to know how to show 5 tallies. SAY: This is how you show it. Write ||||.
**Exercises:** Transfer the data into a chart. Cross out each number as you mark it in the chart.

a) 3, 2, 5, 3, 5, 5  

b) 11, 10, 12, 15, 12, 10, 10, 11, 10

**Answers**

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>###</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>I</td>
<td>1</td>
</tr>
</tbody>
</table>

**Calculating the mean using frequency.** Write on the board:

3, 4, 9, 8, 6, 3, 9, 9, 3, 6

ASK: Up to now, what have we done to calculate the mean of a set? (added the numbers and divided by how many there are) SAY: Now we’re going to calculate the mean using frequencies. Draw the chart below but without any data.

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

ASK: Based on the set, what numbers go in the Data Value column? (3 to 9) Write them in. Have students help you fill in the chart. SAY: If we added all the numbers one by one, we would have a really long addition that would look like this. Write the addition: \(3 + 4 + 9 + 8 + 6 + 3 + 9 + 9 + 3 + 6\). SAY: Instead we can add all the 3s, then all the 4s, and so on. Write:

\[
\begin{align*}
(\_ \times 3) + (\_ \times 4) + (\_ \times 5) + (\_ \times 6) + (\_ \times 7) + (\_ \times 8) + (\_ \times 9) \\
= \_ + \_ + \_ + \_ + \_ + \_ + \_ + \_
\end{align*}
\]

SAY: The frequency of 3 is 3. In other words, there are 3 threes, so we write “3” in the first blank. Continue this way until you have written in all the frequencies. ASK: What is \(3 \times 3\)? (9) Write “9” in the first blank after the equal sign, and continue until all the blanks are full. Ask students to do the addition. SAY: Doing it this way means you get to do a much shorter addition. ASK: What is the total? (60) How many data points are there in the set? (10) How do we find the mean? (60 \(\div\) 10) What is the mean? (6)
Exercises: Put the data into a chart and calculate the mean.

a) 4, 4, 6, 5, 5, 6  
b) 6, 2, 7, 6, 2, 1, 4

Answers

a) 4, 4, 6, 5, 5, 6  
b) 6, 2, 2, 7, 6, 6, 2, 1, 4

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>I</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
(2 \times 4) + (2 \times 5) + (2 \times 6) + (1 \times 1) + (3 \times 2) + (0 \times 3) + (1 \times 4) + (0 \times 5) + (3 \times 6) + (1 \times 7) + 1 + 6 + 0 + 4 + 0 + 18 + 7 = 36
\]

\[
36 \div 9 = 4
\]

Bonus: (students should make a chart counting by 100s)

600, 400, 600, 600, 400, 1,000

Answer: 600

Extensions

(MP.6) 1. a) Find the mean using frequency.

i) 11, 0, 12, 6, 4, 1, 8, 10, 8, 0  
ii) 24, 23, 26, 27, 23, 25, 25, 27

b) Use one of the examples in part a) to explain what frequency means and explain the difference between the value of a data point and its frequency.

Sample answers: a) i) \((2 \times 0) + 1 + 4 + 6 + (2 \times 8) + 10 + 11 + 12 = 60\), and \(60 \div 10 = 6\), so the mean is 6. ii) \((2 \times 23) + 24 + (2 \times 25) + 26 + (2 \times 27) = 200\), and \(200 \div 8 = 25\), so the mean is 25.

b) Frequency means the number of times a data value occurs in a data set. For example, in a) ii), there are two data points with value 27, so the value of the data points is 27, while the frequency of the data value 27 is 2.

(MP.7) 2. Use the expression for the calculation of the mean to figure out the frequencies. Write the frequencies in the blanks.

\[
(\_ \times 1) + (\_ \times 2) + (\_ \times 3) + (\_ \times 4) + (\_ \times 5) + (\_ \times 6) + (\_ \times 7) = 8 + 4 + 12 + 4 + 0 + 24 + 35
\]

Answers

\[
(8 \times 1) + (2 \times 2) + (4 \times 3) + (1 \times 4) + (0 \times 5) + (4 \times 6) + (5 \times 7)
\]
Goals

Students will create and interpret dot plots, and calculate their means.

Prior Knowledge Required

Can calculate the mean of a set of data
Can identify line-symmetric figures

Materials

BLM Stacks (p. P-20)
BLM Dot Plots 1 (p. P-24), BLM Dot Plots 2 (p. P-25)
BLM Dot Plots 3 (p. P-26), BLM Dot Plots 4 (p. P-27)
BLM Dot Plots Bonus Questions (p. P-28)
BLM Dot Plot and Tally Chart Symmetry (p. P-29)
pencil
straightedge
BLM Calculating Means from Dot Plots (p. P-30)

Introduce dot plots. Project BLM Stacks on the board. Draw the picture shown in the margin onto it. SAY: The diagram shows a set with 5 data points. ASK: What are the points? (8, 8, 8, 5, and 2) SAY: When we have large data sets, these diagrams aren’t very good to use. If we had a set with 100 data points, we would have to draw 100 stacks of beads. ASK: Can you draw 100 stacks of beads quickly? (no) SAY: So the first reason we don’t use these diagrams for large sets is because they take too long to draw. But there’s another reason too. Explain that for a large data set that has, for example, twenty 8s we would have to draw 20 stacks of 8 beads. That’s 160 beads, just to show the 8s in the set. SAY: We need a type of diagram where we can draw 1 symbol, like a dot, for every data point in the set. In the case of the twenty 8s, instead of drawing 160 beads, we would draw only 20 dots. In past grades, you may have used line plots to do this. That’s what we’re going to use here too, but now we’ll call them dot plots.

Project BLM Dot Plots 1 on the board. Point to the title. ASK: What is this dot plot about? (the number of stories each student wrote) SAY: The number line shows that students wrote anywhere from 1 to 6 stories. Circle the column of data above 6. SAY: This shows that 2 students wrote 6 stories. ASK: How many students wrote 1 story? (8) 2 stories? (6) Altogether, how many students wrote stories? (28) What was the largest number of stories any student wrote? (6) How many more students wrote 1 story than 2? (2) How many students wrote either 5 or 6 stories? (4)

Give students BLM Dot Plots 2. Ask them to plot the data from the chart. Wait until they are done. ASK: How many people picked 1 plum? (5) How many people picked 3 or more plums? (5) What was the largest number of
plums any one person picked? (5) Did anyone pick exactly 4 plums? (no)
How many more people picked 1 plum than 5 plums? (2) Altogether, how
many people picked plums? (14)

Project BLM Dot Plots 3 on the board. SAY: Sometimes you'll see plots
that look like dot plots but aren't, like B and C. ASK: How are plots A and B
different? (one has a number line, the other has the names of trees) What
about A and C—how are they different? (in C, the gaps between the numbers
along the bottom are not the same) SAY: All dot plots must have a number
line at the bottom, and to be a number line, the gaps between the numbers
all have to be the same. A plot without a number line is not a dot plot.

Project BLM Dot Plots 4 on the board. SAY: Just as we create dot plots
from tally charts, we can create tally charts from dot plots. Point to the first
Data Value cell. ASK: What number goes in here? (1,003) Continue until all
the Data Value cells have been filled in. Have students help you to fill in the
tallies and frequencies.

<table>
<thead>
<tr>
<th>Data Value</th>
<th>1,003</th>
<th>1,004</th>
<th>1,005</th>
<th>1,006</th>
<th>1,007</th>
<th>1,008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tally</td>
<td>I</td>
<td>II</td>
<td>I</td>
<td>I</td>
<td>III</td>
<td>III</td>
</tr>
<tr>
<td>Frequency</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Ask students to do Questions 1–5 on AP Book 6.2 pp. 129–130. Students
who finish early can work on BLM Dot Plots Bonus Questions.

Symmetry in dot plots. SAY: Recall that a shape is symmetrical if, when
folded along a line of symmetry, the sides fit exactly over one another.

**ACTIVITY**

Give each student a strip from BLM Dot Plot and Tally Chart
**Symmetry.** Students will need a pencil and straightedge for the activity.
Explain the instructions.

1. Use pencil to shade in the dots. Make pencil marks very dark.
2. Fold the strip along the dotted line, with the dots on the inside.
   Place the folded strip onto your desk.
3. Using your thumbnail, rub the dots very hard until their images
   appear on the other half of the strip.
4. Unfold the paper to see where the new dots are. (Draw students’
   attention to the fact that the dots on the opposite sides of the center
   line are positioned identically.)
5. Repeat for the tally chart, making very dark pencil lines over each
   existing tally.
The mean and median of symmetrical data sets. Draw the chart below on the board:

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>III</td>
<td>3</td>
</tr>
</tbody>
</table>

Ask students to find the total, and then the mean using the frequencies. (total: $30 + 22 + 26 + 42 = 120$, mean: 12) Then ask students to find the median. Remind them to write the data in order. ASK: What is the median? (12) What do you notice about the median and the mean? (they’re the same) SAY: In symmetrical sets, the median and mean are always equal.

**Exercises:** Put the data in a chart. Look at the arrangement of tallies and decide if the mean equals the median.

a) 27, 26, 27, 26, 26, 29, 28, 28  
   b) 9, 13, 15, 9, 12, 15, 12, 9, 11, 15  
   c) 2, 7, 9, 5, 7, 11, 7, 11, 7, 2  

**Bonus:** 160, 160, 140, 100, 140, 160, 160 (Students should make a chart that goes up by 10s, not 1s.)

**Answers:** a) no, b) yes, c) no, Bonus: no

**Calculate the mean of a dot plot.** SAY: You can find the mean of a symmetrical dot plot just by finding the center value, but for dot plots that aren’t symmetrical, you have to calculate the mean. Draw the dot plot shown in the margin with dots and number line, but no calculations, on the board. Point to the column of dots above 1. ASK: How many dots are above 1? (5) Write “$\times 5$” under the 1 as shown. SAY: The 5 means there are five 1s in the set. To calculate the mean, we add 1 five times. ASK: What is $5 \times 1$? (5) Write “$5$” under “$\times 5$.” Point to the 2 on the number line. ASK: How many dots are above the 2? (0) Write “$\times 0$” under the 2 as shown. ASK: What is $0 \times 2$? (0) Write “0” under “$\times 0$.” SAY: There are no 2s in this data set, so there should be no 2s in the calculation of the mean. Continue until you have done this for all numbers on the number line. Have students calculate the mean and then practice on **BLM Calculating Means from Dot Plots.**

**Identically shaped, symmetrical dot plots on different parts of a number line.** Project a piece of lined paper onto the board, and draw dot plots A and B as shown in the margin. ASK: Can we tell what the mean and median are going to be? (no) Why not? (there are no numbers) Can we tell if the mean and median of dot plot A will be the same as each other? (yes) How? (in symmetrical dot plots the mean and median are the same) What about B? (they’ll be the same as each other too) Write the numbers 10, 20, and 30 on the number line of A and 20, 30, and 40 on B. ASK: What are the mean and median of A? (20) What about B? (30)
Exercises: Have students complete the exercises using dot plots A and B below.

a) Without calculating, say what the mean and median of A will be and if they will equal each other.

b) Without calculating, say whether the mean of A will equal the mean of B.

c) Without calculating, say whether the median of A will equal the median of B.

Answers: a) 40, 40, yes; b) no, c) no

Identically shaped, but not symmetrical dot plots on different parts of a number line. Project a piece of lined paper onto the board, and draw dot plots A and B as shown below.

ASK: Can we tell what the means and medians are going to be? (no) Why not? (there are no numbers) Can we tell if the mean and median of dot plot A will be the same as each other? (no) Why not? (in dot plots that aren’t symmetrical, the mean and median aren’t necessarily the same) What about B? (no, for the same reason as A) Write the numbers 10, 20, 30, and 40 on the number line of A and 20, 30, 40, and 50 on B. Have students calculate the mean of A and of B. (A is 20 and B is 30) Circle 20 on the number line of A and 30 on the number line of B. ASK: What do you notice about the position of the mean of A compared to the position of the mean of B? (they’re in the same position) Have students find the median of A. (10) ASK: Do you think the median of B will be in the same position as the median of A? (yes) Have students find the median of B to verify their guess.

Exercise: Draw the four plots below on the board. Have students calculate the mean of A, and then, without calculating, say what the mean of B will be. Repeat for C and D.

Answers: A: 3, B: 40, C: 60, D: 9
Extensions

1. Draw three very different symmetrical dot plots with lowest value 4, highest value 8, and mean and median 6.

Sample answers

```
<table>
<thead>
<tr>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

2. a) Convert the stack-and-bead diagram into a dot plot.

```
1 8 8 5 2 5 3 6 6
```

b) Why is it easier to calculate the mean, median, and range using the dot plot?

Answers

a) 

```
1 2 3 4 5 6 7 8
```

b) In the dot plot, the data values are in order.

(MP.2, MP.6) 3. a) Make up a situation where the numbers 6, 3, 7, 9, 11, 12 form the data set.

b) Explain the terms “data point” and “mean” using this example. Calculate the mean and explain what it represents in your example.

Sample answer: a) The number of board games that six friends own are 6, 3, 7, 9, 11, and 12. b) Each of the numbers in the list is called a data point, and in this example, each data point is the number of board games one of the friends owns. The mean is the average number of board games owned by each friend. It can be calculated by adding the data point values and dividing by the number of data points in the data set: \( 6 + 3 + 7 + 9 + 11 + 12 = 48, \frac{48}{6} = 8 \). If the friends were to share their board games equally, each friend would get 8 board games.
Moving Beads to Find the Mean

Move beads to find the mean. Cross out the beads you move and draw shaded beads in new positions.

- a) 8 5 2
- b) 7 2 5 2
- c) 1 3 9 3

Mean: ___  Mean: ___  Mean: ___
Finding the Correct Position for the Mean Line

Draw “×”s in the spaces under the line, write the number of beads that are above the line, and write the number of spaces that are below the line. Circle the one picture in each row that has the line in the correct position. Use a ruler to draw the lines in the last row. Check your answers by calculating the mean of each set.

Beads above line:

Spaces below line:

Beads above line:

Spaces below line:

Beads above line:

Spaces below line:

Beads above line:

Spaces below line:
The Mean and Median When the Highest Value Increases

1. Write the number of beads above each stack. Write the median. Move beads to find the mean.
   a) Median: _______
   b) Median: _______
   c) Median: _______

2. Did the median change as the last number in the set became greater?
3. Did the mean change as the last number in the set became greater?
Mean and Median
Dot Plots 1

Number of Stories Each Student Wrote

Number Line

Label

Title
Dot Plots 2

<table>
<thead>
<tr>
<th>Number of Plums Picked</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of People</td>
<td>IIII</td>
<td>IIII</td>
<td>II</td>
<td>III</td>
<td></td>
</tr>
</tbody>
</table>

Number of Plums  
Each Person Picked

<table>
<thead>
<tr>
<th>Number of Plums</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Plums</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Dot Plots 3

A

10 11 12 13 14

maple pine oak willow birch

B

10 11 13 17 18

C

10 11 13 17 18
### Data Value

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,008</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Dot Plots Bonus Questions

1. The data is plotted incorrectly. Cross out all dots that should not be there and draw in the dots that are missing.

| 12 | IIII  |
| 13 | II    |
| 14 | III   |
| 15 | IIII  |
| 16 | I     |

2. Several people were asked to record the number of miles they walk in a week.
   a) Use the chart below to make a number line for the dot plot.
   b) Plot the data from the chart.
   c) How many people walked at least 2 miles? _____
   d) What answer was given most frequently? _____
   e) How many more people walked 1 mile than 3 miles? _____

<table>
<thead>
<tr>
<th>Miles Walked</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>IIII</td>
</tr>
<tr>
<td>1</td>
<td>IIII</td>
</tr>
<tr>
<td>2</td>
<td>IIII</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>III</td>
</tr>
</tbody>
</table>

3. Use the clues to plot the missing data for 1 glass and 3 glasses.
   a) The number of people who drank 1 glass is twice the number who drank 0 glasses.
   b) The number of people who drank 3 glasses is two more than the number who drank 1 glass.
   c) How many people answered the question? _____
Dot Plot and Tally Chart Symmetry

II III I

II III I

II III I
Calculating Means from Dot Plots

Multiply each data value by its frequency. Add the results of the multiplications. Divide by the number of data points in the dot plot.

\[
\begin{array}{c}
\times 5 \\
\times 0 \\
\times \ \\
\times \ \\
\times \\
\times \\
\end{array}
\]

\[
25 + 0 + + + + = \frac{\text{total}}{\text{total number of data points}} = \text{mean}
\]
Unit 8  Geometry: Volume and Surface Area

This Unit in Context

In 6.1 Unit 6, students learned how to find the area of two-dimensional shapes. In this unit, students will learn to find the volume and surface area of three-dimensional shapes. They will apply the formulas \( V = \ell \text{wh} \) and \( V = bh \) to find volumes of right rectangular prisms with fractional edge lengths in order to solve real world and mathematical problems (6.G.A.2). They will represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures (6.G.A.3).

In Grade 7, students will use their knowledge of three-dimensional shapes to solve real world and mathematical problems involving angle measurement, area, surface area, and volume (7.G.B.4). In Grade 8, the study of three-dimensional shapes will extend to include finding the volume of cylinders, cones, and spheres (8.G.C.9).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP1 to MP4, MP6, and MP7. Here are some examples of how students can show that they have met a standard.

**MP1:** In G6-41 Extension 1, students make sense of a challenging problem about finding the possible dimensions of a rectangular prism when they notice that only factors of the given areas are possibilities. Students persevere to solve the problem when they use an organized method to check all the possibilities and when they use the properties of factors and multiples to eliminate possibilities.

**MP3:** In G6-40 Extension 2, students construct a viable argument when they explain why nets will not fold into pyramids.

**MP4:** In G6-37 Extension 1, students model mathematically when they use equations to represent and solve a real-world problem about determining which of several chests will have the smallest surface area.

**MP7:** In G6-39 Extension 2, students make use of structure when they notice that the surface area of a shape can be broken into 14 square faces, and that these faces come from 2 cm cubes, and so the area of each of those square faces must be 4 cm².
Unit 8  Geometry: Volume and Surface Area

Introduction
In this unit, students will learn about volume and surface area. As they find the volumes of prisms and the surface areas of prisms and pyramids, students will add, subtract, multiply, and divide numbers, sometimes including fractions. They will also need to solve equations. For details, see the prior knowledge required for each lesson.

Materials
Throughout the unit, collect various boxes whose volume and surface area students can measure and calculate. Invite students to add to your collection of boxes by bringing some in from home.

In Lessons G6-35 and G6-38, students will need to use four different shapes (two prisms and two pyramids) each. Make sure that each student gets at least one triangular pyramid or prism with a base that is a scalene triangle. To get a wide variety of such shapes, have students glue shapes from nets on BLM Nets (1–16) (pp. Q-42–Q-57) before starting this unit.
**Goals**

Students will find the number of cubes in 3-dimensional stacks.

**STANDARDS**

preparation for 6.G.A.2

**VOCABULARY**

column
height
layer
length
row
width

**PRIOR KNOWLEDGE REQUIRED**

Can find the area of a rectangle
Understands that the number of squares in a rectangle can be found by multiplication
Knows that multiplication is commutative

**MATERIALS**

connecting cubes

Review the area of a rectangle as an array. Draw a rectangle on a grid (or subdivide a rectangle into equal squares). Ask students to write the addition and multiplication statements needed to calculate the area of this rectangle.

Counting cubes in a stack using horizontal layers. Build a $2 \times 4$ "rectangle" using connecting cubes. ASK: How many cubes are in this rectangular layer? Ask students to explain how they calculated the number of cubes—did they count the cubes one by one or did they count them another way? You can use addition or multiplication to count the total number of cubes:

- $2 + 2 + 2 + 2 = 8$ (2 cubes in each of 4 columns)
- $2 \times 4 = 8$ (2 rows of 4 cubes OR length $\times$ width)

Add 1 layer to the stack so that it is 2 cubes high. ASK: How many cubes are in the new stack? Prompt students to use the fact that the stack has 2 horizontal layers. Remind them that they already know the number of cubes in one layer. Ask students to write the addition and multiplication statements for the number of cubes in the new stack using layers. ($8 + 8 = 16, 2 \times 8 = 16$)

Add a third layer to the stack and repeat. ($8 + 8 + 8 = 24; 3 \times 8 = 24$)

Counting cubes in a stack using a vertical layer. Invite students to look at this last stack and calculate the number of cubes by adding vertical layers instead of horizontal layers. ASK: How many cubes are at the end of the stack? ($3 \times 2 = 6$) How many cubes are in each vertical layer? (6) How many vertical layers are in the stack? (4) Invite volunteers to write the addition and multiplication statements for the total number of cubes using the number of cubes in the vertical layer. ASK: Does this method produce a different result than the previous method? (no, it’s the same answer) Why should we expect the same answer? (it is the same stack, only counted differently)

Determine the number of cubes in a stack as a product of length, width, and height. Review the terms length and width. Remind students that to find the number of squares in a rectangle, they can multiply length by width.
Now they have a 3-dimensional stack, but it is very similar to a rectangle. Remind students that the vertical dimension in 3-D objects is called the **height**. Identify the length, width, and height of the stack in the margin. Then use the terms "length," "width," and "height" to label the multiplication statement that gives the number of cubes in the stack:

\[ 4 \times 2 \times 3 = 24 \]

\[
\begin{array}{ccc}
\text{length} & \text{width} & \text{height}
\end{array}
\]

Draw several stacks on the board and ask students to find the number of cubes in each stack by multiplying length, width, and height.

Remind students that order does not matter in multiplication, so the number of cubes can be found in other ways, such as height \( \times \) length \( \times \) width or width \( \times \) length \( \times \) height.

**Extension**

Use connecting cubes.

a) In how many ways can you build a rectangular stack of 36 cubes with a height of 3 cubes?

b) In how many ways can you build a rectangular stack with 48 cubes?

**Answers**

a) 3; 12 \( \times \) 1 \( \times \) 3, 6 \( \times \) 2 \( \times \) 3, 4 \( \times \) 3 \( \times \) 3 (dimensions in order length, width, height)

b) 9; 48 \( \times \) 1 \( \times \) 1, 24 \( \times \) 2 \( \times \) 1, 16 \( \times \) 3 \( \times \) 1, 12 \( \times \) 4 \( \times \) 1, 12 \( \times \) 2 \( \times \) 2,
    8 \( \times \) 3 \( \times \) 2, 8 \( \times \) 6 \( \times \) 1, 6 \( \times \) 4 \( \times \) 2, 4 \( \times \) 4 \( \times \) 3 (different arrangements of the same dimensions omitted)
STANDARDS
preparation for 6.G.A.2

VOCABULARY
cubic unit
dimension
height
length
rectangular prism
volume
width

Goals
Students will find the volume of rectangular prisms with whole-unit lengths.

PRIOR KNOWLEDGE REQUIRED
Can find the number of blocks in a rectangular prism made of blocks using multiplication

MATERIALS
piece of thread
centimeter cubes for demonstration
old newspapers rolled into tubes 1 unit long (see Activity 2)
tape
BLM Nets (8) (p. Q-49)
BLM Cube Skeleton (p. Q-58)

Introduce volume. Hold up a piece of thread. Explain that it has only one dimension—length. ASK: What units do we measure length in? (cm, m, km, in, ft, yd, mi) Point to the top of a desk or table and explain that it has length and width—that’s 2 dimensions. Anything 2-dimensional has area. ASK: What units do we measure area in? (cm², m², km², in², ft², yd², mi²) Emphasize that all these units are squares with sides that measure 1 length unit.

Explain that a 3-dimensional object, such as a cupboard or a box, has length, width, and height. The space taken up by the box or cupboard or any 3-dimensional object is called volume, and we measure it in cubic units: units that are cubes.

Introduce standard cubic units. Draw several cubes on the board and mark the sides as 1 cm for one cube, 1 in for another cube, 1 m for another cube, and so on. Explain that these are different cubic units—cubic centimeter, cubic inch, cubic meter, etc. Show the abbreviated form of each unit.

ACTIVITY 1
Divide students into two groups and assign each group a unit: in³ or ft³. Provide BLM Nets (8) and have students draw a net of a cube to scale, cut it out, and make the cube from the net. Students working to make a cubic foot can use old newspapers or wrapping paper.
ACTIVITY 2

Divide students into three groups. Assign each group a unit: ft³, yd³, or m³. Ahead of time, roll old newspapers or wrapping paper into tubes slightly longer than each unit, to allow for binding at the ends. Mark the unit on each tube as shown. (You might need to combine several newspapers to produce a longer tube.) Give each group 12 tubes, tape, and BLM Cube Skeleton with the instructions. Have students use tape to bind the tubes together to create skeletons of the units to scale.

Compare the relative sizes of the units (use a centimeter cube to show 1 cm³). Point out that cubic meters and cubic yards are very large. For example, to fill an aquarium with a volume of one cubic meter or one cubic yard, you would need about 100 large pails of water!

Finding the volume of rectangular prisms. Show a $3 \times 4 \times 2$ stack of centimeter cubes, and review how to find the number of cubes in a stack (by multiplying length, width, and height). ASK: How many cubes are in the stack? ($3 \times 4 \times 2 = 24$) Explain that the volume of this stack is 24 cubic centimeters.

Draw several stacks of cubes on the board and explain that these cubes are unit cubes. Mark the size of each cube, using different units for different pictures. Have students find the volume of each stack.

Draw a rectangular prism. Explain that mathematicians call rectangular boxes rectangular prisms. ASK: What is the length of this prism? (5 m) Repeat with width and height, and record the information on the board. ASK: If this were a stack of cubes 1 m by 1 m by 1 m, how many cubes would fit along the length of this cube? (5) Width? (3) Height? (4) How many cubes in total would be in the stack? (60) How do you know? (3 \times 5 \times 4 = 60; volume is length \times width \times height) Have students write the multiplication statement. What is the volume of the prism? (60 m³)

Point out that students found the volume of the prism even though you did not draw the cubes that made it. ASK: What did you do to find the volume? (multiplied the dimensions: length \times width \times height) Write the formula $V = l \times w \times h$. On the board, draw the prism in the margin. Have students write the multiplication statement for its volume. Explain that, when finding volume, it helps to write the units for each measurement in the multiplication statement to prevent mistakes. For example, for this prism, the multiplication statement should be:

$Volume = 2\text{ in} \times 3\text{ in} \times 6\text{ in} = 36\text{ in}^3$

Point out that the raised 3 that appears in the cubic units is the number of length measurements that were multiplied together to make the cubic unit:
you multiplied three measurements (length, width, and height) to get the volume. All three measurements were in inches, so the result is in cubic inches (point to the raised 3 in in³).

**ASK:** Where have you seen a similar notation with numbers? (in exponents)

Point out the similarity:

\[ 2 \times 2 \times 2 = 2^3 \]

\[ = 8 \]

\[ 2 \text{ in} \times 2 \text{ in} \times 2 \text{ in} = 8 \text{ in}^3 \]

**Exercises:** Find the volume of the rectangular prism. Include the units in your calculation and answer.

a) \[ 10 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm} \]

b) \[ 8 \text{ ft} \times 2 \text{ ft} \times 7 \text{ ft} \]

c) \[ 3 \text{ km} \times 4 \text{ km} \times 7 \text{ km} \]

d) \[ 4 \text{ m} \times 3 \text{ m} \times 15 \text{ m} \]

e) length 11 in, width 6 in, height 11 in

f) length 3 ft, width 3 ft, height 3 ft

g) length 12 in, width 12 in, height 12 in

h) length 100 cm, width 100 cm, height 100 cm

**Answers:** a) 120 cm³, b) 112 ft³, c) 84 km³, d) 180 m³, e) 726 in³, f) 27 ft³, g) 1,728 in³, h) 1,000,000 cm³

**Bonus:** Use some of your answers above to convert these units.

a) \[ 1 \text{ yd}^3 = ____ \text{ ft}^3 \] 
b) \[ 1 \text{ m}^3 = ____ \text{ cm}^3 \] 
c) \[ 1 \text{ ft}^3 = ____ \text{ in}^3 \]

**Answers:** a) 1 yd³ = 27 ft³, b) 1 m³ = 1,000,000 cm³, c) 1 ft³ = 1,728 in³

**Show that volume does not depend on orientation.** Show students a prism that has different measurements for the length, width, and height (e.g., a tissue box). Write its dimensions to the nearest centimeter on the board. Have students find the volume of the prism. Then turn the prism.

**ASK:** Did the height of the prism change? (yes) The length? (yes) The width? (yes) Do you expect the volume to change? (no) Why not? (the order of the dimensions changed, not the dimensions themselves; we multiply the same numbers but in a different order)

**Extension**

Find the volume of the shape in the margin.

**Answer:** \[ 3 \text{ cm} \times 2 \text{ cm} \times 5 \text{ cm} + 3 \text{ cm} \times 2 \text{ cm} \times 3 \text{ cm} = 48 \text{ cm}^3 \]
**Goals**

Students will find the volume of rectangular prisms with fractional side lengths.

**Prior Knowledge Required**

- Can find the volume of a rectangular prism with whole-unit side lengths
- Can use multiplication to find the number of cubes in a 3-D stack
- Can multiply and divide fractions
- Can divide whole numbers to produce a fraction
- Can convert between units of length in the same system

**Materials**

- 2 cm cubes
- 1 cm cubes
- 1 in cube constructed in Lesson G6-31
- 1 ft cube constructed in Lesson G6-31
- Blocks with sides that measure fractions of an inch
- Inch rulers

**Reviewing Finding the Volume of Rectangular Prisms with Whole-Number Lengths.** Draw several prisms and write beside them the length, width, and height. Use different units for different prisms. Have students find the volume of each prism.

**Comparing the Volume of 2 cm Cubes and 1 cm Cubes.** Give students several 2 cm cubes and 1 cm cubes. Ask them to check how many small cubes fit along the length of a large cube. (2) How many small cubes make 1 large cube? (8) Have students make a cube from 8 small cubes and check that it is the same size as a large cube. ASK: If we use the length of a small cube as a unit, what are the dimensions of the large cube? (2 units by 2 units by 2 units) What is the volume of the large cube? (8 cubic units)

ASK: If we use the length of the large cube as a unit, what is the length of the small cube? (1/2 unit) Draw a cube on the board and mark the sides as 1/2 unit. ASK: How do we find the volume of a cube? (multiply the side length by itself 3 times) What do we expect the volume to be? (1/2 unit × 1/2 unit × 1/2 unit) Remind students how to multiply a fraction by a fraction, and have them do the multiplication. (1/8) Point out that since the volume of the large cube is 1 cubic unit, and 8 small cubes have the same volume as the large cube, the volume of the small cube is 1/8 of the cubic unit.

**Finding the Volume of a 1 in Cube in Cubic Feet.** Show a 1 in cube and a 1 ft cube from Lesson G6-31. ASK: How many inches are in a foot? (12) How many 1 in cubes will fit along the length of the large cube? (12) Width? (12) Height? (12) How many small cubes will fit in one large cube?
What is the volume of the small cube in cubic feet? 
(1/1,728 ft³) What are the dimensions of the small cube in feet? (length, width, and height are all 1/12 ft) What is the volume of the small cube if we multiply length, width, and height? 
(1/12 ft × 1/12 ft × 1/12 ft = 1/1,728 ft³)
Did we get the same answer? (yes)

**ACTIVITY**

If you have a collection of blocks with a length of a fraction of inches, such as 3/4 in cubes, give students a lot of identical rectangular prisms or cubes. Ask them to measure the sides of the block with an inch ruler. Have them try to make a rectangular prism with side lengths in whole inches. ASK: How many blocks are needed to make the prism? Have students find the volume of the prism in whole inches and find the volume of one block by multiplying side lengths. ASK: Did you get the same answer? (yes)

Finding the volume of prisms with fractional side lengths. Draw a 3 × 5 × 2 prism made of cubes on the board. Explain that this prism is made from 1/2 in cubes. Have students find the number of cubes in the prism. (30 cubes) Remind students that a cube that has side lengths of 1/2 unit has a volume of 1/8 cubic unit. So these cubes have a volume of 1/8 in³. ASK: If there are 30 cubes, what is the volume of the prism? (15/4 in³) How do you know? (30 × 1/8 in³ = 30/8 in³ = 15/4 in³)

Now have students find the length, width, and height of the prism in inches (3/2 in × 5/2 in × 1 in) and find the volume using the regular formula. ASK: Did you get the same volume? (yes)

Have students find the volume of a cube that is 1/4 in long (1/64 in³), then find the volume of another stack (e.g., 3 × 4 × 5), this time made from 1/4 in cubes. (15/16 in³)

On the board, draw the prism in the margin and have students find the volume by multiplying side lengths. (Students will need to convert 2 1/4 to an improper fraction.) The volume is 9/4 in × 3/4 in × 3/4 in = 81/64 in³. Then discuss how many 1/4 in cubes would fit along the length, width, and height of the prism, and find the number of cubes that would fit inside the whole prism. (9 × 3 × 3 = 81 cubes) ASK: If the volume of one cube is 1/64 in³, what is the volume of the prism? (81/64 in³) Did you get the same answer? (yes)

Repeat the exercise above with a prism that is 1/2 ft by 3/2 ft by 5/2 ft. Suggest that students think of the prism as made from 1/2 ft cubes. (volume = 15/8 ft³) Then have students find the volume of a cube that has side lengths of 0.1 cm. (0.001 cm³) Repeat the exercise above with a prism that measures 3 cm by 3.5 cm by 2.1 cm. (volume = 22.05 cm³)

ASK: Which is easier: to imagine the prism each time as made from small unit cubes, or to multiply the side lengths? Do both methods give the same answer? (yes) This means you can use the easier method of multiplying to find the volume.
Exercises: Find the volume.

a) \( \frac{3}{4} \text{ in} \times \frac{3}{2} \text{ in} \times 2 \text{ in} \)

b) \( 3.5 \text{ m} \times 2 \text{ m} \times 2.3 \text{ m} \)

c) length 11.4 cm, width 6 cm, height 4.5 cm

d) length 3 \( \frac{1}{2} \) ft, width 3 \( \frac{1}{2} \) ft, height 3 \( \frac{1}{2} \) ft

Answers

a) \( \frac{182}{8} \text{ in}^3 = 22 \frac{3}{4} \text{ in}^3 \)

b) 16.1 m\(^3\)

c) 307.8 cm\(^3\)

d) \( \frac{343}{8} \text{ ft}^3 = 42 \frac{7}{8} \text{ ft}^3 \)

Extensions

(MP3, MP7)

1. a) Kim wants to use cubes that are all the same size to fill a box that measures \( 3 \frac{1}{2} \text{ ft} \times 3 \frac{1}{4} \text{ ft} \times 2 \text{ ft} \) without gaps. He has \( \frac{1}{2} \text{ ft} \) cubes, \( \frac{1}{4} \text{ ft} \) cubes, and 1 ft cubes. Which cubes should he use? Explain.

b) Alyssa wants to use cubes that are all the same size to fill a box that measures \( 3 \frac{1}{2} \text{ ft} \times 2 \frac{1}{4} \text{ ft} \times 2 \text{ ft} \) without gaps. She has \( \frac{1}{2} \text{ ft} \) cubes, \( \frac{1}{4} \text{ ft} \) cubes, and \( \frac{1}{8} \text{ ft} \) cubes. Which cubes should she use? Explain.

c) Pedro wants to use cubes that are all the same size to fill a box that measures \( 2 \frac{1}{2} \text{ ft} \times 3 \frac{1}{3} \text{ ft} \times 2 \frac{1}{2} \text{ ft} \) without gaps. He has \( \frac{1}{2} \text{ ft} \) cubes, \( \frac{1}{4} \text{ ft} \) cubes, and 1 ft cubes. Will any of the cubes work? Explain.

d) When can you divide a fraction by a unit fraction and get a whole-number answer? Explain using denominators of the fractions. Use this to explain why Pedro’s cubes do not work.

e) Carmelita thinks Pedro needs cubes with sides measuring \( \frac{1}{6} \text{ ft} \). Is she correct? Explain.

f) Ilan filled Pedro’s box with cubes with sides \( \frac{5}{6} \text{ ft} \). How many cubes did he use?

g) Tegan thinks she can fill Pedro’s box with cubes with sides \( \frac{7}{6} \text{ ft} \). Is she correct? Explain.

Answers

a) \( \frac{1}{4} \text{ ft cubes} \), because the others will leave gaps. For example, for \( \frac{1}{2} \text{ ft cubes} \), \( 3 \frac{1}{4} \div \frac{1}{2} = \frac{13}{4} \div \frac{1}{2} = \frac{13}{2} \), which is not a whole number. So no whole number of cubes will fit along that side.
b) \( \frac{1}{4} \) ft cubes and \( \frac{1}{8} \) ft cubes will both work, because both numbers divide into the dimensions of the box and the result is a whole number. \( \frac{1}{2} \) does not divide evenly into \( 2 \frac{1}{4} \).

c) None of Pedro’s cubes will work, because none of the numbers produce a whole number when divided into the dimensions of the box.

d) Dividing by a unit fraction is the same as multiplying by the denominator of the unit fraction. For the answer to be a whole number, the denominator of the divisor (the unit fraction) needs to be a multiple of the denominator of the dividend. The length of the cubes needs to be a fraction with denominator that is a common multiple of the denominators of the dimensions.

e) Carmelita is correct. 6 is the LCM of 2 and 3, so \( \frac{1}{6} \) will divide evenly into the dimensions of the cube.

f) \( 2 \frac{1}{2} \div \frac{5}{6} = \frac{5}{2} \div \frac{5}{6} = \frac{5}{2} \times \frac{6}{5} = \frac{30}{10} = 3 \) and  
\( 3 \frac{1}{2} \div \frac{5}{6} = \frac{10}{3} \div \frac{5}{6} = \frac{10}{3} \times \frac{6}{5} = \frac{60}{15} = 4 \)

Ilan used \( 3 \times 4 \times 3 = 36 \) cubes.

g) Cubes of \( \frac{7}{6} \) ft will not work:

\( 2 \frac{1}{2} \div \frac{7}{6} = \frac{5}{2} \div \frac{7}{6} = \frac{5}{2} \times \frac{6}{7} = \frac{30}{14} \) is not a whole number, and  
\( 3 \frac{1}{3} \div \frac{7}{6} = \frac{10}{3} \div \frac{7}{6} = \frac{10}{3} \times \frac{6}{7} = \frac{60}{21} \) is not a whole number either.

(MP1, MP4)

2. During an excavation of an old fort, an underground ammunition storage room was discovered. The room is 10 ft long, 6 ft wide, and 4 ft deep. Several crates of shells measuring 1.5 ft by 1 ft by 1 ft each were found in the room.

a) What is the greatest number of crates that would fit in the room?

b) Experts think that the crates were stacked on either side of a 2.5 ft wide passage as shown. What is the greatest number of crates that would fit in the storage?

Answers: a) The crates should be packed so that the 1.5 ft side is parallel to the 6 ft wall to achieve maximum efficiency, so 4 crates will go along that wall. The greatest number of crates that would fit in the room is \( 4 \times 10 \times 4 = 160 \) crates. b) Arrange the crates so the passage divides the room into two sections for crates, one with length 3 ft, the other with length 4.5 ft. Then all the space can be filled as shown (see margin). One side fits \( 4 \times 3 \times 4 = 48 \) crates, while the other side fits \( 6 \times 3 \times 4 = 72 \) crates, for a total of \( 48 + 72 = 120 \) crates.
**Goals**

Students will use the area of the base to develop and use a formula for the volume of rectangular prisms and solve problems involving the volume of rectangular prisms.

**PRIOR KNOWLEDGE REQUIRED**

Can use the formula \( V = l \times w \times h \) to find the volume of a rectangular prism

Can multiply and divide fractions and decimals

Can divide whole numbers to produce a fraction

**MATERIALS**

- rectangular boxes
- rectangular waterproof container, such as a large juice carton with a ruler attached to the inside (see Activities)
- small fruit, such as a mandarin orange or a strawberry
- several waterproof blocks (not connecting cubes)
- water

**Introduce faces.** Explain that the flat surfaces of 3-D shapes are called *faces*. Hold up a box and have students count how many faces it has. (6)

Explain that, in this lesson, two faces will be most important—the *top face* and the *bottom face*. Show which faces you mean.

ASK: In a rectangular box, or in a rectangular prism, can the top face be larger than the bottom face? (no) Can the bottom face be larger than the top face? (no) Can they have a different shape? (no) The top face and the bottom face are always the same rectangular prisms. If some students have trouble seeing that the top face and the bottom face match exactly, give them a box, have them trace the bottom face of the box, and check that the top face matches the bottom face exactly.

**Review the formula** \( V = l \times w \times h \). Ask students how they can find the volume of their boxes. ASK: What do you need to measure? (length, width, height) Remind students that a formula is a short way to write the instructions for how to calculate something. ASK: What is the formula for the volume of a rectangular prism? (Volume = length \( \times \) width \( \times \) height)

How do we write it the short way? \( (V = l \times w \times h) \)

Write both formulas on the board.

Give each student a rectangular box. Have students measure and record the dimensions of the boxes to the nearest millimeter. Remind students how to convert millimeters to centimeters (divide by 10 by moving the decimal point 1 place value to the left). Ask students to write the dimensions of their boxes in centimeters (e.g., 10.2 cm).
Remind students that they used a formula to find areas of rectangles. They recorded the solution for a problem such as “Find the area of a rectangle with length 3 cm and width 2 cm” as shown in the margin. Keep the formula on the board for future use.

Demonstrate how to record the solution to the problem, “Find the volume of a rectangular prism with length 5.1 cm, width 3.3 cm, and height 2 cm.” Remind students how to multiply decimals. Have students use the formula for the volume of a rectangular prism to find the volume of their boxes. If students use a calculator to solve the problem, they need to estimate the result before entering the numbers. For example, in the prism above, the volume should be about $5 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 30 \text{ cm}^3$, and, since we rounded down both the length and the width, the actual number should be larger.

Review multiplying fractions, and then have students swap boxes with partners, measure the boxes to the nearest eighth of an inch, and find the volume of the new box. To help students check whether their answers are reasonable, tell them that the answer their partners got in cubic centimeters should be about 16 times the answer in cubic inches.

Develop the formula $\text{Volume} = \text{area of bottom face} \times \text{height}$. Write the formula for the volume of a rectangular prism, and ask students whether they see the formula for the area of a rectangle hidden in that formula. (length $\times$ width) ASK: What rectangle is that? Where is it on the prism? The length and width of a rectangle are the lengths of its sides, so which faces of the prism have sides along which you measured the length and width of the prism? (top face and bottom face)

Write on the board:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

area of top face

Explain that we have a new formula: $\text{Volume} = \text{area of top face} \times \text{height}$. Write that formula on the board as well.

ASK: Can we replace the area of the top face by the area of the bottom face in the formula? (yes) Why? (because the top face and the bottom face are identical) Write down the other variation of the formula as well.

Using the formula to find the volume. Solve the first exercise as a class, and then have students work individually. Point out that area is given in square units. Square units mean that there were two length units multiplied to get the square unit, which we can see in the raised 2 in square units. So when we multiply square units by a length unit, we get cubic units.
Exercises: Find the volume.

a) height $\frac{3}{4}$ in  
area of top face $6\frac{3}{4}$ in$^2$  

b) height 2.5 m  
area of top face 9 m$^2$  

c) area of top face 12.8 cm$^2$  
height 4.5 cm  

d) area of top face $20\frac{1}{4}$ ft$^2$  
height $4\frac{1}{2}$ ft

Solution: a) Volume = area of top face $\times$ height  

\[
\frac{3}{2} \text{ in} \times \frac{63}{4} \text{ in}^2 = \frac{7}{2} \text{ in} \times \frac{27}{4} \text{ in}^2 = \frac{189}{8} \text{ in}^3 = 25\frac{5}{8} \text{ in}^3
\]

Answers: b) 22.5 m$^3$, c) 57.6 cm$^3$, d) $\frac{729}{8}$ ft$^3 = 91\frac{1}{8}$ ft$^3$

Finding volume in other ways. Draw the picture in the margin on the board. Ask students how they can find the volume of the prism. (multiply 25 cm$^2 \times 4$ cm) Confirm that this is the right answer. ASK: Do you have the height given? (no) Do you have the area of the top face or the bottom face given? (no) Then how do you know that this product is the volume? If the following two explanations do not arise, lead students to think about them: If you turn this prism so that the front face becomes the top face, the given length becomes the height, and the formula works. Or: We are given the width of the prism and the area of the front face. The given area is length $\times$ height, so we still have length $\times$ height $\times$ width, which is the volume of the prism. We only multiply the dimensions in a different order.

Exercises

1. Find the volume of the prism.

a) length $8\frac{1}{2}$ in  
area of left-side face $12\frac{3}{4}$ in$^2$  

b) width 1.5 m  
area of front face 3 m$^2$
c) area of right-side face \(9.1 \text{ cm}^2\)
   
   length 4.7 cm

2. Can you find the volume of the prism? Find the volume or explain why you cannot.

   a) length 2 in
      
      area of front face 8 in\(^2\)

   b) width 3 m
      
      area of left-side face 9 m\(^2\)

   c) area of top face 4 cm\(^2\)

   d) area of front face 15 ft\(^2\)

   length 2 cm

   width \(\frac{1}{2}\) ft

   width 3 ft

\text{Answers: 1. } a) 108 \frac{3}{8} \text{ in}^3, \quad b) 4.5 \text{ m}^3, \quad c) 42.77 \text{ cm}^3, \quad d) 1 \frac{1}{6} \text{ ft}^3

2. a) length is part of the dimensions of the front face, so there is not enough data to find the volume, b) no, c) no, d) yes, 45 ft\(^3\)

\text{Review solving equations with a missing factor. Work through a few simple equations as a class, while reminding students how to record the solution.}

\text{Exercises}

a) \(w \times 5 = 35\)  
   
   b) \(2 \times h = 25\)  
   
   c) \(a \times 6 = 20\)

\text{Bonus: } 35,000 \times w = 70,000

\text{Answers: a) } w \times 5 = 35  
\hspace{2cm} w = 35 \div 5  
\hspace{2cm} w = 7

b) \(h = 12.5\), c) \(a = \frac{20}{6} = 3 \frac{1}{3}\), Bonus: \(w = 2\)

\text{Find the area of the base or the height given the volume. Tell students you want to make a box that is 30 cm long and 20 cm wide. You need your box to have a volume of 21,000 cm}\(^3\). Record the data on the board. Ask: What height should the box be? Let students think about how to solve this problem and encourage multiple solutions. If the idea of using}
an equation does not arise, present it as another solution: Explain that you know the length and the width, so you can find the area of the top face or the bottom face. ASK: What is the area of the top face? (20 cm × 30 cm = 600 cm²) What formula do we know for the volume that uses this area? (Volume = area of top face × height) Record the formula on the board, together with the area of the top face. Then remind students how to substitute the data into the formula, and that we can write \( h \) instead of the height, as a short form. Finally, have students solve the equation 600 × \( h \) = 21,000. (\( h = 35 \) cm) Emphasize the importance of writing the units in the answer and checking that the answer makes sense: when you are looking for height, you divide the volume (in cubic centimeters) by the area of the face (in square centimeters), so the answer should be in centimeters. Students who have trouble with the units can benefit from the following exercises.

**Exercises:** Write the missing units.

a) \( 5 \text{ cm} \times 6 \text{ _____} = 30 \text{ cm}^3 \)  

b) \( 5 \text{ in} \times 6 \text{ in}^2 = 30 \text{ _____} \)  

c) \( 5 \text{ _____} \times 6 \text{ ft} = 30 \text{ ft}^3 \)  

d) \( 5 \text{ m}^2 \times 6 \text{ _____} = 30 \text{ m}^3 \)

**Answers:** a) cm², b) in³, c) ft², d) m

Then have students practice finding the missing dimension with the exercises below. Students can use a method of their choice to solve the problem. Encourage multiple solutions, and encourage students to check their answers by substitution. If students use a calculator, as they will need to do when working on Question 5 on AP Book 6.2 p. 139, they will need to estimate to make sure the answer they obtained on a calculator makes sense. Students should always substitute to check that the answer is correct.

**Exercises**

a) area of top face = 3 m², volume 12 m³, height = ? 

b) area of top face = 3 cm², volume 4.5 cm³, height = ? 

c) height = \( \frac{1}{2} \) ft, volume = 5 ft³, area of top face = ? 

d) length = 5 in, width = 3 in, volume = 45 in³, height = ? 

e) length = 8 in, height = 3 in, volume = 72 in³, width = ?

**Bonus:** The volume of the shape in the margin is 20 cm³. What is the missing height?

**Answers:** a) 4 m, b) 1.5 cm, c) 2 ft², d) 3 in, e) 3 in, Bonus: 3 cm

**ACTIVITIES**

You will need a waterproof rectangular container, such as a juice carton with the top cut off, with a centimeter ruler attached to the inside of the container. Have a volunteer measure the dimensions of the container.
1. **Finding the volume of an object submerged in water.** Pour water into the container until it reaches a height of 10 cm. ASK: What is the height of the water in millimeters? (100 mm) How much water is in the container in cubic millimeters? Have students find the volume of the water.  

Now show students a small fruit, such as a mandarin orange or a strawberry. Explain that you want to find the volume of the fruit. You will do it the same way the ancient Greek scientist Archimedes found the volume of a crown for a king. (You might tell students the story of Archimedes and the crown.) Place the fruit in the water, and have a volunteer measure the height of the water in the container. Have students find the volume of the water with the fruit. ASK: I know the volume of the water, and the volume of the water and the fruit together. What is the volume of the fruit? Have students find the volume of the fruit.  

2. **Predicting the height of the water after submerging objects.** Use the same container as in Activity 1. Pour water to a height of 10 cm. Show students several waterproof cubes and have volunteers measure them (or provide the dimensions). Have students find the volume of the cubes. ASK: If I drop 5 cubes into the water, what will the volume of the water and the cubes together be? How do you know? What will the height of the water be? Have students figure out the answer on a calculator, rounding to whole numbers. Drop the cubes into the water and check the answer.  

**Extension**  

A wealthy king ordered a new treasure chest. The old chest was 6 in long, 4 in wide, and 5 in high. The king called four carpenters and asked them each to make a chest. The first carpenter’s chest was twice as long as the old chest, but had the same width and height. The second carpenter’s chest was twice as high as the old chest, but had the same width and length. The third carpenter’s chest was twice as long and twice as wide as the old chest, but was only half the height. The fourth carpenter’s chest was the same height as the old chest, but it had a different length and width, and the area of its base was twice the area of the old chest’s base.  

a) Which carpenter made the chest with the largest volume? Explain.  

b) The fourth carpenter’s chest had length and width in whole inches. What might the dimensions of the new treasure chest be?  

**Answers:** a) All chests had a volume that was twice the volume of the old chest: 240 in\(^3\). b) The area of the base of the old chest was 6 in \(\times\) 4 in = 24 in\(^2\). So the area of the base of the new chest was 48 in\(^2\). If both the length and the width of the fourth carpenter’s chest were different from those of the old chest, the chest might have been 3 in wide and 16 in long. Any other pair of whole numbers that multiply to 48 would also work (e.g., 8 in \(\times\) 6 in).
Introduce vertices and edges. Remind students that flat faces of 3-D shapes are called faces, and explain that the line segments along which faces meet are called edges. Hold up a cube and run your finger along the edges. Explain that the corners of the shape, where edges meet, are called vertices. Show the vertices on the cube.

Before this lesson, prepare skeletons of a rectangular pyramid and a triangular prism made with modeling clay and toothpicks. Explain that edges and vertices together make a skeleton of a 3-D shape. Display the skeletons and introduce each shape, then display it together with its name.

ACTIVITY

Have students use modeling clay and toothpicks of different lengths to make skeletons of pyramids and prisms. To make a pyramid, students need to make a triangle or rectangle first, then add an edge to each vertex of the base and join the edges at a point. To make a prism, students need to make two copies of a triangle or rectangle, and join them with edges similar to what they did to make a cube from rolled newspapers. Encourage students to use different triangles and rectangles of different dimensions to ensure a variety of shapes. To make a cube, have students make a rectangular prism using 12 toothpicks of the same length.
Have students start a table with headings “Number of Vertices,” “Number of Edges,” and “Number of Faces.” Students should fill in the table for all the 3-D shapes they made. Students who have trouble counting the edges or the vertices can mark every edge or vertex they count with a marker and keep a tally. Students who have trouble counting the faces can cut out rectangles or triangles and attach them to the skeleton as faces.

**Introduce the names of different faces.** Before this lesson, prepare a cube skeleton and two paper squares and four transparency squares to act as faces. Label the paper squares “bottom face” and “back face,” and the transparency squares “front face,” “top face,” “left-side face,” and “right-side face.” During the lesson, display the skeleton and add the square faces one at a time, emphasizing each face’s position and name.

**Identifying hidden edges.** Ask students to identify the edges of the cube that they see only through the transparent face. ASK: If the transparent faces were made of paper, would you see these edges? (no) The edges that would be invisible if all the faces were nontransparent are called the hidden edges.

Draw a picture of a skeleton of a cube. Explain to students that hidden edges are often drawn using dashed lines. Have volunteers draw the edges, showing the hidden edges as dashed lines. The dashed lines and the solid lines intersect in the picture. But the point of intersection is not a vertex because there is no real intersection between the edges at that point.

**Sketching cubes.** Show students how to sketch a cube, and then have them sketch a cube themselves.

**Step 1:** Draw a square that will become the front face.

**Step 2:** Draw another square of the same size, so that the center of the first square is a vertex of the second square.

**Step 3:** Join the vertices with lines as shown.

**Step 4:** Turn the lines that represent hidden edges into dashed lines by erasing a few parts of those lines.

Struggling students will find it useful to draw cubes on grid paper.

**Sketching right rectangular prisms.** On the board, sketch the two diagrams in the margin. ASK: What is the difference between these two shapes? (the length or the width of the shape—the dimension perpendicular to the front face) How is Step 2 performed differently in each drawing? (the vertex of the back face is not at the center of the front face—it is closer to the respective vertex of the front face in the shorter shape, and farther from that vertex in the longer shape)
ASK: What would you do in Step 2 to draw a very long rectangular prism? Invite a volunteer to draw it. If the drawing is difficult to understand (i.e., if edges overlap), suggest to students that the vertex of the back face should not sit on the diagonal (see margin).

Sketch a prism as shown. ASK: Could these be the dimensions of this prism? (no) What is wrong? (the dimensions are labeled incorrectly) Have a student rearrange the dimensions to better fit the picture. Then have students find the volume of the prism. (131.2 cm³)

Ask students to sketch a rectangular prism and add some dimensions so at least one of them is a decimal or a fraction. Students can then swap their sketches with a partner. Have them find the volume of the prisms.

Extensions

1. Use the table you made in the lesson. For each shape, add the number of faces to the number of vertices, then subtract the number of edges. ASK: What do you get? (2)

The result is 2 for every shape in the table. The formula \( V + F - E = 2 \) is known as Euler’s formula. It is true for most 3-D shapes, and all 3-D shapes that students encounter this year.

2. Place two copies of a skeleton base down on a table and shift the top base of one of them so that the whole prism is tilted to the side. Explain that the prism you have created is called a skew prism. The original prism is called a right prism. Ask students to identify the shape of the side faces of both prisms. (rectangles for right prisms, parallelograms for skew prisms) Point out that a rectangle is a special type of parallelogram. Display several toothpicks-and-clay skeletons of prisms (see margin), and have students work as a class to sort them into right prisms and skew prisms.

3. a) Make up a situation where the numbers 6, 10, 8, 1, 8, 9, 7 form the data set.

b) Explain the terms “median” and “range” using this example.

Sample answers: a) A student’s test scores in one course are 6, 10, 8, 1, 8, 9, 7. b) The range is the difference between the lowest data point and the highest. Putting the data set in order gives: 1, 6, 7, 8, 8, 9, 10. In this example, the range is 10 − 1 = 9. The median is the central value when the data is put in order. In this example, the median is 8. Half of the students test scores were less than or equal to 8, and half of the scores were greater than or equal to 8.
**Goals**

Students will analyze prisms and pyramids

**Prior Knowledge Required**

- Can identify squares, different types of triangles, and rectangles
- Can identify the vertices, edges, and faces of a 3-D shape

**Materials**

- tape
- **BLM Nets (1–16)** (pp. Q-42–Q-57)

**Introduction**

Introduce **pyramids**. Start with a riddle: “Make 4 triangles with 6 toothpicks, without breaking or bending the toothpicks. The toothpicks must touch each other only at the ends.” Let students try to solve the riddle using toothpicks and modeling clay to hold the toothpicks together at the vertices of the triangles. Hint: The solution is 3-dimensional.

**Answer:**

<table>
<thead>
<tr>
<th>Triangular Prisms</th>
<th>Rectangular Prisms</th>
<th>Triangular Pyramids</th>
<th>Rectangular Pyramids</th>
</tr>
</thead>
</table>

**Activity**

Give each student four pages from **BLM Nets (1–16)** (see table below; each student needs one page from each column; make sure all shapes from the table are produced, and make sure every student has at least one shape that does not have a square or an equilateral triangle as a base). Have students cut out the nets and produce two prisms and two pyramids. They will use the prisms and pyramids during Lesson G6-38 as well.

Remind students how they produced skeletons of different 3-D shapes in Lesson G6-34. Remind them that, for some of the shapes, they made a triangle or a rectangle, added an edge at each vertex, and joined these edges together. Have students find the two objects made that way. Explain that these objects are called **pyramids**, and the triangle or the rectangle they made first is called a base of the pyramid. The vertex opposite the base is called the **apex**. Ask students to place the pyramids base down.
The other faces are called side faces. ASK: What is the shape of the side faces in a pyramid? (triangles)

**Naming pyramids.** Explain that the base gives the pyramid its name. For example, if the base is a rectangle, the pyramid is called a rectangular or rectangle-based pyramid. Write the names on the board and have volunteers circle the parts that are the same in the words “rectangle” and “rectangular.” Then ask students to hold up a rectangular pyramid. Repeat with “triangle” and “triangular” and a triangular pyramid.

**In a triangular pyramid, any face can be called a base.** Have students place the rectangular pyramid so that different faces are down. Each time, ask what is on top of the pyramid—a vertex, a face, or an edge? (an edge for side faces, a vertex [the apex] for the base) Repeat with the triangular pyramid. This time, there is always a vertex on top. Explain that for a triangular pyramid, any face can be called a base; then the opposite vertex will be the apex.

**Prisms have two bases and side faces that are rectangles.** Have students look at the other two shapes in their sets. How did they make the skeletons for these shapes? (started with two rectangles or triangles, then joined them with edges) Explain that the two rectangles or triangles they started with are called “bases,” and the other faces are called “side faces.” Explain that, like pyramids, prisms are named according to the shape of their bases. For example, if its base is a rectangle, the prism is called a rectangular or rectangle-based prism. Have students identify the prisms they have. They can also switch prisms with partners and identify the prisms their partners have.

**In a rectangular prism, any pair of opposite faces can be called bases.** Have students place the prisms so different faces are down. If the prism stands on the base, the other base becomes the top face. With a triangular prism, if the bottom face is not the base, the triangular prism has an edge on top and there is no top face or vertex. ASK: How is the situation with a rectangular prism different? (there is always a top face, and it is always the same as the bottom face) Explain that this means that in a rectangular prism, any pair of opposite faces can be called bases and the rest will be called side faces. ASK: What is the shape of the  side faces in prisms? (rectangles)

**Recognizing pyramids and prisms in real life.** Discuss with students where they can see examples of pyramids and prisms in real life. Church spires and tents are sometimes triangular pyramids, and chocolate boxes (e.g., Toblerone), door stops, and tents are sometimes triangular prisms.

(MP6) **Exercise:** Explain the differences between a triangular prism and a triangular pyramid.

**Answer:** A triangular prism has 2 triangular bases and 3 rectangular faces, for a total of 5 faces, while a triangular pyramid has 4 triangular faces, any of which can be called the base. A triangular pyramid has an apex (the vertex opposite the base), while a triangular prism does not.
**Goals**

Students will identify lines parallel or perpendicular to a flat surface.

**Prior Knowledge Required**

Can identify squares, different types of triangles, and rectangles
Can identify parallel and perpendicular lines
Is familiar with the standard markings for parallel and perpendicular lines
Can identify the vertices, edges, and faces of a 3-D shape

**Materials**

- tape
- BLM Nets (2) (p. Q-43)

**Introduce parallel faces.** Ask students to explain what parallel lines are. (straight lines that never meet, even if extended) Explain that things are more complicated in 3-dimensional space, but what they just said works well for flat surfaces or faces. For example, the floor and ceiling are (usually) parallel surfaces because they never meet, even if you extend them. ASK: What other parallel surfaces can you see around? (opposite walls, the top of a desk is parallel to the floor, etc.)

Have students look at the 3-D shapes they used earlier and find as many pairs of parallel faces as they can. Are there any shapes that do not have any parallel faces? (pyramids do not have parallel faces) How many pairs of parallel faces does a triangular prism have? (one pair, the bases) Rectangular prism? (3 pairs) Place the shape on different faces. ASK: Is there a face parallel to the bottom face?

**Introduce parallel edges.** Point to the line where the floor meets a wall and the line where the ceiling meets an adjacent wall. SAY: These lines never meet, but they are not parallel. You cannot run a train along tracks that are set up that way! How can we describe parallel lines in 3-dimensional space? Explain that parallel lines are lines that never meet and that travel in the same direction.

Place a triangular prism base down on the table and have students do the same. Have them pick a vertical edge and ask them to show the edges that are parallel to it. ASK: How many vertical edges are there? (3, all parallel to each other) Choose a horizontal edge and repeat. This time there is only one edge parallel to the given edge, and these edges belong to the same rectangular side face. Repeat with a rectangular prism. (there are 4 parallel edges in each direction)

**Introduce edges parallel to a face.** Explain that you can also talk about lines parallel to a surface. Hold a pen up horizontally. Explain that it is parallel to the floor. If you extend the line the pen represents, it will never
meet the floor, even if you extend the floor too. There are many lines parallel to a surface. Hold up another pen, horizontally as well, at an angle to the first one. Both lines are parallel to the floor, but they are not parallel to each other. They do not go in the same direction. Point to a wall and ask students to hold a pen parallel to the wall. Then ask them to point out other examples of lines parallel to the same wall. (any line on the opposite wall, or any vertical line are good examples)

Ask students to place a triangular prism base down and ask them to show all edges that are parallel to the top face. (all edges of the bottom face) Then have them show all edges that are parallel to one of the side faces. (only the opposite edge) Repeat with a rectangular prism. ASK: How are the answers different? (the edges parallel to the top face are all the edges of the bottom face, as in the triangular prism, but the edges parallel to a side face are all the edges of the opposite side face) Are any of these edges parallel to each other? Have students point to a pair of parallel edges.

Have students look at pyramids. ASK: Are there any edges parallel to the base? (no) Are there any edges parallel to the side faces in a triangular pyramid? (no) In a rectangular pyramid? (yes) To prompt students to see the last answer, have them place a rectangular pyramid on a side face and look for edges parallel to the bottom face. The edge on top is horizontal, so it is parallel to the bottom face.

**Introduce right angles in three dimensions.** Review the concept of perpendicular lines. Have students identify all perpendicular edges on triangular prisms. (side edges are perpendicular to all edges they meet at vertices and, if the base triangle is a right triangle, two edges of each base will be perpendicular too) Repeat with rectangular prisms. Point out that on a picture of a prism, some edges that are perpendicular in a model might not look perpendicular. Have students sketch a rectangular prism or a cube and mark all the right angles.

**Talk about edges perpendicular to faces.** Attach a cutout point to a wall on the line where the wall meets the floor. Using masking tape, make several rays on the floor and one vertical line on the wall from that point. The line on the wall is vertical and the lines on the floor are horizontal, so the wall line is perpendicular to all the floor lines. Remind students that to check that an angle is a right angle, they can use a benchmark—a shape with square corners, such as a postcard or a book. Have volunteers check the angle between the lines on the floor and the line on the wall. ASK: What do you notice? (all angles are right angles)

Explain that if a line is perpendicular to all lines on a surface, then we say that it is perpendicular to the whole surface, so the vertical line is perpendicular to the floor. We cannot check all lines, but it appears that it is enough to check two different lines. So if a line is perpendicular to two adjacent edges on one face, it is perpendicular to the whole face. Have students point to edges that are perpendicular to the bottom face of the rectangular prism. Repeat with one of the side faces.
**ACTIVITY**

Give each student a triangular prism with a scalene right triangle as the base (e.g., from BLM Nets (2), students can cut out the net and produce the prism). Have students place the prism base down. Have them show the edges that are perpendicular to the bottom face. Then ask them to turn the prism as shown in the margin. ASK: Which edges are perpendicular to the bottom face now? (the vertical edges) Trace the edge thickened in the picture in the margin. ASK: Is this edge perpendicular to the bottom face? (no) How can we prove it? Have students check that the slanted edge still makes a right angle with one of the horizontal edges (the angle marked in the picture in the margin). Explain that if an edge does not make a right angle with even one edge on the bottom face, it is not perpendicular to the face. Have students check that the angle between the thick edge and the third edge is not a right angle, so the edge is not perpendicular to the bottom face.

**Extension**

Have students use the prisms from the lesson to determine, then explain, their answers.

a) Palila says that if two edges of a prism are parallel, they always belong to the same face. Is she correct? (no; the bolded edges in the picture in the margin are parallel, but they do not belong to the same face)

b) Dimitry says that if two faces of a prism are parallel and they both have edges that belong to the same third face, then these edges are parallel and equal in length. Is he correct? (yes)
Goals
Students will find the surface area of right rectangular prisms.

Prior Knowledge Required
Can find the area of a rectangle using the correct formula
Can perform basic operations with decimals and fractions
Is familiar with square units of measurement
Can measure length to the closest millimeter
Is familiar with the standard order of operations for the four basic operations

Materials
empty boxes
centimeter rulers
inch rulers

Introduce surface area. Review area of rectangles. Have students find the area of a rectangle that measures 3 in by 4 in. (12 in²) Repeat with 2.3 cm by 4.1 cm to remind students how to multiply decimals. (9.43 cm²) Tell students that the surface area of a prism is the sum of the areas of all the faces of the shape. Ask students when they might need to know the surface area of a prism. (e.g., to calculate the amount of paper needed to make a box)

Finding the surface area of a rectangular prism by adding the areas of all faces. Give each student a box. Have them measure the sides of the prism to the nearest tenth of a centimeter. (Students might need a reminder that a tenth of a centimeter is 1 mm, so, if an edge is 78 mm long, it is 7.8 cm long.) Ask students to sketch the faces of the prism and to label the dimensions of each face. Have students check that they drew all the faces. ASK: How many should be there? (6) Ask students to write a multiplication statement for the area of each face and to add the results for all the faces. ASK: What units would you use to measure surface area? (cm², m², in², etc.) Point out that even though a prism is a 3-dimensional shape, surface area is measured in square units because it is the sum of the areas of all the faces.

Identifying identical faces and using multiplication to find the surface area. On the board, draw the picture in the margin. Ask students to name pairs of opposite faces. ASK: Which face is the same as the front face? (back) Write “back” on the copy of the front face. Repeat with the top and the right side. ASK: How can we use this to shorten the calculation of the surface area? (find the area of three different faces, then add them and double the result; or double first, then add the results)
Exercises: Find the area of the top, front, and right-side faces. Then find the surface area of the prism.

a) 3 cm 4 cm 5 cm
b) 3 ft 2 ft 1/2 ft
c) 4 cm 2.3 cm 1.2 cm

Bonus: Find the surface area of the prism using as few calculations as you can.

Answers: a) 94 cm², b) 17 ft², c) 33.52 m², Bonus: 52.9 cm²

Have students swap boxes, measure them to the nearest quarter of an inch, and find the surface area in square inches. Tell students that the area in square centimeters that their partners obtained should be about 6.5 times the area in square inches. Have them check that the answers make sense.

Extensions

In the extensions below, encourage students to model the problems using equations (MP.4) and to explain how their models answer the questions (MP.2).

1. a) A wealthy king has a treasure chest in the shape of a rectangular prism measuring 40 cm long, 30 cm wide, and 25 cm high. He ordered his carpenters to make a chest that can hold twice as much treasure.

The first carpenter doubled the length of the box and left the width and the height the same. The second carpenter doubled the width of the box and left the length and the height the same. The third carpenter doubled the height of the box and left the length and the width the same. Whose box used the least amount of wood, and so was the least expensive?

b) The three carpenters did not get the job of making the chest for the king’s treasure. He ordered the chest from a fourth carpenter, who suggested a chest 39 cm long, 39 cm wide, and 40 cm high. What were the volume and the surface area of the fourth chest? Why do you think the king chose the fourth carpenter?
Answers

a) First carpenter: The box dimensions were 80 cm × 30 cm × 25 cm.
2 × (80 cm × 30 cm + 80 cm × 25 cm + 30 cm × 25 cm) = 10,300 cm²

Second carpenter: The box dimensions were 40 cm × 60 cm × 25 cm.
2 × (40 cm × 60 cm + 40 cm × 25 cm + 60 cm × 25 cm) = 9,800 cm²

Third carpenter: The box dimensions were 40 cm × 30 cm × 50 cm.
2 × (40 cm × 30 cm + 40 cm × 50 cm + 30 cm × 50 cm) = 9,400 cm²

The third carpenter’s box had the lowest surface area, and so used the least amount of wood, and so was the least expensive.

b) Volume of the first three chests = 60,000 cm³; Volume of the fourth chest = 39 cm × 39 cm × 40 cm = 60,840 cm³, which is 840 cm³ more than the first three chests.

Surface area of the fourth chest.
= 2 × (39 cm × 39 cm + 39 cm × 40 cm + 39 cm × 40 cm)
= 9,282 cm²

The surface area of the chest made by the fourth carpenter is 118 cm² less than the surface area of the chest made by the third carpenter. So the fourth chest uses the least wood of all the chests, so it is the cheapest. It also has a greater volume than the others, so it can hold more. I think the King chose the fourth carpenter because his chest was the cheapest and could hold the most.

2. Zara bought a standalone closet at a garage sale. The closet is 1 m deep, 2 m wide, and 2.5 m high. She wants to paint the outside of the closet—the walls, the door, and the top—but not the bottom. How much paint does she need?

One liter of paint covers 7 m². Write your answer to the nearest tenth and explain why you rounded your answer up or down.

Solution: The surface area of the closet, not including the bottom face, is 2 × (1 m × 2.5 m + 2 × 2.5 m) + 1 m × 2 m = 17 m². Since one liter of paint covers 7 m², I can write the equivalent ratio equation $1 \text{ L} : 7 \text{ m}^2 = x \text{ L} : 17 \text{ m}^2$. Solving for $x$ gives $x = 17/7$. Zara needs about 2.5 L of paint. I rounded my answer up, to make sure Zara has enough paint.
ACTIVITY

Identifying faces of shapes. Give each student two prisms and two pyramids—rectangular and triangular. Have them fill in the table below for their prisms. Students need to sketch the shapes of the faces.

<table>
<thead>
<tr>
<th>Name</th>
<th>Base or Bases</th>
<th>Side Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular Prism</td>
<td>△ △</td>
<td></td>
</tr>
</tbody>
</table>

Remind students that, in mathematics, hatch marks are used to mark equal line segments on sketches. Have students use hatch marks to show equal sides on the faces they sketched in the table. Point out that there are sides of the same length on different faces, because they came from the same edge, and they need to be marked with the same number of hatch marks on all the equal sides. See the example in the margin.

Identifying 3-D shapes from their faces. Draw the picture in the margin on the board. SAY: I drew all the faces of a 3-D shape. What is it? (triangular pyramid) If students have trouble identifying the 3-D shape, ask them to circle the base(s) first.

Exercises: Identify the 3-D shape.

a) △ △ △  b)  c)
Answers: a) triangular pyramid, b) triangular prism, c) triangular prism, d) cube, e) rectangular prism, f) rectangular prism

Making nets for prisms. Hold up a triangular prism. ASK: Which two types of faces does it have? (triangular and rectangular) How many bases does it have? (2) What is the shape of the side faces? (rectangles) How many side faces does it have? (3) Explain that an easy way to make a net for this prism is to start with the band of rectangles for side faces (created by rolling a prism and tracing each side face in turn) and then to draw the bases. Illustrate this on the board, using the prism, as shown in the margin. Have students use this method to draw nets for the two prisms they have.

Making nets for pyramids. Repeat the questions above with a rectangular pyramid. Explain that an easy way to make a net for a pyramid is to start with the base (trace the base on the board), then attach a triangle of the correct size to each edge of the base. Show how to do it by rolling the pyramid over the edges of the base, tracing the faces. Point out that since the prism has a rectangle as a base, the triangles that make the side faces are of two different sizes. Then ask a volunteer to mark the equal edges on the net you created (see margin).

Have students use this method to draw nets for the two pyramids they have. Then ask students to swap the four nets they produced with a partner who has different shapes. Ask students to identify the shapes the nets were made from.

Matching edges on nets. Have students count the edges on the nets. For example, on the net of the pyramid above, students can count 12 edges. Then ask them to take the actual shape and count the edges on it. (8 edges) ASK: Are the two numbers the same? (no) Why not? (some of the edges were counted more than once) Point out that the outer edges of the net were counted twice, once on each face that meets at the edge. ASK: Which edges match to which edges? Ask students to mark the edges that will be glued together with the same number. Then have them cut out the nets and fold them to check their predictions.

ACTIVITY

Have students choose one of the nets they cut out. Then have them cut off all the faces (one at a time) and reattach the faces at different places. Will the new net fold into the same prism? Which edges are places where you would want to reattach the faces and which are not? Students should work with at least two different shapes each. Students will thus explore various ways to create nets for the same solid, rather than memorize a single net shape.
Finally, have students swap shapes with another partner. Have them choose a shape they have not used before and try to sketch a net for it. Ask students to mark equal edges with hatch marks. Have partners check each other’s work.

**Extensions**

1. Match the shape to its set of faces.

   ![Shapes A and B with faces labeled](image)

   a) $2 \times 8 \times \triangle$
   b) $6 \times 8 \times \triangle$

   **Answers:** a) A, b) B

2. a) Helen cuts a face off the net of a cube and glues it in a different place. Which nets will fold into a cube?

   ![Nets](image)

   i) ![Net](image)
   ii) ![Net](image)
   iii) ![Net](image)
   iv) ![Net](image)

   **Answers:** a) i) no, ii) yes, iii) yes, iv) no

   b) Noah cuts a face off a net and glues it in a different place. Can he still fold the net into a 3-D shape? What shape will it be?

   ![Shapes](image)

   i) ![Shape](image)
   ii) ![Shape](image)
   iii) ![Shape](image)
   iv) ![Shape](image)

   **Answers:** a) i) no, ii) yes, iii) yes, iv) no; b) i) no, ii) yes, rectangular prism, iii) yes, square pyramid, iv) no
G6-39  Nets and Surface Area of Rectangular Prisms

Pages 150–151

STANDARDS
6.G.A.4, 6.NS.B.3, 6.EE.A.2, 6.EE.A.4

VOCABULARY
face
height
length
net
rectangular prism
surface area
width

Goals
Students will find the surface area of right rectangular prisms.

PRIOR KNOWLEDGE REQUIRED
Can use the correct formula to find the area of a rectangle
Can identify prisms and pyramids
Can perform basic operations with decimals and fractions
Is familiar with the standard order of operations for the four
basic operations
Is familiar with square units of measurement

MATERIALS
rectangular box

Review sketching nets for rectangular prisms and the names of the
faces. Hold up a rectangular box. Review the names of the faces and
write them on the faces of the box. Ask students how they would sketch a net for
the rectangular prism. Remind students that the net can be made by drawing
a band of four rectangles as though they were rolling the prism over, then
attaching two faces for the side faces as shown in the margin. Demonstrate
the process on the board by rolling the prism and tracing the faces.

On the board, sketch a box as shown below:

```
Top face
Back face
Left-side face
Bottom face
Right-side face
Front face
```

4 in 6 in 2 in

ASK: What are the dimensions of the top face? (6 in by 4 in) Have students
sketch a rectangle that is 6 squares long and 4 squares wide on grid paper
and label it “top face.” ASK: What faces touch the top face on the prism?
(front, back, and both side faces) What face does not touch the top face?
(bottom face) If I now draw an identical rectangle right beside the top face
and label it “bottom face,” would that be correct? (no) I want to draw the
front face now. Where should I attach it? (to the top or to the bottom of the
top face) What should the length of the rectangle be? (6 squares) The width?
(2 squares) How do you know? (the front face dimensions are the length
and the height of the prism)

Continue with the other faces. Then have students find the area of each
face of the net. Remind students that surface area is the total of the areas
of the faces. Have them find the surface area of the prism. (88 in²)
Exercises: Draw a net for the prism on grid paper. Use 1 square for 1 cm. Then find the surface area of the prism.

a) 

b) 

Answers: a) 34 cm², b) 52 cm²

Sketching nets for prisms with given dimensions. On the board, draw the prism in the margin and ask students to sketch the net for it. Remind them that a sketch means the drawing does not need to be perfect, only roughly to scale. Then ask students to mark on the nets which face is which (top, bottom, right side, etc.). Go through the faces on the net one by one and have students identify their dimensions (see answer in margin). Then have them find the area of each rectangle and the surface area of the prism. (front and back faces: 50 ft², top and bottom faces: 20 ft², left-side and right-side faces: 10 ft², surface area: 160 ft²)

Exercises: Find the surface area using nets.

a) 

b) 

Answers: a) 234 m², b) 118.2 cm²

Finding the surface area using shortcuts. Ask: To find the surface area of a rectangular prism, do you need to find the area of all six faces separately? Is there a shortcut? Prompt: Which faces always have the same area? Ask students to try to write a formula for the surface area using the areas of fewer than 6 faces. Write down all the correct variations that students suggest. Then present the following list of formulas and ask students to tell which of them are correct.

A. (area of top + area of bottom + area of right side) × 2
B. (area of top + area of left side + area of back) × 2
C. (area of top + area of right side + area of front) × 2
D. (area of bottom + area of right side + area of left side) × 2
E. (area of bottom + area of left side + area of front) × 2
F. (area of bottom + area of front + area of top) × 2

Answers: B, C, and E are correct.
Then have students find the surface area of the prism below. (72 m²)

Finally, draw a prism and mark the length, width, and height with the variables \( l, w, \) and \( h \). Ask students to write the areas of three faces that would allow them to find the surface area of the prism. Have students produce a formula for the surface area of the prism. Answers will vary by the order of products in the sum and the order of factors.

Surface area = \((l \times h + h \times w + l \times w) \times 2\)

**Exercises:** Find the surface area of the rectangular prism using the formula you produced.

a) length = 3 cm, width = 2.5 cm, height = 4 cm
b) length = \( 3 \frac{3}{4} \) in, width = \( \frac{7}{8} \) in, height = 2 in
c) length = 3.75 m, width = 2.5 m, height = 4.2 m

**Answers:** a) 59 cm², b) \( \frac{401}{16} \) in² = \( 25 \frac{1}{16} \) in², c) 71.25 m²

**Extensions**

1. Find the surface area of a rectangular prism with the given dimensions. Convert the dimensions to the same units first.
   a) length = 1 yd, width = 1 ft, height = 1 in
   b) length = 1 m, width = 1 m, height = 1 cm

**Answers:** a) 960 in² or \( 6 \frac{2}{3} \) ft², b) 20,400 cm² = 2.04 m²

(MP.7)

2. The shape below was made with 2 cm cubes. Find its surface area.

**Sample solution:** Since there are 3 cubes used and they are all 2 cm cubes, the area of any square face will be \( 2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2 \). This shape’s surface area is made up of 14 of these square faces. So the surface area will be 56 cm² \((14 \times 4 = 56)\).

Redirecting students: If students struggle, ASK: What information are you given? How does that help you find the area of one square face? What else do you need to know to find the surface area of the whole shape?
Review area of triangles. Remind students how to find the area of a triangle. Review the concept of height—the distance from a vertex to the opposite side, measured along a perpendicular line.

**Exercises:** Find the area.

a) \[ \text{base: } 3 \text{ ft, height: } 13 \text{ ft} \]

b) \[ \text{base: } 3 \text{ m, height: } 12.4 \text{ m} \]

c) \[ \text{base: } 2 \frac{1}{2} \text{ in, height: } 6 \frac{3}{4} \text{ in} \]

**Answers:** a) 19.5 ft², b) 18.6 m², c) \( \frac{7}{16} \) in²

**Finding the surface area of a pyramid given its net.** Display BLM Surface Area of Triangular Pyramid. As a class, find the area of each triangle on the net, then find the total surface area. (10,504.5 mm²)

Then display the first pyramid on BLM Two Pyramids and a Prism with Nets, together with the net. Discuss how the line segments marked as perpendicular on the drawing of the pyramid are not actually drawn perpendicularly. Show that the angle between line segments on the picture is not a right angle using the corner of a sheet of paper. This is what markings of right angles are for—to show where right angles should be in the actual shape. Have students copy the net; mark the equal line segments, the length of the bases, and the heights; and find the surface area of the pyramid.
Repeat with the other two shapes on the same BLM. Answers are shown below.

Answers: triangular pyramid: 674 in$^2$, rectangular pyramid: 4,104 cm$^2$, triangular prism: 107.4 cm$^2$

ACTIVITY

Give each student a net of a triangular pyramid from BLM Net (10) or BLM Net (13), so that there will be at least one scalene triangular face. Have them draw a height on each face. Have them measure the heights to the nearest tenth of a centimeter and write the measurements beside the heights. ASK: Which side of each triangle is the base? Have students measure and label the base of each triangle. Then have them find the area of each face and the total surface area of the pyramid.

Compare the answers. Discuss reasons why people working with the same pyramid got different answers. (The measurements are approximations, so the results are approximate.) ASK: What could we do to check that our answers make sense? One way to do this is to estimate all the lengths to the nearest centimeter. The surface area to the nearest square centimeter should be very close for everyone.
Extensions

1. Give students a net of a triangular prism BLM Nets (1–7) (pp. Q42–Q-48), and have them find the surface area. Students should measure to the nearest tenth of a centimeter and round the answer to the nearest whole square centimeter.

2. Will these nets fold into pyramids? Explain.

![Nets](image)

**Answers:** a) The triangles will fold into the square, so the pyramid will have no height. b) The sides of the triangles with horizontal bases are shorter than the sides of the triangles with vertical bases, so the edges will not match. c) The edges marked with the same letter on the sketch below should be glued together. However, these edges do not match.

![Sketch](image)

3. Another way to check that the results students obtained in the Activity are good is to use the fact that we have 10 answers and can find the average (mean).

Discuss what the options are to find the mean. ASK: How would you do it by hand? (add all answers and divide by the number of answers) Does doing all the calculations by hand make sense? (no, too much work, lots of chances for mistakes) Does using a calculator make sense? (yes) Could there still be mistakes with a calculator? (yes, we could type some of the numbers incorrectly) Is there a better way? (yes, statistics software) Show students how to find the average using statistics software. Then look at the data entries as a class. Are there values that are much greater or much less than the rest of the set? What do these values represent? (mistakes) Recalculate the mean without the extreme values. In the meantime, have students who created an outlier recalculate the answer, without identifying those students.
Review finding the areas of rectangles and triangles.

Review the formulas for finding the areas of triangles and rectangles. Review finding missing dimensions of rectangles when given the area. (Example: A rectangle has area 13.5 cm² and height 3 cm. What is its width? (4.5 cm))

Review finding the volume and surface area of rectangular prisms. For rectangular prisms, review how to find the volume (Volume = area of the top/bottom face × height and V = ℓ × w × h) and the surface area (add areas of all faces; nets can help you keep track of faces). Remind students that volume is measured in cubic units and surface area is measured in square units. Remind students that writing the units at every step of the calculation helps to make sure that you multiply two dimensions for area and three dimensions for volume.

Finding the volume of rectangular prisms when some of the dimensions are not known. Draw the picture in the margin on the board. Have students identify the missing linear dimension. PROMPT: Which dimensions are you given: length, width, or height? (width and height) You can also ask a volunteer to mark which edges show the length, the width, and the height.

ASK: For which face are you given the area? (front face) Which dimensions do you multiply to find the area of the front face? (length and height) What is the missing length? (4 cm) Have students find the volume and the surface area of the prism.

Volume = 12 cm³
Surface area = 2 × (2 cm × 4 cm + 1.5 cm × 4 cm + 1.5 cm × 2 cm)
= 2 × (8 cm² + 6 cm² + 3 cm²) = 34 cm²

Exercises: Find the missing dimension of the prism in the margin. Then find the volume and the surface area of the prism.

Answers: width = 3.5 cm, volume = 29.4 cm³, surface area = 59.5 cm²
Finding the volume and surface area of 3-D shapes with dimensions in different units. Sketch the prism in the margin on the board. Discuss with students how to find the volume of the prism. Lead them to the idea of converting all measurements to the same units before finding the volume and the surface area. Emphasize how writing the units when substituting the measurements in the formula helps avoid multiplying different units, such as meters and centimeters.

**Length** = 125 cm or **length** = 1.25 m  
**Width** = 100 cm or **width** = 1 m  
**Height** = 70 cm or **height** = 0.7 m  
**Volume** = \( l \times w \times h \) or **Volume** = \( l \times w \times h \)  
= 875,000 cm\(^3\) or = 0.875 m\(^3\)  
**Surface area** = 56,500 cm\(^2\) or = 5.65 m\(^2\)

**Exercises**

1. Find the volume and the surface area of the 3-D shape.
   
   - **a)**
     
     ![Diagram](image1.png)
     
     **Answers:** a) volume = 270 in\(^3\) (or 5/32 ft\(^3\)), surface area = 306 in\(^2\) (or 2 1/8 ft\(^2\))
   
   - **b)**
     
     ![Diagram](image2.png)
     
     **Answers:** b) volume = 0.252 m\(^3\) (or 252,000 cm\(^3\)), surface area = 2.7 m\(^2\) (or 27,000 cm\(^2\))

2. Find the surface area.
   
   - **a)**
     
     ![Diagram](image3.png)
     
     **Answers:** a) 1,732 m\(^2\), b) 288 in\(^2\) = 2 ft\(^2\)
3. A box without a lid has volume 40,000 cm$^3$. It is 40 cm wide and 50 cm long.

   a) What is the height of the box?
   b) What are the dimensions of the lid (the missing face)?
   c) What is the box's surface area? (Remember, there is no lid.)
   d) The material for the box costs $12.35 per m$^2$. How much will the box cost?

   **Answers:**
   a) 20 cm, b) 40 cm $\times$ 50 cm, c) 40 cm $\times$ 50 cm $\times$ (40 cm $\times$ 20 cm + 50 cm $\times$ 20 cm) = 5,600 cm$^2$, d) 5,600 cm$^2$ = 0.56 m$^2$, so the cost for the box will be 0.56 m$^2$ $\times$ $12.35/\text{m}^2$, which rounds to $6.92.

**Bonus:** Find the volume and the surface area of a rectangular prism that is 1 m by 1 cm by 1 km. (volume = 10 m$^3$, surface area = 2,020.02 m$^2$)

**Review exponential notation.** Remind students that an expression such as $3 \times 3 \times 3 \times 3$ can be written using exponents as $3^4$. Have students write in short form the expressions $5 \times 5$ and $6 \times 6 \times 6$. (5$^2$, 6$^3$) Then have students evaluate such expressions as $4^3$, $\left(\frac{1}{2}\right)^2$, and $0.2^3$. (64, $\frac{1}{4}$ or 0.25, 0.008)

**Exponential notation in formulas.** Ask students how they would find the area of a square 3 cm long. Have them write the multiplication equation. (3 cm $\times$ 3 cm = 9 cm$^2$) Repeat with a square 3 in long (9 in$^2$) and a square 6 m long (36 m$^2$). ASK: What is a formula for the area of a square? If students answer “length $\times$ width,” ask them to use the fact that length and width are the same in a square, so the possible formula is area = length $\times$ length. Remind them that formulas are often written as shortcuts, in this case, $A = l \times l$. ASK: How can we write this formula in exponential notation? ($A = l^2$)

Have students use the formula above to find the area of a square with length $\frac{1}{3}$ ft ($\frac{1}{9}$ ft$^2$ or 16 in$^2$), and a square with length 0.5 m (0.25 m$^2$).

Present a cube. Remind students that a cube has equal length, width, and height. ASK: What shape do the faces of a cube have? (squares) How many faces does a cube have? (6) How do you find the surface area of a cube? (find the area of one square and multiply by 6) If a cube has length $l$, what will the area of one face be? ($l^2$) What is the surface area of the cube? ($6 \times l^2$)

Remind students that exponents are done before multiplication, so when you have the expression $6 \times l^2$, and you substitute, for example, 3 cm into it, you find 32 first, then multiply by 6. So the surface area of a cube with length 3 cm is $6 \times (3 \text{ cm} \times 3 \text{ cm}) = 6 \times 9 \text{ cm}^2 = 54 \text{ cm}^2$. 

**Geometry 6-41**
Exercises: Find the surface area of the cube.

a) length = 5 in  

b) length = 3.1 cm  

c) length = $2\frac{1}{4}$ in

Answers: a) 150 in², b) 57.66 cm², c) $\frac{243}{8}$ in² = $30\frac{3}{8}$ in²

Have students identify the mistake in this calculation of the surface area of a cube with sides measuring 1 cm:

\begin{align*}
I &= 1 \text{ cm} \\
\text{Surface area} &= 6 \times \ell^2 \\
&= (6 \times 1 \text{ cm})^2 \\
&= 6 \text{ cm} \times 6 \text{ cm} \\
&= 36 \text{ cm}^2
\end{align*}

(MP3) (exponent should be done before multiplication)

(MP2) Ask students how surface area and the area of a net are connected. (surface area is the area of a net) Ask students to sketch a net for a cube.

ASK: How does the net agree with the formula? (there are 6 squares of the same area) Ask students to draw the net for a 1 cm cube to scale. Then have students draw a few rectangles that have an area of 36 cm²: a 6 cm by 6 cm square, a 3 cm by 12 cm rectangle, a 4 cm by 9 cm rectangle. ASK: Are the rectangles the same size as the net? (no, they are much larger than the net) Point out that this is another way to tell that the calculation above is the wrong way to find the surface area.

Extensions

(MP1, MP7) 1. Edges $a$, $b$, and $c$ have lengths that are whole inches. The area of each face is written directly on the face. What are the possible lengths for edges $a$, $b$, and $c$?

Solution: Since $c$ has to be a common factor of 6 (in) and 12 (in), it must be 1, 2, 3, or 6 (in). If $c = 1$, then $a = 12$, which is not a factor of 18. If $c = 3$, then $a = 4$, which is not a factor of 18. If $c = 6$, then $a = 2$ and $b = 1$, but then $a \times b = 2$, not 18. So the only possible lengths are $a = 6$ in, $b = 3$ in, $c = 2$ in. Explain.

2. Find the surface area and volume of the tissue box shown below. Remember to leave out the opening on the top.

Answers: volume = 1,848 cm³, surface area = 944 cm²
3. You are designing a box for a cereal company. The cereal box needs to have a volume of 2,000 cm³. There are many possible boxes you could make that have this volume.

a) Verify that these 3 sets of measurements for boxes each have a volume of 2,000 cm³.
   - 1 cm × 1 cm × 2,000 cm
   - 2 cm × 25 cm × 40 cm
   - 5 cm × 25 cm × 16 cm

b) Calculate the surface area of each box in part a).

c) If the material to make the box costs 0.025¢ per cm², which box from part a) is the least expensive? How much does it cost?

d) Find three more boxes with the same volume and calculate the surface area of each. Now which box would you recommend?

e) The cereal company wants the front of the box to be at least 20 cm wide and 20 cm high and the depth of the box to be at least 4 cm. Find two boxes satisfying these conditions that each have a volume of 2,000 cm³. Which box would you recommend and how much would it cost?

Answers: b) 8,002 cm², 2,260 cm², 1,210 cm²; c) the third, $0.30; e) 4 cm × 20 cm × 25 cm, surface area = 1,360 cm², cost = $0.34; and 5 cm × 20 cm × 20 cm, surface area = 1,200 cm², cost = $0.30. The second box has the smallest surface area so it’s the least expensive to make.

4. a) Amit wants to solve \( \frac{2}{3}x = \frac{3}{5} \). He says that he can solve \( 10x = 3 \) instead, and his answer will be the same. Do you agree or disagree with Amit? Explain. Use math words.

b) In pairs, discuss your answers to part a). Do you agree with each other? Discuss why or why not.

Answer: Yes, I agree, because Amit just multiplied both sides of the equation by 5: \( 5 \times 2x = 10x, 5 \times \frac{3}{5} = 15/5 = 3, \) and when you multiply both sides of an equation by the same number, in this case 5, the value of \( x \) that makes both sides the same doesn’t change.

NOTE: In part b), encourage partners to ask questions to understand and challenge each other’s thinking—see p. A-49 for sample sentence and question stems).

Look for students to demonstrate that they understand that solving an equation means finding the value of \( x \) that makes it true (MP6).
Nets (1)
Nets (3)
Nets (5)
Nets (6)
Nets (8)
Nets (9)
Nets (10)
Nets (12)
Nets (13)
Nets (15)
Nets (16)
Cube Skeleton

You will need 12 tubes and tape to make a skeleton of a cube.

1. Use tape to bind tubes together as shown.

2. Make two squares with the tubes. Make sure the squares are 1 unit long and 1 unit wide on the inside.

3. Bind the four leftover tubes to one of the squares as shown.

4. Add the other square to the top.
Two Pyramids and a Prism with Nets

15 cm
80 cm
24 cm
7 cm
6.5 cm
7 cm
5.4 cm
2.4 cm
1.6 cm
20 in
14 in
30 in
8 in
17 in
41 cm
80 cm
24 cm
20 in
14 in
30 in
8 in
17 in
41 cm
80 cm
24 cm

Blackline Master — Geometry — Teacher Resource for Grade 6
Q-59
Surface Area of Triangular Pyramid
Unit 9  Statistics and Probability: Distribution

This Unit in Context

In 6.2 Unit 7, students learned to calculate the mean, median, and range of a collection of data. In this unit, they will extend their knowledge of how to summarize numerical data sets to include measures of variability, such as interquartile range and mean absolute deviation (6.SP.B.5c). In Grade 7, students will extend their knowledge of statistics by using random samplings to draw inferences about single populations (7.SPA.1) and draw informal comparative references about two populations (7.SPB.3). In Grade 8, their study of statistics will deal with bivariate data (8.SPA.1).

Mathematical Practices in This Unit

In this unit, you will have the opportunity to assess MP2, MP4, and MP6. Here are some examples of how students can show that they have met a standard.

MP2: In SP6-5 Extension 2, students reason abstractly and quantitatively when they represent a real-world situation using equations and variables, and when they explain how the math relates to the real-world context. In the Exercises in SP6-9, students reason abstractly and quantitatively when they make up a real world situation to represent a data set and when they use the real-world example to explain the term “interquartile range.”

MP4: In SP6-5 Extension 2, students model mathematically when they use equations with variables to represent and solve a non-routine, real-world problem.

MP6: In the Exercises in SP6-9, students attend to precision when they use an example to explain the term “interquartile range.”
**Introduce histograms.** Explain that data sets are often very large; they could have hundreds of points. ASK: Why would it be difficult to make dot plots for such large sets? (too many dots to draw) Explain that with very large data sets, instead of plotting every single data point, we can break the data up into smaller parts and plot the parts. We do this using histograms.

**Intervals (width of bars).** Draw the histogram shown in the margin but without brackets, arrows, or dollar ranges. Explain that students at an elementary school raised money to buy a new slide for their playground. They only collected whole dollar amounts such as $5 or $7, but not amounts such as $5.20 or $7.45. In this histogram, the units used to measure the data are dollars. Point at the bars one by one. SAY: Each bar represents an interval, which is a part of the whole set of data.

Draw a bracket under the first bar. Write $0, $1, $2, $3, $4, $5, $6, $7, $8, $9 beside the histogram. SAY: If a student raised any one of these amounts, it would go in the first interval. We can’t tell exactly how much each student raised but, if it was in this interval, it was one of those amounts. ASK: How many different amounts are in this interval? (10) SAY: So the first interval spans a range of ten dollars. Write “$0–$9” under the first bracket. SAY: In histograms, data that falls right on the boundary between two intervals goes in the higher interval. Anyone who raised exactly $10 is counted in the next interval. Draw a bracket under the second bar. Write $10, $11, $12, $13, $14, $15, $16, $17, $18, $19 beside the histogram. SAY: If a student raised any one of these amounts, it goes in the second interval. ASK: Can we tell exactly how much each student raised? (no) How many different amounts are in this interval? (10) SAY: So, like the first interval, the second interval also spans a range of ten dollars. Write “$10–$19” under the second bracket. ASK: If a student raised exactly $20, which interval would that go in? ($20–$29) Repeat for the remaining intervals. ASK: What range do all five intervals span? (ten dollars) SAY: Just like in this histogram, interval widths in any one histogram are always equal to the widths of the other intervals.
Exercises: Draw histograms a) and b) shown below, but don't have students draw them. Also draw the table with bold headings included. Have students state the number of intervals, then state the units, interval widths, and lowest and highest values in the first and last intervals.

<table>
<thead>
<tr>
<th>Number of Intervals</th>
<th>Units</th>
<th>Width of Intervals</th>
<th>Lowest and Highest Values in the First Interval</th>
<th>Lowest and Highest Values in the Last Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>5 weeks</td>
<td>4 weeks</td>
<td>0 and 3</td>
<td>16 and 19</td>
</tr>
<tr>
<td>b)</td>
<td>8 degrees</td>
<td>3 degrees</td>
<td>0 and 2</td>
<td>21 and 23</td>
</tr>
</tbody>
</table>

Frequencies (height of bars) and totals. Draw the histogram shown in the margin with brackets, but without arrows, blanks, and numbers. Explain that the histogram represents how many minutes it took people to solve a Rubik's Cube®. Explain that only whole minutes, not seconds, were recorded. Point to the top of the 0–60 bar. SAY: This bar goes up to 25, which means it shows data for 25 people out of the total number of people whose times were recorded. Draw the first arrow and write “25 people” below it. ASK: How many people took from 60 to 119 minutes? (50) Draw the second arrow and write “50 people” below it. Repeat for the remaining intervals. ASK: Which intervals contain the largest amount of data? (the last two) ASK: How can we use the numbers 25, 50, 75, 125, and 125 to figure out the total number of people? (add them) Write the equation on the board:

\[
25 + 50 + 75 + 125 + 125 = 400 \text{ people}
\]

ASK: How many people’s data does this histogram represent altogether? (400) Explain that you now want to put the data in a frequency table. Draw the table below with bold headings included and have students help you fill in the frequencies.

<table>
<thead>
<tr>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–59</td>
</tr>
<tr>
<td>60–119</td>
</tr>
<tr>
<td>120–179</td>
</tr>
<tr>
<td>180–239</td>
</tr>
<tr>
<td>240–299</td>
</tr>
</tbody>
</table>

Exercises: Have students work on BLM Frequency.

Answers: a) \(40 + 30 + 20 + 0 + 10 = 100\),
b) \(80 + 120 + 0 + 240 + 160 + 200 = 800\)
Interpreting histograms. Draw the histogram in the margin and the frequency table below with bold headings included. Explain that the histogram shows how many whole minutes Grade 6 students at West Lake Middle School spent on the internet yesterday. Have students help you fill in the table.

<table>
<thead>
<tr>
<th>Interval</th>
<th>0–19</th>
<th>20–39</th>
<th>40–59</th>
<th>60–79</th>
<th>80–99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>60</td>
<td>45</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

ASK: What is the total number of students represented by this histogram? (60 + 45 + 15 + 15 + 15 = 150) What was the longest time interval? (80–99) How many students spent at least 40 minutes on the internet yesterday? (15 + 15 + 15 = 45) Which interval contains the largest number of students? (0–19) How many students spent less than an hour on the Internet? (60 + 45 + 15 = 120) Would a student who spent exactly 60 minutes on the internet be in the third or fourth interval? (fourth) What is the maximum number of minutes that can be represented in this histogram? (99) Could a student who spent 100 minutes on the internet be represented in this histogram? (no)

Have students work on Questions 1 and 2 on AP Book 6.2 p. 155.

Extra Practice: Have students complete BLM Interpreting Histograms.

Answers

a) | Interval   | 0–1 | 2–3 | 4–5 | 6–7 | 8–9 | 10–11 | 12–13 | 14–15 | 16–17 | 18–19 | 20–21 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>40</td>
<td>24</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

b) 20 – 21, c) 10 – 11, d) 4 + 8 + 12 + 24 = 48, e) 24 + 8 + 4 + 4 + 4 = 44, f) 4 + 8 + 12 + 24 + 36 + 40 + 24 + 8 + 4 + 4 + 4 = 168, g) Set 2: 17, 16, 17, 17 and Set 3: 16, 17, 17, 16

Drawing histograms from data. Explain that students have to be able to draw histograms from the data given. Draw the frequency table below on the board with bold headings included. Explain that it shows how far people were able to throw a one-pound ball, and that the distances were measured to the closest whole foot.

<table>
<thead>
<tr>
<th>Interval</th>
<th>0–4</th>
<th>5–9</th>
<th>10–14</th>
<th>15–19</th>
<th>20–24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>20</td>
<td>60</td>
<td>50</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

Draw the histogram shown in the margin. ASK: How many people are represented in the first interval? (20) Ask for volunteers to draw the other bars and explain what each means. Have students work on Question 3 on AP Book 6.2 p. 156.

Exercises: Have students complete the questions on BLM Drawing Histograms.

Extra Practice: Have students complete BLM From Data to Dot Plots to Histograms.
Extensions

1. Draw the histogram.

![Histogram](image)

Money Students Collected for Charity

<table>
<thead>
<tr>
<th>Money Collected (dollars)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

a) What is the least amount of money a student could have collected? Explain.

b) What is the greatest number of whole dollars a student could have collected? Explain.

**Answers:**
a) $0 because that’s the lowest amount on the histogram,
b) $5 because that’s the highest amount on the histogram. You may need to remind students that although “$6” appears on the histogram, $6 would be recorded in the next interval, which is not marked on the histogram.

2. Marco is four times as old as his eldest child, who shares the same birthday as Marco. If the sum of their ages is 45, how old was Marco when he became a father?

Model the situation using equations and variables. Explain how the math relates to the real world situation.

**Sample solution:** Let $d$ be the age of Marco’s child. Then Marco’s age is $4d$. I wrote the sum of their ages as $4d + d$, which equals $(d + d + d + d) + d = 5d$. So I wrote an equation to solve for $d$: $5d = 45$. Dividing each side of the equation by 5, I found $d = 9$. So the child’s age is 9, and Marco’s age is $9 \times 4 = 36$. Marco became a father 9 years ago, so Marco was $36 - 9 = 27$.

**NOTE:** If Marco and his child did not share the same birthday, the answer might have been off by 1.
Collecting data. Have students work in pairs. Give each pair one copy of BLM Five-Part Spinner. Each pair of students is to collect two sets of data. A set of data is made up of ten spins of the spinner. For the first set, one student spins and the other records the results. For the second set, students switch roles. If the spinner lands on a line between two segments, they repeat the spin.

Reminder about finding the mean. Remind students that to find the mean of a data set, they must add all the data points, then divide by the number of data points. Have students find the mean of 1, 6, 20. \( \frac{1 + 6 + 20}{3} = 9 \)

Reminder about finding the range. Write the data set and the equation in the margin. Remind students that when you find how far the highest and lowest values in a set are from each other, you are finding the range. Have students find the range of the set. (9)

Statistical questions. ASK: Did you expect the spinner to land on the same number every time you spun it? (no) Before spinning it, could you predict where it would land? (no) SAY: When we asked the question "where will the spinner land when we spin it 10 times?" there were some things we could expect and others we couldn’t. We could expect the results to vary, but we could not expect to be able to predict what number each spin would land on. Questions that have a variety of possible results are statistical questions. Questions that have only one predictable answer, such as "what day is it today?" are not statistical questions. Explain that you are going to ask a number of questions and you would like students to decide whether they are statistical questions. Have them put up their hand when they think a question is statistical. ASK: What season are we in now? (not statistical) What is the weather usually like this season? (statistical) What is my age? (not statistical) What are the ages of the people at my school? (statistical) How many months are there in a year? (not statistical) How tall are the players on the team? (statistical) How tall is the coach of the team? (not statistical) How long is the Mississippi River? (not statistical) How long are the longest rivers in the world? (statistical)
Another tool for describing sets. Explain that when we work with sets of data, as in the spinner activity, we need to be able to describe them. We do this in several ways: we explain how the data was collected, count how many points there are, find the center, and find the range. We also need to describe how the results vary, in other words, their *variability*. Draw and vertically align the two dot plots in the margin. SAY: We’ll pretend these two dot plots are from the spinner activity; the first one shows that students spun a 1 four times, a 3 two times, and a 5 four times. Draw the table without the fourth column. Have students help you fill in the first three columns.

<table>
<thead>
<tr>
<th>Dot Plot 1</th>
<th>Number of Dots</th>
<th>Mean</th>
<th>Range</th>
<th>Mean Absolute Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>3</td>
<td>4</td>
<td>1 3/5</td>
</tr>
<tr>
<td>Dot Plot 2</td>
<td>10</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

ASK: Do the dot plots have the same number of points? (yes) Do they have the same mean? (yes) Do they have the same range? (yes) Cover up (but don’t erase) the dot plots. ASK: If you had not seen the dot plots at all, could you tell, just by looking at the table, that the dot plots are different? (no) Explain that the dot plots clearly are different and, since sometimes we only see tables, we need to put something in the table that shows the difference. SAY: There is a number we calculate that does this. It shows how tightly bunched or *clustered*, or widely spread out, the data points are from the mean; in this case, 3. Circle the 3 on both number lines of the dot plots. ASK: Would you say the points in one of these dot plots are more spread out from the mean than in the other dot plot? (yes, they’re more spread out in Dot Plot 1) Leave the dot plots and table on the board.

**Reminder about absolute value.** Draw the number line, but without dots or arrows. Remind students that the absolute value of a number is its *distance from 0* without regard to whether it is left or right of 0. Put a dot on −3 and mark in the arrow between 0 and −3.

ASK: How many units is −3 from 0? (3 units) Write | −3 | = 3. SAY: We use this symbol—a line on both sides of the number—to mean we are finding its absolute value, or its distance from 0. Repeat for +4.

**Exercises:** Find the absolute value.

- a) | −24 |
- b) | +24 |
- c) | 237 |
- d) | −34,999 |

**Answers:** a) 24, b) 24, c) 237, d) 34,999

**Absolute deviation.** SAY: Unlike absolute value, which is a number’s *distance from 0*, the *absolute deviation* is a data point’s *distance from the mean* without regard to whether it is left or right of the mean. Draw the number line, but without the data or lines above it. Have students find the
mean of 3, 4, 8. (5) Mark the points on the number line and circle the 5 to show the mean.

\[
\begin{array}{c}
\text{absolute deviation} = 1 \\
\text{absolute deviation} = 2 \\
\text{absolute deviation} = 3 \\
\end{array}
\]

Draw the line from 3 to 5. ASK: What is the distance from the data point at 3 to the mean at 5? (2) SAY: We say the “absolute deviation of 3 is 2.” Draw an arrow pointing to that line. Write “absolute deviation = 2.” Repeat for 8, then 4. SAY: What we are doing is finding the distance each data point in the set is from the mean. Write the set 1, 2, 7, 10 on the board and have students find the mean. (5) Draw the number line. Have a volunteer mark on the points and circle the mean. Have volunteers draw the absolute deviation lines and arrows showing what the absolute deviations are.

\[
\begin{array}{c}
\text{absolute deviation} = 4 \\
\text{absolute deviation} = 3 \\
\text{absolute deviation} = 2 \\
\end{array}
\]

**Exercises:** Find the mean of the set 3, 8, 10. Draw a number line, mark on the points, circle the mean, draw the absolute deviation lines, and write the absolute deviation for each point.

**Answers:**

\[
\begin{array}{c|c|c}
\text{Data Point} & \text{Mean} & \text{Absolute Deviation} \\
6 & 10 & 4 \\
7 & 10 & 3 \\
17 & 10 & 7 \\
\hline
\text{Total deviation} & \frac{14}{3} = 4 \frac{2}{3} \\
\end{array}
\]

**Mean absolute deviation.** SAY: Knowing the absolute deviation for every point is the same as knowing how far every point is from the mean. When we then find the average of all these distances, or absolute deviations, we are finding the **mean absolute deviation**, or MAD. The mean absolute deviation is the average distance of all the points from the mean. It helps us describe the variability in a set by showing us how tightly clustered or widely spread out the data in a set is from the mean. For instance, if all the points are close to the mean, the average of the distances will be small. If all the points are far from the mean, the average will be large. And if some points are close and others are far, the average will be somewhere in between. Explain that points can be any distance from the mean, including right on it. Write 6, 7, 17. Draw the table in the margin. Fill in the “data point” column. Have students find the mean. (10) Fill in the “mean” column. ASK: What is the absolute deviation of 6? (4) Write 4 in the “absolute deviation” column. If students have trouble with this, draw a number line. Repeat for 7 and 17. ASK: How do we find the average of the three absolute deviations? (add them up and divide by 3) Write the equation in the margin.
Exercises: Complete the table. Then find the mean absolute deviation.

a) | Data Point | Mean | Absolute Deviation |
---|---|---|---|
2 | 5 | 3 |
6 | 5 | 1 |
7 | 5 | 2 |
Total deviation = \( \frac{3 + 1 + 2}{3} = \frac{6}{3} = 2 \)

b) | Data Point | Mean | Absolute Deviation |
---|---|---|---|
1 | 5 | 4 |
3 | 5 | 2 |
11 | 5 | 6 |
Total deviation = \( \frac{4 + 2 + 6}{3} = \frac{12}{3} = 4 \)

What the mean absolute deviation actually is. Explain that now you’re going to make number lines for exercises a) and b) above to show what the mean absolute deviation actually is. Draw the number lines so that they are vertically aligned. Ask for volunteers to mark on the data points and circle the mean. Beside each number line, write the mean absolute deviation: 2 for part a) and 4 for part b).

ASK: Which data set has the larger mean absolute deviation? (b) In which set are the points more widely spread out around the mean? (b) SAY: Sets of data that are more widely spread out around their mean have a larger mean absolute deviation. Data sets that are more tightly clustered around the mean have a smaller mean absolute deviation.

Which dot plot has the larger MAD? Refer students back to the two dot plots and the table from the beginning of the lesson, and add the fourth column. Explain that now you want the class to help you calculate the mean absolute deviation for these two dot plots. Draw empty versions of the tables in the margin beside their respective dot plots. ASK: What goes in the “mean” column for both dot plots? (3) Write it in. Start with dot plot 1 and go from left to right. Point to each dot and ask what its value is. Once the data points are in the table, have students help you complete the “absolute deviation” column. Point out that since the two data points at 3 on the number line are right on the mean, the distance from each dot to the mean is 0.

Dot Plot 1

| Data Point | Mean | Absolute Deviation |
---|---|---|
1 | 3 | 2 |
2 | 3 | 2 |
3 | 3 | 0 |
4 | 3 | 2 |
5 | 3 | 2 |
Total deviation = 16

Dot Plot 2

| Data Point | Mean | Absolute Deviation |
---|---|---|
1 | 3 | 2 |
2 | 3 | 1 |
2 | 3 | 1 |
3 | 3 | 0 |
4 | 3 | 1 |
4 | 3 | 1 |
5 | 3 | 2 |
Total deviation = 10
Write in all the absolute deviations. Have students find the total of the deviations. (16) ASK: What do we divide 16 by to get the mean? (10, because there are 10 data points) Write $\frac{16}{10} = 1.6 = \frac{3}{5}$. Have students help you fill in the table for Dot Plot 2. Write $\frac{10}{10} = 1$. ASK: Which dot plot has the larger mean absolute deviation? (Dot Plot 1) So which set of data is more widely spread out around its mean? (Dot Plot 1) Write the two mean absolute deviations in the original table. ASK: Now if we could see only the table, not the dot plots, what more would we understand about how the dot plots are different? (Dot Plot 2 has a smaller mean absolute deviation, so the data points in it are more tightly clustered around the mean.)

**Exercises**

1. The table shows the absolute deviations for three data sets.
   a) Find the mean absolute deviation of each set.
   b) Which set is most spread out around the mean?

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Set 1 (0, 1, 8) and Set 2 (26, 27, 34) have very different data points, but the same mean absolute deviation. Explain how this is possible.

   Set 1
   0  1  2  3  4  5  6  7  8

   Set 2
   26 27 28 29 30 31 32 33 34

   **Answers**
   1. a) Set 1: MAD = 2, Set 2: MAD = 3, Set 3: MAD = 0; b) Set 2
   2. The three data points in Set 1 are exactly the same distances from the mean as the three data points in Set 2. The mean absolute deviation depends on distance from the mean no matter where the mean is.

**Extension**

Create a data set with three data points, mean = 5, and the total of the three absolute deviations = 6. Use the number line as a guide.

   0  1  2  3  4  5  6  7  8  9  10

   **Sample answers:** 2, 6, 7; 2, 5, 8; or 3, 4, 8
**SP6-7 Interquartile Range**

**Pages 159–161**

**STANDARDS**
6.SP.A.1, 6.SP.A.3, 6.SP.B.5a, 6.SP.B.5b, 6.SP.B.5c, 6.SP.B.5d

**VOCABULARY**
- first quartile (Q1)
- interquartile range
- lower half
- third quartile (Q3)
- upper half

**Set 1:** 1 4 4 6 9 11 12

**Set 2:** 1 4 4 6 9 11

**Goals**
Students will calculate and interpret the interquartile range of data sets.

**PRIOR KNOWLEDGE REQUIRED**
Can find the median of a data set

**Reminder about the median.** Remind students that the median is the number in the center position of an ordered set. Write the sets in the margin. ASK: When there is an odd number of points, like in Set 1, how do we find the median? (it's the number in the center position) What is the median of Set 1? (6) ASK: How do we find the median in sets with an even number of points, like Set 2? (add the two middle numbers and divide by 2) What is the median of Set 2? (5)

**Interquartile range.** Explain that there is another number similar to mean absolute deviation that shows the variability in a set of data. SAY: We used the mean absolute deviation to help describe the variability of data sets that were symmetrical. For sets that are not symmetrical, we use the interquartile range to help us understand the variability in the set.

**Dividing data sets in half.** Explain that to find the interquartile range of data sets, we divide the sets in half, then divide the halves in half again. We will do this with sets that have an even or an odd number of data points. SAY: We use the median to divide sets in half, creating a lower half and an upper half. Write 1, 2, 4, 9. SAY: This set has an even number of data points. ASK: What is the median? (3) How many data points are in each half? (2) Show the breakdown into halves as shown in the margin. SAY: Now we'll do this with a set that has an odd number of data points. Write 1, 2, 4, 9, 13. ASK: What is the median? (4) ASK: Can we divide 5 points into 2 equal halves? (no) Explain that when sets have an odd number of data points, we divide them into equal halves by not including the median in either half. Show the breakdown as shown in the margin. Point out that the median 4 isn't in either half. ASK: When we leave out the median, how many data points are in each half? (2)

**Exercises:** Write the set. State the median. Underline the halves and label them "lower half" and "upper half."

a) 6 32 45 49 51 66 104

b) 13  23 33 43 53 63

**Bonus:** 650  1,272  1,338  2,601  4,039

**Answers:** a) median = 49, 6 32 45 49 51 66 104;  
b) median = 38, 13 23 33 43 53 63;  
Bonus: median = 1,338, 650 1,272 1,338 2,601 4,039
Finding the first and third quartiles. Write the set without quartile labels and arrows, but with everything else.

lower half  upper half

\[
\begin{array}{cccccccc}
3 & 4 & 6 & 7 & 9 & 19 & 19 & 25 \\
\end{array}
\]

first quartile, Q1  third quartile, Q3

\[
Q1 = 5 \\
Q3 = 19
\]

SAY: “M” stands for “median.” ASK: What is the median? (8) Write it in. Explain that the lower and upper halves of a set each have a median at their center. ASK: Looking at the lower half, where is its median or center? (between 4 and 6) Draw the arrow for the first quartile. SAY: The median of the lower half is between 4 and 6, so it’s 5. We call the median of the lower half the first quartile, or Q1. Label it. ASK: Looking at the upper half, where is its median? (between 19 and 19) Draw the arrow for the third quartile. SAY: The median of the lower half is between 19 and 19, so it’s 19. SAY: We call the median of the upper half the third quartile, or Q3. Label it.

Exercises: Write the set. Mark the position of the median (M), the first quartile (Q1), and the third quartile (Q3). State their values.

a) 15 25 35 45 55 65 75 85 95 105 115 125

\[
Q1 = 40 \\
M = 70 \\
Q3 = 100
\]

b) 1 4 6 8 8 11 11 17 19 23 30 30 34 37 41 42

\[
Q1 = 8 \\
M = 18 \\
Q3 = 32
\]

Write the set shown in the margin with no labels, arrows, or brackets. ASK: What is the median of this set? (9) Label it. What numbers are in the lower half? (1, 2, 4, 6, 6) ASK: What is the first quartile, or the median of the lower half? (4) Label it. What numbers are in the upper half? (12, 14, 17, 29, 29) ASK: What is the third quartile? (17) Label it.

Exercises: Write the set. Mark the positions of the median (M), the first quartile (Q1), and the third quartile (Q3). State their values.

a) 2 11 13 14 21 23 24 28 32

\[
Q1 = 12 \\
M = 21 \\
Q3 = 26
\]

b) 0 4 5 10 16 17 18 18 21 22 22

\[
Q1 = 5 \\
M = 17 \\
Q3 = 21
\]
Interquartile range. SAY: So far in this lesson, we have found the median, and first and third quartiles of sets. Write the set in the margin and have students help you find and label the median, Q1, and Q3 as shown. Draw the line from Q1 to Q3. SAY: This distance between Q1 and Q3 is the interquartile range. Write the top equation.

\[ \text{interquartile range} = Q_3 - Q_1 \]

\[ \text{IQR} = 18 - 4 \]

\[ \text{IQR} = 14 \]

SAY: We find the value of the interquartile range by subtracting Q1 from Q3. Have students help you fill it in and solve it as shown. It is the number we use to tell us how the data in the set is arranged around the median. The wider this distance is, the more widely spread out the data is around the median. Explain that it is like the range but, instead of showing the distance between the highest and lowest values in the set, it shows the distance between Q1 and Q3.

Exercises:
Find the median, Q1, Q3, and the interquartile range.
Set 1: 5 5 6 7 7 7 17 18 18
Set 2: 3 4 8 8 8 13 22 24 25
Bonus: 167 245 267 379 381 604 899 907 987 1,291 1,314
Answers:
Set 1: Q1 = 6, M = 7, Q3 = 17, IQR = 11; Set 2: Q1 = 6, M = 8, Q3 = 23, IQR = 17; Bonus: Q1 = 267, M = 604, Q3 = 987, IQR = 720

Finding the unknown when you know any two of IQR, Q1, and Q3.
Explain that if you know any two of the values for IQR, Q1, or Q3, you can find the third. Write the known and unknown values, and the equation for IQR in the margin on the board. ASK: Where in the equation do you put 45? (IQR) Write it in.

Exercises: Find the unknown value.
a) Q3 = 8, IQR = 5, Q1 = ? b) Q1 = 22, IQR = 16, Q3 = ?
Bonus: Q1 = 421, IQR = 579, Q3 = ?
Answers: a) Q1 = 3, b) Q3 = 38, Bonus: Q3 = 1,000

How we use the interquartile range. Draw the table.

<table>
<thead>
<tr>
<th>Group 1 Quiz Marks</th>
<th>1</th>
<th>11</th>
<th>13</th>
<th>14</th>
<th>16</th>
<th>17</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1 Median</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 1 Q1</td>
<td>12</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 1 Q3</td>
<td>18</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 1 IQR</td>
<td>6</td>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Explain that these are the marks two groups of students got on a quiz that was out of 20. SAY: We’ll use the interquartile ranges to help us understand how the two groups’ quiz marks compare with each other. ASK: Did the two groups get the same highest and lowest marks? (yes) Draw the table in the margin and have students help you complete it. Draw a number line as below. You will build the diagram gradually. Start by having volunteers plot the Group 1 data and mark in the median, Q1, and Q3.
Draw an undivided box around the entire set as shown above. SAY: This box shows the data as a whole set. Explain that you are going to divide the set into the lower and upper halves by drawing a line at the median. Draw it in, put a bracket over each half, and label the lower and upper halves.

ASK: Do the halves contain the same number of data points? (yes) How many? (4) SAY: So even though the two halves are different lengths they still each contain half of the data. In other words, we’re not dividing the set in half by length; we’re dividing it in half by number of data points. Explain that now you’re going to divide the lower and upper halves in half again at the quartiles. Draw in the vertical lines at the quartiles. SAY: I can tell whether I’ve divided the lower half in half correctly if the two sections have the same number of data points. Point to the section to the left of Q1. ASK: How many data points are in this section? (2) Repeat for the section between Q1 and M. Repeat these steps for dividing the upper half at Q3. ASK: Now, how do I find the interquartile range? (IQR = 18 – 12) What is the IQR? (6)

Repeat all these steps for Group 2. Start by drawing a number line right under, and vertically aligned with, that of Group 1. Then, ASK: Which interquartile range is smaller? (Group 1) SAY: That means Group 1’s marks are more closely grouped around the median and more like each other than Group 2’s marks.

**Exercises:** Graph the data. Find the median, Q1, Q3, and IQR.

**Set 1:** 8  9  11  24  26  27  29  30

**Set 2:** 10  20  20  30  40  50  90  100  100  110  130

**Answers:** Set 1: Q1 = 10, M = 25, Q3 = 28, IQR = 18; Set 2: Q1 = 20, M = 50, Q3 = 100, IQR = 80
Extensions

1. Use the dot plot in the margin to explain how a set of data can have a median that is equal to Q1. (Hint: Write the data as a list.)

2. For extra practice, have students complete BLM Comparing IQRs (p. R-24).

3. For extra practice, teach the following lesson using BLM Grade 6 Math Test Dot Plots (p. R-25).

Project BLM Grade 6 Math Test Dot Plots, covering up the two tables on the right side. Explain that the dot plots represent the marks that two classes got on a Grade 6 math test. One is for a Grade 7 class and the other is for a Grade 9 class. Have students help you fill in the table at the bottom of the BLM. This involves finding the median. Have them help you make lists of the data in the dot plots, then use the lists to find the median. (90)

<table>
<thead>
<tr>
<th></th>
<th>Number of Students</th>
<th>Median</th>
<th>Range</th>
<th>Interquartile Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dot Plot 1</td>
<td>23</td>
<td>90</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Dot Plot 2</td>
<td>23</td>
<td>90</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

ASK: Since the numbers in the table for the two dot plots are identical, can the table alone show how the sets are different? (no) SAY: We need another way of describing the data sets that will show us the difference between a Grade 7 class and a Grade 9 class. We’ll do this by finding the interquartile range. Circle “90” on both number lines and label it “median.” ASK: In which dot plot are the data points more widely spread out around the median? (Dot Plot 1)

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q3</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dot Plot 1</td>
<td>75</td>
<td>95</td>
<td>20</td>
</tr>
<tr>
<td>Dot Plot 2</td>
<td>85</td>
<td>95</td>
<td>10</td>
</tr>
</tbody>
</table>

Hand out BLM Grade 6 Math Test Dot Plots. Have students find Q1 and Q3 for both dot plots by using the two lists of test marks. ASK: What is the interquartile range for Dot Plot 1? (20) What is the interquartile range for Dot Plot 2? (10) Which dot plot has the larger interquartile range? (Dot Plot 1) SAY: The data set with the larger IQR is more widely spread out around the median. ASK: Which one is that? (Dot Plot 1) Write the two IQRs in the original table. ASK: Now if we could see only the table, not the dot plots, what more would we understand about how the dot plots are different? (the data points in Dot Plot 1 are more spread out around the median than in Dot Plot 2) Explain that, overall, the marks for the Grade 9 class should be more alike and probably near the high end because they know more math, which means Dot Plot 2 represents the Grade 9 class’s marks.
Introduce the idea of comparing sets of data. Remind students that in the last lesson, they learned how to find the interquartile range of a set. They also learned how to compare interquartile ranges from different sets to learn more about how the sets are different from each other. SAY: But we often need to compare more than just the interquartile range. One way to do that is by using box plots.

**Box plots.** SAY: Box plots are diagrams that summarize five important features of data sets and give us information about the variability in a set. Write the set in the margin and draw only a number line with a horizontal line above it, as shown below.

```
0 1 3 4 6 7 9 19 20
```

Have volunteers plot the data and label Q1, M, and Q3. SAY: We’re going to divide this set up the way we did in the last lesson. Draw the box around the set and divide it up by drawing vertical lines at Q1, M, and Q3. Tell students that box plots look a lot like these diagrams. Draw the actual box plot above the diagram you just drew. SAY: In box plots, the box part doesn’t go around the entire set, it only goes around the interquartile range. It extends from Q1 to Q3, and gives us information about the arrangement of the data around the center. Point to the horizontal lines that extend from Q1 to the lowest value, and from Q3 to the highest value. SAY: Since the highest and lowest values are important to know, we show them with these two horizontal lines. One is from Q1 to the lowest value in the set and the other is from Q3 to the highest value. Point out that the ends of these lines are precisely where the highest and lowest data values are.
Finding the median from a dot plot. Remind students that they can find the median of a dot plot by writing the dots as a list and counting to the middle. Draw the dot plot shown in the margin. Have students find the median. (2)

But why use box plots when sets are easy to draw? Explain that with very small sets like the one you just looked at, it’s hard to understand how a box plot is any more convenient than just plotting the data itself. Project BLM Box Plots for Large Data Sets.

Tell students that you’re going to need their help to create a box plot for this data set. Ask them to count the data points. (31) You are going to ask students to help you find the median, Q1, and Q3. If you think they need to see the data as a list first, have them help you make one. Otherwise, ASK: If there are 31 data points, where is the median? (16th data point) Mark it as shown. ASK: Where is Q1? (8th data point) Have students find it and tell you its value. (12) Mark it. ASK: Where is Q3? (the center of the upper half) What is Q3? (16) Mark it. ASK: What is the lowest value? (0) Mark it. ASK: What is the highest value? (17) Mark it. Draw in the vertical dotted lines. Explain that you are drawing them so you can draw the box plot in the right position above the dot plot. Start by drawing the box that extends from Q1 to Q3. Draw the median, the line from Q1 to the lowest value, and finally, the line from Q3 to the highest value. ASK: If you just look at the box plot, not the dot plot, can you tell where all the individual data points are? (no) SAY: That’s not a problem because we use box plots to summarize some of the most important features of data sets, not to show all the data as a dot plot would. Have students complete Questions 1 and 4 on AP Book 6.2 pp. 162–163.

Extra Practice: BLM Reading Box Plots

Extra Practice: Have students complete BLM Matching Dot Plots with Box Plots.

Extension

Create two different sets of data. Both should have seven data points, lowest value $= 4$, highest value $= 13$, median $= 10$, and IQR $= 3$. Use the diagram below as a guide.

Sample Answer

| 4 | 7 | 8 | 10 | 10 | 10 | 13; |
| 4 | 9 | 9 | 10 | 11 | 12 | 13 |

R-16
Introduce distributions. Note: The concept of a distribution is very advanced. In this lesson, the word “distribution” is used to mean the overall shape of the data set or the overall arrangement of its points. 

SAY: We normally describe things by using numbers and words. For example, we describe a person’s height, age, and weight using numbers, but we don’t use numbers to describe their personality. For personality, we use words such as “friendly” or “honest.” It would make no sense to describe a person’s personality as, for example, “67.” This is also true for sets of data. So far, we have mostly used numbers such as the mean, median, range, quartiles, mean absolute deviation, and interquartile range to describe sets of data. But when we describe the overall shape of a set, or the overall arrangement of its points, we are talking about its distribution. Since the results of statistical questions vary a lot, the shapes of those results also vary when graphed. The distribution of a set is the arrangement of the results of a statistical question. We do not use numbers to describe the distribution of a set. Draw the dot plot in the margin. Leave it on the board. ASK: What word do we use to describe this shape? (symmetrical) “Symmetrical” is an example of the kinds of words we use to describe distributions of data sets.

Common shapes and features of distributions. Explain to students that they have already worked with two common shapes of distributions, one of which is called symmetrical. Draw the dot plot and the histogram below.

Leave them on the board. SAY: Sets of data graphed as dot plots or histograms that lean to one side or the other, and are not symmetrical, are called skewed.
Draw the dot plot and the histogram below.

SAY: Sometimes skewed distributions have extreme values that are far from the rest of the data. We call these extreme values outliers. Point to the dot at 16 and explain that it is an outlier. SAY: As we learned earlier, outliers such as this one cause the mean to change but not the median to change. Unlike in the other dot plots and histogram we just looked at, this dot plot has a space in it where there are no data points at all. We call a space between points, like this one, a gap. Sets of data can have one or more gaps.

Draw the two dot plots below side by side.

Point to the dot plot with two peaks. SAY: Some sets of data have one peak around which the data points are clustered. Some sets have more than one peak, and others have none. One of these dot plots has two peaks and the other has none. ASK: Which is which? (the one on the right has two peaks, the one on the left has no peaks)

Exercises: Have students complete Questions 3 and 5 on AP Book 6.2 pp. 165 and 166.

(MP2, MP6) Exercise: Make up a story problem where the numbers 6, 10, 8, 1, 8, 9, 7 form the data set. Explain the term “interquartile range” using this example.

Sample answer: The heights of plants in centimeters were 6, 10, 8, 1, 8, 9, 7. Putting the data set in order gives: 1, 6, 7, 8, 8, 9, 10. The median, or central value, is 8, which divides the data set into an upper part (8, 9, 10) and lower part (1, 6, 7). The first quartile or Q1 is the median of the lower part (6), the third quartile or Q3 is the median of the upper part (9), and the interquartile range or IQR is the difference between the first and third quartiles. In this example, IQR = Q3 − Q1 = 9 − 6 = 3.
Frequency

Complete the frequency table. Find the total number of data points in the set.

a)  

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>40</td>
</tr>
<tr>
<td>3–5</td>
<td></td>
</tr>
<tr>
<td>6–8</td>
<td></td>
</tr>
<tr>
<td>9–11</td>
<td>0</td>
</tr>
<tr>
<td>12–14</td>
<td></td>
</tr>
</tbody>
</table>

\[ \_ + \_ + \_ + \_ + \_ = \_ \]

b)  

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–4</td>
<td></td>
</tr>
<tr>
<td>5–9</td>
<td></td>
</tr>
<tr>
<td>10–14</td>
<td></td>
</tr>
<tr>
<td>15–19</td>
<td></td>
</tr>
<tr>
<td>20–24</td>
<td></td>
</tr>
<tr>
<td>25–29</td>
<td></td>
</tr>
</tbody>
</table>

\[ \_ + \_ + \_ + \_ + \_ + \_ = \_ \]
Interpreting Histograms

Length of Time People Were Sick with a Cold

<table>
<thead>
<tr>
<th>Interval</th>
<th>0–1</th>
<th>2–3</th>
<th>4–5</th>
<th>6–7</th>
<th>8–9</th>
<th>10–11</th>
<th>12–13</th>
<th>14–15</th>
<th>16–17</th>
<th>18–19</th>
<th>20–21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Complete the frequency table.
b) The longest time interval during which people had a cold was ____ – ____ days.
c) The largest number of people had a cold in the interval ____ – ____ days.
d) How many people had a cold for less than 8 days? _______
e) How many people had a cold for 12 or more days? _______
f) How many people altogether are represented by the data in this histogram? _______
g) The interval 16–17 represents data for four people. Below are four sets of data that could possibly represent how long those four people had colds. Put a check mark in the box beside the sets that correctly represent the data in the interval 16–17.

<table>
<thead>
<tr>
<th>Number of Days with a Cold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
</tr>
<tr>
<td>Set 2</td>
</tr>
<tr>
<td>Set 3</td>
</tr>
<tr>
<td>Set 4</td>
</tr>
</tbody>
</table>
## Drawing Histograms

1. Three hundred drivers recorded how many whole tanks of gas they used in November. The results are shown in a frequency table. Use the data to draw a histogram.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–1</td>
<td>20</td>
</tr>
<tr>
<td>2–3</td>
<td>80</td>
</tr>
<tr>
<td>4–5</td>
<td>110</td>
</tr>
<tr>
<td>6–7</td>
<td>50</td>
</tr>
<tr>
<td>8–9</td>
<td>30</td>
</tr>
<tr>
<td>10–11</td>
<td>10</td>
</tr>
</tbody>
</table>

![Histogram](chart.png)

2. Complete the frequency table. Graph the histogram.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Data Points</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–9</td>
<td>0, 1, 1, 3, 4, 5, 7, 8, 9, 9, 9</td>
<td>12</td>
</tr>
<tr>
<td>10–19</td>
<td>11, 11, 11, 12, 14, 14, 14, 14, 15, 17, 18, 18, 19, 19</td>
<td></td>
</tr>
<tr>
<td>20–29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30–39</td>
<td>31, 33, 33, 34, 35, 37, 38, 38, 38, 39, 39, 39, 39</td>
<td></td>
</tr>
<tr>
<td>40–49</td>
<td>43, 44, 45, 45, 45, 45, 45, 46, 49</td>
<td></td>
</tr>
<tr>
<td>50–59</td>
<td>52, 55, 56, 56, 58, 59</td>
<td></td>
</tr>
</tbody>
</table>

![Histogram](chart.png)

3. Put the data into the table. Graph the histogram.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>70–79</td>
<td>III</td>
<td></td>
</tr>
<tr>
<td>80–89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90–99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From Data to Dot Plot to Histogram

1. Plot the data points on the dot plot. Cross them off as you plot them.

<table>
<thead>
<tr>
<th>Interval</th>
<th>0–10</th>
<th>10–20</th>
<th>20–30</th>
<th>30–40</th>
<th>40–50</th>
<th>50–60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Points</td>
<td><img src="image" alt="Data Points" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Data Points</td>
<td><img src="image" alt="Number of Data Points" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Plot “Number of Data Points” from the table on the histogram. Cross them off as you plot them.
Five-Part Spinner

1. Spin 10 times and record the results in the table called “Data Set 1.”

2. Spin 10 times and record the results in the table called “Data Set 2.”

3. Make a dot plot for Data Set 1.

4. Make a dot plot for Data Set 2.

Data Set 1

<table>
<thead>
<tr>
<th>Spin Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin Result</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data Set 2

<table>
<thead>
<tr>
<th>Spin Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin Result</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Comparing IQRs

1. First, plot all four sets of data below in dot plots and try to rank them from highest to lowest IQR just by looking at them. Hint: A low IQR means the data points are more tightly clustered around the median.

(highest IQR) __________ __________ __________ __________ (lowest IQR)

2. Use the sets of data to find the median, Q1, and Q3. Calculate the IQR. Do your answers match the order in which you ranked the IQRs in part 1?

Set 1 0 4 10 11 17 18 20 20

\[ \text{IQR} = Q_3 - Q_1 \]

Set 2 0 2 10 11 17 16 18 20

\[ \text{IQR} = Q_3 - Q_1 \]

Set 3 0 6 16 17 17 17 19 20

\[ \text{IQR} = Q_3 - Q_1 \]

Set 4 0 1 3 16 18 20 20 20

\[ \text{IQR} = Q_3 - Q_1 \]
Grade 6 Math Test Dot Plots

Dot Plot 1:
Grade _____ Marks on a Grade 6 Math Test

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Q1</th>
<th>Q3</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dot Plot 2:
Grade _____ Marks on a Grade 6 Math Test

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Q1</th>
<th>Q3</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Data Points</th>
<th>Median</th>
<th>Range</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dot Plot 1: Grade _____</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dot Plot 2: Grade _____</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Box Plots for Large Data Sets
### Reading Box Plots

Use the box plots to complete the table.

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Q1</th>
<th>Q3</th>
<th>IQR (not part of Box Plots)</th>
<th>Highest Value</th>
<th>Lowest Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box Plot A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Box Plot B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Box Plot C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Box Plot D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Box Plot E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Matching Dot Plots with Box Plots

Match each dot plot with one box plot.

Received dot plots and box plots are not provided in this text. Please refer to the visual representations in the document for the matching task.
PS6-9 Choosing Between Strategies

Teach this lesson after: 6.2 Unit 9

Standards: 6.NS.A.1, 6.NS.B.2, 6.EE.B.5

Goals:
Students solve a variety of applied problems across the Grade 6 curriculum using a variety of problem-solving strategies, such as doing a systematic search, using structure, finding a pattern, using logical reasoning, and more.

Prior Knowledge Required:
Can write equivalent fractions
Can add and subtract two-digit numbers
Can multiply 2 two-digit numbers
Can use long division to divide up to four-digit numbers by one- or two-digit numbers
Can find the perimeter of a shape by adding the side lengths
Can represent fractions by using lengths and areas
Can add and subtract fractions and mixed numbers with unlike denominators
Can multiply fractions
Can add, subtract, and multiply decimals to hundredths
Can write an expression for a given term in a pattern
Can evaluate expressions at a given value for the variable

Vocabulary: altitude (of a triangle), average, constant speed, factor tree, guess-check-revise, power, prime factorization, ratio, reciprocal, remainder, search systematically, sequence, square, structure, term

Materials:
calculators
BLM Adding Reciprocals (pp. R-33–34)
BLM International Text Cost (pp. R-36–38, see Performance Task)
BLM Volume and Area of a Storage Unit (pp. R-40–42, see Performance Task)

Review problem-solving strategies. Remind students of the problem-solving strategies they have learned. You might ask students to suggest clues in a problem that would alert them to try a specific strategy. Post the strategies (along with any clues) on the board so students can refer to them while completing the Problem Bank questions.

Problem Bank
1. Anna, Bob, and Carl are running a 600 m race. They each run at their own constant speed throughout. Anna finishes 200 m ahead of Bob and 300 m ahead of Carl.
   a) When Anna finishes the 600 m race, how far will Bob have run? How far will Carl have run?
   b) When Bob finishes the 600 m race, how far will Carl have run in total?
Answers: a) Bob will have run 400 m, Carl will have run 300 m; b) 450 m
2. a) In how many ways can 5 people line up in a straight line?
b) In how many ways can 5 people sit at a round table? (Rotations are not counted as different, but reflections are.)

**Solutions:**
a) There are 5 possibilities for the first position. Once a person takes the first position, there are 4 possibilities for the second position, 3 possibilities for the third position, 2 possibilities for the fourth position, and just 1 possibility for the fifth position. \(5 \times 4 \times 3 \times 2 \times 1 = 120\) possibilities

b) The first person only has 1 possible position because every seat around the table is identical. Since the first position isn’t determined (like it is in a straight line), it isn’t until you place the first person that the other positions have significance. The second person can choose any of the remaining positions, so they have 4 possibilities. The third person has 3 possibilities, the fourth person has 2 possibilities, and the fifth person has just 1 possibility. \(1 \times 4 \times 3 \times 2 \times 1 = 24\) possibilities

3. How many decimals have 10 decimal places, with the 10th decimal place not zero, that round to 4.36?

**Solution:** Create a pattern by answering the same question for 3 decimal places, 4 decimal places, and so on:

- 3 decimal places: \(10 - 1 = 9\) (4.355, 4.356, ..., 4.364, but not 4.360 because the last decimal place is a zero)
- 4 decimal places: \(100 - 10 = 90\)
- 5 decimal places: \(1,000 - 100 = 900\)
- 10 decimal places: \(90,000,000\)

4. The shape below has 1 angle that is 270° and 5 angles that are 90° angles.

Another shape has 17 angles that are 270°. All the other angles are 90°. How many angles does the other shape have altogether?

**Solution:** Draw examples with different numbers of 90° angles and 270° angles. The number of 90° angles is always 4 more than the number of 270° angles. The other shape has 21 90° angles, so it has 17 + 21 = 39 angles altogether.

5. The third chapter in a book starts at page 561 and ends at page 802. Is it longer or shorter than the average of the first two chapters?

**Answer:** shorter

6. A triangle has three altitudes and sides \(x\), \(y\), and \(z\), with \(x < y < z\). If the shortest altitude is \(h\), what is the longest altitude? Hint: Find the area using two different sides and altitudes.

**Answer:** \(zh/x\)
7. A path alternates going north or south, and then east or west. It starts by going 1 unit east and then every time it changes direction, it goes 1 unit farther. Find the length of the shortest path that ends where it starts. Hint: Start by looking at the parts that go east and west. Then look separately at the paths that go north and south.

**Examples:**

**Sample solution:** The east and west lengths are odd numbers, 1, 3, 5, ..., and the north and south lengths are even numbers, 2, 4, 6, .... To make the east and west lengths cancel out using the shortest path possible, you need to use 1 and 7 in one direction (for example, east) and 3 and 5 in the other (for example, west). \((1 + 5 = 3 + 3)\) would be shorter, but you are not allowed to use two 3s.) To make the north and south lengths cancel out, you can use \(2 + 4 = 6\). So, the path has lengths of 1, 2, 3, 4, 5, 6, 7, going in the directions, for example: E, N, W, N, W, S, E. The length is \(1 + 2 + 3 + 4 + 5 + 6 + 7 = 28\).

8. Complete BLM Adding Reciprocals.

**Selected answers:**
1. a) more, 1, less; b) more, 3, less, 2, more; c) less, 4, less, 1, less; d) less, 70, 1, more, 30, 3, more, 70 + 30 = 100
2. b) less, 2/7, more, 2/5, more, more; c) less, 3/8, more, 3/5, more, more; d) less, 3/11, more, 3/8, more, more
3. a) 61/30 or 2 1/30, b) 74/35 or 2 4/35, c) 89/40 or 2 9/40, d) 185/88 or 2 9/88

**Bonus:**
\(b - a, x/b, x/a, \text{more}; \text{we are given } a < b \text{ so } x/a > x/b \text{ because a smaller denominator makes the fraction bigger since the numerators are the same}; \text{more than } 1 + 1 = 2 \text{ because the amount more than 1 is more than the amount less than 1}

9. Evaluate.

**a)** \(1 \frac{1}{2} \times 1 \frac{1}{3} \times 1 \frac{1}{4} \times 1 \frac{1}{5} \times 1 \frac{1}{6} \times 1 \frac{1}{7} \times 1 \frac{1}{8} \times 1 \frac{1}{9}\)

**Answers:** a) 10/2 or 5, b) 63/3 or 21

**MP.1** 10. Find the next two numbers in the sequence by first writing each number as a square.

\(4, 16, 36, 64, 100, 144, \ldots, \ldots\)

**Answer:** \(2^2, 4^2, 6^2, 8^2, 10^2, 12^2\). So, the next numbers are \(14^2 = 196\) and \(16^2 = 256\).

**MP.1** 11. Find the next two fractions in the sequence by first writing each fraction as the power of a quotient.

\(\frac{1}{4}, \frac{8}{27}, \frac{81}{256}, \ldots, \ldots\)

**Answer:** \((1/2)^2, (2/3)^3, (3/4)^4\), so the next two fractions are \((4/5)^5 = 1,024/3,125\) and \((5/6)^6 = 15,625/46,656\)
(MP.1) 12. Find a sequence of rectangles with a width of 5 cm. What is the ratio of length to area? Can you find a ratio between the length and the perimeter?

**Answer:** The area of a rectangle with a width of 5 cm is $5\ell$, where $\ell$ is the length of the rectangle in centimeters. The ratio of length to area is $1 : 5$. The perimeter in centimeters in this case will be $2\ell + 10$ and will not be proportional to the length.
Adding Reciprocals (1)

If a fraction \( \frac{a}{b} \) is less than 1, its reciprocal \( \frac{b}{a} \) is more than 1. How does the sum \( \frac{a}{b} + \frac{b}{a} \) compare to 2?

1. Start with a simpler problem: Without adding the whole numbers, decide whether the sum is more than 100 or less than 100.

   a) \( 38 + 61 \)
   
   38 is ______ than 40 by ______
   
   61 is ______ than 60 by ______
   
   So 38 + 61 is ______ than 40 + 60 = 100

   b) \( 53 + 48 \)
   
   53 is ______ than 50 by ______
   
   48 is ______ than 50 by ______
   
   So 53 + 48 is ______ than 50 + 50 = 100

   c) \( 76 + 19 \)
   
   76 is ______ than 80 by ______
   
   19 is ______ than 20 by ______
   
   So 76 + 19 is ______ than 80 + 20 = 100

   d) \( 69 + 33 \)
   
   69 is ______ than ______ by ______
   
   33 is ______ than ______ by ______
   
   So 69 + 33 is ______ than ______

2. Describe what worked for the simpler problems: How did you decide whether the sum is greater or less than 100 without calculating the sum?

   ____________________________________________
   ____________________________________________
   ____________________________________________

3. Use the method that worked for the simpler problem for the problem we are investigating.

   a) Is \( \frac{5}{6} + \frac{6}{5} \) more or less than 2?
   
   \( \frac{5}{6} \) is ______ than 1 by ______
   
   \( \frac{6}{5} \) is ______ than 1 by ______
   
   The amount more than 1 is ______
   
   So \( \frac{5}{6} + \frac{6}{5} \) is ______ than \( 1 + 1 = 2 \).

   b) Is \( \frac{5}{7} + \frac{7}{5} \) more or less than 2?
   
   \( \frac{5}{7} \) is ______ than 1 by ______
   
   \( \frac{7}{5} \) is ______ than 1 by ______
   
   The amount more than 1 is ______
   
   So \( \frac{5}{7} + \frac{7}{5} \) is ______ than \( 1 + 1 = 2 \).
Adding Reciprocals (2)

c) Is $\frac{5}{8} + \frac{8}{5}$ more or less than 2?

$\frac{5}{8}$ is ______ than 1 by ____

$\frac{8}{5}$ is ______ than 1 by ____

The amount more than 1 is __________

than the amount less than 1,

so $\frac{5}{8} + \frac{8}{5}$ is ______ than $1 + 1 = 2$.

d) Is $\frac{8}{11} + \frac{11}{8}$ more or less than 2?

$\frac{8}{11}$ is ______ than 1 by ____

$\frac{11}{8}$ is ______ than 1 by ____

The amount more than 1 is __________

than the amount less than 1,

so $\frac{8}{11} + \frac{11}{8}$ is ______ than $1 + 1 = 2$.

4. Check your answers to Question 3 by calculating the sums.

a) $\frac{5}{6} + \frac{6}{5}$

b) $\frac{5}{7} + \frac{7}{5}$

c) $\frac{5}{8} + \frac{8}{5}$

d) $\frac{8}{11} + \frac{11}{8}$

Bonus: Generalize the result for any $a < b$: What expression for $x$ works for both equations?

If $\frac{a}{b} + \frac{x}{b} = \frac{b}{b}$ and $\frac{a}{a} + \frac{x}{a} = \frac{b}{a}$, then $x =$ ________.

How much less than 1 is $\frac{a}{b}$? ______ How much more than 1 is $\frac{b}{a}$? ______

The amount more than 1 is ________ than the amount less than 1. How do you know?

Is $\frac{a}{b} + \frac{b}{a}$ more or less than $1 + 1 = 2$? Explain.
Performance Task: International Text Cost

Materials:
BLM International Text Cost (pp. R-36–38)

Performance Task: International Text Cost. Provide students with BLM International Text Cost. In this task, students plot data from a table of values and find the equation of a line representing the data. Students read and extrapolate values from the graph. Students extend a pattern in a table and make predictions about the sequence.

Answers:
1. a) (2, 0.40), (3, 0.60), (4, 0.80); b) teacher to check; c) dependent variable: cost (in dollars), independent variable: number of messages; d) \( y = 0.2x \); e) $0.2(3) = $0.60; f) 6 ÷ 0.2 = 30
2. a) 5.20, 5.30, 5.40; b) $5.00; c) 5.20, 5.30, 5.40, 5.50, 5.60, 5.70, 5.80, 5.90, 6.00, so it costs $6 to send 10 messages
3. a) Company A: $4.00, Company B: $7.00; b) Company A: $20.00, Company B: $15.00; Bonus: 50 text messages
International Text Cost (1)

1. Phone Company A charges $0.20 for each international text message.
   a) Complete the table of values for the cost of sending 2, 3, and 4 international text messages. Write a list of ordered pairs for the table.

<table>
<thead>
<tr>
<th>Number of Messages</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   (1, 0.20), (2, ), (3, ), (4, )

   b) Plot the ordered pairs from the table of values in part a) on the grid below.

   [Cost of International Text Messages diagram]

   c) What are the dependent and independent variables?
International Text Cost (2)

d) Write an equation to represent the cost of sending $x$ international text messages.

e) Substitute $x = 3$ in the equation to find the cost sending 3 international text messages.

f) Kevin has $6 in his wallet. Use the equation to find the number of international text messages he can send.

2. Phone Company B charges $5.00 per month, plus $0.10 for each international text message.

a) Complete the table of values for the cost of sending 2, 3, and 4 international text messages.

<table>
<thead>
<tr>
<th>Number of Messages</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.10</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

b) How much would you pay before you have even sent a message? ________

c) Write the cost of sending international text messages using company B in a sequence. Then, find the cost of sending 10 messages.

3. a) Kevin wants to send 20 international text messages. How much would he pay each company to send 20 messages?

b) How much would Kevin pay each company to send 100 messages?

Bonus ► For how many text messages is the cost the same for both companies?
Performance Task: Volume and Area of a Storage Unit

Materials:
BLM Volume and Area of a Storage Unit (pp. R-40–42)

Provide students with BLM Volume and Area of a Storage Unit. In this task, students calculate the dimensions of a storage unit, including the height, the area of the sides, and the volume. Students use a given painter’s rate to determine the cost of having the storage unit painted, and as a bonus, they compare the cost savings of painting the storage unit by calculating percentages of a budget.

Answers:
1. a) \(6 \frac{2}{3} + 1 \frac{1}{3} = 8\) yd, b) \(42 \frac{2}{3} \text{yd}^2\), c) \(2,560/9 \text{yd}^3\) or \(284 \frac{4}{9} \text{yd}^3\)
2. \(256/9 \text{yd}^3\) or \(28 \frac{4}{9} \text{yd}^3\)
3. \(312 \frac{8}{9} \text{yd}^3\)
4. \(177 \frac{7}{9} \text{yd}^2\)
5. \$2,800.00
Bonus: \$2,800.00 – \$2,100.00 = \$700.00
Volume and Area of a Storage Unit (1)

The storage unit shown has a rectangular base with width $5 \frac{1}{3}$ yd and length 8 yd. The height of the rectangular prism part is $6 \frac{2}{3}$ yd.

The storage unit has a slant roof with a height of $1 \frac{1}{3}$ yd.

1. a) What is the height of the highest part of the storage unit roof from the ground?

b) Find the area of the base of the storage unit.

c) Find the volume of the rectangular prism part of the storage unit.
Volume and Area of a Storage Unit (2)

2. John wants to find the volume of the top part of the silo under the slant roof.

John’s teacher suggests that he finds the volume of the rectangular prism in the second picture and then halve that to get the volume of the part under the slant roof. Use this method to find the volume of the part under the slant roof.

3. Find the total volume of the storage unit.

4. Find the total area of the four side faces of the rectangular prism part.
Volume and Area of a Storage Unit (3)

5. A painter charges $1.75 per square foot. How much will it cost to have all four side faces of the rectangular prism part painted? Hint: Convert all measurements to feet. One yard is 3 feet, so each square yard is 9 square feet.

Bonus ► The owner of the storage unit decides to paint the storage unit with his friends. He has $5,000 and spends 20% of it on paint and painting equipment, and 25% of the remaining money to rent scaffolds. He spends $100 on lunch for 2 days for his friends. How much does he save compared to hiring the painter in Question 5?
1 cm Grid Paper
**Answer Keys for AP Book 6.2**

**AP Book G6-20**

**Geometry: Coordinate Grids – AP Book 6.2: Unit 5**

1. Teacher to check.
2. Teacher to check.
3. a) Teacher to check.
   b) Teacher to check.
   c) Teacher to check.
   d) Teacher to check.
4. a) $27$ mm  $13$ mm
   b) Teacher to check.
   c) $B (2,0.5)$
   d) Teacher to check.
   e) $C (-0.8,1.0)$
   f) $D (-1.5,-0.8)$
   g) $E (-2.9,0.7)$
   h) $F (2.1,-1.6)$

5. a) $II$
   b) $IV$
   c) $III$
   d) $I$
   e) $IV$
   f) $II$
   g) $III$
   h) $I$

6. Teacher to check.
7. Answers may vary.
   Teacher to check.

**AP Book G6-21**

**Page 95**

1. Teacher to check.
2. a) $B (-5,-3)$
   b) $C \left( \frac{3}{2}, -\frac{1}{2} \right)$
   c) $D \left( 0, -\frac{3}{2} \right)$
   d) $E \left( \frac{3}{2}, \frac{1}{2} \right)$
   e) $F \left( \frac{1}{2}, 0 \right)$
   f) $G \left( 0, 0 \right)$
   g) $H \left( -\frac{1}{2}, \frac{3}{2} \right)$

3. b) $B \left( 1, \frac{1}{2} \right)$
   c) $C \left( -\frac{1}{4}, -1 \right)$
   d) $D \left( \frac{1}{2}, \frac{1}{2} \right)$
   e) $E \left( \frac{3}{4}, -\frac{3}{4} \right)$
   f) $F \left( -\frac{3}{4}, -\frac{1}{4} \right)$

**AP Book G6-22**

**Page 97**

1. Teacher to check points.
   a) $y$-axis
   b) $y$-axis
   c) $x$-axis

2. a) Vertical
   Teacher to check additional points.
   All points on line $PQ$ have same coordinate on $x$-axis (3).
   b) Teacher to check.

3. a) $|4| + |-2| = 6$ units
   b) $|4| + |0| = 4$ units
   c) $|1| + |1| = 2$ units
   d) $|3| + |1| = 4$ units
   e) $|-1| + |1| = 2$ units
   f) $|-4| - |-2| = 2$ units

4. a) 4

**AP Book G6-23**

**Page 98**

1. b) 5
2. Subtract the $x$-coordinates.
3. b) 3 units, because $6 - 3 = 3$
   c) 4 units, because $7 - 3 = 4$
   d) 5 units, because $6 - 1 = 5$

4. b) 7 units, because $5 + 2 = 7$
   c) 7 units, because $5 + 2 = 7$
   d) 5 units, because $4 + 1 = 5$
   e) 4 units, because $1 + 3 = 4$
   f) 9 units, because $7 + 2 = 9$

5. b) $|4| + |-2| = 6$ units
   c) $|1| + |1| = 2$ units
   d) $|3| + |1| = 4$ units
   e) $|-1| + |1| = 2$ units
   f) $|-4| - |-2| = 2$ units

6. a) 4
Geometry: Coordinate Grids – AP Book 6.2: Unit 5

AP Book G6-27
page 105
1. b) $P (-3, -2)$
   $P' (-3, 2)$
   c) $P (0, 2)$ $P' (0, -2)$
2. a) $P (3, -1) P' (3, 1)$
   $Q (1, 1) Q' (1, -1)$
   $R (-2, -2)$ $R' (-2, 2)$
   b) $P (2, 1) P' (2, -1)$
   $Q (-3, 1)$
   $Q' (-3, -1)$
   $R (0, 2) R' (0, -2)$
   c) $P (-2, 1)$
   $P' (-2, -1)$
   $Q (2, 3) Q' (2, -3)$
   $R (0, 1) R' (0, -1)$
3. The $y$-coordinate changes.
The $x$-coordinate stays the same.
4. $A' (-2, -1)$
   $B' (-3, 2)$
   $C' (0, 1)$
   $D' (2, -3)$
   $E' (3, 0)$

AP Book G6-25
page 102
1. a) $D (2, 1)$
   b) $Z (-5, 4)$
2. a) $(-2, 1), (1, 1)$
   $(-2, -3), (1, -3)$
   b) $(-2, 4), (-2, -1)$
   $(4, 4), (4, -1)$
3. a) $3, D (-2, -3)$
   b) $2, D (3, -3)$
   or $D (3, 1)$

AP Book G6-26
page 103
1. a) Teacher to check.
   b) Teacher to check.
   c) $(-1, 4)$ and $(-1, -1)$
   5 units
   d) Teacher to check line.
   distance = 5
2. a) 65
   b) 49.5
3. a) $BC$ and $AD$
   Height: 5
   b) $FE$ and $GH$
   Height: 5
4. a) $AB = 1$
   $CD = 1$
   Parallelogram
   Height: 2
   b) Parallel: $EF$ and $GH$
   $EF = 247$
   $GH = 155$
   $EFGH$ is a trapezoid
   Height: 487
   c) Parallel: $IL$ and $KJ$
   $IL = 2.46$
   $KJ = 6.83$
   $IJKL$ is a trapezoid
   Height: 5.61

BONUS
25.3

AP Book G6-27
page 105
1. b) $P (-3, -2)$
   $P' (-3, 2)$
   c) $P (0, 2)$ $P' (0, -2)$
2. a) $P (3, -1) P' (3, 1)$
   $Q (1, 1) Q' (1, -1)$
   $R (-2, -2)$ $R' (-2, 2)$
   b) $P (2, 1) P' (2, -1)$
   $Q (-3, 1)$
   $Q' (-3, -1)$
   $R (0, 2) R' (0, -2)$
   c) $P (-2, 1)$
   $P' (-2, -1)$
   $Q (2, 3) Q' (2, -3)$
   $R (0, 1) R' (0, -1)$
3. The $y$-coordinate changes.
The $x$-coordinate stays the same.
4. $A' (-2, -1)$
   $B' (-3, 2)$
   $C' (0, 1)$
   $D' (2, -3)$
   $E' (3, 0)$
5. b) \( P (-3, -2) \)
   \( P' (3, -2) \)
   c) \( P (2, 0) \) \( P' (-2, 0) \)

6. a) \( P (2, 3) \) \( P' (-2, 3) \)
   \( Q (3, 1) \) \( Q' (-3, 1) \)
   \( R (1, -2) \)
   \( R' (-1, -2) \)
   b) \( P (-3, 1) \) \( P' (3, 1) \)
   \( Q (-3, 3) \) \( Q' (3, 3) \)
   \( R (-1, -1) \)
   \( R' (1, -1) \)
   c) \( P (1, 3) \) \( P' (-1, 3) \)
   \( Q (-2, -2) \) \( Q' (2, -2) \)
   \( R (-3, 0) \)
   \( R' (3, 0) \)

7. The x-coordinate changes. The y-coordinate stays the same.

8. \( A' (2, 1) \)
   \( B' (3, -2) \)
   \( C' (0, -1) \)
   \( D' (-2, 3) \)
   \( E' (-3, 0) \)

9. a) \( (-3, 1) \) \( (3, 1) \)
    y-axis as mirror line, opposite x-coordinates.
   b) \( (-2, 2) \) \( (-2, -2) \)
    x-axis as mirror line, opposite y-coordinates

AP Book G6-28
page 107
1. a) Teacher to check.
   b) 2 squares, 200 m
   c) 2,700 m
   d) 2,700 m = 2.7 km
    \( 2 \text{ km} = 10 \text{ min} \)
    \( 2.7 \text{ km} = 13.5 \text{ min} \)
   e) \( 13.5 + (5 \times 3) \)
    \( = 13.5 + 15 \)
    \( = 28.5 \text{ min} \)
   f) 1 km²
   g) 10.7 km²

2. a) Teacher to check.
   b) \( 11.7 + 3 = 3.9 \)
   c) Teacher to check.
   d) \( (-2, 3.1) \) \( (1, 3.1) \)
   e) \( 3 + 3 + 3.9 + 3.9 \)
    \( = 13.8 \text{ km} \)
   f) 1 km²
   g) \( 10.7 \text{ km}² \)

3. a) i) Bear Cave
    ii) Treasure
    iii) Clear Spring
    iv) Fang Cliff
   b) i) \( (2, -3) \)
    ii) \( (0, 0) \)
    iii) \( (3, -0.5) \)
   c) i) Swamp
    ii) Treasure
    iii) Fang Cliff
   d) i) 3.5 km
    ii) 0.5 km

BONUS
Ear Wood

AP Book G6-29
page 108

1. a) Teacher to check.
   b) 2 squares, 200 m
   c) 2,700 m
   d) 2,700 m = 2.7 km
    \( 2 \text{ km} = 10 \text{ min} \)
    \( 2.7 \text{ km} = 13.5 \text{ min} \)
   e) \( 13.5 + (5 \times 3) \)
    \( = 13.5 + 15 \)
    \( = 28.5 \text{ min} \)
   f) 1 km²
   g) \( 10.7 \text{ km}² \)

2. a) Teacher to check.
   b) \( 11.7 + 3 = 3.9 \)
   c) Teacher to check.
   d) \( (-2, 3.1) \) \( (1, 3.1) \)
   e) \( 3 + 3 + 3.9 + 3.9 \)
    \( = 13.8 \text{ km} \)
   f) 1 km²
   g) \( 10.7 \text{ km}² \)

3. a) i) Bear Cave
    ii) Treasure
    iii) Clear Spring
    iv) Fang Cliff
   b) i) \( (2, -3) \)
    ii) \( (0, 0) \)
    iii) \( (3, -0.5) \)
   c) i) Swamp
    ii) Treasure
    iii) Fang Cliff
   d) i) 3.5 km
    ii) 0.5 km

BONUS
Ear Wood
Expressions and Equations: Algebraic Equations – AP Book 6.2: Unit 6

### AP Book EE6-16

**page 110**

1. a) \(3: 3(3) = 9\)
   \[3 + 3 + 3 = 9\]
   \(4: 3(4) = 12\)
   \[2 \times (3 + 1) = 8\]
   \[2 \times (4 + 1) = 10\]
   Yes.

   b) \(2: 2(2) + 2 = 6\)
   \[2 \times (2 + 1) = 6\]
   \(3: 2(3) + 2 = 8\)
   \[2 \times (3 + 1) = 8\]
   \(4: 2(4) + 2 = 10\)
   \[2 \times (4 + 1) = 10\]
   Yes.

2. a) \(1 \times 3\)
   \(2 \times 3\)
   \(3 \times 3\)
   \(7 \times 3\)
   \(n \times 3\)

   b) \(2 + 2\)
   \(2 + 3\)
   \(2 + 9\)
   \(2 + n\)
   \(3 \times 2\)
   \(3 \times 3\)
   \(3 \times 8\)
   \(3 \times n\)

   c) \(2\)
   \(3\)
   \(4\)
   \(n\)

   b) \(8x\)
   \(c) 8x\)
   \(d) 11x\)
   \(e) 20x\)

BONUS

\(11x\)

4. b) \(5x = 15\)
   \(x = 3\)
   \(7x = 28\)
   \(x = 4\)
   \(9x = 18\)
   \(x = 2\)
   \(11x = 22\)
   \(x = 2\)

BONUS

\(7x = 0\)
   \(x = 0\)

5. a) \(0\)
   \(b) 0\)
   \(c) 0\)
   \(d) 0\)
   \(e) 0\)
   \(f) 0\)
   \(g) 0\)
   \(h) 0\)
   \(i) 0\)
   \(j) 0\)
   \(k) \frac{3}{5}\)
   \(l) x\)
   \(m) \frac{1}{4}\)
   \(n) 7\)

6. a) \(3 \times (x + 2)\)

### AP Book EE6-17

**page 112**

1. Teacher to check circles.

   b) \(0.5 \times 11\)
   \(c) 2 \times 5\)
   \(d) -6.1 \times \frac{3}{4}\)

2. \(6x = 2x + 4x\)
   \(6x = 1x + 5x\)

   Yes.

3. b) \(8x\)
   \(c) 8x\)
   \(d) 11x\)
   \(e) 20x\)

BONUS

\(11x\)

5. a) \(0\)
   \(b) 0\)
   \(c) 0\)
   \(d) 0\)
   \(e) 0\)
   \(f) 0\)
   \(g) 0\)
   \(h) 0\)
   \(i) 0\)
   \(j) 0\)
   \(k) \frac{3}{5}\)
   \(l) x\)
   \(m) \frac{1}{4}\)
   \(n) 7\)

6. a) \(3 \times (x + 2)\)

### AP Book EE6-18

**page 114**

1. \(x + 2\)
   \(x + 3\)
   \(x - 2\)
   \(4x\)
   \(x - 3\)

   b) \(x - 8.5\)
   \(c) x + 8\)
   \(d) x - 2\)
   \(e) x + 2.9\)
   \(f) x - 4\)
   \(g) 5x\)
   \(h) x + 6\)
   \(i) 7x\)
   \(j) 2x\)
   \(k) x + 4.7\)
   \(l) 3.3x\)

3. a) \(x + 4 = 18\)
   \(b) x = 6\)
   \(c) 5x = 30\)
   \(d) 6x = 42\)
   \(e) 6x = 12.4\)
   \(f) x = 17.4\)
   \(g) 6x = 14\)
   \(h) 2x = 18\)
   \(i) x = 9\)
   \(j) 2x = 9\)
   \(k) 3x = 18\)
   \(l) x = 6\)

   b) \(x = 3\)
   \(c) 5x = 2\)
   \(d) 6x = 7\)
   \(e) 6x = 1.5\)
   \(f) 5x = 2\)
   \(g) 2x = 2\)
   \(h) 3x = 18\)
   \(i) x = 6\)
   \(j) 2x = 10.6\)
   \(k) 3x = 3.6\)
   \(l) x = 1.2\)
Expressions and Equations:
Algebraic Equations – AP Book 6.2: Unit 6

BONUS

1. a) \( \frac{1}{2} x = 4 + 1 \)
\( \frac{1}{2} x = 5 \)
\( x = 10 \)

4. a) i) \( x + x + x + x = 4x \)
ii) \( x + x + x + x + x = 5x \)
iii) \( \frac{x}{3} + \frac{x}{3} + \frac{x}{3} + \frac{x}{3} = 5x \)

b) i) \( x = 4 \)
ii) \( x = 3 \)
iii) \( x = 2.4 \)

5. a) \( 2x = 24 \)
\( x = 12 \)

b) \( 4x = 16 \)
\( x = 4 \)

c) \( 2(2x) + 2x = 72 \)
\( 6x = 72 \)
\( x = 12 \)

6. a) \( x = \) Mark
\( 3x = x + 24 \)
\( 2x = 24 \)
\( x = 12 \)

Mark is 12 years old.

7. \( 42 + 6 = x \)
\( x = 7 \)

7 days.

8. \( x + 2x = 45 \)
\( 3x = 45 \)
\( x = 15 \)
\( 2x = 30 \)

The numbers are 15 and 30.

9. \( x + 5x = 42 \)
\( 6x = 42 \)
\( x = 7 \)

Chandra is 35 and Rita is 7.

BONUS

2x + x = 51
3x = 51
\( x = 17 \)

2x = 34
Regular price is $34.

Answer Keys for AP Book 6.2
6. b) \( w > 50 \)
   c) \( w > -5 \)
   d) \( w < 5 \)

7. b) \( x < $7.50 \) and \( x > 0 \)
   c) \( x < 12 \)
   d) \( x < 17 \) and \( x > 0 \)

8. Teacher to check.

BONUS

\( x > $37 \)

(Teacher to check number line.)
Statistics and Probability:
Mean and Dot Plots – AP Book 6.2: Unit 7

Answer Keys for AP Book 6.2

1. a) 3 
b) 2 
c) 2 
d) 3 
e) 4 
f) 2 
2. a) 3 
b) 4 
c) 4 
3. a) i) Mean: 4 
ii) Beads: 12 
Stacks: 4 
Mean: 3 
b) No, she would not want the teacher to count all 4 because it would decrease her mean quiz mark. 
4. a) i) Beads: 15 
Stacks: 5 
Mean: 3 
ii) Beads: 10 
Stacks: 5 
Mean: 2 
iii) Beads: 12 
Stacks: 6 
Mean: 2 
b) i) 15 + 5 = 3 
ii) 10 + 5 = 2 
iii) 12 + 6 = 2 
5. 
<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Stacks</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>30</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>c</td>
<td>45</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>d</td>
<td>40</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>e</td>
<td>1,000</td>
<td>4</td>
<td>250</td>
</tr>
<tr>
<td>f</td>
<td>3,000</td>
<td>3</td>
<td>1,000</td>
</tr>
</tbody>
</table>
6. Teacher to check lines. 
   b) 12 + 3 = 4 
c) 16 + 4 = 4 
7. a) ii) 4 below mean 
   4 above mean 
   iii) 5 below mean 
   5 above mean 
   b) Same number of spaces below and above mean. 
c) i) Low 
   ii) Low 
   iii) High 
   8. a) ii) Mean: 5 
   Set: 6, 10, 13, 7 
   New Mean: 9 
   iii) Mean: 2 
   Set: 4, 5, 5, 8, 9, 5 
   New Mean: 6 
b) 4 
9. a) Beluga Calf 
Mean: 180 
185 is greater than the mean 
Seal Pup 
Mean: 25 
25 is equal to the mean 
   b) The 200 pound elephant 
   i) Tap | Bottle |
   ii) 15¢ | $300 |
   iii) 20¢ | $400 |
   b) ii) $300 − 15¢ = $299.85 
   iii) $400 − 20¢ = $399.80 
10. a) Nathan: 5 
   Shyla: 5 
   b) They are the same. 
   c) Teacher to check, answers may vary. 
   d) No. 
   BONUS 
   a) Mean: 41 
   Median: 3 
   b) It would be difficult to find the mean of this data using beads because of the large number (231) of beads that would have to be physically moved to find the mean. 
   4. a) Highest: 477 
   Lowest: 44 
   Range: 433 
   b) Highest: 632 
   Lowest: 236 
   Range: 396 
   5. a) 8 
   Highest: 8 
   Lowest: 2 
   Range 6 
   b) 39 
   Highest: 39 
   Lowest: 16 
   Range: 23 
   6. Teacher to check, explanations may vary. 
   Higher range in all students versus only kindergarten, because all students would include a higher ‘highest’ and lower ‘lowest’ than in one grade, producing a greater range. 
   BONUS 
   Teacher to check. 
   AP Book SP6-3 
   page 128 
   1. a) Teacher to check. 
   b) i) 7 
   ii) 11 
   c) i) 7 
   ii) 11 
   d) i) 7 
   ii) 11 
   2. a) 18 
   b) 0, 3, 3, 4, 4, 4 
   c) 1 × 0 
   2 × 3 
   3 × 4 
   = 0 + 6 + 12 
   = 18 
   d) Yes. 
   e) Mean = 18 + 6 
   = 3 
   3. a) Mean: 3 
   b) Mean: 5 
   BONUS 
   Mean: 50 
   AP Book SP6-4 
   page 129 
   1. a) 6 
   b) 3
Statistics and Probability:
Mean and Dot Plots – AP Book 6.2: Unit 7

(continued)

c) 3
d) 30
e) 7

2. a) 1 2 3 4
   b) Teacher to check.
c) 1
d) 10

3. Graph B, because it
does not have a number
line.

4. a)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

b) Teacher to check.
c) Teacher to check.

5.

<table>
<thead>
<tr>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

6. Teacher to check.

7. a) Yes
   b) 14
   c) Mean: 14
      Median: 14
   d) The mean, median, and center are all
      the same in this data
      set.

8. a)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>MD</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>S</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>E</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

b) In symmetrical
   graphs, the mean
   and median are
   the same.

9. a) No, because the
    mean and median
    for B is greater
    than A, so they will
    not be the same.
b) Yes, because it is
   symmetrical.

c) No, because the
   mean and median
   for D is greater
   than C, so they will
   not be the same.

10. Mean of Graph A: 4
    Mean of Graph B: 40
    Mean of Graph C: 10
    Mean of Graph D: 9
1. b) $4 + 4 = 8$
   $2 \times 4 = 8$
   $4 + 4 + 4 + 4 + 4 = 20$
   $5 \times 4 = 20$

c) $4 + 4 + 4 + 4 + 4 = 20$
   $5 \times 4 = 20$

d) $4 + 4 + 4 + 4 + 4 = 20$
   $5 \times 4 = 20$

2. a) $3$
   
   b) $3$
   
   c) $3$

3. a) $4 + 4 + 4 = 12$
   
   b) $3 \times 4 = 12$

4. a) $6$
   
   b) $6 + 6 + 6 + 6 = 24$
   
   c) $6 \times 4 = 24$

5. a) $2 + 2 + 2 = 6$
   $2 \times 3 = 6$
   $10 + 10 + 10 + 10 = 40$
   $10 \times 4 = 40$
   
   b) $12 + 12 + 12 = 36$
   $12 \times 3 = 36$

6. a) $3 \times 2 = 6$ blocks
   
   b) i) $3 \times 2 \times 2 = 12$
   ii) $3 \times 2 \times 3 = 18$
   iii) $3 \times 2 \times 4 = 24$

7. a) $3 \times 4 \times 2 = 24$
   
   b) $4 \times 5 \times 3 = 60$
   
   c) $4 \times 5 \times 2 = 40$

8. a) $3 \times 2 \times 4 = 24$
   
   b) $4 \times 2 = 8$
   $4 \times 2 \times 3 = 24$
   
   c) $4 \times 3 = 12$
   $4 \times 3 \times 2 = 24$

AP Book G6-31

page 134

1. a) $3$
   
   b) $11$
   
   c) $12$

2. b) $8$ cm
   
   c) $7$ mm
   
   d) $24$ ft
   
   e) $12$ in
   
   f) $36$ m

AP Book G6-32

page 136

1. a) $l = 2$
   $w = 2$
   $h = 2$
   
   b) $8$
   
   c) $1$ in

2. $8$ cm

AP Book G6-33

page 138

1. Teacher to check shading.
   
2. $8$ cm

Answer Keys for AP Book 6.2
4. a) Triangular pyramid
   \[
   \begin{array}{ccc}
   V & E & F \\
   6 & 9 & 5 \\
   
   \end{array}
   \]
4. Teacher to check.
4. Teacher to check markings.
4. Teacher to check.
5. a) Back: 6 cm²
   Bottom: 12 cm²
   Left side: 8 cm²
   b) Back: 15 cm²
   Bottom: 6 cm²
   Right side: 10 cm²
   c) Back: 12 m²
   Bottom: 18 m²
   Left side: 6 m²
6. b) Top + Bottom: 2 × 15 cm² = 30 cm²
   Front + Back: 2 × 6 cm² = 12 cm²
   Right + Left Sides: 2 × 10 cm² = 20 cm²
   c) i) Top + Bottom: 15 cm² × 2 = 30 cm²
   Right + Left: 10 cm² × 2 = 20 cm²
   SA: 12 cm² + 30 cm² + 20 cm² = 62 cm²
   ii) Front + Back: 12 cm² × 2 = 24 cm²
   Top + Bottom: 16 cm² × 2 = 32 cm²
   Right + Left: 12 cm² × 2 = 24 cm²
   SA: 24 cm² + 32 cm² + 24 cm² = 80 cm²

AP Book G6-38
page 150
1. b) Teacher to check.
2. a) Front: 2 × 3 = 6 cm²
   Back: 2 × 3 = 6 cm²
   Right side: 1 × 2 = 2 cm²
   Left side: 1 × 2 = 2 cm²
   Top: 1 × 3 = 3 cm²
   Bottom: 1 × 3 = 3 cm²
   SA: 22 cm²
   b) Front: 1 × 4 = 4 cm²
   Back: 1 × 4 = 4 cm²
   Right side: 1 × 2 = 2 cm²
   Left side: 1 × 2 = 2 cm²
   Top: 2 × 4 = 8 cm²
   Bottom: 2 × 4 = 8 cm²
   SA: 28 cm²
3. Teacher to check.
4. a) Teacher to check.
   b) i) Top: 12 cm²
      Back: 3 cm²
      Front: 3 cm²
      Right: 4 cm²
      Left: 4 cm²
      ii) Top: 12 cm²
         Back: 3 cm²
         Front: 3 cm²
         Right: 4 cm²
         Left: 4 cm²

AP Book G6-39
page 144
1. Teacher to check.
2. Teacher to check.
3. Teacher to check.
4. Teacher to check.
5. Teacher to check.
6. B D A C
7. Teacher to check markings.
8. Teacher to check.
9. Teacher to check.
10. Teacher to check markings.
11. Rectangular prism
12. Rectangular prism
13. Triangular pyramid
14. Triangular pyramid
15. Cube
16. Square pyramid
17. Cube
18. Square pyramid
20. Teacher to check markings.
21. Rectangular prism
22. Rectangular prism
23. Triangular pyramid
24. Triangular pyramid
25. Cube
26. Square pyramid
27. Cube
28. Square pyramid
29. B D A C
30. Teacher to check markings.
31. Rectangular prism
32. Rectangular prism
33. Triangular pyramid
34. Triangular pyramid
35. Cube
36. Square pyramid
37. Cube
38. Square pyramid
39. B D A C
40. Teacher to check markings.
41. Rectangular prism
42. Rectangular prism
43. Triangular pyramid
44. Triangular pyramid
45. Cube
46. Square pyramid
47. Cube
48. Square pyramid
49. B D A C
50. Teacher to check markings.
51. Rectangular prism
52. Rectangular prism
53. Triangular pyramid
54. Triangular pyramid
55. Cube
56. Square pyramid
57. Cube
58. Square pyramid
59. B D A C
60. Teacher to check markings.
61. Rectangular prism
62. Rectangular prism
63. Triangular pyramid
64. Triangular pyramid
65. Cube
66. Square pyramid
67. Cube
68. Square pyramid
ii) Top: 125 in^2  
Bottom: 125 in^2  
Front: 50 in^2  
Back: 50 in^2  
Right: 40 in^2  
Left: 40 in^2  
c) i) 38 cm^2  
ii) 430 in^2  
5. a) Alexandra is correct because there are two of the same face, and four of the same face.  
b) 336 cm^2  
6. Ariel can multiply 20 cm^2 by 2, because each of the faces she has that add up to 20 cm^2 are doubled for total surface area (e.g. front + back, top + bottom, right + left).  
7. Teacher to check.  
8. 2(l \times w) + 2(l \times h) + 2(h \times w)  

AP Book G6-40  
page 152  
1. a) SA: 60 m^2  
b) SA: 282.5 ft^2  
2. a) SA: 564 cm^2  
b) SA: 22.35 m^2  

AP Book G6-41  
page 153  
1. a) 94 cm^2  
b) 37 ft^2  
c) 42.75 in^2  
d) 61,500 m^2  
e) 116.07 mm^2  
f) 43 \frac{1}{3} in^2  
2. a) Teacher to check.  
b) Top + Bottom: 750 cm^2  
Front + Back: 500 cm^2  
Left + Right Side: 600 cm^2  

b) Yes, he is correct because each face of a cube is the edge multiplied by itself, being a square. There are six faces, so the area of each face times six will result in the total surface area.  
c) V: 0.125 ft^3 or \frac{1}{8} ft^3  
SA: 1.5 ft^2  
d) i) V: 27 m^3  
SA: 54 m^2  
ii) V: 21.952 cm^3  
SA: 47.04 cm^2  
iii) V: 20.796875 in^3  
SA: 45.375 in^2  
10. 1,890 m^2  

BONUS  
4.3 \times 5 = 21.5  
21.5 + 2 = 10.75  
10.75 \times 8 = 86 \text{ cm}^2  
5 \times 5 = 25  
6 \times 25 = 150 \text{ cm}^2  
86 \text{ cm}^2 + 150 \text{ cm}^2 = 236 \text{ cm}^2  

Statistics and Probability: Distribution – AP Book 6.2: Unit 9

AP Book SP6-5
page 155

1. a) 5
   b) 3 minutes
   c) 2
   d) 3–6 minutes long
   e) 12–15 minutes long
   f) 30
   g) 12

2. a) Years
   b) Teacher to check, explanations may vary. The intervals are marked in consistent intervals of 15 years.
   c) No, because the intervals are across a span of 15 years.
   d) Yes, because each dot would represent a specific user’s age.

3. a) Teacher to check.
   b) ii) 8–12
      iii) 16–20

4. a) Teacher to check.
   b) Teacher to check.
   c) Different
   d) Same
   e) Explanations may vary. Sample: The intervals used can group the data into similar looking histograms, even when actual data sets are different.

5. Teacher to check.

AP Book SP6-6
page 157

1. a) 3
   b) 2
   c) 0

2. b) 16
   c) 3
   d) 124

3. Teacher to check.

4. a) Set 1 Mean: 6
   Set 2 Mean: 6
   b) Set 2
   c) Set 2 MAD: 3.33
   Set 2 MAD: 2
   d) Teacher to check, explanations may vary. The intervals are marked in consistent intervals of 15 years.
   e) No, because the intervals are across a span of 15 years.
   f) Lower MAD

5. Teacher to check, explanations may vary. Class 6-B should have a higher MAD because the data is not clustered around the mean, but in two distant intervals. Their distance from the mean is greater, which results in a higher MAD.

AP Book SP6-7
page 159

1. b) M: Between 18–18
   Q1: Between 6–6
   Q3: Between 21–23
   c) M: Between 33–35
   Q1: 22
   Q3: 39
   d) M: Between 4–4
   Q1: 3
   Q3: 7

2. a) Median: 45
   Q1: 25
   Q3: 65
   b) Median: 16
   Q1: 7
   Q3: 28

3. a) Median: 21
   Q1: 13
   Q3: 38
   b) Median: 37
   Q1: 30
   Q3: 49

BONUS
Median: 24,399
Q1: 7,999
Q3: 66,944

4. a) i) 0 2 2 4 4 6 6 8 8
   8 10 10 10 10
   10 12 12 12 12
   14 14 14 14 14
   ii) 0 4 8 8 10 10 10
   10 10 10 10 10
   12 12 12 12 12
   14 14 14 14 14

BONUS
Q1: 4,532
Q3: 68,756
IQR: 64,224

6. a) Set 1: 10
   Set 2: 6
   Set 3: 16
   b) Set 2
   c) Set 3

7. | Q1 | Q3 | IQR |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>27</td>
<td>12</td>
</tr>
<tr>
<td>152</td>
<td>153</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>102</td>
<td>100</td>
</tr>
<tr>
<td>30</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>5</td>
</tr>
<tr>
<td>361</td>
<td>379</td>
<td>18</td>
</tr>
</tbody>
</table>

BONUS
Teacher to check, explanations may vary. Seamus is incorrect, because two completely different data sets can have same interquartile range if the difference between Q3 and Q1 happens to be the same, regardless of data points in the set.

AP Book SP6-8
page 162

1. a) Median: 2
   IQR: 3 – 1 = 2
   Range: 5 - 0 = 5
   b) Median: 2.0
   IQR: 3.5 – 1.5 = 2
   Range: 4.0 – 0.5 = 3.5
   c) Median: \( \frac{5}{2} \)
   IQR: \( \frac{3}{2} - \frac{3}{2} = \frac{3}{2} \)
   Range: \( \frac{7}{2} - \frac{1}{2} = 3 \)
   d) Median: 340
   IQR: 375 – 275 = 100
   Range: 425 – 150 = 275

2. a) A
   b) A
   c) B

3. a) | Q1 | Q3 | IQR |
     |----|----|----|
     | 13 | 18 | 5  |
     | 9  | 14 | 5  |

b) No, because the IQRs are the same for both box plots.

b) No, because the IQRs are the same for both box plots.

b) No, because the IQRs are the same for both box plots.

b) No, because the IQRs are the same for both box plots.

b) No, because the IQRs are the same for both box plots.
d) A

AP Book SP6-9 page 164

1. a) Teacher to check.
   b) Symmetrical
   c) Mean: 3

2. a) Teacher to check.
   b) One gap
   c) Skewed

3. b) D
   c) F
   d) C
   e) A
   f) E

4. d) and e) because they are both symmetrical

5. a) E
   b) A
   c) C
   d) D
   e) B
   f) F

6. a) Histogram 1 because the data is clustered more closely.
   b) Histogram 2 because the highest interval will affect the mean.

7. a) Teacher to check.
   b) Mean: 12
   c) Median: 6
   d) Teacher to check.
   e) 10
   f) 6
   g) 28
   h) 6
   i) Yes
   j) No
   k) Decrease
   l) 7.5
   m) 6

n) The median, the middle number, is still the same when we left out the outlier, because there were multiples of the same number (6) around the median.
Unit 5: Geometry
Quiz (Lessons 20 to 23)

1. a) Find the coordinates of the points.
   \[ A(\quad, \quad) \quad B(\quad, \quad) \]
   \[ C(\quad, \quad) \quad D(\quad, \quad) \]
   \[ E(\quad, \quad) \quad F(\quad, \quad) \]

   b) Plot and label the points
   \[ G(-1, -3) \quad H(2, 4) \]
   \[ I\left(\frac{1}{2}, \frac{1}{2}\right) \quad J(4, -2.5) \]

   c) Which points are in quadrant 3?

   ______________________________________

2. a) \( S(-4, 1) \) and \( U(1, 1) \) are two points on a line. Is the line horizontal or vertical? _____________

   b) \( Y(5, -7) \) and \( Z(5, 3) \) are two points on a line. Is the line horizontal or vertical? _____________

   **BONUS** What is another point on the line \( YZ? \) ( ,   )

3. Find the distance between the points using the diagram.

   a) \((-2, -1) \) and \((-2, -2) \) ________ unit

   b) \((1, 1) \) and \((6, 1) \) ________ units

4. Add or subtract the absolute values of the \( y \)-coordinates to find the distance between the points.

   a) \((2, 10) \) and \((2, 8) \) ________ units

   b) \((-14, 6) \) and \((-14, -14) \) ________ units
Unit 5: Geometry

Quiz (Lessons 20 to 23)

1. a)  
   A (3,4)
   B (−4, −3)
   C (−4, 2)
   D (2.5, −2)
   E (−0.5, 0)
   F (−2.5, −1.5)

   b) Teacher to check.

   c) B, F, G

2. a) Horizontal

   b) Vertical

   BONUS
   Answers may vary; teacher to check. The point must have x-coordinate 5.

3. a) 1

   b) 5

4. a) 2

   b) 20
1. a) Find the coordinates of the points.

\[ L(\quad,\quad)\quad M(\quad,\quad)\quad N(\quad,\quad)\quad O(\quad,\quad)\quad P(\quad,\quad)\quad Q(\quad,\quad) \]

b) Plot and label the points.

\[ F(-5,1)\quad G(3,2.5)\]
\[ H(0,5)\quad I(4,0)\]
\[ J\left(-\frac{4}{2},-\frac{1}{2}\right)\quad K(0,0) \]

**BONUS** ▶ Sort the points by location.

<table>
<thead>
<tr>
<th>First Quadrant</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Second Quadrant</td>
<td></td>
</tr>
<tr>
<td>Third Quadrant</td>
<td></td>
</tr>
<tr>
<td>Fourth Quadrant</td>
<td></td>
</tr>
<tr>
<td>On the x-axis</td>
<td></td>
</tr>
<tr>
<td>On the y-axis</td>
<td></td>
</tr>
<tr>
<td>At the Origin</td>
<td></td>
</tr>
</tbody>
</table>

2. Add or subtract the absolute values of the coordinates to find the distance between the points.

a) \((-3, 2)\) and \((-6, 2)\) ________ units

b) \((4, 2)\) and \((4, -5)\) ________ units

c) \((5, 6)\) and \((-3, 6)\) ________ units

d) \((3, -4)\) and \((3, 5.5)\) ________ units

e) \left(-8,\frac{1}{2}\right)\) and \((-8, -8)\) ________ units

**BONUS** ▶

(1,850, -28) and (1,600, -28) ________ units

3. Fill in the table.

<table>
<thead>
<tr>
<th>Point</th>
<th>((-2, 5), (-2, 8))</th>
<th>((-4, 3), (5, 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same coordinate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direction of line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Same quadrant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unit 5: Geometry

Test (Lessons 20 to 29)

4. A rectangle has vertical and horizontal sides.
   One side length and the coordinates of two adjacent vertices are given. Find two possible sets of coordinates for the other two vertices of the rectangle.

\[ W(2, -1), X(6, -1), \text{ height } = 3 \text{ units} \]

\[ Y(\ , \ ), Z(\ , \ ) \text{ or } Y(\ , \ ), Z(\ , \ ) \]

5. Shape \(ABCD\) has vertices: \(A(1,\ 3), B(7,\ 3), C(-3,\ -3), D(5,\ -3)\).
   a) Identify the vertical or horizontal sides that are parallel and find their lengths.
      parallel sides: \(\) and \(\)
      lengths: \(=\) \(=\)
   b) What shape is \(ABCD\)? \(ABCD\) is a \(\)
   c) What is the height of the shape? height: \(\)
   d) What is the area of the shape? area: \(\)

6. A meteorologist recorded temperatures in Barrow, Alaska.

<table>
<thead>
<tr>
<th>Month</th>
<th>April</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>−5°F</td>
<td>35°F</td>
</tr>
</tbody>
</table>

a) Plot the data on the grid and join the points with a line.
   b) Assume the temperature changes at the same rate.
      What is the temperature in May?
      
   c) Assume the temperature changes at the same rate.
      In what month would the temperature be −25°F? 
      How do you know?
ADVANCED

7. Reflect points A, B, C, and D through the y-axis and then through the x-axis. Label the resulting image points \(A', B', C',\) and \(D'\) and write the coordinates.

\[
\begin{align*}
A(\quad,\quad) & \quad\quad\longrightarrow\quad\quad A'(\quad,\quad) \\
B(\quad,\quad) & \quad\quad\longrightarrow\quad\quad B'(\quad,\quad) \\
C(\quad,\quad) & \quad\quad\longrightarrow\quad\quad C'(\quad,\quad) \\
D(\quad,\quad) & \quad\quad\longrightarrow\quad\quad D'(\quad,\quad)
\end{align*}
\]
1. a) $L (5, 5)$  
    $M (-2, 4)$  
    $N (-3.5, -4.5)$  
    $O (0, -3)$  
    $P (-4, 0)$  
    $Q (4, -2.5)$  

   b) Teacher to check.

   **BONUS**
   
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>L, G</td>
</tr>
<tr>
<td>II</td>
<td>M</td>
</tr>
<tr>
<td>III</td>
<td>N, J</td>
</tr>
<tr>
<td>IV</td>
<td>Q</td>
</tr>
<tr>
<td>x-axis</td>
<td>P, I, K</td>
</tr>
<tr>
<td>y-axis</td>
<td>O, H, K</td>
</tr>
<tr>
<td>Origin</td>
<td>K</td>
</tr>
</tbody>
</table>

2. a) 3  
   b) 7  
   c) 8  
   d) 9.5  
   e) 9.5  

   **BONUS**

   250

3. $x$  
   $y$  
   vertical  horizontal  
   yes  no  
   $8 - 5 = 3$  $4 + 5 = 9$

4. $Y (6, -4), Z (2, -4)$  
   or  
   $Y (6, 2), Z (2, 2)$

5. a) Parallel sides: $AB$ and $CD$  
   Lengths: $AB = CD = 8$  
   b) Parallelogram  
   c) Height: 6  
   d) Area: 48 sq. units

6. a) Teacher to check.  
   b) $15^\circ$F

7. $A (-1, 4)$  
   $B (4, 2)$  
   $C (-4, 1)$  
   $D (-2, -4)$

   **Sample answer:**

   March. I extended the line on the graph toward negative temperatures. Graph should show extension of line.

   **ADVANCED**

   $A (-1, 4)$  
   $B (4, 2)$  
   $C (4, 1)$  
   $D (2, 4)$
Unit 6: Expressions and Equations

Quiz (Lessons 16 to 18)

1. Match each expression in the top row to its equivalent in the bottom row.

\[(x - 4) \times 3 \quad 4 \times (x - 3) \quad 12 \times (x - 1) \quad (x - 12) \times 1\]

\[12x - 12 \quad 3x - 12 \quad 4x - 12 \quad x - 12\]

2. Group the x's together, then solve the equation for x.

a) \[6x + 2x = 24\]  
b) \[9x + x = 50\]  
c) \[8x + 3x = 44\]

3. Solve for x. Check your answer.

a) \[x + 0.6 = 1.9\]  
b) \[2x = 6.2\]  
c) \[0.7 = x - 1.4\]

4. Write an algebraic expression for the description.

a) 3 times a number ____________________

b) 8 divided into a number ____________________

c) the sum of a number and 3.2 ____________________

d) a number increased by 1.8 ____________________

5. Carmen’s cousin is three times older than Carmen. The difference between their ages is 12 years. How old is Carmen?
1. \((x - 4) \times 3 = 3x - 12\)
   \(4 \times (x - 3) = 4x - 12\)
   \(12 \times (x - 1) = 12x - 12\)
   \((x - 12) \times 1 = x - 12\)

2. a) \(6x + 2x = 24\)
   \(8x = 24\)
   \(x = 3\)

   b) \(9x + x = 50\)
   \(10x = 50\)
   \(x = 5\)

   c) \(8x + 3x = 44\)
   \(11x = 44\)
   \(x = 4\)

3. a) \(x + 0.6 = 1.9\)
   \(x = 1.9 - 0.6\)
   \(x = 1.3\)
   To check:
   \(1.3 + 0.6 = 1.9\)

   b) \(2x = 6.2\)
   \(x = 3.1\)
   To check:
   \(2 \times 3.1 = 6.2\)

   c) \(0.7 = x - 1.4\)
   \(x = 0.7 + 1.4\)
   \(x = 2.1\)
   To check:
   \(0.7 = 2.1 - 1.4\)

4. a) \(3x\)

   b) \(\frac{x}{8}\)

   c) \(x + 3.2\)

   d) \(x + 1.8\)

5. Carmen = \(x\)
   Cousin = \(3x\)
   \(3x - 12 = x\)
   \(3x - x = 12\)
   \(2x = 12\)
   \(x = 6\)

   Carmen is 6 and her cousin is 18.
1. Use the distributive property to write an equivalent expression.
   a) $5 \times (x - 2)$
   b) $3 \times (n + 4)$
   c) $(2x - 5) \times 4$

2. Group the $x$'s together and then solve for $x$.
   a) $6x + x - x - 2x = 10$
   b) $8x + 3x - 4x = 14$
   c) $2.4 = 5x - 4x + 4x + 5x - 4x$

3. Solve the problem by first writing an equation.
   a) 3 times a number is 24.
   b) 4 divided into a number is 0.5
   c) Twice a number is 14.2

4. Write an equation to find the length of the missing side.
   a) $x$
      \[ \text{Area} = 30 \]
   b) $x$
      \[ \text{Perimeter} = 36 \]

5. Ricardo is four times older than Angela. The sum of their ages is 25. How old are Ricardo and Angela?
6. Make a table for each set of points on the coordinate grid. Write a rule for each table that tells you how to calculate \( y \) from \( x \).

<table>
<thead>
<tr>
<th>Graph A</th>
<th>Graph B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
</tbody>
</table>

Rule: \( y = \) ______  Rule: \( y = \) ______

7. The graph shows the cost of renting a scooter from Bernard’s store.

a) What is the independent variable? __________________
   What is the dependent variable? __________________

b) How much would you pay to ride a scooter for ...
   1 hour? __________
   2 hours? __________
   4 hours? __________

c) How much do you have to pay for the scooter before you have even ridden it? __________

8. a) Write an inequality for the number line.
   i) \[ 23 \ 24 \ 25 \ 26 \ 27 \]
   ii) \[ 30 \ 40 \ 50 \ 60 \ 70 \]

b) Draw a thick line to show the solution of each inequality on the given number line.
   i) \( x > 37 \)
   ii) \( x < 12 \)

c) Write an inequality for the story “Kevin has $10 and Sam has less money than Kevin.” Let \( x \) represent the unknown and indicate if the solution is always positive.
1. a) $5x - 10$
   b) $3n + 12$
   c) $8x - 20$
2. a) $4x = 10$
   \[ x = \frac{5}{2} \]
   b) $7x = 14$
   \[ x = 2 \]
   c) $2.4 = 6x$
   \[ x = 0.4 \]
3. a) $3x = 24$
   \[ x = 8 \]
   b) $\frac{x}{4} = 0.5, x = 2$
   c) $2x = 14.2$
   \[ x = 7.1 \]
4. a) $10x = 30$
   \[ x = 3 \]
   b) $2x + 2x + x + x = 36$
   \[ 6x = 36 \]
   \[ x = 6 \]
5. Angela = $x$
   Ricardo = $4x$
   \[ x + 4x = 25 \]
   \[ 5x = 25 \]
   \[ x = 5 \]
   Angela = 5 years old
   Ricardo = 20 years old
6. | Graph A |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>
   Rule: $y = 3x$
7. a) Independent variable: Time
   Dependent variable: Total cost
   b) 1 hour: $6$
      2 hour: $9$
      3 hour: $12$
   c) $3$
8. a) i) $25 < x$
   ii) $x < 50$
   b) Teacher to check.
   c) $x < 10$ and $x > 0$
1. a) Write the total number of beads and stacks. Move beads on the stacks to find the mean.

   i)  
   Beads: ______  Stacks: ______  Mean: ______

   ii) 
   Beads: ______  Stacks: ______  Mean: ______

   b) Use the three numbers from each picture in part a) to write a complete division equation.

   i) _______________________________  ii) _______________________________

2. Order the data from lowest to highest. Find the median of the set of data.

   a) 9 1 4 3 3  
   b) 12 2 10 7 5 3

   ____ ____ ____ ____ ____ ____  ____ ____ ____ ____ ____ ____

   Median: ______  Median: ______

3. The dot plot shows the number of TV shows watched in a week by students in Abe’s class. Complete the statements about the data.

   a) The most shows watched by any student was: ______.

   b) ______ students watched exactly 5 shows.

   c) The largest number of students watched ______ shows.

   d) The number of students in the class is: ______.

   e) ______ students watched 1 or 2 shows.
4. Find the median and range of the quiz marks for each student.

<table>
<thead>
<tr>
<th>Student</th>
<th>Quiz 1</th>
<th>Quiz 2</th>
<th>Quiz 3</th>
<th>Median</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shelley</td>
<td>12</td>
<td>8</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nadia</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Transfer the data into the tally chart. Find the mean.

Data: 2, 1, 3, 1, 2, 3, 1, 2, 1, 4

<table>
<thead>
<tr>
<th>Data Value</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean: ______

6. Use the tally chart to draw a dot plot. Answer the questions about the data.

a) What is the median of the data? ______

b) What is the mean of the data? ______

c) Is the graph symmetrical? Explain why it is or is not symmetrical.

BONUS ▶ Answer the question for this data set: 90, 10, 50, 50, 50, 50, 90, 10, 50, 50

a) What is the median of this data? ______

b) What is the mean of this data? ______
1. a) i) Beads: 12
    Stacks: 4
    Mean: 3
   ii) Beads: 15
    Stacks: 3
    Mean: 5
 b) i) $12 \div 4 = 3$
 ii) $15 \div 3 = 5$

2. a) 1 3 3 4 9
    Median: 3
 b) 2 3 5 7 10 12
    Median: 6

3. a) 6
 b) 4
 c) 4
 d) 20
 e) 4

4. 9, 4
   10, 6

5. | Data Value | Tally | Fr. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
 Mean: $20 \div 10 = 2$

6. | 2 3 4 5 6 |
 a) Median: 4
 b) Mean: 4
 c) Yes, it is symmetrical. The dot plot is balanced with the center at 4.

BONUS
 a) $10, 10, 50, 50, 50, 50, 50, 50, 90$
    Median is 50.
 b) $500 \div 10 = 50$
    Mean is 50.
    (NOTE: The data is symmetrical, and this could be used to find the mean.)
Unit 8: Geometry
Quiz (Lessons 30 to 36)

1. Find the volume.
   a) Length = _______  
      Width = _______  
      Height = _______  
      Volume = _______________________ = _______  
   b) Length = _______  
      Width = _______  
      Height = _______  
      Volume = _______________________ = _______  

2. Find the volume. Do not use a calculator.  
   BONUS ▶
   Area of top face = 6.5 ft²  
   Height = 2 ft  
   Volume = _______________  
            Volume = _______________  

3. A rectangular tank is 32 cm long and 16 cm wide. The water height is 10.2 cm.  
   What is the volume of the water? You can use a calculator.

4. Complete the table.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Name</th>
<th>Number of ...</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Vertices</td>
<td>Edges</td>
<td>Faces</td>
<td></td>
</tr>
<tr>
<td>![Cube]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Pyramid]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Unit 8: Geometry

Quiz (Lessons 30 to 36)

1. a) 12 in$^3$
   b) 57 ft$^3$
2. 25.2 cm$^3$
   BONUS 13 ft$^3$
3. 5,222.4 cm$^3$
4. 

<table>
<thead>
<tr>
<th>Name</th>
<th>v</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>cube</td>
<td>8</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>rectangular pyramid</td>
<td>5</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>
1. Find the volume of the rectangular prism. Do not use a calculator.
   a) length 15 m, width 5 m, height 8 m
   b) length 13 cm, width 4 cm, height 2 cm

   Volume = ____________________

2. A rectangular tank is 100 cm long and 75 cm wide. The water height is 12 cm. You can use a calculator to solve the following questions.
   a) What is the volume of the water?

   b) Jude placed two cubes into the water. They are each 10 cm × 10 cm × 10 cm. What is the new volume of contents of the tank?

   c) What is the height of the water in the tank in centimeters after Jude added the cubes? Round to one decimal place.

3. The area of each visible face is given. What is the total surface area? Do not use a calculator. Show your work.

   top 14 cm²
   front 18 cm²
   right side 6 cm²

   23 3/4 cm²
Unit 8: Geometry
Test (Lessons 30 to 41)

4. a) Sketch the net for the prism and label each face.

b) Marco says that he only needs to find the area of two faces of this prism to calculate the surface area. Is he correct? Explain.

c) What is the surface area of the prism? Do not use a calculator.

5. Felix says that the volume of this box is 1650 m$^3$. Is he correct? Explain why or why not.

BONUS ► Change the measurements of the box into meters and find the volume and the surface area. Do not use a calculator. Hint: 1 m = 100 cm.

6. Write the height and the base of the triangles on the net. Find the area of each face. Then find the surface area of the pyramid. Do not use a calculator.

Surface area = ____________________________
1.  a) 600 m³  
    b) 95 cm³  
2.  a) 90,000 cm³  
    b) 92,000 cm³  
    c) 12.3 cm  
3.  76 cm²  
4.  a) 
   
    b) Yes. All of the faces are either 3 cm × 3 cm or 3 cm × 2 cm. You only need to find the surface area of these faces and then multiply each by the amount of times each face appears on the shape.  
    c) 42 cm²  
5. No, because the width of the shape is given in centimeters, not in meters. Felix has to convert the centimeters to meters before calculating the volume.  
   **BONUS**  
   16.5 m³  
6. 354 cm²
1. Use the frequency table to draw a histogram.

<table>
<thead>
<tr>
<th>Number of Whole Laps Run</th>
<th>0–3</th>
<th>3–6</th>
<th>6–9</th>
<th>9–12</th>
<th>12–15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

2. Use the histogram to fill in the blanks.
   a) How many intervals are shown? _____
   b) How many letters are in each interval? _____
   c) How many names are between 3 and 6 letters long? ______
   d) The longest names are between ______ and _____ letters long.
   e) How many students are represented by this histogram? ______

3. The points of a set are shown on the number line.
   a) Find the mean and mark it on the number line.
   b) Draw 3 lines above the number line to show the 3 absolute deviations.
   c) Find the mean absolute deviation.

Do your calculations here.
Unit 9: Statistics and Probability

Test (Lessons 5 to 9)

4. a) Label the first quartile (Q1), the median (M), and the third quartile (Q3), and mark each with an arrow.

Set 1: 1 2 3 4 5 9 16 17 18 20 22

Set 2: 1 2 6 7 8 9 10 11 12 20 22

b) Find the interquartile range.

Set 1: IQR __________ Set 2: IQR __________

c) Use your answer from part b) to decide which set is more spread out around the median? Explain.

___________________________________________________________

___________________________________________________________

5. Find the range and interquartile range.

A: Range _______, IQR _______

B: Range _______, IQR _______

6. Label each histogram as “symmetrical” or “skewed.”

---

Sample Unit Quizzes and Tests for AP Book 6.2
1. Teacher to check.

2. a) 4  
   b) 3  
   c) 12  
   d) 9 and 12  
   e) 28 students  

3. a) 8  
   b) Teacher to check.  
   c) \( \frac{6 + 4 + 2}{3} = \frac{12}{3} = 4 \)  

4. a) Set 1:  
   Q1 = 3  
   M = 9  
   Q3 = 18  
   Set 2:  
   Q1 = 6  
   M = 9  
   Q3 = 12  
   b) Set 1: IQR = 15  
      Set 2: IQR = 6  
   c) Set 1 has a larger IQR than Set 2.  
      That means Q1 and Q3 are further apart in Set 1 than in Set 2 and further apart means more spread out. So Set 1 is more spread out.  

5. A:  
   Range = 80  
   IQR = 40  
   B:  
   Range = 70  
   IQR = 30  

6. Skewed, symmetrical
**Scoring Guides for Sample Unit Quizzes and Tests**

**Unit 5: Geometry**

---

**Quiz (Lessons 20 to 23), p. U-28**

**Common Core State Standards Emphasized:** 6.G.A.3, 6.NS.B.3, 6.NS.C.6b, 6.NS.C.6c, 6.NS.C.8

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct answer for $A$</td>
<td>$(3, 4)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $B$</td>
<td>$(-4, -3)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $C$</td>
<td>$(-4, 2)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $D$</td>
<td>$(2.5, -2)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $E$</td>
<td>$(-0.5, 0)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for $F$</td>
<td>$(-2.5, -1.5)$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Plots $G$ correctly</td>
<td>Teacher to check.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Plots $H$ correctly</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Plots $I$ correctly</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Plots $J$ correctly</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$B, F, G$</td>
<td>1</td>
<td>/11</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>horizontal</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>vertical</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>any point with $x$-coordinate 5</td>
<td>yes / no</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>5</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>20</td>
<td>1</td>
<td>/2</td>
</tr>
</tbody>
</table>

**Total Points** /17
Scoring Guides forSample Unit Quizzes and Tests
Unit 5: Geometry
(continued)

Test (Lessons 20 to 29), p. U-30


<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 1        | a) Gives correct answer for \( L \)  
          |   | \((5, 5)\)  
          |   | 1  
          |   | b) Plots \( F \) correctly  
          |   | Teacher to check.  
          |   | 1  
          |   | Bonus: Gives correct answer  
          |   | \( L, G \)  
          |   | \( M \)  
          |   | \( N, J \)  
          |   | \( Q \)  
          |   | \( P, I, K \)  
          |   | \( O, H, K \)  
          |   | \( K \)  
| 2        | a) Gives correct answer  
          |   | 3  
          |   | 1  
          |   | b) Gives correct answer  
          |   | 7  
          |   | 1  
          |   | c) Gives correct answer  
          |   | 8  
          |   | 1  
          |   | d) Gives correct answer  
          |   | 9.5  
          |   | 1  
          |   | e) Gives correct answer  
          |   | 9.5  
          |   | 1  
          |   | Bonus: Gives correct answer  
          |   | 250  
          |   | yes / no  
| 3        | Gives correct answers for the same coordinate  
          |   | \( x \) \( y \)  
          |   | \( \text{vert. horiz.} \)  
          |   | \( \text{yes no} \)  
          |   | 3 \( 9 \)  
| 4        | Gives two correct sets of answers  
          |   | \( (2, 2), (6, 2) \) or \( (2, -4), (6, -4) \)  
          |   | 2  
          |   | Gives one correct set of answers  
          |   | \( (1) \)  
| 5        | a) Correctly identifies parallel sides  
          |   | \( AB, CD \)  
          |   | \( AB = CD = 8 \)  
          |   | 1  
          |   | b) Gives correct answer  
          |   | parallelogram  
          |   | 1  
          |   | c) Gives correct answer  
          |   | 6  
          |   | 1  
          |   | d) Gives correct answer  
          |   | 48 units²  
          |   | 1  

Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 6.2

U-73
### Question: How to Score

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
</table>
| 6        | a) Correctly plots both points  
           Correctly plots one point | Teacher to check. | 2 (1) | (1) |
|          | b) Gives correct answer | 15°F | 1 | |
|          | c) Gives correct answer  
           Gives correct explanation | Sample answer: 
I extended the line on the graph towards negative temperatures. | 1 | |
| Advanced | 7 | Gives correct answer for A and A'  
(−1, 4) → (1, −4)  
(4, 2) → (−4, −2) | 1 | |
|          | Gives correct answer for B and B'  
(−4, 1) → (4, −1) | 1 | |
|          | Gives correct answer for C and C'  
(−2, −4) → (2, 4) | 1 | |
|          | Gives correct answer for D and D' | 1 | |
|          | **Total Points** | **/5** | **/33** | **/33** |
### Rubric for Unit 5: Geometry

Test (Lessons 20 to 29), p. U-30

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td>6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</td>
<td>Can answer some questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</td>
<td>Can answer some questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.NS.C.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</td>
<td>Can answer some questions accurately and independently.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Rubric for Unit 5: Geometry
Test (Lessons 20 to 29), p. U-30

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</td>
<td>6</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

**Comments**
### Scoring Guides for Sample Unit Quizzes and Tests

**Unit 6: Expressions and Equations**

**Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 6.2**

#### Quiz (Lessons 16 to 18), p. U-34

**Common Core State Standards Emphasized:** 6.EE.A.2, 6.EE.A.2a, 6.EE.A.3, 6.EE.A.4, 6.EE.B.6, 6.EE.B.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Matches four expressions correctly</td>
<td>((x - 4) \times 3 = 3x - 12)</td>
<td>3</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>Matches two expressions correctly</td>
<td>(4 \times (x - 3) = 4x - 12)</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Matches one expression correctly</td>
<td>(12 \times (x - 1) = 12x - 12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>((x - 12) \times 1 = x - 12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Groups x’s correctly Correctly solves equation</td>
<td>(8x = 24)</td>
<td>1</td>
<td>/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 24 ÷ 8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Groups x’s correctly Correctly solves equation</td>
<td>(10x = 50)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 50 ÷ 10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Groups x’s correctly Correctly solves equation</td>
<td>(11x = 44)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 44 ÷ 11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a) Correctly solves equation</td>
<td>(x = 1.9 - 0.6)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 1.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.3 + 0.6 = 1.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Correctly solves equation</td>
<td>(x = 6.2 ÷ 2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 3.1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2 \times 3.1 = 6.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Correctly solves equation</td>
<td>(x = 0.7 + 1.4)</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 2.1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.7 = 2.1 - 1.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Gives correct answer</td>
<td>(3x)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>(x \div 8)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>(x + 3.2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>(x + 1.8)</td>
<td>1</td>
<td>/4</td>
</tr>
<tr>
<td>5</td>
<td>Defines variables</td>
<td>Carmen’s age: (x)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cousin’s age: (3x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>(3x - x = 12)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2x = 12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 12 ÷ 6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(x = 6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Groups x’s correctly</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>Carmen is 6 years old.</td>
<td>1</td>
<td>/5</td>
</tr>
</tbody>
</table>

**Total Points** 24
## Test (Lessons 16 to 21), p. U-36


<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Gives correct answer</td>
<td>$5x - 10$</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>b) Gives correct answer</td>
<td>$3n + 12$</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>c) Gives correct answer</td>
<td>$8x - 20$</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| a) Groups $x$'s correctly  
Correctly solves equation | $4x = 10$  
$x = \frac{10}{4} = \frac{5}{2}$ | 1  
1 |              | 3/6 |
| b) Groups $x$'s correctly  
Correctly solves equation | $7x = 14$  
$x = 14 \div 7$  
$x = 2$ | 1  
1 |              | 1/6 |
| c) Groups $x$'s correctly  
Correctly solves equation | $2.4 = 6x$  
$x = 2.4 \div 6$  
$x = 0.4$ | 1  
1 |              | 1/6 |
| **3**    |              |        |                 |              |
| a) Gives correct equation  
Correctly solves equation | $3x = 24$  
$x = 24 \div 3$  
$x = 8$ | 1  
1 |              | 1/6 |
| b) Gives correct equation  
Correctly solves equation | $x = 0.5$  
$x = 0.5 \times 4$  
$x = 2$ | 1  
1 |              | 1/6 |
| c) Gives correct equation  
Correctly solves equation | $2x = 14.2$  
$x = 14.2 \div 2$  
$x = 7.1$ | 1  
1 |              | 1/6 |
| **4**    |              |        |                 |              |
| a) Gives correct equation  
Correctly solves equation | $10x = 30$  
$x = 30 \div 10$  
$x = 3$ | 1  
1 |              | 1/5 |
| b) Gives correct equation  
Groups $x$'s correctly  
Correctly solves equation | $x + 2x + x + 2x = 36$  
$6x = 36$  
$x = 36 + 6$  
$x = 6$ | 1  
1 |              | 1/5 |
### Scoring Guides for Sample Unit Quizzes and Tests
#### Unit 6: Expressions and Equations

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Defines variables</td>
<td>Angela’s age:</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ricardo’s age:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct equation</td>
<td>x + 4x = 25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Groups x’s correctly</td>
<td>5x = 25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correctly solves equation</td>
<td>x = 25 ÷ 5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>x = 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Angela is 5 years old.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ricardo is 20 years old.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Completes table for Graph A correctly</td>
<td>Graph A</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Makes one error</td>
<td>1 3</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Completes table for Graph B correctly</td>
<td>Graph B</td>
<td>2</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Makes one error</td>
<td>1 1</td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct rule for Graph A</td>
<td>y = 3x</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives correct rule for Graph B</td>
<td>y = x</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td>7</td>
<td>a) Gives correct answer for independent variable</td>
<td>time</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for dependent variable</td>
<td>total cost</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer for 1 hour</td>
<td>$6</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 2 hours</td>
<td>$9</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer for 3 hours</td>
<td>$12</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>$3</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td>8</td>
<td>a) i) Gives correct answer</td>
<td>25 &lt; x</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) ii) Gives correct answer</td>
<td>x &lt; 50</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) i) Gives correct answer</td>
<td>Teacher to check.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) ii) Gives correct answer</td>
<td>Teacher to check.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct inequality to show “Sam has less money that Kevin”</td>
<td>x &lt; 10 and x &gt; 0</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td>Gives correct inequality to indicate that the solution is always positive</td>
<td>x &gt; 0</td>
<td>1</td>
<td>/6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Points</td>
<td>43</td>
<td></td>
</tr>
</tbody>
</table>
Rubric for Unit 6: Expressions and Equations
Test (Lessons 16 to 21), p. U-36

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.EE.A.2</td>
<td>Write, read, and evaluate expressions in which letters stand for numbers.</td>
<td>3, 4, 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.A.2a</td>
<td>Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation &quot;Subtract y from 5&quot; as 5 − y.</td>
<td>3, 4, 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.A.4</td>
<td>Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.</td>
<td>1, 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.B.6</td>
<td>Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.B.7</td>
<td>Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.</td>
<td>2, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Rubric for Unit 6: Expressions and Equations

Test (Lessons 16 to 21), p. U-36

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.EE.B.8</td>
<td>8</td>
</tr>
<tr>
<td>Write an inequality of the form ( x &gt; c ) or ( x &lt; c ) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form ( x &gt; c ) or ( x &lt; c ) have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</td>
<td></td>
</tr>
<tr>
<td>6.EE.C.9</td>
<td>7</td>
</tr>
<tr>
<td>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation ( d = 65t ) to represent the relationship between distance and time.</td>
<td></td>
</tr>
</tbody>
</table>

**Comments**

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 7: Statistics and Probability

**Test (Lessons 1 to 4), p. U-39**

<table>
<thead>
<tr>
<th>Common Core State Standards Emphasized: 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5a, 6.SP.B.5b, 6.SP.B.5c</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Bonus</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Total Points</strong></td>
</tr>
<tr>
<td>Common Core State Standard</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>6.SP.A.2</td>
</tr>
<tr>
<td>Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</td>
</tr>
<tr>
<td>6.SP.A.3</td>
</tr>
<tr>
<td>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td>
</tr>
<tr>
<td>6.SP.B.4</td>
</tr>
<tr>
<td>Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</td>
</tr>
<tr>
<td>6.SP.B.5a</td>
</tr>
<tr>
<td>Reporting the number of observations.</td>
</tr>
<tr>
<td>6.SP.B.5b</td>
</tr>
<tr>
<td>Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</td>
</tr>
</tbody>
</table>
### Rubric for Unit 7: Statistics and Probability
Test (Lessons 1 to 4), p. U-39

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.SP.B.5c</td>
<td></td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td><strong>Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</strong></td>
<td>1, 2, 3, 4, 5, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments**
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 8: Geometry

**Quiz (Lessons 30 to 36), p. U-42**

**Common Core State Standards Emphasized:** 6.G.A.2, 6.G.A.4, 6.NS.B.3, 6.EE.B.7

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct length</td>
<td>6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct width</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct height</td>
<td>2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct expression for volume</td>
<td>$6 \times 1 \times 2$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 12 \text{ in}^3$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct length</td>
<td>3</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct width</td>
<td>2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct height</td>
<td>9.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct expression for volume</td>
<td>$3 \times 2 \times 9.5$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 57 \text{ ft}^3$</td>
<td>0.5</td>
<td>/5</td>
</tr>
<tr>
<td>2</td>
<td>Gives correct expression for volume</td>
<td>$12.6 \times 2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 25.2 \text{ cm}^3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonus: Gives correct answer</td>
<td>$6.5 \times 2 = 13 \text{ ft}^3$</td>
<td>yes / no</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Gives correct expression for volume</td>
<td>$32 \times 16 \times 10.2$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 5,222.4 \text{ cm}^3$</td>
<td>1</td>
<td>/2</td>
</tr>
<tr>
<td>4</td>
<td>Gives correct name in top row</td>
<td>cube</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct numbers in top row</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct name in bottom row</td>
<td>rectangular pyramid</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gives correct numbers in bottom row</td>
<td>8</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total Points /13**
<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a) Gives correct expression for volume</td>
<td>$15 \times 5 \times 8$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 600 \text{ m}^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct expression for volume</td>
<td>$23.75 \times 4$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 95 \text{ cm}^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct expression for volume</td>
<td>$100 \times 75 \times 12$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 90,000 \text{ cm}^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct expression for volume</td>
<td>$90,000 + 2 \times 1,000$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 92,000 \text{ cm}^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct expression</td>
<td>$92,000 \div 7,500$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer</td>
<td>$\approx 12.3 \text{ cm}^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Gives correct expression for surface area</td>
<td>$(2 \times 14) + (2 \times 18) + (2 \times 6)$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td>$= 76 \text{ cm}^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a) Correctly draws a net</td>
<td>Teacher to check.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Correctly labels each face</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>Yes. All faces are either $2 \times 3$ or $3 \times 3$ so you only need to find the area of these faces and then multiply.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer with units</td>
<td>$42 \text{ cm}^2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct answer</td>
<td>No. The width is given in cm. Felix has to convert the cm to m before calculating the volume.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct explanation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonus:</td>
<td>Gives correct answer</td>
<td>$16.5 \text{ m}^3$</td>
<td>yes / no</td>
<td>2</td>
</tr>
<tr>
<td>Question</td>
<td>How to Score</td>
<td>Answer</td>
<td>Number of Points</td>
<td>Total Points</td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
<td>--------</td>
<td>------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>6</td>
<td>Writes height and base of triangles on net</td>
<td>Teacher to check.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Finds area of one triangular face</td>
<td>$16 \times 7.5 \div 2 = 60 \text{ cm}^2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Finds area of the other triangular face</td>
<td>$9 \times 10 \div 2 = 45 \text{ cm}^2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Finds area of base</td>
<td>$16 \times 9 = 144 \text{ cm}^2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Gives correct expression for surface area</td>
<td>$2 \times 60 + 2 \times 45 + 144 = 354 \text{ cm}^2$</td>
<td>1</td>
<td>1 /6</td>
</tr>
<tr>
<td></td>
<td>Gives correct answer with units</td>
<td></td>
<td></td>
<td>1 /6</td>
</tr>
</tbody>
</table>

Total Points /25
### Rubric for Unit 8: Geometry
**Test (Lessons 30 to 41), p. U-44**

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s)</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.G.A.2</td>
<td>2, 4, 5</td>
<td>Can answer few, if any, questions accurately and independently.</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
<tr>
<td><strong>6.G.A.2</strong> Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas ( V = l w h ) and ( V = b h ) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.G.A.4</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>6.G.A.4</strong> Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.NS.B.3</td>
<td>3, 5, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.B.3</strong> Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.A.2</td>
<td>1, 2, 3, 4, 5, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>6.EE.A.2</strong> Write, read, and evaluate expressions in which letters stand for numbers.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common Core State Standard</td>
<td>Assessed by Question(s) ...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>-----------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s^3 and A = 6 s^2 to find the volume and surface area of a cube with sides of length s = 1/2.</td>
<td>1, 2, 3, 4, 5, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.</td>
<td>4, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comments
## Scoring Guides for Sample Unit Quizzes and Tests
### Unit 9: Statistics and Probability

**Test (Lessons 5 to 9), p. U-47**

**Common Core State Standards Emphasized:** 6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5a, 6.SP.B.5b, 6.SP.B.5c, 6.SP.B.5d

<table>
<thead>
<tr>
<th>Question</th>
<th>How to Score</th>
<th>Answer</th>
<th>Number of Points</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gives correct answer Makes one error</td>
<td>Teacher to check.</td>
<td>2</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>a) Gives correct answer</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer</td>
<td>12</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) Gives correct answer</td>
<td>9 and 12</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e) Gives correct answer</td>
<td>28</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>a) Gives correct answer</td>
<td>(2 + 10 + 12) / 3 = 8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct answer</td>
<td>Teacher to check.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer Makes one computational error</td>
<td>6 + 2 + 4 = 12 / 12 + 3 = 4</td>
<td>2</td>
<td>1/4</td>
</tr>
<tr>
<td>4</td>
<td>a) Correctly labels Q1 for Set 1 Correctly labels M for Set 1 Correctly labels Q3 for Set 1 Correctly labels Q1 for Set 2 Correctly labels M for Set 2 Correctly labels Q3 for Set 2</td>
<td>3, 9, 18, 6, 9, 12</td>
<td>1, 1, 1, 1, 1, 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Gives correct IQR for Set 1 Gives correct IQR for Set 2</td>
<td>15, 6</td>
<td>1, 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Gives correct answer Gives correct explanation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Set 1 is more spread out around the median. It has a higher IQR.</td>
<td>1</td>
<td>1/10</td>
</tr>
<tr>
<td>5</td>
<td>Gives correct range for A Gives correct IQR for A Gives correct range for B Gives correct IQR for B</td>
<td>80, 40, 70, 30</td>
<td>1, 1, 1, 1/4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Gives correct answer for graph on left Gives correct answer for graph on right</td>
<td>skewed symmetrical</td>
<td>1, 1</td>
<td>1/2</td>
</tr>
</tbody>
</table>
## Rubric for Unit 9: Statistics and Probability

### Test (Lessons 5 to 9), p. U-47

Name: ____________________

### Scoring Guides and Rubrics to accompany Sample Unit Quizzes and Tests for AP Book 6.2 U-91

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1 Can answer few, if any, questions accurately and independently.</th>
<th>Level 2 Can answer some questions accurately and independently.</th>
<th>Level 3 Can answer most questions accurately and independently.</th>
<th>Level 4 Can answer all or almost all questions, including bonuses, accurately and independently.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.SP.A.2</td>
<td></td>
<td>3, 4, 5, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.SP.A.3</td>
<td></td>
<td>3, 4, 5, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.SP.B.4</td>
<td></td>
<td>1, 2, 5, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.SP.B.5a</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reporting the number of observations.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.SP.B.5c</td>
<td></td>
<td>3, 4, 5, 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Rubric for Unit 9: Statistics and Probability

Test (Lessons 5 to 9), p. U-47

<table>
<thead>
<tr>
<th>Common Core State Standard</th>
<th>Assessed by Question(s) …</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.SP.B.5d</td>
<td>Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</td>
<td>3, 4, 5, 6</td>
<td>Can answer some questions accurately and independently.</td>
<td>Can answer most questions accurately and independently.</td>
<td>Can answer all or almost all questions, including bonuses, accurately and independently.</td>
</tr>
</tbody>
</table>

**Comments**
**Grade 6 Common Core State Standards Curriculum Correlations**

**NOTE:** The italicized gray JUMP Math lessons contain prerequisite material for the Common Core standards.

### Domain

- **RP** Ratios and Proportional Relationships
- **NS** The Number System
- **EE** Expressions and Equations
- **G** Geometry
- **SP** Statistics and Probability

### Cluster

#### 6.RP Ratios and Proportional Relationships

<table>
<thead>
<tr>
<th>6.RPA</th>
<th>Understand ratio concepts and use ratio reasoning to solve problems.</th>
<th>JUMP Math Grade 6 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.RPA.1</td>
<td>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 1 RP6-6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 5 RP6-8, 9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 2 RP6-11, 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 2 RP6-29</td>
</tr>
<tr>
<td>6.RPA.2</td>
<td>Understand the concept of a unit rate $a/b$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per burger.”</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 5 RP6-15, 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 2 RP6-23</td>
</tr>
<tr>
<td>6.RPA.3</td>
<td>Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td>JUMP Math Grade 6 Lessons</td>
</tr>
<tr>
<td>6.RPA.3a</td>
<td>Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td>Part Unit Lessons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 1 RP6-1 to 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 5 RP6-11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 6 G6-6, 16, 17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 2 RP6-23 to 25, 29</td>
</tr>
</tbody>
</table>
6.R.P.A.3b  Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 1    | 1    | RP6-3, 4  
      |      | RP6-6 to 10 |
| 1    | 5    | RP6-11  
      |      | RP6-12 to 15  
      |      | RP6-16  
      |      | RP6-17 to 20  
      |      | RP6-21  
      |      | RP6-22 |
| 2    | 2    | RP6-23 to 25, 26, 29 |

6.R.P.A.3c  Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 1    | 1    | RP6-3, 4  
      |      | RP6-6 to 10 |
| 1    | 5    | RP6-11  
      |      | RP6-12 to 15  
      |      | RP6-16  
      |      | RP6-17 to 20  
      |      | RP6-21  
      |      | RP6-22 |
| 2    | 2    | RP6-23 to 25, 29, 30 |

6.R.P.A.3d  Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
</table>
| 1    | 1    | RP6-3, 4  
      |      | RP6-6 to 10 |
| 1    | 5    | RP6-11  
      |      | RP6-12 to 15  
      |      | RP6-16  
      |      | RP6-17 to 20  
      |      | RP6-21  
      |      | RP6-22 |
| 2    | 2    | RP6-23 to 25, 27 to 29 |
### 6.NS The Number System

#### 6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

**6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \((2/3) \div (3/4)\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \((2/3) \div (3/4) = 8/9\) because \(3/4 \text{ of } 8/9 \text{ is } 2/3\). (In general, \((a/b) \div (c/d) = ad/bc\).) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS6-6 to 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-10, 12</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>NS6-37, 38</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>NS6-45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-49 to 55</td>
</tr>
</tbody>
</table>

#### 6.NS.B Compute fluently with multi-digit numbers and find common factors and multiples.

**6.NS.B.2** Fluently divide multi-digit numbers using the standard algorithm.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS6-6, 7</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>NS6-37</td>
</tr>
</tbody>
</table>

**6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS6-1, 2, 6, 7</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>NS6-24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-31 to 33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-34, 35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-38 to 41</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>NS6-46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-48, 57 to 62</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>G6-23, 24, 26</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>G6-32 to 34, 39 to 41</td>
</tr>
</tbody>
</table>

**6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express \(36 + 8\) as \(4 \times (9 + 2)\).

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>NS6-14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NS6-16, 17, 19, 21, 22</td>
</tr>
<tr>
<td>6.NS</td>
<td>The Number System</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C</strong></td>
<td><strong>Apply and extend previous understandings of numbers to the system of rational numbers.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C.5</strong></td>
<td>Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</td>
<td></td>
</tr>
<tr>
<td><strong>JUMP Math Grade 6 Lessons</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
<td><strong>Lessons</strong></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>NS6-3, 4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>NS6-63, 65</td>
</tr>
<tr>
<td><strong>6.NS.C.6</strong></td>
<td>Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</td>
<td></td>
</tr>
<tr>
<td><strong>JUMP Math Grade 6 Lessons</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>6.NS.C.6a</strong></td>
<td>Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $\text{-(}(-3)\text{)} = 3$, and that 0 is its own opposite.</td>
<td></td>
</tr>
<tr>
<td><strong>Part</strong></td>
<td><strong>Unit</strong></td>
<td><strong>Lessons</strong></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
| 1 | 2 | NS6-4, 5  
NS6-7  
NS6-9, 11 to 13, 20 |
<p>| 1 | 4 | NS6-27, 29 |
| 2 | 4 | NS6-66, 68 |
| 2 | 5 | G6-24, 25, 27 to 29 |
| <strong>6.NS.C.6b</strong> | Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. |</p>
<table>
<thead>
<tr>
<th><strong>Part</strong></th>
<th><strong>Unit</strong></th>
<th><strong>Lessons</strong></th>
</tr>
</thead>
</table>
| 1 | 2 | NS6-4, 5  
NS6-7  
NS6-12, 20 |
<p>| 1 | 4 | NS6-27, 29 |
| 2 | 5 | G6-20, 21, 24, 25, 27 to 29 |
| <strong>6.NS.C.6c</strong> | Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. |</p>
<table>
<thead>
<tr>
<th><strong>Part</strong></th>
<th><strong>Unit</strong></th>
<th><strong>Lessons</strong></th>
</tr>
</thead>
</table>
| 1 | 2 | NS6-3, 5  
NS6-7  
NS6-9, 11 to 13 |
<p>| 1 | 4 | NS6-27, 29 |
| 2 | 5 | G6-20, 21, 24, 25, 27 to 29 |</p>
<table>
<thead>
<tr>
<th>6.NS.C.7</th>
<th>Understand ordering and absolute value of rational numbers.</th>
<th>JUMP Math Grade 6 Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.NS.C.7a</td>
<td>Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <em>For example, interpret</em> $-3 &gt; -7$ <em>as a statement that</em> $-3$ <em>is located to the right of</em> $-7$ <em>on a number line oriented from left to right.</em></td>
<td><strong>Part</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

| 6.NS.C.7b | Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3° C > -7° C$ *to express the fact that* $-3° C$ *is warmer than* $-7° C$. | **Part** | **Unit** | **Lessons** |
| | | 1 | 2 | NS6-3, NS6-7, NS6-11, 12, 20 |
| | | 1 | 4 | NS6-24, 25, NS6-26, 27, 29, 30, 36 |
| | | 2 | 4 | NS6-69 |

| 6.NS.C.7c | Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* $-30$ *dollars, write* $| -30 | = 30$ *to describe the size of the debt in dollars.* | **Part** | **Unit** | **Lessons** |
| | | 1 | 2 | NS6-7, NS6-12 |
| | | 1 | 4 | NS6-24, 25, NS6-26, 27, 29, 30, 36 |
| | | 2 | 4 | NS6-68, 69 |

| 6.NS.C.7d | Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* $-30$ *dollars represents a debt greater than 30 dollars.* | **Part** | **Unit** | **Lessons** |
| | | 1 | 2 | NS6-7, NS6-12 |
| | | 1 | 4 | NS6-24, 25, NS6-26, 27, 29, 30, 36 |
| | | 2 | 4 | NS6-64, 69 |

| 6.NS.C.8 | Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. | **Part** | **Unit** | **Lessons** |
| | | 1 | 6 | G6-3, G6-4 to 6, G6-7 |
| | | 2 | 4 | NS6-67 to 69 |
| | | 2 | 5 | G6-22, 23, 25, 26 |
## 6.EE  Expressions and Equations

### 6.EE.A  Apply and extend previous understandings of arithmetic to algebraic expressions.

<table>
<thead>
<tr>
<th>6.EE.A.1</th>
<th>Write and evaluate numerical expressions involving whole-number exponents.</th>
</tr>
</thead>
<tbody>
<tr>
<td>JUMP Math Grade 6 Lessons</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.EE.A.2</th>
<th>Write, read, and evaluate expressions in which letters stand for numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.EE.A.2a</td>
<td>Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract ( y ) from 5” as ( 5 - y ).</td>
</tr>
<tr>
<td>JUMP Math Grade 6 Lessons</td>
<td>Part</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

| 6.EE.A.2b | Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression \( 2 (8 + 7) \) as a product of two factors; view \( (8 + 7) \) as both a single entity and a sum of two terms. |
| JUMP Math Grade 6 Lessons | Part | Unit | Lessons |
| | 1 | 3 | EE6-2 EE6-3, 4 |
| | 2 | 1 | NS6-42, 43 |
| | 2 | 6 | EE6-16, 17 |
| | 2 | 8 | G6-39 |

| 6.EE.A.2c | Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas \( V = s^3 \) and \( A = 6 s^2 \) to find the volume and surface area of a cube with sides of length \( s = 1/2 \). |
| JUMP Math Grade 6 Lessons | Part | Unit | Lessons |
| | 1 | 3 | EE6-2 EE6-3, 4 |
| | 1 | 6 | G6-10 to 15, 18, 19 |
| | 2 | 1 | NS6-42, 43 NS6-56 |
| | 2 | 3 | EE6-13 |
| | 2 | 6 | EE6-16, 17 |
| | 2 | 8 | G6-39, 41 |

| 6.EE.A.3 | Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression \( 3 (2 + x) \) to produce the equivalent expression \( 6 + 3x \); apply the distributive property to the expression \( 24x + 18y \) to produce the equivalent expression \( 6 (4x + 3y) \); apply properties of operations to \( y + y + y \) to produce the equivalent expression \( 3y \). |
| JUMP Math Grade 6 Lessons | Part | Unit | Lessons |
| | 2 | 1 | NS6-43 |
| | 2 | 6 | EE6-16, 17 |

| 6.EE.A.4 | Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions \( y + y + y \) and \( 3y \) are equivalent because they name the same number regardless of which number \( y \) stands for. |
| JUMP Math Grade 6 Lessons | Part | Unit | Lessons |
| | 2 | 6 | EE6-16, 17 |
| | 2 | 8 | G6-39 |
### 6.EE Expressions and Equations

#### 6.EE.B Reason about and solve one-variable equations and inequalities.

<p>| 6.EE.B.5 | Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>EE6-2</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>EE6-5</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>EE6-6, 7</strong></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>EE6-10</td>
</tr>
</tbody>
</table>

<p>| 6.EE.B.6 | Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td><strong>EE6-5</strong></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>EE6-10 to 12, 14, 15</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>EE6-18</td>
</tr>
</tbody>
</table>

<p>| 6.EE.B.7 | Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>EE6-7</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td><strong>G6-5</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>G6-7</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>G6-10 to 15, 18</strong></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>EE6-8 to 12, 14, 15</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>EE6-17</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>G6-33, 41</td>
</tr>
</tbody>
</table>

<p>| 6.EE.B.8 | Write an inequality of the form $x &gt; c$ or $x &lt; c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x &gt; c$ or $x &lt; c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>EE6-21</td>
</tr>
</tbody>
</table>

#### 6.EE.C Represent and analyze quantitative relationships between dependent and independent variables.

<p>| 6.EE.C.9 | Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time. |</p>
<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>EE6-19, 20</td>
</tr>
<tr>
<td>6.G.A</td>
<td>Solve real-world and mathematical problems involving area, surface area, and volume.</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>--------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>6.G.A.1</td>
<td>Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>JUMP Math Grade 6 Lessons</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6.G.A.2</td>
<td>Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell , w , h$ and $V = b , h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>6.G.A.3</td>
<td>Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6.G.A.4</td>
<td>Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Part</td>
<td>Unit</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>
### 6.SP ▶ Statistics and Probability

#### 6.SPA ▶ Develop understanding of statistical variability.

**6.SPA.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
<td>SP6-6, 7, 9</td>
</tr>
</tbody>
</table>

**6.SPA.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>SP6-1 to 4</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>SP6-5, 9</td>
</tr>
</tbody>
</table>

**6.SPA.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>SP6-1 to 3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>SP6-6 to 8</td>
</tr>
</tbody>
</table>

#### 6.SP ▶ Statistics and Probability

#### 6.SPB ▶ Summarize and describe distributions.

**6.SPB.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

<table>
<thead>
<tr>
<th>Part</th>
<th>Unit</th>
<th>Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>SP6-4</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>SP6-5, 8</td>
</tr>
</tbody>
</table>

**6.SPB.5** Summarize numerical data sets in relation to their context, such as by:

- **6.SPB.5a** Reporting the number of observations.
  - | Part | Unit | Lessons |
  - | 2    | 7    | SP6-1 to 3 |
  - | 2    | 9    | SP6-5 to 7 |

- **6.SPB.5b** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  - | Part | Unit | Lessons |
  - | 2    | 7    | SP6-1, 2, 4 |
  - | 2    | 9    | SP6-5 to 7 |

- **6.SPB.5c** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
  - | Part | Unit | Lessons |
  - | 2    | 7    | SP6-1 to 3 |
  - | 2    | 9    | SP6-5 to 9 |

- **6.SPB.5d** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
  - | Part | Unit | Lessons |
  - | 2    | 9    | SP6-6, 7 |